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DYNAMIC MODELING OF CLOSED CELL CUSHIONING MATERIAL

presented by

DMITRI V. POKUDIN

has been accepted towards fulfillment
of the requirements for

MASTERS degree in **PACKAGING**

GARY BURGESS

Major professor

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**DYNAMIC MODELING OF CLOSED CELL
CUSHIONING MATERIAL**

By

Dmitri V. Pokudin

THESIS

**Submitted to
Michigan State University
in partial fulfillment of the requirements
for the degree of**

MASTER OF PACKAGING SCIENCE

School of Packaging

1998

ABSTRACT

DYNAMIC MODELING OF CLOSED-CELL CUSHIONING MATERIAL

By

Dmitri V. Pokudin

Closed cell cushioning materials are clearly a multi-phase material system, thus it is not surprising that all attempts to describe their behavior by Hooke's Law and the Gas Law which idealize a single phase material response have failed. The approach taken in this paper separates the polymer matrix and gas contributions, applying appropriate idealizations to each of them. The model does not try to preconceive the simplest idealizations and parameters but rather is used to study the system to find out which phenomena are the most significant and to measure the values of the associated variables.

The study showed that representation of the trapped air as an ideal gas with some heat transfer to the walls and the polymer matrix as a plastic material with an additional rate dependent resistance gives the best results. The model's search algorithm allows all the required system parameters to be found from a single experimental acceleration profile. These are used to build a modeled acceleration profile aimed at replacing the Fourier filtering technique and answer all of the other questions about the cushion behavior. Since the measured coefficients are fairly stable for a given material, cushion thickness, drop height and weight, more drops can be processed to obtain and tabulate their average values. Obtained data is similar in its description of the material properties to the cushion curves which predict the peak G, but it is capable of providing the entire impact profile and complete information about it.

ACKNOWLEDGEMENTS

This thesis would not have been possible the great input from Dr. Gary Burgess. His knowledge of packaging dynamics, physical concepts, patience and miraculous ability to dismiss my wrongful but very convincing arguments has led to the successful completion of this thesis.

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CHAPTER 1

INTRODUCTION AND LITERATYRE REVIEW

Various types of cushioning materials are used in the packaging industry to protect fragile products from damaging levels of acceleration. The mechanism of their actions is to provide separation and some degree of independent movement to the product inside its individual shipping container. By doing so, it is ensured that the shocks experienced by the container are transmitted to the product through the cushion which both absorbs and spreads part of the shock energy over time. Depending on the severity of the expected shocks and the product fragility, different amounts of different cushion materials provide the lowest cost for an adequate protection level. One of the most popular and frequently used materials are cellular polymers.

Cellular polymers used as cushion materials are multi-phase systems that consist of a polymer matrix and a fluid phase. The fluid is generally gas (air) and is either trapped or continuous in the polymer matrix [3]. Thus two major types of polymer cushions exist: closed-cell and opened-cell. The two types of material differ mainly in the gas phase contribution to the cushion reaction force during its compression. Closed-cell air is compressed and reacts back with increased pressure whereas an open-cell air escapes and provides little or no resistance. There is much variation in the material properties even within the types themselves. Cushion properties depend on the exact composition of the polymer matrix, average cell size and wall thickness. Hundreds of different cushion grades are available.

In order to utilize the advantages of one or the other material effectively and minimize the cost, packaging engineers have to consider and evaluate a lot of different

package configurations. It is hardly ever practical to make and test prototypes for all the possible choices. Instead, preliminary estimations are made to narrow the field of cushion materials. Because of the complexity of closed-cell polymers, which I will now concentrate on, none of the attempts to describe their properties in terms of a few idealized phenomena and associated constants were successful. Hooke's Law, which is useful for most solid state materials, does not work [4]. Neither does purely adiabatic or isothermal compression of the trapped air [1]. So in their calculations engineers have to rely on the material properties supplied by the cushion manufacturer in the form of cushion curves. Cushion curves relate the peak G transmitted by the cushion to its thickness, product weight and bearing area for different expected drop heights. Since engineers are faced with hundreds of similar tables and because no automation of the process can be implemented, it becomes very difficult to use them.

As described in [5], each point on the cushion curve has to be obtained from an average of 5-10 drops of a flat massive plate (drop head) directed squarely by guide rods onto the sample cushion. In all, thousands of drops have to be performed and drop head acceleration profiles analyzed. So the cushion curves are very difficult to generate too.

A typical acceleration profile recorded by the accelerometer mounted on the drop head is shown in Figure 1.

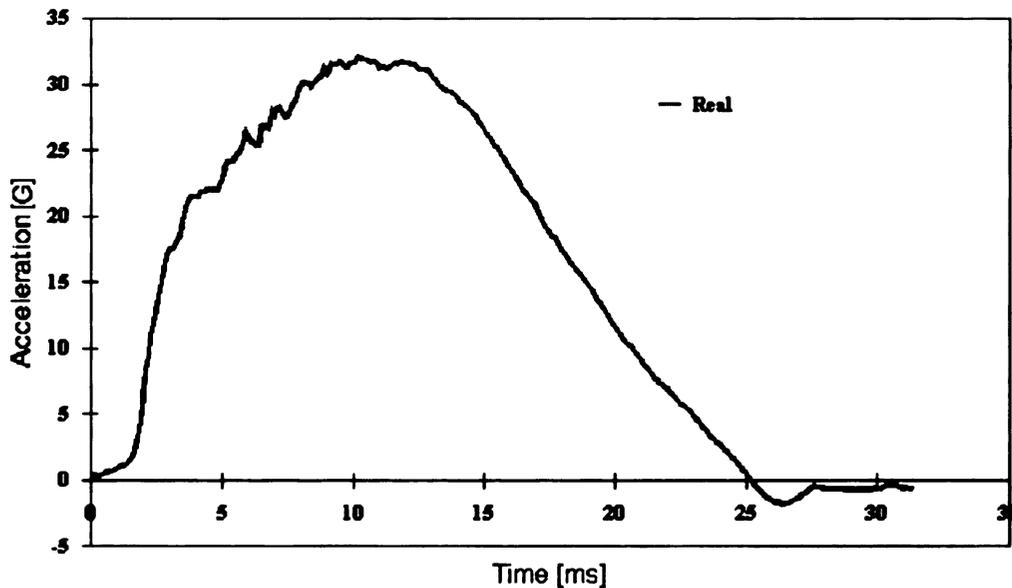


Figure 1. Typical acceleration profile.

The only parameter of interest for the cushion curves is peak acceleration.

Although the general shape of the signal is mainly ignored, much attention is given to its fine structure (ripples). Their presence makes it difficult to determine the exact location and value of the peak acceleration. Moreover since the real acceleration of the massive drop head is expected to be smoother than that, the ripples are attributed to the equipment noise and are usually filtered out before any of the impact parameters are measured. Some of the recorders used in industry have hardware filtering and others do it in their software. Both methods are very similar in their physical nature. Software algorithms expand the real profile into a Fourier series and drop the trailing terms starting from some preselected filtering frequency [6]. Hardware attenuates high frequencies by analog filters. The result produced is very similar and obviously depends greatly on the choice of the filtering frequency. The mathematical apparatus of Fourier filtering does not have anything to do

with the physical phenomena of the process at hand which consequently does not provide any information on the proper filtering frequency to use. Excessively high frequencies do not change the signal very much and may leave ambiguity in peak G readings (Figure 2.) Whereas low filtering frequencies distort the signal too much and may render its other parameters such as impact duration, restitution coefficient and area under the curve needed by some more advanced applications useless (Figure 3.)

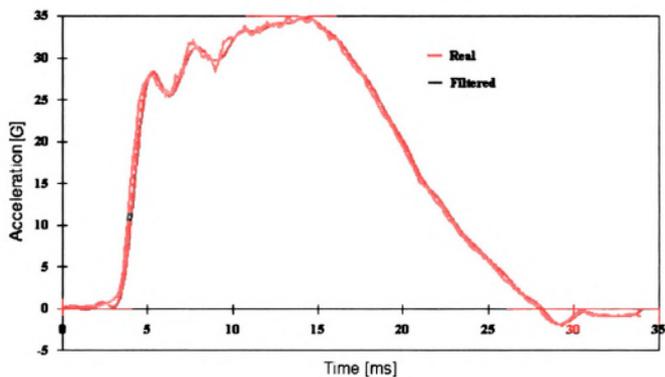


Figure 2. Under-filtering of the accelerometer signal.

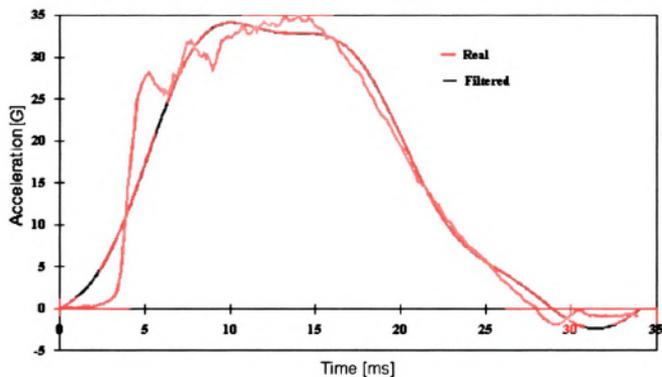


Figure 3. Over-filtering of the accelerometer signal.

As a result, data on the cushion material properties provided by existing signal analysis techniques is incomplete and possibly unreliable.

The questions which prompted this thesis are whether or not the process of collecting and presenting the data on cushion material properties can be simplified.

I wanted to avoid conventional techniques of signal processing and look the entire unfiltered profile in order get more information about the cushion properties, to possibly develop a better method of filtering and loss of important signal features.

CHAPTER 2

MATERIALS AND METHODS

2.1 Philosophy and justification of the modeling approach

In earlier investigations of closed-cell materials, cushions were modeled as a volume of trapped air [1], and as solid elastic and plastic materials [7] and [8]. Theoretical formulas to determine peak accelerations and impact durations were worked out. Obtained results and generated cushion curves were reasonably close to the experimental data. It should be pointed out that in spite of the apparent success, neither of the models in fact accurately described the process and it was pointed out that some complicated approach involving continuous heat transfer has to be taken.

In my approach, I am separating the polymer matrix and gas contributions and applying appropriate idealizations to each of them. To maximize the amount of information obtained from an experimental signal, I am looking not only at the peak G's but at its entire profile. The model dynamically traces all the cushion parameters during the entire impact duration. In order to do this a computer model utilizing a small time step approach was created.

The philosophy behind modeling approach consists of the following: the model attempts to describe the entire process of impacting a cushion with a drop head in terms of known physics laws and the cushion physical characteristics. In order to do this, the physical system is reasonably simplified and defined by a set of model assumptions. Logical guesses and assumptions are made about the unknown phenomena and some characteristics which are unavailable or difficult to measure. Subsequent running of the model with one or another set of unknown variables and comparing the result to the real

shock profile verifies whether the assumptions were right and provides reasonable ranges for the unknown values of model variables. Finally, when the model is well studied and is believed to predict real system behavior with sufficient accuracy, a search algorithm is employed to find a set of variables values which minimizes the mean square difference between the model and real profile. The obtained curve is essentially a least square model fit and can be used in place of the filtered curve to analyze the impact. On the other hand, the model parameters fully describe everything and the curve is not really needed. The approach is similar to the linear regression where some presumably linear data is fitted a line to obtain its slope which is then used in place of the original data.

2.2 Model assumptions

- 1) the basic system set up is shown in Figure 4.

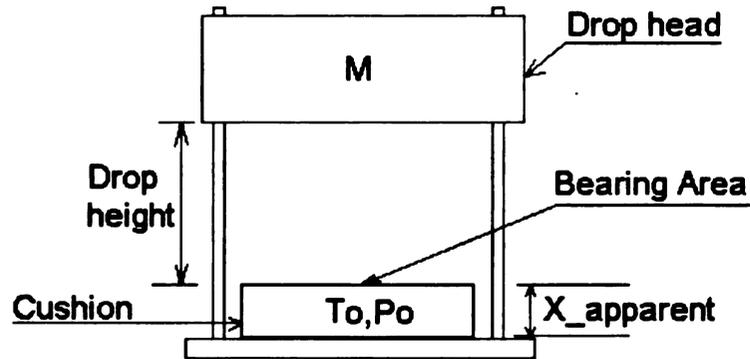


Figure 4. Basic system set up

- 2) The object of investigation is a closed cell cushioning material. Its force producing components are trapped air and matrix of PE bubble walls.
- 3) Cushion cross section area stays constant during compression, thus reducing the problem to one dimension.
- 4) There is no loss of thermal energy to the environment. The low air-to-PE convection heat transfer coefficient (order of $10\text{W/m}^2\text{K}$) and the fact that average duration of the impact event is only 20- 40 milliseconds makes the loss negligible.
- 5) Initially, internal air is in thermal equilibrium with its environment and has normal atmospheric pressure.
- 6) All trapped air is considered to be a single volume of ideal gas. Van der Waals corrections for initial and maximum compression states result in less than 0.1% difference compared to the Ideal Gas Law.

- 7) The heat transfer to the PE walls is not negligible because of the big contact area between internal air and walls[1].
- 8) The internal energy and thus the temperature of the walls changes as a result of the heat transfer and work done by the friction forces. The walls are much more massive than the air and have greater heat capacity, thus the change is small. (Even if all the energy of the falling mass is absorbed by the PE only its temperature will be raised only by approximately 1 °C. For the purpose of air behavior, this can be disregarded, but is kept later to estimate cushion recovery times.)
- 9) The matrix of PE walls resist change of their height with forces proportional to the rate and the rate squared during the entire impact, designated by $F^V(t)$. They also resist with some additional force as a function of relative compression itself, $F^X(t)$.
- 10) The guide rods of the testing equipment can also have rate dependent friction, which is indistinguishable from the force inside the cushion and is taken in account by the same $F^V(t)$ term.
- 11) Some of the cushion thickness consists of damaged cells. This is taken into account by decreasing effective thickness.
- 12) For the entire time of the impact drop head face and the cushion surface stay together. The end of an impact is defined by the moment of drop head and cushion separation. If some of the friction comes from equipment, head acceleration at that moment will not be equal to zero. So the cushion condition for the end point is $P(t_{end}) = P_0$.
- 13) Model input data consists of system constants describing major parameters of the physical set up including test cushion dimensions:
 - X_{apparent} - Cushion thickness.

- P_0 - Initial equilibrium air pressure.
- **Area** - Cushion area.
- M - Mass of the drop head.
- m - Cushion or PE mass.
- H - effective drop height calculated from experimental impact velocity.
- T_0 - Initial equilibrium temperature.
- C_v, ρ - Heat capacity and density for air.
- C_{pe} - Heat capacity for PE.

14) The search algorithm needs $A^{\text{experimental}}$ vs time - experimental drop head acceleration profile.

15) The following model variables are inputted or searched for by the algorithm for each individual test:

- F^X, F^V - Resistance forces of the PE matrix.
- h - PE-to-Air convection heat transfer coefficient.

16) For given system constants and model variables, the model generates:

- $A(t)$ Predicted acceleration profile. Time positions of any point of interest (peak G, duration, etc.) can be precisely read right from the graph.
- **Fit coefficient** - mean square deviation between $A(t)$ and $A^{\text{experimental}}(t)$.
- G_{max} - Peak acceleration in G.
- $t(G_m)$ - Time position of G_{max}
- X_{min} - Drop head position above the base plate at the moment of maximum compression.

- X_{final} - drop head position at the moment of its separation from the cushion.
- $t(T_a=T_w)$ - time when temperature of the air is equal to the temperature of the walls. Walls start to cool after that.
- $t(V=0)$ - time of the moment when direction of velocity reverses (same as maximum compression).
- dT_{max} - given for air and walls as the maximum temperature change reached by them over the initial conditions.
- dT_{final} - given for air and walls as the temperature change over the initial conditions at the end of the intact.
- **Restitution Coefficient**- Final velocity of drop head divided by its initial velocity.

15) The model uses dt - variable time step in order to find a trade off between the model accuracy, execution time and round off error.

2.3 Derivation of the model's physical foundation

The physics of the small step model is based on starting with initial conditions and propagating them in time by adding changes incurred on them over the small period of time. The major parameters of the system are the position, velocity and acceleration of the cushion drop head boundary. The initial values are:

$$\begin{aligned} A(0) &= -g \\ V(0) &= -\sqrt{2 \cdot g \cdot H} \\ X(0) &= X_{\text{apparent}} \end{aligned} \quad (1)$$

Where g is acceleration due to gravity. Due to extreme smallness of chosen time step dt , the total force on the drop head over this period is considered to be constant.

Acceleration, velocity and position of the drop head at any moment of time t can be expressed using Newton's second law and its basic integral forms together with acceleration, velocity, position and total force at the $t-dt$ moment of time. It is convenient to denote position $t-dt$ with an integer n and t as $(n+1)$.

$$\begin{aligned} A_{n+1} &= \frac{F_n^{\text{total}}}{M} \\ V_{n+1} &= V_n + A_n \cdot dt \\ X_{n+1} &= X_n + V_n \cdot dt + \frac{A_n \cdot dt^2}{2} \end{aligned} \quad (2 \text{ a, b, c})$$

Repeating the calculations the necessary amount of times, we can obtain position, velocity and acceleration as a discrete functions of $t=0, dt, 2dt, 3dt, \dots, ndt$ for the entire impact duration. The total upward force is

$$F_n^{\text{total}} = F_n^{\text{air}} + F_n^V + F_n^X - P_0 \cdot \text{Area} \quad (3)$$

where F_n^{air} is the force due to the compressed air and F_n^V, F_n^X are due to the resistance of PE walls as a function of rate and position. According to the assumptions made:

$$\begin{aligned}
F_n^V &= C1^V \cdot V_n + C2^V \cdot V_n^2 \\
F_n^X &= \text{Some positive function of } X
\end{aligned}
\tag{4 a, b}$$

where $C1^V$, $C2^V$ and F^X are unknown variables of the model. Later some specific forms of F^X are introduced.

Let me next address $F^{air}(t)$, the last calculable unknown needed to complete the model:

$$F_n^{air} = -P_n \cdot Area \tag{5}$$

P_n is propagated similarly from its initial value P_0 . The necessary iterative relationship similar to (2) is based on the assumption that air compression follows the Ideal Gas Law.

In one time interval dt , air undergoes a transition between the two states:

$$\{P_n, V_n^{air}, T_n^{air}\} \rightarrow \{P_{n+1}, V_{n+1}^{air}, T_{n+1}^{air}\}$$

For any process involving ideal gas, the relationships

between absolute temperature, pressure and volume are [9]:

$$P_{n+1} = P_n \cdot \frac{T_{n+1}^{air} \cdot X_n}{T_n^{air} \cdot X_{n+1}} \tag{6}$$

Here the assumption of constant cross-sectional area and thus $V^{air} = Area \cdot X$ is used. X_n could be replaced by its expression from (2,c) or left like it is noting that it should be evaluated before P_{n+1} is needed. The expression for T^{air} is worked out below and presented in (12 a).

Using the assumption that the system does not lose energy to the environment, the first law of thermodynamics applied to the whole cushion can be used to write an energy balance for every transition from n to the $n+1$ states:

$$dU_n^{air} + dU_n^{walls} + dE_n^{walls} = dW_n^{air} + dW_n^{walls} \tag{7}$$

Here dU_n is a change of internal energy of the air and thermal energy in the walls and dW_n

is the work done by external forces on the air and walls air and walls. dE_n is the mechanical energy associated with the wall compression (later in the case of elastic compression it will be equal to the work of F^X and zero for plastic deformation) At the same time, some thermal energy dQ_n was exchanged between air and walls. So (7) can be separated in two equations:

$$\begin{aligned} dU_n^{air} &= dW_n^{air} - dQ_n^{a \rightarrow w} \\ dU_n^{walls} &= dW_n^{walls} - dE_n^{walls} + dQ_n^{a \rightarrow w} \end{aligned} \quad (8 \text{ a, b})$$

The heat term dQ_n is proportional to the convection heat transfer coefficient, contact area, process duration and average wall-to-air temperature difference. For small dt , both T^{air} and T^{walls} can be considered to be constant and equal to their respective values in the beginning of the transition.

$$\begin{aligned} dQ_n^{a \rightarrow w} &= h \cdot A^{a \rightarrow w} \cdot (T_n^{air} - T_n^{wall}) \cdot dt \\ A^{a \rightarrow w} &= \frac{6 \cdot X_{\text{apparent}} \cdot \text{Area}}{D} \end{aligned} \quad (9 \text{ a, b})$$

Here air to wall contact area $A^{a \rightarrow w}$ is estimated from consideration of the PE average cell dimension, $D = 0.00114$ m, representing them as cubes or spheres. It stays constant through out the entire compression in spite of the change in cushion geometry. Estimation of the convection heat transfer coefficient h represents the most difficulty. One estimate on it can come from considering the average dimensions of the PE cells and static heat transfer coefficients for PE and air . Careful consideration of the situation is presented in [1]. It gives $h=44.8\text{W}/(\text{m}^2\text{K})$. Another estimation is based on experimental data for free convection in spherical cavities and is also given in [1] but actually comes from [10]. After consideration of the average cell size and a reasonable range of temperature differences gives $h=21.9\text{W}/(\text{m}^2\text{K})$. In view of such differences and the fact that heat transfer is likely

to be accelerated by the rapid change of the cells geometry, h will be left as an unknown model parameter.

Using similar considerations of small dt and constancy of pressure and resistance forces, the work done by the air is $dW^{air} = P \cdot dV$ and by the walls is $dW^{walls} = (F^v + F^x) dx$.

$$\begin{aligned} dW_n^{air} &= P_n \cdot (V_n^{air} - V_{n+1}^{air}) = P_n \cdot (X_n - X_{n+1}) \cdot Area \\ dW_n^{walls} &= (F_n^v + F_n^x) \cdot (X_n - X_{n+1}) \end{aligned} \quad (10 \text{ a, b})$$

The changes of internal energy of the air and thermal energy of the walls are proportional to the changes in their absolute temperatures.

$$\begin{aligned} dU_n^{air} &= C_v \cdot (T_n^{air} - T_{n+1}^{air}) \cdot Area \cdot X_o \cdot \rho \\ dU_n^{walls} &= C_{PE} \cdot (T_n^{walls} - T_{n+1}^{walls}) \cdot m \end{aligned} \quad (11 \text{ a, b})$$

Specific heats C_v and C_{PE} could be considered constant for the given temperature and compression ranges. Their respective values are 717.3 and 1003.8 J/(kg*K) [11].

Equations (8),(9), (10) and (11) can be put together to obtain iterative expressions for T^{air} and T^{walls} ,

$$\begin{aligned} T_{n+1}^{air} &= T_n^{air} - \frac{dW_n^{air} - dQ_n^{a \rightarrow w}}{C_v \cdot Area \cdot X_o \cdot \rho} \\ T_{n+1}^{walls} &= T_n^{walls} - \frac{dW_n^{walls} - dE_n^{walls} + dQ_n^{a \rightarrow w}}{C_{PE} \cdot m} \end{aligned} \quad (12 \text{ a, b})$$

It should be mentioned that (12 b) is worked out for the most general case of some part of friction work being converted to heat. Later when the nature of friction is discussed, additional conditions of $dE^{walls} = F^X \cdot dx$ (elastic) and $dE^{walls} = 0$ (plastic) will be taken into consideration.

Now all the formulas can be implemented in the computer program shown in Appendix.

2.4 Study and verification of the model's assumptions

2.4.1 Heat transfer considerations

Running the model with no F^y or F^x and setting the heat transfer coefficient h to zero gives the adiabatic model described in [1] and shown below by Model 1. Setting $h = 1000000$ (a reasonable number for infinity in computer applications) gives the isothermal model. It is shown below by Model 2 in Figure 5. Maximum air and wall temperatures there are raised only by 2.43 and 1.07 °C above $T_0 = 295$ K (room temperature) which is negligible.

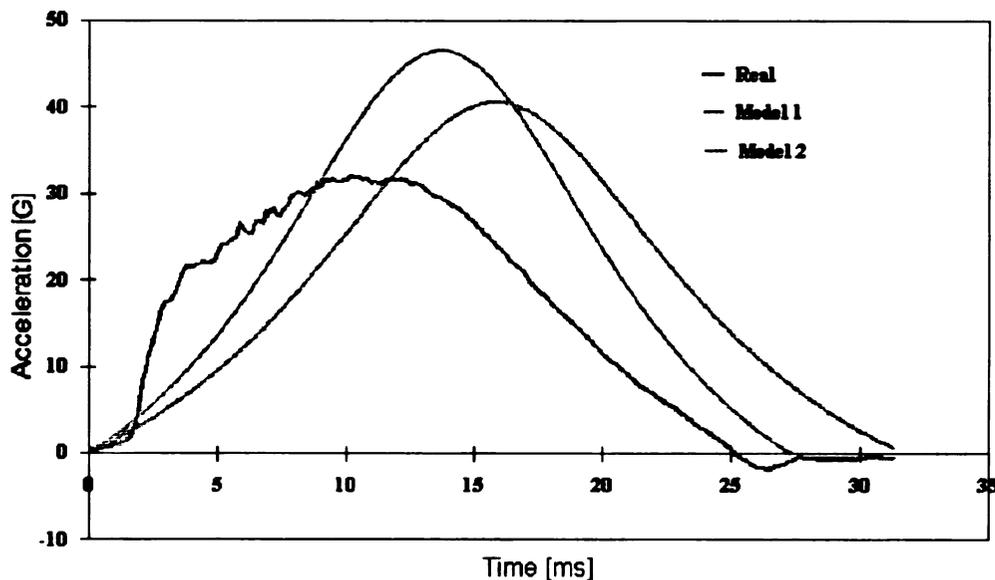


Figure 5. Adiabatic and Isothermal model checks

Both peak accelerations agree well with the theoretical formulas derived for them in [1]. This fact proves the validity of the small time step approach. However, both models give an overestimation of peak G when in [1] adiabatic was an over estimation and isothermal underestimation of cushion curve data. This happens first of all because my real signal is a

profile of a first drop on the cushion, not the average of five consecutive drops as it is done for the cushion curves and secondly it is very likely that the manufacturer put some safety factor in their curves.

The major differences between the model signal profiles and the real signal do not leave any doubt that neither isothermal nor adiabatic processes are dominant contributors in cushion reaction forces, especially during the first half of it. To see whether the introduction of heat exchange improves the situation, the model was run with $h = 22$ and $43 \text{ W}/(\text{m}^2\text{K})$, shown in Figure 6 by Model 1 and Model 2 respectively.

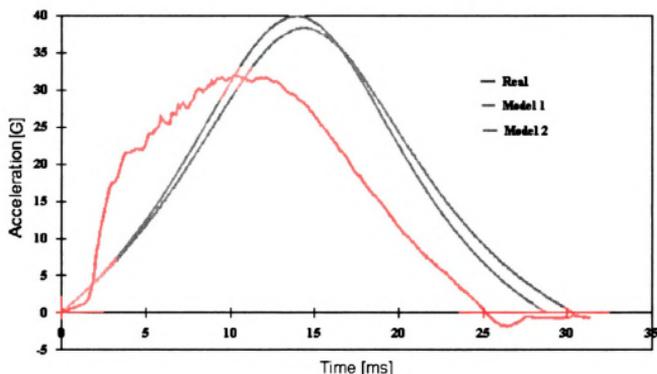


Figure 6. Introduction of heat transfer coefficient $h=22$ and $h=43$.

The good thing is that the peak G and the general shape of the signal does not appear to be very sensitive to the changes in h . A two fold change in h produced only a 5% difference in peak G and the results are just somewhere between the limiting cases of the purely adiabatic and isothermal models (not that it was unexpected). But introduction of

heat transfer did not make the model profile more like the real signal. Its peak G's and general shape are still wrong.

But heat transfer it is not altogether insignificant. It helps to address the mechanism of residual cushion compression at the end of the impact and consequently its recovery times. Up to the moment of maximum compression both the air and walls are heated up by compression heating and heat transfer. Then on the way up, the air expands and cools down, but for some time it is still warmer than the walls and continues to heat them. At some moment later, the air temperature catches up with the wall temperature and the walls start to cool as well. The condition of the impact end $P(t_{\text{end}}) = P_0$ is reached well before the walls have time to reach, but enough for the air to cool below T_0 . In the models shown above with $h = 22$ and $43 \text{ W/(m}^2\text{K)}$, the maximum temperature reached by air was respectively 49 and $31 \text{ }^\circ\text{C}$ above T_0 . In the end, it cooled down to 31 and $22 \text{ }^\circ\text{C}$ below T_0 . The walls reached 0.82 and $0.99 \text{ }^\circ\text{C}$ at 23.3 and 18.6 ms respectively, which is 82% and 62% of the impact time, after which it had enough time to cool only to 0.58 and $0.43 \text{ }^\circ\text{C}$ above T_0 . The fact that the air is colder than the initial conditions means that the cushion volume and thickness are reduced, referred to as "permanent set" in [5]. Starting from 0.054m thickness the models ended up at 0.0481 and 0.0501 . To observe the situation, the same plots shown above can be integrated to produce acceleration as the function of cushion thickness.

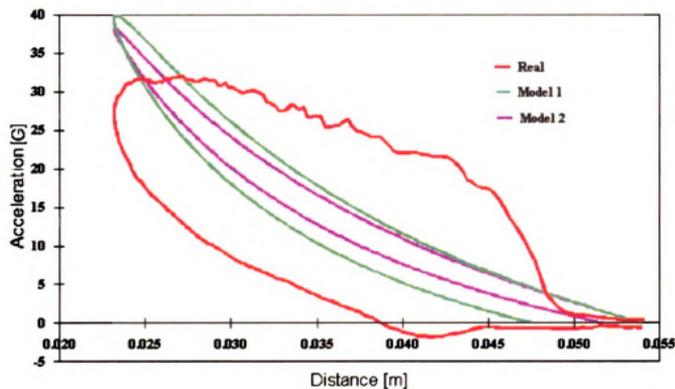


Figure 7. Heat transfer models acceleration vs. X

Here it looks like only Model 1 intersects the X axis at 0.0481 m exactly right where it is supposed to, whereas Model 2 is off its 0.0501 m mark mentioned above. This is caused by the fact that actual axis conversion has been made on the basis of Model 1. Repeating the same for Model 2 gives the right result too.

2.4.2 Defining the real time of cushion contact

Further investigating the shock profile, one should notice that for the first 1.5 ms or 0.007m of travel, the real acceleration is considerably lower than predicted by any of the models. This can not possibly be remedied by the addition of F^V or F^X as they would only increase the model response force. Careful consideration of the possible causes for the phenomenon leads to the realization that this section is due to the viscous forces resulting from displacement of thin layer of air by the rapidly oncoming drop head. Both the drop head and cushion are really experiencing this force. As a result, right before the

actual impact, the cushion is slightly presqueezed and the drop head decelerated. In order to take these affects into account, the real time of impact should be placed at the intersection of x-axis and a linear extension of the impact leading edge, the deceleration of drop head calculated and the effective cushion thickness should be measured under slight pressure.

2.4.3 F^V considerations

Now the real cushion response at time of contact is not equal to zero. There is no air compression and F^X has hardly started yet so the force had to have come from rate dependent forces. This condition can be used to estimate C1 or C2 or their combination needed for F^V in (4 a). Making such estimations for C1 and taking C2=0 in Model 1.

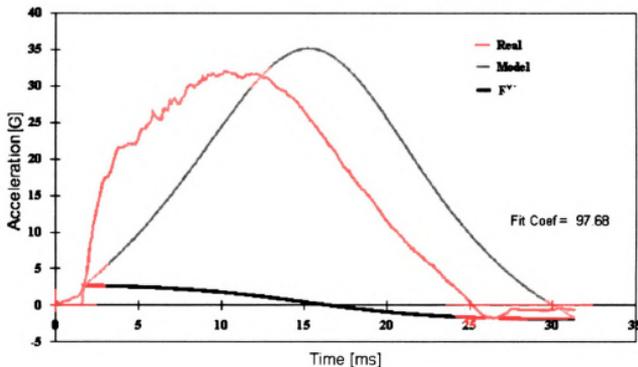


Figure 8. Addition of F^V to explain the reaction force at the time of contact.

F^V pushes Model in the right direction towards better fitting the real signal, shown in Figure 8. It also explains the negative acceleration on the trailing edge of the real signal profile. But the effect is not nearly powerful enough. Estimating C2 or the C1,C2 combination gives very similar results. So the contact time force condition is probably not useful. Letting the computer find the best fit in terms of C1 and C2 gives Figure 9.

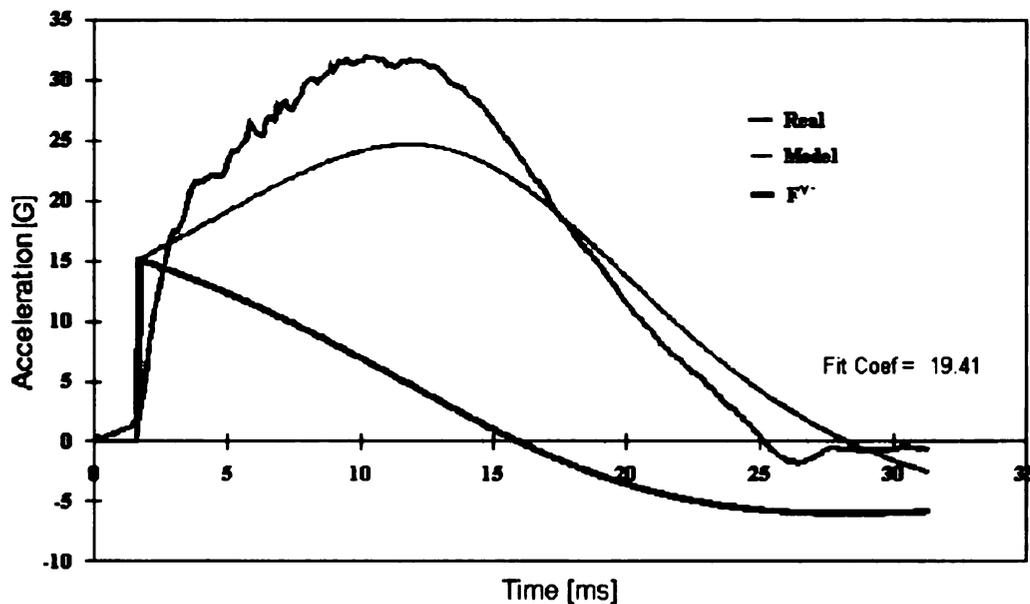


Figure 9. F^V adjusted for the best fit

This model behaves much more like the real signal. The fit coefficient went down to 19.4, meaning that the average error in the predicted and actual accelerations is 4.3 G. The fact that the leading edge of the model is vertical is discussed later.

2.4.4 F^X and X_0 considerations

The next variable which can improve the model is F^X mentioned in (4 b). Since F^X is unknown, the approach is to first look at what F^X profile is required to make the model

behave exactly as the real signal does. Since F^X is a function of position, it helps to look at acceleration vs. position as in Figure 10.

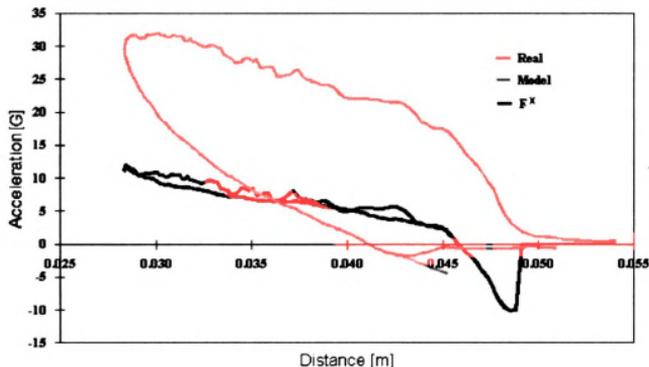


Figure 10. First estimation of possible F^X (Elastic)

I should disregard the leading edge up to 0.045m there F^X is negative as it happens only because it is compensating for too much F^Y there. Otherwise it looks almost linear, increasing as compression grows. Linear extension of the profile intersects the X axis at position of contact. This is exactly the behavior one would expect from any elastic material. So one way to represent F^X is as an elastic force that obeys Hooke's Law: $F^X = \text{Area} * Y * dx / X_0$ where Y is Young's modulus for the material: unfortunately it is not available for the complex structure at hand, but I can let the model find its value for the best fit. Remember that model should be adjusted to reflect the fact that elastic compression does not heat the walls: all of the work goes to the mechanical energy

(condition $dW^X = dE^X$ in (12 b)). There dW^{wallis} is not separated in dW^X and dW^V but it is easily done.

Another F^X form which I think could be applicable to such a material as PE especially at high compression ratios is elastic - perfectly plastic . Its behavior is elastic up to some critical value of relative compression (strain). Then the force stays constant (perfectly plastic) all the way until maximum compression and becomes elastic on the way back. Young's modulus and thus the stress vs. strain curve slopes stay the same during expansion as it was during compression. This approach can only improve the model because it includes the previous one in it. If the model does not prefer the elastic - perfectly plastic approach, it will find during the search that the length of plastic plateau is equal to zero thus returning to the elastic representation. This by the way is exactly what happens.

While changing F^V in the process of looking for best fit (and varying X_{apparent} usefulness of which will be shown later) I saw that another shape of an arbitrary F^X can work too, as in Figure 11.

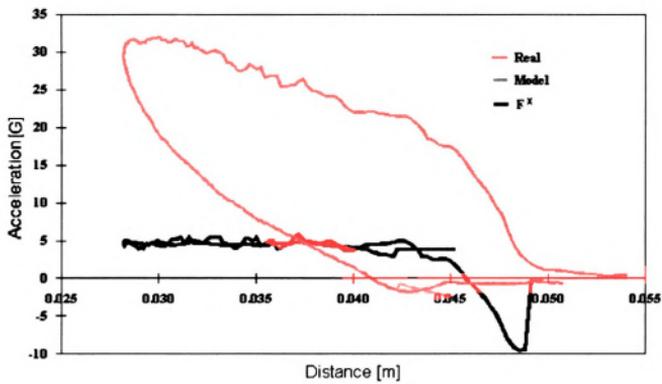


Figure 11. Second estimation of possible F^X (plastic)

This tells me that another form of F^X as a constant (perfectly plastic) should not be ignored.

Introduction of an elastic F^X improved both the general appearance and the fit coefficient.

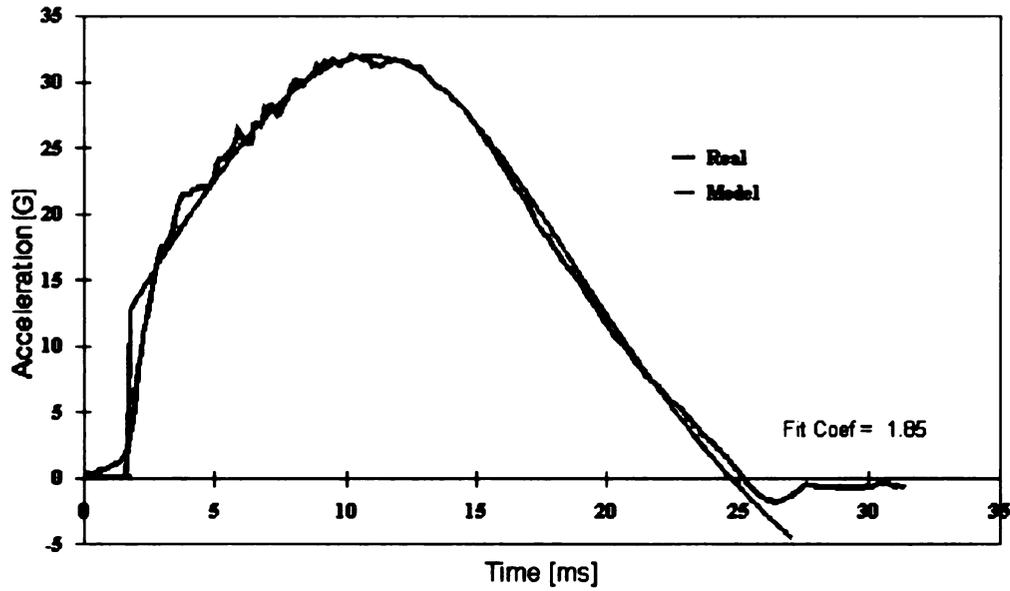


Figure 12. Fitted model curve with elastic F^x

Fitting all five experimental sets of data and tabulating the results obtained gives Table 1.

Table 1. Fitted results for Elastic F^x model.

	Mass, Drop Height	Fit coefficient	$C1^V$ [N*s/m]	$C2^V$ [N*s ² /m ²]	Young's modulus [kN/m ²]
1	17.8 kg 0.4577m	1.9	706	0	79950
2	17.8 kg 0.2543m	1.1	756	0	166909.5
3	17.8 kg 0.6611m	2.9	600	0	38502.38
4	8.7 kg 0.6611m	6.72	564	0	224516.7
5	8.7 kg 0.4577m	9.32	538	0	310016.7

Even the worst fit coefficient of 9.32 looks fairly good as shown in Figure 13.

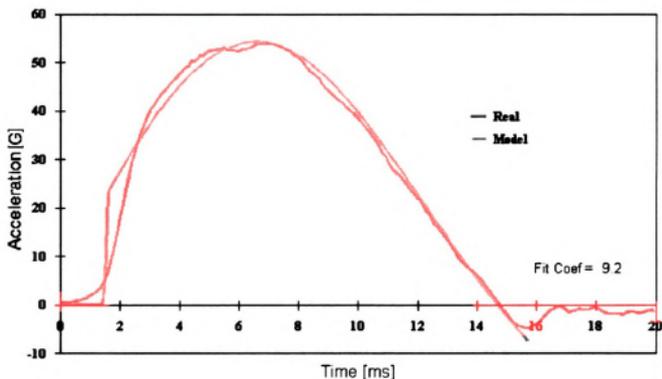


Figure 13. Worst fit for the model curve with elastic F^X

So apparently the model with an elastic F^X has enough flexibility to be used for filtering. But the fact that Young's modulus varies so much means that it is not physical and thus not acceptable. Trying the elastic-plastic approach gives identical results because the search algorithm finds best fits when the length of plastic plateau is equal to zero leaving only elastic part.

Going back to the possibility of perfectly plastic F^X and fitting it to all five experimental sets of data and tabulating the results gives Table 2.

Table 2. Fitted results for Plastic F^X model.

	Mass, Drop Height	Fit coefficient	$C1^V$ [N*s/m]	$C2^V$ [N*s ² /m ²]	F^X [N]
1	17.8 kg 0.4577m	14.56	709	0	892.6472
2	17.8 kg 0.2543m	28.08	780	0	2280.511
3	17.8 kg 0.6611m	6.34	580	0	523.854
4	8.7 kg 0.6611m	72.6	612	0	869.6859
5	8.7 kg 0.4577m	120	700	0	1043.794

This is not an improvement at all. As a matter of fact, fits to data 2,3 and 5 are clearly not physical. Their shape is not what one would expect which is reflected in the fit coefficient too. This is shown below in Figure 14.

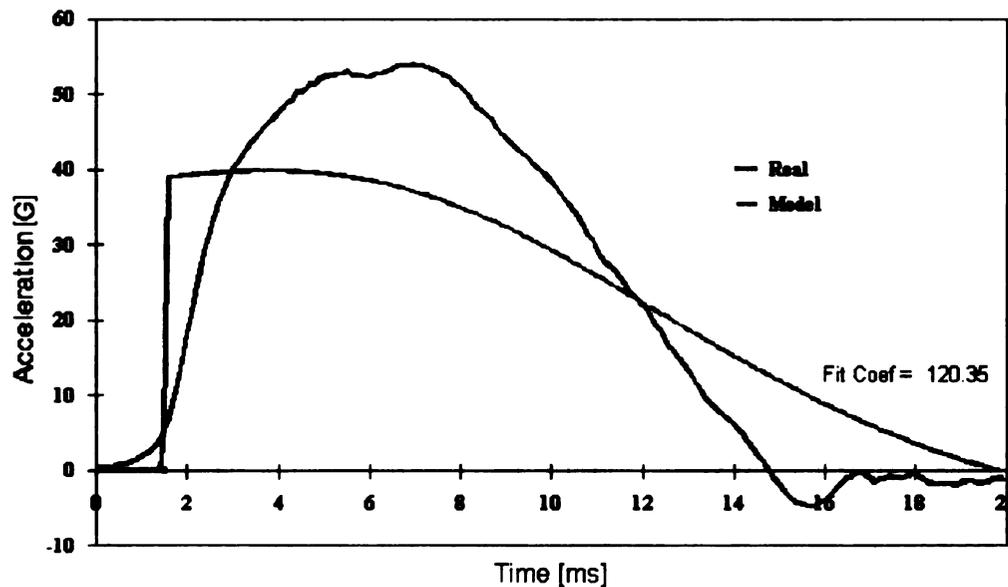


Figure 14. Non physical result for plastic F^x

But the fact that the two drops with the most energy are still in semi acceptable shape says that the factor that is not taken into consideration by the model is connected to the energy of the drops. One of the possible things this factor can be is X_o , the effective thickness of the cushion. As of now it is taken as the apparent cushion thickness minus the PE Volume/Area ($X_o = 0.054m$). Physical observations of cushion compression under static load shows evidence that the central regions of the cushion experience smaller relative compression than the ones next to the cushion faces. Moreover, smaller than average size

cells if they happen to have thicker than average walls get compressed less or hardly at all. All this justifies the reduction of cushion thickness in the model, but does not give any realizable estimates of how much. It essentially introduces another unknown variable in the model. X_o can not be greater than the apparent thickness and is probably not less than 10-20% of it.

Conducting a search for the best X_o using the elastic and elastic plastic F^X does not lead anywhere. Fit coefficients there are too small already and the search sets $X_o = X_{\text{apparent}}$ to produce the results identical to the ones shown in Table 1. The plastic F^X approach gives the results in Table 3.

Table 3. Fitted results for Plastic F^X model with adjusted X_o

	Mass, Drop Height	Fit coefficient	$C1^V$ [N*s/m]	$C2^V$ [N*s ² /m ²]	X_o/X_{apparent}	F^X [N]
1	17.8 kg 0.4577m	2.58	625	0	0.7822	668.7
2	17.8 kg 0.2543m	2.53	633	0	0.53	590.2
3	17.8 kg 0.6611m	4.21	545	0	0.88	576.2
4	8.7 kg 0.6611m	13.66	544	0	0.65	483.0
5	8.7 kg 0.4577m	14.42	508	0	0.543	507.8

Now the fit coefficients are much better and there is some consistency in F^X , $C1^V$ and good logic in X_o (it is decreasing for smaller weights and smaller heights). Fit coefficients and the general shape of the curves are similar to the ones shown in Figures 12 and 13.

Relative success of the plastic F^X does not mean that elastic can not do better. The difficulty there is that X_o is acting as a parameter by itself and affects Young's modulus too. It so happens that the latter effect is stronger and pulls the model towards $X_o = X_{\text{apparent}}$ and away from a consistent Young's modulus. A massive search algorithm which takes

several sets of data, puts first priority on Young's modulus consistency and looks for the minimum square deviation across all the data simultaneously can separate that dependency. Unfortunately this is not straightforward and has to be left for later. For now I can insist that the same type of X_0 reduction should happen during both model approaches. I can verify the possibility by forcing X_0 to be equal to its values in Table 3. Running a set of fittings for the elastic F^X again gives the results in Table 4. The numbers marked by a * in the table were given the value to keep overall consistency of entire table. They were estimated using an iteration procedure of putting initial values for X_0 to get best Young's moduli, then using its average to estimate new best values for X_0 and over again.

Table 4. Fitted results for Elastic F^X model with adjusted X_0

	Mass, Drop Height	Fit coefficient	$C1^V$ [N*s/m]	$C2^V$ [N*s ² /m ²]	$\frac{X_0}{X_{apparent}}$	Young's modulus
1	17.8kg 0.457m	4.28	720	0	0.78	34013.6
2	17.8kg 0.254m	3.49	618	92	0.53	32074.8
3	17.8kg 0.661m	7.78	572	30	0.88	34013.6*
4	8.7kg 0.661m	15.89	471	37.5	0.65*	35430.8
5	8.7kg 0.457m	17.94	585	2.87	0.50*	44977.3

So Young's modulus can be constant with out introducing too much disturbance in other parameters. It means that the elastic F^X can be as good as plastic. Individual differences between different drops of the same weight from the same height are greater than the difference between the two models. As a matter of fact, in reality it is most likely that a combination of the two is working. Knowing the limiting cases is enough to get a good fit and estimate all the other parameters. The plastic F^X model has slightly better fit

coefficients and is much more stable during the automatic searches, so later it is more convenient to use it for filtering and tabulation of average values for the model coefficients.

2.4.5 Justification of the model's use as a filtering tool

The stability of the filtering technique by fitting the model to the real shock pulse is verified by checking if it can filter out artificial noise in the form of a sine wave.

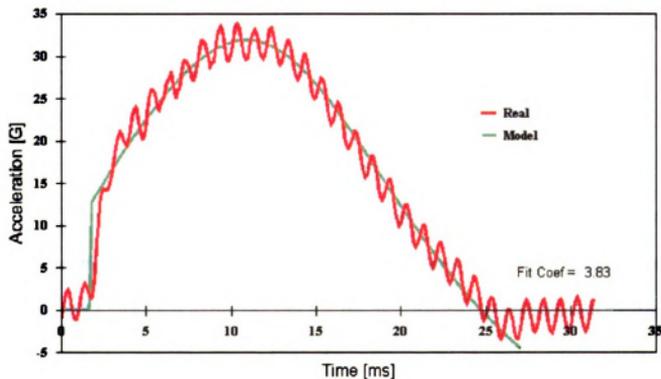


Figure 15. Fit filtering of a noisy signal.

In the example shown in Figure 15, a 1000 Hz sine wave (noise) was added to the acceleration profile # 1 from Table 3. Running the model's search algorithm and bringing the coefficients together in Table 5, we can see that the results of the fit filtering are practically identical.

Table 5. Results of fitting original and noisy signals

	Signal fitted	Fit coefficient	$C1^V$ [N*s/m]	X_o/X_{apparent}	F^X [N]
1	Original	2.58	625	0.7822	668.7
2	Noisy	4.64	620	0.7834	671

The fit coefficient was expectedly higher, but over all, the model was not confused by the additional noise. Increasing the noise frequency gives even better results. Higher or lower noise amplitudes appears not to have any affect at all. Frequencies below 500 Hz should be avoided, but those usually have low amplitudes and do not matter any way. So the model appears not to be sensitive to sine wave noise too much and probably it is as effective in filtering out natural noise in the signal as well.

2.4.6 Verification of suggested cushion recovery time

There is still no real handle on the heat transfer coefficient h because the difference in fit coefficients for $h=22$ and $h=43$ with all the other coefficients kept the same is on the order of 5%, which is easily absorbed in the uncertainties of all other coefficients. Heat transfer coefficients taken outside the 5 to 50 range consistently give worse fit coefficients which cannot be compensated by any other parameters. For the absence of any better choice, I will take $h=32$ for the fit filtering and results tabulation. Now for this fixed value of h the model gives final temperatures for the air and walls for example for the #1 drop in Table 3 of 0.56 °C and -23.24 °C above initial temperature respectively. It will take heat transfer and work of air expansion against P_o only 18.7 ms to warm up the air and cool the walls to reduce the difference by a factor of ten. Unsteady state heat transfer is dominating the work factor in the energy loss process and it is an exponential

process, so it is not surprising that the next factor of ten reduction is reached at 37.5ms (roughly double the time). Thus by the cushion recovery time recommended in [5] of one minute between the consecutive test drops is over the air-to-wall the temperature difference would be reduced by about 3000 orders of magnitude (overkill). On the other hand, both temperatures will converge on the value which is 0.5 °C higher than the initial temperature. The time to dissipate this energy can be estimated. Because of poor heat conductivity of the cushioning material, it will clearly be more than a few minutes. The energy of consecutive drops will accumulate even if the one minute recommendation is followed. After a dozen or so drops, the elevated cushion temperature could be felt by hand. Fortunately, raising the initial temperature by even 10 °C does not introduce too much difference. For the above mentioned example, peak G would increase by 0.5 G which is only 1.5 %.

2.4.7 Consideration of model's vertical leading edge

The fact that model has vertical leading edge and real signals never do can be explained by several factors:

- Cushions can not possibly have perfectly uniform thickness so they meet the drop head with the high point first and then rapidly increase the area of contact.
- The top layer of cells is no doubt damaged more than the center so the F^V coefficient is smaller while these are compressed and is rapidly reaching a constant value. This explanation has a supporting evidence of slight and not explained by anything else increase in cushion response right before the end of impact.

- **Some of the restricting force comes from the slider rods. It is practically zero when the head is freely sliding. As the head comes into contact with the cushion, the reaction force is likely not to be passing exactly through its center of mass, gradually creating a rotational moment which jams the rods and increases their friction.**

No matter which factor or what combination of them is really working, this can be taken into account by adding just one more variable. My estimations show that it will drastically improve the fit coefficient but does not change any other important parameters by more than 1-2%, so I will not complicate the model.

CHAPTER 3

RESULTS AND CONCLUSIONS

3.1 Corrections and additions to the model assumptions.

As a result of the model study, the following corrections and additions to the original model assumptions have been made:

- X_0 -effective cushion thickness is introduced as a model unknown.
- C_2^V Should be completely dropped from the model since it is consistently found that best fits have it equal to zero indicating no F^V dependence on second order of compression rate. This does not contradict any physics of the system because there is no considerable air flow with which the second order of rate resistance is usually associated.
- Plastic F^X is preferable to elastic not only because of the ease of use but because of better fits, stability of the model coefficients, and the plastic nature of PE. The condition $dE_n=0$ is used in (12 b). It should be mentioned that the latter condition practically affects only the temperature of the PE walls and not that much either since $F^V > F^X$ for the most of time.
- The heat transfer coefficient is taken to be $32 \text{ W}/(\text{m}^2\text{K})$, the middle of its reasonable range for the lack of better choice. Fortunately, because of the model's extreme insensitivity to h , this does not render the model inaccurate.
- The vertical leading edge of the modeled profile is ignored for the lack of verifiable physical explanation and its little overall effect on the model.

3.2 Model use for tabulating the material properties

Now with the model studied and complete, given a new cushion it is enough to make several drops to get averages on F^x , X_o , and $C1^v$. Within reasonable limits, they could be interpolated for any other real situation. Using the interpolated values and running the model we can receive an estimated acceleration profile and predict the peak G, impact duration, restitution coefficient, maximum compression as well as almost anything else one can think of. For example, using Table 3 we can interpolate model coefficients for a {17.8 kg 0.557m} and {13.25 kg 0.6611m} drops. They are given in Table 6.

Table 6. Interpolated parameters for an arbitrary drops.

	Mass, Drop Height	$C1^v$ [N*s/m]	X_o/X_{apparent}	F^x [N]
1	17.8 kg 0.557m	585	0.83	615
2	13.25 kg 0.6611m	545	0.765	529

Predicted acceleration profiles together with some other useful information are shown in Figure 16, Models 1 and 2 respectively.

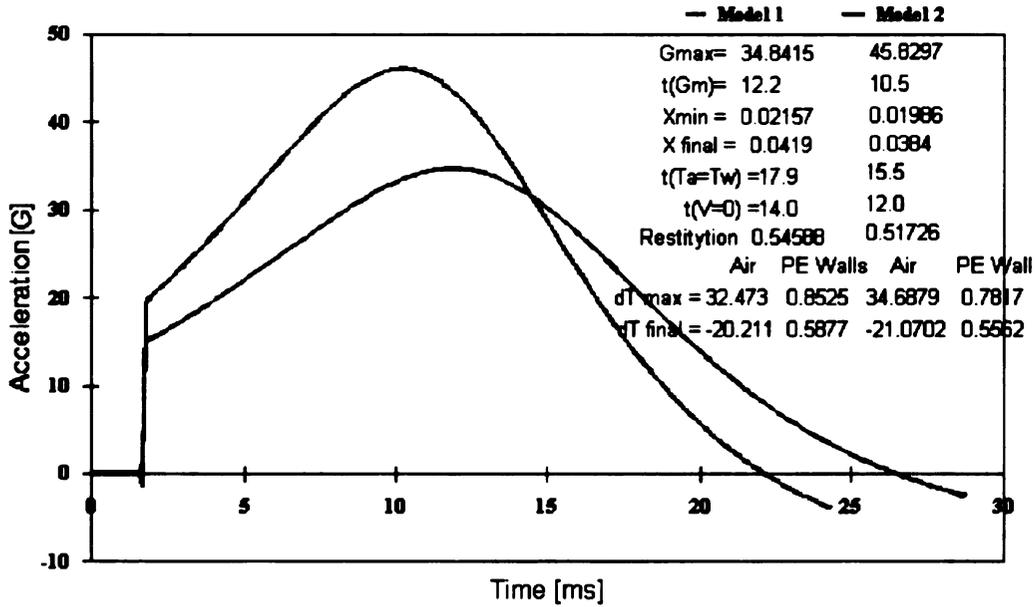


Figure 16. Model Predictions for {17.8 kg 0.557m} and {13.25 kg 0.6611m} drops.

3.3 Model use for fit filtering

The model can be used to fit itself to the real signal profile essentially filtering it and finding the coefficients for this particular drop. The only input parameters required by the model are drop height, drop head mass and cushion thickness which are usually known. The result of such an approach is shown in Figure 17.

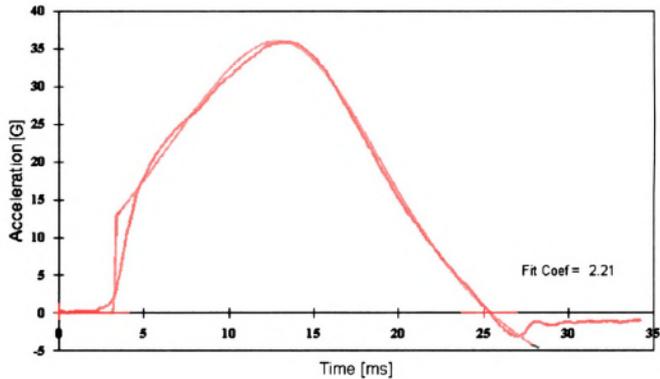


Figure 17. Unknown signal fit filtering

The real signal in Figure 17 is the result of {25.6kg 0.508m} drop on the same cushion but the which was beaten more than cushions used for the data in Table 3. So the model not only gives a good fit but predicts a decrease in F^x and $C1^v$, an increase in X_o , peak G , restitution coefficient and other logical changes in temperatures reached due to the fact that the cushion is worn out.

CONCLUSIONS

The model of the cushion compression process in conjunction with the best fit search algorithm allows the user to process experimental data on cushion tests more accurately and efficiently than using existing techniques.

Because fit filtering as opposed to Fourier filtering is based on the real physics of the process, it can recognize and partially cancel some experimental systematic errors. It also deals with random errors (noise) at least as well as the Fourier filtering, but does not cause a dilemma of filtering frequency choice. The process of filtering simultaneously provides information on all useful drop parameters.

The type of material constants generated as the result of such data analysis carries more information about the cushion properties and is much more compact than the standard cushion curves. A new ASTM standard for generation and application of these properties can be easily designed. It will require the use of a computer and a standard algorithm, but in our time it is fairly easy to set up. The advantage of using the computer is that the search for the material capable of providing appropriate product protection levels at the minimum cost can be automated.

Extension of the model search capabilities and abilities to work with multiple sets of experimental data is not very difficult to implement. It most likely will lead to better approximation of the heat transfer coefficient and incorporation of a different apparent cushion thickness dependence, which in turn will make the model more accurate and further simplify the material properties tabulation.

APPENDIX

Computer Program Code

```
Public Sub Build_model(Index As Integer)
Index gives ability to use the same function for different variable sets
  col = (Index + 3) * 2
  Sum = 0
  VnotDone = True
  TnotDone = True
  Temp_FofX = 0
  dt = Tstep / Niterations
  XN = Xo(Index)
  XaN = XN
  VN = -Sqr(2 * g * DropHeight)
  AN = 0
  PN = Po
  TN_Air = InitialTemp + 273.15
  TN_wall = TN_Air
  CurTime = TContact(Index)
  M_air = 1.225 * Xo(Index) * Area For my volume
  N = M_air / 28.9644 'number of moles
  Cp = 1003.8 * M_air 'for my mass
  Cv = 717.29875 * M_air For my volume
  Cpe = 1901 * Mpe For my mass
  h = Ktherm(Index) * 5263 * Area * X_apparent
  Convective Heat Transfer coefficient for my area of contact
  With FFiltering.Graph1.DataGrid
  ContactTimeInteger = IntRangeStart + CInt(TContact(Index) / Tstep) + 1
  'Will start impact there
  If TContact(Index) > 0 Then
  Aend = 0
    For i = 1 To ContactTimeInteger 'no cishion compression yet
      .GetData i, 2, temp, 0
      .SetData i, col, 0, 0
      .SetData i, 10, 0, 0
      Abeg = Aend
      Aend = temp * g
      For j = 1 To Niterations
        'before the contact Xn=Xo
          VN = VN + (Abeg + (Aend - Abeg) * j / Niterations) * dt
          'Velocity of the mass
        Next j
      Next
    AN = Aend
  End If
  VContact(Index) = VN
```

```

counter = IntRangeStart + CInt(T_modelStart(Index) / Tstep) + 1
*Will start impact there
If (counter > IntRangeStart + 1) And (counter > ContactTimeInteger) Then
    Temp_FofV = FofV(Index, VN)
    Temp_FofX = AN * mass - Temp_FofV - Area * (PN - Po)
    Abeg = AN
    For i = ContactTimeInteger To counter
        If FFiltering.View_FofV(Index).Checked Then .SetData i, 10, FofV(Index, VN) /
g / mass, 0
        If FFiltering.View_FofX(Index).Checked Then .SetData i, 10, (AN * mass -
Temp_FofV - Area * (PN - Po)) / g / mass, 0
        .SetData i, col, 0, 1
        .GetData i, 2, temp, 0
        Abeg = Aend
        Aend = temp * g
        For j = 1 To Niterations
            step (Index)
            AN = Abeg + (Aend - Abeg) * j / Niterations
        Next j
    Next
End If
CurTime = T_modelStart(Index)

Temp_FofV = FofV(Index, VN) ' It is used later to show the graphs
Temp_FofX = FofX(Index, XaN, VN, FofX_type_Number(Index))

VN_modelStart(Index) = VN
AN_modelStart(Index) = AN
Temp_FofV_modelStart = Temp_FofV
Temp_FofX_modelStart = Temp_FofX

Do While (PN >= Po) And (counter <= IntRangeEnd)
    .SetData counter, col, AN / g, 0 Recording next point
    .GetData counter, 2, temp, 0
    Sum = Sum + (temp - AN / g) ^ 2
    If FFiltering.View_FofV(Index).Checked Then .SetData counter, 10, Temp_FofV / g
/ mass, 0
    If FFiltering.View_FofX(Index).Checked Then .SetData counter, 10, Temp_FofX / g
/ mass, 0
    For i = 1 To Niterations
        step (Index)
        AN = (Area * (PN - Po) + Temp_FofX + Temp_FofV) / mass
    Next i

```

```

    counter = counter + 1
Loop

For i = counter To IntRangeEnd
    .SetData i, col, 0, 1 'clearing the rest of the points
    .SetData i, 10, 0, 1
Next i
FFiltering.XfBox = Format(XN, "##0.0###")
FFiltering.TfBox = Format(TN_Air - 273.15 - InitialTemp, "##0.0###")
FFiltering.TmaxBox = Format(T_Air_Max - 273.15 - InitialTemp, "##0.0###")
T_Air_Max = 0
FFiltering.TwallmaxBox = Format(T_wall_Max - 273.15 - InitialTemp, "##0.0###")
T_wall_Max = 0
FFiltering.TwallFinalBox = Format(TN_wall - 273.15 - InitialTemp, "##0.0###")
FFiltering.GmaxBox = Format(Amax / g, "##0.0###")
Amax = 0
FFiltering.FitCoefBox = Format(Sum / (counter - (IntRangeStart +
CInt(T_modelStart(Index) / Tstep) + 1)), "####0.0#")
FFiltering.RestitutionBox = Format(-VN / VContact(Index), "0.#####")
End With
End Sub

```

Public Sub step(Index As Integer) ' Is called from the main body of Build_model

```

    dX = VN * dt + AN * dt * dt / 2
    XaN = XN + dX
    VN = VN + AN * dt
    If VN > 0 And VnotDone Then
        Xmin = XN
        FFiltering.XminBox = Format(XN, "0.0####")
        FFiltering.TimeOfVis0Box = Format(CurTime * 1000, "##0.0###")
        VnotDone = False
    End If

    'work done by gas
    dW_air = PN * Area * dX 'exact adiabatic PN * Area * XN / (gam - 1) * (1 -
(XN / XaN) ^ (gam - 1))

    'work by forces of friction done on the walls
    dW_Friction = -dX * (Temp_FofV + Temp_FofX) ' no Temp_FofX if it was a
spring
    ' heat given to the air
    dQ = h * (TN_wall - TN_Air) * dt

```

```

TaN_Air = TN_Air + (dQ - dW_air) / Cv TN_Air * (XN / XaN) ^ (gam - 1)
TN_wall = TN_wall + (dW_Friction - dQ) / Cpe
PN = PN * (XN / XaN) * (TaN_Air / TN_Air)
Temp_FofV = FofV(Index, VN) ' It is used later to show the graphs
Temp_FofX = FofX(Index, XaN, VN, FofX_type_Number(Index))
CurTime = CurTime + dt
XN = XaN
TN_Air = TaN_Air

```

```

If TN_Air < TN_wall And TnotDone Then
    FFiltering.TimeOfTisTwallBox = Format(CurTime * 1000, "##0.0###")
    TnotDone = False
End If
If AN > Amax Then
    Amax = AN
    VofAmax = VN
    XofAmax = XN
    FFiltering.TimeOfAmaxBox = Format(CurTime * 1000, "##0.0###")
End If
If TN_Air > T_Air_Max Then
    T_Air_Max = TN_Air
End If
If TN_wall > T_wall_Max Then
    T_wall_Max = TN_wall
End If

```

End Sub

Private Sub Minimize_Click(Index As Integer)' used for variable search and fit minimization.

```

Dim Smalest_variable As Double
Dim VarStep As Double
Dim Error As Double
Dim Smalest_Error As Double
Dim N_Intervals As Integer
Dim Error_Valyes(0 To 20) As Double

Dim max, min As Double
Dim Best_Smalest_Error As Double
Dim best_Xo As Double
Dim best_FofV_C1 As Double
Dim best_FofV_C2 As Double
Dim best_Ktherm As Double
Dim best_FofX_Valyes(0 To 4) As Double
Dim best_FofX_PosFrac(0 To 4) As Double

```

If T_modelStart(Index) = TContact(Index) Then

 Randomize ' Initialize random-number generator.

 CoefficientsOK_Click (Index) ' get the numbers in and run the model first time to get the constants

 Best_Smalest_Error = 1E+300 ' get first best error to be big

 For k = 1 To N_RandomStarts N of random (10 to 20) numbers of intervals applied.

 N_Intervals = Int((N_Intervals_above_5 * Rnd) + 5) ' Generate random value between 10 and 20.

 For l = 1 To N_variable_cycles N times to cycle all variables

 Xo minimization

 If XoAsFractionBox(Index).BackColor = 12632256 Then 'variable is marked for minimization

 Smalest_Error = 1E+300 ' get first reference error to be big

 max = Xo_max(Index)

 min = Xo_min(Index)

 For m = 1 To N_Step_Reductions N times to reduce the step and interval

 VarStep = (max - min) / N_Intervals

 For p = 0 To N_Intervals

 Xo(Index) = min + p * VarStep

 Error = Quick_model_Sum(Index)

 If Error < Smalest_Error Then

 min_number = p

 Smalest_Error = Error

 Smalest_variable = Xo(Index)

 End If

 Next p

 If min_number > 0 Then min = min + VarStep * (min_number - 1)

 If min_number < N_Intervals Then max = min + VarStep * 2

 Next m

 Xo(Index) = Smalest_variable

 End If

Same thing for FofV_C1

 If F1(Index).BackColor = 12632256 Then 'variable is marked

 Smalest_Error = 1E+300 ' get first reference error to be big

 max = FofV_C1_max(Index)

 min = FofV_C1_min(Index)

 For m = 1 To N_Step_Reductions N times to reduce the step and interval

 VarStep = (max - min) / N_Intervals

 For p = 0 To N_Intervals

```

FofV_C1(Index) = min + p * VarStep
Error = Quick_model_Sum(Index)
If Error < Smalest_Error Then
    min_number = p
    Smalest_Error = Error
    Smalest_variable = FofV_C1(Index)
End If
Next p
If min_number > 0 Then min = min + VarStep * (min_number - 1)
If min_number < N_Intervals Then max = min + VarStep * 2
Next m
FofV_C1(Index) = Smalest_variable
End If

```

Same thing for FofV_C2

```

If F2(Index).BackColor = 12632256 Then `variable is marked
    Smalest_Error = 1E+300 `get first refference error to be big
    max = FofV_C2_max(Index)
    min = FofV_C2_min(Index)
    For m = 1 To N_Step_Reductions `N times to reduce the step and interval
        VarStep = (max - min) / N_Intervals
        For p = 0 To N_Intervals
            FofV_C2(Index) = min + p * VarStep
            Error = Quick_model_Sum(Index)
            If Error < Smalest_Error Then
                min_number = p
                Smalest_Error = Error
                Smalest_variable = FofV_C2(Index)
            End If
        Next p
        If min_number > 0 Then min = min + VarStep * (min_number - 1)
        If min_number < N_Intervals Then max = min + VarStep * 2
    Next m
    FofV_C2(Index) = Smalest_variable
End If

```

Same thing for Ktherm

```

If K_therm(Index).BackColor = 12632256 Then `variable is marked
    Smalest_Error = 1E+300 `get first refference error to be big
    max = h_max(Index)
    min = h_min(Index)
    For m = 1 To N_Step_Reductions `N times to reduce the step and interval

```

```

VarStep = (max - min) / N_Intervals
For p = 0 To N_Intervals
    Ktherm(Index) = min + p * VarStep
    Error = Quick_model_Sum(Index)
    If Error < Smalest_Error Then
        min_number = p
        Smalest_Error = Error
        Smalest_variable = Ktherm(Index)
    End If
Next p
If min_number > 0 Then min = min + VarStep * (min_number - 1)
If min_number < N_Intervals Then max = min + VarStep * 2
Next m
Ktherm(Index) = Smalest_variable
End If

```

Same thing for 5 FofX_Valyes and FofX_PosFrac

```

For r = 0 To 4
    If BValuesBox(r + 5 * Index).BackColor = 12632256 Then 'variable is marked
        Smalest_Error = 1E+300 'get first refference error to be big
        max = FofX_Valyes_max(Index, r)
        min = FofX_Valyes_min(Index, r)
        For m = 1 To N_Step_Reductions 'N times to reduce the step and interval
            VarStep = (max - min) / N_Intervals
            For p = 0 To N_Intervals
                FofX_Valyes(Index, r) = min + p * VarStep
                Error = Quick_model_Sum(Index)
                If Error < Smalest_Error Then
                    min_number = p
                    Smalest_Error = Error
                    Smalest_variable = FofX_Valyes(Index, r)
                End If
            Next p
            If min_number > 0 Then min = min + VarStep * (min_number - 1)
            If min_number < N_Intervals Then max = min + VarStep * 2
        Next m
        FofX_Valyes(Index, r) = Smalest_variable
    End If

```

```

If BPositionBox(r + 5 * Index).BackColor = 12632256 Then 'variable is marked
    Smalest_Error = 1E+300 'get first refference error to be big
    max = FofX_PosFrac_max(Index, r)
    min = FofX_PosFrac_min(Index, r)
    For m = 1 To N_Step_Reductions 'N times to reduce the step and interval

```

```

    VarStep = (max - min) / N_Intervals
    For p = 0 To N_Intervals
        FofX_PosFrac(Index, r) = min + p * VarStep
        Error = Quick_model_Sum(Index)
        If Error < Smalest_Error Then
            min_number = p
            Smalest_Error = Error
            Smalest_variable = FofX_PosFrac(Index, r)
        End If
    Next p
    If min_number > 0 Then min = min + VarStep * (min_number - 1)
    If min_number < N_Intervals Then max = min + VarStep * 2
Next m
FofX_PosFrac(Index, r) = Smalest_variable
End If
Next r
Next l
    If Best_Smalest_Error > Smalest_Error Then
        best_Xo = Xo(Index)
        best_FofV_C1 = FofV_C1(Index)
        best_FofV_C2 = FofV_C2(Index)
        best_Ktherm = Ktherm(Index)
        For r = 0 To 4
            best_FofX_Valyes(r) = FofX_Valyes(Index, r)
            best_FofX_PosFrac(r) = FofX_PosFrac(Index, r)
        Next r
    End If
Next k
Xo(Index) = best_Xo
FofV_C1(Index) = best_FofV_C1
FofV_C2(Index) = best_FofV_C2
Ktherm(Index) = best_Ktherm
For r = 0 To 4
    FofX_Valyes(Index, r) = best_FofX_Valyes(r)
    FofX_PosFrac(Index, r) = best_FofX_PosFrac(r)
Next r
Write_To_Boxes
Build_model (Index) 'run the model to update graph
Else
    MsgBox ("Time of modelstart should be the same as time of contact to run
minimization")
End If

End Sub

```

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