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SOCIAL SECURITY – WHAT IS IT GOOD FOR?

By

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ABSTRACT

SOCIAL SECURITY – WHAT IS IT GOOD FOR?

By

Edward Peter Van Wesep

I develop three life cycle models that are designed to examine under what circumstances social security can improve the welfare of societies in the Pareto sense: can at least one person be made better off, without harming anyone else? All three models abstract from intragenerational redistribution – all young and all old people are assumed to be identical. Rather, the focus is on intergenerational redistribution, which can occur three ways: 1) via private intergenerational transfers, 2) via private saving, and 3) via social security.

Externalities play a prominent role in two of the models. In chapter 1, young people make private transfers to their parents, but the quantity of the total transfer from children to parents is not optimal because of an externality. Social security plays a Pigovian role in that model, setting up an incentive system designed to cause private behavior to yield the optimum result. In chapter 3, pay-as-you-go social security is revealed to be a system of intergenerational externalities. Because of this, there is avoidance of social security taxes by the young. This phenomenon of social security tax avoidance leads to a stricter welfare test for pay-as-you-go social security than the traditional Samuelsonian test.

There are no externalities in chapter 2, since social security is treated there as an investment plan for workers: workers thus internalize all of the utility impacts of social security tax payments, just as they internalize the utility impacts of their private saving decisions. The welfare impact of social security in that model is a pure "moneysworth," or rate of return effect. The simplicity of the welfare criterion developed in the model of chapter 2 makes it possible to develop an empirical test of the welfare properties of the United States' Social Security system, using data from the Health and Retirement Study.

To my advisor, Lawrence W. Martin, and to my family.

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INTRODUCTION

The focus of this dissertation is on the impact social security can have on a society's welfare *in the Pareto sense*: can at least one person be made better off, without harming anyone else? The American Social Security system has performed a well-known intragenerational income redistribution role, and it is a matter for debate whether that role alone would justify continuing the American pay-as-you-go Social Security system. Such a debate, however, is outside the province of my work here.

This dissertation keeps the Pareto standard in mind at all times. Taxation of a young worker can be justified if that worker's total lifetime utility is at least not lowered as a result of the taxation. In a traditional life cycle social security model, the marginal utility cost of being taxed when young is offset by the marginal utility benefit of a reward when retired that is at least as great as what the worker could have earned through private saving. The spirit of the traditional model is preserved in the model of chapter 2, and a simple welfare criterion emerges: the Social Security system must return at least as much as private savings in order to be Pareto improving. Using the third wave (1996) of the Health and Retirement Study, I develop an empirical welfare test using this criterion.

Things get more complicated in chapter 3, since people can increase their utility by avoiding the social security tax. As a result, we end up with a stricter standard for welfare improvement than in the traditional model, since we must allow for the marginal utility impacts of enforcement.

Chapter 1 attempts to answer the following question: In a life cycle model with two-sided altruism, is there a role for social security? When we include altruism in a life cycle model, we now have the motive for a private net intergenerational transfer. If that transfer were optimal, there would be no role for social security. But in chapter 1, there is an asymmetry in how people care about their children vs. how they care about their parents. That asymmetry leads to parents having more children than is optimal, to compensate for a perceived deficiency in the total transfer they will receive from their children when they (the parents) retire. Social security in the model of chapter 1 is an incentive scheme designed to encourage people to invest less in children -- and more in the private capital market -- for their old-age security.

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Chapter 1

SOCIAL SECURITY AS A PIGOVIAN SUBSIDY

I. INTRODUCTION

In any society in which people become materially unproductive when they are old, transfers from the younger members of the society are inevitable, whether those transfers occur across time (i.e., saving) or across generations (i.e., transfers to the elderly from the concurrent generation of young workers), or both.[1] If savings and intergenerational transfers were always at optimal levels, there would be no need for social security. This paper presents a theoretical explanation for why societies depart from the optimal levels of savings and intergenerational transfers, and proposes a specific social security system designed to restore the optimum.

Because people in my model will always be able to use savings to achieve the desired balance between consumption-when-young and consumption-when-old, the focus of our attention will be on transfers to the elderly from the concurrent generation of young workers – specifically, on intrahousehold transfers. The idea of depending on one's own children for support in retirement is well established: "That adult children are expected to, and do in fact, provide financial support and informal care giving to their old and dependent parents…has been observed in all civilized societies and in most of the

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1 In the spirit of Samuelson (1958), I rule out borrowing by the elderly.

less developed countries to this day."[2] One way to view this dependence upon support from one's children in old age is that it is part of a contract (whether implicit or explicit) between children and their parents: parents incur expenses in time and material to raise children, and children, in turn, pay their parents a return on that investment, in the form of the intergenerational transfer. This contract is sometimes referred to as a "cultural norm."[3]

But even if a transfer from children is culturally mandated, that transfer could fall short of parents' *desired* transfer. Indeed, Nugent (1985) discusses "studies showing that children do not always transfer as much to their elderly parents as the latter would like…"[4] Suppose it was known that each child would make a transfer of a given amount to his parents upon the parents' retirement. If, when planning the size of their family, the parents *perceived* that the future per-child transfer would be insufficient, it would be reasonable to conclude that parents would have more children, as long as each additional child justified the additional child rearing expense at the margin. Or, as Nugent puts it, "…a low rate of return on children may in certain circumstances make high fertility more necessary than it would otherwise be."[5]

What might lead to parents' expectation of an insufficient transfer from children? In their paper on uncertain altruism, Chakrabarti, Lord and Rangazas (1993) mention the possibility of parents discovering that "their own well-being is less important to their

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2 Isaac Ehrlich and Francis T. Lui, "Intergenerational Trade, Longevity, and Economic Growth," The Journal of Political Economy 99 (1991) 1030 - 1031.

3 See, for example, Ehrlich and Lui (1991).

4 Jeffrey B. Nugent, "The Old-Age Security Motive for Fertility," Population and Development Review 11.1 (1985) 79.

5 Nugent, p. 91 (n. 17).

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2 Isaac Ehrlich and Francis T. Lui, "Intergenerational Trade, Longevity, and Economic Growth," The Journal of Political Economy 99 (1991) 1029 - 1059.

3 See, for example, Ehrlich and Lui (1991).

4 Jeffrey B. Nugent, "The Old-Age Security Motive for Fertility," Population and Development Review 11.1 (1985) 79.

5 Nugent, p. 91 (n. 17).

children than was expected."[6] While altruism is certain in my model, this passage from Chakrabarti et al., touches on the central assumption of this paper: people care differently about their children than they do about their parents. Support for this assumption can be found in the work of the great evolutionary biologist, W.D. Hamilton (1964a), who asserts that there is "a certain fundamental asymmetry" in the parent-offspring relationship.[7] In evolutionary biology, the continuation of favorable genetic traits is of paramount importance. In a typical overlapping generations model with endogenous fertility, the parent of a young, child-bearing adult of generation i has already passed down his genetic material. Thus, from the point of view of the young person of generation i, the passing down of genetic traits from generation (i-1) to generation i is predetermined, and therefore exogenous. But the young person of generation i has a choice to make about how many children to create – i.e., she has a choice to make about how many ways to pass down her own genetic material. In the overlapping generations model I develop in this paper, fertility is indeed endogenous, and Hamilton's "fundamental asymmetry" is manifested in the following way: The model features two-sided altruism, but this two-sided altruism is asymmetric in the sense that a person's utility function incorporates her children's total utility but only the level of consumption of her parent, rather than the total utility of her parent.

The original inspiration for my model was the model of endogenous fertility developed in Becker and Barro (1988) and Barro and Becker (1989); I will refer to this model going forward as "B & B." While my model owes its existence to theirs, there are

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6 Subir Chakrabarti, William Lord, and Peter Rangazas, "Uncertain Altruism and Investment in Children," The American Economic Review 83.4 (1993) 994.

7 W.D. Hamilton, "The Genetical Evolution of Social Behaviour. I," The Journal of Theoretical Biology 7 (1964) 2.

important differences in assumptions between the two models that lead to fundamentally different results. In B & B, parents cannot reap a pecuniary benefit from children, since parents are dead by the time their children become productive. In order to motivate fertility, the authors are forced to include what I call an "altruism effect:" as the number of children one produces increases, so does total altruism toward children. But we will see later that the presence of the altruism effect leads to a counterintuitive result: if a young worker's wage rises relative to what it cost to raise him, his lifetime consumption *decreases*. This conundrum is solved in my model by giving children both the motive (backward altruism) and the opportunity (two periods of economic life) to make transfers to their parents, i.e., by allowing the presence of a pecuniary rate of return from children to be a sufficient motive for having children. Another way to say this is that people choose to have children, knowing that their children will *want* to make a transfer to them in their retirement.

In section II, I present the model, derive the Pareto optimal first order conditions, and compare the optimal equilibrium in my model to the optimal equilibrium in B & B. In section III, I analyze the private equilibrium and develop a role for social security. Section IV concludes the chapter.

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II. THE PARETO OPTIMUM

THE MODEL

All young people are identical, as are all old people, within and across generations.

Following B & B, we consider a dynasty, founded by the "dynastic head" at time zero.

The utility of a member of generation i of the dynasty is:

$$(1) \quad U_i = v(c_i(i), c_i(i+1)) + \alpha U_{i+1} + p(c_{i-1}(i)),$$

where:

- α ≡ Total altruism toward children; $0 < \alpha < 1$.[8]
- $p(\cdot)$ ≡ Young person's felicity from consumption of his/her parent, when old.
- Subscripts indicate the generation affected by a variable or parameter, while time arguments indicate the time period. The time argument will be suppressed if no ambiguity is created by doing so.
- Consumption is of a composite good ("goods"), and $c_i(i) > 0$, $c_i(i+1) > 0$, $\forall i$.
- v and p are strictly concave.

The dynastic utility function is:

$$(2) \quad U_o = \sum_{i=0}^{\infty} \alpha^i [v(c_i(i), c_i(i+1)) + p(c_{i-1}(i))],$$

where $p(c_{-1}(0)) = 0$.[9]

The dynastic budget constraint is:

$$(3) \quad k_o + \sum_{i=0}^{\infty} d(i) N_i w_i = \sum_{i=0}^{\infty} [d(i) N_i c_i(i) + d(i+1) N_i c_i(i+1) + d(i) N_{i+1} \pi_i],$$

---

8 Note that the total altruism constant is a simplification of altruism per child, α / n_i, times the number of children produced by the member of generation i, n_i.

9 The sole member of generation 0 (i.e., the dynastic head) has no parent. In the tradition of this literature, I assume only one parent per nuclear family.

where:

k_o ≡ the initial level of capital

w_i ≡ wage earned by a member of generation i (labor is supplied inelastically)

π_i ≡ cost of raising a child

r ≡ the interest rate, assumed to be constant across periods. The interest rate is earned by investing savings in the private capital market, which exists outside the model.[10]

$d(i)$ ≡ $(1+r)^{-i}$

n_i ≡ number of children chosen by a member of generation i. Those children then become members of generation (i+1), whose total population is N_{i+1}.

N_i ≡ $\prod_{j=0}^{i-1} n_j$, for i = 1, 2, ...(and $N_o = 1$)

Assume that the transversality condition, $\lim_{i \to \infty} d(i) N_i k_i = 0$, is satisfied. I ignore the integer restriction on n_i. Finally, assume that the government has complete information, balances its budget each period, and that the administration of any social security system is costless.

ANALYSIS OF THE OPTIMAL EQUILIBRIUM

Maximizing the dynastic utility function with respect to consumption and fertility, subject to the dynastic budget constraint, gives us a set of first order conditions that characterizes the Pareto optimum:[11]

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[10] The interest rate is thus exogenous, as are the cost of child rearing and the wage.

[11] It is worth clarifying that:

$p'(c_i(i+1))$ = the marginal utility that a member of generation (i+1) gets from the consumption of his/her parent (during retirement), whereas:

$\frac{\partial v}{\partial c_i(i+1)}$ = the marginal utility that a member of generation i gets from his/her own consumption during retirement.

where:

k_0 ≡ the initial level of capital

w_i ≡ wage earned by a member of generation i (labor is supplied inelastically)

π_i ≡ cost of raising a child

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$\frac{\partial v}{\partial c_i(i+1)}$ = the marginal utility that a member of generation i gets from his/her own consumption during retirement.

(4) $MRS_i = 1 + r$,
where:

$$MRS_i \equiv \frac{\dfrac{\partial v}{\partial c_i(i)}}{\dfrac{\partial v}{\partial c_i(i+1)} + \alpha p'(c_i(i+1))}$$

(5) $MRS(i) = n_{i-1}$,
where:

$$MRS(i) \equiv \frac{\dfrac{\partial v}{\partial c_i(i)}}{p'(c_{i-1}(i)) + \left(\dfrac{1}{\alpha}\right)\dfrac{\partial v}{\partial c_{i-1}(i)}}$$

(6) $c_i(i) + \dfrac{c_i(i+1)}{1+r} = w_i - (1+r)\pi_{i-1}$

Equation (4) is the condition for the optimal intragenerational, interperiod allocation of goods, and may be thought of as the condition for optimal saving by a member of generation i for her own retirement. When choosing the amount to save, a member of generation i equates the marginal rate of substitution between consumption-when-young and consumption-when-old (MRS_i) to the marginal rate of transformation available in the capital market ($1+r$). MRS_i optimally incorporates the marginal utility cost to a member of generation i of giving up consumption when she is young, and the marginal utility benefits both she and her children derive from her consumption when she is old.

Equation (5) is the condition for the optimal intergenerational, intraperiod allocation of goods, and may be thought of as the condition for the optimal transfer from a young person to his parent, during period i. The marginal rate of substitution between the

(4) $MRS_i = 1 + r$,

where:

$$MRS_i \equiv \frac{\dfrac{\partial u}{\partial c_i(i+1)} + \alpha \rho'(c_i(i+1))}{\dfrac{\partial u}{\partial c_i(i)}}$$

(5) $MRS(i) = \pi_{i-1}$,

where:

$$MRS(i) \equiv \frac{\rho'(c_{i-1}(i)) + \left(\dfrac{1}{\alpha}\right)\dfrac{\partial u}{\partial c_{i-1}(i)}}{\dfrac{\partial u}{\partial c_i(i)}}$$

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Equation (5) is the condition for the optimal intergenerational, intraperiod allocation of goods, and may be thought of as the condition for the optimal transfer from a young person to his parent, during period i. The marginal rate of substitution between the consumption of the young person and the consumption of his parent ($MRS(i)$) is equated to the marginal rate of transformation, which in this case is the total number of children making transfers to the parent (n_{i-1}). $MRS(i)$ optimally incorporates the marginal utility cost to the young person of giving up consumption, and the marginal utility benefits both the young person and his parent derive from the parent's consumption.

When we compare MRS_i and $MRS(i)$, we observe that there is symmetry between forward and backward altruism in the optimal equilibrium: in addition to her own marginal utility benefits, a young person of generation i optimally incorporates the marginal utility benefits both her children and her parent receive from her saving and transfer decisions.

The fertility first order condition, (6), contains the intuitively appealing result that it is optimal for each member of the dynasty to consume his net material contribution to the dynasty, which consists of his wage less what it cost to raise him, expressed in units of period i goods. This result flows directly from my assumption that total altruism is perfectly inelastic with respect to the number of children a person chooses to have. The intuition is that if increasing the number of one's children does not increase utility in a non-material way (i.e., via increased total altruism toward the next generation), then we are left with only a pecuniary fertility first-order condition: at the margin, increasing the number of children increases utility only via the material "return on investment" the dynasty receives from those children. It turns out to be important to distinguish between these two effects, which I will call the "altruism effect" (the impact on dynastic utility through altruism as the number of children increases) and the "return on investment

effect" (the impact on dynastic utility through the material contribution from children as the number of children increases).

To analyze the impact of these two effects on the dynasty, it will be useful to compare B & B with my model. The analogous equations to (1) through (3) in B & B are as follows: [12]

The individual's utility function is:

$$(7_{B\&B}) \quad U_i = v(c_i) + \alpha(n_i)^{1-\varepsilon} U_{i+1},$$

where:

- the "B&B" subscript indicates that the equation is from their model.
- economic life lasts only one period in their model. The consumption variable, c_i, thus represents lifetime consumption by a member of generation i in period i.
- Note that total altruism toward children is now $\alpha(n_i)^{1-\varepsilon}$, where -ε = the elasticity of altruism per child with respect to the number of children born (and so (1 - ε) is the elasticity of *total* altruism with respect to the number of children born).

The dynastic utility function then becomes:

$$(8_{B\&B}) \quad U_o = \sum_{i=0}^{\infty} \alpha^i (N_i)^{1-\varepsilon} v(c_i),$$

and the dynastic budget constraint is:

$$(9_{B\&B}) \quad k_o + \sum_{i=0}^{\infty} d(i) N_i w_i = \sum_{i=0}^{\infty} d(i)[N_i c_i + N_{i+1}\pi_i],$$

The analogous first order conditions to my (4) through (6) are:

[12] I have made two slight modifications from B&B, that do not impact the fundamental analysis. Those modifications are: 1) I have continued my constant interest rate assumption and 2) I have substituted π for their β to represent the child rearing cost.

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- Note that total altruism toward children is now $a(n_i)^{1-\varepsilon}$, where $-\varepsilon$ = the elasticity of altruism per child with respect to the number of children born (and so $(1-\varepsilon)$ is the elasticity of total altruism with respect to the number of children born).

The dynastic utility function then becomes:

$$(8_{B\&B}) \quad U_0 = \sum_{i=0}^{\infty} a^i (N_i)^{1-\varepsilon} v(c_i),$$

and the dynastic budget constraint is:

$$(9_{B\&B}) \quad k_0 + \sum_{i=0}^{\infty} d(i) N_i w_i = \sum_{i=0}^{\infty} d(i)[N_i c_i + N_{i+1} \pi_i]$$

The analogous first order conditions to my (4) through (6) are:

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[12] I have made two slight modifications from B&B that do not impact the fundamental analysis. Those modifications are: 1) I have continued my constant interest rate assumption and 2) I have substituted π for their β to represent the child rearing cost.

$$(10_{B\&B})\quad \frac{v'(c_{i-1})}{v'(c_i)} = \alpha\left[\frac{1+r}{(n_{i-1})^{\varepsilon}}\right]$$

$$(11_{B\&B})\quad v'(c_i)[c_i] = (1-\varepsilon)v(c_i) + v'(c_i)[w_i - (1+r)\pi_{i-1}]\ ,$$

Note that $(10_{B\&B})$ is a condition for the marginal rate of substitution between the consumption of a member of generation i and the consumption of a member of generation (i-1). We can obtain an essentially identical condition in my model by setting p(·) equal to zero to reflect the lack of backward altruism in B & B,[13] and then dividing (4) by (5), from which we obtain:

$$(12)\quad \frac{\dfrac{\partial v}{\partial c_{i-1}(i)}}{\dfrac{\partial v}{\partial c_i(i+1)}} = \alpha\left[\frac{1+r}{n_{i-1}}\right]$$

The differences between $(10_{B\&B})$ and (12) are not important ones. On the left-hand side of (12), we still have an intergenerational marginal rate of substitution (between consumption-when-old by a member of generation (i-1) and consumption-when-old by a member of generation i), and we still have an intergenerational marginal rate of transformation on the right-hand side, where (12) reflects my assumption that $\varepsilon = 1$.

The important departure of my model from B & B can be discovered by comparing $(11_{B\&B})$ to its counterpart in my model, (6). In $(11_{B\&B})$, the left-hand side is the marginal utility cost of a small increase in the size of generation i, and the right-hand side

13 The presence of backward altruism in my model is not important for the present discussion; suspending it for the time being makes for a clearer comparison of the two models.

is the marginal utility benefit of a small increase in the size of generation i. Note that the marginal utility benefit has two terms: the first term is the altruism effect, and the second term is the return on investment effect, where $w_i - (1+r)\pi_{i-1}$ is a member of generation i's net material contribution to the dynasty. With $\varepsilon < 1$, as B & B must assume,[14] $(11_{B\&B})$ gives us their result that the optimal consumption of a member of generation i exceeds her net material contribution to the dynasty, i.e., that:

$$(13_{B\&B}) \qquad c_i > w_i - (1+r)\pi_{i-1}$$

Using the specification, $v(c_i) = (c_i)^{\sigma}$ $(0 < \sigma < 1-\varepsilon)$,[15] we can solve for consumption:[16]

$$(14_{B\&B}) \qquad c_i^{**} = \frac{\sigma}{1-\varepsilon-\sigma}[(1+r)\pi_{i-1} - w_i]$$

Note in $(14_{B\&B})$ that a positive shock to the wage of a member of generation i results in *reduced* lifetime consumption by that person! This is a strange result, and one which the authors are at some pains to justify. Even if we allow for a positive functional relationship between the child rearing cost, π_{i-1}, and the wage, w_i (interpreting all or part of the child-rearing cost as an investment in human capital as the authors do in Becker and Barro (1988)), an increase in the wage due to a change in any *other* variable

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[14] See Becker and Barro (1988) for a discussion of the necessary conditions to induce fertility. Intuitively: when economic life lasts only one period and the individual undertakes to solve his private utility maximization problem, the only way to get a "payoff" from children to offset the marginal utility cost of raising them is if there is an altruism effect.

[15] The restriction on σ is from B & B. See also n. 14.

[16] Two asterisks denote the level of a variable obtained by solving the dynastic utility maximization problem (i.e., the Pareto optimal level). A single asterisk will denote the level of a variable obtained by solving the private utility maximization problem.

is the marginal utility benefit of a small increase in the size of generation i. Note that the marginal utility benefit has two terms: the first term is the altruism effect, and the second term is the return on investment effect. Since $w_i - (1+r)z_{i-1}$ is a member of generation i's net material contribution to the dynasty. With $\alpha < 1$, as B & B must assume,[15] $(11_{B\&B})$ gives us their result that the optimal consumption of a member of generation i exceeds her net material contribution to the dynasty, i.e., that:

$$(13_{B\&B}) \quad c_i > w_i - (1+r)z_{i-1}$$

Using the specification, $v(c_i) = c_i^{\sigma}$ $(0 < \sigma < 1)$, we can solve for consumption:[16]

$$(14_{B\&B}) \quad c_i^{**} = \frac{\sigma}{1-\sigma}[(1+r)z_{i-1} - w_i]$$

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on the right-hand side of the wage equation (e.g., an increase in productivity caused by investments in physical capital) will cause a decrease in consumption in B & B, ceteris paribus.

In my model it is mathematically (and economically) possible to eliminate the altruism effect (i.e., to set $\varepsilon = 1$) because the presence of backward altruism and two periods of economic life means that when an individual undertakes to solve her private utility maximization problem, the return on investment effect is sufficient justification for having children: parents get a return on their investment in children in the form of a transfer received from their children in retirement. Children perform the same role here as physical savings, which is another way of saying that my model incorporates the transfer-from-children form of old-age insurance that we often observe in less industrially developed cultures.[17] And note in (6) that positive wage shocks now give us the expected result: the member of generation i whose wage rises, enjoys the fruit of that increase in the form of increased lifetime consumption, regardless of the reason for the wage shock.

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[17] See Nugent (1985) for a survey of research done on the old-age insurance motive for fertility.

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III. DEPARTURE FROM THE OPTIMUM: A ROLE FOR SOCIAL SECURITY

In section II, we were concerned solely with the Pareto optimum, and not with private behavior. We now bring private behavior into the picture, which requires the addition of two variables that will act as the vehicles by which transfers are made. The two types of transfers suggested by section II are private saving (the intragenerational, interperiod transfer), which will be designated as k_i,[18] and the private transfer to the parent (the intergenerational, intraperiod transfer), which will be designated as x_{i-1}^{i}, where the superscript indicates the generation choosing the level of the transfer that affects the subscript generation. Thus, x_{i-1}^{i} represents the level of transfer chosen by a member of generation i to give to her parent.

The private budget constraint is:

$$(15)\quad c_i(i) = w_i - n_i\pi_i - k_i - x_{i-1}^{i}$$

$$(16)\quad c_i(i+1) = (1+r)k_i + n_i x_i^{i+1}$$

At times it will be convenient to refer to the budget constraint in the lifetime consumption form obtained by adding (15) to the present value of (16):

$$(17)\quad c_i(i) + \frac{c_i(i+1)}{1+r} = w_i - x_{i-1}^{i} - n_i\left[\pi_i - \left(\frac{x_i^{i+1}}{1+r}\right)\right]$$

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[18] Borrowing at rate r is permitted.

ANALYSIS OF THE PRIVATE EQUILIBRUM

The private analogs to equations (4) through (6) are obtained by maximizing U_i with respect to k_i, x_{i-1}^i, and n_i, subject to the private budget constraint, from which we obtain:

$$(18) \quad MRS_i = 1 + r$$

$$(19) \quad \frac{\frac{\partial v}{\partial c_i(i)}}{p'(c_{i-1}(i))} = n_{i-1}$$

$$(20) \quad MRS_i = \frac{x_i^{i+1}}{\pi_i}$$

Note that the private saving condition, (18), is identical to the optimal saving condition, (4). This is no accident. The private saving variable, k_i, appears in felicities of generations i and (i+1). So, because children's utility (U_{i+1}) is an argument of their parent's utility function (U_i), a member of generation i internalizes all of the dynastic utility effects of her private saving decision and thus behaves optimally.

But the private transfer-to-parent condition, (19), differs from its optimal counterpart, (5). When a young person of generation i makes his decision about the transfer he will make to his parent, there is an externality: the young person fails to consider the marginal utility his parent receives from the transfer, since young people do not internalize their parent's utility, as they do their children's utility. In equations (18) and (19) we see the result of the asymmetry that we have assumed between forward and

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(18) $MRS_i = 1 +$ [illegible]

(19) [illegible]

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In the private fertility condition, (20), a member of generation i equates the marginal rate of substitution between his consumption-when-young and his consumption-when-old to the marginal rate of transformation from investing another unit of goods into child rearing, $\left[\frac{x_i^{i+1}}{\pi_i}\right]$ (each child costs π_i units of goods to raise, whereas the material reward of having that child is a transfer of x_i^{i+1} units of goods received by the parent in his old age).

The solution for the equilibrium private intergenerational transfer is immediately apparent from a comparison of (18) and (20):

$$(21) \quad x_i^{i+1^*} = (1+r)\pi_i$$

In the private equilibrium, the optimal transfer from a young person to her parent consists of returning the cost of raising her, plus interest. If we substitute from (21) into (17), we get (6). We thus have two out of three of the optimal first order conditions ((4) and (6)) holding in the private equilibrium.

The departure of the private equilibrium from the optimum comes about because of the departure of (19) from (5), i.e., as a direct result of the asymmetry that we have assumed between forward and backward altruism and the resultant externality discussed above. This leads to our first proposition:

PROPOSITION ONE

Recall that (4) and (6) hold in both the optimal and the private equilibria, and assume that v and p are such that we can solve explicitly for consumption-when-young and consumption-when-old using (4) and (6). Then each member of the dynasty has a larger than optimal number of children in the private equilibrium.

PROOF:

Since we can obtain explicit solutions for optimal consumption-when-young and consumption-when-old using (4) and (6), we can solve for n_{i-1} in the optimal and private equilibria by using (5) and (19) respectively:

$$(22)\quad n_{i-1}^{**} = \frac{\dfrac{\partial v}{\partial c_i(i)^{**}}}{p'\left(c_{i-1}(i)^{**}\right) + \left(\dfrac{1}{\alpha}\right)\dfrac{\partial v}{\partial c_{i-1}(i)^{**}}}$$

$$(23)\quad n_{i-1}^{*} = \frac{\dfrac{\partial v}{\partial c_i(i)^{**}}}{p'\left(c_{i-1}(i)^{**}\right)}$$

Since optimal consumption levels are the same in both equilibria (because (4) and (6) hold in both equilibria), it follows from a comparison of (22) and (23) that $n_{i-1}^{*} > n_{i-1}^{**}$.

Q.E.D.

In this model, people save for their retirement in two ways: 1) by having children, and 2) by investing in the private capital market. Proposition One gives us the result that people invest too much in children in the private equilibrium, and an immediate corollary is that they invest too little in the private capital market:

PROPOSITION ONE

Recall that (4) and (6) hold in both the optimal and the private equilibria, and assume that v and p are such that we can solve explicitly for consumption-when-young and consumption-when-old using (4) and (6). Then each member of the dynasty has a larger than optimal number of children in the private equilibrium.

PROOF:

Since we can obtain explicit solutions for optimal consumption-when-young and consumption-when-old using (4) and (6), we can solve for n_{t-1} in the optimal and private equilibria by using (5) and (19) respectively.

$$(22) \quad n_{t-1}^{**} = \frac{\dfrac{\partial \gamma}{\partial c_t(t)^{**}}}{p'\left(c_{t+1}(t)^{**}\right) + \left(\dfrac{1}{\alpha}\right)\dfrac{\partial \gamma}{\partial c_{t+1}(t)^{**}}}$$

$$(23) \quad n_{t-1}^{*} = \frac{\dfrac{\partial \gamma}{\partial c_t(t)^{**}}}{p'\left(c_{t+1}(t)^{**}\right)}$$

Since optimal consumption levels are the same in both equilibria (because (4) and (6) hold in both equilibria), it follows from a comparison of (22) and (23) that $n_{t-1}^{*} > n_{t-1}^{**}$.

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COROLLARY TO PROPOSITION ONE

In the private equilibrium, the level of per capita savings is smaller than is optimal.

PROOF:

We substitute the optimal level of consumption obtained from (4) and (6), $c_i(i)^{**}$, the private transfer from (21) and the levels of fertility from (22) and (23) into (15) and then solve for the optimal and private levels of saving, respectively:

(24) $k_i^{**} = w_i - (1+r)\pi_{i-1} - c_i(i)^{**} - n_i^{**}\pi_i$

(25) $k_i^{*} = w_i - (1+r)\pi_{i-1} - c_i(i)^{**} - n_i^{*}\pi_i$

By Proposition One, $n_i^* > n_i^{**}$, and so it follows from a comparison of (24) and (25) that $k_i^* < k_i^{**}$.

Q.E.D.

The intuition behind the results in Proposition One and its corollary is that, from the point of view of a young person of generation i, the optimal *total* transfer from her children, $n_i^{**}(1+r)\pi_i$, would be too small. She believes this because she knows that her children do not consider the marginal utility she receives from transfers when making their transfer decision. She compensates for this deficiency by having more children than is optimal. We may therefore view her *desired* (i.e., privately optimal) total transfer as being the larger amount, $n_i^*(1+r)\pi_i$. Of course, having more children adds additional child rearing cost, and the young person of generation i ends up saving less than is optimal as a result.

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We substitute the optimal level of consumption obtained from (4) and (6), $c_i(t)^{**}$, the private transfer from (21) and the levels of fertility from (22) and (23) into (15) and then solve for the optimal and private levels of savings respectively.

(24) $k_i^{**} = w_i -$ [illegible]

(25) $k_i^{*} = w_i -$ [illegible]

By Proposition One, $n_i^{*} > n_i^{**}$, and so it follows from a comparison of (24) and (25) that

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EXAMPLE ONE

Let $v(c_i(i), c_i(i+1)) = \ln(c_i(i)) + \delta \ln(c_i(i+1))$, and let $p(c_{i-1}(i)) = \varphi \ln(c_{i-1}(i))$, where φ parameterizes the level of altruism of a child toward his/her parent ($\varphi > 0$) and δ is the time discount factor ($0 < \delta \le 1$). We use (4) and (6) to solve for optimal consumption:

$$(26) \quad c_i(i)^{**} = \left[\frac{1}{1+\delta+\alpha\varphi}\right]\left[w_i - (1+r)\pi_{i-1}\right]$$

$$(27) \quad c_i(i+1)^{**} = \left[\frac{(1+r)(\delta+\alpha\varphi)}{1+\delta+\alpha\varphi}\right]\left[w_i - (1+r)\pi_{i-1}\right]$$

We now use (22), (23), (26) and (27) to obtain the optimal and private levels of fertility for a member of generation i:

$$(28) \quad n_i^{**} = \alpha(1+r)\left[\frac{w_i - (1+r)\pi_{i-1}}{w_{i+1} - (1+r)\pi_i}\right]$$

$$(29) \quad n_i^{*} = \left(\alpha + \frac{\delta}{\varphi}\right)(1+r)\left[\frac{w_i - (1+r)\pi_{i-1}}{w_{i+1} - (1+r)\pi_i}\right]$$

Finally, we use (24) through (29) to obtain the optimal and private levels of savings:

$$(30) \quad k_i^{**} = \left[\frac{\delta+\alpha\varphi}{1+\delta+\alpha\varphi}\right]\left[w_i - (1+r)\pi_{i-1}\right] - \alpha(1+r)\left[\frac{w_i - (1+r)\pi_{i-1}}{w_{i+1} - (1+r)\pi_i}\right]\pi_i$$

$$(31) \quad k_i^{*} = \left[\frac{\delta+\alpha\varphi}{1+\delta+\alpha\varphi}\right]\left[w_i - (1+r)\pi_{i-1}\right] - \left(\alpha + \frac{\delta}{\varphi}\right)(1+r)\left[\frac{w_i - (1+r)\pi_{i-1}}{w_{i+1} - (1+r)\pi_i}\right]\pi_i$$

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Let $u(c_i(t), c_i(t+1)) = \ln(c_i(t)) + \delta \ln(c_i(t+1))$, and let $\gamma(c_{i-1}(t)) = \varphi \ln(c_{i-1}(t))$, where φ parameterizes the level of altruism of a child toward his/her parent ($\varphi > 0$) and δ is the time discount factor ($0 < \delta \le 1$). We use (4) and (6) to solve for optimal consumption.

$$(26)\quad c_i(t)^{**} = \left[\frac{1}{1+\delta+\alpha\varphi}\right]\left[w_i - (1+r)\pi_{i-1}\right]$$

$$(27)\quad c_i(t+1)^{**} = \left[\frac{(1+r)(\delta+\alpha\varphi)}{1+\delta+\alpha\varphi}\right]\left[w_i - (1+r)\pi_{i-1}\right]$$

We now use (22), (23), (26) and (27) to obtain the optimal and private levels of fertility for a member of generation i.

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Since the departures of fertility and savings from their optimal levels are caused by an externality, it is only natural to seek a Pigovian remedy, which we will do in a moment. But first, a comment about the impact of industrial progress on fertility and private saving. Let us suppose that industrial progress leads to increasingly productive workers, which is manifested in this model as an increase in $w_{i+1} - (1+r)\pi_i$ relative to $w_i - (1+r)\pi_{i-1}$. Observe in (28) and (29) that in both the optimal and private equilibria, industrial progress leads to reduced fertility. This fits with what we know about the evolution of fertility as societies become more industrialized and workers become more productive.[19] Furthermore, observe in (30) and (31), that in both equilibria, industrial progress leads to increased investment in the private capital market. This fits with Enke's (1971) observation that, "Low savings *per capita* are associated with...high fertility" (his emphasis).[20] Equations (28) through (31) tell us that as societies progress industrially, people rely less on children and more on the private capital market for their old-age insurance.

We now turn to the role the government can play in restoring the Pareto optimum:

PROPOSITION TWO

Let the Pigovian subsidy, s_{i-1}, be defined as:

(32) $s_{i-1} \equiv \overset{*}{n}_{i-1} - \overset{**}{n}_{i-1}$,

where $\overset{**}{n}_{i-1}$ and $\overset{*}{n}_{i-1}$ are from equations (22) and (23) respectively.

19 See, for example, Rosenzweig (1990).

20 Stephen Enke, "Economic Consequences of Rapid Population Growth," The Economic Journal 81.324 (1971) 801.

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Then the following fully funded social security program will restore the Pareto optimum:

1.) A lump sum tax on each member of generation (i-1) when he is young in the amount of $\frac{s_{i-1}x_{i-1}^{i^*}}{1+r}$, which is invested in the private capital market.

2.) A total transfer to the member of generation (i-1) in retirement of $s_{i-1}x_{i-1}^{i}$.[21]

PROOF:

With the social security program in place, the private budget constraint becomes:

$$(33)\quad c_i(i) = w_i - n_i\pi_i - k_i - x_{i-1}^{i} - \frac{s_i x_i^{i+1^*}}{1+r}$$

$$(34)\quad c_i(i+1) = (1+r)k_i + x_i^{i+1}(n_i + s_i)$$

For convenience, the budget constraint for consumption-when-old of a member of generation (i-1) is also shown below:

$$(35)\quad c_{i-1}(i) = (1+r)k_{i-1} + x_{i-1}^{i}(n_{i-1} + s_{i-1})$$

Note that nothing has occurred to change conditions (18) and (20), which means that (21) continues to hold, and so (6) will hold as well. We thus have (4) and (6) holding in the private equilibrium, as before. But the private transfer-to-parent first order condition is now:

$$(36)\quad \frac{\partial v}{\partial c_i(i)} = p'(c_{i-1}(i))(n_{i-1} + s_{i-1})$$

Recognizing that the solutions for consumption will be the same as before (since (4) and

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[21] Note that the total transfer is not a function of the number of children making transfers – it is a function of the per-child transfer only.

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PROOF:

With the social security program in place, the private budget constraint becomes:

$$(33)\quad c_i(t) = w_i - a_i g_i - \text{[illegible]} - \frac{s_{i-1}x_{i-1}^{*}}{1+r}$$

$$(34)\quad c_i(t+1) = (1+r)a_i + s_i^{t+1}\text{[illegible]}$$

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$$(36)\quad \frac{\partial v}{\partial c_i(t)} - p'(c_{i-1}(t))(n_{i-1} + s_{i-1})$$

Recognizing that the solutions for consumption will be the same as before (since (4) and

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21 Note that the total transfer is not a function of the number of children making transfers -- it is a function of the per-child transfer only.

(6) continue to hold), and substituting from (32) into (36) gives us:

$$(37)\quad \frac{\partial v}{\partial c_i(i)^{**}} = p'\left(c_{i-1}(i)^{**}\right)\left(n_{i-1} + n_{i-1}^{*} - n_{i-1}^{**}\right)$$

Solving for n_{i-1} in (37) gives:

$$(38)\quad n_{i-1} = \frac{\dfrac{\partial v}{\partial c_i(i)^{**}}}{p'\left(c_{i-1}(i)^{**}\right)} - n_{i-1}^{*} + n_{i-1}^{**}$$

Substituting from (23) into (38) gives us:

$$(39)\quad n_{i-1} = n_{i-1}^{**}$$

Thus far, with the social security system in place, we have seen that consumption and fertility will be at their optimal levels. It remains to show that the level of investment in the private capital market will also be optimal, but this follows from our assumption that the lump-sum tax will be invested in the capital market. To see this, note that thus far in the proof we have the following private budget constraint for a member of generation (i-1) when young (using (33)):

$$c_{i-1}(i-1)^{**} = w_{i-1} - (1+r)\pi_{i-2} - n_{i-1}^{**}\pi_{i-1} - k_{i-1} - \frac{s_{i-1}x_{i-1}^{i^{*}}}{1+r}$$

The last two terms on the right-hand side of the above equation represent the total investment in the private capital market. Solving for those two terms gives:

$$(40)\quad k_{i-1} + \frac{s_{i-1}x_{i-1}^{i^{*}}}{1+r} = k_{i-1}^{**} = w_{i-1} - (1+r)\pi_{i-2} - c_{i-1}(i-1)^{**} - n_{i-1}^{**}\pi_{i-1}$$

Q.E.D.

(6) continue to hold), and substituting from (35) into (36) gives us:

$$(37)\quad \frac{\partial V}{\partial c_t(t)^{**}} = [illegible]$$

Solving for n_{t+1} in (37) gives:

$$(38)\quad n_{t+1} = [illegible]$$

Substituting from (23) into (38) gives us:

$$(39)\quad n_{t+1} = n_{t+1}^{**}$$

Thus far, with the social security system in place, we have seen that consumption and fertility will be at their optimal levels. It remains to show that the level of investment in the private capital market will also be optimal, but this follows from our assumption that the lump-sum tax will be invested in the capital market. To see this, note that thus far in the proof we have the following private budget constraint for a member of generation (t-1) when young (using (33)):

$$c_{t-1}(t-1)^{**} = w_{t-1} - (1+r)\pi_{t-2} - n_{t-1}^{**}\pi_{t-1} - k_{t-1} - \frac{s_{t-1}\lambda_{t-1}^{*}}{1+r}$$

The last two terms on the right-hand side of the above equation represent the total investment in the private capital market. Solving for those two terms gives:

$$(40)\quad k_{t-1} + \frac{s_{t-1}\lambda_{t-1}^{*}}{1+r} = k_{t-1}^{**} = w_{t-1} - (1+r)\pi_{t-2} - c_{t-1}(t-1)^{**} - n_{t-1}^{**}\pi_{t-1}$$

Q.E.D.

The intuition behind why the social security system introduced in Proposition Two works is as follows: Recall that the young person of generation i desires a total transfer from her children of $n_i^*(1+r)\pi_i$. The Pigovian subsidy increases the transfer she receives *per child* (from both her children and the government) in the private equilibrium from $(1+r)\pi_i$ to $\left(1+\left[\frac{n_i^* - n_i^{**}}{n_i}\right]\right)(1+r)\pi_i$, where $\left[\frac{n_i^* - n_i^{**}}{n_i}\right]$ indicates that the Pigovian subsidy is divided by the number of children she chooses to have, to arrive at the per child subsidy. She will arrive at her desired total transfer of $n_i^*(1+r)\pi_i$ by choosing $n_i = n_i^{**}$.

EXAMPLE TWO

Continuing with the specification of utility in Example One, we get the Pigovian subsidy by substituting from (28) and (29) into (32):

$$(41)\quad s_i = \left(\frac{\delta}{\varphi}\right)(1+r)\left[\frac{w_i-(1+r)\pi_{i-1}}{w_{i+1}-(1+r)\pi_i}\right]$$

We will have the same consumption functions, (26) and (27), as before, and so substituting from (26), (27) and (41) into the transfer-to-parent first order condition, (37), gives us:

$$\left[\frac{1+\delta+\alpha\varphi}{w_{i+1}-(1+r)\pi_i}\right]=\left[\frac{\varphi(1+\delta+\alpha\varphi)}{(1+r)(\delta+\alpha\varphi)[w_i-(1+r)\pi_{i-1}]}\right]\left(n_i+\left(\frac{\delta}{\varphi}\right)(1+r)\left[\frac{w_i-(1+r)\pi_{i-1}}{w_{i+1}-(1+r)\pi_i}\right]\right)$$

which simplifies to:

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$$\left[\frac{1+\delta+\alpha\varphi}{w_{i+1}-(1+r)\pi_i}\right]=\left[\frac{\varphi(1+\delta+\alpha\varphi)}{(1+r)(\delta+\alpha\varphi)\left[w_i-(1+r)\pi_{i-1}\right]}\right]\left(n_i+\left(\frac{\delta}{\varphi}\right)(1+r)\left[\frac{w_i-(1+r)\pi_{i-1}}{w_{i-1}-(1+r)\pi_i}\right]\right)$$

which simplifies to:

$$\left(\alpha+\frac{\delta}{\varphi}\right)(1+r)\left[\frac{w_i-(1+r)\pi_{i-1}}{w_{i+1}-(1+r)\pi_i}\right]=n_i+\left(\frac{\delta}{\varphi}\right)(1+r)\left[\frac{w_i-(1+r)\pi_{i-1}}{w_{i+1}-(1+r)\pi_i}\right],$$

Solving the above expression for n_i gives us:

$$(42)\quad n_i=n_i^{**}=\alpha(1+r)\left[\frac{w_i-(1+r)\pi_{i-1}}{w_{i+1}-(1+r)\pi_i}\right]$$

Note that the total transfer received by the member of generation i in retirement is:

$$(43)\quad \left(n_i^{**}+s_i\right)(1+r)\pi_i=\left(\alpha+\frac{\delta}{\varphi}\right)(1+r)\left[\frac{w_i-(1+r)\pi_{i-1}}{w_{i+1}-(1+r)\pi_i}\right]=n_i^{*}(1+r)\pi_i\ ,$$

where, again, $n_i^{*}(1+r)\pi_i$ is the member of generation i's desired total transfer to be received in retirement.

We can observe in (41) that industrial progress leads to a reduction in the size of the Pigovian subsidy; i.e., as society progresses, the social security system "withers away." This is to be expected: we have seen that as society progresses, people rely less on children and more on private saving for their consumption-when-old. Therefore, a social security system designed to correct for a suboptimal number of children becomes less and less important as society progresses.

IV. SUMMARY AND POLICY IMPLICATIONS

The model developed in this chapter achieves the following:

- It corrects the anomalous result in B & B that positive wage shocks, holding child rearing costs fixed, lead to reduced consumption by the person earning that wage. The anomaly is corrected by eliminating the altruism effect and allowing children to serve their well-known role in developing countries as a form of investment.
- In both the optimal and sub-optimal equilibria, the model gives the well-known result that industrial development leads to reduced fertility and increased investment in the private capital market.
- It develops a role for social security that is motivated by the need to correct for asymmetric altruism shown by young people toward their parents and children. This social security system decreases in importance as society progresses.

The policy implications of the model have to do with the structure of the social security system. What emerges from the model is that the social security system should not be a pay-as-you-go system, but rather, should be a fully funded, privatized system in which there is compulsory saving. If the compulsory savings extracted from a young person were simply returned in lump-sum form when the young person retires, we could reasonably expect that the young person's total savings would not change, given the ability to borrow at the capital market interest rate. What makes compulsory saving effective in this model is that the lump-sum savings extracted by the government from a young person are not simply returned to her in lump-sum form when she is old. Rather, those savings are used to fund the Pigovian incentive scheme. A young person ends up giving the same amount to her parent as in the decentralized private equilibrium, but the presence of the Pigovian subsidy leads to the young person having fewer children, thus allowing her to save more than she would have in the decentralized private equilibrium, due to lower child rearing costs. Thus, compulsory saving actually does lead to an increase in private savings by the young in this model, despite the ability to borrow at the capital market interest rate.

Chapter 2

PERCEIVED FUTURE SOCIAL SECURITY GENEROSITY: AN EMPIRICAL WELFARE TEST

I. INTRODUCTION

Is the current generation of American workers better off with or without Social Security? It is common knowledge that the Social Security program in this country is "in trouble." The ratio of workers to Social Security beneficiaries is expected to average about 2.0 through most of the 21st century, down from 3.4 currently.[22] A population trend like this has obvious negative implications for a pay-as-you-go social security[23] system. But in order to answer the question of whether today's workers are *better off* with or without Social Security, we must first explore the true nature of the system: from the point of view of the worker, is Social Security an investment plan, or is it a social insurance plan? How we answer this question will determine important assumptions to be made in developing a theoretical model of the system, and will ultimately lead to different criteria by which to judge the impact of Social Security on the welfare of today's workers.

According to the United States Supreme Court, "The Social Security system may be accurately described as a form of social insurance..."[24] The court goes on to make it

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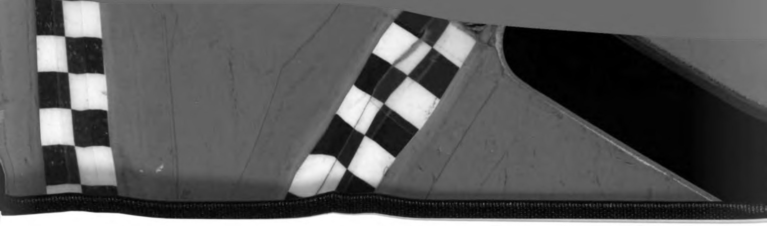
[22] Social Security Advisory Board, "Agenda for Social Security: Challenges for the New Congress and the New Administration" February 2001, p. 3, downloaded from the Social Security Advisory Board's website (http://www.ssab.gov/reports.html).

[23] I will capitalize Social Security when referring to the current U.S. system. When referring to social security in a generic sense, I will use lower-case letters.

[24] Flemming v. Nestor (363 U.S. 603). Justice Harlan wrote for the court.



clear that as a social insurance plan, Social Security's primary function is to help prevent a situation in which current retirees live in poverty and also, importantly, to help alleviate current workers' fear that they might end up in poverty in the future, after they retire. For such a system to work, the benefits paid to current retirees must be sufficient to lift most of them out of poverty, and workers paying into the system must be assured that the same will be done for them, *regardless of the amount they have paid into the system.* Indeed, the court makes it very clear that it explicitly does not view Social Security as an investment plan:

> Each worker's [Social Security] benefits...are not dependent on the degree to which he was called upon to support the system by taxation. It is apparent that the noncontractual interest of an employee covered by the [Social Security] act cannot be soundly analogized to that of the holder of an annuity, whose right to benefits is bottomed on his contractual premium payments.[25]

But according to the Social Security Administration ("SSA"), the American pay-as-you-go Social Security system, "...is designed so that there is a link between how much a worker pays into the system and how much he or she will get in benefits."[26] The 1994-1996 Advisory Council on Social Security (the "Advisory Council") stated that, "Social Security should provide benefits to each generation of workers that bear a reasonable relationship to total taxes paid, plus interest."[27] Official statements like these provide a

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[25] Flemming v. Nestor (363 U.S. 603)

[26] Social Security Administration, "The Future of Social Security," Publication No. 05-10055, August 2000, downloaded from the Social Security Administration website (http://www.ssa.gov/pubs/10055.html) on June 12, 2001 – no page number is available.

[27] 1994-1996 Advisory Council on Social Security, "Findings, Recommendations and Statements," downloaded from the Social Security Administration website (http://www.ssa.gov/history/reports/adcouncil/report/findings.htm) on June 5, 2001 – no page number is available. The 1994-1996 Advisory Council, chaired by Edward Gramlich, Ph.D., was the last of the Advisory Councils, which had been required by law to be formed every four years. The Advisory Councils were replaced by the Social Security Advisory Board, which is a standing body.

strong argument for thinking of Social Security as an investment plan, in which each worker's benefits are indeed "dependent on the degree to which he was called upon to support the system by taxation."

In this chapter and in chapter 3, I present life cycle models in which workers can avoid paying a portion of their total social security tax bill by choosing to take some of their wages in the form of nontaxable fringe benefits.[28] The models in each chapter differ in one, and only one, way: in this chapter, social security is treated as an investment plan, while in chapter 3, social security is treated as a social insurance plan. The impact on social welfare of this difference in the two models is felt through the social security benefit. In chapter 3, the assumption that social security is a social insurance plan enters the model in the form of a benefit that is exogenous. Because there is no altruism in the model, workers pay their social security taxes solely to avoid the penalty they would suffer if they were to take all of their wages in the form of fringe benefits. In this setting, pay-as-you-go social security embodies a system of intergenerational externalities: workers pay taxes to reduce disutility from a penalty, and the concurrent cohort of retirees enjoys the fruit of those social security tax payments as a positive externality.

In this chapter, since social security is viewed as an investment plan, social security taxes paid are a form of saving, and the social security benefit is a return on that saving. Workers in this setting thus internalize all of the utility impacts of their social security tax-paying decision. This would be the appropriate model for a social security system consisting entirely of mandatory private retirement savings accounts, but a pay-as-you-go system that is viewed strictly as an investment plan can also be modeled in this way.

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[28] See chapter 3 for a discussion of the rationale for such a model.

As we will see, the criterion we use to judge the impact of social security on social welfare differs, depending on whether we conceive of social security as a social insurance plan or an investment plan. In section II of this chapter, I present the model and develop the welfare criterion for social security as an investment plan. In section III, I present an empirical welfare analysis for the current generation of workers ("generation t"), using the welfare criterion of section II. Section IV concludes the chapter.

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II. THE THEORETICAL FRAMEWORK

For a representative worker whose economic life lasts two periods, beginning in period t, let:[29]

$Y_f(t)$ ≡ Nontaxable wages in the form of fringe benefits.

$Y_m(t)$ ≡ Taxable wages that can be used for any kind of consumption (i.e., "cash").

$Y(t)$ ≡ Total wages = $Y_f(t) + Y_m(t)$. I assume $Y(t)$ to be exogenous.

$q(t)$ ≡ The social security tax rate. The amount of social security tax the representative worker pays in period t is $q(t)Y_m(t)$.

$C_f(t)$ ≡ Consumption of fringe benefits. (For convenience, I assume fringes are all consumed when the worker is young.)

$C_m(t)$ ≡ The amount of cash income consumed when young. Payment of taxes other than the social security tax is included in $C_m(t)$.

$S(t)$ ≡ The amount of cash wages saved when young. Savings earn the real interest rate. Assume that $S(t)$ can be less than zero; i.e., borrowing at the real interest rate is permitted.

$C(t+1)$ ≡ Consumption when old.

$r(t+1)$ ≡ The real interest rate from period t to period (t+1), which is known with certainty in period t. The real interest rate is set outside the model, in the private capital market.

$B(t+1)$ ≡ The worker's expected social security benefit (paid in cash). Assume that the worker's forecast is accurate, so that $B(t+1)$ is also the actual benefit received in period (t+1).

The worker's choice variables are $S(t)$ and $Y_f(t)$ (from which $Y_m(t)$ is also determined).

The social insurance and investment plan roles social security can be called upon to play are both included in the following benefit structure:

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[29] Time arguments always indicate the period, rather than the generation. For notational convenience, I am suppressing generation subscripts -- I will specify which generation is involved, if that is not clear from the context. The worker here represents "generation t."

(1) $B(t+1) = \gamma(t+1) + \mu(t+1)[1 + r(t+1)]q(t)Y_m(t)$,

where $\gamma(t+1) \geq 0$ and $\mu(t+1) \geq 0$.

In (1), $\gamma(t+1)$ and $\mu(t+1)$ are both government-controlled parameters. Because $\gamma(t+1)$ is not a function of social security taxes paid, it represents the exogenous, social insurance role of the social security program. Taking $r(t+1)$ as given, $\mu(t+1)$ controls the rate of return workers will receive on their social security "investment" (i.e., taxes paid), and will therefore be called the "generosity multiplier." In this chapter and in chapter 3, we will examine two special cases: 1) $\gamma(t+1) \geq 0$; $\mu(t+1) = 0$ (chapter 3), and 2) $\gamma(t+1) = 0$; $\mu(t+1) \geq 0$ (this chapter). At this juncture, it is important to understand that "generality" in this setting is a generality in how we conceptualize social security's role in society. In special case number 1 above, social security is strictly a social insurance plan and in special case number 2, it is strictly an investment plan. But each of these special cases is completely general in that it can account for all possible social security outflows from, and inflows to, generation t. In the general case ($\gamma(t+1) \geq 0$; $\mu(t+1) \geq 0$), we would view the social security system as performing both the social insurance and investment plan roles, to varying degrees.

Going forward, then, let us maintain the assumption that $\gamma(t+1) = 0$ and $\mu(t+1) \geq 0$. Consider a utility model similar to that of S.A. Drakopoulos (1994), who specifies a separable utility function in which the arguments are cash wages and fringe benefits.[30] The worker's problem will then be to:

[30] Drakopoulos assumes not only that utility is separable, but that it is quasilinear in cash wages. Not surprisingly, workers in his model desire any increase in total income to be in the form of cash wages.

(1) $B(t+1) = \gamma(t+1) + \mu(t+1)$[illegible]

where $\gamma(t+1) \geq 0$ and $\mu(t+1) \geq 0$.

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$$\max_{Y_f(t), S(t)} U(t) = \ln(C_m(t)) + \phi \ln(C_f(t)) + \delta \ln(C(t+1)) ,$$

subject to:

(2) $C_m(t) = [1 - q(t)][Y(t) - Y_f(t)] - S(t)$

(3) $C_f(t) = Y_f(t)$

(4) $C(t+1) = [1 + r(t+1)]S(t) + B(t+1) = [1 + r(t+1)]\{S(t) + \mu(t+1)q(t)[Y(t) - Y_f(t)]\}$

In the utility function, δ is the factor workers use to discount their second-period felicity ($0 < \delta \leq 1$). There is a penalty for noncompliance with social security taxation that arises out of the separability of the felicity from total first period consumption (C_m(t) + C_f(t)). This penalty is parameterized by the taste for fringe benefits, ϕ. From the point-of-view of the government, ϕ is an exogenous parameter that must be taken into account when formulating policy. The government's social security policy tools affecting generation t are thus: 1) setting the level of q(t) and 2) setting the level of μ(t+1).

The first order conditions of the maximization problem are:

(5) $S(t):$ $$\frac{1}{[1 - q(t)][Y(t) - Y_f(t)] - S(t)} = \frac{\delta}{S(t) + \mu(t+1)q(t)[Y(t) - Y_f(t)]}$$

(6) $Y_f(t):$ $$\frac{1 - q(t)}{[1 - q(t)][Y(t) - Y_f(t)] - S(t)} + \frac{\delta\mu(t+1)q(t)}{S(t) + \mu(t+1)q(t)[Y(t) - Y_f(t)]} = \frac{\phi}{Y_f(t)}$$

The right-hand side of (6) represents the marginal utility benefit of reducing the amount of the worker's wages that are exposed to the social security tax, while the left-hand side represents the marginal utility cost of doing so. Equation (6) helps to clarify the sense in

which ϕ parameterizes the penalty for noncompliance: For any given level of q(t) > 0, as ϕ approaches 0, the penalty for noncompliance approaches infinity -- if payroll workers have no taste for fringe benefits, they have no choice but to expose all of their wages to the social security tax. As ϕ rises, the penalty for noncompliance falls.

Solving the system (2) through (6) gives the following consumption, saving and wage functions in the social security ("S") economy:

$$(7)\quad C_m(t)^* = \left(\frac{1}{1+\delta+\phi}\right)\{1-[1-\mu(t+1)]q(t)\}Y(t)$$

$$(8)\quad C_f(t)^* = Y_f(t)^* = \left(\frac{\phi}{1+\delta+\phi}\right)Y(t)$$

$$(9)\quad C(t+1)^* = \left(\frac{\delta}{1+\delta+\phi}\right)[1+r(t+1)]\{1-[1-\mu(t+1)]q(t)\}Y(t)$$

$$(10)\quad S(t)^* = \left\{\frac{\delta[1-q(t)]-\mu(t+1)q(t)}{1+\delta+\phi}\right\}Y(t)$$

$$(11)\quad Y_m(t)^* = \left(\frac{1+\delta}{1+\delta+\phi}\right)Y(t)$$

Note in (11) that the optimal level of wages the worker exposes to the social security tax, $Y_m(t)^*$, is invariant with respect to both of the social security policy parameters, μ(t+1) and q(t). This would not, in general, be true with γ(t+1) > 0.[31] The behavior of $Y_m(t)^*$ in

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[31] In the case with γ(t+1) > 0, we would obtain

$$C_f(t)^* = \left(\frac{\phi}{1+\delta+\phi}\right)\left[Y(t)+\left(\frac{\gamma(t+1)}{\{1-[1-\mu(t+1)]q(t)\}\{1+r(t+1)\}}\right)\right]$$

In this case, $C_f(t)^*$ would be invariant with respect to the social security tax rate only if μ(t+1) = 1, and invariant with respect to μ(t+1) only if q(t) = 0.

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$$C_f(t)^* = \left(\frac{\phi}{1+\delta+\phi}\right)\left[Y(t) + \left(\frac{\gamma(t+1)}{\{1-[1-\mu(t+1)]q(t)\}[1+r(t+1)]}\right)\right]$$

In this case, $C_f(t)^*$ would be invariant with respect to the social security tax rate only if $\mu(t+1) = 1$, and invariant with respect to $\mu(t+1)$ only if $q(t) = 0$.

this model thus depends critically on our conception of social security as an investment plan. Treating social security tax payments as a form of investment means that we essentially have a system of mandatory private retirement savings accounts (even if the system is nominally pay-as-you-go) and young people thus save in two ways: 1) via the private capital market, and 2) via the government. Let G(t) denote the latter form of saving, and so:

$$(12)\quad G(t)^{*} = q(t)Y_m(t)^{*} = q(t)\left(\frac{1+\delta}{1+\delta+\phi}\right)Y(t)$$

Therefore, we can consider only two diversions that reduce cash consumption when the worker is young: 1) wages diverted to fringe benefits, $Y_f(t)$, and 2) total saving ($S(t)+G(t)$).[32] The amount the worker devotes to total saving will no more affect the level of fringes in this model than it would in a model in which there is no social security program – the only thing that would change from one model to the other is the nature of saving and the rate of return on saving, but the two models would not be different in any fundamental sense. This model is simply more general.

To drive this point home, let Z(t) denote total saving, and so we have:

$$(13)\quad Z(t)^{*} = S(t)^{*} + G(t)^{*} = \left(\frac{\delta+[1-\mu(t+1)]q(t)}{1+\delta+\phi}\right)Y(t)$$

In a system in which the social security benefit was strictly exogenous, the effective rate of return on savings would always be the real interest rate, r(t+1). But in this system, let

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$$(12)\quad G(t)^* = \phi(t)Y_w(t) \text{ [illegible]}$$

Therefore, we can consider only two diversions that reduce cash consumption when the worker is young: 1) wages diverted to fringe benefits, $F_y(t)$, and 2) total saving ($S(t) + G(t)$).[32] The amount the worker devotes to total saving will no more affect the level of fringes in this model than it would in a model in which there is no social security program – the only thing that would change from one model to the other is the nature of saving and the rate of return on saving, but the two models would not be different in any fundamental sense. This model is simply more general.

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[32] In a model in which the social security benefit is strictly exogenous, total saving would just be S(t), and a third diversion would be social security taxes paid.

$\rho(t+1)^*$ denote the effective rate of return on optimal total savings ("savings"). We can solve for this rate of return as follows:

$$(14)\quad 1+\rho(t+1)^* = \frac{C(t+1)^*}{Z(t)^*} = [1+r(t+1)]\left(\frac{\delta\{1-[1-\mu(t+1)]q(t)\}}{\delta+[1-\mu(t+1)]q(t)}\right)$$

In the special case in which there is no social security program (q(t) = 0), the effective rate of return on savings would be the real interest rate. Of course, the government can deliver the same result in the general case (q(t) > 0) by setting μ(t+1) = 1. As μ(t+1) rises (falls), the effective rate of return on savings rises (falls).

We can analyze the welfare effects of the government's manipulation of μ(t+1) by first writing down the indirect utility function for a representative member of generation t in the S economy, $V(t)_S$:

$$(15)\quad V(t)_S = \ln\left[\left(\frac{1}{1+\delta+\phi}\right)\{1-[1-\mu(t+1)]q(t)\}Y(t)\right] + \phi\ln\left[\left(\frac{\phi}{1+\delta+\phi}\right)Y(t)\right]$$
$$+\delta\ln\left[\left(\frac{\delta}{1+\delta+\phi}\right)[1+r(t+1)]\{1-[1-\mu(t+1)]q(t)\}Y(t)\right]$$

In the economy with no social security ("NS"), we get the indirect utility function, $V(t)_{NS}$, by setting q(t) = 0:

$$(16)\quad V(t)_{NS} = \ln\left[\left(\frac{1}{1+\delta+\phi}\right)Y(t)\right] + \phi\ln\left[\left(\frac{\phi}{1+\delta+\phi}\right)Y(t)\right]$$
$$+\delta\ln\left[\left(\frac{\delta}{1+\delta+\phi}\right)[1+r(t+1)]Y(t)\right]$$

The S economy will be Pareto inferior to the NS economy if, for any generation t, $V(t)_S < V(t)_{NS}$. The S economy will be (weakly) Pareto superior to the NS economy if, for all generations t, $V(t)_S \geq V(t)_{NS}$. By inspection of (15) and (16), we can derive:

The Welfare Criterion

Given all of our previous assumptions, the S economy will be Pareto inferior to the NS economy if, for any generation t:

$$\mu(t+1) < 1$$

The S economy will be (weakly) Pareto superior to the NS economy if, for all generations t:

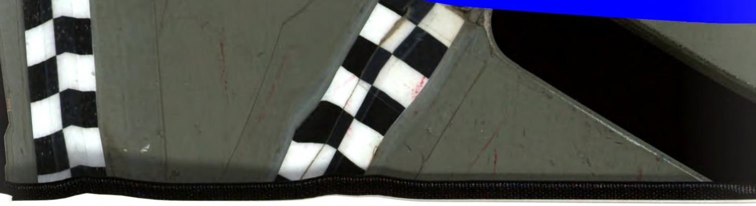
$$\mu(t+1) \geq 1$$

Our welfare criterion fits with the Advisory Council's recommendation cited earlier that, "Social Security should provide benefits to each generation of workers that bear a reasonable relationship to total taxes paid, plus interest." It is also the same as the traditional result from the Samuelson (1958, 1975[a&b])/Diamond (1965) life cycle/growth model: the social security system must deliver a return to the worker which is at least as good as the capital market return (i.e., the real interest rate), in order to have any hope of being Pareto improving. But we got to the result in a non-traditional way, by including enforcement. Including enforcement reduces the level of social security taxes paid vis-à-vis an equivalent Samuelson/Diamond model.[33] But the presence of enforcement in this model does not affect *welfare* when the government changes the social security parameters, q(t) and $\mu(t+1)$, since workers treat all cash outflows when

[33] To see this, simply compare (12) with $\phi = 0$ (Samuelson/Diamond) to (12) with $\phi > 0$.

young as the outcome of a saving decision. The welfare impact of changes in q(t) and $\mu(t+1)$ is thus felt via changes in the effective rate of return on workers' savings.[34]

[34] The results are different when $\gamma(t+1) > 0$. In that case, the presence of enforcement does affect welfare when the government increases the social security tax rate, since there is a tax shield advantage to increasing consumption in the form of fringes. I discuss this further in chapter. 3.



III. EMPIRICAL WELFARE ANALYSIS

The empirical mandate from the previous section is clear: determine the level of $\mu(t+1)$. In that section, I assumed that the expected and actual levels of $\mu(t+1)$ were the same. Assumptions like this seem more reasonable than ever in the internet age, when so much data is readily and cheaply available. Indeed, the Social Security Advisory Board (the "Advisory Board") has stated publicly that SSA should "increase public understanding of social security."[35] In October 1999, SSA took a significant step toward increasing public understanding by starting annual mailings of *Your Social Security Statement* to all American workers aged 25 and older.[36] In a recent edition of the *Statement*, Larry G. Massanari, Acting Commissioner of Social Security, wrote:

> Will Social Security be there when you retire? Of course it will. But changes will be needed to meet the demands of the times...[By 2038] payroll taxes collected will be enough to pay only about 73% of benefits owed.[37]

The mass mailing of Mr. Massanari's words means we may reasonably conclude that virtually every American worker aged 25 and older (i.e., generation t) has at least had the opportunity to learn the government's current beliefs about social security: In the language of our theoretical model, the government believes $\mu(t+1)$ to be greater than zero ("Of course," social security will "be there when you retire"), but also that, without changes, $\mu(t+1)$ might be less than one ("payroll taxes collected will be enough to pay only about 73% of benefits owed"). Further support for believing that $\mu(t+1)$ could be less than one comes from the Advisory Council, whose report stated as fact that "under

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35 Social Security Advisory Board, "Agenda for Social Security: Challenges for the New Congress and the New Administration" February 2001, p. 21.

36 Social Security Administration, "The Future of Social Security" – no page number is available.

present law," many young workers will be paying, "...taxes that add to considerably more than the present value of their anticipated benefits."[38]

Such widely-available views on the part of the government could reasonably be expected to imbue generation t as a whole with a certain skepticism about the future of their social security benefit. Indeed, many surveys have confirmed such skepticism. A 1999 survey conducted by SSA found that, "...more than half of non-retirees who were polled were only a little confident or not confident at all that Social Security retirement benefits will be there for them when they retire."[39] Another survey, produced jointly by the Employee Benefit Research Institute and The Gallup Organization, Inc. in 1995, revealed that 82% of respondents either "agreed or strongly agreed with the statement that working Americans are beginning to lose faith in whether Social Security benefits will be available when they retire."[40] Surveys such as these are useful in capturing the general mood of pessimism about the future of social security. But they lack precision in the sense that they present the respondent with only a stark choice (social security benefits will or will not "be there" for them when they retire), and a limited menu of qualitative probabilities from which to choose ("a little confident," strongly agree," etc.).

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[37] Mr. Massanari's statement appears in "Your Social Security Statement – Prepared especially for Edward P. Van Wesep, Jr.," published by the Social Security Administration on April 25, 2001.

[38] 1994-1996 Advisory Council on Social Security, "Findings, Recommendations and Statements" – no page number is available.

[39] Social Security Advisory Board, "Agenda for Social Security: Challenges for the New Congress and the New Administration" February 2001, p. 22.

[40] Pamela Ostuw, "Public Attitudes on Social Security Reform," Assessing Social Security Reform Alternatives, ed. Dallas L. Salisbury (Washington, D.C.: Employee Benefit Research Institute, 1997) 127.

Enter the Health and Retirement Study ("HRS"), the extensive panel study begun in 1992.[41] In the third wave of the HRS ("HRS 3"), conducted in 1996, respondents were asked to estimate the probability on an integer scale from 0 to 100%, that Congress will change Social Security so that it "becomes less generous than now".[42] This question is impressive, in that it allowed people to face a less drastic choice than usual (social security will or will not become "less generous," vs. not being there at all), and gave them a large menu of quantitative probabilities from which to choose. The wording of the question is also impressive in that it did not ask respondents to predict the future of their *personal* Social Security benefit, but rather to predict the future of benefits *in general*. This avoids a host of estimation errors due to incomplete information about future income, how Social Security benefits are calculated, etc. In fact, the wording of this question will allow us to develop a methodology for estimating the value of $\mu(t+1)$, which will in turn allow us to evaluate the existing Social Security system using the welfare criterion of the previous section.

We start with some definitions:

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[41] Information on HRS from the HRS website (www.umich.edu/~hrswww):

- Sponsoring Organization: National Institute on Aging
- Data Collection Organization: Institute for Social Research, University of Michigan
- Principal Investigator: Robert J. Willis
- National panel study
- Initial sample of over 12,600 persons in 7,600 households

Source: www.umich.edu/~hrswww/overview/hrsover.html

[42] HRS Wave 3: Section H (Expectations), question H16. The full text of the HRS Wave 3 questionnaire is available at the HRS website.

Definition One

The Social Security system will be as generous for generation t as it is now if:

$$\frac{B(t+1)}{[1+r(t+1)]} = B(t)$$

In words: the Social Security system will be as generous for generation t as it is now, if the present value of generation t's future per capita benefit is the same as generation (t-1)'s current per capita benefit.

Definition Two

The Social Security system will be less generous for generation t than it is now if:

$$\frac{B(t+1)}{[1+r(t+1)]} = \lambda B(t), \text{ where } 0 \leq \lambda < 1$$

Let us first determine the level of $\mu(t+1)$, assuming that current generosity continues. Substituting from Definition One into (1) (with $\gamma(t+1) = 0$, and using $Y_m(t) = Y_m(t)^*$) gives us that, if the Social Security system were to be as generous for generation t as it is now:

(17) $$\mu(t+1) = \frac{B(t)}{q(t)Y_m(t)^*}$$

In 1995 (the year before HRS 3 and thus "current" at the time of the survey), the aggregate annual current benefit-to-current tax ratio was 0.96.[43] Let us assume a ratio of retirement years to working years of ½. Taking 1995 as our representative year and

[43] I arrived at this figure as follows: In calendar 1995, the Old Age and Survivors Insurance (the retirement portion of the social security system) trust fund took in payroll taxes of $304.6 billion. The trust fund paid out $291.6 billion in benefit payments. $291.6 billion / $304.6 billion = 0.96. Source: 1996 OASDI Trustees Report, Table I.D1, downloaded from the SSA website (http://www.ssa.gov/history/reports/trust/1996/tbid1.html) on June 4, 2001.

projecting forward, we thus get an aggregate pro forma lifetime current benefit-to-current tax ratio that is half of the annual ratio, or 0.48. As we have seen, the current ratio of workers to retirees is approximately 3.4. This ratio multiplied by the aggregate pro forma lifetime current benefit-to-current tax ratio, gives us the per capita pro forma lifetime current benefit-to-current tax ratio:

(18) $\frac{B(t)}{q(t)Y_m(t)^*}$ = [Aggregate Pro Forma Lifetime Current Benefit-to-Current Tax Ratio] x [Current Worker-to-Retiree Ratio] = [0.48] x [3.4] = **1.6**

Thus, by (17), if the Social Security system were predicted to be as generous for generation t as it currently is for generation (t-1), we would have:

(19) $\mu(t+1) = 1.6$

Note the importance of (19) for welfare: given our previous assumptions, and given current generosity, the existing Social Security system would improve the welfare of generation t (since $\mu(t+1) > 1$).

Of course, the empirical issue to be decided is what the value of $\mu(t+1)$ is, given that Social Security generosity may be reduced in the future. As framed by the question asked of HRS 3 respondents, we have two possible states of nature for future Social Security generosity: 1) Social Security will be less generous than now, or 2) Social Security will not be less generous than now. If state (1) were realized when generation t retires, Definition Two would apply, and we would have $\frac{B(t+1)}{[1+r(t+1)]} = \lambda B(t)$. The realization of state (2) merits some discussion. How do we interpret the phrase "will not

be less generous than now"? Does it mean that Definition One applies, i.e., that the system will be equally as generous as it is now when generation t retires? Or does it mean that the system will be *at least* as generous as it is now? I can only offer a conjecture: given the current climate of pessimism, I think it unlikely that most people would assign a probability greater than zero to the event, "Social Security becomes *more* generous than it is now." But the question was not asked in HRS 3, so this remains a conjecture. Let us nevertheless assume that state (2) is characterized by Definition One, and so we get the following general equation for $\mu(t+1)$:

$$(20)\quad \mu(t+1) = \theta\left[\frac{B(t)}{q(t)Y_m(t)^*}\right] + (1-\theta)\left[\frac{\lambda B(t)}{q(t)Y_m(t)^*}\right],$$

where:

$1 - \theta$ ≡ The probability that Social Security will be less generous for generation t than it is now.[44]

Equation (20) is general, in the sense that we can get (17) as a special case by forcing λ to be one, i.e., by assuming that current generosity will continue. Substituting from (18) into (20) gives us:

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[44] Equation (20) is derived as follows:

We have two possible realizations of the random variable, $\frac{B(t+1)}{[1+r(t+1)]}$: It will take the value B(t) with probability θ and it will take the value λB(t) with probability $(1-\theta)$. So the expected value is: $E\left[\frac{B(t+1)}{[1+r(t+1)]}\right] = \theta B(t) + (1-\theta)\lambda B(t)$. Substituting from (1) (with $\gamma(t+1) = 0$, and using $Y_m(t) = Y_m(t)^*$) gives: $E\left[\mu(t+1)q(t)Y_m(t)^*\right] = \theta B(t) + (1-\theta)\lambda B(t)$.

But q(t) and $Y_m(t)^*$ are constants, and so: $E[\mu(t+1)] = \theta\left[\frac{B(t)}{q(t)Y_m(t)^*}\right] + (1-\theta)\left[\frac{\lambda B(t)}{q(t)Y_m(t)^*}\right]$.

Finally, we have assumed perfect foresight, i.e., that $E[\mu(t+1)] = \mu(t+1)$, from which we get (20).

(21) $\mu(t+1)=[\lambda+\theta(1-\lambda)]1.6$

My inference from HRS 3 will focus on θ in (21). I will then perform sensitivity analysis to assess the combined impact of λ and θ on welfare.

Using HRS 3 for inference on θ entails two important advantages. The first of these is the enormous sample size: 7,518 usable observations in my case. The second advantage of using HRS 3 for inference is the huge variety of questions that were asked in the survey. Hundreds of detailed questions were asked in a large variety of categories, including income, employment, health, etc. Thus, inference using HRS 3 is less likely to encounter the omitted variables problem than it would be using a data set that is not as rich.

HRS 3's target population was "all adults in the contiguous United States, born during the years 1931-1941, who reside in households.[45] Thus, the target population was limited to those people who were 55 to 65 years old at the time of the survey.[46] While this means that a significant portion of generation t is not represented, our primary empirical goal here is not to conduct a public opinion poll of generation t, but to get an accurate estimate of the probability that Social Security benefits will become less generous in the future. It is a matter for debate whether including all of generation t in the target population would help or hinder that goal.

[45] "Survey Design," downloaded from the HRS website (www.umich.edu/~hrswww/studydet/design.html) on February 9, 2001 – no page number is available.

[46] The reason for this age restriction on the target population was that the HRS was designed to follow people "as they made the transition from active worker into retirement." Source: "Survey History and Design," downloaded from the HRS website (www.umich.edu/~hrswww/studydet/develop/history.html), on February 9, 2001 – no page number is available.

Let us define "optimism" as the per cent probability that social security will not become less generous in the future, i.e.,

(22) $\text{optimism} \equiv 100(\theta)$

Our primary empirical goal is to obtain a point estimate of optimism from the sample as a whole. An important secondary goal is to explore the empirical determinants of optimism in the target population, which will in turn lead to an analysis of point estimates of optimism from certain subsamples of interest. As to the primary goal, I obtained the following point estimate from the sample as a whole:[47]

(23) Point estimate of optimism = 36.9%[48]

Substituting from (23) into (21) gives us:

(24) $\hat{\mu}(t+1) = [\lambda + 0.369(1-\lambda)]1.6 = 0.59 + 1.01\lambda$

The welfare impact of the optimism estimate thus depends critically on the value of λ. In the starkest case, in which Social Security would not "be there" in the reduced-benefit state of nature (i.e., $\lambda = 0$), the S economy would be Pareto inferior to the NS economy ($\hat{\mu}(t+1) < 1$). We can obtain a benchmark value of λ ("λ_b") by recalling that the Advisory Board projects the worker-to-retiree ratio to fall to an average of about 2.0 for

[47] The HRS is a complex survey. The complexities include oversampling (e.g., of blacks and hispanics) and the presentation of responses from non-age-eligible respondents (e.g., spouses of age eligible respondents) in respondent-level sections of the survey. HRS therefore provides sampling weights to use in inference. I have used those weights in all of my inferences, using the "svy" command set in Stata. Non-age-eligible respondents were all supposed to have respondent-level weights of zero. However, I discovered a handful of non-age-eligible respondents with nonzero respondent-level weights, and I deleted those observations.

[48] The 95% confidence interval for this estimate is (36.1%, 37.7%).

most of the 21st century. We can solve for λ_b by first developing an equation similar to (18), but now the left-hand side will be the *projected* per capita lifetime benefit-to-tax ratio, given no changes in the Social Security program. The right-hand side will assume the same aggregate pro forma current benefit-to-current tax ratio as before, but will use the projected worker-to-retiree ratio, rather than the current worker-to-retiree ratio:

(25) $\dfrac{\lambda_b B(t)}{q(t)Y_m(t)^*}$ = [Aggregate Pro Forma Lifetime Current Benefit-to-Current Tax Ratio] x [Projected Worker-to-Retiree Ratio] = [0.48] x [2.0] = **0.96**

We now obtain λ_b by simply dividing (25) by (18):

(26) $\lambda_b = 0.96 / 1.6 = 0.6$

Using λ_b in (24) gives us a value of $\hat{\mu}(t+1)$ of 1.2. Generation t would thus be better off with Social Security in this case, even though in the reduced-benefit state of nature their benefit would only be 0.6 times that of generation (t-1)'s current benefit. It is clear from (24) that the "break-even" value of λ is approximately 0.41. Assigning a value to λ is a matter for further discussion and, perhaps, future survey questions.

Let us turn now to our secondary empirical goal: to explore the empirical determinants of optimism in the target population, which will in turn lead to an analysis of point estimates of optimism from certain subsamples of interest. My main hypothesis going into this research was that optimism would decrease as people got younger, ceteris paribus. A secondary hypothesis was that optimism would decrease with greater education. Both of these hypotheses have been strongly confirmed, as we will see.

I felt comfortable using ordinary least squares since optimism could take 101 different values, and thus can be treated as a continuous variable. The regression equation for a given respondent i is:

$$
\begin{aligned}
(27)\quad optimism_i = \beta_o + \beta_1 age_i + \beta_2 highsch_i + \beta_3 college_i + \beta_4 postcoll_i \\
+ \beta_5 black_i + \beta_6 hispanic_i + \beta_7 hhinc000_i + \beta_8 receivess_i \\
\beta_9 receivepen_i + \beta_{10} female_i + \beta_{11} married_i + \varepsilon_i \ ,
\end{aligned}
$$

where we make all of the usual OLS assumptions, and where:

age	≡ 1996 – the respondent's year of birth
highsch	≡ dummy = 1 if the respondent graduated from high school
college	≡ dummy = 1 if the respondent graduated from college
postcoll	≡ dummy = 1 if the respondent did graduate work
black	≡ dummy = 1 if the respondent was black
hispanic	≡ dummy = 1 if the respondent was hispanic
hhinc000	≡ the respondent's total household income from all sources, in thousands
receivess	≡ dummy = 1 if the respondent or his/her spouse was receiving income from Social Security at the time of the interview
receivepen	≡ dummy = 1 if the respondent or his/her spouse was receiving income from a retirement pension at the time of the interview
female	≡ dummy = 1 if the respondent was female
married	≡ dummy = 1 if the respondent was married or otherwise partnered

After cleaning up the data (deleting don't knows, etc.), I dropped 60 observations for which household income was less than $1,000 in calendar 1995, which left me, as stated previously, with a sample size of 7,518 observations. The regression results are summarized in Table 1.

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$$\begin{aligned}(27)\quad \textbf{optimism}_i = {} & \beta_0 + \beta_1\textbf{age}_i + \beta_2\textbf{highsch}_i + \beta_3\textbf{college}_i + \beta_4\textbf{postcoll}_i \\ & + \beta_5\textbf{black}_i + \beta_6\textbf{hispanic}_i + \beta_7\textbf{hhinc000}_i + \beta_8\textbf{receivess}_i \\ & + \beta_9\textbf{receivepen}_i + \beta_{10}\textbf{female}_i + \beta_{11}\textbf{married}_i + \varepsilon_i\end{aligned}$$

where we make all of the usual OLS assumptions, and where:

age	≡ 1996 – the respondent's year of birth
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After cleaning up the data (deleting don't knows, etc.), I dropped 60 observations for which household income was less than $1,000 in calendar 1995, which left me, as stated previously, with a sample size of 7,518 observations. The regression results are summarized in Table 1.

TABLE 1
REGRESSION RESULTS

Variable	Coefficient	Standard Error	P Value
age	0.69	0.1477	0.000
highsch	-3.04	1.1832	0.010
college	-4.43	1.2429	0.000
postcoll	-8.84	1.5171	0.000
black	5.75	1.3746	0.000
hispanic	5.15	1.7339	0.003
hhinc000	-0.01	0.0051	0.006
receivess	2.83	1.0614	0.008
receivepen	-1.42	0.9396	0.131
female	1.16	0.8370	0.167
married	-0.52	1.0222	0.609

The coefficients on age and education are highly significant and have the expected signs. All else equal, another year of age increases optimism by 0.69 percentage point, and being highly educated reduces optimism by a full 8.84 percentage points when compared to having less than a high school education.

Some interesting results emerge from some of the control variables. The race and ethnicity variables are highly significant: being either black or hispanic increases optimism by over 5 percentage points.[49] Currently receiving income from the Social Security Administration is a significant cause of increased optimism, but currently receiving income from a retirement pension is not a significant explanatory variable.

The coefficient on household income is essentially zero, but it is an important control variable. It is worth noting that in the HRS, total household income includes

[49] Only 15 people in the sample were both black and hispanic.

income from all sources, including assets. Whether or not the respondent was female or married did not have significant explanatory power.

Observing the significant difference in optimism caused by changes in education level, it is tempting to conduct the following experiment: Let us now suppose that the most highly educated segment of the target population (postcoll = 1) is the best informed about current events and is the best able to understand probability theory, and hence is the segment best qualified to assign a value to θ. There were 810 post college-educated people in the subsample, and the point estimate of optimism from this group is 29.8%.[50] Substituting this point estimate into (21) gives us:

(28) $\hat{\mu}(t+1)_{he} = [\lambda + 0.298(1-\lambda)]1.6 = 0.48 + 1.12\lambda$,

where the "he" subscript indicates that this is the point estimate from the highly-educated subsample.

Using $\lambda_b = 0.6$ in (28) gives us $\hat{\mu}(t+1)_{he} = 1.15$, which means that generation t is still better off with social security, even if we accept as truth the greater pessimism of the highly educated. Table 2 contains the results of several experiments like this one. It contains the point estimates of optimism from the whole sample as well as various subsamples of interest, and the levels of $\hat{\mu}(t+1)$ that result from various values of λ, including the benchmark value, λ_b.

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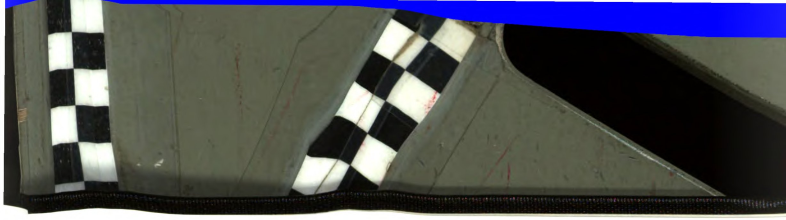
[50] The 95% confidence interval for this estimate is (27.7%, 31.9%).

TABLE 2
SENSITIVITY ANALYSIS

	Some Grad School	Age 55	Whole Sample	Age 65	Less than High School
Number of Observations	810	633	7,518	451	1,755
Point Estimate of Optimism	29.8%	32.0%	36.9%	41.9%	42.1%
95% Confidence Interval	± 2.1%	± 2.3%	± 0.8%	± 3.0%	± 1.9%
$\hat{\mu}$ (t+1) with λ = **0.0**	**0.48**	**0.51**	**0.59**	**0.67**	**0.67**
$\hat{\mu}$ (t+1) with λ = 0.3	0.82	0.84	0.90	0.95	0.95
$\hat{\mu}$ (t+1) with $\lambda = \lambda_b$ = **0.6**	**1.15**	**1.16**	**1.20**	**1.23**	**1.23**
$\hat{\mu}$ (t+1) with λ = 0.9	1.49	1.50	1.50	1.51	1.51

The columns of Table 2 identify the particular segment of the sample to be examined. Moving from left to right those segments are: 1) the most highly educated (postcoll = 1), 2) the youngest members of the target population (age = 55), 3) the sample as a whole, 4) the oldest members of the target population (age = 65), and 5) the least educated (highsch = 0, college = 0, and postcoll = 0). Note that optimism increases moving from left to right. At low values of λ (0.0 and 0.3), generation t is better off without social security, even at relatively high levels of optimism. At higher values of λ (including the benchmark value, λ_b), generation t is better off with social security, even at relatively low



levels of optimism. Of course, at very high values of λ, $\hat{\mu}$ (t+1) is quite robust with respect to changes in the point estimate of optimism.[51]

[51] Just differentiate (21) with respect to θ, from which we obtain: $\frac{\partial \mu(t+1)}{\partial \theta} = (1-\lambda)1.6$. As λ approaches 1, $\frac{\partial \mu(t+1)}{\partial \theta}$ approaches zero.

IV. CONCLUSION

We are reaching a point in the history of the Social Security program in the United States, at which we must call into question whether the program as it exists now continues to be welfare-enhancing for the current generation of workers. One can imagine a time in the past, when the probability of Social Security benefits being less generous in the future would have been low enough that knowing the value of λ would not have been very important in a welfare analysis of the program. But as we saw in the previous section, knowing ***how much*** the real Social Security benefit would be reduced in the reduced-benefit state of nature is now critically important in assessing the impact of the Social Security program on the welfare of today's workers.

HRS 3 opened an important door when it structured its Social Security pessimism question to allow for the possibility that λ could be something other than zero in the reduced-benefit state of nature. The mystery is what respondents to the survey thought "reduced" meant. To be able to determine whether the S economy continues to improve welfare vis-à-vis the NS economy, we must develop methods to determine ***both*** λ and θ. Consider the following possible survey question, for example:

> Let's suppose there are two possibilities for Social Security generosity in the future: either Social Security will become 40% less generous than it is now, or it will remain equally as generous as it is now. What do you think are the chances that it will become 40% less generous?

This question would give researchers a reference point to use in nailing down the meaning of "less generous." The number I chose corresponds to $\lambda_b = 0.6$, but other choices are certainly possible, as is asking two (or more) such questions, reflecting different values of λ.

IV. CONCLUSION

We are reaching a point in the history of the Social Security program in the United States, at which we must call into question whether the program as it exists now continues to be welfare-enhancing to the current generation of workers. One can imagine a time in the past when the probability of Social Security benefits being less generous in the future would have been low enough that knowing the value of λ would not have been very important in a welfare analysis of the program. But as we saw in the previous section, knowing *how much* the real Social Security benefit would be reduced in the reduced-benefit state of nature is critically important in assessing the impact of the Social Security program on the welfare of today's workers.

HRS 3 opened an important door when it structured its Social Security pessimism question to allow for the possibility that λ could be something other than zero in the reduced-benefit state of nature. The mystery is what respondents to the survey thought "reduced" meant. To be able to determine whether the S economy continues to improve welfare vis-á-vis the NS economy, we must develop methods to determine *both* λ and θ. Consider the following possible survey question, for example:

> Let's suppose there are two possibilities for Social Security generosity in the future: either Social Security will become 40% less generous than it is now, or it will remain equally as generous as it is now. What do you think are the chances that it will become 40% less generous?

This question would give researchers a reference point to use in nailing down the meaning of "less generous." The number I chose corresponds to $\lambda_S = 0.6$, but other choices are certainly possible, as is asking two (or more) such questions, reflecting different values of λ.

As we have seen, when we model Social Security as an investment plan, setting the generosity multiplier ($\mu(t+1)$) equal to 1 delivers a Pareto neutral result: workers are equally as well off in the S economy as in the NS economy. And the generosity multiplier is *guaranteed* to be 1 with a system of mandatory private retirement accounts that earn the capital market interest rate. Should future research find that the generosity multiplier is reliably less than one, then our welfare criterion suggests that pay-as-you-go Social Security is no longer welfare-enhancing and we should thus consider the alternative: the guaranteed Pareto neutrality that would come from privatizing Social Security in the United States, vs. the Pareto inferiority of continuing the existing pay-as-you-go system.

Chapter 3

THE WELFARE EFFECT OF SOCIAL SECURITY WHEN TAX AVOIDANCE IS CONSIDERED

I. INTRODUCTION

In a recent edition of *Your Social Security Statement*, a document that is mailed to all American workers aged 25 and older,[52] Larry G. Massanari, Acting Commissioner of Social Security, wrote that, "[By 2038] payroll taxes collected will be enough to pay only about 73% of [Social Security] benefits owed."[53] A key reason for the impending Social Security crisis is, of course, the well-documented decline in the United States' population growth rate. In a pay-as-you-go social security[54] system such as we have in the United States, all else equal, the greater the population growth rate, the greater the chance that a given generation of young people will have sufficient benefits when they retire. Of course, the converse of this (i.e., the declining population growth rate) is a significant contributing factor to the current pessimism on the part of the young about the future of their benefits: "From a post-World War II high of 3.7 children born per woman in 1957, the national fertility rate had fallen to 1.8 in 1976."[55] Those 1976 births are now 24 to 25

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[52] Social Security Administration, "The Future of Social Security," Publication No. 05-10055, August 2000, downloaded from the Social Security Administration website (http://www.ssa.gov/pubs/10055.html) on June 12, 2001 – no page number is available.

[53] Mr. Massanari's statement appears in "Your Social Security Statement – Prepared especially for Edward P. Van Wesep, Jr.," published by the Social Security Administration on April 25, 2001.

[54] I will capitalize Social Security when referring to the current U.S. system. When referring to social security in a generic sense, I will use lower-case letters.

[55] Edward Cowan, "Background and History: The Crisis in Public Finance and Social Security," The Crisis in Social Security: Problems and Prospects, ed. Michael J. Boskin (San Francisco: Institute for Contemporary Studies, 1978) 7.

years old and starting to enter the workforce, just as the "baby boom" generation is starting to retire. Both cohorts face a grim prospect: "...because of the declining birthrate, the number of social security beneficiaries for every hundred workers paying social security taxes [is forecast to] climb from 31 in 1976 to more than 50 by the middle of the twenty-first century."[56] Because of this decline in the population growth rate, baby boomers have good reason to worry about whether they will get the full amount of promised benefits when they retire, and newer entrants into the workforce have good reason to worry about how burdensome their payroll taxes might become.

Because the raising of Social Security tax revenue has become such a critical issue, it is important to delve into the little-discussed issue of social security tax compliance. While it may seem odd to talk about compliance in a payroll tax setting (since employers routinely extract Social Security taxes and there are no deductions or exemptions which would create opportunities for evasion), I define an employee's compensation as wages *plus fringe benefits*. Under this definition of compensation, workers' social security tax compliance decision consists not only of deciding to make their payroll tax payments, but also what fraction of their total compensation to expose to social security taxation. In this setting, compliance becomes a very important issue indeed. What, then, is the relevance of the existing tax compliance and social security literatures to the issue of social security tax compliance?

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[56] Cowan, p. 7. He cites as his source, "A. Hasworth Robertson, 'The Cost of Social Security: 1976 – 2050,' speech (mimeo), Social Security Administration, Baltimore (n.d.), p. 5." Cowan's forecast has been confirmed by more recent actuarial research, which has the worker-to-retiree ratio falling below 2.0 in 2061 (Source: Social Security Advisory Board, "Agenda for Social Security: Challenges for the New Congress and the New Administration" February 2001, p. 3; downloaded from the Social Security Advisory Board's website (http://www.ssab.gov/reports.html)).

The Relevance of the Tax Compliance Literature

In the United States, the complexity of the income tax creates numerous opportunities for evasion and avoidance, and it is the income tax that receives the primary focus in the tax evasion literature. [57] But the Social Security tax is a payroll tax, and thus quite different from the income tax. In fact, a number of economists have proposed abandoning the income tax in favor of "a tax on labor income [i.e., a payroll tax] *on grounds of simplicity and administrative feasibility*"[58] (my emphasis). While payroll workers in the United States can evade part of their income tax liability in a myriad of ways (e.g., deductions for charitable contributions they never made, etc.), they cannot feasibly evade any part of the Social Security tax liability arising from their payroll wages.

The Social Security payroll tax rate and taxable ceiling have been rising over the years: In 2001, the combined employee and employer Social Security tax rate is 12.4% of wages up to a ceiling of $80,400.00 per year. Combined with the rising Social Security tax rate and ceiling has been an erosion of confidence in the Social Security system:

> ...public confidence in the future of Social Security has declined so much in recent years that workers are expressing a desire to withhold and invest their Social Security contributions elsewhere. Many younger workers believe that the risk of financial loss in such an investment is less than the risk of not receiving future Social Security benefits.[59]

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[57] "Avoidance" will denote the process of lowering one's tax liability through legal means, while "evasion" will denote the process of lowering one's tax liability through illegal means where the probability of detection is strictly less than one.

[58] Joseph A. Pechman, "The Future of the Income Tax," The American Economic Review 80 (1990): 9.

[59] G. Lawrence Atkins, "Closing Back Doors out of Social Security," Checks and Balances in Social Security, eds. Yung-Ping Chen and George F. Rohrlich (Lanham, MD: University Press of America, 1986) 240.

If evasion is not a reasonable option, how will younger workers manifest this desire "to withhold and invest their Social Security contributions elsewhere"? If it is possible for them to do so, how will they decide *how much* of their social security tax liability to withhold?

The existing tax compliance models are not well suited to the task of answering the questions posed above. Take, for example, the celebrated tax compliance model of Michael G. Allingham and Agnar Sandmo (1972). In their model, people choose how much of their income tax liability to pay by maximizing a static expected utility function in which the two possible states of nature are that they are either audited or not audited. If they are audited and discovered to have paid less than 100% of their tax liability, they are penalized (the penalty is restricted to taking the form of a fine).

The Allingham-Sandmo model is not applicable to the social security setting for two principal reasons. First, evasion of a payroll tax in a country where firms are routinely audited is not feasible. It would never occur to most employees or their employers -- once the employees are on the payroll -- to try to dodge paying a portion of their Social Security tax.[60] Once an employee is on the payroll of a firm, it is virtually impossible for him or her to choose to evade any portion of his or her Social Security tax, since the employer pays the tax on the employee's behalf. Employers that underreport Social Security taxes face a severe penalty (100% of the unreported amount),[61] and so we may safely assume that illegal underreporting of the Social Security tax liabilities of

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60 It is particularly important to distinguish here between evasion and avoidance. The social security tax base has been *legally* eroding over the years, a point that I will explore more fully later.

61 Internal Revenue Service Code Section 6672, as reported in "The Texas A&M University System Tax Manual," downloaded from the Texas A&M website (http://sago.tamu.edu/soba/TaxManual/FedTaxPen.html) on April 7, 1999.

payroll employees is rare.[62] Thus, the uncertainty that drives people's behavior in the Allingham-Sandmo model -- whether they will be audited -- is essentially irrelevant in a payroll setting. Meanwhile, the Allingham-Sandmo model is silent with respect to the crucial uncertainty that drives the decision of many of today's young people to try to reduce their Social Security taxes: whether there will be sufficient benefits for them when they retire.[63]

The preceding discussion leads us to the second reason that the Allingham-Sandmo model is not applicable to social security tax compliance: the social security tax problem is inherently a dynamic one, while the Allingham-Sandmo model is static. The United States' Social Security system is essentially a "pay-as-you-go" system: benefits paid to the current group of retirees are primarily financed by taxes on the current group of workers. (There is a trust fund, but it is small compared to the present value of pension obligations: "The so-called 'trust fund' could more accurately be termed a 'petty cash' fund." [64]) Any pay-as-you-go social security system thus contains an intrinsic system of intergenerational externalities: part of the utility of retirees is dependent upon the behavior of young workers, e.g., their decision regarding how much of their social security tax liability to pay. A static model would fail to capture this important system of externalities: during any given period of time, the young suffer disutility to support the

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62 It is of course tempting for firms to treat employees as independent contractors to avoid paying social security taxes. In this section of the paper, I am specifically discussing firms' and employees' behavior *given that the decision has been made to add the employee to a firm's payroll.*

63 A 1999 survey conducted by the Social Security Administration found that, "...more than half of non-retirees who were polled were only a little confident or not confident at all that Social Security retirement benefits will be there for them when they retire." Social Security Advisory Board, "Agenda for Social Security: Challenges for the New Congress and the New Administration," February 2001, p. 22, downloaded from the Social Security Advisory Board's website (http://www.ssab.gov/reports.html)

64 Milton Friedman, "Payroll Taxes, No; General Revenues, Yes," The Crisis in Social Security: Problems and Prospects, ed. Michael J. Boskin (San Francisco: Institute for Contemporary Studies, 1978) 26.

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old. With only one period modeled, there can be no justification for such a system, at least on Pareto grounds. We must allow the current young to become old and (possibly) receive in turn their positive externality from the next generation of young.

In their comprehensive review of the tax compliance literature, James Andreoni, Brian Erard and Jonathan Feinstein ["AEF"] (1998) discuss a number of models, including the Allingham-Sandmo model discussed above. In the Allingham-Sandmo model, the probability of audit is constant. AEF discuss more complicated models in which the probability of audit is variable, and which include both principal-agent and game-theoretic type models, but again the focus is on evasion and the uncertainty of audit, which is not helpful in the case of social security. AEF mention a number of other models that deal with various issues, such as the effects of income and tax rates on compliance, etc., but "compliance" throughout the article is virtually always taken to mean "non-evasion." The common thread running through these models is that there is an opportunity for people to evade a portion of their tax liability with a probability of detection by the tax authority that is less than one.

So, the existing tax compliance literature deals extensively with tax evasion, but when discussing social security taxes, we must reorient our thinking away from tax evasion toward tax avoidance. And there has been avoidance:

> The proportion of the total compensation paid to workers that is taxable for Social Security purposes has shrunk considerably over the last 30 years, due largely to the growth in nonwage forms of compensation. In 1950, fringe benefits accounted for 5 percent of total compensation, and Social Security taxes were levied on 95 percent of compensation. By 1980, fringe benefits had grown to 16 percent of compensation, leaving only 84 percent taxable for Social Security.[65]

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[65] Atkins, p. 239.

Firms and their employees have thus found perfectly legal ways to reduce the fraction of employees' compensation (including fringe benefits) paid in Social Security taxes. This has taken place in full view of the tax authority -- that is to say, when analyzing social security taxation, the probability of audit is not an important parameter, and may safely be assumed to be constant and equal to 1.

The Social Security tax base continues to erode due to avoidance: using Social Security actuaries' assumptions, Atkins (1986) forecasted that by the year 2056, "only 62 percent of total compensation will remain taxable for Social Security purposes."[66] But while the Social Security tax base is eroding, it obviously has not disappeared entirely. There must be a penalty structure that prevents people from avoiding the Social Security tax entirely, but which has been increasingly relaxed through the past few decades to permit the increased avoidance of Social Security taxes. I will propose one such penalty structure in the next section of this chapter.

Summarizing thus far: any pay-as-you-go social security model attempting to explain a worker's choice regarding how much of his or her wages to expose to the social security tax will include the following necessary components:

1. an enforcement mechanism.
2. a dynamic representation of the intergenerational externalities inherently present in a pay-as-you-go social security system.

The existing tax-compliance literature, with its focus on tax evasion and typically static setting, is not the place to go for insight into what promises to be an ever more important issue over the next few years: the future of Social Security. Let us now examine the

[66] Atkins, p. 239.

social security literature to see if its models contain any or all of the necessary components outlined above.

The Relevance of the Social Security Literature

Early dynamic models of social security were developed by Robert Barro (1974) and Paul Samuelson (1975b). Both of these models employ a two-generation overlapping generations framework. Barro assumes that each generation is of the same size, so there is no population growth advantage to pay-as-you-go social security vs. private investment. But people care about the utility of the next generation of young and so view private bequests and pay-as-you-go social security taxes as substitutes (people are identical, and thus are indifferent as to which particular household gets their bequest). But in the real world, people are heterogeneous and care very much about who gets their private bequest. People in the real world surely do not view the payment of social security taxes (which flow to unknown recipients) as being a perfect substitute for saving with the goal of making a private bequest to their own children. Barro's model thus will not help us to understand social security tax compliance in the real world.

Samuelson's model, which I will refer to later as the "traditional model," also has identical people, but there is no intergenerational altruism in his model. Unlike Barro, Samuelson does include a labor force growth rate parameter, which is the key to the possibility of social security being a Pareto-improving program. In a model like Samuelson's, the rate of growth of the labor force performs the same role as the rate of return on private investment: if the former is greater than the latter, society's welfare is improved by implementing a social security program. But in Samuelson's model, while

the representative worker's level of private saving is endogenous, that same worker's social security tax payment is an exogenous parameter. Samuelson thus ignores the free-rider problem inherent in a system of intergenerational transfers (in which there is no altruism): why should a given young worker – whose expected social security benefit is exogenous[67] – pay any social security tax at all? In the enforcement model I develop in the next section, Samuelson's exogenous social security tax payment can be derived as a special case in which the penalty for noncompliance is infinite.

Alan Auerbach and Laurence Kotlikoff (1987) have developed what they call a "dynamic general equilibrium numerical simulation model" to study various fiscal policy problems, including social security reform. The model consists of 55 overlapping generations of households, plus firms and the government. They include a population growth rate parameter, and specifically model the collection of social security taxes and paying out of social security benefits in a pay-as-you-go system. But, again, there is no mention of the enforcement mechanism that lies behind people's decisions about social security tax compliance.

Eytan Sheshinski and Yoram Weiss (1981) developed a two-generation overlapping generations model of social security, private savings and private annuities. They recognize the intergenerational externality inherent in pay-as-you-go social security, and also hint at the need for enforcement: "Since society is committed to provide a minimum standard of living to the aged, a *compulsory* payment system is required to avoid the 'free-rider' problem"[68] (my emphasis). But even though Sheshinski and Weiss recognize

[67] In the Samuelson model, the exogenous benefit is (1 + exogenous population growth rate) x (exogenous per capita social security tax payment).

[68] Eytan Sheshinski and Yoram Weiss, "Uncertainty and Optimal Social Security Systems," The Quarterly Journal of Economics 96.2 (1981) 190.

that social security payments must be compulsory, they also do not include an enforcement mechanism in their model.

So, even though the social security literature consistently uses dynamic overlapping-generations models, which capture the important intergenerational externality inherent in pay-as-you-go social security, to date the literature has not emphasized the other necessary component mentioned above: an enforcement mechanism.

In the next section I develop a dynamic social security model with enforcement, and develop a set of welfare conditions by which an economy with social security can be judged against an economy with no social security system.

II. WELFARE ANALYSIS

For a representative worker whose economic life lasts two periods, beginning in period t, let:[69]

$Y_f(t)$ ≡ Nontaxable wages in the form of fringe benefits.

$Y_m(t)$ ≡ Taxable wages that can be used for any kind of consumption (i.e., "cash").

$Y(t)$ ≡ Total wages = $Y_f(t) + Y_m(t)$. I assume that labor is supplied inelastically and that the wage rate is fixed; Y(t) is thus exogenous.

$q(t)$ ≡ The social security tax rate; $0 < q(t) < 1$. The amount of social security tax the representative worker pays in period t is $q(t)Y_m(t)$.

$C_f(t)$ ≡ Consumption of fringe benefits. (For convenience, I assume fringes are all consumed when the worker is young.)

$C_m(t)$ ≡ The amount of cash income consumed when young. Payment of taxes other than the social security tax is included in $C_m(t)$.

$S(t)$ ≡ The amount of cash wages saved when young. Savings earn the real interest rate. Assume that S(t) can be less than zero; i.e., borrowing at the real interest rate is permitted.

$C(t+1)$ ≡ Consumption when old.

$r(t+1)$ ≡ The real interest rate from period t to period (t+1), which is known with certainty in period t. The real interest rate is set outside the model, in the private capital market.

$B(t+1)$ ≡ The worker's expected social security benefit (paid in cash). Assume that the worker's forecast is accurate, so that B(t+1) is also the actual benefit received in period (t+1).

The worker's choice variables are S(t) and $Y_f(t)$ (from which $Y_m(t)$ is also determined).

The general benefit structure in chapters 2 and 3 is as follows:

$$(1) \quad B(t+1) = \gamma(t+1) + \mu(t+1)[1 + r(t+1)]q(t)Y_m(t) \ ,$$

where $\gamma(t+1) \geq 0$ and $\mu(t+1) \geq 0$.

In (1), $\gamma(t+1)$ and $\mu(t+1)$ are both government-controlled parameters. Because $\gamma(t+1)$ is not a function of the social security taxes paid by generation t, it is the parameter that would represent a true pay-as-you-go social security program. In a true pay-as-you-go program, the benefit paid to generation t is a function of the social security taxes that generation (t+1) chooses to pay, a decision that enters generation t's utility function as an exogenous parameter. More generally, $\gamma(t+1)$ can be any exogenous social security benefit; e.g., it could be partly funded out of general tax revenues collected during period (t+1). A strict pay-as-you-go system is thus a special case of this more general exogenous benefit structure, where we specify that the system is strictly pay-as-you-go via the government's budget constraint. The general exogeneity of $\gamma(t+1)$ captures the role of social security as a social insurance program: the level of one's benefit is not a function of how much he or she has paid into the system.

Taking r(t+1) as given, $\mu(t+1)$, the controls the rate of return workers receive on social security taxes paid when those are considered as an investment, and is thus called the "generosity multiplier" in chapter 2. When social security is modeled purely as an investment plan, workers internalize all of the utility impacts of their compliance decision. Since this chapter is concerned with the need for enforcement due to the system of intergenerational *externalities* inherent in a true pay-as-you-go social security system, I will assume $\mu(t+1) = 0$ going forward.

The worker's problem is to:

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[69] Time arguments always indicate the period, rather than the generation. For notational convenience, I am suppressing generation subscripts -- I will specify which generation is involved, if that is not clear from the context. The worker here represents "generation t."

$$\max_{Y_f(t), S(t)} U(t) = \ln(C_m(t)) + \phi \ln(C_f(t)) + \delta \ln(C(t+1)) \ ,$$

subject to:

(2) $$C_m(t) = [1 - q(t)][Y(t) - Y_f(t)] - S(t)$$

(3) $$C_f(t) = Y_f(t)$$

(4) $$C(t+1) = [1 + r(t+1)]S(t) + B(t+1) = [1 + r(t+1)]S(t) + \gamma(t+1)$$

In the utility function, δ is the factor workers use to discount their second-period felicity ($0 < \delta \leq 1$). The penalty for noncompliance with social security taxes arises out of the separability of the felicity from total first period consumption ($C_m(t) + C_f(t)$). This penalty is parameterized by the taste for fringe benefits, ϕ. From the point-of-view of the government, ϕ is an exogenous parameter that must be taken into account when formulating policy. The government's policy tools are thus: 1) setting the level of q(t) and 2) setting the level of γ(t+1).

The first order conditions of the maximization problem are:

(5) $$\frac{1}{[1 - q(t)][Y(t) - Y_f(t)] - S(t)} = \frac{\delta[1 + r(t+1)]}{[1 + r(t+1)]S(t) + \gamma(t+1)}$$

(6) $$\frac{1}{[1 - q(t)][Y(t) - Y_f(t)] - S(t)} = \frac{\phi}{[1 - q(t)]Y_f(t)}$$

Equation (5) is the familiar saving first order condition from an overlapping generations life-cycle model.[70] Equation (6) is the fringe benefit first order condition, rearranged a bit to emphasize that the social security tax rate has a role to play in enforcement,

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[70] See, for example, Samuelson (1958, 1975[a&b]) and Diamond (1965).

separate from its role in raising revenue for the social security program. On the left-hand side of (6), we see q(t) playing its revenue-raising role, the only role it would play if taxable income were exogenous. On the right-hand side of (6), enforcement is parameterized by both ϕ and q(t). For any given level of q(t) > 0, as ϕ approaches 0, the penalty for noncompliance approaches infinity: if payroll workers have no taste for fringe benefits, they have no choice but to expose all of their wages to the social security tax. For any given level of ϕ, as q(t) is lowered the *effective* penalty is increased, since lowering q(t) raises the price of fringe benefit consumption relative to the price of cash consumption.[71] These effects are analogous to the income and substitution effects of a wage increase in a labor-leisure choice model, since lowering the social security tax rate raises the worker's take-home wage. In this model, there is no change in the consumption of leisure due to the increase in take-home pay (since labor is supplied inelastically), but consumption of fringes steps into the leisure role: the increase in take-home pay increases the price of fringes in this model, just as it increases the price of leisure.[72]

Solving the system (2) through (6) gives the following consumption, saving and wage functions:

$$(7) \quad C_m(t)^* = \left(\frac{1}{1+\delta+\phi}\right)\left\{[1-q(t)]Y(t) + \left[\frac{\gamma(t+1)}{1+r(t+1)}\right]\right\}$$

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[71] To see this, rewrite the budget constraint, (2) – (4), in the somewhat more abstract form:

$$C_m(t) + [1-q(t)]C_f(t) + \left[\frac{1}{[1+r(t+1)]}\right]C(t+1) = [1-q(t)]Y(t) + \left[\frac{1}{1+r(t+1)}\right]\gamma(t+1).$$

As q(t) falls, the price of fringe benefit consumption rises.

[72] An obvious step, which I defer to a later date, would be to generalize the model to allow for any type of tax-free consumption, including leisure, to play the tax-shelter role.

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$$(7) \quad C_m(t)^* = \left(\frac{1}{1+\delta+\phi}\right)\left\{[1-q(t)]w(t) + \left[\frac{y(t+1)}{1+r(t+1)}\right]\right\} \text{ [illegible]}$$

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[72] An obvious step, which I defer to a later date, would be to generalize the model to allow for any type of tax-free consumption, including leisure, to play the tax-shelter role.

$$(8)\quad C_f(t)^* = Y_f(t)^* = \left(\frac{1}{1-q(t)}\right)\left(\frac{\phi}{1+\delta+\phi}\right)\left\{[1-q(t)]Y(t) + \left[\frac{\gamma(t+1)}{1+r(t+1)}\right]\right\}$$

$$(9)\quad C(t+1)^* = [1+r(t+1)]\left(\frac{\delta}{1+\delta+\phi}\right)\left\{[1-q(t)]Y(t) + \left[\frac{\gamma(t+1)}{1+r(t+1)}\right]\right\}$$

$$(10)\quad S(t)^* = \left(\frac{1}{1+\delta+\phi}\right)\left\{\delta[1-q(t)]Y(t) - [1+\phi]\left[\frac{\gamma(t+1)}{1+r(t+1)}\right]\right\}$$

$$(11)\quad Y_m(t)^* = \left(\frac{1}{1-q(t)}\right)\left(\frac{1}{1+\delta+\phi}\right)\left\{[1+\delta][1-q(t)]Y(t) - \phi\left[\frac{\gamma(t+1)}{1+r(t+1)}\right]\right\}$$

Note in (11), that in the special case of $\phi = 0$ (i.e., an infinite penalty), cash wages would simply equal total wages. We can thus derive Samuelson's exogenous payment of the entire social security tax bill (mentioned in the Introduction) as a special case. But in the general case ($\phi > 0$), optimal cash wages are reduced in several ways. First, even in the absence of a social security program, the taste for fringes causes a reduction in cash wages vis-à-vis the case in which $\phi = 0$. Second, holding ϕ and q(t) fixed, an increase in the expected social security benefit would reduce cash wages, since the worker could afford to divert more of her wages to fringes, using the benefit to replace her lost cash wages.[73] Finally, holding ϕ and γ(t+1) fixed, an increase in the social security tax rate would reduce cash wages, since the worker would want to take advantage of the

[73] We have as a necessary condition to avoid the corner solution, $Y_m(t)^* = 0$ (and hence, the breakdown of the social security system), that $[1+\delta][1-q(t)]Y(t) > \phi\left[\frac{\gamma(t+1)}{1+r(t+1)}\right]$. The social security benefit must not become so generous that workers avoid paying the social security tax altogether, diverting all of their wages to fringe benefits. I will assume throughout the remainder of the paper that this necessary condition holds.

increasingly attractive tax shield provided by fringes (i.e., the effective penalty is reduced).

Substituting (7) through (9) into the utility function gives:

$$(12)\quad V(t)_S = \ln\left[\left(\frac{1}{1+\delta+\phi}\right)\left\{[1-q(t)]Y(t)+\left[\frac{\gamma(t+1)}{1+r(t+1)}\right]\right\}\right]$$

$$+\ \phi\ln\left[\left(\frac{1}{1-q(t)}\right)\left(\frac{\phi}{1+\delta+\phi}\right)\left\{[1-q(t)]Y(t)+\left[\frac{\gamma(t+1)}{1+r(t+1)}\right]\right\}\right]$$

$$+\ \delta\ln\left[[1+r(t+1)]\left(\frac{\delta}{1+\delta+\phi}\right)\left\{[1-q(t)]Y(t)+\left[\frac{\gamma(t+1)}{1+r(t+1)}\right]\right\}\right],$$

where $V(t)_S$ indicates indirect utility in the social security ("S") economy. In an economy with no social security ("NS"), in which q(t) and γ(t+1) are both set to zero, we would have the following indirect utility:

$$(13)\quad V(t)_{NS} =$$

$$\ln\left[\left(\frac{1}{1+\delta+\phi}\right)Y(t)\right]+\phi\ln\left[\left(\frac{\phi}{1+\delta+\phi}\right)Y(t)\right]+\delta\ln\left[[1+r(t+1)]\left(\frac{\delta}{1+\delta+\phi}\right)Y(t)\right]$$

Generation t will be better off in the S economy vis-à-vis the NS economy if $V(t)_S > V(t)_{NS}$. We can more readily compare $V(t)_S$ to $V(t)_{NS}$ by rearranging the arguments in (12):

$$(14)\quad V(t)_S = \ln\left[\left(\frac{1}{1+\delta+\phi}\right)Y(t) + \left(\frac{1}{1+\delta+\phi}\right)\left\{\left[\frac{\gamma(t+1)}{1+r(t+1)}\right] - q(t)Y(t)\right\}\right]$$

$$+\ \phi\ln\left[\left(\frac{\phi}{1+\delta+\phi}\right)Y(t) + \left(\frac{1}{1-q(t)}\right)\left(\frac{\phi}{1+\delta+\phi}\right)\left\{\left[\frac{\gamma(t+1)}{1+r(t+1)}\right]\right\}\right]$$

$$+\delta\ln\left[[1+r(t+1)]\left(\frac{\delta}{1+\delta+\phi}\right)Y(t) + [1+r(t+1)]\left(\frac{\delta}{1+\delta+\phi}\right)\left\{\left[\frac{\gamma(t+1)}{1+r(t+1)}\right] - q(t)Y(t)\right\}\right]$$

In (14), the first term in each of the arguments of $V(t)_S$ is identical to the corresponding argument of $V(t)_{NS}$ in (13). A comparison of (13) and (14) reveals one clear way for the government to ensure that generation t is better off in the S economy vis-à-vis the NS economy: simply set generation t's benefit equal to the future value of its total tax bill; i.e.,

$$(15)\quad \gamma(t+1) = [1+r(t+1)]q(t)Y(t)$$

Substituting (15) into (14) gives us:

$$(16)\quad V(t)_S = \ln\left[\left(\frac{1}{1+\delta+\phi}\right)Y(t)\right]$$

$$+\phi\ln\left[\left(\frac{\phi}{1+\delta+\phi}\right)Y(t) + \left(\frac{1}{1-q(t)}\right)\left(\frac{\phi}{1+\delta+\phi}\right)\left\{\left[\frac{\gamma(t+1)}{1+r(t+1)}\right]\right\}\right]$$

$$+\delta\ln\left[[1+r(t+1)]\left(\frac{\delta}{1+\delta+\phi}\right)Y(t)\right] > V(t)_{NS}$$

Note the departure in (16) from the traditional Samuelsonian result. In the traditional model, a social security program with a benefit that returns only the capital market rate of

return to the total tax bill would deliver only a neutral result: generation t would be no better off in the S economy than it would be in the NS economy. But in this model, such a benefit improves the welfare of generation t. The reason for this result is that the benefit is pegged to the total tax bill, but the worker can, and does, avoid a portion of that total tax bill, which we can see by substituting from (15) into (11):

$$(17)\quad Y_m(t)^* = \left(\frac{1}{1-q(t)}\right)\left\{\left[\frac{1+\delta}{1+\delta+\phi}\right] - q(t)\right\}Y(t)$$

It can easily be shown that we will always get $Y_m(t)^* < Y(t)$ in (17), as long as $\phi > 0$.

Of course, if we assume a strict pay-as-you-go social security system, the question arises of how the government will pay the benefit in (15). Let us now assume a steady state in which workers are identical across generations, q, Y and r are constant across generations, and the population grows at the exogenous rate of n per period. In a strict pay-as-you-go system, the government would balance its social security budget each period, and we thus obtain the following per capita budget constraint for the government:

$$(18)\quad \gamma = (1+n)qY_m^*$$

Substituting the steady state versions of (15) and (17) into (18), and simplifying a bit, gives us:

$$(19)\quad 1+r = (1+n)\left(\frac{1}{1-q}\right)\left\{\left[\frac{1+\delta}{1+\delta+\phi}\right] - q\right\}$$

return to the total tax bill would [illegible] generation t would be no better off in the S economy than it would be in the NS economy. But in this model, such a benefit improves the welfare of [illegible] results that the benefit is pegged to the total tax bill that [illegible] avoid a portion of that total tax bill, which we can see [illegible] (15) and (17):

$$\text{(17)}\quad Y_m(t)^* = \left(\frac{1}{1-q(t)}\right)\left[\ldots\right] \text{[illegible]}$$

It can easily be shown that [illegible] as long as $\phi > 0$.

Of course, if we assume [illegible] social security system, the question arises of how the government will pay the [illegible]. Let us now assume a steady state in which workers are identical across generations, [illegible] are constant across generations, and the population grows at the exogenous rate of n per period. In a strict pay-as-you-go system, the government would balance its social security budget each period, and we thus obtain the following per capita budget constraint for the government:

$$\text{(18)}\quad \gamma = (1+n)qY_m^*$$

Substituting the steady state versions of (15) and (17) into (18), and simplifying a bit, gives us:

$$\text{(19)}\quad 1+r = (1+n)\left(\frac{1}{1-q}\right)\left\{\left[\frac{1+\delta}{1+\delta+\phi}\right] - q\right\}$$

Note that if n = r, (19) will not hold, and so (18) will not hold; i.e., the government will not be able to pay a benefit equal to the future value of the entire social security tax bill, since the actual social security tax revenue collected from a given generation will fall short of that generation's entire social security tax bill. We must have n sufficiently larger than r to compensate for social security tax avoidance by the workers of each generation. Assuming n to be sufficiently large for the equality in (19) to hold allows us to solve for the level of q that will allow the government to pay the promised benefit:

$$(20)\quad q = \frac{\left(\dfrac{1+\delta}{1+\delta+\phi}\right) - \left(\dfrac{1+r}{1+n}\right)}{1-\left(\dfrac{1+r}{1+n}\right)}$$

In (20), any level of r and n will give us q < 1, but to avoid the corner in which the government cannot afford the promised benefit (i.e., q ≤ 0), we must have:

$$(21)\quad 1+n > \left(1+\left[\frac{\phi}{1+\delta}\right]\right)[1+r]$$

Thus, while a social security benefit that pays the capital market rate of return to the total tax bill is welfare-improving in this model (while in the traditional model it is not), the necessary condition for the population growth rate is a more stringent one in this model. In the traditional model, ϕ would be zero and we would only need for n to be slightly greater than r for a strict pay-as-you-go social security system to be welfare-improving.

The benefit in (15) is, of course, a special case – chosen both for its tractability, and because it clearly demonstrates the differences between a social security model with enforcement and the traditional social security model. To develop a general welfare

criterion for the social security benefit, it will be useful to take a first-degree Taylor approximation of each of the felicities in (14) around the decentralized consumption functions, $\left(\frac{1}{1+\delta+\phi}\right)Y(t)$, $\left(\frac{\phi}{1+\delta+\phi}\right)Y(t)$, and $[1+r(t+1)]\left(\frac{\delta}{1+\delta+\phi}\right)Y(t)$, and then to substitute these approximations into (14) to obtain:

$$(22)\quad V(t)_S \cong V(t)_{NS} + \left(\frac{1}{Y(t)}\right)\left\{\left[\frac{\gamma(t+1)}{1+r(t+1)}\right] - q(t)Y(t)\right\}$$
$$+ \left(\frac{1}{1-q(t)}\right)\left(\frac{\phi}{Y(t)}\right)\left\{\left[\frac{\gamma(t+1)}{1+r(t+1)}\right]\right\}$$
$$+ [1+r(t+1)]\left(\frac{\delta}{Y(t)}\right)\left\{\left[\frac{\gamma(t+1)}{1+r(t+1)}\right] - q(t)Y(t)\right\}$$

Let us now generalize the benefit in (15) as follows:

$$(23)\quad \gamma(t+1) = \eta(t+1)[1+r(t+1)]q(t)Y(t)\ ,$$

where $\eta(t+1)$ is a multiplier that is controlled by the government.

In (23), $\eta(t+1)$ plays a role similar to that of the generosity multiplier in chapter 2, except that here the government scales the benefit against total taxes due, while in chapter 2, the government scales the benefit against taxes actually paid. Note that we can derive (15) as a special case by setting $\eta(t+1)$ equal to 1. Substituting from (23) into (22) gives us:

$$(24)\quad V(t)_S \cong V(t)_{NS} + \left\{[\eta(t+1)-1](1+\delta[1+r(t+1)]) + \eta(t+1)\left[\frac{\phi}{1-q(t)}\right]\right\}q(t)$$

From (24) we can develop:

The Welfare Criterion

Given all of our previous assumptions, the S economy will be Pareto inferior to the NS economy if, for any generation t, $V(t)_S < V(t)_{NS}$, which is equivalent to:

$$\eta(t+1) < \left(\frac{1+\delta[1+r(t+1)]}{1+\delta[1+r(t+1)]+\left[\frac{\phi}{1-q(t)}\right]} \right)$$

The S economy will be Pareto superior to the NS economy if, for all generations t, $V(t)_S > V(t)_{NS}$, which is equivalent to:

$$\eta(t+1) > \left(\frac{1+\delta[1+r(t+1)]}{1+\delta[1+r(t+1)]+\left[\frac{\phi}{1-q(t)}\right]} \right)$$

The welfare criterion tells us the necessary condition for any exogenous benefit to be Pareto-improving. Let us now specify once again that the social security system is strictly pay-as-you-go and assume a steady state; i.e., we assume that (18) holds. We can derive the steady state level of η that always satisfies (18) (the "affordable multiplier") by first substituting (23) into (11) (using the steady state versions of those equations) to obtain the generalized steady state cash wage function:

$$(25)\quad Y_m^* = \left(\frac{1}{1-q}\right)\left\{\left[\frac{1+\delta}{1+\delta+\phi}\right]-\left[\frac{1+\delta+\eta\phi}{1+\delta+\phi}\right]q\right\}Y$$

Note in (25) that, just as in our special case, there will be avoidance of the total tax bill as long as $\phi > 0$.[74] Now, substitute (25) and the steady state version of (23) into (18) to obtain the affordable multiplier:

$$(26)\quad \eta = \frac{1+n}{1+r+\left[\frac{\phi}{(1+\delta)(1-q)}\right]\left[(1+r)+q(n-r)\right]}$$

If ϕ were zero in (26), we would obtain the intuitive result that the government could afford to pay a benefit that exceeded the future value of the total tax bill ($\eta > 1$) if the population growth rate were to exceed the real interest rate. But with $\phi > 0$, we get distortions due to tax avoidance. Note in (26) that the social security tax rate appears in two different places, reflecting two different roles that it plays in shaping the affordable multiplier. The first of these roles is one we have already discussed: the enforcement role, which it plays in what I will call the "effective penalty factor," $\left[\frac{\phi}{(1+\delta)(1-q)}\right]$. As the effective penalty factor increases from zero, the penalty for noncompliance with the social security tax decreases from infinity. Thus, increasing q in this role lowers the penalty for noncompliance, which in turn reduces the affordable multiplier as a partial effect.

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74 We want to show that $Y_m^* < Y$, which is equivalent to showing that:
$\left[\frac{1+\delta}{1+\delta+\phi}\right]-\left[\frac{1+\delta+\eta\phi}{1+\delta+\phi}\right]q < 1-q$. Simplification of this expression gives us $[1-\eta]q < 1$, which will be true for any level of η as long as $q < 1$, which we have assumed.

There is a second, more subtle, role for q in (26) which arises due to a second appearance by the population growth rate in the denominator of η. An increase in the population growth rate would increase the representative worker's lifetime income in the strict pay-as-you-go social security system that we assumed to get to (26). With $\phi = 0$, this income effect of an increase in n would be the only effect. But with $\phi > 0$, the increase in tax-free cash income (in the form of the social security benefit) due to an increase in n (holding r fixed), causes the representative worker to shelter more of his wages in the form of an increase in fringe benefit consumption. Because q scales the impact on the social security benefit of any change in n, it appears in this substitution effect.

Since a declining population growth rate seems to be the principal cause of anxiety about the future of Social Security among the current generation of American workers (as discussed in the Introduction), the main welfare result of this paper will focus on n. What steady state level of n will be Pareto-improving? To get to this result, let us first write down the necessary condition for the steady state multiplier to be Pareto improving:[75]

$$(27)\quad \eta > \left(\frac{1+\delta(1+r)}{1+\delta(1+r)+\left[\dfrac{\phi}{1-q}\right]} \right)$$

Substituting from (26) into (27) and solving for n gives us the necessary condition for the population growth rate to be Pareto-improving in a strict pay-as-you-go social security system:

[75] This follows immediately from the welfare criterion.

$$(28)\quad n > r\left\{\frac{1+\delta(1+r)+\phi\left(1-\left[\dfrac{q\delta r-\delta(1+r)}{(1+\delta)(1-q)}\right]\right)}{1+\delta(1+r)+\phi\left(1-\left[\dfrac{q\delta r}{(1+\delta)(1-q)}\right]\right)}\right\}$$

With ϕ equal to zero, we again get the traditional result that n need only be slightly larger than r for a strict pay-as-you-go social security system to be Pareto-improving. But because of distortions due to social security tax avoidance ($\phi > 0$), the end result is that we have a stricter welfare test for the population growth rate than in the traditional model: the numerator in the bracketed factor on the right-hand side of (28) is always larger than the denominator.

III. CONCLUSION

Value in a pay-as-you-go social security system is primarily added by the population growth rate, and when that rate declines relative to the rate-of-return available from the private capital market, so does the value added by the system. But the value added by Social Security in the United States is declining for another reason as well: the erosion of the Social Security tax base due to increasing avoidance of the Social Security tax. The government might try to make up for the erosion of the tax base by increasing the Social Security tax rate (as indeed it has tried to do in the past), but we have seen that when enforcement is modeled, an increase in the tax rate can no longer be seen as having only an income effect. There is also a substitution effect: increasing the social security tax rate lowers the effective penalty for noncompliance, causing people to substitute away from cash wages toward fringe benefits, thus eroding the tax base further.

There was worry enough about the declining population growth rate when the standard was only that it had to be slightly greater than the capital market rate of return. The end result of all the distortions caused by Social Security tax avoidance is that the population growth rate standard is stricter than had originally been thought. There is a very strong possibility that the current United States economy with the existing Social Security system is Pareto inferior to an equivalent economy without such a system. Of course, very few people are saying that we ought to continue the current pay-as-you-go system without any changes. The results in this chapter support efforts currently under way to privatize the Social Security system.

SUMMARY

The model developed in chapter 1 suggests that, rather than being pay-as-you-go, social security should be designed as a fully-funded Pigovian incentive scheme: private transfers to retirees from children would be augmented by the government in the form of a subsidy, where the subsidy is funded by social security taxes paid when young. The results are two-fold: less reliance on children (and hence, lower fertility) and more reliance on private savings (and hence, greater investment in the private capital market) for retirement income. This type of system is particularly applicable to less developed countries.

The empirical welfare test of chapter 2 calls into question whether the current American Social Security system is Pareto improving. But we cannot answer this question conclusively without knowing more about the reduced-benefit state of nature. I proposed a survey question designed to provide further insight into what "less generous" means.

In chapter 3, we saw that erosion of the Social Security tax base has become a serious problem: fringe benefits have increased substantially as a fraction of total compensation over the past few decades. Changing from pay-as-you-go Social Security to a program with private retirement savings accounts would end the current system of intergenerational externalities, thus removing the incentive to avoid Social Security taxes.

REFERENCES

1994-1996 Advisory Council on Social Security, "Findings, Recommendations and Statements," downloaded from the Social Security Administration website (http://www.ssa.gov/history/reports/adcouncil/report/findings.htm) on June 5, 2001.

1994-1996 Advisory Council on Social Security, "Membership of the 1994-1996 Advisory Council on Social Security," downloaded from the Social Security Administration website (http://www.ssa.gov/history/reports/adcouncil/report/member.htm) on June 14, 2001.

Abel, Andrew B. and B. Douglas Bernheim, "Fiscal Policy with Impure Intergenerational Altruism," Econometrica 59 (1991) 1687 – 1711.

Abel, Andrew B. and Mark Warshawsky, "Specification of the Joy of Giving: Insights from Altruism," The Review of Economics and Statistics 70.1 (1988) 145 – 149.

Allingham, Michael G. and Agnar Sandmo, "Income Tax Evasion: A Theoretical Analysis," Journal of Public Economics 1 (1972) 323 - 338.

Andreoni, James, "Giving with Impure Altruism: Applications to Charity and Ricardian Equivalence," The Journal of Political Economy 97 (1989) 1447 - 1458.

Andreoni, James, Brian Erard and Jonathan Feinstein, "Tax Compliance," Journal of Economic Literature 36 (1998) 818 - 860.

Atkins, G. Lawrence, "Closing Back Doors out of Social Security," Checks and Balances in Social Security, eds. Yung-Ping Chen and George F. Rohrlich. Lanham, MD: University Press of America (1986) 239 - 244.

Auerbach, Alan J. and Laurence J. Kotlikoff, "Evaluating Fiscal Policy with a Dynamic Simulation Model," The American Economic Review 77.2 (1987) 49 - 55.

Barro, Robert J., "Are Government Bonds Net Wealth?" The Journal of Political Economy 82 (1974) 1095 - 1117.

Barro, Robert J. and Gary S. Becker, "Fertility Choice in a Model of Economic Growth," Econometrica 57 (1989) 481 – 501.

Becker, Gary S., "A Theory of Social Interactions," The Journal of Political Economy 82 (1974) 1063 - 1093.

Becker, Gary S. and Robert J. Barro, "A Reformulation of the Economic Theory of Fertility," The Quarterly Journal of Economics 103.1 (1988) 1 – 25.

Burnstein, Eugene, Christian Crandall, and Shinobu Kitayama, "Some Neo-Darwinian Decision Rules for Altruism: Weighing Cues for Inclusive Fitness as a Function of the Biological Importance of the Decision," The Journal of Personality and Social Psychology 67.5 (1994) 773 – 789.

Chakrabarti, Subir, William Lord, and Peter Rangazas, "Uncertain Altruism and Investment in Children," The American Economic Review 83.4 (1993) 994 – 1102.

Clay, Daniel C. and Jane E. Vander Haar, "Patterns of Intergenerational Support and Childbearing in the Third World," Population Studies 47.1 (1993) 67 – 83.

Cowan, Edward, "Background and History: The Crisis in Public Finance and Social Security," The Crisis in Social Security: Problems and Prospects, ed. Michael J. Boskin. San Francisco: Institute for Contemporary Studies (1978) 1 - 15.

Daly, Martin, Catherine Salmon, and Margo Wilson, "Kinship: The Conceptual Hole in Psychological Studies of Social Cognition and Close Relationships," Evolutionary Social Psychology, eds. Jeffry A. Simpson and Douglas T. Kenrick. Mahwah, NJ: Lawrence Erlbaum Associates (1997) 265 - 296.

Datta, S.K. and J.B. Nugent, "Are Old-Age Security and the Utility of Children in Rural India Really Unimportant?" Population Studies 38.3 (1984) 507 - 509.

Diamond, Peter A., "National Debt in a Neoclassical Growth Model," The American Economic Review 55.5 (1965) 1126 – 1150.

Drakopoulos, S.A., "A Note on the Substitution Between Wage and Non-Wage Benefits in Spot Labour Markets," Economics Letters 45 (1994) 495 - 499.

Drazen, Allan, "Government Debt, Human Capital, and Bequests in a Life-Cycle Model," The Journal of Political Economy 86 (1978) 505 - 516.

Ehrlich, Isaac and Francis T. Lui, "Intergenerational Trade, Longevity, and Economic Growth," The Journal of Political Economy 99 (1991) 1029 - 1059.

Enke, Stephen, "Economic Consequences of Rapid Population Growth," The Economic Journal 81.324 (1971) 800 - 811.

Friedman, Milton, "Payroll Taxes, No; General Revenues, Yes," The Crisis in Social Security: Problems and Prospects, ed. Michael J. Boskin. San Francisco: Institute for Contemporary Studies (1978) 25 - 30.

Hamilton, W.D., "The Genetical Evolution of Social Behaviour. I," The Journal of Theoretical Biology 7 (1964) 1 - 16.

Hamilton, W.D., "The Genetical Evolution of Social Behaviour. II," The Journal of Theoretical Biology 7 (1964) 17 - 52.

Hansson, Ingemar and Charles Stuart, "Social Security as Trade among Living Generations," The American Economic Review 79.5 (1989) 1182 – 1195.

Health and Retirement Study, "HRS Overview," downloaded from the Health and Retirement Study website (http://www.umich.edu/~hrswww/overview/hrsover.html) on June 16, 2001.

Health and Retirement Study, "HRS Wave 3 Questionnaire," downloaded from the Health and Retirement Study website (https://ssl.isr.umich.edu/hrs/).

Health and Retirement Study, "Survey Design," downloaded from the Health and Retirement Study website (http://www.umich.edu/~hrswww/studydet/design.html) on February 9, 2001.

Health and Retirement Study, "Survey History and Design," downloaded from the Health and Retirement Study website (http://www.umich.edu/~hrswww/studydet/develop/history.html) on February 9, 2001.

Internal Revenue Service Code Section 6672, as reported in "The Texas A&M University System Tax Manual," downloaded from the Texas A&M website (http://sago.tamu.edu/soba/TaxManual/FedTaxPen.html) on April 7, 1999.

Kimball, Miles S., "Making Sense of Two-Sided Altruism," Journal of Monetary Economics 20 (1987) 301 - 326.

Lagerlot, Nils-Petter, "Endogenous Fertility and the Old-Age Security Hypothesis: A Note," Journal of Public Economics 64 (1997) 279 - 286.

Long, James E. and Frank A. Scott, "The Income Tax and Nonwage Compensation" The Review of Economics and Statistics 64.2 (1982) 211 - 219.

Malik, Arun S. and Robert M. Schwab, "The Economics of Tax Amnesties," Journal of Public Economics 46 (1991) 29 - 49.

Nugent, Jeffrey B., "The Old-Age Security Motive for Fertility," Population and Development Review 11.1 (1985) 75 - 97.

Olsen, Kelly, "Keeping Track of Social Security Reform Proposals: A Summary," Assessing Social Security Reform Alternatives, ed. Dallas L. Salisbury. Washington, D.C.: Employee Benefit Research Institute (1997) 113 - 122.

Ostuw, Pamela, "Public Attitudes on Social Security Reform," Assessing Social Security Reform Alternatives, ed. Dallas L. Salisbury. Washington, D.C.: Employee Benefit Research Institute (1997) 127 - 129.

Pechman, Joseph A., "The Future of the Income Tax," The American Economic Review 80.1 (1990) 1 – 20.

Rosenzweig, Mark R., "Population Growth and Human Capital Investments: Theory and Evidence," The Journal of Political Economy 98 (1990) S38 – S70.

Samuelson, Paul A., "An Exact Consumption-Loan Model of Interest with or without the Social Contrivance of Money," The Journal of Political Economy 66 (1958) 467 - 482.

Samuelson, Paul A., "The Optimum Growth Rate for Population" International Economic Review 16 (1975) 531 - 538.

Samuelson, Paul A., "Optimum Social Security in a Life-Cycle Growth Model" International Economic Review 16 (1975) 539 - 544.

Seidman, Laurence S., "Taxes in a Life Cycle Growth Model with Bequests and Inheritances," The American Economic Review 73.3 (1983) 437 – 441.

Sheshinski, Eytan and Yoram Weiss, "Uncertainty and Optimal Social Security Systems," The Quarterly Journal of Economics 96.2 (1981) 189 - 206.

Slemrod, Joel, ed. Why People Pay Taxes. Ann Arbor: The University of Michigan Press, 1992.

Snavely, Keith, "Governmental Policies to Reduce Tax Evasion: Coerced Behavior versus Services and Values Development," Policy Sciences 23 (1990) 57 - 72.

Social Security Administration, "1996 OASDI Trustees Report, Section I.B," downloaded from the Social Security Administration website (http://www.ssa.gov/history/reports/trust/1996/triaa.html) on June 5, 2001.

Social Security Administration, "1996 OASDI Trustees Report, Table I.D.1," downloaded from the Social Security Administration website (http://www.ssa.gov/history/reports/trust/1996/tbid1.html) on June 4, 2001.

Social Security Administration, "The Future of Social Security," Publication No. 05-10055, August 2000, downloaded from the Social Security Administration website (http://www.ssa.gov/pubs/10055.html) on June 12, 2001.

Social Security Administration, "OASDI Tax Rate Table," downloaded from the Social Security Administration website (http://www.ssa.gov/OACT/COLA/oasdiRates.html) on July 16, 2001.

Social Security Administration, "Supreme Court Case: Flemming vs. Nestor," downloaded from the Social Security Administration website (http://ftp.ssa.gov/history/nestor.html) on May 23, 2001.

Social Security Administration, "Your Social Security Statement – Prepared especially for Edward P. Vanwesep, Jr.," April 25, 2001.

Social Security Advisory Board, "Agenda for Social Security: Challenges for the New Congress and the New Administration," February 2001, downloaded from the Social Security Advisory Board's website (http://www.ssab.gov/reports.html).

Social Security Advisory Board, "Social Security Advisory Board" [an overview of the work of the Board, its membership, etc.], downloaded from the Social Security Advisory Board's website (http://www.ssab.gov/) on June 13, 2001.

Waud, Roger N., "Tax Aversion, Optimal Tax Rates, and Indexation," Public Finance 43 (1988) 310 - 325.

Willis, Robert J., "The Direction of Intergenerational Transfers and Demographic Transition: The Caldwell Hypothesis Reexamined," Population and Development Review 8 (Issue Supplement: Income Distribution and the Family) (1982) 207 - 234.

Woodbury, Stephen A., "Substitution Between Wage and Nonwage Benefits," The American Economic Review 73.1 (1983) 166 – 182.

Zhang, Junsen and Kazuo Nishimura "The Old-Age Security Hypothesis Revisited," Journal of Development Economics 41 (1993) 191 - 202.