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THREE ESSAYS ON UNCERTAINTY AND LEARNING BY ECONOMIC AGENTS

By

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ABSTRACT

THREE ESSAYS ON UNCERTAINTY AND LEARNING BY ECONOMIC AGENTS

By

Hilde Patron

Based on the assumption that agent's decisions affect their understanding of their environments I introduce Bayesian updating of beliefs in three different economic models to study how the agent's ability to learn affects the decisions of rational agents.

In Chapter 1 I study a two-period model with an incumbent firm threatened with entry. Demand is unknown and stochastic, and prices contain statistical information about demand. The incumbent's first period decision affects the informativeness of the price level and through it, the probability of entry. Hence the incumbent can manipulate its quantity to discourage entry. In equilibrium, unless the possible demand functions differ by a constant, the incumbent always manipulates first period output to reduce the probability of entry, i.e. limit prices. In particular, if given its prior information the entrant is currently not entering, then the incumbent limit prices by concealing information from the entrant; if at current beliefs the entrant is entering, then the incumbent limit prices by revealing information.

In Chapter 2 I study a model in which fiscal policy determines that for a short period of time the government must rely exclusively on the income from issuing money, but it is uncertain about the way in which monetary policy influences the public's demand for money. I assume that at any point in time the government uses all the

information available to it to make the best possible decision, and that as new observations become available the government updates its beliefs about the demand for money using Bayes' rule. The three results of the model are: First, the government values more information about the demand for money as this allows it to make a more accurate decision. Second, if the government can affect the informational content of the demand for money (that is, unless the possible demand functions differ by a constant) it will adjust the rate of growth of the money supply to increase information. Third, under some parameter specifications the government induces a hyperinflation to learn about demand.

In Chapter 3 I study the design of incentive contracts for central bankers when the government and the private sector are imperfectly informed about the central banker's preferences. Since the contract affects the inflation rate set by the bankers, which in turn contains statistical information about the banker's loss function, the government can manipulate the contract to increase information. However, bankers have incentives to manipulate the government's and the public's perception of their preferences and hence they manipulate the inflation rate accordingly, which affects the informational content of each contract and the expected variability of the inflation rate. The interaction of these two effects determines the gains and losses from inducing more transparent monetary policies, and whether bankers should be appointed to one or two periods.

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INTRODUCTION

When rational, optimizing agents have to make decisions without fully knowing their environments they must use all the information available to them at any point in time to make the best decision possible, they must revise their beliefs as new information becomes available and they must adjust their actions according to what they learn. Based on these premises I study how the agent's ability to learn, and furthermore to generate information affect the decisions of rational optimizing agents in three different models.

In Chapter 1 I present a two period model of two rational, risk neutral, profit maximizing firms, the incumbent and the entrant, that face an unknown and stochastic demand function. There are two possible states of demand but firms do not know which is the true state. In the first period the incumbent sets a maximizing output, and a price and output are realized. Firms use this information to update their beliefs. If given these beliefs the expected profits of the entrant in the second period are positive the entrant enters and both firms compete in quantities, if they are negative the entrant does not enter and the incumbent stays a monopolist.

Since the incumbent's first period output affects its posterior beliefs, the incumbent can induce more informative outputs. With more information it can make a better informed decision in the second period and increase expected profits. However, the entrant starts out with the same priors as the incumbent does, observes the same variables, and updates beliefs in the same way, thus whatever the incumbent learns, the entrant learns as well. More informative outputs then also affect the expected profitability of the entrant. The incumbent can use this to its advantage to try to discourage entry.

I find that the expected profits of the incumbent firm are decreasing in the probability of entry, and thus that unless the incumbent cannot manipulate the information of the entrant (that is unless each output level is equally informative), it manipulates the first period quantity to deter entry.

In Chapter 2 I study a model in which the government has accumulated large deficits or has large spending needs, but it can only finance itself with the income from money creation (seignorage), for example because of the inefficiency of the tax system combined with low credit ratings.

The government is uncertain about the parameters of the money demand function, and can learn about them in a Bayesian fashion. Knowing the demand function accurately is important because this helps the government determine what proportion of money issue turns into inflation and what into real seignorage revenues.

I find that the government always gains from better knowledge of the demand function and thus it seeks to learn and it adapts to new information. Constant money growth rates are thus suboptimal (unless the possible demand functions differ only by a constant). Furthermore, I find that for some parameter values the government creates a hyperinflation in order to learn about demand.

In Chapter 3 I study the design of linear contracts for central bankers when the central banker's preferences for output over inflation stabilization are not fully known. Bankers are hired for one or two periods and given a contract that stipulates a punishment if the inflation rate is above the socially desirable rate. If they are hired for two periods the contract is revised after the first period.

After the bankers are hired, the private sector forms expectations of inflation for the period and sets wages accordingly. A supply shock is then realized and observed by the bankers who then set the inflation rate.

Based on the inflation rate, which is a noisy observation of the banker's preferences, the government updates its beliefs and adjusts the second period contract (or punishment) accordingly. The private sector also updates its expectations of inflation and wages after observing the inflation rate.

The three results of the model are: First, there exists a parameter space for which the government benefits from more transparent monetary policy (more information). For these parameters however, the bankers might not benefit from more transparency. Moreover, as long as the banker can affect the beliefs of the government it will do so, that is, bankers act strategically. In particular if the banker's value of signaling its type is positive (negative) it will try to increase (decrease) the government's amount of information. Second, the strategic behavior of the central banker has two effects on welfare. On the one hand it increases or reduces the informativeness of the inflation rate, and on the other hand it increases or reduces the variability of the inflation rate (and of output). Third, the interaction of these two effects determines whether the government sets contracts that increase information, and whether bankers should be hired to serve one or two periods.

CHAPTER 1

MONOPOLY EXPERIMENTATION AND ENTRY DETERRENCE: LIMIT PRICING THROUGH LEARNING

1.1 Introduction

Firms often face uncertainty about their environments. For example, they may not fully know their cost structure, the quality of their product, or the demand they face. If so, they might take deliberate measures to increase their information, thereby sacrificing current period profits for more information and thus future higher profits.

When there is more than one firm and when information is a public good the first period decision becomes a complicated issue, as it can be used not only to learn (experimentation) but also to alter the rival firms' perception of profits (limit pricing or predatory pricing). Thus considering two firms introduces all sorts of dynamics into the decision making process in the presence of uncertainty. One particular case is a monopolist faced with the threat of entry, which is the topic of this paper.

I present a two period model of two rational, risk neutral, profit maximizing firms, the incumbent and the entrant, that face an unknown and stochastic demand function. There are two possible states of demand but firms do not know which is the true state.

In the first period the incumbent sets a profit maximizing output, and a price and output are realized. Firms use this information to update their beliefs. If given these beliefs the expected profits of the entrant in the second period are positive, the entrant enters and both firms compete in quantities, if they are negative, the entrant does not enter and the incumbent stays a monopolist.

Since the incumbent's first period output affects its posterior beliefs, the incumbent can induce more informative outputs. With more information it can make a better informed decision in the second period and increase expected profits. However, the entrant starts out with the same priors as the incumbent does, observes the same information, and updates beliefs in the same way, thus whatever the incumbent learns, the entrant learns as well. More informative outputs then also affect the expected profitability of the entrant. The incumbent can use this to its advantage to try to discourage entry.

I find that unless the established firm is not able to affect the informativeness of the price level, it limit prices, that is, it manipulates output to reduce the probability of entry. This result is similar to Fudenberg and Tirole (1986), who found that a firm might price below monopoly to reduce the probability of other firms joining the market. In my paper however, limit pricing can mean increasing or decreasing quantity.

Under some circumstances entry deterrence takes the form of concealing information (less precise beliefs), and under other circumstances it takes the form of revealing information (increasing the chance of bad news). The intuition of this result is as follows: the entry decision is based on what the entrant learns after the first period. If for example the entrant is currently considering entry, new information might reveal some bad news with more precise beliefs. Thus the incumbent can deter entry by increasing the entrant's information.

The form of uncertainty and the information generation process I study in this paper have already been studied in monopoly and duopoly situations. Among the many papers in the literature, two that are particularly important to my study are Mirman, Samuelson and Urbano (1993) and Mirman, Samuelson and Schlee (1994)

Mirman, Samuelson and Urbano (1993) study the conditions under which a monopolist faced with an uncertain and stochastic demand function experiments. They find that as long as the monopolist can affect the informativeness of the price level it experiments in the direction that increases information, except for the case in which the value of information is zero[1].

Mirman, Samuelson and Schlee (1994) study the same problem for a duopolistic market, and find that since the value of information for the duopolist could be negative, information could take the direction of reducing information. Otherwise, firms experiment so as to increase information.

In this paper I study a firm that can be either a monopolist as in Mirman, Samuelson and Urbano (1993) or a duopolist as in Mirman, Samuelson and Schlee (1994). The firm has a belief concerning whether it will be one or the other but does not know for sure which one. The formulation of my model is similar to their models, however I study how the decision to increase or decrease information depends on how the probability of entry is affected with more or less information.

The rest of the paper is organized as follows. In Section 1.2 I review the limit pricing literature. In Section 1.3 I set up the model, and in Section 1.4 describe the sequential equilibrium of the game. In Section 1.5 I solve for the equilibrium of an

[1] One such case is a zero marginal cost monopolist that faces two possible linear demand functions that cross at the quantity axis.

example. In Section 1.6 I study the conditions that determine whether the incumbent limit prices or not, and in Section 1.7 I conclude and discuss several extensions of this paper.

1.2 Related Literature on Limit Pricing

In a seminal paper Bain (1949) suggested that prices in an industry could reveal future pricing policies or the character of the industry. Subsequent papers on limit pricing can be divided into one of these two groups. Papers that study limit pricing as a credible threat (e.g. Bain (1949), Modigliani (1958), Friedman (1979), Salop (1979) and Spulber (1981)), and papers that study limit pricing as revealing something about an industry characteristic known to the established firm but unknown to the entrant.

Among the latter, the preeminent example is Milgrom and Roberts (1982). Other papers are Matthews and Mirman (1983), Harrington (1986) and Jain, Jeitschko and Mirman (2000). My paper is among this second group and for this reason I concentrate on reviewing this strand of the literature.

Milgrom and Roberts (1983) study a signaling model with two firms. Each firm is uncertain about its rival's unit costs, which can be low or high. Moreover, the price set by the incumbent contains information about its costs. Thus a price below the monopoly price could be a signal of low costs. Since the incumbent firm wants the entrant to think it is low cost, limit pricing occurs in equilibrium. Entry however is not necessarily deterred. For example, in a separating equilibrium the entrant is not fooled and entry is not deterred.

Matthews and Mirman (1983) study a similar model of asymmetric information. They assume that firms know all cost functions, but that there is a persistent industry profitability parameter known to the established firm but unknown to the entrant. Moreover, there are unobservable random shocks to demand, which occur after the established firm has made its decision, and which prevent the price from perfectly revealing the industry characteristic. They find that limit pricing always occurs in equilibrium.

Harrington (1986), also in a model of asymmetric information, assumes that the incumbent is uncertain about the entrant's costs and the entrant is uncertain about the costs of both firms. If the two cost functions are correlated, the first period price conveys information about both cost structures. For example, a high price signals that the incumbent firm has high costs and that the entrant is likely to have high costs as well. Decreasing price thus does not necessarily deter entry.

For Milgrom and Roberts (1983) and Matthews and Mirman (1983) limit pricing means charging a price below the short run monopoly price. Unlike them, Harrington finds that if the incumbent's and the entrant's cost are sufficiently correlated, which is reasonable if both firms have access to the same technology, limit pricing can take the form of charging a price above monopoly price.

Following Harrington's result, I will define limit pricing as any action taken by the incumbent firm with the intent to reduce the probably of entry, whether it means increasing or decreasing prices.

Jain, Jeitschko and Mirman (2000) study an incumbent firm threatened with entry, but unlike my paper the incumbent firm must contract with a bank for outside funds. In their model uncertainty arises because the bank and the entrant are not sure about the incumbent's type (costs) but observe a noisy signal of it. They study the design of contracts between the bank and the incumbent firm, and how these are affected when the incumbent is threatened with entry. They show that the bank structures the first period contract so as to reduce the probability of entry, because deterring entry increases the profits of the incumbent and thus the payments that the bank can extract[2].

My model is different from theirs because I do not model contracts between financial institutions and firms, and thus I do not concentrate on learning in principal-agent models. Instead I concentrate on the pure entry deterrence model among producing firms.

In the above papers, as well as in other models of limit pricing, the results depend crucially on how information is modeled. In particular, it is the incentive to preserve its informational advantage which determines the incumbent's incentives to limit price. In my model I relax the assumption of asymmetric information of any kind. I relax this assumption to study the more realistic question of limit pricing when the incumbent is still learning about its environment (demand). This adds richness to the problem because now the incumbent faces a real dilemma: given the public nature of information, if it wants its rival to learn less it has to learn less as well.

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[2] Jain, Jeitschko and Mirman (2000) is a modified version of the paper by Jeitschko and Mirman (2000) who study short term contracting in a principal-agent model where the principal does not know the agent's type. They however do not consider the threat of entry.

I find that even in the absence of an informational advantage, as long as firms do not fully know the characteristics of the market demand, limit pricing almost always occurs in equilibrium[3].

1.3 The Model

I analyze a two period game. There are two rational, profit maximizing, risk neutral firms. In period one there is only a single firm, an incumbent (I) that can produce at a marginal cost k_I. The incumbent sets an output and observes the resulting price. There is a second firm, the entrant (E) that observes the incumbent's output and the resulting price.

In period two the entrant can choose to produce an identical product with marginal cost k_E. The entrant incurs a small entry cost ξ if it chooses to produce. If expected profits are negative, the entrant will not enter and the incumbent remains a monopolist. If it enters, each firm independently and simultaneously sets output. Given the total output, a market clearing price results.

In each period the inverse demand function is given by $p = g(Q,\gamma) + \varepsilon$, where p is the market price, Q is the total quantity produced, and ε is the realization of a random variable, whose distribution is characterized by the density function $f(\varepsilon)$. I assume that the expected value of ε is zero, that $f(\varepsilon)$ is continuous and positive and that it satisfies the monotone likelihood ratio property (MLRP), i.e. $\dfrac{f'(\varepsilon)}{f(\varepsilon)}$ is strictly decreasing in ε.

[3] Only when the incumbent firm cannot affect the informativeness of the price level, that is when the possible demand functions only differ by a constant, limit pricing does not occur in equilibrium

γ is an unknown parameter that can take on two possible values, $\gamma \in \{\bar{\gamma}, \underline{\gamma}\}$, where the upper bar means the "good" demand and the lower, the "bad" demand, i.e, $\forall Q$ $g(Q,\bar{\gamma}) \geq g(Q,\underline{\gamma})$. The function g is non-increasing in Q, and for convenience, twice continuously differentiable in Q. Finally assume that there exist numbers $\bar{Q}$ and $\underline{Q}$ with $0 < \underline{Q} < \bar{Q}$ such that $g(\underline{Q},\gamma) > g(\bar{Q},\gamma) \geq min(k_I, k_E)$, that is assume that there is positive production in equilibrium.

The firms do not know which is the true state of demand. However, in period one both firms have common beliefs about demand. That is, for a given output, both firms expect the same price. In particular, they have a belief ρ^0 that γ takes on the value $\bar{\gamma}$.

In period one, firm I chooses an output Q_I. After the first period quantity is chosen, a random shock is realized (although not observed) and hence a price is realized. Firm I's expected profit in period one is,

$$\pi_I(Q_I, \rho^0) = [\ g(Q_I, \bar{\gamma})\rho^0 + g(Q_I, \underline{\gamma})(1-\rho^0) - k_I]Q_I .$$

Assume that there are no fixed costs, that $\pi_I(Q_I, \rho^0)$ is strictly quasiconcave, and that the function g is such that $Q\ g' + 2g$ is strictly decreasing in Q. These last two assumptions are technical assumptions that are needed in order to ensure the existence of an equilibrium (the proof is in Appendix A).

Both firms observe p and Q_I but not ε. With these commonly observed variables and common priors, firms update their beliefs about demand according to Bayes rule and have the same posterior. Let ρ denote the posterior belief that γ takes on the value $\bar{\gamma}$, then, by Bayes rule,

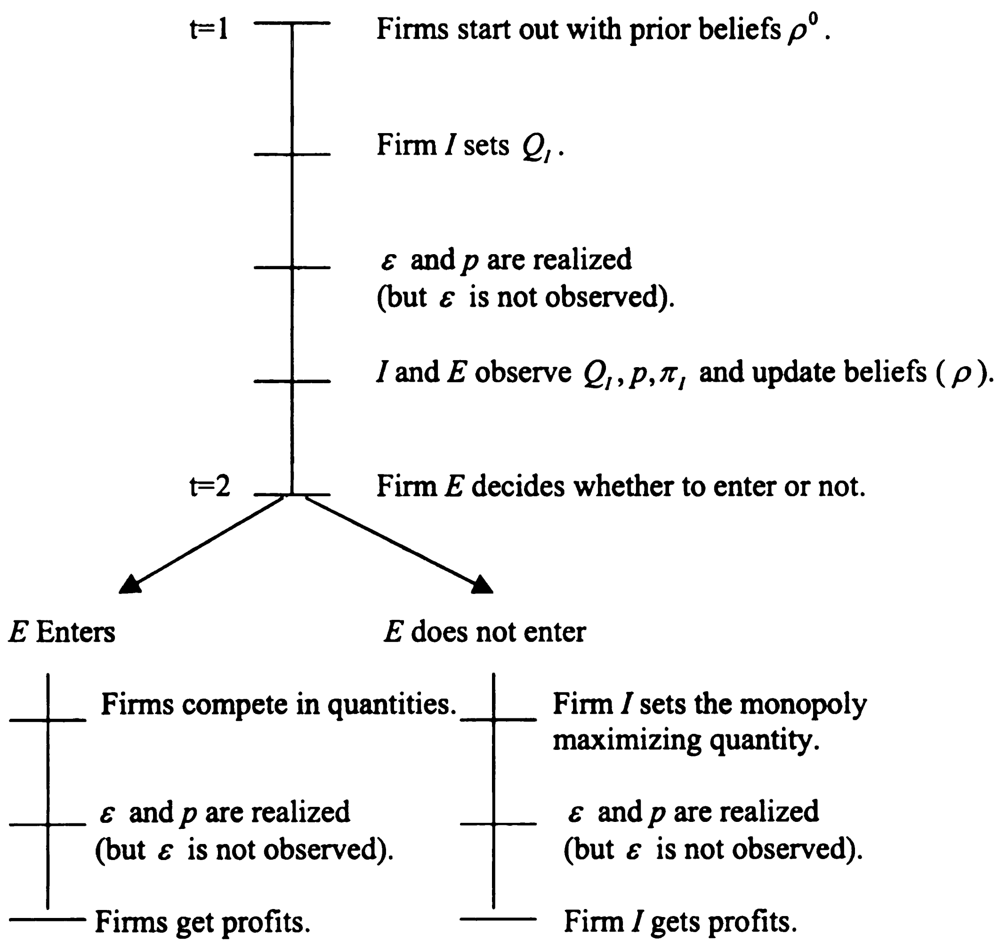
$$\rho(p,Q_I)=\frac{\rho^0\overline{f}}{\rho^0\overline{f}+(1-\rho^0)\underline{f}},$$

where $\overline{f}=f(p-\overline{g})=f(p-g(Q_I,\overline{\gamma}))$ and $\underline{f}=f(p-\underline{g})=f(p-g(Q_I,\underline{\gamma}))$.

In period two, after observing the quantity chosen in period one (Q_I) and the resulting price, firms update their beliefs and the entrant decides whether to enter or not. If it enters, both firms compete in quantities; if it does not enter, the incumbent sets the quantity that maximizes monopoly profits.

The timing of the model can be summarized in the following diagram.

Time Line



1.4 The equilibrium

I am interested in a sequential equilibrium and thus I solve for the second period equilibrium first.

1.4.1 The Second Period Equilibrium

After the first period, having observed price and quantity, firms update their beliefs and the entrant makes a decision of whether to enter the market or not. This decision is made by forecasting the duopoly equilibrium (q_I, q_E), and calculating expected profits (π_I, π_E). Formally, each firm solves the following maximization problem,

$$\underset{q_i}{Max}\ \pi_i(q_I, q_E, \rho)\ \ s.t.\ \ q_j = \underset{q_j}{\arg\max}\ \ \pi_j(q_I, q_E, \rho),\ i = I, E, \tag{1.1}$$

where

$$\pi_I(q_I, q_E, \rho) = [g(q_I + q_E, \bar{\gamma})\rho + g(q_I + q_E, \underline{\gamma})(1-\rho) - k_I]q_I,$$

$$\pi_E(q_I, q_E, \rho) = [g(q_I + q_E, \bar{\gamma})\rho + g(q_I + q_E, \underline{\gamma})(1-\rho) - k_E]q_E - \xi.$$

Let $q_I{}^*(\rho)$ and $q_E{}^*(\rho)$ denote the outputs, as a function of beliefs that maximize the above profits. Finally let total quantity be denoted by $q^*(\rho) \equiv q_I{}^*(\rho) + q_E{}^*(\rho)$.

Let $V_E(\rho)$ denote the resulting value function of the entrant, and let $V_I(\rho)$ denote the resulting value function of the incumbent (if entry occurs), i.e.

$$V_E(\rho) = [g(q^*(\rho), \bar{\gamma})\rho + g(q^*(\rho), \underline{\gamma})(1-\rho) - k_E]q_E{}^*(\rho) - \xi,$$

$$V_I(\rho) = [g(q^*(\rho), \bar{\gamma})\rho + g(q^*(\rho), \underline{\gamma})(1-\rho) - k_I]q_I{}^*(\rho).$$

If the value function of the entrant is positive, i.e. if $V_E(\rho) \geq 0$, E will enter and the two firms will compete in quantities. If $V_E(\rho) < 0$ the entrant will not enter and the incumbent will remain a monopolist. I.e. for some beliefs, $q_E^*(\rho)$ may be zero or close to zero so that profits would be strictly negative, as the entrant must pay an entry cost ξ.

Since the value function of the entrant is non-decreasing in prices (Lemma 1.1 below), the entry decision of the incumbent can be written in relation to a minimum price and a minimum associated posterior belief.

Lemma 1.1: The value function of the entrant is non-decreasing in prices, $\frac{dV_E(\rho)}{dp} \geq 0.$

Proof: Differentiating the value function of the entrant with respect to first period price yields,

$$\frac{dV_E(\rho)}{dp} = \frac{d([g(q^*(\rho),\bar{\gamma})\rho + g(q^*(\rho),\underline{\gamma})(1-\rho) - k_E]q_E^*(\rho) - \xi)}{dp},$$

$$\frac{dV_E(\rho)}{dp} = \left[(g(q^*(\rho),\bar{\gamma}) - g(q^*(\rho),\underline{\gamma}))q_E^*(\rho)\right]\frac{d\rho}{dp}.$$

By assumption $(g(q^*(\rho),\bar{\gamma}) - g(q^*(\rho),\underline{\gamma})) \geq 0$, then the sign of $\frac{dV_E(\rho)}{dp}$ equals the sign of $\frac{d\rho}{dp}$. To find $\frac{d\rho}{dp}$ remember that by Bayes rule $\rho(p,Q_I) = \frac{\rho^0 \bar{f}}{\rho^0 \bar{f} + (1-\rho^0)\underline{f}}$, and thus taking the derivative with respect to p yields,

$$\begin{aligned}\frac{d\rho}{dp} &= \frac{\rho^0 \bar{f}'(\rho^0 \bar{f} + (1-\rho^0)\underline{f}) - \rho^0 \bar{f}(\rho^0 \bar{f}' + (1-\rho^0)\underline{f}')}{(\rho^0 \bar{f} + (1-\rho^0)\underline{f})^2} \\ &= \frac{\rho^0(1-\rho^0)(\bar{f}'\underline{f} - \bar{f}\underline{f}')}{(\rho^0 \bar{f} + (1-\rho^0)\underline{f})^2}\end{aligned},$$

which is non-negative given the monotone likelihood ratio property (i.e. according to the likelihood ratio property $\frac{f'}{f}$ is strictly decreasing in ε, and thus $\frac{\overline{f}'}{\overline{f}} > \frac{\underline{f}'}{\underline{f}} \leftrightarrow \overline{f}'\underline{f} > \overline{f}\underline{f}'$). Thus $\frac{dV_E(\rho)}{dp} \geq 0$.

□

Since entry happens for high enough prices (Lemma 1.1), there is a minimum price $\psi(Q_I)$ such that if the first period price is higher than this price, the entrant enters, if it is lower, the entrant does not enter. The minimum price that induces entry is determined implicitly through Bayes rule as follows,

$$\rho^*(p,Q_I) \equiv \frac{\rho^0 f(\psi(Q_I)-g(Q_I,\overline{\gamma}))}{\rho^0 f(\psi(Q_I)-g(Q_I,\overline{\gamma}))+(1-\rho^0)f(\psi(Q_I)-g(Q_I,\underline{\gamma}))}, \tag{1.2}$$

where $\rho^*(p,Q_I)$ is the minimum value of the posterior belief needed for the entrant to enter, i.e. $\rho^*(p,Q_I)$ is defined by $V_E(\rho^*) \equiv 0$. Alternatively, we can define the minimum price that induces entry $\psi(Q_I)$ by the identity $V_E(\rho(\psi(Q_I),Q_I)) \equiv 0$.

When first period price is not high enough, the entrant does not enter and the incumbent remains a monopolist. Let $V_I^m(\rho)$ denote the value function of the incumbent when there is not entry, i.e.

$$V_I^m(\rho) = [g(q^m(\rho),\overline{\gamma})\rho + g(q^m(\rho),\underline{\gamma})(1-\rho) - k_I]q^m(\rho),$$

where $q^m(\rho)$ is the quantity that maximizes monopoly profits for the incumbent, i.e. $q^m(\rho) \in \arg\max_q \; [g(q,\overline{\gamma})\rho + g(q,\underline{\gamma})(1-\rho) - k_I]q$.

In summary the second period equilibrium is given by the quantity $q^{m}(\rho)$ if $\rho<\rho^{*}(p,Q_{I})$, and by the quantities $q_{I}^{*}(\rho)$ and $q_{E}^{*}(\rho)$ if $\rho\geq\rho^{*}(p,Q_{I})$.

1.4.2 The First Period Equilibrium

In period one the incumbent accounts for the effect of its first period decision on the informativeness of the price level and thus on future profits. Thus in period one the incumbent maximizes first period profits plus the expected discounted value of second period profits

I have to take expectations of second period profits because in period one the posterior belief ρ is a random variable whose distribution depends upon the incumbent's first period output and price. At the time of the maximization problem, p is random variable and thus I have to take expectations of second period profits over all possible prices.

Formally, the incumbent chooses Q_{I} to maximize,

$$\Pi_{I}(Q_{I},\rho^{0})=\left(\begin{array}{c}\pi_{I}(Q_{I},\rho^{0})\\ +\delta\left[\int_{-\infty}^{\psi(Q_{I})}V_{I}^{m}(\rho(p,Q_{I}))h(p,Q_{I})dp+\int_{\psi(Q_{I})}^{\infty}V_{I}(\rho(p,Q_{I}))h(p,Q_{I})dp\right]\end{array}\right)\quad(1.3)$$

where δ is the discount factor, and $h(p,Q_{I})$ is the posterior distribution of first period price implied by posterior beliefs and by the shock ε.

To find $h(p,Q_{I})$ let $\bar{\varepsilon}(p,Q_{I})$ be the value of ε that must result if the period one price is p, the first period quantity is Q_{I}, and the true state of demand is $\bar{\gamma}$; and let $\underline{\varepsilon}(p,Q_{I})$ be the value of ε that must result if the period one price is p, the first period quantity is Q_I, and the true state of demand is $\underline{\gamma}$. The density of p can be written as $h(p,Q_I) = \rho^0 \,\overline{f} + (1-\rho^0)\underline{f}$.

Intuitively, the incumbent firm maximizes first period profits plus the discounted expected value of second period profits. Expected second period profits are expected duopoly profits conditional on entry (i.e. conditional on p being at least $\psi(Q_I)$), plus expected monopoly profits conditional on no entry (i.e. conditional on p being less than $\psi(Q_I)$).

Let $Q_I{}^*(\rho^0)$ be the resulting first period equilibrium quantity. The existence of equilibrium is proven in Lemma 1.2 and Proposition 1.1. For continuity of exposition I prove these statements in Appendix A.

Lemma 1.2: If a period two equilibrium exists, then that equilibrium is unique and at least one firm produces a positive quantity[4].

Proposition 1.1: Let g(Q,γ) be defined and continuous on $\mathbb{R}_+$ for $\gamma \in \{\overline{\gamma}, \underline{\gamma}\}$ and let there be $\hat{Q}$ such that $g(\hat{Q},\gamma) = 0$ for $\gamma \in \{\overline{\gamma}, \underline{\gamma}\}$. Then a (possibly mixed strategy) sequential equilibrium exists[5].

In the next section I develop an example to illustrate the model and the equilibrium.

1.5 An Example

For illustration, the following linear demand example is used throughout the paper. Consider the following two linear demand functions:

---

[4] Mirman, Samuelson and Schlee (1994), Lemma 2, page 368.

[5] Mirman, Samuelson and Schlee (1994), Proposition 1, page 370.

$$p = 20 - Q + \varepsilon$$
$$p = 20 - 1.5Q + \varepsilon$$

Since for all $Q > 0$ and all ε, $20 - Q + \varepsilon \geq 20 - 1.5Q + \varepsilon$, then $p = 20 - Q + \varepsilon$ is the "good demand". Graphically these two demand functions are shown in Figure 1.1.

Assume that ε is uniformly distributed in the interval [-15,15]. The density function of ε is thus given by $f(\varepsilon) = \dfrac{1}{15-(-15)} = \dfrac{1}{30}$, and the expected value of ε by

$E(\varepsilon) = \int_{-15}^{15} \dfrac{\varepsilon}{30} d\varepsilon = \dfrac{\varepsilon^2}{60}\Big|_{-15}^{15} = \dfrac{(15)^2 - (-15)^2}{60} = 0$[6].

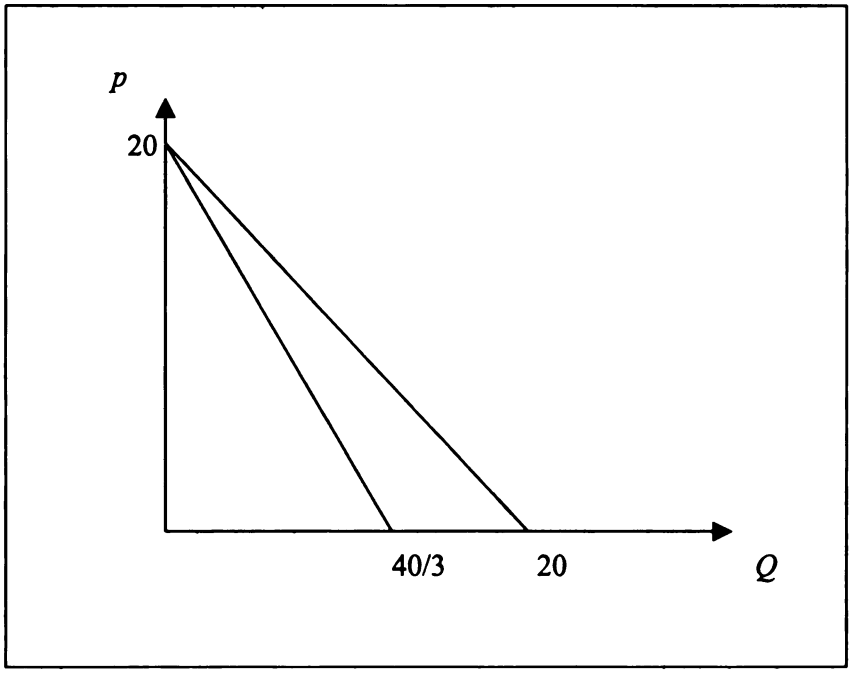


Figure 1.1. Expected Demand Functions of the Example

[6] In general, if ε is uniformly distributed in the interval $[a,b]$, its density function is given by $f(\varepsilon) = \dfrac{1}{b-a}$ and its expected value by $E(\varepsilon) = \int_a^b \dfrac{\varepsilon}{b-a} d\varepsilon = \dfrac{\varepsilon^2}{2(b-a)}\Big|_a^b = \dfrac{(b)^2-(a)^2}{2(b-a)} = \dfrac{(b-a)(b+a)}{2(b-a)} = \dfrac{a+b}{2}$.

This example violates two of the assumptions of the model: first, that the shock has unbounded support, and second that its density function is differentiable. Although these assumptions are violated, the example retains their general implications, namely that there is no learning with probability one after the first period, and that higher prices increase the beliefs that demand is the good one (that is that beliefs are monotonic).

Finally assume that $\delta = 1$, $k_I = 4$, $k_E = 8$, $\xi = 5.5$, and $\rho^0 = 0.5$.

To find the equilibrium I first find the second period equilibrium. If there is no entry, the incumbent chooses q^m to maximize monopoly profits, which are given by $\pi_I^m(\rho) = \left[(20 - q^m)\rho + \left(20 - 1.5q^m\right)(1-\rho) - 4\right]q^m$, which gives maximizing quantity $q_m{}^*(\rho) = \dfrac{8}{1.5 - 0.5\rho}$, and profits $V_I^m(\rho) = \dfrac{64}{1.5 - 0.5\rho}$.

If the entrant enters, firms compete in quantities, and thus solve (1.1), which in the example is given by

$$\begin{gathered}\underset{q_I}{Max}\,[\rho(20 - q_I - q_E) + (1-\rho)(20 - 1.5(q_I + q_E)) - 4]q_I \\ \underset{q_E}{Max}\,[\rho(20 - q_I - q_E) + (1-\rho)(20 - 1.5(q_I + q_E)) - 8]q_E - 5.5\end{gathered},$$

which gives the Cournot equilibrium quantities $q_I{}^*(\rho) = \dfrac{20}{3(1.5 - 0.5\rho)}$, $q_E{}^*(\rho) = \dfrac{8}{3(1.5 - 0.5\rho)}$, and value functions for the incumbent, $V_I(\rho) = \dfrac{400}{9(1.5 - 0.5\rho)}$, and for the entrant $V_E(\rho) = \dfrac{64}{9(1.5 - 0.5\rho)} - 5.5$.

To find the incumbent's expected second period value function I need to integrate second period profits over all possible posterior beliefs. With a uniform distribution, there are only three possible posterior beliefs, zero, the same prior, or one, as follows. If the true demand is given by $p = 20 - 1.5Q_I + \varepsilon$, then the first period price is within the following interval $p \in [20 - 1.5Q_I - 15, 20 - 1.5Q_I + 15]$; If the true demand is given by $p = 20 - Q_I + \varepsilon$, then the first period price is within the following interval $p \in [20 - Q_I - 15, 20 - Q_I + 15]$. If the first period price is above $20 - 1.5Q_I - 15$ but strictly below $20 - Q_I - 15$, then the firms learn that the true demand is the "bad" demand. If the first period price is strictly above $20 - 1.5Q_I + 15$, then the firms learn that the true demand is the "good" one. Formally posterior beliefs are given as follows,

$$\rho = \begin{cases} 0 & \text{if } \ 20 - 1.5Q_I - 15 \le p < 20 - Q_I - 15 \\ \rho^0 = 0.5 & \text{if } \ 20 - Q_I - 15 \quad \le p \le 20 - 1.5Q_I + 15, \\ 1 & \text{if } \ 20 - 1.5Q_I + 15 < p \le 20 - Q_I + 15 \end{cases}$$

and each posterior happens according to the following distribution function[7],

$$h(p,q) = \begin{cases} \dfrac{1-\rho^0}{30} & \text{if } \ 20 - 1.5Q_I - 15 \le p < 20 - Q_I - 15 \\ \dfrac{1}{30} & \text{if } \ 20 - Q_I - 15 \le p \le 20 - 1.5Q_I + 15 \\ \dfrac{\rho^0}{30} & \text{if } \ 20 - 1.5Q_I + 15 < p \le 20 - Q_I + 15 \end{cases}.$$

Since the value function of the entrant $V_E(\rho)$ is only positive for values of ρ above 0.41, if the entrant believes that ρ is below this value, he will not enter. That is $\rho^* = 0.41$ and the minimum price that induces entry is given by,

[7] To find the density function note that $h(p,q) = \rho^0 \overline{f} + (1-\rho^0)\underline{f} = 0.5(\overline{f} + \underline{f})$, and that for a uniform distribution function $\overline{f} = \underline{f} = \dfrac{1}{(15-(-15))} = \dfrac{1}{30}$, and thus $h(p,q) = 0.5\left(\dfrac{1}{30} + \dfrac{1}{30}\right)$. Finally, from Bayes' rule, if $\rho = 0$ then $0 = \dfrac{\rho^0 \overline{f}}{\rho^0 \overline{f} + (1-\rho^0)\underline{f}} \Leftrightarrow \rho^0 \overline{f} = 0$; and if $\rho = 1$, then $1 = \dfrac{\rho^0 \overline{f}}{\rho^0 \overline{f} + (1-\rho^0)\underline{f}} \Leftrightarrow (1-\rho^0)\underline{f} = 0$.

$\psi(Q_I) = 20 - Q_I - 15 = 5 - Q_I$. Therefore the entrant's decision can be represented with the following rule,

$$\textit{Entry Rule}\begin{cases} \textit{Entry} & \textit{if} \quad \rho \in \{\rho^0, 1\} \\ \textit{No Entry} & \textit{if} \quad \rho = 0 \end{cases},$$

or by,

$$\textit{Entry Rule}\begin{cases} \textit{Entry} & \textit{if} \quad p \geq 5 - Q_I \\ \textit{No Entry} & \textit{if} \quad \textit{Otherwise} \end{cases}.$$

For prices above $5 - Q_I$ the incumbent expects duopoly profits $V_I(\rho)$, and for prices below $5 - Q_I$, it expects monopoly profits $V_I^m(\rho)$. Second period expected profits are thus given by,

$$W(Q_I) = \begin{pmatrix} \int_{20-1.5Q_I+15}^{20-Q_I+15} V_I(\rho=1)h(\rho=1)dp + \int_{20-Q_I-15}^{20-1.5Q_I+15} V_I(\rho=0.5)h(\rho=0.5)dp \\ + \int_{20-1.5Q_I-15}^{20-Q_I-15} V_I^m(\rho=0)h(\rho=0)dp \end{pmatrix},$$

thus,

$$W(Q_I) = \begin{pmatrix} \int_{35-1.5Q_I}^{35-Q_I} \frac{400}{9(1.5-0.5(1))}\frac{1}{60}dp + \int_{5-Q_I}^{35-1.5Q_I} \frac{400}{9(1.5-0.5(0.5))}\frac{1}{30}dp \\ + \int_{5-1.5Q_I}^{5-Q_I} \frac{64}{1.5-0.5(0)}\frac{1}{60}dp \end{pmatrix} = 35.6 + 0.13Q_I.$$

Expected second period profits are an increasing function of first period quantity, intuitively because higher quantities both increase information and reduce entry. (More on this in the next section.)

To obtain the sequential equilibrium the incumbent chooses quantity Q_I to maximize $[0.5(20-Q_I)+0.5(20-1.5Q_I)-4]Q_I+W(Q_I)$, which yields the first period equilibrium quantity Q_I *=6.45.

1.6 If the Incumbent Could, Would it try to Discourage Entry?

So far I described the model and equilibrium, and illustrated it with an example. Now I address the following natural questions. Given that this is a model with a threat of entry, how does the incumbent react to the threat of entry? That is, does the incumbent limit price? What does limit pricing look like? Does the incumbent limit price by concealing information from the entrant? Or instead, by increasing the entrant's knowledge of the industry? And exactly, what is the definition of limit pricing?

1.6.1 General Discussion

In the first period the incumbent sets a quantity Q_I, which affects the function $g(Q_I,\gamma)$, and through it the price level $p=g(Q_I,\gamma)+\varepsilon$. Although the incumbent or the entrant does not observe the shock ε, the quantity Q_I affects the possible distribution functions of prices $\overline{f}(\varepsilon)=f(p-g(Q_I,\overline{\gamma}))$ and $\underline{f}(\varepsilon)=f(p-g(Q_I,\underline{\gamma}))$, and through them the minimum price that induces entry $\psi(Q_I)$, implicitly defined by,

$$\rho^*(p,Q_I)\equiv\frac{\rho^0 f(\psi(Q_I)-g(Q_I,\overline{\gamma}))}{\rho^0 f(\psi(Q_I)-g(Q_I,\overline{\gamma}))+(1-\rho^0)f(\psi(Q_I)-g(Q_I,\underline{\gamma}))},$$

posterior beliefs ρ,

$$\rho(p,Q_I)=\frac{\rho^0\overline{f}}{\rho^0\overline{f}+(1-\rho^0)\underline{f}},$$

their distribution,

$$h(p,Q_I) = \rho^0 \,\overline{f} + (1-\rho^0)\underline{f},$$

and therefore the probability of entry, G, formally given by $G(Q_I) = \int_{\psi(Q_I)}^{\infty} h(p,Q_I)dp$.

If the posterior distribution of beliefs is riskier in the Rothschild-Stiglitz sense, then information increases[8]. Thus if changes in Q_I increase information (lead to more accurately beliefs) and through more information deter entry (reduce the probability of entry), then the incumbent limit prices by enhancing the entrant's information. If changes in Q_I decrease information and through less information deter entry, then the incumbent limit prices by concealing information from the entrant.

In the example of Section 1.5, entry occurs for $\rho \geq \rho^* = 0.41$, or prices above $\psi(Q_I) = 5 - Q_I$. Thus the probability of entry is given by,

$$G(Q_I) = \int_{20-Q_I-15}^{20-1.5Q_I+15} \frac{1}{2\varepsilon}dp + \int_{20-1.5Q_I+15}^{20-Q_I+15} \frac{\rho^0}{2\varepsilon}dp = 1 - \frac{1}{120}Q_I.$$

Since the entrant enters for prices above $\psi(Q_I) = 5 - Q_I$, an increase of one in first period quantity decreases the minimum price that induces entry by one. All else constant, there are more price observations that induce entry, and thus one would expect an increase in the probability of entry. The probability of entry however goes from $G(Q_I) = 1 - \frac{1}{120}Q_I$ to $G(Q_I+1) = 1 - \frac{1}{120}(Q_I+1)$, which means that G decreases by $\frac{1}{120}$,

[8] This comes from the definition of a more informative experiment as defined by Blackwell (1951) and (1953). An experiment is a pair of measures $[f(p-g(Q_I,\underline{\gamma})), f(p-g(Q_I,\overline{\gamma}))]$, one for each state of nature that gives the distribution of the first period price. An experiment $z_1 = [f(p-g(Q_I{}'',\underline{\gamma})), f(p-g(Q_I{}'',\overline{\gamma}))]$ is *more informative* than an experiment $z_2 = [f(p-g(Q_I{}',\underline{\gamma})), f(p-g(Q_I{}',\overline{\gamma}))]$, if $\int \chi(\rho(Q_I{}'',p))h(p,Q_I)dp \geq \int \chi(\rho(Q_I{}',p))h(p,Q_I)dp$ for every continuous, convex function $\chi(\rho(Q_I,p)):[0,1] \rightarrow \Re$.

not increases by one. In this example, if the incumbent wants to limit price it must increase quantity.

Note that not all changes in Q_I that affect information necessarily affect the probability of entry, hence, when studying limit pricing I must abstract from everything other than changes in $\overline{f}$ and $\underline{f}$ that translate into changes in the probability of entry. In particular I will say that *if the incumbent sets a quantity Q_I* so as to reduce the probability of entry, then the incumbent limit prices.*

Traditionally, the standard definition in the literature is that the incumbent limit prices if it charges a price below the short run monopoly price with the intent to discourage other firms from joining the market (as in Bain (1949), Matthews and Mirman (1983), or Harrington (1986) for example). I do not want to use this definition however, because as explained before the difference of my model and the short run monopoly might include manipulation of information that does not affect the likelihood of other firms joining the market. That is, since information may be of value to the incumbent firm, the firm might manipulate information to make a better informed decision even if this does not affect the entrant's decision to enter or not. It seems natural thus to define limit pricing by its purpose, by any action aimed at reducing the likelihood of entry.

1.6.2 Conditions Under which the Incumbent Limit Prices

To study the incentives of the incumbent to reduce the probability of entry I compare its maximization problem with a hypothetical incumbent that takes the probability of entry as constant.

An incumbent firm that takes the probability entry as given chooses Q_I to maximize equation (1.3) subject to $\frac{dG(Q_I)}{dQ_I}=0$. Using Leibniz rule to differentiate $G(Q_I)$ with respect to Q_I yields[9],

$$\frac{dG(Q_I)}{dQ_I} = -\psi'(Q_I)h(\psi(Q_I),Q_I) + \int_{\psi(Q_I)}^{\infty} \frac{dh(p,Q_I)}{dQ_I}dp.$$

Hence the incumbent that takes entry as given solves the following problem,

$$\begin{aligned} &\underset{Q_I}{Max}\ \pi_I(Q_I,\rho^0)+\delta\left[\int_{-\infty}^{\psi(Q_I)} V_I^m(\rho(p,Q_I))h(p,Q_I)dp + \int_{\psi(Q_I)}^{\infty} V_I(\rho(p,Q_I))h(p,Q_I)dp\right] \\ &s.t.\ \frac{dG(Q_I)}{dQ_I}=0 \Leftrightarrow \psi'(Q_I)h(\psi(Q_I),Q_I) = \int_{\psi(Q_I)}^{\infty} \frac{dh(p,Q_I)}{dQ_I}dp \end{aligned}.$$

That is, an incumbent that takes the probability of entry as given maximizes first period profits plus the discounted expected value of second period profits, subject to a fixed probability of entry[10]. Let Q_I^{FE} be the solution to this problem.

In general, given some assumptions that introduced next I can show that the incumbent threatened with entry will always limit price. This result is presented in

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[9] According to Leibniz' rule if $y(x)=\int_{\alpha(x)}^{\beta(x)} z(x,t)dt$, $\frac{dy}{dx}=\frac{d\beta}{dx}z(x,\beta(x))-\frac{d\alpha}{dx}z(x,\alpha(x))+\int_{\alpha(x)}^{\beta(x)}\frac{\partial z}{\partial x}dt$.

[10] The probability of entry could change in two ways, through the minimum price that induces entry and through the distribution of prices above this minimum price. When assuming that the probability of entry is fixed, the correct assumption is that $\psi'(Q_I)h(\psi(Q_I),Q_I)=\int_{\psi(Q_I)}^{\infty}\frac{dh(p,Q_I)}{dQ_I}dp$, but not necessarily that $\psi'(Q_I)=0$ and $\int_{\psi(Q_I)}^{\infty}\frac{dh(p,Q_I)}{dQ_I}dp=0$. That is, the incumbent limit prices if it engages in any behavior such that $\psi'(Q_I)h(\psi(Q_I),Q_I)\neq\int_{\psi(Q_I)}^{\infty}\frac{dh(p,Q_I)}{dQ_I}dp$.

Lemma 1.2, and the proof, in Appendix A. Before I present the lemma however, I will present and discuss the following assumptions.

I will assume for the remainder of the paper that if at current beliefs the entrant is entering, i.e. if $\rho^0 > \rho^*$, then increases in information reduce the probability of entry; if at current beliefs the entrant is not entering, i.e. if $\rho^0 < \rho^*$, then increases in information increase the probability of entry[11]. Formally these assumptions imply the following:

Assumption A1.1: In any of the following two scenarios the probability of entry is non-increasing in quantity, i.e. $\frac{dG(Q_I)}{dQ_I} \leq 0$.

A1.1.1 If increasing quantity decreases information, $(\bar{g}' - \underline{g}') < 0$, and if at current beliefs the entrant is not entering, $\rho^0 < \rho^*$.

A1.1.2 If increasing quantity increases information, $(\bar{g}' - \underline{g}') > 0$, and if at current beliefs the entrant is entering, $\rho^0 > \rho^*$.

Assumption A1.2: In any of the following two scenarios the probability of entry is non-decreasing in quantity, i.e. $\frac{dG(Q_I)}{dQ_I} \geq 0$.

A1.2.1 If increasing quantity decreases information, $(\bar{g}' - \underline{g}') < 0$, and if at current beliefs the entrant is entering, $\rho^0 > \rho^*$.

[11] This is an assumption and not a result of the model. For example consider the example from Section 1.5. If prior beliefs had been assumed to be given by $\rho^0 = 0.1$ then at prior beliefs the entrant would not enter and thus increases in information should increase the probability of entry. However the probability of entry in this case is given by $G(Q_I) = 1 - \frac{0.9}{60} Q_I$, and hence $\frac{dG(Q_I)}{dQ_I} < 0$. Hence mathematically it is possible to construct models that violate thia assumption. Since these cases do not make intuitive sense I will rule them out.

A1.2.2 If increasing quantity increases information, $(\bar{g}'-\underline{g}')>0$, and if at current beliefs the entrant is not entering, $\rho^0 < \rho^*$.

The term $(\bar{g}'-\underline{g}')$ gives the distance between the means of the two possible distribution functions of prices, and it is thus a measure of the amount of information. If $(\bar{g}'-\underline{g}')$ is positive, it means that as quantity increases, the distance between the means of the two possible distribution functions of prices increases, and thus information increases (as in Figure 1.2).

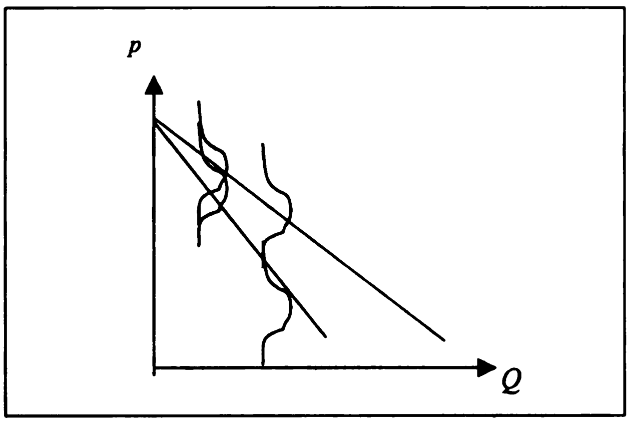

Figure 1.2. Increasing Quantity Increases Information

If $(\bar{g}'-\underline{g}')$ is negative, as quantity increases the two possible posterior distributions of prices move closer together and become more undistinguishable (Figure 1.3). Increasing quantity thus decreases information.

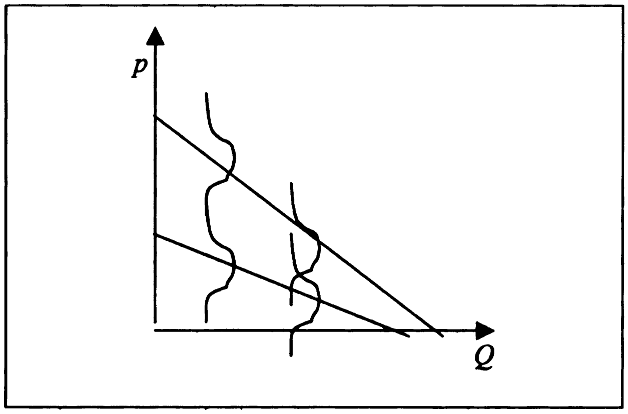

Figure 1.3. Increasing Quantity Decreases Information

The role of $(\bar{g}'-\underline{g}')$ in the assumptions (and in Lemma 1.3) is twofold. First, it determines whether limit pricing means increasing or decreasing quantity. Second it establishes a relation between the entry decision at priors and at posterior beliefs as follows. For example, if $(\bar{g}'-\underline{g}')\leq 0$ (as in Figure 1.3), increases in quantity decrease information. If at prior beliefs the entrant does not enter (i.e. $\rho^0<\rho^*$), increases in quantity do not reveal any new information, and thus should not change the decision of the entrant. Decreasing quantity however might reveal some good news with more precise beliefs and increase the profitability of the entrant. One would thus expect the probability of entry to increase as quantity decreases, i.e. $\frac{dG(Q_I)}{dQ_I}\leq 0$.

Now assume that at current beliefs the entrant would enter (i.e. $\rho^0>\rho^*$). Decreasing quantity, which increases information, might reveal some bad news for the entrant with more precise beliefs. One would expect the probability of entry to decrease as quantity decreases, $\frac{dG(Q_I)}{dQ_I} \geq 0$.

These assumptions, although arbitrary, have intuitive appeal. This is a model where the only reason why an entrant's profitability might increase or decrease is through the beliefs of the entrant. Thus the only reason why an entrant might change its mind about entering or not entering is if it learns something hurtful or helpful. If given priors the entrant is entering, no news should not change its mind; bad news might.

Lemma 1.3: Given assumptions A1.1 and A1.2, an incumbent firm threatened with entry limit prices, that is $G(Q_I*) \leq G(Q_I^{FE})$. In particular,

(a) If reductions in quantity increase information, and through more information increase the probability of entry $\left((\bar{g}' - \underline{g}') < 0 \quad and \quad \frac{dG(Q_I)}{dQ_I} \leq 0 \right)$, the incumbent sets a quantity higher than the incumbent that takes entry as given, i.e. $Q_I* \geq Q_I^{FE}$. Since this reduces the probability of entry and decreases information, then the incumbent limit prices by concealing information from the entrant.

(b) If increases in quantity increase information, and through more information increase the probability of entry $\left((\bar{g}' - \underline{g}') > 0 \quad and \quad \frac{dG(Q_I)}{dQ_I} \geq 0 \right)$, then the incumbent sets a quantity lower than the incumbent that takes entry as given, i.e.

$Q_I{*} \leq Q_I^{FE}$. Since this reduces the probability of entry and decreases information, then the incumbent limit prices by concealing information from the entrant.

(c) If increases in quantity increase information, and through more information reduce the probability of entry $\left((\overline{g}' - \underline{g}') > 0 \quad and \quad \frac{dG(Q_I)}{dQ_I} \leq 0 \right)$, then the incumbent sets a quantity higher than the incumbent that takes entry as given, i.e. $Q_I{*} \geq Q_I^{FE}$. Since this reduces the probability of entry and increases information, then *the incumbent limit prices by revealing information to the entrant.*

(d) If decreases in quantity increase information, and through more information reduce the probability of entry $\left((\overline{g}' - \underline{g}') < 0 \quad and \quad \frac{dG(Q_I)}{dQ_I} \geq 0 \right)$, the incumbent sets a quantity lower than the incumbent that takes entry as given, i.e. $Q_I{*} \leq Q_I^{FE}$. Since this reduces the probability of entry and increases information, then *the incumbent limit prices by revealing information to the entrant.*

Proof: In Appendix A.

Lemma 1.3 states that the incumbent threatened with entry manipulates first period quantity to reduce the probability of entry. The only case in which the incumbent does not limit price is when the demand functions differ only by a constant because in this case the incumbent cannot affect the informativeness of the price level. The lemma also characterizes what limit pricing looks like. In particular, if the incumbent can reduce the probability of entry by concealing information from the entrant $\left(i.e. \quad (\bar{g}' - \underline{g}') \frac{dG(Q_I)}{dQ_I} \geq 0 \right)$, the incumbent limit prices by decreasing information; If enhancing the entrant's knowledge of demand reduces the probability of entry $\left(i.e. \quad (\bar{g}' - \underline{g}') \frac{dG(Q_I)}{dQ_I} \leq 0 \right)$, the incumbent limit prices by increasing the entrant's information.

Of cases *a* through *d* of Lemma 1.3, case *a* is the traditional result of the limit pricing literature. That is, in case *a* the incumbent limit prices by increasing quantity, which conceals information from the entrant. Unlike the previous literature however, this implies that the incumbent's information is reduced as well.

In case *b* the incumbent limit prices by reducing quantity, but also by concealing information. Harrington's (1986) result could be adapted to fit this case. For Harrington, if the entrant and the incumbent's costs are sufficiently correlated, higher quantities might reveal good news to the entrant, namely that it is likely to be the low cost type. The incumbent wants to limit price by setting a lower quantity and thus concealing this information from the entrant. Once again, unlike Harrington's result, reducing the entrant's information also reduces the incumbent's.

More surprisingly however is the result that limit pricing can mean revealing information, cases *c* and *d*. Since the incumbent uses information as a strategic tool, and since more information can mean bad news to the entrant with more precise beliefs, limit pricing can very well mean enhancing the entrant's knowledge of demand.

The example of section 1.5 corresponds to case *c* (Figure 1.1): increasing quantity decreases the probability of entry $\frac{dG(Q_I)}{dQ_I} = -\frac{1}{120} < 0$, and thus by Lemma 1.3 the incumbent limit prices by increasing quantity, i.e. $Q_I{*} > Q_I^{FE}$. Since increasing quantity increases information, i.e. since $(\bar{g}' - \underline{g}') = -1 - (-1.5) = 0.5 > 0$, then the incumbent limit prices by increasing the entrant's information about demand. The intuition of this result is as follows. At the current beliefs the entrant enters, i.e. $\rho^0 = 0.5 > \rho{*} = 0.41$. By increasing output the incumbent increases the chance of bad news for the entrant, and thus the probability of entry goes down.

As an example of case *d*, assume that the two possible demand functions are given by $p = 30 - Q + \varepsilon$ and $p = 20 - \frac{2}{3}Q + \varepsilon$ (Figure 1.4). Assume that ε is uniformly distributed in the interval [-20,20], that $\delta = 1$, $k_I = 4$, $k_E = 8$, $\xi = 20$, and $\rho^0 = 0.7$, which yields the first period sequential equilibrium is $Q{*} = 10.09$. In this example the minimum posterior that induces entry is $\rho{*} = 0.4$, thus at prior beliefs the entrant is entering. The probability of entry is given by $G(Q_I) = 0.23 + .01Q_I$, and thus a reduction in quantity, which increases information, reduces the probability of entry. According to Lemma 1.3 then the incumbent wants to increase information to increase the chance of bad news, $Q^{FE} > Q*$. Thus the incumbent limit prices by reducing quantity, which increases information.

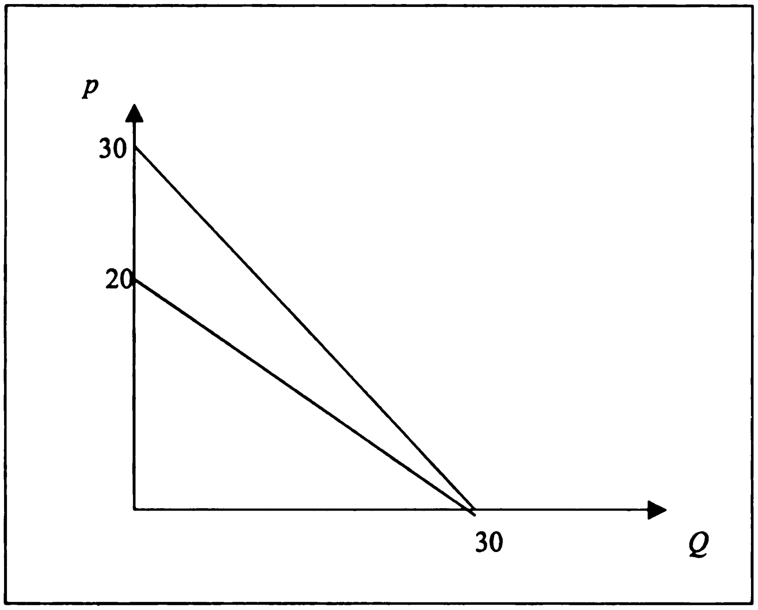

Figure 1.4. Expected Demand Functions: Case (d)

1.7 Conclusions

Going back to the idea first proposed by Bain (1949) that prices embody information about the industry I study limit pricing under a set up in which prices clearly embody statistical information about the demand function. Firms do not know the demand function and learn about it through the actions of an incumbent firm that can increase or reduce the informational content of the price level to limit price.

The incumbent's costs are fully known and so its actions only reveal information about the market, both to itself and to others observing the market and contemplating entry, which implies that if it wants to learn, its rival learns as well; if it wants its rival to learn less, it learns less itself.

I find that in equilibrium unless the demand functions differ by a constant the incumbent firm limit prices. More importantly however is the result that limit pricing can mean revealing information to the entrant. In particular, if given current beliefs the entrant is entering the market, more information might reveal some bad news with more precise beliefs and thus reduce the probability of entry. In this case the incumbent limit prices by increasing information.

It would be interesting to study the incentives to limit price in the presence of symmetric information, such as in this model, but uncertainty about different things, such as the cost functions of firms, or a general industry parameter. I would expect the results to hold: even if information is symmetric, the incumbent can manipulate the probability of good or bad news to try to deter entry.

I would expect the results to also hold in a model of information jamming such as Fudenberg and Tiroles's (1986). That is, if the entrant cannot observe the first period quantity, only the price, then it is said that the incumbent limit prices by jamming its rival's information. Assuming that the entrant cannot observe quantity implies that the entrant and the incumbent update beliefs in a different manner and thus learn differently. The intuition behind the results should not change however. That is, even if the firms learn in different ways, the incumbent has incentives to jam the entrant's learning so as to make entry appear less profitable.

Allowing for different initial priors (asymmetric information) should also extend the results of this model, although once again, the intuition behind the results should not change. That is, even if firms have different information to start with, as long as both the incumbent and the entrant are not fully informed about demand, the incumbent still has incentives to manipulate (or jam) the entrant's learning so as to make entry appear less profitable.

CHAPTER 2

UNCERTAINTY, LEARNING, AND SEIGNORAGE MAXIMIZING GOVERNMENTS

2.1 Introduction

The relationship between the instruments of monetary policy such as the money growth rate or the interest rate, and policy targets such as the inflation rate is not fully certain. Governments or central bankers must then conduct monetary policy less than the fully informed. To aid themselves, policy makers gather data, run econometric models, and simulate responses of the economy to various policy shocks. Moreover, they update these models regularly after new data becomes available, which means that they are trying to learn or to update their beliefs about the economic environment.

This is a simultaneous control and estimation problem that affects the optimal decisions of policy makers. The policy maker tries to reach its target as best as possible given its current beliefs, but it must also revise its understanding about the relationship between instruments and targets, and as it revises its beliefs it must adjust monetary policy accordingly.

The relevance of learning in monetary economics has already been recognized in the literature. Most papers however study the role of the private sector's learning about the economic environment, but there are not many papers that study the complementary problem of learning by the monetary authority. I review the literature of learning by monetary policy makers in Section 2.2 of this essay.

The policy maker's need to learn about the economy is particularly important during abnormal periods of time during which the government is very uncertain about the reaction of the economy to different monetary actions. One such period of time is a hyperinflation, or a high inflation episode. During episodes of high inflation the velocity of money increases significantly, and eventually people substitute bad (inflating) money for good, stable money (such as the dollar for example), or they simply turn to barter. In order for the government to meet its targets it must learn quickly how the demand for money will react to monetary policy. For example, if a significant portion of the government's finances must come from seignorage, knowing to what extent people will actually use the national currency determines which portion of money issue will translate into inflation and which portion into real seignorage revenues. This in turn determines how much money the government should issue.

In this paper I study the impact of learning by the monetary authorities on the optimal supply of money and the income from seignorage in one of these highly inflationary economies. I assume that the government relies heavily on money issue to finance its budget and thus its objective is to maximize seignorage.

The reason why a government comes to rely largely on income from money issue can be explained using Sargent and Wallace's (1981) unpleasant monetarist arithmetic scenario. They showed that if fiscal policy precedes monetary policy, if there is a limit to the debt that the government can issue, and if the government cannot raise taxes, then the tighter monetary policy is today, the looser it will be in the future. If the government accumulates a lot of debt and if tax collections are not significant sources of income then the government will have to issue a lot of money.

The conditions that lead a government to accumulate debt and to run large deficits can be as varied as the financing of a war or the inefficiency of the tax system.

Deficits can soar for example during periods of civil unrest during which the government's expenditure rises to combat the rebellious groups. Tax revenues also fall as this portion of the population stops paying taxes[11]. Moreover, wars in general (not only civil wars) are very difficult to finance with traditional taxes. Hamilton (1977) for example points to "the unwillingness of our political leaders in both parties to attempt to pay the cost of the (Vietnam) war through taxation"[12] as the cause of the U.S. inflation of the 1970's. He argues that "this method of payment would have revealed the true cost, and thus ended the war."[13]

Low reliance on tax income is not exclusive to wars or social unrest. Inefficient or unsophisticated tax systems, very common in countries with high political instability and polarization, or in countries with large rural areas and poor infrastructure, result in too little resources invested in tax collection or in tax avoidance and thus in little tax revenue[14].

Low credit ratings and high transitory spending also increase the need for seignorage[15]. Transitory spending is not usually financed with tax reforms, which are meant to be permanent, but with domestic or foreign debt. If the government cannot get any loans, it turns to printing money.

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[11] Capie (1986).
[12] Hamilton (1977), p. 17-18.
[13] Hamilton (1977), p. 17-18.
[14] Cukierman et. al (1992).
[15] Click (1998).

Finally, the inability to reach a policy decision can also lead to deteriorating deficits and to seignorage as the major temporary source of income[16]. As an example consider France during the first half of the 1920's. France needed to pay for the reparations of the war, and it was clear that the Germans could not keep up with the reparation payments imposed by the Versailles treaty. The French tried invading the region of the Ruhr in Germany to collect their production as payment but the German government promoted a policy of passive resistance during which German workers were paid not to work[17]. The French effort proved unsuccessful and did not solve (nor help) the budget's problem. A tax reform was imminent but the Chamber of Deputies could not reach an agreement for several years because of the opposition between Conservatives and Socialists. This led to an 18-month period of complete fiscal inaction during which the government relied on printing money to finance itself[18].

All of the conditions that are likely to lead to the maximization of seignorage by the government are in essence short-lived or transitory. For this reason I work with a short horizon. In particular, I work with a two-period model. In the first period the government chooses a growth rate of money given some prior beliefs about the demand function. There are stochastic shocks to demand, a demand for money is realized, and the government collects S_1 in seignorage; the government then updates its beliefs according to Bayes rule and sets a new growth of the money supply for the second period, and collects S_2 in seignorage. A riskier distribution of posterior beliefs in the Rothschild-

[16]Alesina and Drazen (1991).

[17] A policy such as this also increases the government's need for seignorage. In particular, the German policy of passive resistance and the reparation payments imposed on Germany after the war are considered to be the two leading causes of the German hyperinflation of the 1922-1923.

[18] Alesina and Drazen (1991) suggest France and other European countries as examples of this type of war of attrition where the different political parties that made up the government could not agree on fiscal reforms following the war.

Stiglitz sense means that the government learns or that information increases. Intuitively, a riskier distribution of beliefs makes the possible demand functions easier to distinguish. The description of this model and the definition of the equilibrium are presented in Section 2.3.

In Sections 2.4 and 2.5 I present the three results from the model. I find that the government can increase real seignorage collections by learning about the demand for money (Proposition 2.1). Hence information is valuable. Intuitively, learning about the demand for money allows the government to make more accurate and efficient decisions.

Given that the government gains from learning, the government actively seeks to learn (Proposition 2.2), and thus constant money growth rates are not optimal, except when the possible demand functions differ by a constant because in this case every quantity of money demanded is equally informative.

Finally, for some parameter values the government induces a hyperinflation in order to learn about the demand for money (Proposition 2.3). That is, for some parameter values, if the government does not update beliefs the seignorage maximizing money growth rate is not hyperinflationary, but if the government updates beliefs the money growth rate generates a hyperinflation. If this is the case then the hyperinflation is the result of a rational decision by the government, which is interesting because as most people would agree hyperinflations should (if possible) be avoided. I should note however that this is not the result of monetary conditions but instead of fiscal policy, which determines the conduct of monetary policy.

In Section 2.6 I illustrate the model and the three results with an example. I show how learning can more than triple the inflation rate and trigger a hyperinflation.

In Section 2.7 I present an intuitive explanation of our model, and in Section 2.8, a short case study for the Bolivian hyperinflation of the 1980's. At last I conclude.

2.2 Literature Review

In this section I review how the policy maker's uncertainty and learning affect certain monetary problems as studied previously in other papers.

The idea about learning and monetary policy goes back to the 1960's: Brainard (1967) studied the making of monetary policy when there is uncertainty about the effect of the policy on the targets, and Poole (1970) studied the choice of policy tool under uncertainty about output. Neither one of them however studied learning. The more recent literature focus on the effects of the monetary authority's and the public's ability to reduce their uncertainty about the economy, e.g. Sack (1999), Kasa (1999), Hardy (1997), Caplin and Leahy (1996), Balvers and Cosimano (1994) and Bertocchi and Spagat (1993) among many others[19].

Bertocchi and Spagat (1993) consider a central bank that wants to minimize output variability in an infinite horizon with unknown structure of the economy. They find that Friedman's argument (1968) for a fixed money supply rule is only optimal in very specific circumstances, because learning is valuable, and it implies periodic adjustments of the optimal monetary actions.

Balvers and Cosimano (1994) study the debate of gradualism in monetary policy when the central bank minimizes the variability of inflation and is uncertain about the

[19] Learning in monetary economics has concentrated mostly on learning by the public, not by the government, unlike my paper. The papers by Marcet and Nicollini (1998) and Marcet and Sargent (1987) are for example two papers that study the dynamics of hyperinflations given different learning mechanisms by the public, for example least squares learning.

parameters of the aggregate demand function. By assuming that inflation changes stochastically with money growth, they develop an optimal way to learn about the inflation process and find that gradual convergence of money growth is optimal, because, although sharp reductions in money growth increase information about the economy, they also increase its variability, which makes it harder to forecast future inflation.

Caplin and Leahy (1996) study a central bank that learns via the reactions of the investors in a recession to the choice of monetary policy. They find that very gradual reductions in interest rates are ineffectual whereas aggressive polices are more successful, because when faced by small cuts in interest rates investors have new information that implies that further cuts may be in order and thus wait to invest. Aggressive policies on the other hand, reveal that there will probably be no further cuts and induce higher investment.

Hardy (1997), by studying how different central banks intervene in different ways in the financial market to keep interest rates near their targets, argues that the design of instruments has an important effect on the informational content of market prices. Since information is valuable in the determination of the operational target, by not intervening the central bank allows fluctuations that give rise to valuable information. One implication of Hardy's result is that since information increases the value function of the central bank, monetary policy actually reduces market efficiency.

Kasa (1999) considers a central bank that attempts to stabilize output and inflation but is unsure about the relationship between these two variables. He derives conditions on parameter values that lead the central bank to erroneously believe that there is an exploitable relation between output and inflation, which provides a new explanation to the inflation bias.

Finally, Sack (1999) sets up a model of monopoly experimentation when the Federal Reserve sets an interest rate rule, is uncertain about the policy multiplier and learns through the reaction of the economy to monetary policy. He finds that since the Fed faces greater uncertainty about the impact of its policy as it moves away from the previous interest rate level, the optimal monetary rule is gradual adjustments towards the target.

The previous papers have in common that they study a central bank seeking to maximize a social benefit over an infinite time horizon, whether it is a trade off between output and inflation, the minimization of inflation variability, maximization of output, etc, and they do it in the presence of uncertainty and learning. In my paper however, as a result of fiscal policy the government needs to maximize seignorage. Since this is likely a short term objective, I restrict my analysis to a short horizon.

This model can be used to determine the conditions (as in Bertocchi and Spagat (1993)) under which Friedman's argument is true in a scenario in which the government needs to finance itself strictly with money creation, but can also be used to determine what learning does to money growth and to seignorage, and the conditions under which learning leads to hyperinflations, which has not been studied before.

2.3 Model and Equilibrium

2.3.1 The Basic Model

There are two periods. At time t=1 the state of the economy is as follows: the government has a high level of desired spending, past inflation is high, people are not buying bonds and the government cannot raise taxes, and thus to finance public expenditures it must issue money. Given the stock of high-powered money, the government selects a growth rate for the money supply for the period, g_1, and collects seignorage, S_1.

Seignorage in period t is given by the rate of growth of money times the supply of real balances,

$$S_t = \frac{\dot{M}_t}{P_t} = \frac{\dot{M}_t}{M_t}\frac{M_t}{P_t} = g_t \frac{M_t}{P_t},$$

where M_t is the nominal supply of money, P_t is the price level, and g_t is the growth rate of money supply in period t. The condition for equilibrium in the money market is given by,

$$\frac{M_t}{P_t} = L(i_t, Y_t),$$

where $L(i_t, Y_t)$ is the demand for real balances, i_t is the nominal interest rate and Y_t is real output. Assume a traditional money demand function, i.e. $\frac{dL(i_t,Y_t)}{di_t} \leq 0$ and $\frac{dL(i_t,Y_t)}{dY_t} \geq 0$. Also assume that the interest rate and the output level are functions of the money growth rate.

According to Fisher's theorem (1930), the nominal interest rate can be written as follows,

$$(1+i_t) = (1+r_t)(1+\pi_t^e),$$

where r_t is the real rate and π_t^e are the expectations of inflation, and where both r_t and π_t^e are functions of g_t. Thus the reduced form equation for the interest rate can be written as,

$$i_t = \alpha + \beta g_t \tag{2.1}$$

α is a positive parameter, and β can be positive or negative. If an increase in the money growth rate reduces the interest rate, which is known as the liquidity effect, then $\beta < 0$, otherwise, $\beta > 0$.

The traditional explanation given to the liquidity effect is that without flexible prices, an increase in money growth requires a reduction in real rates. (Think of the IS-LM model. An increase in money growth shifts the LM curve to the right along the downward sloping IS curve.) Additionally, an increase in money growth increases inflation expectations. If the reduction in the real rate more than offsets the increase in expectations, we say that we have the liquidity effect.

In highly inflationary economies I would expect the effect of expectations of inflation to dominate over the interest rate effect, and thus that $\beta > 0$.

Output is also a function of money growth,

$$Y_t = \hat{Y} + \phi g_t \tag{2.2}$$

where $\hat{Y}$ is the (known) level of output associated with full employment.

If an increase in the money growth rate increases output (i.e. $\phi > 0$), I have the Tobin effect; if it reduces output (i.e. $\phi < 0$), the anti-Tobin effect.

The Tobin effect is the effect that an increase in the rate of growth of money supply has on the demand for assets. If there is a Tobin effect, an increase in the rate of growth of money, which increases the cost of holding cash balances decreases the demand for real balances; individuals switch out of money and into equities, which increases the demand for capital and thus increases output.

Increasing the money growth rate also reduces the demand for bank deposits, which means that bank lending is reduced to firms that want to purchase capital goods. The anti-Tobin effect results is the reduction in investment from firms dominates over the individuals' increased demand for assets[20].

In highly inflationary economies the effect of money growth on the real variables is expected to be small compared to the nominal changes in the economy and thus that $\phi \approx 0$.

Let $m_t = \frac{M_t}{P_t}$, then, based on equations (2.1) and (2.2) assume that the money demand equation can be written as a function of the rate of growth of money as follows[21],

$$m_t = l(g_t, \Omega) + \varepsilon_t \tag{2.3}$$

where ε_t is a random shock to demand, distributed with a continuous, differentiable distribution function $f(\varepsilon_t)$ in $-\infty < \varepsilon < \infty$, and with zero expectation. The shock ε_t is

[20] Gale (1983), pages 8-9.

[21] An extension of this model is to consider heteroskedastic shocks as follows, $m_t = l(g_t, \Omega) + g_t \varepsilon_t$.

meant to capture all the variables excluded from the analysis, as well as the stochastic nature of the economy[22].

Ω is a set of parameters unknown to the government, which include among others α, β and ϕ.

$l(g_t, \Omega)$ is a function of the rate of growth of money such that $\frac{dl(g_t, \Omega)}{dg_t} = l'(g_t, \Omega) \leq 0$ and $l(g_t, \Omega) + g_t l'(g_t, \Omega)$ is strictly decreasing in g_t. That is, assume that the demand function is downward sloping[23], and that marginal revenue from seignorage is decreasing. The latter assumption is satisfied for example if $l(g_t, \Omega)$ is concave or not too convex.

The reason why I want to introduce the latter assumption about the demand for money (that $l(g_t, \Omega) + g_t l'(g_t, \Omega)$ is decreasing in g_t) is that I want to have a Laffer-curve such as in Figure 2.1, as I explain next. Remember that seignorage is given by the demand for real balances times the rate of growth of money, $S_t = g_t l(g_t, \Omega)$. Since demand is downward sloping, as g_t increases, $l(g_t, \Omega)$ decreases, and hence the two terms in the seignorage equation move in opposite directions, i.e.

$$\frac{dS_t}{dg_t} = l(g_t, \Omega) + g_t l'(g_t, \Omega),$$

[22] Since the distribution of ε is independent of g, and since I assume that $-\infty < \varepsilon < \infty$, my formulation allows the possibility that $m < 0$. This assumption significantly simplifies the analysis, and it is traditionally used in the learning literature given the transparency of the results. When dealing with examples I will however assume that negative quantities are not allowed.

[23] It is in principle possible to obtain positively sloped demand functions by assuming a Tobin effect and a liquidity effect. This possibility however does not appear to be very attractive, specially during hyperinflations when one would expect the effect of expectations of inflation to be large compared to changes in the real side of the economy.

where the first term is positive, and the second term is negative. As g_t approaches zero $g_t l'(g_t,\Omega)$ approaches zero (as long as $l'(g_t,\Omega)$ does not go to minus infinity too fast). Since $l(g_t,\Omega)$ is positive, then for low values of g_t increasing the rate of growth of money increases seignorage. However, it is reasonable to assume that as g_t becomes very large, the second effect dominates the first. This means that it is reasonable to assume that the seignorage function is concave in g_t, or that demand function is such that $l(g_t,\Omega)+g_t l'(g_t,\Omega)$ is decreasing in g_t.

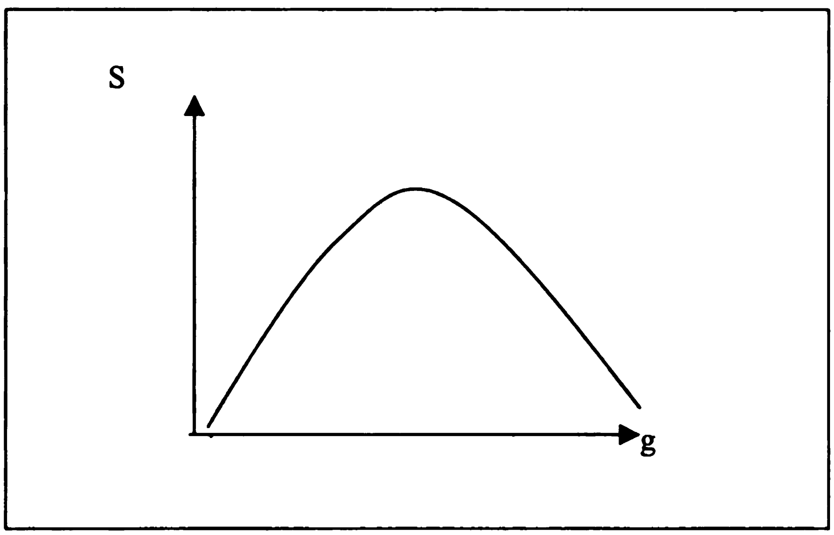

Figure 2.1. Seignorage Function with Laffer Curve Property

There are several possible representations for the money demand function, e.g. Cagan's (1956), Blanchard and Fischer's (1993), and Lucas' (1994).

Cagan (1956) suggested that a good representation for the demand for money during hyperinflations is given by $m_t = e^{a-bi_t+\ln Y_t}$, hence,

$l(g_t, \Omega) = e^{a - bi_t + \ln Y_t} = e^{a - \alpha b - b\beta g_t + \ln(\hat{Y} + \phi g_t)}$, where a and b are positive parameters. Cagan's formulation is usually used for convenience, although it appears to resemble high inflation economies very well.

Blanchard and Fischer (1993 – p. 513-517) suggest a more general demand function for money of the form $\frac{m_t}{Y_t} = \theta(r_t + \pi_t^e)$, where $\theta'(\cdot) < 0$[24]. In this case, $l(g_t, \Omega) = \theta(r_t + \pi_t^e) Y_t$.

Another representation of the demand for money (although not necessarily a good fit for a hyperinflation) is given by $l(g_t, \Omega) = Ai^{-\eta} Y$, where A is a constant, and where both the nominal interest rate and real income can be functions of the money growth rate. This function is used by Lucas (1994) to characterize the U.S. data[25].

Under either Cagan's (1956) or Blanchard and Fischer's (1993) specifications, Y and r are assumed to be constant. The argument traditionally given is that during hyperinflations the changes in the real variables in the economy are either null or very small compared to the large changes in money supply and prices. A notable exception to this hypothesis has been suggested by Pickersgill (1968) for the hyperinflationary episode in the Soviet Union during 1921-1926.

I will not follow any particular form for the demand for money, but instead work with the general specification $l(g_t, \Omega)$.

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[24] If for example $\theta(r_t + \pi_t^e) = e^{a - b(r_t + \pi_t^e)}$ and $i_t = r_t + \pi_t^e$, Blanchard and Fisher's specification becomes Cagan's.

[25] Of the three demand functions reviewed only Lucas' specification satisfies the restrictions of this model, given some additional restrictions on the signs of β and ϕ.

To introduce uncertainty, assume that the demand function can take on one of the two following representations,

$$m_t = l(g_t, \overline{\Omega}) + \varepsilon_t, \tag{2.4}$$

$$m_t = l(g_t, \underline{\Omega}) + \varepsilon_t, \tag{2.5}$$

and assume that $\forall g,\ l(g_t, \overline{\Omega}) \geq l(g_t, \underline{\Omega})$. This means that one demand function is always above the other, and thus I will refer to $l(g_t, \overline{\Omega})$ as the "high" demand curve and to $l(g_t, \underline{\Omega})$ as the "low" demand curve. I explain the meaning of this assumption in a later section.

In the first period, the government has a prior belief μ_1 that the demand for money is equation (2.4), i.e. it believes that the correct parameters are $\overline{\Omega}$ with prior probability μ_1, and with prior probability 1-μ_1 that they are $\underline{\Omega}$.

In period one, the government chooses a growth rate for the money supply $g_1(\mu_1)$, and a value of ε and hence m_1 are realized. The government's expected seignorage in period one is then,

$$E_{\mu_1} S_1 = \left[\mu_1 l(g_t, \overline{\Omega}) + (1-\mu_1) l(g_t, \underline{\Omega})\right] g_1 .$$

The government does not observe the realization of ε, but ε_t prevents the government from perfectly learning the parameters of the demand function after the first period. The observation of m_1, together with knowledge of $g_1(\mu_1)$, leads the government to revise its beliefs about Ω. Let μ_2 be the posterior belief that the correct parameters are $\overline{\Omega}$, then, through Bayes rule write μ_2 as follows,

$$\mu_2 = \frac{\mu_1 f(m_1 - l(g_1,\overline{\Omega}))}{(1-\mu_1)f(m_1 - l(g_1,\underline{\Omega})) + \mu_1 f(m_1 - l(g_1,\overline{\Omega}))}.$$

With the new beliefs, the government sets a growth rate of money for the second period, $g_2(\mu_2)$ and collects $S_2(\mu_2)$ in seignorage.

2.3.2 The Equilibrium

Since posterior beliefs are a function of prior beliefs and of the first period money growth rate, I solve for the second period equilibrium first. In period 2, the government maximizes expected seignorage given posterior beliefs,

$$E_{\mu_2} S_2 = \left[\mu_2 l(g_2,\overline{\Omega}) + (1-\mu_2) l(g_2,\underline{\Omega})\right] g_2,$$

with respect to g_2. The first order condition is given by,

$$\left[\mu_2 l(g_2,\overline{\Omega}) + (1-\mu_2) l(g_2,\underline{\Omega})\right] + \left[\mu_2 l'(g_2,\overline{\Omega}) + (1-\mu_2) l'(g_2,\underline{\Omega})\right] g_2 = 0,$$

and the second order condition by,

$$2\left[\mu_2 l'(g_2,\overline{\Omega}) + (1-\mu_2) l'(g_2,\underline{\Omega})\right] + \left[\mu_2 l''(g_2,\overline{\Omega}) + (1-\mu_2) l''(g_2,\underline{\Omega})\right] g_2 \leq 0.$$

Let $g_2(\mu_2)$ denote the second period optimal money growth rate. Given the assumptions about $l(g_t,\Omega)$, the second order condition is satisfied with strict inequality, and thus $g_2(\mu_2)$ denotes a unique maximum.

Let $S_2{}^*(\mu_2)$ denote the resulting value function of the government, that is,

$$S_2{}^*(\mu_2) = \left[\mu_2 l(g_2(\mu_2),\overline{\Omega}) + (1-\mu_2) l(g_2(\mu_2),\underline{\Omega})\right] g_2(\mu_2).$$

In the first period the government chooses a rate of growth of money by maximizing expected seignorage over the two periods. Since in period one the posterior beliefs μ_2 are a random variable whose distribution function depends upon the first period money growth rate and upon the distribution of m_1 implied by ε, I take the expectation of the second period seignorage over all possible beliefs and all possible amounts of money demanded. Assuming that the time discount parameter is given by δ, the government's first period problem is as follows,

$$\underset{g_1}{Max} \ E_{\mu_1} S_1 \ + \ \delta \int_{-\infty}^{\infty} S_2 * (\mu_2) h(m,g) dm,$$

where the distribution m implied by ε is given by,

$$h(m,g) = (1-\mu_1) f(m - l(g, \underline{\Omega})) + \mu_1 f(m - l(g, \overline{\Omega})).$$

Let $g_1(\mu_1)$ be the equilibrium money growth rate in period one.

2.3.3 The Myopic Equilibrium

In this dynamic problem periods one and two are connected by posterior beliefs. Thus when the government chooses $g_1(\mu_1)$ it accounts for the effect of period one money growth on posterior beliefs, and of posterior beliefs on second period seignorage. I now want to compare $g_1(\mu_1)$ with the money growth rate that the government would set if it did not account for the effect of $g_1(\mu_1)$ on posterior beliefs. Therefore, I compare $g_1(\mu_1)$ with the optimal myopic money growth rate, $g_{myopic}(\mu_1)$, which is chosen to maximize first period expected seignorage, $\underset{g_1}{Max} \ E_{\mu_1} S_1$.

(Note that $g_{myopic}(\mu_1)$ and $g_2(\mu_2)$ only differ in the value of μ. The first and second order conditions are also the same except for the value of μ.)

If the government issues a rate $g_1(\mu_1)$ different than the myopic rate $g_{myopic}(\mu_1)$, then the government experiments; if this leads to a riskier distribution of beliefs, then it increases information.

2.4 The Value of Information

In the previous section I defined the equilibrium money growth rate $g_1(\mu_1)$ and the myopic money growth rate $g_{myopic}(\mu_1)$, and defined the difference between the two as the amount of experimentation. Thus experimentation is the deviation from myopic policies in order to increase or reduce information, where more information means a riskier distribution of posterior beliefs. In this section I study whether the government benefits from more information.

Blackwell (1951, 1953) defined more informative experiments and the sufficient conditions for a more informative experiment to be valuable. Intuitively, an experiment z_1 is more informative than an experiment z_2, if every distribution of beliefs that can be generated from z_2, can also be generated by z_1. Formally, an experiment is a pair of measures $[f(m-l(g,\underline{\Omega})), f(m-l(g,\overline{\Omega}))]$, one for each state of nature that gives the distribution of the first period money demand.

Definition: An experiment $z_1=[f(m-l(g'',\underline{\Omega})), f(m-l(g'',\overline{\Omega}))]$ is *more informative* than an experiment $z_2=[f(m-l(g',\underline{\Omega})), f(m-l(g',\overline{\Omega}))]$, if

$$\int \chi(\mu_2(g'',m))h(m,g)dg \geq \int \chi(\mu_2(g',m))h(m,g)dm$$

for every continuous, convex function $\chi(\mu_2(g'',m)):[0,1]\rightarrow \Re$.

In the spirit of Blackwell (1951, 1953), the second derivative of the second period value function with respect to beliefs, $\frac{d^2 S_2 *}{d\mu_2^2}$, is called the value of information, and more information is said to be good if the second period value function is convex in beliefs, which comes from the definition of a more informative experiment.

In my model, experiments are rates of growth of the money supply, observations are money demanded, and more informative experiments are determined by the demand functions as follows. Remember that $\forall g_t$, $l(g_t, \overline{\Omega}) \geq l(g_t, \underline{\Omega})$, which implies that the two demand functions look like one of the two panels in Figure 2.2 (although they do not have to be straight lines). In Figure 2.2(a) the possible distribution functions of m_1 implied by g_1 move further apart and overlap less as g_1 increases. Thus a greater g_1 produces a more informative signal. On panel (b), the possible distributions of m_1 move closer together and overlap more as g_1 increases, and thus a greater g_1 produces a less informative signal.

Intuitively, more informative money growth rates spread the means of the two possible demand functions apart making them more distinguishable.

Proposition 2.1: The government that maximizes expected seignorage values information, i.e. $\frac{d^2 S_2 *(\mu_2)}{d\mu_2^2} \geq 0$.

Proof: See Appendix B.

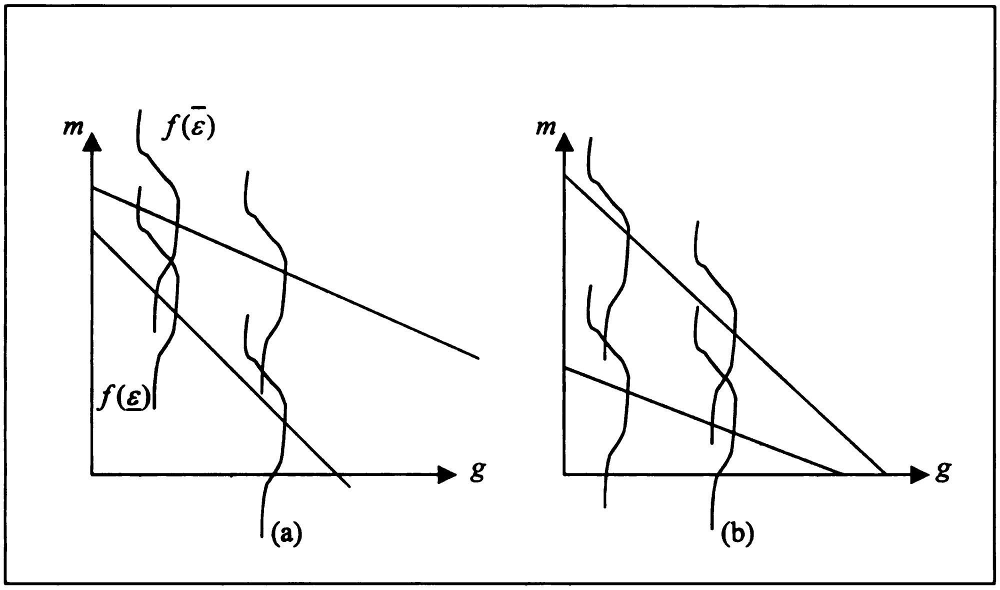

Figure 2.2. Informativeness of the Inflation Rate

Proposition 2.1 shows that the government values more information. Mathematically, since expected seignorage is a linear function of posterior beliefs, the value function is the supremum of a collection of linear functions, and it is thus convex. Intuitively, more information is valuable because it allows the government to make a better informed decision.

2.5 Effects of Experimentation in High Inflation Economies

In this section I study the government's incentives, created by the effect of period one money growth rates on the informativeness of money demanded, for the government to adjust the rate of growth of money supply away from the myopically optimal rates. To do so, I compare the first order conditions of the myopic and non-myopic problems.

The non-myopic government maximizes $E_{\mu_1} S_1 + \delta \int_{-\infty}^{\infty} S_2{*}(\mu_2) h(m,g) dm$, and the myopic government, $E_{\mu_1} S_1$, both with respect to g_1. Therefore, if $\frac{d \int_{-\infty}^{\infty} S_2{*}(\mu_2) h(m,g) dm}{dg_1} > (<) 0$, in order for the non-myopic first order condition to be satisfied, $\frac{dE_{\mu_1} S_1}{dg_1} < (>) 0$. This means that the myopic objective function is decreasing (increasing) at $g_1(\mu_1)$. Since expected seignorage is a concave function, it is decreasing (increasing) only for values of g_1 above (below) $g_{myopic}(\mu_1)$. Graphically this argument is represented in Figure 2.3.

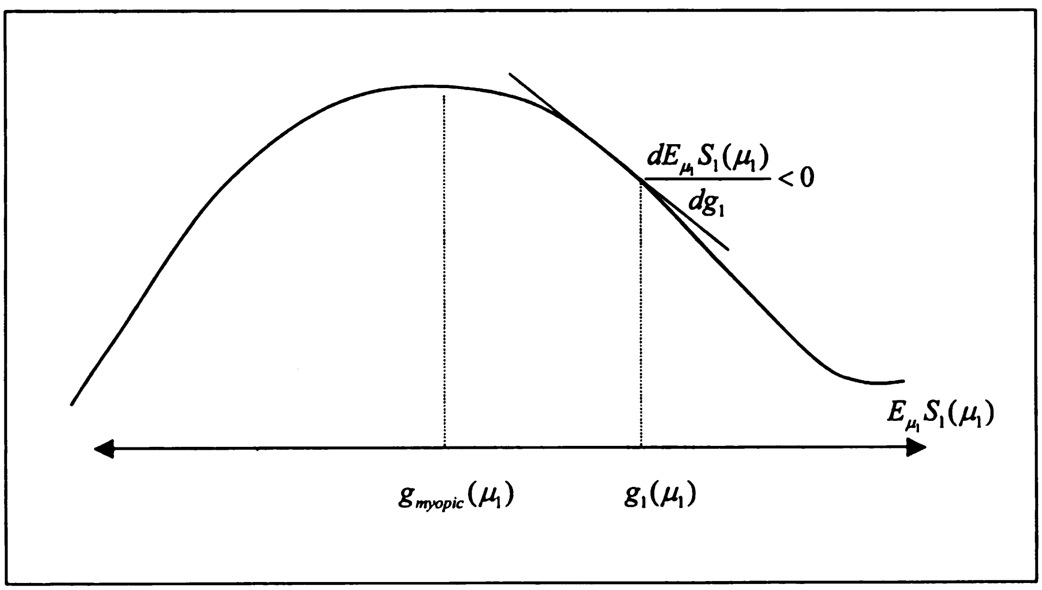

Figure 2.3. Conditions under which the Non-Myopic Inflation Rate is Higher than the Myopic Rate

After some algebra (in Appendix B) $\dfrac{d\int_{-\infty}^{\infty} S_2{*}(\mu_2)h(m,g)dm}{dg_1}$ can be written as a function of the value of information and of the amount of information. In particular,

$$\frac{d\int_{-\infty}^{\infty} S_2{*}(\mu_2)h(m,g)dm}{dg_1} = (l'(g_1,\overline{\Omega}) - l'(g_1,\underline{\Omega})) \int_{-\infty}^{\infty} \frac{d^2 S_2{*}}{d\mu_2{}^2}\mu_2(1-\mu_1)\frac{d\mu_2}{dm}\underline{f}dm \,. \quad (2.6)$$

The relation $l'(g_1,\overline{\Omega}) > (<) l'(g_1,\underline{\Omega})$ indicates whether increasing the money growth rate increases (decreases) information. In Figure 2.2(a) $l'(g_1,\overline{\Omega}) > l'(g_1,\underline{\Omega})$, hence, increasing the rate of growth of money spreads the means of the two demand functions apart so that they become more distinguishable. Higher money growth rates are thus more informative. Since information is valuable (Proposition 2.1), one would expect the government to increase the rate of growth of money above the myopic rate to increase information. Alternatively, if $l'(g_1,\overline{\Omega}) < l'(g_1,\underline{\Omega})$, e.g. as in Figure 2.2(b), lower money growth rates are more informative, and thus one would expect the government to set $g_1(\mu_1)$ below the myopic rate. Formally, these arguments are summarized in the next Proposition. Before presenting the proposition I state one assumption commonly used in the information literature.

Assumption A2: Assume that $f(\varepsilon)$ satisfies the monotone likelihood ratio property (MLRP): $\dfrac{f'(\varepsilon)}{f(\varepsilon)}$ is strictly decreasing in ε.

An implication of the MLRP is that a higher demand for money leads to a higher belief that the demand is the "high" one, i.e. $\frac{d\mu_2}{dm} \geq 0$. (The proof to this statement is in Appendix B.)

Proposition 2.2: Let the MLRP hold. Then the optimal money growth rate $g_1(\mu_1)$ is higher (lower) than the myopic rate $g_{myopic}(\mu_1)$, if increasing the rate of growth of money increases (decreases) information. That is, $g_1(\mu_1) \geq (\leq) g_{myopic}(\mu_1)$ if $l'(g_1, \overline{\Omega}) \geq (\leq) l'(g_1, \underline{\Omega})$.

Proof: See section Appendix B.

The relation between the myopic money growth rate $g_{myopic}(\mu_1)$, the non-myopic money growth rate $g_1(\mu_1)$, and the second period money growth rate $g_2(\mu_2)$, is presented in Figure 2.4. In the graph, by assumption, there are three possible posterior beliefs, $\mu_2 > \mu_1$, $\mu_2 = \mu_1$ and $\mu_2 < \mu_1$. Figure 2.4(a) represents the three possible time paths for the growth rate of money when $l'(g_1, \overline{\Omega}) \geq l'(g_1, \underline{\Omega})$. For presentation purposes I assume that before period one the prevailing rate of growth is the myopic rate. In period one, the growth rate jumps up to $g_1(\mu_1)$, because this increases information. In period two it can go either up or down again depending on posterior beliefs. For example, if the posterior beliefs are the same as the prior beliefs, the second period money growth rate is exactly the same myopic rate, i.e. $g_2(\mu_1) = g_{myopic}(\mu_1)$.

Figure 2.4(b) shows the possible paths when $l'(g_1, \overline{\Omega}) \leq l'(g_1, \underline{\Omega})$, which corresponds to Figure 2.2(b). In period one the rate is lower than the myopic rate because this increases information, but in period two it can go up or down depending on posterior beliefs.

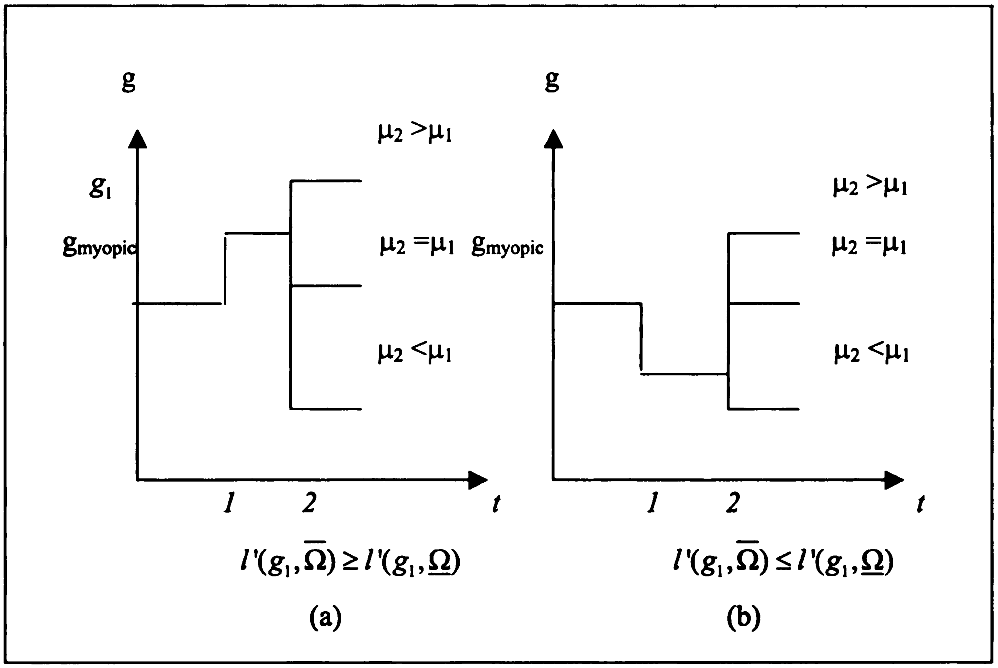

Figure 2.4: Money Growth Paths

Finally, I can identify the conditions under which experimentation leads to a hyperinflation, which I present formally in proposition 2.3. The definition of a hyperinflation is arbitrary. Cagan, for example, who was the first to study hyperinflations, defined a hyperinflationary episode as starting in the month the inflation rate is above 50% and ending in the month when it has fallen below that rate and has stayed there for at least a year. His definition is arbitrary, but it served the purpose of his study of seven hyperinflations of the 1920's and 1940's. Other definitions are slacker, or cover longer periods of time. I will not specify a particular inflation rate as hyperinflationary, thus when I say that the money growth rate is hyperinflationary, I mean that it leads to a very high inflation rate.

Definition: The money growth rate g_1 is hyperinflationary if it leads to an inflation rate above *I*, where the inflation rate is given by,

$$\pi(g_1) = \left(\frac{P_1(g_1)}{P_0} - 1 \right),$$

and the price level is given by,

$$P_1(g_1) = \frac{(1+g_1)M_0}{l(g_1, \Omega) + \varepsilon_1}.$$

To obtain a hyperinflation as the result of experimentation, the myopic rate must not be hyperinflationary (i.e. it must lead to yearly inflation rates below *I* or below 8,000% if we were to follow Cagan's definition), but the optimal amount of experimentation must be large enough to generate a hyperinflation. This case is represented in Figure 2.5 and in Proposition 2.3.

To establish Proposition 2.3 I compare the myopic and nonmyopic problems for the cases where higher money growth rates are more informative[26]. I assume that $g_{myopic}(\mu_1)$ is not conducive to a hyperinflation but that $g_1(\mu_1)$ is, and I look at the conditions that make the difference between these two rates, which is given by equation (2.6), large (hyperinflationary).

[26] I could apply Proposition 2.3 to hyperdeflations. In this case I would study cases where decreasing the money growth rate increases information, such as Figure (2b).

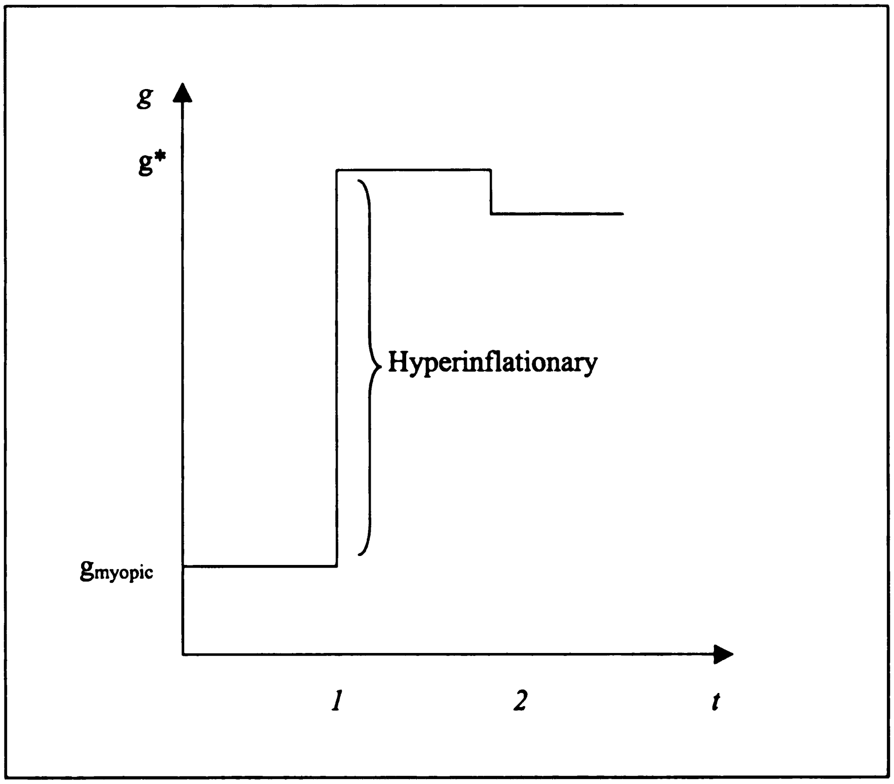

Figure 2.5. Hyperinflationary Money Growth Paths

Proposition 2.3: Assume that the myopic inflation rate is not hyperinflationary, i.e.

$$\pi(g_{myopic}(\mu_1)) = \left(\frac{(1+g_{myopic}(\mu_1))M_0}{l(g_{myopic}(\mu_1),\Omega)+\varepsilon_1} - 1 \right) = I - n,$$

where $n \in \mathbb{R}_+$, and where P_0 is normalized to one. Then if the following two conditions are true, hyperinflations are the optimal result of uncertainty and learning:

(3.1) Increasing the rate of growth of money increases information, i.e. $l'(g_1,\overline{\Omega}) > l'(g_1,\underline{\Omega})$, and

$$(3.2)\ \delta(l'(g_1,\overline{\Omega}) - l'(g_1,\underline{\Omega})) \int_{-\infty}^{\infty} \frac{d^2 S_2^*}{d\mu_2^{\,2}} \mu_2 (1-\mu_1) \frac{d\mu_2}{dm} \underline{f} dm \geq \frac{(l(g_{myopic},\Omega)+\varepsilon_1)n}{M_0}.$$

Proof:

Since $l'(g_1,\overline{\Omega}) > l'(g_1,\underline{\Omega})$ then the non-myopic inflation rate is above the myopic rate (Proposition 2.2), and thus given that $l'(g_1,\Omega) < 0$, then,

$$\left(\frac{(1+g_{myopic}(\mu_1))M_0}{l(g_{myopic}(\mu_1),\Omega)+\varepsilon_1}\right) \leq \left(\frac{(1+g_1(\mu_1))M_0}{l(g_1(\mu_1),\Omega)+\varepsilon_1}\right),$$

and thus $\pi(g_{myopic}(\mu_1)) \leq \pi(g_1(\mu_1))$.

Condition 3.2 is a sufficient condition for the difference between these two rates to be larger than *n*, that is,

$$\pi(g_1(\mu_1)) - \pi(g_{myopic}(\mu_1)) = \frac{(1+g_1(\mu_1))M_0}{l(g_1(\mu_1),\Omega)+\varepsilon_1} - \frac{(1+g_{myopic}(\mu_1))M_0}{l(g_{myopic}(\mu_1),\Omega)+\varepsilon_1}.$$

Note that since $l'(g_1,\Omega) < 0$ and $g_{myopic}(\mu_1) \leq g_1(\mu_1)$, then,

$$\frac{(1+g_1(\mu_1))M_0}{l(g_1(\mu_1),\Omega)+\varepsilon_1} \geq \frac{(1+g_1(\mu_1))M_0}{l(g_{myopic}(\mu_1),\Omega)+\varepsilon_1},$$

and thus that,

$$\pi(g_1(\mu_1)) - \pi(g_{myopic}(\mu_1)) \geq \frac{(1+g_1(\mu_1))M_0}{l(g_{myopic}(\mu_1),\Omega)+\varepsilon_1} - \frac{(1+g_{myopic}(\mu_1))M_0}{l(g_{myopic}(\mu_1),\Omega)+\varepsilon_1}.$$

Hence, if

$$\frac{(1+g_1(\mu_1))M_0}{l(g_{myopic}(\mu_1),\Omega)+\varepsilon_1} - \frac{(1+g_{myopic}(\mu_1))M_0}{l(g_{myopic}(\mu_1),\Omega)+\varepsilon_1} \geq n,$$

which is equivalent to,

$$(g_1(\mu_1) - g_{myopic}(\mu_1)) \geq \frac{(l(g_{myopic}(\mu_1),\Omega)+\varepsilon_1)n}{M_0},$$

then $\pi(g_1(\mu_1)) - \pi(g_{myopic}(\mu_1)) \geq n$, and thus $\pi(g_1(\mu_1)) \geq I$.

Now note that the non-myopic inflation rate equals the myopic inflation rate plus the optimal amount of experimentation, where the latter is given by equation 2.6 (see proof in Appendix B), thus,

$$g_1(\mu_1) - g_{myopic}(\mu_1) = \delta(l'(g_1, \overline{\Omega}) - l'(g_1, \underline{\Omega})) \int_{-\infty}^{\infty} \frac{d^2 S_2{}^*}{d\mu_2{}^2} \mu_2(1-\mu_1) \frac{d\mu_2}{dm} \underline{f} dm .$$

Thus, if

$$\delta(l'(g_1, \overline{\Omega}) - l'(g_1, \underline{\Omega})) \int_{-\infty}^{\infty} \frac{d^2 S_2{}^*}{d\mu_2{}^2} \mu_2(1-\mu_1) \frac{d\mu_2}{dm} \underline{f} dm \geq \frac{(l(g_{myopic}, \Omega) + \varepsilon_1) n}{M_0},$$

then $\pi(g_1(\mu_1)) \geq I$.

□

For experimentation to lead to hyperinflations I need that higher money growth rates be more informative, and that the incentives to experiment (equation (2.6)) be large. Among other things, the larger the discount parameter δ, the larger the value of information $\frac{d^2 S_2{}^*}{d\mu_2{}^2}$, and the more spread out the demand functions, that is the larger $l'(g_1, \overline{\Omega}) - l'(g_1, \underline{\Omega})$, the larger the amount of experimentation. In particular, the more spread apart the means of the two possible distribution functions of posterior beliefs, the more there is to be learned, and thus the more likely is the government to hyperinflate. That is, the probability that the non-myopic money growth rate is hyperinflationary equals the following probability,

$$\Pr\left(\varepsilon_1 \le \frac{M_0\delta(l'(g_1,\overline{\Omega}) - l'(g_1,\underline{\Omega})) \displaystyle\int_{-\infty}^{\infty} \frac{d^2 S_2{}^*}{d\mu_2{}^2}\mu_2(1-\mu_1)\frac{d\mu_2}{dm} \underline{f} dm}{n} - l(g_{myopic},\Omega) \right),$$

which given $\frac{d^2 S_2{}^*}{d\mu_2{}^2} \ge 0$, increases as $l'(g_1,\overline{\Omega}) - l'(g_1,\underline{\Omega})$ increases. Thus, the larger $l'(g_1,\overline{\Omega}) - l'(g_1,\underline{\Omega})$, the larger the probability that learning leads to a hyperinflation.

In Figure 2.6(b) for example, $l'(g_1,\overline{\Omega}) - l'(g_1,\underline{\Omega})$ is larger than in Figure 2.6(a), thus a hyperinflation is more likely under Figure 2.6(b) than under Figure 2.6(a).

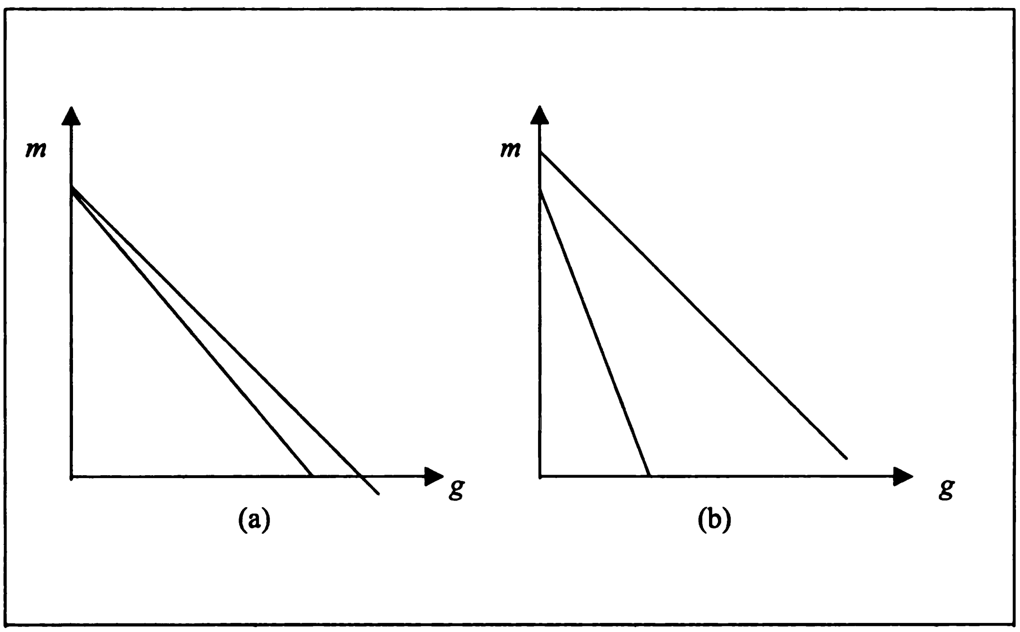

Figure 2.6. Spread of the Possible Demand Functions

I should emphasize that although in this model the introduction of learning can lead to large (hyperinflationary) money growth rates, this is the result of fiscal decisions which precede monetary policy and which determine the need for large seignorage.

2.6 Solution for Uniformly Distributed Shocks and Linear Demand Functions

To illustrate the results from the model I solve for the simplest possible case, linear demand functions and uniformly distributed random shocks. This example is meant to illustrate how significant the effect of learning can be on the inflation rate and on the seignorage level, and how a rational government can induce hyperinflations to learn about demand.

Assume that the random shocks ε_t are distributed uniformly on the interval $[-2.115, 2.115]$. The demand function is linear in the nominal interest rate and output level, $m_t = a - bi_t + Y_t + \varepsilon_t$, which using equations (2.1) and (2.2) leads to $m_t = (a - b\alpha + \hat{Y}) + (\phi - b\beta)g_t + \varepsilon_t$. That is $l(g_t, \Omega) = (a - b\alpha + \hat{Y}) + (\phi - b\beta)g_t$ and $\Omega = \{a, b, \alpha, \beta, \phi, \hat{Y}\}$.

Assume that prior beliefs are given by μ_1=0.5, and that $\overline{\phi} - \overline{b\beta} = -0.08$, $\underline{\phi} - \underline{b\beta} = -0.19$, and $\overline{a} - \overline{b\alpha} + \hat{Y} = \underline{a} - \underline{b\alpha} + \hat{Y} = 9$. Graphically, these types of demand functions correspond to Figure 2.2(a). Finally, assume starting values $g_0 = g_{myopic}(\mu_1)$ and $P_0 = 1$.

I am assuming that $(\phi - b\beta) < 0$ for both types of demand functions. This is a sign restriction that must be imposed to satisfy the restrictions on money demand ($l' < 0$ and $2l' + gl'' < 0$). Assuming that $(\phi - b\beta) < 0$ is a reasonable assumption for highly inflationary economies where it is expected that the effect of the money growth rate on the real variables of the economy be negligible or at least small compared to the effects on the monetary variables (the expectations of inflation). It would therefore be expected that $\beta > 0$ and ϕ to be close to zero, and thus $(\phi - b\beta) < 0$.

Since $9-0.08g+\varepsilon > 9-0.19g+\varepsilon \ \ \forall g$, then $9-0.0.8g+\varepsilon$ is called the "high" demand, and $9-0.19g+\varepsilon$, the "low" demand. $\mu_0 = 0.5$ is then the prior belief that the demand is the "high" one.

With a uniform distribution function there are only three possible posterior beliefs as follows. Since $\varepsilon \in [-2.115, 2.115]$, then $m_1 \in (9-0.19g_1-2.115, 9-0.19g_1+2.115)$ if $9-0.19g_1+\varepsilon$ is the true demand, and $m_1 \in (9-0.08g_1-2.115, 9-0.08g_1+2.115)$ if $9-0.08g_1+\varepsilon$ is the true demand function. If observed money demand is at least $9-0.19g_1-2.115$ but strictly less than $9-0.08g_1-2.115$, then only the "low" demand could have generated this observation. In this case the government learns that the true demand function is $9-0.19g_1+\varepsilon$. The posterior belief is thus given by $\mu_2 = 0$. If observed money demand is between $9-0.19g_1+2.115$ and $9-0.08g_1+2.115$, then only the "high" type of demand function could have generated the observation. The government thus learns that this is the true demand, $\mu_2 = 1$. Otherwise, the government does not learn anything. In summary, the three possible posterior beliefs are,

$$\mu_2 = \begin{cases} 0 & if \ \ 9-0.19g_1-2.115 \le m_1 < 9-0.08g_1-2.115 \\ \mu_1 & if \ \ 9-0.08g_1-2.115 \ \le m_1 \le 9-0.19g_1+2.115 \\ 1 & if \ \ 9-0.19g_1+2.115 < m_1 \le 9-0.08g_1+2.115 \end{cases}.$$

Posterior beliefs are a function of first period money demand. To find the distribution of first period money demand implied by the random shock, let $\bar{\varepsilon}(m_1, g_1)$ be the value of ε that must appear if the money demand is m_1, the money growth rate in period one is g_1, and the true state of demand is $\overline{\Omega}$; with $\underline{\varepsilon}(m_1, g_1)$ being the value of ε that must appear if the money demand is m_1, the money growth rate in period one is g_1, and the true state of demand is $\underline{\Omega}$. Then the density of m_1 is given by $h(m,g)=(1-\mu_1)f(m-l(g,\underline{\Omega}))+\mu_1 f(m-l(g,\overline{\Omega}))$.

Since by assumption $\mu_1=0.5$, then the distribution of m_1 is given by $h(m,g)=0.5f(m-l(g,\underline{\Omega}))+0.5f(m-l(g,\overline{\Omega}))$. Note however that from Bayes' rule if the posterior belief is given by $\mu_2=0$ then,

$$0=\frac{0.5f(m_1-l(g_1,\overline{\Omega}))}{0.5f(m_1-l(g_1,\underline{\Omega}))+0.5f(m_1-l(g_1,\overline{\Omega}))}\Leftrightarrow 0=0.5f(m_1-l(g_1,\overline{\Omega})),$$

and for $\mu_2=1$,

$$1=\frac{0.5f(m_1-l(g_1,\overline{\Omega}))}{0.5f(m_1-l(g_1,\underline{\Omega}))+0.5f(m_1-l(g_1,\overline{\Omega}))}\Leftrightarrow 0=0.5f(m_1-l(g_1,\underline{\Omega})),$$

thus,

$$h(m,g)=\begin{cases}\dfrac{f(m-l(g,\underline{\Omega}))}{2} & \text{if } 9-.19g_1-2.115\le m_1<9-.08g_1-2.115\\ \dfrac{f(m-l(g,\underline{\Omega}))+f(m-l(g,\overline{\Omega}))}{2} & \text{if } 9-.08g_1-2.115\le m_1\le 9-.19g_1+2.115\\ \dfrac{f(m-l(g,\overline{\Omega}))}{2} & \text{if } 9-.19g_1+2.115<m_1\le 9-.08g_1+2.115\end{cases}$$

Finally, since I assumed that the shocks are uniformly distributed in $[-\hat{\varepsilon},\hat{\varepsilon}]$, where $\hat{\varepsilon}=2.115$, then $f(\varepsilon)=\dfrac{1}{2\hat{\varepsilon}}=\dfrac{1}{4.23}$. Hence,

$$h(m,g)=\begin{cases}\dfrac{1}{8.46} & \text{if } 9-0.19g_1-2.115\le m_1<9-0.08g_1-2.115\\ \dfrac{1}{4.23} & \text{if } 9-0.08g_1-2.115\le m_1\le 9-0.19g_1+2.115\\ \dfrac{1}{8.46} & \text{if } 9-0.19g_1+2.115<m_1\le 9-0.08g_1+2.115\end{cases}.$$

Having defined all the preliminaries I now move on to find the equilibrium money growth rates.

Given posterior beliefs and given that the expected value of ε is zero, in the second period the government maximizes,

$$\begin{aligned} S_2(\mu_2) &= ((9-0.08g_2)\mu_2 + (9-0.19g_2)(1-\mu_2))g_2 \\ &= [9-(0.19-0.11\mu_2)g_2]g_2 \end{aligned},$$

with respect to g_2. The first order condition is that $9-(0.38-0.22\mu_2)g_2 = 0$, which yields the second period equilibrium money growth rate $g_2(\mu_2) = \dfrac{9}{0.38-0.22\mu_2}$.

Plugging this rate in the seignorage function gives the second period value function as a function of posterior beliefs,

$$S_2*(\mu_2) = \left(\left(9-0.08\frac{9}{0.38-0.22\mu_2}\right)\mu_2 + \left(9-0.19\frac{9}{0.38-0.22\mu_2}\right)(1-\mu_2)\right)\frac{9}{0.38-0.22\mu_2},$$

thus $S_2*(\mu_2) = \dfrac{40.5}{0.38-0.22\mu_2}$.

Given the three possible posterior beliefs and the probability of each occurring, the expected value of $S_2*(\mu_2)$ is given by,

$$E_m S_2*(\mu_2) = \begin{pmatrix} \displaystyle\int_{9-0.19g_1-2.115}^{9-0.08g_1-2.115} S_2*(\mu_2=0)\frac{1}{8.46}dm \quad + \int_{9-0.08g_1-2.115}^{9-0.19g_1+2.115} S_2*(\mu_2=\mu_1)\frac{1}{4.23}dm \\ \displaystyle + \int_{9-0.19g_1+2.115}^{9-0.08g_1+2.115} S_2*(\mu_2=1)\frac{1}{8.46}dm \end{pmatrix},$$

and thus,

$$E_m S_2{}^*(\mu_2) = \left(\begin{array}{l} \int_{9-0.19g_1-2.115}^{9-0.08g_1-2.115} \frac{40.5}{0.38-0.22(0)} \frac{1}{8.46} dm \\ + \int_{9-0.08g_1-2.115}^{9-0.19g_1+2.115} \frac{40.5}{0.38-0.22(0.5)} \frac{1}{4.23} dm \\ + \int_{9-0.19g_1+2.115}^{9-0.08g_1+2.115} \frac{40.5}{0.38-0.22(1)} \frac{1}{8.46} dm \end{array} \right) = 150 + 0.78 g_1 .$$

Since increasing the rate of growth of money increases (valuable) information, second period expected seignorage increases as the rate of money growth increases.

In the first period the government maximizes expected first period seignorage given prior beliefs $\mu_1 = 0.5$, plus the discounted expected value of second period seignorage,

$$((9-0.08g_1)0.5 + (9-0.19g_1)(0.5))g_1 + \delta(150+0.78g_1),$$

with respect to g_1. Assuming that $\delta = 1$, the first order condition is that $9 - 0.27g_1 + 0.78 = 0$, which yields $g_1(0.5) = 36.22 = 3{,}622\%$.

Total (undiscounted by assumption) expected seignorage over the two periods is thus,

$$((9-0.08*36.22)0.5 + (9-0.19*36.22)(1-0.5))(36.22) + 150 + 0.78(36.22) = 327 .$$

To find the maximizing myopic money growth rate and seignorage level I differentiate myopic expected seignorage $((9-0.08g_t)0.5 + (9-0.19g_t)(1-0.5))g_t$ with respect to g_t in each period, which yields myopic money growth rate $g_{myopic} = 33.33 = 3333\%$. Myopic seignorage over the two periods is thus,

$$2*[(9-0.08(33.33))0.5 + (9-0.19(3.33))0.5](33.33) = 300 .$$

Thus when the government accounts for the effect of the first period money growth rate on posterior beliefs it sets a money growth rate of 3,622%, which is 8.7% higher than the rate that it would set if it did not account for this effect (3,333%). This increases seignorage by 9% from 300 to 327. Real seignorage then increases by more than the increase in the rate of growth of money.

To calculate the effect of the government's ability to learn on the inflation rate I first need to find the initial money stock, M_0. Since $P_0 = \dfrac{M_0}{9-0.19g_{myopic}+\varepsilon}$ or $P_0 = \dfrac{M_0}{9-0.08g_{myopic}+\varepsilon}$, and since $P_0 = 1$, then $M_0 = 9-0.19g_{myopic}+\varepsilon$ or $M_0 = 9-0.08g_{myopic}+\varepsilon$. A reasonable assumption is thus that the initial money stock is an average of these two, i.e. $M_0 = 0.5(9-0.19g_{myopic})+0.5(9-0.08g_{myopic}) = 4.5$.

With the initial money stock and the rate of growth of money I can calculate the stock of money in period one, $M_1 = (1+g_1)M_0$, and the price level in period one. The price level in period one will be $P_1 = \dfrac{4.5(1+g_1)}{9-0.19g_1+\varepsilon_1}$ or $P_1 = \dfrac{4.5(1+g_1)}{9-0.08g_1+\varepsilon_1}$ depending on which is the true demand function for money. The government does not know which is the true demand function, nor does it know the value of the shock ε, but it can calculate an expected price level. To find the expected price level, integrate over ε and over beliefs. Let $\tilde{P}_1$ be the expected price level in period one, then $\tilde{P}_1$ is found according to the following formula,

$$\tilde{P}_1 = \int_{-2.115}^{2.115} \left(\frac{4.5(1+g_1)}{9-0.19g_1+\varepsilon_1} 0.5 + \frac{4.5(1+g_1)}{9-0.08g_1+\varepsilon_1} 0.5 \right) f(\varepsilon) d\varepsilon$$
$$= \int_{-2.115}^{2.115} \left(\frac{4.5(1+g_1)}{9-0.19g_1+\varepsilon_1} 0.5 + \frac{4.5(1+g_1)}{9-0.08g_1+\varepsilon_1} 0.5 \right) \frac{1}{4.23} d\varepsilon.$$

This integral can be solved to yield,

$$\tilde{P}_1 = \frac{0.5*4.5*(1+g_1)}{4.23} \begin{bmatrix} Ln(9-0.19g_1+2.115) + Ln(9-0.08g_1+2.115) \\ -Ln(9-0.19g_1-2.115) - Ln(9-0.08g_1-2.115) \end{bmatrix}.$$

Substituing the value of $g_1 = 36.22$ and taking the natural logarithm gives $\tilde{P}_1 = 156.62$. Doing the same for the myopic money growth rate $g_{myopic} = 33.33$, yields expected price $\tilde{P}_1^{myopic} = 52.1$. The expected inflation rates are thus $\pi_1 = \frac{(\tilde{P}_1 - P_0)}{P_0} 100\% = 15{,}562\%$ for the nonmyopic money growth rate, and $\pi_1^{myopic} = \frac{(\tilde{P}_1^{myopic} - P_0)}{P_0} 100\% = 5{,}110\%$ for the myopic money growth rate. The expected inflation rate increases by 205% when the government updates its beliefs about the demand function. Moreover, according to Cagan's definition of a hyperinflation, which is roughly 8,000% a year, the myopic money growth rate is not hyperinflationary but the nonmyopic (rational) money growth rate is.

The results from this example, and for different discount parameters are summarized in Table 2.1.

In this example the government's ability to learn implies an increase in the expected inflation rate of about 205% for a discount parameter of one. As the discount parameter decreases, the non-myopic maximization problem resembles more and more the myopic problem, for example, if the discount parameter is 0.5 the increase in inflation is of 28%; if the discount parameter is 0.1 the increase in inflation is only of 4%.

Table 2.1 - Summary of Example

δ	g_1*	g_{myopic}	% Increase	Inflation	Myopic Inflation	% Increase
1	3,622%	3,333%	8.7%	15,562%	5,110%	**205%**
0.9	3,592%	3,333%	7.7%	9,693%	5,110%	**90%**
0.8	3,563%	3,333%	6.9%	8,369%	5,110%	**64%**
0.7	3,535%	3,333%	6.1%	7,588%	5,110%	**49%**
0.6	3,506%	3,333%	5.2%	7,003%	5,110%	**37%**
0.5	3,477%	3,333%	4.3%	6,544%	5,110%	**28%**
0.4	3,448%	3,333%	3.5%	6,167%	5,110%	**21%**
0.3	3,420%	3,333%	2.6%	5,858%	5,110%	**19%**
0.2	3,391%	3,333%	1.7%	5,578%	5,110%	**9%**
0.1	3,362%	3,333%	0.9%	5,331%	5,110%	**4%**
0.0	3,333%	3,333%	0%	5,110%	5,110%	**0%**

Table 2.1 showed the government's expected inflation rates. These are however not necessarily the true inflation rates. Depending on the value of the shocks and of the true demand function the increase in prices can be higher or lower. Table 2.2 for example presents the largest and smallest possible inflation rates under both myopic and

non-myopic behavior. To find these inflation rates I first find the largest and smallest possible prices. The largest possible price $\left(\overline{P_1}\right)$ will result when the true demand function is $9-0.19g_1+\varepsilon$ and the shock is $\varepsilon=-2.115$, and the lowest price $\left(\underline{P_1}\right)$, when the true demand is $9-0.08g_1+\varepsilon$ and the shock is $\varepsilon=2.115$, that is,

$$\overline{P_1}=\frac{M_0(1+g_1)}{9-0.19g_1-2.115},$$

and

$$\underline{P_1}=\frac{M_0(1+g_1)}{9-0.08g_1+2.115},$$

with the lowest and highest prices under the myopic rate $\left(\underline{P_1^{myopic}},\overline{P_1^{myopic}}\right)$ being analogous for the myopic money growth rate.

The lowest and highest inflation rates are thus given by $\overline{\pi_1}=\frac{(\overline{P_1}-P_0)}{P_0}$ and $\underline{\pi_1}=\frac{(\underline{P_1}-P_0)}{P_0}$. This minimum and maximum inflation rates are shown in Table 2.2.

In the absence of discounting ($\delta=1$), the increase in the inflation rate can be as little as 12% and as high as 18,000%, thus depending on the correct model (i.e. on the true demand function), learning can lead to drastic increases in the inflation rate.

Although the numbers on Table 2.2 can be rather large, they are not necessarily unrealistic. In Hungary for example between March of 1923 and February of 1924 the total increase in prices was 4,400%, and between August of 1945 and July of 1946 the inflation rate was about $3.81x10^{27}$%[27].

Table 2.2 - Minimum and Maximum Inflation Rates[28]

δ	$\overline{\pi_1}$	$\overline{\pi_1^{myopic}}$	% increase	$\underline{\pi_1}$	$\underline{\pi_1^{myopic}}$	% increase
1	$5x10^6$%	27,871%	18,679%	1,938%	1,729%	12%
0.9	275,880%	27,871%	890%	1,916%	1,729%	11%
0.8	142,862%	27,871%	413%	1,894%	1,729%	10%
0.7	96,977%	27,871%	248%	1,874%	1,729%	8%
0.6	72,472%	27,871%	160%	1,853%	1,729%	7%
0.5	57,656%	27,871%	107%	1,832%	1,729%	6%
0.4	47,731%	27,871%	71%	1,811%	1,729%	5%
0.3	40,830%	27,871%	46%	1,790%	1,729%	4%
0.2	35,434%	27,871%	27%	1,770%	1,729%	2%
0.1	31,233%	27,871%	12%	1,749%	1,729%	1%
0.0	27,781%	27,871%	0%	1,729%	1,729%	0%

To complete the analysis of the impact of learning on the model I want to measure the gains of learning on seignorage. In Table 2.3 I present the total collection of

[27] Cagan (1956), Table 1, p. 26.
[28] These numbers can vary significantly depending on how many decimal places I work with. For example if I work with $g = 36.2222$, then $\overline{\pi_1} = 6x10^6\%$, if I use instead $g = 36.22$, then $\overline{\pi_1} = 5x10^6\%$.

seignorage over the two periods, and the ratio of the percentage increase in seignorage over the percentage increase in the money growth rate. Formally the measure is given by

$$e_{S,g} = \frac{\frac{S_1 + \delta S_2}{(1+\delta)S_{myopic}} - 1}{\frac{g_1}{g_{myopic}} - 1}.$$

This last measure can be interpreted as the elasticity of real seignorage to a 1% experiment, that is, $e_{S,g}$ measures the percentage increase in seignorage from a 1% increase in the money growth rate, where both increases are measured with respect to the myopic levels and rates.

Table 2.3 - Gains in Seignorage from the Experiment

δ	$S_1 + \delta S_2$	$(1+\delta)S_{myopic}$	g_1*	g_{myopic}	$e_{S,g}$
1	327	300	3,622%	3,333%	1.04
0.9	309	285	3,592%	3,333%	1.09
0.8	291	270	3,563%	3,333%	1.15
0.7	274	255	3,535%	3,333%	1.21
0.6	256	240	3,506%	3,333%	1.28
0.5	238	225	3,477%	3,333%	1.36
0.4	221	210	3,448%	3,333%	1.45
0.3	203	195	3,420%	3,333%	1.56
0.2	185	180	3,391%	3,333%	1.68
0.1	168	165	3,362%	3,333%	1.83
0.0	150	150	3,333%	3,333%	-

Increasing the money growth rate by 1% above the myopic rate increases total real seignorage collections by at least 1.04%. Moreover, the lower the discount parameter the higher the elasticity of real seignorage to the experiment. This happens because for low discount parameters the myopic and non-myopic money growth rates are more similar, and relatively low compared to the nonmyopic money growth rates that result from high δ's. Since marginal seignorage is decreasing in g, the higher the money growth rate (i.e. the higher δ) the smaller the increase in seignorage from a 1% increase in the money growth rate.

2.7 Some Intuition on the Model

All the results of the paper hinge on what is the relationship between the two possible demand functions, whether they fan in or out. There is nothing in the theory that says which way is the correct assumption. An alternative way to present the problem is to assume that for a very long time the government has set a constant money growth rate, and thus one point in the demand function is observed, e.g. m_0, g_0. The two possible demand functions then look like Figure 2.7.

Given this assumption, the government will always find it optimal to experiment because it can always generate information. Since highly inflationary economies are characterized by increasing money growth rates, it seems reasonable to assume that these countries operate in region II of Figure 2.7. According to Proposition 2.3 then for these highly inflationary economies, the more spread out the demand functions, the more likely a hyperinflation.

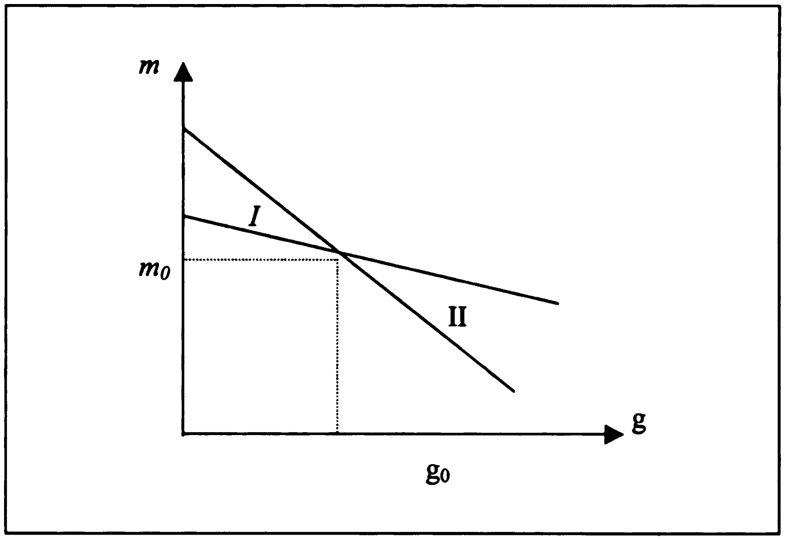

Figure 2.7. Intuitive Representation of the Model

In the next section I present a case study for Bolivia, and characterize the growing uncertainty in this country at the time the hyperinflation process was getting started as a possible application of my model.

2.8 Case Study: The Bolivian Hyperinflation of 1984-1985

Between April 1984 and September 1985, Bolivia experienced a 26,000 percent inflation rate. Its immediate cause was the government's loss of international creditworthiness in the 1980's. In this short case study however, I allege that Bolivia's hyperinflation could have been in part the result of, or at least could have been worsened by, the government's uncertainty about the economy during the first half of the 1980's.

From 1964 to 1978 Bolivia was ruled under an uninterrupted military regime. During this time the government financed itself mostly by accumulating debt. From 1978 to 1982 there was significant political instability, in particular, eleven heads of state in four years, violent coups, one accidental death, interim governments, and fraudulent elections. During this period, the various Bolivian governments relied heavily on foreign borrowing to finance government expenditures. However, the combination of the buildup of international debt, the poor macroeconomic management, weak tax system, poor export prospects, and the preceding political instability, precluded the Bolivian government from obtaining new international loans after 1981. When the borrowing stopped, instead of raising taxes or cutting expenditures, the government expanded money supply rapidly, setting off the hyperinflationary process.

Proposition 2.3 shows that the degree of uncertainty about money demand, output and interest rate determination affects the optimal money growth (and inflation) rate, and since some specific aspects of the Bolivian economy make it reasonable to argue that the knowledge that the government had of the economy was deteriorating precisely at the time that the hyperinflation process started, I believe that uncertainty might have lead (at least in part) to the Bolivian hyperinflation.

"The political chaos between 1978 and the end of 1982 had a paralyzing effect on the economy. The uncertainties that arose from this situation delayed recognition of the external disturbances that the national economy faced and obstructed the process of decision-making needed to take appropriate action. Political antagonists attributed the effects of external shocks to their political foes, instead of looking at the true causes... Bolivia remained largely isolated from the ongoing discussion in academic and official circles about the way to cope with the crisis. Some of the macroeconomic mistakes that were made can be attributed to this isolation." (Morales and Sachs (1990), p. 185.)

After 1982, the government was not as well informed as it predecessors: there is in general a lack of fiscal data, and what exists is mostly for the central government, and not the consolidated public sector. The lack of data can lead to misdiagnosis of the economy, under or overestimations of policy effects, and to inadequate econometric analysis.

In 1982 the coca market boomed and the share of coca exports increased, making the predictions of exports and output, and thus money demand hard to estimate.

In 1983 wage indexation was introduced for the first time but eliminated two years later, an indication that the policy did not achieve the expected outcome, and also an indication of the government's lack of understanding of the economy.

In 1984 an important black market for dollars is developed, smuggling becomes significant, and a large portion of imports go unrecorded; the official demand for dollars goes down dramatically and unexpectedly, and capital flight reached new highs, all of which makes predictions of money demand more complicated.

It is also reasonable to assume that money demand shifted in this period in its sensitivity to expected inflation, inflation uncertainty and the underlying monetary disequilibria (Asilis et. al. (1993)).

For all the reasons mentioned above, together with Propositions 2.1 through 2.3, it is reasonable to argue that the uncertainty, which reached its peak in the first half of the 1980's, had an important effect on the Bolivian hyperinflation of 1984-1985.

2.9 Conclusions

In this paper I assumed that the government maximizes seignorage in a two-period model with uncertainty about the money demand function and about the relationship between interest rates and money growth and, real output and money growth. The model is aimed at resembling unstable governments or highly inflationary economies.

The conditions under which the government experiments are similar to the conditions of monopoly experimentation found before by Mirman, Samuelson and Urbano (1993): As long as the value of information is non zero, if experimentation increases information, the growth of money in period one is greater then the myopic rate; if it decreases information, it is lower.

Proposition 2.2 implies that constant money growth rates are in general suboptimal. They are only optimal in the very specific case in which the demand functions differ only by a constant.

Proposition 2.3 implies that if increasing the rate of growth of money increases information, the more spread out the possible demand functions, the higher the probability that experimentation can lead to a hyperinflation.

Marcet and Nicolini (1998), in a study of hyperinflations under bounded rationality have said that "...the reduction in seignorage that is needed to achieve an inflation equal to $\overline{\beta}$ [the maximum tolerable] is often quite moderate, which raises the issue of why governments have used ERR [exchange rate rules] instead of lowering the fiscal deficit (and seignorage) sufficiently. One possible answer is that lowering seignorage by the exact amount requires much more information: it can only be implemented when the government knows exactly the model and all the parameter values, including those that determine the (boundedly rational) expectations [of prices] P^e_{t+1}, and all the shocks." (Marcet and Nicolini (1998), p. 10). They have however, provided no answer to this question. My paper is a first approximation as it shows that the lack of information about the determinants of seignorage (including those that determine the public's expectations) can produce hyperinflations. An extension of my model that includes exchange rate regimes should answer this question in more detail, and should provide more insights in to the process of hyperinflations.

My paper is also similar to Marcet and Nicolini (1998) because both papers study hyperinflations under learning. In their paper the public learns using a switching mechanism that consists of least squares learning during low and stable seignorage levels, and a "tracking" learning mechanism when seignorage becomes high (possibly because of stochastic shocks). Since the rational expectations equilibrium is harder to learn under high levels of seignorage, hyperinflations become more likely. In my paper I do not explicitly model the learning of the public, instead I focus on the government's learning. Furthermore, I do not assume any particular learning mechanism for the public, only that it be a function of monetary policy. An interesting question is how my results would change if I explicitly model the public's expectations. I leave these topics for further research.

CHAPTER 3

MULTIPLE YEAR INCENTIVE CONTRACTS FOR CENTRAL BANKERS: INFORMATIONAL IMPLICATIONS OF LINEAR CONTRACTS

3.1 Introduction

Low and stable inflation is believed to help the economy function better, to prevent fluctuations in employment and output, and erosions in income. However, distortions such as taxes or unemployment benefits, or the existence of a monopolistic competitive sector, which lead to an equilibrium output level that is too low, create incentives for the government or the central bank to pursue an output target above the natural rate, and hence to set inflation above the socially desirable rate.

In order to avoid or reduce the inflation bias two different paths of academic research have emerged, which in turn have led to major structural changes in monetary institutions. The two approaches are the *legislative* approach and the *contracting* or *targeting* approach.

According to the *legislative* approach monetary policy should be delegated to a central bank that is by law free to conduct monetary policy without the interference of the government (instrument independent) but that has the exclusive mandate of attaining price stability (goal dependent). The most important academic contribution in this area is Rogoff (1985) who showed that the government should choose a banker with a weight on inflation stabilization that is higher than society's although not infinite. If this is the case, the bank's reactions to supply shocks are not as radically distorted as when the banker

only cares about inflation and supply shocks are passed entirely through output. Moreover, the benefit of output stabilization outweighs the costs from excess inflation.

Within the same approach Lohmann (1992) showed that the government could do even better than this by appointing a conservative central banker such as Rogoff's but overriding him when shocks were too large.

In the *contracting or targeting* approach the government imposes a target on the banker or allows the banker to explicitly set his targets. The banker is then held accountable for the success of monetary policy through linear contracts, inflation targets, combinations of inflation targets and contracts, or dismissal rules. Walsh (1995) and Svensson (1997) are pioneering studies in this area.

Walsh (1995) showed that if the central banker and the government's preferences were the same there exists a linear contract that leads to the precommitment outcome. The precommitment outcome is found by assuming that the government can credibly precommit to a zero inflation rate (or to the socially desirable rate), which yields the lowest welfare losses of all possible outcomes, but is time inconsistent. Time inconsistency means that after the public's expectations are formed it is no longer optimal for the government to implement the announced rate.

Svensson (1997) found that (in the absence of unemployment persistence) inflation targets are equivalent to Walsh's linear inflation contracts. They both eliminate the inflation bias without a cost in output stabilization making inflation targeting (or linear contracts) superior to Rogoff's conservative banker.

The previous papers assume that both the government and the public know the preferences of the central banker, however, there are at least two reasons why the preferences of the head of the central bank may not be fully known. First, if the government or society selects the central banker at random from the population (or from a group of the population) and if there is disagreement within this group regarding the costs of inflation, the preferences of the banker are a random variable. Second, if the preferences of individuals are a function of the state of the economy (as proposed by Cukierman and Meltzer (1986) and Ball (1995)), which is subject to random shocks, then the preferences themselves follow stochastic processes and are hence not fully known at any point in time.

The unknown nature of the central banker's preferences has been part of academic, political and even judicial discussions. Goodfriend (1986) for example reports on a lawsuit brought against the Federal Open Market Committee (FOMC) by a graduate law student under the Freedom of Information Act of 1966 to make the minutes from the meetings available as soon as these were over, whilst the directives of the Federal Reserve argued for secrecy on the grounds that "such disclosure would be injurious to its function and the nation's monetary and economic status"[29] and that "uncertainty in monetary markets best serves (FOMC) needs."[30,31]

In Goodfriend (1986) there is also a reference to a 1983 report released by the House Banking Committee in which the Democratic majority's opinion characterized the Fed's directives as "having a 'near obsession' with secrecy about its goals and actions"[32].

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[29] This excerpt comes from the lawsuit itself as reported by Goodfriend (1986) p.67.
[30] Goodfriend (1986) p. 68.
[31] After several years and several appeals the Court eventually ruled in favor of the FOMC.
[32] Goodfriend (1986) p. 63.

More recently Alan Greenspan said in Congress, "If I've made myself clear, you've misunderstood me"[33].

Assuming that the preferences of the banker are unknown, Beetsma and Jensen (1998), Muscatelli (1998a), (1998b), (1999) and Schaling, et. al. (1998) address the accountability problem previously studied by Rogoff, Walsh and Svensson under conditions of certainty. The three main findings of these studies are, first that uncertainty about the banker's preferences affects the variability of output and inflation, and hence welfare. Second, that under uncertain central banker's preferences the trade-off between the inflation bias and output stabilization reemerges resurrecting a possible role for Rogoff's conservative central banker. Third, that the equivalence between Svensson's inflation targets and Walsh's linear contracts breaks down. In particular inflation contracts are superior to targets. For example in Beetsma and Jensen (1998) the reason is that contracts help to stabilize the output and inflation variability induced by the uncertainty, while targets only affect the mean inflation rate and output level.

Two correlated issues also arise from these models. On the one hand, the degree of uncertainty about the banker's preferences affects the welfare of society, the optimal delegation agreement, and the losses of the central banker. On the other hand, the inflation rate contains statistical information about the banker's preferences, that is, for any given delegation arrangement and for every set of preferences[34] there is a corresponding inflation rate. In the absence of supply shocks this would imply that after the first period the government and the public fully learn the preferences of the banker. In a stochastic model although learning is not complete (that is there is always a positive

[33] As reported in David Smith's (October 22, 2000) Sunday Times column "Dim Wim's darkest hour".
[34] The set of preferences is determined by the relative weighs the banker sets on output and inflation stabilization.

probability that the banker has a given set of preferences), the expected value and the variance of the parameters that characterize the banker's preferences changes. Hence the degree of uncertainty about the banker's preferences evolves over time. Hence a rational government, private sector and central banker should account for the effect of the inflation rate on their beliefs (i.e. on the expected value and variance of the parameters that characterize the banker's preferences) and on their expected losses. However, I do not know of any paper that incorporates the government's and the public's updating of beliefs in the study of central bank accountability. My paper is the first to study the performance of linear contracts (or any other delegation agreement for that matter) when the government and the public learn about the banker's preferences based on the observed inflation rate.

The paper is divided in two parts. The purpose of Section 3.2 is to introduce the general model and related literature. The bulk of this section is not original research but mostly a review of the literature that motivates the model. A textbook exposition can be found in Walsh (2000), and also most of the mathematical derivations can be found in Persson and Tabellini (1993). In this section I show that linear contracts that reproduce the optimal precommitment outcome only exist contingent on the bank's private information. Contingent contracts are either very costly or impossible to enforce. I then show the best alternative contract that the government can design, the contract that minimizes social welfare losses subject to the bank's inflation rate and the public's expectations. This contract however might be inferior to the contract that accounts for the government's learning process.

In Section 3.3 I extend the model to add learning, which is the point of departure of this paper. I assume that there is a limit to the number of periods that the bankers can be in office, which seems to be corroborated by several real life examples, two of which are the U.S. Federal Reserve Bank and the European Central Bank. In the U.S. the seven members of the board of the Federal Reserve can serve up to a full term of 14 years. One term begins every two years. A member who serves a full term may not be reappointed, but a member who completes an unexpired portion of a term may be reappointed[35]. The directors of the European Central Bank are appointed to eight-year nonrenewable contracts[36].

For simplicity I will assume that the bankers can be hired for two periods at the most. I assume that the government offers one period contracts to the central bankers[37] – the myopic, static, or one-shot government – or two period contracts, where the terms of the contract are determined at the beginning of each period[38]. If bankers are offered one period contracts, they cannot be reappointed.

If the government offers one period contracts and hires a new banker in the second period the government does not care about learning about the banker's preferences. However, since the inflation rate has information about the bankers'

[35] The chairman of the board of governors is chosen from among the members of the board to serve for four-year terms. This appointment may be renewed several times.
[36] According to Article 11 of the Protocol of the Statute of the European System of Central Banks and the European Central Bank (ECB), the term of office of the Executive Board of the ECB (the president, vice president and the other four members) shall be eight years, and shall not be renewable. The initial appointment of the Executive Board (Article 50) shall be for eight years for the president, four years for the vice president, and fixed terms between five and eight years for the other four members. None of these terms of office are renewable.
[37] Note that a period can be interpreted as one year, 18 months or two years, depending on how long it takes monetary policy to take effect.
[38] An example of multiple year contracts in which the terms of the contract are negotiated yearly is New Zealand where the inflation targets (the basis for dismissing the governor) are negotiated between the government and the bank while the governor is under contract. A simpler example is a wage increase set by the government at the beginning of each year.

preferences the government can induce more informative inflation rates than the myopic rates. The government can thus offer two-period contracts and use the new knowledge about the banker's preferences in future decisions. In particular I assume that the government learns using a Bayes rule mechanism.

Finally, I assume that there are only two types of bankers; a simplifying assumption that allows me to study the bank's incentives to manipulate information and the government's to reply accurately, and the effect of learning on the optimal number of years that the banker should be appointed to.

For some parameter values I find that the government, which I assume is the faithful agent of society, and thus society benefits from more information about the banker's preferences. Given that it is said that monetary policy is transparent if there is little uncertainty about the banker's preferences[39], transparency of monetary policy can thus increase the welfare of society.

For the parameter space for which the government values information, the banker does not necessarily benefit from a better informational state of the government. Moreover, as long as the banker can affect the beliefs of the government it will do so. That is, the central banker acts strategically by increasing or reducing the likelihood as perceived by the government of its type being revealed.

Since the government forecasts the behavior of the bank when it designs the contract, the optimal contract accounts for the bank's expected manipulation of information.

[39] There are actually two aspects to monetary transparency: transparency of policy goals (which I study) and transparency of policy apparatus and procedures.

The degree of transparency that can be achieved through multiple year contracts is also a contributing factor in the optimal length of time of the contract. Studies about the optimal length of time to which bankers are appointed have focused on the relation between central bank independence, inflation, and turnover rate of central bankers. It is believed that the optimal length should be long enough to insulate the bank from political pressures, timed rightly so as to reduce the uncertainty from the turnover of political leadership, but not long enough to lock in bad policies. See for example O'Flaherty (1990), Waller (1992), Waller and Walsh (1996) and Garcia de Paso (2000).

In my model the results are driven by the government's and the central banker's value of more transparent monetary policy, and by the excess variability in the inflation rate induced by the banker's manipulation of information, which are the two effects of the strategic behavior of the banker on the government's loss function.

In Section 3.4 of the paper I summarize the most important results and discuss several possible extensions of the model.

3.2. The Model

The notation and general set up of the model follows Beetsma and Jensen (1998).

The government has preferences over output and inflation according to the function,

$$L_t^G = \frac{1}{2}\left[\lambda(y_t - \bar{y})^2 + (\pi_t - \bar{\pi})^2\right], \tag{3.1}$$

where $(y_t - \bar{y})$ are deviations of output around a target level, and $(\pi_t - \bar{\pi})$ are deviations of inflation around a target rate. These targets are the government's preferred values for output and inflation.

The parameter λ determines the government's preferences for output over inflation stabilization.

Output is determined according to the following supply equation,

$$y_t = \pi_t - \pi_t^e + \varepsilon_t, \tag{3.2}$$

where π_t^e are the public's expectations of inflation, and thus $(\pi_t - \pi_t^e)$ is unexpected inflation. ε_t are serially uncorrelated supply shocks with zero mean, variance σ_ε^2, and density function $f(\varepsilon)$.

Equation (3.2) can be motivated as arising from the presence of nominal wage contracts that are signed prior to the setting of the money growth rate based on the public's expectations of the inflation rate. If actual inflation is higher than expected, real wages decrease, and firms demand for labor, and hence output, increase. If actual inflation falls short of the expected rate, real wages increase and firms reduce employment.

The public's expectations are formed before ε is observed. However, before the central bank (CB) sets inflation it observes the shock ε. The role of private information is to emphasize that the government often has problems knowing what central bankers know. For example, a monetary expansion could be the central bank's answer to its forecast about the velocity of money; or it could be a desire to expand output. The government however, has no means to tell one reason from the other. This assumption is discussed in more detail in Canzoneri (1985) and in Garfinkle and Oh (1995).

An implication of equation (3.2) is that the natural rate of unemployment is zero (a convenient normalization). This formulation also implies that the government's target for output is above the natural output level, $\bar{y}>0$. This assumption is crucial in the literature as the existence of this difference gives rise to the government's (and the bank's) incentives to inflate above the inflation target.

Barro and Gordon (1983) and Calvo (1978) suggest that in the presence of labor market distortions such as unemployment compensation and income taxation, the natural output level will tend to be inefficiently low. Another motivation (Walsh (2000)) can be provided by the presence of a monopolistic competitive sector that leads to an equilibrium output level that is too low. If this is the case the government might pursue an output level above the natural level.

The value of $\bar{y}$ is determined from the government's fiscal side optimization problem. I will not address this problem in this paper. Instead I will assume that monetary policy is carried out given a predetermined value for $\bar{y}$.

Precommitment

I first present the precommitment outcome, when the government is able to commit to the announced inflation rate. Since in this case welfare losses are at a minimum it serves as the benchmark case.

The sequence of events in the model is as follows. First the public sets expectations π_t^e and based on these expectations the public sets nominal wages. Then the supply shock ε_t is realized. The government, who functions as CB, observes ε_t and sets inflation. Hence output is realized.

The government sets inflation by minimizing the expected present discounted value of (3.1) subject to (3.2) and subject to the restriction that average inflation equals announced inflation on average (the precommitment assumption). The resulting inflation rate is given by[40],

$$\pi_t^{precommitment} = \bar{\pi} - \frac{\lambda}{1+\lambda}\varepsilon . \tag{3.3}$$

The optimal response of inflation to a supply shock is negatively proportional to ε. A negative aggregate supply shock lowers output, and to stabilize output, inflation should be increased. The more the government cares about output stabilization relative to inflation stabilization, i.e. the larger λ, the larger $\frac{\lambda}{1+\lambda}$, and hence the larger the adjustment in inflation.

Expected losses for the government under precommitment are given by $E(L^{G,precommitment}) = \frac{\lambda}{2(1+\lambda)}\sigma_\varepsilon^2 + \frac{\lambda}{2}\bar{y}^2$.

It is well known that this policy is time inconsistent and therefore not credible. Once expectations are formed according to the policy rule given by (3.3) and nominal wages set, the government has incentives to deviate from (3.3) to stimulate output.

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[40] Given that the government acts as the bank, it knows all the variables at the time of its minimization problem. Thus it minimizes (1) with respect to π_t and π_t^e, subject to the restriction that expected inflation equals announced inflation on average. I thus minimize the Lagrangian,

$$\frac{1}{2}\left[\lambda(\pi_t - \pi_t^e + \varepsilon_t - \bar{y})^2 + (\pi_t - \bar{\pi})^2\right] + \omega\left[\pi_t^e - E(\pi_t)\right],$$

with respect to π_t, π_t^e and ω, where ω is the Lagrangian multiplier associated with the constraint that π_t^e equal the mathematical expectation of π_t, conditional on the public's information.

The first order conditions are (i) $\lambda(\pi_t - \pi_t^e + \varepsilon_t - \bar{y}) + (\pi_t - \bar{\pi}) - \omega\frac{dE(\pi_t)}{d\pi_t} = 0$, (ii) $-\lambda(\pi_t - \pi_t^e + \varepsilon_t - \bar{y}) + \omega = 0$ and (iii) $\pi_t^e - E(\pi_t) = 0$. Taking the expectation of (ii) and combining it with (iii) yields $\omega = -\lambda\bar{y}$. Taking the expectation of (i) and combining with $\omega = -\lambda\bar{y}$ and $\pi_t^e = \bar{\pi}$ yields $\frac{dE(\pi_t)}{d\pi_t} = 1$. Substituting $\frac{dE(\pi_t)}{d\pi_t} = 1$ and $\omega = -\lambda\bar{y}$ in (i) yields equation (3.3). More detailed explanations of this precommitment scenario can be found in Persson and Tabellini (1993).

Therefore it is more realistic to assume that the government is not able to precommit to an announced inflation rate.

Discretion without delegation

If the government is not able to bind itself to its announcements, the optimal inflation rate is found by minimizing (3.1) with respect to π_t, subject to (3.2) and taking expectations of inflation as given. The first order condition of the government yields,

$$\pi_t^{discretion\ w/o\ delegation} = \frac{\lambda(\pi_t^e + \bar{y} - \varepsilon) + \bar{\pi}}{(1+\lambda)}. \tag{3.4}$$

Since the public is aware that the government sets inflation according to the rule (3.4) expectations of inflation are given by,

$$\pi_t^e = E(\pi_t^{discretion\ w/o\ delegation}) = E\left(\frac{\lambda(\pi_t^e + \bar{y} - \varepsilon) + \bar{\pi}}{(1+\lambda)}\right) \Rightarrow \pi_t^e = \lambda\bar{y} + \bar{\pi}, \tag{3.5}$$

where E represents the expectation given the public's information.

Expected inflation exceeds the optimal target $\bar{\pi}$ by $\lambda\bar{y}$ because of the incentives to create an inflation surprise in order to stimulate output.

Substituting (3.5) in (3.4) gives the discretion equilibrium inflation rate in the absence of delegation,

$$\begin{aligned}\pi_t^{discretion\ w/o\ delegation} &= \bar{\pi} - \frac{\lambda}{1+\lambda}\varepsilon + \lambda\bar{y} \\ &= \pi_t^{precommitment} + \lambda\bar{y}\end{aligned}.$$

The government sets an inflation rate above the optimal precommitment rate by $\lambda\bar{y}$ in order to bring output closer to the target.

Expected losses of the government under discretion without delegation are $E(L^{G,discretion\,w/o\,delegation}) = \frac{\lambda}{2(1+\lambda)}\sigma_{\varepsilon}^{2} + \frac{\lambda(1+\lambda)}{2}\bar{y}^{2}$, which are higher than expected losses under precommitment. The government could thus try to delegate monetary policy to an independent central bank to try to reduce these losses.

Delegation with discretion

Assume now that monetary policy is conducted by an instrument-independent central bank, whose preferences are uncertain at the moment of delegation. Assume that the CB's loss function is given by,

$$L_t^{CB} = \frac{1}{2}\left[(\lambda-\alpha)\ (y_t-\bar{y})^2 + (1+\alpha)(\pi_t-\bar{\pi})^2\right], \tag{3.6}$$

where $\alpha \in (-1,\lambda)$ is a stochastic parameter unobserved by the government and the private sector.

Assume that in the first period the government and the public have some beliefs about α, such that the expected value of α is given by $E(\alpha)$, and its variance by σ_{α}^{2}. Finally, assume that α is independent of ε, hence $E(\alpha\varepsilon)=0$ [41].

Note that if $\alpha=-1$, the banker is an "employment nutter" – according to Mervyn King's (1997) jargon – a banker that only cares about output; if $\alpha=\lambda$, the bank is an

[41] The way uncertainty is modeled in Beetsma and Jensen (1998) is referred to as "pure" uncertainty. It is called pure uncertainty because the sum of the coefficients of the banker's function equals the sum of the coefficients of the government's function, i.e. $(\lambda-\alpha)+(1+\alpha)=1+\lambda$, which implies that the preferences of the banker on average coincide with the preferences of the government (or society), and that monetary policy will on average coincide with monetary policy in the absence of uncertainty about the banker's preferences. A way to relax this assumption is to assume the following objective function for the banker, $L_t^{CB} = \frac{1}{2}\left[\lambda\ (y_t-\bar{y})^2 + (1+\chi)(\pi_t-\bar{\pi})^2\right]$, where χ is a stochastic parameter.

"inflation nutter" because it only cares about inflation; if $\alpha = 0$ the banker's and the government's preferences are the same.

The central bank chooses inflation by minimizing the discounted value of (3.6) subject to (3.2) and taking the expectations of inflation as given, which results in an inflation rate, $\pi_t^{discretion}$ given by,

$$\pi_t^{discretion} = \frac{(\lambda - \alpha)(\pi_t^e + \bar{y} - \varepsilon) + (1+\alpha)\bar{\pi}}{(1+\lambda)}. \tag{3.7}$$

Since the public is aware that inflation is set to satisfy (3.7), inflation expectations are given by,

$$\pi_t^e = E(\pi_t^{discretion}) = E\left(\frac{(\lambda - \alpha)(\pi_t^e + \bar{y} - \varepsilon) + (1+\alpha)\bar{\pi}}{(1+\lambda)}\right)$$

$$\Rightarrow (1+\lambda)\pi_t^e = (\lambda - E(\alpha))(\pi_t^e + \bar{y}) + (1+E(\alpha))\bar{\pi}.$$

Canceling terms and rearranging yields,

$$\pi_t^e = \bar{\pi} + \frac{(\lambda - E(\alpha))}{(1+E(\alpha))}\bar{y}. \tag{3.8}$$

Substituting (3.8) in (3.7) gives the equilibrium inflation rate when monetary policy is delegated to the CB,

$$\begin{aligned}\pi_t^{discretion} &= \bar{\pi} - \frac{(\lambda - \alpha)}{1+\lambda}\varepsilon + \frac{(\lambda - \alpha)}{1+E(\alpha)}\bar{y} \\ &= \pi_t^{precommitment} + \frac{(\lambda - \alpha)}{1+E(\alpha)}\bar{y} + \frac{\alpha\varepsilon}{1+\lambda}\end{aligned}. \tag{3.9}$$

The term $\frac{\alpha\varepsilon}{(1+\lambda)}$ represents the distortions on the reactions to supply shocks relative to what is socially optimal. The term $\frac{(\lambda-\alpha)}{1+E(\alpha)}\bar{y}$ represents the bank's incentives to create output surprises.

A negative aggregate supply shock, which lowers output, induces an increase in inflation to stabilize output. The CB increases inflation by $-\frac{(\lambda-\alpha)}{1+\lambda}\varepsilon$, which is less than the increase that the government would like for positive values of α, and more than the government would like for negative values of α.

Finally, the larger α, i.e. the less the central banker cares about output, the smaller the incentives of the CB to create output surprises.

The expected losses of the government under delegation are given by, $E(L^{G,discretion}) = \frac{\lambda+E(\alpha^2)}{2(1+\lambda)}\sigma_\varepsilon^2 + \frac{(\lambda+E(\alpha^2))(1+\lambda)}{2(1+E(\alpha))}\bar{y}^2$. Since delegation of monetary policy to the central bank does not bring inflation down to the precommitment level, nor does it improve on welfare, the government can try to design contracts that give incentives to the CB to set the precommitment inflation rate. Walsh (1995) showed that as long as the government and the bank have the same preferences such a contract exists. We now study whether it also exists when the bank does not necessarily share the government's preferences over output and inflation, and furthermore, the government is unsure about these preferences.

Linear contracts

Assume that the government wants to design a contract to try to induce the CB to set the precommitment inflation rate. Let the new loss function of the CB be given by,

$$L_t^{CB} + m_t(\pi_t - \bar{\pi}), \tag{3.10}$$

where m_t defines the contract: If actual inflation is above the target the banker incurs a penalty of size m_t for each additional unit of inflation above the target.

The purpose of the contract is to increase the marginal cost of inflation of the banker (by m_t). The government therefore wants to artificially generate conservative central bankers (such as Rogoff's) without inducing distortions in the reactions to supply shocks (if possible).

Note that the contract is not meant to reflect the preferences of the government over inflation stabilization. That is, the government dislikes inflation above or below $\bar{\pi}$, but bankers are not punished for setting inflation below $\bar{\pi}$. The purpose of the contract is to increase the banker's marginal cost of inflation to induce lower inflation rates (if possible to induce the precommitment inflation rate, $\bar{\pi} - \frac{\lambda}{1+\lambda}\varepsilon$)[42].

In order to set the contract m_t the government forecasts the bank's reaction function. The central bank minimizes the discounted expected value of (3.10) subject to (3.2) with respect to π_t, and taking the expectations of inflation as given, which is

[42] This issue also arises when contracts are defined as is traditionally done in the literature, as $m_t \pi_t$. In this case, bankers are punished by setting any positive inflation rate, even the socially desirable inflation rate $\bar{\pi}$, or the precommitment inflation rate. The relevant point however is that the marginal cost of inflation also increases by m_t. Furthermore, if the bankers and the government have the same preferences, then the government can manipulate the banker's marginal cost of inflation, inducing the precommitment inflation rate without inducing distortions in the stabilization of supply shocks (Walsh (1995)).

equivalent to solving a sequence of single period decision problems[43]. The first order condition of this problem is given by,

$$(\lambda - \alpha)(\pi_t - \pi_t^e + \varepsilon_t - \bar{y}) + (1 + \alpha)(\pi_t - \bar{\pi}) + m_t = 0.$$

Collecting π_t,

$$\pi_t = \frac{(\lambda - \alpha)(\pi_t^e - \varepsilon_t + \bar{y}) + (1 + \alpha)\bar{\pi} - m_t}{(1 + \lambda)}.$$

The public is aware of CB's reaction function and thus the expectations of inflation are given by,

$$\pi_t^e = E\left(\frac{(\lambda - \alpha)(\pi_t^e - \varepsilon_t + \bar{y}) + (1 + \alpha)\bar{\pi} - m_t}{(1 + \lambda)}\right).$$

Since the expected value of and $\alpha \varepsilon_t$ is zero,

$$\pi_t^e = \frac{(\lambda - E(\alpha))(\pi_t^e + \bar{y}) + (1 + E(\alpha))\bar{\pi} - m_t}{(1 + \lambda)}.$$

Solving for π_t^e,

$$\pi_t^e = \bar{\pi} + \frac{\lambda - E(\alpha)}{1 + E(\alpha)}\bar{y} - \frac{1}{1 + E(\alpha)} m_t.$$

Plugging π_t^e in the CB's reaction function yields the CB's inflation rate for each contract *m*,

$$\begin{aligned} \pi_t &= \bar{\pi} + \frac{(\lambda - \alpha)}{1 + E(\alpha)}\bar{y} - \frac{(\lambda - \alpha)}{(1 + \lambda)}\varepsilon_t - \frac{(1 + \lambda - \alpha + E(\alpha))}{(1 + \lambda)(1 + E(\alpha))} m_t \\ \pi_t &= \pi_t^{precommitment} + \frac{(\lambda - \alpha)}{1 + E(\alpha)}\bar{y} + \frac{\alpha}{(1 + \lambda)}\varepsilon_t - \frac{(1 + \lambda - \alpha + E(\alpha))}{(1 + \lambda)(1 + E(\alpha))} m_t \end{aligned} \quad . \tag{3.11}$$

[43] Since time periods are independent of each other the term of office to which bankers are appointed is irrelevant so far.

The government would like to set a contract that induces the precommitment outcome, i.e. choose m_t so that $\pi_t^{contract} = \pi_t^{precommitment}$, which is given by $m_t = \left(\frac{(\lambda - \alpha)}{1+E(\alpha)} \bar{y} + \frac{\alpha}{(1+\lambda)} \varepsilon_t \right) \left(\frac{(1+\lambda)(1+E(\alpha))}{(1+\lambda-\alpha+E(\alpha))} \right)$, but this contract is a function on the bank's private information about supply shocks ε and on the unknown parameter α, but neither of these is verifiable. If contingent contracts are not enforceable the government's best alternative (so far) is to minimize the discounted expected value of $L_t^G - m_t(\pi_t - \bar{\pi})$ subject to (3.2) and (3.11) and subject to the public's expectations. Formally the government solves the following problem,

$$\begin{aligned} &\underset{m_t}{Min} \quad E\left[\frac{1}{2} \left[\lambda(\pi_t - \pi_t^e + \varepsilon_t - \bar{y})^2 + (\pi_t - \bar{\pi})^2 \right] - m_t(\pi_t - \bar{\pi}) \right] \\ &s.t. \quad \pi_t = \frac{(\lambda - \alpha)(\pi_t^e - \varepsilon_t + \bar{y}) + (1+\alpha)\bar{\pi} - m_t}{(1+\lambda)} \\ &\& \quad \pi_t^e = \bar{\pi} + \frac{\lambda - E(\alpha)}{1+E(\alpha)} \bar{y} - \frac{1}{1+E(\alpha)} m_t \end{aligned} \quad . \tag{3.12}$$

The approach I use to solve this problem is as follows. I first square the terms inside the brackets and then take the expectation. Finally I differentiate.

Squaring the terms in brackets,

$$E\left[\begin{aligned} &\frac{1}{2}\left[\lambda(\pi_t^2 - 2\pi_t\pi_t^e + 2\pi_t\varepsilon_t - 2\pi_t\bar{y} + \pi_t^{e2} - 2\pi_t^e\varepsilon_t + 2\pi_t^e\bar{y} + \varepsilon_t^2 - 2\varepsilon_t\bar{y} + \bar{y}^2) \right] \\ &+ \frac{1}{2}(\pi_t^2 - 2\pi_t\bar{\pi} + \bar{\pi}^2) - m_t(\pi_t - \bar{\pi}) \end{aligned} \right].$$

Taking expectation using the facts that $E\varepsilon^2 = \sigma_\varepsilon^2$ and $E\pi_t = \pi_t^e$,

$$\left[\frac{\lambda}{2}(E\pi_t^2 - \pi_t^{e2} + 2E(\pi_t\varepsilon_t) + \sigma_\varepsilon^2 + \bar{y}^2) + \frac{1}{2}(E\pi_t^2 - 2\pi_t^e\bar{\pi} + \bar{\pi}^2) - m_t(\pi_t^e - \bar{\pi}) \right],$$

where $E(\pi_t \varepsilon_t) = E\left(\frac{(\lambda-\alpha)(\pi_t^e-\varepsilon_t+\bar{y})+(1+\alpha)\bar{\pi}-m_t}{(1+\lambda)}\varepsilon_t\right) = -\frac{(\lambda-E(\alpha))\sigma_\varepsilon^2}{(1+\lambda)}$

The government's objective function can be written as follows,

$$\frac{1+\lambda}{2}E\pi_t^2+\frac{\left[\lambda(-\pi_t^{e2}+\sigma_\varepsilon^2+\bar{y}^2)\right]}{2}+\frac{\left[-2\pi_t^e\bar{\pi}+\bar{\pi}^2\right]}{2}-m_t\left[\pi_t^e-\bar{\pi}\right]-\frac{\lambda[\lambda-E(\alpha)]\sigma_\varepsilon^2}{(1+\lambda)}, \tag{3.13}$$

where

$$E\pi_t^2 = E\left(\frac{(\lambda-\alpha)(\pi_t^e-\varepsilon_t+\bar{y})+(1+\alpha)\bar{\pi}-m_t}{(1+\lambda)}\right)^2,$$

$$\Rightarrow E\pi_t^2 = E\left(\begin{array}{l}\frac{(\lambda-\alpha)^2(\pi_t^e-\varepsilon_t+\bar{y})^2+2(\lambda-\alpha)(1+\alpha)(\pi_t^e-\varepsilon_t+\bar{y})\bar{\pi}}{(1+\lambda)^2}\\ +\frac{-2(\lambda-\alpha)(\pi_t^e-\varepsilon_t+\bar{y})m_t+(1+\alpha)^2\bar{\pi}^2-2(1+\alpha)\bar{\pi}m_t+m_t^2}{(1+\lambda)^2}\end{array}\right),$$

$$\Rightarrow E\pi_t^2 = \left(\begin{array}{l}\frac{(\lambda^2-2\lambda E(\alpha)+E(\alpha^2))(\pi_t^{e2}+2\pi_t^e\bar{y}+\sigma_\varepsilon^2+\bar{y}^2)}{(1+\lambda)^2}\\ +\frac{2(\lambda-E(\alpha)+\lambda E(\alpha)-E(\alpha^2))(\pi_t^e+\bar{y})\bar{\pi}}{(1+\lambda)^2}\\ +\frac{-2(\lambda-E(\alpha))(\pi_t^e+\bar{y})m_t+(1+2E(\alpha)+E(\alpha^2))\bar{\pi}^2}{(1+\lambda)^2}\\ +\frac{-2(1+E(\alpha))\bar{\pi}m_t+m_t^2}{(1+\lambda)^2}\end{array}\right)^{44}.$$

Differentiating the government's loss function (equation (3.13)) with respect to m_t,

$$\frac{1+\lambda}{2}\frac{dE\pi_t^2}{dm_t}-\left[\lambda\pi_t^e+\bar{\pi}+m_t\right]\frac{d\pi_t^e}{dm_t}-\pi_t^e+\bar{\pi}=0, \tag{3.14}$$

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[44] When taking the expectation $E\pi_t^2$ I use the assumptions $E\varepsilon=0$ and $E(\alpha\varepsilon)=0$.

which, after substituting $\pi_t^e = \bar{\pi} + \frac{\lambda - E(\alpha)}{1+E(\alpha)}\bar{y} - \frac{1}{1+E(\alpha)}m_t$, $\frac{d\pi_t^e}{dm_t} = -\frac{1}{1+E(\alpha)}$, and

$$\frac{dE\pi_t^2}{dm_t} = \left(\begin{array}{c} -\frac{2}{(1+E(\alpha))}\bar{\pi} - \frac{2(\lambda+\lambda^2 - E(\alpha) - \lambda E(\alpha) + E(\alpha^2) - E(\alpha)^2)}{(1+\lambda)(1+E(\alpha))^2}\bar{y} \\ + \frac{2(1+2\lambda+\lambda^2 + E(\alpha^2) - E(\alpha)^2)}{(1+\lambda)^2(1+E(\alpha))^2} m_t \end{array} \right)$$ yields the

following first order condition,

$$\left(\begin{array}{l} \frac{-(1+\lambda)}{(1+E(\alpha))}\bar{\pi} - \frac{(\lambda - E(\alpha))(1+\lambda) + E(\alpha^2) - E(\alpha)^2}{(1+E(\alpha))^2}\bar{y} + \frac{(1+\lambda)^2 + E(\alpha^2) - E(\alpha)^2}{(1+\lambda)(1+E(\alpha))^2} m_t \\ + \frac{\left(\lambda\left(\bar{\pi} + \frac{\lambda - E(\alpha)}{1+E(\alpha)}\bar{y} - \frac{m_t}{1+E(\alpha)}\right) + \bar{\pi} + m_t\right)}{1+E(\alpha)} - \bar{\pi} - \frac{\lambda - E(\alpha)}{1+E(\alpha)}\bar{y} + \frac{m_t}{1+E(\alpha)} + \bar{\pi} \end{array} \right) = 0.$$

Simplifying and solving for m_t yields the optimal punishment,

$$m_t = \frac{(1+\lambda)(2\lambda - 2E(\alpha) + \lambda E(\alpha) - 2E(\alpha)^2 + E(\alpha^2))}{3 + 3\lambda + E(\alpha^2) - E(\alpha)^2 + 2(1+\lambda)E(\alpha)}\bar{y}. \qquad (3.15)$$

Plugging the optimal contract in equation (3.13) yields the expected (myopic) loss function of the government.

Up to this point this paper has mostly been a review of the state of the literature of linear contracts. In the following section I expand this literature to include the natural possibility of the government and the public learning about the bankers. Since the observed inflation rate is a function of α, and since different contracts (different m's) result in different inflation rates, the government can improve upon the contract m^{myopic} by learning about the parameter α. In particular, if the government sets a contract that induces a riskier distribution of posterior beliefs we say that information increases. In the next section I study how the government's learning affects the delegation problem.

3.3 Delegation with Learning

3.3.1 The Model

Assume now that the government hires the bankers for two periods in order to improve its information about their preferences and thus be able to design more accurate incentives. For simplicity I will work with a simplified model as follows. The central bank and the government contract over the inflation rate, which is unknown at the time the contracts are set. Assume that in general the inflation rate has two components: a deterministic component $\pi(\kappa,\alpha)$ and a stochastic component $g(\alpha,\kappa)\varepsilon_t$, where the government is uncertain about α and ε_t. κ is a set of commonly known parameters that include $\bar{\pi},\bar{y},\lambda,\pi_t^e$ and m_t. Formally,

$$\pi_t(\alpha,\varepsilon_t)=\pi_t(\alpha,\kappa)+g(\alpha,\kappa)\varepsilon_t.$$

The parameter α determines the preferences of the banker, and the function $g(\alpha,\kappa)$, which is a function of the preferences of the bank determines by how much the inflation rate is adjusted in the presence of a supply shock. This function comes from the central banker's minimization problem and is given by $g(\alpha,\kappa)=-\dfrac{\lambda-\alpha}{1+\lambda}$.

I assume that there are only two types of central bankers, thus α can take on two values $\alpha\in\{\alpha_L,\alpha_H\}$, where $\alpha_L<\alpha_H$. That is, the banker of type *H* cares more about inflation stabilization than the type *L* banker.

The first time that an individual is hired μ_0 is the prior belief that the individual is type *L*, i.e. that $\alpha=\alpha_L$; $1-\mu_0$ is the prior that $\alpha=\alpha_H$.

At the beginning of period one, and given prior beliefs μ_0, the government hires a banker and sets the contract m_1. Given this contract the banker sets the first period inflation rate π_1. At the outset of the second period the CB and the government know the first period contract and the first period inflation rate. The government and the public use this information to update beliefs about the CB's preferences using Bayes' rule. Let μ_1 denote the posterior belief that $\alpha = \alpha_L$, then by Bayes rule,

$$\mu_1 = \frac{\mu_0 f_L}{\mu_0 f_L + (1-\mu_0) f_H}, \tag{3.16}$$

where $f_L = f\left(\dfrac{\pi_1 - \pi_1(\alpha_L, \kappa)}{g(\alpha_L, \kappa)}\right)$ and $f_H = f\left(\dfrac{\pi_1 - \pi_1(\alpha_H, \kappa)}{g(\alpha_H, \kappa)}\right)$.

Finally, the random shock ε_t is distributed according to the density function $f(\varepsilon)$ on $-\eta < \varepsilon < \eta$, where $\eta \in \mathbb{R}$ and assume that $E(\varepsilon) = 0$, and that ε_t is distributed independently from α and thus $E(\varepsilon\alpha) = 0$. Three final characteristics of the random shocks are that η must be "large enough" with little weight on the tails of $f(\varepsilon)$ to avoid learning of the banker' preferences in the myopic model, but that η must not be infinite, i.e. $\eta < \infty$. Otherwise the losses of central banking would be unbounded. I explain these assumptions in the next paragraphs.

Since the inflation rate is a stochastic variable, if I assume that the random shocks are appropriately large and that there is little weight on the tails of the density function, that is if $f(-\eta) = f(\eta) \cong 0$, then in equilibrium the type of the banker is not fully revealed after the first period. To illustrate what I mean by "appropriately large" consider the static (myopic) problem of section 3.2 (Beetsma and Jensen's (1998) model), and assume the following parameters, $\mu_0 = 0.5, \lambda = 1, \alpha_L = 0.1, \alpha_H = 0.2, \bar{\pi} = 0$ and $\bar{y} = 0.1$. Using equations (3.11) and (3.15) the inflation rate set in the static model by the L type is given by $\pi_1(\alpha_L, \varepsilon_1) = 0.0289 - 0.45\varepsilon_1$, and the inflation set by H type is given by $\pi_1(\alpha_H, \varepsilon_1) = 0.0226 - 0.4\varepsilon_1$. If the random shock is distributed in the interval $[-\eta, \eta]$, then if the banker is the L type the inflation rate is within the interval $\mathrm{I}_L = [0.0289 - 0.45\eta, 0.0289 + 0.45\eta]$ and if the banker is the H type then the inflation rate is within the interval $\mathrm{I}_H = [0.0226 - 0.4\eta, 0.0226 + 0.4\eta]$. Now assume that $\eta = 0.001$, then $\mathrm{I}_L = [0.028, 0.029]$ and $\mathrm{I}_H = [0.022, 0.023]$. Since the interception between these intervals is empty, i.e. since $\mathrm{I}_L \cap \mathrm{I}_H = \varnothing$, then the type of the banker is fully revealed in the first period with probability one. Shocks are therefore not appropriately large, and my model reduces to Beetsma and Jensen's. Now assume that $\eta = 0.056$. In this case $\mathrm{I}_L = [0.004, 0.054]$ and $\mathrm{I}_H = [0.000, 0.045]$. Then only if the first period inflation rate is very low (below 0.4%) or high (above 4.5%) the government learns that the banker is the L type. If additionally the probability of observing the very high or very low rates (i.e. if there is little weight on the tails of $f(\varepsilon)$) then the probability that the type of banker is revealed is low. If this probability goes to zero then it is safe to assume that the banker's type is not fully revealed in the first period in the myopic model. In this case shocks are appropriately large making this a model of learning.

The last assumption regarding the random shocks is that these must be *only* appropriately large but no larger, and definitely, not infinite. This assumption is necessary in order to avoid unbounded central bankers' losses, which would prevent individuals from accepting the job of central banking. In the example, if $\eta = 0.056$ static losses for the bankers are no higher than 0.02 (for either banker). If however $\eta = \infty$, these losses are infinite. A maximum size of the shock of $\eta = 0.056$ therefore seems appropriate in this example as it satisfies both assumptions: bankers' losses are bounded and there is no full learning in the myopic model.

Note that this model could not be applied to an environment in which to avoid learning shocks must be infinite.

This last assumption also implies that the model is only applicable to stable environments or to economies not subject to extremely large disturbances. For example, this model is not applicable to hyperinflations.

3.3.2 The Equilibrium

Since the decision in period two depends on the expected value of α given posterior beliefs and since posterior beliefs are a function of the first period inflation rate, and hence of the first period contract, the government must take into account when choosing the contract the effect of m_1 on its future beliefs μ_1. The government's objective is thus to minimize expected discounted losses over the two periods. Therefore the model must be solved through backward induction beginning with the second period equilibrium, and within each period beginning with the problem of the central banker.

I will assume that within each period the individual rationality constraint of the central banker is satisfied. This implies that whatever losses the central banker gets in each period are lower than its reservation losses, that is I assume first, that there exists a number $\overline{m} \in \mathbb{R}$ such that if $m_t > \overline{m}$ the bankers do not accept the contract, and second, that the equilibrium contract in every period is lower than the limit contract $\overline{m}$, that is

$m_t{*} < \overline{m}$. With this assumption I rule out (or I do not consider my model to be applicable to) cases in which equilibrium punishments are too large (such as $m_t{*} = \infty$).

Assuming that the participation constraint of bankers is satisfied has been called previously in the literature the "ego rents" of central bankers. This is a sensible assumption in this model given that I have assumed that shocks are only appropriately large to avoid learning but not infinite, and therefore that the losses of bankers are bounded as explained in the previous subsection.

The second period

Given the posterior beliefs the government designs the second period contract. To do so, the government first forecasts the central bank's behavior.

The central bank

The central bank of type *i* (*i=L,H*) sets inflation by minimizing $L_2^{CB,i} + m_2(\pi_2 - \overline{\pi})$ with respect to π_2, where these losses are defined by,

$$L_2^{CB,i} = \frac{1}{2}(\lambda - \alpha_i)\left(\pi_2 - \pi_2^e + \varepsilon_2 - \overline{y}\right)^2 + \frac{1}{2}(1+\alpha_i)\left(\pi_2 - \overline{\pi}\right)^2,$$

for *i=L,H.* As a result the *i*-type's first order condition is given by,

$$(\lambda - \alpha_i)\left(\pi_2 - \pi_2^e + \varepsilon_2 - \overline{y}\right) + (1+\alpha_i)\left(\pi_2 - \overline{\pi}\right) + m_2 = 0.$$

Solving for π_2 yields the reaction function of the banker of type *i*. In particular, the type-*L* banker sets inflation according to,

$$\pi_2(\alpha_L,\varepsilon_2)=\frac{(1+\alpha_L)\bar{\pi}+(\lambda-\alpha_L)(\pi_2^e+\bar{y})-m_2}{(1+\lambda)}-\frac{(\lambda-\alpha_L)}{(1+\lambda)}\varepsilon_2 \\ =\pi_2^L(m_2,\pi_2^e)-\frac{(\lambda-\alpha_L)}{(1+\lambda)}\varepsilon_2 \quad , \tag{3.17}$$

and the *H*-type according to,

$$\pi_2(\alpha_H,\varepsilon_2)=\frac{(1+\alpha_H)\bar{\pi}+(\lambda-\alpha_H)(\pi_2^e+\bar{y})-m_2}{(1+\lambda)}-\frac{(\lambda-\alpha_H)}{(1+\lambda)}\varepsilon_2 \\ =\pi_2^H(m_2,\pi_2^e)-\frac{(\lambda-\alpha_H)}{(1+\lambda)}\varepsilon_2 \quad . \tag{3.18}$$

Since the public knows the reaction function of each banker, it forms expectations rationally forecasting equations (3.17) and (3.18),

$$\pi_2^e=\mu_1 E\pi_2(\alpha_L,\varepsilon_2)+(1-\mu_1)E\pi_2(\alpha_H,\varepsilon_2) \\ =\mu_1\pi_2^L(m_2,\pi_2^e)+(1-\mu_1)\pi_2^H(m_2,\pi_2^e) \quad .$$

Solving for π_2^e,

$$\pi_2^e(m_2,\mu_1)=\bar{\pi}+\frac{(\lambda-\mu_1\alpha_L-(1-\mu_1)\alpha_H)\bar{y}-m_2}{(1+\mu_1\alpha_L+(1-\mu_1)\alpha_H)} . \tag{3.19}$$

Note that the public forms its expectations already knowing the punishment m_2, and thus expectations are a function of the contract designed by the government. Since the CB responds in an optimal way to inflation expectations, we can write the reaction functions of the bankers as functions of the optimal contract as follows,

$$\pi_2(\alpha_L,\varepsilon_2)=\pi_2^L(m_2,\pi_2^e(m_2,\mu_1))-\frac{(\lambda-\alpha_L)}{(1+\lambda)}\varepsilon_2 \\ =\pi_2^L(m_2,\mu_1)+g^L\varepsilon_2 \quad ,$$

and

$$\pi_2(\alpha_H,\varepsilon_2)=\pi_2^H(m_2,\pi_2^e(m_2,\mu_1))-\frac{(\lambda-\alpha_H)}{(1+\lambda)}\varepsilon_2 \\ =\pi_2^H(m_2,\mu_1)+g^H\varepsilon_2 .$$

The government

Given $\pi_2(\alpha_L,\varepsilon_2)=\pi_2^L(m_2,\mu_1)+g^L\varepsilon_2$ and $\pi_2(\alpha_H,\varepsilon_2)=\pi_2^H(m_2,\mu_1)+g^H\varepsilon_2$ the government minimizes its expected losses G_2 with respect to m_2, where

$$G_2(m_2,\mu_1)=\mu_1G_2^L(m_2,\mu_1)+(1-\mu_1)G_2^H(m_2,\mu_1),$$

$$G_2^i(m_2,\mu_1)=L_2^{G,i}(m_2,\mu_1)-m_2[\pi_2^i(m_2,\mu_1)-\bar{\pi}],$$

and

$$L_2^{G,i}(m_2,\mu_1)=E\left[\frac{\lambda}{2}[\pi_2^i(m_2,\mu_1)+g^i\varepsilon_2-\pi_2^e(m_2,\mu_1)+\varepsilon_2-\bar{y}]^2+\frac{1}{2}[\pi_2^i(m_2,\mu_1)+g^i\varepsilon_2-\bar{\pi}]^2\right],$$

for $i=H, L$.

To eliminate the expectation's operator I square the expressions inside the brackets and take the expected value with respect to ε, setting $E\varepsilon=0$, $E(\alpha\varepsilon)=0$, and $E\varepsilon^2=\sigma_\varepsilon^2$,

$$L_2^{G,i}=\begin{pmatrix}\frac{1}{2}\bar{\pi}^2+\frac{(1+\lambda)}{2}\pi_2^i(m_2,\mu_1)^2-\pi_2^i(m_2,\mu_1)[\lambda\pi_2^e(m_2,\mu_1)+\lambda\bar{y}+\bar{\pi}] \\ +\frac{\lambda(g^i+1)^2+g^{i2}}{2}\sigma_\varepsilon^2+\frac{\lambda\pi_2^e(m_2,\mu_1)^2}{2}+\lambda\pi_2^e(m_2,\mu_1)\bar{y}+\frac{\lambda\bar{y}^2}{2}\end{pmatrix}. \quad (3.20)$$

Let $m_2(\mu_1)$ be the contract that minimizes G_2. (Note that the optimal contract $m_2(\mu_1)$ corresponds to the static optimal contract found in the previous section, equation (3.15), with $E(\alpha)=\mu_1\alpha_L+(1-\mu_1)\alpha_H$ and $E(\alpha^2)=\mu_1\alpha_L{}^2+(1-\mu_1)\alpha_H{}^2$).

Finally, let $V^G(\mu_1)$ denote the resulting value function of the government, i.e.

$$\begin{aligned} V^G(\mu_1) &= \underset{m_2}{Min}\, G_2(m_2,\mu_1) \\ &= G_2(m_2(\mu_1),\mu_1) \\ &= \mu_1 G_2^L(m_2(\mu_1),\mu_1) + (1-\mu_1) G_2^H(m_2(\mu_1),\mu_1) \end{aligned},$$

and let $V_i^{CB}(\mu_1)$ denote the value function of the central bank of type i, i.e.

$$V_i^{CB}(\mu_1) = L_2^{CB,i}(m_2(\mu_1),\mu_1) + m_2(\mu_1)(\pi_2^i(m_2(\mu_1),\mu_1) + g_2^i\varepsilon_2 - \bar{\pi}).$$

Period 1

The CB's problem

The banker's second period decision (π_2) is a function of the second period contract $m_2(\mu_1)$, which is a function of the first period inflation rate, and hence the banker's second period losses $(V_i^{CB}(\mu_1))$ are a function of the government's posterior beliefs, and thus in period one the central bank of type i chooses π_1 to minimize the discounted value of the two-period losses,

$$L_1^{CB,i} + m_1(\pi_1 - \bar{\pi}) + \rho E V_i^{CB}(\mu_1), \tag{3.21}$$

taking the expectations of inflation as given, where ρ is the discount parameter,

$$L_1^{CB,i} = \frac{1}{2}(\lambda - \alpha_i)\left(\pi_1 - \pi_1^e + \varepsilon_1 - \bar{y}\right)^2 + \frac{1}{2}(1+\alpha_i)\left(\pi_1 - \bar{\pi}\right)^2,$$

and

$$E V_i^{CB}(\mu_1) = \int V_i^{CB}(\mu_1) f(\varepsilon) d\varepsilon .$$

That is, I must take expectations of the banker's second period losses with respect to ε_2, which is unknown to the CB in period one. Since at the time of its decision the banke

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banker knows all the variables that determine the posterior beliefs μ_1, I do not need to integrate with respect to posterior beliefs.

Let $\pi_1(\alpha_i, \varepsilon_1) = \pi_1^i(m_1) + g^i\varepsilon_1$ be the resulting reaction function of the banker of type i. When choosing the first period contract m_1 the government forecasts these reaction functions and minimizes the discounted expected value of the two-period losses, $G_1(m_1, \mu_0) + \rho EV^G(\mu_1)$, where

$$G_1(m_1, \mu_0) = \mu_0 G_1^L(m_1, \mu_0) + (1-\mu_0)G_1^H(m_1, \mu_0),$$

$$G_1^i(m_1, \mu_0) = L_1^{G,i}(m_1, \mu_0) - m_1[\pi_1^i(m_1, \mu_0) - \bar{\pi}],$$

$$L_1^{G,i}(m_1, \mu_0) = E\left[\frac{\lambda}{2}[\pi_1^i(m_1, \mu_0) + g^i\varepsilon_1 - \pi_1^e(m_1, \mu_0) + \varepsilon_1 - \bar{y}]^2 + \frac{1}{2}[\pi_1^i(m_1, \mu_0) + g^i\varepsilon_1 - \bar{\pi}]^2\right].$$

and

$$EV^G(\mu_1) = \int V^G(\mu_1)\left[\mu_0 f_L + (1-\mu_0)f_H\right]d\pi,$$

that is, when I take the expectation of the government's second period losses $V^G(\mu_1)$ I must integrate with respect to μ_1, which is unknown to the government in the first period. I know from Bayes' rule that for a given value of m_1, μ_1 is a function of the first period inflation rate, which is a random variable in period one (see equation (3.16)). To find the density of this random variable, let $\varepsilon^L(\pi_1, m_1)$ be the value of ε that must appear if the first period inflation rate is π_1, the contract set in the first period is m_1, and the banker is type L; with $\varepsilon^H(\pi_1, m_1)$ being analogous for the H type, then the posterior distribution of π_1 is given by

$$\mu_0 f(\varepsilon^L(\pi_1, m_1)) + (1-\mu_0) f(\varepsilon^H(\pi_1, m_1)).$$

Let m_1 * be the resulting equilibrium contract in period one.

I now want to study how the model with learning differs from the static model. In particular I want to study what the government's ability to learn implies in terms of the behavior of the banker, how the government reacts to the banker's behavior, and for how many periods should the banker be appointed to office.

3.3.3 The Value of "More Transparent" Monetary Policy (The Value of Information)

At the outset of the second period the government knows the first period inflation rate and contract (but not the random shock) and updates beliefs using Bayes rule. If the distribution of beliefs in the second period is riskier in the Rothschild-Stiglitz sense then information increases[45]. If the expected losses of the government are reduced with more information i.e. if the value function of the government is concave in posterior beliefs, then the value of information to the government is positive.

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[45] The definition of more informative experiments comes from Blackwell (1951) and (1953). Formally, an experiment is a pair of measures $\left[f\left(\frac{\pi_1-\pi_1(\alpha_L,\kappa(m_1))}{g^L(\alpha_L,\kappa)}\right), f\left(\frac{\pi_1-\pi_1(\alpha_H,\kappa(m_1))}{g^H(\alpha_H,\kappa)}\right)\right]$, one for each state of nature that gives the distribution of the first period inflation rate. An experiment $\left[f\left(\frac{\pi_1-\pi_1(\alpha_L,\kappa(m_1''))}{g^L(\alpha_L,\kappa)}\right), f\left(\frac{\pi_1-\pi_1(\alpha_H,\kappa(m_1''))}{g^H(\alpha_H,\kappa)}\right)\right]$ is *more informative* than an experiment $\left[f\left(\frac{\pi_1-\pi_1(\alpha_L,\kappa(m_1'))}{g^L(\alpha_L,\kappa)}\right), f\left(\frac{\pi_1-\pi_1(\alpha_H,\kappa(m_1'))}{g^H(\alpha_H,\kappa)}\right)\right]$, if for every continuous, convex function $\chi(\mu_1(m_1,\pi_1)):[0,1]\rightarrow\Re$, then $\int\chi(\mu_1(m_1'',\pi_1))h(m_1,\pi_1)d\pi \geq \int\chi(\mu_1(m_1',\pi_1))h(m_1,\pi_1)d\pi$, where $h(m_1,\pi_1)$ is the distribution o f π_1 implied by m_1 and ε_1.

This is a complex model, and showing that the value function is concave is not straightforward. The reason for this is that information enters into the government's expected loss function in several ways: directly through the inflation rate, but also through the public's expectations of inflation, which in turn feed into the inflation rate themselves. The value of information to the government depends on the interaction between all of these channels. More information for example allows the government to make a better informed decision, reducing its net expected losses (the losses due to the variance of the inflation rate and output level, plus the transfer to bankers - or minus the transfer from bankers[46]).

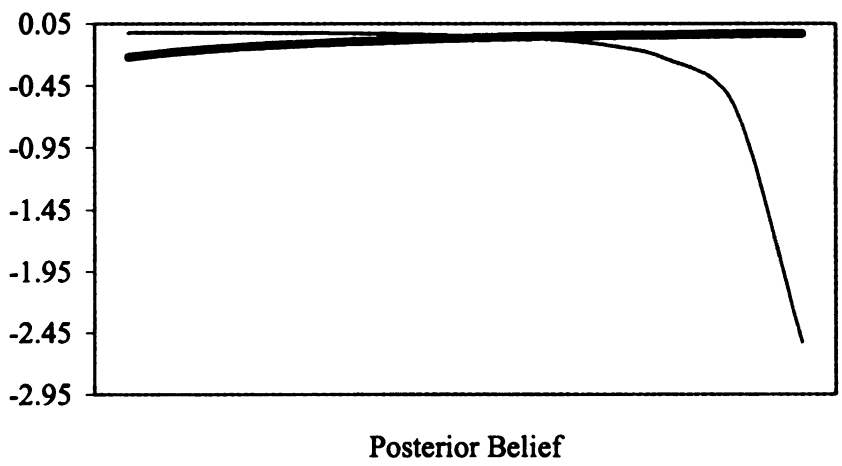
More information also allows the public to make more accurate predictions, reducing surprise inflation and therefore the effectiveness of monetary policy.

The difficulty of the flow of information in the model arises from the public's expectations of inflation. To study the complexity of the value of information consider the example depicted in Figure 3.1. In the Figure I show how the value of information varies between the restricted model in which the public does not update beliefs and the unrestricted model in which it does. The thin curve in the figure shows the second derivative of the government's value function with respect to μ_1. The thick curve shows the same derivative for the restricted model in which the public cannot update beliefs.

For both models and for any value of the posterior beliefs the value of information to the government is positive, that is the second derivative of the value function is negative.

[46] This is a model in which the government cares about the transfer per se, which implies that the government may be willing to trade off the socially optimal inflation rate for costly rents to bankers. These rents are affected by the flows of information as well.

For $\mu_1 < 0.57$ the value of information under the restricted model is higher, but for $\mu_1 > 0.57$ the ordering is reversed. An intuitive explanation is as follows. Starting at $\mu_1 = 0.57$, assume that μ_1 increases. As the probability that the banker is the L type increases, then the expected losses due to higher variance of the inflation rate around $\bar{\pi}$ increase, and the rents to bankers are reduced (or alternatively, their punishment increased). Since information is always valuable, the gains are higher than the losses. However, if the expectations of inflation are fixed, according to the thick curve, as μ_1 increases the gains from higher rents do not increase as rapidly as the losses, that is, as the government learns that the banker is the low type its value of information is reduced (it gets closer and closer to zero).



$\mu_0 = 0.9, \alpha_L = -0.99$, $\alpha_H = 0.99$, $\lambda = 1$, $\bar{\pi} = 0$ and $\bar{y} = 0.1$.

Figure 3.1. Value of Information: Restricted vs. Unrestricted Models

When the public updates beliefs (thin curve), as μ_1 increases, the expectations of inflation increase, which leads bankers to increase the inflation rate. This increases the variability of the inflation rate (further than in the restricted model) but at the same time it also increases the rents to the government. Since according Figure 3.1 the value of information increases as μ_1 increases, then rents increase more rapidly.

The reverse argument would hold for low values of μ_1.

What is most important to grasp from this example is that when information affects the expectations of the public, as the government and the public learn that the banker is the low type, the losses due to more variability in the inflation rate, and the gains from higher rents from bankers intensify, but the gains increase more rapidly, increasing the value of information. When they learn that the banker is the high type, the gains from less variability in the economy and the losses from lower rents intensify as well, but the losses grow more rapidly reducing the value of information to the government.

Although information flows in this model are complex, it is possible to show that for some (reasonable) parameter values the value of information to the government is still positive, as we show in the next proposition.

Proposition 3.1: There exists a (sensible) parameter space for which the value function of the government $V^G(\mu_1)$ is concave in posterior beliefs.

Proof: The value function of the government is given by $V^G(\mu_1) = \underset{m_2}{Min}\, G_2(m_2, \mu_1) = \mu_1 G_2^L(m_2(\mu_1), \mu_1) + (1-\mu_1) G_2^H(m_2(\mu_1), \mu_1)$ where $m_2(\mu_1)$ is the optimal contract. To show that this function is concave in μ_1 I first show that for some parameter values,

$$V^G(\mu_1) \leq \mu_1 G_2^L(m_2(\hat{\mu}_1), \hat{\mu}_1) + (1-\mu_1) G_2^H(m_2(\hat{\mu}_1), \hat{\mu}_1),$$

for all $\hat{\mu}_1 \neq \mu_1$. In a second step I show that since $V^G(\mu_1)$ is the lower envelope of a set of linear functions it must be concave.

Step 1: There exists a sensible parameter space for which $V^G(\mu_1) \leq \mu_1 G_2^L(m_2(\hat{\mu}_1), \hat{\mu}_1) + (1-\mu_1) G_2^H(m_2(\hat{\mu}_1), \hat{\mu}_1)$ for all $\hat{\mu}_1 \neq \mu_1$.

Proof:

By contradiction assume that $V^G(\mu_1) > \mu_1 G_2^L(m_2(\hat{\mu}_1), \hat{\mu}_1) + (1-\mu_1) G_2^H(m_2(\hat{\mu}_1), \hat{\mu}_1)$ for some $\hat{\mu}_1 \neq \mu_1$. Then, provided that there exists an admissible $\tilde{m}$ that is a solution to the following quadratic equation,

$$\mu_1 G_2^L(\tilde{m}, \mu_1) + (1-\mu_1) G_2^H(\tilde{m}, \mu_1) \equiv \mu_1 G_2^L(m_2(\hat{\mu}_1), \hat{\mu}_1) + (1-\mu_1) G_2^H(m_2(\hat{\mu}_1), \hat{\mu}_1),$$

there exists a contract, call it $\tilde{m}(\mu_1, \hat{\mu}_1)$, which yields strictly lower losses than $m_2(\mu_1)$. Since $m_2(\mu_1)$ is the loss minimizing contract this is a contradiction.

The parameter space for which the value function is concave is determined by the existence of a real-numbered solution to the quadratic equation.

Step 2: Given step 1, $\theta V^G(\mu_1') + (1-\theta) V^G(\mu_1'') \leq V^G(\theta\mu_1' + (1-\theta)\mu_1'')$ for $\mu_1', \mu_1'' \neq \hat{\mu}_1$.

Proof:

Since $V^G(\mu_1) \leq \mu_1 G_2^L(m_2(\hat{\mu}_1), \hat{\mu}_1) + (1-\mu_1) G_2^H(m_2(\hat{\mu}_1), \hat{\mu}_1)$, for all $\hat{\mu}_1 \neq \mu_1$, then

$$\theta V^G(\mu_1') \leq \theta \left[\mu_1' G_2^L(m_2(\hat{\mu}_1), \hat{\mu}_1) + (1-\mu_1') G_2^H(m_2(\hat{\mu}_1), \hat{\mu}_1)\right]$$

and

$$(1-\theta)V^G(\mu_1'') \leq (1-\theta)[\mu_1'' G_2^L(m_2(\hat{\mu}_1),\hat{\mu}_1) + (1-\mu_1'')G_2^H(m_2(\hat{\mu}_1),\hat{\mu}_1)]$$

for any two beliefs $\mu_1', \mu_1'' \neq \hat{\mu}_1$ and $\theta \in (0,1)$.

Since this is true for all $\hat{\mu}_1$, it is true for $\hat{\mu}_1 = \theta\mu_1' + (1-\theta)\mu_1''$, in which case adding these two inequalities yields,

$$\theta V^G(\mu_1') + (1-\theta)V^G(\mu_1'') \leq V^G(\theta\mu_1' + (1-\theta)\mu_1''),$$

hence $V^G(\mu_1)$ is concave in μ_1.

□

Note that Proposition 3.1 does not necessarily apply for all possible values of the parameters, i.e. it is possible that for some parameter values the quadratic equation may not have a real solution. In this case Proposition 3.1 cannot be applied.

Given that monetary policy is transparent if there is little uncertainty about the central banker's preferences, and for the parameter space for which the government's (society's) expected losses decrease as the information about the bankers' preferences increases, then more transparent monetary policies are better in terms of welfare. The bankers however do not necessarily benefit from a more transparent monetary policy as shown next.

The definition of the value of information to the CB is a little different from the government's, in particular, the value of information to the central banker is positive if its second period losses decrease as the central banker differentiates itself from the other type of banker. For example, if $\pi_t(\alpha_L, \varepsilon_t) > \pi_t(\alpha_H, \varepsilon_t)$ for all m_t, then the L type banker distinguishes itself from the H type by increasing π_t; the H type distinguishes itself by decreasing π_t (see Figure 3.2a)[47]. The reverse would be true if $\pi_t(\alpha_L,\varepsilon_t) < \pi_t(\alpha_H,\varepsilon_t)$ as seen in Figure 3.2b. Finally, if $\pi_t(\alpha_L,\varepsilon_t) > \pi_t(\alpha_H,\varepsilon_t)$ for some values of m_t and $\pi_t(\alpha_L,\varepsilon_t) < \pi_t(\alpha_H,\varepsilon_t)$ for other values of m_t, then the bankers differentiate themselves from each other by rotating their reaction functions as shown in Figure 3.2c.

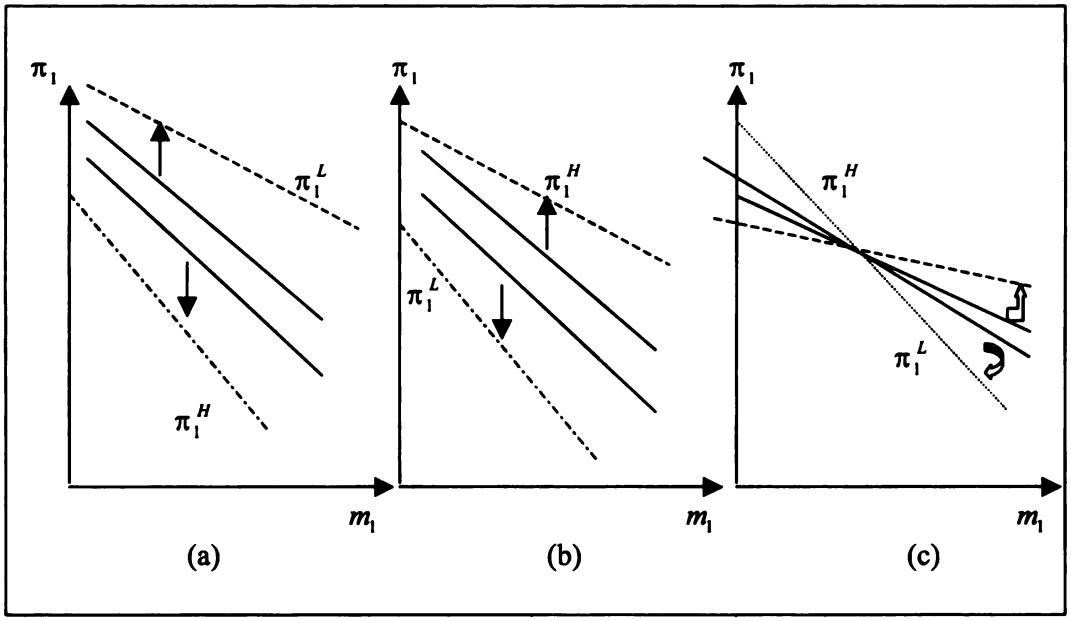


Figure 3.2. How Bankers Signal their Type

[47] In figures 1a through 1c, the reaction functions of the central bankers are negatively sloped. To show why this is so note that the reaction functions are found by minimizing $L_1^{CB,i} + m_1[\pi_1 - \bar{\pi}] + \rho EV_t^{CB}(\mu_1)$ with respect to π_1. Hence the first order condition is given by $\frac{dL_1^{CB,i}}{d\pi_1} + m_1 + \rho\frac{dEV_t^{CB}(\mu_1)}{d\pi_1} = 0$. To find the slope of the reaction function, differentiate the first order condition with respect to m_1 and solve for $\frac{d\pi_1}{dm_1}$, which yields $\frac{d\pi_1}{dm_1} = -\frac{1}{\frac{d^2L_1^{CB,i}}{d\pi_1^2} + \rho\frac{d^2EV_t^{CB}(\mu_1)}{d\pi_1^2}}$, which is negative if $L_1^{CB,i} + m_1[\pi_1 - \bar{\pi}] + \rho EV_t^{CB}(\mu_1)$ is a convex function. The same reasoning holds for the non-myopic reaction functions.

The correct relation (or ordering) between $\pi_t(\alpha_L, \varepsilon_t)$ and $\pi_t(\alpha_H, \varepsilon_t)$ depends on the size of the shocks ε_t. To see this consider first the second period reaction functions given by equations (3.17) and (3.18). Subtracting (3.18) from (3.17) it can be shown that $\pi_2(\alpha_L, \varepsilon_2) > \pi_2(\alpha_H, \varepsilon_2)$ only if

$$\frac{(\alpha_L - \alpha_H)(\bar{\pi} - \pi_2^e - \bar{y} + \varepsilon_2)}{(1+\lambda)} \geq 0 \Leftrightarrow \bar{\pi} - \pi_2^e - \bar{y} + \varepsilon_2 \leq 0 \Leftrightarrow \varepsilon_2 \leq -\bar{\pi} + \pi_2^e + \bar{y}.$$

Substituting for π_2^e from equation (3.19),

$$\begin{aligned} \varepsilon_2 \leq -\bar{\pi} + \pi_2^e + \bar{y} &\Leftrightarrow -\bar{\pi} + \left[\bar{\pi} + \frac{(\lambda - \mu_1\alpha_L - (1-\mu_1)\alpha_H)\bar{y} - m_2}{(1+\mu_1\alpha_L + (1-\mu_1)\alpha_H)}\right] + \bar{y} \\ &\Leftrightarrow \varepsilon_2 \leq \frac{(1+\lambda)\bar{y} - m_2}{1+\mu_1\alpha_L + (1-\mu_1)\alpha_H} \end{aligned}.$$

Since $\dfrac{\bar{m}}{1+\mu_1\alpha_L + (1-\mu_1)\alpha_H} > \dfrac{m_2}{1+\mu_1\alpha_L + (1-\mu_1)\alpha_H}$, where $\bar{m}$ is the maximum punishment that bankers accept, then assuming that $\varepsilon_2 \leq \dfrac{(1+\lambda)\bar{y} - \bar{m}}{1+\mu_1\alpha_L + (1-\mu_1)\alpha_H}$ yields that for all m, $\pi_2(\alpha_L, \varepsilon_2) > \pi_2(\alpha_H, \varepsilon_2)$.

However, if shocks to the economy are "very large" it is possible that $\pi_2(\alpha_L, \varepsilon_2) < \pi_2(\alpha_H, \varepsilon_2)$. Intuitively, since the banker of type *L* cares more about output than the banker of type *H*, its reaction to supply shocks is more distorted, i.e. $|g^L| > |g^H|$. The larger the shocks, the larger the distortion, and thus the larger the possibility that $\pi_2(\alpha_L, \varepsilon_2) < \pi_2(\alpha_H, \varepsilon_2)$[48].

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[48] A similar argument can be used to show that if $\varepsilon_1 \leq \dfrac{(1+\lambda)\bar{y} - \bar{m}}{1+\mu_0\alpha_L + (1-\mu_0)\alpha_H}$ then myopic reaction function of the type *L* banker is always above that the type *H*'s.

Now consider the strategic reaction functions in period 1. The reaction function of each banker is found by minimizing $L_1^{CB,i} + m_1(\pi_1 - \bar{\pi}) + \rho EV_i^{CB}(\mu_1)$ with respect to π_1, which yields the following first order condition for the banker of type *i*,

$$FOC^i = (\lambda - \alpha_i)(\pi_1 - \pi_1^e + \varepsilon_1 - \bar{y}) + (1 + \alpha_i)(\pi_1 - \bar{\pi}) + m_1 + \rho \frac{dEV_i^{CB}(\mu_1)}{d\mu_1}\frac{d\mu_1}{d\pi_1} = 0.$$

Note that the first order condition of banker of type *L* can be expressed in terms of type *H*'s,

$$FOC^L = \begin{pmatrix} FOC^H + (\alpha_H - \alpha_L)(\pi_1 - \pi_1^e + \varepsilon_1 - \bar{y}) + (\alpha_L - \alpha_H)(\pi_1 - \bar{\pi}) \\ +\rho\left(\dfrac{dEV_L^{CB}(\mu_1)}{d\mu_1} - \dfrac{dEV_H^{CB}(\mu_1)}{d\mu_1}\right)\dfrac{d\mu_1}{d\pi_1} \end{pmatrix} = 0$$

Therefore the first period reaction function of banker of type *L* is above that of banker of type *H* only if

$$(\alpha_H - \alpha_L)(\bar{\pi} - \pi_1^e + \varepsilon_1 - \bar{y}) + \rho\left(\frac{dEV_L^{CB}(\mu_1)}{d\mu_1} - \frac{dEV_H^{CB}(\mu_1)}{d\mu_1}\right)\frac{d\mu_1}{d\pi_1} \le 0^{49},$$

which once again depends on the size of the shock[50]. In footnote 49 I show the difficulty in actually pinning down the exact restrictions that ε must satisfy in order for

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[49] If $(\alpha_H - \alpha_L)(\bar{\pi} - \pi_1^e + \varepsilon_1 - \bar{y}) + \rho\left(\frac{dEV_L^{CB}(\mu_1)}{d\mu_1} - \frac{dEV_H^{CB}(\mu_1)}{d\mu_1}\right)\frac{d\mu_1}{d\pi_1} \le 0$ then it must be the case that $FOC^H > 0$ otherwise the first order condition of the type *L* banker would not be satisfied. This means that the loss function of the type *H* banker evaluated at the *L* inflation rate has positive slope. Since the loss function of the bankers is convex, this means that the reaction function of banker of type *L* is above that of banker of type *H*.

[50] It is very difficult to clearly identify the restrictions on ε implied by the non-strategic reaction functions for two reasons: I do not know the explicit expression for the function $\frac{d\mu_1}{d\pi_1}$, and it is very hard to find a neat expression for $\left(\frac{dEV_L^{CB}(\mu_1)}{d\mu_1} - \frac{dEV_H^{CB}(\mu_1)}{d\mu_1}\right)$. For example assume that a good approximation for $\frac{d\mu_1}{d\pi_1}$ is given by $\frac{d\mu_1}{d\pi_1} = b\pi_1$, where b is a real number. This allows me to find an expression for the inflation expectations by taking the expectations of the first order condition and solving for π_1^e,

$\pi_1(\alpha_L,\varepsilon_1) > \pi_1(\alpha_H,\varepsilon_1)$, or vice versa. However, it is important to note that either Figure 3.2(a) or Figure 3.2(b) are feasible depending on the size of the shocks. However, I will assume throughout the rest of the paper that $\pi_t(\alpha_L,\varepsilon_t) > \pi_t(\alpha_H,\varepsilon_t)$ (as in Figure 3.2(a)), that is that shocks are not too large. The reason I do so is two-fold. First, as I explained before in the paper, I want the losses of the bankers to be bounded, and second, because this allows me to assume that beliefs are monotonic. Formally my assumptions are,

Assumption A3.1. ε_t is not too large, in particular since $-\eta \le \varepsilon_t \le \eta$, assume that η is such that $\pi_t(\alpha_L,\eta) > \pi_t(\alpha_H,\eta)$ and $\pi_t(\alpha_L,-\eta) > \pi_t(\alpha_H,-\eta)$ for all t and all m_t.

$$(\lambda-\alpha_i)(\pi_1-\pi_1^e+\varepsilon_1-\bar{y})+(1+\alpha_i)(\pi_1-\bar{\pi})+m_1+\rho\frac{dEV_i^{CB}(\mu_1)}{d\mu_1}(b\pi_1)=0$$

$$\Rightarrow (\lambda-E(\alpha))(\pi_1^e-\pi_1^e-\bar{y})+(1+E(\alpha))(\pi_1^e-\bar{\pi})+m_1+\rho\left(\mu_1\frac{dEV_L^{CB}(\mu_1)}{d\mu_1}+(1-\mu_1)\frac{dEV_H^{CB}(\mu_1)}{d\mu_1}\right)(b\pi_1^e)=0.$$

$$\Rightarrow \pi_1^e=\frac{(\lambda-E(\alpha))\bar{y}+(1+E(\alpha))\bar{\pi}-m_1}{\left(1+E(\alpha)+b\rho\left(\mu_1\frac{dEV_L^{CB}(\mu_1)}{d\mu_1}+(1-\mu_1)\frac{dEV_H^{CB}(\mu_1)}{d\mu_1}\right)\right)}$$

Then $(\alpha_H-\alpha_L)(\bar{\pi}-\pi_1^e+\varepsilon_1-\bar{y})+\rho\left(\frac{dEV_L^{CB}(\mu_1)}{d\mu_1}-\frac{dEV_H^{CB}(\mu_1)}{d\mu_1}\right)\frac{d\mu_1}{d\pi_1}\le 0$ is equivalent to,

$$\varepsilon_1 \le -\left(\begin{array}{l}-\frac{\rho}{(\alpha_H-\alpha_L)}\left(\frac{dEV_L^{CB}(\mu_1)}{d\mu_1}-\frac{dEV_H^{CB}(\mu_1)}{d\mu_1}\right)\frac{d\mu_1}{d\pi_1}\\[2ex] +\frac{\left(1+\lambda+b\rho\left(\mu_1\frac{dEV_L^{CB}(\mu_1)}{d\mu_1}+(1-\mu_1)\frac{dEV_H^{CB}(\mu_1)}{d\mu_1}\right)\right)\bar{y}}{\left(1+E(\alpha)+b\rho\left(\mu_1\frac{dEV_L^{CB}(\mu_1)}{d\mu_1}+(1-\mu_1)\frac{dEV_H^{CB}(\mu_1)}{d\mu_1}\right)\right)}\\[2ex] -\frac{b\rho\left(\mu_1\frac{dEV_L^{CB}(\mu_1)}{d\mu_1}+(1-\mu_1)\frac{dEV_H^{CB}(\mu_1)}{d\mu_1}\right)\bar{\pi}+m_1}{\left(1+E(\alpha)+b\rho\left(\mu_1\frac{dEV_L^{CB}(\mu_1)}{d\mu_1}+(1-\mu_1)\frac{dEV_H^{CB}(\mu_1)}{d\mu_1}\right)\right)}\end{array}\right).$$

Assumption A3.2. Higher inflation rates lead to higher beliefs that the banker is the L type, $\frac{d\mu_1}{d\pi_1} \geq 0$[51].

Putting assumptions A1 and A2 together leads to the following definition regarding the value to the central bankers of signaling their type.

Definition: The banker of type L benefits (is harmed) by signaling its type if its second period expected losses decrease (increase) as the first period inflation rate π_1 increases (decreases). Hence, given that $\frac{dEV_L^{CB}(\mu_1)}{d\pi_1} = \frac{dEV_L^{CB}(\mu_1)}{d\mu_1}\frac{d\mu_1}{d\pi_1}$ and given Assumption A3.2, the type L-banker benefits (is harmed) by signaling its type if $\frac{dEV_L^{CB}(\mu_1)}{d\mu_1} < (>) 0$.

Definition: The banker of type H benefits (is harmed) by signaling its type if its second period expected losses decrease (increase) as the first period inflation rate π_1 decreases (increases). Hence, the type H-banker benefits (is harmed) by signaling its type if $\frac{dEV_H^{CB}(\mu_1)}{d\pi_1} = \frac{dEV_H^{CB}(\mu_1)}{d\mu_1}\frac{d\mu_1}{d\pi_1} > (<) 0$, which given Assumption A3.2 is equivalent to $\frac{dEV_H^{CB}(\mu_1)}{d\mu_1} > (<) 0$.

Unlike the government who gains from more transparent monetary policy the banker does not necessarily benefit from signaling its type as we show in the next example.

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[51] Assumption A3.2 is the result of Assumption A3.1 together with the maximum likelihood property according to which $f'(\varepsilon)/f(\varepsilon)$ is strictly decreasing in ε.

Example: central bank's value of signaling its type

Assume that $\alpha_L = 0.1$, $\alpha_H = 0.9$, $\lambda = 1$, $\bar{\pi} = 0.03$ and $\bar{y} = 1$. In Figure 3.3 I present the value of $\frac{dEV_i^{CB}(\mu_1)}{d\mu_1}$ for both types of bankers (using the assumption that $E\varepsilon = 0$), which is a function of the posterior belief μ_1.

For the L type $\frac{dEV_L^{CB}(\mu_1)}{d\mu_1} > 0$, and thus its losses are increased by signaling its type. Since this function reaches a maximum at $\mu_1 = 1$, then the value of information to the banker of type L is minimized when its type is fully revealed.

For the H type $\frac{dEV_H^{CB}(\mu_1)}{d\mu_1} < 0$ for most values of μ_1, and thus its losses are increased by signaling its type as well. Unlike for the other banker, the type H banker's value of information is positive and maximized when its type is revealed $\mu_1 = 0$.

For these parameter values, the value of information to the government is positive as seen in the third panel of Figure 3.3. Thus the government and the banker might act at cross purposes. This leads to the question of whether the government can induce more informative inflation rates, and if so, at what cost. The answers to these questions depend on the type of the bankers, on whether they benefit from revealing their type, and on what this implies in terms of the variability of the inflation rate around the target. I address these questions in the next sections.

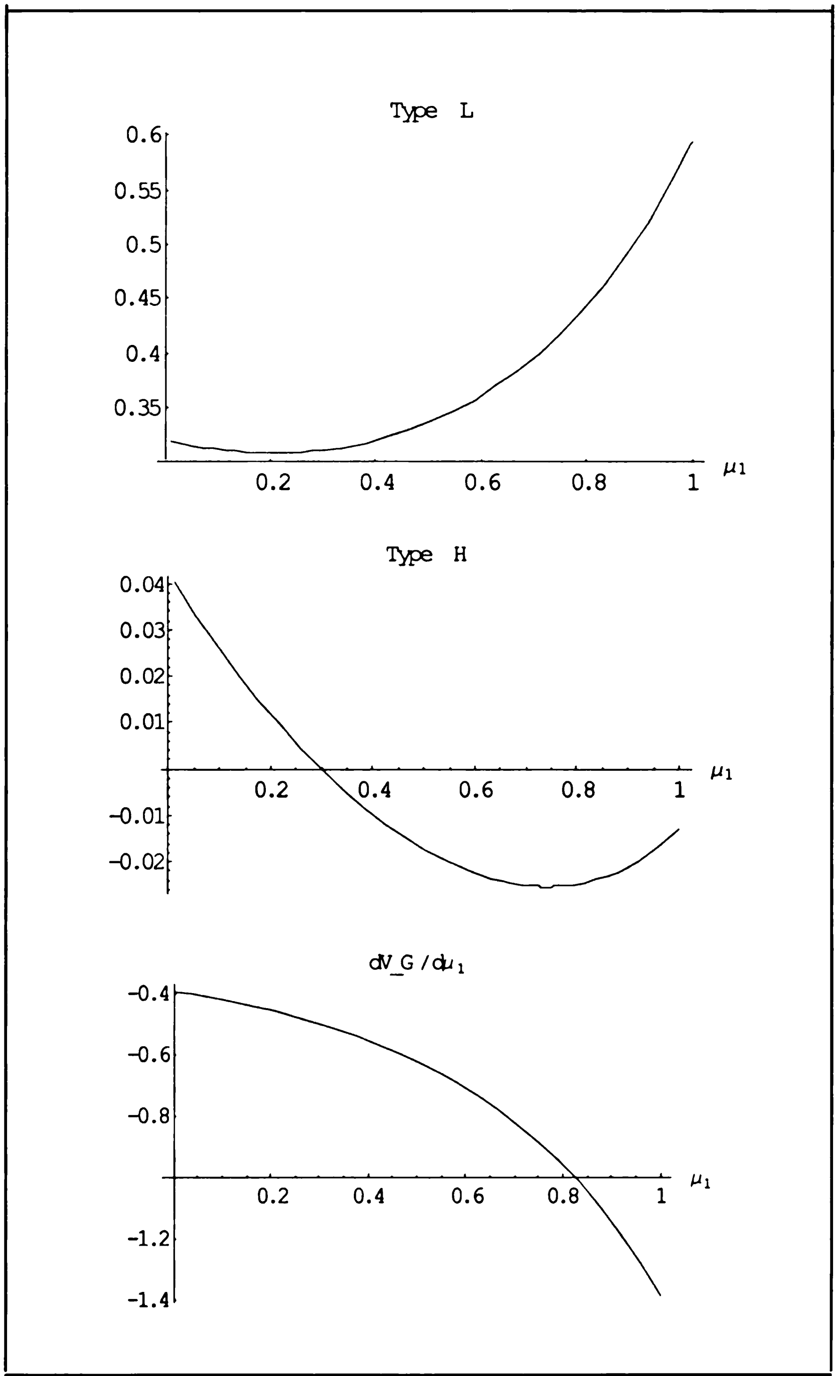

Figure 3.3. Value to the Central Banker of Signaling its Type

3.3.4 Central Banker's Incentives to Manipulate Information

Since the second period losses of the central banker are a function of the government's posterior beliefs then the banker has incentives to manipulate the government's beliefs. In this section I show that if the value to the banker of revealing its type is positive, then the banker acts strategically to signal its type. The reverse holds if the value to the banker of signaling its type is negative. Before I prove this statement I formalize a series of definitions.

Definition (strategic banker): Assume that the banker is appointed to two periods. If the banker accounts for the effect of the first period inflation rate on the government's posterior beliefs and thus on its second period losses, then the banker acts *strategically.* A strategic banker minimizes (3.21) with respect to π_1. The resulting reaction function is given by $\pi_1(\alpha_i, \varepsilon_1) = \pi_1^i(m_1) + g^i\varepsilon_1$.

Definition (myopic banker): Assume that the banker is appointed to two periods. If the banker minimizes $L_1^{CB} + m_1(\pi_1 - \overline{\pi})$ with respect to π_1 without regards to the effect of the inflation rate on posterior beliefs and thus on its second period losses, then the banker acts *myopically* or *non-strategically.* Let $\pi_1^{myopic}(\alpha_i, \varepsilon_1) = \pi_1^{i,myopic} + g^i\varepsilon_1$ be the solution to the myopic problem.

Definition (information manipulation): If when appointed to two periods the strategic central banker sets an inflation rate that differs from the myopic inflation rate, that is if $\pi_1(\alpha_i, \varepsilon_1) \neq \pi_1^{myopic}(\alpha_i, \varepsilon_1)$, then the banker *manipulates information.*

To study the CB's incentives to manipulate information I compare the first order conditions of the CB's problem when it minimizes equation (3.21), and when it acts myopically and minimizes $L_1^{CB} + m_1(\pi_1 - \bar{\pi})$.

Proposition 3.2: Given Assumption A3.2, a strategic central banker will adjust the inflation rate away from the myopic rate in order to influence the government's posterior beliefs if the banker's value of signaling its type is not zero, i.e. if $\frac{dEV_i^{CB}(\mu_1)}{d\mu_1} \neq 0$. In particular,

3.2.1 If the value to the central banker of signaling its type is positive (negative) and the banker is the L type, then the banker increases (decreases) the inflation rate to increase (decrease) the likelihood of its type being the true type: I.e. if $\frac{dEV_L^{CB}(\mu_1)}{d\mu_1} < (>) 0$ then $\forall m_1 \; \pi_1(\alpha_L, \varepsilon_1) > (<) \pi_1^{myopic}(\alpha_L, \varepsilon_1)$.

3.2.2 If the value to the central banker of signaling its type is positive (negative) and the banker is the H type, then the banker decreases (increases) the inflation rate to increase (decrease) the likelihood of its type being the true type: I.e. if $\frac{dEV_H^{CB}(\mu_1)}{d\mu_1} > (<) 0$ then $\forall m_1 \; \pi_1(\alpha_H, \varepsilon_1) < (>) \pi_1^{myopic}(\alpha_H, \varepsilon_1)$.

Proof: In Appendix C.

According to Proposition 3.2 if the banker's losses are reduced through transparency, then the banker wants to increase information or to signal its type. To signal its type each banker separates its signal from the other type's signal. Proposition 3.2 can be represented graphically in the next set of figures. In Figure 3.4 the reaction function of each type of banker is the same whether the banker acts myopically or strategically. This is the result of a zero value of information.

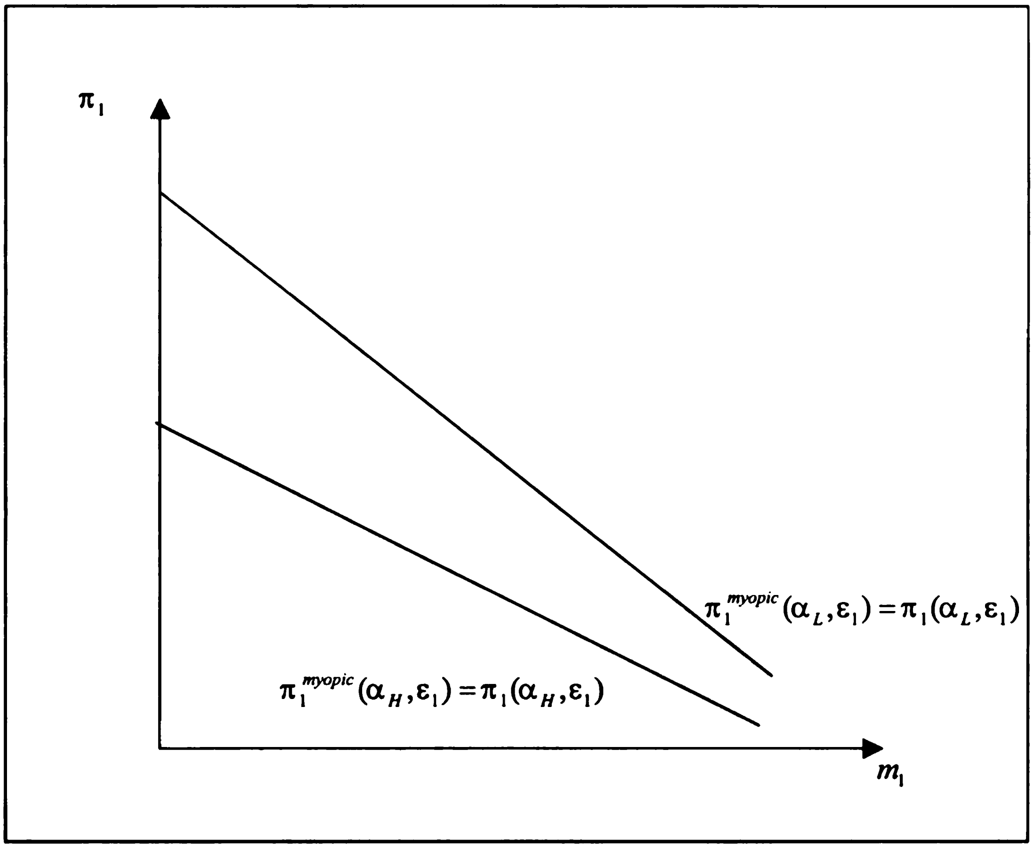

Figure 3.4. The Central Banker does not Manipulate Information

In Figure 3.5 the strategic behavior of the bankers implies that their reaction functions are more spread out than the myopic reaction functions. In this example, for any given contract m_1 the strategic behavior of the bankers increases information. Intuitively the reaction functions are more spread and are easier to distinguish[52].

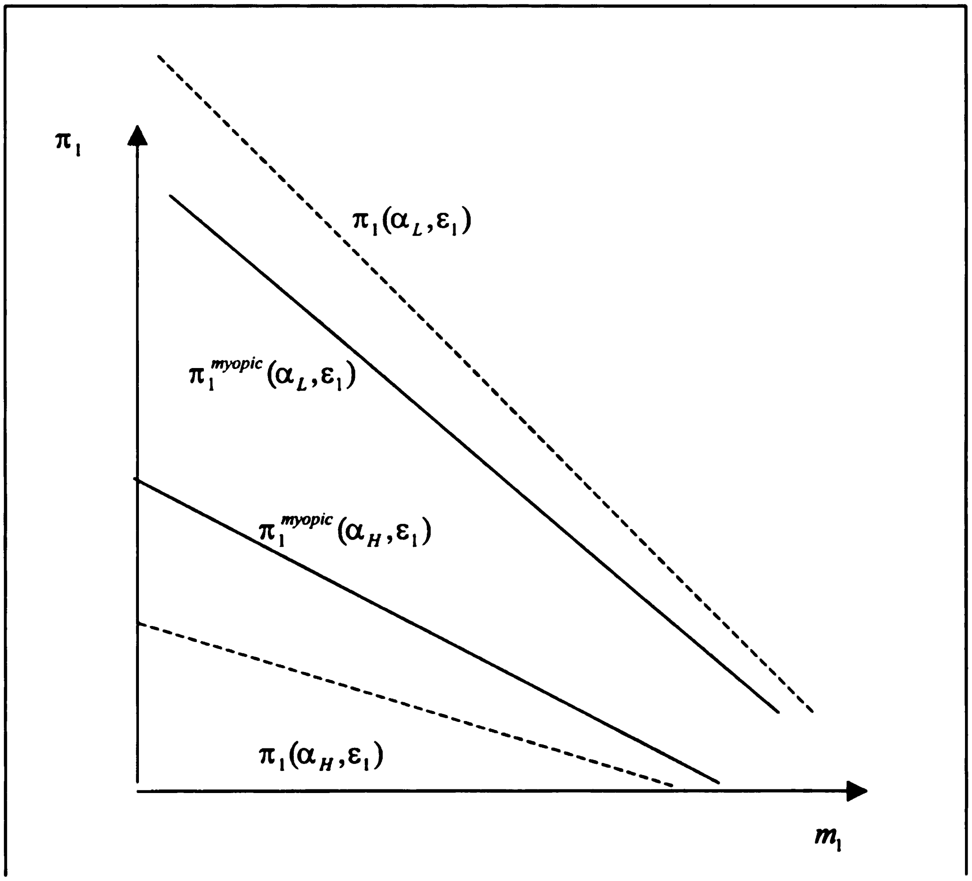

Figure 3.5. The Central Banker Increases Information.

In Figure 3.6 the strategic behavior of the bankers implies that their reaction functions move closer together, which means that each inflation rate is now less

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[52] The movement of the reaction functions is not a parallel movement. In particular, the myopic reaction function has slope $\frac{d\pi_1^{I,myopic}}{dm_1} = -\frac{1}{\frac{d^2L_1^{CB}}{d\pi_1^2}}$ and the strategic reaction function has slope $\frac{d\pi_1}{dm_1} = -\frac{1}{\frac{d^2L_1^{CB}}{d\pi_1^2} + \rho\frac{d^2EV_i^{CB}(\mu_1)}{d\pi_1^2}}$.

informative. This happens because the value to the bankers of signaling their type is negative.

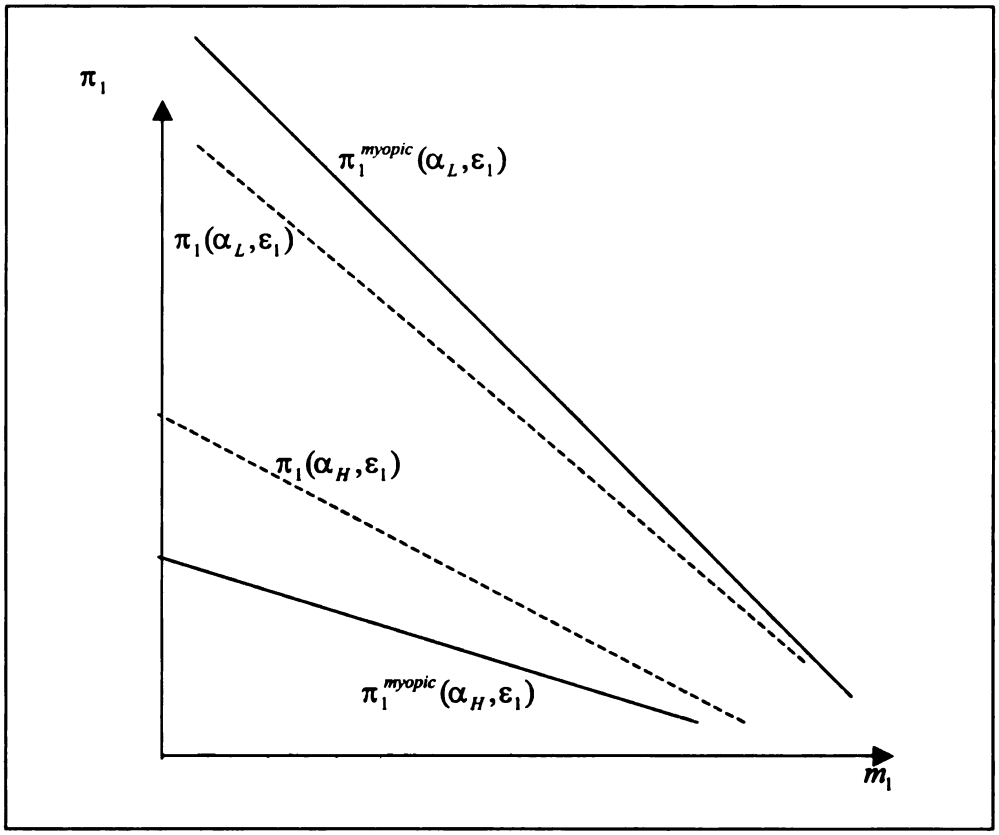

Figure 3.6. The Central Banker Decreases Information.

There are finally two other cases in which one type of banker's value of signaling its type is positive and the other type's is negative. These cases are presented in Figure 3.7. In either of these cases it is not clear a priori whether the strategic inflation rate is more or less informative.

Note that the assumptions imposed on the random shocks, which lead to Assumption A3.1, limit my model to the cases presented in Figures 3.4 through Figure 3.7. That is, the two reaction functions at most converge to the same reaction function but it is never the case that $\pi_1(\alpha_H,\varepsilon_1)>\pi_1(\alpha_L,\varepsilon_1)$.

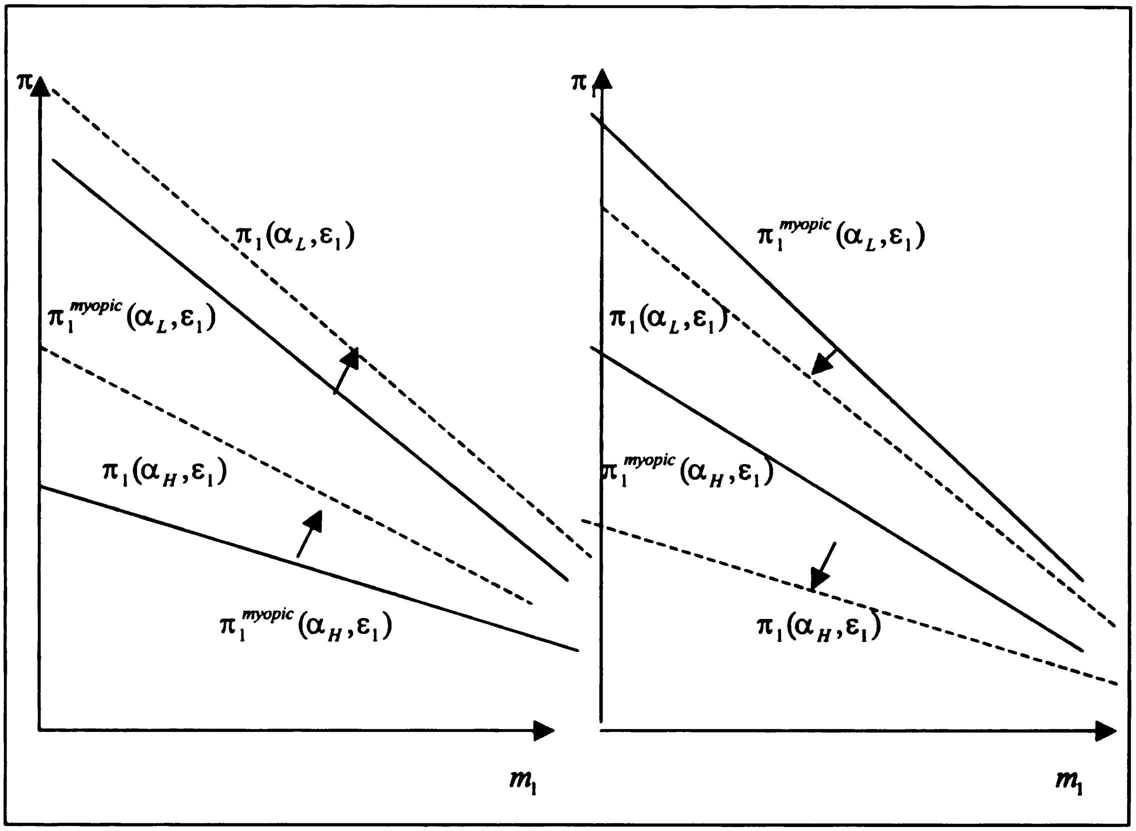

Figure 3.7: One Type of Central Banker Decreases Information, One Type Increases Information

Since the banker, regardless of its type has incentives to manipulate the government's informational state by increasing or decreasing the inflation rate I now study the government's reaction to the strategic behavior of the banker. Before going on note however that the strategic behavior of the banker, which is aimed exclusively at manipulating the posterior beliefs of the government, has two effects on the government's optimization problem. The first effect, which was discussed in detail in Proposition 3.2 and in Figures 3.4 through 3.7, is that the strategic behavior of the banker increases or reduces the informativeness of each inflation rate by spreading the two reaction functions apart or moving them closer together. The second effect of the strategic behavior of the banker is to move the inflation rates closer to, or away from the target $\bar{\pi}$. For example, in Figure 3.5 $\pi_1(\alpha_L, \varepsilon_1) - \bar{\pi}$ increases, and $\pi_1(\alpha_H, \varepsilon_1) - \bar{\pi}$ decreases. Depending on the magnitude of these changes and on prior beliefs μ_0, this implies that the expected variability of the inflation rate around $\bar{\pi}$, which is given by $\mu_0(\pi_1(\alpha_L, \varepsilon_1) - \bar{\pi})^2 + (1 - \mu_0)(\pi_1(\alpha_H, \varepsilon_1) - \bar{\pi})^2$, increases or decreases. Thus even if the strategic behavior of the central banker increases information, it might also increase (or decrease) the variability of the inflation rate and of output, which affects the government's first period total and marginal expected losses. Hence, to study the government's reaction to the banker's strategic behavior I will study the interaction of these two effects.

3.3.5 Government's Incentives to Induce More Informative Inflation Rates (Incentives to Experiment)

In this section I study the government's incentives to increase information, and the government's reaction to the manipulation of information by the central banker. I will do so in two parts. I first assume that the banker does not act strategically. This will allow me to study the pure gains from learning that the government can generate when bankers do not manipulate information, and it will help me differentiate the government's response to the two effects of the strategic behavior of the banker discussed previously.

Incentives to experiment when the banker does not act strategically

If the banker does not act strategically the government's problem is summarized as follows,

$$\underset{m_1}{Min}\ G_1 + \rho EV^G(\mu_1) \quad s.t. \quad \pi_1^{myopic}(\alpha_L,\varepsilon_1) \ \ \& \ \ \pi_1^{myopic}(\alpha_H,\varepsilon_1). \tag{3.22}$$

Note that the reaction functions $\pi_1^{myopic}(\alpha_i,\varepsilon_1)$ correspond to $\pi_1^{myopic}(\alpha_i,\varepsilon_1) = \bar{\pi} + \frac{(\lambda-\alpha_i)}{(1+E_1\alpha)}\bar{y} - \frac{(1+\lambda+E_1\alpha-\alpha_i)}{(1+E_1\alpha)(1+\lambda)}m_1 - \frac{(\lambda-\alpha_i)}{(1+\lambda)}\varepsilon_1$, which is the result of the static minimization problem given prior beliefs.

Let $m_1^{non-strategic}$ be the solution to (3.22).

If the punishment (contract) that minimizes the discounted two-period expected losses of the government $m_1^{non-strategic}$ differs from the contract that minimizes first period expected losses, then the government experiments. The latter contract, call it m_1^{myopic}, comes from solving the following optimization problem,

$$\underset{m_1}{Min}\ G_1 \qquad s.t. \quad \pi_1^{myopic}(\alpha_L,\varepsilon_1) \ \ \& \ \ \pi_1^{myopic}(\alpha_H,\varepsilon_1).$$

Thus m_1^{myopic} is set according to the first order condition,

$$\frac{dG_1}{d\pi_1^{L}}\frac{d\pi_1^{L,myopic}}{dm_1} + \frac{dG_1}{d\pi_1^{H}}\frac{d\pi_1^{H,myopic}}{dm_1} - [\mu_0\pi_1^{L,myopic} + (1-\mu_0)\pi_1^{H,myopic} - \bar{\pi}] = 0, \tag{3.23}$$

which are the first period marginal expected losses of the government.

The contract $m_1^{non-strategic}$ is set according to the first order condition

$$\frac{dG_1}{d\pi_1^{L}}\frac{d\pi_1^{L,myopic}}{dm_1} + \frac{dG_1}{d\pi_1^{H}}\frac{d\pi_1^{H,myopic}}{dm_1} - [\mu_0\pi_1^{L,myopic} + (1-\mu_0)\pi_1^{H,myopic} - \bar{\pi}] + \rho\left[\frac{dEV^G(\mu_1)}{dm_1}\right] = 0,$$

which are the first period marginal expected losses plus the marginal gains or losses in the second period due to more or less information.

If the CB does not act strategically the government will experiment to influence the degree of transparency of monetary policy (information) if the first order conditions of the two problems differ, that is if,

$$\frac{dEV^G(\mu_1)}{dm_1} \neq 0.$$

To find this expression first note that,

$$\Omega = \frac{dEV^G(\mu_1)}{dm_1} = \frac{dEV^G(\mu_1)}{d\pi_1^{\,L}} \frac{d\pi_1^{\,L,myopic}}{dm_1} + \frac{dEV^G(\mu_1)}{d\pi_1^{\,H}} \frac{d\pi_1^{\,H,myopic}}{dm_1}.$$

Differentiating $EV^G(\mu_1) = \int V^G(\mu_1)[\mu_0 f_L + (1-\mu_0) f_H] d\pi$ inside the integral with respect to $\pi_1^{\,i}$ yields,

$$\frac{dEV^G(\mu_1)}{d\pi_1^{\,i}} = \int \frac{dV^G(\mu_1)}{d\mu_1} \frac{d\mu_1}{d\pi_1^{\,i}} [\mu_0 f_L + (1-\mu_0) f_H] d\pi + \int V^G(\mu_1) \frac{d[\mu_0 f_L + (1-\mu_0) f_H]}{d\pi_1^{\,i}} d\pi .$$

After some algebraic manipulation (in Appendix C), $\frac{dEV^G(\mu_1)}{d\pi_1^{\,L}}$ can be written as,

$$\frac{dEV^G(\mu_1)}{d\pi_1^{\,L}} = \int \frac{d^2V^G(\mu_1)}{d\mu_1^{\,2}} \frac{d\mu_1}{d\pi_1} \mu_1(1-\mu_0) f_H d\pi ,$$

and $\frac{dEV^G(\mu_1)}{d\pi_1^{\,H}}$ can be written as,

$$\frac{dEV^G(\mu_1)}{d\pi_1^{\,H}} = -\int \frac{d^2V^G(\mu_1)}{d\mu_1^{\,2}} \frac{d\mu_1}{d\pi_1} \mu_1(1-\mu_0) f_H d\pi .$$

The reason for the asymmetry on the signs of $\frac{dEV^G(\mu_1)}{d\pi_1^H}$ and $\frac{dEV^G(\mu_1)}{d\pi_1^L}$ is as follows. If π_1^L increases, then information increases which reduces the government's expected losses. However, if π_1^H increases, then information decreases which increases the government's expected losses.

The expression Ω can therefore be written as a function of the government's value of information and of the expected amount of information as follows,

$$\Omega = \left(\frac{d\pi_1^{L,myopic}}{dm_1} - \frac{d\pi_1^{H,myopic}}{dm_1} \right) \int \frac{d^2V^G(\mu_1)}{d\mu_1^2} \frac{d\mu_1}{d\pi_1} \mu_1(1-\mu_0) f_H d\pi \ .$$

The term $\left(\frac{d\pi_1^{L,myopic}}{dm_1} - \frac{d\pi_1^{H,myopic}}{dm_1} \right)$ is a measure of the expected amount of information as it gives the distance between the means of the two possible distributions of beliefs f_L and f_H. Since $\left(\frac{d\pi_1^{L,myopic}}{dm_1} - \frac{d\pi_1^{H,myopic}}{dm_1} \right) = \frac{\alpha_L - \alpha_H}{1+\lambda} < 0$, then decreasing the punishment increases the expected amount of information. This can be seen as f_L and f_H move further apart and overlap less as m_1 increases (as in Figure 3.8).

Since decreasing m_1 increases information and since the government's value of information is positive, then the government decreases the punishment to the banker to induce learning. This result is formally stated in Proposition 3.3.

Proposition 3.3: Given Assumption A3.2, if the central banker does not act strategically then if the government's value of information is positive, it will decrease the punishment compared to the myopic punishment to increase the degree of transparency of monetary policy, that is, $m_1^{non-strategic} < m_1^{myopic}$.

Proof: See Appendix C.

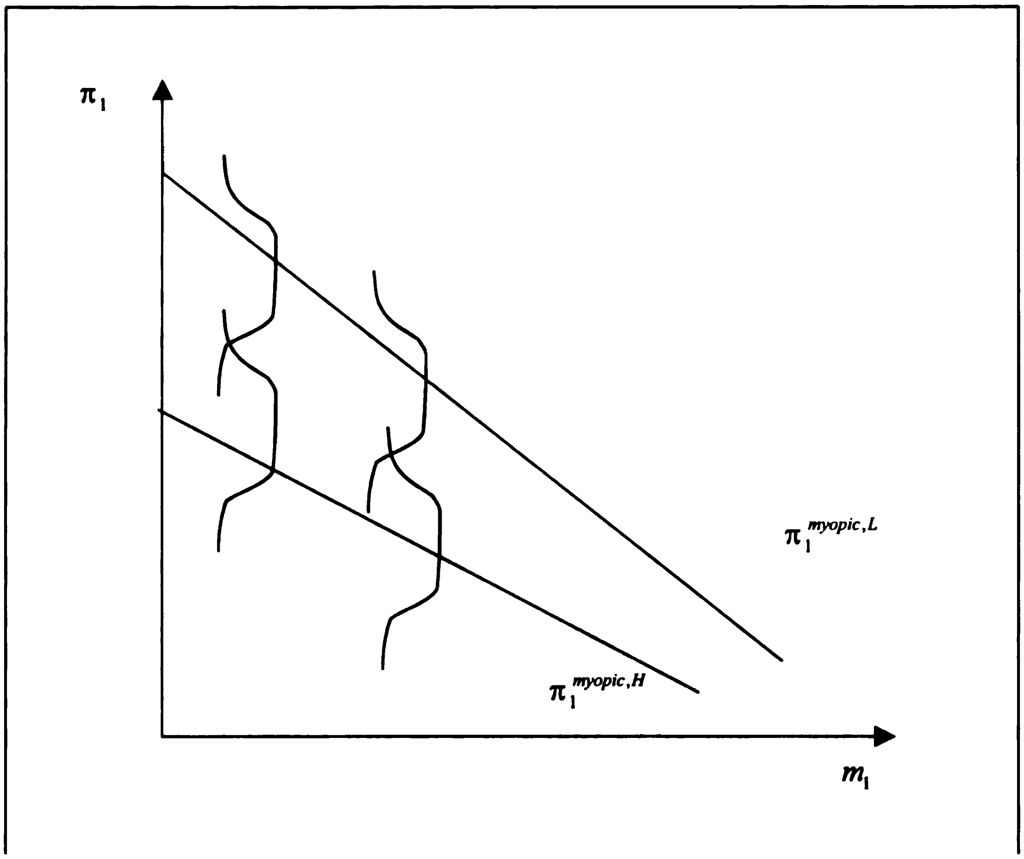

Figure 3.8. Myopic reaction functions

Not surprisingly, if the government's expected losses decrease with more information or more transparent monetary policy, the government will increase the degree of transparency of monetary policy by decreasing the monetary punishment to the banker. The government of course will only deviate from the myopic contract if the gains from information overcome the losses from experimentation.

Proposition 3.4: If the central banker does not act strategically then the government is better off or at least not worse off hiring the bankers for two periods instead of one.

Proof:

If m_1^{myopic} is a feasible contract i.e. if it satisfies the individual rationality constraint but the government chooses $m_1^{non-strategic}$, then it must be that the expected two period losses are lower under $m_1^{non-strategic}$ than under m_1^{myopic}.

□

Effect of the strategic behavior of the central banker

The central banker does not necessarily act myopically. In particular, if $\frac{dEV_i^{CB}(\mu_1)}{d\mu_1} \neq 0$ the banker will act strategically by deviating from the myopic reaction functions. If for example $\left(\frac{d\pi_1^{\ L}}{dm_1} - \frac{d\pi_1^{\ H}}{dm_1}\right) < 0$, the strategic behavior of the banker implies that the reaction functions move outwards as in Figure 3.5, or inwards, as in Figure 3.6. If $\left(\frac{d\pi_1^{\ L}}{dm_1} - \frac{d\pi_1^{\ H}}{dm_1}\right) > 0$, then the strategic behavior of the bank implies more complicated movements, such as in Figure 3.9 (full lines represent myopic reaction functions, dashed lines, strategic reaction functions).

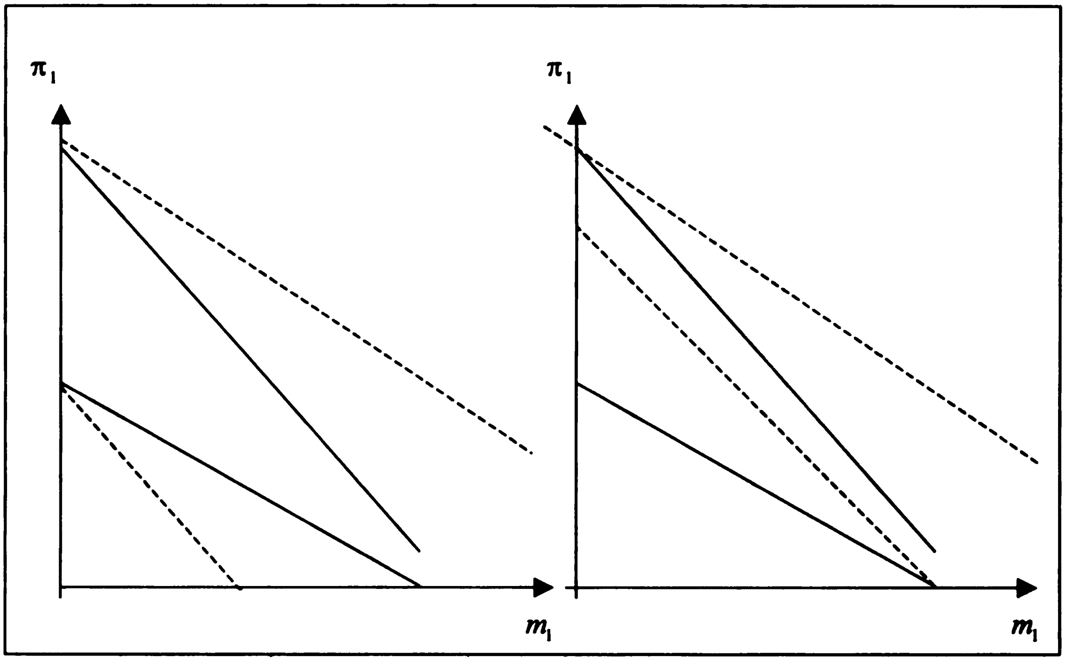

Figure 3.9: Given the Strategic Behavior of the Bankers, Increasing the Punishment Increases Information

For simplicity, I will present our results for the case in which decreasing the punishment increases information, although the intuition of the results that follow remains the same when $\left(\frac{d\pi_1^{\ L}}{dm_1}-\frac{d\pi_1^{\ H}}{dm_1}\right)>0$.

Assumption A3.3: Given the strategic behavior of the bank, decreasing the unit punishment increases information, $\left(\frac{d\pi_1^{\ L}}{dm_1}-\frac{d\pi_1^{\ H}}{dm_1}\right)<0$.

The government's optimization problem when the banker acts strategically, which we described in detail previously, can be summarized as follows,

$$\underset{m_1}{Min}\ G_1+\rho EV^G(\mu_1)\quad s.t.\quad \pi_1(\alpha_L,\varepsilon_1)\quad \&\quad \pi_1(\alpha_H,\varepsilon_1). \qquad (3.24)$$

Remember that I called the equilibrium m_1*.

To study how the optimal contract and the degree of transparency of monetary policy are affected when the banker acts strategically I study the two effects of the strategic behavior of the central bank on the government's expected losses. First, I study how the strategic behavior of the central banker spreads the reaction functions apart or moves them closer together. Second, how it increases or reduces the distance between each signal and the inflation target. The first effect increases or reduces the informativeness of the inflation rate, and the second effect increases or reduces the variability of the inflation rate around $\bar{\pi}$.

The effect of the strategic behavior of the central bank on the optimal contract $m_1* - m_1^{non-strategic}$ can be decomposed as follows,

$$m_1* - m_1^{non-strategic} = (m_1* - m_1^?) - (m_1^{non-strategic} - m_1^{myopic}) + (m_1^? - m_1^{myopic}),$$

where $m_1^?$ is the contract that minimizes the government's first period expected losses subject to the strategic behavior of the central bank, that is $m_1^?$ is found by minimizing

$$\underset{m_1}{Min}\ G_1 \quad s.t. \quad \pi_1(\alpha_L,\varepsilon_1) \quad \& \quad \pi_1(\alpha_H,\varepsilon_1), \tag{3.25}$$

with respect to m_1. Hence $(m_1^? - m_1^{myopic})$ shows the deviation in the period-one loss-minimizing contract due to the higher or lower variability of the inflation rate around its target implied by the strategic behavior of the bank.

The term $(m_1^{non-strategic} - m_1^{myopic})$ shows the optimal amount of experimentation when the bankers act myopically (Proposition 3.3), and the term $(m_1* - m_1^?)$ shows the deviation from the first period loss minimizing punishment given that the bankers use the strategic reaction functions (i.e. the net incentives to increase information)[53]. These terms are explained in Propositions 3.6 and 3.7.

Proposition 3.5: The punishment that minimizes the government's first period expected losses is lower (higher) when the banker acts strategically ($m_1^? < (>) m_1^{myopic}$), if the first period marginal losses of the government are higher (lower) when the banker acts strategically. That is,

$$\text{If } \Theta = \begin{pmatrix} \dfrac{dG_1}{d\pi_1^{\ L}}\left[\dfrac{d\pi_1^{\ L}}{dm_1} - \dfrac{d\pi_1^{\ L,myopic}}{dm_1}\right] + \dfrac{dG_1}{d\pi_1^{\ H}}\left[\dfrac{d\pi_1^{\ H}}{dm_1} - \dfrac{d\pi_1^{\ H,myopic}}{dm_1}\right] \\ -[\mu_0(\pi_1^{\ L} - \pi_1^{\ L,myopic}) + (1-\mu_0)(\pi_1^{\ H} - \pi_1^{\ H,myopic})] \end{pmatrix} > (<) 0 \Rightarrow m_1^? < (>) m_1^{myopic}.$$

Proof: For the proof see Appendix C.

Proposition 3.6: When the central banker acts strategically, if the government's value of information is positive, then the government deviates from the punishment that minimizes its first period losses in order to increase information. In particular, given Assumption A3.3, $m_1{*} < m_1^?$.

Proof: See Appendix C.

By comparing how the strategic behavior of the central bank affects the first period minimum through higher or lower variance of the inflation rate around the target (Proposition 3.5) and the gains from information (Propositions 3.3 and 3.6) I can thus study the effect of the strategic behavior of the central bank on the optimal punishment

[53] Alternatively I could say that the effect of the strategic behavior of the central bank is equal to the difference between the incentives to experiment when the banker act strategically and when it does not, i.e. $m_1{*} - m_1^{non-strategic} = (m_1{*} - m_1^{myopic}) - (m_1^{non-strategic} - m_1^{myopic})$. The incentives to experiment when the banker acts strategically can then be decomposed in the gains from more information and from less variability in the inflation rate, $m_1{*} - m_1^{myopic} = (m_1{*} - m_1^?) - (m_1^? - m_1^{myopic})$.

$m_1 * - m_1^{non-strategic}$, and on the degree of transparency of monetary policy. In particular I define the effect of the strategic behavior of the banker on the degree of transparency of monetary policy as follows,

Definition: If $m_1 *$ is more informative than $m_1^{non-strategic}$, that is if $m_1 * < m_1^{non-strategic}$, then the strategic behavior of the central bank implies more transparent monetary policy. Alternatively, if $m_1 * < m_1^{non-strategic}$ the government's reaction to the strategic behavior of the central bank is to induce a more transparent monetary policy.

Proposition 3.7: Given Assumption A3.3 then if the government's value of information is positive, the strategic behavior of the central banker implies more transparent monetary policy (i.e. $m_1 * < m_1^{non-strategic}$), if any of the following conditions are true,

i) The gains from information to the government are higher when the banker acts strategically, $(m_1 * - m_1^? < m_1^{non-strategic} - m_1^{myopic})$, and the strategic behavior of the central bank implies higher first period marginal losses due to more variability of the inflation rate around its target $(m_1^? < m_1^{myopic})$.

ii) The gains from information to the government are higher when the banker acts strategically, $(m_1 * - m_1^? < m_1^{non-strategic} - m_1^{myopic})$, the strategic behavior of the central bank implies lower first period marginal losses due to less variability of the inflation rate around its target $(m_1^? > m_1^{myopic})$, but the extra gains from information compensate for higher first period losses, i.e. $(m_1 * - m_1^?) - (m_1^{non-strategic} - m_1^{myopic}) < m_1^{myopic} - m_1^?$.

iii) The gains from information to the government are lower when the banker acts strategically, $(m_1{*}-m_1^{?} > m_1^{non-strategic} - m_1^{myopic})$ but the strategic behavior of the banker implies much higher first period marginal losses due to more variability in the inflation rate, i.e. $m_1^{?} < m_1^{myopic}$, but $(m_1{*}-m_1^{?}) - (m_1^{non-strategic} - m_1^{myopic}) < m_1^{myopic} - m_1^{?}$.

Otherwise the strategic behavior of the bank implies less transparent monetary policy, i.e. $m_1{*} > m_1^{non-strategic}$.

Proof: For the algebraic proof see Appendix C.

An intuitive explanation of Proposition 3.7 can be given in Figure 3.10. In this figure the value to the bankers of signaling their type is positive, hence the reaction functions are more spread out than the myopic functions, and the incentives to experiment are higher when bankers act strategically, i.e. $\left(\frac{d\pi_1{}^{L}}{dm_1} - \frac{d\pi_1{}^{H}}{dm_1}\right) - \left(\frac{d\pi_1{}^{L,myopic}}{dm_1} - \frac{d\pi_1{}^{H,myopic}}{dm_1}\right) < 0$. Thus $\left|m_1{*}-m_1^{?}\right| > \left|m_1^{non-strategic} - m_1^{myopic}\right|$ and $m_1{*}-m_1^{?} < m_1^{non-strategic} - m_1^{myopic}$.

The strategic behavior of the bank affects the marginal losses in period one as well through more or less variability in the inflation rate, in particular according to Figure 3.10, $\left(\frac{d\pi_1{}^{L}}{dm_1} - \frac{d\pi_1{}^{L,myopic}}{dm_1}\right) < 0$, $\left(\frac{d\pi_1{}^{H}}{dm_1} - \frac{d\pi_1{}^{H,myopic}}{dm_1}\right) > 0$, $\pi_1{}^{L} > \pi_1{}^{L,myopic}$ and $\pi_1{}^{H} < \pi_1{}^{H,myopic}$. Hence depending on priors μ_0 and on the set of parameters κ, the first period marginal losses implied by the strategic behavior of the bank may be higher or lower than the expected marginal losses implied by the banker that does not act strategically. Thus both $m_1^? > m_1^{myopic}$ and $m_1^? < m_1^{myopic}$ are feasible. If $m_1^? < m_1^{myopic}$ (part *i* of Proposition 3.7), then $m_1^* < m_1^{non-strategic}$ i.e. the strategic behavior of the bank implies more transparent monetary policy. If however $m_1^? > m_1^{myopic}$ the relation between m_1^* and $m_1^{non-strategic}$ depends then magnitude of the gains and losses implied by the strategic behavior of the bank.

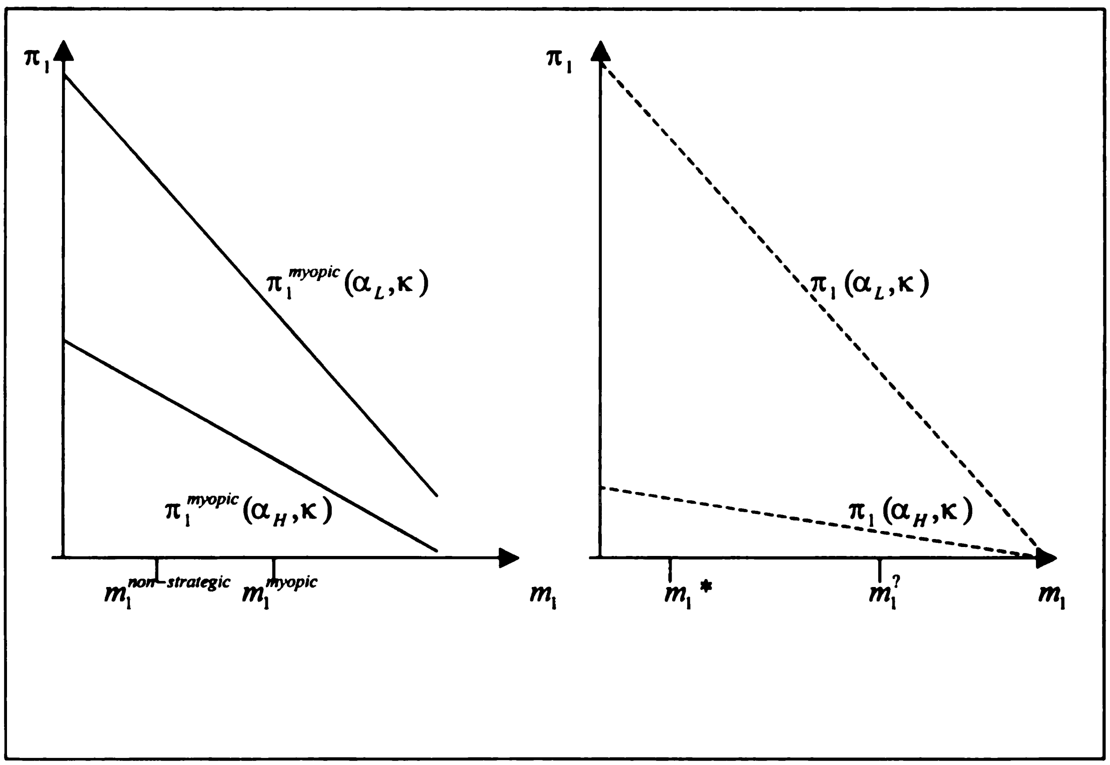

Figure 3.10. The strategic behavior of the banker induces more transparent monetary policy.

When the central banker acts strategically, even if the banker benefits from signaling its type, it might be in the best interest of the government not to induce more transparent monetary policies, unlike when the banker acts myopically.

It is also possible (part *iii* of Proposition 3.7) that even if the value to the bankers of signaling their type is negative, their behavior implies more transparent monetary policy. This will happen if the strategic behavior increases the first period marginal losses of the government significantly.

The combination of these two effects also determines the optimal length to which bankers should be appointed. That is, Proposition 3.4 does not necessarily hold when bankers act strategically. For example, if the strategic behavior of the central banker implies that for every punishment m_1 the static loss function of the government is always above the government's loss function given the myopic reaction functions of the bankers, then one-year appointments are not necessarily inferior any longer.

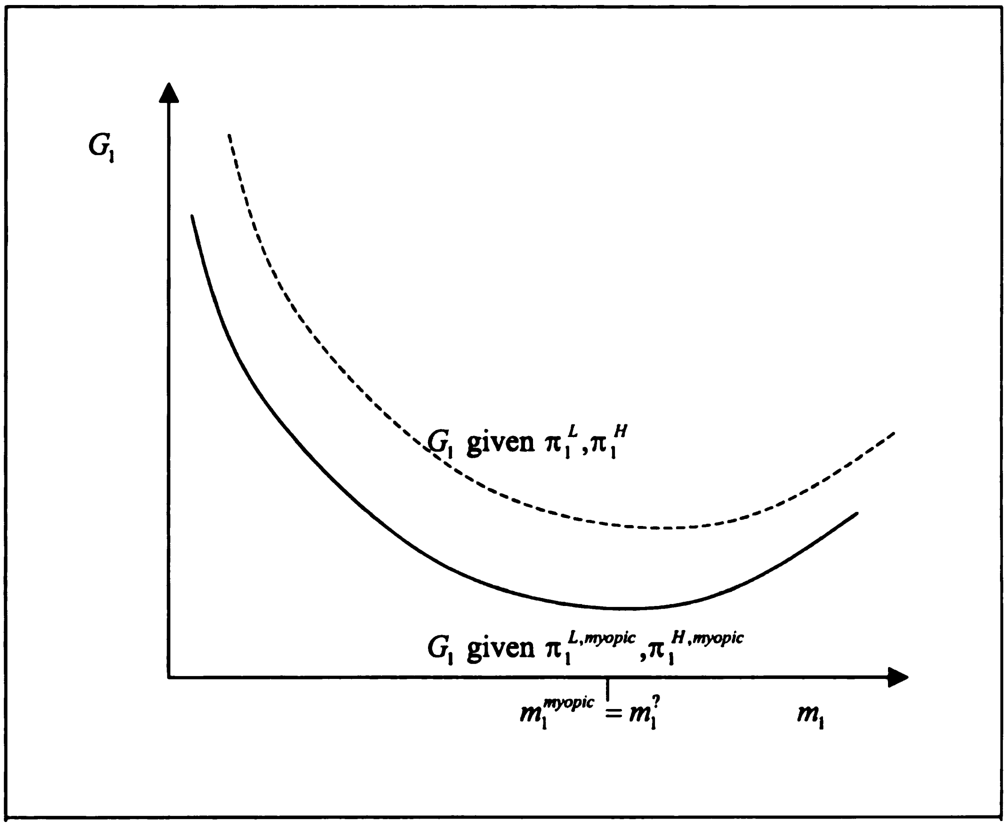

Figure 3.11. Static loss function of the government

In the hypothetical situation of Figure 3.11, the period-one loss-minimizing contract is the same under both reaction functions, $m_1^{myopic} = m_1^?$, but the static losses of the government are lower under m_1^{myopic}. In this case, two-year appointments will only reduce the government's losses below one-year appointments if the government is able to induce large gains from information. This of course depends on the parameters of the model.

3.4 Conclusions

Based on the assumption that distortions such as taxes or unemployment benefits make the natural rate of unemployment inefficiently high, the government has incentives to create surprise inflation to stimulate output. Some of the institutional solutions to this problem deal with the type of individual appointed to the central bank (Rogoff (1985), Lohman (1992)), or with the proper incentives that bankers should face, such as linear contracts (Walsh (1995), Beetsma and Jensen (1998)), inflation targets (Svensson (1997), Beetsma and Jensen (1998)) or dismissal rules (Walsh (1999)).

In this paper I studied the performance of linear contracts in the case in which the government is uncertain about the banker's preferences but can learn about them in time. When the government is uncertain about the preferences of the central bank, the government, based on the observed inflation rate, which is a noisy observation of the banker's type, can design contracts to enhance its information in order to reduce welfare losses. The banker also has incentives to act strategically to affect the government's perception of its type being the true type.

Neither the strategic behavior of the banker nor the reaction of the government has been studied before in the literature. I found that as long as the banker can affect the beliefs of the government it will do so, that is, bankers act strategically. In particular if the banker's value of signaling its type is positive (negative) it will try to increase (decrease) the government's amount of information. Since the government forecasts the behavior of the bank when it designs the contract, the optimal contract accounts for the bank's manipulation of information.

For a given parameter space, the government, unlike the bankers, values information about the bankers' preferences, and thus expected losses are reduced with a better informational state. Transparency of monetary policy, defined as little uncertainty about the banker's preferences can thus increase the welfare of society. If the bankers do not act strategically then two-year terms are superior to one-year terms because the government can always induce more transparency of monetary policy.

If bankers act strategically the latter result is not necessarily true. In particular the optimal length of time in office will depend on the effect of the strategic behavior of the bank on the losses due to excess variability in the economy, and on the extra amount of information that can result from the strategic behavior of the bankers.

Studies about the optimal length of time to which bankers are appointed have focused on the relation between central bank independence, inflation, and turnover rate of central bankers. I add to this list the degree of transparency of monetary policy that can be achieved through multiple year contracts, and the cost of transparency in terms of the variance of the inflation rate around its target. In particular, if the strategic behavior of the central bank implies more transparency of monetary policy and less variability in the expected inflation rate, two-year appointments are superior to one-year appointments. However if it also implies higher losses due to excess variability in the inflation rate, then two-year appointments are not necessarily optimal.

There are several possible extensions to this paper. For example, in this paper I only considered two types of bankers. The model should be generalized to a continuum of types.

I must also study how uncertainty and learning about the bankers' preferences affects the design and performance of inflation targets and of more sophisticated arrangements such as dismissal rules and nonlinear contracts.

My model should also be extended to allow for the possibility of reappointment as a function of learning. If bankers are always appointed to one-year terms and can serve up to two years, then reappointment in the second period will be a function of what the government learns. This adds dynamics to the model because now the banker needs to think not only of minimizing future losses but also of getting reappointed.

Another extension of the model is to assume that bankers are appointed for a fixed number of years but that their appointments can be renewed. This model would then capture better the US system in which the chairman of the Fed is appointed for four year renewable terms.

Finally, virtually any contractual arrangement between the government and the central bank is subject to learning of some kind, not necessarily of the banker's preferences but also of economic phenomena such as the existence of a Phillips curve for example[54]. Particular studies that would be enriched by including learning are for example Svensson's (1997) and Lockwood's (1997) papers of inflation contracts and

[54] Patron (2000) presents a review of the literature of learning and monetary economics.

targets with persistent unemployment in which the dynamic dimension they emphasize naturally gives rise to the possibility of learning.

CONCLUSIONS

Based on the assumption that agent's decisions affect their understanding of their environment I introduce Bayesian learning in three different economic models.

In Chapter 1 I study a two-period model with an incumbent firm threatened with entry. Demand is unknown and stochastic, and prices contain statistical information about demand. I find that, unless the possible demand functions differ by a constant, the incumbent firm always manipulates the entrant's information to discourage entry.

The model of Chapter 1 could be generalized first by assuming asymmetric information in the form of different initial priors for the incumbent and entrant, and second, by studying a model of information jamming, in which the entrant cannot observe the first period quantity, only the price level. In both cases the intuition behind the results should not change. That is, even if firms have different information to start with or learn differently, as long as the incumbent and the entrant are not fully informed about demand, the incumbent has incentives to manipulate or jam the entrant's learning so as to make entry appear less profitable.

In Chapter 2 I study a government that maximizes seignorage over two periods but is uncertain about the money demand function and about the relationship between interest rates and money growth and, real output and money growth. This model is aimed at resembling unstable governments or highly inflationary economies.

I find that unless the possible demand functions differ by a constant, a constant money supply rule is suboptimal. That is, the government should seek to increase its information about the demand function and should adapt to new information. Moreover, if the two possible demand functions are very spread apart, the value to the government of more information, and hence the incentives to experiment, are large, making hyperinflations more likely.

This model leads naturally to the question of how learning affects the end of a hyperinflation, and in particular to the role that learning plays in the reduction of seignorage in the presence of uncertainty.

Another extension is to allow for different learning mechanisms of the public, such as adaptive expectations or least squares learning for example, which would enrich the dynamics of the model by allowing us to study the interaction of learning of the monetary authority and the public.

In Chapter 3 I study the design of linear contracts for central bankers in a two-period model when the government and the public are uncertain about the bankers preferences and about the stochastic variables that constrain their choices, given that they can learn about the bankers in time. I find that there exists a sensible parameter space for which the government values information about the bankers preferences but for which the bankers might not. Moreover the bankers will act strategically to increase or reduce the government's information depending on their own value of signaling their type. The strategic behavior of the central banker has two effects. It increases or reduces information but it might also increase or reduce the variability of the inflation rate around its target. The combination of these effects determines whether two-period contracts are superior to one-period appointments, and whether the government's response to the strategic behavior of the banker implies more or less transparent monetary policies.

This model should be extended to study how uncertainty and learning about the bankers' preferences affects the design and performance of inflation targets and of more sophisticated arrangements such as dismissal rules.

Finally, another extension of the model is to assume that bankers are appointed for a fixed number of years but that their appointments can be renewed. This model would then capture better the US system in which the chairman of the Fed is appointed for four year renewable terms.

APPENDICES

APPENDIX A

APPENDIX TO CHAPTER 1

Proof of existence of equilibrium.

To show that in general that an equilibrium exists, I use a Lemma 1.2 and Proposition 1.1 established in Mirman, Samuelson and Schlee (1994).

Lemma 1.2: If a period two equilibrium exists, then that equilibrium is unique and at least one firm produces a positive quantity.

Proof:

I first show that if the entrant enters and thus if the firms compete in quantities, then if an equilibrium exists, then that equilibrium is unique and at least one firm produces a positive quantity[56].

Let

$$\hat{g}(q,\rho) \equiv \rho g(q,\bar{\gamma}) + (1-\rho) g(q,\underline{\gamma}). \tag{A.1}$$

In equilibrium, q_i must satisfy the first order condition:

$$\hat{g}'(q_I + q_E, \rho) q_i + \hat{g}(q_I + q_E, \rho) - k_i \leq 0. \tag{A.2}$$

However, I assumed that there was positive production in equilibrium, then, the first order condition must be satisfied with equality for at least one firm. If it is satisfied with equality for both firms, adding the two first order conditions yields,

$$\hat{g}'(q_I + q_E, \rho)(q_I + q_E) + 2\hat{g}(q_I + q_E, \rho) - (k_I + k_E) = 0 \tag{A.3}$$

One of my assumptions was that $qg' + 2g$ is strictly decreasing in q, then at most one $q = q_I + q_E$ satisfies (A.3).

There are then two possible equilibria, one in which both firms satisfy their first order condition with equality and one in which only the low cost firm produces. There cannot be an equilibrium in which only the high cost firm produces. If this were true, the

[56] Mirman, Samuelson and Schlee (1994), Lemma 2, page 368.

price would exceed the marginal cost of the high cost firm and also the marginal cost of the low cost firm, but then, production of zero is not optimal for the low cost firm.

If firm I is the low cost firm, the first order condition for the equilibrium in which only the low cost firm produces are,

$$\hat{g}'(q_I^c,\rho)q_I^c + 2\hat{g}(q_I^c,\rho) - k_I = 0 \tag{A.4}$$

$$\hat{g}(q_I^c,\rho) - k_E < 0 \tag{A.5}$$

The first order conditions for the equilibrium in which the first order conditions of both firms are satisfied with equality are:

$$\hat{g}'(q_I^I + q_E^I,\rho)q_I^I + \hat{g}(q_I^I + q_E^I,\rho) - k_I = 0 \tag{A.6}$$

$$\hat{g}'(q_I^I + q_E^I,\rho)q_E^I + \hat{g}(q_I^I + q_E^I,\rho) - k_E = 0 \tag{A.7}$$

Since $\hat{g}'(q_I^I + q_E^I,\rho) < 0$ conditions (A.5) and (A.7) can hold only if $q_I^c > q_I^I + q_E^I$.

Using $\hat{g}' < 0$, equations (A.4) and (A.6) give

$$\begin{aligned}&\hat{g}'(q_I^I + q_E^I,\rho)(q_I^I + q_E^I) + \hat{g}(q_I^I + q_E^I,\rho) \le \hat{g}'(q_I^I + q_E^I,\rho)q_I^I + \hat{g}(q_I^I + q_E^I,\rho)\\ &= k_I = \hat{g}'(q_I^c,\rho)q_I^c + \hat{g}(q_I^c,\rho)\end{aligned} \tag{A.8}$$

The first and last term in equation (A.8) are the marginal revenue of firm I. The inequality of these two terms given $q_I^c > q_I^I + q_E^I$, contradicts quasiconcavity of expected profits. Thus there is a unique aggregate output in equilibrium. Individual outputs are then determined by the first order conditions.

If there is no entry and the incumbent sets the monopoly quantity, then by the assumption of strict quasiconcavity, the monopoly-profits maximizing quantity is unique.

(Q.E.D.)

Proposition 1.1: Let $g(Q,\gamma)$ be defined and continuous on $\mathbb{R}_+$ for $\gamma \in \{\overline{\gamma}, \underline{\gamma}\}$ and let there be $\hat{Q}$ such that $g(\hat{Q},\gamma)=0$ for $\gamma \in \{\overline{\gamma}, \underline{\gamma}\}$. Then a (possibly mixed strategy) sequential equilibrium exists.

Proof (sketch): For a complete proof, see Mirman, Samuelson and Schlee (1994). As a sketch consider the following proof.

First, restrict the strategy space to the closed interval $[0,\hat{Q}]$. By the assumptions of quasiconcavity and continuity of π_i, there exists an equilibrium of the game played in the second period, $(q_I^*(\rho),\ q_E^*(\rho))$. This equilibrium (these two reaction functions) is continuous in ρ. The functions $h(p,q)$, $V_I(\rho)$ and $V_I^m(\rho)$ are continuous in ρ as well, which is continuous in Q_I, thus the entire equation $\pi_I(Q_I,\rho^0)+\delta\ [W(Q_I)]$, where

$$W(Q_I)=\int_{-\infty}^{\psi(Q_I)} V_I^m(\rho(p,Q_I))h(p,Q_I)dp+\int_{\psi(Q_I)}^{\infty} V_I(\rho(p,Q_I))h(p,Q_I)dp,$$

is continuous in Q_I. Applying Glicksberg's (1952) extension of Kakuthani's fixed-point theorem completes the proof. Then a solution to the entire game exists.

(Q.E.D)

Lemma 1.3: Given assumptions A1.1 and A1.2, an incumbent firm threatened with entry always limit prices, that is $G(Q_I*)\le G(Q_I^{FE})$. In particular,

(a) If reductions in quantity increase information, and through more information increase the probability of entry $\left((\overline{g}'-\underline{g}')\le 0 \ \ and \ \ \frac{dG(Q_I)}{dQ_I}\le 0\right)$, the incumbent sets a quantity higher than the incumbent that takes entry as given, i.e. $Q_I^* \geq Q_I^{FE}$. Since this reduces the probability of entry and decreases information, then the incumbent limit prices by concealing information from the entrant.

(b) If increases in quantity increase information, and through more information increase the probability of entry $\left((\bar{g}' - \underline{g}') \geq 0 \quad and \quad \frac{dG(Q_I)}{dQ_I} \geq 0 \right)$, then the incumbent sets a quantity lower than the incumbent that takes entry as given, i.e. $Q_I^* \leq Q_I^{FE}$. Since this reduces the probability of entry and decreases information, then the incumbent limit prices by concealing information from the entrant.

(c) If increases in quantity increase information, and through more information reduce the probability of entry $\left((\bar{g}' - \underline{g}') \geq 0 \quad and \quad \frac{dG(Q_I)}{dQ_I} \leq 0 \right)$, then the incumbent sets a quantity higher than the incumbent that takes entry as given, i.e. $Q_I^* \geq Q_I^{FE}$. Since this reduces the probability of entry and increases information, then *the incumbent limit prices by revealing information to the entrant.*

(d) If decreases in quantity increase information, and through more information reduce the probability of entry $\left((\bar{g}' - \underline{g}') \leq 0 \quad and \quad \frac{dG(Q_I)}{dQ_I} \geq 0 \right)$, the incumbent sets a quantity lower than the incumbent that takes entry as given, i.e. $Q_I^* \leq Q_I^{FE}$. Since this reduces the probability of entry and increases information, then *the incumbent limit prices by revealing information to the entrant.*

Proof:

If the incumbent sets a quantity above or below the quantity of a monopolist that takes the probability of entry as fixed (Q_I^{FE}) so as to reduce the probability of entry then the incumbent limit prices.

To study the incumbent's incentives to deviate from Q_I^{FE} I look at how the first order conditions of the two problems differ. I first look at the maximization problem of the incumbent that does not take entry as fixed, which maximizes $\pi_I(Q_I, \rho^0) + \delta W(Q_I)$, where

$$W(Q_I) = \int_{-\infty}^{\psi(Q_I)} V_I^m(\rho(p,Q_I))h(p,Q_I)dp + \int_{\psi(Q_I)}^{\infty} V_I(\rho(p,Q_I))h(p,Q_I)dp.$$

The first order condition is given by $\frac{d\pi_I(Q_I,\rho^0)}{dQ_I} + \delta\frac{dW(Q_I)}{dQ_I} = 0$, which using Leibniz' rule and looks like,

$$\frac{d\pi_I(Q_I,\rho^0)}{dQ_I} + \delta\left[\begin{array}{l} -\psi'(Q_I)h(\psi(Q_I),Q_I)\left[V_I(\rho(\psi(Q_I),Q_I)) - V_I^m(\rho(\psi(Q_I),Q_I))\right] + \\ \int_{\psi(Q_I)}^{\infty} V_I'(\rho)\frac{d\rho}{dQ_I}h(p,Q_I)dp + \int_{-\infty}^{\psi(Q_I)} V_I^m{}'(\rho)\frac{d\rho}{dQ_I}h(p,Q_I)dp + \\ \int_{\psi(Q_I)}^{\infty} V_I(\rho)\frac{dh(p,Q_I)}{dQ_I}dp + \int_{-\infty}^{\psi(Q_I)} V_I^m(\rho)\frac{dh(p,Q_I)}{dQ_I}dp \end{array}\right] \equiv 0.$$

Since $G(Q_I) = \int_{\psi(Q_I)}^{\infty} h(p,Q_I)dp$, then using Leibniz rule,

$$\frac{dG}{dQ_I} = -\psi'(Q_I)h(\psi(Q_I),Q_I) + \int_{\psi(Q_I)}^{\infty} \frac{dh(p,Q_I)}{dQ_I}dp,$$

then the first order condition can be written as follows,

$$\frac{d\pi_I(Q_I,\rho^0)}{dQ_I} + \delta \begin{bmatrix} \left[\frac{dG}{dQ_I} - \int_{\psi(Q_I)}^{\infty} \frac{dh(p,Q_I)}{dQ_I}dp\right]\left[V_I(\rho(\psi(Q_I),Q_I)) - V_I^m(\rho(\psi(Q_I),Q_I))\right] + \\ \int_{\psi(Q_I)}^{\infty} V_I'(\rho)\frac{d\rho}{dQ_I}h(p,Q_I)dp + \int_{-\infty}^{\psi(Q_I)} V_I^m{}'(\rho)\frac{d\rho}{dQ_I}h(p,Q_I)dp + \\ \int_{\psi(Q_I)}^{\infty} V_I(\rho)\frac{dh(p,Q_I)}{dQ_I}dp + \int_{-\infty}^{\psi(Q_I)} V_I^m(\rho)\frac{dh(p,Q_I)}{dQ_I}dp \end{bmatrix} \equiv 0 \tag{A.9}$$

The incumbent that takes entry as given (or that naively thinks that it cannot affect the probability of entry) solves the same problem but assumes that $\frac{dG}{dQ_I} = 0$. Alternatively, assume that the incumbent that takes entry as given maximizes $\pi_I(Q_I,\rho^0) + \delta W(Q_I)$ subject to the restriction that $\frac{dG}{dQ_I} = 0$. This firm then maximizes,

$$\pi_I(Q_I,\rho^0) + \delta W(Q_I) + \lambda\frac{dG}{dQ_I},$$

where λ is the Lagrange multiplier associated with the restriction that $\frac{dG}{dQ_I} = 0$. The two first order conditions are thus given by,

$$
\begin{aligned}
&\frac{d\pi_I(Q_I,\rho^0)}{dQ_I} \\
&\left[\begin{array}{l}
\left[\dfrac{dG}{dQ_I}-\displaystyle\int_{\psi(Q_I)}^{\infty}\dfrac{dh(p,Q_I)}{dQ_I}dp\right]\left[V_I(\rho(\psi(Q_I),Q_I))-V_I^m(\rho(\psi(Q_I),Q_I))\right]+ \\
+\delta\left[\begin{array}{l}
\displaystyle\int_{\psi(Q_I)}^{\infty}V_I\,'(\rho)\frac{d\rho}{dQ_I}h(p,Q_I)dp+\int_{-\infty}^{\psi(Q_I)}V_I^m\,'(\rho)\frac{d\rho}{dQ_I}h(p,Q_I)dp+ \\
\displaystyle\int_{\psi(Q_I)}^{\infty}V_I(\rho)\frac{dh(p,Q_I)}{dQ_I}dp+\int_{-\infty}^{\psi(Q_I)}V_I^m(\rho)\frac{dh(p,Q_I)}{dQ_I}dp
\end{array}\right] \\
+\lambda\dfrac{d^2G}{dQ_I^{\,2}}
\end{array}\right]\equiv 0
\end{aligned}
$$

and

$$\frac{dG}{dQ_I}\equiv 0.$$

Plugging the second equation in the first gives,

$$
\begin{aligned}
&\frac{d\pi_I(Q_I,\rho^0)}{dQ_I} \\
&\left[\begin{array}{l}
\left[-\displaystyle\int_{\psi(Q_I)}^{\infty}\dfrac{dh(p,Q_I)}{dQ_I}dp\right]\left[V_I(\rho(\psi(Q_I),Q_I))-V_I^m(\rho(\psi(Q_I),Q_I))\right]+ \\
+\delta\left[\begin{array}{l}
\displaystyle\int_{\psi(Q_I)}^{\infty}V_I\,'(\rho)\frac{d\rho}{dQ_I}h(p,Q_I)dp+\int_{-\infty}^{\psi(Q_I)}V_I^m\,'(\rho)\frac{d\rho}{dQ_I}h(p,Q_I)dp+ \\
\displaystyle\int_{\psi(Q_I)}^{\infty}V_I(\rho)\frac{dh(p,Q_I)}{dQ_I}dp+\int_{-\infty}^{\psi(Q_I)}V_I^m(\rho)\frac{dh(p,Q_I)}{dQ_I}dp
\end{array}\right] \\
+\lambda\dfrac{d^2G}{dQ_I^{\,2}}
\end{array}\right]\equiv 0.
\end{aligned}
$$

Since this hypothetical incumbent assumes that the probability of entry is a constant, then it must also be true that $\frac{d^2G}{dQ_I^{\,2}}=0$. (Alternatively, since $\frac{dG}{dQ_I}\equiv 0$ it can be differentiated to yield $\frac{d^2G}{dQ_I^{\,2}}=0$.) Its first order condition thus reduces to,

$$\frac{d\pi_I(Q_I,\rho^0)}{dQ_I}+\delta\left[\begin{array}{l}\left[-\int\limits_{\psi(Q_I)}^{\infty}\frac{dh(p,Q_I)}{dQ_I}dp\right]\left[V_I(\rho(\psi(Q_I),Q_I))-V_I^m(\rho(\psi(Q_I),Q_I))\right]+\\ \int\limits_{\psi(Q_I)}^{\infty}V_I'(\rho)\frac{d\rho}{dQ_I}h(p,Q_I)dp+\int\limits_{-\infty}^{\psi(Q_I)}V_I^m{}'(\rho)\frac{d\rho}{dQ_I}h(p,Q_I)dp+\\ \int\limits_{\psi(Q_I)}^{\infty}V_I(\rho)\frac{dh(p,Q_I)}{dQ_I}dp+\int\limits_{-\infty}^{\psi(Q_I)}V_I^m(\rho)\frac{dh(p,Q_I)}{dQ_I}dp\end{array}\right]\equiv 0 \tag{A.10}$$

The difference between the first order conditions of the two incumbents is thus given by (A.9) minus (A.10),

$$\delta\frac{dG}{dQ_I}\left[V_I(\rho(\psi(Q_I),Q_I))-V_I^m(\rho(\psi(Q_I),Q_I))\right] \tag{A.11}$$

Since $V_I(\rho)$ is decreasing in q^*, then $\left[V_I(\rho(\psi(Q_I),Q_I))-V_I^m(\rho(\psi(Q_I),Q_I))\right]\leq 0$. Thus equation (A.11) is positive (negative) if $\frac{dG}{dQ_I}\leq(\geq)0$. If $\frac{dG}{dQ_I}\leq(\geq)0$, then for the incumbent's first order condition (equation (A.9)) to be satisfied, it must be true that

$$\frac{d\pi_I(Q_I,\rho^0)}{dQ_I}+\delta\left[\begin{array}{l}\left[-\int\limits_{\psi(Q_I)}^{\infty}\frac{dh(p,Q_I)}{dQ_I}dp\right]\left[V_I(\rho(\psi(Q_I),Q_I))-V_I^m(\rho(\psi(Q_I),Q_I))\right]+\\ \int\limits_{\psi(Q_I)}^{\infty}V_I'(\rho)\frac{d\rho}{dQ_I}h(p,Q_I)dp+\int\limits_{-\infty}^{\psi(Q_I)}V_I^m{}'(\rho)\frac{d\rho}{dQ_I}h(p,Q_I)dp+\\ \int\limits_{\psi(Q_I)}^{\infty}V_I(\rho)\frac{dh(p,Q_I)}{dQ_I}dp+\int\limits_{-\infty}^{\psi(Q_I)}V_I^m(\rho)\frac{dh(p,Q_I)}{dQ_I}dp\end{array}\right]\leq(\geq)0,$$

which is the incumbent's that takes entry as given first order condition. This means that Q_I* evaluated at the incumbent's that takes entry as given objective function yields negative (positive) slope, and thus if this is a concave function, it must be the case that $Q_I*\geq(\leq)Q_I^{FE}$.

To characterize the lemma, pair up the possible cases of $(\bar{g}'-\underline{g}')$ and $\frac{dG}{dQ_I}$. For example if $(\bar{g}'-\underline{g}')>0$ and $\rho^0>\rho^*$, then $\frac{dG}{dQ_I}\leq 0$. Hence according to (A.11), $Q_I^*\geq Q_I^{FE}$; if $(\bar{g}'-\underline{g}')>0$ and $\rho^0<\rho^*$, then $\frac{dG}{dQ_I}\geq 0$. Hence $Q_I^*\leq Q_I^{FE}$.

(Q.E.D.)

APPENDIX B

APPENDIX TO CHAPTER 2

Proof of Proposition 2.1

Proposition 2.1: The government that maximizes expected seignorage values information, i.e. $\frac{d^2 S_2*(\mu_2)}{d\mu_2^2} \geq 0$.

Proof: This proof originally comes from Mirman, Samuelson and Urbano (1993) (p. 555).

The proof is a variation of an envelope argument. Let $g_2(\mu_2)$ be the optimal period-two money growth rate as a function of μ_2. Fix $\hat{\mu}_2$. Define:

$$F(\mu_2) = \mu_2 l(g_2(\hat{\mu}_2), \overline{\Omega}) g_2(\hat{\mu}_2) + (1-\mu_2) l(g_2(\hat{\mu}_2), \underline{\Omega}) g_2(\hat{\mu}_2). \tag{B.12}$$

Then $F(\mu_2)$ is linear in μ_2 and $S_2*(\mu_2) \geq F(\mu_2)$. Since $S_2*(\mu_2)$ is the supremum of a set of linear functions, it must be convex (Rockafeller (1970) Theorem 5.5, p. 35)).

Q.E.D.

Proof of Proposition 2.2

Proposition 2.2: Let the MLRP hold. Then the optimal money growth rate $g_1(\mu_1)$ is higher (lower) than the myopic rate $g_{myopic}(\mu_1)$, if increasing the rate of growth of money increases (decreases) information. That is, $g_1(\mu_1) \geq (\leq) g_{myopic}(\mu_1)$ if $l'(g_1, \overline{\Omega}) \geq (\leq) l'(g_1, \underline{\Omega})$.

Proof:

First note that the first order condition of the non-myopic problem is given by $\frac{dE_m S_2*(\mu_2)}{dg_1} + \frac{dE_{\mu_1} S_1(\mu_1)}{dg_1} = 0$, while the myopic first order condition is given by

$\frac{dE_{\mu_1} S_1(\mu_1)}{dg_1} = 0$. If $\frac{dE_m S_2 *(\mu_2)}{dg_1} \geq (\leq) 0$ then, in order for the non-myopic first order condition to be satisfied, it must be true that $\frac{dE_{\mu_1} S_1(\mu_1)}{dg_1} \leq (\geq) 0$. Given that $E_{\mu_1} S_1(\mu_1)$ is a concave function, this implies that $g(\mu_1) \geq (\leq) g_{myopic}(\mu_1)$. Graphically this argument is represented in Figure 2.3.

I now show that,

$$\frac{d\int_{-\infty}^{\infty} S_2 *(\mu_2) h(m,g) dm}{dg_1} = (l'(g_1, \overline{\Omega}) - l'(g_1, \underline{\Omega})) \int_{-\infty}^{\infty} \frac{d^2 S_2 *}{d\mu_2{}^2} \mu_2 (1-\mu_1) \frac{d\mu_2}{dm} \underline{f} dm .$$

Since $E_m S_2 *(\mu_2) = \int_{-\infty}^{\infty} S_2 *(\mu_2) h(m,g) dm$, I differentiate inside the integral,

$$\frac{dE_m S_2 *(\mu_2)}{dg_1} = \int_{-\infty}^{\infty} \left(\frac{dS_2 *}{d\mu_2} \frac{d\mu_2}{dg_1} h(m,g) + S_2 *(\mu_2) \frac{dh(m,g)}{dg_1} \right) dm \tag{B.13}$$

Let $\overline{f} = f(m_1 - l'(g_1, \overline{\Omega}) g_1)$ and $\underline{f} = f(m_1 - l'(g_1, \underline{\Omega}) g_1)$, and let $\overline{l}' = l'(g_1, \overline{\Omega})$ and $\underline{l}' = l'(g_1, \underline{\Omega})$ then, since $h(m,g) = (1-\mu_1)\underline{f} + \mu_1 \overline{f}$,

$$\frac{dh(m,g)}{dg_1} = -\mu_1 \overline{f}' \overline{l}' - (1-\mu_1) \underline{f}' \underline{l}' . \tag{B.14}$$

Plugging equation (B.14) in (B.13),

$$\frac{dE_m S_2 *(\mu_2)}{dg_1} = \int_{-\infty}^{\infty} \left(\frac{dS_2 *}{d\mu_2} \frac{d\mu_2}{dg_1} h(m,g) - S_2 *(\mu_2) [\mu_1 \overline{f}' \overline{l}' + (1-\mu_1) \underline{f}' \underline{l}'] \right) dm . \tag{B.15}$$

Integrating the second term in the rhs of equation (B.15) by parts as follows,

$$\int_{-\infty}^{\infty} S_2 *(\mu_2) [\mu_1 \overline{f}' \overline{l}' + (1-\mu_1) \underline{f}' \underline{l}'] dm = \begin{pmatrix} S_2 *(\mu_2) (\mu_1 \overline{f} \overline{l}' + (1-\mu_1) \underline{f} \underline{l}') \big|_{-\infty}^{\infty} - \\ \int_{-\infty}^{\infty} \frac{dS_2 *}{d\mu_2} \frac{d\mu_2}{dm} [\mu_1 \overline{f} \overline{l}' + (1-\mu_1) \underline{f} \underline{l}'] dm \end{pmatrix} , \tag{B.16}$$

and since $S_2*(\mu_1\bar{f}\bar{l}'+(1-\mu_1)\underline{f}\underline{l}')|_{-\infty}^{\infty}=0$ because $\bar{f}(\infty)=\underline{f}(\infty)=\bar{f}(-\infty)=\underline{f}(-\infty)=0$, plugging (B.16) in (B.15) leads to,

$$\frac{dE_m S_2*(\mu_2)}{dg_1}=\int_{-\infty}^{\infty}\left(\frac{dS_2*}{d\mu_2}\frac{d\mu_2}{dg_1}h(g,m)+\frac{dS_2*}{d\mu_2}\frac{d\mu_2}{dm}[\mu_1\bar{f}\bar{l}'+(1-\mu_1)\underline{f}\underline{l}']\right)dm\,. \tag{B.17}$$

Now I want to write $\frac{d\mu_2}{dg_1}$ as a function of $\frac{d\mu_2}{dm}$ to plug in (B.17). To do so, I need a few manipulations. First of all, taking the derivative of $h(m,g)$ with respect to m,

$$\frac{dh}{dm}=\mu_1\bar{f}'+(1-\mu_1)\underline{f}' \tag{B.18}$$

and combining it with (B.14),

$$\frac{dh}{dg_1}=-\bar{l}'\frac{dh}{dm}-(1-\mu_1)\underline{f}'(\underline{l}'-\bar{l}'). \tag{B.19}$$

I also need the following manipulations. From Bayes rule, $\mu_2=\frac{\mu_1\bar{f}}{(1-\mu_1)\underline{f}+\mu_1\bar{f}}=\frac{\mu_1\bar{f}}{h(m,g)}$, thus $\mu_2\,h(m,g)=\mu_1\bar{f}$. Then,

$$\frac{d\mu_2}{dm}=\frac{\mu_1\bar{f}'}{h}-\frac{\mu_1\bar{f}\left(\frac{dh}{dm}\right)}{h^2} \tag{B.20}$$

but since $\mu_1\bar{f}=\mu_2\,h(m,g)$ then $\frac{\mu_1\bar{f}\left(\frac{dh}{dm}\right)}{h^2}=\frac{\mu_2\left(\frac{dh}{dm}\right)}{h}$, thus equation (B.20) becomes,

$$\frac{d\mu_2}{dm}=\frac{\mu_1\bar{f}'-\mu_2\left(\frac{dh}{dm}\right)}{h} \tag{B.21}$$

Using $\mu_1\bar{f}=\mu_2\,h(m,g)$ once more,

$$\frac{d\mu_2}{dg_1}=-\frac{\mu_1\bar{f}'\bar{l}'}{h}-\frac{\mu_1\bar{f}\left(\frac{dh}{dg_1}\right)}{h^2}=\frac{-\mu_1\bar{f}'\bar{l}'-\mu_2\left(\frac{dh}{dg_1}\right)}{h} \tag{B.22}$$

I now can write equation (B.22) as a function of $\frac{d\mu_2}{dm}$ using equations (B.14), (B.18), (B.19) and (B.21). First, plugging (B.14) in (B.22),

$$\frac{d\mu_2}{dg_1} = \frac{-\mu_1 \bar{f}'\bar{l}'}{h} - \frac{\mu_2\left(-\mu_1 \bar{f}'\bar{l}' - (1-\mu_1)\underline{f}'\underline{l}'\right)}{h},$$

collecting $\bar{l}'$,

$$\frac{d\mu_2}{dg_1} = -\bar{l}'\left(\frac{\mu_1 \bar{f}'}{h} - \frac{\mu_2\mu_1 \bar{f}'}{h}\right) + \frac{\mu_2(1-\mu_1)\underline{f}'\underline{l}'}{h},$$

adding and subtracting $\frac{\bar{l}'\mu_2(1-\mu_1)\underline{f}'}{h}$,

$$\frac{d\mu_2}{dg_1} = -\bar{l}'\left(\frac{\mu_1 \bar{f}'}{h} - \frac{\mu_2}{h}\left(\mu_1 \bar{f}' + (1-\mu_1)\underline{f}' - (1-\mu_1)\underline{f}'\right)\right) + \frac{\mu_2(1-\mu_1)\underline{f}'\underline{l}'}{h},$$

using (B.18) to replace $\mu_1 \bar{f}' + (1-\mu_1)\underline{f}'$ above with $\frac{dh}{dm}$,

$$\frac{d\mu_2}{dg_1} = -\bar{l}'\left(\frac{\mu_1 \bar{f}'}{h} - \frac{\mu_2}{h}\frac{dh}{dm} + \frac{\mu_2(1-\mu_1)\underline{f}'}{h}\right) + \frac{\mu_2(1-\mu_1)\underline{f}'\underline{l}'}{h},$$

using (B.21) to replace $\frac{\mu_1 \bar{f}'}{h} - \frac{\mu_2}{h}\frac{dh}{dm}$ above with $\frac{d\mu_2}{dm}$,

$$\frac{d\mu_2}{dg_1} = -\bar{l}'\frac{d\mu_2}{dm} - \frac{\mu_2(1-\mu_1)\underline{f}'\bar{l}'}{h} + \frac{\mu_2(1-\mu_1)\underline{f}'\underline{l}'}{h},$$

and collecting $\frac{\mu_2(1-\mu_1)\underline{f}'}{h}$, I finally arrive at,

$$\frac{d\mu_2}{dg_1} = -\bar{l}'\frac{d\mu_2}{dm} + \frac{\mu_2(1-\mu_1)\underline{f}'(\underline{l}'-\bar{l}')}{h} \qquad \text{(B.23)}$$

Plugging (B.23) in equation (B.17),

$$\frac{dE_m S_2^*(\mu_2)}{dg_1} = \begin{pmatrix} \displaystyle\int_{-\infty}^{\infty} \frac{dS_2^*}{d\mu_2}\left(-\bar{l}'\frac{d\mu_2}{dm} + \frac{\mu_2(1-\mu_1)\underline{f}'(\underline{l}'-\bar{l}')}{h} \right) h(g,m)dm + \\ \displaystyle\int_{-\infty}^{\infty} \frac{dS_2^*}{d\mu_2}\frac{d\mu_2}{dm}[\mu_1 \bar{f}\bar{l}' + (1-\mu_1)\underline{f}\underline{l}']dm \end{pmatrix} \tag{B.24}$$

I now replace $h(g,m)$ by $(1-\mu_1)\underline{f} + \mu_1\bar{f}$ in (B.24),

$$\frac{dE_m S_2^*(\mu_2)}{dg_1} = \begin{pmatrix} \displaystyle\int_{-\infty}^{\infty} \frac{dS_2^*}{d\mu_2}\left(-\bar{l}'\frac{d\mu_2}{dm} \right)((1-\mu_1)\underline{f} + \cancel{\mu_1\bar{f}})dm + \\ \displaystyle\int_{-\infty}^{\infty} \frac{dS_2^*}{d\mu_2}\left(\mu_2(1-\mu_1)\underline{f}'(\underline{l}'-\bar{l}') \right)dm + \\ \displaystyle\int_{-\infty}^{\infty} \frac{dS_2^*}{d\mu_2}\frac{d\mu_2}{dm}[\cancel{\mu_1\bar{f}\bar{l}'} + (1-\mu_1)\underline{f}\underline{l}']dm \end{pmatrix}.$$

Canceling the terms shown,

$$\frac{dE_m S_2^*(\mu_2)}{dg_1} = \begin{pmatrix} \displaystyle\int_{-\infty}^{\infty} \frac{dS_2^*}{d\mu_2}\left(-\bar{l}'\frac{d\mu_2}{dm} \right)((1-\mu_1)\underline{f})dm + \\ \displaystyle\int_{-\infty}^{\infty} \frac{dS_2^*}{d\mu_2}\left(\mu_2(1-\mu_1)\underline{f}'(\underline{l}'-\bar{l}') \right)dm + \\ \displaystyle\int_{-\infty}^{\infty} \frac{dS_2^*}{d\mu_2}\frac{d\mu_2}{dm}(1-\mu_1)\underline{f}\underline{l}'dm \end{pmatrix},$$

and collecting $(\underline{l}'-\bar{l}')$,

$$\frac{dE_m S_2^*(\mu_2)}{dg_1} = (\underline{l}'-\bar{l}')\left[\int_{-\infty}^{\infty} \frac{dS_2^*}{d\mu_2}\frac{d\mu_2}{dm}(1-\mu_1)\underline{f}dm + \int_{-\infty}^{\infty} \frac{dS_2^*}{d\mu_2}\mu_2(1-\mu_1)\underline{f}'dm \right] \tag{B.25}$$

Integrating by parts the second term in the rhs of (B.25),

$$\int_{-\infty}^{\infty} \frac{dS_2^*}{d\mu_2}\mu_2(1-\mu_1)\underline{f}'dm = \begin{pmatrix} \displaystyle\frac{dS_2^*}{d\mu_2}\mu_2(1-\mu_1)\underline{f}\Big|_{-\infty}^{\infty} - \\ \displaystyle\int_{-\infty}^{\infty} \frac{d^2S_2^*}{d\mu_2^{\,2}}\frac{d\mu_2}{dm}\mu_2(1-\mu_1)\underline{f}dm - \\ \displaystyle\int_{-\infty}^{\infty} \frac{dS_2^*}{d\mu_2}\frac{d\mu_2}{dm}(1-\mu_1)\underline{f}dm \end{pmatrix} \tag{B.26}$$

Since $\underline{f}(\infty) = \underline{f}(-\infty) = 0$, plugging (B.26) in (B.25) gives,

$$\frac{dE_m S_2^*(\mu_2)}{dg_1} = (\underline{l}'-\bar{l}')\begin{bmatrix} \int_{-\infty}^{\infty} \cancel{\frac{dS_2^*}{d\mu_2}\frac{d\mu_2}{dm}(1-\mu_1)\underline{f}\,dm} - \\ \int_{-\infty}^{\infty} \frac{d^2S_2^*}{d\mu_2^{\,2}}\frac{d\mu_2}{dm}\mu_2(1-\mu_1)\underline{f}\,dm - \\ \int_{-\infty}^{\infty} \cancel{\frac{dS_2^*}{d\mu_2}\frac{d\mu_2}{dm}(1-\mu_1)\underline{f}\,dm} \end{bmatrix},$$

and canceling terms,

$$\frac{dE_m S_2^*(\mu_2)}{dg_1} = (\bar{l}'-\underline{l}')\int_{-\infty}^{\infty} \frac{d^2S_2^*}{d\mu_2^{\,2}}\mu_2(1-\mu_1)\frac{d\mu_2}{dm}\underline{f}\,dm \tag{B.27}$$

Given that $\frac{d^2S_2^*}{d\mu_2^{\,2}} \geq 0$ (Proposition 2.1), if $\frac{d\mu_2}{dm} \geq 0$, then $\frac{dE_m S_2^*(\mu_2)}{dg_1} \geq (\leq) 0$ whenever $(\bar{l}'-\underline{l}') \geq (\leq) 0$. Thus, given $\frac{d\mu_2}{dm} \geq 0$, $g_1(\mu_1) \geq (\leq)\, g_{myopic}(\mu_1)$ if $(\bar{l}'-\underline{l}') \geq (\leq) 0$.

I now show that the MLRP assumption implies $\frac{d\mu_2}{dm} \geq 0$.

By Bayes rule, $\mu_2 = \frac{\mu_1\bar{f}}{(1-\mu_1)\underline{f}+\mu_1\bar{f}}$, thus

$$\frac{d\mu_2}{dm} = \frac{\mu_1\bar{f}'\left[(1-\mu_1)\underline{f}+\cancel{\mu_1\bar{f}}\right]-\mu_1\bar{f}\left[(1-\mu_1)\underline{f}'+\cancel{\mu_1\bar{f}'}\right]}{\left[(1-\mu_1)\underline{f}+\mu_1\bar{f}\right]^2} = \frac{\mu_1(1-\mu_1)(\underline{f}\bar{f}'-\bar{f}\underline{f}')}{\left[(1-\mu_1)\underline{f}+\mu_1\bar{f}\right]^2}.$$

The sign of $\frac{d\mu_2}{dm}$ is thus equal to the sign of $(\underline{f}\bar{f}'-\bar{f}\underline{f}')$. From the MLRP I know that $\frac{f'}{f}$ is decreasing in ε, where $\varepsilon = m - l(g,\Omega)$. Since by assumption $\bar{l} \geq \underline{l}$, then $m-\bar{l} \leq m-\underline{l}$, and thus $\frac{\bar{f}'}{\bar{f}} \geq \frac{\underline{f}'}{\underline{f}}$. Cross multiplying gives us, $\bar{f}'\underline{f} \geq \underline{f}'\bar{f}$. Hence $(\underline{f}\bar{f}'-\bar{f}\underline{f}') \geq 0$, and $\frac{d\mu_2}{dm} \geq 0$. Q.E.D.

Algebraic solution for the uniformly distributed shocks on linear demand functions.

In this appendix I find the non-myopic and myopic money growth rates under linear demand functions of the type used in the example of section 2.6. For notational brevity, assume that the two possible demand functions are $m_t = x + \bar{z}g_t + \varepsilon_t$ and $m_t = x + \underline{z}g_t + \varepsilon_t$, that is, assume that only the slope of the demand function is unknown. By letting $x = a - b\alpha + \hat{Y}$ and $z = \phi - b\beta$, we get back the model of section 2.6. Assume starting values $g_0 = g_{myopic}, P_0 = 1$ and $\mu_1 = 0.5$.

Note that since $P_0 = \dfrac{M_0}{x + \bar{z}g_{myopic} + \varepsilon}$ or $P_0 = \dfrac{M_0}{x + \underline{z}g_{myopic} + \varepsilon}$, and since $P_0 = 1$, then $M_0 = x + \bar{z}g_{myopic} + \varepsilon$ or $M_0 = x + \underline{z}g_{myopic} + \varepsilon$, then it is reasonable to assume an initial money stock of $0.5(x + \bar{z}g_{myopic}) + 0.5(x + \underline{z}g_{myopic})$.

The random shocks ε_t are distributed uniformly on the interval $[-\hat{\varepsilon}, \hat{\varepsilon}]$, and thus the three possible posterior beliefs are,

$$\mu_2 = \begin{cases} 0 & if \;\; x + \underline{z}g_1 - \hat{\varepsilon} \le m_1 < x + \bar{z}g_1 - \hat{\varepsilon} \\ 0.5 & if \;\; x + \bar{z}g_1 - \hat{\varepsilon} \le m_1 \le x + \underline{z}g_1 + \hat{\varepsilon} \\ 1 & if \;\; x + \underline{z}g_1 + \hat{\varepsilon} < m_1 \le x + \bar{z}g_1 + \hat{\varepsilon} \end{cases}.$$

The distribution of first period money demand implied by the random shock is

$$h(m,g) = \begin{cases} \dfrac{1}{4\hat{\varepsilon}} & if \;\; x + \underline{z}g_1 - \hat{\varepsilon} \le m_1 < x + \bar{z}g_1 - \hat{\varepsilon} \\ \dfrac{1}{2\hat{\varepsilon}} & if \;\; x + \bar{z}g_1 - \hat{\varepsilon} \le m_1 \le x + \underline{z}g_1 + \hat{\varepsilon} \\ \dfrac{1}{4\hat{\varepsilon}} & if \;\; x + \underline{z}g_1 + \hat{\varepsilon} < m_1 \le x + \bar{z}g_1 + \hat{\varepsilon} \end{cases}.$$

Having defined all the preliminaries, I find the equilibrium money growth rates.

Given posterior beliefs and given that the expected value of ε is zero, in the second period the government maximizes,

$$((x+\bar{z}g_2)\mu_2+(x+\underline{z}g_2)(1-\mu_2))g_2,$$

with respect to g_2. The first order condition is that $x+2(\bar{z}\mu_2+\underline{z}(1-\mu_2))g_2=0$, which yields the second period equilibrium money growth rate $g_2(\mu_2)=\dfrac{-x}{2(\bar{z}\mu_2+\underline{z}(1-\mu_2))}$.

Plugging this rate in the seignorage function gives the second period value function as a function of posterior beliefs,

$$S_2*(\mu_2)=\left[\begin{array}{l}\mu_2\left(x+\bar{z}\left(\dfrac{-x}{2(\bar{z}\mu_2+\underline{z}(1-\mu_2))}\right)\right)\left(\dfrac{-x}{2(\bar{z}\mu_2+\underline{z}(1-\mu_2))}\right)+\\(1-\mu_2)\left(x+\underline{z}\left(\dfrac{-x}{2(\bar{z}\mu_2+\underline{z}(1-\mu_2))}\right)\right)\left(\dfrac{-x}{2(\bar{z}\mu_2+\underline{z}(1-\mu_2))}\right)\end{array}\right]=\frac{-x^2}{4(\bar{z}\mu_2+\underline{z}(1-\mu_2))}$$

To find $E_mS_2*(\mu_2)$ note that if μ_2=0, then $S_2*(\mu_2)=\dfrac{-x^2}{4\underline{z}}$, if $\mu_2=\mu_1=0.5$, then $S_2*(\mu_2)=\dfrac{-x^2}{2(\bar{z}+\underline{z})}$, and if μ_2=1, then $S_2*(\mu_2)=\dfrac{-x^2}{4\bar{z}}$. Then, given $h(m,g)$,

$$E_mS_2*(\mu_2)=\left(\int_{x+\underline{z}g_1-\hat{\varepsilon}}^{x+\bar{z}g_1-\hat{\varepsilon}}\frac{-x^2}{4\underline{z}}\frac{1}{4\hat{\varepsilon}}dm+\int_{x+\bar{z}g_1-\hat{\varepsilon}}^{x+\underline{z}g_1+\hat{\varepsilon}}\frac{-x^2}{2(\bar{z}+\underline{z})}\frac{1}{2\hat{\varepsilon}}dm+\int_{x+\underline{z}g_1+\hat{\varepsilon}}^{x+\bar{z}g_1+\hat{\varepsilon}}\frac{-x^2}{4\bar{z}}\frac{1}{4\hat{\varepsilon}}dm\right),$$

thus,

$$E_mS_2*(\mu_2)=\left(\begin{array}{c}\dfrac{-x^2}{4\underline{z}}\dfrac{1}{4\hat{\varepsilon}}[x+\bar{z}g_1-\hat{\varepsilon}-x-\underline{z}g_1+\hat{\varepsilon}]+\dfrac{-x^2}{2(\bar{z}+\underline{z})}\dfrac{1}{2\hat{\varepsilon}}[x+\underline{z}g_1+\hat{\varepsilon}-x-\bar{z}g_1+\hat{\varepsilon}]\\+\dfrac{-x^2}{4\bar{z}}\dfrac{1}{4\hat{\varepsilon}}[x+\bar{z}g_1+\hat{\varepsilon}-x-\underline{z}g_1-\hat{\varepsilon}]\end{array}\right),$$

which, after simplifying looks like,

$$E_mS_2*(\mu_2)=\frac{-x^2(\bar{z}-\underline{z})^3}{16\underline{z}\bar{z}(\bar{z}+\underline{z})\hat{\varepsilon}}g_1-\frac{x^2}{2(\bar{z}+\underline{z})}.$$

In the first period the government maximizes expected first period seignorage given prior beliefs $\mu_1 = 0.5$, plus the discounted expected value of second period seignorage,

$$(0.5(x+\bar{z}g_1)+0.5(x+\underline{z}g_1))g_1-\delta\left(\frac{x^2(\bar{z}-\underline{z})^3}{16\underline{z}\bar{z}(\bar{z}+\underline{z})\hat{\varepsilon}}g_1+\frac{x^2}{2(\bar{z}+\underline{z})}\right) \tag{B.28}$$

with respect to g_1. The first order condition is that $x+(\bar{z}+\underline{z})g_1-\frac{\delta x^2(\bar{z}-\underline{z})^3}{16\underline{z}\bar{z}(\bar{z}+\underline{z})\hat{\varepsilon}}=0$, which yields $g_1=-\frac{x}{(\bar{z}+\underline{z})}+\frac{\delta x^2(\bar{z}-\underline{z})^3}{16\underline{z}\bar{z}(\bar{z}+\underline{z})^2\hat{\varepsilon}}$, which after plugging in (B.28) yields total seignorage.

To find the myopic values I first maximize the myopic seignorage function with respect to g_1, where the myopic seignorage function is $(0.5(x+\bar{z}g_1)+0.5(x+\underline{z}g_1))g_1$, which yields $g_{myopic}=-\frac{x}{(\bar{z}+\underline{z})}$. Plugging back in the seignorage function, yields the myopic seignorage level over the two periods is $-\frac{(1+\delta)x^2}{2(\bar{z}+\underline{z})}$.

To find the non-myopic average inflation rate I plug $g_1=-\frac{x}{(\bar{z}+\underline{z})}+\frac{\delta x^2(\bar{z}-\underline{z})^3}{16\underline{z}\bar{z}(\bar{z}+\underline{z})^2\hat{\varepsilon}}$ in the following formula, $\pi=0.5(\bar{\pi}+\underline{\pi})$, where $\bar{\pi}=\frac{(1+g_1)M_0}{x+\bar{z}g_1}-1$ and $\underline{\pi}=\frac{(1+g_1)M_0}{x+\underline{z}g_1}-1$. To find the average myopic inflation rate I plug $g_{myopic}=-\frac{x}{(\bar{z}+\underline{z})}$ in formula, $\pi=0.5(\bar{\pi}+\underline{\pi})$, where $\bar{\pi}=\frac{(1+g_{myopic})M_0}{x+\bar{z}g_{myopic}}-1$ and $\underline{\pi}=\frac{(1+g_{myopic})M_0}{x+\underline{z}g_{myopic}}-1$. The formula for M_0 is given by $M_0=0.5(x+\bar{z}g_{myopic})+0.5(x+\underline{z}g_{myopic})$.

APPENDIX C

APPENDIX TO CHAPTER 3

Proof of Proposition 3.2

Proposition 3.2: Given Assumption A3.2, a strategic central banker will adjust the inflation rate away from the myopic rate in order to influence the government's posterior beliefs if the banker's value of signaling its type is not zero, $\frac{dEV^{CB}(\mu_1)}{d\mu_1} \neq 0$. In particular,

3.2.1 If the value to the central banker of signaling its type is positive (negative) and the banker is the L type, then the banker increases (decreases) the inflation rate to increase (decrease) the likelihood of its type being the true type: I.e. if $\frac{dEV_L^{CB}(\mu_1)}{d\mu_1} < (>) 0$ then $\forall m_1$ $\pi_1(\alpha_L, \varepsilon_1) > (<) \pi_1^{myopic}(\alpha_L, \varepsilon_1)$.

3.2.2 If the value to the central banker of signaling its type is positive (negative) and the banker is the H type, then the banker decreases (increases) the inflation rate to increase (decrease) the likelihood of its type being the true type: I.e. if $\frac{dEV_H^{CB}(\mu_1)}{d\mu_1} > (<) 0$ then $\forall m_1$ $\pi_1(\alpha_H, \varepsilon_1) < (>) \pi_1^{myopic}(\alpha_H, \varepsilon_1)$.

Proof:

If the banker acts strategically it chooses the inflation rate π_1 according to the following first order condition, $\frac{d[L_1^{CB} + m_1(\pi_1 - \bar{\pi})]}{d\pi_1} + \rho \frac{dEV^{CB}(\mu_1)}{d\pi_1} = 0$, whereas the reaction function of the myopic banker is given by $\frac{d[L_1^{CB} + m_1(\pi_1 - \bar{\pi})]}{d\pi_1} = 0$. Thus if

$\frac{dEV^{CB}(\mu_1)}{d\pi_1} > (<)0 \Leftrightarrow \frac{dEV^{CB}(\mu_1)}{d\mu_1} > (<)0$, then for the CB's first order condition to be satisfied it must be the case that $\frac{d[L_1^{CB} + m_1(\pi_1 - \bar{\pi})]}{d\pi_1} < (>)0$. Since $L_1^{CB} + m_1(\pi_1 - \bar{\pi})$ is a convex function of π_1, then $\pi_1^i < (>)\pi_1^{i,myopic}$.

□

Proof of

$$\frac{dEV^G(\mu_1)}{d\pi_1^L} = \int \frac{d^2V^G(\mu_1)}{d\mu_1^2}\frac{d\mu_1}{d\pi_1}\mu_1(1-\mu_0)f_H d\pi$$

and

$$\frac{dEV^G(\mu_1)}{d\pi_1^H} = -\int \frac{d^2V^G(\mu_1)}{d\mu_1^2}\frac{d\mu_1}{d\pi_1}\mu_1(1-\mu_0)f_H d\pi$$

Proof:

Throughout the proof I integrate over the support $(-\eta,\eta)$, and I use the assumption that $f(-\eta) = f(\eta) = 0$.

First note that,

$$\frac{dEV^G(\mu_1)}{d\pi_1^L} = \int \frac{dV^G(\mu_1)}{d\mu_1}\frac{d\mu_1}{d\pi_1^L}(\mu_0 f_L + (1-\mu_0)f_H)d\pi - \int V^G(\mu_1)\frac{\mu_0 f_L{}'}{g^L}d\pi \quad \text{(C.29)}$$

Integrating the second term by parts,

$$\int V^G(\mu_1)\frac{\mu_0 f_L{}'}{g_L}d\pi = \cancel{V^G(\mu_1)\mu_0 f_L\Big|_{-\eta}^{\eta}} - \int \frac{dV^G(\mu_1)}{d\mu_1}\frac{d\mu_1}{d\pi_1}\mu_0 f_L d\pi,$$

and plugging in (C.29),

$$\frac{dEV^G(\mu_1)}{d\pi_1^L}=\begin{bmatrix}\int\frac{dV^G(\mu_1)}{d\mu_1}\frac{d\mu_1}{d\pi_1^L}(\mu_0 f_L+(1-\mu_0)f_H)d\pi\\+\int\frac{dV^G(\mu_1)}{d\mu_1}\frac{d\mu_1}{d\pi_1}\mu_0 f_L d\pi\end{bmatrix} \tag{C.30}$$

Now note that from Bayes' rule $\mu_1=\frac{\mu_0 f_L}{D}$ where $D=\mu_0 f_L+(1-\mu_0)f_H$, $f_L=f\left(\frac{\pi_1-\pi_1^L}{g^L}\right)$ and $f_H=f\left(\frac{\pi_1-\pi_1^H}{g^H}\right)$, then

$$\frac{d\mu_1}{d\pi_1}=\frac{\mu_0 f_L'}{g^L D}-\frac{\mu_0 f_L}{D^2}\frac{dD}{d\pi_1}=\frac{\mu_0 f_L'}{g^L D}-\frac{\mu_1}{D}\frac{dD}{d\pi_1},$$

$$\frac{d\mu_1}{d\pi_1^L}=-\frac{\mu_0 f_L'}{g^L D}-\frac{\mu_1}{D}\frac{dD}{d\pi_1^L}=-\frac{d\mu_1}{d\pi_1}-\frac{\mu_1}{D}\left(\frac{dD}{d\pi_1^L}+\frac{dD}{d\pi_1}\right),$$

$$\frac{dD}{d\pi_1}=\frac{\mu_0 f_L'}{g^L}+\frac{(1-\mu_0)f_H'}{g^H},$$

$$\frac{dD}{d\pi_1^L}=-\frac{\mu_0 f_L'}{g^L}=-\frac{dD}{d\pi_1}+\frac{(1-\mu_0)f_H'}{g^H},$$

and thus,

$$\frac{d\mu_1}{d\pi_1^L}=-\frac{d\mu_1}{d\pi_1}-\frac{\mu_1(1-\mu_0)f_H'}{g^H D}.$$

Plugging $\frac{d\mu_1}{d\pi_1^L}=-\frac{d\mu_1}{d\pi_1}-\frac{\mu_1(1-\mu_0)f_H'}{g_H D}$ in (C.30),

$$\frac{dEV^G(\mu_1)}{d\pi_1^L}=\begin{pmatrix}\int\frac{dV^G(\mu_1)}{d\mu_1}\left(-\frac{d\mu_1}{d\pi_1}-\frac{\mu_1(1-\mu_0)f_H'}{g^H D}\right)(\mu_0 f_L+(1-\mu_0)f_H)d\pi\\+\int\frac{dV^G(\mu_1)}{d\mu_1}\frac{d\mu_1}{d\pi_1}\mu_0 f_L d\pi\end{pmatrix}.$$

Canceling terms,

$$\frac{dEV^G(\mu_1)}{d\pi_1^L} = \begin{pmatrix} -\int \frac{dV^G(\mu_1)}{d\mu_1}\frac{d\mu_1}{d\pi_1}(\cancel{\mu_0 f_L} + (1-\mu_0)f_H)d\pi \\ -\int \frac{dV^G(\mu_1)}{d\mu_1}\left(\frac{\mu_1(1-\mu_0)f_H{'}}{g^H}\right)d\pi \\ +\cancel{\int \frac{dV^G(\mu_1)}{d\mu_1}\frac{d\mu_1}{d\pi_1}\mu_0 f_L d\pi} \end{pmatrix},$$

thus,

$$\frac{dEV^G(\mu_1)}{d\pi_1^L} = \begin{pmatrix} -\int \frac{dV^G(\mu_1)}{d\mu_1}\frac{d\mu_1}{d\pi_1}((1-\mu_0)f_H)d\pi \\ -\int \frac{dV^G(\mu_1)}{d\mu_1}\left(\frac{\mu_1(1-\mu_0)f_H{'}}{g^H}\right)d\pi \end{pmatrix} \tag{C.31}$$

Integrating $\int \frac{dV^G(\mu_1)}{d\mu_1}\left(\frac{\mu_1(1-\mu_0)f_H{'}}{g^H}\right)d\pi$ by parts,

$$\int \frac{dV^G(\mu_1)}{d\mu_1}\left(\frac{\mu_1(1-\mu_0)f_H{'}}{g^H}\right)d\pi = \begin{pmatrix} \cancel{\frac{dV^G(\mu_1)}{d\mu_1}\mu_1(1-\mu_0)f_H\Big|_{-\eta}^{\eta}} \\ -\int \frac{d^2V^G(\mu_1)}{d\mu_1^2}\frac{d\mu_1}{d\pi_1}\mu_1(1-\mu_0)f_H d\pi \\ -\int \frac{dV^G(\mu_1)}{d\mu_1}\frac{d\mu_1}{d\pi_1}(1-\mu_0)f_H d\pi \end{pmatrix},$$

and plugging it in (C.31),

$$\frac{dEV^G(\mu_1)}{d\pi_1^L} = \begin{pmatrix} \cancel{-\int \frac{dV^G(\mu_1)}{d\mu_1}\frac{d\mu_1}{d\pi_1}(1-\mu_0)f_H d\pi} \\ +\int \frac{d^2V^G(\mu_1)}{d\mu_1^2}\frac{d\mu_1}{d\pi_1}\mu_1(1-\mu_0)f_H d\pi \\ \cancel{+\int \frac{dV^G(\mu_1)}{d\mu_1}\frac{d\mu_1}{d\pi_1}(1-\mu_0)f_H d\pi} \end{pmatrix},$$

thus,

$$\frac{dEV^G(\mu_1)}{d\pi_1^L} = \int \frac{d^2V^G(\mu_1)}{d\mu_1^{\ 2}} \frac{d\mu_1}{d\pi_1} \mu_1(1-\mu_0) f_H d\pi .$$

Similarly I now show that $\frac{dEV^G(\mu_1)}{d\pi_1^H} = -\int \frac{d^2V^G(\mu_1)}{d\mu_1^{\ 2}} \frac{d\mu_1}{d\pi_1} \mu_0(1-\mu_1) f_L d\pi$. I first differentiate $EV^G(\mu_1)$ inside the integral with respect to π_1^H,

$$\frac{dEV^G(\mu_1)}{d\pi_1^H} = \int \frac{dV^G(\mu_1)}{d\mu_1} \frac{d\mu_1}{d\pi_1^H} (\mu_0 f_L + (1-\mu_0) f_H) d\pi - \int V^G(\mu_1) \frac{(1-\mu_0) f_H{}'}{g^H} d\pi . \text{(C.32)}$$

Using again Bayes rule $\mu_1 = \frac{\mu_0 f_L}{D}$ where $D = \mu_0 f_L + (1-\mu_0) f_H$, then

$$\frac{d\mu_1}{d\pi_1} = \frac{\mu_0 f_L{}'}{g^L D} - \frac{\mu_0 f_L}{D^2} \frac{dD}{d\pi_1} = \frac{\mu_0 f_L{}'}{g^L D} - \frac{\mu_1}{D} \frac{dD}{d\pi_1},$$

$$\frac{d\mu_1}{d\pi_1^H} = -\frac{\mu_0 f_L}{D^2} \frac{dD}{d\pi_1^H} = -\frac{\mu_1}{D} \frac{dD}{d\pi_1^H},$$

$$\frac{dD}{d\pi_1} = \frac{\mu_0 f_L{}'}{g^L} + \frac{(1-\mu_0) f_H{}'}{g^H},$$

$$\frac{dD}{d\pi_1^H} = -\frac{(1-\mu_0) f_H{}'}{g^H} = -\frac{dD}{d\pi_1} + \frac{\mu_0 f_L{}'}{g^L},$$

and thus,

$$\frac{d\mu_1}{d\pi_1^H} = -\frac{d\mu_1}{d\pi_1} + \frac{\mu_0(1-\mu_1) f_L{}'}{g^L D}.$$

Plugging $\frac{d\mu_1}{d\pi_1^H} = -\frac{d\mu_1}{d\pi_1} + \frac{\mu_0(1-\mu_1) f_L{}'}{g^L D}$ in (C.32),

$$\frac{dEV^G(\mu_1)}{d\pi_1^H} = \begin{pmatrix} \int \frac{dV^G(\mu_1)}{d\mu_1} \left(-\frac{d\mu_1}{d\pi_1} + \frac{\mu_0(1-\mu_1) f_L{}'}{g^L D} \right) (\mu_0 f_L + (1-\mu_0) f_H) d\pi \\ + \int \frac{dV^G(\mu_1)}{d\mu_1} \frac{d\mu_1}{d\pi_1} (1-\mu_0) f_H d\pi \end{pmatrix}.$$

Canceling terms,

$$\frac{dEV^G(\mu_1)}{d\pi_1^H}=\begin{pmatrix} -\int\frac{dV^G(\mu_1)}{d\mu_1}\frac{d\mu_1}{d\pi_1}(\mu_0 f_L+\cancel{(1-\mu_0)f_H})d\pi \\ +\int\frac{dV^G(\mu_1)}{d\mu_1}\left(\frac{\mu_0(1-\mu_1)f_L'}{g^L}\right)d\pi \\ \cancel{+\int\frac{dV^G(\mu_1)}{d\mu_1}\frac{d\mu_1}{d\pi_1}(1-\mu_0)f_H d\pi} \end{pmatrix},$$

thus

$$\frac{dEV^G(\mu_1)}{d\pi_1^H}=\begin{pmatrix} -\int\frac{dV^G(\mu_1)}{d\mu_1}\frac{d\mu_1}{d\pi_1}(\mu_0 f_L)d\pi \\ +\int\frac{dV^G(\mu_1)}{d\mu_1}\left(\frac{\mu_0(1-\mu_1)f_L'}{g^L}\right)d\pi \end{pmatrix}. \tag{C.33}$$

Integrating $\int\frac{dV^G(\mu_1)}{d\mu_1}\left(\frac{\mu_0(1-\mu_1)f_L'}{g^L}\right)d\pi$ by parts,

$$\int\frac{dV^G(\mu_1)}{d\mu_1}\left(\frac{\mu_0(1-\mu_1)f_L'}{g^L}\right)d\pi=\begin{pmatrix} \cancel{\frac{dV^G(\mu_1)}{d\mu_1}\mu_0(1-\mu_1)f_L\Big|_{-\eta}^{\eta}} \\ -\int\frac{d^2V^G(\mu_1)}{d\mu_1^2}\frac{d\mu_1}{d\pi_1}\mu_0(1-\mu_1)f_L d\pi \\ +\int\frac{dV^G(\mu_1)}{d\mu_1}\frac{d\mu_1}{d\pi_1}\mu_0 f_L d\pi \end{pmatrix},$$

plugging in (C.33),

$$\frac{dEV^G(\mu_1)}{d\pi_1^H}=\begin{pmatrix} \cancel{-\int\frac{dV^G(\mu_1)}{d\mu_1}\frac{d\mu_1}{d\pi_1}\mu_0 f_L d\pi} \\ -\int\frac{d^2V^G(\mu_1)}{d\mu_1^2}\frac{d\mu_1}{d\pi_1}\mu_0(1-\mu_1)f_L d\pi \\ \cancel{+\int\frac{dV^G(\mu_1)}{d\mu_1}\frac{d\mu_1}{d\pi_1}\mu_0 f_L d\pi} \end{pmatrix},$$

thus,

$$\frac{dEV^G(\mu_1)}{d\pi_1^H} = -\int \frac{d^2V^G(\mu_1)}{d\mu_1^2}\frac{d\mu_1}{d\pi_1}\mu_0(1-\mu_1)f_L d\pi .$$

Finally, since from Bayes rule $\mu_1 D = \mu_0 f_L \Leftrightarrow \mu_1(\mu_0 f_L + (1-\mu_0)f_H) = \mu_0 f_L$, then $\mu_1(1-\mu_0)f_H = \mu_0(1-\mu_1)f_L$, and hence $\dfrac{dEV^G(\mu_1)}{d\pi_1^H} = -\displaystyle\int \frac{d^2V^G(\mu_1)}{d\mu_1^2}\frac{d\mu_1}{d\pi_1}\mu_1(1-\mu_0)f_H d\pi$.

□

Proof of Proposition 3.3

Proposition 3.3: Given Assumption A3.2, if the central banker does not act strategically then if the government's value of information is positive, it will decrease the punishment compared to the myopic punishment to increase the degree of transparency of monetary policy, that is, $m_1^{non-strategic} < m_1^{myopic}$.

Proof:

Given Assumption A3.2 $\left(\dfrac{d\mu_1}{d\pi_1} > 0\right)$ and proposition one $\left(\dfrac{d^2V^G(\mu_1)}{d\mu_1^2} < 0\right)$ the sign of Ω equals the negative of the sign of $\left(\dfrac{d\pi_1^{L,myopic}}{dm_1} - \dfrac{d\pi_1^{H,myopic}}{dm_1}\right)$. Since $\left(\dfrac{d\pi_1^{L,myopic}}{dm_1} - \dfrac{d\pi_1^{H,myopic}}{dm_1}\right) = \dfrac{\alpha_L - \alpha_H}{1+\lambda} < 0$, then $\Omega > 0$ and thus in order for the first order condition to be satisfied, it must be true that $\dfrac{dG_1}{d\pi_1^L}\dfrac{d\pi_1^{L,myopic}}{dm_1} + \dfrac{dG_1}{d\pi_1^H}\dfrac{d\pi_1^{H,myopic}}{dm_1} - [\mu_0\pi_1^{L,myopic} + (1-\mu_0)\pi_1^{H,myopic} - \bar{\pi}] < 0$. Hence the myopic function G_1 evaluated at $m_1^{non-strategic}$ has negative slope. Since G_1 is convex in m_1 this implies that $m_1^{non-strategic} < m_1^{myopic}$.

□

Proposition 3.5: The punishment that minimizes the government's first period expected losses is lower (higher) when the banker acts strategically ($m_1^? < (>) m_1^{myopic}$), if the first period marginal losses of the government are higher (lower) when the banker acts strategically. That is,

$$\text{If } \Theta = \left(\begin{array}{l} \frac{dG_1}{d\pi_1{}^L}\left[\frac{d\pi_1{}^L}{dm_1} - \frac{d\pi_1{}^{L,myopic}}{dm_1}\right] + \frac{dG_1}{d\pi_1{}^H}\left[\frac{d\pi_1{}^H}{dm_1} - \frac{d\pi_1{}^{H,myopic}}{dm_1}\right] \\ -[\mu_0(\pi_1{}^L - \pi_1{}^{L,myopic}) + (1-\mu_0)(\pi_1{}^H - \pi_1{}^{H,myopic})] \end{array} \right) > (<) 0 \Rightarrow m_1^? < (>) m_1^{myopic}.$$

Proof:

$m_1^?$ is found by minimizing equation (3.25) with respect to m_1, which yields the following first order condition,

$$\frac{dG_1}{d\pi_1{}^L}\frac{d\pi_1{}^L}{dm_1} + \frac{dG_1}{d\pi_1{}^H}\frac{d\pi_1{}^H}{dm_1} - [\mu_0\pi_1{}^L + (1-\mu_0)\pi_1{}^H - \bar{\pi}] = 0. \tag{C.34}$$

Subtracting the first order condition of the government given the myopic behavior of the banker equation (3.23) from (C.34) yields Θ. Given convexity of G_1, if $\Theta > (<) 0 \Rightarrow m_1^? < (>) m_1^{myopic}$.

□

Proof of Proposition 3.6

Proposition 3.6: When the central banker acts strategically, if the government's value of information is positive, then the government deviates from the punishment that minimizes its first period losses in order to increase information. In particular, given Assumption A3.3, $m_1{}^* < m_1^?$.

Proof: The difference between the first order conditions from equation (3.24) and equation (3.25) implicitly determines $m_1 * - m_1^?$. Using the algebra from Appendix C, this difference (call it Φ) is given by,

$$\Phi = \left(\frac{d\pi_1^L}{dm_1} - \frac{d\pi_1^H}{dm_1} \right) \int \frac{d^2V^G(\mu_1)}{d\mu_1^2} \frac{d\mu_1}{d\pi_1} \mu_1 (1-\mu_0) f_H d\pi .$$

Given Assumption A3.2 $\left(\frac{d\mu_1}{d\pi_1} > 0 \right)$ and Proposition 3.1 $\left(\frac{d^2V^G(\mu_1)}{d\mu_1^2} < 0 \right)$ the sign of Φ equals the negative of the sign of $\left(\frac{d\pi_1^L}{dm_1} - \frac{d\pi_1^H}{dm_1} \right)$. Hence if $\left(\frac{d\pi_1^L}{dm_1} - \frac{d\pi_1^H}{dm_1} \right) < 0$ then $\Phi > 0$ and thus $m_1 * < m_1^?$.

□

Proof of Proposition 3.7

Proposition 3.7: Given Assumption A3.3 then if the government's value of information is positive, the strategic behavior of the central banker implies more transparent monetary policy (i.e. $m_1 * < m_1^{non-strategic}$), if any of the following conditions are true,

i) The gains from information to the government are higher when the banker acts strategically, $(m_1 * - m_1^? < m_1^{non-strategic} - m_1^{myopic})$, and the strategic behavior of the central bank implies higher first period marginal losses due to more variability of the inflation rate around its target $(m_1^? < m_1^{myopic})$.

ii) The gains from information to the government are higher when the banker acts strategically, $(m_1 * - m_1^? < m_1^{non-strategic} - m_1^{myopic})$, the strategic behavior of the central bank implies lower first period marginal losses due to less variability of the inflation rate around its target $(m_1^? > m_1^{myopic})$, but the extra gains from information compensate for higher first period losses, i.e. $(m_1 * - m_1^?) - (m_1^{non-strategic} - m_1^{myopic}) < m_1^{myopic} - m_1^?$.

iii) The gains from information to the government are lower when the banker acts strategically, $(m_1 * - m_1^? > m_1^{non-strategic} - m_1^{myopic})$ but the strategic behavior of the banker implies much higher first period marginal losses due to more variability in the inflation rate, i.e. $m_1^? < m_1^{myopic}$, but $(m_1 * - m_1^?) - (m_1^{non-strategic} - m_1^{myopic}) < m_1^{myopic} - m_1^?$.

Otherwise the strategic behavior of the bank implies less transparent monetary policy, i.e. $m_1 * > m_1^{non-strategic}$.

Proof:

The difference between the first order conditions of the government's problem given that the banker acts strategically (problem 3.24) and we it acts non-strategically (problem 3.22) is given by,

$$\Upsilon = \left(\begin{array}{c} \dfrac{dG_1}{d\pi_1^{\,L}} \left[\dfrac{d\pi_1^{\,L}}{dm_1} - \dfrac{d\pi_1^{\,L,myopic}}{dm_1} \right] + \dfrac{dG_1}{d\pi_1^{\,H}} \left[\dfrac{d\pi_1^{\,H}}{dm_1} - \dfrac{d\pi_1^{\,H,myopic}}{dm_1} \right] \\ -[\mu_0(\pi_1^{\,L} - \pi_1^{\,L,myopic}) + (1-\mu_0)(\pi_1^{\,H} - \pi_1^{\,H,myopic})] \\ + \left[\left(\dfrac{d\pi_1^{\,L}}{dm_1} - \dfrac{d\pi_1^{\,H}}{dm_1} \right) - \left(\dfrac{d\pi_1^{\,L,myopic}}{dm_1} - \dfrac{d\pi_1^{\,H,myopic}}{dm_1} \right) \right] \displaystyle\int \frac{d^2 V^G(\mu_1)}{d\mu_1^{\,2}} \frac{d\mu_1}{d\pi_1} \mu_1 (1-\mu_0) f_H \, d\pi \end{array} \right).$$

If $\Upsilon > 0$, then $m_1 * < m_1^{non-strategic}$.

The sign of Υ depends on the one hand on the sign of $\frac{dG_1}{d\pi_1{}^L}\left[\frac{d\pi_1{}^L}{dm_1}-\frac{d\pi_1{}^{L,myopic}}{dm_1}\right]+\frac{dG_1}{d\pi_1{}^H}\left[\frac{d\pi_1{}^H}{dm_1}-\frac{d\pi_1{}^{H,myopic}}{dm_1}\right]$, which in turn determines the sign $-[\mu_0(\pi_1{}^L-\pi_1{}^{L,myopic})+(1-\mu_0)(\pi_1{}^H-\pi_1{}^{H,myopic})]$ and magnitude of $m_1^{myopic}-m_1^?$, and on the other hand on the sign of $\left(\frac{d\pi_1{}^L}{dm_1}-\frac{d\pi_1{}^H}{dm_1}\right)-\left(\frac{d\pi_1{}^{L,myopic}}{dm_1}-\frac{d\pi_1{}^{H,myopic}}{dm_1}\right)$, which determines the sign and magnitude of $(m_1*-m_1^?)-(m_1^{non-strategic}-m_1^{myopic})$.

□

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