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FIRM SIZE, EXIT COSTS, AND THE CAPITAL BUDGETING DECISION

By

Domingo Castelo Joaquin

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ABSTRACT

FIRM SIZE, EXIT COSTS, AND THE CAPITAL BUDGETING DECISION

By

Domingo Castelo Joaquin

One of the important factors to consider in capital budgeting analysis is the option to abandon a project because its presence can substantially limit the project's downside risk. In this dissertation, we examine the capital budgeting implications of the hypothesis that it is costly to abandon a project and that this exit cost is increasing in firm size. Our finding is that a firm would optimally absorb more negative cash flows if exit is costly than if it is not. We refer to this phenomenon as cannibalization since it involves cross-subsidization among projects that would not occur if exit was costless. If exit cost is increasing in firm size, then larger firms would optimally absorb more negative cash flows before exiting. Because of this, a larger firm would be more cautious in its entry decision than the smaller firm.

It is shown that because negative cash flows of some projects can cannibalize the positive cash flows of other projects, the value and riskiness of an identical project will be different for firms with different assets in place. Thus, the value of a project as stand alone may be irrelevant for a firm's investment decisions. The cannibalization phenomenon that follows from the exit cost hypothesis may help explain the recently documented observations that there is value loss from diversification, value gain from splitting up an existing firm, and that value loss is higher in the case of diversification

into unrelated industries. Moreover, it helps to explain why Tobin's q of a merged firm may be lower than the weighted average of the q's of its individual units as independent firms.

The dissertation is divided into three chapters. Chapter 1 examines the capital budgeting implications of the exit cost hypothesis in a single period setting. Chapter 2 extends the analysis to a simple dynamic setting. The final chapter studies the implications of the exit cost hypothesis on investment timing decisions when there is threat of potential competition.

To my wife Maria Victoria

and my daughter Anna Michaela

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1.1 Introduction

A number of recent papers report that diversification *per se* is a value reducing activity and more so when diversification is into an unrelated industry. See, for instance, John and Ofek (1995), Berger and Ofek (1995), and Comment and Jarrell (1995). A possible explanation proposed by these authors is that diversification leads to a decrease in managerial focus. The presumption is that there are diseconomies of managerial scope and the greater the managerial responsibility the less efficient the managers become. Thus, two firms, especially in unrelated businesses, are worth more as stand alone entities rather than as a merged unit.

Further support for diversification being bad comes from studies in related fields. Wernerfelt and Montgomery (1988) and Lang and Stulz (1994) report that Tobin's q for combined firms is lower than the weighted average of the q's of stand alone firms. Lang and Stulz also do not find evidence of synergy gains from acquisitions. Conversely, John and Ofek (1995) document that both sellers's stock price reaction and post sale operating profits are higher with focus-increasing divestitures. Bhagat, Shleifer and Vishny (1990) and Kaplan and Weisbach (1990) report that most divested units are previous acquisitions and the new purchasers are usually in the unit's line of business. Finally, Meyer, Milgrom and Roberts (1992) suggest negative influence costs as a possible rationale for why firms are more likely to dispose of poorly performing assets, and why someone else would buy these units for more than they are worth to the selling firm.

While decrease in focus is a very plausible explanation, it leaves much unexplained.

Why don't, for instance, acquired projects/firms come with their own set of managers? If they do, the issue is more of compatibility and coexistence than spreading managerial talent too thin. What about large firms? If there are dis-economies of managerial scope, these should be less efficient, a conclusion not supported by the apparent success of large firms all over the world.[1] More perplexing is that even though these studies report substantial value loss from diversification, it is not evident in the stock price reaction to acquisitions. Numbers reported in Jensen and Ruback (1983) from numerous studies show that the combined gains to target and acquiring shareholders are positive. More recently, Michael Jensen (in Ross, Westerfield and Jaffe, 1996) claims that for the period 1977-1988, selling shareholders of acquired firms gained over $500 billion those of the buying-firm shareholders gained $50 billion. There is also substantial literature that compares operating performances of diversified and non-diversified firms without finding any significant difference (See, for example, Williamson (1981) and Ravenscraft and Scherer (1987)).

In this chapter, we exploit Scott's (1977) and Galai and Masulis's (1976) insight about the link between cannibalization and limited liability to explain most of the above observations.[2] We show that there can be value loss even when focus is preserved. This occurs because in the presence of the limited liability constraint there is an important economic difference between business operating cash flows of a project and its equity cash flows. While limited liability protects the value of a stand alone project from negative cash

[1] There are, of course, issues of synergies, of economies of scale, and of size and market power. However, the methodologies of these studies control for these. The value loss is apparently net of synergies.

[2] Scott (1977) establishes the link between cannibalization and the weakenning of the limited liability property of equity in a single period state preference framework. In this chapter, we employ the single period capital asset pricing model. Galai and Masulis (1976) demonstrate the link using the Black-Scholes-Merton option pricing framework. In the next chapter, we employ the real options framework to analyze the problem in a dynamic setting.

flow, when two firms or projects are combined, the negative cash flows of each project can cannibalize the contemporaneous cash flows (if positive) of the other project before limited liability kicks in for the merged or diversified firm. In other words, the limited liability constraint is weakened for the merged or diversified firm.[3] This naturally reduces the value of the combined entity to less than the sum of the values of the two firms/projects as stand alone entities. Thus, one will observe a value loss from diversification even when there is no loss in focus or in the operating efficiency of the individual units as a combined entity.

Cannibalization risk has a number of important implications for diversification decisions in particular and capital budgeting decisions in general. For instance, even if synergy exists between a new project and existing projects of the firm, if the amount of synergy is not substantial enough the stand alone value of a project will be greater than or equal to the incremental value it generates when combined with existing projects of a firm.[4] In this case, the shareholders are *better off* diversifying on their own rather than have the firm diversify for them. The reason is that when shareholders diversify it is the equity cash flows that are combined and, thus, there is no loss from cannibalization since the limited liability constraint is not weakened. On the other hand, diversifying firms will have to absorb negative *operating cash flows* from some projects when they the contemporaneous *operating cash flows* from other projects are positive. Another important implication of cannibalization risk is that the value of a project will vary for different firms depending

[3] One needs to worry about not only the negative cash flows of the new investment and how it affects the value of assets in place, but also how the negative cash flows of existing assets affect the value of a new investment.

[4] An important implication of this is that the sum of excess returns to targets and bidders will underestimate synergy gains from a merger to the extent of loss in value through potential cannibalization.

on the firm's existing operating cash flows. Thus, for existing firms, it is inappropriate to equate the value added by a project to a firm to the stand alone NPV of the project. In other words, the commonly employed value additivity principle fails under most circumstances. Similarly, it is inappropriate to equate the new project's asset beta to the equity beta of a comparable pure play all-equity firm , since *the* project asset beta depends not only on nondiversifiable risk of its stand alone cash flow but also on the interaction between the project's operating cash flows and that of the existing assets of the firm. Thus, one cannot equate the beta of a diversified firm to the beta of a portfolio of pure play firms which, together, mimic the activities of the diversified firm.[5]

Cannibalization risk also has implications for Tobin's q. Just as for betas, Tobin's q of a diversified firm cannot be equated with Tobin's q of the comparable portfolio of pure play firms. In fact, with little or no synergies, the q of the diversified firm will be bounded above by the weighted sum of the qs of the pure play firms of the comparable portfolio. This may, at least in part, explain the results in Lang and Stulz and Wernerfeldt and Montgomery.

Since cannibalization occurs when the negative cash flow of one project occurs at the same time as a positive cash flow from another project or from assets in place, we expect the amount of cannibalization to be inversely related to the correlation between the two cash flows. Since cash flows of firms in related industries are likely to display higher correlations, we expect lower cannibalization risk for such acquisitions. This may explain why the reported diversification penalty is lower for more focused diversification. On the flip side, it also may explain why divested units are more likely to be sold to firms in

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[5] Hereafter, we refer to this portfolio of pure play firms as a comparable portfolio of pure play firms.

related industries as they will be able to, on average, offer higher prices. Thus, our approach provides a natural explanation for what Meyer, Milgrom and Roberts refer to as a puzzle: that some buyers may value the divested unit higher than the seller.

This chapter is divided into four sections. In Section 1.2 , we employ the single period capital asset pricing model to study the cannibalization phenomenon. We argue that there is no such thing as *the* project risk. We also identify a sufficient condition for cannibalization to result in value reduction and show that the loss in value from cannibalization is equal to the erosion in the value of limited liability. We also show that if the *operating cash flows* are additive, then Tobin's q of the more diversified firm is less than the weighted average of the Tobin's q of separate firms, each specialized in a division of the diversified firm. Section 1.3 provides a numerical example. Section 1.4 contains a brief summary.

1.2 Valuation and the cannibalization phenomenon

Consider a single period model satisfying the usual assumptions of the capital asset pricing model of Sharpe (1964), Lintner (1965), and Mossin (1966) where agents have quadratic utility functions. In period 0, firms hire production inputs at a fixed cost, payable in the next period. In period 1, they sell the resulting output at a random non-negative price, pay input suppliers, distribute the residual to the shareholders, and dissolve. We assume that all firms are all-equity firms.[6]

Consider two production contracts. One calling for the production of good A, the other

[6] It is true that limited liability has value only because under certain circumstances the stockholders can walk away from contractual arrangements. However, leverage is only one way limited liability gets value. All other fixed contracts with suppliers, buyers, employees, and penalties imposed by courts or other governmental agencies, also give value to limited liability. When we refer to leverage, we use it to mean debt in the traditional way. By an all-equity firm we mean an unlevered firm with only one form of non-debt capital: common equity.

for the production of good B.[7] Let P_j represent the random sales proceeds and C_j represent the nonstochastic contractually fixed cost of producing good j. Then, the random period 1 *operating cash flow* from a contract to produce good j is

$$X_j = P_j - C_j. \tag{1.1a}$$

Since the shareholders are protected by limited liability, the corresponding random *equity cash flow* for a firm that produces only good j is

$$X_j^+ = \max\{P_j - C_j, 0\}. \tag{1.1b}$$

It follows from the capital asset pricing model that the equilibrium value of the firm is given by

$$V_j = \frac{E[X_j^+] - \lambda * cov(X_j^+, r_m)}{1+r} \tag{1.2a}$$

where r_m is the random return on the market portfolio, r is the risk-free return, and λ is the market price of risk, and is equal to:

$$\lambda = \frac{E[r_m] - r}{\sigma_m^2} \tag{1.2b}$$

where σ_m^2 is the variance of the return on the market portfolio.[8]

Suppose now that firm A which is currently engaged in the production of good A is considering adding good B to its product line. Then, the operating cash flow X_{ab} of the diversified firm can be decomposed as follows:

$$X_{ab} \equiv (X_a + X_b) + \{X_{ab} - (X_a + X_b)\} \tag{1.3a}$$

[7] Hereafter, we refer to a firm specializing in the production of good A and good B as firm A and firm B, respectively. We refer to a firm that produces both goods as firm AB. Also, the subscripts a, b, ab will be used to identify firms A, B, and AB, respectively.

[8] See, for example, Hamada (1969) and Rubinstein (1973).

$(X_a + X_b)$ gives us the *operating cash flow* of the diversified firm if there are no synergies resulting from the addition of the new line and managerial focus is just preserved. The impact of technical and managerial synergies on the *operating cash flow* is given by $\{X_{ab} - (X_a + X_b)\}$. Since the basic point we wish to make is that value can be lost through the mere act of diversification, we assume that there are no changes in operating efficiency and managerial focus in the diversified firm. Thus,

$$X_{ab} - (X_a + X_b) = 0 \tag{1.3b}$$

In this case, the *operating cash flow* of the diversified firm is simply $X_a + X_b$. In other words, the *operating cash flows* are additive. However, the additivity property does not extend to *equity cash flows* because of the limited liability constraint. In other words, even though $X_{ab} = X_a + X_b$, this does *not* imply that the equity cash flows X_{ab}^+ of the diversified firm is equal to the sum $X_a^+ + X_b^+$ of the equity cash flows of the corresponding combination of pure play firms. In fact,

$$X_{ab}^+ = (X_a + X_b)^+ \leq X_a^+ + X_b^+.$$

And so,

$$X_{ab}^+ - X_a^+ \leq X_b^+. \tag{1.4}$$

In other words, given that the *operating cash flows* are additive, the incremental *equity cash flow* to firm A from adding good B to its product line can differ from and is bounded above by the *equity cash flow* of a firm specializing in the production of good B.

Since the market value of an all-equity firm is just the value of its equity, the value of a new project to a firm is equal to the incremental value in equity that is generated by the adoption of the project.[9] If V_{ab} represents the value of the diversified firm (or the firm with the new project in place), then the value of adding project B to firm A's existing projects is

$$V_b^a = V_{ab} - V_a \tag{1.5a}$$

or, given the additivity of *operating cash flows,*

$$V_b^a = \frac{E\left[(X_a + X_b)^+ - X_a^+\right] - \lambda * cov\left((X_a + X_b)^+ - X_a^+, r_m\right)}{1+r}. \tag{1.5b}$$

We note from Eq.(1.4) that the incremental *equity cash flow* from the project is less than or equal to the *equity cash flow* of a firm specializing in the project. If strict inequality holds with positive probability, that is, if there is risk of cannibalization, then we expect the strict inequality to hold for values as well, i.e., $V_b^a < V_b$. However, nothing in the certainty equivalence form of the capital asset pricing model assures us that this will be the case. The reason is that CAPM prices only systematic risk so it is at least theoretically possible that $V_b^a > V_b$. The first proposition gives a sufficient condition for cannibalization risk to result in value reduction. Pr stands for probability.

Proposition 1.1 *Suppose that* $\Pr\left(-\frac{1}{\lambda} \le r_m \le \frac{1}{\lambda}\right) = 1$ *where* $\lambda = \frac{E[r_m]-r}{\sigma_m^2} > 0$ *and* $E[r_m] > 0$. *If* $P\left(X_a^+ + X_b^+ - (X_a + X_b)^+ > 0\right) > 0$, *then* $V_b^a < V_b$. *In other words, a positive probability of cannibalization implies value loss.*

Proof. Since $X_a^+ + X_b^+ - (X_a + X_b)^+ \geq 0$ and, by hypothesis, $\Pr(-\frac{1}{\lambda} \le r_m \le \frac{1}{\lambda}) = 1$, we get $X_a^+ + X_b^+ - (X_a + X_b)^+ r_m \le \left(X_a^+ + X_b^+ - (X_a + X_b)^+\right) \frac{1}{\lambda}$ with probability 1. It follows that

$$E\left[X_a^+ + X_b^+ - (X_a + X_b)^+\right] r_m \le E\left[X_a^+ + X_b^+ - (X_a + X_b)^+\right] \frac{1}{\lambda}.$$

[9] Stapleton (1971) also has this starting point. But, by failing to recognize the cannibalization phenomenon, he assumes value additivity and equates the value of a new project to its stand alone value.

Now, by hypothesis, we have $E[r_m] > 0$ and $P\left(X_a^+ + X_b^+ - (X_a + X_b)^+ > 0\right) > 0$. Hence $E\left[X_a^+ + X_b^+ - (X_a + X_b)^+\right] E[r_m] > 0$ and we get

$$E\left[\left(X_a^+ + X_b^+ - (X_a + X_b)^+\right) r_m\right]$$
$$< E\left[X_a^+ + X_b^+ - (X_a + X_b)^+\right] \tfrac{1}{\lambda} + E\left[X_a^+ + X_b^+ - (X_a + X_b)^+\right] E[r_m].$$

It follows that

$$\lambda * cov\left(X_a^+ + X_b^+ - (X_a + X_b)^+, r_m\right) < E\left[X_a^+ + X_b^+ - (X_a + X_b)^+\right]$$

or

$$E\left[(X_a + X_b)^+ - X_a^+\right] - \lambda * cov\left((X_a + X_b)^+ - X_a^+, r_m\right) < E\left[X_b^+\right] - \lambda * cov\left(X_b^+, r_m\right).$$

Dividing both sides by $1 + r$ yields the desired result. □

If there is positive probability of cannibalization of cash flows, then Proposition 1 tells us that this will translate into loss in value only if the return on the market portfolio is essentially bounded by the inverse of the market risk premium, i.e., $\Pr\left(|r_m| \leq \frac{1}{\lambda}\right) = 1$. To check the practical significance of this condition, we compare it to historical data. Ibbotson and Sinquefild (1994) find that, between 1926 and 1993, the average premium on the market is 8.6% and the standard deviation of the market return was 20.5%. This gives a λ of about 2 or $\frac{1}{\lambda}$ of about 50%. If one looks at the historical record, the incidence of the market return exceeding 50% or falling below -50% is quite rare. And so, it is reasonable to presume that, in the absence of synergies, the value of a new project to a firm cannot exceed the project's stand alone value. Hereafter, we assume that the required bounds on the market return holds and thus cannibalization risk implies value loss.

An important implication of Proposition 1 is that shareholders are better off diversifying for themselves than have the firm engage in a different lines of business. This runs counter to common conception in finance which holds that, in the absence of synergy and with no effect on efficiency, shareholders should be indifferent between the two ways of diversification. The reason is that when shareholders diversify there is no loss from cannibalization since, because of limited liability, *equity cash flows* are non-negative. On the other hand, diversifying firms will have to absorb negative *operating cash flows* from some projects when they are accompanied by positive *operating cash flows* from other projects. Thus, the value of the diversified firm can be less than the sum of the stand alone values of its separate projects so that the analogy to mutual funds does not necessarily hold. The difference between the two $(V_a + V_b) - V_{ab} = V_b - V_b^a$ represents the value loss from cannibalization due to diversification. The next proposition shows that the value loss from cannibalization is exactly equal to the erosion in the value of limited liability, which we measure by $V - U$, where V denotes the value of equity with limited liability and U denotes the value of ownership capital without the protection of limited liability. Thus, V is the certainty equivalent of the firm's *equity cash flow*, while U is the certainty equivalent of the firm's *operating cash flow*.[10]

Proposition 1.2 *If* $X_{ab} = X_a + X_b$, *then* $(V_a + V_b) - V_{ab} = ((V_a + V_b) - (U_a + U_b)) - (V_{ab} - U_{ab})$. *In other words, if the operating cash flows are additive, then the value loss from cannibalization is equal to the difference between the value of limited liability for combined firm AB and the sum of the stand alone values of limited liability for firm A and firm B.*

Proof. The additivity of *operating cash flows* implies that $U_{ab} = U_a + U_b$.□

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[10] If Z represents the firm's *operating cash flow*, then U is obtained by substituting Z for X^+ in Eq.(1.2). The value of the firm's equity V is obtained by substituting the firm's *equity cash flow* $Z^+ = \max\{Z, 0\}$ for X^+ in Eq.(1.2).

A fact from Eq.(1.4) which the *mutual fund analogy* misses is that the incremental equity cash flows from adding good B depend on the *operating cash flow* X_a from the existing assets of firm A. Hence, the value of a project can differ for different firms. When viewed from the perspective of risk, an immediate corollary is that there is no such thing as *the* project risk. The riskiness of a project depends on the incremental cash flows and, consequently, can vary from firm to firm. In particular, the asset beta of pure play firms can be used legitimately as proxy for the asset beta of the project *only when* the project is implemented on a stand alone basis. It is inappropriate to use if the new project being added to existing projects of the firm.[11] We formalize this claim as

Proposition 1.3 *Let β_j be the beta of firm j, $j = a, b$, and β_b^a be the beta of the incremental equity cash flow generated by adding good B to firm A's products. Let I_b be the investment required to set up firm B, and I_b^a be the investment required to add good B to firm A's products. Suppose that $I_b^a = I_b$. If there is risk of cannibalization, then $\beta_b^a \neq \beta_b$.*

Proof. $\beta_b^a = \frac{cov\left(X_{ab}^+ - X_a^+, r_m\right)}{I_b^a \sigma_m^2}$ and $\beta_b = \frac{cov\left(X_b^+, r_m\right)}{I_b \sigma_m^2}$. Since, by hypothesis, $I_b^a = I_b$, it follows that $\frac{cov\left(X_b^+, r_m\right)}{I_b \sigma_m^2} = \frac{cov\left(X_b^+, r_m\right)}{I_b^a \sigma_m^2}$. The desired inequality follows from the fact that if there is risk of cannibalization, then Pr $\left(X_{ab}^+ - X_a^+ < X_b^+\right) > 0.\square$

The next proposition gives a decomposition of firm betas which takes into consideration the possible impact of cannibalization.

Proposition 1.4 *Let β_j be the beta of firm j, $j = a, b, ab$ and β_b^a be the beta of the incremental equity cash flow generated by adding good B to firm A's products. Let I_j be the investment required to set up firm j, and I_b^a be the investment required to add good B to*

[11] The beta of asset j is from a one factor model, i.e., $\beta_j = \frac{cov(r_j, r_m)}{\sigma_m^2}$, where r_j is the (random) return on asset j. The conclusions of this paper extend to multifactor models.

firm A's products. Then $\beta_{ab} = \frac{I_a}{I_a+I_b^a}\beta_a + \frac{I_b^a}{I_a+I_b^a}\beta_b^a$.

Proof. $\beta_{ab} = \frac{cov\left(X_{ab}^+, r_m\right)}{I_{ab}\sigma_m^2}$

$= \frac{cov\left(X_a^+, r_m\right)}{I_{ab}\sigma_m^2} + \frac{cov\left(\left(X_{ab}^+ - X_a^+\right), r_m\right)}{I_{ab}\sigma_m^2}$

$= \frac{I_a}{I_{ab}}\frac{cov\left(X_a^+, r_m\right)}{I_a\sigma_m^2} + \frac{I_b^a}{I_{ab}}\frac{cov\left(\left(X_{ab}^+ - X_a^+\right), r_m\right)}{I_b^a\sigma_m^2}$

$= \frac{I_a}{I_a+I_b^a}\beta_a + \frac{I_b^a}{I_a+I_b^a}\beta_b^a$. □

Proposition 4 is at variance with the common practice of identifying the beta of a diversified firm with the value-weighted sum of the divisional betas, or, equivalently, with the portfolio beta of a comparable mutual fund of pure play firms. Under this scheme, $\beta_{ab} = \frac{I_a}{I_a+I_b^a}\beta_a + \frac{I_b}{I_a+I_b}\beta_b$ or $\beta_{ab} = \frac{V_a}{V_a+V_b}\beta_a + \frac{V_b}{V_a+V_b}\beta_b$ if $I_j = V_j$. There are two problems with this procedure. First, the stand-alone value-weights fail to reflect the fact that diversified firms (but not diversified mutual funds) are exposed to cannibalization risk. This is the point of Proposition 1. Second, the stand-alone betas also fail to recognize cannibalization risk. This is the point of Proposition 3.

Another important implication of cannibalization has to do with the ratio of an asset's market value to its replacement cost, also known as the asset's Tobin's q.[12] As Lang and Stulz (1994) report, the q of a diversified firm is less than that of a comparable portfolio of pure play firms. The next proposition shows that this result is a natural consequence of cannibalization.

Proposition 1.5 *Let* $I_a, I_b, I_{ab} > 0$ *be the cost of replacing firm A, firm B, and firm AB's assets, respectively. Suppose that* $X_{ab} = X_a + X_b$ *and* $I_{ab} = I_a + I_b$*. It follows that if* $P\left(X_a^+ + X_b^+ - X_{ab}^+ > 0\right) > 0$*, then* $q_{ab} < \frac{I_a}{I_a+I_b}q_a + \frac{I_b}{I_a+I_b}q_b$*. In other words, if there is risk*

[12] Tobin's $q = \frac{V}{I}$, where V is the market value of an asset and I is its replacement cost.

of cannibalization and the cost of replacing the firm's assets are additive, then Tobin's q of the combined firm AB is less than the weighted sum of the Tobin's q of firm A and firm B, with the weights proportional to the cost of replacing the respective firm's assets.

Proof. Since there is risk of cannibalization, it follows from Proposition 1 that $V_{ab} < V_a + V_b$. Divide both sides by $I_a + I_b$, use the assumption that $I_{ab} = I_a + I_b$, then reexpress in terms of the definition of Tobin's q to obtain the desired result.□

1.3 Numerical example

Insert Table 1.1 here.

A numerical example would be useful to illustrate the various propositions. Consider the joint distribution of cash flows given in Table I. Cannibalization occurs in the first nine states. These are the states in which there is a cannibal, the project with negative *operating cash flow*, and there is some one to cannibalize, the project with the positive *operating cash flow*. Cannibalization does not occur in the last five states. When both cash flows are positive, there is no cannibal; and when both are negative, again there is no one to cannibalize. Consistent with the above analysis, *operating cash flows* are assumed to be additive. So, $X_{ab} = X_a + X_b$. The *equity cash flow* X_j^+ is obtained by taking the maximum of X_j and 0. Given the additivity of *operating cash flows*, $X_{ab}^+ \leq X_a^+ + X_b^+$ or $X_{ab}^+ - X_a^+ \leq X_b^+$. Now, $X_{ab}^+ - X_a^+ < X_b^+$ in precisely those states in which cannibalization occurs. The expected value of the operating and equity cash flows are obtained using Eq.(1.2). Since the probability of cannibalization is positive, we obtain a loss in value from cannibalization of $V_a + V_b - V_{ab} = 7.93$. This loss is exactly equal to the shortfall between the project's value $V_b = 14.43$ as a pure play firm and the value $V_b^a = V_{ab} - V_a = 6.50$ of adding the project to firm A's existing projects. The value of limited liability $V_j - U_j$ for firms A, B, and AB are 8.32, 6.05, and 6.44, respectively. The erosion in the value of limited liability from diversification is equal to $8.32 + 6.05 - 6.44 = 7.93$ which is exactly the value loss from cannibalization. This means that it would be desirable to split off the diversified firm into firm A and firm B if the split off cost is less than 7.93. $\beta_b^a = \frac{cov(X_{ab}^+ - X_a^+, r_m)}{\sigma_m^2 I_b^a} = \frac{0.73}{(.04)(10)} = 1.825$ is about double that of $\beta_b = \frac{cov(X_b^+, r_m)}{\sigma_m^2 I_b} = \frac{0.36}{(.04)(10)}$ =0.90. This means that, given that $I_b^a = I_b$, the appropriate beta for valuation purposes is about double that of the counterpart pure play firm. Finally, the value weighted combination of β_a and β_b is 2. 8339. This underestimates the beta of the expanded firm β_{ab} which is equal to 3.47.

1.4 Conclusion

Limited liability is valuable because it provides equity holders the option to exit when faced with negative cash flows. However, when two firms combine it is less likely that the aggregate cash flows will be negative since the negative cash flows of one firm may be offset by the contemporaneous (if positive) cash flows of the other firm. This reduces the value of the exit option. This loss translates into a gain to stakeholders with fixed contractual arrangements with the individual firms. To the extent that equity holders are unable to recover these gains (transfers), they are worse off by merging. This may at least partly explain recently documented observations that there is value loss from diversification, value gain from splitting up an existing firm, and that the value loss is higher in the case of diversification into unrelated industries. It may also explain why the Tobin's q of a merged firm may be lower than the weighted average of the *q*'s of its individual units as independent firms. In addition, because negative cash flows of some projects can cannibalize the positive cash flows of other projects, the value and riskiness of an identical project will be different for firms with different assets in place. Thus, the value of a project as stand alone may be irrelevant for a firm's investment decisions.

2. Cannibalization Risk in a Simple Dynamic Setting

2.1 Introduction

In this chapter, we analyze the cannibalization phenomenon in a dynamic setting using the real options framework of Brennan and Schwartz (1985), McDonald and Siegel (1985), Pindyck (1988), Dixit (1989) and Abel and Eberly (1995). We show that all the basic conclusions remain. In addition, we show that there are also some real effects. For instance, the optimal entry and exit timing decisions depend on a firm's assets in place. Thus, the same new project can have different value for different firms not only because of the possibility of cannibalization but also because this possibility affects optimal entry and exit decisions. Thus, the cash flow stream of the new projects will be different for different firms. In addition, we are able to show that diversification loss is higher when a firm diversifies into unrelated business against when it diversifies into related business. However, we also show that cannibalization can still occur even when there is perfect correlation between the operating cash flows of the new project and that of the existing assets.

2.2 Valuation of a pureplay firm

Consider an all-equity firm, say, firm j with a stochastic *operating cash flow* generated from a contract which calls for the production of Q units of good j per year. The output can be produced at a constant unit cost C_j and can be sold at a unit price of $P_{jt} = P^{e_j}$ at time t, where P_t, the price of a related traded commodity, follows the geometric Brownian motion:

$$dP_t = \alpha P_t dt + \sigma P_t dz_t. \tag{2.1}$$

α is a constant growth parameter, σ a constant proportional variance parameter, and z_t a standard Brownian motion, and $\varepsilon_j = 0$ or 1. When $\varepsilon_j = 0$, then $P_{jt} = 1$ and P_{jt} is, therefore, independent of P_t. When $\varepsilon_j = 1$, then $P_{jt} = P_t$ and P_{jt} is, therefore, perfectly correlated with P_t. We will use this property to study the special cases of diversifying into related and unrelated industries.

The firm j can be conceived of as a derivative asset whose value V_j depends on the price P of the related traded commodity. It follows from Ito's lemma that the value function satisfies

$$dV_j = \left(\alpha P\frac{\partial V_j}{\partial P} + \frac{1}{2}\sigma^2 P^2 \frac{\partial^2 V_j}{\partial P^2} + \frac{\partial V_j}{\partial t}\right) dt + \sigma P \frac{\partial V_j}{\partial P} dz. \tag{2.2}$$

And so, by a standard replication argument as in Black and Scholes (1973) or by the consumption CAPM as in Sick (1989) or by the risk neutral valuatlion technique of Cox and Ross (1976), we obtain the following differential equation that must be satisfied in equilibrium by the value function V_j:

$$\frac{1}{2}\sigma^2 P^2 \frac{\partial^2 V_j}{\partial P^2} + \frac{\partial V_j}{\partial t} + (r - \delta) P \frac{\partial V_j}{\partial P} - rV_j + Q\left(P^{\varepsilon_j} - C_j\right) = 0. \tag{2.3}$$

P denotes the current output price, r is the (assumed) constant instantaneous risk free rate, and δ is a constant denoting the net convenience yield from holding a dollar's worht of output. We assume that the firm can renege on its production contract at any time it deems optimal. In this case, the production project has no fixed termination date and so the value function depends on only current output price but not on the current date. It follows that $\frac{\partial V_j}{\partial t} = 0$ and the solution to Eq.(2.3) is given by:

$$V_j(P) = N_j P^{\theta_1} + D_j P^{\theta_2} + \frac{QP^{\varepsilon_j}}{\delta_j} - \frac{QC_j}{r} \tag{2.4a}$$

where N_j and D_j are constants to be determined from the relevant boundary conditions,

$$\delta_j = r - (r - \delta)\,\varepsilon_j - \frac{1}{2}\sigma^2 \varepsilon_j\,(\varepsilon_j - 1)\,, \tag{2.4b}$$

and θ_1 and θ_2 are, respectively, the positive and the negative roots of the quadratic form associated with the general solution to Eq.(2.3):

$$\begin{aligned} \theta_1 &= \tfrac{1}{2} - \tfrac{r-\delta}{\sigma^2} + \sqrt{\left(\tfrac{r-\delta}{\sigma^2} - \tfrac{1}{2}\right)^2 + \tfrac{2r}{\sigma^2}} \;>\; 1 \\ \theta_2 &= \tfrac{1}{2} - \tfrac{r-\delta}{\sigma^2} - \sqrt{\left(\tfrac{r-\delta}{\sigma^2} - \tfrac{1}{2}\right)^2 + \tfrac{2r}{\sigma^2}} \;<\; 0 \end{aligned} \tag{2.4c}$$

The last two terms represent of Eq.(2.4a) represent the value of the firm's *operating cash flows* if the firm has *no* option to exit. The first two terms may be interpreted as the value of the option to exit or renege on the production contract. If $\varepsilon_j = 0$, the firm essentially has a perpetuity whose market value is given by $\frac{Q(1-C_j)}{r}$.[13] In this case, it is never optimal to exit and the exit option is worthless and so we set $N_j = D_j = 0$. Now suppose $\varepsilon_j = 1$. As the output price P increases the less likely it becomes that this option will be exercised, making the exit option less valuable. As P approaches ∞, the expected value of the exit option drops to zero and so we set $N_j = 0$ (since $\theta_1 > 0$). Hence, the value function for firm j is given by:

$$V_j(P) = \begin{cases} \frac{Q(1-C_j)}{r} & if \quad \varepsilon_j = 0 \\ D_j P^{\theta_2} + \frac{QP}{\delta} - \frac{QC_j}{r} & if \quad \varepsilon_j = 1 \end{cases} \tag{2.5}$$

Since this production project generates a negative cash flow when the output price falls below unit cost, the firm might consider abandoning the project and paying the cost of

[13] If $\varepsilon_j = 0$ then, to be interesting, we assume that $1 \geq C_j$.

reneging on the production contract for low enough prices. We assume that the project has zero scrap value and that reentry is so prohibitively expensive that any exit is final. We also assume that there are two ways of getting out of the contract. The firm can pay an exit cost E and obtain a terminal payoff of $V_j - E$ or alternatively, it can elect to forfeit all future cash flows and obtain a terminal payoff of zero. Given the uncertainty about future prices, the firm, in effect, waits for the output price to fall to a certain level, say P_L^j, before it abandons the project.[14] This yields the following value matching and smooth pasting conditions:

$$V_j\left(P_L^j\right) = \max\left\{V_j\left(P_L^j\right) - E, 0\right\} \quad (2.6a)$$

$$V_j'\left(P_L^j\right) = \frac{\partial}{\partial P}\max\left\{V_j\left(P_L^j\right) - E, 0\right\} \quad (2.5b)$$

If we subtract $V_j\left(P_L^j\right)$ from both sides of Eq.(2.6a), we find that it is optimal for the firm to continue operations until it becomes worthless. It follows that at the optimal time of exit, the firm's value is equal to zero and Eq.(2.6a) Eq.(2.6b) reduce to

$$V_j\left(P_L^j\right) = 0 \quad (2.6c)$$

$$V_j'\left(P_L^j\right) = 0 \quad (2.6d)$$

If we substitute Eq.(2.5) into Eqs.(2.6c)-(2.6d), for $P \geq P_L^j$ we obtain:

[14] Even before the output price hits P_x, the project would already be generating negative cash flow. If necessary, additional funding will be provided by the shareholders. The shareholders will be willing to sustain this type of cannibalization (directly from their pockets) so long as it remains optimal to continue operations.

$$V_j(P) = \begin{cases} \frac{Q(1-C_j)}{r} & if \quad \varepsilon_j = 0 \\ D_j P^{\theta_2} + \frac{QP}{\delta} - \frac{QC_j}{r} & if \quad \varepsilon_j = 1 \end{cases} \tag{2.7a}$$

where

$$D_j = \begin{cases} 0 & if \quad \varepsilon_j = 0 \\ \frac{-QP_L^j}{\theta_2 \delta} & if \quad \varepsilon_j = 1 \end{cases} \tag{2.7b}$$

$$P_L^j = \begin{cases} \infty & if \quad \varepsilon_j = 0 \\ \frac{-\theta_2 \delta}{1-\theta_2} \frac{C_j}{r} & if \quad \varepsilon_j = 1 \end{cases}$$

We see from Eq.(2.7a) that the value of equity can be decomposed into the value U_j of ownership capital in the absence of the option to exit and the value of the exit option. That is, we can rewrite:

$$V_j(P) = U_j(P) + [V_j(P) - U_j(P)]$$

where

$$U_j(P) = \frac{QP^{\varepsilon_j}}{\delta_j} - \frac{QC_j}{r}.$$

Also, from Eq.(2.7c) we see that if $\varepsilon_j = 1$, then the optimal exit price P_L^j is increasing in unit production cost C_j.

Suppose that we have the option of starting firm j from scratch and on a stand alone basis at any time we deem optimal, by investing I_j. If the current output price is $P \geq P_x$, then the net present value of setting up firm B right now is simply

$$NPV_j(P) = V_j(P) - I_j.$$

The present value of the option to build firm j when output price reaches $P_e \geq P$ is given by

$$NPV_j(P_e;P) = E\left(V_j(P_e) - I_j\right)e^{-rT} \tag{2.8a}$$

where T is a random variable indicating the first time the output price hits P_e given that the initial price is P.[15]

$$T = T(P_e;P) = \inf\{t \geq 0 : P_t \geq P_e,\ P_0 = P\}. \tag{2.8b}$$

From Krylov (1980, Chapter 1) or Dixit and Pindyck (1994, pp. 315-316), we obtain $Ee^{-rT} = \left(\frac{P}{P_e}\right)^{\theta_1}$.Hence, the optimal entry price $\bar{P}_j$ at which to start firm j, conditional on the current output price being P, is given by

$$\bar{P}_j = \underset{P_e \geq P}{\arg\max}\left(D_j P^{\theta_2} + \frac{QP^{\epsilon_j}}{\delta_j} - \frac{QC_j}{r} - I_j\right)\left(\frac{P}{P_e}\right)^{\theta_1}. \tag{2.8c}$$

2.3 Valuation of a diversified firm

Next we derive the value of the diversified firm. Assuming the additivity of *operating cash flows*, we obtain the following value function for the combined firm AB by employing the same procedure as above:

$$V_{ab}(P) = DP^{\theta_2} + \left(\frac{QP^{\epsilon_a}}{\delta_a} - \frac{QC_a}{r}\right) + \left(\frac{QP^{\epsilon_b}}{\delta_b} - \frac{QC_b}{r}\right) \tag{2.9a}$$

where δ_j and θ_2 are as defined in Eq.(2.4) and D is a constant to be determined from the relevant boundary conditions.

Since production generates negative cash flows when the output price falls below unit cost, the firm might consider abandoning operations for low enough output price. Again we

[15] Market spanning allows us to apply the risk-neutral valuation technique of Cox and Ross (1976), according to which we can conduct the analysis as if the agents are risk-neutral so long as we also replace the drift coefficient α by $r - \delta$. This justifies our use of the risk-free rate as the discount rate. A more general treatment of risk-neutral valuation is given in Harrison and Kreps (1979).

assume that the project has zero scrap value and that reentry is so prohibitively expensive that any exit is final. We also assume that there are two ways of getting out of the contract. The firm can split up the firm into two, firm A and firm B, at a fixed cost S and obtain a terminal payoff of $V_a + V_b - S$. Alternatively, it can elect to forfeit all future cash flows and obtain a terminal payoff of zero. Given the uncertainty about future prices, the firm waits for the output prices to fall to a certain level , P_x before it abandons operations as a diversified firm. The optimal exit price P_x is obtained from the following value matching and smooth pasting conditions:

$$V_{ab}\left(P_x\right) = \max\left\{V_a\left(P_x\right) + V_b\left(P_x\right) - S, 0\right\} \quad (2.9b)$$

$$V'_{ab}\left(P_x\right) = \frac{\partial}{\partial P}\max\left\{V_a\left(P_x\right) + V_b\left(P_x\right) - S, 0\right\} \quad (2.9c)$$

If we subtract $V_{ab}\left(P_x\right)$ from both sides of Eq.(2.9b), we find that at the optimal exit price P_x, either the firm is worthless or the value $V_a\left(P_x\right) + V_b\left(P_x\right) - V_{ab}\left(P_x\right)$ lost from cannibalization (or, equivalently, the value to be gained from splitting up) exactly equals the split up cost S. We now show that Proposition 1.1 and Proposition 1.3 hold for the case in which $\varepsilon_a = 0$ and $\varepsilon_b = 1$ and the case in which $\varepsilon_a = 1$ and $\varepsilon_b = 1$.[16] In both cases, the idea behind obtaining the value function is very simple. First, solve Eq.(2.9) for the value function conditional on exit via bankruptcy. Then, solve again conditional on exit via a split off. The unconditional value function, which depends on the split up cost, is the maximum of the two conditional

[16] As in the static setting, dynamic counterpart of Proposition 1.2 follows from the fact that the additivity of operating cash flows implies $U_{ab} = \left(\frac{QP^{\varepsilon_a}}{\delta_a} - \frac{QC_a}{r}\right) + \left(\frac{QP^{\varepsilon_b}}{\delta_b} - \frac{QC_b}{r}\right) = U_a + U_b$. The dynamic counterpart of Proposition 1.4 is obtained by replacing X_j^+ by $d(\ln V_j)$ where dV_j is given by Eq.(2.2) and r_m is replaced by its continuous version. The dynamic counterpart of Proposition 1.5 immediately follows from the dynamic counterpart of Proposition 1.1.

value functions.

2.3.1 Valuation when the cash flows of A and B are uncorrelated

When $\varepsilon_a = 0$ and $\varepsilon_b = 1$, firm AB consists of a perpetuity which generates an annual fixed income of $Q(1 - C_a)$ and a project generating a risky operating cash flow of $Q(P_t - C_b)$ at time t. Its value function, defined by Eqs.(2.9), can be written as:

$$V_{ab}(P) = DP^{\theta_2} + \frac{Y}{r} + \left(\frac{QP}{\delta} - \frac{QC_b}{r}\right) \quad (2.10a)$$

where

$$Y = Q(1 - C_a) \quad (2.10b)$$

$$D = \max\{D_o, D_s\} \quad (2.10c)$$

$$D_o = \begin{cases} \frac{-QP_o^{1-\theta_2}}{\theta_2 \delta} & if \quad P_o \geq 0 \\ 0 & if \quad P_o < 0 \end{cases} \quad (2.10d)$$

$$D_s = \begin{cases} \frac{-QP_s^{1-\theta_2}}{\theta_2 \delta} & if \quad P_s \geq 0 \\ 0 & if \quad P_s < 0 \end{cases} \quad (2.10e)$$

$$P_o = \frac{-\theta_2}{1 - \theta_2} \frac{\delta}{rQ} (QC_b - Y) \quad (2.10f)$$

$$P_s = \frac{-\theta_2}{1 - \theta_2} \frac{\delta}{rQ} (QC_b - rS) \quad (2.10g)$$

P_o is the optimal exit price conditional on exit via bankruptcy while P_s is the optimal exit price conditional on exit via a split off. From Eq.(2.7c) and Eq.(2.10d), we conclude that

$$\max\{P_o, P_s\} < P_L^b.$$

This means that the diversified firm will abandon the risky project later than a pure play firm (firm B) would optimally want. Recall from Eq.(2.10) that at output price P_L^b firm B would already be worthless. And so exiting at a price lower than this implies further cannibalization by the risky division before the diversified firm optimally decides to exit. This, by itself, would be sufficient to make the value of the diversified firm less than the sum of the stand alone values of its constituent divisions.

The value of project B to firm A is given by

$$V_b^a(P) = V_{ab}(P) - V_a(P) = DP^{\theta_2} + \left(\frac{QP}{\delta} - \frac{QC_b}{r}\right). \tag{2.11}$$

Project B will generate negative cash flow and cannibalize the firm's fixed income flow when output price falls below unit cost and this can occur with positive probability. Because of this risk of cannibalization, we expect the value of project B to firm A to be less than the stand alone value of firm B. From Eq.(2.7) and Eqs.(2.10)-(2.11) we find that $D < D_b$ which implies that $V_b^a(P) < V_b(P)$. Thus, Proposition 1.1 holds in this dynamic setting. We use the following relationship between the beta of a derivative asset whose value function is given by V and the beta of the underlying asset whose price is given by P for the dynamic counterpart of Proposition 1.3:[17]

$$\beta_V = \beta\frac{PV'(P)}{V(P)} \tag{2.12}$$

[17] See Dixit (1989) for a derivation.

where β_V is the beta of the derivative asset and β is the beta of the underlying asset, which in this case is good B. Substituting Eq.(2.11) into Eq.(2.12) yields the following expression for the beta of project B from the point of view of firm A:

$$\beta_b^a = \beta \frac{\theta_2 D P^{\theta_2} + \frac{QP}{\delta}}{D P^{\theta_2} + \left(\frac{QP}{\delta} - \frac{QC_b}{r}\right)} \tag{2.13a}$$

Similarly, we obtain the following expression for the stand alone beta of firm B:

$$\beta_b = \beta \frac{\theta_2 D_b P^{\theta_2} + \frac{QP}{\delta}}{D_b P^{\theta_2} + \left(\frac{QP}{\delta} - \frac{QC_b}{r}\right)} \tag{2.13b}$$

We can conclude that $\beta_b^a \neq \beta_b$ from the fact that $\theta_2 < 0$ and $D < D_b$. In particular, if the $\beta > 0$, then project B is riskier as an addition to firm A's existing assets than as a stand alone entity, i.e., $\beta_b^a > \beta_b$. This makes sense since, in this case, project B would have more downside risk when combined with firm A's assets in place.

2.3.2 Investment timing decision

If we back up one step and suppose that the firm A has not yet implemented project B, but can do so, at any time it deems optimal, by investing I_b^a. If the current output price is $P \geq P_x$, then the value to firm A of undertaking the project B right now is simply its net present value

$$NPV_b^a(P) = V_{ab}(P) - \frac{Y}{r} - I_b^a.$$

While this may be positive, the firm may be better off postponing project implementation until prices get even more favorable. More generally, the net present value of the commitment today by firm A to invest in project B when output price reaches $P_e \geq P$

is given by

$$NPV_b^a(P_e;P) = E\left(V_{ab}(P_e) - \frac{Y}{r} - I_b^a\right)e^{-rT} \tag{2.14a}$$

Hence, if the current output price is $P \geq P_x$, then the value of the commitment today to implement the project when output price reaches $P_e \geq P$ is given by

$$NPV_b^a(P_e;P) = \left(DP_e^{\theta_2} + \frac{QP_e}{\delta} - \frac{QC_b}{r} - I_b\right)\left(\frac{P}{P_e}\right)^{\theta_1} \tag{2.14b}$$

The firm is better off postponing project implementation until the price reaches $P_e > P$ if $NPV_b^a(P_e;P) > NPV_b^a(P) = NPV_b^a(P;P)$. The optimal entry price $\bar{P}_b^a$ by firm A into project B, conditional on the current output price being P, is given by

$$\bar{P}_b^a = \arg\max_{P_e \geq P}\left(DP_e^{\theta_2} + \frac{QP_e}{\delta} - \frac{QC_b}{r} - I_b\right)\left(\frac{P}{P_e}\right)^{\theta_1}. \tag{2.15}$$

We note that $\bar{P}_b^a$ is increasing in Y for $\bar{P}_b^a \in (P, \infty)$. This follows from a straightforward application of the implicit function theorem, taking note of the fact that $\theta_1 > 1, \theta_2 < 0$, and $\frac{\partial D}{\partial Y} < 0$. This is to be expected since the larger the fixed income flow, the larger the size of the asset that the firm wants to protect from cannibalization by the investment project in the event that it starts to make losses and, consequently, the higher the required entry price compared to a pure play firm.

A numerical example would be useful to illustrate the impact of assets in place on the investment timing decisions of the firm. We consider the copper mining example of Dixit and Pindyck (1994, pp. 224-5). They study a facility that produces $Q = 10$ million pounds of refined copper per year. In 1992 constant dollars, the estimated cost of building the facility is $I = \$20$ million, and the estimated variable cost is $C = \$0.80$ per pound. The estimated exit cost $E =$ \$2 million. We assume that split up cost S is also equal to this value. The estimated convenience yield is $\delta = .04$, the estimated volatility parameter is $\sigma = .20$ and the estimated risk free rate is $r = .04$. As our initial price P, we take the 1992 average copper price of \$1.00 per pound. From Eq.(2.7), the value function for a firm, say, Firm B, specializing in the copper refinery project is given by

$$V_b(P) = 40P^{-1} + 10\frac{P}{.04} - 10\frac{0.8}{.04}$$

So, at the current price of $P =$ \$1.00 per pound, the net present value of undertaking the project immediately is \$70.0 million. Given the uncertainty about the evolution of prices, it is optimal to wait until the price reaches $\bar{P}_b =$ \$1.4226 before sinking in the required \$20 million investment to set up the refinery from scratch. The expected NPV at such (random) point in time is $NPV_b(\bar{P}_b) = NPV_b(\bar{P}_b; \bar{P}_b) =$ \$163.77 million. The present value of this expected NPV is $NPV_b(\bar{P}_b, P) =$ \$80.92 million. We interpret this as the present value of the commitment today to invest in the copper refinery project when the output price hits \$1.42 per pound. Since this value exceeds the net present value of implementing the project immediately, it follows that investors are better off waiting for the price to reach \$1.4226 per pound before investing in the copper refinery firm. Once invested, it is optimal to exit when the price goes down to $P_L^b =$\$0.40 per pound.

Consider now the situation in which a firm, say, firm A, with existing assets is contemplating on expanding into the copper refinery business. Suppose that firm A generates \$100,000 annually from existing assets.[18] From Eq.(2.11), the value to firm A

[18] Thus, firm A's value function is given by the constant function $V_a(P) = \frac{0.10 \text{ million}}{.04} =$ \$2.5 million.

of the copper refinery project (project B) is

$$V_b^a(P) = 39.204P^{-1} + 10\frac{P}{.04} - 10\frac{0.8}{.04}.$$

At the current price of $P = \$1.00$ per pound, the net present value of undertaking the project immediately is \$69.204 million. Given the uncertainty about the evolution of prices, it is optimal to wait until the price reaches the $\bar{P}_b^a = \$1.4313$ before sinking in the required \$20 million investment to set up the refinery. Note that this is higher than $\bar{P}_b = \$1.4226$ per pound, the optimal entry price for a firm specializing in copper refinery using process B. The expected NPV at such a (random) point in time is $NPV_b^a(\bar{P}_b) = NPV_b^a(\bar{P}_b; \bar{P}_b) = \$165.22$ million with a present value of $NPV_b^a(\bar{P}_b, P) = \$80.647$ million. Given the different optimal entry price, this is smaller than the NPV of B as stand alone by $80.92 - 80.647 = 0.273$. Also, once invested, it is optimal to split up the firm when the price goes down to P_s =\$0.396 per pound, which is lower than the optimal exit price of $P_L^b = \$0.40$ per pound for a firm specializing in the copper refinery project.[19]

2.3.3 Valuation when the cash flows of A and B are perfectly correlated

When $\varepsilon_a = \varepsilon_b = 1$, firm AB essentially has two methods of producing the same good. The unit costs are C_a and C_b under methods A and B, respectively. For specificity, we assume $C_a \geq C_b$. The value function of firm AB, defined by Eqs.(2.9), can be written as:

$$V_{ab}(P) = DP^{\theta_2} + \frac{2QP}{\delta} - \frac{QC_a + QC_b}{r} \tag{2.16a}$$

[19] In this example, $D_s = 39.204 > D_o = 30.625$. Hence it is optimal for the firm to exit via a split up.

where

$$D = \max\{D_o, D_s\} \tag{2.16b}$$

$$D_o = \frac{-2QP_o^{1-\theta_2}}{\theta_2\delta} \tag{2.16c}$$

$$D_s = D_b - \frac{QP_s^{1-\theta_2}}{\theta_2\delta} \tag{2.16d}$$

$$P_o = \frac{-\theta_2}{1-\theta_2}\frac{\delta}{r}\frac{C_a+C_b}{2} \tag{2.16e}$$

$$P_s = \frac{-\theta_2}{1-\theta_2}\frac{\delta}{rQ}(QC_a - rS) \tag{2.16f}$$

D_b is defined in Eq.(2.7c). From this and Eqs.(2.16e)-(2.16f), we conclude that if $C_a > C_b$, then

$$\max\{P_o, P_s\} < P_L^a.$$

Thus, the diversified firm ends up holding to its less efficient division longer than a pure play firm (firm A) would optimally want to.[20] This implies that a diversified firm ends up sustaining more hemorrhaging than its pure play counterpart.

The value of project B to firm A is given by

$$V_b^a(P) = V_{ab}(P) - V_a(P) = (D - D_a)P^{\theta_2} + \left(\frac{QP}{\delta} - \frac{QC_b}{r}\right). \tag{2.17}$$

[20] Note that $V_a(P) = 0$ for all $P \leq P_L^a$.

If the two processes are equally efficient in the sense that $C_a = C_b$ then there would not be any cannibalization since the *operating cash flows* from the two processes will always be of the same sign. In this case we expect that $V_b^a(P) = V_b(P)$ since $D - D_a = D_b$ if and only if $C_a = C_b$. When $C_a > C_b$, then there is positive probability that output price will be strictly between the maximum and the minimum of C_a and C_b. When this happens the positive cash flows from the more efficient process will be cannibalized by the negative cash flow of the less efficient process whence we expect $V_b^a(P) < V_b(P)$. This is in fact the case since $D - D_a < D_b$ when $C_a > C_b$.

Substituting Eq.(2.17) into Eq.(2.12) yields the following expression for the beta of project B from the point of view of firm A:

$$\beta_b^a = \beta \frac{\theta_2 (D - D_a) P^{\theta_2} + \frac{QP}{\delta}}{(D - D_a) P^{\theta_2} + \left(\frac{QP}{\delta} - \frac{QC_b}{r}\right)}. \tag{2.18}$$

We recall the following expression for the beta of firm B:

$$\beta_b = \beta \frac{\theta_2 D_b P^{\theta_2} + \frac{QP}{\delta}}{D_b P^{\theta_2} + \left(\frac{QP}{\delta} - \frac{QC_b}{r}\right)}. \tag{2.13b}$$

If the two processes are not equally efficient, then we conclude that $\beta_b^a \neq \beta_b$ from the fact that $\theta_2 < 0$ and $(D - D_a) < D_b$ when $C_a > C_b$. In particular, if $\beta > 0$, then project B is riskier as an addition to firm A's existing assets than as a stand alone entity. This is true *even if* process B is more efficient since the juxtaposition of the two processes unleashes the cannibalization potential of the less efficient process.

2.3.4 Investment timing decision

Let us again back up one step and study the optimal entry decision of firm A into project B. If the current output price is $P \geq P_x$, then the value of the commitment today to implement the project when output price reaches $P_e \geq P$ is given by

$$NPV_b^a(P_e; P) = \left((D - D_a) P_e^{\theta_2} + \frac{QP_e}{\delta} - \frac{QC_b}{r} - I_b \right) \left(\frac{P}{P_e} \right)^{\theta_1} \tag{2.19}$$

The optimal entry price $\bar{P}_b^a$ of firm A into project B, conditional on the current output price being P, is given by

$$\bar{P}_b^a = \underset{P_e \geq P}{\arg\max} \left((D - D_a) P_e^{\theta_2} + \frac{QP_e}{\delta} - \frac{QC_b}{r} - I_b \right) \left(\frac{P}{P_e} \right)^{\theta_1} \tag{2.20}$$

The optimal entry price $\bar{P}_b$ of a pure play firm into project B, conditional on the current output price being P, is given by

$$\bar{P}_b = \underset{P_e \geq P}{\arg\max} \left(D_b P_e^{\theta_2} + \frac{QP_e}{\delta} - \frac{QC_b}{r} - I_b \right) \left(\frac{P}{P_e} \right)^{\theta_1}. \tag{2.21}$$

We note that $\bar{P}_b$ is decreasing in D_b. This follows from a straightforward application of the implicit function theorem, taking note of the fact that $\theta_1 > 1$ and $\theta_2 < 0$. If $C_a > C_b$ then $D - D_a < D_b$ and we get $\bar{P}_b^a > \bar{P}_b$. This means that, to reduce the risk of cannibalization, firm A would optimally want to wait for output price to get higher before expanding into project B.

Let us return to the copper mining example. Suppose that instead of having a perpetuity, firm A's existing assets are also a copper refinery producing 10 million pounds per year but at a higher unit cost of $C_a = \$0.90$ per pound as opposed to $C_b = \$0.80$ under the new process. Suppose that the economic parameters are the same as before. From Eq.(2.7), the value of firm A is given by

$$V_a(P) = 50.625P^{-1} + 10\frac{P}{.04} - 10\frac{0.9}{.04}$$

Consider now the situation in which firm A is contemplating on expanding its copper refinery business using the process B for its additional 10 million pounds of output. From Eq.(2.17), the value of the new copper refinery project to firm A is

$$V_b^a(P) = (90.3125 - 50.625)\, P^{-1} + 10\frac{P}{.04} - 10\frac{0.8}{.04}.$$

At the current price of $P = \$1.00$ per pound, the net present value of undertaking the project immediately is \$69.688 million. Given the uncertainty about the evolution of prices, it is optimal to wait until the price reaches the $\bar{P}_b^a = \$1.426$ before sinking in the required \$20 million investment to set up the refinery.[21] The expected NPV at such (random) point in time is $NPV_b^a(\bar{P}_b^a) = NPV_b^a(\bar{P}_b^a; \bar{P}_b^a) = \$164.33$ million. The present value of this expected NPV is $NPV_b^a(\bar{P}_b^a, P) = \$80.813$ million. Once the new refinery project is adopted, it is optimal for expanded firm AB to exit at an output price $P_o = \$0.425$ per pound, which is lower than the optimal exit price of $P_L^a = \$0.45$ per pound if firm A were stand alone.[22]

A few observations can be made. One, cannibalization is still possible when cash flows are perfectly correlated as long as one project is less efficient than the other. Two, the extent

[21] This is higher than $\bar{P}_b = \$1.4226$ per pound, the optimal entry price for a firm specializing in copper refinery using process B.

[22] In this example, $D_s = 89.729 < D_o = 90.3125$. Hence it is optimal for the firm AB to exit via bankruptcy.

of cannibalization is, however, higher when cash flows are uncorrelated as in the earlier section. The maximum NPV from acquiring B is $80.813 million when $\varepsilon_a = 1$ and $\varepsilon_b = 1$ while it is $80.647 million when the cash flows are uncorrelated. In both cases, value is lost from cannibalization since the NPV of B as a stand alone firm is $80.92 million. Thus, non-synergistic diversifications will usually result in value loss from cannibalization, and this loss will be higher when diversification is in unrelated industries.

2.4 Conclusion

The five propositions in the previous chapter continue to hold in a simple dynamic setting. Moreover, we are able to obtain new insights from the dynamic analysis. First, the possibility of cannibalization affects the optimal timing of investment for a firm with existing assets. Thus, the same new investment can have different values for different firms for this reason as well. Second, the value loss from diversification is higher when the cash flows are uncorrelated than when they are perfectly correlated. However, perfect correlation between cash flows can still result in value loss through cannibalization and an indirect loss through its effect on the investment timing decision.

3. Cannibalization Risk in a Simple Strategic Model

3.1 Introduction

In Chapter 2, among others, we studied the impact of cannibalization risk on the investment timing decisions of the firm. In this chapter, we extend this analysis by allowing the firm's payoff to depend partly on the investment timing decisions of another firm. We consider the setting of Section 2.2 in which the firm consists of a perpetuity which generates an annual fixed income equal to Y and a project generating a risky operating cash flow $Q_t\,(P_t - C)$, where P_t follows the same geometric Brownian motion as in the previous chapter. We begin by developing a strategic model of entry and exit. We then present an example in which there exists a unique sub-game perfect equilibrium in which the smaller firm invests earlier than and exits before a firm with larger assets-in-place.[1] This occurs because larger assets-in-place translate into higher exit costs which rationally make larger firms more cautious in their entry decision. Consequently, a firm with larger assets in place waits until output price reaches a level higher than the threshold price at which the competing smaller firm will enter.[2] In the process, the larger firm foregoes the opportunity to capture monopoly rents which the first entrant would enjoy until the entry of the competitor.

This chapter is divided into four sections. In Section 3.2, we develop a strategic model of entry and exit. In Section 3.3, we present a numerical example. Section 3.4 contains a brief summary.

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[1] The numerical example is an extension of one in Dixit and Pindyck (1994).

[2] It is possible for the larger firm not to invest at all (if the higher threshold price is never reached).

3.2 A strategic model of entry and exit

We consider two firms which are identical in all respects except for the size of their annual fixed income flow. We assume that if both firms are supplying the market, each supplies $\frac{Q}{2}$ units of output, where Q is some fixed positive number.[25] If one of the firms exits, then the remaining firm supplies Q units. Thus, Q_t is given by

$$Q_t = \begin{cases} 0 & if \quad n(t) = 0 \\ Q & if \quad n(t) = 1 \\ \frac{Q}{2} & if \quad n(t) = 2 \end{cases} \tag{3.1}$$

where $n(t)$ is the number of firms that have invested in the investment project and have not exited as of time t. As is the previous chapter, there are two ways of getting out of the production contract. The firm can elect to forfeit all of its future cash flows and obtain a terminal payoff of zero. Alternatively, it can split up the firm at a cost S and then sell the resulting firms at their stand alone market values. We assume that the small firm's fixed income flow Y_s is so small that $\frac{Y_s}{r}$ is less than S, where r is the instantaneous risk free rate.[26] In other words, the capitalized value of the small firm's perpetuity is less than the split up cost. This means that the small firm is better off exiting via bankruptcy than via a split up.

We assume that reentry is prohibitively expensive and so any exit is final and the remaining firm becomes a monopolist facing no threat of potential competition. Both

[25] This is for simplification. This assumption is harmless since the qualitative results continue to hold if, in the presence of a competitor, both firms supply Q^* units, where $\frac{Q}{2} \leq Q^* \leq Q$. The critical feature is that both firms supply the *same* quantity of output, a feature which is possessed by a Cournot equilibrium.

[26] The big firm's fixed income flow Y_b can be any number greater than Y_s.

firms make investment timing decisions simultaneously at each point in time and each firm is perfectly informed about both firms' previous actions at each point in the game. The problem for each firm is to maximize its discounted expected cash flows by choosing the appropriate entry and exit prices taking into account the impact of the other firm's entry and exit strategies. This is solved by backward induction as is usual with dynamic games.

3.2.1 The decision to be the second investor

Suppose that both firms have already invested, each producing $\frac{Q}{2}$ units of output. Then, when the output price goes down far enough, at least one of the firms will optimally decide to exit. Since the small firm has less fixed income flow, it has to lose from exiting. It follows that the small firm exits first and the big firm eventually becomes a monopolist.[27] And so, a small firm contemplating on becoming the second investor after observing that the big firm has already invested in the project expects to be producing $\frac{Q}{2}$ units of output until it exits. This means that its optimization problem is essentially the same as the one discussed in Chapter 2 with the output level Q replaced by $\frac{Q}{2}$. By appropriately modifying Eqs.(2.10), (2.14), and (2.15), we obtain the following corresponding equations for the small firm as a second investor

$$V_{ss}(P) \quad = \quad D_{ss}P^{\theta_2} + \tfrac{Y_s}{r} + \left(\tfrac{QP}{2\delta} - \tfrac{QC}{2r}\right) \quad for \quad P \geq P_{0ss} \qquad (3.2a)$$

$$NPV_{ss}(P_e; P) \quad = \quad \left(D_{ss}P_e^{\theta_2} + \frac{QP_e}{2\delta} - \frac{QC}{2r} - I\right)\left(\frac{P}{P_e}\right)^{\theta_1} \qquad (3.2b)$$

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[27] Note from Eqs.(2.10) and (2.10g) of Chapter 2 that, since $Y_s < rS$ and $Y_s < Y_b$, it follows that the exit price of the small firm is higher than that of the big firm. In other words, the small firm will exit earlier than the big firm.

$$P_{Hss} = \arg\max_{P_e \geq P} \left(D_{ss} P_e^{\theta_2} + \frac{QP_e}{2\delta} - \frac{QC}{2r} - I \right) \left(\frac{P}{P_e} \right)^{\theta_1} \quad (3.2c)$$

where

$$D_{ss} = \begin{cases} \frac{-QP_{Lss}^{1-\theta_2}}{2\theta_2\delta} & if \quad P_{Lss} \geq 0 \\ 0 & if \quad P_{Lss} < 0 \end{cases}$$

$$P_{Lss} = \frac{-\theta_2}{1-\theta_2} \frac{2\delta}{rQ} \max \left\{ \left(\frac{QC}{2} - Y_s \right), \left(\frac{QC}{2} - rS \right) \right\}.$$

The exit price has a subscript L and the entry price has a subscript H. The additional subscript ss identifies the firm as a *small second* investor. The subscript bs will be used to identify a firm as a *big second* investor. The subscript bm will be used to identify the firm as a *big monopolist*. Thus, P_{Lss} denotes the exit price of the small firm as a second investor. $NPV_{ss}(P_e; P)$ denotes the value of committing today to invest an amount I in the risky project, as a small second investor, when the output price reaches $P_e \geq P$, where P is the current output price.

The corresponding equations for the big firm as a second investor have the same form as the ones for a small firm as a second investor for $P \geq P_{Lss}$.

$$V_{bs}(P) = \frac{Y_b}{r} + D_{bs} P^{\theta_2} + \frac{QP}{2\delta} - \frac{QC}{2r} \quad \text{for} \quad P \in [P_{Lss}, \infty) \quad (3.3a)$$

$$NPV_{bs}(P_e; P) = \left(D_{bs} P_e^{\theta_2} + \frac{QP_e}{2\delta} - \frac{QC}{2r} - I \right) \left(\frac{P}{P_e} \right)^{\theta_1} \quad (3.3b)$$

$$P_{Hbs} = \arg\max_{P_e \geq P} \left(D_{bs} P_e^{\theta_2} + \frac{QP_e}{2\delta} - \frac{QC}{2r} - I \right) \left(\frac{P}{P_e} \right)^{\theta_1} \tag{3.3c}$$

where P_{Hbs} denotes the entry price of the big firm as a second investor and D_{bs} is a constant to be determined from the relevant boundary condition. It is the boundary condition which distinguishes the big firm from the small firm as a second investor. In the case of the small firm, the boundary condition characterizes what happens to the small firm when it exits. In the case of the big firm, the boundary condition specifies what happens to the big firm when the small firm exits, namely, it is transformed into a monopolist supplying the full Q units of output per year. This yields the following value matching condition:

$$V_{bs}(P_{Lss}) = V_{bm}(P_{Lss}) \tag{3.3d}$$

where V_{bm} is the value function for the big firm as a monopolist. V_{bm} is exactly the value function that we considered in the previous chapter with $Y = Y_b$. From Eq. (2.10), we get

$$V_{bm}(P) = \begin{cases} \frac{Y_b}{r} + D_{bm} P^{\theta_2} + \frac{QP}{\delta} - \frac{QC}{r} & if \quad P \geq P_{Lbm} \\ 0 & if \quad P < P_{Lbm} \end{cases} \tag{3.4a}$$

where

$$D_{bm} = \begin{cases} \frac{-QP_{Lbm}^{1-\theta_2}}{\theta_2 \delta} & if \quad P_{Lbm} \geq 0 \\ 0 & if \quad P_{Lbm} < 0 \end{cases} \tag{3.4b}$$

$$P_{Lbm} = \frac{-\theta_2}{1-\theta_2} \frac{\delta C}{rQ} \max\left(QC - Y_b\right), \left(QC - rS\right). \tag{3.4c}$$

where P_{Lbm} denotes the exit price of a big monopolist.

3.2.2 The decision to be the first investor

Suppose now that both firms have not yet invested in the project. The small firm contemplating on being the first investor has to recognize that the big firm will invest when the price gets high enough. Thus, the small firm can be the sole supplier only until the price hits P_{Hbs}, which is the optimal entry price for the big firm as a second investor. By applying the same argument as in Chapter 2, we obtain the following differential equation that must be satisfied by the value function V_{sf} of the small firm if it is the first investor:

$$\frac{1}{2}\sigma^2 P^2 V''_{sf} + (r-\delta) P V'_{sf} - rV_{sf} + Q(P-C) + Y_s = 0. \qquad (3.5)$$

The solution to Eq.(3.5) is given by

$$V_{sf}(P) = \frac{Y_s}{r} + N_{sf} P^{\theta_1} + D_{sf} P^{\theta_2} + \frac{QP}{\delta} - \frac{QC}{r} \quad \text{for} \quad P \in [P_{Lsf}, P_{Hbs}] \qquad (3.6a)$$

where N_{sf}, D_{sf}, and P_{Lsf} are constants to be determined from the relevant boundary conditions, and θ_1 and θ_2 are, respectively, the positive and the negative roots of the quadratic form associated with the general solution to Eq.(3.5).[28]

Given the uncertainty about the evolution of prices, it is optimal for the small firm to wait for output price to fall to a certain level, say P_{Lsf}, before it abandons the project, at which point the firm forfeits all cash flows including its annual fixed income flows. This yields the following value matching and smooth pasting conditions:

$$V_{sf}(P_{Lsf}) = 0 \qquad (3.6b)$$

$$V'_{sf}(P_{Lsf}) = 0. \qquad (3.6c)$$

[28] In this subsection, the subscript sf is used to identify a firm as a *small first* investor. The subscript bf is used to identify a firm as a *big first* investor.

When the price hits P_{Hbs}, entry by the big firm will occur. This effectively transforms the position of the small firm to that of a small second investor. This yields the following value matching condition:

$$V_{sf}\left(P_{Hbs}\right) = V_{ss}\left(P_{Hbs}\right). \tag{3.6d}$$

By making the appropriate substitutions, we are able to solve for N_{sf}, D_{sf}, and P_{Lsf}.

If the current output price is P and the small firm has not yet invested in the project, then the value of the commitment today to invest in the project as a first investor when output price reaches $P_e \geq P$ is given by

$$NPV_{sf}\left(P_e; P\right) = \left(N_{sf}P^{\theta_1} + D_{sf}P_e^{\theta_2} + \frac{QP}{\delta} - \frac{QC}{r} - I\right)\left(\frac{P}{P_e}\right)^{\theta_1}. \tag{3.6e}$$

The analysis for the case of the big firm as a first investor is similar to that of the small firm as a first investor. For economy, we simply list the final results. For $P \in [P_{Lbf}, P_{Hss}]$,we have

$$V_{bf}\left(P\right) = \frac{Y_b}{r} + N_{bf}P^{\theta_1} + D_{bf}P_e^{\theta_2} + \frac{QP}{\delta} - \frac{QC}{r} \tag{3.7a}$$

$$V_{bf}\left(P_{Lbf}\right) = 0 \tag{3.7b}$$

$$V_{bf}'\left(P_{Lbf}\right) = 0 \tag{3.7c}$$

$$V_{bf}\left(P_{Hss}\right) = V_{bs}\left(P_{Hss}\right) \tag{3.7d}$$

$$NPV_{bf}\left(P_e; P\right) = \left(N_{bf}P^{\theta_1} + D_{bf}P_e^{\theta_2} + \frac{QP}{\delta} - \frac{QC}{r} - I\right)\left(\frac{P}{P_e}\right)^{\theta_1} \tag{3.7e}$$

3.3 Equilibrium entry and exit

Given that the only reason a firm optimally would want to exit via a split up (as opposed to bankruptcy) as a mode of exit is to protect as much of its annual fixed income as possible, it follows that it will choose a split up as a mode of exit if and only if $\frac{Y}{r} > S$, regardless of what the other firm does. Since our derivation of the value functions presumes the adoption of this dominant exit strategy, our game is reduced to an entry game.

An outcome of the entry game with an initial output price P is a pair (P_{N_s}, P_{N_b}) consisting of the entry price $P_{N_s} \geq P$ at which small firm invests in the investment project and the entry price $P_{N_b} \geq P$ at which the big firm makes the investment. The firm with the lower entry price invests ahead of the one with the higher entry price. Thus, if we let $NPV_j\,(P_{Ns}, P_{Nb}; P)$ denote firm j's payoff corresponding to the profile $(P_{N_s}, P_{N_b}; P)$, then

$$NPV_s\,(P_{N_s}, P_{N_b}; P) = \begin{cases} NPV_{ss}\,(P_{N_s}; P) & if \quad P_{N_s} \geq P_{N_b} \\ NPV_{sf}\,(P_{N_s}; P) & if \quad P_{N_s} < P_{N_b} \end{cases} \tag{3.8a}$$

$$NPV_b\,(P_{N_s}, P_{N_b}; P) = \begin{cases} NPV_{bs}\,(P_{N_b}; P) & P_{N_s} \leq P_{N_b} \\ NPV_{bf}\,(P_{N_b}; P) & P_{N_s} > P_{N_b} \end{cases} \tag{3.8b}$$

$NPV_i\,(P_{Ns}, P_{Nb}; P)$ gives us the present value to firm i of the commitment to invest in the project when the output price reaches P_{N_i} given that the current output price is P and that firm j, $j \neq i$, is committed to invest in the project when the output price reaches P_{N_j}.

For convenience, we shall refer to the firm with the lower entry price as the first investor and denote its entry price by P_1, and refer to the firm with the higher entry price as the second investor and denote its entry price by P_2. Thus, $P_1 = \min\{P_{N_s}, P_{N_b}\}$ and $P_2 = \max\{P_{N_s}, P_{N_b}\}$. To be considered as an *equilibrium profile*, we require $(P_{N_s}, P_{N_b}; P)$ to satisfy four properties. These requirements are quite natural. First, the first investor should not prefer to invest later than the second investor. Second, the second investor should not prefer to invest earlier than the first investor. In other words, the first investor's entry price should be preemption-proof. Third, the first investor's entry price should be the best among preemption-proof entry prices. Finally, the second investor's entry price should be a best second investor response to the first investor's entry price.

For an equilibrium profile to be stable, neither firm should have an incentive to deviate from its equilibrium strategy with an unexpected early entry when the output price has moved closer to but not quite reached either of the equilibrium threshold prices that prevailed at a lower initial output price. We demonstrate this is indeed the case. Consider an equilibrium profile (P_{N_s}, P_{N_b}) for an entry game with initial output price P. Suppose that $P_{Ns}, P_{Nb} > P' > P$. Then, for $j = s, b$, we have

$$\begin{aligned} NPV_j\left(P_{N_s}, P_{N_b}; P'\right) &= NPV_j\left(P_{N_s}, P_{N_b}; P\right)\left(\frac{P'}{P}\right)^{\theta_1} \\ &\geq NPV_j\left(P_s, P_{N_b}; P\right)\left(\frac{P'}{P}\right)^{\theta_1} \\ &= NPV_j\left(P_s, P_{Nb}; P'\right) \end{aligned}$$

for all $P_s \geq P'$. Similarly, we have $NPV_j\left(P_{N_s}, P_{N_b}; P\right) \geq NPV_j\left(P_{N_s}, P_b; P\right)$ for all $P_b \geq P'$. Hence, (P_{Ns}, P_{Nb}) remains an equilibrium outcome for an entry game starting at P'.[29]

3.4 Numerical example

At this point, it is instructive to return to the copper mining example. Recall that $Q = 10$ million pounds of refined copper per year, the estimated cost of building the facility is

[29] Together with the four properties of an equilibrium profile, this stability property implies that our equilibrium profile is a sub-game perfect Nash equilibrium.

I = \$20 million, and the estimated variable cost is C = \$0.80 per pound. The estimated convenience yield is $\delta = .04$, the estimated volatility parameter is $\sigma = .20$ and the estimated risk free rate is $r = .04$. As our initial price P, we take the 1992 average copper price of \$1.00 per pound. We let the small firm be a single project firm with no other sources of cash flow and let the big firm have an annual fixed income flow of \$1 million. So, $Y_s = 0$ and $Y_b = 1$. We also assume a split up cost S = \$10 million. For comparison, we first consider the investment timing decisions of both firms in the absense of actual and potential competition. The relevant equations are Eq. (2.10) and Eq. (2.14).

3.4.1 Investment timing without potential competition

For the small firm, we have:

$$V(P) = 40P^{-1} + 10\frac{P}{.04} - 10\frac{0.8}{.04}$$

$$NPV(P_e; P) = \left(40P_e^{-1} + 10\frac{P_e}{.04} - 10\frac{0.8}{.04} - 20\right)\left(\frac{P}{P_e}\right)^2.$$

The $NPV(P_e; P)$ function for $Y = 0$ and Y = \$1 million (and S = \$10 million) are graphed in Figure 1. At the current price of P = \$1.00 per pound, the net present value of undertaking the project immediately is \$70.0 million, the vertical intercept in Figure 3.1. Of this total, \$40.0 million represents the value of the exit option. Given the uncertainty about the evolution of prices, it is optimal for the small firm to exit when the price goes down to P_L =\$0.40 per pound. It is also optimal for the small firm to wait until the price reaches the optimal entry price P_H = \$1.4226 before sinking in the required \$20 million investment.

The expected NPV at such (random) point in time is $NPV(P_H) = NPV(P_H; P_H) =$ \$163.77 million. The present value of this expected NPV is $NPV(P_H, P) =$ \$80.92 million. We interpret this as the present value of the commitment to invest in the investment project when the output price hits \$1.42 per pound. Since this value exceeds the net present value of implementing the project immediately, it follows that the small firm is better off waiting for the price to reach \$1.4226 per pound before making the investment.

Insert Figure 3.1 here.

For the big firm, we have:

$$V(P) = \frac{1.0}{.04} + 36.1P^{-1} + 10\frac{P}{.04} - 10\frac{0.8}{.04}$$

$$NPV(P_e; P) = \left(36.1P_e^{-1} + 10\frac{P_e}{.04} - 10\frac{0.8}{.04} - 20\right)\left(\frac{P}{P_e}\right)^2.$$

The net present value to the big firm of immediately implementing the project is only \$66.1 million compared to \$70.0 million for the small firm. The reason is that the value of the exit option has dropped to \$36.1 million. With an annual fixed income flow from \$1 million, the big firm has an asset exposed to potential cannibalization from losses generated by the investment project when production revenue falls below cost. The effective exit cost is thus higher. Consequently, the optimal entry price P_H increases to \$1.4641 for an $NPV(P_H, P)$ of \$79.624 million. Thus, the bigger the firm, the later the entry. With more sources of income, the bigger firm can also absorb more production losses before it becomes optimal to split up or declare bankruptcy. And so, the big firm has a lower exit price P_L of \$0.38

per pound. Let us now consider the impact of potential competition.

3.4.2 Investment timing with potential competition

In this case, the components of the payoff functions are as follows:

$$NPV_{sf}(P_e;P) = \left(-43.775P_e^2 + 43.367P_e^{-1} + 10\frac{P_e}{.04} - 10\frac{.8}{.04} - 20\right)\left(\frac{P}{P_e}\right)^2$$

$$NPV_{ss}(P_e;P) = \left(20P_e + 5\frac{P_e}{.04} - 5\frac{.8}{.04} - 20\right)\left(\frac{P}{P_e}\right)^2$$

$$NPV_{bf}(P_e;P) = \left(-44.396P_e^2 + 39.005P_e^{-1} + 10\frac{P_e}{.04} - 10\frac{.8}{.04} - 20\right)\left(\frac{P}{P_e}\right)^2$$

$$NPV_{bs}(P_e;P) = \left(16.1P_e^{-1} + 5\frac{P_e}{.04} - 5\frac{.8}{.04} - 20\right)\left(\frac{P}{P_e}\right)^2.$$

The component functions corresponding to an initial output price $P = \$1.00$ per pound are graphically illustrated in Figure 3.2. From the bottom graph, we see that, if the small firm is already in, it is optimal big firm to invest as a second investor when the output price hits \$1.6916. At this point, the payoff to the small first investor coincides with that of a small second investor. This is illustrated by the payoff curve for the small first investor coinciding with the payoff curve for the small second investor for $P_e \geq \$1.6916$. Consider the case in which the small firm invests first at an entry price of \$1.2835 and the big firm invests second at an entry price of \$1.6916. In this case, the payoff to the small firm is \$37.969 million and the payoff to the big firm is \$35.285 million.

Insert Figure 3.2 here.

We now verify that the profile ($P_{N_a} = \$1.2835, P_{N_b} = \$1.6916; P = \$1.00$) constitutes an equilibrium profile. First, note that the payoff to the small firm will be less than \$37.969 million if it invests at a price greater than \$1.6916. Thus, it does not prefer to invest later than the second investor. Second, the big firm will have a lower payoff if it invests at a price lower than \$1.2835 even if it is the first investor. Thus, it does not prefer to invest earlier than the small firm. In other words, the small firm's entry price of \$1.2835 is preemption-proof. Third, \$1.2835 is the highest preemption proof entry price for the small firm. If the small firm invests at a price $P' > \$1.2835$, then the big firm is better off investing first at some price P^* greater than \$1.2835 but less than P'. On the other hand, there is no advantage gained by the big firm at any entry price less than or equal to \$1.2835. Thus, the set of preemption proof entry prices for the small firm consists of all prices less than or equal \$1.2835. Among these preemption proof prices, the entry price of \$1.2835 yields the highest payoff. Fourth, it is optimal for the big firm to invest as a second investor when the output price hits \$1.6916. Hence, ($P_{N_a} = \$1.2835, P_{N_b} = \$1.6916; P = \$1.00$) is indeed an equilibrium profile.

It is interesting to note that, in the presence of potential competition, the combined payoffs for both firms, \$37.969 million + \$35.285 million = \$73.254 million, less than what either of the firms would have had in the absence of competition. In the absence of competition, the net payoff would have been \$80.92 million if the small firm makes the investment, and \$79.624 million if the big firm makes the investment. The consumers benefit from having the output becoming available at a lower price when the small firm invests. With potential competition, the small firm invests even earlier at an output price hits $1.2835 compared to $1.4226 without potential competition. The exit prices of both firms with competition are respectively the same as their optimal exit prices in the absence of competition. Thus, with competition, the product will be available to consumers for at least as long as it would have in the absence of competition. The cost to society is that the investment may have to be sunk twice: first by the small firm and (if the output price is high enough) second by the big firm, while in the absence of competition, the investment needs to be made only once. However, since the larger firm invests later in the presence of competition, the present value of the cost to society from a duplication of investment is reduced. Thus, the combined net payoff in the presence of competition is lower than the net payoff in the absence of competition by less than the amount of investment which the big firm has to make if it decides to enter.

3.5 Conclusion

In this chapter, we study the effect of potential competition on the investment timing decisions of firms. We establish that these decisions are dependent on the value of existing assets of the firm. This result violates the value additivity principle commonly used in finance. We show that the firm with the smaller assets in place has an advantage with respect to a new investment project since its exit costs are lower. This allows it to optimally enter first and exit before the larger firm, making the project more valuable to the smaller firm. This occurs because larger assets-in-place translate into higher exit costs which rationally make larger firms more cautious in their entry decision. Consequently, a firm with larger assets in place waits until output price reaches a level higher than the threshold price at

which the competing smaller firm will enter. In the process, the larger firm foregoes the opportunity to capture monopoly rents which the first entrant would enjoy until the entry of the competitor.

Table 1.1 Cannibalization Risk and Project Valuation.

Xa, Xb, and Xab respectively denote the random operating cash flow of Firm A, Firm B, and of the expanded firm AB. Xa^+, Xb^+, and $(Xa+Xb)^+$ denote the corrsponding random equity cash flow. Rm denotes the random return on the market portfolio. Rf is the risk free rate of return The certainty equivalent is obtained using the single-period capital asset pricing model. Ua, Ub, and Uab respectively denote the value of operating cash flow of Firm A, Firm B, and of the expanded firm AB. Va, Vb, and Vab denote the corresponding value of equity cash flow.

Probability	X_a	X_b	X_{ab}	X^+_a	X^+_b	X^+_{ab}	$X^+_{ab} - X^+_a$	R_m	R_f
1/14	60	-6	54	60	0	54	-6	0.32	0.04
1/14	40	-4	36	40	0	36	-4	0.25	0.04
1/14	15	-20	-5	15	0	0	-15	-0.14	0.04
1/14	20	-30	-10	20	0	0	-20	0.17	0.04
1/14	-10	15	5	0	15	5	5	0.31	0.04
1/14	-10	50	40	0	50	40	40	-0.10	0.04
1/14	-5	40	35	0	40	35	35	0.30	0.04
1/14	-30	25	-5	0	25	0	0	-0.13	0.04
1/14	-20	10	-10	0	10	0	0	0.30	0.04
1/14	40	30	70	40	30	70	30	0.32	0.04
1/14	35	10	45	35	10	45	10	0.25	0.04
1/14	25	40	65	25	40	65	40	0.21	0.04
1/14	-10	-8	-18	0	0	0	0	-0.22	0.04
1/14	-15	-7	-22	0	0	0	0	-0.13	0.04
Expected value	9.64	10.36	20.00	16.79	15.71	25.00	8.21	0.12	0.04
Standard deviation	26.49	23.32	31.25	19.51	17.31	26.00	19.24	0.20	0.00
Cov(*,Rm)	2.80	0.83	3.64	2.04	0.36	2.78	0.73	0.04	0.00
Required investment	10.00	10.00	10.00	10.00	10.00	20.00	10.00		
Beta	7.01	2.08	9.09	5.11	0.90	3.47	1.84		
Certainty	3.94	8.38	12.32	12.26	14.43	18.76	6.50		
Equivalent	U_a	U_b	U_{ab}	V_a	V_b	V_{ab}	$V_{ab} - V_a$		

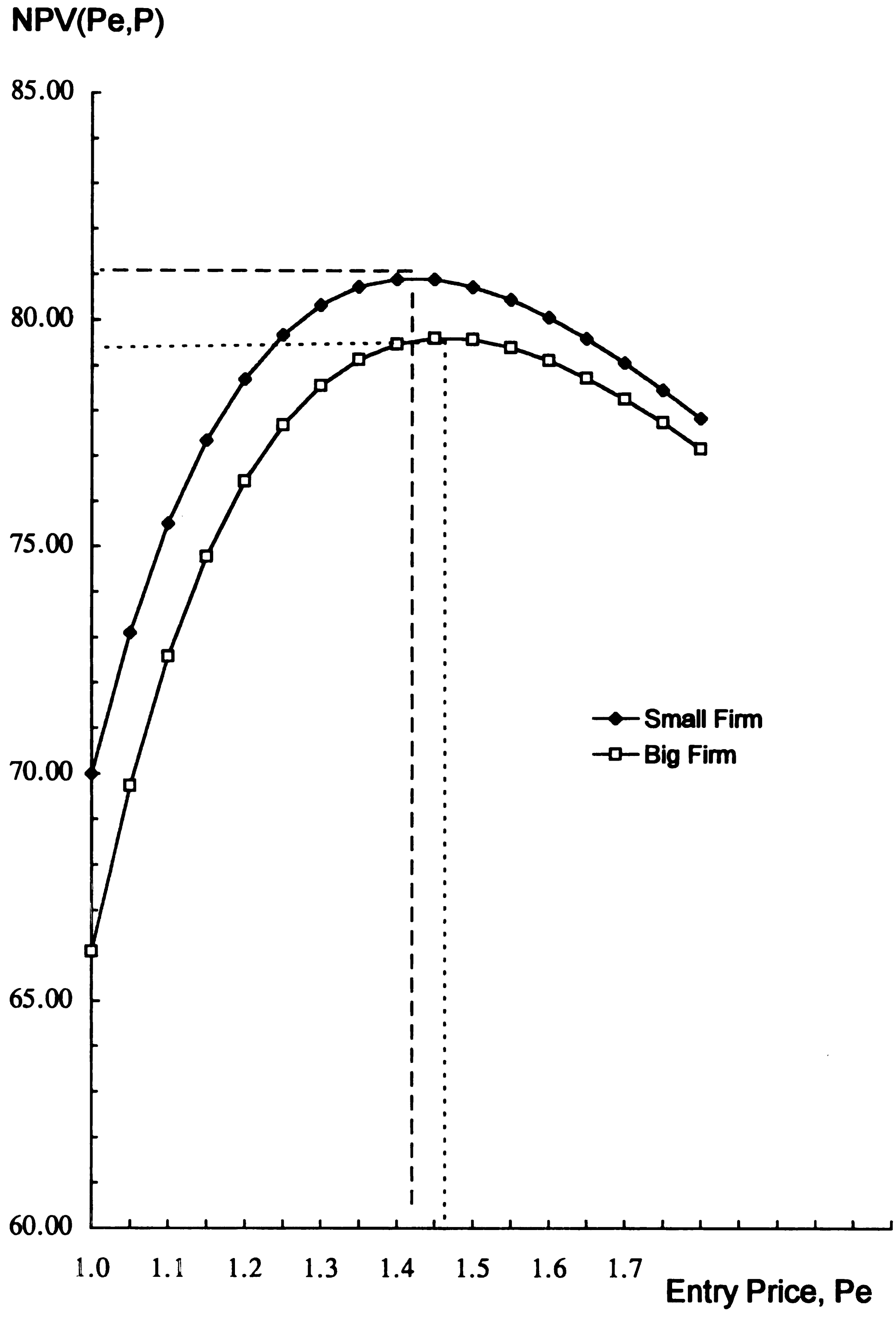

Figure 3.1 Copper Mining Example: Basic Model

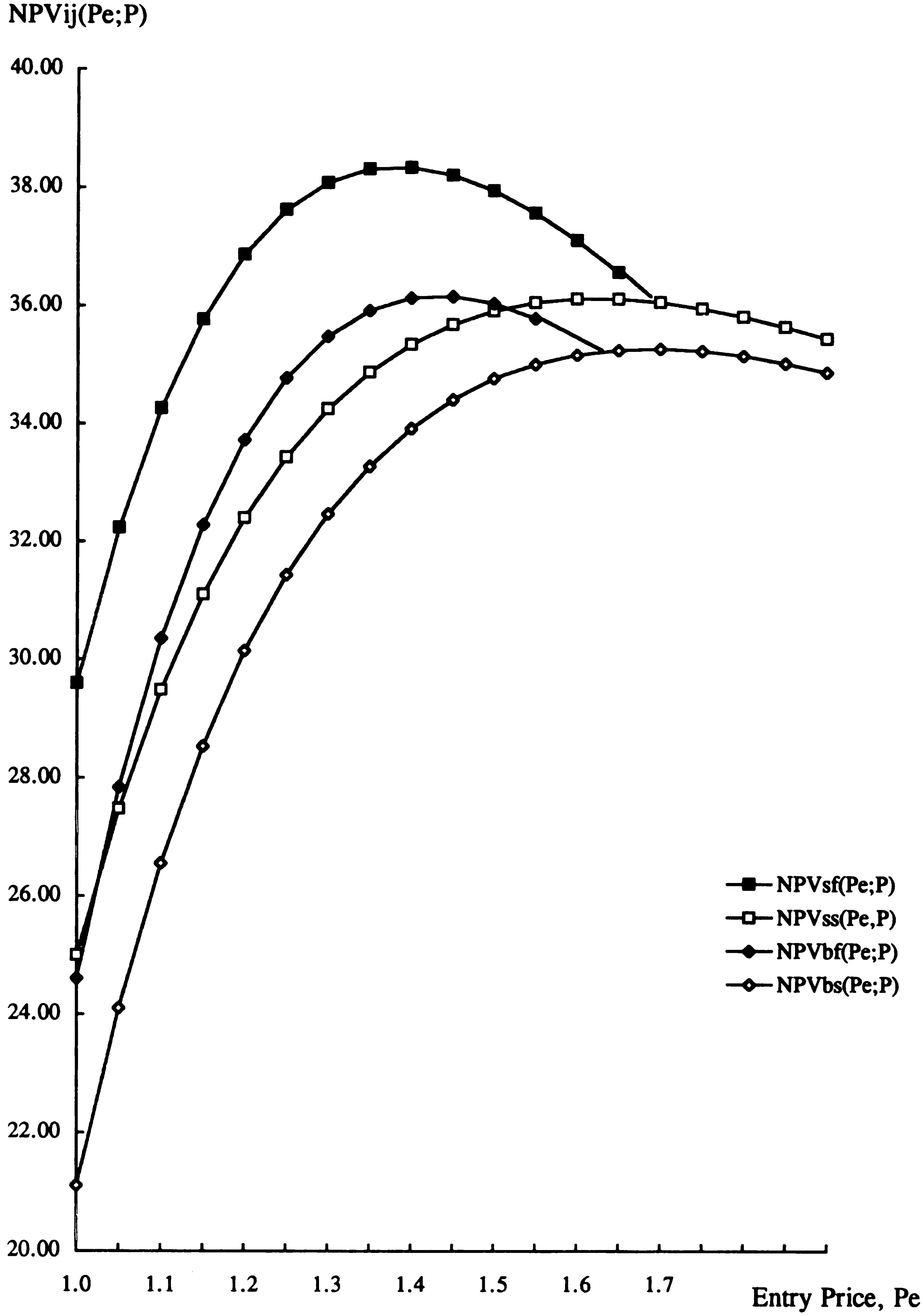

Figure 3.2. Copper Mining Example: Strategic Model

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