THE INFLUENCE OF OPPORTUNITY TO LEARN TO TEACH MATHEMATICS ON PRE-SERVICE TEACHERS' KNOWLEDGE AND BELIEFS: A COMPARATIVE STUDY

By

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ABSTRACT

THE INFLUENCE OF OPPORTUNITY TO LEARN TO TEACH MATHEMATICS ON PRE-SERVICE TEACHERS' KNOWLEDGE AND BELIEFS: A COMPARATIVE STUDY

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This study used the Teacher Education and Development Study in Mathematics (TEDS-M) to examine the differences in opportunities to learn to teach elementary school mathematics and the learning outcomes of final year pre-service elementary teachers in seventeen countries. The study also examined the influence of the pedagogical approaches used in the preparation of final year elementary pre-service teachers' knowledge for teaching and beliefs about learning mathematics in three countries: Poland, Russia, and the United States using a multi-level modeling approach. The study found that there were significant differences in pre-service teachers' knowledge for teaching, beliefs about learning mathematics and opportunities to learn to teach mathematics.

Most of the pre-service teachers from the participating European countries and Chinese Taipei experienced lecture presentations often, while most of the pre-service teachers from the American geographical region and Singapore experienced group work often during their mathematics methods courses. The analysis of teaching and learning mathematics through video analysis, readings and live classrooms was not a common practice across most of the participating countries. Nonetheless, pre-service teachers in Botswana, Russia, Thailand, and the United States experienced this pedagogical practice occasionally. The opportunities to learn how to plan mathematics instruction for conceptual understanding that include the analysis of learning goals, introductions to standards-based curriculum, and learning meaningful learning experiences were often experienced by the pre-service teachers in the Philippines, Singapore, Russia, and the

United States. The opportunities to learn mathematics instruction for conceptual understanding were mostly experienced by countries in the American region and those countries whose elementary teachers are prepared as mathematics specialists (e.g. Thailand and Malaysia).

The opportunities to learn to teach mathematics that made a difference in the pre-service teachers knowledge and beliefs in the three selected countries showed similarities in the three countries. Notably, experiencing models of reform-oriented instruction had a positive within-institution influence on pre-service teachers inquiry beliefs about learning mathematics. Additionally, opportunities to learn how to plan mathematics instruction for conceptual understanding that include analysis of learning goals and introductions to standards-based curriculum had a positive within-institution influence on pre-service teachers' inquiry beliefs about learning mathematics. The opportunities to learn mathematic instruction for conceptual understanding that include learning to show why mathematics procedures work had a positive within-institution influence on pre-service teachers' inquiry beliefs about learning mathematics in the generalist programs in the three countries.

Some differences were found across the three countries. In the East Asian countries, opportunities to ask questions and engage in whole group discussion during their mathematics-related courses had a positive within-institution influence on pre-service teachers' pedagogical content knowledge. Also, opportunities to learn to teach mathematics by teaching using methods demonstrated by instructors had a positive within-institution influence on PSTs inquiry beliefs. Lecture presentations had a negative within and between-institution influence on PSTs' knowledge for teaching mathematics in the United States.

These findings suggest the need for pre-service teachers to take more active roles in their learning to teach mathematics during their teacher preparation

Dedication to my parents, Gershom Nyabwa and the late Tabitha Nyabwa, and husband, Miltone Ayieko, whose hard work and life stories inspired me to be the best that I can be.

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TABLE OF CONTENTS

LIST OF TABLES

LIST OF FIGURES

KEY TO ABBREVIATIONS

MCK	Mathematics knowledge for teaching
MPCK	Mathematics pedagogical content knowledge
PSTs	Pre-service teachers
IDB analyzer	International data base analyzer
ISCED	International Standard Classification of Education
IEA	International Association for the Evaluation of Educational Achievement
TIMSS	Trends in International Mathematics and Science Study
TEDS-M	Teacher Education and Development Study in Mathematics
MORETHAN	More than 100 books in the home
YEARSOF	Parental level of education (given by number of years spent in school)
ASKQUES	Ask questions during class sessions
WHOLDISC	Whole class discussion
GRPWK	Group work
TCHMEDINS	Teaching sessions using methods demonstrated by the instructor
ANATL	Analysis of teaching and learning
INTR_SD	Introduction to standards-based curriculum
MEAN_LR	Meaningful learning experiences
ANA_LRN	Analysis of learning goals
MKDPROC	Make distinctions between procedural and conceptual knowledge
PROWRK	Learn how to show a procedure works
MSOOSTR	Learning how to explore multiple solution strategies

CHAPTER 1

INTRODUCTION

1.1. Purpose of the Study: Connecting Learning Outcomes to Teachers' Knowledge and Beliefs

Reforms in the teaching and learning of mathematics indicate that teachers should teach in ways that few of them experienced when learning mathematics. These reform methods of teaching and learning, introduced more than two decades ago, emphasize that teachers should use an inquiry approach instead of the didactic approaches they experienced while learning mathematics in school. Inquiry methods for teaching and learning mathematics include problem solving, encouraging reasoning and proof, using discourse in mathematics, connecting ideas, using appropriate representations, applying mathematics to real life situations, solving open-ended tasks, and group work (Boaler, 2002; Common Core Standards of Mathematics, 2010; National Council of Teachers of Mathematics, 2000). In contrast, didactic approaches for teaching mathematics, as described by Boaler (2002), focus on explaining mathematics procedures, requiring students to solve numerous exercises to practice the procedures introduced to them, having students working individually on tasks, emphasizing the correct answers to given exercises, providing students with methods of solving given tasks, and introducing topics in mathematics as disconnected units. Despite many years of introducing and encouraging reform in teaching mathematics, teachers' conceptions about teaching mathematics using a didactic approach are still prevalent (Barlow & Reddish, 2006).

Studies show that approaches to teaching mathematics greatly influence students' understanding of mathematics. For example, students taught through inquiry approaches are more successful at performing mathematics tasks and are able to apply or relate their mathematics knowledge to new tasks (Boaler, 2002; Skemp, 1976). Didactic approaches to teaching and learning have been documented as not very effective for teaching mathematics for understanding (Boaler, 2002; Putnam, Heaton, Prawat, & Remillard, 1992; Skemp, 1976). "Effective mathematics teaching requires understanding what students know and need to learn and then challenging and supporting them to learn it well" (NCTM, 2000, p. 15) . It is therefore imperative that teachers need to be encouraged and supported to use more inquiry approaches while teaching mathematics.

One of the issues that has been studied and discussed in mathematics education suggests that student learning outcomes are related to teacher knowledge (Darling-Hammond, 2006; Putnam et al., 1992). Further, scholars argue that teachers' knowledge and beliefs influence their instructional practices (Thompson, 1984; Thompson, 1992). For example, Kuhs and Ball (1986) pointed out that inquiry-oriented instructional practices require teachers to have an in-depth knowledge of mathematics content and mathematics pedagogy. However, Thompson (1992) noted through review of the literature that the context and curriculum, among other factors, also influence teachers' instructional practices. As such, the study of knowledge and beliefs for teaching mathematics effectively remains a huge research agenda because of its connections to the instructional practices that teachers adopt and the learning outcomes of their students.

The knowledge for teaching mathematics has been defined and studied by scholars in an effort to understand the knowledge domains needed for effective teaching of mathematics. These scholars have made many attempts to investigate and define teacher knowledge (e.g., Ball, Thames, & Phelps, 2008; Carpenter, 1986; Hill, Rowan, & Ball, 2005; Shulman, 1986, 1987) Although these studies offer insights on the components of teacher knowledge, it is not clear from these studies how to support teachers to develop knowledge for teaching mathematics using the reform methods of instruction.

The didactic approaches to teaching mathematics, which are experienced by most students in mathematics classrooms globally, have had a strong influence on the knowledge and beliefs about teaching and learning mathematics (Boaler, 2002; Putnam et al., 1992) that pre-service teachers (PSTs) bring to their teacher preparation programs. For these reasons, many mathematics educators believe that if the teaching and learning of mathematics is to be improved globally, then there is a need to challenge the knowledge and beliefs about teaching and learning mathematics that pre-service and in-service teachers have. Proponents of this notion (e.g., Borko & Putnam, 1996; Conference Board of Mathematics and Science, 2001; Feiman-Nemser, 2001) have posited that teacher educators should design ways that can challenge the knowledge and beliefs that pre-service teachers bring to their teacher preparation programs.

Preparing teachers to teach mathematics using reform methods of instruction can be influenced by opportunities to learn to teach in either teacher preparation programs or professional development forums. For example, Schram, Wilcox, Lanier, and Lappan (1988) noted that after a ten-week course with an intensive inquiry-oriented approach for teaching, PSTs who initially had a transmission view of teaching were challenged. Similarly, in-service teachers who were introduced to studies of students' thinking made significant changes in their teaching (e.g., Carpenter, Fennema, Peterson, Chiang, & Loef, 1989), which perhaps were influenced by changes in their beliefs about learning mathematics. Other scholars have discussed opportunities to learn to teach mathematics that show promising learning outcomes in teacher preparation, which include teaching mathematics-related courses using reform-oriented pedagogical practices (e.g., Bartell, Webel, Bowen, & Dyson, 2012; Ma, 1999), introducing critical reflections of teaching practices (Morris & Hiebert, 2009; Carpenter et al., 1989), planning mathematics instruction (Hiebert, Morris, Berk, & Jansen, 2007), and learning mathematics instruction (Charalambous, Hill, & Ball, 2011), among others. Although these studies of the preparation of teachers to teach mathematics show promising approaches that can influence future teachers' knowledge and beliefs about teaching and learning mathematics, they are mostly small-scale interventions or interpretive case studies and are based on studies mostly done in the United States. There is a need to examine the affordances of these practices on a larger scale and in other countries about which there is scant literature, as was also suggested by Adler, Ball, Krainer, Lin, and Novotna (2005) in their review of research on mathematics teacher education.

This study uses empirical evidence from a large-scale cross-national sample of PSTs' experiences in learning to teach elementary school mathematics, who are in their final stages of their teacher preparation, to examine the extent to which differences in reform-based pedagogical approaches in teacher education programs are related to differences in learning outcomes. This study addresses two questions. First, what are the patterns of differences that exist in the pedagogical approaches experienced in teacher preparation and in learning outcomes across the 17 countries that participated in the Teacher Education and Development Study in Mathematics (TEDS-M) survey? Second, how do the patterns of relationship between the pedagogical approaches and the learning outcomes differ within three countries with the largest number of teacher education institutions that participated in the TEDS-M survey? Specifically, the study examined the influence of opportunities to learn (OTL) (i) through pedagogical practices used in teacher preparation, (ii) through studying how to plan mathematics instruction, and (iii) through learning mathematics instruction for conceptual learning, on pre-service teachers' Mathematics

Content Knowledge (MCK), Mathematics Pedagogical Content Knowledge (MPCK), and beliefs about learning mathematics.

1.2. Significance of the Study

The technology shift of the 21st century requires that graduates be able to apply mathematics to fields such as advanced technology, commerce, medicine, and health (National Mathematics Advisory Panel, 2008). In order to meet these demands, students' performance in mathematics and mathematics-related courses needs to be improved (NCTM, 2000). For this to happen, teachers should have a rich knowledge of mathematics and beliefs about teaching and learning mathematics in order to teach in ways that allow their students to graduate with mathematical knowledge that can be applied to novel situations. In other words, they should have adequate knowledge to teach using the reform methods that have been shown to be related to learning mathematics for understanding (e.g., Boaler, 2002; Skemp, 1976). Teacher education is one crucial arena in which pre-service teachers can broaden their knowledge for teaching mathematics and can begin to shift their ingrained beliefs about ways that mathematics can be taught for conceptual understanding. Therefore, studying promising opportunities to learn to teach mathematics using a large sample can provide empirical evidence about what teacher educators have done cross-nationally to prepare PSTs to embrace methods of reform and enrich their knowledge for teaching mathematics. Using the findings from this study, reports can be developed about promising approaches to mathematics teacher preparation, and theories about learning to teach mathematics can be developed or further explored using other research methodologies.

This large-scale comparative study of mathematics teacher preparation across varying contexts, both between and within the countries that took part in the TEDS-M survey, allows for a wider consideration of influencing factors and a wider generalization of findings than using only one context. Countries can adapt/adopt promising practices and approaches that can be used in preparing elementary teachers to teach mathematics using information from this study. Since most of the approaches to teaching elementary mathematics that were examined in this study conform to the methods of reform, the findings can be used to gauge the extent to which teacher education programs are introducing their elementary PSTs to the methods of reform in mathematics, and also if the PSTs are joining the profession with beliefs about teaching mathematics that might promote the teaching of mathematics using inquiry approaches. Also, similarities and differences in terms of factors influencing PSTs' knowledge and beliefs about teaching and learning mathematics, which are not usually obvious when studied in a single context, can be illuminated using such an approach.

The TEDS-M database, based on the first cross-national study of mathematics teacher education, was selected because it has variables that can be used as indicators of the key features of promising approaches to learn to teach. Further, this database complements other cross-national databases that have been used to examine students' learning outcomes in mathematics because it uses similar mathematics content and cognitive domains for PSTs achievement.

Three countries with the largest number of teacher education institutions that participated in the TEDS-M survey were selected for the within-country analysis: Poland, the Russian Federation, and the United States. The large number of institutions allowed for similar analyses to be conducted for each of the three countries, so that the relationships could be compared. Further, the selected countries allow for comparison within the West *group*, which includes teacher education institutions in Europe and in the American geographical region (Blömeke &

Kaiser, 2012). Further, the teacher education institutions in Poland and the United States prepare PSTs through two distinct program types, which are classified as generalist[1] and specialist[2] programs for providing specific information on the opportunities to learn to teach mathematics in different program types. Also, the three countries provide information about the different grade spans that pre-service teachers are prepared to teach. The United States PSTs are prepared to teach grades 1-6, while the PSTs from the Russian Federation are prepared to teach grades1-4. The PST generalist teachers from Poland are prepared to teach grades 1-4, while the specialist PSTs are prepared to teach from grade 4 onwards.

Another compelling difference among the selected countries is the governments' control of the number of PSTs to be enrolled in their teacher preparation programs varies. The universities in Poland have full control of the selection of their PSTs (Sitek, 2013). The United States government, on the other hand, has weaker control on the selection of PSTs, and the number of PSTs to be enrolled in teacher education programs is determined by the universities. The Russia Federation has mixed control of PSTs selection (Tatto et al., 2012). Thus, the three countries selected have a wide range of differences that provide rich information to teacher educators and policy makers about the opportunities to learn to teach mathematics.

[1] These are teachers who are prepared to teach three or more subjects(Tatto et al., 2012)
[2] These are teachers who are prepared to teach mathematics (Tatto et al., 2012)

CHAPTER 2

CONCEPTUAL FRAMEWORK

2.1. Literature Review

Considering that the reform movement for teaching and learning mathematics is intended to improve the teaching and learning of mathematics, it is imperative that teachers are supported to learn to teach in these ways. Reforms in the teaching and learning of mathematics were introduced more than a decade ago, yet the didactic approaches to teaching are still prevalent (Barlow & Reddish, 2006). Scholars have suggested that some teachers still use these approaches to teaching because of gaps in their mathematics knowledge (Putnam, et al., 1992) or because their beliefs about teaching and learning mathematics are influenced by the models of teaching they experienced while learning mathematics (Boaler, 2002).

Teachers' prior knowledge influences their knowledge and beliefs about teaching and learning mathematics. Grossman (1990) posited that one of the key sources of knowledge for teaching is *apprenticeships of observation,* which are influenced by models of how the teachers were taught in school. Further, teacher education courses that are focused on subject matter knowledge influence knowledge for teaching mathematics (Grossman, 1990). These experiences inform teachers of students' understandings, interests, and abilities (Grossman, 1990). There is need for teacher education programs to provide reform-based knowledge for teaching mathematics, in order to expand on what pre-service teachers already know about teaching for conceptual understanding, or to challenge the beliefs and knowledge that they bring to their teacher preparation programs.

Putman and colleagues (1992) argued that it is possible to change teachers' beliefs and knowledge using pre-service and in-service programs that help teachers to rethink their views about teaching. Evaluating the knowledge and beliefs of pre-service teachers at the end of their teacher preparation program can inform education stakeholders about (i) the opportunities that teacher preparation programs offer their pre-service teachers to learn to teach elementary school mathematics, and (ii) what professional development programs graduating teachers need for continued support in developing their knowledge and beliefs about teaching and learning mathematics for conceptual understanding. This review of literature outlines some findings of studies of opportunities that teacher educators have provided their pre-service teachers and the outcomes of these interventions.

The question used to frame this review was “What are the knowledge and beliefs needed for teaching mathematics effectively, and how can teachers be supported to develop these competencies in their teacher preparation programs?” The support in the teacher preparation programs reviewed included case study analysis, action research, simulations, video analysis, practical experiences, use of technology, and pedagogical practices used in the teacher preparation programs.

To address this question, extensive analysis was conducted on studies done from 1986 to 2014, including some earlier studies from the early 1900s, and from the 1960s, and 1970s, because of the important frameworks proposed that recent studies have further developed, as well as their significant contributions to our understanding of teacher knowledge and teaching. A variety of techniques were used to search for relevant articles, including ERIC, JSTOR, Google Scholar, Web of Science, a hand search of some national peer reviewed journals and monographs, and published reviews of related literature. A hand search was done of the *Association of Mathematics Teacher Educators* monographs, the *Journal of Research in Mathematics Teacher Education,* the *Journal of Mathematics Teacher Education,* and the

Journal of Teacher Education. Other journals, including the *Journal of Research in Mathematics Education, The Teacher Educator,* the *Review of Research in Education,* the *American Education Research Journal, Teaching Children Mathematics, The Elementary School Journal, Mathematics Teaching in the Middle Schools, Educational Researcher, The Work of Mathematics Teacher Educators, Mathematics Teacher Education,* and *The International Journal on Mathematics Education* were accessed through the search engines indicated as well as other through professional websites (e.g. NCTM website). Key words used in the search engines included teacher education and the following: case studies, video analysis, action research, pedagogical practices, teaching for conceptual knowledge, planning lessons, modeling, and practical experiences. These key words were informed by literature reviews on teacher education and approaches used in particular teacher preparation programs .The searches were done for the mathematics subject area in order to select instruction specific to the learning of mathematics. Previous reviews in handbooks, such as the *Handbook of Research in Mathematics Teaching and Learning,* the *Handbook of Educational Psychology, Studying Teacher Education,* the *Handbook of Educational Psychology,* and the *Handbook of Research on Teaching* provided useful syntheses of this literature. Finally, some conceptual pieces that discussed mathematics teacher education, cross-national studies on mathematics teacher education, and knowledge for teaching from other journals and books were used in the review.

The review did not include articles on student knowledge and beliefs, or articles that discussed teacher knowledge and beliefs or the preparation of teachers in other subject areas because the main focus was on mathematics. The articles selected were grouped by themes: knowledge for teaching mathematics, beliefs about teaching mathematics, beliefs about learning mathematics, cross-national teacher education studies in mathematics, and opportunities to learn to teach mathematics in teacher preparation. I divided the studies into four topical groups: opportunities to learn to teach mathematics (pedagogical practices, opportunities to learn how to plan mathematics instruction, and opportunities to learn mathematics instruction), beliefs about teaching and learning mathematics, knowledge for teaching mathematics, and relationships between these OTL to teach mathematics and the teacher competencies, as defined as knowledge and beliefs.

This review is not exhaustive in terms of the interventions used for learning to teach mathematics. Rather, the intention of the review is to provide a framework that offers some insights on some pedagogical approaches that have been used in the preparation of teachers to teach mathematics and their influence on teacher competencies as defined as knowledge for teaching mathematics and beliefs about the teaching and learning of mathematics.

The review is organized by literature related to the variables I selected to use in the study (teacher competencies and the opportunities to learn to teach), followed by studies on the relationships among these variables. Specifically, the review first discusses the conceptions and definitions of the knowledge for teaching mathematics. Second, the conceptualization of beliefs about mathematics and the teaching and learning of mathematics is analyzed. Third, the review outlines some discussions of opportunities to learn to teach mathematics and how different scholars have conceptualized them. Following the discussions of the variables used in the study is a discussion of the types of opportunities that have been shown to have an influence on PSTs' knowledge and beliefs, which include case studies, cross-national data, as well as studies using the TEDS-M data.

2.1.1. Knowledge for Teaching Mathematics

Teacher knowledge has been of particular interest to teacher educators, policy makers, and mathematics education researchers. Scholars have proposed and re-examined the categories of the knowledge needed for teaching as one of the important factors for improving the teaching and learning of mathematics. For example, Shulman (1986, 1987) described the domains of knowledge that constitute the knowledge base for teaching. The *knowledge base*, according to Shulman, is composed of three domains: content knowledge, pedagogical content knowledge, and curricular knowledge. In addition, the knowledge base includes knowledge of the learners and their characteristics, knowledge of the educational context, and knowledge of educational foundations (Shulman, 1987). These categories of knowledge, have been re-examined by other scholars (e.g., Ball, 1993; Ball, Thames, & Phelps, 2008; Fan & Cheong, 2002; Grossman, 1990; Hill et al., 2005; Ma, 1999), and subcategories and redefinitions of teacher knowledge have been proposed. A look at the categories for knowledge of teaching mathematics as defined by some scholars is briefly outlined below.

Content knowledge.

Content knowledge (CK) is “the amount and organization of knowledge per se in the mind of the teacher” (Shulman, 1986, p. 9). This is knowledge that is gained by anyone who has gone through formal learning of subject matter. Using Schwab's (1978) definition of knowledge, Shulman described this knowledge as comprised of knowing the substantive and syntactic structures of a subject. The substantive structures, on the one hand, are the “basic concepts and principles organized to incorporate facts” (Shulman, 1987, p. 9). Syntactic structures, on the other hand, are the rules in the discipline (Shulman, 1986, 1987). The syntactic structures include the language, symbols, and axioms used and organized to ascertain truisms and validity in the discipline. Similarly, *procedural knowledge*, referred to by Grossman (1990) and Carpenter (1986) as knowing how to perform procedures or following pre-determined steps to solve a problem, would fall within the CK domain as defined by Shulman.

Pedagogical content knowledge

Pedagogical content knowledge (PCK) is the "amalgam of content and pedagogy" (Shulman, 1986, p.9). The PCK domain of knowledge includes the representation of ideas and the use of metaphors, analogies, and strategies that teachers draw on to make learning accessible to students (Shulman, 1986, 1987). It is the way of "representing and formulating the subject matter to make it comprehensible to others" (Shulman, 1986, p. 9). In addition, having PCK is being able to anticipate the errors, conceptions, and misconceptions that students could have in the learning process (Shulman, 1986). In other words, it is the knowledge needed to transform the subject matter in a way that is familiar and appealing to students, instead of requiring learners to learn the hard facts of the discipline on their own (Dewey, 1902). Dewey (1902) used the term "psychologizing" the curriculum for transforming it to suit the child's level.

Curricular knowledge

This knowledge includes both knowledge of the topics to be taught and knowledge of the different curricula available (Shulman, 1986, 1987). Also, knowledge of the lateral as well as the vertical curriculum is included in this knowledge domain. Knowledge of the lateral curriculum is the knowledge needed to connect topics in one lesson to those "lessons or topics or issues being discussed simultaneously in other classes" (Shulman, 1986, p.10). Vertical curriculum knowledge is having the knowledge of what is to be taught in the future and what has already been taught (Shulman, 1986). Further, curricular knowledge includes teachers' flexibility in learning ever-changing curriculum materials (Shulman, 1987). This knowledge domain is important for its use in decision-making about the instructional practices to be used and the content to be taught (Ball, et al., 2008).

More conceptualizations for knowledge for teaching mathematics

Other scholars have built on the knowledge domains defined by Shulman to provide alternative, similar and/or a finer categorization of knowledge needed for teaching mathematics. Ma (1999) conceptualized knowledge for teaching mathematics as profound understanding of fundamental mathematics (PUFM). According to Ma, “fundamental” refers to foundations, and “profound” is the understanding of mathematics that is “deep, broad, and thorough” (p. 120). In this definition for knowledge for teaching mathematics, Ma provides an expanded definition of the content knowledge domain proposed by Shulman, which is the understanding of mathematics that is connected and includes the knowledge of multiple perspectives for solving mathematical tasks. In addition, Ma includes longitudinal coherence of the curriculum in her definition for knowledge for teaching mathematics, which is similar to the definition of curricular knowledge as defined by Shulman.

Others scholars have argued that Shulman’s knowledge domains are not well understood and need more development (e.g. Ball, Thames & Phelps, 2008). In studies that focused on the skills needed for teaching, Ball and colleagues (2008) proposed that the content knowledge domain proposed by Shulman be further divided into *common content knowledge* and *specialized content knowledge*. Common content knowledge is the “mathematical knowledge and skills used in settings other than teaching” (Ball et al., 2008, p. 399). Specialized content knowledge, on the other hand, is the unique knowledge and skill needed to teach mathematics effectively (Ball et al., 2008). Additionally, Ball and colleagues (2008) proposed that the pedagogical content knowledge introduced by Shulman be divided into two domains: knowledge of content and students and knowledge of content and teaching. The domain referred to as “knowing about students and knowing about mathematics” (p.401) involves knowing students’ thought processes when learning mathematics, and also anticipating students’ errors, misconceptions, and difficulties with particular mathematical concepts (Ball et al., 2008). The knowledge of content and teaching includes knowing how to select examples, what representations to use, and which ideas students have that can be expanded (Ball, et al., 2008). These additional domains of PCK give explicit attention to the distinct knowledge categories that underlie PCK.

Fan and Cheong (2002) suggested other subcategories of pedagogical knowledge, including pedagogical curricular knowledge, knowledge of ways of instruction, and PCK (as cited in Tatto et al., 2009). The pedagogical curricula knowledge and PCK domains are similar to those proposed by Shulman. However, the pedagogical instructional knowledge domain is added to the pedagogical knowledge categories and is similar to the specialized content knowledge domain proposed by Ball and colleagues. The knowledge domains proposed by Ball and her colleagues (2008), Shulman (1986), Ma (1999), Fan and Cheong (2002) were aligned to the knowledge domains tested in the TEDS-M achievement test. The PUFM proposed by Ma (1999) is more particular about the affordances of this understanding to teaching mathematics effectively[3] and gives a relationship between knowledge and instruction that is examined in this study.

Cross national studies of knowledge for teaching mathematics

The proposed domains of the knowledge for teaching mathematics have been examined in cross-national studies of teacher preparation in mathematics. The cross-national studies

[3] Effective teaching of mathematics here means that mathematics is taught for conceptual understanding. In other words the students are not only taught procedures, but they are taught why procedures work and they have a deep understanding of mathematics because of the teaching employed.

include the Mathematics Teaching in the 21st century (MT21) and previous studies done with the Teacher Education and Development Study of Mathematics.

The cross-national study, Mathematics Teaching in the 21st century (MT21), which examined teacher preparation in mathematics in six countries, analyzed knowledge for teaching mathematics using two main categories: content knowledge and pedagogical content knowledge. The pedagogical content knowledge construct comprised “instructional planning knowledge, knowledge about student learning, and curricular knowledge” (Schmidt et al., 2011, p. 61). The PCK used by Schmidt and his colleagues (2011) includes the knowledge about students’ learning (similar to Shulman’s categorization), but includes the curricular knowledge that is a separate dimension in Shulman’s proposal of the knowledge base for teachers. Different from Shulman’s PCK dimension, the instructional planning category is included as part of PCK. Similar to the MT21 conceptualization of knowledge for teaching mathematics, the TEDS-M conceptualization comprises mathematics content knowledge and pedagogical content knowledge. The pedagogical content domain as defined in the TEDS-M database includes “curricular knowledge, knowledge about the planning for teaching, and knowledge related to enacting teaching” (Tatto, 2012, p.131). As such, the knowledge domains in TEDS-M borrow from and modify Shulman’s proposed knowledge domains and Fan and Cheong’s suggested categories of PCK, and they build on the MT21 study. The domains of knowledge used in the TEDS-M study focused on the domains of knowledge that can be studied cross-nationally and are general knowledge domains that are common to all countries.

A view of the findings from cross-national data, including the TEDS-M data, on knowledge for teaching mathematics shows significant differences within programs, within countries, and between countries. Schmidt and colleagues (2011), for example, showed significant differences in knowledge for teaching mathematics among middle school pre-service teachers within and between the six countries that participated MT21 study. (It should be noted that the PSTs samples in the MT21 study were not randomly sampled, and therefore caution should be taken when generalizing to the whole country (Schmidt et al., 2011).) Similarly, Blömeke & Kaiser (2014b) noted that there were significant differences in PSTs' scores for the domains of teacher knowledge within and between the countries that participated in the TEDS-M survey. Specifically, within Russia and Poland PSTs had higher MCK scores as compared to their MPCK scores, while within the United States PSTs had higher MPCK scores as compared to their MCK scores (Blömeke & Kaiser, 2014b). Senk and colleagues (2014) showed that more than half of the PSTs sampled in Russian institutions that participated in the TEDS-M study had an MCK score above anchor point[4] 2. Further, more than 90% of PSTs in Chinese Taipei, with teaching specialization in the 1-6 grade span, had the highest mean score; they were classified at an anchor point of 2 as compared to the other grade span teaching specializations. Between the countries, Senk and colleagues (2014) also noted that PSTs from countries that had programs in which more math classes were offered, or in which teachers were prepared in specialist programs, showed higher MCK scores. Finally, within the programs, the MCK scores varied between "100-200 score points: that is between one to two standard deviations of the full population"(Senk et al., 2014, p.76). This finding confirms that there is great variation in PSTs' performance within programs cross-nationally.

[4] The anchor points were used to describe the PSTs performances using specific points on the MCK and MPCK scales. For MCK, anchor point 1, representing a lower level, corresponds to a scale score of 432 (Tatto et al., 2012, p.136). PSTs classified at this anchor point were likely to be successful at answering questions categorized on the knowing cognitive domain. These include the likelihood of answering questions on basic understanding, straight forward arithmetic, and simples computations (Tatto et al., 2012). For MCK anchor point 2, a scale score of 516 (Tatto et al., 2012, p.136), PSTs were successful at extracting the mathematics from story problems or familiar contexts, had knowledge of algebra to be successful at correctly answering questions on linear expressions, but not enough knowledge to be successful in connecting this knowledge to more complex algebra questions. In other words, anchor point 2 had knowledge in the cognitive domains of knowing, applying, but still showed weaknesses on questions on reasoning (Tatto et al., 2012).

For the MPCK, in which the primary[5] anchor point was defined, there was a significant variation across the countries. PSTs' score which were classified with an anchor point of one were able to analyze teaching strategies, as well as analyze some students' work on given single-step story problems of primary level content (Tatto et al., 2012). Senk and colleagues (2012, 2014) noted large differences between the lowest and highest scores of PSTs MPCK within each of the participating countries. Further, PSTs in the specialist programs had higher MPCK scores than those from the generalist programs within the countries. In short, Senk and colleagues identified large variations in MPCK scores across countries, within a country, and within programs.

Scholars have argued that knowledge for teaching mathematics is closely related to beliefs about mathematics, and about teaching and learning mathematics. Thompson (1992) explained that knowledge and beliefs have a close connection that has created difficulty in distinguishing between the two. Thompson (1992) further suggested that studies of teacher knowledge are only complete if teachers' beliefs are included. The next section gives a description of beliefs about mathematics and mathematics teaching, and of studies of beliefs about teaching and learning mathematics.

2.1.2. Beliefs about Teaching and Learning Mathematics

Teacher beliefs is becoming a popular area of study in education because of its connectedness to knowledge for teaching (Thompson, 1992). Thompson (1992) pointed out in his review that there was a paradigm shift towards studying teachers' thought processes and their

[5] PSTs who scored above the MPCK primary anchor point could interpret given conventional students' work from the primary grades, were able to recognize a correct teaching strategy when given a concrete example, could identify arithmetic elements from single step problems (Senk et al., 2014). However, at the primary anchor point the PSTs were not likely to (i) " use concrete representations to support student learning or recognize how a students thinking is related to a particular algebraic representation" (p.317), (ii) understand measurement and probability concepts to "reword or design a task", (iii) " know why a particular teaching strategy made sense", (iv) be aware of "common misconceptions " devise" useful representations of numerical concepts" (Senk et al., 2014, p. 317)

decision-making, moving away from the previous studies of the relationships between teaching skills and student. Similarly, Shavelson (1988) suggested in his American Educational Research Association presidential address the need for studies to focus on the *mind frames* of practitioners, in order to influence teachers' conceptions. Therefore, Thompson's extensive review of research on teaching and Shavelson's address that focused on how research can contribute to education policy and practice show that studying teachers' knowledge and beliefs gives a more holistic view of what influences teachers' actions, which can then lead to positive learning outcomes.

Beliefs, unlike knowledge, can be categorized in the form of systems (Thompson, 1992). Some unique features of beliefs, as outlined in Thompson's review, include that beliefs "can be held with varying degrees of conviction," are "not consensual," and "can be organized as belief systems" (p. 129). With a particular focus on the teaching and learning of mathematics, Ernest (1989a, 1989b) categorized the belief systems that teachers have into three categories: (i) views about the nature of mathematics; (ii) views about the nature of teaching mathematics; and (iii) views about the process of learning mathematics. In the categories of views about the nature of mathematics, Ernest discussed the *Instrumentalist,* the *Platonist,* and the *Problem Solving* views. In the instrumentalist view, mathematics is believed to be an accumulation of facts, rules, and skills that are unrelated. The Platonist views mathematics as a unified body of knowledge that is discovered and not created. Finally, the problem solving view considers mathematics to be dynamic and learning mathematics to involve a process of inquiry, with the results open to critique (Ernest, 1989a, 1989b). Put hierarchically in terms of the cognitive levels, instrumentalism is the lowest level, followed by the Platonist view, and finally problem solving as the highest view.

In the system of beliefs about the nature of teaching mathematics, Ernest considered the role of the teacher, the teacher's actions, and classroom activities. He categorized beliefs about the teacher's role into (i) the instructor, who emphasizes mastery of skills and corrects performance; (ii) the explainer, who emphasizes conceptual understanding with unified knowledge; and (iii) the facilitator, who is confident in problem posing and solving. Focusing on the teacher's views about the learning of mathematics, Ernest dichotomized this belief system as (i) active construction of knowledge versus passive reception of knowledge, and (ii) development of autonomy in the child's interest versus the learner as submissive and compliant (Ernest, 1989b). Views about teaching and learning mathematics are related to views about the nature of mathematics (Ernest, 1989a). A teacher with a Platonist view about the nature of mathematics may likely have the view that teaching mathematics involves explaining, and that learning is receiving the knowledge provided by the teacher. On the other hand, teachers with a view of mathematics as problem solving view their role in the teaching process as a facilitator and that learning is then an active construction of knowledge. Cooney, Shealy, and Arvold (1998) conceptualized the belief structures of pre-service teachers as the (i) *naïve idealist view*, (ii) *isolationist view*, (iii) *naïve connectionist view*, and the (iv) *reflective connectionist* view. The PST with an *isolationist view* joins their teacher preparation program with the view that they know the right way to teach and are resistant to change because their views and the views in their teacher education programs are separated. Such teachers are not willing to accommodate new views about teaching and learning mathematics. The *naïve idealist* PST is one who takes in other views without critically analyzing the perspectives offered. In contrast to the *naïve idealist* perspective, the *naïve connectionist* PST reflects on previously held views when \ encountering new views but do not resolve the conflicts of the two views, while the *reflective connectionist* reflects on the conflicts between previous views and those of others and is able to adapt to new ideas. Cooney (1999) suggested that the programs offered in teacher preparation should seek ways that can enable their PSTs to develop a reflective stance that then allows them to adapt to new perspectives for the teaching and learning of mathematics.

Studies of the beliefs about teaching and learning mathematics show that beliefs influence teachers' actions. Ernest (1989a) argued that the belief systems that teachers hold about mathematics, mathematics teaching, and the learning of mathematics influences their interaction with texts, their role in the classroom, and their model of what teaching should be. Similarly, Prawat (1992) suggested that the teachers' views about the discipline influence their instruction. Further, some teachers' beliefs about teaching are influenced by their views about whom their students are and views about the subject (Prawat, 1992). Ernest (1989a), however, cautioned that the context and the opportunities available in the context might influence the relationship between the teachers' belief systems and actions. Further, the belief structures of PSTs have a strong influence on whether they adapt to new ideas or retain the ideas that they have from previous experiences.

Cross-national studies of teacher beliefs

Cross-national studies about teacher beliefs show that there are universal patterns of beliefs about the nature of mathematics, while there also exist differences in teachers' beliefs across countries. The MT21 study of the beliefs about mathematics explored four views about mathematics, mathematics as: (i) a creative science; (ii) a useful science; (iii) a formal and logical science with an axiomatic basis, and (iv) an algorithmic science consisting of a collection of terms, formula, and rules (Schmidt et al., 2011, p.168). The study found that there is widely held patterns of PSTs' beliefs about the teaching and learning of mathematics. The study found that the pre-service teachers in all six countries reported having a dynamic[6] view of mathematics. The study also found some variation across countries. The PSTs from South Korea and Taiwan had similar views about mathematics and the nature of teaching and learning mathematics, but these views differed from those held by teachers in other countries in the study (Schmidt et al., 2011). Further, Taiwanese PSTs showed a strong view of mathematics as formal and algorithmic (Schmidt et al., 2011). In sum, though the PSTs beliefs show some similar patterns, there was a strong indication that beliefs about teaching and learning mathematics varied by country.

In the TEDS-M survey, the pre-service teachers responded to questions that aimed to examine their belief systems about the nature of mathematics as rule bound versus a unified body of knowledge and their views about the nature of learning mathematics as teacher directed versus problem solving, using the dichotomies from the beliefs about mathematics and beliefs about learning mathematics given by Ernest. The TEDS-M survey used the terms active learning and teacher-directed to describe beliefs about teaching and learning mathematics.

Scholars who have used the TEDS-M data found differences in views about the nature of mathematics within and between countries. Felbrich, Kaiser, and Schmotz (2012) reported that PSTs' beliefs about the nature of mathematics varied strongly within and between categories of countries, namely collectivist or individualistic. In collectivism culture, "people from birth onwards are integrated into strong cohesive groups"(Hofstede, 2001, p.225). Learning is to fulfill the duty that is expected by the teacher, family or society, who in turn should support the learner. Failure from the learner is attributed to lack of effort of those supporting the learner (Felbrich et al., 2012). In individualistic cultures, " the tie between individuals is loose. Everyone is expected to look after himself" (Hofstede, 2001, p.225). Knowledge is acquired by oneself and failure is caused, therefore, by a mismatch of the conditions for learning and the individual

[6] A dynamic view of mathematics is a belief that mathematics is useful and a creative science (Schmidt et al., 2011)

(Felbrich et al., 2012). Felbrich and team noted that PSTs from highly collectivist countries (Malaysia, Russia, Thailand, and the Philippines) are more inclined to have a static view than a dynamic view of mathematics. However, PSTs from the highly individualistic countries (Norway, Switzerland, and Germany) have a dynamic view of mathematics, while PSTs from Spain, Chinese Taipei, and Singapore PSTs showed both views about mathematics (Felbrich et al., 2012). The static view of mathematics considers that mathematics is based on axioms, is a collection of terms, formulas and rules. These are formalist and schematic views of mathematics, respectively (Felbrich et al., 2012). The dynamic view of mathematics is related to the problem solving process (process-related) and to usefulness to the society (application-related) (Felbrich et al., 2012). These differences across the countries that participated in the TEDS-M showed that the beliefs about mathematics are indeed influenced by culture.

Tang and Hsieh (2014) used a categorization of beliefs about the nature of mathematics, comparing the open and creative nature of mathematics versus the conservative and rigorous nature of the mathematics beliefs of lower secondary mathematics teachers across categories of countries (see categories proposed by Blömeke and Kaiser (2012)) that participated in the TEDS-M survey. Their study showed that PSTs' beliefs in all the countries, except those categorized as *developing Asia*, showed a consistent pattern of beliefs about the nature of mathematics (Tang & Hsieh, 2014). The consistent pattern across the countries showed the open and creative nature of mathematics and that mathematics is best learned through the consideration of student initiatives as the strongest PSTs' belief about the nature of mathematics and the nature of learning mathematics, and utilitarianism in teaching in their beliefs about teaching mathematics as the weakest. Further, PSTs from the categories of countries in *developed Europe*, *Confucian Asia*, and the *American group* showed negative views for teacher instruction and explanation to students, while *developing Asia* and *East Europe* showed positive views (Tang & Hsieh, 2014). In addition, Tang and Hsieh (2014) found that PSTs from the United States, Poland, and Taiwan showed negative views of teacher instruction and explanation to students.

Other studies (e.g., Wang & Hsieh, 2014) showed that the belief profiles of teacher educators showed similarities to those of the PSTs. Similar to MT21 study findings, belief about the inquiry view and the active learning view was a common belief among the PSTs and teacher educators in all the participating countries (Wang & Hsieh, 2014). These findings show that the beliefs about mathematics and about teaching and learning mathematics across all the countries that related to reform-oriented instruction are indeed becoming a global feature.

It still remains apparent from these cross-national studies that there are differences between countries, and the features of groups of countries need further investigation to understand what factors might be influencing these differences in their belief systems.

2.1.3. Opportunity to Learn

The term opportunity to learn (OTL) in educational research was initially used by Carroll (1963), as one of the five variables that can be used to explain differences in students' learning. According to Carroll, three of the five elements that influence learning are individual factors, while two are external factors. The individual factors proposed are (i) aptitude and the time needed to learn a task; (ii) ability to understand the learning task; and (iii) perseverance or the time students spend on a task (Carroll, 1963; 1989). The external conditions that influence learning are (iv) the opportunity to learn, such as the time allowed for learning; and (v) the quality of instruction. Opportunity for learning, in particular, can also refer to the "school schedule … the time allowed for learning" (p. 26). Carroll (1963) argued that influencing or changing a student's aptitude can prove a challenge, but that it is possible to introduce interventions to modify students' perseverance, teachers' quality of instruction, and the opportunity for learning. Carroll noted that previous scholars had not explored the dimension of opportunity to learn.

This model of learning, proposed by Carroll (1963), has received a lot of attention, and is highly regarded among educational researchers. Specifically, the OTL dimension of the model of learning been redefined in different ways, following the initial definition given by Carroll (1963). For example, Schmidt, Cogan, and Houang (2001) framed OTL in two ways. One definition looks at OTL as the instructional time given to a topic. In this case, OTL is the time intended in national curriculum guides for teaching a topic, or the time teachers report they spend teaching a topic. The second way of framing OTL is the proportion of teachers in a country covering the topic (Schmidt et al., 2001). Similarly, Floden (2002), in his review, showed that OTL measures had been defined as the time spent on a topic(s), the relative emphasis of the topic in relation to other topics, and if the topic had been taught or will be taught. Thus, OTL can be compared in terms of either curricular differences or differences in teachers' instructional time on content or subjects (Floden, 2002; Schmidt et al., 2001; Törnroos, 2005). In all these definitions, the OTL discussed focus on the quantity of time, with no focus on the quality of instruction, which is part of the opportunity to learn initially proposed by Carroll.

Together these studies show that opportunity to learn (OTL) has also become a popular area of study in international comparisons (Floden, 2002), and it has found to be positively related to student achievement (Floden, 2002; Schmidt et al., 2001; Törnroos, 2005). OTL, therefore, is important in comparative studies to examine the variations in curriculum across countries and to represent the diversity in content provided by the different programs between and within groups, and to examine the factors influencing differences in the levels of knowledge and, (Tatto et al., 2008). Indeed, Carroll (1989) acknowledged that the model proposed for learning offered a “broader, more theoretical basis for explaining and interpreting school effects” (p. 27) and seeks to achieve “equality of opportunity” rather than “equality of attainment” (p.30). Thus, this variable introduces an aspect of fairness when studying learning outcomes.

Cross-national studies on opportunity to learn in teacher education

The previous studies examined OTL among students who were in the K-12 classrooms. Cross-national studies have also used opportunity to learn in teacher education to explain differences in the PSTs’ competencies that exist across countries and the curricula offered to PSTs. The study of middle school mathematics teacher preparation in six countries (MT21) examined the opportunities to learn to teach mathematics using (i) the time spent in teacher preparation; (ii) topics covered in applied mathematics specific to middle and high school mathematics; (iii) experiences of the PSTs that are related to the teaching of mathematics, including practical experiences; (iv) instructional time for each of the mathematics strands; and (v) the total number of courses covered in mathematics, mathematics pedagogy, and general pedagogy (Schmidt et al., 2011). The study showed that there are variations in the number of courses taken by PSTs across six countries, with PSTs in Taiwan taking four more courses in total, during their teacher preparation, as compared to the PSTs in the United States. Further, the variation in the number of courses taken in mathematics was highest in the United States, with the majority of variation of mathematics course taking found within institutions. Of the countries that participated in this study, PSTs from the United States had taken fewer courses in formal mathematics but more in general pedagogy (Schmidt et al., 2011) as compared to those in other countries. These findings offer the strong possibility of opportunity to learn being a factor that influences the differences in knowledge of teaching mathematics that is found cross-nationally.

In the TEDS–M survey, the learning experiences of PSTs in their teacher preparation are considered opportunities to learn. OTL is framed as the content coverage, the extent of learning particular topics in teacher preparation, the frequency of activities in the content and methods courses, and the teaching methods used (Blömeke & Delaney, 2012; Tatto et al., 2008). As such, the opportunity to learn in the TEDS-M survey includes an aspect of the quality of instruction that was somewhat missing in the studies done on OTL among K-12 students. To examine opportunity to learn, the survey asked teacher educators the frequency with which they used particular activities and teaching strategies to teach mathematics and mathematics methods courses in the teacher preparation program. (See Measures in the methods chapter.) Thus, information about opportunity to learn obtained from the teacher educators gives information about the planned and/or the enacted[7] curriculum. The survey also asked the pre-service teachers the extent to which they engaged in a number of activities and experienced learning strategies during their teacher preparation mathematics related courses. In this case the OTL examined is the experienced curriculum[8].

OTL to teach mathematics from the TEDS-M data showed variation across countries and programs, and was related to the cultural context. Countries in which PSTs were prepared in

[7] The planned curriculum is the teacher educators' response that gives an account of formulating "what they will do". The enacted curriculum is the response about what they actually did (Gehrke, Knapp, & Sirotnik, 1992, p.55). Note that not all the teacher educators in the sample had taught the pre-service teachers mathematics or mathematics methods. For this reason, the responses of the teacher educators could be either the planned or the enacted curriculum.

[8] This response relates to what the PSTs reported that they got from their courses and aligns to the definition of the experienced curriculum defined by Gehrke and colleagues (1992),

mathematics specialist programs had the highest OTL in the coverage of tertiary[9] mathematics topics (Tatto et al., 2012; Blömeke & Kaiser, 2014a). A profile of the OTL across the countries showed that the category of topics classified as *Number* was most studied by the PSTs, while *Calculus* seemed to be the least studied topical area of mathematics. Further, the OTL basic university mathematics was reported common across all countries, while the OTL tertiary and school mathematics varied across the countries (Blömeke & Kaiser, 2012). Similarly, the OTL mathematics pedagogy varied across the countries, and the only OTL that was common across the countries was learning about teaching methods (Blömeke & Kaiser, 2014a). Blömeke and Kaiser (2014a) found that PSTs from the "East" including Taiwan, Singapore, Poland and parts of Russia, had OTL basic mathematics and functional mathematics (p. 323). These results show only particular dimensions of opportunities to learn to teach mathematics that focus on the topics covered that are related to mathematics and mathematics pedagogy and their variations across the countries. However, there are other opportunities that include the aspects of the quality of instruction for learning to teach mathematics that were not explored in these cross-national studies, but have been documented as promising opportunities to learn to teach mathematics.

To understand further the link between teacher competencies and OTL to teach mathematics, the next section reviews studies that relate more components of opportunities to learn to teach mathematics to PSTs' knowledge and beliefs about teaching and learning mathematics.

[9] The tertiary level mathematics areas were: geometry, discrete structures and logic, continuity and functions, and probability and statistics (Tatto et al., 2012).

2.1.4. Relating OTL to Knowledge and Beliefs about Teaching and Learning Mathematics

In discussing ways of supporting teachers to develop mathematics knowledge for teaching and beliefs about teaching and learning mathematics that are related to reform-oriented instruction, it is important to consider what studies have shown to be the important sources of the knowledge that teachers need to have for teaching. Such insights are important in informing teacher preparation programs of the practices and activities that will benefit teachers' development of their knowledge and beliefs for teaching mathematics effectively.

One of the key sources of knowledge for teaching is through the *apprenticeship of observation* (Grossman, 1990; Lortie, 1975). Knowledge is acquired from images of how people are taught in school and the models of teaching teachers observed during teacher preparation. Teachers use these apprenticeship experiences to inform themselves of students' understandings, interests, and abilities (Grossman, 1990). Some of the models of teaching that teachers are familiar with follow the transmission model of teaching and the use of memorization and procedures to solve tasks, which have been shown not to be effective for teaching and learning mathematics for understanding (Putnam et al., 1992; Skemp, 1976). The images of instruction that teachers bring to their teacher preparation programs are influential in determining the classroom practices they adopt. Teacher preparation programs, therefore, need to design ways to challenge and influence the knowledge and beliefs about mathematics teaching and learning that pre-service teachers bring to their teacher preparation programs (Borko & Putnam, 1996; CBMS, 2001). Putnam and colleagues (1992) posited that it is possible to change teachers' beliefs and knowledge using pre-service programs that help teachers to rethink their views of teaching. In the same vein, Feiman-Nemser (2001) emphasized that teacher education programs should provide a curriculum in which the teachers' beliefs are critically examined and should be developed to challenge or amend these beliefs. Only after the beliefs are challenged can teachers embrace the new visions of teaching (Feiman-Nemser, 2001).

Pre-service teachers require models of teaching that they can refer to as exemplary for teaching mathematics. Borko and Putnam (1996), however, contended that disciplinary courses for pre-service teachers do not stress teaching for understanding, and are instead taught using high levels of abstraction. In addition, pre-service teachers get inconsistent messages when exposed to different models of teaching (Borko & Putnam, 1996); the courses that are taught in different departments, by different faculty, sometimes are disconnected and do not form a coherent whole (Feiman-Nemser, 2001). Moreover, the way the faculty suggested teaching is different from what they actually do in their teacher preparation classrooms (Feiman-Nemser, 2001). Cranton (2002) argued that there is a need for pre-service teachers to see models of instruction that they can emulate in their own teaching.

One of the ways teacher preparation programs might influence pre-service teachers' knowledge and beliefs about teaching and learning mathematics is through the pedagogical practices used to teach the content and pedagogy of mathematics. These pedagogical practices provide important images of effective teaching practices that pre-service teachers can learn from. Some studies suggest that the approaches used to help PSTs learn to teach mathematics can influence them to embrace the methods of reform or reaffirm their existing beliefs about teaching and learning mathematics. Other studies suggest that approaches used to learn how to plan mathematics instruction for conceptual understanding and to learn mathematics instruction for conceptual understanding influence teachers' knowledge and beliefs about teaching and learning mathematics.

Following is a brief summary of some of the approaches that have been used in pre-service teacher preparation for teaching mathematics and the outcomes from those approaches. The review outlines (i) pedagogical practices used in teacher preparation programs; (ii) approaches to learning how to plan mathematics instruction for conceptual understanding instruction; and (iii) approaches to learning mathematics instruction for conceptual understanding.

Pedagogical practices used in teacher preparation

Teacher education programs often aim to encourage professional collegiality. Feiman-Nemser (2001) stated that during pre-service teacher preparation, teachers can be encouraged to form "habits and skills that are necessary for studying teaching while in the company of colleagues" (p.1019). Pre-service teachers might be prepared to work in teams by giving them collaborative assignments, and by encouraging activities developed by pre-service teachers (Grossman, 1990). These collaborative activities enabled novice teachers to gain a clearer understanding of certain mathematics concepts and teaching practices (Charalambous et al., 2011; Ma, 1999). Feiman-Nemser (2001) gave examples of collaborative activities: analysis of student work, critical analysis of curriculum materials, interviewing students to uncover their thinking, and studying how other teachers work towards the same goals. Darling-Hammond (2006) posited that sharing ideas is of benefit in the growth of knowledge for teaching.

Models of reform-oriented pedagogical practices

Based on the literature that is available, some promising pedagogical practices that have been used in teacher education are group work, whole group discussion, and the analysis of teaching and learning. Group work, for example, in which PSTs work collaboratively on assignments and assess their peers' responses, as well as engage in whole group discussion (e.g., Bartell et al., 2012; Coffey, 2004; Lloyd 2006; Ma, 1999; Tarr & Papick, 2004), are pedagogical practices that have been found to allow PSTs to get a clearer understanding of mathematics concepts and teaching practices. Studies of individual courses in which data was collected from PSTs' reports, responses to interviews, and journal entries showed that they appreciated that learning mathematics is different from what they are used to.

Models of non-reform oriented pedagogical practices

However, there are pedagogical practices that may have an unintended influence on pre-service teachers beliefs about teaching and learning. For example, if teachers are introduced to learning to teach mathematics using a didactic approach, this may reinforce their transmission views about teaching and learning. Using lecture presentations and demonstrations during teacher preparation was associated with PSTs developing procedural images of learning (Eisenhart Borko, Underhill, Brown Jones, & Agard, 1993). The step-by-step process and the single method of instruction that the pre-service teachers experienced during their learning of mathematics content encouraged them to memorize procedures instead of using the sessions to understand the explanations given (Eisenhart et al., 1993). Though the pedagogical practices encouraged the pre-service teachers to work in groups, the focus was on memorizing what had been demonstrated. Therefore, in this pedagogical practice, the work done in groups did not encourage creative ideas but instead focused on learning through didactic approaches.

Analysis of teaching and learning

The analysis of teaching is another pedagogical practice used in teacher preparation that is related to positive outcomes in teacher preparation. When pre-service teachers have opportunities that allow for reflection to improve their practice, it influences their knowledge for teaching and their beliefs about teaching and learning mathematics (Cooney et al., 1998). The analysis of teaching and learning through audio analysis of mathematics lessons, case study analysis, reading research on teaching and learning, analysis of lesson plans, or any appropriate situation that provides a supportive environment based on the set principles and standards of teaching and learning, have been suggested as examples of supportive contexts for reflection of practice and learning to teach (Hiebert et al., 2007; Morris & Hiebert, 2009). Specifically, audio analysis has been documented for its power as a reflective tool (Taylor & O'Donnell, 2004), as have case study analysis of teaching (Schifter & Bastable, 2008; Silver, Clark, Gosen, & Mills, 2008; Henningsen, 2008) and readings about research on teaching and learning (Carpenter et al., 1989; Van Zoest, Stockero, & Edson, 2010). Another supportive practice that have been used in teacher preparation is the introduction to *children's mathematical teaching experience* (CMTE) in which a mathematics course was integrated with children's mathematics thinking. This practice was found to have helped PSTs to develop more sophisticated beliefs (Philipp, Thanheiser, & Clement, 2002), as compared to the control group who were learning mathematics without an introduction to children's mathematical thinking. Such contexts allow for PSTs to reflect on the conflicts between their previous beliefs and improved in their content knowledge for teaching and learning mathematics and their experiences of alternative teaching methods introduced in various ways during their teacher preparation (Cooney et al., 1998). In these studies, the PSTs' beliefs about learning and teaching were noted to have shifted to thinking more about problem solving practices that help children improve in their learning. Also, the PSTs' responses to interviews and researchers' observations of their discussions in mathematics education courses showed that they focused more on students' thinking. More importantly, they developed a rich understanding of mathematics and its teaching.

The learning outcomes of the PSTs reported from the studies on the pedagogical practices used in teacher preparation suggest that if PSTs experience practices that are similar to the teaching practices recommended by the reforms, their knowledge and beliefs about teaching and learning mathematics may improve. The literature described cases in which these different pedagogical practices used for teaching the PSTs were related to the improvement of the PSTs' beliefs about teaching and learning, and consequently their views at the end of the courses favored the inquiry approach for teaching and learning mathematics.

These studies of the practices used in teacher preparation programs are based on the findings of small-scale interventions conducted by authors of the studies or by instructors in a given mathematics course in the PSTs' teacher preparation. Also, the PSTs' competencies that are reported are mostly action research and interpretive in nature and thus limit the development of a theory about a method of instruction. Although the studies indicate the data they used in their interpretations, the methods of analysis are unclear in most cases. Missing from the literature are large-scale analyses of these practices across different teacher preparation programs in which a clear and appropriate method of analysis is conducted. Such analyses would provide evidence about the generalizability of these small-scale studies of pedagogical practices in mathematics teacher preparation.

Opportunity to learn how to plan mathematics instruction for conceptual understanding

In planning instruction, teachers' beliefs and knowledge about teaching and learning influence the decisions they make. For example, the selection and critique of curriculum materials are influenced by the teachers' knowledge and beliefs (Putnam et al., 1992). A teacher must carefully think through what resources are most appropriate for the learning process of their students, rather than just using the textbook verbatim (Putnam et al., 1992). In teacher education, therefore, it is imperative that PSTs are supported to develop the knowledge and beliefs for planning mathematics instruction for conceptual understanding if we intend to graduate teachers with the needed competencies.

In the literature on mathematics teacher preparation, three different approaches to helping pre-service teachers learn about planning have been proposed: the development and analysis of learning goals (e.g., Morris & Hiebert, 2009), introductions to standards-based curriculum materials (e.g., Lloyd, 2006; Tarr & Papick, 2004), and introductions to meaningful learning experiences that include engaging with rich mathematical tasks, and selecting models for teaching mathematics topics (e.g. Flowers and Rubenstein, 2006; Steele, 2006).

The development and analysis of learning goals

Key to successful teaching is the identification of the learning goals of a lesson or a unit. Emphasis on learning goals is important for planning as well as for the revision and improvement of teaching (Morris & Hiebert, 2009). Knowing the sub-concepts of mathematical ideas, according to Morris and Hiebert (2009), helps teachers identify and think about the mathematical concepts that should fit together in order for a learning goal to be met. In their study of 30 K-8 pre-service teachers who had opportunities to learn to (i) explain and represent mathematical ideas in relation to their embedded sub-concepts, (ii) unpack mathematical concepts into their constituent sub-concepts, and (iii) justify algorithms that work, the pre-service teachers were able to identify sub-concepts of given tasks when the environment was supportive, as suggested from the analysis of their responses to written tasks (Morris & Hiebert, 2009). Some examples of supportive environments were tasks related to evaluating student responses that contained errors.

Teachers who develop focused learning goals join the field with a specialized knowledge of mathematics for teaching. They can use the identification of sub-concepts to evaluate student learning and to identify errors in students' work, and thus improve their instruction based on the evaluations they do while teaching (Hiebert et al., 2007; Morris & Hiebert, 2009). Providing opportunities for PSTs to analyze and reflect on mathematical ideas and student learning improves their knowledge for teaching. Further, it also shows that such opportunities can be used to influence the beliefs that future teachers bring to their teacher preparation programs.

The use of mathematics standards and standards-based curriculum

In planning mathematics instruction, the PSTs should be familiar with the tasks and standards needed for conceptual understanding. The implementation of standards-based curriculum that follows the reform movement in mathematics is new and unfamiliar to beginning teachers. Studies show that practicing teachers have challenges implementing these curriculum materials (Remillard, 2005). Therefore, professional development that focuses on curriculum is important in helping teachers know how to use a novel curriculum that is effective for learning (Hill et al., 2008; Putnam et al., 1992). Pre-service teachers should also have some experience in using these curricula and be familiar with the standards-based curriculum that is novel to many teachers, if teacher educators expect to improve on the knowledge and beliefs that pre-service teachers come with to their teacher preparation program.

Introducing PSTs to standards-based curricula, with which many of them are not familiar, can expand their knowledge and beliefs about teaching and learning. The use of standards-based curriculum, in teaching content and pedagogy courses in which PSTs worked collaboratively on open-ended tasks, was found to reveal PSTs' knowledge deficiencies, to build on these knowledge deficiencies, and to challenge their beliefs about mathematics learning (e.g. Lloyd, 2006; Ma, 1999; Tarr & Papick, 2004). In particular, the use of the standards-based curriculum tasks revealed the deficiencies in knowledge the pre-service teachers had (Tarr & Papick, 2004), built on their knowledge for teaching mathematics, and helped them appreciate the investigations and discussions that the standards-based materials afforded (Lloyd, 2006). These studies show that when pre-service teachers have experiences with the standards-based curriculum when learning to teach, their knowledge gaps in mathematics are foregrounded. Further, the investigative nature of the questions in standards-based curricula provides rich experiences that might influence their beliefs about learning mathematics. These studies, however, are just single cases studied by instructors in their own courses.

Creating meaningful learning experiences for all students

In preparing teachers to develop knowledge for teaching mathematics, the selection of tasks to be solved in class is central when planning for instruction. High-level cognitively-demanding tasks have been documented to be useful in accommodating diverse learning needs in the classroom (e.g.,Kabiri & Smith, 2003; Robert, 2002; Wilkins, Wilkins, & Oliver, 2006). Cognitively demanding tasks allow students to engage in meaningful learning that focuses on the central mathematical concepts and the use of prior knowledge (Polly & Orrill, 2012; Smith & Stein, 1998). Therefore, for pre-service teachers to know the richness of such tasks, they need to have opportunities that allow them to analyze, critique, and solve these tasks during their teacher preparation. The process of the analysis of tasks allows PSTs to learn to develop, modify, and select tasks that are meaningful for their students' various understandings, experiences, and needs.

If PSTs are introduced to rich mathematical tasks, they have opportunities to re-think their beliefs about teaching and learning mathematics and to develop strategies of teaching that show a shift to thinking in more abstract terms. (Flowers & Rubenstein, 2006; Steele, 2008). Other meaningful learning experiences that have been introduced to PSTs for use in K-12 classrooms that show positive teaching competencies are analyzing, selecting, and learning to sequence models, manipulatives, and applets for teaching mathematics (Hjalmarson & Suh, 2008), which are important in planning of a mathematics lesson for conceptual understanding. These studies reveal potentially important OTL to plan mathematics instruction that deserves investigation with a larger population to examine if the effects of these approaches to learning to teach are generally effective. Findings currently in the literature are based on instructors' reports on what they did in individual courses. Most of the studies were analyzed using an insider's perspective, which then allows room for questions about the circumstances under which the data was collected and the teacher/researcher and student relationships (Grossman, 2005). There is a need to examine further these findings, using data that can allow analyses from an outsider's perspective.

Opportunities to learn mathematics instruction for conceptual understanding

In teaching using the methods recommended by reform, teachers have to negotiate, transform, and adapt their content knowledge so that it fits the reform methods of instruction. Teachers should involve their students in problem solving, encourage students to use multiple strategies to solve tasks, encourage reasoning and proof, and use appropriate representations to communicate their ideas (NCTM, 2000). The Conference Board of Mathematics Sciences (2000) pointed out that teachers should believe that mathematics is not rules and procedures, but is instead about ideas that make sense. To prepare PSTs to be able to teach effectively in the methods of reform, opportunities to learn classroom instruction that is specific to learning mathematics for conceptual understanding is of utmost importance during their teacher preparation.

Some approaches that have been used by individual instructors have emphasized ways that PSTs can identify and teach for conceptual understanding, as well as reason mathematically. Providing opportunities to develop skills for providing explanations (Charalambous, Hill, & Ball, 2011), providing opportunities to learn to differentiate procedural and conceptual knowledge in students' work (Bartell et al., 2012), and using approaches that emphasize the affordances of multiple representations and multiple strategies to solve problems (e.g., Crespo, 2000; Grant & Lo, 2009; Ryken, 2009). Some of the competencies that the PSTs developed from these studies were recognizing conceptual understanding from students' responses, perceiving teaching and learning mathematics as focused on reasoning about mathematics (Bartell et al., 2012), appreciating the influence of the problem context on students' reasoning (Grant & Lo, 2009), providing mathematics explanations (Charalambous et al., 2011), focusing on student thinking, and critically analyzing different representations (Ryken, 2009). At the beginning of the courses, PSTs focused more on right and wrong and had a rule-bound perception of mathematics. At the end of the courses, the PSTs were more critical in analyzing their students' responses with a focus that reflected some of the recommendations[10] made by the Conference Board of Mathematical Sciences.

As is the case in other studies of promising pedagogical approaches used in preparing teachers to teach mathematics, the studies shown here are small–scale interventions that are based in the United States. Grossman (2005) pointed out that small- scale teacher education

[10] The recommendations from the CBMS include (i) providing opportunities for PSTs to learn to appreciate and understand alternative strategies that students use when solving problems, (ii) developing flexible knowledge that is important for teaching and understanding students' thinking, (iii) using various modes of representations (iv) working with manipulatives that help to create images and better understanding of mathematics (CBSM, 2000).

studies do not have a standard way of assessing teacher competencies. Further, there is scant literature on large-scale studies of the promising approaches mentioned. Adler and colleagues (2005), in their review of mathematics teacher education, also found that there was a lack of large-scale studies on teacher education. Large-scale studies in which these approaches are examined using a similar analytical tool can provide empirical evidence that shows if these OTL and their relationships to PSTs' knowledge and beliefs can be generalized to a wider population.

2.1.5. Cross-National Studies on the Relationships between OTL in Teacher Education and PSTs' Competencies

Cross-national studies on teacher preparation in mathematics suggest that there may be significant relationships between opportunities to learn to teach mathematics and PSTs' competencies at the end of their teacher preparation.

Findings from MT21

Schmidt and colleagues (2011) found that the relationships between OTL and middle school PSTs' knowledge for teaching mathematics were significant in the MT21 teacher education study. In particular, OTL calculus and advanced mathematics [11] were significantly related to the PSTs' scores on algebra, functions, number geometry, and data, within and between institutions (Schmidt, Blömeke, et al., 2011). In other words, OTL higher-level mathematics was related to PSTs content knowledge at the end of their teacher preparation for this sample.

Schmidt also reported that PSTs' MPCK was related to OTL higher-level mathematics and practical teaching experiences. The categories of MPCK examined were curricular

[11] Advanced mathematics topics included topics in "multivariate calculus, analysis, differential equations, topology, and differential geometry (Schmidt, Blömeke, et al., 2011) .

knowledge[12], knowledge for teaching[13], and knowledge of students[14]. The findings in the MT21 study found that, for this sample, programs in which the PSTs had more opportunities to learn the history of mathematics had higher curricular knowledge. In addition, the study found a significant relationship between the number of practical experiences and PSTs' knowledge for teaching and knowledge of students across programs. Further, OTL mathematics pedagogy, how students learn, and methods of solving problems were significantly related to PSTs' knowledge of student learning across programs. Finally, programs in which PSTs had OTL instructional interactions showed significant relationships with their knowledge for teaching mathematics (Schmidt et al., 2011).

Schmidt and colleagues (2011) examined PSTs' beliefs that were closely associated with the nature of mathematics[15], namely those on the nature of mathematics and those that emphasize the objectives[16] needed for teaching. The study found that the set of beliefs about the nature of mathematics referred to as the algorithmic scale was significantly related to PSTs' knowledge of middle school mathematics and the belief of the " role of the teacher as a knowledge provider" (p.246). Further, Schmidt and colleagues found no differences on the beliefs about the objectives that should be emphasized in the classes across the participating countries The types of learning experiences, examples which include (i) how algorithms and proofs are taught, (ii) exploring student thinking, (iii) principles of mathematics instruction, and (iv) a deeper

[12] Curricula knowledge is the knowledge of the informal curriculum, and the reasons why the topics are arranged in a certain order (Schmidt, Blömeke, et al., 2011). Shulman (1986; 1987) described the curricula knowledge as knowledge of the vertical and the lateral curriculum (see more on Curricula knowledge)
[13] Knowledge for teaching are the instructional interactions that takes place during the process of teaching (Schmidt, Blömeke, et al., 2011).
[14] The knowledge of students includes how the students learn, common errors they make and the misconceptions they have (Schmidt, Blömeke, et al., 2011).
[15] The beliefs scales focus on the nature of mathematics. In particular,(i) mathematics is useful and (ii) mathematics is algorithmic , made of many definitions, procedures, rules, and strategies(Schmidt, Blömeke, et al., 2011).
[16] The beliefs on the objectives include (i) the development of students reasoning and problem solving and (ii) have students learn the algorithms, rules etc. (Schmidt et al., 2011, p.241)

understanding of school level mathematics, however, showed significant relationships with the PSTs' beliefs (Schmidt, Blömeke, et al., 2011). Schmidt and colleagues (2011) concluded that these findings point to experiences that teacher educators should emphasize when teaching pre-service teachers.

Findings from TEDS-M

Scholars have used the TEDS-M survey to explore OTL and PSTs' knowledge and beliefs for teaching and learning mathematics across countries and within countries. Blömeke and Kaiser (2012), for example, found that PSTs who had experienced a functional curriculum in mathematics pedagogy, which is a class with a "narrow focus on mathematics instruction", had higher PCK scores when compared to PSTs who experienced a broader curriculum (p. 313). Further, PSTs' content knowledge and pedagogical content knowledge were found to be significantly related to the number of mathematics-related topics taken (Schmidt, Cogan, & Houang, 2011). Wong, Boey, Lim-Teo, and Dindyal (2012) found similar results when they examined OTL measures in relation to the content knowledge and pedagogical content knowledge in the single teacher education institution in Singapore. They considered the number of tertiary mathematics topics and the number of mathematics pedagogy that the PSTs reported having studied as the OTL measure (Wong et al., 2012). Further, Blömeke and Delaney (2012), in their study analyzing teacher knowledge across countries, found that Taiwanese and Singaporean PSTs performed "relatively better with respect to mathematical language, including representing mathematical entities and handling mathematics symbols or formalisms, than with respect to modeling and reasoning" (p.236). They also reported that the ranking of countries' performances in the TEDS-M knowledge scores was very similar to the TIMSS scores, and found that gender differences, language effects, prior knowledge, and motivation are associated with PSTs' professional knowledge scores in selected countries (Blömeke & Delaney, 2012).

Most of the cross-national studies on teacher education have found relationships between OTL different topics and the time spent in teacher preparation and teachers knowledge. Missing from the cross-national studies is the nature of the pedagogical approaches used in the mathematics or the mathematics methods courses for learning to teach mathematics. In addition, most of the cross-national studies do not discuss the extent to which the pre-service teachers are introduced to reform methods of instruction. Wong and colleagues (2012) used the OTL variables for mathematics pedagogy in their examination of direct and indirect instruction in teacher education. Their results on the relationships between these OTL measures and PSTs' knowledge for teaching are not clear, because they only provided results of the Pearson correlation coefficient with the OTL mathematics methods, which is not specific to the pedagogical practices experienced in teacher preparation. The cross-national study completed here examines the relationships between the pedagogical practices used in teacher preparation and the extent of PSTs' experiences with reform methods of discussion, and their knowledge and beliefs about teaching and learning mathematics. The study also builds on the previous studies that have examined promising approaches in teacher preparation, by using one approach to examine these pedagogical approaches in a larger context, and by using an outsiders' perspective. By so doing, the study will be able to compare the extent of the opportunities to learn across the countries that participated in the TEDS-M study. This study examines the frequency of the activities and experiences the PSTs reported as OTL to teach mathematics, unlike the previous studies that examined the number of topics and curriculum offered as measures of OTL.

2.2. Conceptual Framework and Questions Guiding the Study

Based on the cited literature, I hypothesize that there is a relationship between the extent to which different pedagogical approaches are used in teacher preparation and i) content knowledge, (ii) pedagogical content knowledge, and (iii) beliefs about learning mathematics. The pedagogical approaches considered in this study are pedagogical practices experienced in mathematics and mathematics methods courses, approaches to learning the planning of mathematics instruction for conceptual understanding, and approaches to learning mathematics instruction for conceptual understanding.

Specifically, I hypothesize that variation in three sets of teacher preparation practices will predict differences in three sets of outcome measures. The three sets of teacher preparation practices are listed in the left-hand boxes of Figure 1: pedagogical practices in content and methods courses, OTL to *plan* mathematics instruction for conceptual understanding, and OTL mathematics instruction for conceptual understanding. Each of these three sets of teacher preparation practices has several components, drawn from the literature reviewed in this chapter.

The three types of outcome measures, also drawn from this literature review are mathematics content knowledge, mathematics pedagogical content knowledge, and beliefs about the nature of learning mathematics. These are represented in the right hand boxes of Figure 1.

Based on the literature, I hypothesize that there will be a positive relationship between most of the teacher preparation practices in the left-hand boxes and the outcomes in the right hand boxes. The exceptions to this general pattern are items in each column that are associated with non-reform oriented practices (for teacher preparation practices) and for non-inquiry oriented beliefs (for outcome measures). For these an association in the opposite direction is expected.

In particular, the following indicates how the literature supports these expected associations. The practices that model reform methods of instruction included pedagogical practices that allow for PSTs to communicate their ideas, assess their peers responses, and discuss their thoughts by asking questions, whole classroom discussions, and small group discussions (e.g., Lloyd, 2006; Ma, 1999). These pedagogical practices were associated with PSTs' appreciating that learning was different from what they were used to and a deeper knowledge for teaching mathematics (e.g., Coffey, 2004; Tarr & Pappick, 2004). This relationship is represented in Figure 1 with arrows from models of reform-oriented instruction to MCK, MPCK and inquiry beliefs. The relationship between non-reform oriented pedagogical practices and the unintended outcomes (e.g., Eisenhart et al., 1993) are represented in Figure 1 with arrows from the pedagogical practices to the three teacher competencies. The analysis of teaching and learning using audio, video, case studies, lesson plans, supported PSTs to focus more on student thinking, shifted thinking to more problem solving practices, and deeper understanding of mathematics (e.g., Henningsen, 2008; Taylor &O'Donnell, 2004) and is represented on Figure 1 using an arrow from the pedagogical practices to the PSTs' competencies which were identified as MCK, MPCK and beliefs about teaching and learning.

Learning how to plan mathematics for conceptual understanding through analysis of learning goals (Hiebert et al., 2007; Morris & Hiebert, 2009), using standards-based curriculum (e.g., Putnam et al., 1992), creating meaningful learning experiences such as selection development of tasks, selection of models (e.g., Flowers & Rubinstein, 2006) was related to PSTs competencies allowed PSTs to think more abstractly and rethinking their beliefs about teaching and learning mathematics. These relationships that define the relationships of learning how to plan mathematics instruction and the knowledge and beliefs about teaching and learning mathematics shown were represented on Figure 1 with arrows representing these relationships with the PSTs competencies.

Learning mathematics instruction for conceptual understanding through developing skills to provide explanations or learning to explain why procedures work (e.g., Charalambous et al., 2011), differentiating procedural and conceptual knowledge in students' work, and the affordances of the introduction to multiple representations (e.g., Crespo, 2000; Ryken, 2009), were related to PSTs being able to recognize conceptual understanding on students' responses, developing their reasoning in mathematics tasks, provided them opportunities to analyze multiple representations, which all showed more thinking about problem solving strategies and expansion of their knowledge. These relationships are represented using arrows from the interventions to the PSTs' competencies identified as MCK, MPCK, and beliefs about teaching and learning mathematics. A diagrammatic representation reflecting the research questions and the design of the research is shown in Figure 1.

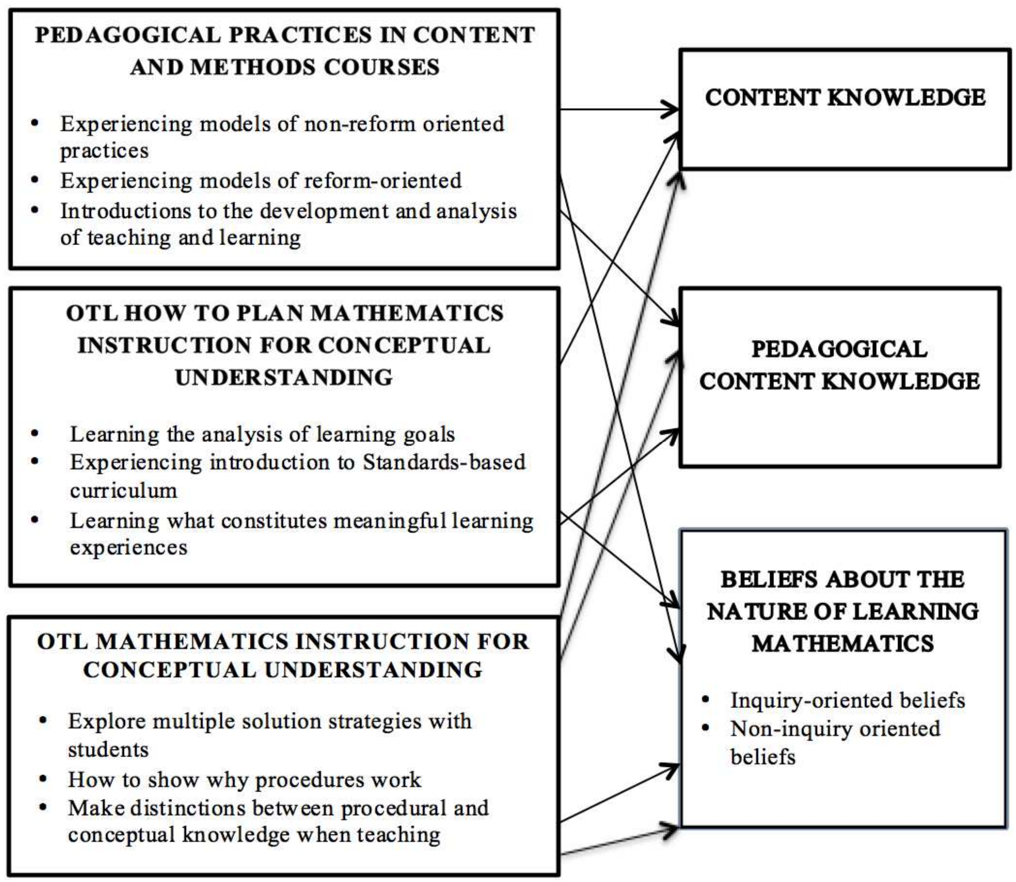

Figure 1. A conceptual framework showing the hypothesized relationships between the opportunities to learn to teach mathematics and the outcome variables.

The questions guiding the study are:

1. What are the differences in PSTs' content knowledge, pedagogical content knowledge, and nature of beliefs about learning mathematics across the 17 countries that participated in the TEDS-M survey?
2. What are the differences in the 17 countries included in the TEDS-M database in the extent of the opportunities to learn to teach through (i) experiencing different pedagogical practices in mathematics education or teacher education courses; (ii) introducing approaches to learning how to plan mathematics instruction for conceptual

understanding; and (iii) introducing mathematics instruction for conceptual understanding?

3. What is the relationship between OTL to teach through experiencing different pedagogical practices in PSTs' mathematics education or teacher education courses and their (i) content knowledge, (ii) pedagogical content knowledge, and (iii) beliefs about the nature of learning mathematics:
   - Within and between teacher education institutions in Poland?
   - Within and between teacher education institutions in Russia?
   - Within and between teacher education institutions in the United States?
4. What is the relationship between OTL to teach through different approaches to learning how to plan mathematics instruction for conceptual understanding and PSTs' (i) content knowledge, (ii) pedagogical content knowledge, and (iii) beliefs about the nature of learning mathematics:
   - Within and between teacher education institutions in Poland?
   - Within and between teacher education institutions in Russia?
   - Within and between teacher education institutions in the United States?
5. What is the relationship between the OTL to teach through the introduction to learning mathematics instruction for conceptual understanding and PSTs' (i) content knowledge, (ii) pedagogical content knowledge, and (iii) beliefs about the nature of learning mathematics:
   - Within and between teacher education institutions in Poland?
   - Within and between teacher education institutions in Russia?
   - Within and between teacher education institutions in the United States?

CHAPTER 3

DATA AND METHODS

3.1. Description of the Data

3.1.1 Sample design

The study used data from the Teacher Education and Development Study in Mathematics (TEDS-M) international survey. This survey was conducted by the national research centers in each of the participating countries with, support from the International Association for the Evaluation of Educational Achievement (IEA). The TEDS-M survey, which was conducted in 2008, had the following research goals (Tatto et al., 2008):

1. To examine the level and depth of mathematics content knowledge and the related knowledge for teaching that future elementary and lower secondary teachers attained, and how these variables varied across the countries that took part in this survey.
2. To examine how teacher education programs varied across countries regarding (i) the learning opportunities available, (ii) the structures available, (iii) the content taught, and (iv) the organization of the programs.
3. To examine the variation across countries on recruitment, curriculum, quality assurance, and funding for teacher education.

The TEDS-M study was designed to examine the interrelationships among the characteristics of teacher education programs, future teacher characteristics, the characteristics of teacher educators, and future teachers' knowledge and beliefs.

Samples of future primary teachers, future secondary teachers, and their teacher educators were obtained from each of the seventeen countries[17] that participated in the TEDS-M

[17] The countries that participated in the TEDS-M survey were "Botswana, Canada (four provinces), Chile, Chinese Taipei, Georgia, Germany, Malaysia, Norway, Oman (lower secondary teacher education only), the Philippines,

survey. The sampling design used in the TEDS-M survey provided nationally representative[18] samples of the pre-service teachers in each of the participating countries. Sources of data used in the TEDS-M survey included individual case study reports, questionnaires, and interviews to examine polices that govern the preparation of primary and secondary teachers. To examine the opportunities to learn to teach mathematics, the pre-service teachers were surveyed using questions about their experiences on the frequency of different pedagogical practices, opportunities to learn to plan mathematics instruction and mathematics instruction, their beliefs about mathematics, and their beliefs about teaching and learning mathematics. The teacher educators were also surveyed using similar questions about the opportunities they provide to their pre-service teachers. Finally, the pre-service teachers' knowledge for teaching mathematics was assessed just before they graduated from their teacher preparation programs (Tatto, 2013).

A stratified multistage probability sampling design was used to conduct surveys of PSTs, teacher educators, and institutions preparing the PSTs. The participating countries were asked to provide a sampling frame of all the routes[19] for teacher preparation, and to highlight the major routes, which then provided the target population (Tatto, 2013). The target population was all the PSTs in the routes identified in the sampling frame who were in their final year of their teacher preparation. The PSTs were sampled after considering the total number of institutions, the size of the institutions, and the sampling method from the target populations, using a two-stage

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Poland, the Russian Federation, Singapore, Spain (primary teacher education only), Switzerland (German speaking cantons only), Thailand, and the United States (public institutions, concurrent and consecutive teacher education program routes only)"(Tatto, 2013, p. 13).

[18] Pre-service teachers were sampled from all routes that lead to teacher certification in the countries that participated in the survey. They were identified and country reports prepared and data produced that allowed for comparison of the institutions across the countries (Tatto, 2013). The target population was any secondary or post-secondary institution that offered a structured teacher preparation program that included mathematics-related units in their curriculum (Tatto et al., 2009). Random samples were drawn to ensure that the PSTs selected represented the population of PSTs in the teacher education institutions in the country.

[19] Route is a "sequence of opportunities to learn …the prescribed pathway through which the teacher education programs are made available in a given country (Tatto et al., 2009, p.23)

sampling design. The institutions were classified “into subgroups defined by the level, route and program combinations” (p.86), called teacher preparation units (TPU) (Tatto, 2013). Systematic random sampling using explicit[20] and implicit[21] stratification was used to select the institutions. Probability proportional to size (PPS) sampling of the institutions was used if the size of the institutions was available, and in cases of small institutions or those without the institutional size available, the institutions were sampled with equal probabilities. The sampling of the institutions was the first stage of sampling (Tatto, 2013).

The second stage of sampling was of the PSTs and the teacher educators within the institutions. Whole-sessions groups[22] of PSTs and individual teachers were selected from the institutions. Whole-sessions were sampled for organizational purposes and it was “desirable and convenient “ for the sampling teams to use this sampling in large institutions instead of sampling individual PSTs (p. 90). Since the selection of whole-session groups is less efficient because of clustering effects, the teams in charge of the selection process either increased the sample sizes or assigned PSTs to the session groups, which were then randomly selected. Examples of countries that sampled whole-session groups are Germany and Russia (Tatto, 2013). For the selection of individual PSTs, the team conducted random sampling or surveyed all the PSTs if the institution’s teacher preparation unit (TPU) had less than 30 PSTs (Tatto, 2013). Examples of countries that sampled individual PSTs are Singapore and Botswana.

[20] Explicit stratification “involves separating the population into strata and then drawing a separate sample from each one”(Tatto, 2013, p.88)
[21] Implicit stratification involves” ordering the sampling frame before sampling according to specified stratification categories so as to ensure an approximately proportional allocation of the entire sample” (Tatto, 2013, p.88).
[22] When whole groups were sometimes selected instead of individual PSTs in some large institutions (e.g. in Chinese Taipei and Russia), they were referred to as “session groups”.

3.1.2 Weights and Participation Rates

The PSTs in the different countries were sampled using different sampling designs, all of which allowed for accurate estimation of responses for the country as a whole. Analyses were done in light of these differences in the sampling designs to achieve unbiased estimates of the population in each country (Winship & Radbill, 1994). Weights "reflect and compensate the different selection probabilities and the different non-response patterns at the various sampling stages" (Brese & Tatto, 2012, p.33). The weights provided with the TEDS-M data were calculated using information from the sampling design and the non-response of institutions, pre-service teachers, and teacher educators. As indicated earlier, the sampling design was a two stage process, in which the institutions were first sampled and then the pre-service teachers and teacher educators. The final institution weight, INSWGT, was calculated as a product of the institution base weight[23] (WGTFAC) and institution non-response factor adjustment[24] (WGTADJ). The final primary teacher weight, FINWGTP, was calculated as a product of final institution weight (INSWGTP), session group weight[25] ($WGTFAC_2P$), PSTs' base weight[26] ($WGTFAC_3P$, the PSTs non-response adjustment factor [27]($WGTADJ_3P$), and future teacher level weight[28] ($WGTFAC_4P$) (Tatto, 2013).

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[23] The institution base weight was given as 1 if the sampling was a census of the institutions. However, for systematic random sampling of equal probability of probability proportional to size it was calculated "for each institution…and each explicit stratum" (Tatto, 2013, p.132).

[24] For each stratum a non-response adjustment factor was calculated to compensate for the non-participating institutions that had been sampled (Tatto, 2013).

[25] Some of the teacher preparation units had session groups that had to be accounted in the weight calculation. However, if the PSTs were selected from a list of all PSTs in the institution, then the session group weight was set to 1 (Tatto, 2013).

[26] In institutions where session groups were not sampled, the individual PSTs were sampled through a systematic random sampling with equal probability. However, in institutions with session groups, the base weight was set at 1 (Tatto, 2013).

[27] The adjustment factor was included to compensate for non-participating PSTs who were already sampled (Tatto, 2013).

The participation rates for the target population was set at a minimum value (85%) to ensure that there were no biases due to non-response (Tatto, 2013).

Seventeen countries participated in the Teacher Education and Development of Mathematics (TEDS-M) survey. The PSTs used in this study were sampled from consecutive and concurrent program structures. In the consecutive programs the PSTs take an undergraduate course in other disciplines together with the teacher education courses, while in concurrent programs the PSTs take courses in teacher education after their undergraduate education in a subject-matter area. A description showing the number of institutions and individual pre-service elementary teachers that participated in the study is summarized in Table 1.

Table 1: *Number of Institutions and Elementary Pre-service Teachers Participating in the TEDS-M Study*

Country	Number of PSTs	Number of Institutions	Program Structures	% Female
Botswana	86	4	concurrent	59.3
Chile	657	31	concurrent	84.9
Chinese Taipei	923	11	concurrent	72.3
Georgia	506	9	concurrent	99.2
Germany*	1032	1	concurrent , consecutive	91.6
Malaysia	576	23	concurrent, consecutive	62.3
Philippines	592	33	concurrent , consecutive	78.5
Poland	2112	78	concurrent	94.8
Russia	2266	49	concurrent	92.2
Singapore	380	1	concurrent, consecutive	74.2
Spain	1093	45	concurrent	79.9
Switzerland	936	14	concurrent	85
Thailand	660	45	concurrent, consecutive	74.8
United States	1501	51	concurrent, consecutive	88.6
Norway (ALU)	392	12	concurrent	75.5
Norway (ALU+)	159	14	concurrent	67.3

*Note: Canada is not shown in the summary because it did not reach the required participation rate set by IEA*Germany had more than one institution participating in the study but the institutions were de-identified in the international TEDS-M data base. That is, the institution ID provided in the data is the same for all the institutions in Germany.*

[28] The future teacher level weight is set at 1 if the PSTs were preparing to teach for primary level only. However, if the PSTs were prepared to teach at the primary and secondary level, then they were randomly assigned to one of the two surveys and adjustments made for this sampling process (Tatto, 2013).

Table 1 shows that Poland, Russia, and the United States had the highest number of institutions that participated in the TEDS-M survey. Females are the majority of the elementary PSTs surveyed in all of the countries. The male representation of PSTs preparing to teach elementary school mathematics is highest in Botswana and Malaysia. Additionally, all the participating countries have concurrent teacher preparation programs, with a few countries also having consecutive teacher preparation programs. This study examined teachers from the concurrent and consecutive elementary teacher preparation programs.

Primary (elementary) PSTs differ in the grade levels of education that they are prepared to teach. In the countries surveyed in the TEDS-M study, the future teacher primary groups were lower primary generalists (grade 4 maximum), primary generalists (grade 6 maximum), primary/lower secondary generalists (grade 10 maximum) and primary mathematics specialists. TEDS-M categorized these specializations as *grade spans*. Across the countries, the PSTs are distributed as shown on Table 2.

Table 2 shows that PSTs who are prepared to teach grades 1 to grade 4 were mostly from the European countries, except Spain and Norway. The East Asian countries and the United States prepare their elementary teachers to teach grades 1 to grade 6. Thailand and Malaysia prepare their elementary teachers as mathematics specialists only. Poland, Singapore, and the United States, however, prepare teachers for specific grade levels and also have programs that prepare mathematics specialists.

Table 2: *Number of PSTs Sampled according to their Teaching Grade Level Specialization (Grade Span)*

Country	Grades 1- 4	Grades 1- 6	Grades 1-10	Specialists
Botswana			86	
Chile			654	
Chinese Taipei		923		
Georgia	506			
Germany	907			97
Malaysia				574
Philippines		594		
Poland	1812	-	-	300
Russia	2260		-	-
Singapore		262		117
Spain		1093		
Switzerland	936			
Thailand				660
United States	-	1310	-	191
Norway (ALU)			392	
Norway (ALU+)			159	

3.1.3. Units of Analysis

This study examined the relationships between the opportunities to learn to teach elementary school mathematics and the knowledge and beliefs about teaching and learning mathematics of the PSTs within and between institutions within the participating countries. The design of the study therefore examined these relationships at the PSTs' level and the institutional level. For this reason, there were two levels for the units of analysis, the pre-service teacher level and the institution level. The PSTs were clustered within the institutions, and thus the PSTs were the level 1 unit of analysis, and the institutions were the level 2 unit of analysis.

3.2. Dependent Variables: Learning Outcomes

The learning outcomes considered in this study were knowledge for teaching mathematics and beliefs about learning mathematics. The knowledge for teaching mathematics included content knowledge and pedagogical content knowledge. The beliefs about learning mathematics were inquiry beliefs and non-inquiry beliefs.

3.2.1. Content Knowledge

The PSTs' depth and level of their mathematics content knowledge was measured using a content knowledge achievement test (Tatto, et al., 2012). Content knowledge domain of teacher knowledge was informed by Shulman's (1986; 1987) content knowledge dimension that includes the substantive and syntactic structures of mathematics. The content knowledge domains were selected from the content subdomains of "number and operations[29], geometry and measurement[30], algebra and functions[31], and data and chance[32]" (p. 32), and they spanned the cognitive subdomains of knowing[33] , applying[34], and reasoning[35]. This achievement test had questions that ranged from curricular-levels categorized as "novice, intermediate, and advanced" (Tatto, 2013, p.32). Novice mathematics is school level mathematics of the PSTs' specialization, while the intermediate level is content that is one or two grades levels above the PSTs'

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[29] Numbers and operations include the content areas " whole numbers, fractions and decimals, number sentences, patterns and relationships, integers, ratios, proportions and percentages, irrational numbers, and number theory" (Tatto, 2013, p.33).
[30] Geometry and measurement include the content areas "geometric shapes, geometric measurements, and location and movement" (Tatto, 2013, p.33).
[31] Algebra and functions include the content areas of "patterns, algebraic expressions, equations/formulas and functions" (Tatto, 2013, p.33).
[32] Data and chance include the content areas of data organization and representation, data reading and interpretation, and chance" (Tatto, 2013, p.33).
[33] The cognitive subdomain of knowing includes being able to (i) "recall definitions, terminology, notation, mathematical conventions, number properties, geometric properties"; (ii) "recognize entities that are mathematically equivalent"; (iii) compute or "carry out algorithmic procedures"; and (iv) "retrieve information from graphs, tables, or other sources" (Garden et al., 2006, p.19).
[34] The cognitive subdomain of applying includes being able to (i) "select an efficient / appropriate method of strategy for solving a problem; (ii)"generate alternative equivalent representations for a given mathematical entity, relationship, or set of information"; (iii) generate an appropriate model such as an equation or diagram for solving a routine problem; and (iv) solving routine problems" (Garden et al., 2006, p.20).
[35] The reasoning domain includes being able to analyze a given problem and "select the mathematical facts necessary to solve a particular problem…make valid references from given information;(ii) generalize or "restate results in more general and widely applicable terms"; and (iii) synthesize or "make connections between different elements of knowledge and related representations… and related mathematical ideas" (Garden et al., 2006, p.22).

specialization. The advanced level content is three or more grade levels higher than the PSTs' specialization (Tatto, 2013). These subdomains were adopted from the Trends in International Mathematics and Science Study (TIMSS) in 2007.

3.2.2. Mathematics Pedagogical Content Knowledge

The PSTs' pedagogical content knowledge was measured using an achievement test that spanned three main subdomains. These subdomains were *mathematics curricular knowledge, knowledge for planning mathematics teaching and learning*, and *knowledge for enacting mathematics for teaching and learning* (Tatto et al., 2012). *Mathematics curricular knowledge* includes knowing the (i) mathematics curriculum, (ii) "key ideas in the learning programs," (iii) "connections within the curriculum," (iv) different assessment formats, and (v) "appropriate learning goals" (Tatto, 2013, p.35). The *knowledge for planning mathematics teaching and learning* comprises (i) the "planning or selection" of activities, (ii) "choosing assessment formats," (iii) a repertoire of students' responses, misconceptions, and ideas, (iv) planning teaching methods for representing mathematical ideas, (iv) linking the "methods and instructional designs," (v) identifying multiple solution strategies, and (vi) "planning mathematics lessons" (Tatto et al., 2012, p. 131; Tatto, 2013, p. 35). *Enacting mathematics* covers (i) understanding the multiple solution strategies used by students, (ii) analyzing the questions students pose, (iii) recognizing responses students make and their common misconceptions, (iv) "explaining or representing mathematical concepts and procedures," (v) posing questions, (vi) "responding to unexpected mathematical issues," and (vii) providing feedback (Tatto et al., 2012, p. 131; Tatto, 2013, p. 35). These mathematics pedagogical content knowledge domains were developed from previous studies of teacher competency. Examples include Shulman's PCK domain, Ball and colleagues (2008) dimensions of PCK, MT21 pilot study, the COACTIV[36] German study, as well as the Learning Mathematics for Teaching study (Tatto, 2013).

3.2.3. Beliefs about Learning Mathematics

The pre-service teachers were asked to respond to questions about their beliefs about learning mathematics. Table 3 gives a summary of the questions related to inquiry and non-inquiry beliefs to which the PSTs responded, as categorized by the TEDS-M team. The response options were given on an ordinal scale of "strongly disagree, disagree, slightly disagree, slightly agree, and strongly agree." These responses were assigned numerical values from one through six, respectively. A composite variable consisting of all the responses that were inquiry beliefs or active learning was used as the outcome variable "inquiry beliefs about learning mathematics." Likewise, a composite of the non-inquiry or teacher-directed beliefs was used as the outcome variable "non-inquiry beliefs about learning mathematics." These belief dichotomies were informed by Ernest's (1989a) dichotomy of the views about the process of learning mathematics. The belief outcomes from the studies cited include a shift in the PSTs beliefs about learning, an appreciation that learning mathematics is different from what they were used to, views about learning focused more on developing conceptual knowledge (e.g., Carpenter et al., 1989; Lloyd, 2006). These beliefs can only be adaptable to change if the programs help their PSTs to develop a desirable belief structure that is adaptable to change (Cooney et al., 1998). The correspondence between questions asked in the TEDS-M instrument and beliefs described in the literature are shown in Table 3.

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[36] The German COACTIV study examined teacher competencies needed for effective mathematics instruction. The study's goals were to investigate ways in which teacher competencies can be identified using empirical evidence, how the teachers' competencies are related to classroom instruction and student learning outcomes, and reasons why teachers' competencies differ (https://www.mpib-berlin.mpg.de/coactiv/en/study/).

Table 3: *Beliefs about Learning Mathematics*

Main Variables	**Variables**	**Description**
Beliefs about learning mathematics	Inquiry beliefs / Active learning (MFD002G, MFD002H, MFD002K, MFD002L, MFD002M, MFD002N)	Pre-service teachers were asked whether they agree with these statements, representing inquiry beliefs: (i) in addition to getting a right answer in mathematics, it is important to understand why the answer is correct; (ii) teachers should encourage students to figure their own ways to solve mathematics problems even if the solutions are inefficient; (iii) the time spent on investigating why a solution to a mathematics problem works is time well spent; (iv) pupils can figure out how to solve mathematics without teachers' help; (v) it is helpful for pupils to discuss different ways to solve problems; and (vi) teachers should allow students to figure out their own ways to solve mathematical problems.
	Non-inquiry beliefs/Teacher directed (MFD002A MFD002B, MFD002C, MFD002D, MFD002E, MFD002F, MFD002I)	Pre-service teachers were asked whether they agree with these statements, representing non-inquiry beliefs: (i) learning mathematics involves the memorization of formulas; (ii) students should be taught exact procedures; (iii) it does not really matter if you understand the problem, if you can get the answer right; (iv) learning mathematics involves solving equations quickly; (v) students learn best by attending to teachers explanations; (vi) emphasis should be on getting the correct answer; (vii) non-standard procedures interfere with learning correct procedure; and (viii) hands-on procedures are not worth the time and expense.

Adapted from the TEDS-M user guide supplement 1 page 87

3.3. Independent Variables

3.3.1 Primary Independent Variables

The pre-service teachers responded to questions about their opportunities to learn to teach mathematics. These opportunities included what they did[37] in their mathematics learning, mathematics methods, and teaching courses, the activities they engaged in, and what they learned to do. The PSTs' rated all these opportunities to learn on an ordinal scale in which they were required to select from "never, rarely, occasionally and often." These responses were numerically assigned numbers from one to four, respectively. The opportunities to learn to teach were categorized into one of three categories: (i) *the pedagogical practices experienced*, (ii*)*

[37] PSTs reported what they did which is also referred to as what they experienced in other parts of the report because their responses are rated on the frequency that they had the given OTL.

learning how to plan mathematics instruction for conceptual understanding, and (iii) *learning mathematics instruction for conceptual understanding*.

Pedagogical practices

The pedagogical practices selected for this study were divided into three categories. These categories were OTL in which the PSTs experienced models of reform-oriented practices and non-reform oriented practices, and the analysis and readings of teaching and learning mathematics[38].

These variables were selected because they correspond fairly well to those interventions for learning to teach mathematics in the literature cited. The variables "work together in groups during class" and "participate in whole class discussion" and "ask questions during class time", in which PSTs work collaboratively on assignments and have opportunities to assess their peers responses are pedagogical practices that prior studies suggest will allow PSTs to get a clearer understanding of mathematics concepts and teaching practices (e.g., Bartell et al., 2012; Coffey, 2004; Lloyd, 2006; Ma, 1999; Tarr & Papick, 2004). Other studies indicated that using lecture presentations and demonstrations during teacher preparation was associated with PSTs developing procedural images of learning (e.g. Eisenhart et al., 1993). The variables from the TEDS-M database that closely correspond to these variables are "listen to lectures" and " teach a class session using the methods demonstrated by the instructor". The analysis of teaching and learning through audio analysis of mathematics lessons (Taylor & O'Donnell, 2004), case study analysis (Shifter & Bastable, 2008; Silver et al., 2008; Henningsen, 2008), reading and research on teaching and learning (Carpenter et al., 1989; Van Zoest et al., 2010), or any appropriate situation that provides a supportive environment based on set principles and standards of teaching and learning, have been suggested as examples of supportive contexts for learning to

[38] The analysis and reading of teaching and learning could be consistent or inconsistent with reform practices.

teach (e.g., Hiebert et al., 2007; Morris & Hiebert, 2009). In the TEDS-M database the variables that are closely related to these documented interventions are "read about research on mathematics education," "read about research on teaching and learning," and "analyze examples of teaching (film, video, transcript of lesson etc.)." A summary of the pedagogical practices variables selected from the TEDS-M database is shown on Table 4.

Table 4: *Pedagogical Practices Experienced in PSTs' Mathematics-related Courses*

Main Variables	Variables	Description
Opportunities to learn to teach mathematics	Reform-oriented pedagogical practices (MFB005B, MFB005C, MFB005G)	Pre-service teachers reported how frequently they (i) ask questions during class time; (ii) participate in a whole class discussion; and (iii) work together in groups during class.
	Non-reform oriented pedagogical practices (MFB005A, MFB005F)	PSTs reported how frequently they (i) listen to lectures and (ii) teach a class session using the methods demonstrated by the instructor.
	Analysis and reading about teaching and learning (MFB005I, MFB005J, MFB005K MFB005H)	PSTs reported how frequently they (i) read about research on mathematics education; (ii) read about research on teaching and learning; and (iii) analyze examples of teaching (film, video, transcript of lesson etc.).

Adapted from TEDS-M User Guide supplement 1 page 74.

Planning mathematics instruction

The opportunities to learn how to plan mathematics instruction for conceptual understanding were selected from the questions related to the activities the PSTs reported they engaged in during their teacher preparation. These activities examined the extent to which the pre-service teachers engaged in learning in-depth planning of an effective and meaningful mathematics lesson. These activities were classified as the composite measures "analysis of learning goals," "meaningful learning experiences," and "introduction to standards and standards-based curriculum."

"Analysis of learning goals" is a construct created from the composite of the variables listed as *assess higher and low level goals*, *use students misconceptions to plan instruction*, *create meaningful learning experiences,* and *set appropriately challenging learning expectations for pupils*. This selection and grouping of these variables as *Analysis of learning goals* is informed by literature from Morris and Hiebert (2009), which stated that "to be clear about learning goals means to identify the learning required to achieve the goals… Clarity about learning goals requires unpacking learning goals into constituent parts" or "sub-concepts (p.493)." Thus, to think about the learning goals during the planning of the lesson requires that the teacher analyzes teaching by planning activities that align with the sub-concepts identified, anticipating students' responses that include both ideal and those with errors, and thinking through possible evidence that show students understanding of the sub-concepts (Morris & Hiebert, 2009). Smith and Stein (1998), in outlining the five practices necessary for teaching, also emphasize that when identifying learning goals teachers should think about how to make the learning intended meaningful by thinking through how to organize the instruction based on the responses made by students on the selected tasks. The information from these studies justifies the selection and grouping of these variables from the TEDS-M as shown above.

"Introduction to standards-based materials" is a construct created from the composite of the variables listed in the TEDS-M data as *analyze and use national or state standards of framework for school mathematics, locate suitable curriculum materials and teaching resources,* and *locate instructional materials that builds on students' experience, interest, and abilities*. These variables correspond fairly well with the use of curriculum materials and studying materials intensively that were related to positive outcomes in the literature cited (e.g., Lloyd, 2006; Ma, 1999; Tarr &Papick, 2004). Further, the open-ended nature of cognitively demanding tasks which allow for multiple approaches to solving them are suitable for students with varying ability levels, interests, and prior knowledge (Smith & Stein, 1998). For this reason the variable *locate instructional materials that builds on students' experience, interest, and abilities* was selected from the TEDS-M database as closely related to the definition of cognitively demanding tasks found in suitable curricula materials. These variables were the best match to this feature of teacher preparation, given that the TEDS-M data does not have a question that directly asks the PSTs about the use or introduction to cognitively demanding tasks.

Finally, "meaningful learning experiences" is a construct created from a composite of the variables listed as *identify appropriate resources needed for teaching*, *build on students' existing mathematical knowledge and thinking skills*, *create projects that motivate all students to participate*, *accommodate a wide range of abilities in each lesson*, *explore how to apply mathematics to real-world problems*, and *explore the use of manipulative or concrete objects to solve mathematics problems* (Brese & Tatto, 2012, p.76). The opportunities to plan instruction using models, manipulatives, or applets, as shown in the available literature (e.g., Hjalmarson & Suh, 2008), corresponds reasonably well to the variables that were used to develop this measure in the mathematics classroom, because these activities require that the teacher identify what is appropriate, allow the students to be creative, accommodate the wide range of students abilities, and relate to real life situations.

Learning mathematics instruction for conceptual understanding

The "opportunities to learn mathematics instruction for conceptual understanding" were also selected from the activities that PSTs reported they engaged in. This category of opportunities to learn to teach mathematics included the variables *learn how to explore multiple solution strategies*, *learn how to show a mathematics procedure works*, and *make distinctions between procedural and conceptual knowledge when teaching mathematics concepts and* *operations to pupils* (Brese & Tatto, 2012, p.76). The OLT mathematics instruction identified in the literature as promising are learning: (i) to show why a procedure works, (ii) to use multiple solution strategies, and (iii) to make distinctions between procedural and conceptual knowledge in the literature (e.g., Bartell et al., 2012; Crespo, 2000; Grant and Lo, 2009; Ryken, 2009). These features of teacher preparation instruction match reasonably well with the variables from the TEDS-M database.

3.3.2 Control Variables

The study used gender and socio-economic status (SES) as the control variables. The gender variables were recoded into binary codes with females coded as a 1 and males coded as 0. This coding was needed for the model analysis in which the gender variable was introduced in the model as a dummy variable.

Socio-economic status has been measured using an index of economic, social, and cultural background (OECD, 2008; Thomson, De Bortoli, Nicholas, Hillman, & Buckley, 2010). Recently, Cowan and colleagues (2012) recommended that SES be defined as “one’s access to financial, social, cultural, and human capital resources” (p.14). This index includes the highest level of parents’ education, the wealth the family has, and the education and other resources present in the home (OECD, 2010). Similarly, other scholars have emphasized that the educational level of the parents and an index of home possessions influence student achievement (e.g., Marks, McMillan, Jones, & Ainley, 2000). The TEDS-M socio-economic indicators are similar to those that have been used in the Program for International Student Assessment (PISA). These variables include parental level of education, number of books in the home, and resources in the parents’ or guardians’ home.

Highest level of education in the household

This SES variable was the highest level of the parental education in the household. Using the UNESCO Institute of Statistics (UIS) mapping, the household income was recoded so that it gave the cumulative years of schooling of the parent with the highest level of education. This is based on the 1997 International Standard Classification of Education (ISCED), because at the time the data was collected the new 2011 ISCED had not been released.

Resources in the home

The number of books in the home was selected to represent this measure. This variable was recoded into a dummy variable, such that 1 represented "having more than one hundred books in the home" and 0 represented having less than one hundred books in the home. In many countries, the variable showed some variation using this categorization. Other resources, such as a calculator, a desk, a DVD, and a computer, did not vary within the countries, and therefore were not used in the study. As stated by Cowan and colleagues (2012), family possessions may not be an accurate measure of SES. In sum, the study used the household level of education and the number of books in the home as the SES measure.

3.4. Analytical Approaches

The TEDS-M data set involved disproportionate sampling, and therefore the analysis took into consideration this complex sampling method. The International Association of Evaluation of Educational Achievement International Database (IEA IDB) analyzer, which has an SPSS plugin, was used to merge data from the PSTs' files with data from the institutional files in all the 17 countries that took part in the survey. The IDB analyzer selects the appropriate sampling weights for the analysis, and corrects for sampling error by using a "balanced repeated replication algorithm" (Brese & Tatto, 2012).

A preliminary analysis was conducted to examine outliers, univariate distributions of each variable, correlations among variables, reliability of the composite variables, and to check whether the data supported the methodological model assumptions. Using the findings from the descriptive analysis, decisions were made about the variables to be used for the composite measures. The measures created were tested using a confirmatory factor analysis to check for suitable relationships (Raykov & Marcoulides, 2006). Proposed composites of variables that gave an internal consistency of $\alpha<0.7$ were left as single variables. Multiple indicators were used in the study, because if a single indicator was used to represent a construct or measure, it would give limited information about the construct to be studied. Further, a single variable can be prone to error because of the possibility of assessing something else or not measuring precisely what it is supposed to measure (Raykov & Marcoulides, 2012). For example, the measure "analysis of learning goals" was created from variables informed by the literature (e.g., Hiebert 2009) that defines what PSTs do when analyzing a learning goal. This measure was not explicitly listed in the TEDS-M database, but a composite measure could be created from the available variables.

For the first research question, which examined the differences in pre-service teacher competencies, a descriptive analysis is shown using box plots to compare the distributions of each variable across nations.

For the second research question which examined the differences in the opportunities to learn to teach mathematics, descriptive analyses were conducted using percentages, means of individual PSTs responses, and means of PSTs responses by institutions. These descriptive analyses were presented in the form of bar graphs and tables with means, standard deviations, and standard errors. These graphs and tables were generated using the IDB analyzer. The countries with the highest and lowest median scores were identified and the median scores of the three focal countries highlighted in comparison to the other countries.

For the within-country analyses associated with the third, fourth, and fifth research questions, which examined the relationships between the opportunities to learn to teach mathematics and the outcome variables within and between the three focal countries, a multilevel regression was used for each focal country. The analysis was such that the relationships were examined within the institutions with individual pre-service teachers, in which the variables were introduced at level 1, or the pre-service teacher level of analysis. A simultaneous analysis was conducted that examined these relationships between the institutions within the three focal countries and it was referred to as the level two or institutional level analysis. This is a suitable model because it took clustering of the PSTs within institutions into consideration, and computed the correct standard errors. In other words, it took the context into consideration when the regression analysis was applied. The multilevel regression was analyzed with the hierarchical linear modeling software.

The equation of the model is shown below (adapted from Lee and Bryk, 1989).

Pre-service teacher level (within the institution):

$$Y_{ij} = \beta_{oj} + \beta_{1j} X_{1ij} \dots + B_{p-1j} X_{p-1ij} + \varepsilon_{ij}$$

The equation is for the pre-service teacher i in institution j, assuming p predictors, and Y is the outcome, with separate analyses for each of the four outcome variables: content knowledge, pedagogical content knowledge, inquiry beliefs, and non-inquiry beliefs.

At the institution level:

$$B_{pj} = \gamma_{po} + \gamma_{p1} W_{1j} \dots + \gamma_{pq} W_{qj} + \vartheta_{pj}$$

The level 1 coefficient was modeled at level 2 (with q institutional level predictors). This analysis approach was used for the relationship using three sets of predictor variables: (i) pedagogical practices experienced in the teacher preparation, (ii) approaches to learning how to plan mathematics instruction for conceptual understanding, and (iii) approaches to learning mathematics instruction for conceptual understanding with the knowledge and beliefs for teaching and learning mathematics. The analyses shown were conducted in stages, and they were based on the previous results. Summaries of the level 1 and level 2 descriptions of variables for the three countries are shown in Table 5 and Table 6.

Table 5: *Mean scores of the OTL and Learning Outcomes for Poland, Russia, and the United States (LEVEL 1)*

	RUSSIA			POLAND (GEN)			POLAND (SPEC)			USA (GEN)		
VARIABLE NAME	**N**	**MEAN**	**SD**	**N**	**MEAN**	**SD**	**N**	**MEAN**	**SD**	**N**	**MEAN**	**SD**
MCK	2260	536.37	96.4	1487	454.89	5.92	281	613.46	94.46	1083	515.44	69.53
MPCK	2260	511.53	87.84	1487	450.76	8.34	281	571.24	76.45	1083	543.07	67.53
Non-inquiry beliefs	2143	24.86	6.25	1396	24.68	6.32	272	20.4	5.41	1139	20.75	5.43
Inquiry beliefs	2160	24.3	3.1	1434	24.2	3.72	273	25.51	2.95	1134	24.24	3.32
More than 100 bks in the home	2246	0.54	0.5	1494	0.41	0.49	281	0.48	0.5	1185	0.59	0.49
Years parents spent in school	2105	14.6	1.92	1266	13.45	1.72	281	27.76	31.62	1451	15.98	2.42
Gender	2260	0.92	0.26	1496	0.98	0.14	280	0.76	0.43	1499	0.89	0.32
Listening to lecture presentation	2254	3.92	0.33	1491	3.68	0.62	281	3.46	0.83	1167	3.3	0.81
Asking questions during class time	2245	3.08	0.68	1491	2.63	0.9	281	2.6	0.9	1167	3.29	0.76
Participate in whole class discussion	2222	3.01	0.79	1487	2.76	0.85	281	2.89	0.81	1165	3.47	0.69
Teach a session using methods demonstrated by instructor	2230	2.01	0.98	1476	2.1	0.99	280	2.09	0.92	1162	2.46	0.97
Work together in groups during class time	2246	3.27	0.81	1484	3.33	0.83	279	2.82	0.88	1162	3.58	0.67
Analysis and reading teaching and learning	2188	10.91	2.79	1462	7.31	2.59	281	9.34	13.53	681	10.62	3.08
Analysis of learning goals	2177	9.32	1.94	1417	6.76	2.02	268	7.06	2.07	1147	9.66	1.78
Introduction to Standards-based curriculum	2210	9.9	1.91	1465	8.43	2.21	277	8.09	2.2	1160	9.91	1.79
Meaningful learning experiences	2230	12.86	2.32	1467	10.25	2.87	280	9.88	2.71	1161	13.64	2.1
Learning how to explore multiple solution strategies with pupils	2243	2.86	0.85	1476	2.63	0.93	277	2.76	0.9	1160	3.17	0.81
Learning how to show why a mathematics procedure works	2226	2.72	0.98	1475	2.34	0.97	277	2.66	0.92	1161	3.05	0.84
Learning to make distinctions between procedural and conceptual knowledge	2208	2.66	0.9	1469	2.41	0.97	276	2.53	0.92	1159	2.87	0.89
FINWGTP	2266	3.78	3.01	1497	2.78	2.36	281	4.46	3.24	1501	17.5	11.21

Table 6: *Mean scores of the OTL and Learning Outcomes for Poland, Russia, and the United States (LEVEL 2)*

	RUSSIA			POLAND (GEN)			POLAND (SPEC)			USA (GEN)		
VARIABLE NAME	**N**	**MEAN**	**SD**	N	MEAN	SD	N	MEAN	SD	N	MEAN	SD
Lecture	49	3.56	0.16	43	3.61	0.18	24	3.53	0.18	51	3.3	0.25
Asking questions	49	2.96	0.16	43	2.87	0.23	24	2.79	0.33	51	3.29	0.24
Whole class discussion	49	3.03	0.15	43	2.95	0.19	24	3	0.3	51	3.46	0.24
Teaching using methods demonstrated by instructor	49	2.25	0.23	43	2.21	0.24	24	2.26	0.31	51	2.46	0.34
Group work	49	3.34	0.15	43	3.33	0.2	24	3.15	0.34	51	3.56	0.32
Analysis of teaching and learning	49	9.64	0.68	43	8.92	1.2	24	8.97	1.39	51	10.75	1.55
Analysis of learning goals	49	8.48	0.5	43	8.03	0.87	24	8.11	0.83	51	9.63	0.63
Introduction to standards based curriculum	49	9	0.53	43	8.85	0.68	24	8.87	0.67	51	9.88	0.7
Meaningful learning experiences	49	11.97	0.61	43	11.55	1.03	24	11.42	1.08	51	13.63	0.73
Learn why a procedure works	49	2.69	0.16	43	2.6	0.25	24	2.82	0.33	51	3.03	0.31
Make distinctions between procedural and conceptual knowledge	49	2.61	0.18	43	2.58	0.27	24	2.63	0.28	51	2.83	0.39
Exploring multiple solutions	49	2.88	0.26	43	2.65	0.31	24	2.78	0.39	51	3.16	0.29
INSWGTP	49	3.28	3.83	43	1.19	0.01	24	1.12	0	51	7.55	7.81
More than 100 bks in the home	49	0.51	0.51	43	0.47	0.5	24	0.5	0.51	51	0.55	0.64
Gender	49	0.96	0.2	43	0.98	0.15	24	0.79	0.41	51	0.8	0.4
Parents years spent in school	49	13.1	6.77	43	11.67	7.91	24	20.75	24.17	51	16.51	3.87

CHAPTER 4

DESCRIPTIVE AND PRELIMINARY ANALYSES

Two approaches are used to present and interpret the data in this study: global and within-country views of the knowledge for teaching mathematics and beliefs about learning mathematics, and opportunities to learn (OTL) to teach elementary mathematics. In the global approach, descriptive results across all of the countries that participated in the TEDS-M survey are first presented and a brief interpretation of the results is given. The descriptive analyses of the opportunities to learn to teach mathematics include (i) pedagogical practices, (ii) learning how to plan mathematics instruction, and (iii) learning mathematics instruction for conceptual understanding. These descriptive results represent an overview of the PSTs' knowledge and beliefs and their OTL to teach across the participating countries, with a particular emphasis on the three focal countries, Poland, Russia, and the United States in comparison to all the participating countries. A focus on the three countries provides a context for examining how these countries' teacher preparation programs with varying OTL to teach mathematics and outcome variables compare with all of the 17 participating countries, which is useful for interpreting the within-country analyses. The descriptive results answer research questions (RQs) 1 and 2:

RQ1: What are the differences in PSTs' content knowledge, pedagogical content knowledge, and beliefs about learning mathematics among the 17 countries that participated in the TEDS-M survey?

RQ2: What are the differences among the 17 countries in the extent of the opportunities to learn to teach through (i) experiencing different pedagogical practices in mathematics education and teacher education courses; (ii) introducing different approaches to planning mathematics instruction for conceptual understanding; and (iii) introducing mathematics instruction for conceptual understanding?

These competencies are shown for the participating countries, highlighting the program types in which PSTs are prepared. The competencies for the individual PSTs are shown to illustrate the differences that exist amongst the PSTs in the countries or the variability at the individual level for the country as a whole. The PSTs' competencies are also shown between the institutions to examine if there are differences in the mean responses of PSTs between the teacher preparation institutions within the countries. These between-program differences will be used in later analysis that examines the effects of between-program variation in teacher preparation practices on the between-program differences in mean outcomes.

The within-country approach is then presented in separate chapters, with the results from the three countries: Poland, Russia, and the United States.

4.1. Descriptive Analyses of the PSTs' Competencies

Knowledge for teaching mathematics is related to beliefs about teaching and learning mathematics. According to Thompson (1992), distinguishing between knowledge and beliefs about teaching and learning mathematics is not easy because these two constructs are intertwined. For this reason, a more holistic analysis includes beliefs about learning mathematics because of its possible influence on PSTs' knowledge for teaching mathematics.

4.1.1. Knowledge for Teaching Mathematics across the Countries

The knowledge for teaching mathematics includes two domains in the TEDS-M data: mathematics content knowledge (MCK) and mathematics pedagogical content knowledge (MPCK). Figure 2 shows the patterns of the PSTs' MCK scores across the countries, while Figure 3 shows the patterns of the PSTs' scores by average PSTs scores in the institutions. These patterns are shown for the different teacher education program types found within the countries, either the generalist[39] or mathematics specialist[40].

[39] Generalists pre-service teachers are "prepared to teach three or more subjects"(Tatto,. et al., 2012, p.35).
[40] Specialist teachers are prepared to teach one or two subjects, (Tatto, et al., 2012). In this study the mathematics specialists were those who were prepared to teach mathematics.

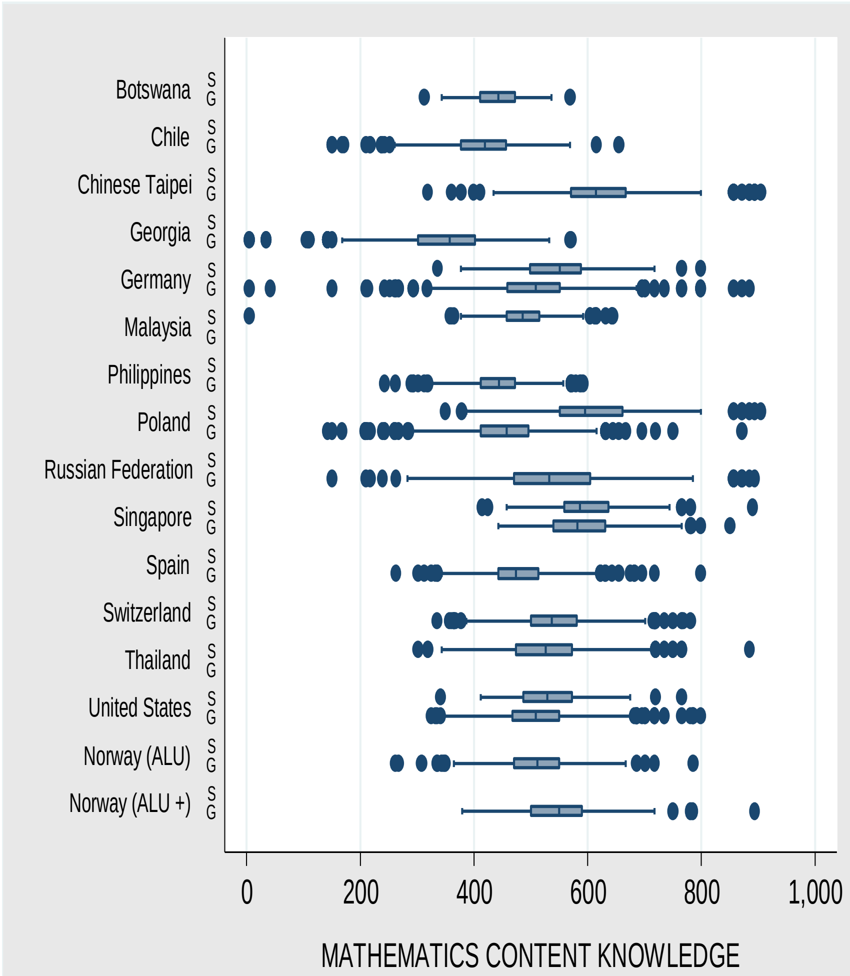

Figure 2. MCK scores of individual PSTs across the participating countries. Note that S represents the specialist programs, while G represents the generalist programs.

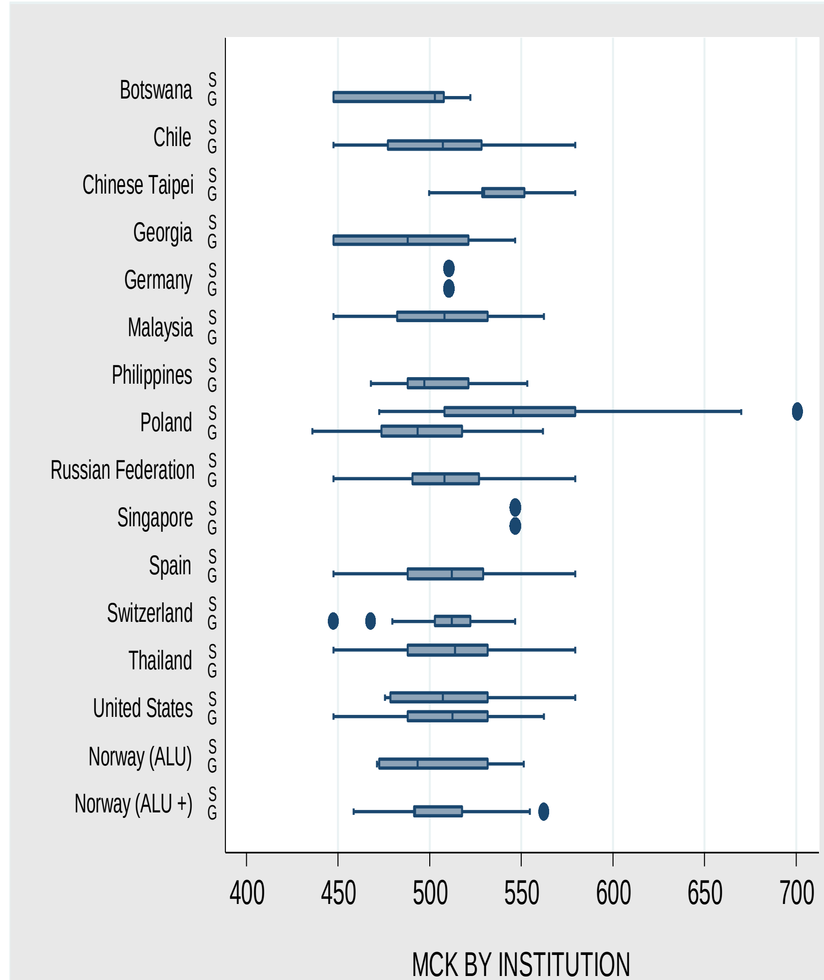

Figure 3. Average PSTs' MCK scores by institutions across the participating countries. Note that S represents the specialist programs, while G represents the generalist programs.

MCK scores of individual PSTs

Figure 2 shows that the PSTs in Chinese Taipei had the highest median MCK score across the countries, with the minimum score below 500 (the international average) and the maximum score at about 800 (excluding the outliers), while the PSTs in Georgia had the lowest median score, below 400

A focus on the three countries of interest, Poland, Russia and the United States, shows that their MCK scores varied and were comparable to the countries with the highest and lowest MCK scores. In Poland the median MCK score for the specialist program was about 600, a similar median score to the MCK score for the PSTs in Singapore and slightly lower than Chinese Taipei, while the median score for the generalist program was below 500 (the international average median score), but above the median score of the PSTs in Georgia, which had the lowest MCK median scores among the participating countries. This distinct difference in scores from the two programs calls for a further analysis within Poland to investigate factors that could be related to these median scores, which were among the highest and the lowest in the participating countries.

The PSTs from Russia had a median score above 500. This median score was above the international median and was close to the median MCK of the specialist programs in the United States and Germany, and higher score than most of the participating countries. Similarly, the MCK median scores of the PSTs in the United States were above 500. Specifically, the MCK scores of the PSTs from the two program types in the United States showed that the scores of the specialist PSTs ranged from 400 to 700, while the range of MCK scores of the generalist PSTs was 350 to 700 (excluding the outliers).

Comparing the three focal countries, the MCK median scores of the PSTs in the specialist programs in Poland and the United States were higher than the median scores of the PSTs in the generalist programs. Further, the median score of the PSTs in Poland generalist programs was the lowest, while the median score of the PSTs in the specialist program was the highest among the three countries. Finally, the PSTs from Russia, a generalist program, had a higher MCK median score than the PSTs from generalist programs in the United States, but close to those from specialist programs in the United States. These results from the countries with the largest number of teacher preparation programs included in the TEDS-M samples showed that there were differences in median MCK scores between program types and also indicated that the Russian generalist programs were comparable to the United States specialist programs. These results warrant further analysis of the factors that could be influencing these differences in the generalist and specialist programs in the three countries.

Average MCK scores of PSTS by institutions

The variation of scores across countries when the institutions were compared show that the specialist PSTs programs in Poland had the highest median average PSTs' MCK score. Conversely, the institutions in Georgian and Polish generalist programs showed the lowest median average PSTs' MCK score (below 500) across the participating countries. In Russia and the United States however, the median average PSTs' MCK scores in the institutions were above 500 and showed a similar average median PSTs' MCK score to those in Chile, Malaysia, Switzerland, Spain, and Thailand.

Individual PSTs' MPCK scores

In this section the MPCK scores across the participating countries are presented within and between institutions. Figure 4 shows the scores of the PSTs' MPCK scores, while Figure 5 shows the PSTs average MPCK scores in the institutions across the countries.

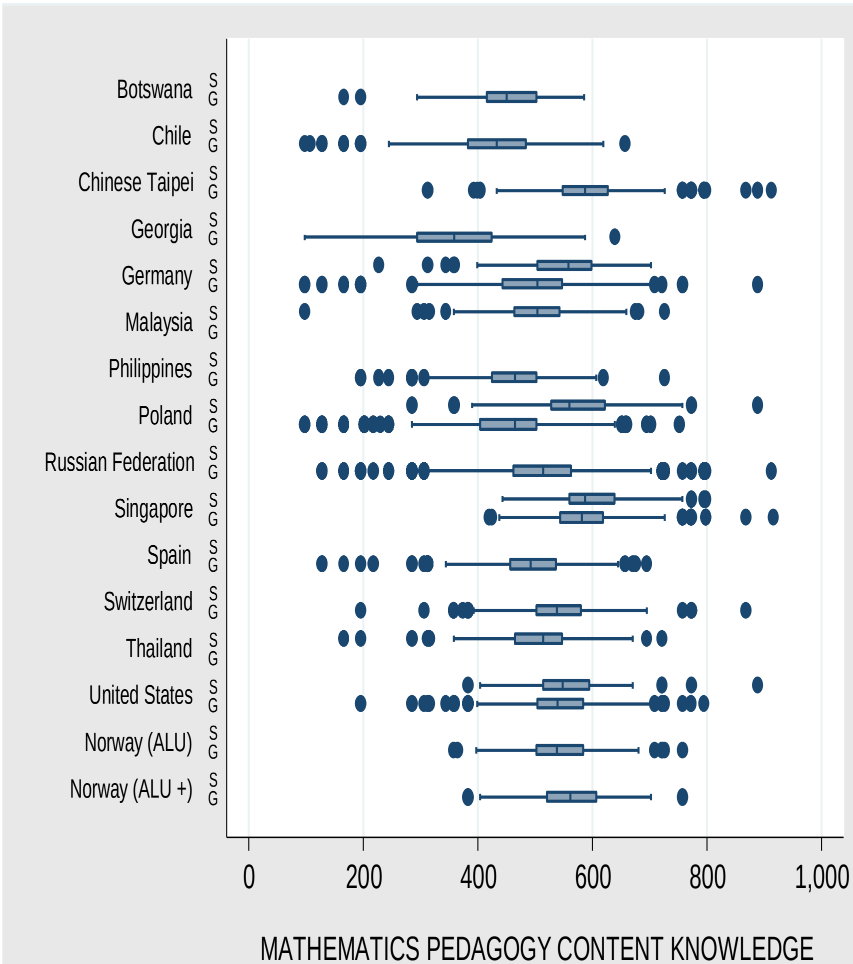

Figure 4 .PSTs' MPCK scores across the participating countries. Note that S represents the specialist programs, while G represents the generalist programs.

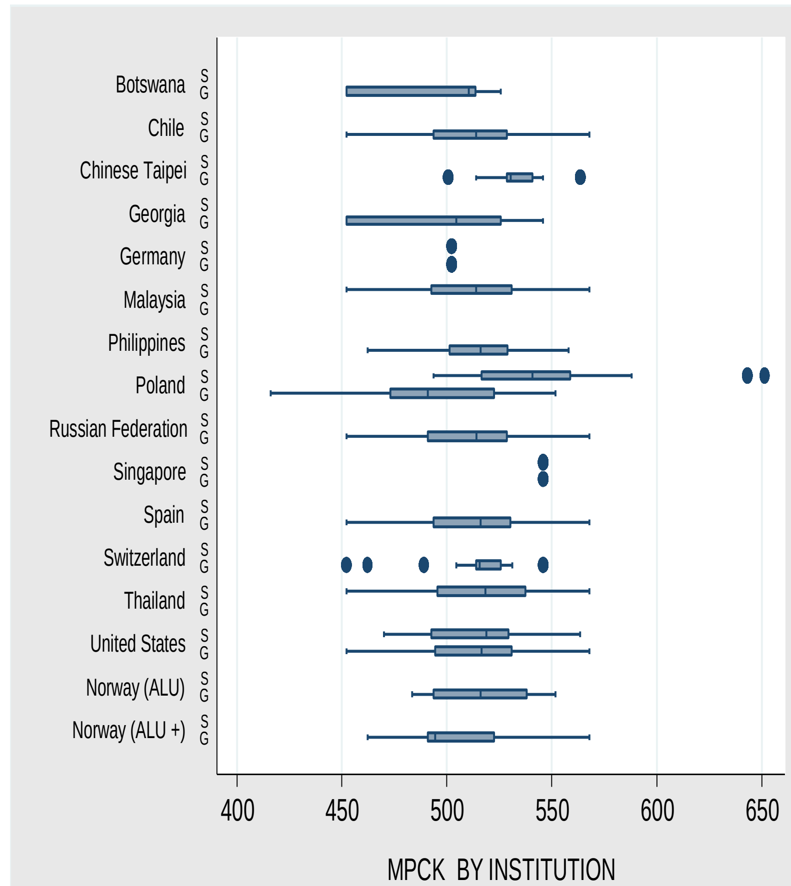

Figure 5. PSTs MPCK average scores in institutions across the participating countries. Note that S represents the specialist programs, while G represents the generalist programs.

Figure 4 shows variations in the PSTs' MPCK scores across the countries that participated in the TEDS-M survey. Specifically, the PSTs in Singapore and Chinese Taipei had the highest median MPCK, with a minimum score that was lower than the international average of 500, and maximum above 700, excluding the outliers. The PSTs in Georgia, on the other hand, showed the lowest median PSTs' MPCK when compared across all the countries that participated in the study but had a large range, with a minimum at about 100, while the maximum is almost 600.

Focusing on the three countries of interest, Poland, Russia, and the United States, the results show differences as well as similarities in the PSTs' MPCK scores. For instance, the PSTs from Polish specialist programs showed a median MPCK score that was close to the PSTs' median scores from the higher achieving countries (Singapore and Chinese Taipei) and the specialist program in Germany, while the PSTs' median MPCK score from generalist programs in Poland had a median score of about 500 (the international average), which was similar to the median MPCK score for the PSTs in the Philippines and Chile, and only higher than the median score for the PSTs in Georgia. Conversely, the PSTs from Russia and the United States (both programs) had a MPCK median score above 500, although the range of scores for the PSTs in Russia was from 300 to 700, while the range of scores for the PSTs from the United States was from 400 to 700 (excluding the outliers). Further, the median MPCK score of the PSTs in the United States was above that for most countries, except for the scores of the PSTs in the East Asian countries, and the specialist programs in Poland and Germany.

In the three countries, the findings showed that the PSTs specialist programs had the highest median MPCK scores. The PSTs from Polish specialist programs had the highest median score, while the PSTs from Russia had a lower score than that of the PSTs in the United States (both programs). The PSTs in the Poland generalist program had the lowest MPCK median score.

The MPCK scores in the three countries highlighted differences in MPCK scores between program types and showed that the PSTs from the United States had higher median MPCK scores than PSTs from most of the other participating countries. These results show that there is need to investigate the factors that could be related to the high median scores of the PSTs in the Poland specialist program and the similarities in the PSTs scores across programs in the United States. Additionally, the factors that could be influencing the relatively low MPCK score in Russia despite MCK scores higher than those of the PSTs in the United States, could be investigated.

Average MPCK scores of PSTs by institution

Figure 5 shows that across the institutions, Polish specialist program had the highest median MPCK score, while Norway ALU+ and the Poland generalist programs showed the lowest median score. The median MPCK scores for the generalist and specialist programs showed no difference for PSTs in the United States. The average median MPCK score across the institutions in Russia was higher than for Polish (generalist), but was very similar to the average median MPCK scores in the United States.

The individual PSTs scores across the countries showed wide variations in MPCK. For example, although the MPCK mean scores for the PSTs in Chinese Taipei and Singapore were high, the results presented showed that there were some PSTs in the United States whose scores were close to the highest scores in the East Asian countries. Further, in the East Asian countries, there were some PSTs MPCK scores that were lower than the international average (500).

In the three focal countries, there were differences in the median MPCK scores between the program types. Specifically, the two programs in Poland showed distinct differences in median MPCK scores. Further, the two program types in the United States had only slight differences in the median MPCK scores. The median scores of the (generalist) programs in Russia were lower than those for the United States programs. These differences in program types and differences in generalist programs across three countries call for further analyses of the factors that could be related to the differences in the generalist programs' median MPCK as well as the features in the specialist programs that influenced their higher MPCK scores.

The knowledge for teaching mathematics, as previously discussed, is comprised of other dimensions of knowledge that include but are not limited to PSTs' MCK and MPCK. The TEDS-M study examined the PSTs on these two dimensions of knowledge for teaching mathematics.

4.1.2. Beliefs about Learning Mathematics

The individual and average (across institutions) PSTs' non-inquiry and inquiry beliefs about learning mathematics within and between the institutions across the seventeen countries are discussed. Figure 6 and Figure 7 present the non-inquiry beliefs of the PSTs and the average non-inquiry beliefs about learning mathematics in the institutions across the participating countries. A score that is close to 8 indicates that the PSTs strongly disagree with non-inquiry beliefs, while a score that tends towards 48 indicates that the PSTs strongly agree with the non-inquiry beliefs.

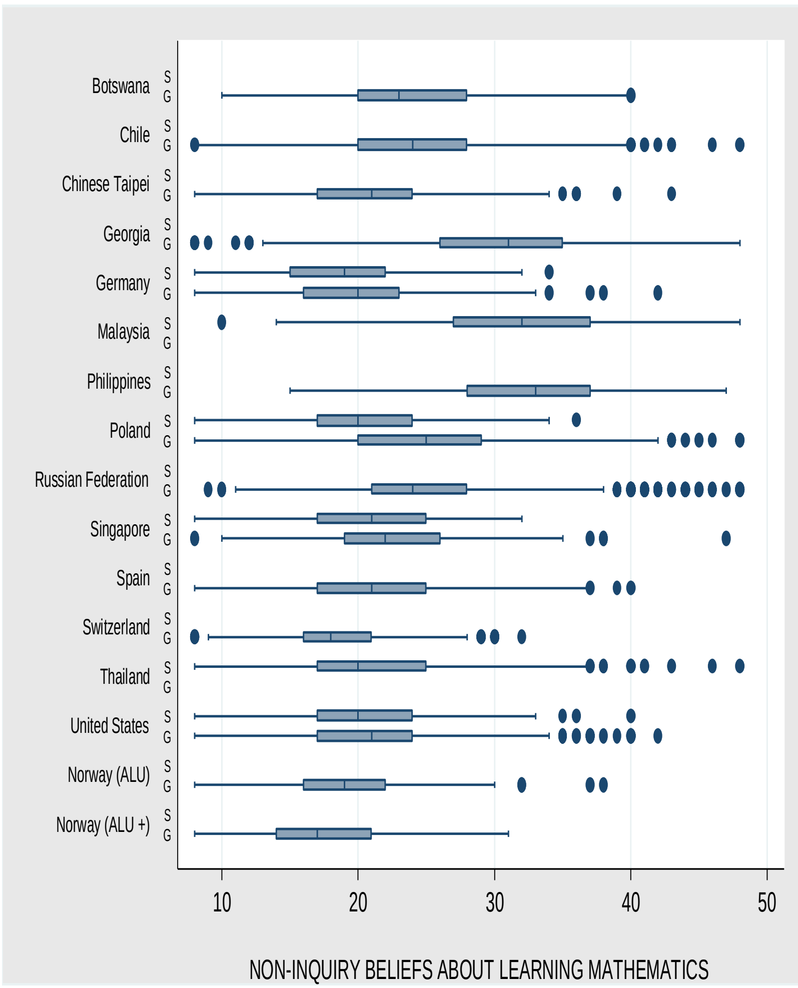

Figure 6. PSTs' non-inquiry beliefs about learning mathematics across the participating countries. Note that S represents the specialist programs, while G represents the generalist programs.

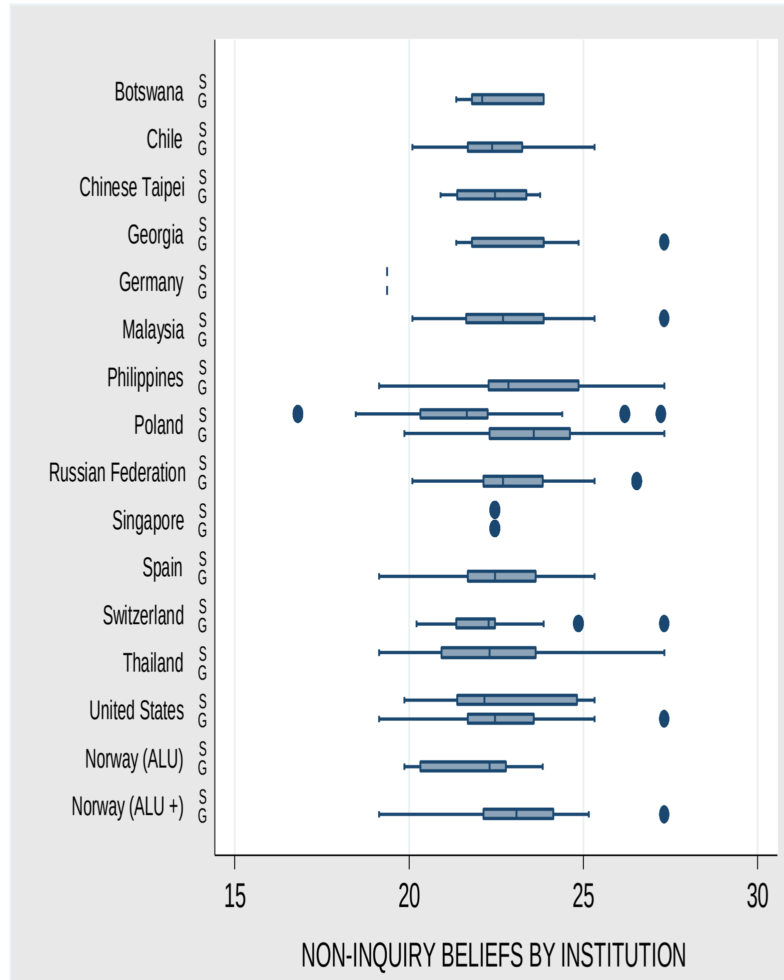

Figure 7. Average PSTs' non-inquiry beliefs about learning mathematics in institutions across the participating countries. Note that S represents the specialist programs, while G represents the generalist programs

Individual PSTs' non-inquiry beliefs about learning mathematics

Figure 6 shows that there were large variations in PSTs' non-inquiry beliefs about learning mathematics in all the countries in the TEDS-M database. PSTs in Malaysia, Georgia, and the Philippines had the highest median non-inquiry belief scores across all the countries shown. In contrast, the PSTs' lowest median non-inquiry belief scores were from Norway and Switzerland. The PSTs' non-inquiry belief median scores from the specialist programs in the United States were similar to those of the PSTs in the specialist programs in Thailand and Poland and in the German generalist program. However, the median scores of the PSTs from the generalist program in the United States was higher and similar to those of the PSTs from Chinese Taipei, Spain, and the Singapore specialist program. The median non-inquiry belief score for the PSTs in Russia was higher than that for most other countries, but lower than the three countries with the highest median score.

Focusing on the three focal countries, Poland, Russia, and the United States, there were differences in the non-inquiry beliefs scores between the generalist and specialist programs within the countries. Specifically, the PSTs in the Polish and United States specialist programs had a median score of 20, while the PSTs' non-inquiry belief scores in the Poland generalist program were 25 and United States about 23, respectively. The PSTs median non-inquiry belief scores in Russia was higher than the PSTs median scores in the United States (from both programs), but still lower than the median score for non-inquiry beliefs of the PSTs from the Polish generalist programs.

Non-inquiry beliefs about learning mathematics between the institutions

The average median PSTs' non-inquiry belief scores were less varied across the institutions, when compared to the variations of the individual PSTs' non-inquiry beliefs within the institutions. The Polish generalist programs had the highest average median non-inquiry beliefs scores when compared to all the countries. On the other hand, the Poland specialist program had the lowest average non-inquiry beliefs. Finally, the average PSTs' non-inquiry belief scores in the institutions in Russia were higher than the average scores in the United States.

In sum, there were variations in the non-inquiry beliefs about learning mathematics between the program types in the countries, as shown in Figure 6 and Figure 7. Further, the generalist programs in the three countries had distinctly different median scores, which showed that there could be differences in the generalist programs that should be further investigated.

Individual PSTs' inquiry beliefs about learning mathematics

From the descriptive results of the non-inquiry beliefs, the information about the PSTs' beliefs about learning mathematics is incomplete if inquiry beliefs are not explored. Figure 8 presents the individual PSTs' and the average inquiry beliefs of the PSTs in the institutions (Figure 9) across the countries in the TEDS-M survey. A score that tends towards a 6 indicates that the PSTs strongly disagree with the inquiry beliefs, while a score that tends towards 36 indicates that the PSTs strongly agree with the inquiry beliefs. Figure 8 shows that the inquiry beliefs were quite similar across the countries in the TEDS-M. The maximum inquiry beliefs for all the countries were 30. Georgia, however, the inquiry belief scores' median value was lower than for all the participating countries. Further, the range of the PSTs inquiry beliefs score in Georgia was the widest when compared to the other countries that participated in the survey: The minimum PSTs' inquiry beliefs for Georgia were 5, while the maximum inquiry beliefs were 30. On the other hand the countries that showed inquiry beliefs with a median score above 25 were from Switzerland (generalist program), Poland (specialist program), Germany, (both programs), and Chile. The median inquiry belief scores of PSTs in the United States programs (both programs) and Russia were higher than those of most countries, and only second to those of the PSTs in Switzerland and Polish specialist programs. The inquiry beliefs of the PSTs in Polish generalist programs were among the lowest in the participating countries.

Notable differences were shown for the PSTs' inquiry beliefs about learning mathematics in Poland, Russia, and the United States. The PSTs in the Poland specialist program had the highest median inquiry belief score, while the PSTs in the Poland generalist program had the lowest median inquiry belief score. Russia and the United States had median scores that were at about 25, with Russia score slightly lower.

Inquiry beliefs about learning mathematics between the institutions

An exploration of the average inquiry beliefs in the institutions in all the countries showed that specialist programs in Poland had the highest average median scores for inquiry beliefs (and had the widest variation when compared to the other countries), while the institutions in Georgia, the Polish generalist programs, and the Russian and the United Stated generalist programs had the lowest median inquiry belief scores.

The institutional-level inquiry belief scores for the generalist programs in the United States, Russia, and Poland had similar median scores. However, the median score for the specialist programs in the United States had a slightly higher median score.

These median inquiry-belief scores showed that there were differences between the program types as well as differences across the generalist programs. The three focal countries offer a context in which the factors that could be influencing these differences could be further explored because of the differences they show in the inquiry and non-inquiry beliefs.

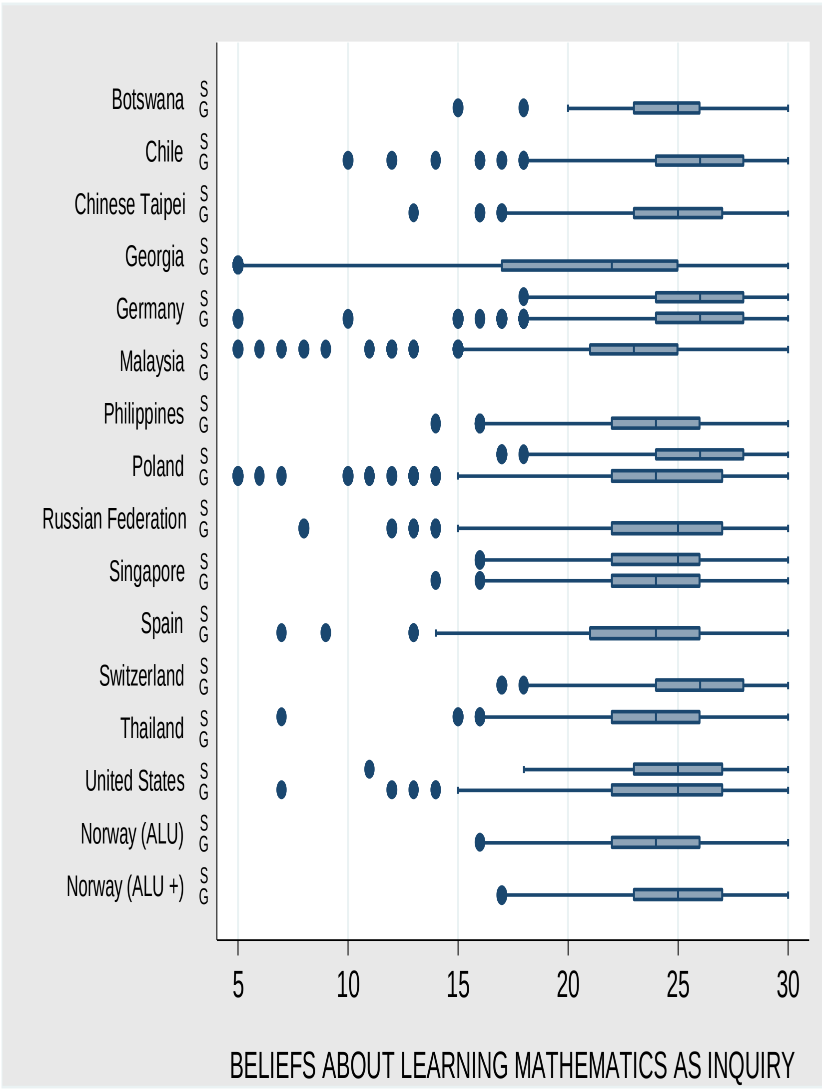

Figure 8. PSTs' inquiry beliefs across the participating countries. Note that S represents the specialist programs, while G represents the generalist programs.

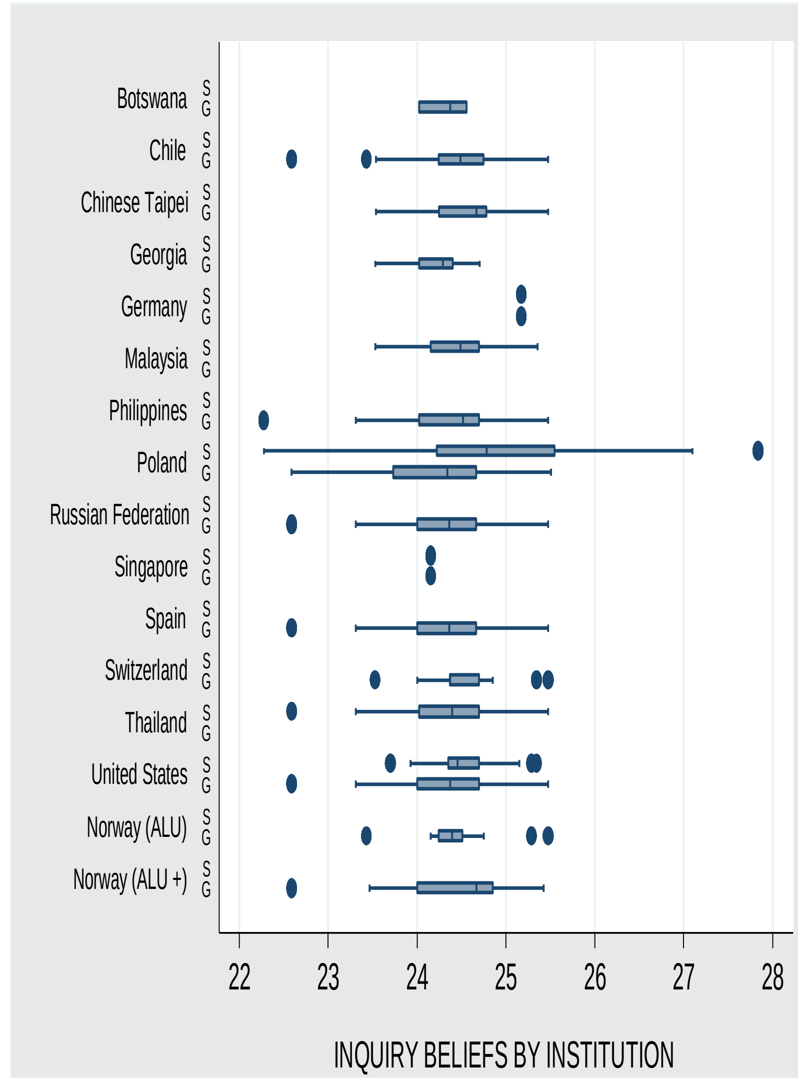

Figure 9. Inquiry beliefs across the participating countries. Note that S represents the specialist programs, while G represents the generalist programs.

4.2. Opportunities to Learn to Teach Mathematics

4.2.1. Pedagogical Practices Experienced in Mathematics-Related Courses

The pedagogical practices used in the descriptive analysis are *reform-oriented pedagogical practices*, *non-reform oriented pedagogical practices*, and the *analysis and reading about the teaching and learning of mathematics*. The first descriptive analysis shows the variation across the countries from the PSTs' reports on the reform and non-reform oriented practices. In particular, these descriptive results show PSTs' reports on experiencing these pedagogical practices *often* in their teacher preparation. The descriptive analyses also show the mean of individual PSTs' responses and the average of the PSTs' responses in the institutions across the participating countries. The second descriptive analysis gives the mean of the composite measure *analysis and reading about teaching and learning of mathematics*.

Figure 10 presents a description of PSTs reports about experiencing often the modeling of reform and non-reform oriented pedagogical practices.

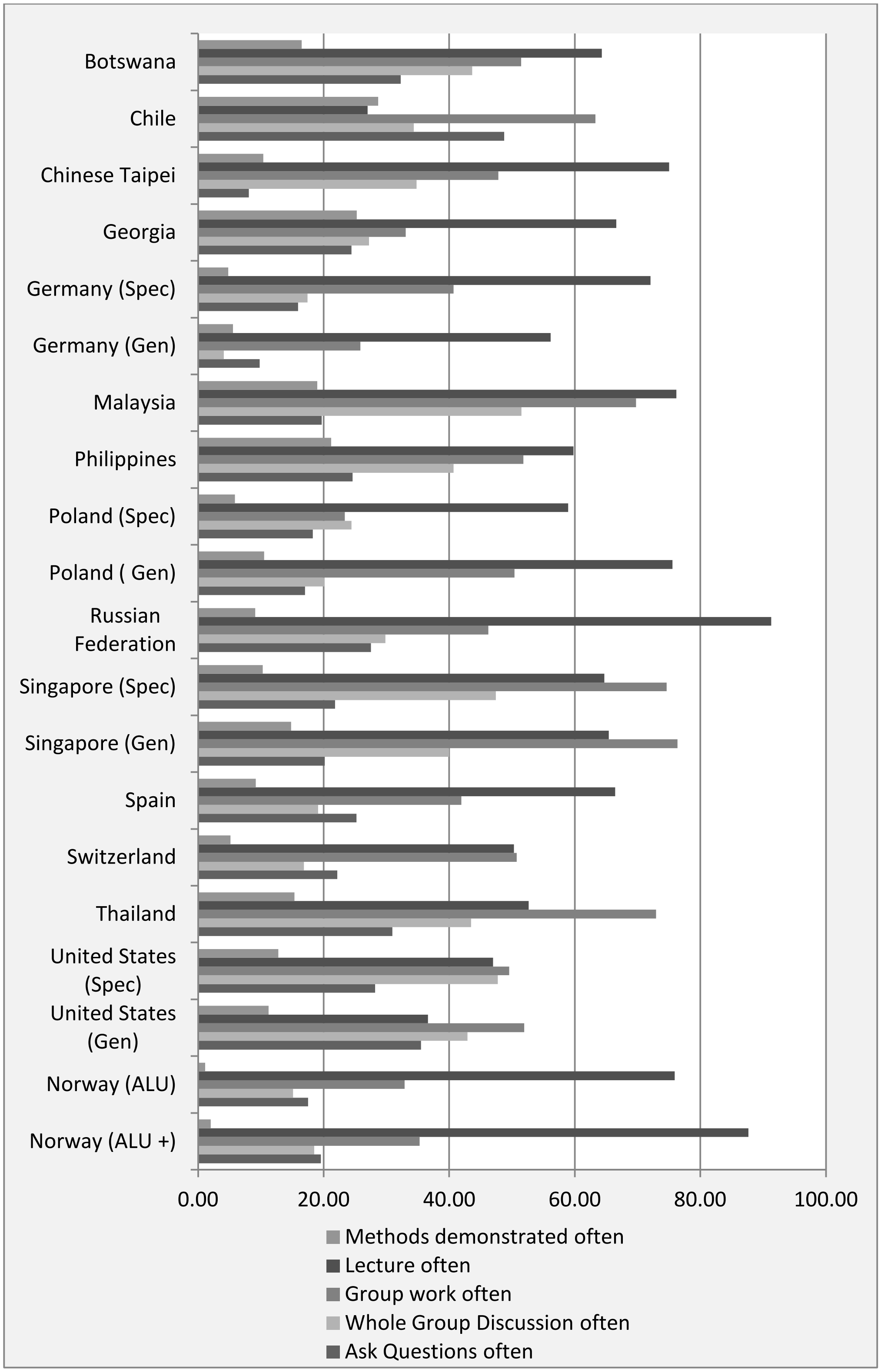

Figure 10. Percentage of PSTs reports about pedagogical practices experienced often in their mathematics-related courses across the participating countries.

Lecture methods often

Figure 10 shows that in 11 countries, the highest percentage of the PSTs reports was *listening to lecture presentations often*. Using the cultural lens[41] proposed by Hsieh, Chu, Hsieh, and Lin (2014) revealed interesting patterns in the PSTs' reports about experiencing lecture methods often during their mathematics-related courses. For example, over 60% of the PSTs from each of the participating countries classified as from the *Confucian Asia* and *Developing Asia* (Malaysia and the Philippines), and from *Eastern Europe* and *Developed Europe* reported that they experienced lecture methods often. Chile and the United States, which are geographically in *the Americas* (Hsieh et al., 2014), but of Western culture, had less than 40% of their PSTs reporting that they experienced listening to lecture presentations often.

The findings in the three focal countries, Russia, Poland and the United States showed that the highest percentage of PSTs in Russia and Poland reported experiencing lecture methods often but in the United States the highest percentage of PSTs reported was not on this pedagogical practice.

Group work often

Over 50% of the PSTs in the *American group* (Chile and the United States) and the participating East Asian countries with *Confucian tradition* (Singapore and Chinese Taipei) that participated in this survey reported having experienced group work often in their mathematics methods courses. Similarly, over 50% of PSTs from Malaysia and Thailand (*developing Asia*), reported experiencing group work often. On the other hand, less than 40% of PSTs from the participating countries in *Europe* (Norway, Germany, and Georgia) reported that they

[41] Hsieh, Chu, Hsieh, and Lin (2014) categorized the countries into West and East based on culture and geographical regions. In the East the countries were further classified as (i) Confucian Asia-Singapore and Chinese Taipei, (ii) Developing Asia-Malaysia, Thailand, and the Philippines. The group categorized as West group comprised (i) East European –Poland, Russia, and Georgia, (ii) Developed Europe –Germany, Norway, Spain, and Switzerland, (iii) American group- Chile and the United States of America (Hsieh, Chu, Hsieh, & Lin, 2014).

experienced group work often in their mathematics-related courses. Notably, Switzerland, a country classified by Hsieh and team (2014) as belonging to *Developed Europe*, was the only European country that had over 50% of PSTs reported experiencing group work in their mathematics-related methods courses.

In Poland, Russia, and the United States, the descriptive results showed that group work was a common practice experienced by PSTs. In particular, the PSTs in the United States had the highest percentage (about 50%) of their PSTs that reported experiencing this practice often. In Russia (46%) and the Poland generalist program (50%), this practice had the second highest percentage of the PSTs reporting they experienced it often.

Whole class discussion often

This pedagogical practice was the third most commonly experienced by the PSTs in most of the participating countries. The patterns of the PSTs' responses showed that over 40% of PSTs from the countries classified as *Developing Asia* reported that they experienced whole class discussion often. Similarly, over 40% of the PSTs in the United States and Singapore reported having experienced whole class discussion often. In contrast, less than 20% of the PSTs in the four countries classified as *Developed Europe* reported experiencing whole class discussion often.

In the three focal countries, PSTs in the United States reported experiencing this practice often (over 40%), while in Russia and Poland less than 40% of the PSTs reported experiencing this practice often. In fact, about 20% of the PSTs in Polish generalist program reported that they experienced this practice often.

Asking questions often

Over 30% of the PSTs in the participating *American grouped* countries reported asking questions often, but none of the participating European countries showed at least 30 % of the PSTs reported asking questions often in their mathematics methods courses. Less than 20% of the PSTs in two of the participating *European* countries (Germany and Norway) and one Asian country (Malaysia) reported asking questions in their mathematics-related courses.

In the three focal countries, the findings showed that just over 25% of the PSTs in the United States and Russia reported that they experienced this practice often. However, about 18% of the PSTs in Polish generalist and the specialist programs experienced this practice often.

Teaching sessions using methods demonstrated by the instructor often

PSTs' reports from twelve of the participating countries showed that this was the least experienced pedagogical practice.

Summary

With specific reference to the three countries with the largest number of teacher preparation institutions, the descriptive results showed that there was variation in the extent the PSTS experienced the pedagogical practices. The variation was related to the program type and country. Most of the PSTs in Poland and Russia reported experiencing lecture presentation often, while most of the PSTs in the United States reported experiencing group work often. In addition, a higher percentage of PSTs from generalist programs in Poland reported experiencing group work as compared to those from specialist programs. However, the PSTs in the generalist and specialist programs in the United States reported experiencing similar pedagogical practices.

To further analyze the degree to which pedagogical practices were experienced by the PSTs, the mean of the individual PSTs' reports and the average of their reports by institutions within each participating country were examined. Table 7 presents a summary of the means and standard deviations of the individual PSTs reports across the countries that participated in the survey.

Table 7: *Mean of Experiencing Models of Reform-Oriented Pedagogical Practices as Reported by the PSTs*

	PSTs						**By Institutions**					
	Ask questions		Whole Group Discussion		Group Work		Ask questions		Whole Group Discussion		Group Work	
	Mean	SD	Mean	SD	Mean	SD	Mean	SD	Mean	SD	Mean	SD
Botswana	3.03	0.81	3.39	0.58	3.43	0.64	2.86	0.08	2.86	0.14	3.23	0.11
Chile	3.3	0.78	2.96	0.94	3.53	0.7	2.94	0.13	3.01	0.12	3.37	0.11
Chinese Taipei	2.36	0.79	3.09	0.82	3.16	0.99	2.75	0.09	3.04	0.12	3.37	0.17
Georgia	2.79	0.96	2.78	1.01	2.87	1.08	2.88	0.08	2.96	0.14	3.3	0.14
Germany (Spec)	2.73	0.84	2.33	1.04	3.2	0.82	2.51	0	1.89	0	2.77	0
Germany (Gen)	2.26	0.96	1.73	0.88	2.56	1.16	2.51	0	1.89	0	2.77	0
Malaysia	2.84	0.76	3.41	0.7	3.61	0.69	2.9	0.16	3.03	0.12	3.36	0.15
Norway (ALU+)	2.79	0.84	2.83	0.79	3.19	0.75	3.02	0.1	3.07	0.12	3.4	0.09
Norway (ALU)	2.68	0.85	2.65	0.83	3.18	0.69	2.95	0.09	3	0.07	3.37	0.09
Philippines	2.97	0.74	3.29	0.68	3.42	0.66	2.9	0.12	3.08	0.11	3.4	0.14
Poland (Spec)	2.61	0.9	2.88	0.84	2.84	0.86	2.81	0.29	3	0.25	3.2	0.28
Poland (Gen)	2.63	0.89	2.78	0.84	3.31	0.83	2.87	0.21	2.96	0.17	3.32	0.18
Russian Federation	3.11	0.67	3.04	0.79	3.29	0.8	2.96	0.17	3.03	0.15	3.34	0.15
Singapore (Spec)	3	0.69	3.39	0.66	3.69	0.58	2.82	0	3.06	0	3.51	0
Singapore (Gen)	2.93	0.71	3.31	0.64	3.73	0.54	2.82	0	3.06	0	3.51	0
Spain	2.95	0.81	2.73	0.89	3.11	0.96	2.95	0.15	3.04	0.14	3.35	0.14
Switzerland	2.8	0.84	2.59	0.91	3.35	0.78	2.88	0.12	2.91	0.16	3.32	0.15
Thailand	3.13	0.71	3.34	0.65	3.68	0.58	2.93	0.15	3.03	0.15	3.36	0.14
United States (Spec)	3.18	0.76	3.49	0.75	3.55	0.7	2.98	0.14	3.04	0.14	3.34	0.22
United States (Gen)	3.3	0.75	3.46	0.68	3.6	0.65	2.93	0.17	3.05	0.15	3.36	0.14

Experiencing models of reform-oriented pedagogical practices

Table 7 shows that the PSTs' reports from the *American group* had the highest means[42] for experiencing models of the reform-oriented pedagogical practice, *ask questions*, while the PSTs' reports for the same practice in the German generalist program had the lowest means. Similarly, the PSTs from the United States reports on *whole class discussion* had the highest means, while the PSTs' reports from Germany showed the lowest means. Additionally, PSTs in the United States had the highest means for *group work* and the lowest means shown with the German PSTs. In sum, PSTs in Germany showed the lowest means for reform-oriented pedagogical practices: in other words they hardly had any experience with whole group discussion but the PSTs in the specialist program reported occasionally engaging in both asking questions and group work.

Similar patterns were shown for the means of the pedagogical practices across the institutions, except for *group work*. Germany had the lowest means for all three pedagogical practices, while the highest means for *asking questions* and *whole class discussions* were from the reports given by the PSTs in the United States. However, the PSTs' reports from Singapore had the highest means for experiencing group work. In addition, Norway (ALU+) and the Philippines were among the countries that showed the highest means for *asking questions* and *whole group discussions,* respectively.

A focus on Poland, Russia, and the United States showed that *group work* was the reform-oriented pedagogical practice that the PSTs in all three countries reported experiencing most, as compared to the other reform-oriented pedagogical practices. In the United States PSTs had group work often, while Polish and German PSTs had group work occasionally.

[42]A mean value of 4 indicates that the PSTs reported experiencing the practice often, while a mean of 3 indicates they experienced the practice occasionally. A value of 2 or 1 indicates they experienced the practice rarely or never respectively.

Experiencing models of non-reform oriented pedagogical practices

Table 8 shows the highest mean of non-reform oriented pedagogical practices was *listening to lecture presentations*. This was reported by PSTs in Russia, while the lowest mean of the PSTs' reports for the same pedagogical practice was from Chile. In all the participating countries, the mean for listening to lecture presentations was between 3 and 4. This indicated that listening to lecture presentations is a practice experienced by PSTs more often than not in all of the countries that participated in this survey. *Teaching using methods demonstrated by the instructor*, however, was experienced less often in all the countries.

Across the institutions, the highest means for lecture presentation were in Singapore (3.63), the United States specialist programs, and the Polish generalist programs (3.61). These values indicate that lecture presentation was a pedagogical practice that these countries experienced often. The means for teaching using methods demonstrated by the instructor across the institutions showed that it was a pedagogical practice that was experienced rarely or never in most countries.

Table 8: *Mean of Models of Non-reform Oriented Pedagogical Practices reported by the PSTs*

	PSTs				**By Institutions**			
	Lecture Presentation		Teaching using Methods Demonstrated by Instructor		Lecture Presentation		Teaching using Methods Demonstrated by Instructor	
	Mean	SD	Mean	SD	Mean	SD	Mean	SD
Botswana	3.64	0.53	2.58	0.94	3.37	0.09	2.39	0.14
Chile	2.44	1.19	2.82	1.01	3.43	0.13	2.32	0.15
Chinese Taipei	3.70	0.57	2.24	0.96	3.57	0.07	2.31	0.12
Georgia	3.24	1.24	2.70	1.08	3.37	0.12	2.45	0.13
Germany (Spec)	3.68	0.59	2.12	0.91	3.24	0.00	1.89	0.00
Germany (Gen)	3.04	1.25	1.68	0.93	3.24	0.00	1.89	0.00
Malaysia	3.68	0.66	2.74	0.86	3.49	0.11	2.32	0.14
Norway (ALU+)	3.88	0.38	1.70	0.81	3.47	0.12	2.30	0.18
Norway (ALU)	3.73	0.54	1.61	0.81	3.49	0.17	2.24	0.13
Philippines	3.53	0.62	2.89	0.79	3.50	0.13	2.30	0.12
Poland (Spec)	3.36	0.89	2.07	0.91	3.53	0.14	2.22	0.27
Poland (Gen)	3.69	0.62	2.12	1.01	3.61	0.16	2.20	0.21
Russian Federation	3.91	0.34	2.08	0.99	3.57	0.16	2.25	0.23
Singapore (Spec)	3.60	0.57	2.71	0.75	3.63	0.00	2.53	0.00
Singapore (Gen)	3.56	0.67	2.73	0.79	3.63	0.00	2.53	0.00
Spain	3.42	0.97	2.07	1.03	3.53	0.13	2.31	0.21
Switzerland	3.26	0.90	1.87	0.93	3.43	0.10	2.25	0.16
Thailand	3.39	0.72	2.79	0.79	3.51	0.14	2.27	0.19
United States (Spec)	3.47	0.79	2.62	0.91	3.61	0.12	2.35	0.26
United States (Gen)	3.27	0.82	2.45	0.97	3.51	0.14	2.28	0.20

Analysis and reading of the teaching and learning of mathematics

The pedagogical practice *analysis and reading of teaching and learning mathematics* is a composite measure comprising the variables (i) read about research on mathematics, (ii) read about research on mathematics education, (iii) read about research on teaching and learning, and (iv) analyze examples of teaching was examined. This measure is not categorized as a model of reform-oriented or a non-reform oriented practice. Also, the TEDS-M data does not specify the specific kinds of teaching and learning that the PSTs analyze and read. However, it is an important pedagogical practice that has been documented as a supportive context for learning to teach (e.g., Hiebert et al., 2007). For this measure, a mean value of 16 indicated that PSTs experienced this practice often, while a value that tends towards 4 indicated that the PSTs experienced these opportunities rarely or never. A summary of the means of this composite measure across the countries is shown in Table 9.

Table 9 shows that PSTs in United States specialist programs experienced analysis and reading about teaching and learning occasionally (11.56), while PSTs in Germany experienced these practices least (7.22), as compared to all the countries that participated in the study.

Comparing the PSTs' reports in the three countries, Poland, Russia and the United States, PSTs in Russia and the United States experienced these practices occasionally, while the PSTs in Poland, in both programs, experienced these practices rarely or never. The average PSTs' reports across the institutions showed that the PSTs in the United States experienced these practices more often than the PSTs in the institutions in Poland and Russia.

Table 9: *Analysis and Reading of Teaching and Learning in Mathematics across the Participating Countries*

Country	**PSTs**		**Institutions**	
	Mean	SD	Mean	SD
Botswana	10.14	2.83	9.3	0.96
Chile	9.13	3.02	9.44	0.64
Chinese Taipei	8.53	2.87	9.21	0.31
Georgia	9.09	3.17	9.48	0.69
Germany (Spec)	7.22	2.36	6.86	0
Germany (Gen)	6.44	2.7	6.86	0
Malaysia	10.88	2.52	9.57	0.56
Norway (ALU +)	8.68	2.71	9.69	0.59
Norway (ALU)	7.61	2.38	9.08	0.27
Philippines	11.2	2.35	9.8	0.63
Poland (Spec)	7.36	2.69	9.04	1.13
Poland (Gen)	7.25	2.52	8.98	1.03
Russian Federation	11.05	2.79	9.66	0.7
Singapore (Spec)	9.18	2.95	9.35	0
Singapore (Gen)	9.23	2.55	9.35	0
Spain[43]	-	-	9.56	0.72
Switzerland	8.4	2.67	9.16	0.77
Thailand	10.15	2.37	9.53	0.65
United States (Spec)	11.56	2.46	10.09	0.69
United States (Gen)	10.57	3.11	9.48	0.68

4.2.2. OTL how to Plan Mathematics Instruction for Conceptual Understanding

Analysis of learning goals

These opportunities to learn are a composite measure of the variables (i) assess higher level goals, (ii) use pupils' misconceptions to plan instruction, and (iii) set appropriately challenging learning expectations for pupils. For this measure, a score of 12 indicated that the PSTs reported having opportunities to analyze learning goals often, while a score of 3 indicated

[43] Results are missing for the analysis and reading about mathematics teaching and learning in mathematics because the composite measure created had missing values on the variable "read research in mathematics education." By institutions, the mean value replaced the missing values. For this reason, analysis using this variable is only at level 2.

that these opportunities to learn were never experienced. A summary of the means of this composite measure across the participating countries is shown on Table 10.

Table 10 shows that the PSTs from the Philippines, Russia, and the United States reported having opportunities to learn how to plan mathematics instruction for conceptual understanding by the analysis of learning goals occasionally. The PSTs' reports in Poland showed that these opportunities to learn were experienced the least, as compared to the PSTs in other countries. The average reports of these practices by institution between the participating countries showed that there was not much difference, and indicated that this practice was experienced occasionally across most of the participating countries.

Table 10: *OTL How to Plan Mathematics Instruction by Learning the Analysis of Learning Goals*

Country	**PSTs**		**Institutions**	
	Mean	SD	Mean	SD
Botswana	8.45	1.86	8.42	0.3
Chile	8.77	2.07	8.41	0.33
Chinese Taipei	8.02	1.85	8.3	0.34
Georgia	7.67	2.38	8.45	0.29
Germany (Spec)	7.86	1.63	8.27	0
Germany (Gen)	7.89	2.23	8.27	0
Malaysia	8.36	1.82	8.43	0.32
Norway (ALU +)	7.63	1.57	8.53	0.28
Norway (ALU)	7.37	1.61	8.1	0.31
Philippines	9.43	1.66	8.54	0.45
Poland (Spec)	6.88	2.03	8.17	0.66
Poland (Gen)	6.72	2.03	8.01	0.8
Russian Federation	9.48	1.87	8.55	0.51
Singapore (Spec)	8.49	1.69	8.79	0
Singapore (Gen)	9.15	1.61	8.79	0
Spain	7.52	2.14	8.4	0.42
Switzerland	8.69	1.7	8.49	0.27
Thailand	8.5	2.04	8.38	0.36
United States (Spec)	9.47	1.94	8.54	0.5
United States (Gen)	9.69	1.78	8.41	0.43

Meaningful learning experiences

These opportunities to learn how to plan mathematics instruction are a composite measure of the variables (i) accommodate a wide range of abilities in each lesson, (ii) create projects that motivate all students to participate, (iii) explore how to use manipulative (concrete) materials or physical models to solve mathematical problems, and (iv) explore how to apply mathematics to real world problems. A mean value of 16 from the PSTs' reports indicated that these opportunities were experienced often, while a value of 4 indicated that these opportunities were never experienced. A summary of the OTL to how to plan mathematics instruction using meaningful learning experiences across the participating countries is shown on Table 11.

Table 11 shows that PSTs in the Philippines, Russia, and the United States (both programs) had the highest scores for opportunities to learn meaningful learning experiences, while PSTs in Germany (both programs) had the lowest mean for this measure. In Poland, however, the means for both programs indicated that these opportunities were experienced rarely.

The means for this measure across the institutions showed the highest means in Russia suggesting that this OTL was experienced occasionally. The OTL, meaningful learning experiences, did not show much difference across the other participating countries by institution.

Table 11: *OTL how to Plan Mathematics Instruction using Meaningful Learning Experiences*

	PSTs		**Institutions**	
Country	Mean	SD	Mean	SD
Botswana	12.68	2.1	11.28	0.43
Chile	12.91	2.46	11.89	0.44
Chinese Taipei	11.81	2.47	11.9	0.35
Georgia	9.77	3.09	11.47	0.57
Germany(Spec)	9.64	2.18	10.04	0
Germany(Gen)	9.26	3.1	10.04	0
Malaysia	12.1	2.31	11.89	0.38
Norway (ALU +)	11.26	2.16	11.96	0.32
Norway (ALU)	11.5	2.22	11.84	0.33
Philippines	13.3	1.85	12.03	0.38
Poland (Spec)	9.78	2.68	11.56	0.89
Poland(Gen)	10.25	2.88	11.53	0.9
Russian Federation	13.04	2.26	12.06	0.62
Singapore (Spec)	12.22	1.69	12.22	0
Singapore(Gen)	12.57	1.61	12.22	0
Spain	10.86	2.81	11.89	0.45
Switzerland	10.86	2.37	11.63	0.5
Thailand	10.96	2.71	11.86	0.42
United States (Spec)	13.16	2.44	12.34	0.57
United States (Gen)	13.7	2.04	11.91	0.48

Introduction to standards-based curriculum

This is a composite measure of the variables (i) analyze and use national and state standards or frameworks for school mathematics; (ii) develop instructional materials that build on pupils experiences, interests, and abilities; and (iii) locate suitable curriculum materials and teaching resources. A mean score of 12 from the PSTs' reports indicated that they experienced all the three opportunities often, while a score of 3 indicated that the PSTs never experienced these opportunities. Table 12 is a summary of the mean scores of the opportunities to learn how to plan mathematics for conceptual understanding by introduction to standards-based curricula.

Table 12: *Introduction to Standards-based Curriculum across the Participating Countries*

Country	**PSTs**		**By Institutions**	
	Mean	SD	Mean	SD
Botswana	8.6	1.61	8.54	0.22
Chile	9.41	1.96	8.83	0.36
Chinese Taipei	9	1.86	8.99	0.26
Georgia	7.87	2.63	8.68	0.33
Germany(Spec)	8.28	1.82	8.37	0
Germany(Gen)	7.5	2.53	8.37	0
Malaysia	9.05	1.8	8.89	0.27
Norway (ALU +)	6.98	1.71	8.84	0.23
Norway (ALU)	7.27	1.82	8.55	0.31
Philippines	9.55	1.52	8.97	0.33
Poland (Spec)	7.78	2.17	8.84	0.57
Poland (Gen)	8.51	2.16	8.85	0.56
Russian Federation	10.06	1.82	9.09	0.55
Singapore (Spec)	8.93	1.67	9.13	0
Singapore (Gen)	9.28	1.69	9.13	0
Spain	7.4	2.19	8.9	0.4
Switzerland	8.2	1.86	8.69	0.29
Thailand	8.57	1.95	8.81	0.35
United States (Spec)	9.47	1.93	9.19	0.57
United States (Gen)	9.94	1.8	8.87	0.4

Table 12 shows that PSTs in Russia experienced learning how to plan mathematics instruction for conceptual understanding through introductions to standards-based curriculum occasionally, while the PSTs from Norway ALU+ had these opportunities rarely or never. For the focal countries, the mean scores for the PSTs in Poland showed that these opportunities to learn how to plan mathematics instruction for conceptual understanding were experienced occasionally, the PSTs from the United States had opportunities for introductions to standards-based curricula more often than the PSTs in Poland.

The means of this measure across the institutions show the values of the mean scores between 8 and 9, but the variation within each of the participating countries differed. This suggests that the mean score of the institutions within the countries varied.

4.2.3. Opportunities to Learn Mathematics Instruction for Conceptual Understanding

Opportunities to learn mathematics instruction for conceptual understanding included being able to (i) show why a procedure works, (ii) distinguish between procedural and conceptual knowledge, and (iii) know how to explore multiple solution strategies with pupils. In this descriptive analysis, the PSTs reports about whether they often had opportunities to learn mathematics instruction on the given dimensions are presented across the participating countries. Figure 11 shows the percentage of PSTs who reported experiencing these opportunities to learn mathematics instruction for conceptual understanding often.

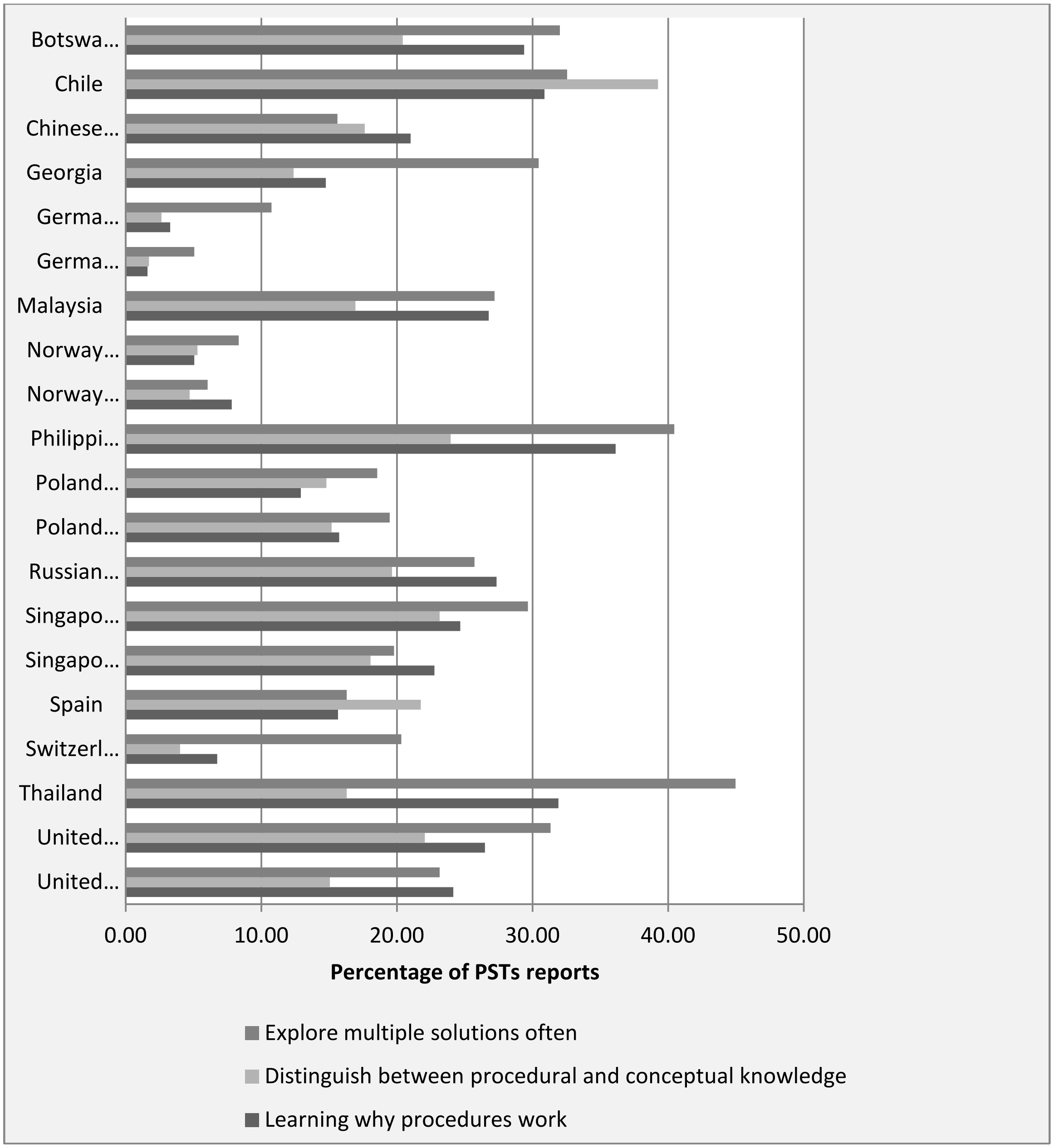

Figure 11. Percentage of PSTs' reports about opportunities to learn mathematics instruction for conceptual understanding often in their mathematics-related courses across the participating countries

Figure 11 shows that in nine countries the greatest percentage of PSTs' reports for learning mathematics instruction for conceptual understanding experienced often were in *learning how to explore multiple solution strategies with pupils.* In contrast, in Spain and Chile the highest percentage of PSTs' reports for learning mathematics instruction often was for *learning to make distinctions between procedural and conceptual knowledge*.

Using the classification of countries by culture done by Hsieh and team (2014), the results from the PSTs reports about learning mathematics instruction for conceptual understanding showed that less than 15% of PSTs in *Developed Europe* (i.e., Germany, Spain, and Norway) that participated in the study learned how to explore multiple solution strategies often in their mathematics-related courses. Further, less than 15% of the PSTs in most of the participating *European* countries (i.e., Georgia, Germany, Poland, Spain, and Norway) reported that they learned to show how a procedure works often in their mathematics-related courses. In addition, less than 15% of PSTs from some of the European countries (Georgia, Germany, Poland, and Norway) reported that they often learned to make distinctions between procedural and conceptual knowledge. In contrast, more than 20% of the PSTs from the American region, Singapore, and the Philippines reported that they often learned each of the three categories of mathematics instruction for conceptual understanding.

A further analysis of these opportunities to learn mathematics instruction for conceptual understanding showed the means of the PSTs' reports about learning mathematics instruction for conceptual understanding in the participating countries. A summary of these analyses is shown on Table 13.

Table 13 shows that the PSTs' reports from the United States had the highest mean for *learning why procedures work*, while PSTs reports from Germany had the lowest mean. PSTs in the United States experienced this OTL occasionally, while the PSTs in Germany experienced this OTL rarely. Comparing the means across institutions showed similar findings.

The opportunity to learn to *make distinctions between procedural and conceptual knowledge* showed the highest mean from the PSTs' reports in Chile, while the lowest mean for this opportunity to learn mathematics instruction was from PSTs' reports in Germany. However, the average reports across institutions showed Singapore to have the highest frequency of learning to make *distinctions procedural and conceptual knowledge*.

Learning multiple solution strategies was an opportunity to learn mathematics instruction that was most experienced in Thailand and the United States, and least experienced in Germany. There were, however, negligible differences in the means of this opportunity to learn mathematics instruction when the means across the institutions were compared.

A focus on Poland, Russia, and the United States showed that PSTs in the United States experienced all three OTL on average more frequently than PSTs in Russia. The PSTs in Poland experienced these practices less frequently than PSTs in both Russia and the United States.

Table 13: *Mean OTL Mathematics Instruction across the Participating Countries*

	PSTs						By Institutions					
	A		B		C		A		B		C	
	Mean	SD	Mean	SD	Mean	SD	Mean	SD	Mean	SD	Mean	SD
Botswana	3.07	0.79	2.83	0.88	3.05	0.89	2.59	0.04	2.52	0.15	2.86	0.05
Chile	2.98	0.89	3.12	0.86	2.98	0.9	2.68	0.12	2.63	0.13	2.85	0.11
Chinese Taipei	2.77	0.86	2.66	0.86	2.66	0.83	2.77	0.13	2.68	0.15	2.86	0.11
Georgia	2.43	1.04	2.37	1.03	2.87	1.11	2.67	0.11	2.63	0.16	2.88	0.08
Germany (Spec)	2.18	0.66	2.12	0.72	2.27	0.77	2.01	0	1.83	0	2.56	0
Germany (Gen)	1.78	0.87	1.6	0.81	2.38	0.95	2.01	0	1.83	0	2.56	0
Malaysia	2.98	0.82	2.79	0.8	2.97	0.84	2.73	0.13	2.64	0.14	2.88	0.09
Norway (ALU+)	2.3	0.79	2.22	0.82	2.45	0.79	2.7	0.15	2.62	0.13	2.88	0.1
Norway (ALU)	2.31	0.85	2.11	0.81	2.4	0.79	2.66	0.13	2.56	0.15	2.78	0.12
Philippines	3.25	0.67	3.01	0.73	3.29	0.68	2.78	0.13	2.68	0.13	2.9	0.11
Poland (Spec)	2.62	0.89	2.49	0.91	2.71	0.9	2.78	0.27	2.58	0.22	2.87	0.17
Poland (Gen)	2.35	0.97	2.42	0.98	2.64	0.94	2.6	0.23	2.57	0.23	2.79	0.2
Russian Federation	2.78	0.98	2.69	0.91	2.93	0.84	2.72	0.16	2.64	0.16	2.88	0.13
Singapore (Spec)	2.85	0.82	2.84	0.75	2.94	0.71	2.91	0	2.92	0	2.99	0
Singapore (Gen)	2.9	0.81	2.95	0.74	3.1	0.73	2.91	0	2.92	0	2.99	0
Spain	2.51	0.96	2.62	0.99	2.61	0.91	2.7	0.17	2.62	0.18	2.85	0.14
Switzerland	2.27	0.8	2.01	0.82	2.86	0.8	2.63	0.14	2.5	0.19	2.88	0.08
Thailand	3.03	0.85	2.69	0.86	3.28	0.78	2.71	0.15	2.61	0.15	2.87	0.12
United States (Spec)	3.02	0.81	2.82	0.8	3.07	0.75	2.73	0.11	2.7	0.16	2.87	0.11
United States (Gen)	3.06	0.86	2.89	0.91	3.17	0.84	2.72	0.17	2.62	0.17	2.87	0.13

Note: A is learning to show *why a mathematics procedure works*, B is learning to make distinctions *between procedural and conceptual knowledge* and C is learning *multiple solution strategies*.

4.3. Summary of the Findings about the Learning Outcomes and OTL

The findings from the cross-national comparisons showed that there were notable differences in the PSTs' knowledge and beliefs about teaching and learning mathematics across the nations. In general the PSTs from the specialist programs had higher knowledge for teaching mathematics and had more inquiry beliefs about learning mathematics than the PSTs in the generalist programs in most of the participating countries that have both programs.

The comparison of the pedagogical practices showed that, across the countries, the two pedagogical practices experienced most often were *listening to lecture presentations* and participating in *group work*. In particular, *listening to lecture presentations* was the pedagogical practice experienced most in the participating countries. Of particular importance is the finding that the PSTs from the participating American countries (Hsieh et al., 2014) reported *group work* as the most common pedagogical practice used in their mathematics-related courses. In addition, *group work* was a pedagogical practice that was not common in the participating European countries (i.e., not more than 30% of the PSTs reported experiencing this practice often). Similarly, *asking question*s *during class sessions* was found to be a common practice among PSTs from the participating American countries (Hsieh et al., 2014). *Analysis of teaching and learning*, a common practice in the United States, was not a common practice in the developed European countries, with the exception of Russia, which is classified among the Eastern European participating countries.

The opportunity to learn how to plan mathematics instruction for conceptual understanding showed distinct differences between the some of the European countries and the countries in the American geographical region. The reports from the PSTs in the United States and Chile showed *meaningful learning experiences* and *analysis of learning goals* to be opportunities to learn how to plan mathematics instruction that were introduced occasionally in mathematics-related teacher education courses, while these practices were least experienced by PSTs in Germany, Georgia, Spain, and Norway.

In most of the participating European countries, notably *developed Europe* (Hsieh et al., 2014), learning mathematics instruction for conceptual understanding was not as frequently experienced as it was with the PSTs from the American group. Less than 15% of PSTs in

Georgia, Germany, Spain, and Norway reported experiencing learning to *make distinctions between procedural and conceptual knowledge.* Also, less than 15% of PSTs in Germany, Georgia, Poland, and Norway reported experiencing learning *multiple solution strategies*.

In sum, the opportunities to learn to teach mathematics showed that the PSTs from the American region and Singapore had more opportunities to learn to teach mathematics through experiencing models of reform-oriented practices and introductions to ways of instruction that promote conceptual understanding, as compared to the PSTs from the participating European countries.

A focus on Poland, Russia and the United States, showed that there were significant differences among these three nations in PSTs' knowledge and beliefs about teaching and learning mathematics. Differences were further shown between the specialist and the generalist programs, with the specialist programs having higher MCK and MPCK median scores, lower non-inquiry belief scores, and higher inquiry belief scores. The descriptive results showed that the generalist programs in the three countries had median scores that were significantly different from one another. The PSTs from the specialist and generalist programs in the United States showed very small differences from each other in their knowledge for teaching mathematics and beliefs about learning mathematics. These similarities found between the two types of United States teacher education programs suggest that the extra mathematics courses taken by the specialist PSTs contribute to a small difference between specialist and the generalist PSTs' competencies. Related to this similarity between the programs is the fact that the extent to which PSTs experience pedagogical practices and activities for planning mathematics for conceptual understanding are not very different between the two program types for teacher preparation in the United States.

In the three countries, the most common practices were *listening to lecture presentations* and *group work*, with *lecture presentations* experienced often by most of the PSTs in Russia and Poland. Further, PSTs in the United States had the most frequent opportunities to *analyze and read about the teaching and learning of mathematics,* for introductions to *meaningful learning experiences,* and for *the analysis of learning goals.* On the other hand, the PSTs in Russia had the most frequent opportunities for *introduction to standards-based curriculum*. Finally, the PSTs in the United States had the most frequent opportunities to learn mathematics instruction for conceptual understanding. The PSTs in both Polish programs experienced these OTL less often.

These findings suggest that different cultural groups emphasize different pedagogical practices for learning to teach mathematics. Bishop (1988) argued that the approaches to teaching mathematics are related to the cultural values of the countries and the institutional norms. The PSTs' reports from the participating European countries suggest experiences that lean more towards *control,* while PSTs in Singapore and from the American region had experiences that lead towards *progress* and *rationalization* and relating mathematics to their experiences. *Control,* according to Bishop (2001), includes the use of rules, procedures, and mastery in mathematics, while *progress* involves questioning, the development of knowledge, and generalizations. Bishop (1988) emphasized that the mathematics learned should be meaningful and be related to the cultural values in the society. Teacher education is one key to the preservation of values in the different cultures (Bishop, 1988). However, the use of certain activities such as group work, project-based learning, and classroom discussions can be used as one way to balance the values that different cultures hold (Bishop, 1988).

CHAPTER 5

WITHIN-COUNTRY ANALYSES OF RELATIONSHIPS BETWEEN OPPORTUNITIES TO LEARN TO TEACH AND PRE-SERVICE TEACHERS' KNOWLEDGE FOR TEACHING MATHEMATICS

5.1. Model specification

This chapter presents the results of the relationships between opportunities to learn to teach and the PSTs' knowledge for teaching mathematics. This chapter and the next chapter answer parts of the research questions 3, 4, and 5. The research questions addressed in this chapter are:

RQ3.What is the relationship between the OTL to teach through experiencing different pedagogical practices in PSTs' mathematics education courses and their (i) content knowledge and (ii) pedagogical content knowledge:

- Within and between the teacher education institutions in Poland?
- Within and between the teacher education institutions in Russia?
- Within and between the teacher education institutions in the United States?

RQ4.What is the relationship between the OTL to teach through different approaches to learning how to plan mathematics instruction for conceptual understanding and PSTs' (i) content knowledge and (ii) pedagogical content knowledge:

- Within and between the teacher education institutions in Poland?
- Within and between the teacher education institutions in Russia?
- Within and between the teacher education institutions in the United States?

RQ5.What is the relationship between the OTL to teach through the introduction to learning mathematics instruction for conceptual understanding and PSTs' (i) content knowledge and (ii) pedagogical content knowledge:

- Within and between the teacher education institutions in Poland?
- Within and between the teacher education institutions in Russia?
- Within and between the teacher education institutions in the United States?

In particular, this chapter outlines the resulting models of the multi-level analyses of the significant relationships between the three main opportunities to learn to teach mathematics, according to the pre-service teachers' reports, and (i) their mathematics content knowledge and (ii) their mathematics pedagogical content knowledge. The relationships of the pedagogical practices experienced by the PSTs and their knowledge for teaching mathematics are discussed first. Second, the relationships between opportunities to learn how to plan mathematics instruction for conceptual understanding and knowledge for teaching mathematics are shown. Finally, the relationships between the opportunities to learn mathematics instruction for conceptual understanding and knowledge for teaching mathematics are discussed. These models are discussed showing the relationships first with MCK as the outcome and then with MPCK as the outcome. All of these analyses are shown for the PSTs within and between the institutions in each of the three countries, Poland, Russia, and the United States. The results include the control variables in the discussions, which are shown first before the models are presented and discussed.

This study focused on examining the teacher preparation programs within Poland, Russia, and the United States because they were the countries with the highest number of teacher preparation institutions in the TEDS-M study. Further, Poland was important for inclusion in the within-country analysis because it was one of the few countries in the TEDS-M study that offered separate specialist programs for elementary teacher preparation, which could provide useful information on the OTL to teach mathematics in such programs. The large number of institutions in these countries made it possible to use variability *among programs* to estimate the effects of OTL on the outcome variables in these countries.

The predictive analyses used a multi-level approach so that the relationship between the opportunities to learn to teach mathematics and the learning outcomes within and between the institutions could be examined. This approach is appropriate because it takes the context and the clustered nature of the data into consideration. The relationships within the institutions were set at level 1, while the relationships between the institutions were set at level 2. That is, level 1 represented the relationships between the OTL and the outcome measures for the individual pre-service teachers, as they differ from others at their institution (the pre-service teachers as the unit of analysis), while level 2 represented the relationships between the OTL and the outcome measures between the institutions or how the institutions differed from other institutions (institutions as the unit of analysis). Therefore, in the interpretation of the findings, the results at level 1 were interpreted with respect to the individual PSTs as they differ from other PSTs within an institution, while the results at level 2 were interpreted with respect to differences among the institutions in the mean values for PSTs at each institution. For example, the relationship between listening to lecture presentation and MCK at level 1 that gives a significant result can be interpreted as "the more the PSTs experience lecture presentation *compared to other PSTs within the institution,* the higher their MCK *compared to other PSTs within the institution*, when other factors are held constant." However, when *listening to lecture presentation* is set at level 2, then the interpretation is "the institutions with a high mean value for "experienced lecture presentations," *compared to the mean value at other institutions*, had higher average MCK scores, *compared to other institutions*, when other factors are held constant."

The models shown in the results include the unconditional[44] model or one-way ANOVA model, models of the background or control variables (gender and socio-economic status), and models showing the opportunities to learn to teach mathematics including the control variables. The control and the OTL variables, which are at level 1, were group-mean centered, while the level 2 variables, which are the OTL and control variables between institutions, were grand-mean centered. The analyses were random-intercept models with fixed and random effects. The fixed effects were the intercept[45] and the slopes[46] (or the coefficients of the independent variables), while the random effects were the variances estimated. Note that the parameters in the random part of the model equation can vary (Raudenbush & Bryk, 2002).

In the creation of the multivariate data matrix (mdm), a list-wise deletion was performed when the level 1 variables were included in the mdm file. The list-wise deletion dropped the cases that had missing values in at least one of the specified variables. This procedure was conducted automatically while using the hierarchical linear modeling software (HLM) because data cannot be introduced into the HLM software with missing values at level 1. The missing values at level 2 were replaced with the means of the variables, and therefore all cases were included at this level. One of the measures created, *analysis of teaching and learning*, had a large percentage of missing values at level 1 and therefore the variable was only included at level 2

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[44] The unconditional model provides "useful preliminary information about how much variation in the outcome lies within and between" institutions as well as providing the reliability of each institution's "sample mean as an estimate of its true population mean" (Raudenbush & Bryk, 2002, p. 69)

[45] The intercepts give the value of the average institution mean.

[46] The value of the slope provides information about the direction of the relationship and is a coefficient that provides the difference in the predicted value for any unit change in the predictor when other factors are held constant. If the variable is categorical, the value of the slope represents the average difference of the predicted value between the reference and comparison group, when other factors are held constant. The slope values are shown on the tables with the models. The slopes are highlighted with a dagger symbol or stars to represent whether the relationship is statistically significant.

where the missing values were replaced with the means of the variable in the institutions. The findings, therefore, in chapters 5 and 6 are for the pre-service teachers in the three countries with no missing values at level 1.

The significance levels that were used for the interpretation of the significant relationships were †p<.10 as marginally significant, **p<.05 as significant, and ***p<.001 as highly significant. The tables presented, therefore, show the values of the slopes (see footnote 46) and the standard errors[47] (given in parenthesis), and they highlight the significant relationships and their levels of significance.

5.2. Results

5.2.1. Unconditional MCK model.

The unconditional models for the MCK and MPCK are presented before the other relationships are discussed. Table 14 shows the unconditional model for MCK. The equation that represents the unconditional model is:

Unconditional model:

$$MCK_{ij} = \gamma_{00} + u_{0j} + r_i$$

γ_{00} is the average MCK across the institutions, u_{0j} is the group level effect and r_i is the individual (PST) level effect.

[47] The standard error is a measure of the variability and it shows how accurately the sample parameter estimates the population parameter (McHugh, 2008).

Table 14: *Unconditional models for MCK for the three Countries*

Variables	United States (Gen)	Russia	Poland (Gen)	Poland (Spec)
INTERCEPT	512.74*** (6.45)	525.66*** (9.30)	456.55*** (4.07)	614.21*** (9.53)
Variance components				
Intercept u_0[48]	837.64	2911.88	588.21	1504.65
Level 1 r	4161.41	4400.13	4045.33	7111.11
ICC[49]	0.167	0.398	0.127	0.175
Reliability coefficient	0.771	0.966	0.811	0.689

†*p*<.10, ***p*<.05, ****p*<.001

Table 14 shows that the average MCK score of the PSTs in the United States was 512.74, with 16.7% of the variation across institutions, and 83.3% of the variation within institutions. In Russia, the average MCK score of the PSTs was 525.66, with 39.8% of the MCK variation across institutions, and 60.2% of the variation within institutions. The average MCK score of the PSTs in the generalist program in Poland was 588.21, with 12.7% of the MCK variation across institutions, and 87.3% of the variation within institutions. Finally, the average MCK score for the PSTs across the institutions in the Poland specialist program was 614.21, with the proportion of variance between institutions 17.5% and within institutions 82.5%. In sum, specialist programs in Poland posted the highest MCK score, while the Polish generalist programs posted the lowest average MCK score.

5.2.2. Background Variables and MCK

Background factors were included in the study so that the differences in the outcome that are influenced by these factors are controlled. The findings of the significant relationships

[48] This value in the multi-level output is the variation of the intercept. It is the variance component of the intercept. In this study it is the variation of the average MCK score.

[49] The intra class correlation (ICC) is the value that gives the percentage of total variation that is between groups. In this case it gives the percentage of variation between the teacher education institutions (Raudenbush & Bryk, 2002). If the value is high that indicates that the variation is mostly composed of the differences that exist between the groups, while a small group difference indicates that most of the variation is due to individual differences within groups.

between the opportunities to learn to teach mathematics and the knowledge for teaching showed that there was a gender difference in MCK score across all the countries, with female PSTs scoring significantly lower than the male PSTs within the institutions in the three countries. Further, there were significant relationships between SES and MCK, but these relationships differed across the three countries. The equation for the multi-level models of the background variables is:

Equation of the background variables (model 1)

Level-1 Model 1

$$MCK_{ij} = \beta_{0j} + \beta_{1j}*(FEMALE_{ij}) + \beta_{2j}*(YEARSOFS_{ij}) + \beta_{3j}*(MORETHAN_{ij}) + r_{ij}$$

Level-2 Model

$$\beta_{0j} = \gamma_{00} + u_{0j}$$
$$\beta_{1j} = \gamma_{10}$$
$$\beta_{2j} = \gamma_{20}$$
$$\beta_{3j} = \gamma_{30}$$

Equation of the background variables (model 2)

Level-1 Model

$$MCK_{ij} = \beta_{0j} + r_{ij}$$

Level-2 Model

$$\beta_{0j} = \gamma_{00} + \gamma_{01}*(MORETHAN_{j}) + \gamma_{02}*(FEMALE_{j}) + \gamma_{03}*(YEARSOFS_{j}) + u_{0j}$$

The background variables considered in the study were the PSTs' gender and their socio-economic status, as indicated by their reports of parental level of education (given in number of years spent in school) (YEARSOF) and the number of books in the home (MORETHAN). Table 15 presents a summary of the relationships between the background variables and MCK. Compared to the male PSTs' MCK scores the female PSTs' average MCK scores were significantly lower by 29.31 points, when other factors were held constant in the United States. In Russia the average MCK score for the females was 16.52 points lower than that for the males. The same patterns were found for the PSTs' MCK scores in Poland, in that the females in the generalist program had a significant 45.11 points lower than the males, and females in the specialist programs had a significant 45.87 points lower than the males.

For the SES proxy variables, the results showed no significant relationships with MCK for parental level of education in any of the three countries. The number of books in the home, however, showed a significant positive relationship with MCK in Poland (both programs). In other words, PSTs in Poland who reported having more than 100 books in the home had significantly higher MCK scores. Specifically, on average, the PSTs in Poland's generalist program who reported that they had more than 100 books in the home had MCK scores 14.27 higher than those who had less than 100 books, while the PSTs in the specialist program who reported the same scored on average 39.96 points higher than their counterparts.

Table 15: *Multi-level Model for the relationship between the background variables and MCK*

Variables	**United States**		**Russia**		**Poland (generalists)**		**Poland (specialists)**	
	Model 1	Model 2	Model 1	Model 2	Model 1	Model 2	Model 1	Model 2
INTERCEPT	517.52*** (7.10)	514.35*** (4.38)	526.25*** (12.17)	525.55*** (9.41)	456.84*** (4.24)	451.44*** (8.70)	615.31*** (10.13)	613.99*** (8.40)
Level 1								
GENDER	-29.31*** (7.43)		-16.52** (6.89)	-	-45.11** (21.66)	-	-45.87** (17.06)	-
MORETHAN	-4.50 (6.55)		1.72 (4.57)	-	14.27*** (4.23)	-	39.96** (13.77)	-
YEARSOF	2.12 (1.06)		0.94 (1.21)	-	-0.48 (1.16)	-	3.76† (2.20)	-
Level 2								
GENDER	-	36.93† (18.38)	-	37.27 (38.27)	-	27.81 (30.68)	-	0.49 (21.70)
MORETHAN	-	-4.13 (9.62)	-	-	-	-9.60 (10.26)	-	41.96** (17.68)
YEARSOF	-	1.70 (4.43)	-	-	-	2.54 (3.16)	-	0.11 (0.31)
Variance Components								
Intercept u_0	861.59	813.80	2924.23	2859	633.03	647.02	1682.88	1017.91
Level 1 r	4188.88	4292.72	4371	4400	3927.93	4171.33	6673.36	7112.73
ICC	0.170	0.159	0.401	0.394	0.139	0.134	0.201	0.125
Reliability coefficient	0.772	0.778	0.965	0.97	0.802	0.819	0.682	0.604

†p<.10,**p<.05,***p<.001

Between the programs, females had higher MCK scores across institutions, with the United States showing that the average MCK for females across the institutions was significantly higher than that for the male PSTs.

5.2.3. Opportunities to Learn to Teach Mathematics and MCK

The relationship between the OTL variables and the PSTs' MCK were examined; the results showed that across the three countries whole group discussion was positively related to PSTs' MCK within the institutions in some of the countries. The experiences that modeled non-reform practices were negatively related to the PSTs' MCK in some of the countries. The analysis shown in the following table summarizes the relationships between (i) pedagogical practices and the MCK, (ii) opportunities to learn how to plan mathematics instruction and MCK, and (iii) opportunities to learn mathematics instruction and MCK, in the three countries. These analyses were used to test hypothesis A1, and sought to answer sections of research questions 3, 4, and 5.

Hypothesis A1: The more pre-service teachers experience opportunities to learn to teach mathematics that model reform-oriented instruction in their mathematics-related courses, the more likely they will have higher mathematics content knowledge, and the more they experience models of non-reform oriented practices, the lower their mathematics content knowledge.

Pedagogical practices and MCK

A summary of the models showing the significant relationships from the multi-level analysis of the variables within and between the countries is shown in Tables 16 and 17. The research question related to this analysis is "What is the relationship between the OTL to teach through experiencing different pedagogical practices in PSTs' mathematics education courses and their content knowledge within and between the institutions?" The equations representing these models are similar to those used for the relationships of the background variables. For this analysis, the equations were:

Model 1

Level-1

$MCK_{ij} = \beta_{0j} + \beta_{1j}*(MORETHAN_{ij}) + \beta_{2j}*(GENDER_{ij}) + \beta_{3j}*(YEARSOFS_{ij}) + \beta_{4j}*(ASKQUES_{ij}) + \beta_{5j}*(WHOLDISC_{ij}) + r_{ij}$

Level-2

$\beta_{0j} = \gamma_{00} + u_{0j}$

$\beta_{1j} = \gamma_{10}$

$\beta_{2j} = \gamma_{20}$

$\beta_{3j} = \gamma_{30}$

$\beta_{4j} = \gamma_{40}$

$\beta_{5j} = \gamma_{50}$

Model 2

Level 1

$MCK_{ij} = \beta_{0j} + \beta_{1j}*(MORETHAN_{ij}) + \beta_{2j}*(FEMALE_{ij}) + \beta_{3j}*(YEARSOFS_{ij}) + \beta_{4j}*(WHOLDISC_{ij}) + \beta_{5j}*(GRPWK_{ij}) + \beta_{6j}*(TCHMEDINS_{ij}) + r_{ij}$

Level-2

$\beta_{0j} = \gamma_{00} + u_{0j}$

$\beta_{1j} = \gamma_{10}$

$\beta_{2j} = \gamma_{20}$

$\beta_{3j} = \gamma_{30}$

$\beta_{4j} = \gamma_{40}$

$\beta_{5j} = \gamma_{50}$

$\beta_{6j} = \gamma_{60}$

A description of Model 2 is such that level 1 model represents the relationships between MCK with regards to the pedagogical practices, and the PSTs background characteristics within the institutions and a random error r_{ij}. The parameters are interpreted as follows:

β_{0j} = Mean MCK achievement of the PSTs in instituion *j*.

β_{1j}=Mean difference between the MCK score of PSTs who have more than 100 books in their home and those with less than 100 books.

β_{2j}=Mean difference between the MCK score of the males and females.

β_{3j} =The degree to which differences in parental level of education is related to the PSTs MCK scores.

β_{4j} =The degree to which differences in OTL to teach mathematics through whole class discussion is related to the PSTs MCK scores.

β_{5}=The degree to which differences in OTL to teach mathematics through group work experiences.

β_{6j} =The degree to which differences in OTL to teach mathematics through teaching sessions using methods demonstrated by the instructor.

For the level 2 models u_{0j} represents the residual (β_{0j} - γ_{00}).

These variables were presented using different models because of the correlations that were found among some of the variables. By using separate models it was possible to "separate out the predictive effects of each of the variables" so that the resulting models did not have collinear variables (Ott & Longnecker, 2001, p. 708). If the variables introduced in the same model were highly correlated, then the overall test would show significance, but the individual tests would show no significance. This positive result of the overall test would show that there exists significance somewhere but the results would not show which of the variables were significant (Ott & Longnecker, 2001).

Table 16: *Multi-level Models of the Relationships between Pedagogical Practices and MCK within the Institutions*

Variables	**United States**		**Russia**		**Poland (Gen)**		**Poland (Spec)**	
	Model1	Model 2	Model 1	Model 2	Model 1	Model 2	Model 1	Model2
INTERCEPT	513.92***	513.60***	526.27***	527.10***	457.00***	457.91***	614.29***	614.59***
	(5.99)	(6.14)	(9.21)	(9.15)	(4.23)	(4.26)	(9.58)	(9.52)
GENDER	-28.92**	-27.50**	-16.52	-16.88	-45.76**	-43.44**	-43.75***	-42.19***
	(9.70)	(10.55)	(10.33)	(10.52)	(21.97)	(21.87)	(12.60)	(12.39)
YEARSOF	1.77	2.25	0.98	0.77	-0.56	-0.69	-0.15	-0.21
	(1.43)	(1.39)	(0.96)	(0.93)	(1.21)	(1.09)	(0.16)	(0.17)
MORETHAN	-5.50	-6.00	1.79	2.40	14.19***	13.21	33.89**	31.89**
	(4.69)	(4.30)	(3.70)	(3.76)	(4.18)	(4.04)	(10.38)	(9.92)
Modeling reform-oriented practices								
ASKQUES	4.33	-	-0.12	-	2.00	-	4.09	-
	(3.56)		(3.04)		(2.21)		6.15)	
WHOLDISC	-	12.04**	-	2.13	-	6.28†	-	12.46
		(2.45)		(2.83)		(3.27)		(8.00)
GRPWK	-	-4.06	-	-4.83	-	-0.26	-	1.17
		(4.03)		(3.00)		(2.96)		(5.54)
Modeling non-reform oriented pedagogical practices								
TCHMEDINS	-	-1.84	-	2.53	-	1.48	-	-15.40**
		(2.64)		(2.28)		2.19)		(6.82)
LECTURE	-6.30**	-	4.78	-	2.67	-	4.92	-
	(2.86)		(3.94)		(2.22)		(5.47)	
Variance components								
Intercept u_0	773.40	786.79	2926.12	2846.69	626.97	639.61	1570.12	1543.75
Level 1 r	3966.16	3987.82	4354.36	4355.93	3948.62	3815	6472.70	6366.69
ICC	0.16	0.164	0.40	0.40	0.137	0.14	0.20	0.20
Reliability coefficient	0.761	0.761	0.964	0.962	0.798	0.804	0.716	0.713

†*p*<.10, ***p*<.05, ****p*<.001

Modeling reform-oriented pedagogical practices within institutions

The reform-oriented practices that were included in this analysis were (i) *asking questions (*ASKQUES), (ii) *whole class discussion (*WHOLDISC), and (iii) *group work* (GRPWK). Table 16 shows that in the United States, the more PSTs experienced *whole class discussion* in their mathematics-related courses, the higher their MCK scores. Similar findings are shown for reports from the PSTs in the Polish generalist programs. In particular, for every unit increase in frequency of experiencing whole group discussion, there was a corresponding increase of 12.04 marks in PSTs MCK ($p<0.05$) in the United States. For the reform-oriented pedagogical practice *group work* and *asking questions*, no significant relationships were found with MCK within the programs in all three countries.

Modeling non-reform oriented instruction within institutions

The relationship between experiencing models of non-reform oriented practices and MCK shows that *listening to lecture presentations* had a significant negative relationship for PSTs in the United States. In other words, the more PSTs *listened to lecture presentations* the lower their MCK; or for every unit increase in the frequency of listening to lecture presentations, there was a corresponding decrease of 6.3 points in the PSTs' MCK ($p<0.05$). This finding is unique to the United States PSTs; the relationship between listening to lecture presentation and MCK in the other two countries was non-significant and positive. *Teaching a session using methods demonstrated by their instructors (*TCHMEDINS), showed a significant negative relationship with MCK for the specialist program in Poland, but was a non-significant positive relationship for the generalist program. This means that the more the specialist PSTs in Poland experienced having opportunities in which they taught a session using methods demonstrated by their instructor, the lower their MCK. In the United States the relationship was negative, while in Russia it was positive and not significant.

A summary of the relationships between the pedagogical practices and MCK between institutions is shown on Table 17. The equations for the relationships of the pedagogical practices and MCK between the programs were:

Model 1

Level-1

$$MCK_{ij} = \beta_{0j} + \beta_{1j}*(MORETHAN_{ij}) + \beta_{2j}*(FEMALE_{ij}) + \beta_{3j}*(YEARSOFS_{ij}) + r_{ij}$$

Level-2

$$\beta_{0j} = \gamma_{00} + \gamma_{01}*(CLADISS_{j}) + \gamma_{02}*(GRPWK_{j}) + u_{0j}$$
$$\beta_{1j} = \gamma_{10}$$
$$\beta_{2j} = \gamma_{20}$$
$$\beta_{3j} = \gamma_{30}$$

Model 2

Level-1

$$MCK_{ij} = \beta_{0j} + \beta_{1j}*(MORETHAN_{ij}) + \beta_{2j}*(FEMALE_{ij}) + \beta_{3j}*(YEARSOFS_{ij}) + r_{ij}$$

Level-2

$$\beta_{0j} = \gamma_{00} + \gamma_{01}*(ASKQST_{j}) + \gamma_{02}*(MTHDTCHN_{j}) + \gamma_{03}*(ANATL_{j}) + u_{0j}$$
$$\beta_{1j} = \gamma_{10}$$
$$\beta_{2j} = \gamma_{20}$$
$$\beta_{3j} = \gamma_{30}$$

Table 17: *Multi-level Models of the Relationships between Pedagogical Practices and MCK between the Institutions*

Variables	**United States**		**Russia**		**Poland (generalists)**		**Poland (specialists)**	
	Model 1	Model 2	Model 1	Model 2	Model 1	Model 2	Model 1	Model 2
INTERCEPT	514.85***	513.14***	522.90***	528.96***	456.95***	456.90***	614.38***	614.06***
	(5.32)	(6.50)	(8.84)	(7.37)	(4.19)	(4.04)	(9.15)	(9.27)
Level 1								
GENDER	-28.48**	-28.34	-16.52	-16.51	-45.23**	-45.36**	-42.23**	-43.19**
	(10.06)	(10.03)	(10.32)	(10.32)	(21.66)	(21.63)	(13.33)	(13.30)
YEARSOF	2.20†	2.20†	0.94	0.94	-0.47	-0.47	-0.16	-0.16
	(1.32)	(1.31)	(0.93)	(0.93)	1.15)	(1.15)	(0.17)	(0.17)
MORETHAN	-6.03	-6.01	1.72	1.72	14.24***	14.30***	34.25**	34.04**
	(4.35)	(4.34)	(3.57)	(3.57)	(4.24)	(4.25)	(10.64)	10.63)
Level 2								
Modeling reform oriented								
ASKQUES	-	-6.10	-	23.70	-	46.36**	-	8.63
		(19.98)		(59.64)		(18.77)		(42.91)
WHOLDISC	-47.32†	-	12.64	-	25.81	-	0.09	-
			79.27)		(21.26)		(34.71)	
GRPWK	43.43**	-	91.65		6.61	-	25.91	-
	(21.53)		(70.18)		(15.90)		(34.78)	
Modeling non-reform								
TCHMEDINS	-	-2.27		73.96**	-	-28.95	-	-32.30
		(21.59)		(31.35)		(19.39)		(36.37)
LECTURE	-10.67		75.92	-	25.34	-	-47.21	-
	(15.77)		(50.13)		(22.97)		(67.99)	
ANATL	-	1.02		12.32	-	-4.31	-	5.08
		(5.42)		(16.33)		(4.22)		(13.16)
Variance components								
Intercept u_0	570.48	796.691	2640.29	2408.90	596.44	545.55	1380.36	1466.50
Level 1 r	4046.16	4035.93	4371.29	4371.61	3928.36	3928.23	6506.38	6504.17
ICC	0.12	0.16	0.38	0.35	0.13	0.12	0.18	0.18
Reliability coefficient	0.704	0.764	0.961	0.957	0.792	0.778	0.689	0.702

†p<.10, **p<.05, ***p<.001

Modeling reform-oriented pedagogical practices and MCK between institutions

Table 17 shows that in the United States *group work* had a positive significant relationship with PSTs' MCK between the institutions. Although the results in other countries showed positive relationships between group work and MCK, they were not significant. The results further showed a significant negative relationship between *whole group discussion* and MCK. In other words, the more institutions modeled *whole class discussion*, the less the average MCK for the generalist PSTs in the United States. The results for the other two countries showed a positive relationship between whole group discussion and MCK. For the pedagogical practice, *ask questions*, the results showed a positive significant relationship with MCK between institutions for the generalist programs in Poland. For the other countries the relationships were non-significant and were negative in the United States.

Modeling non-reform oriented pedagogical practice and MCK between the institutions

For the PSTs in Russia, the more the pedagogical practice *teaching using methods demonstrated by the instructor* is experienced by PSTs between institutions, the higher the average MCK. This positive result was seen only for the PSTs in Russia. For the United States and the two programs types in Poland, the results showed a negative relationship, although not significant.

OTL how to plan mathematics instruction for conceptual understanding and MCK

The OTL how to plan mathematics instruction for conceptual understanding includes three composite measures: *(i) the analysis of learning goals (ANA_LRG), (ii) introduction to standards-based curriculum* (INTR_SD), and *(iii) meaningful learning experiences (*MEAN_LRN). The relationships between these measures and MCK are summarized in Table 18 and Table 19, for within-institution relationships and between-institution relationships, respectively. The equations for these relationships were parallel to those shown in the previous models.

The findings shown in Table 18, about the within-institution relationship between PSTs reports about learning how to plan mathematics instruction and their MCK, showed similar patterns in the United States, Russia, and Poland (generalist programs). Specifically, the relationship between the composite measure, *introduction to standards-based curriculum,* and MCK was significant and positive for the generalist PSTs in Russia within institutions In other words, the more the PSTs were introduced to standards-based curriculum, the higher their MCK. Also, the more the PSTs in the Poland generalist program had *meaningful learning experiences*, the higher their MCK. These relationships were positive in the United States and in the Polish generalist programs but were not significant. For the specialist PSTs in Poland, the relationships between these measures about learning to plan instruction showed a negative relationship with MCK.

Table 18: *Multilevel Models of OTL how to plan Mathematics Instruction and MCK within the Institutions*

Variables	United States		Russia		Poland (generalists)		Poland (specialists)	
	Model 1	Model 2	Model 1	Model 2	Model 1	Model 2	Model 1	Model 2
INTERCEPT	513.45***	513.27***	526.49***	526.07***	458.43***	458.26***	615.18***	614.86***
	(6.23)	(6.20)	(9.18)	(9.21)	(4.14)	(4.23)	(10.11)	(9.70)
Level 1								
GENDER	-28.47**	-28.36**	-17.51†	-16.27	-50.40**	-49.21**	-42.74**	-44.34**
	(10.00)	(9.94)	(10.45)	(10.43)	(21.91)	(21.71)	(16.63)	(16.55)
YEARSOF	2.17†	2.19†	0.74	0.88	-0.47	-0.27	4.53**	3.47
	(1.31)	(1.32)	(0.92)	(0.97)	(1.18)	(1.21)	(2.12)	(2.18)
MORETHAN	-6.35	-6.18	0.21	1.32	14.78***	13.69**	41.18**	43.81**
	(4.38)	(4.41)	(3.73)	(3.60)	(4.27)	(4.21)	(12.97)	(13.84)
OTL how to plan mathematics instruction								
INTR_SD	1.01	-	1.73†	-	0.97	-	-2.55	-
	(1.12)		(1.06)		(1.14)		(2.56)	
MEAN_LRN	-	0.61		0.89		2.43***		-4.59***
		(1.66)-		(0.69)		(0.73)		(1.30)
Variance components								
Intercept u_0	817.09	798.28	2935.54	2925	595.98	627.91	1670.76	1450.14
Level 1 r	4027.96	4043.13	4310.10	4304.25	3872.67	3837.92	6597.46	6595.47
ICC	0.17	0.16	0.41	0.40	0.13	0.14	0.20	0.18
Reliability coefficient	0.767	0.763	0.964	0.965	0.79	0.800	0.681	0.652

†p<.10, **p<.05, ***p<.001

Table 19 shows the relationships between OTL how to plan mathematics instruction and MCK between institutions. The results show that there was a significant positive relationship between average PSTs reports on *analysis of learning goals*, and *introduction to standards based curriculum,* and their MCK scores between programs in Russia. In other words, the more the programs provide opportunities for PSTs to *analyze learning goals* and have received *introductions to standards-based curriculum*, the higher their MCK. The relationships of variables related to OTL how to plan mathematics instruction and MCK showed no significant between-institution relationships in the United States and Poland.

Table 19: *Multilevel models of OTL how to Plan Mathematics Instruction and MCK between the Institutions*

Variables	**United States**		**Russia**		**Poland (generalists)**		**Poland (specialists)**	
	Model 1	Model 2	Model 1	Model 2	Model 1	Model 2	Model 1	Model 2
INTERCEPT	513.55***	513.24***	527.41***	524.24***	456.82***	456.87***	615.32***	615.37***
	(5.89)	(6.01)	(8.36)	(8.48)	(4.29)	(4.23)	(10.13)	(9.89)
Level 1								
GENDER	-28.32**	-28.34**	-16.52	-16.52	-45.12**	-45.10**	-45.89**	-45.74**
	(10.02)	(10.03)	(10.32)	(10.32)	(21.66)	(21.67)	(17.02)	(15.56)
YEARSOF	2.20†	2.20†	0.94	0.94	-0.48	-0.48	3.77†	3.76†
	(1.32)	(1.32)	(0.93)	(0.93)	(1.16)	(1.16)	(2.20)	(2.20)
MORETHAN	-6.01	-6.01	1.72	1.72	14.27***	14.27***	39.97**	39.81**
	(4.35)	(4.35)	(3.57)	(3.57)	(4.24)	(4.24)	(13.74)	(13.78)
Opportunities to learn how to plan mathematics instruction								
Level 2								
ANA_LRG	-14.31	-	28.90†	-	-1.04	-	0.95	-
	(9.16)		(15.59)		(3.62)		(10.33)	
INTR_SD	-	4.00	-	25.30†	-	2.42	-	-16.92
		(10.05)		(13.65)		(4.64)		(15.56)
MEAN_LRN	-	-	-		-	-	-	-
Variance components								
Intercept u_0	723.33	795.74	2689.41	2705.27	632.56	627.79	1683.61	1573.04
Level 1 r	4036.12	4036.11	4370.86	4370.90	3927.87	3928.40	6672.95	6667.75
ICC	0.16	0.16	0.38	0.38	0.14	0.14	0.20	0.20
Reliability coefficient	0.748	0.764	0.962	0.962	0.801	0.800	0.682	0.668

†p<.10, **p<.05, ***p<.001

OTL mathematics instruction and MCK

Within the programs, the results showed no significant relationships between the opportunities to learn mathematics instruction and MCK. Between the programs there were some significant relationships, which are shown in Table 20. The results show that across the institutions there were significant relationships between the average PSTs' reports of *learning to make distinctions between procedural and conceptual knowledge* for the PSTs in the Poland specialist program. These relationships were negative, showing that an increase in the frequency of *learning to make distinctions between procedural and conceptual knowledge* was related to a decrease in the average PSTs' MCK between the specialist programs in Poland. In the United States there was a positive and significant relationship between the average PSTs' reports about *learning to explain multiple strategies* and the PSTs MCK. Similar findings for this relationship were found for the Poland generalist program.

Table 20: *Multi-level Models of Relationships between Opportunities to Learn Mathematics Instruction and MCK between the Institutions*

Variables	**United States**		**Russia**		**Poland (generalists)**		**Poland (specialists)**	
	Model 1	Model 2	Model 1	Model 2	Model 1	Model 2	Model 1	Model 2
INTERCEPT	515.82***	512.80***	522.13***	525.28***	456.96***	456.80***	614.15***	613..48***
	(5.89)	(5.88)	(8.65)	(9.09)	(4.07)	(4.23)	(9.54)	(8.33)
Level 1								
GENDER	-28.25**	-28.30**	-16.52	-16.51	-45.26**	-45.09**	-43.12**	-43.04**
	(10.04)	(10.03)	(10.32)	(10.32)	(21.63)	(21.65)	(13.32)	(13.37)
YEARSOF	2.20+	2.21+	0.94	0.94	-0.47	-0.48	-0.16	-0.16
	(1.32)	(1.32)	(0.93)	(0.93)	(1.16)	(1.15)	(0.17)	(0.17)
MORETHAN	-6.02	-6.00	1.72	1.72	14.27***	14.26***	34.05**	33.61**
	(4.36)	(4.35)	(3.57)	(3.57)	(4.23)	(4.24)	(10.62)	(10.66)
Level 2								
Opportunities to learn mathematics instruction for conceptual understanding								
AVEREXPL	36.25†	-	43.33	-	28.91**	-	-8.68	-
	(20.01)-		(30.05)		(13.38)		(19.31)	
MKDIS	-	16.53	-	49.99	-	-5.64	-	-85.63**
		(11.92)		(48.56)		(11.36)		(35.81)
Variance components								
Intercept u_0	651.44	730.41	2771.32	2844.61	552.42	632.99	1552.00	1062.32
Level 1 r	4038.75	4038.71	4371.21	4371.04	3928.58	3927.55	6506.21	6501.79
ICC	0.14	0.15	0.39	0.39	0.12	0.14	0.19	0.14
Reliability coefficient	0.729	0.749	0.963	0.964	0.780	0.802	0.713	0.633

†*p*<.10, ***p*<.05, ****p*<.001

5.2.4. Unconditional MPCK models

The other component of the knowledge for teaching mathematics was mathematics pedagogical content knowledge. The relationships between the opportunities to learn to teach mathematics and MPCK explored in this section include those between (i) pedagogical practices and MPCK, (ii) opportunities to learn how to plan for mathematics instruction for conceptual understanding and MPCK, and (iii) opportunities to learn mathematics instruction and MPCK. The unconditional models for MPCK and the relationships of the background variables are first shown within and between institutions, and then each of the relationships with MPCK are discussed. These models were multi-level models in which variables were introduced at level 1 and level 2, as in the previous section. Table 21 presents results for the unconditional model of the MPCK within the three countries.

Table 21: *Unconditional Models of MPCK across the three Countries*

Variables	**United States**	**Russia**	**Poland (generalists)**	**Poland (specialists)**
INTERCEPT	543.65*** (5.02)	507.15*** (6.46)	452.69*** (5.69)	573.92*** (7.35)
Variance components				
Intercept u_0	335.51	1910.03	1159.98	855.92
Level 1 r	4186.61	4205.03	7200.77	4663.56
ICC	0.0002	0.312	0.139	0.155
Reliability coefficient	0.595	0.952	0.825	0.659

†*p*<.10, ***p*<.05, ****p*<.001

Table 21 shows that the average MPCK score for PSTs in the United States was 543.65, with 0.02% of the proportion of variation between institutions and 99.98% of the variation within institutions. In Russia the average MPCK score of the PSTs across the institutions was 507.15, with 31.2% of the proportion of variance between the institutions and 68.8% of the variation within institutions. The average MPCK score of PSTs in the generalist program in Poland was 452.69, with 13.9% of the MPCK variation between the institutions and 86.4% of the variation within the institutions. Finally, the average MPCK score of the PSTs across the specialist programs in Poland was 573.93, with 15.5% of the proportion of variance between the institutions and 84.5% of the variation within the institutions.

5.2.5. Background variables and MPCK.

A summary of the within-institution relationships between the background variables and MPCK is shown in the top half of Table 22. The results show that there was no significant relationship, between gender and PSTs' MPCK across the three countries. For the SES factors, in the United States and the Polish specialist programs, there was a significant positive relationship between the parental level of education and the PSTs' MPCK, while in Russia, it was a negative significant relationship. In other words, the higher the PSTs' parents level of education, the higher their MPCK in the United States and the Polish specialist programs. For the SES proxy variable *number of books in the home*, however, the results show a positive significant relationship with MPCK in Russia and the generalist PSTs in Poland and positive non-significant relationships in the United States and the Polish specialist programs.

Table 22: *Multilevel models of Relationship between Background Variables and MPCK within and between Institutions*

Variables	**United States**		**Russia**		**Poland (generalists)**		**Poland (specialists)**	
	Model 1	Model 2	Model 1	Model 2	Model 1	Model 2	Model 1	Model 2
INTERCEPT	541.16***	541.18***	507.32***	507.08***	452.76***	453.78***	575.12***	573.95***
	(4.72)	(3.34)	(6.47)	(7.05)	(5.89)	(11.43)	(8.00)	(7.06)
Level 1								
GENDER	-5.18	-	3.70	-	-25.46		-18.02	-
	(6.31)		(10.40)		(18.87)		(12.80)	
YEARSOF	2.59**	-	-1.55**	-	-0.63		4.06†	-
	(1.23)		(0.78)		(1.74)		(2.30)	
MORETHAN	3.07	-	6.86**	-	11.02**		3.08	-
	(3.73)		(3.16)		(5.60)		(8.88)	
Level 2								
GENDER	-	23.84†	-	-	-	72.24†	-	13.63
		(12.09)				(40.29)		(15.46)
YEARSOF	-	0.79	-	1.65	-	0.10	-	0.01
		(2.78)		(2.31)		(4.14)		(0.14)
MORETHAN	-	2.60	-	22.83	-	-4.30	-	16.84
		(8.19)		(13.81)		(13.48)		(15.09)
Variance components								
Intercept u_0	366.10	285.88	1925.09	1615.06	1230.89	1127.04	980.52	738
Level 1 r	3831.01	4309.18	4076.87	4134.82	6939.27	6940.54	4760.58	4666.50
ICC	0.08	0.06	0.32	0.29	0.15	0.14	0.17	0.14
Reliability coefficient	0.625	0.574	0.951	0.945	0.816	0.825)	0.639	0.626

†p<.10, **p<.05, ***p<.001

Between the institutions, the average female MPCK score was significantly higher than the average male score in the United States and in the Poland generalist program.

5.2.6. Opportunities to Learn to Teach Mathematics and MPCK

Similar to the relationships examined for OTL and MCK, this section shows the relationships between OTL to teach mathematics and MPCK. The hypothesis tested for these relationships sought to answer parts of questions 3, 4, and 5:

Hypothesis A2: The more pre-service teachers experience frequent opportunities to learn to teach mathematics that focus on reform-oriented approaches in their mathematics-related courses, the more likely they will have a higher mathematics pedagogical content knowledge and the more they have opportunities to learn to teach that focus on models of non-reform oriented approaches the lower their mathematics pedagogical content knowledge.

Pedagogical practices and MPCK

To test the relationships between the pedagogical practices the pre-service teachers reported they experienced and their relationships to MPCK, and to answer the of research question 3, "What is the relationship between the OTL to teach through experiencing different pedagogical practices in PSTs' mathematics education courses and their mathematics pedagogical content knowledge," multi-level models were used. The results of the within-institution analyses are shown on Table 23.

Table 23: *Multi-level Model of the Relationships between the Pedagogical Practices and MPCK within the Institutions*

Variables	**United States**		**Russia**		**Poland (generalists)**		**Poland (specialists)**	
	Model 1	Model 2	Model 1	Model 2	Model 1	Model 2	Model 1	Model 2
INTERCEPT	541.69***	542.09***	507.20***	507.34***	453.07***	453.45***	574.25***	574.52***
	(4.52)	(4.62)	(6.60)	(6.37)	(5.87)	(5.88)	(7.35)	(7.35)
Level 1								
GENDER	-5.25	-3.86	3.37	3.69	-26.70	-22.69	-19.72	-21.14†
	(6.18)	(6.11)	(10.48)	(10.59)	(18.09)	(18.26)	(12.68)	(12.59)
YEARSOF	2.42**	2.16†	-1.37	-1.56**	-0.67	-1.06	-0.12	-0.13
	(1.11)	(1.19)	(0.89)	(0.73)	(1.74)	(1.60)	(0.12)	(0.13)
MORETHAN	3.51	2.78	6.06†	5.41	11.06†	9.63	7.93	5.93
	(3.78)	(3.97)	(3.28)	(3.52)	(5.71)	(5.96)	(7.53)	(6.91)
Level 2								
Reform oriented								
ASKQUES	0.03	-	5.44**	-	2.23	-	10.98**	-
	(3.18)		(1.82)		(3.56)		(4.52)	
WHOLDISC	-	-2.42	-	6.86**	-	7.67**	-	18.69**
				(2.21)		(3.64)		(7.13)
GRPWK	-	-0.81	-	-0.97	-	-2.47	-	-1.70
		(4.79)		(2.79)		(3.35)		(5.02)
Non-reform								
TCHMEDINS	-	-2.33	-	4.40	-	2.78	-	-5.51
		(2.93)		(3.32)		(3.37)		(5.48)
LECTURE	-1.76	-	1.93	-	4.67	-	3.34	-
	(4.60)		(4.84)		(4.97)		(3.84)	
ANATL	-	-	-	-	-		-	
Variance components								
Intercept u_0	346.94	350.53	1968.80	1872.89	1217.18	1219.30	875.84	875.51
Level 1 r	3807.06	3800.62	4042.77	4005.15	6934.31	6867.48	4445.79	4381.99
ICC	0.08	0.08	0.32	0.32	0.15	0.15	0.16	0.17
Reliability coefficient	0.615	0.614	0.952	0.948	0.813	0.812	0.674	0.674

†p<.10, **p<.05, ***p<.001

Table 23 shows that, within the institutions, the pedagogical practice *whole group discussion* had a positive significant relationship with the PSTs' MPCK in Russia, and in both programs in Poland: the more the PSTs had such opportunities, the higher their MPCK. In addition, the PSTs' reports about the frequency of *asking questions* in their mathematics methods courses showed a significant relationship with PSTs' MPCK in Russia. Similar findings were shown for the specialist programs in Poland. The non-reform oriented practices, however, showed no significant within-institution relationships with the PSTs' MPCK across the three countries.

An analysis of the relationships between the pedagogical practices and MPCK by institution was done using multilevel models, in which the background variables were set at level 1 and the pedagogical practices were set at level 2. A summary of the models with significant relationships is shown in Table 24.

Modeling reform-oriented pedagogical practices within the institutions

Different relationships were significant in the three countries. Specifically, for PSTs in Russia, reports showed that an increase in the frequency of asking questions corresponds to a decrease in the PSTs' MPCK between the institutions. Conversely, for the generalist PSTs in Poland, the results showed that the more the PSTs were able to ask questions in their mathematics-related courses, the higher their average MPCK scores. In the United States the relationships in the between-institution models of reform oriented practices and MPCK were non-significant and for whole class discussion, the sign was opposite to that of the other countries.

The pedagogical practice *analysis of teaching and learning* showed that, looking at the variability across the programs in Russia, there was a positive significant relationship between experiencing this practice and their MPCK. In other words, the more the programs included opportunities for analyzing and reading teaching and learning mathematics, the higher the average MPCK scores.

Table 24: *Multi-level models of Relationships between the Pedagogical Practices and MPCK between the Institutions*

Variables	United States		Russia		Poland (generalists)		Poland (specialists)	
	Model 1	Model 2	Model 1	Model 2	Model 1	Model 2	Model 1	Model 2
INTERCEPT	540.49***	541.33***	508.34***	505.53***	452.79***	452.74***	573.83***	574.14***
	(4.93)	(4.02)	(6.00)	(6.83)	(5.72)	(5.93)	(7.08)	(7.35)
Level 1								
GENDER	-5.15	-5.27	3.71	3.71	-25.74	-25.50	-19.57	-19.70
	(6.31)	(6.35)	(10.40)	(10.40)	(18.88)	(18.87)	(12.94)	(12.97)
YEARSOF	2.59**	2.58**	-1.55**	-1.55	-0.62	-0.63	-0.13	-0.13
	(1.24)	(1.24)	(0.78)	(0.78)	(1.74)	(1.74)	(0.11)	(0.11)
MORETHAN	3.08	3.09	6.87**	6.87**	11.05**	11.04**	9.27	9.52
	(3.73)	(3.74)	(3.16)	(3.16)	(5.61)	(5.61)	(7.24)	(7.24)
Level 2								
Modeling reform oriented practices								
ASKQUES	10.10		-74.91**	-	51.88†	-	-39.34	-
	(12.70)		(34.80)		(27.51)		(26.03)	
WHOLDISC	-	-24.99	-	10.74	-	12.64	-	3.25
		(16.30)		(79.11)		(28.24)		(23.10)
GRPWK	-	27.85	-	69.44	-	-20.50	-	1.86
		(19.28)		(56.63)		(18.87)		(24.28)
Modeling non-reform oriented practices								
TCHMEDINS	-9.65		10.90	-	31.86	-	-5.74	-
	(16.07)		(23.62)		(22.74)		(25.24)	
LECTURE	-	-21.52†	-	20.67	-	4.04	-	-13.58
		(11.89)		(39.52)		(29.27)		(52.32)
ANATL	1.88		34.96**	-	-6.06	-	6.07	-
	(4.10)		(11.78)		(5.29)		(8.18)	
Variance components								
Intercept u_0	351.02	217.03	1616.43	1796.97	1127.13	1220.58	805.77	863.65
Level 1 r	3830.34	3848.53	4077.12	4077.44	6938.66	6938.79	4545.61	4547.18
ICC	0.08	0.05	0.28	0.30	0.14	0.15	0.15	0.16
Reliability coefficient	0.616	0.508	0.942	0.948	0.803	0.815	0.651	0.666

†$p<.10$, **$p<.05$, ***$p<.001$

Modeling non-reform oriented pedagogical practices between institutions

Table 24 shows different between-institution relationships between *listening to lecture presentations* and MPCK in the three countries. In the United States, for example, the relationship between this variable and PSTs' MPCK was significant and negative. Similar patterns of this relationship were seen with the specialist programs in Poland. However, in Russia and the generalist PSTs' programs in Poland, the results showed that the relationships between *listening to lecture presentation* and MPCK were positive (though not significantly so) across the teacher preparation programs.

Planning mathematics instruction and MPCK

A summary of the models with the significant within-institution relationships between the opportunities to learn how to plan mathematics instruction for conceptual understanding and MPCK are presented in Table 25.

Table 25: *Multi-level Models of Relationships between OTL how to Plan Mathematics Instruction and MPCK within the Institutions*

Variables	United States		Russia		Poland (generalists)		Poland (specialists)	
	Model 1	Model 2	Model 1	Model 2	Model 1	Model 2	Model 1	Model 2
INTERCEPT	541.37***	541.02***	508.08***	507.18***	456.06***	454.11***	574.86***	574.71***
	(4.59)	(4.73)	(6.41)	(6.64)	(5.78)	(5.85)	(8.12)	(7.95)
Level 1								
GENDER	-5.33	-4.69	1.53	2.96	-27.71	-28.88	-15.98	-17.43
	(6.46)	(6.53)	(10.87)	(10.83)	(20.36)	(19.37)	(12.85)	(13.00)
YEARSOF	2.78**	2.59**	-1.63	-1.44†	-0.71	-0.40	4.73**	5.00**
	(1.21)	(1.25)	(0.80)	(0.86)	(1.74)	(1.81)	(2.04)	(2.12)
MORETHAN	2.77	3.11	6.35†	6.72	11.57†	11.07†	4.95	3.13
	(3.72)	(3.70)	(3.32)	(3.18)	(5.96)	(5.85)	(9.53)	(9.21)
ANA_LRG	2.32	-	0.14	-	-0.32	-	1.22	-
	(1.25)-		(1.44)		(1.28)		(2.21)	
INTR_SD	-	-	-	-	-	-	-	-
MEAN_LRN	-	0.36	-	1.57**	-	3.24**	-	1.41
		(1.10)		(0.76)		(1.02)		(1.73)
Variance Components								
Intercept u_0	343.29	365.68	1909.29	1967.02	1158.15	1216.20	1004.17	958.20
Level 1 r	3803.90	3833.71	3979.54	4023.35	6808.79	6734.96	4794.70	4762.17
ICC	0.08	0.09	0.32	0.33	0.15	0.15	0.17	0.18
Reliability coefficient	0.609	0.624	0.950	0.952	0.798	0.815	0.636	0.632

†p<.10, **p<.05, ***p<.001

Table 25 shows that in the United States, the opportunity to learn how to plan mathematics instruction composite measure, *analysis of learning goals*, showed a significant positive relationship with the PSTs' MPCK within the institutions. In other words, the more the PSTs had opportunities to learn the *analysis of learning goals* in their mathematics methods courses, the higher their MPCK. In Russia and the Poland generalist programs, there was a significant positive relationship between reports about learning *meaningful learning experiences* and the PSTs MPCK: the more the PSTs' learned how to plan mathematics instruction for conceptual understanding by having frequent opportunities to learn *meaningful learning experiences*, the higher their MPCK scores.

The relationships between the OTL how to plan mathematics instruction and MPCK between the institutions is shown in Table 26. Table 26 shows that between the institutions in the United States, the opportunities to learn to plan mathematics instruction were significantly related to the PSTs' MPCK. In particular, the average PSTs' reports about frequency of the *analysis of learning goals* and *meaningful learning experiences* showed that the more often the PSTs in the United States had such opportunities, the higher their MPCK.

Table 26: *Multi-level Relationships of OTL how to Plan Mathematics Instruction and MPCK between the institutions*

Variables	**United States**		**Russia**		**Poland (generalists)**		**Poland (specialists)**	
	Model 1	Model 2	Model 1	Model 2	Model 1	Model 2	Model 1	Model 2
INTERCEPT	541.29***	541.38***	507.93***	506.69***	452.67***	452.75***	575.06***	575.03***
	(4.30)	(4.35)	(6.64)	(6.21)	(5.92)	(5.93)	(7.89)	(7.59)
Level 1								
GENDER	-5.10	-5.05	3.71	3.70	-25.50	-25.47	-17.91	-17.81
	(6.30)	(6.30)	(10.40)	(10.40)	(18.87)	(18.87)	(12.80)	(12.82)
YEARSOF	2.59**	2.58**	-1.55**	-1.55**	-0.63	-0.63	4.05†	4.06†
	(1.23)	(1.24)	(0.79)	(0.78)	(1.74)	(1.74)	(2.30)	(2.31)
MORETHAN	3.09	3.12	6.86**	6.87**	11.03**	11.03**	2.97	2.80
	(3.73)	(3.73)	(3.16)	(3.16)	(5.59)	(5.60)	(8.91)	(8.92)
Level 2								
ANA_LRG	12.44†	-	16.02	-	-3.83	-	-6.21	-
	(6.59)		(11.31)		(3.91)		(7.94)	
INTR_SD	-	-	-	-	-	-	-	-
MEAN_LRN	-	9.79†	-	8.71	-	-0.44	-	-10.82
		(5.44)		(8.72)		(3.28)		(7.49)
Variance components								
Intercept u_0	295.30	305.95	1849.80	1895.83	1219.26	1230.89	948.83	840.47
Level 1 r	3835.44	3834.68	4076.98	4076.92	6939.36	6939.24	4762.43	4761.76
ICC	0.07	0.07	0.31	0.32	0.15	0.15	0.17	0.15
Reliability coefficient	0.578	0.586	0.949	0.950	0.815	0.816	0.632	0.605

†p<.10, **p<.05, ***p<.001

Opportunities to learn mathematics instruction for conceptual understanding and MPCK

Multi-level models of the significant relationships between OTL mathematics instruction for conceptual understanding and MPCK within and between institutions in the three countries are shown in Tables 27 and 28. These models sought to answer part of research question 5, " What is the relationship between the OTL to teach mathematics instruction for conceptual understanding and PSTs' mathematics pedagogical content knowledge within and between the institutions in the three countries?"

Table 27: *Multi-level Models of the Relationships between OTL, Mathematics Instruction for Conceptual Understanding and MPCK within the Institutions*

Variables	United States		Russia		Poland (generalists)		Poland (specialists)	
	Model 1	Model 2	Model 1	Model 2	Model 1	Model 2	Model 1	Model 2
INTERCEPT	513.38***	513.21***	507.92***	507.76***	454.89***	453.58***	573.31***	573.71***
	(6.22)	(6.19)	(6.48)	(6.49)	(5.57)	(5.82)	(7.34)	(7.30)
Level 1								
GENDER	-28.12**	-28.34**	0.37	2.17	-27.57	-30.57	-19.18	-18.97
	(10.56)	(10.06)	(10.58)	(10.38)	(19.40)	(19.07)	(12.95)	(13.31)
YEARSOF	2.28†	2.27†	-1.72**	-1.64**	-0.72	-0.58	-0.11	-0.14
	(1.34)	(1.34)	(0.81)	(0.79)	(1.77)	(1.73)	(0.11)	(0.12)
MORETHAN	-6.18	-6.07	6.96**	6.89**	12.92**	12.45**	10.19	9.73
	(4.51)	(4.57)	(3.32)	(3.22)	(5.86)	(5.84)	(7.24)	(7.67)
MKDPROC	-2.55	-	-1.24	-	6.42**	-	-3.36	-
	(4.54)		(2.52)		(3.03)		(4.67)	
PROWRK	-	1.95	-	0.81	-	6.23**	-	5.38
		(5.14)		(1.78)		(2.36)		(3.74)
MSOSTR	-1.64	-	0.35	-	-3.41	-	-4.36	-
	(3.90)		(2.05)		(3.49)		(5.57)	
Variance components								
Intercept u_0	809.16	799.71	1915.26	1924.10	1074.30	1197.23	865.06	851.41
Level 1 r	4032.82	4041.19	4037.73	4025.36	6753.10	6864.03	4517.32	4548.76
ICC	0.17	0.17	0.32	0.32	0.14	0.15	0.16	0.16
Reliability coefficient	0.764	0.763	0.950	0.951	0.796	0.811	0.655	0.661

†*p*<.10, ***p*<.05, ****p*<.001

Table 27 shows that there were significant positive within-institution relationships between learning mathematics instruction and MPCK in the Poland generalist program for the opportunities for (i) *learning why procedures work* and (ii) *learning to make distinctions between procedural and conceptual knowledge*.. In particular, the more the PSTs had these experiences, the higher their MPCK within the institutions in the Polish generalist programs. The relationships between *learning why procedures work* and MPCK were positive but non-significant, and the relationship between the OTL to *make distinctions between procedural and conceptual knowledge* were negative and non-significant in the other countries within the institutions.

The relationships between institutions are presented in Table 28. In Poland (both programs) between the institutions, the more opportunities the PSTs had for learning to *make distinctions between procedural and conceptual knowledge*, the lower their MPCK Unlike In contrast, the other countries showed positive relationships between this OTL and MPCK, although they were not significant.

In the United States, the findings from the average PSTs reports between the institutions showed that the more the PSTs had opportunities to learn to *explain multiple solution strategies*, the higher their average MPCK score. The same positive relationships were consistent in all three countries although the relationships were not significant in Russia and Poland.

Table 28: *Multi-level models of the Relationships between OTL Mathematics Instruction for conceptual understanding and MPCK between the Institutions*

Variables	**United States**		**Russia**		**Poland (generalists)**		**Poland (specialists)**	
	Model 1	Model 2	Model 1	Model 2	Model 1	Model 2	Model 1	Model 2
INTERCEPT	515.82***	512.79***	504.978***	506.97***	452.85***	452.56***	574.13***	573.55***
	(5.89)	(5.88)	(5.90)	(6.16)	(5.82)	(5.81)	(7.31)	(6.62)
Level 1								
GENDER	-28.24**	-28.30**	3.71	3.71	-25.57	-25.41	-19.78	-19.52
	(10.03)	(10.03)	(10.40)	(10.40)	(18.86)	(18.85)	(12.94)	(12.99)
YEARSOF	2.20†	2.20†	-1.55**	-1.55**	-0.63	-0.62	-0.13	-0.13
	(1.32)	(1.32)	(0.78)	(0.78)	(1.74)	(1.73)	(0.11)	(0.11)
MORETHAN	-6.02	-6.00	6.87**	6.87**	11.03**	11.02**	9.51	9.21
	(4.36)	(4.36)	(3.16)	(3.16)	(5.59)	(5.59)	(7.23)	(7.25)
Level 2								
MKDPROC	-	16.53	-	17.87	-	-28.01†	-	-55.64**
		(11.91)		(44.56)		(14.08)		(26.01)
MSOSTR	36.25†	-	24.70	-	23.92	-	10.10	-
	(20.01)		(28.31)		(20.64)		(10.90)	
Variance Components								
Intercept u_0	651.44	730.40	1870.70	1914.00	1175.77	1188.62	853.22	669.72
Level 1 r	4038.75	4038.71	4077.21	4076.93	6939.62	6937.10	4549.03	4542.81
ICC	0.14	0.15	0.31	0.32	0.14	0.15	0.16	0.13
Reliability coefficient	0.729	0.749	0.949	0.951	0.809	0.811	0.663	0.610

†p<.10, **p<.05, ***p<.001

5.3. Discussion

For the United States, two significant results – one within institutions and one between institutions --are consistent with the notion that, the more PSTs experience approaches to learning to teach that focus on reform-oriented practices through modeling and introduction to related activities, the more likely their mathematics content knowledge is increased (Hypothesis A1). For this nation, there was a positive significant relationship between whole group discussion and MCK, and a negative relationship between listening to lecture presentations and MCK within the institutions. Also, between the institutions there was a positive significant relationship between *group work* and MCK.

Between-institution results for some practices in the United States also showed support for the notion that the more the OTL to teach mathematics focused on reform oriented instruction through modeling these practices and introducing the PSTs to related activities, the more likely the PSTs mathematics pedagogical content knowledge was increased (Hypothesis A2). There was a negative significant relationship between *lecture presentations* and MPCK between institutions. Further, the significant relationships between opportunities to learn how to plan mathematics instruction for conceptual understanding by learning the analysis of learning goals and learning meaningful learning experiences showed positive between-institution relationships with MPCK between the institutions. Finally, the significant results for the relationships between learning mathematics instruction and MPCK showed support for Hypothesis A2 for the variable *explain multiple solutions strategies* in the United States.

In Russia, the results supporting hypothesis A1 varied by the program features and were inconclusive. The pedagogical practices that modeled non-reform oriented practices between the institutions showed evidence against Hypothesis A1. The significant measures used to represent opportunities for learning how to plan mathematics instruction for conceptual understanding, however, showed positive between-institution relationships with MCK, thus showing support for Hypothesis A1.

In Russia the OTL to teach mathematics and MPCK showed support for Hypothesis A2 within the institutions, while some of the relationships between the institutions contradicted it. The pedagogical practices *whole group discussion, asking questions* and learning *meaningful learning experiences* in the planning of mathematics instruction within institutions were positively related to MPCK. Further, the practice of *analysis and reading of teaching and learning mathematics* supported Hypothesis A2, but the modeling of the reform-oriented pedagogical practice *asking questions* was negatively related to the average MPCK between institutions. These findings call for further analysis of the relationships between pedagogical practices and MPCK so as to ascertain what models of teaching the PSTs have opportunities to read and analyze the teaching and learning of mathematics and the mode of asking questions in their mathematics education courses.

In Poland the support for hypothesis A1 varied by the program type and program features. Different relationships between the pedagogical practices and MCK were found for PSTs in the generalist and specialist programs. In the generalist program the significant findings showed support for Hypothesis A1, because the modeling of the reform-oriented practice, *whole class discussion,* was positively related to MCK within the institutions. Further, the significant relationships between the measures for learning how to plan mathematics instruction showed support for Hypothesis A1 between the institutions, because the measure *learning multiple solution strategies* in the Poland generalist program was positively related to the average MCK. In the specialist programs, the findings for modeling non-reform oriented practices showed support for Hypothesis A1, because the practice, *teaching a session using methods demonstrated by the instructor*, showed a negative relationship with MCK. For the Poland specialist program, however, the significant results for the opportunities to plan and learn mathematics instruction for conceptual understanding between institutions showed evidence against Hypothesis A1, because *learning to make distinctions between procedural and conceptual knowledge* and planning mathematics for conceptual understanding through *opportunities to learn meaningful learning experiences* showed a relationship that was opposite to Hypothesis A1. In both programs, the significant findings showed some support for Hypothesis A2 on the pedagogical practices modeling of the reform-oriented practice, *whole class discussion,* that was positively related to MPCK within the institutions. The OTL how to plan mathematics instruction through *meaningful learning experiences* and learning mathematics instruction for conceptual understanding through *learning to explain why procedures work* and *learning to make distinctions between procedural and conceptual knowledge* showed support for Hypothesis A2 because of the significant positive relationships to MPCK within generalist programs. Likewise, the Polish specialist programs, the modeling of the reform-oriented practice *asking questions* showed support for the hypothesis A2 within the institutions, while in the Poland generalist program the relationship between asking questions and MPCK showed support for hypothesis A2 between the institutions. However, the OTL mathematics instruction through *learning to make distinctions between procedural and conceptual knowledge* showed evidence against Hypothesis A2 because of the negative relationship to MPCK between institutions for both programs.

CHAPTER 6

WITHIN-COUNTRY ANALYSES OF RELATIONSHIPS BETWEEN OPPORTUNITIES TO LEARN TO TEACH MATHEMATICS AND PRE-SERVICE TEACHERS' BELIEFS ABOUT LEARNING MATHEMATICS

6.1. Model Specification

In this chapter, multi-level models of the relationships between opportunities to learn to teach and pre-service teachers' beliefs about learning mathematics within and between institutions within three countries are presented and discussed. This chapter reports on answers to parts of the research questions 3, 4, and 5 that focus on pre-service teachers' beliefs about learning mathematics. The research questions addressed in this chapter are:

RQ3: What is the relationship between the OTL to teach through experiencing different pedagogical practices in PSTs' mathematics education and teacher education courses and their beliefs about the nature of learning mathematics:

- Within and between the teacher education institutions in Poland?
- Within and between the teacher education institutions in Russia?
- Within and between the teacher education institutions in the United States?

RQ4: What is the relationship between the OTL to teach through different approaches to learning how to plan mathematics instruction for conceptual understanding and PSTs' beliefs about the nature of learning mathematics:

- Within and between the teacher education institutions in Poland?
- Within and between the teacher education institutions in Russia?
- Within and between the teacher education institutions in the United States?

RQ5: What is the relationship between the OTL to teach through the introduction to learning mathematics instruction for conceptual understanding and PSTs' beliefs about the nature of learning mathematics:

- Within and between the teacher education institutions in Poland?
- Within and between the teacher education institutions in Russia?
- Within and between the teacher education institutions in the United States?

The opportunities to learn to teach mathematics as presented in the research questions include (i) pedagogical practices experienced; (ii) learning how to plan mathematics instruction; and (iii) learning mathematics instruction according to the pre-service teachers' reports. The nature of the pre-service teachers' beliefs about learning mathematics, were classified as either inquiry beliefs or non-inquiry beliefs. PSTs whose beliefs about learning mathematics were inquiry-oriented would indicate agreement with statements such as (i) it is important to understand why the answer is correct, (ii) students should be encourage and allowed to figure out ways to solve mathematics problems even if the solutions are inefficient, (iii) time spent investigating why a solution in mathematics works is well spent, (iv) students can learn to solve mathematics tasks without the teacher's help, and (v) it is helpful for students to discuss different ways to solve problems (Brese & Tatto, 2012). In contrast, PSTs whose beliefs are non-inquiry oriented would indicate agreement with statements such as (i) mathematics involves memorization of formulas, (ii) students should be taught exact procedures, (iii) it does not matter if students do not understand the problem if they can get the right answer, (iv) students learn best by attending to the teacher's explanations, (v) emphasis should be on getting the correct answer, (vi) non-standard procedures interfere with the learning of the correct procedure, (vii) solving equations quickly is important, and (viii) hands-on procedures are not worth the time and expense (Brese & Tatto, 2012).

The models presented are the models developed after testing that the OTL to teach mathematics variables and measures that are introduced in the models are not highly correlated at both level 1 and level 2. The models created are similar to those created in the previous chapter, but the outcomes are the belief measures instead of knowledge for teaching mathematics. The chapter first shows the relationships between OTL to teach mathematics and non-inquiry beliefs about learning mathematics. Second, similar relationships are presented and discussed between the OTL to teach mathematics and inquiry beliefs. In both these sections, the unconditional models, the models between the relationships of the background variables and PSTs' beliefs, the hypotheses to be tested, and the models between OTL to teach mathematics and the PSTs' beliefs measures will be discussed for the three countries, Poland, Russia, and the United States. In each case, results are presented for both the relationships within institutions and between institutions (i.e., prediction of relationships between individual PSTs and individual institutions, respectively).

6.2. Results

6.2.1. Unconditional Non-inquiry Beliefs about Learning Mathematics model.

The unconditional models are presented and discussed for the non-inquiry beliefs and the inquiry beliefs about learning mathematics. Table 29 shows the unconditional model for the non-inquiry beliefs about learning mathematics. The average non-inquiry beliefs about learning mathematics were lower for the PSTs in the United States and the Poland specialist programs compared to the average non-inquiry beliefs about learning for the PSTs in Russia and the Poland generalist programs.

Table 29 shows that the average non-inquiry belief score across the institutions in the United States was 20.94, with 13.7% of the proportion of variance between institutions, and 86.3% of the variance within institutions. In Russia, the average non-inquiry belief score across the institutions was 24.96, with 11.8% of the proportion of variance between institutions, and 88.2% of the variance within institutions. For the generalist PSTs in Poland, the average non-inquiry belief score was 24.75, with 13% of the proportion of variance between institutions, and 87% of the variance within institutions. The average non-inquiry belief score for the PSTs in the Poland specialists programs was 20.17, with 10% of the proportion of variance between institutions and 90% of the variance within institutions. In sum, PSTs in the United States and in the Polish specialist programs had much lower non-inquiry beliefs than the PSTs in Russia and in the Polish generalist programs.

Table 29: *Unconditional Models for Non-inquiry beliefs about learning mathematics in the three countries*

Variables	**United States (generalists)**	**Russia**	**Poland (generalists)**	**Poland (specialists)**
INTERCEPT	20.94***	24.96***	24.75***	20.17***
	(0.43)	(0.34)	(0.39)	(0.47)
Variance components				
Intercept u_0	4.27	4.11	5.24	2.86
Level 1 r	26.90	29.87	34.06	25.65
ICC	0.1370	0.118	0.13	0.100
Reliability coefficient	0.762	0.856	0.810	0.537

†p<.10, **p<.05, ***p<.001

6.2.2. Background Variables and Non-Inquiry Beliefs about Learning Mathematics

The background factors considered were gender and socio-economic status (SES). The variables representing SES were the number of books in the home (a dummy variable in which the 1 represents PSTs reports of having more than 100 books in the home and 0 represents PSTs reports that they had less than 100 books in the home) and parental level of education. The models are shown on Tables 30 and 31. There were no significant gender differences in non-inquiry beliefs about learning mathematics in any of the three countries. For the SES proxy measures, there were no significant relationships between the parental level of education and non-inquiry beliefs. However, in the Polish generalist programs, the findings showed that PSTs who had more than 100 books in the home had significantly lower non-inquiry beliefs when compared to those who reported having fewer than 100 books in the home. Similar relationships were shown for the United States and Russia, although the differences were not significant.

Between the institutions, the significant findings showed that the average non-inquiry beliefs of female PSTs was significantly lower than the male non-inquiry beliefs in the United States and Russia. Also, between the institutions in the United States, the significant findings showed that institutions with PSTs who had more than 100 books in the home had significantly higher non-inquiry beliefs when compared to institutions with PSTs who had fewer than 100 books in the home.

Table 30: *Multi-level Model of the Relationships between the Background Variables and Non-inquiry Beliefs about Learning Mathematics*

Variables	**United States (generalists)**		**Russia**		**Poland (generalists)**		**Poland (specialists)**	
	Model 1	Model 2	Model 1	Model 2	Model 1	Model 2	Model 1	Model 2
INTERCEPT	21.02***	20.87***	24.96***	24.99***	24.80***	23.89***	20.07***	20.18***
	(0.59)	(0.40)	(0.34)	(0.21)	(0.39)	(0.82)	(0.50)	(0.44)
Level 1								
GENDER	0.02		0.21	-	0.76		-0.09	
	(1.06)		(0.51)		(1.29)		(0.81)	
YEARSOF	-0.07		0.01	-	0.17		-0.23	
	(0.16)		(0.07)		(0.12)		(0.14)	
MORETHAN	-0.19				-1.30**		-0.74	
	(0.55)				(0.45)		(0.60)	
Level 2								
GENDER	-	-2.38†	-	-5.52***	-	-1.11	-	-0.96
		(1.30)		(0.79)		(2.88)		(0.93)
YEARSOF	-	-0.11	-	-	-	0.27	-	0.03
		(0.20)				(0.30)		(0.01)
MORETHAN	-	2.00**	-	-	-	-1.16	-	-1.32
		(0.71)				(0.96)		(0.91)
Variance components								
Intercept u_0	5.89	2.35	4.18	3.07	5.20	5.70	2.91	2.04
Level 1 r	27.41	26.17	29.57	29.86	34.41	33.90	26.86	25.63
ICC	0.176	0.082	0.123	0.09	0.131	0.14	0.10	0.07
Reliability coefficient	0.804	0.668	0.850	0.819	0.781	0.825	0.481	0.457

†*p*<.10, ***p*<.05, ****p*<.001

6.2.3. Opportunities to Learn to Teach Mathematics and Non-inquiry Beliefs about Learning Mathematics

The multi-level regressions for the relationships between OTL to teach mathematics and non-inquiry beliefs varied across the countries. The analyses show models with significant relationships for (i) pedagogical practices and non-inquiry beliefs about learning mathematics; (ii) OTL how to plan mathematics instruction for conceptual understanding and non-inquiry beliefs; and (iii) OTL mathematics instruction for conceptual understanding and non-inquiry beliefs. These analyses were used to test hypothesis A3 and to answer parts of Research Questions 3, 4, and 5.

Hypothesis A3: The more pre-service teachers experience opportunities to learn to teach mathematics that focus on reform-oriented instruction in their mathematics and mathematics methods courses, the more likely they will have lower non-inquiry beliefs about learning mathematics, and the more they experience models of non-reform oriented practices, the higher their non-inquiry beliefs.

Pedagogical practices and non-inquiry beliefs about learning mathematics

These analyses sought to answer the research question, "What is the relationship between the OTL to teach through experiencing different pedagogical practices in the PSTs mathematics education courses and their beliefs about the nature of learning mathematics within and between the institutions?" Table 31 presents summaries of the models with significant relationships between the pedagogical practices experienced by the PSTs, as per their reports, within the institutions in the three countries.

Table 31: *Multi-level Model of Pedagogical Practices and Non-inquiry Beliefs about Learning Mathematics between the Institutions*

Variables	**United States**		**Russia**		**Poland (generalists)**		**Poland (specialists)**	
	Model 1	Model 2	Model 1	Model 2	Model 1	Model 2	Model 1	Model 2
INTERCEPT	20.95***	21.04***	24.96***	24.95***	24.78***	24.78***	20.17***	20.10***
	(0.56)	(0.58)	(0.34)	(0.35)	(0.39)	(0.39)	(0.47)	(0.48)
Level 1								
GENDER	0.05	-0.06	0.32	0.45	0.71	0.71	0.01	-0.36
	(1.03)	(1.02)	(0.52)	(0.51)	(1.31)	(1.27)	(0.78)	(0.80)
YEARSOF	-0.01	-0.06	0.02	0.03	0.18	0.22†	0.01	0.01†
	(0.14)	(1.03)	(0.08)	(0.08)	(0.12)	(0.12)	0.01)	(0.01
MORETHAN	-0.22	-0.15	-0.01	-0.18	-1.30**	-1.26**	-1.46**	-1.53
	(0.53)	(0.53)	(0.31)	(0.29)	(0.45)	(0.47)	(0.54)	(0.48)
Modeling Reform oriented Practices								
ASKQUES	-0.94**	-	-0.77**		-0.33		0.29	
	(0.40)		(0.33)		(0.22)		(0.28)	
WHOLDISC	-	-1.14**	-	-0.16	-	-0.32	-	0.23
		(0.45)		(0.29)		(0.24)		(0.41)
GRPWK	-	-0.89**	-	-0.60**	-	0.15	-	0.04
		(0.45)		(0.22)		(0.19)		(0.41)
Modeling non-reform oriented Practices								
TCHMEDINS	-	-0.03	-	0.21	-	0.21	-	0.98**
		(0.24)		(0.26)		(0.21)		(0.40)
LECTURE	0.51**	-	-0.01	-	0.02	-	0.05	-
	(0.25)		(0.55)		(0.26)		(0.30)	
Variance components								
Intercept u_0	5.38	5.84	4.20	4.28	5.07	5.15	2.95	3.10
Level 1 r	26.33	26.50	29.40	29.07	34.12	33.78	25.07	24.04
ICC	0.169	0.18	0.125	0.128	0.13	0.13	0.105	0.114
Reliability coefficient	0.796	0.806	0.851	0.851	0.776	0.780	0.549	0.567

†*p*<.10, ***p*<.05, ****p*<.001

Modeling reform oriented practices and non-inquiry beliefs within institutions

The findings on Table 31 show that experiencing models of reform oriented practices (*group work, whole group discussion,* and *asking questions)* were negatively related to non-inquiry beliefs about teaching and learning mathematics for the PSTs in the United States, looking at variability within institutions. In other words, the more frequently the PSTs experienced these practices in their mathematics-related courses, as compared to other PSTs in their institution,, the less their non-inquiry beliefs, again as compared to others in their institution. Similar findings on two of the variables (group work and asking questions) were found for the Russian PSTs. In the Polish specialist programs, the relationships between experiencing reform-oriented practices and non-inquiry beliefs were positive though non-significant within the institutions.

Modeling non-reform oriented practices and non-inquiry beliefs within institution

The within-institution relationships between experiencing models of non-reform oriented practices or transmission methods of instruction and non-inquiry beliefs about learning mathematics showed a positive relationship for the PSTs in the United States and Poland. Specifically, the findings of the relationship between experiencing models of non-reform oriented instruction (*listening to lecture presentations)* and non-inquiry beliefs about teaching and learning were positive and significant. This means that the more the PSTs had lecture presentations, as compared to others in their institution, the more they had non-inquiry beliefs in the United States. Similarly, in the specialist program in Poland, the more, relative to others in their institution, the PSTs had experiences in which they were required to teach sessions using methods demonstrated by their instructors, the more their non-inquiry beliefs about teaching and learning mathematics.

A summary of the relationships of the pedagogical practices and non-inquiry beliefs about learning mathematics between the institutions is shown in Table 32. Between the programs in the United States, *group work* was negatively related to PSTs non-inquiry beliefs. In other words, the more programs had experiences for *group work* in the mathematics-related courses, the lower the average PSTs' non-inquiry beliefs about learning mathematics in the United States.

Table 32: *Multi-level Models of Pedagogical Practices and Non-inquiry Beliefs about Learning Mathematics between the Institutions*

Variables	United States (generalists)	Russia	Poland (generalists)	Poland (specialists)
INTERCEPT	20.96***	24.98***	24.80***	20.17***
	(0.49)	(0.33)	(0.40)	(0.43)
Level 1				
GENDER	0.03	0.21	0.76	-0.01
	(1.06)	(0.51)	(1.28)	(0.79)
YEARSOF	-0.07	0.01	0.17	0.01
	(0.16)	(0.07)	(0.12)	(0.01)
MORETHAN	-0.19	-0.10	-1.30**	-1.44
	(0.55)	(0.31)	(0.45)	(0.52)
Level 2				
Modeling Reform-oriented Practices				
WHOLDISC	0.29	-1.14	-0.20	1.43
	(2.39)	(3.20)	(1.86)	(1.22)
GRPWK	-3.49†	-0.69	2.61	-1.34
	(1.79)	(3.70)	(1.89)	(1.59)
Modeling non-reform oriented practices				
LECTURE	**-0.35**	**-0.48**	**1.66**	**4.54**
	(1.58)	**(2.97)**	**(1.94)**	**(3.21)**
Variance components				
Intercept u_0	4.13	4.09	4.99	2.34
Level 1 r	27.43	29.57	34.40	25.03
ICC	0.13	0.12	0.13	0.09
Reliability coefficient	0.745	0.848	0.774	0.494

†*p*<.10, ***p*<.05, ****p*<.001

OTL how to plan mathematics instruction for conceptual understanding and non-inquiry beliefs

The opportunities to learn how to plan mathematics instruction for conceptual understanding included three composite measures: (i) *analysis of learning goals*; (ii) *introduction to standards-based curricula*; and (iii) *introduction to meaningful learning experiences*.

Summaries of the models with significant relationships within the institutions in the three countries are presented in Table 33. The models created were used to answer part of Research Question 4, "What is the relationship between the OTL to teach through different approaches to learning how to plan mathematics instruction and PSTs beliefs about the nature of learning mathematics?"

In the United States, the PSTs reports on the frequency of experiences in which they were involved in the *analysis of learning goals* and *introduction to standards-based curriculum* were negatively related to their non-inquiry beliefs. In other analyses (in the appendix), introductions to meaningful learning experiences also showed a negative significant within-institution relationship with PSTs' non-inquiry beliefs in the United States. In contrast, the Polish generalist programs, the relationship between the opportunities to learn *the analysis of learning goals* showed a positive and significant within-institution relationship. Similarly, opportunities for introduction to standards-based curriculum showed a positive significant within-institution relationship to the PSTs' non-inquiry beliefs about learning mathematics in the Polish specialist programs. The results in Russia were non-significant for the three composite measures.

Table 33: *Multi-level Models of OTL how to plan Mathematics Instruction and Non-inquiry Beliefs about Learning Mathematics within the Institutions*

Variables	United States (generalists)		Russia		Poland (generalists)		Poland (specialists)	
	Model 1	Model 2	Model 1	Model 2	Model 1	Model 2	Model 1	Model 2
INTERCEPT	21.01***	20.98***	24.96***	24.92***	24.80***	24.74***	20.13***	20.04***
	(0.59)	(0.59)	(0.34)	(0.32)	(0.40)	(0.40)	(0.49)	(0.51)
Level 1								
GENDER	0.21	0.32	0.11	0.24	0.82	0.84	-0.19	-0.24
	(1.07)	(1.06)	(0.53)	(0.53)	(1.41)	(1.35)	(0.88)	(0.86)
YEARSOF	-0.06	-0.05	0.01	0.003	0.20	0.21†	-0.17	-0.13
	(0.15)	(0.16)	(0.08)	(0.08)	(0.12)	(0.12)	(0.15)	(0.14)
MORETHAN	-0.13	-0.16	-0.10	-0.11	-1.29**	-1.32**	-0.80	-0.85
	(0.53)	(0.50)	(0.31)	(0.30)	(0.46)	(0.46)	(0.62)	(0.60)
ANA_LRG	-0.47**	-	0.03	-	0.34**	-	0.17	-
	(0.15)		(0.10)		(0.11)		(0.16)	
INTR_SD	-	-0.44**	-	-0.05	-	0.06	-	0.35**
		(0.20)		(0.11)		(0.08)		(0.16)
MEAN_LRN	-	-	-	-	-	-	-	
Intercept u_0	5.88	5.91	4.11	4.03	5.32	5.23	2.54	2.91
Level 1 r	26.73	26.69	29.45	29.46	34.38	34.38	26.94	26.25
ICC	0.18	0.18	0.12	0.12	0.13	0.13	0.08	0.10
Reliability coefficient	0.804	0.807	0.845	0.844	0.775	0.779	0.442	0.483

†*p*<.10, ***p*<.05, ****p*<.001

Table 34 presents between-institution models of the relationships of the opportunities to learn mathematics instruction for conceptual understanding and PSTs reports about their non-inquiry beliefs about learning mathematics. The significant findings in Table 34 show that *analysis of learning goals* between institutions in the United States was significant and negatively related to non-inquiry beliefs. In contrast, for the Polish generalist programs, the results for a similar relationship between the institutions were positive. The average PSTs reports about *meaningful learning experiences* between the institutions showed a negative significant relationship with non-inquiry beliefs in Russia. The relationships for the average PSTs reports in Russia on the *introductions to meaningful learning experiences* was in the same direction as the United States, and were significant, while a similar relationship were positive with the non-inquiry beliefs between the institutions in Poland (both programs), although it was not significant.

Table 34: *Multi-level Models of OTL how to plan Mathematics Instruction and Non-inquiry Beliefs about Learning Mathematics between the Institutions*

Variables	**United States (generalists)**		**Russia**		**Poland (generalists)**		**Poland (specialists)**	
	Model 1	Model 2	Model 1	Model 2	Model 1	Model 2	Model 1	Model 2
INTERCEPT	21.00***	21.00***	24.93***	25.03***	24.82***	24.81***	20.07***	20.07***
	(0.54)	(0.55)	(0.34)	(0.31)	(0.38)	(0.39)	(0.50)	(0.49)
Level 1								
GENDER	0.02	0.01	0.21	0.21	0.78	0.77	-0.1	-0.11
	(1.06)	(1.05)	(0.51)	(0.51)	(1.28)	(1.28)	(0.82)	(0.82)
YEARSOF	-0.07	-0.07	0.01	0.01	0.17	0.17	-0.23	-0.23
	(0.16)	(0.16)	(0.07)	(0.07)	0.13)	(0.12)	(0.14)	(0.14)
MORETHAN	-0.19	-0.19	-0.10	-0.10	-1.30**	-1.30**	-0.73	-0.72
	(0.55)	(0.55)	(0.31)	(0.31)	(0.45)	(0.45)	(0.60)	(0.60)
Level 2								
ANA_LRG	-1.07†	-	-0.97	-	0.91**	-	0.28	-
	(0.64)		(0.58)		(0.37)		(0.45)	
INTR_SD	-		-	-	-		-	-
MEAN_LRN	-	-0.80	-	-0.98**	-	0.51	-	0.51
		(0.77)		(0.43)		(0.35)		(0.45)
Variance components								
Intercept u_0	5.37	5.55	3.92	3.84	4.63	4.97	2.83	2.53
Level 1 r	27.42	27.41	29.57	29.57	34.39	34.40	26.88	26.90
ICC	0.16	0.17	0.12	0.11	0.12	0.13	0.10	0.09
Reliability coefficient	0.79	0.795	0.842	0.84	0.761	0.774	0.474	0.447

†*p*<.10, ***p*<.05, ****p*<.001

OTL mathematics instruction for conceptual understanding and PSTs non-inquiry beliefs

The relationships of the OTL mathematics instruction for conceptual understanding and PSTs' non-inquiry beliefs about learning mathematics were in opposing directions in the United States and Poland when compared between the institutions. A summary of the significant between-institution relationships is shown in Table 35. The models created sought to answer part of research question 5, "What is the relationship between the OTL to teach mathematics through the introduction to learning mathematics instruction and PSTs' beliefs about the nature of learning mathematics?" The relationships within the institutions showed no significant relationships, and therefore are not presented.

Table 35: *Multi-level Models of OTL Mathematics Instruction and Non-inquiry Beliefs between the Institutions*

Variables	**United States**		**Russia**		**Poland (generalists)**		**Poland (specialists)**	
	Model 1	Model 2	Model 1	Model 2	Model 1	Model 2	Model 1	Model 2
INTERCEPT	21.10***	20.90***	24.96***	24.93***	24.82***	24.81***	20.20***	20.17***
	(0.47)	(0.53)	(0.33)	(0.33)	(0.37)	(0.37)	(0.45)	(0.47)
Level 1								
GENDER	0.01	0.01	0.21	0.21	0.75	0.76	-0.01	-0.01
	(1.05)	(1.06)	(0.51)	(0.51)	(1.28)	(1.28)	(0.80)	(0.79)
YEARSOF	-0.07	-0.07	0.01	0.01	0.17	0.17	0.01	0.01
	(0.16)	(1.16)	(0.07)	(0.07)	(0.12)	(0.12)	(0.01)	(0.01)
MORETHAN	-0.20	-0.19	-0.10	-0.10	-1.30**	-1.30**	-1.41**	-1.42**
	(0.55)	(0.55)	(0.31)	(0.31)	(0.45)	(0.45)	(0.52)	(0.53)
Level 2								
MKDPROC	-2.72**	-	-0.08	-	3.05**	-	2.47	-
	(0.94)		(1.64)		(1.22)-		(1.91)	
MSOSTR	-	-	-	-	-	-	-	-
PROWRK	-	-2.07**	-	0.71		3.47**	-	0.70
		(0.91)		(1.82)		(1.21)		(1.37)
Variance Components								
Intercept u_0	4.13	5.17	4.17	4.16	4.63	4.54	2.45	2.94
Level 1 r	27.46	27.41	29.57	29.57	34.39	34.4	25.17	25.12
ICC	0.13	0.16	0.12	0.12	0.12	0.12	0.09	0.10
Reliability coefficient	0.745	0.784	0.850	0.850	0.762	0.758	0.504	0.548

†$p<.10$, **$p<.05$, ***$p<.001$

Table 35 shows that between the institutions in the United States, the average PSTs' reports about opportunities to learn *to show why procedures work*, *make distinctions between procedural and conceptual knowledge*, and *learn to explore multiple solution strategies* were negatively related to their non-inquiry beliefs. In particular, the more the programs in the United States offered these learning opportunities, the lower the average non-inquiry beliefs about teaching and learning across the institutions. In contrast, for the Poland generalist programs, the more the programs provided opportunities in which the PSTs *learned to show why procedures work* and *learned to make distinctions between procedural and conceptual knowledge,* the more their non-inquiry beliefs**.** In Russia, the relationships between the OTL mathematics instruction for conceptual understanding and non-inquiry beliefs were not significant**.**

6.2.4. Unconditional Inquiry Beliefs about Learning Mathematics models

The inquiry beliefs when compared across the countries showed averages that were similar across the three countries. Table 36 presents a summary of the unconditional models of the inquiry beliefs about learning mathematics reported by the pre-service teachers.

Table 36: *Unconditional Models of Inquiry beliefs about Learning Mathematics across the three Countries*

Variables	**United States (generalists)**	**Russia**	**Poland (generalists)**	**Poland (specialists)**
INTERCEPT	24.20*** (0.27)	24.51*** (0.17)	24.21*** (0.15)	25.60*** (0.25)
Variance components				
Intercept u_0	1.23	0.71	0.58	0.831
Level 1 r	9.47	8.10	13.53	7.58
ICC	0.115	0.081	0.041	0.099
Reliability coefficient	0.723	0.797	0.563	0.536

†*p*<.10, ***p*<.05, ****p*<.001

Table 36 shows that the average inquiry belief scores about learning mathematics across the generalist programs in all three countries were very similar to one another. The average inquiry belief score in the Poland specialist program, however, was slightly higher than in the other countries. In the United States, the proportion of variance of the non-inquiry belief scores between institutions was 11.5% and within institutions the proportion of variance was 88.5%. In Russia the proportion of variance was 8.1% across the institutions, and 91.9% within institutions. In Poland the proportion of variance of the inquiry belief scores was 4.1% between institutions, and 95.9% within institutions for the generalist programs, while for the specialist programs the proportion of variance of the inquiry beliefs was 9.9% between institutions, and 90.1% within institutions.

6.2.5. Background Variables and Inquiry Beliefs about Learning Mathematics

The multi-level models showing the relationships between the background variables and inquiry beliefs showed that gender and SES relationships with PSTs inquiry beliefs differed across the three countries. Table 37 presents a summary of the relationships of the background factors between and within the institutions in the three countries.

Table 37: *Multi-level Model of Background Factors and Inquiry Beliefs about Learning Mathematics*

Variables	**United States (generalists)**		**Russia**		**Poland (generalists)**		**Poland (specialists)**	
	Model 1	Model 2	Model 1	Model 2	Model 1	Model 2	Model 1	Model 2
INTERCEPT	24.21***	24.11***	-	24.49***	24.23***	24.23***	25.61***	25.60***
	(0.28)	(0.22)		(0.08)	(0.16)	(0.33)	(0.27)	(0.25)
Level 1								
GENDER	-0.57		-		1.95**		0.15	
	(0.57)				(0.78)		(0.49)	
YEARSOF	0.02		-		-0.12**		0.26***	
	(0.07)				(0.06)		(0.07)	
MORETHAN	0.25		-		0.13		0.40	
	(0.26)				(0.21)		(0.42)	
Level 2								
GENDER	-	1.35†	-	0.47†	-	0.79	-	-0.53
		(0.71)		(0.25)		(1.16)		(0.67)
YEARSOF	-	0.18	-	-0.04		-0.02		0.01
		(0.13)		(0.05)		(0.12)		(0.01)
MORETHAN	-	-1.11**	-	0.34+		0.20		-0.23
		(0.42)		(0.19)		(0.39)		(0.53)
Variance components								
Intercept u_0	1.16	0.82		4.72	0.54	0.59	0.86	0.78
Level 1 r	9.66	9.29		3.90	12.85	13.67	7.50	7.56
ICC	0.107	0.08		0.547	0.04	0.04	0.10	0.09
Reliability coefficient		0.663		0.547	0.518	0.564	0.498	0.523

†*p*<.10, ***p*<.05, ****p*<.001

Table 37 shows that there was a significant gender difference in the inquiry beliefs about learning mathematics of the PSTs in the generalist programs in Poland. That is, the female PSTs had significantly higher inquiry beliefs about learning mathematics than the male PSTs. For the SES proxy measure, *number of books in the home*, the PSTs in the Polish specialist programs who reported that they had more than 100 books in the home had significantly higher inquiry beliefs about learning mathematics when compared to those who reported that they had fewer than one hundred books in the home. The results of the relationships between SES and inquiry beliefs showed similar patterns in the United States and Russia, although the relationships were not significant.

Between the institutions in the United States, the significant findings showed that the average inquiry beliefs of females were significantly higher than the males. The results also showed that programs in which the average PSTs reported that they had more than 100 books in the home had significantly lower inquiry beliefs about learning mathematics when compared to those who reported that they less than 100 books in the home in the United States. In Russia, the female PSTs had significantly higher inquiry beliefs than the male PSTs. In Poland the gender differences were not significant between the institutions.

6.2.6. Opportunities to Learn to Teach Mathematics and Inquiry Beliefs about Learning Mathematics

The relationships of the opportunities to learn to teach mathematics and inquiry beliefs were examined and the results showed that the significant relationships in the generalist programs in the three countries had similar patterns between the relationships of inquiry beliefs with (i) pedagogical practices that modeled reform oriented instruction experienced, (ii) planning mathematics instruction and learning, and (iii) learning mathematics instruction. In this section the multilevel models of the significant relationships within and between the institutions are presented and discussed briefly. These analyses were used to test Hypothesis A4, and sought to answer questions parts of Research Questions 3, 4, and 5.

Hypothesis A4: The more pre-service teachers experience opportunities to learn to teach mathematics that focus on reform-oriented instruction in their mathematics and mathematics methods courses, the more likely they will have higher inquiry beliefs about learning mathematics, and the more they experience models of non-reform oriented practices, the lower their inquiry beliefs.

Pedagogical practices and inquiry beliefs about learning mathematics

. The pedagogical practices examined were those that model reform-oriented instruction and those that model non-reform oriented instruction. The multi-level models presented in Table 38 showed significant positive relationships for the models that included the pedagogical practices, *group work, whole group discussions*, and *opportunities to ask questions,* which are all categorized as practices that model reform-oriented instruction. The models of non-reform oriented practices presented were *listening to lecture methods* and *teaching using methods demonstrated by the instructors,* which showed varying patterns when the relationships with inquiry beliefs were compared across the three countries.

Models of reform-oriented pedagogical practices and inquiry beliefs about learning mathematics within institutions

Table 38 shows that, looking at variation within institution, practices that model reform-oriented practices (*group work often, whole class discussion often*, and *asking questions*) were positive and significantly related to the inquiry beliefs about learning mathematics for the PSTs in the United States. Similar findings were shown for PSTs in the generalist programs in Poland. In the Polish specialist programs, positive significant relationships between inquiry beliefs and *whole group discussion* and *asking questions* were found. For the PSTs in Russia, *group work* and *asking questions* were positive and significantly related to inquiry beliefs. The pedagogical practice of opportunities to *ask questions*, showed a positive significant within-institution relationship with inquiry beliefs in all three countries.

Table 38: *Multi-level Models of Pedagogical Practices and Inquiry Beliefs about Learning Mathematics within the Institutions*

Variables	**United States**		**Russia**		**Poland (generalists)**		**Poland (specialists)**	
	Model 1	Model 2	Model 1	Model 2	Model 1	Model 2	Model 1	Model 2
INTERCEPT	24.18***	24.22***	24.52***	24.51***	24.23***	24.24***	25.60***	25.59***
	(0.29)	(0.26)	(0.17)	(0.17)	(0.16)	(0.16)	(0.25)	(0.25)
Level 1								
GENDER	-0.59	-0.55	0.24	0.25	1.94**	1.96**	-0.04	-0.07
	(0.57)	(0.58)	(0.22)	(0.22)	(0.83)	(0.79)	(0.37)	(0.38)
YEARSOF	0.03	0.01	-0.05	-0.05	-0.11	-0.10	-0.0004	-0.002
	(0.08)	(0.07)	(0.04)	(0.04)	(0.07)	(0.06)	(0.004)	(0.004)
MORETHAN	0.16	0.21	0.12	0.11	0.07	0.06	0.80**	0.78**
	0.24)	(0.26)	(0.15)	(0.17)	(0.22)	(0.21)	(0.38)	(0.38)
Reform oriented								
ASKQUES	0.45**	-	0.25†	-	0.57***	-	0.35†	-
	(0.17)		(0.13)		(0.14)		(0.20)	
WHOLDISC	-	0.44†	-	0.13	-	0.61***	-	0.43**
		(0.24)		(0.11)		(0.16)		(0.22)
GRPWK	-	0.55**	-	0.34**	-	0.32**	-	0.21
		(0.22)		(0.12)		(0.14)		(0.19)
Non-reform								
TCHMEDINS		-0.17		0.18**	-	-0.12	-	-0.24
		(0.15)		(0.07)		(0.13)		(0.19)
LECTURE	-0.13	-	-0.01	-	0.21	-	0.32	-
	(0.14)		(0.38)		(0.22)		(0.24)	
Variance components								
Intercept u_0	1.30	1.00	0.70	0.70	0.55	0.58	0.83	0.83
Level 1 r	9.44	9.37	7.99	7.94	12.63	12.55	7.23	7.26
ICC	0.12	0.10	0.08	0.08	0.04	0.04	0.10	0.10
Reliability coefficient	0.727	0.674	0.783	0.78	0.526	0.535	0.547	0.541

†p<.10, **p<.05, ***p<.001

Models of non-reform oriented pedagogical practices and inquiry beliefs about learning mathematics within the institutions

The within-institution analyses in Table 38 show that the PSTs in Russia who reported having experiences in which they *teach sessions using methods demonstrated by their instructors* showed a positive relationship with their inquiry beliefs about learning mathematics. The same relationships in the other countries were negative and not significant.

A summary of the between-institution relationships between the pedagogical practices and inquiry beliefs about learning mathematics are presented in Table 39.

Table 39: *Multi-level Models of Pedagogical Practices and Inquiry Beliefs about Learning Mathematics between the Institutions*

Variables	**United States**		**Russia**		**Poland (generalists)**		**Poland (specialists)**	
	Model 1	Model 2	Model 1	Model 2	Model 1	Model 2	Model 1	Model 2
INTERCEPT	24.19***	24.24***	24.53***	24.44***	24.26***	24.25***	25.59***	25.58***
	(0.25)	(0.19)	(0.16)	(0.14)	(0.13)	(0.14)	(0.25)	(0.24)
Level 1								
GENDER	-0.56	-0.60	0.28	0.28	1.92**	1.96**	-0.01	-0.004
	(0.57)	(0.57)	(0.21)	(0.21)	(0.77)	(0.78)	(0.36)	(0.36)
YEARSOF	0.02	0.023	-0.05	-0.05	-0.12**	-0.12**	-0.001	-0.001
	(0.80)	(0.08)	(0.04)	(0.04)	(0.06)	(0.06)	(0.004)	(0.005)
MORETHAN	0.26	0.26	0.15	0.15	0.13	0.13	0.81**	0.80**
	(0.26)	(0.26)	(0.16)	(0.16)	(0.21)	0.21)	(0.38)	(0.38)
Level 2								
Reform oriented								
ASKQUES	0.87	-	0.29	-	2.41**	-	-0.65	-
	(0.81)		(1.39)		(0.96)		(1.26)	
WHOLDISC	-	-0.48	-	1.32	-	0.90	-	0.17
		(1.01)		(1.37)		(0.86)		(1.29)
GRPWK	-	2.48***	-	1.85†	-	0.41	-	-0.75
		(0.61)		(1.05)		(0.94)		(0.80)
Non-reform								
TCHMEDINS	-0.39	-	1.13	-	-1.48**	-	0.31	-
	(0.83)		(0.69)		(0.61)		(0.92)	
LECTURE	-	-0.04	-	1.84**	-	-0.85	-	1.54
		(0.61)		(0.77)		(0.91)		(1.20)
ANATL	0.25	-	0.002	-	-0.14	-	0.10	-
	(0.20)		(0.37)		(0.23)		(0.28)	
Variance components								
Intercept u_0	0.98	0.43	0.61	0.49	0.30	0.44	0.80	0.71
Level 1 r	9.65	9.65	8.01	8.00	3.58	12.85	7.41	7.39
ICC	0.09	0.04	0.07	0.06	0.08	0.03	0.10	0.09
Reliability coefficient	0.665	0.483	0.761	0.722	0.382	0.467	0.533	0.504

†p<.10, **p<.05, ***p<.001

Modeling reform-oriented practices and beliefs about learning mathematics between institutions

. Table 39 shows that in the United States *group work* was significantly and positively related to PSTs' inquiry beliefs between the institutions. Similar findings were found for the relationship between the average PSTs' reports on *group work* and inquiry beliefs in Russia between the institutions. In Poland the relationships between group work and inquiry beliefs were not significant. *Asking questions* in the mathematics methods courses and inquiry beliefs also showed significant positive relationships between the institutions in the Poland generalist program. In the Russia and the United States the relationships were not significant. In contrast, the relationship between asking questions and inquiry beliefs was negative and non-significant in the Polish specialist programs between the institutions.

Modeling non-reform oriented practices and beliefs about learning mathematics between institutions

. The findings of the non-reform oriented instruction showed no significant relationships with inquiry beliefs in the United States and the Polish specialist programs between the institutions. Differing patterns were shown in Russia, in which the relationship between listening *to lecture presentations* and inquiry beliefs showed a positive significant between-institution relationship with inquiry beliefs. The non-reform oriented instruction *teaching a session using methods demonstrated by the instructor* showed a negative between-institution relationship with the average inquiry beliefs for the institutions from the Poland generalist programs. In the United States, Polish specialist programs, and Russia, the relationship between teaching methods using methods demonstrated by the instructor and the inquiry beliefs were not significant between the institutions.

OTL how to plan mathematics instruction for conceptual understanding and inquiry beliefs about learning mathematics

. The relationships between the OTL how to plan mathematics instruction and inquiry beliefs showed strong within-institution relationships with inquiry beliefs in all the generalist programs in the three countries. However, in the Polish specialist programs, the patterns of similar relationships were found to be in the same direction, although they were not significant. The measures of the OTL how to plan mathematics instruction for conceptual understanding were (i) analysis of learning goals, (ii) introduction to standards-based curriculum, and (iii) introduction to meaningful learning experiences. These relationships were investigated using a multi-level model in separate models because of the collinearity between the measures. Table 40 presents the models with significant relationships within the institutions.

The findings from the PSTs' reports in the United States, Russia and the generalist program in Poland showed that the more, relative to others at their institution, they had opportunities in which they were involved in *analyzing learning goals,* were *introduced to standards based curriculum* and were *introduced to meaningful learning experiences,*[50] the more they agreed with inquiry beliefs about learning mathematics, relative to others at their institution. (See Table 40) Similarly, for the PSTs in the Poland specialist program, the results were positive, though were not significantly related to their inquiry beliefs.

[50] The multi-level model showing this relationship is on Table A12 in the appendix.

Table 40: *Multi-level Models of Learning how to Plan Mathematics Instruction and Inquiry Beliefs about Learning Mathematics within the Institutions*

Variables	United States		Russia		Poland (generalists)		Poland (specialists)	
	Model 1	Model 2	Model 1	Model 2	Model 1	Model 2	Model 1	Model 2
INTERCEPT	24.23*** (0.27)	24.21*** (0.28)	24.51*** (0.17)	24.51*** (0.17)	24.23*** (0.16)	24.23*** (0.16)	25.64*** (0.27)	25.6*** (0.27)
Level 1								
GENDER	-0.72 (0.53)	-0.74 (0.53)	0.17 (0.24)	0.22 (0.23)	1.65** (0.68)	1.96** (0.77)	0.29 (0.51)	0.08 (0.50)
YEARSOF	0.004 (0.08)	-0.001 (0.08)	-0.05 (0.04)	-0.05 (0.04)	-0.12+ (0.06)	-0.11+ (0.06)	0.23** (0.07)	0.26*** (0.08)
MORETHAN	0.22 (0.23)	0.24 (0.22)	0.04 (0.14)	0.10 (0.15)	0.14 (0.22)	0.13 (0.22)	0.54 (0.45)	0.32 (0.43)
ANA_LRG	0.44*** (0.72)	-	0.29*** (0.05)	-	0.24** (0.08)	-	0.12 (0.09)	-
INTR_SD	-	0.40*** (0.1)	-	0.17** (0.07)	-	0.23*** (0.05)	-	0.16 (0.10)
Variance Components								
Intercept u_0	1.11	1.19	0.713	0.69	0.621	0.57	0.92	0.88
Level 1 r	9.01	9.18	7.72	7.84	12.84	12.69	7.11	7.46
ICC	0.11	0.11	0.10	0.10	0.05	0.04	0.11	0.11
Reliability coefficient	0.701	0.713	0.787	0.782	0.537	0.528	0.520	0.502

†*p*<.10, ***p*<.05, ****p*<.001

The analyses of the relationships between the institutions showed that the United States had different patterns when compared to the other two countries. A summary of the multi-level models showing the between-institution relationships of OTL how to plan mathematics instruction and inquiry beliefs are presented in Table 41.

The average PSTs' reports about *analysis of learning goals, introduction to standards-based curriculum*[51], and *meaningful learning experiences* were positively and significantly related to inquiry beliefs in the United States between the institutions. Similarly, in Russia and the Polish generalist program the patterns of the same relationships were positive. In contrast, the reports of the PSTs in the Polish specialist programs showed that the more the PSTs were introduced to the *analysis of learning goals* and identifying *meaningful learning experiences*, the less their inquiry beliefs about learning mathematics.

[51] The multi-level model for this relationship is in Table A13 in the appendix.

Table 41: *Multi-level Models of Pedagogical Practices and Inquiry Beliefs about Learning Mathematics between the Institutions*

Variables	United States		Russia		Poland (generalists)		Poland (specialists)	
	Model 1	Model 2	Model 1	Model 2	Model 1	Model 2	Model 1	Model 2
INTERCEPT	24.23***	24.23***	24.52***	24.50***	24.24***	24.24***	25.60***	25.60***
	(0.21)	(0.23)	(0.17)	(0.17)	(0.15)	(0.16)	(0.27)	(0.27)
Level 1								
GENDER	-0.56	-0.56	0.28	0.28	1.94**	1.94**	0.16	0.15
	(0.57)	(0.56)	(0.21)	(0.21)	(0.78)	(0.78)	(0.37)	(0.48)
YEARSOF	0.02	0.02	-0.04	-0.05	-0.12**	-0.12	0.25***	0.25***
	(0.08)	(0.08)	(0.04)	(0.04)	(0.06)	(0.06)	(0.07)	(0.07)
MORETHAN	0.26	0.26	0.15	0.15	0.13	0.13	0.40	0.40
	(0.26)	(0.26)	(0.16)	(0.16)	(0.21)	(0.21)	(0.42)	(0.42)
Level 2								
ANA_LRG	1.07***	-	0.10	-	0.09	-	-0.16	-
	(0.25)		(0.34)		(0.19)		(0.37)	
MEAN_LRN	-	0.76**	-	0.23	-	0.04	-	-0.12
		(0.31)		(0.25)		(0.15)		(0.27)
Variance components								
Intercept u_0	0.744	0.88	0.68	0.66	0.53	0.54	0.84	0.84
Level 1 r	9.65	9.65	8.00	8.00	12.85	12.85	7.51	7.51
ICC	0.07	0.08	0.07	0.08	0.04	0.04	0.10	0.10
Reliability coefficient	0.607	0.644	0.781	0.776	0.514	0.517	0.491	0.492

†p<.10, **p<.05, ***p<.001

OTL mathematics instruction for conceptual understanding and inquiry beliefs

The relationships between OTL mathematics instruction for conceptual understanding and inquiry beliefs about learning mathematics showed similar patterns in all three countries within the institutions. However, between the institutions the Poland generalist programs showed differing patterns of relationships between the OTL mathematics instruction and inquiry beliefs. The relationships were analyzed using a multi-level model in which the OTL variables were (i) learning why procedures work, (ii) learning multiple solution strategies, and (iii) learning to make distinctions between procedural and conceptual knowledge. A summary table showing the models of the relationships between the institutions is shown in Table 42.

Table 42: *Multi-level Models of OTL Mathematics Instruction and Inquiry Beliefs about Learning Mathematics within the Institutions*

Variables	**United States**		**Russia**		**Poland (generalists)**		**Poland (specialists)**	
	Model 1	Model 2	Model 1	Model 2	Model 1	Model 2	Model 1	Model 2
INTERCEPT	24.21***	24.21***	24.51***	24.5***	24.22***	24.25***	25.60***	25.60***
	(0.28)	(0.28)	(0.17)	(0.17)	(0.16)	(0.16)	(0.26)	(0.26)
Level 1								
GENDER	-0.58	-0.63	0.33	0.29	1.87**	2.01**	-0.02	-0.001
	(0.54)	(0.52)	(0.23)	(0.21)	(0.75)	(0.74)	(0.36)	(0.35)
YEARSOF	0.002	0.01	-0.05	-0.04	-0.13**	-0.11**	-0.001	-0.001
	(0.08)	(0.08)	(0.04)	(0.04)	(0.06)	(0.06)	(0.004)	(0.004)
MORETHAN	0.22	0.21	0.09	0.14	0.17	0.07	0.76**	0.74+
	(0.23)	(0.24)	(0.16)	(0.16)	(0.21)	(0.21)	(0.38)	(0.40)
MKDPROC	0.59**	-	0.09	-	0.18	-	0.23	-
	(0.18)		(0.09)		(0.15)		(0.15)	
MSOSTR	0.38	-	0.33**	-	0.42**	-	0.08	-
	(0.25)		(0.11)		(0.16)		(0.18)	
PROWRK	-	0.53**	-	0.22**	-	0.49***	-	0.28
		(0.23)		(0.09)		(0.14)		(0.19)
Variance Components								
Intercept u_0	1.16	1.16	0.72	0.70	0.58	0.54	0.85	0.85
Level 1 r	9.18	9.46	7.92	7.97	12.69	12.67	7.40	7.38
ICC	0.11	0.11	0.08	0.08	0.04	0.04	0.10	0.10
Reliability coefficient	0.71)	0.703	0.788	0.782	0.534	0.522	0.543	0.545

†*p*<.10, ***p*<.05, ****p*<.001

Table 42 shows that learning mathematics instruction for conceptual understanding was positively related to PSTs' inquiry beliefs within the institutions. That is, the more, as compared to others in their institution, the PSTs in Russia and those in the generalist programs in Poland had opportunities to learn *why procedures work* and to learn *how to explain multiple solution strategies*, the more their inquiry beliefs, relative to others in their institution. The relationships for the PSTs in the United States and in the specialist programs in Poland showed positive relationships with inquiry beliefs although the relationships were not significant.

Table 43 presents a summary of the models showing the relationships of learning mathematics instruction and inquiry beliefs between the institutions. In the United States, the variables selected for learning mathematics instruction were positively and significantly related to the inquiry beliefs between institutions. Specifically, the significant results indicated that the more institutions offered their PSTs opportunities to learning mathematics instruction for conceptual understanding in which they (i) *learned why procedures work, (ii) made distinctions between procedural and conceptual knowledge*, and (iii) learned to *explain multiple solutions strategies*[52], the more their students endorsed inquiry beliefs on average between the institutions in the United States. In Russia, the findings between institutions showed that the more programs offered *learning multiple solution strategies* in learning mathematics instruction, the more the PSTs' inquiry beliefs. Similarly, for the Poland specialist PSTs, the more the programs had opportunities for *learning multiple solutions strategies*, the more their inquiry beliefs.

[52] The models showing this significant relationship to the inquiry beliefs is not shown but is in Table A14 the appendix.

Table 43: *Multi-level Models of OTL Mathematics Instruction and Inquiry Beliefs about Learning Mathematics between the Institutions*

Variables	**United States (generalists)**		**Russia**		**Poland (generalists)**		**Poland (specialists)**	
	Model 1	Model 2	Model 1	Model 2	Model 1	Model 2	Model 1	Model 2
INTERCEPT	24.29***	24.14***	24.50***	24.48***	24.22***	24.32***	25.69***	25.59***
	(0.21)	(0.18)	(0.17)	(0.16)	(0.15)	(0.15)	(0.26)	(0.25)
Level 1								
GENDER	-0.56	-0.55	0.28	0.28	1.95**	1.95**	-0.01	-0.01
	(0.57)	(0.57)	(0.21)	(0.21)	(0.77)	(0.78)	(0.35)	(0.35)
YEARSOF	0.02	0.02	-0.05	-0.05	-0.12**	-0.12**	-0.001	-0.001
	(0.08)	(0.08)	(0.04)	(0.04)	(0.06)	(0.06)	(0.005)	(0.005)
MORETHAN	0.26	0.26	0.15	0.15	0.13	0.13	0.81**	0.82**
	(0.26)	(0.26)	(0.16)	(0.16)	(0.21)	(0.21)	(0.38)	(0.38)
MKDPROC	1.73***	-	0.75	-	0.54	-	0.28	-
	(0.38)		(1.22)		(12.84)		(1.01)	
MSOSTR	-	-	-	-	-	-	-	-
PROWRK	-	1.73***	-	1.28	-	-0.32	-	0.83
		(0.35)		(1.25)		(0.63)		(0.85)
Variance Components								
Intercept u_0	0.72	0.54	0.67	0.64	0.54	0.54	0.84	0.80
Level 1 r	9.65	9.66)	8.00	8.00	12.84	12.85	7.40	7.39
ICC	0.07	0.05	0.08	0.07	0.04	0.04	0.11	0.10
Reliability coefficient	0.601	0.532	0.776	0.768	0.517	0.519	0.544	0.533

†*p*<.10, ***p*<.05, ****p*<.001

6.3. Discussion

The models in the United States showed support for Hypothesis A3 because two significant results- one within and one between institutions are consistent with the notion that the more PSTs experience approaches to learning to teach that focus on reform-oriented pedagogical approaches through modeling and introductions to related activities, the less their non-inquiry beliefs about learning mathematics. For this nation, there were negative significant relationships between opportunities to *ask questions*, engage in *whole class discussion*, and *group work*, and their non-inquiry beliefs. Also, there were negative relationships between OTL how to plan mathematics instruction for conceptual understanding through the *analysis of learning goals* and *introductions to standards-based curriculum*, and non-inquiry beliefs about learning mathematics within institutions.

Between-institution relationships for some of the pedagogical approaches in the United States also showed support for the notion that the more the OTL to teach mathematics focused on reform-oriented approaches through modeling practices and introduction to related activities, the lower the non-reform oriented practices (Hypothesis A3). There were negative relationships between *group work* pedagogical practices and non-inquiry beliefs between institutions in the United States. Further, there were negative relationships between the OTL how to plan mathematics instruction through the *analysis of learning goals* and non-inquiry beliefs about learning mathematics between the institutions. In this nation, there were also negative relationships between the OTL how to plan mathematics instruction that included *learning to distinguish conceptual and procedural knowledge* and a learning to *show how procedures work*, and non-inquiry beliefs between institutions.

The significant results in the United States showed support for the notion that the more PSTs experienced approaches to learning to teach that focused on reform-oriented instruction through modeling and introductions to related activities, the higher their inquiry beliefs about learning mathematics within and between institutions (Hypothesis A4). Within the institutions, having opportunities to engage in *group-work* activities, *whole class discussion*, and *ask questions* in mathematics related courses were positively related to their inquiry beliefs about learning mathematics. Additionally, the OTL how to plan mathematics instruction for conceptual understanding through learning about the *analysis of learning goals* and *introductions to standards-based curriculum* were positively related to the inquiry beliefs of PSTs within institutions. Also, the OTL to learn mathematics instruction for conceptual understanding through learning to *show how procedures work* and learning to show *distinctions between procedural and conceptual knowledge* was positively related to PSTs inquiry beliefs about learning mathematics within the institutions.

Between the institutions, having opportunities to engage in *group work*, OTL how to plan mathematics instruction through *analysis of learning goals* and *introductions to meaningful learning experiences*, and OTL mathematics instruction that included learning to *distinguish conceptual and procedural knowledge* and *showing how procedures work* were all positively related to inquiry beliefs between institutions in the United States.

In Russia, the significant relationships showed support for Hypothesis A3 and A4 within and between institutions. In this nation, there were negative relationships between the opportunities to engage in *group work* and *asking questions* in the PSTs mathematics-related courses and their non-inquiry beliefs within institutions. Additionally, the OTL how to plan mathematics instruction that included *meaningful learning experiences* were negatively related to PSTs non-inquiry beliefs about learning mathematics within the institutions. The relationships between the OTL how to plan mathematics instruction for conceptual understanding that included the *analysis of learning goals* and *introductions to standards-based curriculum* were positively related to the inquiry beliefs within the institutions. Further, there were positive relationships between the OTL to learn mathematics instruction for conceptual understanding that included learning to *show why procedures work* and learning to *show multiple solution strategies* and their inquiry beliefs about learning mathematics within the institutions in Russia.

Between the institutions in Russia the significant results showed some support for Hypothesis A4, while other findings showed evidence against this hypothesis. There were positive significant relationships between engaging in *group work* and *listening to lecture presentation*, and PSTs inquiry beliefs of the PSTs between institutions. In other words, pedagogical practices that model reform and non-reform oriented instruction were positively related to the inquiry beliefs about learning mathematics between the institutions.

In the Polish programs program the significant findings showed evidence against support for Hypothesis A3, but some support for Hypothesis A4 within and between institutions. There were positive relationships between the OTL to teach mathematics through having experienced models of non-reform oriented pedagogical practices that included *teaching using methods demonstrated by the instructor* and non-inquiry beliefs within the institutions in the Polish specialist programs. Also, there were positive relationships between the OTL how to plan mathematics instruction for conceptual understanding that included the *analysis of learning goals* and PSTs' non-inquiry beliefs within the institutions in the Polish generalist programs, while *introductions to standards-based curriculum* was positively related to the PSTs non-inquiry beliefs in the Polish specialist programs.

Between the institutions, the significant results for the Polish generalist programs showed evidence against Hypothesis A3. There were positive relationships between the OTL to learn how to plan mathematics instruction through *analysis of learning goals* and non-inquiry beliefs about learning mathematics. There were also positive relationships between the OTL mathematics instruction that included *learning to distinguish between conceptual and procedural knowledge* and *showing why procedures work*, and PSTs non-inquiry beliefs about learning mathematics.

Across the Polish programs, the significant findings showed support for Hypothesis A4 within and between institutions. There were positive relationships between the pedagogical practices that provided opportunities for the PSTs to *ask questions* and engage in *whole class discussion*, and their inquiry beliefs within the institutions in both Polish programs. Also, there were positive relationships between the opportunities to engage in *group work* and inquiry beliefs about learning mathematics within the institutions in the Polish generalist programs. There were positive relationships between the OTL how to plan mathematics instruction for conceptual understanding that included the *analysis of learning goals* and the *introduction to standards-based curriculum* within institutions in the Polish generalist programs. Further, there were positive relationships between the OTL mathematics instruction for conceptual understanding that included learning to *show multiple solution strategies* and showing *how procedures work* and inquiry beliefs of the PSTs within institutions in the Polish generalist programs.

Between the institutions the significant results showed support for Hypothesis A4 in the Polish generalist programs. There was a positive relationship between the OTL mathematics by experiencing opportunities to *ask questions* in their mathematics related courses and inquiry beliefs about learning mathematics between institutions. Also, there were negative relationships between the OTL to teach mathematics in which PSTs *taught sessions using methods demonstrated by their instructors* and inquiry beliefs about learning mathematics between institutions in the Polish generalist programs.

The patterns of the relationships of OTL and beliefs about learning mathematics showed evidence against Hypothesis A3 and, but support for Hypothesis A4 within and between the programs in the Polish specialist programs. Although most of these relationships were not significant, the findings call for further studies to ascertain if the OTL to learn are related to the non-inquiry or inquiry beliefs about learning mathematics. These findings for the Polish specialist programs show some ambiguity allowing room for further analysis.

CHAPTER 7

DISCUSSION AND CONCLUSIONS

7.1. Summary of Findings

International comparisons are important for informing policy and practice in education. Careful consideration should be taken when interpreting the results, however. First, it should be noted that countries differ, and therefore any relationships explored should be within-country (Givvin, Hiebert, Jacobs, Hollingsworth, & Gallimore, 2005). For example, patterns of teaching differ due to cultural patterns, values, educational structures, beliefs, curricular differences, opportunities to learn, institutional norms, and expectations across countries (Bishop, 1988; Bishop, 2001; Desimone, Smith, Baker, & Ueno, 2005; Schmidt et al., 2001; Schmidt, Cogan, & Houang, 2011). In consideration of these important issues, the descriptions of the variables were given for each of the seventeen countries and the models developed in the relational analysis were done within the three selected countries, and the within-country relationships compared.

7.1.1. Differences in Opportunities to Learn and Teacher Competencies

The study examined the differences in the knowledge and beliefs about teaching and learning mathematics and the extent (i) to which particular pedagogical practices were used in teacher preparation, (ii) of opportunities to learn how to plan mathematics instruction for conceptual understanding, and (iii) of opportunities to learn mathematics instruction for conceptual understanding across the countries that participated in the TEDS-M survey. Using data from three countries, the study then examined the influence of opportunities to learn to teach mathematics on pre-service teachers' knowledge and beliefs about learning mathematics.

Knowledge for teaching mathematics

. The results showed that there were significant differences in the knowledge for teaching mathematics across the 17 countries. Previous cross-national studies of pre-service teachers' knowledge for teaching also found significant differences between and within countries (Blömeke & Kaiser, 2014; Senk et al., 2012; Schmidt et al., 2011). The findings from these studies pointed to the possibility of different features in the teacher preparation programs that could be contributing to the differences in the knowledge that PSTs have at the end of their teacher preparation.

Beliefs about learning mathematics

The nature of beliefs that pre-service teachers had about learning mathematics showed that there were patterns in the nature of PSTs beliefs about learning mathematics that were similar across most countries, although some countries still had distinct beliefs, separating them from other countries. The beliefs about learning mathematics showed significant differences in the patterns of the PSTs' beliefs about the nature of learning mathematics in the focal countries. Similar to findings from previous cross-national studies, inquiry beliefs about learning mathematics were found among pre-service teachers in most countries (Wang & Hsieh, 2014). Further, previous studies showed that the nature of PSTs' beliefs about learning mathematics is influenced by culture (Schmidt et al., 2011; Tang & Hsieh, 2014). In this study some of the distinct differences shown could be due to the cultural differences in the participating countries. Notably, most countries that participated in the TEDS-M survey showed that they valued student-centered approaches for teaching mathematics, which showed that this approach for teaching is now becoming a global feature (Schmidt et al., 2011; Wang & Hsieh, 2014).

Opportunities to learn to teach mathematics

. The opportunities to learn to teach mathematics across countries provided valuable insights that can inform policy and practice in mathematics teacher education. Across the countries that participated in the TEDS-M survey, important differences were found in the extent of (i) the pedagogical practices experienced during teacher preparation, (ii) learning how to plan mathematics instruction for conceptual understanding, and (iii) learning mathematics instruction for conceptual understanding. Below is a brief discussion of the differences shown in the opportunities to learn across the countries categorized under the three opportunities to learn.

Differences in pedagogical practices experienced

Comparing the extent to which the PSTs experienced models of reform-oriented and non-reform oriented practices across the countries showed distinct patterns worth considering. For instance, the findings showed that most PSTs reported that they often experienced lecture presentation, a model of non-reform oriented practice. PSTs in a few countries reported group work to be the most frequently experienced practice in their teacher education. Specifically, most of the PSTs from the participating European countries reported experiencing lecture presentations often, while most of the PSTs from the American geographical region that participated in this study experienced group work often during their mathematics methods courses. The PSTs from the participating East Asian countries showed significant differences in the pedagogical practices they experienced most often: Chinese Taipei showed more PSTs experienced lecture presentations often as compared to PSTs in Singapore, and more Singaporean PSTs reported experiencing group work often as compared to PSTs in Chinese Taipei.

Experiencing whole group discussion was not as common across the countries experiencing lecture presentations and group work. However, it is worth noting that the countries in which a significant percentage of PSTs reported experiencing this practice often were the United States, Chile, Malaysia, and Singapore. Similarly, a larger percentage of PSTs from the participating countries from the American regions, Botswana, and Thailand reported experiencing instances in which they could ask questions in their methods courses as compared to the other participating countries.

Finally, analyzing teaching through readings, video analysis, and live classrooms was not a common pedagogical practice across the countries that participated in the TEDS-M survey. Countries in which this pedagogical practice was experienced occasionally were Botswana, Russia, Thailand, and the United States.

These differences in the pedagogical practices experienced often in teacher preparation to teach mathematics could be attributed to recent changes made in teacher preparation in some of the participating countries. For example, a group of authorities from private and public institutions in Chile produced a report on changes that were then endorsed by the heads of the teacher education institutions (Davidson, 2013). Similarly, teacher educators in the Singapore National Institute of Education (NIE) developed an innovative feature in their curriculum, which is a combination of core courses in education studies, content courses for teaching, academic studies, a practicum component, a language enhancement course, and the development of academic discourse (Schwille, Ingvarson, & Holdgreve-Resendez, 2013). In Singapore, the content courses in teacher education had shifted to more practically-oriented approaches from their original psychological orientations such that they are now similar to methods courses in the United States (Lim-Teo, 2010). This may explain the similarity in PSTs' experience of the group work pedagogical practices between the United States and Singapore. In the United States, the "states are responsible for establishing the content guidelines in their teacher preparation programs" (Youngs & Grogan, 2013, p.264), and the requirements for content guidelines in teacher preparation vary across states. These differences in the pedagogical practices used during teacher preparation could be attributed to the innovative features introduced in the various programs, which could be at the institution level, the state level, as well as the country level.

Opportunities to learn how to plan to teach mathematic for conceptual understanding

The differences across the countries in the opportunities to learn how to plan mathematics instruction could be related to differences in the autonomy of teacher education across the countries and to innovations introduced in teacher education in the different countries. PSTs in the United States, Russia, Singapore, and the Philippines experienced approaches to learning to plan mathematics instruction often. These opportunities to learn to plan mathematics instruction for conceptual understanding included the OTL the analysis of learning goals, OTL to learn meaningful learning experiences, and introducing the use of standards-based curriculum. These opportunities could be due to a number of country-specific factors. The 2004 competency-based reform titled "Policies and Standards for Undergraduate Teacher Education curriculum" that was revised and implemented in the 2005/2006 year in the Philippines, which aimed to improve teachers' competency in lesson planning and innovative teaching approaches, among others (Ogena, Brawner, & Ibe, 2013). In Russia teacher education programs have the autonomy to select suitable sections from the state standards to develop their own curricula, but the choice of standards has to be approved by the Ministry of Education. Additionally, PSTs in Russia were required to take three terms of pedagogy and psychology as well as didactics and mathematics methods in their second and third years of their teacher preparation (Schwille et al., 2013). In the

United States, the autonomy given to the teacher education institutions allowed for innovative emphases in planning mathematics instruction that are based on research done in teacher education and discussion forums in professional organizations that encourage teacher educators to share innovative features that have been used in different institutions. These emphases could explain the findings of Blömeke, Suhl, and Kaiser (2014), in which they noted the strength of the United States teacher education programs in planning instruction and identifying student misconceptions. Because Singapore and Chile followed the proposed teaching strategies that are used in some United States curriculum, the PST reports showed similar patterns in the emphasis on components of learning how to plan mathematics instruction. Further, similar to the findings of Hsieh, Chu, Hsieh, and Lin (2014) the results showed that, except for Singapore, the participating Eastern countries put less emphasis on connections to real life situations.

Opportunities to learn mathematics instruction for conceptual understanding

PSTs from the countries in the American region, the Philippines, Singapore, and those countries whose programs are fully specialist programs (Thailand and Malaysia) had opportunities to learn mathematics instruction for conceptual understanding often. Notably, PSTs from most of the participating Developed European countries (Germany, Norway, and Switzerland) had fewer experiences to learn to distinguish between procedural and conceptual knowledge and how to show why a procedure works. The United States and those countries that follow closely the American model of teaching (i.e. Chile and Singapore), however, showed that their PSTs had more opportunities to learn mathematics instruction for conceptual understanding.

In sum, the important differences highlighted in the opportunities to learn to teach mathematics in this study further contribute to the OTL to learn mathematics pedagogy for conceptual understanding that was not much explored in previous cross-national teacher education studies. The previous cross-national studies on teacher preparation focused more on the OTL mathematics content, time spent in teacher education, practical experiences, the total number of courses taken by the PSTs, and the frequency of the activities done in content and methods courses (Blömeke & Delaney, 2014; Blömeke & Kaiser, 2014; Tatto et al., 2009; Tatto et al., 2012). This study showed that the extent of the OTL to teach mathematics that PSTs had for learning the process of teaching elementary mathematics and the models that they experienced in teacher education differed across the countries. These findings further confirm that the values in mathematics teacher education are influenced by the culture of the countries and the institutional norms (Bishop, 2001).

7.1.2. Opportunities to Learn to Teach Mathematics that Makes a Difference

The opportunities to learn to teach mathematics that this study focused on are (i) the pedagogical practices used in their mathematics-related courses, (ii) learning how to plan mathematics instruction for conceptual understanding, and (iii) learning mathematics instruction for conceptual understanding. These opportunities to learn to teach mathematics for conceptual understanding can challenge the beliefs and build on the knowledge that PSTs bring to their teacher preparation programs. Identifying the relationships that exist between these opportunities to learn and knowledge and beliefs about teaching and learning is the center of this study. The results showed that within the three selected countries there were significant relationships that are key findings for teacher preparation.

Background factors

Before examining the variables of interest, I discuss some significant findings about the PSTs' background factors. A brief look at the PSTs' backgrounds, which were the control variables for the three countries, showed that gender and socio-economic status influenced the knowledge and beliefs about teaching and learning mathematics, yet their importance as factors varied across the three countries.

Gender

The findings showed that within the institutions there was a gender gap, with female PSTs having lower MCK in the United States, Russia, and Poland. In addition, the female PSTs in the Poland generalist program had significantly higher inquiry beliefs about learning mathematics as compared to their male peers.

Between the institutions, the differences between the institutions showing female PSTs' average MCK and MPCK as significantly higher than the male PSTs could be due to the higher percentages of females in the institutions (e.g., 94.8% in Poland, 92.2% Russia; 88.6% United States). The number of males participating in the study was small and therefore may have been too small to support inferences about the differences. Gonzales and colleagues (2008), for example, found gender differences such that some countries had males performing higher, while other countries had females performing higher. These results show that cross-national gender differences in mathematics achievement (Mullis et al., 1998) are mirrored in teacher preparation programs.

Socio-economic factors

The SES factors used in this study, parental education and number of books in the home, showed significant relationships with PSTs' competencies within the three countries. PSTs from homes with more than 100 books had significantly (i) higher MCK scores in Poland, (ii) higher MPCK scores in Russia and the Poland generalist program, (iii) higher inquiry beliefs about learning among Poland specialist PSTs, when compared to those who had fewer books in the home. The parental level of education was a significant factor related to (i) PSTs' MPCK in the

United States, (ii) PSTs' MCK in the Polish specialist programs, and (iii) PSTs' inquiry beliefs about learning mathematics in the Polish programs. In sum, SES is a factor that was related to the PSTs' knowledge for teaching mathematics in the three countries and their beliefs about learning mathematics in Poland and the United States.

Factors influencing PSTs' mathematics content knowledge

The findings from the study showed that OTL to learn to teach mathematics that include models and activities for reform-oriented pedagogical approaches to learning to teach mathematics were related differently to PSTs' content knowledge across the three countries.

Pedagogical practices within the institutions

For instance, looking at variability within institutions, whole class discussion was significantly related to higher MCK in the United States and the Polish generalist programs. Whole class discussion is a pedagogical practice that has been shown to aid students in developing "mathematical argumentation … and mathematical sophistication" which have been shown to improve students learning of mathematics (Yackel & Cobb, 1996, p.461). Similarly, PSTs get a clearer understanding of mathematical concepts when they engage in discussions (e.g., Bartell et al., 2012; Charalambous et al., 2011).The more the PSTs had opportunities to listen to lecture presentations, the less was their content knowledge in the United States. Thus, this pedagogical practice, if used often, did not expand content knowledge for PSTs in the United States, although in Russia and Poland, it was the most common practice used and was positively related to the PSTs content knowledge though the relationship was not significant. Perhaps listening to lecture presentations denies PSTs the opportunity to develop their mathematical ideas through discussion, but may still be of some benefit in developing PSTs content knowledge.

These findings suggest that in the United States the values of progress[53] were of benefit to PSTs' development of mathematics content knowledge while more control[54] values hindered the PSTs development of mathematics content knowledge.

Planning mathematics instruction within the institutions

Some aspects of learning how to plan mathematics instruction for conceptual understanding is a pedagogical approach that showed significant within-institution relationships with PSTs' content knowledge in some of the countries. In particular, the more PSTs had opportunities to work with standards-based curriculum, the higher their content knowledge in Russia. Previous studies have showed that the use of standards- based curriculum expands PSTs content knowledge (Lloyd, 2006) when they engage in solving high level tasks in which multiple solutions strategies can be used. Further, if their incompetency is foregrounded when exposed to tasks in standards-based curricula (Tarr & Papick, 2004), they can then work to fill the gaps in their knowledge. Among the three countries, PSTs in Russia had the highest frequency of opportunities in which they were introduced to standards-based curriculum. The more the PSTs within a program were introduced to meaningful learning experiences, the higher their content knowledge (relative to others in their program) in Polish generalist programs, but the lower their content knowledge in the Polish specialist programs. The emphasis in the development of PSTs' content knowledge in the two program types are different in that the specialist programs focus more on building mathematics knowledge, while the generalist programs focus more on the pedagogy (e.g., Blömeke & Kaiser, 2014a). Meaningful learning experiences were rarely experienced by PSTs in both program types in Poland. However, the few who had opportunities

[53] Progress is a value fostered in Western mathematics education that focuses on the development of knowledge, generalizing, and questioning in mathematics classrooms (Bishop, 1988; 2001).

[54] Control values are related to the use of values, prediction, and mastery which are predominant in practices that aim at telling rather than reasoning (Bishop, 1988; 2001).

to learn meaningful learning often, showed higher content knowledge, compared to others within their institution. Studies have shown that introducing PSTs to meaningful learning experiences assists them to develop their abstract thinking (e.g., Flowers & Rubenstein, 2006; Steele, 2008).

Between-institution factors

The pedagogical approaches showed different patterns across the three countries. Institutions in which PSTs had more group work activities had higher MCK scores across the three countries with a significant relationship in the United States. Institutions in which there were more whole class discussions had higher MCK in Russia and Poland, but in the United States more whole class discussions were related to a significantly lower MCK. This inconsistent finding in the relationships between practices that involve classroom talk and MCK in the United States institutions calls for further examining how these practices are conducted. Scholars have proposed that whole group discussion should involve introducing socio-mathematical norms which allow for productive discussions (e.g.,Yackel & Cobb, 1996). Others scholars have introduced practices that aid in productive discussions (Smith & Stein, 2011). Productive discussions have been shown to be useful for aiding PSTs to learn from each other and is associated with improvement in their content knowledge (Bartell et al., 2012; Coffey, 2004). Further, opportunities to ask questions often aid in the development of content knowledge, as shown in the positive finding in the Polish generalist programs. Across the institutions in Russia, institutions in which few PSTs reported teaching using methods demonstrated by their instructors had higher average MCK scores. These findings suggest that, in Russia, the institutions value for practices that model control are related to PSTs' development of content knowledge.

Institutions in which PSTs had more opportunities to analyze learning goals and were more often introduced to standards-based curriculum had higher average MCK scores in Russia.

Introduction to standards-based curriculum was positively related to PSTs' MCK in the United States and the Polish generalist programs. Morris and Hiebert (2009) found that PSTs could be supported to identify the sub-concepts of a given task as well as learn to explain and represent the mathematical ideas in relation to the embedded sub-concepts. Such opportunities developed their content knowledge, as shown from the findings in Russia. In the United States and the Polish generalist programs, the relationships between the analysis of learning goals and MCK were negative. These results differ with the previous findings done by Morris and Hiebert (2007). It is possible that these OTL to teach mathematics were related to other dimensions of knowledge for teaching and not the content knowledge between the institutions in the United States and Poland. Also, these differing relationships across the three countries suggest that the approaches used to introduce these OTL how to plan mathematics instruction for conceptual understanding and the values emphasized in the teacher education institutions were related to different teaching competencies across the three countries.

Factors influencing PSTs' mathematics pedagogical content knowledge

. In the three focal countries, the findings showed that reform-oriented pedagogical approaches influence the MPCK of PSTs within the institutions in Russia and Poland, and between the institutions in the three countries.

Pedagogical practices within the institutions

The pedagogical practice asking questions had a positive within-institution influence on PSTs' MPCK in Russia and in the Polish specialist programs. Whole class discussions had a positive influence of PSTs' MCK in Russia and the Polish programs. Previous smaller case study findings indicate that experiencing models of reform-oriented pedagogical practices such as whole group discussion and asking questions had a positive influence on PSTs pedagogical content knowledge (e.g., Coffey, 2004; Lloyd, 2006; Tarr & Papick, 2004). As such, the findings from this study corroborate that asking questions and whole class discussions during mathematics-related courses in pre-service teacher preparation positively influences PSTs' MPCK.

Learning how to plan mathematics instruction for conceptual understanding

Within-institution differences in opportunities that allow for learning how to plan mathematics instruction and mathematics instruction influenced PSTs' MPCK in Russia and the Polish generalist programs. Meaningful learning experiences, which have been shown to help teachers analyze, select, and sequence models, applets, and manipulatives for teaching (Hjalmarson & Suh, 2008), and develop strategies for teaching (Flowers & Rubenstein, 2006), showed positive significant relationships with PSTs' MPCK in Russia and the Polish generalist programs.

Learning mathematics instruction for conceptual understanding

Opportunities to learn mathematics instruction by learning why procedures work and learning to make distinctions between procedural and conceptual knowledge had a positive within-institution influence on the MPCK of the PSTs in the Polish generalist programs. The OTL mathematics instruction for conceptual understanding in which PSTs learned to show why procedures work had a positive within-institution influence on PSTs' MPCK in all the three countries. Although learning to show why procedures work was not experienced often by some PSTs, it is a promising practice that can build on PSTs' MPCK. Such instruction enables PSTs to expand their repertoire of solution strategies and gain conceptual knowledge that is needed for reform-oriented teaching, and is also needed for teachers to help them understand possible students' responses as expected in such classrooms. Studies have shown that approaches to teaching that provide opportunities for providing explanations develop PSTs' competencies in providing mathematics explanations (e.g., Charalambous et al., 2011), aid them in learning to focus on student thinking, and strengthen their ability to analyze critically different representations used by students (Ryken, 2009). In sum, the OTL mathematics instruction that include learning to show why procedures work had a positive within-institution influence on PSTs' MPCK.

Between-institution factors

Some reform-oriented pedagogical approaches in the Polish generalist programs had positive between-institution influence on the PSTs' mathematics pedagogical content knowledge, but had a negative influence on the average MPCK in Russia across the institutions. In particular, the opportunities to ask questions in the mathematics-related courses had a positive influence on PSTs' MPCK in the Polish generalist programs, but had a negative influence on the Russian PSTs' MPCK across the institutions. Having opportunities to ask questions in the mathematics-related courses was a practice that was experienced occasionally yet the influence in these two countries varied. The findings suggest that the institutions in Russia and the Polish generalist programs have varying institutional norms for the practices used in their teacher preparation programs. Pedagogical practices with an emphasis on lecture presentations had a negative influence on PSTs' average MPCK across the institutions in the United States, while in Russia and the Polish institutions these relationships were positively related to MPCK though not significant.

Analysis of teaching and learning of mathematics is a pedagogical practice that has been documented to serve as a supportive context that can assist PSTs to learn to reflect on teaching practices and improve their pedagogical content knowledge (e.g., Henningsen, 2008; Schifter &

Bastable, 2008; Taylor & O'Donnel, 2009). In this study the analysis of teaching and learning had a positive between-institution influence on PSTs' MPCK in Russia. The patterns of relationships in the other countries suggest that analysis of teaching and learning mathematics had a positive influence on PSTs' pedagogical content knowledge.

In the United States, the more the programs emphasized learning the analysis of learning goals and learning about meaningful learning experience, the more the PSTs gained in their mathematics pedagogical content knowledge across the institutions. Similar to previous findings, emphasizing the analysis of learning goals helped PSTs to be able to identify sub-concepts of the learning goals and use the identified sub-concepts to evaluate students' learning and identify student errors (Hiebert et al., 2007; Morris & Hiebert, 2009). Additionally, meaningful learning experiences have been found to help PSTs develop how to analyze, select, and sequence models, manipulatives, and tasks so that students get meaningful experiences during their learning (Hjalmarson & Suh, 2008). As such, these documented competencies that are related to OTL how to plan mathematics instruction for conceptual understanding are part of the specialized knowledge for teaching mathematics and are part of mathematics pedagogical content knowledge needed for teaching mathematics.

Across the three countries, the OTL mathematics instruction for conceptual understanding by emphasizing multiple solution strategies had a positive influence on PSTs knowledge across the institutions. These findings are parallel to those found by Charalambous and colleagues (2011), Ryken (2009), Bartell and colleagues (2012), and Crespo (2000), in which PSTs developed important dimensions of pedagogical content knowledge such as recognizing conceptual understanding from students' responses and analyzing different representations and strategies used by students. In the Polish programs learning mathematics for conceptual understanding by having opportunities to distinguish conceptual and procedural knowledge had a negative influence on the MPCK across the institutions.

The significant findings suggest that the institutions in the United States had values that focus more on process than on control. In Russia the values from the significant findings are mixed, while in the Polish programs, the values lean towards control than process.

Factors influencing PSTs' beliefs about learning mathematics

Within institution factors

Reform-oriented pedagogical approaches had a positive influence on the beliefs that PSTs have about learning mathematics. In particular, within the institutions, models of reform-oriented practices had a positive influence on PSTs' inquiry beliefs in the United States and Russia. The models of non-reform oriented practices had a positive influence on PSTs' inquiry beliefs, although lecture presentations also had a positive influence on PSTs' non-inquiry beliefs in the United States .

The OTL the planning of mathematics instruction by analysis of learning goals, and introduction to standards-based curriculum were found to be significant factors that influenced PSTs inquiry beliefs about learning mathematics in all three countries, although in the Polish programs analysis of learning goals and introductions to standards-based curriculum had a positive influence on non-inquiry beliefs in the Polish generalist and Polish specialist programs, respectively. The OTL mathematics instruction for conceptual understanding that included, (i) learning to distinguish conceptual and procedural knowledge, (ii) learning to show why procedures work, (iii) and learning to multiple solution strategies, had a positive influence on the PSTs' inquiry beliefs for learning mathematics.

Between institutions factors

Opportunities to learn to teach mathematics instruction for conceptual understanding had a positive influence on PSTs' beliefs that had about learning mathematics in the United States and Russia. In the United States, the institutions in which PSTs experienced group work, learned to plan mathematics instruction for conceptual understanding by the analysis of learning goals and meaningful learning experiences, and learned mathematics instruction for conceptual understanding that included distinguishing conceptual and procedural knowledge and learning to show why procedures work, had a positive influence on PSTs' inquiry beliefs about learning mathematics. In Russia, institutions in which the PSTs experienced models of group work, had lecture presentations, and had opportunities to learn about meaningful learning experienced, had a positive influence on PSTs' inquiry beliefs. In Polish generalist programs, teaching using methods demonstrated by the instructors, OTL to learn the planning of mathematics instruction and mathematics instruction, had a negative influence on the inquiry belief about learning, but opportunities to ask questions in their mathematics-related courses had a positive influence on their inquiry beliefs. These findings suggest that in the Unites States and Russia, process values have a greater effect than control values on PSTs inquiry beliefs about learning mathematics. In contrast, in Poland the values that influence inquiry beliefs are more of control than progress.. Similar to the results shown in the two of the three countries, Tang and Hsieh (2014) found that the PSTs from East European countries had positive beliefs about teacher instruction and using explanations in teaching students, while PSTs from the American region (the United States and Chile) had negative views about teacher instruction and explanations to students. However, for Russia, the significant results are not consistent with findings of Tang and Hsieh (2014).

The findings about the factors that influence PSTs' beliefs are similar to findings from previous smaller case studies done in teacher education. PSTs who experienced models of reform-oriented pedagogical practices in their teacher education appreciated that learning mathematics was different from what they were used to (e.g., Lloyd, 2006; Tarr & Papick, 2004). However, non-reform oriented practices can reaffirm the beliefs about teaching and learning that PSTs bring to their teacher preparation (Eisenhart et al., 1993). Other studies showed that PSTs shifted to thinking more about problem solving strategies, and appreciated the investigations and tasks in the standards-based curricula, when they were introduced to learning how to plan mathematics instruction for conceptual understanding (e.g., Morris & Hiebert, 2009; Lloyd, 2006). Finally, PSTs appreciated the influence of the context used in tasks that offered multiple approaches and a focus on student thinking(e.g., Carpenter, 1986; Ryken, 2009). In sum, if PSTs have opportunities in which they engage in the process of teaching as well as in observing modeling of desired practices, they experience the richness that these opportunities offer. Such powerful opportunities can challenge their pre-existing beliefs about students learn mathematics for conceptual understanding.

A summary of the key findings in this study were:

- The opportunities to ask questions during PSTs' mathematics-related courses and engage in whole class discussions had a positive within-institution influence on pedagogical content knowledge of PSTs in the two East European countries and on the content knowledge of the PSTs in the generalist programs in the United States and Poland.
- Having opportunities to experience models of reform-oriented practices had a positive within-institution influence on PSTs' inquiry beliefs about learning mathematics in all three countries.

- Opportunities in which PSTs learned to teach mathematics by teaching using methods demonstrated by the instructor had a positive within-institution influence on PSTs' inquiry beliefs about learning in the two participating Eastern European countries.
- Opportunities to learn how to plan mathematics instruction for conceptual understanding that include the analysis of learning goals and the introductions to standards-based curriculum had a positive within-institution influence on inquiry beliefs about learning mathematics in the generalist programs in all three countries.
- Opportunities to learn mathematics instruction for conceptual understanding that include learning to show why mathematics procedures work had a positive within-institution influence on PSTs' inquiry beliefs about learning mathematics in the generalist programs of the three countries, while PSTs who learned mathematics instruction that included learning to explore multiple solution strategies with pupils had a positive within-institution influence on inquiry beliefs about learning for the generalist programs in the participating East European countries.
- Opportunities to learn to teach mathematics that promote reform-oriented teaching practices had positive between-institution influence on some, but not all, dimensions of PSTs' knowledge and beliefs in the generalist programs in the three countries.
- Lecture presentations had a negative within and between-institution influence on PSTs ' knowledge for teaching mathematics in the United States.

The patterns of the significant relationships within and between institutions are summarized on Figures 12 -19.

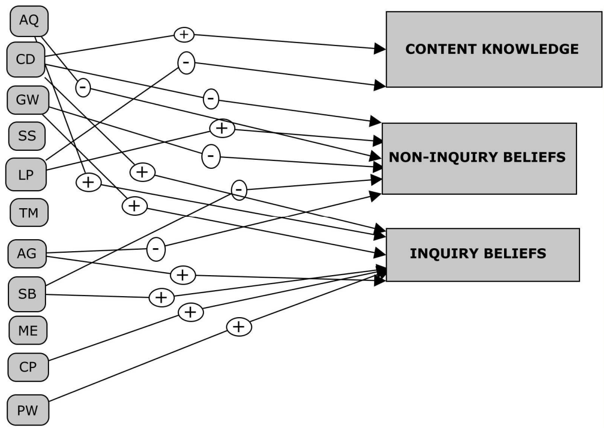


Figure 12. A conceptual framework of significant within-institution relationships in the United States. AQ= asking questions; CD=whole class discussion; LP=lecture presentation; AQ= asking questions; GW=group work; AG= analysis of learning goals; SB= introductions to standards-based curriculum; CP= make distinctions between procedural and conceptual knowledge; PW= show how procedures work.

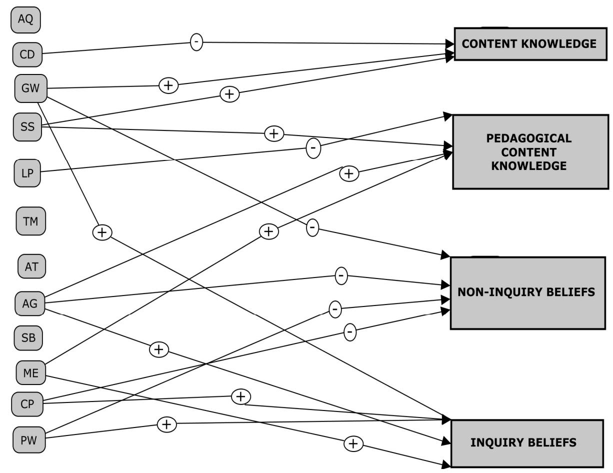


Figure 13. A conceptual framework of significant between-institution relationships in the United States. CD=whole class discussion; GW=group work; AQ= asking questions; SS=learn to explore multiple solution strategies; LP=lecture presentation; AG= analysis of learning goals; ME = meaningful learning experiences; CP= make distinctions between procedural and conceptual knowledge; PW= show how procedures work.

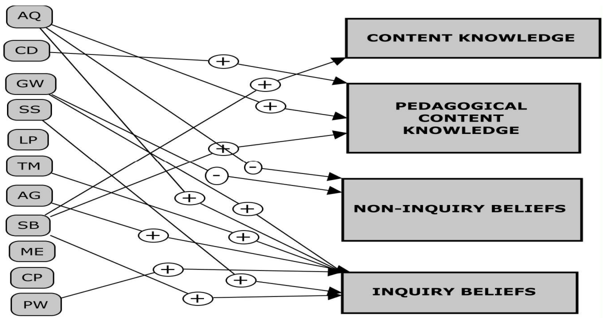


Figure 14. A conceptual framework of the significant within-institution relationships in the Russian Federation. CD=whole class discussion; TM=teaching using methods demonstrated by instructor; AQ= asking questions; GW=group work; AG= analysis of learning goals; SB=introductions to standards-based curriculum; PW= show how procedures work; SS=learn to explore multiple solution strategies.

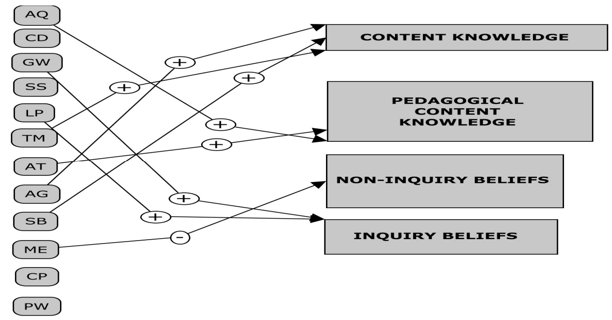


Figure 16. A conceptual framework of the significant between-institution relationships in the Russian Federation. AG= analysis of learning goals; SB=introductions to standards-based curriculum; TM=teaching using methods demonstrated by instructor; AT= analysis of teaching and learning; AQ= asking questions; ME = meaningful learning experiences; LP=lecture presentation; GW=group work.

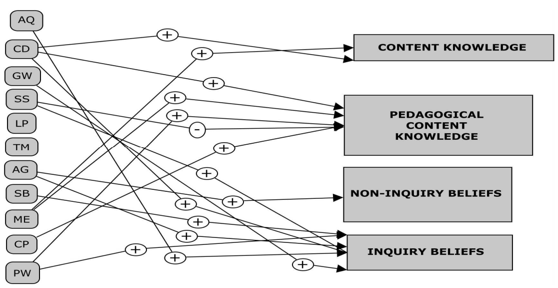


Figure 15. A conceptual framework of the significant within-institution relationships in Poland (generalist programs). CP= make distinctions between procedural and conceptual knowledge; ME = meaningful learning experiences; PW= show how procedures work; SS=explore multiple solution strategies; SB=introductions to standards-based curriculum; AG= analysis of learning goals; CD=whole class discussion; AQ= asking questions; GW=group work.

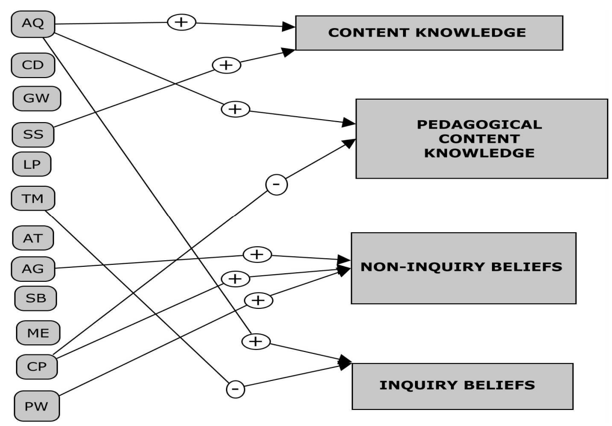


Figure 17. A conceptual framework of the significant between-institution relationships in Poland (generalist programs). SS=explore multiple solution strategies; CP= make distinctions between procedural and conceptual knowledge; ME = meaningful learning experiences; AQ= asking questions; PW= show how procedures work; AG= analysis of learning goals; TM=teaching using methods demonstrated by instructor.

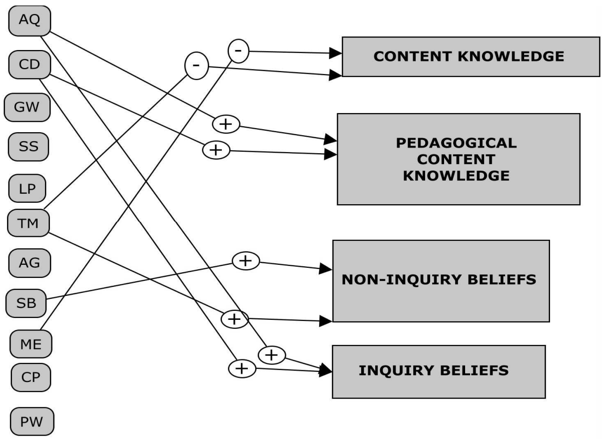


Figure 18. A conceptual framework of the significant within-institution relationships in the Polish specialist programs. ME = meaningful learning experiences; AQ= asking questions; SB= introductions to standards-based curriculum; TM= teaching using methods demonstrated by instructor; CD; whole class discussion.

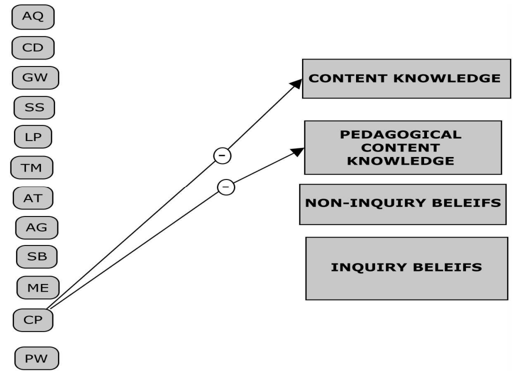


Figure 19. A conceptual framework of the significant within-institution relationships in the Polish specialist programs. CP= make distinctions between procedural and conceptual knowledge.

7.1.3. Reflections on the Interpretations

The analysis described involved some limitations, including (i) when the data was collected; (ii) the analysis was limited to pre-service teacher reports; (iii) some countries did not reach the percentage participation rate expected by the IEA; (iv) lack of variables on prior MCK and MPCK and beliefs about learning; (v) missing values in some country data, (vi) SES indicators used.

First, the data was collected in 2008, and therefore the opportunities to learn and the knowledge and beliefs about teaching and learning mathematics amongst the PSTs in their last year of their teacher preparation program may now be different. Teacher preparation programs are always working to improve, and therefore it is important to note this ever-changing situation when considering the present situation and when making recommendations for the future. However, the changes in programs need not affect the associations between pedagogical features of programs and the teacher preparation outcomes, which are at the center of this study.

Second, the analysis was limited to the PSTs' self-report. The data used in this study was only from the PSTs' perspective, which may not accurately represent characteristics of the programs. Results must be interpreted as reflecting degrees of association with PST perceptions, rather than association of the program features enacted by the teacher educators.

Third, in the United States, the results are only for the public institutions. However, because most teachers in the US are prepared in public institutions, the results represent the experiences of the majority of US elementary school teachers.

Fourth, the survey did not include the PSTs' prior knowledge and beliefs about teaching and learning elementary school mathematics. The TEDS-M survey was the first large-scale cross-sectional study exploring the knowledge and beliefs of PSTs towards the end of the teacher preparation. The study, therefore, did not control for prior knowledge and beliefs so that the influence of the teacher preparation could be shown more clearly. The SES and the gender were some of the factors controlled in this study. The findings of the study like other large-scale studies provide information of the relationships at the time the data was collected.

Fifth, the presence of missing values in some of the variables necessitated using imputation methods, and a list-wise deletion. The list-wise deletion was only used at level 1 because not more than 10% of the data was missing. Also, at level 2 the missing values were replaced with the mean scores in the institutions. This treatment of missing data made it possible to include variables with high missing values at level 2 instead of leaving the measures out entirely. This treatment of missing values represents common analytic practice, so is not a major cause for concern.

Although there are limitations in this study, the significant findings obtained in the study should be given careful consideration because of the careful and rigorous data collection procedures conducted by the national teams, the research coordinators of each of the participating countries, and data processing centers in all the participating countries with the support of the IEA. The rigorous processes as well as the analytic procedures used in this study produced results that can provide important findings.

7.2. Implications for Mathematics Teacher Education and Policy

As previously mentioned, debates in mathematics teacher education have focused on the need to design teacher education programs that challenge the knowledge and beliefs about teaching and learning mathematics that pre-service teachers bring to their teacher preparation

(Borko & Putnam, 1996; CBMS, 2001; Feiman-Nemser, 2001). During pre-service teacher preparation, varied pedagogical approaches have been used in teaching different courses in different disciplines, which then send mixed messages about what good teaching is (Borko & Putnam, 1992). For instance, some of the mathematics courses are taught with a high level of abstraction (Borko & Putnam, 1996) and the courses as a whole are not taught using connected pedagogical approaches (Feiman-Nemser, 2001). Perhaps these approaches to teaching have contributed to some extent to the prevalent non-reform oriented pedagogical practices in our mathematics classrooms today. In sum, it is important that PSTs experience unified pedagogical approaches to teaching when learning how to teach mathematics for conceptual understanding.

Scholars have shown that thoughtfully designed pedagogical approaches lead to promising outcomes (e.g., Bartell et al., 2012; Charalambous et al., 2011; Eisenhart et al., 1993; Hiebert et al., 2007; Schram et al., 1988). The findings from these studies in which varied tools have been used to identify the competencies of pre-service teachers, indicate that PSTs' beliefs are challenged more by doing and experiencing these methods of teaching during their pre-service teacher preparation. As Grossman (1990) emphasized, students learn through the apprenticeship of observation. In other words, what they observe can influence their beliefs about what good teaching is.

This study provided empirical evidence that modeling pedagogical practices that focus on reform-oriented instruction can have an influence on PSTs' knowledge for teaching mathematics in diverse ways across the three countries. That is, providing PSTs with opportunities to ask questions and engage in whole group discussion had a positive within-institution influence on the pedagogical content knowledge in Russia and Poland, and on the PSTS' content knowledge in the United States. Further, programs that provided opportunities for PSTs to experience models of reform-oriented practices such as asking questions in the Polish generalist program and group work in the United States had a positive between-institution influence on the PSTs knowledge for teaching and content knowledge, respectively. Providing opportunities for the analysis of teaching and learning mathematics had a positive influence on PSTs' pedagogical content knowledge in Russia.

The findings suggest that experiencing models of non-reform oriented pedagogical practices can have diverse influences on PSTs' knowledge, as was found across the three countries. In the United States listening to lecture presentations had a negative within-institution influence on PSTs' knowledge for teaching mathematics, and learning to teach by teaching using methods demonstrated by the instructors had a negative within-institution influence on PSTs' content knowledge in the Polish specialist programs. In contrast, OTL to teach mathematics that included teaching using methods demonstrated by the instructor had a positive between-institution influence on PSTs' content knowledge in the Russian institutions.

The study also provided empirical evidence that providing opportunities to experience models of reform-oriented pedagogical practices, OTL how to plan mathematics instruction for conceptual understanding, and OTL mathematics instruction for conceptual understanding can influence PSTs' beliefs about learning mathematics in comparable ways across the three countries. That is, providing opportunities to ask questions, engage in whole class discussion, and work in groups during their mathematics-related courses had a positive within-institution influence on PSTs' inquiry beliefs about learning mathematics in all the three countries. In addition, having opportunities to learn how to plan mathematics instruction for conceptual understanding that included the analysis of learning goals and introductions to standards-based curriculum had a positive between-institution influence on PSTs inquiry beliefs about learning mathematics across the three generalist programs in all three countries. OTL mathematics instruction for conceptual understanding that include learning to show why procedures work had a positive within-institution influence on PSTs' inquiry beliefs about learning mathematics in the generalist programs in all three countries, while learning to explore multiple solutions strategies with pupils had a positive within-institution influence on PSTs' inquiry beliefs about learning mathematics for the generalist programs in Poland and Russia.

Pedagogical approaches used in teacher education can influence particular PSTs' beliefs in specific countries. In the United States experiences that model reform oriented practices that included opportunities to ask questions, engage in whole class discussion, and group work activities had a negative within-institution influence on PSTs' non-inquiry beliefs about learning mathematics. Also, the OTL the analysis of learning goals and introductions to standards-based curriculum had a negative influence on PSTs' inquiry beliefs about learning mathematics. In the United States, group work, analysis of learning goals, and OTL mathematics instruction for conceptual understanding that include learning to distinguish conceptual and procedural knowledge and multiple solution strategies, all had a negative between-institution influence on PSTs' non-inquiry beliefs. In contrast, engaging in group work, learning to plan mathematics instruction for conceptual understanding that included analysis of learning goals, meaningful learning experiences, and OTL mathematics instruction for conceptual understanding that included distinguishing between conceptual and procedural knowledge, and learning to show why procedures work, all had a positive between-institution influence in PSTs' inquiry beliefs.

In addition, experiencing models of non-reform oriented pedagogical practices that include listening to lecture presentations had a negative influence on PSTs knowledge for teaching mathematics in the United States. In the Polish generalist programs listening to lecture presentation, the analysis of learning goals, and introductions to standards-based curriculum, learning to distinguish between conceptual and procedural knowledge, and learning to show why procedures work, all had a positive between-institution influence on PSTs' non-inquiry beliefs. In Russia, group work and listening to lecture presentations had a positive between-institution influence on PSTs' inquiry beliefs.

These findings in which some relationships vary and others are comparable across the three countries show that there are OTL to teach mathematics that hold in a large-scale context and that have positive influences on PSTs' knowledge and beliefs about teaching and learning mathematics. As such, teacher education programs in these three countries can evaluate how their programs influence their PSTs competencies needed for teaching mathematics and also increase the frequency on the OTL to teach mathematics that show positive influences within their countries.

Policies in teacher preparation should encourage more activities in which PSTs are actively participating in their learning to teach mathematics and learning by experiencing modeling of best practices during teacher preparation. Further, the curriculum should include discussions for planning mathematics lessons in which the PSTs are engaged in analyzing the learning goals, analyze curriculum and use standards-based curriculum extensively, and have in-depth discussions about what meaningful learning involves. Finally, the curriculum should include ways that introduce PSTs to mathematics instruction for conceptual understanding through discussions and classroom activities. In sum, the policies should emphasize the need for PSTs to take a more active role in their learning to teach mathematics during mathematics teacher preparation.

7.3. Implications for Further Research

This study used the Teacher Education and Development Study in Mathematics to examine the relationships between opportunities to learn to teach mathematics and knowledge and beliefs for teaching and learning mathematics among pre-service teachers in the final stage of their teacher preparation. Longitudinal studies that can inform the influencing factors during teacher preparation can provide more information than using a cross-sectional study such as that used in the TEDS-M study. This is because a longitudinal study will provide more information about the prior knowledge and beliefs that PSTs have when they join their teacher preparation and the competencies they have developed during different stages and at the end of their teacher preparation. Also, studies on the influence of practical experiences are important for informing policy for teacher preparation to add on to the influencing factors in mathematics teacher preparation.

This study also serves as a guide for further in-depth qualitative studies in teacher preparation. For example, in Chinese Taipei, the results showed that listening to lecture presentations was the pedagogical practice most experienced in teacher preparation, yet this country has consistently performed very well in international assessments and was among the highest in knowledge for teaching mathematics in the TEDS-M study. A study of their teacher preparation programs might provide further insights into ways that lecture presentations in teacher preparation can be conducted with rewarding outcomes. More qualitative studies are needed that use similar methodologies and data analysis procedures that outline the successful pedagogical approaches that have been used to prepare pre-service teachers to teach mathematics. Finally, longitudinal studies are needed that monitor pre-service teachers' transition to their in-service teaching, to examine if their beliefs and knowledge about teaching and learning mathematics are transferred to their future teaching and the challenges of enacting the practices learned during their teacher preparation.

APPENDICES

Appendix A: Sample Items of Institutional Questions

[Source: User guide TEDS-M Supplement 1 (Brese & Tatto, 2012, p. 12; p, 19)].

5.

How many years and months does it take for a typical <future teacher> to complete the academic or subject-matter preparation of this consecutive program?

MIA005A ____years and

MIA005B ____months

2.

Which of the following characteristics or sources of information are used in selecting entering <future teachers> for this teacher preparation program? Please indicate below how important each of the characteristics or sources is in the selection process.

Check one box in each row.

			Not considered	Not very important	Somewhat important	Very important
MIC002A	A.	The candidates' overall level of attainment in their final year of <secondary> schooling, as measured by school marks or grades	☐1	☐2	☐3	☐4
MIC002B	B.	The candidates' performance at the end of their final year of <secondary> schooling, as measured by their performance on a national or state examination	☐1	☐2	☐3	☐4

Appendix B: Sample Items from Teacher Educators Questionnaire on Opportunity to Learn

[Source: User Guide TEDS-M Supplement 1(Brese and Tatto, 2012, p.54; p.57)]

2.

In the <course> you selected, how often do you give your <future teachers> the <u>opportunity</u> to do the following?

Check *one* box in each *row*.

			Never	Rarely	Occasionally	Often
MEG002A	A.	Analyze and use national or state standards or frameworks for school mathematics	☐1	☐2	☐3	☐4
MEG002B	B.	Build on pupils' existing mathematics knowledge and thinking skills	☐1	☐2	☐3	☐4
MEG002C	C.	Explore how to apply mathematics to real-world problems	☐1	☐2	☐3	☐4

1.

In the <course> you selected above, to what extent are your <future teachers> expected to do each of the following?

Check *one* box in each *row*.

			Never	Rarely	Occasionally	Often
MEI001A	A.	Listen to a lecture	☐1	☐2	☐3	☐4
MEI001B	B.	Ask questions during class time	☐1	☐2	☐3	☐4

Appendix C: Sample Items on Pre-Service Teachers' Beliefs about Learning

[Source: User Guide TEDS-M Supplement 1 (Brese and Tatto, 2012, p. 85)].

2.

From your perspective, to what extent would you agree or disagree with each of the following statements about <u>learning mathematics</u>?

Check one box in each row.

			Strongly disagree	Disagree	Slightly disagree	Slightly agree	Agree	Strongly agree
MFD002A	A.	The best way to do well in mathematics is to memorize all the formulas.	☐1	☐2	☐3	☐4	☐5	☐6
MFD002B	B.	Pupils need to be taught exact procedures for solving mathematical problems.	☐1	☐2	☐3	☐4	☐5	☐6
MFD002G	G.	In addition to getting a right answer in mathematics, it is important to understand why the answer is correct.	☐1	☐2	☐3	☐4	☐5	☐6
MFD002H	H.	Teachers should allow pupils to figure out their own ways to solve mathematical problems.	☐1	☐2	☐3	☐4	☐5	☐6

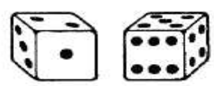
Appendix D: Sample Items used to Test PSTs Content Knowledge and Pedagogical Content Knowledge

Content Knowledge

[Source: User Guide TEDS-M Supplement 4 (Brese and Tatto, 2012, p. 6; p.11)

ID: MFC106	MS Booklet: PM1, PM5	MS Block: B1PM	Item Format: MC	Max Points: 1
Knowledge Dimension: MCK	Content Domain: Data		Sub-domain: Applying	

MFC106 Two fair six-sided number cubes are thrown in a probability game and the two numbers at the top are recorded.



[Josie] wins if the difference between the two numbers is 0, 1 or 2.
[Farid] wins if the difference between the two numbers is 3, 4 or 5.

The students discuss whether the game is fair.

Which of the following statements is correct?

Check one box.

A. Both have an equal chance of winning. ☐1

B. [Josie] has the greater chance of winning. ☐2

C. [Farid] has the greater chance of winning. ☐3

D. As the game involves number cubes, it's not possible to say who has the greater chance of winning. ☐4

Figure 20. Sample item

ID: MFC206A	MS Booklet: PM1, PM2	MS Block: B2PM	Item Format: MC	Max Points: 1
Knowledge Dimension: MCK	Content Domain: Number		Sub-domain: Applying	

MFC206A (a) A machine uses 2.4 litres of fuel for every 30 hours of operation.
How many litres of fuel will the machine use in 100 hours if it continues to use fuel at the same rate?

Check one box.

A. 7.2 ☐[1]

B. 8.0 ☐[2]

C. 8.4 ☐[3]

D. 9.6 ☐[4]

ID: MFC206B	MS Booklet: PM1, PM2	MS Block: B2PM	Item Format: CR	Max Points: 1
Knowledge Dimension: MPCK	Content Domain: Number		Sub-domain: Planning	

MFC206B

(b) Create a different problem of the same type as the problem in (a) (same processes/operations) that is **EASIER** for <primary> children to solve.

Appendix E: Distributions of the Outcome Variables in the Three Countries

Figure 21. Variation of PSTs' MCK scores within the three countries.

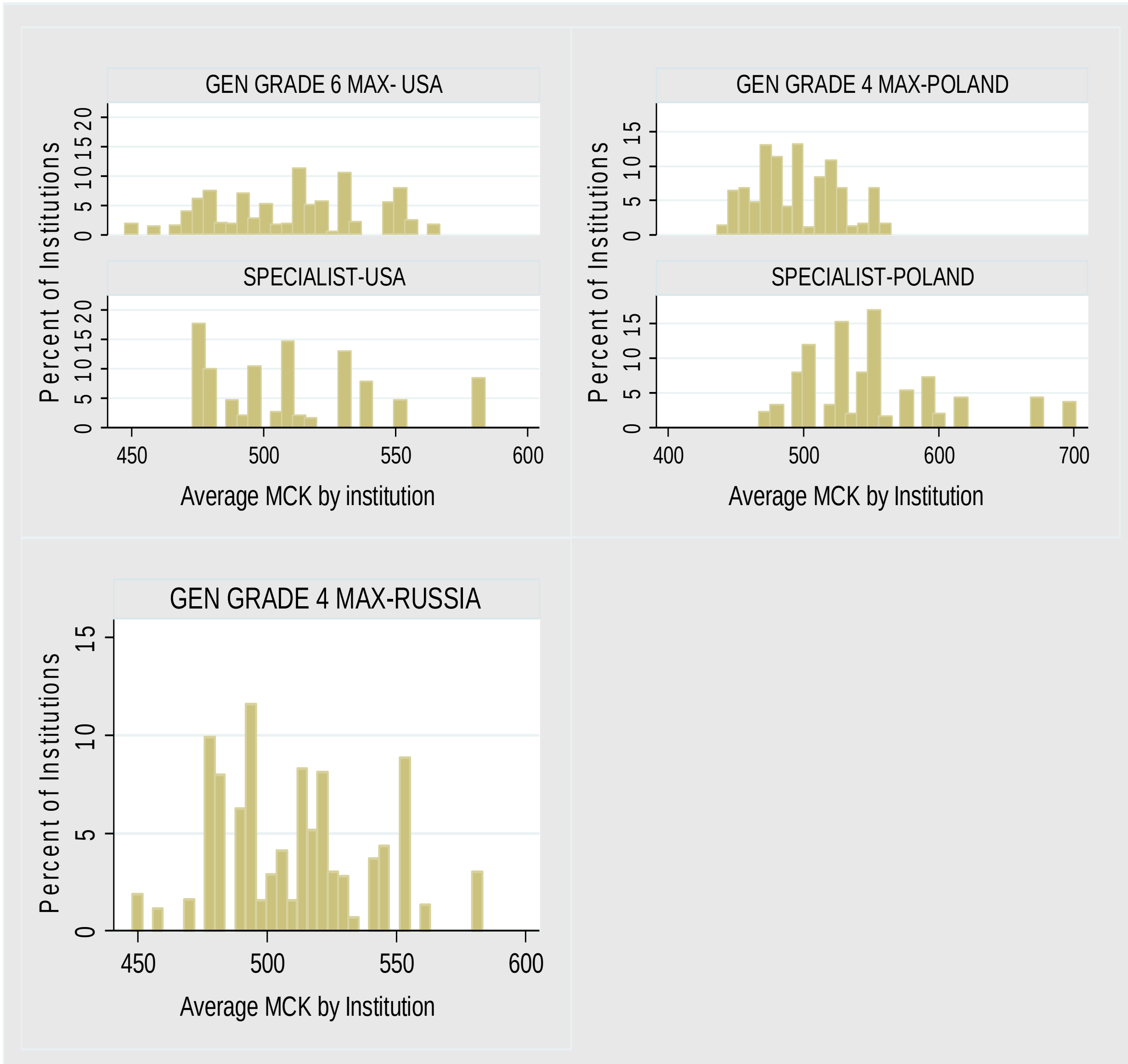


Figure 22. PSTs' average MCK scores by institution within the three countries.

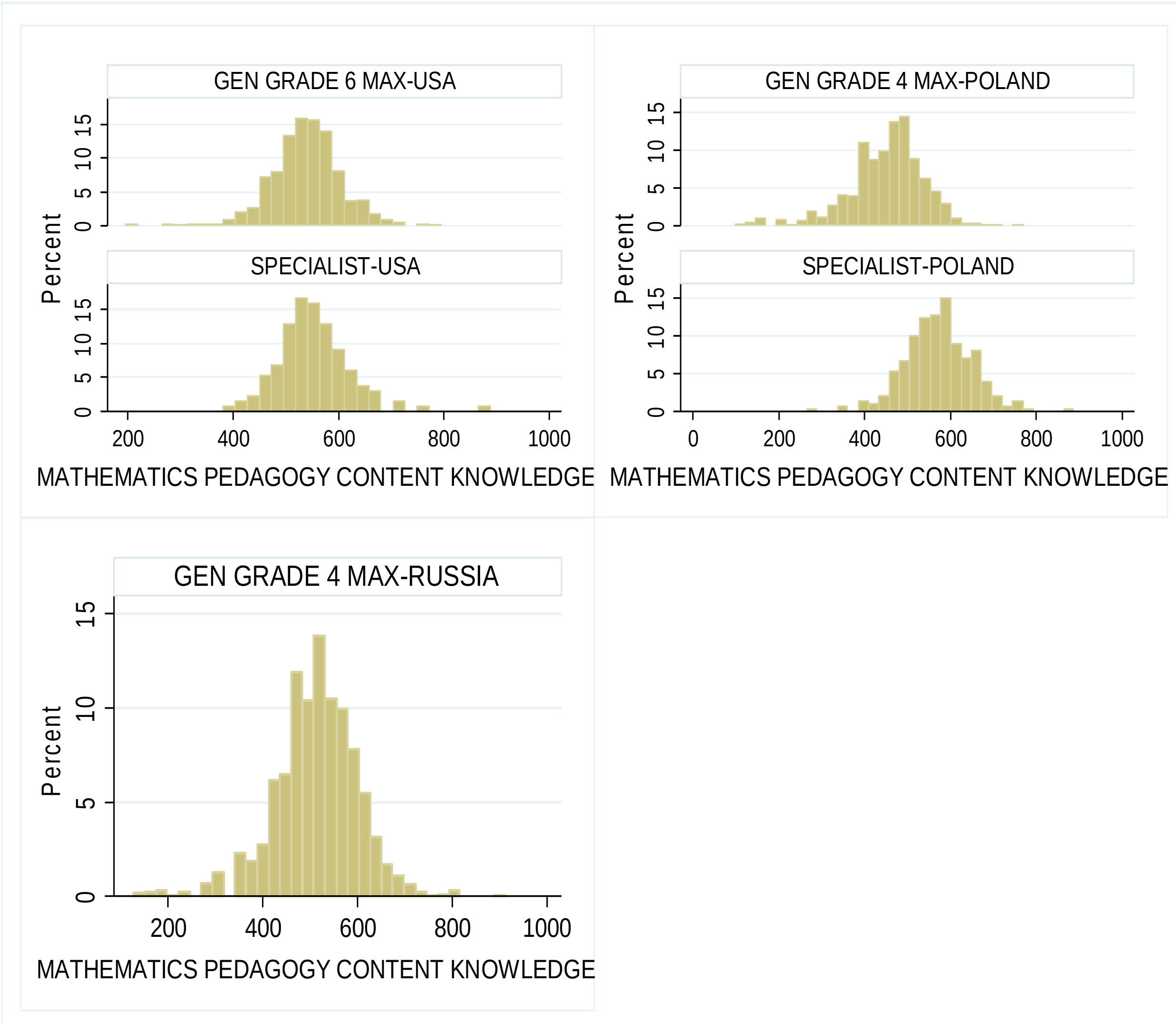


Figure 23. PSTs' MPCK scores within the three countries.

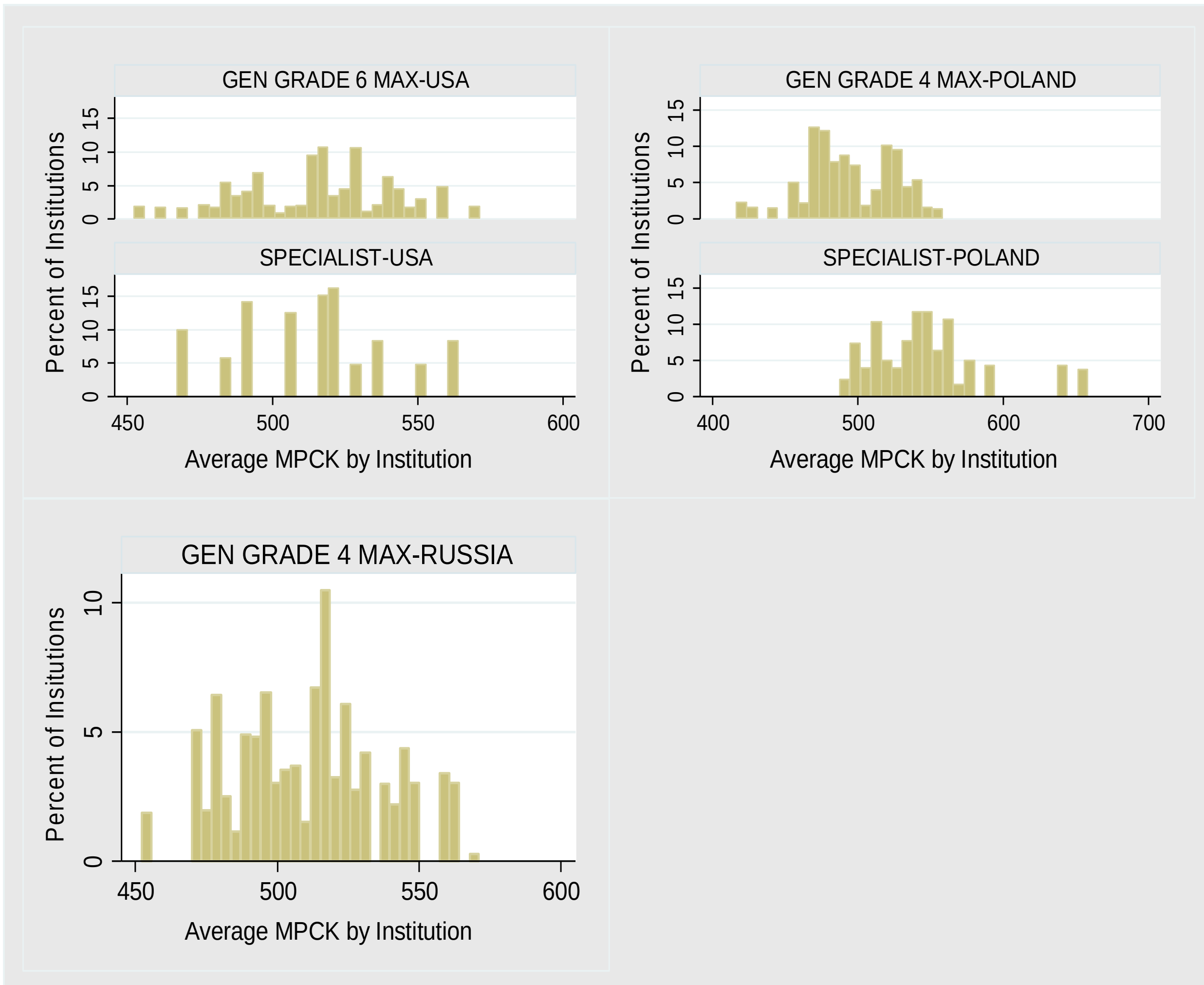


Figure 24. PSTs' average MPCK by institution within the three countries.

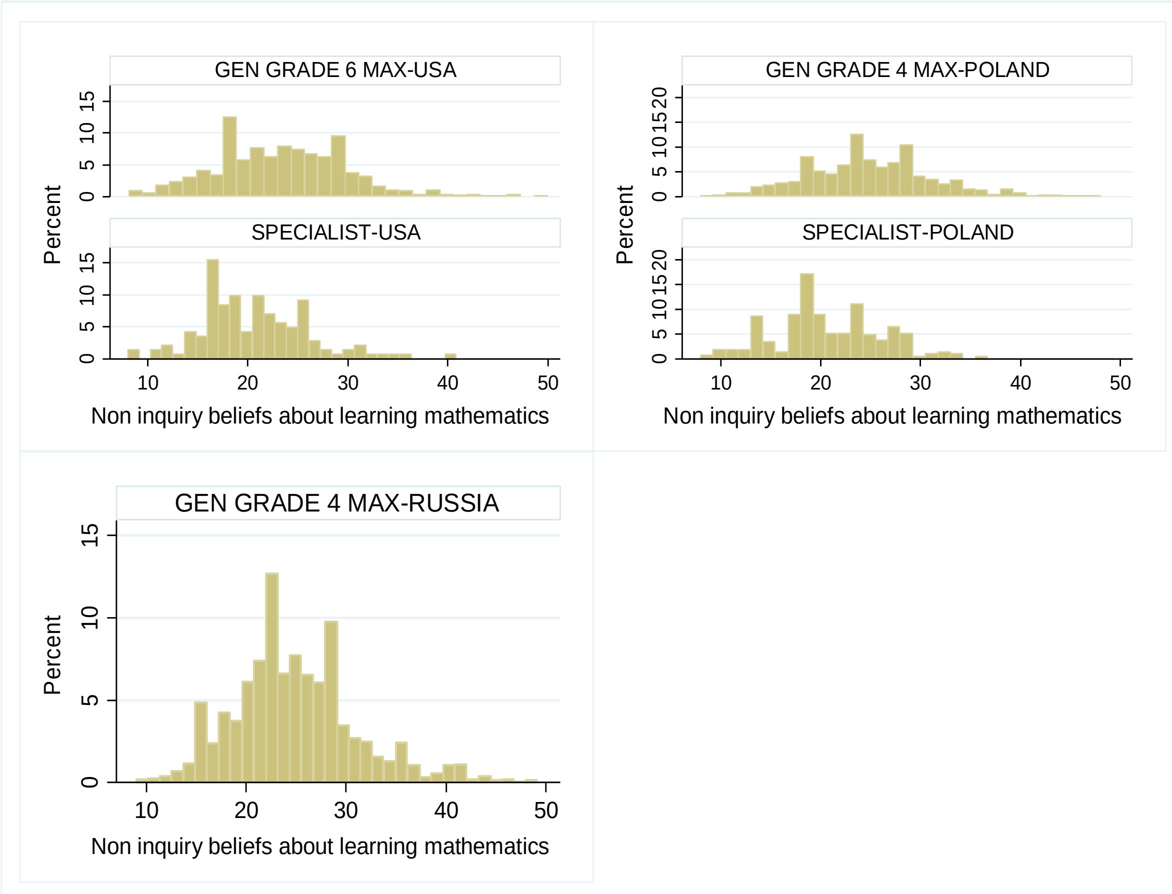


Figure 25. PSTs' non-inquiry beliefs about learning mathematics within the three countries.

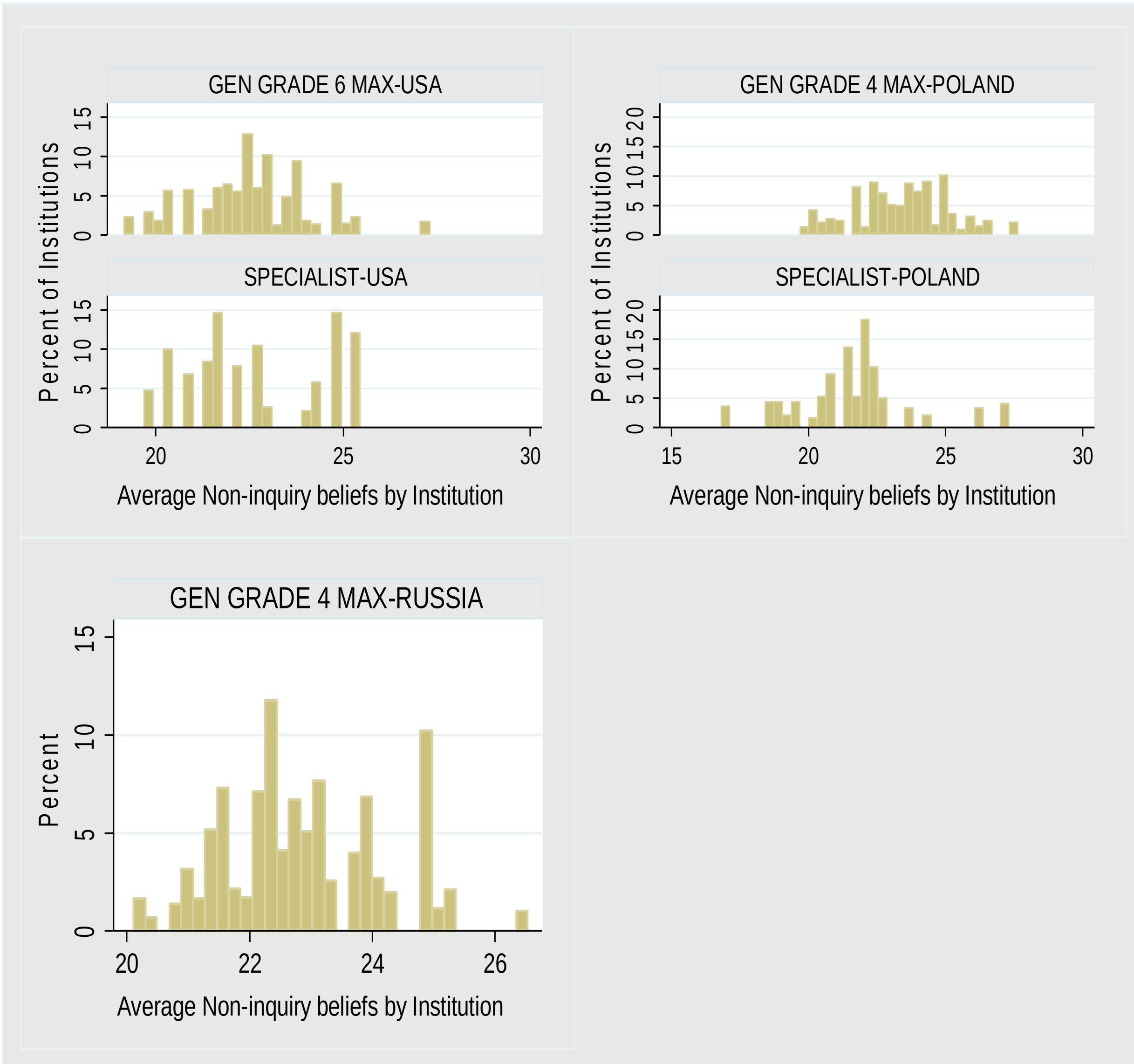


Figure 26. PSTs' non-inquiry beliefs by institution within the three countries.

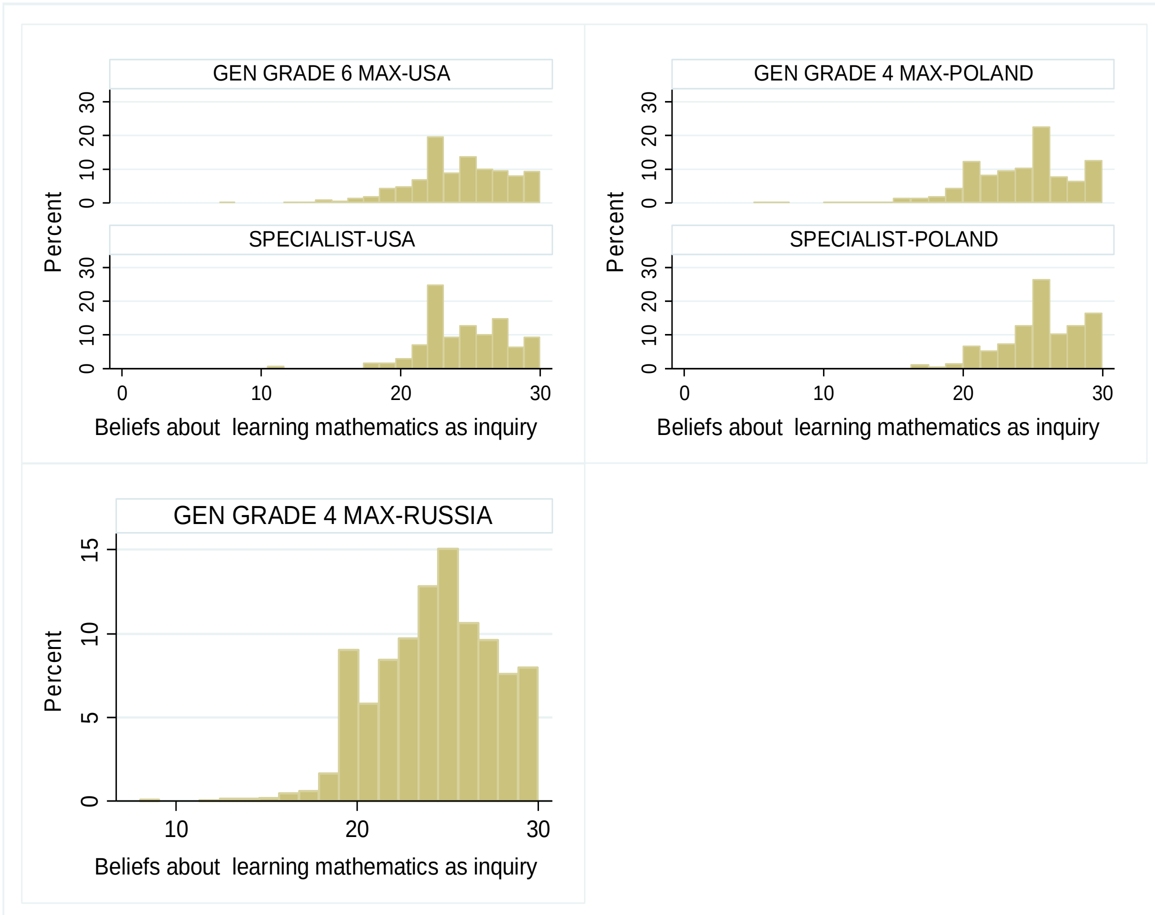


Figure 27. PSTs' inquiry beliefs about learning mathematics scores within the three countries.

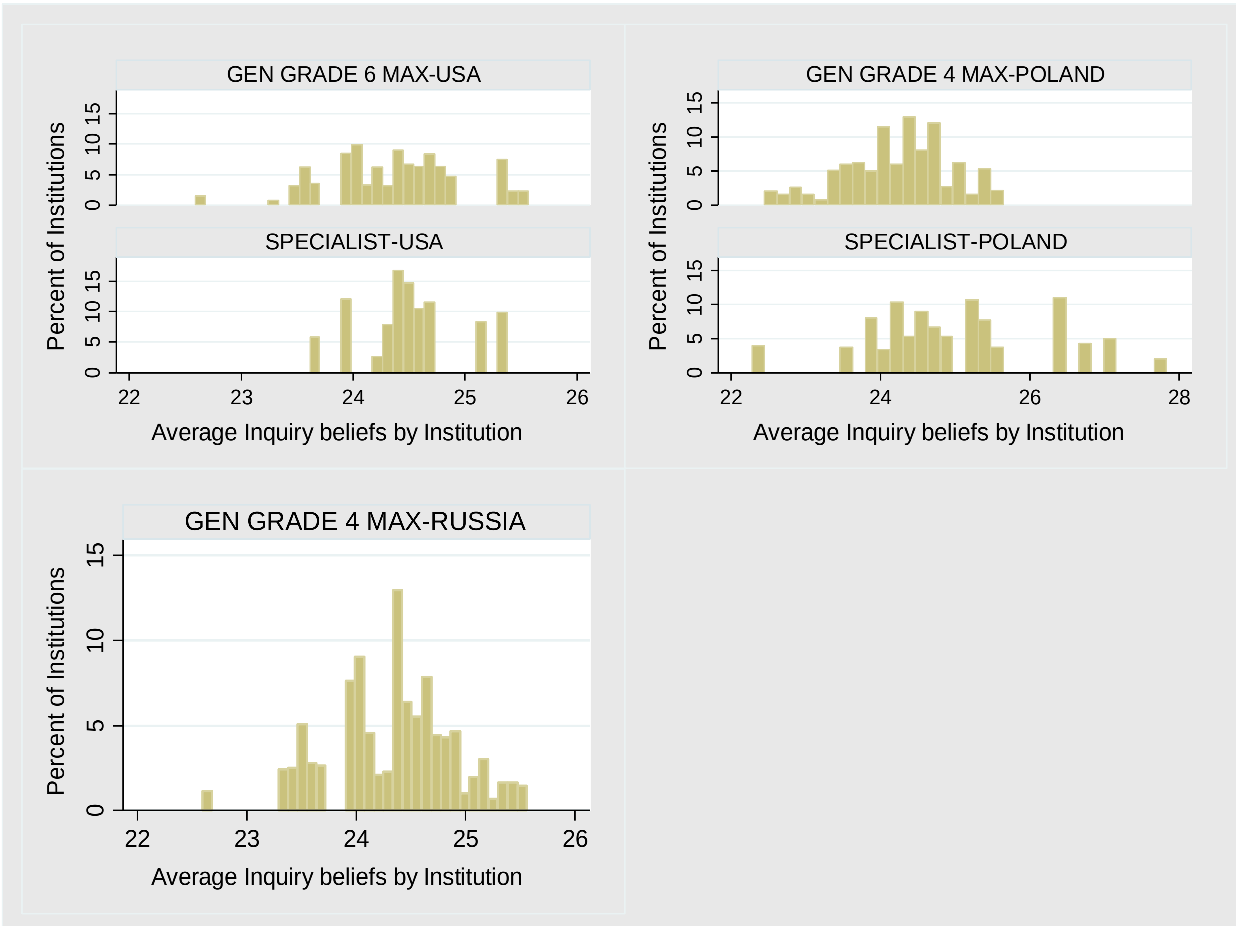


Figure 28. PSTs' inquiry beliefs between the institutions within the three countries.

Appendix F: Correlational Analysis of the Relationships between the Variables in the Study

Table 44: *United States (generalists) -Correlational analysis of variables used in the models*

Correlations

		female teachers	MORE_100_BKS	years of schooling	GEN\DO IN MATH COURSE\LISTEN	GEN\DO IN MATH COURSE\ASK QUESTIONS	GEN\DO IN MATH COURSE\CLASS DISCUSSION	GEN\DO IN MATH COURSE\TEACH METHOD DEMON	GEN\DO IN MATH COURSE\WORK IN GROUPS	analysis of learning goals PSTs coninuous	Introduction to standards-based materials PSTs Cont	meaningfull learning experince PSTs cont	GEN\ACT OPP TO LEARN\MULT SOLUTION STRAT	GEN\ACT OPP TO LEARN\WHY MAT PROC WORKS	GEN\ACT OPP TO LEARN\MAKE DISTINCTIONS
female teachers	Pearson Correlation	1	.038	-.017	-.057*	-.040	-.056	-.031	.005	.051	.040	.046	.031	.015	-.025
	Sig. (2-tailed)		.194	.525	.050	.172	.057	.293	.871	.082	.179	.121	.290	.615	.391
	N	1499	1185	1449	1165	1165	1163	1160	1160	1145	1158	1159	1158	1159	1157
MORE_100_BKS	Pearson Correlation	.038	1	.240**	-.010	.000	-.007	-.025	-.008	.018	.015	.030	.050	.014	-.002
	Sig. (2-tailed)	.194		.000	.737	.999	.823	.389	.776	.554	.612	.302	.092	.634	.947
	N	1185	1185	1159	1164	1164	1162	1159	1159	1144	1157	1158	1157	1158	1156
years of schooling	Pearson Correlation	-.017	.240**	1	-.047	-.008	-.017	-.041	.049	.045	.046	.031	.076*	.000	.047
	Sig. (2-tailed)	.525	.000		.113	.787	.557	.165	.101	.135	.117	.294	.010	.996	.116
	N	1449	1159	1451	1144	1144	1142	1139	1139	1126	1138	1140	1138	1139	1137
GEN\DO IN MATH COURSE\LISTEN	Pearson Correlation	-.057*	-.010	-.047	1	-.014	.006	.039	-.013	.012	.035	.010	-.006	.002	.018
	Sig. (2-tailed)	.050	.737	.113		.622	.828	.180	.657	.675	.233	.746	.830	.936	.537
	N	1165	1164	1144	1167	1166	1164	1161	1161	1145	1158	1159	1158	1159	1157
GEN\DO IN MATH COURSE\ASK QUESTIONS	Pearson Correlation	-.040	.000	-.008	-.014	1	.634**	.196**	.243**	.207**	.188**	.264**	.178**	.199**	.220**
	Sig. (2-tailed)	.172	.999	.787	.622		.000	.000	.000	.000	.000	.000	.000	.000	.000
	N	1165	1164	1144	1166	1167	1164	1161	1161	1145	1159	1159	1158	1159	1157
GEN\DO IN MATH COURSE\CLASS DISCUSSION	Pearson Correlation	-.056	-.007	-.017	.006	.634**	1	.226**	.335**	.241**	.218**	.303**	.206**	.184**	.241**
	Sig. (2-tailed)	.057	.823	.557	.828	.000		.000	.000	.000	.000	.000	.000	.000	.000
	N	1163	1162	1142	1164	1164	1165	1160	1159	1143	1156	1157	1156	1157	1155
GEN\DO IN MATH COURSE\TEACH METHOD DEMON	Pearson Correlation	-.031	-.025	-.041	.039	.196**	.226**	1	.154**	.221**	.237**	.264**	.230**	.195**	.232**
	Sig. (2-tailed)	.293	.389	.165	.180	.000	.000		.000	.000	.000	.000	.000	.000	.000
	N	1160	1159	1139	1161	1161	1160	1162	1156	1140	1153	1154	1153	1154	1152
GEN\DO IN MATH COURSE\WORK IN GROUPS	Pearson Correlation	.005	-.008	.049	-.013	.243**	.335**	.154**	1	.228**	.273**	.327**	.244**	.233**	.239**
	Sig. (2-tailed)	.871	.776	.101	.657	.000	.000	.000		.000	.000	.000	.000	.000	.000
	N	1160	1159	1139	1161	1161	1159	1156	1162	1141	1153	1154	1153	1154	1152
analysis of learning goals PSTs coninuous	Pearson Correlation	.051	.018	.045	.012	.207**	.241**	.221**	.228**	1	.628**	.662**	.558**	.518**	.507**
	Sig. (2-tailed)	.082	.554	.135	.675	.000	.000	.000	.000		.000	.000	.000	.000	.000
	N	1145	1144	1126	1145	1145	1143	1140	1141	1147	1145	1146	1145	1146	1145
Introduction to standards-based materials PSTs Cont	Pearson Correlation	.040	.015	.046	.035	.188**	.218**	.237**	.273**	.628**	1	.714**	.523**	.520**	.525**
	Sig. (2-tailed)	.179	.612	.117	.233	.000	.000	.000	.000	.000		.000	.000	.000	.000
	N	1158	1157	1138	1158	1159	1156	1153	1153	1145	1160	1159	1158	1159	1157
meaningfull learning experince PSTs cont	Pearson Correlation	.046	.030	.031	.010	.264**	.303**	.264**	.327**	.662**	.714**	1	.555**	.530**	.472**
	Sig. (2-tailed)	.121	.302	.294	.746	.000	.000	.000	.000	.000	.000		.000	.000	.000
	N	1159	1158	1140	1159	1159	1157	1154	1154	1146	1159	1161	1158	1159	1157
GEN\ACT OPP TO LEARN\MULT SOLUTION STRAT	Pearson Correlation	.031	.050	.076*	-.006	.178**	.206**	.230**	.244**	.558**	.523**	.555**	1	.590**	.461**
	Sig. (2-tailed)	.290	.092	.010	.830	.000	.000	.000	.000	.000	.000	.000		.000	.000
	N	1158	1157	1138	1158	1158	1156	1153	1153	1145	1158	1158	1160	1159	1157
GEN\ACT OPP TO LEARN\WHY MAT PROC WORKS	Pearson Correlation	.015	.014	.000	.002	.199**	.184**	.195**	.233**	.518**	.520**	.530**	.590**	1	.539**
	Sig. (2-tailed)	.615	.634	.996	.936	.000	.000	.000	.000	.000	.000	.000	.000		.000
	N	1159	1158	1139	1159	1159	1157	1154	1154	1146	1159	1159	1159	1161	1158
GEN\ACT OPP TO LEARN\MAKE DISTINCTIONS	Pearson Correlation	-.025	-.002	.047	.018	.220**	.241**	.232**	.239**	.507**	.525**	.472**	.461**	.539**	1
	Sig. (2-tailed)	.391	.947	.116	.537	.000	.000	.000	.000	.000	.000	.000	.000	.000	
	N	1157	1156	1137	1157	1157	1155	1152	1152	1145	1157	1157	1157	1158	1159

*. Correlation is significant at the 0.05 level (2-tailed).

**. Correlation is significant at the 0.01 level (2-tailed).

Table 45: *Russia- Correlation analysis of variables used in the models*

Correlations

		MORE_100_BKS	years of schooling	female teachers	Analysis of teaching and learning and mathematics	GEN\DO IN MATH COURSE\LISTEN	GEN\DO IN MATH COURSE\ASK QUESTIONS	GEN\DO IN MATH COURSE\CLASS DISCUSSION	GEN\DO IN MATH COURSE\TEACH OWN METHOD	GEN\DO IN MATH COURSE\WORK IN GROUPS	analysis of learning goals PSTs coninuous	Introduction to standards-based materials PSTs Cont	meaningfull learning experince PSTs cont	GEN\ACT OPP TO LEARN\MULT SOLUTION STRAT	GEN\ACT OPP TO LEARN\WHY MAT PROC WORKS	GEN\ACT OPP TO LEARN\MAKE DISTINCTIONS
MORE_100_BKS	Pearson Correlation	1	.253**	-.008	.132**	.044*	.114**	.129**	.104**	.057**	.118**	.085**	.100**	.071**	.053*	.025
	Sig. (2-tailed)		.000	.716	.000	.036	.000	.000	.000	.007	.000	.000	.000	.001	.013	.245
	N	2246	2091	2244	2169	2234	2227	2202	2214	2226	2157	2191	2210	2223	2207	2188
years of schooling	Pearson Correlation	.253**	1	-.043*	.036	.030	.038	.045*	.075**	.031	-.014	-.016	.002	-.001	-.007	-.046*
	Sig. (2-tailed)	.000		.048	.109	.164	.085	.039	.001	.152	.540	.466	.939	.975	.758	.038
	N	2091	2105	2101	2038	2095	2088	2065	2078	2087	2026	2056	2075	2085	2071	2051
female teachers	Pearson Correlation	-.008	-.043*	1	.082**	.076**	.072**	.038	.048*	.067**	.109**	.172**	.105**	.080**	.072**	.059**
	Sig. (2-tailed)	.716	.048		.000	.000	.001	.072	.024	.002	.000	.000	.000	.000	.001	.006
	N	2244	2101	2260	2182	2248	2240	2216	2228	2240	2171	2205	2224	2237	2221	2202
Analysis of teaching and learning and mathematics	Pearson Correlation	.132**	.036	.082**	1	.136**	.332**	.378**	.369**	.365**	.444**	.467**	.465**	.360**	.356**	.401**
	Sig. (2-tailed)	.000	.109	.000		.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000
	N	2169	2038	2182	2188	2186	2179	2155	2169	2179	2116	2148	2168	2176	2162	2151
GEN\DO IN MATH COURSE\LISTEN	Pearson Correlation	.044*	.030	.076**	.136**	1	.191**	.136**	.032	.135**	.084**	.137**	.131**	.037	.035	.042*
	Sig. (2-tailed)	.036	.164	.000	.000		.000	.000	.128	.000	.000	.000	.000	.079	.098	.050
	N	2234	2095	2248	2186	2254	2241	2220	2230	2241	2174	2206	2227	2238	2221	2204
GEN\DO IN MATH COURSE\ASK QUESTIONS	Pearson Correlation	.114**	.038	.072**	.332**	.191**	1	.501**	.262**	.228**	.270**	.286**	.285**	.194**	.181**	.180**
	Sig. (2-tailed)	.000	.085	.001	.000	.000		.000	.000	.000	.000	.000	.000	.000	.000	.000
	N	2227	2088	2240	2179	2241	2245	2210	2224	2233	2165	2197	2219	2229	2212	2196
GEN\DO IN MATH COURSE\CLASS DISCUSSION	Pearson Correlation	.129**	.045*	.038	.378**	.136**	.501**	1	.326**	.305**	.307**	.290**	.314**	.252**	.209**	.232**
	Sig. (2-tailed)	.000	.039	.072	.000	.000	.000		.000	.000	.000	.000	.000	.000	.000	.000
	N	2202	2065	2216	2155	2220	2210	2222	2202	2211	2144	2175	2196	2206	2191	2175
GEN\DO IN MATH COURSE\TEACH OWN METHOD	Pearson Correlation	.104**	.075**	.048*	.369**	.032	.262**	.326**	1	.253**	.303**	.281**	.300**	.282**	.281**	.263**
	Sig. (2-tailed)	.000	.001	.024	.000	.128	.000	.000		.000	.000	.000	.000	.000	.000	.000
	N	2214	2078	2228	2169	2230	2224	2202	2234	2223	2156	2188	2210	2219	2202	2186
GEN\DO IN MATH COURSE\WORK IN GROUPS	Pearson Correlation	.057**	.031	.067**	.365**	.135**	.228**	.305**	.253**	1	.270**	.284**	.255**	.194**	.178**	.151**
	Sig. (2-tailed)	.007	.152	.002	.000	.000	.000	.000	.000		.000	.000	.000	.000	.000	.000
	N	2226	2087	2240	2179	2241	2233	2211	2223	2246	2165	2198	2219	2229	2216	2197
analysis of learning goals PSTs coninuous	Pearson Correlation	.118**	-.014	.109**	.444**	.084**	.270**	.307**	.303**	.270**	1	.632**	.648**	.556**	.466**	.471**
	Sig. (2-tailed)	.000	.540	.000	.000	.000	.000	.000	.000	.000		.000	.000	.000	.000	.000
	N	2157	2026	2171	2116	2174	2165	2144	2156	2165	2177	2150	2163	2174	2162	2148
Introduction to standards-based materials PSTs Cont	Pearson Correlation	.085**	-.016	.172**	.467**	.137**	.286**	.290**	.281**	.284**	.632**	1	.683**	.495**	.482**	.484**
	Sig. (2-tailed)	.000	.466	.000	.000	.000	.000	.000	.000	.000	.000		.000	.000	.000	.000
	N	2191	2056	2205	2148	2206	2197	2175	2188	2198	2150	2210	2197	2208	2194	2178
meaningfull learning experince PSTs cont	Pearson Correlation	.100**	.002	.105**	.465**	.131**	.285**	.314**	.300**	.255**	.648**	.683**	1	.516**	.454**	.478**
	Sig. (2-tailed)	.000	.939	.000	.000	.000	.000	.000	.000	.000	.000	.000		.000	.000	.000
	N	2210	2075	2224	2168	2227	2219	2196	2210	2219	2163	2197	2230	2224	2208	2192
GEN\ACT OPP TO LEARN\MULT SOLUTION STRAT	Pearson Correlation	.071**	-.001	.080**	.360**	.037	.194**	.252**	.282**	.194**	.556**	.495**	.516**	1	.530**	.475**
	Sig. (2-tailed)	.001	.975	.000	.000	.079	.000	.000	.000	.000	.000	.000	.000		.000	.000
	N	2223	2085	2237	2176	2238	2229	2206	2219	2229	2174	2208	2224	2243	2221	2205
GEN\ACT OPP TO LEARN\WHY MAT PROC WORKS	Pearson Correlation	.053*	-.007	.072**	.356**	.035	.181**	.209**	.281**	.178**	.466**	.482**	.454**	.530**	1	.555**
	Sig. (2-tailed)	.013	.758	.001	.000	.098	.000	.000	.000	.000	.000	.000	.000	.000		.000
	N	2207	2071	2221	2162	2221	2212	2191	2202	2216	2162	2194	2208	2221	2226	2192
GEN\ACT OPP TO LEARN\MAKE DISTINCTIONS	Pearson Correlation	.025	-.046*	.059**	.401**	.042*	.180**	.232**	.263**	.151**	.471**	.484**	.478**	.475**	.555**	1
	Sig. (2-tailed)	.245	.038	.006	.000	.050	.000	.000	.000	.000	.000	.000	.000	.000	.000	
	N	2188	2051	2202	2151	2204	2196	2175	2186	2197	2148	2178	2192	2205	2192	2208

**. Correlation is significant at the 0.01 level (2-tailed).

*. Correlation is significant at the 0.05 level (2-tailed).

Table 46: *Poland (generalists)-Correlation analysis of the variables used in the models*

Correlations

		female teachers	years of schooling	MORE_100_BKS	GEN\DO IN MATH COURSE\LISTEN	GEN\DO IN MATH COURSE\ASK QUESTIONS	GEN\DO IN MATH COURSE\CLASS DISCUSSION	GEN\DO IN MATH COURSE\TEACH METHOD DEMON	GEN\DO IN MATH COURSE\WORK IN GROUPS	Analysis of teaching and learning and mathematics	analysis of learning goals PSTs coninuous	Introduction to standards-based materials PSTs Cont	meaningfull learning experince PSTs cont	GEN\ACT OPP TO LEARN\MULT SOLUTION STRAT	GEN\ACT OPP TO LEARN\WHY MAT PROC WORKS	GEN\ACT OPP TO LEARN\MAKE DISTINCTIONS
female teachers	Pearson Correlation	1	-.052	-.030	.084**	-.041	-.033	-.025	.044	-.042	-.034	-.013	-.033	.009	-.033	-.023
	Sig. (2-tailed)		.065	.245	.001	.109	.199	.341	.088	.110	.198	.612	.207	.724	.207	.387
	N	1496	1266	1493	1490	1490	1486	1475	1483	1461	1416	1464	1466	1475	1474	1468
years of schooling	Pearson Correlation	-.052	1	.188**	-.066*	.033	.013	.023	-.020	.016	.000	-.012	-.023	.014	-.028	-.019
	Sig. (2-tailed)	.065		.000	.019	.242	.650	.414	.484	.564	.996	.685	.416	.616	.315	.497
	N	1266	1266	1266	1260	1260	1258	1246	1257	1236	1192	1239	1241	1249	1248	1243
MORE_100_BKS	Pearson Correlation	-.030	.188**	1	-.017	.096**	.104**	.051	-.020	.041	.010	.060*	.063*	.028	.066*	-.014
	Sig. (2-tailed)	.245	.000		.510	.000	.000	.050	.449	.116	.700	.023	.016	.282	.011	.603
	N	1493	1266	1494	1488	1488	1485	1473	1481	1459	1416	1464	1466	1475	1474	1468
GEN\DO IN MATH COURSE\LISTEN	Pearson Correlation	.084**	-.066*	-.017	1	.081**	.088**	.042	.097**	.088**	.071**	.063*	.051	.026	.062*	.063*
	Sig. (2-tailed)	.001	.019	.510		.002	.001	.105	.000	.001	.007	.016	.053	.310	.017	.015
	N	1490	1260	1488	1491	1486	1481	1470	1478	1456	1411	1459	1461	1470	1469	1463
GEN\DO IN MATH COURSE\ASK QUESTIONS	Pearson Correlation	-.041	.033	.096**	.081**	1	.664**	.259**	.148**	.172**	.212**	.201**	.263**	.176**	.191**	.136**
	Sig. (2-tailed)	.109	.242	.000	.002		.000	.000	.000	.000	.000	.000	.000	.000	.000	.000
	N	1490	1260	1488	1486	1491	1482	1472	1479	1458	1412	1461	1463	1471	1469	1464
GEN\DO IN MATH COURSE\CLASS DISCUSSION	Pearson Correlation	-.033	.013	.104**	.088**	.664**	1	.290**	.224**	.257**	.240**	.224**	.290**	.201**	.182**	.143**
	Sig. (2-tailed)	.199	.650	.000	.001	.000		.000	.000	.000	.000	.000	.000	.000	.000	.000
	N	1486	1258	1485	1481	1482	1487	1468	1475	1454	1411	1459	1461	1469	1467	1462
GEN\DO IN MATH COURSE\TEACH METHOD DEMON	Pearson Correlation	-.025	.023	.051	.042	.259**	.290**	1	.270**	.309**	.309**	.306**	.308**	.220**	.188**	.193**
	Sig. (2-tailed)	.341	.414	.050	.105	.000	.000		.000	.000	.000	.000	.000	.000	.000	.000
	N	1475	1246	1473	1470	1472	1468	1476	1468	1448	1407	1454	1455	1461	1458	1454
GEN\DO IN MATH COURSE\WORK IN GROUPS	Pearson Correlation	.044	-.020	-.020	.097**	.148**	.224**	.270**	1	.289**	.216**	.224**	.213**	.182**	.147**	.167**
	Sig. (2-tailed)	.088	.484	.449	.000	.000	.000	.000		.000	.000	.000	.000	.000	.000	.000
	N	1483	1257	1481	1478	1479	1475	1468	1484	1456	1409	1458	1460	1469	1466	1461
Analysis of teaching and learning and mathematics	Pearson Correlation	-.042	.016	.041	.088**	.172**	.257**	.309**	.289**	1	.398**	.358**	.387**	.252**	.266**	.254**
	Sig. (2-tailed)	.110	.564	.116	.001	.000	.000	.000	.000		.000	.000	.000	.000	.000	.000
	N	1461	1236	1459	1456	1458	1454	1448	1456	1462	1389	1438	1440	1447	1445	1439
analysis of learning goals PSTs coninuous	Pearson Correlation	-.034	.000	.010	.071**	.212**	.240**	.309**	.216**	.398**	1	.558**	.564**	.515**	.471**	.475**
	Sig. (2-tailed)	.198	.996	.700	.007	.000	.000	.000	.000	.000		.000	.000	.000	.000	.000
	N	1416	1192	1416	1411	1412	1411	1407	1409	1389	1417	1405	1405	1412	1409	1408
Introduction to standards-based materials PSTs Cont	Pearson Correlation	-.013	-.012	.060*	.063*	.201**	.224**	.306**	.224**	.358**	.558**	1	.690**	.517**	.519**	.536**
	Sig. (2-tailed)	.612	.685	.023	.016	.000	.000	.000	.000	.000	.000		.000	.000	.000	.000
	N	1464	1239	1464	1459	1461	1459	1454	1458	1438	1405	1465	1457	1462	1459	1457
meaningfull learning experince PSTs cont	Pearson Correlation	-.033	-.023	.063*	.051	.263**	.290**	.308**	.213**	.387**	.564**	.690**	1	.555**	.527**	.493**
	Sig. (2-tailed)	.207	.416	.016	.053	.000	.000	.000	.000	.000	.000	.000		.000	.000	.000
	N	1466	1241	1466	1461	1463	1461	1455	1460	1440	1405	1457	1467	1463	1460	1456
GEN\ACT OPP TO LEARN\MULT SOLUTION STRAT	Pearson Correlation	.009	.014	.028	.026	.176**	.201**	.220**	.182**	.252**	.515**	.517**	.555**	1	.582**	.482**
	Sig. (2-tailed)	.724	.616	.282	.310	.000	.000	.000	.000	.000	.000	.000	.000		.000	.000
	N	1475	1249	1475	1470	1471	1469	1461	1469	1447	1412	1462	1463	1476	1471	1465
GEN\ACT OPP TO LEARN\WHY MAT PROC WORKS	Pearson Correlation	-.033	-.028	.066*	.062*	.191**	.182**	.188**	.147**	.266**	.471**	.519**	.527**	.582**	1	.577**
	Sig. (2-tailed)	.207	.315	.011	.017	.000	.000	.000	.000	.000	.000	.000	.000	.000		.000
	N	1474	1248	1474	1469	1469	1467	1458	1466	1445	1409	1459	1460	1471	1475	1463
GEN\ACT OPP TO LEARN\MAKE DISTINCTIONS	Pearson Correlation	-.023	-.019	-.014	.063*	.136**	.143**	.193**	.167**	.254**	.475**	.536**	.493**	.482**	.577**	1
	Sig. (2-tailed)	.387	.497	.603	.015	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	
	N	1468	1243	1468	1463	1464	1462	1454	1461	1439	1408	1457	1456	1465	1463	1469

**. Correlation is significant at the 0.01 level (2-tailed).

*. Correlation is significant at the 0.05 level (2-tailed).

Table 47: *Poland (specialists)-Correlation analysis of variables used in the analysis*

Correlations

		female teachers	years of schooling	MORE_100_BKS	GEN\DO IN MATH COURSE\LISTEN	GEN\DO IN MATH COURSE\ASK QUESTIONS	GEN\DO IN MATH COURSE\CLASS DISCUSSION	GEN\DO IN MATH COURSE\TEACH METHOD DEMON	GEN\DO IN MATH COURSE\WORK IN GROUPS	Analysis of teaching and learning and mathematics	analysis of learning goals PSTs coninuous	Introduction to standards-based materials PSTs Cont	meaningfull learning experince PSTs cont	GEN\ACT OPP TO LEARN\MULT SOLUTION STRAT	GEN\ACT OPP TO LEARN\WHY MAT PROC WORKS	GEN\ACT OPP TO LEARN\MAKE DISTINCTIONS
female teachers	Pearson Correlation	1	-.049	.035	.119*	.011	.110	.159**	.160**	.023	.032	.096	.074	.066	.008	.029
	Sig. (2-tailed)		.417	.564	.046	.852	.066	.008	.008	.705	.600	.112	.219	.272	.898	.630
	N	280	280	280	280	280	280	279	278	280	268	277	279	277	277	276
years of schooling	Pearson Correlation	-.049	1	.032	-.062	.035	.004	-.150*	.077	.014	.013	.055	.032	.025	.025	.007
	Sig. (2-tailed)	.417		.588	.302	.564	.944	.012	.201	.814	.831	.358	.589	.683	.677	.907
	N	280	281	281	281	281	281	280	279	281	268	277	280	277	277	276
MORE_100_BKS	Pearson Correlation	.035	.032	1	-.055	.106	.136*	-.012	.012	-.046	.086	.084	.095	.003	.070	.001
	Sig. (2-tailed)	.564	.588		.355	.077	.023	.848	.839	.440	.161	.165	.114	.956	.243	.983
	N	280	281	281	281	281	281	280	279	281	268	277	280	277	277	276
GEN\DO IN MATH COURSE\LISTEN	Pearson Correlation	.119*	-.062	-.055	1	.116	.089	.074	.022	.087	.090	.160**	.124*	.089	.025	-.003
	Sig. (2-tailed)	.046	.302	.355		.052	.135	.215	.709	.146	.142	.008	.038	.141	.680	.964
	N	280	281	281	281	281	281	280	279	281	268	277	280	277	277	276
GEN\DO IN MATH COURSE\ASK QUESTIONS	Pearson Correlation	.011	.035	.106	.116	1	.686**	.233**	.078	.169**	.221**	.216**	.195**	.151*	.137*	.216**
	Sig. (2-tailed)	.852	.564	.077	.052		.000	.000	.193	.005	.000	.000	.001	.012	.022	.000
	N	280	281	281	281	281	281	280	279	281	268	277	280	277	277	276
GEN\DO IN MATH COURSE\CLASS DISCUSSION	Pearson Correlation	.110	.004	.136*	.089	.686**	1	.253**	.182**	.177**	.282**	.300**	.261**	.207**	.212**	.237**
	Sig. (2-tailed)	.066	.944	.023	.135	.000		.000	.002	.003	.000	.000	.000	.001	.000	.000
	N	280	281	281	281	281	281	280	279	281	268	277	280	277	277	276
GEN\DO IN MATH COURSE\TEACH METHOD DEMON	Pearson Correlation	.159**	-.150*	-.012	.074	.233**	.253**	1	.147*	.151*	.182**	.336**	.326**	.062	.160**	.149*
	Sig. (2-tailed)	.008	.012	.848	.215	.000	.000		.014	.011	.003	.000	.000	.302	.008	.013
	N	279	280	280	280	280	280	280	278	280	267	276	279	276	276	275
GEN\DO IN MATH COURSE\WORK IN GROUPS	Pearson Correlation	.160**	.077	.012	.022	.078	.182**	.147*	1	.031	.323**	.298**	.317**	.228**	.147*	.172**
	Sig. (2-tailed)	.008	.201	.839	.709	.193	.002	.014		.610	.000	.000	.000	.000	.015	.004
	N	278	279	279	279	279	279	278	279	279	266	275	278	275	275	274
Analysis of teaching and learning and mathematics	Pearson Correlation	.023	.014	-.046	.087	.169**	.177**	.151*	.031	1	.198**	.217**	.188**	.095	.120*	.119*
	Sig. (2-tailed)	.705	.814	.440	.146	.005	.003	.011	.610		.001	.000	.002	.116	.045	.048
	N	280	281	281	281	281	281	280	279	281	268	277	280	277	277	276
analysis of learning goals PSTs coninuous	Pearson Correlation	.032	.013	.086	.090	.221**	.282**	.182**	.323**	.198**	1	.660**	.685**	.596**	.588**	.528**
	Sig. (2-tailed)	.600	.831	.161	.142	.000	.000	.003	.000	.001		.000	.000	.000	.000	.000
	N	268	268	268	268	268	268	267	266	268	268	268	268	268	268	267
Introduction to standards-based materials PSTs Cont	Pearson Correlation	.096	.055	.084	.160**	.216**	.300**	.336**	.298**	.217**	.660**	1	.713**	.525**	.547**	.492**
	Sig. (2-tailed)	.112	.358	.165	.008	.000	.000	.000	.000	.000	.000		.000	.000	.000	.000
	N	277	277	277	277	277	277	276	275	277	268	277	277	277	277	276
meaningfull learning experince PSTs cont	Pearson Correlation	.074	.032	.095	.124*	.195**	.261**	.326**	.317**	.188**	.685**	.713**	1	.528**	.486**	.425**
	Sig. (2-tailed)	.219	.589	.114	.038	.001	.000	.000	.000	.002	.000	.000		.000	.000	.000
	N	279	280	280	280	280	280	279	278	280	268	277	280	277	277	276
GEN\ACT OPP TO LEARN\MULT SOLUTION STRAT	Pearson Correlation	.066	.025	.003	.089	.151*	.207**	.062	.228**	.095	.596**	.525**	.528**	1	.606**	.425**
	Sig. (2-tailed)	.272	.683	.956	.141	.012	.001	.302	.000	.116	.000	.000	.000		.000	.000
	N	277	277	277	277	277	277	276	275	277	268	277	277	277	277	276
GEN\ACT OPP TO LEARN\WHY MAT PROC WORKS	Pearson Correlation	.008	.025	.070	.025	.137*	.212**	.160**	.147*	.120*	.588**	.547**	.486**	.606**	1	.496**
	Sig. (2-tailed)	.898	.677	.243	.680	.022	.000	.008	.015	.045	.000	.000	.000	.000		.000
	N	277	277	277	277	277	277	276	275	277	268	277	277	277	277	276
GEN\ACT OPP TO LEARN\MAKE DISTINCTIONS	Pearson Correlation	.029	.007	.001	-.003	.216**	.237**	.149*	.172**	.119*	.528**	.492**	.425**	.425**	.496**	1
	Sig. (2-tailed)	.630	.907	.983	.964	.000	.000	.013	.004	.048	.000	.000	.000	.000	.000	
	N	276	276	276	276	276	276	275	274	276	267	276	276	276	276	276

*. Correlation is significant at the 0.05 level (2-tailed).

**. Correlation is significant at the 0.01 level (2-tailed).

Appendix G: Reliability Analysis

Table 48: *Reliability analysis for creating composite measure analysis of teaching and learning*

	Valid	Percent	Missing	Percent	Total	Percent	Reliability Statistics	using 4 variables.							Kolmogorov-Smirnova			Shapiro-Wilk		
							Cronbach's	Cronbach's	Mean	Median	Std. Error	standard d(	Skewness	Kurtosis	Statistic	df	Sig.	Statistic	df	Sig.
all countries																				
Botswana	81	94.20%	5	5.80%	86	100.00%	0.79	0.787	10.1481	10	0.31726	2.85531	-0.52434	-0.65784	0.111	81	0.015	0.975	81	0.119
Chile	634	96.50%	23	3.50%	657	100.00%	0.772	0.775	9.1293	9	0.11994	3.02005	2.886598	-2.72165	0.096	178	0	0.968	178	0
Chinese Tai	915	99.10%	8	0.90%	923	100.00%	0.813	0.821	8.5716	9	0.0946	2.86151	2.950617	-2.41358	0.102	634	0	0.97	634	0
Georgia	408	80.60%	98	19.40%	506	100.00%	0.745	0.755	9.1348	9	0.15639	3.15901	1.371901	-3.49793	0.085	915	0	0.966	915	0
Germany	1003	97.20%	29	2.80%	1032	100.00%	0.76	0.775	6.8604	6	0.08399	2.66013	9.116883	-2.01299	0.098	408	0	0.966	408	0
Malaysia	569	98.80%	7	1.20%	576	100.00%	0.825	0.829	10.8858	11	0.10574	2.52229	-1.18627	-0.64706	0.151	1003	0	0.899	1003	0
Philippines	577	97.50%	15	2.50%	592	100.00%	0.794	0.798	11.026	11	0.09984	2.39814	-1.83333	-1.30049	0.095	569	0	0.976	569	0
Poland	2065	97.80%	47	2.20%	2112	100.00%	0.703	0.721	7.3429	7	0.05683	2.58267	11.14815	-2	0.113	577	0	0.978	577	0
Russian Fec	2188	96.60%	78	3.40%	2266	100.00%	0.784	0.782	10.9127	11	0.05971	2.79283	-3.76923	-4.90476	0.127	2065	0	0.938	2065	0
Singapore	378	99.50%	2	0.50%	380	100.00%	0.804	0.805	9.2063	9	0.13795	2.68205	1.096	-0.636	0.088	2188	0	0.977	2188	0
Spain																				
Switzerlanc	922	98.50%	14	1.50%	936	100.00%	0.741	0.752	8.3883	8	0.08813	2.67595	3.395062	-3.52795	0.107	378	0	0.975	378	0
Thailand	654	99.10%	6	0.90%	660	100.00%	0.764	0.784	10.1575	10	0.09281	2.37353	0.729167	-1.24084	0.103	922	0	0.968	922	0
United Stat	681	45.40%	820	54.60%	1501	100.00%	0.811	0.813	10.6182	11	0.1181	3.08191	-1.25532	-3.21925	0.119	654	0	0.977	654	0
Norway (Al	386	98.50%	6	1.50%	392	100.00%	0.778	0.784	7.6036	8	0.12218	2.40041	2.145161	-1.44758	0.072	681	0	0.972	681	0
Norway (Al	154	96.90%	5	3.10%	159	100.00%	0.818	0.818	8.7662	9	0.21953	2.72436	1.230769	-0.27506	0.104	386	0	0.955	386	0
				Note that Spain PSTs did not answer MFB005I(read research in mathematics education)											0.091	154	0.003	0.97	154	0.002

Note that in the United States the missing and 9 is from PSTs not responding to MFB005I(read research on mathematics education).

Table 49: *Reliability analysis for creating composite measure introduction to standards-based curriculum*

preservice teachers introduction to standards based curriculum									
	alpha	standardized alp	valid percent	Mean	Median	SE	SD	Skewness	Kurtosis
all countries	.660	.663	92.0	8.8567	9	0.01933	2.15669	22.9545	7
Botswana	.368	.377	87.2	8.6133	9	0.18782	1.62658	3.61	0.01
Chile	.576	.578	96.5	9.4211	10	0.07789	1.96114	6.38	0.9227
Chinese Taipei	.608	.614	99.7	9.025	9	0.06122	1.8568	6.593	0.633
Georgia	.684	.685	72.5	7.8747	8	0.13757	2.63553	1.63	6.98
Germany	.714	.720	97.7	8.373	9	0.07777	2.46922	5.5	3.968
Malaysia	.675	.676	96.2	9.0523	9	0.07638	1.79787	5.779	1.89
Philipines	.583	.586	96.8	9.4869	10	0.06444	1.54261	4.716	0.392
poland	.640	.643	98	8.4219	9	0.04802	2.18429	6.592	4.09
Russian	.620	.623	97.5	9.9	10	0.04069	1.91263	19.46	5.932
Singapore	.579	.581	99.5	9.164	9	0.08697	1.69085	3.4	0.128
Spain	.634	.633	97.4	7.3756	7	0.06433	2.09945	1.387	3.64
Switzerland	.530	.531	97.8	8.1869	8	0.06183	1.97026	1.543	2.16
Thailand	.631	.638	97.9	8.5728	9	0.07678	1.95138	3.26	1.536
United States	.630	.630	77.3	9.9112	10	0.05246	1.7867	12.51	5.444
Norway(ALU)	.532	.531	98.5	7.2617	7	0.09305	1.82804	0.548	1.069
Norway(ALU+)	.501	.489	98.1	7	7	0.13642	1.70389	0.696	0.645

Table 50: *Reliability analysis for creating composite measure meaningful learning experiences*

preservice teachers -meaningful learning experiences									
	alpha	standardized alp	valid percent	mean	median	Se	SD	Skewness	kurtosis
all countries	.718	.718	92.5	11.7006	12	0.02498	2.78757	22.59	7.022
Botswana	.565	.582	91.9	12.6709	13	0.23745	2.11054	3.3	4.447
chile	.668	.676	97.4	12.9016	13	0.09736	2.46312	7.928	0.207
chinese taipei	.664	.662	99.3	11.8168	12	0.08137	2.46398	5.481	0.696
georgia	.631	.632	76.9	9.8175	10	0.15649	3.0864	0.395	3.583
germany	.734	.735	96.9	10.04	10	0.09235	2.92043	2.896	3.4774
malaysia	.755	.757	97.2	12.1	12	0.09777	2.31368	6.01	2.941
philipines	.667	.668	96.8	13.3089	14	0.07769	1.85973	5.618	0.3676
poland	.678	.680	98.1	10.2231	10	0.06271	2.85367	1.667	6.065
russian federatio	.650	.655	98.4	12.8601	13	0.04916	2.32164	15.17	2.269
singapore	.599	.602	99.5	12.455	12	0.09638	1.87383	2.576	0.988
spain	.699	.698	98.2	10.8686	11	0.08266	2.70752	3.72	2.8322
switzerland	.622	.622	98.8	10.8551	11	0.07805	2.37378	2.388	2.503
thailand	.713	.718	98.6	10.9601	11	0.10638	2.71434	3.76	2.39
United States	.695	.694	77.3	13.6374	14	0.06168	2.10179	12.08	3.238
norway(ALU)	.644	.647	98.5	11.4922	12	0.11439	2.24735	1.94	1.052
Norway(ALU+)	.644	.645	98.7	11.2739	11	0.16775	2.10193	2.15	2.0467

Table 51: *Reliability analysis for creating composite measure analysis of learning goals*

preservice teachers analysis of learning goals									
	alpha	standardized alp	valid percent	mean	median	Standard error	standard deviati	Skewness	Kurtosis
all countries	0.646	0.647	87.3	8.3524	9	0.01921	2.14353	14.13	10.68
botswana	.445	.444	67.4	8.4655	9	0.24988	1.9	0.631	0.408
chile	.560	.563	87.7	8.7882	9	0.08605	2.0651	4.57	1.02
chinese taipei	.577	.579	99.2	8.0033	8	0.06139	1.85808	2.6	1.633
georgia	.555	.554	48.0	7.6296	8	0.15312	2.38683	0.97	2.91
Germany	.659	.658	94.7	8.2733	8	0.06805	2.12712	3.28	3.67
malaysia	.616	.618	95	8.3492	9	0.07794	1.82288	3.75	0.02
philipines	.559	.561	71.6	9.2594	9	0.08047	1.65704	2.85	1.16
poland	.588	.589	94.7	6.7946	7	0.04516	2.01997	2.72	4.77
russian federatio	.620	.620	96.1	9.31	10	0.04154	1.93816	14.06	2.51
singapore	.593	.595	98.7	8.9413	9	0.08598	1.66492	2.444	0.19
Spain	.629	.629	90.3	7.4397	7	0.06639	2.08568	1.218	3.21
switzerland	.505	.508	94.3	8.863	9	0.05731	1.70304	2.75	1.21
thailand	.650	.651	95.2	8.5016	9	0.08146	2.04144	3.337	1.605
United States	.631	.635	76.4	9.6565	10	0.0527	1.78482	9.07	1.465
Norway (ALU)	.369	.367	92.6	7.3774	7	0.08508	1.62108	0.0625	0.078
Norway(ALU+)	.500	.501	87.4	7.6691	8	0.1325	1.56218	1.689	0.122

Table 52: *Reliability analysis for creating composite measure inquiry beliefs about learning mathematics*

	Cases														Tests of Normality					
	Valid		Missing		Total										Kolmogorov-Smirnova			Shapiro-Wilk		
	N	Percent	N	Percent	N	Percent	cronbach a	standardize	Mean	Median	Std. Error	Std. Deviati	Skewness	kurtosis	Statistic	df	Sig.	Statistic	df	Sig.
all countrie	13184	91.2	1267	8.8	14451	100	0.714	0.721	24.3369	25	0.03073	3.52842	-46.4286	61.04651	0.095	13184	0			
Botswana	79	91.90%	7	8.10%	86	100.00%	0.358	0.448	24.8228	25	0.32864	2.92101	-1.49815	1.357009	0.111	79	0.018	0.964	79	0.023
Chile	612	93.20%	45	6.80%	657	100.00%	0.649	0.659	25.8562	26	0.12998	3.21549	-8.78788	6.091371	0.105	612	0	0.933	612	0
Chinese Tai	919	99.60%	4	0.40%	923	100.00%	0.704	0.722	24.8803	25	0.09527	2.88804	-3.02469	-1.26087	0.095	919	0	0.976	919	0
Georgia	435	86.00%	71	14.00%	506	100.00%	0.869	0.87	20.1977	22	0.29368	6.12523	-7.23932	-0.53419	0.164	435	0	0.916	435	0
Germany	972	94.20%	60	5.80%	1032	100.00%	0.736	0.747	25.4856	26	0.10397	3.24153	-13.7051	20.44586	0.109	972	0	0.931	972	0
Malaysia	558	96.90%	18	3.10%	576	100.00%	0.747	0.757	22.6165	23	0.17011	4.01834	-10.165	12.45146	0.1	558	0	0.938	558	0
Philippines	572	96.60%	20	3.40%	592	100.00%	0.546	0.576	24.0297	24	0.12806	3.06283	-2.96078	-0.45588	0.088	572	0	0.98	572	0
Poland	2016	95.50%	96	4.50%	2112	100.00%	0.763	0.77	24.3457	25	0.08129	3.65008	-14.4364	17.11927	0.089	2016	0	0.953	2016	0
Russian Fec	2160	95.30%	106	4.70%	2266	100.00%	0.682	0.69	24.3042	25	0.06666	3.098	-9.41509	5.4	0.097	2160	0	0.973	2160	0
Singapore	376	98.90%	4	1.10%	380	100.00%	0.616	0.633	23.9096	24	0.14935	2.89594	-0.89683	0.450199	0.089	376	0	0.982	376	0
Spain	1072	98.10%	21	1.90%	1093	100.00%	0.664	0.665	23.7071	24	0.10516	3.44302	-5.36	2.073826	0.083	1072	0	0.978	1072	0
Switzerlanc	927	99.00%	9	1.00%	936	100.00%	0.635	0.644	25.6354	26	0.08562	2.60684	-4.6375	-1.8375	0.096	927	0	0.97	927	0
Thailand	647	98.00%	13	2.00%	660	100.00%	0.601	0.631	24.2411	24	0.11772	2.99442	-6	7.083333	0.085	647	0	0.97	647	0
United Stat	1134	75.50%	367	24.50%	1501	100.00%	0.686	0.701	24.2425	25	0.09852	3.3176	-7.19178	3.593103	0.1	1134	0	0.971	1134	0
Norway (Al	375	95.70%	17	4.30%	392	100.00%	0.701	0.707	24.3333	24	0.15403	2.98272	-0.55556	-1.16335	0.077	375	0	0.979	375	0
Norway (Al	154	96.90%	5	3.10%	159	100.00%	0.701	0.711	24.8506	25	0.23808	2.95452	-0.75897	-1.23907	0.087	154	0.006	0.974	154	0.005

Table 53: *Reliability analysis for creating composite measure non-inquiry beliefs about learning mathematics*

	Cases														Tests of Normality					
	Valid		Missing		Total										Kolmogorov-Smirnova			Shapiro-Wilk		
	N	Percent	N	Percent	N	Percent	cronbach a	standardize	Mean	Median	Std. Error	Std. Deviati	Skewness	kurtosis	Statistic	df	Sig.	Statistic	df	Sig.
all countrie	13029	90.20%	1422	9.80%	14451	100.00%	0.805	0.81	23.1009	22	0.06043	6.89721	27.38095	6.302326	0.073	13029	0			
Botswana	79	91.90%	7	8.10%	86	100.00%	0.564	0.57	23.962	23	0.66352	5.89751	1.073801	0.293458	0.096	79	0.066	0.983	79	0.395
Chile	597	90.90%	60	9.10%	657	100.00%	0.688	0.69	24.3769	24	0.26679	6.51871	3.55	1.01	0.057	597	0	0.99	597	0
Chinese Tai	915	99.10%	8	0.90%	923	100.00%	0.731	0.74	20.7224	21	0.16872	5.10368	2.740741	2.388889	0.05	915	0	0.991	915	0
Georgia	411	81.20%	95	18.80%	506	100.00%	0.729	0.72	29.8005	31	0.34942	7.08376	-5.6	2.591667	0.09	411	0	0.968	411	0
Germany	959	92.90%	73	7.10%	1032	100.00%	0.682	0.68	19.6361	20	0.15447	4.78363	4.35443	2.829114	0.05	959	0	0.987	959	0
Malaysia	550	95.50%	26	4.50%	576	100.00%	0.786	0.79	31.8055	32	0.29029	6.80782	-1.53846	-0.625	0.047	550	0.005	0.994	550	0.04
Philippines	562	94.90%	30	5.10%	592	100.00%	0.717	0.72	32.5	33	0.25679	6.08767	-3.34951	-0.11165	0.075	562	0	0.988	562	0
Poland	1977	93.60%	135	6.40%	2112	100.00%	0.754	0.76	24.1381	24	0.14201	6.31404	5.618182	2.090909	0.048	1977	0	0.991	1977	0
Russian Fec	2143	94.60%	123	5.40%	2266	100.00%	0.808	0.81	24.8581	24	0.13504	6.25146	12.4717	6.783019	0.077	2143	0	0.972	2143	0
Singapore	370	97.40%	10	2.60%	380	100.00%	0.739	0.74	22.0189	22	0.28481	5.47843	2.685039	3.434783	0.05	370	0.027	0.988	370	0.003
Spain	1071	98.00%	22	2.00%	1093	100.00%	0.693	0.70	21.1447	21	0.16233	5.31235	2.133333	-0.6443	0.044	1071	0	0.994	1071	0
Switzerlanc	914	97.60%	22	2.40%	936	100.00%	0.599	0.60	18.2812	18	0.13264	4.00995	1.45679	-0.16049	0.061	914	0	0.992	914	0
Thailand	634	96.10%	26	3.90%	660	100.00%	0.779	0.78	21.0994	20	0.25695	6.46985	6.546392	4.994845	0.08	634	0	0.974	634	0
United Stat	1139	75.90%	362	24.10%	1501	100.00%	0.732	0.73	20.7454	21	0.16088	5.4297	5.527778	3.406897	0.055	1139	0	0.987	1139	0
Norway (AL	381	97.20%	11	2.80%	392	100.00%	0.678	0.68	18.9633	19	0.24109	4.70596	2.168	2.811245	0.057	381	0.005	0.986	381	0.001
Norway (AL	152	95.60%	7	4.40%	159	100.00%	0.722	0.72	17.5461	17	0.37755	4.65475	1.284264	-0.65985	0.071	152	0.059	0.985	152	0.097

Appendix H: Models for Multi-level Regressions Analysis Included

Table 54: *Multi-level models between OTL how to plan mathematics instruction and non-inquiry beliefs about learning mathematics within institutions (meaningful learning experiences)*

Variables	United States	Russia
Intercept	20.99***	24.92***
	(0.59)	(0.34)
Level 1		
GENDER	0.33	0.20
	(1.08)	(0.51)
YEARSOF	-0.07	0.003
	(0.15)	(0.08)
MORETHAN	-0.14	-0.11
	(0.52)	(0.32)
MEAN_LRN	-0.52***	0.004
	(0.15)	(0.072)
Variance Components		
Intercept u_0	5.88	4.17
Level 1 r	26.23	29.57
ICC	0.18	0.12
Reliability coefficient	0.809	0.848

†*p*<.10, ***p*<.05, ****p*<.001

Table 55: *Multi-level models between OTL how to plan mathematics instruction and inquiry beliefs about learning mathematics within institutions (meaningful learning experiences)*

Variables	United States	Russia	Poland (Gen)	Poland (Spec)
Intercept	24.20***	24.53***	24.23***	25.60***
	(0.28)	(0.17)	(0.15)	(0.27)
Level 1				
GENDER	-0.74	0.14	2.09	0.15
	(0.55)	(0.22)	(0.76)	(0.46)
YEARSOF	0.01	-0.04	-0.10+	0.28***
	(0.07)	(0.04)	(0.06)	(0.07)
MORETHAN	0.19	0.10	0.08	0.28
	(0.23)	(0.15)	(0.21)	(0.42)
MEAN_LRN	0.38***	0.24***	0.17***	0.20**
	(0.08)	(0.03)	(0.04)	(0.07)
Variance Components				
Intercept u_0	1.18	0.69	0.52	0.88
Level 1 r	3.01	7.79	12.67	7.28
ICC	0.28	0.08	0.04	0.11
Reliability coefficient	0.715	0.784	0.511	0.511

†*p*<.10, ***p*<.05, ****p*<.001

Table 56: *Multi-level models between OTL how to plan mathematics instruction and inquiry beliefs about learning mathematics between institutions (introduction to standards-based curricula)*

Variables	United States	Poland (gen)	Poland (Spec)
Intercept	24.19***	24.23***	25.60***
	(0.26)	(0.16)	(0.27)
Level 1			
GENDER	-0.57	1.94**	0.15
	(0.57)	(0.77)	(0.49)
YEARSOF	0.02	-0.12**	0.26***
	(0.08)	(0.06)	(0.07)
MORETHAN	0.25	0.13	0.40
	(0.26)	(0.21)	(0.42)
Level 2			
INTR_SD	0.47†	-0.24	0.20
	(0.29)	(0.22)	(0.34)
Variance Components			
Intercept u_0	1.07	0.52	0.85
Level 1 r	9.66	12.84	7.50
ICC	0.10	0.03	0.10
Reliability coefficient	0.686	0.511	0.495

†*p*<.10, ***p*<.05, ****p*<.001

Table 57: *Multi-level models between OTL mathematics instruction and inquiry beliefs about learning mathematics between institutions*

Variables	United States	Russia	Poland (gen)	Poland (Spec)
Intercept	24.29***	24.36**	24.24***	25.59***
	(0.21)	(0.12)	(0.15)	(0.22)
Level 1				
GENDER	-0.56	0.28	1.94**	-0.02
	(0.57)	(0.21)	(0.78)	(0.36)
YEARSOF	0.02	-0.04	-0.12†	-0.001
	(0.08)	(0.04)	(0.06)	(0.004)
MORETHAN	0.26	0.15	0.13	0.82**
	(0.26)	(0.16)	(0.21)	(0.38)
Level 2				
MSOSTR	2.49***	1.78**	0.50	1.42**
	(0.61)	(0.53)	(0.50)	(0.68)
Variance Components				
Intercept u_0	0.54	0.42	0.51	0.54
Level 1 r	9.65	8.01	12.85	7.43
ICC	0.05	0.04	0.04	0.07
Reliability coefficient	0.536	0.695	0.505	0.434

†*p*<.10, ***p*<.05, ****p*<.001

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