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**TRANSIENT ANALYSIS OF PLANE WAVE
SCATTERING IN A LAYERED MEDIUM**

By

Jungwook Suk

A DISSERTATION

Submitted to
Michigan State University
in partial fulfillment of the requirements
for the degree of

DOCTOR OF PHILOSOPHY

Department of Electrical and Computer Engineering

2000

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ABSTRACT

TRANSIENT ANALYSIS OF PLANE WAVE SCATTERING IN A LAYERED MEDIUM

By

Jungwook Suk

The transient scattering of plane electromagnetic waves from a dispersive layered medium has been a difficult problem to solve, even though its frequency domain behavior is well known. In this study, analytical transient solutions for the electromagnetic waves scattered from a multi-layered medium excited at oblique incidence by a uniform plane wave are derived for both TE and TM polarizations. It is assumed that each layer has infinite width in space but finite thickness, and isotropic, homogeneous and frequency independent electrical parameters.

First, the time-domain reflection coefficient for a single interface in the medium is derived using the inverse Fourier transform of the frequency domain formulation. Then, the overall transient scattered field is found for a layered medium by combining the individual transient reflection coefficients using a series expansion and convolution integrals. The derived expressions are verified by comparison with data measured from laboratory experiments.

The results obtained in this study may be used as a basis for material parameter estimation by transient probing.

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To my Father and Mother

There are many
of all my graduates
has motivated me to
about to do my
this thesis. I have
to Dr. Dennis
work during my
Dr. Byron E. Cook
guidance and
I appreciate
and Chris C. Jones
study. From the
valuable experience
happy to study with
University.
Finally, I would
constant love and

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Finally, I would like to say my deep thanks to my parents and sister. Their constant love and care have always encouraged me through my life.

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30

Figure 2.11 Time-
value
2.50

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value
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Figure 2.14 Trans-
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 $\sigma_{n-1} =$

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Figure 3.3 Num-
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Figure 3.8 Time
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Figure 4.3 A d

Figure 4.4 Num.
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Figure 4.5 Num.
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Figure 4.6 Multip
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Figure 4.7 Multip
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 $\mu_2 =$
 $\epsilon_3 =$
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Figure 5.1 Experi

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Figure 5.4 (b) 1
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Figure 5.5 (a) 1
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Figure 5.5 (b) 1
plate

Figure 5.6 (a) 1
Sp

Figure 5.6 (b) 1
Sp

Figure 5.7 (a) 1
spectr

Figure 5.7 (b) 1
spectr

Figure 5.8 (a) 1
angle

Figure 5.8 (b) 1
angle

Figure 5.9 (a) 1
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Figure 5.9 (b) 1
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Figure 5.10 (a) 1
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det.

Figure 5.13 (b) T
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Figure 5.14 (a) C
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Figure 5.14 (b) C
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Figure 5.15 $-\epsilon^2$

Figure 5.16 Trial
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Figure 5.17 (a) T
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inc

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The topic of this
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and magnetic wave
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magnetic TM po

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problem has a wave
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is available. The
equation directly
Straton form
magnetic wave
used to solve the

CHAPTER 1

INTRODUCTION

The topic of this study is to find an analytical solution for the transient scattering of electromagnetic waves from a multi-layered medium, excited at oblique incidence by a uniform plane wave. The geometry for the structure of concern is shown in Figure 1.1. It is assumed that each layer of the multi-layered medium consists of a homogeneous, isotropical material, and has frequency independent permeability, permittivity and conductivity. Also, each layer is assumed to have infinite width, but finite thickness. The first layer is assumed to be lossless. Transient solutions are obtained for both polarizations of the incident plane wave, i.e. transverse electric (TE) and transverse magnetic (TM) polarization.

Although this research is motivated by a desire to estimate the parameters of each layer in the medium by using its time domain scattered field, the solution to this problem has a wide area of practical applications, such as industrial non-destructive inspection, geophysical probing and subsurface communication. Therefore, it is not surprising that there have been many studies on transient scattering from a lossy medium, or on related topics in the field of electromagnetics. While the frequency domain solution to this problem in terms of Fresnel's reflection coefficient is already well known, and can be found in many text books and papers, to the best of our knowledge the exact analytical transient solution for an arbitrary input waveform is not available. This is partly because of the difficulty in solving the time domain wave equation directly.

Stratton formulated the inverse Laplace transform pair for propagation of electromagnetic wave in a dispersive medium [1], and since then many researchers have tried to solve the scattering problem by formulating the solution in the frequency do-

main and then to
use certain types
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the problem due
to solve it [4]. [5]
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main and then finding the exact analytical inverse transform. Many of these studies use certain types of approximations; e.g., the diffusion approximation obtained by neglecting the displacement current term in Maxwell's equations [2], or restrictions on the incident angle and material parameters [3]. Other researchers have formulated the problem directly in the time domain, and have employed numerical techniques to solve it [4], [5]. Various contour integral techniques in the Laplace domain had been developed and closed form transient solutions for both polarizations had been obtained for a double exponential and unit step input waveform excitation using incomplete Lipschitz-Hankel integrals (ILHI's) in [6], [7]. Though these solutions are restricted to specific input waveforms and are valid for a single interface, they may have been the first closed form transient solutions to the problem. Therefore, the results derived in this study are verified by those previous works.

In this thesis, the problem is formulated in the frequency domain first, in the same manner as the previous studies. The constant offset value in the derived frequency domain reflection coefficient is then identified and subtracted from the original expression and its inverse Fourier transform is determined. Next, the reduced form of the original reflection coefficient is manipulated so as to use known inverse Fourier transform pairs. This method was developed for a single interface in [8]. This same approach is taken, but extended to treat the more general case of a single interface in a multi-layered medium. Proper branch cuts are derived and applied for this more rigorous solution. The results for a single interface reflection are used to derive the overall transient reflection from a multi-layered medium. Finally, the derived solutions are verified by laboratory experiments.

In the following chapter, the frequency domain reflection coefficients for a single interface in a multi-layered medium are formulated and classified for TE polarization. The inverse Fourier transform of each classified reflection coefficient is performed to obtain the transient reflection coefficient. The various aspects of the transient forms,

including causal
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obtain the trans
transient reflecti
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The descriptions
derived formulas

including causality problems, are discussed, and approximate forms, which might be useful in practical applications, are derived. The same procedures are taken to obtain the transient reflection coefficients for TM polarization in Chapter 3. The transient reflection coefficients for multi-layered medium are derived in Chapter 4 using series expansion in the frequency domain and applying the convolution theorem. The descriptions for procedures and results of experiments performed to verify the derived formulas are provided in Chapter 5.



Figure 1.1.
multi-layered medium

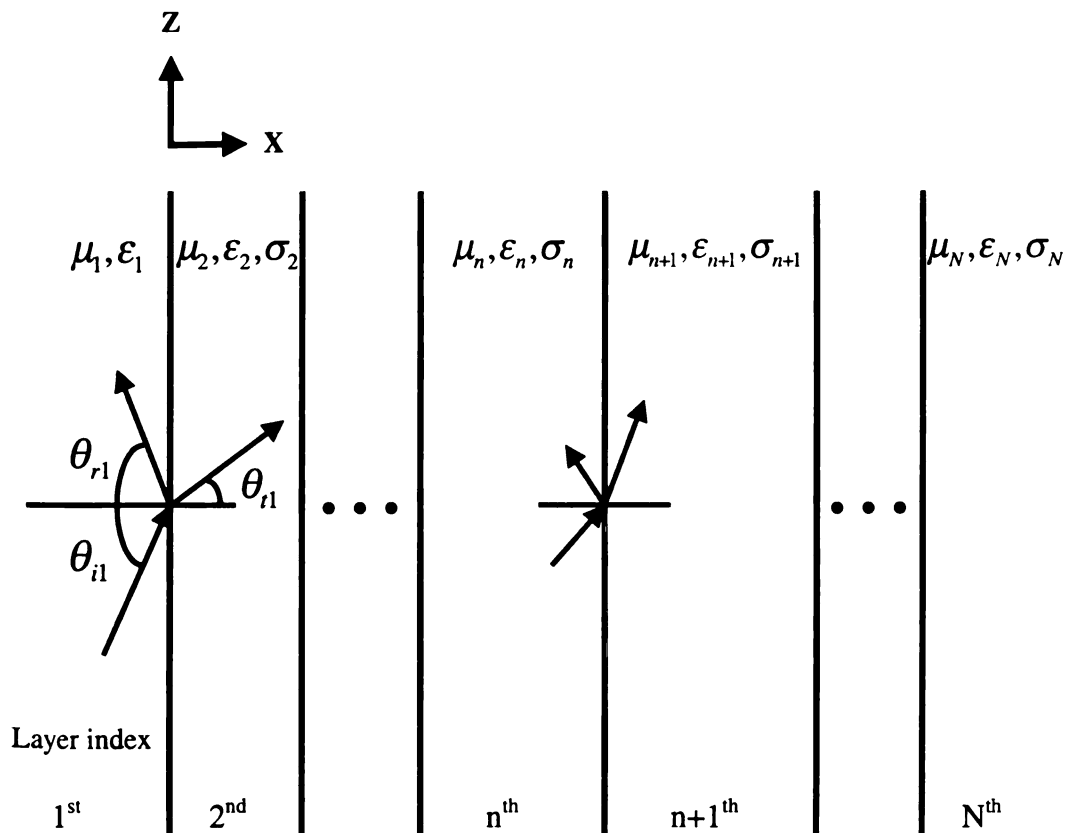


Figure 1.1. The geometry for analysis of transient plane wave scattering from a multi-layered medium.

INTERFERENCE

2.1 Introduction

The interfacial region

to incident wave

infinitely extended

transient scattering

stationary interference

forms of transition

form before observation

transient interference

coefficient, which is

form is obtained

some algebraic

reflection coefficient

2.2 Frequency

2.2.1 Derivation

Consider an incident

Figure 2.1. which

whole region

from region 1

CHAPTER 2

INTERFACIAL REFLECTION COEFFICIENTS FOR TE-POLARIZATION

2.1 Introduction

The interfacial reflection coefficient is defined as the ratio of reflected wave amplitude to incident wave amplitude at an interface between two layers, each of which has infinitely extended depth as shown in Figure 2.1. Because, in this study, the overall transient scattering from a multi-layered medium is derived from a combination of the individual interfacial coefficients for the layers (as discussed in chapter 4), the closed forms of transient interfacial reflection coefficients for TE-polarized plane waves are found before obtaining the total transient scattered field from the medium. To find the transient interfacial reflection coefficient, the frequency domain interfacial reflection coefficient, which is known as Fresnel's coefficient, is found first. Then the transient form is obtained using the inverse Fourier transform from a transform table after some algebraic manipulation. Finally, approximate forms of the transient interfacial reflection coefficients are derived.

2.2 Frequency Domain Formulation of Interfacial Reflection Coefficient

2.2.1 Derivation

Consider an interface between two homogeneous, isotropic materials as shown in Figure 2.1, where region n has time-independent constitutive parameters $(\mu_n, \epsilon_n, \sigma_n)$ while region $(n+1)$ has $(\mu_{n+1}, \epsilon_{n+1}, \sigma_{n+1})$. A plane wave is assumed to be incident from region 1 onto interface N between the two regions. From Maxwell's equations,

$$\nabla \times \bar{E}(x, z, \omega) = -j\omega\mu_n\bar{H}(x, z, \omega) \quad (2.1)$$

Region n

Reflected W

Incident

Figure 2.1. 1
Interface.

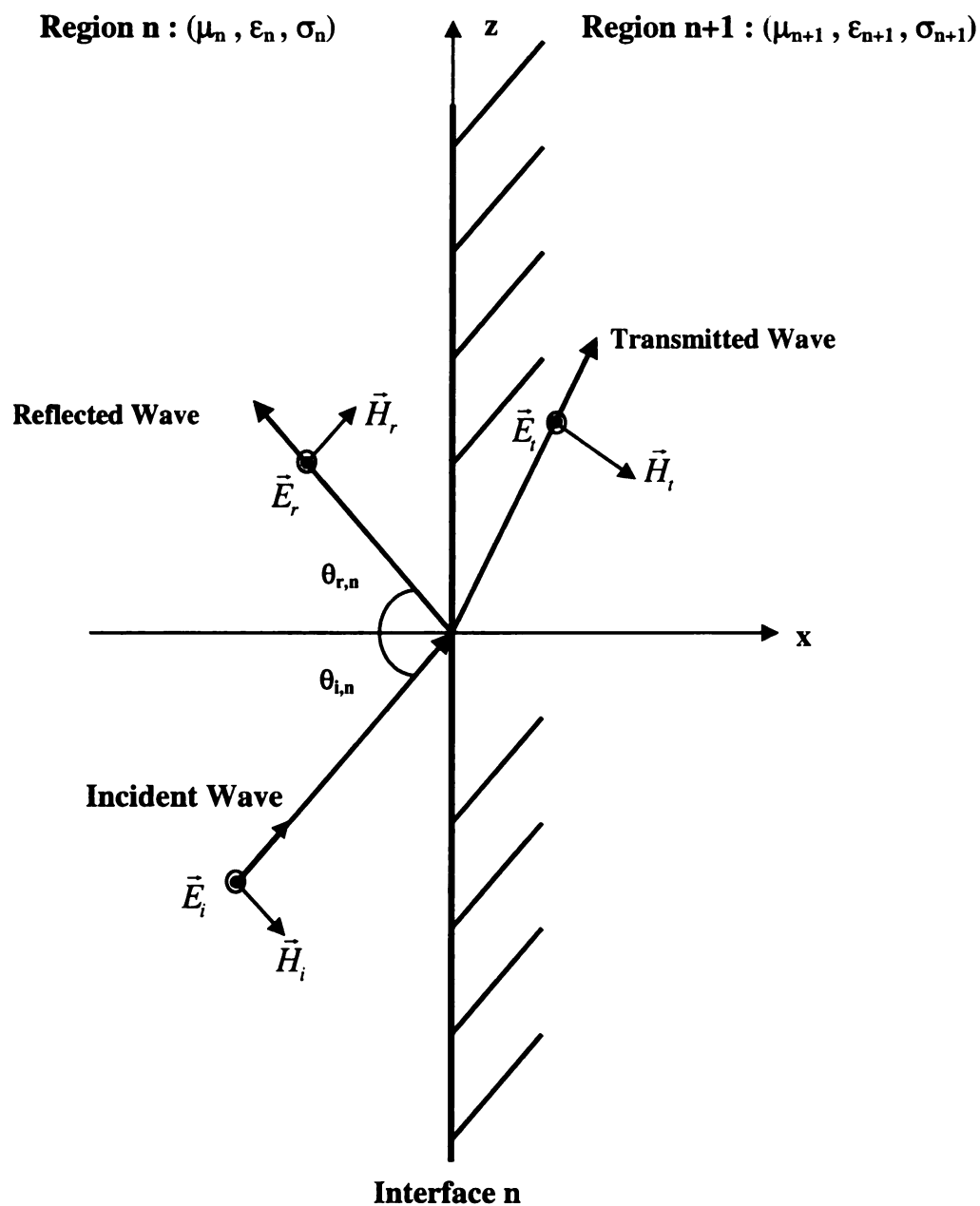


Figure 2.1. The incident, reflected and transmitted TE-polarized plane wave at an interface.

and the vector $\mathbf{H}$

where the propa-

For TE polarization

The solution for

where the comp-

Only if region n

will we speak of

only use $\mathbf{e}_1, \mathbf{e}_2$

$$\nabla \times \bar{H}(x, z, \omega) = (\sigma_n + j\omega\epsilon_n)\bar{E}(x, z, \omega) \quad (2.2)$$

and the vector Helmholtz equation for the electric field of region n is

$$\nabla^2 \bar{E}(x, z, \omega) - \gamma_n^2 \bar{E}(x, z, \omega) = 0 \quad (2.3)$$

where the propagation constant for the n-th layer is given by

$$\gamma_n^2 = j\omega\mu_n(\sigma_n + j\omega\epsilon_n) = -\omega^2\mu_n\epsilon_n + j\omega\mu_n\sigma_n. \quad (2.4)$$

For TE polarization $\bar{E} = \hat{y}E_y$ and (2.3) can be rewritten as

$$\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial z^2} \right) E_y(x, z, \omega) - \gamma_n^2 E_y(x, z, \omega) = 0. \quad (2.5)$$

The solution for the electric field is

$$\bar{E}(x, z, \omega) = \hat{y}E_o(\omega)e^{\gamma_{nx}x + \gamma_{nz}z} \quad (2.6)$$

where the components of the propagation constants are defined by

$$\gamma_n^2 = \gamma_{nx}^2 + \gamma_{nz}^2. \quad (2.7)$$

Only if region n is the first region of a multi-layered material and this region is lossless will we speak of an angle of incidence and an angle of reflection. Otherwise, we will only use $\gamma_{nx}^i, \gamma_{nz}^i, \gamma_{nx}^r$ and γ_{nz}^r . For the incident wave,

$$\gamma_{nx} = \gamma_{nx}^i = \gamma_n \cos \theta_{in} \quad (2.8)$$

$$\gamma_{nz} = \gamma_{nz}^i = \gamma_n \sin \theta_{in}, \quad (2.9)$$

and while for

Here θ_{ij} is the

21. The matrix

$\hat{H}$ is

In summary, the

$\hat{E}$ is

$\hat{H}$ is

$\hat{E}$ is

$\hat{H}$ is

In region

and, while for the reflected wave

$$\gamma_{nx} = \gamma_{nx}^r = -\gamma_n \cos \theta_{rn} \quad (2.10)$$

$$\gamma_{nz} = \gamma_{nz}^r = \gamma_n \sin \theta_{rn}. \quad (2.11)$$

Here $\theta_{i,n}$ is the angle of incidence and $\theta_{r,n}$ is the angle of reflection, as shown in Figure

2.1. The magnetic field in region n is obtained by

$$\begin{aligned} \bar{H}(x, z, \omega) &= -\frac{1}{j\omega\mu_n} \nabla \times \bar{E}(x, z, \omega) \\ &= -\frac{1}{j\omega\mu_n} \left\{ -\hat{x} \frac{\partial E_y}{\partial z} + \hat{z} \frac{\partial E_y}{\partial x} \right\} \\ &= -\frac{E_o(\omega)}{j\omega\mu_n} \left\{ -\hat{x} \gamma_{nz} e^{\gamma_{nx}x + \gamma_{nz}z} + \hat{z} \gamma_{nx} e^{\gamma_{nx}x + \gamma_{nz}z} \right\} \\ &= \hat{x} \frac{\gamma_{nz}}{j\omega\mu_n} E_o(\omega) e^{\gamma_{nx}x + \gamma_{nz}z} - \hat{z} \frac{\gamma_{nx}}{j\omega\mu_n} E_o(\omega) e^{\gamma_{nx}x + \gamma_{nz}z}. \end{aligned} \quad (2.12)$$

In summary, the fields in region n can be expressed as

$$\begin{aligned} \bar{E}_i(x, z, \omega) &= \hat{y} E_{io}(\omega) e^{\gamma_{nx}^i x + \gamma_{nz}^i z} \\ \bar{H}_i(x, z, \omega) &= \hat{x} \frac{\gamma_{nz}^i}{j\omega\mu_n} E_{io}(\omega) e^{\gamma_{nx}^i x + \gamma_{nz}^i z} - \hat{z} \frac{\gamma_{nx}^i}{j\omega\mu_n} E_{io}(\omega) e^{\gamma_{nx}^i x + \gamma_{nz}^i z} \end{aligned} \quad (2.13)$$

$$\begin{aligned} \bar{E}_r(x, z, \omega) &= \hat{y} E_{ro}(\omega) e^{\gamma_{nx}^r x + \gamma_{nz}^r z} \\ \bar{H}_r(x, z, \omega) &= \hat{x} \frac{\gamma_{nz}^r}{j\omega\mu_n} E_{ro}(\omega) e^{\gamma_{nx}^r x + \gamma_{nz}^r z} - \hat{z} \frac{\gamma_{nx}^r}{j\omega\mu_n} E_{ro}(\omega) e^{\gamma_{nx}^r x + \gamma_{nz}^r z}. \end{aligned} \quad (2.14)$$

In region (n+1), (2.3) is

$$\nabla^2 \bar{E}(x, z, \omega) - \gamma_{n+1}^2 \bar{E}(x, z, \omega) = 0 \quad (2.15)$$

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$$\gamma_{n+1}^2 = j\omega\mu_{n+1}(\sigma_{n+1} + j\omega\epsilon_{n+1}) = -\omega^2\mu_{n+1}\epsilon_{n+1} + j\omega\mu_{n+1}\sigma_{n+1}. \quad (2.16)$$

Then, the vector Helmholtz equation for the electric field in region (n+1) (i.e., the transmitted electric field) is given by

$$\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial z^2} \right) E_y(x, z, \omega) - \gamma_{n+1}^2 E_y(x, z, \omega) = 0. \quad (2.17)$$

This has the solution

$$\bar{E}_t(x, z, \omega) = \hat{y} E_{to}(\omega) e^{\gamma_{n+1,x}x + \gamma_{n+1,z}z} \quad (2.18)$$

where the components of propagation constants are defined by

$$\gamma_{n+1}^2 = \gamma_{n+1,x}^2 + \gamma_{n+1,z}^2. \quad (2.19)$$

When both regions n and (n+1) are lossless, we may define a transmission angle θ_t such that

$$\gamma_{n+1,x} = \gamma_{n+1} \cos \theta_t \quad (2.20)$$

$$\gamma_{n+1,z} = \gamma_{n+1} \sin \theta_t. \quad (2.21)$$

The transmitted magnetic field is given by

$$\begin{aligned} \bar{H}_t(x, z, \omega) &= -\frac{1}{j\omega\mu_{n+1}} \nabla \times \bar{E}_t(x, z, \omega) \\ &= -\frac{1}{j\omega\mu_{n+1}} \left\{ -\hat{x} \frac{\partial E_y}{\partial z} + \hat{z} \frac{\partial E_y}{\partial x} \right\} \\ &= -\frac{E_{to}(\omega)}{j\omega\mu_{n+1}} \left\{ -\hat{x} \gamma_{n+1,z} e^{\gamma_{n+1,x}x + \gamma_{n+1,z}z} + \hat{z} \gamma_{n+1,x} e^{\gamma_{n+1,x}x + \gamma_{n+1,z}z} \right\} \end{aligned}$$

In summary, the

$$\hat{E}_i(x, z, \omega) =$$

$$\hat{H}_i(x, z, \omega) =$$

To obtain the

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must be satisfied

$$= \hat{x} \frac{\gamma_{n+1,z}}{j\omega\mu_{n+1}} E_{to}(\omega) e^{\gamma_{n+1,x}x + \gamma_{n+1,z}z} - \hat{z} \frac{\gamma_{n+1,x}}{j\omega\mu_{n+1}} E_{to}(\omega) e^{\gamma_{n+1,x}x + \gamma_{n+1,z}z}. \quad (2.22)$$

In summary, the fields in region (n+1) can be expressed as

$$\begin{aligned} \bar{E}_t(x, z, \omega) &= \hat{y} E_{to}(\omega) e^{\gamma_{n+1,x}x + \gamma_{n+1,z}z} \\ \bar{H}_t(x, z, \omega) &= \hat{x} \frac{\gamma_{n+1,z}}{j\omega\mu_{n+1}} E_{to}(\omega) e^{\gamma_{n+1,x}x + \gamma_{n+1,z}z} - \hat{z} \frac{\gamma_{n+1,x}}{j\omega\mu_{n+1}} E_{to}(\omega) e^{\gamma_{n+1,x}x + \gamma_{n+1,z}z}. \end{aligned} \quad (2.23)$$

To obtain the ratio of the reflected and transmitted electric field amplitudes, two boundary conditions are applied. The first one is the boundary condition for continuity of the tangential components of the electric field at the interface. That is, for all z ,

$$\bar{E}_i(x = 0^-) + \bar{E}_r(x = 0^-) = \bar{E}_t(x = 0^+)$$

or,

$$E_{io}(\omega) e^{\gamma_{nz}^i z} + E_{ro}(\omega) e^{\gamma_{nz}^r z} = E_{to}(\omega) e^{\gamma_{n+1,z} z}. \quad (2.24)$$

Then, to satisfy the above boundary condition, the equations

$$\gamma_{nz}^i = \gamma_{nz}^r = \gamma_{n+1,z} \quad (2.25)$$

$$E_{io}(\omega) + E_{ro}(\omega) = E_{to}(\omega) \quad (2.26)$$

must be satisfied. For lossless materials, (2.25) becomes

$$\gamma_n \sin \theta_i = \gamma_n \sin \theta_r = \gamma_{n+1} \sin \theta_t \quad (2.27)$$

and thus

which are the wave
and considering

The second boundary
on the interface

or

$$\frac{A_1}{A_2} = \frac{Z_2}{Z_1}$$

Eq. 2.30

Now multiply

Therefore, the

and thus

$$\theta_i = \theta_r \quad (2.28)$$

$$\frac{\gamma_n}{\gamma_{n+1}} = \frac{\sin \theta_t}{\sin \theta_i} \quad (2.29)$$

which are the well-known Snell's law of reflection and refraction. From (2.7), (2.25) and considering the direction of propagation,

$$\gamma_{nx}^i = -\gamma_{nx}^r = \gamma_{nx}. \quad (2.30)$$

The second boundary condition requires the continuity of tangential magnetic field on the interface. That is, for all z ,

$$\bar{H}_{iz}(x = 0^-) + \bar{H}_{rz}(x = 0^-) = \bar{H}_{tz}(x = 0^+)$$

or,

$$-\frac{\gamma_{nx}^i}{j\omega\mu_n} E_{io}(\omega) e^{\gamma_{nz}^i z} - \frac{\gamma_{nx}^r}{j\omega\mu_n} E_{ro}(\omega) e^{\gamma_{nz}^r z} = -\frac{\gamma_{n+1,x}}{j\omega\mu_{n+1}} E_{to}(\omega) e^{\gamma_{n+1,z} z}. \quad (2.31)$$

Using (2.30), this equation simplifies to

$$\frac{\gamma_{nx}}{\mu_n} E_{io}(\omega) - \frac{\gamma_{nx}}{\mu_n} E_{ro}(\omega) = \frac{\gamma_{n+1,x}}{\mu_{n+1}} E_{to}(\omega). \quad (2.32)$$

Now, multiplying (2.26) by $\frac{\gamma_{n+1,x}}{\mu_{n+1}}$ and subtracting (2.32) yields

$$\left(\frac{\gamma_{n+1,x}}{\mu_{n+1}} - \frac{\gamma_{n,x}}{\mu_n} \right) E_{io}(\omega) + \left(\frac{\gamma_{n+1,x}}{\mu_{n+1}} + \frac{\gamma_{n,x}}{\mu_n} \right) E_{ro}(\omega) = 0. \quad (2.33)$$

Therefore, the frequency domain interfacial reflection coefficient for TE-polarized

plane waves is

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This is a general

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2.2.2 Branch

Because (2.34) i

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holds across ϵ_0

It is assumed t

where θ_1 is th

the propagation

plane waves is

$$\begin{aligned}
 R^{TE}(\omega) = \frac{E_{ro}(\omega)}{E_{io}(\omega)} &= \left(\frac{\gamma_{nx}}{\mu_n} - \frac{\gamma_{n+1,x}}{\mu_{n+1}} \right) / \left(\frac{\gamma_{nx}}{\mu_n} + \frac{\gamma_{n+1,x}}{\mu_{n+1}} \right) \\
 &= \frac{\mu_{n+1}\gamma_{nx}(\omega) - \mu_n\gamma_{n+1,x}(\omega)}{\mu_{n+1}\gamma_{nx}(\omega) + \mu_n\gamma_{n+1,x}(\omega)}.
 \end{aligned} \tag{2.34}$$

This is a generalized form of the Fresnel's reflection coefficient for TE-polarized incident and reflect waves at an interface in a layered structure. Note that the form of $R^{TE}(\omega)$ is the same as that given in [1].

2.2.2 Branch-cuts

Because (2.34) includes square root functions of complex argument, proper branch-cuts (or, square root rules) must be set. To do this properly, it must be assumed that the interface is the n 'th interface in a multi-layered material. Since Snell's law (2.25) holds across each interface, we must have

$$\gamma_{n+1,z} = \gamma_{nz} = \dots = \gamma_{1z}. \tag{2.35}$$

It is assumed that region 1 is lossless and thus

$$\gamma_{1z} = \omega^2 \mu_1 \epsilon_1 \sin \theta_{i1} \tag{2.36}$$

where θ_{i1} is the incident angle of the plane wave in region 1. The x -components of the propagation constants are

$$\begin{aligned}
 \gamma_{nx}(\omega) &= \sqrt{\gamma_n^2 - \gamma_{nz}^2} \\
 \gamma_{n+1,x}(\omega) &= \sqrt{\gamma_{n+1}^2 - \gamma_{n+1,z}^2}.
 \end{aligned} \tag{2.37}$$

Tr-

The condition
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propagation con-
The z-component
 $Re\{\omega_z\} = 0$. H
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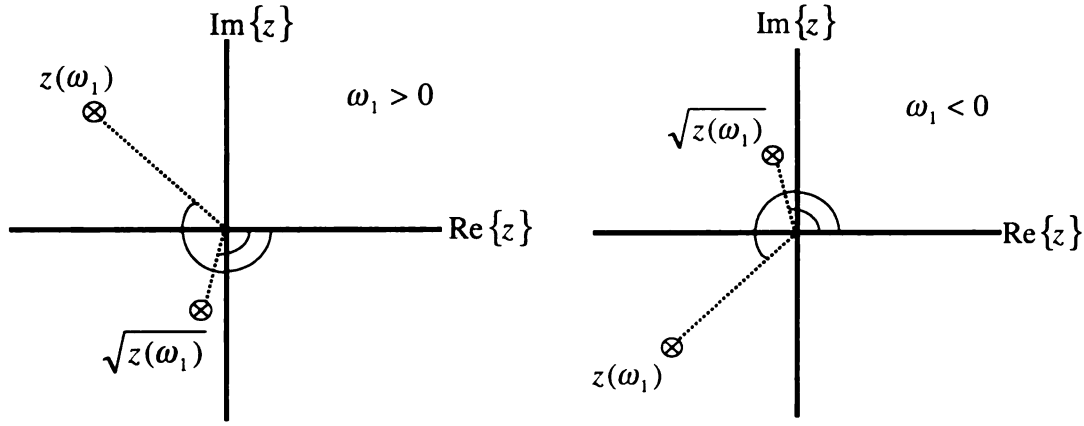
Using (2.35), this becomes

$$\begin{aligned}
\gamma_{nx}(\omega) &= \sqrt{\gamma_n^2 - \gamma_{1z}^2} \\
&= \sqrt{(-\omega^2 \mu_n \epsilon_n + j\omega \mu_n \sigma_n) - (-\omega^2 \mu_1 \epsilon_1 \sin^2 \theta_{i1})} \\
&= \sqrt{-\omega^2 (\mu_n \epsilon_n - \mu_1 \epsilon_1 \sin^2 \theta_{i1}) + j\omega \mu_n \sigma_n} \\
\gamma_{n+1,x}(\omega) &= \sqrt{\gamma_{n+1,z}^2 - \gamma_{1z}^2} \\
&= \sqrt{-\omega^2 (\mu_{n+1} \epsilon_{n+1} - \mu_1 \epsilon_1 \sin^2 \theta_{i1}) + j\omega \mu_{n+1} \sigma_{n+1}}. \tag{2.38}
\end{aligned}$$

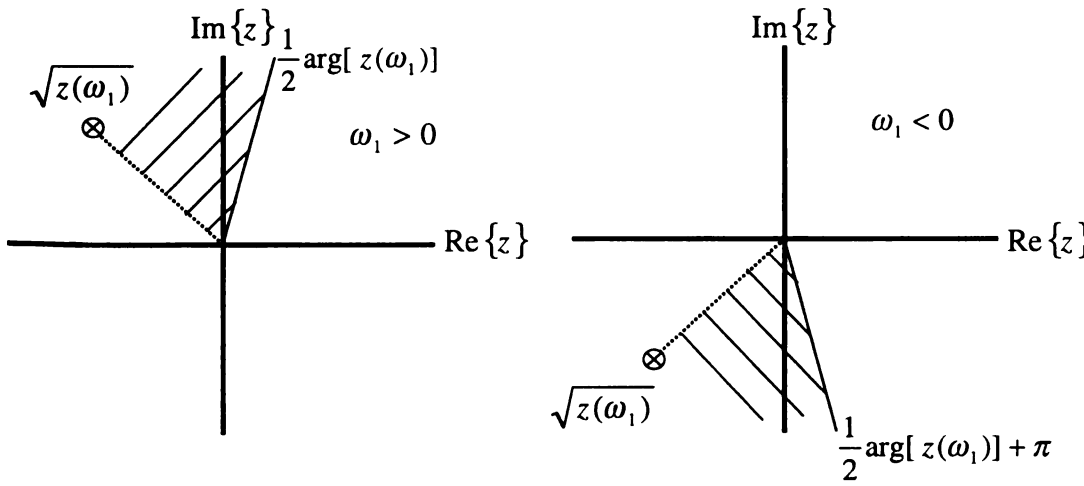
The condition that region 1 must be lossless confines the locations of the resulting propagation constants on the complex plane, and contributes to the definition of the branch cuts. The condition for setting branch cuts is the radiation condition, which says that, by the energy conservation law [9], an isolated wave cannot increase during its propagation. To satisfy this condition, the real part of each component of the propagation constant must not be positive for both positive and negative frequency. The z -component always satisfies this condition since $Re\{\gamma_{nz}\} = Re\{\gamma_{n-1,z}\} = \dots = Re\{\gamma_{1z}\} = 0$. However, two different branch cuts must be applied to $\gamma_{n,x}$, since the imaginary part inside of the square root has two different signs according to the sign of the frequency variable ω , as shown in Figure 2.2 (a). The resulting branch cuts to evaluate a square root of complex value $\sqrt{z(\omega = \omega_1)}$ are given by

$$\begin{aligned}
\omega_1 > 0 &: \frac{1}{2}arg[z(\omega_1)] < \theta_B < arg[z(\omega_1)] \\
\omega_1 < 0 &: arg[z(\omega_1)] < \theta_B < \frac{1}{2}arg[z(\omega_1)] + \pi
\end{aligned} \tag{2.39}$$

where θ_B and $arg[z(\omega_1)]$ indicate the argument angles of a branch cut line and $z(\omega_1)$ respectively, and $0 \leq arg[z(\omega_1)] < 2\pi$. The branch cuts of (2.39) should be applied consistently through the entire derivation procedure, and Appendix A shows the operations of square root functions used in this study, which are obtained by applying



(a)



(b)

Figure 2.2. The branch cut setting. (a) Evaluation of $\sqrt{z(\omega_1)}$ and (b) allowed region of branch cuts.

the branch cut

2.2.3. Classification

The Fresnel's

some constant

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Note that $\hat{B}_i$ is

appropriate bound

and $\hat{B}_{i+1}$. All p

Denoting $s =$

and (2.34) may

Now, the frequency

forms according

layer parameters

used in the derivative

the branch cuts.

2.2.3 Classification of frequency domain coefficients

The Fresnel's coefficient given by (2.34) can be classified according to the signs of some constant values included within it. Therefore, layer constants may be defined as

$$\tilde{D}_n = (\mu_n \epsilon_n - \mu_1 \epsilon_1 \sin^2 \theta_{i1}) / \mu_n^2 \quad (2.40)$$

$$\tilde{B}_n = \sigma_n / \mu_n. \quad (2.41)$$

Note that $\tilde{B}_n$ is always positive. In order to compute the square root functions, the appropriate branch cuts must be applied. This depends on the signs of $\tilde{D}_n$, $\tilde{B}_n$, $\tilde{D}_{n+1}$ and $\tilde{B}_{n+1}$. All possibilities are considered next.

Denoting $s = j\omega$ the terms in (2.34) become

$$\begin{aligned} \frac{\gamma_{nx}}{\mu_n} &= \sqrt{s(\tilde{D}_n s + \tilde{B}_n)} \\ \frac{\gamma_{n+1,x}}{\mu_{n+1}} &= \sqrt{s(\tilde{D}_{n+1} s + \tilde{B}_{n+1})} \end{aligned}$$

and (2.34) may be rewritten as

$$R^{TE}(s) = \frac{\sqrt{s(\tilde{D}_n s + \tilde{B}_n)} - \sqrt{s(\tilde{D}_{n+1} s + \tilde{B}_{n+1})}}{\sqrt{s(\tilde{D}_n s + \tilde{B}_n)} + \sqrt{s(\tilde{D}_{n+1} s + \tilde{B}_{n+1})}}. \quad (2.42)$$

Now, the frequency domain reflection coefficient can be classified to four different forms according to which branch cut is needed. This depends on the signs of the layer parameters $\tilde{D}_n$ and $\tilde{D}_{n+1}$. The algebraic manipulations with the branch cuts used in the derivation processes are given by (A.1) and (A.2).

$$(1) \tilde{D}_{n-1} > 0 \text{ and}$$

$$R^T$$

Simplifying gives

$$(2) \tilde{D}_{n-1} < 0 \text{ and}$$

$$R^T E_{n-1}$$

Simplifying gives

$$(3) \tilde{D}_{n-1} > 0 \text{ and}$$

$$R^T E_{n-1}$$

Simplifying gives

(1) $\tilde{D}_{n+1} > 0$ and $\tilde{D}_n > 0$

$$R^{TE}(s) = \frac{-\sqrt{\tilde{D}_n}\sqrt{s}\sqrt{s + \frac{\tilde{B}_n}{\tilde{D}_n}} + \sqrt{\tilde{D}_{n+1}}\sqrt{s}\sqrt{s + \frac{\tilde{B}_{n+1}}{\tilde{D}_{n+1}}}}{-\sqrt{\tilde{D}_n}\sqrt{s}\sqrt{s + \frac{\tilde{B}_n}{\tilde{D}_n}} - \sqrt{\tilde{D}_{n+1}}\sqrt{s}\sqrt{s + \frac{\tilde{B}_{n+1}}{\tilde{D}_{n+1}}}}.$$

Simplifying gives

$$R^{TE}(s) = \frac{\sqrt{s + \frac{\tilde{B}_n}{\tilde{D}_n}} - \sqrt{\frac{\tilde{D}_{n+1}}{\tilde{D}_n}}\sqrt{s + \frac{\tilde{B}_{n+1}}{\tilde{D}_{n+1}}}}{\sqrt{s + \frac{\tilde{B}_n}{\tilde{D}_n}} + \sqrt{\frac{\tilde{D}_{n+1}}{\tilde{D}_n}}\sqrt{s + \frac{\tilde{B}_{n+1}}{\tilde{D}_{n+1}}}}. \quad (2.43)$$

(2) $\tilde{D}_{n+1} < 0$ and $\tilde{D}_n < 0$

$$R^{TE}(s) = \frac{\pm j\sqrt{|\tilde{D}_n|}\sqrt{s}\sqrt{s - \frac{\tilde{B}_n}{|\tilde{D}_n|}} \mp j\sqrt{|\tilde{D}_{n+1}|}\sqrt{s}\sqrt{s - \frac{\tilde{B}_{n+1}}{|\tilde{D}_{n+1}|}}}{\pm j\sqrt{|\tilde{D}_n|}\sqrt{s}\sqrt{s - \frac{\tilde{B}_n}{|\tilde{D}_n|}} \pm j\sqrt{|\tilde{D}_{n+1}|}\sqrt{s}\sqrt{s - \frac{\tilde{B}_{n+1}}{|\tilde{D}_{n+1}|}}}.$$

Simplifying gives

$$R^{TE}(s) = \frac{\sqrt{s - \frac{\tilde{B}_n}{|\tilde{D}_n|}} - \sqrt{\frac{|\tilde{D}_{n+1}|}{|\tilde{D}_n|}}\sqrt{s - \frac{\tilde{B}_{n+1}}{|\tilde{D}_{n+1}|}}}{\sqrt{s - \frac{\tilde{B}_n}{|\tilde{D}_n|}} + \sqrt{\frac{|\tilde{D}_{n+1}|}{|\tilde{D}_n|}}\sqrt{s - \frac{\tilde{B}_{n+1}}{|\tilde{D}_{n+1}|}}}. \quad (2.44)$$

(3) $\tilde{D}_{n+1} > 0$ and $\tilde{D}_n < 0$

$$R^{TE}(s) = \frac{\pm j\sqrt{|\tilde{D}_n|}\sqrt{s}\sqrt{s - \frac{\tilde{B}_n}{|\tilde{D}_n|}} + \sqrt{|\tilde{D}_{n+1}|}\sqrt{s}\sqrt{s + \frac{\tilde{B}_{n+1}}{|\tilde{D}_{n+1}|}}}{\pm j\sqrt{|\tilde{D}_n|}\sqrt{s}\sqrt{s - \frac{\tilde{B}_n}{|\tilde{D}_n|}} - \sqrt{|\tilde{D}_{n+1}|}\sqrt{s}\sqrt{s + \frac{\tilde{B}_{n+1}}{|\tilde{D}_{n+1}|}}}.$$

Simplifying gives

$$R^{TE}(s) = \frac{\sqrt{s - \frac{\tilde{B}_n}{|\tilde{D}_n|}} \mp j\sqrt{\frac{|\tilde{D}_{n+1}|}{|\tilde{D}_n|}}\sqrt{s + \frac{\tilde{B}_{n+1}}{|\tilde{D}_{n+1}|}}}{\sqrt{s - \frac{\tilde{B}_n}{|\tilde{D}_n|}} \pm j\sqrt{\frac{|\tilde{D}_{n+1}|}{|\tilde{D}_n|}}\sqrt{s + \frac{\tilde{B}_{n+1}}{|\tilde{D}_{n+1}|}}}. \quad (2.45)$$

$$+ \hat{D}_{\sigma\sigma} < 0$$

$$R^{+1}$$

Simplifying gives

In each of these

to $\alpha < 0$. Note

through the whole

22.4 Reduction

Observing 2.43

frequency domain

efficient by the

be integrable, i.e.

transform, the con

To make the

$$D$$

$$B$$

and take the limit

(4) $\tilde{D}_{n+1} < 0$ and $\tilde{D}_n > 0$

$$R^{TE}(s) = \frac{-\sqrt{|\tilde{D}_n|}\sqrt{s}\sqrt{s + \frac{\tilde{B}_n}{|\tilde{D}_n|}} \mp j\sqrt{|\tilde{D}_{n+1}|}\sqrt{s}\sqrt{s - \frac{\tilde{B}_{n+1}}{|\tilde{D}_{n+1}|}}}{-\sqrt{|\tilde{D}_n|}\sqrt{s}\sqrt{s + \frac{\tilde{B}_n}{|\tilde{D}_n|}} \pm j\sqrt{|\tilde{D}_{n+1}|}\sqrt{s}\sqrt{s - \frac{\tilde{B}_{n+1}}{|\tilde{D}_{n+1}|}}}. \quad (2.46)$$

Simplifying gives

$$R^{TE}(s) = \frac{\sqrt{s + \frac{\tilde{B}_n}{|\tilde{D}_n|}} \pm j\sqrt{\frac{|\tilde{D}_{n+1}|}{|\tilde{D}_n|}}\sqrt{s - \frac{\tilde{B}_{n+1}}{|\tilde{D}_{n+1}|}}}{\sqrt{s + \frac{\tilde{B}_n}{|\tilde{D}_n|}} \mp j\sqrt{\frac{|\tilde{D}_{n+1}|}{|\tilde{D}_n|}}\sqrt{s - \frac{\tilde{B}_{n+1}}{|\tilde{D}_{n+1}|}}}. \quad (2.47)$$

In each of these expressions the upper sign corresponds to $\omega > 0$ and lower sign does to $\omega < 0$. Note again that the branch cuts defined in section 2.2.2 have been used through the whole manipulation steps for the square root functions.

2.2.4 Reduction of the interfacial reflection coefficients

Observing (2.43)-(2.47), it is realized that there is a constant offset value inside of each frequency domain expression. Since we are supposed to find the transient reflection coefficient by the inverse Fourier transform, and the frequency domain function must be integrable (i.e. $\int_{-\infty}^{\infty} |R^{TE}(\omega)|^2 d\omega \leq \infty$) for the existence of the inverse Frouier transform, the constant offset value should be extracted first.

To make the (2.43)-(2.47) more readable, let

$$\begin{aligned} D_n &= \sqrt{\frac{|\tilde{D}_{n+1}|}{|\tilde{D}_n|}} = \sqrt{\frac{\mu_n^2}{\mu_{n+1}^2} \frac{|\mu_{n+1}\epsilon_{n+1} - \mu_1\epsilon_1 \sin^2 \theta_{i1}|}{|\mu_n\epsilon_n - \mu_1\epsilon_1 \sin^2 \theta_{i1}|}} \\ B_n &= \frac{\tilde{B}_n}{|\tilde{D}_n|} = \frac{\mu_n\sigma_n}{|\mu_n\epsilon_n - \mu_1\epsilon_1 \sin^2 \theta_{i1}|}. \end{aligned} \quad (2.48)$$

and take the limit values at infinite frequency. Then,

$\{D_{n-1} > 0\}$

$\{L_{n-1} < 0\}$

$\{L_{n-1} > 0\}$

$\{L_{n-1} < 0\}$

In these ex

$\lambda < 0$, N

R^2 L

(1) $\tilde{D}_{n+1} > 0$ and $\tilde{D}_n > 0$

$$R^{TE}(s) = \frac{\sqrt{s+B_n} - \sqrt{D_n}\sqrt{s+B_{n+1}}}{\sqrt{s+B_n} + \sqrt{D_n}\sqrt{s+B_{n+1}}}, \quad (2.49)$$

$$\lim_{\omega \rightarrow \infty} R^{TE}(s) = \frac{1 - \sqrt{D_n}}{1 + \sqrt{D_n}}. \quad (2.50)$$

(2) $\tilde{D}_{n+1} < 0$ and $\tilde{D}_n < 0$

$$R^{TE}(s) = \frac{\sqrt{s-B_n} - \sqrt{D_n}\sqrt{s-B_{n+1}}}{\sqrt{s-B_n} + \sqrt{D_n}\sqrt{s-B_{n+1}}}, \quad (2.51)$$

$$\lim_{\omega \rightarrow \infty} R^{TE}(s) = \frac{1 - \sqrt{D_n}}{1 + \sqrt{D_n}}. \quad (2.52)$$

(3) $\tilde{D}_{n+1} > 0$ and $\tilde{D}_n < 0$

$$R^{TE}(s) = \frac{\sqrt{s-B_n} \mp j\sqrt{D_n}\sqrt{s+B_{n+1}}}{\sqrt{s-B_n} \pm j\sqrt{D_n}\sqrt{s+B_{n+1}}}, \quad (2.53)$$

$$\lim_{\omega \rightarrow \infty} R^{TE}(s) = \frac{1 \mp j\sqrt{D_n}}{1 \pm j\sqrt{D_n}}. \quad (2.54)$$

(4) $\tilde{D}_{n+1} < 0$ and $\tilde{D}_n > 0$

$$R^{TE}(s) = \frac{\sqrt{s+B_n} \pm j\sqrt{D_n}\sqrt{s-B_{n+1}}}{\sqrt{s+B_n} \mp j\sqrt{D_n}\sqrt{s-B_{n+1}}}, \quad (2.55)$$

$$\lim_{\omega \rightarrow \infty} R^{TE}(s) = \frac{1 \pm j\sqrt{D_n}}{1 \mp j\sqrt{D_n}}. \quad (2.56)$$

In these expressions the upper sign corresponds to $\omega > 0$ and lower sign does to $\omega < 0$. Next, the obtained constant values have to be subtracted from the original $R^{TE}(s)$. Let's define $\tilde{R}^{TE}(s) = R^{TE}(s) - R^{TE}(s = \pm j\infty)$, then

$$(\dot{D}_{k+1} > 0)$$

Eliminating th.

$$\tilde{R}^{FE}_{(s)}$$

$$= \frac{2\lambda}{1 -}$$

$$= \frac{1 -}{1 -}$$

The $s = B_1$ term
of 2.58), giving

$$\tilde{R}^{FE}$$

(1) $\tilde{D}_{n+1} > 0$ and $\tilde{D}_n > 0$

$$\begin{aligned}\tilde{R}^{TE}(s) &= \frac{\sqrt{s+B_n} - \sqrt{D_n}\sqrt{s+B_{n+1}}}{\sqrt{s+B_n} + \sqrt{D_n}\sqrt{s+B_{n+1}}} - \frac{1 - \sqrt{D_n}}{1 + \sqrt{D_n}} \\ &= \frac{2\sqrt{D_n}}{1 + \sqrt{D_n}} \frac{\sqrt{s+B_n} - \sqrt{s+B_{n+1}}}{\sqrt{s+B_n} + \sqrt{D_n}\sqrt{s+B_{n+1}}}\end{aligned}\quad (2.57)$$

Eliminating the square roots in the denominator of (2.57), gives

$$\begin{aligned}\tilde{R}^{TE}(s) &= \frac{2\sqrt{D_n}}{1 + \sqrt{D_n}} \frac{\{\sqrt{s+B_n} - \sqrt{s+B_{n+1}}\}\{\sqrt{s+B_n} - \sqrt{D_n}\sqrt{s+B_{n+1}}\}}{(s+B_n) - D_n(s+B_{n+1})} \\ &= \frac{2\sqrt{D_n}}{(1-D_n)(1+\sqrt{D_n})} \times \\ &\quad \frac{(1+\sqrt{D_n})(s+B_n) - (1+\sqrt{D_n})\sqrt{s+B_n}\sqrt{s+B_{n+1}} - \sqrt{D_n}(B_n - B_{n+1})}{s + \left(\frac{B_n - D_n B_{n+1}}{1-D_n}\right)}.\end{aligned}\quad (2.58)$$

The $s + B_n$ term can be factored out of the first and second term in the numerator of (2.58), giving

$$\begin{aligned}\tilde{R}^{TE}(s) &= \frac{2\sqrt{D_n}}{1-D_n} \frac{(s+B_n) \left\{1 - \frac{\sqrt{s+B_n}\sqrt{s+B_{n+1}}}{(s+B_n)}\right\}}{s + \left(\frac{B_n - D_n B_{n+1}}{1-D_n}\right)} \\ &\quad - \frac{2D_n(B_n - B_{n+1})}{(1-D_n)(1+\sqrt{D_n})} \frac{1}{s + \left(\frac{B_n - D_n B_{n+1}}{1-D_n}\right)} \\ &= \frac{2B_n\sqrt{D_n}}{1-D_n} \frac{1}{s + \left(\frac{B_n - D_n B_{n+1}}{1-D_n}\right)} \left\{1 - \sqrt{\frac{s+B_{n+1}}{s+B_n}}\right\} \\ &\quad + \frac{2\sqrt{D_n}}{1-D_n} \frac{s}{s + \left(\frac{B_n - D_n B_{n+1}}{1-D_n}\right)} \left\{1 - \sqrt{\frac{s+B_{n+1}}{s+B_n}}\right\} \\ &\quad - \frac{2D_n(B_n - B_{n+1})}{(1-D_n)(1+\sqrt{D_n})} \frac{1}{s + \left(\frac{B_n - D_n B_{n+1}}{1-D_n}\right)}\end{aligned}\quad (2.59)$$

where (A.3) is

of (2.59). For

procedure must

0 and $\tilde{D}_n < 0$,

2. $\tilde{D}_{n+1} < 0$ and

$$\tilde{R}^{(i)}$$

3. $\tilde{D}_{n+1} > 0$,

$$\tilde{R}^{(i)}$$

4. $\tilde{D}_{n+1} < 0$,

$$\tilde{R}^{(i)}$$

where (A.3) is used. Known inverse Fourier transform pairs are available for all terms of (2.59). For the other cases, the quite similar steps are taken, except that the procedure must be done separately for $\omega > 0$ and $\omega < 0$ in the cases of $\tilde{D}_{n+1} > 0$ and $\tilde{D}_n < 0$, and $\tilde{D}_{n+1} < 0$ and $\tilde{D}_n > 0$. These are

(2) $\tilde{D}_{n+1} < 0$ and $\tilde{D}_n < 0$

$$\begin{aligned}
\tilde{R}^{TE}(s) &= \frac{\sqrt{s-B_n} - \sqrt{D_n}\sqrt{s-B_{n+1}}}{\sqrt{s-B_n} + \sqrt{D_n}\sqrt{s-B_{n+1}}} - \frac{1 - \sqrt{D_n}}{1 + \sqrt{D_n}} \\
&= -\frac{2B_n\sqrt{D_n}}{1-D_n} \frac{1}{s + \left(\frac{B_n-D_nB_{n+1}}{1-D_n}\right)} \left\{ 1 - \sqrt{\frac{s-B_{n+1}}{s-B_n}} \right\} \\
&\quad + \frac{2\sqrt{D_n}}{1-D_n} \frac{s}{s + \left(\frac{B_n-D_nB_{n+1}}{1-D_n}\right)} \left\{ 1 - \sqrt{\frac{s-B_{n+1}}{s-B_n}} \right\} \\
&\quad - \frac{2D_n(B_{n+1}-B_n)}{(1-D_n)(1+\sqrt{D_n})} \frac{1}{s + \left(\frac{B_n-D_nB_{n+1}}{1-D_n}\right)}. \tag{2.60}
\end{aligned}$$

(3) $\tilde{D}_{n+1} > 0$ and $\tilde{D}_n < 0$

$$\begin{aligned}
\tilde{R}^{TE}(s) &= \frac{\sqrt{s-B_n} \mp j\sqrt{D_n}\sqrt{s+B_{n+1}}}{\sqrt{s-B_n} \pm j\sqrt{D_n}\sqrt{s+B_{n+1}}} - \frac{1 \mp j\sqrt{D_n}}{1 \pm j\sqrt{D_n}} \\
&= \mp \frac{2jB_n\sqrt{D_n}}{1+D_n} \frac{1}{s - \left(\frac{B_n-D_nB_{n+1}}{1+D_n}\right)} \left\{ 1 - \sqrt{\frac{s+B_{n+1}}{s-B_n}} \right\} \\
&\quad \mp \frac{2j\sqrt{D_n}}{1+D_n} \frac{s}{s - \left(\frac{B_n-D_nB_{n+1}}{1+D_n}\right)} \left\{ 1 - \sqrt{\frac{s+B_{n+1}}{s-B_n}} \right\} \\
&\quad - \frac{2D_n(B_{n+1}+B_n)}{(1+D_n)(1 \mp j\sqrt{D_n})} \frac{1}{s - \left(\frac{B_n-D_nB_{n+1}}{1+D_n}\right)}. \tag{2.61}
\end{aligned}$$

(4) $\tilde{D}_{n+1} < 0$ and $\tilde{D}_n > 0$

$$\begin{aligned}
\tilde{R}^{TE}(s) &= \frac{\sqrt{s+B_n} \pm j\sqrt{D_n}\sqrt{s-B_{n+1}}}{\sqrt{s+B_n} \mp j\sqrt{D_n}\sqrt{s-B_{n+1}}} - \frac{1 \pm j\sqrt{D_n}}{1 \mp j\sqrt{D_n}} \\
&= \mp \frac{2jB_n\sqrt{D_n}}{1+D_n} \frac{1}{s + \left(\frac{B_n-D_nB_{n+1}}{1+D_n}\right)} \left\{ 1 + \sqrt{\frac{s-B_{n+1}}{s+B_n}} \right\}
\end{aligned}$$

and A_4, A_5

corresponds to

$$R^{EE}(s) = \pm$$

and is equivalent

depends on the

may be called a

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media has some

The physical r

discussed later

2.3 Deriva

2.3.1 The

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given by the

$$R^{EE}(s) = \pm$$

given as def

$$\text{in } \tilde{D}_{n-1} >$$

$$\begin{aligned} & \mp \frac{2j\sqrt{D_n}}{1+D_n} \frac{s}{s + \left(\frac{B_n - D_n B_{n+1}}{1+D_n}\right)} \left\{ 1 + \sqrt{\frac{s - B_{n+1}}{s + B_n}} \right\} \\ & + \frac{2D_n(B_{n+1} + B_n)}{(1+D_n)(1 \mp j\sqrt{D_n})} \frac{1}{s + \left(\frac{B_n - D_n B_{n+1}}{1+D_n}\right)}, \end{aligned} \quad (2.62)$$

and (A.4), (A.5), and (A.6) are used respectively. In these expressions the upper sign corresponds to $\omega > 0$ and lower sign does to $\omega < 0$.

$R^{TE}(s = \pm j\infty)$ is the reflection coefficient obtained at infinitely high frequency, and is equivalent to the coefficient when the medium has no conductivity. Therefore, it depends on the displacement current term only in the Maxwell's equation of (2.2), and may be called an asymptotic reflection coefficient, since the value of $R^{TE}(s)$ approaches this (or these) value(s). In contrast, $\tilde{R}^{TE}(s)$ exists only when at least one of the two media has some conductivity. Let's call it a reduced interfacial reflection coefficient. The physical meaning of the two reflection coefficients in transient analysis will be discussed later.

2.3 Derivation of Transient Interfacial Reflection Coefficients

2.3.1 The transient forms

The transient forms of the frequency domain interfacial reflection coefficients are given by the inverse Fourier transform. Because the asymptotic reflection coefficients $R^{TE}(s = \pm j\infty)$ are constants in the frequency domain, their transforms are simply given as delta-functions multiplied by the constants. Let's denote it by $R_\infty^{TE}(t)$. Then,

(1) $\tilde{D}_{n+1} > 0$ and $\tilde{D}_n > 0$

$$\begin{aligned} R_\infty^{TE}(t) &= F^{-1} \left\{ \lim_{\omega \rightarrow \infty} R^{TE}(s) \right\} \\ &= \frac{1 - \sqrt{D_n}}{1 + \sqrt{D_n}} \delta(t) \end{aligned} \quad (2.63)$$

$$(2) \hat{D}_{n-1} < 0$$

where F^{-1} denotes

$$(3) \hat{D}_{n-1} > 0$$

As shown by

to the sign of

where G^* denotes

unit step function

transform rate

where $a(t) = t$

with a delta function

Notice that r

be in the physical

(2) $\tilde{D}_{n+1} < 0$ and $\tilde{D}_n < 0$

$$\begin{aligned} R_{\infty}^{TE}(t) &= F^{-1} \left\{ \lim_{\omega \rightarrow \infty} R^{TE}(s) \right\} \\ &= \frac{1 - \sqrt{D_n}}{1 + \sqrt{D_n}} \delta(t) \end{aligned} \quad (2.64)$$

where F^{-1} denotes the inverse Fourier transform.

(3) $\tilde{D}_{n+1} > 0$ and $\tilde{D}_n < 0$

As shown by (2.54), there are two different asymptotic values in this case according to the sign of ω . Therefore, letting $G = \frac{1 - j\sqrt{D_n}}{1 + j\sqrt{D_n}}$ then (2.54) can be rewritten as

$$\lim_{\omega \rightarrow \infty} R^{TE}(\omega) = GU(\omega) + G^*U(-\omega) \quad (2.65)$$

where G^* denotes complex conjugate value of G , and $U(\omega)$ is a frequency domain unit step function defined by $U(\omega) = 1$ for $\omega > 0$ and $U(\omega) = 0$ for $\omega < 0$. From the transform table given in (B.2),

$$\begin{aligned} R_{\infty}^{TE}(t) &= F^{-1} \{ GU(\omega) + G^*U(-\omega) \} \\ &= \frac{1}{2}(G + G^*)\delta(t) + \frac{j}{2\pi} \frac{1}{t} * (G - G^*)\delta(t) \\ &= \frac{1 - D_n}{1 + D_n} \delta(t) + \frac{2\sqrt{D_n}}{\pi(1 + D_n)} \left\{ \frac{1}{t} * \delta(t) \right\} \end{aligned} \quad (2.66)$$

where $a(t) * b(t)$ is the convolution of $a(t)$ and $b(t)$. The convolution of a function with a delta function gives the function itself. Thus,

$$R_{\infty}^{TE}(t) = \frac{1 - D_n}{1 + D_n} \delta(t) + \frac{2\sqrt{D_n}}{\pi(1 + D_n)t}. \quad (2.67)$$

Notice that the transient reflection coefficient is a real valued function as it should be in the physical world.

4) $\tilde{D}_{\alpha\alpha} < 0$

The transition

with case (3)

The inverse

be found direct

following consi

Then for

1) $\tilde{D}_{\alpha\alpha} > 0$ a

2.59) is re-writ

This form has

and the inverse

(4) $\tilde{D}_{n+1} < 0$ and $\tilde{D}_n > 0$

The transient form for this case is easily obtained by applying the same procedure with case (3). Thus,

$$R_{\infty}^{TE}(t) = \frac{1 + D_n}{1 - D_n} \delta(t) - \frac{2\sqrt{D_n}}{\pi(1 + D_n)t}. \quad (2.68)$$

The inverse Fourier transforms of the reduced interfacial reflection coefficient can be found directly from the transform table given in Appendix B. Let's define the following constants

$$P_1 = (B_{n+1} + B_n)/2 \quad (2.69)$$

$$P_2 = (B_n - D_n B_{n+1})/(1 - D_n) \quad (2.70)$$

$$\beta = (B_{n+1} - B_n)/2. \quad (2.71)$$

Then for

(1) $\tilde{D}_{n+1} > 0$ and $\tilde{D}_n > 0$

(2.59) is rewritten as

$$\begin{aligned} \tilde{R}^{TE}(s) = & \frac{2B_n\sqrt{D_n}}{1 - D_n} \frac{1}{s + P_2} \left\{ 1 - \sqrt{\frac{s + B_{n+1}}{s + B_n}} \right\} \\ & + \frac{2\sqrt{D_n}}{1 - D_n} \frac{s}{s + P_2} \left\{ 1 - \sqrt{\frac{s + B_{n+1}}{s + B_n}} \right\} \\ & - \frac{2D_n(B_n - B_{n+1})}{(1 - D_n)(1 + \sqrt{D_n})} \frac{1}{s + P_2}. \end{aligned} \quad (2.72)$$

This form has two different inverse Fourier transforms according to the sign of P_2 , and the inverse Fourier transform for each term can be found in (B.3)-(B.4). Thus,

when $P_2 > 0$.

$$\begin{aligned} \hat{R}^{\text{FE}}(t) &= \left[\frac{2}{1} \right] \\ &= \frac{2\lambda}{1 -} \\ &+ \left\{ \right. \end{aligned}$$

where $I_r(t)$ is the

property of the

$$\hat{R}^{\text{FE}}(t) =$$

=

It is necessary

$$e^{-P_2 t} \int_0^t e^{-P_2 \tau} d\tau$$

When t is large
 $t \approx t$, since I .

when $P_2 > 0$,

$$\begin{aligned}
\tilde{R}^{TE}(t) &= \left[\frac{2B_n\sqrt{D_n}}{1-D_n} e^{-P_2t} u(t) + \frac{2\sqrt{D_n}}{1-D_n} e^{-P_2t} \{\delta(t) - P_2 u(t)\} \right] * \\
&\quad \left[-\beta e^{-P_1t} \{I_1(\beta t) + I_0(\beta t)\} u(t) \right] - \frac{2D_n(B_n - B_{n+1})}{(1-D_n)(1+\sqrt{D_n})} e^{-P_2t} u(t) \\
&= \frac{2\sqrt{D_n}}{1-D_n} e^{-P_2t} \delta(t) * (-\beta) e^{-P_1t} \{I_1(\beta t) + I_0(\beta t)\} u(t) \\
&\quad + \left\{ \frac{2B_n\sqrt{D_n}}{1-D_n} - \frac{2\sqrt{D_n}}{1-D_n} P_2 \right\} e^{-P_2t} u(t) * (-\beta) e^{-P_1t} \{I_1(\beta t) + I_0(\beta t)\} u(t) \\
&\quad - \frac{2D_n(B_n - B_{n+1})}{(1-D_n)(1+\sqrt{D_n})} e^{-P_2t} u(t) \tag{2.73}
\end{aligned}$$

where $I_n(t)$ is the first kind modified Bessel function of order n . Using the convolution property of the delta function, (2.73) becomes

$$\begin{aligned}
\tilde{R}^{TE}(t) &= \frac{(B_n - B_{n+1})\sqrt{D_n}}{1-D_n} e^{-P_1t} \{I_1(\beta t) + I_0(\beta t)\} u(t) \\
&\quad + \frac{2D_n\sqrt{D_n}(B_{n+1} - B_n)}{(1-D_n)^2} \frac{(B_n - B_{n+1})}{2} e^{-P_2t} u(t) * \\
&\quad e^{-P_1t} \{I_1(\beta t) + I_0(\beta t)\} u(t) - \frac{2D_n(B_n - B_{n+1})}{(1-D_n)(1+\sqrt{D_n})} e^{-P_2t} u(t) \\
&= \frac{(B_n - B_{n+1})\sqrt{D_n}}{1-D_n} e^{-P_1t} \{I_1(\beta t) + I_0(\beta t)\} u(t) \\
&\quad - \frac{(B_n - B_{n+1})^2 D_n \sqrt{D_n}}{(1-D_n)^2} e^{-P_2t} u(t) \int_0^t e^{-(P_1-P_2)x} \{I_1(\beta x) + I_0(\beta x)\} dx \\
&\quad + \frac{2D_n(B_{n+1} - B_n)}{(1-D_n)(1+\sqrt{D_n})} e^{-P_2t} u(t). \tag{2.74}
\end{aligned}$$

It is necessary to check whether the integral term from time domain convolution $e^{-P_2t} u(t) \int_0^t e^{-(P_1-P_2)x} \{I_1(\beta x) + I_0(\beta x)\} dx$ exists as $t \rightarrow \infty$. Let

$$I(t) = \int_0^t e^{-P_2t} e^{-(P_1-P_2)x} \{I_1(\beta x) + I_0(\beta x)\} dx.$$

When t is large, the integrand varies as $e^{-(P_1-P_2+\beta)x} e^{-P_2t} = e^{-P_2(t-x)} e^{-(\beta+P_1)x}$ for $x \approx t$, since $I_v(x) \approx \frac{e^x}{\sqrt{2\pi x}}$ for large x . When $P_2 > 0$, since $t - x > 0$ and $\beta + P_1 > 0$,

when $P_2 > 0$,

$$\begin{aligned}
\tilde{R}^{TE}(t) &= \left[\frac{2B_n\sqrt{D_n}}{1-D_n} e^{-P_2 t} u(t) + \frac{2\sqrt{D_n}}{1-D_n} e^{-P_2 t} \{\delta(t) - P_2 u(t)\} \right] * \\
&\quad [-\beta e^{-P_1 t} \{I_1(\beta t) + I_0(\beta t)\} u(t)] - \frac{2D_n(B_n - B_{n+1})}{(1-D_n)(1+\sqrt{D_n})} e^{-P_2 t} u(t) \\
&= \frac{2\sqrt{D_n}}{1-D_n} e^{-P_2 t} \delta(t) * (-\beta) e^{-P_1 t} \{I_1(\beta t) + I_0(\beta t)\} u(t) \\
&\quad + \left\{ \frac{2B_n\sqrt{D_n}}{1-D_n} - \frac{2\sqrt{D_n}}{1-D_n} P_2 \right\} e^{-P_2 t} u(t) * (-\beta) e^{-P_1 t} \{I_1(\beta t) + I_0(\beta t)\} u(t) \\
&\quad - \frac{2D_n(B_n - B_{n+1})}{(1-D_n)(1+\sqrt{D_n})} e^{-P_2 t} u(t) \tag{2.73}
\end{aligned}$$

where $I_n(t)$ is the first kind modified Bessel function of order n . Using the convolution property of the delta function, (2.73) becomes

$$\begin{aligned}
\hat{R}^{TE}(t) &= \frac{(B_n - B_{n+1})\sqrt{D_n}}{1-D_n} e^{-P_1 t} \{I_1(\beta t) + I_0(\beta t)\} u(t) \\
&\quad + \frac{2D_n\sqrt{D_n}(B_{n+1} - B_n)}{(1-D_n)^2} \frac{(B_n - B_{n+1})}{2} e^{-P_2 t} u(t) * \\
&\quad e^{-P_1 t} \{I_1(\beta t) + I_0(\beta t)\} u(t) - \frac{2D_n(B_n - B_{n+1})}{(1-D_n)(1+\sqrt{D_n})} e^{-P_2 t} u(t) \\
&= \frac{(B_n - B_{n+1})\sqrt{D_n}}{1-D_n} e^{-P_1 t} \{I_1(\beta t) + I_0(\beta t)\} u(t) \\
&\quad - \frac{(B_n - B_{n+1})^2 D_n \sqrt{D_n}}{(1-D_n)^2} e^{-P_2 t} u(t) \int_0^t e^{-(P_1-P_2)x} \{I_1(\beta x) + I_0(\beta x)\} dx \\
&\quad + \frac{2D_n(B_{n+1} - B_n)}{(1-D_n)(1+\sqrt{D_n})} e^{-P_2 t} u(t). \tag{2.74}
\end{aligned}$$

It is necessary to check whether the integral term from time domain convolution $e^{-P_2 t} u(t) \int_0^t e^{-(P_1-P_2)x} \{I_1(\beta x) + I_0(\beta x)\} dx$ exists as $t \rightarrow \infty$. Let

$$I(t) = \int_0^t e^{-P_2 t} e^{-(P_1-P_2)x} \{I_1(\beta x) + I_0(\beta x)\} dx.$$

When t is large, the integrand varies as $e^{-(P_1-P_2+\beta)x} e^{-P_2 t} = e^{-P_2(t-x)} e^{-(\beta+P_1)x}$ for $x \approx t$, since $I_v(x) \approx \frac{e^x}{\sqrt{2\pi x}}$ for large x . When $P_2 > 0$, since $t - x > 0$ and $\beta + P_1 > 0$,

the integrand does

When $P_2 < 0$,

$$\tilde{R}^E_{(t)} = \left[-\frac{2}{\gamma} \right]$$

$$= \frac{(B)}{}$$

$$\left\{ - \right.$$

$$= \frac{B}{}$$

This expression

However, it is

expression, a

$t < 0$.

$$\tilde{R}^E_{(t)}$$

Let $a = \gamma P$

$$R_{\left\{ \frac{\gamma}{\gamma} \right\}} >$$

the integrand decays exponentially, and thus the integral converges between $x = 0 \sim t$.

When $P_2 < 0$, the inverse Fourier transform of (2.72) is

$$\begin{aligned}
\tilde{R}^{TE}(t) &= \left[-\frac{2B_n\sqrt{D_n}}{1-D_n}e^{-P_2t}u(-t) + \frac{2\sqrt{D_n}}{1-D_n}e^{-P_2t}\{\delta(t) + P_2u(t)\} \right] * \\
&\quad [-\beta e^{-P_1t}\{I_1(\beta t) + I_0(\beta t)\}u(t)] + \frac{2D_n(B_n - B_{n+1})}{(1-D_n)(1+\sqrt{D_n})}e^{-P_2t}u(-t) \\
&= \frac{(B_n - B_{n+1})\sqrt{D_n}}{1-D_n}e^{-P_1t}\{I_1(\beta t) + I_0(\beta t)\}u(t) \\
&\quad \left\{ -\frac{2B_n\sqrt{D_n}}{1-D_n} + \frac{2\sqrt{D_n}}{1-D_n}P_2 \right\} (-\beta)e^{-P_2t}u(-t) * e^{-P_1t}\{I_1(\beta t) + I_0(\beta t)\}u(t) \\
&\quad + \frac{2D_n(B_n - B_{n+1})}{(1-D_n)(1+\sqrt{D_n})}e^{-P_2t}u(-t) \\
&= \frac{(B_n - B_{n+1})\sqrt{D_n}}{1-D_n}e^{-P_1t}\{I_1(\beta t) + I_0(\beta t)\}u(t) \\
&\quad + \frac{(B_n - B_{n+1})^2D_n\sqrt{D_n}}{(1-D_n)^2}e^{-P_2t} \int_{\max(t,0)}^{\infty} e^{-(P_1-P_2)x}\{I_1(\beta x) + I_0(\beta x)\}dx \\
&\quad + \frac{2D_n(B_n - B_{n+1})}{(1-D_n)(1+\sqrt{D_n})}e^{-P_2t}u(-t). \tag{2.75}
\end{aligned}$$

This expression apparently includes non-causal terms in its second and third terms. However, it is shown that these non-causal terms can be removed by reforming the expression, and there are only causal terms remaining in the result. That is, when $t < 0$,

$$\begin{aligned}
\tilde{R}^{TE}(t) &= \frac{(B_n - B_{n+1})^2D_n\sqrt{D_n}}{(1-D_n)^2}e^{-P_2t} \int_0^{\infty} e^{-(P_1-P_2)x}\{I_1(\beta x) + I_0(\beta x)\}dx \\
&\quad + \frac{2D_n(B_n - B_{n+1})}{(1-D_n)(1+\sqrt{D_n})}e^{-P_2t}u(-t) \tag{2.76}
\end{aligned}$$

Let $\alpha = (P_1 - P_2) = \frac{(1+D_n)(B_{n+1} - B_n)}{2(1-D_n)}$ and check the requirements that $Re\{\alpha\} - |Re\{\beta\}| > 0$ to use 6.611.4 of [10] given by

$$\int_0^{\infty} e^{-\alpha x} I_v(\beta x) dx = \frac{\beta^{-v} \left[\alpha - \sqrt{\alpha^2 - \beta^2} \right]^v}{\sqrt{\alpha^2 - \beta^2}}. \tag{2.77}$$

For $B_{n+1} > B_n$,

$$R\{a\} = R\{b\}$$

For $B_{n+1} < B_n$,

$$R\{a\} = R\{b\}$$

Therefore, to us

Now, consider t

B_{n+1} and thus

and thus $D_n >$

$$\int$$

v

and

gives

$$\tilde{R}^E(t) =$$

For $B_{n+1} > B_n$,

$$Re\{\alpha\} - |Re\{\beta\}| = \frac{(1 + D_n)(B_{n+1} - B_n)}{2(1 - D_n)} - \frac{(B_{n+1} - B_n)}{2} = \frac{D_n(B_{n+1} - B_n)}{1 - D_n}. \quad (2.78)$$

For $B_{n+1} < B_n$,

$$Re\{\alpha\} - |Re\{\beta\}| = \frac{(1 + D_n)(B_{n+1} - B_n)}{2(1 - D_n)} + \frac{(B_{n+1} - B_n)}{2} = \frac{(B_{n+1} - B_n)}{1 - D_n}. \quad (2.79)$$

Therefore, to use 6.611.4 of [10] $\begin{cases} B_{n+1} > B_n & \Leftrightarrow & 0 < D_n < 1 \\ B_{n+1} < B_n & \Leftrightarrow & D_n > 1 \end{cases}$ must be satisfied.

Now, consider the case $P_2 = \frac{B_n - D_n B_{n+1}}{1 - D_n} < 0$. Then $1 - D_n > 0 \Leftrightarrow B_n < D_n B_{n+1} < B_{n+1}$ and thus $0 < D_n < 1 \Leftrightarrow B_{n+1} > B_n$. Also, $1 - D_n < 0 \Leftrightarrow B_n > D_n B_{n+1} > B_{n+1}$ and thus $D_n > 1 \Leftrightarrow B_n > B_{n+1}$. Using,

$$\int_0^\infty e^{-\alpha x} I_1(\beta x) dx = \frac{1}{\beta} \left\{ \frac{\alpha}{\sqrt{\alpha^2 - \beta^2}} - 1 \right\} = \frac{(\alpha/\beta)}{\sqrt{\alpha^2 - \beta^2}} - \frac{1}{\beta}, \quad (2.80)$$

$$\begin{aligned} \sqrt{\alpha^2 - \beta^2} &= \left\{ \frac{(1 + D_n)^2 (B_{n+1} - B_n)^2}{4(1 - D_n)^2} - \frac{(B_{n+1} - B_n)^2}{4} \right\}^{\frac{1}{2}} \\ &= \frac{(B_{n+1} - B_n) \sqrt{D_n}}{1 - D_n} \end{aligned}$$

and

$$\int_0^\infty e^{-\alpha x} I_0(\beta x) dx = \frac{1}{\sqrt{\alpha^2 - \beta^2}} \quad (2.81)$$

gives

$$\tilde{R}^{TE}(t) = \frac{(B_n - B_{n+1})^2 D_n \sqrt{D_n}}{(1 - D_n)^2} e^{-P_2 t} \left\{ \frac{(\alpha/\beta)}{\sqrt{\alpha^2 - \beta^2}} - \frac{1}{\beta} + \frac{1}{\sqrt{\alpha^2 - \beta^2}} \right\}$$

$$= e^{-t}$$

$$= e^{-t}$$

Therefore, (2.7)

$$R^{TE}(t) =$$

$$=$$

$$=$$

where $\int_0^\infty$
that (2.83)

$$2 \tilde{D}_{n-1} <$$

$$\begin{aligned}
& + \frac{2D_n(B_n - B_{n+1})}{(1 - D_n)(1 + \sqrt{D_n})} e^{-P_2 t} u(-t) \\
= & e^{-P_2 t} \left[\left\{ \frac{1 - D_n}{(B_{n+1} - B_n)\sqrt{D_n}} \left(\frac{(1 + D_n)(B_{n+1} - B_n)}{2(1 - D_n)} \frac{2}{(B_{n+1} - B_n)} + 1 \right) \right. \right. \\
& \left. \left. - \frac{2}{(B_{n+1} - B_n)} \right\} \times \frac{(B_n - B_{n+1})^2 D_n \sqrt{D_n}}{(1 - D_n)^2} + \frac{2D_n(B_n - B_{n+1})}{(1 - D_n)(1 + \sqrt{D_n})} \right] \\
= & e^{-P_2 t} \left[\frac{2D_n(B_{n+1} - B_n)(1 - \sqrt{D_n})}{(1 - D_n)(1 - \sqrt{D_n})(1 + \sqrt{D_n})} + \frac{2D_n(B_n - B_{n+1})}{(1 - D_n)(1 + \sqrt{D_n})} \right] = 0.
\end{aligned} \tag{2.82}$$

Therefore, (2.75) consists of pure causal terms, and can be rewritten as, when $t > 0$

$$\begin{aligned}
\tilde{R}^{TE}(t) &= \frac{(B_n - B_{n+1})\sqrt{D_n}}{1 - D_n} e^{-P_1 t} \{I_1(\beta t) + I_0(\beta t)\} u(t) \\
&+ \frac{(B_n - B_{n+1})^2 D_n \sqrt{D_n}}{(1 - D_n)^2} e^{-P_2 t} u(t) \int_t^\infty e^{-(P_1 - P_2)x} \{I_1(\beta x) + I_0(\beta x)\} dx \\
= & \frac{(B_n - B_{n+1})\sqrt{D_n}}{1 - D_n} e^{-P_1 t} \{I_1(\beta t) + I_0(\beta t)\} u(t) \\
&+ \frac{(B_n - B_{n+1})^2 D_n \sqrt{D_n}}{(1 - D_n)^2} e^{-P_2 t} u(t) \int_0^\infty e^{-(P_1 - P_2)x} \{I_1(\beta x) + I_0(\beta x)\} dx \\
&- \frac{(B_n - B_{n+1})^2 D_n \sqrt{D_n}}{(1 - D_n)^2} e^{-P_2 t} u(t) \int_0^t e^{-(P_1 - P_2)x} \{I_1(\beta x) + I_0(\beta x)\} dx \\
= & \frac{(B_n - B_{n+1})\sqrt{D_n}}{1 - D_n} e^{-P_1 t} \{I_1(\beta t) + I_0(\beta t)\} u(t) \\
&- \frac{(B_n - B_{n+1})^2 D_n \sqrt{D_n}}{(1 - D_n)^2} e^{-P_2 t} u(t) \int_0^t e^{-(P_1 - P_2)x} \{I_1(\beta x) + I_0(\beta x)\} dx \\
&+ \frac{2D_n(B_{n+1} - B_n)}{(1 - D_n)(1 + \sqrt{D_n})} e^{-P_2 t} u(t)
\end{aligned} \tag{2.83}$$

where $\int_0^\infty e^{-(P_1 - P_2)x} \{I_1(\beta x) + I_0(\beta x)\} dx = \frac{2(1 - D_n)}{(B_{n+1} - B_n)\sqrt{D_n}}$ was used. Notice that (2.83) is exactly same with (2.74).

(2) $\tilde{D}_{n+1} < 0$ and $\tilde{D}_n < 0$

$$\tilde{R}^{TE}(s) = -\frac{2B_n\sqrt{D_n}}{1 - D_n} \frac{1}{s - P_2} \left\{ 1 - \sqrt{\frac{s - B_{n+1}}{s - B_n}} \right\}$$

When $P_1 > 0$,

$$\tilde{R}^{EE}_{it} =$$

=

=

and, when P_1

$$\tilde{R}^{EE}_{it} =$$

=

$$\begin{aligned}
& + \frac{2\sqrt{D_n}}{1-D_n} \frac{s}{s-P_2} \left\{ 1 - \sqrt{\frac{s-B_{n+1}}{s-B_n}} \right\} \\
& + \frac{2D_n(B_n-B_{n+1})}{(1-D_n)(1+\sqrt{D_n})} \frac{1}{s-P_2}.
\end{aligned} \tag{2.84}$$

When $P_2 > 0$, (2.84) has the inverse Fourier transform (see B.7 as

$$\begin{aligned}
\tilde{R}^{TE}(t) &= \left[\frac{2B_n\sqrt{D_n}}{1-D_n} e^{P_2 t} u(-t) + \frac{2\sqrt{D_n}}{1-D_n} e^{P_2 t} \{\delta(t) - P_2 u(-t)\} \right] * \\
& \quad \left[\beta e^{P_1 t} \{I_1(\beta t) - I_0(\beta t)\} u(-t) \right] - \frac{2D_n(B_n-B_{n+1})}{(1-D_n)(1+\sqrt{D_n})} e^{P_2 t} u(-t) \\
&= \frac{2B_n\sqrt{D_n}}{1-D_n} \beta e^{P_1 t} \{I_1(\beta t) - I_0(\beta t)\} u(-t) \\
& \quad + \beta \left\{ \frac{2B_n\sqrt{D_n}}{1-D_n} - P_2 \frac{2\sqrt{D_n}}{1-D_n} \right\} e^{P_2 t} u(-t) * e^{P_1 t} \{I_1(\beta t) - I_0(\beta t)\} u(-t) \\
& \quad - \frac{2D_n(B_n-B_{n+1})}{(1-D_n)(1+\sqrt{D_n})} e^{P_2 t} u(-t) \\
&= \frac{(B_{n+1}-B_n)\sqrt{D_n}}{1-D_n} e^{P_1 t} \{I_1(\beta t) - I_0(\beta t)\} u(-t) \\
& \quad + \frac{(B_{n+1}-B_n)^2 D_n \sqrt{D_n}}{(1-D_n)^2} e^{P_2 t} u(-t) \int_t^0 e^{(P_1-P_2)x} \{I_1(\beta x) - I_0(\beta x)\} dx \\
& \quad + \frac{2D_n(B_{n+1}-B_n)}{(1-D_n)(1+\sqrt{D_n})} e^{P_2 t} u(-t)
\end{aligned} \tag{2.85}$$

and, when $P_2 < 0$,

$$\begin{aligned}
\tilde{R}^{TE}(t) &= \left[-\frac{2B_n\sqrt{D_n}}{1-D_n} e^{P_2 t} u(t) + \frac{2\sqrt{D_n}}{1-D_n} e^{P_2 t} \{\delta(t) + P_2 u(t)\} \right] * \\
& \quad \left[\beta e^{P_1 t} \{I_1(\beta t) - I_0(\beta t)\} u(-t) \right] - \frac{2D_n(B_{n+1}-B_n)}{(1-D_n)(1+\sqrt{D_n})} e^{P_2 t} u(t) \\
&= \frac{(B_{n+1}-B_n)\sqrt{D_n}}{1-D_n} e^{P_1 t} \{I_1(\beta t) - I_0(\beta t)\} u(-t) \\
& \quad \left\{ -\frac{2B_n\sqrt{D_n}}{1-D_n} + \frac{2\sqrt{D_n}}{1-D_n} P_2 \right\} \beta e^{P_2 t} u(t) * e^{P_1 t} \{I_1(\beta t) - I_0(\beta t)\} u(-t) \\
& \quad - \frac{2D_n(B_{n+1}-B_n)}{(1-D_n)(1+\sqrt{D_n})} e^{P_2 t} u(t) \\
&= \frac{(B_{n+1}-B_n)\sqrt{D_n}}{1-D_n} e^{P_1 t} \{I_1(\beta t) - I_0(\beta t)\} u(-t)
\end{aligned}$$

In contrast to
and these terms
becomes

$$\tilde{R}^{IE}(t) =$$

Letting $x = -z$,

$$\int_{-\infty}^0 e^{P_1 - P_2 z}$$

Then, since $t >$

$$\tilde{R}^{IE}(t) =$$

Consequently

$$\tilde{R}^{IE}(t) =$$

$$\begin{aligned}
& -\frac{(B_{n+1} - B_n)^2 D_n \sqrt{D_n}}{(1 - D_n)^2} e^{P_2 t} \int_{-\infty}^{\min(t, 0)} e^{(P_1 - P_2)x} \{I_1(\beta x) - I_0(\beta x)\} dx \\
& -\frac{2D_n(B_{n+1} - B_n)}{(1 - D_n)(1 + \sqrt{D_n})} e^{P_2 t} u(t).
\end{aligned} \tag{2.86}$$

In contrast to the $\tilde{D}_{n+1} > 0$, and $\tilde{D}_n > 0$ case, (2.86) has apparent causal terms, and these terms can be converted to pure non-causal terms. When $t > 0$, (2.86) becomes

$$\begin{aligned}
\tilde{R}^{TE}(t) &= -\frac{(B_{n+1} - B_n)^2 D_n \sqrt{D_n}}{(1 - D_n)^2} e^{P_2 t} \int_{-\infty}^0 e^{(P_1 - P_2)x} \{I_1(\beta x) - I_0(\beta x)\} dx \\
&+ \frac{2D_n(B_n - B_{n+1})}{(1 - D_n)(1 + \sqrt{D_n})} e^{P_2 t} u(t).
\end{aligned} \tag{2.87}$$

Letting $x = -z$, the integral term is rewritten as

$$\begin{aligned}
\int_{-\infty}^0 e^{(P_1 - P_2)x} \{I_1(\beta x) - I_0(\beta x)\} dz &= \int_{\infty}^0 e^{-(P_1 - P_2)z} \{I_1(-\beta z) - I_0(-\beta z)\} dz \\
&= -\int_{\infty}^0 e^{-(P_1 - P_2)z} \{-I_1(\beta z) - I_0(\beta z)\} dz \\
&= -\int_0^{\infty} e^{-(P_1 - P_2)z} \{I_1(\beta z) + I_0(\beta z)\} dz.
\end{aligned} \tag{2.88}$$

Then, since $t > 0$

$$\begin{aligned}
\tilde{R}^{TE}(t) &= e^{P_2 t} \left[\frac{(B_{n+1} - B_n)^2 D_n \sqrt{D_n}}{(1 - D_n)^2} \int_0^{\infty} e^{-(P_1 - P_2)x} \{I_1(\beta x) + I_0(\beta x)\} dx \right. \\
&\quad \left. + \frac{2D_n(B_n - B_{n+1})}{(1 - D_n)(1 + \sqrt{D_n})} \right] \\
&= 0
\end{aligned} \tag{2.89}$$

Consequently, when $t < 0$, (2.86) becomes

$$\tilde{R}^{TE}(t) = \frac{(B_{n+1} - B_n) \sqrt{D_n}}{1 - D_n} e^{P_1 t} \{I_1(\beta t) - I_0(\beta t)\} u(-t)$$

which includes

$$(3) \dot{D}_{t-1} > 0$$

Using the frequency

as

where

If we let g

$$\begin{aligned}
& -\frac{(B_{n+1}-B_n)^2 D_n \sqrt{D_n}}{(1-D_n)^2} e^{P_2 t} u(-t) \int_{-\infty}^t e^{(P_1-P_2)x} \{I_1(\beta x) - I_0(\beta x)\} dx \\
= & \frac{(B_{n+1}-B_n) \sqrt{D_n}}{1-D_n} e^{P_1 t} \{I_1(\beta t) - I_0(\beta t)\} u(-t) \\
& -\frac{(B_{n+1}-B_n)^2 D_n \sqrt{D_n}}{(1-D_n)^2} e^{P_2 t} u(-t) \int_{-\infty}^0 e^{(P_1-P_2)x} \{I_1(\beta x) - I_0(\beta x)\} dx \\
& +\frac{(B_{n+1}-B_n)^2 D_n \sqrt{D_n}}{(1-D_n)^2} e^{P_2 t} u(-t) \int_t^0 e^{(P_1-P_2)x} \{I_1(\beta x) - I_0(\beta x)\} dx \\
= & \frac{(B_{n+1}-B_n) \sqrt{D_n}}{1-D_n} e^{P_1 t} \{I_1(\beta t) - I_0(\beta t)\} u(-t) \\
& +\frac{(B_{n+1}-B_n)^2 D_n \sqrt{D_n}}{(1-D_n)^2} e^{P_2 t} u(-t) \int_t^0 e^{(P_1-P_2)x} \{I_1(\beta x) - I_0(\beta x)\} dx \\
& +\frac{2D_n(B_{n+1}-B_n)}{(1-D_n)(1+\sqrt{D_n})} e^{P_2 t} u(-t)
\end{aligned} \tag{2.90}$$

which includes non-causal terms only.

$$(3) \quad \tilde{D}_{n+1} > 0 \text{ and } \tilde{D}_n < 0$$

Using the frequency domain unit step function $U(s) = U(j\omega)$, (2.61) can be expressed as

$$\tilde{R}^{TE}(s) = G(s)U(s) + G^*(-s)U(-s) \tag{2.91}$$

where

$$\begin{aligned}
G(s) = & -\frac{2jB_n\sqrt{D_n}}{1+D_n} \frac{1}{s-P_2} \left\{ 1 - \sqrt{\frac{s+B_{n+1}}{s-B_n}} \right\} \\
& +\frac{2j\sqrt{D_n}}{1+D_n} \frac{s}{s-P_2} \left\{ 1 - \sqrt{\frac{s+B_{n+1}}{s-B_n}} \right\} \\
& -\frac{2D_n(B_n+B_{n+1})}{(1+D_n)(1+j\sqrt{D_n})} \frac{1}{s-P_2}.
\end{aligned} \tag{2.92}$$

If we let $g(t) = F^{-1}\{G(s)\}$, then, from (B.2),

$$\begin{aligned}
\tilde{R}^{TE}(t) &= \frac{1}{2} \{g(t) + g^*(t)\} + \frac{j}{2\pi t} * \{g(t) - g^*(t)\} \\
&= Re\{g(t)\} - \frac{1}{\pi t} * Im\{g(t)\}.
\end{aligned} \tag{2.93}$$

When P_2

$\mathcal{G}'$

where

$f_{\lambda}(t)$

from

and

I_r

End

When $P_2 > 0$,

$$\begin{aligned}
g(t) &= \frac{2jB_n\sqrt{D_n}}{1+D_n}e^{P_2t}u(-t) * f_3(t) + \frac{2j\sqrt{D_n}}{1+D_n}\{\delta(t) - P_2u(-t)\}e^{P_2t} * f_3(t) \\
&\quad - \frac{2D_n(B_n + B_{n+1})}{(1+D_n)(1-j\sqrt{D_n})}e^{P_2t}u(-t)
\end{aligned} \tag{2.94}$$

where

$$\begin{aligned}
f_3(t) &= F^{-1} \left\{ 1 - \sqrt{\frac{s+B_{n+1}}{s-B_n}} \right\} \\
&= \frac{B_n}{2}e^{\frac{B_n}{2}t} \left\{ I_1\left(\frac{B_n}{2}t\right) + I_0\left(\frac{B_n}{2}t\right) \right\} u(-t) \\
&\quad - \frac{B_{n+1}}{2}e^{-\frac{B_{n+1}}{2}t} \left\{ I_1\left(\frac{B_{n+1}}{2}t\right) + I_0\left(\frac{B_{n+1}}{2}t\right) \right\} u(t) \\
&\quad + \frac{B_{n+1}B_n}{4}e^{\frac{B_n}{2}t} \int_{\max(t,0)}^{\infty} e^{-\frac{(B_{n+1}+B_n)}{2}x} \left\{ I_1\left[\frac{B_n}{2}(t-x)\right] + I_0\left[\frac{B_n}{2}(t-x)\right] \right\} \\
&\quad \times \left\{ I_1\left(\frac{B_{n+1}}{2}x\right) + I_0\left(\frac{B_{n+1}}{2}x\right) \right\} dx
\end{aligned} \tag{2.95}$$

from (B.10). Now,

$$Re\{g(t)\} = \frac{2D_n(B_n + B_{n+1})}{(1+D_n)^2}e^{P_2t}u(-t) \tag{2.96}$$

and,

$$\begin{aligned}
Im\{g(t)\} &= \frac{2B_n\sqrt{D_n}}{1+D_n}e^{P_2t}u(-t) * f_3(t) + \frac{2\sqrt{D_n}}{1+D_n}\{\delta(t) - P_2u(-t)\}e^{P_2t} * f_3(t) \\
&\quad - \frac{2D_n\sqrt{D_n}(B_n + B_{n+1})}{(1+D_n)^2}e^{P_2t}u(-t).
\end{aligned} \tag{2.97}$$

Finally, for $P_2 > 0$, the transient interfacial reflection coefficient is

$$\begin{aligned}
\tilde{R}^{TE}(t) &= Re\{g(t)\} - \frac{1}{\pi t} * Im\{g(t)\} \\
&= \frac{2D_n(B_n + B_{n+1})}{(1+D_n)^2}e^{P_2t}u(-t) - \frac{2\sqrt{D_n}}{1+D_n}f_3(t) * \frac{1}{\pi t}
\end{aligned}$$

$$\begin{aligned}
& -\frac{2D_n\sqrt{D_n}(B_n+B_{n+1})}{(1+D_n)^2}e^{P_2t}u(-t)*f_3(t)*\frac{1}{\pi t} \\
& +\frac{2D_n\sqrt{D_n}(B_n+B_{n+1})}{(1+D_n)^2}e^{P_2t}u(-t)*\frac{1}{\pi t}.
\end{aligned} \tag{2.98}$$

Alternatively,

$$\begin{aligned}
\tilde{R}^{TE}(t) &= \frac{2D_n\sqrt{D_n}(B_n+B_{n+1})}{(1+D_n)^2} \left[\frac{1}{\sqrt{D_n}}e^{P_2t}u(-t) + \{1-f_3(t)\}*\frac{1}{\pi t}*e^{P_2t}u(-t) \right] \\
& -\frac{2\sqrt{D_n}}{1+D_n}*\frac{1}{\pi t}*f_3(t).
\end{aligned} \tag{2.99}$$

Similarly, for $P_2 < 0$,

$$Re\{g(t)\} = -\frac{2D_n(B_n+B_{n+1})}{(1+D_n)^2}e^{P_2t}u(t) \tag{2.100}$$

and,

$$\begin{aligned}
Im\{g(t)\} &= -\frac{2B_n\sqrt{D_n}}{1+D_n}e^{P_2t}u(t)*f_3(t) + \frac{2\sqrt{D_n}}{1+D_n}\{\delta(t)+P_2u(t)\}*f_3(t) \\
& +\frac{2D_n\sqrt{D_n}(B_n+B_{n+1})}{(1+D_n)^2}e^{P_2t}u(t).
\end{aligned} \tag{2.101}$$

Therefore, the resulting transient interfacial reflection coefficient for $P_2 < 0$ is given by

$$\begin{aligned}
\tilde{R}^{TE}(t) &= Re\{g(t)\} - \frac{1}{\pi t}*Im\{g(t)\} \\
&= -\frac{2D_n(B_n+B_{n+1})}{(1+D_n)^2}e^{P_2t}u(t) - \frac{2\sqrt{D_n}}{1+D_n}f_3(t)*\frac{1}{\pi t} \\
& +\frac{2D_n\sqrt{D_n}(B_n+B_{n+1})}{(1+D_n)^2}e^{P_2t}u(t)*f_3(t)*\frac{1}{\pi t} \\
& -\frac{2D_n\sqrt{D_n}(B_n+B_{n+1})}{(1+D_n)^2}e^{P_2t}u(t)*\frac{1}{\pi t}.
\end{aligned} \tag{2.102}$$

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$$\begin{aligned}\tilde{R}^{TE}(t) = & -\frac{2D_n\sqrt{D_n}(B_n+B_{n+1})}{(1+D_n)^2} \left[\frac{1}{\sqrt{D_n}}e^{P_2t}u(t) + \{1-f_3(t)\} * \frac{1}{\pi t} * e^{P_2t}u(t) \right] \\ & -\frac{2\sqrt{D_n}}{1+D_n} * \frac{1}{\pi t} * f_3(t).\end{aligned}\quad (2.103)$$

(4) $\tilde{D}_{n+1} < 0$ and $\tilde{D}_n > 0$

Using frequency domain unit step function $U(\omega)$, (2.62) can be expressed as

$$\tilde{R}^{TE}(s) = G(s)U(s) + G^*(-s)U(-s) \quad (2.104)$$

where

$$\begin{aligned}G(s) = & -\frac{2jB_n\sqrt{D_n}}{1+D_n} \frac{1}{s+P_2} \left\{ 1 + \sqrt{\frac{s-B_{n+1}}{s+B_n}} \right\} \\ & -\frac{2j\sqrt{D_n}}{1+D_n} \frac{s}{s+P_2} \left\{ 1 + \sqrt{\frac{s-B_{n+1}}{s+B_n}} \right\} \\ & +\frac{2D_n(B_n+B_{n+1})}{(1+D_n)(1-j\sqrt{D_n})} \frac{1}{s+P_2}.\end{aligned}\quad (2.105)$$

If we let $g(t) = F^{-1}\{G(s)\}$, then, from (B.2),

$$\begin{aligned}\tilde{R}^{TE}(t) = & \frac{1}{2}\{g(t) + g^*(t)\} + \frac{j}{2\pi t} * \{g(t) - g^*(t)\} \\ = & Re\{g(t)\} - \frac{1}{\pi t} * Im\{g(t)\}.\end{aligned}\quad (2.106)$$

When $P_2 > 0$,

$$\begin{aligned}g(t) = & -\frac{2jB_n\sqrt{D_n}}{1+D_n}e^{-P_2t}u(t) * f_4(t) - \frac{2j\sqrt{D_n}}{1+D_n}\{\delta(t) - P_2u(t)\}e^{-P_2t} * f_4(t) \\ & +\frac{2D_n(B_n+B_{n+1})}{(1+D_n)(1-j\sqrt{D_n})}e^{-P_2t}u(t)\end{aligned}\quad (2.107)$$

where

$$\begin{aligned}
f_4(t) &= F^{-1} \left\{ 1 + \sqrt{\frac{s - B_{n+1}}{s + B_n}} \right\} \\
&= \frac{B_{n+1}}{2} e^{\frac{B_{n+1}}{2}t} \left\{ I_1\left(\frac{B_{n+1}}{2}t\right) - I_0\left(\frac{B_{n+1}}{2}t\right) \right\} u(-t) \\
&\quad - \frac{B_n}{2} e^{-\frac{B_n}{2}t} \left\{ I_1\left(\frac{B_n}{2}t\right) - I_0\left(\frac{B_n}{2}t\right) \right\} u(t) + \frac{(B_{n+1}B_n)}{4} e^{\frac{B_{n+1}}{2}t} \times \\
&\quad \int_{\max(t,0)}^{\infty} e^{-\frac{(B_{n+1}+B_n)}{2}x} \left\{ I_1\left[\frac{B_{n+1}}{2}(t-x)\right] - I_0\left[\frac{B_{n+1}}{2}(t-x)\right] \right\} \\
&\quad \times \left\{ I_1\left(\frac{B_n}{2}x\right) - I_0\left(\frac{B_n}{2}x\right) \right\} dx
\end{aligned} \tag{2.108}$$

from (B.11). Now,

$$Re\{g(t)\} = \frac{2D_n(B_n + B_{n+1})}{(1 + D_n)^2} e^{-P_2t} u(t) \tag{2.109}$$

and,

$$\begin{aligned}
Im\{g(t)\} &= -\frac{2B_n\sqrt{D_n}}{1 + D_n} e^{-P_2t} u(t) * f_4(t) - \frac{2\sqrt{D_n}}{1 + D_n} \{\delta(t) - P_2u(t)\} e^{-P_2t} * f_4(t) \\
&\quad + \frac{2D_n\sqrt{D_n}(B_n + B_{n+1})}{(1 + D_n)^2} e^{-P_2t} u(t).
\end{aligned} \tag{2.110}$$

Thus, for $P_2 > 0$, the transient interfacial reflection coefficient is

$$\begin{aligned}
\tilde{R}^{TE}(t) &= Re\{g(t)\} - \frac{1}{\pi t} * Im\{g(t)\} \\
&= \frac{2D_n(B_n + B_{n+1})}{(1 + D_n)^2} e^{-P_2t} u(t) + \frac{2\sqrt{D_n}}{1 + D_n} f_4(t) * \frac{1}{\pi t} \\
&\quad + \frac{2D_n\sqrt{D_n}(B_n + B_{n+1})}{(1 + D_n)^2} e^{-P_2t} u(t) * f_4(t) * \frac{1}{\pi t} \\
&\quad - \frac{2D_n\sqrt{D_n}(B_n + B_{n+1})}{(1 + D_n)^2} e^{-P_2t} u(t) * \frac{1}{\pi t}.
\end{aligned} \tag{2.111}$$

Alternatively,

$$\begin{aligned}\tilde{R}^{TE}(t) &= \frac{2D_n\sqrt{D_n}(B_n + B_{n+1})}{(1 + D_n)^2} \left[\frac{1}{\sqrt{D_n}}e^{-P_2t}u(t) + \{f_4(t) - 1\} * \frac{1}{\pi t} * e^{-P_2t}u(t) \right] \\ &\quad + \frac{2\sqrt{D_n}}{1 + D_n} * \frac{1}{\pi t} * f_4(t).\end{aligned}\quad (2.112)$$

Similarly, for $P_2 < 0$,

$$Re\{g(t)\} = -\frac{2D_n(B_n + B_{n+1})}{(1 + D_n)^2}e^{-P_2t}u(-t) \quad (2.113)$$

and,

$$\begin{aligned}Im\{g(t)\} &= \frac{2B_n\sqrt{D_n}}{1 + D_n}e^{-P_2t}u(-t) * f_4(t) - \frac{2\sqrt{D_n}}{1 + D_n}\{\delta(t) + P_2u(-t)\} * f_4(t) \\ &\quad - \frac{2D_n\sqrt{D_n}(B_n + B_{n+1})}{(1 + D_n)^2}e^{-P_2t}u(-t).\end{aligned}\quad (2.114)$$

Therefore, the resulting transient interfacial reflection coefficient for $P_2 < 0$ is given by

$$\begin{aligned}\tilde{R}^{TE}(t) &= Re\{g(t)\} - \frac{1}{\pi t} * Im\{g(t)\} \\ &= -\frac{2D_n(B_n + B_{n+1})}{(1 + D_n)^2}e^{-P_2t}u(-t) + \frac{2\sqrt{D_n}}{1 + D_n}f_4(t) * \frac{1}{\pi t} \\ &\quad - \frac{2D_n\sqrt{D_n}(B_n + B_{n+1})}{(1 + D_n)^2}e^{-P_2t}u(-t) * f_4(t) * \frac{1}{\pi t} \\ &\quad + \frac{2D_n\sqrt{D_n}(B_n + B_{n+1})}{(1 + D_n)^2}e^{-P_2t}u(-t) * \frac{1}{\pi t}.\end{aligned}\quad (2.115)$$

Alternatively,

$$\begin{aligned}\tilde{R}^{TE}(t) &= -\frac{2D_n\sqrt{D_n}(B_n + B_{n+1})}{(1 + D_n)^2} \left[-\frac{1}{\sqrt{D_n}}e^{-P_2t}u(-t) + \{1 - f_4(t)\} * \frac{1}{\pi t} * e^{-P_2t}u(-t) \right] \\ &\quad + \frac{2\sqrt{D_n}}{1 + D_n} * \frac{1}{\pi t} * f_4(t).\end{aligned}\quad (2.116)$$

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2.3.2 Causality

In the transient forms for the cases of (2), (3) and (4), there are non-causal terms involving $u(-t)$ or $\frac{1}{t}$. These are unexpected results, because the convolution of any causal (thus, physically realizable) input waveform with these terms will produce non-causal transient reflected waveform components that appears to disobey Einstein's causality, which says that nothing can travel faster than the speed of light. Also, it should be noted that this is 'pure' non-causality which is different from the non-causality caused by the diffusion approximation referred in [11]. The existence of the non-causal terms depends on the sign of $\tilde{D}_{n+1}$ and $\tilde{D}_n$. That is, the non-causality happens when the sign of either one of the two parameters is negative. When the sign is negative, the real part of γ_x is always negative by the branch cut; therefore the wave propagates in an evanescent mode, regardless of the existence of conductivity. A good example of this case is total internal reflection at the interface of lossless dielectric material layers.

The conventional geometrical ray optics approach can not explain this non-causality. Consider the geometry shown in Figure 2.3, where total internal reflection of an impulse incident plane wave is assumed to occur at the interface between two lossless dielectric layers. The reflected wave packet from the point Q1 arrives at the observation plane at $t = 0$, at the same moment as the reflection of the incident wave from Q2, because their propagation velocities are the same in this same region, and the lengths of travel along paths P1 and P2 are identical. Consider a surface wave in region 2, which starts at Q1 and propagates along the z-axis with propagation velocity equal to or less than $v_{2z} = \frac{\omega}{\gamma_{2z}} = \frac{\omega}{\gamma_{1z}} = \frac{\omega}{\gamma_1 \sin \theta_{i1}}$. Then, the travel time of the surface wave is $t_d \geq \frac{d}{v_{2z}} = \frac{d\gamma_1 \sin \theta_{i1}}{\omega} = \frac{P2}{\sin \theta_{i1} \frac{\omega}{\gamma_1 \sin \theta_{i1}}} = \frac{P2}{(\omega/\gamma_1)}$. Therefore, the surface wave arrives at Q2 at the same or later moment at which the incident wave impinges on this point.

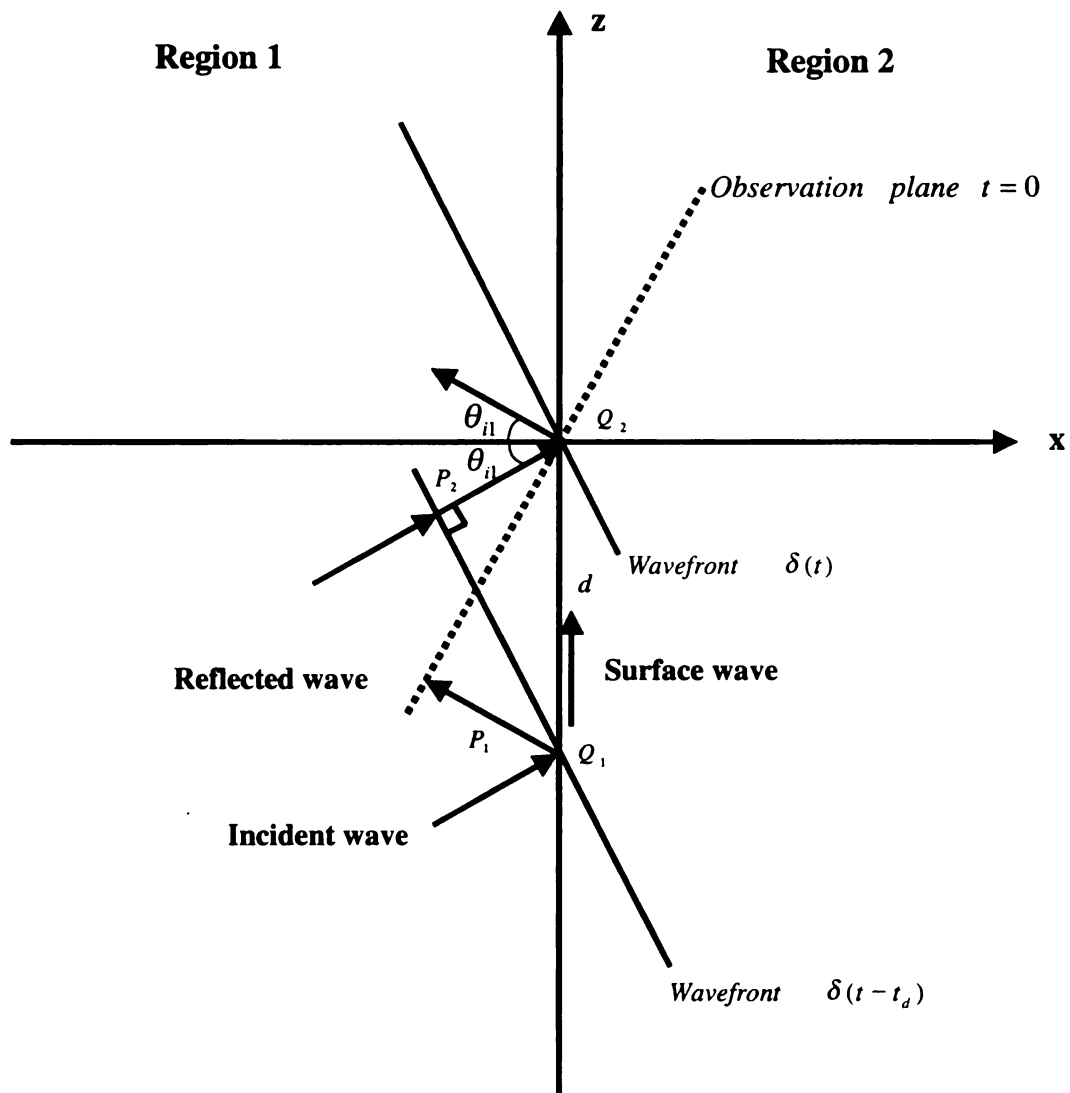


Figure 2.3. The time relationship of wavefronts.

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Next, consider the possible existence of a precursor effect. Any wave front of the spherical wave excited at Q1 that arrives at a point on the observation plane faster than the wave reflected from Q2 might be a precursor and thus a source of the non-causality. From Figure 2.4, consider path 1 with $l_1 = \sqrt{h^2 + l_3^2} = \sqrt{(d+x)^2 + \left(\frac{x}{\tan\theta_{i1}}\right)^2}$ and path 2 with $l_2 + l_3 = d \sin\theta_{i1} + \frac{x}{\sin\theta_{i1}}$. To be a precursor, path 1 must be less than path 2. However

$$\begin{aligned}
l_1^2 - (l_2 + l_3)^2 &= \left\{ (d+x)^2 + \left(\frac{x}{\tan\theta_{i1}}\right)^2 \right\} - \left(d \sin\theta_{i1} + \frac{x}{\sin\theta_{i1}} \right)^2 \\
&= d^2 + x^2 + x^2 \left(\frac{1}{\tan^2\theta_{i1}} - \frac{1}{\sin^2\theta_{i1}} \right) - d^2 \sin^2\theta_{i1} \\
&= d^2 \cos^2\theta_{i1} > 0
\end{aligned} \tag{2.117}$$

and thus, there is no precursor.

In this study, a reasonable explanation of the non-causality has not been found. The non-causality might originate from the impractical assumption of infinite layer width because the incident plane wave interacts with the interface an infinite amount time ago, i.e. as far back as $t = -\infty$, although the relationship between this assumption and non-causality could not be found using ordinary ray optics approach as explained above.

There are several interesting ongoing research activities that might help explain this phenomenon. In physics, there have been some trials to measure or compute the exact tunneling time of a particle through a potential barrier, which is important in modern microelectronic tunneling devices [12], [13]. It turns out that the tunneling of a particle (or a wavepacket) is quite similar to the transmission of an evanescent electromagnetic wave, e.g. the propagation of a wave having a frequency less than the cut-off frequency in a waveguide [14]. Therefore, there were many experiments performed by using optical pulses (e.g. [15] and [16]), or microwave propagation in a

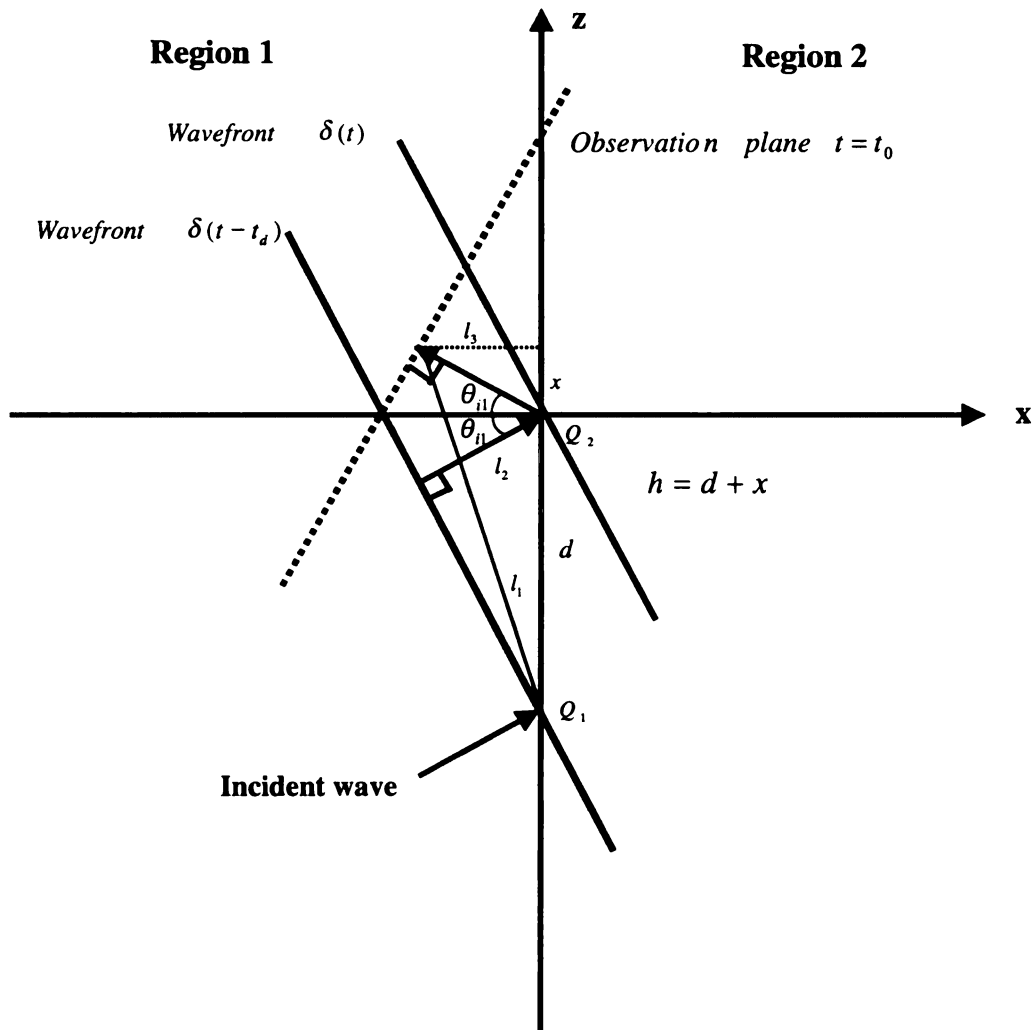


Figure 2.4. The consideration of the existence of a precursor.

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In these experiments, it has been commonly observed that the pulse peak propagation velocities seem to be independent of the thickness of a potential barrier (or an evanescent region, equivalently), which consequently implies so-called “super luminal” propagation velocity of the wave modes. For example, if the width of region were to become infinite, then the speed of wave propagation would increase infinitely, therefore faster than the speed of light, which leads to apparent contradiction of the Einstein causality. Although this phenomenon had been theoretically predicted in the past [18], the exact theoretical explanation has yet been given. Many have attempted to explain this phenomenon as a reshaping of the pulse [15], [16]. That is, attenuation of the pulse in the barrier shifts the peak of the pulse forward. But little is known about why the barriers attenuate pulses unevenly [19]. Also, there is a question about the validity of ‘peak’ measurement methodology in dispersive media, since it is sometimes hard to define the pulse peak in that case.

It should be noted that there is some possibility that the “super luminal phenomenon” of those experiments might be different from the non-causal tail in the impulse response waveform described in this study. In this study the pulse peak point is causal (located at the time origin of $t = 0$), while the “super luminal” velocities were measured only using the location of the pulse peak, and did not consider the shape of the waveform since very short optical pulse waveforms were used in the past experiments. Nevertheless, there might be a relationship between the observed results and the non-causality in the transient interfacial reflection coefficients, considering the fact that both happen in the same situation, i.e. only in evanescent propagation mode.

As a result, only the causal case will be considered in the remainder of this dissertation, simply because there is no reliable explanation for the non-causality. The non-causal case can be studied in separate research. Note that the non-causal case is

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2.4 Numerical

2.4.1 Verification

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unlikely to occur in a practical situation. That is, to be a non-causal case, the layer parameter $\tilde{D}_n = (\mu_n \epsilon_n - \mu_1 \epsilon_1 \sin^2 \theta_{i1}) / \mu_n^2$ must be negative, and thus

$$\frac{\mu_n}{\mu_1} < \frac{\epsilon_1}{\epsilon_n} \sin^2 \theta_{i1}. \quad (2.118)$$

The first layer can be any lossless dielectric, but is typically free space in practice because it has to have infinite depth and width. Then, $\epsilon_1 = \epsilon_0$ and $\mu_1 = \mu_0$. Considering that $\sin^2 \theta_{i1} < 1$ and $\epsilon_0 < \epsilon_n$ always, it can be recognized that satisfying the condition of (2.118) is improbable.

2.4 Numerical Examples

2.4.1 Verification of theoretical expressions

The previously derived transient forms for interfacial reflection coefficients may be compared to direct numerical computation to ensure that the transform are correct. The most common method to do this is simply to compute the numerical inverse Fast Fourier transforms (IFFT) from the frequency domain forms. For the reasons mentioned earlier, only the causal case ($\tilde{D}_{n+1} > 0$ and $\tilde{D}_n > 0$) will be discussed.

Since the asymptotic interfacial reflection coefficients appear as delta functions in the time domain, and are thus difficult to express numerically, only the reduced interfacial reflection coefficients are compared. Figure 2.5 (a) and (b) shows the comparison of the derived transient interfacial reflection coefficients with the transient forms obtained by using a 3072 point IFFT, for the $P_2 > 0$ and $P_2 < 0$ cases, respectively. The material parameters for Figure 2.5 (a) are those for an interface of free-space and typical sea water at low frequency as described in [20], while those for Figure 2.5 (b) are for an interface of plexiglass and ethyl alcohol 10GHz, as described in [21]. Note that for the non-magnetic materials, $P_2 < 0$ case happens when $\epsilon_n > \epsilon_{n+1}$ and $(\sigma_{n+1}/\sigma_n) > (\tilde{D}_{n+1}/\tilde{D}_n) > 1$, or $\epsilon_n < \epsilon_{n+1}$ and $(\sigma_{n+1}/\sigma_n) < (\tilde{D}_{n+1}/\tilde{D}_n) < 1$.

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The transient reflection coefficient curves from the two different methods are nearly identical, and thus correctness of the derived transient interfacial reflection coefficients for TE polarization have been verified.

To verify the combination of the asymptotic and the reduced transient reflection coefficient, the time domain convolution of the interfacial reflection coefficient with an input waveform is needed, i.e.

$$\begin{aligned}
S(t) &= X(t) * R^{TE}(t) \\
&= X(t) * \left\{ R_{\infty}^{TE} \delta(t) + \tilde{R}^{TE}(t) \right\} \\
&= R_{\infty}^{TE} X(t) + \tilde{R}^{TE}(t) * X(t)
\end{aligned} \tag{2.119}$$

where $X(t)$ denotes the input waveform and $S(t)$ is the resulting reflected waveform. One of the most commonly used input waveforms for this purpose is a double exponential pulse waveform, given as

$$X(t) = K (e^{-at} - e^{-bt}) \tag{2.120}$$

where K , a and b are arbitrary positive constants. Figure 2.6 shows an example of the input waveform where $K = 30.71$, $a = 2 \times 10^9$ and $b = 4 \times 10^9$. Figure 2.7 (a) and Figure 2.7 (b) shows the comparison of the time domain reflected waveforms with the input waveform shown in Figure 2.6 for the two different layer parameter sets ($P_2 > 0$ and $P_2 < 0$). Again, in both cases, the derived expressions and the IFFT results are well matched.

As mentioned in chapter 1, exact theoretical expressions are available from the previous research [11] for transient interfacial reflection due to a unit step input waveform. A comparison with those results also helps to verify the work in this study. Figure 2.8 (a) and Figure 2.8 (b) shows that the re-produced results of the

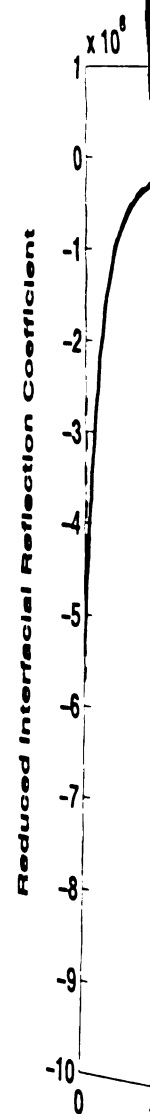


Figure 2.5. (a)
reflection coefficient
 $\sigma_n = 0, \bar{U}_n, m_n, \mu_n$

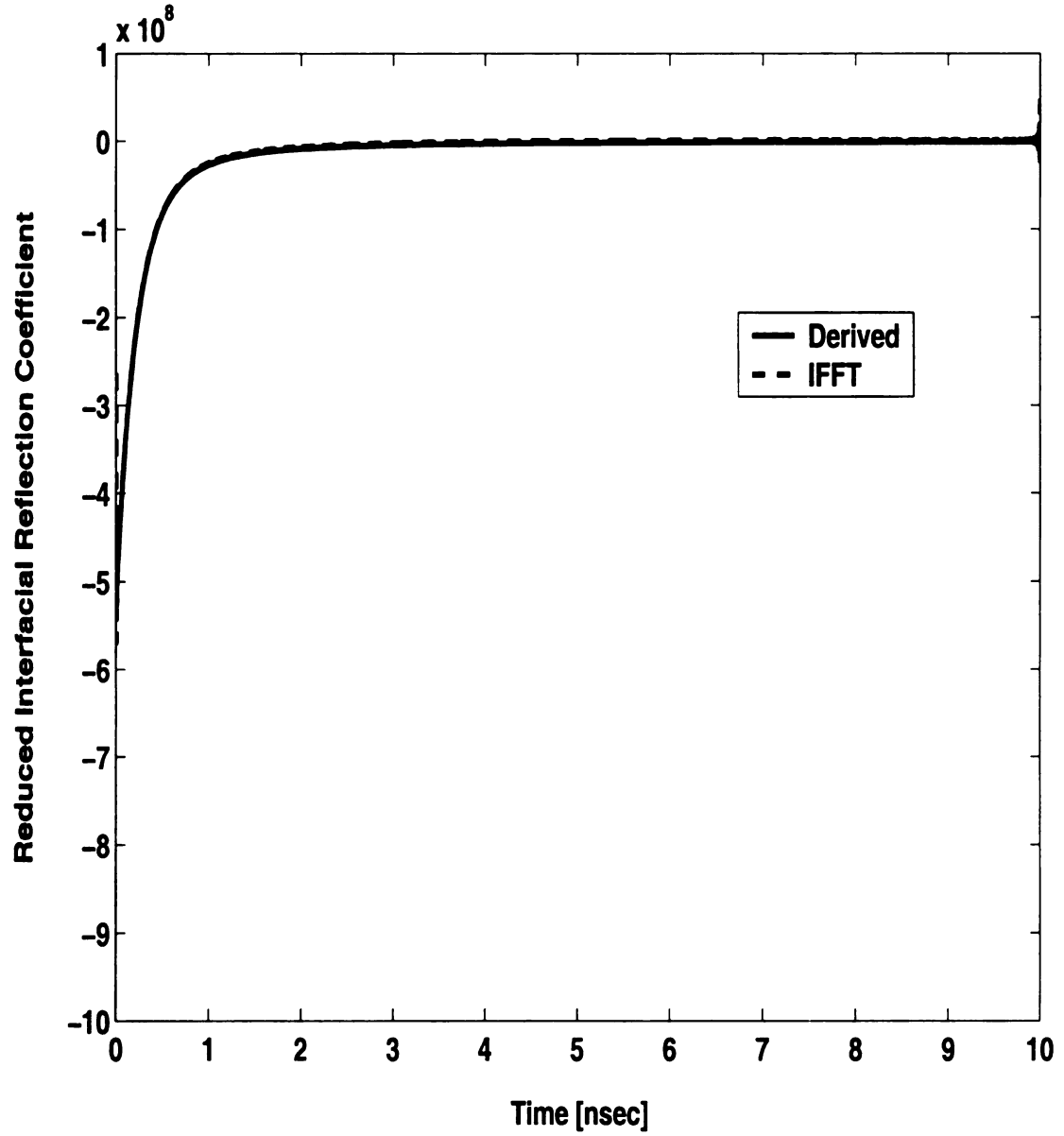


Figure 2.5. (a) Numerical comparison of the derived transient reduced interfacial reflection coefficient with that from the IFFT (TE polarization) : $\mu_n = \mu_0$, $\epsilon_n = \epsilon_0$, $\sigma_n = 0[\mathcal{U}/m]$, $\mu_{n+1} = \mu_0$, $\epsilon_{n+1} = 72\epsilon_0$, $\sigma_{n+1} = 4[\mathcal{U}/m]$, $\theta_{i1} = 30^\circ$, $P_2 = 0.64 \times 10^{10}$.



Figure
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 $\sigma_n = 9$
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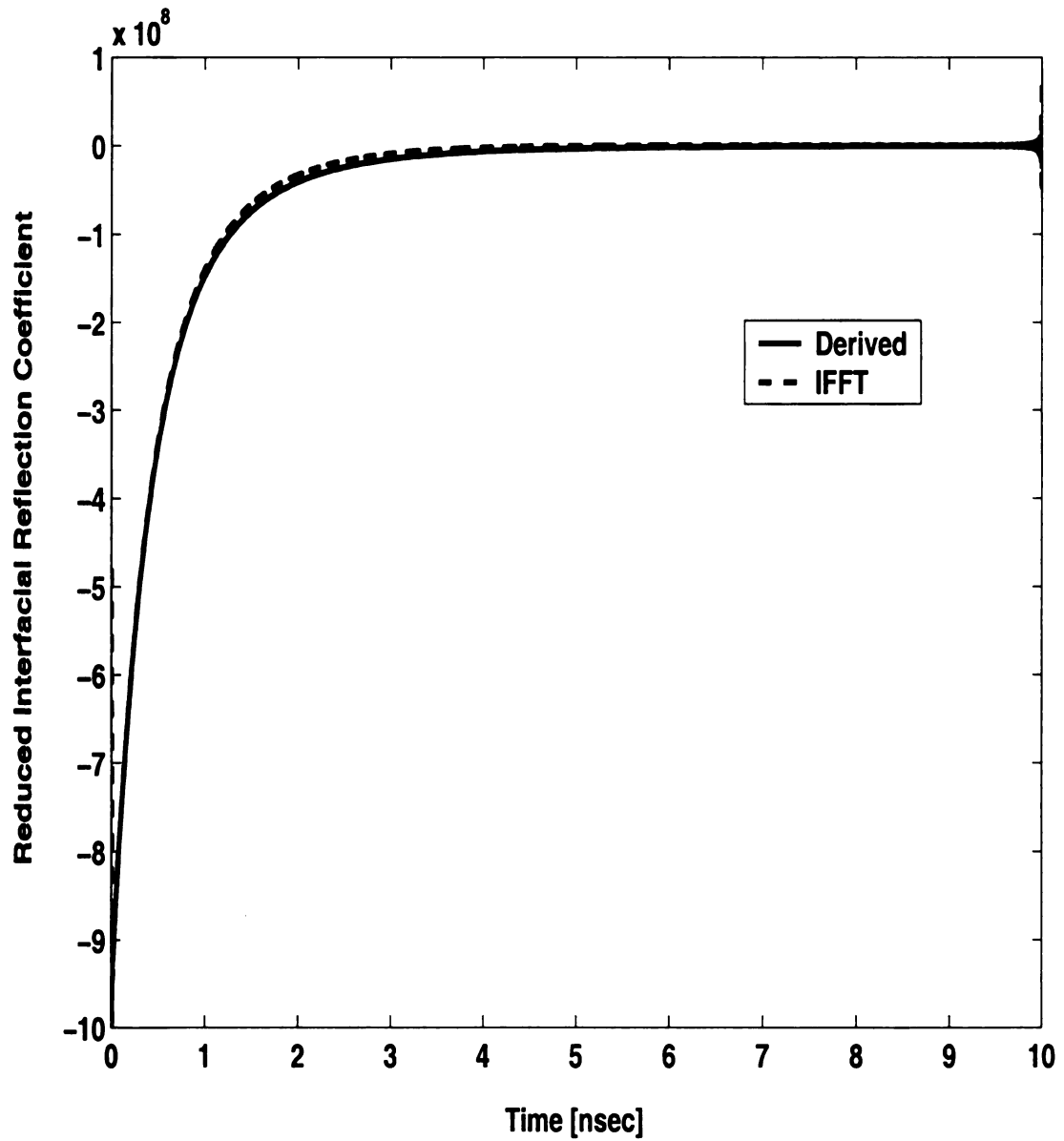
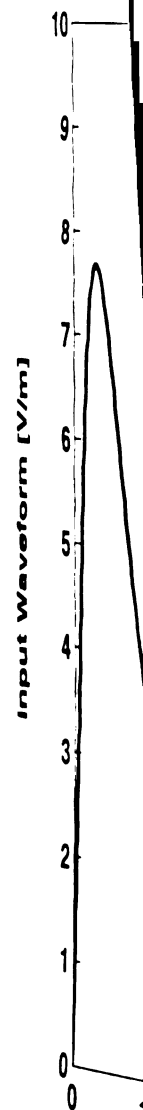


Figure 2.5. (b) Numerical comparison of the derived transient reduced interfacial reflection coefficient with that from the IFFT (TE polarization) : $\mu_n = \mu_0$, $\epsilon_n = 2.59\epsilon_0$, $\sigma_n = 9.73 \times 10^{-3} [\text{U}/m]$, $\mu_{n+1} = \mu_0$, $\epsilon_{n+1} = 1.70\epsilon_0$, $\sigma_{n+1} = 5.60 \times 10^{-2} [\text{U}/m]$, $\theta_{i1} = 30^\circ$, $P_2 = -0.59 \times 10^{10}$.



Figure

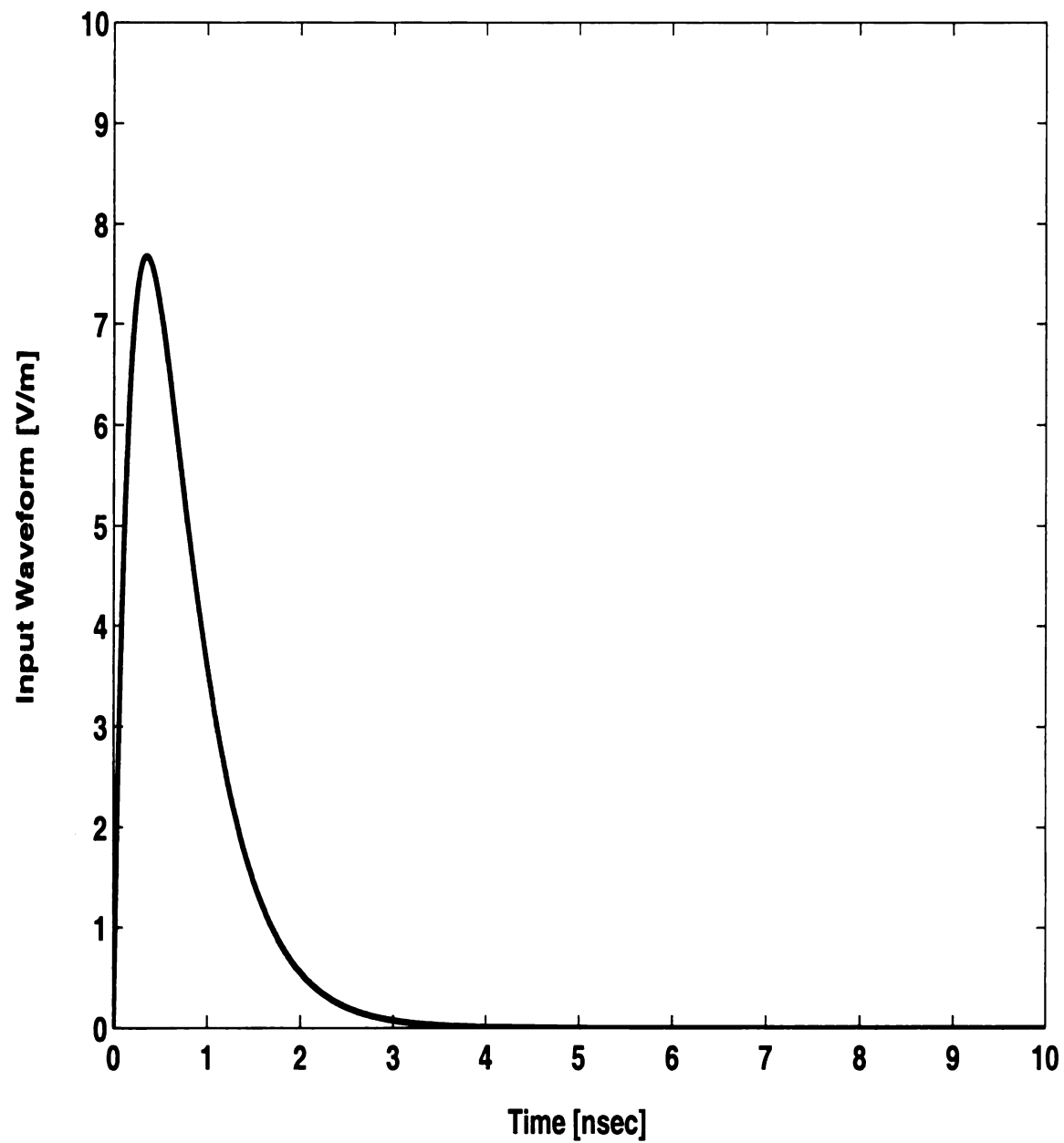


Figure 2.6. An example of double exponential input waveform.

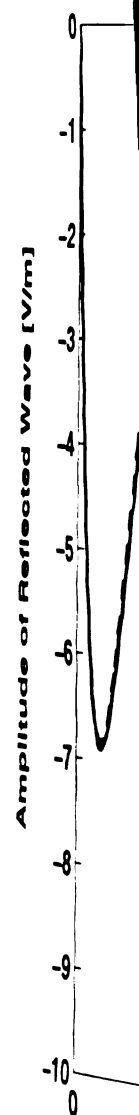


Figure 2.7. (a)
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 $\mu_n = \mu_0$, $\epsilon_n = \epsilon_0$
 $P_2 = 0.64 \times 10^{10}$

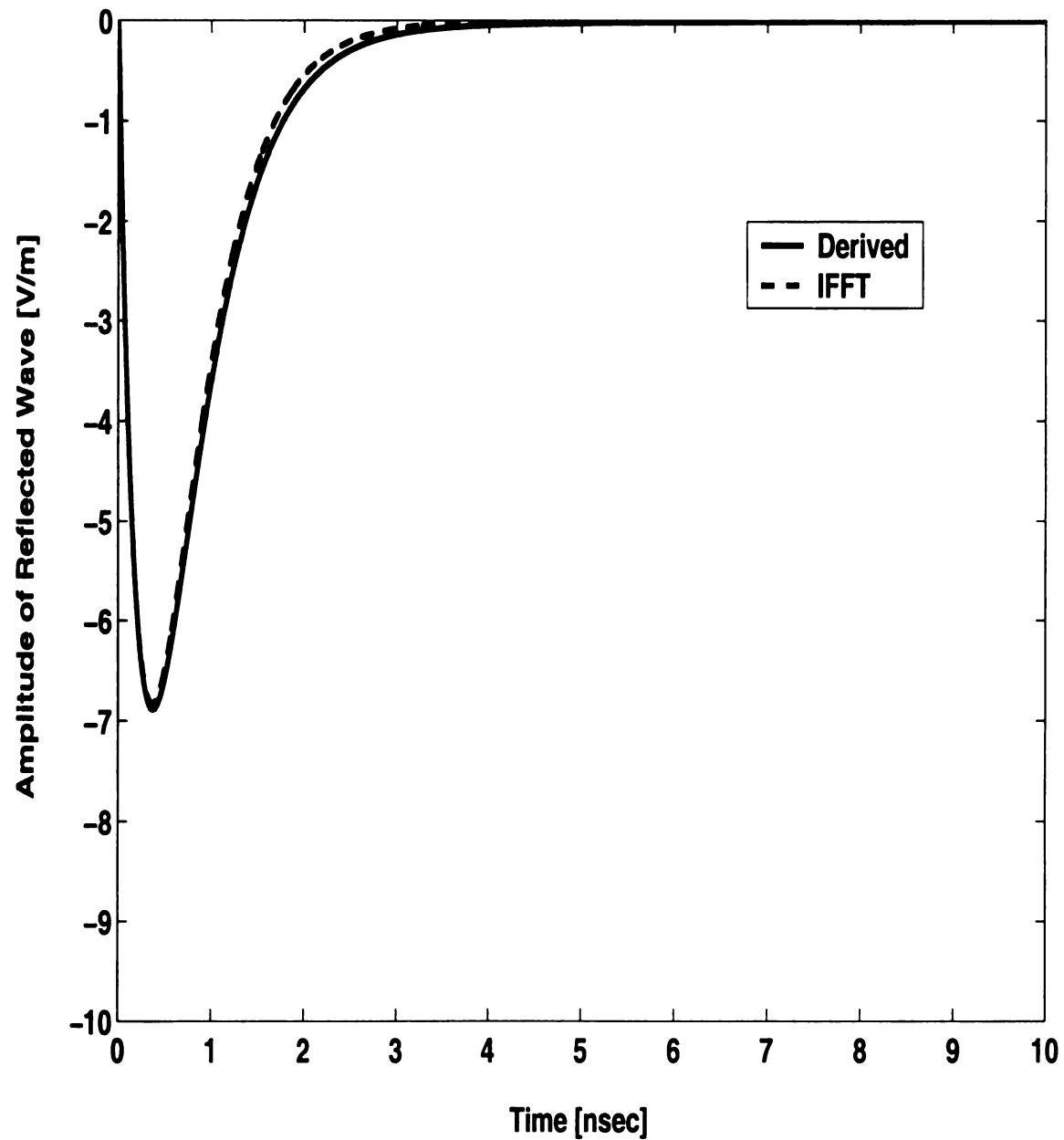


Figure 2.7. (a) Numerical comparison of the transient reflected electric field waveform for the input waveform shown in Figure 2.6 with the IFFT (TE polarization) : $\mu_n = \mu_0$, $\epsilon_n = \epsilon_0$, $\sigma_n = 0[\text{U}/m]$, $\mu_{n+1} = \mu_0$, $\epsilon_{n+1} = 72\epsilon_0$, $\sigma_{n+1} = 4[\text{U}/m]$, $\theta_{i1} = 30^\circ$, $P_2 = 0.64 \times 10^{10}$.



Figure 2.7.
 waveform for
 polarization :
 $\epsilon_{r-1} = 1.70\epsilon_0$

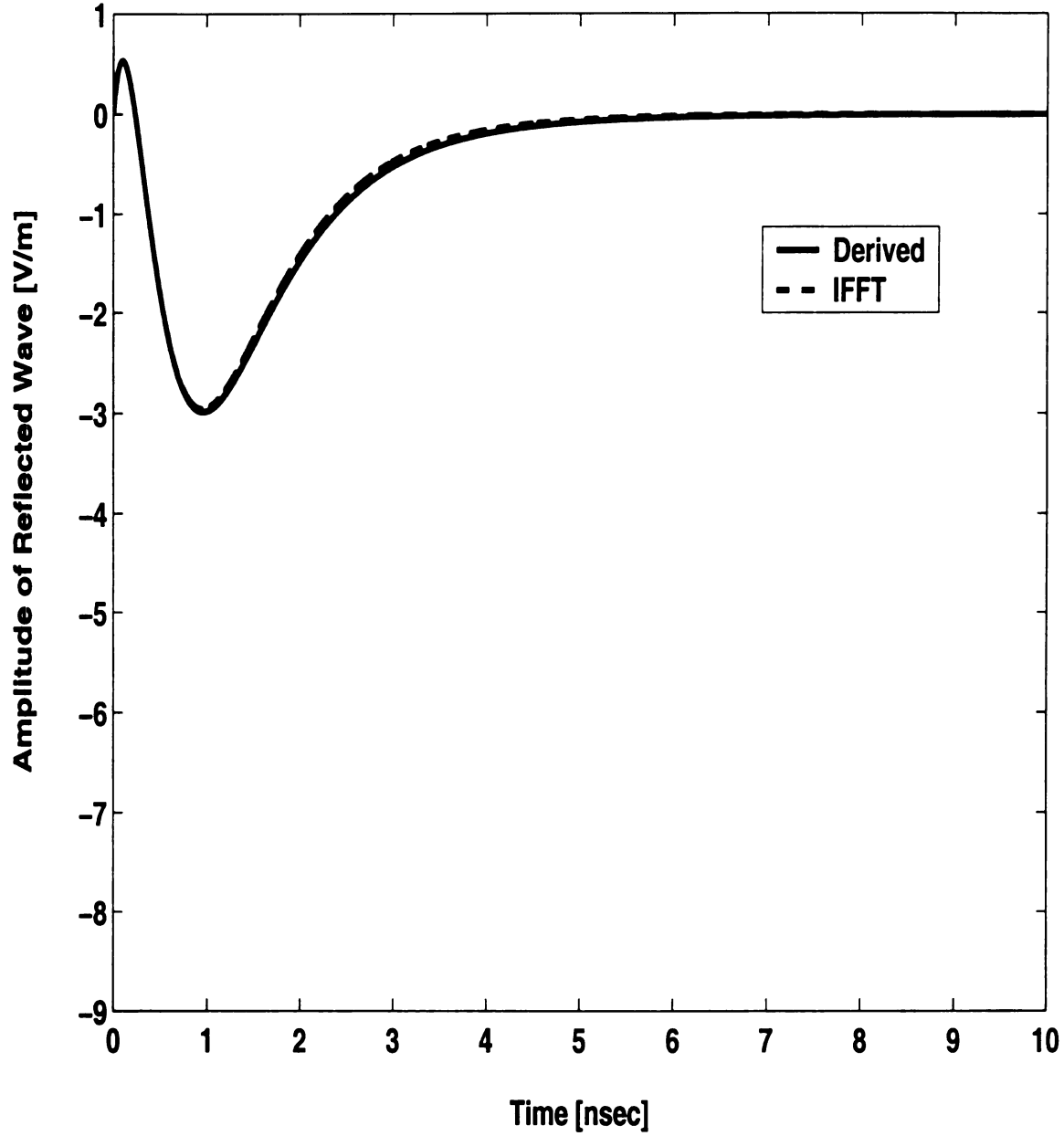


Figure 2.7. (b) Numerical comparison of the transient reflected electric field waveform for the input waveform shown in Figure 2.6 with the IFFT (TE polarization) : $\mu_n = \mu_0$, $\epsilon_n = 2.59\epsilon_0$, $\sigma_n = 9.73 \times 10^{-3}[\text{U}/m]$, $\mu_{n+1} = \mu_0$, $\epsilon_{n+1} = 1.70\epsilon_0$, $\sigma_{n+1} = 5.60 \times 10^{-2}[\text{U}/m]$, $\theta_{i1} = 30^\circ$, $P_2 = -0.59 \times 10^{10}$.

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electric fields to a unit step input waveform excitation obtained by using the transient interfacial reflection coefficient and transmission coefficient of this study, are exactly same as those found in Figure 4.5 and Figure 4.4 in [11]. Note that these plots are drawn with the nomalized time axis given by

$$at = \frac{\sigma_{n+1}t}{2\epsilon_{n+1} \left(1 - \frac{\mu_1\epsilon_1 \sin^2 \theta_{i1}}{\mu_{n+1}\epsilon_{n+1}} \right)}.$$

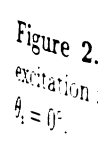
It can be easily realized that the early time portion of the scattered electrical field waveform is dominated by the high frequency component of the interfacial reflection coefficient, i.e. the asymptotic interfacial reflection coefficient. The late time portion is due to the lower frequency components of the interfacial reflection coefficient.

2.4.2 The transient responses for various parameter sets

It is interesting to study the dependence of the transient reflection coefficient on the various parameter values, i.e. permittivity, permeability, conductivity and aspect angle. Comparisons of the reduced reflection coefficients are shown in Figure 2.9 - Figure 2.12 which are plotted using semilog scales for easier identification. Note that the first two curves in each plot, except Figure 2.12, are for $P_2 < 0$ cases, so that the effects of parameter change can be observed for both cases of $P_2 > 0$ and $P_2 < 0$.

(1) Permittivity

The curves in Figure 2.9 are plotted for $\epsilon_{r,n+1} = 1.7, 2.5, 10$ and 72 . The corresponding values of R_∞^{TE} are $0.12, 0.0098, -0.34$ and -0.69 , respectively. Note that the phase reversal of the asymptotic reflection coefficient happens when ϵ_{n+1} becomes greater than ϵ_n . It can be seen that the slopes of different $\tilde{R}^{TE}(t)$ curves are inversely proportional to the increasing permittivity values, while the asymptotic reflections are proportional to the increasing permittivity values except the case $\epsilon_{r,n+1} = 2.5$, where the material constants of the two layers approach each other, and therefore



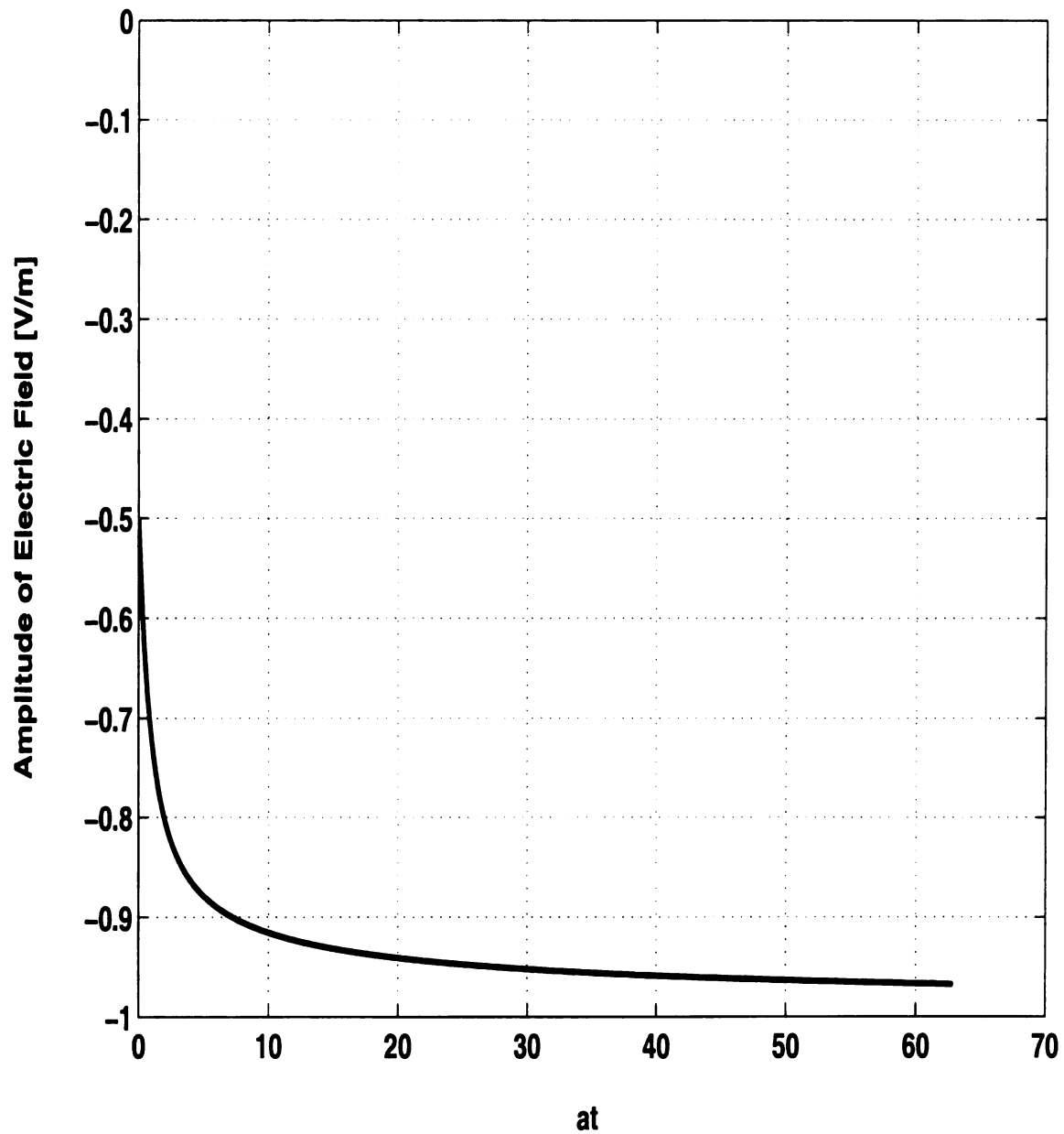


Figure 2.8. (a) The transient reflected electrical field waveform for a unit step excitation: $\mu_n = \mu_0$, $\epsilon_n = \epsilon_0$, $\sigma_n = 0$, $\mu_{n+1} = \mu_0$, $\epsilon_{n+1} = 9\epsilon_0$, $\sigma_{n+1} = 1.00 \times 10^{-3} [\text{U}/m]$, $\theta_i = 0^\circ$.



Figure 2.8. ()
excitation : μm
 $H_1 = 30^\circ$.

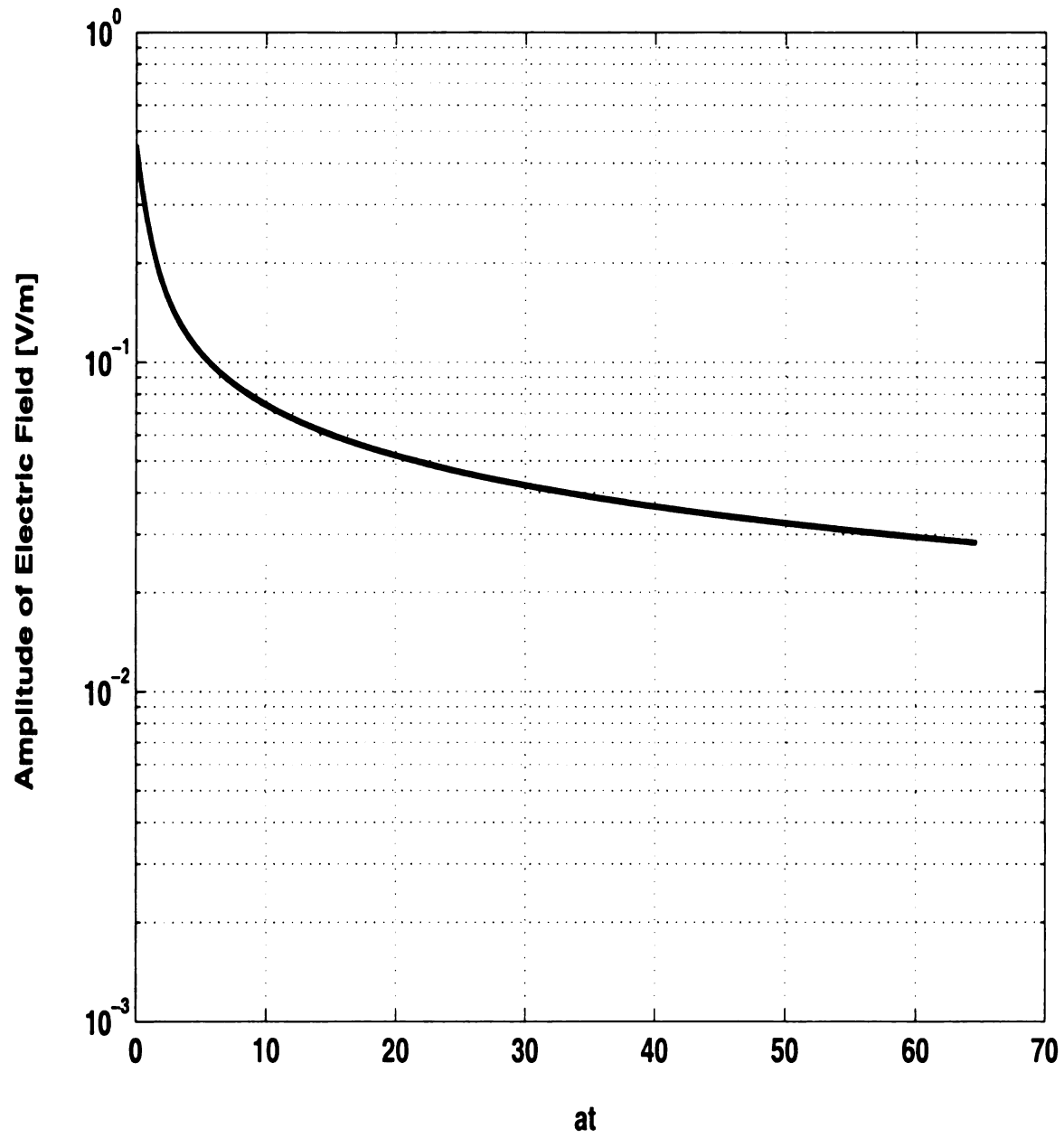


Figure 2.8. (b) The transient transmitted electrical field waveform for a unit step excitation : $\mu_n = \mu_0$, $\epsilon_n = \epsilon_0$, $\sigma_n = 0$, $\mu_{n+1} = \mu_0$, $\epsilon_{n+1} = 9\epsilon_0$, $\sigma_{n+1} = 1.00 \times 10^{-3} [\text{U}/m]$, $\theta_i = 30^\circ$.

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2.5 Approxin

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the overall reflection itself is weak. This means that the overall transient reflection increases when the permittivity contrast of both layers is sharper.

(2) Permeability

The curves in Figure 2.10 are plotted for $\mu_{r,n+1} = 1, 5, 10$ and 75 . The corresponding values of R_{∞}^{TE} are $0.027, 0.38, 0.51$ and 0.79 respectively. The slopes of $\tilde{R}^{TE}(t)$ are seen to be inversely proportional to the increasing permeability values, while the asymptotic reflections are proportional to the increasing permeability values. Therefore, the amount of overall reflection increases with the increasing permeability.

(3) Conductivity

Because an increase in conductivity causes attenuation in wave propagation, the transient response $\tilde{R}^{TE}(t)$ suffers a more rapid decrease with time, while the asymptotic component is independent of the change. This can be seen explicitly in Figure 2.11 where $\sigma_n = 0.01, 0.03, 4$ and 10 , respectively.

(4) Aspect angle

The curves shown in Figure 2.12 are plotted for $\theta_{i1} = 0^\circ, 30^\circ, 45^\circ$ and 75° , while the corresponding values of R_{∞}^{TE} are $0.11, 0.12, 0.14$ and 0.19 respectively. Therefore, the slope of the reduced reflection coefficient as well as the amplitude of the asymptotic component increases with increasing aspect angle, and the reflection becomes more dependent on its early time portion coming from the asymptotic reflection term.

2.5 Approximation of Interfacial Reflection Coefficients

It may be useful to obtain approximate forms of the previously derived transient reflection coefficients, as long as accuracy is maintained. Because the numerical computation of repeated time domain convolution integrals (needed for obtaining multi-layer scattering as discussed in chapter 4) requires a significant amount of time, the total computation time could be burdensome in real time applications. Also, the process of finding the approximate forms gives some insight into the roles of previously defined

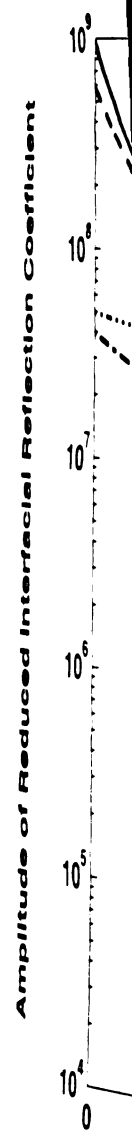


Figure 2.9. The
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 $\sigma_{n-1} = 5.60 \times 10$

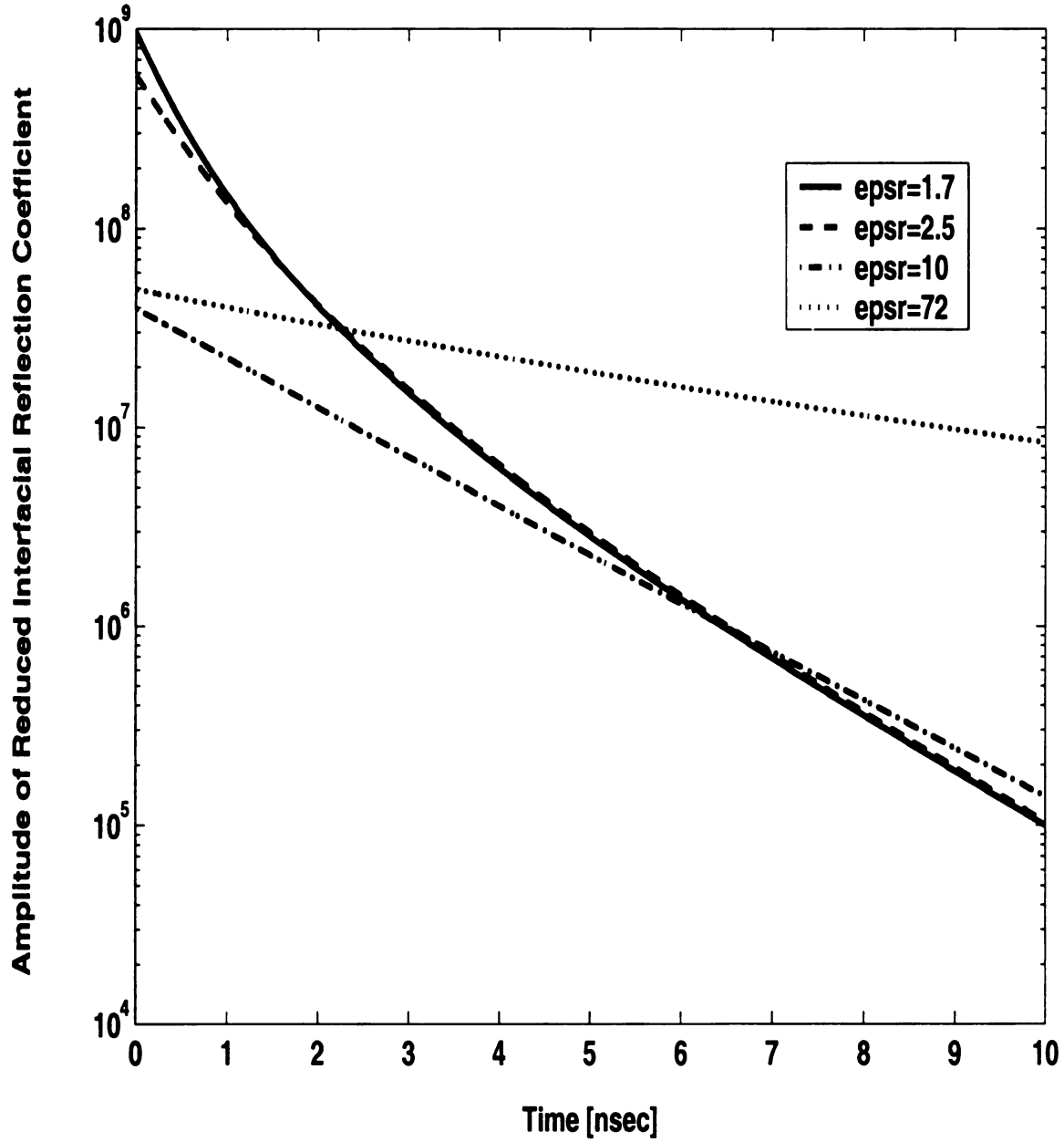


Figure 2.9. Time domain reduced interfacial reflection coefficients for various values of permittivity (TE polarization) : $\mu_n = \mu_{n+1} = \mu_0$, $\epsilon_n = 2.59\epsilon_0$, $\sigma_n = 9.73 \times 10^{-3}$, $\sigma_{n+1} = 5.60 \times 10^{-2}[\text{U}/m]$, $\theta_{i1} = 30^\circ$.

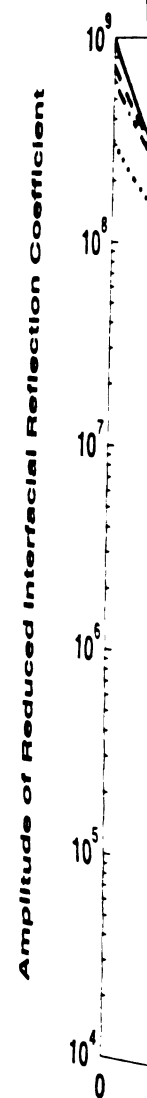


Figure 2.10.
values of permea
 $\epsilon_{t-1} = 1.70\epsilon_0$, σ_{t-1}

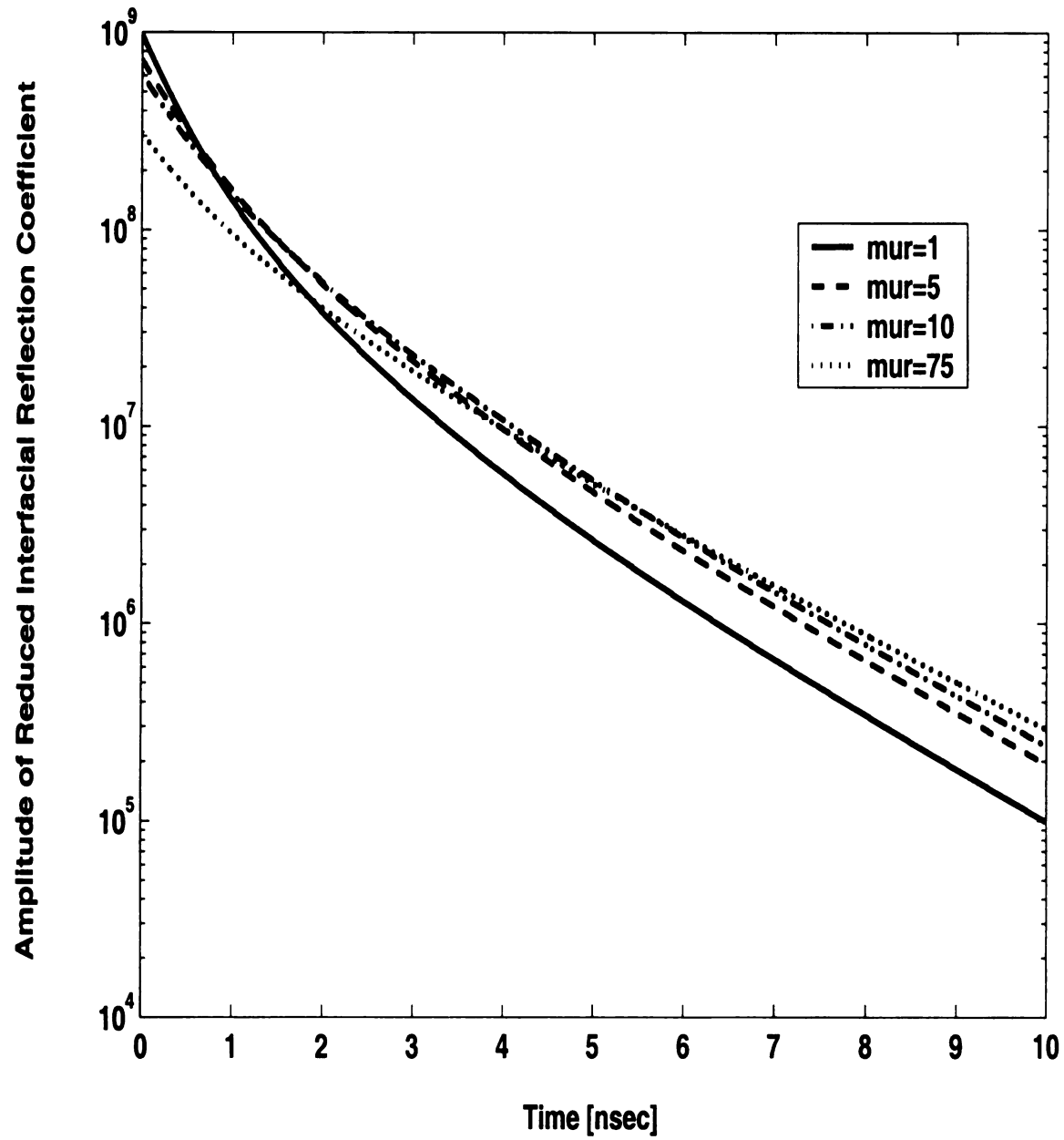


Figure 2.10. Time domain reduced interfacial reflection coefficients for various values of permeability (TE polarization) : $\mu_n = 1.5\mu_0$, $\epsilon_n = 2.59\epsilon_0$, $\sigma_n = 9.73 \times 10^{-3}$, $\epsilon_{n+1} = 1.70\epsilon_0$, $\sigma_{n+1} = 5.60 \times 10^{-2}[\text{U}/m]$, $\theta_{i1} = 30^\circ$.

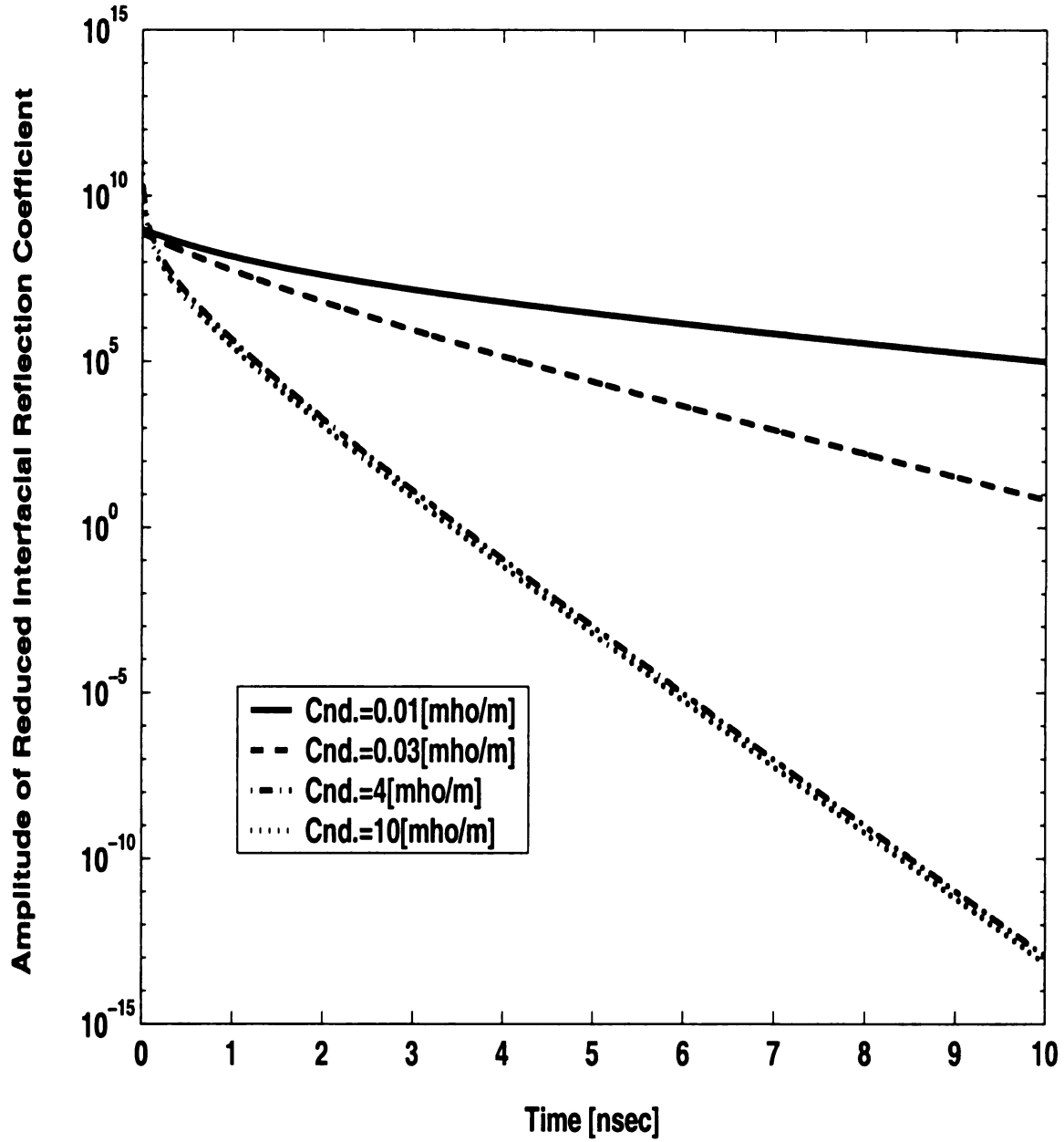


Figure 2.11. Time domain reduced interfacial reflection coefficients for various values of conductivity (TE polarization): $\mu_n = \mu_{n+1} = \mu_0$, $\epsilon_n = 2.59\epsilon_0$, $\epsilon_{n+1} = 1.70\epsilon_0$, $\sigma_{n+1} = 5.60 \times 10^{-2}[\text{U}/m]$, $\theta_{i1} = 30^\circ$.

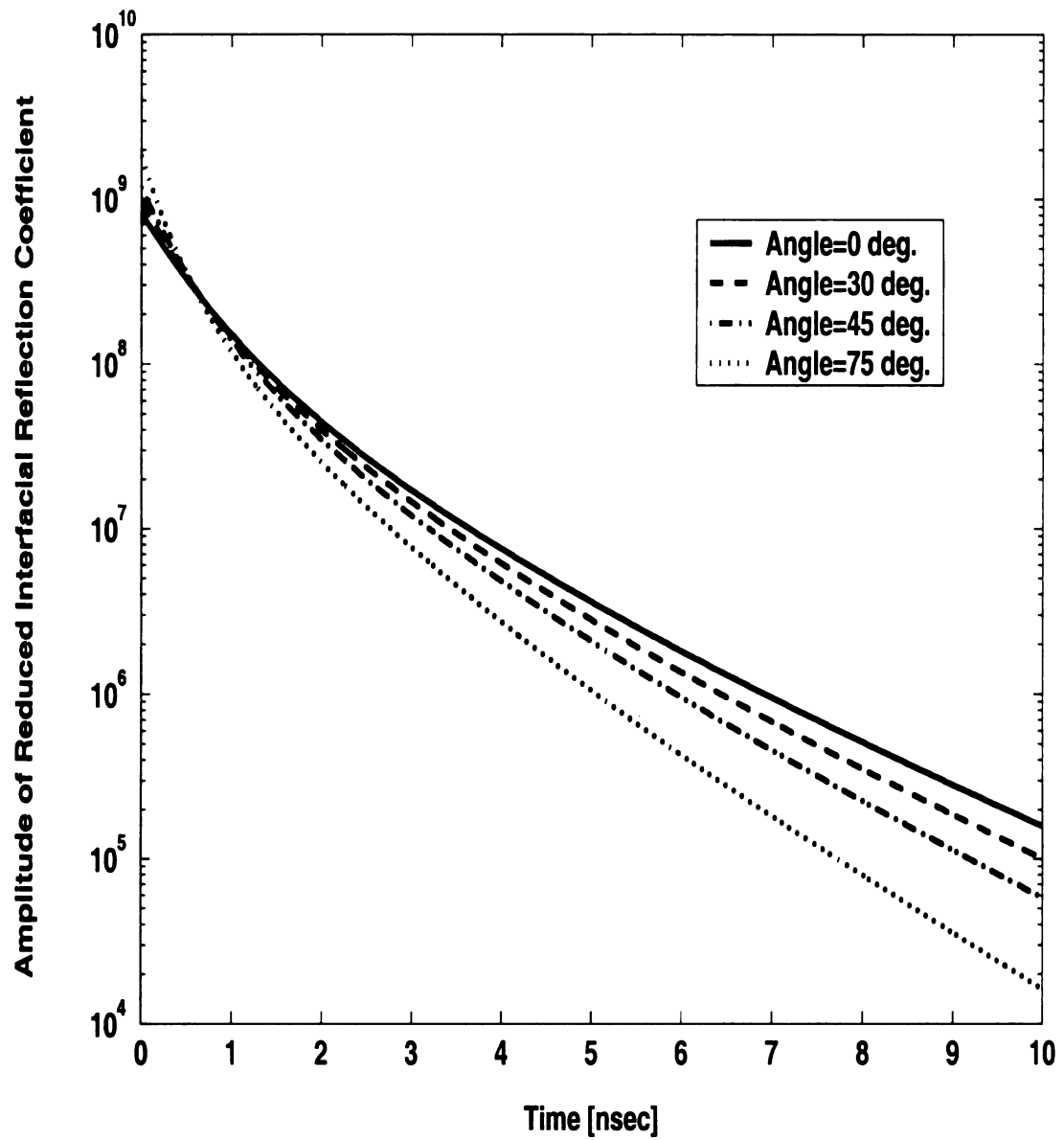


Figure 2.12. Time domain reduced interfacial reflection coefficients for various values of incident angle (TE polarization) : $\mu_n = 1.5\mu_0$, $\epsilon_n = 2.59\epsilon_0$, $\sigma_n = 9.73 \times 10^{-3}$, $\mu_{n+1} = \mu_0$, $\epsilon_{n+1} = 1.70\epsilon_0$, $\sigma_{n+1} = 5.60 \times 10^{-2}[\text{U}/m]$.

constants in the transient interfacial reflection coefficient expressions. In this section, therefore, the approximate forms for the transient interfacial reflection coefficients ($\tilde{D}_{n+1} > 0$ and $\tilde{D}_n > 0$ case) are derived, and the accuracy of the approximation method is discussed.

There are two different approximate forms since the $\tilde{D}_{n+1} > 0$ and $\tilde{D}_n > 0$ case has two different transient reflection coefficients according to the sign of the constant P_2 . For each transient reflection coefficient, The approximations for large values of the time variable t and small values of the time variable t are derived. Then they are combined by using a pair of weighting functions to produce a smooth curve. For convenience, the simpler reflection coefficient (for $P_2 < 0$) will be discussed first.

2.5.1 Case I : $P_2 < 0$

(1) Large t approximation

For simpler notation, let's define the constants

$$\begin{aligned} C_1 &= \frac{(B_n - B_{n+1})\sqrt{D_n}}{(1 - D_n)} \\ C_2 &= \frac{(B_n - B_{n+1})^2 D_n \sqrt{D_n}}{(1 - D_n)^2} \\ C_3 &= \frac{2D_n(B_{n+1} - B_n)}{(1 - D_n)(1 + \sqrt{D_n})} \end{aligned} \quad (2.121)$$

and a function

$$Q(t) = e^{-P_1 t} \{I_1(\beta t) + I_0(\beta t)\}, \quad (2.122)$$

then (2.83) is rewritten as

$$\tilde{R}^{TE}(t) = C_1 Q(t)u(t) + C_2 u(t) \int_t^\infty e^{-P_2(t-x)} Q(x) dx. \quad (2.123)$$

Using the change of variables $u = P_2(t - x)$, then $x = -\frac{u}{P_2} + t$, and $dx = -\frac{du}{P_2}$. Therefore $x = \infty$ leads to $u = \infty$, $x = t$ leads to $u = 0$ and (2.123) becomes

$$\begin{aligned}\tilde{R}^{TE}(t) &= C_1 Q(t)u(t) - C_2 \frac{1}{P_2} u(t) \int_0^\infty e^{-u} Q(-\frac{u}{P_2} + t) du \\ &= C_1 Q(t)u(t) - \frac{C_2}{P_2} u(t) \int_0^\infty e^{-u} \left\{ Q(t - \frac{u}{P_2}) - Q(t) \right\} du \\ &\quad + \frac{C_2}{P_2} Q(t)u(t) \int_0^\infty e^{-u} du\end{aligned}\tag{2.124}$$

In (2.124),

$$\begin{aligned}\int_0^\infty e^{-u} \left\{ Q(t - \frac{u}{P_2}) - Q(t) \right\} du &= \int_0^\infty e^{-u} \frac{\left\{ Q(t - \frac{u}{P_2}) - Q(t) \right\}}{(-\frac{u}{P_2})} (-\frac{u}{P_2}) du \\ &\approx -\frac{1}{P_2} Q'(t) \int_0^\infty u e^{-u} du.\end{aligned}\tag{2.125}$$

This approximation is possible since for large P_2 , most of the contribution to the integration occurs when u is small due to e^{-u} term. Also, $\int_0^\infty u e^{-u} du = 1$. Finally, for large t ,

$$\begin{aligned}\tilde{R}^{TE}(t) &\approx \frac{(B_n - B_{n+1})B_n \sqrt{D_n}}{(B_n - D_n B_{n+1})} Q(t) + \frac{(B_n - B_{n+1})^2 D_n \sqrt{D_n}}{(B_n - D_n B_{n+1})^2} Q'(t) \\ &= \frac{(B_n - B_{n+1})B_n \sqrt{D_n}}{(B_n - D_n B_{n+1})} e^{-P_1 t} \{I_1(\beta t) + I_0(\beta t)\} u(t) - \\ &\quad \frac{(B_n - B_{n+1})^2 D_n \sqrt{D_n}}{(B_n - D_n B_{n+1})^2} e^{-P_1 t} \left\{ (B_n + \frac{1}{t}) I_1(\beta t) + B_n I_0(\beta t) \right\} u(t).\end{aligned}\tag{2.126}$$

(2) Small t approximation

Recall that (2.83) originates from (2.75), then

$$\begin{aligned}\tilde{R}^{TE}(t) &= C_1 e^{-P_1 t} \{I_1(\beta t) + I_0(\beta t)\} u(t) - C_3 e^{-P_2 t} u(-t) \\ &\quad - C_2 e^{-P_2 t} u(-t) * e^{-P_1 t} \{I_1(\beta t) + I_0(\beta t)\} u(t)\end{aligned}\tag{2.127}$$

For $|P_2| \gg |P_1|$, the $e^{-P_2 t} u(-t)$ term behaves like a δ -function by its peaking property. Therefore,

$$\begin{aligned}\tilde{R}^{TE}(t) &\approx C_1 e^{-P_1 t} \{I_1(\beta t) + I_0(\beta t)\} u(t) - C_2 e^{-P_1 t} \{I_1(\beta t) + I_0(\beta t)\} u(t) + 0 \\ &\approx (C_1 - C_2) e^{-P_1 t} \{0 + 1\} u(t) = A e^{-P_1 t} u(t)\end{aligned}\quad (2.128)$$

for small t , where A is a constant which will be determined later.

For $|P_2| \approx |P_1|$, $-P_2 \approx P_1$ and

$$\begin{aligned}\tilde{R}^{TE}(t) &= C_1 e^{-P_1 t} \{I_1(\beta t) + I_0(\beta t)\} u(t) \\ &\quad - C_2 e^{+P_1 t} u(-t) * e^{-P_1 t} \{I_1(\beta t) + I_0(\beta t)\} u(t) + C_3 e^{P_1 t} u(-t) \\ &\approx C_1 e^{-P_1 t} \{0 + 1\} u(t) - C_2 e^{P_1 t} u(-t) * e^{-P_1 t} \{0 + 1\} u(t) + C_3 e^{P_1 t} u(-t) \\ &\approx C_1 e^{-P_1 t} u(t) - \frac{C_2}{2P_1} e^{P_1 t} + C_3 e^{P_1 t} u(-t)\end{aligned}$$

for small t . However, it has already been shown that the exact form does not have any non-causal term. Therefore,

$$\tilde{R}^{TE}(t) \approx C_1 e^{-P_1 t} u(t) - \frac{C_2}{2P_1} e^{P_1 t} u(t) \approx A e^{-P_1 t} u(t) \quad (2.129)$$

For $|P_2| \ll |P_1|$, the $e^{-P_1 t} u(t)$ term has the role of the δ -function, and

$$\begin{aligned}\tilde{R}^{TE}(t) &\approx C_1 e^{-P_1 t} u(t) + C_2 e^{-P_2 t} u(-t) + C_3 e^{-P_2 t} u(-t) \\ &\approx A C_3 e^{-P_1 t} u(t)\end{aligned}\quad (2.130)$$

where the second and third terms must be discarded to maintain causality. In a conclusion, the small t approximation for the $P_2 < 0$ case is given by

$$\tilde{R}^{TE}(t) \approx A e^{-P_1 t} u(t). \quad (2.131)$$

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(3) Combined approximation

From the large and small t approximations derived above, a direct combination is

$$\begin{aligned}\tilde{R}_{approx}^{TE}(t) &= \frac{(B_n - B_{n+1})B_n\sqrt{D_n}}{(B_n - D_nB_{n+1})}e^{-P_1t} \{I_1(\beta t) + I_0(\beta t)\} u(t) - \\ &\quad \frac{(B_n - B_{n+1})^2D_n\sqrt{D_n}}{(B_n - D_nB_{n+1})^2}e^{-P_1t} \left\{ \left(B_n + \frac{1}{t}\right)I_1(\beta t) + B_nI_0(\beta t) \right\} u(t) \\ &\quad + Ae^{-P_1t}u(t).\end{aligned}\tag{2.132}$$

The most intuitive way to determine the constant A is using the function value of the reduced reflection coefficient at $t = 0$, i.e. by letting $\tilde{R}_{approx}^{TE}(t)\big|_{t=0} = \tilde{R}_{exact}^{TE}(t)\big|_{t=0}$ and

$$\begin{aligned}\tilde{R}_{approx}^{TE}(t)\big|_{t=0} &= \frac{(B_n - B_{n+1})B_n\sqrt{D_n}}{(B_n - D_nB_{n+1})} \{0 + 1\} - \\ &\quad - \frac{(B_n - B_{n+1})^2D_n\sqrt{D_n}}{(B_n - D_nB_{n+1})^2} \left\{ \left(0 + \frac{\beta}{2}\right) + B_n \right\} + A \\ &= A + \frac{(B_n - B_{n+1})B_n\sqrt{D_n}}{(B_n - D_nB_{n+1})} \\ &\quad - \frac{(B_n - B_{n+1})^2D_n\sqrt{D_n}}{(B_n - D_nB_{n+1})^2} \frac{(B_{n+1} + 3B_n)}{4}\end{aligned}\tag{2.133}$$

$$\begin{aligned}\tilde{R}_{exact}^{TE}(t)\big|_{t=0} &= \frac{(B_n - B_{n+1})\sqrt{D_n}}{(1 - D_n)} - \frac{(B_n - B_{n+1})^2D_n\sqrt{D_n}}{(1 - D_n)^2} \times 0 \\ &\quad + \frac{2D_n(B_{n+1} - B_n)}{(1 - D_n)(1 + \sqrt{D_n})} \\ &= \frac{(B_n - B_{n+1})\sqrt{D_n}}{(1 + \sqrt{D_n})^2}.\end{aligned}\tag{2.134}$$

To evaluate A , equate (2.133) and (2.134) to give

$$\frac{(B_n - B_{n+1})\sqrt{D_n}}{(1 + \sqrt{D_n})^2} = \frac{(B_n - B_{n+1})B_n\sqrt{D_n}}{(B_n - D_nB_{n+1})}$$

$$-\frac{(B_n - B_{n+1})^2 D_n \sqrt{D_n} (B_{n+1} + 3B_n)}{(B_n - D_n B_{n+1})^2} \frac{1}{4} + A.$$

So,

$$\begin{aligned} A = & \frac{(B_n - B_{n+1})\sqrt{D_n}}{(1 + \sqrt{D_n})^2} - \frac{(B_n - B_{n+1})B_n\sqrt{D_n}}{(B_n - D_n B_{n+1})} \\ & + \frac{(B_n - B_{n+1})^2 D_n \sqrt{D_n} (B_{n+1} + 3B_n)}{(B_n - D_n B_{n+1})^2} \frac{1}{4} \end{aligned} \quad (2.135)$$

Therefore, for $P_2 < 0$, an approximate form is given by

$$\begin{aligned} \tilde{R}^{TE}(t) \approx & \frac{(B_n - B_{n+1})B_n\sqrt{D_n}}{(B_n - D_n B_{n+1})} e^{-P_1 t} \{I_1(\beta t) + I_0(\beta t)\} u(t) - \\ & \frac{(B_n - B_{n+1})^2 D_n \sqrt{D_n}}{(B_n - D_n B_{n+1})^2} e^{-P_1 t} \left\{ \left(B_n + \frac{1}{t}\right) I_1(\beta t) + B_n I_0(\beta t) \right\} u(t) \\ & + \left\{ \frac{(B_n - B_{n+1})\sqrt{D_n}}{(1 + \sqrt{D_n})^2} - \frac{(B_n - B_{n+1})B_n\sqrt{D_n}}{(B_n - D_n B_{n+1})} \right. \\ & \left. + \frac{(B_n - B_{n+1})^2 D_n \sqrt{D_n} (B_{n+1} + 3B_n)}{(B_n - D_n B_{n+1})^2} \frac{1}{4} \right\} e^{-P_1 t} u(t) \end{aligned} \quad (2.136)$$

Let's consider the accuracy of the approximate form (2.136). Actually the large t approximation is exact except for the term in (2.125), which is valid when $t \gg u/|P_2|$ (or $|P_2|t \gg u$) is satisfied. Note that the constant power P_1 is not involved with the large t approximation. For the small approximation, of course, the δ -function approximation of exponential term contributes to the error, especially when the ratio $|P_1|/|P_2|$ is not sufficiently big or small. However, the selection of the constant A using $\tilde{R}_{approx}^{TE}(t)\Big|_{t=0} = \tilde{R}_{large\ t\ approx}^{TE}(t)\Big|_{t=0} + \tilde{R}_{small\ t\ approx}^{TE}(t)\Big|_{t=0} = \tilde{R}_{exact}^{TE}(t)\Big|_{t=0}$ is the main source, because the function values from the large t approximation, which should be excluded in the small t interval (including $t = 0$), is imposed on the early time period. Therefore, the constant A should be given by $\tilde{R}_{exact}^{TE}(t)\Big|_{t=0} = \tilde{R}_{small\ t\ approx}^{TE}(t)\Big|_{t=0} = \frac{(B_n - B_{n+1})\sqrt{D_n}}{(1 + \sqrt{D_n})^2}$ only. However, this choice of A has trouble

as well, since it will introduce a discontinuity at the junction of the small t and large t approximations. Therefore, in this study, a pair of weighting functions are introduced to avoid the discontinuity. From observing the derivation process the approximate transient reflection coefficient, it can be realized that the constant P_2 behaves like a ‘switch’ which turns ‘on’ and ‘off’ the validity of the large t approximation, as a power factor of an exponential function. Also, note that P_2 does not appear in the small t approximation. In addition to these, the product $P_2 t$ provides a normalized time axis. As a result of these observations, a pair of weighting functions is selected to combine the large and small t approximation functions as

$$\begin{aligned} W_L(t) &= (1 - e^{-a|P_2|t}) \\ W_S(t) &= e^{-a|P_2|t} \end{aligned} \quad (2.137)$$

In this definition, $W_L(t)$ is a weighting function for the large t approximation, while $W_S(t)$ is for the small t approximation. The constant transition factor a determines the transition time from small t approximation to large t approximation. That is, the bigger value of a means an earlier transition from the small t to the large t approximation. Although the choice of a has some flexibility, in this study, the selection method is as follows. First, set the value of reflection coefficient at the ‘transition’ time as the exact transition reflection coefficient which is given by, for example, $0.5 \tilde{R}_{exact}^{TE}(t) \Big|_{t=0}$. Next find the time corresponding to the function value, say, t_{trans} . Then a is obtained by letting $e^{-a|P_2|t_{trans}} = 0.5$.

In summary, the complete approximation for $P_2 < 0$ is given by

$$\begin{aligned} \tilde{R}^{TE}(t) \approx & W_L(t) \left[\frac{(B_n - B_{n+1})B_n\sqrt{D_n}}{(B_n - D_n B_{n+1})} e^{-P_1 t} \{I_1(\beta t) + I_0(\beta t)\} u(t) - \right. \\ & \left. \frac{(B_n - B_{n+1})^2 D_n \sqrt{D_n}}{(B_n - D_n B_{n+1})^2} e^{-P_1 t} \left\{ \left(B_n + \frac{1}{t}\right) I_1(\beta t) + B_n I_0(\beta t) \right\} u(t) \right] \end{aligned}$$

$$\begin{aligned}
& +W_S(t) \frac{(B_n - B_{n+1})\sqrt{D_n}}{(1 + \sqrt{D_n})^2} e^{-P_1 t} u(t) \\
= & W_S(t) \frac{2\beta\sqrt{D_n}}{(1 + \sqrt{D_n})^2} e^{-P_1 t} u(t) \\
& -W_L(t) \frac{\beta^2\sqrt{D_n}}{P_2(1 - D)} \left[\frac{B_n}{\beta} e^{-P_1 t} \{I_1(\beta t) + I_0(\beta t)\} \right. \\
& \left. + \frac{4D}{P_2(1 - D)} e^{-P_1 t} \left\{ (B_n + \frac{1}{t}) I_1(\beta t) + B_n I_0(\beta t) \right\} \right] u(t). \quad (2.138)
\end{aligned}$$

A example of the transient reduced interfacial reflection coefficient for this case is plotted along the normalized time axis of $a|P_2|t$ in Figure 2.13. It can be observed that the largest error occurs in the transition region as expected. Small t approximation shows relatively poorer performance than large t approximation, since it relies on one exponential term only. Also, it is observed that the large t approximation is well matched with the exact reflection coefficient curve in large t interval.

2.5.2 Case II : $P_2 > 0$

(1) Large t approximation

Using previous definitions for the constants and the function $Q(t)$, (2.74) is rewritten as

$$\tilde{R}^{TE}(t) = C_1 Q(t) u(t) - C_2 u(t) \int_0^t e^{-P_2(t-x)} Q(x) dx + C_3 e^{-P_2 t} u(t). \quad (2.139)$$

Let $u = P_2(x - t)$, then $x = \frac{u}{P_2} + t$ and $dx = \frac{du}{P_2}$. Also, $x = t$ for $u = 0$ and $x = 0$ for $u = -P_2 t$. Thus (2.139) becomes

$$\begin{aligned}
\tilde{R}^{TE}(t) &= C_1 Q(t) u(t) - C_2 u(t) \int_{-P_2 t}^0 e^u Q(x) dx + C_3 e^{-P_2 t} u(t) \\
&= C_1 Q(t) u(t) - \frac{C_2}{P_2} u(t) \int_{-P_2 t}^0 e^u Q(t + \frac{u}{P_2}) du + C_3 e^{-P_2 t} u(t) \\
&= C_1 Q(t) u(t) - \frac{C_2}{P_2} u(t) \int_{-P_2 t}^0 e^u \left\{ Q(t + \frac{u}{P_2}) - Q(t) \right\} du
\end{aligned}$$

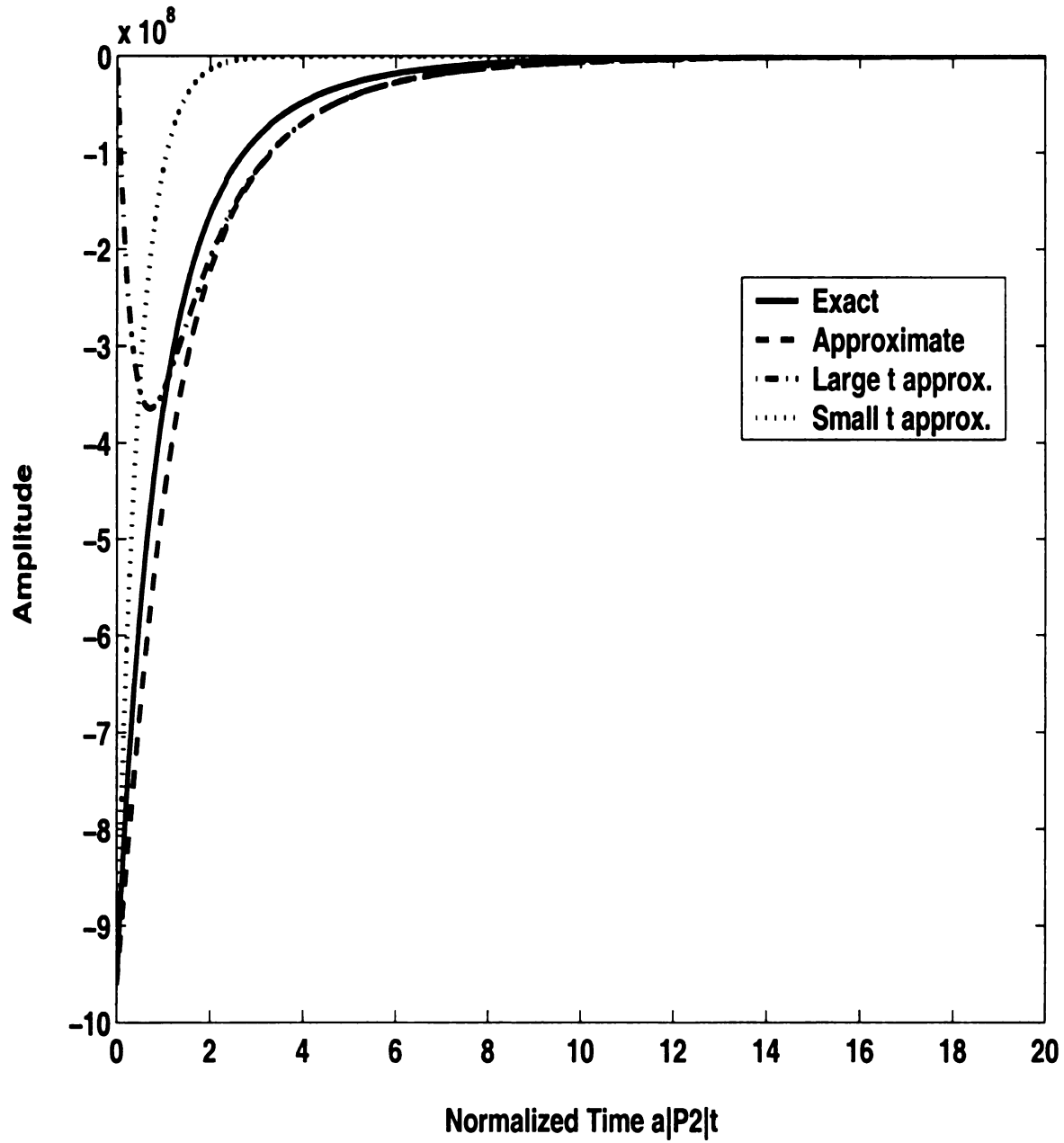


Figure 2.13. Transient interfacial reflection coefficient for $P_2 < 0$ (TE polarization) : $\mu_n = \mu_0$, $\epsilon_n = 2.59\epsilon_0$, $\sigma_n = 9.73 \times 10^{-3}[\text{U}/m]$, $\mu_{n+1} = \mu_0$, $\epsilon_{n+1} = 1.7\epsilon_0$, $\sigma_{n+1} = 5.60 \times 10^{-2}[\text{U}/m]$, $\theta_{i1} = 30^\circ$, $P_2 = -0.59 \times 10^{10}$, and $a = 0.37$.

$$-\frac{C_2}{P_2}u(t) \int_{-P_2t}^0 e^u Q(t) du + C_3 e^{-P_2t} u(t), \quad (2.140)$$

and the approximate form is

$$\begin{aligned} \tilde{R}^{TE}(t) \approx & C_1 Q(t) u(t) - \frac{C_2}{P_2} (1 - e^{-P_2t}) Q(t) u(t) \\ & - \frac{C_2}{P_2^2} u(t) \int_{-P_2t}^0 e^u Q'(t) du + C_3 e^{-P_2t} u(t) \end{aligned} \quad (2.141)$$

for large t. This can be rewritten as

$$\begin{aligned} \tilde{R}_{approx}^{TE}(t) &= (C_1 - \frac{C_2}{P_2}) Q(t) u(t) + \frac{C_2}{P_2} e^{-P_2t} Q(t) u(t) \\ &\quad - \frac{C_2}{P_2^2} Q'(t) u(t) \int_{-P_2t}^0 e^u du + C_3 e^{-P_2t} u(t) \\ &= (C_1 - \frac{C_2}{P_2}) Q(t) u(t) + \frac{C_2}{P_2} e^{-P_2t} Q(t) u(t) + C_3 e^{-P_2t} u(t) \\ &\quad + \frac{C_2}{P_2^2} Q'(t) u(t) \{1 - (P_2t + 1)e^{P_2t}\} \end{aligned}$$

where

$$\begin{aligned} Q'(t) &= -P_1 e^{-P_1t} \{I_1(\beta t) + I_0(\beta t)\} + \beta e^{-P_1t} \left\{ I_1(\beta t) - \frac{1}{\beta t} I_1(\beta t) + I_0(\beta t) \right\} \\ &= -e^{-P_1t} \left\{ (B_n + \frac{1}{t}) I_1(\beta t) + B_n I_0(\beta t) \right\}. \end{aligned} \quad (2.142)$$

Therefore, the large t approximation of transient interfacial reflection coefficient for $P_2 > 0$ is

$$\begin{aligned} \tilde{R}^{TE}(t) \approx & \left\{ \frac{(B_n - B_{n+1})B_n \sqrt{D_n}}{(B_n - D_n B_{n+1})} + \frac{(B_n - B_{n+1})^2 D_n \sqrt{D_n}}{(1 - D_n)(B_n - D_n B_{n+1})} e^{-P_2t} \right\} \times \\ & e^{-P_1t} \left\{ I_1\left(\frac{(B_{n+1} - B_n)}{2}t\right) + I_0\left(\frac{(B_{n+1} - B_n)}{2}t\right) \right\} u(t) \\ & - \frac{(B_n - B_{n+1})^2 D_n \sqrt{D_n}}{(B_n - D_n B_{n+1})^2} \{1 - (P_2t + 1)e^{P_2t}\} e^{-P_1t} \times \end{aligned}$$

$$\left\{ \left(B_n + \frac{1}{t} \right) I_1 \left(\frac{(B_{n+1} - B_n)}{2} t \right) + B_n I_0 \left(\frac{(B_{n+1} - B_n)}{2} t \right) \right\} u(t) + \frac{2D_n(B_{n+1} - B_n)}{(1 - D_n)(1 + \sqrt{D_n})} e^{-P_2 t} u(t). \quad (2.143)$$

(2) Small t approximation

It has been shown that (2.74) can be written in exactly the same form as (2.83), and therefore also as (2.75), as long as 6.611.4 in [10] is applicable. Let's consider the required condition for this conversion. As mentioned earlier, to use 6.611.4 in [10], $B_{n+1} > B_n$ and $0 < D_n < 1$, or $B_{n+1} < B_n$ and $D_n > 1$ must be satisfied. But, the given assumption for this case $P_2 = (B_n - D_n B_{n+1})(1 - D_n) > 0$ does not produce any relation for B_n and B_{n+1} that makes 6.611.4 applicable, since it is equivalent to either one of $1 > D_n$ and $B_n > D_n B_{n+1}$, or $1 < D_n$ and $B_n < D_n B_{n+1}$. On the other hand, considering

$$\begin{aligned} P_1 - P_2 &= \frac{(B_{n+1} + B_n)}{2} - \left(\frac{B_n - D_n B_{n+1}}{1 - D_n} \right) \\ &= \frac{(1 + D_n)(B_{n+1} - B_n)}{2(1 - D_n)}, \end{aligned} \quad (2.144)$$

if $P_1 > P_2$ then 6.611.4 is applicable. Then,

$$\begin{aligned} \tilde{R}^{TE}(t) &= C_1 e^{-P_1 t} \{I_1(\beta t) + I_0(\beta t)\} u(t) \\ &\quad + C_2 e^{-P_2 t} u(-t) * e^{-P_1 t} \{I_1(\beta t) + I_0(\beta t)\} u(t) \\ &\quad - C_3 e^{-P_2 t} u(-t). \end{aligned} \quad (2.145)$$

If the ratio P_1/P_2 is sufficiently large, then the $e^{-P_1 t}$ term, by its peaking property, provides a good approximation of a δ -function. Thus

$$\tilde{R}^{TE}(t) \approx C_1 e^{-P_1 t} \{I_1(\beta t) + I_0(\beta t)\} u(t)$$

$$\begin{aligned}
& +C_2e^{-P_2t}u(t) * \delta(t) - C_3e^{-P_2t}u(-t) \\
& \approx C_1e^{-P_1t}u(t) + C_2e^{-P_2t}u(-t) - C_3e^{-P_2t}u(-t)
\end{aligned} \tag{2.146}$$

Previously, it has been proved that $\tilde{R}^{TE}(t)$ is purely causal for $\tilde{D}_{n+1} > 0$ and $\tilde{D}_n > 0$, and thus the non-causal components may be neglected. Consequently $\tilde{R}^{TE}(t) \approx Ae^{-P_1t}u(t)$ for small t , where A is a constant which will be determined later.

When $P_1 < P_2$, 6.611.4 is not applicable. Since $P_2 > P_1$, $e^{-P_2t} \approx \delta(t)$ and (2.74) is approximated as

$$\begin{aligned}
\tilde{R}^{TE}(t) &= C_1e^{-P_1t} \{I_1(\beta t) + I_0(\beta t)\} u(t) \\
&\quad - C_2e^{-P_2t}u(t) * e^{-P_1t} \{I_1(\beta t) + I_0(\beta t)\} u(t) \\
&\quad + C_3e^{-P_2t}u(-t) \\
&\approx C_1e^{-P_1t} \{0 + 1\} u(t) - C_2\delta(t) * e^{-P_1t} \{0 + 1\} u(t) + C_3 \cdot 0 \\
&\approx Ae^{-P_1t}u(t)
\end{aligned} \tag{2.147}$$

where A is a constant. Consequently, the small t approximation for $P_2 > 0$ includes the $Ae^{-P_1t}u(t)$ term only, and the constant A is obtained by letting $\tilde{R}_{exact}^{TE}(t)\Big|_{t=0} = \tilde{R}_{small\ t\ approx}^{TE}(t)\Big|_{t=0}$, and

$$A = \frac{2D_n(B_{n+1} - B_n)}{(1 - D_n)(1 + \sqrt{D_n})}. \tag{2.148}$$

As a result, in a way similar to the $P_2 < 0$ case, the approximation of a transient interfacial reflection coefficient for $P_2 > 0$ may be constructed from the combination of the large and small t approximations using the same pair of weighting functions

$$\begin{aligned}
\tilde{R}^{TE}(t) &\approx W_L(t) \times \\
&\quad \left\{ \frac{(B_n - B_{n+1})B_n\sqrt{D_n}}{(B_n - D_nB_{n+1})} + \frac{(B_n - B_{n+1})^2D_n\sqrt{D_n}}{(1 - D_n)(B_n - D_nB_{n+1})} e^{-P_2t} \right\} \times \\
&\quad e^{-P_1t} \left\{ I_1\left(\frac{(B_{n+1} - B_n)}{2}t\right) + I_0\left(\frac{(B_{n+1} - B_n)}{2}t\right) \right\} u(t)
\end{aligned}$$

$$\begin{aligned}
& - \frac{(B_n - B_{n+1})^2 D_n \sqrt{D_n}}{(B_n - D_n B_{n+1})^2} \{1 - (P_2 t + 1)e^{P_2 t}\} e^{-P_1 t} \times \\
& \left\{ \left(B_n + \frac{1}{t}\right) I_1\left(\frac{(B_{n+1} - B_n)}{2} t\right) + B_n I_0\left(\frac{(B_{n+1} - B_n)}{2} t\right) u(t) \right\} \\
& + \frac{2D_n(B_{n+1} - B_n)}{(1 - D_n)(1 + \sqrt{D_n})} e^{-P_1 t} u(t) + W_S(t) \frac{2D_n(B_{n+1} - B_n)}{(1 - D_n)(1 + \sqrt{D_n})} e^{P_1 t} u(t)
\end{aligned} \tag{2.149}$$

where $W_L(t)$ and $W_S(t)$ are given in (2.137). An example of the approximation for $P_2 > 0$ is shown in Figure 2.14. In comparison with that shown in Figure 2.13, the combined approximation shows relatively better result in this example. The individual errors from each approximation are cancelled out each other in transition region, and thus the combined approximate function provides a good match with the exact reflection coefficient curve.

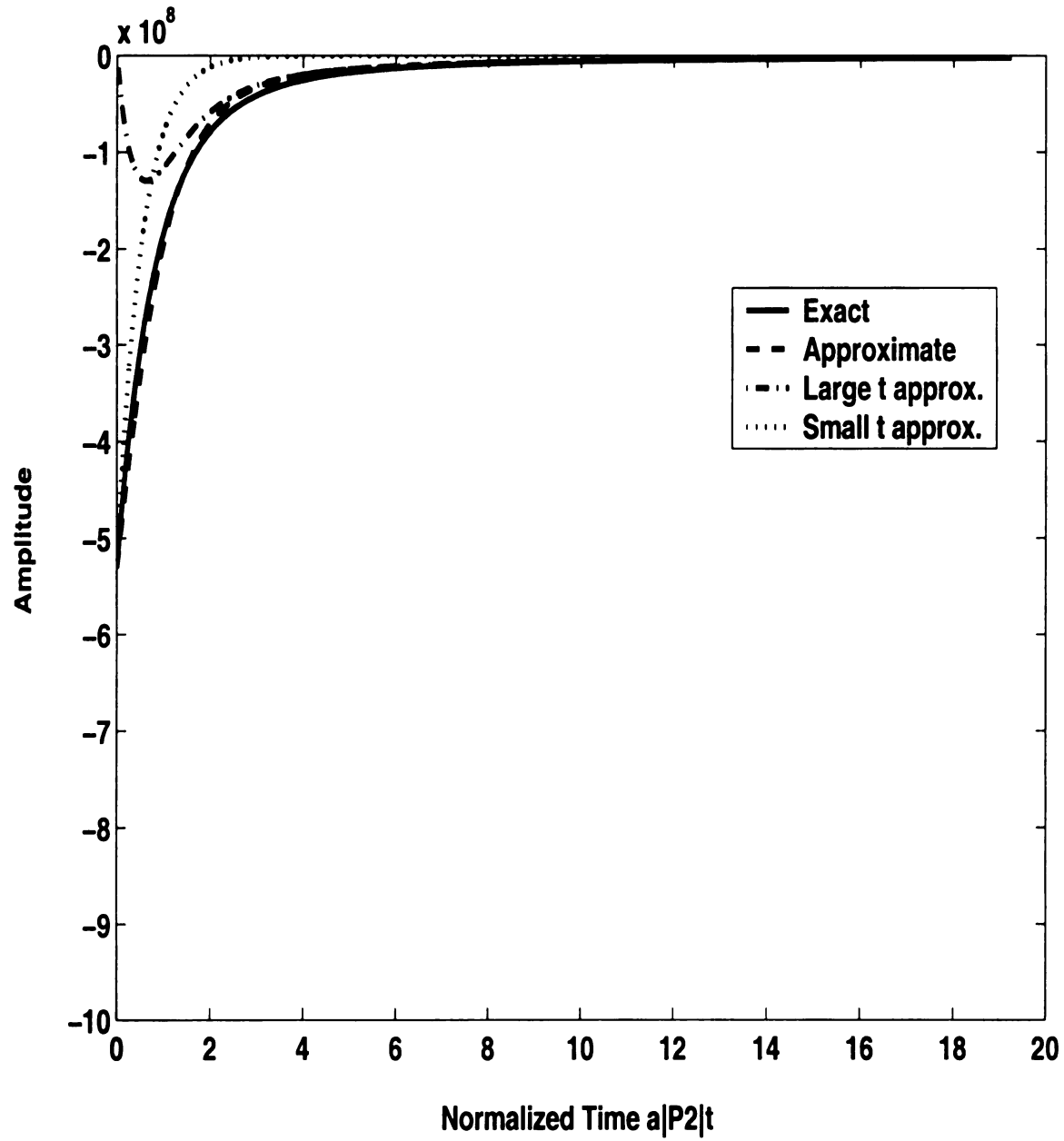


Figure 2.14. Transient interfacial reflection coefficient for $P_2 > 0$ (TE polarization)
: $\mu_n = \mu_0$, $\epsilon_n = \epsilon_0$, $\sigma_n = 0[\text{U}/m]$, $\mu_{n+1} = \mu_0$, $\epsilon_{n+1} = 72\epsilon_0$, $\sigma_{n+1} = 4[\text{U}/m]$, $\theta_{i1} = 30^\circ$,
 $P_2 = 0.64 \times 10^{10}$ and $a = 0.65$.

CHAPTER 3

INTERFACIAL REFLECTION COEFFICIENTS FOR TM-POLARIZATION

3.1 Introduction

In this chapter, the transient interfacial reflection coefficient for a transverse magnetic (TM) polarized incident plane wave is derived. For a TM polarized uniform plane wave, the direction of the magnetic field, rather than that of electric field as for TE polarization, is parallel to the interface. By a similar approach to that for TE polarization, the frequency domain formula is derived first, and its inverse Fourier transform is obtained after reducing and classifying the frequency domain reflection coefficients using algebraic manipulation with the previously defined branch cuts. The resulting reflection coefficients for the ratio of incident and reflected electric field are similar to those for TE polarization but more complicated as expected. The theoretically derived transient forms are verified by numerical comparisons with the IFFT of the frequency domain expressions. Finally, the dependence on various parameter sets is discussed.

3.2 Frequency Domain Formulation of Interfacial Reflection Coefficient

3.2.1 Derivation

The geometry for the frequency domain formulation is shown in Figure 3.1. From Maxwell's equations,

$$\nabla \times \bar{E}(x, z, \omega) = -j\omega\mu_n\bar{H}(x, z, \omega) \quad (3.1)$$

$$\nabla \times \bar{H}(x, z, \omega) = (\sigma_n + j\omega\epsilon_n)\bar{E}(x, z, \omega) \quad (3.2)$$

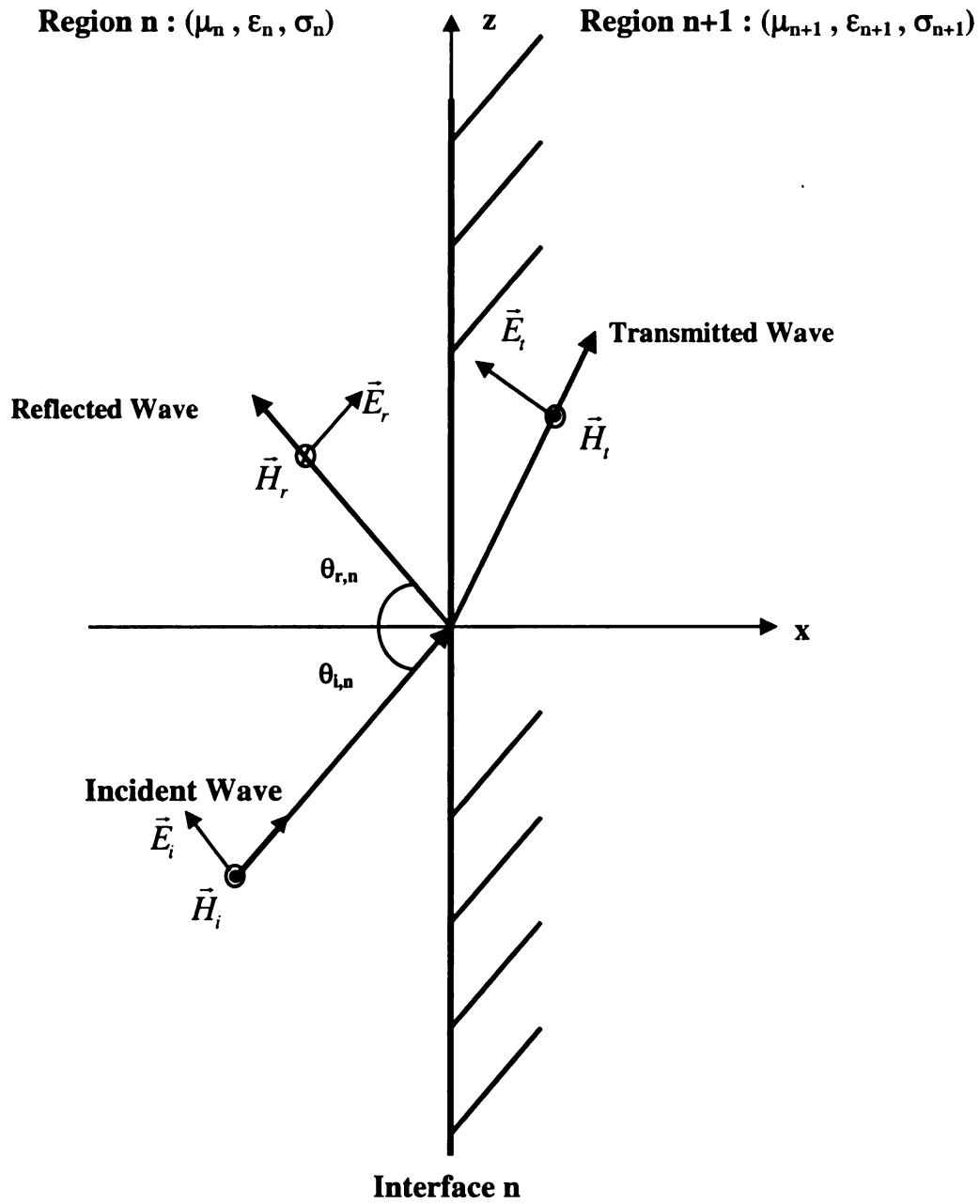


Figure 3.1. The incident, reflected and transmitted TM-polarized plane wave at an interface.

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and the vector Helmholtz equation for the magnetic field of region n is

$$\nabla^2 \bar{H}(x, z, \omega) - \gamma_n^2 \bar{H}(x, z, \omega) = 0 \quad (3.3)$$

where the propagation constant for the n-th layer is given by

$$\begin{aligned} \gamma_n^2 &= j\omega\mu_n(\sigma_n + j\omega\epsilon_n) \\ &= -\omega^2\mu_n\epsilon_n + j\omega\mu_n\sigma_n. \end{aligned} \quad (3.4)$$

For TM polarization $\bar{H} = \hat{y}H_y$ and (3.3) can be rewritten as

$$\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial z^2} \right) H_y(x, z, \omega) - \gamma_n^2 H_y(x, z, \omega) = 0. \quad (3.5)$$

The solution for the magnetic field is

$$\bar{H}(x, z, \omega) = \hat{y}H_o(\omega)e^{\gamma_{nx}x + \gamma_{nz}z} \quad (3.6)$$

where the components of the propagation constants are defined by

$$\gamma_n^2 = \gamma_{nx}^2 + \gamma_{nz}^2. \quad (3.7)$$

Only if region n is the first region of a multi-layered material and this region is lossless will we speak of an angle of incidence and an angle of reflection. Otherwise, we will use $\gamma_{nx}^i, \gamma_{nz}^i, \gamma_{nx}^r$ and γ_{nz}^r . For the incident wave,

$$\gamma_{nx} = \gamma_{nx}^i = \gamma_n \cos \theta_{in} \quad (3.8)$$

$$\gamma_{nz} = \gamma_{nz}^i = \gamma_n \sin \theta_{in}, \quad (3.9)$$

while for the reflected wave

$$\gamma_{nx} = \gamma_{nx}^r = -\gamma_n \cos \theta_{rn} \quad (3.10)$$

$$\gamma_{nz} = \gamma_{nz}^r = \gamma_n \sin \theta_{rn}. \quad (3.11)$$

Here $\theta_{i,n}$ is the angle of incidence and $\theta_{r,n}$ is the angle of reflection, as shown in Figure 3.1. The electric field in region n is obtained by

$$\begin{aligned} \bar{E}(x, z, \omega) &= \frac{1}{\sigma_n + j\omega\mu_n} \nabla \times \bar{H}(x, z, \omega) \\ &= \frac{j\omega\mu_n}{\gamma_n^2} \left\{ -\hat{x} \frac{\partial H_y}{\partial z} + \hat{z} \frac{\partial H_y}{\partial x} \right\} \\ &= \frac{j\omega\mu_n H_o(\omega)}{\gamma_n^2} \left\{ -\hat{x} \gamma_{nz} e^{\gamma_{nx}x + \gamma_{nz}z} + \hat{z} \gamma_{nx} e^{\gamma_{nx}x + \gamma_{nz}z} \right\} \\ &= -\hat{x} \frac{j\omega\mu_n \gamma_{nz}}{\gamma_n^2} H_o(\omega) e^{\gamma_{nx}x + \gamma_{nz}z} + \hat{z} \frac{j\omega\mu_n \gamma_{nx}}{\gamma_n^2} H_o(\omega) e^{\gamma_{nx}x + \gamma_{nz}z}. \end{aligned} \quad (3.12)$$

In summary, the fields in region n can be expressed as

$$\begin{aligned} \bar{H}_i(x, z, \omega) &= \hat{y} H_{io}(\omega) e^{\gamma_{nx}^i x + \gamma_{nz}^i z} \\ \bar{E}_i(x, z, \omega) &= -\hat{x} \frac{j\omega\mu_n \gamma_{nz}^i}{\gamma_n^2} H_{io}(\omega) e^{\gamma_{nx}^i x + \gamma_{nz}^i z} \\ &\quad + \hat{z} \frac{j\omega\mu_n \gamma_{nx}^i}{\gamma_n^2} H_{io}(\omega) e^{\gamma_{nx}^i x + \gamma_{nz}^i z} \end{aligned} \quad (3.13)$$

$$\begin{aligned} \bar{H}_r(x, z, \omega) &= -\hat{y} H_{ro}(\omega) e^{\gamma_{nx}^r x + \gamma_{nz}^r z} \\ \bar{E}_r(x, z, \omega) &= \hat{x} \frac{\gamma_{nz}^r}{j\omega\mu_n} H_{io}(\omega) e^{\gamma_{nx}^r x + \gamma_{nz}^r z} - \hat{z} \frac{\gamma_{nx}^r}{j\omega\mu_n} H_{io}(\omega) \end{aligned} \quad (3.14)$$

Notice that, unlike with TE polarization, the direction of the reflected magnetic field is reversed from that of the incident field while the electric field maintains the same direction at the interface.

In region (n+1), (3.3) is

$$\nabla^2 \bar{H}(x, z, \omega) - \gamma_{n+1}^2 \bar{H}(x, z, \omega) = 0 \quad (3.15)$$

where the propagation constant is

$$\begin{aligned} \gamma_{n+1}^2 &= j\omega\mu_{n+1}(\sigma_{n+1} + j\omega\epsilon_{n+1}) \\ &= -\omega^2\mu_{n+1}\epsilon_{n+1} + j\omega\mu_{n+1}\sigma_{n+1}. \end{aligned} \quad (3.16)$$

Then, the vector Helmholtz equation for the magnetic field in region n+1 (i.e., the transmitted magnetic field) is given by

$$\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial z^2} \right) H_y(x, z, \omega) - \gamma_{n+1}^2 H_y(x, z, \omega) = 0. \quad (3.17)$$

This has the solution

$$\bar{H}_t(x, z, \omega) = \hat{y} H_{to}(\omega) e^{\gamma_{n+1,x}x + \gamma_{n+1,z}z} \quad (3.18)$$

where the components of the propagation constants are defined by

$$\gamma_{n+1}^2 = \gamma_{n+1,x}^2 + \gamma_{n+1,z}^2. \quad (3.19)$$

When both regions n and n+1 are lossless, we may define a transmission angle θ_t such that

$$\gamma_{n+1,x} = \gamma_{n+1} \cos \theta_t \quad (3.20)$$

$$\gamma_{n+1,z} = \gamma_{n+1} \sin \theta_t. \quad (3.21)$$

The transmitted magnetic field is given by

$$\begin{aligned}
\bar{E}_t(x, z, \omega) &= -\frac{j\omega\mu_{n+1}}{\gamma_{n+1}^2} \nabla \times \bar{H}_t(x, z, \omega) \\
&= -\frac{j\omega\mu_{n+1}}{\gamma_{n+1}^2} \left\{ -\hat{x} \frac{\partial H_y}{\partial z} + \hat{z} \frac{\partial H_y}{\partial x} \right\} \\
&= \frac{j\omega\mu_{n+1} H_{to}(\omega)}{\gamma_{n+1}^2} \left\{ -\hat{x} \gamma_{n+1,z} e^{\gamma_{n+1,x}x + \gamma_{n+1,z}z} + \hat{z} \gamma_{n+1,x} e^{\gamma_{n+1,x}x + \gamma_{n+1,z}z} \right\} \\
&= -\hat{x} \frac{j\omega\mu_{n+1}}{\gamma_{n+1}^2} \gamma_{n+1,z} H_{to}(\omega) e^{\gamma_{n+1,x}x + \gamma_{n+1,z}z} \\
&\quad + \hat{z} \frac{j\omega\mu_{n+1}}{\gamma_{n+1}^2} \gamma_{n+1,x} H_{to}(\omega) e^{\gamma_{n+1,x}x + \gamma_{n+1,z}z}.
\end{aligned} \tag{3.22}$$

In summary, the fields in region (n+1) can be expressed as

$$\begin{aligned}
\bar{H}_t(x, z, \omega) &= \hat{y} H_{to}(\omega) e^{\gamma_{n+1,x}x + \gamma_{n+1,z}z} \\
\bar{E}_t(x, z, \omega) &= -\hat{x} \frac{j\omega\mu_{n+1}}{\gamma_{n+1}^2} \gamma_{n+1,z} H_{to}(\omega) e^{\gamma_{n+1,x}x + \gamma_{n+1,z}z} \\
&\quad + \hat{z} \frac{j\omega\mu_{n+1}}{\gamma_{n+1}^2} \gamma_{n+1,x} H_{to}(\omega) e^{\gamma_{n+1,x}x + \gamma_{n+1,z}z}
\end{aligned} \tag{3.23}$$

To obtain the ratio of the reflected and transmitted electric field amplitudes, two boundary conditions are applied. The first one is the boundary condition for continuity of the tangential components of the magnetic field at the interface. That is, for all z ,

$$\bar{H}_i(x = 0^-) + \bar{H}_r(x = 0^-) = \bar{H}_t(x = 0^+)$$

or,

$$H_{io}(\omega) e^{\gamma_{n,z}^i z} - H_{ro}(\omega) e^{\gamma_{n,z}^r z} = H_{to}(\omega) e^{\gamma_{n+1,z} z}. \tag{3.24}$$

Then, to satisfy the above boundary condition, the equations

$$\gamma_{nz}^i = \gamma_{nz}^r = \gamma_{n+1,z} \quad (3.25)$$

$$H_{io}(\omega) - H_{ro}(\omega) = H_{to}(\omega) \quad (3.26)$$

must be satisfied. For lossless materials, (3.25) becomes

$$\gamma_n \sin \theta_i = \gamma_n \sin \theta_r = \gamma_{n+1} \sin \theta_t \quad (3.27)$$

and thus

$$\theta_i = \theta_r \quad (3.28)$$

$$\frac{\gamma_n}{\gamma_{n+1}} = \frac{\sin \theta_t}{\sin \theta_i} \quad (3.29)$$

which are Snell's law of reflection and refraction, consistent with the result derived in Chapter 2. From (3.7), (3.25) and considering the direction of propagation,

$$\gamma_{nx}^i = -\gamma_{nx}^r = \gamma_{nx}. \quad (3.30)$$

The second boundary condition requires the continuity of tangential electric field on the interface. That is, for all z ,

$$\bar{E}_{iz}(x = 0^-) + \bar{E}_{rz}(x = 0^-) = \bar{E}_{tz}(x = 0^+)$$

or,

$$\gamma_{nx}^i \frac{j\omega\mu_n}{\gamma_n^2} H_{io}(\omega) e^{\gamma_{nz}^i z} - \gamma_{nx}^r \frac{j\omega\mu_n}{\gamma_n^2} H_{ro}(\omega) e^{\gamma_{nz}^r z} = \gamma_{n+1,x} \frac{j\omega\mu_{n+1}}{\gamma_{n+1}^2} H_{to}(\omega) e^{\gamma_{n+1,z} z}. \quad (3.31)$$

Using (3.30), this equation simplifies to

$$\frac{\mu_n}{\gamma_n^2} \gamma_{nx} H_{io}(\omega) + \frac{\mu_n}{\gamma_n^2} \gamma_{nx} H_{ro}(\omega) = \frac{\mu_{n+1}}{\gamma_{n+1}^2} \gamma_{n+1,x} H_{to}(\omega). \quad (3.32)$$

Alternatively,

$$H_{io}(\omega) + H_{ro}(\omega) = \frac{\mu_{n+1} \gamma_{n+1,x} \gamma_n^2}{\mu_n \gamma_{nx} \gamma_{n+1}^2} H_{to}(\omega). \quad (3.33)$$

Now, multiplying (3.26) by $\frac{\mu_{n+1} \gamma_{n+1,x} \gamma_n^2}{\mu_n \gamma_{nx} \gamma_{n+1}^2}$ and subtracting (3.33) yields

$$\left(\frac{\mu_{n+1} \gamma_{n+1,x} \gamma_n^2}{\mu_n \gamma_{nx} \gamma_{n+1}^2} - 1 \right) H_{io}(\omega) - \left(\frac{\mu_{n+1} \gamma_{n+1,x} \gamma_n^2}{\mu_n \gamma_{nx} \gamma_{n+1}^2} + 1 \right) H_{ro}(\omega) = 0 \quad (3.34)$$

Therefore, the frequency domain interfacial reflection coefficient for TM-polarized plane waves is

$$\begin{aligned} R^{TM}(\omega) &= \frac{H_{ro}(\omega)}{H_{io}(\omega)} \\ &= \left(\frac{\mu_{n+1} \gamma_{n+1,x} \gamma_n^2}{\mu_n \gamma_{nx} \gamma_{n+1}^2} - 1 \right) / \left(\frac{\mu_{n+1} \gamma_{n+1,x} \gamma_n^2}{\mu_n \gamma_{nx} \gamma_{n+1}^2} + 1 \right) \\ &= \frac{\mu_{n+1} \gamma_{n+1,x}(\omega) \gamma_n^2(\omega) - \mu_n \gamma_{nx}(\omega) \gamma_{n+1}^2}{\mu_{n+1} \gamma_{n+1,x}(\omega) \gamma_n^2(\omega) + \mu_n \gamma_{nx}(\omega) \gamma_{n+1}^2} \end{aligned} \quad (3.35)$$

This is a generalized form of Fresnel's reflection coefficient for TM-polarized incident and reflected waves at an interface in a layered structure. Note that the form of $R^{TM}(\omega)$ is the same as that given in [1].

3.2.2 Classification of frequency domain coefficients

The Fresnel's coefficient given by (3.35) can be classified according to the signs of some constant values included within it. Therefore, layer constants may be defined as

$$A_n = \sigma_n / \epsilon_n \quad (3.36)$$

$$\tilde{D}_n = (\mu_n \epsilon_n - \mu_1 \epsilon_1 \sin^2 \theta_{i1}) / \epsilon_n^2 \quad (3.37)$$

$$\tilde{B}_n = \mu_n \sigma_n / \epsilon_n^2. \quad (3.38)$$

Note that A_n and $\tilde{B}_n$ are always positive, and the definitions of layer parameters are slightly different from those for TE polarization. To compute the square root functions, the same branch cuts described in the TE case are applied. This depends on the signs of A_n , $\tilde{D}_n$, $\tilde{B}_n$, A_{n+1} , $\tilde{D}_{n+1}$ and $\tilde{B}_{n+1}$.

Denoting $s = j\omega$ and using the defined layer constants, (3.35) may be rewritten as

$$R^{TM}(s) = \frac{(s + A_n) \sqrt{s(\tilde{D}_n s + \tilde{B}_n)} - (s + A_{n+1}) \sqrt{s(\tilde{D}_{n+1} s + \tilde{B}_{n+1})}}{(s + A_n) \sqrt{s(\tilde{D}_n s + \tilde{B}_n)} + (s + A_{n+1}) \sqrt{s(\tilde{D}_{n+1} s + \tilde{B}_{n+1})}}. \quad (3.39)$$

Now, the frequency domain reflection coefficient can be classified to four different forms according to which branch cut is needed. This depends on the signs of the layer parameters $\tilde{D}_n$ and $\tilde{D}_{n+1}$. The algebraic manipulations with the branch cuts used in the derivation processes are given by (A.1) and (A.2).

(1) $\tilde{D}_{n+1} > 0$ and $\tilde{D}_n > 0$

$$R^{TM}(s) = \frac{-(s + A_n) \sqrt{\tilde{D}_{n+1}} \sqrt{s} \sqrt{s + \frac{\tilde{B}_{n+1}}{\tilde{D}_{n+1}}} + (s + A_{n+1}) \sqrt{\tilde{D}_n} \sqrt{s} \sqrt{s + \frac{\tilde{B}_n}{\tilde{D}_n}}}{-(s + A_n) \sqrt{\tilde{D}_{n+1}} \sqrt{s} \sqrt{s + \frac{\tilde{B}_{n+1}}{\tilde{D}_{n+1}}} - (s + A_{n+1}) \sqrt{\tilde{D}_n} \sqrt{s} \sqrt{s + \frac{\tilde{B}_n}{\tilde{D}_n}}}.$$

Simplifying gives

$$R^{TM}(s) = \frac{(s + A_n) \sqrt{s + \frac{\tilde{B}_{n+1}}{\tilde{D}_{n+1}}} - (s + A_{n+1}) \sqrt{\frac{\tilde{D}_n}{\tilde{D}_{n+1}}} \sqrt{s + \frac{\tilde{B}_n}{\tilde{D}_n}}}{(s + A_n) \sqrt{s + \frac{\tilde{B}_{n+1}}{\tilde{D}_{n+1}}} + (s + A_{n+1}) \sqrt{\frac{\tilde{D}_n}{\tilde{D}_{n+1}}} \sqrt{s + \frac{\tilde{B}_n}{\tilde{D}_n}}}. \quad (3.40)$$

(2) $\tilde{D}_{n+1} < 0$ and $\tilde{D}_n < 0$

$$R^{TM}(s) = \frac{\pm j(s + A_n) \sqrt{|\tilde{D}_{n+1}|} \sqrt{s} \sqrt{s - \frac{\tilde{B}_{n+1}}{|\tilde{D}_{n+1}|}} \mp j(s + A_{n+1}) \sqrt{|\tilde{D}_n|} \sqrt{s} \sqrt{s - \frac{\tilde{B}_n}{|\tilde{D}_n|}}}{\pm j(s + A_n) \sqrt{|\tilde{D}_{n+1}|} \sqrt{s} \sqrt{s - \frac{\tilde{B}_{n+1}}{|\tilde{D}_{n+1}|}} \pm j(s + A_{n+1}) \sqrt{|\tilde{D}_n|} \sqrt{s} \sqrt{s - \frac{\tilde{B}_n}{|\tilde{D}_n|}}}.$$

Simplifying gives

$$R^{TM}(s) = \frac{(s + A_n) \sqrt{s - \frac{\tilde{B}_{n+1}}{|\tilde{D}_{n+1}|}} - (s + A_{n+1}) \sqrt{\frac{|\tilde{D}_n|}{|\tilde{D}_{n+1}|}} \sqrt{s - \frac{\tilde{B}_n}{|\tilde{D}_n|}}}{(s + A_n) \sqrt{s - \frac{\tilde{B}_{n+1}}{|\tilde{D}_{n+1}|}} + (s + A_{n+1}) \sqrt{\frac{|\tilde{D}_n|}{|\tilde{D}_{n+1}|}} \sqrt{s - \frac{\tilde{B}_n}{|\tilde{D}_n|}}}. \quad (3.41)$$

(3) $\tilde{D}_{n+1} > 0$, and $\tilde{D}_n < 0$

$$R^{TM}(s) = \frac{-(s + A_n) \sqrt{|\tilde{D}_{n+1}|} \sqrt{s} \sqrt{s + \frac{\tilde{B}_{n+1}}{|\tilde{D}_{n+1}|}} \mp j(s + A_{n+1}) \sqrt{|\tilde{D}_n|} \sqrt{s} \sqrt{s - \frac{\tilde{B}_n}{|\tilde{D}_n|}}}{-(s + A_n) \sqrt{|\tilde{D}_{n+1}|} \sqrt{s} \sqrt{s + \frac{\tilde{B}_{n+1}}{|\tilde{D}_{n+1}|}} \pm j(s + A_{n+1}) \sqrt{|\tilde{D}_n|} \sqrt{s} \sqrt{s - \frac{\tilde{B}_n}{|\tilde{D}_n|}}}.$$

Simplifying gives

$$R^{TM}(s) = \frac{(s + A_n) \sqrt{s + \frac{\tilde{B}_{n+1}}{|\tilde{D}_{n+1}|}} \pm j(s + A_{n+1}) \sqrt{\frac{|\tilde{D}_n|}{|\tilde{D}_{n+1}|}} \sqrt{s - \frac{\tilde{B}_n}{|\tilde{D}_n|}}}{(s + A_n) \sqrt{s + \frac{\tilde{B}_{n+1}}{|\tilde{D}_{n+1}|}} \mp j(s + A_{n+1}) \sqrt{\frac{|\tilde{D}_n|}{|\tilde{D}_{n+1}|}} \sqrt{s - \frac{\tilde{B}_n}{|\tilde{D}_n|}}}. \quad (3.42)$$

(4) $\tilde{D}_{n+1} < 0$, and $\tilde{D}_n > 0$

$$R^{TM}(s) = \frac{\pm j(s + A_n) \sqrt{|\tilde{D}_{n+1}|} \sqrt{s} \sqrt{s - \frac{\tilde{B}_{n+1}}{|\tilde{D}_{n+1}|}} + (s + A_{n+1}) \sqrt{|\tilde{D}_n|} \sqrt{s} \sqrt{s + \frac{\tilde{B}_n}{|\tilde{D}_n|}}}{\pm j(s + A_n) \sqrt{|\tilde{D}_{n+1}|} \sqrt{s} \sqrt{s - \frac{\tilde{B}_{n+1}}{|\tilde{D}_{n+1}|}} - (s + A_{n+1}) \sqrt{|\tilde{D}_n|} \sqrt{s} \sqrt{s + \frac{\tilde{B}_n}{|\tilde{D}_n|}}}.$$

Simplifying gives

$$R^{TM}(s) = \frac{(s + A_n) \sqrt{s - \frac{\tilde{B}_{n+1}}{|\tilde{D}_{n+1}|}} \mp j(s + A_{n+1}) \sqrt{\frac{|\tilde{D}_n|}{|\tilde{D}_{n+1}|}} \sqrt{s + \frac{\tilde{B}_n}{|\tilde{D}_n|}}}{(s + A_n) \sqrt{s - \frac{\tilde{B}_{n+1}}{|\tilde{D}_{n+1}|}} \pm j(s + A_{n+1}) \sqrt{\frac{|\tilde{D}_n|}{|\tilde{D}_{n+1}|}} \sqrt{s + \frac{\tilde{B}_n}{|\tilde{D}_n|}}}. \quad (3.43)$$

In each of these expressions the upper sign corresponds to $\omega > 0$ and the lower sign

to $\omega < 0$. Note again that the branch cuts defined in section 2.2.2 have been used throughout for the square root functions.

3.2.3 Reduction of the interfacial reflection coefficients

Observing (3.40)-(3.43), it is realized that there is a constant offset value inside of each frequency domain expression, as there was with the TE polarization. Therefore, the constant offset value, the asymptotic reflection coefficient, should be extracted first.

To make the (3.40)-(3.43) more readable, let

$$\begin{aligned} D_n &= \sqrt{\frac{|\tilde{D}_n|}{|\tilde{D}_{n+1}|}} = \sqrt{\frac{\epsilon_{n+1}^2}{\epsilon_n^2} \frac{|\mu_n \epsilon_n - \mu_1 \epsilon_1 \sin^2 \theta_{i1}|}{|\mu_{n+1} \epsilon_{n+1} - \mu_1 \epsilon_1 \sin^2 \theta_{i1}|}} \\ B_n &= \frac{\tilde{B}_n}{|\tilde{D}_n|} = \frac{\mu_n \sigma_n}{|\mu_n \epsilon_n - \mu_1 \epsilon_1 \sin^2 \theta_{i1}|}. \end{aligned} \quad (3.44)$$

and take the limiting values at infinite frequency. Then,

(1) $\tilde{D}_{n+1} > 0$ and $\tilde{D}_n > 0$

$$R^{TM}(s) = \frac{(s + A_n)\sqrt{s + B_{n+1}} - \sqrt{D_n}(s + A_{n+1})\sqrt{s + B_n}}{(s + A_n)\sqrt{s + B_{n+1}} + \sqrt{D_n}(s + A_{n+1})\sqrt{s + B_n}}, \quad (3.45)$$

$$\lim_{\omega \rightarrow \infty} R^{TM}(s) = \frac{1 - \sqrt{D_n}}{1 + \sqrt{D_n}}. \quad (3.46)$$

(2) $\tilde{D}_{n+1} < 0$ and $\tilde{D}_n < 0$

$$R^{TM}(s) = \frac{(s + A_n)\sqrt{s - B_{n+1}} - \sqrt{D_n}(s + A_{n+1})\sqrt{s - B_n}}{(s + A_n)\sqrt{s - B_{n+1}} + \sqrt{D_n}(s + A_{n+1})\sqrt{s - B_n}}, \quad (3.47)$$

$$\lim_{\omega \rightarrow \infty} R^{TM}(s) = \frac{1 - \sqrt{D_n}}{1 + \sqrt{D_n}}. \quad (3.48)$$

(3) $\tilde{D}_{n+1} > 0$ and $\tilde{D}_n < 0$

$$R^{TM}(s) = \frac{(s + A_n)\sqrt{s + B_{n+1}} \pm j\sqrt{D_n}(s + A_{n+1})\sqrt{s - B_n}}{(s + A_n)\sqrt{s + B_{n+1}} \mp j\sqrt{D_n}(s + A_{n+1})\sqrt{s - B_n}}, \quad (3.49)$$

$$\lim_{\omega \rightarrow \infty} R^{TM}(s) = \frac{1 \pm j\sqrt{D_n}}{1 \mp j\sqrt{D_n}}. \quad (3.50)$$

(4) $\tilde{D}_{n+1} < 0$ and $\tilde{D}_n > 0$

$$R^{TM}(s) = \frac{(s + A_n)\sqrt{s - B_{n+1}} \mp j\sqrt{D_n}(s + A_{n+1})\sqrt{s + B_n}}{(s + A_n)\sqrt{s - B_{n+1}} \pm j\sqrt{D_n}(s + A_{n+1})\sqrt{s + B_n}}, \quad (3.51)$$

$$\lim_{\omega \rightarrow \infty} R^{TM}(s) = \frac{1 \mp j\sqrt{D_n}}{1 \pm j\sqrt{D_n}}. \quad (3.52)$$

In these expressions the upper sign corresponds to $\omega > 0$ and lower sign to $\omega < 0$.

Next, the obtained asymptotic reflection coefficient have to be subtracted from the original $R^{TM}(s)$. Let's denote the asymptotic reflection coefficient as R_∞^{TM} and define

$\tilde{R}^{TM}(s) = R^{TM}(s) - R_\infty^{TM}$, then

(1) $\tilde{D}_{n+1} > 0$ and $\tilde{D}_n > 0$

$$\begin{aligned} \tilde{R}^{TM}(s) &= \frac{(s + A_n)\sqrt{s + B_{n+1}} - (s + A_{n+1})\sqrt{D_n}\sqrt{s + B_n}}{(s + A_n)\sqrt{s + B_{n+1}} + (s + A_{n+1})\sqrt{D_n}\sqrt{s + B_n}} - \frac{1 - \sqrt{D_n}}{1 + \sqrt{D_n}} \\ &= \frac{2\sqrt{D_n}}{1 + \sqrt{D_n}} \frac{(s + A_n)\sqrt{s + B_{n+1}} - (s + A_{n+1})\sqrt{s + B_n}}{(s + A_n)\sqrt{s + B_{n+1}} + (s + A_{n+1})\sqrt{D_n}\sqrt{s + B_n}} \end{aligned} \quad (3.53)$$

Eliminating the square roots in the denominator of (3.53), gives

$$\begin{aligned} \tilde{R}^{TM}(s) &= \frac{2\sqrt{D_n}}{1 + \sqrt{D_n}} \frac{(s + A_n)\sqrt{s + B_{n+1}} - (s + A_{n+1})\sqrt{s + B_n}}{(s + A_n)^2(s + B_{n+1}) - D_n(s + A_{n+1})^2(s + B_n)} \\ &\quad \times \{(s + A_n)\sqrt{s + B_{n+1}} - \sqrt{D_n}(s + A_{n+1})\sqrt{s + B_n}\}. \end{aligned} \quad (3.54)$$

In the above equation, the numerator is given as

$$\begin{aligned} & (s + A_n)^2(s + B_{n+1}) + \sqrt{D_n}A_{n+1}^2(s + B_n) \\ & - (1 + \sqrt{D_n})(s + A_n)(s + A_{n+1})\sqrt{s + B_n}\sqrt{s + B_{n+1}} \end{aligned} \quad (3.55)$$

while the denominator can be arranged as

$$\begin{aligned} & (1 - D_n)s^3 + \{2(A_n - 2D_nA_{n+1}) + (B_{n+1} - B_n)\}s^2 \\ & + \{A_n(A_n + B_{n+1}) - D_nA_{n+1}(A_{n+1} + B_n)\}s + (A_n^2B_{n+1} - D_nA_{n+1}^2B_n) \end{aligned} \quad (3.56)$$

Zeros of the denominator exist, therefore (3.54) can be expanded using partial fractions and a standard inverse Fourier transform is available for each of those fraction terms. However, that would be quite complex. So, the special but simpler cases of (3.54), i.e. when either one of the both layers has no conductivity, and $D_{n+1} > 0$ and $D_n > 0$ will be discussed in this study.

Assume that $\sigma_n = 0$, then $A_n = B_n = 0$ and (3.45) becomes

$$R^{TM}(s) = \frac{\sqrt{s}\sqrt{s + B_{n+1}} - \sqrt{D_n}(s + A_{n+1})}{\sqrt{s}\sqrt{s + B_{n+1}} + \sqrt{D_n}(s + A_{n+1})}. \quad (3.57)$$

Now, the asymptotic value still remains the same and is subtracted from the reflection coefficient (3.57) to obtain the reduced form as

$$\begin{aligned} \tilde{R}^{TM}(s) &= \frac{\sqrt{s + B_{n+1}} - \sqrt{D_n}(s + A_{n+1})}{\sqrt{s + B_{n+1}} + \sqrt{D_n}(s + A_{n+1})} - \frac{1 - \sqrt{D_n}}{1 + \sqrt{D_n}} \\ &= \frac{2\sqrt{D_n}}{1 + \sqrt{D_n}} \frac{\sqrt{s}\sqrt{s + B_{n+1}} - (s + A_{n+1})}{\sqrt{s}\sqrt{s + B_{n+1}} + \sqrt{D_n}(s + A_{n+1})}. \end{aligned} \quad (3.58)$$

Eliminating the square root terms in the denominator of (3.58), gives more convenient

form to use partial fraction expansion as

$$\begin{aligned}
& \tilde{R}^{TM}(s) \\
&= \frac{2\sqrt{D_n}}{1 + \sqrt{D_n}} \frac{\{\sqrt{s}\sqrt{s+B_{n+1}} - (s+A_{n+1})\}\{\sqrt{s}\sqrt{s+B_{n+1}} - \sqrt{D_n}(s+A_{n+1})\}}{s(s+B_{n+1}) - D_n(s+A_{n+1})^2} \\
&= \frac{2\sqrt{D_n}}{(1 + \sqrt{D_n})(1 - D_n)} \frac{(B_{n+1} - A_{n+1} + A_{n+1}\sqrt{D_n})s + A_{n+1}^2\sqrt{D_n}}{s^2 - \frac{2D_n A_{n+1} - B_{n+1}}{1 - D_n}s - \frac{D_n A_{n+1}^2}{1 - D_n}} \\
&+ \frac{2\sqrt{D_n}}{1 - D_n} \frac{s(s + A_{n+1})}{s^2 - \frac{2D_n A_{n+1} - B_{n+1}}{1 - D_n}s - \frac{D_n A_{n+1}^2}{1 - D_n}} \left\{ 1 - \sqrt{\frac{s+B_{n+1}}{s}} \right\}. \tag{3.59}
\end{aligned}$$

Known inverse Fourier transform pairs are available for all terms of (3.59). To make it more readable, denote the constants in the numerator as

$$\begin{aligned}
E &= B_{n+1} - A_{n+1} + A_{n+1}\sqrt{D_n} \\
F &= A_{n+1}^2\sqrt{D_n}
\end{aligned} \tag{3.60}$$

and the costants in the denominator

$$\begin{aligned}
b &= \frac{2D_n A_{n+1} - B_{n+1}}{1 - D_n} \\
c &= \frac{D_n A_{n+1}^2}{D_n - 1}.
\end{aligned} \tag{3.61}$$

Also, the common denominator of (3.59) can be expressed as $(s - P_1)(s - P_2)$ using the two roots of the equation $s^2 - bs + c = 0$. It is necessary to check the signs of the constants before finding P_1 and P_2 , because if they have certain restriction on the signs, then that will confine the location of the roots in the complex plane. Let's examine

$$b = \frac{\sigma_{n+1}\{\epsilon_n(2\mu_n\epsilon_{n+1} - \mu_{n+1}\epsilon_n) - 2\epsilon_{n+1}\mu_1\epsilon_1 \sin^2 \theta_{i1}\}}{\epsilon_{n+1}\epsilon_n(\mu_{n+1}\epsilon_n - \mu_n\epsilon_{n+1}) - (\epsilon_n^2 - \epsilon_{n+1}^2)\mu_1\epsilon_1 \sin^2 \theta_{i1}}$$

$$c = \frac{D_n \mu_{n+1} \epsilon_{n+1} - \mu_1 \epsilon_1 \sin^2 \theta_{i1}}{\epsilon_{n+1} \epsilon_n (\mu_{n+1} \epsilon_n - \mu_n \epsilon_{n+1}) - (\epsilon_n^2 - \epsilon_{n+1}^2) \mu_1 \epsilon_1 \sin^2 \theta_{i1}} \frac{\epsilon_n^2 \sigma_{n+1}^2}{\epsilon_{n+1}^2} \quad (3.62)$$

Thus b and c do not have any restriction on their signs even for the special case of $\mu_n = \mu_{n+1} = \mu_1$, and P_1 and P_2 can be any type of number. Using the partial fraction expansion, the frequency domain reflection coefficient (3.59) is rewritten as

$$\begin{aligned} \tilde{R}^{TM}(s) &= \frac{2\sqrt{D_n}}{(1 + \sqrt{D_n})(1 - D_n)} \frac{Es + F}{(s - P_1)(s - P_2)} + \frac{2\sqrt{D_n}}{1 - D_n} \frac{s(s + A_{n+1})}{(s - P_1)(s - P_2)} \\ &= \frac{2\sqrt{D_n}}{(1 + \sqrt{D_n})(1 - D_n)} \left\{ \frac{K_1}{s - P_1} + \frac{K_2}{s - P_2} \right\} \\ &\quad + \frac{2\sqrt{D_n}}{1 - D_n} \left\{ \frac{sK_3}{s - P_1} + \frac{sK_4}{s - P_2} \right\} \left\{ 1 - \sqrt{\frac{s + B_{n+1}}{s}} \right\}. \end{aligned} \quad (3.63)$$

and the constant coefficients are defined by

$$\begin{aligned} K_1 &= \left. \frac{Es + F}{s - P_2} \right|_{s=P_1} = \frac{EP_1 + F}{P_1 - P_2} \\ K_2 &= \left. \frac{Es + F}{s - P_1} \right|_{s=P_2} = \frac{EP_2 + F}{P_2 - P_1} \\ K_3 &= \left. \frac{s + A_{n+1}}{s - P_2} \right|_{s=P_1} = \frac{P_1 + A_{n+1}}{P_1 - P_2} \\ K_4 &= \left. \frac{s + A_{n+1}}{s - P_1} \right|_{s=P_2} = \frac{P_2 + A_{n+1}}{P_2 - P_1}. \end{aligned} \quad (3.64)$$

Notice that $K_2 = K_1^*$ and $K_4 = K_3^*$ where K^* indicates a complex conjugate of K .

The roots P_1 and P_2 are obtained as

$$\begin{aligned} P_1 &= (b + \sqrt{b^2 - 4c}) \\ P_2 &= (b - \sqrt{b^2 - 4c}). \end{aligned} \quad (3.65)$$

It should be noticed that the roots are in the relation of a complex conjugate pair, i.e.

$P_2 = P_1^*$. It has been already shown that b has no restriction on its sign. Consider

the sign of

$$\begin{aligned}
b^2 - 4c &= \frac{(2D_n A_{n+1} - B_{n+1})^2}{(1 - D_n)^2} - \frac{4A_{n+1}^2}{D_n - 1} \\
&= \frac{4(D_n^2 - D_n + 1)A_{n+1}^2 + B_{n+1}(B_{n+1} - 4D_n A_{n+1})}{(1 - D_n)^2}
\end{aligned} \tag{3.66}$$

where $4(D_n^2 - D_n + 1)A_{n+1}^2 > 0$ always, and

$$\begin{aligned}
B_{n+1}(B_{n+1} - 4D_n A_{n+1}) &= \frac{\mu_{n+1}\sigma_{n+1}}{|\mu_{n+1}\epsilon_{n+1} - \mu_1\epsilon_1 \sin^2 \theta_{i1}|} \\
&\quad - \frac{4\sigma_{n+1}}{\epsilon_{n+1}} \frac{\epsilon_{n+1}^2}{\epsilon_n^2} \left| \frac{\mu_n\epsilon_n - \mu_1\epsilon_1 \sin^2 \theta_{i1}}{|\mu_{n+1}\epsilon_{n+1} - \mu_1\epsilon_1 \sin^2 \theta_{i1}|} \right| \\
&= \frac{\sigma_{n+1} \{ \mu_{n+1}\epsilon_n^2 - 4\epsilon_{n+1} |\mu_n\epsilon_n - \mu_1\epsilon_1 \sin^2 \theta_{i1}| \}}{\epsilon_n |\mu_{n+1}\epsilon_{n+1} - \mu_1\epsilon_1 \sin^2 \theta_{i1}|}.
\end{aligned} \tag{3.67}$$

The definition (3.67) can have any value, even for $\mu_{n+1} = \mu_n = \mu_1$. As a result, $b^2 - 4c$ can have any sign, and therefore P_1 and P_2 can be any type of number.

Assume that $\sigma_{n+1} = 0$, then $A_{n+1} = B_{n+1} = 0$ and (3.45) is simplified to

$$R^{TM}(s) = \frac{(s + A_n) - \sqrt{D_n} \sqrt{s} \sqrt{s + B_n}}{(s + A_n) + \sqrt{D_n} \sqrt{s} \sqrt{s + B_n}}. \tag{3.68}$$

Observing this expression carefully, it can be recognized that by replacing A_n , B_n and D_n with A_{n+1} , B_{n+1} and $1/D_n$ respectively, $R^{TM}(s)$ for $\sigma_{n+1} = 0$ is equivalent to $-R^{TM}(s)$ for $\sigma_n = 0$. That is, the reflection coefficients for both cases are in ‘dual’ relationship. Therefore, it is expected that the transient reflection coefficients for both cases have the same relationship.

3.3 Derivation of Transient Interfacial Reflection Coefficients

The transient forms of the frequency domain interfacial reflection coefficients are given by the inverse Fourier transform. Because the asymptotic reflection coefficients R_∞^{TM}

are constants in the frequency domain, their transforms are simply given as delta-functions multiplied by the constants. Let's denote them by $R_\infty^{TM}(t)$. For $\tilde{D}_{n+1} > 0$ and $\tilde{D}_n > 0$,

$$\begin{aligned} R_\infty^{TM}(t) &= F^{-1} \left\{ \lim_{\omega \rightarrow \infty} R^{TM}(s) \right\} \\ &= \frac{1 - \sqrt{D_n}}{1 + \sqrt{D_n}} \delta(t). \end{aligned} \quad (3.69)$$

and the resulting transient interfacial reflection coefficient is given by

$$\begin{aligned} R^{TM}(t) &= F^{-1} \left\{ \lim_{\omega \rightarrow \infty} R^{TM}(s) \right\} + F^{-1} \left\{ \tilde{R}^{TM}(s) \right\} \\ &= \frac{1 - \sqrt{D_n}}{1 + \sqrt{D_n}} \delta(t) + \tilde{R}^{TM}(t). \end{aligned} \quad (3.70)$$

By the 'dual' relationship of $\sigma_n = 0$ and $\sigma_{n+1} = 0$ mentioned earlier, only the transient interfacial reflection coefficients for $\sigma_n = 0$ will be derived. Then, what is needed to obtain the transient form for the dual $\sigma_{n+1} = 0$ case is just taking negative sign of the obtained transient reflection coefficient for $\sigma_n = 0$. The transform pair for each term in (3.63) can be found in Appendix B.

Since P_1 and P_2 can be any number, real or complex, the transient reflection coefficient has five different forms depending on the roots of the frequency domain denominator. Let's denote

$$\begin{aligned} P_r &= \text{Re}\{P_1\} = \text{Re}\{P_2\} \\ P_i &= \text{Im}\{P_1\} = -\text{Im}\{P_2\}. \end{aligned} \quad (3.71)$$

From (3.64), the constants K_1 , K_2 , K_3 and K_4 are generally complex numbers as

well, and using their complex conjugate relationship, let

$$\begin{aligned}
K_{1r} &= \operatorname{Re}\{K_1\} = \operatorname{Re}\{K_2\} \\
K_{1i} &= \operatorname{Im}\{K_1\} = -\operatorname{Im}\{K_2\} \\
K_{3r} &= \operatorname{Re}\{K_3\} = \operatorname{Re}\{K_4\} \\
K_{3i} &= \operatorname{Im}\{K_3\} = -\operatorname{Im}\{K_4\}
\end{aligned} \tag{3.72}$$

and denote $\beta = \frac{B_{n+1}}{2}$.

(1) When P_1 and P_2 are complex numbers, and $P_r < 0$,

the transform of (3.63) is given by

$$\begin{aligned}
\tilde{R}^{TM}(t) &= \frac{2\sqrt{D_n}}{(1 - D_n)(1 + \sqrt{D_n})} \{K_1 e^{P_1 t} + K_2 e^{P_2 t}\} u(t) + \\
&\quad \frac{2\sqrt{D_n}}{1 - D_n} [K_3 e^{P_1 t} \{\delta(t) + P_1 u(t)\} + K_4 e^{P_2 t} \{\delta(t) + P_2 u(t)\}] * \\
&\quad (-\beta) e^{-\beta t} \{I_1(\beta t) + I_0(\beta t)\} u(t).
\end{aligned} \tag{3.73}$$

Here $I_n(t)$ is the first kind modified Bessel function of order n . Each term in this expression can be simplified using (3.71) and (3.72) as

$$\begin{aligned}
K_1 e^{P_1 t} + K_2 e^{P_2 t} &= \operatorname{Re}\{K_1\} e^{P_r + jP_i t} + j\operatorname{Im}\{K_1\} e^{P_r + jP_i t} + \\
&\quad \operatorname{Re}\{K_1\} e^{P_r + jP_i t} - j\operatorname{Im}\{K_1\} e^{P_r + jP_i t} \\
&= K_{1r} e^{P_r t} (e^{jP_i t} + e^{-jP_i t}) + jK_{1i} e^{P_r t} (e^{jP_i t} - e^{-jP_i t}) \\
&= 2e^{P_r t} (K_{1r} \cos P_i t - K_{1i} \sin P_i t),
\end{aligned} \tag{3.74}$$

$$K_3 e^{P_1 t} \delta(t) + K_4 e^{P_2 t} \delta(t) = K_3 \delta(t) + K_4 \delta(t) = 2K_{3r} \delta(t), \tag{3.75}$$

and

$$\begin{aligned}
K_3 P_1 e^{P_1 t} + K_4 P_2 e^{P_2 t} &= (K_{3r} + jK_{3i})(P_r + jP_i)e^{P_r + jP_i t} + \\
&\quad (K_{3r} - jK_{3i})(P_r - jP_i)e^{P_r - jP_i t} \\
&= e^{P_r t} \{ K_{3r} P_r (e^{jP_i t} + e^{-jP_i t}) - K_{3i} P_i (e^{jP_i t} + e^{-jP_i t}) \\
&\quad jK_{3r} P_i (e^{jP_i t} - e^{-jP_i t}) + jK_{3i} P_r (e^{jP_i t} - e^{-jP_i t}) \} \\
&= 2e^{P_r t} \{ (K_{3r} P_r - K_{3i} P_i) \cos P_i t - (K_{3r} P_i + K_{3i} P_r) \sin P_i t \}.
\end{aligned} \tag{3.76}$$

Finally, the transient interfacial reflection coefficient is

$$\begin{aligned}
&\tilde{R}^{TM}(t) \\
&= \frac{4\sqrt{D_n}}{(1 - D_n)(1 + \sqrt{D_n})} e^{P_r t} (K_{1r} \cos P_i t - K_{1i} \sin P_i t) u(t) + \frac{4\sqrt{D_n}}{1 - D_n} [K_{3r} \delta(t) \\
&\quad + e^{P_r t} \{ (K_{3r} P_r - K_{3i} P_i) \cos P_i t - (K_{3r} P_i + K_{3i} P_r) \sin P_i t \} u(t)] * \\
&\quad (-\beta) e^{-\beta t} \{ I_1(\beta t) + I_0(\beta t) \} u(t) \\
&= \frac{4\sqrt{D_n}}{(1 - D_n)(1 + \sqrt{D_n})} e^{P_r t} (K_{1r} \cos P_i t - K_{1i} \sin P_i t) u(t) \\
&\quad - \frac{4K_{3r} \beta \sqrt{D_n}}{1 - D_n} e^{-\beta t} \{ I_1(\beta t) + I_0(\beta t) \} u(t) \\
&\quad - \frac{4\beta \sqrt{D_n}}{1 - D_n} (K_{3r} P_r - K_{3i} P_i) e^{P_r t} u(t) \times \\
&\quad \int_0^t \cos P_i(t - x) e^{-(P_r + \beta)x} \{ I_1(\beta x) + I_0(\beta x) \} dx \\
&\quad + \frac{4\beta \sqrt{D_n}}{1 - D_n} (K_{3r} P_r + K_{3i} P_i) e^{P_r t} u(t) \times \\
&\quad \int_0^t \sin P_i(t - x) e^{-(P_r + \beta)x} \{ I_1(\beta x) + I_0(\beta x) \} dx \\
&= \frac{4\sqrt{D_n}}{(1 - D_n)(1 + \sqrt{D_n})} e^{P_r t} (K_{1r} \cos P_i t - K_{1i} \sin P_i t) u(t) \\
&\quad - \frac{4K_{3r} \beta \sqrt{D_n}}{1 - D_n} e^{-\beta t} \{ I_1(\beta t) + I_0(\beta t) \} u(t) - \frac{4\beta \sqrt{D_n}}{1 - D_n} e^{P_r t} u(t) \times
\end{aligned}$$

$$\int_0^t \{(K_{3r}P_r - K_{3i}P_i) \cos P_i(t-x) - (K_{3r}P_r + K_{3i}P_i) \sin P_i(t-x)\} \times e^{-(P_r+\beta)x} \{I_1(\beta x) + I_0(\beta x)\} dx. \quad (3.77)$$

(2) When P_1 and P_2 are complex numbers, and $P_r > 0$,

the transform of (3.63) is given by

$$\begin{aligned} \tilde{R}^{TM}(t) &= -\frac{2\sqrt{D_n}}{(1-D_n)(1+\sqrt{D_n})} \{K_1 e^{P_1 t} + K_2 e^{P_2 t}\} u(-t) + \\ &\quad \frac{2\sqrt{D_n}}{1-D_n} [K_3 e^{P_1 t} \{\delta(t) - P_1 u(-t)\} + K_4 e^{P_2 t} \{\delta(t) - P_2 u(-t)\}] * \\ &\quad (-\beta) e^{-\beta t} \{I_1(\beta t) + I_0(\beta t)\} u(t) \\ &= -\frac{4\sqrt{D_n}}{(1-D_n)(1+\sqrt{D_n})} e^{P_r t} (K_{1r} \cos P_i t + K_{1i} \sin P_i t) u(-t) + \frac{4\sqrt{D_n}}{1-D_n} \times \\ &\quad [K_{3r} \delta(t) + e^{P_r t} \{(K_{3r}P_r - K_{3i}P_i) \cos P_i t - (K_{3r}P_i + K_{3i}P_r) \sin P_i t\} \times \\ &\quad u(-t)] * (-\beta) e^{-\beta t} \{I_1(\beta t) + I_0(\beta t)\} u(t) \\ &= \frac{4\sqrt{D_n}}{(1-D_n)(1+\sqrt{D_n})} e^{P_r t} (K_{1i} \sin P_i t - K_{1r} \cos P_i t) u(-t) \\ &\quad - \frac{4K_{3r}\beta\sqrt{D_n}}{1-D_n} e^{-\beta t} \{I_1(\beta t) + I_0(\beta t)\} u(t) \\ &\quad - \frac{4\beta\sqrt{D_n}}{1-D_n} e^{P_r t} \int_{\max(t,0)}^{\infty} \{(K_{3r}P_r - K_{3i}P_i) \cos P_i(t-x) \\ &\quad - (K_{3r}P_r + K_{3i}P_i) \sin P_i(t-x)\} e^{-(P_r+\beta)x} \{I_1(\beta x) + I_0(\beta x)\} dx. \quad (3.78) \end{aligned}$$

(3) When P_1 and P_2 are real numbers, and $P_1 > P_2 > 0$,

the transform of (3.63) is given by

$$\begin{aligned} \tilde{R}^{TM}(t) &= -\frac{2\sqrt{D_n}}{(1-D_n)(1+\sqrt{D_n})} \{K_1 e^{P_1 t} + K_2 e^{P_2 t}\} u(-t) + \\ &\quad \frac{2\sqrt{D_n}}{1-D_n} [K_3 e^{P_1 t} \{\delta(t) - P_1 u(-t)\} + K_4 e^{P_2 t} \{\delta(t) - P_2 u(-t)\}] \end{aligned}$$

$$\begin{aligned}
& *(-\beta)e^{-\beta t} \{I_1(\beta t) + I_0(\beta t)\} u(t) \\
= & -\frac{2\sqrt{D_n}}{(1-D_n)(1+\sqrt{D_n})} \{K_1e^{P_1t} + K_2e^{P_2t}\} u(-t) \\
& -\frac{2\beta\sqrt{D_n}}{1-D_n} (K_3 + K_4)e^{-\beta t} \{I_1(\beta t) + I_0(\beta t)\} u(t) \\
& +\frac{2K_3P_1\beta\sqrt{D_n}}{1-D_n} e^{P_1t} \int_{\max(t,0)}^{\infty} e^{-(P_1+\beta)x} \{I_1(\beta x) + I_0(\beta x)\} dx \\
& +\frac{2K_4P_2\beta\sqrt{D_n}}{1-D_n} e^{P_2t} \int_{\max(t,0)}^{\infty} e^{-(P_2+\beta)x} \{I_1(\beta x) + I_0(\beta x)\} dx. \quad (3.79)
\end{aligned}$$

(4) When P_1 and P_2 are real numbers, and $P_1 > 0 > P_2$,

the transform of (3.63) is given by

$$\begin{aligned}
& \tilde{R}^{TM}(t) \\
= & \frac{2\sqrt{D_n}}{(1-D_n)(1+\sqrt{D_n})} \{-K_1e^{P_1t}u(-t) + K_2e^{P_2t}u(t)\} + \\
& \frac{2\sqrt{D_n}}{1-D_n} [K_3e^{P_1t} \{\delta(t) - P_1u(-t)\} + K_4e^{P_2t} \{\delta(t) + P_2u(t)\}] \\
& *(-\beta)e^{-\beta t} \{I_1(\beta t) + I_0(\beta t)\} u(t) \\
= & \frac{2\sqrt{D_n}}{(1-D_n)(1+\sqrt{D_n})} \{-K_1e^{P_1t}u(-t) + K_2e^{P_2t}u(t)\} \\
& -\frac{2\beta\sqrt{D_n}}{1-D_n} (K_3 + K_4)e^{-\beta t} \{I_1(\beta t) + I_0(\beta t)\} u(t) \\
& +\frac{2K_3P_1\beta\sqrt{D_n}}{1-D_n} e^{P_1t} \int_{\max(t,0)}^{\infty} e^{-(P_1+\beta)x} \{I_1(\beta x) + I_0(\beta x)\} dx \\
& -\frac{2K_4P_2\beta\sqrt{D_n}}{1-D_n} e^{P_2t}u(t) \int_0^t e^{-(P_2+\beta)x} \{I_1(\beta x) + I_0(\beta x)\} dx. \quad (3.80)
\end{aligned}$$

(5) When P_1 and P_2 are real numbers, and $0 > P_1 > P_2$,

the transform of (3.63) is given by

$$\begin{aligned}
& \tilde{R}^{TM}(t) \\
= & \frac{2\sqrt{D_n}}{(1-D_n)(1+\sqrt{D_n})} \{K_1e^{P_1t} + K_2e^{P_2t}\} u(t) +
\end{aligned}$$

$$\begin{aligned}
& \frac{2\sqrt{D_n}}{1-D_n} [K_3 e^{P_1 t} \{\delta(t) + P_1 u(t)\} + K_4 e^{P_2 t} \{\delta(t) + P_2 u(t)\}] \\
& * (-\beta) e^{-\beta t} \{I_1(\beta t) + I_0(\beta t)\} u(t) \\
= & \frac{2\sqrt{D_n}}{(1-D_n)(1+\sqrt{D_n})} \{K_1 e^{P_1 t} + K_2 e^{P_2 t}\} u(t) \\
& - \frac{2\beta\sqrt{D_n}}{1-D_n} (K_3 + K_4) e^{-\beta t} \{I_1(\beta t) + I_0(\beta t)\} u(t) \\
& - \frac{2K_3 P_1 \beta \sqrt{D_n}}{1-D_n} e^{P_1 t} u(t) \int_0^t e^{-(P_1+\beta)x} \{I_1(\beta x) + I_0(\beta x)\} dx \\
& - \frac{2K_4 P_2 \beta \sqrt{D_n}}{1-D_n} e^{P_2 t} u(t) \int_0^t e^{-(P_2+\beta)x} \{I_1(\beta x) + I_0(\beta x)\} dx. \quad (3.81)
\end{aligned}$$

Notice that, for TM polarization, there are non-causal terms even for the $D_{n+1} > 0$ and $D_n > 0$ case.

3.4 Numerical Examples

3.4.1 Verification of theoretical expressions

The previously derived transient forms of the interfacial reflection coefficients may be compared to direct numerical computation to ensure that the transforms are correct. Again, the numerical IFFT, is used for this purpose. For the reasons explained in the previous chapter, only the causal case will be discussed.

Since the asymptotic interfacial reflection coefficients appear as delta functions in the time domain, and are thus difficult to express numerically, only the reduced interfacial reflection coefficients are compared. Figure 3.2 shows the comparison of the derived transient interfacial reflection coefficient with the transient forms obtained by using a 3072 point IFFT for $\sigma_n = 0$. The material parameters for Figure 3.2 are those for free-space and typical sea water at low frequency as described in [20]. The transient reflection coefficient curves from the two different methods are nearly identical, and thus the derived transient interfacial reflection coefficient for TM polarization has been verified.

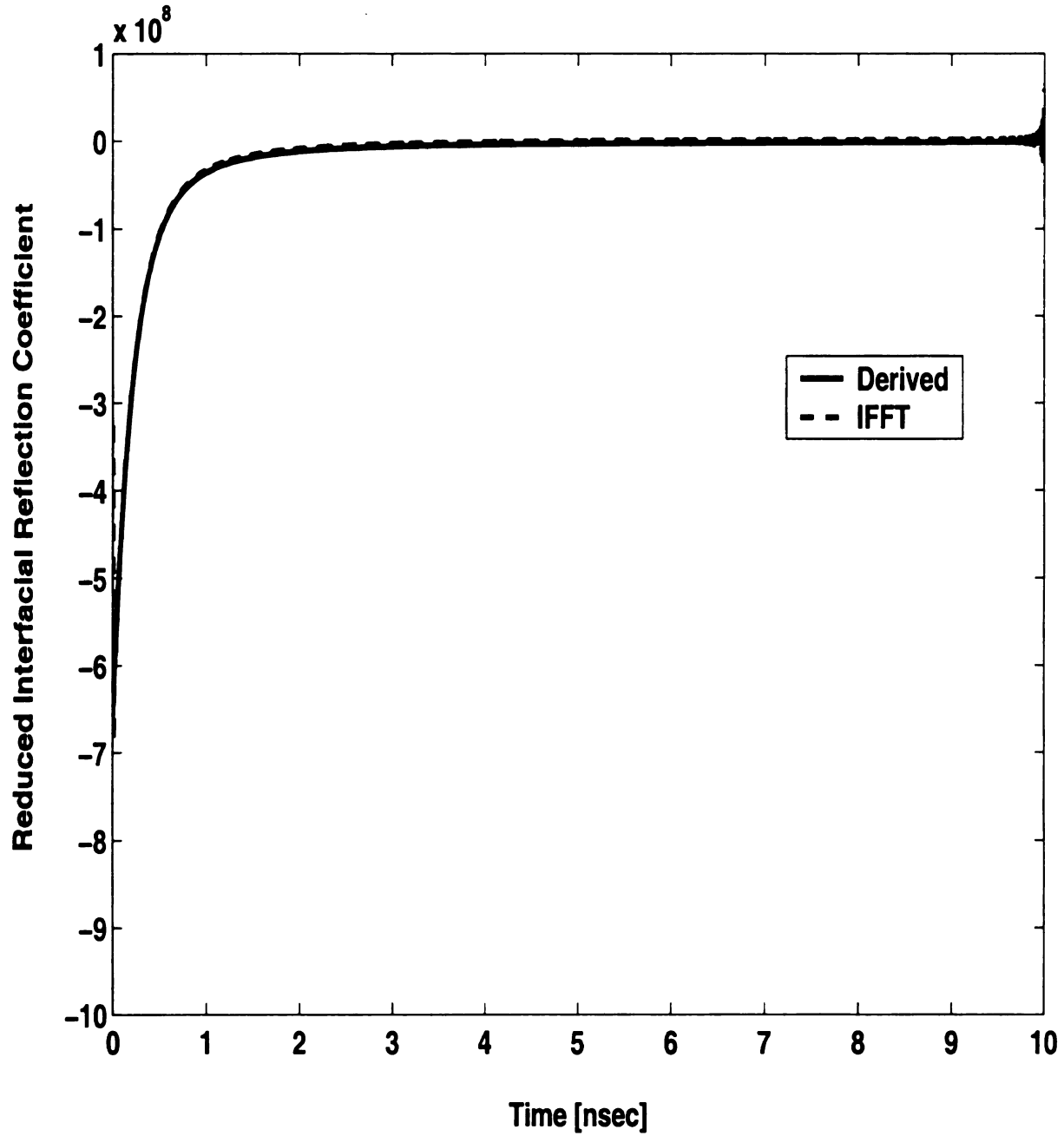


Figure 3.2. Numerical comparison of the derived transient reduced interfacial reflection coefficient with that from the IFFT (TM polarization) : $\mu_n = \mu_0$, $\epsilon_n = \epsilon_0$, $\sigma_n = 0[\text{U}/m]$, $\mu_{n+1} = \mu_0$, $\epsilon_{n+1} = 72\epsilon_0$, $\sigma_{n+1} = 4[\text{U}/m]$, $\theta_{i1} = 30^\circ$.

To verify the combination of the asymptotic and the reduced transient reflection coefficient, the time domain convolution of the interfacial reflection coefficient with an input waveform is needed, i.e.

$$\begin{aligned}
S(t) &= X(t) * R^{TM}(t) \\
&= X(t) * \left\{ R_{\infty}^{TM} \delta(t) + \tilde{R}^{TM}(t) \right\} \\
&= R_{\infty}^{TM} X(t) + \tilde{R}^{TM}(t) * X(t).
\end{aligned} \tag{3.82}$$

Here $X(t)$ denotes the input waveform and $S(t)$ is the resulting reflected waveform. The same input waveform given in Figure 2.6 is used for TM polarization. Figure 3.3 shows the comparison of the time domain reflected waveforms with the input waveform shown in Figure 2.6 for $\sigma_n = 0$. Again, the derived expression and the IFFT results are well matched.

Exact theoretical expressions are available from the previous research [11] for TM polarized transient interfacial reflection due to a unit step input waveform. A comparison with those results also helps to verify the work in this study. Figure 3.4 (a) and (b) show that the re-produced results of the electric fields to a double exponential input waveform excitation as shown in Figure 4.1 in [11], obtained by using the transient interfacial reflection coefficient and transmission coefficient of this study, are exactly same as those found in Figure 5.3 and Figure 5.2 in [11].

3.4.2 The transient responses for various parameter sets

It is interesting to study the dependence of the transient reflection coefficient on various parameter values, i.e. permittivity, permeability, conductivity and aspect angle. Comparisons of the reduced reflection coefficients are shown in Figure 3.5 - Figure 3.8, which are plotted using semilog scales for easier identification. Note that the curves in each plot are for the $\sigma_n = 0$ case.

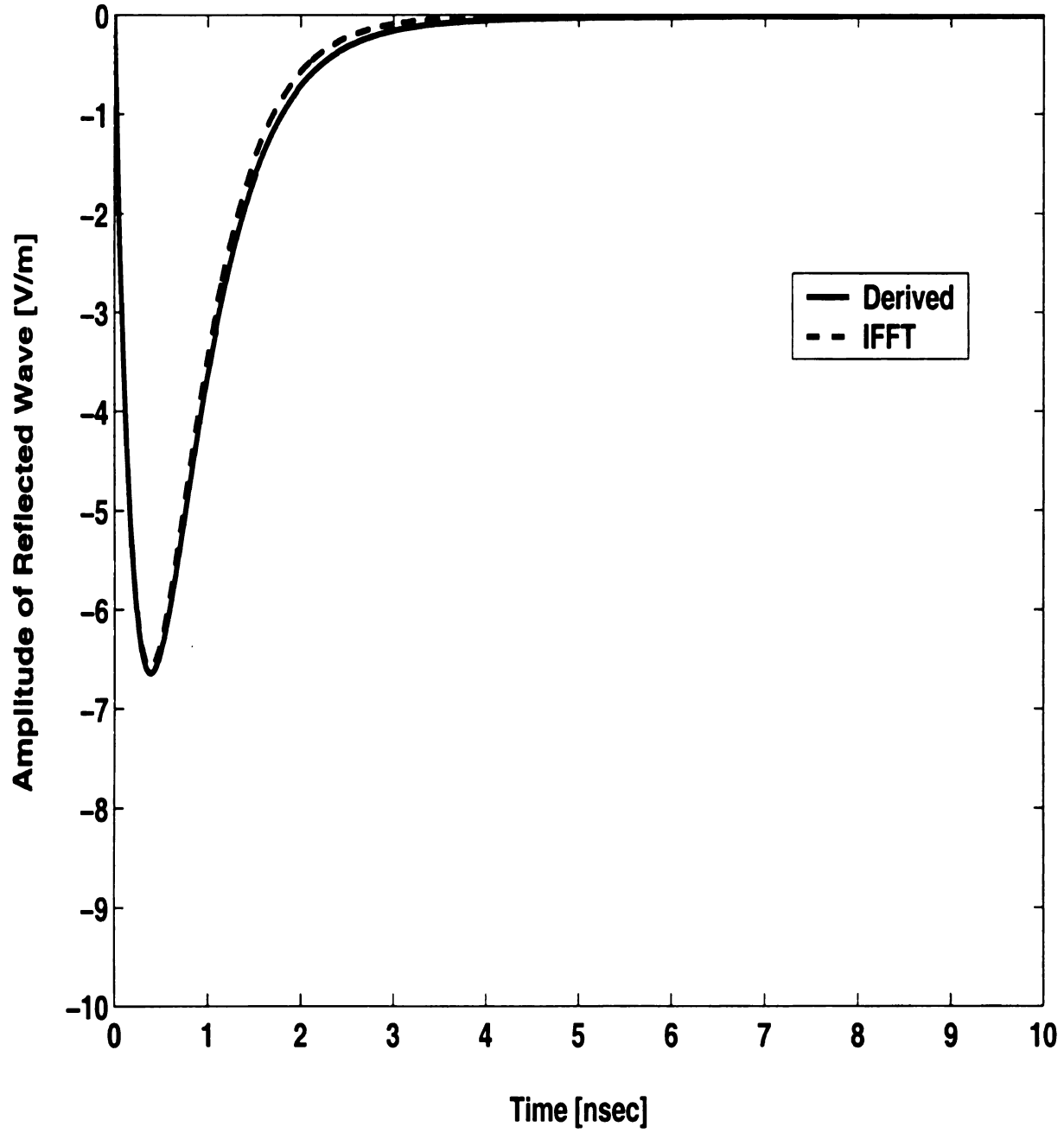


Figure 3.3. Numerical comparison of the derived transient reflected electric field waveform for the input waveform shown in Figure 2.6 with the IFFT (TM polarization) : $\mu_n = \mu_0$, $\epsilon_n = \epsilon_0$, $\sigma_n = 0[\text{U}/m]$, $\mu_{n+1} = \mu_0$, $\epsilon_{n+1} = 72\epsilon_0$, $\sigma_{n+1} = 4[\text{U}/m]$, $\theta_{i1} = 30^\circ$.

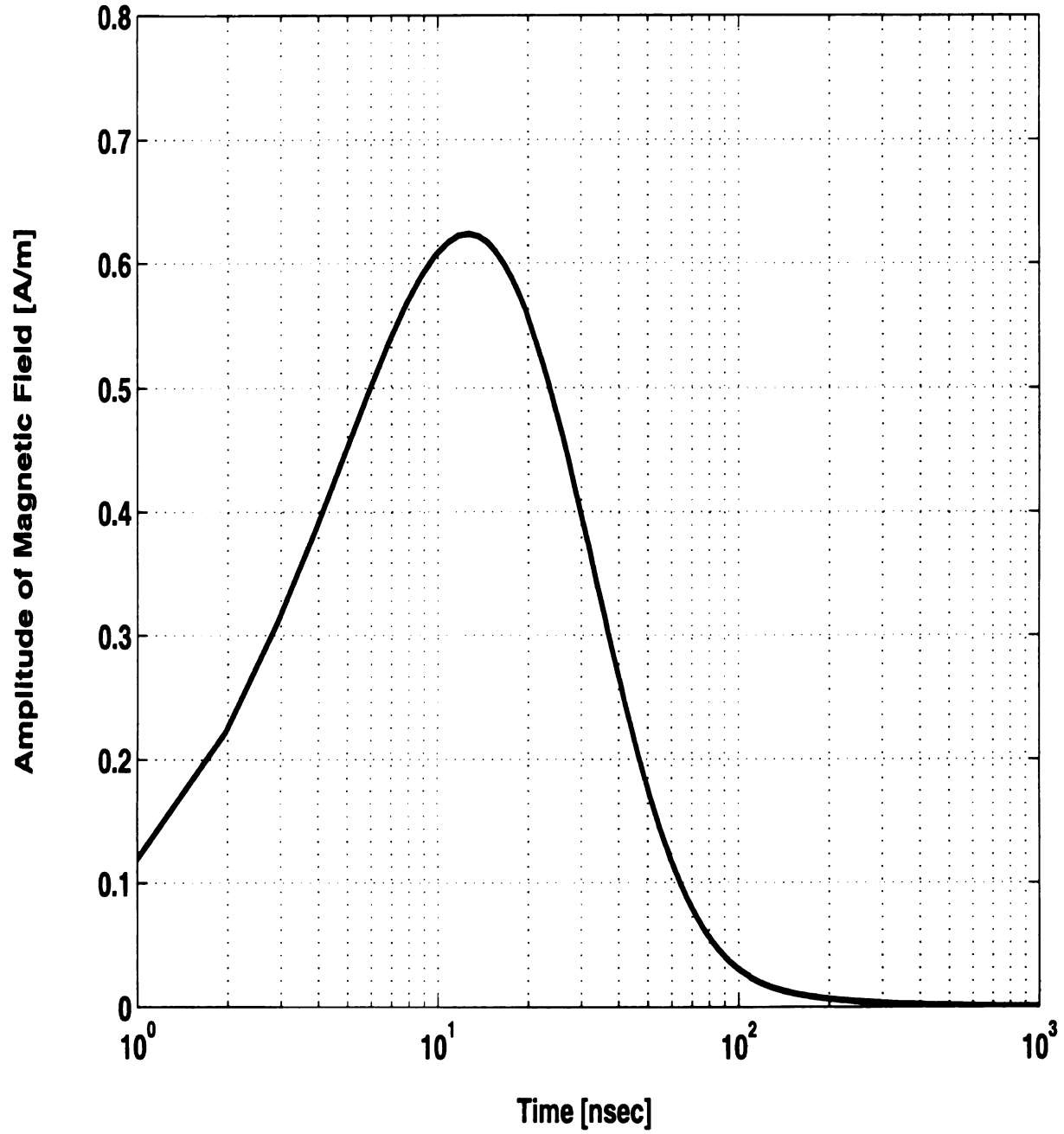


Figure 3.4. (a) The transient reflected magnetic field waveform for a double exponential excitation : $\epsilon = 10\epsilon_0$, $\mu = \mu_0$, $\sigma = 2 \times 10^{-2}[\text{U}/m]$, $\theta_i = 45^\circ$.

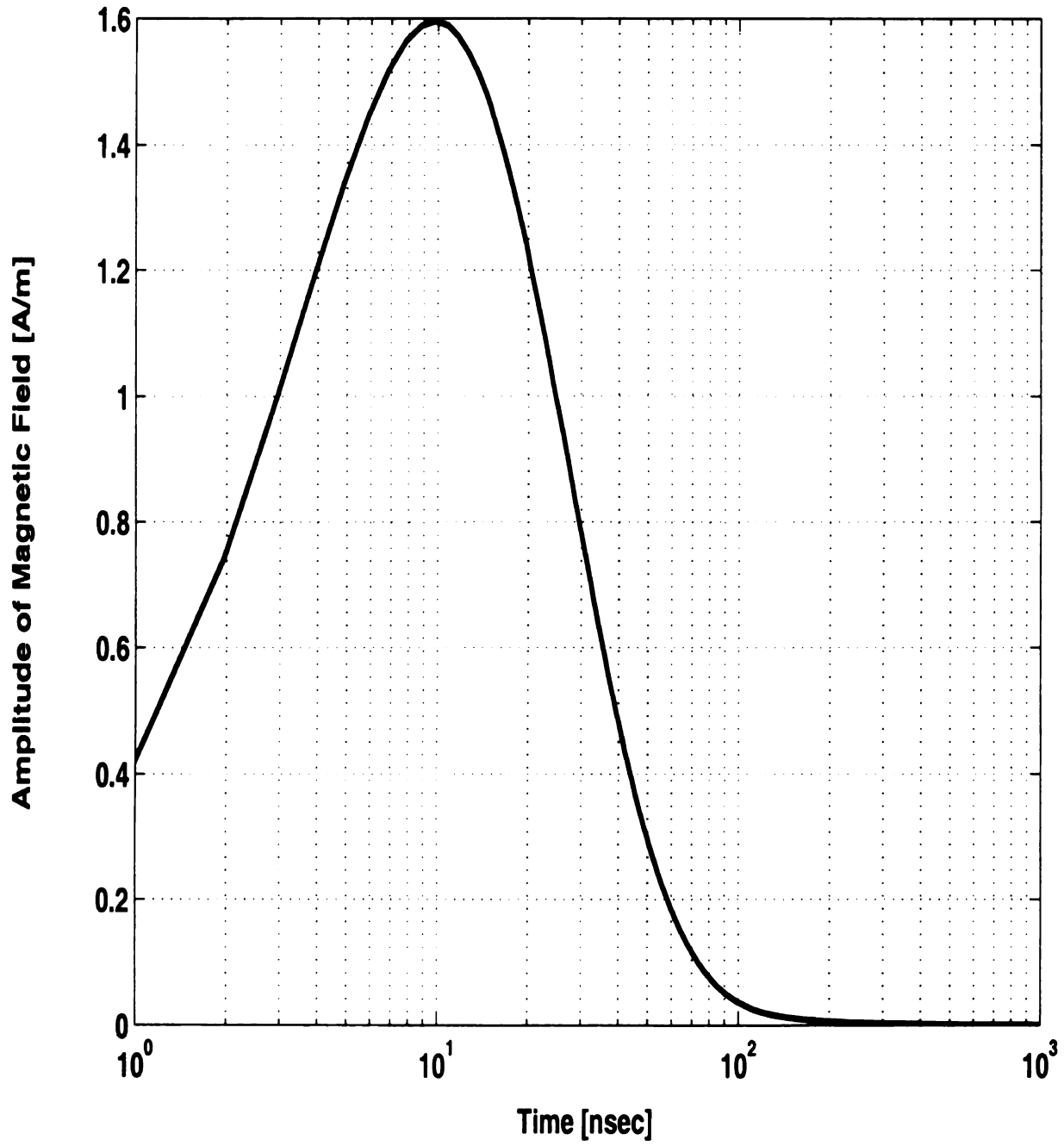


Figure 3.4. (b) The transient transmitted magnetic field waveform for a double exponential excitation : $\epsilon = 10\epsilon_0$, $\mu = \mu_0$, $\sigma = 2 \times 10^{-2}[\text{U}/m]$, $\theta_i = 45^\circ$.

(1) Permittivity

The curves in Figure 3.5 are plotted for $\epsilon_{r,n+1} = 1.7, 2.5, 10$ and 72 . The corresponding values of R_{∞}^{TM} are $-0.10, -0.18, -0.47$ and -0.76 , respectively. It can be seen that the slopes of different $\tilde{R}^{TM}(t)$ curves are proportional to the increasing permittivity values as well as increasing asymptotic reflection coefficient R_{∞}^{TM} . This means that the transient reflection becomes more dependent on its early time portion coming from the asymptotic reflection term when the permittivity contrast of both layers is sharper.

(2) Permeability

The curves in Figure 3.6 are plotted for $\mu_{r,n+1} = 1, 5, 10$ and 50 . The corresponding values of R_{∞}^{TM} are $-0.76, -0.53, -0.40$ and -0.019 respectively. The slopes of $\tilde{R}^{TM}(t)$ are seen to be inversely proportional to increasing permeability values, and to increasing amplitude of the asymptotic reflection coefficient. This indicates that the transient reflection becomes more ‘relaxed’ with increasing values of permeability.

(3) Conductivity

Because an increase in conductivity causes attenuation in wave propagation, the transient response $\tilde{R}^{TM}(t)$ suffers a more rapid decrease with time as conductivity is increased, while the asymptotic component is independent of the change. This can be seen explicitly in Figure 3.7 where $\sigma_n = 0.10, 1, 4$ and 10 , respectively.

(4) Aspect angle

The curves shown in Figure 3.8 are plotted for $\theta_{i1} = 0^\circ, 30^\circ, 45^\circ$ and 75° , while the corresponding values of R_{∞}^{TM} are $-0.79, -0.76, -0.72$ and -0.38 respectively. Therefore, the slope of the reduced reflection coefficient as well as the amplitude of the asymptotic component decreases with increasing aspect angle, and the transient reflection is less dependent on its early time portion.

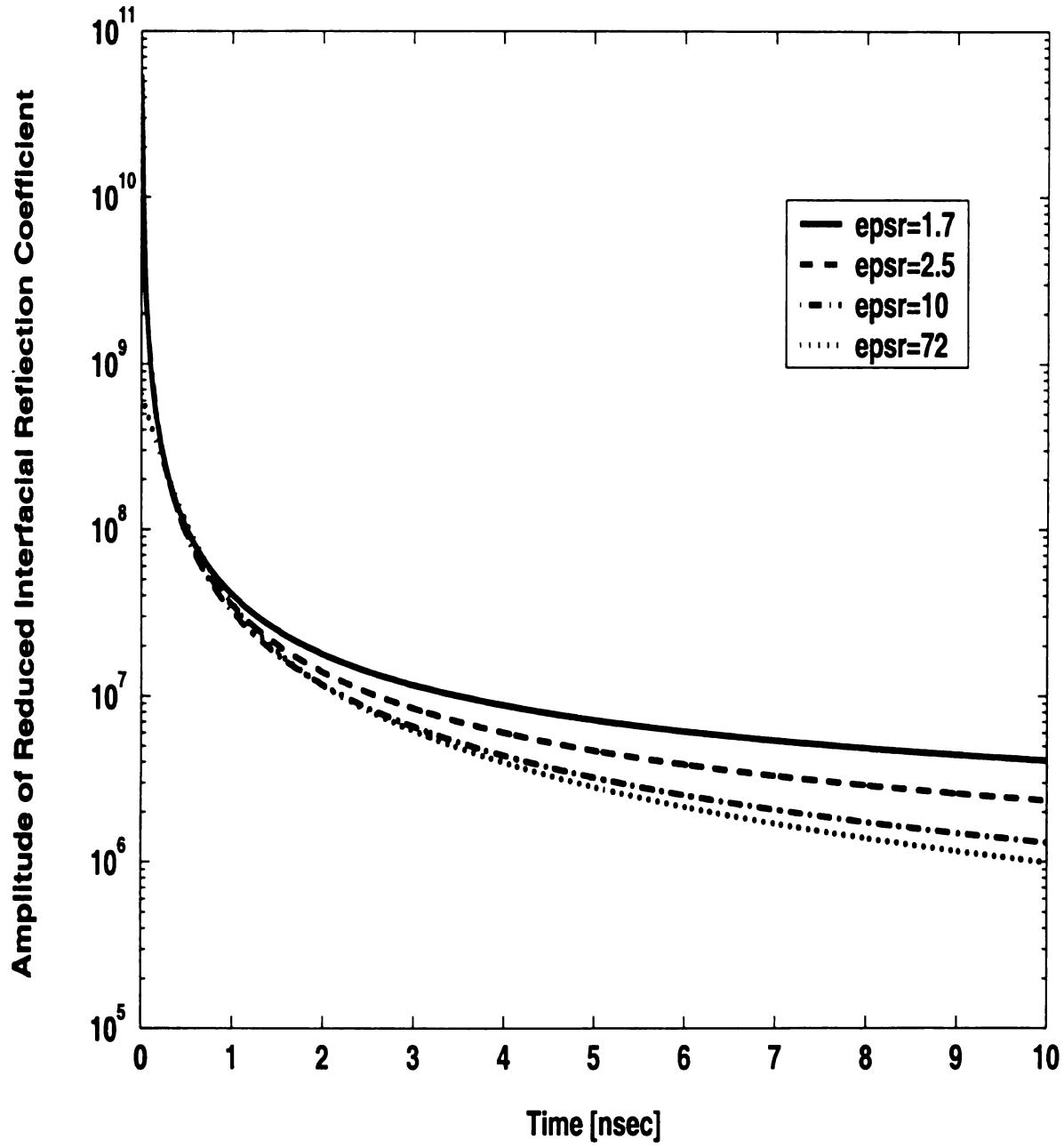


Figure 3.5. Time domain reduced interfacial reflection coefficients for various values of permittivity (TM polarization) : $\epsilon_n = \epsilon_0$, $\mu_n = \mu_{n+1} = \mu_0$, $\sigma_n = 0$, $\sigma_{n+1} = 4[\mathcal{V}/m]$, $\theta_{i1} = 30^\circ$.

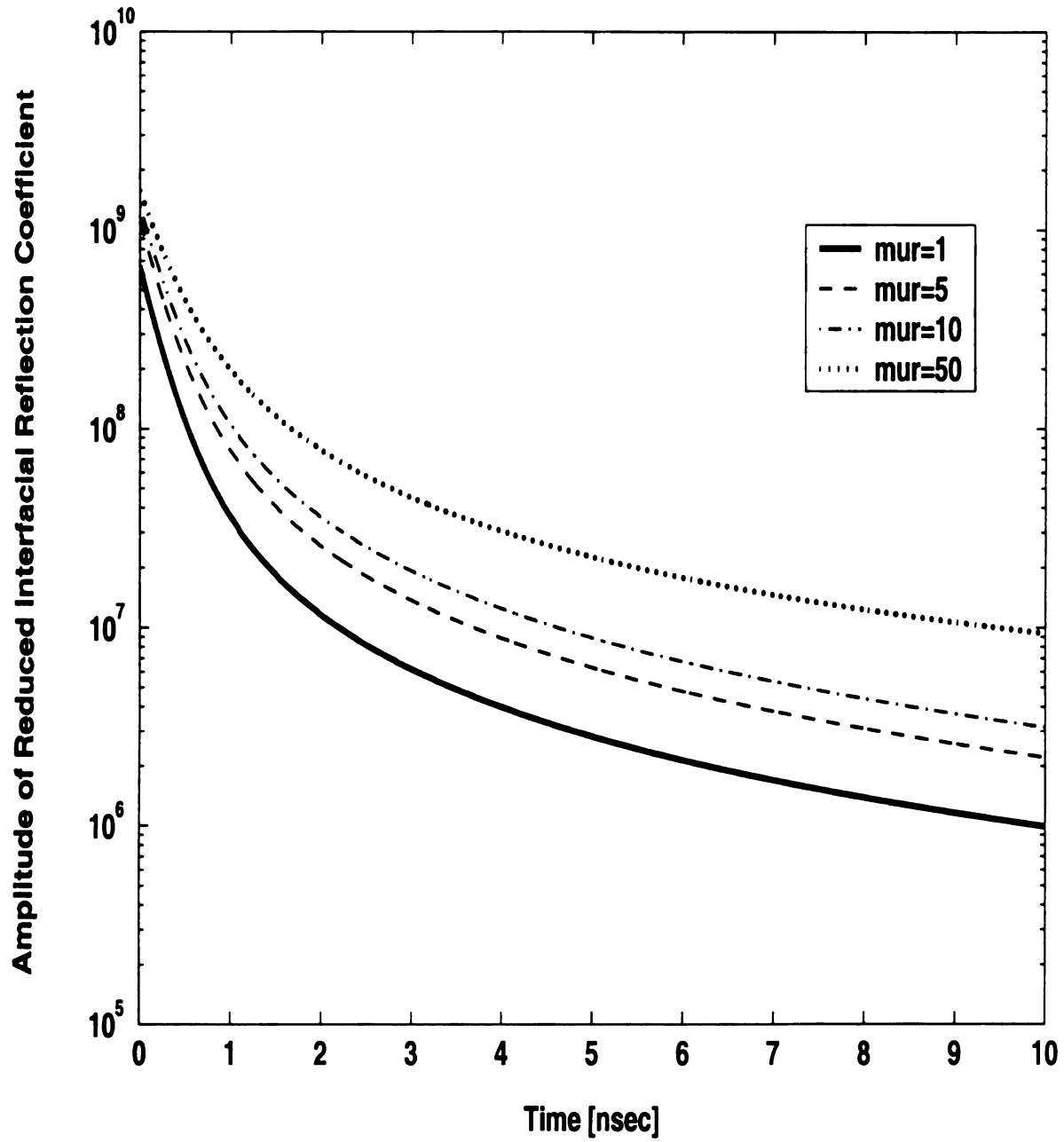


Figure 3.6. Time domain reduced interfacial reflection coefficients for various values of permeability (TM polarization) : $\epsilon_n = \epsilon_0$, $\epsilon_{n+1} = 72\epsilon_0$, $\mu_n = \mu_0$, $\sigma_n = 0$, $\sigma_{n+1} = 4[\mathcal{U}/m]$, $\theta_{i1} = 30^\circ$.

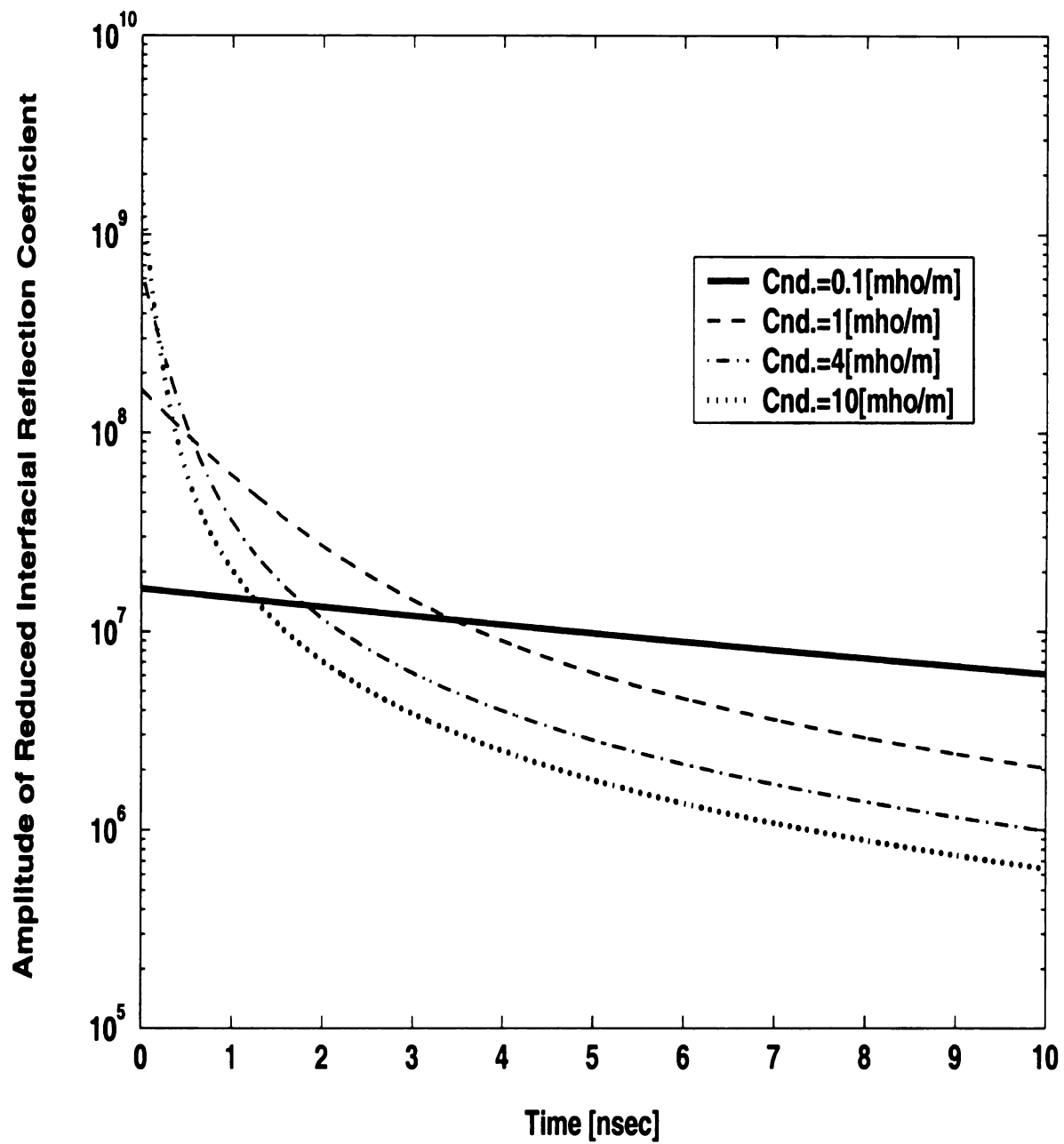


Figure 3.7. Time domain reduced interfacial reflection coefficients for various values of conductivity (TM polarization) : $\epsilon_n = \epsilon_0$, $\epsilon_{n+1} = 72\epsilon_0$, $\mu_n = \mu_{n+1} = \mu_0$, $\sigma_n = 0[\text{U}/m]$, $\theta_{i1} = 30^\circ$.

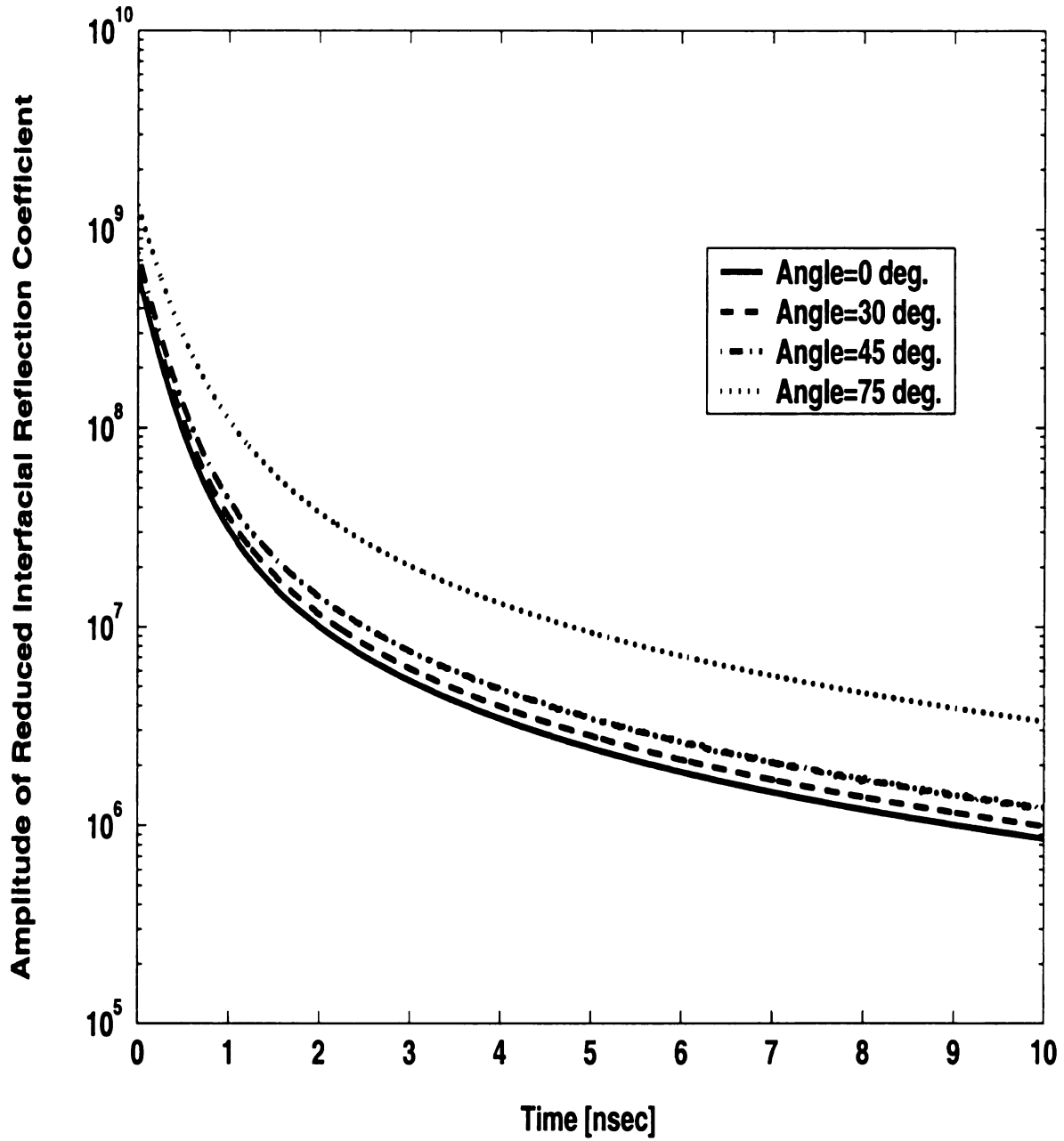


Figure 3.8. Time domain reduced interfacial reflection coefficients for various values of incident angle (TM polarization) : $\epsilon_n = \epsilon_0$, $\epsilon_{n+1} = 72\epsilon_0$, $\mu_n = \mu_{n+1} = \mu_0$, $\sigma_n = 0$, $\sigma_{n+1} = 4[\text{U}/m]$.

CHAPTER 4

SCATTERING FROM A MULTI-LAYERED MEDIUM

4.1 Introduction

The overall reflection coefficient is defined as the ratio of the reflected wave amplitude to the incident wave amplitude at the first interface of a multi-layered medium, for which the effect of scattering from the other layers must be considered. In this study, the overall transient scattering from a multi-layered medium is derived from a combination of propagation terms for the layers and the individual interfacial coefficients for which closed forms have been derived in the chapters 2 and 3 for each polarization. To obtain the transient formulation for the overall reflection coefficient, its frequency domain formula is derived first using the wave matrix method, and then time domain expressions are found using a series expansion and the convolution theorem. For this derivation, the analytical form of the transient propagation term is found. Finally, the obtained transient expression is verified by numerical computation examples, and various aspects of the formula are discussed.

4.2 Formulation of Transient Overall Reflection Coefficient

The overall reflection coefficient for a multi-layered medium (assumed to have N layers) can be computed in the frequency domain using wave matrices [22]. Several other techniques can also be used, but this is the most common technique. Figure 4.1 shows the geometry of the n^{th} layer of a multi-layered configuration. The incident and reflected waves immediately to the left of the $(n-1 | n)$ interface are c_n and b_n respectively. Similarly, the waves immediately to the left and right of the $(n | n+1)$ interface are c_{n+1} , b_{n+1} and c_{n+2} . The overall reflection coefficients immediately to the left of the $(n-1 | n)$ and $(n | n+1)$ interfaces are $\Gamma_n = b_n/c_n$ and $\Gamma_{n+1} = b_{n+1}/c_{n+1}$,

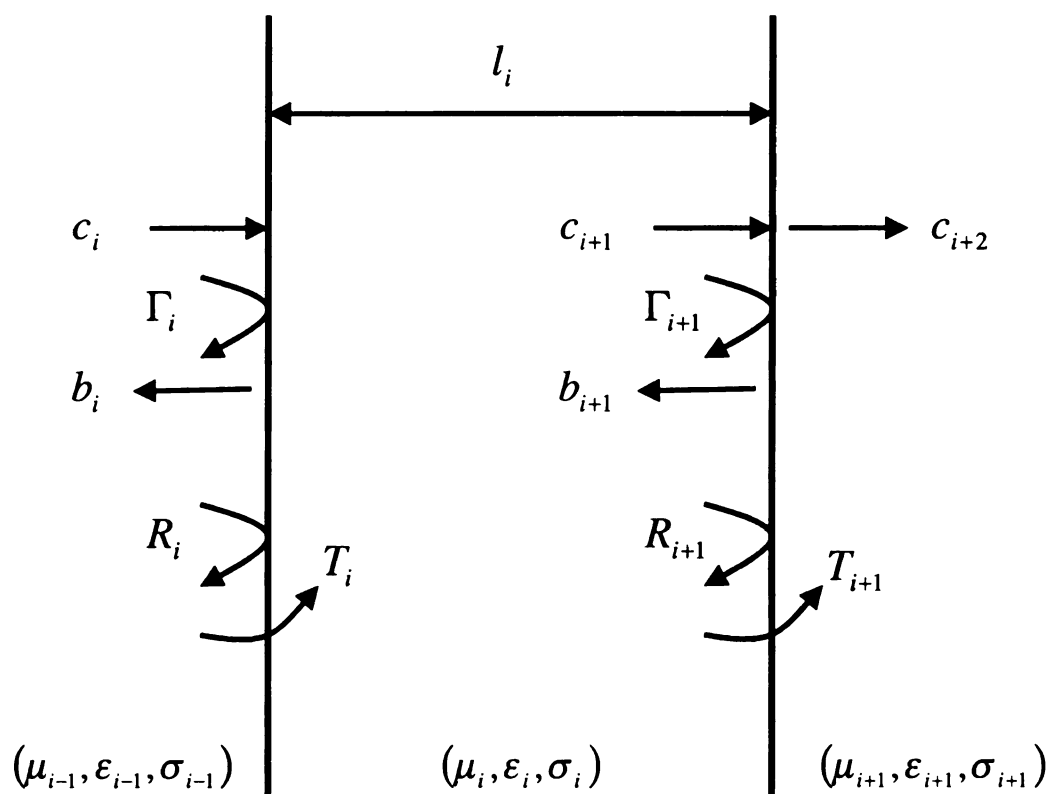


Figure 4.1. The n^{th} layer of a multi-layered environment.

respectively. The interfacial reflection and transmission coefficients are R_n , T_n and R_{n+1} , T_{n+1} . Wave matrices are used here to relate Γ_n to Γ_{n+1} . By doing so, we can develop a recursive relation that will ultimately relate the reflection coefficient at the front of the interface (i.e. the first layer) to the reflection coefficients of the underlying layers.

Using Collin's wave matrix result [22], c_n and b_n are related to c_{n+1} and b_{n+1} by the following relation

$$\begin{bmatrix} c_n \\ b_n \end{bmatrix} = \frac{1}{T_n} \begin{bmatrix} P_n^{-1}(\omega) & R_n P_n(\omega) \\ R_n P_n^{-1}(\omega) & P_n(\omega) \end{bmatrix} \begin{bmatrix} c_{n+1} \\ b_{n+1} \end{bmatrix} \quad (4.1)$$

where $P_n(\omega) = e^{\gamma_n l_n}$ is the frequency domain propagation term, and l_n is the length of layer n. Then, the recursive formula for Γ_n in terms of Γ_{n+1} can be computed using Collin's wave matrix result as follows

$$\Gamma_n(\omega) = \frac{b_n}{c_n} = \frac{\frac{1}{T_n} \{ R_n P_n^{-1}(\omega) c_{n+1} + P_n(\omega) b_{n+1} \}}{\frac{1}{T_n} \{ P_n^{-1}(\omega) c_{n+1} + R_n P_n(\omega) b_{n+1} \}} \quad (4.2)$$

Factoring out $P_n^{-1}(\omega) c_{n+1}$ from the numerator and denominator and using $\Gamma_{n+1} = b_{n+1}/c_{n+1}$ produces the desired frequency domain result

$$\Gamma_n(\omega) = \frac{R_n(\omega) + P_n^2(\omega) \Gamma_{n+1}(\omega)}{1 + R_n(\omega) P_n^2(\omega) \Gamma_{n+1}(\omega)}. \quad (4.3)$$

To obtain the transient form of overall reflection coefficient, (4.3) is expanded as a series summation, because it is difficult to find directly the inverse Fourier transform of the formula. For example, assume the three layer case. Then $\Gamma_2 = R_2$ and

$$\Gamma_1(\omega) = \frac{R_1(\omega) + P_1^2(\omega) R_2(\omega)}{1 + R_1(\omega) P_1^2(\omega) R_2(\omega)}$$

$$\begin{aligned}
&= \{R_1(\omega) + P_1^2(\omega)R_2(\omega)\} [1 - P_1^2(\omega)R_1(\omega)R_2(\omega) + \\
&\quad \{P_1^2(\omega)R_1(\omega)R_2(\omega)\}^2 - \{P_1^2(\omega)R_1(\omega)R_2(\omega)\}^3 + \cdot \cdot \cdot] \\
&= R_1(\omega) + P_1^2(\omega)R_2(\omega)\{1 - R_1^2(\omega)\} - P_1^4(\omega)R_1(\omega)R_2^2(\omega)\{1 - R_1^2(\omega)\} \\
&\quad + P_1^6(\omega)R_1^2(\omega)R_2^3(\omega)\{1 - R_1^2(\omega)\} - P_1^8(\omega)R_1^3(\omega)R_2^4(\omega)\{1 - R_1^2(\omega)\} + \cdot \cdot \cdot
\end{aligned} \tag{4.4}$$

This series expansion is possible because of radiation condition referred in section 2.2.2. To be able to expand by a series summation, the amplitude of $R_n(\omega)$ and $P_n(\omega)$ must be equal to or less than unity. This is satisfied due to radiation condition (leading to the branch cut definition), which requires a non-positive real part of the propagation constant. That is,

$$|P(\omega)| = |e^{\gamma_{nx}l_n}| = e^{\text{Re}\{\gamma_{nx}\}l_n} \leq 1 \tag{4.5}$$

and

$$\begin{aligned}
R^{TE}(s) &= \frac{\sqrt{s+B_n} - \sqrt{D_n}\sqrt{s+B_{n+1}}}{\sqrt{s+B_n} + \sqrt{D_n}\sqrt{s+B_{n+1}}} \\
&= \frac{\{-a(\omega) \pm jb(\omega)\} - \{-c(\omega) \pm jd(\omega)\}}{\{-a(\omega) \pm jb(\omega)\} + \{-c(\omega) \pm jd(\omega)\}} \\
&= \frac{-\{a(\omega) - c(\omega)\} \pm j\{b(\omega) - d(\omega)\}}{-\{a(\omega) + c(\omega)\} \pm j\{b(\omega) + d(\omega)\}}
\end{aligned} \tag{4.6}$$

where $a(\omega)$, $b(\omega)$, $c(\omega)$ and $d(\omega)$ are arbitrary positive functions of frequency. Thus,

$$|R^{TE}(s)| = \frac{\sqrt{\{a(\omega) - c(\omega)\}^2 + \{b(\omega) - d(\omega)\}^2}}{\sqrt{\{a(\omega) + c(\omega)\}^2 + \{b(\omega) + d(\omega)\}^2}} \leq 1. \tag{4.7}$$

The overall reflection coefficient for a medium having more layers can be found replacing $R_2(\omega)$ with $\Gamma_2(\omega)$ in (4.4) and using recursive relation given by (4.3). To obtain the transient overall reflection coefficient, the convolution theorem is used,

which says that the multiplication in frequency domain is equivalent to convolution in time domain. Therefore, the transient form of (4.4) is given by

$$\begin{aligned}\Gamma_1(t) = & R_1(t) + P_1(t) * P_1(t) * R_2(t) * \{\delta(t) - R_1(t) * R_1(t)\} - \\ & P_1(t) * P_1(t) * P_1(t) * P_1(t) * R_1(t) * R_2(t) * R_2(t) * \{\delta(t) - R_1(t) * R_1(t)\} \\ & + \dots\end{aligned}\quad (4.8)$$

And finally, the reflection $S(t)$ from the first interface of a multi-layered medium due to an input waveform $X(t)$ can be obtained as

$$S(t) = X(t) * \Gamma_1(t). \quad (4.9)$$

4.3 Transient Propagation

To compute the transient overall reflection given by (4.9), the transient propagation terms must be found. By letting

$$\begin{aligned}d_n &= \mu_n \epsilon_n - \mu_1 \epsilon_1 \sin^2 \theta_{i1} \\ b_n &= \mu_n \sigma_n,\end{aligned}\quad (4.10)$$

the definition of the propagation constant in (2.38) is rewritten as

$$\gamma_{nx} = \sqrt{d_n s^2 + b_n s} = -\sqrt{d_n} \sqrt{s^2 + \frac{l_n}{d_n} s} \quad (4.11)$$

for $d_n > 0$, and

$$\gamma_{nx} = \sqrt{d_n s^2 + b_n s} = \pm j \sqrt{|d_n|} \sqrt{s^2 - \frac{l_n}{|d_n|} s} \quad (4.12)$$

for $d_n < 0$, and where the upper sign is used for $\omega > 0$ and the lower sign for $\omega < 0$.

Only the $d_n > 0$ case will be considered due to the causality difficulties mentioned in

Chapter 2.

Now the frequency domain propagation term is written as

$$\begin{aligned} P_n(\omega) &= e^{\gamma_x l_n} \\ &= e^{-l_n \sqrt{d_n} \sqrt{s^2 + \frac{l_n}{d_n} s}}. \end{aligned} \quad (4.13)$$

To obtain the inverse Fourier transform of (4.13), the transform pair in [23],

$$\begin{aligned} e^{-\frac{x}{v} \sqrt{(s+\rho)^2 - \sigma^2}} &\Leftrightarrow e^{-\frac{\rho x}{v}} \delta\left(t - \frac{x}{v}\right) + \\ &\frac{\sigma x}{v \sqrt{t^2 - \left(\frac{x}{v}\right)^2}} e^{-\rho t} I_1\left(\sigma \sqrt{t^2 - \left(\frac{x}{v}\right)^2}\right) u\left(t - \frac{x}{v}\right) \end{aligned} \quad (4.14)$$

where $Re\{\rho, \sigma\} \geq 0$, is used. When (4.13) and (4.14) are compared, it can be realized that $\frac{b_n}{2d_n} > 0$ corresponds to ρ , and $\sigma = \rho > 0$. Then,

$$\begin{aligned} e^{-\frac{\rho l_n}{v}} \delta\left(t - \frac{l_n}{v}\right) &\rightarrow e^{-\frac{b_n}{2d_n} l_n \sqrt{d_n}} \delta(t - l_n \sqrt{d_n}) \\ \frac{\sigma l_n}{v \sqrt{t^2 - \left(\frac{l_n}{v}\right)^2}} &\rightarrow \frac{\frac{b_n}{2d_n} l_n \sqrt{d_n}}{\sqrt{t^2 - l_n^2 d_n}} \\ e^{-\rho t} I_1\left(\sigma \sqrt{t^2 - \left(\frac{l_n}{v}\right)^2}\right) u\left(t - \frac{l_n}{v}\right) &\rightarrow e^{-\frac{b_n}{2d_n} t} I_1\left(\frac{b_n}{2d_n} \sqrt{t^2 - l_n^2 d_n}\right) u(t - l_n \sqrt{d_n}). \end{aligned} \quad (4.15)$$

As a result, the transient propagation term is given by

$$\begin{aligned} P_n(t) &= e^{-\frac{b_n}{2\sqrt{d_n}} l_n} \delta(t - l_n \sqrt{d_n}) \\ &+ \frac{(b_n l_n / 2 \sqrt{d_n})}{\sqrt{t^2 - l_n^2 d_n}} I_1\left(\frac{b_n}{2d_n} \sqrt{t^2 - l_n^2 d_n}\right) u(t - l_n \sqrt{d_n}). \end{aligned} \quad (4.16)$$

Also, it can be easily shown that

$$\begin{aligned} \{P_n(s)\}^k &\Leftrightarrow e^{-\frac{b_n}{2\sqrt{d_n}(kl_n)}} \delta(t - kl_n \sqrt{d_n}) \\ &+ \frac{(kl_n b_n / 2\sqrt{d_n})}{\sqrt{t^2 - l_n^2 d_n}} I_1 \left(\frac{b_n}{2d_n} \sqrt{t^2 - (kl_n)^2 d_n} \right) u(t - kl_n \sqrt{d_n}). \end{aligned} \quad (4.17)$$

The transient form (4.17) should be verified by numerical inverse Fourier transform, but it is difficult to compute directly the transient form because it includes the δ -function. Therefore, a modified form obtained by removing the δ -function term is compared with its corresponding frequency domain form. That is, by being modified as

$$\tilde{P}_n(t) = P_n(t) - e^{-\frac{b_n}{2\sqrt{d_n}} l_n} \delta(t - l_n \sqrt{d_n}), \quad (4.18)$$

the corresponding frequency domain transform becomes

$$\begin{aligned} \tilde{P}_n(s) &= P_n(s) - F \left\{ e^{-\frac{b_n}{2\sqrt{d_n}} l_n} \delta(t - l_n \sqrt{d_n}) \right\} \\ &= e^{-\sqrt{d_n} \sqrt{s^2 + \frac{b_n}{d_n}} s l_n} - e^{-\frac{b_n}{2\sqrt{d_n}} l_n} e^{-(l_n \sqrt{d_n}) s} \\ &= e^{-\sqrt{d_n} \sqrt{s^2 + \frac{b_n}{d_n}} s l_n} - e^{-(\frac{b_n}{2\sqrt{d_n}} + \sqrt{d_n} s) l_n}, \end{aligned} \quad (4.19)$$

and the transform pair of (4.18) and (4.19) may be compared using numerical inverse Fourier transform. The plot in Figure 4.2 shows an example of such a comparison obtained using a 2048 point IFFT with a 20 nano second interval. The transient form and the IFFT result are well matched, and therefore the correctness of the transient propagation term has been verified.

4.4 Numerical Examples of Overall Reflection

The previously derived transient overall reflection form may be verified using numerical computation to ensure its correctness. Considering there are δ -function terms included in both the transient propagation and interfacial reflection terms,

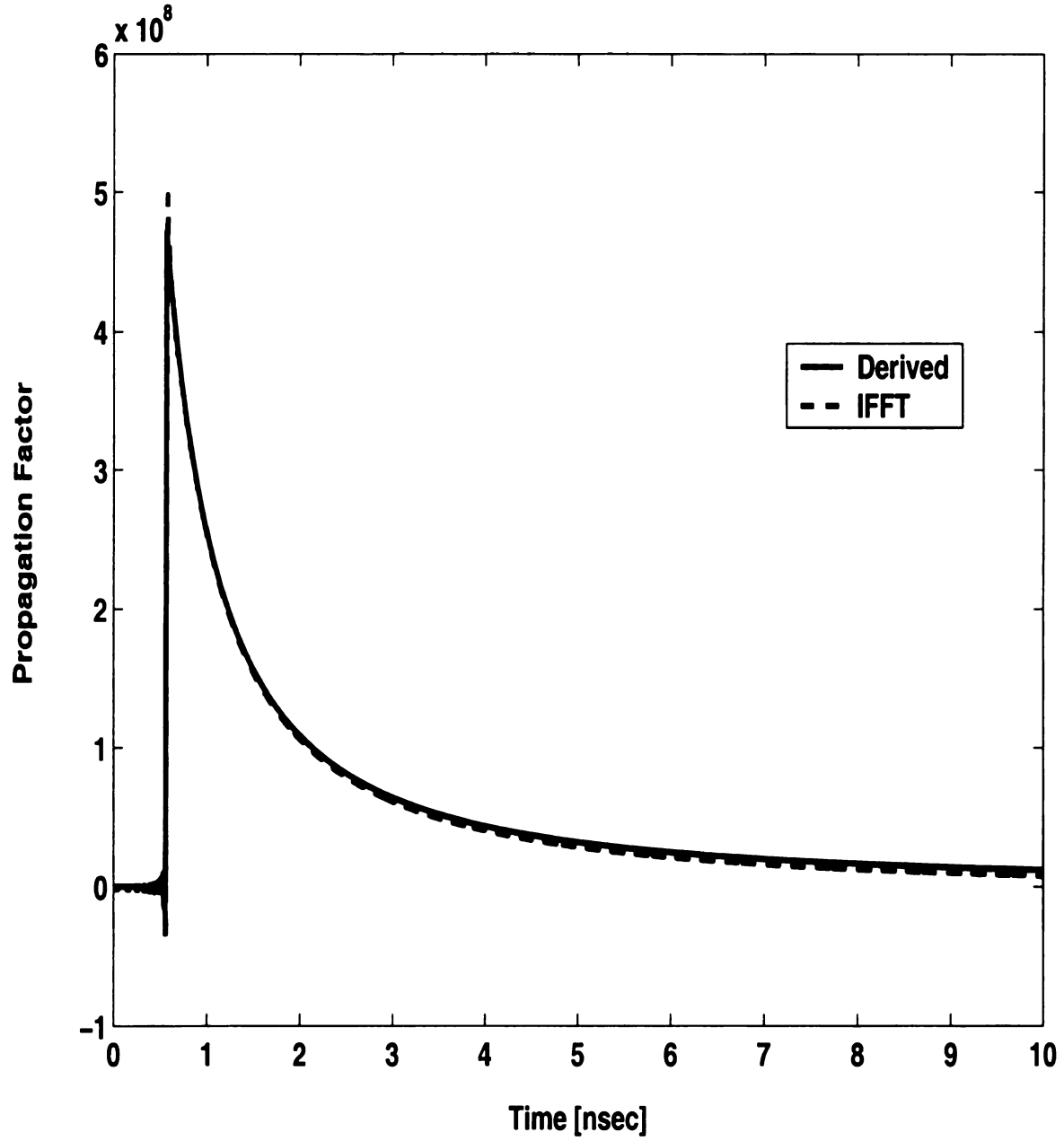


Figure 4.2. Numerical comparison of the derived transient propagation term with that from the IFFT : $\mu_n = \mu_0$, $\epsilon_n = 72\epsilon_0$, $\sigma_n = 4[\text{U}/m]$, $l_n = 0.01[\text{m}]$, $\theta_{i1} = 30^\circ$.

direct numerical computation of (4.8) may not produce an acceptable result. Therefore, the following numerical implementation is used in this study. When assuming $P(t) = A\delta(t - t_d) + \tilde{P}(t - t_d)$ and $R(t) = R_\infty\delta(t) + \tilde{R}(t)$ for example, the convolution becomes

$$\begin{aligned}
P(t) * R(t) &= \{A\delta(t - t_d) + \tilde{P}(t - t_d)\} * \{R_\infty\delta(t) + \tilde{R}(t)\} \\
&= AR_\infty\delta(t) * \delta(t - t_d) + A\delta(t - t_d) * \tilde{R}(t) \\
&\quad + R_\infty\delta(t) * \tilde{P}(t - t_d) + \tilde{P}(t - t_d) * \tilde{R}(t) \\
&= AR_\infty\delta(t - t_d) + A\tilde{R}(t - t_d) + R_\infty\tilde{P}(t - t_d) + \tilde{P}(t - t_d) * \tilde{R}(t).
\end{aligned} \tag{4.20}$$

The amplitude and position of the δ -function in (4.20) are stored separately for use in subsequent computations. To compute $\tilde{P}(t - t_d) * \tilde{R}(t)$, several different numerical algorithms are available [24], but the simple discrete convolution algorithm is used in this study, and it turns out that the method provides sufficient accuracy when compared with IFFT results. Detailed programming source codes for numerical implementation can be found in Appendix C.

Numerical examples of scattering excited by the input waveform shown in Figure 4.3 from the simplest 3 lossless layer (free space, 10cm thick-plexiglass and free space) cases for both polarizations are shown in Figure 4.4 and Figure 4.5. The double exponential input waveform shown in Figure 4.3 is produced as

$$X(t) = 10.24 \times \left(e^{-8 \times 10^9 t} - e^{-16 \times 10^9 t} \right). \tag{4.21}$$

The permittivity value at 10GHz for the plexiglass layer, $2.59\epsilon_0$, is found in [21]. The results from the transient formulas are compared with those obtained from a 2048 point IFFT with a 20 nano second range, although only the first 10 nano seconds

are shown. The comparison shows good agreement in both polarizations. The phase reversed reflected waveforms at each edge are seen in the both plots.

To see explicitly multiple reflections due to time-delaying propagation terms, the scattering from a 5-layered lossless medium (free space, 10cm plexiglass, 10cm free space, 10cm plexiglass and free space) is computed, and shown in Figure 4.6. Because this medium consists of lossless layers, the reflected waveforms are controlled only by the asymptotic reflection coefficients, and there is no attenuation suffered by the reflected waves. Instead, the amplitudes of reflected waves decrease after each reflection because the amplitude of each asymptotic reflection coefficient must be less than unity.

The overall reflections from a lossy layered medium are plotted Figure 4.7 and Figure 4.8 for both polarizations respectively. The 5 layered medium is identical to the previous case except that the third layer is distilled water with parameter values taken at 10 GHz, and the equivalent conductivity used to represent the dielectric loss of water at that frequency. As expected, the multiple reflections suffer severe attenuation, and the reflections after a short period peter out.

It may be interesting and practical to identify each reflected waveform in the multiple reflections shown in Figure 4.6 - Figure 4.8, by associating them with each term in (4.8). The time domain waveform identification makes it possible to remove unwanted signals, for an example, multi-path echos. Figure 4.9 shows the individual reflections in Figure 4.7 due to the first through the fourth term in (4.8). The first peak obviously is the reflection from the first interface of air to plexiglass, and depends on the asymptotic reflection coefficient only. Also, notice that the sign of the amplitude is inverted since the wave is reflected from a electrically denser layer. The second peak comes from the reflection at the plexiglass-water interface. Similarly, the sign of amplitude is reversed, but the large reflection occurs because the permittivity profile difference between the two layers is larger than for the previous interface. Now the

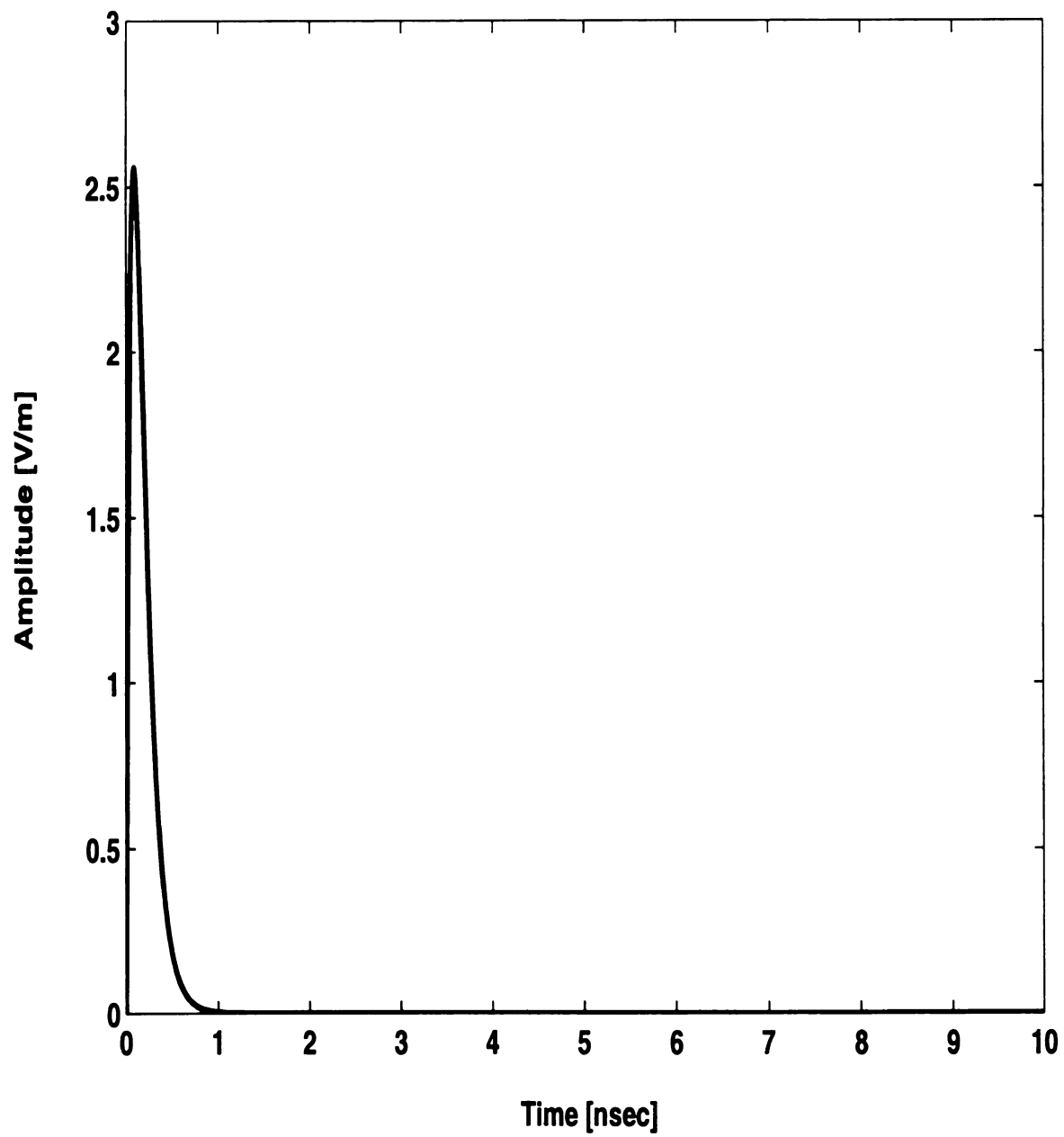


Figure 4.3. A double exponential input waveform

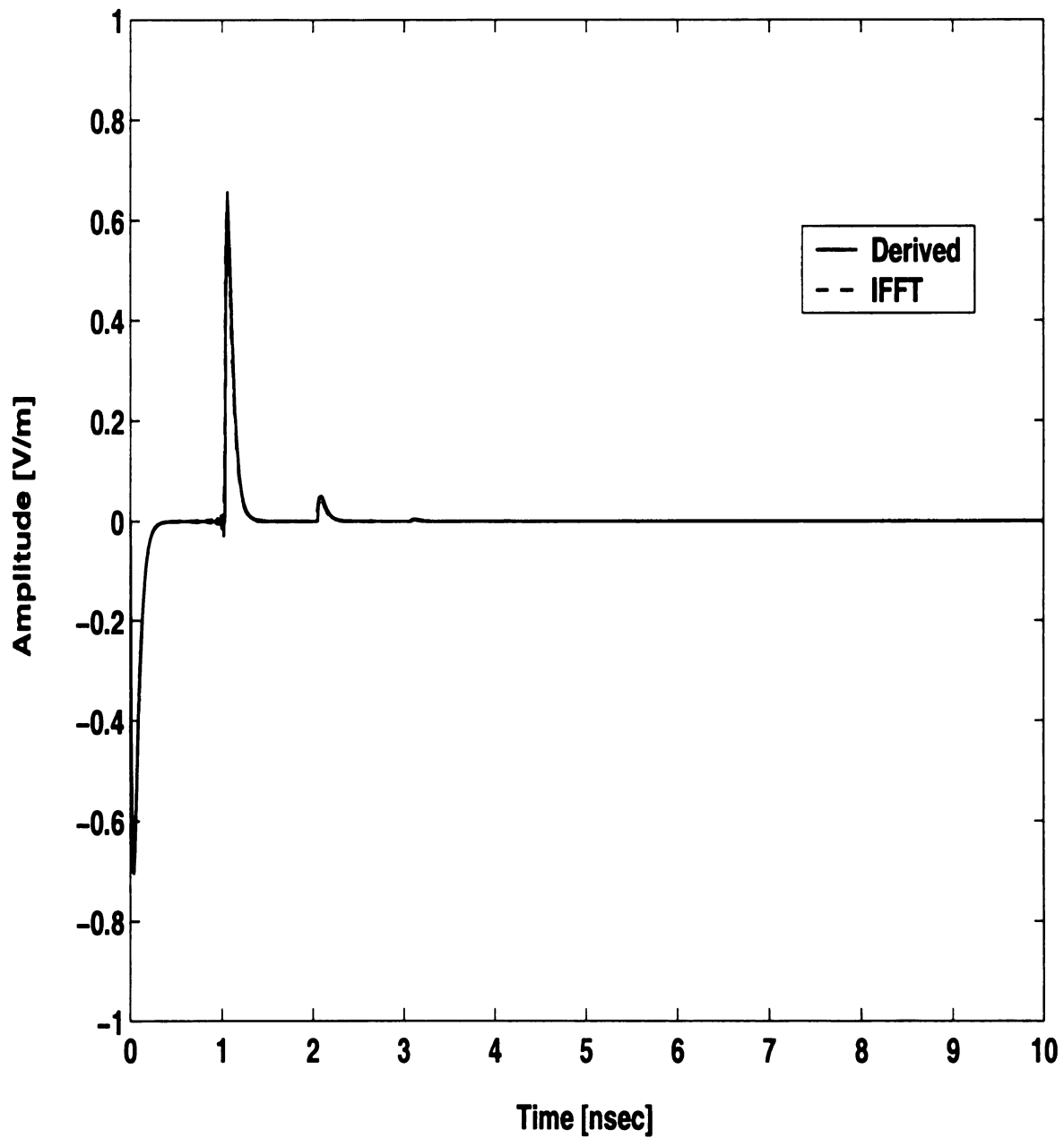


Figure 4.4. Numerical comparison of the derived overall reflection from a 3 layered medium with that from the IFFT (TE polarization) : $\mu_2 = \mu_0$, $\epsilon_2 = 2.59\epsilon_0$, $\sigma_2 = 0[\text{U}/m]$, $l_2 = 0.1[\text{m}]$, $\theta_{i1} = 30^\circ$.

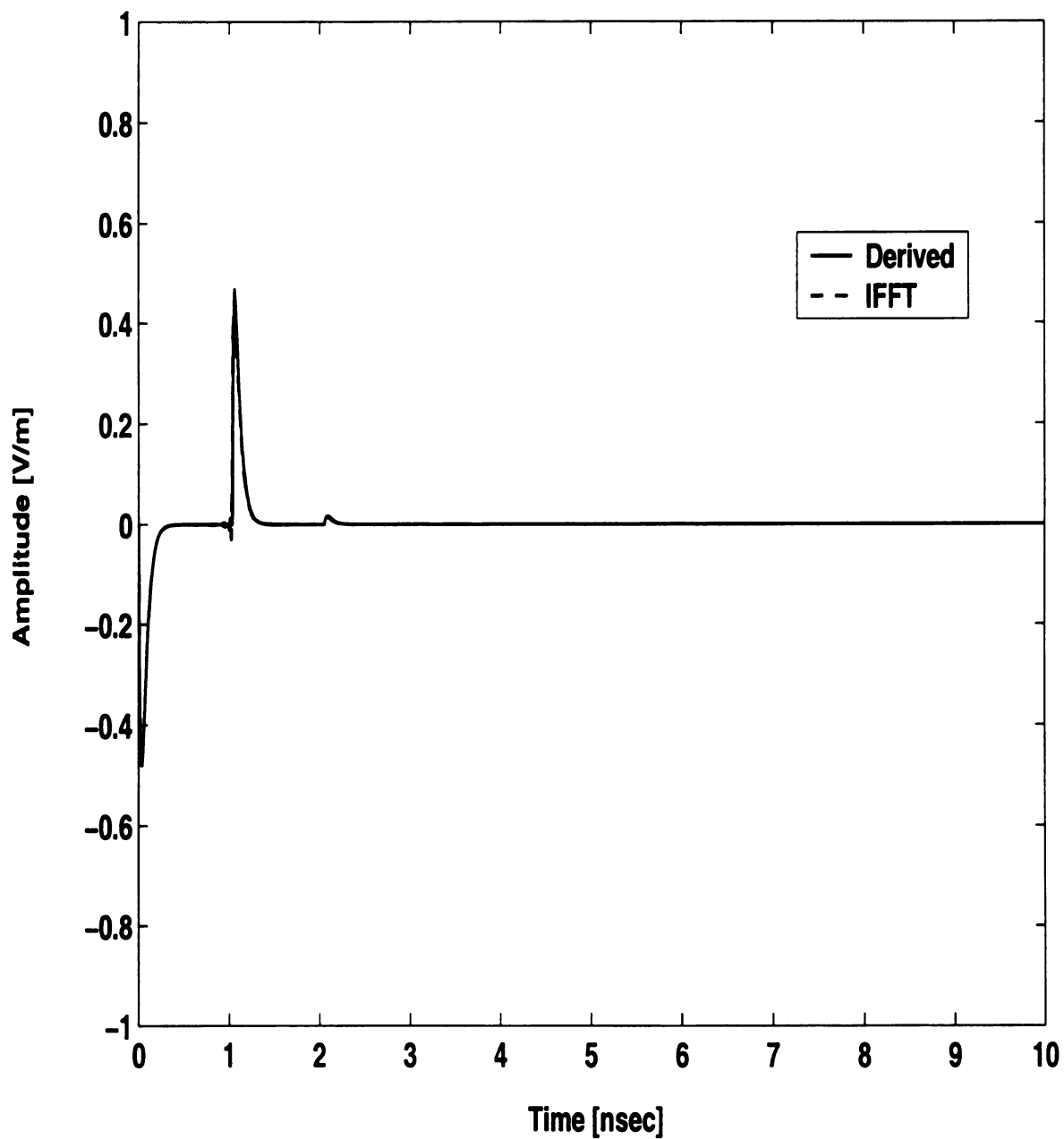


Figure 4.5. Numerical comparison of the derived overall reflection from a 3 layered medium with that from the IFFT (TM polarization) : $\mu_2 = \mu_0$, $\epsilon_2 = 2.59\epsilon_0$, $\sigma_2 = 0[\text{S/m}]$, $l_2 = 0.1[\text{m}]$, $\theta_{i1} = 30^\circ$.

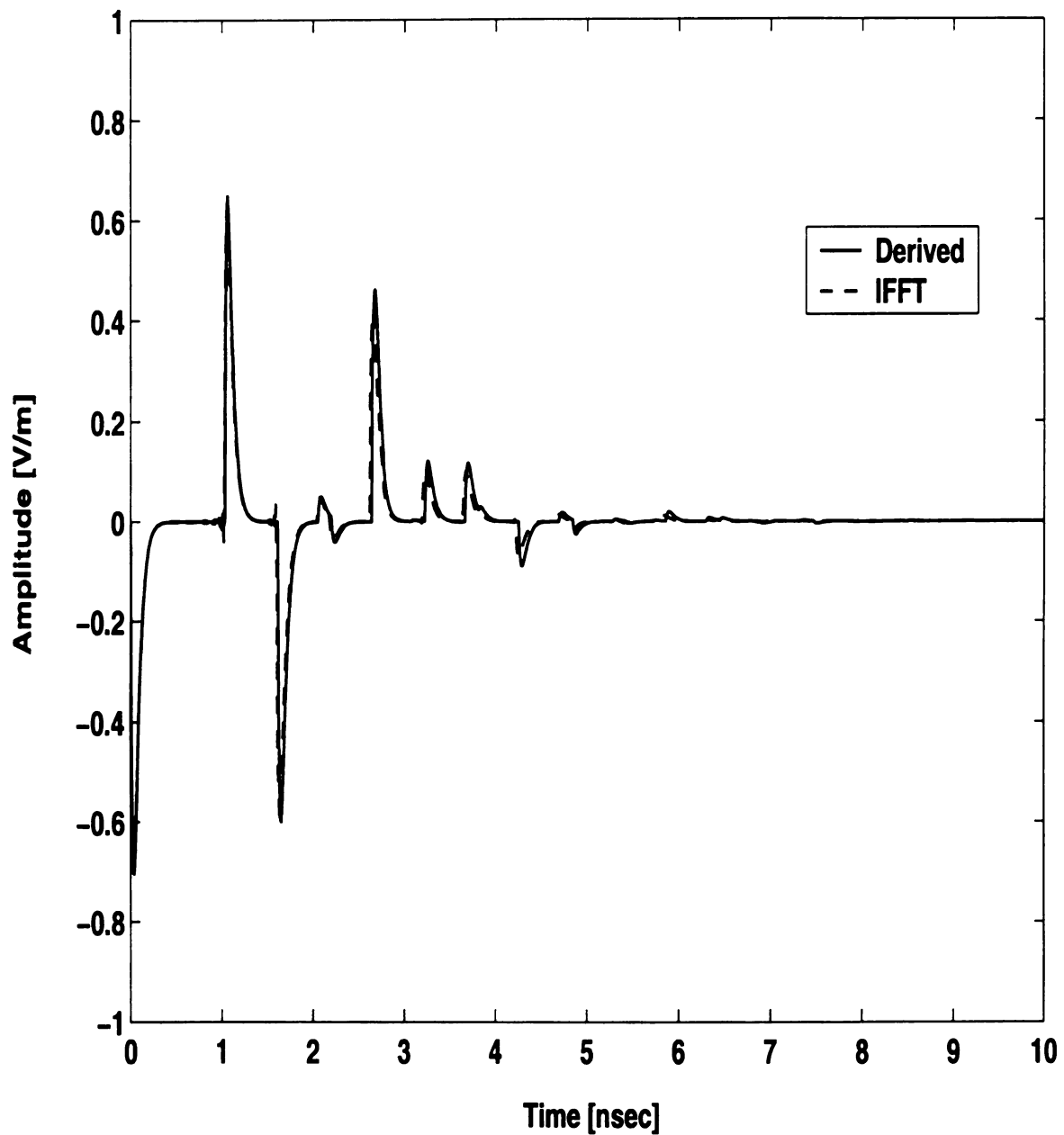


Figure 4.6. Multiple reflection from a lossless 5 layered medium (TE polarization)
: $\mu_2 = \mu_0$, $\epsilon_2 = 2.59\epsilon_0$, $\sigma_2 = 0[\text{U/m}]$, $l_2 = 0.1[\text{m}]$, $\mu_3 = \mu_0$, $\epsilon_3 = \epsilon_0$, $\sigma_3 = 0[\text{U/m}]$,
 $l_3 = 0.1[\text{m}]$, $\mu_4 = \mu_0$, $\epsilon_4 = 2.59\epsilon_0$, $\sigma_4 = 0[\text{U/m}]$, $l_4 = 0.1[\text{m}]$, $\theta_{i1} = 30^\circ$.

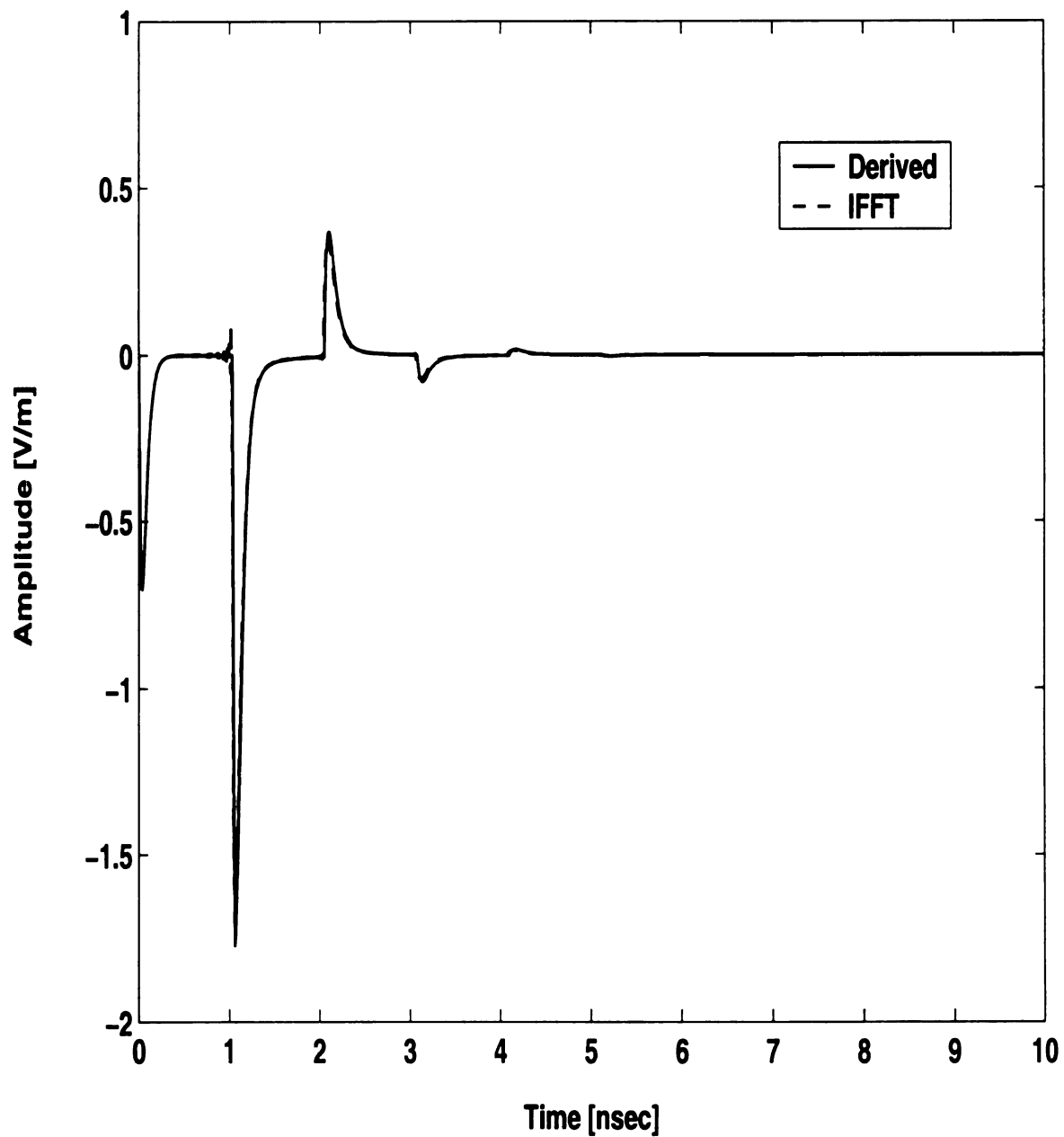


Figure 4.7. Multiple reflection from a lossy 5 layered medium (TE polarization) : $\mu_2 = \mu_0$, $\epsilon_2 = 2.59\epsilon_0$, $\sigma_2 = 0[\text{U/m}]$, $l_2 = 0.1[\text{m}]$, $\mu_3 = \mu_0$, $\epsilon_3 = 55\epsilon_0$, $\sigma_3 = 16.7[\text{U/m}]$, $l_3 = 0.1[\text{m}]$, $\mu_4 = \mu_0$, $\epsilon_4 = 2.59\epsilon_0$, $\sigma_4 = 0[\text{U/m}]$, $l_4 = 0.1[\text{m}]$, $\theta_{i1} = 30^\circ$.

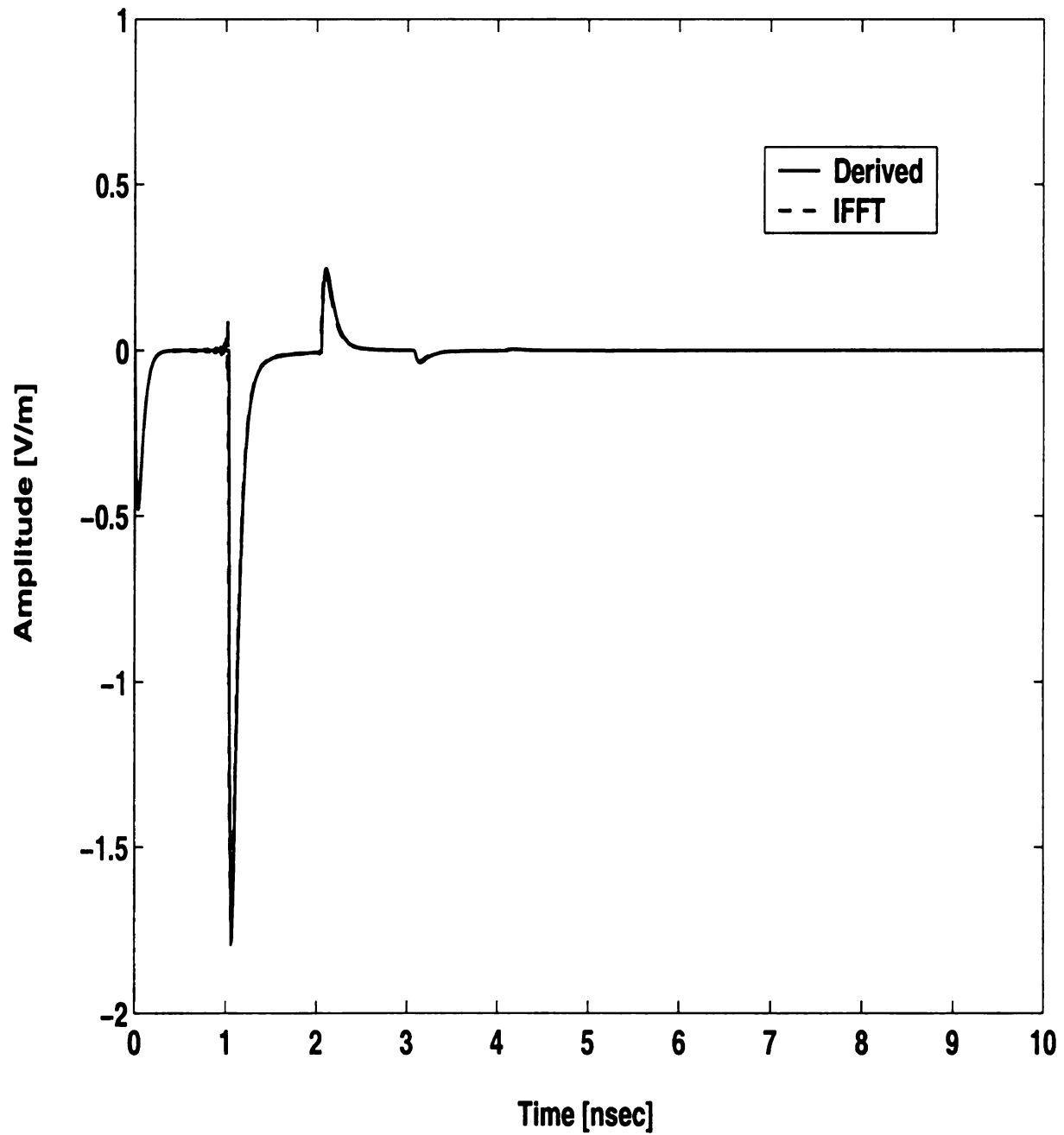


Figure 4.8. Multiple reflection from a lossy 5 layered medium (TM polarization) : $\mu_2 = \mu_0$, $\epsilon_2 = 2.59\epsilon_0$, $\sigma_2 = 0[\text{U/m}]$, $l_2 = 0.1[\text{m}]$, $\mu_3 = \mu_0$, $\epsilon_3 = 55\epsilon_0$, $\sigma_3 = 16.7[\text{U/m}]$, $l_3 = 0.1[\text{m}]$, $\mu_4 = \mu_0$, $\epsilon_4 = 2.59\epsilon_0$, $\sigma_4 = 0[\text{U/m}]$, $l_4 = 0.1[\text{m}]$, $\theta_{i1} = 30^\circ$.

reflected waveform is contributed by the reduced reflection coefficient as well as the asymptotic component. The polarity of the third peak is the same as that of the input waveform. Therefore, there is a possibility that it comes from the reflection at the water-plexiglass interface. However, considering the relatively high permittivity profile of water which makes the waveform propagation much slower than in plexiglass, as well as the heavy attenuation that the wave suffers due to the dielectric loss of water, this possibility must be discarded. Therefore, the third peak originates from three reflections at the interfaces of plexiglass-water, air-plexiglass, plexiglass-water and transmission through the air plexiglass interface. The fourth peak comes from an additional two reflections at the air-plexiglass and plexiglass-water interfaces. Of course, this rigorous geometrical ray optics approach is possible because of the exact timing analysis.

The incident angle dependence of the overall reflection is shown in Figure 4.10 for the same 5 layered lossy medium used in Figure 4.9. In the first reflected waveform set, the largest angle produces the largest peak waveform because the asymptotic reflection coefficient is proportional to angle as described in section 2.4.2. In the second reflected waveform set, the smallest angle produces the largest peak, due to its large reduced reflection coefficient components (see Figure 2.12). The same explanation can be applied to the other reflected waveform sets. Note that the multiple reflections with the largest incidence angle occur earliest. Although this phenomenon may be intuitively confusing since a longer travel path for a ray inside of a layer is expected for a larger incidence angle, it can be justified by considering the time delay in (4.16), or by the rigorous geometrical ray optics research found in [25].

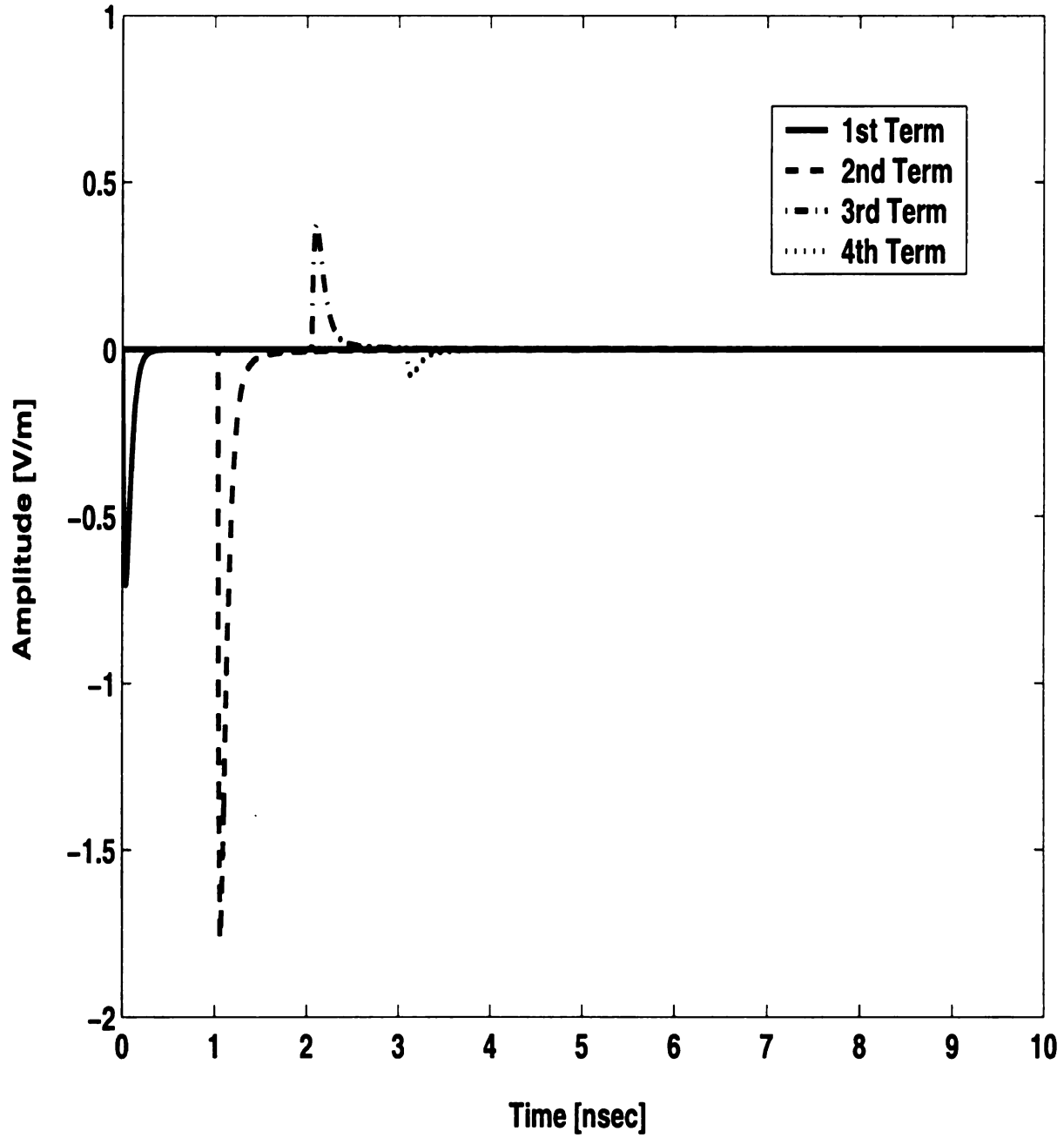


Figure 4.9. Identification of individual reflection in overall scattering from a 5 layered medium (TE polarization) : $\mu_2 = \mu_0$, $\epsilon_2 = 2.59\epsilon_0$, $\sigma_2 = 0[\text{U}/m]$, $l_2 = 0.1[\text{m}]$, $\mu_3 = \mu_0$, $\epsilon_3 = 55\epsilon_0$, $\sigma_3 = 16.7[\text{U}/m]$, $l_3 = 0.1[\text{m}]$, $\mu_4 = \mu_0$, $\epsilon_4 = 2.59\epsilon_0$, $\sigma_4 = 0[\text{U}/m]$, $l_4 = 0.1[\text{m}]$, $\theta_{i1} = 30^\circ$.

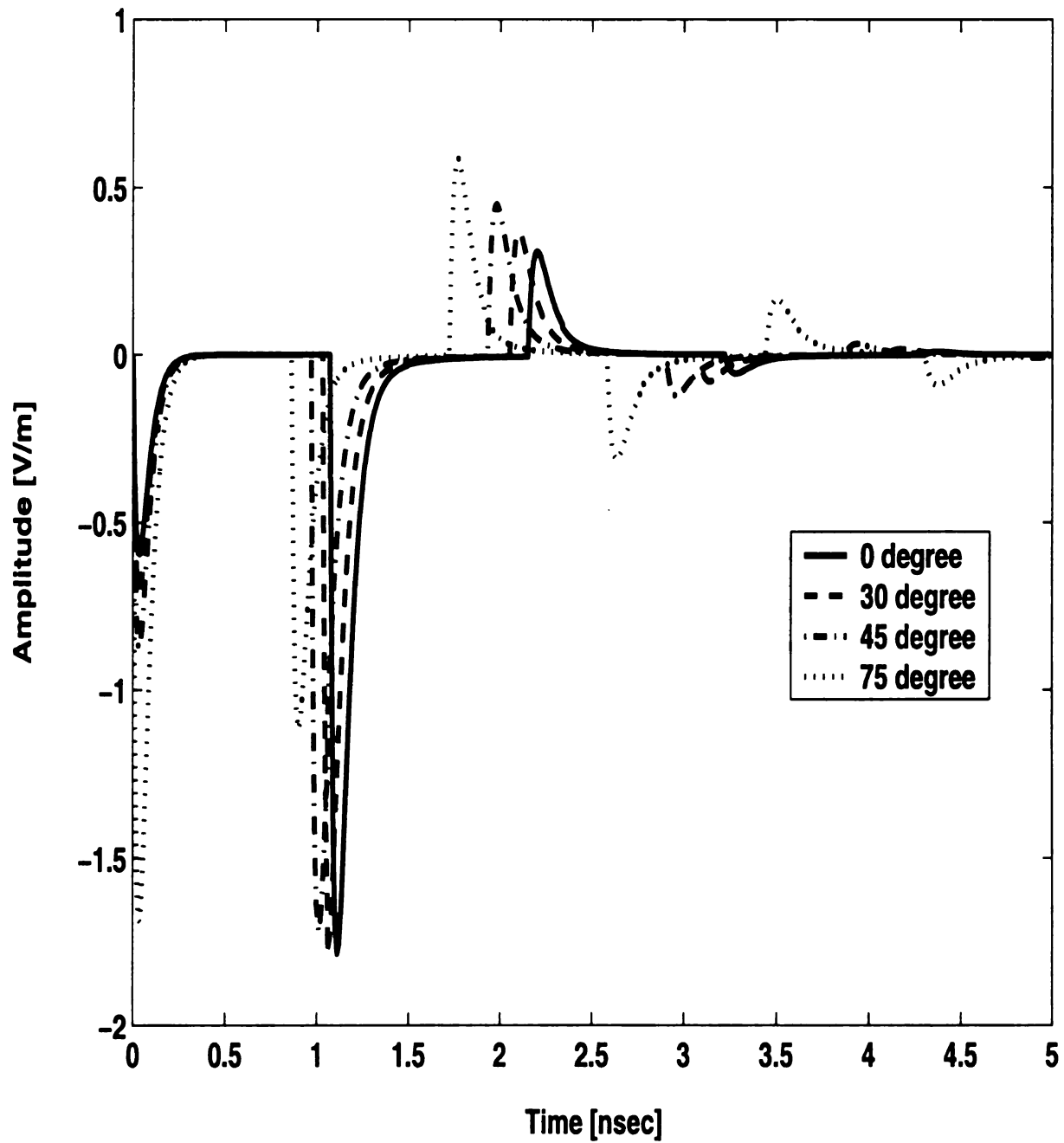


Figure 4.10. Time domain overall reflection for various values of incident angle (TE polarization) : $\mu_2 = \mu_0$, $\epsilon_2 = 2.59\epsilon_0$, $\sigma_2 = 0[\text{U}/m]$, $l_2 = 0.1[\text{m}]$, $\mu_3 = \mu_0$, $\epsilon_3 = 55\epsilon_0$, $\sigma_3 = 16.7[\text{U}/m]$, $l_3 = 0.1[\text{m}]$, $\mu_4 = \mu_0$, $\epsilon_4 = 2.59\epsilon_0$, $\sigma_4 = 0[\text{U}/m]$, $l_4 = 0.1[\text{m}]$.

CHAPTER 5

EXPERIMENTS

5.1 Introduction

The derived formulas in the previous chapters have so far been verified only by numerical computation. However, verification by actual experiments may provide additional valuable knowledge such as practical limits in real world applications. In this chapter, the description and results of the measurement processes which have been used to verify previously derived expressions are provided. Also, various aspects of the information obtained from the experiments are discussed. The descriptions of the experimental set up and the equipment are given first, and the calibration procedure which is needed to obtain ‘refined’ results from the raw measurement data in order to compare with theory are described. Finally, the experiment results from actual measurements are provided and discussed.

5.2 Experimental Set Up

For experiments, the set up depicted in Figure 5.1 is used. The arch range at Michigan State University consists of two 90° steel rail arcs on to which are attached movable transmitting and receiving antenna mounts. The radius of the rail is 120” (3.05m), and the height of the rail is 47” (1.19m), while the center axes of the antennas are placed at 59” (1.50m) height. Note that although several EM wave absorbers are used to reduce reflection from surrounding environments (e.g. wall, door, or metal rails), this arch range is fundamentally NOT an ‘anechoic’ chamber like those used in frequency domain radiation measurements. A target object (material plates fixed on a metal mounter in this study) is placed at the center of the range. More detailed descriptions on the arch range facility in Michigan State University can be found in

[26].

For time domain measurements, Hewlett Packard's digital sampling oscilloscope HP54750A and its HP54753A time domain transmission/reflectionmetry (TDT/TDR) plug-in module, providing 20GHz and 18GHz channels, are used. The TDR unit has a built-in step waveform generator that creates pulse trains with 5msec pulse width, 20msec period, and 190mV amplitude. The rising edge of each pulse is used to trigger a Picosecond Pulse Labs (PSPL) 4015B pulse generator. This instrument creates another step using a remote pulse head, which is connected to the input of PSPL 5208 Impulse Forming Network (IFN). The secondary step waveform has a leading edge fall time less than about 15psec, and an amplitude of -9V. An impulse forming network generates the impulse shaped waveform shown in Figure 5.2, using the step as an input. The impulse waveform is fed through connecting cables to a transmitting horn antenna mounted on the arch rail through connecting cables. The plots for the intermediate waveforms can be found in [27]. The equipment is prone to time-axis drifting, which causes degrading of the measurements. Therefore, pre-measurement warm-up of equipment is required several hours prior to the measurements.

Both the transmitting and receiving horn antennas used for these experiments have a 2 - 18GHz bandwidth, and the polarization of transmitted wave may be changed by rotating the antennas. Dielectric lenses are used to collimate the transmitted spherical wavefront to create an incident plane wave, and also to ensure that a major amount of the transmitted energy is projected on the target. There is a considerable amount of coupling between the two antennas observed in the measured waveform, but most of the coupling can be removed by the calibration process which will be explained later.

Several different target objects have been used for the experiments. Polystyrene ($\mu = \mu_0$, $\epsilon_r = 2.55$, and $\sigma = 0$ at 10GHz) and plexiglass ($\mu = \mu_0$, $\epsilon_r = 2.59$, and $\sigma = 0$ at 10GHz) are used for 3 lossless layer (free space, the material and free space)

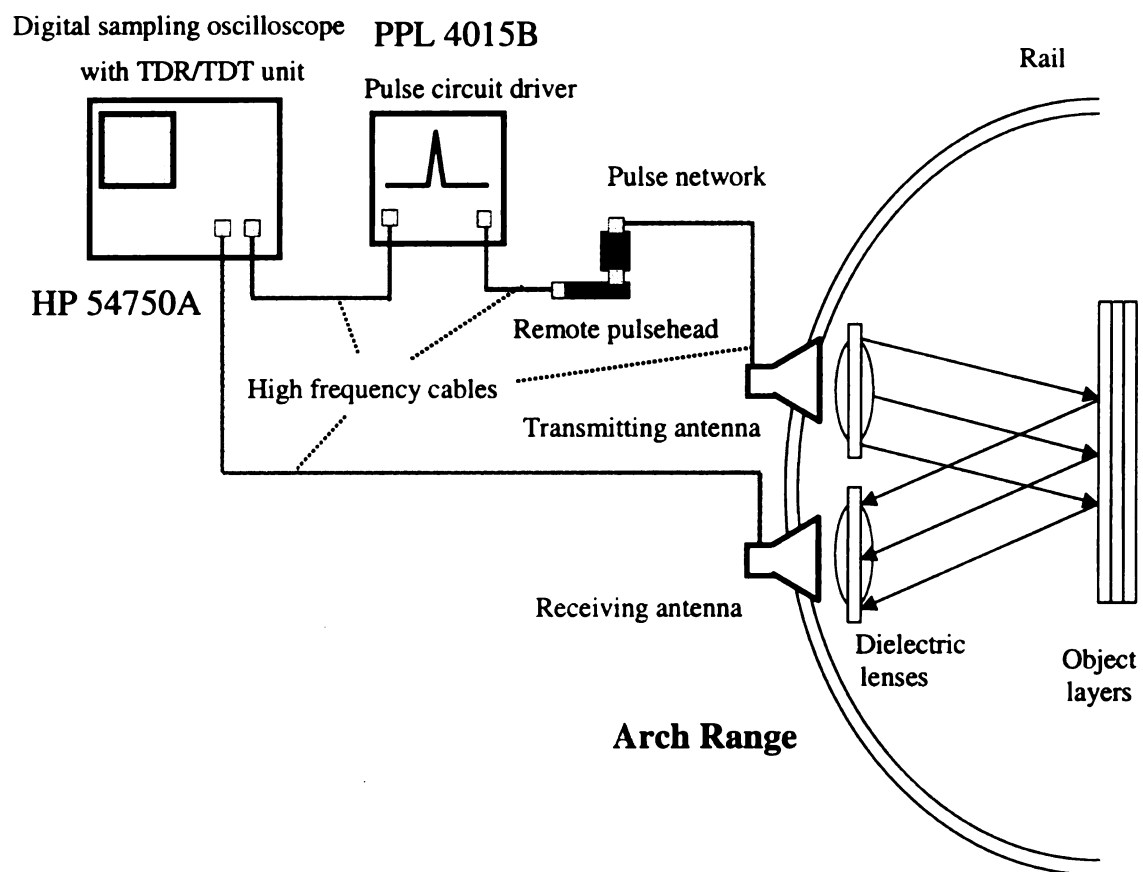


Figure 5.1. Experimental set up.

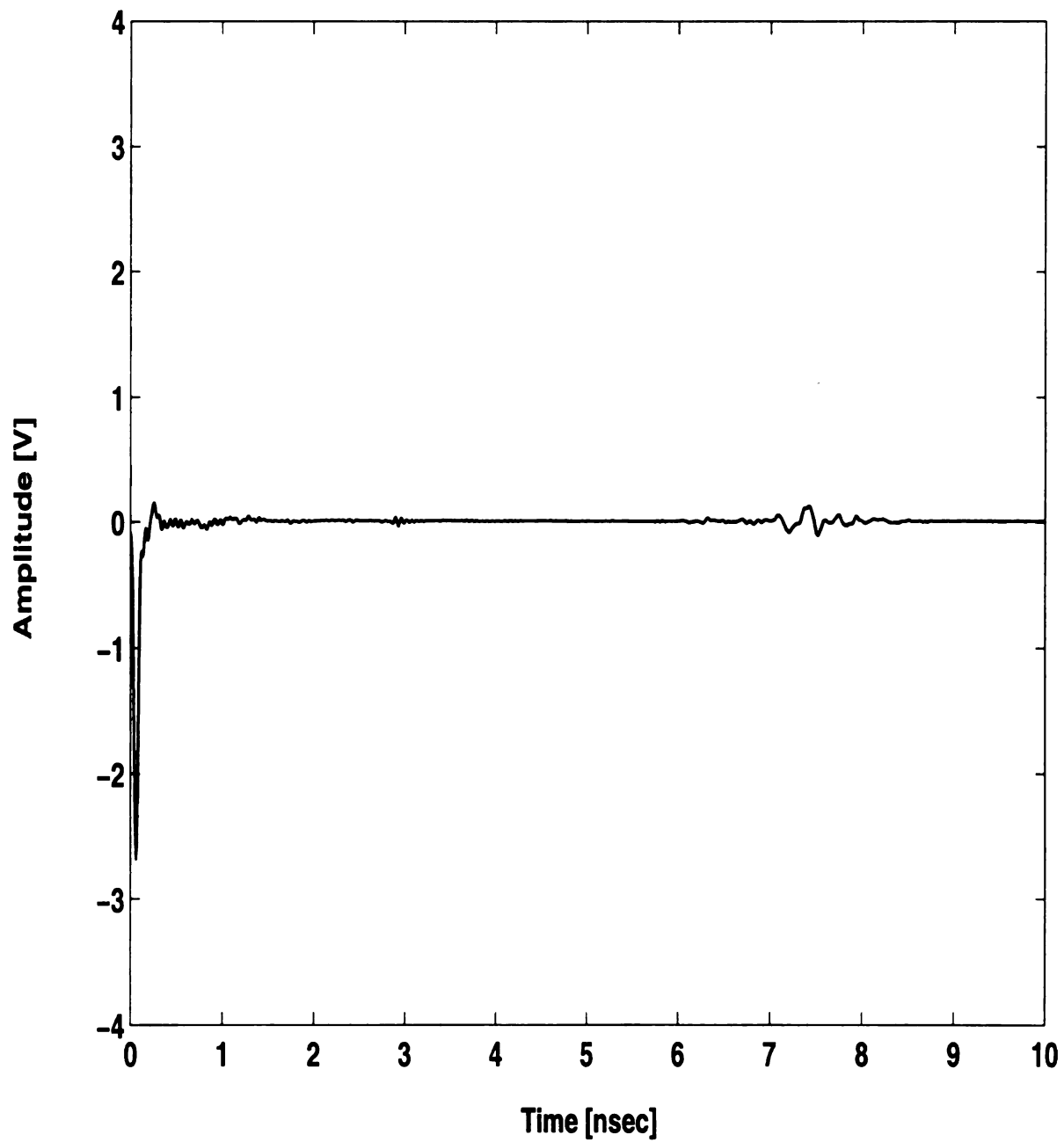


Figure 5.2. (a) Impulse shaped waveform transmitted from pulse forming network.

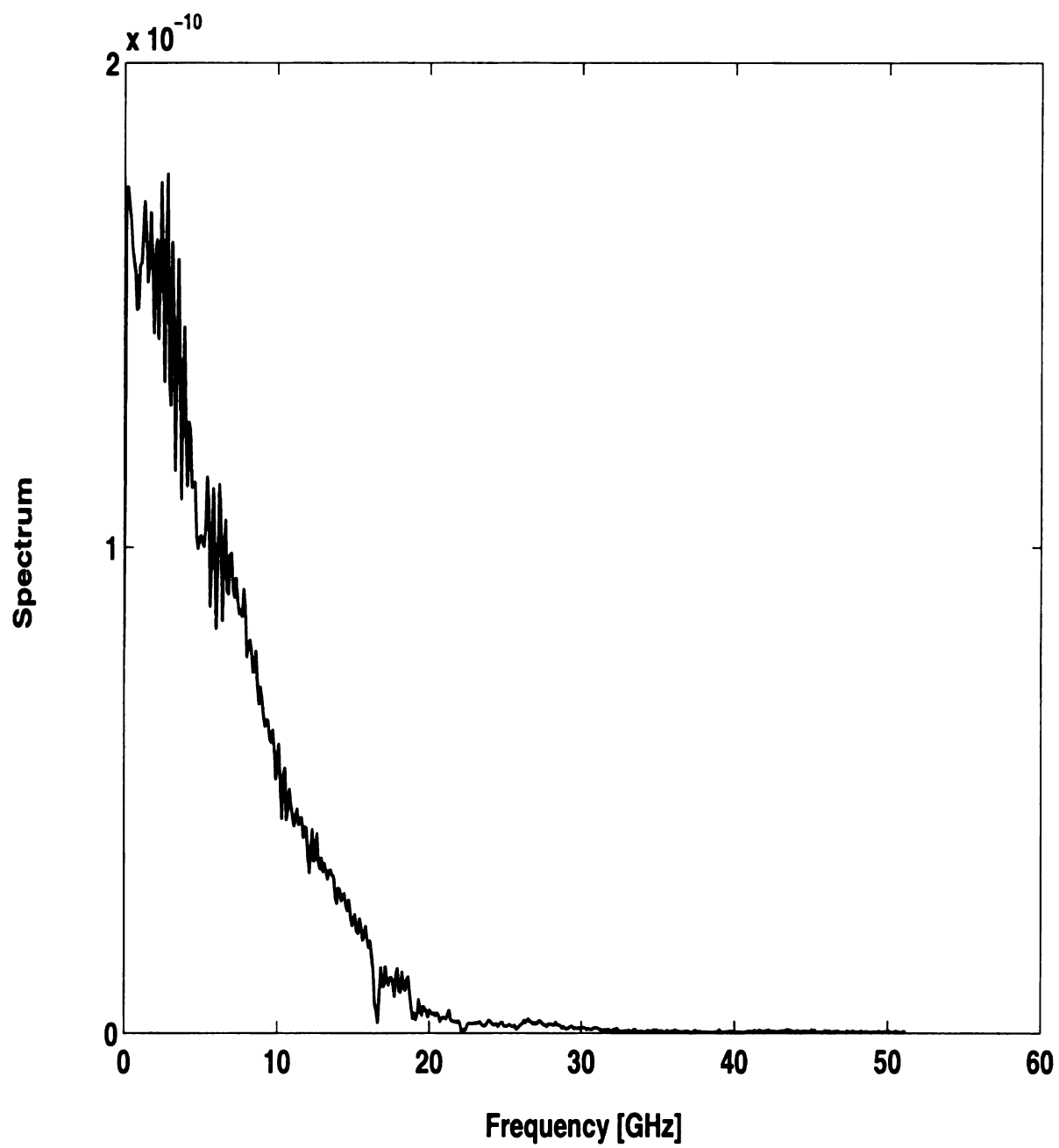


Figure 5.2. (b) Spectrum of impulse shaped waveform transmitted from pulse forming network.

measurements, while a water container made of plexiglass plates is used for 5 lossless or lossy (by filling it with distilled water) measurements. All the plates are 2 feet by 2 feet square in size, and have an approximate thickness of 4.7mm for polystyrene and 5.3mm for plexiglass. Also, an aluminum plate is used as a perfect conductor (PEC) for calibration measurements.

5.3 Calibration

Each intermediate component (e.g. cables, antennas) of the experimental set up shown in Figure 5.1 has its own system function due to dispersion, propagation time delay, amplitude attenuation, etc., and each of these changes the original shape of transmitted waveform. Therefore, a calibration procedure is required to isolate and eliminate the effects of the intermediate system functions from the measured waveform.

Let's denote the original transmitted input waveform by $x(t)$, the impulse response of target object by $r(t)$, the intermediate system impulse response including component effects and time delays in both transmitting and receiving paths by $h_{sys}(t)$, and the received waveform at the scope by $s(t)$, and denote their Fourier transforms as $X(\omega)$, $R(\omega)$, $H_{sys}(\omega)$, $S(\omega)$ respectively. Then, in a measurement, it is expected that

$$S(\omega) = X(\omega)H_{sys}(\omega)R(\omega), \quad (5.1)$$

and equivalently in the time domain

$$s(t) = x(t) * h_{sys}(t) * r(t). \quad (5.2)$$

To isolate the unwanted system response $H_{sys}(\omega)$, a measurement is performed using an object with a known response. One of the appropriate choices for the object would be a PEC plate, for which the theoretical frequency domain reflection coefficient is

-1. Let's denote the scattering measurement from a PEC plate as $C(\omega)$. Then

$$C(\omega) = -X(\omega)H_{sys}(\omega). \quad (5.3)$$

Therefore the intermediate system function is simply given by $H_{sys}(\omega) = -C(\omega)/X(\omega)$ and the calibrated scattering from a target object is

$$\begin{aligned} S_{cal}(\omega) &= S(\omega)/H_{sys}(\omega) \\ &= X(\omega)R(\omega) \end{aligned} \quad (5.4)$$

and $s_{cal}(t) = F^{-1}\{S_{cal}(\omega)\}$.

There is another factor to be considered in actual calibration procedure. Because of the bandwidth limit of the transmitting and receiving antennas, the measurement data outside of the bandwidth range are unreliable and should be discarded by truncating the frequency domain data in outside of the 2-18GHz band and restoring the reduced data size by interpolation. All the data manipulation for the calibration process is executed by using the software WAVECULATOR, written by Dr. J. E. Ross.

5.4 Measurements

5.4.1 Three lossless layer measurements

Using the previously described experimental set up and calibration procedure, actual measurements for several different object layers are performed. For each measurement, scattering from an object is measured first, and then the noise signal from the background environment is measured and subtracted from the target signal. A test measurement of the background noise is plotted in Figure 5.3. Due to absorbers used at several critical spots in these experiments, there are no large undesired reflections

and the remaining background noise level is usually much smaller than that of the target signal. Also, appropriate time windowing helps remove unwanted strong reflections from in background objects or mutipath signals. Next, the reflection from PEC plate is measured for calibration purposes and the background noise signal is measured and subtracted again. Ideally, once the system response has been obtained, the calibration measurement does not have to be repeated. But, in practice, it is necessary to repeat it for each measurement, so as to minimize the noise effect from time drifting of the equipment. For the same reason, the time consumed by each measurement must be minimized. Therefore, as a trade off between these considerations, 1024 points of data are taken within 10 nano second time range with 256 time averages for each data point.

A square polystyrene plate is selected as the first object layer to be measured, because its permittivity ($\epsilon = 2.55\epsilon_0$) has been already verified by a frequency domain measurement at Michigan State University. Figure 5.4 (a) and Figure 5.4 (b) show the time domain object measurement data and its frequency spectrum obtained using a 1024 point fast Fourier transform (FFT), while Figure 5.5 (a) and Figure 5.5 (b) show those of PEC plate measurement data for calibration. These measurements were performed at a 6° aspect angle, which is the closest to normal that is allowed by the experiment system. Note that the amplitude of the reflected wave from the object is quite a bit smaller than that of the transmitted impulse shown in Figure 5.2, so it is plotted on a different scale. Also, it can be recognized that the reflection from the conductor plate is changed significantly from its original transmitted impulse waveform due to the effect of intermediate system function. The system function $H_{sys}(\omega)$ is obtained by the method described in previous section 5.2, and shown in Figure 5.6 (a) and (b). The big peaks in the high frequency region ($> 20\text{GHz}$) come from division errors due to lack of signal content in this band. This fact shows the necessity of truncation of the data spectrum. Finally, the calibrated target responses

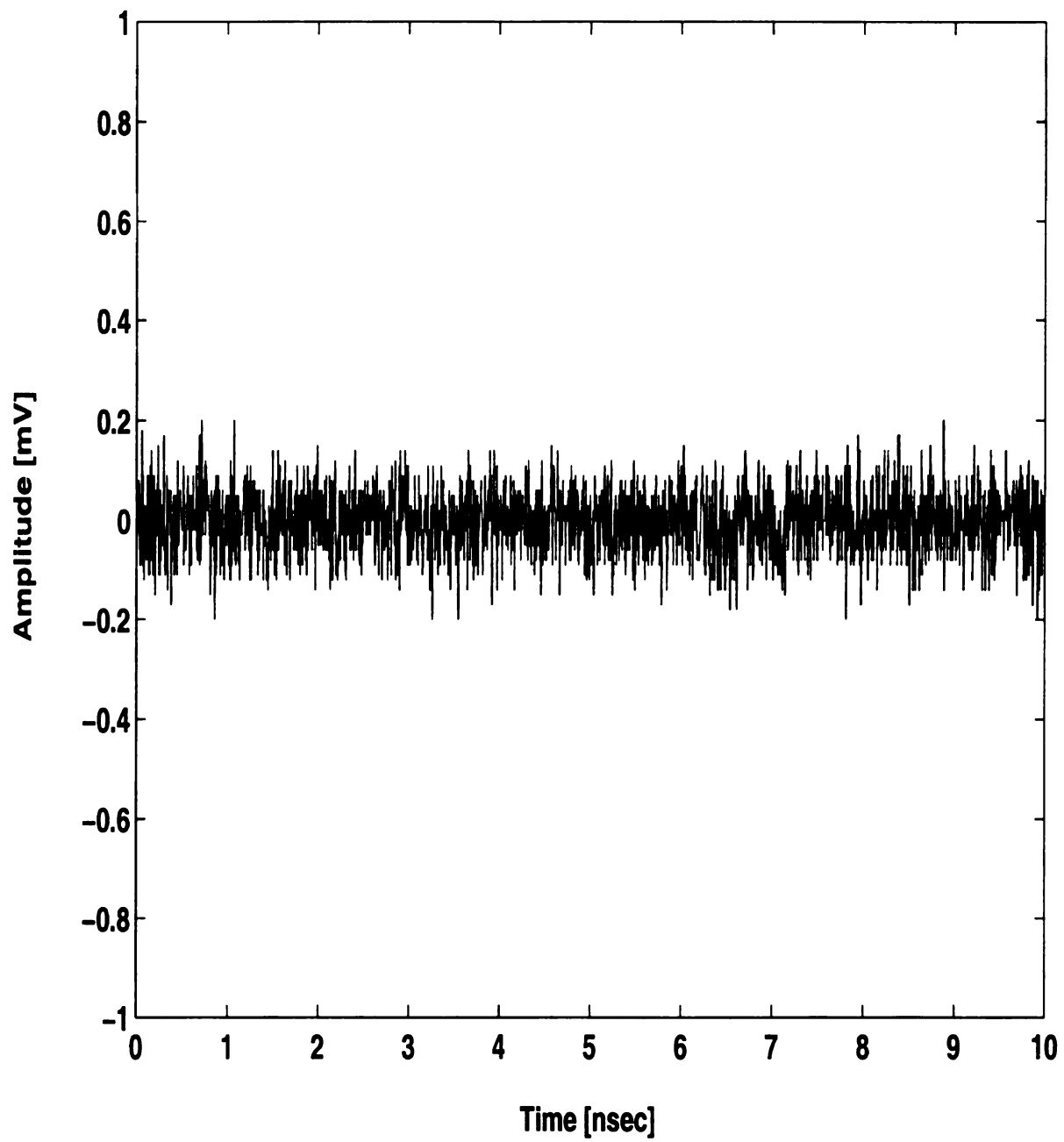


Figure 5.3. Test measurement of background noise level for the arch range.

using the system function are shown in Figure 5.7 (a) and (b) for the results without and with the spectrum truncation, respectively. It is obvious that the out-band noise enlarged by division errors produces significant ringing in the calibrated waveform. The calibrated waveform after truncation processing matches well with that from theoretical computation. The slight mismatches occurring at the third and fourth peaks are mainly due to tilt angle error of the mounted target. The electromagnetic wave propagation velocity inside of polystyrene is $\frac{1}{\sqrt{2.55\mu_0\epsilon_0}} = 1.88 \times 10^8$ m/s. Thus, the distance between the third peaks of the measured and theoretical results of about 10 pico seconds corresponds to a two way propagation distance of 1.88mm. Therefore, a slight tilt error or warping of the surface of less than 1mm can make that difference.

Similar measurements are performed using a plexiglass (thickness=5.3mm, $\epsilon = 2.59\epsilon_0$) layer for both polarizations and three different incidence angles of 6°, 15° and 30°. The results are compared with the corresponding theoretical computations and shown in Figure 5.8, Figure 5.9, and Figure 5.10, respectively. Again, all the results show good agreement with the theoretical curves.

5.4.2 Five lossless layer measurements

An empty plexiglass container was built to be used as a three lossless layered medium, consisting of three even thickness (5.3mm) layers (plexiglass, free space and plexiglass). Then, scattering from the five layered object (free space, plexiglass, free space, plexiglass and free space) can be measured. The calibrated results for both polarizations and three different incidence angles are shown in Figure 5.11, Figure 5.12 and Figure 5.13 respectively. All the measurements are well matched with the theoretical results, although they are slightly worse than those from the three layer measurements. This is probably because of the crude construction of the container, with layer thicknesses different from the designed values. But, still multiple reflections at the layer interfaces can be clearly identified.

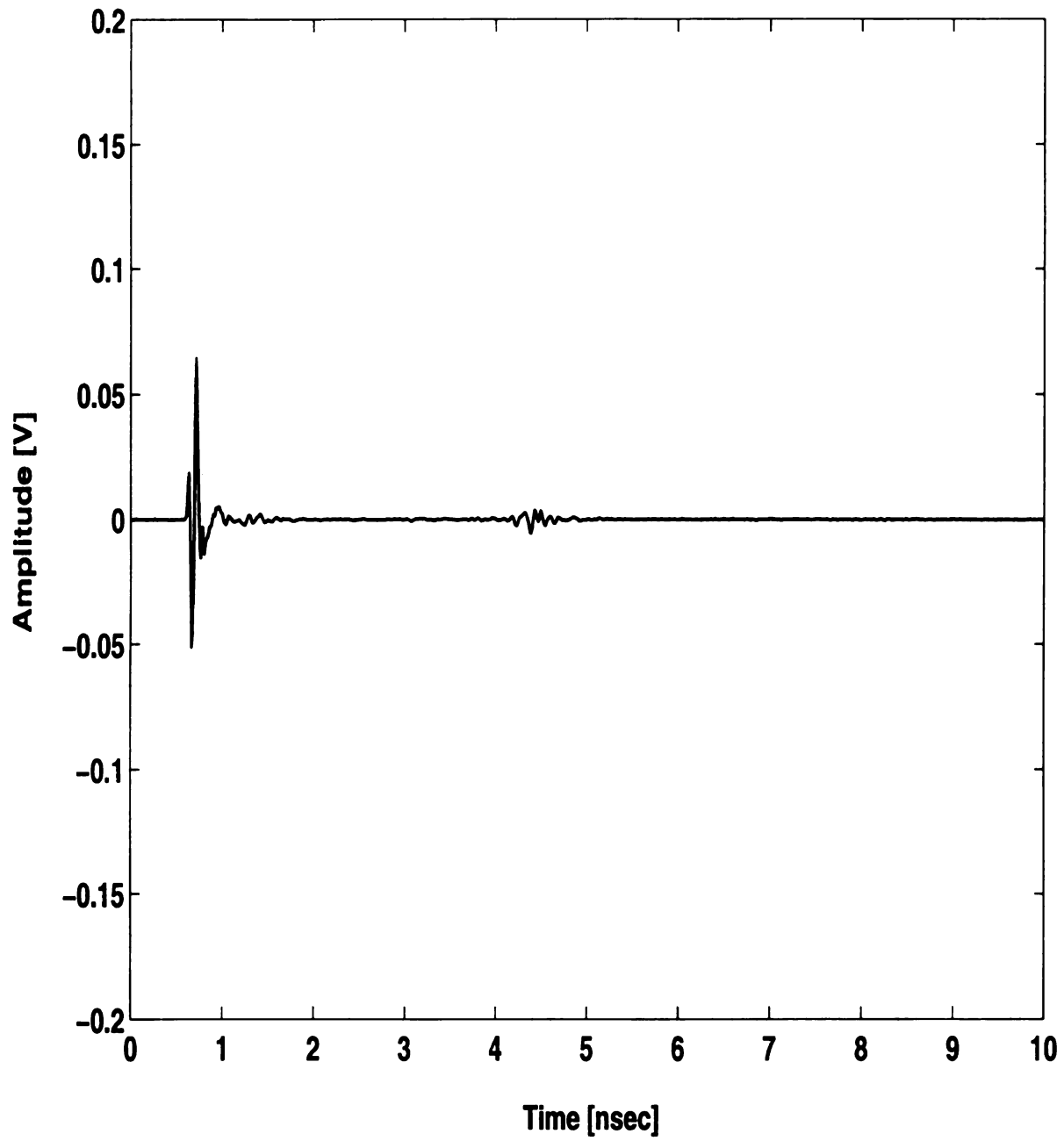


Figure 5.4. (a) Time domain raw measurement data of a polystyrene plate with $\theta_{i1} = 6^\circ$ (TE polarization).

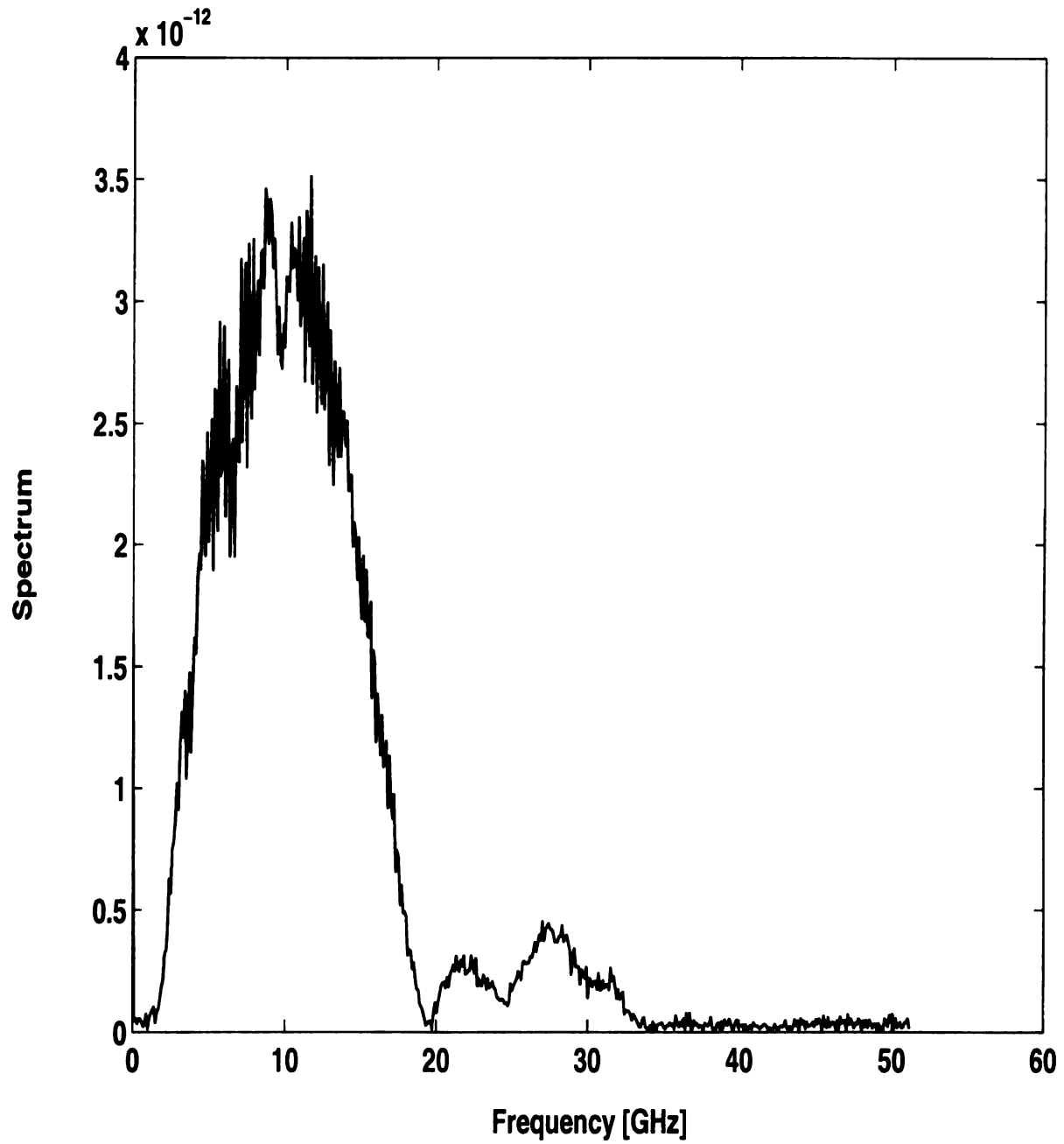


Figure 5.4. (b) Frequency domain spectrum of raw measurement data of a polystyrene plate with $\theta_{i1} = 6^\circ$ (TE polarization).

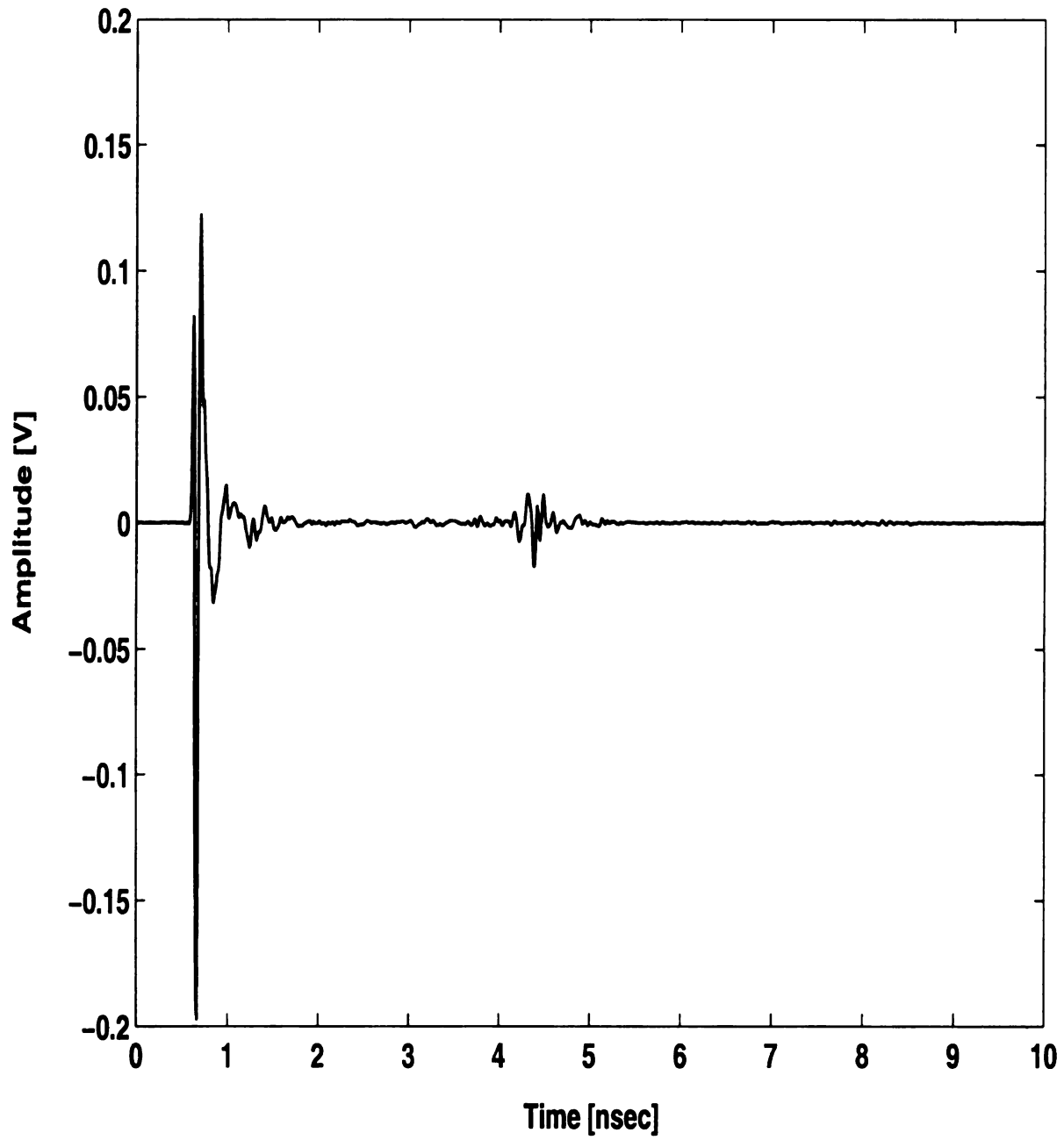


Figure 5.5. (a) Time domain raw measurement data of a PEC plate with $\theta_{i1} = 6^\circ$ (TE polarization).

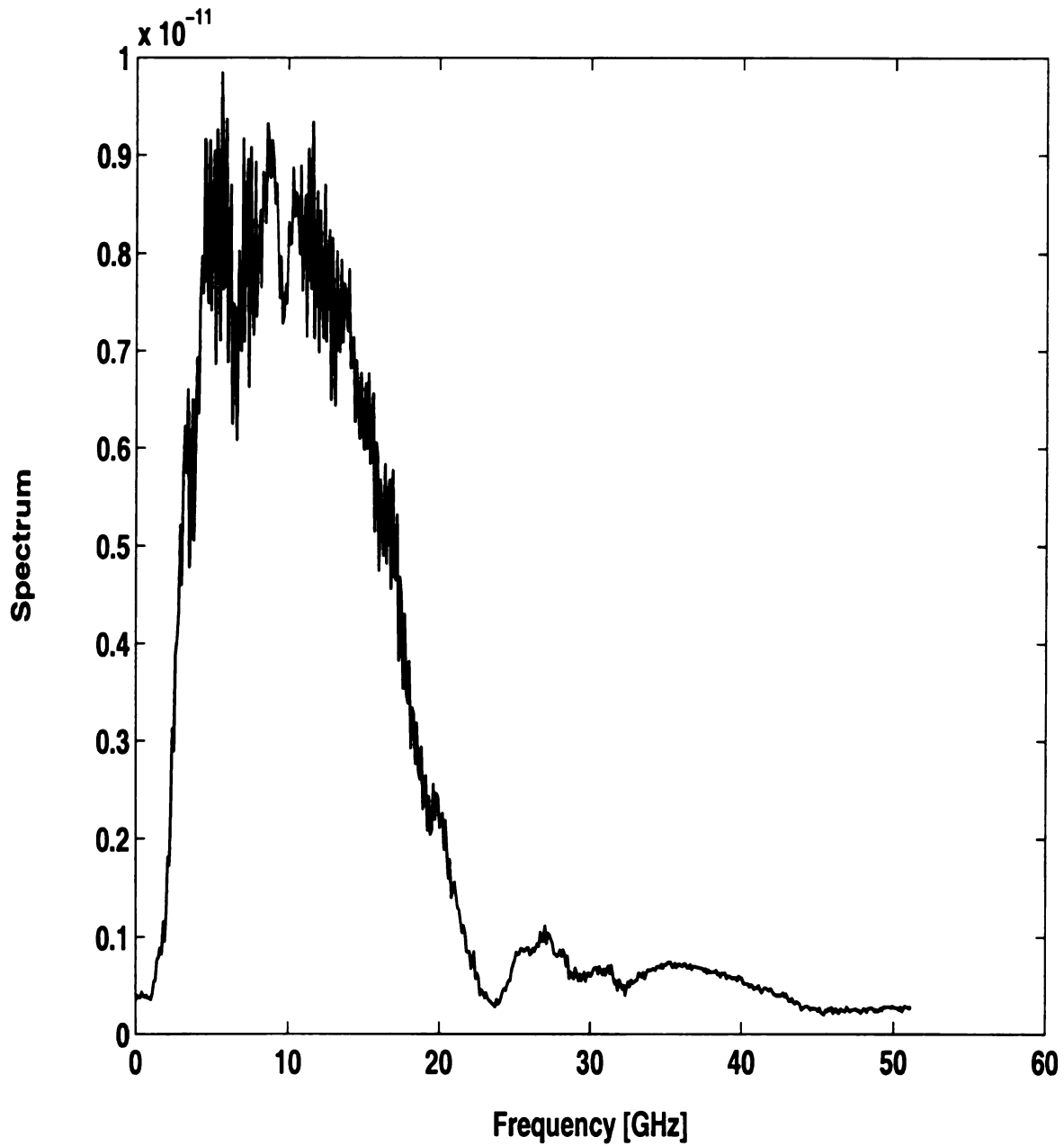


Figure 5.5. (b) Frequency domain spectrum of raw measurement data of a PEC plate with $\theta_{i1} = 6^\circ$ (TE polarization).

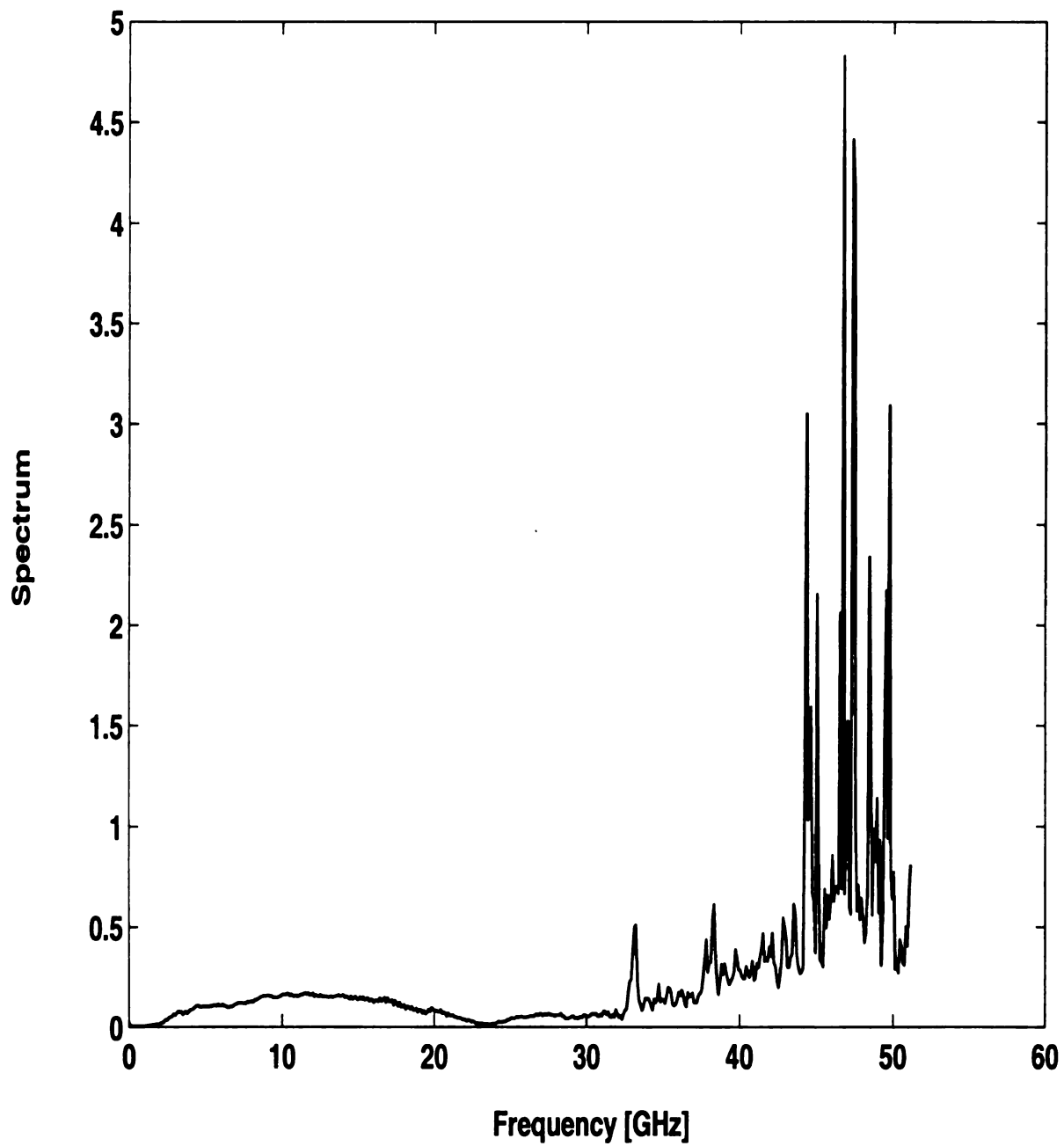


Figure 5.6. (a) Spectrum of intermediate system function.

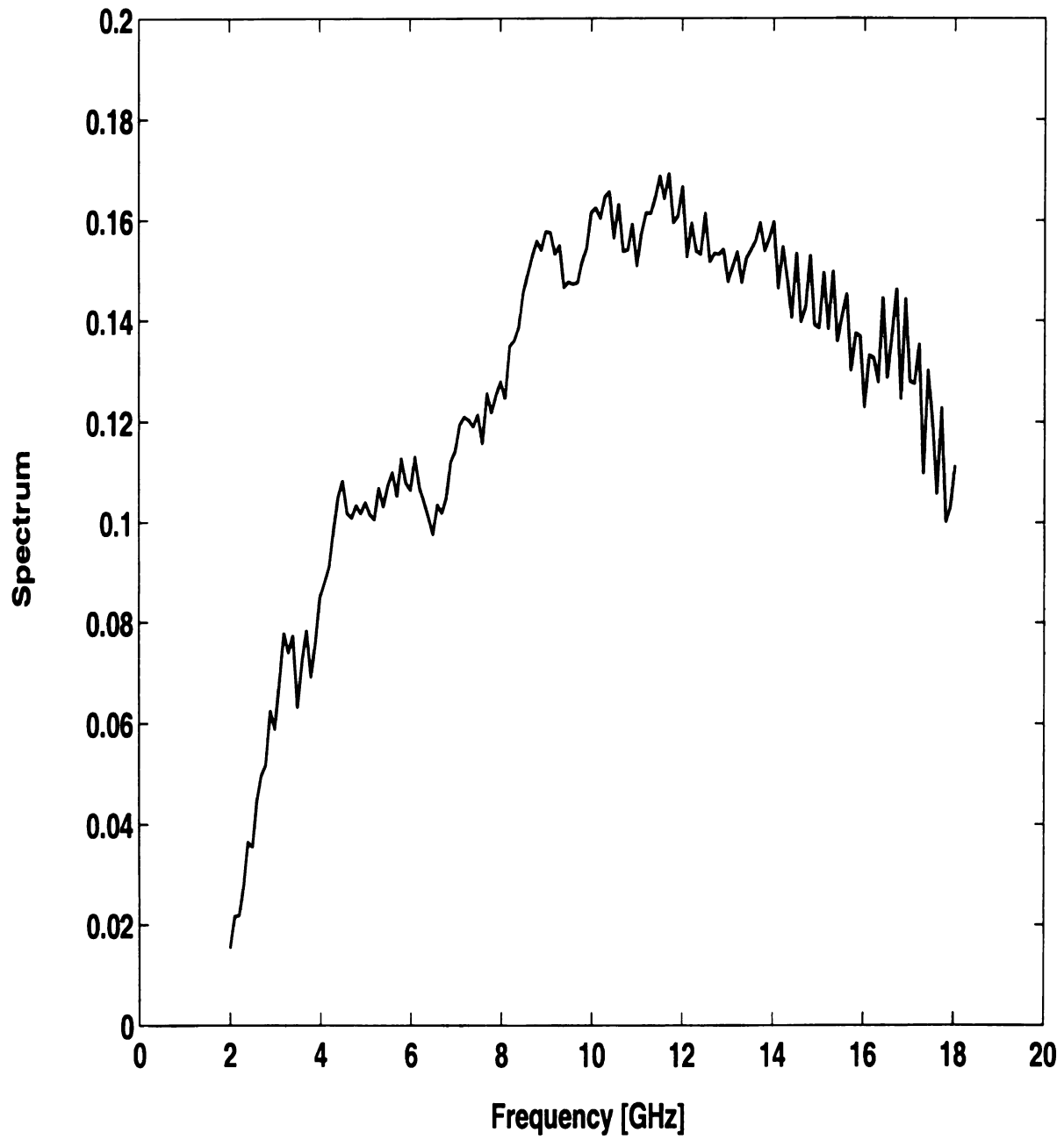


Figure 5.6. (b) Spectrum of intermediate system function after truncation.

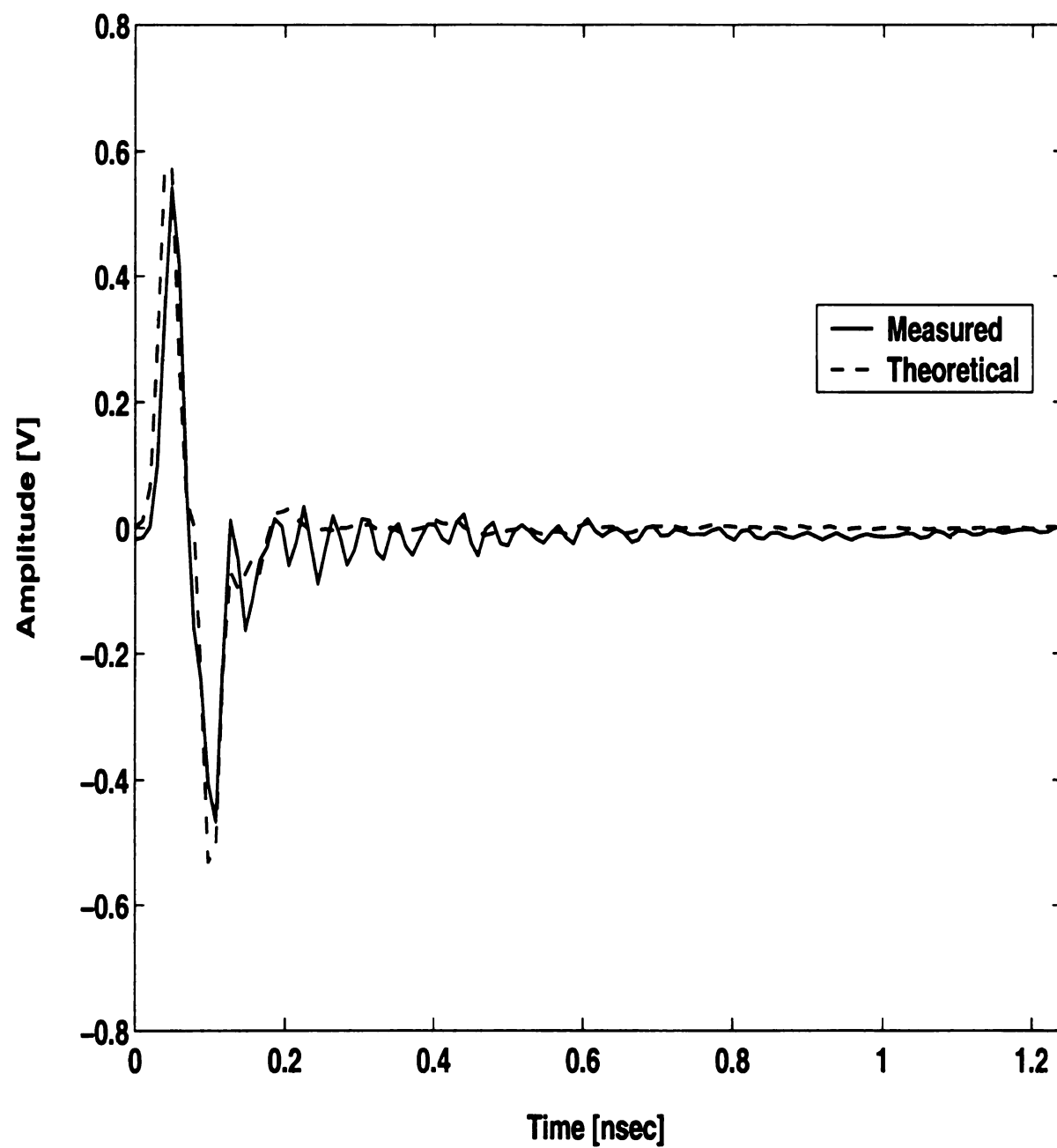


Figure 5.7. (a) Calibrated transient scattering from a polystyrene layer without spectrum truncation (TE polarization).

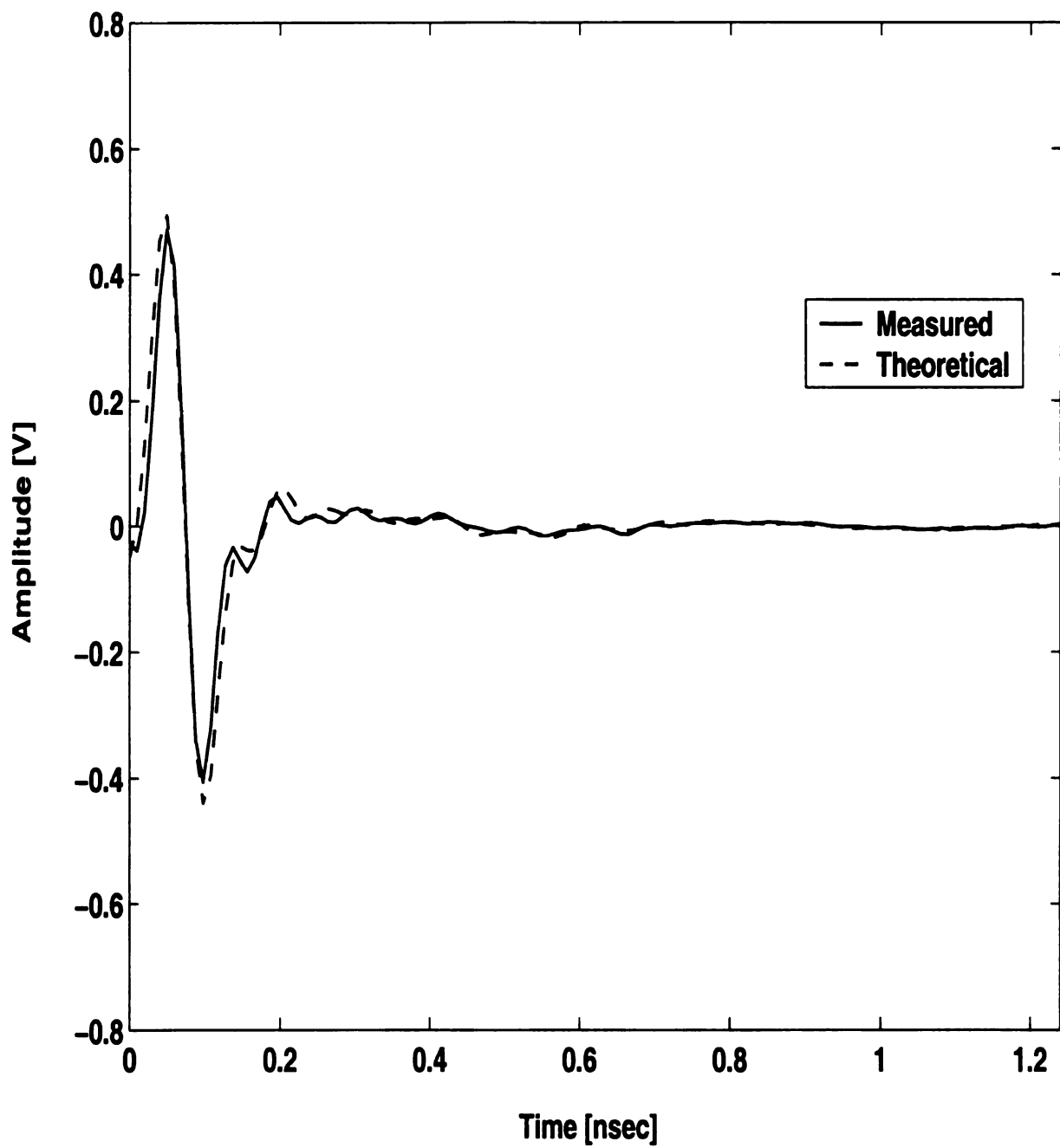


Figure 5.7. (b) Calibrated transient scattering from a polystyrene layer with spectrum truncation (TE polarization).

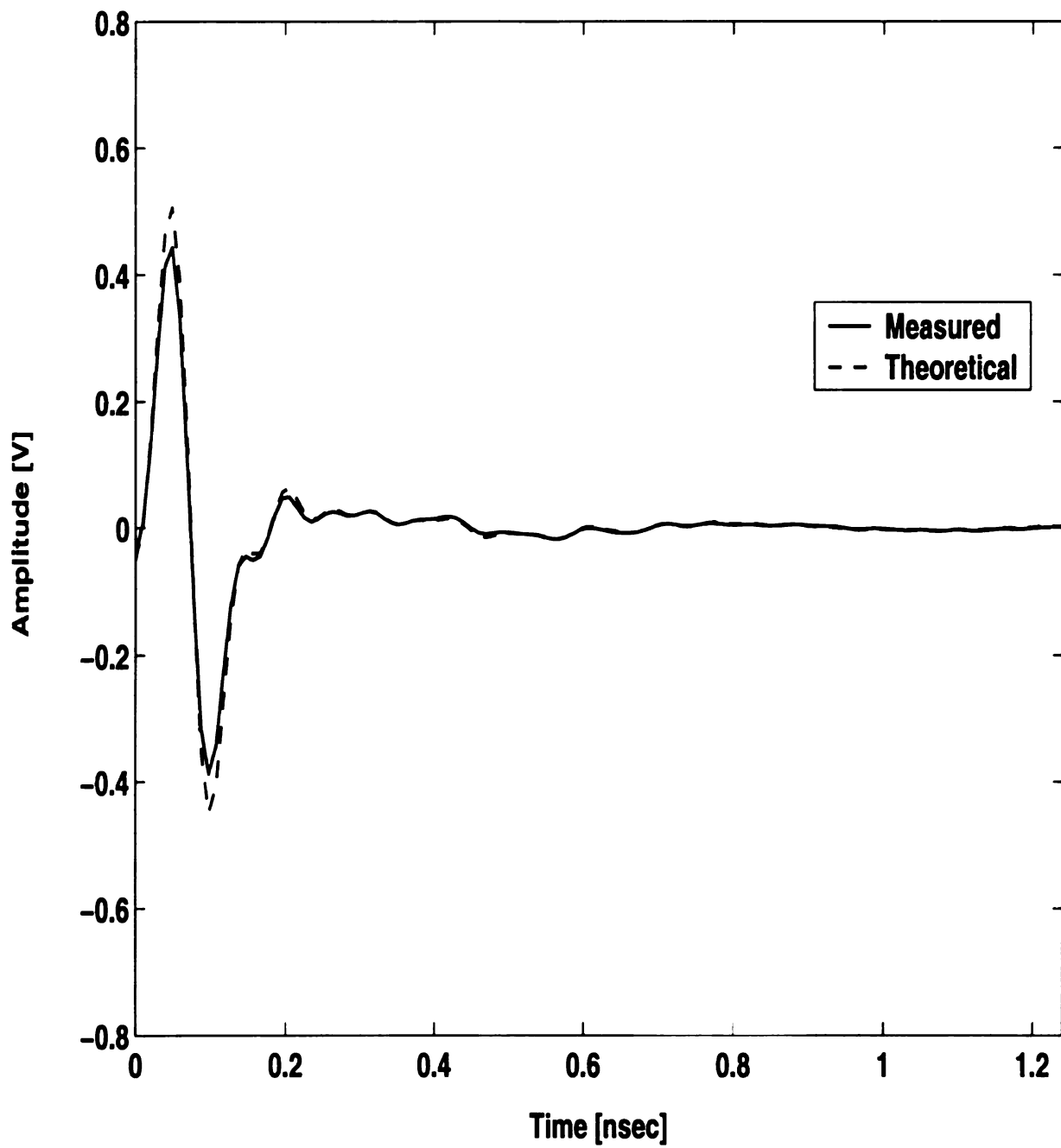


Figure 5.8. (a) Transient scattered field from a plexiglass layer with incidence angle 6° (TE polarization).

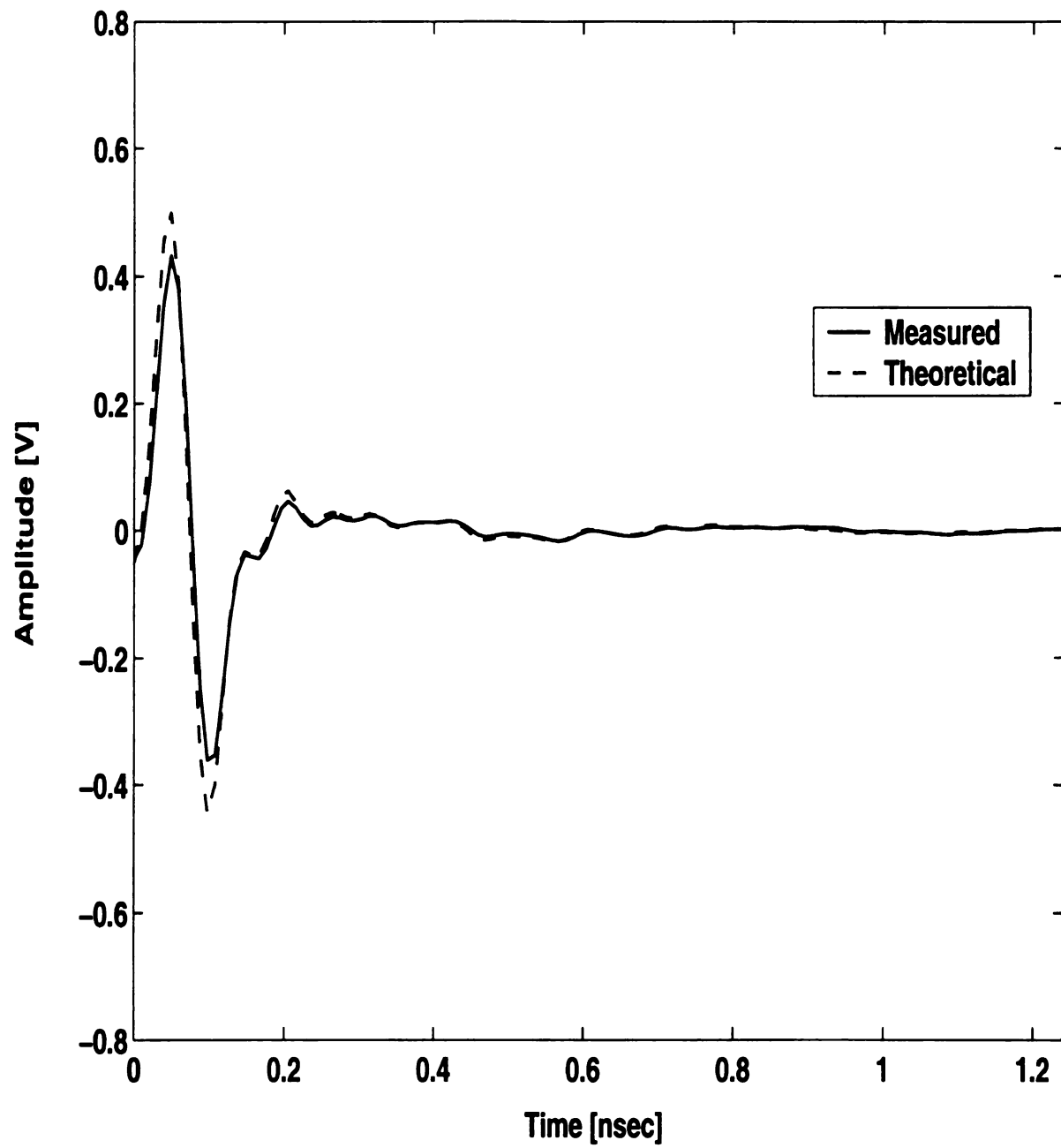


Figure 5.8. (b) Transient scattered field from a plexiglass layer with incidence angle 6° (TM polarization).

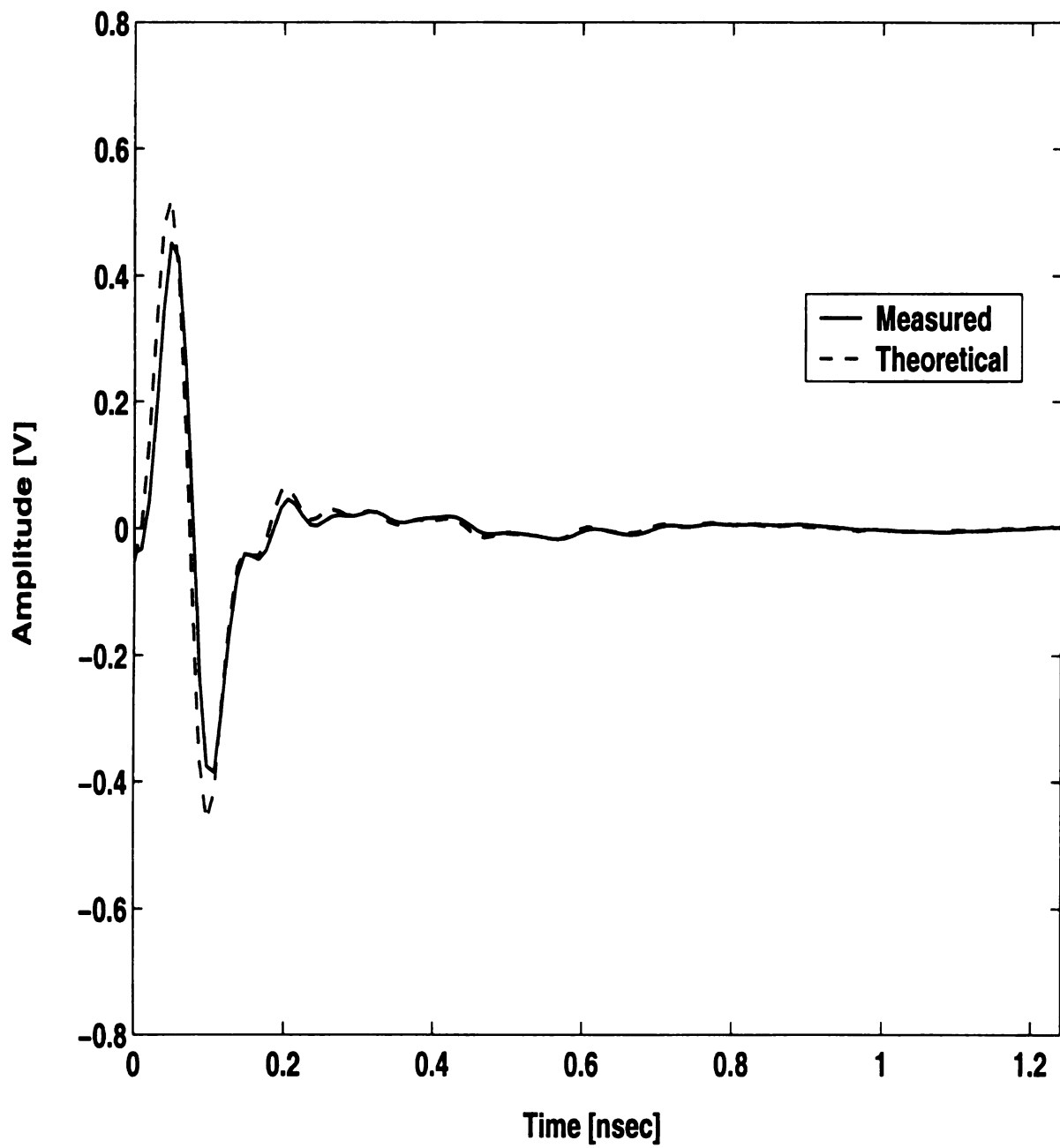


Figure 5.9. (a) Transient scattered field from a plexiglass layer with incidence angle 15° (TE polarization).

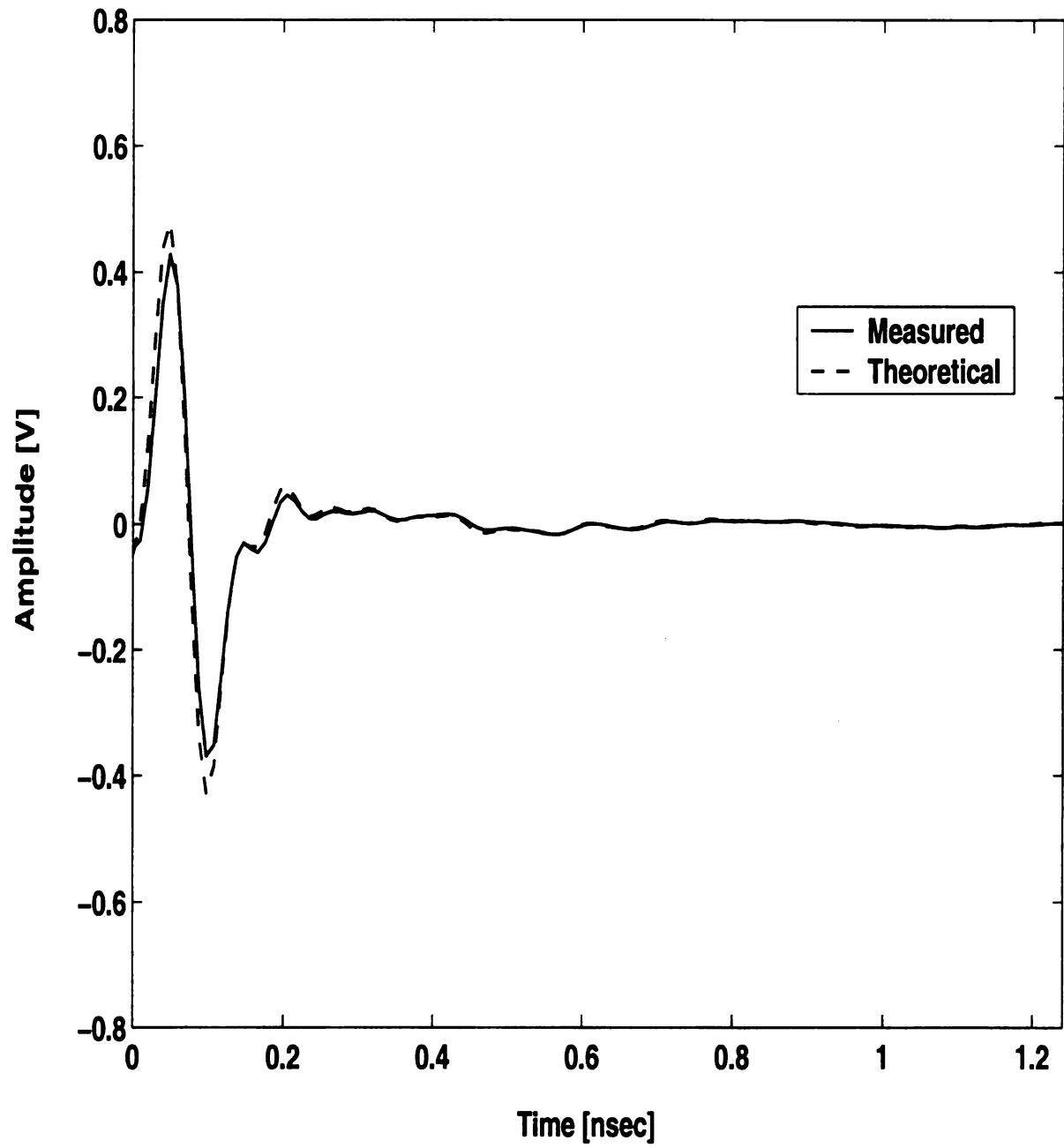


Figure 5.9. (b) Transient scattered field from a plexiglass layer with incidence angle 15° (TM polarization).

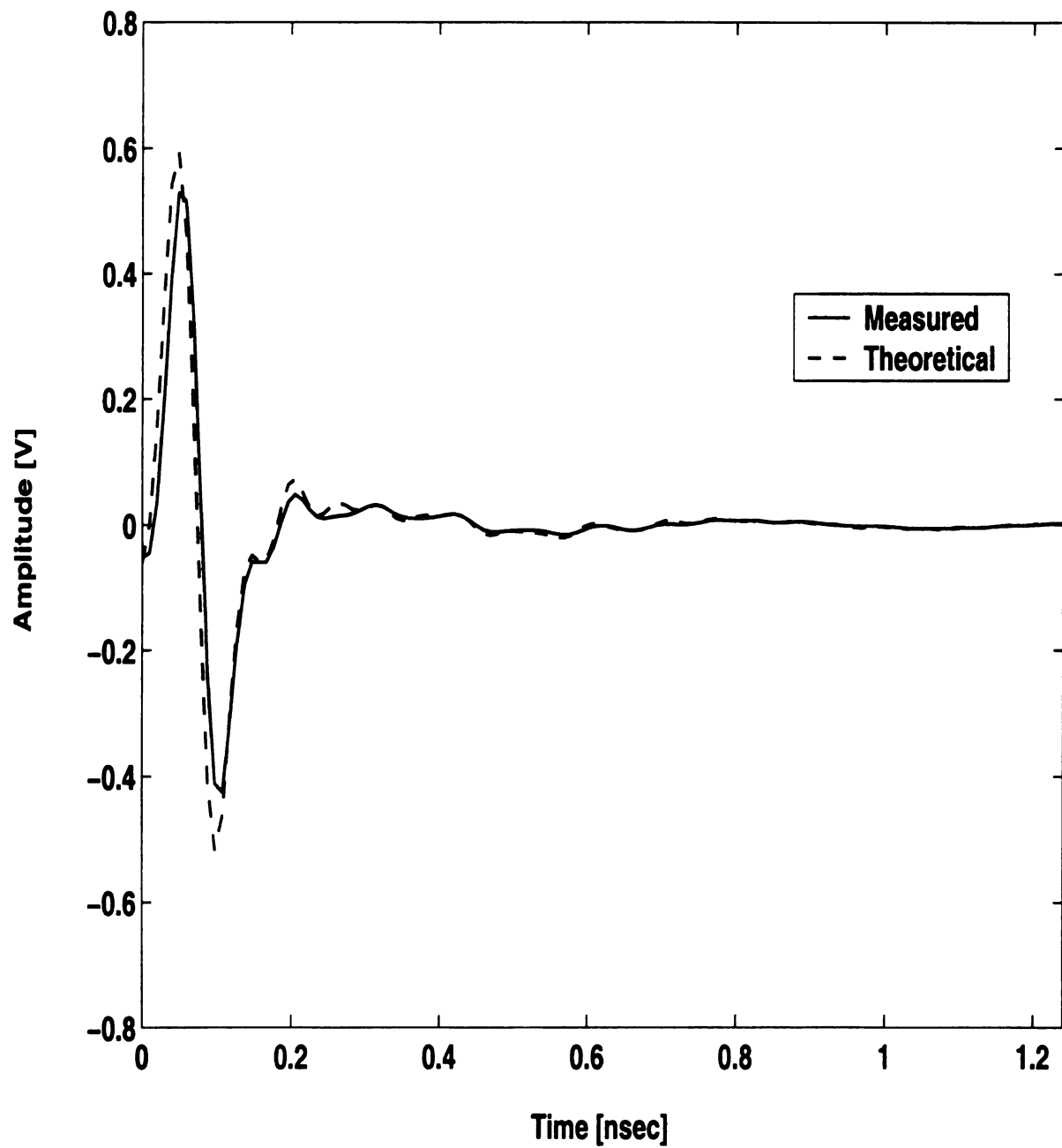


Figure 5.10. (a) Transient scattered field from a plexiglass layer with incidence angle 30° (TE polarization).

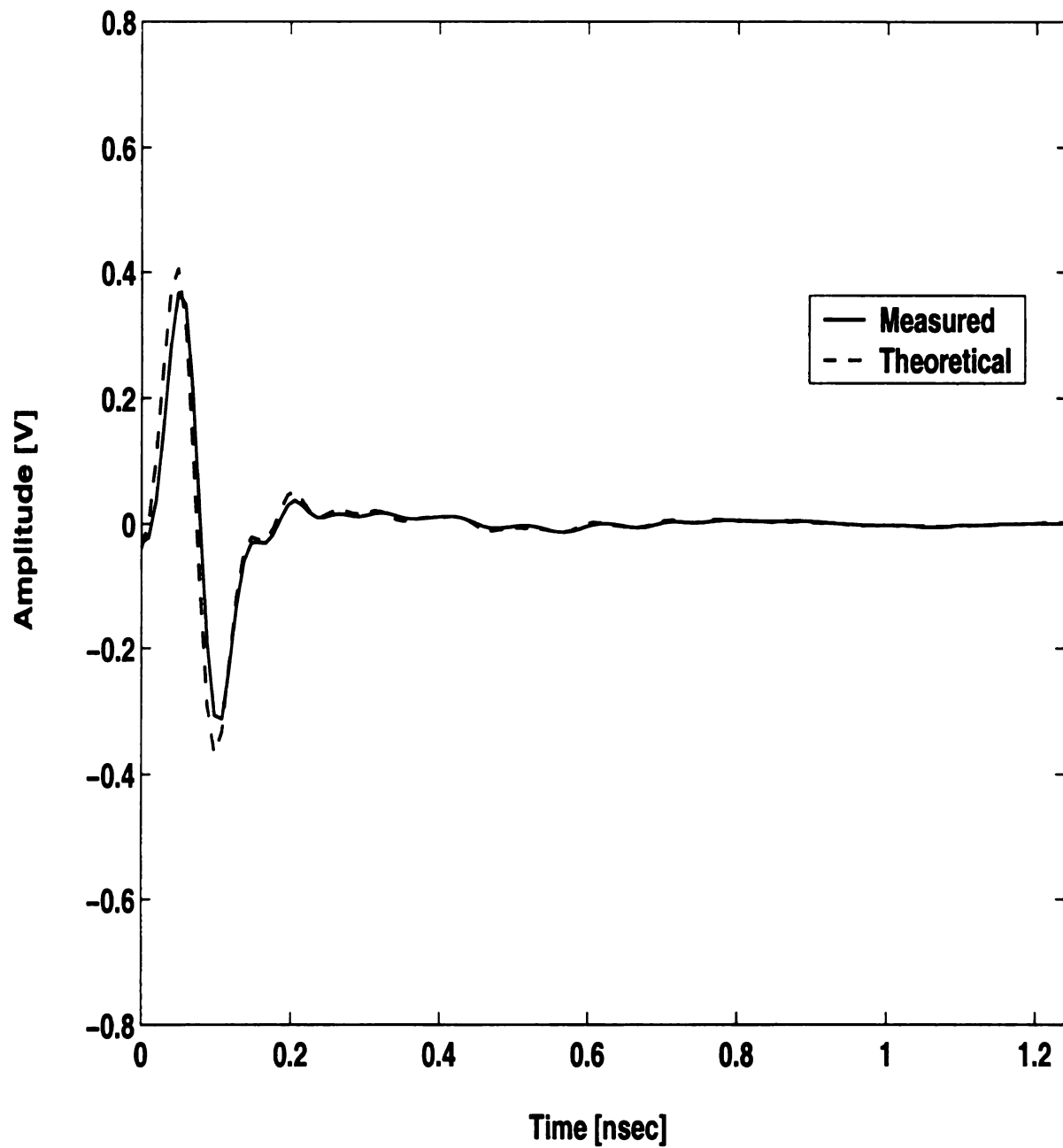


Figure 5.10. (b) Transient scattered field from a plexiglass layer with incidence angle 30° (TM polarization).

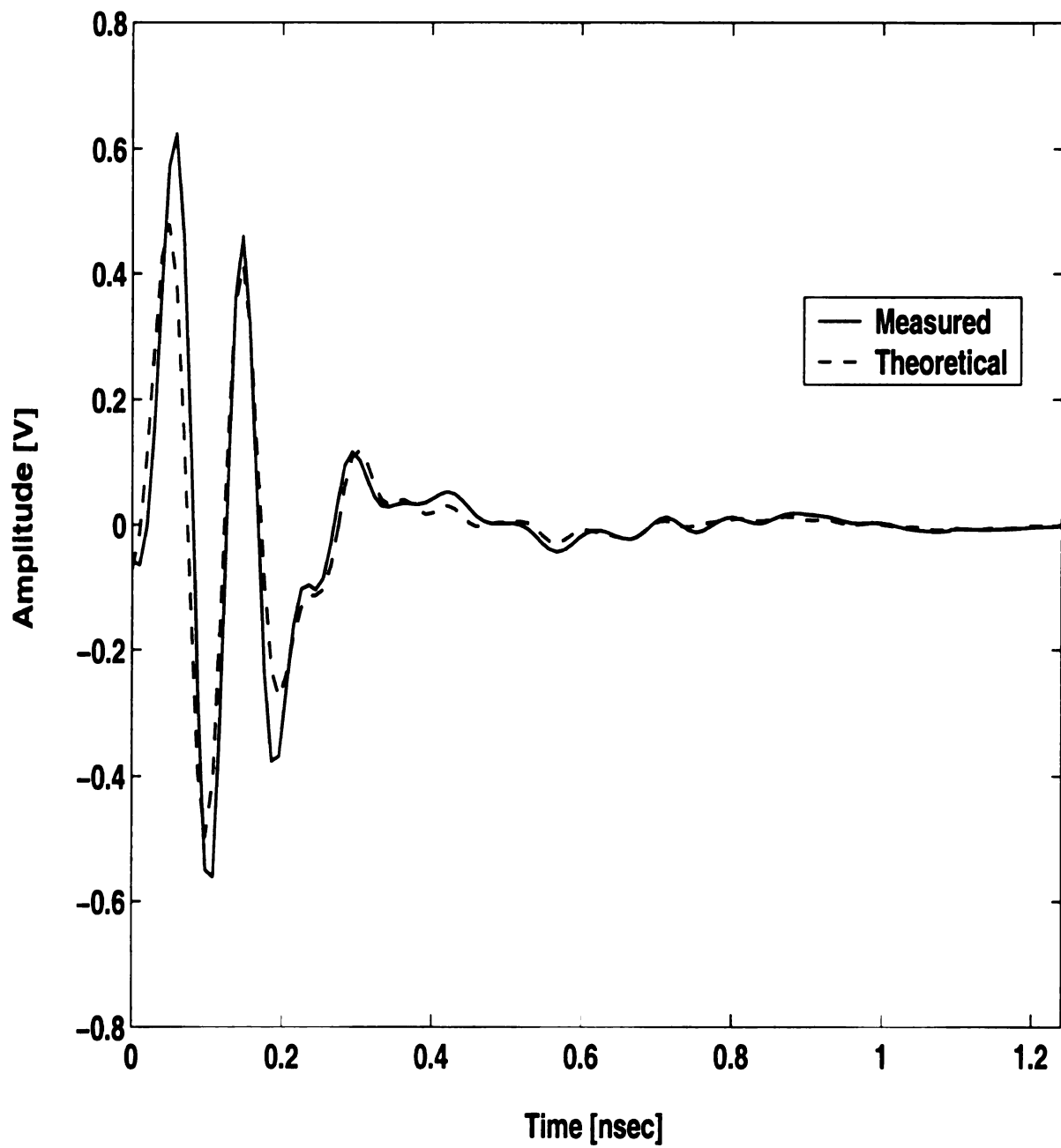


Figure 5.11. (a) Transient scattered field from a plexiglass container with incidence angle 6° (TE polarization).

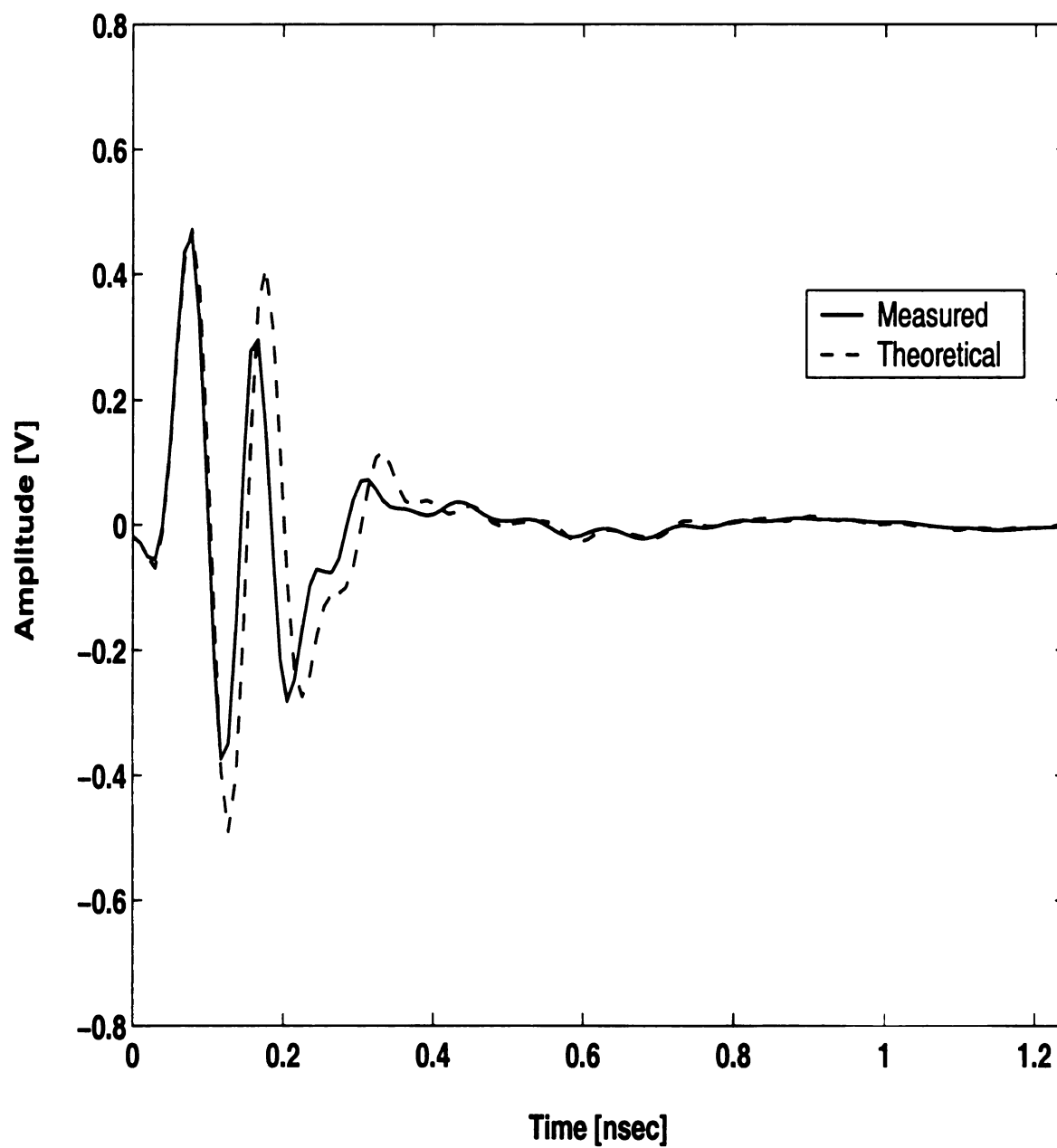


Figure 5.11. (b) Transient scattered field from a plexiglass container with incidence angle 6° (TM polarization).

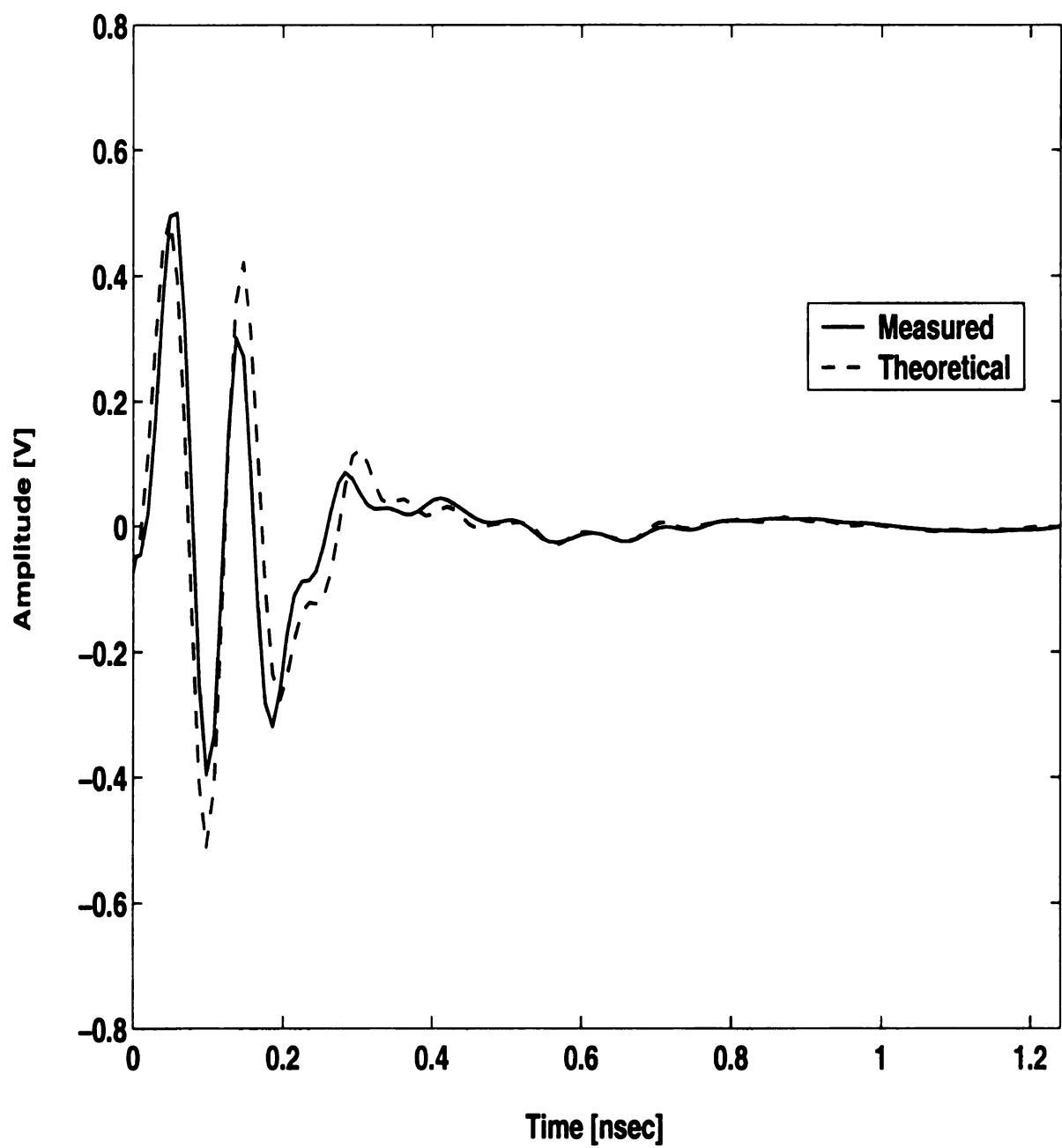


Figure 5.12. (a) Transient scattered field from a plexiglass container with incidence angle 15° (TE polarization).

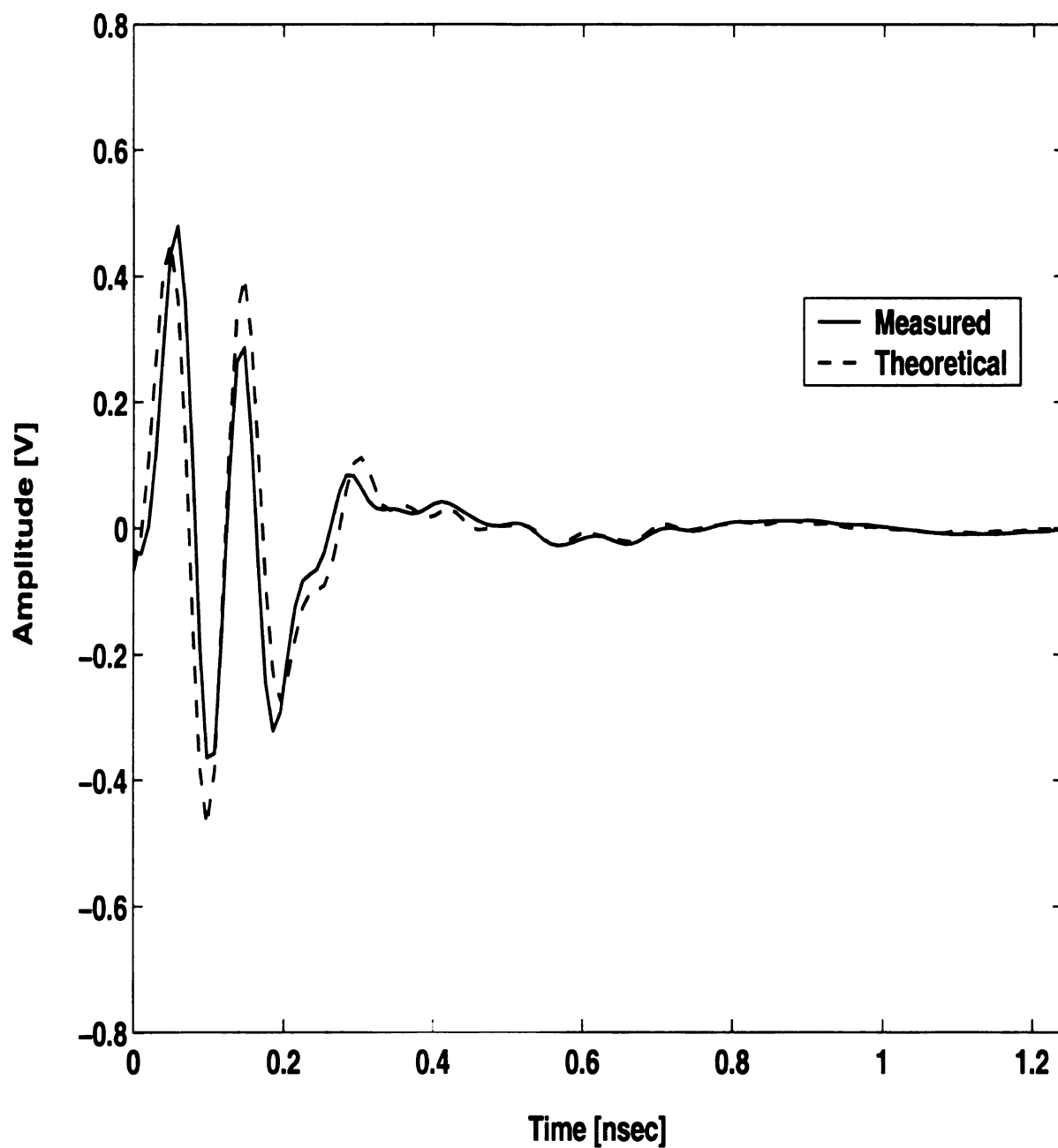


Figure 5.12. (b) Transient scattered field from a plexiglass container with incidence angle 15° (TM polarization).

Another measurement that may have practical interest is performed. Two plates of plexiglass are ‘mechanically’ adhered with some pressure, and the total thickness is 9.8mm. Then, the scattering from the object is measured and plotted in Figure 5.14 (a). What is expected is a reflection from a ‘single’ plate of thickness 9.8mm, and is a waveform consisting of two isolated peaks. But the actual measurement is shows quite a different waveform. Therefore, the theoretical reflection from a three layered object (the five layered medium case) is computed under the assumption that there is a small ‘invisible’ air gap of 0.05mm thickness between the two plexiglass plates, and compared with the measured data. As shown in Figure 5.14, the modified model gives a good match with the measuree data. This is a good example of ‘non destructive’ TDR inspection which is being widely used in industry, and suggests the possibility that more accurate information (size and kind) of mechanical defects smaller than the transmitted pulse width can be obtained, since the exact theoretical response can be found for even lossy materials from the theory derived in this study.

5.4.3 Lossy layered medium measurements

The plexiglass container used in the previous measurement is filled with distilled water for the measurement of scattering from a lossy five layered object. Note that water is a strong polar material, and therefore has significant dielectric loss at high frequency. The theoretical response is computed by converting this dielectric loss to an equivalent conductivity σ_{eq} . Unfortunately, the dielectric loss is a function of frequency, while frequency independent parameters are assumed in this study. The frequency dependence of permittivity which is usually denoted using complex permittivity $\epsilon = \epsilon' - j\epsilon''$ where $\epsilon'' = \frac{\sigma_{eq}}{\omega}$, can be modelled by various functions, including Debye equation [28] as shown in Figure 5.15 for distilled water at 20°C. Considering the available distilled water parameters at several frequency points found in [21], the bandwidth range of 2-18GHz, and the maximum dielectric loss frequency in the Debye model, the equivalent constant conductivity is chosen to be 16.7 Ω/m , the

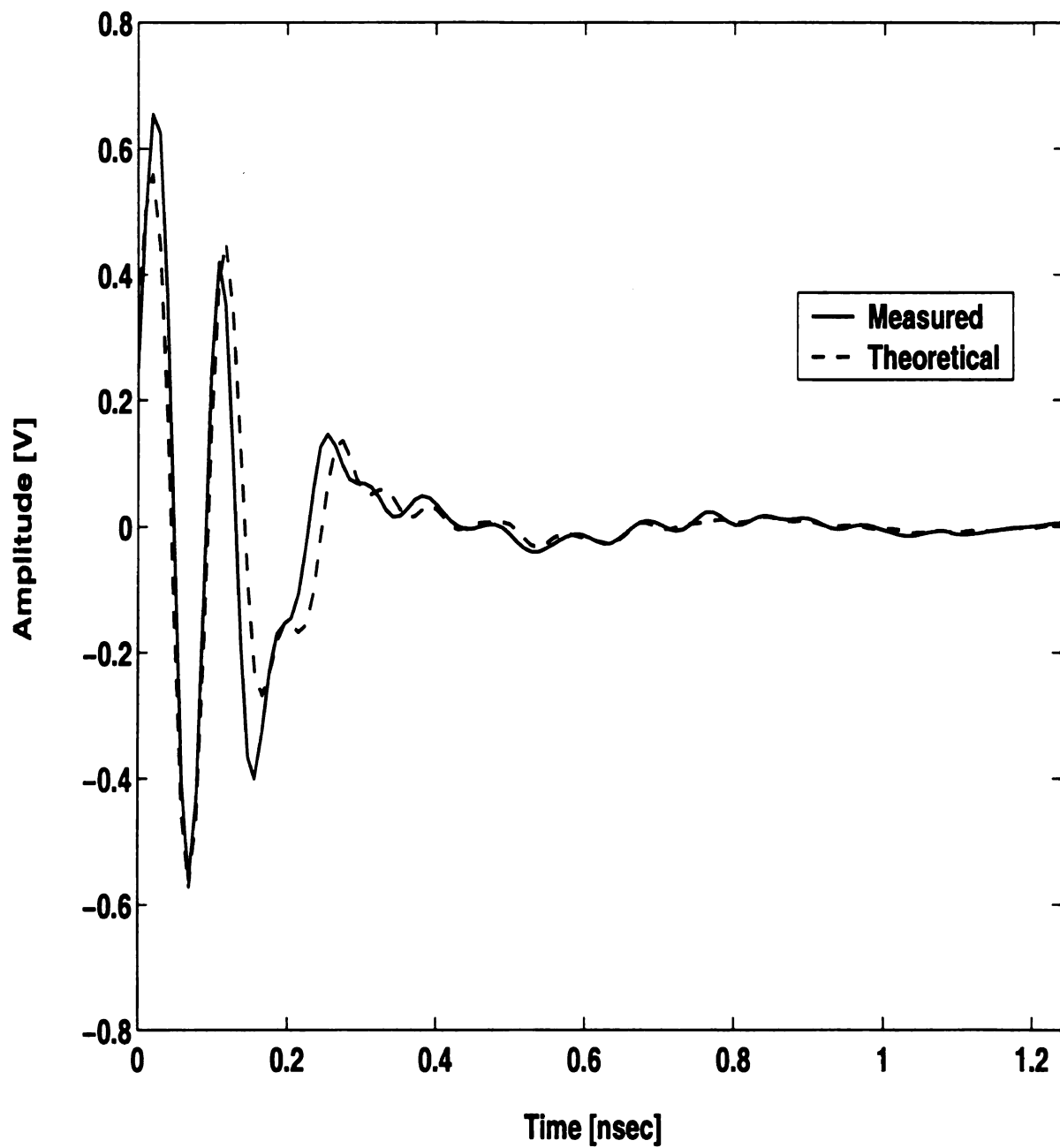


Figure 5.13. (a) Transient scattered field from a plexiglass container with incidence angle 30° (TE polarization).

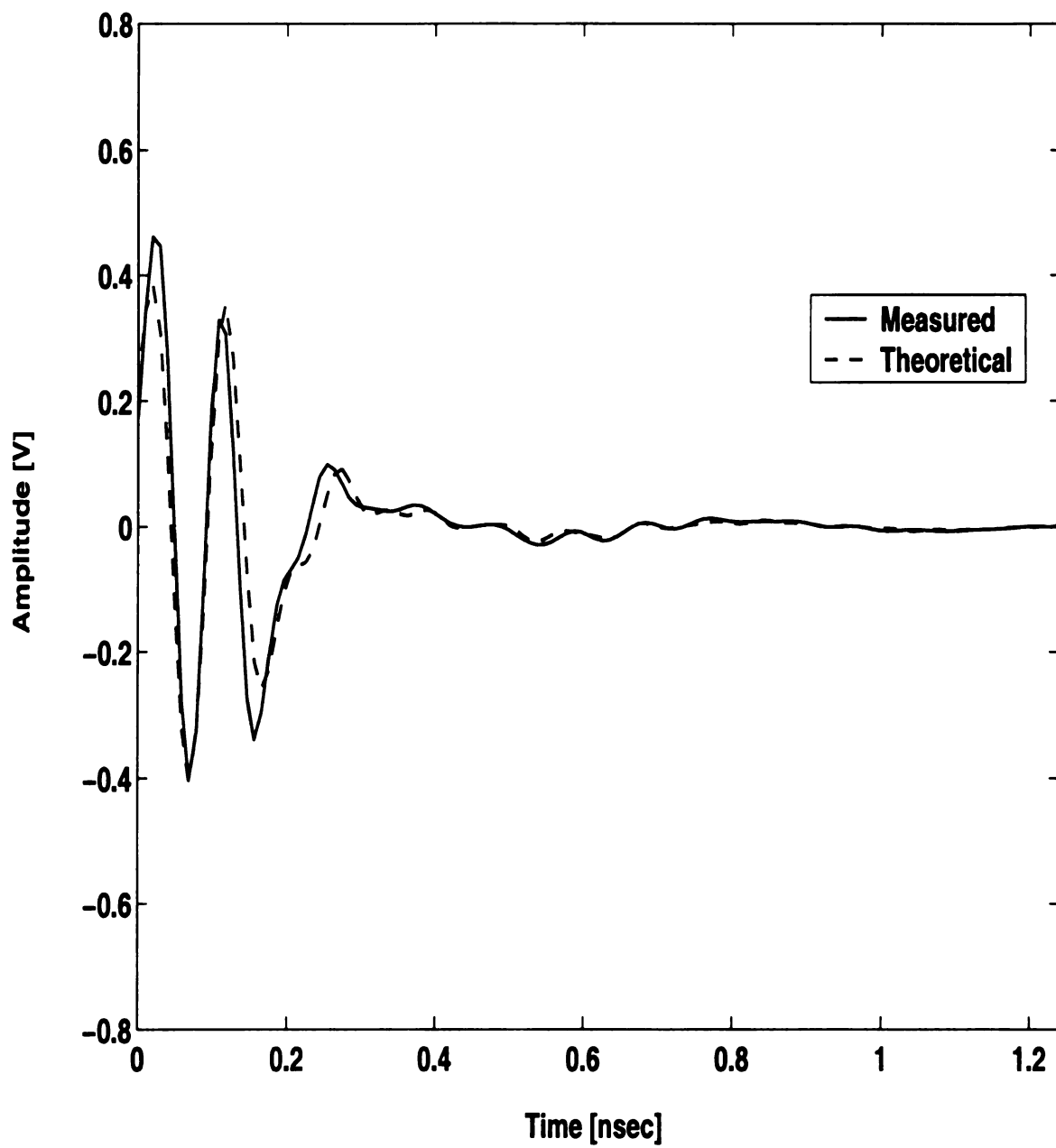


Figure 5.13. (b) Transient scattered field from a plexiglass container with incidence angle 30° (TM polarization).

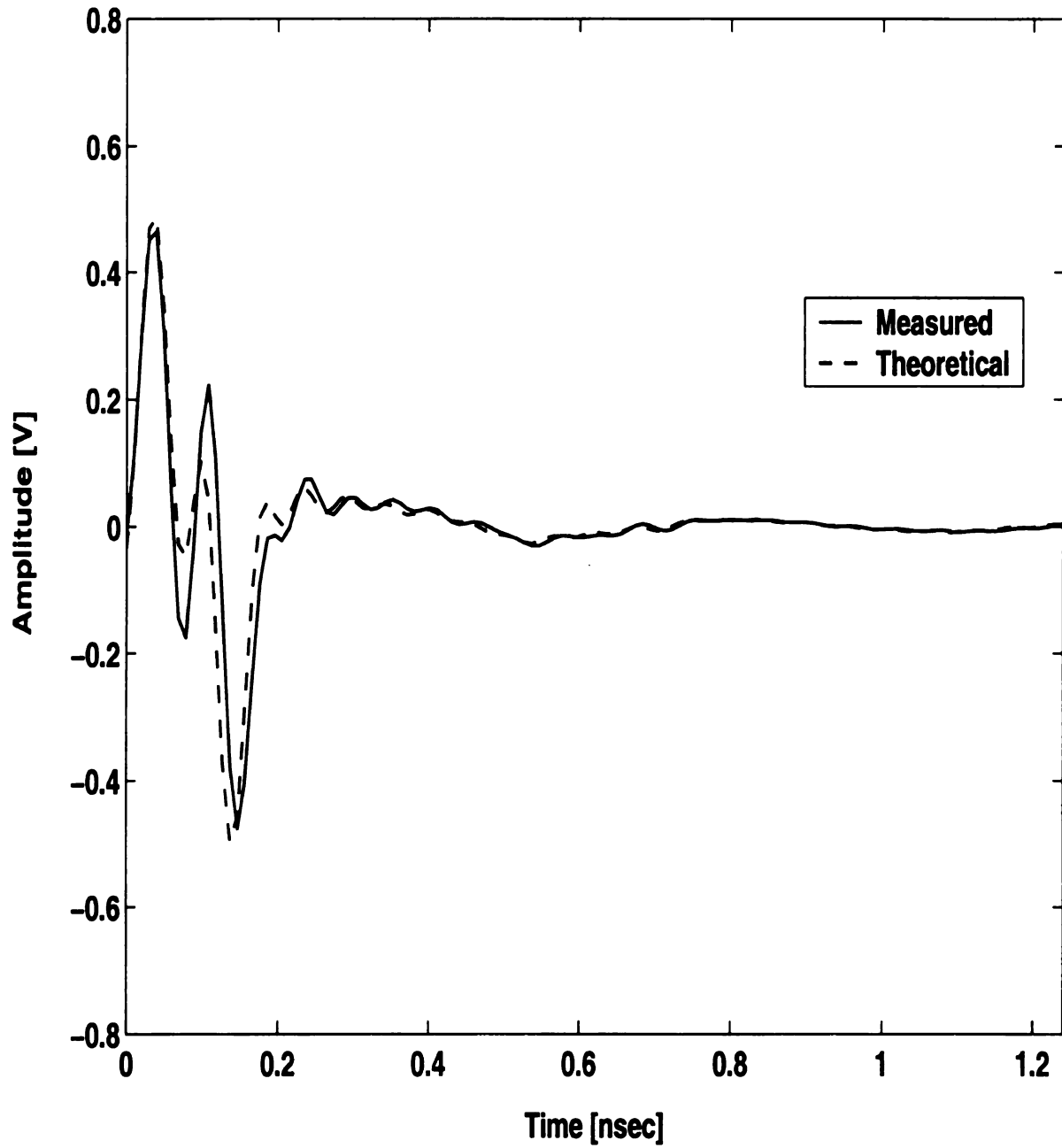


Figure 5.14. (a) Comparison of transient reflection from a mechanically adhered plexiglass plates with that for a single material ($\theta_{i1} = 6^\circ$, TE polarization).

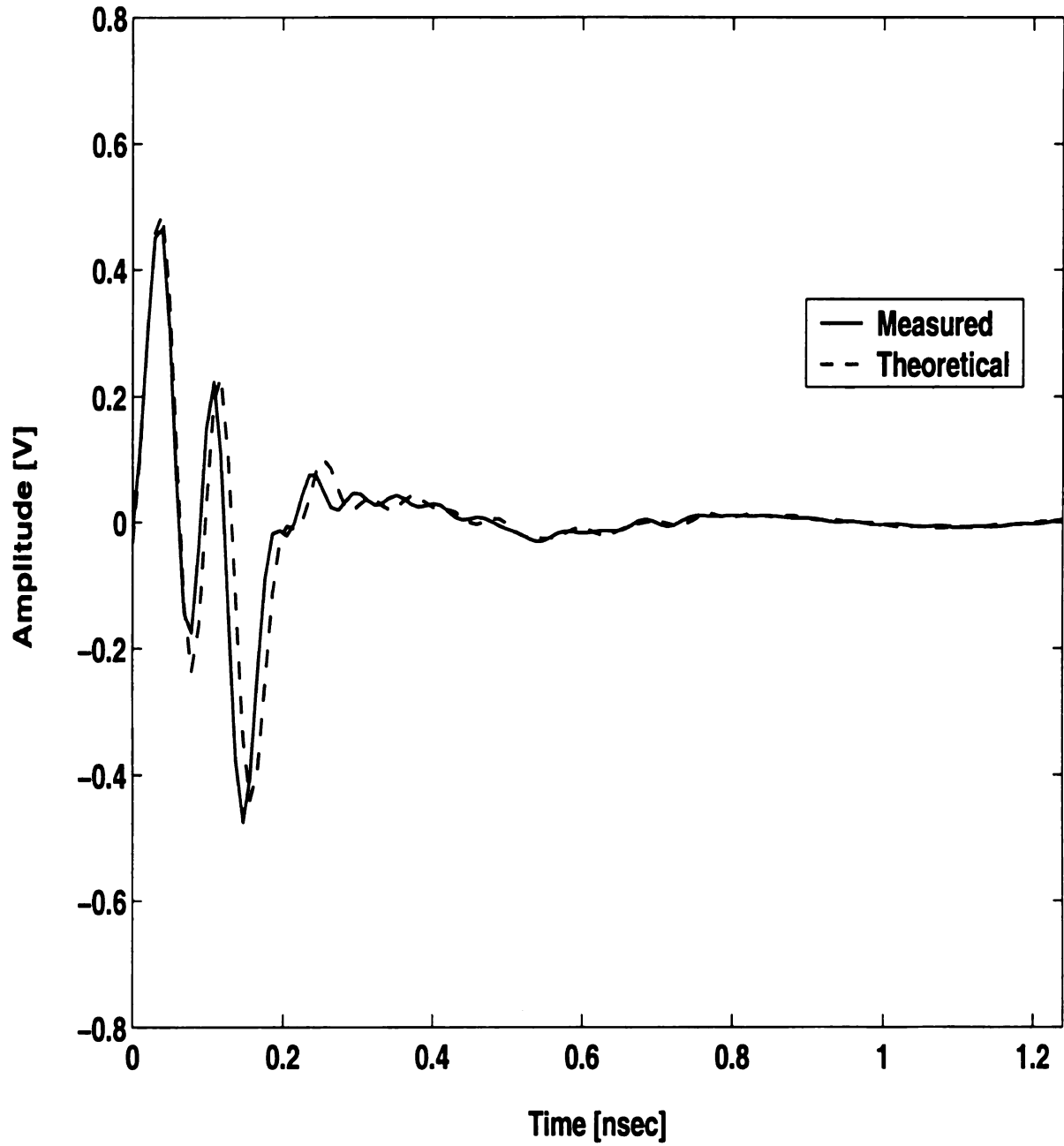


Figure 5.14. (b) Comparison of transient reflection from a mechanically adhered plexiglass plates with that for an air gap inserted model ($\theta_{i1} = 6^\circ$, TE polarization).

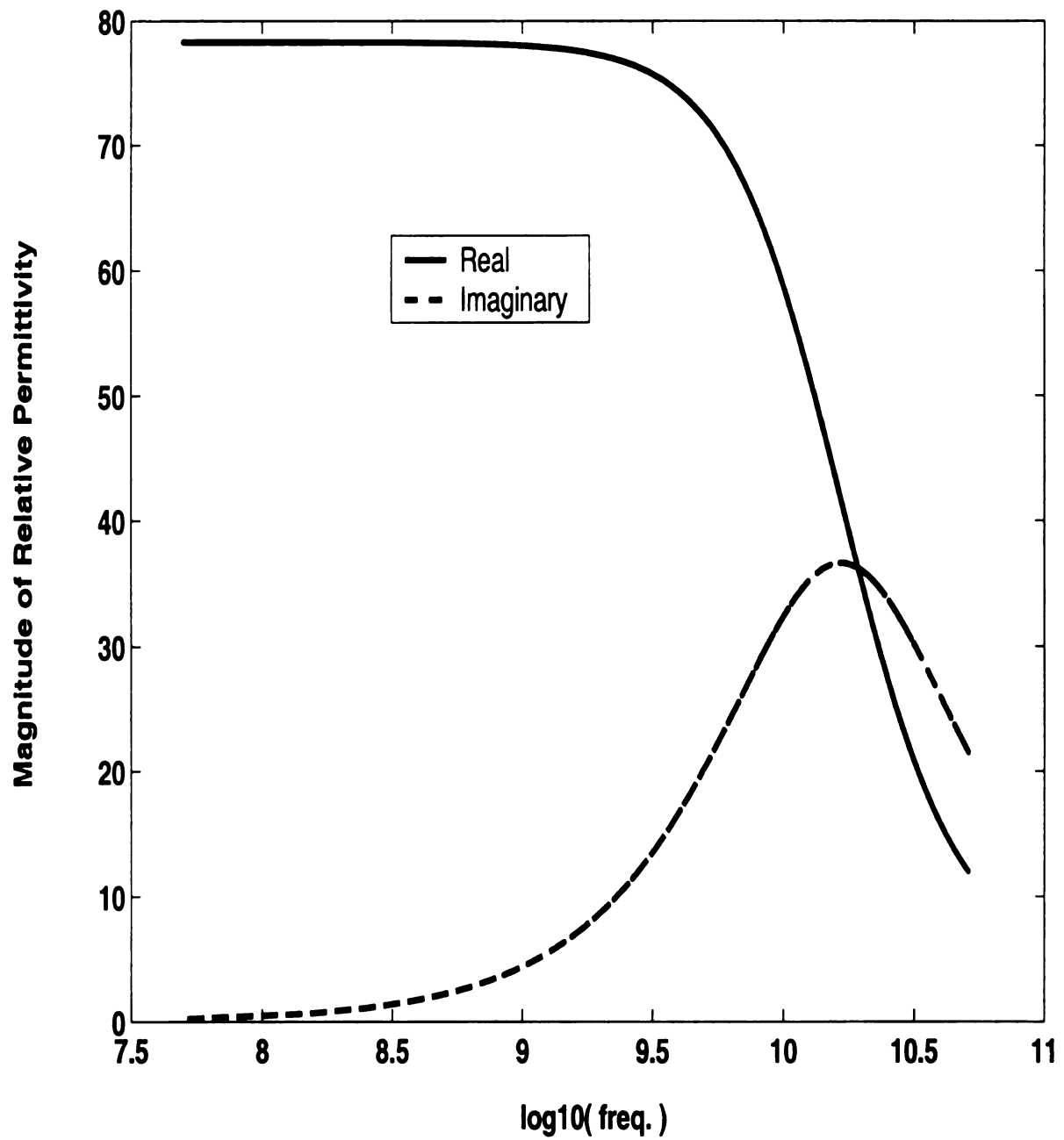


Figure 5.15. $-\epsilon''/\epsilon_0$ curve for water at 20°C given by the Debye model.

value at 10GHz. Therefore the resulting material parameters for the layers are given by $\mu_2 = \mu_0$, $\epsilon_2 = 2.59\epsilon_0$, $\sigma_2 = 0\text{U}/m$, $l_2 = 5.3\text{mm}$, $\mu_3 = \mu_0$, $\epsilon_3 = 55\epsilon_0$, $\sigma_3 = 16.7\text{U}/m$, $l_3 = 5.3\text{mm}$, $\mu_4 = \mu_0$, $\epsilon_4 = 2.59\epsilon_0$, $\sigma_4 = 0\text{U}/m$, $l_4 = 5.3\text{mm}$.

A measurement result for $\theta_{i1} = 6^\circ$ incidence angle is shown in Figure 5.16. As seen in the figure, there is considerable mismatch in the theoretical and measured data. However, by careful observation, it can be realized that there is a constant scaling relationship between the two curves. It turns out that the cause of this mismatch is ‘mechanical’ rather than ‘electrical’. That is, the shape of the water filled container has been changed by water pressure and gravity, and it causes a tilt angle which result in a reduction of the received wave amplitude. This explanation can be justified from the observation that the first reflection peak from air and plexiglass interface, which can not be effected by the third layer material, has been reduced. By considering this phenomenon, all the measurements for the distilled water container are scaled by the ratio of the first peak amplitudes of the measured data to that of the theoretical result. The scaled measurements are shown in Figure 5.17-Figure 5.19, and all of them show a good match between measurements and theoretical expectations.

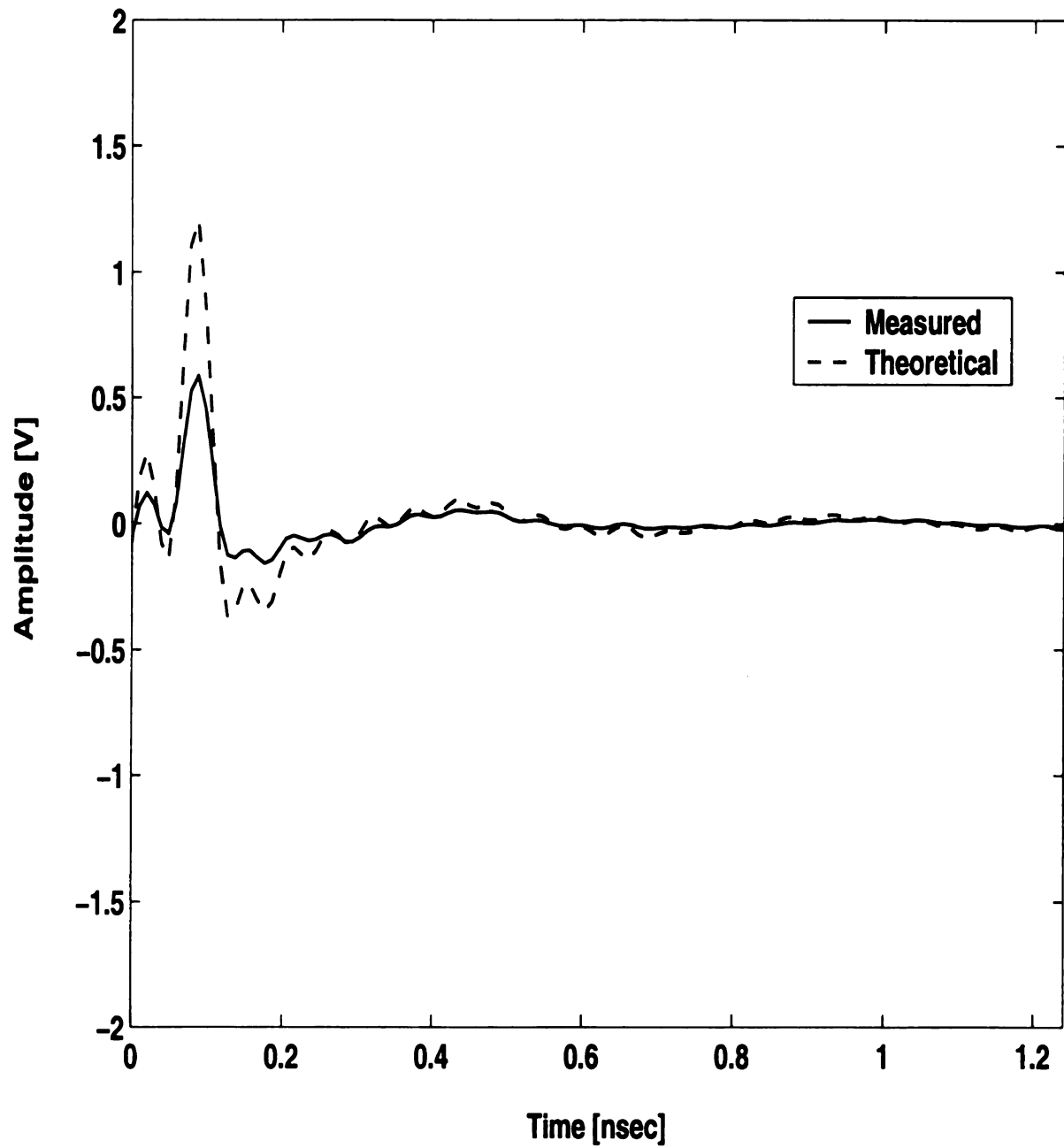


Figure 5.16. Trial measurements for transient scattered field from a 5 lossy layered medium with incidence angle 6° (TE polarization).

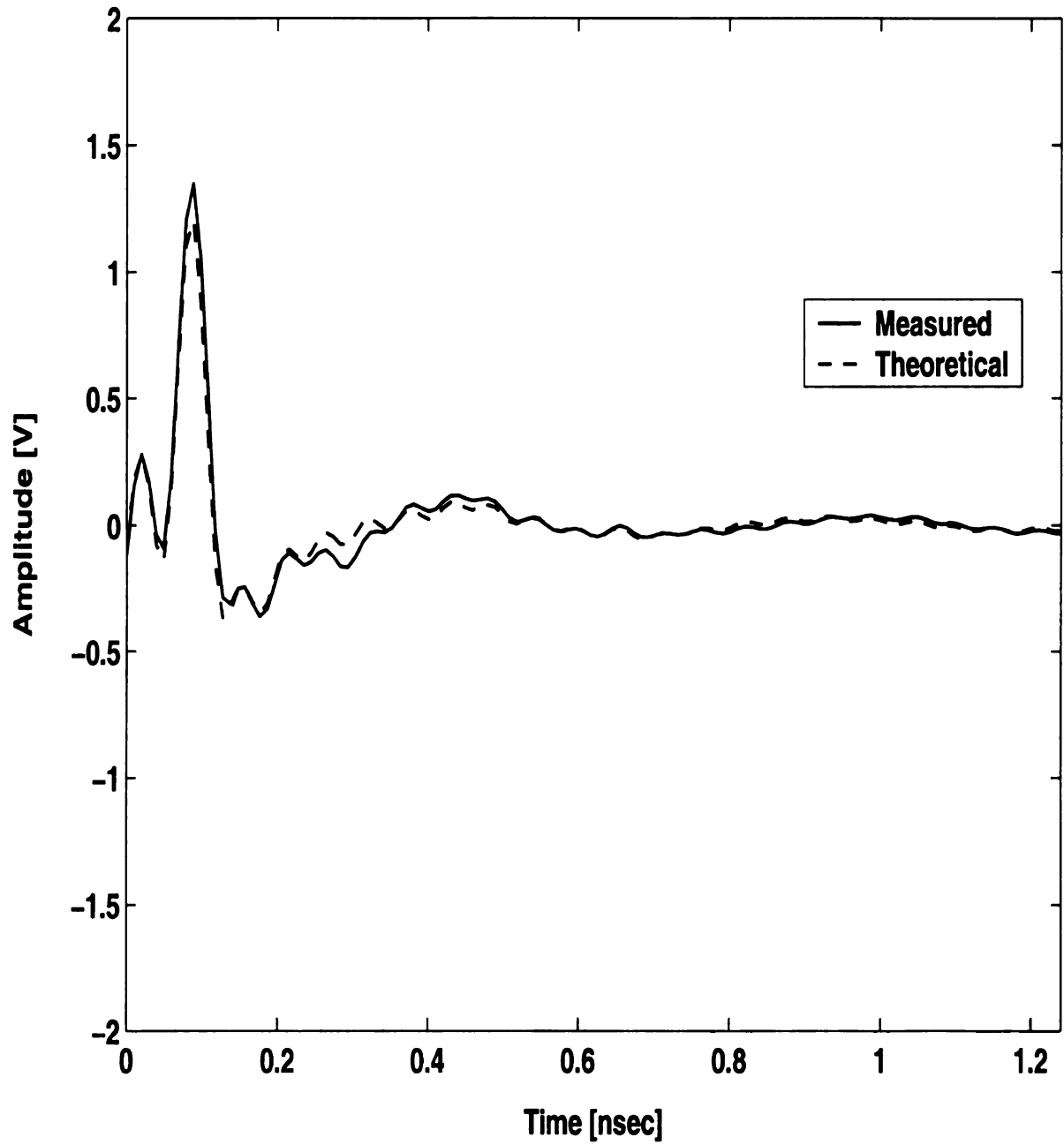


Figure 5.17. (a) Transient scattered field from a 5 lossy layered medium with incidence angle 6° (TE polarization).

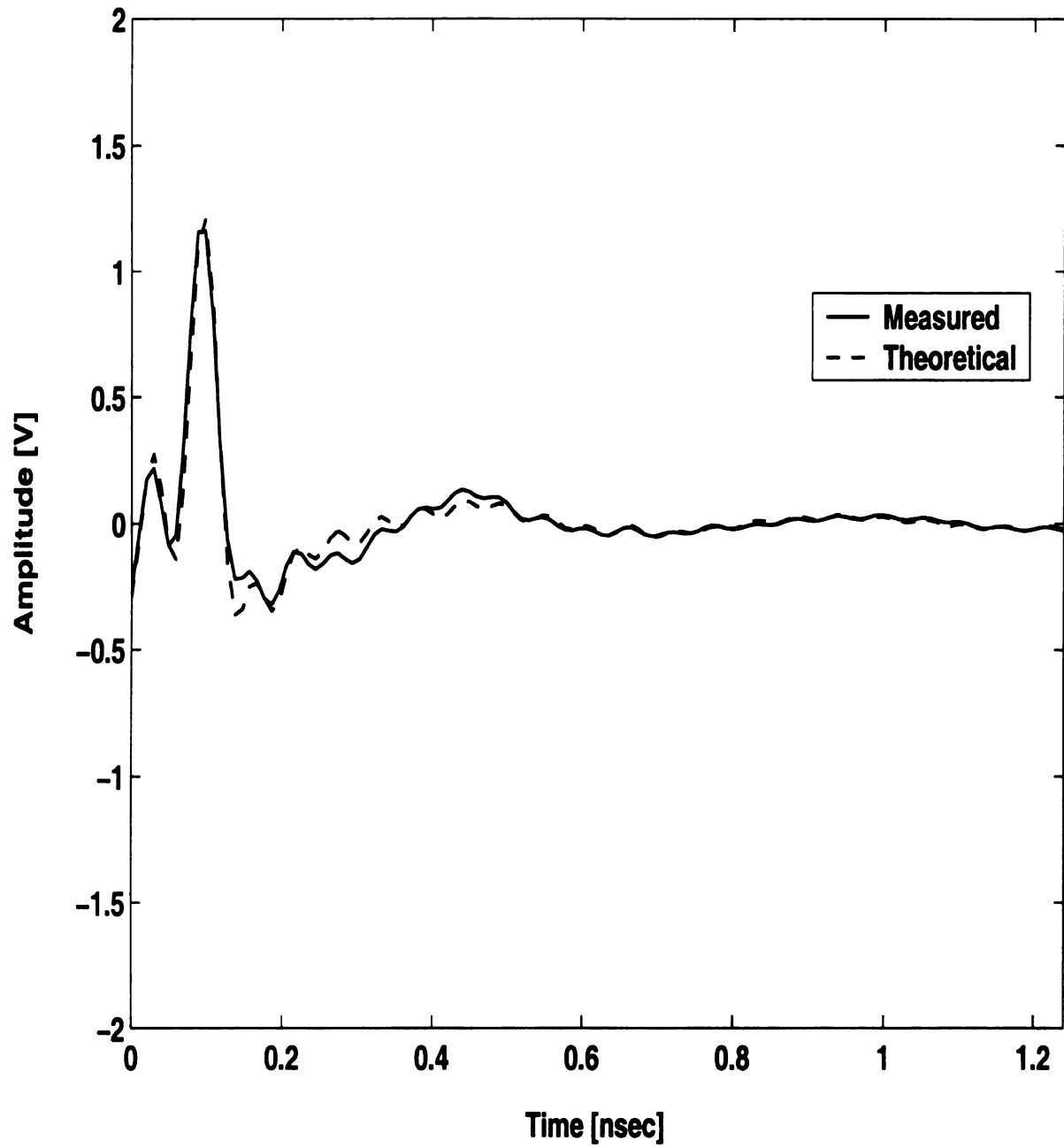


Figure 5.17. (b) Transient scattered field from a 5 lossy layered medium with incidence angle 6° (TM polarization).

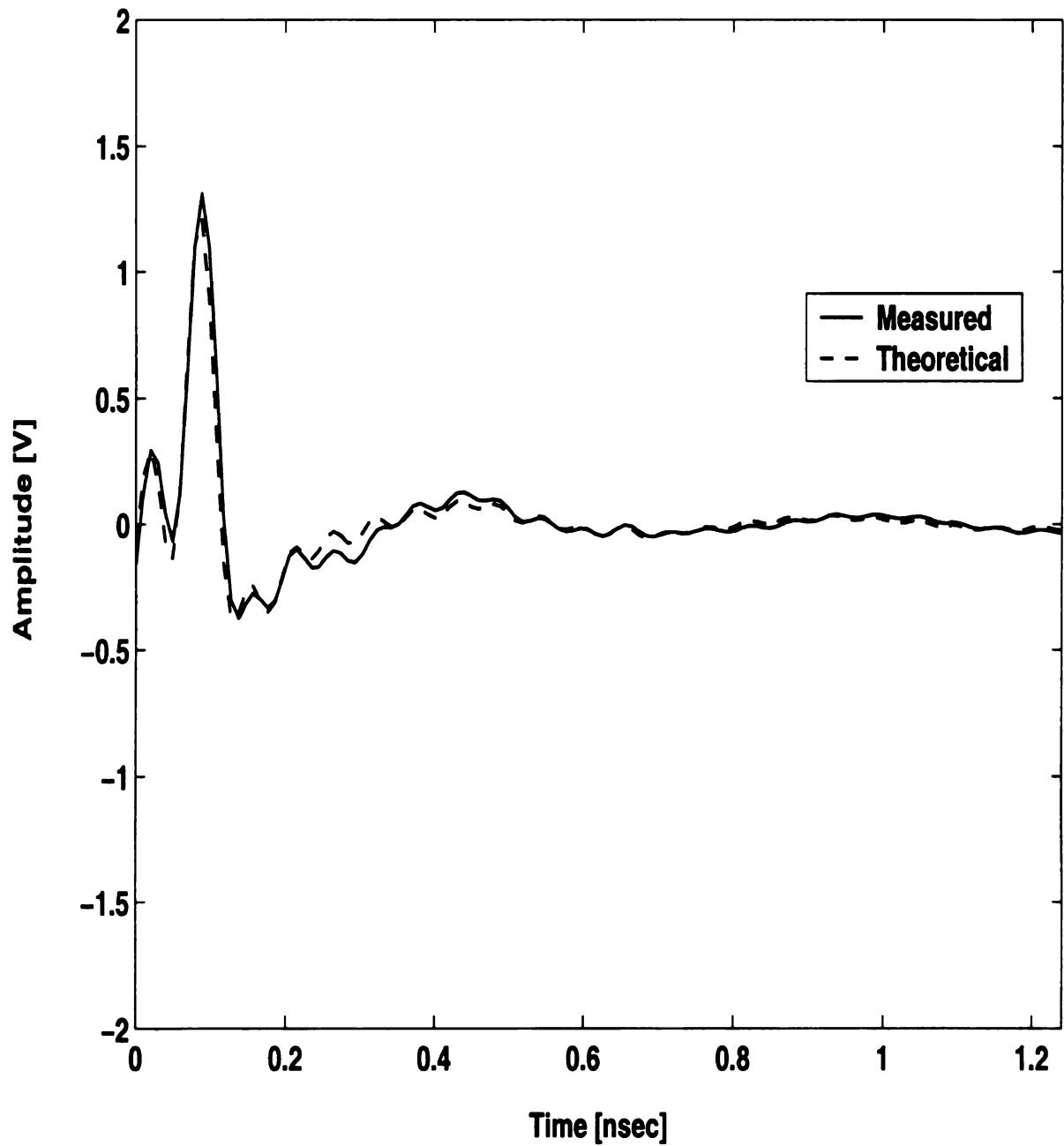


Figure 5.18. (a) Transient scattered field from a 5 lossy layered medium with incidence angle 15° (TE polarization).

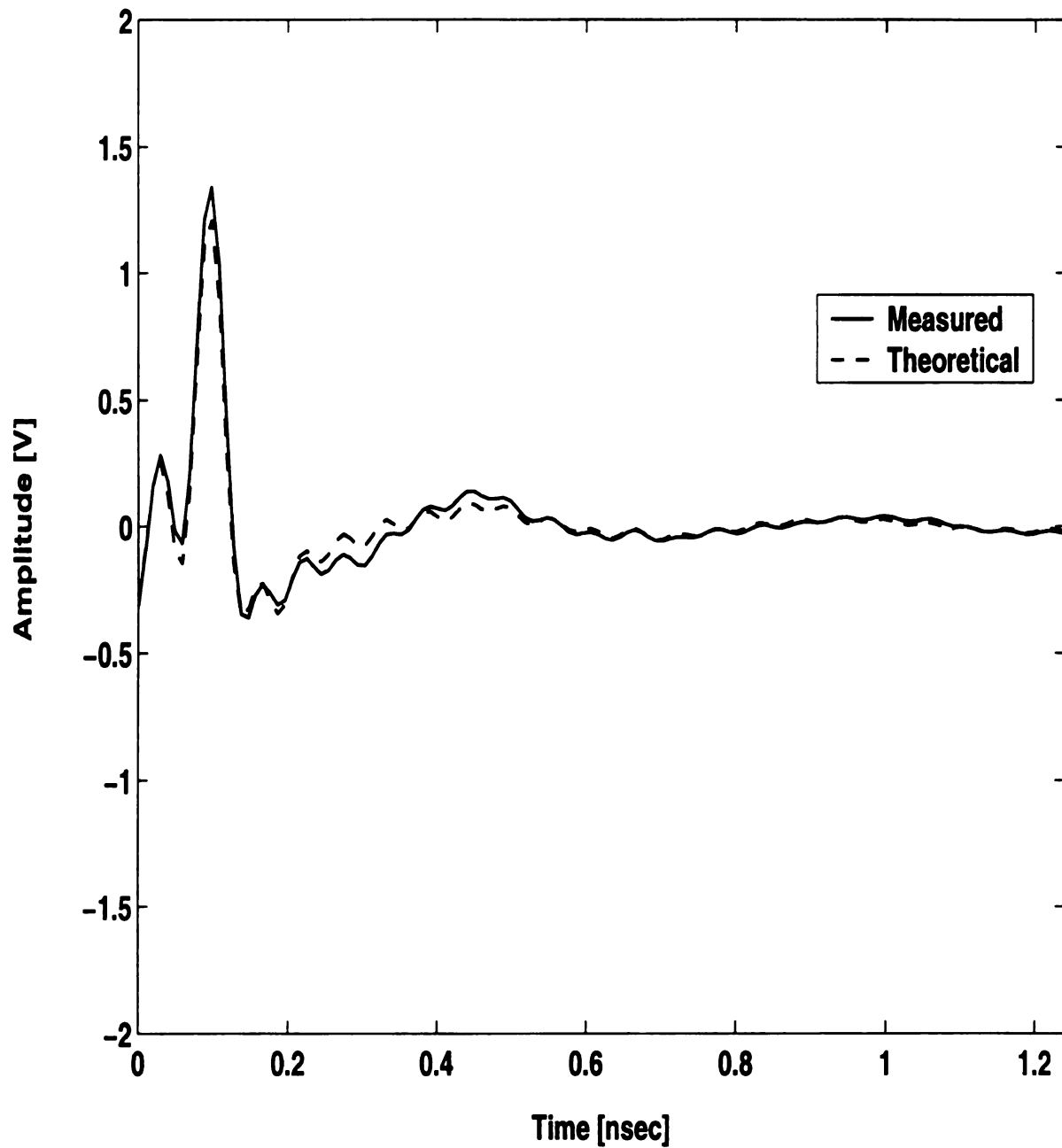


Figure 5.18. (b) Transient scattered field from a 5 lossy layered medium with incidence angle 15° (TM polarization).

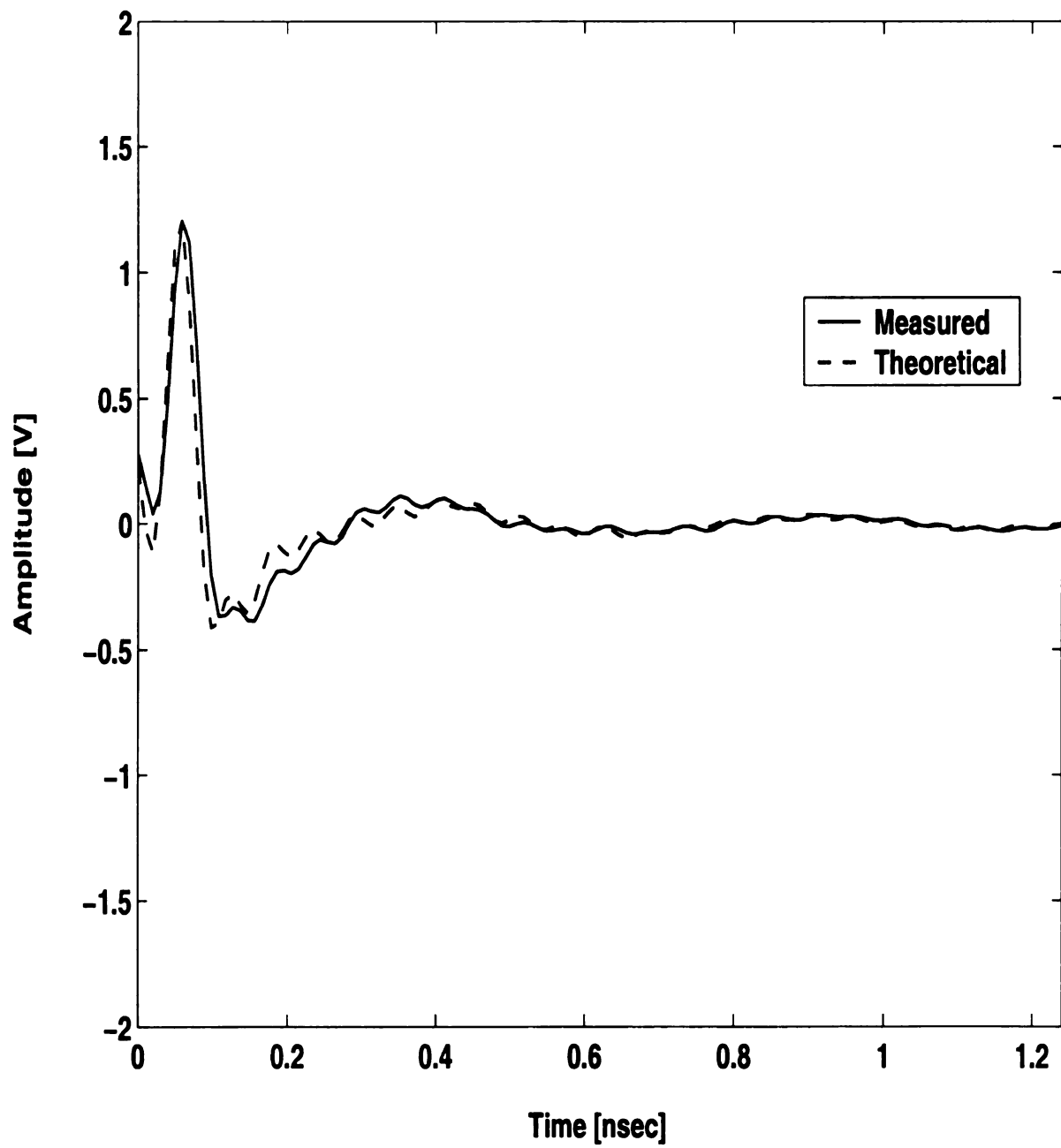


Figure 5.19. (a) Transient scattered field from a 5 lossy layered medium with incidence angle 30° (TE polarization).

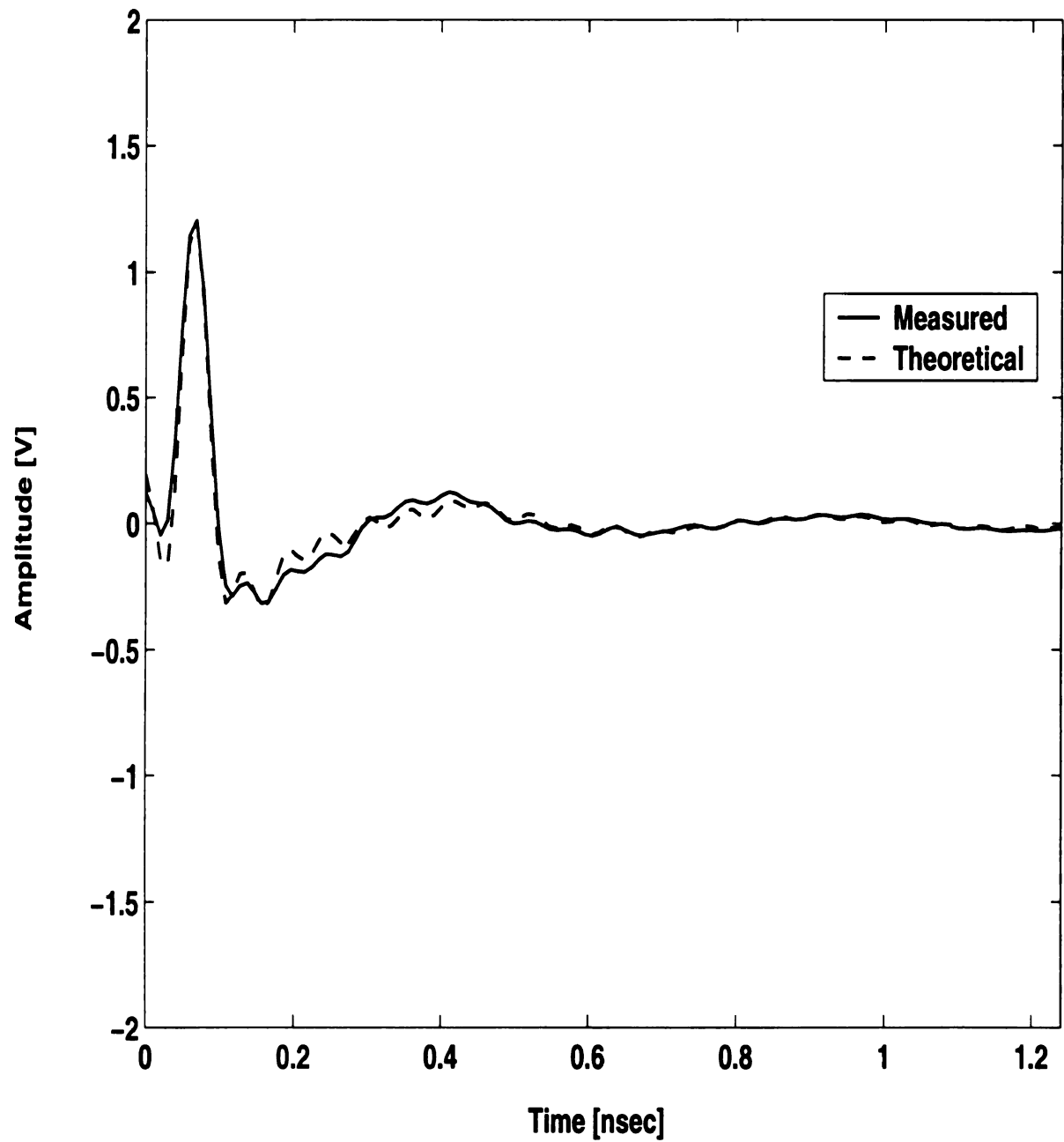


Figure 5.19. (b) Transient scattered field from a 5 lossy layered medium with incidence angle 30° (TM polarization).

CHAPTER 6

CONCLUSIONS

Transient scattering from a multi-layered medium by an oblique uniform plane wave has been studied in this thesis. The frequency domain reflection coefficients were classified according to their different corresponding inverse Fourier transform pairs through careful algebraic manipulation and by using appropriate branch cuts. The exact analytical transient reflection coefficients of a single interface for TE and TM polarizations have been developed from inverse Fourier transforms of the frequency domain reflection coefficients. Also, approximate forms of the reflection coefficients were suggested for the TE polarization case as an example. It was found that all coefficients are causal for certain case. A reasonable explanation for the non causality of other cases could not be found. However, several reports about the occurrence of non causality in theoretical or experimental developments observed by other researchers were introduced. The expressions for the overall transient reflection coefficients were derived using a series expansion and the convolution theorem. The derived interfacial and overall reflection coefficients were verified by numerical computation using the IFFT. Also, actual time domain reflection measurements were performed to compare with the theoretically derived results, using lossless and lossy layers as the objects. All the measured and calibrated results have matched well with computed results based on the derived theory, and therefore verified the correctness of the derived transient expressions.

As discussed in Chapter 1, the purpose of this study is to provide fundamental background for parameter estimation of a multi-layered object using the transient scattered field. Except for the unexplained non causal cases, which should be studied separately in the future, the exact theoretical solutions for the transient scattered

field have been made available by the results of this study. Because each reflection from each layer has the information about the constitutive parameters, and because their functional relationship with the input waveform has been determined by this study, the next step is to find reliable methods to extract the information from the measured transient waveform.

APPENDICES

APPENDIX A

OPERATIONS OF COMPLEX VALUED SQUARE ROOT FUNCTIONS USING THE BRANCH CUTS

The derivation processes of frequency domain expressions explained in Chapter 2 and Chapter 3 require algebraic manipulations of square roots of which arguments are complex numbers. Sometimes, the results of algebraic manipulations using conventional negative real axis branch cut are different from those using the branch cuts derived section 2.2.2 from radiation condition. Because the branch cuts from radiation condition are consistently used in this thesis, it is helpful to introduce selected algebraic manipulations for derivation of frequency domain expressions using the branch cuts.

When $D > 0$ and $B > 0$

$$\sqrt{Ds^2 + Bs} = -\sqrt{D}\sqrt{Ds + B} = -\sqrt{D}\sqrt{s}\sqrt{s + \frac{B}{D}} \quad (\text{A.1})$$

while for $D < 0$ and $B > 0$

$$\sqrt{Ds^2 + Bs} = \sqrt{s}\sqrt{Ds + B} = \begin{cases} +j\sqrt{|D|}\sqrt{s}\sqrt{s - \frac{B}{|D|}} & : (\omega > 0) \\ -j\sqrt{|D|}\sqrt{s}\sqrt{s - \frac{B}{|D|}} & : (\omega < 0) \end{cases} \quad (\text{A.2})$$

By denoting a square root from the conventional negative real axis branch cut as $\dot{\sqrt{()}}$,

$$\frac{\sqrt{s + B_{n+1}}}{\sqrt{s + B_n}} = \pm \sqrt{\frac{s + B_{n+1}}{s + B_n}} = \dot{\sqrt{\frac{s + B_{n+1}}{s + B_n}}} \quad (\text{A.3})$$

where the upper sign corresponds to $B_{n+1} > B_n$ and lower sign does to $B_{n+1} < B_n$. The relationship between the two different branch cut square roots is needed because its inverse Fourier transform pair in reference material is given by using the negative

real axis branch cut. Similarly,

$$\frac{\sqrt{s - B_{n+1}}}{\sqrt{s - B_n}} = \pm \sqrt{\frac{s - B_{n+1}}{s - B_n}} = \dot{\sqrt{\frac{s - B_{n+1}}{s - B_n}}} \quad (\text{A.4})$$

where the upper sign corresponds to $B_{n+1} < B_n$ and lower sign does to $B_{n+1} > B_n$.

On the other hand, to find its inverse Fourier transform more easily, each of the other fraction of square root terms can be rewritten as

$$\begin{aligned} \frac{\sqrt{s + B_{n+1}}}{\sqrt{s - B_n}} &= \sqrt{\frac{s + B_{n+1}}{s - B_n}} \\ &= \sqrt{\frac{s + B_{n+1}}{s}} \sqrt{\frac{s}{s - B_n}} \\ &= \dot{\sqrt{\frac{s + B_{n+1}}{s}}} \dot{\sqrt{\frac{s}{s - B_n}}} \end{aligned} \quad (\text{A.5})$$

and

$$\begin{aligned} \frac{\sqrt{s - B_{n+1}}}{\sqrt{s + B_n}} &= -\sqrt{\frac{s - B_{n+1}}{s + B_n}} \\ &= -\sqrt{\frac{s - B_{n+1}}{s}} \sqrt{\frac{s}{s + B_n}} \\ &= -\dot{\sqrt{\frac{s - B_{n+1}}{s}}} \dot{\sqrt{\frac{s}{s + B_n}}}. \end{aligned} \quad (\text{A.6})$$

APPENDIX B

INVERSE FOURIER TRANSFORM PAIRS

In this appendix, the inverse Fourier transform pairs which are used to derive transient expressions, are provided. Some of these can be directly found from [23], while the others are derived from the given pairs.

Assume $R(\omega) = G(\omega)U^*(\omega) + G(-\omega)U(-\omega)$ where $G^*(\omega)$ is a complex conjugate of $G(\omega)$, and $U(\omega)$ is a frequency domain unit step function. By letting $g(t) = F^{-1}\{G(\omega)\}$, $g^*(t) = F^{-1}\{G^*(\omega)\}$ and using the duality theorem for Fourier transform that says, $F(t) = F^{-1}\{2\pi f(-\omega)\}$ when $f(t) = F^{-1}\{F(\omega)\}$,

$$\begin{aligned} \pi\delta(t) + \frac{1}{jt} &= 2\pi F^{-1}\{U(-\omega)\} \\ \frac{\delta(t)}{2} - \frac{1}{2j\pi t} &= 2\pi F^{-1}\{U(\omega)\} \end{aligned} \tag{B.1}$$

and

$$\begin{aligned} R(t) &= g(t) * \left\{ \frac{\delta(t)}{2} - \frac{1}{2j\pi t} \right\} + g^*(t) * \left\{ \frac{\delta(t)}{2} + \frac{1}{2j\pi t} \right\} \\ &= \{g(t) + g^*(t)\} * \frac{\delta(t)}{2} + \{g(t) - g^*(t)\} * \frac{1}{2j\pi t} \\ &= \frac{1}{2}\{g(t) + g^*(t)\} + \frac{j}{2\pi t} * \{g(t) - g^*(t)\}. \end{aligned} \tag{B.2}$$

From the inverse transform pair of

$$\frac{1}{s + P} \Leftrightarrow e^{-Pt}u(t) \tag{B.3}$$

for $P > 0$, and $-e^{-Pt}u(-t)$ for $P < 0$, its time derivative form is given by

$$\begin{aligned}
\frac{s}{s+P} &\Leftrightarrow \frac{d}{dt} \{e^{-Pt}u(t)\} \\
&= -Pe^{-Pt}u(t) + e^{-Pt}\delta(t) \\
&= e^{-Pt} \{\delta(t) - Pu(t)\}
\end{aligned} \tag{B.4}$$

for $P > 0$, and

$$\begin{aligned}
\frac{s}{s+P} &\Leftrightarrow \frac{d}{dt} \{-e^{-Pt}u(-t)\} \\
&= Pe^{-Pt}u(-t) + e^{-Pt}\delta(t) \\
&= e^{-Pt} \{\delta(t) + Pu(t)\}
\end{aligned} \tag{B.5}$$

for $P < 0$.

From 561.0 of [23],

$$\begin{aligned}
1 - \sqrt{\frac{s+B_{n+1}}{s+B_n}} &\Leftrightarrow \\
-\frac{B_{n+1}-B_n}{2}e^{-\frac{(B_{n+1}+B_n)}{2}t} &\left\{ I_1\left(\frac{B_{n+1}-B_n}{2}t\right) + I_0\left(\frac{B_{n+1}-B_n}{2}t\right) \right\} u(t),
\end{aligned} \tag{B.6}$$

and it can be derived that

$$\begin{aligned}
1 - \sqrt{\frac{s-B_{n+1}}{s-B_n}} &\Leftrightarrow \\
\frac{B_{n+1}-B_n}{2}e^{\frac{(B_{n+1}+B_n)}{2}t} &\left\{ I_1\left(\frac{B_{n+1}-B_n}{2}t\right) - I_0\left(\frac{B_{n+1}-B_n}{2}t\right) \right\} u(-t).
\end{aligned} \tag{B.7}$$

Also, from (A.5),

$$\dot{\sqrt{\frac{s+B_{n+1}}{s}}} \Leftrightarrow \delta(t) + \frac{B_{n+1}}{2} e^{-\frac{B_{n+1}}{2}t} \left\{ I_1\left(\frac{B_{n+1}}{2}t\right) + I_0\left(\frac{B_{n+1}}{2}t\right) \right\} u(t) \quad (\text{B.8})$$

and

$$\dot{\sqrt{\frac{s}{s-B_n}}} \Leftrightarrow \delta(t) - \frac{B_n}{2} e^{-\frac{B_n}{2}t} \left\{ I_1\left(\frac{B_n}{2}t\right) + I_0\left(\frac{B_n}{2}t\right) \right\} u(-t), \quad (\text{B.9})$$

the inverse Fourier transform of $1 - \sqrt{\frac{s+B_{n+1}}{s-B_n}}$ is given by

$$\begin{aligned} \delta(t) &= \left[\delta(t) + \frac{B_{n+1}}{2} e^{-\frac{B_{n+1}}{2}t} \left\{ I_1\left(\frac{B_{n+1}}{2}t\right) + I_0\left(\frac{B_{n+1}}{2}t\right) \right\} u(t) \right] \\ &\quad * \left[\delta(t) - \frac{B_n}{2} e^{\frac{B_n}{2}t} \left\{ I_1\left(\frac{B_n}{2}t\right) + I_0\left(\frac{B_n}{2}t\right) \right\} u(-t) \right] \\ &= \frac{B_n}{2} e^{\frac{B_n}{2}t} \left\{ I_1\left(\frac{B_n}{2}t\right) + I_0\left(\frac{B_n}{2}t\right) \right\} u(-t) \\ &\quad - \frac{B_{n+1}}{2} e^{-\frac{B_{n+1}}{2}t} \left\{ I_1\left(\frac{B_{n+1}}{2}t\right) + I_0\left(\frac{B_{n+1}}{2}t\right) \right\} u(t) \\ &\quad + \frac{B_{n+1}B_n}{4} e^{\frac{B_n}{2}t} \int_{\max(t,0)}^{\infty} e^{-\frac{(B_{n+1}+B_n)}{2}x} \left\{ I_1\left[\frac{B_n}{2}(t-x)\right] + I_0\left[\frac{B_n}{2}(t-x)\right] \right\} \\ &\quad \times \left\{ I_1\left(\frac{B_{n+1}}{2}x\right) + I_0\left(\frac{B_{n+1}}{2}x\right) \right\} dx. \end{aligned} \quad (\text{B.10})$$

By similar approach, the inverse Fourier transform of $1 + \sqrt{\frac{s-B_{n+1}}{s+B_n}}$ can be found as

$$\begin{aligned} &\frac{B_{n+1}}{2} e^{\frac{B_{n+1}}{2}t} \left\{ I_1\left(\frac{B_{n+1}}{2}t\right) - I_0\left(\frac{B_{n+1}}{2}t\right) \right\} u(-t) \\ &- \frac{B_n}{2} e^{-\frac{B_n}{2}t} \left\{ I_1\left(\frac{B_n}{2}t\right) - I_0\left(\frac{B_n}{2}t\right) \right\} u(t) \\ &+ \frac{(B_{n+1}B_n)}{4} e^{\frac{B_{n+1}}{2}t} \int_{\max(t,0)}^{\infty} e^{-\frac{(B_{n+1}+B_n)}{2}x} \left\{ I_1\left[\frac{B_{n+1}}{2}(t-x)\right] - I_0\left[\frac{B_{n+1}}{2}(t-x)\right] \right\} \\ &\times \left\{ I_1\left(\frac{B_n}{2}x\right) - I_0\left(\frac{B_n}{2}x\right) \right\} dx. \end{aligned} \quad (\text{B.11})$$

APPENDIX C

PROGRAM SOURCE CODES IN FORTRAN 77

A computer program to compute theoretical TE polarization overall reflection from a multi-layered medium is provided. This program is written in standard Fortran 77 programming language, except a subroutine called Q2AGI from MICROSOFT IMSL package, which is used to compute infinite integral term in interfacial reflection coefficient when $P_2 < 0$ (see 2.83).

This program reads an input waveform data from a text file 'inwave.dat', the material parameters of each layer (thickness, permeability, permittivity, conductivity) and initial incident angle from a text file 'layers.dat', and computes the layer parameter sets defined in this thesis, and writes them in a text file 'param.buf' for verification purpose. Also, the information about time range, number of data points and number of convolution terms are read from command line inputs. After the input stage, program computes the interfacial reflection coefficients for each interface, overall reflection and finally writes the outputs in a text output file 'gammax.out'. The structure of the programme to compute TM polarization is basically same. The differences are subroutines to compute interfacial reflection coefficients, and layer parameter definitions.

(1) An example of input data file 'layers.dat'

The value of incident angle is computed using the value in the data file as a denominator of a fraction, of which numerator is fixed by π radian. That is, for an example, when given "thetai 30.0" in the data file, the incident angle will be $\frac{\pi}{30}$ radian = 6° . Metric unit is used for thickness input. The first and the last layer have infinite thicknesses, therefore their values are meaningless.

thetai 30.0
mur1 1.0
er1 1.0
sigma1 0.0
thick1 0.0
mur2 1.0
er2 2.59
sigma2 0.0
thick2 0.0053
mur3 1.0
er3 55.0
sigma3 16.7
thick3 0.0053
mur4 1.0
er4 2.59
sigma4 0.0
thick4 0.0053
mur5 1.0
er5 1.0
sigma5 0.0
thick5 0.0

(2) Main program

Because the products of an exponential function and a modified Bessel function appear frequently, the product is computed altogether using a subroutine. The expressions for numerical computation of the modified Bessel functions can be found in [29].


```

      PROGRAM GAMXTE
      USE MSIMSL
C *****
C      This program computes transient overall reflection
C      using iterative convolutions, about the case for
C      DN1>0 .AND. DN >0.
C
C      An input waveform is obtained from a text data file which
C      contains actual measurement data of transmitting impulse,
C      called "inwave.dat"
C
C      Text data file "layers.dat" is required as an input
C      parameter description for each layer.
C
C      Text data file "gammamax.out" will be output.
C
C *****
      CHARACTER*8 TMP$
      INTEGER ITCONV
      INTEGER NT, SMAX, LMAX, I, K, NL
      PARAMETER (SMAX=10250)
      PARAMETER (LMAX=10)
      REAL MU(LMAX), EN(LMAX), SIG(LMAX), THCK(LMAX), B(LMAX), DN(LMAX)
      REAL C1(LMAX-1), C2(LMAX-1), C3(LMAX-1), P1(LMAX-1), P2(LMAX-1)
      REAL BTA(LMAX-1), D(LMAX-1), CD(LMAX-1)
      REAL T(SMAX), X(SMAX)
      REAL R(LMAX-1, SMAX), P(LMAX-1, SMAX), GAM(LMAX-1, SMAX)
      REAL THETA1, G, H, TMAX, DT
      REAL TMP, PSZERO
      REAL PI, E0, MU0
      COMMON /PWRS/C1, C2, C3, P1, P2, BTA, CD
      COMMON /ARRS/R, P, GAM
      COMMON /TARRS/T
      COMMON /INDX/I
      COMMON /TINDX/K, NT
      COMMON /LINDX/ITCONV, NL

      PI=4.*ATAN(1.)
      E0=1.0E-9/(36.0*PI)
      MU0=4.0*PI*1.0E-7
      PSZERO=1.0E-25

      OPEN(5, FILE='layers.dat', STATUS='unknown')
      OPEN(6, FILE='gammamax.out', STATUS='unknown')
      OPEN(7, FILE='param.buf', STATUS='unknown')
      OPEN(8, FILE='inwave.dat', STATUS='unknown')

```

```

C *****
C READ AND COMPUTE CONSTANTS OF EACH LAYER
C *****
      WRITE(*,*) 'Enter the number of layers in <<layers.dat>> :'
      READ(*,*) NL

C      *****
C      READ INCIDNET ANGLE
C      *****
11     FORMAT(A8,F12.4)
12     FORMAT(3X, F12.4,2X, A8,'= ',E12.4)
20     FORMAT(/)
      READ(5,11) TMPS,TMP
      THETAI = PI/TMP
      WRITE(*,12) TMP,TMPS,THETAI
      WRITE(7,12) TMP,TMPS,THETAI

C      *****
C      READ MATERIAL PARAMETERS
C      *****
      DO 21 I=1,NL
        READ(5,11) TMPS,TMP
        MU(I)=TMP*MU0
        WRITE(*,12) TMP,TMPS,MU(I)
        WRITE(7,12) TMP,TMPS,MU(I)

        READ(5,11) TMPS,TMP
        EN(I)=TMP*E0
        WRITE(*,12) TMP,TMPS,EN(I)
        WRITE(7,12) TMP,TMPS,EN(I)

        READ(5,11) TMPS,SIG(I)
        WRITE(*,12) SIG(I),TMPS,SIG(I)
        WRITE(7,12) SIG(I),TMPS,SIG(I)

        READ(5,11) TMPS,THCK(I)
        WRITE(*,12) THCK(I),TMPS,THCK(I)
        WRITE(7,12) THCK(I),TMPS,THCK(I)

        TMP=MU(1)*EN(1)*((SIN(THETAI))**2)
        DN(I)=MU(I)*EN(I)-TMP
        B(I)=MU(I)*SIG(I)/ABS(DN(I))
        WRITE(*,13) DN(I),B(I)
        WRITE(7,13) DN(I),B(I)
13     FORMAT(5X,'Dn=',E10.3,2X,'Bn=',E10.3,/)
21     CONTINUE

```

```

C *****
C COMPUTE LAYER PARAMETERS FOR EACH INTERFACE
C *****

      DO 22 I=1,NL-1
        WRITE(*,14) I
        WRITE(7,14) I
14      FORMAT(/,3X,'Interface Index =',I3)

        D(I)=( (MU(I)**2)*DN(I+1)) / ( (MU(I+1)**2)*DN(I) )
        CD(I)=(1.0-SQRT(D(I))) / (1.0+SQRT(D(I)))
        WRITE(*,15) D(I),CD(I)
        WRITE(7,15) D(I),CD(I)
15      FORMAT(3X,'D=',E10.3,3X,'CD=',E10.3)

        BTA(I)=(B(I+1)-B(I))/2.0
        P1(I)=(B(I+1)+B(I))/2.0
        P2(I)=(B(I)-D(I)*B(I+1))/(1.0-D(I))
        WRITE(*,16) P1(I),P2(I),BTA(I)
        WRITE(7,16) P1(I),P2(I),BTA(I)
16      FORMAT(3X,'P1=',E10.3,2X,'P2=',E10.3,2X,'BETA=',E10.3)

        C1(I)=(B(I)-B(I+1))*SQRT(D(I))/(1.0-D(I))
        C2(I)=( (B(I)-B(I+1))**2)*D(I)*SQRT(D(I)) / ( (1.0-D(I))**2 )
        C3(I)=2.0*D(I)*(B(I+1)-B(I)) / ( (1.0-D(I))*(1.0+SQRT(D(I))) )
        WRITE(*,17) C1(I),C2(I),C3(I)
        WRITE(7,17) C1(I),C2(I),C3(I)
17      FORMAT(3X,'C1=',E10.3,2X,'C2=',E10.3,2X,'C3=',E10.3)
22      CONTINUE

C *****
C READ TIME RANGE, NUMBER OF PARAMETERS AND CONVOLUTIONS
C *****

      WRITE(*,*) 'Enter the Time range in seconds : '
      READ(*,*) TMAX
      WRITE(*,*) 'Enter the Number of Points ( SMAX<=10250) '
      WRITE(*,*) 'EVEN number recommended to avoid zero freq. : '
      READ(*,*) NT
      WRITE(*,*) 'Enter the Number of Convolution Sum : '
      READ(*,*) ITCONV

C *****
C DEFINE TIME INTERVAL OF EVALUATION
C *****

      NT=NT-1
      DT=TMAX/REAL(NT)
      DO 31 K=1,NT+1
        T(K)=DT*(K-1)
31      CONTINUE

```

```

      DO 32 I=1,NL-1
        DO 33 K=1,NT+3
          R(I,K)=0.0
          P(I,K)=0.0
          GAM(I,K)=0.0
          X(K)=0.0
33      CONTINUE
32      CONTINUE
      WRITE(7,18) TMAX,(NT+1),DT,(1.0/DT)
18      FORMAT(/,3X,'TMAX=',E10.3,' [sec]',2X,I7,' Pts.',2X/,
+          3X 'DT=',E10.3,' [sec]',2X, '1/DT=',E10.3)
C *****
C READ INPUT WAVEFORM FROM DATA FILE
C *****
      DO 61 K=1,NT+1
        READ(8,*) X(K)
61      CONTINUE
C *****
C COMPUTE INTERFACIAL REFL. COEFF.'S AND PROPAGATION TERMS
C *****
      DO 89 I=1,NL-1
        CALL REFLCOEFF(R(I,:),DT)
        G=DN(I)
        H=MU(I)*SIG(I)
        CALL PROPA(P(I,:),THCK(I),G,H,2,DT)
89      CONTINUE
C *****
C COMPUTE THE FINAL IMPULSE RESPONSE OF LAYERS
C *****
      DO 87 K=1,NT+3
        GAM(NL-1,K)=R(NL-1,K)
87      CONTINUE

      CALL GAMX(1,X,DT)

C *****
C WRITE TIME, INPUT AND OUTPUT WAVEFORM DATA
C *****
      DO 50 K=1,NT+3
        IF (ABS(GAM(1,K)).LT.PSZERO) GAM(1,K)=0.0
        WRITE(6,99) T(K),X(K),GAM(1,K)
99      FORMAT(1X,E10.3,1X,E13.5,1X,E13.5)
50      CONTINUE

      ENDFILE 5
      ENDFILE 6
      ENDFILE 7
      ENDFILE 8

```

```

      END
C *****
C SUBROUTINE TO COMPUTE OVERALL REFLECTION OF EACH LAYER
C *****
      RECURSIVE SUBROUTINE GAMX(I,XIN,DT)
      INTEGER NT,I,SMAX,LMAX,NL,ITCONV,K,N
      REAL DT
      PARAMETER (SMAX=10250)
      PARAMETER (LMAX=10)
      REAL XIN(SMAX),BUF(LMAX-2,SMAX),GBUF(SMAX)
      REAL R(LMAX-1,SMAX),P(LMAX-1,SMAX),GAM(LMAX-1,SMAX)
      COMMON /ARRS/R,P,GAM
      COMMON /TINDX/K,NT
      COMMON /LINDX/ITCONV,NL

      IF (I.EQ.(NL-2)) THEN
        CALL CONVOL(R(I,:),XIN,GAM(I,:),DT)
        CALL CONVOL(R(I,:),R(I,:),GBUF,DT)
        GBUF(NT+2)=1.0/DT-GBUF(NT+2)
        DO 41 K=1,NT+1
          GBUF(K)=-GBUF(K)
41      CONTINUE
        CALL CONVOL(GBUF,P(I+1,:),GBUF,DT)
        CALL CONVOL(GBUF,GAM(I+1,:),GBUF,DT)
        CALL CONVOL(GBUF,XIN,GBUF,DT)
        DO 42 K=1,NT+1
          GAM(I,K)=GAM(I,K)+GBUF(K)
42      CONTINUE
        DO 43 N=1,ITCONV
          CALL CONVOL(GBUF,R(I,:),GBUF,DT)
          CALL CONVOL(GBUF,GAM(I+1,:),GBUF,DT)
          CALL CONVOL(GBUF,P(I+1,:),GBUF,DT)
          DO 61 K=1,NT+2
            GBUF(K)=-GBUF(K)
61      CONTINUE
          DO 44 K=1,NT+1
            GAM(I,K)=GAM(I,K)+GBUF(K)
44      CONTINUE
43      CONTINUE
        WRITE(*,19) I
19      FORMAT(3X,'IN=',I3)
      ELSE
        CALL CONVOL(R(I,:),XIN,GAM(I,:),DT)
        CALL GAMX(I+1,XIN,DT)
        CALL CONVOL(R(I,:),R(I,:),BUF(I,:),DT)
        BUF(I,NT+2)=1.0/DT-BUF(I,NT+2)
        DO 55 K=1,NT+1
          BUF(I,K)=-BUF(I,K)

```

```

55      CONTINUE
        DO 65 K=1,NT+3
          GBUF(K)=BUF(I,K)
65      CONTINUE
        CALL CONVOL(GBUF,GAM(I+1,:),GBUF,DT)
        CALL CONVOL(GBUF,P(I+1,:),GBUF,DT)
        DO 56 K=1,NT+1
          GAM(I,K)=GAM(I,K)+GBUF(K)
56      CONTINUE
        DO 57 N=1,ITCONV
          CALL CONVOL(GBUF,R(I,:),GBUF,DT)
          CALL CONVOL(GBUF,P(I+1,:),GBUF,DT)
          DO 62 K=1,NT+2
            BUF(I,K)=-GBUF(K)
62      CONTINUE
            BUF(I,NT+3)=-GBUF(NT+3)
            CALL GAMX(I+1,BUF(I,:),DT)
            DO 58 K=1,NT+1
              GAM(I,K)=GAM(I,K)+GAM(I+1,K)
58      CONTINUE
            DO 63 K=1,NT+3
              GBUF(K)=GAM(I+1,K)
63      CONTINUE
57      CONTINUE
        WRITE(*,59) I
59      FORMAT(3X,'IN=',I3)
      ENDIF

      RETURN
      END

```

```

C *****
C SUBROUTINE TO COMPUTE INDIVIDUAL TIME DOMAIN REFLECTION
C *****
      SUBROUTINE REFLCOEFF(R,DT)
      EXTERNAL FUNCP,FUNCN
      REAL SI0,SI1
      INTEGER K,NT,I,L,SMAX,LMAX
      PARAMETER (SMAX=10250)
      PARAMETER (LMAX=10)
      REAL C1(LMAX-1),C2(LMAX-1),C3(LMAX-1),P1(LMAX-1),P2(LMAX-1)
      REAL BTA(LMAX-1),CD(LMAX-1)
      REAL TMP,S,Q1,Q2,Q3,DT
      REAL T(SMAX),R(SMAX)
      COMMON /PWRS/C1,C2,C3,P1,P2,BTA,CD
      COMMON /TARRS/T
      COMMON /INDX/I
      COMMON /TINDX/K,NT

```

```

REAL ERRREL,RESULT,ERREST
INTEGER MAXSUB,NEVAL,NSUBIN
PARAMETER (MAXSUB=2000)
REAL ALIST(MAXSUB),BLIST(MAXSUB),RLIST(MAXSUB),ELIST(MAXSUB)
INTEGER IORD(MAXSUB)
ERRREL=1.0E-03

IF (P2(I).GE.0.) THEN
  DO 31 K=1,NT+1
    TMP=-P1(I)*T(K)
    Q1=C1(I)*(SI1(BTA(I)*T(K),TMP)+SI0(BTA(I)*T(K),TMP))
    S=0.0
    IF (K.GT.1) THEN
      Q2=0.0
      DO 32 L=2,K
        CALL QTRAP(FUNCP,T(L-1),T(L),S)
        Q2=Q2+S
32      CONTINUE
      Q2=C2(I)*Q2
    ELSE
      Q2=0.0
    ENDIF
    Q3=C3(I)*EXP(-P2(I)*T(K))
    R(K)=Q1-Q2+Q3
    WRITE(*,34) I,K
34    FORMAT(3X,'I=',I3,2X,'K=',I5)
31    CONTINUE
  ELSE
    DO 51 K=1,NT+1
      TMP=-P1(I)*T(K)
      Q1=SI0(BTA(I)*T(K),TMP)
      Q1=Q1+SI1(BTA(I)*T(K),TMP)
      Q1=C1(I)*Q1
      CALL Q2AGI(FUNCN,0.,1,0.,ERRREL,RESULT,ERREST,MAXSUB,
+              NEVAL,NSUBIN,ALIST,BLIST,RLIST,ELIST,IORD )
      Q2=C2(I)*RESULT/P2(I)
      R(K)=Q1-Q2
      WRITE(*,54) I,K
54    FORMAT(3X,'I=',I3,2X,'K=',I5)
51    CONTINUE
  ENDIF
C *****
C ADD AMPLITUDE AND POSITION OF DELTA FUNCTION
C *****
R(NT+2)=CD(I)/DT
R(NT+3)=1.0
RETURN
END

```

```

C *****
C SUBROUTINE TO COMPUTE TIME DOMAIN PROPAGATION TERM
C *****
      SUBROUTINE PROPA(P,THCK,G,H,N,DT)
      REAL SI1
      INTEGER K,NT,SMAX,N,M
      PARAMETER (SMAX=10250)
      REAL Z,W1,W2,TRIG,THCK,G,H,DT
      REAL T(SMAX),P(SMAX)
      COMMON /TARRS/T
      COMMON /TINDX/K,NT

      THCK=N*THCK
      TRIG=THCK*SQRT(G)
      W1=0.5*H/SQRT(G)
      W2=0.5*H/G

      DO 71 M=1,NT+1
        IF (T(M).GT.TRIG) THEN
          Z=SQRT(T(M)**2-G*(THCK**2))
          P(M)=W1*THCK*SI1(W2*Z,-W2*T(M))/Z
        ENDIF
71    CONTINUE
C *****
C ADD AMPLITUDE AND POSITION OF DELTA FUNCTION
C *****
      DO 72 M=1,NT+1
        IF (T(M).GT.TRIG) THEN
          P(NT+2)=EXP(-W1*THCK)/DT
          P(NT+3)=REAL(M)
          GO TO 73
        ENDIF
72    CONTINUE
73    RETURN
      END

C *****
C SUBROUTINE TO COMPUTE TIME DOMAIN CONVOLUTION
C *****
      SUBROUTINE CONVOL(IN1,IN2,OUT,DT)
      REAL DT,S,A,B,PSZERO
      INTEGER K,NT,SMAX,N,M,L
      PARAMETER (SMAX=10250)
      REAL IN1(SMAX),IN2(SMAX),OUT(SMAX),LOBUF(SMAX)
      COMMON /TINDX/K,NT
      PSZERO=1.0E-25

      DO 80 M=1,NT+3
        LOBUF(M)=0.0

```



```

80    CONTINUE
      LOBUF (NT+2) = IN1 (NT+2) * IN2 (NT+2) * DT
      LOBUF (NT+3) = (IN1 (NT+3) - 1.0) + (IN2 (NT+3) - 1.0) + 1.0
      L = INT (IN1 (NT+3))
      IF (L.LE.(NT+1)) THEN
        DO 81 M=1, (NT+2-L)
          LOBUF (M+L-1) = LOBUF (M+L-1) + IN1 (NT+2) * IN2 (M) * DT
81      CONTINUE
        ENDIF
      L = INT (IN2 (NT+3))
      IF (L.LE.(NT+1)) THEN
        DO 82 M=1, (NT+2-L)
          LOBUF (M+L-1) = LOBUF (M+L-1) + IN2 (NT+2) * IN1 (M) * DT
82      CONTINUE
        ENDIF
      DO 83 M=2, NT+1
        S = 0.0
        DO 84 N=2, M
          A = IN1 (M-N+1) * IN2 (N)
          B = IN1 (M-N+2) * IN2 (N-1)
          IF (ABS (A*B) .GT. PSZERO) S = S + 0.5 * DT * (A+B)
84      CONTINUE
          LOBUF (M) = LOBUF (M) + S
83      CONTINUE
        DO 85 M=1, NT+3
          OUT (M) = LOBUF (M)
85      CONTINUE
        RETURN
      END
C *****
C INNER INTEGRATION ROUTINE USING SIMPSON'S RULE
C *****
      SUBROUTINE qtrap (FUNC, a, b, s)
      EXTERNAL FUNC
      INTEGER JMAX
      REAL a, b, EPS
      REAL olds, s, FUNC
      PARAMETER (EPS = 1.e-3, JMAX = 20)
C      USES trapzd
      INTEGER j
      olds = 0.0
      do 41 j=1, JMAX
        call trapzd (FUNC, a, b, s, j)
        if (abs (s-olds) .lt. EPS * abs (olds)) return
        if (s.eq.0..and.olds.eq.0..and.j.gt.6) return
        olds = s
41      continue
      WRITE (*, 43)

```

```

43      FORMAT('Too many steps in qtrap. Press any number to
continue...')
      READ(*,*) TI
      END
C *****
C TRAPEZOIDAL RULE AS A SUBROUTINE FOR INTEGRATION
C *****
      SUBROUTINE trapzd(FUNC,a,b,s,n)
      EXTERNAL FUNC
      INTEGER n
      REAL a,b,x
      REAL del,sum,tnm,s
      REAL FUNC
      INTEGER it,j
      if (n.eq.1) then
        s=0.5*(b-a)*(FUNC(a)+FUNC(b))
      else
        it=2**(n-2)
        tnm=it
        del=(b-a)/tnm
        x=a+0.5*del
        sum=0.
        do 41 j=1,it
          sum=sum+FUNC(x)
          x=x+del
41      continue
        s=0.5*(s+(b-a)*sum/tnm)
      endif
      return
      END
C *****
C INTEGRAND FUNCTION DEFINITION FOR P2 >= 0
C *****
      REAL FUNCTION FUNCP(U)
      INTEGER NT,I,K,SMAX,LMAX
      PARAMETER (SMAX=10250)
      PARAMETER (LMAX=10)
      REAL U,DTMP
      REAL SI0,SI1
      REAL T(SMAX)
      REAL C1(LMAX-1),C2(LMAX-1),C3(LMAX-1),P1(LMAX-1),P2(LMAX-1)
      REAL BTA(LMAX-1),CD(LMAX-1)
      COMMON /PWRS/C1,C2,C3,P1,P2,BTA,CD
      COMMON /TARRS/T
      COMMON /INDX/I
      COMMON /TINDX/K,NT

      DTMP=- (P2(I)*T(K)+(P1(I)-P2(I))*U)

```

```

        FUNCP=SI1(BTA(I)*U,DTMP)+SI0(BTA(I)*U,DTMP)
        RETURN
    END
C *****
C INTEGRAND FUNCTION DEFINITION FOR P2 < 0
C *****
    REAL FUNCTION FUNCN(U)
    INTEGER NT,I,K,SMAX,LMAX
    PARAMETER (SMAX=10250)
    PARAMETER (LMAX=10)
    REAL U,DTMP
    REAL SI0,SI1
    REAL T(SMAX)
    REAL C1(LMAX-1),C2(LMAX-1),C3(LMAX-1),P1(LMAX-1),P2(LMAX-1)
    REAL BTA(LMAX-1),CD(LMAX-1)
    COMMON /PWRS/C1,C2,C3,P1,P2,BTA,CD
    COMMON /TARRS/T
    COMMON /INDX/I
    COMMON /TINDX/K,NT

    DTMP=-P1(I)*T(K)+(P1(I)/P2(I)-1.0)*U
    FUNCN=SI0(BTA(I)*(T(K)-U/P2(I)),DTMP)+
+        SI1(BTA(I)*(T(K)-U/P2(I)),DTMP)
    RETURN
    END
C *****
C PRODUCT OF EXPONENTIAL AND MODIFIED BESSEL
C FUNCTION OF ZEROth ORDER
C *****
    REAL FUNCTION SI0(X,OFST)
    REAL X,T,Y,OFST

    T=ABS(X)/3.75

    IF (ABS(X).LT.3.75) THEN
        Y=EXP(OFST)
        SI0=1.0*Y
        Y=Y*(T**2)
        SI0=SI0+3.5156229*Y
        Y=Y*(T**2)
        SI0=SI0+3.0899424*Y
        Y=Y*(T**2)
        SI0=SI0+1.2067492*Y
        Y=Y*(T**2)
        SI0=SI0+0.2659732*Y
        Y=Y*(T**2)
        SI0=SI0+0.0360768*Y
        Y=Y*(T**2)

```

```

        SI0=SI0+0.0045813*Y
ELSE
        Y=EXP (ABS (X) +OFST) /SQRT (ABS (X) )
        SI0=0.39894228*Y
        Y=Y/T
        SI0=SI0+0.01328592*Y
        Y=Y/T
        SI0=SI0+0.00225319*Y
        Y=Y/T
        SI0=SI0-0.00157565*Y
        Y=Y/T
        SI0=SI0+0.00916281*Y
        Y=Y/T
        SI0=SI0-0.02057706*Y
        Y=Y/T
        SI0=SI0+0.02635537*Y
        Y=Y/T
        SI0=SI0-0.01647633*Y
        Y=Y/T
        SI0=SI0+0.00392377*Y
ENDIF
RETURN
END
C *****
C PRODUCT OF EXPONENTIAL AND MODIFIED BESSEL
C FUNCTION OF FIRST ORDER
C *****
      REAL FUNCTION SI1(X,OFST)
      REAL X,T,Y,OFST

      T=ABS(X)/3.75

      IF (ABS(X).LT.3.75) THEN
        Y=X*EXP(OFST)
        SI1=0.5*Y
        Y=Y*(T**2)
        SI1=SI1+0.87890594*Y
        Y=Y*(T**2)
        SI1=SI1+0.51498869*Y
        Y=Y*(T**2)
        SI1=SI1+0.15084934*Y
        Y=Y*(T**2)
        SI1=SI1+0.02658733*Y
        Y=Y*(T**2)
        SI1=SI1+0.00301532*Y
        Y=Y*(T**2)
        SI1=SI1+0.00032411*Y
      ELSE

```

```

Y=EXP (ABS (X) +OFST) /SQRT (ABS (X) )
SI1=0.39894228*Y
Y=Y/T
SI1=SI1-0.03988024*Y
Y=Y/T
SI1=SI1-0.00362018*Y
Y=Y/T
SI1=SI1+0.00163801*Y
Y=Y/T
SI1=SI1-0.01031555*Y
Y=Y/T
SI1=SI1+0.02282967*Y
Y=Y/T
SI1=SI1-0.02895312*Y
Y=Y/T
SI1=SI1+0.01787654*Y
Y=Y/T
SI1=SI1-0.00420059*Y
IF (X.LT.0.0) SI1=-SI1
ENDIF
RETURN
END

```

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