A COMPARATIVE STUDY OF THE RHETORIC OF POLICYMAKERS AND MATHEMATICS TEACHERS IN THE WESTERN CAPE, SOUTH AFRICA

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ABSTRACT

A COMPARATIVE STUDY OF THE RHETORIC OF POLICYMAKERS AND MATHEMATICS TEACHERS IN THE WESTERN CAPE, SOUTH AFRICA

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This study is an examination of the similarities and differences between the rhetoric of policymakers and the rhetoric of mathematics teachers in the Western Cape in South Africa. The data corpus consists of two kinds of data, namely, transcribed interviews—observational data with three middle school and three high school teachers—and a set of policy documents on *Curriculum 2005,* the post-apartheid South African government's outcomes-based curriculum policy. Interviews focused on teachers' views about the nature of mathematics and the teaching and learning of mathematics.

An analysis of the interviews indicates that the teachers' rhetoric is heavily influenced by their beliefs and knowledge about the nature of mathematics and the teaching and learning thereof. For example, internal to the school, they encounter the school mathematics tradition with its fragmented mathematics curriculum and modal practice with its familiar routine of checking answers to the previous day's homework and an emphasis on mathematics as an isolated collection of rules, facts, and procedures.

An analysis of the rhetoric in the curriculum policy, *Curriculum 2005,* shows that it wishes to transcend and merge education and training. It has "learner-centeredness" as a key principle in curriculum development. Learners are defined in broad terms and include out-of-school youth and

adults, especially those who have been underdeveloped through apartheid education policies. The curriculum policy focuses on "outcomes," which is what learners should know at the end of a period of teaching and learning, as opposed to "content." Rhetoric in the mathematics component of *Curriculum 2005* shows a continuity in a desire to merge and transcend education and training. There are also several educational slogans from various mathematics education traditions such as the school mathematics tradition, constructivism, disciplinary approaches to mathematics teaching and learning, ethnomathematics, critical mathematics education, and a variation of the last two. The mathematics component has no curricular examples, such as teaching vignettes to illustrate the rhetoric associated with different mathematics education traditions.

On the one hand the findings of the comparison between teacher rhetoric and policymaker rhetoric reveal the strength of the school mathematics tradition. On the other hand the comparison also shows variations in the teacher rhetoric. For example, contact points in the comparison indicate that there are instances of "similar words and similar understandings," and "similar words and different understandings." The results of the study highlight the great challenges that lie ahead in terms of transcending and merging a "rigid division" like education and training in school mathematics reform in the Western Cape and in South Africa. Also, the results allude to challenges in the areas of teacher learning and curriculum development in mathematics content, in order to produce instruction that reflects the mathematics component of *Curriculum 2005.*

To Ayesha, Qudsiya, Obeidullah and my parents

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LIST OF TRANSCRIPTION SYMBOLS

PN:	A middle school teacher's turns
AS:	A middle school teacher's turns
NJ:	A middle school teacher's turns
HY:	A high school teacher's turns
EM:	A high school teacher's turns
DS:	A high school teacher's turns
Faaiz:	My turns

CHAPTER 1

INTRODUCTION

A focus statement of the problem

The South African government's National Department of Education launched an "outcomes based education" reform package popularly known as *Curriculum 2005* on March 24, 1997. In February 2000, the Minister of Education, Kader Asmal, appointed an 11-member review committee of independent educationists to research the key aspects of *Curriculum 2005.* A major criticism leveled against *Curriculum 2005* is that it is jargon-laden, poorly designed and inadequately understood by teachers (*The Teacher,* June 6, 2000). This reform package constructs mathematics in school and the teaching and learning thereof in particular ways. There thus arise possibilities where there could be points of contact, potential confusion and shared understandings between the ways in which mathematics teachers think and talk about their classroom practice and the arguments about the teaching and learning of mathematics in the mathematics component of *Curriculum 2005.* My study is about describing similarities and differences in the two discourse communities, namely mathematics teachers and policymakers. I am interested in finding out how the mathematics components of *Curriculum 2005* might unfold at the micro level, namely, the classroom.

Research questions:

The following research questions drive my study.

- What are the matches and mismatches[1] in the rhetoric of teachers and the rhetoric of policymakers as described in the mathematics component of *Curriculum 2005*?

[1] The matches and mismatches or similarities and differences occur at the points of contact in the two rhetorics.

- How can these similarities and differences inform subsequent thinking on school mathematics reform?

Overview

The first chapter in this study has several goals. The first is to present a statement of the problem and the research questions. This will be followed by background, such as details about the state of education and school mathematics in South Africa, the current national curriculum policy, *Curriculum 2005*, and the relationship between the policy and formal schooling. There will also be a description of the Western Cape province and its importance as a site in the study. Finally, there will be a rationale for the study in terms of the site in relation to the rest of the country and for comparing the ways in which teachers speak and policymakers write. Special reference will be made to the teaching and learning of school mathematics in the South African context.

Background to the present study

One of the main reasons for sketching a background to my study is to provide a context. This background begins with a note on education before and after apartheid, then moves to a description of educational reform in post - 1994 South Africa where we have *Curriculum 2005*. In order to identify the curriculum policy, ample detail about it in relation to similar curricular initiatives in the United Kingdom will be given. Finally, there will be a description of the Western Cape, the site of the present study and a note on school mathematics in South Africa.

A note on education in apartheid South Africa

To gain an idea of education under apartheid in South Africa, we need some background on the institutionalization of apartheid. In the elections of 1948 mostly white South Africans were eligible to vote, in a country that had a large black majority. The results of these elections brought the "whites-only" Afrikaner

Nationalist Party (NP) to power with a narrow majority. The NP then went about enforcing its policies of apartheid, or racial segregation. Apartheid policies affected almost every aspect of the lives of South Africans. For example, the NP-led government enforced, through formal legislation, separate residential areas, separate hospitals, separate hotels, separate movie theaters, separate schools and created separate "races." It devised schemes for defining the racial category of every person in the country. It set up a special government agency to carry out this task. South Africans were classified as "white," "Coloured," "Indian" and "Africans." "Coloureds," broadly speaking, are those who are of a "mixed" racial background. "Indians" are those who have some ethnic connections with the Indian subcontinent. For the majority of black South Africans, or "Africans" as they were referred to at times, racial classifications were based on the different vernaculars in the different regions of the country. For example, black South Africans were classified as Zulu, Xhosa, Tswana and Sotho, to name but a few. Besides being so-called racial categories, these are also vernaculars peculiar to different regions within the country. Today we still have these "racial" groups that make up the South African nation.

Apartheid policies guaranteed preferential treatment for "white" South Africans in almost every facet of political and public life. For example, better-equipped and –resourced schools were for "white" students. Schools for students of "Indian" and "Coloured" racial backgrounds had and still have fewer resources compared to schools for "whites." Schools for the majority black South African students were and still are appallingly poor in resources. These same schools were in turn subdivided according to the various vernaculars and ethnic groups, for example Zulu, Xhosa, Tswana and seven other vernacular groups. Under apartheid rule, these schools were located in the different "homelands"—for example, KwaZulu, for the Zulus, Ciskei and Transkei for the Xhosas and

Bophuthatswana for the Tswanas. The expressed policy of apartheid education was to keep black South Africans in their own "homelands" with their own schools and universities. Today we still have the phenomenon of historically - black universities that were established under apartheid. A divided education system was a cornerstone of the "old" South African regime's educational policy.

A note on the Western Cape province, site of the present study

The region known as the Western Cape in South Africa is the first part of the country that was colonized by western European colonial powers, beginning around the middle of the 17th century. Of these powers, the Dutch were the first to settle there, in the year 1652. They established a seaport they called the Cape of Good Hope, which later became the city of Cape Town. Their idea was to have a refreshment station for their ships that were *en route* to the Indonesian archipelago. Later other western European colonial powers, the French and finally the British took control of the western Cape, making the province the base for colonial expansion in southern Africa.

The demographic patterns in the province differ vastly from national trends. The province has the fifth largest population. The most salient feature of the population of the province is the high concentration of "Coloureds," artificially maintained in the apartheid era when the Western Cape was designated as a "Coloured Labour Preference" area. A strict influx control policy was enforced until 1983, limiting the growth of a permanent black or African population. A relatively low concentration of Africans in the province contrasts with a high concentration of whites. The majority in the province, 59.7%, were classified Coloured , 22.1% white, 17.8% African, and a small 0.7% were classified Indian[1].

[1] (Education Coordination Service (1994) Salient Features of the Proposed Western Cape Province (Pretoria: Internal Working Document).

The "liberal Cape" is the only province in which the previous ruling National Party (NP) retained political power following the 1994 democratic elections. The NP is the same party that legislated and enforced apartheid when it came to power in the 1948 elections. According to Glenda Kruss (1997a, p. 91), the NP in the Western Cape presents itself as the spearhead for "good and responsible" government. She is a South African education researcher with an interest in comparative education. From a political perspective, the province is thus interesting because it has a semblance of a "white rule" enclave in a country that has a black majority.

A note on the state of education in the Western Cape province within South Africa

I shall describe the Western Cape by first focusing on the current state of education in Cape Town, the biggest city in the Western Cape. This will be followed by sketching a picture of the state of education in the rest of the province. Finally, there will be focus on tensions between the province and the central government in relation to education policy. The reason for this note is to indicate that caution should be exercised when one wants to transfer research findings in Western Cape to the rest of South Africa.

It is true that today the diversity in the population of Cape Town and its surrounding suburbs is mirrored in its public schools. Urban public schools in Cape Town are more likely to be representative of a "new" South Africa, where racial groups are not so divided as during the days of apartheid. This claim has to be qualified. There is a brutal reality about the conditions of schools in the Western Cape. The following excerpt from an article called "Worlds apart" by Philippa Garson in *The Teacher* (October 5th, 1998) captures the state of affairs in public schools in the province:

The 2 000 schools in the province fall under one administrative structure, the differences between them are so vast they may as well be hemispheres apart. One school has abundant resources, good access to the provincial Department of Education, and sophisticated management and teaching skills that allow it to forge ahead with fund-raising in a period of budget cuts. A few kilometers away, another school has none of these. Instead, it has violence, gangsterism, overcrowded classrooms and ill-trained teachers in short supply. "Many of these differences are of course inherited from apartheid, and one can argue that school profiles, what schools look like and what they do, are fundamentally problems of our history," wrote Crain Soudien, whose paper *School Realities in the Western Cape: Some Observations* was presented as a framework for discussion. Soudien stressed however the significance of the fact that even now, in 1998, such startling gaps between haves and have - nots are so apparent. "With white and middle-class parents able to subsidise their children's education and working-class and black parents unable to do so, there is a wide difference between what schools look like, feel like and what goes in inside them," he said. Public schools are in a financial squeeze, with provincial funding at an all-time low.

Why are there overcrowded classrooms in former coloured and African schools in the Western Cape? The most significant action from the post – 1994 national Department of Education (DE) that emerged in the field of governance was the decision to move towards the allocation of educational finances on the basis of equity defined in geographical terms. That decision was made against the remorseless logic of the search for equity and redress. One consequence was that the government faced the tough task of equalizing the spending on education in each of the nine provinces over a five–year period, according to Peter Kallaway (1997). He is a South African education researcher with research interests in the history of education, comparative education and education and development. In reality equalizing education spending meant the allocation of funds from the better - off provinces to the worse - off provinces, with the Western Cape facing the harsh consequences of this policy in the form of cutbacks and rationalization. Thus, a study in the Western Cape is interesting in terms of being grounded in the reality of schools that are experiencing the effects of a central Ministry of Education that is out to address equity - related issues.

In the Western Cape and elsewhere in the country, one finds the phenomenon of the so-called Model C schools. These are semi-privatized state schools in the former "whites only" sector, usually located in middle - class suburbs. During the late 1980s under the apartheid regime, parent communities at these schools were forced to take full responsibility for the upkeep, finances and governance of their schools (Kallaway, 1997, p. 46). The only responsibility retained by the state was the payment of teachers' salaries and for the time-being those arrangements have continued. These schools are virtually entrenched by special constitutional provisions, according to Glenda Kruss (1997a, p. 88). They have managed to change the racial profile of the students. Because of their location in the suburbs of Cape Town, the majority of student population is still middle-class, suburban and white who live in the neighborhood of these schools. Many of the students who attend these schools in the Cape Town southern suburbs area are from "Coloured" professional homes and have come from relatively advantaged "Coloured" schools. This process of "creaming off" the most talented from the former coloured schools is a further cause of frustration and anger for many "Coloured" teachers. Thus a study conducted with teachers at Model C schools is interesting because it will show findings in one of the nation's "best" schools.

Despite these harsh realities in the different schools, Western Cape enjoys the highest matriculation (12th grade) examination results in the country for the past few years. These results are evident from available statistics[2]:

[2]Source: Department of Education, 7 March 1997
(Edusource, June, 1997)

Table 1

Matriculation pass rates and university exemptions for the provinces of South Africa, 1994 to 1996

		Total passes %	University Exemptions %
Mpumalanga	1994	47	11
	1995	38	7
	1996	47	10
North West	1994	70	23
	1995	66	17
	1996	69	16
Northern Cape	1994	78	21
	1995	74	17
	1996	73	17
Eastern Cape	1994	57	14
	1995	48	11
	1996	49	11
Gauteng	1994	61	21
	1995	58	19
	1996	58	19
Northern Province	1994	44	12
	1995	38	7
	1996	38	7
Western Cape	**1994**	**86**	**29**
	1995	**83**	**27**
	1996	**80**	**35**
Free State	1994	56	15
	1995	50	12
	1996	51	12
KwaZulu-Natal	1994	68	26
	1995	69	25
	1996	62	23
NATIONAL	1994	58	18
	1995	53	15
	1996	54	15

As can be seen, during 1996 the Western Cape had an 80 percent matric pass rate while the national average was 54 percent. The "matric," as it is known locally and nationally, is a gate keeping examination for entrance to higher education institutions. A student who passes matric with an "exemption," can gain entrance into a university to continue with higher education. An "exemption" pass means that the student's grades in school subjects like the official languages, mathematics, physics and biology have met the requirements set by universities in order to gain entrance. Securing financial assistance for higher education would still be the onus of the student. A pass with an exemption makes it easier on the student to obtain financial assistance. During the same year, 1996, the Western Cape had a 35 percent university exemption rate compared to a national rate of 15 percent. This kind of education statistics would make one conclude that the Western Cape is an exemplary province for the nation.

The Western Cape province appears determined to go its own way in terms of education policy. At the time of my study, May through August 1998, the Western Cape's provincial legislature and education ministry were dominated by a National Party (NP) majority. Historically this is the same party that enforced apartheid when it came to power in 1948. According to Kruss (1997a, p. 91), the NP majority appears determined to assert maximum independence from the central government and provide maximum evidence of the superiority of its policies. During post-1994 deliberations on education policy with the central Ministry of Education, the province argued that it had the competence to develop curriculum and wanted to proceed independently from the central government (Kruss, 1997b, p. 8). There is thus tension between the national Department of Education (DE) and the Western Cape Education Department, with the latter out to maintain its "superior" status in education. This status has much to do with the apartheid order of the past, an order that privileged white South Africans.

In conclusion, the Western Cape province is an interesting site for a study about comparing the rhetoric of *Curriculum 2005* and teachers because it might be a place where one would especially expect tensions. Particular tensions could arise for the following reasons:

(a) its schools have very diverse contexts which that current education policy at work, such as the disparity in material resources

(b) it has the highest matriculation pass rate in the country and thus enjoys a certain leadership in education in the country.

(c) its education department appears determined to go its own way claiming it has the capacity to "implement" education reform.

The description of the Western Cape also gives a picture of external and internal teaching conditions. The high "matric" pass rate in the province implies that it will have particular reactions such as resistance to the central government's curriculum policy initiative, *Curriculum 2005.* A brief background to this curriculum policy is in order.

Background to *Curriculum 2005*

In this section there will be a short description of the origins of *Curriculum 2005* and how the political climate in South Africa, gave rise to this policy. The goal is to give a clear picture of the significance of this policy in light of the situation of a post–1994 South Africa.

In April of 1994, for the first time in South African history, democratic elections were held. All South Africans of legal voting age, irrespective of race, class or gender, became eligible to vote. The African National Congress (ANC) under the leadership of Nelson Mandela, came to power with an overwhelming majority, to form a "Government of National Unity" (GNU). The ANC-led GNU formed a political alliance with the South African Communist Party (SACP) and the Congress of South African Trade Unions (COSATU), the biggest trade union alliance in the country. The ANC, SACP and COSATU are historically part of the "congress" movement, which has a long history of opposing white rule and the apartheid regime. COSATU, the biggest representative of labor is an important player in education policy in post–1994 South Africa.

During the months following these historic elections, most government ministries in the new GNU developed policy and discussion documents relating to a vision of the reconstruction of South Africa. Several were directed at

education. The *White Paper on Education and Training* (WPET, 1995) an important document, promoted a new education policy framework in a country that "has never had a truly a national system of education and training" (p. 2). "White papers" are usually written by government officials in conjunction with consultants and their particular expertise. An integrated approach to education and training is a key feature in the new government's education policy. One year later, on April 1 1995, South Africa's nineteen racially - divided education administrations became a unitary authority (Donn, 1995). Under the auspices of the national ministry of education in Pretoria, all nine provinces established departments of education.

In the post-1994 South Africa, for any educational reform to carry through it had to be sanctioned centrally, by the national Department of Education (DE), which falls under the central ministry of education in Pretoria. This department is responsible for establishing a national policy framework of "norms and standards" for education in general, and has direct control over education at universities and technical colleges, locally referred to as "technikons," which are somewhat similar to community colleges in the United States. The nine provinces have power to legislate on and manage all aspects of basic schooling and education, including teacher education, subject to this national policy framework. The DE has argued that its role is to "establish norms and standard, exams, and certification in terms of the National Educational Policy Act 1996" (OBE in SA, 1997). One may argue that a strong reason for this central control is to ensure that there is never a reversion to the dark days of education under apartheid, which was centrally enforced by an all-white racist, undemocratic regime. Today the central government is more representative of all South Africans.

From 1994 onwards the ANC-led DE in the GNU was faced with a legacy of an education system that was "not working" (Kruss, 1997b). The majority of

South Africans had suffered under apartheid education. Through legislation published in the *White Paper on Education and Training* (WPET, 1995) the GNU assumed responsibility for formulating education policy in the context of a declining economy and an unskilled labor force, represented largely by the Congress of South African Trade Unions (COSATU). According to the South African comparative education researcher, Glenda Kruss (1997b), unemployment rates at the time were high and there was (and is) gross inequality based on race, gender and urban - rural divides. These were and are unique and local conditions peculiar to South Africa.

Kruss (1997b) continues:

> [The] global economic and social order, with the dominance of international capital and a neo-liberal ideology of the market promoting privatization, deregulation and fiscal discipline has exerted strong pressure on the policy. All contributed to a growing consensus among influential policy actors that economic development requires a generally well-educated population equipped with the competencies and skills required by the economy as well as the qualities of flexibility and capacity to learn. Thus, the commitment to a National Qualifications Framework (NQF) and a curriculum grounded in a philosophy of outcomes (p. 3).

From this one can gauge how global and local economic conditions have shaped post-1994 education policy in South Africa. One notices a link between "competencies" and "outcomes." Several questions arise, namely, what are "outcomes," "competencies," and the NQF? Where do these concepts come from and how can we begin to understand them?

Jonathan Jansen (1998), a noted South African education researcher with a deep interest in current educational reform in South Africa, notes that the concept of "outcomes" requires an understanding of competencies, unit standards, learning programmes, curriculum, assessment criteria, range statements, bands, levels, phases, curriculum frameworks, and their relationship to the South African Qualifications Authority (SAQA), and the National Qualifications Framework (NQF). He notes that there are at least 50 different concepts and labels. A focus on the NQF will be helpful in terms of understanding some of the above - mentioned concepts associated with *Curriculum 2005.*

Understanding* Curriculum 2005 *through the National Qualifications Framework (NQF)

Understanding the National Qualifications Framework (NQF) is at the heart of understanding *Curriculum 2005.* A description of the origins of the NQF will illuminate the nature of the present educational reform in South Africa.

The NQF is a structure that merges education and training. It is a means whereby qualifications in training have equivalents in education. For example, different grade levels in formal schooling have equivalents in the training of artisans and other occupations in the area of labor. The columns in table 2 illustrate NQF "levels" and their equivalents in grade levels in school :

Table 2

Equivalent grade levels in formal schooling and in training

School Grades	NQF Level
Doctorates & Certificates Degree, Diploma	8 7 6 5
Further Education and Training Certificates	
12	4
11	3
10	2
General Education and Training Certificates	
9 8 7 6 5 4 3 2 1 R	1

(from DE, 1997b, p. 30).

The ninth grade level in school corresponds to level 1 in the NQF. At this point, General Education and Training Certificates are issued. Grades 10, 11 and 12 correspond to the NQF levels of 2, 3 and 4 respectively. At the 12th grade Further Education and Training Certificates are obtained. Degrees, diplomas and certificates are equivalent to NQF levels 5, 6 and 7. The South African Qualifications Authority (SAQA) is the body that will determine which levels are equivalent to which degrees, diplomas or certificates. At the 8th NQF level, equivalent degrees are doctorates and further research degrees (see DE, 1997b, p. 30).

The NQF thus offers a means for those who have no or little formal schooling to obtain equivalent qualifications. It is a way to get out-of-school youth and adults into the mainstream of academic qualifications. More in-depth explanations of the different NQF levels will follow.

Paula Ensor (1997), a South African mathematics educator, gives a concise description of the aims of the NQF. She writes from a sociological perspective influenced by Basil Bernstein. Her research interests include school mathematics and "everyday life," and induction processes of novice mathematics teachers. According to her, the NQF promises a bold new educational dispensation; "learner-centeredness," the integration of education and training, the facilitation of "lifelong learning" and the developments of knowledge and skills required to carry South Africa into prosperity in the 21st century (p. 36). "Dispensation" in the South African context is similar to "reform package" in the United States. The NQF is thus the mechanism that is going to usher in this new educational reform package. It is a new structure that aims to improve the quality of education in South Africa (DE, 1997b, p. 4). It provides

> [learning] opportunities for learners regardless of age, circumstances, gender, level of education and training. It allows learners to learn on an on-going basis. This is called lifelong learning and is central to the NQF (p. 4).

The vision of the NQF is that those who have been neglected under apartheid education, will be given opportunities to learn. Hence there is a focus on "learners" in a broad sense. Learners were defined in very broad and non-discriminatory terms. This makes sense given the fact that a first-ever democratic elections had just taken place. Learners definitely include adults and youth of different genders, who had experienced discrimination in education under apartheid. What is striking about the NQF is that it will ensure that education and training are brought together. In the past education was seen as an area where knowledge is gained and training as an area where skills are obtained. The NQF will join education and training. It will enable "learners" to move from one place of learning to another, thereby ensuring mobility. NQF hopes to ensure are that

- Learning is recognized whether it takes place in formal or informal settings
- Learners are able to move between the education and working environments
- Credits and qualifications are easily transferable from one learning situation to another,
- Needs of learners and the nation are addressed and met
- Qualifications obtained by learners are recognized and accepted nationally and internationally (p. 5).

These aims also indicate economic imperatives of the "new" South Africa. For example, according to new legislation there will be a close cooperation between the ministers of education and labor (WPET, 1995, p. 9) to encourage and develop common interests in an integrated approach to education and training and a NQF. The central Department of Education contends that the only way for the NQF to be effective is to change the existing educational system—from a content-based to an "outcomes-based" approach (DE, 1997b, p. 5).

To summarize, the NQF is a means to ensure that a "learner's" education or qualification is connected to a world of future employment. There are 8 NQF levels. Level 1 is equivalent to grade 9 in the formal school system. A "learner," who could be an adult or out-of-school youth, who goes beyond grade 9 or level 1 obtains a "General Education and Training Certificate." Levels 2, 3 and 4 are equivalent to grades 10, 11 and 12 in the formal school system. Upon completing level 4 a "learner" will obtain a "Further Education and Training Certificate." At levels 5, 6 and 7 degrees, diplomas and certificates are issued. The 8th level is the equivalent of doctorates and further research degrees.

These levels are in the higher education and training band. Adults can obtain their certificates and qualifications through ABET (Adult and Basic Education and Training) from levels 1 through 4, with level 4 being equivalent to an NQF level 1 and a formal school grade level of 9. ABET 1 and 2 are equivalent to formal school grade levels 2 through 5, representing the "foundation phase." ABET 3, representing the "intermediate phase," is equivalent to formal school grade levels 6 through 8. ABET 4, representing the "senior phase," is equivalent to a formal school grade level 9.

Table 3

A more detailed table showing the equivalent levels in formal schooling and in training through the National Qualifications Framework (NQF)

School Grades	**NQF Level**	**Band**	**Types of qualifications & certificates**	
doctorates	8	Higher Education and Training Band	Doctorates and further research degrees	
Degrees diplomas & certificates	7 6 5		Degrees, Diplomas and Certificates	
Further Education and Training Certificates				
12	4	Further Education and Training Band	School/Colleges/NGOs Training certificates, mix of units	
11	3		School/Colleges/NGOs Training certificates, mix of units	
10	2		School/Colleges/NGOs Training certificates, mix of units	
General Education and Training Certificates				
9	1	General Education and Training Band	Senior Phase	ABET 4
8 7 6			Intermediate Phase	ABET 3
5 4 3 2			Foundation Phase	ABET 2 ABET 1
1 R			Pre-School	

(from DE, 1997b, p. 30).

The NQF bridges education and training even if it is only on paper. Combining education and training overcomes a "rigid division" that *Curriculum 2005* rejects (DE, 1997a, p. 1; 1997c, p. 11). Analogous "rigid divisions" or dichotomies that the curriculum policy wants to transcend are academic and applied knowledge, theory and practice, and knowledge and skills (DE, 1997a, p. 1).[1]

"Learners" will achieve different qualifications and certificates at different levels in the NQF through "learning programmes" that will consist of units of learning. A "unit standard" states the "specific outcomes" that needs to be achieved to make up a credit on the NQF (DE, 1997b, p. 32). Specific outcomes are context-specific. They describe the competence that learners should be able to demonstrate in specific contexts and particular areas of learning at certain levels. These are the outcomes that "should serve as the basis for assessing the progress of learners, and thus indirectly the effectiveness of learning processes and learning programmes" (DE, 1997a, p. 35-36). "Unit standards" are defined as nationally and internationally agreed upon statements of "outcomes." The South African Qualifications Authority (SAQA) is the body that specifies what it will consist of (DE, 1997a, p. 36). A unit standard will consist of, a unit standard title, a SAQA approval logo, the issue date, the review date, "assessment criteria," "range statements," and so on. Unit standards will be registered and qualifications awarded to those who have completed a combination of unit standards. In the same vein, in the language of *Curriculum 2005* traditional school subjects are referred to as "learning areas."

The practice of "continuous assessment" will underpin all assessment across all education and training bands at all levels (DE, 1997a, p. 37). According to policymakers there shall be a "paradigm shift" from promotions based on the results of a single test of examination to "ongoing formative assessment" of the learner. This kind of assessment will take the form of recording the learners' strengths and weaknesses through portfolios of learners' work, documented

[1] Lisa Delpit (1988), in *The Silenced Dialogue: Power and Pedagogy in Educating Other People's Children*, writes about the resolution of an analogous dichotomy, namely, the skills/process debate.

records of teachers' appraisals, both verbal and scored on which promotion decisions shall be based and other methods (p. 37).

The above paragraphs describe specific methods policymakers in the country want to employ to solve the problems of apartheid education. In order to overcome practical problems that apartheid education caused, policymakers have decided to focus their attention on overcoming the education and training divide. This is a noble goal if one takes into consideration the deliberate underdevelopment under apartheid.

Where do concepts such as specific outcomes, learning areas, learning programmes, continuous assessment, unit standards, national qualifications framework, assessment criteria, and range statements come from? This is what one needs to know to see "outcomes-based" *Curriculum 2005* in the South African context.

The National Qualifications Framework (NQF) and the (Scottish) National Vocational Qualifications (NVQs/SVQs)

The purpose for doing a comparison between the NQF and the National Vocational Qualifications (NVQs) is to give some insights into the NQF and some of its many associated concepts such as outcomes, learning, continuous assessment, unit standard, assessment criteria, and range statements. In Scotland, NVQs are referred to as Scottish Vocational Qualifications (SVQs). A clue for this comparison comes from research done by Soudien and Baxen (1997), two South African education researchers whose interests include shifts in education policy. In their research they interviewed a Western Cape Education Department official who attended a meeting on curriculum policy making organized by the central Department of Education. At the meeting this official told how he was given a guideline document wherein a reference was made to "the Scottish Model" (p. 453).

The "Scottish Model" reveals the context of the debate on aligning and relating academic and vocational education/training in the United Kingdom in the 1980s and early 1990s. The debate centered on achieving parity of esteem between academic and vocational qualifications, according to Gilbert Jessup, (1995, p. 33). He is a noted researcher on issues related to education and training in the United Kingdom. It is evident from the rhetoric in *Curriculum 2005* that post-apartheid South Africa faced a similar problem of merging education and training. Jessup (1991) cites a "National Certificate" introduced in August 1984, in Scotland, as a basis in successful educational practices. Thus, the debate in the United Kingdom about achieving parity between academic and vocational qualifications has a success story in Scotland's "National Certificate." This certificate was based on a range of educational practices set through outcomes and methods of assessment, rather than from prescriptive courses (Jessup, 1991, p. 154). Thus the adoption of the "Scottish Model," in the South African context, explains the focus on "outcomes."

It can be inferred that "the Scottish Model" referred to by Soudien and Baxen (1997) is a conceptualization of Gilbert Jessup. In Jessup's (1991) *Outcomes: NVQs and the Emerging Model of Education and Training,* he proposes that the "outcomes" model of defining qualifications and learning is applicable to all forms of learning. "Outcomes" are statements of competence and attainment, which have to be specified in order to be developed and to know when they have been achieved (Jessup, 1991, p. 136). Outcomes have similar meanings in the case of *Curriculum 2005.* They are defined as results of learning processes and learners should be able to demonstrate that they understand and can apply desired outcomes within a certain context (DE, 1997b, p. 32). Jessup defines "learning" in very broad terms and suggests that it can happen in a wide range of locations and by different methods. In his view it should not be equated in the minds of people with "academic," "classrooms," "boredom" or "failure" (Jessup, 1991, p. 36). The idea behind such a broad description of learning is to recognize learning that happens outside of formal academic settings such as classrooms. This notion of learning makes it possible to include youth who are not in school and adults who wish to go to school and obtain qualifications. Similarly, in the case of *Curriculum 2005*, "learners" are defined in very broad terms—for example, as out-of-school youth and adults, two important segments in the South African population that experienced underdevelopment in apartheid South Africa.

It was argued that, if adopted, Jessup's model would open access to learning to far more individuals of all ages, and that it would lead to more efficient and cost-effective learning. It would provide a means of relating and aligning academic and vocational education/training (Jessup, 1995, p. 33). National Vocational Qualifications (NVQs) are central to understanding Jessup's model.

What are NVQs? Criteria for NVQs were first established in 1988 in the United Kingdom. NVQs have a primary focus on qualifications and employment. NVQs allow and encourage people to acquire competence through a variety of modes of learning, formal and informal, full-time or part-time and in a variety of locations. The structure of NVQs includes an agreed "statement of competence " or "outcomes" which is accompanied by "performance criteria" that identify only the essential aspects of performance necessary for "competence" and "range statements" that express the various circumstances in which the competence must be applied." These are summaries from research done by Hyland (1994), who takes a critical perspective on the notion of competence and the NVQs. Similarly, in the South African case, the NQF is a central mechanism for integrating education and training, by specifying "outcomes" rather than "competence." May be the idea of specifying "outcomes" implies somehow that "competence" has been achieved.

Another concept in NVQs is "unit credits," which are made up of a number of units, based upon requirements for employment. Here the idea is that a national system of credit accumulation and transfer is gradually established upon a common currency of units (Jessup, 1995, p. 37). In the South African case, "unit standards" are the equivalent of "unit credits," which are defined as nationally and internationally agreed upon statements of "outcomes."

According to Jessup (1995, p. 35), the concept of the National Vocational Qualifications (NVQs) framework is that qualifications will be designed to cover comprehensively all occupations and professions. NVQs are allocated to one of five levels in the framework with the most basic occupations at level 1 and the senior professions at level 5. Level 2 corresponds roughly to a "GCSE" (General Certificate of Secondary Education) level. Level 3 corresponds to Advanced Supplementary levels, deemed as half an "A" level in the GCE. Level 4 corresponds to bachelor's degrees and certificates in higher education. Level 5 corresponds to higher degrees such as doctorates and research degrees.

Table 4

Equivalent levels between formal schooling and the National Vocational Qualifications (NVQs)

Post-Graduate	NVQ level 5
Degrees/Higher Education	NVQ level 4
A/AS levels (Advanced & Advanced Supplement Levels)	NVQ level 3
	NVQ level 2
	NVQ level 1
National Curriculum (General Certificate in Secondary Education, GCSE)	

(see Jessup, 1991, p. 86).

Here the claim or assertion is that NVQs levels, which are about employment or training, have equivalents in formal education. The figure above is a synopsis of Jessup's model, which shows different NVQ levels. The National Curriculum is the school curriculum in the United Kingdom with its school subjects.

In the South African case, there are eight levels in the NQF, with Level 1 beginning with General Education and Training Certificates, equivalent to grade 9 in the schooling system. NQF Levels 2, 3 and 4 are equivalent to grades 10, 11 and 12 in the formal school system. Levels 5, 6 and 7 correspond to degrees, diplomas and certificates. Level 8 corresponds to doctorate and further research degrees (DE, 1997a, 1997b).

In conclusion, the NVQs framework is certainly about training or employment, and the assertion is that the latter have equivalents in the formal education system in Scotland and thus the United Kingdom. Similarly the NQF is a mechanism whereby qualifications are awarded to out - of - school youth and adults in order to make them "competent" through "life-long learning" in and for the world of work. Thus *Curriculum 2005* and its concepts is an adaptation of a model from Scotland with influences from Australia and New Zealand, but one that is unique to the South African context. Central to *Curriculum 2005* is the idea of merging the world of work and formal schooling, that is, education and training.

Curriculum 2005 and formal schooling

How would *Curriculum 2005* be introduced in the schools? There was a tentative timetable for introducing *Curriculum 2005* in phases, starting with grades 1 and 7 in 1998. The table below shows the implementation dates:

Table 5

Implementation dates for *Curriculum 2005*

Grades	Year of Implementation
1 and 7	1998
2 and 8	1999
3 and 9	2000
4 and 10	2001
5 and 11	2002
6 and 12	2003

The period 2004 - 2005 will be devoted to an evaluation of the new curriculum with a view to improving and refining it (DE, 1997a, p. 18).

What about traditional school subjects? These will be referred to as "learning areas," a concept that is consistent with a constant reference to "learners," "learning" and "outcomes" in *Curriculum 2005* literature. There will be eight learning areas. These will form the basis of all education up to the Further Education and Training Certificate (DE, 1997a, p. 16):

1. Language Literacy and Communication
2. Mathematical Literacy, Mathematics and Mathematical Sciences (MLMMS)
3. Human and Social Sciences
4. Natural Sciences
5. Technology
6. Arts and Culture
7. Economics and Management Sciences
8. Life Orientation

For these learning areas there will be four phases:

Foundation Phase (Grades 1 through 3); Intermediate Phase (Grades 4 through 6); Senior Phase (Grades 7 through 9) and Further Education and Training (Grades 10 - 12 in school education; out of - school youth and adult learners) (DE, 1997a, p. 19).

Each of the eight learning areas has "learning outcomes" that were drawn up by Learning Area Committees (LACs). The outcomes are of two kinds, namely "critical cross-field outcomes" and "specific outcomes" (DE, 1997a, p. 33). Critical cross-field outcomes are generic, cross-curricular, broad outcomes that focus on the capacity to apply knowledge, skills and attitudes in an integrated way (OBE in SA, March, 1997, p.3). They underpin the learning in all its facets (DE, 1997a, p. 34). Specific outcomes are context-specific. They describe the competence learners should be able to demonstrate in specific contexts and particular areas of learning at certain levels. These are the outcomes that "should serve as the basis for assessing the progress of learners, and thus indirectly the effectiveness of learning processes and learning programmes" (pp. 35-36).

In conclusion, the curriculum policy, *Curriculum 2005* , is concerned about merging and transcending "rigid divisions" such as academic and applied knowledge, knowledge and skills and theory and practice. It also has a strong emphasis on integrating education and training, that is, bringing the world of work closer to school. This emphasis is motivated by the need of the South African state and thus policymakers, to address the political, social and economic conditions of the post-apartheid period. It is therefore important to examine the ways in which teachers talk about their practice and the ways in which policymakers write.

Why compare the rhetoric of policymakers and teachers?

The research questions that drive this study are:

- What are the matches and mismatches in the rhetoric of teachers and the rhetoric of policymakers as described in the mathematics component of *Curriculum 2005* ? and
- How can these similarities and differences inform subsequent thinking on school mathematics reform in the South African context?

This section will discuss the importance of teachers in education policy and outline how the desire of merging the dichotomy between education and training in *Curriculum 2005* calls for serious rethinking of education policy and thus teaching and learning. Most of the argument will be based on Darling – Hammond's (1990) "Instructional Policy into Practice: The power of the Bottom over the Top." The choice of this article will become evident. This study is significant in the context of mathematics education in South Africa.

Education and teachers are part of the larger society. Education policy enacted through schools has the task of selecting people for rare positions in society and socializing everyone to become citizens within that society. These rare positions include professions that not everyone can attain. Thus, there are others in society who have to assume positions that are much needed to make the society function. These positions include occupations such as bricklayers, carpenters and so on, that require skills. In other words, schools have a dichotomous task of selecting and socializing. Through education policy policymakers attempt to influence teachers to think about these different positions in society and what an education would be like for these positions. Thus there is a debate between education and training. Education is conceived in a liberal arts sense as education for future schooling whereas training is thought of as in a vocation or training for a particular job. As mentioned earlier on, in the South African context, *Curriculum 2005* wishes to merge education and training.

Teachers are important when it comes to education policy.
Linda Darling – Hammond (1990) uses the metaphor of a "black box" to describe the relationship between policy enactment and policy outcomes (p. 234). A black box is defined as "a complicated electronic device that can be inserted in or removed as a unit from a larger assembly of parts" or "an electronic device whose internal mechanism is hidden from or mysterious to the user" (Webster's New Collegiate Dictionary, 1974). Her metaphor is striking for two reasons. First, it alerts us to the importance of understanding teaching in the "larger assembly of parts," namely, the educational enterprise. Second, it draws attention to internal mechanisms such as teacher knowledge and belief, and teaching circumstances or the social context of teaching when it comes to policy enactment and outcome.

Curriculum 2005 is a curriculum policy that is poking its head into the inner workings of schooling. For this policy to have an effect, what teachers at the classroom level say and do is vital. Teachers' prior learning, beliefs and attitudes are an essential ingredient in the process of teaching. This is often hidden from the rest of the education system, which makes the black box metaphor appropriate. Policy about teaching assume local meanings, indicating the "power of the bottom over the top." By talking to teachers about their practice and working lives, one could learn how they will understand and interpret the intentions of *Curriculum 2005* in the context of their knowledge and beliefs. By interviewing teachers in South Africa, in the Western Cape province, one could determine from their talk how they will come to understand a curriculum policy that wishes to transcend dichotomies like education and training, academic and applied knowledge, knowledge and skills, and theory and practice.

Reformers, through policy documents, call on teachers to change their practice in novel ways that they themselves have not experienced. Thus, teachers very often do not understand or have not seen reform practices. This is evident from their apprenticeship of observation (Lortie, 1975). There are many examples in the literature of the very different ways in which teachers come to understand reform rhetoric in school mathematics — for example, the study of teachers and the *California Frameworks* (1985) (see Cohen and Ball, 1990). A comparative study of the rhetoric of the two communities, teachers and policymakers, will reveal contact points and mismatches. It will reveal places where teachers are already thinking and even doing the reform and places where there are particular difficulties.

In describing South African mathematics teaching, two South African mathematics educators, Brodie and Strauss (1994), observe that "seldom are students asked to justify their thinking, i. e. communicate mathematically or to explore mathematical connections." Also, Jill Adler (1993), another South African mathematics educator, notes:

> 'Tell and drill' remains the dominant practice in mathematics classrooms. Textbook-based teaching and rule-bound learning styles constitute pupils' mathematical diet.

In several ways these excerpts illustrate modal practice in the country. For example, "seldom are students asked to justify their thinking, i. e. communicate mathematically or to explore mathematical connections," implies an "implicit conception of mathematics" (Confrey, 1988) as a collection of isolated rules and procedures to be memorized. 'Tell and drill' points to instructional practices that separate teaching from content. These excerpts do not reveal particular struggles that teachers are engaged in as they try to move beyond modal practice to transcend another well - known dichotomy, the child and the curriculum (Dewey, 1902/1992), for example. A rhetoric-rhetoric comparison will illuminate how teachers would understand the ambitious educational reforms in *Curriculum 2005* that have been put in place to enact "redress," "reconstruction," and issues of education and training, in the present South African context.

Mathematics as a school subject in South Africa

In South African high schools students do not take algebra, geometry, trigonometry and calculus as separate classes, as in the United States. In other words, a teacher does not teach an algebra- or a geometry-only class. He or she teaches "mathematics," at any grade level in school. The written examinations for mathematics in high school consist of two papers, normally. In the first paper, there will be an algebra and a calculus "section." The second paper typically has a geometry and a trigonometry "section." At any particular time during the week, the teacher may be teaching algebra for a few days and geometry or trigonometry for the rest of the week. This structure is arranged by the mathematics department in the school. Two South African mathematics educators, Kulubya and Glencross (1997), describe the "traditional" high school mathematics curriculum as consisting of algebra, trigonometry, geometry and calculus. The middle grades curriculum consists of number theory, algebra, geometry, mensuration and measurement and statistics and probability (Western Cape Education Department, Junior Secondary Course, 1997).

Although this curriculum is for the Western Cape middle grades, one can assume it is more or less the same for the rest of the country.

There is a lot of similarity between the school mathematics curriculum in South Africa and the United States. For example, in the United States, Steen (1990) describes mathematics in the school curriculum as follows:

> It picks very few strands (e.g., arithmetic, geometry, algebra) and arranges them horizontally to form the curriculum: first arithmetic, then simple algebra, then geometry, then more algebra, and finally as if it were the epitome of mathematical knowledge, calculus (p. 4).

The mathematics that the elementary student is concerned with does tend to look linear, or at least the arithmetical part does. Wheeler (1980) notes that " geometry gives trouble because it isn't clear where it fits into the structure -- and indeed, it doesn't fit in any linear, hierarchical sense." Steen's description is not very different from the one given in the previous paragraph. Ensor (1996), a South African mathematics educator, eloquently describes mathematics in the school curriculum as strongly classified, brutally sequenced and ruthlessly paced. Her description is consonant with the different "sections" in the curriculum and Steen's description.

CHAPTER 2

REVIEW OF RELEVANT LITERATURE

Overview

In part this study focuses on how teachers talk about the teaching and learning of mathematics. When they do so, they reveal their knowledge, belief and practice, which could be related to mathematics education traditions. Hence, this chapter will begin with a review of literature related to teacher knowledge, belief and action. Second, there will be an explanation of the notion of "traditions" in relation to a set of beliefs about teaching mathematics. Third, there will be a review of research by Cobb *et. al.* (1992) and Gregg (1995) on the school mathematics tradition with Active Mathematics Teaching (AMT) (Good *et. al.*, 1983) as an instantiation of it. Finally, there will be a less than exhaustive review of mathematics education traditions such as constructivist traditions, disciplinary traditions, ethnomathematics and critical mathematics education traditions and a combination of the two.

Literature on teacher belief, knowledge and action

This section reviews of literature on teacher belief, knowledge and practice, beginning with general remarks and then becoming more specific to the teaching and learning of mathematics.

Nespor (1987, p. 323) notes that "to understand teaching from teachers' perspectives we have to understand the beliefs with which they define their work." For the most part, studies of the relationship between teachers' beliefs about teaching and instructional practice have examined the congruence between teachers' professed beliefs and their observed practice (Thompson, 1992, p. 137). Paul Ernest (1987) argues that while teachers' knowledge is important, it alone is not enough to account for differences among mathematics teachers. He argues that two teachers may have similar knowledge, but while one teaches mathematics with an emphasis on activities aimed at engaging students in the generative processes of mathematics, the other has a more traditional and didactic approach. He cautions that teaching reform cannot take place unless teachers' deeply held beliefs about mathematics and its teaching and learning change. He argues that the practice of teaching mathematics depends on a number of key elements, most notably:

- the teacher's mental contents or schemes, particularly the system of beliefs concerning mathematics and its teaching and learning;
- the social context of the teaching situation, particularly the constraints and opportunities it provides; and the teacher's level of thought processes and reflection (Ernest, 1987, p. 249)

He identifies the key belief components of the mathematics teacher as, the teacher's:

- view or conception of the nature of mathematics
- model or view of the nature of mathematics teaching
- model or view of the process of learning mathematics (p. 250)

Thom (1973) asserts a clear connection between what a teacher thinks mathematics is, and his or her teaching practice:

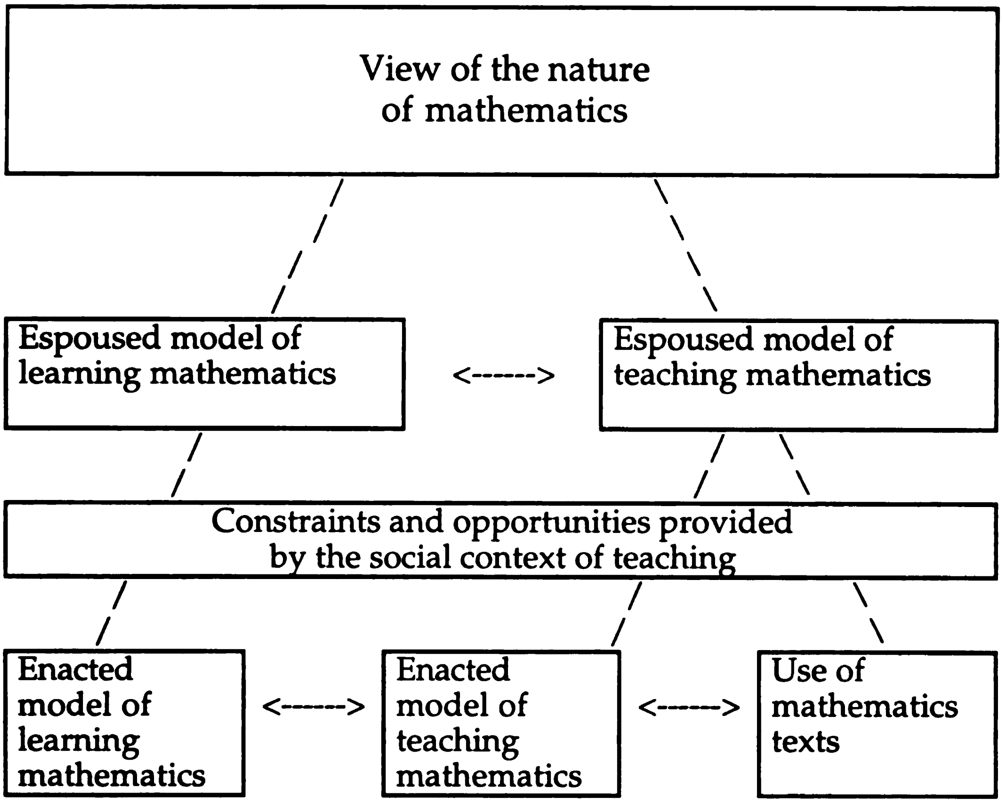
> In fact, whether one wishes it or not, all mathematical pedagogy, even if scarcely coherent, rests on a philosophy of mathematics (p. 204).

Ernest (1987) distinguishes three "philosophies" because of their occurrence in the teaching of mathematics (Thompson, 1984). Ernest (1987, p. 250) describes these philosophies, as, the instrumentalist view that mathematics is an accumulation of facts, rules, and skills to be used in persuance of some external end. Thus, mathematics is viewed as a set of unrelated but utilitarian rules and facts. Second, there is the Platonist view of mathematics as a static but unified body of certain knowledge. Mathematics is discovered, not created. Third, there is a problem-solving view of mathematics as a dynamic, continually expanding field of human creation and invention, a cultural product. Mathematics is a process of inquiry and coming to know, not a finished product, for its results remain open to revision (p. 250).

Ernest (1987, p. 252) depicts relationships between beliefs and their influence on practice as shown in Figure 1.

Figure 1

Relationships between beliefs and their influence on practice



This chart shows the relationships between teachers' views about the nature of mathematics (see the rectangular box at the top) and their models of teaching and learning. Teachers' views of the nature of mathematics provide a basis for the teachers' mental models of teaching and learning mathematics, as indicated by the downward arrows. For example, an instrumental view of mathematics is most likely to be associated with "teaching as telling" and a strict following of the text. It is also associated with the student's compliant behavior and a mastery of skills model of learning. Ernest makes the espoused-enacted distinction in models of learning and teaching mathematics, because there can be disparities between the two.

Reasons for the disparities are, first of all, the powerful influence of the social context. This results from the expectations of others, including students, parents, peers (fellow teachers) and superiors. It also results from the institutionalized curriculum: the adopted text or curricular scheme, the system of assessment and the overall nature of the system of schooling (Ernest, 1987, p. 252).

Schifter's (1995) research on mathematics teaching illustrates relationships between teachers' knowledge and current reform efforts. She analytically isolates one "strand" (p. 18) of the complex process: changes in the conception of school mathematics enacted by teachers working to transform their practice along the lines of current reforms in school mathematics. She characterizes conception of mathematics that teachers enact in practice as

- an ad hoc accumulation of facts, definitions and computational routines;
- Student-centered activity, but with little or no systematic inquiry into issues of mathematical structure and validity;
- Student-centered activity, directed toward systematic inquiry into issues of mathematical structure and validity;
- Systematic mathematical inquiry organized around investigation of "big" mathematical ideas, the organizing principles of mathematics.

Each conception or stage entails an understanding of what counts as "doing mathematics," which should be regarded as "developmental" (Schifter, 1995, p. 21). By "developmental" she means that as teachers' mathematical understandings evolve in the context of their in-service work, boundaries between one stage and another become very indistinct (p. 22). She also makes a connection between conceptions of the nature of mathematics and the teaching of mathematics, a move consonant with Thom (1973) and Ernest (1987).

A note on the use of "traditions"

The word "tradition(s)" appears in several places in this chapter and elsewhere in this study. It is therefore important to clearly explain what is meant by it. Its use closely relates to notions of teacher belief, knowledge and practice. A similar use becomes evident in the case of a mathematics education "tradition." The relationship between traditions and individual practitioners will also be spelled out.

In *The Practice of Teaching,* Phillip Jackson (1986) writes about the "mimetic" and "transformative" teaching *traditions.* The "mimetic" tradition gives central place to the transmission of factual and procedural knowledge from one person to another, through essentially an *imitative* process (p. 117). This tradition places importance on "method" in a narrow sense. On the other hand, teachers working in the "transformative" tradition seek to change their students (and themselves) through discussion, demonstration and argumentation (p. 127). Teaching that is solely mimetic in orientation may be as rare as that which is solely transformative (p. 129). He justifies the use of "traditions" because each has a long and respectable history, going back at least several hundred years and possibly beyond. Each has more than an intellectual argument. Each provokes feelings of partisanship and loyalty toward a particular point of view and thus beliefs; each entails commitment to a set of related practices. In short, each comprises what might be called a "form of life," a relatively coherent and unified way of thinking, feeling, and acting within a particular domain - in this instance, the sphere of education (p. 116). In the domain of educational thought and practice, one can think of a "tradition" as involving beliefs, attachment, and commitment and thus relate it to notions of teacher belief, knowledge and practice. Some traditions which draw on the works of Paulo Freire make explicit their relationship to the social context of teaching such as sociopolitical conditions.

A distinction must be made between traditions and the work of individual practitioners. Individuals might be characterized as exhibiting characteristics of a tradition, but would not be captured completely by that characterization. For example, a teacher might claim to be working in a constructivist tradition, but might at times be drawn to traditions that are different from constructivism. In other words, individuals are multi-dimensional and more complex than traditions. This point will be revisited in the case of mathematics education traditions.

What is the school mathematics tradition?

Cobb *et. al.* (1992, p. 598) refer to the taken-as-shared beliefs and practices in teacher-centered classrooms, as the "school mathematics tradition" (SMT). This "tradition" will be described in detail by referring to the research of Cobb *et. al.* (1992) and Gregg (1995), in order to show its associated beliefs and practices. They conducted their studies in elementary and high school classroom settings, respectively. A description of the SMT will also link it to modal practice or teacher - centered instruction in the United States (see Cuban, 1984) and South Africa, independent of grade level. The tradition will be described with a focus on its views of mathematics and the teaching and learning of mathematics.

Skemp's (1978) discussion of instrumental learning can be interpreted as a characterization of normative mathematical activity in the SMT (Cobb *et. al.*, p. 597). This observation illustrates the model of learning that characterizes this tradition. Cobb *et. al.* (1992) further note that in the SMT students typically experience "mathematical understanding when they can follow procedural instruction successfully" (p.598). This observation gives an idea of the model of teaching that characterizes the tradition, i. e. teaching explicitly or directly focuses on giving students a set of procedures, rules and facts.

Gregg's (1995) examples of practices and taken-as-shared beliefs associated with the SMT are not very different from the findings of Cobb *et. al.* (1992). They are more detailed:

- Emphasizing form and procedures (p. 452).
- Employing tests as measures of student understanding (p. 453)
- Maintaining control and the constitution of mathematics (p. 456)
- Producing the answers and procedures in the textbook (p. 456)
- Proceduralizing and decomposing the mathematics (p. 458)
- Accounting for students' difficulty by referring to their ability (p. 461).
- A separation of teaching and learning (p. 464).

Gregg (1995) conducted an ethnographic study of a beginning high school teacher's acculturation into the school mathematics tradition. While the last two examples about students' ability and a separation of teaching and learning are about taken-as-shared beliefs, the rest are examples of practices. In fact, a close reading of his study shows it is very difficult to separate practices from taken-as-shared beliefs. For example, the teacher's practice of "emphasizing form and procedures" indicates her emphasis on students obtaining correct answers to routine homework exercises, with no likelihood that students would develop anything beyond a fragmented, procedural understanding (p. 452). Her practice of "maintaining control and the constitution of mathematics (p. 456), reveals her belief that the content of school mathematics was fixed and unchangeable (p. 456). Similarly, her practice of "producing the answers and procedures in the textbook" (p. 456) meant that students' reasoning was subordinated to the need to produce the procedures specified in the textbook" (p. 457). In her geometry class, notions of "explain" and "justify" were constituted to mean naming a rule or a procedure or stating a fact or a theorem (p. 451). This reflects her conception of the nature of mathematics, that is, mathematics as a collection of isolated rules, procedures and facts. It also shows her belief in the separation of teaching and learning, which hinders her efforts to assess how well students are prepared for tests (p. 463). Her practice of "proceduralizing and decomposing the mathematics" (p. 458) reflects her belief in keeping tests "straightforward," where there is a close correspondence between homework exercises and test questions (p. 458). She explains students' difficulty in the proceduralized mathematics by referring to their ability (p. 461). She describes individual student's difficulties in terms of "an inability to remember ideas or simply inability to understand, but never in terms of any specific mathematical constructions" (p. 460). This last remark is another indication of the teacher's conception of the nature of mathematics, in relation to her practice of separating teaching from mathematical content.

The cultural assumptions we hold about mathematics further strengthen the school mathematics tradition, as described by Cobb *et. al.* (1992) and Gregg (1995). For example, Lampert's (1990) writing about the ways of knowing and doing in the "school experience" complements the practices and beliefs in the school mathematics tradition. For example, in school and in general, mathematics is commonly associated with certainty and knowing it —being able to get the right answer, quickly (Schoenfeld, 1985; Ball, 1988 and Stodolsky, 1985, quoted in Lampert, 1990). Lampert (1990, p. 32) asserts that these cultural assumptions are shaped by school experience, in which *doing* mathematics means when they can follow the rules laid down by the teacher; *knowing* mathematics means remembering and applying the correct rule when a teacher asks a question; and mathematical *truth is determined* when the answer is ratified by the teacher or the textbook. This tight control of the teaching and learning of mathematics gives a sense of stability and order to the mathematics. It gives the mathematics an "institutional impact" (Bauersfeld, 1980; Popkewitz, 1988)

In conclusion, in the SMT there is a conception of mathematics as a collection of isolated rules, procedures, facts and conventions. Teaching in this tradition takes on the form of "direct instruction" with an emphasis on form, conventions, rules and memorization, where rules, formulas and facts are treated as if they were arguments (Lampert, 1990, p. 56). Learning mathematics amounts to memorizing rules without reasons, from one "section to another" (Gregg, 1995, p. 452). The textbook and teacher are the mathematical authorities. In order to be competent in this tradition, students are said to have ability that is considered as fixed and final.

Active Mathematics Teaching (AMT) as an instantiation of the school mathematics tradition

This section will review—Active Mathematics Teaching (AMT) of Good, Grouws and Ebmeier (1983), a mathematics education program that has its origin in process-product research. The focus in this section will be on key practices and beliefs associated with AMT, and their relation to the SMT (Cobb *et. al.* 1992; Gregg, 1995). Also, from the key practices and beliefs of AMT inferences will be made about its views of the nature of mathematics, espoused and enacted models of teaching and learning of mathematics and the use of mathematics texts.

Research on teaching, known as "effective teaching," blossomed in the early 1970s with the adoption of the "process-product" research tradition. The proponents of this research tradition sought to discover stable relationships between teacher behaviors and strategies—process variables—and measures of student outcome, most typically scores on standardized tests —product variables (Putnam, Lampert, Peterson, 1990, p.124). In process - product research the prototypical study was a correlational analysis of the relation between observed patterns of teacher classroom behavior and student performances on tests of achievement and attitude (Shulman and Quinlan, 1996, p. 402). The classroom behaviors explored were for the most part, straightforward and easily quantifiable, for example, time spent on questioning, "active learning time," amount of praise, amount of feedback (Schoenfeld, 1988, p. 147). According to Putnam, Lampert and Peterson (1990), there are several assumptions that go along with "effective teaching," and thus process-product research :

- Separation of teaching and content (p. 124).
- Mathematics knowledge defined as achievements on standardized tests (p. 125)
- Knowledge as separable into discrete parts (p. 127)
- Students' engagement as a measure of learning (p. 128)

These are the key practices and beliefs associated with effective teaching of the 1970s.

When Good, Grouws and Ebmeier (1983) developed "Active Mathematics Teaching" (AMT) through their Missouri Mathematics Program, they continued in the process-product research tradition by assuming that the best indicator of effective teaching is student achievement test scores and by focusing on instructional behavior that were virtually free of mathematics content (Putnam, Lampert, Peterson, 1990). Their resulting recommendations about teaching, such as "focus on meaning and promoting student understanding," or "assess student comprehension," might apply as well to the teaching of history or reading as to the teaching of mathematics. These rather generic recommendations suggest that teaching is a delivery system for content that is determined by others and specified in the curriculum (p. 125).

In their program Good *et. al.* (1983) made the assumption that knowledge can be decomposed into discrete entities, or fragments. This thinking reveals their view of the nature of mathematics. Mathematics as algorithms and procedures is an implicit conception of mathematics in AMT (Confrey, 1986, p. 354). They measured student learning with an achievement test that separated mathematics knowledge into three subtests: knowledge, skills and problem solving (Putnam, Lampert and Peterson, 1990 p. 128). The assumption is that within a grade level, students need "knowledge," followed by "skills" and then "problem solving," in that particular order. Such a view fits in well with the hierarchy of the different grade levels in school. Students' curricular experience with mathematics is thus premised on the notion of "prerequisites," which are almost always about rules and procedures taught in earlier grade levels. The understanding is that these rule, facts and procedures will help students "later on" in higher grade levels (Gregg, 1995, p. 461).

Related to the notion of "prerequisites" as procedures, rules and facts is the assumption that mathematics is linear. Mathematical knowledge is thought of as a "material" stock of "prerequisites" consisting of single elements that must first be learned in a cumulative way in one grade level in order to make later discoveries of the underlying structural relations and meanings possible (Steinbring, 1989, p. 29).

Confrey (1986) cites AMT as a model of direct instruction. Teachers teach on the basis of explicit procedures, rules and operational techniques, hoping to elaborate the mathematical concept and it meaning (Steinbring, 1989, p. 29). This is the espoused and enacted model of teaching mathematics in AMT. This kind of instruction is characterize by highly structured lessons. The espoused and enacted model of teaching in AMT is one where "teaching is a delivery system of content that is determined by others and specified in the curriculum" (Putnam, Lampert and Peterson, 1990, p. 125). Also, this separation of teaching and content in AMT implies the use of mathematics texts where there is no attention to the nature of mathematics. For example, there would be no emphasis on the "big ideas" within mathematics texts. The espoused and enacted model of learning mathematics in AMT has little to do with relational understanding (Skemp, 1978) because achievement on standardized tests are indicators of student learning.

In summary, AMT is a crystallization of the school mathematics tradition. The table below gives a summary of beliefs and practices associated with the school mathematics tradition and ATM.

Table 6

A summary of beliefs and practices associated with the school mathematics tradition

Nature of mathematics	**Teaching of mathematics**	**Learning of mathematics**	**Use of texts**	**Sociopolitical context**
A collection of isolated rules, facts, procedures and algorithms mathematics is linear prerequisites: mainly procedures knowledge, skills, problem solving mathematics knowledge decomposed into discrete parts Mathematics knowledge defined as achievement on standardized tests	direct instruction Lessons are highly structured lessons in order to maintain control and the constitution of mathematics objectives specified A separation of teaching and content	instrumental understanding rote learning Achievement tests are measures of student understanding and student learning Belief in students <u>ability</u> as fixed and final A separation of teaching and learning	Uncritical use of texts; Textbook as mathematical authority Producing the answers and procedures in the textbook	Issues related to the sociopolitical context within mathematics and the teaching and learning of mathematics are not explicitly addressed.

Other mathematics education traditions

The goal of this section is to review some of the literature on different mathematics education traditions. There are several such traditions that have the goal of affecting students' curricular experiences with mathematics. An examination of a mathematics education tradition's key ideas will reveal its views about

- the nature of mathematics,
- its espoused models of teaching and learning mathematics,
- its enacted models of learning and teaching mathematics, and
- the influence of the social context of teaching
- what it sees as an appropriate use of mathematics texts (Ernest,1987, p. 252)

The above is an adaptation of Ernest's (1987, p. 252) description of the relationships between beliefs and their influence on practice. The influence of the social context of teaching could include references to the sociopolitical context. A mathematics education tradition is an idealized state. This means that some traditions might not really be that distinct from others. They might have overlapping educational slogans or rallying symbols (Scheffler, 1960/1964). Individual practitioners might subscribe to a particular mathematics education tradition, but might not be captured completely by that tradition.

The following mathematics education traditions will be reviewed using an adaptation of Ernest's (1987, p. 252) chart: constructivist traditions, disciplinary traditions, ethnomathematics and critical mathematics. This list of traditions is in no way exhaustive. The traditions have been chosen partly because of their presence and absence in the South African context.

Constructivist traditions

In this section we will outline constructivism, which helps to illustrate what I call constructivist traditions. Second, we will examine constructivist traditions in relation to the nature of mathematics, espoused and enacted models of teaching and learning mathematics. Third, attention will be given to what constructivist traditions have to say about constraints and opportunities in the social context of teaching. This will include any reference to the use of mathematics texts. There will also be a focus on associated key phrases and words. Attention will be directed to individual constructivist practitioners whose work is not entirely captured by constructivism.

According to Zevenbergen (1996), constructivism espouses a view that knowledge is a private, individual construction. It is a perspective on knowledge and learning that suggests any one person's knowledge is not acquired from outside, but is constructed by that individual. It focuses on the individual construction of meaning (p. 97). It began as the epistemological position associated with Jean Piaget, namely, that a learner both incorporates novel experiences into existing mental structures (assimilation) and recognizes those structures to handle more problematic experience (accommodation). According to Simon (1995), we construct our knowledge of our world from our perceptions and our experiences, which are themselves mediated through our previous knowledge. The concern is whether it *works* (fits our experiential world) (p. 115). Von Glaserfeld (1995) refers to this as "viability," in keeping with the biological model of learning as adaptation developed by Piaget (1970).

Constructivist traditions are thus traditions that make intellectual arguments for, and that are partisan to, constructivism. It is a theory of knowledge acquisition, meaning it is about espoused and enacted models of learning. For example, a constructivist view of mathematics learning (Cobb & Steffe, 1983; Confrey, 1985) focuses on a learner's personal construction of mathematical knowledge. In such traditions there are also espoused and enacted models of teaching. According to Kuhs & Ball (1986, p. 2), a constructivist model of learning typically underlies a "learner-focused" view of mathematics teaching. Thus "learner-focused," or "learner-centered" is a key word or educational slogan (Scheffler, 1960/1964) in constructivist traditions. The idea is that all teachers should continually make a conscious attempt to "see" both their own and the children's actions from the children's point of view (Cobb & Steffe, 1983, p. 85). A constructivist teacher would initiate activities, and the child would reflect on and abstract patterns or regularities from these (Koehler & Grouws, 1992, p. 119). This differs from the school mathematics tradition and its taken-as-shared belief and practice that achievement on standardized tests are measures of student understanding (Gregg, 1995, p. 453). In the school mathematics tradition there is a separation of teaching and learning (Gregg, 1995, p. 464). Linking learning and teaching in constructivist traditions is thus a concerted move away from the school mathematics tradition. A related key phrase is "from a children's point of view" because of the need to focus on how the individual child constructs mathematical knowledge. Also, in constructivist traditions the teacher is viewed as a "facilitator" and a "stimulator" of student learning, posing interesting questions and situations for investigation, challenging students to think and to uncover the inadequacies in their own thinking (Kuhs and Ball, 1986). "Facilitator" and "stimulator" are thus key words in identifying constructivist traditions. They serve as rallying symbols (Scheffler, 1960/1964) regarding the role of the teacher. This role merges teaching and learning, which is not the case in the school mathematics tradition.

Views of the nature of mathematics in constructivist traditions differ from the school mathematics tradition. For example, Steffe (1991) stresses that the teachers' plans must be informed by the "mathematics *of* students." The most basic responsibility of constructivist teachers is to learn the mathematical knowledge of their students and how to harmonize their teaching methods with the nature of that mathematical knowledge (Steffe & Wiegel, 1992, p. 17). From these two quotes it is clear that in constructivist traditions mathematics is not viewed as a set of "cut and dried" procedures and algorithms. Steffe (1990) writes about the need for teachers "to develop an encompassing network of mathematical concepts that could deepen, unify, and extend *their* conceptions of school mathematics" (p. 167). This is similar to the notion of searching for "big ideas" within the fragmented school curriculum. The quotes also give an idea of how teachers who come from constructivist traditions might use mathematics texts when they teach. For example, they would make the mathematics texts "relevant" to the learning of individual students.

According to Zevenbergen (1996, p. 97) constructivism ignores "real-life" or "real world" conditions, like the wider socio-political context within which learning and teaching occurs and the implication of that learning beyond the formal school context. O'Loughlin (1992, p. 791) makes a similar observation when he notes that constructivism has limitations when it comes to issues of "culture, power, and discourse in the classroom." Constructivist approaches stress the ability to adapt to rather than to be critical towards or change society (Christiansen, 1999, p.20). Culture, power, and discourse in the classroom are thus not rallying symbols in constructivist approaches. The image of an individual knower is rife in constructivism, a point that is evident in the National Council of Teachers of Mathematics (NCTM, 1989) *Standards*. In this regard Bishop (1990, p. 367) makes the following comment:

> To read the *Standards* one would think that the *only* interaction that is important for a learner's mathematical development (and therefore the nation's mathematical development) is that it takes place between teacher and learner. (italics in original)

He goes further:

> The lived reality for the participants in the pedagogical interaction needs to be addressed in any educational reform process. It is ignored in the *Standards*; indeed the whole sociopolitical dimension is never referred to.

There are, however individual practitioners within constructivist traditions who have concerns about sociopolitical dimensions. For example, Confrey (1982) writes about the need to give students "access to knowledge" and how this access is closely related to how the content is selected and presented in classrooms by teachers. For example, she is concerned about how the question of teaching students more applications in mathematics or science "is seldom connected to any sociological argument of equity" (p. 15). She sees the instruction in a subject matter as closely related to the maintenance or change of the *status quo*. Hence, she advocates the merging of "instructional and social questions in a subject-matter-focused way" (p. 16).

In summary, in constructivist traditions there would be concerted efforts to focus on the sense-making of the individual student, with teacher as "facilitator" and "stimulator." These are examples of espoused and enacted models of teaching and learning mathematics. The nature of mathematics would be viewed from the perspective of the individual student and a concern for a network of unifying mathematical concepts. This same perspective would be adopted when it comes to the use of mathematics texts in teaching and learning situations. To some individual practitioners within constructivist traditions, the wider socio-political context would be considered as relevant in content and pedagogy.

Disciplinary traditions

In this section we will outline what I call disciplinary traditions. These traditions will be examined in relation to the ones I have discussed earlier and the school mathematics tradition. Attention will be given to views on the nature of mathematics, espoused and enacted models of teaching and learning mathematics, the influence of the social context of teaching and the use of mathematics texts. Specific approaches within disciplinary traditions in relation to algebra and calculus in the school curriculum will be pointed out. As in earlier mathematics education traditions, there will be a focus on related educational slogans (Scheffler, 1960/1964).

One can argue that mathematics education traditions characterized as disciplinary traditions have their origins in Dewey's famous phrase "psychologizing the subject matter" as their rallying cry (Scheffler, 1960/1964). Therefore in disciplinary traditions there would be intellectual arguments based on "psychologizing the subject matter." John Dewey (1902/1992) distinguished between the *logical* and *psychological* aspects of a subject. He began to think about the bridge the between the subject matter in the mind of the mature expert and the subject matter as it is prepared for the pupil. For Dewey, the subject matter was not something *other than* what humans beings learn, think, and explore; it was precisely the record and result of human voyages of discovery (quoted in Shulman and Quinlan, 1996, p. 402). Lampert and Ball as mathematics educators and in particular as scholar-practitioners have based their research programs on Dewey's notion of "psychologizing the subject matter."

Lampert (1988a; 1988b; 1990) contends that "psychologizing the subject matter" implies that any discourse about the teaching and learning of mathematics must first attend to the nature of mathematics itself. She thus makes a tight link between the nature of mathematics and the teaching and learning of mathematics. This is not the case with AMT espoused by Good Grouws and Ebmeier (1983), whose recommendations such as a "focus on meaning and student understanding and "assess student comprehension" might as well apply to the teaching of history or reading as to the teaching of mathematics (Putnam, Lampert & Peterson, 1990). Lampert first analyzes the subject matter, the mathematics, and then asks what kind of a psychology would suit it. Like Bruner (1960) and Schwab (1962), she is concerned with both the *substance* of mathematics—the skills, concepts, and strategies that constitute the "stuff" of the discipline —and the *syntax* of mathematics—the rules and procedures that mathematicians use to test, critique, and extend their knowledge (quoted in Shulman and Quinlan, 1996, p. 413). Lampert (1990) believes that new mathematics is brought about through "a process of 'conscious guessing' about relationships among quantities and shapes with proof following a 'zig-zag' path starting from conjectures and moving to the examination of premises through the use of counterexamples or 'refutations'"(p. 30). She bases her notion of mathematics as a discipline on her readings of Pólya and Lakatos.

Examples of some key phrases and words, and thus rallying symbols (Scheffler, 1960/1964) in Lampert's disciplinary tradition are "conscious guessing," "reasoning," "conjectures," "argument," "counterexamples" and the notion of "proof." Lampert (1990) articulates a position on the teaching of mathematics that deliberately alters roles and responsibilities of teacher and students, however. In her classroom students are encouraged to make conjectures, abstract mathematical properties, explain their reasoning, validate their assertions, and discuss and question their own thinking and the thinking of others (p. 33). These are examples of espoused and enacted models of teaching and learning mathematics. The model of teaching and learning she espouses is analogous to mathematics "in the making" (Pólya (1945/1988, p. vii). These ways of "doing mathematics" are not exclusive to disciplinary traditions. They overlap with constructivist traditions. These models of teaching and learning mathematics are very different from direct instruction and rote learning, as in the school mathematics tradition.

"Problem solving" is another key phrase in disciplinary traditions. It is also a phrase that is common to all mathematics education traditions. This educational slogan aims at bringing knowing and doing mathematics in school closer to the discipline. A related key word in the literature on mathematics as problem solving is "heuristics," or the process of discovering solutions to mathematics problems (Putnam, Lampert, Peterson, 1990). Pólya (1957, p. 30) explains that "heuristics endeavors to understand the process of solving problems, especially in the mental operations typically useful in this process. Knowledge of heuristics is elusive and explains why "problem solving" and "heuristics" function as educational slogans or rallying calls (Scheffler, 1960/1964).

Views on the nature of mathematics in disciplinary traditions are well-developed. For example, as a counter to the over-emphasis on procedures and discrete sections in the school mathematics tradition, many mathematics educators have promoted the notion of a network of "big ideas" or "unifying ideas" within the curriculum. This is a marked departure from the notion of "knowledge as separable into discrete parts" (Good *et. al.* 1983) or "proceduralizing and decomposing" (Gregg, 1995) the mathematics as in the school mathematics tradition. Much earlier Dewey called for a curricular focus around a few basic ideas, methods and dispositions (see Shulman & Quinlan, 1996, p. 416). On the same point, Whitehead (1911/1964), a prominent mathematician and philosopher, notes, with concern:

> The reason for the failure of this science [mathematics] to live up to its reputation is that its fundamental ideas are not explained to the student disentangled from the technical procedure which has been invented to present their exact presentation in particular instances. Accordingly the unfortunate learner finds himself (sic) struggling to acquire a knowledge of mass details which are not illuminated by any general conception (p. 1).

Students and teachers are thus unable to adequately answer a question like, "what are the 'big ideas' or 'general conception' in the mathematics?" because of the many sections and procedures that predominate.[1]

At the elementary level some examples of "big ideas" in developing a principled understanding of multidigit multiplication include the notion (a) that the way digits are lined up in a number has meaning, (b) that all quantities are compositions of other quantities, (c) that one can recompose problems into sets of more easily manipulated subproblems (e.g. 78 - 33 converted to a more solvable [70 - 30] + [8 - 3], (d) that the order in which multiplication is done does not affect the final product, and so on (Lampert, 1986).

A conception of "school algebra" with its focus on algebra as the study of structures (Usiskin, 1988, p. 15) is common in the school curriculum in South Africa and in the United States. Students typically learn through memorization different and often difficult rules and procedures for factoring in algebra as the study of structures. One possible reason for teaching and learning algebra in this way is that consensus and stability (Popkewitz, 1988) are easily reached. In other words, it becomes easy to reach agreement about the "right" factors or procedures in general. Mathematics educators like Pimm (1995) and Sfard (1995) promote the study of equations as a "big idea" in school algebra. Chazan (2000), on the other hand, makes an argument for a relationships between quantities, or functions, as the mathematical object of study in school algebra. He proposes that equations be viewed as the comparison between two functions, of the form $f(x) = g(x)$? (see Chazan, 1993a). In his "functions approach" Chazan (1993b) has worked on ways of making school algebra "relevant" to his low-achieving track students' "real possibilities and interests." In the school mathematics tradition,

[1] The notion of "big ideas" is also prominent in the philosopher of the Coalition of Essential Schools which, has "less is more" as one of its "ten common principles" (Sizer, 1984)

functions are trivialized and usually presented to students in definitional form, with the characteristic uniqueness of range-domain pairing (Confrey, 1981). Curricular moves that draw on disciplinary traditions, such as a focus on "big ideas" in school algebra, should be seen as a way toward helping students identify mathematical objects they are working with. They are a response to what Whithead (1911/1964) calls a "general conception" and certainly a way to counter practices and beliefs associated with the school mathematics tradition. School algebra mathematics texts in disciplinary traditions would take on a "psychologizing the subject matter" perspective (Chazan, 1993b; 2000).

In the case of calculus, Schnepp *et. al.* (2000), drawing on the work of Thompson (1994), take *rate* and *accumulation* as a "story line" or "big idea" for an experimental and inductive way of teaching and learning of calculus. In the school mathematics education tradition, in the case of calculus there is a focus on what Tall (1987, p. 54) calls the "surface structure" of symbolic manipulation to obtain formal derivatives and integrals. Schnepp *et. al.* (2000) view calculus as dealing with the interplay between the amount of something and its rate of change. The "differential" direction goes from amounts to rates. The "integral" direction goes from rates to amount. More specifically, the Fundamental Theorem is a statement to the effect that one can "recover" the amount by accumulating its rates of change (Bill Rosenthal, 2000, personal communication). In this way Schnepp *et. al.* (2000) "psychologize the subject matter," calculus, by representing it in ways that are "relevant" to where students are and what they know. These researchers base much of their argument on the historical development of calculus (see Grabiner, 1983). Also, from a disciplinary and historical perspective, calculus developed through a dialectic sequence in which geometric approaches were contrasted to arithmetic approaches (Thom, 1971). Their approach is synonymous to calculus "in the making" (Pólya (1945/1988, p.

vii). Schnepp *et. al.* (2000) downplay formalized rules for finding limits, derivatives and integrals. Their espoused and enacted models of teaching and learning (Ernest, 1987) calculus fall "outside" the practices and beliefs in the school mathematics tradition.

In summary, the disciplinary traditions outlined draw on Dewey's "psychologizing the subject matter" as a rallying cry (Scheffler, 1960/1964). They aim at a tight relationship between the nature of mathematics and its historical and disciplinary development, and espoused and enacted models of teaching and learning of mathematics. They emphasize that mathematics texts be mindful of a disciplinary development of mathematics. Individual practitioners within disciplinary traditions direct their attention to the wider socio-political context (see Rosenthal, 1995; Chazan, 1993b & 2000). It cannot be claimed that disciplinary traditions *per se* are attentive to sociopolitical conditions.

Ethnomathematics and critical mathematics education traditions

Many mathematics educators have expounded on the notions of ethnomathematics and critical mathematics education and their implications for the school curriculum. What follows is an elaboration of the two traditions with the goal of identifying their key phrases and words or educational slogans (Scheffler, 1960/1964). A third and related tradition, namely a combination of the two called a "social, cultural and political approach" (SCPA) (Vithal, 1997), will also be identified through key phrases and words. Attention will be given to the nature of mathematics, espoused and enacted models of teaching and learning mathematics, the use of mathematics texts, and the influence of the wider sociopolitical context. These mathematics education tradition are prominent in South Africa.

According to Borba (1990), "ethno" refers to socio-cultural groups and not to the anachronistic concept of race, and "mathematics" should be seen as a set of activities such as ciphering, measuring, classifying, ordering, inferring and modeling (p. 40). D'Ambrosio (1985) defines ethnomathematics as the mathematics practiced among identifiable cultural groups, such as national-tribal societies, labor groups, children of a certain age bracket, professional classes, and so on. Borba (1990) includes the mathematics produced by professional mathematicians as a form of ethnomathematics because it is produced by an identifiable cultural group, and it is not the only mathematics that has been produced. D'Ambrosio and Borba give a very broad conceptualization of ethnomathematics and one that has implications for the school curriculum, especially where learners have diverse backgrounds. Likewise, Gerdes (1988) another proponent of ethnomathematics, has argued that the mathematics curriculum should be imbedded into the cultural environment of the pupils (p. 35). His view matches what some call the "real world" of students. It is part of a broader framework that elaborates the social and political dimensions of mathematics and mathematics education, but especially dimensions of "culture" (Vithal & Skovsmose, 1997). What is thus evident in these traditions is the primary concern for the wider sociopolitical context.

One recognizes an ethnomathematics tradition through its key words and phrases, for example, "culture," "cultural contexts, environment and artifacts," and "socio-cultural groups." These phrases and words are definitive in identifying an ethnomathematics tradition from the works of D'Ambrosio (1985; 1990), Borba (1990), Gerdes (1988) and others. They are used as rallying symbols to draw attention to "real world," "real life" or "relevance." These educational slogans should be seen as particular reactions to the stable, static and strongly classified nature of mathematics in the school mathematics tradition.

Skovsmose (1994) is an important advocate of what he calls critical mathematics education. It has also been developed as a school mathematics education tradition with particular curricular experiences in South Africa (see Vithal, 1997). For example "thematization" and "project organization," to use Skovsmose's (1985) terminology, are ways many authors have found of breaking the atomization of traditional curricula as in the school mathematics tradition. This terminology illustrates a view of the nature of mathematics (Borba, 1990, p. 42) and espoused and enacted models of teaching and learning mathematics. In "thematization" and "project organization," the themes and/or projects to be developed are decided by both students and teachers. The themes are not necessarily "mathematical" or "biological" or "artistic." One important emphasis in the critical mathematics education is to avoid "mystification" (Skovsmose, 1985). In other words, students should not see mathematics as an oppressive and all-powerful realm of knowledge disconnected from their daily experiences.

Two key notions in critical mathematics education are "critical citizenship" and the "formatting power of mathematics" (Skovsmose, 1994; Vithal & Skovsmose, 1997). The first is about educating students in schools to become "critical citizens," prepared to take risks and to challenge and to believe that their actions can make a difference in the larger society (Skovsmose, 1997, p. 24). "Critical citizenship" thus ties in with the idea of avoiding "mystification." The second is about understanding how mathematics provides a way of "looking" and a way of "doing," i.e. it has "formatting power." This is said to happen especially in a highly technological society, as in the "first world." As societies become more technological, people are having to act in a system that imbeds increasingly in more complex mathematics without explicitly knowing the underlying mathematical abstraction processes (Vithal & Skovsmose, 1997, p. 143). The result is that one has to trust a machine, a specialist, an "expert" or an institution. Mathematical abstractions such as tax formulas, interest rates and the "minimum wage" are developed and determined by lawmakers and "experts." These abstractions involve a fair amount of mathematics that is known to only a few. Many people's lives are affected by such abstractions. In this sense mathematics is formatting society. Thus key phrases and words in identifying a critical mathematics education tradition are "critical," "critical mathematics," and "critical understanding." Also included are any references that call into question social, economic and political conditions, not necessarily through "mathematical" means because the notion of "mathematical" and its "formatting power" in society is called into question (Skovsmose, 1994).

While ethnomathematics has an overt focus on "culture," critical mathematics education has a focus on the "political." As separate mathematics education traditions they have been questioned by Vithal and Skovsmose (1997) in their relevance to the South African context. Under apartheid South Africans were legally segregated and divided along racial and ethnic lines, which affected every aspect of their lives. An ethnomathematics tradition that valorizes the mathematics of different cultural groups, has been called into question, because it is seen as a reminder of racial and other divisions under apartheid. For this reason Vithal (1997) has proposed a "social, cultural and political approach" (SCPA) to the school mathematics curriculum. This approach should be seen as a merger between critical mathematics education and ethnomathematics traditions (see Vithal, 1997; Vithal & Skovsmose, 1997).

Although images of ethnomathematics and critical mathematics education point in the direction of "culture" and sociopolitical contexts, there are individual practitioners in these traditions who are active in "pure" mathematics. For example, practitioners like Ubiritan D'Ambrosio, Marcelo Borba, and Paulus Gerdes are active in research in mathematics.

In summary, ethnomathematics, critical mathematics traditions and Vithal's SCPA (1997) address issues in the curriculum that constructivist and disciplinary traditions do not necessarily address. They focus on the wider cultural and socio-political context within which teaching and learning occurs and the implications of that learning beyond the formal school context. Thus they stress that espoused and enacted models of teaching and learning mathematics be firmly situated in the sociopolitical context. They would place a similar emphasis on mathematics texts and their use. Practices and taken-as-shared beliefs in the school mathematics tradition do not address such issues. These traditions make a serious argument for a mathematical *education* and downplay mathematical training (see Bishop, 1990).

Summary

Ernest's (1987) illustration of the relationships between beliefs and their relationships to practice are helpful in organizing this chapter. A review of literature on mathematics teaching (Schifter, 1995) shows relationships between teacher knowledge, teacher belief and the school mathematics tradition. In the school mathematics tradition mathematics is viewed as an isolated collection of rules, algorithms and procedures. Mathematics knowledge is separated into discrete entities. In this tradition the espoused and enacted model of teaching is direct instruction. The espoused and enacted model of learning is rote learning and thus instrumental understanding (Skemp, 1978; 1987). There is a separation of teaching and learning, and content and teaching. No serious attention is given to mathematics texts as representative of disciplinary knowledge. Reviewing this body of literature enables one to see teachers' talk and classroom practice in relation the school mathematics tradition.

A review of the literature on constructivist traditions shows their emphasis on the knowledge acquisition of the individual learner. There is also an emphasis on the teacher as "facilitator" and "stimulator" and on possible unifying ideas in school mathematics. Concerns for sociopolitical contexts come mainly from individual practitioners who subscribe to constructivist traditions. In disciplinary traditions, connections between the student, the teacher and the mathematics are key. These traditions can be traced to Dewey's "psychologizing the subject matter." Individual practitioners of disciplinary traditions do show interest in sociopolitical contexts, as in the case of constructivist traditions. A review of the literature on ethnomathematics and critical mathematics education traditions indicates there foci are on "cultural" and "political" contexts, respectively. Individual practitioners in these two traditions do direct their attention to "pure" mathematics. In the case of Vithal's work, the focus is on the "social, cultural and political." It is difficult to claim that Vithal's (1997) work incorporates the nature of mathematics in a way that say disciplinary traditions do. Throughout the review of these mathematics education traditions, there is the tension between the work of individual practitioners and the tradition.

There are many mathematics education traditions that have not been reviewed. For example, there is the work of the Freudenthal Institute, called Realistic Mathematics, and traditions that draw on anthropological outlooks.

CHAPTER 3
DATA COLLECTION AND METHODS

Overview

The main goal of this chapter is to give details about the methods that were used to answer the central research question in this study: What are the similarities and differences between the rhetoric of teachers and policymakers? This chapter begins with a description of how the study evolved from its proposal to its present form. There are two sets of data in this study, namely, transcribed interviews with the subjects and a set of policy documents on *Curriculum 2005*. In the interview data there is the teacher rhetoric and in the policy documents there is the policymaker rhetoric. The methods are of two kinds, namely, data collection methods used while in the field and methods of analysis.

Data collection methods used in the field for the original study include notes on who the teachers in the sample were, how they were selected, and descriptions of the interviews with the subjects. Methods of analysis and data presentation include descriptions of how the texts—the mathematics component of *Curriculum 2005* and the transcribed interviews with the teachers—were analyzed. Details will be given about what is meant by teacher rhetoric and how it was identified in the interview data. Second, details will be given about what is meant by policymaker rhetoric, and how it was recognized in the mathematics component of *Curriculum 2005* and in different mathematics education traditions. Third, there will be an analysis of my own rhetoric, which made it possible to do an indirect juxtaposition of teacher and policymaker rhetorics. These three points illustrate what the three core chapters in the study are about.

Evolution of the study

The original research question in this study was, what conceptions of mathematics and theories of teaching and learning mathematics do qualified and experienced South African mathematics teachers hold? The original study was thus a cognitive approach about the beliefs and knowledge of individual teachers. Upon returning from the field, the focus of the study shifted to a comparison between mathematics teachers' rhetoric and the rhetoric in *Curriculum 2005* policy documents on "Mathematical Literacy, Mathematics and Mathematical Sciences."

When and why did the focus of the study change? On November 11, 1998 at a meeting of the Mathematics Learning and Research Group (MLRG) at Michigan State University, I presented an initial analysis of excerpts from my data on teacher interviews. In the presentation I shared excerpts of individual teachers' thinking about the teaching and learning of mathematics. MLRG members and attendees recognized the teachers' language in the excerpts as common to the ways mathematics teachers speak about their work. For example, in the excerpts there were words and phrases like, products, factors, application, "discovery methods," students' ability, exercises, investigation, sections, real world and "show your work." From there evolved the notion of "teacher rhetoric." MLRG members and attendees pointed out these excerpts are about the discourse and "traditions" of teachers.

The mathematics component of *Curriculum 2005* has many educational slogans (Scheffler, 1960/1964) key phrases and words that are associated with different mathematics education traditions. Thus, I realized that it was possible to eventually do a comparison between the discourse of the teachers and policymakers. Such a comparison would be useful in terms of seeing how the rhetoric in *Curriculum 2005* would be understood by teachers at the classroom level. The significance of doing a comparison between teacher rhetoric and policymaker rhetoric in the mathematics component of *Curriculum 2005* was spelled out in chapter 1. One limitation in such an approach to data analysis is that I could not say what individual teachers were saying and thinking. Besides, claims about individual teachers are problematic because of the short period in the field and the fact that I conducted only four interviews per teacher. Individuals are multidimensional, meaning that my claims would be specific only to the moment when I conducted the interview.

Who were the subjects in the study?

The rows in table 7 give details about each of the six subjects. Teachers A, B and C are high school teachers, while C, D and E are middle school or junior primary school teachers. The teachers' academic qualifications and teacher certification, their number of years as teachers and the location of their particular schools, relative to Cape Town, are given. A "teaching diploma" is the formal certification that is required by education authorities at the end of a bachelor of arts or a bachelor of science degree. It amounts to one year of courses in the sociology, philosophy and psychology of education, and subject "methods" courses. This is the case with high school teachers. Many middle school teachers obtain their "teaching diploma" or "certificate" through a "teacher training college," something similar to a "normal school," as in the earlier part of the nineteenth century in the United States. Model C schools are former "whites only" schools that are usually well-funded and located in the suburbs of Cape Town (see Chapter 1).

Table 7

Details about the teachers in the sample

Subject	Gender	Type of School	Qualification	Number of years teaching	Location of school, relative to Cape Town
A	Male	High School	BS and teaching diploma	7	Urban, former "Coloured" school
B	Male	High School	BS and teaching diploma	14	suburb Model C
C	Male	High School	BS no teaching diploma	7	Suburb, private/ religious in a "Coloured" and "Indian" area
D	Female	Middle School	BS and teaching diploma	4	Suburb, private/ religious Model C
E	Female	Junior Primary School	Teachers college (normal school)	7	Suburb, "Coloured"
F	Male	Junior Primary School	Teachers college (normal school)	8	Suburb, "Coloured"

How were the subjects contacted?

During 1997, about a year before going to Cape Town, South Africa, I negotiated with workers at a mathematics curriculum project to help me find subjects for my study. Although I had assurances from the project workers about possible subjects, things did not work out as planned. When I arrived in Cape Town, around the middle of May 1998, I spent several weeks trying to find teachers who would be willing to participate in my study. I personally called principals and asked friends who are in mathematics education to help me. My efforts proved fruitful around the beginning of June and I managed to recruit three middle school or "junior primary" and three high school mathematics teachers.

Data collection methods while in the field

In this section are details about the interviews with the teachers, such as how many and the nature of the interviews, and details about the kinds of education policy documents I collected.

I conducted four interviews with each of the six teachers. The first interview focused on biographical details about the teachers and their experiences with mathematics. I also had them describe someone who is good and not good at mathematics (see Appendix A). The second interview was structured around their responses to Likert scale items that I had adapted from *A study package for examining and tracking changes in teachers' knowledge* (Kennedy, Ball & McDiarmid, 1993) (see Appendix B). The third interview was structured around any examination papers that they had drawn up (see Appendix C). With two of the middle school teachers I used reform-consistent middle grades mathematics curriculum materials from the United States, the *Connected Mathematics Project,* as a means to have teachers talk about the teaching and learning of mathematics. The fourth and final interview involved a classroom observation and a post-observation conference (see Appendix D).

In the first interview, the reason for having the teachers share their experiences with mathematics was to get familiar with them and have them espouse their thinking about mathematics as a cultural phenomenon and as a school subject. Whatever they said during the remaining interviews gave me ideas about their views of mathematics and their thinking about the teaching and learning of mathematics. When they described someone who is good and not good at mathematics, I gained insights into their thinking about the nature of mathematics. For example, do they see mathematics a set of isolated collection of rul

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rules, facts and procedures, or as a domain of inquiry, where students should have opportunities to see relationships between mathematical ideas and reconstruct mathematical ideas? In the second interview on their responses to Likert scale items, I had another opportunity to probe and find out their views about the nature of mathematics and the teaching and learning of mathematics. I had a similar opportunity in the third interview, which focused on their examination papers. In the case of the reform-minded curriculum materials from the United States, another opportunity arose where they espoused their views about the nature of mathematics and the teaching and learning of mathematics. The same is the case with the fourth interview where I observed the teachers and then interviewed them afterwards about their teaching.

While in the field I learned that the South African government had recently launched its new curriculum policy, *Curriculum 2005.* I collected the following education policy documents:

- *Curriculum 2005, Lifelong Learning for the 21st Century. A User's Guide,* printed in 1997
- *Curriculum 2005, Lifelong Learning for the 21st Century,* printed in 1997

These are informational booklets that aim at popularizing *Curriculum 2005.* Another document printed during the same year is,

- *Outcomes Based Education in South Africa. Background Information for Educators, Discussion Document,* March 1997

This document gives an explanation of terminology related to outcomes based education.

- *Government Gazette,* no. 18051, of June 6th, 1997, issued by the South African government printers.

This policy document has an in-depth elaboration on the "specific outcomes," "assessment criteria" and "range statements" related to "Mathematical Literacy, Mathematics and Mathematical Sciences" (MLMMS), and the remaining seven school subjects, referred to as "learning areas."

The data consists of two kinds , namely, transcribed interviews with three middle school and three high school teachers and a set of policy documents on *Curriculum 2005.*

Methods of analysis

In this section are descriptions of the analysis and writing parts of this study. There are notes on the notions, teacher rhetoric and policymaker rhetoric, and how they were identified in the interview data and policy documents, respectively. Second, is an analysis of my own rhetoric as found in the interview data. This analysis enabled me to do an indirect juxtaposition of teacher rhetoric and policymaker rhetoric.

What is meant by teacher rhetoric?

For the purposes of this study the notion of "teacher rhetoric" includes any talk in the transcribed interviews where teachers make references to the following :

- their views of the nature of mathematics
- their espoused models of learning mathematics
- their espoused models of teaching mathematics
- their talk about constraints and opportunities provided by the social context of teaching
- their enacted models of learning mathematics
- their enacted models of teaching mathematics
- their use of mathematics texts

This list comes from Ernest's work (1987) on the relationship between beliefs and their influence on practice (p. 252).

Teachers' talk about their views of the nature of mathematics, their espoused and enacted models of learning mathematics and their espoused and enacted models of teaching mathematics would seem to mean that representing their rhetoric is "obvious." So why include all of the above in the notion of teacher rhetoric ? Bauersfeld (1980) writes about deficient areas of research, which he calls mathematics education's "hidden dimensions." One such area is the fact that the teaching and learning of mathematics is realized in institutions such as schools and classrooms, which society has set up explicitly to produce shared meanings among their members. Institutions are represented and reproduced through their members and that is why they have characteristic impacts on human interactions inside of the institutional (Bauersfeld, 1980, p. 35). Schools and classrooms have "hidden dimensions," for example, the school mathematics tradition. When teachers wish to break away from the taken-as-shared beliefs and practices of the school mathematics tradition, they potentially come into conflict with the social context of teaching. They could either get support or could face challenges. It is therefore imperative to include in the notion of "teacher rhetoric" teachers' talk about constraints and opportunities provided by the social context of teaching. For example, through their talk, teachers could reveal difficulties they encounter in realizing and bridging their espoused and enacted models of learning and teaching mathematics. They could reveal tensions, frustrations, struggles and innovations they have thought of as they try to move beyond the school mathematics tradition.

A word has to be said about the use of the word "rhetoric" as in teacher rhetoric. In this case it should not be viewed as in the usual "rhetoric and reality" distinction. It is about teachers talking about their practice and their work. In *Life in Classrooms*, Jackson (1968/1990) notes that one of the most notable features of teachers talk is the absence of a technical vocabulary. Unlike professional encounters between lawyers, engineers and garage mechanics, when teachers talk together almost any reasonably intelligent adult can listen in and

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comprehend what is being said (p. 143). Similarly, Lortie (1966) states that teachers have no "technical rhetoric" available to them (p. 62). Members of the Mathematics Learning and Research Group (MLRG) meeting on November 18, 1998, were able to identify a "technical rhetoric" in the teacher talk in my presentation as the talk of mathematics teachers.

What is meant by policymaker rhetoric?

For purposes of this study, policymaker rhetoric is the arguments and language reformers use when they want to impact school practice and implement education policy. It refers to all those warmly and habitually repeated words and phrases that are found in education policy documents. I shall illustrate what it is by referring to several education policy documents and the literature on educational slogans (Scheffler, 1960/1964).

Education policy documents are usually issued by central, state and local governments, education departments and education organizations. Examples in the United States are *Curriculum and Evaluation Standards for School Mathematics* (NCTM, 1989) and *Professional Standards for Teaching Mathematics* (NCTM, 1991), two documents commonly referred to as the *Standards*. These documents are issued by a national organization of mathematics teachers and teacher educators. Many states in the United States have developed their own policy documents that they have aligned with the *Standards*. In the state of California there is the policy document, "Mathematics Framework for California Public Schools," generally referred to as the *California Frameworks* (1985). In the United Kingdom there is *National Curriculum*, which has a mathematics component called *Mathematics in the National Curriculum* (DES, 1988). In South Africa one finds at present several education policy documents on *Curriculum 2005*, the central government's new curriculum policy. These documents pertain to the teaching and learning of school subjects, or "learning areas," such as history, mathematics, biology, and so on.

Education policy documents can be regarded as a *slogan system* (Komisar & McClellan, 1961) with a certain appealing rhetoric. Slogan systems use key phrases and words that capture a broad interest and have a popular appeal, making it difficult for outsiders to disagree with the arguments in them. Thus, by searching for key phrases and words within education policy documents, one can identify "policymaker rhetoric." For example, in the case of *Curriculum 2005,* there is the educational slogan (Scheffler, 1960/1964) "learner-centeredness." In the South African context this slogan is persuasive, has emotive force and is closely related to the "climate of opinion" (Eastwood, 1964). For example, in 1994 South Africa had its first-ever democratically held elections. Thus, the focus in education policy on "learner-centeredness" where learners are broadly defined suits the "climate of opinion." It signals a focus on another rallying symbol (Scheffler, 1960/1964, p. 37) in the post-apartheid South Africa, namely, "redress and reconstruction." In a sense these slogans "grab us" (Apple, 1992). "Learner-centeredness" is at one and the same time *vague* enough to attract theorists and sufficiently *specific* to direct the practitioner (Komisar & McClellan, 1961, p. 212). This explains a proliferation in current research in education on what this slogan might mean in the South African context.

Educational slogans when repeated become influential among teachers in popularized versions (Scheffler, 1960/1964, p. 37). The literature on educational slogans (Scheffler 1960/1964 ; Komisar & McClellan, 1961) sharpens one's focus on those habitually and warmly repeated phrases and words in policy documents. For example, many mathematics teachers talk about teaching mathematics as "problem solving." Educational slogans in policy documents enable one to identify particular mathematics education traditions (see Chapter 2).

How were different mathematics education traditions defined and recognized?

Ernest's (1987, p. 252) chart enabled me to characterize different mathematics education traditions through their views on the views of mathematics, espoused and enacted models of teaching and learning mathematics, their ideas about mathematics texts and any comments they have on the sociopolitical context. Also, the research of Cobb *et. al.* (1992) and Gregg (1995) was helpful in outlining the practices and taken-as-shared beliefs of the different mathematics education tradition in relation to the school mathematics tradition. These traditions were recognized through a search for key phrases and words that were identified as educational slogans (Scheffler 1960/1964 ; Komisar & McClellan, 1961) in the different traditions (see Chapter 4).

An analysis of my own rhetoric

A careful examination of my questions and prompts during the interviews shows that my rhetoric lies in the direction of disciplinary traditions (see Chapter 2). This tradition is characterized by a concern for "big ideas" within the curriculum (Dewey, 1902/1992; Whitehead, 1911/1958; Lmpert, 1986; Ball, 1988; Chazan, 2000; Schnepp *et. al.* 2000), mathematics "in the making," (Pólya (1945/1988), relational understanding (Skemp, 1978) and teaching in ways where students make conjectures, test the validity of their assertions, and discuss and question their own thinking and the thinking of others (Lampert, 1990). Disciplinary traditions have "psychologizing the subject matter" as a focus, thereby attempting to transcend the dichotomy between school mathematics and mathematics as a discipline.

Below is a list of questions and prompts I used during interviews with my subjects. They have been categorized in a way to show particular aspects of what I called disciplinary traditions (see Chapter 2) :

Transforming mathematics for the purposes of teaching, a focus on mathematics "in the making"(Pólya (1945/1988) :

- What do you use as indicators for you to really work at the mathematics in order for the students to understand?

A concern for relational understanding (Skemp, 1978; 1987) :

- So what do you think of that as a way of learning rules and why?
- Do you think it would help the lower track students when they learn mathematics that they be able to provide reasons to support their answers?
- To be good at mathematics in school, how important do you think it is for students to be able to provide reasons to support their solutions?

A concern for "big ideas" and multiple representations within the strongly classified curriculum (Dewey, 1902/1992; Whitehead, 1911/1958; Lampert, 1986; Ball, 1988; Chazan, 2000; Schnepp *et. al.* 2000):

- Okay. In grade seven, what are some of the big ideas, the big mathematical ideas that students are exposed to?
- Have you thought of alternate ways to address those problems?
- Have you thought of alternate ways of approaching the very same one, $2x + 5 = 13$?

Awareness of mathematical relationships, similar to "big ideas" :

- When you say "practice" for the student, should the practice be one kind of so-called problems and then another kind or should they a kind that runs through and shows some deeper connections between these different ones?
- I'm also thinking of a case where a student asks you what previous mathematics does this bit of work connect with?
- What's the prior knowledge that students need to do this?

- What do you want your students to take away from calculus? Calculus as what? What do you want them to understand calculus to be all about?

Awareness of mathematics as a school subject and mathematics as a discipline:

- You said school math and math, do you make a distinction?

In summary, it is evident that my rhetoric lies mainly in the direction of disciplinary traditions.

How was the juxtaposition between teacher rhetoric and policymaker rhetoric done?

During interviews with the teachers I never had conversations about *Curriculum 2005* policy documents. The juxtaposition between the two discourse communities, teachers and policymakers, is therefore indirect. This juxtaposition was done by first doing an analysis of my own rhetoric. My prompts and probes during the teacher interviews reveal that my rhetoric lies in the direction of disciplinary traditions. The indirect juxtaposition occurs thus at those excerpts where my rhetoric points in the direction of disciplinary traditions. These traditions are also present in the mathematics component of *Curriculum 2005.* For example, "mathematical relationships" or "big ideas" and "mathematics as reasoning" are central notions in disciplinary traditions (see Chapter 6).

Data presentation and analysis

The core chapters will be organized in the following order:

- Chapter 4: Policymaker rhetoric in the mathematics component of *Curriculum 2005*
- Chapter 5: Evidence of the strength of the school mathematics tradition among the teachers in the sample and constraints on teaching
- Chapter 6: An indirect juxtaposition: Teachers and talk about mathematical relationships and mathematics as reasoning

The central research question, what are the differences and similarities between the two rhetoric communities, is answered in this order.

Chapter four begins with a description of the mathematics component of *Curriculum 2005.* The different mathematics education traditions are identified through key phrases and words. Chapter five deals with the differences in the two rhetoric communities. In this chapter we will first examine of teacher rhetoric about the mathematics curriculum. This is followed by teacher rhetoric about ability, and then, the social context of teaching. In this chapter high school and middle school teacher rhetoric is not analyzed separately because the mathematics curriculum is strongly classified across and within grade levels. Teachers' talk about ability is also common across grade levels. There is no particular advantage in separating high school and middle school teachers when it comes to the social context of teaching because constraints in bureaucracies such as schools are common. Chapter six illuminates the nature of the similarities in the two rhetoric communities. In this chapter high and middle school teacher rhetoric on mathematical relationships and reasoning is analyzed separately, the being to eventually challenge the notion of "prerequisites" as procedures in the school mathematics curriculum. Another reason for a separate analysis of high school and middle school rhetoric is that *Curriculum 2005* only applies up to the end of the ninth grade. The analysis of high school rhetoric is therefore more future-oriented regarding the curriculum policy *Curriculum 2005.*

The last chapter has several parts—first a summary of the nature of the results of the present study. This takes the form of claims about the rhetoric in the mathematics component of *Curriculum 2005,* the rhetoric of the teachers, and the effect of the interview process. Second, there is a reflection on the nature of the results. This reflection takes the form of an conversation among an education researcher, a mathematician, an ethnomathematics educator, and a critical mathematics educator and me. The goal of the reflection is to highlight the significance of the results and therefore the challenges that lie ahead in school mathematics reform in South Africa.

CHAPTER 4

POLICYMAKER RHETORIC IN THE MATHEMATICS COMPONENT OF *CURRICULUM 2005*

Overview

The data corpus in this study consists of a set of policy documents on *Curriculum 2005* and a set of teacher interviews. The research question that drives this study is, what are the differences and similarities between the rhetoric of policymakers and teachers? To this end, the goal of this chapter is to analyze the rhetoric in the policy document, namely, the mathematics component of *Curriculum 2005.* The chapter begins with a description of the mathematics component of *Curriculum 2005.* Then follows a brief comparison between the mathematics component and mathematics in the National Curriculum in the United Kingdom. Thereafter, the mathematics component is analyzed by identifying key words and phrases that are peculiar to different mathematics education traditions. These words and phrases show different directions that policymakers imagine the curriculum policy to go. This is followed by an examination of what policymakers are not saying in the mathematics component. Finally, there is an overall summary.

An analysis of the mathematics component of *Curriculum 2005*

This section describes and analyses Mathematical Literacy, Mathematics and Mathematical Sciences (MLMMS), the mathematics component of *Curriculum 2005.* Key words and phrases are examined to show their origins, meanings and implications for classroom practice.

The mathematics component of *Curriculum 2005* is called the "learning area," Mathematical Literacy, Mathematics and Mathematical Sciences (MLMMS). In the *Government Gazette* (6 June, 1997, no. 18051, pp. 113 - 135) there is a detailed outline of MLMMS that includes a "definition" of m

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mathematics, a "rationale," and ten "specific outcomes" with "assessment criteria" and "range statement" for every specific outcome. Each of the ten specific outcomes has assessment criteria and several range statements for the "Foundation Phase," (Grades 1 - 3), "Intermediate Phase" (Grades 4 - 6) and "Senior Phase" (Grades 7 - 9). Grades 10 - 12 falls under "Further Education and Training." Although the learning area is only applicable up to grade 9, *Curriculum 2005* in fact goes up to the year 2005, when it will be introduced at the 12th grade level (see DE 1997a; 1997b)

Concepts like "learning area," "specific outcomes," "assessment criteria" and "range statement" are key in terms of identifying the origins of *Curriculum 2005* in the literature on "outcomes," "learning" and the "curriculum," and aligning education and training (Jessup, 1991; 1995). Specific outcomes describe the "competence" that learners should be able to demonstrate in specific contexts and in particular areas of learning at certain levels. They focus on learners and express levels of outcomes in relation to skills and knowledge that learners should attain. They serve as the basis for assessing the effectiveness of learning processes (DE, 1997b). Learners are broadly defined and include out-of-school youth and adult learners and, of course, students in school. Specific outcomes are the products of learning, the outcomes. Assessment criteria are derived directly from the specific outcome and state the evidence an educator, usually referred to as a teacher, should look for in order to decide whether a specific outcome has been achieved. Range statements indicate the scope, depth, and parameters of a learner's achievement. They give some idea of the level of complexity in the teaching and learning situation (DE, 1997b).

What could be the intent behind such a long name, Mathematical Literacy, Mathematics and Mathematical Sciences? "Mathematical literacy" evokes images of adult learners and out - of - school youth struggling to become literate in mathematics in a broad sense. This mathematics could include ideas and activities that these learners encounter and do outside of formal schooling or in the workplace, where one deals with issues related to "training." "Mathematical sciences" evokes images of mathematics in the "hard sciences," like physics and astronomy, where statistics and probability, numerical analysis and so on are used. One normally associates these areas with issues related to "education." Thus a defensible inference one can make is to say that the intent of policymakers in calling the learning area Mathematical Literacy, Mathematics and Mathematical Sciences is to bridge a "rigid division" between academic and applied knowledge, theory and practice and knowledge and skills (see DE, 1997a, p. 1; 1997c, p. 9). Such a "bridge" is consistent with the focus on "education and training," two concepts that are always mentioned together in *Curriculum 2005* publications.

The learning area, MLMMS, has a "definition" for mathematics:

> Mathematics is the construction of knowledge that deals with qualitative and quantitative relationships of space and time. It is a human activity that deals with patterns, problem solving, logical thinking, etc., in an attempt to understand the world and make use of that understanding. This understanding is expressed, developed and contested through language, symbols and social interaction (*Government Gazette* (6 June, 1997, no. 18051, p. 115).

This definition contains several educational slogans (Scheffler, 1960/1964), for example, "human activity," "patterns," "problem solving," "logical thinking," "understanding" and "social interaction." Mathematics education traditions such as disciplinary traditions, constructivism, ethnomathematics and critical mathematics, have similar slogans. An acknowledgement on the part of policymakers that these are educational slogans or rallying symbols

(Scheffler, 1960/1964) would be useful because it is difficult to pinpoint what these phrases really mean. These educational slogans could be interpreted narrowly and instrumentally and be made to fit with the school mathematics tradition (Cobb *et. al*. 1992; Gregg, 1995). The fact that there is a "definition" implies that there is an answer to the question, what is mathematics? There is no single view of "what mathematics is." The following part of the "definition" comes close to mathematics "in the making" (Pólya (1945/1988, p. vii) :

> This understanding is expressed, developed and contested through language, symbols and social interaction.

A note on "the nature of mathematics," as opposed to a "definition," would be more useful in terms of a reader's professional development. Such a note could point out how mathematical ideas evolve and develop informally, experimentally and inductively before they become purely formal, deductive and symbolic, as we generally know them to be.

The "rationale" for MLMMS gives further evidence of the intentions of policymakers to address issues related to "redress and reconstruction" (DE, 1997a; 1997b). It states that MLMMS should "provide powerful numeric, spatial, temporal, symbolic, communicative and other conceptual tools, skills, knowledge, attitudes and values to :

- analyse
- make and justify critical decisions; and

take transformative action,

thereby empowering people to :

- work toward the reconstruction and development of South African society;
- develop equal opportunities and choice;
- contribute towards the widest development of South African society;
- participate in their communities and in the South African society as a whole in a democratic, non-racist and non-sexist manner;
- act responsibly in protecting the total environment;

- interact in a rapidly-changing technological global context;
- derive pleasure through the pursuit of rigour, elegance and the analysis of patterns and relationships;
- understand the contested nature of mathematical knowledge; and
- engage with political organisational systems and socio-economic relations.

(*Government Gazette* (6 June, 1997, no. 18051, p. 115).

Most of these goals could be put on the agenda of such other "learning areas" as:

- Language Literacy and Communication
- Human and Social Sciences
- Natural Sciences
- Technology
- Arts and Culture
- Economics and Management Sciences
- Life Orientation

(DE, 1997a, p. 16)

and thus still serve issues related to "redress and reconstruction."

The parts of the "rationale" that lean most towards mathematics are:

- derive pleasure through the pursuit of rigour, elegance and the analysis of patterns and relationships;
- understand the contested nature of mathematical knowledge.

MLMMS has ten "specific outcomes," each having "assessment criteria" and "range statements." The table below gives an outline of the number of assessment criteria and range statements per specific outcome that are applicable to the "foundation," (grades 1 - 3), "intermediate," (grades 4 - 6) and "senior" phases (grades 7 - 9). The columns give the number of assessment criteria and range statements, while the rows indicate the phases :

Table 8
Number of assessment criteria and range statements for specific outcomes 1 through 4

Specific Outcome # 1: Demonstrate understanding about ways of working with numbers		
Phases	# of Assessment Criteria	# of Range Statements
Foundation	6	14
Intermediate	5	12
Senior	7	26
Specific Outcome # 2: Manipulate number patterns in different ways		
Foundation	4	16
Intermediate	5	18
Senior	6	21
Specific Outcome # 3: Demonstrate understanding of the historical development of mathematics in various social and cultural contexts		
Foundation	1	3
Intermediate	4	9
Senior	4	10
Specific Outcome # 4: Critically analyse how mathematical relationships are used in social, political and economic relations		
Foundation	3	6
Intermediate	4	7
Senior	6	15

Table 9

Number of assessment criteria and range statements for specific outcomes 5 through 10

Specific Outcome # 5: Measure with competence and confidence in a variety of contexts		
Foundation	5	13
Intermediate	6	17
Senior	5	15
Specific Outcome # 6: Use data from various context to make informed judgements		
Foundation	9	21
Intermediate	9	17
Senior	9	18
Specific Outcome # 7: Describe and represent experiences with shape, space, time and motion, using available senses.		
Foundation	4	11
Intermediate	4	11
Senior	4	11
Specific Outcome # 8: Analyse natural forms, cultural products and processes as representations of shape, space and time		
Foundation	4	7
Intermediate	4	7
Senior	4	7
Specific Outcome # 9: Use mathematical language to communicate mathematical ideas, concepts, generalisations and thought processes.		
Foundation	5	10
Intermediate	6	13
Senior	6	13
Specific Outcome # 10: Use various logical processes to formulate, test and justify conjectures		
Foundation	3	8
Intermediate	3	8
Senior	3	9

There is thus a total of 392 range statements, which gives an average of 13 range statements per specific outcome per phase.

Below is a layout of the first specific outcome, complete with assessment criteria and range statements. The remaining nine specific outcomes have the same structure, i. e. a statement of the specific outcome followed by an elaboration of the specific outcome, and related assessment criteria, each with accompanying range statements. The purpose for showing this layout is to give the reader a sense of how each of the specific outcomes is organized.

Figure 2

Specific outcome # 1

1 Demonstrate understanding about ways of working with numbers
The development of the number concept is an integral part of mathematics. All learners have an intuitive understanding of the number concept. This outcome intends to extend that understanding. Its aim is to enable students to know the history of the development of numbers and number systems and to use numbers as part of their tool kits when working with other outcomes. Solving problems and handling information, attitudes and awareness may depend crucially on a confident use and understanding of number.

FOUNDATION PHASE

ASSESSMENT CRITERIA	RANGE STATEMENT
1. Evidence of use of heuristics to understand number concept	1.1 Use personal experiences to show the significance of number 1.2 Express numbers in words and symbols
2. Evidence of knowledge of number history	2.1 Understand counting as an historical activity 2.2 Show knowledge of the history of counting in their own communities, history of Roman numerals and the history of Arabic numerals 2.3 Understand importance of place value
3. Estimation as a skill	3.1 Estimate lengths, heights, volume, mass and time 3.2 Use calculators to check
4. Performance of basic operations	4.1 Add and subtract positive whole numbers 4.2 Multiply and divide positive whole numbers 4.3 Do simple mental calculations

5. Knowledge of fractions	5.1 Share and divide as an introduction to fractions 5.2 Use decimal fractions and place value 5.3 Do operations on money
6. Solving of real life and simulated problems	6.1 Solve real life or simulated problems

INTERMEDIATE PHASE

ASSESSMENT CRITERIA	RANGE STATEMENT
1. Evidence of use of heuristics to understand number concept	1.1 Demonstrate the use of a personal set of referents for large and small numbers
2. Evidence of knowledge of number history	2.1 Understand counting as an historical activity 2.2 Show knowledge of the history of counting in their own communities, history of Roman numerals and the history of Arabic numerals 2.3 Understand importance of place value
3. Estimation as a skill	3.1 Estimate lengths, heights, volume, mass and time 3.2 Estimate the square roots of numbers up to hundred
4. Performance of basic operations	4.1 Add and subtract positive whole numbers 4.2 Multiply and divide positive whole numbers 4.3 Perform operations on decimal fractions and money 4.4 Perform operations mentally 4.5 Use available technologies
5. Solving of real life and simulated problems	6.1 Solve real life or simulated problems

SENIOR PHASE

ASSESSMENT CRITERIA	RANGE STATEMENT
1. Evidence of some knowledge of rational and irrational numbers, including the properties of rational numbers	1.1 Demonstrate knowledge of the difference between rational and irrational numbers and the idea of recurring decimals 1.2 Commit to memory the decimal equivalents of commonly used fractions 1.3 Use and understand negative numbers in context 1.4 Commit to memory the approximate decimal equivalents of and 1.5 Illustrate properties of rational numbers
2. Evidence of knowledge of number history	2.1 Show knowledge of the history of counting in their own communities, history of Roman numerals, history of Arabic numerals 2.2 Show knowledge of the activity of mathematics and mathematicians from Africa, Asia, Middle East and South America

3. Evidence of estimation approaches	3.1 Recognise the difference between exact and approximate values 3.2 Estimate multiplication of rational numbers 3.3 Estimate square and cube roots of numbers 3.4 Estimation of heights and distances using a variety of approaches and technologies 3.5 Use a variety of mental maths techniques and check the reasonableness of results
4. Performance of operations accurately	4.1 Use rules of order of operations 4.2 Recognise significant digits 4.3 Show understanding of standard index form 4.4 Work with exponents, developing laws of exponents from numerical cases 4.5 Use a calculator to perform a sequence of numerical operations 4.6 Substitute numbers into formulae
5. Evidence of knowledge of percent, rate and ratio	5.1 Use algebraic techniques to solve problems involving percent, rate, and ratio 5.2 Solve problems involving proportions
6. Solving of real life and simulated problems	6.1 Perform basic financial computations 6.2 Perform general tax and sales tax computations 6.3 Pick and analyse authentic problems from newspapers and journals 6.4 Critically analyse at least two investment scenarios 6.5 Pick and analyse at least one local developmental problem
7.Demonstration of skills of investigative approaches within mathematics	7.1 Investigate open-ended questions 7.2 Ask and respond to questions like "what would happen if ...?" 7.3 Apply approaches that demonstrate reflective capabilities.

(*Government Gazette* (6 June, 1997, no. 18051, pp. 118-9)

In the second sentence of the first specific outcome one finds the phrase, "all learners." This is phrase is consistent with "learner-centeredness," which is a key principle in curriculum development in *Curriculum 2005* (DE, 1997a). It is also an educational slogan (Scheffler, 1964/1960) that has a popular appeal. No teacher can openly say that she or he would ignore "learners" or students when teaching. For the first time in the South African school curriculum, learners in a broad sense are referred to in reassuring terms. This has not been the case in the past under apartheid. The introduction of the slogan "learner-centeredness" is thus closely related to the "climate of opinion" (Eastwood, 1964) of the post-1994, post-apartheid era. The word "learners" is repeated in each of the statements of the ten specific outcomes, thereby capturing wide interest and popular support among the population.

A key word in the first specific outcome is "demonstrate understanding" because it is a rallying symbol of the key ideas of many mathematics education traditions. It is thus also an educational slogan (Scheffler, 1964/1960) because mathematics education as a field *per se* has "demonstrate understanding" as a rallying symbol of its key idea. Teachers and mathematics educators who are of the "Active Mathematics Teaching" (Good, Grouws and Ebmeier, 1983) tradition, which is a crystallization of the school mathematics tradition, who subscribe to constructivist traditions (Cobb and Steffe, 1993; Confrey, 1985), who subscribe to disciplinary traditions (Lampert, 1990; Ball, 1988; Schoenfeld, 1988 and others), all write about students or learners having to "demonstrate understanding." Similarly teachers and mathematics educators who are from ethnomathematics (D'Ambrosio,1985, 1990 & Borba, 1990) and critical mathematics (Skovsmose, 1994) traditions and/or a combination of the two, i.e. a social, cultural and political approach (SCPA) (Vithal and Skovsmose, 1997; Vithal, 1997), speak about students or learners having to "understand." The traditions that have been mentioned are not exhaustive of all the mathematics education traditions.

Besides "demonstrate understanding" there are several other key words and phrases in the specific outcomes through which one can identify different mathematics education traditions. To further illustrate the nature of the mathematics component of *Curriculum 2005*, it will be compared with the mathematics component of the National Curriculum (DES, 1989) in England and Wales.

Mathematics in the National Curriculum and the mathematics component of *Curriculum 2005*

Historically the United Kingdom and South Africa are connected, the former having been a key colonial power in South Africa. In the United Kingdom the "National Curriculum" (NC) also has a mathematics component. A comparison between Mathematics in the NC (DES, 1989) and MLMMS is justified from a historical perspective. Such a comparison will give a better sense of similarities and differences between the two mathematics components.

Both sets of policy documents have been published by national governments. In the case of South Africa, MLMMS is published by the National Department of Education, and in the United Kingdom, Mathematics in the National Curriculum is published by the Department of Education and Science (DES). Rhetoric related to issues of "education and training, redress and redistribution" is very strong and noticeable in *Curriculum 2005* in general and MLMMS in particular. Similar rhetoric is absent in the mathematics component of the National Curriculum. What is striking is that Dowling and Noss (1990) describe the National Curriculum as a product of a "government of privatization" in which the logic of the market holds sway. The notion of "privatization" of goes counter to the notion of "redress and redistribution" in MLMMS.

The National Curriculum in Mathematics consists of 14 "attainment targets," each of which is broken down into a series of "statements of attainment" (stoats). The stoats are classified into ten levels. The use of the term "level" implies an order, and as a rough guide, a level is meant to correspond to "two years of pupil progress" (quoted from Küchemann, in Dowling and Noss, 1990). There are 296 stoats in Mathematics in the National Curriculum that works out to an average of just over two stoats per level per attainment target.

The ten "specific outcomes" in MLMMS are the closest to 14 "attainment targets." The total number of range statements is 392, which averages to just over 13 range statements per specific outcome per phase. As mentioned earlier, range statements indicate the scope, depth, and parameters of a learner's achievement and phases refer to "foundation," "intermediate" and "senior" grade levels (DE, 1997b).

In the "definition" of mathematics in MLMMS is a reference to the contested nature of mathematics:

> This understanding (of mathematics) is expressed, developed and contested through language, symbols and social interaction. (*Government Gazette* (6 June, 1997, no. 18051, p. 115).

A view of the nature of mathematics in the mathematics component of NC can be inferred from an implicit view of mathematical activity:

> Mathematics is the most abstract of subjects. Attainment targets in mathematics have to be very tightly defined to avoid ambiguity, and the degree of precision required gives very clear indication of the "content skills and processes" associated with the targets (quoted in Dowling and Noss, 1990, p. 18).

From a cultural perspective, mathematics is thought to be the most abstract of subjects. The reality is that this abstract nature of mathematics is "expressed, developed and contested through language, symbols and social interaction." For example, a mathematical idea such as zero does have ambiguity to it. Nowhere in the "specific outcomes" in MLMMS are there attempts to "very tightly define" mathematical ideas as in the case of the "attainment targets." "Specific outcomes" are "specific" and about "outcomes" and therefore "very tightly defined." A cursory look at the ten "specific outcomes" indicate that they are not about mathematics content as such. Similarly in the case of Mathematics in the NC, mathematics content and the nature of mathematics are called into question. For example, Dowling and Noss (1990) titled their book "Mathematics *versus* the National Curriculum." In addition, they call the last chapter in this book "Multiplying by Zero," as a way to underscore their analysis of the nature of mathematics in the NC.

In summary, the MLMMS is overtly concerned with issues related to "redress and reconstruction," which it wants to address via "specific outcomes." In the Mathematics in the National Curriculum there is no reference to issues about "redress and reconstruction." In MLMMS there is a slight acknowledgement of the controversial nature of mathematics, although no coherent focus on mathematics content. In the National Curriculum there is an overt move to "very tightly define" attainment targets which gives an implicit view of the nature of mathematics as something that has no ambiguity.

Mathematics education traditions in *Mathematical Literacy, Mathematics and Mathematical Sciences* (MLMMS)

In this section we examine contact points between key words and phrases in various mathematics education traditions and in the mathematics component of *Curriculum 2005.* The traditions are identified through key words and phrases. The mathematics education traditions examined include the school mathematics tradition (Cobb *et. al.*, 1992; Gregg, 1995) as instantiated by Active Mathematics Teaching (Good *et. al.*, 1983), followed by constructivist traditions, disciplinary traditions, ethnomathematics and critical mathematics and/or a combination of the two. In each tradition there is a brief description of its key phrases and words. This is not an exhaustive list of all possible traditions in MLMMS.

Evidence of the school mathematics tradition and Active Mathematics Teaching (AMT)

In "Active Mathematics Teaching" there is "an implicit conception of mathematics" as algorithms and procedures, because they "can be analyzed most easily into small pieces, can have objectives stated precisely, and require practice" Confrey (1986, p. 354). Achievement tests (or similarly constructed measures) are viewed as measure of instructional "success" (Schoenfeld, 1988, p. 164). Learning is operationally defined as performance on achievement tests —tests which may fail in significant ways to measure subject-matter understanding. Teaching is separated from the content being taught and a major assumption is that knowledge can be decomposed into discrete entities and takes on the form of "direct instruction." Teaching is via recitation, drill, practice or application activities (Brophy & Good, 1986, p. 366). Assessing evidence of learners "knowing" mathematics in direct instruction involves learners correctly stating definitions, performing algorithms, obtaining correct answers, doing practice problems. These are taken as evidence of learners "demonstrating understanding," which leans in the direction of instrumental understanding (Skemp, 1978).

Teachers who work within the school mathematics tradition will be comfortable with the first specific outcome, "demonstrate understanding about ways of working with numbers." Taken literally, this "outcome" has to be *specified* and cited, like an objective, in advance of any teaching and learning (see DE, 1997b). Teaching and learning could thus take the form of clear, well-structured presentations. The same argument holds for the remaining nine specific outcomes—

- Manipulate number patterns in different ways.
- Demonstrate understanding of the historical development of mathematics in various social and cultural contexts.
- Critically analyze how mathematical relationships are used in social, political and economic relations
- Measure with competence and confidence in a variety of contexts.
- Use data from various contexts to make informed judgements.
- Describe and represent experiences with shape, space, time and motion, using all available senses.
- Use mathematical language to communicate mathematical ideas, concepts, generalisations and thought processes.
- Use various logical processes to formulate, test and justify conjectures.

—simply because they have to specified, as in "specific outcomes," in advance of teaching and learning. It may not necessarily be the case with specific outcomes where one finds phrases such as "understanding of the historical development of mathematics in various social and cultural contexts," "critically analyze how mathematical relationships are used in social, political and economic relations," and "use various logical processes to formulate, test and justify conjectures," because these are not key phrases in the literature on Active Mathematics Teaching (Good *et. al.*, 1983; Brophy & Good, 1986).

In conclusion, teachers working within the school mathematics tradition will be comfortable with most of the specific outcomes and the means to assess them because these have to be specified. Assessment in each outcome amounts to looking for narrow mathematical techniques and procedures because each of the specific outcomes are over-specified and narrowed down in minute detail.

Evidence of constructivist traditions

A constructivist view of mathematics learning (Cobb & Steffe, 1983; Confrey, 1985), according to Kuhs & Ball (1986, p. 2), typically underlies a "learner-focused" view of mathematics teaching that focuses on the learner's personal construction of mathematical knowledge. The fact that "learner-centeredness" is a key principle in curriculum development in *Curriculum 2005* implies that there will be ample evidence of constructivism in *Mathematical Literacy, Mathematics and Mathematical Sciences.*

In the detailed outlines of all the specific outcomes there are prescriptions through "outcomes" about what learners should do and should know. In constructivist traditions teachers should continually make a conscious attempt to "see" both their own and the children's actions from the children's point of view (Cobb & Steffe, 1983, p. 85). The phrase "from a children's point of view" necessarily implies tapping into children's "intuitive understanding," which is a phrase in the first specific outcome (*Government Gazette*, 6 June, 1997, no. 18051, p. 116). This first specific outcome, which sets the tone for the rest of the specific outcomes, has "demonstrate understanding" as a rallying symbol (Scheffler, 1960/1964). In the range statement that indicates the level and complexity of the teaching and learning of the same specific outcome, learners can be asked to respond to questions like "what would happen if...?" This phrase is also consonant with constructivist teaching and learning because it indicates that teachers have to "see" both their own view and that of the learners. In this way learners are able to "demonstrate understanding."

Teachers who subscribe to constructivist traditions will not necessarily be comfortable with the following specific outcomes

- Demonstrate understanding of the historical development of mathematics in various social and cultural contexts.

- Critically analyze how mathematical relationships are used in social, political and economic relations

(*Government Gazette*, 6 June, 1997, no. 18051, p. 116)

As individuals, teachers may subscribe to these specific outcomes. This point shows the tension between individual practitioners and a tradition. An individual practitioner is more complex and multi-dimensional than a mathematics education tradition. Specific outcome (SO) 3 explicitly states that the view of mathematics as a European product must be challenged and that learners must be able to understand the historical background of their communities' use of mathematics. SO 4 explicitly advocates a mathematics curriculum that addresses "critical thinking about how social inequalities, particularly concerning race, gender and class, are created and perpetuated." This phrase falls outside the traditional discourse of constructivist traditions.

In conclusion, there is ample evidence of constructivist traditions to the teaching and learning of mathematics in the mathematics component of *Curriculum 2005*. This observation can readily be inferred from the educational slogan, "learner-centeredness," a key principle in curriculum development in *Curriculum 2000* (DE, 1997a, 1997b).

Evidence of disciplinary traditions

Disciplinary traditions in mathematics teaching and learning have a focus on what it might mean to know and understand mathematics from the perspective of the discipline. Key phrases and words in such traditions are, "big ideas," "conscious guessing," "reasoning," "conjectures," "argument" and "counterexamples." These are key phrases and words in the writings of Lampert (1990) who draws on the work of Lakatos and Pólya who have written much on what doing mathematics is like in the discipline.

In the tenth specific outcome one finds direct evidence of Lampert's (1990) disciplinary approach. For example, this specific outcome states that learners should "use various logical processes to formulate, test and justify conjectures." In the same specific outcome one finds the following sentences:

> Reasoning is fundamental to mathematical activity. Mathematics programmes should provide opportunities for learners to develop and employ their reasoning skills. Learners need varied experiences to construct convincing arguments in problem settings and to evaluate the arguments of others (*Government Gazette*, no. 18051, p. 135).

One can read in the first sentence a strong assertion of how learners should "demonstrate understanding." Besides knowing procedures, there should be a focus on reasoning, for example, knowing why procedures or rules work. "Varied experiences" implies that learners should have opportunities to make and question their conjectures as ways to learn mathematics. In their "varied experiences" they should also have opportunities to see and make "counterexamples," as they "evaluate the arguments of others." In the range statement of this same specific outcome one finds the phrase, "demonstrate respect for different reasoning approaches" of others (*Government Gazette*, no. 18051, p. 135). Other learners in the classroom may have different reasoning approaches, which means that learners should "evaluate the arguments of others." Thus in a classroom setting this implies the use of such notions as "intellectual courage," where one should be ready to revise any of one's beliefs; "intellectual honesty," where one changes a belief when there is a good reason to change it; and "wise restraint," where one would not change a belief wantonly, without good reason or without serious examination (Pólya, 1954, pp. 7 - 8) among learners and the teacher. It is evident from this outcome that learners should engage in mathematical argument in the classroom (Schifter, 1995). This implies that teaching should take on the form of "systematic mathematical inquiry" Lampert (1990).

A "natural" follow-up of a teacher encouraging students to engage in mathematical argument in the classroom is a practice that aspires toward students doing problems that involve the exploration of mathematical connection, of underlying mathematical order , or "big ideas" (Schifter, 1995, p. 19). This is the fourth stage in Schifter's model of teachers' changing conception of the nature of mathematics and its enactment in the classroom. The second specific outcome, "manipulate number patterns in different ways," points in the direction of "the exploration of mathematical connection, of underlying mathematical order." It reads further :

> Mathematics involves observing, representing and investigating patterns in social and physical phenomena and *within mathematical relationships* (*Government Gazette*, no. 18051, p. 120) [italics added]

In conclusion, there is evidence of disciplinary traditions to mathematics teaching and learning in the mathematics component of *Curriculum 2005*. These go beyond modal practice in South African classrooms (see Adler, 1993; Vithal, 1997) and is another instance of learners "demonstrate understanding."

Evidence of ethnomathematics and critical mathematics education traditions.

These traditions are characterized by explicit phrases and words that refer to issues of "culture, power, and discourse in the classroom" (O'Loughlin (1992, p. 791). They try to incorporate the cultural, social and political realities that learners may experience in their daily lives (see Vithal & Skovsmose, 1997; Vithal, 1997; Borba (1990; D'Ambrosio, 1985; Gerdes, 1988).

One recognizes an ethnomathematics tradition through its key words and phrases, for example "culture," "cultural contexts, environment and artifacts," and "socio-cultural groups." The third specific outcome explicitly states that learners should "demonstrate an understanding of the historical development of mathematics in various social and cultural contexts." (*Government Gazette*, no. 18051, p. 120). This outcome calls on learners to demonstrate understanding in ways that take into consideration their cultural environment and its mathematical content. Examples can include artifacts and instruments a teacher can use to organize curricular experiences for learners. In the range statement of this outcome one finds an elaboration of related teaching and learning experiences. For example, learners have to "show a knowledge of counting in different cultures," and "analyse mathematical ideas from their own culture." Also, the eighth specific outcome favors an ethnomathematics tradition because it has the key words "cultural products" in it. It states that learners should "analyse natural forms, cultural products and processes as representation of shape, space and time." In the range statement of this specific outcome, teaching and learning examples include instances where learners have to "represent cultural products and processes in various mathematical forms, in two and three dimensions (*Government Gazette*, no. 18051, p. 131). These products could be particular artifacts that learners bring from home into the classroom.

Key phrases and words in identifying a critical mathematics education tradition are "critical," "critical mathematics," and "critical understanding." Also included are any references that call into question social, economic and political conditions, not necessarily through "mathematical" means because the notion of "the mathematical" and its "formatting power" (Skovsmose, 1985; 1994) in society is called into question. The fourth specific outcome explicitly advocates a critical mathematics education tradition. This can be inferred from references to "social, political and economic relations," a "critical" outlook, and a "critical mathematics" curriculum in its elaboration. It states that learners should [Critically] analyse how mathematical relationships are used in social, political and economic relations.

An elaboration of this specific outcome reads as follows:

> Mathematics is used as an instrument to express ideas from a wide range of other fields. The use of mathematics in these fields often creates problems. This outcome aims to foster a critical outlook to enable learners to engage with issues that concern their lives individually, in their communities and beyond. A critical mathematics curriculum should develop critical thinking about how social inequalities, particularly concerning race, gender and class are created and perpetuated. (*Government Gazette*, no. 18051, p. 123)

In the assessment criteria and range statement of this specific outcome, one finds more key phrases related to critical mathematics education, for example, "critical understanding of mathematics use in the media," "analyse income distribution in South Africa," "compare the financing of education under apartheid and after 1994." These examples indicate assessment practices and teaching and learning activities. Constructs such as income distribution and the financing of education have mathematical formulations imbedded in them and are thus about the formatting power of mathematics in society. These constructs are key in "project organization" or "thematization" (Skovsmose, 1985, 1994).

There is also evidence of a "social, cultural and political approach" (SCPA) (Vithal (1997; Vithal & Skovsmose, 1997) to the school mathematics education curriculum. For example, in the third specific outcome, which states that learners should "demonstrate an understanding of the historical development of mathematics in various social and cultural contexts," one finds in the range statements the following suggested teaching and learning activities:

- Critically analyse the role of mathematics as a tool for differentiation and
- Critically analyse mathematics as a predominantly European activity.

(*Government Gazette*, no. 18051, p. 122)

Range statements indicate the scope, depth, and parameters of a learner's achievement and give some idea of the level of complexity in the teaching and learning situation (DE, 1997b). Under apartheid, "differentiation" along lines of race and ethnicity was certainly a feature in the political lives of all South Africans. For example, it involved a fair amount of mathematics to determine resource distribution and allocation under apartheid that favored "white" South Africans. In addition it is fair to infer that the majority of South Africans under apartheid education came to believe that "mathematics is a predominantly European activity" and their own culture had no or little mathematics. This outcome aims at developing a critical approach to issues that are simultaneously "political" and "cultural." Also, in the eighth specific outcome one finds a merging between critical mathematics education and ethnomathematics traditions. For example, one of the range statements suggests teaching and learning activities where learners "critically analyse the misuse of nature and cultural products and processes" (*Government Gazette*, no. 18051, p. 131). In a highly technological society and even in developing countries one finds examples where nature is exploited for the purposes of economic development. Thus in a "critical analysis" one would imagine teaching and learning that would take the form of "project organization" or "thematization" (Skovsmose, 1994). Rhetoric such as "cultural products and processes" reminds one of an ethnomathematics tradition. On the other hand, "misuse" reminds one of a critical mathematics education tradition, because it points to political consequences such as playing off different people against each other.

In conclusion, there is evidence of the rhetoric of mathematics education traditions such as ethnomathematics, critical mathematics and a "social, cultural, political approach" in the mathematics component of *Curriculum 2005.*

What is not mentioned in the mathematics component of Curriculum 2005?

Several things are not mentioned in the mathematics component of *Curriculum 2005.* For example, there is no acknowledgement of the social context of teaching (Ernest, 1987), like the conditions that teachers labor under, their beliefs, the institutionalized curriculum such as the adopted text or curricular scheme, the system assessment, i.e. role of high stakes testing such as the "matric" in education in South Africa. As for mathematics itself, no attention in the document is drawn to the non-linearity of mathematics, possible "unifying ideas" or "big ideas" within the school mathematics curriculum such as a general notion of scale, functions, linearity and proportionality. In this section each of these is discussed in detail and their importance in any talk about introducing the mathematics component of *Curriculum 2005* is pointed out.

For *Curriculum 2005* to make any noticeable impact on the school mathematics tradition (Cobb *et. al.* & Gregg, 1995) depends on the teachers' belief and knowledge. In particular, any inroads on the part of *Curriculum 2005* hinges on the teacher's conception of the nature of mathematics and his or her mental models of teaching and learning mathematics (Ernest, 1987, p. 249). Nowhere in the component is there a discussion of the kind of professional development that would have to complement a path toward policy outcomes.

Any conversation about changing teachers' practice has to take into consideration the social context of teaching (Ernest, 1987). This would include teaching conditions such as classroom sizes and material resources such a textbooks, manipulatives and other types of classroom materials that will facilitate the kind of teaching and learning that is envisioned in the mathematics component. Also, there need to be opportunities for teachers to learn to teach the kind of mathematics espoused in the mathematics document. According to Jansen (1998, p. 330) there is already a weak culture of teaching and learning in South African schools. He notes that rationalization, a feature of post-1994 South Africa, further limits the human resource capacity for managing change (see "A note on the state of education in the Western Cape province in South Africa" in Chapter 1). Like the NCTM (1989; 1991) *Standards*, the mathematics component ignores teachers' reality, the mathematics department's reality and the institution's reality (see Bishop, 1990, p. 367). A section such as "how to get change going in your school" might help. Vithal (quoted in Jansen, 1998) paints a picture of the reality in the mathematics departments in schools in South Africa:

> In mathematics education in South Africa there is arguably a tradition of defining narrow behavioural objectives derived from the content that mathematics teachers are expected to teach (pp. 1 - 2)

Mathematics teachers are left to figure out how to "implement" the mathematics component on their own. In fact, nowhere in the mathematics component of *Curriculum 2005* is there a reference to teachers.

High stakes examinations at the twelfth grade level, the "matric," remains an important feature of and gatekeeper in the education of all South African students who wish to enter into higher education. Mathematics as a school subject is entrenched in these examinations, complete with its traditional divisions, algebra, geometry, trigonometry and calculus. The mathematics in the "matric" remains strongly classified. There is a good deal of overlap between the nature of the "matric" mathematics and vague and fuzzy phrases like "demonstrate understanding" as in many of the "specific outcomes." This probably is a reason policymakers have made MLMMS only applicable up to grade nine.

Specifics about mathematics content in the document is vague. For example, the seventh specific outcome, which is about "describing and representing experiences with space, shape, time and motion using all available senses," has a range statement that reads, "show links between Algebra and Geometry," (*Government Gazette*, no. 18051, p. 129). What these "links" should look like or ought to be are not spelled out in ways that inform teachers about complexities and possibilities. In South Africa the school mathematics curriculum is strongly classified into traditional divisions such algebra, geometry and trigonometry as well as within these divisions (Ensor, 1996). In the MLMMS there are no in-depth examples of possible conceptual connections or links between" arithmetic," "algebra" or "geometry," such as a study of linearity, proportionality and/or functions. The latter are examples of possible "unifying ideas" within school mathematics.

The "specific outcomes" are about "processes," such as demonstrating understanding, manipulating symbols, critically analyzing, recognizing number patterns, measuring with accuracy, describing and representing, generalizing, abstracting and conjecturing. They are not about content. A cursory look at the phraseology of the specific outcomes supports this claim :

- 1 # Demonstrate understanding about ways of working with numbers.
- 2 # Manipulate number patterns in different ways.
- 3 # Demonstrate understanding of the historical development of mathematics in various social and cultural contexts.
- 4 # Critically analyse how mathematical relationships are used in social, political and economic relations.
- 5 # Measure with competence and confidence in a variety of contexts.

- 6 # Use data from various contexts to make informed judgements.
- 7 # Describe and represent experiences with shape, space, time and motion, using all available senses.
- 8 # Analyse natural forms, cultural products and processes as representations of shape, space, and time.
- 9 # Use mathematical language to communicate mathematical ideas, concepts, generalisations and thought processes.
- 10 # Use various logical processes to formulate, test and justify conjectures.

As a result they are open to wide range of interpretations by mathematics teachers. This means, for example that "demonstrate understanding" could be interpreted in a variety of ways. It could mean one thing in a classroom where "tell and drill" (see Adler, 1993) is rife and another where constructivist approaches is a tradition. There is nothing in the mathematics component to prevent such a latitude of interpretations.

Nowhere in the mathematics component is the non-linearity of mathematics challenged because it is about "outcomes" and not content. The "definition" of mathematics (*Government Gazette*, no. 18051, p. 115) *per se* gives the illusion that there is an answer to the question, what is mathematics? This definition is particularly problematic when it comes to teachers' professional development about the nature of mathematics and policy enactment (see Schifter, 1995). An explicit reference to the non-linearity of mathematics in the "definition," at least, is certainly an important aspect of professional development. It is not true to claim that "arithmetic" precedes "geometry," and that "trigonometry" follows "algebra." These are certainly school ways of dividing and sequencing mathematics. Steen (1990) describes mathematics in the school curriculum as follows:

> It picks very few strands (e.g., arithmetic, geometry, algebra) and arranges them horizontally to form the curriculum: first arithmetic, them simple algebra, then geometry, then more algebra, and finally as if it were the epitome of mathematical knowledge, calculus (p. 4).

The mathematics that the elementary student is concerned with does tend to look linear, or at least the arithmetical part does. Wheeler (1980) notes that:

> geometry gives trouble because it isn't clear where it fits in into the structure -- and indeed, it doesn't fit in any linear, hierarchical sense.

What Steen is saying is confirmed in the South African case by Ensor (1996).

Summary of the overall rhetoric in the mathematics component of *Curriculum 2005*

Issues related to "redress and reconstruction" are very strong because they are given in the "rationale" for Mathematical Literacy, Mathematics and Mathematical Sciences" (MLMMS). This long name of the mathematics component also suggests the seriousness of policymakers to bridge education and training. In MLMMS there is the rhetoric of many mathematics education traditions, ranging from the school mathematics tradition and constructivist traditions to approaches that draw on ethnomathematics and critical mathematics. This phenomenon shows different currents in mathematics education in South Africa. The mathematics component is thus a "consensus" document that aims at being "representative" of mathematics education traditions in the country.

It is very difficult to ignore influence of "objectives" and thus "specific outcomes" in MLMMS that would certainly favor the school mathematics tradition (Cobb, *et. al.* 1992; Gregg, 1995). On the other hand, rhetoric related to ethnomathematics and critical mathematics and a combination of the two offers experiences for reflecting on mathematics and reflecting about mathematics. The whole sociopolitical dimension of mathematics education in MLMMS is referred to in ways that makes the document very different from the NCTM (1989) *Standards,* where it is never referred to (Bishop, 1990). Nowhere in the document is there an attempt to deal with mathematics content that would be in line with the specific outcomes. There are no curricular examples that would complement the specific outcomes. Nowhere in the document are teachers and their day - to - day experiences referred to. The constructivism in the document is thus certainly for "learners as opposed to teachers" (Julie, 1998). The words of the director-general of education rings true:

> The emphasis will be on what learners should know at the end of a course of learning and teaching, instead of the *means* which are to be used to achieve those results. (DE, 1997b) [italics added]

The means or the process is where the crucial role of teachers come into play in the education process. Teachers' practices and beliefs point to the "black box" between policy enactment and policy outcomes (Darling - Hammond, 1990). What thus follows an analysis of what teachers in the sample have to say about their practice.

CHAPTER 5
EVIDENCE OF THE STRENGTH OF THE SCHOOL MATHEMATICS TRADITION AMONG THE TEACHERS AND CONSTRAINTS ON TEACHING

Overview

The research question that drives this study is, what are the differences and similarities between the rhetoric of teachers and policymakers? The data corpus for this study consists of two kinds of data, namely, a set of policy documents containing policymaker rhetoric on *Curriculum 2005* and transcribed interviews with and observational data involving three middle school and three high school teachers. Interviews focus on teachers' views about the nature of mathematics and the teaching and learning of mathematics. Policymaker rhetoric in the mathematics component of *Curriculum 2005* does not acknowledge or refer to the school mathematics tradition (Cobb *et. al.*, 1992; Gregg, 1995) nor the social context of teaching resulting from the institutionalized curriculum such as the adopted text, the system of assessment and the overall system of schooling.

The data in this chapter come from the interview data. The first goal of this chapter is to examine teacher rhetoric about the school mathematics tradition by focusing on data categories such as teacher rhetoric about the mathematics curriculum and teachers' conceptions of ability (Gregg, 1995). The second is to examine teacher rhetoric about the social context of teaching such as the internal and external conditions of teaching, which is also prominent in the ways teachers talk about their classroom practice.

Teacher rhetoric and the school mathematics tradition

This section examines middle and high school rhetoric in relation to the "school mathematics tradition" (Cobb *et. al.* 1992; Gregg, 1995). Below is a summary of the practices and beliefs associated with the school mathematics tradition :

- There is a separation of teaching and content, and teaching and learning.
- Mathematics knowledge is defined as achievement on standardized tests
- Achievement on standardized tests is regarded as the measure of students' understanding.
- Mathematics knowledge is separable into discrete parts and thus considered linear, with the idea that there are prerequisites in order to understand the mathematics in higher grade levels.
- Teachers proceduralize and decompose the mathematics.
- Form, facts and procedures are learned mechanically and mechanically applied.
- Textbooks and the teacher are the mathematical authorities
- Teachers account for students' difficulty by referring to their ability.

These practices and beliefs indicate the thinking and talking within the school mathematics tradition. "Direct instruction" (Good, Grouws & Ebmeier, 1983) as a theory of teaching best illustrates this tradition in which mathematics is presented as a collection of facts and procedures. Also in this tradition, mathematics is learned via instrumental understanding (Skemp, 1978). In the teaching and learning of "algebra," "geometry," "trigonometry," and "calculus" there are variations of "direct instruction" with an overt focus on rules and procedures.

The order in which the school mathematics tradition is examined in this section is, teacher rhetoric about the school mathematics curriculum, the nature of mathematics and finally the notion of ability. Beginning with teacher rhetoric about the school mathematics curriculum gives a sense of how teachers talk about the school mathematics in South Africa.

Teacher rhetoric about the school mathematics curriculum

In the excerpts that follow first middle school and then high school teachers describe the school mathematics curriculum in general. There is no particular reason for going in this order, other than the assumed linearity in the grade level mathematics in school. "Sections" turns out to be a key word in the way the teachers talk about the mathematics curriculum. Also, there will be specific examples of teacher rhetoric about algebra, geometry, trigonometry, and calculus, the traditional divisions in the mathematics curriculum.

During the first interviews I asked the middle school teachers to tell me what mathematics in a particular grade level was all about. I used prompts such as, tell me what are the "big ideas" and what is the grade level mathematics a "story" about, and what are the "mathematical ideas"?

In the following excerpt I prompted AS to tell me about the big ideas in the grade level he teaches:

Faaiz	Let me come back to my question, what are the big ideas in the standard (grade level) that you teach?
AS	You mean in terms of sections that we cover.
Faaiz	Yes, I mean (reluctantly)
AS	I'll give you all the sections. I've given you what is *new* in the section. This is the way they discuss (pointing to a Western Cape Education Department syllabus bulletin, 1996), they go into this whole discussion of how we should teach maths, the actual aims of the subject, which is good to a certain extent. [emphasis in original] (interview # 1)

When AS notes that the syllabus outline is "good," he is acknowledging with caution ("good to a certain extent") the value he sees in it. In its outline this syllabus has different "sections," which turns out to be a key word in describing the school mathematics curriculum. PN also uses "sections" to describe the mathematics in the grade level she teaches:

Faaiz	The grade seven mathematics, what is it a story about?
PN	It's compartmentalized, into your different sections. I can actually give you a copy of our syllabus and you can see.
Faaiz	What are some mathematical ideas that students have to get into?
PN	We start off with number patterns, we do the calculator, calculator skills, area and perimeter, we do percentages, ratio, fractions, decimals, all the different properties, distributive, (interview # 1)

PN gives specifics about the different "sections," which appears to be a mere list of topics. NJ, another middle school teacher, gives similar descriptions of the mathematics in the grade level she teaches:

Faaiz	Okay. In grade seven, what are some of the big ideas, the big mathematical ideas that students get exposed to?
NJ	They, well most of what they've learned in grade seven they've touched on in grade six, it's taking them just a bit further with the arithmetic and rounding off the arithmetic in grade seven. We do a lot of work on fractions, just rounding off the whole of adding and subtracting fractions and so on. Then we do a large part of work based on the geometrical figures, quadrilaterals especially. We put them in touch with the basic properties of those various geometrical figures preparing them for the next year.

NJ So they learn the basic properties of a square, the basic properties of a trapezium, *et cetera*. Those are the main things that we do. We include quite a lot of word problems so that they are now able to relate the arithmetic to real life situations. So that is basically it.
(interview # 1)

She mentions a serial list of topics, for example, fractions, geometrical figures, word problems, arithmetic, and real life situations. One can think of these as the different "sections." The sections are strongly classified, or "compartmentalized," as PN notes. There appears little reasoning that connects the "sections" in any coherent way. The accounts of all three middle school teachers indicate that the mathematics curriculum is strongly classified and brutally sequenced (Ensor, 1996). As in direct instruction, the middle school teacher rhetoric about mathematics teaching is separate from content (Good, Grouws & Ebmeier, 1983). It is evident from the excerpts that the curriculum operates on the assumption that the mathematics knowledge is separable into discrete parts (Putnam, Lampert and Peterson, 1990, p. 127)

"Sections" is also a key word in the case of the high school teacher rhetoric. One finds similar evidence of a strongly classified curriculum. For example, HY talks about "separate sections":

HY Things aren't much linked in mathematics. It's like separate sections. You have to link them by means of your teaching.
(interview # 1)

He is also talking about how he teaches, i.e. he "links" "separate sections" as a way to make the curriculum more coherent. DS also talks about "graphs" as a "different section" in the curriculum. In the second interview he went in a different direction when I asked him about graphing calculators. I had in mind different representations of functions in school algebra (Usiskin, 1988):

Faaiz	Do you use graphing calculators, not the normal ones?
DS	Scientific ones?
Faaiz	I'm talking about the ones with wide screens.
DS	No, no.
Faaiz	Have you laid your hands on one?
DS	I think I have seen one. No, we don't use those. Seeing that you mention graphs I wonder whether it is possible to use graphs to get this across. *No, graphs is a different section.* Don't you think that will be too heavy? Normally the factors of a quadratic turns out to be the x-intercepts of a graph. You see at the standard eight level (10th grade) we first teach factorization then we teach graphs. [emphasis added] (interview # 1)

In this excerpt he is in fact giving details about the order in which the "separate sections" are taught. For example, in algebra teachers teach "factors" first, in one grade level, then they teach "graphs" in the next grade. My mention of graphing calculators also seemed to trigger some of DS's thinking about linking "graphs" and the "factors of a quadratic." In the school mathematics tradition these are "separate sections." Evidence of "separate sections" is also apparent in EM's rhetoric.

It is worth pointing out how the strongly classified mathematics curriculum brings about very distorted images of "school algebra" (Usiskin, 1988) because it is seen as one of many "sections." It is recognized by procedures such as "factoring" and "finding products." Within school algebra there is a further classification of "sections." This is evident from AS's rhetoric:

Faaiz	Do you get into some algebra with the seventh graders?

AS	Yes, we do in terms of solving for x and stuff like that. Basically the algebraic part is solving for x. I explain that in terms of unknowns. Instead of an x they put in a "what." Three times what equals that plus that, then they understand it very easily. If high school students would think of it in those terms they wouldn't have all those problems that they experience.
Faaiz	What have you done on number patterns, I see that in the curriculum outline?
AS	We completed that section. The supervisor doesn't require us to go into the types of numbers. (last interview # 4)

"Number patterns" becomes another "section" and loses all potential as an entry into intuitive and informal thinking about possible "big ideas" such as linearity within the curriculum. The strong classification of mathematics in school into isolated "sections" is at times knowingly and sometimes unknowingly fostered by powerful school people like supervisors and heads of mathematics departments. If one is an advocate for algebra as the study of relationships among quantities (Usiskin, 1988, p.16), one would find the above rhetoric discouraging. The manner in which disciplined knowledge—for example, "school algebra" (Usiskin, 1988)—is brought into school, gives emphasis to consensus and stability (Popkewitz, 1988). Doing many drill and application problems as in direct instruction on factoring makes school algebra stable and static and something that is central to school mathematics tradition. The following excerpt from a high school teacher supports this point:

High School (HS) Teacher	School maths is taught *totally* in isolation of what happens outside. There are only certain sections you can link up with what happens outside. I think trigonometry is one of the main sections you can link up with what is happening in the real world.
Faaiz	And algebra?

HS Teacher No, not algebra, I mean if you talk about factors, there is no way you can link factors to what happens outside.
[emphasis in original] (interview # 2)

In school algebra as the study of relationships among quantities there could be problems where there are variables as part of "factors," which one relates "to what happens outside" or "real-life" contexts. There are mathematics education traditions that have rhetoric about "relevance" and "real-life" contexts (see ethnomathematics and critical mathematics education). One of the middle school teachers made a similar comment about school algebra:

Middle school Teacher In real life they're probably not going to come across algebraic problems where they've got to have all the different rules in their mind and apply them. *It's only really maths teachers who use algebra in their jobs.*
[emphasis added] (interview # 2)

From this excerpt it is evident that the teacher thinks of algebra mainly in terms of its "different rules" and procedures and how they "apply."

In the case of calculus in the school mathematics tradition, there is a focus on formal rules and procedures or what Tall (1987, p. 54) calls the "surface structure" of symbolic manipulation to obtain formal derivatives and integrals. It becomes extremely difficult for high school teachers to see any relationship that calculus has with "geometry," "trigonometry" or "algebra" or to think of it in informal and intuitive ways, with a focus on "big ideas" (see Thompson, 1995; Schnepp *et. al.*, 2000). In the following Likert scale item response by HY, one notices the formalized way in which calculus is viewed:

In math learning students must master one topics and skills at one level before going on

HY School maths is the stepping stone maths. You need one section in order to go on to another section. For example, if you want to know calculus, you can't find the derivative, and this is based on school, you can't find the derivative if you don't know exponents. You see at university we have this quotient rule for calculus. Now at school, we don't have this rule. Now what the pupils have to do, they have to get rid of the denominator. For example, if you take $x^2 + x + 3$ divided by $x + 1$ and ask to find $f'(x)$. They do not know the quotient rule. They first have to factorize and cancel out. We also don't have the product rule. If you say $y = (x + 3)(2x - 4)$ and I ask them find $f'(x)$. At university you can use the product rule. At school they first have to multiply out. So you must know how to multiply out before you can do that. (interview # 1)

There is a certain amount of truth in saying that "you can't find the derivative if you don't know exponents," which connects algebra and calculus. However, "if you want to know calculus," with an emphasis on "know," then there are other places to start where one does not need to know exponents if one wants to find a derivative or a rate (Thompson, 1995). An overt focus on procedures is reinforced by HY's experience with calculus at university level. This compressed and formal view of calculus is even a focus at the school level, where typically the 12th grade level end-of-the-year examination is rife with questions that require students to recall a rule, as in instrumental understanding (Skemp, 1987) in order to answer a question. Thus, it is not surprising when HY says, "you must know how to multiply out before you can do that," a procedure that precedes an application of the product or quotient rule. In this instance his language is squarely within the school mathematics tradition.

Teacher rhetoric about ability

The excerpts in this section come from places in the interview data where teachers reveal their rhetoric about students' ability in relation to the mathematics curriculum. For example, I asked them to describe someone who is good and not good at mathematics. Their response to Likert scale items on "being good at mathematics" during the second interview is another source to reveal their rhetoric about the nature of mathematics, and thus ability. Their responses show aspects of mathematics that they think are important. For example, is mathematics a set of rules or formulas that must be applied, meaning those who are good at mathematics are those who have memorized all the relevant rules? Or is it a set of concepts and procedures that have meaning? Finally, is it a domain of inquiry that involves invention, argument, and application? Within the school mathematics tradition teachers often talk about the notion of ability. By locating students' problems "outside" of the school mathematics tradition, the conception of ability as capacity allows teachers to believe that they are teaching as effectively as possible (Gregg, 1995, p. 462).

Middle school teachers are analyzed first, followed by high school teachers. The reason for going in this order is that competence and thus ability in the school mathematics tradition means being able to follow procedures and conventions specified in lower grade in order to do mathematics in higher grade levels.

In the following excerpt the middle school teacher, AS, talks about being good or not good at mathematics:

AS	Being good and not so good are relative terms in the first place. These are subjective words. I have a boy, Doug, in my class. His maths is exceptional.
Faaiz	Why?

AS Because he's able to apply. The problem that I find in mathematics, anybody can add and subtract. Anybody can be taught a recipe. There's got to be that understanding. What has unfortunately happened is that people have unfortunately been taught recipes. I feel that the child is able to understand if he *applies* what he has been taught. He's your better maths pupil. In other words we're talking about the application of what has been taught.
[emphasis in original] (interview # 1)

What is evident from the excerpt that is AS aware of drill and practice procedures so common in direct instruction (Good, Grouws & Ebmeier, 1983) and in the school mathematics tradition. For example, he notes "anyone can be taught a recipe," which is obviously about rules and procedures. He sees such kind of teaching and learning as "unfortunate" and thus as something that needs to change. He is in a way criticizing instrumental understanding (Skemp, 1978)—that is, learning rules without reasons. He sees the "better pupil" as the one who is able to "apply" what he understands. Later on in the interview he gives an example of what he means:

AS If you taught, for example, let's take odd and even numbers. *Everybody* will know what an odd number is. I tell my pupils if there's a 2 here, a 4 here, or a zero here, then the numbers are even. If you for example ask the students, work out the 12th even number. The child who is able to do that is a better maths pupil. If you give them 2, 4, 6, 8, 10 ...
[emphasis in original] (interview # 1)

The example he gives is about "rules" and formal definitions of odd and even numbers, and an "application" of those definitions. SA's rhetoric falls within the school mathematics tradition with its emphasis on formalized mathematics—that is, mathematics presented as a collection of facts and procedures. In this tradition, understanding is instrumental (Skemp, 1978) and amounts to simply replicating or "applying" procedures or facts taught in class.

There were times when his rhetoric did not neatly fall within the school mathematics tradition. For example, the following excerpt comes from his response to a Likert scale item about being good at mathematics. The item reads, "being good at mathematics means being able to:

2. Think in a logical step by step manner.

AS	Now that is also subjective. Logical is a subjective term.
Faaiz	Say a little bit more.
AS	Logical to me is not necessarily logical to you. Teachers have fallen into this false perception that what their logic is, is the child's logic. The child's logic is totally different. If you say think in a logical step by step manner, logical to whom? There's nothing wrong with your question. (interview # 2)

His awareness of differences between "teachers' logic" and the "child's logic" is a step toward thinking about how students develop their understandings of mathematics. Teacher rhetoric or technical rhetoric (Lortie, 1966) about the "child's logic" points in a direction where students' intuitions are regarded as important in terms of teaching and learning mathematics in school.

AS also made a specific comment about ability :

AS	Another thing about mathematics, what comes into play is your ability. You have to be able to understand or comprehend the problem. I feel that with my weaker pupils, they are the ones that are unable to understand a concept, for a lot of reasons.

AS They may not be able to understand the language. The other problem I find is that their retention is low.
(interview # 1)

Talk about "ability" in mathematics amounts to being competent "inside" the school mathematics tradition (SMT) (Gregg, 1995) where mathematics is presented as a collection of isolated rules and procedures and where it is strongly classified and brutally sequenced. In the SMT students' understanding is assessed almost exclusively through memorization or "retention." In the SMT they have few if any opportunities to "figure out" what mathematical ideas they are learning and how they are related.

In the following excerpt PN, the second middle school teacher, describes Ali, who is good at mathematics:

Faaiz So Ali is good because somehow?

PN He has that insight, that extra little bit. He will find a way.

Faaiz What about some of his intuitive ideas, do you see that in his work?

PN Oh yes, definitely. I think the fact that he doesn't give up, and he tells me "teacher believe me I want to do it." I can't now think of very, very specific things but he really stands out when it comes to this type of thing. We do multiplication of fractions. I think he is also well-read, does a little bit of research because he could immediately explain to me, why. We didn't do reciprocals, but he could explain to me, you know, we turn it around and I asked him why? And he said to me, "teacher because multiplication is opposite of division." We're still trying to get there but I cannot safely say that it is all his own, he could have done this in the past or worked with someone. It could be that he worked with someone and that he just remembers. Those are the little things that we want our kids to be able to do
(interview # 1)

"Intuitive ideas" comes from my rhetoric and shows my bias towards what I call disciplinary traditions. I somehow saw it as appropriate to use during the interview, as a prompt. From PN's response it is clear that she does not really understand what I mean. When I used the notion of "intuitive ideas" I wanted to find out from her whether what Ali was doing involved testing and assessing intuitions and conjectures. Her response gives an idea of what Ali did. In her example, the algorithm for dividing fractions, "We didn't do reciprocals, but he could explain to me, you know, we turn it around and I asked him why? And he said to me, teacher because multiplication is opposite of division" is treated as if it were argument (Lampert, 1990). This rule is very much a feature of the school mathematics tradition and is an example of instrumental understanding (Skemp,1978).

However PN does have professional development ideas about mathematics teaching and learning. She attended and organized in- service workshops and is aware of *Curriculum 2005.*

PN	The Department (Western Cape Education Department, WCED) trained teachers from grade one upwards. They never reached us because something else changed. We had to go out and find people who were doing this. I found a place that I could go to but what about the others? You're looking at eight schools alone. Many of us really don't *understand* what it is all about. You have a faint idea, but the *application* thereof.
Faaiz	So from what you're saying, I guess there's an informal network of professional development that is going on.
PN	The Mathematics Project at the University, they were working with us.
Faaiz	So they ran a couple of workshops?

PN Yes, the Mathematics Project ran a couple of workshops at school here, after I had attended a few there. But they leave you there with all that information and you have to run around and go and find things that you have to adapt for the classroom because ultimately *in* the classroom is what is going to make the difference.
[emphasis in original] (interview # 2)

PN is talking about an in-service series of workshops on the mathematics component of the curriculum policy *Curriculum 2005,* which is being introduced by the Western Cape Education Department (WCED). She has made concerted efforts to get herself and colleagues acquainted with the new curriculum policy. She also talks about difficulty she has in making sense of the policy. For example, she notes, "Many of us really don't *understand* what it is all about." Her rhetoric, "in the classroom is what is going to make the difference," shows her awareness of the importance of the classroom teaching and learning situation.

NJ, the other middle school teacher, spoke about a good student as one who has no "blockages" (interview # 1). This is an interesting description and should be viewed in light of the "strongly classified and brutally sequenced" curriculum (Ensor, 1996). Thus the student who has "no blockages" is competent in the school mathematics tradition where doing mathematics amounts to being able to follow a set of procedures and conventions specified by the teacher and the textbook. It is therefore not surprising that NJ also describes good students as having "natural ability" and thus "no blockages." The latter are examples of the taken-as-shared beliefs of the school mathematics tradition (Gregg, 1995, p. 462) .

In summary, middle school teacher rhetoric on the nature of mathematics is heavily influenced by the school mathematics tradition where mathematics was formally presented as a collection of isolated rules and procedures to be memorized and applied. Their talk about ability is closely related to being competent in this tradition. Two of the three middle school teachers do have ideas about ways to develop professionally as they struggle against the school mathematics tradition.

High school teacher rhetoric about the nature of mathematics appears to be different from middle school teacher rhetoric. All three high school were asked to describe someone who is good and not good at mathematics. ME said that he honestly has no idea why students are good at mathematics. In the case of someone who is not good at mathematics, he has some interesting thoughts:

Faaiz	So looking at the flip side, a student who is not good at mathematics?
EM	I'd say the majority of pupils are exactly that, but there is something missing. I don't know what it is. I mean you can't get classes full of, I'll put in inverted commas, morons, you know. *It's impossible for me, I can't see why you'll get a class that is completely non-mathematical.* I've thought about it, looking at the socioeconomic structure. I just don't know why that is the case. I just finished marking some standard grade matric papers (12th grade students) and I want to show you the results. [emphasis added] (interview # 1)

EM had just graded or "marked" "standard grade" or "lower track" students' papers. He knows that there is "something missing" or problematic with the traditional mathematics curriculum but is unable to pinpoint what it is. For example, he notes with frustration, "I can't see why you'll get a class that is completely non-mathematical." He cites "the socioeconomic structure" as a possible reason why his students are so "completely non-mathematical," which is another way of talking about ability. Being "mathematical" or competent in the school mathematics tradition amounts to being able to replicate a set of isolated procedures that was taught in class. One way to address his frustration is to have him think about what "mathematical" means. I missed an opportunity to ask him what he meant by "non-mathematical" and "the socioeconomic structure." It is fair to claim that the assessment he graded took on the form of students having to replicate procedures and facts taught in class as is characteristic of the school mathematics tradition. It is also clear that his students did not score well, which is why he was frustrated.

DS had something interesting to say about someone who is not good at mathematics:

DS To me I don't think that there is any student who cannot do mathematics. *It's a matter of how they were introduced to it*, how students were taken through it, the development phases, the necessary skills, to do mathematics. To me that is more important.
[emphasis added] (interview # 2)

This excerpt reveals SD's espoused theory of mathematics teaching and learning. Here he makes no reference to students' ability (Gregg, 1995). He was not asked what he meant by "development phases" and "necessary skills," which would have been useful and illuminating information. In disciplinary traditions, practitioners like Ball (1988) and Lampert (1990) and others, drawing on philosophers of mathematics like Davis and Hersh (1981) and Lakatos (1976) emphasize the importance of "doing mathematics" and thus how one introduces or presents it.

At a rhetorical level DS is aware of the importance of how mathematics should be introduced or presented. From this one can glean that he has some notion of doing mathematics which falls at the edge of the school mathematics tradition. For example, he makes the following comment about school mathematics:

DS The subject lacks color and vibrancy. It really scares you when you look at those x's and y's and you don't know what they stand for. It can be done in a different way. I am sure there are ways to address this issue. The x's and y's should come as a *result* of other activity, as a need to deal with it in that way, even if I go back to the x's and y's in the end. But in introducing the pupil to these (x's and y's), don't confront him with that directly.
[emphasis in original] (interview # 3)

His technical rhetoric (Lortie, 1966) is unusual because it is certainly "outside" the school mathematics tradition. Here he is espousing a model of mathematics teaching and learning that points in a direction of constructivist and disciplinary traditions. His comment is a striking critique of the strongly classified, purely formal and symbolic mathematics imbedded in schooling. When he says that the x's and y's should come as a result of other activity, he could mean to downplay of formalized mathematics and say that deductive, purely formal and symbolic mathematics is not a good place to begin. His rhetoric seems to go against teaching mathematics as a collection of rules and procedures, as in direct instruction. He is however not saying that formalism should completely disappear (even if I go back to the x's and y's in the end). A modest claim is that his rhetoric is moving in a direction where mathematics is seen as a domain of inquiry that involves invention, argument and application.

HY gives the following description of someone who is good at mathematics :

HY I taught a boy once at City High School, and I asked a question and that boy gave me an answer, and I said you are sharp, just by the answer he gave me. He looked at the question, and he gave the answer just like that. And I could see, I expected about a few minutes for him to come up with an answer, but I could see he has good maths skills.

Faaiz So he gave you an answer quickly?

HY Yes. When I just finished the question, he gave me an answer. I wrote up the question on the board and just as I turned around, he gave me the answer.
(interview # 1)

Commonly, mathematics is associated with certainty and knowing it, being able to get to the right answer, quickly (Schoenfeld, 1985; Ball, 1988 and Stodolsky, 1985). This is a widely - held cultural assumption and is closely connected to the notion of ability. HY's description of someone who is good at mathematics is no different from this assumption. He does not get specific about the mathematics itself.

In the following excerpt HY gives another explanation of why people are good at mathematics. His reasoning is rather repetitive and more about the notion of a "mathematical mind," and thus, ability.

HY Normally the pupils who do well at maths in school, those are the ones that can do maths. Those who fail maths, they can't do maths. They have problems with maths. They are the weaker pupils. I see some correlation between the pupil who can do maths, he has the ability to go further and solve problems at a higher level, whereas a pupil who is battling with maths, it's like an in-built problem within his system.

Faaiz What is his "system"?

HY In his brain, he hasn't got a mathematical mind.
(interview # 1)

Later on in a second interview on Likert scale items on "being good at mathematics," I asked him to say more about a "mathematical mind."

HY	To be good at math you need to have a "mathematical mind." (reads to himself)
Faaiz	Do you want to say more about a mathematical mind ?
HY	A mathematical mind is what you're borne with. It's in your genes. You can't develop it in school maths.
Faaiz	Why not?
HY	You must be borne with it. I really believe it. You can have a mathematical mind and still not enjoy school maths. To be good at maths, and here I am talking about maths on the higher grade level, you have to have a mathematical mind. You are really going to be good. (interview # 2)

There is an element of truth in saying that one can have a "mathematical mind" and still not enjoy "school maths," which is the traditional, fragmented curriculum with its many rules and procedures. It also depends on what is meant by a "mathematical mind." In this instance HY associates the notion of a "mathematical mind" with "higher grade" or "top lane" mathematics. Students in South African high schools are tracked into "higher grade" and "standard grade" mathematics classes, the equivalent of "top lane" and "low lane" classes, respectively. Students who do "higher grade" mathematics are normally college - bound. Colleges and universities where professional mathematicians, the "experts and elites," reside, exert great influence on the school mathematics curriculum. A cultural assumption many of us hold is that these mathematicians have "mathematical minds." HY is suggesting that "higher grade" students who are good have "mathematical minds." This kind of rhetoric could be as a result of the way we think of mathematics, i.e. as deductive, purely formal, symbolic and abstract, as one thinks of the mathematics of professional mathematicians. This is certainly *one* aspect of mathematics (Pólya (1945/1988 p. vii).

In summary, as for the high school teacher rhetoric, EM is aware that there is something problematic with the mathematics curriculum because he finds it impossible to believe that his whole class is so "completely non-mathematical." He has not given enough thought as to why this is the case with his students. At a rhetorical level DS is aware that introducing mathematics in a formalized way is problematic. This amounts to small but conscious critique of the school mathematics tradition. In fact he gives an eloquent critique of mathematics as a school subject ("the subject lacks color and vibrancy"). HY's rhetoric about doing mathematics "quickly" and having a "mathematical mind" makes the nature of mathematics appear mystical and inexplicable. HY is unable to ponder over the mathematics curriculum in ways like EM or critique it like DS. However, MY earlier on talked about how he "links" different "sections" in the mathematics curriculum. His rhetoric about a "mathematical mind" feeds into the notion of ability as is the case with two of the middle school teachers, AS and NJ.

Teacher rhetoric about the social context of teaching.

This section discusses how teachers in the study talk about the different external and internal conditions they face. From the ways they talk about these conditions, one can deduce constraints and opportunities in the teaching and learning of mathematics. There is no point in considering middle and high school teacher rhetoric separately because the goal is to get a general idea of how teachers talk about the social context of teaching. External conditions will be considered first, followed by internal ones.

Examples of external conditions are rationalization or "downsizing" in teaching staff in public schools, resulting in overcrowded classrooms; a lack of government support for the new curriculum policy; "school fees" that students in public schools have to pay; and the pressure of 12th grade external examinations, the "matriculation" or "matric," in the case of high school teachers. These are links between these external conditions and teacher rhetoric.

At the time of the interviews, teachers in public schools faced "down-sizing" in teaching staff. Under apartheid there was a significant, unequal spending in education for the country's different "population groups." PN is a middle school teacher at a former "coloured" school in the suburbs of Cape Town. At the time of the interviews, her school was in the midst of rationalization or "down-sizing" in teaching staff, which is part of the central government's plan to equalize spending in education. Her school had more staff members in the past. I secured my first interview with her and had shown her my university-approved permission to interview her. She then said the following:

PN I like this (pointing to my UCRIHS-approved advertisement about my project), "acquiring some different insights into the nature of mathematics." *We* have come to a stage where we don't know where we are going any more.
[emphasis in original] (interview # 1)

A UCRIHS reviewer insisted that I state in my advertising for recruits for my study some benefits for participants. In it I wrote that participants may acquire some different insights into the nature of mathematics. PN looked at the advert when she made this comment. One gets an impression that she is in a collective way aware of problems of practice, for example, her emphasis on "we." Her concerns with classroom practice become more evident from in a subsequent comment:

PN We don't have the kind of support system from government or from any NGO [non-governmental organization] where we can really build. We went into this problem-solving thingy and it was just left there and nowhere is it spoken about anymore. Everybody just goes on happily doing their own thing. (interview # 1)

PN is noting a lack of support from the central government for professional development at the classroom level. She found inadequate curricular initiative on problem solving she had attended. It is evident that she is also saying how alone she feels alone in her struggles with classroom practice. The new democratic, post-1994 central government had introduced many changes in education such as its outcomes based-education (OBE) curriculum policy, *Curriculum 2005*, more popularly referred to as OBE by South Africans. At the same time, the post-1994 central government shifted toward a policy of rationalization or "down-sizing" in the education departments. The Western Cape province has been particularly affected by this policy. In particular it meant that more funding for education was now allocated to poorer provinces. Also, teachers not necessarily of retirement age were offered severance packages, as a way to cut spending on education. PN's comment, "everybody just goes on happily doing their own thing," shows her concern that there was no external, coordinated follow up on an initiative on "problem solving" which she attended. This implies she has little opportunity to share with or to find out what other teachers and perhaps other colleagues who took part in the curricular initiative on problem solving are doing in terms of mathematics teaching and learning.

AS is another middle school teacher at a former "coloured" school in the suburbs that is also in the midst of a rationalization or "down-sizing" in teaching staff. During the first interview he expressed similar concerns about directions in which education is going. For example, in the following excerpt I prompted him:

Faaiz	How would you describe some of the obstacles you face?
AS	A major obstacle, is the policies that the state is making. To teach mathematics in an ideal way, you need about 25 children in a class. To group children not only homogeneously but heterogeneously as well would be ideal. So in a class of 25 children, you can have groups of five. In that sense you have a good sample to work with. Nothing is being done to promote mathematics. How does OBE or how does rationalization, the kinds of problems that we are facing today, how does that affect maths teaching ? If you say to me listen, you've got to sit with sixty kids in a class. How can that be equated to having progress in the subject? So just in terms of policy, it is not conducive to teaching *any* subject, if you think about it. [emphasis in original] (interview # 1)

One major effect of rationalization is that when teachers opt out of the education system, their students are then divided among those teachers who remain in the school. The remaining teachers in a particular school thus have many more students than would normally be the case. AS is one such a teacher. His reality seems totally ignored by education authorities. It is evident that his overcrowded classes are caused by rationalization which in turn affects his rhetoric.

EM is teaching under similar conditions. He is a high school teacher at a prestigious former "coloured" high school in Cape Town , which is also experiencing the effects of rationalization. Students at his school are unable to afford "school fees," which is an amount of money that students at public schools have to pay at the beginning of the school year in January. Rationalization means that schools have to pay out of their own funds if they want to hire extra teachers to reduce class sizes:

EM You were talking about graphing calculators the other day. It's quite cheap, but our pupils here cannot even afford their school fees and we have one of the lowest school fees around. (interview # 2)

Some principals keep school fees low and affordable by organizing fund raising activities throughout the school year. EM's school does that such. Yet a picture of an embattled school comes to mind, struggling to make it as an institution. An external factor such as students being able or not able to afford school fees affects the way EM speaks about mathematics teaching and learning. In the excerpt he makes me aware of a constraint he faces in teaching mathematics.

The external pressure of the matric as a national system of assessment greatly affects what and how teachers teach. It regulates the mathematics curriculum in many ways and certainly impacts teachers' espoused and enacted models of teaching and learning mathematics (Ernest, 1987). An example is produced by HY, who teaches at a prestigious, former "whites-only" high school, known as "Model C schools," in the posh suburbs of Cape Town. Such schools are nowadays open to all "races." The following excerpt comes from a conversation around the 12th grade, end-of-the-year examinations, or the matric.

HY Previously they used to ask a lot to motivate, but now lately they don't ask students to motivate anymore or why this is so.
Faaiz When you say "they" whom are you referring to ?
HY The examiners, the external examiners
Faaiz Okay.
(interview # 1)

The excerpt shows the pressure of "external examiners" who grade the matric. They are usually teachers and administrators selected by education authorities to draw up and grade the matric. They are "external" to the school in the sense that teachers in general do not know who they are. The influence of external examinations, such as the matric is a reality that all high school mathematics teachers have to contend with. It greatly influences and determines how teachers view mathematics teaching and learning. In the excerpt I prompted HY to find out what he understands by "reasoning." He then referred to the matric and recalled that some of questions in the mathematics component of the matric in the past required students to "motivate." The fact that he referred to the matric is telling about its influence on his and other teachers' discourse about mathematics teaching and learning.

In conclusion, the ways teachers talk about their working lives include salient categories such as, a lack of support from the central government, rationalization, overcrowded classes, the payment of school fees and the influence of the matric. These categories are intimately linked to the teaching and learning of mathematics, but they are the only factors that constrain and enable mathematics teaching and learning. Teachers also face conditions internal to the school.

Examples of conditions internal to the school are the hierarchy of relations within the school such as the mathematics department, problematic collegial relationships, and organizational issues such as time, the lack of professional development opportunities and the tracking of students into "higher grade" and "standard grade" classes or "top set" and "bottom set." These are links between these internal conditions and the way the teachers speak about mathematics teaching and learning.

In the following excerpt AS talks about how difficult he finds it to have conversations with his colleagues about his mathematics teaching:

Faaiz	Do you speak with other teachers, your colleagues, about the maths that you teach?
AS	People are not very flexible in terms of speaking about the work, how can I say, it is because they, a case where they think that you want to dictate to them.
Faaiz	I understand.
AS	Eventually you go on your own. (last interview, # 4)

I asked him about whether he had conversations about his teaching with his colleagues. Lortie (1975) notes that teachers are often alone in their reflection about teaching and learning. AS does not appear to be an exception to this finding. The problematic collegial relationship he describes makes opportunity for discussions about mathematics teaching and learning limited.

AS has interesting professional development ideas, for example, he notes:

AS	To tell you the truth, we don't have a lot of *brainstorming* about problems. The ideal situation would be everybody teaches his own class. Class teaching can work if we get together to discuss the problems and work on strategies, but everybody doesn't have that time. [emphasis in original] (interview# 1)

His notion of "brainstorming" is about sharing ideas and strategies about mathematics teaching and learning. As a middle school teacher he teaches other subjects in addition to mathematics. He longs for professional development opportunities that do not seem forthcoming because of organizational constraints internal to the school. He says "everybody does not have the time," which makes it almost impossible for teachers to come together to discuss ideas about teaching and learning.

Time is another internal condition that impacts teacher rhetoric. NJ spoke about the importance of time to allow her students to "explore." I had asked her about students learning rules and reasons:

NJ Well, I think there should be more of that at school level. So often it is almost every week, someone asks me what's the point of this, because they just see it as going through the rules and applying the rules they've learned and solving the problem. It will be nice to actually have the time in the classroom, to have them *explore* a lot more, like you do at university, you have to explore to find out what you've learned. It will be nice in the classroom to have that kind of time.
[emphasis in original] (interview # 1)

Her comment about a lack of time illustrates the way mathematics in the school mathematics tradition is "ruthlessly paced" (Ensor, 1996). Teachers spend a lot of time teaching isolated "sections" in the school mathematics curriculum.

In schools teachers are given a syllabus or a curricular scheme, which lists the order in which the mathematics will be taught in each grade level. The syllabus gives a sense of stability and order to the mathematics. It gives the mathematics an "institutional impact" (Bauersfeld, 1980; Popkewitz, 1988). In the following excerpt SA talks about how the relationship between the syllabus textbooks and how his principal sees that relationship:

AS You see (pause), we have a guideline in the syllabus that we have to cover. The textbooks are written that way (laughs).

AS I would love the idea of being able to draw up my own system, where I could do that in the class. But all these things are monitored (laughs). The principal would look at your book and say this section of the syllabus has not been covered, you know, why is that so. It's that kind of situation in our schools. (interview # 1)

This excerpt comes from the first interview where I brought up the idea of possible changes in the mathematics curriculum. AS said he would love to be able to draw up his own system, where he could initiate changes in his class. The reality is that curricular changes such as his "own system" must be supported and understood by his principal, colleagues, parents and of course, students. It is evident that the principal as colleague and mentor would need to understand and respect AS's professional judgment in changes in the "sections" in the curriculum. One notices how internal conditions such as the syllabus and its relation to the textbook and the role of the principal can constrain and enable teacher rhetoric.

DS shared a similar experience in relation to the syllabus and his mathematics department head and issues of conformity and uniformity. He is a high school teacher at a private, modestly-funded, religious high school in the suburbs of Cape Town. During the first interview he spoke about some insights he had acquired in terms of teaching algebra. He then spoke about how he was forced to conform by his head of department:

DS In fact this year I approached it this year, I did it like that, I started out, saying to my standard nines (eleventh graders), before I did the remainder/factor theorem, I'll go with you to solve a cubic equation.

DS They did one on the board. We're going to do it the same way as we solve a quadratic equation, we must factorize. Now how exactly do we solve a cubic equation? And then I came down to the factor theorem and the remainder theorem, and we started with the remainder theorem, we started with the remainder theorem. And then I got rudely interrupted by my department head, he said no, that should be taught in the second quarter. And then I was lagging behind him, so we could not set a similar paper, and the whole thing just went off track.
(interview # 1)

This excerpt tells one about the powerful influence of the mathematics department head in regulating the mathematics curriculum and thus placing a constraint on teachers' enacted models of teaching and learning mathematics (Ernest, 1987, p. 252). The mathematics department's reality is that it is situated in an institution where uniformity and conformity are emphasized (Popkewitz, 1988). The excerpt does inform one about problematic internal collegial relationships. SD's professional judgment about the order of teaching the remainder and factor theorem was forced to the wayside for the sake of keeping conformity within the fragmented mathematics curriculum.

In conclusion it is evident that internal to the school, the hierarchy of relations in the school, the workings of the mathematics department, the issue of time, the syllabus or curricular scheme and the lack of professional development opportunities are salient categories in the way teachers talk about their espoused and enacted models of teaching and learning mathematics (Ernest, 1987).

Summary

The school mathematics tradition appears to be very strong in the rhetoric of all six teachers in the study. There are instances where they espouse about the teaching and learning of mathematics that go beyond the school mathematics tradition. For example, there is the case of DS where he says that "the subject lacks color and vibrancy." During the interviews some of them show their frustration as they struggle to try to articulate ways of teaching mathematics in ways that are different from the school mathematics tradition. In particular, their beliefs about students' ability and the teaching and learning of mathematics are major obstacles. The ways in which they talk about the social context of teaching show that it further constrains their espoused and enacted theories of teaching and learning mathematics. This is especially the case with conditions internal to the school, such as a lack of collegial discussion about the teaching and learning of mathematics, over-crowded classrooms and the fragmented curriculum. In the case of the high school teachers there is the pressure of the matric.

A further analysis is needed to see how the teachers in the sample react when they are pushed beyond the school mathematics tradition. This is a way to compare their rhetoric with the reform rhetoric in *Mathematical Literacy, Mathematics and Mathematical Sciences* (MLMMS).

CHAPTER 6
AN INDIRECT JUXTAPOSITION: TALK ABOUT MATHEMATICS AS REASONING AND MATHEMATICAL RELATIONSHIPS AMONG TEACHERS IN THE SAMPLE

Overview

The research question that drives this study is, what are the similarities and differences in the ways teachers speak and policymakers write? The data corpus consists of two kinds, transcribed interviews and observations with three middle school and three high school teachers and a set of policy documents on *Curriculum 2005.* The interviews also include observations and post-observation conferences. The previous chapter shows that there is strong evidence of the school mathematics tradition (SMT) (Cobb *et. al.* 1992; Gregg, 1995) in the way teachers talk about their practice. The goal of this sixth chapter is to do an analysis of the contact points between the rhetoric of teachers and policymakers that goes beyond the SMT.

In a study of the state of California's *Mathematics Framework* (California State Department of Education, 1985), Darling - Hammond (1990) coins the phrase "the power of the bottom over the top" to underscore the importance of teachers' understanding of policymaker rhetoric. What teachers understand and do in the classroom is what counts as instructional policy. This study is about finding out how teachers understand policymaker rhetoric about school mathematics reform. It is about finding out whether policymakers through their rhetoric adequately address conditions at the classroom level. The latter includes constraints and opportunities in the social context of teaching. More importantly, it includes teacher belief and knowledge about the nature of mathematics and the teaching and learning of mathematics. Teachers therefore have power "over the top, " namely, the policymakers, because their understanding and interpretation or lack thereof is the education policy. In summary, this study is about finding out how the two communities, policymakers and classroom practitioners, impact one another.

Teacher interviews focused on the nature of mathematics and the teaching and learning of mathematics. There are places in my prompts where I used some of the language in the policymaker rhetoric about "mathematical relationships" and "mathematics as reasoning," two "specific outcomes" in the mathematics component of *Curriculum 2005*. As a result the teachers in the sample came up against this language and were able to interact with it. This therefore makes a place to learn about the contact points between the two sets of rhetoric. The juxtaposition between the two discourse communities, teachers and policymakers, is indirect because it was not my intention to do one while I conducted the interviews.

First is a brief description of the two specific outcomes on mathematical relationships and reasoning. This is followed by analysis of data categories from the middle school and then high school teacher rhetoric in relation to the two specific outcomes. The data categories include observational data. In the case of the high school teachers' talk in relation to the two specific outcomes, the data categories are algebra, geometry and calculus, the traditional divisions in the mathematics curriculum in South Africa. The reason for separating middle and high school teacher rhetoric is because the school mathematics tradition and the mathematics curriculum operate on the assumption that middle school mathematics is a "prerequisite" to high school mathematics, i. e. mathematics is somehow linear. This is a belief common to the school mathematics tradition and something that causes friction in terms of students' understanding, as will be seen.

Specific outcomes on mathematical relationships and reasoning

"An analysis of my own rhetoric" in chapter three shows that my own rhetoric in the interview data comes mainly from disciplinary traditions to mathematics education. In the interview data are excerpts where I prompt teachers to find out whether they ask their students to "reason" about the mathematics and whether they point out "mathematical relationships" between different parts of the mathematics curriculum. The corresponding policymaker rhetoric in the mathematics component of *Curriculum 2005,* Mathematical Literacy, Mathematics and Mathematical Sciences (MLMMS), are specific outcome 2 :

> 2. Manipulate number patterns in different ways.
> Mathematics involves observing, representing and investigating patterns in social and physical phenomena and *within mathematical relationships.* Learners have a natural interest in investigating relationships and making connections between phenomena. Mathematics offers ways of thinking, structuring, organizing and making sense of the world (p. 116).
> [Mathematical Literacy, Mathematics and Mathematical Sciences, *Government Gazette,* June 6, 1997, no. 18051] (italics added)

and specific outcome 10, which is about reasoning:

> 10. Use various logical processes to formulate, test and justify conjectures:
> Reasoning is fundamental to mathematical activity. Active learners question, conjecture and experiment. Mathematics programs should provide opportunities for learners to develop and employ their reasoning skills. Learners need varied experiences to construct convincing arguments in problem settings and to evaluate the arguments of others. (p. 117)
> [Mathematical Literacy, Mathematics and Mathematical Sciences, *Government Gazette,* June 6, 1997, no. 18051]

These two outcomes are central in the writings of Dewey (1902/1992), Ball (1988a; 1988b), Lampert (1988a; 1988b; 1990; 1992), Chazan (1993a; 1993b; 2000), Schnepp *et. al.* (2000) and Thompson (1994) among others. They emphasize disciplinary approaches to the teaching and learning of mathematics, specifically on what it means to "do" and "know" mathematics. Thus a statement like "mathematics involves observing, representing and investigating patterns in social and physical phenomena and within mathematical relationships" illustrates what it means to do and to know mathematics in a disciplinary tradition. This key statement in the second "specific outcome" I refer to as policymaker rhetoric about "mathematical relationships." Similar disciplinary activity in the teaching and learning of mathematics is implied in rhetoric such as "mathematics programs should provide opportunities for learners to develop and employ their reasoning skills," and "learners need varied experiences to construct convincing arguments in problem settings and to evaluate the arguments of others." These are key statements in the tenth specific outcome in policymaker rhetoric about "mathematics as reasoning."

An earlier analysis of my own rhetoric in all the interviews with the teachers indicates that I often use such prompts as:

- What prior mathematical knowledge do you rely on?
- Should there be some deeper connection between different exercises?
- What previous mathematics does this bit of work connect with?
- Have you thought of alternate ways of approaching the very same ideas?
- Have you thought of re-phrasing or re-presenting the problem for the students?
- Are you saying that students' explanations should be discouraged?

These prompts point in the direction of the specific outcomes on mathematical relationships and mathematics as reasoning. When I used these prompts my goal was to find out whether the teachers had any thinking about the mathematics that takes them beyond the school mathematics tradition (SMT).

Middle school teacher rhetoric about mathematics as reasoning

In this section is an analysis of the rhetoric of three middle school teachers, PN, AS and NJ, with regard to the specific outcome on mathematics as reasoning. Their rhetoric comes from the interview data. Also, in the case of AS there is an analysis of observational data. The three teachers had different reactions when I prompted them about whether they ask their students to reason or to explain their answers. Teachers' understanding of the notion mathematics as reasoning is important because they hold the "power of the bottom over the top" (Darling - Hammond, 1990) when it comes to education policy at the classroom level.

In the first interview PN spoke of her background as a mathematics teacher and in general about her student assignments. She talked about how she makes questions in students' assignment "realistic" and "practical." It was at this point that I prompted her about reasoning:

Faaiz	Do you ask them to reason also, within any particular question?
PN	Yes, I think that is what comes up in word problems. (interview # 1 of 4)

I did not ask her to give a specific example. By asking her "do you ask them to reason ?" I wanted to know whether she allows her students to explain why different rules in general in mathematics work, or why it is legitimate to use a rule in any particular instance in the mathematics classroom (Lampert, 1990, p. 56). I do not think of reasons or reasoning in terms of repeating a rule or perhaps making a statement like "my teacher said so."

Many "word problems" or "story problems" in the mathematics curriculum do require students to reason. They do allow students "to develop and employ their reasoning skills" [*Government Gazette*, June 6, 1997, no. 18051(p. 117)]. Here is an example of a similarity between teacher rhetoric and policymaker rhetoric. The rhetoric about reasoning in MLMMS does not include in-depth illustrations of "word problems" that involve reasoning. On the other hand "word problems" is not an only area in the mathematics curriculum where students need to reason. The fact that the teacher in this instance thinks of reasoning in terms of "word problems" means that she probably has not had opportunities to see other examples of mathematics as reasoning. For example, students could work on numerical calculations in another medium, (Lampert, 1990, p. 56) as for example pictorial representations of the addition or subtraction of fractions, which are not necessarily "word problems." Here students could be asked to explain how they shaded different fractions in order to add or subtract them.

In the second interview that focused on one of her question papers, I had another opportunity to talk about mathematics as reasoning. The particular question I looked at required students to find the perimeter of a triangle. At first she explained to me how she graded her students' work:

Faaiz	Do you explicitly ask your students to reason, to give reasons for their answers? For example if they say that the perimeter is 3 cm, do you probe them by asking them how they arrived at 3?
PN	I always tell them if you just give me an answer, you only get one mark. So I don't accept just an answer, they have to show *how* they arrived at the answer. I always say in a question, the answer would be worth three or four marks, if you *only* give me an answer, I can only give you one mark.

PN The rest of the marks are specifically for the way in which you worked it out. That is the standard rule here, nobody just ever gives an answer even in a test.
[emphasis in original] (interview # 2 of 4)

The example she gives about the perimeter of 3cm is more specific compared to the more general case of mathematics as reasoning in the first excerpt. By prompting her about reasoning I wanted to know whether she allows her students to distinguish between perimeter and area, for example. In the school mathematics tradition, the difference between perimeter and area is formalized in terms of formulas for each, perimeter and area. Her question to her students did not read in a way where they were asked to reason about perimeter and area. It required the mere application of a rule for perimeter. In her response PN is referring to her tests and how she grades student work. "Mark" in a United States setting is the equivalent of a "point" in grading. Her rhetoric, "how they arrive at answers" and "the way they worked it out" is about reasoning and thus a contact point with "learners need experiences to construct arguments in problem settings" [*Government Gazette*, June 6, 1997, no. 18051, p. 117]. However, reasoning that involves "varied experiences," "convincing arguments" and "evaluate the arguments of others" could also demand instruction and examination questions where students are pushed in such directions. Such teaching and examination questions do not require a repetition of a rule as a reason (Lampert, 1990) for perimeter, for example. A student in the class might have thought of area and perimeter as something similar. Students could thus be asked to "evaluate the arguments of others" in the classroom as in standard instruction as well as in a graded test.

Her question about perimeter to her students was about the repetition of a rule for perimeter and most likely influenced by the school mathematics tradition. Also, her rhetoric :

> "So I don't accept just an answer, they have to show *how* they arrived at the answer. I always say in a question, the answer would be worth three or four marks, if you *only* give me an answer, I can only give you one mark. The rest of the marks are specifically for the way in which you worked it out. That is the standard rule here, nobody just ever gives an answer even in a test."

indicates what she understands about students being explicitly ask to reason and to give reasons for their answers. Here she is probably talking about her students making their reasoning visible. On the other hand reasoning such as "how they arrived at the answer" might also be about procedures and rules, as in the school mathematics tradition. More detail is needed to uncover how she thinks about mathematics as reasoning.

During the same interview she showed me one of her multiple choice questions, which had a drawing of a square with a length of 1 centimeter. Students are asked to give the area of the particular square. I then asked her whether there were students who said that the answer is $4cm^2$? In the mathematics curriculum, area and perimeter are two concepts that students often confuse. Thus a teaching moment could be about having students "construct convincing arguments" about differences between area and perimeter :

Faaiz	The reason why I ask this is, what would be your reaction if I look at this question, I make the drawing and I say, Mary said that the area of this square is 4 cm^2. Do you agree or disagree and why? How would you react to such a question? Would you give that to the students? Would that be helpful?
PN	You mean would I ask the class? Oh, yes I see what you're saying.
Faaiz	I'm asking what you think about such a question?
PN	I think, (pause), *it would be something different for us.* You don't really work, it would also make the child realize, it would make the child think more (pause). It would definitely get the child to think. Now he has to make choice and he has to decide whether the answer is correct or incorrect.
Faaiz	And also say why.
PN	And why. I should actually try that (laughs) [emphasis added] (interview # 2 of 4)

My prompt, "Mary said that the area of this square is 4 cm^2. Do you agree or disagree and why? How would you react to such a question? Would you give that to the students?" in this excerpt illustrates what I understand by mathematics as reasoning. In such a question there would be an attempt to transcend instruction and assessment or teaching and learning because the question tries to capture what happened in a teaching episode. Such a question would allow students "varied experiences," opportunities to construct "convincing arguments" and have them "evaluate the arguments of others" as in the policymaker rhetoric. The question is more narrowed down and detailed compared to the policymaker rhetoric which reads :

> Reasoning is fundamental to mathematical activity. Active learners question, conjecture and experiment.
> [*Government Gazette*, June 6, 1997, no. 18051, p. 117]

My prompt is more specific to "question and conjecture" and "construct convincing arguments in problem settings" than the policymaker rhetoric. From the excerpt above it appears that she has an idea of what I mean by mathematics as reasoning. For example, she says, "It would definitely get the child to think." This seeming realization of what I mean by mathematics as reasoning cannot be taken seriously because it is only her talk.

Excerpts from the third and fourth interviews indicate her rhetoric about reasoning is different. The notion of mathematics as reasoning in geometry is easier than other curricular areas. For example, in the third interview she talked about asking her students the area of a triangle:

PN For example this morning, they were doing triangles. How do you find the area of triangle? The students said, teacher, base times height divided by two and not one of them could explain to me *why, where* that came from, you know. Then I had to take them back to the square and the rectangle, and divided it in half and gave them some idea of where it came from. That is what I believe should be done in every aspect of the work, it's not just learning something because it has to be known, but it is *understanding*. And that is why I would insist on going back to the squares and the rectangles.
[emphasis in original] (interview #3 of 4)

In this excerpt PN makes a connection between knowing a rule, the area of a triangle, and why, when she refers to "the square and the rectangle." This is interesting because students are able to see how one arrives at the formula for the area of a triangle, and why it so. She could have expanded on rectangles and squares by including trapezium and parallelograms. In this instance her example points in the direction of relational understanding (Skemp, 1978). Here her rhetoric makes a positive contact and is similar to policymaker rhetoric about providing opportunities for learners to develop and employ their reasoning skills [*Government Gazette*, June 6, 1997, no. 18051(p. 117)].

In another excerpt from the fourth and last interview, PN recounts the exact example about the one by one centimeter square:

PN	What was the one example that you gave me the last time. Oh, something that they got incorrect and the 1 cm square area we used and I put it on the board
Faaiz	Oh, with the perimeter being 4 cm.
PN	Yes. They had to find the area and somebody gave an answer of 4 cm^2. I put it on the board and I said I got this answer, what do they think? *Why* do you think somebody gave this answer? And somebody could actually tell me "teacher, they worked out the perimeter instead of the area." They saw the connection immediately. I put it on the board the very next day after you spoke about it. [emphasis in original] (last interview, # 4)

The fact that she recounts the same example could be an indication of how new the notion of mathematics as reasoning is to her. On the other hand, her rhetoric here could just be a case of good interviewee behavior, that is, saying what she thinks I want to hear. She may actually have put the problem about the 1 by 1 cm square up on the board the next day and ask her class whether the area is 4 cm^2 and asked them to explain their answers. What she says in this excerpt matches the policymaker rhetoric about mathematics as reasoning. The problem with the latter is that it is too broad and not specific in terms of curricular examples about perimeter and area problems. My earlier prompts are more focused on particular instances of mathematics as reasoning in terms of perimeter and area, something which is not in the printed policymaker rhetoric in MLMMS.

AS, the second middle school teacher, resisted the idea of asking students to give reasons for their answers. In the following excerpt he talks about a "number sequences" question in one of his assignments. I prompted him to find out whether he asks his students for reasons:

AS	For example this one, write down the next number in the sequence below. You've got to let them see the sequence in order for them to understand that. So you cater for everybody.
Faaiz	And do you ask them why? Suppose students say that the next number is 15, for example, do you ask them why? Do you ask them to explain how they arrive at 15 or whatever?
AS	In the class while we're doing it yes, but not in a question paper.
Faaiz	Why not?
AS	No, I don't do it in a question paper. I haven't done it. You're going to throw the kids out, I think, by asking the question why. The explanation in the class is fine. (interview # 1 of 4)

This excerpt is about "number sequences" in which typically students explore sequences of numbers that have different patterns. For example, the number sequences could be "arithmetic" where there is the addition or subtraction of a certain number or numbers. They could also be "geometric," where each consecutive number is multiplied by a certain number, and a pattern develops.

Lampert (1990) found that when students work with number sequences in the school mathematics tradition they keep their thinking and thus their reasoning "implicit and private" (p. 56). The teacher would ask them to "show their work" by only having them give or write down "the answer" or the next number in a number sequence, as in this case. My prompt

> "And do you ask them why? Suppose students say that the next number is 15, for example, do you ask them why?"

points in the direction of policymaker rhetoric about students or "learners developing and employing their reasoning skills" and "constructing convincing arguments in problem settings" [*Government Gazette,* June 6, 1997, no. 18051]. My prompt is also more specific in terms of a curricular example about number sequences compared to the policymaker rhetoric.

AS's response to my prompt is about keeping students' thinking and thus their reasoning implicit and private. For example, he says "in the class while we're doing it yes, but not in a question paper." Here he rejects the idea of explicitly asking students in the form of a written question how they arrive at their answers or how they reason because he seems to fear that such a test might be too hard. It might also be that he believes that one arrives at answers in mathematics quickly, or that one keeps one's thinking and reasoning private and implicit. The latter is common to the school mathematics tradition.

"The explanation in the class is fine" sort of matches with policymaker rhetoric about "mathematics programs should provide opportunities for learners to develop and employ their reasoning skills." By not putting something important on a test one gives the message that it is not important.
However, the policymaker rhetoric also states that "learners need varied experiences to construct convincing arguments in problem settings and to evaluate the arguments of others [*Government Gazette,* June 6, 1997, no. 18051(p. 117)]. An example of "varied experiences" could certainly be about having students write out their explanations in number sequence questions in an examination to stress the importance of having students explain their answers.
As much as his rhetoric about explanation is similar to the policymaker rhetoric,

it is in fact a disconnect from the way I interpret "varied experiences to construct convincing arguments." Here his rhetoric potentially indicates how he will make sense of the policymaker rhetoric. Observational data about AS will give more details about how he understands policymaker rhetoric about mathematics as reasoning.

In an observation of a lesson on decimals numbers, AS wrote the problem up on the board:

> *Calculate the following, then check the answer with a pocket calculator and explain to your friend what you observed.*
>
> (a) 10 x 4,7
> (b) 10 x 2,61
> (c) 100 x 2,61
> (d) 10 x 0,246
> (e) 10 x 0,0513
> (f) 100 x 3,8
> (g) 100 x 0,871
> (h) 100 x 0,0195
> (i) 1000 x 0,0426

This lesson I observed was part of the third interview I had with him. What he wrote up on the board is about providing his students "opportunities to develop and employ their reasoning skills" [*Government Gazette,* June 6, 1997, no. 18051(p. 117)]. For example, he asks them "to explain to your friend what you observed." This seems very close to making a conjecture. There is thus a similarity between what he wrote on the board and the policymaker rhetoric. Prior to observing this lesson I had two interviews with him in which I talked about reasoning. His note "explain to your friend" might be a way of pleasing me or apprehending and enacting what he understands by reasoning.

In a traditional sense the answers to the questions he posed on decimals are about "shifting the comma" or (shifting the dot in the United States), a procedure. Thus students will have to "explain to a friend" that in their answers they noticed the comma shifts to the right as they multiply by powers of ten. In their answers they will not be able to notice a continuity between the (a) and (b) parts of the problem on the board, although there is between the (b) and (c) part :

(a) 10 x 4,7

(b) 10 x 2,61

(c) 100 x 2,61

The remaining questions, (d) through (h), look as if they have been chosen at random. His students might therefore miss an opportunity to reason about the centrality of size or scale operating on the same quantity when it is comes to decimals. By shifting the comma to the right, they might not be able to see a growth in the numbers that could be quantities like distances they were working with. His students could have plotted the growth of a single number say, 2,61 and notice how it increase in scale and size. In this way his students would have "varied" experiences to construct convincing arguments in a "problem setting" [*Government Gazette,* June 6, 1997, no. 18051(p. 117)] that focuses on the centrality of size and scale when it comes to decimals. Although his note to his students is similar to the policymaker rhetoric, it is in fact a disconnect because of his overt focus on the procedure of "shifting the comma to the right."

By the fourth and last interview I had given AS a few middle grades reform-consistent mathematics curriculum materials from the United States, the *Connected Mathematics Project* (CMP), to browse through a week before the actual interview. Here is an excerpt of his rhetoric :

AS Our problem is that we tend to teach methods. Our books are designed like that, showing lengthy arithmetic procedures. Whereas they don't.

AS When I read through this book I understood why you asked me every time, Do you ask the children to explain this to you? because their books are structured this way, explain your reasoning, explain your reasoning. We would ask the students verbally and orally, like you saw in my class but here they ask the students everywhere explain your reasoning, explain your reasoning.
(last interview # 4)

From the excerpt above it seems apparent that AS picked up some of the rhetoric about mathematics as reasoning. Here his rhetoric makes a positive contact with the policymaker rhetoric about reasoning.

When he states "methods" he is obviously referring to the overt focus on isolated procedures (lengthy arithmetic procedures), so common to the school mathematics tradition. What he "verbally and orally" asked his students to explain when I observed him was about procedures. His repetition of "explain your reasoning" does not mean that he thinks about reasoning in ways where one knows a reason behind a rule or why a it works or why it is legitimate to use it in a particular case (Lampert, 1990, p. 56). It merely means that he has seen questions in the *CMP* materials in which students are exhorted to make arguments that do not require the repetition of a rule as a reason. What he says cannot be taken as evidence that he will change his classroom practice.

His comment implies that texts and other curriculum materials can be important agents of change (Cohen and Ball, 1990, p. 251). Textbooks can be important agents that mediate between policymaker rhetoric and teacher rhetoric. For example, the *Connected Mathematics Project* (CMP) is an instantiation of the policy statements in the *Standards* (NCTM, 1989; 1991). This project has developed elaborate curriculum materials that show what learning and teaching to the *Standards* look like. Curriculum materials that portray policymaker rhetoric are therefore powerful ways of influencing what teachers say and not necessarily what they do.

Later on during the same interview he reflects on the challenges of *Curriculum 2005* itself:

AS We actually teach the child how to think, which is a lengthy process. I think we'll have to do, for lack of a better word, an *unlearning* of what we've done in order to do this (referring to a reform-minded middle school curriculum unit from the United States) and with the *Curriculum 2005*, I think this is what we're striving for.
[emphasis in original] (last interview, # 4)

His rhetoric about "unlearning" is noteworthy because it implies calling into question how one teaches and learns mathematics and the whole fragmented mathematics curriculum. Ball (1988a) makes a similar observation about "unlearning to teach mathematics" in light of the school mathematics reform according to the NCTM *Standards* (1989; 1991). The title of her article is significant because it draws attention to the need to transcend a dichotomy such as teaching and learning mathematics. In the school mathematics tradition teachers do the teaching and students do the learning. Current school mathematics reform in the United States aims at getting teachers to learn to teach mathematics. That is, it has an overt focus on transcending the teaching-learning dichotomy. AS is making a similar comment about bringing together learning and teaching in order to understand *Curriculum 2005.*

An examination of NJ's rhetoric shows a picture of students learning rules without reasons. She even notes that students who are good at mathematics are able to do mathematics without giving reasons to support their answers:

NJ So I think in order to be good at maths, you don't have to be able to give reasons to support your answers.

Her rhetoric indicates a disconnect with similar policymaker rhetoric about reasoning. Her response typifies what it means to be good in the school mathematics tradition, where the mathematics is formalized. Gregg (1995, p. 458) in his study found that the emphasis on formalized school mathematics appeared to result in a situation in which answers could not be justified mathematically. Also, from the excerpt it seems mostly likely that she prefers her students to keep their thinking and thus their reasoning "implicit and private" (Lampert, 1990). This means that she is going to find it difficult to think about policymaker rhetoric about mathematics as reasoning where "active learners question, conjecture and experiment" and have "varied experiences to construct convincing arguments in problem settings and to evaluate the arguments of others" [*Government Gazette*, June 6, 1997, no. 18051(p. 117)].

So far we looked at contact points between teacher and policymaker rhetoric with regard to mathematics as reasoning. The next comparison looks at contact points between the two rhetoric communities with respect to "mathematical relationships."

Middle school teacher rhetoric about mathematical relationships

This section begins with a focus on PN's rhetoric in relation to the specific outcome on observing, representing and investigating patterns within mathematical relationship [*Government Gazette*, June 6, 1997, no. 18051, p. 116]. This is followed by the rhetoric of the two other middle school teachers, AS and NJ. PN's rhetoric is the most interesting because at the beginning of the study she talks about how she "integrates" in her teaching. In the last interview it becomes clearer what she means by "integrating."

During the first interview I asked PN to tell me what she is doing about the "compartmentalized" curriculum :

Faaiz	If you look at the seventh grade syllabus, have you thought of any ways—you mentioned that it is compartmentalized—do you have any different thinking on that? Are there some of the so-called sections that you've thought of in a different way?
PN	You see this is the first year that I'm teaching grade seven after a couple of years. So the syllabus has changed as such. Like computers, I mean the calculator skills, I integrate with most of the other work. I feel I don't want to just teach the properties of, the distributive property and those other properties all in isolation. Those should also be integrated. Percentages we normally latch up with decimals. Oh, all of them. You have to let the child see the relation, all the way, *not* teach things in isolation. I think that is important. Because the child learns about common fractions, and now when you do decimal fractions, it is something so different, so new. They need to see it together. They need to be able to relate it to each other. Decimals and percentages I feel have so much in common, you know. So we try and work those two together. Your angles, and your triangles, those types of things. [emphasis in original] (interview # 1 of 4)

The traditional mathematics curriculum is fragmented into several "sections." In the words of the South African mathematics educator, Paula Ensor (1996), it is strongly classified, brutally sequenced and ruthlessly paced. Many mathematics educators have written and continue to write about and explore unifying ideas within the curriculum. With my prompt I wanted to find out whether PN had any thinking about possible unifying ideas within the curriculum. A study of relationships between quantities (Chazan, 2000) is an example of a "big idea" or a unifying idea that ties together many of the "sections" in the fragmented curriculum. My prompt points in the direction of policymaker rhetoric about "mathematics involves observing, representing and investigating patterns in social and physical phenomena and within mathematical relationships" [*Government Gazette,* June 6, 1997, no. 18051(p. 116)].

PN is aware that teaching "calculator skills," the "distributive property" and "those other properties" in isolation is not good for her teaching practice. She therefore talks about how she "integrates." She gives very definite examples of what she thinks should be "integrated." For example, she mentions percents, decimals, and how they "relate" to each other and how students need to see them "together." She is therefore talking about investigating mathematical relationships. [*Government Gazette,* June 6, 1997, no. 18051(p. 116)]. Here her rhetoric is similar to and makes contact with the policymaker rhetoric.

Copies of PN mid-year (June) examinations and other tasks I saw which she gave her students do not have any questions where percentages are "latched up" with decimals. At a rhetorical level she realizes that in her teaching she could "integrate" different "sections" in the fragmented mathematics curriculum. Later on in the same interview she uses the word "integrate" again. A close reading of the related excerpt gives one a better sense of what she means by "we integrate":

PN We use a variety of textbooks, whatever we feel the child will benefit from. I like my games, like we play cards to get around area. I've got quite a few games up there.

PN I haven't started on tessellations, because of the backlog that they had. We had a very difficult time during the first term. We *integrate*, we do not just work on one thing per week. We try to put in a variety, because you can teach fractions this week and do something the next week. You need to keep it all alive all the time.
(interview # 1 of 4) [emphasis added]

The cards she plays with her students "to get around area" could be a possible way to have her students "observe, represent and investigate patterns within mathematical relationships." The cards could also "offer ways of thinking, structuring, organizing and making sense of the world" [*Government Gazette*, June 6, 1997, no. 18051(p. 116)]. Her use of cards is therefore a positive contact point with policymaker rhetoric about mathematical relationships.

By "integrate" she appears to mean teaching a "variety" of "sections" in the curriculum during one particular week. For example, she states, "we just do not work on one thing per week." It is difficult to make the claim that there are mathematical relationships between the "variety" because "fractions" and the "something" she mentions, are not linked via a "big idea," like scale as a way to explore mathematical relationships. Although her rhetoric matches with the policymaker rhetoric about mathematical relationships, it is in fact a disconnect.

In the second interview PN again talks about "integrating" :

PN I did decimals and then we went over to area, no perimeter and used those decimals numbers in the perimeter as well. Kids really don't see the connection themselves, you know, you have to guide them. And then we did the pie chart and we put fractions in and we put one decimal in. The child realizes that they're not very much different. I hope I'm doing the right thing.
(interview # 2 of 4)

In the excerpt she talks about "the connection" between fractions and a decimal and how she puts them in a pie-chart. She is in a way saying something related to observing, representing and investigating patterns within mathematical relationships [*Government Gazette,* June 6, 1997, no. 18051(p. 116)]. However this contact between her rhetoric and the policymaker rhetoric is a disconnect because she does not stress mathematical relationships between concepts like decimal numbers, and quantities such as areas, and perimeter. For example, she could have students explore areas and perimeters of squares and rectangles by reducing and/or increasing their values by decimals. Students could be asked to make conjectures about lengths of the sides of the squares and rectangles. Her rhetoric, "I hope I'm doing the right thing," is an indication that teaching in ways that emphasize mathematical relationships in a strongly classified mathematics curriculum is something novel to her practice.

By the fourth and last interview, PN talked about how her students are still thinking in "compartments." This interview was conducted around a reform-consistent middle grades curriculum unit from the *Connected Mathematics Project* from the United States. I had given her the unit to browse through a week before the actual interview. For the interview PN could basically talk about any part of the book. From the excerpt below it seems she realizes some value in stressing mathematical relationships in the curriculum:

PN When you are giving the child a ratio, why can't the ratio not be given in the form of a percentage and a fraction, which will consolidate what they already know but will also make them realize that, like for instance this morning, I gave them an exercise with perimeter and area of triangles and area of rectangles, et cetera.

PN But in the exercises we came to conversions from meters to centimeters, because some of measurements were in meters and some were in centimeters and they had to put the final answer in meters. And we were converting and I said to them that we have to multiply by 100,
and you know because we've *done* multiplication by 100, 10 and 1 000, in the first term, the kids couldn't use that and transfer it because now we're busy with area. They still thinking in *compartments*. That hasn't been broken down yet. Our kids are still compartmentalizing in their minds, so their transfer of knowledge is not easy, they cannot easily do that._ That only pertains to that.
(emphasis in original) [last interview # 4)

A comment like "why can't the ratio not be given in the form of a percentage and a fraction" is in some ways faulty because some "ratios" are in the realm of irrational numbers, like pi, while others like fractions are more in the realm of rational numbers. Here her rhetoric, however, makes contact with policymaker rhetoric about observing and investigating possible mathematical relationships between ratios, percentages and fractions. A similar observation can be made about "multiplication by 100, 10 and 1 000" although she might have treated this multiplication as just another procedure. Her rhetoric about fractions and decimals seems to indicate that she wants to bridge her espoused model of teaching mathematics with enactment in the classroom. On the other hand it could also be a case of good interviewee behavior. What she emphasizes about her students thinking in "compartments" is not an indication that she will or has changed her practice. Teachers construct their practices gradually out of their own experience as students, their professional education, and their previous encounters with policies designed to change their practice (Cohen and Ball, 1990).

In the school mathematics traditions, ratios, decimals and percentages are "strongly classified" (Ensor, 1996). Her rhetoric about relating decimals and quantities such as the perimeter and area of a triangle is a good opportunity in her practice to talk about a general concept of a scale of a notation in relation to quantities. A drawback in stressing mathematical relationships such as a general concept of scale may be the way she introduced "multiplication by 10, 100 and 1 000, in the first term" as a set of isolated procedures. Her remarks that students are thinking in "compartments" and "that only pertains to that" are possible indications of her seeming awareness of the fragmented mathematics curriculum.

NJ's rhetoric about mathematical relationships is a disconnect with the policymaker rhetoric. She did not see the strongly classified (Ensor, 1996) curriculum as something to overcome or to counter. She sees it important for students to master "each section of the work on its own" (interview # 2), as in the notion of "prerequisites" as being mainly procedures. She also "fully agreed" that in mathematics learning students should master topics and skills at one level before going on. Her rhetoric seems channeled by the logic behind the strongly classified curriculum in the school where in school subjects like mathematics, emphasis is placed on stability and consensus (Popkewitz, 1988). For example, after I observed her algebra class on "factors" I prompted her to find out what previous mathematics "factors" related to. She gave this reply:

NJ Well that was a whole new concept, something different to what they've done. I couldn't relate it to anything that they've done in the past.
(interview # 3 of 4)

NJ is articulating a conception of school algebra (Usiskin, 1988) as the study of structures, where students typically do a great deal of factoring. According to

Adler (1998), a South African mathematics educator, there is a "rush into abstract algebra" in the middle grades curriculum in South Africa. A major result is that students and teachers have difficult in seeing fundamental ideas in school algebra because they are not "disentangled from the technical procedure which has been invented to present their exact presentation in particular instances" (Whitehead, 1911/1958). Her rhetoric, "I couldn't relate it to anything that they've done in the past" indicates how strongly classified the curriculum really is. Having students do lots of factoring is a stable activity and also one where consensus about the "right" answer could be reached easily. It also implies that there will be tremendous challenges in having her think about "mathematics involves observing, representing and investigating patterns in social and physical phenomena and within mathematical relationships" [*Government Gazette*, June 6, 1997, no. 18051, p. 116)].

Summary of middle school teacher rhetoric

An important point to note is that there is variation in the contact points between the teacher rhetoric and the policymaker rhetoric. There are two sorts of things that can happen when the teachers in the sample and the policymakers use similar words but mean different things. For example, at times there is positive contact and at times, negative contact between the two rhetoric communities. For example, PN talks about "word problems" as a place where students can employ their reasoning skills. This is a positive contact and yet, not the only place where students can employ their reasoning skills. An example of a negative contact is the case where NJ says that to be good at math one does not need to give reasons. PN uses the words "integrate" and "integrating," which are similar to policymaker rhetoric about mathematical relationships. The analysis indicates that what she "integrates" in fact makes a negative contact with the policymaker rhetoric about mathematical relationships. Positive and negative contacts are mirror image terminology and fall "inside" and "outside" the school mathematics tradition, respectively.

Juxtaposing high school teacher and policymaker rhetorics

In the interview data there are places where the high school teachers HY, DS and EM talk at length about the teaching and learning of algebra, geometry, and calculus, the traditional divisions in the high school mathematics curriculum in South Africa. In the policy document, *Mathematical Literacy, Mathematics and Mathematical Sciences* (MLMMS) there are places where policymaker rhetoric has implications for the teaching and learning of algebra, geometry and calculus. The goal of this section is to analyze "contact points," or similarities between the way high school teachers speak and policymakers write. Such an analysis answers the research question, what are the similarities and differences between the two sets of rhetoric? Teachers' interpretation of policymaker rhetoric at the classroom level is what counts and is the education policy.

At this point it is important to reiterate the nature of the high school mathematics curriculum in South Africa. In South African high schools, students do not take algebra, geometry, trigonometry and calculus as separate classes, as in the United States. A teacher does not teach an algebra-, geometry- or calculus-only class. He or she teaches "mathematics," at any grade level in the high school. The written examinations for mathematics in the high school consist of two papers, normally. In the first paper, there may be an algebra and a calculus "section." The second paper normally has a geometry and a trigonometry "section." At any particular time during the week, the teacher may be teaching algebra for a few days and geometry or trigonometry for the rest of the week. This structure is arranged by the mathematics department in the school.

Algebra, geometry and calculus, are therefore separate "sections" in the South African high school curriculum and I analyze the data in this section accordingly. Another reason is to show how these divisions or "sections" narrow down richer and multiple representations of mathematics. Within algebra, however, a teacher may also speak about different "sections," by which he or she means the different chapters within a prescribed textbook.

Within each of the data categories in the high school teacher rhetoric there is a juxtaposition of teacher and policymaker rhetoric in relation to mathematical relationships and/or mathematics as reasoning, the two "specific outcomes" in the mathematics component of *Curriculum 2005*. In the selected excerpts the policymaker rhetoric is evident in my prompts.

Mathematical relationships and reasoning in algebra

Each of the three high school teachers, HY, DS and EM, speak about ways they teach mathematics, and in particular, algebra. HY's talk reveals his knowledge and belief about the strongly classified curriculum and what he does to make it more coherent. He uses the word "section(s)" to describe the prescribed mathematics curriculum. Typically, South African high school algebra is divided into "sections" such as linear and quadratic equations, straight lines, parabolas and their graphs. These "sections" could be seen as chapters in a prescribed textbook that would also have separate chapters on exponents, indices, logarithms and their graphs. Within these chapters there are usually "sections" on "word problems," "enrichment" and "challenges."

HY gave interesting examples of how he teaches algebra. He talked about "linking" various "sections" within algebra in the strongly classified mathematics curriculum :

HY	Things aren't linked much in mathematics. It's like separate sections. You have to link them by means of your teaching. Take an example, if you teach equations. Now you've done equations but you have to link equations to graphs.
Faaiz	Right.
HY	because, what I normally tell students is that when you're solving two equations, you are solving two graphs simultaneously. But that alone, to explain that with adequate examples, it will take 4 to 5 periods before the kids really believe you. I mean if you explain to a kid, solve for x in $x + 2 = -x$, right. He will solve that quickly, but if you now show by means of graphs what you're doing. The graphs is lagging behind at all levels at school. Pupils have problems with graphs. (interview # 2 of 4)

HY's words, "you have to link equations to graphs" goes beyond the school mathematics tradition in which the curriculum is strongly classified, that is, "equations" and "graphs" are "separate sections." This rhetoric comes from his own interaction with the curriculum and is an example of his espoused and enacted model of teaching (Ernest, 1987). Also, by linking "equations" to "graphs" he is emphasizing mathematical relationships between the two. In this instance his rhetoric makes positive contact with "mathematics involves observing, representing and investigating patterns within mathematical relationships" (*Government Gazette*, no. 18051, p. 120).

HY's example of $x + 2 = -x$ with its graphical solution is an interesting case of $f(x) = g(x)$? (Chazan, 1993). Evidence for this claim is in the line where he states, "you now show them by means of graphs what you're doing." His comment, "graphs is lagging behind at all levels at school" is a poignant observation about a lack of mathematical relationships within the fragmented curriculum. In a curricular sense what he is saying and doing goes "beyond what is prescribed" to use his words, and thus falls outside the school mathematics tradition.

When I observed his algebra lesson on graphs, HY talked about the formalized "standard form," y = mx + c (y = 3x - 6). During the lesson his students reasoned about this graph purely by referring to the formalized, y = mx + c. They also looked at mathematical relationships in the graph of y = 3x - 6. Here is an excerpt from this particular lesson that I observed:

HY	We can deduce two things when the graph is written in the standard form. James, what are the two things?
James	It's going up hill, sir
HY	If you look at the value of m, it is positive 3. That gives you the direction of the graph, the sign of the gradient gives you the direction of the graph. What's the second thing you can determine?
Student 1	The steepness of the graph
HY	Okay, the steepness. But what else can you deduce?
Student 2	The y - intercept
	In other words, just by looking at the graph, y = 3x - 6, you can deduce that the y - intercept is what?
Student 3	- 6

When HY says that "we can deduce two things when the graph is written in the standard form," he is "providing an opportunity for his students to develop and employ their reasoning skills. [*Government Gazette,* June 6, 1997, no. 18051 (p. 117)]. Throughout the excerpt it is clear that his students are reasoning. They are, however, reasoning about formalized rules and the formula for the graph, y =

$mx + c$. In the school mathematics tradition, the m is thought of as the "slope" which gives the "steepness" of the graph, and the 6 is referred to as the "y - intercept." This excerpt is not about a case of mathematical argument (Lampert, 1990, p. 56) because his students are not asked to explain or say why or how the notion of "steepness" arises. They are, however, reasoning in the sense that he asks them to "deduce." From the excerpt one learns that the students are not finding out why the 3 and the 6 in $y = 3x + 6$ give different information.
In this way, there is a mismatch or negative contact with policymaker rhetoric about "learners need varied experiences to construct convincing arguments in problem settings" [*Government Gazette,* June 6, 1997, no. 18051 (p. 117)].

In the case of mathematical relationships, his students are not finding out how the graph $y = 3x + 6$, could be composed of different graphs like, $y = 3$, $y = x$ and $y = 6$. Also, for the complete lesson, I did not see his students looking at a table of values for the function, $y = 3x + 6$, whereby they could see the 3 and the 6 appear in the table of values. The students, however, were looking at relationships between "steepness" and the "y - intercept" but in a narrow, restricted way. Although HY uses language that is similar to policymaker rhetoric about "observing, representing and investigating phenomena within mathematical relationships" [*Government Gazette,* June 6, 1997, no. 18051 (p. 116)], the contact is a negative one. During my observation I did not see a continuity in his earlier rhetoric about $f(x) = g(x)$? (Chazan, 1993). The question is therefore how seriously should one consider the outlier instance of $f(x) = g(x)$?

In the following excerpt I tried to find out what HY had to say about the quadratic function, $y = x^2 - 9$. I wanted to find out whether he has a variety of ways of thinking about $y = x^2 - 9$. The first line in the excerpt is my prompt to find out whether MY sees possible mathematical relationships within $y = x^2 - 9$. There are other relationships—for example, the multiplication of $y = x$ and $y = x$

and then followed by the addition of $y = -9$, giving the resulting function, $y = x^2 - 9$. This excerpt is another example of an indirect juxtaposition of policymaker and teacher rhetoric because my prompt, "have you thought of different ways" points towards "observing, representing and investigating patterns in social and physical phenomena and within mathematical relationships (*Government Gazette,* no. 18051, 1997, p. 120). :

Faaiz	Have you thought of different ways to look at $y = x^2 - 9$?
HY	You mean in terms of factorizing it?
Faaiz	Well, in terms of "what could this be"?
HY	You know I would link it up to products
Faaiz	Okay.
HY	And show them, know if you multiply $(x + 3)$ and $(x - 3)$ then you end up with $x^2 - 9$. That's how the section links up. But in teaching them to factorize it, it's basically understanding, and know the method, that's all. If I'm going to bring in areas and then try to factorize, it's going to be too much for them to understand because factorizing is a simple procedure.
Faaiz	You don't regard bringing in factors as being helpful ?
HY	No.
Faaiz	Why not?
HY	Because factorize is, identify and there's a method, just like that. Bringing in extra stuff might confuse them because ultimately what they have to do is, identify and know the method. (interview #3 of 4)

After my first prompt, he mentions "products," which in the traditional algebra curriculum is another "section" that is almost purely about procedures—for example, the multiplication of two or more factors. The mathematics curriculum in South African schools (Adler, 1998) has an overt focus on school algebra as the study of algebraic structures (Usiskin, 1988), where students typically do a great deal of factoring. I used a further prompt, "what could this be" to find out further what he had to say. Toward the end of the excerpt where he says "the method," he means being able to factor, a procedure. His remark, "factorizing is a simple procedure," meshes well with the notion of school as an institution and a place where school subjects take on a static and stable quality (Popkewitz, 1988). His remark about "a simple procedure" points in the direction of the school mathematics tradition. He is thus not saying that function, $y = x^2 - 9$, consists of the multiplication of two linear functions or the addition of two functions, $y = x^2$ and $y = -9$. His comment about "bringing in extra stuff might confuse them because ultimately what they have to do is, identify and know the method," indicates a disconnect with the policymaker rhetoric about "observing, representing and investigating patterns within mathematical relationships" and giving students "varied experiences to construct convincing arguments in problem settings" [*Government Gazette*, June 6, 1997, no. 18051 (pp. 117-120)]. It could be the case that he has thought deeply about certain sections of the algebra curriculum and not others.

There were, however, instances when HY discussed how he teaches the graphing of quadratic and other functions as "transformations," or more particularly, as horizontal and vertical "shifts." Here his language makes a positive contact with policymaker rhetoric about opportunities for students to observe, represent and investigate patterns within mathematical relationships (*Government Gazette*, no. 18051, 1997, p. 120). Such an approach is a departure from the traditional curriculum, or the "normal way," to use his words, where students follow a set of procedures when they plot "parabolas" or quadratic functions. :

HY When I teach parabolas and absolute values, you have the normal way of teaching them and you have the transformation way. The normal way is to find the x-intercepts and the y-intercept, the turning point and so on.

You can teach the transformation way as well. Let's say you start with $y = x^2$, then you take the graph $y = (x - 3)^2$ which shifts three units to the right. Then you can show them how to sketch $y = (x - 3)^2 + 9$ by shifting it vertically. That gives them another approach to solving that type of problem. Then it is up to them in the exams. If you ask them to sketch the graph, they can use either method, the traditional method or the transformation way. But once again, their method will give their reasoning.

Faaiz In other words you're saying that you don't explicitly ask them for their reasoning?

HY No, you don't need to. It's an open-ended question. All the questions, it is what is mathematically correct, whichever method the pupil has used, that's mathematically correct, you accept it. (interview # 2 of 4)

This excerpt illustrates another indirect juxtaposition of policymaker and teacher rhetoric because of my prompt about mathematics as reasoning. I was interested to find out whether he would give his students "varied experiences to construct convincing arguments in problem settings" [*Government Gazette,* June 6, 1997, no. 18051 (p. 117)]. I wanted to know whether he draws students' reasoning to the symbol manipulations involved. Here I interpret reasoning to mean giving students opportunities to investigate the role of the negative and positive sign and resulting mathematical relationships. Such opportunities I understand as "mathematically correct," to use his language. Students could develop and employ their reasoning skills by making conjectures, raising questions, and examining the role of the negative sign. HY could have his students make conjectures about the position of the negative sign in a quadratic function and resulting graphical changes. For example, an assignment could read, make conjectures about possible changes that occur when one has the negative sign on the "outside" of the brackets as in $y = -(x - a)^2$ and $y = (x - a)^2 - b$, or on the "inside" as in $y = [x -(- a)]^2 + b$.

Through the "traditional way" he seems to prefer to have his students keep their reasoning implicit and private (Lampert, 1990). For example, he states that he would not ask them explicitly to show their reasoning and also notes that "their method will give their reasoning." Here his rhetoric is a disconnect with the way I interpret policymaker rhetoric, because in "the traditional way," students' "method" as reasoning is about restating rules (Lampert, 1990) about the negative sign in this case, as argument. On the other hand his talk about "transformation way" and "open - ended" question is a positive contact with policymaker rhetoric about "learners need varied experiences to construct convincing arguments in problem settings" and "observing, representing and investigating patterns within mathematical relationships."

In the following excerpt HY espouses further what he means by "the transformation way." I asked him how he sees his students taking the straight line idea into the parabola and what previous knowledge like the straight line comes into the picture here :

Faaiz	Taking this straight line further, not at the same standard seven (ninth grade) level, at the standard eight level, where the next so-called graph they do, is the parabola, right?
HY	Yes.
Faaiz	How do you see students taking the straight line idea into the parabola?
HY	Okay, the way I introduce that again is by means of a set of points, but now the rule changes a bit.
Faaiz	The what?

HY	The rule. I call it the rule, in other words the equation changes to $y = x^2$ and so on.
Faaiz	When you say set of points you mean?
HY	A set of points obeying a fixed rule, whatever the rule might be and I'll then introduce it the same way as I introduced the straight line.
Faaiz	How?
HY	I give them a set of points and plot the points and they will start to see that the points do not form a straight line, if they join the points and again give them x-values and they calculate the y-values based on the pattern in the points and ask them can you relate x to y? What's the relationship? Can you form an equation relating x and y?
Faaiz	Okay.
HY	And from there build up. What I normally do in standard 8 (tenth grade) level, I do transformations of the graphs because they do the equation $y = ax^2 + c$. So instead of talking about an axis of symmetry, I show them what happens if you take $y = x^2$ and $y = -3$, what happens to the graph. Then they see that it shifts down.
Faaiz	Right.
HY	So I rather teach on that basis. Once they come to standard 9, (eleventh grade) then we introduce something like $y = (x - 2)^2$
Faaiz	And what previous knowledge like the straight line comes into the picture here?
HY	Again it comes into the picture where they draw a parabola and a straight line, on the same set of axes, and they find the points of intersection between the two.
Faaiz	Okay.
HY	That's where the straight line links up with the parabola. It's always points of intersection. We always calculate points of intersection. We don't do parabola only, we do points of intersection of a parabola and points of intersection of a straight line and a parabola. (last interview, # 4)

Rhetoric such as "a set of points," "the rule, $y = x^2$, the pattern in the points, $y = x^2$ and $y = -3$, what happens to the graph, and "shifts down" are about "varied experiences" and "observing, representing and investigating patterns within mathematical relationships" [*Government Gazette*, June 6, 1997, no. 18051 (pp. 116 - 117)]. Here HY's rhetoric makes a positive contact with the policymaker rhetoric. In the traditional curriculum, there is often an overt focus on the formalized quadratic function of the form $y = ax^2 + c$ of which $y = x^2 - 3$ is an example. In the excerpt HY is explaining how he allows his students to arrive at a formalized quadratic function by investigating and observing mathematical relationships within a quadratic function.

The line

> We don't do parabola only, we do points of intersection of a parabola and points of intersection of a straight line and a parabola.

in the excerpt is interesting, because one can think of a "parabola" and a "straight line" as functions, f(x) and g(x), respectively. Thus, it would be fair to infer that HY's espoused and enacted model of teaching (Ernest, 1987) is about having his students examine graphical solutions, or "points of intersections," where $f(x) = g(x)$? (Chazan, 1993). The data I have is however only about his espoused model of teaching which is definitely a departure from terse directions for symbolic manipulations in algebra (Factor, Simplify, Solve...). In this instance his explanation of how he teaches matches policymaker rhetoric about mathematics involves observing, representing and investigating patterns within mathematical relationships" (*Government Gazette*, no. 18051, p. 120).

In another example in algebra HY talks about how he highlights mathematical relationships between linear and quadratic functions in a lower track classroom. The example he gives are the two functions, $y = x^2 + 3x + 4$ and y = -10 and finding their points of intersection. He even has his class look at the graphs of these functions:

Faaiz — Do you show them the two graphs?

HY — Yes. Let's just say both graphs don't intersect, with the parabola turning here and the straight line is right at the bottom. So they can see that there are no points of intersection. I would then equate the equations.

Faaiz — Right

HY — Now once they equate them, we still end up with a quadratic equation.

Faaiz — Right.

HY — And then we calculate delta. It's only when you ask them explain, why did you do this, then you can really understand what is their level of maths.

Faaiz — Do you push this kind of thing in your own teaching?

HY — Not necessarily with exam questions.

Faaiz — Why not?

HY — Again it depends on the level that you teach. For standard grade (lower track) I would promote it in class, ask reasons and so on, but when I set a paper, then it is a straightforward type of question.

Faaiz — Straightforward meaning you know it or you don't ?

HY — That's it, or the application. There won't be "motivate" your answer or give a reason for your answer or something like that. (interview # 1 of 4)

In the above excerpt his own rhetoric, "it's only when you ask them explain, why did you do this, then you can really understand what is their level of maths," is another positive contact point with policymaker rhetoric about giving learners varied experiences to construct convincing arguments in problem settings

[*Government Gazette,* June 6, 1997, no. 18051(p. 117)]. More excerpts need to be examined to find out what he means by "ask them to explain."

When he says "it depends on the level that you teach," he is implying that there is no need to have lower track students give explanations or reasons when one assesses their understanding. Here he seems to be stating a belief about the "ability" of lower track students. He is saying that his lower track students get "a straightforward type of question" that requires them to know a rule as in instrumental understanding (Skemp, 1987). This is a disconnect with policymaker rhetoric because "varied experiences to construct convincing arguments in problem settings" could include test questions where students have come up with reasons behind rules. Such questions would link instruction and assessment or teaching and learning. Policymaker rhetoric in MLMMS never refers to students' or learners' ability.

It may be more the case that HY treats rules, formulas and facts as if they were argument (Lampert, 1990, p. 56). For example, in his response to the Likert item in the second interview:

The most important issue is not whether the answer to any math problem is correct, but whether students can explain their answers

he says that students explain their answers when they "show their work." In fact he sees reasoning as something peculiar to geometry:

HY I think most of the reasoning is in the geometry.
(interview # 2 of 4)

EM, like HY, does not have the notion of reasoning in his rhetoric about mathematics in general and algebra in particular. During the second interview he wondered just what reasoning in algebra could be like. In a response to the following Likert scale item, EM talks about teaching algebra:

> To be good at mathematics in school, how important do you think it is for students to :
>
> *Be able to provide reasons to support their solutions*

EM	So you are obviously talking about geometry maybe or trigonometry?
Faaiz	Why do you mention those two?
EM	In algebra, how would one support your solutions with reasons?
Faaiz	Do you try to do that?
EM	Well in algebra it's like a procedure basically. And if you don't follow the procedure then you'll obviously not get the answer. So reasoning, I'm trying to think now, how would one give a reason for your answer? (interview # 2 of 4)

Policymaker rhetoric about reasoning matches the prompt "be able to provide reasons to support their solutions," in the Likert scale item. In this interview EM's response about algebra as a "procedure basically" indicates a disconnect with policymaker rhetoric about reasoning being fundamental to mathematics. It could be the case that he is pulled by his past pedagogical experience with the fragmented school curriculum, his apprenticeship of observation (Lortie, 1975) and the fact that he may not have seen vignettes or curriculum materials in algebra with a focus on mathematics as reasoning.

During the third interview I had another opportunity to prompt EM about reasoning in algebra. This time he seems receptive to the notion of reasoning. It may also a case where of good interviewee behavior where he realized that I allude to reasoning during every interview. The excerpt comes from a conversation around $f(x) = ax^2 + bx + c$, the formalized general formula for a quadratic function :

EM	When "a" is positive, the parabola is smiling.
EM	They accept that. In other words I am going strictly according to the textbook. So I am telling them, look here, if "a" is positive you are going to have a smiling graph.
Faaiz	And do they ask you why?
EM	(laughs). Unfortunately.
Faaiz	Unfortunately?
EM	They should actually.
Faaiz	Do you think it is a useful way to go?
EM	Yes, it would be.
	(interview # 3 of 4)

His comment "they should actually" shows a possible realization that there could be benefits in asking students "why ?" or having them think of reasons behind rules. In this instance his rhetoric makes a positive contact with similar policymaker rhetoric. For example, investigating, observing, experimenting and examining changes in the sign of the values of a, b and c and in the graphing of the quadratic function $f(x) = ax^2 + bx + c$ will reveal interesting patterns within mathematical relationships. Teaching could therefore focus on having "learners question, conjecture and experiment" with the idea that "reasoning is fundamental to mathematical activity" (*Government Gazette*, no. 18051, 1997, p. 116 - 117).

When DS was prompted about asking students to explaining their reasoning, he noted that "we *never* do that." His immediate comments were about how his subject was "too focused on the actual symbolism," which could refer to algebra. I did not ask for specifics. At the time I was interested in hearing and finding out what he had to say about students explaining their answers:

DS I think this opens up a whole new thingy here. So I am looking at myself,

DS my subject is too focused on the actual symbolism and stuff like that that's there, and mathematics like you said, could be directed in explain, or here's a solution, I'm giving you a problem and solution, can you discuss or explain or what's happening? That will be nice, to discuss this or whatever, or how would you use this or where else you think you can use it. That will be a real step forward, I think, into thinking, mathematical thinking, you know. (interview # 2 of 4)

In this excerpt DS's language, "mathematics like you said, could be directed in explain, or here's a solution, I'm giving you a problem and solution, can you discuss or explain or what's happening?" is interesting because it reveals a seeming realization of the importance of stressing mathematical relationships and reasoning. Here his rhetoric makes a positive contact with policymaker rhetoric that mathematics programs should provide opportunities for learners to develop and employ their reasoning skills and "varied experiences for learners to construct convincing arguments and to evaluate the arguments of others" (*Government Gazette*, no. 18051, 1997, p. 116 - 117), which could be in algebra.

In summary, HY has interesting rhetoric about mathematical relationships in the teaching and learning of algebra. His talk about "shifts" and "transformations" in graphing quadratic functions falls "outside" the school mathematics tradition. Here his language makes a positive contact and is similar to policymaker rhetoric about learners observing and investigating mathematical relationships. Also, his example of f(x) = g(x) ? (Chazan, 1993), where he talks about "linking separate sections" like equations and graphs is more detailed compared to the policymaker rhetoric, which are mere statements about "outcomes." HY's example is another difference between his rhetoric and the policymaker rhetoric about mathematical relationships.

His rhetoric about reasoning in the teaching and learning of algebra is a disconnect with the policymaker rhetoric in the sense that he treats rules, facts and formulas as argument (Lampert, 1990). For example, he states that "their method will show their reasoning," referring to the "traditional way" or the school mathematics tradition. Also, his lesson about $y = 3x + 6$ had no explain or "why" questions that are "outside" the school mathematics tradition. During this lesson his students were reasoning about formalized rules. The strongly classified and formalized curriculum certainly works against him. Also, he probably never saw curriculum materials where reasons behind rules are a focus. Another factor could be his beliefs about the necessity for a fragmented mathematics curriculum. This may not be true because at times he stresses the need to "link" different "sections" of the algebra in the school curriculum. For example, he notes that "graphs are lagging behind." In the case of the two other high school teachers, DS and EM, one can make a similar claim about a disconnect with the related policymaker rhetoric about mathematics as reasoning. For example, EM, as well as HY, see students explaining their answers as something peculiar to geometry. Also, DS notes that asking students to explain their answers is something teachers "never do," which shows a disconnect with the policymaker rhetoric about reasoning. EM and DS seem to have picked up on the idea of having students explain their answers in algebra. This is probably due to the interview process, or that we happened to talk about the same ideas during interviews or simply good interviewee behavior. Thus the variation in the high school teacher rhetoric in the sample about reasoning cannot be taken seriously because teachers change their practice gradually over time (Cohen and Ball, 1990).

In conclusion, there is variation in the contact points between the policy statement "reasoning is fundamental to mathematical activity" and high school teacher rhetoric about the teaching and learning of algebra.

Mathematical relationships and reasoning in geometry

A long excerpt comes from a final interview as a post-observation conference I had with EM after he taught a geometry lesson. In the lesson he introduced a proportionality theorem in geometry that states that if two triangles are similar then the ratios of their lengths are proportional. In the institutionalized curriculum this lesson is part of "similarity," a "section" in geometry. In the excerpt, my prompts, "How do you want them (the students) to learn it? How do you want them to see it," point in the direction of policymaker rhetoric about mathematical relationships (*Government Gazette*, no. 18051, 1997, p. 116 - 117). In the excerpt the "it" refers to the proportionality theorem.

Faaiz	How do you expect your students to learn this "section" of the work, called similarity?
EM	Are you talking about the theorem or its application?
Faaiz	Well, both, similarity as some mathematical idea, call it a concept.
EM	Can you give me some input on your question?
Faaiz	You obviously want them to learn this (pointing to the theorem on the board). How do you want them to learn it? How do you want them to see it. Okay this is all the mathematics that we have to know for Mr. EM, similarity is here but it is flowing into whatever.
EM	I don't understand the question.

From the excerpt it seems that my question did not come across very clearly. This could be due to the fragmented curriculum and hence the disconnect between "sections" in the curriculum. I then spoke about the ratios of the sides of the triangles in the theorem on the board and referred to trigonometry:

Faaiz	If I look at similarity right, toward the end you came up with something that is really a common thread in my opinion, ratios. Ratios takes you right into trigonometry, that very graph, that very trigonometric function, f(x) = sin x, those points there are really ratios. For me evidence of understanding would be when the student looks at similarity and he connects it to ratios, that's very visible here and he then thinks about the ratios in the trigonometry, because the trigonometry evolves out of similar triangles, isn't it?

In the excerpt above I point out mathematical relationships across geometry, trigonometry and algebra, the traditional "sections" in the curriculum. Here my prompts match policymaker rhetoric and they are also more detailed in terms of a curricular example. He then acknowledged that what I was saying makes sense:

EM	Yes.
Faaiz	I mean you cannot get to the height of the tower but you have some sort of set of ratios that you call sine. And then you generate all those tables. In other words it's really a little similar triangle and a big tower up there.
EM	I think I know what you are trying to say. You are trying to ask me maybe if I want them at the end of the day to see the correlation between that over there (pointing to the proportionality theorem on the board) and trig graphs possibly or may be a straight line for that matter.
Faaiz	Yes.
EM	I would actually have to say no.
Faaiz	Why?

EM	There are instances where I would like them to see, for example where you have a product may be a + b and p + q, as in algebra.
Faaiz	You are saying no, right, because?
EM	There are certain things that I would like them to obviously see, but then there are certain things that if you want them to see, it would be a burden on them, it's extra work, maybe more time involved. The moment I did "products," the teacher didn't show them the correlation over here. The moment I drew this on the board [pointing to a spatial representation of (a+b)(c+d)] there were pupils that took this down, and they asked "sir, how did you get that?" You get an answer nicely in terms of area. In your specific example, if you ask me let them see the correlation between that
Faaiz	Between similarity and the sine function?
EM	That is a little "too far-fetched" (making inverted commas). I wouldn't really want them to see that.
Faaiz	Why do you say that?
EM	Maybe things that are more easier maybe too, I'll use the word in brackets be, easier to see, which don't involve much explanation also. For example, when you were showing them that sine is actually a ratio?
Faaiz	Yes.
EM	Right, now that is another approach that maybe I didn't follow that approach where you use graph paper and you draw a line and you measure it and you draw another angle and you measure it, take the opposite over the hypotenuse. That approach is obviously a practical approach. That is a route that I don't think many teachers do follow.

EM resists the idea of deepening students' understanding of a mathematical relationship ("the correlation") between the theorem he taught and trigonometric graphs. We must bear in mind that the curriculum he works with is fragmented into several "sections." His response to my prompts is therefore understandable.

Also, his example about "products" and their representation matches policymaker rhetoric about reasoning and mathematical relationships. For example, a different representation of "products" could give learners "varied" experiences to reason about mathematics. On the other hand "products" could also be mainly about procedures, common to the school mathematics tradition. An implication is that the fragmented curriculum itself is problematic in terms of making sense of the policymaker rhetoric about mathematical relationships and reasoning. EM appears, however, to be amenable to having his students see some mathematical relationships. For example, he notes:

EM There are certain things that I would like them to obviously see, but then there are certain things that if you want them to see, it would be a burden on them, it's extra work, may be more time involved.

He is inclined to let his students see "certain things," but is aware of the time constraint ("maybe more time involved"). His remark, "it's extra work" could be explained by the ruthless pace of the mathematics in the curriculum (Ensor, 1996), which is an example of the social context of teaching (Ernest, 1987). By focusing on a "big idea" like relationships between quantities, for example ratios, in algebra, trigonometry and geometry he might be less inclined to say that the "correlation" between similarity and the sine function is a "little too far fetched" or that it is "extra work."

From this excerpt it also appears that EM has not thought of geometry, trigonometry and algebra in a way that I brought up in the interview:

EM To tell you the truth, the reason why I didn't follow that is because I wasn't taught that way also.

EM So I followed something I could rely on. I obviously looked at this and I obviously planned that this is going to take too much time. That is the route that I did not follow. So I relied on something that I knew would work and that took less time.

The pull of his past experience, his apprenticeship of observation (Lortie, 1976) and the school mathematics tradition where mathematics is stable and static (Popkewitz, 1988) are evident in his words "So I followed something I could rely on" and "So I relied on something that I knew would work and that took less time." Presenting the mathematics in ways that are formalized, static and stable does take "less time." The following excerpt is most telling showing that he has not looked at the same mathematics in ways where mathematical relationships are stressed:

EM I mean for example, when you said similarity is related to sine graphs, you had me puzzled for a moment, because I thought, "how are they linked?" But when you mentioned ratio then obviously I knew, look here this is what you're talking about. But I am sure there are many teachers if you were to tell them that these two are linked (similar triangles and the sine function) they won't understand what you're saying. I mean I did not understand what you were saying but the moment you put the two together.

In this case his rhetoric about the fragmented curriculum alludes to the challenges ahead in terms of seeing "unifying ideas," like ratios as an example of relationships between quantities.

Later in the interview he acknowledges that it "nice to link things together" but resists doing so because he thinks of all the curricular time that will be consumed. The pull of the institutionalized curriculum such as the 12th grade end-of-the-year matric examination that confront him in the school most likely causes him say that "it is actually a waste of time because, that time you can use to revise may be an entire syllabus or something like that."

EM	You know I also mean it is nice to link things together. It is going to take a week to link the two together (laughs), then I think it is actually a waste of time because, that time you can use to revise may be an entire syllabus or something like that.
Faaiz	The bits, the bits you mean, bits of this and bits of that?
EM	Something like that (laughs).

I did my best to point out the strongly classified curriculum when I said, "the bits, the bits you mean, bits of this and bits of that?"

There is variation in EM's rhetoric in the above excerpts. He makes positive contact with the policymaker rhetoric after I brought a specific curricular example about the sine function in trigonometry or algebra and the geometry he was teaching that day. It seems very hard to have EM talk about observing, representing and investigating patterns within mathematical relationships between trigonometry, algebra and geometry (*Government Gazette*, no. 18051, 1997, p. 116). Also, the line in the *Government Gazette* "mathematics involves observing, representing and investigating patterns within mathematical relationships," appears so new to him because of his apprenticeship of observation (Lortie, 1975). His reaction to my prompts implies that the realization of policy statements in the *Government Gazette*, will be very hard to talk about and to imagine as possible policy outcomes.

The other two high school teachers in the sample teach the "same" institutionalized curriculum. One can therefore speculate that the same policymaker rhetoric about learners observing and investigating patterns within mathematical relationships between trigonometry and geometry will be as difficult for the other high school teachers in the sample. Instances where EM's rhetoric makes positive contact with the policymaker rhetoric implies that support in the form of curriculum materials and teacher development will be helpful in terms of realizing the policymaker rhetoric in the *Government Gazette* as I understand it.

Mathematical relationships and mathematics as reasoning in calculus

Excerpts that are analyzed come from two of the three high school teachers, EM and DS. The indirect juxtaposition of policymaker and teacher rhetoric is in the lines where I, for example, prompt EM to find out whether he has any thinking about "rephrasing or re-presenting" the calculus questions and why students have problems. DS espouses a model of teaching calculus in the future, that is different from the school mathematics tradition. My prompts point in the direction of "investigating and observing patterns within mathematical relationships" and "reasoning being fundamental to mathematical activity" (*Government Gazette*, no. 18051, 1997, p. 116-7).

In the school mathematics tradition there is a focus on what Tall (1987) calls the "surface structure" of symbolic manipulation, or rules and procedures to obtain formal derivatives and integrals. Very different from such a traditional approach is the notion of calculus as the interplay between rate and accumulation. Schnepp *et. al.* (2000, p. 2) poignantly summarize calculus as being about "the back-and-forth between rate-to-amount and amount-to-rate questions and these questions' resolutions." They are inspired by Thompson's (1994) *images of rate* as a "big idea" in calculus :

A primary theme I will develop is that students' difficulties with the Fundamental Theorem of Calculus can be traced to impoverished images of rate...

By "image" I mean much more than a mental image. Rather I mean "image" as a kind of knowledge that enables one to walk into a room full of friends and expect to know how events will unfold (p. 125).

In the second interview EM acknowledges that when he teaches calculus, it is a "procedural thing." He then talked about making it more "meaningful" to his students. This is evidence of his frustration with traditional approaches of following rules to find derivatives. In the excerpt below I ask him to articulate what it is that his students struggle with :

Faaiz	What are some of the problems that students have within calculus say, that you've observed from your teaching? What are the things that they are confused about?
EM	The notation actually, the f(x) notation is very confusing to them.
Faaiz	Why?
EM	To tell you the truth I don't know. I've explained it over and over. May be it's my explanation. When I came to university, that was the first time I saw a notation like that. It sort of haunted me as well.
EM	[writes down $f'(x) = \frac{\lim}{h \to 0} \frac{f(x+h) - f(x)}{h}$ And also, say $f(x) = 2x^2$, for them to find f(x+h) which is a substitution problem. They struggle with that. I've actually noticed that the Department has taken this out of the standard 7 (ninth grade) syllabus. This was actually first in the standard seven syllabus.
Faaiz	Okay, just a minute, $f(x) = 2x^2$ and they have to determine f(x+h). Is that the question as it is?
EM	No, it is not the actual question but it is part of the question.
Faaiz	Why do you think students have difficulty in coming to grips with that question?

EM I think they haven't been exposed long enough to this notation. A good question. I am busy with my standard 8s (10th graders) and I'm doing Pythagoras. This is the first time they have done Pythagoras in their lives, so they have got now half of standard 8 and then standard 9 and 10 to do Pythagoras, whereas I have started Pythagoras in standard 6 already. So it is a two-year difference. Look at the exposure there and it is actually in the standard 6 syllabus. I don't know whether they have taken it out because there is some changes, like, silly things that they've done. Oh yes, this is the one that I was talking about where this was out of the standard 7 syllabus. Now I think this should be included because what you're testing here is you're testing substitution as well. That is something that you're eventually going to do in standard 10 (12th grade), and now you're leaving it out in standard 7 (9th grade).
(interview # 3 of 4)

His overt focus on the "surface structure" (Tall, 1987) is evident from the line where he says, "the notation actually, the f(x) notation is very confusing to them." In the school mathematics tradition there is a focus on formalized mathematics, on symbols and notation, which are often compact in meaning. In the tradition of direct instruction common to the school mathematics tradition, teachers teach "on the basis of explicit procedures, rules, and operational techniques, hoping to elaborate the mathematical concept and it meaning" (Steinbring, 1989, p. 29).

It is clear from the excerpt that EM is frustrated, hence he says, "I've explained it over and over." When he says, "maybe it's my explanation," it seems that he is aware that there could be a problem with the way he is teaching calculus. In this case his language matches policymaker rhetoric about mathematics as reasoning but is in fact a disconnect from the way I understand reasoning in calculus. His explanation focuses on "prerequisites" as procedures taught in earlier grade levels. My understanding of reasoning and mathematical relationships in the calculus is based on a Schnepp *et. al.* (2000, p. 2) idea that calculus is about "the back-and-forth between rate-to-amount and amount-to-rate questions and these questions' resolutions."

His belief, fostered by his own experience with school mathematics, is that mathematical knowledge is a "material" stock of "prerequisites," consisting of single elements that must first be learned in a cumulative way in order to make later discoveries of the underlying structural relations and meanings possible (Steinbring, 1989, p. 29). For this reason he talks about "exposure" and points to the syllabus and says,

> [Oh yes,] this is the one that I was talking about where this was out of the standard 7 (ninth grade) syllabus. Now I think this should be included because what you're testing here is you're testing substitution as well. That is something that you're eventually going to do in standard 10 (12th grade), and now you're leaving it out in standard 7 (ninth grade).

An example of the "material" stock in this case is the "substitution" procedure taught in the seventh grade and then applied in the question on the calculus. "Testing substitution" alone is not going to unravel to students the "big idea" about images of rate in the calculus (Thompson, 1994), and calculus as being about "the back-and-forth between rate-to-amount and amount-to-rate questions and these questions' resolutions" (Schnepp, *et. al.*, 2000). In calculus there has to be a simultaneous examination of numerical, graphical and symbolic viewpoints. From a disciplinary tradition this is assumed to be a sound way to arouse students' interests in "investigating relationships and making connections within mathematical relationships" (*Government Gazette,* no. 18051, p. 116).

Later on in the same interview, I prompted him about rephrasing or re-presenting the same compact, symbolic question that reads, find f'(x) where f(x) = $2x^2$:

Faaiz	Have you thought of rephrasing it or re-presenting it to the students so that they can get into it in a way that they can see where this is all leading to? I am speaking about f(x) = $2x^2$.
EM	How would one do that? As I said this one I taught in 1994, and when I heard next, it was out of the syllabus. I didn't actually really think about that. Normally we get questions and I do think why they don't understand, trying may be to explain in a different method, you know, it just cannot surface or come to anything. Sometimes I rely too heavily on the method that I was taught in a particular manner which I thought was a good method,
EM	now I think look here, you must also understand, because I think that was sort of a speedy way of transferring the knowledge. Maybe I don't actually look for an alternative or something like that or maybe I do have an alternative. (interview # 3 of 4)

Through my prompt I wanted to find out whether he had any thinking about ways for students to investigate a pattern within the mathematical relationship between f'(x) and f(x). My prompt points in the direction of policymaker rhetoric, "mathematics involves observing, representing and investigating patterns within mathematical relationships," and the need for learners to have varied experiences to construct convincing arguments (*Government Gazette*, no. 18051, p. 116 - 117). When he says "How would one do that?" it implies he has not thought or heard of calculus with a focus on images of rate and accumulation (Thompson, 1994) as "big ideas."

"Sometimes I rely too heavily on the method that I was taught in" is a positive statement because it seems to indicate a realization that "the method," which focuses on procedures and rules, is not helpful when it comes to students not understanding. The "compactness" of the question, if $f(x) = 2x^2$, find $f'(x)$, is a source of tension for his students. The "exposure" or "prerequisite" he has in mind, "substitution," as a procedure does not turn out to be helpful in terms of developing his students' understanding. This frustrates EM ("I've explained it over and over").

The focus on the "surface structure" (Tall, 1987) is also evident elsewhere in the interview with EM. A student had asked him why one sets the derivative equal to zero when finding maxima or minima. He thought the question to be peculiar to the student. A focus on images of rate (Thompson, 1994) as a "big idea" would be helpful in developing in students a deeper understanding of maxima and minima, and why one sets the derivative equal to zero to find them. Such an approach points in the direction of policy rhetoric about "reasoning being fundamental to mathematical activity," (*Government Gazette*, no. 18051, p. 117) and thus, relational understanding (Skemp, 1987). EM's response to the student's question is worth examining :

EM	Somebody asked me, why do you set the derivative equal to naught?
Faaiz	So what did you say ?
EM	So I said when the derivative is equal to naught, that is where you possibly get your maxima or minima. Then I used the sketch to show him, look here on this turning point and at that turning point,

EM you have your derivative equal to naught and in this one you get your minimum and on that one you get your maximum. So at least he appreciated why he set it equal to naught. Some teachers, they just use an algorithm, get the equation, find the derivative and set it equal to naught and solve for x. There is no understanding in that actually, you're just following an algorithm blindly. Whereas if you draw something and you say there's your maximum and your minimum, and obviously at that point x, what is your maximum volume or area, substitute that into the equation as well as the coordinates of the turning points. (interview # 3 of 4)

EM's question, "Why do you set the derivative equal to naught?" is a positive contact with policymaker rhetoric about "reasoning is fundamental to mathematical activity." His response to this question is also a positive contact with policymaker rhetoric about mathematical relationships and mathematics as reasoning within the calculus. For example, here his rhetoric is about "mathematics offers ways of thinking, structuring, organizing and making sense of the world [*Government Gazette,* June 6, 1997, no. 18051, (p. 116)]. I cannot say whether he teaches the calculus in the way he describes in the excerpt. He may just see the student's question about why the derivative is set equal to zero as an unusual one, not really worthwhile in terms of teaching and learning calculus.

What is markedly absent in this excerpt is a notion of calculus as being about change, or "images of rate" (Thompson, 1994). The "derivative," "maxima," and "minima" are all linked through "the back-and-forth between rate-to-amount and amount-to-rate questions and these questions' resolutions" " (Schnepp *et. al.,* 2000). It is the image of the rate-to-amount that becomes "naught" or zero where one finds "your maximum volume of area."

DS's espoused model (Ernest, 1987) about calculus teaching and learning is "outside" the school mathematics tradition. In the following excerpt he talks about specifics with regard to calculus teaching to the 12th grade students, the "matrics," if he were to teach them the following year:

DS	A lot can be done at high school level, for example I have this one example, but I haven't taught matric yet, seeing our matrics are required to do a project. What they do nowadays is a project on some aspect of mathematics which they just put on a cardboard. It's a poster type of thing.
Faaiz	Are they expected to delve into the mathematics?
DS	If it's a higher grade project. One would think that they should go a bit further than the syllabus. What I picked up in one of the books in the library, "An invitation to mathematics," where the author explains that we're dealing with symbols. Basically first of all symbol manipulation, you just need to know how to manipulate symbols according to a set of rules. That's the frame with which he starts out with. In that particular book he gives examples of how one can explain concepts of limits, very concretely, instead of having it abstractly. He looks at formulas that the pupils should know, like the area of a circle or even the area of a triangle.
DS	All the pupils know the area of a triangle, but using calculus and limits, to find that equation of the area a triangle. A simple case would be to take a triangle and divide it into little rectangles, stepwise and let the steps then become smaller and smaller so the concept of limit comes across in a very practical way. So that is what I feel if I have an opportunity again to teach matric, hopefully next year, I would use something like that and ask them to do a project on calculus in that way. (interview # 3 of 4).

A "higher grade" project means one that is intended for "top lane" students. From the excerpt it is clear that he wishes to go beyond symbol manipulation in calculus. In this case his rhetoric makes positive contact with policymaker rhetoric about "mathematics programs should provide opportunities for learners to develop and employ their reasoning skills" and "learners need varied experiences to construct convincing arguments in problem settings [*Government Gazette,* June 6, 1997, no. 18051(p. 117)]. Also included is the policymaker rhetoric about "mathematics offers ways thinking, structuring, organizing and making sense of the world" [*Government Gazette,* June 6, 1997, no. 18051 (p. 116)]. For example, DS states "all the pupils know the area of a triangle, but using calculus and limits, to find that equation of the area a triangle." He then talks about calculus in a way that points toward the "deep structure" of the calculus (Tall, 1987). For example, one could argue that a phrase like "take a triangle and divide it into little rectangles, stepwise and let the steps then become smaller and smaller," is about examining the rate of accumulation of the little rectangles to form the triangle under consideration. It also matches with the notion that mathematics involves observing, representing and investigating patterns in physical phenomena and within mathematical relationships (*Government Gazette,* no. 18051, p. 117).

In the following excerpt DS brings up more examples of how he would like to teach calculus:

DS I'd like to teach them about finding the formulas of volumes for certain shapes, because there is a need to calculate the volumes of a certain shape in real life, quickly. Do we dump it in water and measure the water? That's physical science. How would mathematicians approach it?

DS Can we chop it up in layers, chop it again, take out the cubes, and these are the bits and pieces, estimate this and estimate that and get a rough estimate. How would you approach it? That's where the calculus comes in and all sorts of things. That will be like a global thing, you want to set out and calculate the volume and know whatever mathematics comes out of that, down to the basic symbol manipulation, that would be a nice approach I think, much better, built into a unit of work.
(interview # 3 of 4)

In this excerpt he is talking about giving students varied experiences to construct convincing arguments and mathematics being about observing, representing and investigating patterns in physical phenomena and within mathematical relationships (*Government Gazette*, no. 18051, pp. 116 - 117). For example, he wants his students to find out how different "formulas of volumes for certain shapes, because there is a need to calculate the volumes of a certain shape in real life." The calculus is excellent in terms of having students investigate and observe the mathematical relationships between volume and surface area formulas for different three dimensional objects like, cubes, cones, cylinders and pyramids. In the school mathematics tradition students acquire an instrumental understanding (Skemp, 1978) about these particular formulas.

His words "Can we chop it up in layers, chop it again," match with Tall's (1987) notion of the "deep structure" of calculus, which also matches the policymaker rhetoric. In addition, DS is interested in developing a relational understanding (Skemp, 1987) of different volume and area formulas via calculus. What is most interesting is that he positions symbol manipulation for volume and area formulas at the end of his approach. For example, he says, "and calculate the volume and know whatever mathematics comes out of that, *down* to the basic symbol manipulation, that would be a nice approach I think." In the school mathematics tradition, formal symbol manipulation is a major focus of teaching and learning.

In this excerpt his rhetoric, in the form of his espoused model of teaching (Ernest, 1987), matches both specific outcomes on investigating and observing patterns in physical phenomena like area and volume formulas and within mathematical relationships and having students see, question, examine, conjecture and experiment with the reasons behind these formulas. What is interesting is that he sees his approach only for "higher grade" or "top lane" students. This comment could be due to the reality of tracking of mathematics students in the school system or because he has not seriously thought about mathematical ability, common to the school mathematics tradition.

In conclusion, there are positive and negative contacts between the two sets of rhetoric on the teaching and learning of calculus. For example, EM relies on formalized notation and "prerequisites" as rules and procedures taught in earlier grade levels as ways to have his students understand calculus. His rhetoric about mathematics as reasoning and mathematical relationships makes a negative contact or is a disconnect with similar policymaker rhetoric. DS espouses a model of teaching and learning calculus that makes a positive contact with the same policymaker rhetoric.

Summary of high school teacher rhetoric about mathematical relationships and mathematics as reasoning in the calculus

There is variation in the high school teacher rhetoric. At times they make positive contact with similar policymaker rhetoric in algebra, geometry and calculus. In other words, their language and the policymaker language mean the same things. Also, at times there is a disconnect between the two sets of rhetoric. In this case teacher rhetoric and policymaker rhetoric overlap but the meanings are fundamentally different.

Beyond a summary of the results

In this section there is first a brief note about a major finding in this study. To understand this finding, special attention is paid to terminology I developed in this chapter where I compared the two rhetoric communities, teachers and policymakers. Eventually the terminology will serve as a means to talk more clearly about the implications of the study. The latter is tied to the second research question in the study, namely, how can similarities and differences between policymaker and teacher rhetoric inform subsequent thinking on school mathematics reform? Such similarities and differences have implications and point to enormous challenges that lie ahead in terms of bridging the education policy and classroom practice divide.

An important finding in this study is that people use similar language but mean different things at times. For this reason I focus on the terminology I have been using in this study. In comparing the rhetoric of teachers and policymakers I have used the words, similarities, differences, matches, mismatches, points of contact, positive contact, negative contact and disconnects. Similarities, matches and points of contact mean the same thing. They are about overlaps in the language of the teachers in the sample and the ways policymakers write. Positive contact and negative contact are words that describe the nature of the overlaps. These words describe mirror images, meaning that they consistent and inconsistent respectively, with the way I understand the reform rhetoric in *Mathematical Literacy, Mathematics and Mathematical Sciences* (MLMMS). The terminology is therefore a means to distinguish between reform and non-reform overlaps. At times I have also used the word disconnect to describe a negative contact between the two rhetoric communities. In particular, a disconnect means that the teachers and policymakers use similar language but there is a fundamental difference in meaning. There are therefore these two kinds of cases, positive contacts and negative contacts. In other words, there is variation in the teacher rhetoric. Similarly, there is also a kind of variation in the policymaker rhetoric. These variations in contact points between the two rhetoric communities will affect how one thinks about the implications of the results of this study.

To illustrate the nature of the contact points I shall examine how one teacher in the sample uses similar words but mean different things at times. For example, at times PN's rhetoric makes a positive contact with policymaker rhetoric about mathematical relationships :

> I feel I don't want to just teach the properties of the distributive property and those other properties all in isolation. Those should also be integrated.

and

> Because the child learns about common fractions, and now when you do decimal fractions, it is something so different, so new. They need to see it together. They need to be able to relate it to each other.

This is a case where PN uses language ("integrated," "together," and "relate to each other") that has similar meanings to the policymaker rhetoric about patterns within mathematical relationships:

> Mathematics involves observing, representing and investigating patterns within mathematical relationships." [*Government Gazette,* June 6, 1997, no. 18051 (p. 116)]

This is therefore a case of a positive contact between the two rhetoric communities, because there is a match in meaning. In particular, the words PN uses, "integrated," "together," and "relate to each other," match the policymaker rhetoric, "within mathematical relationships."

The excerpt below is an example of a weak positive contact with the same policymaker rhetoric:

> We integrate, we do not just work on one thing per week. We try to put in a variety, because you can teach fractions this week and do something else the next week.

Here her rhetoric is moving in the direction of a negative point of contact. "We do not just work on one thing per week" could involve "observing, representing and investigating patterns within mathematical relationships." "Fractions" and "something," however, as a contact point is weak because "something" is not clearly spelled out. Similarly, in the case of the cards that she uses:

> I like my games, like we play cards to get around area.

This rhetoric is a weak positive point of contact because she does not clearly say how playing cards helps "to get around area." On the other hand, the cards could be used as a way of " observing, representing and investigating patterns within mathematical relationships," in area in this case.

In the following excerpt a match with the policymaker rhetoric becomes weak and moves in the direction of a negative point of contact :

> And then we did the pie chart and we put fractions in and we put one decimal in. The child realizes that they're not very much different. I hope I'm doing the right thing.

Here PN is talking about her students not seeing "the connection" when she puts fractions and "one decimal" in a pie chart. The contact point between her rhetoric and similar policymaker rhetoric is therefore negative because there is a mismatch between the similar language. We thus have a disconnect. Observing and investigating patterns within mathematical relationships does not involve putting in "one decimal" and a few fractions in a pie chart. Decimals, plural, and fractions a could be connected through an investigation or an observation of size as a focus and the role that decimals and fractions play in size. Also, such an investigation would have to be done over a number of class periods.

In conclusion, the notion of positive and negative points of contact between the two rhetoric communities is important when it comes to talking about the implications of the findings of results of this study. In a later discussion these points of contact will be characterized further as weak or strong in order to talk more effectively about their implications.

CHAPTER 7
CONCLUDING DISCUSSION

Overview

This dissertation is about comparing ways in which policymakers write and mathematics teachers speak in the Western Cape, South Africa. The research questions in this study are: What are the similarities and differences in the rhetoric of teachers and the rhetoric of policymakers? How can these similarities and differences inform subsequent thinking on school mathematics reform? The first part of this seventh chapter is a summary of the nature of the results of the study. The second part of the chapter addresses the implications of the results of the study. It ends with reflections on those results.

The results of the study takes the form of claims about the rhetoric in *Curriculum 2005* documents and the rhetoric of the teachers in the study. This is followed by a discussion of the implications of the results of the study. The reflections take the form of an imaginary dialogue between an education researcher (ER), an ethnomathematics educator (EME), a critical mathematics educator (CME), a mathematician (M), and me (MFG). These participants are "imaginary" because they are potential colleagues I imagine myself in conversation with in the South African educational reform context.

Results of the present study

Results of the present study is about each of the analyses I completed in the various chapters. They take on the form of summaries from chapters one and four—on the nature of the policymaker rhetoric—and chapters five and six —on the nature of the teacher rhetoric. There will also be a methodological note on the interview process which does not fit in with the overall structure of the dissertation. It is, however, worth noting how the interview process affected the rhetoric of the teachers. This is a point that will be taken up further.

A summary of the policymaker rhetoric

In the first chapter of this dissertation is a general introduction to the curriculum policy, *Curriculum 2005* and its rhetoric that points mostly in the direction of issues related to "redress, reconstruction and training" (DE, 1997a; 1997b). A careful reading of the National Qualifications Framework (NQF) in *Curriculum 2005* shows that it wishes to overcome "rigid divisions" such as academic and applied knowledge, knowledge and skills, theory and practice, and education and training (DE, 1997a; 1997b). For example, according to the NQF, grade levels in formal schooling will have equivalent "levels" for "learners" who could be out-of-school and adults (DE, 1997b, p. 30). These learners could obtain "General Education and Training Certificates" at the ninth grade, and "Further Education and Training Certificates" at the tenth grade level (see p. 17). These certificates should be viewed as a means to offer out-of-school youth and adults, who have been systematically underdeveloped by apartheid education, qualifications that have equivalent levels in formal schooling. More importantly, the certificates have the goal of bringing together education and training. They are therefore central to the idea of bridging the education and training divide.

"Learner-centeredness" a key educational slogan in *Curriculum 2005,* is consonant with the idea of transcending education and training and the other "rigid divisions" (DE, 1997a; 1997b). The choice of this slogan enables policymakers to include out-of-school youth and adults who have been systematically underdeveloped by apartheid education. A slogan like "student-centeredness" does not capture the spirit of *Curriculum 2005* .

The first chapter of this study also shows that there is a close relationship between the rhetoric of the NQF and the (Scottish) National Vocational Qualifications (NVQs/SVQs), which is premised on Jessup's (1991; 1995) "Scottish Model" (see p. 19). This model also has the goal of aligning academic and vocational education, through a focus on "outcomes" and a broad definition of "learners" and "learning."

There are five key points in chapter four of this study in which I analyzed policymaker rhetoric in the mathematics component of *Curriculum 2005.* From this analysis it is clear that there is a continuation of the general desire to focus on "redress and reconstruction and training." An analysis of the name of the mathematics component, *Mathematical Literacy, Mathematics and Mathematical Sciences,"*(MLMMS) in this chapter, reveals the seriousness of policymakers' intention to transcend education and training (see p. 77). For example, the "rationale" that is given for MLMMS, contains phrases such as :

- work toward the reconstruction and development of South African society;
- develop equal opportunities and choice;
- contribute towards the widest development of South African society;

(*Government Gazette,* 6 June, 1997, no. 18051, p. 115)

These phrases indicate that policymakers are not thinking of education and training as separate notions.

Second, from chapter four it is also clear that in MLMMS the ten "specific outcomes" are stated in a way that they merge or bring together formal school grade levels with "phases." For example, each of the "specific outcomes" has a "foundation phase" (grades 1 - 3), an "intermediate phase" (grades 4 - 6) and a "senior phase" (grades 7 - 9) (DE, 1997a; 1997b; *Government Gazette,* 6 June, 1997, no. 18051). The different "phases," therefore, make it possible for out-of-school youth and adults to obtain certifications that have equivalent grade levels in the formal schooling system. This move should be seen in light of the rhetoric about "redress, reconstruction and training" and bridging "rigid divisions" such as education and training, knowledge and skills and theory and practice (DE, 1997a; 1997b). Each of the "specific outcomes" has "range statements" and "assessment criteria," which are terms that have their origins in Jessup's (1991; 1995) model of aligning education and training through "outcomes."

Third, in the ten "specific outcomes" in MLMMS one finds rhetoric from mathematics education traditions such as constructivism, disciplinary traditions, ethnomathematics, critical mathematics, and variations of these two. There are also fuzzy phrases such as "demonstrate understanding," and "show understanding" in all of the ten "specific outcomes." These educational slogans or rallying symbols (Scheffler, 1960/1964) appeal to a wide range of people and can equally apply to the school mathematics tradition (SMT) (see Cobb *et. al.* 1992; Gregg, 1995). Also, the goal behind such diverse rhetoric could be to placate different mathematics education constituencies within South Africa. This helps make the mathematics component of *Curriculum 2005* a consensus document.

Fourth, clearly the situation in South Africa is more cognizant of the sociopolitical context than the NCTM *Standards* (1989; 1991) (see Bishop, 1990; Apple, 1992). For example, policymaker rhetoric in MLMMS is explicit about the sociopolitical context:

> A critical mathematics curriculum should develop critical thinking about how social inequalities, particularly concerning, race, gender and class, are created and perpetuated.
> (*Government Gazette*, 6 June, 1997, no. 18051, p. 122)
>
> [Learners should]
> Critically analyze mathematics as a tool for differentiation
>
> Critically analyze mathematics as a predominantly European activity.
> (*Government Gazette*, 6 June, 1997, no. 18051, p. 123)

Another example of cognizance of the sociopolitical context is in the eighth "specific outcome" which favors an ethnomathematics tradition because it has a key word, "cultural products" :

> [Learners] should analyse natural forms, cultural products and processes as representation of shape, space and time.
>
> [Learners] should represent cultural products and processes in various mathematical forms, in two and three dimensions (*Government Gazette*, 6 June, 1997, no. 18051, p. 131).

Fifth, in MLMMS there are no curricular examples in the form of particular mathematical content like in the NCTM *Standards* (1989; 1991). Nor are there any vignettes of for example, teachers teaching in the spirit of MLMMS in overcrowded classrooms. Curricular examples and vignettes of teaching and learning are possible because the "specific outcomes" have slogans peculiar to different mathematics education traditions. A teacher who is steeped in the school mathematics tradition (SMT) (Cobb *et. al.* 1992; Gregg, 1995), as crystallized in Active Mathematics Teaching (AMT) (Good, *et. al.*, 1983), can claim to be teaching in the spirit of *Curriculum 2005.* A teacher could also teach a highly structured lesson with specified objectives, which could, for example, be in the spirit of ethnomathematics and critical mathematics.

A summary of the teacher rhetoric

Beliefs, practices and knowledge in the school mathematics tradition (SMT) were key tools in analyzing teacher rhetoric. These beliefs, practices and knowledge can be categorized into the nature of mathematics, the teaching of mathematics, the learning of mathematics, the use of textbooks and the social context of teaching. Gregg (1995) associates the following practices and beliefs in the SMT:

- Emphasizing form and procedures (p. 452).

* Proceduralizing and decomposing the mathematics (p. 458)
* Employing tests as measures of student understanding (p. 453)
* Maintaining control and the constitution of mathematics (p. 456)
* Producing the answers and procedures in the textbook (p. 456)
* Accounting for students' difficulty by referring to their ability (p. 461)
* A separation of teaching and learning (p. 464)

From here one can guage the nature of mathematics and the teaching and learning of mathematics in the SMT. For example, teaching takes the form of "direct instruction" with the textbook as the mathematical authority. Lessons are highly structured with specified objectives and rote learning as the goal. There is a separation of teaching and content and teaching and learning. Mathematics is viewed as a collection of isolated rules, facts, procedures and algorithms. It is also viewed as linear, consisting of "prerequisites," that are mainly procedures taught in previous grade levels. It is decomposed into discrete parts. Learners are said to need "knowledge, skills, problem solving," in that particular order. No particular attention is paid to issues related to the sociopolitical context within mathematics and the teaching and learning of mathematics.

The SMT is very strong in the rhetoric of the six teachers in the sample (see pp. 104 - 112). For example in SMT, mathematics knowledge is seen as separable into discrete parts and thus considered linear. This assumed linearity takes the form of "prerequisites" as mostly procedures and rules that are needed in order to understand the mathematics in higher grade levels (Cobb *et. al.* 1992; Gregg, 1995). Both middle and high school teachers spoke about the school mathematics curriculum as consisting of isolated and strongly classified "sections." For example, one middle school teacher referred to the curriculum as being "compartmentalized," and as a list of topics, such as "arithmetic," "fractions," "geometrical figures," and "story problems." Also, one high school teacher spoke about "school maths as stepping stone maths," where you "need one section in order to go on to another section" (see p. 111). The overt focus on proceduralizing and decomposing the mathematics in the SMT seems to have the effect of teachers talking about "sections." The effect of the fragmented and proceduralized curriculum is strong to the point where one middle teacher said that "it's only really maths teachers who use algebra in their jobs" (see p. 110).

In chapter five of the study the teachers also spoke about a mathematical mind and "natural ability" (see pp. 112 - 115). The notion of natural ability is part of the taken-as-shared beliefs of the SMT (Gregg, 1995). Two of the middle school teachers referred to students' ability at applying rules without reasons. One spoke about students who are not good at mathematics as having "blockages" when it comes to the fragmented school mathematics curriculum (p. 117). One high school teacher referred to a student who is not good at mathematics as not having a "mathematical mind." The other two high school teachers were less inclined to talk about a mathematical mind and seemed puzzled about their students' difficulties with the fragmented mathematics curriculum.

In chapter five teachers also talked about external and internal constraints on teaching (see pp. 123 - 132). For example, two middle school teachers mentioned a lack of professional development opportunities and the effects of "down-sizing," resulting in overcrowded classrooms. High school teachers spoke about students being unable to afford "school fees," the pressure of the "matric" and a lack of collegial discussions about changing the order of the "syllabus" or the fragmented mathematics curriculum (see p. 131).

In chapter six, (see pp. 134 - 192) there is an analysis of an indirect juxtaposition of policymaker rhetoric and teacher rhetoric about "mathematical relationships" :

> Mathematics involves observing, representing and investigating patterns in social and physical phenomena and within mathematical relationships.
> [*Government Gazette*, June 6, 1997, no. 18051, p. 116]

and "mathematics as reasoning" in specific outcome 10 :

> Use various logical processes to formulate, test and justify conjectures: Reasoning is fundamental to mathematical activity. Active learners question, conjecture and experiment. Mathematics programs should provide opportunities for learners to develop and employ their reasoning skills. Learners need varied experiences to construct convincing arguments in problem settings and to evaluate the arguments of others.
> [*Government Gazette*, June 6, 1997, no. 18051, p. 117]

In this chapter middle school and high school teacher rhetoric about "mathematical relationships" and "mathematics as reasoning" were analyzed separately.

It is apparent from the rhetoric of the three middle school teachers that they had a different understanding of what "mathematics as reasoning" means. For example, one of them said that "in order to be good at maths, you don't have to be able to give reasons to support your answers" (p. 151). It is also apparent that they did not have a well-developed sense of "mathematical relationships." For example, one of the teachers is aware of the "compartmentalized" curriculum and thus sees the need to "integrate" decimals, fractions, perimeter and area. She "integrates" decimals and fractions in area and perimeter problems by simply replacing length measures with decimals and fractional values. (pp. 154 - 156). She could have paid attention to decimals and fractions as measures of scale and explored their effects on area and perimeter problems. Another middle school teacher sees it important for students to "master each section of the work on its own" and she "fully agreed" that students should master topics and skills at one level before going on" (p. 157). The third middle school teacher spoke of the chapter on rates and ratio as "a subsection of number patterns" (p. 145). Here it seems that he was talking about "mathematical relationships."

High school teacher rhetoric about "mathematics as reasoning" and "mathematical relationships" focused on algebra, geometry and and calculus (pp. 159 -193). One of the high school teachers talked about how he "links" the "separate sections" in algebra in the fragmented curriculum. His rhetoric points in the direction of relations between equations and their graphs (p. 161). It matches well policymaker rhetoric about "observing, representing and investigating patterns within mathematical relationships. [*Government Gazette,* June 6, 1997, no. 18051, p. 116]. The notion of "mathematics as reasoning" in algebra is absent in the rhetoric of all three high school teachers. Two of the high teachers thought of "reasoning" as referring to geometry only (p. 172). The influence of the fragmented mathematics in the end-of-the year "matric" examination, with its focus on procedures seems most likely to be a barrier to having the one high school teacher see and thus talk about "mathematical relationships" and "mathematics as reasoning" in the case of similarity in geometry (see pp. 169 - 174). The same is the case with high school teacher rhetoric about "mathematical relationships" and "mathematics as reasoning" in calculus. There was, however, one high school teacher whose talk came close to a disciplinary approach to calculus as the study of "images of rate and accumulation" (Thompson, 1995) and thus about "mathematical relationships" and "mathematics as reasoning." For example, when he spoke about the area of a triangle he used the words "take a triangle and divide it into little rectangles, stepwise, and let the steps then become smaller and smaller so the concept of limit comes across in a very practical way "(p. 189). Here his rhetoric points in the direction of Tall's (1987) work on the "deep structure" as opposed to the "surface structure" in the calculus. The latter is common to the SMT because of its focus on rules and procedures for teaching and learning calculus.

From the overall teacher rhetoric in chapter six, it is clear that there is tension between "prerequisites" as mainly procedures and rules in lower grade levels and understanding the mathematics at higher grade levels (pp. 177 - 178). "Prerequisites" as procedures and rules as in the SMT do not help students to understand the mathematics in the higher grade levels. Here is another instance of how entrenched the SMT is in the teacher rhetoric overall.

So far I have summarized what is in chapters four, five and six. At this point it is important to note that both policymakers and teachers share a rhetoric that evokes the SMT. For example, fuzzy phrases such as "demonstrate understanding," and "show understanding," which appear in all of the ten "specific outcomes," can equally apply to the school mathematics tradition (SMT). It is also important to point out a methodological note about the interview process.

A note about the interview process

The interview process seems to indicate a recognition of differences in my rhetoric and theirs. In chapter six, which is about an indirect juxtaposition of teacher and policymaker rhetorics, it appears that two middle school and two high school teachers picked up on the notion of "mathematical relationships" and "mathematics as reasoning." These two notions are related to points of contact between the teachers and the rhetoric of disciplinary traditions.

Toward the end of the interview process two middle school teachers began to espouse ways of teaching mathematics in which these two notions feature. For example, during the fourth and last interview one middle school teacher said to me,

> "what was that example you gave me the last time" (p. 144).

She was referring to her students' conceptual difficulties with area and perimeter I had pointed out during a previous interview. This same teacher also said, "when you give the child a ratio, why can't the ratio not be given in the form of a percentage and a fraction" (p. 156). This seems like an example of how is she is thinking about "mathematical relationships." It seems most likely that this kind of rhetoric was not there before. The other middle school teacher said the following, during the last interview:

> When I read through this book I understood why you asked me every time, do you ask the children to explain this to you, because their books are structured this way, explain your reasoning, explain your reasoning (p. 149).

"This book" refers to curriculum units from the *Connected Mathematics Project* (CMP) which I used as a way to have the teachers espouse their rhetoric about mathematics and the teaching and learning of mathematics.

Also, it seems that two of the high school teachers became aware of notions of "mathematical relationships" and "mathematics as reasoning." For example, one of them said:

> So I am looking at myself, my subject is too focused on the actual symbolism and stuff like that that's there, and mathematics like you said, could be directed in explain (p. 174)

during one interview. About "mathematical relationships," the other high school teacher said the following :

> I mean for example, when you said similarity is related to sine graphs, you had me puzzled for a moment, because I thought how

are they linked? But when you mentioned ratio then I obviously I knew, look here this is what you're talking about (p. 180).

In summary, it appears that the interview process seems to have the teachers think and talk in slightly different ways about the teaching and learning of the fragmented mathematics curriculum.

So far I have summarized what I learned about the nature of the results of the study. What are the implications?

Implications of the results of the present study.

Implications of the results of this study are tied to the second research question, how can similarities and differences between teacher rhetoric and policymaker rhetoric inform subsequent thinking on school mathematics reform? This study indicates that there are a number of different sorts of challenges in supporting change in mathematics instruction as envisioned in *Mathematical Literacy, Mathematics and Mathematical Sciences* (MLMMS), one of the "learning areas" in the curriculum policy, *Curriculum 2005.* In the context of these challenges policy documents like MLMMS work differently as vehicles for reform. For example, teachers can sometimes identify with the rhetoric in MLMMS and at times the very document will not even provide a start as a way to "produce instruction" (Cohen and Ball, 1990, p. 250).

First is a general comment on the implications of policy documents on *Curriculum 2005* as vehicles for reform. Then I shall address challenges associated with more complicated places where there are points of contact between teacher rhetoric and policymaker rhetoric as in MLMMS. When this happens, two possibilities arise, namely, same words and same understandings, and same words and different understandings. In each case, policy documents play a different role. A noteworthy complexity is that the rhetoric of a single teacher can at times be a case of similar words and similar understanding in one instance and similar words and different understanding during another.

Implications of policymaker rhetoric in *Curriculum 2005* documents

In chapter one I noted that a major criticism leveled against *Curriculum 2005* is that it is jargon-laden, poorly designed and inadequately understood by teachers (*The Teacher*, June 6, 2000). A similar argument holds in the case of MLMMS as a messenger of its vision of mathematics instruction in schools (Cohen and Ball, 1990; p. 249). Thus as a vehicle for reform, MLMMS has one particular limitation because is open to multiple interpretations. For instance, in chapter four, I show rhetorical traces of educational slogans (Scheffler, 1960/1964) from various mathematics education traditions. In many ways it contains a bundle of disparate ideas, many vaguely stated, and is thus vulnerable to many different constructions. For example, it contains recurring, fuzzy phrases such as "demonstrate understanding" and "show understanding." What people mean by these phrases can —and does —differ wildly, as Cohen and Ball (1990) note in the case of the *California Frameworks*. From one perspective such vagueness in the rhetoric of policy is a strength because it may broaden the appeal of the curriculum policy. This implies that practitioners of various mathematics tradition can identify with the policymaker rhetoric. On the other hand such vagueness is a defect because it may draw in contrary tendencies and persuasions under a single banner (Cohen and Ball, 1990).

At a particular level, a key educational slogan in *Curriculum 2005* and thus in MLMMS is "learner-centeredness." Earlier I made the point that an educational slogan like "student-centeredness" does not capture the spirit of *Curriculum 2005.* This implies that teachers have to think about out-of-school youth and adults as possible students. An important implication is that teachers have to broaden their understanding about those whom they teach or will be teaching. They would definitely have to change their belief about "mathematical ability" as being fixed and final, as in the school mathematics tradition (Gregg, 1995, p. 452), when it comes to "learners." Two middle school teachers in the sample speak about "ability" and students having "blockages" with regard to the fragmented mathematics curriculum (see pp. 117 -123). Also, one high school teacher speaks about being good in mathematics as having a "mathematical mind" (see p. 127). It is important to point out that the comment about "blockages" comes from a teacher at a "Model C" school, which is regarded as an "exemplary" school in the Western Cape and the "best" in the nation. From this comment it seems it is going to be very challenging to have this teacher think differently about "learners" as described in *Curriculum 2005*, and thus her teaching.

"Learner-centeredness" as an educational slogan is consonant with the idea of transcending education and training and the other "rigid divisions" (DE, 1997a; 1997b) such as academic and applied knowledge, knowledge and skills, theory and practice. The implications are therefore that teachers and "learners" are exhorted to become serious and independent thinkers and to understand big mathematical ideas. It implies that teachers should encourage learners to come up with divergent approaches to solving problems. It also means that in instruction, learners often would be puzzled, even stuck (Cohen and Ball, 1990; p. 252), as a way to transcend, for example, theory and practice. These are ambitious and noble school reform ideals on the part of the policymakers.

The policymakers are asking teachers in the Western Cape and in South Africa to make great changes in mathematics teaching. This implies that teachers on the whole have to be offered many of the resources that support such changes. The apparent strength of the school mathematics tradition that I show in chapter five implies that the teachers in the sample have had few opportunities to see examples of the sort of teaching that the South African state thinks it wants. As do Cohen and Ball (1990, p. 254) in the case of the *California Frameworks,* I raise the question, Will the South African Government and other educational agencies help teachers capitalize on the changes by deploying resources to support and advance them? Some of the teachers in the sample are at schools that face rationalization or "down-sizing" in teaching staff, a lack of support from the central government and educational agencies or non-governmental organizations (see pp. 125 - 126). As a result some of the teachers in the sample have overcrowded classrooms. There is also a lack of collegial discussions about teaching and learning, or "brainstorming" as one of the middle school teachers in the sample notes (see p. 129). As in the case of California, real pedagogical change in the Western Cape, and thus in South Africa, would thrive on the creation of a rich conversation, in and around classrooms, about mathematics, teaching and learning (Cohen and Ball, 1990; p. 255).

It is therefore necessary to address implications and related challenges of the points of contact that teachers in the sample make with specific reform rhetoric in MLMMS. Teachers might read the policy document MLMMS and think they understand what they are being asked to do. Such a situation can lead to two different kinds of challenges. First, there can be cases of similar words and similar understandings. Second, there can be cases of similar words and different understandings.

Implications of contact points involving similar words and similar understandings

In the case of points of contact involving similar words and similar understandings, there is a fit between how the teachers in the sample use words and the words of the policy. For example, in the case of mathematics as reasoning and mathematical relationships there are points of contact between the teacher rhetoric and the policymaker rhetoric in MLMMS. The contact points vary from being strong to weak at times.

Policymaker rhetoric about mathematics as reasoning reads as follows:
[Learners should]

> Use various logical processes to formulate, test and justify conjectures: Reasoning is fundamental to mathematical activity. Active learners question, conjecture and experiment. Mathematics programs should provide opportunities for learners to develop and employ their reasoning skills. Learners need varied experiences to construct convincing arguments in problem settings and to evaluate the arguments of others.
> [*Government Gazette,* June 6, 1997, no. 18051, p. 117]

PN, one middle school teacher, thinks of mathematics as reasoning in terms of "word problems." Her reference to "word problems" is similar in meaning to the policymaker rhetoric, "learners need varied experiences to construct convincing arguments in problem settings and to evaluate the arguments of others." This point of contact is partial because the notion of mathematics as reasoning is applicable to much larger parts of the mathematics curriculum. From her rhetoric it seems that she has some sort of idea of the notion of mathematics as reasoning. The challenge is therefore to support teachers like PN with texts and curriculum materials and learning opportunities where there is a focus on mathematics as reasoning in other parts of the mathematics curriculum. In his teaching of decimals, the other middle school teacher, AS, comes close to having his students make a conjecture from an observation. Similar policymaker rhetoric reads "active learners question, conjecture and experiment" [*Government Gazette,* June 6, 1997, no. 18051, p. 117]. For example, during one observation he wrote on the board the following:

> *Calculate the following, then check the answer with a pocket calculator and explain to your friend what you observed.*

(a) 10 x 4,7
(b) 10 x 2,61
(c) 100 x 2,61
(d) 10 x 0,246
(e) 10 x 0,0513
(f) 100 x 3,8
(see page 153)

(g) 100 x 0,871
(h) 100 x 0,0195
(i) 1000 x 0,0426

The part of his note "explain to your friend what you observed" comes close to making a conjecture and also to the policy statement, "mathematics programs should provide opportunities for learners to develop and employ their reasoning skills" [*Government Gazette*, June 6, 1997, no. 18051, p. 117]. It is difficult to say whether he consciously thought of "conjectures" as an example of the policymaker rhetoric about mathematics as reasoning. It is therefore likely—as opposed to certain—that AS will understand the relevant policymaker rhetoric.

By the last interview AS seems to realize that there is value in "explain your reasoning" (see p. 149), which is another point of contact with similar policymaker rhetoric, namely, "reasoning is fundamental to mathematical activity" [*Government Gazette*, June 6, 1997, no. 18051, p. 117]. His remarks come during the fourth interview. For this interview I had him browse through the *Connected Mathematics Project* (CMP) middle grades reform –consistent curriculum materials that present mathematics with a focus on reasoning.

The challenge is therefore to support teachers like AS to change their teaching in ways that reflect the notion of mathematics as reasoning. Toward the last interview with AS he seemingly realizes what the notion of mathematics as reasoning could be about. Thus the kind of interaction that I had with him through the interview process at least, is an example of the kind of interaction that could produce instruction in the spirit of MLMMS. The document MLMMS on its own is not sufficient to help in "producing instruction" (Cohen and Ball, 1990, p. 250). It can only be a start. Teachers like AS need many more opportunities to learn to teach (Ball, 1988a) and interactions with curriculum materials and other players like teacher educators, where their thinking about mathematics as reasoning may develop.

There are similar instances of similar words and similar understandings in the points of contact in the case of the high school teacher rhetoric. For example, at one point DS says:

> [and] mathematics like you said, could be directed in explain, or here's a solution, I'm giving you a problem and solution, can you discuss or explain what's happening? That will be nice, to discuss this or whatever, or how would you use this or where else you think you can use it. That will be a real step forward, I think, into thinking, mathematical thinking, you know. (see p. 174)

I base my point on the line where he says, "like you said." His comment implies that somehow there is a lack of resources such as reform–minded curriculum materials and opportunities to learn to teach in collaboration with reform–minded teacher educators. Also, EM notes that asking students "why" would be a useful way to go in algebra :

EM	When "a" is positive, the parabola is smiling. They accept that. In other words I am going strictly according to the textbook. So I am telling them, look here, if "a" is positive you are going to have a smiling graph.
Faaiz	And do they ask you why?
EM	(laughs). Unfortunately.
Faaiz	Unfortunately?
EM	They should actually.
Faaiz	Do you think it is a useful way to go?

EM Yes, it would be.
(interview # 3 of 4)

From this excerpt it appears that EM is amenable to the idea of having his students know reasons behind rules—"they should actually." From the interview I infer he has not seen school algebra texts and curriculum material where mathematics as reasoning is a focus. The policy document MLMMS on its own, with its focus on "outcomes," will not be adequate in terms of moving EM to teach in ways where there is a focus on mathematics as reasoning.

The challenge for policymakers is therefore to support teachers like EM and DS with resources that focus on mathematics as reasoning that will hopefully "produce instruction" (Cohen and Ball, 1990, p. 250). In addition, such teachers as EM need to interact with reform-minded teacher educators and curriculum materials. I base my argument about interaction on the fact that EM makes these statements about reasoning during the third interview.

HY's rhetoric makes contact with similar policymaker rhetoric about mathematics relationships (see p. 174) :

> Mathematics involves observing, representing and investigating patterns in social and physical phenomena and within mathematical relationships.
> [*Government Gazette*, June 6, 1997, no. 18051, p. 116]

For example, he espouses in-depth models of teaching and learning algebra :

HY because, what I normally tell students is that when you're solving two equations, you are solving two graphs simultaneously. But that alone, to explain that with adequate examples, take, it will take 4 to 5 periods before the kids really believe you. I mean if you explain to a kid, solve for x in x + 2 = -x, right. He will solve that quickly, but if you now show them by means of graphs what you're doing. The graphs is (sic) lagging behind, at all levels at school. Pupils have problems with graphs.
(interview # 2 of 4)
(see p. 161).

His rhetoric "now show them by means of graphs what you're doing" is similar in understanding to "observing, representing and investigating patterns within mathematical relationships." His example of x + 2 = -x with its graphical solution is an interesting case of f(x) = g(x)? (Chazan, 1993). He also describes other curricular examples about "shifts" and "transformations" in graphing quadratic functions (f(x) = g(x)?; Chazan, 1993).

DS's rhetoric about the teaching and learning of calculus makes a strong point of contact with similar policymaker rhetoric in MLMMS about mathematics as reasoning and mathematical relationships. In other words he is most likely able to understand the policy statements about mathematics as reasoning and mathematical relationships. For example, he espouses detailed models of teaching and learning (Ernest, 1987) calculus (see pp. 190 - 191). He spoke about "a unit of work" that he would like to teach the next year. In this unit he wants to let students see reasons behind the formulas for volume and area of certain shapes. In this instance his rhetoric makes positive contact with the "deep structure" of the calculus (Tall, 1987), which falls "outside" the school mathematics tradition. The challenge in this instance is therefore to provide DS and teachers like him with resources, such as curriculum materials and learning opportunities with reform–minded teacher educators, that could lead to enactment in the classroom of the policy statements. The idea of learning opportunities comes through in the words of AS. He talked about an "unlearning of what we've done" as a way to understand *Curriculum 2005* (see p. 156). Here his rhetoric makes contact with the idea of teacher learning or learning to teach (see Ball's (1988a) *Unlearning to teach mathematics*). This implies that there is the challenge of building an ethos of teacher learning in conjunction with reform-consistent curriculum materials. MLMMS as a policy document even if it had curricular illustrations are not sufficient on its own for teacher learning to happen. It can only be a start.

Implications of contact points involving similar words and different understandings

There are many instances in the interview and observational data where the points of contact between the rhetoric communities can be described as similar words but different understanding. Thus in this instance the policy document MLMMS, does not even provide a start because the teachers in the sample think they understand what is being asked.

One middle school teacher, PN, thinks of mathematics as reasoning in terms of the way she grades and how students "worked it out" (see p. 146). When I asked her whether she explicitly asks her students to give reasons for their answers, she replied:

PN I always tell them if you just give me an answer, you only one mark. So I don't accept just an answer, they have to show *how* they arrived at the answer.

PN I always say in a question, the answer would be worth three or four marks, if you *only* give me an answer, I can only give you one mark. The rest of the marks are specifically for the way in which you worked it out. That is the standard rule here, nobody just ever gives an answer even in a test.
[emphasis in original] (interview # 2 of 4)

(see p. 141)

From this excerpt it seems most likely that she understands mathematics as reasoning to be about "procedures" as in "the way in which you worked it out." When AS was asked about reasoning during the first interview he responded as follows:

AS No, I don't do it in a question paper. I haven't done it. You're going to throw the kids out, I think, by asking the question why. The explanation in the class is fine.
(interview # 1 of 4)
(see p. 145)

In this instance he seems to prefer that his students keep their thinking and thus their reasoning private and implicit (Lampert, 1990). NJ, the other middle school teacher, said that "in order to be good at maths, you have to be able to give reasons to support your answers." (see p. 157).

In all the three examples the middle school teacher rhetoric about mathematics as reasoning is different from the policymaker rhetoric. They do not realize that there is a different intent. In the same way, the policymakers may have not realized that their use of language about mathematics as reasoning does not fit with what is used by teachers like those in the sample. The document MLMMS on its own is therefore not a way to communicate what is being asked. In this way it is like places where the teachers simply do not understand the policymaker rhetoric. The middle school teachers think they understand what is being asked.

HY, one of the high school teachers, thinks most of the reasoning is in the geometry. According to him, students explain their answers or show their reasoning when they "show their work," as in traditional rote memorization-type tasks (see, p. 171). Also, EM appeared puzzled about supporting solutions with reasons in algebra (see p. 172). DS was quick to note that as teachers they "never do that," i.e. ask their students for reasons (see p. 173). Except for perhaps DS, teacher rhetoric about reasoning is fundamentally different and thus a disconnect from the policymaker rhetoric:

> Reasoning is fundamental to mathematical activity.
> [Mathematical Literacy, Mathematics and Mathematical Sciences, *Government Gazette*, June 6, 1997, no. 18051(p. 117)]

EM relies solely on "procedures" such as "substitution" taught in earlier grade levels, notation and rules, as ways to stress mathematical relationships and mathematics as reasoning in the teaching and learning of calculus in the 12th grade (see pp. 183 - 186). Therefore he ascribes his students' difficulty with the calculus as due to a lack of "exposure" to procedures taught in earlier grade levels. In this instance he most likely thinks of mathematical knowledge as a "material" stock of "prerequisites," consisting of single elements. They must first be learned in a cumulative way in order to make later discoveries of the underlying structural relations and meanings possible (Steinbring, 1989, p. 29). For this reason he talks about "exposure" and points to an education department syllabus and says,

> [Oh yes,] this is the one that I was talking about where this was out of the standard 7 (ninth grade) syllabus. Now I think this should be included because what you're testing here is you're testing substitution as well. That is something that you're eventually going to do in standard

10 (12th grade), and now you're leaving it out in standard 7 (ninth grade).

The so-called mathematical relationships and mathematics as reasoning, the "prerequisites," implies that he has a different understanding from what is being asked. "Testing substitution" alone is not going to unravel to students the "big idea" about images of rate in calculus (Thompson, 1994), and calculus as being about "the back-and-forth between rate-to-amount and amount-to-rate questions and these questions' resolutions" (Schnepp *et. al.*, 2000).

What is to be done in cases of contact points where one has similar words but different understandings? The first step is for both policymakers and teachers to realize that they are using similar words in different ways. This is the least that can be done. MLMMS as a document cannot do this. The kind of interaction I had in my interviews with the teachers is therefore important. The teachers on their own cannot find out that they are using similar words that have different understandings compared to the policymakers. There has to be the "creation of a rich conversation, in and around classrooms, about mathematics, teaching and learning" (Cohen and Ball, 1990; p. 255). For example, teachers have to made aware that the mathematics curriculum is fragmented or "strongly classified" (Ensor, 1996). During the interview process with the teachers, I tried to point this out. Their reactions to such a claim can hopefully be the beginning of a "rich conversation."

Teachers need to see that they are using similar words in different ways and therefore appreciate the different ways in intent that are below the surface. For example, here is a clarifying excerpt from an interaction I had with EM about his geometry lesson. I probed him about how his geometry lesson fits in with other parts of the curriculum. After a while I gave him some input to my question. He then replied:

EM I mean for example, when you said similarity is related to sine graphs, you had me puzzled for a moment, because I thought how are they linked? But when you mentioned ratio then I obviously I knew, look here this is what you're talking about. But I am sure there are many teachers if you were to tell them that these two are linked, (similar triangles and the sine function) they won't understand what you're saying. I mean I did not understand what you were saying but the moment you put the two together.
(see p. 180).

There are also instances in the interview process that did not clarify cases of similar words and different understandings. For example, in the excerpt below, PN talks about mathematical relationships:

PN When you are giving the child a ratio, why can't the ratio not be given in the form of a percentage and a fraction, which will consolidate what they already ...
[last interview # 4] (see p. 162)

Her comment "why can't the ratio not be given in the form of a percentage and a fraction" is in some ways faulty because some "ratios" are in the realm of irrational numbers, like pi, while others like fractions are more in the realm of rational numbers. Thus my interaction with her and the *Connected Mathematics Project* (CMP) curriculum materials did not clarify the nature of possible mathematical relationships between ratios, percentages and fractions. A much more concerted interaction is needed with teachers like PN to surface different understandings about mathematical relationships that are below the surface.

In conclusion, in using policy documents as vehicles for reform, it is important to identify situations of similar words and similar understanding and then similar words and different understanding. There will then still be the challenges of supporting teachers in visions of the reform that they understand but do not have experience carrying out. Ultimately, real pedagogical change would thrive on the creation of a rich conversation, in and around classrooms, about mathematics, teaching and learning (Cohen and Ball, 1990, p. 255).

Next I reflect on the significance of these results and address people's reactions to the ways that I framed my arguments.

Reflecting on the results in the study

Most of the reflections on the present study take the form of an imaginary dialogue with selected participants I have mentioned earlier. The education researcher was chosen because viewpoints of education researchers in general are relevant when it comes to school reform. Practitioners of ethnomathematics and critical mathematics education have a significant presence in the South African educational context. The choice of a mathematician is because research mathematicians teaching at the university level push for an abstract, purely formal, deductive and symbolic form of mathematics when it comes to school reform. The chosen participants in the imaginary dialogue are therefore important in educational reform.

The fact that I am inclined toward a disciplinary tradition in mathematics education might mean that the imaginary dialogue participants will take issue with my claim to be addressing key issues like the education and training divide in *Curriculum 2005.* The ethnomathematics educator (EM) and critical mathematics educator (CME), in particular, will question the relevance of a disciplinary tradition in the South African mathematics education context. This makes the participants in the imaginary dialogue all the more important. The dialogue is a means to highlight the challenges that lie ahead in school mathematics reform in South Africa. I have been outside South Africa for some time. I was outside the country when the first-ever democratic elections were held in1994. I witnessed from outside the transition from a white minority government to a more representative black government. More importantly, I have followed the launching of *Curriculum 2005*, which started in 1997. The writing of this dissertation has made me more attuned to what is happening in educational reform in South Africa. I have come to learn a great deal about the curriculum policy itself. Also, I have come to realize the importance of taking education policy seriously.

When I grew up in South Africa, I was often dismissive of the government's education policy. I had this attitude simply because the South African state at the time was illegal and illegitimate in many respects. I was not the only one with such a frame of mind. Education policy was not something that interested me *per se* because I saw it as another means of the state to underdevelop the majority of South Africans. I remember during my first full committee meeting, one of my then committee members talked about policy-practice studies. At the time, I simply did not understand what she meant. One of her research interests is in merging education policy and practice, in a way where both affect each other. Looking back at that meeting, I realize the policy-practice divide is probably my first awakening to one of many dichotomies.

ER How does the policy-practice divide or dichotomy, to use your words, address the fact that *Curriculum 2005* is on its way out? I refer you to an article titled, "Scrap *Curriculum 2005*,"which appeared in *The Teacher* in the *Weekly Mail and Guardian*, dated, June 6, 2000.

MFG *Curriculum 2005* may be history soon but the challenges and the underlying situation of South Africa remain. There is still a desire to merge or bring together education and training. My dissertation isn't really about *Curriculum 2005* . It is rather about the situation that South Africa finds itself in. This situation is about transcending dichotomies such as theory and practice, knowledge and skills and education and training (DE, 1997a; 1997b).

The fact that I have studied a particular policy that is on its way out means that I have gained insights from having studied that policy. These insights might be useful in reflecting on education policy down the line.

M As a mathematician, the words education and training don't seem so different. Why are you distinguishing between the two ?

MFG Education and training are regarded as different by the public in general. Governmental education departments, for example, have separate divisions that deal with "education" and "training."

MFG Often higher education institutions are also separated along "education" and "training" divisions. We know that somehow education enjoys "higher" status compared to training.

ER I agree.

MFG When I read the National Qualifications Framework (NQF), I realized the extent of policymakers' seriousness when it comes to equalizing education and training. I read the intent of the NQF to be about making education less esoteric and removed from people and at the same time training will be less narrowing in a sense that people getting education and training will have more opportunities rather than narrowly getting slotted into particular job openings.

MFG The goal of *Curriculum 2005* is to give out-of-school youth and adults opportunities to obtain qualifications that transcend "rigid divisions" or dichotomies such a theory and practice, knowledge and skills, and education and training. For example, for out-of-school youth and adults, the NQF has "General Education and Training Certificates" and "Further Education and Training Certificates" that have equivalents with the formal grade levels in schools. Recall that there are generations of South Africans, which include adults and out-of-school youth, who have been systematically underdeveloped under apartheid. At one level I think the NQF would like to see education institutions merge education and training.

At another level, to transcend the education and training dichotomy implies that we move away from it as a relevant way of talking about it. In fact, it's more than purely a curricular issue.

EME What do you mean by that?

MFG I recall Lisa Delpit (1988) arguing for a resolution of the skill/process debate. I interpret her to mean transcending the skills/process debate.

CME I'm familiar with that piece.

MFG In her argument she points out that those who are most skillful at teaching black and poor children do not allow themselves to be placed in "skills" or "process" boxes. She notes that :

> They understand the need for both approaches, the need to help students to establish their own voices, but to coach those voices to produce notes that will be heard clearly in the larger society (Delpit, 1988, p. 296).

To "produce notes that will be heard clearly in the larger society" implies addressing the dichotomy of education and training and the other dichotomies I have mentioned. I therefore extend her argument about transcending the skills/process debate to out-of-school youth and thus to education and training.

CME Do you see any way to transcend the education and training dichotomy or is that dichotomy always with us?

EME I have a similar question. In your dissertation you never write about the education and training divide. In your interviews with the mathematics teachers you focused on education, for example, their theories about the teaching and learning of mathematics.

MFG That's true. My focus on education allows me to talk about transcending the education and training divide. Let me explain. In the literature review of this study I give details about the focus on unifying "big ideas" in disciplinary traditions in mathematics education. This focus aims at transcending the dichotomy between mathematics as a discipline and the fragmented mathematics in school. I therefore propose that in the case of the education and training debate there be a focus on "big ideas" in mathematics, in the case of adults, out-of-school and in-school youth.

M You use the word "tradition." Could you say a little bit about why you use that word?

MFG I see tradition as a relatively coherent and unified way of thinking, feeling, and acting within a particular domain. I think mathematicians have a tradition into which they get inducted. Mathematics is developed and shared between the minds of mathematicians and practitioners over the centuries. Mathematicians feel a certain partisanship and loyalty to mathematics that is rigorous, purely formal and deductive. Even to the public in general, mathematics is associated with rigor, formalism, truth, certainty and even infallibility.

M I have to admit that.

MFG Here is where another dichotomy comes up.

ER What do you mean?

M Yes, I, too, would like to know what you mean ?

MFG Mathematics "in the making" has never been made known to the student, or to the teacher, or to the public in general. I am referring to mathematics "in the process of being invented." Let's face it, in practice a mathematician's thought is never a formalized one. What I am saying is there should a be healthy balance, that we should transcend the division between mathematics that is deductive, purely formal and symbolic and mathematics that appears as an experimental, inductive science.

MFG In chapter six where I do an indirect juxtaposition of policymaker and teacher rhetoric, I discuss the example of one of my subjects who wishes to approach calculus in an inductive and experimental way. He wants his students to know why formulas for volumes and areas of objects are what they are. I think it's one thing to know all those formulas, and quite powerful to know the "why" and the how of those formulas. Besides, in South African mathematics classrooms, there is very seldom a focus on "why."

CME How is knowing a rule and why, helpful? I am curious.

MFG By making explicit about the how and why of rules and formulas in mathematics when one is teaching, I would argue that students, in this case out-of-school and in-school youth, learn what it means to do and to know mathematics. By knowing the reason behind a rule we make the mathematics less mysterious to learners and everyone. For example, in the case of dividing fractions, the rules is that you "flip and multiply," a well-known rule in the school mathematics tradition (SMT).

MFG Briefly, in this tradition mathematics knowledge is decomposed and proceduralized, rote learning is common, and teaching takes on the form of direct instruction. Teaching is separated from content and from learning. Learners need opportunities to see why that rule works. In my study one high school teacher talked about developing opportunities for his students to see why we have the rules for the areas and volumes of three- and two-dimensional figures. Such opportunities make these rules less mysterious. It amounts to transcending mathematics and mathematics "in the making," knowing the how and why behind rules.

CME But you come from what you call a disciplinary tradition. When I read through your dissertation I realized how United States-centric your notion of a "disciplinary tradition" is.

EME Yes.

CME Therefore, how can we take you seriously?

MFG I don't think there should be a monopolization of the current debate in education reform in South African by any particular mathematics education tradition. In chapter four I indicate that there is evidence of many mathematics education traditions in the mathematics component of *Curriculum 2005.* The determination of policymakers to merge the system that provide education and training remains.

MFG I am saying I would like to contribute to the education and training debate. In the dissertation I have talked about many dichotomies, such as policy and practice, teaching and content, mathematics and mathematics "in the making." Disciplinary traditions attempt to transcend these dichotomies. Another divide is what Dewey (1902/1992) eloquently refers to as the child and the curriculum. He speaks about the "evils" that happen when we separate the child from the curriculum. When learners, to use the relevant South African term, encounter mathematics for the first time, they have intuitions that we have to explore and use when we teach. In the school mathematics tradition there is no opportunity for that to happen because "methods" as procedures tend to be reduced to "cut and dried" routine.

CME But how would you, coming from a disciplinary tradition, speak to the situation on the ground, in South Africa?

MFG The situation on the ground is that in the majority of South African mathematics classrooms rote learning is rife and teaching as "telling" is rife. As you know mathematics is thought of as purely formal, deductive and symbolically stated information, intelligible only to the initiated. My interview data shows how entrenched the school mathematics tradition is. Students labor under the yoke of procedures, facts, and rules and have no inkling of the "big ideas" within mathematics. I could develop a mathematics program for out-of-school youth that aims at striking a balance between technical and symbolically stated information and being

MFG able to trace its connections with the objects and operations with which they are familiar. In other words, I mean striking a balance between mathematics, and mathematics "in the making." While manual skill and technical efficiency and immediate satisfaction is found in work, these things would have to be subordinated to intellectual results and the forming of a socialized disposition. The goal of teaching in such a program would be to develop in out-of-school youth a socialized disposition. I would argue similarly in the case of working with teachers who are interested in transcending the education and training divide.

CME Why is a "socialized disposition" appropriate for both out-of-school and in-school youth?

MFG Since the mass of learners, the out-of-school youth and adults and of course students in school, are never going to become scientific

MFG specialists, it is much more important that they should get some insight into what "scientific method" means than that they should copy at long range and second hand the results that scientific men and women have reached. The latter is typically what happens in school and is instantiated in the school mathematics tradition.

EME Would you consider an ethnomathematics tradition as a possible route to what you are advocating?

CME And a critical mathematics education tradition, or a combination of the two? After all, these traditions speak more to the situation on the ground in South Africa, and thus more likely to education and training.

MFG The answer is yes. I have said earlier I am attracted to a disciplinary tradition. I have to make clear the distinction between an individual practitioner and a tradition. The latter is about a set of ideas that are unified and coherent. Then there is how a person enacts those ideas and a person is multidimensional. There are ethnomathematics educators who are cognizant of mathematics in general and the mathematics in the school curriculum from a historical and philosophical perspective. For example, I am thinking of Ubitaran D'Ambrosio and Paulus Gerdes.

I am concerned about the mathematics in the curriculum that goes by "unnoticed." I am wary that when teachers are introduced to ethnomathematics and critical mathematics education traditions, they might see them as "extras." There is the notion of "the day's work," and then there's "extras."

EME Say a little more about "the day's work."

MFG I am talking about teachers' routines, where there is a focus on rules, procedures and facts and nothing more. For example, in one of the interviews with the teachers I gave my subjects some of the *Connected Mathematics Project* curriculum materials to browse through. I then constructed individual interviews with them around these materials. From their responses I started to feel that in their responses there is evidence of some things that initially were not there. They began to ask, Faaiz, what do you mean? Then over time they started to understand, or at least I think there might be the beginnings of some understandings of what I meant. Most of the teachers said something like, I'll use this and that from the curriculum materials, at the beginning of the lesson, just before I begin the "day's work." Thus there is the challenge of teacher development when it comes to school mathematics reform and transcending the education and training divide.

Transcending the education and training divide is a very ambitious task. If policymakers are really serious, then I would point out what teachers say about the school mathematics curriculum. They all confirmed the school mathematics tradition and the mathematics curriculum as consisting of different "sections" and as a collection of isolated rules, facts and procedures that need to be memorized.

MFG The school mathematics tradition is very strong in the teacher rhetoric. Except for one high school teacher, the teachers never talked about changing the school curriculum. This implies that their rhetoric points in a direction where there is a separation of teaching from content and where achievement on standardized tests amounts to understanding. This is a trivialization of learning and understanding. I wonder how the teachers would react to out-of-school youth learning mathematics in the same institutions where they will be teaching? The subjects in my study talk about natural ability, a mathematical mind, "prerequisites" as procedures, algebra as something that only teachers use in their jobs," "reasoning" as pertaining to geometry only. Such teachers are not going to be able to work effectively with out-of-school youth, nor with in-school youth. They may see these learners as people who do not have natural ability and mathematical minds. These are certainly barriers when it comes to transcending the education and training divide.
It takes a special kind of "listening" to transcend the education and training divide, a listening that requires open hearts and minds from the larger society. At this point I would like to restate the great challenges that lie ahead for mathematics education in South Africa. I will include comments on the work of policymakers in the area of curriculum development, teaching and teacher development and the site of the study.

Work has to be done in the area of curriculum development.

MFG Any new policy document on school mathematics reform must have multiple illustrations of what is meant by various educational slogans or rallying symbols that come from different mathematics education traditions. In the present document, *Mathematical Literacy, Mathematics and Mathematical Sciences* (MLMMS), there are no illustrations. When I brought up curricular examples during the interviews, the teachers seem to have a better idea of what I was talking about. The results of my study reveal that teacher rhetoric about content like algebra, geometry, and calculus, the traditional "sections" in the school curriculum, is heavily influenced by the SMT. Multiple illustrations of the entire mathematics curriculum need to be revised in ways that transcend the education and training divide. This implies a radical overhaul of the current "sections" in the mathematics curriculum.

In MLMMS there is rhetoric from constructivism, ethnomathematics, critical mathematics, and combinations of them. The challenge is thus to include in a new policy document in-depth and varied illustrations that come from these traditions. These illustrations could include vignettes of teachers teaching out-of-school and in-school youth.

Within the area of curriculum development, the "definition" of mathematics in MLMMS alone does not help in terms of transcending the education and training divide. The "definition" has to be balanced with an equal reference to mathematics "in the making," which helps in focussing attention on transcending the dichotomy between mathematics and mathematics "in the making" and thus on the education and training divide.

MFG For example, mathematics "in the making" could include vignettes of classroom teaching that illustrate educational slogans like "mathematical thinking," "mathematical communication," "mathematical ideas," and "mathematical tools and techniques," as in the *California Frameworks* (1992). These educational slogans expand on the narrow understanding and beliefs about mathematics in the SMT.

In the area of teacher development, any new policy document on transcending the education and training divide must make nods to the conditions that teachers labor under. There is the case of overcrowded classrooms in the public schools in the Western Cape and elsewhere in the country. One middle school teacher was very upfront about the fact that his classrooms are so overcrowded. The inclusion of vignettes of teaching in overcrowded classrooms in a new policy document would speak to the conditions of mathematics education on the ground. The challenge is to produce such vignettes.

Teacher development has to take into consideration the strength of the SMT. Teachers' "experience" or years in the classroom does not help in breaking with their apprenticeship of observation (Lortie, 1975).

Their "experience" in the classroom most likely entrenches the SMT, which is a barrier to transcending the education and training divide. In order to transcend this divide there is a lot of "unlearning" that needs to occur.

MFG This was a word that of one of the middle school teachers used during the last interview. The challenge is to have all teachers see multiple curricular illustrations of what those educational slogans in MLMMS that aim at transcending the education and training divide mean.

To transcend the education and training divide also implies that teacher development include in-service opportunities for all teachers to critique the strongly classified curriculum. In my study there is the case of one high school teacher who questioned the order of the mathematics he was teaching and got "rudely interrupted" by his department head (see p. 133). The challenge is thus to convince education departments and school bureaucracies about the need for teachers to critique the fragmented curriculum.

With regard to the site of the present study, the Western Cape, there is a great challenge that awaits policymakers when it comes to transcending the education and training divide. This is a province that has the highest end-of-the-year 12th grade "matric" results. Students with good "matric" passes move on to institutions of higher education. It is therefore most likely that education authorities in the province will resist attempts by the central government to merge and transcend the education and training divide. Earlier on I spoke about Lisa Delpit's idea that we need a special kind of "listening" when it comes to transcending the skill/process debate. I would like to say something that continues in that vein and is more specific to demystifying mathematics in schools.

MFG The school mathematics tradition does little to transcend the education and training debate, as I have shown. Henrici (1972), an applied mathematician, reflecting as a teacher of mathematics, writes the following:

> [Could] it be that in mathematics, too, we need a new Consciousness? A Consciousness where mathematics is told in human words rather than a mass of symbols, intelligible only to the initiated; a Consciousness where mathematics is experienced as an enlightening intellectual activity rather than an almost fully automated logical robot, ardently performing simultaneously a large number of unrelated tasks (p. 38)?

On an almost daily basis in the fragmented mathematics curriculum students do not experience mathematics as an enlightening intellectual activity. They do so many unrelated tasks, thereby widening "rigid divisions" such as education and training, policy and practice, content and teaching, teaching and learning and mathematics and mathematics "in the making."

APPENDICES

APPENDIX A

Interview # 1a

Personal Histories and Orientations: Baseline Questions

A1.I'd like to start out by learning a little about what brings you to teaching.
When did you first start thinking that you might want to teach?
Why are you interested in teaching?
[Probe the teacher's intellectual interests and perspective s/he holds as a teacher]

A2.My research focuses specifically on how teachers think about subject matter, mathematics in this case and how they teach it and how they think one learns it.

(a) What do you remember about learning mathematics in elementary school?

(b) What about the high school level?
What do remember about learning math in high school?

(c) What about college? Have you taken math in college?

If yes: What stands out to you about math in college?

if no: Why not?

Interview # 1b

Conceptions about the nature of mathematics[1]

I shall break mathematics up into *algebra, geometry and trigonometry* in the case of high school teachers and *algebra and geometry* in the case of middle school teachers.

A1. I'd like you to think of someone you know who is good at algebra...Who is it? Why do you think of__________as good at algebra/geometry? What does she/he do?

(note the issue here is what it means to be "good at algebra/geometry/trigonometry")

What's your hunch about why this person is good at algebra/geometry?

(Note: the issue here is what the source of success in algebra/geometry is)

Probe for specificity: What do you mean? Can you give me an example? What does x have to do with being good at algebra/geometry?

A2. What about the flip side of the coin -- do you know anyone whom you consider not to be very good at algebra/geometry?

(Note: the issue here is what it means to be "not good in algebra/geometry")

Why do you think of ___________as not very good in algebra/geometry?

Do you have any ideas about why__________is not very good in algebra/geometry?

(Note: Many people identify themselves with this question. Ask, What's the explanation you give yourself about why you aren't so good in algebra/geometry?)

A3. Do you have a favorite subject or perhaps a favorite area within a particular subject? What do you enjoy about it?

A4. Are there some things in algebra/geometry or about algebra/geometry that you especially like/enjoy? Are there some things in algebra/geometry or about algebra/geometry that you especially dislike?

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[1] Adapted from *A Study Package for Examining and Tracking Changes in Teachers' Knowledge,* Technical Series 93-1, National Center for Research on Teacher Learning, by M. Kennedy, D. L. Ball and G. W. McDiarmid

APPENDIX B

Questionnaire

Being good at mathematics

For the statements below indicate your agreement or disagreement by circling the number that best expresses what you think about the statement. Your replies to these statements can range from **strongly agree (SA or 1) to strongly disagree (SD or 7)**

1 2 3 4 5 6 7

<----o---------o---------o---------o--------o--------o----------o--->

To be good at mathematics in school, how important do you think it is for students to do the following

1. Remember formulas, principles and procedures	1	2	3	4	5	6	7
2. Think in a logical step by step manner.	1	2	3	4	5	6	7
3. Understand mathematical concepts, principles and strategies	1	2	3	4	5	6	7
4. Be able to think flexibly and creatively	1	2	3	4	5	6	7
5. Understand how mathematics is used in the real world?	1	2	3	4	5	6	7
6. Have a "mathematical mind"	1	2	3	4	5	6	7
7. Work hard at it.	1	2	3	4	5	6	7
8. Be able to provide reasons to support their solutions	1	2	3	4	5	6	7

APPENDIX B

Questionnaire (cont'd)

Learning mathematics

Remember 1 means strongly agree (SA) and 7 means strongly disagree (SD)

9. When students can't solve problems, it's usually because they can't remember the right formula or rule.	1	2	3	4	5	6	7
10. For students to get better at math they need to practice a lot	1	2	3	4	5	6	7
11. One can learn a lot by watching an expert mathematician "think aloud" while solving problems	1	2	3	4	5	6	7
12. If students get into an arguments about ideas or procedures in math class, it can impede their learning.	1	2	3	4	5	6	7
13. In math learning students must master topics and skills at one level before going on.	1	2	3	4	5	6	7
14. A teacher should wait before students are developmentally ready before introducing new ideas and skills.	1	2	3	4	5	6	7
15. It is important for students to master the basic computational skills before studying topics like probability and logic	1	2	3	4	5	6	7
16. Math is a subject in which natural ability matters more than effort	1	2	3	4	5	6	7
17. Since older students can reason abstractly, the use of models and other visual aids become less necessary.	1	2	3	4	5	6	7

APPENDIX B

Questionnaire (cont'd)

Teaching mathematics

Remember **1 means strongly agree (SA) and 7 means strongly disagree (SD)**

18. Students should never leave the math class (or end the math period) feeling confused or stuck	1	2	3	4	5	6	7
19. Teachers should not necessarily answer students' questions but should let them puzzle things out for themselves	1	2	3	4	5	6	7
20. Students should "show their work" when they solve math problems	1	2	3	4	5	6	7
21. If students are having difficulty in math, a good approach is to give them more practice in the skills they lack.	1	2	3	4	5	6	7
22. If a students is confused in math the teacher should go over the material more slowly	1	2	3	4	5	6	7
23. The most important issue is not whether the answer to any math problem is correct, but whether students can explain their answers.	1	2	3	4	5	6	7
24. To do well, students must learn facts, principles, and formulas in mathematics.	1	2	3	4	5	6	7
25. Teachers should follow the math textbook that is used in their school	1	2	3	4	5	6	7

APPENDIX C

Semi-structured interview focusing on a task that the teacher developed.

1.Tell me about your unit on ________?

How did you introduce it?

What are your goals for the unit?

What kinds of things did you take into consideration in planning the unit?

Can you tell me something about the discussions/lessons?

How long did the unit take?

Was the text required by your school or did you choose it?

If you chose it, why did you choose this text?

2. Tell me about the students in your class
[probe for number of students, homogeneity or heterogeneity of class]

Tell me about any assignments or activities that you used in the unit. Do you have any copies I could see?

Did you have any final paper or test associated with the unit? What was it like?

Tell me what you thought the students got out of the unit.

Tell me how you thought the unit went. How could you change the unit if you taught it again?

How might you change the unit if you taught a much stronger group of students? How about a weaker group of students?

APPENDIX D

Post-Observation Interviews

1. How did you feel things went in class today?

    How did things compare with what you expected?

        Did anything surprise you ?

    Was there anything you were particularly pleased about?

        What? Why?

    Did anything disappoint you?

        What? Why?

2. How did you decide whom to call on?

    (To work at the board, to answer questions, etc.)

    (Probe about any pattern of calling you noticed)

3. One thing I am interested in is how teachers select activities, tasks, explanations, examples, that they use or how they explain things to learners.

I noticed that you said/did_______________

Where did this example, story, task come from?

Why did you decide to do this?

Does it have any particular advantages or disadvantages (benefits or drawbacks)?

4. As usual, I've only been able to observe this one day. Was this session typical of what you're doing these days?

    If yes: Did you do anything special because you knew I would be here?

    If no: How was today's session different from the usual?

5. Are you talking with any other teachers in the school?

    How frequently?

    What did you talk about?

6. Are there any other questions I have not asked that you think I should ask?

APPENDIX E

Consent Form

To be read to the subject:

You are being asked to participate in a project "Conceptions of mathematics and the teaching and learning of mathematics: The case of experienced and qualified South African teachers." This project is about experienced and qualified mathematics teachers' ways of knowing and doing mathematics. The researcher, Mohammad Faaiz Gierdien, wishes to find out how you know , think about, and do mathematics. More specifically the researcher is interested in how you change or transform the mathematics for the purposes of teaching and learning. He hopes to find all of these out by observing you teaching mathematics and by having post-observation conversations with you. These conversations will last a maximum of half an hour . Also, he will conduct conversations with you around homework tasks you assign, around particular units you have developed, and around chapters you will be teaching or have just taught. You are expected to give the researcher some of these homework tasks, units, tests, and examinations, as part of his data. He will also ask you to sort through some cards that have mathematical ideas or topics written on them and to arrange them in ways that they make sense to you. Wherever possible he will audio tape conversations and interviews. There will also be a Likert scale questionnaire on what being "good at mathematics" entails, the teaching of mathematics and the learning of mathematics, which you will fill in. This you can do at home and it will be collected the following day. The researcher will later on conduct a few follow up interviews with you, after has done the Likert Scale questionnaire, done some classroom observations, post-observation interviews and the card sort tasks. An estimate of the amount of time you will asked to contribute is about five (5) weeks, beginning some time in May and ending in early August, 1998. Out of every week of the five weeks only four days will involve interviews and/or classroom observations. The interviews themselves will not exceed an hour.

You will be given a printed copy of the above explanation of the project.

You are being asked to participate in this project. Your participation is entirely voluntary and you realize that there is no penalty for refusal to participate. Further, you realize that you have the right to withdraw your participation in the study at any time during the data collection, without explanation. You have the right not to answer any question you do not want to answer during the interviews. It would not be possible for you to withdraw from the study after Mr. Gierdien has done all the write up of the study.

In order to protect your identity no names will be used, meaning that you will not be identified in the report of the research. In the case of your school a pseudonym will be used. You do recognize that although all efforts will be made to protect your confidentiality, there are limitations to those protections because those who know you are participating in the project will be able to identify you in published reports.

---------------- Yes, I give permission to participate in the study. Mr. Gierdien may contact me to set up different times to participate in his research project.

-- ------------------------------ -----------------------------

(name) (date) (phone number)

If you have any questions, please contact:

M Faaiz Gierdien
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Phone: 23 6802
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BIBLIOGRAPHY

BIBLIOGRAPHY

Adler, J. (1993). "Beyond 'Tell And Drill' Maths," *The Weekly Mail & Guardian*, November 1993.

Adler, J. (1994). Mathematics teachers in the South African transition, *Mathematics Education Research Journal, 6* (2), 101-112.

Adler, J. (1997). Professionalism in process: Mathematics teacher as researcher from a South African perspective, *Educational Action Research 5* (1), 87-103.

Anyon, J. (1981). Social class and school knowledge, *Curriculum Inquiry, 11,* (1), 3 - 41.

Apple, M. W. (1992). Do the standards go far enough? Power, policy and practice in mathematics education, *Journal for Research in Mathematics Education, 23* (5), 412 - 431.

Ball, D. L. (1988a). *Unlearning to teach mathematics.* Issue Paper 88 - 1. East Lansing, Michigan: National Center for Research on Teacher Learning, Michigan State University.

Ball, D. L. (1988b). *Knowledge and reasoning in mathematical pedagogy: Examining what prospective teachers bring to teacher education.* Unpublished doctoral dissertation, Michigan State University, East Lansing.

Bauersfeld, H. (1980). Hidden dimensions in the so-called reality of a mathematics classroom. *Educational Studies in Mathematics, 11,* 23 - 41.

Bishop, A. J. (1990). Mathematical power to the people, *Harvard Educational Review, August, 1990, 60* (3), 357 - 369.

Bishop, A. J. (1985). The social construction of meaning: a significant development in mathematics education? *For the learning of mathematics, 5* (1), 24 - 28.

Borba, M. C. (1990). Ethnomathematics and education, *For the learning of mathematics, 10* (1), 39 -43

Brodie, K. (1998). "Learner-centered" teaching and possibilities for learning in South African mathematics classrooms. In A. Olivier and K. Newstead (Eds.). *Proceedings of the 22nd Conference of the International Group for the PME,* vol. 4. Stellenbosch: University of Stellenbosch, pp. 160 - 167.

Brodie, K., & Strauss, J. (Eds.) (1994). *Proceedings of the First National Congress of AMESA.* Johannesburg: University of Witwatersrand.

Brophy, J. & Good, T. (1986). Teacher behavior and student achievement. In M. C. Wittrock (Ed.) Handbook of research on teaching (3rd. ed.,pp. 328 - 375). New York: Macmillan.

Bruner, J. S. (1960). *The process of education.* Cambridge, MA: Harvard University Press.

California State Department of Education (1992). *Mathematics Curriculum Framework for California Public Schools, Grades Kindergarten through 12.* Sacramento, Calif.: The Department.

Chazan, D. (1993a). F(x) = G(x)? : An approach to modeling with algebra, *For the learning of mathematics, 13* (3), 22 - 26.

Chazan, D. (1993b). *Algebra for all students ?* Unpublished manuscript. Michigan State University.

Chazan, D. (n.d). *What are these x's and y's? Intellectual engagement in the Algebra One classroom.* Unpublished manuscript. Michigan State University.

Chazan, D. (2000). *Beyond formulas in mathematics and teaching: Dynamics of the high school algebra classroom.* New York: Teachers College Press.

Christiansen, I. (1999). Are theories in mathematics education of any use to practice?, *For the learning of mathematics, 19* (1) 20 - 23.

Christie, P. (1995). Global trends in local contexts: a South African perspective on competency debates, unpublished paper, University of the Witwatersrand, Johannesburg.

Cobb, P., & Steffe, L. P. (1983). The constructivist researcher as teacher and model – builder. *Journal for research in mathematics education, 14,* 83 – 94.

Cobb, P., Wood, T., Yackel, E., & McNeal, B. (1992). Characteristics of classroom mathematics traditions: An interactional analysis. *American Educational Research Journal, 29,* 573 - 604.

Cohen, D. K. & Ball, D. L .(1990). Policy and Practice: An Overview *Educational Evaluation and Policy Analysis, Fall 1990, 12* (3) 233 - 241.

Confrey, J. (1981). Conceptual change analysis: implications for mathematics and curriculum, *Curriculum Inquiry, 11* (3) 243 - 257.

Confrey, J. (1982). Content and pedagogy in secondary schools, *Journal of Teacher Education, 33* (1) 13 - 16.

Confrey, J. (1985*). A constructivist view of mathematics instruction. Part I: A theoretical perspective.* Paper presented at the annual meeting of the American Educational Research Association, Chicago, IL.

Confrey, J. (1986). A critique of teacher effectiveness research in mathematics education, *Journal For Research In Mathematics Education, 17* (5) 347 - 360.

Cuban, L. (1984). *How teachers taught: Constancy and change in American classrooms, 1890 - 1980.* New York: Longman

D'Ambrosio, U. (1985). Ethnomathematics and its place in the history and pedagogy of mathematics. *For the learning of mathematics, 5* (1), 44 - 48.

D'Ambrosio, U. (1990). The role of mathematics education in building a just and democratic society, and its place in the history and pedagogy of mathematics. *For the learning of mathematics, 10* (3), 20 - 23.

Darling-Hammond, L. (1990). Instructional policy into practice: "The power of the bottom over the top," *Educational Evaluation and Policy Analysis, Fall 1990, 12* (3), 233 - 241.

Davis, R. B. (1967). Mathematics teaching with special reference to epistemological problems, *Journal of Research and Development in Education, Fall, 1967.* monograph No. 1, Athens, GA.: University of Georgia.

Davis, P. J. & Hersh, R. (1981). *The mathematical experience.* Boston: Houghton Mifflin.

Delpit, L. D. (1988). The silenced dialogue: power and pedagogy in educating other people's children. *Harvard Educational Review, August, 1988, 58* (3), 280 - 298.

Department of Education (March, 1997). *Outcomes Based Education in South Africa. Background Information for Educators, Discussion Document,.*

Department of Education (1995). *Education and Training in South Africa. First Steps to develop a new system. The White Paper on Education and Training.* Government Gazette No. 196, Pretoria: Government Printers.

Department of Education (1997a). *Outcomes Based Education in South . Africa: Background Information for Educators,* Discussion Document, March 1997.

Department of Education (1997b). *Curriculum 2005, Lifelong Learning for the 21st Century, A User's Guide.* Pretoria: Department of Education.

Department of Education (1997c). *Curriculum 2005, Lifelong learning for the 21st Century.* Pretoria: Department of Education

DES and Welsh Office (1989). *Mathematics in the National Curriculum,* London: HMSO.

Dewey, J. (1902/1992). *The child and the curriculum and the school and*

society. Chicago: University of Chicago Press.
Dewey, J. (1964). The relation of theory to practice in education. In R. Archambault (Ed.). *John Dewey on education* (pp. 313 - 338). Chicago: University of Chicago Press. (Original work published 1904).
Dewey, J. (1966): *Democracy and education.* New York: Free Press
Dieudonné, J. A. (1963). Should we teach modern mathematics ?*American Scientist, 61,* 16-19.
Dieudonné, J. A. (1971). Modern axiomatic mathematics and the foundations of mathematics, In *Great currents of mathematical thought,* Vol. II, 251-266, New York: Dover.
Donn, G. (1995). Education policy in the new South Africa: centre-province relations, *Occasional Papers, No. 56.* Centre for African Studies, Edinburgh University.
Dowling, P. C. & Noss, R. (Eds.) (1990). *Mathematics versus the National Curriculum.* Falmer Press: London.
Eastwood, R. G. (1964). Observations on slogan systems, *Canadian Education and Research Digest* (September, 1964), 208 - 218.
Ensor, P. (1997). School mathematics, everyday life and the NQF: a case of non-equivalence?, *Pythagoras 42* (April), 36 - 44.
Ensor, P. (1996). From Student to Teacher: Continuity or Rupture?, In Robertson S. A. (Ed.) *In Pursuit of Equality,* Kenwyn: Jutas.
Ernest. P. (1987). The impact of beliefs on the teaching of mathematics (pp. 249 - 252). In Ernest. P. (Ed.) *Mathematics teaching: The state of the art,.* Falmer Press: London.
Gierdien, M. F. (1998). A mathematics teacher's conceptual orientation (CO). In A. Olivier and K. Newstead (Eds.). *Proceedings of the 22nd Conference of the International Group for the PME,* vol. 4. Stellenbosch: University of Stellenbosch, p. 333.
Good, T. L., Grouws, D. A. & Ebmeier, H. (1983). *Active mathematics teaching,* New York: Longman
Good, T. L. & Grouws, D. A. (1987). Increasing teachers' understanding of mathematical ideas through inservice training. *Phi Delta Kappan* (June), 778 - 783.
Gordon, M. (1978). Conflict and liberation: Personal aspects of the mathematics experience. *Curriculum Inquiry, 8* (3) 251 - 271.
Government Gazette, no. 18051, of June 6th, 1997, issued by the

South African government printers.
Gowar, N. (1979). *An invitation to mathematics.* Oxford: Oxford University Press.
Grabiner, J. V. (1983). The changing concept of change: The derivative from Fermat to Weierstrass, *Mathematics Magazine, 56* (4), 195 - 206.
Gregg, J. (1995). The tensions and contradictions of the school mathematics tradition. *Journal for Research in Mathematics Education, 26* (5), 467 - 486.
Hawkins, D. (1993). Early roots, *For the learning of mathematics, 13* (1), 15 - 18.
Hawkins, D. (1985). The edge of Platonism, *For the learning of mathematics, 5* (2), 2 - 6.
Hawkins, D. (1974a). *Curriculum research: A policy proposal,* National Institute of Education (DHEW), Washington D.C. Task Force on Curriculum Development.
Hawkins, D. (1974b). Nature, man, and mathematics. In *The informed vision,* (pp. 109-131). New York: Agathon. (Original work published 1972).
Hawkins, D. (1969). I, Thou and It. *Mathematics Teaching, 47,* 22 - 28.
Heuvel-Panhuizen, M. (1998). Realistic Mathematics Education work in progress (Available at **http://www.fi.uu.nl/en/rme/welcome.html**)
Henrici, P. (1972). Reflections of a teacher of applied mathematics, *Quarterly of Applied Mathematics* , (April, 1972), 31-39.
Hyland, T. (1994). *Competence, education and the NVQs: Dissenting perspectives.* London: Cassell.
Jackson, P. W. (1968/1990). *Life in classrooms.* New York, NY: Teachers College Press.
Jackson, P. W. (1986/1989). *The practice of teaching.* New York, NY: Teachers College Press.
Jansen, J. D. (1997). Can Policy Learn? Reflections on "Why OBE Will Fail" In *Outcomes Based Education and the New Curriculum,* West Campus, Wits University.
Jansen, J. D. (1998). Curriculum Reform in South Africa: a critical analysis of Outcomes-based Education, *Cambridge Journal of Education, 28* (3) 321 - 331.
Jessup, G. (1991). *Outcomes: NVQs and the Emerging Model of Education and Training,* London: Falmer Press.

Jessup, G. (1995). Outcome Based Qualifications and the Implications for Learning, In J. Burke (Ed.). *Outcomes, Learning and the Curriculum,* London: Falmer Press.

Julie, C. (1998). The Production of Artefacts As a Goal for School Mathematics? In A. Olivier and K. Newstead (Eds.). *Proceedings of the 22nd Conference of the International Group for the PME,* vol. 1. Stellenbosch: University of Stellenbosch, 49 - 65.

Kallaway P., Kruss, G. Donn, G. and Fataar, A. (Eds.) (1997). *Education after Apartheid: South African Education in Transition,* Cape Town: UCT Press.

Keddie, N. (1984). Classroom knowledge, In A. Hargreaves & P. Woods (Eds.) *Classrooms and staffrooms: the sociology of teachers and teaching* (pp. 108 - 122), Milton Keynes: Open University Press.

Kennedy, M. ; Ball, D. ; McDiarmid, G. (1993). *A study package for examining and tracking changes in teachers' knowledge.* Technical Series 93 - 1. East Lansing, Michigan: National Center for Research on Teacher Learning, Michigan State University.

Khuzwayo, H. (forthcoming) Occupation of our minds: a dominant feature in the history of mathematics education in South Africa, presented at the *First Mathematics Education and Society Conference,* Nottingham, University of Nottingham.

Kilpatrick, J. (1997). Confronting Reform. *American Mathematical Monthly, 104* (10), 955 - 962.

Kline, M. (1970). Logic versus pedagogy, *American Mathematical Monthly, 77* (3), 264 - 281.

Koehler, M. S. & Grouws, D. A. (1992). Mathematics teaching practices and their effects. In Grouws, D. A. (Ed.), *Handbook of Research in Mathematics Teaching and Learning* (pp. 115 - 126). New York, NY: Macmillan.

Komisar, B. P. & McClellan, J. E. (1961). The logic of slogans. In B. O. Smith & R. H. Ennis (Eds.) *Language and concepts in education* (pp. 195 - 214). Chicago: Rand McNally

Kruss, G. (1997a) Restructuring education at the provincial level: the case of the Western Cape, In *Education after apartheid: South African Education in transition,* Kallaway P., Kruss, G. Donn, G. and Fataar, A., (Eds.), Cape Town: UCT Press.

Kruss, G. (1997b) *Teachers, Curriculum 2005 and the Education Policy-*

Making Process, Bellville: University of the Western Cape.

Küchemann, D. (1990). Ratio in the National Curriculum, In P. C. Dowling & R. Noss (Eds.) *Mathematics versus the National Curriculum* (pp. 98 - 111). Falmer Press: London.

Kuhs, T. M., & Ball, D. L. (1986). *Approaches to teaching mathematics: mapping the domains of knowledge, skills and dispositions.* East Lansing: Michigan State University, Center on Teacher Education.

Kulubya, M. M. & Glencross, M. J. (1997). Mathematics achievement and attitudes of senior secondary - school students in Transkei, South Africa, *Psychological Reports, 80,* 915 - 919.

Lakatos, I. (1976). *Proofs and refutations.* Cambridge, U.K.: Cambridge University Press.

Lampert, M. (1984). Teaching about thinking and thinking about teaching, *Journal of Curriculum Studies, 16,* 1 - 18.

Lampert, M. (1985). How do teachers manage to teach? Perspectives on problems in practice, *Harvard Educational Review, 55* (2), 178 - 194.

Lampert, M. (1986). Knowing, doing and teaching multiplication, *Cognition and Instruction, 3,* 305 - 42.

Lampert, M. (1988). What can research tells us about improving quality in mathematics education, *Teaching and Teacher Education, 4* (2) 157 - 170.

Lampert, M. (1988). The teacher's role in reinventing the meaning of mathematical knowing in the classroom. In M. J. Behr, C. B Lacampagne, & M. M. Wheeler (Eds.), *PME-NA: Proceedings of the tenth annual meeting* (pp. 433-480). DeKalb, Il: Northern Illinois University.

Lampert, M. (1990, Spring). When the problem is not the question and the solution is not the answer: Mathematical knowing and teaching, *American Educational Research Journal 27* (1), 26-63.

Lampert, M. (1992) Practices and Problems in Teaching Authentic Mathematics, In Jean-Luc Patry (Ed.), *Effective and responsible teaching,* San Francisco: Jossey- Bass.

Lappan, G., Fey, J., Fitzgerald, W., Friel, S., & Phillips, E. (1998). *The connected mathematics project.* Palo Alto, CA: Dale Seymour Publications.

Laridon, P. (1993) Towards transforming mathematics education in South Africa, *Pythagoras, 31,* 40 - 45.

Lortie, D. (1966). Teacher socialization: The Robinson Crusoe model (pp. 54 - 66), In *The Real World of the Beginning Teacher*, Washington DC.: National Education Association of America

Lortie, D. (1975). *Schoolteacher: A sociological study*. Chicago: University of Chicago Press.

Mitchell, L. (1995) Outcomes and the National (Scottish) Vocational Qualifications (pp. 96 - 113). In *Outcomes, Learning and the Curriculum* , Burke, J. (Ed.) London: The Falmer Press.

McGarth, S. (1997) Reading the NQF: Toward an analytical framework, In *Education after Apartheid: South African Education in Transition*, Kallaway P., Kruss, G. Donn, G. and Fataar, A. (Eds). Cape Town: UCT Press.

McNeil, L. M. (1988) *Contradictions of control: School structure and school knowledge*, New York: Routledge.

NCTM (1989). *Curriculum and evaluation standards for school mathematics*. Reston, VA: National Council of Teachers of Mathematics.

_____ (1991). *Professional standards for teaching mathematics*. Reston, VA: National Council of Teachers of Mathematics.

Nespor, J. (1987). The role of beliefs in the practice of teaching. *Journal of Curriculum Studies, 10,* 317-328.

Nyabanyaba, T. (1999). Whither Relevance? Mathematics Teachers' Discussion of the Use of "Real-Life" Contexts in School Mathematics, *For the learning of mathematics, 19* (3), 10 - 14.

O'Loughlin, M. (1992). Rethinking science education: beyond Piagetian constructivism - toward a sociocultural model of teaching and learning, *Journal of Research in Science Teaching 29* (8), 791 - 820.

Panel to review OBE. (2000, February, 8). *Daily Mail & Guardian: The Teacher*.

Piaget, J. (1970). *Genetic epistemology*. New York: Columbia University Press.

Pimm, D. (1995). *Symbols and meanings in school mathematics*. London: Routledge.

Pólya, G. (1957). *How to solve it* . (2nd. ed.). Princeton, NJ: Princeton University Press.

Pólya, G. (1945; 2nd edition, 1988). *How to solve it* . Princeton, NJ: Princeton University Press.

Popkewitz, T. S. (1988). Institutional issues in the study of school mathematics: curriculum research, *Educational Studies in Mathematics, 19,* 221 - 249.

Popkewitz, T. S. (1979). Educational reform and the problem of institutional life, *Educational Researcher, 8* (3), 3 - 8.

Putnam, R. T., Lampert, M., & Peterson, P. L. (1990). Alternative perspectives on knowing mathematics in elementary schools. In C. B. Cazden (Ed.), *Review of research in education 16,* (pp. 57 - 150). Washington D. C: American Educational Research Association.

Richards, J. (1991). Mathematical discussions. In E. von Glaserfeld (Ed.), *Radical constructivism in mathematics education* (pp. 13 - 51). Dodrecht, The Netherlands: Kluwer.

Rosenthal, W. (1995). *Culturally Relevant Mathematics Pedagogies and Curricula for Flint Children*, A proposal to SUMMA (unpublished).

Scheffler, I. (1964). *The language of education.* Springfield, IL: Charles C. Thomas. (Original work published 1960).

Schifter, D. (1998). Learning mathematics for teaching: From a teachers' seminar to the classroom. *Journal of mathematics teacher education, 1,* 55 - 87.

Schifter, D. (1995). Teachers' changing conception of the nature of mathematics: Enactment in the classroom. In Nelson, B. S. (Ed.) *Inquiry and the Development of Teaching: Issues in the Transformation of Mathematics Teaching.* Newton: MA: Center for the Development of Teaching, Education Development Center, Inc.

Schnepp, M. J., Bachman, K. C., Povlitz, K., Rosenthal, B. & Sandow, D. (April, 2000). Images of high school calculus in flux: Confluent perspectives. A Roundtable presented at the Annual Meeting of the American Educational Research Association, New Orleans.

Schoenfeld, A. (1988). When good teaching leads to bad results: The disasters of "well-taught" mathematics courses. *Educational Psychologist 23* (2), 145 – 166.

Schwab, J. J. (1979). Education and the structure of the disciplines. In I. Westbury and N. Wilkof (Eds.) *Science, curriculum, and liberal education: Selected essays* (pp. 229- 272). Chicago: University of Chicago Press. (Original work published in 1960).

Schwab, J. J. (1959). The "impossible" role of the teacher in progressive education, *School Review, 62,* 139-159

'Scrap Curriculum 2005' (2000, June 6). *Daily Mail and Guardian:The Teacher.* (Available at **http://www.mg.co.za/news/2000jun1/6jun-educ.html**)

Sfard, A. (1995). The development of algebra - confronting historical and psychological perspectives. *Journal of Mathematical Behavior, 14* (1), 15 - 40.

Shulman, L. & Quinlan, K. (1996). The comparative psychology of school subjects. In D. Berliner & R. Calfree (Eds.) *Handbook of educational psychology* 399 - 422, NY: Macmillan.

Shulman, L. (1987). Knowledge and teaching: Foundation for the New Reform, *Harvard Educational Review, 57* (1), 1 - 22.

Shulman, L. (1986). Those who understand: Knowledge growth in teaching, *Educational Researcher, 15* (2), 4 - 14.

Simon, M. A. (1995). Reconstructing mathematics pedagogy from a constructivist perspective. *Journal for Research in Mathematics Education, 26* (2), 114 - 145.

Skemp, R. R. (1987). *The psychology of learning mathematics,* (Revised American edition), Hillsdale, NJ: Lawrence Erlbaum.

Skemp, R. R. (1978). Relational understanding and instrumental understanding. *Arithmetic Teacher, 26* (3) 9 - 15.

Skovsmose, O. (1994). *Toward a critical philosophy of mathematics education,* Kluwer Academic Publishers, Dordrecht, Netherlands.

Soudien, C., & Baxen, J. (1997). Transformation and outcomes-based education in South Africa: Opportunities and challenges, *Journal of Negro Education, 66* (4), 449 - 459.

Steen, L. A. (Ed.) (1990). *On the shoulders of giants.* Washington DC: National Research Council.

Steffe, L. (1991). The constructivist teaching experiment: Illustrations and implications. In E. von Glaserfeld (Ed.), *Radical constructivism in mathematics education* (pp. 177 - 194). Dordrecht, The Netherlands: Kluwer.

Steffe, L., & Gale, J. (Eds.) (1995). *Constructivism in education.* Hillsdale, NJ: Lawrence Erlbaum.

Steffe, L., & Wiegel, H. (1992). On reforming practice in mathematics edcation. *Educational Studies in Mathematics, 23,* 445 - 465

Steffe, L. P. (1990). On the knowledge of mathematics teachers, In Davis, R. B.; C. A. Maher, & N. Noddings (Eds.), *Constructivist views on the teaching and learning of mathematics, Journal for Research on Mathematics Education Monograph No. 4* (167 - 184). Reston, VA: National Council of Mathematics Teachers.

Steffe, L. P. (1986). *Mathematics teacher education in a constructivist framework.* Unpublished manuscript.

Steinbring, H. (1989). Routine and meaning in the mathematics classroom. *For the learning of mathematics, 9* (1), 24 - 33.

Tall. D. (1987). Whither calculus? *Mathematics Teaching, 118,* 50 - 54.

Thom, R. (1973). Modern mathematics: Does it exist?. In A. G. Howson (Ed.), *Developments in mathematical education: Proceedings of the second international congress on mathematical education* (pp. 194-209). Cambridge, U.K.: Cambridge University Press.

Thompson, A. (1984). The relationship of teachers' conceptions of mathematics to instructional practice. *Educational Studies in Mathematics, 15,* 105-127.

Thompson, A. (1992). Teachers' beliefs and conceptions: A synthesis of the research. In Grouws, D. A. (Ed.), *Handbook of Research in Mathematics Teaching and Learning,* New York, NY, Macmillan, 127 - 146.

Thompson, P. (1994). Images of rate and operational understanding of the fundamental theorem of calculus. *Educational Studies in Mathematics, 26,* 229-274.

Toeplitz, O. (1963). *The calculus: A genetic approach.* Chicago: University of Chicago Press. (Original work published in German in 1949)

Usiskin, Z. (1988). Conceptions of algebra and uses of variables. In A. Coxford & A. Schulte (Eds.), *The ideas of algebra, K - 12* . Reston, VA: National Council of Teachers of Mathematics.

von Glaserfeld, E. (1995). A constructivist approach to teaching. In Steffe, L. & J. Gale (Eds.), *Constructivism in education* (pp. 3 - 16). Hillsdale, NJ: Lawrence Erlbaum.

Vithal, R. (1997). Exploring student teachers' understanding of a theoretical perspective in mathematics education, In M. Sanders (Ed.), *Proceedings of the Fifth Annual Meeting of the South African Association of Research in Mathematics and Science Education.* University of the Witswatersrand, January 23 - 27, 331-340

Vithal, R. & Skovsmose, O. (1997). The end of innocence: A critique of "ethnomathematics." *Educational studies in mathematics 34,* 131 - 157.

Volmink, J. (1994). Mathematics by all, in Lerman, S. (Ed.), *Cultural perspectives on the mathematics classroom.* (pp. 51 - 67). Dordrecht, Kluwer.

Wheeler, D. (1980). An askance look at remediation in mathematics. *Outlook, 38,* 41-50.

Whitehead, A. N. (1958). *An introduction to mathematics.* New York: Oxford University Press. (Original work published in 1911.)

Wits, EPU: *Quarterly Review of Education and Training in South Africa, 4* (4)(15 June 1997).

'Worlds Apart' (1998, October 6).*Daily Mail and Guardian: The Teacher.* (Available at **http://www.mg.co.za/news/2000oct1/6oct-educ.html**)

Zevenbergen, R. (1996). Constructivism as a liberal bourgeois discourse. *Educational Studies in Matematics, 31,* 95-113.