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TO HOME AND BACK:
THE INFLUENCE OF STUDENTS' CONVERSATIONS
ON THEIR COMPLETION OF SCHOOL MATHEMATICS TASKS

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James Weldon Reineke

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Ralph Putnam
Major professor

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TO HOME AND BACK:
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by

James Weldon Reineke

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ABSTRACT

TO HOME AND BACK:
THE INFLUENCE OF STUDENTS' CONVERSATIONS
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This study explored the influence of various practices on students' mathematics learning. Sociocultural theories of learning hold that people develop skills and knowledge as they carry out routine everyday activities or participate in socially-defined practices. As people move among practices it is possible that skills and knowledge helpful in one practice may conflict with the requirements of other practices. Mathematics instruction, students' homework conversations, and other out-of-school practices such as grocery shopping represent potentially conflicting practices. Embedding various practices in students' classwork, juxtaposing traditional elementary-school mathematics assignments and those consistent with the reforms in mathematics education, and watching students participate in conversations at home and in school provided an opportunity to explore the influence of various practices on students' completion of their mathematics schoolwork.

To explore the influence of these practices, ongoing whole-class instruction was video taped, small group interactions were video taped or audio taped, and students and their parents recorded their conversations at home. Students' interactions in each of these settings were transcribed and analyzed to trace the development of ideas, strategies, and answers as the students moved among the conversations.

The students' and their parents' quickly completed tasks that were consistent with their elementary mathematics experience. When the tasks were inconsistent with their experience, no one practice determined what or how students learned. Instead, students, their parents, and classmates brought together different conceptions of mathematics and other experiences to form an amalgamated practice. Students conversations at home

often had a greater influence than those in school. In school, students sought to prove that their answers were correct and others were not and their arguments often rested on warrants that were not mathematical. In light of these findings, mathematics educators can no longer assume that school provides students' sole exposure to mathematical ideas or that students naturally make connections among mathematical ideas. Implementing the reforms in mathematics education may need to include helping parents understand reform mathematics.

To

June, Caitlin, and Rebekka

ACKNOWLEDGMENTS

One week from today and seven years from the start of my doctoral studies, I will file this dissertation. Over those seven years I have been influenced by many things. My professors at Michigan State, teachers in local elementary schools, fellow graduate students, and the writings of past masters, among other things have been tremendously influential. Nothing, however, has influenced me as much as completing this dissertation. From the original idea, through two small pilot studies and a series of refinements, a data collection period spent clenching my teeth waiting for students to return tapes of conversations recorded at home, and the ever daunting task of making sense of it all, this study has provided endless intellectual and logistic challenges that I could not have met alone. In this space I want to thank some of the people who contributed to this study.

First, I want to thank Ms. Smith, the teacher in whose class this study was conducted, and her students. Ms. Smith's dedication to this study and our three-year collaboration proved invaluable in devising tasks, communicating with parents, and collecting data. Her students' willingness to discuss mathematical ideas while being taped was essential to the success of this study.

There is one group of students and parents that I particularly need to thank. These six families recorded their conversations at home. Asking them to record their conversations was an imposition; agreeing to record their conversations was a sacrifice. Taking on another responsibility--one that does not contribute to an otherwise smooth-running system--is risky business. Things may be inadvertently recorded or conversation may be awkward with recording equipment staring you in the face. These families accepted the risks and responsibilities of recording their conversations and they, alone, are responsible for documenting the conversations at home.

I also want to thank my program and dissertation committee: Ralph Putnam, Deborah Ball, King Beach, Jim Gavelek, and Penelope Peterson. Each of these people contributed greatly to my education and to the completion of this study. By providing research opportunities and guidance, thoughtfully prepared courses, thoughtful critiques of early conceptions of this study, and constructive comments on early drafts of my dissertation and other writing projects, they contributed to a exceptional educational experience at Michigan State. Although each of these members have been influential, I want to thank two members of my committee in particular.

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CHAPTER 1

INTRODUCTION

In this study I investigated the influence of students' participation in conversations at home and in school on their completion of a series of elementary school mathematics[1] tasks. Investigating this phenomenon offered an opportunity to address issues raised both by theorists traditionally interested in human learning (e.g., educational psychologists, anthropologists, and sociologists) and by mathematics educators who have traditionally focused on providing the best possible classroom instruction in mathematics.

Recent research on learning (Beach, 1993; Beach, 1990; Lave, 1988; Lave & Wenger, 1991; Scribner, 1984) has suggested that what and how people learn is heavily influenced by the chores, vocations, and avocations that mark their daily lives. Each of these activities require that participants know certain things that allow them to be effective participants in the activities. Many of the activities people engage in involve mathematics. The participants in these activities construct mathematical systems that allow them more efficiently to fulfill the requirements of the tasks they are performing. Dairy workers (Scribner, 1984), candy sellers (Carraher, Carraher, & Schliemann, 1985), and grocery shoppers (Lave, Murtaugh, & de la Rocha, 1984), for instance, have all been shown to construct mathematical systems that allow them to assess situations they encounter and carry out the necessary mathematical operations. But, at the same time they are able to compute complex problems in the field, their knowledge of school mathematics is limited. In several studies, the participants could not compute school-like mathematics problems that included the same mathematical ideas with which they were fluent in the field. The researchers concluded from these

[1] I use the term "elementary school mathematics" to distinguish between what mathematicians do in their professions and what students in elementary mathematics classes do. Whereas mathematicians construct new mathematical knowledge, mathematics students are learning what past mathematicians have done.

findings that the mathematics people learn is inextricably tied to the activities they perform routinely and that these activities should become the focus of psychological investigation (Scribner, 1984).

This research has not been lost on mathematics educators (Cobb, Wood, & Yackel, 1993; Schoenfeld, 1992). Accepting the idea that the mathematics people learn is shaped by the things they do and where they do them, mathematics educators began looking at classrooms as having a similar influence on the mathematics students learn (Bishop, 1991; Cobb et al., 1993; Lampert, 1990). Students in classrooms in which computation was emphasized learned to compute; students in classrooms where mathematical thinking was emphasized learned to reason through situations using mathematics as a system of inquiry. The latter of these characterizations is the focus of recent reforms in mathematics education (National Council of Teachers of Mathematics, 1989; National Research Council, 1989; National Research Council, 1990). As a result of participating in a community of practice in which students are required to assess situations mathematically, flexibly apply mathematical tools, and justify their responses, it is believed that students will develop the ability to "think mathematically" and solve problems (National Council of Teachers of Mathematics, 1989; National Research Council, 1990).

Both the psychological research and the work in mathematics education focused on how the immediate surroundings in which the people were participating influenced what and how they learned. Recent theorizing, however, has suggested that learning is not restricted to the immediate situation. Learning, it is posited, includes extending what is learned beyond the immediate situation to other situations where what was learned can be beneficial (Lave, 1993). As a result, to understand what a person has learned we need to watch them as they participate in various practices and attempt to document the flow of ideas as they move among the practices.

In investigations of learning in mathematics classrooms, looking beyond the immediate situation might include looking at how students construct and use mathematical knowledge in out-of-school settings. In particular, looking at how students are influenced by both their classroom instruction and by working on their school work with their parents at home would provide a glimpse at the mutual influence of these two practices. At the same time that studying this phenomenon can inform mathematics education, it also provides an opportunity to explore the larger theoretical issues posited in recent psychological theory. A study of this type has not been attempted in the past. As a result, new methodology and methods needed to be devised. These three things are represented in the questions I posed at the beginning of this study:

- How do students' interactions in school and at home influence their performance on a series of school math tasks and what are the implications of this influence for our understanding of children's mathematics learning?
- What are the implications of simultaneous participation in the home and school for our understanding of learning and development and educational practices?
- Will a methodology tracing the influences of interactions in school and at home contribute to our understanding of how children learn?

In addressing these questions I accepted two challenges. First, addressing two audiences--educational psychologists and mathematics educators--presents a substantial challenge of providing adequate explanations to each group. Scholars representing these two fields of inquiry bring different assumptions to the study of learning in mathematics classrooms that makes an attempt to address both audiences particularly challenging. Historically, the relationship between mathematics education and educational psychology has been characterized by tensions that have made it difficult to bring the two fields together. Although successful research agendas have bridged the two fields (Fuson, 1988; Leinhardt & Greeno, 1986; Leinhardt & Smith, 1985; Nesher, 1989; Romberg & Carpenter, 1986) tensions have remained. Educational psychologists' recent interest in the sociocultural influences on human learning and mathematics education, however, provides a reason to attempt another union between the two fields of study. Second, to investigate the influence of participating in conversations at home and in school I relied on the subjects to record the home conversations. The reliance on subjects to record their conversations created a potentially incomplete record of the conversations students had with their parents and, perhaps, created inauthentic conversations. In this chapter I look at these two challenges and how I attempted to address them in this study.

Challenge 1: Finding a Niche

Research on teaching and learning in mathematics classrooms has its roots in the disciplines of mathematics and psychology. At the turn of the century, as they began to contemplate what ought to be included in mathematics instruction, scholars from these two disciplines drew on the accumulated work of their disciplines. Translating findings or beliefs about subject matters into prescriptions for practice is a difficult task. As they transcribe their beliefs, scholars cautiously commit the "naturalistic fallacy" of moving from "is" statements (statements that represent the scholars' current explanation for the phenomena they study) to "ought" statements (statements that represent prescriptions for practice) (Kohlberg, 1971; Bronfenbrenner, Kessel, Kessen, & White, 1986). The assumption held by scholars is that their beliefs--their is statements--are true and should be reflected in practice. Even within disciplines, however, scholars disagree about what is true and what practice should include. The scholars that began researching teaching and learning in mathematics classrooms came from disparate backgrounds, all of which had their own set of is statements they wanted the newly constructed field to reflect.

Psychologists brought is statements that reflected beliefs about human learning or behavior that transcended subject matters and ought statements that were instructional interventions to help people develop appropriate behaviors. Having modeled psychological inquiry after the natural sciences where the goal is to discover basic, universal laws, the psychologists believed that people learn everything in the same way and the study of learning can be "content independent;" that is, you could replace mathematics with any other subject and people would learn it the same way. As Kohlberg (1971) described this belief,

> The critical category of the Stimulus-response approach was "learning" not "knowing," where the concept of "learning" did not imply "knowing." Accordingly, S-R theory assumes that the process of learning truths is the same as the processes of learning lies or illusions. It explains the learning of logical operations or "truths" in terms of the same processes as those involved in learning a social dance step (which is cognitively neutral), or those involved in "learning" a psychosis or a pattern of maze errors (which are cognitively erroneous). (pp. 151-152)

The psychologists also believed that only observable behavior was worthy of psychological study. As a result, they needed to define mathematics, or at least what they would consider evidence of having learned mathematics, behaviorally. Students' performance on written computation exercises that could be reduced to a series of discrete behaviors taught independent of each other became the definition of mathematics.

Mathematicians started with a different set of beliefs. The mathematicians' is statements focused on mathematical processes--both mental and mechanical--that mathematicians used while *doing* mathematics. To develop the is statements they looked to practicing mathematicians and how they constructed new mathematical knowledge. Mathematicians' ought statements focused on the content and activities that should be emphasized in mathematics instruction to help students develop the processes associated with doing mathematics. Mathematicians argued that instruction needed to include ways of thinking mathematically, rather than focusing exclusively on computation (Hadamard, 1945/1954; Kilpatrick, 1992).

Bringing the psychologists' behavioral conceptions of learning and mathematicians' conceptions of mathematics together into one coherent agenda of research on teaching and learning in mathematics classrooms proved difficult, if not impossible. The researchers could not agree on what should be investigated or how it should be investigated. As a result, two parallel fields of study emerged: educational psychology and mathematics education. Educational psychologists were devoted to the application of psychological principles to classroom instruction (e.g., Thorndike and Judd). Although mathematics was used as a site for much of their research (Kilpatrick, 1992), these psychologists believed their conclusions and subsequent prescriptions were generic and contributed to their universal understanding of human learning.

In contrast, mathematics educators continued to emphasize mathematical content. Although they disagreed on what aspects of mathematics ought to be represented in classroom instruction, they agreed that the study of teaching and learning mathematics needed to be specific to the discipline. They rejected their psychological contemporaries' belief that all things were learned in the same way and raised the question: "Where's the math?" in response to research in educational psychology.

Over time, however, psychological thought, and consequently educational psychology, changed. Following the cognitive revolution of the 1950s, there were reasons to believe that educational psychology and mathematics education had something to offer each other. Psychologists had opened behaviorism's "black box" and had begun investigating how people process information as they encounter new situations. Psychologists came to believe that, although the basic processes of encoding, storing, and retrieving information were generic (Shuell, 1986), the organization of knowledge was specific to the discipline being studied (Resnick, 1987). In response to these beliefs, many researchers began successful agendas looking at the cognitive requirements of teaching and learning mathematics. Researchers compared and contrasted the knowledge of expert and novice mathematics teachers (Leinhardt & Greeno, 1986; Leinhardt & Smith, 1985). They studied expert and novice problem-solvers (Schoenfeld, 1983), searched for the best instructional representations for mathematical ideas (Nesher, 1989) and sought to define how a person's knowledge of specific domains ought to be organized (Carpenter, Peterson, Chiang, & Loef, 1988; Romberg, 1982). Each of these research agendas contributed to a growing understanding of teaching and learning in mathematics classrooms.

But even the seminal research that grew out of the cognitive revolution did not resolve all of the differences between the psychological and mathematical viewpoints. Indeed some of the same concerns--the differing perspectives on what mathematics is, how it should be studied, and what instruction should include--remained. Long, Meltzer, and Hilton (1970) summarized the remaining tension this way:

> There seems to be a consensus that cognitive psychology at the present time is not oriented toward problems or approaches which are likely to be useful to mathematics education. . . . A substantial problem seems to be to find psychologists (mathematicians) who understand mathematics (psychology) in any but a shallow manner. (p. 451)

Recently, however, many educational psychologists, drawing largely from research in developmental psychology (Laboratory of Comparative Human Cognition, 1983; Rogoff, 1990; Wertsch, 1991), anthropology (Lave, 1988; Lave & Wenger, 1991), and sociology (Eckert, 1989), have broadened their view of learning to account for the social structures within which people work, play, and study. Recent theorizing in these disciplines has suggested that what and how people learn is greatly influenced by their participation in particular forms of socially-defined practices. Socially-defined practices are the routine activities in which people participate every day or nearly every day. They have their own histories and can be discussed--or studied--independent of the people who participate in them. They encompass bodies of knowledge, selected technologies, and sets of rules that guide or shape the participants' actions and interactions. Practices can be as large as the practice of law or as narrow as the practice of writing a memo and one may be nested within another.

Researchers interested in the influence of socially defined practices on cognition argue that, as participants in the practices become more experienced, they gradually adopt accepted ways of thinking and behaving in the practice. Part of this adoption includes developing ways of communicating that assume a shared understanding of the problems and solutions in the practice. As a result, practitioners are enculturated into ways of assessing situations for their worth as problems, and determining intelligent ways of solving the identified problems.

The influence of practice on a person's thoughts and actions can be seen in many areas. Scientists working within a given paradigm look upon their subject matter in specific ways. The unique way of looking shapes the questions the scientists ask, how they go about investigating phenomena, and how they interpret their results. Trades- and craftspeople also learn to work within specific boundaries that define participation in their practices. In the construction trades, which require a working knowledge of contemporary technology and local building codes, or recreational activities such as technical rock climbing where a sophisticated knowledge of the rock on which they are climbing is imperative to the climbers' safety, the knowledge necessary to be successful is embedded in the practice or activity in which the person is participating.

Mathematics educators have taken notice of this research in situated cognition and have begun thinking of classrooms as communities of practice (Bishop, 1991; Cobb, 1986; Cobb et al., 1993; Schoenfeld, 1992); that is as groups of people simultaneously engaged in a practice. Mathematics classrooms, and the way mathematics is done in the classrooms, represent socially-defined practices that function the same way as other communities of practice. Participation in mathematics classrooms not only results in learning *mathematics*, but in learning *about* mathematics. Students in mathematics classrooms learn conceptions, origins, and uses of mathematics as well as mathematical skills and abilities. These things include whether mathematics is thought to be a collection of memorized facts and procedures or the "science of patterns" emphasized in recent reform documents that emphasize seeking solutions, exploring patterns, and formulating conjectures (National Research Council, 1990). Students in classrooms in which one of these conceptions of mathematics is emphasized will emerge with a set of beliefs consistent with that conception (Schoenfeld, 1992; Skemp, 1976).

Both the research on situated cognition and on classroom communities of practice have assumed that practices are discrete entities. The practice in which people are participating, it is assumed, shapes--in a rather behavioral use of the term--what and how the participants learn. Although most of the situated cognition research has compared the mathematics used in practice to the mathematics usually taught in western schools, the emphasis is on the differences between the two mathematical systems not on the relationship between the two practices. In mathematics education, it has long been believed that what students are exposed to in their classrooms is their primary, if not sole, exposure to mathematics (Kilpatrick, 1987; Lampert, 1990). The emphasis on how practices shape participants' thoughts and actions has led situated learning researchers to argue that "The practices themselves need to become objects of cognitive analysis." (Scribner, 1984, p. 14). Mathematics educators have also focused on classroom practices. Cobb asked the mathematics education question this way: "With regard to school mathematics, the question that arises is how do social interactions in the classroom influence the overall goals that students establish and thus their beliefs?" (Cobb, 1986, p. 6).

If knowledge was determined solely by the practices in which we participate, however, people would be involved in a never ending string of "code switches" (Edwards & Mercer, 1987) among a series of reproductionist communities of practice. As we entered each different practice we would leave behind all our experiences and become an unquestioning member of the community--all carpenters would think and do the same things as would all psychologists and all teachers. But all members of a community are not the same. People bring ways of knowing and specific bodies of knowledge with them as they begin to participate in practices. It would seem that their participation would depend as much on what they bring as on what is demanded by the practice. Indeed, their participation shapes the practice at the same time the practice shapes their thoughts and actions. This idea has not escaped mathematics educators. In 1993, Cobb and his colleagues wrote:

> We find ourselves in agreement with a basic tenet of Leont'ev's activity theory as interpreted by Minick (1989): that an individual's psychological development is profoundly influenced by his or her participation in particular forms of social practice. We do, however, reject any analysis that takes a particular social practice such as schooling or the mathematics education of young children as pregiven . . . [and] we question . . . [the] metaphor of either students or teachers being embedded or included in a social practice. Such metaphors, tend to reify social practices, whereas we believe that they do not exist apart from and are interactively constituted by the actions of actively interpreting individuals. (Cobb et al., 1993, p. 96)

The agreement between mathematics educators and situated learning theorists that an investigation of learning must strive to understand what individual participants bring to the practice and the social structure in which they participate offers another opportunity for addressing the fields of psychology and mathematics education at the same time. Watching a person participate in separate, but related, socially-defined practices can help us understand what a person brings to a practice. In mathematics education, watching students work on school math tasks both in school and at home (i.e., in different practices) may shed light on the how students come to complete and understand their school math work and, as a result, contribute to classroom practice. At the same time, and in agreement with Bob Davis (1967) and others (Moll & Whitmore, 1993), classroom practice often surpasses theory and an understanding of classroom practice may contribute to the growing theory of situated learning.

Challenge 2: Data Collection Methods

Investigating the influence of participating in conversations in school and at home also presents a methodological challenge. To document completely students' participation it might be necessary to follow them as they move among practices. It was not possible, however, to follow the students from their classroom to their home and back to school in person. As a result, students were asked to record their conversations at home themselves. The resulting conversations may not be authentic or complete. The students and their parents may have talked about the tasks without turning on the tape recorder or they may have had conversations that would not otherwise have occurred. In either case, the conversations might not represent authentic participation in the practice of doing homework in their homes.

Beyond the concern over authenticity, however, it is still possible to see ideas as they emerge in one conversation and reappear in subsequent conversations. As ideas emerge in the conversations, it is possible to infer experiences that may have given rise to the idea or shaped it in some way. Tracing ideas from conversation to conversation also provides a glimpse of how students' ideas are formed by their participation in subsequent conversations--whether or not those conversations represent authentic participation in socially-defined practices.

Goals and Purposes of This Study

The purpose of this study was to investigate the influence of various practices on students' completion of school math tasks. Investigating the influence would elaborate contextual theories of learning and help explain the relationship between the individual and social structure in elementary school mathematics classrooms and in socially-defined practices in general. This goal required that I first look at what students take away from the practices in which they participate; that is, how are they enculturated into a given practice.

Enculturation into Practices

A hallmark of Vygotsky's sociohistorical theory (Vygotsky, 1978) is that more knowledgeable others--teachers, parents, or peers--gradually transfer the authority for completing the task to the learners. As students gradually develop their ability to complete the tasks independently, teachers or parents contribute less. The transfer of authority can occur within a conversation or across conversations in which participants work on similar tasks.

At the end of this process, students have appropriated ways of completing the task that are consistent with the practice in which they are participating and, in effect, are enculturated into the practice. At the same time, the rules used to complete similar tasks might change to reflect the experiences of the participants.

With respect to elementary mathematics, I believed that the conversations students and their parents have at home would change over time to reflect the transfer of authority from the adult to the child. In tasks that contain similar questions, this transfer might occur in the conversation, or the transfer of authority might also be seen in later tasks that include similar questions.

In light of the recent calls for reform in mathematics education, the parents may also change their participation in the practice of homework. Their knowledge of mathematics may not include an understanding of the tasks with which they are asked to help their children--at least not the first time they are exposed to them. Including similar questions in later tasks would, I hoped, provide a glimpse at parents' changing participation in their children's' math homework.

The Influence of Socially-Defined Practices

To investigate the influence of participating in various practices on students' completion of their school math tasks, different practices were brought together with the goal of illuminating the influence each had on students' completion of the tasks. The practices were brought together in three ways, each of which is described in more detail below. First, potentially conflicting practices were embedded in the tasks presented to students. Second, conflict among practices can also result from the evolution of a practice. Students in this study were presented with tasks that represented both traditional mathematics instruction and tasks that represented teaching in the spirit of the recent calls for reform in mathematics education. Third, the tasks moved back and forth between practices--from the classroom to students' homes back to the classroom.

Embedded Tasks: Bringing the Out-of-School World into School. Bringing two separate practices together can create conflicts that need to be resolved while completing a task. Practices are characterized by sets of rules that govern working within those practices. If two practices and, therefore, two sets of rules clash when brought together, participants must decide which set of rules to use or negotiate a new set of rules, drawing on the merging practices.

One tenet of the recent calls for reform in mathematics education (National Council of Teachers of Mathematics, 1989; National Council of Teachers of Mathematics, 1991; National Research Council, 1989; National Research Council, 1990) holds that students should understand out-of-school uses of mathematics. This emphasis is shown in this quote from *Reshaping School Mathematics*:

> Students need to experience mathematical ideas in the context in which they naturally arise--from simple counting and measurement to applications in business and science. . . Appealing applications should be drawn from the world in which the child lives, from community events, or from other parts of the curriculum. . . . The primary goal of instruction should be for students to learn to use mathematical tools in contexts that mirror their use in actual situations. (National Research Council, 1990, p. 38)

The different situations called for in this document represent different socially-defined practices that may include rules other than those traditionally taught in school math classes. To complete tasks that reflect out-of school practices, students, their parents, and classmates must determine the set of rules they will use to govern the task as they work together. As a result, bringing out-of-school practices into school creates a situation which includes tremendous potential for conflict among practices. Observing students and their coworkers as they work together on school math tasks that include out-of-school practices can illuminate the mutual influence of various practices on the completion of a task.

Evolving Practices: Reforms in Mathematics Education. Conflicts between practices can also result from changes within a practice. Thoughts and actions that are accepted procedures at one time may fall into disfavor as the practice evolves. Although change is an aspect of growth in every practice, it is likely to meet resistance when it is first introduced (for a description of this phenomenon in the sciences, see Kuhn (1962)). The resulting conflict represents two colliding practices in much the same way as bringing together two separate practices did. Each of the practices represents different sets of rules and the differences between the rules must be reconciled in some way when the practices are brought together.

Recent calls for reform in mathematics education have suggested that teachers drop their emphasis on arithmetic computation and develop what has been referred to as "mathematical pedagogy" (Ball, 1988). Ball describes the goal of mathematical pedagogy this way:

> Rooted in mathematics itself, the goal of mathematical pedagogy is to help students develop mathematical power and to become active participants in mathematics as a system of human thought. To do this, pupils must learn to make sense of and use concepts and procedures that others have invented--the body of accumulated knowledge in the discipline--but they also must have experience doing mathematics, developing and pursuing mathematical hunches themselves, inventing mathematics, and learning to make mathematical arguments for their ideas (see (Romberg, 1988). Good mathematics teaching according to this view, should eventually result in meaningful understandings *about* mathematics: what it means to "do" mathematics and how one establishes the validity of answers, for instance. (Ball, 1988, pp. 3-4)

Richards (1991) has referred to the sort of classroom activity described by Ball as "inquiry math," and has contrasted it with the "school math" associated with traditional classroom instruction focusing on arithmetic computation in the elementary grades. School math and inquiry math constitute different socially defined--and potentially conflicting--practices (Cobb et al., 1993). Each practice has its own set of rules that guide participation in the practice. Whereas in school math arriving at a correct answer using an established algorithm may signify success, in inquiry math the emphasis is equally placed on creation and justification of new or different solutions and the construction of mathematical knowledge.

As happened with the "new math" of the 1960s, inquiry math is likely to conflict with parents' previous school math experience. In reform classrooms, students are likely to receive instruction in inquiry math while receiving instruction in school math at home. As a result, students, parents, and classmates will need to reconcile the differences between the colliding practices as they complete their school math tasks.

Moving among Practices

The instruction students receive in school and at home represents different practices that may be governed by different rules. This possibility increases in the wake of the recent reforms in mathematics education. As mentioned above, parents' experience in school math may have focused on arithmetic computation that has characterized elementary mathematics. The emphasis on assessing situations mathematically, flexibly applying mathematical tools, and justifying solutions may only be part of students' mathematics instruction in school. Students receive instruction while participating in both practices. Trailing the tasks as they moved from school to home and back can illuminate the influence different practices had on students' completion of the tasks.

Summary

Each of these ways of bringing practices together are included in this study. Although working on school math tasks at home and in school will always influence how students complete those tasks, the changes in mathematics education provide an exciting opportunity to investigate the mutual influence of various practices. As a result, watching students work at home and in school on tasks that bring together various socially defined practices can contribute both to a understanding of the impact of changing mathematics instruction and to the development of situated learning theory.

Overview of the Dissertation

Chapter 2

In Chapter 2 I look more closely at the research that informed this study. Sociocultural theories of learning begin with the premise that learning is a fundamentally social process. Drawing on the work of Vygotsky, it is argued that all higher psychological functions appear first interpsychologically (between people) and then intrapsychologically (within people). The gradual progression from interpsychological plane to intrapsychological plane is characterized by two processes: the semiotic negotiation of tasks and the direction of development as defined by the practice within which people are working. These two processes provide the framework for Chapter 2. I begin by reviewing research on negotiation in the zone of proximal development (ZPD) (Vygotsky, 1978). Although much of this research has focused on expert-novice dyads in a laboratory setting, some recent work has looked at classroom interaction and negotiation of tasks among peers.

The review of research looking at the direction of development begins with a look at research in situated cognition that led researchers to conclude that what and how we learn is determined by the practices in which we participate. Each of the studies reviewed compared the mathematical systems *in situ* and the mathematics usually taught in western schools. Although these studies provided the foundation for a host of illuminating research on human learning, they failed to account for participation in various practices at the same time. Looking at the relationship between students' participation in school and at home provides an opportunity to look at students participating in more than one practice at the same time.

Although little research has been conducted on the mutual influence of interactions in the home and school, many researchers have looked at the relationship between activities at home and success or failure at school and at homework as a instructional tool. In this section I review research that focused on the relationship between the school and home. This section is divided into two subsections. First, I review literature on in-home activities that influence children's development of literacy skills and the discontinuity in interactional norms between the home and the school. Second, I review the research on homework. this section is also divided into two subsections. The first focuses on homework's effects on student achievement, the second on what homework sessions look like in different homes.

Chapter 3

In Chapter 3 I discuss the methodology and methods used in this study. I begin this chapter by revisiting some of the theoretical issues associated with research on learning in practice and developing a conceptual framework within which I define the term "practice" and outline a methodology for investigating mutually constitutive practices.

I begin the methods section by introducing Ms. Smith, the teacher in whose classroom this study was conducted, and mathematics instruction in her class. The introduction includes a brief history of my work in Ms. Smith's classroom and her vision of teaching in the spirit of the recent reforms in mathematics education. Following Ms. Smith's introduction, I introduce the students who participated in the study and their families. These introductions include the makeup of each family, the students' past experience in school, their parents' school math experiences, and how the families organize homework sessions at home.

After introducing the students and their families, I describe the tasks Ms. Smith and I presented to the class. To describe the tasks, I use a framework that includes the instructional, mathematical, and research reasons for the tasks. It was important that the tasks fit with ongoing classroom instruction, addressed some of the concerns of mathematics educators, and addressed some of the theoretical issues described earlier. How the tasks accomplished these things is addressed in these descriptions.

Following the description of the tasks, I describe the data collection path and the technology used in each setting to record and document the activities. This section also includes the response rate of students and their families. Finally, I discuss the methods I used to analyze the data.

The next three chapters present the findings of this study. Each of the chapters focuses on one of the ways of bringing potentially conflicting practices together discussed above.

Chapter 4

Chapter 4 focuses on how students, their parents, and their classmates resolved dilemmas created when two conflicting practices--school math and grocery shopping--were brought together. Although the task was designed to see which set of rules the students and their parents would choose, it became clear that more than the practices represented in the tasks and how math was done in Ms. Smith's classroom influenced how the students and their parents approached and completed the task. Parents' and students' experience in mathematics classes and in other out-of-school practices greatly influenced how they answered the questions.

Chapter 5

Chapter 5 focuses on continuous and discontinuous mathematical practices. Tasks two and three reflect the two types of mathematics instruction described by Richards (1991). Task two included a series of word problems similar to those found in traditional elementary school math texts.[2] This task represented what Richards called "school math." The questions posed in Task Three presented were consistent with the recent calls for reform in mathematics education and represented what Richards called "inquiry math." In Task Three students were asked to determine the number of three- or four-digit numbers possible using a given set of numerals and to explore the value of each digit in the numbers they found. The questions posed in Task Two were believed to be continuous with students' parents' school math experience, whereas the questions asked in Task Three were thought likely to be discontinuous with the parents' school math experience.

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[2]For a more complete description of problems presented in elementary mathematics textbooks, see (Remillard, 1990)

Parent-student interaction changed depending on the task on which they were working. While working on tasks that were consistent with parents' and students' school mathematics experience, they quickly agreed on the arithmetic operation to use and how it should be computed. The parents' role consisted of monitoring the students' work and correcting them when they made mistakes. While working on tasks that were inconsistent with their school mathematics experience, parents and students roles changed. Parents, rather than remaining in charge of the conversation, deferred to the students and their understanding of what was required in the task. As the students worked their parents watched and listened intently, learning what was expected in the tasks. As they learned more and their confidence grew, they contributed more to the students' homework until, during their conversations for the second part Task 3, parents' contributions began to resemble their contributions on earlier tasks.

Chapter 6

Chapter 6 looks across conversations to see how ideas develop as students worked on the tasks both at home and in school. Looking again at the conversations from Tasks 1 and 3 points out two things. Students do not internalize what they learn in conversation and use the intact knowledge in subsequent conversations. Rather, with the help of conventional ways of documenting their work, students reconstruct their answers when they return to school. Although their may reflect the conversations they had at home, they do not mimic them. Students take part of what they discussed in conversations at home back to school and reinterpret what they have written down. When the documentation is lost, students have tremendous difficulty reconstructing their answers without the help of their parents.

Task 4 supports the assertion that students do not internalize everything they discuss in conversation for use later on. Task 4 included four questions similar to those in the first three tasks with the belief that parents and students would use the things they discussed earlier to help them answer the questions in this task. Neither parents nor students, however, referred back to the earlier tasks.

The second assertion supported by the conversations from Task 1 and 3 holds that learning is a gradual process. Students and their parents developed ways of thinking and acting that allowed them to complete the tasks on which they were working. In subsequent conversations, the students and their parents changed the ways of thinking and acting to take into account new data. As their actions changed, so did the students understanding of the mathematics in the situations.

Chapter 7

In Chapter 7, I revisit the findings from the previous three chapters and discuss the implications of this study for classroom instruction and the development of situated learning theories. Many of the findings of this study support the work of other researchers working in a sociocultural tradition. As a result, this study may beg the question "So what?" But pointing out the tremendous number of influences on students' thinking in school mathematics classes can never be done too often. Even in its repetition, this study offers refinements or extensions of previously conducted research on elementary students' mathematics learning and the influence of various socially-defined practices on what and how people learn. Those refinements and extensions are discussed in the final chapter.

CHAPTER 2

A REVIEW OF THE LITERATURE

At the end of "The Internalization of Higher Psychological Functions," an essay from his seminal book *Mind in Society*, Vygotsky (1978) wrote: "The internalization of socially rooted and historically developed activities is the distinguishing feature of human psychology, the basis of the qualitative leap from animal to human psychology. As yet, the barest outline of this process is known" (p. 57).

Although Vygotsky believed the process of internalizing activities of this sort was not well known, he had gone a long way in explaining this complex phenomenon. The process of internalization,[1] according to Vygotsky, is an extended process through which children fundamentally change the way they think and behave. Very young children's earliest attempts to perform higher psychological processes such as memorizing items on a list begin as an unaided, eidetic internal activity. These methods prove difficult and often unsuccessful. To assist children in learning to memorize things, an adult, or more knowledgeable peer in some cases, mediates sociohistorically determined methods of recording the items to be memorized. They might, for example, show them how to notch a stick, tie a string around their fingers, or write a list. Eventually the child begins to perform these activities on his or her own and reconstructs them internally; that is, the child internalizes them. The interaction between the child and the more knowledgeable other casts the foundation on which Vygotsky's theory of human development rests:

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[1]There is an ongoing debate among psychologists about the internalization of concepts. The term "internalization" suggests to some that people accept the ideas and ways of thinking of a culture without, in some way, tailoring those ideas for their own use. Taken to an extreme, this would lead to the mere reproduction of a culture from one generation to the next. Those who oppose the term "internalization" suggest instead that people "appropriate" knowledge in a way that makes sense to them. This conceptualization of this process allows for manipulating and changing culturally accepted ideas and allows the person to take ownership of his or her own thoughts. Although this second conceptualization more adequately describes the process of coming to know something, I have chosen to use the term "internalization" here to be consistent with Vygotsky's writings.

> Every function in the child's cultural development appears twice: first, on the social level, and later on the individual level; first *between* people (*interpsychological*), and then *inside* the child (*intrapsychological*). This applies equally to voluntary attention, to logical memory, and to the formation of concepts. All the higher functions originate as actual relations between human individuals. (Vygotsky, 1978, p. 57)

The transformation of interpersonal actions into intrapersonal abilities occurs gradually. During their first attempt to write a list, children may need to be shown every step. While writing subsequent lists they will gradually complete more of the task without assistance, finally completing the list on their own. Because internalization is a gradual process, Vygotsky argued that two levels of development needed to be understood--the level at which a person can function by his or herself and the level at which they can function with the help of another person. These two developmental levels became the boundaries of Vygotsky's construct of the "zone of proximal development" (ZPD). Vygotsky defined the ZPD as "the distance between the actual developmental level as determined by independent problem solving and the level of potential development as determined through problem solving under adult guidance or in collaboration with more capable peers." (Vygotsky, 1978, p. 86).

Learning within the zone of proximal development is evidenced by the learner's movement--both cognitive and behavioral--toward the socioculturally determined ways of accomplishing tasks. Adults, or more knowledgeable others in Vygotsky's words, have the responsibility of facilitating this change. This conception of learning begs two questions: How do more knowledgeable others facilitate change in learners' thoughts and actions? And, what determines what is to be learned, or the direction of development?

As Vygotsky argued, movement within the ZPD is a fundamentally social process and is negotiated in conversation. The direction of development, it can be argued, is determined by the accepted methods of completing the task--or by the practice within which the participants are engaged. In the next sections I describe research that has explored these two components of learning within the zone of proximal development and other work that has contributed to this project.

Negotiation within the ZPD

Vygotsky's definition of the zone of proximal development suggests the ZPD is an individual trait. People have two different levels of functioning--an actual level and a potential level that remain constant no matter where they are or with whom they interact. This definition of the zone of proximal development as an individual trait, however, is somewhat shortsighted. Wertsch (Wertsch, 1984) tells the stories of two students--one in fifth grade and one in second grade--who work with the same more knowledgeable other. In both of these stories, the learner is able to complete a complex division problem. Even though both students were able to complete the problem with assistance from the more knowledgeable person, Wertsch suggests they do not have the same level of potential development. Rather, he has argued that the zone of proximal development, rather than being a trait of an individual, is constructed through the interaction of two or more people. And, to understand the zone as a product of interaction we need to understand three mechanisms of the zone of proximal development: (a) *situation definitions* and their corresponding *action patterns*; (b) *the intersubjective situation definition* on which the participants agree; and, (c) the *semiotic moves* used to negotiate the intersubjective situation definition.

People construct intrapsychological situation definitions and bring them to bear in conversation. Although these situation definitions include the spatial and temporal settings in which the two people are working, they also include much more. They include the participant's representation of the problem and objects in the problem and cannot be separated from the actions in which the people take part; that is, their action patterns. To understand and analyze the zone of proximal development, then, we need an explicit account of each person's action pattern.

Wertsch and his colleagues (1984; Wertsch, 1979) have looked at the action patterns of adults and children as they replicate a model of a barnyard scene. The children in these studies often choose a piece to put in the model without looking at the original. Once they have chosen a piece, they place it in the model without looking to match the original. This action pattern could be written: (a) choose a piece; and, (b) place the piece in the model. Adults working with these children, however, often chose the next piece while looking at the original and then placed the piece in the model in its appropriate place. This action pattern could be written: (a) Check the original to identify the next piece needed; (b) Find the piece identified in the first step; and, (c) Place the piece in its appropriate place in the model.

Movement from one action pattern to the other marks a cognitive change. In most models of the zone of proximal development there is an assumption that the adult holds to a representation of the situation that is appropriate to their social and cultural background and is better or more advanced than the child's. As a result, the cognitive change in these models occurs as the child's action pattern comes to resemble the action pattern of the adult. This change cannot be considered a quantitative shift or addition to the pattern. In the action patterns mentioned above, moving from the child's action pattern to the adult's action pattern would take more then adding an extra step. Indeed, it would require a qualitative shift in the child's representation of the situation. The steps in the child's action pattern would need to be redefined. Making this kind of change requires the child give up his or her previous situation definition and replace it with the new definition--some instantiation of the adult's situation definition. Wertsch (Wertsch, 1984) summarizes his discussion this way:

> Thus, in order to understand the way in which an individual defines a situation, we have seen that two interrelated issues are involved: the representation of objects and the representation of action patterns for operating on those objects. Furthermore, we have seen that a defining property of the zone of proximal development is that the participants involved in collaborative problem solving have different situation definitions. Finally, we have seen that we cannot account for growth in the zone of proximal development solely in terms of quantitative increments to an existing situation definition. Rather, we must recognize that a fundamental characteristic of such growth is what one might term *situation redefinition*--something that involves giving up a previous situation definition in favor of a qualitatively new one. (p.)

Although the situation definitions of each participant, and changes in those definitions, help us see growth within the ZPD, there is a third situation definition that must be accounted for in adult-child interactions. Along the way to the child's acceptance of the adult's situation definition, they work together to complete an agreed upon, or intersubjective, task. The agreed upon task constitutes a third situation definition that initially may mirror the child's and gradually becomes a combination of the child's and the adult's situation definitions. As a result, adults often work with a situation definition that is different from the one they would use if they were working independently. Adults, even though they may work on a situation that reflects both their definition and the child's, do not change their "ideal" definition. The only lasting change is on the part of the child.

Intersubjectivity hinges on using language in ways both participants understand within a situation in which they agree to work. Luria (1976), in a study conducted in the early 1930s, compared subjects who had no schooling with those who had several years of formal instruction. The subjects with no schooling were unable to categorize items in ways the researchers thought were correct. Instead of using theoretical classifications, they grouped the objects on the basis on concrete situations or practical uses. In this study, Luria presented the subjects with pictures of four items--a hammer, a hatchet, a saw, and a log--and asked them which of the four items did not belong to the group. Although the researcher and the subjects were able to communicate, they often understood the task in very different ways; that is, they did not have an intersubjective situation definition. One 39 year old, nonliterate subject had this conversation with the researcher.

1	Subject:	They're all alike. I think all of them have to be here. See, if you're going to saw, you need a saw, and if you have to split something you need a hatchet. So they're all needed here.
2	Researcher:	Which of these things could you call by one word?
3	Subject:	How's that? If you call all three of them a "hammer," that won't be right either.

4 Researcher: But one fellow picked three things--the hammer, saw and hatchet--and said that they were alike.

5 Subject: Probably he's got a lot of firewood, but if we'll be left without firewood, we won't be able to do anything.

6 Researcher: True, but a hammer, a saw, and a hatchet are all tools.

7 Subject: Yes, but even if we have tools, we still need wood--otherwise, we can't build anything.

In this interaction the subject and the researcher interpreted the situation in different ways. The researcher brought with him a situation definition and a set of responses he deemed correct. The responses resulted from a classification system based on what Wertsch and Minnick (Wertsch, 1985; 1990) refer to as "sign type--sign type" relations that exist independent of any specific situation. In these relations a saw, hatchet, or hammer are always tools, but a log is never a tool. The subject, in lines six and seven, rejected the use of a general term for the things in the set. Instead, he grouped the items in terms of a situation in which he might use them--cutting firewood. In this situation, unless the person has "a lot of firewood," he or she would need all four items. So, although the researcher and the subject were able to communicate--that is, the words they used signified the same objects--they did not agree on the "sense" of the situation. Intersubjectivity goes beyond sharing word meanings and includes sharing a situation definition.

Adults and children negotiate the intersubjective situation definition. To do this, they offer "bids" to each other that may or may not be accepted. According to Wertsch, adults are more conscious of their bids than are the children. They use different bids to shape the way the children view the situation. These bids constitute the third mechanism within the zone of proximal development: semiotic mediation.

While working with children, adults often overtly direct the children to do a certain thing. For example, if an adult and child were working on the modeling task described above, the adult may tell the child which piece to choose and where to put it in the model.

In this case, the child would not need to understand anything about the task, only that by following the adult's instruction they can complete the task. In a different scenario, the adult might ask the child which piece they need next. Depending on the child's response, the adult may revert back to a more overt directive to get the child to move closer to the adult's situation definition or change their temporary situation definition to more closely reflect the child's. Finally, the adult might refer to the model to gently nudge the child closer to their own situation definition. In each of these cases, however, the child can either accept or refuse the adult's bid and continue using their own situation definition to complete the task.

Wertsch's (1984; 1985) model of semiotic negotiation within the zone of proximal development attempted to describe the scaffolding process in greater detail. It focused on what the adult does to entice the child to see the situation as they do and to mimic their actions as they work through it. The changes in the child's actions during the conversation represents a transformation from "other-regulated" activity to "self-regulated" activity [Wertsch, 1985 #643. The adult, Wertsch argued, uses a set of semiotic "moves" (Wertsch, 1984) to direct the child's actions toward the desired endpoint. The moves grow increasingly directive as the adult assesses the child's needs.

Wertsch's early work--and scaffolding in general--while being criticized for portraying adult-child interaction as being determined solely by the adult, has spawned a host of research on learning within the ZPD (Elbers, 1991; Elbers, Maier, Hoekstra, & Hoogsteder, 1992; Griffin & Cole, 1984). The child in early models, the critics allege, is represented as a passive being who is willing to obey any adult wish. The end product of the conversation is the child working alone, using the strategies displayed by the adult in the conversation, to solve the problem. In this model, there is no room for the creation of new knowledge by the child or altering what is presented by the adult.

Elbers and his colleagues (1991; 1992) have argued that children are not passive recipients, but, instead, actively participate in the interactions they have with adults.

They help to define and complete tasks and contribute to what Elbers and his colleagues refer to as the "mode of conversation." The mode of conversation is a "contract" that the participants negotiate. It includes the "norms, mutual obligations and expectations" (Elbers et al., 1992, p. 106) that guide the interaction. The researchers describe four modes that characterize conversations: task oriented, instructional, playful, or affective. How the participants interact depends on how they perceive the conversation. To illustrate this point the authors point to a study in which researchers compared the interaction between young children and their mothers and the young children and their teachers (Olthof, Goudena, & Groenendaal, 1989). The researchers found that children interacting with teachers took much less initiative then they did in their interactions with their mothers. The authors contend that the students expected their conversations with teachers to include more direct instruction; that is, they expected to be told and shown what to do and to copy the demonstration. As a result, they did little until they were told what to do.

Even in the school-like interactions where the children said little, they actively negotiated their role in the conversation. Taking less initiative was a conscious choice based on their expectations for conversations with teachers. This led Elbers and his colleagues to suggest that the observable change in the child's actions represents a shift from "joint-regulation" to " self-regulation" (Elbers et al., 1992) rather than the "other-regulation" to "self-regulation" shift posited by earlier conceptions of learning within the ZPD.

Sociocultural Influence on the Direction of Development

As we have seen, new ideas and psychological processes begin to develop through social interaction. Fundamental to this conception of learning is that adults know more than children and are able to mediate a given body of knowledge to them. As the children or adult learners continue to interact, the newcomer's ways of thinking and acting begin to resemble those of the more experienced person. But, where do the more experienced person's thoughts and actions come from? One response to this question is that they are the product of participating in various socially-defined practices. Practices and the tools and symbols associated with them shape the way participants think and behave while engaged in the practice. As a result, thoughts about important questions and intelligent ways of answering them are situated in the practices and the practices, themselves, determine the direction of development. This deterministic conception of practices has led to a growing body of research on "situated cognition."

In situated cognition studies involving mathematics, participants undertook and solved complicated tasks involving mathematical calculation. The methods they used to complete the calculations, however, were sometimes different from the mathematics normally taught in school. Perhaps some of the best known of this research is Sylvia Scribner's (1984) work with diary workers, Jean Lave's (1988; Lave et al., 1984) work with grocery shoppers, and Nunes, Schliemann, and Carraher's (Carraher et al., 1985; Carraher, Carraher, & Schliemann, 1987; 1993) work with Brazilian street vendors.

In Scribner's (1984) dairy research, the workers constructed a mathematical system that allowed them to fill orders and load trucks without resorting to written computation. Each day the "preloaders" were given "load-out" order forms that listed the products and amounts the delivery drivers had ordered. The load-out order forms were produced by a computer that "cases out" each order by converting units into cases. If the number of units ordered did not equal a full case, the form listed the number of cases and an additional amount of units. If the extra number of units was less than half of the case, the additional amount was listed "+ x," and represented the number of units that needed to be added to the full cases. If the additional amount was more than half a case, the number of cases was increased by one and the additional amount was expressed as "- x"--the number of units that needed to be removed from the extra case. For example, because half gallons of milk came nine to a case, 21 half gallons would be represented "2+3" in the half-gallon column and 49 pints, which came in cases of 32, would be written "2-15."

Scribner found that the workers used the "least-physical-effort solution" to each of the problems. For example, in the diary quarts of milk came sixteen to a case. If a preloader received an order for 1-6, he or she would need to end up with one full case and another with 10 quarts of milk in it (16-6=10). If the preloader had the option of using a full case and removing 6 quarts from a second case (the literal strategy) or using a case with 2 quarts already in it and adding 8, the literal strategy requires less physical energy as only six units needed to be moved as opposed to eight in the alternative strategy. If, in another situation, the partial case had 8 quarts and only 2 quarts needed to be added, filling the order as 8+2 would be the least-physical-effort solution saving four moves.

The dairy workers were able to do these problems very quickly. They did not count the number of cartons in the partial cases. Rather they shortcut the arithmetic and did it merely by looking at the case. One preloader explained how he filled an order for 1-8 quarts of milk (half a case) this way:

> I walked over and I visualized. I knew the case I was looking at had ten out of it, and I only wanted eight, so I just added two to it. I was throwing my self off, counting the units. I don't never count when I'm making the order. I do it visual, a visual thing, you know. (Scribner, 1984, p. 26)

Lave and her colleague's (1988; Lave et al., 1984) grocery shoppers constructed mathematical systems in which they accounted for factors other than savings or mere numbers of items needed. The product or brand that represented the best value was often one that met requirements other than its cost. Smaller, more expensive packages were chosen because they fit into available kitchen storage. Smaller packages that cost more per unit, but less in total cost, were chosen to stay within a monthly budget. Large quantities of perishable items, even though they cost less, were not purchased if there was a chance of them spoiling before being eaten. In general, arithmetic calculation of cost was used only as a last resort--when the shoppers had narrowed their choice to a couple brand names and had no preference among them.

When the shoppers did use arithmetic calculations, they, on average, did 2.5 calculations. Although the early computations were often not correct, the shoppers' final calculation and decision concerning the best value was correct an astonishing 98% of the time. In doing these calculations, the shoppers nearly always simplified the computation. For example, one shopper compared the cost of different sized boxes of spaghetti: a four pound box cost $1.98 and a two pound box $1.12. The shopper divided $2.00 by four getting $.50 and $1.20 by two getting $.60 to compare the cost. On the basis of these calculations, the shopper determined that the four pound box was the better deal (Lave et al., 1984, pp. 84-88).

In Brazil, Nunes, Schliemann, and Carraher (Carraher et al., 1985; Carraher et al., 1987; 1993) found that young street vendors computed the price of their produce in ways quite disparate from those taught in western classrooms. For example, one street vendor (M.) computed the price of ten coconuts this way:

> Customer: How much is one coconut?
>
> M: Thirty-five.
>
> Customer: I'd like ten, how much is that?
>
> M: [pause] Three will be one hundred and five; with three more, that will be two hundred and ten. [Pause] I need four more. That is . . . [pause] three hundred and fifteen . . .I think it is three hundred and fifty.
>
> (Nunes et al., 1993, pp. 18-19)

Nunes et al hypothesized that the computations these children did in the streets was beyond their knowledge of formal algorithms. As a result, they would have difficulty using school routines to solve the problems. To test this hypothesis they administered "informal" and "formal" tests to the street vendors. The informal tests were conducted in the streets. Participant observers asked the vendors a series of questions in which they needed to compute a price. The example given above comes from one of the informal tests. The formal tests included two sections. In the first section the researchers wrote out similar questions in formal notation. Most of these questions contained the exact content asked in the informal test. There were some instances, however, where the researchers asked the inverse question (e.g., 350 ÷ 10 rather than 35 x 10) or moved a decimal point to slightly change the problem. In the second section of the formal test, the researchers embedded the questions in story problems similar to the situations the vendors found in the streets.

As was expected, the street vendors computed the prices with remarkable accuracy in the streets and had poorer results on the formal tests. Across the five subjects, the interviewers asked 63 informal test questions and the vendors answered 61 of them correctly (96.8%). The vendors answered 45 of 61 questions correctly in the formal test word problems (73.7%) and 14 of the 38 questions asked in formal notation (36.8%).

In each of these examples the knowledge necessary to be successful is embedded in the practice or activity in which the person is participating. To understand what a person knows and how that knowledge influences their actions, it seems necessary to watch the person participate in the practice or activity; psychological investigations of knowing and learning need to be situated. Because of this association between practices and the cognitive skills necessary to participate in them, researchers have suggested "the practices themselves need to become objects of cognitive analysis." (Scribner, 1984, p. 14)].[2]

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[2]Mathematics educators also have shown a great interest in the mathematics used in out-of-school settings--but for different reasons. Whereas situated learning theorists focused on mathematical systems used in the course of performing other tasks and how those systems differed from the mathematics taught in school, mathematics educators investigated mathematics in out-of-school situations to develop an elementary school curriculum that would benefit students throughout their lives. Through surveys of businessmen (Wilson, 1911/1922); analyses of cookbooks, factory payrolls, retail sales advertisements, and general hardware catalogs (Mitchell, 1918); analyses of various occupations (business, insurance, plastering, painting, masonry, salesmanship, and banking) (Wise, 1919; Woody, 1922); analyses of the mathematics contained in newspapers and journals (Adams, 1924); and analyses of freshman college courses (Gallaway, 1923; William, 1921); mathematics educators developed an elementary

As in the practices investigated by these researchers, the direction of development in classrooms is determined by the structure of classroom practice. Vygotsky (1986; 1994a; 1994b) wrote that in schools, the direction of development is largely determined by academic disciplines and student learning is a product of connecting the concepts students brought with them to class and the concepts accepted within the various disciplines. Building on Piaget's distinction between spontaneous and non-spontaneous concepts, he suggested there are two types of concepts: *spontaneous* or everyday concepts and *scientific* concepts which represent the academic disciplines. Van der Veer and Valsiner (1991) have summarized Vygotsky's discussion of these two types of concepts this way:

> By spontaneous concepts he meant concepts that are acquired by the child outside of the context of explicit instruction. In themselves these concepts are mostly taken from adults, but they never have been introduced to the child in a systematic fashion and no attempts have been made to connect then with other related concepts. Because Vygotsky explicitly acknowledged the role of adults in the formation of these so-called spontaneous concepts he preferred to call them "everyday" concepts, thus avoiding the idea that they had been spontaneously invented by the child. . . . By "scientific" concepts Vygotsky meant concepts that had been explicitly introduced by a teacher at school. Ideally such concepts would cover the essential aspects of an area of knowledge and would be presented as a system of interrelated ideas." (p.)

In 1935, Shif (van der Veer & Valsiner, 1991), a student of Vygotsky's, conducted a study in which she investigated the development of scientific concepts in second- and fourth-grade classrooms where students were studying the history of the communist movement in the Soviet Union. Drawing on studies conducted by Piaget, Shif designed statements she believed illuminated the child's development of both everyday and scientific concepts. These statements, as had Piaget's, all ended with the conjunctions *because* or *although*. In the study the students were asked to complete each sentence. Piaget's questions had all dealt with what Vygotsky termed everyday concepts and were similar to "The boy fell off his bicycle because . . ." or "The boy fell off his bicycle,

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school curriculum that included basic arithmetic operations that were found to be prominent in these practices.

although . . . ." Shif included questions similar to those used by Piaget as well as those she believed represented scientific concepts. Among Shif's scientific questions were "The police shot the revolutionaries because . . ." and "There are still workers who believe in God, although . . ." (van der Veer & Valsiner, 1991, p. 271).

In the study, the students were asked to complete four different types of statements. The statements can be thought of as a two by two matrix with *everyday* and *scientific* across the top and *because* and although" down the side with each of the four cells representing a different statement. The statements were either scientific/because, scientific/although, everyday/because, or everyday/although.

Shif found that the second-grade students responded significantly better to scientific questions ending in "because." The discrepancy among the scores, she argued, was due to the classroom instruction. In class, the students are required to answer questions about the scientific concepts being presented. These questions, represented specific causal relations among the ideas being taught. As a result, they developed what Shif and Vygotsky have referred to as a "conscious realization" of the relationship among ideas; that is, an explicit understanding of how the ideas are related to each other. In everyday activities, children are not explicitly instructed in the relationship among ideas and are not often able to articulate them. This lack of a conscious realization of everyday concepts, Shif argued, led students to respond incorrectly more often to the everyday questions.

To explain the discrepancy between the second-grade students' scores on the *because* and *although* statements, Shif again turned to the classroom instruction. The relationships among ideas taught in the classroom, she argued, could best be characterized as causal. This relationship is best represented by the *because* conjunction. Students' poorer showing on the *although* questions could be attributed, she argued, to their lack of exposure to the term *although* and to the fact that the *although* conjunction was more difficult for young children to grasp.

Although the second grade students responded correctly to most of the scientific/because questions and were able to provide an explanation of their response, their answers were often echoes of what they were told in class and they were unable to provide concrete examples of the answers they gave. Shif argued that these "schematic" responses were evidence that the students did not truly understand the answers they were giving, but had memorized the correct response from their classroom instruction.

The fourth-grade students displayed an entirely different pattern of responses. The fourth-grade students answered a high percentage of each of the four types of questions. Shif interpreted this finding as evidence that students had mastered the causal way of thinking emphasized in their classroom instruction. She also pointed out that the students' answers had lost the stereotypical character displayed in the second-grade students' responses. The increase in correct answers to the everyday questions, Shif argued, was evidence that explicit classroom instruction leads to certain ways of thinking, that those ways of thinking will gradually spread to other areas and raise the child's thinking to new levels, and that, eventually, children meld their everyday concepts with the scientific concepts presented in formal instruction.

Classroom instruction, in this example, creates a zone of proximal development that is bounded on one side by the child's everyday concepts and the other by the scientific concepts being introduced in school. Students' everyday concepts shape the situation definitions they bring to class and the formal ways of thinking represent the situation definitions of the teacher. Through extended negotiation, they gradually bring together these sometimes disparate conceptions of the same task or concept until they reach a common knowledge or intersubjectivity (Au, 1990; Edwards & Mercer, 1987; Panofsky, John Steiner, & Blackwell, 1990).

Mathematics educators have suggested their classrooms function much like those described by Shif (Bishop, 1991). Students in mathematics classes, they argue, learn mathematics in ways specific to the classroom in which they are participating. In this way, mathematics classes form their own culture, which includes a set of beliefs and values that shape how and what mathematics is taught and learned (Bishop, 1991). The beliefs and values include informal theories of the origins of mathematical knowledge, the goals of mathematics, and the object of mathematical study. Based on different sets of beliefs, teachers might choose to emphasize computation, problem solving, logic, inquiry, or other aspects of mathematics. Schoenfeld described the relationship between beliefs and instruction this way:

> Goals for mathematics instruction depend on one's conceptualization of what mathematics is and what it means to understand mathematics. . . . At one end of the spectrum, mathematics is seen as a body of facts and procedures dealing with quantities, magnitudes, and forms, and the relationships among them; knowing mathematics is seen as having mastered these facts and procedures. At the other end of the spectrum, mathematics is conceptualized as the "science of patterns," an (almost) empirical discipline closely akin to the sciences in its emphasis on pattern-seeking on the basis of empirical evidence. (Schoenfeld, 1992, pp. 334-335)

Based on these arguments, it is believed that changing teachers' beliefs about mathematics, and consequently their instructional goals, will change students' conceptions and knowledge of mathematics.

The deterministic conception of practices and classrooms ignores the fact that different people contribute different things to the practices in which they participate. As a result, the practices that shape the participants' thinking and behavior are, in turn, shaped by the participants; to understand a person's participation, we must understand his or her contribution to the practice. Both mathematics educators and situated learning theorists have rejected the deterministic conception of contexts or practices (Cobb et al., 1993; Lave, 1993) and have begun looking at the relationship between individuals and the social structure that surrounds them. One way of conceptualizing this relationship is that people bring with them their experiences in other socially defined practices. These experiences constitute their contribution to the immediate practice. To understand a person's participation in a practice, it is necessary to look beyond the immediate practice to the other practices in which a person participates. This influence has recently become the focus of research.

Saxe (1990a; 1990b), while investigating the mathematics used by candy sellers on the streets in Racife, Brazil, sought to explain the influence of school mathematics on the sellers' conception and computation of mathematical problems that emerged in their practice. At the same time, he sought to explain the influence of selling candy on the sellers' performance on school-math tasks.

To investigate the influence of school mathematics classes on the sellers *in situ* computations, Saxe compared the pricing and selling strategies used by sellers with different amounts of school experience. Through a series of observations and interviews, he found that candy sellers with more school experience used different--although no more efficient or mathematically sound--strategies to price the candy in practice. Saxe characterized the strategic differences this way:

> The first grader conceptualizes the mathematical relation with respect to the selling convention--3 bars for Cr$1000--a convention linked to the retail transaction itself. The first grader's solution has a direct mapping on the actual operations of exchange in that each count of three by Cr$1000 represents a transaction. In contrast, the schooled child conceptualizes the mathematical relation with reference to an intermediate value that is not a part of any aspect of the actual transaction itself--the wholesale price per unit, a value that is accessible by the use of the standard division or repeated application of a multiplication algorithm. Using this approach, the schooled seller marks up the derived wholesale price and performs another computation to translate the mark-up into the street convention. Such a solution strategy is both distant from the seller's anticipated transactions and it is also powerful; the solution strategy is one that can be used across many types of problems regardless of the particular selling convention. (Saxe, 1990b, pp. 225-226)

Candy sellers with more school experience were also better at recognizing written numerals, such as those listed on price posters in the wholesale markets. There were no differences, however, among the groups in their ability to recognize and differentiate between different currency denominations.

To investigate the influence of selling candy on the sellers' approach to school math problems, Saxe presented a series of arithmetic problems to second- and third-grade sellers and nonsellers. The candy sellers computed more problems correctly than did nonsellers and used more flexible strategies to complete the computation. The candy sellers more often used what Saxe referred to as "regrouping strategies." In these strategies, the sellers broke numbers up into like parts and performed the operation on those parts. For example, to add 28 + 26, the sellers might have added 20 + 20 = 40 and 8 + 6 = 14, then added 40 + 14 = 54.

Beach (1990; 1990) followed high school students as they moved from school to the workplace in Nepal. At the time of this study, Nepal had just instituted a public education system which included mathematics classes in which western algorithms were taught. The introduction of western mathematics into the village resulted in a growing contempt for traditional mathematical systems. The high school students--and sometimes the shopkeepers themselves--referred to the mathematics used in local shops as "guesstimate" or "dumb math." In spite of this belief, shopkeepers continued to use the older mathematical systems in their businesses. The high school graduates Beach followed from school to work were part of the first class to graduate from the new Nepali school system and the first to take western mathematics into the shops.

Following graduation, students were placed in apprenticeships in local shops. As they began their new jobs the students tried implementing the mathematics they learned in school. To determine the price of different wares, the recent graduates wrote out the prices and quantities in traditional western algorithms. This process proved too time consuming and cumbersome. In its place, shopkeepers instructed the apprentices in the ways they computed prices in the shop. These procedures were done mentally and emphasized speed over accuracy. Although the shop procedures were not as accurate as the traditional algorithms, they estimated measurements and costs accurately enough to guarantee profits in the shop. In the end, the students combined the two mathematical systems to construct a system that incorporated both their school mathematics and traditional shopkeeping.

Both Saxe's and Beach's studies point out the influence participating in one practice has on participation in subsequent practices. In each case participants drew on their experiences in one practice--schooling--to assist them in completing tasks another practice--shopkeeping and selling candy. In each case the skills learned in one practice did not remain intact as they moved from one practice to the next. Rather, the apprentice's skills were modified in some way to accommodate the exigencies of the new practice.

But at the same time these studies point out this influence, they conceptualized the influence as unidirectional--what was learned in one practice influenced what was done in a subsequent practice. In Saxe's study, for instance, the sellers left school before beginning to sell the candy. As a result, Saxe observed sellers in the streets, but did not observe students in schools; participation in schools and how sellers might approach school-like problems was simulated in interviews. In Beach's study, the students finished high school before entering the shops. Neither study looked at people engaged in two practices simultaneously, yet we are never engaged in only one practice. To fully understand the mutual influence of practices it is necessary to observe a person's participation in two related practices.

Students in elementary mathematics classes work on their school math tasks both in school and at home and, as a result, these tasks provide an opportunity to explore the mutual influence of various practices. The conversations students have in each of these settings constitute different socially defined practices--each of which would have its own set of beliefs and values about mathematics and homework. Mathematics may, for example, be understood as a set of computational rules or as a system of inquiry. Homework may, for example, be interpreted as a way for parents to monitor what their children are doing in school, or it may be interpreted as an ongoing assessment of student ability, or it may represent an infringement on parents' time and energy. Each of these conceptions of mathematics and homework may influence how students complete their school math tasks. And, because students work on the same task in the two settings, I believed it would be possible to see the influence the two practices have on students' completion of their school-math tasks.

The home-school relationship and mathematics homework has been the focus of a tremendous amount of research. I turn now to a discussion of that work.

The Home-School Relationship

Educators have long been concerned that underprivileged schoolchildren fail at a greater rate than other students. Much of the research devoted to explaining this disparity has focused on the activities or characteristics of students' homes and environments that lead to either school success or failure. The research focused on school success has looked at the characteristics of students' homes that reliably predict high levels of school achievement and have resulted in prescriptions for improving the environments of "culturally deprived" children. Because middle-class students performed better in school, it was believed that the characteristics of middle-class homes would provide a window onto school success. Programs such as Head Start, which were designed to provide at-risk students with the experiences necessary to succeed in school, grew out of this research tradition.

The research seeking an explanation of school failure has focused on the notion that schools do not adequately meet the needs of their students. In general, this research suggests that students "achieve" school failure by actively choosing to "not learn" in order to save face or preserve their own cultural traditions (Kohl, 1984; McDermott, 1987). The need to save face, these researchers argue, comes about as a result of schools' ignoring students' cultural backgrounds and the ways they learn things at home. Much of this research has focused on discontinuities in the ways language is used in students' homes and their schools. Researchers focusing on discontinuities between the home and school have suggested that, if teachers were more knowledgeable about how their students interact in their homes, they would better understand their participation in the classroom. What are often interpreted as behavioral problems could instead be understood as culturally appropriate behavior. This understanding would lead to increased communication and higher student achievement.

In this section I review both bodies of literature. All of the research reported in this section focuses on the development of literacy skills. I found no research looking at the characteristics of students' homes that predict success in school math or research that suggested a discontinuity between the mathematics used in the home and that taught in school. Although the research focuses on literacy, there is much to be gained for an investigation of mathematics learning. If different ways of interacting in school and at home exist and result in difficulty in school, it is likely that different ways of thinking about mathematics, especially in the wake of the recent reforms in mathematics education, do exist at home and in school and influence what and how students learn in school math classes.

The Influence of the Home on the Development of Literacy Skills

Catherine Snow (Snow, Barnes, Chandler, Goodman, & Hemphill, 1991) and her colleagues at Harvard undertook a massive study in which they investigated the influence of students' home activities on their scores on literacy measures. Snow and her colleagues reviewed much of the research addressing these issues and, from it, constructed three models of home influence on students' development of literacy skills. The study they undertook was an attempt to test the adequacy of the three models in explaining students' scores on four different literacy outcomes.

The study, conducted over two years, included 31 families with children in 11 classrooms representing four schools.[3] Snow and her colleagues collected data that illuminated three areas: Literacy activities in students' homes, literacy instruction in school, and a measure of students' literacy.

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[3]The number of classrooms grew to 25 in the second year of the study as students were not kept together as they moved from one grade to the next.

In the homes, Snow and her colleagues conducted interviews with family members. Although an attempt was made to interview all family members, the interviews often included only the mother and the children. The interviews focused on seven content areas: (a) basic demographics; (b) educational level of the parents and their literacy practices; (c) children's educational histories and their literacy practices; (d) parents' view of their children's school and teachers and their relationship with the school; (e) children's educational aspirations and parents' educational expectations for their children; (f) the financial and emotional stress family members were experiencing; and, (g) the family's television viewing habits and other leisure time activities.

Students in each family were asked to fill out a "time allocation diary" twice each year of the study--once during the school year and once in the summer. The diaries included a log in which students documented what they did, where they did it, and with whom they did it each 15 minutes from 6:00 AM to 11:00 PM. Each diary also included a seasonal list of activities in which the students may have participated; the school year diary included a checklist of activities students may have done while in school and the summer diary included a similar list of out-of-school activities. Students were asked to check off all of the activities in which they participated each day.

The researchers also observed each family for 30 minutes while they worked on a "homework-like" task. During the task the students, along with one or both of their parents, completed one page of the time allocation diary. The researchers used Bale's (1979) System of Multiple Level Observation of Groups (SYMLOG) to rate the participants in these conversations on three structural dimensions of group interactions--negative/positive, dominant/submissive, and task oriented/emotionally expressive. Although the researchers found these interviews illuminating, they focused mainly on the data collected in the interviews conducted with the families to test the three models of home influences on students' literacy skills.

In the schools, the researchers looked at school records to document each student's history, determine if the students had, at any time, required special assistance from the school, or had other things in their histories (e.g., health problems or absenteeism) that may have contributed to their achievement level. The researchers collected, among other things, students' grades and standardized test scores, the number of times the students had changed schools, and whether they had ever repeated a grade.

Snow and her colleagues also administered questionnaires to teachers. The questionnaires focused on two broad areas: their teaching methods and the thoughts on the focal students. The questionnaire asked them about their classroom activities, instructional emphases, contact with parents, focal students' strengths and weaknesses, and their expectations for students. Finally, each classroom was observed for a total of three hours. The three hours were broken down into three one-hour observations of reading instruction, other teacher-led instruction, and one hour of less structured time.

The researchers used three measures to derive four literacy scores for each student. First, the researchers administered the Diagnostic Assessment of Reading and Teaching Strategies (DARTS). Although they administered the entire battery to remain consistent with the test administration instructions, they used the scores from only two sections of the test. They used the students' scores on the word recognition and reading comprehension sections of the test. Second, they collected two writing samples from each student each year of the study. Each year the students were asked to write a narrative describing a picture of an elderly woman holding a package of three tomatoes and an expository paper about a person whom they admired. Their writing quality was measured by the number of words in the sample. Length scores, the authors claimed "correlated highly with holistic scoring results, with measures of syntactic complexity, and with measures of the quality of content." (Snow et al., 1991, p. 57). Third, to measure the students' vocabulary skills, they administered the vocabulary subtest of the

WISC-R. As with the DARTS test, the entire WISC-R was administered to remain consistent with the administration directions.

From the data collected from these activities the researchers developed rating scales and formed summary variables that were grouped together in different ways to test three models of home influence that represent the abundant literature on the home-school relationship. Each of these models was based on an explanation of student literacy success or failure commonly found in the literature. The three are models are: (a) the family as educator; (b) the resilient family; and, (c) the parent-school relationship.

The family as Educator. This model is based on the argument that families that serve as educating agents better facilitate their children's success in school (Bloom, 1976; Bradley & Caldwell, 1984; Iverson & Walberg, 1982; Walberg & Marjoribanks, 1976; Ware & Garber, 1972). Although the titles are similar, this notion represents a fundamental difference from the research conducted at the Elbenwood Center for Research on the Family as Educator. Elbenwood researchers assume that education is a fundamental social process that infuses all family activities. As such, all families are educating agents. The "family as educator" model presented by Snow and her colleagues, however, compares those families that educate their children and those that do not.

To construct this model of home influence, the researchers drew on previous work in which researchers attempted to identify educational characteristics of families and households that predicted school success. They constructed a model that includes five components. First, the literacy environment of the home was measured by two summary variables: the observer's ratings of parental literacy and the observer's ratings of the provision of literacy. The second component of the model assessed the amount of direct teaching done by the parents. This component comprised two variables, how frequently parents helped with homework and the positive/negative dimension of the SYMLOG rating scale of parent-child interactions during the homework-like task. The third component included whether and how often parents created opportunities for their children to learn.

This component was measured by three variables, the number of outings with adults in a typical week, whether parents restricted television watching time, and whether the content of television was monitored by the parents. The fourth component measured the level of parental education. Finally, the fifth component measured the educational expectations parents had for their children.

The Resilient Family. The resilient family model of home influence is based on the notion that different families experience different amounts of stress, and the amount of stress a family experiences has an effect on the children's development of literacy skills (Belle, 1982). At the base of this argument is the belief that working-class families tolerate greater amounts of financial stress than do middle-class families. Often this stress is compounded by marital discord, the absence of one marital partner, alcoholism or drug abuse, or other factors that contribute to a stressful household. Many families overcome these obstacles to provide an environment which supports and encourages literacy development. The characteristics of the families able to overcome such hardship constitute the resilient family model.

To test the resilient family model, Snow and her colleagues developed three measures. The first of these measures focused on the organization of family activities. The variables in this component include whether parents monitored what their children watched on television, whether they controlled the amount of time allowed for television viewing, and an overall organization score. The overall organization score was a composite score that included "the presence of rules for behavior, some predictability in scheduling of daily events, reliability of family members in meeting responsibilities, punctuality, physical neatness, and cleanliness of the house." (Snow et al., 1991, p. 92). The second component of this model measured the emotional climate of the household. The component included the children's perspectives on their relationships with their parents, how punitive or nurturing their parents were and the frequency of opportunities for having fun with either their parents or with other children. This component includes things like the number of outings with parents, the amount of time parents spend with their children, parent-child relationships, and a punishment scale. Finally, the model included a measure of family stress. This component included the family's income, any life changes that may have influenced the children's literacy development and a summary rating of the stress on the family and the individual family members.

Parent-School Partnership. The parent-school partnership model sought to test the argument that parents' consistent involvement in their children's education will lead to increased student achievement (Epstein, 1983; Epstein, 1986a; Epstein, 1986b; Epstein, 1988; Marjoribanks, 1979; Walberg, 1984). This model included measures of the parents' formal involvement with their children's education. Parents who were formally involved often were members of the parent teacher organization, served as a volunteer aid, or went along as chaperones on school field trips. Another variable included in this model was the amount of help parents gave their children with their homework. Parents who helped their children with their homework exemplified this model in two ways. First, they supported the kind of learning that goes on in school. Second, they accepted the responsibility for their children completing their school work. Related to parents' helping with their children's homework is the nature of parent-child interaction during the homework-like task. Parents have the power to make homework a positive or negative experience. Students whose parents earned a positive rating on the SYMLOG scale had higher literacy scores. The parent-school partnership model also included the number of contacts parents had with teachers and students' tardiness records.

Each of the three models explained most of the variance on one or more of the four literacy scores. The family as educator model explained 45 percent of the variance in children's word recognition skills and 60 of the variance in the children's vocabulary scores. Individual variables in the family as educator model that accurately predicted student literacy scores include: (a) the home literacy environment; (b) mother's educational level; and, (c) mother's educational expectations for their children. The resilient family model accounted for 43 percent of the variance in students' writing scores. The variables most closely associated with high writing scores included: (a) parent-child relationship during the homework-like task; (b) observer ratings of organization in the home; (c) presence of television viewing rules; and, (d) the number of activities listed in the time allocation diaries. The parent-school partnership model explained "respectable amounts" (Snow et al., 1991, p. 118) of variance on each of the four literacy measures (21 percent on reading comprehension, 32 percent on word recognition and writing, and 38 percent on vocabulary). Of the variables included in this model, parents' formal involvement accounted for the majority of the variance as it correlated highly with each of the four literacy scores.

Summary. The research conducted by Snow and her colleagues, and those that came before them, is valuable in pointing out that many factors contribute to a students' development of literacy skills. Different familial activities contribute in different ways to a child's success in school. Taken to perhaps a flippant extreme, problems with differential school achievement could be solved if we could make sure all families of school age children have characteristics similar to those identified with school success; that is, we can change the rate of school success by changing households. At the same time, however, findings that associate more formal education for parents, higher educational expectations, families that emphasize literacy activities, homes where the rules are similar to those in school, a wide variety of educational activities, and parental involvement in school functions with increased literacy scores seem to suggest that schools are serving the needs of only a small segment of the population. This finding is supported by many other reports of life in schools (Powell, Farrar, & Cohen, 1985; Sizer, 1992). Is it possible to change schools in order to accommodate the ways of thinking, knowing, and doing that exist in other segments of our population? What happens to the other students whose lives don't match up well with their classrooms? These questions are addressed by the researchers whose work is presented in the next section.

The Discontinuity between Home and School

As I mentioned above, the research reviewed in this section has focused on the discontinuity between language use at home and in school. Much of this work has been conducted by anthropologists and sociolinguists using a form of participant observation as their methodology.

In perhaps the best known of these studies, Shirley Brice Heath (1982; 1983) spent several years in two communities in the Piedmont Carolinas. Heath spent five years working with teachers and students in classrooms and in their homes to better understand the interaction patterns of the region. She found that the questions teachers asked in school were very similar to the interaction in their homes. The teachers' questions were characterized by two things. First, teachers often asked questions to which they already knew the answers. Second, the questions were often "about things being about themselves" (Heath, 1982, p. 105), as one student described them. These questions asked students for information about the labels, attributes, or characteristics of objects or events removed from their naturally occurring contexts. As did the homework sessions investigated by the researchers with the Elbenwood center (McDermott, Goldman, & Varenne, 1984; Varenne, Hamid-Buglione, McDermott, & Morison, 1982), these questions required students to display their knowledge publicly. Both of these patterns were in conflict with the interaction patterns in students homes.

In their homes, Trackton children learned to interact with adults in specific ways. They were not considered "information-givers" (Heath, 1982, p. 119) nor were they expected to answer questions to which the adults already knew the answer. The questions Trackton adults asked were about whole activities or objects. They compared events or objects, asked about the uses for an object, explored causal relationships, or started stories. In each of these situations it was wrong to ask questions to which the questioner already knew the answers.

The discontinuities between how interaction patterns in school and at home led students to remove themselves from classroom conversations. Their removal often was interpreted as failure or poor performance in school. Similar relationships between interactions at home and in school have been documented by other researchers.

Susan Urmston Philips (Philips, 1993) spent two academic years and the summer in between on the Warm Springs Indian Reservation in central Oregon. Philips had two items on her agenda during this time. First, she sought to document culturally distinctive uses of the English language, the primary language of all tribal members. Second, students' test scores at the Warm Springs high school were traditionally lower than their anglo counterparts in nearby schools. Although the difference in scores existed across all academic subjects, the difference was greatest on the verbal sections of the tests. Philips wanted to explore the notion that the cultural differences in language use may contribute to the difficulty students were having in school.

Philips found that Warm Springs students learned socially appropriate ways of learning and interacting that were very different than what was expected in school. She found that children in Warm Springs families learned through what she termed the "visual channel." Children often watched their elders while they cooked, chopped wood, sewed, or engaged in other tribal activities. The elders offered very little instruction during these observations. The children would replicate what they watched until they believed they did it correctly. Then they would display their newly developed skill by doing whatever it was they had learned.

Verbalizing was left to the tribal elders who were believed to be much wiser as a product of their age. At Tribal administration meetings, only the elders would discuss important Tribal decisions. The younger members of the tribe were expected to contribute physically. They would carry out the mandates of the tribal elders. Also at the Tribal meetings, members would move their chairs out of rows to the perimeter of the room. This enabled them to see the people with whom they were talking and stopped anyone from sitting behind them.

Most of the conversations the Warm springs Indians had were centered around a specific activity or task. In these conversations they used much less exaggerated actions than did anglos. They did not, for example, maintain a gaze into either the speaker's or listener's eyes and they seldom nodded in agreement or offered "Uh-huhs" or "Um-hmms" to show they were paying attention as anglos often do. They used fewer changes in volume or inflection to reestablish their listeners' attention.

In school, Warm Springs students exhibiting behaviors consistent with Philips' description of tribal conversations were often scolded or punished in some other way. In school they were expected to show competency and understanding by offering answers to teachers' questions. This was in direct conflict with the teaching and learning they experienced before coming to school. These students were accustomed to listening to the elders speak and carrying out what they said and did at tribal meetings. They were also accustomed to learning through observation and demonstrating their competence by performing the task. Offering their ideas was in direct conflict with the ways of interacting common in tribal meetings and in their previous learning experiences. Students were often scolded for not paying attention because they would not maintain a steady gaze at the teacher or nod their head as a sign of understanding. These gestures were culturally determined ways of conveying attention that were not part of the Warm Springs culture.

The discontinuity between children's uses of language in and out of school was also the focus of a study conducted by Sara Michaels (1981; 1986). As part of a larger ethnographic study of communication in the home and school, Michaels focused on the narrative structures students used during "sharing time" in a first-grade classroom. Sharing time, sometimes referred to as "show and tell," was a time in class when during which students could tell the class something they thought was important. The teacher was actively involved in the students' stories, maintaining the floor for them and interjecting questions and comments throughout the story.

Michaels found that the students' narratives fit into one of two categories: (a) topic centered; or, (b) topic associating. In topic centered narratives, students developed the story or theme through a linear presentation of information that described on one event or object. In topic associating narratives, students presented a sequence of personally associated personal anecdotes. At first glance, these narratives appeared to have no beginning, middle, or end and, as a result, no point. After more in-depth analysis, however, the collections of anecdotes in these narratives were found to be related to one event. Thus, students employing a topic associating narrative structure developed their themes through anecdotal association rather than linear description.

Michaels described the class in which she conducted this study as comprising "half white and half black children" (Michaels, 1981, p. 426). The different narrative styles were aligned with the different ethnic groups in the classroom. Topic centered narratives best characterized the stories told by white students and topic associating narratives best describe African-American students', particularly girls', stories. The teacher--an excellent teacher Michaels carefully points out--was skillful at helping the white students craft their stories, but had difficulty working with students who told topic-associated stories. While working with the white students she quickly picked up the topic of the story and helped the students expand it through a series of well-timed questions and comments. As a result, the teacher and student engaged in a synchronized conversation that allowed the student to maintain the focus or direction of his or her story.

With the black students, however, the teacher had difficulty discerning the topic of the story and asked questions that seemed ill-timed and threw the students off course. The difficulties led the teacher to institute a set of rules governing the conversations.

These rules included choosing an important topic to talk about and only talking about one thing--things on which the teacher believed the black students needed to work. The different narrative structures and misinterpretation of prosodic clues led to gross misunderstanding about the children and their abilities.

In another study, researchers at the Kamehameha Early Education Program sought to explain the disparity between the reading scores of children of Hawaiian ancestry and those of majority ethnic groups. When early attempts based on increasing student motivation and questioning students' general intellect failed, these researchers began to contemplate other reasons for the disparity. The hypothesis they posited suggested that there were differences between the ways students learned out of school and what they were asked to do in school and that the differences might impede the students' learning.

In response to their hypothesis, a team of teachers, psychologists, anthropologists, and linguists worked together to develop a culturally congruent reading program. The reading program includes elements of "Talk Story," a Hawaiian story telling tradition. Interactions in talk story are characterized by "mutual participation." The teacher, rather than explaining the story to her students or merely quizzing them on the story, works together with the students to reconstruct the story; that is, they "co-narrate" the story. Co-narration is the joint narration of a story by two or more people that is common to Hawaiian story telling traditions. In the KEEP program, the teacher and students co-narrate the story, adding comments whenever they believe it is appropriate. According to western standards, the children in the KEEP program talk out of turn and aggressively butt into the conversation. But, allowing them to co-narrate stories like this has greatly improved their reading scores.

In each of these studies, students' classroom instruction was vastly different from the ways in which they spoke and learned at home. At home they talked, listened, and conveyed attention in ways that were interpreted as misbehaving or as evidence as a deficit in their classrooms. In the instances where these differences were reconciled, students' achievement increased. Although, there is no evidence that discontinuities similar to these exist in mathematics, the research conducted by these researchers does provide insight into a study of home and school influences on students' completion of school math tasks. As in other classes, students in school math classes will use the interactional patterns they have grown accustomed to at home. As a result, it is important to look for these differences while listening to students' interactions. Second, the existence of discontinuities like these suggest that there may be differences in the conceptions of mathematics used in school and at home. This possibility increases as teachers begin to teach math in the spirit of the current reforms in mathematics education. As mathematics instruction gradually changes, parents' conceptions of math may grow more and more different from those expressed in schools and, eventually, they may conflict. It is equally important to keep an eye open for conceptual differences between the home and the school.

Homework

Over the years attitudes toward homework have oscillated. In the early twentieth century, homework was an integral part of schooling. Two wide-spread beliefs may account for this emphasis. First, people imagined the mind as a large muscle. Taking part in certain intellectual activities exercised the muscle, increased its strength, and disciplined the mind (Brink, 1937; Cooper, 1989). One of these activities--memorizing names, dates, and facts--was easily accomplished in the home. As a result, teachers frequently sent memorization tasks home.

The second belief may have arisen in response to the first one. Around the turn of the century, Thorndike and Woodworth (1901) rejected the notion that the brain is a muscle and that academic practices such as memorizing dates and facts exercise it. In its place, they suggested that knowledge comprised a series of relationships between a person and the environment that was represented by a collection of bonds between environmental stimuli and appropriate responses. Once a person established a bond, he or she would exhibit the same response each time they came across the stimulus. Educators needed to provide their students with tasks that would strengthen the appropriate stimulus-response bonds. Students could strengthen their stimulus response bonds by practicing the appropriate responses as part of their homework.

The sentiment surrounding homework changed often during the first three quarters of the century. At times educators were calling for reforms that are being echoed today (Cooper, 1989). In these calls, educators asked teachers to emphasize problem solving rather than the drill and practice advocated by earlier theorists. And, like the calls we hear today, educators questioned the usefulness of homework. Indeed, the life adjustment movement suggested that homework intruded on students' rights to pursue other out-of-school interests (LaConte, 1981). At other times educators believed teachers needed to assign more homework. For example, after the Russians launched Sputnik Americans became concerned that schools were not preparing students for the future. In response, educators looked to homework as a way of accelerating students' acquisition of subject matter knowledge.

In the 1960s, students' "home environments" led to another controversy around homework (Austin, 1979; Cooper, 1989). In homes where academic work was valued, students were expected to complete their homework and perform well. This expectation, educators argued, had mixed effects. On one hand, working hard on their homework assignments could develop students' self-discipline and good attitudes toward school. On the other hand, the emphasis on performing well might get parents to move beyond the role of tutor or assistant and complete the students' work or students may copy from other students. In other homes, it was believed noneducated parents could not help their children learn the material they needed to know.

Although the sentiment toward homework has changed over time, the research on homework has focused on the same small group of questions. Reviewers (Austin, 1979; Coulter, 1979; Keith, 1987) have identified three major paradigms of research on homework. The first of these paradigms investigated the effects of homework on students' school achievement. Cooper (1989) has further refined this category to include subcategories of homework versus no treatment, homework versus in-class supervised study, and time spent on homework assignments. In all of these studies students' achievement was measured either by their scores on standardized achievement tests or on teacher constructed tests. The major findings of this group of studies tended to support the view that regularly assigned homework increased student achievement. Somewhat surprisingly, some researchers in this paradigm drew other conclusions (unsupported according to Coulter (1979)) and suggested the abolition of homework.

For example, in 1937, DiNapoli (1937) investigated the influence homework had on fifth- and seventh-grade students in nine New York elementary schools. The design consisted of two conditions: compulsory homework and no compulsory homework. Fifth-grade students in the compulsory group were given up to one hour's worth of homework each evening; seventh-grade students were given up to one and a half hours. Students in the no compulsory homework condition were not required to do any homework although many chose to. DiNapoli found the compulsory group of fifth-grade students scored higher on achievement tests, but the differences between the conditions in seventh grade were insignificant. From these findings DiNapoli concluded homework was not a good idea.

In another study, Crawford and Carmichael (Crawford & Carmichael, 1937) spent six years investigating the effects of homework on students' school achievement. The researchers in this study, rather than developing an experimental situation, investigated the change during a district wide policy shift in the El Segundo, California schools. During the first three years of this study students were assigned mandatory homework. The second three years no homework was assigned. Each year the researchers administered the Stanford achievement test to fifth-, sixth-, seventh-, and eighth-grade students. Crawford and Carmichael found that the SAT scores were better the first three years than they were the second three. This difference was attributed to compulsory homework.

Maertens and Johnson (1972) conducted a study in which the parents of 400 fourth-, fifth-, and sixth-grade students were trained to provide feedback on arithmetic assignments. The study included three conditions. In the first condition teachers assigned no homework and did not allow students to complete unfinished work at home. In the second condition teachers assigned homework daily and parents were instructed to provide feedback immediately after each problem. In the third condition, students were assigned homework and parents provided feedback but only when the student had completed the entire assignment. The researchers found no difference between conditions two and three, but students in those two conditions scored better than students in condition one. From these findings the researchers concluded that homework structured to include parental involvement and feedback enhances student achievement.

More recently, Gray and Allison (1971) looked at the relationship between homework and mathematics achievement and found the differences between the homework and nonhomework treatments to be insignificant. In a series of interviews conducted after the study, these researchers found that no student understood the fractional concepts and computation being taught. From this study the researchers concluded we need to be careful attributing success or failure to homework.

In the late sixties and early seventies there was a shift in the focus of research in this group to looking at the effects of certain kinds of homework on certain types of students. This research compared students in homework conditions to students in supervised study halls and compared the effects of homework on high and low achieving students. Doane (1972), for example, found a significant relationship between homework and high achievers and an insignificant relationship between homework and low achievers. Ten Brink (1967) found that higher achieving students thrived under traditional homework conditions, but not under supervised study. The results were reversed for low achieving students.

In all of the studies included in the first group, homework was conceptualized quantitatively (i.e., amount of time spent working, whether or not is was assigned). In general, the researchers concluded that homework increases students achievement and ought to be an integral part of classroom instruction. In the second group of studies, the question shifted from whether or not homework should be assigned to how homework assignments ought to be structured and introduced in the classroom. Researchers working in this paradigm investigated how homework should be organized to accomplish different tasks such as introducing new topics or reviewing topics already covered.

One group of researchers (Butcher, 1975; Laing, 1970; Urwiller, 1971) looking at these questions compared the effects of distributed or spiral assignments and massed or traditional homework assignments. In the distributed assignment condition students were assigned sets of exercises that included things taught that day as well as exercises reviewing topics covered in past lessons. In the massed or traditional condition students were assigned exercises pertaining only to the topic discussed on that day. Although none of the finding of this research were statistically significant, the results tended to support the spiral or distributed homework condition.

Another group of researchers (Friesen, 1975; Peterson, 1971) looked at the effects of exploratory homework assignments on students' achievement. These researchers concluded exploratory homework is a good introduction to new material as it provides an intuitive base for instruction.

The third group of researchers looked at feedback and grading strategies teachers might use when homework assignments are brought back to school. In general, they found that when teachers determined students' grades by from number of correct answers, rather than only checking that the homework was completed, led to increased performance.

In sum, homework has been a research topic for quite some time. The research done has focused on three different questions. The first question, should homework be assigned, has generated the largest literature. Conclusions from this work, along with the life adjustment movement and concern over students' home environments led to a large amount of controversy over homework. Although the data suggested homework was a good practice, researchers often claimed the benefits were small enough to warrant the abolition of homework (Coulter, 1979). Later research accepted homework as a worthwhile practice and focused on the structure, uses, and feedback on the work students did. In each of these traditions homework *per se* was not investigated; that is, no one looked at what homework looked like in different homes or explored the nature of the influences it had on students' completion of the tasks--only that it did or did not influence student achievement. There is, however, one group who has looked at homework in different homes. This work out of the Elbenwood Center for Research on the Family as Educator is the subject of the next section.

Families as Educators

To discuss the research on families as educators, I draw heavily on the work of Hope Jensen Leichter (1979; 1985) and her colleagues at the Elbenwood Center for Research on Families as Educators. Outside of the work done at this center, research on families as educators is sparse. The research conducted at the Elbenwood Center, however, provides an excellent view of educational practices within families. As most researchers investigating the relationship between schools and homes have done, researchers at the Elbenwood Center have focused on the development of literacy skills. To investigate this influence they have developed three strands of research: (a) the influence of family activities on the development of literacy skills; (b) families' routines surrounding television viewing; and, (c) adolescents moving through, engaging in, and making sense of educative experiences throughout their social networks.

The work at the Elbenwood Center has led to a set of characteristics of familial life and education. The characteristics, rather than simplifying the study of families as educators, describe the difficulties inherent to such an agenda of study. Perhaps the most telling of these characteristics is that family life consists of simultaneous streams of activity within which are embedded other activities. Family members, for example, may be preparing dinner, answering homework questions, watching television, and cleaning a scraped knee, all at the same time. These embedded activities make the onset and conclusion of action sequences difficult, if not impossible, to identify. As a result, educative patterns are virtually inseparable from other activities.

Interaction among adults and their children change throughout the life cycle of the family. These changing relationships also render educative patterns difficult to predict. Whereas the changing life cycle of the family makes some things difficult, the families share a history throughout the cycle that influences the ways in which they communicate. This history often leads to the development of a shorthand family members use to communicate with each other. Written communication often becomes incomprehensible to others when written in this shorthand and verbal communication is sometimes only accessible to the initiated.

These difficulties in documenting educative situations in families became more apparent as a group of researchers from the Elbenwood center began to look at homework sessions in different households (McDermott et al., 1984; Varenne et al., 1982). This group of researchers spent time with 12 families while they participated in activities that contributed to their children's development of literacy skills necessary for school success. The researchers documented ongoing household activities including housework, doing and checking homework, watching television, conversations, meals, and family outings. They also interviewed both nuclear and extended family members, neighbors, and teachers and other school personnel. The interviews focused on the family histories, past and current literacy practices, each family's history of contact with the school, other family members' school literacy successes, and family members' perceptions of the functions of literacy. After the researchers had established a rapport with the families, they videotaped one event. In each family, the researchers tried to record a formally defined (by the family) homework session. Although some families refused and suggested instead that they record a meal or other family event, the tapes of the homework sessions provided an insight into homework in different students' homes.

Most education arises from the children's participation in activities deeply embedded in the "flow of everyday goals and possibilities" (Varenne et al., 1982, p. 11). Homework, however, was an exception. Unlike other educational activities in the home, homework was a special event. The families had well specified routines for how, when, and where the work would be done. For Joe Kinney, for instance, homework included two separate, but related routines. First, Joe went to his grandmother's house immediately after school where he sat at the kitchen table and worked. His grandmother often helped him with his homework and, at times, did some of it for him. The second routine began when Joe's mother picked him up after work. After picking him up, Joe and his mother went home where Joe finished his homework at the dining room table. Joe and his mother looked over what needed to be done and his mother devised a strategy for completing the work. Once the strategy was determined, Joe worked alone on his homework while his mother prepared dinner and took care of Joe's younger sister.

Most of the recorded homework sessions were divided into two subsections where one of the participants worked while the other waited for his or her turn. In the Kinney family, Joe's mother went first, setting up things for Joe to do. While she did this, Joe actively waited to do what whatever his mother set up. While his mother determined what to do, Joe attended closely so when his turn came he could begin immediately. In another home, the Farrells, the two subsections were reversed. Sheila, the student, did her homework alone in the first subsession. In the second subsession, Sheila's mother checked her work and asked questions to determine how Sheila arrived at the answers on the paper. Although the work was completed by different people at different times, the work represents a collaboration between the students and their family members. The collaboration, rather than being on the actual homework task, comes in the coordination of the different subsessions of the homework session.

Regardless of the sequence of activity in the different households, the families knew how to act "school like" and the interactions between students and family members closely resembled the interaction teachers and students have in school. The "canonical form" (Varenne et al., 1982, p. 104) of these conversations had the adult asking questions to which the student supplied an answer. The questions/answer exchanges were similar to the classroom exchanges identified by other educational researchers (Cazden, 1988; Edwards & Mercer, 1987; Mehan, 1979) where teachers ask students a question to which they already know the answer, the student responds, and the teacher evaluates what the students said. The Elbenwood center researchers identified two types of questions: (a) eliciting questions where the students displayed their knowledge of a certain domain and (b) elaborating questions where the student was asked to explain how they arrived at a certain answer.

Most of the families in which the children were successful in school put great effort into the homework sessions. Their goal for these sessions may not have been for the student to learn something, however, but for them to take correct answers back to school. Through the students' work and their question and answer sessions with family members, the families could ensure correct answers to the homework. This emphasis, the researchers argued, comes from the spotlight placed on students and their family by homework. Teachers and schools not only grade students, but families. As a result, if a family member feels competent, they are unlikely to let their children take wrong answers back to school. As a result, the homework sessions were not instructional sessions, but instead are opportunities for students to display what they have learned in school or other places. The researchers summarized their argument this way:

> Homework is organized as a school knowledge display scene for purposes of evaluation. There is no definite suggestion that in homework children learn. At best they display a knowledge that they have acquired elsewhere and "elsewhen." In no sense can we say that our children learn through their families by doing homework. (Varenne et al., 1982, p. 101)

The work of the Elbenwood center was the first to look at what homework was and meant to different families. At the same time that it illuminated the practice of homework within the different families, it also pointed out that homework creates a link or conduit between the home and school. Through the conduit, school work comes home, and families become responsible for the completion and quality of the homework when it returns to school. The social relationship developed between families and schools shapes how families do homework--what their goals are--and the value they place on it. This relationship, however, is not a direct link between the school and home, but is nested within the other relationships that influence how families operate. Job demands, sibling needs, and everyday household chores, among other things, influence how families organize homework sessions.

Summary

The research presented in this chapter presents a portrait in which all learning results from interactions with more knowledgeable people. By mimicking the more-knowledge-other's actions, we internalize their situation definitions in ways that allow us to use them in subsequent interactions. The direction of development (e. g., what is learned) is determined, or at least shaped, by the socially-defined practices within which the conversations take place.

Elementary school mathematics and homework are socially-defined practices that shape how students think about elementary school mathematics. But the relationship between the home and school is complex. Whereas teachers may send mathematics assignments home to supplement the work they have done in class, parents may see homework as serving different purposes. They may see homework as a way of keeping parents informed of classroom activities, or they may see it as an imposition on their time, or they may see it as a way of monitoring parental obligations. As a result, the goal of homework sessions in students' homes may not be to help students learn mathematics, but to send them back to school with correct answers in order for the family to save face. How homework sessions are organized, the goals families have for the sessions, and the roles children and their parents play in them must be taken into account in an investigation of the influence of homework on students' completion of elementary school mathematics tasks.

CHAPTER 3

METHODOLOGY AND METHODS

The title of this chapter, *Methodology and Methods,* reflects a distinction between the two sections that make it up. The Oxford English Dictionary defines methodology as "the branch of knowledge that deals with method and its application in a particular field." Following from this definition, the section on methodology addresses questions about the phenomenon being investigated and what might characterize an appropriate way of investigating it. Stephen Toulmin (1982) has argued that there is nothing about a given subject matter that dictates the methods that should be used to investigate it. Rather, the methods need to be consistent with the researchers' conception of the subject they are investigating. In the first section, I lay out my conception of learning in practice and the mutually constitutive nature of participating in multiple practices. The discussion focuses on the unit of analysis appropriate for an investigation of learning in practice. Following that I discuss the characteristics of a method that might allow researchers to investigate the mutual influence of participating in various practices.

Method, according to the Oxford Dictionary, is "a mode or procedure; a (defined or systematic) way of doing a thing, especially in accordance with a particular theory or as associated with a particular person." In the methods sections I describe the specific data collection and analysis activities--the methods--used in this study and how they represent the characteristics of the methodology developed in the first section. Specifically, I introduce the people who took part in this study, the neighborhoods in which they lived, and the classroom in which the study was conducted. I discuss the tasks that were used in the study and how the classroom teacher and I developed them. Finally, I discuss the analytic framework I used to begin analyzing the data.

Methodology

Recent advances in psychological investigation have included the notion that psychologists need to adopt a unit of analysis that is larger than, but inclusive of the individual (Cole, 1990; Rogoff, 1990). A unit of analysis of this type departs from psychology's traditional focus on the individual to include people *doing things*. A basic tenet of this new approach is that what and how people know is inseparable from the things they do and where they do them. The search for a more encompassing unit of analysis has led researchers to look at learning in context, in practice, as an event, in activities, and in other settings that appear to influence a person's thinking.

The current emphasis on situated learning or learning in context was foreshadowed in writing of John Dewey (1938/1991). Dewey suggested that the things we do are always connected to the other things we have done; they are part of an experienced world. He referred to the relationship among the things we do as *situations*, which he defined this way:

> What is designated by the word "situation" is *not* a single object or event or set of objects or events. For we never experience nor form judgments about objects and events in isolation, but only in connection with a contextual whole. This latter is what is called a "situation." (Dewey, 1938/1991, p. 72)

Dewey goes on to suggest that psychological investigations have ignored the situated nature of learning and knowing and, instead, have focused on isolated skills or events. He wrote:

> Psychological treatment takes a singular object or event for the subject-matter of its analysis. In actual experience, there is never any such isolated singular object or event; *an* object or event is always a special part, phase, or aspect of an environing experienced world--a situation. (Dewey, 1938/1991, p. 72)

Dewey's call for a new unit of analysis that takes into account the situations people encounter while carrying out a definite purpose (Dewey, 1895/1964) is echoed in the work of many contemporary psychologists and other social scientists (Laboratory of Comparative Human Cognition, 1983; Lave & Wenger, 1991; Leontiev, 1981; Rogoff, 1990; Scribner, 1984; Wertsch, 1981). Consistent to all the calls for change in psychology is the notion that investigations of learning and knowing need to include the things people do; that people learn new things in order to fulfill a goal or for some other purpose and that these things are completed within an expansive, elaborate setting.

Researchers searching for a unit of analysis similar to Dewey's situations use many different terms to express the situated nature of learning. Along with other terms, these researchers often talk about practice, activity, or context. In choosing a term or phrase to describe the things people do, I wanted to accomplish two things. First, and foremost, I wanted the term to adequately describe the idea that people do many different things in many different places that influence what and how they know and learn. At the same time, I wanted a term that was consistent with the way other educators and psychologists have thought about these issues. The second part was quite difficult as researchers seem to use the terms *practice* and *context* in different and sometimes confusing ways.

I chose the term *practice* rather than *context* for one reason. Whereas contexts are ever-expanding networks of connections unique to a particular time and place, socially defined practices have their own histories which allows them to be discussed independent of the participants. I can talk, for instance, about the practice of farming without talking about a specific farmer. I can talk about how the practice varies based on geographical settings--the differences between dry-land farming in the Western United States and the farming in the lush soils of the midwestern states, for example. Or, we can talk about the history of farming and how technological advancements have changed the practice.

Practice, as I use it here, refers to the routine activities in which people participate. Practices comprise both a socially determined structure of activity and the meaning participants attach to the activities. Practices can be as large as the practice of medicine, law, or farming. Or, they can be a small as reading x-rays, writing a brief, or selling grain. Each of these practices is shaped by a fuzzy set of rules or guidelines and expectations that set out certain elements of the practice. The rules and expectations surrounding a practice form the socially constructed structure of the practice.

Within the structure of a practice, people participate in different ways. How a person participates in a given practice is influenced by their knowledge, skill, and the technology to which they have access (Scribner & Cole, 1981). Writers who use word processors, for example, may write very differently than those who use pen and paper, as would an experienced writer as compared with a novice.

To understand a person's participation in practice, it is necessary to understand both the socially constructed structure of the practice and the meaning the participant attaches to different parts of the practice. But how does participating in two or more socially-constructed practices at the same time influence what and how we know and learn? This question is somewhat problematic for two reasons.

First, although the question asks about the mutually constitutive nature of participation, it seems to present a situation in conflict with our everyday experience--we do not usually participate in different practices at the same time. If a person is a Methodist, a plumber, a softball player, and a father, he seldom goes to church, fixes a leaky faucet, fields a ground ball, and disciplines his child simultaneously. Yet, how he plays softball--whether he plays one or four nights a week, for instance--might be influenced by his being a father. A person with kids might choose to play less frequently so that more time could be spent with family. The influence might go the other way as well; playing softball might become a part of his interaction with his kids. They might play catch, hit "fly balls," or the father might provide more formal instruction on how to play the game. In this way the influence is two way: being a father influences how and when he plays softball and playing softball influences how he interacts with his kids. So, while we participate in many different activities each of which influences how we participate in other activities, we do not participate in all those activities at the same time. Rather, we move from one practice to another at different times. But, as we move among practices, we don't leave behind the other things we do. It might be best to say one practice dominates the others at different times, or even that the practice changes in response to the setting in which the person finds himself.

The second problem arising from this question is, given the definition of mutually constitutive practices, we can never find the origin of "ways of thinking" or ideas. Rather, everything we learn or do is influenced by the other things we have learned or have done. In Chapter One, I suggested that researchers have traditionally conceptualized the relationship among practices as unidirectional. For this conceptualization to be useful, the researchers needed to identify the origin of ideas or knowledge in one practice, for example, the practice of school mathematics, and see how that knowledge influenced (i.e., whether or not it was used or how it changed) a person's participation in another practice, filling dairy orders, for example. Locating the origin of knowledge is problematic in two ways. First, if we believe that collateral participation in socially constructed practices influences how we participate in different practices, then whether or not the strategies used to fill the dairy orders are identical to the traditional mathematical algorithms taught in school, the mathematics the dairy workers learned in school influenced the strategies they constructed in the dairy. Second, in most studies comparing in- and out-of-school math, the relationship between practices has been investigated by documenting the mathematics used in various situations and then comparing the workers performance *in situ* to their performance on a school-like math test. The people in these studies are, however, no longer in school. The studies look at what these people have remembered years after their school days and compare that knowledge to the mathematical strategies they constructed in the workplace. To truly investigate the influence of participating in one socially-constructed practice on another, we need to look at practices in which people are participating at roughly the same time; that is, at collateral practices.

A methodology for investigating collateral participation needs to take into consideration three things. First, the methodology needs to reflect the everyday experience of moving among various practices; investigating collateral participation needs to follow or trace a person moving among various practices. Second, the methodology needs to focus on something other than the origin of a person's knowledge; investigating collateral participation needs to focus on influences rather than origins. Third, it needs to look at people participating in two or more related practices during the same time period. Finally, I would like to add one more necessary component: a study of collateral participation requires we look at related practices; the practices must include a shared domain of knowledge. For instance, all of the practices must be mathematical practices or include a mathematical component.

To look at the two-way influence described above and to meet the criteria laid out in the previous paragraph it is necessary to find something that crosses boundaries among practices; something that people *do* while participating in different settings or practices.

The things people do--the tasks they undertake--often begin and end in the same practice. They may, however, move among practices for short periods of time. For example, a person who sets out to complete a household landscaping project might take the project from her home to the hardware store, to the nursery, and back home before finishing the project. While the initial plans for the project may have been discussed and drawn at home and the project is completed at home, the interactions the woman has at the hardware store and the nursery will change the project. She might find out about a new tool at the hardware store, or, at the nursery, she might learn that the plants she chose won't do well in the shade around her house, so she needs to choose shrubs better suited to her environment. In either case, the finished project will have been influenced by the people with whom she talked. By looking closely at the interactions people have while participating in various practices and understanding how the project or task changes over time we can begin to understand how participating in different activities influences what a person does.

In this study I looked closely at the practices of school mathematics and homework. The tasks were school-math homework assignments. Like the landscaping example I presented above, homework begins and ends in the same practice--school math--but crosses boundaries and is discussed and worked on at home. The tasks for this project started in the classroom, went to students' homes, and returned to the classroom where students talked in small groups and then in whole class discussions about the tasks. I documented interactions around the tasks through observation, videotaping, and audiotaping in the classroom, and by audiotaping interactions around homework in the students' homes. By following the tasks across the different settings we can see changes in the tasks and infer the influence the different conversations had on the students' completion of the tasks.

Understanding how participating in the two practices influenced students' completion of the tasks required understanding the structure of the practices and the meaning the students, their parents, and teacher attached to them. School math in this classroom was dominated by small-group and whole-class discussions. Homework in each of the students' homes also included conversations between the students and a family member. In the next section I report the specific data collection methods used to document students' conversations in school and in their homes. Along the way, I will introduce the school, classroom, teacher, students, and families that took part in this study.

Methods

The Classroom and the Teacher

The research I report here was conducted in Alice Smith's[1] fifth-grade classroom. My collaboration with Ms. Smith began as part of a study conducted by researchers with

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[1]The teacher's and the students' names used in this dissertation are pseudonyms.

the Center for Learning and Teaching of Elementary Subjects at Michigan State University. In 1989, researchers with the Elementary Subjects Center were interested in collaborating closely with a small number of teachers and supporting them in making meaningful changes in their teaching. Much of this research was conducted in professional development schools affiliated with the Partnership for the New American Education housed at Michigan State University. Based on the model presented by the Holmes Group (1990), professional development schools were intended to provide teachers with opportunities to work with each other and in collaboration with university researchers to examine and make changes in their teaching as well as play a larger role in the administration of their school. Teachers in these schools were given release time to meet and reflect with university researchers and there was a general expectation at the schools that teachers would enter into collaborative research projects with university researchers. Smith was one of four teachers who responded to a request we made to teachers to form a working group to think about mathematics teaching and learning and changes they might make in their teaching. As a part of our collaboration, we studied the difficulties involved in changing interactional norms in the classroom and attending to students' mathematical thinking (Perry, 1981; Wertsch, 1984).

In the fall of 1992, the upper elementary level teachers decided to departmentalize the intermediate grades. Smith was scheduled to teach four mathematics classes--two fourth-grade classes and two fifth-grade classes. Smith wanted to teach these classes in a way that reflected the spirit of current reforms in mathematics education. (National Center for Research on Teacher Education, 1989; National Commission on Excellence in Education, 1983; National Council of Teachers of Mathematics, 1989; National Council of Teachers of Mathematics, 1991; National Research Council, 1989). To help her reach this goal, she attended summer seminars on mathematics education and joined a group of teachers and university personnel who were reading and talking about alternatives to traditional mathematics instruction. Each of these things contributed to the pedagogy Ms. Smith used in her classroom.

Ms. Smith presented her students with many different types of tasks. The previous Spring, the school district had adopted a new mathematics textbook. Smith was interested in using the text as much as possible. As a result, Ms. Smith's students were presented with tasks that sometimes represented more traditional school-math work and others that reflected the problem-solving emphasis of the recent reforms. In each case, students were expected to complete the tasks and justify what they had done to complete them. Often, new tasks were introduced as homework. Students were expected to take the task home to work on and return to school with either a justification of their solution or a list of questions their classmates and teacher could help them answer.

In the classroom, students worked in "math groups" made up of three or four students. In these groups, the students would discuss the tasks and their solutions or questions. The ultimate goal of these groups was to construct a consensus solution to the task and a justification for the solution. While the students worked, Smith wandered from group to group "putting out fires," entertaining questions and rewarding groups that remained on task.

When most of the groups were finished with the task, Smith usually brought the class together to discuss their solutions. These discussions began with one group presenting their solution to the task and often ended in exciting conversations about different mathematical ideas--conversations which sometimes lasted for days. During these conversations, students were encouraged to write the different solutions in their math notebooks.

In short, although Smith was ultimately concerned with answer to the task, she stressed understanding why the solutions her students used arrived at the correct answer and that there may be many solutions to the same task. In her teaching, she worked diligently to make her students' thinking public through conversation and writing.

The Students and their Families

The students and their families that took part in this study represented the social make-up of the classroom, the school, and the neighborhood. The students lived in one of two neighborhoods. One feature that divided the two neighborhoods was whether students walked to school or were bused from a "downtown" neighborhood. The students who walked to school came from the neighborhood immediately surrounding the school. This neighborhood comprised mainly Caucasian working-class families. Although Ms. Smith described virtually all of her class as having "working-class backgrounds" and "blue-collar home environments," the neighborhood surrounding the school was the more attractive of the two. Two of the six students in this project came from this neighborhood.

The downtown neighborhood comprised mostly minority families and was considered something less than attractive. Ms. Smith described it as an "unprotected neighborhood with old cars in the driveways and lawns." In the middle of this undesirable neighborhood there was an area called "Capital Commons," a subsidized housing community where rent costs were based on the family's income. Ms. Smith described this area as a protected island in the middle of an otherwise dangerous neighborhood. The people living in Capital Commons, Ms. Smith told me, came from middle-class backgrounds and had middle-class values. One student who participated in this project lived in Capital Commons, the other three students came from the downtown neighborhood surrounding Capital Commons.

Of the six students who took part in this study, three were boys, three girls; three were African American and three were Caucasian; four of the students came from single parent families, two from two parent families. Each family was supplied with an audiotape recorder, tapes, and instructions on how to use the recorders to tape homework conversations. At the conclusion of the project, the tape recorders were given to the students for their participation.

Although the students share many characteristics (e.g., they are all in fifth grade and have the same teacher) they differ in many ways as well. Each student represents different social and ethnic backgrounds. They bring with them both academic and out-of-school histories that influence their completion of the tasks. Among a number of other things, there are three things that stand out as particularly important to this study. These characteristics include parents' experiences with math in school and in their "everyday lives," the students' success or failure in school math, and the normal homework routine in each student's home. In this section I look at the characteristics of the students and their families that might influence their completion of the school math tasks. These characteristics are summarized in table 3.1.

Kathy

Kathy, according to Ms. Smith, was and always had been a good math student. She was popular among a group of students Ms. Smith called "potential leaders." These students were characterized by their middle-class backgrounds and values and involved parents. All of the students in this group participated in the "Math-a-Rama" program--a program in which students from schools throughout the district competed in mathematical contests which emphasized computational speed and accuracy.

Kathy lived in Capital Commons with her father and younger brother. Kathy's father was in school at a nearby university, majoring in chemistry. At the time of this project he was applying to medical schools across the country. To put himself through college he worked as a medical chemist in a local laboratory. He told me he used mathematics constantly in his work.

> Well I work as a medical chemist and we use, ah, all the way from extremely complex polynomials to figure things out to, you now, just simple ratio problems. Oh, the concentration of an unknown is equal to the, ah, the percent concentration of unknown, you know, divided by the, ah, the concentration of a standard. Different things like that, you know, just very simple things all the way to very complex. And then, everyday in class, ah, I'm taking a class right now where it's a, um,

physiologic, ah, study of physiological problems with a hand-held computer and so we're taking complex equations and, and manipulating them in a computer and using them to calculate cerebral blood flow and um, ah, metabolic rates and different things like that, so, it's a real common thing.

Table 3.1

Characteristics of Student Participants and their parents

Student	Neighborhood	Ethnicity	Academic history	Household makeup	Parents' background
Karen	Surrounding	Caucasian	High average	Father, Brother	Auto worker, Geometry II
Kathy	Capital Commons	Caucasian	Leader, Math-a-Rama	Father, Brother	Medical chemist, pre-med student
Pete	Surrounding	Caucasian	Emotionally impaired, temper tantrums	Mother, Aunt, Grandfather, Grandmother, two siblings	Grandmother helped with homework
Ronnie	Downtown	African-American	Good student, college material, Math-a-Rama	Mother	Good math student, school math had changed
Shaundra	Downtown	African-American	Good language arts student, difficulty in math	Mother, Father	Father interested in engineering and math.
Tony	Downtown	African-American	Not motivated, athletically oriented	Mother, Step-father, Brother, Step-brother	Mother confident in school math.

Although he had taken many math courses, Kathy's father was not completely comfortable with what he called the "new math." His discomfort came from the "creative part of math." He recounted math classes where he was given a textbook with pages of computational problems. Now, he suggested, students are asked to develop their own problems and formulas. The expectation to create unique problems and formulas was part of his university math classes as well. In those classes, he felt somewhat disadvantaged in relation to recent high school graduates. They had experience doing these sorts of things; he did not.

His discomfort did not result in scorn for the creative part of math. Rather, he thought it was important and he was disappointed it was not part of his mathematics experience. It did, however, make him a bit uncomfortable when helping Kathy with her math. Ms. Smith sometimes asked the class to write problems they thought their classmates could solve. Kathy's father told me he was sometimes uncomfortable helping her with this part of her homework. He said, "In the story writing aspect she might be a little bit better than I am already just because I wasn't exposed to it as early."

Studying or doing homework was a family activity in Kathy's home. Each person had his or her own work station. Kathy's father sat at the dining room table, Kathy sat at her desk and Kathy's brother, Bobby, sat at a smaller table nearby. On most nights they listened to music while working. They took turns choosing the evening's music. Because they all had work to finish, there were rules for when Kathy or her brother could interrupt their father. He needed, he told me, to maintain a train of thought to complete his homework and couldn't always be interrupted. Kathy and Bobby were asked to go on to other problems and keep any questions they had until their father was at a place in his work where he could take a break. When he was between problems or came to a break in his work, Kathy's father asked Kathy and her brother if they needed any help.

Tony

Tony lived downtown with his mother and stepfather and many children. One brother had recently been released from prison on an educational program. Ms. Smith characterized another brother as being "pretty temperamental." Tony's stepfather also had two children from previous marriages. His son, Tony's stepbrother, was in the other fifth-grade class in this school. His daughter was in Ms. Smith's class at the beginning of the year, but soon moved out of state to live with her mother.

Ms. Smith described Tony as "not motivated and athletically oriented." Tony had not been academically successful in his years in this school. "He can surprise you, though," Ms. Smith told me. Tony read at a third grade level and was weak in math, but he "knows his facts." Tony, Ms. Smith told me, was not very attentive in class, although he would ask some questions. He had a slight hearing problem that may have influenced his being perceived as inattentive.

Tony's mother worked with him on his homework. She told me she was comfortable helping him. she made it clear that she stressed school work in their home. Tony, she said, came home from school and sat at the kitchen table to do his work. They worked together while she prepared dinner. Throughout the homework sessions, she was available to answer questions. During our conversation, she asked if it was okay if she raised her voice during the recorded conversations.

Karen

Karen lived with her father and brother in the neighborhood surrounding the school. Karen's mother left many years before. It was a difficult experience for Karen and her brother. Ms. Smith told me of a "heart wrenching" story Karen wrote about her mother leaving and that her brother had been labeled "explosive" during that time. In spite of this turmoil, both Karen and her brother had adjusted very well. Karen made the first quarter honor roll as did her brother in middle school.

Karen had spent most of her academic career at this school. She did, however, transfer to another school for a short time. While at this other school her teacher thought Karen had academic problems and referred her to the special education program. Her father became very upset and pulled her out of the school and brought her back to this school. Ms. Smith told me that in this school population, Karen, although she has "some weaknesses," is a strong average student. Indeed, in Ms. Smith's classroom she is an above average student and receives mostly "Bs" on her report card. Ms. Smith suggested some of this success might be attributed to her "tremendous level of motivation." Ms. Smith used some examples to illustrate Karen's abilities and weaknesses. She told me Karen did fine on her weekly spelling lists, but had problems with common words in writing; she did well in verbal performance, but her punctuation was weak. Ms. Smith characterized Karen as a "pretty good math student." She is, Smith told me, sometimes confused by mathematical concepts but she perseveres until she understands. Karen contributed substantially to nearly every class discussion.

Karen's father told me he had gone through Geometry II in high school. He told me students were expected to do more these days than when he was in high school. His son was in an accelerated algebra class in seventh grade. This surprised Karen's father because he remembered algebra as a ninth-grade subject. He told me he still uses some of the things he learned in math. As part of his job at a local auto assembly plant, he needs to calculate how many man-hours and parts are needed for jobs described on assignment sheets. He told me:

> I have to take care of parts and make sure the parts are going to be here so as far as multiplying and dividing by days, you know, and how many we're going to use a day, that kind of thing. I use it a couple times a week. . . I make sure we have enough, you know, to finish the week.

There was a breakfast bar in the kitchen at Karen's house where they sat to do the homework. Her father described this area as having "stools, good lighting, and a hard surface" all of which made it a good place to work. The kitchen was adjacent to the living room where the television was usually on during homework time. Karen usually worked alone at the breakfast bar asking questions only when she struggled with her work. In one conversation Karen's father told me he usually helps with her spelling, but Karen doesn't need much help with her math homework. Karen's father told me he was comfortable helping Karen with her math homework, but that Karen often did not need help.

Pete

Pete had a history of violent temper tantrums. Ms. Smith told me he had been in the principal's office nearly every day the previous year. He was placed in special education classes and had been labeled emotionally impaired. Pete often refused to participate in class. When asked questions or to demonstrate something at the overhead projector, Pete would often refuse to talk or get out of his desk. His participation in class increased the year of this project. Although he would still refuse to participate at times, he contributed substantially to both small group and whole class discussions.

Along with his refusal to participate in class, Pete was generally not engaged in school subjects. Ms. Smith, however, pointed to two things that made her believe Pete was interested in what she called "material with substance." The class had recently taken a trip to a local museum. Pete took the trip very seriously, preparing questions for museum guides and learning much about state history before going. During the trip, he was quite engaged, asking the questions he had prepared and follow-up questions to the responses given by museum guides. The second example Ms. Smith used to support the assertion that Pete was interested in substantive material came in class. Ms. Smith often reads to her class just after lunch. In conjunction with a unit on American Indians, the class read a book entitled "I Sailed on the Mayflower" by Roger Pilkington. One day, Ms. Smith told the class they could read more after lunch. Some class members objected; they wanted her to read from a book they had been reading after lunch. Pete argued quite strongly for reading the "Mayflower" book.

Pete lived in the neighborhood surrounding the school with his mother, his mother's sister, his mother's parents, and two younger siblings. Because his mother sometimes felt uncomfortable helping Pete with his math homework, he often worked with one of the other adults in his household. Elementary math, she told me, is much more advanced than it was when she went to school. When I asked her if the math she learned in school was helpful when she worked with Pete she told me:

> At the beginning it was, now it's, doesn't really do any good because I don't know what he is talking about. . . Whatever it is that he's doing right now. I've just, I've given up because I, the first time he brought it home I said 'take it into papa.' (laughs) I don't even know what you're dealing with there.

Pete's mother thought some of the math she learned in school was useful in her everyday life. She used an example of buying meat in the grocery store to show how she needed to calculate prices and the total cost of groceries to stay within a budget.

Doing homework was a daily activity in Pete's house. He and the person helping him would usually sit at the dining room table. In the background Pete's grandfather would watch television and listen to the conversation at the table, adding insight as needed. As it turned out Pete worked primarily with his grandmother.

Shaundra

Shaundra came to this school in the middle of the previous year. Her mother reported that Shaundra "went on hold" when they moved to this area and started having lapses in her school work. Ms. Smith told me the lapses, while sometimes still a problem, were much fewer and farther between than they had been the previous year. She told me a story of Shaundra missing an assembly because of unfinished work. While Shaundra was sitting doing her work she looked up and said, "If you think I'm not doing well this year, you should have seen me last year." This year Shaundra was completing her work and participating in class discussions.

Ms. Smith told me she was quite impressed with Shaundra's ability to make connections and draw sophisticated conclusions in language arts. This ability, however, was not as apparent in math; math was difficult for Shaundra Some of the difficulty may have come from her father's interest in mathematics. He had wanted to be an engineer and really enjoyed math. As a result, he spent a lot of time working with Shaundra on her math. Ms. Smith, trying to teach in a way that reflected the current calls for reform in math education, emphasized mathematical reasoning and problem solving in her class.

Shaundra would often say her father was telling her to do it a different way. Ms. Smith thought the different explanations may have led to Shaundra's confusion.

Ronnie

Although Ronnie was quiet and didn't often volunteer his thoughts, he was one of Ms. Smith's best students. She referred to him as "college material" and described him as "Usually quite calm, very self-controlled, diligent with his work, performs well." But, like Shaundra, Ms. Smith thought "this kind of math" was difficult for Ronnie; that he would be more comfortable if math were taught in a more traditional fashion stressing computation rather than reasoning. Like Kathy, Ronnie was quite successful in the Math-a-Rama program where students compete in computational speed and accuracy.

Ronnie, his mother, and brother moved to the area just before the school year started. His mother had enrolled in a court reporting program at a local community college. Ronnie's mother told Ms. Smith they had moved frequently before they moved here. As a result, Ronnie had not completed a school year in one school. Because Ronnie liked this school so much, she told Ms. Smith she would make a concerted effort to keep him in this school.

Ronnie's mother had been a good math student. Math came easily to her and, as a result, she became the math expert in her family--even her older sisters went to her for help. she told me of times in her math classes where she would finish an assignment "like that" (snapped her fingers) and then just sit and "waste time." Even with the mathematical history, Ronnie's mother worried that she had forgotten nearly all the math she had learned and was a little concerned about the help she could provide Ronnie. She told me a story of one assignment where Ronnie got "everything wrong."

> I helped him one time and he got everything wrong, okay, and it was my fault. He was very upset with me and told me he would never ask me to help him with his homework. . . . It had to do with fractions and I was telling him to reduce them to the

lowest terms the wrong way. . . . So, after that, it's like he doesn't want my help unless he needs it, then he'll ask.

She went on to say that school math had changed so much that there were things she didn't know about.

I'm embarrassed to say this, but, he brought, he had his homework one day, I didn't understand it at all. . . . I don't remember what it was, it had something to do with, ah, I don't know if it was rounding off or something you asked me. No it wasn't rounding off cause I know that. It was something and it had to do with some kind of problem that I'd never seen before, you know and I was like 'I don't understand this' you know and he's like 'come on mom' and I say 'I don't' you know and I think that's kinda, I need brushing up on the, the more up-to-date math--how they do it, you know, the way they do their problems and solve them so that I could know because it makes it easier for both of us. If I don't know, it's embarrassing for me to tell him I don't know, I don't understand it.

Her feeling that she had forgotten the math she learned in school and that different things were being asked of elementary math students that she knew nothing about contributed to Ronnie's mother's concern for her ability to help Ronnie with his homework.

In spite of the one bad experience and Ronnie's mother's concerns about her knowledge of mathematics, Ronnie and his mother worked together on his homework. Ronnie's mother was a self proclaimed "TV. freak" and that Ronnie worked better in front of the television. She described their homework sessions this way:

He'll get his books and he'll come sit with them on the floor and I'll be on the couch; I'm a T. V. freak. It seems like he'll work better when the T. V.'s on. . . . He does his work, ya know, and then if he has a problem he can just come up on the couch and tell "Well, I don't understand this or that" you know. That's basically how it goes.

The Tasks

The tasks used in this project were designed by the teacher, Ms. Smith, and myself. Three things were taken into consideration in designing the tasks: instructional considerations, mathematical considerations, and research considerations. Instructional considerations included how comfortable Ms. Smith felt about the content and how the task fit with her ongoing classroom instruction. At the beginning of the year, the school district in which this project was conducted adopted a new mathematics textbook. Although teachers were not required to use the text, Ms. Smith wanted to incorporate the text into her instruction in some way. Some of the tasks we sent home during this project began as lessons from the new textbook. None, however, came directly from the textbook. Each task, in some way, grew out of a previous activity or led into a new mathematical area into which Ms. Smith wanted to move.

Once the topic of the tasks was decided, we looked more closely at the mathematical ideas we wanted students to explore. Three of the four tasks included an introductory paragraph followed by four or five questions about the paragraph. The questions following the introductory paragraph were designed to address specific mathematical ideas we deemed important. These ideas ranged from making sure both quotative and partitive models of division were represented in the first task to including zero as a choice when working with combinations and permutations in the third task. So, although the topic for the task usually came from Ms. Smith's classroom instruction, the specific questions asked were chosen based on different mathematical considerations.

Research and theoretical considerations also led us to use certain tasks and questions. For instance, recent psychological theories have suggested adults *scaffold* (Piaget, 1966; Stipek, 1988) children as they work together on tasks. In an attempt to illuminate this phenomenon, the tasks often included some repetition of ideas. It was believed if parents referred back to previous problems or provided less support or assistance in subsequent questions or tasks they were "tearing down or building up the scaffolding." Another research consideration concerned the mathematical content of the tasks. In the initial interviews with parents, they told of math classes they had in school. In general, the classes they described emphasized a computational understanding of mathematics. Calls for reform in mathematics education are suggesting students learn to assess situations mathematically, flexibly apply mathematical tools and justify their solutions using accepted canons of evidence (National Council of Teachers of Mathematics, 1989; National Research Council, 1990). Tasks were chosen that reflected one of these disparate perspectives to see if the mediation in or out of school changed in relation to the content being discussed.

Instructional, mathematical, and research considerations influenced each task in different ways. In the next section I look more closely at each of the tasks used in this project. What may already be clear is that these considerations are inseparable. The instructional considerations include mathematical thought, mathematical considerations involve both instructional and research/theoretical considerations, and research/theoretical considerations also include both instructional and mathematical considerations. For the purposes of analysis, however, it is necessary to distinguish among the three considerations.

Task 1

Instructional Considerations. Just before we started this project, Ms. Smith gave each student in her class a shopping guide from a local grocery store. Along with the guide, she gave them a series of math problems[2] that could be solved using the information provided on the guide. Students computed amounts for various items based on their advertised prices. The problems Ms. Smith assigned focused, for the most part, on multiplication and division. We decided the first task should reflect both shopping, a

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[2]The term problem is used here in its traditional sense; that is, as a computational exercise or a short story problem. This usage is consistent with the teacher's use of the term.

"real-life" situation, and multiplication and division like the shopping guide activity had.

As a result, we designed a task in which Willie, a student like the students in Ms. Smith's class, agreed with two friends to take turns packing lunches. Willie was going to pack lunches for the first week. Below the introductory paragraph in which Willie and his task are introduced, there were four questions the students were asked to solve to help Willie pack the lunches. The assignment sheet is shown in figure 3.1.

<u>Task #1</u>

Willie and two of his friends decided they would take turns packing lunch for each other. They talked about what the lunches should include and decided on a sandwich, a piece of fruit, and a dessert for each person. Willie volunteered to pack lunch for the first week. When he checked his refrigerator he found he needed some food in order to pack lunches for the week. So, Willie went to the market. When he got there he found buying groceries was a bit more confusing than he thought and he is asking for your help. Please help him solve the following problems.

1. Willie believed that all sandwiches should have tomatoes on them. The supermarket advertised tomatoes at 3 for $1.00. Willie decided he only needed two tomatoes for the week.
   - How much would the tomatoes cost?

2. Willie decided he wanted to put apples in the lunches two days in the first week. The store sold five apples for $2.00.
   - How many apples will Willie need?
   - How much will Willie pay for the apples?

3. Twinkees are Willie's favorite dessert. He wanted to put them in as many lunches as possible. At the store Willie found that Twinkees come in packages of 12 and cost $4.56.
   - How many days would one package of Twinkees last?
   - How much would it cost to put Twinkees in the lunches for one day?

4. Willie and his friends decided they would split the cost of the lunches each week.
   - How much did Willie spend at the supermarket?
   - How much did it cost each person?

Figure 3.1: Task 1 Assignment Sheet

Mathematical Considerations. Like the shopping guide problems Ms. Smith assigned earlier, division can be used to answer the questions we chose for this task. Division can be thought of in two ways. First, in a partitive model of division the divisor represents the number of sets among which the dividend is divided. Partitive division answers the question: "How many items will be in each group if I divide XX into X groups?" An example of partitive division would be dealing a deck of cards (52) to four people and counting the number of cards each person received (13). Second, in a quotative model of division the divisor represents the number of items in each set and answers the question "How many groups of X are there in XX?" An example of quotative division would be taking the same deck of cards and, if you play a game where each player gets four cards, how many people can play at one time? Both partitive and quotative interpretations of division were represented in the questions for this task.

Questions one, two, and four each can be answered using a partitive model of division. In question number one the cost of three tomatoes ($1.00) is divided into three groups with the quotient representing the amount of money in each group (i.e., the cost per tomato). One step in a possible solution for question number two would be to divide the total cost of five apples by five to get the amount of a single apple. Like question one, the total cost of the apples is divided into five groups with the quotient representing the amount of money in each group. In question four, students were asked to calculate the total cost of groceries Willie bought and figure out how much each person should pay. The total cost is divided among three people with the quotient representing the cost each person needs to pay. Question three can also be answered using division, but a quotative model of division. In this question students are asked how many days a box of twelve Twinkees would last if Willie put them in each lunch. Since there are three lunches being packed each day, the question asks students how many groups of three Twinkees are included in a box of twelve.

Research Considerations. One of the questions we were interested in during this project was whether the practice within which people were participating influenced the way they solved problems. In this task, if the task is interpreted as a "real life" problem, the practice of grocery shopping could constrain the solutions students construct. Because the numerals represent money in this task, solutions like thirty-three and one third cent are not appropriate. Thus, when working on the first question, students needed to negotiate a remainder and round to the nearest cent. Students who divided one dollar by three came up with either three groups of thirty-three cents with one cent left over or they wanted to add two cents so that they could have three groups of thirty-four cents. If, however, the problem was interpreted as a school math task and the numbers were taken out of the practice of grocery shopping, answers such as thirty-three and one third might be acceptable.

A second research consideration arose from the notion of scaffolding mentioned above. Three of the questions following the introductory paragraph could be solved using a partitive model of division. These questions were written this way to see if parents or other family members would refer to previous problems while helping the students with their math homework.

Task 2

Instructional Considerations. When we began discussing task number two, Ms. Smith suggested we use a set of questions she had already constructed. She wanted to use these questions to get students thinking about place value and base ten. The assignment would provide a segue from the material covered in task one to a project in the new textbook in which students would rewrite numbers in expanded notation. The assignment sheet is shown in figure 3.2:[3]

Mathematical Considerations. As I mentioned in the section on instructional considerations, Ms. Smith wanted to get her students thinking about place value and base ten. Because of this, she wrote questions which her students could solve by multiplying or dividing by multiples of ten. Questions 1, 3, 4, 5, and 6 fit this model. Question five is the only question students might use division to solve. In this question they might

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[3]The sheet was actually hand written by Ms. Smith.

divide 1,420 by 100 to get the number of books in a stack. Computing this problem, however, requires students to manipulate a remainder. Students solving this problem without taking into account the fact that books cannot be divided up into pieces of books to even out the stacks might answer the question saying there would be 14 remainder 20 books in each stack or 14.2 books per stack. An answer for the question taking into account the nature of books might be eighty stacks of fourteen books and twenty stacks of fifteen books.

Math

1. Keisha had 142 books. Her mom told her she'd give her $10 for every book she read. How much money would she receive if she read all of her books?

2. If she received $3 for each book, how much would she receive?

3. If, over three years, she received 100 times as many books as she had in problem one, how many books would she have?

4. If she received 1000 times as many books, how many would she have?

5. If Keisha had 1420 books and she put them in 100 stacks, how many books would be in each stack?

6. How many groups of 10 equal 100?
   " " " " 10 " 1,000?
   " " " " " " 10,000?
   " " " " " " 100,000?

Figure 3.2: Task 2 Assignment Sheet

Research Considerations. The research considerations in choosing this task are twofold. First, it was important to me that the classroom teacher feel a part of this project. This importance was manifested in a couple of ways. As I mentioned before, the tasks needed to reflect ongoing classroom instruction. I did not want to construct tasks that would require Ms. Smith to teach something other than what she would teach if I were not in the room. To accomplish this, is was important for her to be involved in the construction of the tasks. She constructed this task with specific reasons in mind--reasons that were consistent with her ongoing classroom instruction. Second, the interviews with parents suggested they were accustomed to school math in which they were asked to compute problems like those included in this task. Other tasks on which students and their family members would work would ask them to think about mathematics in a different way. These questions provided a "baseline" with which to compare other conversations.

Task 3

Instructional Considerations. As I mentioned above, Ms. Smith wanted her students to start thinking about place value. In the district's newly adopted textbook, Ms. Smith found an activity in which students were given a set of three notecards, each with a digit written upon it. In the activity the students were asked to arrange the cards to show as many three digit numbers as possible and write each number they found in expanded notation. Ms. Smith thought this activity would give the students an opportunity to think about place value. We used the activity as a starting point and designed a task in which students were asked to arrange the cards and write each number in expanded notation, as it was described in the textbook. We expanded the activity to include two other things we thought were important. The students were asked to determine when they had found all the possible numbers, and, in the end, determine a way to predict how many numbers could be found from a set of digits.

This task is the only one that went home twice. The assignment sheets are shown in figure 3.3.

Mathematical Considerations. Among the major emphases of the recent reforms in mathematics education (National Commission on Excellence in Education, 1983; National Council of Teachers of Mathematics, 1989; National Research Council, 1989) are the students' understanding of number systems and number theory and their ability to reason through various mathematical problems. We hoped this task would require students to do those things.

Math

1. Compile a list of three digit numbers that can be made using the digits 3, 4, and 7.

    A. How many numbers did you come up with?

    B. How do you know when you've found all the numbers?

2. List all the different numbers in order from smallest to largest

    A. Tell why you ordered the numbers as you did.

3. Write each of the numbers you find in expanded notation.

    A. Explain what each digit means in each number.

Math

1. Write down as many four digit numbers as you can using the digits 0, 2, 5, and 9.

    A. How many numbers did you find?

    B. How do you know when you've found all the numbers?

2. List all the different numbers in order from smallest to largest.

    A. Tell why you ordered the numbers as you did.

3. Write each of the numbers you find in expanded notation.

    A. Explain what each digit means in each number.

4. Now that you have worked with both three and four digit combinations, can you think of a way to predict the number of combinations you can make with a certain number of digits?

Figure 3.3: Task 3 Assignment Sheet

The task asks students to explore two different mathematical areas. First, students were asked to arrange the three digits in as many ways as possible, which provided them with an opportunity to think about permutations and combinations. Second, students explored place value by ordering the numbers and writing them in expanded notation. The task also provided two opportunities for students to reason mathematically. First, in traditional mathematics classes, students have completed their problems or assignments when there are no more problems to compute and they know they are right or wrong when the teacher tells them so. Yet, in mathematics there is no one available to tell you when you are finished or whether your solution is adequate. Developing the ability to assess the reasonableness of a solution is one of the goals of mathematics education. After students were asked to record the three digit numbers they found, they were asked to explain how they knew they had included all the possible permutations; that is, how they knew their solution was adequate. The second part of this task was given to students after they had discussed their answers to part one. Part two was designed to give students an opportunity to gather similar data with a different number of digits and make sense of the entire corpus of data. This part of the task differed in two important ways. First, zero was included in the four digits. A four digit number with zero in the thousands place (0295) represents the same value as the three digit number without the zero (295). Students , we believed, might question whether zero could be placed in the thousands place in a four digit number. Second, part two culminated in with a question about a general method for estimating the number of permutations that could be made with a given number of digits.

Research Considerations. As with the other tasks, the different considerations overlapped for third task. This task was designed to address two research questions. First, the content of this task, although consistent with the recent calls for reform, was perhaps at odds with parents' mathematics experiences. The task was, in part, designed to see if the content of the task changed the conversations between parents and children. Second, this task, like the first task, was designed to see if parents or students would refer back to earlier problems or tasks during the conversations.

Task 4

Instructional Considerations. Task number four was the last assignment before the Winter Holidays. Because of approaching holidays, Ms. Smith wanted to "wrap things up." The task included something from each of the three previous tasks. It began with a story about Nick and his family's holiday celebration. Following the story was a series of questions about the festivities. Each of the questions reflected, in some way, one of the first three tasks.

Math

Nick and his family always celebrate the holidays in the same way. They gather together at Nick's house and exchange a few gifts. After that they sing holiday songs and eat cookies and fruit. Nick decided this year he was going to give his brother and sister some presents. He would like you to help him with his Holiday plans.

1. Nick has two sisters and two brothers. He wanted to give each of them a pair of socks, so he bought four pairs--a red pair, a blue pair, a green pair, and a purple pair. Nick couldn't decide what color to give each person. How many different ways can Nick arrange the socks for his brothers and sisters?

2. Nick was also in charge of choosing the songs they were going to sing. He decided, based on what they had done in the past, they would sing for two hours. Each song, he thought, would take about ten minutes. How many songs did Nick need to choose?

3. As was their tradition, Nick wanted to buy his brothers and sisters some fruit. He thought he would get each of them an apple and an orange. At the market apples cost 3 for 99¢ and oranges cost 2 for 60¢. How much would Nick need to pay for the fruit?

4. Nick also wanted to give each of brothers and sisters some holiday cookies. He bought 20 cookies. How many cookies would each person get?

Figure 3.4: Task 4 Assignment Sheet

Mathematical Considerations. Students were asked in the first question for the number of permutations Nick could make with four pairs of socks. This first question is similar to what the class did in task number three, but asks only for the number of permutations and not the permutations themselves. The second, third, and fourth questions reflect tasks number one and two. In the second question students were asked to help Nick decide how many songs he would need to choose. This question represents a quotative model of division; that is, it asks "How many groups of ten minutes are there in two hours?" The third question asks the students to compute the cost of fruit at a certain price. This problem reflects the connection to a "real life" situation from task number one. Finally, question number four asks students to compute a partitive division problem.

Research Considerations. The major research consideration for this task was to see if students, their parents, or anyone else would make connections among the problems they were working on for this task and those they had worked on earlier.

The Data Collection Path

The data collection path is shown in figure 3.1. The path started in school where Ms. Smith introduced each task during large group discussions. Each of the introductions, as was all large group interaction in the classroom, was videotaped or audiotaped.

Collecting the data for this project depended heavily on the cooperation and assistance of the volunteer families. These people all agreed to record the conversations they had with their children about their math homework. If they chose not to record these tapes, no data could have been collected for the in-home portion of this project. Four tasks were sent home as part of this project. The overall return rate for the tapes recorded in students' homes was 79 percent (19 of 24 tapes). For the first task, all six families returned the tapes. On the second task, which was recorded the Tuesday before Thanksgiving, three of six tapes were returned. All six families returned tapes for the third task which was the only task that lasted more than one evening. For this task, students and their family members discussed related tasks on two consecutive evenings. These conversations were recorded on opposite sides of the same tape. All families recorded conversations around both parts of the task. The fourth task resulted in four of six tapes. The tapes returned by each student is documented in table 3.2.

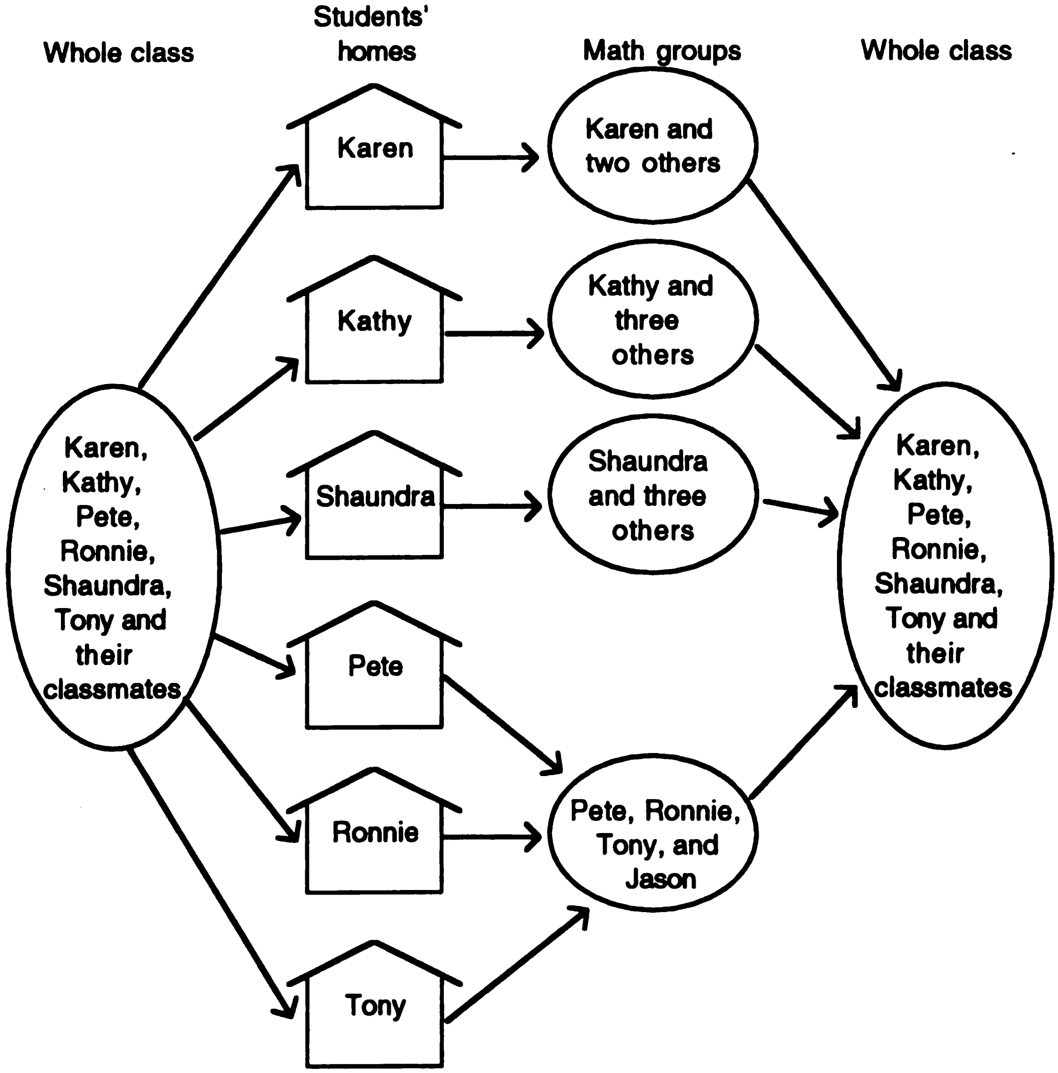

Figure 3.5: The Data Collection Path

After the students worked on the tasks at home, they brought them back to the classroom. The normal routine in the classroom included a short time during which students met and talked in small groups. The six students who participated in this project were members of four different small groups. Tony, Pete, and Ronnie, were in the same group. The other three students, Shaundra, Kathy, and Karen were members of different groups. The small groups including the student participants were either video or audio taped. Each day, one group was video taped and three others were audiotaped. Which group was videotaped depended in part on which students were in class on a given day, and which groups had been video taped in the recent past. An effort was made to video tape each group the same number of times. After students met in small groups, they would come back together as a large group to discuss their solutions to the tasks. Representatives from different groups presented the solution their group agreed upon and other students would comment on the solution. Usually as a result of these discussions, students would deem solutions either acceptable or unacceptable. These discussions were all videotaped.

Table 3.2

Returned tapes

	Students					
Task	Karen	Kathy	Pete	Ronnie	Shaundra	Tony
1	No tape	X	X	X	X	X
2	X	No tape	X	No tape	X	No tape
3	X	X	X	X	X	X
4	X	No tape	No tape	X	X	X

In both settings, at home and in school, students were encouraged to record all their work on paper. They were asked to leave all marks on their papers so that their work could be followed while listening to the conversations they had while working on the tasks. Students' written work was collected and photocopied following each task.

Methods of Analysis

The analysis followed the data collection path. At each stop along the path, I asked different questions to better understand the interactions at home or in school and how the interactions influenced the students' work. At the same time I looked at individual conversations, I looked for strands of influence on the students' work; that is, was there evidence that their interactions in one setting influenced their completed task. The researcher's task is an analytic one. Varenne, Hamid-Buglione, and McDermott, and

Morison describe it this way: The researcher's task "consists in guiding an audience through the artifact to highlight selected properties. In the process, it is not only permissible, but actually required, that the analysis substitute theoretically meaningful words for those of the informants." (Varenne et al., 1982, p. 63). Different theoretical perspectives contributed to the data analysis in this study. In what follows I explain in more detail the analysis at each stop along the data collection path.

Ms. Smith's introductions to the tasks were analyzed for their contribution to the students' completion of the tasks. Ms. Smith told me she was concerned that the students might not answer the same questions if they did not clarify things in the classroom before taking the tasks home. As I looked at the introductions, I tried to understand how Ms. Smith worked to alleviate ambiguity in the tasks.

Wertsch's model of negotiation in the zone of proximal development served as a starting point from which to analyze the interactions students had at home. As mentioned in Chapter Two, Wertsch's model includes three components: (a) the situation definitions participants bring to the conversations; (b) the intersubjective situation definition; and, (c) the semiotic moves the participants use to gain intersubjectivity. For each of the conversations I looked for evidence of the participants' situation definitions. The situation definitions participants brought to bear in the conversations were analyzed in terms of the socioculturally defined practices they represented. Practices, it was believed, could influence what students' work in two ways. First, defining the problem in terms of different practices might lead to using different rules to guide the completion of the task. Second, the participants may bring discontinuous conceptions of school math to the conversations. I looked for evidence of either of these influences in the conversations.

I also looked for evidence of an intersubjective situation definition in the conversations and how the participants negotiated intersubjectivity. Wertsch's model included three semiotic moves adults use in conversation to negotiate intersubjectivity.

These moves include (a) directly telling the child what to do; (b) asking the child to determine the next move and responding to their suggestion in one of two ways: directing them to a more appropriate move or changing the way they defined the situation to more closely match the child's; or, (c) referring to a completed example for the child to model. I looked at the conversations for evidence of each of these moves. Although Wertsch developed his model to explain adult-child interactions, the model also served as a framework for analyzing the interactions students had with their classmates.

Learning in the Wertsch's model is evidenced by the child "taking on" the adult's situation definition during the interaction. Learning from a situated cognition perspective (Lave, 1993) includes using what has been done in one practice to aid participation in other practices. To investigate the influence of students' interactions in one setting on their interactions in other settings and on their completion of the tasks, I looked for evidence of both definitions of learning.

Whereas using the theoretical perspectives as a starting point for the data analysis in this study helped me understand the conversations I was listened to and the ongoing classroom instruction, it also provided an opportunity to look closely at the theories and their adequacy in describing the data. The analysis led to a series of questions about the theories themselves. Does Wertsch's model of negotiation within the zone of proximal development adequately describe the conversations recorded in this study? what is intersubjectivity and what does it mean to gain intersubjectivity? Does the content influence which of the semiotic moves family members used in the conversations? How do the answers to these questions refine or expand the theoretical perspectives used to analyze the data?

CHAPTER 4

BRINGING THE OUT-OF-SCHOOL WORLD INTO SCHOOL

Situated learning theorists have suggested that socially-defined practices shape the way people think and learn. Candy sellers, grocery shoppers, and weight watchers all developed mathematical systems other than the formal mathematics taught in schools that allowed them to accomplish goals situated in their particular practices. So strong was the belief that practices shaped cognition that theorists argued the practices themselves, rather than the individuals who make up the practices, should become the objects of psychological investigation (Scribner, 1984). But, people participate in more than one practice at the same time. What happens when two or more practices with conflicting conventions convene? How do people resolve the conflict?

One way to explore the influence of various practices is to embed potentially conflicting practices in the tasks presented to different people. In this chapter, I look at the results of embedding an out-of-school practice, grocery shopping, in a elementary school mathematics task. Embedding out-of-school practices in the tasks was meant to illuminate issues of interest to both mathematics educators and educational psychologists interested in situated learning. Situated learning theorists have argued that the practices in which we participate shape our thoughts and behavior. Following from this, it is likely that students, when presented with tasks representing an out-of-school practice, would work within the rules of that practice. The influence of out-of-school practices on systems of mathematics is also emphasized in the recent calls for reform in mathematics education (National Council of Teachers of Mathematics, 1989; National Research Council, 1989; National Research Council, 1990). The reform documents have argued that elementary school students, and other mathematics students, should recognize the situated nature of mathematics. Simple calculation rules traditionally taught in mathematics classes are not sufficient in all situations. Rather, different levels of precision, different computational conventions, and other slight differences change the mathematical procedures used in different practices. Bringing two practices with somewhat different mathematical rules together, we hoped, would create a conflict that the students and their parents would need to resolve as they worked on the task. In an attempt to design a task that was consistent with the reforms in mathematics education and issues in situated learning, we brought the out-of-school world into the classroom by creating a school-math task about grocery shopping.

Task 1

Task 1 began with a story in which the students were asked to help Willie make some decisions and compute the prices of different items during a grocery-shopping trip. The assignment sheet is shown in figure 4.1. The practices of school math and of grocery shopping have different sets of rules that govern computation (Lave et al., 1984). Whereas traditional mathematics instruction has emphasized computational accuracy and stringent rules that govern computation, grocery shoppers often loosely estimate the cost of products to monitor how much money they are spending, to choose among different brands of the same product, or to make other decisions associated with grocery shopping. Because Task 1 was a school math task about grocery shopping it was likely that the two practices would conflict. The conversations the students had with their parents pointed out that more than these two practices influenced how they approached and answered the questions.

Question 1: The Rules of Practice

Students in elementary mathematics classrooms are taught to round off numbers to a certain significant place by following a specific set of rules. James and James (1976) define these rules this way:

> When the first digit dropped is less than 5, the preceding digit is not changed; when the first digit is greater than 5, or 5 and some succeeding digit is not zero, the preceding digit is increased by 1; when the first digit dropped is 5, and all succeeding digits are zero, the commonly accepted rule (computer's rule) is to make the preceding digit even, *i.e.*, add 1 to it if it is odd, and leave it alone if it is already even.

Task #1

Willie and two of his friends decided they would take turns packing lunch for each other. They talked about what the lunches should include and decided on a sandwich, a piece of fruit, and a dessert for each person. Willie volunteered to pack lunch for the first week. When he checked his refrigerator he found he needed some food in order to pack lunches for the week. So, Willie went to the market. When he got there he found buying groceries was a bit more confusing than he thought and he is asking for your help. Please help him solve the following problems.

1. Willie believed that all sandwiches should have tomatoes on them. The supermarket advertised tomatoes at 3 for $1.00. Willie decided he only needed two tomatoes for the week.
   - How much would the tomatoes cost?

2. Willie decided he wanted to put apples in the lunches two days in the first week. The store sold five apples for $2.00.
   - How many apples will Willie need?
   - How much will Willie pay for the apples?

3. Twinkees are Willie's favorite dessert. He wanted to put them in as many lunches as possible. At the store Willie found that Twinkees come in packages of 12 and cost $4.56.
   - How many days would one package of Twinkees last?
   - How much would it cost to put Twinkees in the lunches for one day?

4. Willie and his friends decided they would split the cost of the lunches each week.
   - How much did Willie spend at the supermarket?
   - How much did it cost each person?

Figure 4.1: Task 1 Assignment Sheet

Grocery shoppers, in contrast, use a different set of rules: when confronted with prices including a fraction of a cent, grocery shoppers round the price up to the nearest cent. Using this rule accomplishes two things. First, grocery stores, as do other stores, round prices up at the checkout counter.[1] Mimicking the procedure used by the store will

[1]During one presentation of this study, an audience member who had spent 20 years in the grocery business questioned my conception of grocery shopping. With the advent of scanners, he told me, grocery stores no longer always round up to the nearest cent. Rather, it depends on the order groceries go through the check out lane. If three cans of soup were sold for one dollar, they would cost thirty three cents a piece if they went over the scanner at different times. He cited public relations and legal reasons for this pricing: If the store rounded up and a customer bought three cans of soup priced at three for a dollar and had them scanned at different times, the cost would come out to $1.02. The two cent difference between the advertised price and the actual cost of the soup could anger customers or even lead to law suits. To avoid these problems, grocery stores

provide a more accurate estimate of the total cost of the items in the shopper's cart. Second, many grocery shoppers have budgets which constrain their spending. Rounding prices up helps insure they remain within their budgets.

Question 1 was designed to bring together these two sets of rules and see which set the students and their parents--and classmates--would choose to follow. The question asked the students to determine the price of two tomatoes if three tomatoes cost one dollar. If the students and their parents used the rules of school math, the tomatoes would cost 33 cents each and 66 cents for two. If they used the rules of grocery shopping, the tomatoes would cost 34 cents each and 68 cents for two. The answers each student arrived at for Question 1 are summarized in table 4.1.

Five of the six students (83%) answered Question 1, saying two tomatoes cost 66 cents. Although only Kathy and her father listed the cost of the tomatoes at 68 cents, the choice between sets of rules was apparent in three of the five conversations recorded.[2] In each conversation different decisions were made about how to answer the questions that reflect different practices and, perhaps, other influences on how students and their parents complete school math tasks.

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round the price down to the nearest cent. So, three cans of soup scattered throughout your groceries would cost 99 cents. If, however, you sent two cans of soup through the check out at the same time, the price would be added together (33 1/3+33 1/3=66 2/3) and rounded up to 67 cents (see Shaundra's father's explanation in the next conversation) and a third can coming through later would cost thirty three cents for a total cost of one dollar. Finally three cans going through at one time would cost one dollar. Being a grocer, though, is different from being a grocery shopper. Although, my new understanding of the grocery business will undoubtedly change the way I buy groceries, Ms. Smith and I used the understanding of grocery shopping described in the text to design Task One. And, it was Kathy's father's conception of grocery shopping, not what grocers do, that was brought to bear in their conversation. As inauthentic as these conceptions might be, they did influence the students' completion of the task. Newman, Griffin and Cole (1989, p. 15) have agreed that "Not all change is for the better. In many cases, the interactions [between teachers and students] can have detrimental effects or result in changes that can not be assessed as better or worse." Perhaps we participated in such an interaction.

[2]Karen and her father did not record their conversation. Karen's answers were taken from her written work.

Table 4.1

Solution Table for Task 1, Question 1

Student	Cost of the tomatoes	Rules
Karen	$1.00 ÷ 3 = .33	School math
	.33 + .33 = .66	
Kathy	$1.00 ÷3= .34	Grocery shopping
	.34 x 2= .68	
Pete	$1.00 ÷ 3 = $33\frac{1}{3}$	School math
	.33 + .33 = .66	
Ronnie	$1.00 ÷3= .33	School math
	.33 x 2= .66	
Shaundra	$1.00 ÷3= .33 r. 1	School math
	.33 x 2= .66	
Tony	$1.00 ÷3= .33 r. 1,	School math
	.33+.33=.66	

In Kathy and her father's conversation, Kathy had trouble answering the question within the rules of school math. When she arrived at a quotient with a remainder, she was unsure of how to interpret it. Her father, using rules consistent with grocery shopping, directed her to an answer.

1 F Okay, so in the first question, um, it says, that Willie thought they had to have tomatoes. Okay, he thought you had to have tomatoes in a sandwich and that they were three for a dollar. And, ah, so Willie decided he only needed two for the week. So, if they were three for a dollar and he only had to buy two, how much would they cost?

2 KG I, I don't know because do, thirty-three cents it would be three times and it would go into ninety-nine cents though. It wouldn't go into a dollar. So you have to go up one, thirty-four times three is a, is a dollar two. So it would have to be thirty-four cents remainder two. But, but, but the st, but

3 F But now hold it. How much is each tomato?

4 KG thirty-three cents or thirty-four.

5 F And your saying thirty-four cents, right? So, and he wants to buy two, so how much is it going to cost him?

6 KG Wait, oh thirty-three cents, sixty-six.

7 F Yeah, but you said thirty-four cents, right? You said you had to go up one.

8 KG Oh, oh yeah, thirty-four. then it would have to be sixty-eight. A dollar sixty eight, no, just sixty-eight cents.

9 F Right, that's the answer to your first question, right?

10 KG Yup.

Prior to this conversation Kathy had done some preliminary computation. When her father asked her how much the tomatoes would cost she responded with two possibilities for the price of one tomato. She was a bit confused, though, because one price--33 cents--when multiplied by three equals 99 cents and the other, 34 cents, when multiplied equals $1.02. In her thinking at this point, neither of these seems right because the multiplication she used to check her division should come out to exactly 100 cents and neither of the answers do that.

Kathy seemed to solve the problem without considering the conventions of the practice in which the task is situated. Instead, she dealt only with the numbers she extracted from the assignment sheet and without considering the situation neither--or both--responses made sense. Looking at the problem this way provided no guidance in figuring out which way to round the quotient and, as a result, she found herself confronted with a dilemma.

Her father, however, saw the task as grocery shopping. Although his definition of the situation was not explicitly stated, he did and said some things that suggested his definition was consistent with the practice of grocery shopping. In line twelve, after Kathy told him the tomatoes could cost either 33 or 34 cents apiece, he, by asking a pointed question, guided her to agree that each tomato costs 34 cents. Furthermore, in the conversation Kathy's option of 34 remainder two[3] was stripped of the remainder; that is, the extra two cents were no longer an important consideration--a move that can be interpreted as being consistent with the practice of grocery shopping. Once they decided that 34 cents was the correct answer, Kathy added that amount to itself and she and her dad agreed, after a bit of confusion over the computation, that two tomatoes cost 68 cents.

Although Kathy and her father's conversation was the only one in which the rules of grocery shopping determined the correct answer, the choice between two sets of rules--and consequently two possible answers--arose in two other conversations. In Shaundra's house there appeared to be no question about which set of rules to follow while answering the question, but a dilemma emerged as they recorded their answer.

After Shaundra read the introduction to the first task, her father read the first question and wondered aloud about how to answer it. The question asked how much Willie would pay for two tomatoes if the store sold three tomatoes for one dollar. His wondering led him to ask how much one tomato cost.

1	D	Okay, lets take a look here. Problem number one. Okay, Willie believed that all sandwiches should have tomatoes on them. The supermarket advertised tomatoes at three for a dollar. So, Willie decided he needed two tomatoes for the week.
2	SQ	Problem.
3	D	Uh-huh, that's a problem. He needs only two tomatoes but they're three for a dollar. So how much would he pay for the two tomatoes that he bought?
4	SQ	Um, I see now.
5	D	So, if there's three tomatoes for a dollar, then how much would one tomato cost?
6	SQ	Um,

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[3]Kathy's used the term "remainder" nontraditionally in this example. Rather than signifying the left over portion of the dividend, Kathy used the term to represent the number of cents over one dollar that results from multiplying 34 x 3. Multiplying 34 cents by three results in a product of $1.02.

7 D That'd be three divided into one dollar.

8 SQ Three divided into one dollar.

9 D Right.

Shaundra did the division narrating each step as she went along. "Three goes into one no times. Three goes into ten three times. Three times three, nine . . . . " When she paused, even for the shortest of times, her father joined in asking her questions about what to do next or telling her what to do. Together Shaundra and her father worked through the division and came up with an answer of 33 remainder one. Although her father pointed out the remainder and made sure Shaundra had it written down, they did not discuss it or what to do about it. Instead, they dropped it and determined the cost of one tomato to be 33 cents with no mention of the remainder--until they had answered the question. After they had agreed that Willie would have paid 66 cents for the tomatoes and Shaundra had written her answer down on her paper, her father said:

> The thing I was wondering is that, you know, when you go to a store a lot of times, well, this won't be rounded off. A lot of times they round it off to the next number. Which it might not be sixty-six cents it might be sixty-seven. Because they're gonna make sure that they get the advantage with that penny that's left over. They always charge you that extra penny. Whether you want to pay it or not. So, that way they never lose any money.

In Shaundra and her father's conversation, they used rules that are consistent with those taught in traditional elementary mathematics classrooms. But, when they finished, Shaundra's father wondered aloud about an alternative answer that was consistent with the rules of grocery shopping.[4] Although he recognized the choice

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[4]Shaundra's father's conceptualized the rules of grocery shopping slightly different than the conceptualization used to develop this task. Rather than rounding the price of each tomato to the nearest cent, Shaundra's father rounded the price of two tomatoes to the nearest cent. This conception is similar to the rules explained to me by the audience member experienced in grocery shopping.

between the two sets of rules--and the resulting difference in answers--they did not change their answer. What they, or he, believed was expected in school held more authority than did the situation in which the task was set. Tony's mother also pointed out different possible answers, but chose the answer consistent with school math in their conversation.

Tony began the conversation by reading the assignment sheet aloud. As he read, he stumbled over words such as "refrigerator" and "confusing" causing his mother to become upset; both words were included in Tony's spelling list and, she suggested, he should recognize them. Despite their initial difficulty with the assignment, Tony and his mother worked their way through Question 1.

Tony and his mother decided that the cost of one tomato could be determined by dividing one dollar by three. They worked through the algorithm together: Tony narrated the steps as he worked and his mother questioned him about the reason for each step. When they finished the division, they arrived at an answer of 33 with a "1" at the bottom of the division bracket. Tony's mother asked what they should do about the "1."

1 M This is dollars and cents, remember, so is you going to round one off or you going to keep on going?

2 TW Keep on going to cents. (pause) No, round it off.

3 M So, how much would each tomato cost? How much would it cost for two tomatoes?

4 TW Uh, each would cost thirty-four cents?

5 M Apiece? Or, together?

6 TW Apiece.

7 M It couldn't actually cost 33 cent, I mean 34 cent apiece then that would make it be more than a dollar. If you add thirty-four up three times

8 TW Let me think, yeah. It's 33 cents.

Recognizing his difficulty interpreting the remainder, Tony's mother suggested they "put the decimal point behind the three" and "try adding a zero on." This strategy reflects another heuristic or rule often taught in mathematics classrooms: carrying a division problem out to more places may result in an quotient with no remainder. Tony added the decimal point and a zero and carried out the division. Again he arrived at a quotient with a remainder which his mother helped him interpret.

9 M So in order to round it off, you can take the one and leave it as the remainder, but we would say they cost thirty three and a half cents apiece, right?

10 TW Um-hmm, we would get (pause)

11 M For two. Which you would get if . . . Now you gotta do what, add?

12 TW Um-hmm

13 M And, what are ya gonna add? (pause)

14 TW Thirty-three (pause) two times

15 M All right then, so, what did you come out with?

16 TW That'd be sixty-six cents.

17 M All right then. Actually sixty-seven cents but you're not that far advanced.

Tony's mother, as did Shaundra's father, recognized that different solutions were possible for this question. One possible solution included dropping or ignoring the remainder and adding 33 + 33 to arrive at a cost of 66 cents for two tomatoes. The other solution involved rounding 33 r. 1 to 33.5 and adding 33.5 + 33.5 for a cost of 67 cents. Although it is not clear that she was following the rules of grocery shopping[5] the two possible solutions reflect the different sets of rules common to grocery shopping and school math. But, the decision about which set of rules to follow was not influenced as much by the setting in which the question was situated or originated as it was by Tony's mother's beliefs about his ability. She saw the first solution (33 + 33 = 66) as a

[5]Tony's mother may be using a "buggy" set of school math rules in which she interprets any remainder as one half. At the beginning of her explanation, she used the same rules Kathy did in the conversation presented above. She ruled out 34 cents as the answer because 34 times three equals $1.02. That rule was attributed to school math classes. It may be that she, as was Kathy, was searching for a school math rule that fit the situation.

developmentally prior solution to the "more advanced" solution of 33.5 + 33.5 = 67 cents.

In the conversations Kathy, Shaundra, and Tony had with their parents, they appeared to choose the practice to which to attach the questions. Depending on their choice, different sets of rules governed how the question was answered. The choice of which rules to follow was determined by different things in different conversations. In one conversation presented here, the choice was determined by the practice in which the task was set--rules consistent with the practice of grocery shopping were used to determine an appropriate answer. In the second conversation, the set of rules was determined by the practice in which the task originated and where the students' answers would be assessed--rules consistent with the practice of school math were used to guide their work. In the final conversation, the rules reflected the parent's determination of the student's ability level. In this conversation the rules used to shape their answer changed as the parent better understood the student's ability.

The conversations about Task 1, Question 1, point out that things other than the practices embedded in the tasks influenced how students and their parents answered the questions. The influence of other practices became evident in the conversations the students and their parents' had as they worked on the rest of Task 1. In their conversations about Question 2, the assumptions the students and their parents made while answering the questions shaped their answers and may have masked the mathematical thought in the conversations.

Question 2: Situation Definitions

Question 2 was similar to the Question 1. In both questions the students needed to determine the cost of one item and, using that cost, determine the cost of a group of items. Whereas in the first question they were asked to determine the cost of a smaller number of items than were advertised, in Question 2 they were asked to determine the cost of more items than were included in the advertised cost. The similarity between the questions was intended to see if parents would refer to the earlier question while assisting their children in answering Question 2. Referring back to a completed model or past experience is consistent with instruction within the zone of proximal development (Newman et al., 1989; Wertsch, 1984). None of the parents referred to the previous question.

The first part of Question 2 asked the students to determine the number of apples Willie would need if he wanted to pack them in the lunches on two days. Five of the six students who participated in this study agreed that Willie would need six apples. How they arrived at this answer varied slightly but, in essence, they took into account the number of days Willie wanted to pack apples in the lunches (2) and how many lunches were being packed each day (3). The strategies the students used are summarized in table 4.2.

The second part of Question 2 asked the students to determine the cost of six apples if the store sold five apples for two dollars. Two similar strategies emerged among the five students who agreed that Willie needed six apples to pack the lunches. In each of the strategies, the students and their parents first determined the cost of one apple by dividing two dollars by five arriving at a cost of 40 cents. To determine the cost of the six apples Willie needed, the students either multiplied .40 x 6 or added .40 to $2.00 (the cost of five apples) to arrive at $2.40. Either strategy concluded that six apples cost $2.40. The strategies students used are summarized in table 4.2.

Although five of the students agreed that Willie needed six apples and they would cost $2.40, one student (Tony) did not agree with those answers. He answered the questions saying Willie needed four apples and they would cost fifty cents. Although his answers are different, there are some similarities between Tony's solutions and those of the other students. To understand these similarities we need to look at the conversation Tony had with his mother as they answered the questions.

Table 4.2

Solution Table for Task 1, Question 2

Student	Number of apples needed	Cost of the apples
Karen	6	\$2.00 + 5 = .40
		.40 x 6 = \$2.40
Kathy	Intuited 6	\$2.00 + 5 = .40
		.40 x 6 = \$2.40
		and
		\$2.00+.40 = \$2.40
Pete	3 x 2 = 6	\$2.00 + 5 = .40
		\$2.00 + .40 = \$2.40
Ronnie	Directed to 6	\$2.00 + 5 = .40
		\$2.00+.40= \$2.40
Shaundra	3 + 3 = 6	\$2.00 + 5 = .40
		.40 x 6 = \$2.40
Tony	2 + 2 = 4	\$2.00 + 4 = .50

Tony's mother read Question 2 aloud: "Now problem number two. Willie decided he wanted to put apples in the lunches two days in the first week. The store sold apples, sold five apples for two dollars." When she had finished reading the question, she summarized it and helped Tony come up with an answer.

1 M Okay, he only wanted to put apples in the lunches two days in the first week. [Um-hmm] Okay, so the apples are five for two dollars, so what [are you] going to do with that problem? . . . The key is right here. He wanted to put apples in the lunches two days in the first week.

2 TW That means he needs two apples.

3 M You gotta keep remembering that Willie is making lunch for himself and his friend. . . two days in the week.

4 TW He gonna need four.

5 M He gonna need four, all right.

Tony and his mother determined the number of apples Willie needed by first determining the number of lunches being packed (2) and the number of days Willie was packing apples in the lunches (2). Using these figures they determined Willie needed four apples. Although the answer Tony and his mother arrived at was different than the other students' answers, they used a similar strategy. They, as did the other students, multiplied the number of apples needed per day by the number of days Willie needed the apples or added the number of apples used each day together the number of days they were being packed in the lunches. What differed were the assumptions they used to define the situation and to determine what would constitute an appropriate answer. Rather than Willie packing lunches for himself and two friends, they defined the question (and the entire task) as Willie packing lunches for himself and one friend.

Tony and his mother's answer to the second part of Question 2 is harder to explain. In the beginning of their conversation, Tony's mother pointed out that five apples sold for $2.00, but to answer the second part of the question, Tony and his mother divided $2.00 by four--the number of apples they determined Willie would need--to determine the cost of one apple. From, this calculation they determined one apple would cost 50 cents. This quotient was the only answer discussed in their conversation and the only answer recorded on Tony's assignment sheet. Their answer might reflect another redefinition of the question. They may have redefined the question to read: Willie paid $2.00 to put apples in the lunches on two days. How many apples would he need? How much would each apple cost? If these were the questions they answered, their answers would have been correct.

Although Tony and his mother's answer might be explained by understanding the assumptions they used while answering the question, how those assumptions changed is not explained. Looking more closely at their conversation and the practice of homework in Tony's household, provides some insight on how changes like this may come about.

Tony began their work on the second part of Question 2 by writing out the division bracket. His mother saw what he had written and offered her approval by saying "All right, talk to me brother." As Tony began to do the computation, the loud snap of a mouse trap being sprung sounded in the background and a voice announced "Well, you got 'em" indicating a long sought mouse had just been caught. When she heard this, Tony's mother excused herself from the table saying "call me when you get ready to do the problem, but right now, I got to do something."

Tony continued to compute his answer, narrating each step. His mother monitored the computation from a distance. Tony quickly suggested that four went into 20 two and then six times. Believing that Tony had merely guessed, his mother threatened to make him write his "times tables" if he did not stop guessing. Tony offered four as the number of times four went into twenty which elicited the response his mother had promised. Tony turned his paper over and wrote out the multiplication facts from 4 x 1 to 4 x 5. When he realized 4 x 5 = 20, he offered five as the answer to the second part of Question 2. His mother agreed and together they completed the computation arriving at an answer of fifty cents. When they finished, Tony's mother instructed him to write his answers on his paper. Tony wrote: four apples, 50¢.

The subject of their conversation had changed. In the earlier conversation, Tony and his mother talked about the number of apples Willie needed and other things that shaped the questions they were answering. But as they began working on the second part of Question 2, Tony's mother's attention was diverted away from Tony's homework and directed toward other household tasks. Although their conversation continued, it focused on Tony's computation and not on whether that computation fit the question they were answering. The computation Tony did was correct--$2.00 + 4 does equal 50 cents--and we can judge Tony and his mother's conversation successful if we accept the topic they chose for their interaction. But Tony's answers were wrong and their conversation was unsuccessful if we situate it in the larger picture of Tony's math assignment; his answers were determined incorrect when he returned to school.

Homework, itself, represents a socially-defined practice that represents only one of many streams of activity that occur simultaneously in students' homes (McDermott et al., 1984; Varenne et al., 1982). At the same time that they assist with homework, family members fold laundry, make dinner, discipline children, clean house, pay bills, and participate in other household chores. Each of the practices, except maybe homework, contributes to the overall operation of the household. Homework may, in fact, take away from the otherwise smooth operation of the home. As a result, homework may seem less important than other activities that more directly contribute to the household. Confronted with a household activity that needs attention, parents may need to divert attention away from their children's homework. This appears to be what happened in Tony and his mother's conversation. Although she continued their conversation, her divided attention may have allowed the conversation to change direction just enough to miss its target. As a result, Tony's computation, although done correctly, did not address the question in a way that was accepted in school.

Looking ahead to the conversation Tony and his mother had about Question 3 suggests that the practice of homework is also characterized by accepted norms of interaction. In their conversation about Question 2, it became apparent Tony's mother assumed two lunches were being packed rather than three. During their discussion of Question 3, Tony questioned that assumption and received a warning from his mother about the consequences of doing so.

Tony began their conversation about Question 3 by reading it aloud. The question asked how many days a package of Twelve Twinkees would last. When he finished reading, his mother instructed him to answer the first part of the question. "Solve the first one first, child. How many days? It says as many days as possible, now, keep that in mind." Tony told his mother the Twinkees would last four days--an answer consistent with the assumption that three lunches were being packed. His mother, however, disagreed and directed Tony to a different answer. She said, "Now, so there's two people, so what would you do?" Tony, recognizing that his mother had defined the problem differently than he had, objected. "I thought it was Willie and his two friends" to which his mother responded: "No it's just Willie and one friend. Willie only got one friend, it's two people. If Willie gets another friend I'm gonna go to bed." Faced with the prospect of losing his mother's assistance, Tony accepted her situation definition and never again mentioned his alternative. Tony and his mother completed their work on Question 3 and the rest of Task 1 assuming two lunches were being packed. This is how their conversation about Question 3 ended:

1	M	Now, what Willie trying to do is put Twinkees in lunch for hisself [sic] and his friend for as many days as he can. Okay, there's twelve Twinkees. Now, if two people ate a Twinkee everyday, how many days or how many times could they have Twinkees per week? (pause) Count by twos.
2	TW	Six
3	M	Right!

In Tony's household, he was expected to accept his mother's contribution to the conversation without question. His attempt to question her assumptions resulted in his being warned about the consequences of continued questioning--his mother would leave him to work alone. Although Tony's mother's reaction might be interpreted as inappropriate or even rude, it may also characterize culturally appropriate norms of interaction in their household. As in the Warm Springs tribe (Philips, 1993) where council elders are not to be challenged, it may be that Tony's role in his household does not afford him the option of questioning his mother's position. All of the students who participated in this study interacted in ways that represented other conversations they had in their homes. Indeed, Tony's mother, in our first conversation, asked if it was okay if she raised her voice while they worked on Tony's homework as that characterized the conversations they had in the past.

Tony and his mother's answers to the questions in Task 1 were influenced by two forces other than those associated with the practices of school math or grocery shopping. First, they, or perhaps Tony's mother, defined the problem in a way that was inconsistent with the definition expected when Tony returned to school. In their definition, Willie was packing lunches for two, rather than three people. Although this assumption caused different values to be used, Tony and his mother used the same strategies as the other students to determine the number of apples Willie needed. As a result, Tony's answers make sense in the context of their conversations although they were not accepted when he returned to school.

The practice of homework also influenced Tony's answers. Homework is a complex practice. It is completed within a tangle of household activities, many of which demand attention at unpredictable times. As a result, the goals of understanding the mathematics in the task may subordinated to the goals of other household chores. When Tony's mother's attention was diverted to other necessary household activities, the topic of their conversation changed. Rather than talking about the fit between the computation and the question they were answering, they began talking only about the computation. This slight change resulted in an incorrect answer that Tony took back to school. The practice of homework also includes conversational norms within which students and their parents agree on definition of the question they are asking and appropriate ways to answer it. The interactional norms that characterize these conversations may represent ways of interacting common to other conversations students have with their parents. As a result, the practice of homework is different in different students' homes and will influence their completion of school-math tasks in different ways. In Kathy's home, as we shall see below, questioning her father's contribution is accepted and even encouraged.

As the conversations students had about Questions One and Two suggest, many things contribute to Students' completion of school math tasks. But, along with the influence of the practice of school math, the practice in which the task is situated, and the practice of homework, students' previous experience in school math classes--and, perhaps their parents' experience in school math--also influence how they approach tasks. This influence is illuminated in the conversations students had as they answered Question 3.

Question 3: Differing Assumptions

Question 3 asked the students to determine the number of days a box of twelve Twinkees would last and the cost of putting Twinkees into the lunches each day. Taking the number of Twinkees and determining how many groups of three were in the box represented a different model of division than was used in Questions One and Two.[6]

Only Tony's mother mentioned the difference between the earlier two questions and this one. The difference, however, was not in the models of division, but in the question's level of difficulty. During their conversation Tony was having difficulty decided what operation he needed to use and what values to place in the algorithm. As he struggled, Tony's mother interrupted him saying:

> This one here is kind of difficult for you then. You did the first step. How many days would a package of Twinkees last? Okay, we determined that two people is eating out of it so we determined that it will last six days. Now we got the answer six. Six, now we got to divide into the cost. How much do Twinkees cost?

Tony and his mother went on to divide $4.56 by 6, the number of days the Twinkees would last. From their calculation, they determined the daily cost to be 76 cents. As happened in their conversation about Question 2, Tony and his mother used a strategy used by other students. But, because of their assumption that two, rather than three, lunches were being packed, they ended up with incorrect answers.

The other five students all determined the Twinkees would last four days. In each case the students and their parents determined the number of days by dividing 12 (the number of Twinkees in the box) by 3 (the number of Twinkees packed each day). The students answers to Question 3 are summarized in table 4.3.

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[6]See chapter three for a more complete description of the different models of division.

Table 4.3

Solution Table for Task 1, Question 3

Student	Number of days	Cost
Karen	12 ÷ 3 = 4	\$4.56 ÷ 12 = .38
		.38 x 3 = \$1.14
Kathy	12 ÷ 3 = 4	\$4.56 ÷ 4 = \$1.14
Pete	12 ÷ 3 = 4	\$4.56 ÷ 12 = .38
		.38 x 3 = \$1.14
Ronnie	12 ÷ 3 = 4	\$4.56 ÷ 12 = .38
		.38 x 3 = \$1.14
Shaundra	12 ÷ 3 = 4	\$4.56 ÷ 4 = \$1.14
Tony	6	\$4.56 ÷ 6 = .76

Although the students all ended up using the same strategy to determine the number of days the Twinkees would last, two of them--Ronnie and Shaundra--began answering the question in a way that illuminates the influence of their experience in traditional elementary mathematics classrooms on how they completed the task.

In traditional mathematics classrooms, assignments often include a series of problems which focus on a particular aspect of computation. To complete these assignments students must figure out the focal procedure and repeat it each time they compute a problem. The same format is often used in story-problem assignments: students must figure out the strategy for solving the first problem and apply that same strategy to the remainder of the problems in the assignment. Participating in classrooms where assignments are formatted in this way would lead students to develop conceptions of school math that reflect that structure. Conceptions consistent with this model may explain Ronnie and Shaundra's initial attempts to answer the first part of

Question 3 and provide a reason for one of the strategies used to answer the second part of Question 3.

Both Questions One and Two could be answered using a two step strategy. In the first step, students determined the cost of one item (tomatoes or apples) by dividing the advertised amount by the number of items sold for that amount. After the item cost was determined, the total cost could be computed by adding the cost for each item needed or by multiplying the cost by the number of items needed. This strategy could be carried out by merely pulling the numbers out of the text of the problem and applying the sequence of operations to those numbers. Most students answered the questions this way: out of 12 answers to the two questions, 11 (92%) reflected this strategy. If students held a conception of mathematics instruction similar to the one described above, and had answered both questions using the same strategy, it is likely they would attempt to use the strategy to answer the remaining questions. Ronnie and Shaundra did just that.

After Ronnie read Question 3 aloud, his mother asked him how he could figure out the number of days the Twinkees would last. Ronnie suggested the answer could be determined by dividing "Twelve into four-fifty-six." Carrying out this computation would determine the cost of one item (a Twinkee) and reflects the first step of the strategy commonly used in Questions One and Two. The numbers Ronnie used were the only numbers included in Question 3 on the assignment sheet. Ronnie, it appears, had taken the numbers from the story and began to apply the sequence of operations that successfully answered Questions One and Two.

Ronnie's mother interpreted his response as jumping ahead to the second part of Question 3 where the cost of putting the Twinkees into the lunches for one day was computed. She redirected him to another strategy and reserved his strategy for the second part of Question 3.

1 M No, no, no, no. Not yet. Don't, I'm not, that's not what I'm sayin' first. Okay, you, you serving lunch for three kids, right? [yeah] So, what's three to make twelve? How many times does three go into twelve?

2 RB Three goes into four times.

3 M Okay, so you can get, they can get a Twinkee in their lunch for how many days?

4 RB Four.

In a similar conversation, Shaundra's father asked her how she would determine the number of days the Twinkees would last. Shaundra was more explicit about the strategy she wanted to use.

1 SQ Well, first we gotta find out how much one cost.

2 D Oh, you do?

3 SQ Right

4 D Okay, well let's see. let's see, I'm trying to figure. Okay, First off, I want to do the first problem of how many days would one package of Twinkees last. Okay, we got three people eating Twinkees, right?

5 SQ Right.

6 D We've got twelve Twinkees. Twelve Twinkees, three people are eating them, eating these twelve Twinkees. They could, they going to eat three Twinkees, three Twinkees are going to be gone for each day.

7 SQ Right

8 D Right? So, you have to divide that three those three Twinkees into the twelve Twinkees and you'll get, and that'll tell you how many days the Twinkees will last. Do you know why? Because you're using three Twinkees a day. And you have to use, you have to divide the amount of Twinkees you're using up.

9 SQ Right.

At this point, Shaundra's father used an example to explain the situation to Shaundra. He told her the Twinkees were like the packages of bologna her parents buy for her and her sibling's lunches. Each day, he told her, they use three slices of bologna and can determine how many days a package of bologna will last by dividing the number of slices used each day into the number of slices in the package. At the end of the example, Shaundra began dividing three (the number of Twinkees used each day) into twelve (the number of Twinkees in a box).

In each of these conversations the students began to answer Question 3 by using the strategy that worked to answer the first two questions. Their parents, in both cases, directed them to a more appropriate strategy for answering the first part of the question. The conversations, however, illustrate that the students' experience in mathematics classes influence how they approach school math tasks. Not only are their answers influenced by the immediate practices in which they are participating (homework, grocery shopping, and Ms. Smith's classroom), but by the practices in which they have previously participated.

The influence of previous school math experience may also have contributed to the choice of strategies students and their parents used to solve the second part of Question 3. Students and their parents used one of two strategies to answer the second part. The strategies students and their parents used are summarized in table 4.3. In the first of these strategies--the strategy Tony and his mother used--the students and their parents determined the number of days the Twinkees would last and divided the total cost of the Twinkees by that number. Three of the six students (including Tony) used this strategy. The other three students answered Question 3 by determining the cost of one Twinkee (38 cents) by dividing the total cost of the Twinkees ($4.56) by the number of Twinkees in a box (12) and multiplying the cost by three (the number of Twinkees packed each day). This strategy is the same strategy students and their parents used to answer Questions One and Two. For each question, the dyads determined the cost of one item and then, by multiplying or adding, determined the cost of the number of items needed. Using this strategy may reflect students' and parents' previous school math experience where sets of questions could be answered using the same strategy.

In sum, students and parents brought to bear their previous school math experience as they worked on Task 1. This experience may have included a conception of school math tasks in which all the questions included in the assignment could be answered using the same algorithm or strategy. Part of completing the assignment was to figure out the strategy and apply it to all the questions.

Math in Ms. Smith's classroom, however, was different. In Ms. Smith's classroom students were encouraged to explore different ways of answering questions. Ms. Smith's conception of school math also influenced how the students answered the questions in Task 1. This influence is particularly apparent in the answers the class accepted for Question 4.

Question 4: Accepted Answers

Question 4 asked the students to pull together what they had done to answer the previous three questions and determine the total cost of the groceries Willie bought and the amount each person needed to contribute. Although the students' answers varied (there were four different answers), their responses represented two strategies. The students' answers are summarized in table 4.4.

Table 4.4

Solution table for Task 1, Question 4

Student	Total cost	Cost per person
Karen	.66+2.40+4.56 = 7.62	$7.62 + 3 = $2.54
Kathy	.68+2.40+4.56 = $7.64	$7.64 + 3 = $2.54 r2
Pete	.66+2.40+1.14 = 4.20	$4.20 + 3 = $1.40
Ronnie	.66+2.40+4.56 = 7.62	$7.62 + 3 = $2.54
Shaundra	.66+2.40+4.56 = 7.62	$7.62 + 3 = $2.54
Tony	.66+.50+.76 = $1.92	$1.92 + 2 = .96

In a sense, all six students used a similar strategy to answer Question 4. Each student extracted amounts from the first three questions, added those amounts together, and divided the sum by the number of lunches being packed. The amounts students chose to add together, however, differed and reflected two different strategies and, perhaps, different conceptions of school math assignments.

Two of the students (Pete and Tony) added the final answers to the first three questions together and divided that amount by the number of lunches they believed were being packed. Tony's answers are conspicuous because of the assumptions he and his mother made while answering the questions. Pete, however, answered Questions One, Two, and Three in ways that were consistent with the other students' answers. Taking the final answer to each of the first three questions led Pete to include the cost of putting Twinkees in the lunches one day rather than the cost of an entire box. As a result, Pete computed a total cost of $4.20 and an individual cost of $1.40. The strategy Pete and Tony used may represent a conception of school math assignments where the last story problem draws together the answers from earlier problems and a arithmetic operation is applied to produce a final answer. Pete's conversations with his grandmother as they answered this questions supports this interpretation.

Pete's grandmother coached him through each of the steps in the strategy described above. As they began to work on Question 4, she instructed him to look through his answers to the previous questions. Pete listed their answers to Questions One and Two and with each answer he received approval from his grandmother; "Okay," she said after Pete wrote down each number. After collecting the answers to the first two questions, Pete's grandmother interrupted his work and asked him how he might answer the question. Pete told her which arithmetic operation he intended to use by responding "add it." His grandmother agreed and told him to write down "the cost of your third day." Pete wrote "1.14" on his assignment sheet and his grandmother approved and asked him for the total cost to which Pete replied "Four twenty." His grandmother agreed and Pete carried out the division to determine the cost per person.

The other four students (Karen, Kathy, Ronnie,[7] and Shaundra) added the cost of the items Willie bought at the store and divided that cost by three to determine the cost per person. Although Kathy used the same strategy as the other three students, she arrived at a different total cost and cost per person. The difference resulted from the rules she and her father used to answer Question 1. Kathy and her father used rules consistent with the practice of grocery shopping and determined the cost of two tomatoes to be 68 cents. In the other three students' solutions, the tomatoes cost 66 cents. This difference led the class to accept a list of different appropriate answers.

In class the students worked in their math groups to compare their answers to the questions. As was common in Ms. Smith's classroom, each group arrived at a consensus cost for each item, the total cost of the groceries, and how much each person needed to pay. When all the groups had reached consensus, each group presented their answers to class which debated them until appropriate answers for each question were determined.

The class accepted three possible answers to Question 4. The answers the class accepted were based on different assumptions the students made while completing Task 1. The answers the class accepted are summarized in table 4. 5. The class agreed that the apples cost $2.40 and the Twinkees cost $4.56. The difference among the answers stemmed from the rules students followed while answering Question 1 and, if the rules of grocery shopping were followed, what to do with the "r.2." In the end, the students decided if they went to a "nice store" that charged 33 cents for each tomato, the total cost of the groceries was $7.62 and each person needed to pay $2.54 . If the students went to a "greedy store" they would have paid 34 cents for each tomato and the groceries would have cost $7.64 which, when divided, left a cost per person of $2.54 r. 2. The

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[7]Ronnie began to answer the question using the prices listed in the questions on the assignment sheet. To determine the total cost, he initially added $1.00 (the price of three tomatoes), $2.00 (the price of five apples), and $4.56 (the price of a box of Twinkees). His mother quickly stopped him and redirected him to the strategy presented here. His initial attempt may also signify a conception of school math assignments similar to the one used by Pete and Tony. Ronnie and his mother's conversation about this question is explored in greater detail in chapter six.

students determined two ways to interpret the remainder. The first, which was attached to the greedy store solution, was to round the cost up to $2.55. The second way of dealing with the remainder led to a third solution the class called the "work solution." In this solution Willie's two friends each paid $2.55 and, because he had done the shopping, Willie paid $2.54. Neither Tony nor Pete's answers were included in the list of appropriate answers.

Table 4.5

Solutions Accepted by Ms. Smith's Class

Solution	Computation
"Nice" store	.66 + 2.40+ 4.56 = 7.62
	7.62+3=2.54
	Cost per person: $2.54
"Greedy" store	.68 + 2.40 + 4.56 = 7.64
	7.64+3=2.54 r2
	Cost per person: $2.55
"Work" solution	.68 + 2.40 + 4.56 = 7.64
	7.64+3=2.54 r2
	Willie's cost: $2.54,
	Two friends' cost: $2.55

The nice store solution used the rules of school math, the greedy store used the rules of grocery shopping, and the work solution provided a pragmatic way of solving a real problem. In Ms. Smith's classroom none of these sets of rules was given primacy. Instead, Ms. Smith encouraged and expected students to think about different ways to answer the questions and justify their solutions. Many times the class agreed on a set of correct answers rather than one correct answer. It may be that Tony's answers would have been accepted had he presented a justification for his answers in class.

Summary

The conversations the students and their parents had as they worked on Task 1 suggest that no one practice determines how students approach and complete school math tasks. In the conversations described above, the practice embedded in the task, the practice in which it originated, parents' perception of students' ability, students' conceptions of school math, the practice of inquiry math in Ms. Smith's classroom, the intertwined nature of homework in students' homes, cultural norms of interaction, and the assumptions made by parents and students as they worked, all influenced how students and their parents interpreted and answered the questions.

All of these influences contributed to a new set of rules the students and their parents used to guide their conversations. The new set of rules were amalgams of different practices and beliefs the students and their parents brought to bear in the conversations. Although these different practices were combined to produce a new set of rules--a new or amalgamated practice--the different practices did not contribute equally. Instead, the different practices competed for prominence. In the conversations the students recorded, school math was weighted more heavily than the other practices represented in the conversations.

Kathy and her father's conversation in the next section provides an example of the different influences on their completion of school math tasks. In the conversation both Kathy and her father bring many things to bear that shape their answers to Task 1, question 3.

The Practice of Math Homework in Kathy's Household

Kathy began the conversation by reading the question aloud.

> Number three says Twinkees are Willie's favorite desert. He wanted to put them in as many lunches as . . . possible. At the store Willie found that Twinkees come in packages of twelve and cost four fifty-six. Hold it, he wanted to put them in as many lunches as possible. (pause) [inaudible] They can have each one in each group. They can have four in each group. Oh, Ms. Coleman, Ms. Coleman said that there's single packs. She said that there is twelve Twinkees there are twelve Twinkees in the package.

In the middle of reading the question, Kathy paused and drew a diagram on her paper that looked like this:

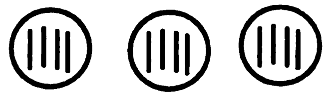

Figure 4.2 Kathy's Diagram

Through out her instruction, Ms. Smith encouraged her class to draw pictures of the questions they were trying to answer. Drawing pictures, she believed , helped the students better understand what they are doing in the tasks. In this case the division could be done using a picture rather than the division algorithm. Thus, Kathy began the problem in a way consistent with her school math class. At the same time, she recognized a discrepancy between her conception of grocery shopping and the one presented on the assignment sheet--Twinkees don't really come in packages of one, she thought, but because Ms. Coleman told her they do, it was okay to use packages of one for this question. This inconsistency led to a discussion about whether Twinkees come in packages of twelve in which each Twinkee is wrapped individually and whether or not this question authentically represented the practice of grocery shopping.

1 D Yeah, you can buy a box of twelve. That's what they're talkin' about.

2 KG That's stupid though, because when they have 'em, they have regular Twinkees though there's two in each pack.

3 D Right, yeah.

4 KG And there's twelve packages. Like, okay, you buy a big box . . .

5 D No, there's not twenty-four

6 KG Yes there is, Yes there are.

7 D Well I've never seen 'em that way, but anyway it doesn't matter we have to do with what they're giving us here. Okay?

8 KG But, Ms. Smith . . .

9 D So Twinkees are Willie's favorite desert and he wanted to put them into as many lunches as possible. At the store Willie found that Twinkees come in packages of twelve and cost four fifty-six. How many days would one package of Twinkees last? (pause)

10 KG Four

11 D Um-hmm, because he'd use three a day.

12 KG Three a day?

13 D Yes. Where do you think you got four?

14 KG Oh. Oh, I see, a day.

15 D You'd use three a day divided by twelve is going to give you four days. So, a package will last four days. Okay, so the first answer is four.

16 KG Okay. Okay, I got to label. Six

17 D Oh yeah, that's right. Make sure you put your units on there. Very common mistake in story problems not to put your units.

18 KG Six. How many apples will Willie need? (pause) Four days. Four, How many days will one package of Twinkees cost--last. Four days, days.

19 D You probably don't need to record this all.

Although Kathy's father told her he had seen packages of twelve individually wrapped Twinkees (his conception of grocery shopping), in the end he invoked the rules of school math saying "it doesn't matter we have to do with what they're giving us here" (line 7). In the face of Kathy's protest, her father reread the question and Kathy provided the number of days the Twinkees would last--four. Kathy, although she agreed on the answer, questioned her father's explanation that Willie would use three Twinkees per day. Kathy's diagram showed four Twinkees for each person, not four days. In order for her to understand her father's explanation, she needed to reinterpret her diagram.

After they resolved this dilemma, Kathy began to document their work on her assignment sheet. In line 16, Kathy mentioned that she needed to "label" her answers. Her father agreed and told her to "make sure you put your units on there." The term "units" was never discussed in Ms. Smith's class. Rather, it appears to be a remnant of Kathy's father's university math classes. Kathy's father often brought up things from his classes while they discussed these tasks. During their conversations about Task 1, Question 2 he questioned the assumptions Kathy was making about the task--was Willie making a lunch for himself and two friends or just his two friends. He told her that the assumptions she made would influence how she would answer the questions. Kathy told him Willie was packing three lunches. Her father told her she "probably was right," and they would *assume* that through the rest of the questions, but that she always needed to be aware of the assumptions she was making. Like the term "units," the idea that the students need to make their assumptions explicit was not discussed in Ms. Smith's classroom.

As Kathy documented their work, her father suggested that she did not need to "record this all" (line 35). This is the first time the "rules" associated with this study are explicitly discussed by Kathy and her father. The collection of practices that influenced Kathy and her father's completion of this task now included grocery shopping, Kathy's conception of school math, her father's conception of school math, and this study. Each of these practices contributed to what would count as a correct answer and an appropriate presentation of the answer.

Both the rules of this study and Kathy's father's school math experience came up throughout their conversations. In the next segment of this conversation for example, in line 23, Kathy's father reminded her to write everything on her paper; another rule of this study. Near the end of this conversation, Kathy's father again brought to bear his school math experience. In line 37 he told Kathy "there's many ways to do it" and later he translates four times four into "four squared" (line 53). Both of these statements are likely to have come from his school math experience.

20	KG	How much would it cost to put Twinkees in the lunch for one day? Okay, so, four days it would cost four fifty-six.
21	D	So you can divide four fifty-six by . . .
22	KG	Four.
23	D	By four, if you would like to do that. No, no, no, no. Remember what he said. Do all work on the paper you're going to hand in.
24	KG	Okay. Four fifty-six

25 D Do it right here.

26 KG Four into four fifty-six.

27 D Um-hmm

28 KG No, I don't think so. Cause it said how much would one day [um-hm], so one day...

29 D How many days? How many days did you say up here?

30 KG Four days.

31 D Four days.

32 KG But, wait, it says 'how much will Willie pay for,' for wait, okay 'how much would it cost to put Twinkees in the lunches for *one* day.

33 D But we already know what it's gonna cost to put them in for four days [T/ for four days]. So, if we divide that number by four that'll tell ya, tell us what it is for one, right?

34 KG Oh, okay.

35 D Right?

36 KG Four

37 D There's many ways to do it.

38 KG 'kay. Four goes into four one time, one times four, I need a favor, I need a better background.

39 D You're right you do. Use this for now.

40 KG 'Kay, so. one into four once is four, zero bring down five, four goes into five, that's one, four goes into one, zero, zero bring down your six, four goes into six . . . It's not going to cost eleven something. I did something wrong. It's divide

41 D No, not eleven something. Oh, have you done division with decimals yet?

42 KG I don't think so.

43 D I don't think you have either. Well instead make it four hundred and fifty-six pennies then instead of four dollars and fifty-six cents. I mean I could show you how to do decimal division but if you haven't covered it yet it's probably just better to learn what you know. Four hundred fifty-six cents now. Remember, so you're answer is going to come out in cents.

44 KG Okay. (pause)

45 D You should even label that right out here. (pause)

46 KG Cents. Okay. Four days, four days goes into that sounds dumb, "four days goes into . . ." four hundred fifty-six cents.

47 D Yeah, it's better just to leave that un . . ., you know, you don't have to worry about it because it's not going to come out

48 KG 'Kay

49 D Actually it should.

50 KG Four one, four that's zero bring down the five, one goes into four four times, that's one bring down your six, four goes into sixteen . . . Is it four times four equals sixteen?

51 D Hmm?

52 KG Four times four is sixteen isn't it?

53 D Um-hmm, four squared.

54 KG Four goes into sixteen four times. four times four is sixteen, zero. Exactly one fourteen.

55 D Now wanna hold it. Exactly one hundred and fourteen, right?

56 KG Yup

57 D Remember, we have this in cents so it's exactly one hundred and fourteen cents.

58 KG Okay

59 D Which you know is a dollar fourteen but still you have to leave it in the same unit. You can't change that.

60 KG Okay.

61 D Where's the graph paper Kathy?

62 KG That I gave you? Up in your room. I gave you a whole pile of graph paper up in your room. There's no graph paper down here.

63 D Well, we don't need to record this, okay? [okay] anyway, go ahead, go on to the next one.

64 KG Okay. Number four. Okay so each one would cost one fourteen. fourteen a day. Ah, boy, okay. Now . . .

65 D Hold it that's one fourteen *per* day.

Throughout the last part of this conversation, Kathy's father brought to bear a rich knowledge of units. After he and Kathy agreed that they needed to divide $4.56 cents by four to determine the cost of putting Twinkees in the lunches for one day, Kathy began to compute the answer. After working a short time, Kathy announced she came up with "eleven something," but that could not be correct. In line 41, her father asked her if she had "done division with decimals yet?" When Kathy answered that she had not, her father told her "its better to learn what you know" and suggested they change the units from dollars to cents. By doing that, Kathy could divide 456 by 4--a division problem with which she was already familiar. Kathy's father reminded her often throughout the rest of their conversation that they had changed the units and she needed to document that.

In essence, Kathy first checked to see if the question represented an authentic grocery shopping task. If it did not, then the discontinuity could be resolved by invoking the rules of a different practice. In this case, whether or not the question was authentic was overridden by the rules of school math where it often does not matter if the questions are authentic. Documenting their work on the task included drawing a picture of the solution in a way that was consistent with doing math in Ms. Smith's classroom, making sure that the units they used were clearly marked on the paper in a way that represented Kathy's father's university math classes, and putting all documentation on one sheet for this study. All of these things represent a collection of practices that contributed to the amalgamated practice of "math homework in Kathy's household."

Summary and Conclusions

Completing Task 1 in each student's household represented a unique practice--an amalgamated practice--that reflected the participants' experiences in other socially-defined practices and different conceptions of the practice of elementary school mathematics. Although the students all worked on the same tasks and were held to the same assessment standards when they returned to school, their responses to the questions in Task 1 fulfilled requirements beyond those of Ms. Smith's class. The requirements of each of the practices brought to bear in the conversations were at least partially met in the conversations.

Amalgamated practices are likely to be developed in any household where students and parents work together on school math tasks. In any conversation, students and their family members are going to bring to bear experiences in various, related practices. Their attempts to fulfill the requirements of those practices will always lead to the development of an amalgamated practice. This developmental process is magnified in instances where the out-of-school world is brought into schools and homes as more practices are likely to contribute to the amalgamated practice.

The development of amalgamated practices as the students worked on Task 1 both refutes and supports the contentions of situated learning theorists. On one hand, the notion of amalgamated practices refutes early contentions that practices determine what and how people think and do. Although different practices clearly influence people, they are not deterministic as suggested by Scribner (1984) and other theorists. As Cobb (1993) and his colleagues have suggested, practices, while they have their own history, are made up of the people who participate in them and the experiences they bring to bear while participating. To understand a practice, and a person's participation in a practice, it is necessary to understand the experiences the participants bring to the practice as well as the practice's own history.

The development of amalgamated practices, on the other hand, supports the more contemporary contention that learning involves extending what is learned beyond the immediate situation. In each of these conversations, the students and their parents used things learned while participating in other practices. What they brought with them to these conversation reflected their own experience in school mathematics classes and in other socially defined practices. Their participation was not limited to the things in their immediate surroundings.

But, just as bringing different practices together in a conversation can lead to conflict, so to can changes within a practice. The conversations presented in this chapter have foreshadowed the influence different conceptions of the same practice can have on students' work. Recent calls for reform in mathematics education have suggested that teachers change their instruction to focus on conceptual understanding rather than the traditional focus on computation. The calls for reform have created a situation in which different conceptions of elementary school mathematics are likely to collide. In chapter five I look at the influence the evolving practice of mathematics education has on students' conversations with their parents.

CHAPTER 5

THE INFLUENCE OF EVOLVING PRACTICES ON STUDENTS' COMPLETION OF THE TASKS

The conversations presented in Chapter 4, pointed out that the students and their parents brought to bear various experiences that shaped how they approached and answered questions. In their conversations they constructed a new set of rules--an amalgamated practice--that drew on and often satisfied the sometimes disparate experiences the participants brought to bear. To construct the amalgamated practices, the students and their parents needed to resolve discrepancies among the various practices. As a result, practices often competed for prominence and, as a result, collided in ways that illuminated the differences among the practices and how the participants resolved those differences.

The evolution of a practice can also create situations in which practices, or conceptions of the same practice, compete. Technological advancements, changing beliefs and values, and other conditions that demand the practice adapt can all lead to the evolution of a practice. In tool and die shops, for instance, the development of computer numeric control (CNC) lathes have changed what it means to be a machinist (Martin & Beach, 1992). No longer can machinists rely on their mechanical knowledge of a metal lathe and their ability to precisely turn a piece manually. They now must understand the abstract connections between the computer image of the piece they are fashioning and the mechanical performance of the machine. As a result, what it means to be a machinist has changed over time; machinists are now technicians as well as craftsmen.

If two machinists--one a traditional craftsman and the other a newly trained technician--sat down to solve a problem, they would bring to bear different experiences that would shape how they defined and solved the problem. The traditional craftsman might look for the problem in the lathe, whereas the newly trained technician might look for the problem in the computer. As they worked, their different perspectives might compete for prominence as did the various practices brought together in Task 1. This sort of competition occurs in any evolving practice.

Mathematics education is an evolving practice. Recent changes in beliefs about the mathematical skills and abilities students should develop while participating in mathematics classes (i.e., what it means to do mathematics) have led to different ideas about mathematics instruction. Reform documents (National Council of Teachers of Mathematics, 1989; National Council of Teachers of Mathematics, 1991; National Research Council, 1989; National Research Council, 1990) have argued that teachers need to reduce their emphasis on isolated computational skills and include opportunities for students to recognize mathematical elements in situations, flexibly apply appropriate mathematical tools, and engage in mathematical reasoning. Richards (1991) has labeled mathematics instruction in the spirit of the reforms *inquiry math* and traditional school mathematics instruction *school math*. Inquiry math classrooms, he argued, emphasize the mathematical skills called for in the reform documents. In inquiry classrooms, computation, although it still plays an important role in mathematics education, is not the sole focus of instruction. Students spend time playing with numbers, debating mathematical definitions, and exploring various mathematical situations.

Just as the "new math" in the late 1950s was incompatible with parents' school math experience, current calls for reform suggest a conception of mathematics that often differs from the experience of parents of many school-aged children. Kathy's father told me in an early interview that:

> The math when I was a kid was, you know, they give you a book and you have a set of problems, you do those problems. Now I think it's an awful lot more, um, where math is set up to make children, ah, formulate their own problem, a lot of story problems. I think they start at a much earlier age now then they did when I was in school. In fact, Ms. Smith and I were talking about this. Ah, when I took Algebra and

> Trig for the first time in college, when they introduced us to creating our own story problems, writing our own formulas and stuff like that, I was completely inept at it. Whereas people who had just come out of high school were quite a bit better at it. So I think that it is slowly changing.

The evolution of elementary mathematics instruction can have disconcerting consequences. Pete's mother felt the changes were quite intimidating and stopped her from working with Pete. She told me:

> We didn't start math as early as they do in grade school. . . . It was, I don't know, a lot different. Nothing was as required as it is now. Like, whatever it is they're doing now in math, I have absolutely no idea how to do any of it. So, it's always someone else in the house that's helping him with that. . . . We got as far as fractions. That's how far I went in school.

Although some parents, such as Kathy's father, may embrace the changes in mathematics instruction, others, like Pete's mother may shy away from being involved in their children's mathematics homework. In either case the changes will have an impact on how students and their parents interact while completing school math tasks. As occurred with the hypothetical machinists, these different conceptions of the same practice may compete when parents and their children work together on inquiry math tasks. This chapter looks at the influence different conceptions of elementary mathematics had on students' and their parents' conversations.

Tasks 2 and 3 presented students with mathematical tasks that reflected the two views of mathematics education presented by Richards. In Task 2 the students were presented with a series of word problems reminiscent of those found in traditional elementary mathematics textbooks and in school-math classes. In Task 3 the students were asked to determine how many three digit numbers could be formed from the digits 3, 4, and 7, how many four digit numbers could be formed from the digits 0, 2, 5, and 9, and, based on their work, how they might determine how many numbers are possible from any set of numerals. Both of these tasks focused on place value, the next topic in Ms. Smith's ongoing classroom instruction. In Task 2 the students explored the impact of multiplying or dividing numbers by multiples of ten. Task 3 asked the students what digits stood for in different places in the numerals and how they helped determine the value of numbers.

The conversations students had with their parents as they completed Task 2 were quite different from those they had while working on Task 3. One explanation of the differences in the conversations centers around the parents' familiarity with the tasks students were asked to complete. When the tasks were continuous with the parents' past experience, they were much more willing to offer assistance and guide the students to an answer. When the tasks contrasted with their experience--that is, when the practices were discontinuous--parents contributed less to the conversation and deferred to the students or classroom teacher more often. In the following sections, I look closely at the conversations students and their parents had while working on Tasks 2 and 3 and the solutions the students brought with them to school.

Task 2

Task 2 included a series of story problems about a girl named Keisha, some books, and an agreement she made with her mother to earn a certain amount of money for reading each book. In the first two questions the amount of money Keisha would receive for reading each book changed and the students were to determine how the change would effect Keisha's situation. In Questions 3 and 4 the number of books Keisha had increased by a multiple of ten. Students were asked to compute the number of books Keisha now had. In Question 5 Keisha divided her books into 100 stacks. The students were asked to determine how many books would be in each stack. Question 6 asked the students to divide various multiples of ten by ten. The assignment sheet is shown in figure 5.1.

Math

1. Keisha had 142 books. Her mom told her she'd give her $10 for every book she read. How much money would she receive if she read all of her books?

2. If she received $3 for each book, how much would she receive?

3. If, over three years, she received 100 times as many books as she had in problem one, how many books would she have?

4. If she received 1000 times as many books, how many would she have?

5. If Keisha had 1420 books and she put them in 100 stacks, how many books would be in each stack?

6. How many groups of 10 equal 100?
   " " " " 10 " 1,000?
   " " " " " " 10,000?
   " " " " " " 100,000?

Figure 5.1: Task 2 Assignment Sheet

Students' answers to Task 2 are summarized in table 5.1.[1] The table includes students' work as it appeared on their assignment sheets and was discussed in their conversations at home. All six students (100%) arrived at the same answers for Questions 1 and 2. Although Pete's computation for Question 1 looks slightly different than the other students', it is likely that they all used the same rules to guide their computation. The rules commonly taught in school math classes hold that writing the largest number (i.e., the one with the most digits) above the smaller number makes multiplication easier. The difference in Pete's computation might be explained by the inclusion of a the decimal point and two zeros indicating cents in his work. By adding the decimal point and zeros to ten dollars, the number becomes bigger than 142. Following the rule, $10.00 should be placed above 142 in the multiplication algorithm.

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[1]Three of the students (Karen, Pete, and Shaundra) recorded their conversations for Task Two. The other answers included in table 5.1 were taken from students' written work.

Table 5.1

Solution Table for Task 2

Ques. #	Karen	Kathy	Pete
1.	142 x 10 000 + 1,420 $1,420.00	142 x 10 000 + 1,420 $1,420	$10.00 x 142 2000 4000 10000 $1,420.00
2.	1 142 x 3 $426.00	142 X 3 $426	1 142 x 3 $426.00
3.	142 x 100 000 0000 +14200 14,200	142 x 100 000 0000 +14200 14,200	300 + 142 442 books
4.	1,000 x 142 2000 40000 100000 142,000	1,000 x 142 2000 40000 100000 142,000	1,000 x 142 2,000 4,000 10000 142,000
5.	14 R20 100)1420 100 420 400 020	14 r2 100)1420 10 42 40 2 14 in each stack	142 100)1420 100 420 400 200 200 0
6a	10	10	10
6b	100	100	100
6c	1,000	1,000	1,000
6d	10,000	10,000	10,000

Table 5.1 cont.

Solution Table for Task 2

Ques. #	Ronnie	Shaundra	Tony
1.	142 x 10 1,420	142 x 10 000 + 1,420 $1,420	142 x 10 1,420
2.	$426	142 X 3 $426	142 X 3 $426
3.	142 x 100 000 000 142 14,200 + 100 14,300	142 x 100 000 000 +142 14,200	142 x 100 14,200
4.	142,000	142 x 1000 000 000 +142 142,000	1,000 x 142 142,000
5.	122 r20 100)1420 10 22 20 20 0 20 14 stacks	14.20 100)1420 -100 420 -400 20	1420 + 100 14.20
6a	10	10	10
6b	100	100	10 hundreds
6c	1,000	1,000	100 hundreds
6d	10,000	10,000	10,000

Four of the six students (67%) answered Question 3 saying Keisha now had 14,200 books. All four of these students multiplied 142 by 100 to answer the question and their written work was virtually identical. Ronnie, although he began his solution in the same way as the other three students and arrived at 14,200 midway through his computation, determined that Keisha now had 14,300 books.

Pete's answer of 442 books can only be explained by looking at the conversations he had both at home and in school. While Pete worked with his grandmother they had this conversation.

1 G Number three. If, over three years, she received a hundred times as many books as she had in problem one, how many books would she have? [pause] What do you have to do with that problem? Over three years she received a hundred as many, hundred times as many books as she had in problem one. How many books would she have? [pause] What do you have to do to this problem?

2 PC Add it. [pause]

3 G Right. Okay, number four.

After Pete suggested that he add in line 2, he wrote 300 + 142 on his paper and added the values together arriving at an answer of 442. Although there is no explanation of where the values Pete added together came from, his grandmother supported his choice of addition as the appropriate way of answering the question and Pete returned to school believing his answer was correct. The origin of the values Pete had added together became more clear when he discussed the questions in his math group the next day.

When Pete, Tony, Ronnie, and Jason compared their answers to Questions 1 and 2 they found they all answered the questions the same way. That was not true for Question 3; no one in their math group had the same answer. As they compared their answers they were joined by a special education teacher who was observing students in their class. The special education teacher asked the group questions about their answers that illuminated the origin of the values Pete had added and Jason's answer of 56,800 books as well.

What caused the different answers was the students' different interpretations of "over three years" in the question. Entering the numbers into his calculator, Jason explained his answer this way:

> One-four-two times one-zero-zero, one thousand four hundred and two.[2] Plus you need it for over three years, times four equal five, see . . .

Jason showed the special education teacher his calculator with 56,800 written in the display.

Jason had interpreted "over three years" to mean that the number of books increased one hundred fold each year for four years. As a result, he multiplied Keisha's initial 142 books by 100 and then by four to arrive at an answer of 56,800 books.

As Jason entered the numbers into his calculator, Ronnie, who Pete had convinced that 442 books was the correct answer, explained their solution this way: "Take one-forty-two, add a hundred to it, two-forty-two. Add another hundred to it, three-forty-two. Add another hundred to it, four-forty-two." Ronnie[3] and Pete had interpreted "over three years" to mean that Keisha added one hundred books a year for three years to her total number of books.

To these students all of the information in the question needed to be used in the solution; their answers needed to reflect "over three years" in some way. As was pointed out in chapter four, the students' previous experience in school math classes may have led them to believe that story problems included no irrelevant information.

The students answered Question 4, which did not include a time span, but asked the students to determine the numbers of books Keisha would have if she received 1,000 times as many as she had in Question 1, with no difficulty. All six of the students who participated in this study and Jason, the only member of Pete's math group who did not

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[2]Jason misread his calculator here. It read 14,200.

[3]The paper Ronnie turned in at the end of this task--the one that determined his grade--listed the answer to this question as 14,300. Ronnie arrived at that answer by multiplying 142 by 100 getting 14,200 and then adding 100 arriving at a final answer of 14,300. That answer is as yet unexplained.

record conversations at home, listed 142,000 as the number of books Keisha would receive. Their ease in answering this Question 4 supports the notion that the time span in Question 3 confused the students as they answered that question.

Question 5 asked how many books would be in each of 100 piles if Keisha had 1,420 books. Only two students, Ronnie and Kathy, labeled their answers. Ronnie, although his computation was incorrect, wrote that Keisha would have 14 stacks. Kathy, whose computation also is incorrect, wrote there would be 14 in each stack. In both of these answers the student dropped the remainder from the quotient. This may reflect their understanding that the remainder could not be evenly divided among the piles. The other four students used no labels and reported answers of 14 r 20 or 14.2. Both answers were computationally correct, but neither interpreted the remainder in the quotient--there could not be 14.2 books in each pile.

All the students answered Question 6 the same way. They each listed the answers to the division problems and showed no computation. The students' answers were all correct.

The Conversations around Task 2

In the conversations for Task 2, parents were quite directive. For each of the six questions, the students used the traditional multiplication and long division algorithms--often talking aloud throughout the computation. The parents' role was to guide the students to the appropriate operation and to monitor the students' computation. In an illustrative conversation, Shaundra and her father were trying to determine how much money Keisha would receive if she was paid 10 dollars for each of the 142 books she read.

1 SQ Keisha had a hundred and forty two books. Her mom told her she'd give her ten dollars for every book she read. How much money would she receive if she read all of her books?

2 F Okay, so we have a hundred and forty two books to read. And, if you're going to give somebody ten dollars for every book, that means you're gonna do what to it

3 SQ You could times it or you can just count by tens up to a hundred and forty two. Well,

4 F You'll also end up . . . It's always better to multiply. That saves you a lot of time than adding up ten four hundred and forty two times. So . . .

5 SQ I'm talking about if you put like ten, twenty, thirty, . . .

6 F Oh. that's the long way. that's the long way and if you do that, you'll be you'll be way behind anybody in the class trying to finish that. That's why you have to multiply. Because it cuts it short. You can get through a lot faster multiplying, that's why you multiply. That's the only way. So, you multiply a hundred and forty two times what?

7 SQ Ten.

8 F A hundred forty two times ten. And, what do you get?

9 SQ Zero times two is zero. Zero times four is zero. Huh-mm [no] You do that and you gotta search [inaudible] Cause if you go, cause when you do that you always times it from the bottom number and if you go zero times two is zero, zero times four is zero, and zero times one is zero.

10 F You always do the first number first. It'd be one times two, one times four, and one times one and then you bring the zero over one. You did it backwards. Go right ahead and tell me how you're going to do it.

11 SQ What I'm going to do is, all that is times zero is zero.

12 F All of what is times zero?

13 SQ Anything times zero is zero. Cause a hundred and forty two times ten you get [Okay] zero times two is zero and zero times four is zero and zero times one is zero. [okay] So, what I'm going to do is I'm gonna go over to the next number and times it by one.

14 F Okay.

15 SQ All right, one times two is two, one times four is four, and one times one is one and my answer is one thousand four hundred and twenty.

16 F One thousand four hundred and twenty is your answer. One thousand four hundred and twenty what?

17 SQ Dollars

Shaundra began this conversation by reading the question aloud and attempting to answer it with no assistance from her father. In line 3, she offered her father a choice of two different operations. Her father, citing the efficiency of multiplication, steered her away from addition as the correct choice. In lines 4 and 6, he told her multiplication was always better than addition because it saves time and will help her stay up with the rest of her class. Once they agreed that multiplication was the correct choice, Shaundra began the computation. As she worked, her father paid close attention to what she wrote and asked questions when he was unsure of her actions. Finally, Shaundra's father made sure she labeled her answer correctly.

Although Shaundra carried out the steps in the computation, her father remained in control while answering this question. He directed her to a specific operation--one he believed was the best suited for the question--and guided her through the computation making sure she carried it out correctly and labeled her answer in an appropriate way.

The direction Shaundra's father provided in the conversation was apparent in all of the conversations the students had while working on Task 2 at home. In the conversation between Shaundra and her father, Shaundra made choices that did not always fit with her father's conception of the problem. When that occurred, he redirected her to a more appropriate way of answering the question. In other conversations, the students' responses fit closely with the parents conception of the questions. Conversations in which the students and their parents agreed on the what needed to be done were very short and to the point. In the following conversation, Pete and his grandmother computed the amount of money Keisha would receive if she read 142 books and was paid three dollars for each book she read.

1 G Okay, number two. If she received three dollars for each book, how much would she receive? (pause) I'll read it again, the dogs kind of interrupted us. If she received three dollars for each book, how much would she receive? So, how many books did she have?

2 PC A hundred and forty two.

3 G And the number is three dollars so you have to, what do you have to do to figure that one out?

4 PC Times it.

5 G Okay. So it's three times a hundred and forty two. [pause] Let's see. Um, three times two is . . .

6 PC Six.

7 G Okay. Three times four is . . .

8 PC Twelve

9 G Okay, three times one is . . .

10 PC Four.

11 G No.

12 PC Three.

13 G plus one

14 PC Four

15 G Okay. [pause] So, she would receive, that's right. Four hundred and twenty-six dollars.

Pete's grandmother began this conversation by reading the question aloud. After reading the question, she asked Pete a series of questions that led to using the traditional multiplication algorithm to compute the amount of money Keisha earned. By asking the questions she gradually talked Pete through the multiplication algorithm as he did the computation, correcting him when he made mistakes. Finally, she told Pete he was right and rephrased the answer to include dollars.

Pete and his grandmother's conversation was similar to Shaundra and her father's in important ways. Both conversations, and all the conversations students had with their parents during Task 2, comprised four sections: (a) read the question aloud; (b) choose the appropriate operation; (c) compute the answer; and, (d) present the answer in an appropriate way. In each of these four sections, except perhaps the first, the students initially worked by themselves. When the parents recognized an error or that the student was having trouble, they intervened and directed the student to the correct answer.

The direction shown by the parents in these conversations was also apparent in the conversations they had while working on Task 1. In order for the parents to direct the students as they did in these two tasks, they needed to be familiar with what was being asked of the students. Tasks 1 and 2 likely represented practices with which the parents were familiar--grocery shopping and a traditional conception of elementary school mathematics. Task 3, although it too represented the practice of school math, included things the parents were less likely to have encountered in their school math experience or used in their daily lives; that is, it represented a different conception of the practice of elementary school mathematics. As a result, the conversations students had with their parents were also different from their conversations on the previous tasks.

Task 3

Task 3 was assigned on two consecutive nights. On the first night, the students were asked to compile a list of three-digit numbers using the digits 3, 4, and 7 and explain how they knew when they found them all; to write those numbers in order from largest to smallest and explain why they ordered them as they did; and to write them in expanded notation and explain what each digit meant in the numbers. On the second night, they repeated the assignment, but with four digits--0, 2, 5, and 9, and a fourth question that asked the students if they could predict how many numbers could be generated from a set of digits. The assignment sheets are shown in figure 5.2

Task 3 resulted in more incomplete or unanswered questions than did the other tasks. Although all six students recorded conversations for this task, the structure of the conversations was different than those for the other tasks and they differed among the households as well. The structure also changed from the first night to the second.

Each night, Karen had a short conversation with her father as she read her answers into the tape recorder. They did not, however, work together to answer the questions. Although the conversations they recorded often seemed to be cursory inspections of Karen's completed work, Karen recounted their conversations in great detail as she worked in her math group in school. Karen, for instance, taught the class an alternative way of writing numbers in expanded notation that she claimed her father taught her. That conversation was not recorded.

Math

1. Compile a list of three digit numbers that can be made using the digits 3, 4, and 7.

   A. How many numbers did you come up with?

   B. How do you know when you've found all the numbers?

2. List all the different numbers in order from smallest to largest

   A. Tell why you ordered the numbers as you did.

3. Write each of the numbers you find in expanded notation.

   A. Explain what each digit means in each number.

Math

1. Write down as many four digit numbers as you can using the digits 0, 2, 5, and 9.

   A. How many numbers did you find?

   B. How do you know when you've found all the numbers?

2. List all the different numbers in order from smallest to largest.

   A. Tell why you ordered the numbers as you did.

3. Write each of the numbers you find in expanded notation.

   A. Explain what each digit means in each number.

4. Now that you have worked with both three and four digit combinations, can you think of a way to predict the number of combinations you can make with a certain number of digits?

Figure 5.2: Task 3 Assignment Sheets

Kathy worked with her father both nights. The first night they worked together through all three questions. But in spite of her father's presence, Kathy completed the task alone after having the directions clarified or the task prepared in some other way. The second evening, Kathy began working with her father, but, after they reconstructed the algorithm that predicted how many numbers were possible,[4] Kathy's father left her

[4]Kathy and her father's conversations are analyzed more closely in chapter 6.

to complete the task by herself. As Kathy expanded the numbers in her list, her father watched a television show about Malcom X and commented on his discontent with the federal government.

Pete was ill the first night, so he and his grandmother completed both parts of Task 3 the second night. They worked together on the first two questions of Task 3a, but when the third question asked the students to write the numbers in expanded notation, Pete's grandmother suggested he leave his paper blank and ask Ms. Smith when he returned to school.[5] Their conversation for the second part was similar. Pete and his grandmother answered the questions together. When they were to write the numbers in expanded notation, Pete's grandmother read the question and said: "Write each of the numbers you find in expanded notation and we don't know what that means." Pete did not include an answer to that question and they went on to answer the last question together.

Ronnie and his mother also set out to complete both parts of Task 3 the second night, but they worked on the assignments in reverse order. After completing the second part of Task 3, Ronnie suggested they begin working on the first part. His mother, however, told him it was too late and that he needed to get some sleep. Ronnie never did complete the first part of the task.

Shaundra and her mother also worked on both parts of Task 3 on the second evening. Shaundra's mother, as were many of the parents, was uncertain about the content of the task. Rather than leaving Shaundra to work alone, however, Shaundra's mother worked closely with her as she completed the first part. She eagerly watched Shaundra as she compiled her list of numbers, wrote them in ascending order, and wrote them in expanded notation. What she learned from the first part of Task 3, she used to provide direction on part two. But, when Shaundra began "expanding the numbers" on the second part, her mother left her to work alone. Before she left, she instructed Shaundra to write out four numbers and a note for Ms. Smith saying that 24 was too many

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[5]Pete never asked Ms. Smith about expanded notation.

numbers to expand. Shaundra did not complete the second part of Task 3. As she worked the tape recorder shut off and Shaundra came back and recorded a message to me on the tape: "I won't get to the last problems. Sorry."

Tony and his mother began working on Task 3 together, but when the assignment asked them to write the numbers in expanded notation, Tony's mother shut the tape recorder off and the remainder of their conversation was not recorded. The second night, Tony read his incomplete answers into the tape recorder alone.

Table 5.2

Solution Table for Task 3a

#	Karen	Kathy	Pete
1a	6	6	6
1b	KM: How do *I* know? F: You used your threes, fours, and sevens.	KG: I've used all the numbers. F: Every number has occupied every position.	G: you have two that start with three, two that start with four, and two that start with seven.
2a	347, 374, 437, 374, 734, 743	347, 374, 437, 374, 734, 743	347, 374, 437, 374, 734, 743
2b	F: you put the smallest first because that's the easiest.	Looked at the Hundreds, then tens, then ones.	G: you wrote it down because this number came before this number?
3	Explained how she did one and said she did the same for the rest of the numbers.	No response	G: told Pete to get help when he went back to school.
3a	Hundreds, tens, and ones.	KG: I did three equals three hundred, the four equals forty, and the seven equals seven. I did that to all of 'em.	Ones, tens, hundreds

Table 5.2 (cont.)

Solution Table for Task 3a

#	Ronnie	Shaundra	Tony
1a	6	6	6
1b	No response	SQ: I've used all the numbers because I've mixed all the numbers up as many ways that are possible	RB: I used them all and I can't find no kinda way.
2a	No response	347, 374, 437, 374, 734, 743	347, 374, 437, 374, 734, 743
2b	No response	from lowest to highest	No response
3	No response	Constructed number sentences that included equations for each digit. (E.g., 347 could be written as : (3x1)+(7-3)+(6+1)	No response
4	No response	the three stands in the one hundreds place and it would become three hundred and the seven stands in the tens place it becomes seventy now go to the four . . .the ones place And, in that place stands the four, which is just plain four.	No response

In general, the students worked alone more the second night than the first. The parents' decreased role in these conversations might be explained by the repetition of the two parts of Task 3. Many of the first evening's answers could be expanded for the second evening's questions. The parents may have believed the students did not need the amount of assistance they needed the first night. An alternative explanation suggests the parents' uncertainty with the content led them to withdraw from the conversations. Many of the parents stopped contributing when the students began writing numbers in expanded notation the second night. The parents often explicitly stated that they did not know how to write numbers in expanded notation and told the students to work alone. Despite working alone, the students all completed Task 3.

The students' responses for Task 3 are shown in tables 5.2 and 5.3. Students' responses were derived from their conversations and their written work. The students' written work for Task 3, more than any other task, failed to match what was said in the conversations. At times the conversations included important aspects of the students' answers that were not written down. As a result, the students or their parents are sometimes quoted in the tables. When quotes are used, the students' initials or the initial of their parent is given to credit the speaker.

Task 3 A

Question 1. On the first evening of Task 3, all of the students and their parents found six three-digit numbers. The students' explanations of how they knew they were finished all focused--at least initially--on the idea that they had used all the digits. In three conversations the parents expanded the explanation to describe a more systematic approach to their search for the numbers. Kathy's father included the number of positions a digit could occupy in their explanation. Throughout their conversation, Kathy's father tried to determine how many numbers they should be able to find. Although he was not completely convinced, at the end of the conversation, he concluded the number could be determined by adding the number of digits available to fill each position in the three digit numbers (3+2+1=6). Pete's grandmother recognized that each digit appeared in the hundreds place in two of the numbers in their list. She expanded Pete's explanation to include multiplying the number of digits available by how many numbers begin with each digit (3x2=6). Shaundra's mother also recognized that each number appeared in the hundreds place twice. She told Shaundra about her discovery this way:

> But you know what else I learned? Okay, in each one of the numbers, like three forty seven, three seventy four, you can only mix them up twice on each one. See

> the sevens, you have seven forty three and seven thirty four. And the fours, you have four thirty seven and four seventy three. So, there's only certain ways you can turn it around.

Each of these extensions influenced how the students and parents approached and answered the questions the second night of Task 3. In the conversation between Kathy and her father, for instance, Kathy's father continued his search for the way to predict how many numbers were possible. In Shaundra and her mother's conversation, Shaundra's mother drew on the patterns she saw in Task 3 A to devise a strategy that would make Shaundra's work more efficient.

Question 2. All five of the students who answered this question (100%) ordered the numerals in the same way. Four of the five (80%) justified their order saying they ordered the list from "lowest to highest." Although Ms. Smith had talked about place value in class before the students took Task 3 home, none of the students explicitly mentioned place value as a reason for ordering the numbers.

Question 3. Question 3 asked the students to write the numbers they found in expanded notation. Sensing the students might be unfamiliar with this topic, Ms. Smith had introduced expanded notation and asked the students how they might write their numbers. The students who were familiar with expanded notation went to the board and demonstrated how the numbers could be written. The class agreed that 48,265 should be written 40,000+8,000+200+60+5 as the numbers in this representation reflect the value of each digit in the original number. Despite this discussion, the students and their parents found this question difficult as they worked at home: five of the six parents were uncertain about how to answer this question.

Four of the five parents (80%) who were unsure of expanded notation asked the student how it was done. The fifth parent, Tony's mother, shut off the tape recorder and they did not record the rest of their conversation. In three of the four recorded conversations, the students showed their parents how to write the numbers in expanded notation.[6] Once the student did one or two, the parents became more willing to help them finish writing the numbers.

All of the students who recorded conversations answered the last question same way. They all used place value labels to explain what the different digits meant. They did not, however, make any connection between the expanded notation and the last question.

Task 3b

For the second part of Task 3, the students repeated the activity they had done the previous evening, but with four digits (0, 2, 5, and 9). At the end of the second part of Task 3, the students were asked how they might predict how many numbers could be derived from a given set of digits. The students responses to Task 3 B are summarized in table 5.3. the lists of numbers the students compiled and the lists of the numbers in order from smallest to largest are not included in the table to preserve room.

Question 1. The students all compiled different lists of numbers the second evening. Even though they did not agree on how many numbers were possible, all but one student, Kathy, held fast to the notion that the best way to know they had found all the possible numbers was to "use all the numbers." Kathy and her father, in contrast, determined that there should be twenty-four numbers based on an algorithm Kathy's father remembered from his school-math experience. After a lengthy discussion during which they sought out a way to explain the twenty-four numbers Kathy found, Kathy's father remembered that the number of possible permutations could be determined by multiplying the number of positions available for each digit (4x3x2x1=24).

Question 2. The students' lists of numbers in order from smallest to largest varied depending on the numbers included in their original lists. All of them, however, listed the numbers in the proper order. Only Ronnie mentioned place value while explaining why he ordered the numbers as he did. Other than Tony, who said "because if I didn't I'd

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[6]In the fourth conversation, Pete's grandmother asked him if he understood expanded notation. Pete told her he did not and she suggested he talk to his teacher about it when he returned to school. Their conversation ended there.

get the wrong answer," all the students (67%) used the same answer they used the previous evening.

Question 3. All of the students wrote the numbers out in expanded notation using the method Ms. Smith introduced in class or the method Karen taught the class earlier in school. Karen's father, the only parent who was completely sure of how to answer this question, taught her a different way to write expanded notation that Karen brought back and presented to the class. The class agreed that the method worked and accepted it as an alternative to the method on which they had they agreed. Using Karen's father's notation, 347 would be written (3x100)+(4x10)+(7x1).

Table 5.3

Solution Table for Task 3b

#	Karen	Kathy	Pete
1a	16	24	25
1b	KM: I did the thousands, the hundreds, the tens, and the ones.	Multiplying 4x3x2x1=24 helped them determine how many numbers they should find.	PC: I checked over it. I checked all the combinations.
2b	KM: Smallest to the largest.	Looked at the thousands, hundreds, tens, then ones.	PC: smallest to largest
3a	thousands, hundreds, tens, and ones	KG: It means it has the thousands, hundreds, the ones and tens.	Ones, tens, hundreds, thousands
4	You can make sixteen.	KG: If you have three, you can make six. If you have four, you can make twenty-four.	Multiply the number of combinations possible for each number by the number of digits you have. E.g., for three digits multiply two times three, for four digits multiply 4 times six.

Table 5.3 (cont.)

Solution chart for Task 3b

#	Ronnie	Shaundra	Tony
1a	18	23	12
1b	M: 4x4=16 RB: I checked all of them. I checked to see if I could make any more numbers.	SQ: I know I found the numbers because I worked with every numeral possible.	TW: by switching them, by scrambling them up, and looking.
2b	RB: I used my place value.	SQ: to find how many different possible ways I could use the numbers	TW: Because if I didn't I wouldn't get the answer.
3a	Ones, tens, hundreds, and thousands.	No response	You take two thousand, plus zero plus fifty plus nine and get two thousand fifty nine.
4	M: I would think that you would take the number and, you know like, four digits times the same amount.	No response	No response

Four of the five students who answered Question 3a listed the place value of each number when describing what each digit meant in the numbers. Tony read an example of the number sentences he had written down. Although he did not explicitly mention place value in his answer, he did read the places correctly in his number sentences.

Question 4. Of the four students who answered Question 4, two only reported how many numbers they found each evening. Although Kathy and her father agreed that the algorithm determined how many numbers were possible, Kathy was one of the students who reported how many numbers she had found while completing the two parts of Task 3. The other two, Ronnie and Pete, answered Question 4 saying that the number could be calculated in some way. Ronnie and his mother suggested they could predict four-digit numbers by multiplying 4 x 4=16. Pete's grandmother summarized their answers saying they could multiply the number of combinations possible for each number by the numbers of digits to determine how many were possible. Pete and his grandmother's solution is described in more detail below.

The conversations about Task 3

In contrast to the conversations the students and their parents had for Task 2, the parents were much less directive as they worked on the first part of Task 3. In the conversations for Task 2, the parents were able to recognize difficulties the students were having and direct them to a more appropriate strategy for answering the questions. The students seldom questioned the parents' strategies and the parents only corrected the students when they were mistaken. The adults clearly directed the conversations.

In Task 3a, however, the parents were often uncertain about the tasks and more often asked for and accepted students' explanations--even when their explanations were wrong. The parents' uncertainty about the tasks led to two reactions. In two households, the parents left the students to answer the questions on their own or ended the conversation and shut off the tape recorder. In the other three households, the students and their parents worked together to answer the questions.[7] When this occurred, the students, rather than their parents, led the conversations. In extreme situations they switched roles: the student, based on his or her conception of what was expected upon returning to school, became the more knowledgeable other and the parent became the

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[7]Only five conversations were recorded for Task 3a. Ronnie did not complete that part of the task. In their conversation for the second part of Task 3, however, Ronnie and his mother worked together to compile the lists and answer the questions. At the end of their conversation, Ronnie's mother contributed to Ronnie's understanding in ways that typify the conversations in other students' homes. Karen's recorded conversations with her father did not exemplify the conversations in other students' homes. Karen did, however, recount unrecorded conversations with her father as she worked in her math group in school. Many of the ideas she recalled from those conversations influenced her group's and the entire class' thinking about this task. These conversations suggest that the conversations in which the parents contributed to the students' thinking, even when parents were uncertain about the content, may be even more prevalent than the data presented here suggest.

learner. The students explained to their parents what expanded notation was and demonstrated how expanded numbers could be written and wrote them down.

After the students finished their instruction, they worked alone. As they worked, their parents watched closely to figure out what the students were doing and began to contribute when they thought they had figured it out. When the students finished, their parents often reviewed their work and elaborated the things the students said or wrote. The parents' elaborations did not seem to reflect their knowledge of mathematics or ability to solve mathematical problems as much as it did a disposition to become engaged in their children's work, even when the work deviated from their conception of elementary mathematics content and instruction. The parents in these conversations, like the students, were trying to figure out the answers as they worked. Their observations led them to ask different questions and offer insight the second evening they were unable to offer at first.

The parents who were actively engaged in the first part of Task 3 were able to contribute to the conversations about Task 3b in one of two ways. Some parents helped the students use the lists of numbers they generated for each part the task, and patterns within those lists, to answer Question 4 on the second part of Task 3. Question 4 asked the students how they might predict how many numbers would be possible with a given set of digits. This contribution is shown in the conversations Pete and his grandmother had as they worked on the two parts of Task 3.

After Pete compiled his list of numbers for Task 3a, his grandmother read the next part of the question. The question asked how they knew they had found all the numbers.

G How do you know when you found all the numbers? (pause) How many combinations of each number do you have.

PC two?

G So you have two that start with three, two that start with four, and two that start with seven.

Pete's grand mother had recognized a pattern in the list. The pattern was that there were two combinations for each number; that is, that each number was in the hundreds place twice. Pete wrote this down as the reason he knew he had found all the numbers.

Pete compiled his list for the second part of Task 3. When he had finished, his grandmother recognized a similar pattern in Pete's list: each number was in the thousands place six times. The two patterns Pete's grandmother found served as the data from which they derived a way of predicting how many numbers would be possible. They had this exchange:

G Okay, it says, now that you've worked with both three and four digit combinations, can you think of a way to predict the number of combinations you can make with a certain number of digits? With your three digit ones you had, how many combinations of each number did you have?

PC What?

G With your three digit numbers--right there on that page--how many combinations did you have? Of each number?

PC Two.

G Okay, so you know that you can get two combinations out of each, each three digit, out of each three, each of the three digits. So you have . . .?

PC Two, four, six.

G That, okay, so you know you can get six combinations out of a three digit. so, on your four digit one here, how many combinations did you get on each number?

PC Six.

G Okay and there's four numbers. And you came up with how many altogether? What did you tell me?

PC Twenty-four.

G Okay. So, six times four is what?

PC Twenty-four.

G Um-kay. So, you have six combinations, six, yeah, six combinations in each number and six times four numbers is twenty-four. So, if you had, um, as long as you know how many combinations you have, like two times three is six, six combinations. Six times four is twenty-four. So, do you understand that you can mul, you simply multiply that number out? If you knew your multiplication numbers better, it'd be easier wouldn't it? We have to work on those. Um-kay, that completes your math for today.

Pete's grandmother was onto something. Her method of determining how many numbers were possible would work in all situations, provided you could determine "how many combinations" were possible for each number. Given an opportunity, they may have gone on to discover that they could use factorial multiplication to determine how many combinations were possible and extend their response to include multiplying together the number of digits available while filling each place in the numbers (i.e., 4x3x2x1=24).

But, Pete and his grandmother's search for the answer may be more important than the answer itself. Students not only learn what is done in interactions with adults, but how it is done as well (Newman et al., 1989). Pete's grandmother insightfully recognized the patterns in each of the lists she and Pete compiled. By juxtaposing the two patterns she was able to generate a rule that predicted how many numbers were possible with a certain set of digits. Pete's grandmother's contributions included many of the characteristics called for in the reform documents in mathematics education. Together she and Pete gathered data and made mathematically sound decisions about how to interpret the data. In essence, she and Pete were exploring mathematics as they worked on the tasks.

The same thing can be said about Shaundra and her mother as they worked on Task 3. As they worked together to compile the lists, Shaundra's mother, as did Pete's grandmother, searched for patterns and struggled to make sense of the content of the tasks. Shaundra's mother's role in these conversations characterizes the second way in which parents contributed to their children's work on Task 3: Parents gradually became more willing to suggest strategies that would assist the students. The parents' initial uncertainty and reliance on the students' conception of what was expected in school and the parents' gradual contribution to the task is evident in these two conversations between Shaundra and her mother. In the first conversation Shaundra's mother was uncertain about expanded notation as they worked on Task 3a and so she asked Shaundra if she knew what it was.[8] Shaundra assured her mother she did and began writing out the numbers. As she watched Shaundra write numbers in expanded notation Shaundra's mother worked to learn the rules within which the Shaundra operated. Once she understood the rules, she was more willing to contribute to the conversation.

1 M What's expanded notation? Do you know?

2 SQ Yeah

3 M Tell me.

4 SQ It's like, um, you do like Karen did, her dad told her that um, that like three hundred forty seven you could write it like this: three times one and then say plus, um,

5 M Oh.

6 SQ four and then, um,

7 M Probably minus

8 SQ Um,

9 M Well, I see you got three times one notation

10 SQ is three

11 M then times, or plus four so that's the problem right there wouldn't it? I mean that equals three, four and seven.

12 SQ No, like three hundred forty seven.

13 M Okay, go ahead and do it cause you understand it.

14 SQ And, plus, um, eight minus one.

15 M Okay good, ah write each of the numbers you find

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[8]Shaundra recorded her conversation for the first part of Task Three a day later than the other students. the day after Task 3a was assigned, Karen taught her method of writing numbers in expanded notation. In this conversation Shaundra refers to Karen's method of writing numbers in expanded notation.

As she talked, Shaundra wrote out a mathematical expression. The expression, (3x1)+4+(8-1), included smaller expressions that, when simplified, would produce the digits in the number with which she started. In this example, 3x1=3, 4=4, and 8-1=7. Put in succession, those numbers read 347, the number Shaundra was writing out in expanded notation. As Shaundra worked, her mother struggled to make sense of what she was doing. In line 11 she believed she understood: 3x1 is 3, plus 4 is 7, and since those are the digits in the original number, they are finished. Shaundra, however, disagreed and continued to write out the number sentence. Her mother accepted her error and told Shaundra to continue on because "you understand it."

Shaundra's mother continued to watch and, at the end of their conversation, contributed to one of Shaundra's number sentences. The last number Shaundra worked on was 374. As she worked she and her mother had this conversation.

16 SQ Problem of number is three hundred and seventy four. So, I'll just go three and eight minus one and the other number was

17 M How about four? Can it be anything like four minus zero?

18 SQ Okay.

19 M It has to equal up to four, right? That'd be something different.

20 SQ Minus zero that'd be four so that make three hundred forty eight, what happened?

21 M No,

22 SQ Seven

23 M Right. Three hundred seventy four.

Shaundra's mother figured out the rules within which Shaundra was working. In line 17 she contributed an expression, 4-0, that fit Shaundra's rules and added to her number sentence. Shaundra accepted her mother's contribution and, after a slight confusion, declared herself finished. Her mother, now understanding the rules, also pronounced the expression "right" and repeated the number to signify their conclusion.

Shaundra's conception of expanded notation was wrong. Even though what Shaundra had written down resembled Karen's method of expanding numbers, her expressions did not represent either of the ways agreed on by Ms. Smith and her class. Shaundra's mother, however, did not know that. And, being unsure of expanded notation herself, she accepted Shaundra's explanation and worked hard to understand it as Shaundra worked.

During this conversation, Shaundra's mother learned something new, albeit something incorrect, in order to help her daughter while she worked on her homework. What began as an assignment that may have been discontinuous with her past experience was now part of her experience. When she and Shaundra worked on the second part of Task 3, the things they had done and learned while working on the first part informed their decisions and actions. Shaundra's mother, armed with newly constructed mathematical knowledge, contributed more to the conversations. This influence can be seen in the conversation Shaundra and her mother had while compiling their list of four-digit numbers.

Whereas Shaundra's mother helped Shaundra compile the list in the first part, she offered no strategy that might assist her in doing so. When she had finished, Shaundra's mother recognized a pattern in the numbers and told Shaundra about it. In their second conversation, Shaundra's mother, recognizing that Shaundra was going to have trouble keeping track of the numbers she found, offered a strategy that would assist her. The strategy she offered was consistent with the pattern she recognized in the earlier conversation. She told her:

> I'm going make this, try and make this easier for you, okay? Why don't you try to work with all your nines first, like how many ways you can turn nine hundred, you got nine, two, five, oh. Then you can do nine five two oh. . . . You know what I mean? Do all the nines first and then do all the ways you can do five and all the ways you can do two. Okay, you know what I mean?

After Shaundra's mother offered this strategy, they easily compiled a list of twenty-two numbers.[9] Shaundra's mother contributed to the search by finding numbers Shaundra had not yet seen and by offering bits of advice as they worked.

They began their search by looking for "the nines"--numbers with the digit 9 in the thousands place. As they started looking, Shaundra guessed they would find only three and quickly compiled a list of three numbers that began with the digit 9. When she showed the list to her mother, her mother remarked, "Is that the only ways you can do it? Nine two five oh, Nine five oh two, Nine oh two five. How about Nine two oh five? Is that a different way?" Shaundra agreed that it was a different way and added 9,205 to their list. With their amended list, they assumed they had found all the possible numbers and went on to find "the twos." Shaundra now predicted they would find four "two numbers" and quickly compiled a list of four numbers with the digit 2 in the thousands place. Her mother, looking over Shaundra's work, saw another number that was not in their list.

1 M I see one you haven't got.

2 SQ Oh yeah! I forgot. Um, yup, five oh nine.

3 M Yeah.

4 SQ Now there were one, two, three, four, five.

5 M So, you got five out of there, you probably should of got five out of the other ones.

6 SQ Well, I don't know. I think Ms. Smith said that they don't all have to be even.

7 M Oh, okay. Well still if you got the same numbers. Just make sure you ain't got two of the same. Two nine five oh, Two nine oh five, Two oh nine five, You got two oh five nine, okay that's it. Okay.

8 SQ And, now, we're going on.

9 M What about, then I see one more then. If you flip those two around it'd be two five nine zero. Is that another way of doing it? You ain't got that one.

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[9]The other two numbers in their list were found when they listed the numbers in order from smallest to largest and were added later in their conversation.

10 SQ Wait, two, nope. Two five nine oh. Now we go on. So we found six for that one. And that was just the maximum for the other one.

Shaundra's mother was more directive in this conversation than she was in part one of Task 3. She had a clear conception of how to answer this question, and when Shaundra began doing things that did not match that conception, she quickly directed her to a strategy that would help her answer the question. As Shaundra began using the strategy, her mother monitored her work. She provided hints about missing numbers and even contributed numbers that Shaundra had missed.

Shaundra and her mother went on to look for the "fives" and the "ohs." In spite of their new belief that each digit would be in the thousands place of six different numbers, they did not go back to complete the list of nines. Rather, they moved ahead to find the six numbers that began with five and the six that began with the digit 0. With each successive search, they refined their strategy. In the end, they no longer searched for fives or ohs, but for "five ohs" or "oh fives." This change represented their understanding that each of these new categories would comprise two numbers and if they found one, they could "flip" the last two digits to find the other one. Using this strategy, Shaundra and her mother easily compiled the list.

Shaundra and her mother's search for the numbers in the first part of Task 3 began as a random gathering of numbers. The different permutations could be intuited easily and "I used them all" was perhaps the best explanation available for knowing when they were finished. But, as they worked over the two parts of Task 3, Shaundra and her mother constructed an understanding of the situation that allowed them to efficiently compile the lists of three and four digit numbers. On first glance, Shaundra's assertion that she knew she was done because she "worked with every numeral possible" does not explain how she knew. But if "worked with every numeral" is rewritten to say "found all six numbers that began with each of the digits presented by recognizing that, within each group of six numbers, each of the other three digits will be in the hundreds place in two different numbers, and, if I found one of those numbers, I could find the other by flipping the numbers in the tens and ones places," it becomes clear that Shaundra and her mother, although they may have begun the activity with little understanding of what was being asked of them, learned how to approach and answer these questions.

Shaundra's mother, however, never overcame her uncertainty about expanded notation. When she and Shaundra started writing numbers in expanded notation, Shaundra's mother told her, "Okay, now I'm going to leave that up to you," and she left the table to take care of some other household needs. She continued to talk with Shaundra from a distance, but their conversation focused on how many of the 24 numbers Shaundra needed to write out in expanded notation. In the end, they decided Shaundra should write out four numbers and a note to Ms. Smith explaining that 24 was too many to write out.

Summary and Conclusions

The ultimate goal of any homework conversation is to send the students back to school with correct answers (Varenne et al., 1982). This goal is difficult, if not impossible, to meet when parents do not understand the content of the assignments. Parents need to know what a correct answer *is* in order to guide their children to one. When both the students and their parents bring similar conceptions of elementary school mathematics to a conversation, there is little or nothing to negotiate. The process of determining what information is required to answer the question or what combination of operations to use to arrive at an answer is obvious to the participants--especially the parents.

The conversations recorded for Task 2 are examples of homework conversations in which parents knew what a correct answer was. It is likely that the story problems presented as part of that task were consistent with their elementary school mathematics experiences and, consequently, their understanding of mathematics. In those conversations the parents quickly determined how to arrive at the correct answer and efficiently guided their children to that answer. When their children's actions deviated from the course they had chosen, they quickly redirected them to more appropriate actions. The assignments were completed quickly and the students answered most of the questions in the same way as their classmates.

When parents do not know what a correct answer is, they have two options. They can stop participating in the conversations and let the students work alone or they can turn to the child to find out what a correct answer is and then use that description, right or wrong, to guide the students while working on subsequent questions or assignments. In Task 3 many of the parents did not know what a correct answer was. Although some parents stopped participating in the conversations, most parents sat with the students as they worked on the first part of Task 3 and sought to learn what the students were doing. When they understood the task, the parents' participation more closely resembled their contributions to the conversations they had for the earlier tasks.

The change in parents' participation was apparent both within and across the conversations. As parents watched their children working, they interjected comments whenever they believed they understood the rules. These small contributions tested their hypotheses about what a correct answer was and how it could be arrived at. If the students accepted their contributions, they were on the right track; if the students rejected their contributions, they needed to rethink what the students were doing and retest their new understandings. By the end of the conversations, the parents and students agreed on the task and how the questions could be answered. When the students and parents began working on the second part of Task 3, parents entered the conversation with a more thorough understanding of what the task entailed. Using their newly constructed understanding they were more directive and their contributions more closely resembled those in the earlier conversations.

The conversations for Task 3 point out that parents, or other adults, do not always have a better, or more sophisticated, conception of what is required in elementary school mathematics tasks. Children can be teachers and parents can be learners and, at least in these conversations, parents are willing learners. They paid close attention to what their children were doing and learned everything they could in order to assist them later on.

The change in focus and content of elementary school mathematics instruction to something inconsistent with their experience did not stop the parents from participating in these conversations. It just changed how they contributed. In Tasks 1 and 2, which were set within practices familiar to the parents, they stayed in command of the students' actions, correcting them as they worked. In Task 3, they became "co-explorers" of the situation who modeled mathematical inquiry in ways that are consistent with the calls for reform in mathematics education. Although they did not always answer the questions correctly, the parents and students gathered data, searched for patterns, devised explanations of the patterns they found, and used what they learned in one conversation to inform their work in subsequent conversations--all of which are dispositions mathematics educators hope their students develop.

In Chapter 4 I argued that the conversations students had at home greatly influenced the work they brought back to the classroom. In their conversations with their parents they were exposed to many different practices and conceptions of the same practice that shaped the way they defined the tasks and influenced the answers they brought back to school. The conversations presented here point out the school's influence on what students do at home. The tasks presented to students have a great impact on how parents interact with their children about their homework. But the influence of the school on the home and of the home on the school occurs at the same time; that is, they are mutual influences. The mutual influence of the school and home is explored in greater depth in Chapter 6.

CHAPTER 6

TO HOME AND BACK

In the beginning of *Life on the Mississippi*, Mark Twain told the physical and "historical" history of the Mississippi River. He presented the historical history as four "epochs" in the river's existence. He wrote:

> Let us drop the Mississippi's physical history, and say a word about its historical history--so to speak. We can glance at its slumbrous first epoch in a couple of short chapters; at its second and wider-awake epoch in a couple more; at its flushest and widest-awake epoch in a good many succeeding chapters; and then talk about its comparatively tranquil present in what shall remain of the book.

In each epoch, people who came to see or lived near the river came to know it in new ways--ways that fit with the things they needed to do or included new technologies (e.g., the riverboat) that allowed them to do new things. Throughout its history, the river changed both in its physical structure and in the meaning given it by its visitors and residents.

In the third epoch, the one he refers to as the "widest-awake" epoch, Twain chronicled his own learning of the river. Twain used the same structure to describe his own learning. Much like the changes in the ways the explorers and residents knew the river, his knowledge of the river underwent qualitative changes with different "epochs" of learning. At the beginning of his apprenticeship on the river, Mr. Bixby, Twain's mentor, instructed him to keep a notebook of the river's features. As a result, he first learned the river, he recounted, as a list of the physical characteristics--the towns, sandbars, points, and bends. Although he could recite the list for all but "ten miles of river in every fifty," Mr. Bixby made sure he did not become complacent and pointed out that such a list was inadequate for captaining a riverboat. To better prepare himself for his future position, Twain, with the help of Mr. Bixby, learned the river as a "dark hallway." Knowing the river in this way allowed him to "feel" his way in the dark constantly knowing where he was and what to expect ahead. This way of knowing the river also proved inadequate, as the river changed constantly. That constant change forced Twain to learn the river in yet another way. In the end, he learned to read the river as if it were an ever-changing novel. The novel, he wrote, was new each time he read it and each new novel demanded a close reading. It was this final way of knowing the river that allowed him to become a highly competent riverboat captain.

Twain goes on to say that he not only changed the way he thought about the river, but he gave up the ways he knew the river before becoming a riverboat captain. He wrote:

> Now when I had mastered the language of this water and had come to know every trifling feature that bordered the great river as familiarly as I knew the letters of the alphabet, I had made a valuable acquisition. But I had lost something, too. All the grace, the beauty, the poetry had gone out of the majestic river! (Twain, 1883/1984, p. 95)

Twain went on to describe a beautiful sunset he remembered from his past and what the patterns of water, the color of the sky, and floating objects meant to him then and what they meant now. Whereas in the past they were the "glories and charms" of the river, they now were predictors of tomorrow's weather, the rise and fall of the river's water, and the changing physical structure of the river. To fulfill his goal of becoming a riverboat captain, it was necessary for Twain to give up old ways of knowing the river and take on new ways that made his task easier.

Twain's knowledge of the river developed gradually and involved both the accrual of new knowledge and qualitative transformations in how he knew the river. He could not forget the list of towns, points, and sandbars he memorized earlier, but as he gained experience on the river, they took on new meanings--meanings that were necessary for him to accomplish the things he set out for himself.

Children learn mathematics and other school subjects in much the same way. They do not merely know or not know how to compute different mathematical operations, solve mathematical problems, or understand mathematical concepts. They develop these abilities over time. Their learning, like Twain's, involves both the accrual of new knowledge and qualitative transformations in the way they assess mathematical situations, use mathematical tools, and make sense of their findings.

The gradual development of concepts and procedures was important as well to Vygotsky (1978). He argued that children, through interaction with more knowledgeable adults or peers, gradually develop the ability to participate in socially defined and historically determined activities. In their initial attempts, children require a large amount of assistance to complete the task. Over time the child assumes more responsibility for the finished product and the adult provides less assistance. Eventually, the child reconstructs the activities internally and completes them independently.

Throughout this process the child's ability increases and the gap between the aspects of the task he or she can do alone and those they can do with assistance shrinks. This gap represents the zone of proximal development. Evidence of learning within the zone of proximal, and within a specific conversation, comes from changes in what children *do*--their actions and thoughts--while working with adults or more experienced peers to solve a problem. The activity is initially displayed in the actions of the more experienced other in the conversation; that is, in the physical manifestation of their situation definitions. The child observes, imitates, or obeys the adult's actions or directions until they can complete the same tasks with no help. Wertsch (1984) referred to the things people do in conversations as their "action patterns." If a child begins to mimic the more experienced participant's action pattern, they have learned.

But, as Lave has pointed out, learning includes "extending what one knows beyond the immediate situation" (1993, p. 13). So, although the changes students made in their action patterns during their conversations at home may be evidence of learning, it might be beneficial to watch the student in subsequent interactions around the same tasks to see how they extend what they did in those conversations to other conversations.

Looking beyond the immediate situation points out that people do not merely internalize the things they are exposed to. Rather, learning, as Vygotsky may have intended, is a active, gradual process in which learners personalize the things they are exposed to. Drawing on the work of Leont'ev (1981), Newman, Griffin, and Cole (1989) have argued that children, rather than internalizing knowledge, "appropriate" it in ways that aid them in doing things necessary in their immediate surroundings. Whereas internalization suggests learners add intact pieces of knowledge to existing cognitive structures, the notion of appropriation suggests that children might use part of what is being taught or change what is being taught to fit their own purposes. Newman, Griffin and Cole summarized this argument saying: "The child has only to come to an understanding that is adequate for using the culturally elaborated object in the novel life circumstances he encounters" (1989, p. 63). In the case of elementary school mathematics, the "culturally elaborated objects" are the concepts, procedures, and other mathematical tools. Students may take part of a concept or mathematical tool and use it to achieve some end only to find that they may need to refine the "crude tools" they have constructed to work in another instance.

In this chapter, I looked beyond the situation in two ways. In the first section, I discuss two cases that illustrate the development and extension of mathematical ideas across conversations. These cases were drawn from Tasks 1 and 3 that were discussed in more detail in the two previous chapters. In the first case, "It's Two Something," Ronnie and his mother answered a question at home only to have Ronnie return to school where he failed to fully reconstruct the answer. Although he could not recount the work he and his mother did, it is still clear that their conversation influenced Ronnie's interaction in school. This case also points out the role culturally defined tools play in recollecting past actions. In the second case, "Reconstructing a tool," Kathy and her father figured out a way of predicting how many three- and four-digit numerals can be made from a given set of digits. Although their initial "tool" worked to determine the number of three-digit numerals, they found it did not work for four-digit numerals. They needed to, and did, refine the tool to work in other situations and, at the same time, still work in the initial situation. Kathy brought the tool they constructed back to the classroom and convinced her classmates and teacher that it did, indeed, predict the number of numerals that can be made with a given number of digits--but not without some resistance from her classmates. The cases begin in the students' homes where they began working on the tasks. As a result, each case revisits many of the issues discussed in chapters four and five. Both cases also look beyond the immediate conversation to help explain the movement seen within the conversations in students' homes.

In the second section, I look at the students' conversations as they completed Task 4. Task 4 was made up of questions that included content similar to that contained in the first three tasks. Although the students had successfully completed the previous three tasks, neither the students nor the parents referred back to the earlier tasks as they worked on task 4. Although this may be explained by the students' ability to complete many of the questions without assistance, it also suggests, as does "It's Two Something," that what is learned in one setting, may not be available in its entirety to the student in subsequent conversations.

It's Two Something

At Home

Ronnie and his mother sat down to work on his homework. As usual, they sat in front of the television, Ronnie on the floor, his mother on the couch behind him (interview, 11/12/92). This night they were working on Task 1 which asked the students to assist Willie, who was packing lunches for two friends, on a trip to the grocery store. Ronnie and his mother discussed each of the questions as they worked on them. For the first two questions, they agreed on the unit price of the different items and the number of each item Willie would need to pack the lunches. When they had figured out these values, Ronnie multiplied the two numbers to find the total price and his mother checked his computation. The third question involved a different model of division.[1] In spite of this difference, the interaction between Ronnie and his mother remained largely the same. As they discussed each of the first three questions, Ronnie's mother repeated the question as it was written on the assignment sheet and let Ronnie begin answering it. When she recognized Ronnie was having trouble, she would interrupt his work and direct him to an appropriate answer to the question. When they got to the fourth question they had this conversation:

1 RB (Reading the question from the assignment sheet.) Willie and his friends decided they would split the cost of the lunches each week. How much did Willie spend at the supermarket? How much did it cost each person?

2 M Um-hmm

3 RB One dollar (Ronnie began writing on the back of his assignment sheet.)

4 M What do you, do you add it up on that side of the paper? No, no, no, no, wait a minute though. Where you getting four fifty-six from? (pause)

5 RB Four fifty-six cause there's twelve Twinkees in the problem

6 M Oh, you goin' back this way instead of down. [yeah] Okay. (pause)

7 RB [inaudible]

8 M What else do you got down there?

9 RB Seven fifty-six. Willie spent. (pause) I already know that it's two fifty-two.

10 M What's two fifty-two

11 RB The answer.

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[1] Questions one and two involved a partitive model of division and question three a quotative model. A partitive model of division answers questions like "If you twelve items into three groups, how many items will be in each group?" A quotative model answers the question " How many groups of three are there in twelve?" For a more complete description of the division models see chapter three.

12 M No, no, no, no. See it says how much did Willie spend at the supermarket? Okay up here he needed two tomatoes which were sixty-six cents. Okay then he needed six apples but only five of them was for two dollars so he had, you had to put how much more with it?

13 RB Two forty

14 M Right. So sixty-six plus two forty. Okay then down here he bought, he had to get the Twinkees, right? [Um-hmm] which was four fifty-six. And then, then, then you add those up. So, you'd be adding up the sixty-six cents that he spent [uh-huh] and the two . . .

15 RB The two forty

16 M Right. and then the . . .

17 RB The four fifty-six.

18 M Right.

19 RB So, I can do that here. Four fifty-six, two forty . . .

20 M With the sixty-six cents he paid for the tomatoes. (pause)

21 RB [inaudible] Seven sixty two.

22 M Let me see. (pause) carry the one, six plus ten is sixteen, put down the six carry the one, yup seven sixty two. [So, three] That's how much he spent, no, no, no that's how much he spent at the grocery store for his week. For his . . . (pause)

23 RB Okay

24 M Okay, you gotta make sure you write that down for the answer though.

25 RB I don't need to.

26 M Yes you do.

27 RB Uh-uh, she knows it's on the back.

28 M Oh, okay. And how much did it cost each person?

29 RB Now I gotta average. Three goes into seven sixty-two

30 M Um-hmm. (pause) You must use these up here. (pause)

31 RB That's what that'll be.

32 M What?

33 RB Two fifty-two, four.

34 M Two dollars and fifty-four cents.

Ronnie began the conversation by posing the question as it was stated on the assignment sheet and began answering it on his own. He started by writing down the cost of the Twinkees in Question 3. In line 4 his mother stopped him and asked him "Where you getting four fifty-six from?" Although Ronnie was only going in a different direction (i.e., listing the values from Question 3, Question 2, and then Question 1 rather than the reverse order), Ronnie's mother thought he was doing something different from the procedure she thought was appropriate. When she understood what he was doing, she recognized it as an alternative way of answering the question and changed her situation definition to match his.

Ronnie then listed amounts from the first three questions--not Willie's cost, but the prices listed in the questions on the assignment sheet and added them together arriving at a total cost of "seven fifty-six" ($4.56+2.00+1.00=7.56). Ronnie, apparently having done some work before the conversation, announced that he already knew the cost per person was $2.52. As he wrote down his answer, his mother (line 12) recognized that he had listed amounts that were inconsistent with her situation definition and became more directive. She stopped him and told him he needed to use different amounts--the amounts on which they had agreed earlier in their conversation--and that he needed to add those amounts to find out how much Willie spent at the grocery store. After her explanation, Ronnie and his mother said the amounts together as he wrote them down. Ronnie added the amounts and arrived at a sum of "seven sixty-two." His mother checked his computation by talking aloud through the addition algorithm and explained that the number represented the total cost of the shopping trip.

In lines 24-28, it became clear that Ronnie's and his mother's situation definitions included an appropriate presentation of the answer as well as the answer itself. Ronnie did all the calculations on the back side of his paper. His mother, perhaps drawing on her conception of school math, told him to write the sum on the front of his paper.

Ronnie declined, saying he could write it there because his teacher would know where to find it. His mother accepted his explanation--again changing her situation definition to match his--and they went on to the next part of the question: How much would each person need to pay? Ronnie announced he needed to "average," divided the total cost by three--the number of people eating the lunches--and ended up with a cost of $2.54 per person.

To ascertain Ronnie's zone of proximal development, we need to determine what he was able to do on his own and what he was able to do with his mother's assistance. Ronnie knew that he needed to collect amounts from the first three questions, add them together, and "average" them. Ronnie also knew how to complete all of the computation necessary to answer the question. This information, however, was not enough for him to answer the question correctly. To accomplish that, his mother needed to assist him.

Ronnie's situation definition differed only slightly from his mother's. She too believed they needed to gather amounts from each of the previous three questions, add them together, and divide by three. But Ronnie and his mother may not have agreed on which amounts needed to be collected and the reason the sum needed to be divided by three. With his mother's help, Ronnie was able to answer the question in an appropriate way. Ronnie's mother was able to assist him in determining which amounts to gather and, perhaps, understanding why they needed to divide the sum by three. And, in the end, when Ronnie and his mother began reciting the values in unison, it appeared that Ronnie had changed his situation definition to more closely resemble his mother's.

The shift in Ronnie's situation definition suggests that he learned--or appropriated--his mother's strategy for answering the question. The shift represents the qualitative change in his actions that signifies learning in the zone of proximal development. But, even though Ronnie appeared to change his situation definition, there are still pieces of Ronnie's solution that are difficult to understand--unless we look at what he did in his classroom before this conversation. Further, the appearance that

Ronnie has learned how to solve this problem is brought into question if we look at what he did in his classroom after this conversation.

There are many ways in which this conversation reflects mathematics in Ronnie's classroom. The most obvious connection is that Ronnie and his mother are talking about math homework. They probably would not have had this conversation if the task had not been assigned as homework--and perhaps they would not have had this conversation if they were not participating in this study. Second, Ronnie's ideas about where to record the different pieces of his solution reflect the way they "do math" in his classroom. This conception is different from the conception his mother has, but when he invokes the authority of the classroom (line 27: "She knows it's on the back"), Ronnie's mother changes her definition to match Ronnie's conception of math in Ms. Smith's classroom. Finally, Ronnie announced that he needed to average. Taken alone, Ronnie's announcement might not make sense; he was not finding an average cost. But, a few weeks prior to this task, the class completed a short unit on averages. During that unit they learned an algorithm for computing averages. The algorithm was something like: "Add up all the different values, divide the sum by the number of values you added together. The quotient will be the average." In a sense, Ronnie used this algorithm in this task. Although the task was not an average problem, he did add up three values and divided the sum by three--the number of values he added together. So, the algorithm he used is consistent with the one he learned to compute averages and the connection he made seems plausible.

In School

On most days students met in small groups to discuss the tasks before talking about them as a class. When Ronnie returned to the classroom, he met with two other students, Jason and Pete, to discuss Question 4. Tony, who was also a member of this math group, was absent. As they began their discussion, Jason and Pete were trying to see who could get the largest number in the display of their calculators. To increase the size of the number, they entered an addition problem and then repeatedly and quickly pushed the "+" button with the eraser end of their pencils. In the midst of this competition, Ronnie paraphrased Question 4 from the assignment sheet and they had this exchange:

1 RB Now Willie and his friends, how much did they spend at the supermarket altogether?

2 PC How much did they spend altogether?

3 JS Check that out y'all (Shows his groupmates a very large number in the display of his calculator). Get that much.

4 RB (Working on the assignment) One dollar, two dollars, four fifty-six

5 PC Don't push it (the addition button on Jason's calculator) anymore and I'll beat it. (pause)

Jason stopped pushing the addition button, tossed his calculator onto Pete's desk, and turned to face Ronnie's desk. He watched Ronnie work on Question 4. When Ronnie finished, he announced that the total cost of the groceries was "seven fifty-six" (the erroneous total he had gotten before his mother corrected him at home) and he looked to Pete and Jason for confirmation. Jason was looking at his paper and Pete was still punching the addition button on his calculator. Ronnie, in an angry voice, repeated his answer to Pete who, without stopping or looking at Ronnie's work, nodded and repeated his answer, "seven fifty-six," a different total than the one written on Pete's assignment sheet. Jason, however, thought Ronnie's answer was wrong and turned to face his own desk and checked his figures.

As Ronnie and Jason worked on Question 4, Pete picked up the microphone, thrust it at Jason saying "I'll smack you." Pete's action captured the interest of a classmate who asked what the microphone was. Pete told the classmate "it'll record you." Jason then turned to the microphone and said some nonsense words into it. Ronnie, who was still working on the question, turned to both Pete and Jason and reprimanded them for not working on their assignment. In response to Ronnie's reprimand, Jason threatened to leave the group and turned to his own desk. Ronnie turned to Pete. Jason, however, soon returned to the conversation and he and Ronnie discussed the questions as Pete continued to push the buttons on his calculator.

6 RB Three goes into seven . . .

7 JS No, it's supposed to be . . .

8 RB how many times?

9 PC (still pushing the addition button on his calculator) Three goes into seven?

10 RB Two times, right?

11 JS You're supposed to see how much it costs for everything and it comes out to five sixty-four,[2] right?

12 PC What numbers do you have to push to spell 'hell?'

13 JS And you have to times[3] it by three people because three people are splitting the cost. So that equals . . . it ends up for me, it ends up for me a dollar eighty-eight for each person. A dollar eighty-eight for each person.

14 RB That's not right. It's two something.

15 JS A dollar eighty-eight for each person. I'll even do it. (pause)

Although Ronnie was having trouble reconstructing the answer he and his mother agreed on in their conversation, he was sure that Jason's answer, "a dollar eighty-eight," was not correct.[4] Ronnie remembered that the answer he and his mother agreed on was "two something." Jason however, still believed his answer was right. To support it, he began punching the numbers he had used to determine the cost of the

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[2]Jason arrived at his answer of $1.88 by adding $2.04 (34 cents--the cost of one tomato--by 6--the number of tomatoes he believed Willie needed, he then added $2.40 (the cost of six apples)+$1.20 (the cost of putting Twinkees in the lunches for one day) and then dividing the sum ($5.64) by three (the number of people sharing the cost) to arrive at a cost per person of $1.88.

[3]Although Jason used the term "times" to describe what he had done, he divided the total cost ($5.64) by three.

[4]Jason, Tony, and Pete had all used the same strategy to answer Question Four. They all took the answer to the last part of questions one, two, and three, added them together, and divided the sum by three to get the cost per person. As a result, they all took the cost of putting Twinkees into the lunches each day rather than the cost for the week. This strategy might represent their understanding of how to do word problems in school math. Their strategy might include a provision for sets of problems where the last question, in some way, summarizes the earlier questions. In these instances, you take the answers to the earlier questions and manipulate them in some way.

groceries into his calculator. When he finished, he showed Ronnie and Pete the display and said "See, a dollar eighty-eight." Ronnie still was not convinced and asked Jason, "Where did you get five sixty-four from?" to which Jason replied "From everything" and grabbed his paper to show him. As the conversation continued, Ronnie tried to show Jason that the values he (Ronnie) added together resulted in a total cost of $7.56. Jason initially balked at Ronnie's explanation, then reluctantly added the values.

16 RB That was three for one dollar, they paid for that. Then they pay right here, four fifty-six. Then, oh

17 JS I got my answer right, that's final.

18 RB Look and two dollars. Two dollars plus one dollar plus four fifty-six is not no five something.

19 JS Five point sixty four divided by three equals a dollar eighty-eight.

20 PC (Looking at Jason's calculator) Ah, dude, you only got eighteen hundred dollars and ninety-five.

21 RB Look Pete, I'm gonna add it up in front of your face, look.

22 JS (reading the number in the display on his calculator) Eighteen hundred ninety-five.

23 RB Look y'all. One dollar, two dollars, and four fifty-six is seven fifty-six. Tell him it's right y'all. Tell him it's right.

24 JS That's wrong.

25 RB Tell him it's right. You gotta add that one dollar . . .

26 JS Let me see this. I'll add it all up.

27 RB that two dollars and that four fifty-six. And that comes up to seven fifty-six. Ain't that right?

28 JS (Beginning to add the costs together again) One point zero zero plus

29 PC Yes, that's right.

30 RB Yeah, he's comin' up with five something.

31 JS two point zero zero plus

32 PC Do it on your calculator

33 JS four point fifty-six equals seven fifty-six. Seven fifty-six.

34 RB That's what I had, no. You so dumb.

35 JS You so dumb. (pause)

36 PC Seven fifty-six, that's the answer.

37 JS Two fifty-two.

Much like the families during homework sessions, Ronnie and his groupmates were involved in competing activities. Pete, although he occasionally contributed to the discussion of the homework, was equally concerned with the race for the largest number on the calculator and other calculator tricks. He reduced his role in conversation to confirming the results of the other members while Ronnie and Jason discussed and argued out a solution to Question 4. His confirmations, however, seldom involved more than a cursory nod or verbal agreement. Although it appears that Pete was not prepared for this assignment, he and his grandmother successfully answered the questions at home two nights before this conversation. In many instances, the work Pete and his grandmother did while answering the questions may have helped Jason and Ronnie successfully answer the questions.

Throughout the conversation, Ronnie used the situation definition he used when he started working with his mother. He did not use the solution on which they finally agreed. He wrote down the amounts listed in the questions on the assignment sheet: $1.00, $2.00, and $4.56 (lines 16, 18, and 23). When Jason suggested a different solution, Ronnie, in line 14, said the solution made no sense, that it needed to be "two something," but he could not completely reconstruct the strategy he and his mother used two nights before. In line 16, Ronnie talked about the amounts Willie paid for the different items on his list. This, too, was part of the discussion Ronnie and his mother had at home. Each of these things suggests that Ronnie took something away from the conversation he had with his mother. He knew that the answer could be determined by collecting the amount Willie spent for each item, adding the amounts together, and dividing by three. He also knew the answer should be "two something." But, while he appropriated pieces of their solution, he did not internalize it entirely.

Later in class, Ronnie was called on to explain his group's answer. Ronnie again had trouble reconstructing their solution. Ms. Smith asked him if he had explained everything he had done to answer the question. Ronnie told her he had not and explained that he had lost his homework paper and had no written record of his work at home. The only thing he had written down was the computation he had done with his group and his record of that was sketchy.

If Ronnie had his paper it is likely that he could have reconstructed the solution on which he and his mother agreed. Vygotsky (1978) suggested that the development of higher psychological processes was mediated by more knowledgeable others who introduce culturally defined tools or activities designed to facilitate the process. When developing memory, for instance, adults may teach children to notch a stick or write a list. The children can use this method until they develop the capacity to remember without the tool. In Ronnie's case, his written record may have served the same purpose. It may have allowed him to recall processes that were not yet fully developed. Although Ronnie's difficulty reconstructing the strategy in school might suggest he did not change his thinking in a way that more closely reflects that of his mother, another plausible interpretation focuses on the importance of the tools or mnemonic devices Ronnie used to help him think about the question. Had he held on to his homework, the notes and calculations he had written down may have helped him reconstruct the solution he and his mother agreed to. But without them he needed more assistance than was available from his groupmates to answer the question.

The class went on to accept three possible answers to Question 4. The answers the class accepted were based on different assumptions the students made while answering all the questions in Task 1. If students went to a "nice store" that charged 33 cents for each tomato, each person would need to pay $2.54 (.66+2.40+4.56=$7.62.

7.62÷3=$2.54). If the students went to a "greedy store" they would have paid 34 cents for each tomato for a total cost per person of $2.55 (.68+2.40+4.56=7.64. 7.64÷3=$2.54 r2 rounded to $2.55). The third acceptable answer was a variation on the greedy store solution that they termed the "work solution." In this solution Willie's two friends each paid $2.55 and, because he had done the shopping, Willie paid $2.54. The answer Ronnie and his math group had constructed was not included in the list of acceptable answers. The answers Ronnie and his mother agreed to--the nice store solution--was.

Summary

In their description of the zone of proximal development, Newman, Griffin, and Cole (1989) wrote:

> The concept refers to an interactive system within which people work on a problem which at least one of them could not, alone, work on effectively. . . . Methodologically, cognitive change can be observed as children pass through or work within the zone. (Newman et al., 1989, p. 61)

This description fits the conversation between Ronnie and his mother as they worked on Task 1, Question 4. Together they worked to answer a question that Ronnie could not answer on his own. As they worked together, Ronnie changed his action pattern to more closely resemble his mother's. Ronnie's revised action pattern provided evidence of cognitive change within the zone of proximal development.

But Ronnie and his mother's interaction was not that simple. Some important aspects of the conversation are overlooked in this short description. In the conversation, the role of the more knowledgeable other shifted. At the same time Ronnie's mother helped him answer the question, Ronnie helped his mother understand what it meant to do math in his classroom. As in the conversations reported in chapters four and five, this trade off resulted in an amalgamated practice that represented the conceptions of school math and other practices they brought to the conversation. And, again as in the previous chapters, the influence was not evenly distributed across practices. Rather, because the task began and ended in school, what was required for an appropriate presentation in Ronnie's classroom carried more weight than the other practices.

Although Ronnie revised his solution to more closely resemble his mother's, it still reflected his experience in Ms. Smith's classroom. Near the end of their conversation Ronnie announced that he needed to "average" the costs. Had we listened only to the conversation between Ronnie and his mother, we may have questioned his use of that term--perhaps going so far as to say he did not understand the concept of averaging. The recognition that he used a algorithm that had been labeled "the averaging algorithm" in class makes his choice of words reasonable.

Looking across the conversations points out the gradual appropriation of ideas and activities posited by Vygotsky (1978) and other sociocultural theorists (Newman et al., 1989). Although we can argue that we were able to observe cognitive change in Ronnie's conversation with his mother, the change was not lasting. Ronnie's difficulty recounting the solution when he returned to school suggested that he did not completely understand how to answer the question. Yet, his interaction with his math group did include ideas about which he and his mother talked. What he appropriated from the conversation with his mother was a partial understanding that may be filled out in subsequent conversations. Ronnie's difficulty in school also highlights the importance of the cultural tools available to document conversations. During the conversation with his mother, Ronnie wrote down what he and his mother agreed upon and even debated what his documentation of the solution ought to look like. But, without his written record, Ronnie was unable to reconstruct the solution in school. Perhaps if he had his paper he could have.

Newman, Griffin, and Cole (1989) continue their discussion of the zone of proximal development saying, "When cognitive change occurs not only *what* is carried out among participants, but *how* they carry it out appears again as an independent psychological function that can be attributed to the novice." (Newman et al., 1989, p. 61). Students not only learn mathematical ideas in their conversations, but how to *do* math as well. What they learned about doing math in their conversations at home influenced their participation in school. This phenomenon is clear in the second case, Reconstructing a Tool. In the case, Kathy and her father worked together to find a way to predict how many numbers can be formed from a given set of digits. Over two nights' discussion, they arrived at an appropriate way to predict how many numbers were possible. Although this skill is valuable in itself, participating in the process of constructing it may have been even more valuable for Kathy. The case also highlights the gradual development of mathematical tools and ideas.

Reconstructing a Tool

At Home

Kathy and her father worked together at the dining room table. Normally, Kathy, her brother Bobby, and her father all did their homework together. This evening, however, Bobby watched a television program about the "old west" while Kathy and her father worked on Task 3. Although Kathy and her father's answer to the question of how they knew they had all the numbers was similar to the answers of the other students, their conversation was quite different. Kathy and her father were the only people who tried to determine a way of predicting how many numbers were possible while working on the first part of Task 3. The other students generated the list of numbers and then tried to explain how they generated the list.

Kathy read the first question aloud and began answering it. The question asked the students to list as many three-digit numbers as possible using the digits 3, 4, and 7. Kathy found six numbers that she could write using the digits. She showed the list to her father and he his initial response was that there should be more numbers. They had this conversation:

1 F It should be more than six.

2 KG What, it can't be.

3 F There are.

4 KG No, daddy, I've used all the numbers.

5 F Um-hmm

6 KG You have to use all three of them at one time.

7 F I know. Three should be able to take up every position, right? So here it's in this position, that position, and this position. Four should be able to be in every position. So, here it's in second position [347], last position [374], first position [437], first position [473].

8 KG let's see, right here.

9 F Yeah, but if you look at this, three should be able to be in three positions, four should be able to be in four positions, and seven should be able to be in three positions. That's nine total numbers that you should be able to come up with.

10 KG Four shouldn't be able to be in four different positions.

11 F Three different positions.

Although Kathy found only six three-digit numbers, her father believed more numbers were possible. His reasoning about how many three-digit numbers were possible focused on the number of positions each digit could fill. Because each digit could be in three positions, and there were three digits to use, nine numbers were possible (3+3+3=9).

Keeping the idea that nine numbers were possible in mind, Kathy and her father looked for three more numbers to complete their list of all possible numbers. They looked at the list to make sure each digit had occupied each position an equal number of times. When they could not find any more numbers Kathy's father somewhat reluctantly changed the way he predicted the number possible.

12 F Well see you might be right, I can't remember how this works, but, the number in the first position can have three positions, the number in the second can have two, and the number in the last position can have one. That's a total of six.

13 KG See, that's what I got.

14 F Yeah, I know, but I still keep thinking there's more combinations.

15 KG that's seven thirty four. I've got six. So the answer would be . . .

16 F Okay, go ahead and go, go with your answer because I can't come up with any other.

17 KG Okay, we'll just say, we'll just say six numbers.

18 F Remember we're supposed to leave all this work on here.

19 KG I know. Okay, for 'B' it says 'How do you know when you've found all the numbers?' All the letters are like, gone.

20 F All the letters?

21 KG I mean all the numbers are gone.

22 F Every number has occupied every position.

Kathy and her father's strategy changed in this part of their conversation. Rather than predicting how many numbers were possible and then trying to find that many numbers, they accepted the list of six numbers Kathy had written down and tried to figure out a way of predicting that number. Their second attempt at predicting how many numbers were possible still focused on the number of digits and the positions each digit could fill, but took into account the fact that once a position is filled it is no longer available to the other digits. As a result, the first digit can be placed in three positions, the second in two positions and the third in the leftover position. Even though Kathy's father toyed with this method of predicting how many numbers were possible, Kathy and her father were not completely convinced that this strategy was correct. Instead, Kathy wrote down "all the numbers are gone" as the reason for knowing they had found all the numbers.

In School

The next day Kathy took her answers back to school. As on most days, Ms. Smith's class met in math groups to discuss their homework. Kathy met with Elaine and Helen. Their discussion was short, all agreeing there were six possible numbers, how those numbers should be ordered from smallest to largest, and how they could determine the relative size of the numbers. That day each math group produced a chart showing the numbers that could be made with the three digits and the answers to the other questions. Kathy and her group added an answer to the bottom of their chart which said "We never have anything to talk about because we always agree." Kathy and her father's discussion about how to determine how many numbers were possible was not discussed in their math group, only the answers on the assignment sheets. Near the end of class, Ms. Smith passed out the second part of Task 3 and instructed the students to take it home and complete it overnight.

Back at Home

As they had the previous evening, Kathy and her father worked at the dining room table. In the background, Bobby watched a television show about Malcom X. Throughout the conversation, Kathy's, and sometimes her father's attention, shifted back and forth between the television show and Kathy's homework.

Question 1 asked the students to generate as many four-digit numbers as possible using the digits 0, 2, 5, and 9. Kathy had written a list of numbers before they started recording. she started the tape by reading the list aloud. Near the end she asked her father if she could use zero in the thousands place. He told she could, but would not say "zero thousands" when she read the number. Kathy completed the list and announced she had 24 numbers. Her announcement took her father by surprise.

1 KG I have twenty-four.

2 F Twenty-four numbers?

3 KG Yes, that I made out of those. I have two, two oh five nine. two five oh nine. Two oh nine five. Two five oh nine. Two five . . .

4 F Yeah, that's the same number. . .

5 KG nine oh.

6 F I can predict that there'll be . . .

7 KG Two nine oh five, Two nine five oh. See

8 F Fourteen. There should be fourteen numbers.

9 KG One, two, three four, five, six, seven, eight, nine, ten, eleven, twelve, thirteen, fourteen, fifteen, sixteen, seventeen, eighteen, nineteen, twenty, twenty-one, twenty-two, twenty-three, twenty-four. Twenty-four, not fourteen. Cause I used the zero.

10 F No, shouldn't matter.

11 KG Watch, for each one it should have at least six.

12 F Well how many numbers did you come up with for three? Remember when you had three?

13 KG Three?

14 F Yeah, when you had three, how many number combinations did you come up with? You came up with six didn't you?

15 KG Yeah.

16 F Okay. Now think about that. You had three numbers. Okay, lets just call it A, B, and C.

17 KG I know, but now I have . . .

18 F And you come up with six combinations. Right?

19 KG Um-hmm [yes]

20 F So, we could either say that that's two times as many as these or you could look at it that A here can be in three different positions, right, it can be in this position, this position, or this position.

21 KG Um-hmm

22 F But if it's in this position, then these two can only occupy two positions. And if this one is in this position and this one's in this position, this one can only occupy one position, all right? So, we take and we add them up. Three plus two plus one and what number does that give us?

23 KG (pause) Six.

24 F Right. And we came up with six combinations, right? Now let's look at this number. We have four of 'em: A, B, C, and D, all right? Now we have four different possibilities. Let's use the same rule we came up with up here. This one can be in four positions [Um-hmm]; this one can be in three positions; this one can be in two positions; this one can be in one position. How many does that give us?

25 KG Ten.

26 F Oh, you're right.

27 KG seven, eight, nine ten . . .

28 F How did I come up with fourteen? There should only be ten combinations.

29 KG But I can get that many, Daddy! I can't help it.

30 F Are you sure you don't have any repeats like you did there above though?

31 KG Two, ah, you listen. Two oh five nine . . .

32 F No, I have to look at them, I can't just listen.

33 KG Two oh nine five. Two five oh nine. Two five nine oh. Two nine oh five. Two nine five oh. Two five, no, Five two oh nine. Five two nine oh. Five nine oh two. Five nine two oh. Five oh two nine. Five oh nine two. Oh Two hundred ninety-five. Two hundred fifty-nine. Five hundred ninety two. Five hundred two, twenty nine. Nine hundred twenty five. Nine hundred fifty two. Okay, Nine two oh five. Nine two five oh. Nine five oh two. Nine five two oh.

34 F You're right.

35 KG Nine oh two five.

36 F All right, we'll have to rethink this then.

37 KG Nine oh five two.

38 F So, maybe,

39 KG This can be in four positions, that can be in four, that can be in four, and that can be in four. One two, that'd be eight and . . .

40 F That's four to the fourth.

41 KG What?

42 F Four to the fourth. It's four times four times four times four.

43 KG No.

44 F Oh, you're adding them together.

45 KG Yeah. That's four

46 F plus four plus four plus . . .

47 KG Sixteen. I don't know how I got these.

48 F Four times four is sixteen.

49 KG but they're all logic right?

50 F Yeah, but how many, how many do we have?

51 KG Twenty-four.

52 F Twenty-four, lets see how we can come up with twenty-four out of this then. Oh, oh!! Ah-ha!!! I know what it is. Look at this. We can come up with that. This is not plus, this is times. Look three times two is six, times one is six. Now watch. Four times three times two times one. Four times three is twelve [twelve], times two is twenty-four, times one is twenty-four. Dah, da-da-da!!

Kathy's father's initial reaction to her announcement rests on the strategy they used the previous night to predict how many numbers they could make. Although they were not completely convinced of the strategy the previous night, Kathy's father used it here to predict how many numbers were possible. The strategy they used was to add the number of positions in which a digit could be placed. For three digits the strategy was to add 3+2+1 for a total of six numbers. For four digits the strategy was to add 4+3+2+1. Kathy's father's initial thought that there were 14 possible numbers was likely caused by a computational error.[5]

When they determined that Kathy's list included 24 different numbers, they realized they needed to rethink their strategy (line 36). Kathy began this process by trying out the strategy her father first used the night before (lines 39-48). The first digit, she suggested, could be in four positions, the second could be in four positions, and the third and forth digits could also be in four positions. Adding according to this strategy resulted in a prediction of 16 numbers. But they had 24 numbers. Kathy's father told her they needed to figure out how they could get 24. Almost immediately, he said, "this is not plus, this is times" (line 52). To support this idea he pointed out that the strategy worked for both three-digit numbers (3x2x1=6) and four-digit numbers (4x3x2x1=24). Kathy's father celebrated their solution by singing a trumpeter's announcement "Dah, da-da-da."

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[5]Although there is no evidence of his computation, it may be that he added 4+4+3+2+1=14.

The strategy Kathy and her father constructed to predict how many numbers were possible with a certain number of digits developed over two conversations. Over the course of its development the strategy took on three identities, or--to use Twain's term--went through three epochs. Each of the epochs represented a relationship between the number of digits and the positions in which the digit could placed. At first Kathy and her father believed that each digit could be in the same number of positions. To predict how many numbers were possible, they multiplied the number of digits by the number of positions. When the lists of numbers did not match their predictions, Kathy and her father changed their strategy. They started to think that the number of positions in which each digit could be placed depended on its position in the number. In a three-digit number, the first digit could be in three positions, the second in two positions, and the third in one position. By adding these values together, they could predict six possible numbers. This strategy worked for three-digit numbers, but when they used it to predict how many four-digit numbers could be made, they found the strategy was inadequate. In its final epoch, the strategy was to multiply the number of positions in which each digit could be placed together. For four-digit numbers they multiplied 4x3x2x1 for a total of 24 possible numbers. This strategy was supported by the fact that it worked for both three and four-digit numbers.

Back in School

The following day Kathy did not go to school but the events in class form important background for understanding what happened when she did return to school. Elaine was the only member of Kathy's math group in school that day. Ms. Smith asked her to join Mark, Karen, and Joan's group to discuss their answers. As Elaine moved her desk over to the group, she asked Karen how many numbers she found. Karen told her she found sixteen. Elaine said she found fifteen. Karen began comparing the lists to find the number Elaine missed. As Joan entered the group, Karen asked her how many numbers she found. Joan told her "eighteen." Dumbfounded that someone had found more numbers than she, Karen grabbed Joan's paper and began looking at the numbers. She and Elaine compared the lists. Although Karen found numbers on Joan's list that were not on her or Elaine's lists, she was not ready to change what she had written down.

Joan's list did not include numbers that began with zero. She believed that numbers could not start with zero--0259 was the same as 259 and did not count as a four-digit number. Karen was even more concerned that Joan had eighteen numbers and had not used numbers with zero in the thousands place. She argued that her own list was correct because "I have my dad at my house recording me every time we get something like this." "Big deal!" Joan responded and Karen elaborated, saying her father was 36 years old. Neither being convinced of the other person's position, Joan and Karen began a discussion of whether zero was a digit that lasted the entire math period. At the end of the day nothing had been resolved. Karen and Joan each held to their own beliefs about zero and their own lists of four-digit numbers.

When Kathy returned to school the following day, she joined Karen, Joan, and Fred's math group. Mark and Elaine were absent. As she entered her new group, she was met with cheers. Kathy was very influential in class. Her ideas often were the focus of class discussions. Karen's reaction reflects Kathy's position in class.

Fred was not feeling well. The group started their conversation by advising Frank that he should go home. He could not, he told them, because his mother was Christmas shopping and could not come to pick him up. They tried to convince him to go see the school nurse, but Frank refused. Instead, he laid his head on his desk while the other group members discussed the assignment. Karen, Kathy, and Joan began by comparing how many numbers they found. (KM denotes Karen; KG denotes Kathy; JS denotes Joan)

1 KG How many numbers did you get?

2 KM Sixteen.

3 KG You got sixteen, I got twenty-four.

4 JS I got eighteen

5 KM What the heck, where's your paper?

Karen, Kathy, and Joan started comparing the lists of numbers they generated the previous night. But, rather than focusing on the numbers included in each list, they focused on how many numbers should be in the lists. Their conversation, as did many of the conversations in class, focused on supporting and justifying the answers they brought back to class. Kathy started by telling how she and her father determined how many numbers were possible:

> See, watch, okay. There's four numbers. Then you can put zero in four places, you could put two in three places, you can put five in two places, and you can put nine in one. Now four times three is twelve, times--what is that number?--times two is twenty-four, times one is twenty-four. So you should have twenty-four.

Karen disagreed with Kathy saying only sixteen numbers were possible. She supported her contention in the two ways she had the day before. As on the previous day, these arguments did not convince her group. At first she told the group "I get tape recorded" believing that being part of this study added credence to her work. By recording her conversations with her father, her answers gained strength and allowed her to be more forceful when presenting her answers. Kathy also recorded her conversations and told Karen so. In response, Karen looked for another way to substantiate her list.

Karen's second attempt to support her answer came when she told the group that her father had showed her how to predict how many numbers were possible. She told her groupmates, "My dad told me if there's four digits you should have four here, four here, four here, four here." Karen explained this argument more clearly later in the conversation saying "There should be, [for] zero there should be four of them, you got that right. Two should be four numbers, five should be four numbers, and nine should be, have four numbers." Karen was arguing that there should be four numbers beginning with 0, four beginning with 2, four beginning with 5 and four numbers beginning with 9.

Karen had constructed this strategy with her father earlier in the week. The strategy was similar to the first strategy Kathy and her father used. Kathy and her father changed the strategy when the predicted number did not match the numbers Kathy found. In Karen's conversation with her father, the strategy matched the list of numbers she had written down; that is, Karen's list included 16 numbers and the strategy predicted sixteen possible numbers. Given this match, there was no reason to believe the strategy did not work. And, to give up the strategy without some resistance meant giving up what she and her father agreed was the right answer.

Varenne et al. (1982) have argued that, in areas in which parents feel competent, they work very hard to send their children to school with correct answers on their homework. Parents work with their children because they believe, Varenne et al. argue, that teachers assess families as well as their students. Parents who allow their children to come to school with incorrect answers are held in lower esteem then those parents whose children come with their homework done correctly. Students, it appears, also work to save face for their families. They apparently believe that their parents would not allow them to take incorrect answers back to school. As a result, students often used their parents as support for their answers. Kathy used her father's instruction as support later in the conversation when she said, "I understand it and maybe you won't and I don't know how to explain it. My dad just showed me that way and it works."

Joan disagreed with both Kathy and Karen. She had eighteen numbers and pointed at Karen's paper while telling her there should be six there, six there, six there and six there."[6] In the end, Karen, Kathy, and Joan were no closer to agreeing on how many numbers were possible. Their conversation ended this way:

6 KM Well, all I know is that I got sixteen numbers and I think it's right.

7 KG I think twenty-four is right.

---

[6]these numbers add up to 24 numbers. Joan did not have 24 numbers in her list at this point.

8 JS I got the same thing as you [Kathy] but I didn't use the zeros.

Joan's comment that she didn't use the zeros brought back the unresolved issue from the previous day: Could zero be used in the thousands place in the numbers? Although Karen and Kathy agreed that it could, Joan did not. The group turned to this issue before returning to the original topic.

Kathy raised the question of zero in the thousands place with her father two nights before. During their conversation, Kathy's father assured her zero could be used, but that she would not say "zero thousands" when saying the number. Karen and her father did not discuss whether zero could be in the thousands place; they included it without question. In school Kathy and Karen worked together to persuade Joan that zero could be used in the thousands place.

9 KG I didn't put it when I did my expanded notation. I didn't, I didn't put the zero, but, um, when I put it down, I had to put it or else people would complain 'where's the zero?'

10 JS Okay. I understand that zero is a digit, but using zero thousand I don't think that'd really sound right because that . . .

11 KM Its just zero thousand two hundred fifty nine.

12 JS I know, but if you have no thousands . . .

13 KM It's just two hundred fifty nine.

14 JS Why don't you just say two hundred ninety five.

15 KM It's just two hundred fifty-nine. You just put the zero, you just add the zero.

16 KG And you don't say it, you just say 'two fifty nine.' And then, and then if they ask you a question say 'But I didn't want to say the zero.'

17 JS Yeah.

18 KM She understand now?

19 JS So, I can add the zeros if I like. All right.

20 KG It says four digits, so that's how you guys could prove zero is a digit cause it said four-digit numbers.

21 KM That's what I was trying to tell her the other day.

22 JS I know, but I didn't understand because they, zero thousand, I didn't understand that. If you have zero thousand, why not just say two hundred fifty nine?

Kathy and Karen's experience in different practices is evident in this short exchange. Kathy's initial statement that she included zero because "people would complain 'where's the zero'" (line 9) if she did not reflects her understanding of how math is done in her classroom. Kathy's understanding of math in her classroom is evident again in line 16 when she offers a way of supporting the notion that zero could be in the thousands place and not be said when reading the numbers. To explain why zero could be used, Kathy offered her father's explanation: "And you don't say it, you just say 'two fifty-nine.'" Although Karen did not make direct reference to her conversation with her father, she was steadfast in her belief that zero could be in the thousands place, just as she was that sixteen numbers were possible.

At the end of this exchange, Joan was convinced that zero could be used in the thousands place and that she could "add the zeros if I like." As Kathy and Karen talked, Joan wrote out another six numbers all beginning with zero. When she finished, her list had expanded to 24 numbers and included all the numbers on Kathy's list. Now she and Kathy agreed that there were 24 numbers, but Karen held to her list of sixteen. This issue proved more difficult to resolve.

23 KM I got sixteen numbers, that's all I can say.

24 KG I did, I only got six. On number four. I mean on the one before this with three numbers I only got six. That would be right. Three, two and one.

25 JS Yeah six for each one.

26 KG Three times two is, you could put three in three places, you could put two in two places, you could put one in, let's say it was seven, say this is a seven. And you could put, then you could put there in three places, Frank stop, and you can put seven in two places and you can put open in one place. And, three times two is six plus, times one is six. Okay, just

27 KM What, what, what three, two and one.

28 KG [To Frank] Get that out of here. (Frank thrusts a piece of paper in front of Kathy) Three

29 KM Now what, listen to me, Kathy, but, if I remember right, I think I had like twelve numbers

30 KG I don't think you could have twelve

31 KM Or nine or something like that.

32 KG Let's just experiment. Say it's . . . Frank stop it. Get it out of here. Okay, we could do three . . .

33 FN I'll put it [the tape recorder] on pause.

34 KG twenty-one. Three one two. That's all you can do right there that's all. And then you have to go to the twos. Then it'd be two one three. Two three one. Okay so that's one, two, three, four. And, then you go to the ones. One three two. One two three. And, it's six numbers and that's all you can make out of it, see? See, one, two, three, four, five, six.

35 KM What I don't get is . . .

36 JS Yeah, but see you're switching the numbers around.

37 KM I'm talking to Kathy just for a minute. Say here's the same numbers. But here, I don't get what you're doing.

38 JS There's only four digits.

39 KM I only get four numbers.

40 JS I think there's more.

41 KG I think there's six. Why do you think there's . . .Okay, let me see what you have. Two oh . . .Zero two five nine. Then you have zero two nine five. Then you have zero five two nine. But you can also have zero five nine two. From zero nine two five, you can have zero nine five two.

42 JS We had that.

43 KM We just bring . . .

44 KG Fine, I'm not even gonna use this. You guys can do that. Me and Joan will make our own numbers.

Frank came alive as they discussed how many numbers were possible. But, rather than entering into the conversation, he started playing with the recording equipment and the other students' papers. Kathy, Karen, and Joan's attention was divided between the task and Frank's playfulness as they tried to answer the question.

At the beginning of this exchange, Kathy tried to show that her strategy worked for both three and four-digit numbers. Using an example similar to what she and her father had done, Kathy predicted that six three-digit numbers would be possible. Karen objected saying she thought she had found more than six numbers on the first part of Task 3. Although she could not remember exactly how many numbers she found, her second guess--nine--fits the strategy she was using in this conversation: If a four-digit number would have 4+4+4+4=16 numbers, then a three-digit number would have 3+3+3=9 numbers.[7] With Karen still not convinced, Kathy took her paper to look over her list of numbers. In the end, her frustration took over and she suggested that she and Joan would leave and start their own group. With the threat of her group splitting up, Karen asked for help. I had the following exchange with the group.

45	KM	Please help us
46	KG	With four numbers you can make twenty-four combinations can't you?
47	JR	What do you think?
48	KM	I think
49	JS	We're trying to explain to her how we're getting six numbers on each one of the, like nine five two zero. We're trying to explain to her how we're getting six numbers instead of four and she, we can't find a way to explain it.
50	KM	You're trying to explain it to who?
51	KG	She thinks there is only sixteen. I think there is twenty-four.
52	JR	Did you show her you're list?
53	KG, JS	YES!
54	JS	And, we tried to explain it.
55	KG	She doesn't have half the numbers I have.

[7]Karen and her father found six numbers on the first part of Task Three. In their conversation they said they knew they had found all the numbers when Karen "took the threes and you took the fours and you took the sevens."

56 JR So, you don't have all the numbers she has? {KM nods} So do the numbers you don't have work?

57 JS Yes, they all work

58 JR Then you would, would you have to add those numbers to your list?

59 KM I don't know

60 JR What do you think?

61 KM Yeah.

After our conversation, the group compared and combined their lists, eventually ending with a list of 24 numbers. As they did, Ms. Smith called the class back together to discuss the questions.

During the whole class discussion, Kathy told the class her group had found 24 numbers. Kathy's announcement was met with a chorus of surprised reactions. No other group found more than 16 numbers. Ms. Smith asked Kathy to read her group's list and explain how they knew how many numbers were possible. She did, using the same explanation she used in her small group. When constructing the number you have a choice of four positions for the first digit, three positions for the second digit, two positions for the third digit, and one position for the final digit (4x3x2x1=24 possible numbers). After a short discussion, the class accepted Kathy's explanation, agreed on the list of 24 numbers, and accepted Kathy's method as a way of predicting how many numbers were possible.

Summary

In their conversations at home, Kathy and her father worked together to reconstruct a strategy her father had used in the past. Their initial strategy--to add the number of choices for each digit (3+2+1=6)--worked to predict how many three digit numbers were possible. When Kathy returned to school she and her group agreed on the list of possible numbers and the answers to the other questions in the task. As a result, they never questioned the strategy Kathy or the others used to determine how many numbers were possible and it is reasonable to say that Kathy believed the strategy worked.

When they worked on the second part of Task 3, Kathy and her father were confronted with a situation in which their strategy did not work. When they realized that Kathy had found more numbers than their initial strategy predicted, Kathy's father announced they needed to "rethink" what they had done and refine their strategy. After a short time, Kathy's father proclaimed the correct strategy was to multiply the number of choices for each position.

As in the first case, Kathy and her father's conversations highlight the gradual development of mathematical ideas and activities. The strategy they reconstructed went through a series of changes that support the contentions of sociocultural theorists. These theorists argue that people appropriate understandings that allow them to function in their immediate situations. Kathy and her father's initial strategy worked in the situation in which they were working and there was no reason for them to look beyond that situation. As they worked on the second part of Task 3, their situation changed. Faced with this change, they changed their understanding to include both parts of Task 3.

Perhaps even more important than getting the right answer, Kathy was able to see her father's willingness to conjecture about how to predict the numbers and to change his conjecture when it was refuted by new evidence. Kathy's conception of school math, as well as being influenced by her participation in school, was influenced by her conversations at home. Modeling inquiry as Kathy's father did may have helped Kathy participate in school in more sophisticated ways. At the time of this study, Kathy's father was taking university mathematics classes where students were asked to develop what he referred to as the "creative part of math" in which, he explained, students develop their own problems and formulas. Kathy's ability to explain how she and her father determined how many numbers were possible may have come from her extensive involvement in the conversations with her father. Whereas many other students were confined to a role in which they only computed the final answer to the questions, Kathy and her father "developed their own formulas." Her involvement not only allowed her to contribute her answer to the class, but to explain what she and her father had done at home.

Kathy, Karen, and Joan's small group conversation points out that students place a tremendous amount of faith in the things their parents tell them. As a result they support the answer they bring back to school by claiming it was their parent's answer and listing their parent's attributes. Their faith sometimes resulted in group members threatening to leave their group. Rather than rethinking or changing their answers, the students would rather leave to preserve the answer they had written down. As Twain suggested, learning involves giving up old ways of thinking and accepting new ways. Although the students eventually did change the way they answered the questions, the give-and-take aspect of learning was often very difficult for the students.

Finally, other practices, such as this study, influenced how students participated in class. Students who participated in this study used their recorded conversations as support for their answers.

The gradual learning process is also apparent in students' conversations for subsequent tasks that involved similar content. Task 4 included four questions similar to those asked in Tasks 1, 2, and 3. Watching students as they answered these questions supports many of the findings presented above.

Task 4: Making Connections Across Tasks

Looking at students' conversations in different settings as they worked on a task illuminated the idea that concepts and understanding develop gradually. Participating in a conversation does not insure the ability to do the things discussed in another setting. Nor does a solution in one setting insure successful solutions in other settings. But, even though students may not be able to use the things discussed in a conversation in their entirety, they take away pieces that eventually will resemble the whole activity.

Implicit in the belief that learning is gradual is the notion that responsibility for completing tasks is gradually taken on by the student while participating in subsequent tasks involving similar ideas or questions. According to Vygotsky (1978), in subsequent sessions, learners do more on their own whereas more knowledgeable others--parents, in this case--contribute less. Parents, it has been suggested (Wertsch, 1984; Wertsch, Minick, & Arns, 1984) refer to previous attempts to point out things the learners have already done with assistance. The previous attempts serve as models for the student to follow.

Math

Nick and his family always celebrate the holidays in the same way. They gather together at Nick's house and exchange a few gifts. After that they sing holiday songs and eat cookies and fruit. Nick decided this year he was going to give his brother and sister some presents. He would like you to help him with his Holiday plans.

1. Nick has two sisters and two brothers. He wanted to give each of them a pair of socks, so he bought four pairs--a red pair, a blue pair, a green pair, and a purple pair. Nick couldn't decide what color to give each person. How many different ways can Nick arrange the socks for his brothers and sisters?

2. Nick was also in charge of choosing the songs they were going to sing. He decided, based on what they had done in the past, they would sing for two hours. Each song, he thought, would take about ten minutes. How many songs did Nick need to choose?

3. As was their tradition, Nick wanted to buy his brothers and sisters some fruit. He thought he would get each of them an apple and an orange. At the market apples cost 3 for 99¢ and oranges cost 2 for 60¢. How much would Nick need to pay for the fruit?

4. Nick also wanted to give each of his brothers and sisters some holiday cookies. He bought 20 cookies. How many cookies would each person get?

Figure 6.1: Task 4 Assignment Sheet.

Task 4 was designed to explore parents' decreasing assistance and to see whether they referred back to previous tasks while working. All of the questions in Task 4 were similar to those asked in the three earlier tasks. Question 1 asked the students to find out how many ways Nick could distribute gifts to his brothers and sisters. This question was similar to questions in Task 3. Questions two and four included division problems similar to those in Task 2. Question 3 took the students back to the grocery store as in Task 1. The assignment sheet is shown in figure 6.1.

Four of the six students recorded conversations for Task 4 (66%). The students' answers to Task 4 are summarized in table 6.1. Kathy and Pete are not represented in the table as they did not record their conversations or turn in their written work. Task 4 began the last week of school before winter break and continued after the break in January. On the last day before break, Ms. Smith asked her students to clean out their desks. Many students threw their math papers away before they were collected.

Table 6.1

Solution Table for Task 4

#	Karen	Ronnie	Shaundra	Tony
1	16	2	16	16
2	12	12	12	12
3	Apples: $1.32	Apples: $1.32	Apples: $1.32	Apples: $1.32
	Oranges: $1.20	Oranges: $1.20	Oranges: $1.20	Oranges: $1.20
	Total: $2.62	Total: no answer	Total: $2.52	Total: $2.52
4	5	5	5	5

The students answered questions two, three, and four with no difficulty. Their conversations as they worked to answer the questions were very reminiscent of those they had for Tasks One and Two. The students worked with their parents to determine what operation was necessary and then to compute an answer. In no conversation did the parents or students refer back to the questions in the earlier tasks. This short conversation between Shaundra and her father illustrates the conversations the students and their parents had for these questions.

1 SQ Um-hmm, Okay. So, now for number two. It says, "Nick also was in charge of choosing the songs they were going to sing. He decided, based on what they had done in the past, that he would sing for two hours. Each song, he thought, would take about ten minutes. How many songs did Nick need to choose?"

2 F So how many minutes do you have in an hour?

3 SQ Sixty.

4 F Okay, so he's gonna sing for ten minutes. How many times does ten minutes go in sixty? Ten divided into sixty?

5 SQ Um, six times.

6 F Okay, six times. So, ah, that's one hour.

7 SQ Yeah.

8 F So, you got six takes care of one hour. Six, so that's six songs that you can sing in one hour. But we got two hours. So what do you have to do?

9 SQ Just, um, like if you take both of those sixes from, if you divide them, then if you add them both up you get twelve.

10 F No, you don't divide them, you just, you can either multiply that six times two hours, or you can add another six for the second hour, which is, like you said, it equals, ah,

11 SQ Ah, twelve.

12 F So, it equals twelve, so what are you saying, you got twelve songs?

13 SQ Yeah, so he'll need, there's um, sixty minutes in one hour, so he'll need six songs for one hour and there's two hours so that would equal twelve and he would have to think of, um, twelve songs.

In this conversation, Shaundra answered a series of questions that led her to the answer. Her father, although Shaundra did the computation, structured the conversation to guide Shaundra to the right answer.

As was the case with Task 1, none of the parents or students referred back to previous questions that included similar content. Perhaps the best explanation of this is that the students were able to answer these questions with little or no assistance. Therefore, there was no reason for the parents to refer back to the other tasks. In the conversations the students seldom had difficulty choosing an operation or computing an answer. When they did have trouble, their parents quickly directed them to an appropriate operation or corrected their computation.

Question 1, however, was different. Although Karen, Shaundra, and Tony all said sixteen combinations of the socks were possible--an incorrect answer--they arrived at the answer in different ways. Karen multiplied four times four to get sixteen. Shaundra grouped the socks in matching colors. For the "red group" she included "Red, red; red, green; red, blue; and red, purple." For each color group she constructed four matches. Together, the four groups totaled 16 combinations. Tony wrote out this array:

B	R	G	P
R	B	P	G
G	P	B	R
P	G	R	B

Figure 6.2: Tony's Array

Although the matrix represents four permutations of the socks, there are sixteen items in the matrix. Tony counted the items and not the permutations.

As with the other three questions, neither the students or their parents referred back to Task 3 while answering Question 1. This is particularly interesting as Shaundra and Karen had been involved in conversations that led to the correct number of permutations while working on Task 3 and seemed to understand how to generate the lists of permutations.

Karen and Kathy had worked together on Task 3. Their group's work was presented in the vignette above. In their group Karen initially held fast to the list of sixteen numbers she and her father had compiled. But, after a fierce argument, she conceded, wrote down the six missing numbers, and seemingly accepted Kathy's method of determining how many numbers were possible. But, when she answered this question for Task 4 she did not hesitate. She read the questions, said "sixteen" without taking a breath, and went on to the next question. On her paper she wrote "4x4=16." She did not write out a list of permutations or in any other way represent the arrangements, she merely stated her answer and went on. The method she used to determine how many arrangements were possible is the same method she and her father used for Task 3.

Shaundra and her mother had worked hard to compile a list of 24 four digit numbers in Task 3. As they worked, Shaundra's mother pointed out the patterns in the numbers they found. At the end of their conversation, Shaundra used the patterns to make sure she had found all the numbers. But when she and her father started working on Task 4, Shaundra did not refer to the patterns she and her mother found the week before. Rather, they started looking for ways of matching the socks in pairs. Shaundra's list included sixteen different ways of pairing the socks.

As did the two cases presented above, both of these conversations portray the gradual learning process. both of the conversations support the notion that what appears to be learned in one setting may not extend beyond that conversation. Both Shuandra and Karen had accepted and used an appropriate way of determining the possible permutations with a group of four items. But, neither of them used it to answer this question.

Summary

People learn things gradually. Throughout the process they accrue new knowledge and refine things they already know. As a result, the things we know and do go through a series of qualitative changes. Our earliest attempts to do things are often aided by more experienced people and the tools that aid us in completing the tasks. During our interactions with these people and tools, we observe ways of thinking about and doing things that help us better understand our situation. Over time, we become more skilled at the things we have observed and practiced in our interactions. As our abilities increase, our dependence on the tools and people around us diminishes. But along with our growing independence comes the necessity of giving up previous ways of knowing.

The gradual change in a person's thinking, it has been posited, can be seen as he or she works with other people in problem-solving situations. When the person changes what they do to more closely resemble the things they've heard, seen, and practiced in the interaction, they have learned. In the first case presented in this chapter, Ronnie made changes of this sort as he talked with his mother about Task 1. As they talked, Ronnie changed his strategy to one his mother suggested would better answer the question. As they worked, Ronnie wrote down this new way of answering the question. But somewhere between home and school Ronnie lost his paper and, without his written record, was unable to completely reconstruct the strategy in school. Instead he reverted to the strategy he used at the beginning of the conversation with his mother.

Although it looked as though Ronnie learned how to answer the question from his mother, he had not completely appropriated the strategy. What looked like clear evidence of learning in their conversation, turned out to be something slightly less. Ronnie had appropriated parts of the solution--he remembered that the answer was "two something," for instance--but had not appropriated the entire solution.

Recent work in situated learning (Lave, 1993) has suggested that learning includes extending what is appropriated beyond the immediate situation. The tools used within specific practices work as an aid in this extension. Had Ronnie not lost his assignment sheet, he may have been able to reconstruct the solution. But, without the tools and his mother's assistance, he could not.

The second case presented here also portrayed the gradual learning process. This portrayal, however, was slightly different. Whereas the first case pointed out the necessity of looking beyond the immediate situation for evidence of learning, this case highlights two other aspects of the gradual development of mathematical ideas. First, Newman, Griffin, and Cole (1989) argued that people appropriate only what is necessary to make sense of their immediate situation. As they encounter new situations, they change what they think to fit their new circumstances and explain their past experience as well. Kathy and her father worked together to find a strategy to predict how many numbers could be made from a given set of digits. Their efforts provide a clear example of the process of refining mathematical ideas and activities to accommodate new situations.

Second, through participation in conversations, people not only appropriate *what* they talk about, but *how* they talk about it as well. By observing her father conjecture about how to predict how many numbers were possible and revise his thinking when the list of numbers they found did not match their prediction, Kathy may have appropriated the skills that allowed her to reason through different situations.

Both of the cases also point out that many things influence students' development of mathematical thinking ability. As did the conversations reported in chapters four and five, the conversations the students had at home greatly influenced what they did in the classroom. Students placed a great amount of faith in their parents' mathematical abilities. They believe, if for no other reason than their parents' age, that they know the mathematics that is being taught in school. They have, after all, gone through it themselves. As a consequence of their faith, the interaction in the math groups often began with the students sticking up for their parent's answer. If their answer was not accepted by the group, the students would threaten to leave rather than changing their answer.

Parents do not, however, know mathematics in the same way. Kathy's father, for example, may have had a richer--or perhaps just more contemporary--understanding of the mathematical content of the tasks than did the other parents. Through her interactions with him, Kathy developed more sophisticated ways of participating in class. Kathy's father modeled mathematical inquiry in their conversations in a way that was not present in the other conversations. As a result, Kathy brought more than answers back to school.

In school, as in their homes, the students' math groups were involved in competing streams of activity. The students were often involved in other classroom activities that were important to their social standing in class. Whether their ideas were accepted by their group was sometimes as much a social issue as it was a mathematical or academic one. In the end, they would turn to a classroom authority--in this case either Ms. Smith or me--to resolve conflicts. Even then, students would only reluctantly change their answers.

CHAPTER 7

SUMMARY AND IMPLICATIONS

Students' conversations at home and in school have a profound influence on their mathematics learning. In conversations, students, along with their teachers, parents, and classmates, bring together different conceptions of mathematics and other experiences that shape what and how they learn. Students, however, do not easily learn the things they discuss. Rather, learning is a complex process filled with choices, risks, and other social pressures that students and their conversational partners must negotiate as they talk. In the end, students cannot always do or say what appeared so easily learned in their conversations. And yet, the influence of their conversations at home and in school remains great. In this final chapter, I review the findings of this study that led to this brief summary and contemplate their contribution to broader theories of learning and mathematics education.

Students' conversations in different places illuminated different aspects of learning mathematics. In the following sections I revisit students' participation in conversations at home and in school and as they move from one conversation to the next. At the end of each section, I discuss the contributions of the findings to previous research and theory in educational psychology and mathematics education.

Following the summary of the findings, I look back on the data collection methods used in this study. Although these methods provided a glimpse of the influence students' conversations at home and in school had on their completion of elementary school mathematics tasks, there were many uncontrollable gaps in the "data trail." In this section I discuss the gaps that existed and how they may have influenced the data constructed for this study and changes in the methods that might result in a more complete data trail.

Finally, I explore the implications of this study in light of recent calls for reform in education and mathematics education. Calls for reform in education have identified

eight goals (National Education Goals Panel, 1994) that are meant to address and reverse negative trends in America's schools. One of the goals is to increase parents' involvement in their children's schooling. The findings of this study can inform attempts to develop productive home-school partnerships that can help fulfill this goal and ease the implementation of the recent reforms in mathematics education. The current emphasis on home-school partnerships also provides an opportunity to continue the study of the influence of students' homes and schools on what and how they learn. In this final section I look more closely at the goal of increased parental participation and how extensions of this study might contribute to its fulfillment.

Students' Conversations

Conversations at Home

The conversations students and their parents had at home constituted one of many streams of ongoing activity that characterize most households. In all but Shaundra's house, the students and their parents listened to music or watched television as they worked on the homework tasks. Parents sometimes cooked dinner, took care of family pets, or talked with other family members as they worked with their children. Parents sometimes talked to students about their homework from across the room or from adjacent rooms. It was clear that homework at certain times was subordinated to other household practices that required immediate attention. Parents whose attention was divided among household tasks were less able to monitor or contribute to their children's work.

At times, parents' divided attention caused them to overlook errors in the students' work. Although the students in these conversations often used the same strategies as did other students, misunderstandings or confusion over important aspects of the tasks or values used in computation produced incorrect answers. Given the assumptions and values they chose for their calculations, their work was done correctly. When they returned to school, however, their answers were considered wrong by both their teachers and classmates.

The students and their parents interacted in ways that were consistent with normal conversation in their household. Different rules applied in different homes. In one household, for instance, the student was not allowed to question his mother's contribution to the tasks; in another, the student was expected to question her father's contribution. The rules within which the students and their parents worked influenced the students' participation in school.

In general, the students began all of the conversations at home by trying to answer the questions with no assistance from their parents. Their parents, however, sat nearby and intervened at the slightest hint that the students were having difficulty answering the questions. Although this scenario briefly describes the conversations in students' homes, what the students and their parents contributed to the conversations depended greatly on how consistent the task was with their experience in elementary school mathematics classes or other out-of-school practices.

Consistent Tasks. Most of the tasks the students and their parents worked on during this study were consistent with some aspect of their past experience. While completing these tasks, the students and their parents drew on the different practices in which they participated. When these practices were brought together in conversation, they often created conflicts that demanded resolution before the students and their parents could answer the questions.

Tasks that were consistent with students' and parents' past school mathematics experience (Task 2, for instance), presented little or nothing to resolve. As students and their parents completed these tasks, they quickly agreed on what operations were required and how they could be computed. As a result, the parents monitored students' work, only correcting minor computational errors. Tasks that brought out-of-school practices into school drew more practices into the conversation, consequently giving rise to more conflicts. These conflicts were resolved in one of two ways. Conflicts could be resolved by bringing together aspects of different practices to form a new, amalgamated practice. Students and parents then worked within the rules of this new practice, with the answers they constructed meeting the requirements of each of the contributing practices. Conflicts could also be resolved by invoking the rules of a more powerful or influential practice. In many of the home conversations, for example, the practice of inquiry math in Ms. Smith's classroom was weighted most heavily and could override any other practice brought to bear in the conversations.

Inconsistent Tasks. When the tasks presented activities that were inconsistent with the students' or parents' experience, the parents could not provide the sort of guidance they did on the other tasks. Although the conversations around these tasks started out as did those discussed above, when students experienced difficulty, parents initially either deferred to the students and allowed them to work on the tasks alone or ended the conversation. When they deferred to the students one of two things occurred. When the students, too, were uncertain about what to do, the parents deferred to Ms. Smith and the students returned to school without completing the assignments. When the students believed they knew what to do, they worked alone as their parents looked on.

The parents were not passive participants in these conversations. Rather, they watched intently as their children worked to answer the questions. As they watched, they asked questions to check their growing understanding of what the student was doing--even when the student was doing something that led to an incorrect answer. When the parents believed they understood the task, they contributed more to the conversations--even to wrong answers. When the parents felt competent, they offered the students advice about how to answer the questions. They contributed strategies that would increase efficiency, algorithms from their school mathematics experience, and other things that helped students answer and document their answers to the questions.

In conclusion, the conversations students had at home were enmeshed with other household activities that limited participation in the conversations. Even with these limitations, students and parents contributed whatever they believed would aid in answering the questions. What they were able to contribute depended on the connections they made between the immediate task and various aspects of their experience. When parents were certain about the mathematics, they directed the students to correct answers. When parents were uncertain about the mathematics asked for in the tasks, they deferred and attended to the students' knowledge of what was discussed in class. As they watched, they learned about the mathematics and eventually contributed to the conversations. These conversations both supported and refuted contentions of sociocultural theorists, mathematics educators, and other educational researchers. In the next section I review claims of these researchers and how this study contributes to their findings.

Contributions to Theory. McDermott, Varenne, and their colleagues (McDermott et al., 1984; Varenne et al. 1982) have suggested that parents may have different goals for homework sessions than do teachers. Whereas teachers view homework as practice of important skills and ideas raised in their classrooms, parents strive to send students back to school with correct answers. The conversations presented here support this contention. When parents were certain about what the students were being asked to do, they directed students to what they knew were correct answers. When parents were uncertain about how to complete the tasks, some, rather than risking an incorrect answer when the students returned to school, suggested students not complete that part of the task and ask their teacher about it when they returned to school the following day. It was better, it seems, to go back with no answers than with wrong answers. In other households, parents worked hard to understand what it was the students were being asked to do. When they believed they knew what a correct answer was, they again became quite directive in the conversations. In extreme cases, parents avoided telling their children about new mathematical ideas to insure correct answers. Kathy's father told her, "I mean I could show you how to do decimal division but, if you haven't covered it yet, it's probably just better to learn what you know."

Within these conversational structures, students and their parents defined the tasks on which they worked and what constituted an appropriate answer. This process, too, has been the focus of much work in the sociocultural tradition and the conversations recorded here both support and extend this previous research.

Wertsch (1984), in his seminal discussion of negotiation within the zone of proximal development, suggested that, in order for someone to learn something in a conversation, participants must enter the conversation with different situation definitions. A person's situation definition is manifested in the things they do as they participate in the conversation (i.e., in their action patterns). Learning in the conversations can be evidenced in the changes in one person's action pattern to more closely resemble the actions of other participants. The findings of this study support this contention. While the students and their parents worked on tasks that were either inconsistent with their school mathematics experience or drew on more than their school mathematics experience (Tasks 1 and 3), they came to the conversations with different situation definitions that needed to be negotiated as they worked. While the students and their parents worked on tasks that were consistent with their school mathematics experiences they brought similar if not identical situation definitions and there was little or nothing to negotiate (i.e., Task 2). In these conversations it is doubtful that the students--or the parents--learned anything they did not know before entering the conversation.

In Wertsch's (1984; 1985) discussion of learning in the zone of proximal development, he also suggested that adults enter conversations with a more appropriate situation definition than do children. Through the course of their conversations, children take on the adults' action patterns and begin to define the situation as the adult does. Adults, however, make only temporary changes in their definition to accommodate the child's immature approach to the task. Elbers and his colleagues (1991; Elbers, Maier, Hoekstra, & Hoogsteder, 1992) have criticized this transition from "other-regulation to self-regulation" as ignoring the contributions of the child to the conversations. Instead, they argue, the transformation moves from "joint-regulation to self-regulation," with both participants actively contributing to the formation of the task. The conversations recorded for this study support Elbers' claim.

When the students and their parents brought different situation definitions to the conversations, they negotiated a shared definition of the task that drew on the experiences each of them brought to the conversation. At different times the students and their parents assumed the role of the more knowledgeable other. Students brought knowledge of how mathematics was done in Ms. Smith's classroom and their parents brought their own experiences both in and out of school that contributed to the shared definition. In the end, the student completed the task in a way that reflected the different experiences both participants brought to the conversation.

At the same time that these conversations supported many claims of sociocultural theorists, they also call into question other findings and provide empirical support for more contemporary theories of learning. Sociocultural theorists have been concerned with the relationship between the individual and the social structures within which they live, work, and play. At times, theorists have argued that the practices in which people participate determine what and how they learn (Scribner, 1984). More recently, however, theorists have argued that people move among various practices (Cole, 1990; Rogoff, 1992; Valsiner, 1993; Wertsch & Hickman, 1987) and that each of the practices in which they participate influences their participation in other practices.

The conversations students had at home suggest that no one practice determined what or how students learned. Rather, students and their parents drew on their experiences in many different practices to construct a new set of rules--a new practice--within which they worked. Their experience in any practice was available for them to use while working on these tasks and the new practice they constructed included aspects of other practices they believed could contribute to their answers to the questions presented in the tasks.

The different practices did not contribute equally. In most households, what the students and their parents believed Ms. Smith expected in her classroom was given more weight than other practices. The rules of "doing math" in Ms. Smith's classroom, they believed, determined the data they had to work with and appropriate presentation of their answers. Although their varied experience may have led them to question the tasks sent home or what students were doing in school, in the end, parents insisted that the students comply with the expectations in Ms. Smith's classroom.

This finding becomes particularly important in light of the recent calls for reform in mathematics education. Cobb and his colleagues (Cobb, Pateman, & Bednarz, 1993; Cobb, Wood, & Yackel, 1993) have resisted the contention of sociocultural theorists that practices exist independent of the participants in the practice or activity. This study supports their stand. The practices the students and their parents brought to bear reflected *their* experience in school mathematics classes and other out-of-school practices. How they did math homework in students' homes depended on their past experiences and how they fit homework into other ongoing streams of activity.

At the same time that conversations recorded for this study support Cobb and his colleagues, they call into question the assumption that schools provide students' sole exposure to mathematics. Mathematics has often served as a site for psychological research in education. Kilpatrick (1992) has suggested psychology's interest in mathematics may have stemmed from

> perceptions regarding its important role in the school curriculum; its relative independence of nonschool influences; its cumulative, hierarchical structure as a school subject; its abstraction and arbitrariness; and the range of complexity and difficulty in the learning tasks it can provide.

Included in Kilpatrick's list of reasons is the notion that classrooms provide students' sole or primary exposure to mathematics. The conversations recorded here, however, suggest that students' families may have as much or more influence on the mathematics students learn than do teachers and classmates. When the parents felt certain about the content, they were directive in their instruction; students listened to what they said and mimicked their actions. As I will discuss in more detail later, upon returning to school, the students often aggressively defended the things their parents taught them at home; they were reluctant to change their answers--even when their assumptions, strategies, or answers were at odds with other things discussed in class. When the content deviated from parents' elementary school mathematics experience, parents exhibited some of the traits associated with the calls for reform in mathematics education. They explored the situation with their children, tried to figure out what was going on, and, when they felt comfortable, began participating in the conversations in ways that resembled their earlier contributions. What the students learned in the conversations was not always consistent with what Ms. Smith intended. Their answers, however, still adequately answered the questions, in their thinking, and influenced what students brought back to school more so than did their participation in school.

In sum, the conversations in students' homes presented a portrait of homework as a complex practice that is constructed from parents' and students' experiences both in and out of school. What was learned in the homework sessions was determined as much by what happened in other classrooms and out-of-school practices as it was by the immediate practice of elementary school mathematics in the students' present classroom. When the students completed their homework, they brought the products of their homework sessions back to school where they compared their answers to those of other students. What they learned at home and how they arrived at their answers served as preparation for their discussions in school.

Conversations in School

In many ways, the conversations students had in their math groups at school were reminiscent of those they had at home. Students' conversations about mathematics in school represented just one strand of ongoing activity. As they worked in their math groups, the students engaged in conversations about calculators, about the equipment used to video and audio tape their conversations, about athletics, and about other out-of-school activities. Ms. Smith sometimes interrupted their work to ask about assignments they had not completed or other pieces of classroom business. Daily announcements came over the intercom telling students about school activities or calling students to the office. Each of these things represented different strands of activity to which students needed to attend as they worked.

The students each brought different experiences to the conversations that shaped what they believed needed to be done. Their experiences at home heavily influenced what they brought to their math groups in school. But, what they brought to the conversations in school was different from what they brought to the conversations at home. Because the tasks were completed as homework before they were discussed in school, the students came to school armed with answers, not just ideas about what to do.

The students' answers became the initial focus of their conversations in school, and having--not getting--the right answers was the goal of the conversations. The conversations began with the students comparing their answers. If they all had the same answer, they went on to the next question without discussing how they arrived at their answers or the content of the question. If they had different answers, their task changed. Now the goal became selecting the correct answer from among those proposed in the conversation. The students' objective was to defend their answers and convince their group members that they were correct.

The students' first line of defense was to check their computation. Students either wrote out the computation on a new sheet of paper or punched the numbers into a calculator to show how they had computed their answers. When they completed the computation, and the result agreed with the answer they had written down, they presented it to their groupmates as evidence that their answers were correct. When discrepancies were found in the computation, the students willingly changed their answers.

If everybody's computation was done correctly and they still had different answers, the students moved to a second line of defense. Their objective now was to determine who had the correct answer. They set out to support their answers and, at the same time, discount answers other than those they had written down. Just as the parents wanted to send their children back to school with right answers, the students argued vehemently for the answers they constructed with their parents. Even when confronted with convincing arguments for other answers the students would refuse--at least initially--to change the answers they had constructed at home. In a sense, they chose to "not learn" (Kohl, 1991) what other students were telling them in order to preserve their family's position.

The students used many things to support their answers. Their parents, by virtue of their age and having done these sorts of tasks before, were set up as authorities who would not send them back to school with wrong answers. Students told their groups that their answers must be correct because their parents had worked with them the night before.

The students also used their and other students' standing in Ms. Smith's class to support the answers they chose. The students accepted answers from students with higher standing in the classroom more readily than those of lower standing students--even when the lower-standing students' answers were correct. The students who enjoyed higher standing in the classroom participated more and were more instrumental in determining the answers on which the class agreed. Many factors contributed to a student's classroom standing. Students could gain standing for their contributions to class discussions in math class or they could gain standing from their accomplishments in other academic and non-academic activities. The students also used their participation in this study as evidence of their standing in class. Students who recorded their conversations sometimes reminded their math-group partners of their participation in this study as support for their answers. Invoking their participation as evidence did not always work, however, as other students in the math groups often also recorded their conversations.

If the students could not convince their groupmates of their answers or agree that one group member's answer was correct, they sought assistance from a classroom authority--Ms. Smith or me. They asked which answer was correct and Ms. Smith or I talked with the students to help them sort out their difficulties. The students would accept the outcome of those conversations as correct answers and they changed their answers to reflect the conversations.

As did the conversations students had at home, these conversations both support and refute contentions of sociocultural theorists. In the next section, I discuss how these findings support, extend, or refute the contentions of various theorists.

Contributions to Theory. Students' conversations in school support many of the contentions of sociocultural theorists and mathematics educators. The conversations in school, as did those at home, support the contention that practices are made up of the people who participate in them (Cobb et al., 1993). Although the rules of mathematics in Ms. Smith's classroom included exploring alternative answers and changing what students believed based on things said by other students, these rules were not always followed in the small group conversations. Groups saw their task as determining one correct answer and each student believed his or her answer was correct. What counted as warrants for answers in the math group discussions was determined by the students and, often in the heat of argument, students invoked many things other than mathematical reasoning to support their answers.

Vygotsky (1978) argued that more knowledgeable peers assist their coworkers or classmates in learning new things. With this assistance, less knowledgeable people hone their skills and abilities as if they were working with adults. Recent classroom research (Bivens, 1990; Eichinger, 1992; Tudge, 1990), however, suggests that students do not always assist their classmates in such a democratic manner. Rather, students resist working with some people, accept the ideas of popular students over those of more marginalized students, and worry more about their standing with other students than with the teacher. Indeed, good standing with the teacher is often avoided. The conversations recorded for this study support the contentions of these researchers. Students may have tried to help other students, but they would often refuse help. Answers they accepted had as much to do with students' standing in the class as it did with their mathematical worth. Standing, however, was sometimes granted for contributions to the class discussions.

Although the findings from this study support those of other researchers looking at small group interaction, Ms. Smith's class always arrived at mathematically sound answers to the question in their large group discussions. During these discussions, each math group presented their answers and the class was able to ask questions to clarify what was presented. Often students' questions forced other students to rethink the idea they were presenting. In the end, with Ms. Smith's help, the class discussed all of the answers presented and determined which of them were appropriate. The whole class negotiation of appropriate answers might provide a fruitful site for future research.

The conversations students had at home and in school show students changing their thoughts and actions in ways that reflect their conversational partners' situation definitions. Although the change was not always easy, in the end students appeared to mimic the action patterns of the people with whom they talked. But even though students changed the things they did and said as they participated in conversations at home and in school, it became evident as they moved among conversations that changing what they do in one conversation may not be definitive evidence of their learning. In the next section, I look more closely as students move across conversations.

Looking across Conversations.

What students apparently learn in their conversations at home and in school is not always available to them in subsequent conversations. Students often learned only part of what was discussed and did not always recognize situations in which they can use what they learned in previous conversations.

What the students had available to them when they returned to school depended, in part, on their documentation of their conversation at home. When the students lost their documentation or did not document their conversation, they were less able to reconstruct the answers they had agreed upon at home. Even when the students could not reconstruct their entire answer, however, it was clear that their conversations at home influenced their thinking. Students brought incomplete understandings of the questions back to school that included ideas about which answers might be correct. Indeed, when they heard their groupmates' answers, they often knew which ones were incorrect and why.

Even with fully documented and reconstructed answers, the students did not recognize situations in which they could use what they had discussed in previous conversations. At home, the students and their parents approached the tasks as if they were separate, unconnected activities. While working on Tasks 1, 2, and 4, the students and their parents did not refer back to things they had done in previous tasks.[1] This phenomenon was particularly evident as they worked on Task 4, which was specifically designed to explore the connections students and their parents would make to previous conversations. When they did refer to their experience, their references were to rules

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[1]There is one notable exception to this pattern. Pete's grandmother, while working an Task 2, reminded him of computational rules she had told him the week before. In their conversation, she repeated how they computed the answer to a question from Task 1 and told Pete to do the same thing to answer the questions on Task 2. She did not, however, refer back to the earlier tasks while completing Task 4.

of participation--what needed to be included in answers and how they should be presented, when to use labels, rules for the same operation in different practices--but seldom about a conceptual understanding of mathematical ideas.[2]

Only while the students and their parents worked on Task 3, which explicitly asked them to look back at their previous work, was the gradual development of ideas visible. As they worked on the first part of Task 3, students and their parents compiled a list of three-digit numbers and then explained how they compiled the list. As they worked, they did not emphasize the mathematical ideas embedded in the task. Rather, they focused on answering the questions. As a result, mathematically suspect answers were accepted when they believed they had sufficiently answered the questions in the first part of Task 3. The second part of Task 3, an extension of the first part, asked them to compile a list of four-digit numbers, review what they had done in the first part, and devise a method of predicting how many numbers could be made using a given set of digits. As they compiled their lists of four-digit numbers, the students and parents referred back to what they had done on the first part. They used strategies they had devised and revised their thinking about how to predict how many numbers were possible. When they finished, the families were able to conjecture about a way to predict how many numbers were possible with a given set of digits.

In contrast to their conversations for the other three tasks, the students and their parents made connections between mathematical ideas they had constructed earlier, the things they had done in previous conversations, and the immediate questions. As they worked, the students and their parents refined their strategies, honed their thinking and, in essence, *did* mathematics. Indeed, many of their actions were consistent with the calls for reform in mathematics education. They did these things, however, only when

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[2]Ronnie's declaration that he was going to average the cost of the groceries in Task 1 may be an exception to this finding. In that instance, Ronnie recognized a similarity between what he was doing in the immediate task and a mathematical idea he had previously discussed in school. Although he declared the usefulness of that procedure, he did not discuss why the notion of averaging was applicable in that task.

the tasks specifically required them. Perhaps more than anything else, these conversations suggest that making connections between tasks and mathematical ideas is a skill that must be encouraged in mathematics classrooms. Along with the practical implications of these conversations, they also inform recent theories of learning.

Contributions to Theory. Watching students as they participated in this series of conversations provides empirical support for--and clarification or extension of--the claims of sociocultural theorists. One of the major tenets of Vygotsky's (1978) theory of development was that people learn things gradually. Over time, and with the assistance of more experienced others, people develop skills and abilities that allow them to accomplish other, larger socially-defined tasks. Vygotsky (1978), and other soviet psychologists (Luria, 1978) sought to document this process by watching the "microgenetic" development of cognitive abilities. In Western psychology, the microgenetic development of cognitive abilities has been taken to mean that learning can be seen in one observation. Wertsch and Stone (1978) described microgenesis as "the development of a skill, concept or strategy within the context of a single observational session" (p. 8) and Newman, Griffin, and Cole suggested that "Methodologically, cognitive development can be observed as children pass through or work within the zone [of proximal development]." (p. 61).[3]

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[3]Catán (1989) has argued that, although this definition of microgenesis has been widely accepted in the west, it represents a "misuse" of Vygotsky's original intent. Vygotsky used microgenesis to describe a methodology with which he could explore the cultural history and ontogenetic development of psychological functions in a miniature, accelerated form. To document the development of writing in children, for instance, Luria (1978) used children between 5 and 9 years old because he believed they would have the same psychologically salient characteristics of preliterate adults. In a series of short experimental sessions through which, he hypothesized, he could accelerate the course of development, he presented the children with sets of phrases that would be difficult to remember with no means of recording. Asking the children to remember these phrases and sentences, he hypothesized, would require the children to develop a system of recording that would resemble that development of similar systems at more macro levels. Neither the research conducted by Wertsch and Stone, Newman, Griffin, and Cole, nor the research conducted as part of this project would fall into the category of microgenetic research thus defined.

This study, however, suggests that what students have apparently learned in one conversation may not be available to them in subsequent conversations. This finding draws into question the notion that learning can be seen in one observational session. Rather, researchers need to look for evidence of learning in subsequent interactions and to understand how people use what was discussed in the earlier conversations to inform their actions. Traditionally, this issue has been viewed as one of *transfer.*

Lave (1988) and others (Beach, 1990; Pea, 1988) have suggested that to examine this issue we need to reconceptualize *the transfer problem.* The transfer problem arose from Thorndike's rejection of the notion that learning academic subjects such as Latin would benefit people in other academic endeavors (Thorndike & Woodworth, 1901). In its place, Thorndike suggested that skills would transfer from one setting to another if both settings contained *common elements.* This conception of transfer held that the actors within the settings merely responded to environmental stimuli and suggested that, when taught how to respond to a given stimulus, a person's response would be consistent across situations. The common elements theory of transfer, though still apparent in some places, has come under fire from many directions.

Perhaps the greatest fervor is around the notion that individual agency plays a role in the transfer of knowledge from one situation to another. Transfer, it is currently argued, does not rest on the knowledge a person has, but on their ability to access the knowledge at appropriate times. What is transferred is determined by the actor's *perceived similarities* (Brown, Bransford, Ferrara, & Campione, 1983; Pea, 1988) among situations, rather than common elements residing in the situations themselves, and their determination of what is and is not useful as they work in unique situations. In this conception of transfer, the prior knowledge and experience of the learner and the continuities (what is the same across situations), discontinuities (what is different), and transformations (how people change their thinking and actions) across situations (Beach, 1990) in which they are working need to be considered.

The conversations students and their parents had while working on Task 3 support this conception of transfer. The two parts of the task can be described in terms of their continuities (both parts asked the students to compile lists of numbers given a set of digits, both asked students to write the numbers in order and in expanded notation), their discontinuities (the second part included four rather than three digits), and in how the students and their parents transformed their thinking across the conversations. As they worked on the first part of Task 3, the students and their parents constructed incomplete, erroneous, or underdeveloped understandings of permutations that were not always helpful as they worked on the second part. Their recognition of the continuities between the tasks, however, led them to use what they learned while answering the first set of questions. When they determined their strategies did not work, they transformed their strategies into something useful in both situations. Indeed, the discontinuity between the two parts of Task 3 may have led students and their parents to a more complete understanding of permutations.

Students' and parents' construction of incomplete or underdeveloped understandings of permutations as they worked on the first part of Task 3 supports other claims made by scoiocultural theorists. Newman, Griffin, and Cole (1989) have argued that, rather than internalizing skills and abilities intact, children appropriate pieces of what is discussed that they find useful in completing the task at hand without consideration of future tasks. When they encounter unique situations, however, they may find what they have appropriated wanting. Faced with this dilemma, they must change the way they define the situation or change their understanding of the ideas they are bring to bear in their work. The students and their parents did this as they worked on the two parts of Task 3.

In addition, what students appropriate and have available in subsequent situations is influenced by their documentation of the conversations in which they participate. (Remember Ronnie's inability to reconstruct the work he did with his mother when he left his homework paper at home.) This supports Vygotsky's (1978) claim that higher psychological processes are mediated by cultural tools and artifacts developed to facilitate the process.

Although sociocultural theorists agree that researchers must look beyond the immediate situation to document the gradual development of ideas and abilities and that the influence of tools designed to aid in the development of higher psychological processes need to be documented, there has been little empirical support of this contention. This study provides initial empirical evidence of these claims and recent conceptions of transfer. It shows people referring to their documentation of previous conversations (or the results of losing that documentation) to retrieve ideas they began to understand. It also shows them changing their thinking when they discover the things they learned while working on the earlier tasks were not helpful.

Summary.

The conversations in which students participate are only one of many concurrent streams of activity. Both at home and in school, the streams of activity compete for students' attention, sometimes pulling participants' attention away from the immediate task. Each of the participants in the conversations brings varied experiences that contribute to how they define the conversation and, as a result, what students, their parents, and classmates learn by participating in one conversation is heavily influenced by the experiences they and their conversational partners bring to bear in the conversation.

The reasons people choose to invoke different experiences in conversations also vary. Although the majority of things people do and say are done to complete the immediate task, participants sometimes choose to invoke things in order to preserve other aspects of their lives. These things may include saving time for other activities or to save face. Participants accept or deny the contributions of their conversational partners for various reasons. Students' standing in class, parents' or teachers' authority in different settings, and other things contribute to what students accept as appropriate solutions.

Finally, although people appear to learn something in one conversation, they may not be able to use what they learned in subsequent conversations. What is available to students as they participate in successive conversations depends on their documentation of the earlier conversations. Even with complete documentation, however, students often can reconstruct only part of what they did earlier.

Each of these findings contributes to a growing literature on human learning and can contribute to the conversations surrounding the implementation of recent reforms in mathematics education. The findings from this study support the notion that learning cannot be seen in one observational session and raise questions about what evidence we need in order to say a person knows something. Indeed, the ability to use what is discussed in one conversation in subsequent conversations (i.e., extending what is learned beyond the immediate setting) may better illuminate what a person truly knows. The connections people make among conversations and ideas, however, are influenced by many things and may not be a natural characteristic of learning. Rather, the willingness to make productive or appropriate connections may need to be taught.

The findings of this study can also inform the implementation of reforms in mathematics education. Mathematics educators cannot assume that students' sole exposure to mathematical ideas comes in school. Parents contribute greatly to students' mathematical understanding. Although the mutual influence of the home and school has always existed, it is particularly important in the wake of the reforms which include new definitions of what it means to know and do mathematics. Parents and students may have conceptions of school mathematics that conflict with the spirit of the reforms. Parents' and students' unwillingness to change their conceptions of elementary-school mathematics might create barriers to students' mathematical understanding. As a result, successful implementation of the reforms may include parent education as well as changes in elementary- and secondary-school classrooms. Parent education and developing successful home-school partnerships provides a fruitful opportunity for further research.

Methodology and Methods

Although prior to this study theorists had begun suggesting that what and how people learn is influenced by their participation in many socially-defined practices, no research had documented successive conversations to explore how ideas change as people move from practice to practice. Consequently, no established methods existed to follow people among practices and to document their developing ideas. The methods I used in this study were developed as an initial attempt to document cognitive change across a series of conversations. In general, the data collection methods were successful. Listening in on students' conversations at home and watching them participate in school illuminated many of the influences on students' thinking. But even though the methods provided a glimpse at the influence of different conversations on students' completion of the tasks, there are aspects of the methods that can be improved. In this section I review the methods used to collect the data and discuss some of the difficulties that resulted. Finally, I discuss some changes that might lead to a more complete data trail.

The data collection methods I used in this study comprised two components. The first component focused on documenting classroom activity. To collect this data, I videotaped Ms. Smith's ongoing classroom instruction and videotaped or audiotaped small group conversations during class time. Before I began collecting data, I had resigned myself to collecting conversations of only one small group each day. As I began filming and recording in class, the students who recorded their conversations at home also began recording their conversations in school. Many of them brought their tape recorders and turned them on at the beginning of math class and left them on until the end. Taking a hint from them, I started placing audio tape recorders on in the small groups that were not being video taped on a given day. In the end, I produced a series of tapes, both audio and video, that documented whole class instruction and each of the small group conversations in which the students who took part in this study participated. To document the students' written work I collected their assignment sheets after each task.

The second data collection component focused on documenting the conversations students had with their parents as they worked on school math tasks at home. To collect these data, I provided each of the six students with an audio tape recorder and tapes. I asked them to record the conversations they had with their parents and write their work on the assignment sheets Ms. Smith handed out in class. I instructed the students to return the tapes the following morning.

The goal of these methods was to document an uninterrupted trail of ideas moving from school to home and back to school. The trail, however, was beset with gaps. Some of the gaps resulted from unrecorded conversations. In the beginning of the study, primarily, the conversations recorded in school documented only part of what the students discussed. Because of technological limitations (e.g., the number of tape recorders and microphones available), absent students, and other logistical hurdles, every conversation in which the students participated could not be recorded. This caused a problem as illuminating conversations between students and their parents could not always be followed up with their conversations in school.

Collecting students' written work also created a gap of sorts. Part of Ms. Smith's conception of teaching in the spirit of the recent reforms in mathematics education included having students revise their answers based on their small group conversations and then again during whole class discussions if changes were warranted. Although the students were instructed to write down all of their work and asked not to erase earlier work, they often did erase and write over previous answers. In the videotapes of small group interaction, students erased or in some way changed the answers they had written down earlier. Yet, on the assignment sheets collected at the end of each task, the students answers often were the same as those they constructed with their parents at home. This made it difficult, if not impossible to determine what changes were made in the small group interactions.

Other gaps were created in the conversations at home. In each conversation recorded at home there were times when the tape recorder clicked off and back on. There may be many reasons for the recorders being turned off and, at times, hints to the reasons were given on the tape. For example, Kathy's father told her to shut the recorder off while he went to check something as they worked on Task Three. Based on where the tape was stopped and their conversation both before and after, it seems reasonable to conclude that he went to look up something that helped him determine how to predict how many numbers were possible. Kathy's father, however, never alluded to where he went or what he did when the recorder was off. In another example, Kathy's father told her she "didn't need to record all this" as she wrote down her answer to a question on Task One. Although he may have been preserving tape, the gap may also have included some substantive conversation that was not recorded. In yet another example, Tony's mother turned off the recorder when she got "heated up" with Tony. When the recorder was turned back on, she was no longer heated up and they continued to work on the assignment.

Each of these examples points out that parts of the conversations students had in their homes were not recorded. The conversations students had in school also contained evidence that the parts of their in-home conversations were not recorded. While discussing Task Three, for example, Karen explained the strategy she and her father used to predict how many numbers were possible. Her father had told her, she recounted, that for four-digit numbers there would be four numbers that began with each digit. So, to determine how many numbers were possible she needed to multiply four times four arriving at a prediction of 16 numbers. Although this strategy fit the answer Karen had written down, it was not discussed in the conversation she recorded the night before. On the tape Karen spoke alone to the tape recorder. Her response to the question about predictions was only "you can make six numbers and you can make sixteen." These numbers--six and sixteen--were how many three- and four-digit numbers Karen and her father found while working on the two parts of Task Three.

In many cases an account of what the student had done at home could be constructed from listening to the conversations they had in school and at home. But, the missing conversations and written work meant providing an incomplete picture of how the students completed school-math tasks in their homes, and, as a result, an incomplete picture of the influence the different interactions had on students' completion of the tasks. Participating in a practice means knowing what resources are available and how to use them. Having mathematical resources like those that may have been available in Kathy's home and using them during the conversations is an important aspect of the practice of doing math in a student's home. Written documentation of students' thinking also helped to piece together the trail of ideas necessary in a study of collateral participation in various practices.

Other gaps were created when as the adults' participation in the recorded conversations decreased. Over the course of the study, the tapes grew progressively shorter and scantier. In some cases, the parents stopped participating in the recordings, leaving the student to narrate what was already written on their papers. On these tapes it is not clear whether the parent had worked with the student on the task or the student completed the task with no help. The shorter tapes might be explained, in part, by the different content in the tasks. The earlier tasks contained questions with which the adults may have been familiar. Task 3, however, contained questions that the parents may have been uncomfortable answering. Parents may be more willing to engage in conversation when they feel comfortable, or competent, with the content.

Although the reasons for the shorter tapes cannot be conclusively determined, they may result from asking parents to record conversations that do not normally occur in their households. The conversations recorded later in this study were often carried on from a distance. In Tony and his mother's conversation for Task Four, for example, Tony's mother talked with him from a different room. Her comments are difficult to hear on the tape and Tony often asked her to repeat what she said. Throughout the conversation, Tony's mother was doing other things important to their family's life. The later tapes in which parents talked from a distance and engaged in other household activities may be a better representation of authentic conversations students and their family members have about their school work. Asking the parents to sit down and record their conversations for this study may have created an artificial conversation--a conversation that took parents away from other important household obligations.

Of the things that created gaps in the data, only a few are controlled by the researcher. The things beyond the control of the researcher include parents' and students' decisions to turn the tape recorder off and on while recording the conversations, the incomplete record of written work, and the parents decreased participation in the conversations. Although many of these difficulties are unique to this study, parents' decreasing participation in the conversations is consistent with Varenne et al.'s (1982) difficulties while videotaping homework sessions.

The controllable aspects of the study include the missed conversations in the classroom and the content of the different tasks. The methods would be stronger if the people who recorded their conversations at home worked together in a math group at school and all of their conversations in each setting, as well as all their contributions to large group discussions were recorded.

In sum, the data collection methods I used in this study illuminated the influences different conversations have on what and how people learn. In this way, they contributed to the development of a method to investigate the mutual influences of various practices on learning. The methods did not, however, provide a complete data trail. Rather, gaps existed in the data that might mask some aspects of the various practices in which the students participated. Although some of the gaps could be eliminated by careful selection of students and placement in their math groups, most of the gaps are created by difficulties inherent in this type of research. Even with these difficulties, however, the methods produced a clear vision of the sociocultural influences on students' completion of a series of elementary-school mathematics tasks with implications for practice and further study.

Implications for Practice

This study provides a detailed look at the influence of students' interactions at home and in school on their mathematical thinking. Although many of the findings support contentions of other theorists and researchers, never before have they been brought together to describe the phenomenon of learning across conversations in this way. As a result, the findings of this study have implications for both learning theory and elementary-school mathematics. I discussed the theoretical implications in the previous sections. In this final sections I explore the practical implications in light of the calls for reform in education and mathematics education.

Recent reports (National Commission on Excellence in Education, 1983; National Education Goals Panel, 1992) have suggested that America's schools are failing--test scores are dropping, violence is increasing, and students are not being prepared for their futures. In response to these reports a bipartisan coalition of educators (National Education Goals Panel, 1994) has formulated a set of eight goals meant to address and reverse the negative trends in our nation's schools. One of the eight goals is to increase parents' participation in their children's academic development through home-school partnerships.

Successful home-school partnerships require more than parents merely asking their children if they have finished their homework. Indeed, the recommendations suggest parents be actively involved in their children's schoolwork. This is meant to accomplish two things. First, parents, it is assumed, can provide additional assistance while their children complete their schoolwork. Second, when both teachers and parents have worked with the student on the same tasks, they will have a shared experience about which they can talk. Although these suggestions, if followed, may assist parents and teachers in their work with their children and students, they rely on an overly simplistic view of students' homes and the home-school relationship.

The research presented here provides an initial look at home-school relationships that might inform the development and maintenance of home-school partnerships. Parents in this study were deeply concerned about their children's academic success and wanted to do whatever they could to help them with their schoolwork. Running a household, however, is a tremendously complex task that requires parents to juggle many different-sized spheres of ongoing activity. Merely asking parents to assume more responsibility for their children's schoolwork may be unsuccessful. Likewise, the failure of parents to fully participate cannot be taken as a rejection of the program.

In the study reported here, parents' participation in their children's schoolwork varied depending on their perceived competence with the material on which the students were working and their own schedules. When it fit into their schedules, the parents worked closely with their children on their schoolwork. When they felt qualified, they efficiently directed their children to answers they believed were correct, but when they were unsure of what was being asked, they shied away from the questions, often telling their children to work alone or, if the student was unsure, to ask the teacher the next day.

The parents' reactions to the content of assignments gains importance in the wake of the calls for reform in mathematics education. As was the case with the "new math" of the 1950s, mathematics assignments in reform classrooms are likely to be inconsistent with parents' mathematics experiences. Consequently, changing mathematics instruction may enlarge the discontinuities that already exist between the home and school and may make it more difficult to form home-school partnerships.

The study reported here is an initial attempt at understanding the home-school relationship and the influence the two institutions have on students' mathematics learning. Using the findings of this study and, perhaps, those of subsequent studies, it may be possible to develop methods for developing and maintaining productive home-school partnerships.

In light of the interest in home-school partnerships, there are two logical extensions of this research. First, I may continue to study the relationship between homes and schools to better understand the influence each has on students' schoolwork. To conduct this research, I may duplicate the study I have reported here taking into account the shortcomings mentioned above to ensure a more complete data trail. An added component of the study would include an instructional component for parents participating in the study. The instructional component would most likely take the form of a newsletter explaining the mathematical ideas being discussed in class. The success or failure of the home-school partnerships could contribute to the development of methods or prescriptions for practicing teachers. A second logical extension would be to teach an elementary school mathematics class myself and attempt to develop productive home-school partnerships with my students' families. This research could contribute methods pitfalls of developing home-school relationships.

Summary and Conclusions

The study reported here, more than anything else, supports recent advancements in learning theory. It provides evidence in support of the notion that learning is not an individual process, but, rather, a fundamentally social process in which primacy can given be given to neither the individual or the social structures within which they function. The study provides empirical support for the notion that people do many things concurrently and each of those things influences how they think about mathematics and, perhaps, school in general. It points out that learning is a gradual process during which learners make choices about what they are to learn or believe within a constrained set of choices offered by the socially-defined practices within which they participate. Finally, it points out that conflicts can arise when various practices containing conflicting rules are brought together in conversation.

These theoretical tenets can be used to describe elementary school students as they think about and discuss mathematics at home and in school. Elementary mathematics students, use and learn mathematics in both settings. When conflicting conceptions of mathematics, homework, or school collide in their conversations, the conflict must be resolved. The resolution, however, entails many risks. Students may be faced with choices about whose rules to follow. Following the wrong rules can result in answers that are inconsistent with those expected when the students return to school.

The recent calls for reform in mathematics education have increased the chances of conflict in students' conversations. The calls for reform have drastically changed what it means to know and do mathematics in elementary-school classrooms. These changes are likely to conflict with parents' and students' previous mathematics experiences. Students exposed to different conceptions of mathematics at home and in school may be slow to change the way they think about mathematics. As a result, teachers will need to probe students' answers to understand the assumptions they used when answering the questions and to remain open to alternative interpretations of mathematical situations and answers. And, successful implementation of the reforms might need to include the development of home-school partnerships in which parents are instructed in the new conceptions of elementary-school mathematics as well as changes in elementary-school mathematics classrooms.

At the same time that the implementation of the reforms can be aided by the findings of this study, the reforms continue to provide an opportunity to explore the relationships among various practices and their influence on learning. The conversations students participate in at home and in school reflect changing practices in which students learn mathematics. Understanding how they change and their contribution to students' mathematics learning can contribute to the growing body of knowledge about human learning. These two opportunities for further study suggest that mathematics educators and researchers interested in human learning have reasons to continue working together.

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