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dissertation entitled

CAPITAL INVESTMENT BY RISK NEUTRAL AGENTS:

MERGING ADJUSTMENT COSTS AND IRREVERSIBILITY

presented by

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has been accepted towards fulfillment
of the requirements for

Ph.D. degree in Economics

Major professor

Date May 10, 1999

CAPITAL INVESTMENT BY RISK NEUTRAL AGENTS: MERGING ADJUSTMENT COSTS AND IRREVERSIBILITY

By

Hirokatsu Asano

A DISSERTATION

Submitted to
Michigan State University
in partial fulfillment of the requirements
for the degree of

DOCTOR OF PHILOSOPHY

Department of Economics

1999

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ABSTRACT

CAPITAL INVESTMENT BY RISK NEUTRAL AGENTS: MERGING ADJUSTMENT COSTS AND IRREVERSIBILITY

By

Hirokatsu Asano

Current capital investment affects future investment by setting conditions upon which a firm makes future investment decisions. This analysis applies option pricing theory to capital investment in order to determine possibilities of future investment, and shows the optimal investment choice for a firm contemplating future investment. The theoretical analysis develops a model for capital investment by risk neutral agents. The model includes costly reversibility and fixed costs of investment. Then, numerical analysis shows that optimal investment is approximately linear in economic parameters such as the discount rate. Empirical analysis shows that actual investment behaves as theoretical analysis predicts.

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ACKNOWLEDGMENTS

Without the help and support from many people, this dissertation could not have been completed. First, I would like to thank Professor Robert Rasche for guidance that he gave me throughout this work. An article that he suggested me initiated this research, and his comments and suggestions improved this work greatly. I would also like to thank the other members of my committee, Professors Jeffery Wooldridge and Gerhard Glomm, for their insight and support. I am truly thankful to all members of my committee for the quality and the speed of their feedback on my work.

I am very fortunate to have good friends at Michigan State University. I especially want to thank Daiji, Heather, Hiroki, Katsushi and Pablo. They made my life and study in Lansing enjoyable, if not pleasant, and some of them also contributed to my work.

Finally, I thank my parents for allowing me to pursue my Ph.D. dream. I am also grateful to my brother and his family for their support. They made my academic achievement possible.

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INTRODUCTION

When a firm invests, its investment decision is based upon its current level of capital stock. Then, firm's current investment affects its future investment decisions by setting conditions upon which the firm makes its future decisions. Therefore, the firm should take into account the possibilities of future investment when it makes its investment decision. This paper shows the optimal investment decision for a firm contemplating future investment, and investigates actual investment in accordance with theoretical analysis.

This analysis applies option pricing theory to capital investment in order to take into account the possibilities of future investment. Currently, Abel and Eberly are merging Tobin's q theory with an adjustment cost function and irreversible investment in order to incorporate the possibility of future investment. Abel and Eberly (1996) use a model with costly reversibility but without fixed costs of investment. Abel and Eberly (1997) use a model with fixed costs of investment and irreversibility. This paper uses a model with both fixed costs of investment and costly reversibility. In general, a closed form solution of this model is unattainable. Therefore, this analysis solves the problem numerically. The theoretical analysis shows that Tobin's q becomes lower, when a firm takes into account its future investment.

Tobin's (marginal) q theory implies that a firm invests when the marginal benefit of capital exceeds the marginal cost of capital. The firm will invest up to the point where the marginal benefit of investment becomes equal to the marginal cost. The marginal benefit of capital is Tobin's q, and equal to the derivative of the present discounted value (PDV) of net cash flow or firm's equity. The marginal cost is the price of capital. Investment is an increasing function of Tobin's q.

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This analysis modifies the friction model to analyze actual investment. The analysis compares five different investment models including one that specifies costly reversible investment with fixed costs. The analysis shows that costly reversible investment or the corresponding econometric model, a generalized version of the friction model (GFM), is the best model to analyze actual investment among the tested models. The analysis also shows that an investment model of partial specification such as irreversible investment can be inferior to a model of reversible investment without fixed costs for empirical research.

1. Irreversibility, Costly Reversibility and Fixed Costs of Investment

If the firm cannot sell its installed capital, investment is irreversible. When the firm invests heavily now, it is more likely that the firm will have too much capital in the future and Tobin's q will be lower than the price of capital. In other words, the marginal benefit of capital will be less than the marginal cost of capital in the future. In such a case, the firm will want to sell its installed capital, but cannot due to the irreversibility of capital. On the other hand, if the firm postpones its investment decision, it will acquire more information about the economy and, then, can adjust its capital stock. Thus, waiting has value for the firm in a stochastic economy, and the firm's gain from waiting is more accurate adjustment of its capital stock in the future. The irreversible investment literature applies option pricing theory in order to evaluate the value of waiting.

The irreversibility literature regards future investment as a call option. A call option is a right, but not an obligation, for investors to buy financial assets or commodities at a predetermined price, or striking price, at a predetermined date in the future. The investors or option buyers exercise their option if the market price of the assets exceeds the striking price at the contract date. If the market price is lower than the striking price, the option buyers will not exercise their option. The gain for the option buyer is difference between the striking price and the market price at the maturity date. Therefore, the gain is kinked at the striking price. The gain is zero if the market price is

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lower than the striking price. If the market price is higher than the striking price, the gain is positive and increasing along with the market price. The premium that the option buyers have to pay to option writers is equal to the expected gain for the option buyer. Option pricing theory evaluates this premium.

In the stochastic economy in this paper, operating profit and cash flows are modeled as functions of capital stock and a stochastic variable, which is called the economic indicator variable. For a given level of capital stock, there is a critical value for investment of the economic indicator variable, which corresponds to the striking price. The firm invests when the economic indicator is higher than the critical value. Otherwise, the firm does not invest, because the marginal benefit of investment is less than the marginal cost of investment.

When the firm can sell its installed capital at a resale price lower than the purchase price of new capital, investment is costly reversible. Costly reversibility introduces another critical value of the economic indicator variable, the value at which disinvestment becomes profitable. The firm disinvests if the economic indicator is lower than the critical value for disinvestment. Then the firm will disinvest until Tobin's q becomes equal to the resale price of capital. The firm does not disinvest if Tobin's q is above the resale price. When Tobin's q is between the purchase price of capital and the resale price of capital, zero investment, or inaction, is optimal. Similar to the gain from investment, the gain from disinvestment is kinked at the critical value. The gain is positive when the economic indicator is below the critical value and zero when the economic indicator is above the critical value. In the analysis, disinvestment is similar to a put option.

When there are fixed costs for investment, fixed costs introduce a minimum amount of optimal investment. Sunk costs or installation costs of investment are examples of fixed costs of investment. Because of fixed costs, a value of Tobin's q higher than the price of capital is not sufficient for optimality. An increase in the PDV of

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net cash flow from investment can be smaller than fixed costs. When the firm invests, the gain from investment should be larger than fixed costs. As a result, optimal investment has a minimum. If there are fixed costs for disinvestment, optimal disinvestment also has a minimum. Fixed costs also introduce a range of inaction in the economic indicator.

2. Prior Work

Abel and Eberly have published several papers about capital investment by risk neutral agents which incorporate Tobin's q theory with an adjustment cost function and irreversibility. Abel and Eberly (1994) formally incorporate fixed costs of investment into their model. They solve a continuous time model with a general form of the investment cost function which includes both fixed costs and the costly reversibility of investment as well as a conventional convex adjustment cost function. Their analysis shows that a range of inaction appears in the firm's investment decision. Both fixed costs and the costly reversibility result in inaction as an optimal investment decision. In one case, a constant returns to scale (CRTS) Cobb-Douglas technology and a perfectly competitive market are assumed. This model results in the firm showing risk-loving-like behavior, i.e., the firm will invest more when the price of output fluctuates more, because the operating profit function is convex in the output price under those assumptions.

When the firm contemplates an investment project, the firm can wait until the firm's market condition becomes more favorable. McDonald and Siegel (1986) employ option pricing theory to evaluate the value of waiting. The irreversible investment literature shows that optimal investment has two states; one is strictly positive investment and the other is zero investment, and one of the two states appears at a time. Bernanke (1983) by employing option pricing theory shows an asymmetric character to irreversible investment. When market conditions for the firm are favorable, the firm will invest and acquire more capital. Because the firm cannot resell its installed capital, the firm should be worried about "bad news." Due to the irreversibility of investment, the firm cannot sell its install

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sell its installed capital, even if a lower capital stock is optimal. In other words, the firm can adjust its capital stock upward but not downward. Thus, the firm's investment decision is concerned only with bad news, because what "irreversible investment is sensitive to is 'downside' uncertainty (Bernanke, pp. 93)."

Currently, Abel and Eberly are merging the adjustment cost function literature and the irreversibility literature. Table 1 shows recent articles and the topics which the articles cover. Abel, Dixit, Eberly and Pindyck (1996) develop a model which incorporates both Tobin's q theory and option pricing theory. The model is a two period model with the costly reversibility, but without fixed costs. One conclusion from that model is that when the firm makes a decision it should take into account the adjustment of its capital stock in the future (the second period). The appropriate q for current investment is the derivative of the PDV of cash flow assuming no future investment or disinvestment, plus the value of the put option for future disinvestment, minus the value of the call option for future investment. Their analysis also shows that an upward shift in the distribution of shocks to the firm's revenue increases the incentive to invest, while a mean-preserving spread in the distribution of the shocks has an ambiguous effect on investment.

Table 1 Recent Literature on Adjustment Cost Function and Irreversibility

Papers*	Adjustment Cost Function			Irreversibility		Model	
	Costly Reversibility	Fixed Costs	Convex Function	Irreversi-bility	Option Pricing	Two Period	Continuous Time
AE (1994)	X	X	X				X
ADEP(1996)	X				X	X	
AE (1996)	X				X		X
AE (1997)		X		X	X		X
Asano(1999)	X	X			X	X	X

* AE and ADEP stand for Abel and Eberly, and Abel, Dixit, Eberly and Pindyck, respectively.

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Abel and Eberly (1996) use an investment model with the costly reversibility, but without fixed costs. The paper shows that the range of inaction is wider than that calculated from the Jorgenson's user cost of capital, i.e., (real interest rate + depreciation rate) × (purchase price or resale price of capital). Their model uses an operating profit function which has constant returns to scale in capital stock and a demand shock variable. The analysis uses a Taylor approximation for its solution, but there are large approximation errors in some cases.

Abel and Eberly (1997) use a model incorporating both irreversibility and fixed costs. The model shows that there are a trigger value and a target value for a composite variable, i.e., a random variable representing the ratio of economic conditions to current capital stock. When the composite variable exceeds the trigger value, which means that the economy is booming, the firm increases its capital stock such that the composite variable becomes the target value. The model uses a Cobb-Douglas technology, and an iso-elastic demand function. There are stochastic shocks to technology, the demand for a firm's output and the price of flow input. The model also includes the level of factor utilization.

The analysis presented here uses a modified Abel and Eberly (1997) model with the costly reversibility. The analysis includes fixed costs, but excludes the factor utilization. The model without fixed costs is a special case of the model with fixed costs; namely fixed costs are zero. The analysis will quantitatively show optimal investment and effects of economic parameters on optimal investment. In order to derive critical values for optimal investment, there are two obstacles: solving a quotient of power functions for the model without fixed costs and solving a simultaneous equation system of power functions for the model with fixed costs. This analysis resorts to numerical solutions.

Since costly reversible investment has three ranges in the economic indicator variable, the analysis employs a generalized version of the friction model. Maddala

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(1983, pp. 162) discusses this model. The friction model is an extension of the Tobit model, which has two ranges. Tobin (1958) studies a model in which a dependent variable is varying in one of the two ranges while it remains constant in the other range. However, the economic model of this analysis has three ranges in the economic indicator: one for investment, zero investment and disinvestment. The economic model requires a modification to allow some explanatory variables to have different coefficients in different parts of the friction model. Thus, the analysis generalizes the friction model for analysis.

The analysis presented here compares five investment models: reversible without fixed costs, irreversible without fixed costs, irreversible with fixed costs, costly reversible without fixed costs, and costly reversible with fixed costs. Since some examined models are non-nested, the LR test is not appropriate for comparison. Voung (1989) proposes a test of model selection, which is an extension of the LR test. The analysis employs Voung's test of model selection.

The empirical analysis investigates three industries: the computer and office equipment industry, the automobile industry, and the airline industry. For all three industries, costly reversible investment with fixed costs or the corresponding generalized friction model is the best among the five investment models, since its likelihood is highest and its estimated coefficients are compatible with the economic model.

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CHAPTER 1

COSTLY REVERSIBLE INVESTMENT WITHOUT FIXED COSTS

This chapter analyzes costly reversible investment without fixed costs. The analysis assumes that the operating profit function, π, has the following functional form.

$$\pi(K_t, Z_t) = A_\pi Z_t^{1-\theta} K_t^\theta \tag{1}$$

Here, Z_t and K_t are the stochastic economic indicator variable and capital stock, respectively. The specification of equation (1) can be derived for a firm with a CRTS Cobb-Douglas technology facing an iso-elastic demand curve. The economic indicator shifts a firm's operating profits and is assumed to follow a geometric Brownian motion with drift μ_Z and volatility σ_Z.

$$dZ_t = \mu_Z Z_t dt + \sigma_Z Z_t dz_Z \tag{2}$$

Here, dz_Z is a standard Wiener process.

The firm invests or disinvests to maximize the firm's expected equity or the PDV of firm's expected net cash flow, $V(K_t,Z_t)$. The investment and disinvestment incur total costs, IC_t, which include payments for investment, and receipts from disinvestment. The functions I_t and D_t are a cumulative investment function and a cumulative disinvestment function, respectively. The function I_t is the sum of all purchases of capital up to time t. Therefore, dI_t is the amount of investment at time t. Similarly, D_t is the sum of all sales of capital up to time t. Both are non-decreasing step functions. And, because capital stock, K_t, is exponentially depreciating at an exogenous rate, δ, while the functions I_t and

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D_t remain the same level until the next investment or disinvestment, in general, $K_t \neq I_t - D_t$. We can write the value function, $V(K_t,Z_t)$, as follows.[1]

$$V(K_t,Z_t) = \max_{\{dI_{t+s},dD_{t+s}\}} E_t\left[\int_0^\infty e^{-\gamma s}\left\{A_\pi Z_{t+s}^{1-\theta} K_{t+s}^\theta ds - IC_{t+s}\right\}\right] \tag{3}$$

$$\text{subject to } dK_{t+s} = dI_{t+s} - dD_{t+s} - \delta K_{t+s}dt,\ K_{t+s} \geq 0 \quad \forall s \geq 0,$$
$$dZ_{t+s} = \mu_Z Z_{t+s}dt + \sigma_Z Z_{t+s}dz_Z, \text{ and}$$
$$IC_{t+s} = p_K^+ dI_{t+s} - p_K^- dD_{t+s}$$

Here, γ, p_K^+ and p_K^- are the discount rate, the purchase price of capital and the resale price of capital, respectively. Parameters γ, p_K^+ and p_K^- are given. At a point in time, t, the firm makes its decision about K_t after the stochastic economic indicator variable, Z_t, is revealed, in order to maximize its expected equity or expected PDV of the net cash flow assuming optimal investment for the future.

1-1. Bellman Equation

By splitting the time period of equation (3) into two: the first period from zero to Δt, and the second period from Δt to infinity, and assuming that Δt is sufficiently small and the spread, $\gamma - \mu_Z$, is strictly positive, we have

$$\begin{aligned}
V(K_t,Z_t) &= E_t\left[\int_0^{\Delta t} e^{-\gamma s} A_\pi Z_{t+s}^{1-\theta} K_{t+s}^\theta ds\right] + \max_{\{dI_{t+s},dD_{t+s}\}} E_t\left[\int_{\Delta t}^\infty e^{-\gamma s}\left\{A_\pi Z_{t+s}^{1-\theta} K_{t+s}^\theta ds - IC_{t+s}\right\}\right] \\
&= A_\pi K_t^\theta \int_0^{\Delta t} e^{-(\gamma+\theta\delta)s} E\left(Z_{t+s}^{1-\theta} \middle| Z_t\right) ds \\
&\quad + e^{-\gamma\Delta t} \max_{\{dI_{t+\Delta t+\tau},dD_{t+\Delta t+\tau}\}} E_t\left[\int_0^\infty e^{-\gamma\tau}\left\{A_\pi Z_{t+\Delta t+\tau}^{1-\theta} K_{t+\Delta t+\tau}^\theta d\tau - p_K^+ dI_{t+\Delta t+\tau} + p_K^- dD_{t+\Delta t+\tau}\right\}\right] \\
&\approx A_\pi Z_t^{1-\theta} K_t^\theta \Delta t + \frac{1}{1+\gamma\Delta t} E_t\left[V\left(K_{t+\Delta t}, Z_{t+\Delta t}\right)\right].
\end{aligned} \tag{4}$$

[1] The main text of this paper solves a continuous time model, while the appendix A of the paper solves a two period model. Although the two period model of this paper shows economic intuition, it is the continuous time model which allows us to derive the optimal investment and show effects of parameter variation on the optimal investment. In the two period model of this paper, for simplicity, a firm makes investment decisions only twice; one at the beginning of each period, so that the second period decision does not concern future investment.

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$$\gamma\Delta t V(K_t, Z_t) = A_\pi Z_t^{1-\theta} K_t^\theta (1+\gamma\Delta t)\Delta t + E_t[V(K_{t+\Delta t}, Z_{t+\Delta t}) - V(K_t, Z_t)]$$
$$= A_\pi Z_t^{1-\theta} K_t^\theta (1+\gamma\Delta t)\Delta t + E_t[\Delta V(K_t, Z_t)]. \tag{5}$$

By dividing both sides by Δt and letting $\Delta t \to 0$, the above equation becomes

$$\gamma V(K_t, Z_t) = A_\pi Z_t^{1-\theta} K_t^\theta + (1/dt)E_t[dV(K_t, Z_t)]. \tag{6}$$

By Ito's lemma, $E_t[dV(\cdot)]$ can be expressed as follows:

$$E_t[dV(K_t, Z_t)] = -\delta K_t V_K dt + V_Z \mu_Z Z_t dt + \frac{1}{2} V_{ZZ} \sigma_Z^2 Z_t^2 dt. \tag{7}$$

Thus, the Bellman equation for $V(\cdot)$ is

$$\gamma V(K_t, Z_t) = A_\pi Z_t^{1-\theta} K_t^\theta - \delta K_t V_K + \mu_Z Z_t V_Z + \frac{1}{2} \sigma_Z^2 Z_t^2 V_{ZZ}. \tag{8}$$

Because $V(\cdot)$ is set up to be homogeneous of degree one in K and Z by choosing the exponents on Z, we can transform the partial differential equation (8) to an ordinary differential equation. The derivative of the expected equity, V_K, is equal to Tobin's q. By defining $y \equiv Z_t / K_t$, $\mu_y \equiv \mu_Z + \delta$, and $\sigma_y \equiv \sigma_Z$, $q(y)$ ($\equiv V_K$) becomes the following second order ordinary differential equation

$$(\gamma + \delta)q(y) = A_\pi \theta y^{1-\theta} + \mu_y y q'(y) + \frac{\sigma_y^2}{2} y^2 q''(y). \tag{9}$$

The solution of equation (9) is

$$q(y) = H y^{1-\theta} + B_N y^{\varphi_N} + B_P y^{\varphi_P}. \tag{10}$$

The first term of equation (10) is the particular solution, and $H = A_\pi \theta / f(1-\theta)$. Then the characteristic function f associated with equation (9) is given by

$$f(\varphi) \equiv -\frac{\sigma_y^2}{2}\varphi^2 - \left(\mu_y - \frac{\sigma_y^2}{2}\right)\varphi + (\gamma + \delta). \tag{11}$$

The second term and the third term of equation (10) are the complementary solution. The second term corresponds to the derivative of the value of future disinvestment as the put option and th

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1-2. Optimal Investment for Costly Reversible Investment without Fixed Costs

The optimal investment decision equates Tobin's q with the price of capital. In the case of costly reversible investment, the firm should buy new capital when q exceeds the purchase price of capital, while it should sell its installed capital when q is lower than the resale price of capital. Then, the firm invests or disinvests until Tobin's q becomes equal to the corresponding price of capital.

When there are no fixed costs for investment, there are two critical values in y: one for investment and the other for disinvestment. At the critical value for investment, y^+, the firm is indifferent between investing and waiting. Above y^+, investment takes place, while below y^+ the firm waits. The critical value for disinvestment, y^- has similar properties. Then, the boundary conditions for equation (10) are fourfold:

$$q(y^+) = p_K^+, \tag{12a}$$

$$q(y^-) = p_K^-, \tag{12b}$$

$$q'(y^+) = 0, \tag{12c}$$

and

$$q'(y^-) = 0. \tag{12d}$$

The first two conditions are the smooth pasting conditions. The last two conditions are the high order contact. For investment without fixed costs, there are four unknowns, y^+, y^-, B_N, and B_P.

Abel and Eberly (1996) derive the solution to equation (10) which satisfies the boundary conditions (12a) to (12d) as

$$B_N = -\frac{(1-\theta)H}{\varphi_N} g(G)(y^-)^{1-\theta-\varphi_N}, \tag{13a}$$

$$B_P = -\frac{(1-\theta)H}{\varphi_P} [1 - g(G)](y^-)^{1-\theta-\varphi_P}, \tag{13b}$$

and

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$$y^{+} = \left[\frac{p_K^{+}}{A_\pi \theta\, h(G^{-1})}\right]^{\frac{1}{1-\theta}}, \tag{13c}$$

and

$$y^{-} = \left[\frac{p_K^{-}}{A_\pi \theta\, h(G)}\right]^{\frac{1}{1-\theta}}. \tag{13d}$$

Here, $G \equiv y^{+}/y^{-}$, $g(x) \equiv (x^{\varphi_P} - x^{1-\theta})/(x^{\varphi_P} - x^{\varphi_N})$, and $h(x) \equiv [1-(1-\theta)g(x)/\varphi_N - (1-\theta)\{1-g(x)\}/\varphi_P]/f(1-\theta)$. Here, G satisfies the following equation

$$J(R,G) \equiv Rh(G) - G^{1-\theta}h(G^{-1}) = 0 \tag{14}$$

where, $R \equiv p_K^{+}/p_K^{-}$.

Abel and Eberly (1996) solve equation (14) by a Taylor approximation at the point ($R = 1$ and $G = 1$). They suggest

$$G \cong 1 + \left[\frac{6\sigma_y^2}{(1-\theta)(\gamma+\delta)}\right]^{1/3} (R-1)^{1/3}. \tag{15}$$

But, usually $R > 1$ and $G > 1$. When the solution is far from the point of the Taylor approximation, there is a large approximation error. Figure 1.1 shows such an example. For example, when $R = 2$, Abel and Eberly suggest that G is approximately 1.51, but G is actually 5.43. An alternative approximation for equation (14) can be derived by choosing a larger term in either the numerator or the denominator in the function g for simplifying the function h. The alternative approximation yields equation (16), which is a better approximation as figure 1.1 shows.[2]

$$G \approx \left[\left(\frac{\varphi_N - 1 + \theta}{\varphi_P - 1 + \theta}\right)\left(\frac{\varphi_P}{\varphi_N}\right)(R-1) + 1\right]^{\frac{1}{1-\theta}} \tag{16}$$

[2] See the appendix B for derivation.

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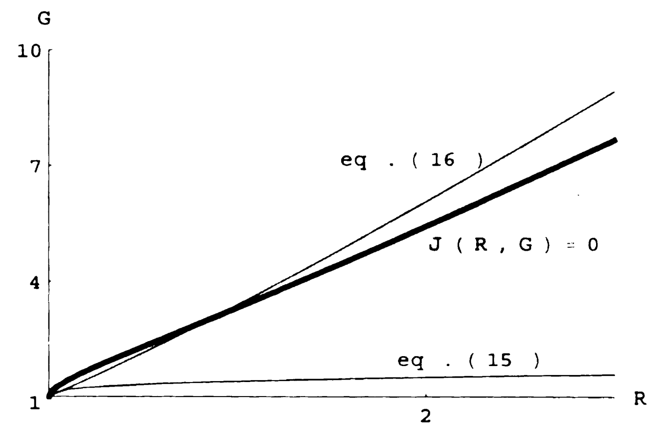

Figure 1.1 Approximation of $J(R,G) = 0$

$(\theta \approx 0.143, A_\pi \approx 0.506, \delta = 0.06, \gamma = 0.07, \mu_Z = 0.05, \sigma_Z = \mu_Z)$

Equation (16) suggests that G is approximately 6.07. However, this analysis solves equation (14) numerically in order to find more accurate results.

Figure 1.2 shows one solution for costly reversible investment without fixed costs. Since $0 < \theta < 1$, $H > 0$ and $0 < 1 - \theta < 1$. Therefore, the first term of the q function, $Hy^{1-\theta}$, which is equal to the so-called naive case, is concave and increasing for positive y. And, the term is zero at $y = 0$. Since $\varphi_N < 0$ and $B_N > 0$, the second term of the q function, $B_N y^{\varphi_N}$, is convex and decreasing for positive y. As the second term is infinite at $y = 0$, it dominates the other terms for small positive y. Since $\varphi_P > 1$ and $B_P < 0$, the third term of the q function, $B_P y^{\varphi_P}$, is concave and decreasing for positive y. The third term equals zero at $y = 0$, and approaches negative infinity as y approaches infinity, so that it dominates the other terms for large positive y. As a result, for positive y, there are a local minimum of the q function at a small value in y and a local maximum of the q function at a large value in y. Figure 1.2 (a) shows the (marginal) q ratio, which is drawn as a function of y. The q function is negatively sloped for $y < y^-$, then positively sloped, peaks at y^+, and again becomes negatively sloped. The q function is (b

Figur

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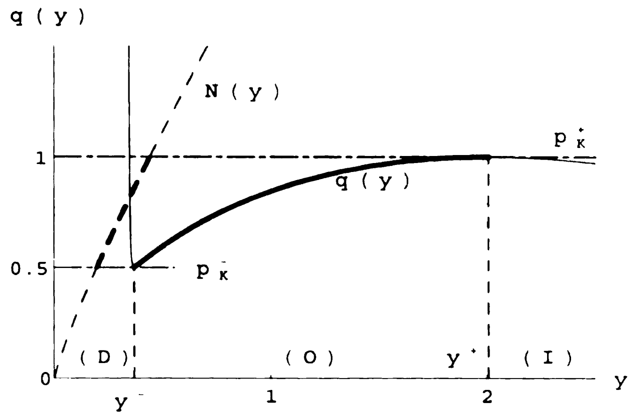
(a) Functions q (y) and $N(y)$



(b) Optimal Investment and Disinvestment

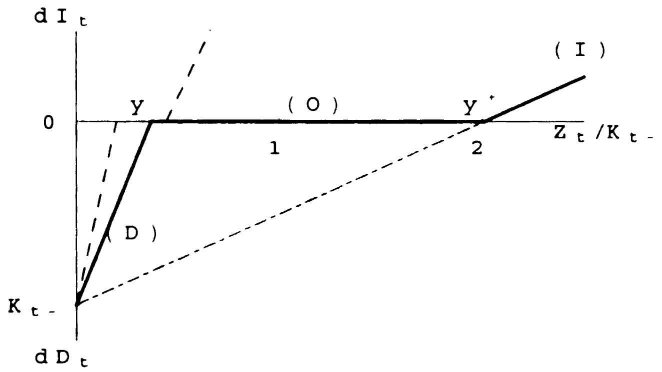


Figure 1.2 Costly Reversible Investment without Fixed Costs

($\theta \approx 0.143, A_\pi \approx 0.506, \delta = 0.06, \gamma = 0.07, \mu_Z = 0.05, \sigma_Z = \mu_Z, G \approx 5.43$)

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tangential to the price lines ($p_K^+ = 1$ and $p_K^- = 0.5$) at the critical values of y. The N function is the so-called naive case in which the firm assumes zero future investment. It is presented as a dashed line in figure 1.2 (a). The bold line of either the q function or the N function is relevant to investment decisions.

For costly reversible investment without fixed costs, whenever the firm observes y above y^+ or below y^-, the firm will invest or disinvest accordingly. When the firm invests or disinvests, y after investment or disinvestment equals the corresponding critical value, y^+ or y^-. When y is between y^+ and y^-, zero investment, or inaction, is optimal, and capital stock remains at the same level. Since this model assumes that installation of investment is instantaneous, we do not observe y above y^+ or below y^-. For the naive case, the optimal investment rule (the bold dashed line) is entirely located left of the costly reversible case (the bold solid line), and two critical values are lower than those for the costly reversible case.

Figure 1.2 (b) shows optimal investment, dI_t, and optimal disinvestment, dD_t, as a function of the ratio of the stochastic variable, Z_t to capital stock before investment, K_{t-}. In figure 1.2 (b), a solid line is optimal investment and disinvestment for the costly reversible investment, and a dashed line is optimal investment for the naive case. Both lines are derived from figure 1.2 (a). When the ratio, Z_t / K_{t-}, is between y^+ and y^-, zero investment is optimal so that $K_{t+} = K_{t-}$, or $dI_t = 0$ and $dD_t = 0$. If the ratio exceeds y^+, optimal investment is positive and given by

$$dI_t = \frac{Z_t}{y^+} - K_{t-}. \tag{17}$$

If the ratio is below y^-, optimal disinvestment is positive and given by

$$dD_t = K_{t-} - \frac{Z_t}{y^-}. \tag{18}$$

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Investment for the naive case is entirely located left of costly reversible investment, so that, when the firm contemplates future investment, its optimal investment is smaller than when it does not consider future investment. Disinvestment for the naive case is also located left of costly reversible investment, so that optimal disinvestment is larger when the firm contemplates future disinvestment than when it does not. In terms of capital stock, the firm chooses lower capital stock when it consider future investment than when it does not.

Figures 1.3 and 1.4 show the optimal rule for investment and disinvestment, respectively. The optimal rule is to bring y back to one of the two critical values whenever y moves beyond the corresponding critical value. Otherwise, inaction or zero investment is optimal. In figure 1.3, the function q approaches the purchase price line (p_K^+) from below, becomes tangent to the purchase price line at the critical point for investment, y^+, and then moves downward. In figure 1.4, the function q shows the opposite move. It approaches the resale price line (p_K^-) from above, becomes tangent to the resale price line at the critical point for disinvestment, y^-, and then moves upward.

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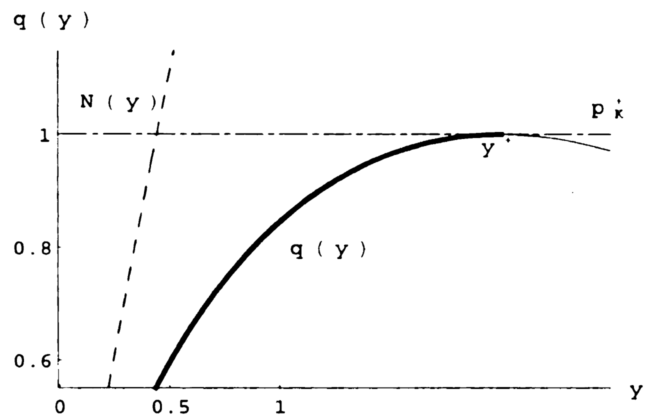

Figure 1.3 Optimal Rule for Investment in
Costly Reversible Investment without Fixed Costs Model

($\theta \approx 0.143, A_\pi \approx 0.506, \delta = 0.06, \gamma = 0.07, \mu_Z = 0.05, \sigma_Z = \mu_Z, y^+ \approx 2.01$)

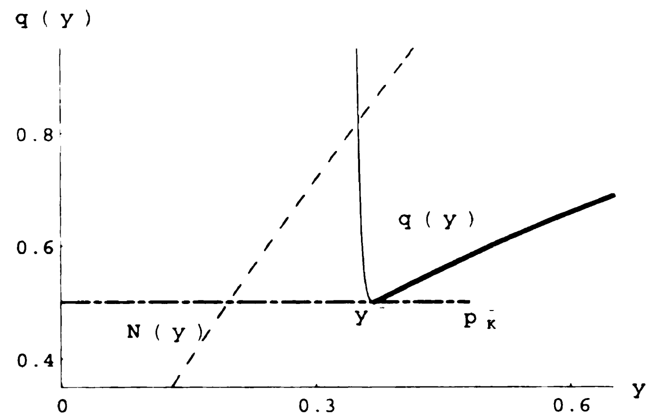

Figure 1.4 Optimal Rule for Disinvestment in
Costly Reversible Investment without Fixed Costs Model

($\theta \approx 0.143, A_\pi \approx 0.506, \delta = 0.06, \gamma = 0.07, \mu_Z = 0.05, \sigma_Z = \mu_Z, y^- \approx 0.369$)

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CHAPTER 2

COSTLY REVERSIBLE INVESTMENT WITH FIXED COSTS

This chapter derives the optimal investment rule when investment has fixed costs. Due to fixed costs, both investment and disinvestment have a trigger value and a target value in y. The investment costs have two parts: payment for new capital or revenue from resale on one hand, and fixed costs on the other hand. Fixed costs, however, do not affect the marginal analysis of investment, so that the Bellman equation is the same as costly reversible investment without fixed costs.

We can write the value function, $V(K_t,Z_t)$, as follows. There is one difference from investment without fixed costs, i.e., a specification of fixed costs.

$$V(K_t,Z_t)=\max_{\{dI_{t+s},dD_{t+s}\}} E_t\left[\int_0^\infty e^{-\gamma s}\left\{A_\pi Z_{t+s}^{1-\theta}K_{t+s}^{\theta}ds-IC_{t+s}\right\}\right] \tag{19}$$

$$\begin{aligned}\text{subject to } dK_{t+s} &= dI_{t+s}-dD_{t+s}-\delta K_{t+s}dt,\ K_{t+s}\geq 0 \quad \forall s\geq 0,\\ dZ_{t+s} &= \mu_Z Z_{t+s}dt+\sigma_Z Z_{t+s}dz_Z, \text{ and}\\ IC_{t+s} &= p_K^+ dI_{t+s}-p_K^- dD_{t+s}+F_I Z_{t+s}+F_D Z_{t+s}\end{aligned}$$

Here, $F_I Z_{t+s}$ and $F_D Z_{t+s}$ are fixed costs for investment and disinvestment, respectively. Coefficients, F_I and F_D are constant and given. Then, equation (19) yields the following Bellman equation.

$$\gamma V(K_t,Z_t)=A_\pi Z_t^{1-\theta}K_t^{\theta}-\delta K_t V_K+\mu_Z Z_t V_Z+\frac{1}{2}\sigma_Z^2 Z_t^2 V_{ZZ} \tag{20}$$

The solution of equation (20) is

$$V_K(\cdot)=q(y)=Hy^{1-\theta}+B_N^{\bullet} y^{\varphi_N}+B_P^{\bullet} y^{\varphi_P}. \tag{21}$$

Here, H, φ_N and φ_P are defined similar to investment without fixed costs.

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2-1. Optimal Investment for Costly Reversible Investment with Fixed Costs

Denoting the trigger values and the target values of investment and disinvestment by y_{Tr}^{+}, y_{Tr}^{-}, y_{Ta}^{+}, and y_{Ta}^{-}, respectively, the boundary conditions for equation (21) become

$$q\left(y_{Tr}^{+}\right) = p_K^{+}, \tag{22a}$$

$$q\left(y_{Ta}^{+}\right) = p_K^{+}, \tag{22b}$$

$$q\left(y_{Tr}^{-}\right) = p_K^{-}, \tag{22c}$$

$$q\left(y_{Ta}^{-}\right) = p_K^{-}, \tag{22d}$$

$$\int_{Z/y_{Tr}^{+}}^{Z/y_{Ta}^{+}} \left[V_K(\cdot) - p_K^{+}\right] dK = ZF_I , \tag{22e}$$

and

$$\int_{Z/y_{Ta}^{-}}^{Z/y_{Tr}^{-}} \left[p_K^{-} - V_K(\cdot)\right] dK = ZF_D . \tag{22f}$$

At the trigger value for investment, the firm is indifferent between investing and not investing. Equation (22a) represents this. When the firm invests, the firm should invest until function q becomes equal to the purchase price of capital. This yields equation (22b). The benefit from investment from the trigger value to the target value is equal to fixed costs of investment. Equation (22e) shows this. Similar arguments for disinvestment yield equations (22c), (22d), and (22f).

Although the function q with fixed costs has the same equation form as the function q without fixed costs, two coefficients for the function q with fixed costs, $B_N^{\bullet}$ and $B_P^{\bullet}$, satisfy the following relation with the corresponding coefficients for the function q without fixed costs, B_N and B_P.[3]

$$B_P < B_P^{\bullet} < 0 < B_N^{\bullet} < B_N \tag{23}$$

By multiplying a positive coefficient smaller than B_N, the minimum of the function q lowers. Similarly, by multiplying a negative coefficient larger than B_P, the maximum of

[3] See the appendix C for proof.

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the function q rises. By choosing appropriate $B_N^{\bullet}$ and $B_P^{\bullet}$, the model yields positive values for the four critical values, y_{Tr}^{+}, y_{Tr}^{-}, y_{Ta}^{+}, and y_{Ta}^{-}, satisfying the boundary conditions (22a) to (22f).

From equations (21) and (22a) to (22f), we have

$$F_1(\xi) = H\left(y_{Tr}^{+}\right)^{1-\theta} + B_N^{\bullet}\left(y_{Tr}^{+}\right)^{\varphi_N} + B_P^{\bullet}\left(y_{Tr}^{+}\right)^{\varphi_P} - p_K^{+} = 0, \tag{24a}$$

$$F_2(\xi) = H\left(y_{Ta}^{+}\right)^{1-\theta} + B_N^{\bullet}\left(y_{Ta}^{+}\right)^{\varphi_N} + B_P^{\bullet}\left(y_{Ta}^{+}\right)^{\varphi_P} - p_K^{+} = 0, \tag{24b}$$

$$F_3(\xi) = H\left(y_{Tr}^{-}\right)^{1-\theta} + B_N^{\bullet}\left(y_{Tr}^{-}\right)^{\varphi_N} + B_P^{\bullet}\left(y_{Tr}^{-}\right)^{\varphi_P} - p_K^{-} = 0, \tag{24c}$$

$$F_4(\xi) = H\left(y_{Ta}^{-}\right)^{1-\theta} + B_N^{\bullet}\left(y_{Ta}^{-}\right)^{\varphi_N} + B_P^{\bullet}\left(y_{Ta}^{-}\right)^{\varphi_P} - p_K^{-} = 0, \tag{24d}$$

$$\begin{aligned} F_5(\xi) = &-\frac{A_\pi}{f(1-\theta)}\left[\left(y_{Tr}^{+}\right)^{-\theta} - \left(y_{Ta}^{+}\right)^{-\theta}\right] + \frac{B_N^{\bullet}}{\varphi_N - 1}\left[\left(y_{Tr}^{+}\right)^{\varphi_N - 1} - \left(y_{Ta}^{+}\right)^{\varphi_N - 1}\right] \\ &+ \frac{B_P^{\bullet}}{\varphi_P - 1}\left[\left(y_{Tr}^{+}\right)^{\varphi_P - 1} - \left(y_{Ta}^{+}\right)^{\varphi_P - 1}\right] + p_K^{+}\left[\left(y_{Tr}^{+}\right)^{-1} - \left(y_{Ta}^{+}\right)^{-1}\right] - F_I = 0, \end{aligned} \tag{24e}$$

and

$$\begin{aligned} F_6(\xi) = &-p_K^{-}\left[\left(y_{Ta}^{-}\right)^{-1} - \left(y_{Tr}^{-}\right)^{-1}\right] + \frac{A_\pi}{f(1-\theta)}\left[\left(y_{Ta}^{-}\right)^{-\theta} - \left(y_{Tr}^{-}\right)^{-\theta}\right] \\ &- \frac{B_N^{\bullet}}{\varphi_N - 1}\left[\left(y_{Ta}^{-}\right)^{\varphi_N - 1} - \left(y_{Tr}^{-}\right)^{\varphi_N - 1}\right] - \frac{B_P^{\bullet}}{\varphi_P - 1}\left[\left(y_{Ta}^{-}\right)^{\varphi_P - 1} - \left(y_{Tr}^{-}\right)^{\varphi_P - 1}\right] - F_D = 0. \end{aligned} \tag{24f}$$

Here, ξ is a vector of six unknowns for investment with fixed costs, where $\xi = \left\{y_{Tr}^{+}, y_{Ta}^{+}, y_{Tr}^{-}, y_{Ta}^{-}, B_N^{\bullet}, B_P^{\bullet}\right\}$. This analysis again numerically solves the system of equations (24a) to (24f).

Figure 2.1 shows one solution of costly reversible investment with fixed costs. The solid line is the function q for investment with fixed costs, while a broken line is q without fixed costs. And, a dashed line is the function N. The difference between q with fixed costs and without fixed costs is only the inclusion of fixed costs, while all other conditions are the same. When there are fixed costs, the peak of the function q moves upward and to the right. As a result, the function q reaches above the purchase price line

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($p_K^+ = 1$). And, the crescent area between the function q and the purchase price line corresponds to fixed costs of investment, or the left hand side of equation (22e). At the same time, the minimum of the function q moves downward and to the left, and the minimum of the function q is below the resale price line ($p_K^- = 0.5$). A triangular area between the function q and the resale price corresponds to fixed costs of disinvestment, or the left hand side of equation (22f). The peak of the function q is flat, while the trough of the function q is steep. When y reaches part (D) in figure 2.1, the firm should disinvest, while when y reaches part (I) in the figure, the firm should invest. If y remains in either part (O), (O′), or (O″), then zero investment is optimal. Also, figure 2.1 shows $y_{Tr}^- < y_{Ta}^- < y_{Ta}^+ < y_{Tr}^+$.

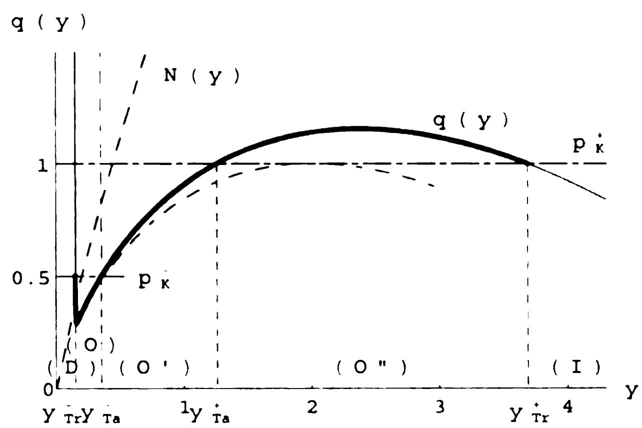

Figure 2.1 $q(y)$ and $N(y)$ for Costly Reversible Investment with Fixed Costs

($\theta \approx 0.143$, $A_\pi \approx 0.506$, $\delta = 0.06$, $\gamma = 0.07$, $\mu_Z = 0.05$, $\sigma_Z = 0.05$, $p_K^+ = 1$, $p_K^- = 0.5$, $F_D = 0.5$, $F_I = 0.05$)

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Figures 2.2 and 2.3 show the optimal rule for investment and disinvestment, respectively. The optimal rule is to bring y to one of two target values, whenever y exceeds the corresponding trigger value. For example, the pre-investment level of y is above y^+_{Tr}, the firm should invest so that the post-investment level of y equals y^+_{Ta}. In figure 2.2, the optimal investment is to bring y to y^+_{Ta} ($\approx$ 1.26), whenever y exceeds y^+_{Tr} ($\approx$ 3.67). Similarly, in figure 2.3, the firm makes y equal to y^-_{Ta} ($\approx$ 0.352), whenever y moves below y^-_{Tr} ($\approx$ 0.150). When y is between y^-_{Tr} and y^+_{Tr}, inaction is optimal.

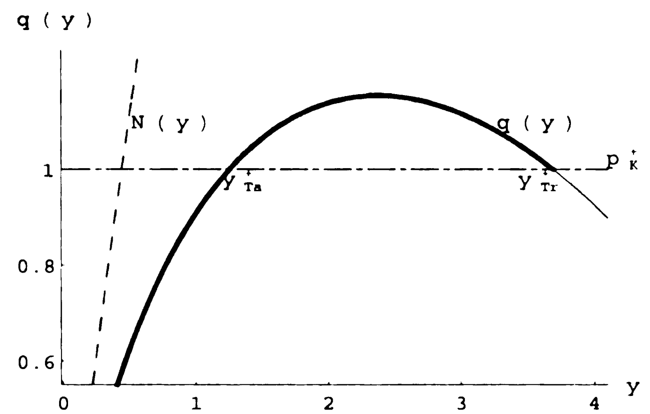

Figure 2.2 Optimal Rule for Investment in
Costly Reversible Investment with Fixed Costs Model

($\theta \approx 0.143$, $A_\pi \approx 0.506$, $\delta = 0.06$, $\gamma = 0.07$, $\mu_Z = 0.05$, $\sigma_Z = 0.05$,
$p^+_K = 1$, $p^-_K = 0.5$, $F_D = 0.5$, $F_I = 0.05$, $y^+_{Ta} \approx 1.26$, $y^+_{Tr} \approx 3.67$)

2-2. User Cost

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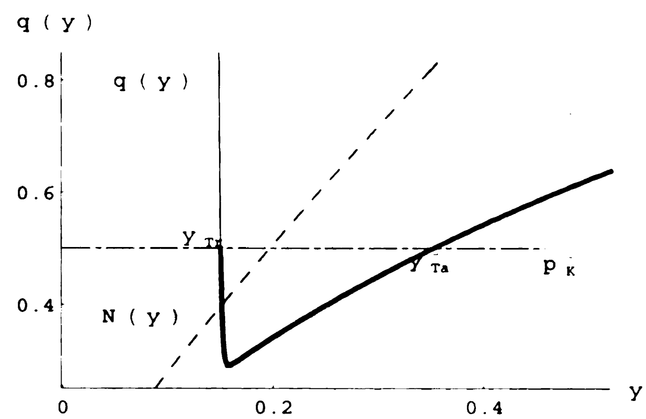

Figure 2.3 Optimal Rule for Disinvestment in
Costly Reversible Investment with Fixed Costs Model

($\theta \approx 0.143$, $A_\pi \approx 0.506$, $\delta = 0.06$, $\gamma = 0.07$, $\mu_Z = 0.05$, $\sigma_Z = 0.05$,
$p_K^+ = 1$, $p_K^- = 0.5$, $F_D = 0.5$, $F_I = 0.05$, $y_{Tr}^- \approx 0.150$, $y_{Ta}^- \approx 0.352$)

2-2. User Cost of Capital

Abel and Eberly (1996) show that, for costly reversible investment without fixed costs, the user cost of capital is higher for investment or lower for disinvestment than Jorgenson's user cost of capital. This section shows that, for costly reversible investment with fixed costs, the user cost of capital is even higher at the trigger for investment and lower at the trigger for disinvestment than those in the Abel and Eberly's model. Jorgenson (1963) show that the user cost of capital, c_J, is

$$c_J^+ = (\gamma + \delta)p_K^+ + \dot{p}_K^+ \text{ for investment, or} \tag{25a}$$

$$c_J^- = (\gamma + \delta)p_K^- + \dot{p}_K^- \text{ for disinvestment.} \tag{25b}$$

Here, $\dot{p}$ is the derivative of p with respect to time. In this analysis, the price of capital is constant over time, so that

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$$c_J^+ = (\gamma + \delta)p_K^+ \text{ for investment, or} \tag{26a}$$

$$c_J^- = (\gamma + \delta)p_K^- \text{ for disinvestment.} \tag{26b}$$

Abel and Eberly show that, for costly reversible investment without fixed costs, the user cost of capital, $c(y)$, is the following.

$$\begin{aligned} c(y) &\equiv (\gamma + \delta)q(y) - \frac{1}{dt}E[dq(y)] \\ &= (\gamma + \delta)q(y) - \mu_y y q'(y) - \frac{1}{2}\sigma_y y^2 q''(y) \\ &= A_\pi \theta\, y^{1-\theta} \end{aligned} \tag{27}$$

Then, the user cost of capital at each trigger value is

$$c^+ = A_\pi \theta (y^+)^{1-\theta} \text{ for investment without fixed costs, or} \tag{28a}$$

$$c^- = A_\pi \theta (y^-)^{1-\theta} \text{ for disinvestment without fixed costs.} \tag{28b}$$

Abel and Eberly also show

$$c^- < c_J^- < c_J^+ < c^+ . \tag{29}$$

Because the fixed costs of investment do not affect the marginal analysis, equation (27) is valid for investment with fixed costs. Thus, at the trigger value of investment or disinvestment, the user cost of capital is

$$c_{Tr}^+ = A_\pi \theta (y_{Tr}^+)^{1-\theta} \text{ for investment with fixed costs, or} \tag{30a}$$

$$c_{Tr}^- = A_\pi \theta (y_{Tr}^-)^{1-\theta} \text{ for disinvestment with fixed costs.} \tag{30b}$$

Therefore, because $y_{Tr}^- < y^- < y^+ < y_{Tr}^+$ as appendix C shows,

$$c_{Tr}^- < c^- \left(< c_J^- < c_J^+\right) < c^+ < c_{Tr}^+ . \tag{31}$$

The user cost of capital at the trigger value for investment is higher with fixed costs than without fixed costs, while the user cost of capital at the trigger value for disinvestment is lower with fixed costs than without fixed costs.

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CHAPTER 3

OPTIMAL INVESTMENT RULE AND EFFECTS OF PARAMETERS

3-1. Optimal Investment Rule

This section discusses the optimal investment in terms of capital stock. Figure 3.1 shows a schematic diagram for costly reversible investment with fixed costs. A solid line in figure 3.1 is optimal post-investment capital stock for the costly reversible investment with fixed costs. At time t, given firm's pre-investment capital stock as K_{t-}, if the ratio of the economic indicator to pre-investment capital stock, Z_t / K_{t-}, is between y_{Tr}^{-} and y_{Tr}^{+}, optimal investment is zero gross investment or inaction, so that capital stock remains at the same level and $K_{t+} / K_{t-} = 1$. The horizontal locus in figure 3.1, which is called inaction locus, shows this.

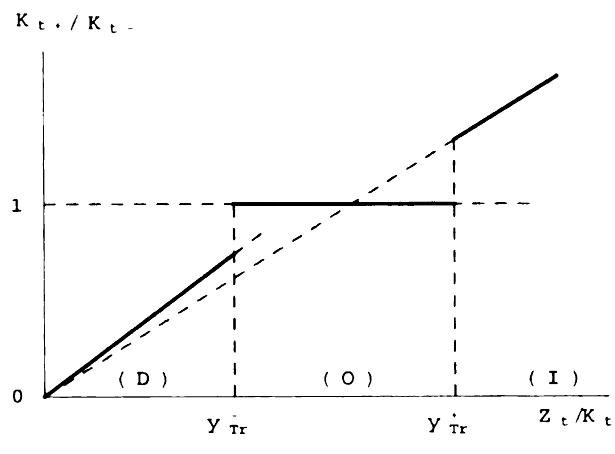

Figure 3.1 Schematic Diagram for Costly Reversible Investment with Fixed Costs

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The upper end of inaction locus is y^{+}_{Tr}, so that higher y^{+}_{Tr} widens the inaction range by increasing its upper limit. When the ratio exceeds y^{+}_{Tr}, the firm will invest such that

$$\frac{K_{t+}}{K_{t-}} = \left(\frac{1}{y^{+}_{Ta}}\right)\frac{Z_t}{K_{t-}}. \tag{32}$$

The slope of K_{t+} / K_{t-} locus for investment is proportional to the reciprocal of y^{+}_{Ta}. A higher value of y^{+}_{Ta} flattens investment locus, so that post-investment capital stock, K_{t+}, is lower and the optimal investment is smaller. Due to fixed costs, there is a gap between inaction locus and investment locus at y^{+}_{Tr}. Optimal investment can be written as follows:

$$dI_t = \frac{Z_t}{y^{+}_{Ta}} - K_{t-} \qquad \text{for } Z_t \geq Z^{+} (= y^{+}_{Tr} K_{t-}). \tag{33}$$

Similarly, when the ratio Z_t / K_{t-} moves below y^{-}_{Tr} the firm disinvests such that

$$\frac{K_{t+}}{K_{t-}} = \left(\frac{1}{y^{-}_{Ta}}\right)\frac{Z_t}{K_{t-}}. \tag{34}$$

The slope of K_{t+} / K_{t-} locus for disinvestment is proportional to the reciprocal of y^{-}_{Ta}. There is a similar gap for disinvestment. The lower end of inaction locus is y^{-}_{Tr}, so that higher y^{-}_{Tr} narrows the inaction range by decreasing its lower limit. A higher value of y^{-}_{Ta} flattens disinvestment locus, so that post-investment capital stock is lower and optimal disinvestment is larger, in contrast to optimal investment. Optimal disinvestment can be written as follows:

$$dD_t = K_{t-} - \frac{Z_t}{y^{-}_{Ta}} \qquad \text{for } Z_t \geq Z^{-} (= y^{-}_{Tr} K_{t-}). \tag{35}$$

When there are no fixed costs of investment, there is no gap among disinvestment locus, inaction locus and investment locus. In other words, for investment without fixed costs, the disi

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costs, the disinvestment locus, inaction locus and investment locus are connected, and $y^+_{Tr} = y^+_{Ta} = y^+$ and $y^-_{Tr} = y^-_{Ta} = y^-$.

The size of the inaction range is measured by $G\left(= Z^+/Z^- = y^+/y^- \text{ or } y^+_{Tr}/y^-_{Tr}\right)$. G is large when the range of inaction is wide in terms of Z_t. For investment with fixed costs, minimum investment and minimum disinvestment are measured by G^+ and G^-, respectively, in terms of the ratio of post-investment capital stock to pre-investment capital stock. G^+ and G^- are defined as K_{t+} / K_{t-} (y^+_{Tr}/y^+_{Ta} for investment, or y^-_{Tr}/y^-_{Ta} for disinvestment). A large G^+ corresponds to a large value of minimum investment. A large G^- corresponds to a smaller value of minimum disinvestment.

3-2. Effects of Parameters on Optimal Investment

This section presents simulations constructed under a wide range of values for the economic parameters in order to investigate their effects on capital investment. The number of simulation cases is more than 90,000 for investment with fixed costs and more than 30,000 for investment without fixed costs. Thus, the conclusions of the simulations are robust. The simulations show that all of the critical values of the optimal investment, except the trigger value for investment, are approximately linear in the economic parameters such as the discount rate, while the trigger value for investment is semi-log linear in the economic parameters.

Table 3.1 lists the economic parameters for the simulations. The simulations vary all of the economic parameters except the depreciation rate and the purchase price of capital. The purchase price of capital is used as the numeraire. In equation (9), which is the differential equation for the theoretical model, trend in the economic indicator variable, μ_Z, the discount rate, γ, and the depreciation rate, δ, appear as $\mu_Z + \delta$ ($\equiv \mu_y$) and ($\gamma + \delta$). In other words, three parameters, μ_Z, γ and δ, yield two coefficients. In order to avoid a redundancy in the simulations, the simulations drop the depreciation rate. The simulations first focus on two parameters: μ_Z, and the spread, which is defined as the

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Table 3.1 Cases for Simulations

Parameter	Values	Number of Points
Elasticity of Profit (θ)*	0.06 ~ 0.71	19
Coefficient of Profit (A_π)*	0.17 ~ 0.70	19
Depreciation Rate (δ)	0.06	1
Volatility in Z_t (σ_Z)	0.01, 0.05 and 0.1	3
Purchase Price of Capital (p_K^+)	1 (numeraire)	1
Resale Price of Capital (p_K^-)	0.25, 0.5 and 0.75	3
Fixed Costs of Investment (F_I, F_D)	0.1, 0.5 and 1	3
Trend in Z_t (μ_Z)	-0.06 to 0.09 by 0.01	16
Spread ($\equiv \gamma - \mu_Z$)	0.001 to 0.091 by 0.01	10

* θ and A_π are determined by an iso-elastic demand curve, $Q_d = (P/X_1)^{-\varepsilon}$, and a Cobb-Douglas production function, $X_2 K^\alpha L^\beta$, where X_1 and X_2 are stochastic coefficients. Values for three parameters, α, β and ε, are determined as $\alpha + \beta = 1$, 0.8 and 0.6, $\beta / \alpha =$ 1, 2, and 3, and $\varepsilon = 1.5$, 3, and 6. As the three parameters, α, β and ε, yield the two simulation parameters, θ and A_π, different combinations of the three parameters result in the same simulation parameters. Thus, the number of the simulation cases is nineteen rather than twenty seven.

difference between the discount rate and trend in the economic indicator variable, μ_Z. When the simulation changes the spread and μ_Z, and keeps other parameters constant, all of the critical values except the trigger value for investment with fixed costs are very linear in the two parameters.

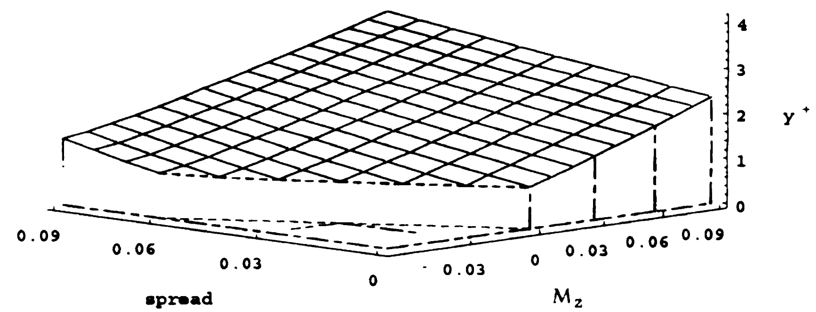
Figure 3.2 shows one example of the critical values for investment without fixed costs and with fixed costs. Each graph in figure 3.2 contains 139 points. In figure 3.2 (a) and (b) for investment without fixed costs, the surfaces of the critical values, y^+ and y^- are very flat. As a result, the linear approximation is a good fit. Similarly, in figure 3.2 (c), (e) and (f), three critical values for investment with fixed costs: the target value for investment, y_{Ta}^+, and the two critical values for disinvestment, y_{Ta}^-, and y_{Tr}^-, are very flat. Therefore, the linear approximation is a good fit for each critical value. However, the trigger value for investment with fixed costs, y_{Tr}^+, is convex in both μ_Z and the spread

Costly Reversible without Fixed Costs

(a) Critical value for investment, y^*

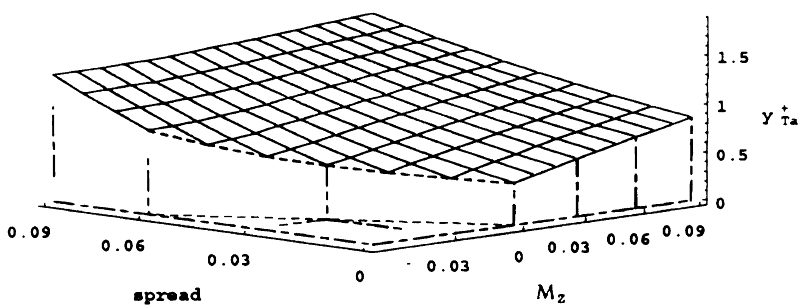
Costly Reversible without Fixed Costs

(a) Critical value for investment, y^+



Costly Reversible with Fixed Costs

(b) Target value for investment, y^+_{Ta}



(c) Trigger value for investment, y^+_{Tr}

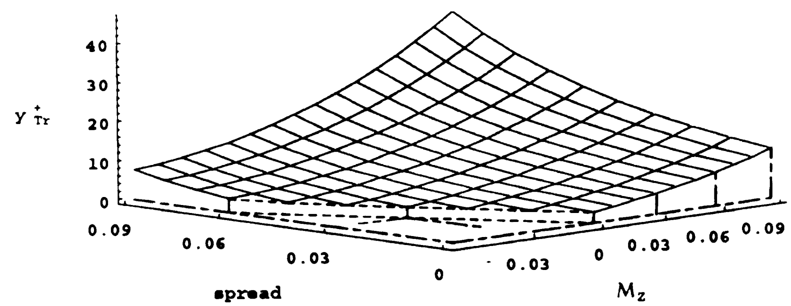


Figure 3.2 Critical Values for Investment and Disinvestment

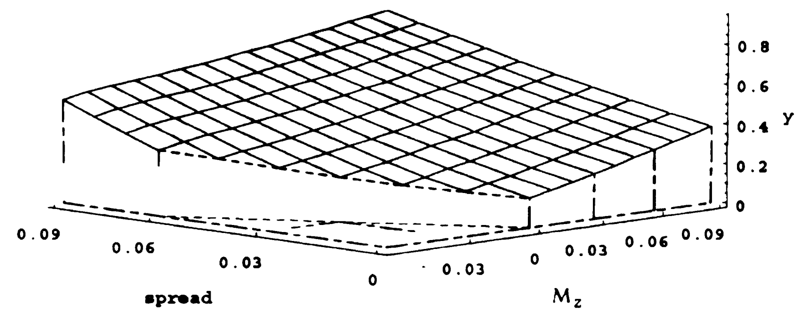
($\theta \approx 0.143$, $A_\pi \approx 0.506$, δ=0.06, $\sigma_Z = 0.05$, $p^+_K = 1$, $p^-_K = 0.5$, $F_I = 0.5$, $F_D = 0.5$)

Costly Reversible without Fixed Costs

(d) Critical value for disinvestment, y

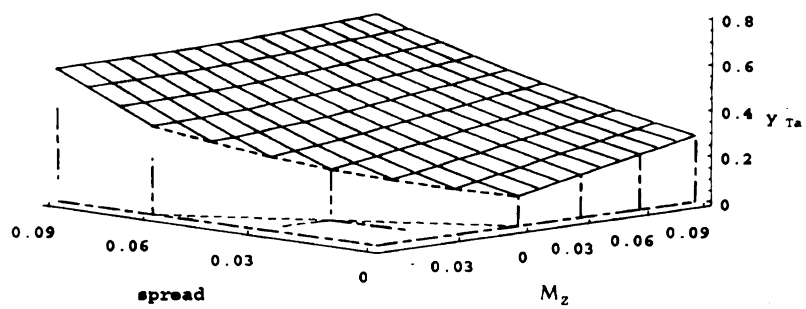
Costly Reversible without Fixed Costs

(d) Critical value for disinvestment, y^-



Costly Reversible with Fixed Costs

(e) Target value for disinvestment, y^-_{Ta}



(f) Trigger value for disinvestment, y^-_{Tr}

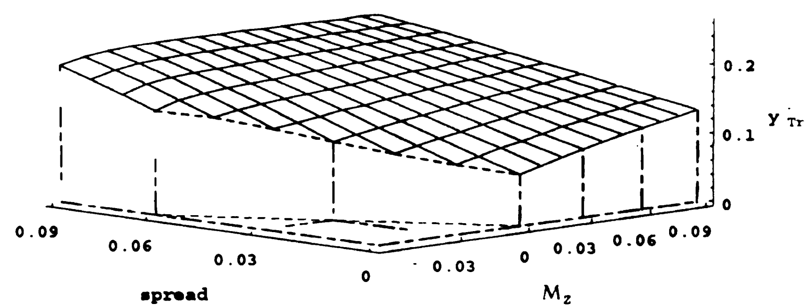


Figure 3.2 (cont'd)

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as figure 3.2 (d) shows. As a result, the semi-log linear approximation is a better fit than the linear approximation. With the parameters for figure 3.2, the linear or semi-log linear approximations of all six critical values and their R^2 are as follows:

$$y^{+} = 0.775 + 18.4 \times \text{spread} + 17.6 \times \mu_Z \qquad (R^2 = 0.999, \text{ data size} = 139)$$
$$(0.00407) \quad (0.0655) \qquad (0.0428)$$

$$y^{-} = 0.109 + 6.12 \times \text{spread} + 2.79 \times \mu_Z \qquad (R^2 = 0.997, \text{ data size} = 139)$$
$$(0.00580) \quad (0.0934) \qquad (0.0610)$$

$$y^{+}_{Tu} = 0.350 + 12.0 \times \text{spread} + 4.61 \times \mu_Z \qquad (R^2 = 0.987, \text{ data size} = 139)$$
$$(0.00937) \quad (0.151) \qquad (0.0985)$$

$$\text{Ln}\, y^{+}_{Tr} = 1.09 + 17.9 \times \text{spread} + 14.9 \times \mu_Z \qquad (R^2 = 0.982, \text{ data size} = 139)$$
$$(0.0198) \quad (0.318) \qquad (0.208)$$

$$y^{-}_{Tu} = 0.0995 + 5.92 \times \text{spread} + 1.65 \times \mu_Z \qquad (R^2 = 0.990, \text{ data size} = 139)$$
$$(0.0107) \quad (0.171) \qquad (0.112)$$

$$y^{-}_{Tr} = 0.0774 + 1.67 \times \text{spread} + 0.488 \times \mu_Z \qquad (R^2 = 0.990, \text{ data size} = 139)$$
$$(0.00742) \quad (0.119) \qquad (0.0780)$$

Figures in parentheses are the standard errors. The R^2 of all critical values are very close to unity, so that the linear approximations are a good fit with regard to μ_Z and the spread.

By testing the hypothesis that the sum of the coefficients of the spread and μ_Z are equal to zero, the analysis investigates the significance of γ. When γ is significant, the hypothesis will be rejected. The test statistics is

$$t = \frac{\hat{\beta}_{\text{spread}} + \hat{\beta}_{\mu_Z}}{se\left(\hat{\beta}_{\text{spread}} + \hat{\beta}_{\mu_Z}\right)} \sim t_{\text{data size}-k} \,. \tag{36}$$

Here, $\hat{\beta}_{\text{spread}}$ and $\hat{\beta}_{\mu_Z}$ are estimated coefficients for the spread and μ_Z, respectively. The denominator is the standard error of ($\hat{\beta}_{\text{spread}} + \hat{\beta}_{\mu_Z}$), and k is the number of explanatory variables. Table 3.2 shows the t-statistics under the hypothesis. All of six tested values reject the hypothesis of insignificant γ at the one percent level. Therefore, the simulations conclude that the discount rate, γ, is statistically significant.

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Tables 3.3 to 3

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Table 3.2 Significance of Discount Rate

	t-Statistics
Critical Value of Investment, y^+	417.9***
Critical Value of Disinvestment, y^-	72.6***
Target Value of Investment, y^+_{Ta}	64.1***
Trigger Value of Investment, Ln y^+_{Tr}	235.5***
Target Value of Disinvestment, y^-_{Ta}	33.6***
Target Value of Disinvestment, y^-_{Tr}	13.8***

***: reject Ho at 1%

Similarly, the critical values show a linear relationship to the other parameters. Tables 3.3 to 3.8 show estimated coefficients of five approximations: two for linear models with and without interaction terms, and three for quadratic models with and without interaction terms. There are eight explanatory variables for the linear approximation of investment with fixed costs: a constant, θ, A_π, p_K^-, fixed costs, the spread, σ_Z, and μ_Z, and seven for investment without fixed costs. Explanatory variables for the quadratic approximation are at most 36 for investment with fixed costs: the constant, the seven variables, squares of the seven variables, and 21 interaction terms, and at most 28 for investment without fixed costs. Even the linear approximations have R^2 higher than 0.7, although they have only seven or eight explanatory variables. As the analysis includes more explanatory variables such as squared terms, the R^2 increases. In the case of the quadratic approximation with all interaction terms, all critical values have R^2 higher than 0.9.

Table 3

constant	3
θ	-5.
θ^2	N
A_z	-3.
A_z^2	N
σ_z	1.
σ_z^2	N
p_K	-0.
p_K^2	N
$\theta \times A_z$	N
$\theta \times \sigma_z$	N
$\theta \times p_K$	N
$A_z \times \sigma_z$	N
$A_z \times p_K$	N
$\sigma_z \times p_K$	N
spread	17.
spread^2	N
$\theta \times$ spread	N
$A_z \times$ spread	N
$\sigma_z \times$ spread	N
$p_K \times$ spread	N
μ_z	17.
μ_z^2	N
$\theta \times \mu_z$	N
$A_z \times \sigma_z$	N
$\sigma_z \times \mu_z$	N
$p_K \times \mu_z$	N
spread $\times \mu_z$	N
R^2	0.

data size: 33.14
standard error i

Table 3.3 Critical Value for Investment without Fixed Costs, y^+

	Linear		Quadratic		Linear with Interaction		Quadratic with Interaction (1)		Quadratic with Interaction (2)	
constant	3.7	(0.03)	7.8	(0.04)	1.4	(0.06)	5.5	(0.05)	-0.7	(0.11)
θ	-5.6	(0.03)	-15.7	(0.07)	-3.0	(0.07)	-13.0	(0.07)	1.4	(0.24)
θ^2	N/A		12.5	(0.09)	N/A		12.5	(0.08)	5.0	(0.14)
A_π	-3.8	(0.04)	-15.0	(0.12)	0.0	(0.08)	-11.3	(0.12)	8.9	(0.34)
A_π^2	N/A		10.7	(0.13)	N/A		10.7	(0.12)	-5.1	(0.27)
σ_Z	1.3	(0.10)	0.2	(0.31)	0.8	(0.21)	-0.2	(0.30)	1.9	(0.48)
σ_Z^2	N/A		9.1	(2.67)	N/A		9.0	(2.36)	9.0	(2.21)
$\bar{p}_K$	-0.0	(0.02)	-0.0	(0.09)	0.0	(0.04)	0.0	(0.08)	-0.0	(0.10)
$\bar{p}_K^2$	N/A		0.0	(0.09)	N/A		0.0	(0.08)	0.0	(0.07)
$\theta \times A_\pi$	N/A		N/A		N/A		N/A		-22.9	(0.36)
$\theta \times \sigma_Z$	N/A		N/A		N/A		N/A		-4.0	(0.48)
$\theta \times \bar{p}_K$	N/A		N/A		N/A		N/A		0.0	(0.09)
$A_\pi \times \sigma_Z$	N/A		N/A		N/A		N/A		-2.3	(0.60)
$A_\pi \times \bar{p}_K$	N/A		N/A		N/A		N/A		0.0	(0.11)
$\sigma_Z \times \bar{p}_K$	N/A		N/A		N/A		N/A		0.0	(0.28)
spread	17.6	(0.13)	17.9	(0.35)	49.3	(0.90)	49.5	(0.66)	46.2	(0.63)
spread^2	N/A		-1.8	(3.56)	N/A		-1.9	(3.14)	15.9	(3.05)
$\theta \times$ spread	N/A		N/A		-34.5	(1.06)	-35.9	(0.69)	-36.2	(0.65)
$A_\pi \times$ spread	N/A		N/A		-52.6	(1.33)	-51.1	(0.87)	-50.8	(0.81)
$\sigma_Z \times$ spread	N/A		N/A		11.7	(3.43)	10.8	(2.24)	11.2	(2.10)
$\bar{p}_K \times$ spread	N/A		N/A		-0.3	(0.62)	-0.4	(0.40)	-0.3	(0.38)
μ_Z	17.3	(0.09)	16.6	(0.09)	47.6	(0.59)	46.9	(0.39)	43.8	(0.39)
μ_Z^2	N/A		17.9	(1.47)	N/A		17.9	(1.30)	28.6	(1.32)
$\theta \times \mu_Z$	N/A		N/A		-35.9	(0.70)	-36.4	(0.46)	-36.4	(0.43)
$A_\pi \times \sigma_Z$	N/A		N/A		-46.2	(0.87)	-46.1	(0.57)	-45.9	(0.53)
$\sigma_Z \times \mu_Z$	N/A		N/A		-5.7	(2.26)	-4.4	(1.47)	-4.5	(1.38)
$\bar{p}_K \times \mu_Z$	N/A		N/A		-0.2	(0.41)	-0.1	(0.27)	-0.1	(0.25)
spread $\times \mu_Z$	N/A		N/A		N/A		N/A		44.9	(2.17)
R^2	0.717		0.862		0.748		0.893		0.906	

data size: 33,143
standard error in parenthesis

Tab

constant	
θ	-
θ^2	N
A_z	-(
A_z^2	N
σ_z	-(
σ_z^2	N
p_K	1
p_K^2	N
$\theta \times A_z$	N
$\theta \times \sigma_z$	N
$\theta \times p_K$	N
$A_z \times \sigma_z$	N A
$A_z \times p_K$	N A
$\sigma_z \times p_K$	N A
spread	5.(
spread^2	N/A
$\theta \times$ spread	N/A
$A_z \times$ spread	N/A
$\sigma_z \times$ spread	N/A
$p_K \times$ spread	N/A
μ_z	2.95
μ_z^2	N/A
$\theta \times \mu_z$	N/A
$A_z \times \mu_z$	N/A
$\sigma_z \times \mu_z$	N/A
$p_K \times \mu_z$	N/A
spread $\times \mu_z$	N/A
R^2	0.709

data size: 33,143
standard error in par

Table 3.4 Critical Value for Disinvestment without Fixed Costs, y^-

	Linear	Quadratic	Linear with Interaction	Quadratic with Interaction (1)	Quadratic with Interaction (2)
constant	0.21 (0.01)	1.22 (0.02)	0.03 (0.02)	1.06 (0.02)	-0.88 (0.04)
θ	-1.51 (0.01)	-4.28 (0.03)	-0.50 (0.02)	-3.28 (0.03)	0.78 (0.07)
θ^2	N/A	3.50 (0.04)	N/A	3.53 (0.03)	2.43 (0.04)
A_π	-0.65 (0.01)	-2.12 (0.05)	0.13 (0.03)	-1.37 (0.05)	2.70 (0.11)
A_π^2	N/A	1.14 (0.06)	N/A	1.13 (0.05)	-1.20 (0.09)
σ_Z	-0.11 (0.03)	0.03 (0.13)	-0.06 (0.07)	0.03 (0.12)	-0.07 (0.15)
σ_Z^2	N/A	-1.23 (1.15)	N/A	-0.88 (0.93)	-0.90 (0.69)
p_K^-	1.18 (0.01)	0.14 (0.04)	0.30 (0.01)	-0.75 (0.03)	1.35 (0.03)
p_K^{-2}	N/A	1.04 (0.04)	N/A	1.05 (0.03)	1.05 (0.02)
$\theta \times A_\pi$	N/A	N/A	N/A	N/A	-3.37 (0.11)
$\theta \times \sigma_Z$	N/A	N/A	N/A	N/A	0.44 (0.15)
$\theta \times p_K^-$	N/A	N/A	N/A	N/A	-3.98 (0.03)
$A_\pi \times \sigma_Z$	N/A	N/A	N/A	N/A	0.20 (0.19)
$A_\pi \times p_K^-$	N/A	N/A	N/A	N/A	-2.27 (0.03)
$\sigma_Z \times p_K^-$	N/A	N/A	N/A	N/A	-0.21 (0.09)
spread	5.03 (0.05)	4.65 (0.15)	8.32 (0.28)	7.87 (0.26)	7.31 (0.20)
spread^2	N/A	4.69 (1.53)	N/A	4.87 (1.23)	7.39 (0.95)
$\theta \times$ spread	N/A	N/A	-16.10 (0.34)	-16.50 (0.27)	-16.40 (0.20)
$A_\pi \times$ spread	N/A	N/A	-11.90 (0.42)	-11.50 (0.34)	-11.40 (0.25)
$\sigma_Z \times$ spread	N/A	N/A	-2.09 (1.09)	-2.27 (0.88)	-2.38 (0.65)
$p_K^- \times$ spread	N/A	N/A	13.00 (0.20)	13.00 (0.16)	13.20 (0.12)
μ_Z	2.95 (0.03)	2.92 (0.04)	3.35 (0.19)	3.28 (0.15)	2.97 (0.12)
μ_Z^2	N/A	0.74 (0.63)	N/A	1.66 (0.51)	2.98 (0.41)
$\theta \times \mu_Z$	N/A	N/A	-8.30 (0.22)	-8.42 (0.18)	-8.56 (0.13)
$A_\pi \times \mu_Z$	N/A	N/A	-7.16 (0.28)	-7.16 (0.22)	-7.22 (0.17)
$\sigma_Z \times \mu_Z$	N/A	N/A	1.87 (0.72)	2.11 (0.58)	2.09 (0.43)
$p_K^- \times \mu_Z$	N/A	N/A	9.81 (0.13)	9.84 (0.10)	9.88 (0.08)
spread $\times \mu_Z$	N/A	N/A	N/A	N/A	5.36 (0.68)
R^2	0.709	0.783	0.784	0.859	0.922

data size: 33,143
standard error in parenthesis

constant	
θ	
θ^2	[illegible]
A_z	[illegible]
A_z^2	[illegible]
σ_z	
σ_z^2	N
p_k^-	-(
p_k^{-2}	N
fixed costs	-(
fc^2	N
$\theta \times A_z$	N
$\theta \times \sigma_z$	N
$\theta \times p_k^-$	N
$\theta \times$ fc	N
$A_z \times \sigma_z$	N
$A_z \times p_k^-$	N A
$A_z \times$ fc	N A
$\sigma_z \times p_k^-$	N A
$\sigma_z \times$ fc	N A
$p_k^- \times$ fc	N A
spread	10.
$spread^2$	N A
$\theta \times$ spread	N A
$A_z \times$ spread	N A
$\sigma_z \times$ spread	N A
$p_k^- \times$ spread	N A
fc $\times$ spread	N A
μ_z	5.
μ_z^2	N A
$\theta \times \mu_z$	N A
$A_z \times \mu_z$	N A
$\sigma_z \times \mu_z$	N A
$p_k^- \times \mu_z$	N A
fc $\times \mu_z$	N A
spread $\times \mu_z$	N A
R^2	0.7

data size: 91,101
standard error in

Table 3.5 Target Value for Investment with Fixed Costs, y^{+}_{Ta}

	Linear	Quadratic	Linear with Interaction	Quadratic with Interaction (1)	Quadratic with Interaction (2)
constant	2.0 (0.01)	3.7 (0.01)	0.7 (0.01)	2.4 (0.01)	0.6 (0.03)
θ	-2.8 (0.01)	-7.2 (0.02)	-1.2 (0.02)	-5.5 (0.02)	-1.2 (0.06)
θ^2	N/A	5.5 (0.02)	N/A	5.3 (0.02)	2.8 (0.03)
A_π	-1.7 (0.01)	-5.8 (0.03)	-0.1 (0.02)	-4.2 (0.03)	2.2 (0.08)
A_π^2	N/A	3.8 (0.03)	N/A	3.8 (0.03)	-1.4 (0.06)
σ_Z	0.3 (0.02)	0.1 (0.08)	0.3 (0.05)	0.0 (0.07)	0.7 (0.11)
σ_Z^2	N/A	2.2 (0.68)	N/A	2.3 (0.55)	2.3 (0.50)
p_K^-	-0.0 (0.00)	-0.0 (0.02)	-0.0 (0.01)	-0.0 (0.02)	-0.0 (0.02)
p_K^{-2}	N/A	0.0 (0.02)	N/A	0.0 (0.02)	0.0 (0.02)
fixed costs	-0.3 (0.00)	-0.8 (0.01)	-0.1 (0.00)	-0.6 (0.01)	-1.1 (0.01)
fc^2	N/A	0.4 (0.01)	N/A	0.4 (0.01)	0.4 (0.01)
$\theta \times A_\pi$	N/A	N/A	N/A	N/A	-7.8 (0.08)
$\theta \times \sigma_Z$	N/A	N/A	N/A	N/A	-0.9 (0.11)
$\theta \times p_K^-$	N/A	N/A	N/A	N/A	-0.0 (0.02)
$\theta \times$ fc	N/A	N/A	N/A	N/A	1.0 (0.01)
$A_\pi \times \sigma_Z$	N/A	N/A	N/A	N/A	-0.4 (0.13)
$A_\pi \times p_K^-$	N/A	N/A	N/A	N/A	-0.0 (0.02)
$A_\pi \times$ fc	N/A	N/A	N/A	N/A	0.5 (0.01)
$\sigma_Z \times p_K^-$	N/A	N/A	N/A	N/A	-0.0 (0.06)
$\sigma_Z \times$ fc	N/A	N/A	N/A	N/A	-0.3 (0.04)
$p_K^- \times$ fc	N/A	N/A	N/A	N/A	0.0 (0.01)
spread	10.4 (0.03)	8.6 (0.09)	30.4 (0.22)	28.3 (0.15)	28.2 (0.14)
spread^2	N/A	23.5 (0.91)	N/A	22.9 (0.72)	21.2 (0.69)
$\theta \times$ spread	N/A	N/A	-30.1 (0.26)	-28.8 (0.16)	-27.1 (0.15)
$A_\pi \times$ spread	N/A	N/A	-25.2 (0.31)	-26.0 (0.20)	-25.7 (0.18)
$\sigma_Z \times$ spread	N/A	N/A	0.7 (0.81)	0.8 (0.52)	0.3 (0.48)
$p_K^- \times$ spread	N/A	N/A	0.2 (0.15)	-0.1 (0.09)	-0.1 (0.09)
fc $\times$ spread	N/A	N/A	-1.7 (0.08)	-1.0 (0.05)	-0.7 (0.05)
μ_Z	5.1 (0.02)	5.3 (0.02)	15.8 (0.15)	15.0 (0.09)	15.4 (0.09)
μ_Z^2	N/A	3.2 (0.39)	N/A	5.4 (0.32)	3.7 (0.31)
$\theta \times \mu_Z$	N/A	N/A	-9.4 (0.17)	-8.2 (0.11)	-7.8 (0.10)
$A_\pi \times \mu_Z$	N/A	N/A	-12.8 (0.21)	-12.4 (0.14)	-12.3 (0.13)
$\sigma_Z \times \mu_Z$	N/A	N/A	-1.4 (0.55)	-0.8 (0.35)	-0.8 (0.32)
$p_K^- \times \mu_Z$	N/A	N/A	-0.1 (0.10)	0.0 (0.06)	-0.0 (0.06)
fc $\times \mu_Z$	N/A	N/A	-4.8 (0.06)	-4.2 (0.04)	-4.3 (0.03)
spread $\times \mu_Z$	N/A	N/A	N/A	N/A	-8.0 (0.50)
R^2	0.763	0.881	0.812	0.924	0.936

data size: 91,101
standard error in parenthesis

Table 3.6

constant	[illegible]
θ	-[illegible]
θ^2	N[illegible]
A_z	-6[illegible]
A_z^2	N[illegible]
σ_z	1[illegible]
σ_z^2	N[illegible]
p_k^-	-0[illegible]
p_k^{-2}	N[illegible]
fixed costs	2[illegible]
fc^2	N[illegible]
$\theta \times A_z$	N[illegible]
$\theta \times \sigma_z$	N[illegible]
$\theta \times p_k^-$	N[illegible]
$\theta \times$ fc	N[illegible]
$A_z \times \sigma_z$	N[illegible]
$A_z \times p_k^-$	N[illegible]
$A_z \times$ fc	N[illegible]
$\sigma_z \times p_k^-$	N[illegible]
$\sigma_Z \times$ fc	N[illegible]
$p_k^- \times$ fc	N/[illegible]
spread	22.[illegible]
spread^2	N/[illegible]
$\theta \times$ spread	N/[illegible]
$A_z \times$ spread	N/[illegible]
$\sigma_z \times$ spread	N/[illegible]
$p_k^- \times$ spread	N/[illegible]
fc $\times$ spread	N/[illegible]
μ_z	19.[illegible]
μ_Z^2	N/A
$\theta \times \mu_Z$	N/A
$A_z \times \mu_Z$	N/A
$\sigma_z \times \mu_Z$	N/A
$p_k^- \times \mu_Z$	N/A
fc $\times \mu_Z$	N/A
spread $\times \mu_Z$	N/A
R^2	0.7[illegible]

data size: 91,101

standard error in

Table 3.6 Trigger Value for Investment with Fixed Costs (Natural Logged), Ln y^{+}_{Tr}

	Linear		Quadratic		Linear with Interaction		Quadratic with Interaction (1)		Quadratic with Interaction (2)	
constant	4.1	(0.02)	8.4	(0.03)	4.4	(0.04)	9.1	(0.03)	-3.7	(0.07)
θ	-7.4	(0.02)	-10.8	(0.06)	-8.8	(0.04)	-12.8	(0.06)	14.3	(0.15)
θ^2	N/A		3.8	(0.07)	N/A		4.3	(0.06)	-9.2	(0.09)
A_π	-6.2	(0.03)	-25.1	(0.10)	-4.6	(0.05)	-24.3	(0.09)	13.2	(0.21)
A_π^2	N/A		20.4	(0.11)	N/A		21.5	(0.09)	-5.6	(0.17)
σ_Z	1.2	(0.07)	0.2	(0.26)	1.6	(0.14)	0.4	(0.22)	0.7	(0.31)
σ_Z^2	N/A		9.7	(2.24)	N/A		9.7	(1.75)	9.6	(1.39)
p_K^-	-0.0	(0.01)	0.0	(0.07)	-0.0	(0.02)	-0.0	(0.06)	0.0	(0.06)
p_K^{-2}	N/A		-0.0	(0.07)	N/A		-0.0	(0.06)	0.0	(0.04)
fixed costs	2.0	(0.01)	2.8	(0.03)	0.7	(0.01)	1.4	(0.02)	4.4	(0.03)
fc^2	N/A		-0.7	(0.02)	N/A		-0.6	(0.02)	-0.6	(0.01)
$\theta \times A_\pi$	N/A		N/A		N/A		N/A		-41.2	(0.23)
$\theta \times \sigma_Z$	N/A		N/A		N/A		N/A		-0.4	(0.30)
$\theta \times p_K^-$	N/A		N/A		N/A		N/A		0.0	(0.05)
$\theta \times$ fc	N/A		N/A		N/A		N/A		-2.9	(0.03)
$A_\pi \times \sigma_Z$	N/A		N/A		N/A		N/A		-0.8	(0.37)
$A_\pi \times p_K^-$	N/A		N/A		N/A		N/A		-0.0	(0.07)
$A_\pi \times$ fc	N/A		N/A		N/A		N/A		-5.1	(0.04)
$\sigma_Z \times p_K^-$	N/A		N/A		N/A		N/A		-0.0	(0.18)
$\sigma_Z \times$ fc	N/A		N/A		N/A		N/A		0.5	(0.10)
$p_K^- \times$ fc	N/A		N/A		N/A		N/A		-0.0	(0.02)
spread	22.4	(0.09)	21.5	(0.29)	17.5	(0.60)	17.4	(0.49)	25.4	(0.40)
spread^2	N/A		19.5	(2.98)	N/A		31.0	(2.34)	-0.8	(1.92)
$\theta \times$ spread	N/A		N/A		17.1	(0.71)	19.2	(0.52)	18.6	(0.42)
$A_\pi \times$ spread	N/A		N/A		-25.8	(0.87)	-32.3	(0.64)	-35.2	(0.51)
$\sigma_Z \times$ spread	N/A		N/A		1.9	(2.26)	2.9	(1.66)	1.8	(1.32)
$p_K^- \times$ spread	N/A		N/A		0.5	(0.41)	0.3	(0.30)	0.0	(0.24)
fc $\times$ spread	N/A		N/A		21.5	(0.23)	22.3	(0.17)	22.4	(0.13)
μ_Z	19.0	(0.06)	20.1	(0.08)	11.3	(0.40)	11.7	(0.30)	17.7	(0.26)
μ_Z^2	N/A		-13.6	(1.29)	N/A		-18.9	(1.01)	-38.4	(0.87)
$\theta \times \mu_Z$	N/A		N/A		32.7	(0.47)	35.4	(0.35)	35.2	(0.28)
$A_\pi \times \mu_Z$	N/A		N/A		-12.8	(0.59)	-13.5	(0.44)	-12.8	(0.35)
$\sigma_Z \times \mu_Z$	N/A		N/A		-14.7	(1.53)	-14.3	(1.13)	-13.6	(0.90)
$p_K^- \times \mu_Z$	N/A		N/A		-0.1	(0.28)	-0.1	(0.20)	-0.0	(0.16)
fc $\times \mu_Z$	N/A		N/A		10.3	(0.15)	11.2	(0.11)	10.9	(0.09)
spread $\times \mu_Z$	N/A		N/A		N/A		N/A		-95.6	(1.40)
R^2	0.779		0.849		0.829		0.907		0.941	

data size: 91,101
standard error in parenthesis

constant	
θ	
θ^2	
A_z	
A_z^2	
σ_z	
σ_z^2	
p_K^-	
p_K^{-2}	
fixed costs	
fc^2	
$\theta \times A_z$	
$\theta \times \sigma_z$	[illegible]
$\theta \times p_K^-$	[illegible]
$\theta \times$ fc	[illegible]
$A_z \times \sigma_z$	[illegible]
$A_z \times p_K^-$	N
$A_z \times$ fc	N
$\sigma_z \times p_K^-$	N
$\sigma_z \times$ fc	N
$p_K^- \times$ fc	N
spread	
spread^2	N
$\theta \times$ spread	N
$A_z \times$ spread	N
$\sigma_z \times$ spread	N
$p_K^- \times$ spread	N
fc $\times$ spread	N
μ_z	[illegible]
μ_z^2	N
$\theta \times \mu_z$	N
$A_z \times \mu_z$	N
$\sigma_z \times \mu_z$	N
$p_K^- \times \mu_z$	N
fc $\times \mu_z$	N/
spread $\times \mu_z$	N/A
R^2	0.7

data size: 91,101
standard error in

Table 3.7 Target Value for Disinvestment with Fixed Costs, y^{-}_{Ta}

	Linear	Quadratic	Linear with Interaction	Quadratic with Interaction (1)	Quadratic with Interaction (2)
constant	0.24 (0.00)	0.96 (0.01)	0.03 (0.01)	0.75 (0.01)	-0.60 (0.02)
θ	-1.25 (0.00)	-3.37 (0.01)	-0.35 (0.01)	-2.45 (0.01)	0.16 (0.03)
θ^2	N/A	2.70 (0.02)	N/A	2.64 (0.01)	1.99 (0.02)
A_π	-0.53 (0.01)	-1.64 (0.02)	0.07 (0.01)	-0.97 (0.02)	1.60 (0.04)
A_π^2	N/A	0.83 (0.03)	N/A	0.75 (0.02)	-0.61 (0.04)
σ_Z	0.08 (0.02)	0.03 (0.06)	0.08 (0.03)	0.02 (0.05)	0.08 (0.06)
σ_Z^2	N/A	0.48 (0.51)	N/A	0.54 (0.40)	0.54 (0.29)
p_K^-	-0.92 (0.00)	0.43 (0.02)	0.24 (0.01)	-0.23 (0.01)	1.44 (0.01)
p_K^{-2}	N/A	0.50 (0.02)	N/A	0.48 (0.01)	0.49 (0.01)
fixed costs	-0.06 (0.00)	-0.14 (0.01)	-0.01 (0.00)	-0.09 (0.01)	-0.07 (0.01)
fc^2	N/A	0.07 (0.01)	N/A	0.06 (0.00)	0.07 (0.00)
$\theta \times A_\pi$	N/A	N/A	N/A	N/A	-2.07 (0.05)
$\theta \times \sigma_Z$	N/A	N/A	N/A	N/A	-0.32 (0.06)
$\theta \times p_K^-$	N/A	N/A	N/A	N/A	-2.96 (0.01)
$\theta \times$ fc	N/A	N/A	N/A	N/A	0.24 (0.01)
$A_\pi \times \sigma_Z$	N/A	N/A	N/A	N/A	-0.13 (0.08)
$A_\pi \times p_K^-$	N/A	N/A	N/A	N/A	-1.71 (0.01)
$A_\pi \times$ fc	N/A	N/A	N/A	N/A	0.05 (0.01)
$\sigma_Z \times p_K^-$	N/A	N/A	N/A	N/A	0.25 (0.04)
$\sigma_Z \times$ fc	N/A	N/A	N/A	N/A	-0.08 (0.02)
$p_K^- \times$ fc	N/A	N/A	N/A	N/A	-0.24 (0.00)
spread	4.54 (0.02)	3.88 (0.07)	7.96 (0.13)	7.18 (0.11)	7.38 (0.08)
spread^2	N/A	8.78 (0.68)	N/A	7.79 (0.54)	7.37 (0.40)
$\theta \times$ spread	N/A	N/A	-16.70 (0.15)	-16.20 (0.12)	-15.60 (0.09)
$A_\pi \times$ spread	N/A	N/A	-9.72 (0.18)	-9.80 (0.15)	-9.74 (0.11)
$\sigma_Z \times$ spread	N/A	N/A	-0.13 (0.47)	-0.02 (0.38)	-0.16 (0.28)
$p_K^- \times$ spread	N/A	N/A	11.50 (0.08)	11.40 (0.07)	10.90 (0.05)
fc $\times$ spread	N/A	N/A	-0.59 (0.05)	-0.31 (0.04)	-0.23 (0.03)
μ_Z	1.59 (0.01)	1.63 (0.02)	3.25 (0.08)	2.82 (0.07)	2.99 (0.05)
μ_Z^2	N/A	2.00 (0.29)	N/A	2.70 (0.23)	2.25 (0.18)
$\theta \times \mu_Z$	N/A	N/A	-4.67 (0.10)	-4.17 (0.08)	-4.23 (0.06)
$A_\pi \times \mu_Z$	N/A	N/A	-4.79 (0.12)	-4.55 (0.10)	-4.59 (0.07)
$\sigma_Z \times \mu_Z$	N/A	N/A	0.05 (0.32)	0.26 (0.26)	0.39 (0.19)
$p_K^- \times \mu_Z$	N/A	N/A	4.72 (0.06)	4.79 (0.05)	4.93 (0.03)
fc $\times \mu_Z$	N/A	N/A	-1.29 (0.03)	-0.99 (0.03)	-1.00 (0.02)
spread $\times \mu_Z$	N/A	N/A	N/A	N/A	-3.32 (0.29)
R^2	0.742	0.808	0.817	0.880	0.938

data size: 91,101
standard error in parenthesis

Ta

constant	0
θ	-0
θ^2	N
A_z	-0
A_z^2	N
σ_z	-0
σ_z^2	N
p_K	0
p_K^2	N
fixed costs	-0.
fc^2	N
$\theta \times A_z$	N
$\theta \times \sigma_z$	N
$\theta \times p_K$	N
$\theta \times$ fc	N
$A_z \times \sigma_z$	N
$A_z \times p_K$	N
$A_z \times$ fc	N
$\sigma_z \times p_K$	N
$\sigma_z \times$ fc	N
$p_K \times$ fc	N
spread	1.
spread^2	N A
$\theta \times$ spread	N A
$A_z \times$ spread	N A
$\sigma_z \times$ spread	N A
$p_K \times$ spread	N A
fc $\times$ spread	N A
μ_z	0.6
μ_z^2	N A
$\theta \times \mu_z$	N A
$A_z \times \mu_z$	N A
$\sigma_z \times \mu_z$	N A
$p_K \times \mu_z$	N A
fc $\times \mu_z$	N A
spread $\times \mu_z$	N A
R^2	0.7

data size: 91,101
standard error in p

Table 3.8 Trigger Value for Disinvestment with Fixed Costs, y^-_{Tr}

	Linear	Quadratic	Linear with Interaction	Quadratic with Interaction (1)	Quadratic with Interaction (2)
constant	0.17 (0.00)	0.44 (0.00)	0.07 (0.00)	0.33 (0.00)	-0.10 (0.01)
θ	-0.50 (0.00)	-1.17 (0.01)	-0.22 (0.00)	-0.87 (0.01)	-0.20 (0.01)
θ^2	N/A	0.85 (0.01)	N/A	0.81 (0.01)	0.60 (0.01)
A_π	-0.20 (0.00)	-0.63 (0.01)	-0.02 (0.01)	-0.42 (0.01)	0.35 (0.02)
A_π^2	N/A	0.35 (0.01)	N/A	0.32 (0.01)	-0.14 (0.01)
σ_Z	-0.09 (0.01)	-0.01 (0.03)	-0.06 (0.01)	0.01 (0.03)	-0.05 (0.03)
σ_Z^2	N/A	-0.73 (0.24)	N/A	-0.65 (0.21)	-0.66 (0.13)
p_K^-	0.39 (0.00)	0.23 (0.01)	0.16 (0.00)	0.00 (0.01)	0.78 (0.01)
p_K^{-2}	N/A	0.16 (0.01)	N/A	0.16 (0.01)	0.16 (0.00)
fixed costs	-0.14 (0.00)	-0.33 (0.00)	-0.03 (0.00)	-0.22 (0.00)	-0.27 (0.00)
fc^2	N/A	0.17 (0.00)	N/A	0.16 (0.00)	0.17 (0.00)
$\theta \times A_\pi$	N/A	N/A	N/A	N/A	-0.78 (0.02)
$\theta \times \sigma_Z$	N/A	N/A	N/A	N/A	0.32 (0.03)
$\theta \times p_K^-$	N/A	N/A	N/A	N/A	-1.08 (0.01)
$\theta \times$ fc	N/A	N/A	N/A	N/A	0.52 (0.00)
$A_\pi \times \sigma_Z$	N/A	N/A	N/A	N/A	0.11 (0.03)
$A_\pi \times p_K^-$	N/A	N/A	N/A	N/A	-0.57 (0.01)
$A_\pi \times$ fc	N/A	N/A	N/A	N/A	0.20 (0.00)
$\sigma_Z \times p_K^-$	N/A	N/A	N/A	N/A	-0.21 (0.02)
$\sigma_Z \times$ fc	N/A	N/A	N/A	N/A	0.06 (0.01)
$p_K^- \times$ fc	N/A	N/A	N/A	N/A	-0.39 (0.00)
spread	1.55 (0.01)	1.54 (0.03)	3.36 (0.06)	3.35 (0.06)	3.26 (0.04)
spread^2	N/A	0.84 (0.32)	N/A	-0.51 (0.28)	-0.87 (0.18)
$\theta \times$ spread	N/A	N/A	-5.30 (0.07)	-5.06 (0.06)	-4.16 (0.04)
$A_\pi \times$ spread	N/A	N/A	-2.78 (0.09)	-2.78 (0.08)	-2.56 (0.05)
$\sigma_Z \times$ spread	N/A	N/A	-1.36 (0.22)	-1.28 (0.20)	-1.29 (0.12)
$p_K^- \times$ spread	N/A	N/A	3.83 (0.04)	3.80 (0.04)	3.44 (0.02)
fc $\times$ spread	N/A	N/A	-1.90 (0.02)	-1.77 (0.02)	-1.61 (0.01)
μ_Z	0.67 (0.01)	0.74 (0.01)	1.21 (0.04)	1.11 (0.04)	1.17 (0.02)
μ_Z^2	N/A	-0.57 (0.14)	N/A	-0.16 (0.12)	-0.72 (0.08)
$\theta \times \mu_Z$	N/A	N/A	-1.51 (0.05)	-1.31 (0.04)	-1.12 (0.03)
$A_\pi \times \mu_Z$	N/A	N/A	-1.44 (0.06)	-1.33 (0.05)	-1.28 (0.03)
$\sigma_Z \times \mu_Z$	N/A	N/A	1.32 (0.15)	1.29 (0.14)	1.28 (0.08)
$p_K^- \times \mu_Z$	N/A	N/A	1.77 (0.03)	1.79 (0.02)	1.85 (0.02)
fc $\times \mu_Z$	N/A	N/A	-0.91 (0.02)	-0.82 (0.01)	-0.85 (0.01)
spread $\times \mu_Z$	N/A	N/A	N/A	N/A	-2.40 (0.13)
R^2	0.730	0.778	0.791	0.834	0.936

data size: 91,101
standard error in parenthesis

Table

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Table 3.9 shows the significance of the quadratic terms and the interaction terms for the approximation. This analysis tests the significance of the quadratic terms or the interaction terms by a test comparing the R^2. Under the null hypothesis that the approximation with the quadratic or the interaction terms and the approximation without the terms are equivalent, the test statistics is

$$F = \frac{\left(R^2_{\text{with terms}} - R^2_{\text{without terms}}\right)/m}{\left(1 - R^2_{\text{with terms}}\right)/(\text{data size} - k)} \sim F(m, \text{data size} - k). \tag{37}$$

Here, m is the number of the quadratic or interaction terms, and k is the number of explanatory variables in the model with the quadratic or interaction terms. Because the data size is large, differences in the R^2 for all cases are statistically significant. For the linear approximation, either the quadratic terms or the interaction terms are added to the linear approximation. Table 3.9 shows that the quadratic terms are more significant than the interaction terms for investment, but both terms are equally significant for disinvestment. For the quadratic approximation, two sets of the interaction terms are added to the quadratic approximation and the second set of interactions includes the first set of interactions and additional terms, i.e., the first set is a proper subset of the second set. As the quadratic approximation includes more interaction terms, its R^2 increases for all critical values. And, the increase of the R^2 is statistically significant.

In addition, the three ratios of the critical values, G^+, G^- and G, are also semi-log linear. Tables 3.10 to 3.12 show their approximations. This result facilitates empirical research.

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2. Costly Rev

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(d) F-Statistics

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(e) F-Statistics

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(f) F-Statistics f

Approximation Model	Q
Linear	
Quadratic	

***: reject Ho at

Table 3.9 Significance of Quadratic Terms and Interaction Terms

1. Costly Reversible Investment without Fixed Costs

(a) F-Statistics for Critical Value of Investment, y^+

Approximation Model	Quadratic Terms	Interactions for Linear Model	Interactions for Quad. Model (1)	Interactions for Quad. Model (2)
Linear	5,802***	509***	N/A	N/A
Quadratic	N/A	N/A	1,200***	1,033***

(b) F-Statistics for Critical Value of Disinvestment, y^-

Approximation Model	Quadratic Terms	Interactions for Linear Model	Interactions for Quad. Model (1)	Interactions for Quad. Model (2)
Linear	1,438***	1,883***	N/A	N/A
Quadratic	N/A	N/A	2,232***	3,934***

2. Costly Reversible Investment with Fixed Costs

(c) F-Statistics for Target Value of Investment, y^+_{Ta}

Approximation Model	Quadratic Terms	Interactions for Linear Model	Interactions for Quad. Model (1)	Interactions for Quad. Model (2)
Linear	12,903***	2,374***	N/A	N/A
Quadratic	N/A	N/A	5,153***	3,727***

(d) F-Statistics for Trigger Value of Investment, Ln y^+_{Tr}

Approximation Model	Quadratic Terms	Interactions for Linear Model	Interactions for Quad. Model (1)	Interactions for Quad. Model (2)
Linear	6,032***	2,663***	N/A	N/A
Quadratic	N/A	N/A	5,680***	6,762***

(e) F-Statistics for Target Value of Disinvestment, y^-_{Ta}

Approximation Model	Quadratic Terms	Interactions for Linear Model	Interactions for Quad. Model (1)	Interactions for Quad. Model (2)
Linear	4,473***	3,733***	N/A	N/A
Quadratic	N/A	N/A	5,465***	9,093***

(f) F-Statistics for Critical Value of Investment, y^-_{Tr}

Approximation Model	Quadratic Terms	Interactions for Linear Model	Interactions for Quad. Model (1)	Interactions for Quad. Model (2)
Linear	2,813***	2,658***	N/A	N/A
Quadratic	N/A	N/A	3,072***	10,706***

***: reject Ho at 1%

Tabl

constant	
θ	-
θ^2	N
A_z	-
A_z^2	N
σ_z	(
σ_z^2	N
p_K^-	-(
p_K^{-2}	N
fixed costs	2
fc^2	N
$\theta \times A_z$	N
$\theta \times \sigma_z$	N
$\theta \times p_K^-$	N
$\theta \times$ fc	N
$A_z \times \sigma_z$	N
$A_z \times p_K^-$	N
$A_z \times$ fc	N
$\sigma_z \times p_K^-$	N/
$\sigma_Z \times$ fc	N/
$p_K^- \times$ fc	N/
spread	9.
$spread^2$	N/
$\theta \times$ spread	N/
$A_z \times$ spread	N/
$\sigma_z \times$ spread	N/
$p_K^- \times$ spread	N/A
fc $\times$ spread	N/A
μ_z	11.7
μ_Z^2	N/A
$\theta \times \mu_z$	N/A
$A_z \times \mu_z$	N/A
$\sigma_z \times \mu_z$	N/A
$p_K^- \times \mu_z$	N/A
fc $\times \mu_z$	N/A
spread $\times \mu_z$	N/A
R^2	0.76

data size: 91,101
standard error in pa

Table 3.10 Measure of Minimum Investment (Natural Logged), $\text{Ln}\, G^{+}$

	Linear		Quadratic		Linear with Interaction		Quadratic with Interaction (1)		Quadratic with Interaction (2)	
constant	2.4	(0.01)	4.9	(0.03)	2.2	(0.03)	4.9	(0.03)	-2.0	(0.07)
θ	-2.5	(0.02)	-5.1	(0.05)	-2.6	(0.03)	-5.6	(0.05)	8.3	(0.14)
θ^2	N/A		3.0	(0.06)	N/A		3.5	(0.05)	-3.0	(0.08)
A_π	-3.3	(0.02)	-13.9	(0.08)	-1.4	(0.04)	-12.4	(0.07)	6.9	(0.20)
A_π^2	N/A		11.3	(0.09)	N/A		12.0	(0.08)	-0.8	(0.16)
σ_Z	0.8	(0.05)	0.1	(0.21)	0.9	(0.10)	0.1	(0.19)	0.4	(0.28)
σ_Z^2	N/A		6.4	(1.80)	N/A		6.5	(1.47)	6.3	(1.27)
$\bar{p}_K$	-0.0	(0.01)	0.0	(0.06)	-0.0	(0.02)	-0.0	(0.05)	0.0	(0.06)
$\bar{p}_K^{2}$	N/A		-0.0	(0.06)	N/A		0.0	(0.05)	0.0	(0.04)
fixed costs	2.3	(0.01)	3.5	(0.02)	1.1	(0.01)	2.2	(0.02)	5.2	(0.03)
fc^2	N/A		-1.0	(0.01)	N/A		-1.0	(0.01)	-1.0	(0.01)
$\theta \times A_\pi$	N/A		N/A		N/A		N/A		-19.8	(0.21)
$\theta \times \sigma_Z$	N/A		N/A		N/A		N/A		-0.7	(0.28)
$\theta \times \bar{p}_K$	N/A		N/A		N/A		N/A		-0.0	(0.05)
$\theta \times$ fc	N/A		N/A		N/A		N/A		-3.0	(0.03)
$A_\pi \times \sigma_Z$	N/A		N/A		N/A		N/A		-1.0	(0.34)
$A_\pi \times \bar{p}_K$	N/A		N/A		N/A		N/A		-0.0	(0.06)
$A_\pi \times$ fc	N/A		N/A		N/A		N/A		-5.1	(0.03)
$\sigma_Z \times \bar{p}_K$	N/A		N/A		N/A		N/A		0.0	(0.16)
$\sigma_Z \times$ fc	N/A		N/A		N/A		N/A		0.7	(0.09)
$\bar{p}_K \times$ fc	N/A		N/A		N/A		N/A		-0.0	(0.02)
spread	9.3	(0.07)	6.8	(0.23)	11.4	(0.49)	8.9	(0.41)	11.2	(0.37)
spread^2	N/A		32.5	(2.39)	N/A		45.8	(1.96)	45.4	(1.76)
$\theta \times$ spread	N/A		N/A		0.9	(0.53)	2.0	(0.44)	-0.0	(0.38)
$A_\pi \times$ spread	N/A		N/A		-28.8	(0.65)	-32.5	(0.54)	-36.0	(0.47)
$\sigma_Z \times$ spread	N/A		N/A		4.4	(1.69)	5.1	(1.39)	5.4	(1.21)
$\bar{p}_K \times$ spread	N/A		N/A		0.3	(0.31)	0.1	(0.25)	0.0	(0.22)
fc × spread	N/A		N/A		19.0	(0.17)	19.4	(0.14)	19.4	(0.12)
μ_Z	11.7	(0.05)	12.5	(0.06)	13.5	(0.30)	14.1	(0.25)	13.9	(0.23)
μ_Z^2	N/A		-12.0	(1.03)	N/A		-17.0	(0.85)	-14.9	(0.80)
$\theta \times \mu_Z$	N/A		N/A		5.7	(0.35)	7.2	(0.29)	7.0	(0.25)
$A_\pi \times \mu_Z$	N/A		N/A		-20.8	(0.44)	-21.4	(0.37)	-20.9	(0.32)
$\sigma_Z \times \mu_Z$	N/A		N/A		-9.5	(1.14)	-9.6	(0.95)	-9.3	(0.82)
$\bar{p}_K \times \mu_Z$	N/A		N/A		-0.0	(0.21)	-0.0	(0.17)	0.0	(0.15)
fc × μ_Z	N/A		N/A		12.4	(0.12)	13.1	(0.10)	13.1	(0.08)
spread × μ_Z	N/A		N/A		N/A		N/A		-1.3	(1.28)
R^2	0.763		0.814		0.819		0.876		0.907	

data size: 91,101
standard error in parenthesis

Table

constant	
θ	-
θ^2	N
A_τ	-(
A_τ^2	N
σ_z	
σ_z^2	N
p_k^-	-(
p_k^{-2}	N
fixed costs	1
fc^2	N
$\theta \times A_\tau$	N
$\theta \times \sigma_z$	N
$\theta \times p_k^-$	N
$\theta \times$ fc	N
$A_\tau \times \sigma_z$	N
$A_\tau \times p_k^-$	N
$A_\tau \times$ fc	N
$\sigma_z \times p_k^-$	N
$\sigma_z \times$ fc	N
$p_k^- \times$ fc	N
spread	22.
spread^2	N/
$\theta \times$ spread	N/
$A_\tau \times$ spread	N/
$\sigma_z \times$ spread	N/
$p_k^- \times$ spread	N/A
fc $\times$ spread	N/A
μ_z	19.0
μ_z^2	N/A
$\theta \times \mu_z$	N/A
$A_\tau \times \mu_z$	N/A
$\sigma_z \times \mu_z$	N/A
$p_k^- \times \mu_z$	N/A
fc $\times \mu_z$	N/A
spread $\times \mu_z$	N/A
R^2	0.7

data size: 91,101

standard error in p

Table 3.11 Measure of Minimum Disinvestment (Natural Logged), Ln G^-

	Linear	Quadratic	Linear with Interaction	Quadratic with Interaction (1)	Quadratic with Interaction (2)
constant	4.05 (0.02)	-1.01 (0.00)	-0.59 (0.01)	-1.00 (0.01)	-0.18 (0.01)
θ	-7.37 (0.02)	2.11 (0.01)	1.18 (0.01)	2.06 (0.01)	-0.33 (0.02)
θ^2	N/A	-1.01 (0.01)	N/A	-1.06 (0.01)	0.05 (0.01)
A_π	-6.22 (0.03)	2.45 (0.01)	0.49 (0.01)	2.26 (0.01)	-1.00 (0.02)
A_π^2	N/A	-1.79 (0.02)	N/A	-1.84 (0.01)	0.36 (0.02)
σ_Z	1.24 (0.07)	-0.14 (0.04)	-1.20 (0.02)	-0.46 (0.03)	-0.53 (0.03)
σ_Z^2	N/A	-6.74 (0.32)	N/A	-6.42 (0.26)	-6.42 (0.15)
p_K^-	-0.01 (0.01)	-0.44 (0.01)	-0.32 (0.00)	-0.44 (0.01)	-0.34 (0.01)
p_K^{-2}	N/A	0.12 (0.01)	N/A	0.12 (0.01)	0.11 (0.00)
fixed costs	1.96 (0.01)	-1.17 (0.00)	-0.41 (0.00)	-0.94 (0.00)	-1.44 (0.00)
fc^2	N/A	0.49 (0.00)	N/A	0.48 (0.00)	0.49 (0.00)
$\theta \times A_\pi$	N/A	N/A	N/A	N/A	3.12 (0.02)
$\theta \times \sigma_Z$	N/A	N/A	N/A	N/A	0.32 (0.03)
$\theta \times p_K^-$	N/A	N/A	N/A	N/A	-0.50 (0.01)
$\theta \times$ fc	N/A	N/A	N/A	N/A	1.08 (0.00)
$A_\pi \times \sigma_Z$	N/A	N/A	N/A	N/A	0.15 (0.04)
$A_\pi \times p_K^-$	N/A	N/A	N/A	N/A	0.30 (0.01)
$A_\pi \times$ fc	N/A	N/A	N/A	N/A	0.58 (0.00)
$\sigma_Z \times p_K^-$	N/A	N/A	N/A	N/A	-0.13 (0.02)
$\sigma_Z \times$ fc	N/A	N/A	N/A	N/A	-0.03 (0.01)
$p_K^- \times$ fc	N/A	N/A	N/A	N/A	-0.14 (0.00)
spread	22.40 (0.09)	-4.34 (0.04)	-4.55 (0.09)	-4.94 (0.07)	-6.66 (0.04)
spread^2	N/A	2.42 (0.42)	N/A	0.29 (0.35)	6.49 (0.21)
$\theta \times$ spread	N/A	N/A	2.60 (0.11)	2.57 (0.08)	4.05 (0.04)
$A_\pi \times$ spread	N/A	N/A	3.76 (0.13)	4.45 (0.10)	4.82 (0.05)
$\sigma_Z \times$ spread	N/A	N/A	0.01 (0.34)	0.17 (0.25)	0.28 (0.14)
$p_K^- \times$ spread	N/A	N/A	0.51 (0.06)	0.56 (0.04)	0.41 (0.03)
fc $\times$ spread	N/A	N/A	-4.20 (0.03)	-4.18 (0.02)	-3.91 (0.01)
μ_Z	19.00 (0.06)	-0.76 (0.01)	-0.82 (0.06)	-0.33 (0.04)	-1.56 (0.03)
μ_Z^2	N/A	-9.84 (0.18)	N/A	-9.22 (0.15)	-5.54 (0.09)
$\theta \times \mu_Z$	N/A	N/A	-1.79 (0.07)	-2.02 (0.05)	-1.60 (0.03)
$A_\pi \times \mu_Z$	N/A	N/A	1.25 (0.09)	1.35 (0.06)	1.43 (0.04)
$\sigma_Z \times \mu_Z$	N/A	N/A	11.70 (0.23)	10.80 (0.17)	0.39 (0.19)
$p_K^- \times \mu_Z$	N/A	N/A	-1.27 (0.04)	-1.31 (0.03)	-1.33 (0.02)
fc $\times \mu_Z$	N/A	N/A	-0.72 (0.02)	-0.85 (0.02)	-0.81 (0.01)
spread $\times \mu_Z$	N/A	N/A	N/A	N/A	16.60 (0.15)
R^2	0.779	0.927	0.906	0.951	0.984

data size: 91,101
standard error in parenthesis

Table 3.1[illegible]

constant	
θ	
θ^2	
A_z	
A_z^2	
σ_z	
σ_z^2	[illegible]
p_K^-	
p_K^{-2}	N[illegible]
fixed costs	
fc^2	N[illegible]
$\theta \times A_z$	N[illegible]
$\theta \times \sigma_z$	N[illegible]
$\theta \times p_K^-$	N[illegible]
$\theta \times$ fc	N[illegible]
$A_z \times \sigma_z$	N[illegible]
$A_z \times p_K^-$	N[illegible]
$A_z \times$ fc	N[illegible]
$\sigma_z \times p_K^-$	N[illegible]
$\sigma_z \times$ fc	N[illegible]
$p_K^- \times$ fc	N[illegible]
spread	9[illegible]
spread^2	N[illegible]
$\theta \times$ spread	N[illegible]
$A_z \times$ spread	N[illegible]
$\sigma_z \times$ spread	N/[illegible]
$p_K^- \times$ spread	N/[illegible]
fc $\times$ spread	N/[illegible]
μ_z	12.[illegible]
μ_z^2	N/A
$\theta \times \mu_z$	N/A
$A_z \times \mu_z$	N/A
$\sigma_z \times \mu_z$	N/A
$p_K^- \times \mu_z$	N/A
fc $\times \mu_z$	N/A
spread $\times \mu_z$	N/A
R^2	0.7[illegible]

data size: 91,101

standard error in p[illegible]

Table 3.12 Measure of Inaction Range (without Fixed Costs, Natural Logged), Ln G

	Linear		Quadratic		Linear with Interaction		Quadratic with Interaction (1)		Quadratic with Interaction (2)	
constant	5.3	(0.01)	9.4	(0.03)	5.1	(0.03)	9.4	(0.03)	1.8	(0.07)
θ	-0.9	(0.02)	-7.7	(0.06)	-0.5	(0.04)	-7.8	(0.05)	7.9	(0.15)
θ^2	N/A		8.4	(0.07)	N/A		8.8	(0.06)	3.5	(0.09)
A_π	-3.2	(0.02)	-14.6	(0.10)	-0.8	(0.05)	-12.7	(0.09)	5.0	(0.21)
A_π^2	N/A		11.4	(0.11)	N/A		12.1	(0.09)	1.7	(0.17)
σ_Z	1.8	(0.01)	0.2	(0.24)	2.3	(0.13)	0.7	(0.22)	0.9	(0.30)
σ_Z^2	N/A		14.7	(2.10)	N/A		14.2	(1.73)	14.1	(1.36)
p_K^-	-3.2	(0.01)	-6.5	(0.07)	-3.6	(0.02)	-6.8	(0.06)	-4.2	(0.06)
p_K^{-2}	N/A		3.2	(0.07)	N/A		3.2	(0.06)	3.3	(0.04)
fixed costs	2.8	(0.01)	4.2	(0.02)	1.3	(0.01)	2.6	(0.02)	6.0	(0.03)
fc^2	N/A		-1.3	(0.02)	N/A		-1.2	(0.02)	-1.2	(0.01)
$\theta \times A_\pi$	N/A		N/A		N/A		N/A		-16.1	(0.23)
$\theta \times \sigma_Z$	N/A		N/A		N/A		N/A		-0.5	(0.29)
$\theta \times p_K^-$	N/A		N/A		N/A		N/A		-6.8	(0.05)
$\theta \times$ fc	N/A		N/A		N/A		N/A		-4.2	(0.03)
$A_\pi \times \sigma_Z$	N/A		N/A		N/A		N/A		-0.8	(0.37)
$A_\pi \times p_K^-$	N/A		N/A		N/A		N/A		-2.2	(0.07)
$A_\pi \times$ fc	N/A		N/A		N/A		N/A		-5.7	(0.04)
$\sigma_Z \times p_K^-$	N/A		N/A		N/A		N/A		-0.1	(0.17)
$\sigma_Z \times$ fc	N/A		N/A		N/A		N/A		0.5	(0.10)
$p_K^- \times$ fc	N/A		N/A		N/A		N/A		0.4	(0.02)
spread	9.6	(0.09)	3.9	(0.27)	10.9	(0.57)	4.4	(0.49)	8.0	(0.39)
spread^2	N/A		70.7	(2.79)	N/A		86.7	(2.30)	85.4	(1.88)
$\theta \times$ spread	N/A		N/A		-7.8	(0.67)	-5.8	(0.51)	-9.4	(0.41)
$A_\pi \times$ spread	N/A		N/A		-36.7	(0.82)	-40.2	(0.63)	-44.2	(0.50)
$\sigma_Z \times$ spread	N/A		N/A		5.1	(2.13)	6.0	(1.64)	6.6	(1.30)
$p_K^- \times$ spread	N/A		N/A		7.6	(0.39)	7.1	(0.30)	6.3	(0.23)
fc $\times$ spread	N/A		N/A		24.8	(0.22)	25.7	(0.17)	25.3	(0.13)
μ_Z	12.9	(0.06)	14.1	(0.07)	16.5	(0.38)	16.5	(0.30)	16.7	(0.25)
μ_Z^2	N/A		-12.4	(1.21)	N/A		-17.8	(1.00)	-15.7	(0.85)
$\theta \times \mu_Z$	N/A		N/A		5.4	(0.44)	7.7	(0.34)	6.6	(0.27)
$A_\pi \times \mu_Z$	N/A		N/A		-22.0	(0.56)	-22.2	(0.43)	-21.9	(0.34)
$\sigma_Z \times \mu_Z$	N/A		N/A		-24.9	(1.45)	-24.9	(1.11)	-24.2	(0.88)
$p_K^- \times \mu_Z$	N/A		N/A		0.3	(0.26)	0.5	(0.20)	0.7	(0.16)
fc $\times \mu_Z$	N/A		N/A		11.5	(0.15)	13.0	(0.11)	13.1	(0.09)
spread $\times \mu_Z$	N/A		N/A		N/A		N/A		-4.8	(1.37)
R^2	0.783		0.849		0.826		0.898		0.936	

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CHAPTER 4

ECONOMETRIC PROCEDURE

4-1. Econometric Model

This chapter investigates actual investment in accordance with the theoretical model of costly reversible investment with fixed costs. The optimal investment rule of the theoretical model is

$$(\text{I})\quad K_{t+} = Z_t / y^+_{Ta}, \quad \text{if } Z_t > Z^+\left(= y^+_{Tr} K_{t-}\right), \tag{38-a}$$

$$(\text{O})\quad K_{t+} = K_{t-}, \quad \text{if } Z^- \le Z_t \le Z^+, \tag{38-b}$$

and

$$(\text{D})\quad K_{t+} = Z_t / y^-_{Ta}, \quad \text{if } Z_t < Z^-\left(= y^-_{Tr} K_{t-}\right). \tag{38-c}$$

Both axes in figure 3.1 are normalized by pre-investment capital stock, K_{t-}, so that firm's size does not affect the analysis. After both axes are natural-logged, the schematic diagram becomes figure 4.1. In figure 4.1, investment locus and disinvestment locus are parallel, and their slope is forty five degree.

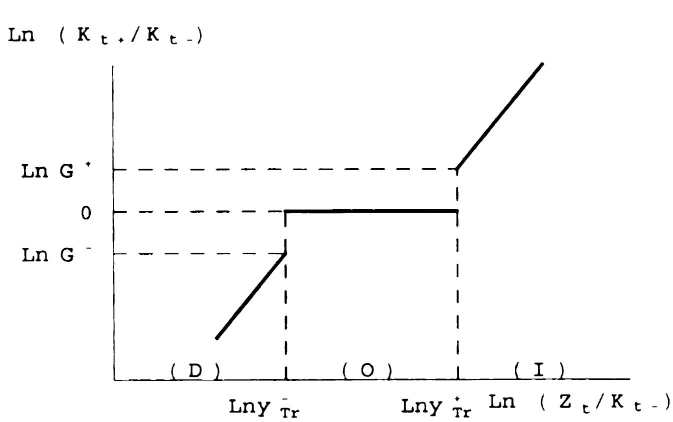

Figure 4.1 - Normalized Investment Function

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This analysis uses annual data. The capital stock at the beginning of the period, K_{t+1}, represents post-investment capital stock, K_{t+}. Pre-investment capital stock, K_{t-}, is represented by $e^{-\delta t} K_t$. By assuming a depreciation rate (δ), the measure of investment for analysis, k_t, is

$$k_t = \mathrm{Ln}\left(K_{t+}/K_{t-}\right) = \mathrm{Ln}\left(K_{t+1}/K_t\right) + \delta . \tag{39}$$

By using the ratios of the critical values as $G^+ = y_{Tr}^+/y_{Ta}^+$ (> 1), $G^- = y_{Tr}^-/y_{Ta}^-$ (< 1), and $G = y_{Tr}^+/y_{Tr}^-$ (> 1), the optimal investment rule becomes as follows:

$$\text{(I)}\quad k_t = -\mathrm{Ln}\, y_{Ta}^+ + \mathrm{Ln}\left(Z_t/K_t\right) + \delta$$
$$= \mathrm{Ln}G^+ - \mathrm{Ln}\, y_{Tr}^+ + \mathrm{Ln}\left(Z_t/K_t\right) + \delta, \qquad \text{if } \mathrm{Ln}\left(Z_t/K_t\right) + \delta > \mathrm{Ln}\, y_{Tr}^+ \tag{40-a}$$

$$\text{(O)}\quad k_t = 0, \qquad \text{if } \mathrm{Ln}\, y_{Tr}^- \left(= \mathrm{Ln}\, y_{Tr}^+ - \mathrm{Ln}G\right) \le \mathrm{Ln}\left(Z_t/K_t\right) + \delta \le \mathrm{Ln}\, y_{Tr}^+ \tag{40-b}$$

$$\text{(D)}\quad k_t = \mathrm{Ln}G^- - \mathrm{Ln}\, y_{Tr}^+ + \mathrm{Ln}G + \mathrm{Ln}\left(Z_t/K_t\right) + \delta,$$
$$\text{if } \mathrm{Ln}\left(Z_t/K_t\right) + \delta < \mathrm{Ln}\, y_{Tr}^+ - \mathrm{Ln}G \tag{40-c}$$

The theoretical model shows that the logarithms of the four values, y_{Tr}^+, G^+, G^-, and G are functions of economic parameters such as the spread and trend of the stochastic variable. They can be approximated by linear equations.

$$\mathrm{Ln}\, y_{Tr}^+ = f_1(\text{ spread}, \mu_Z, \theta, A_\pi, \sigma_Z, \text{fixed costs}, p_K^-) \approx z_t v \tag{41-a}$$

$$\mathrm{Ln}\, G^+ = f_2(\text{ spread}, \mu_Z, \theta, A_\pi, \sigma_Z, \text{fixed costs}, p_K^-) \approx z_t v_g^+ \tag{41-b}$$

$$\mathrm{Ln}\, G^- = f_3(\text{ spread}, \mu_Z, \theta, A_\pi, \sigma_Z, \text{fixed costs}, p_K^-) \approx z_t v_g^- \tag{41-c}$$

$$\mathrm{Ln}\, G = f_4(\text{ spread}, \mu_Z, \theta, A_\pi, \sigma_Z, \text{fixed costs}, p_K^-) \approx z_t v_g \tag{41-d}$$

Here, z_t is a vector of parameters, where $z_t \equiv$ (spread, μ_Z, σ_Z, θ, A_π, fixed costs, p_K^-, constant). This analysis assumes that the depreciation rate (δ), trend and volatility of the stochastic variable (μ_Z and σ_Z), the operating profit function ($A_\pi Z_t^{1-\theta} K_t^\theta$), fixed costs of investment, and the resale price of capital (p_K^-) are all constant over time. Therefore, z_t becomes (γ (= spread + μ_Z), constant).

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There are two additional assumptions about the market demand of a firm's output and technology to measure the economic indicator variable, Z_t. The market demand of firm's output is assumed to be iso-elastic, i.e., $Q_d = (P/X_1)^{-\varepsilon}$ ($\varepsilon > 1$), and, the technology is Cobb-Douglas, i.e., $Q = X_2 K^{\alpha} L^{\beta}$. Both X_1 and X_2 are stochastic coefficients. Then, the stochastic variable, Z_t, is the product of X_1 and X_2, and, by assuming frictionless adjustment of flow inputs, $Z_t = \left[Re_t K_t^{-\alpha_\varepsilon} (wL)_t^{-\beta_\varepsilon}\right]^{1/(1-\alpha_\varepsilon-\beta_\varepsilon)}$. Here, Re_t is the firm's revenue ($= PQ$), $(wL)_t$ is the cost of flow inputs, and α_ε and β_ε are $\alpha(1 - 1/\varepsilon)$ and $\beta(1 - 1/\varepsilon)$, respectively. The stochastic variable, Z_t, is homogeneous of degree one in Re_t, K_t, and $(wL)_t$ under these assumptions.

In addition, there is an error, u_t, which is assumed to be normal with zero mean and variance σ^2, and independent and identically distributed, i.e., $u_t \sim$ i.i.d. N[0, σ^2]. Also, the econometric model includes company dummy variables which represent possible heterogeneity among firms. By defining

$$
\begin{aligned}
y_t^* &\equiv \mathrm{Ln}(Z_t/K_t) + \text{dummy variables} + u_t \\
&= (\mathrm{Ln}(Re_t), \mathrm{Ln}(K_t), \mathrm{Ln}((wL)_t))\,(1/\psi, -(1-\beta_\varepsilon)/\psi, -\beta_\varepsilon/\psi)' + \text{dummies} + u_t \\
&= (\mathrm{Ln}(Re_t), \mathrm{Ln}(K_t), \mathrm{Ln}((wL)_t), d1, d2, d3, \dots)\,\eta + u_t = x_t\,\eta + u_t,
\end{aligned}
$$

$$f_t \equiv \mathrm{Ln}\; y_{Tr}^{+} - \delta \approx z_{t,noc}\, v_f, \qquad z_{t,noc} \equiv (\gamma), \quad \text{and} \quad v_f \equiv (\text{coefficient for } \gamma),$$

the econometric model becomes the following.

$$(\mathrm{I}) \quad k_t = z_t v_g^{+} + x_t\eta - z_{t,noc} v_f + u_t, \qquad \text{if } x_t\eta + u_t > z_{t,noc} v_f \tag{42-a}$$

$$(\mathrm{O}) \quad k_t = 0, \qquad \text{if } z_{t,noc} v_f - z_t v_g \le x_t\eta + u_t \le z_{t,noc} v_f \tag{42-b}$$

$$(\mathrm{D}) \quad k_t = z_t v_g^{-} + x_t\eta - z_{t,noc} v_f + z_t v_g + u_t, \quad \text{if } x_t\eta + u_t < z_{t,noc} v_f - z_t v_g \tag{42-c}$$

The dummy variables absorb the constant term in $z_t v_f$ (= Ln $y_{Tr}^{+} - \delta$), since y_t^* and Ln y_{Tr}^{+} always appear in a pair and we cannot identify their constant terms separately. Then, this analysis uses a generalized version of the friction model (Maddala 1983, pp. 162). The likelihood function of the model is the following.

$$
\begin{aligned}
&L\left[\eta, \nu_f, \nu_g^+, \nu_g^-, \nu_g, \sigma \middle| k_t, x_t, z_t\right] \\
&= \prod_I \frac{1}{\sigma} \phi\left(\frac{k_t - \left\{z_t \nu_g^+\right\} - x_t \eta + z_{t,noc} \nu_f}{\sigma}\right) \\
&\times \prod_O \left[\Phi\left(\frac{z_{t,noc} \nu_f - x_t \eta}{\sigma}\right) - \Phi\left(\frac{z_{t,noc} \nu_f - \left\{z_t \nu_g\right\} - x_t \eta}{\sigma}\right)\right] \\
&\times \prod_D \frac{1}{\sigma} \phi\left(\frac{k_t - \left\{z_t \nu_g^-\right\} - x_t \eta + z_{t,noc} \nu_f - z_t \nu_g}{\sigma}\right)
\end{aligned}
\tag{43}
$$

Here, ϕ and Φ are the probability density function (p.d.f.) and the cumulative distribution function (c.d.f) of the standard normal distribution, respectively. Terms in braces are additions to the friction model. In other words, some explanatory variables, i.e., z_t, have different coefficients in different places.

4-2 Analysis Methods

i. Data Sources

The financial data that this analysis uses are the sales revenue (Re_t), the book value of capital stock (K_t), and the cost of goods sold ($(wL)_t$) in Compustat PC Plus. Although value added and labor costs could be appropriate for Re_t and $(wL)_t$, such data is not available in Compustat PC. Compustat PC Plus contains corporate financial data over 20 years from 1977 to 1996. It has the beginning balance of capital stock as well as the end balance of capital stock. Therefore, we have 21 years data of capital stock from 1977 to1997. This analysis investigates three industries: the computer and office equipment industry (SIC 3570), the automobile industry (SIC 3711), and the airline industry (SIC 4512). Three or four companies were selected from each industry. The choice of the industry is arbitrary, while the choice of the companies is based upon the observations that some companies had decreases in their capital stock during the examined period, and that each company has a complete set of data, i.e., the panel data is balanced.

The economic data that this analysis employs are the producers price index (PPI), the consumers price index (CPI) and the federal funds rate. The PPI is used as the numeraire. The discount rate or a risk free interest rate (γ) is the difference between the federal funds rate and the CPI rate of inflation.

ii. Procedure

(a) Estimation and Measure of Zero Investment

In order to identify firm's annual capital investment as positive, zero or negative, the depreciation rate, δ, and its range, $\Delta\delta$, are assumed. Then, investment is classified as positive, zero or negative by the ratio of capital stock between two periods. The ratio of capital stock is calculated as either

$$\text{(a)}\quad k_t = \text{Ln}(\, K_{t+n} \,/\, K_{t+n-1} \,) + \delta, \tag{44a}$$

or

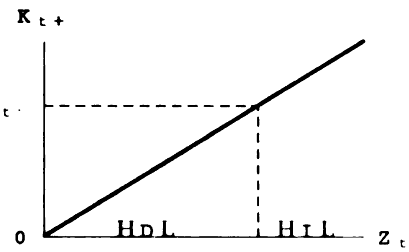
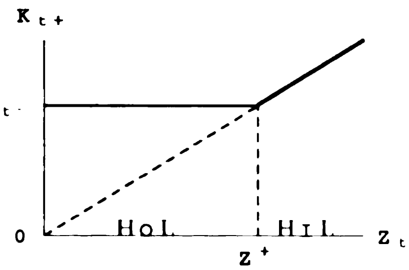
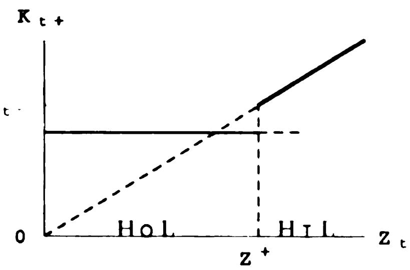
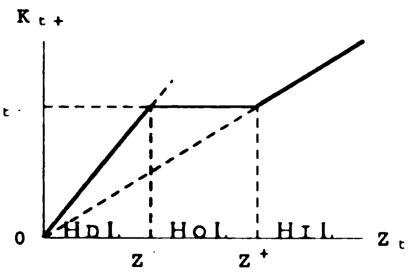
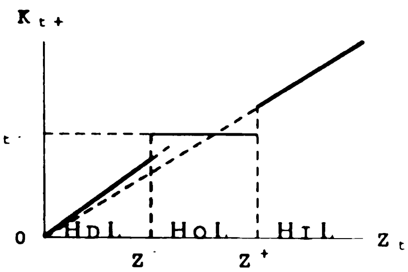
$$\text{(b)}\quad k_t = \text{Ln}(\, K_{t+n} \,/\, K_t \,) \,/\, n + \delta. \tag{44b}$$

For example, when the ratio of capital stock is greater than $+\Delta\delta$, then investment at time t is classified as positive investment. Similarly, if the ratio of capital stocks is between $-\Delta\delta$ and $+\Delta\delta$, or less than $-\Delta\delta$, investment at time t is classified as zero or negative, accordingly. Then, the analysis uses equation (43) for the maximum likelihood estimation.

(b) Test of Model Selection

The analysis compares five investment models by the likelihood of the corresponding econometric models: (1) reversible investment without fixed costs estimated by OLS, (2) irreversible investment without fixed costs estimated by a censored Tobit model, (3) irreversible investment with fixed costs estimated by a generalized version of the Tobit model (G Tobit), (4) costly reversible investment without fixed costs estimated by a ristricted version of the friction model (RFM), and (5) costly reversible investment with fixed costs estimated by a generalized version of the friction model (GFM). By comparing the likelihood of each model, the analysis selects the best model according to the selection criteria of Voung (1989). Table 4.1 shows these models, the corresponding econometric methods, and their likelihood functions.

Table 4.1 Investment Models and Econometric Methods

Capital Investment	Econometric Method and Likelihood Function
Reversible w/o fixed costs 	Ordinary Least Squares (OLS) $L_1 = \prod \frac{1}{\sigma}\phi\left(\frac{k_t - x_t\eta + z_{t,noc}v_f}{\sigma}\right)$
Irreversible w/o fixed costs 	Censored Tobit (Tobit) $L_2 = \prod_I \frac{1}{\sigma}\phi\left(\frac{k_t - x_t\eta + z_{t,noc}v_f}{\sigma}\right)$ $\times \prod_O \left[1 - \Phi\left(\frac{x_t\eta - z_{t,noc}v_f}{\sigma}\right)\right]$
Irreversible with fixed costs 	Generalized Tobit (G Tobit) $L_3 = \prod_I \frac{1}{\sigma}\phi\left(\frac{k_t - x_t\eta + z_{t,noc}v_f - z_t v_g^+}{\sigma}\right)$ $\times \prod_O \left[1 - \Phi\left(\frac{x_t\eta - z_{t,noc}v_f}{\sigma}\right)\right]$
Costly reversible w/o fixed costs 	Restricted Friction Model (RFM) $L_4 = \prod_I \frac{1}{\sigma}\phi\left(\frac{k_t - x_t\eta + z_{t,noc}v_f}{\sigma}\right)$ $\times \prod_O \left[\Phi\left(\frac{x_t\eta - z_{t,noc}v_f + z_t v_g}{\sigma}\right) - \Phi\left(\frac{x_t\eta - z_{t,noc}v_f}{\sigma}\right)\right]$ $\times \prod_D \frac{1}{\sigma}\phi\left(\frac{k_t - x_t\eta + z_{t,noc}v_f - z_t v_g}{\sigma}\right)$
Costly reversible with fixed costs 	Generalized Friction Model (GFM) $L_5 = \prod_I \frac{1}{\sigma}\phi\left(\frac{k_t - x_t\eta + z_{t,noc}v_f - z_t v_g^+}{\sigma}\right)$ $\times \prod_O \left[\Phi\left(\frac{x_t\eta - z_{t,noc}v_f + z_t v_g}{\sigma}\right) - \Phi\left(\frac{x_t\eta - z_{t,noc}v_f}{\sigma}\right)\right]$ $\times \prod_D \frac{1}{\sigma}\phi\left(\frac{k_t - x_t\eta + z_{t,noc}v_f - z_t v_g - z_t v_g^-}{\sigma}\right)$

Voung (1989) proves that the logarithm of the ratio of likelihood, $LR(\cdot)$, converges to the chi-squared distribution when the two competing models: $\boldsymbol{F}_\theta = \{f(y|x;\theta); \theta \in \Theta\}$ and $\boldsymbol{G}_\gamma = \{g(y|x;\gamma); \gamma \in \Gamma\}$, are nested. In other words, when two models are nested, the classic Neyman-Pearson LR test is valid. However, if the two models are non-nested, the ratio converges to a normal distribution. Under the null hypothesis that two models are equivalent,

$2\ LR(\cdot) \xrightarrow{D} \chi^2_{p-q}$, when two models are nested, and

$n^{-1/2}\ LR(\cdot)/\hat{\omega} \xrightarrow{D} \mathrm{N}[\,0,\,1\,]$, when two models are non-nested.

Here, $LR(\hat{\theta},\hat{\gamma}) = \sum \mathrm{Ln}\left[f(Y_t|X_t;\hat{\theta})/g(Y_t|X_t;\hat{\gamma})\right]$ and

$\hat{\omega}^2 = (1/n)\sum\left\{\mathrm{Ln}\left[f(Y_t|X_t;\hat{\theta})/g(Y_t|X_t;\hat{\gamma})\right]\right\}^2 - \left\{(1/n)\sum \mathrm{Ln}\left[f(Y_t|X_t;\hat{\theta})/g(Y_t|X_t;\hat{\gamma})\right]\right\}^2$. Thus, the classical LR test is a special case in Voung's test of model selection.[4]

(c) Test of Serial Correlation

This analysis examines whether residuals, $\hat{u}_t$, are serially correlated. The test is based upon Wooldridge (1995, pp351). The null hypothesis is that there is no serial correlation. This test uses three types of residuals. Two are residuals from OLS and residuals from the GFM. Another is residuals from a two step estimation (TSE) for a limited dependent variable model.[5] The reason that this analysis includes TSE residuals is that the test of serial correlation in this analysis is based upon OLS residuals and the second step of the TSE is OLS. The procedure is two steps.

1st step: regress the lagged OLS residuals or the lagged GFM residuals, $\hat{u}_{i,t-1}$, on x_{it} and z_{it}, or regress the lagged TSE residuals on x_{it}, z_{it} and the estimated inverse Mills ratio. Then, save the residuals, $\hat{r}_{i,t-1}$.

[4] Appendix D discusses the test of model selection under the serially correlated error assumption.

[5] Appendix E shows the TSE and its results.

2nd step: (under homoskedasticity assumption) regress $\hat{u}_{it}$ on $\hat{r}_{i,t-1}$. The *t*-statistics on $\hat{r}_{i,t-1}$ is asymptotically normal under the null hypothesis.

(under heteroskedasticity assumption) regress 1 on $\hat{u}_{it} \times \hat{r}_{i,t-1}$. Then, $\sum_{i=1}^{N}(T_i - 1)$ – SSR is asymptotically chi-squared of degree one under the null hypothesis.

(d) Determination of Depreciation Rate, δ

In order to estimate the depreciation rate, δ, the analysis repeats the procedure with different values of the assumed depreciation rate and its range. The estimate of the depreciation rate is determined by four criteria: (1) high likelihood, (2) homogeneity of degree one for Z_t, (3) the signs of the estimated critical ratios, i.e., $\text{Ln}\,G^+ > 0$, $\text{Ln}\,G^- < 0$, and $\text{Ln}\ G > 0$, and (4) superiority of the GFM.

(e) Testing Hypotheses

a. (Ho) $\eta_{Rc} + \eta_K + \eta_{wL} = 0$; this is a test of whether the ratio, Z_t / K_t, is homogeneous of degree zero in the firm's revenue, capital stock and costs of flow inputs. (Wald test) This substitutes for a test of whether Z_t is homogeneous of degree one in the three variables.

b. (Ho) two competing investment models are equivalent vs (Ha) one model is better than the other. (Voung's Test of Model Selection)

CHAPTER 5

INDUSTRY ANALYSIS

This chapter examines three industries: the computer and the office equipment industry (SIC 3570), the automobile industry (SIC 3711), and the airline industry (SIC 4511), in accordance with the procedure in chapter 4. The analysis chooses three or four firms from each industry. The analysis confirms that actual investment is compatible with the theoretical analysis.

5-1. Computer and Office Equipment Industry

i. Estimation and Measure of Zero Investment

Table 5.1 shows the measure of zero investment for the computer and office equipment industry. According to the four criteria, the case with net capital stock, $\delta = -0.005$, $\Delta\delta = 0.01$, and $n = 1$ is most compatible with the economic model, although cases with δ close to zero show very similar results.[6] Then, the measurement of investment for this analysis is $\text{Ln}(K_{t+1} / K_t) - 0.005$. The case has a high likelihood, the sum of the estimated coefficients for the three financial variables is not significantly different from zero, the signs of the critical ratios are appropriate, and the GFM has the highest

[6] There are two issues with a negative depreciation rate, δ. One is that the depreciation rate used for net capital stock is the accounting rate, rather than the physical or economic depreciation rate. In the cases with net capital stock, δ represents the difference between the accounting depreciation rate and the economic equivalent. The accounting depreciation rate could be larger than the economic rate. Another issue is that of the price of capital. The book value of capital depends upon the acquiring costs of capital, or the purchase price of capital. Even if a firm buys the same capital goods, their prices vary from time to time. In general, old capital goods are cheaper than new ones due to the inflation. Consequently, the book value of capital stock could underestimate older capital, so that the estimated depreciation rate can be negative.

Table 5.1 Measure of Zero Investment (Computer and Office Equipment Industry)

Measure of Capital	δ	$\Delta\delta$	Lags	Econometrics Model					Number of Data			
				OLS	Tobit	G Tobit	RFM	GFM	Total	(D)	(O)	(I)
Net	−0.03	0.01	1	L_1=60.17 Pr[1]=0.00	L_2=36.37 Pr[1]=0.00	L_3=53.83 Pr[1]=0.00 **Pr[2]=0.00**	L_4=46.02 Pr[1]=0.00	L_5=66.11 Pr[1]=0.00 **Pr[2]=0.00**	60	14	3	43
Net	−0.02	0.01	1	L_1=61.22 Pr[1]=0.00	L_2=41.35 Pr[1]=0.00	L_3=57.09 Pr[1]=0.00 **Pr[2]=0.00**	L_4=44.25 Pr[1]=0.01	L_5=68.56 Pr[1]=0.00 **Pr[2]=0.00**	60	11	4	45
Net	−0.01	0.01	1	L_1=62.06 **Pr[1]=0.13**	L_2=43.05 **Pr[1]=0.10**	L_3=58.73 Pr[1]=0.07 **Pr[2]=0.00**	L_4=48.49 **Pr[1]=0.15**	L_5=70.39 **Pr[1]=0.12** **Pr[2]=0.00**	60	11	3	46
Net (Best Case)	−0.005	0.01	1	L_1=62.40 **Pr[1]=0.34**	L_2=45.69 **Pr[1]=0.27**	L_3=59.66 **Pr[1]=0.25** **Pr[2]=0.00**	L_4=52.50 **Pr[1]=0.37**	L_5=71.37 **Pr[1]=0.38** **Pr[2]=0.00**	60	11	2	47
Net	0.00	0.01	1	L_1=62.69 **Pr[1]=0.67**	L_2=49.25 **Pr[1]=0.40**	L_3=60.62 **Pr[1]=0.32** **Pr[2]=0.00**	L_4=57.11 **Pr[1]=0.70**	L_5=70.93 **Pr[1]=0.45** **Pr[2]=0.00**	60	10	1	49
Net	0.005	0.01	1	L_1=62.93 **Pr[1]=0.96**	L_2=48.98 **Pr[1]=0.72**	L_3=60.55 **Pr[1]=0.56** **Pr[2]=0.00**	L_4=57.34 **Pr[1]=0.93**	L_5=70.93 **Pr[1]=0.74** **Pr[2]=0.00**	60	10	1	49
Net	0.01	0.01	1	L_1=63.13 **Pr[1]=0.64**	L_2=48.66 **Pr[1]=0.94**	L_3=60.46 **Pr[1]=0.81** **Pr[2]=0.00**	L_4=53.66 **Pr[1]=0.52**	L_5=70.23 **Pr[1]=0.93** **Pr[2]=0.00**	60	9	2	49
Net	0.03	0.01	1	L_1=63.54 Pr[1]=0.08	L_2=51.88 **Pr[1]=0.20**	L_3=62.88 **Pr[1]=0.36** **Pr[2]=0.00**	L_4=55.31 Pr[1]=0.04	L_5=70.41 **Pr[1]=0.28** **Pr[2]=0.00**	60	7	2	51

Pr[1]=Pr[$\eta_{Re}+\eta_K+\eta_{wL} = 0$]

Pr[2]=Pr[Fixed costs of investment are significant by the classical Neyman-Pearson LM test.]

Bold: Compatible with the economic model.

Table 5.1 (cont'd)

Measure of Capital	δ	$\Delta\delta$	Lags *	Econometrics Model: OLS	Tobit	G Tobit	RFM	GFM	Number of Data: Total	(D)	(O)	(I)
Net	-0.02	0.01	2 (a)	L_1=65.57 Pr[1]=0.09	L_2=46.10 Pr[1]=0.08	L_3=54.89 Pr[1]=0.05 **Pr[2]=0.00**	L_4=50.13 **Pr[1]=0.13**	L_5=64.40 Pr[1]=0.06 **Pr[2]=0.00**	57	11	4	42
Net	-0.01	0.01	2 (a)	L_1=66.11 **Pr[1]=0.81**	L_2=47.40 **Pr[1]=0.70**	L_3=56.74 **Pr[1]=0.38** **Pr[2]=0.00**	L_4=53.90 **Pr[1]=0.88**	L_5=66.18 **Pr[1]=0.46** **Pr[2]=0.00**	57	11	3	43
Net	0.00	0.01	2 (a)	L_1=66.41 **Pr[1]=0.38**	L_2=55.20 **Pr[1]=0.52**	L_3=60.97 **Pr[1]=0.84** **Pr[2]=0.00**	L_4=62.79 **Pr[1]=0.32**	L_5=69.55 **Pr[1]=0.79** **Pr[2]=0.00**	57	10	1	46
Net	0.01	0.01	2 (a)	L_1=66.51 Pr[1]=0.08	L_2=54.55 **Pr[1]=0.12**	L_3=60.81 **Pr[1]=0.40** **Pr[2]=0.00**	L_4=59.97 Pr[1]=0.06	L_5=68.30 **Pr[1]=0.41** **Pr[2]=0.00**	57	9	2	46
Net	-0.02	0.01	2 (b)	L_1=82.68 Pr[1]=0.00	L_2=63.34 Pr[1]=0.00	L_3=75.41 Pr[1]=0.00 **Pr[2]=0.00**	L_4=72.48 Pr[1]=0.00	failed to converge	57	11	2	44
Net	-0.01	0.01	2 (b)	L_1=83.92 **Pr[1]=0.27**	L_2=66.22 **Pr[1]=0.12**	L_3=76.39 Pr[1]=0.06 **Pr[2]=0.00**	L_4=79.35 **Pr[1]=0.31**	L_5=93.68 Pr[1]=0.04 **Pr[2]=0.00**	57	11	1	45
Net	0.00	0.01	2 (b)	L_1=84.77 **Pr[1]=0.65**	L_2=68.24 **Pr[1]=0.93**	L_3=78.19 **Pr[1]=0.59** **Pr[2]=0.00**	L_4=96.09 **Pr[1]=0.59**	L_5=96.09 **Pr[1]=0.59** Pr[2]=1.00	57	11	0	46
Net	0.01	0.01	2 (b)	L_1=85.27 **Pr[1]=0.10**	L_2=66.89 **Pr[1]=0.19**	L_3=78.05 **Pr[1]=0.58** **Pr[2]=0.00**	L_4=96.22 **Pr[1]=0.51**	L_5=96.22 **Pr[1]=0.51** Pr[2]=1.00	57	11	0	46

Pr[1]=Pr[$\eta_{R_e}+\eta_K+\eta_{wL}=0$]
Pr[2]=Pr[Fixed costs of investment are significant by the classical Neyman-Pearson LM test.]
Bold: Compatible with the economic model. * (a) or (b) in the fourth column shows how the ratio of capital stock is calculated.

Table 5.1 (cont'd)

Measure of Capital	δ	$\Delta\delta$	Lags	Econometrics Model					Number of Data			
				OLS	Tobit	G Tobit	RFM	GFM	Total	(D)	(O)	(I)
Net	−0.01	0.02	1	L_1=62.06 **Pr[1]=0.13**	L_2=40.78 **Pr[1]=0.11**	L_3=57.21 Pr[1]=0.07 **Pr[2]=0.00**	L_4=41.95 **Pr[1]=0.16**	L_5=67.49 **Pr[1]=0.12** **Pr[2]=0.00**	60	10	5	45
Net	0.01	0.02	1	L_1=63.13 **Pr[1]=0.64**	L_2=48.66 **Pr[1]=0.94**	L_3=60.46 **Pr[1]=0.81** **Pr[2]=0.00**	L_4=47.45 **Pr[1]=0.41**	L_5=67.70 **Pr[1]=0.94** **Pr[2]=0.00**	60	7	4	49
Net	0.03	0.02	1	L_1=63.54 Pr[1]=0.08	L_2=49.88 **Pr[1]=0.17**	L_3=61.32 **Pr[1]=0.42** **Pr[2]=0.00**	L_4=52.39 Pr[1]=0.03	L_5=68.75 **Pr[1]=0.31** **Pr[2]=0.00**	60	7	3	50
Net	0.05	0.02	1	L_1=63.50 Pr[1]=0.01	L_2=55.91 Pr[1]=0.03	L_3=64.52 **Pr[1]=0.12** **Pr[2]=0.00**	L_4=57.92 Pr[1]=0.00	L_5=71.25 Pr[1]=0.09 **Pr[2]=0.00**	60	5	2	53
Gross	0.02	0.01	1	L_1=80.43 **Pr[1]=0.10**	L_2=65.28 Pr[1]=0.07	L_3=77.93 **Pr[1]=0.69** **Pr[2]=0.00**	L_4=72.36 Pr[1]=0.03	L_5=88.32 Pr[1]=@ **Pr[2]=0.00**	60	8	2	50
Gross	0.03	0.01	1	L_1=80.29 Pr[1]=0.05	L_2=66.02 Pr[1]=0.03	L_3=79.34 **Pr[1]=0.35** **Pr[2]=0.00**	L_4=81.10 Pr[1]=0.01	L_5=89.44 Pr[1]=@ **Pr[2]=0.00**	60	8	1	51
Gross	0.04	0.01	1	L_1=80.10 Pr[1]=0.02	L_2=67.14 Pr[1]=0.02	L_3=80.51 Pr[1]=@ **Pr[2]=0.00**	L_4=77.54 Pr[1]=0.00	L_5=89.28 Pr[1]=@ **Pr[2]=0.00**	60	7	1	52
Gross	0.05	0.01	1	L_1=79.89 Pr[1]=0.01	L_2=65.62 Pr[1]=0.01	L_3=80.15 Pr[1]=@ **Pr[2]=0.00**	L_4=62.95 Pr[1]=0.01	L_5=87.23 Pr[1]=@ **Pr[2]=0.00**	60	4	4	52

Pr[1]=Pr[$\eta_{R_e}+\eta_K+\eta_{wL} = 0$]

Pr[2]=Pr[Fixed costs of investment are significant by the classical Neyman-Pearson LM test.]

Bold: Compatible with the economic model. @ At least one variable has a wrong sign, even though Pr[1] is insignificant.

likelihood in the five competing models. When the analysis uses gross capital stock, the sum of the estimated coefficients, η_{Re}, η_K and η_{wL}, seems to be significantly different from zero.

Table 5.2 shows the estimated coefficients of the best case with net capital stock, $\delta = -0.005$, $\Delta\delta = 0.01$ and $n = 1$. The sum of the estimated coefficients, η_{Re}, η_K and η_{wL}, is –0.019, which is very close to zero. The log likelihood of the GFM exceeds the log likelihood of the next best model, which is OLS, by a margin of about nine. Fixed costs of disinvestment seem larger than fixed costs of investment, since the absolute value of the estimated constant for v_g^- is about three times greater than the estimated constant for v_g^+ for the GFM. Those estimated constants are –0.616 and 0.225, respectively.

Table 5.2 Estimation of Computer and Office Equipment Industry

	OLS	Tobit	G Tobit	RFM	GFM
η_{Re}	0.671(0.12)	0.642(0.13)	0.493(0.13)	0.698(0.13)	0.559(0.13)
η_K	-0.386(0.05)	-0.371(0.06)	-0.327(0.06)	-0.396(0.06)	-0.351(0.06)
η_{wL}	-0.309(0.07)	-0.297(0.08)	-0.190(0.08)	-0.325(0.08)	-0.227(0.07)
$d1$ (DEC)	-0.461(0.30)	-0.405(0.30)	-0.730(63.3)	-0.499(0.31)	-0.591(116)
$d2$ (HWP)	-0.478(0.30)	-0.426(0.31)	-0.774(63.3)	-0.513(0.32)	-0.635(116)
$d3$ (IBM)	-0.407(0.34)	-0.358(0.34)	-0.660(63.3)	-0.446(0.35)	-0.531(116)
$v_I : \gamma$	-0.019(0.01)	-0.019(0.01)	-0.010(15.4)	-0.020(0.01)	0.040(31.6)
$v_g : \gamma$	N/A	N/A	N/A	0.001(0.01)	0.005(46.2)
v_g : _cons	N/A	N/A	N/A	0.015(0.02)	0.764(161)
$v_g^+ : \gamma$	N/A	N/A	0.006(15.4)	N/A	0.057(31.6)
v_g^+ : _cons	N/A	N/A	0.480(63.3)	N/A	0.225(116)
$v_g^- : \gamma$	N/A	N/A	N/A	N/A	0.026(33.8)
v_g^- : _cons	N/A	N/A	N/A	N/A	-0.616(112)
σ	0.086(0.01)	0.078(0.01)	0.068(0.01)	0.088(0.01)	0.071(0.01)
Pr[$\Sigma\eta$=0]	0.345	0.272	0.250	0.372	0.378
Ln L_i	62.397	45.693	59.658	52.502	71.366

Standard errors in parentheses
Parameters: net capital stock, $\delta = -0.005$, $\Delta\delta = 0.01$, and $n = 1$
Number of observations: total 60, investment 47, zero investment 2, and disinvestment 11

ii. Test of Model Selection

Table 5.3 shows the statistics that Voung (1989) proposed for model selection. Two cases: (a) Tobit vs G Tobit and (b) the RFM vs the GFM, are nested so that the classical LR test is valid. All other cases are non-nested. According Voung's test of model selection, the GFM is significantly better than any other model. When the test compares the GFM with any other model, Voung's statistics for the GFM is significant and positive. By comparing the Tobit model with the G Tobit model and the RFM with the GFM, we observe significant and positive values in the statistics, 27.930 and 37.728, respectively, so that we conclude that fixed costs are significantly different from zero. At the same time, by comparing the G Tobit model with the GFM, we also observe significant and positive values in the statistics, 2.446, so that incorporating disinvestment has a significant effect on the analysis of investment.

In addition, OLS is significantly better than the Tobit model and the RFM, since Voung's statistics is significant and negative for two cases. Therefore, the tests suggest that a partial specification of investment model, e.g., costly reversible investment or irreversible investment without fixed costs, is not appropriate to analyze actual investment behavior.

Table 5.3 Model Selection for Computer and Office Equipment Industry

F_θ \ G_γ	OLS	Tobit	G Tobit	RFM	GFM
OLS		0.9333	0.6808	0.5173	0.2483
Tobit	-2.232***		0.3748	1.0034	0.9351
G Tobit	-0.429	27.930***		1.1674	0.3820
RFM	-1.776*	0.878	-0.855		0.7068
GFM	2.324**	3.427***	2.446***	37.728***	

Lower left: Voung's statistics, $(NT)^{-1/2} LR(\cdot)/\hat{\omega}$ or $2\,LR(\cdot)$; a positive value favors F_θ over G_γ

Upper right: sample variances of the log-likelihood ratio, $\hat{\omega}^2$

Parameters: net capital stock, $\delta = -0.005$, $\Delta\delta = 0.01$, and $n = 1$

*: reject Ho at 10 % **: reject Ho at 5 % ***: reject Ho at 1 %

iii. Test of Serial Correlation

Table 5.4 shows the test of serial correlation for the computer and office equipment industry. The test with the TSE residuals and the GFM residuals fails to reject the no-serial-correlation hypothesis, while the test with the OLS residuals rejects the hypothesis at the five percent level. Therefore, the test of serial correlation also favors the GFM. The TSE residuals and the GFM residuals yield essentially the same probability for no serial correlation. In addition, the homoskedasticity case and the heteroskedasticity case yield very close probabilities for no serial correlation.

Table 5.4 Serial Correlation Test for Computer and Office Equipment Industry

	OLS Residuals	TSE Residuals	GFM Residuals
Homoskedasticity Case	t-stat. for $\hat{r}_{t-1} = 2.063$ $\Pr[\cdot] = 0.044$	t-stat. for $\hat{r}_{t-1} = 0.565$ $\Pr[\cdot] = 0.574$	t-stat. for $\hat{r}_{t-1} = 0.584$ $\Pr[\cdot] = 0.562$
Heteroskedasticity Case	$\Sigma(T_i\text{-}1)\text{-SSR} = 4.027$ $\Pr[\cdot] = 0.045$	$\Sigma(T_i\text{-}1)\text{-SSR} = 0.324$ $\Pr[\cdot] = 0.569$	$\Sigma(T_i\text{-}1)\text{-SSR} = 0.345$ $\Pr[\cdot] = 0.557$

Parameters: net capital stock, $\delta = -0.005$, $\Delta\delta = 0.01$, and $n = 1$
Number of data: OLS; $\Sigma (T_i - 1) = 57$, TSE and GFM; $\Sigma (T_i - 1) = 53$ (the difference is due to zero investment.)

5-2. Automobile Industry

i. Estimation and Measure of Zero Investment

Table 5.5 shows the measure of zero investment for the automobile industry. The case with net capital stock, $\delta = -0.02$, $\Delta\delta = 0.01$, and $n = 2$ (b), shows the best result according to the four criteria. Then, the measurement of investment for this analysis is $\text{Ln}(K_{t+2} / K_t) / 2 - 0.02$. When the analysis uses gross capital stock, the program for analysis often fails to converge.

Table 5.5 Measure of Zero Investment (Automobile Industry)

Measure of Capital	δ	$\Delta\delta$	Lags	Econometrics Model					Number of Data			
				OLS	Tobit	G Tobit	RFM	GFM	Total	(D)	(O)	(I)
Net	-0.06	0.01	1	L_1=64.49 Pr[1]=@	L_2=25.43 Pr[1]=@	fail to converge			60	16	6	38
Net	-0.05	0.01	1	L_1=64.31 Pr[1]=@	L_2=33.85 Pr[1]=@	L_3=63.35 Pr[1]=@ **Pr[2]=0.00**	L_4=53.32 Pr[1]=@	L_5=82.58 **Pr[1]=0.77** **Pr[2]=0.00**	60	16	2	42
Net	-0.04	0.01	1	L_1=63.99 Pr[1]=@	L_2=37.22 Pr[1]=@	L_3=64.95 Pr[1]=@ **Pr[2]=0.00**	L_4=54.64 Pr[1]=0.98	L_5=79.88 **Pr[1]=0.74** **Pr[2]=0.00**	60	13	1	44
Net	-0.02	0.01	1	L_1=63.29 Pr[1]=@	L_2=40.46 Pr[1]=0.05	L_3=69.85 Pr[1]=@ **Pr[2]=0.00**	L_4=41.20 Pr[1]=@	L_5=78.24 **Pr[1]=0.97** **Pr[2]=0.00**	60	8	5	47
Net	0.00	0.01	1	L_1=62.71 Pr[1]=@	L_2=50.48 Pr[1]=0.03	L_3=71.51 Pr[1]=@ **Pr[2]=0.00**	L_4=62.41 Pr[1]=0.03	L_5=76.29 Pr[1]=@ **Pr[2]=0.00**	60	6	2	52
Net	-0.04	0.02	1	L_1=63.99 Pr[1]=@	L_2=31.52 Pr[1]=@	L_3=63.60 Pr[1]=@ **Pr[2]=0.00**	L_4=37.18 Pr[1]=@	fail to converge	60	12	6	42
Net	0.00	0.02	1	L_1=62.71 Pr[1]=0.12	L_2=38.06 Pr[1]=0.04	L_3=70.84 Pr[1]=@ **Pr[2]=0.00**	L_4=31.57 Pr[1]=0.09	L_5=75.34 **Pr[1]=0.46** **Pr[2]=0.00**	60	4	8	48
Net	-0.06	0.01	2 (a)	L_1=60.47 Pr[1]=0.02	L_2=24.27 Pr[1]=0.01	fail to converge			57	16	6	35

Pr[1]=Pr[$\eta_{R_e}+\eta_K+\eta_{wL}=0$]

Pr[2]=Pr[Fixed costs of investment are significant by the classical Neyman-Pearson LM test.]

Bold: Compatible with the economic model. @: Some estimated coefficients have a wrong sign, although Pr[1] is insignificant.

Table 5.5 (cont'd)

Measure of Capital	δ	$\Delta\delta$	Lags	Econometrics Model					Number of Data			
				OLS	Tobit	G Tobit	RFM	GFM	Total	(D)	(O)	(I)
Net	-0.04	0.01	2 (a)	L_1=60.47 **Pr[1]=0.10**	L_2=34.34 **Pr[1]=0.13**	L_3=60.16 **Pr[1]=0.16** **Pr[2]=0.00**	L_4=46.76 Pr[1]=0.07	L_5=75.15 Pr[1]=0.04 **Pr[2]=0.00**	57	13	3	41
Net	-0.02	0.01	2 (a)	L_1=59.51 **Pr[1]=0.18**	L_2=36.22 **Pr[1]=0.42**	L_3=64.63 **Pr[1]=0.25** **Pr[2]=0.00**	L_4=37.76 **Pr[1]=0.28**	L_5=72.74 Pr[1]=0.01 **Pr[2]=0.00**	57	8	5	44
Net	-0.06	0.01	2 (b)	L_1=76.56 **Pr[1]=0.13**	L_2=48.86 Pr[1]=0.13	fail to converge			57	15	3	39
Net	-0.04	0.01	2 (b)	L_1=74.52 **Pr[1]=0.69**	L_2=42.72 **Pr[1]=0.98**	fail to converge			57	12	3	42
Net	-0.03	0.01	2 (b)	L_1=73.69 **Pr[1]=0.89**	L_2=39.48 **Pr[1]=0.75**	fail to converge			57	11	4	42
Net (Best Case)	-0.02	0.01	2 (b)	L_1=73.00 **Pr[1]=0.98**	L_2=43.43 **Pr[1]=0.95**	L_3=74.92 Pr[1]=@ **Pr[2]=0.00**	L_4=57.45 **Pr[1]=0.94**	L_5=89.87 **Pr[1]=0.51** **Pr[2]=0.00**	57	9	3	45
Net	-0.01	0.01	2 (b)	L_1=72.45 **Pr[1]=0.91**	L_2=43.43 **Pr[1]=0.87**	L_3=76.39 Pr[1]=@ **Pr[2]=0.00**	L_4=53.28 **Pr[1]=0.79**	L_5=87.16 **Pr[1]=0.53** **Pr[2]=0.00**	57	7	4	46
Net	0.00	0.01	2 (b)	L_1=71.99 **Pr[1]=0.87**	L_2=46.84 **Pr[1]=0.93**	L_3=78.68 Pr[1]=0.06 **Pr[2]=0.00**	L_4=53.72 **Pr[1]=0.73**	L_5=88.92 Pr[1]=0.05 **Pr[2]=0.00**	57	5	4	48

Pr[1]=Pr[$\eta_{Re}+\eta_K+\eta_{wL}=0$]

Pr[2]=Pr[Fixed costs of investment are significant by the classical Neyman-Pearson LM test.]

Bold: Compatible with the economic model. @: Some estimated coefficients have a wrong sign, although Pr[1] is insignificant.

Table 5.5 (cont'd)

Measure of Capital	δ	$\Delta\delta$	Lags	Econometrics Model					Number of Data			
				OLS	Tobit	G Tobit	RFM	GFM	Total	(D)	(O)	(I)
Net	-0.06	0.02	2 (b)	L_1=76.56 **Pr[1]=0.13**	L_2=36.03 **Pr[1]=0.11**	fail to converge			57	15	5	37
Net	-0.04	0.02	2 (b)	L_1=74.52 **Pr[1]=0.69**	L_2=40.33 **Pr[1]=0.92**	fail to converge			57	11	5	41
Net	-0.06	0.01	3 (b)	L_1=92.06 Pr[1]=0.00	L_2=39.38 Pr[1]=0.00	fail to converge			54	12	8	34
Gross	0.00	0.01	1	L_1=82.41 **Pr[1]=0.21**	L_2=75.27 Pr[1]=0.02	L_3=94.73 Pr[1]=0.04 **Pr[2]=0.00**	L_4=64.42 **Pr[1]=0.18**	L_5=99.26 Pr[1]=0.04 Pr[2]=0.00	60	2	4	58
Gross	0.02	0.01	1	L_1=82.03 **Pr[1]=0.20**	L_2=86.39 Pr[1]=0.02	L_3=98.17 Pr[1]=0.00 **Pr[2]=0.00**	L_4=95.53 Pr[1]=0.00	fail to converge	60	1	1	60
Gross	0.00	0.01	2 (b)	L_1=92.05 **Pr[1]=0.82**	L_2=89.29 **Pr[1]=0.87**	L_3=103.31 Pr[1]=0.01 **Pr[2]=0.00**	L_4=108.09 Pr[1]=0.00	fail to converge	57	2	0	55
				L_1= Pr[1]=	L_2= Pr[1]=	L_3= Pr[1]= Pr[2]=	L_4= Pr[1]=	L_5= Pr[1]= Pr[2]=				
				L_1= Pr[1]=	L_2= Pr[1]=	L_3= Pr[1]= Pr[2]=	L_4= Pr[1]=	L_5= Pr[1]= Pr[2]=				

Pr[1]=Pr[$\eta_{R_e}+\eta_K+\eta_{wL}=0$]
Pr[2]=Pr[Fixed costs of investment are significant by the classical Neyman-Pearson LM test.]
Bold: Compatible with the economic model. @: Some estimated coefficients have a wrong sign, although Pr[1] is insignificant.

Table 5.6 is the estimated coefficients of the best case that yields the highest likelihood. The sum of the estimated coefficients for η_{Rc}, η_K and η_{wL}, is 0.029, and very close to zero. And, the log likelihood of the GFM exceeds the log likelihood of the second best model, OLS, by a margin of about seventeen. Fixed costs of investment and fixed costs of disinvestment seem to be of comparable size, since the estimated constant for v_g^+ is as large as the absolute value of the estimated constant for v_g^- for the GFM. Those estimated constants are 0.301 and –0.367, respectively.

Table 5.6 Estimation of Automobile Industry

	OLS	Tobit	G Tobit	RFM	GFM
η_{Rc}	0.519(0.22)	0.597(0.24)	0.128(0.19)	0.563(0.29)	0.163(0.17)
η_K	-0.101(0.03)	-0.112(0.03)	-0.138(0.03)	-0.099(0.03)	-0.110(0.02)
η_{wL}	-0.417(0.27)	-0.481(0.29)	0.065(0.24)	-0.468(0.29)	-0.024(0.21)
$d1$ (C)	-0.113(0.53)	-0.159(0.57)	-1.060(26.81)	-0.062(0.56)	-0.621(28.16)
$d2$ (F)	-0.146(0.59)	-0.206(0.64)	-1.180(26.81)	-0.088(0.63)	-0.687(28.16)
$d3$ (GM)	-0.097(0.61)	-0.145(0.66)	-1.143(26.81)	-0.037(0.65)	-0.662(28.16)
$v_f : \gamma$	-0.000(0.01)	-0.000(0.01)	-0.024(5.77)	0.000(0.01)	-0.000(10.24)
$v_g : \gamma$	N/A	N/A	N/A	-0.002(0.00)	0.002(15.59)
v_g : _cons	N/A	N/A	N/A	0.010(0.01)	0.490(39.78)
$v_g^+ : \gamma$	N/A	N/A	-0.015(5.77)	N/A	0.007(10.24)
v_g^+ : _cons	N/A	N/A	0.457(26.80)	N/A	0.301(28.16)
$v_g^- : \gamma$	N/A	N/A	N/A	N/A	0.014(11.79)
v_g^- : _cons	N/A	N/A	N/A	N/A	-0.367(28.09)
σ	0.067(0.01)	0.071(0.01)	0.046(0.00)	0.071(0.01)	0.046(0.00)
$\Pr[\Sigma\eta = 0]$	0.984	0.954	0.239	0.935	0.507
Ln L_i	73.004	43.434	74.923	57.450	89.875

Standard errors in parentheses

Parameters: net capital stock, $\delta = -0.02$, $\Delta\delta = 0.01$, $n = 2$ (b)

Number of observations: total 57, investment 45, zero investment 3, disinvestment 9

ii. Test of Model Selection

Table 5.7 shows Voung's test of model selection for the automobile industry. Voung's test of model selection ranks the five models as follows:

$$\text{GFM} > \text{OLS} \approx \text{G Tobit} > \text{RFM} > \text{Tobit}.$$

For the automobile industry, the generalized Tobit model is as good as OLS. Also, there are significant fixed costs of investment and disinvestment, since the GFM and the G Tobit model are better than the RFM and the Tobit model, respectively. And, because the GFM and the RFM are better than the G Tobit model and the Tobit model, respectively, disinvestment is important for analysis.

Table 5.7 Model Selection for Automobile Industry

F_θ \ G_γ	OLS	Tobit	G Tobit	RFM	GFM
OLS		0.8998	0.5576	0.8937	0.8283
Tobit	-4.129***		0.6821	0.7782	1.9468
G Tobit	0.340	62.978***		0.7603	0.5120
RFM	-2.179***	2.104**	-2.654***		0.9630
GFM	2.455***	4.409***	2.768***	64.850***	

Lower left: Voung's statistics, $(NT)^{-1/2} LR(\cdot)/\hat{\omega}$ or $2\,LR(\cdot)$; a positive value favors F_θ over G_γ

Upper right: sample variances of the log-likelihood ratio, $\hat{\omega}^2$

Parameters: net capital stock, $\delta = -0.02$, $\Delta\delta = 0.01$, and $n = 2$ (b)

** : reject Ho at 5 % *** : reject Ho at 1 %

iii. Test of Serial Correlation

Table 5.8 shows the test of serial correlation for the automobile industry. The TSE residuals fail to reject the no-serial-correlation hypothesis at the ten percent level. The GFM residuals fail to reject the hypothesis at the six percent level. The OLS residuals reject the hypothesis. The homoskedastic case and the heteroskedastic case yield essentially the same probability for no serial correlation, similar to the other industries.

Table 5.8 Serial Correlation Test for Automobile Industry

	OLS Residuals	TSE Residuals	GFM Residuals
Homoskedasticity Case	t-stat. for $\hat{r}_{t-1}$ = 2.648 Pr[·] = 0.011	t-stat. for $\hat{r}_{t-1}$ = 1.675 Pr[·] = 0.101	t-stat. for $\hat{r}_{t-1}$ = 1.902 Pr[·] = 0.063
Heteroskedasticity Case	$\Sigma(T_i$-1)-SSR = 6.309 Pr[·] = 0.012	$\Sigma(T_i$-1)-SSR= 2.703 Pr[·] = 0.1002	$\Sigma(T_i$-1)-SSR = 3.430 Pr[·] = 0.064

Parameters: net capital stock, $\delta = -0.02$, $\Delta\delta = 0.01$, and $n = 2$ (b)
Number of data: OLS; $\Sigma\ (T_i - 1) = 54$, TSE and GFM; $\Sigma\ (T_i - 1) = 48$ (the difference is due to zero investment.)

5-3. Airline Industry

Since there is an organized international used airplane market, it is easy for airline companies to resell their used planes, so the industry may have small fixed costs of disinvestment. The industry experienced a deregulation in the 1980's. The Airline Deregulation Act of 1978 initiated this deregulation. Thus, this analysis includes a deregulation dummy variable, *dereg* (1 for years ≥1987, 0 for years <1987) in order to incorporate industry's adjustments to the deregulation. Without this dummy variable, the econometric program that this analysis uses often fails to converge. The airline industry, however, shows results similar to the two industries and that there are significant fixed costs of investment and disinvestment.

i. Estimation and Measure of Zero Investment

Table 5.9 shows the measure of zero investment for the airline industry. The case with net capital stock, $\delta = -0.05$, $\Delta\delta = 0.01$, and $n = 2$ (b), shows the best result according to the four criteria. Thus, the measurement of investment for this analysis is $\text{Ln}(K_{t+2} / K_t) / 2 - 0.05$. When the analysis uses the gross capital stock, the sum of the estimated coefficients for η_{Kc}, η_K and η_{wL}, is significantly different from zero, similar to the computer and office equipment industry.

Table 5.9 Measure of Zero Investment (Airline Industry)

Measure of Capital	δ	$\Delta\delta$	Lags	Econometrics Model					Number of Data			
				OLS	Tobit	G Tobit	RFM	GFM	Total	(D)	(O)	(I)
Net	-0.05	0.01	1	L_1=50.56 Pr[1]=0.07	L_2=-1.19 Pr[1]=0.01	L_3=41.37 **Pr[1]=0.77** **Pr[2]=0.00**	L_4=15.19 Pr[1]=0.05	L_5=67.90 Pr[1]=0.84 Pr[2]=0.00	80	22	12	46
Net	-0.04	0.01	1	L_1=50.80 **Pr[1]=0.12**	L_2= 2.00 Pr[1]=0.02	L_3=43.27 **Pr[1]=0.82** **Pr[2]=0.00**	L_4=18.75 Pr[1]=0.09	L_5=68.29 **Pr[1]=0.96** **Pr[2]=0.00**	80	23	8	49
Net	-0.03	0.01	1	L_1=51.01 **Pr[1]=0.20**	L_2=11.09 Pr[1]=0.04	L_3=48.27 **Pr[1]=0.31** **Pr[2]=0.00**	L_4=36.06 **Pr[1]=0.19**	L_5=71.14 **Pr[1]=0.81** **Pr[2]=0.00**	80	22	3	55
Net	-0.02	0.01	1	L_1=51.19 **Pr[1]=0.31**	L_2=13.38 Pr[1]=0.07	L_3=50.52 **Pr[1]=0.40** **Pr[2]=0.00**	L_4=36.38 **Pr[1]=0.30**	L_5=70.98 **Pr[1]=0.96** **Pr[2]=0.00**	80	20	3	57
Net	-0.01	0.01	1	L_1=51.32 **Pr[1]=0.41**	L_2=13.44 **Pr[1]=0.14**	L_3=50.79 **Pr[1]=0.20** **Pr[2]=0.00**	L_4=29.44 **Pr[1]=0.41**	L_5=69.19 **Pr[1]=0.71** **Pr[2]=0.00**	80	17	5	58
Net	0.00	0.01	1	L_1=50.67 **Pr[1]=0.65**	L_2=14.97 Pr[1]=@	L_3=46.41 Pr[1]=@ **Pr[2]=0.00**	L_4=32.52 Pr[1]=@	L_5=63.77 **Pr[1]=0.14** **Pr[2]=0.00**	80	16	4	60
Net	-0.03	0.01	1	L_1=51.01 **Pr[1]=0.20**	Failed to converge	Failed to converge	L_4=10.09 **Pr[1]=0.16**	L_5=64.83 **Pr[1]=0.78** **Pr[2]=0.00**	80	20	11	49
Net	-0.02	0.01	1	L_1=51.19 **Pr[1]=0.31**	L_2=9.61 Pr[1]=0.07	L_3=48.41 **Pr[1]=0.39** **Pr[2]=0.00**	L_4=19.79 **Pr[1]=0.29**	L_5=67.52 **Pr[1]=0.89** **Pr[2]=0.00**	80	17	8	55

Pr[1]=Pr[$\eta_{R_e}+\eta_K+\eta_{wL}$ = 0]

Pr[2]=Pr[Fixed costs of investment are significant by the classical Neyman-Pearson LM test.]

Bold: Compatible with the economic model. @: Some estimated coefficients have a wrong sign, although Pr[1] is insignificant.

Table 5.9 (cont'd)

Measure of Capital	δ	$\Delta\delta$	Lags	Econometrics Model					Number of Data			
				OLS	Tobit	G Tobit	RFM	GFM	Total	(D)	(O)	(I)
Net	-0.01	0.02	1	L_1=51.32 **Pr[1]=0.41**	L_2=11.96 **Pr[1]=0.10**	L_3=50.56 **Pr[1]=0.47** **Pr[2]=0.00**	L_4=23.30 **Pr[1]=0.38**	L_5=68.84 **Pr[1]=1.00** **Pr[2]=0.00**	80	16	7	57
Net	0.00	0.02	1	L_1=51.39 **Pr[1]=0.50**	L_2=11.96 **Pr[1]=0.19**	L_3=50.81 **Pr[1]=0.24** **Pr[2]=0.00**	L_4=13.22 **Pr[1]=0.47**	L_5=61.78 **Pr[1]=0.69** **Pr[2]=0.00**	80	11	11	58
Net	-0.04	0.01	2 (a)	L_1=55.11 Pr[1]=@	L_2= 4.92 Pr[1]=@	L_3=36.67 Pr[1]=@ **Pr[2]=0.00**	L_4=24.15 Pr[1]=@	L_5=63.00 Pr[1]=@ **Pr[2]=0.00**	76	21	8	47
Net	-0.02	0.01	2 (a)	L_1=55.04 Pr[1]=@	L_2=17.10 Pr[1]=@	L_3=42.02 Pr[1]=@ Pr[2]=0.18	L_4=42.89 Pr[1]=@	L_5=64.44 Pr[1]=@ Pr[2]=0.00	76	18	3	55
Net	0.00	0.01	2 (a)	L_1=54.97 Pr[1]=@	L_2=20.45 Pr[1]=@	L_3=42.42 Pr[1]=@ **Pr[2]=0.00**	L_4=41.26 Pr[1]=@	L_5=60.97 Pr[1]=@ **Pr[2]=0.00**	76	15	3	58
Net	-0.06	0.01	2 (b)	L_1=81.42 Pr[1]=@	L_2=31.57 Pr[1]=@	L_3=64.46 Pr[1]=@ **Pr[2]=0.00**	L_4=58.01 Pr[1]=@	L_5=96.55 **Pr[1]=0.11** **Pr[2]=0.00**	76	20	5	51
Net (Best Case)	-0.05	0.01	2 (b)	L_1=81.54 Pr[1]=@	L_2=40.70 Pr[1]=@	L_3=68.77 **Pr[1]=0.85** **Pr[2]=0.00**	L_4=66.29 Pr[1]=@	L_5=97.55 **Pr[1]=0.50** **Pr[2]=0.00**	76	18	3	55
Net	-0.04	0.01	2 (b)	L_1=81.63 Pr[1]=@	L_2=41.49 Pr[1]=@	L_3=69.67 **Pr[1]=0.92** **Pr[2]=0.00**	L_4=66.22 Pr[1]=@	L_5=96.70 **Pr[1]=0.53** **Pr[2]=0.00**	76	17	3	56

Pr[1]=Pr[$\eta_{Re}+\eta_K+\eta_{wL} = 0$]

Pr[2]=Pr[Fixed costs of investment are significant by the classical Neyman-Pearson LM test.]

Bold: Compatible with the economic model. @: Some estimated coefficients have a wrong sign, although Pr[1] is insignificant.

Table 5.9 (cont'd)

Measure of Capital	δ	$\Delta\delta$	Lags	Econometrics Model					Number of Data			
				OLS	Tobit	G Tobit	RFM	GFM	Total	(D)	(O)	(I)
Net	-0.03	0.01	2 (b)	L_1=81.68 Pr[1]=@	L_2=45.24 Pr[1]=@	L_3=71.79 **Pr[1]=0.82** **Pr[2]=0.00**	L_4=83.36 **Pr[1]=0.12**	L_5=96.92 **Pr[1]=0.68** **Pr[2]=0.00**	76	16	2	58
Net	-0.02	0.01	2 (b)	L_1=81.67 Pr[1]=@	L_2=45.99 Pr[1]=@	L_3=72.60 **Pr[1]=0.93** **Pr[2]=0.00**	L_4=66.79 Pr[1]=@	L_5=94.63 **Pr[1]=0.55** **Pr[2]=0.00**	76	14	3	59
Net	-0.04	0.02	2 (b)	L_1=81.63 Pr[1]=@	L_2=38.87 Pr[1]=@	L_3=68.47 **Pr[1]=0.87** **Pr[2]=0.00**	L_4=58.25 Pr[1]=@	L_5=94.14 **Pr[1]=0.61** **Pr[2]=0.00**	76	16	5	55
Net	-0.04	0.01	3 (a)	L_1=57.30 Pr[1]=@	L_2= 9.28 Pr[1]=@	Failed to converge	Failed to converge	Failed to converge	72	21	7	44
Net	-0.04	0.01	3 (b)	L_1=96.70 Pr[1]=0.00	L_2=67.26 Pr[1]=0.00	L_3=85.97 **Pr[1]=0.24** **Pr[2]=0.00**	L_4=85.70 Pr[1]=0.00	L_5=107.44 Pr[1]=0.07 **Pr[2]=0.00**	72	12	2	58
Net	-0.02	0.01	3 (b)	L_1=96.42 Pr[1]=0.00	L_2=69.27 Pr[1]=0.00	L_3=88.47 **Pr[1]=0.22** **Pr[2]=0.00**	L_4=109.56 Pr[1]=0.06	L_5=109.56 Pr[1]=0.06 Pr[2]=1.00	72	12	0	60
Net	0.00	0.01	3 (b)	L_1=96.04 Pr[1]=@	L_2=65.62 Pr[1]=@	L_3=87.95 **Pr[1]=0.18** **Pr[2]=0.00**	L_4=73.45 Pr[1]=@	L_5=98.40 **Pr[1]=0.12** **Pr[2]=0.00**	72	6	6	60
Net	-0.06	0.02	3 (b)	L_1=96.80 Pr[1]=0.00	L_2=39.83 Pr[1]=0.00	Failed to converge	Failed to converge	Failed to converge	72	12	10	50

Pr[1]=Pr[$\eta_{Re}+\eta_K+\eta_{wL} = 0$]
Pr[2]=Pr[Fixed costs of investment are significant by the classical Neyman-Pearson LM test.]
Bold: Compatible with the economic model. @: Some estimated coefficients have a wrong sign, although Pr[1] is insignificant.

Table 5.9 (cont'd)

Measure of Capital	δ	$\Delta\delta$	Lags	Econometrics Model					Number of Data			
				OLS	Tobit	G Tobit	RFM	GFM	Total	(D)	(O)	(I)
Net	-0.05	0.02	3 (b)	L_1=96.78 Pr[1]=0.00	L_2=46.46 Pr[1]=0.00	Failed to converge	Failed to converge	Failed to converge	72	12	10	50
Net	-0.04	0.02	3 (b)	L_1=96.70 Pr[1]=0.00	L_2=50.82 Pr[1]=0.00	L_3=76.55 **Pr[1]=0.23** **Pr[2]=0.00**	L_4=62.45 Pr[1]=0.00	L_5=98.24 Pr[1]=0.07 Pr[2]=0.00	72	12	8	52
Net	-0.02	0.02	3 (b)	L_1=96.42 Pr[1]=0.00	L_2=66.61 Pr[1]=0.00	L_3=86.78 **Pr[1]=0.21** **Pr[2]=0.00**	L_4=82.27 Pr[1]=0.00	L_5=103.96 Pr[1]=0.08 **Pr[2]=0.00**	72	10	3	59
Net	0.00	0.02	3 (b)	L_1=96.04 Pr[1]=@	L_2=65.62 Pr[1]=0.00	L_3=87.95 **Pr[1]=0.18** **Pr[2]=0.00**	L_4=68.94 Pr[1]=0.00	L_5=93.88 Pr[1]=0.13 Pr[2]=0.00	72	3	9	60
				L_1= Pr[1]=	L_2= Pr[1]=	L_3= Pr[1]= Pr[2]=	L_4= Pr[1]=	L_5= Pr[1]= Pr[2]=				
				L_1= Pr[1]=	L_2= Pr[1]=	L_3= Pr[1]= Pr[2]=	L_4= Pr[1]=	L_5= Pr[1]= Pr[2]=				
				L_1= Pr[1]=	L_2= Pr[1]=	L_3= Pr[1]= Pr[2]=	L_4= Pr[1]=	L_5= Pr[1]= Pr[2]=				
				L_1= Pr[1]=	L_2= Pr[1]=	L_3= Pr[1]= Pr[2]=	L_4= Pr[1]=	L_5= Pr[1]= Pr[2]=				

Pr[1]=Pr[$\eta_{R_e}+\eta_K+\eta_{wL} = 0$]

Pr[2]=Pr[Fixed costs of investment are significant by the classical Neyman-Pearson LM test.]

Bold: Compatible with the economic model. @: Some estimated coefficients have a wrong sign, although Pr[1] is insignificant.

Table 5.9 (cont'd)

Measure of Capital	δ	$\Delta\delta$	Lags	Econometrics Model					Number of Data			
				OLS	Tobit	G Tobit	RFM	GFM	Total	(D)	(O)	(I)
Gross	0.05	0.01	1	L_1=71.12 **Pr[1]=0.87**	L_2=64.86 **Pr[1]=0.84**	L_3=77.81 Pr[1]=0.06 Pr[2]=0.00	L_4=81.61 Pr[1]=0.08	L_5=81.61 Pr[1]=0.08 Pr[2]=1.00	80	3	0	77
Gross	0.04	0.01	1	L_1=71.29 **Pr[1]=0.91**	L_2=65.67 Pr[1]=0.83	L_3=77.94 Pr[1]=0.07 Pr[2]=0.00	L_4=81.79 Pr[1]=0.09	L_5=81.79 Pr[1]=0.09 Pr[2]=1.00	80	3	0	77
Gross	0.03	0.01	1	L_1=71.46 **Pr[1]=0.94**	L_2=63.97 Pr[1]=0.90	L_3=77.02 Pr[1]=0.06 Pr[2]=0.00	L_4=68.47 **Pr[1]=0.80**	L_5=80.87 Pr[1]=0.09 **Pr[2]=0.00**	80	3	1	76
Gross	0.04	0.02	1	L_1=71.29 **Pr[1]=0.91**	L_2=63.08 **Pr[1]=0.92**	L_3=76.87 Pr[1]=0.05 **Pr[2]=0.00**	L_4=68.14 **Pr[1]=0.78**	L_5=80.67 Pr[1]=0.07 **Pr[2]=0.00**	80	3	1	76
Gross	0.03	0.02	1	L_1=71.46 **Pr[1]=0.94**	L_2=54.69 **Pr[1]=0.88**	L_3=76.25 Pr[1]=0.05 **Pr[2]=0.00**	L_4=55.22 **Pr[1]=0.90**	L_5=79.96 Pr[1]=0.08 **Pr[2]=0.00**	80	3	4	73
				L_1= Pr[1]=	L_2= Pr[1]=	L_3= Pr[1]= Pr[2]=	L_4= Pr[1]=	L_5= Pr[1]= Pr[2]=				
				L_1= Pr[1]=	L_2= Pr[1]=	L_3= Pr[1]= Pr[2]=	L_4= Pr[1]=	L_5= Pr[1]= Pr[2]=				
				L_1= Pr[1]=	L_2= Pr[1]=	L_3= Pr[1]= Pr[2]=	L_4= Pr[1]=	L_5= Pr[1]= Pr[2]=				

Pr[1]=Pr[$\eta_{R_e}+\eta_K+\eta_{wL} = 0$]
Pr[2]=Pr[Fixed costs of investment are significant by the classical Neyman-Pearson LM test.]
Bold: Compatible with the economic model.

Table 5.10 shows the estimated coefficients of the best case with net capital stock, $\delta = -0.05$, $\Delta\delta = 0.01$, and $n = 2$ (b). The sum of the estimated coefficients for η_{Re}, η_K and η_{wL} is −0.026, and very close to zero. And, the log likelihood of the GFM exceeds the log likelihood of the second best model, OLS, by a margin of about sixteen. Fixed costs of investment and fixed costs of disinvestment seem to be of comparable size, similar to the automobile industry. Those estimated coefficients for ν_g^+ and ν_g^- are 0.417 and −0.389, respectively.

Table 5.10 Estimation of Airline Industry

	OLS	Tobit	G Tobit	RFM	GFM
η_{Re}	-0.016(0.16)	-0.047(0.18)	0.277(0.16)	-0.040(0.16)	0.245(0.13)
η_K	-0.108(0.05)	-0.122(0.06)	-0.099(0.05)	-0.108(0.06)	-0.096(0.04)
η_{wL}	-0.016(0.12)	0.013(0.14)	-0.167(0.13)	-0.002(0.13)	-0.175(0.10)
d1 (AMR)	1.238(0.37)	1.411(0.44)	-0.548(48.54)	1.373(0.39)	-0.112(31.23)
d2 (DAL)	1.191(0.36)	1.311(0.43)	-0.603(48.54)	1.278(0.38)	-0.172(31.23)
d3 (UAL)	1.218(0.39)	1.351(0.45)	-0.605(48.54)	1.308(0.40)	-0.163(31.23)
d4 (U)	1.151(0.33)	1.261(0.39)	-0.474(48.54)	1.228(0.35)	-0.100(31.23)
dereg	0.046(0.04)	0.059(0.05)	0.003(0.05)	0.048(0.05)	0.025(0.04)
$\nu_l : \gamma$	-0.008(0.00)	-0.010(0.01)	0.002(20.26)	-0.008(0.00)	0.001(8.72)
$\nu_g : \gamma$	N/A	N/A	N/A	0.002(0.00)	0.006(20.20)
ν_g : _cons	N/A	N/A	N/A	0.012(0.01)	0.673(55.16)
$\nu_g^+ : \gamma$	N/A	N/A	0.004(20.26)	N/A	0.004(8.72)
ν_g^+ : _cons	N/A	N/A	0.547(48.59)	N/A	0.417(31.23)
$\nu_g^- : \gamma$	N/A	N/A	N/A	N/A	-0.019(18.21)
ν_g^- : _cons	N/A	N/A	N/A	N/A	-0.389(45.54)
σ	0.083(0.01)	0.088(0.01)	0.069(0.01)	0.086(0.01)	0.064(0.01)
$\Pr[\Sigma\eta = 0]$	0.002	0.003	0.849	0.001	0.498
Ln L_i	81.545	40.702	68.770	66.292	97.547

Standard errors in parentheses

Parameters: net capital stock, $\delta = -0.05$, $\Delta\delta = 0.01$, $n = 2$ (b)

Number of observations: total 76, investment 55, zero investment 3, disinvestment 18

ii. Test of Model Selection

Table 5.11 shows Voung's test of model selection for the airline industry. Voung's test of model selection ranks the five models as follows:

$$\text{GFM} > \text{OLS} > \text{G Tobit} \approx \text{RFM} > \text{Tobit}.$$

For the airline industry, the RFM is as good as the G Tobit. Otherwise, the order is the same as the other industries. There are significant fixed costs of investment and disinvestment contrary to expectations. Disinvestment is significant.

Table 5.11 Model Selection for Airline Industry

F_θ \ G_γ	OLS	Tobit	G Tobit	RFM	GFM
OLS		0.7572	0.6010	0.6515	0.4352
Tobit	-5.384***		0.2406	0.7450	1.1552
G Tobit	-1.890*	56.135***		0.8555	0.6732
RFM	-2.168**	3.401***	-0.307		0.6415
GFM	2.783***	6.067***	4.023***	62.52***	

Lower left: Voung's statistics, $(NT)^{-1/2} LR(\cdot)/\hat{\omega}$ or 2 $LR(\cdot)$; a positive value favors F_θ over G_γ

Upper right: sample variances of the log-likelihood ratio, $\hat{\omega}^2$

Parameters: net capital stock, $\delta = -0.05$, $\Delta\delta = 0.01$, and $n = 2$ (b)

*: reject Ho at 10 % ** : reject Ho at 5 % *** : reject Ho at 1 %

iii. Test of Serial Correlation

Table 5.12 shows the test of serial correlation for the airline industry. The TSE residuals and the GFM residuals fail to reject the no-serial-correlation hypothesis at the ten percent level, while the OLS residuals reject the hypothesis. The conclusion is the same as the other industries. The TSE residuals and the GFM residuals yield essentially the same probability for no serial correlation. The homoskedastic case and the heteroskedastic case also yield essentially the same probability for no serial correlation.

Table 5.12 Serial Correlation Test for Airline Industry

	OLS Residuals	TSE Residuals	GFM Residuals
Homoskedasticity Case	t-stat. for $\hat{r}_{t-1}$ = 3.200 Pr[·] = 0.002	t-stat. for $\hat{r}_{t-1}$ = 1.138 Pr[·] = 0.259	t-stat. for $\hat{r}_{t-1}$ = 1.153 Pr[·] = 0.253
Heteroskedasticity Case	$\Sigma(T_i$-1)-SSR = 9.075 Pr[·] = 0.003	$\Sigma(T_i$-1)-SSR= 1.289 Pr[·] = 0.256	$\Sigma(T_i$-1)-SSR = 1.324 Pr[·] = 0.250

Parameters: net capital stock, $\delta = -0.05$, $\Delta\delta = 0.01$, and $n = 2$ (b)
Number of data: OLS; $\Sigma (T_i - 1) = 72$, TSE and GFM; $\Sigma (T_i - 1) = 67$ (the difference is due to zero investment.)

5-4. Summary

The empirical analysis investigates three industries: the computer and office equipment industry(SIC 3570), the automobile industry (SIC 3711), and the airline industry (SIC 4512). Although the choice of the industries is arbitrary, all of the examined industries show many similarities: the homogeneity of the economic indicator variable in the sales revenue, capital stock and the cost of flow inputs, the superiority of costly reversible investment with fixed costs or the corresponding GFM, no serial correlation in the GFM residuals as well as the TSE residuals, and reversible investment without fixed costs or corresponding OLS as the second best model. By comparing likelihood of the five investment models, the analysis concludes that fixed costs are significantly different from zero, and that incorporating disinvestment has significant effects on analysis. Also, as OLS is significantly better than the other models except the GFM and, possibly the G Tobit model, the analysis suggests that partial specification of investment, e.g., investment without the costly reversibility or fixed costs, is not appropriate to analyze actual investment.

However, the estimated coefficients are different among the industries, which suggests heterogeneity among the industries. In addition, for the airline industry, the analysis requires the deregulation dummy variable so that the airline deregulation of the

1980's could have affected industry's investment. For the measure of investment, a net capital stock shows better results than the gross capital stock. The estimated depreciation rate is zero or negative for the examined industries, when the analysis uses the net capital stock.

CONCLUSIONS

This analysis investigated costly reversible investment with fixed costs. The analysis applied option pricing theory to incorporate the impact of the possibility of future investment on current investment. Fixed costs of investment introduced minimum investment, while fixed costs of disinvestment introduced minimum disinvestment. Both fixed costs and the costly reversibility resulted in zero investment as the optimal choice. When the firm contemplated future investment, its investment was lower than when it did not.

The analysis also investigated how the economic parameters such as the discount rate affected the critical values such as the target value and the trigger value of investment. As the theoretical model did not yield closed form solutions, this analysis resorted numerical solutions. More than 90,000 cases for investment with fixed costs and more than 30,000 cases for investment without fixed costs were examined, so that the conclusions were robust. All of the critical values, except the trigger value for investment with fixed costs, were linear in the parameters, while the target value for investment was semi-log linear in the parameters.

The empirical analysis confirmed that actual investment behaved as the theoretical analysis predicted. The analysis examined three industries: the computer and office equipment industry, the automobile industry, and the airline industry. Although each industry had different estimated coefficients, they all showed the similar results: the homogeneity of the stochastic variable, the superiority of costly reversible investment with fixed costs or the corresponding GFM, reversible investment without fixed costs or corresponding OLS as the second best model, and no serial correlation in the GFM residuals. The analysis confirmed that there were the costly reversibility and fixed costs in actual investment.

APPENDICES

APPENDIX A

TWO PERIOD MODEL OF COSTLY REVERSIBLE INVESTMENT WITHOUT FIXED COSTS

This appendix solves the model as a two period model, and shows economic intuition of the model. The model regards future investment and disinvestment as a call option and a put option, respectively. The appendix shows that value of future investment as the call option is equal to the expected value of net gain from the second period investment while value of future disinvestment as the put option is equal to the expected value of net gain from future disinvestment. The appendix also shows that how first period capital stock affects the expectation of the second period gain.

This two period model is a special case of Abel, Dixit, Eberly and Pindyck (abbreviated ADEP) (1996). They use a general form for the profit function, while this model uses the same profit function as the main text. There are no fixed costs of investment in the appendix. There are three differences between this model and the model in ADEP (1996): In ADEP (1996) there is no physical depreciation, the purchase price of new capital is different between two periods,[7] and the expected PDV of net cash flow includes costs of the first period investment.[8]

[7] In the two period model, the derivative of the expected PDV of net cash flow will not reach the (second period) purchase price of capital in many cases. In such cases, the first period price of capital should be lower than the second period price of capital. This section assumes conditions under which the derivative of the expected PDV of net cash flow reaches higher than the second period price of capital. Therefore, the purchase price of capital can be the same for the two periods.

[8] ADEP (1996) adopt the Net Present Value (NPV) rule, i.e., if the (expected) net present value, or the expected PDV of net cash flow less initial investment costs, is positive, the firm should invest. One consequence is a difference in the optimal investment rule. When the NPV rule is used, the derivative of the expected NPV, which is called q°,

A firm makes its first investment decision to choose optimally capital stock for the first period, K_{0+}, for a given level of capital stock, K_{0-}, after the economic indicator at time $t = 0$, Z_0, is revealed. Or, equivalently, the firm determines either dI_0 (= $K_{0+} - K_{0-}$) or dD_0 (= $K_{0-} - K_{0+}$). At time $t = \Delta t$, firm's capital depreciates by the depreciation rate, δ, to its pre-investment level of capital stock, K_{1-} (= $e^{-\delta\Delta t} K_{0+}$). Then, after the economy reveals Z_1, the firm makes its second investment decision to determine its post-investment capital stock, K_{1+}. The firm will not make any further investment after $t = \Delta t$. In addition, there are no fixed costs for both investment and disinvestment. Thus, we can rewrite equation (3) in the main text as follows:

$$V(K_{0+}, Z_0) = \max_{\{dI, dD\}} E_0\left[\left\{A_\pi K_{0+}^{\theta} \int_0^{\Delta t} e^{-(\gamma+\delta\theta)s} Z_s^{1-\theta} ds - p_K^+ dI_0 + p_K^- dD_0\right\}\right.$$
$$\left. + e^{-\gamma\Delta t}\left\{A_\pi K_{1+}^{\theta} \int_0^{\infty} e^{-(\gamma+\delta\theta)\tau} Z_\tau^{1-\theta} d\tau - p_K^+ dI_1 + p_K^- dD_1\right\}\right] \quad (45)$$

subject to $K_{0+} = K_{0-} + dI_0 - dD_0$, K_{0-}; given, and
$K_{1+} = K_{1-} + dI_1 - dD_1 \left(= e^{-\delta\Delta t} K_{0+} + dI_1 - dD_1\right)$.

A-1. Investment Model

First, we solve the second period investment decision, or the maximization of the second brace in equation (45). Assuming $\gamma > \mu_Z$,

$$\max_{\{dI, dD\}} E_0\left[A_\pi K_{1+}^{\theta} \int_0^{\infty} e^{-(\gamma+\delta\theta)\tau} Z_\tau^{1-\theta} d\tau - p_K^+ dI_1 + p_K^- dD_1\right] \quad (46)$$

subject to $K_{1+} = K_{1-} + dI_1 - dD_1 \left(= e^{-\delta\Delta t} K_{0+} + dI_1 - dD_1\right)$, K_{1-}: given.

Defining $A_y \equiv A_\pi \theta / \left[(\gamma + \delta\theta) - (1-\theta)(\mu_Z - \theta\sigma_Z^2/2)\right]$, $y_1 \equiv Z_1 / K_{1+}$,

$y_1^+ \equiv \left(p_K^+ / A_y\right)^{1/(1-\theta)}$, and $y_1^- \equiv \left(p_K^- / A_y\right)^{1/(1-\theta)}$, we can rewrite equation (46) as follows.

---

should equal to zero, i.e., the optimal investment rule is $q^{\circ}=0$. When the expected PDV of net cash flow is used, the derivative of the expected PDV of net cash flow, called q, should equal the purchase price of capital for investment or the resale price of capital for disinvestment if the firm invests or disinvests.

$$\max_{\{dI,dD\}} E\left[\frac{A_y}{\theta} Z_1^{1-\theta} K_{1+}^{\theta} - p_K^+ dI_1 + p_K^- dD_1 \middle| Z_1\right] \tag{47}$$

Its solution is

- If $Z_1/K_{1-} > y_1^+$, then the firm should buy new capital such that $y_1 = y_1^+$, or $dI = (Z_1/y_1^+) - K_{1-}$.
- If $y_1^- \le Z_1/K_{1-} \le y_1^+$, then the firm should neither buy nor sell capital so that $K_{1+} = K_{1-}$.
- If $Z_1/K_{1-} < y_1^-$, then the firm should sell installed capital such that $y_1 = y_1^-$, or $dD = K_{1-} - (Z_1/y_1^-)$.

Here, y_1^+ and y_1^- are two critical values for the ratio, Z_1 / K_{1+}, in the second period. Then, after solving the first term in equation (45) and incorporating the above optimal investment for the second period, we can rewrite equation (45) as follows:

$$\begin{aligned}
V(K_{0+}, Z_0) = {} & A_\pi Z_0^{1-\theta} K_{0+}^{\theta} \Delta t \\
& + e^{-\gamma\Delta t} \int_0^{y_1^- K_{1-}} \left\{ \frac{A_y}{\theta} Z_1^{1-\theta} K_{1+}^{\theta} + p_K^- (K_{1-} - K_{1+}) \right\} dF(Z_1|Z_0) \\
& + e^{-\gamma\Delta t} \int_{y_1^- K_{1-}}^{y_1^+ K_{1-}} \frac{A_y}{\theta} Z_1^{1-\theta} K_{1-}^{\theta} dF(Z_1|Z_0) \\
& + e^{-\gamma\Delta t} \int_{y_1^+ K_{1-}}^{\infty} \left\{ \frac{A_y}{\theta} Z_1^{1-\theta} K_{1+}^{\theta} - p_K^+ (K_{1+} - K_{1-}) \right\} dF(Z_1|Z_0) \\
& - [K_{0+} > K_{0-}] p_K^+ (K_{0+} - K_{0-}) + [K_{0+} < K_{0-}] p_K^- (K_{0-} - K_{0+})
\end{aligned} \tag{48}$$

$$\text{subject to } K_{1+} = \begin{cases} Z_1/y_1^+ \text{, if } Z_1 > y_1^+ K_{1-} \\ K_{1-} \text{, if } y_1^+ K_{1-} \ge Z_1 \ge y_1^- K_{1-} \\ Z_1/y_1^- \text{, if } y_1^- K_{1-} > Z_1. \end{cases}$$

Here, $F(Z_1|Z_0)$ is the cumulative density function of Z_1 conditional on Z_0. Since K_{t-} is equal to $e^{-\delta\Delta t}K_{0+}$ and appears in the limits of the definite integral in equation (48), the firm's choice of capital stock in the first period affects not only the net cash flow for the first period but also the expectation of net cash flow for the second period. This is how the first period decision and the second period decision are linked.

A-2. Values of Future Investment and Disinvestment

From equation (48), we can derive the value of future disinvestment as the put option, $P(K_0, Z_0)$, and the value of future investment as the call option, $C(K_0, Z_0)$ by comparing equation (48) with the so-called naive case in which the firm assumes zero second period investment when it makes its first period investment decision. By rewriting equation (48), we have the following equation.

$$
\begin{aligned}
V(K_{0+}, Z_0) = {} & A_\pi Z_0^{1-\theta} K_{0+}^{\theta} \Delta t + e^{-(\gamma+\delta\theta)\Delta t} \int_0^\infty \frac{A_y}{\theta} Z_1^{1-\theta} K_{0+}^{\theta} dF(Z_1 | Z_0) \\
& + e^{-\gamma\Delta t} \int_0^{y_1^- K_{1-}} \left\{ \left[\frac{A_y}{\theta} Z_1^{1-\theta} K_{1+}^{\theta} - p_K^- K_{1+} \right] - \left[\frac{A_y}{\theta} Z_1^{1-\theta} K_{1-}^{\theta} - p_K^- K_{1-} \right] \right\} dF(Z_1 | Z_0) \\
& + e^{-\gamma\Delta t} \int_{y_1^+ K_{1-}}^{\infty} \left\{ \left[\frac{A_y}{\theta} Z_1^{1-\theta} K_{1+}^{\theta} - p_K^+ K_{1+} \right] - \left[\frac{A_y}{\theta} Z_1^{1-\theta} K_{1-}^{\theta} - p_K^+ K_{1-} \right] \right\} dF(Z_1 | Z_0) \\
& - [K_{0+} > K_{0-}] p_K^+ (K_{0+} - K_{0-}) + [K_{0+} < K_{0-}] p_K^- (K_{0-} - K_{0+}) \qquad (49)
\end{aligned}
$$

$$
\begin{aligned}
& = G(K_{0+}, Z_0) + e^{-\gamma\Delta t} P(K_{0+}, Z_0) - e^{-\gamma\Delta t} C(K_{0+}, Z_0) \\
& \quad - [K_{0+} > K_{0-}] p_K^+ (K_{0+} - K_{0-}) + [K_{0+} < K_{0-}] p_K^- (K_{0-} - K_{0+})
\end{aligned}
$$

Here,

$$G(\cdot) = A_\pi Z_0^{1-\theta} K_{0+}^{\theta} \Delta t + e^{-(\gamma+\delta\theta)\Delta t} \int_0^\infty \frac{A_y}{\theta} Z_1^{1-\theta} K_{0+}^{\theta} dF(Z_1 | Z_0), \qquad (50a)$$

$$P(\cdot) = \int_0^{y_1^- K_{1-}} \left\{ \left[\frac{A_y}{\theta} Z_1^{1-\theta} K_{1+}^{\theta} - p_K^- K_{1+} \right] - \left[\frac{A_y}{\theta} Z_1^{1-\theta} K_{1-}^{\theta} - p_K^- K_{1-} \right] \right\} dF(Z_1 | Z_0), \qquad (50b)$$

and

$$C(\cdot) = \int_{y_1^+ K_{1-}}^{\infty} \left\{ - \left[\frac{A_y}{\theta} Z_{1+}^{1-\theta} K_{1+}^{\theta} - p_K^+ K_{1+} \right] + \left[\frac{A_y}{\theta} Z_1^{1-\theta} K_{1-}^{\theta} - p_K^+ K_{1-} \right] \right\} dF(Z_1 | Z_0). \qquad (50c)$$

$G(\cdot)$ is the PDV of the net cash flow for the so-called naive case. $P(\cdot)$ is the value of future disinvestment as the put option. The first bracket of function P is the net cash flow from the capital stock after disinvestment, K_{1+}. The second bracket of function P is the net cash flow from the capital stock before disinvestment, K_{1-}. Therefore, the brace as a whole is the net gain from second period disinvestment. If the economic indicator variable at the beginning of the second period, Z_1, is below the striking value, $y_1^- K_{1-}$, the firm will disinvest. Otherwise, the firm will not disinvest so that the net gain is zero. Thus, function P is the expected net gain from second period disinvestment conditional upon information available at the beginning of the first period, Z_0. Similarly, if the economic indicator exceeds the striking price, $y_1^+ K_{1-}$, the firm invests. Otherwise, the firm does not invest. Function $C(\cdot)$ is equal to the expected net gain from second period investment conditional upon Z_0, and the value of future investment as the call option.

A-3. Optimal Investment for First Period

The optimal decision for first period investment is to choose K_{0+} to maximize $V(\cdot)$.

$$\max_{\{K_{0+}\}}\left[V\left(K_{0+}, Z_0\right)\right] \tag{51}$$

Equivalently, the optimal K_{0+} is determined from the derivatives of equation (49).

$$q^{\circ}\left(K_{0+}, Z_0\right)\left(\equiv \frac{\partial V}{\partial K_{0+}}\right) = 0$$

Or, by defining $N\left(K_{0+}, Z_0\right) = \partial G\left(K_{0+}, Z_0\right) / \partial K_{0+}$,

$$q^{\circ}\left(K_{0+}, Z_0\right) = N\left(K_{0+}, Z_0\right) + e^{-r\Delta t} P'\left(K_{0+}, Z_0\right) - e^{-r\Delta t} C'\left(K_{0+}, Z_0\right) - p_K^{+} = 0 \tag{52a}$$

for investment, or

$$q^{\circ}\left(K_{0+}, Z_0\right) = N\left(K_{0+}, Z_0\right) + e^{-r\Delta t} P'\left(K_{0+}, Z_0\right) - e^{-r\Delta t} C'\left(K_{0+}, Z_0\right) - p_K^{-} = 0 \tag{52b}$$

for disinvestment.

Letting $\Delta t = 1$, equations $N(\cdot)$, $P'(\cdot)$, and $C'(\cdot)$ can be expressed as follows:

$$N(\cdot) = A_y \left(\frac{Z_0}{K_{0+}}\right)^{1-\theta}, \tag{53a}$$

$$P'(\cdot) = e^{-2\delta}\frac{A_y}{\theta}\left(y_1^-\right)^{2-\theta}\left(1-e^{-2\delta\theta}\right)K_{0+} + e^{-\delta}p_K^-\Phi\left[\frac{\ln\left(y_1^- K_{0+}/Z_0\right)-\delta-\left(\mu_Z-\sigma_Z^2/2\right)}{\sigma_Z}\right]$$

$$-A_y\exp\left[(1-\theta)\left(\mu_Z-\frac{\theta\sigma_Z^2}{2}\right)-\delta\theta\right]\times \tag{53b}$$

$$\Phi\left[\frac{\ln\left(y_1^- K_{0+}/Z_0\right)-\delta-\left\{\mu_Z-(\theta-1/2)\sigma_Z^2\right\}}{\sigma_Z}\right]\left(\frac{Z_0}{K_{0+}}\right)^{1-\theta},$$

and

$$C'(\cdot) = e^{-2\delta}\frac{A_y}{\theta}\left(y_1^+\right)^{2-\theta}\left(1-e^{-2\delta\theta}\right)K_{0+} - e^{-\delta}p_K^+\left\{1-\Phi\left[\frac{\ln\left(y_1^+ K_{0+}/Z_0\right)-\delta-\left(\mu_Z-\sigma_Z^2/2\right)}{\sigma_Z}\right]\right\}$$

$$+A_y\exp\left[(1-\theta)\left(\mu_Z-\frac{\theta\sigma_Z^2}{2}\right)-\delta\theta\right]\times$$

$$\left\{1-\Phi\left[\frac{\ln\left(y_1^+ K_{0+}/Z_0\right)-\left\{\mu_Z-\delta-(\theta-1/2)\sigma_Z^2\right\}}{\sigma_Z}\right]\right\}\left(\frac{Z_0}{K_{0+}}\right)^{1-\theta}.$$

(53c)

When the depreciation is assumed to be zero as in ADEP (1996), the first terms in $P'(\cdot)$ and $C'(\cdot)$, which are functions of K_{0+} rather than Z_0 / K_{0+}, vanish. Then, all of three functions become functions of Z_0 / K_{0+}. By defining y_0 as Z_0 / K_{0+}, and $q(y_0)$ as $N(y_0) + e^{-\gamma}\{P'(y_0)-C'(y_0)\}$, equations (52a) and (52b) can be rewritten as

$$q^{\circ}(y_0) = \left\{N(y_0) - p_K^+\right\} + e^{-\gamma}\left\{P'(y_0) - C'(y_0)\right\} = 0, \text{ or } q(y_0) = p_K^+ \tag{54a}$$

for investment, and

$$q^{\circ}(y_0) = \left\{N(y_0) - p_K^-\right\} + e^{-\gamma}\left\{P'(y_0) - C'(y_0)\right\} = 0, \text{ or } q(y_0) = p_K^- \tag{54b}$$

for disinvestment.

Figure A.1 shows the functions $q(y)$, $N(y)$, $P'(y)$, and $C'(y)$. In figure A.1, a dashed line shows the function $N(y)$, which is concave and monotonically increasing. $N(y)$ is the so-called naive case and the same as the derivative of the expected PDV of net cash flow for the second period since the firm will not make any investment decision after $t = \Delta t$. The function $P'(y)$ appears as a convex function at the lower left corner. Its

y-intercept is close to but lower than p_K^- (= 1), and it is monotonically decreasing and approaches zero. The function $C'(y)$ appears as a concave function in the lower right part of the figure. It starts at the origin and remains close to zero until y approaches about 0.5. Then, it increases as y increases. the function $q(y)$ starts at the $P'(y)$'s y-intercept, and moves horizontally to the right. Then, it moves along $N(y)$, while $P'(y)$ and $C'(y)$ are close to zero. Finally, it moves away from $N(y)$ and crosses the purchase price line ($p_K^+ = 2$) from below. A bold part of $q(y)$ is relevant to optimal investment.

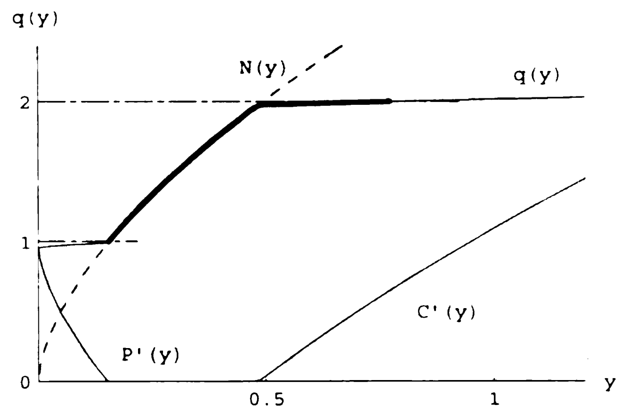


Figure A.1 $q(y)$, $N(y)$, $P'(y)$, and $C'(y)$ for Two Period Model

($\theta = 0.4$, $A_\pi = 0.29$, $\delta = 0$, $\gamma = 0.05$, $\mu_Z = 0.02$, $\sigma_Z = \mu_Z$, $p_K^+ = 2$, $p_K^- = 1$)

Because the firm anticipates high possibilities of future investment when the economy is booming, i.e., Z_0 is high, so is y, $C'(y)$ has higher values when y is high. On the other hand, when the economy is in slump, the firm contemplates disinvestment, so that $P'(y)$ is high when Z_0 is low. As the physical deprecation is assumed zero, $C'(y)$ is constant at zero for a low y, while $P'(y)$ is constant at zero for a high y. When the depreciation rate is non-zero, $C'(y)$ is constant at a positive level rather than zero for low y, while $P'(y)$ is constant at a positive level lower than $C'(y)$ for high y. Therefore, $q(y)$ shifts right when depreciation exists. In other words, optimal capital stock is lower when there is depreciation than when there is no depreciation.

Figures 12 and 13 show the optimal rule for investment and disinvestment, respectively. Optimal investment, given the purchase price of capital, p_K^+, is the amount that makes Z_0 / K_{0+} equals y_0^+, if Z_0 / K_{0-} exceeds y_0^+. As figure A.2 shows, the function q moves apart from $N(y)$ before it reaches the purchase price line. The critical value for $q(y)$, y_0^+, is higher than the critical value for $N(y)$, y_1^+, so that given Z_0, the firm should invest less when it considers the possibility of future investment than when it does not. The optimal disinvestment rule is that the firm should disinvest by the amount that makes Z_0 / K_{0+} equal to y_0^-, when Z_0 / K_{0-} is below y_0^-. Contrary to the optimal investment rule, as figure A.3 shows, $q(y)$ and $N(y)$ cross the resale price at the same point. In other words, the consideration of future disinvestment does not affect the optimal disinvestment decision in this case.

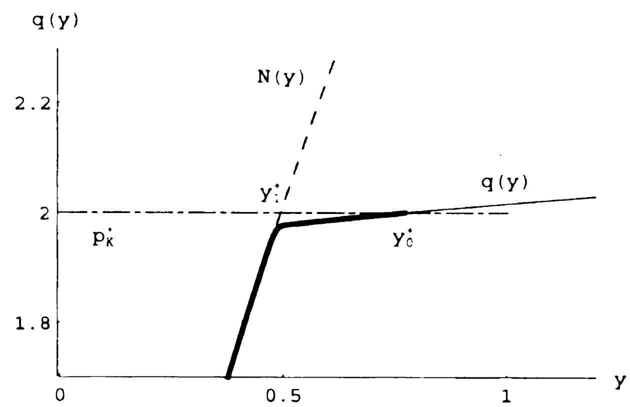


Figure A.2 Optimal Rule for Investment in Two Period Model

($y_1^+ \approx 0.497$, and $y_0^+ \approx 0.778$)

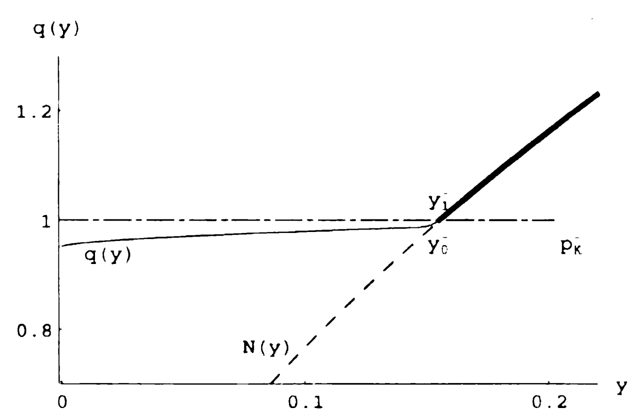


Figure A.3 Optimal Rule for Disinvestment in Two Period Model

($y_0^- \approx 0.158$, $y_1^- \approx 0.158$)

APPENDIX B

APPROXIMATION OF *G* SATISFYING *J(R,G)* = 0

This appendix derives the approximation of G satisfying $J(R,G) = 0$.

$$J(R,G) \equiv R\,h(G) - G^{1-\theta} h(G^{-1}) = 0 \tag{55}$$

Here,

$$h(x) \equiv \frac{1}{f(1-\theta)}\left\{1 - \frac{1-\theta}{\varphi_N} g(x) - \frac{1-\theta}{\varphi_P}[1 - g(x)]\right\},$$

$$g(x) \equiv \frac{x^{\varphi_P} - x^{1-\theta}}{x^{\varphi_P} - x^{\varphi_N}}, \text{ and } 1 - g(x) = \frac{x^{1-\theta} - x^{\varphi_N}}{x^{\varphi_P} - x^{\varphi_N}}.$$

G^* satisfies $J(R,G) = 0$, given R.

B-1. Approximation by Order of Exponents

This section derives an approximation by choosing a greater term from two terms in either the denominator or the numerator in equations $g(x)$ and $1 - g(x)$. Since $G > 1$ and $\varphi_N < 0 < 1 - \theta < 1 < \varphi_P$,

$$0 < G^{\varphi_N} < 1 < G^{1-\theta} < G^{\varphi_P}, \text{ and } \quad 0 < G^{-\varphi_P} < G^{-1+\theta} < 1 < G^{-\varphi_N}.$$

Therefore, by assuming that G^{φ_P} is much greater than $G^{1-\theta}$, $G^{-\varphi_N}$ is much greater than $G^{-1+\theta}$, we have

$$G^{\varphi_P} - G^{1-\theta} \approx G^{\varphi_P}, \quad G^{-\varphi_P} - G^{-\varphi_N} \approx -G^{-\varphi_N}, \text{ and } \quad G^{-1+\theta} - G^{-\varphi_N} \approx -G^{\varphi_N}.$$

Also, assuming that $G^{\varphi_N} \approx 0$, and $G^{-\varphi_P} \approx 0$,

$$G^{\varphi_P} - G^{\varphi_N} \approx G^{\varphi_P}, \quad G^{1-\theta} - G^{\varphi_N} \approx G^{1-\theta}, \text{ and } \quad G^{-\varphi_P} - G^{-1+\theta} \approx -G^{-1+\theta}.$$

Then,

$$g(G) \approx 1, \quad 1 - g(G) \approx \frac{G^{1-\theta}}{G^{\varphi_P}} = G^{1-\theta-\varphi_P} \approx 0,$$

$$g(G^{-1}) \approx \frac{G^{-1+\theta}}{G^{\varphi_N}} = G^{\varphi_N -1+\theta} \approx 0, \quad 1 - g(G^{-1}) \approx 1,$$

$$h(G) = \frac{1}{f(1-\theta)}\left\{1 - \frac{1-\theta}{\varphi_N}\right\}, \text{ and } \quad h(G^{-1}) = \frac{1}{f(1-\theta)}\left\{1 - \frac{1-\theta}{\varphi_P}\right\}.$$

Thus, equation (55) can be approximately written as

$$\frac{R}{f(1-\theta)} \frac{\varphi_N - 1 + \theta}{\varphi_N} - \frac{G^{1-\theta}}{f(1-\theta)} \frac{\varphi_P - 1 + \theta}{\varphi_P} = 0. \tag{56}$$

By rewriting equation (56), we have

$$G^* \approx \left[\left(\frac{\varphi_N - 1 + \theta}{\varphi_P - 1 + \theta}\right)\left(\frac{\varphi_P}{\varphi_N}\right) R\right]^{\frac{1}{1-\theta}}. \tag{57}$$

B-2. Approximation by Binomial Series

The Binomial series is

$$\begin{aligned}(1+x)^{\varphi} &= 1 + \varphi x + \frac{\varphi(\varphi-1)}{2!}x^2 + \frac{\varphi(\varphi-1)(\varphi-2)}{3!}x^3 + \cdots \\ &\approx 1 + \varphi x.\end{aligned} \tag{58}$$

Then,

$$G^{\varphi_P} \approx 1 + \varphi_P(G-1), \quad G^{1-\theta} \approx 1 + (1-\theta)(G-1), \quad G^{\varphi_N} \approx 1 + \varphi_N(G-1),$$
$$G^{-\varphi_P} \approx 1 - \varphi_P(G-1), \quad G^{-1+\theta} \approx 1 - (1-\theta)(G-1), \text{ and } \quad G^{-\varphi_N} \approx 1 - \varphi_N(G-1).$$

Therefore,

$$G^{\varphi_P} - G^{\varphi_N} \approx (\varphi_P - \varphi_N)(G-1), \quad G^{\varphi_P} - G^{1-\theta} \approx (\varphi_P - 1 + \theta)(G-1),$$
$$G^{1-\theta} - G^{\varphi_N} \approx (1 - \theta - \varphi_N)(G-1), \quad G^{-\varphi_P} - G^{-\varphi_N} \approx (-\varphi_P + \varphi_N)(G-1),$$
$$G^{-\varphi_P} - G^{-1+\theta} \approx (-\varphi_P + 1 - \theta)(G-1), \text{ and } \quad G^{-1+\theta} - G^{-\varphi_N} \approx (-1 + \theta + \varphi_N)(G-1).$$

Thus,

$$g(G) \approx \frac{\varphi_P - 1 + \theta}{\varphi_P - \varphi_N} \approx g(G^{-1}), \text{ and } \quad 1 - g(G) \approx \frac{1 - \theta - \varphi_N}{\varphi_P - \varphi_N} \approx 1 - g(G^{-1}), \text{ and}$$

$$h(G) \approx \frac{1}{f(1-\theta)}\left\{1 - \left(\frac{1-\theta}{\varphi_N}\right)\left(\frac{\varphi_P - 1 + \theta}{\varphi_P - \varphi_N}\right) - \left(\frac{1-\theta}{\varphi_P}\right)\left(\frac{1 - \theta - \varphi_N\theta}{\varphi_P - \varphi_N}\right)\right\} \approx h(G^{-1}).$$

As a result,

$$R - G^{1-\theta} = 0. \tag{59}$$

By rewriting equation (59), we have

$$G^* \approx R^{\frac{1}{1-\theta}}. \tag{60}$$

B-3. Refinement of Approximation

Figure B.1 shows approximation of G^* by equations (57) and (60), as well as the approximation proposed by Abel and Eberly (1996) and exact G^*. Abel and Eberly suggest

$$G^* \approx 1 + \left[\frac{6\sigma_y^2}{(1-\theta)(\gamma+\delta)}\right]^{\frac{1}{3}} (R-1)^{\frac{1}{3}}. \tag{61}$$

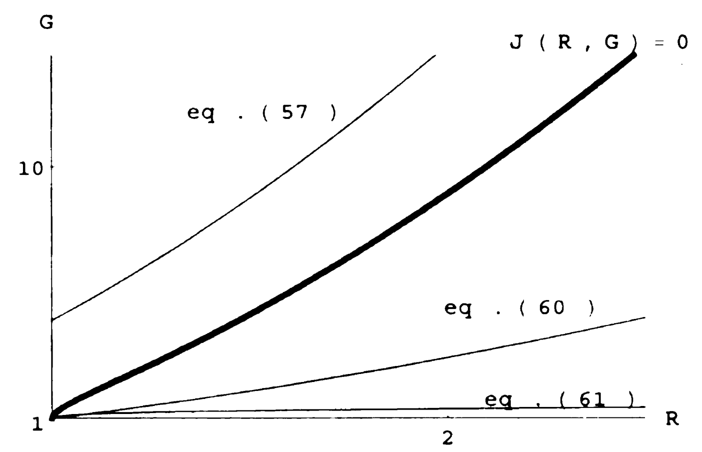


Figure B.1 Approximation of G^* (1)

(θ= 0.4, δ= 0.06, γ= 0.051, μ_Z = 0.05, σ_Z = 0.02)

Equation (57) seems a better fit, although G^* by equation (57) is not equal to unity at $R = 1$. By adding an auxiliary term to equate G^* to 1 at $R = 1$, we have

$$G^* \approx \left[\left(\frac{\varphi_N - 1 + \theta}{\varphi_P - 1 + \theta}\right)\left(\frac{\varphi_P}{\varphi_N}\right) R\right]^{\frac{1}{1-\theta}} - \left[\left(\frac{\varphi_N - 1 + \theta}{\varphi_P - 1 + \theta}\right)\left(\frac{\varphi_P}{\varphi_N}\right)\right]^{\frac{1}{1-\theta}} + 1, \tag{62}$$

or

$$G^* \approx \left[\left(\frac{\varphi_N - 1 + \theta}{\varphi_P - 1 + \theta}\right)\left(\frac{\varphi_P}{\varphi_N}\right)(R-1) + 1\right]^{\frac{1}{1-\theta}}. \tag{63}$$

Figure B.2 shows G^* by equations (62) and (63) as well exact G^*. Equation (63) is a better fit than equation (62). Equation (63) is equation (16) in the main text.

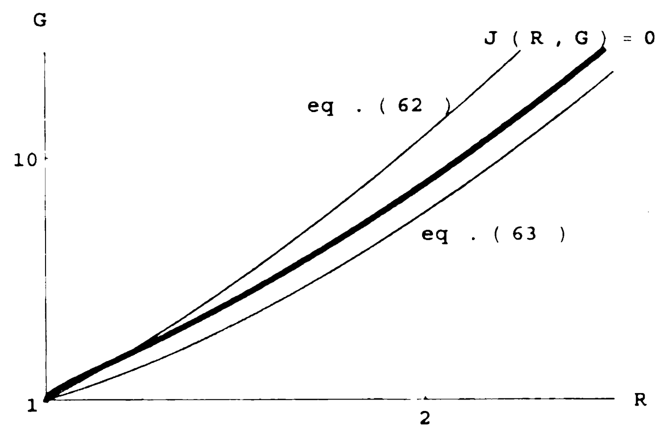


Figure B.2 Approximation of G^* (2)

($\theta = 0.4$, $\delta = 0.06$, $\gamma = 0.051$, $\mu_Z = 0.05$, $\sigma_Z = 0.02$)

APPENDIX C

FUNCTION *q* WITH AND WITHOUT FIXED COSTS AND RANGE OF INACTION

This appendix discusses the coefficients of the function q, B_P and B_N. In particular, this appendix shows $B_p < B_p^* < 0 < B_N^* < B_N$ and $y_{Tr}^- < y^- < y^+ < y_{Tr}^+$, where an asterisk (*) indicates investment with fixed costs.

C-1. Coefficients of Function *q*, B_N and B_P

The function q is written as follows:

$$q = Hy^{1-\theta} + B_N y^{\varphi_N} + B_P y^{\varphi_P}. \qquad (64)$$

$$H > 0, \quad B_N > 0, \quad B_P < 0, \quad 0 < \theta < 1, \quad \varphi_N < 0, \quad \varphi_P > 1$$

Because of fixed costs, there are a triangular area below the resale price line and a crescent area above the purchase price line for the function q^* with fixed costs. On the other hand, the function q without fixed costs are tangential to those price lines. Therefore, the function q^* with fixed costs is below the function q without fixed costs for small y, while the function q^* is above the function q for large y. In other words, $q - q^* > 0$ for small y, while $q - q^* < 0$ for large y. The difference, $q - q^*$, can be written as follows:

$$q - q^* = \left(B_N - B_N^*\right)y^{\varphi_N} + \left(B_P - B_P^*\right)y^{\varphi_P}. \qquad (65)$$

Since $\varphi_N < 0$ and $\varphi_P > 1$, the first term of equation (65) dominates for small y, while the second term of equation (65) dominates for large y. Therefore, $B_N - B_N^* > 0$ to move the local minimum of the function q below the resale price line, and $B_P - B_P^* < 0$ to move the local maximum of the function q above the purchase price line. Thus, the coefficients of the function q has the following relationship.

$$B_p < B_p^{\bullet} < 0 < B_N^{\bullet} < B_N \tag{66}$$

C-2. Range of Inaction

The function q first moves downward, then turns upward, and, moves downward again, as y increases from zero. And, the boundary conditions for investment without fixed costs is

$$q'(y)\left[= (1-\theta)Hy^{-\theta} + \varphi_N B_N y^{\varphi_N - 1} + \varphi_P B_P y^{\varphi_P - 1}\right] = 0 \text{ at } y^- \text{ and } y^+. \tag{67}$$

Therefore, the derivative of q changes its sign from negative to positive to negative at the critical values, y^+ and y^-. As a result, $q'(y)$ is strictly positive for the range of inaction, (y^+, y^-). The derivative of the function $q^{\bullet}$ has the following relationship with the function q.

$$q^{\bullet}{}'(y) - q'(y) = \varphi_N\left(B_N^{\bullet} - B_N\right)y^{\varphi_N - 1} + \varphi_P\left(B_P^{\bullet} - B_P\right)y^{\varphi_P - 1} > 0 \quad \text{for } y > 0 \tag{68}$$

At y^- and y^+, $q^{\bullet}{}'(y) > 0$, since $q'(y) = 0$. Therefore, the positive slope of the function $q^{\bullet}$ is wider than that of the function q. In other words, by defining y_0^- and y_0^+ at which the function $q^{\bullet}$ has the local minimum and the local maximum, respectively, we have $y_0^- < y^- < y^+ < y_0^+$ since the slope of the function $q^{\bullet}$ is strictly positive between y^- and y^+. As a result, when investment has fixed costs, the local minimum for the function q moves to the left and down, while the local maximum for the function q moves to the right and upward. At the same time, at y_{Tr}^- and y_{Tr}^+, the function $q^{\bullet}$ is negatively sloped. Therefore, $y_{Tr}^- < y_0^- < y_0^+ < y_{Tr}^+$. Thus, we have $y_{Tr}^- < y^- < y^+ < y_{Tr}^+$. In other words, the range of inaction for investment with fixed costs, (y_{Tr}^-, y_{Tr}^+), is wider than the inaction range without fixed costs, (y^+, y^-).

APPENDIX D

Serially Correlated Error

This appendix discusses a serially correlated error in the econometric models, and Voung's test of model selection is revised in accordance with a serial correlation assumption. This appendix assumes that an error is an AR(1) process, i.e., $u_t = \rho u_{t-1} + v_t$, where $v_t \sim$ i.i.d. N[0, σ^2]. Discussion focuses on the GFM and OLS and concludes that the GFM is still better than OLS.

D-1. Voung's Test of Model Selection

With the serially correlated error, the likelihood function for the GFM becomes the following.

$$L[u_t | X_t, z_t, \eta', v_g] = \prod_{I,D} \frac{1}{\sigma} \phi\left(\frac{u_t - \rho u_{t-1}}{\sigma}\right) \times \prod_{O}\left[\Phi\left(\frac{X_t \eta' - z_t v_g}{\sigma}\right) - \Phi\left(\frac{X_t \eta'}{\sigma}\right)\right] \quad (69)$$

Here, $X_t = \{x_t, z_{t,noc}\}$ and $\eta' = \{\eta, v_f\}$. OLS, the censored and generalized Tobit models and the RFM in the analysis have their own likelihood functions similar to equation (69). This analysis uses estimates for u_t, η', v_f, and σ under the no-serial-correlation assumption, since those estimates are consistent. This depends upon the fact that the Tobit model under very general assumptions about u_t is consistent (Robinson, 1982). The estimator of ρ for this analysis is the following.

$$\hat{\rho} = \frac{\sum_{i=1}^{3or4} \sum_{t=2}^{T} \hat{u}_{it} \hat{u}_{i,t-1}}{\sum_{i=1}^{3or4} \sum_{t=2}^{T} \hat{u}_{i,t}^2} \quad (70)$$

For $t = 1$ and terms after zero investment observations, a term in equation (69), $u_t - \rho u_{t-1}$, is substituted with $(1 - \rho)u_t$, since we do not have u_{t-1}.

Table D.1 shows the log likelihood of five investment models. The GFM has the highest likelihood among the five models regardless of the assumptions about serial correlation. OLS is the second highest except the automobile industry under the no-serial-correlation assumption. But, for the automobile industry under the no-serial-correlation assumption, OLS is close to the G. Tobit model, which is the second best. We conduct the Likelihood Ratio (LR) test. Its hypothesis is that the serial correlation assumption and the no-serial-correlation assumption are equivalent, which implies that there is no serial correlation in error. The GFM fails to reject the hypothesis at the ten percent level in the computer and office equipment industry, at the five percent in the airline industry, and at the one percent level in the automobile industry. OLS and the RFM strongly reject the hypothesis in two industries, the automobile industry and the

Table D.1 Log Likelihood of Investment Models

	OLS	Tobit	G. Tobit	RFM	GFM
Computer and Office Equipment Industry					
No Serial Correlation	62.397	45.693	59.658	52.502	71.366
Serial Correlation	64.458	47.044	60.270	54.663	71.508
LR Test	4.112 (0.081)	2.702 (0.247)	1.224 (0.611)	4.322 (0.069)	0.284 (0.915)
Automobile Industry					
No Serial Correlation	73.004	43.434	74.923	57.450	89.875
Serial Correlation	78.912	45.918	76.616	62.795	92.338
LR Test	10.816 (0.000)	4.968 (0.039)	3.386 (0.148)	10.690 (0.000)	4.926 (0.040)
Airline Industry					
No Serial Correlation	81.545	40.702	68.770	66.292	97.547
Serial Correlation	91.830	46.968	69.914	76.684	99.554
LR Test	20.590 (0.000)	12.532 (0.000)	2.288 (0.327)	20.784 (0.000)	4.014 (0.089)

Ho for LR test: the serial correlation assumption and the no serial correlation assumption are equivalent.

p-value in parenthesis

airline industry. For the airline industry, the censored Tobit model also rejects the hypothesis. The GFM and the G Tobit model show higher p-values than the other three models.

Table D.2 shows Voung's test of model selection under the serial correlation assumption. Voung's test of model selection generally ranks the five models as follows:

GFM > OLS > G. Tobit > RFM > Tobit.

However, there is one exception, which is the G. Tobit model vs the RFM in the airline industry. Even though this case has an opposite sign, the difference between the two competing models is statistically insignificant. In addition, OLS is as good as the GFM

Table D.2 Model Selection under Serially Correlated Error

1. Computer and Office Equipment Industry

F_θ \ G_γ	OLS	Tobit	G. Tobit	RFM	GFM
OLS		1.1887	0.7705	0.7095	0.2570
Tobit	-2.062**		0.3553	1.1618	0.9699
G. Tobit	-0.616	26.451***		1.2263	0.4262
RFM	-1.501	0.913	-0.654		0.6586
GFM	1.795*	3.207***	2.223**	33.691***	

2. Automobile Industry

F_θ \ G_γ	OLS	Tobit	G. Tobit	RFM	GFM
OLS		1.2654	0.5495	0.9537	0.6463
Tobit	-3.885***		0.6726	1.1400	1.8953
G. Tobit	-0.410	61.396***		0.7479	0.4882
RFM	-2.186**	2.094**	-2.117**		0.7542
GFM	2.212**	4.466***	2.980***	59.085***	

3. Airline Industry

F_θ \ G_γ	OLS	Tobit	G. Tobit	RFM	GFM
OLS		0.8462	0.5286	0.6315	0.3999
Tobit	-5.594***		0.2626	0.8686	1.2781
G. Tobit	-3.458***	45.893***		0.8517	0.6338
RFM	-2.186**	3.658***	0.841		0.6545
GFM	1.401	5.336***	4.271***	45.741***	

Lower left: $2\times LR(\cdot)$ or $(NT)^{-1/2} LR(\cdot)/\hat{\omega}$; a positive value favors F_θ over G_γ.
Upper right: $\hat{\omega}^2$
*: reject Ho at 10 % **: reject Ho at 5 % ***: reject Ho at 1 %

in the airline industry. Although the GFM has a higher likelihood than OLS, the difference between the GFM and OLS is statistically insignificant.

D-2. Estimated ρ and Dutbin-Watson Test of OLS Residuals

Table D.3 shows the estimated ρ which are used for the maximum likelihood estimation under the serial correlation assumption. The GFM and the G. Tobit model show lower estimates, while OLS, the Tobit model and the RFM show higher estimates. The GFM ranks its estimated ρ as: the automobile industry > the airline industry > the computer and office equipment industry. And, OLS ranks the estimated ρ as: the airline industry > the automobile industry > the computer industry. These results are consistent with the test of serial correlation in the main test.

Table D.4 shows the Durbin-Watson (D-W)statistic for the OLS residuals. The D-W statistic is based upon the OLS residuals. The computer and office equipment industry shows the highest statistic, while the airline industry shows the lowest statistic. All of the three industries reject the no-serial-correlation hypothesis at five percent, since their D-W statistic is lower the corresponding critical value, $d_{L,5\%}$. The results are also compatible with the test of serial correlation.

Table D.3 Estimated Coefficient of AR(1) Process, ρ

Industry	OLS	Tobit	G. Tobit	RFM	GFM
Computer	0.2826	0.2915	0.1822	0.2854	0.0781
Automobile	0.5086	0.4918	0.2776	0.5042	0.3129
Airline	0.5563	0.5813	0.2255	0.5693	0.2514

Table D.4 Durbin-Watson Statistic for OLS Residuals

Industry	D-W Statistic	k', n	$d_{L,5\%}$	$d_{U,5\%}$
Computer	1.3527	6, 60	1.372	1.808
Automobile	0.9778	6, 57	1.334	1.814
Airline	0.8491	8, 76	1.399	1.867

D-3. Discussion

Even though this appendix incorporates serial correlation in error into the analysis, the GFM is still likely to be the best among the five models. The GFM residuals seem to be serially uncorrelated. On the other hand, OLS residuals seem to be serially correlated. In addition, OLS with serial correlation could be as good as the GFM when we look at the airline industry. However, there is one reason to conclude that OLS is biased and the GFM is better than OLS. Figure D.1 is a schematic diagram for comparison of the GFM with OLS. In figure D.1, solid lines represent the GFM, while a broken line represents OLS. As figure D.1 shows, OLS overestimates investment, while it underestimates disinvestment. At the same time, $\mathrm{Ln}(Z_t / K_t)$ is assumed to be an arithmetic Brownian motion, or a unit root process, so that, when it is high, it remains high for a while. Since the GFM is assumed to be consistent, observations are located around the GFM, and each GFM residual is either negative or positive regardless of $\mathrm{Ln}(Z_t / K_t)$. On the other hand, when an OLS residual is negative for high $\mathrm{Ln}(Z_t / K_t)$, the next OLS residual is likely to be negative because $\mathrm{Ln}(Z_t / K_t)$ likely remains high due to the unit root process. Thus, the OLS residuals are likely to remain negative as long as $\mathrm{Ln}(Z_t / K_t)$ is high. Therefore, the OLS residuals are serially correlated with positive ρ, while the GFM residuals are serially uncorrelated.

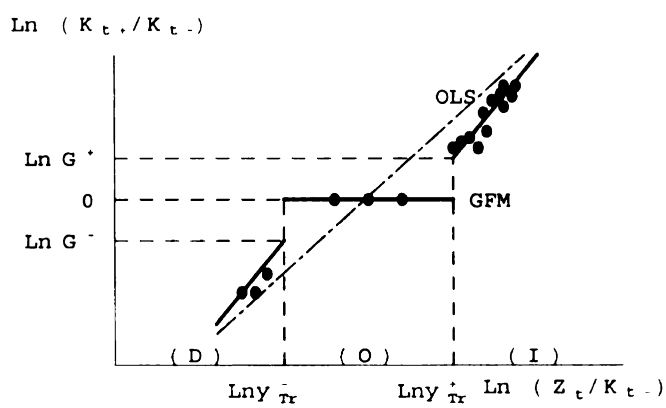


Figure D.1 GFM vs OLS

APPENDIX E

TWO STEP ESTIMATION FOR LIMITED DEPENDENT VARIABLE MODEL

In order to conduct the test of serial correlation, we estimate the limited dependent variable model by a two step estimation (TSE). This section shows its estimates. For comparison, the GFM estimates are reported in the same functional form. Both estimates seem to be close to each other.

For non-limited observations such as investment observations, k_t, their expected value, $E[k_t|x_t, z_t, k_t > 0]$, is written as the following.

$$\begin{aligned} E[k_t|x_t, z_t, k_t > 0] &= z_t v_g^+ + x_t\eta - z_{t,noc}v_f + E[u_t > -(z_t v_g^+ + x_t\eta - z_{t,noc}v_f)] \\ &= z_t v_g^+ + x_t\eta - z_{t,noc}v_f + \sigma\frac{\phi[(z_t v_g^+ + x_t\eta - z_{t,noc}v_f)/\sigma]}{\Phi[(z_t v_g^+ + x_t\eta - z_{t,noc}v_f)/\sigma]} \\ &= z_t v_g^+ + x_t\eta - z_{t,noc}v_f + \sigma\lambda[(z_t v_g^+ + x_t\eta - z_{t,noc}v_f)/\sigma] \end{aligned} \quad (71)$$

Here, ϕ and Φ are the p.d.f. and the c.d.f. of the standard normal distribution, respectively, and $\lambda(\cdot)(\equiv \phi(\cdot)/\Phi(\cdot))$ is the inverse Mills ratio. Disinvestment observations have a similar expected value. The two step estimation used in the analysis first estimates $\lambda(\cdot)$, and, then estimates coefficients in equation (71).

The first step of the two step estimation is a probit-type discrete regression model using all observations, and estimates the inverse Mills ratio, $\lambda(\cdot)$. The likelihood function for the model is the following:

$$\begin{aligned} L = &\prod_I (1 - \Phi[(-x_t\eta + z_{t,noc}v_f)/\sigma]) \\ &\times \prod_O (\Phi[(-x_t\eta + z_{t,noc}v_f)/\sigma] - \Phi[(-x_t\eta + z_{t,noc}v_f - z_t v_g)/\sigma]) \\ &\times \prod_D (\Phi[(-x_t\eta + z_{t,noc}v_f - z_t v_g)/\sigma]) \end{aligned} \quad (72)$$

The second step is OLS of the dependent variable, k_t, on the explanatory variables, x_t, $z_{t,mxc}$, and the estimated inverse Mills ration, $\hat{\lambda}$, using only investment observations and disinvestment observations. Although TSE's standard errors are reported in parentheses, they are invalid since the TSE is a regression on a generated regressor. Brackets in the following equations are an index variable which gives one if observation is disinvestment, or zero otherwise, i.e., [disinvestment] = 1 if an observation is disinvestment, 0 if an observation is investment. The TSE estimates and the GFM estimates are the following.

(a) Computer and Office Equipment Industry

TSE
$$\begin{aligned}\hat{k}_t = {} & \underset{(0.1438)}{0.5867}\, Re_t - \underset{(0.0653)}{0.3654}\, K_t - \underset{(0.0824)}{0.2426}\, (wL)_t \\ & - \underset{(0.3056)}{0.3874}\, d1 - \underset{(0.3093)}{0.4226}\, d2 - \underset{(0.3407)}{0.3272}\, d3 + \underset{(0.0063)}{0.0183}\, \gamma + \underset{(0.0410)}{0.0382}\, \hat{\lambda} \\ & - [\text{disinvestment}](\underset{(0.1742)}{0.2318}\, d1 + \underset{(0.1742)}{0.2318}\, d2 + \underset{(0.1742)}{0.2318}\, d3 + \underset{(0.0213)}{0.0382}\, \gamma)\end{aligned}$$

GFM
$$\begin{aligned}\hat{k}_t = {} & \underset{(0.1290)}{0.5586}\, Re_t - \underset{(0.0582)}{0.3508}\, K_t - \underset{(0.0740)}{0.2265}\, (wL)_t \\ & - \underset{(0.2797)}{0.3661}\, d1 - \underset{(0.2837)}{0.4108}\, d2 - \underset{(0.3121)}{0.3068}\, d3 + \underset{(0.0058)}{0.0178}\, \gamma \\ & - [\text{disinvestment}](\underset{(0.2603)}{0.0758}\, d1 + \underset{(0.2658)}{0.0758}\, d2 + \underset{(0.2950)}{0.0758}\, d3 + \underset{(0.0135)}{0.0260}\, \gamma)\end{aligned}$$
$$\hat{\sigma} = 0.0707\ (0.0066)$$

(b) Automobile Industry

TSE
$$\begin{aligned}\hat{k}_t = {} & \underset{(0.1762)}{0.1689}\, Re_t - \underset{(0.0260)}{0.0998}\, K_t - \underset{(0.2215)}{0.0525}\, (wL)_t \\ & - \underset{(0.4547)}{0.1801}\, d1 - \underset{(0.5085)}{0.2367}\, d2 - \underset{(0.5218)}{0.2042}\, d3 + \underset{(0.0044)}{0.0079}\, \gamma - \underset{(0.0229)}{0.0454}\, \hat{\lambda} \\ & - [\text{disinvestment}](- \underset{(0.1446)}{0.0999}\, d1 - \underset{(0.1446)}{0.0999}\, d2 - \underset{(0.1446)}{0.0999}\, d3 - \underset{(0.0085)}{0.0081}\, \gamma)\end{aligned}$$

GFM
$$\begin{aligned}\hat{k}_t = {} & \underset{(0.1659)}{0.1627}\, Re_t - \underset{(0.0241)}{0.1097}\, K_t - \underset{(0.2083)}{0.0245}\, (wL)_t \\ & - \underset{(0.4232)}{0.3198}\, d1 - \underset{(0.4738)}{0.3866}\, d2 - \underset{(0.4859)}{0.3607}\, d3 + \underset{(0.0042)}{0.0074}\, \gamma \\ & - [\text{disinvestment}](\underset{(0.4395)}{0.1775}\, d1 + \underset{(0.4902)}{0.1775}\, d2 + \underset{(0.5023)}{0.1775}\, d3 - \underset{(0.0091)}{0.0089}\, \gamma)\end{aligned}$$
$$\hat{\sigma} = 0.0458\ (0.0044)$$

(c) Airline Industry

TSE

$$\hat{k}_t = \underset{(0.1422)}{0.2788}\, Re_t - \underset{(0.0467)}{0.0952}\, K_t - \underset{(0.1133)}{0.1964}\, (wL)_t + \underset{(0.3578)}{0.1906}\, d1 + \underset{(0.3483)}{0.1399}\, d2$$
$$+ \underset{(0.3647)}{0.1465}\, d3 + \underset{(0.3161)}{0.2218}\, d4 - \underset{(0.1251)}{0.0086}\, dereg + \underset{(0.0044)}{0.0026}\, \gamma - \underset{(0.0197)}{0.0283}\, \hat{\lambda}$$
$$- [\text{disinvestment}](- \underset{(0.1203)}{0.0318}\, d1 - \underset{(0.1203)}{0.0318}\, d2 - \underset{(0.1203)}{0.0318}\, d3$$
$$- \underset{(0.1203)}{0.0318}\, d4 - \underset{(0.1203)}{0.0318}\, dereg + \underset{(0.0127)}{0.0132}\, \gamma)$$

GFM

$$\hat{k}_t = \underset{(0.1303)}{0.2450}\, Re_t - \underset{(0.0434)}{0.0963}\, K_t - \underset{(0.1043)}{0.1747}\, (wL)_t + \underset{(0.3243)}{0.3046}\, d1 + \underset{(0.3164)}{0.2448}\, d2$$
$$+ \underset{(0.3317)}{0.2541}\, d3 + \underset{(0.2873)}{0.3167}\, d4 + \underset{(34.672)}{0.4427}\, dereg + \underset{(0.0041)}{0.0031}\, \gamma$$
$$- [\text{disinvestment}](\underset{(0.3348)}{0.1333}\, d1 + \underset{(0.3268)}{0.1333}\, d2 + \underset{(0.3418)}{0.1333}\, d3$$
$$+ \underset{(0.2973)}{0.1333}\, d4 + \underset{(33.841)}{0.1333}\, dereg + \underset{(0.0107)}{0.0159}\, \gamma)$$
$$\hat{\sigma} = 0.0636 \ (0.0053)$$

For the three industries, both the TSE estimates and the GFM estimates seem to be close to each other.

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