2
2003

This is to certify that the

dissertation entitled

ESSAYS ON MIXED OLIGOPOLY AND AGRICULTURAL R&D

presented by

Anwar Naseem

has been accepted towards fulfillment
of the requirements for

Ph.D. degree in Agricultural Economics

Richard N. Bernsten
Major professor

Date 7/8/02

ESSAYS ON MIXED OLIGOPOLY AND AGRICULTURAL R&D

By

Anwar Naseem

A DISSERTATION

Submitted to
Michigan State University
in partial fulfillment of the requirements for the degree of

DOCTOR OF PHILOSOPHY

Department of Agricultural Economics

2002

ABSTRACT

ESSAYS ON MIXED OLIGOPOLY AND AGRICILUTURAL R&D

By

Anwar Naseem

Three essays are presented which explore how micro level interactions between public and private research and development (R&D) translate into observable macro effects. The framework of analysis is a mixed oligopoly model where a welfare maximizing public firm is in competition with a profit-maximizing firm. Careful attention is paid to how asymmetric objectives, spillovers and appropriability of research exert strong influences on the behavior of competing firms.

The motivations for the dissertation are set out in the introductory chapter. In the first essay, a simple model of an R&D race between a public and private firm is presented. The essay presents three specifications of the general model, and seeks to define and differentiate between the effects of public research appropriability and research spillovers. In the second essay, the nature of the observed market structure and R&D competition in genomics research is used as the basis for a comparative analysis of research under a mixed oligopoly, pure oligopoly and monopoly when the timing of the innovation outcome is uncertain (as in an R&D race), the winner-take-all assumption is relaxed and the profits in later stages are a function of the R&D expenditures of prior stages. In the third and final essay, cooperative and noncooperative behavior in the context of mixed oligopoly models is examined. The essay attempts to investigate the reasons for the relatively small number of partnerships between public and private firms

in spite of institutional arrangements that encourage them; and explores whether partnerships with private firms are inconsistent with the objective of a welfare maximizing public-sector firm. In particular we consider cooperative and noncoopertive R&D between a public sector and private firm in a two-stage model where research in the first stage is followed by further research and production in the second stage.

The results of the models are used to understand the changing market structure of the agriculture research and development and their implications. The role of a competitive public sector firm in agricultural R&D is highlighted and its welfare effects studied.

Dedicated to my parents
S.M. Naseem and Zarina Naseem
East Lansing, MI, June 2002

ACKNOWLEDGEMENTS

There are numerous individuals that deserve my sincere gratitude for their support throughout my graduate program. In the writing of this dissertation, I have had the great pleasure of working with Professor James Oehmke, who, as the dissertation supervisor, has been a constant source of good ideas, encouragement and patient guidance. Professor Oehmke not only extended to me the freedom to research, but also had the faith and confidence in me to let me complete the dissertation while I worked at Rutgers University. I am grateful to the other committee members for their comments and suggestions. I was privileged to have had Professors Jay Pil Choi and Carl Pray as my external thesis examiners, who graciously agreed to shoulder the burden of this essential task. I am particularly indebted to Professor Pray for providing me the resources and time to work on the dissertation while I was at Rutgers. I thank Professor Christopher Wolf for his detailed reading of the manuscript and constructive criticism. I also appreciate the comments of Professor Dave Weatherspoon, which has led to a sharper intuition of the results in many instances.

The research issues explored in this dissertation were initially proposed and discussed at the weekly meetings of the Biotechnology Interest Group (B.I.G.) at the Department of Agricultural Economics. Members of the group, which included Professors Oehmke, Wolf and Weatherspoon, as well as John Francis, Amie Hightower, Dr. Mywish Maredia and Professor Kellie Raper, generously lent their time to listen, and politely offered criticism, to my initially grand, but often confused research objectives.

I would also like to express my appreciation to Professor Richard Bernsten, my major professor, who has provided invaluable guidance and mentoring ever since I came to Michigan State. From the very beginning, Professor Bernsten has taken a deep interest in my intellectual development, and has provided great ideas, contacts and suggestions. I could not have wished for a more caring major professor.

Finally, I thank the Food Security Project II and the Michigan Agricultural Experiment Station for supporting my graduate studies through various research assistantships. The support provided by the MSU/Ford Foundation pre-dissertation fellowship for a study tour of the Philippines is also appreciated.

My graduate studies would not have been the same without the social and academic challenges and diversions provided by fellow students. Greatly missed will be the conviviality of the many meals shared with Nazmul Chaudhry and Brady Deaton. Also missed will be the Hayford House parties and the company of hosts David Mather and Matthew Schaeffer. With so many other friends at MSU, especially Bocar Diagana, Andrea Jeffers, Chris Penders, Julie Stepanek, and George Young, I have shared many diversely rewarding experiences, and my connection to each has been a continuous source of enjoyment and sustenance. To Monika Tothova, I owe so much of my happiness of the past three years. She has been my greatest distraction and staunchest supporter as I struggled with the challenges of graduate research work.

My final thanks must be reserved for my parents, who have instilled in all their children the importance of education and perseverance; and for my siblings, Farouk and Saba, who have given me all the pleasures that comes from being their "little" brother, as well as uncle to their children. I know of no greater joy than coming home to family.

TABLE OF CONTENTS

List of Tables x

List of Figures xi

Chapter I: Introduction 1

Chapter II: The Effect of Public Research Appropriability and Spillovers on Private Research 9
- II.1. Introduction 9
- II.2. Literature Review 16
- II.3. The Model 17
  - II.3.1. Specification 1: Generalizing Appropriability in D&D and P-T models 17
  - II.3.2. Specification 2: Appropriability With Private Research Capacity 26
  - II.3.3. Specification 3: Public R&D Spillovers 34
- II.4. Comparative Statics and Welfare Analysis 41
- II.5. Conclusions 43

Chapter III: Should the Public Sector Conduct Genomics Research? 46
- III.1. Introduction 46
- III.2. Literature Review 52
- III.3. The Model 54
  - III.3.1. Monopoly equilibrium condition 57
  - III.3.2. Pure duopoly equilibrium condition 57
  - III.3.3. Mixed duopoly equilibrium condition 58
- III.4. Comparative Analysis 61
  - III.4.1. Monopoly vs. Pure Duopoly 65
  - III.4.2. Monopoly vs. Mixed Duopoly 70
  - III.4.3 Mixed Duopoly vs. Pure Duopoly 73
- III.5. Discussion 77
- III.6. Conclusions 81

Chapter IV: Cooperative and Noncooperative R&D in a Mixed Duopoly with Spillovers 85
- IV.1. Introduction 85
- IV.2. The Model 89
  - IV.2.1. Regime 1: Mixed Duopoly in Both Stages 92
    - IV.2.1.1. Non-Cooperative Game 93
    - IV.2.1.2. Cooperative Game 100
  - IV.2.2. Regime 2: Mixed Duopoly in R&D Stage Only 101
    - IV.2.2.1. Non-Cooperative Game 102

III.2.2.2. Cooperative Game 104
IV.3. Specializing the Model 106
IV.3.1. Regime 1: Mixed Duopoly in Both Stages 106
IV.3.1.1. Non-Cooperative Game 106
IV.3.1.2. Cooperative Game 114
IV.3.2. Regime 2: Mixed Duopoly in R&D Stage Only 118
IV.3.2.1. Non-Cooperative Game 119
IV.3.2.2. Cooperative Game 122
IV.4. Welfare 126
IV.5. Conclusions 128

Chapter V: Conclusion 131

Appendices 139
Chapter II 140
Chapter III 146
Chapter IV 155

Bibliography 168

LIST OF TABLES

Table II.1: Effect of Spillover on the Public and Private Research 42

Table IV.1: Overview of the Cooperative and Noncooperative Game 92
Table IV.2: Summary of Results for §IV.3.2 106

Table AIII.1. Sufficient Conditions for Ranking Monopoly and Pure Duopoly Research Effort 151
Table AIII.2. Sufficient Conditions for Ranking Monopoly and Mixed Duopoly Research Effort 152
Table AIII.3a. Sufficient Conditions for Ranking Pure Duopoly and Mixed Duopoly Research Effort 153
Table AIII.3b. Sufficient Conditions for Ranking Pure Duopoly and Mixed Duopoly Research Effort 154

Table AIV.3: Nash Equilibrium Values for the Noncooperative Case in Regime 1 157
Table AIV.4. Nash Equilibrium Values for the Cooperative Case in Regime 1 159
Table AIV.5: Comparing Equilibrium Values Between Cooperative and Noncooperative Cases Under Regime 1 161
Table AIV.6. Nash Equilibrium Values for the Noncooperative Case in Regime 2 162
Table AIV.7: Nash Equilibrium Values for the Cooperative Case in Regime 2 164
Table AIV.8: Comparing Equilibrium Values Between Cooperative and Noncooperative Case Under Regime 2 166
Table AIV.9: Socially Efficient Values at Different β Values. 166
Table AIV.10: Comparison of Nash-Equilibrium Output and Research Across Models 167

LIST OF FIGURES

Figure I.1: Mergers and Acquisition by Diversified Biotechnology Firms 3

Figure II.1: R&D Expenditures by Public and Private Sectors in the United States 10

Figure AII.2: Relationship Between R&D Intensity, Appropriability and Spillovers 140
Figure AII.3: Public and Private Best Response Under Varying Levels of α for Specification 1. 141
Figure AII.4: Public and Private Best Response Under Varying Levels of α for Specification 2. 142
Figure AII.5a: Public and Private Best Response Under Varying Levels of β for Specification 3 (β =0.5). 143
Figure AII.5b: Public and Private Best Response Under Varying Levels of β for Specification 3 (β =1) 144
Figure AII.5c: Public and Private Best Response Under Varying Levels of β for Specification 3 (β =0) 145

Figure A.1: Second Stage Mixed Market if Private Firm is the Winner of the R&D Race 150
Figure A.2: Second Stage Mixed Market if Public Firm is the Winner of the R&D Race 150

CHAPTER I

INTRODUCTION

Research and development (R&D) in agriculture, both by public and private sectors, has been a significant source of productivity improvements over the last century (Huffman and Evenson, 1992; Alston, Pardey and Smith, 1997). New technologies, such as biotechnology and information technology, provide hope that improvement in efficiency will continue well into the new century. What is less clear is what impact, if any, the comprehensive restructuring that has taken place in the agricultural R&D industry, especially over the past decade, will have on market performance, economic growth and social welfare.

The industry has undergone three distinct structural changes in recent years. The most important, perhaps, is the change in relative levels of investments by public and private sectors. Historically, the government has supported agriculture research on the premise that the knowledge that results from research activities is, in general, a public good (i.e., non-rival and non-excludable). Even when this is not the case, and some returns to research can be captured by innovators, the incentives for the private sector R&D investment may be weak. For this reason, public sector[1] involvement in agriculture research has been considerable, and during the first half of the past century (1906 to 1950) has been larger than the private sector's (Huffman and Evenson, 1992). For the latter part of the twentieth century, private sector research investments have risen noticeably, both in absolute terms and relative to public funding. The rate of growth in

[1] References to "public" sector research pertains to all research performed in state agricultural experimental stations, land grant and other universities, and the U.S. Department of Agriculture. "Private" research pertains to all research activities conducted by private sector firms.

public research expenditure has, however, slowed since the mid-1970s, and virtually all the growth in U.S. agricultural research expenditures can be accounted by private sector investments (Fuglie, 2000). By the mid-1990's private sector expenditures exceeded those of the public sector by as much as 55 to 65 percent (Huffman and Evenson, 1992; Fuglie 2000).

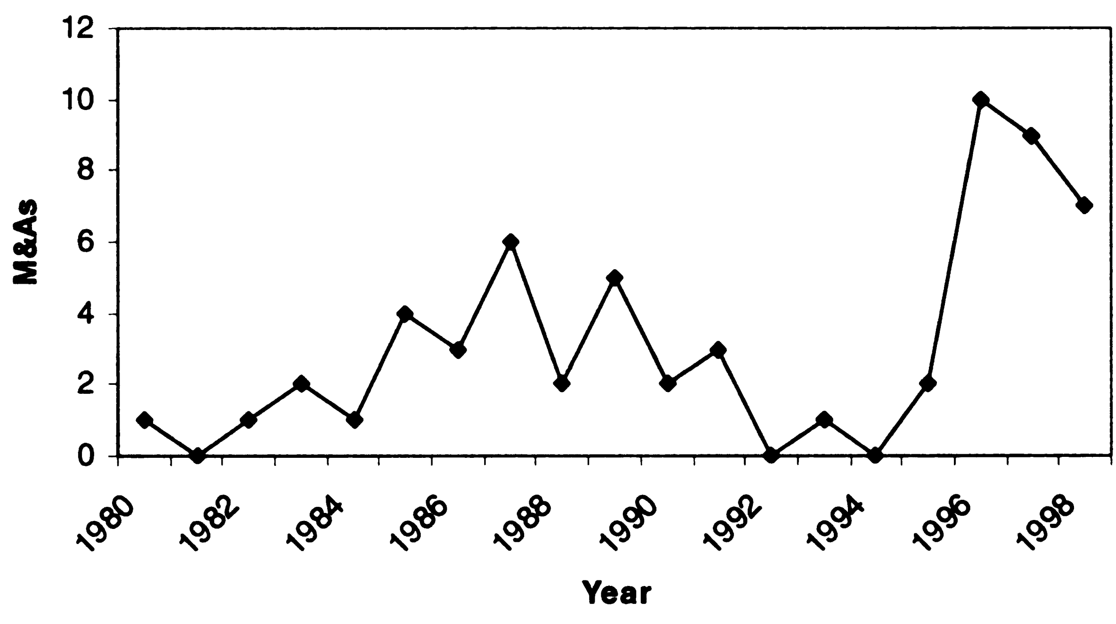
A second structural change occurring in the agricultural R&D market is that the composition of research conducted by the private sector has changed. During the 1960's, 94% of the private sector research focused on machinery, chemicals, and post-harvest and processing technologies. The private-sector focus was determined in large part because the appropriability of research benefits was much easier in machinery, chemicals and post-harvest innovations than in on-farm and agronomic technologies. However, with the strengthening of the intellectual property rights (IPR) system, technological opportunities offered by advances in biotechnology, along with an explicit policy to encourage more public and private sector collaboration, has resulted in more private investment in research areas that once largely fell in the public domain (e.g., plant breeding and livestock improvement) (Fuglie *et al.*, 1996). And while public sector research priorities by technology area have remained relatively unchanged over the years (i.e., a continued emphasis on basic research in crop and animal research), it is increasingly in competition with the private sector in research areas that would be considered "basic."[2] Genomics research, which has many of the attributes of basic research, is an example where both

[2] **As defined by the National Science Foundation, "the objective of basic research is to gain more complete understanding of the subject under study without specific applications in mind. In industry basic research is defined as research that advances scientific knowledge but does not have specific objectives, although it may in fields of present or potential commercial interests." (NSF, 1996)**

private and public sectors compete to sequence the genetic code of several organisms, cooperatively and noncooperatively.

A third structural change, and one which has received much scrutiny among agricultural economists, has been the increasing consolidation in agriculture and related sectors[3]. Mergers and acquisitions (M&As) have been the key drivers for the increased consolidation (Fulton and Gianankas, 2001). Figure I.1 illustrates that the trend in M&A activities by diversified biotechnology firms follows a cyclical pattern, with peaks occurring in the late 80's and 90's.

Figure I.1: Mergers and Acquisition by Diversified Biotechnology Firms



Source: Kalaitzandonakes and Hayenga (2000).

[3] Among studies that have documented, explained and/or related consolidation activity to innovation measures include Kalaitzandonakes and Hayenga (1999), Fulton and Giannakas (2001), Oehmke *et al.* (2000), and Brennan, Pray and Courtmanche (1999).

There is evidence that the consolidation in the vertically integrated seed, biotechnology and chemical markets has resulted in concentration in specific output markets as well as innovation markets. Kalaitzandonakes and Hayenga (2000) report that in 1998, Monsanto and Pioneer Hi-Bred had, respectively, a market share of 15% and 39% of the U.S. seed corn market, and 24% and 17% of the soybean seed market. The cottonseed market is essentially controlled by Delta and Pine Land (with a 71% market share) and Stoneville (with a 16% market share) (Kalaitzandonakes and Hayenga, 2000). Brennan, Pray and Courtmanche (2000) provide preliminary evidence of concentration in the plant biotechnology R&D market. Using firm level data on field trials of genetically modified organisms (GMOs) in the U.S., they construct a four firm concentration ratio for innovation in plant biotechnology. In 1998 the top four firms conducted 87% of all field trials, which declined to 63% in 1995 and then rose to reach a high of 79% in 1998 (Brennan, Pray and Courtmanche, 2000). Whereas the evidence of concentration in the plant biotechnology industry is compelling, its impact on output and innovation market performance is ambiguous.[4]

This dissertation furthers an understanding of the implications of structural changes in agricultural research and development from a theoretical standpoint. The approach adopted is to model the micro-level interaction between innovating firms. In particular, and in light of the changing R&D market, the dissertation seeks to address the following research questions:

[4] Brennan, Pray and Courtmanche did not find a strong causal relationship between concentration and innovation in the plant biotechnology industry.

1. How can we explain the differences in public-private research activities across research areas? In other words, under what market and institutional conditions does one sector conduct more R&D than the other?
2. What are the implications for a decline in public research activities on private sector output? More generally, what, if any, is the nature of the causality between private and public sector research?
3. Will the use of IPRs by both private and public sectors limit innovation? Our underlying hypothesis is that stronger IPR will limit the amount of knowledge sharing between research entities, and therefore may have implications for market performance and welfare.
4. What is the relationship between market concentration (in both product and research markets) and research effort of the public and private sectors?
5. Under what conditions can the public sector affect market structure that ensures competition and innovation? Consider, for example, that following the enactment of the 1970 Plant Variety Protection Act, Butler and Marion (1985) found little evidence of decline in competition in the seed industry which they attribute to public plant breeding institutions that developed and released non-protected varieties, thereby keeping concentration in check. Can one expect that public sector research will be a source of competitiveness and innovation in an increasingly concentrated market, and if so, under what conditions?
6. With the public sector coming under increasing pressure to form collaborative research programs with private firms, what affect can be expected of such cooperative research?

This dissertation addresses the above research questions in three related essays (although no single essay speaks to all the issues). The analytical basis for these essays is a game-theoretic mixed oligopoly framework, wherein profit-maximizing private firms and welfare-maximizing public sector firms compete, non-cooperatively, in product and/or research markets. Due to the diverging objectives of public sector and private sector firms in a mixed oligopoly setting, the outcome predicted by such a modeling framework is different from the pure oligopoly case where only private-sector firms are engaged in strategic behavior. The mixed oligopoly framework offers an approximate representation of the agriculture research market, where both public and private firms complement and compete with each other's activities.[5] The use of the mixed oligopoly modeling as a tool to better understand the market structure and innovation as it relates to agricultural research is the main analytical contribution of this dissertation.

Research questions 1 to 3 are addressed in the first essay. Differentiating between appropriability affects and spillovers we show that these two affects can explain, in a simple one-period innovation race model, the relative level of private and public sector firm research. We first review the existing theoretical literature, which provide contradictory results as to whether public research encourages or discourages private research and later construct a general Cournot type game that includes the existing models as special cases. We show that the contradictory results are due to different interpretations of spillovers and appropriability. Moreover, we show that the mechanisms

[5] There does not exist, to our knowledge, a unified theory of mixed oligopoly. De Fraja and Delbono (1990) and Nett (1991) review the existing models, all of which are a variation of the standard oligopoly models where firms compete in output or prices, except that in a typical mixed oligopoly model, at least one firm is welfare maximizing. In the agricultural sector, mixed oligopoly models have been used to study the behavior of agricultural cooperatives in relationship to private firms (Tennback, 1995; Azzam and Andersson, 2001).

through which public R&D impacts private R&D—whether through greater knowledge flows or the ease with which the private firm can appropriate public sector firm's innovations—matter in determining the outcome of the R&D game. We interpret our results in the context of strengthening of the IPR regime and changing resource allocation of public and private sectors.

The second essay, which addresses research questions 4 and 5, is motivated by the observed market structure and the nature of R&D competition in genomics research. Genomics research (for example that on the human genome and to a lesser extent, rice genome) has been characterized by intense competition between public and private enterprises to sequence DNA. The successful sequencing of genes has profound implications for downstream research, which depends on the accuracy and methodology of sequencing[6]. We therefore model genomics research as a two-stage process, where the research race in the first stage impacts the research of the second stage (the second stage can also be interpreted as an output market). We compare the market performance of the observed mixed oligopoly in genomics research *vis a vis* a monopoly and a pure duopoly. We derive conditions that allow us to rank the amount of research that is performed by each of the three markets and make statements on how competition in genomics research is affected by the presence of a public sector.

Our last essay addresses the issue of cooperative and non-cooperative behavior in the context of mixed oligopoly models. Our interest here is to understand the reasons for the relatively small number of partnerships between public and private firms (relative to

[6] For instance, rice has related proteins for 85 percent of the proteins identified in cereals. The high percentage of related genes could ease the identification of agronomically important genes in cereals. Another prospect resulting from the research on rice genome is an increased ability to reveal the DNA sequences of traditional rice varieties and wild species (Ronald and Leung, 2002)

the a much more frequent occurrence of cooperation among private firms). Further we evaluate the merits of an oft-repeated criticism of partnerships between public and private firms is that it compromises the public mission of the public organization and may also give the partnering firm a competitive advantage over the other firms. We explore this issue under two different types of market settings (one where the public sector firm competes in the product market and another where it does not), and show that the results are sensitive to the level of spillovers between firms. In particular, we find that for high spillovers, profits of the private firm when it collaborates with the public sector firm are actually lower than the noncooperative case or lower than the firm that does not cooperate. Further it is found that cooperation is always a Pareto improvement for the public sector firm, suggesting that incentives for the private firm to cooperate with the private firm would be a welfare improving policy.

The concluding chapter of the dissertation summarizes the key findings of the study and relates them to the structural changes that were described earlier. The contribution of the research is highlighted and future work on empirically testing the hypotheses generated by the models is suggested.

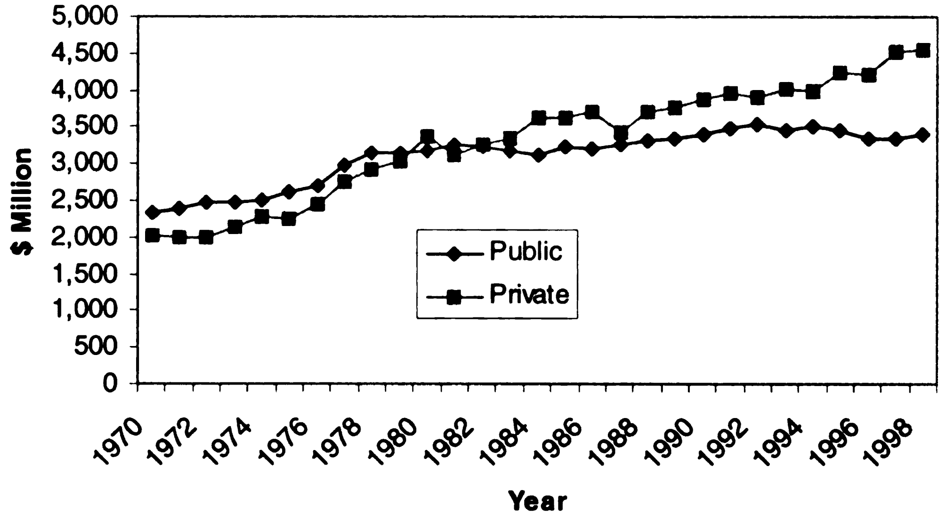
CHAPTER II

THE EFFECT OF PUBLIC RESEARCH APPROPRIABILITY AND SPILLOVERS ON PRIVATE RESEARCH

II.1. Introduction

In this chapter, we present a model to study the impacts of two significant developments in agricultural R&D. First, as mentioned in the introductory chapter, is the observation that there has been a shift over the last two decades in R&D effort from the public sector to the private sector. Figure II.1, which plots the trends in R&D expenditures (in real 1993 dollars) from 1970 to 1998, shows that since the mid 80's private sector R&D has surpassed that of the public sector. In 1992, the private sector spent $3.5 billion (1993 dollars) on agricultural research, nearly one billion dollars more than the public sector (Alston, Pardey and Smith, 1998). In addition, the private sector moved into plant-breeding and veterinary research, areas traditionally dominated by the public sector, spending 12% and 9%, respectively, of its funds in these areas. Since 1992, the real value of public agricultural research expenditures has fallen (Alston, Pardey and Smith, 1998). Today, agricultural R&D activities are dominated by firms such as Monsanto, whose research budget in 1997 of approximately $1.2 billion (Monsanto, 1998) was approximately equal to the *total* amount spent by the U.S. public sector spent on crop related activities.

Figure II.1: R&D Expenditures by Public and Private Sectors in the United States



Source: Economic Research Service (2002)

A second observed phenomenon is the increasing use of intellectual property (IP) protection, especially patents, by the public sector. IP protection, gives innovators, public and private alike, a mechanism to appropriate the returns due to their R&D and hence giving them an incentive to carry out further research. According to the Association of University Technology Managers (AUTM) surveys, patenting activity in U.S. research universities has been increasing since the Bayh-Dole Act of 1980, which sought to encourage the transfer of publicly funded innovations to the private sector with an emphasis on patenting. Prior to the Act, universities were patenting at a rate of 250 patents annually but have dramatically increased from 1600 patents issued to them in 1991 to about 2000 in 1998. For the period 1994-97, new patent applications and licensing of 65 universities that responded to the survey, grew by 20.4% and 9.6% respectively. This growth in the "commercialization" of academic research is seen as by some as evidence of the benefit of university research as well as raising concerns about the conduct and priorities of university research (Thursby and Thursby, 2000).

Standard economic theory suggests that if profit-maximizing firms are better able to appropriate their rents due to R&D (and to the extent that stronger IPRs provide better appropriability conditions), they will increase their research activity. This may not be applicable to public sector organizations (as the casual observation of the data suggests), since public sector organizations (henceforth, public sector firms), such as universities and government-controlled entities, do not, generally, operate under the profit maximizing principle. Instead, public sector firms are considered to be welfare-maximizers and it is not clear, *a priori,* whether the ability to appropriate R&D returns (through patenting, for example) should imply greater R&D intensity on the part of the public sector firm. For example, if the public sector firm is able to appropriate returns of its R&D, it would suggest that the knowledge/innovation that is being patented has "lost" its public goods characteristics (i.e., it is no longer non-excludable) and has attained the attributes of a private good. The more a research output becomes "privatized", the less compelling it becomes for the public sector firm to conduct such research.

A more holistic approach to understanding the incentives for research, for both private and public sector firms, requires that we go beyond just the notion of appropriability but situate it within the context of knowledge flows and spillovers. Before so doing, it is important to be clear on the definition and distinction between the related concepts of research appropriability and research spillovers that will be used in this and subsequent chapters. Appropriability relates to the amount of rents captured by an innovating firm due to its innovation. As such, it is a notion that applies after the discovery process that leads up to the innovation. R&D Spillovers, on the other hand, are most often defined as externalities that arise from research effort of individuals or firms. Geroski (1995) identifies three modes through which spillovers occur:

"they routinely arise when different agents discuss subjects of mutual interest, or when research results are disseminated through publications and seminar presentations. Spillovers can also be created when one agent observes the actions of another and makes inferences about the thinking that lies behind those actions. Last but not least, spillovers occur when a researcher paid by one firm to generate new knowledge transfers to another firm (or creates a spin-off firm) without compensating his/her former employer for the full inventory of ideas that travels with him/her" (Geroski, 1995)

This suggests that spillovers occur during the R&D process and not so much after (Hinloopen, 2000). This is especially true for public R&D, where evidence suggest that public research, especially research on biotechnology, plays an important role in the innovation process of US industry (McMillan, Narin and Deeds, 2000).

R&D spillovers have been recognized to having an important effect on innovative performance and productivity growth (Griliches, 1992). While spillovers may increase the productivity of recipients of R&D, and hence of R&D at the industry level, they may also diminish appropriability (Cohen and Walsh, 2000). That is, the leakage of knowledge flows from a firm may give other firms an edge in the research process and use it to produce better or similar innovations; thereby reducing the market share and profits for the firm from which the spillovers initially arose. However low appropriability does not necessarily imply the presence of large research spillovers. Consider two firms conducting research independently and noncooperatively of each other and without any research spillovers between the two. If one firm innovates first, but does not (or cannot) adequately protect its innovation such that the other firm is able to effectively imitate and reproduce the innovation, the appropriability of rents for the innovating firm are diminished and essentially shared among the two firms as the non-innovating firm free rides off the innovation.

Figure AII.2, adapted from Cohen and Walsh (2000), helps conceptualize the relationship between spillovers, appropriability and R&D intensity. In the schematic representation of Figure AII.2 R&D is undertaken by public and private sector firms. Appropriability is characterized as the level to which different mechanisms, such as IP protection through patents and plant variety protection, trade secrecy and first mover advantage, are effective in protecting returns due to R&D. For the private firm appropriability has a positive effect on the level of research conducted by it, as theory would suggest. For the public sector firm, as suggested earlier, the ability to appropriate may or may not have a positive effect on the public R&D as the public sector firm may leave such research to the private firm and concentrate on research through which its welfare-objectives are better met (i.e., research that may be difficult to appropriate but still is welfare increasing). The relationship between appropriability and spillovers may also be different across public and private firms. If spillovers are large and significant within an industry, then private firms will find it more difficult to appropriate rents to R&D, hence the negative relationship. For the public sector firm, the lack of appropriability due to greater spillovers, while decreasing any pure monetary gains to itself, may in fact be welfare increasing if it results in significant productivity gains in the economy.[1]

---

[1] Since the public sector need not be financially profitable, it can invest in research beyond the point which the private sector can afford, with the express intent of creating spillovers to improve social welfare. Thus, the appropriability problem that arises in private sector research is not an issue for the public sector. The public sector's objective of increasing social welfare may, in fact, be advanced through more public research spillovers.

For both firms, however, the use of different appropriability mechanisms may weaken the extent and value to competitors of spillovers. Some appropriability mechanisms, such as trade secrecy, result in spillovers information flows being less than others, such as patents where one has to disclose the nature of innovation while offering protection. We conjecture, however, that since public sector firm's use of IP protection is a fairly recent phenomenon it is more appropriate to consider spillovers with and without IP protection for the public sector firm. That is, in the case of the public sector firm, the relevant issue is whether IP protection results in more or less spillovers occurring? We surmise that since public sector firms serve the public good, they are more open to the concept of sharing knowledge and explicitly allow spillovers to occur (as opposed to their counterparts in the private sector, who tend to restrict such information flows). When a public sector firm patents, while it still may have to disclose information in that patent, it can restrict the use of the innovation to only a few agents by licensing the technology exclusively.[2] It is in this sense the IP protection by the public sector firm may lead to lower spillovers.

Spillovers may also condition the incentives to conduct R&D by either complementing firm R&D (Levin *et al.*, 1987) or by substituting for it (Spence, 1984)[3].

---

[2] Maredia *et. al.* (2000), provide examples where exclusive licensing of a public sector firm's innovation may indeed increase welfare. However, since only a few firms were licensed the technology, the spillover effects of such patenting and licensing may have been limited (relative to the case where the technology was not patented and anyone could have availed them to it).

[3] Empirical evidence does not always support the view that industries with high spillover rates will have low R&D expenditures or that the latter are inversely related with the degree of spillovers. Bernstein and Nadiri (1989) find that for some industries with high spillovers, firms perform quite well in terms of dynamic efficiency and thus appear not to be discouraged by the implicit knowledge leakage. A similar conclusion is reached in a survey conducted by Levin and colleagues (Levin *et al.*, (1987)), where it was found that the firms with the highest level of spillovers (computers, communications equipment, electronic components, an aircraft) also ranked high in R&D intensity.

Lastly, industry R&D effort itself increases R&D spillovers across firms, as greater amount of R&D implies a greater amount of spillovers.

Although the literature has noted the importance of the magnitude of the knowledge flows for the level of innovative activities that are obtained with competitive or cooperative research among private firms (see DeBondt (1997) for a review), substantially less attention has been given to spillovers from public sector firms within the context of a mixed R&D market. The study of public research spillovers is important especially at a time when publicly funded research institutions are protecting their innovations for a variety of reasons such as increasing revenues through licensing, 'defensive patenting', and protecting and licensing commercially unattractive innovations (Maredia *et al.,* 2000). The gains from protection by a public sector firm however must be weighed against the response that they generate among the private firms. If the protection of innovations implies a restriction in the sharing of innovation and the knowledge embodied in it, then the question of how such restrictions on public research spillover effects the behavioral response of the private firm is not merely an empirical one, but important also for its theoretical stipulations. In this paper we focus our attention on theoretical aspects of public research spillovers to clarify the impacts that are due to greater appropriability for public research and those that are due to greater knowledge flows (spillovers). We present variations of the standard patent race models within the context of a mixed market and under the assumption of asymmetric spillovers.[4]

The plan of this chapter is as follows. The next section briefly reviews the literature on R&D in the context of mixed markets, highlighting key results that have bearing on our modeling. The modeling section follows in which three specifications, variations of the standard R&D race models, are presented. The first specification generalizes two earlier models by parameterizing the degree to which public sector firm's R&D is appropriable. The second specification further develops the first specification, by requiring that the private firm have a research capacity to reap benefits of public sector firm innovations. The last specification models public research spillovers as cost reducing. In section four we provide a comparative analysis and their welfare implications of the results of the three specifications. Section five concludes the paper.

II.2. Literature Review

Before presenting the theoretical framework of this paper, which builds on and owes to the work of earlier researchers, a brief review of recent papers by Nett (1994), Delbono and Denicolo (1993) and Poyago-Theotoky (1998), who have considered the performance of mixed R&D oligopolies, is necessary. Nett's analysis considers research for a process innovation in a mixed duopoly with production. The analysis is cast under certainty, without spillovers, where the objective of the public sector firm is output maximizing (market-share maximizing). Under such a setup, Nett (1994) shows that a private firm has an incentive to operate at lower variable cost but higher fixed costs than a public sector firm. By choosing to shift a higher amount of its resources to the sunk cost category, a private firm has more flexibility in its strategic behavior of the market game than its public rival. Further, welfare in a private (pure) duopoly may exceed welfare in the mixed duopoly. In contrast, Delbono and Denicolo (1993), in their modeling of R&D

[4] The asymmetry is due to spillovers arising only from public sector firms and not private, as our interest in

as a patent race, show that a welfare-maximizing (not output-maximizing) public sector firm actually contributes to an increase in social welfare relative to the private duopoly. Having a public, welfare-maximizing firm compete with a private, profit maximizing firm results in both firms reducing their R&D effort, which alleviates the over-investment problem of the non-cooperative equilibrium (i.e., the case of the private duopoly). Poyago-Theotoky (1998) extends the analysis of D&D to consider the under-investment problem that arises from the 'easy imitation' of the reward. By modeling public sector firm's research as not appropriable (i.e., the benefits of public research are distributed among both private and public sector firms), P-T is able to reverse the results of D&D. In particular she finds that the public sector firm will invest more in R&D than its rival private firm, and when comparing welfare across different institutional settings, welfare can be higher or lower in the mixed duopoly relative to the private duopoly, depending on the size of the reward to innovation.

The D&D and P-T models serve as the point of departure for this chapter. In particular we show that their models and results can be easily generalized by interpreting the models in terms of appropriability as defined earlier. The effect of different levels of public research appropriability on private research and welfare is discussed.

II.3. The Model

II.3.1. Specification 1: Generalizing Appropriability in D&D and P-T models

Consider a one-shot non-cooperative game between a profit-maximizing private firm (*P*) and welfare maximizing public sector firm (*S*), where the firms invest in R&D

---

this chapter is to primarily understand the impacts of appropriability and spillovers of public research.

with the aim of innovating. The firm that innovates first is awarded an exogenously determined prize (W), which is the same for the two firms, i.e., $W_S = W_P = W$.[5] The probability of success by firm i is a function of its R&D expenditure x_i; where x_i is the flow cost that firm i pays until one firm succeeds. The payoff function of the private firm is specified as the present value of expected profits, net of R&D costs:[6]

$$V_P^1 = \int_0^\infty \exp\{-(h[x_P]+h[x_S]+r)t\}\left[\frac{W}{r}\left(h[x_P]+\alpha h[x_S]\right)-x_P\right]dt \qquad \text{(II.1a)}$$

$$=\frac{(W/r)\left(h[x_P]+\alpha h[x_S]\right)-x_P}{h[x_P]+h[x_S]+r} \qquad \text{(II.1b)}$$

where r is the discount rate. Following the literature on innovation races, $h[x_i]$, the hazard function, is the instantaneous probability of innovating (Reinganum, 1989). The hazard function is twice differentiable, strictly increasing and satisfies:

1. $h[0] = 0 = \lim_{x\to\infty} h'[x]$
2. $h'[x_i] > 0$
3. $h''[x_i] < 0$

The parameter α is the appropriability parameter and lies on the closed unit interval $[0,1]$. The interpretation of the appropriability parameter within the present context is the following. Most R&D race models have assumed that the returns to R&D for the winning firm is value of the prize (W), with the losing firm getting nothing (winner take all

[5] D&D justify this assumption on the grounds that "when the private firm is a perfectly discriminating monopolist, whereas the public sector firm maximizes social welfare also in the product market."

assumption). However if we assume that the public sector firm is unable (or unwilling) to appropriate all of the returns to its research, than it is natural to assume that the private firm benefits even if it were to lose the race. By assuming that public research is imperfectly appropriable such that $0 \leq \alpha \leq 1$ then the returns to the private research when it loses are αW. The more appropriable public R&D becomes, due to increased patenting by the public sector firm, the lower the opportunity for the private firm to copy public sector firm's innovation and reap benefits. In the D&D model it is implicitly assumed that the public sector firm can appropriate all the returns to its research, thus α=0. P-T considers the other extreme wherein returns to public research, should the public sector firm win, is shared with the private firm (i.e., α=1.)[7]

Next, we specify the payoff to the public sector firm. The innovation entails a social benefit, which is assumed to be equivalent to the prize obtained by the private firm. Since only one innovation is in prospect, the public sector firm is indifferent as to who wins the race; to the public sector firm the expected date of innovation is what matters. Moreover, the public sector firm takes into account the R&D costs of both firms. It is in these respects that the public sector firm is considered a welfare-maximizing firm[8]. The public sector firm's payoff is specified as

---

[6] A note on notation: to differentiate between the different payoff specification under consideration in this paper, the superscript in V_i^j for j=1,2,3 represents the case being considered. The initial case that generalizes the modeling of D&D and P-T is denoted case i=1, the other two cases (whose description follows) are denoted 2 and 3.

[7] To allow for comparison and consistency among the models, I use D&D's specification as the basis. In P-T's original specification, the instantaneous resource cost of achieving a hazard rate x is given by $\gamma[x]$. The function $\gamma[x]$ is a strictly increasing cost function satisfying the condition $\forall x \geq 0$, $\gamma[x] \geq 0$, $\gamma'[x] \geq 0$. D&D's and P-T's specification are equivalent in that they imply that there are decreasing returns to research everywhere.

[8] The underlying assumption here is that a faster pace of innovation in the economy is welfare increasing.

$$V_S^1 = \int_0^\infty \exp\{-(h[x_P]+h[x_S]+r)t\}\left[\frac{W}{r}\left(h[x_P]+h[x_S]\right)-x_P-x_S\right]dt \qquad \text{(II.2a)}$$

$$= \frac{(W/r)\left(h[x_P]+h[x_S]\right)-x_P-x_S}{h[x_P]+h[x_S]+r} \qquad \text{(II.2b)}$$

It is important to note that appropriability parameter does not enter into the public sector firm's payoff. As a social-welfare maximizer, the public sector firm benefits whether it wins or the private firm wins. Further, the fact that the private firm can easily imitate public research firm when $\alpha = 1$, does not diminish the value of the prize. This is an important point and requires emphasizing. We have already assumed that the value of the innovation is the same across the two innovators and not related to who innovates[9]. Since the 'prize' to the public sector firm is social welfare whereas to the private firm it is private profits when it is a perfectly discriminating monopolist, the value of the prize is equal. The notion of appropriability would suggest that if the benefits of a firm's innovation are appropriated among several users (due to weak patent, for example) then its *profits* should be lower relative to the case when it is able appropriate all the benefits to itself. However, for the public sector firm profits, and more crucially the distribution of profits among firms, are irrelevant, since the public sector firm maximizes public welfare. Since the concept of welfare constitutes individual firm profits, the public sector firm is less concerned with whether it makes more profits (and hence someone else less) due to increased appropriability as total social welfare remains unchanged given our assumptions on equivalence of the prize. Appropriability would, however, matter to all firms if all were profit-maximizing firms. In this case, an increasing ability to appropriate returns by firm i (should it win) would imply decreasing post-innovation profits for firm j (the losing firm)[10].

The two firms acting non-cooperatively and simultaneously choose the R&D expenditure in order to maximize their respective payoffs (i.e. equation II.1 and II.2). From the first order condition for a maximum, one derives the following reaction curves for the private and public sector firms, respectively, in the R&D space (x_P, x_S)

$$R_P^1[x_P, x_S] = h'[x_P](r + (1-\alpha)h[x_S])W - r^2 - rh[x_P] - rh[x_S] + rx_P h'[x_P] = 0 \quad \text{(II.2)}$$

$$R_S^1[x_p, x_S] = h'[x_S](W + x_P) - r - h[x_P] - h[x_S] + x_S h'[x_S] = 0 \quad \text{(II.3)}$$

The reaction functions defined by equations (II.2) and (II.3) are continuous and will have a unique positive equilibrium $x_P^*[x_S]$ and $x_S^*[x_P]$ if $R_i^1[x_P, x_S]$ is decreasing in x_i, with R_i positive at $x_i = 0$ and negative at $x_i = \infty$.[11] Choosing $h'[0] > 1/W$ and $h'[\infty] = 0$, for example, ensures a unique positive solution (x_P^*, x_S^*) for every α. The stability condition for the R&D race is equivalent to $\partial R/\partial x_i$ <0 (Lee and Wilde, 1980). Nti (1999) shows that the stability condition $\partial R/\partial x_i < 0$ is satisfied if the hazard rate elasticity $H[x] = xh'[x]/h[x]$ is differentiable and non-increasing in x.

To characterize the Nash equilibrium in the context of public research appropriability, we first describe the shape of the reaction curves. We limit our attention

---

[9] This assumption fails when the use of the innovation by the public sector firm differs from that of the private firm (D&D).
[10] That is if W is the value of the prize then, under conditions of imperfect appropriability, the winning firm may get αW and the losing firm $(1-\alpha)W$ with $0.5 \leq \alpha \leq 1$ (the lower bound representing lowest appropriability and the upper bound the highest appropriability).

to the private firm, as the public sector firm's payoff structure remains unchanged and its properties have been described by D&D[12]. Our interest here is to examine how equilibrium research varies due to appropriability of public innovation. We formalize the shape characteristics of the private reaction function in the following lemma:

Lemma II.1: Define $R_P^1[x_S, x_P] = 0$ as the private firm's reaction function (II.3), and $R_S^1[x_S, x_P] = 0$ as the public sector firm's reaction function (II.3). Then

a) $R_P^1[0, x_P] = R_S^1[x_S, 0]$, or more compactly, $x_P^{x_S=0} = x_S^{x_P=0}$ where $x_i^{x_j=0}$ is the optimal R&D investment of firm i, given that the other firm does not invest.

b) Define MVP_W as the marginal value product (*MVP*) of research from winning the race, MVP_L as the MVP of research from losing the race, and *MC* as the marginal cost of research, then

   i. Private reaction curve is positively sloped for $MVP_W - MVP_L > MC$

   ii. Private reaction curve is negatively sloped for $MVP_W - MVP_L < MC$

   iii. Private reaction curve has a slope of zero for $MVP_W - MVP_L = MC$

c) $\forall \alpha, 0 \le \alpha \le 1\ \exists\ \alpha^*$, that solves $dx_P[x_S]/dx_S = 0$

d) The private reaction function is bounded

Proof

---

[11] This will be true for all the best response functions that we consider in the paper

[12] The public sector firm's reaction curve is downward sloping if $x_S > x_P$ and upward sloping for $x_S < x_P$. In both D&D case and P-T case, there exists a unique Nash equilibrium of the innovation race, where D&D shows that the public sector firm invests less in R&D than the private one at that equilibrium, whereas P-T shows the opposite.

a) Setting $x_S = 0$ into (II.2) and $x_P = 0$ into (II.3) we get, in both instances,

$$h'[x_i](W + x_i) - h[x_i] - r = 0 \tag{II.4}$$

Hence $x_P^{x_S=0} = x_S^{x_P=0}$.

b) Implicitly differentiating equation (II.2) and exploiting the second order condition we get

$$\text{sign}\left(\frac{dx_P[x_S]}{dx_S}\right) = \text{sign}\left[h'(x_P)(1-\alpha)W - r\right] \tag{II.5}$$

Rearranging the term on the right hand side of equation (II.5), $\frac{dx_P[x_S]}{dx_S} \gtreqless 0$ iff

$$\frac{h'[x_P]W}{r} - \frac{\alpha h'[x_P]W}{r} \gtreqless 1 \tag{II.6a}$$

or

$$MVP_W - MVP_L \gtreqless MC \tag{II.7b}$$

Recall that the cost of research is linear, x, hence the marginal cost of research is 1. What equation (II.7) shows is that when the difference between the marginal value product of winning and losing is greater than the marginal cost of research, then the private reaction curve is upward sloping and is downward sloping when the difference is less than the marginal cost. The D&D result implies that for $\alpha = 0$ $MVP_W > MC$, hence the reaction curve is upward sloping. Similarly, in the P-T framework where ($\alpha = 1$), $MVP_W = MVP_L$ implies $0 < MC$, hence the reaction curve is downward sloping.

c) The solution to $dx_P[x_S]/dx_S=0$ requires $\alpha = 1 - \frac{r}{Wh'[x_P]}$. The zero slope of the private reaction curve implies that $\forall x_S$, $x_P = x_P^{x_S=0}$. Thus, $\alpha^* = 1 - \frac{r}{Wh'[x_P^{x_S=0}]}$, a critical value.

d) From part c of this lemma, we know that $\frac{h'[x_P]W(1-\alpha)}{r} \gtreqless 1$ or after rearranging the terms $h'[x_P] \gtreqless \frac{r}{W(1-\alpha)}$. Hence, $x_P^{x_S=0} < x_P$ cannot exceed the finite value x_P^{∞} $\forall \alpha, 0 \leq \alpha < \alpha^* \leq 1$ implicitly defined by $h'[x_P^{\infty}] = \frac{r}{W(1-\alpha)}$. Similarly $x_P^{x_S=0} < x_P$ cannot be less then the finite value x_P^0 $\forall \alpha, 0 \geq \alpha > \alpha^* \geq 1$ implicitly defined by $h'[x_P^0] = \frac{r}{W(1-\alpha)}$

Figure AII.3 summarizes the main contents of this case, which graphs the public reaction curve and the private reaction curves for five different values of α. These graphs were generated in Mathematica using $h[x] = x/(1+x)$, $W = 100$ and $r = 0.1$[13]. The diagonal line from the origin is the 45-degree line, and the public reaction curve is the curve originating from the xs axis. Five different private reaction curves are graphed, representing, from left to right on the xp axis, the following values for α=1, 0.95, 0.75, 0.5 and 0. For this example, the critical value for alpha (α^*), where $dx_P[x_S]/dx_S = 0$ is calculated to be 0.89.

This case shows that the D&D and P-T models are special cases once we parameterize the appropriability of public innovations. Without appropriability (as in D&D) the private firm reacts to an increase in the public sector firm's R&D investment by increasing its effort for fear of being surpassed in the innovation race. When the private firm can perfectly copy the public sector firm's innovation (as in P-T), there is a free-rider aspect to R&D such that the private firm leaves the R&D to its rival public sector firm to undertake and then benefits through imitation. Such a situation leads to under-investment in research and in equilibrium the public sector firm invests more in research than the private one. By introducing appropriability as a parameter we show that these curvature properties hold; that is for $\alpha > \alpha^*$, the private firm's reaction curve is positively sloped and for $\alpha < \alpha^*$ for the private firm the curve is negatively sloped.

Proposition II.1: For $\alpha > \alpha^*$ the equilibrium public R&D is increasing in α. For $\alpha < \alpha^*$ equilibrium public R&D is decreasing in α for $x_P > x_S$ and increasing for $x_P < x_S$

Proof: The public reaction curve is increasing for $x_P > x_S$ and decreasing for $x_P < x_S$ (For a formal proof refer to D&D). Let $0 \le \alpha 1 < \alpha 2 \le 1$ then $R^1_{P,\alpha 1}[x_S, x_P] < R^1_{P,\alpha 2}[x_S, x_P]$. From Lemma II.1, the private firm's reaction function is positively sloped for $\alpha > \alpha^*$, decreasing for $\alpha < \alpha^*$ and has a slope of zero at $\alpha^* = 0$. Also from Lemma II.1 $x_P^{x_S=0} = x_S^{x_P=0}$. It follows then that for

---

[13] Several other functional forms can be chosen which meet Nti's condition of a non-increasing hazard rate elasticity. These include the power (x^β for $0 < \beta < 1$), logarithmic ($\log[1+x]$), and exponential functions ($1 - e^{-x}$). We settle on the rational functional form as it is the easiest to solve.

$1 \geq \alpha 2 > \alpha 1 > \alpha^{*} > 0, [x_S^{*}, x_P^{*}]_{\alpha 1} < [x_S^{*}, x_P^{*}]_{\alpha 2}$; for $0 \leq \alpha 2 < \alpha 1 < \alpha^{*} < 1$,

$[x_S^{*}, x_P^{*}]_{\alpha 1} > [x_S^{*}, x_P^{*}]_{\alpha 2}$ if $x_P > x_S$ and $\left\{[x_S^{*}]_{\alpha 1} < [x_S^{*}]_{\alpha 2}, [x_P^{*}]_{\alpha 1} > [x_P^{*}]_{\alpha 2}\right\}$ if $x_P < x_S$

Taken together Lemma II.1 and Proposition II.1 have the following interpretation. A decrease in the appropriability of public innovations ($\alpha \rightarrow 1$) decreases the incentive for the private firm to do research since it can free ride off the innovations of the public sector firm. The change in response by the private firm due to appropriability of public innovations affects the Nash equilibrium, with lower appropriability resulting in equilibrium conditions in which private R&D is lower and generally lower public R&D[14].

II.3.2. Specification 2: Appropriability With Private Research Capacity

In this section, we consider a slightly different specification of that presented earlier. We maintain the assumption that some public research may not be appropriable such that the private firm is able to imitate the innovation of the public sector firm, should the public sector firm win the R&D race. In this specification, however, we shall assume that to be able to evaluate and employ the external public innovation, the private firm must possesses a certain amount of internal research capacity that allows it to 'copy' public sector firm's innovation. A weakness of the earlier specification is that it raises the possibility that the private firm can benefit from the public sector firm's innovation without any research effort of its own[15]. As pointed out by Geroski (1995) even if the appropriability is weak, it remains unclear as to the benefits recipient may accrue from

[14] Equilibrium R&D will *increase* with lower appropriability if $x_S > x_P$

the use of its rival's innovations. That is, it is not difficult to imagine scenarios where firms must invest in R&D in order to learn, adapt and use a rival's innovation[16]. Therefore a more realistic condition would require that the firm undertake some research to benefit from the imitation of the public sector firm innovation. Further, the more research the private firm undertakes, the greater is the probability that it will be able to benefit from the spillovers.

Mathematically, the appropriability of public sector firm's R&D and that the private firm maintain a research capacity to benefit from the loss of public research appropriability is specified in the payoff of the private firm:

$$V_P^2 = \int_0^\infty \exp\{-(h[x_P]+h[x_S]+r)t\}\left[\frac{W}{r}\left(h[x_P]+\alpha g[x_P]h[x_S]\right)-x_P\right]dt \qquad \text{(II.7a)}$$

$$= \frac{(W/r)\left(h[x_P]+\alpha g[x_P]h[x_S]\right)-x_P}{h[x_P]+h[x_S]+r} \qquad \text{(II.8b)}$$

Written this way, if the private firm wins the race, the flow of profits accruing to it is W. On the other hand, if it loses the race to the public sector firm, the flow of profits is reduced and becomes a function of the appropriability parameter (α) and how much research capacity the private firm accumulated during the race $g[x_P]$, which has the following properties

1. $g[0]=0$
2. $\lim_{x\to\infty} g[x]=1$
3. $g'[x]>0$

---

[15] Substituting α=1 and x_P=0 in equation I.1 we note that the payoff to the private firm is positive.

4. $g''[x] < 0$

The payoff for the public sector firm remains unchanged from that specified by both D&D and P-T (see equation II.2), as the appropriability is asymmetric.

As in the previous case, to characterize the Nash equilibrium in R&D space (x_P, x_S), we derive, for each firm, the best response function and their shapes. Since the payoff function for the public sector firm has already been considered, the properties of the associated best response function for the public sector firm is not derived in this section. What follows are the derivation for the properties of the private reaction function given the payoff specified in equation (II.7a).

From the first order condition for a maximum, the reaction curve of the private firm is

$$\begin{aligned} R_P^2[x_P, x_S] = &\left(h'[x_P](r + h[x_S])W - r^2 - rh[x_S] - rh[x_P] + rx_P h'[x_P]\right) + \\ &W\alpha h[x_S]\left((r + h[x_S] + h[x_P])g'[x_P] - h'[x_P]g[x_P]\right) = 0 \end{aligned} \tag{II.8}$$

To facilitate interpretation, the reaction curve has been specified such that spillover effect is separated from the no-spillover effect[17]. The term on the first line of equation (II.8) is the private firm's best response function without the appropriability effect. The terms on the second line represent the change to the best response function when the public sector firm's innovation is not perfectly appropriable.

To help us identify the Nash equilibrium in the x_P, x_S space, we state the following lemmas to establish the shape of the reaction curves

---

[16] Geroski (1995) mentions the study of learning in chemical processing industries by Lieberman (1984).
[17] The second line in equation (II.8) being the spillover effect.

Lemma II.2: Define $R_P^2[x_S, x_P]=0$ as the private firm's reaction function (equation (II.8)), and $R_S^2[x_S, x_P]=0$ as the public sector firm's reaction function (equation (II.3)). Then

a) $R_P^2[0, x_P] = R_S^2[x_S, 0]$ or $x_P^{x_S=0} = x_S^{x_P=0}$.

b) $\forall x_S \geq 0, \alpha \in |0,1|$ and $x_P^{x_S=0} < \tilde{x}_P < x_P^{\infty}$ a sufficient condition for $dx_P[x_S]/dx_S > 0$ (private firm's reaction curve positively slope) is $Wh'[\tilde{x}_P](1-\alpha g[\tilde{x}_P]) > r$.

c) $\forall x_S, x_P > 0, \alpha \in |0,1|$ a necessary condition for $dx_P[x_S]/dx_S < 0$ (private firm's reaction curve negatively sloped) is $Wh'[x_P](1-\alpha g[x_P]) > r$.

Proof

a) As in Lemmas II.1, setting $x_S=0$ into (II.8) and $x_P=0$ into (II.3) we get $h'(x_i)(W + x_i) - h(x_i) - r = 0$, which implies $x_P^{x_S=0} = x_S^{x_P=0}$

b) Implicitly differentiating equation I.17, and assuming that the second order condition is satisfied, we get

$$sign\left[\frac{dx_P}{dx_S}\right] = sign\left[\left(Wh'[x_P](1-\alpha g[x_P]) - r\right) + \alpha Wg'[x_P]\left(r + h[x_P] + 2h[x_S]\right)\right] \quad \text{(II.9)}$$

Equation (II.9) breaks down the curvature of the private reaction into two terms. The second term is unambiguously positive $\forall \alpha, 0 \leq \alpha \leq 1$ and $\forall x_S \geq 0$. For the private reaction curve to be unambiguously upward sloping requires that the first term in equation (II.9) be positive. Hence, $\forall \tilde{x}_P$, $0 < \tilde{x}_P < x_P^{\infty}$, a *sufficient* condition for (II.9)>0 is $Wh'[\tilde{x}_P](1-\alpha g[\tilde{x}_P]) > r$

c) $\forall \alpha, 0 < \alpha \leq 1$ and $\forall x_S \geq 0$, equation (II.9)<0 iff $Wh'[x_P](1-\alpha g[x_P]) < r$, $\forall x_P > 0$

The generality of our analysis does not allow for definitive characterization of the private firm's reaction curve. To facilitate interpretation of the private best response function and to examine the forces at play that determine its shape, we can express equation (II.9) in reduced form and in terms of the marginal value product as we did in Lemma II.1:

$$\frac{dx_P}{dx_S} \gtreqless 0 \text{ iff } F[g'[x_P], h[x_S]] + (MVP_W - MVP_L[g[x_P]]) \gtreqless r,$$

where $MVP_W = Wh'[x_P]$ is the undiscounted marginal value product of winning and $MVP_L[g[x_P]] = \alpha Wh'[x_P]g[x_P]$ is the undiscounted marginal value of losing. Here the marginal value of losing is a function of the ease by which the private firm can appropriate the public sector firm's research, which in turn is a function of an exogenous appropriability parameter (α), and the ability of the private firm to imitate the winner's prize. The α parameter is interpreted as the strength of the IPR regime. Whereas the ability to imitate in specification 1 was only a function of the magnitude of the spillover parameter, here it is also a function of how much research is undertaken by the private firm itself during the course of the race that allows it to capture the shared knowledge. The F function is interpreted as the marginal value change in the ability of the private firm to imitate the public sector firm's prize.

To illustrate the ambiguity in the response of the private firm to research by the public sector firm, we provide two examples where the assumptions on functional form and prize value will drive the results. We state these in the following propositions:

Proposition II.2: $\forall x_S \geq 0, \alpha = 1$, and $0 < r$ sufficient conditions for $dx_P[x_S]/dx_S < 0$ for $0 \leq x_S < x_S^*$ and $dx_P[x_S]/dx_S > 0$ for $0 < x_S^* < x_S$ are a) $0 \ll W$ and b) $g'[0] \to \infty$

Proof: Consider $0 \ll \tilde{\tilde{W}}$ which from Lemma II.2a implies $0 \ll \tilde{\tilde{x}}_P^{x_S=0}$. That is the higher the prize value, the greater the initial value of private research effort, given no research by the other firm. If we assume $g[x]$, such that $g'[\tilde{x}_P^{x_S=0}] \approx 0$ and $g[\tilde{x}_P^{x_S=0}] \approx 1$, then $\left[\frac{dx_P^{x_S=0}}{dx_S}\right]$<0 from equation (II.9). If we assume x_P is bounded from below by zero (i.e. $x_S \to \infty$, $x_P \to 0$), then dx_P / dx_S>0 from equation I.18. This implies that there exists x_S^* such that dx_P / dx_S^*=0

What Proposition II.2 says is that it is possible to get a negatively sloped reaction curve for the private sector in specification 3, if we assume that the prize value is "large" and that the private firm can easily imitate the public sector firm's winnings with only the minimal imitative capacity of its own. This result is similar to the P-T case which also showed a downward sloping private reaction curve, but differs in that the private reaction curve is not negatively sloped everywhere. By decreasing its research effort the private firm also decreases its ability to imitate and gain from the benefits of the public sector firm's prize. Thus at high levels of public research, the private will reverse the decline of its own research.

Next, we show that under sufficiently strong assumption on the hazard rate, the g function, and the best response function, the private best response function is increasing in public research and that there exist a unique Nash equilibrium. Matters are simplified further if we assume that $g[x] = h[x]$. Consider for example the rational function $f_1[x] = x/(1+x)$ or the exponential function $f_2[x] = 1 - e^{-x}$, either of which satisfy the concavity properties of the g and h functions. Further, if either f_1 or f_2 are specified as the hazard rate then they also satisfy the stability condition ($R_i < 0$) as both functions have a non-increasing hazard rate elasticity.

Proposition II.3: Assume that $g[x] = h[x]$, then $\forall x_S > 0$ and $0 < \alpha \leq 1$,

$dx_P[x_S]/dx_S > 0$

Proof: With $g[x] = h[x]$, and from the first order condition for a maximum, the reaction curve of the private firm is now

$$\begin{gathered}\left(h'[x_P](r + h[x_S])W - r^2 - rh[x_S] - rh[x_P] + rx_P h'[x_P]\right) + \\ W\alpha h[x_S]\left((r + h[x_S])h'[x_P]\right) = 0\end{gathered} \tag{II.10}$$

which can be re-written as

$$W(1 + \alpha h[x_S])h'[x_P] - r = \frac{r(h[x_P] - x_P h'[x_P])}{r + h[x_S]} > 0 \tag{II.11b}$$

Implicitly differentiating equation. (II.10), we note that it follows from equation II.11b that

$$\text{sign}\left[\frac{dx_P}{dx_S}\right] = \text{sign}\left[W(1 + \alpha(r + 2h[x_S])h'[x_P] - r\right] > 0$$

Proposition II.3 essentially reverses the P-T result, which showed that the private reaction function is unambiguously upward sloping for $\beta = 1$. By requiring that the private firm maintain a capacity to imitate the winnings of the public sector firm, and under sufficiently strong assumptions about the capacity building function and hazard rate, we show that the result will converge to that of D&D. However our intuition differs from that of D&D. Under Proposition II.3, the private firm increases its research effort for two reasons. Firstly it fears the competitive threat from the public sector firm and secondly it realizes that a higher research effort results in greater ease by which it can imitate the public sector firm's winnings.

With the assumptions used to put forth Proposition II.3, we can also show that with an increase in appropriability of public sector firm's innovation, the equilibrium R&D conducted by the public and private firms is lower.

Proposition II.4: Assume $g[x_P] = h[x_P]$ and let $0 \leq \alpha 1 < \alpha 2 \leq 1$, then

$[x_S^*, x_P^*]_{\alpha 1} < [x_S^*, x_P^*]_{\alpha 2}$

Proof: From Proposition II.3 and Lemma II.2, the public and the private reaction functions are both positively sloped for $x_P > x_S$ (the private reaction curve is unambiguously upward sloping). For $0 \leq \alpha 1 < \alpha 2 \leq 1$, $R_P^2[x_S, x_P]_{\alpha 1} < R_P^2[x_S, x_P]_{\alpha 2}$, that is the private reaction curve when the appropriability parameter is $\alpha 1$ lies below the one when the appropriability parameter is $\alpha 2$. Since $x_P^{x_S = 0} = x_S^{x_P = 0}$ it follows than that equilibrium R&D is lower $\alpha 1$ than it is for $\alpha 2$.

The result of Proposition II.4 is much more conclusive than its counterpart for specification 1 (Proposition II.1) It is also contrary to the earlier result where, recall, we showed that if $x_P > x_S$, then equilibrium public R&D is increasing with decreasing appropriability. Under Proposition II.4, we have shown the conditions under which R&D may increase with decreasing appropriability.

Figure AII.4 graphs the public and private reactions curves in the R&D space.

II.3.3. Specification 3: Public R&D Spillovers

Our last specification considers the impact of public research spillovers on the private firm and equilibrium properties of the R&D game.

A simple, yet realistic characterization of the R&D process is one where public research reduces the effective cost of the private firm's R&D. We define the effective cost of the private R&D as the expenditure by the private firm (x_P) less the R&D spillovers that emanate from the public sector firm:

$$C_P = x_P(1-\beta g[x_S]) \geq 0 \tag{II.11}$$

where β is interpreted as the spillover parameter falling in the closed unit interval and the function $g[x_i]$ signifies the amount of public R&D and has the following properties

5. $g[0]=0$
6. $\lim_{x\to\infty} g[x]=1$
7. $g'[x]>0$
8. $g''[x]<0$

From our characterization of the private firm costs as that specified in (II.11), the marginal cost of research is decreasing in both the degree to which spillovers occur and the level of public R&D. To account for cost saving research spillovers from the public sector firm, we re-specify the payoff function of the private firm:

$$V_P^3 = \int_0^\infty \exp\{-(h[x_P]+h[x_S]+r)t\}\left[\frac{W}{r}(h[x_P]) - x_P(1-\beta g[x_S])\right]dt \qquad \text{(II.12a)}$$

$$= \frac{(W/r)(h[x_P]) - x_P(1-\beta g[x_S])}{h[x_P]+h[x_S]+r} \qquad \text{(II.13b)}$$

Cost reducing spillovers affect R&D effort of the public sector firm in an interesting way. Recall that the public sector firm, as social welfare maximizer, takes into account the cost of research by the private firm as well as its own. This implies that more research by the public sector firm reduces the cost for the private firm and hence the public sector firm's total cost as well. The payoff for the public sector firm is specified as

$$V_S^2 = \int_0^\infty \exp\{-(h[x_P]+h[x_S]+r)t\}\left[\frac{W}{r}(h[x_P]+h[x_S]) - x_P(1-\beta g[x_S]) - x_S\right]dt \qquad \text{(II.13a)}$$

$$= \frac{(W/r)h[x_S]+(W/r)h[x_P] - x_P(1-\beta g[x_S]) - x_S}{h[x_S]+h[x_P]+r} \qquad \text{(II.14b)}$$

Note that when $\beta = 0$, this is model is equivalent to the one considered by D&D, but it cannot be generalized to the P-T model for any value of β. The two firms, acting non-cooperatively and simultaneously, choose R&D expenditure (x_i) to maximize their payoffs. Using the first order condition for a maximum, one derives the following reaction curves for firm P and S, respectively:

$$R_P^3[x_P,x_S] = \begin{pmatrix} h'[x_P](r+h[x_S])W - r - h[x_S] - h[x_P] + x_P h'[x_P] + \\ \beta g[x_S](r+h[x_S]+h[x_P]-x_P h'[x_P]) \end{pmatrix} = 0 \qquad \text{(II.14)}$$

$$R_S^3[x_P, x_S] = \begin{pmatrix} h'[x_S](W + x_P) - r - h[x_P] - h[x_S] + x_S h'[x_S] + \\ \beta x_P\left(g'[x_S](r + h[x_P] + h[x_S]) - g[x_S]h'[x_S]\right) \end{pmatrix} = 0 \qquad \text{(II.15)}$$

Equation (II.14) and (II.15) simultaneously determine the equilibrium value of the R&D expenditure x_P^* and x_S^*.[18] The two terms that appear in (II.14) and (II.15) that did not appear in the first order conditions of D&D specification (β=0 in equation (II.2), are $\beta g[x_S](\bullet)$ in (II.14) and $\beta x_P(\bullet)$ in (II.15). Both these terms reflect the presence of spillovers in the technological competition. The private firm unambiguously benefits from public spillovers as it increases instantaneous profits. In this model then, the presence of spillovers induces the private firm to speed up the expected date of innovation. The effect of spillover on the public sector firm is less clear, as there are two competing forces at play. Increased research expenditures by the public sector firm not only increases the probability of success for the public sector firm but also decreases the cost of private research, and hence increasing its instantaneous payoff. However greater research expenditure implies a greater cost for the public sector firm as well as an increase in the discount rate (the denominator in equation II.14b, and hence a lower payoff). Which of these two forces dominate will depend on the relative values of x_P and x_S, as well as the magnitude of the spillover parameter.

We formalize these statements and characterize the shape of the reaction curves (equation (II.14) and (II.15) in the following lemmas:

---

[18] As in specification 1, the continuity of the reaction functions and existence of a unique solution follows from quasiconcavity assumption of the objective function.

Lemma II.3: For specification 3, define $R_P^3[x_S, x_P]=0$ as the private firm's reaction function (equation (II.14)), and $R_S^3[x_S, x_P]=0$ as the public sector firm's reaction function (equation (II.15)). Then

a) $R_P^3[0, x_P, \beta]=R_S^3[x_S, 0]$, or $x_P^{x_S=0}=x_S^{x_P=0}$

b) $\forall \beta \in |0,1|$, the private reaction curve is positively sloped

c) For $\beta=0$, the private reaction curve is bounded. $\forall \beta \in \langle 0,1|$, the private firm's reaction function is unbounded.

Proof:

a) Setting x_S=0 into (II.14) and x_P=0 into (II.15) we get

$$h'[x_i](W+x_i)-h[x_i]-r=0 \tag{II.16}$$

Hence $x_P^{x_S=0}=x_S^{x_P=0}$

b) Implicitly differentiating equation (II.14), and exploiting the second-order condition we get:

$$sign\left[\frac{dx_P}{dx_S}\right]=sign\left[\begin{array}{l} h'[x_S]\left(h'[x_P]W+r\beta g[x_S]-r\right)+ \\ \left(\beta g'[x_S](r+(h[x_P]+h[x_S]-x_P h'[x_P])\right) \end{array}\right] \tag{II.17}$$

The second term on the right hand side of equation (II.17) is always positive due to the concavity of the hazard function ($h[x_P]>x_P h'[x_P]$). Rearranging the terms in the FOC of the private firm (equation I.7), shows that the first term is positive.

$$h'[x_P]W+r\beta g[x_S]-r=\frac{(h[x_P]-x_P h'[x_P])-\beta g[x_S](h[x_P]-x_P h'[x_P])}{r+h[x_S]}>0$$

as $\beta g[x_S]<1$. Hence sign$\left[\frac{dx_P}{dx_S}\right]$>0.

c) To prove the last part of the lemma, consider I.11, which requires that $h'[x_P]W + r\beta g[x_S] > r$ or $h'[x_P] > \frac{r(1-\beta g[x_S])}{W}$. If $\beta = 0$ then x_P cannot exceed the finite value x_P^∞ implicitly equals by $h'[x_P^\infty] > \frac{r}{W}$. For $0 < \beta \leq 1$ the reaction curve is unbounded as the value x_P^∞ is implicitly equal to $h'[x_P^\infty] = \frac{r(1-\beta g[x_S])}{W}$. That is for $h'[x_P] = \varepsilon$, there exists x_S such that $\frac{r(1-\beta g[x_S])}{W} < \varepsilon$

A comparison of Lemmas II.1 and II.3 clearly reveals the opposite effects that the spillover parameter and the appropriability parameter has on the private firm when spillover are modeled as either as cost reducing or increasing the ease in which the spillovers can be imitated. In the former case, spillovers unambiguously increase the amount of research by the private firm, whereas in the later case the private firm reduces its effort and simply free-rides off the effort of the public sector firm.

Unlike in case 1, the public sector's payoff function is also affected as a result of the cost reducing spillovers. Consequently its properties need to be formalized as well.

Lemma II.4: $\forall \beta, 0 < \beta \leq 1$, the public reaction function, defined by $R_S^3[x_S, x_P] = 0$, has the following properties

a) $\lim_{x_P \to 0} \frac{dx_S}{dx_P} < 0$

b) $\lim_{x_P \to \infty} \frac{dx_S}{dx_P} > 0$

c) $\forall\beta, 0 \le \beta_1 < \beta_2 \le 1,\ R_S^2[x_S, x_P]_{\beta_1} \gtreqless R_S^2[x_S, x_P]_{\beta_2}$ iff $\dfrac{g'[x_S]}{g[x_S]} \lesseqgtr \dfrac{h'[x_S]}{r + h[x_P] + h[x_S]}$

Proof:

a) Implicitly differentiating the public sector firm's best response function, we get

$$sign[\frac{dx_P}{dx_S}] = sign\left[\begin{array}{l}(h'[x_S] - g[x_S]h'[x_S])) + \\ \beta\dfrac{g'[x_S]}{g[x_S]}\left(\dfrac{r}{x_P} + \dfrac{h[x_S]}{x_P} + \dfrac{h[x_P]}{x_P}\right) + h'[x_P]\left(\beta\dfrac{g'[x_S]}{g[x_S]} - 1\right)\end{array}\right] \quad \text{(II.18)}$$

Consider the three terms in the right hand side of equation (II.18). The first two terms are positive $\forall x_S, x_P, \beta > 0$. From lemma 2 part a, we know that

$\lim\limits_{x_P \to 0} R_S^2(x_S, x_P, \beta) \to R_S^2(x_S^{x_P=0}, 0, \beta)$ where $x_S^{x_P=0}$>0. If we assume that

$\dfrac{g'[x_S^{x_P=0}]}{g[x_S^{x_P=0}]}$<1 then $\lim\limits_{x_P \to 0} h'[x_P]\left(\beta\dfrac{g'[x_S]}{g[x_S]} - 1\right) \to -\infty$, implying that

$\lim\limits_{x_P \to 0} \dfrac{dx_P}{dx_S} < 0$

b) Here we note that $\lim\limits_{x_P \to \infty} h'[x_P]\left(\beta\dfrac{g'[x_S]}{g[x_S]} - 1\right) \to 0$, implying that $\lim\limits_{x_P \to 0} \dfrac{dx_P}{dx_S} > 0$

c) $R_S^3[x_S, x_P]_{\beta_1} - R_S^3[x_S, x_P]_{\beta_2} = x_P(\beta_1 - \beta_2)g[x_S]\left(\dfrac{g'[x_S]}{g[x_S]}(r + h[x_P] + h[x_S]) - h'[x_S]\right)$

Given that $\beta_1 < \beta_2$, for $R_S^2(x_S, x_P, \beta_1) \gtreqless R_S^2(x_S, x_P, \beta_2)$ requires that

$\left(\dfrac{g'[x_S]}{g[x_S]}(r + h[x_P] + h[x_S]) - h'[x_S]\right) \lesseqgtr 0$.

The message of Lemma II.4 is that given the generality of the exposition, we are unable to definitively infer the shape of the public reaction curve. However as in specification 1, under sufficiently strong assumption on the hazard rate, the g function, and the best response function we can guarantee the existence of a Nash equilibrium. Assuming that $g[x] = h[x]$, we can show that for any β, the public reaction function is positively sloped for $x_P > x_S$.

Proposition II.5: Assume $h[x] = g[x]$, then $\forall \beta \in \langle 0,1|$ and $x_P \geq x_S$, the public reaction curve in specification 2 will be positively sloped.

Proof: With $h[x] = g[x]$, equation (II.18) is re-written as

$$sign\left[\frac{dx_S}{dx_P}\right] = sign\left[h'[x_S]\left(1+\beta(r+h[x_S]+h[x_P])\right)-h'[x_P]\right] \tag{II.19}$$

whence the result follows from the concavity of $h[x]$

The importance of Proposition II.5 is that, together with Lemma II.3, the existence of a unique Nash equilibrium for specification 3 is now guaranteed. Notice that in (II.19), the sign remains ambiguous for $x_S > x_P$, as $\frac{dx_S}{dx_P} \lesseqgtr 0$ if

$h'[x_S]\left(1+\beta(r+h[x_S]+h[x_P])\right) \lesseqgtr h'[x_P]$. What this suggests is that the spillover parameter (β) determines the range at which the public reaction curve is negatively sloped and when it is positively sloped (for $x_S > x_P$). The higher the spillover, the smaller the range of x_P values for which the public reaction curve is negatively sloped.

Lemma II.5: There exists a Nash equilibrium (x_P^*, x_S^*) for the mixed duopoly R&D game when there are cost reducing spillovers present.

Proof: Follows from the continuity of the reaction functions and Lemmas II.2 and II.3.

Figure AII.5.a, b and c illustrate the curvature properties of the public and private reaction curves, for β =0, 0.5 and 1, respectively. As before these graphs were generated on Mathematica where $g[x] = h[x] = x/(1+x)$, W=100 and r=0.1. From the panels of Figure AII.5, it is easily checked that for each specified β there is a unique Nash equilibrium.

II.4. Comparative Statics and Welfare Analysis

Table II.1 provides a comparison of how the spillover and appropriability parameter affects private and public research effort. Under specification 1, appropriability of public sector firm's innovations unambiguously lead to a decline in research by the private firm. When the imitation of public innovation requires some level of private R&D capacity, as in specification 2, a decrease in the appropriability of public innovation has an unambiguous positive effect on private research only when we assume the equivalence of $g[x]$ and $h[x]$. When we model spillovers as cost reducing (specification 3) the private firm increases its research effort to an increase in the spillover. The corresponding result for the public sector firm is ambiguous until we make the simplifying assumption that $g[x] = h[x]$, in which case $\partial x_S / \partial \beta > 0$.

Table II.1: Effect of Spillover on the Public and Private Research

	Model Specification Number		
	1	2	3
$\frac{\partial x_P}{\partial p}$	<0	Ambiguous for $g[x] \neq h[x]$ >0 for $g[x] = h[x]$	>0
$\frac{\partial x_s}{\partial p}$	0	0	Ambiguous for $g[x] \neq h[x]$ >0 for $g[x] = h[x]$

where $p = \alpha$ for specifications 1 and 2 and $p = \beta$ for specification 3.

We next turn to the question of whether presence of a public sector firm in the three specifications is a welfare improvement. We note that a finding of the D&D paper was that social welfare is higher in a mixed duopoly than in a private duopoly (i.e. $\beta = 0$ in specification 1). P-T, in her setup ($\beta = 1$) finds the result to be ambiguous and shows that for low values of welfare prize the social welfare is higher in a mixed duopoly, whereas for higher value of prize welfare in the private duopoly is higher. In specification 1, the parameterization of appropriability allows us to derive that level of appropriability for which social welfare is maximized.

Proposition 4: In case 1, $\forall \alpha, 0 \le \alpha \le 1$, there exists $\tilde{\alpha}$ such that social welfare is maximized .

Proof: A measure of social welfare is the public sector firm's objective function given by equation (II.2a). The social optimum is given by evaluating V_S^1 at $x_S = x_P$. This implies in the mixed duopoly the social optimum is achieved if in the Nash equilibrium

$x_P^* = x_S^* = x^*$. From D&D's analysis we know that at $x_S = x_P$, the public sector firm's reaction curve has a curvature of zero ($dx_S[x_P]/dx_P = 0$). It follows then that if the Nash equilibrium is to be at the social optimum $x_P^* = x_S^* = x^*$, x^* must solve the private firm's reaction best response equation (II.2), $\forall W, r > 0$, and

$$\tilde{\alpha} = \frac{\left(r(W + x^*) + Wh[x^*]\right)h'[x^*] - r(r + 2h[x^*])}{Wh[x^*]h'[x^*]}$$

What the above proposition suggests is that for any prize value, the amount that the public sector firm is allowed to appropriate in the strategic game can be regulated to achieve the social optimum. Consider a social planner that has prior knowledge of the payoffs for the private and public sector firms. Given any *W* and *r*, and the curvature properties of the best response functions, it is relatively straightforward to calculate for $\tilde{\alpha}$ that results in the Nash being the social optimum.

For specification 2 and 3, the ambiguities that arise due to the nature of the best response functions do not allow us to make a similar conclusion with regards to the effect of the parameters on welfare. However, our results for these two cases confirm the D&D findings that under the assumption when $g[x] = h[x]$, the mixed duopoly is welfare increasing compared to the private duopoly.

II.5. Conclusions

We began this chapter with the observation that there has been a notable shift in R&D effort of public and private firms (with the later taking on a more prominent role) and that public sector firms have been patenting and licensing at a much faster pace than they were in years prior to the Bayh-Dole Act. Standard economic theory would suggest that the more a firm is able to appropriate its returns, the more likely it is that it will conduct further research and increase its R&D expenditures. Why then, has the public sector firm not responded to an increasing ability to appropriate the returns from its innovation?

The contribution of this paper is to provide a theoretical model in which the public sector may respond positively to an increase in its ability to appropriate the results of its R&D by reducing its investments in R&D. This result hinges on the objective function of the public-sector firm, namely that of welfare maximization. Recall, that the public sector firm's payoff does not change, either in specification 1 or 2, from the parameterization of the appropriability parameter. It was pointed out that as a welfare maximizer, the prize value and who wins the race was of a lesser concern then the pace of innovation. While the firm itself may get higher returns from an increased ability to appropriate, the welfare value remains unchanged (as it is distributed across *all* firms). With "welfare" as the prize, the behavioral response of the public sector firm is unaffected by the appropriability parameter. The effect of change in the appropriability of public innovation however does impact private R&D. It was shown, in specification 1, that the private firm's response to an increase in the ability of the public sector firm to appropriate its innovation is to increase its (private) R&D. In specification 2, while the general result was ambiguous, specializing the model revealed that the private R&D decreases with higher public R&D appropriability, and that the equilibrium public R&D

was lower with a greater ability to appropriate returns. This result possesses some consistency with the reality of public R&D.

The parameterization of appropriability in specification 1 suggests that if the public sector firm has leverage in how much of the returns it appropriates than it is possible to attain a social optimum. It was found that if the public sector firm appropriates all its returns and effectively barring the private firm from the use of its innovation (through excessive protection of intellectual property), then it over-shoots the social optimum. It also misses the social optimum mark when it allows the technology to be essentially given away, giving rise to the free-rider problem and under-investment by the private firm.

To clarify the distinction between the incentives that arise due to appropriability issues and those that result from pure spillovers, specification 3 was presented to model the latter. Whereas 'appropriability' concerns how much rent the public sector firm is able to capture from its innovation, the notion of spillovers has to do with the degree of knowledge flow during the R&D process. Stated this way, α can also be interpreted as the spillover that occurs after the race and β is the spillovers during the race. Spillovers, in specification 3, were modeled as cost reducing, and under reasonable assumptions about the hazard function, we find that private reaction is unambiguously upward sloping with greater spillovers resulting in higher private research. This result is consistent with empirical evidence that public R&D positively impacts private sector R&D and innovation (McMillan, Narin and Deeds, 2000).

CHAPTER III

SHOULD THE PUBLIC SECTOR CONDUCT GENOMICS RESEARCH?

III.1. Introduction

Over the past two and a half decades biotechnology has revolutionized major portions of the human health and agriculture industries, transforming them into the life sciences industry. Yet, potentially, the most revolutionary biotechnologies are still in the early stages. Perhaps the most potent of these technologies is genomics—the science of sequencing all the genes of a given species to study the structure, function and evolution of diverse organism. Genome research, however, is more than biology; it is also about developing better drugs, foods, industrial products, and, in the case of agriculture, improving plant and animal productivity and quality.

A striking feature of genomic research is the significant levels of investment by a few dominant private firms in competition with an equally well-funded public sector that seek to discover and subsequently patent important gene sequences. This observation is made most pellucid by the private sector's Celera Genomics Group challenge to the longer-lived and more expensive, publicly funded Human Genome Project (HGP). The Department of Energy and National Institutes of Health started HGP in 1990, at a cost of approximately $2.2 billion over the course of the project. In 1992 Craig Venter, a scientist with the HGP, left to form his own private company, Celera Genomics, and claimed that the firm could sequence, using a different technique from the HGP, the whole genome in less time and at a fraction of the cost (approx. $200 million). Celera's challenge to the publicly funded HGP signaled the start of the race for the sequencing of the human genome, which was joined by numerous other start-up companies looking to capitalize on the potential of genomic research. The competition between Celera and HGP so accelerated sequencing efforts that by late 2000 both projects were essentially complete ahead of schedule.

Although less confrontational, the private sector is also at the forefront of characterizing plant and animal genomes, sometimes subcontracting from the public sector. In 1998, a consortium led by the International Rice Genome Research Program (IRGSP) in Tsukuba, Japan, began efforts to sequence the rice genome. The participants, which included primarily government and research foundation sponsored labs, took a traditional approach to genome mapping known as the 'stepwise sequence analysis' (Bennetzen, 2002). This approach, while expensive and slow, provides the most precise and complete sequence with a goal of 99.99% accuracy.[1] Soon after the IRGSP was initiated, the Monsanto Co. began funding research to sequence the same variety of rice as that by IRGSP. The Monsanto sequencing strategy was slightly different from that of IRGSP, allowing it to sequence more of the genome with less time and cost. However, Monsanto's strategy does not provide enough information for highly accurate assembly (Bennetzen, 2002).[2] Much more recently, the Beijing Genomics Institute (BGI) and Syngenta, a Switzerland based agricultural biotechnology company, independently produced draft sequences of the rice genome by the quickest and least costly method, the 'shotgun sequence analysis.' Syngenta obtained 99.8% sequence accuracy, identifying

[1] By early 2002, the project had sequenced 15% of the genome.
[2] In 2000, Monsanto abandoned its rice sequencing effort and donated its data to IRGSP.

more than 99% of the genes at 10% of the cost of the IRGSP strategy (Bennetzen, 2002)[3]. On January 26, 2001 Syngenta and Myriad Genetics announced that they had sequenced the rice genome and planned to provide their database to commercial customers, such as seed companies or agricultural biotechnology companies. The competition from the private sector has in turn spurred the IRGSP into advancing its calendar by almost four years (to 2004) and increasing its budget. The Government of Japan pledged to increase its annual rice genome research $60 million in 2000; a threefold increase over the previous year.

The Human and Rice Genome Projects are classic examples of a research race between firms where the objective is to be the first to discover, and subsequently obtain patents, on important gene constructs. However, current innovation or R&D racing models do not conform well to the type of behavior observed in genomics research for two reasons. First, most models have assumed that the research race is between profit maximizing private agents (Sabido, 1994; Reinganum, 1985). This is appropriate for a variety of industrial research in which the public sector is not involved. However in the agricultural sector, or in genomic research, there is a dominant public research sector whose objective, it can be argued, is to maximize not profits but welfare.[4] Thus, such research is best characterized as a race between a welfare maximizing public organization(s) and a profit maximizing private firm(s). In the context of genomics

[3] A useful analogy in comparing the different strategies is to imagine the whole genome as being a large puzzle. Without the knowledge of what the whole genome (puzzle) resembles, the genome is first broken down into pieces (DNA strands) and the individual pieces subsequently sequenced. After the identification (sequencing) of the smaller pieces has occurred the task of determining how the individual pieces relate to each other and where they fall on the map follows (this is akin to putting puzzle together). While it is easier to sequence smaller DNA strands, it is much more difficult to 'rebuild' the map with so many pieces.
[4] Following the literature on mixed oligopolies, we abstract here from moral hazard and internal organization issues and define a public sector firm to be an entity whose objective is to maximize social welfare, whereas a private firm would aim to maximize profit.

research, a private firm's objective is to patent the genetic code for important proteins and obtain royalties on the patent. Whereas the objective of a publicly funded entity is to promote further innovations (and hence increase welfare) which it does by making the genetic information more widely available, without regard to maximizing royalty revenues. The asymmetric objectives of these two types of firms' gives rise to a different behaviors from the case wherein all the firms in the analysis are private.

A second reason why earlier patent racing models fail to capture the intricacies of research such as genomics is in their assumption that the value of the prize is exogenously determined (Sabido, 1994). Genomics is not simply the identification of a sequence of genes, but also involves understanding the properties and relationships of the genetic code embodied in those genes. How accurately, and in what manner, the genetic sequence has been identified has bearing on the ease of interpreting the functions of the genes in the later stages. The amount of research done and the method employed in the sequencing stage can also influence later stages of genomics research if, for example, one assumes that more expenditure in the sequencing stage will, on average, lower the cost of doing research in the later stages. For example, the scientific knowledge and tools used in the sequencing the plant DNA have the potential of lowering the cost of breeding varieties with agronomicaly desirably traits. These cost reductions primarily result in more precision in transferring desirable genes to crops and reducing the time to breed specific varieties. The implication for the winning firm is that it gains knowledge in the process of the race that is useful and can be productively employed in further research (by it or others). That is, the more research it expends on the race today, the greater is the likelihood of winning the R&D race *and* the greater will be the cost savings on future research or production. Moreover, if we do not assume a winner-take all situations, the endogeneity of the prize value also has repercussions for the losing firm[5]. That is, the losing firm, which moves on to the next stage but uses the now inferior technology, will face lower profits due to a decline in market share. The decline in market share for the losing firm, and by extension profits, is a function then of the cost reduction implied by winning the prize (for the winning firm) and how much research was expended to achieve that prize (by the winning firm)[6].

To capture more accurately the microfoundations of an R&D race observed in genomics research, this chapter characterizes research as a two-stage process. The research effort in the first stage reduces the cost of applied research or production in the second stage. We use this framework to gain insights into a traditional theme in industrial organization research that of the relationship between market structure and innovation. Economists have been interested in this issue ever since Joseph Schumpeter's seminal work hypothesizing a positive correlation between market power and innovation. Schumpeter (1934) argued that a few firms were more likely efficiently to develop and

---

[5] The notion of 'winning' and 'losing' in the context of genome research may seem inappropriate as firms cannot obtain patents for simply sequencing a certain DNA strand nor are there any immediate commercial benefits from the knowledge of such sequences. Nevertheless, it has been observed that private firms are more reluctant to disclose their sequences (e.g., both Syngenta and Celera did not make their discoveries public through a commonly used public database). By effectively using trade secrecy to protect their sequences (especially large assembled sequences), the private firms hope to appropriate any return that may arise at later stages. This is in contrast to public efforts in genomic research, who have made the sequences available to all without strings attached.

[6] For example, assume a strategic game between two firms in the output stage where the cost of production for firm *i* before the innovation is $c_i[q_i, \gamma]$ where q_i is the amount produced and γ a technology parameter. After the innovation race, the winning firm will maximize its profits $\pi_w = P[q_w, q_l]q_w - c[q_w, \gamma_w, x_w]$ choosing q_w (where x_w is the research expended in the racing stage by the winner) and the losing firm maximizes $\pi_l = P[q_w, q_l]q_l - c[q_l, \gamma_l = \gamma]$ choosing q_l (assume $\frac{\partial c_i}{\partial \gamma_i}, \frac{\partial c_w}{\partial x_w} < 0$ and $\gamma_w > \gamma_l$). Solving for the equilibrium properties it can be shown that profits of the winning firm will be greater than that of the losing firm $\pi_w^* = \pi_w^*[\gamma_l^*, \gamma_l^*, x_w^*] > \pi_l^* = \pi_l^*[\gamma_l^*, \gamma_l^*, x_w^*]$

employ more advanced technology than a competitive industry. Formal models of firms' innovation-seeking behavior have evolved, that have either confirmed or refuted the so-called 'Schumpeterian tradeoff.'[7] Similar to the mixed theoretical results, the findings of the vast empirical literature on the Schumpeterian tradeoff are mixed and ambiguous as no obvious relationship between industrial concentration and R&D performance emerges from the data.

Specifically, we ask what kind of market structure, and circumstances, promote R&D when the nature of R&D is as described for genomics. Does the public sector serve a useful purpose by performing genomics R&D when all evidence suggests that there are willing private firms that can do the same type of research more quickly and at lower cost? Does public sector efforts have a role to play in genomics R&D? We show that this question can be answered in the affirmative, if certain sufficient conditions regarding the prize value and the nature of the innovation process are met. In the process of deriving these conditions we also obtain conditions under which a monopoly research market performs more R&D than a pure duopoly market and vice versa. These market structures are chosen as they represent the observed (mixed duopoly) and alternative (duopoly and monopoly) market structures in genomics research[8]. It is not the intention of our analysis to comment on the desirability of a mixed market over the other two (which requires comparison to the first-best outcome), but rather to provide a set of sufficient conditions whereby one would *expect* the public sector to conduct more R&D relative to firms in the

[7] For a review of this literature see Kamien and Schwartz (1982) or van Cayseele (1998). Among the many writers who subscribe to Schumpeterian are Scherer (1980) and Kamien and Schwartz (1982). The claim has been challenged by Arrow (1962) and Dasgupta and Stiglitz (1980).
[8] The high fixed costs associated with genomics R&D serves as an entry barrier, and hence a competitive environment is not analyzed here.

other regimes. It is in this sense that we suggest that the public sector has a role to play in conducting research of the kind observed in genomics R&D.

The paper is organized as follows. The next section briefly reviews earlier R&D models on which our analysis is based. Section three introduces the modeling framework, the assumptions and our approach in the comparing the different market structures. Section four discusses the comparative analysis. Section five concludes the paper.

III.2. Literature Review

Before proceeding to describe our modeling framework, we briefly review the relevant R&D models of Loury (1979), Lee and Wilde (1980) and Delbono and Denicolo (1991). Loury (1979) modeled a one-shot non-cooperative game in which n identical firms invest in R&D to innovate first. The first innovator is awarded an exogenously determined prize. The probability of success by firm i is an exponential function of its hazard rate ($h[x_i]$) given no success to date. The loser of the race gets nothing and thus suffers a loss equal to the cost of its R&D expenditure. The R&D cost for each firm is a lump-sum x, expended at the beginning of the race. Lee and Wilde (1980) reformulated Loury's model assuming that the R&D expenditure is a flow cost that firm i pays until any one firm is successful. The different specification of the cost of R&D results in a different impact of an increase in the number of firms on the equilibrium individual R&D effort. Whereas such an effect is negative in Loury's model, it is positive in Lee and Wilde's model.

The contribution of the aforementioned models has been to illuminate the relationship between the intensity of rivalry and R&D performance. However, since neither Loury (1979) or Lee and Wilde (1980) explicitly modeled the product market, the relationship between the structure of the product market and incentives to invest in R&D was not examined. One rationalization for not allowing for an explicit specification of a product market has been that firms compete in prices in a homogenous product market so that a Bertrand equilibrium results. In such a market, pre-innovation profits are zero (as all firms share the same technology) and post-innovation, the winner, which has reduced its own cost, will be the only active firm (Delbono and Denicolo, 1991).

Delbono and Denicolo (1991) show that when firms make positive profits in the pre- and post-innovation Cournot-equilibrium markets (where the losers also make positive profits, post innovation) the Loury result—that there is a positive relationship between profits and equilibrium R&D effort—holds. This result is significant in that it highlights the relationship between the number of firms and the equilibrium R&D effort to be much more complex than in models where the prize value, or expected returns from R&D, are exogenously given.

Now while the Delbono and Denicolo model allows for pre- and post-innovation profits for all firms, the nature of the innovation and how much of an improvement it is on a prior technology is still exogenously given. That is, at the end of the race, the winner obtains the rights to technology that bestows on it an exogenously determined cost advantage over the prior technology. This cost advantage is exogenously given and has no relationship to the actual amount of research that the firm conducted in the R&D race. As we have argued, there exists the possibility, particularly in genomics research, that the R&D effort of the racing stage of the game results in further cost advantages for later stages, such that a higher amount of research leads to a higher amount of product value for the winning firm. It is this aspect of the R&D process, and the point of departure from other models, that we highlight in our modeling framework and comparative analysis.

III.3. The Model

Consider a two-stage game. In the first stage, firms choose their R&D investment and in the second stage they compete further by conducting more applied R&D or competing in the product market[9]. The first stage is modeled as an innovation race where firms compete for the rights to an infinitely lived patent. The innovation embodied in the patent allows firms to lower the cost of research in the second stage (or the cost of production, if the second stage is modeled as a product market). Through backward induction, the profits from the second stage determine the value of the first stage patent. The firm that innovates first is awarded the patent and gets the exclusive right to use the more productive technology forever. The losing firm, on the other hand, has to continue using the pre-innovation race technology in the second stage and hence accrues a lower profit than the winning firm, and possibly even lower than its own profits prior to innovation.

The research effort employed in the R&D race not only determines the outcome of the race, but also results in the generation of knowledge that is valuable to the winning firm. This knowledge can be used in later stages to complement with the winning technology and lower costs in those stages even further. In this respect the value of the prize for the winner is endogenous and an increasing function of research expenditure in the R&D race. R&D effort thus has a two pronged direct effect on the winning firm;

[9] The specific characteristics of the second stage are not of concern here. We note only that the payoff (or prize) from undertaking R&D in the first stage is function of the market structure in the second stage and the amount of research performed in the first stage.

allowing it to win the race *and* lowering its cost in later stages. The losing firm is also affected by the amount of research effort employed by the winning firm in the first stage (see footnote 6). Since we assume the strategic game in the second stage as well, an increased market share for the winning firm from the lowering of its cost will imply lower profits for the losing firm, *cetris paribus*.

To fix these ideas, assume that two firms play the following two-stage game. In the first stage, firm i independently takes action, denoted x_i, regarding the current research market. In a patent race set-up, x_i represents the flow cost of research where its probability of being successful at or prior to date t is $1-e^{-th[x_i]}$. The instantaneous conditional probability that firm i will be first to innovate at time t, given no success to date, is therefore $h[x_i]$. Firm i's expected benefits after a discovery are determined by both firms actions (b_i,b_j) and are denoted by $W_{Wi}(b_i[x_i],b_j[x_i])$ if the firm emerges as the winner and $W_{Li}(b_i[x_j],b_j[x_j])$ when it losses the race. In a Cournot set-up, b_i would represent output whereas in a Bertrand game it would be price. A strategy of firm i in this entire game can then be written as $s_i \equiv (x_i, b_i[\bullet])$ where $b_i[\bullet]$ is a function specifying firm i's post innovation action conditional on first stage actions, in particular on the amount of research done by the winning firm. Given (s_i, s_j), the payoff to the private firm i is:

$$V_i[W_{Wi}, W_{Li}, \{x_i, x_j\}] =$$

$$\int_0^{\infty} e^{-(h[x_i]+h[x_j]+r)t}(h[x_i]W_{Wi}[x_i]/r + h[x_j]W_{Li}[x_j]/r + W_i - x_i)dt \qquad \text{(III.1a)}$$

$$= \frac{(1/r)\left(h[x_i]W_{Wi}[x_i] + h[x_j]W_{Li}[x_j]\right) + W_i - x_i}{h[x_i] + h[x_j] + r} \qquad \text{(III.1b)}$$

where

r is the discount rate;

x_i is firm i's R&D expenditure;

$h[x_i]$ is firm i's instantaneous probability of innovating or the hazard rate. The hazard rate, $h[x_i]$, is twice differentiable, strictly increasing and satisfies 1) $h[0] = 0 = \lim_{x \to \infty} h'[x]$, 2) $h'[x_i] > 0$, 3) $h''[x_i] < 0$;

$W_{Wi}[x_i]$ is the value of innovation accruing to firm i if it wins the race;

$W_{Li}[x_j]$ is the value of innovation accruing to firm i if it loses the race where j is the winning firm;

W_i is the pre-innovation benefits accruing to firm i.

As we shall see, the value of the innovation will be different for the private and public sector firms. For the public sector firm the value of the an innovation is the total welfare generated by it. For the private firm it is the value of the private benefits or profits. Next, we proceed to characterize the equilibrium condition in R&D for the market structures of interest, namely monopoly, pure duopoly, and mixed duopoly. We make progress by first characterizing the best response function for each firm in the three markets.

III.3.1. Monopoly equilibrium condition

The monopolist maximizes its payoff (equation (III.1)) where i=1=M), by choosing x_M.[10] This defines the first order condition for a maximum for the monopolist, $\frac{\partial V_M}{\partial x_M}$=0, which is true if and only if $R_M[x_M]=0$ where

$$R_M[x_M] \equiv \begin{bmatrix} h'[x_M](W_M[x_M]-W_M)-r-h[x_M]+x_M h'[x_M]+ \\ h[x_M]W'_{WM}[x_M](1+h[x_M]/r) \end{bmatrix} = 0 \qquad \text{(III.2)}$$

Equation (III.2) determines the equilibrium value of the R&D expenditure by the monopolist x_M^*.[11]

Following Delbono and Denicolo (1993), the difference between the profit from winning and the firm's current profit $(W_{WM}[x_M]-W_M)$ measures the incentive to innovate in the absence of rivalry and is referred to as the 'profit incentive.' If we assume that $W_{WM}[x_M]>W_M>0$, then presence of current profits in this model induces firms to delay the expected date of innovation[12].

III.3.2. Pure duopoly equilibrium condition

In the pure duopoly case (two profit maximizing firms), firm 1 chooses x_1 and firm 2 chooses x_2 to maximize the payoff function. Due to symmetry (n=2, therefore,

[10] The monopolist faces no rival, therefore x_j=0.

[11] In equation III.2 and all subsequent equations, prime denotes first derivates and double prime second derivative.

[12] Stated differently, if there were no current profits such that $W_M=0$, then $W_{WM}[x_M]-W_M=W_{WM}[x_M]$, increasing the LHS of equation (III.2), relative to the case where $W_M>0$

x_1=x_2=x_D), the best response function for each firm is defined by the condition $\frac{\partial V_D}{\partial x_D} = 0$, which is true, if and only if, $R_D[x_D] = 0$, where

$$R_D[x_D] \equiv \begin{bmatrix} h'[x_D](W_{WD}[x_D] - W_D) + \\ (1/r)h'[x_D]h[x_D](W_{WD}[x_D] - W_{LD}[x_D]) - \\ r - 2h[x_D] + x_D h'[x_D] + h[x_D]W'_{WD}[x_D](1 + (2/r)h[x_D]) \end{bmatrix} = 0 \quad \text{(III.3)}$$

As in the monopolist case, the duopolist also faces a 'profit incentive' $(W_{WD}[x_D] - W_D)$ which measures the incentive to invest in R&D in the absence of rivalry. However in a duopoly there is rivalry as each firm anticipates research by the other. In the presence of rivalry the incentive to invest in R&D is also reflected in the difference between the flow of profits should it win the race and should it not, $W_{WD}[x_D] - W_{LD}[x_D]$. Call this the 'rivalry incentive.' The presence of both pre-innovation profits and post-innovation profits for the loser induce firms to delay the expected date of innovation, that is higher pre-innovation and loser profits decrease the profit and rivalry incentives. The smaller the profit and rivalry incentive, the more time it will take for innovations to arrive.

III.3.3. Mixed duopoly equilibrium condition

In the mixed duopoly the private and public sector firms choose x_i but maximize different payoffs. The private firm maximization problem remains unchanged from that of a firm in the pure duopoly case. That is, it maximizes equation (III.1) (for i=2), where we denote firm 1 as the private firm (P) and firm 2 as the public (S). Since symmetry no longer holds (as the public sector firm's payoff is different), the private firm's best response function is defined by the condition $\frac{\partial V_P}{\partial x_P}=0$, which is true if and only if

$R_P[x_P,x_S]=0$ where

$$R_P[x_P,x_S]\equiv\begin{bmatrix} h'[x_P](W_{WP}[x_P]-W_P)+ \\ (1/r)h'[x_P]h[x_S](W_{WP}[x_P]-W_{LP}[x_S])-r- \\ h[x_P]-h[x_S]+x_P h'[x_P]+h[x_P]W_{WP}'[x_P]\left(1+(1/r)(h[x_P]+h[x_S])\right) \end{bmatrix}=0$$

(III.4)

The best response function of the private firm in the mixed duopoly is similar to the one in pure duopoly. In both cases, there exists a profit incentive as well as a competitive threat, faced by the private firm. The only difference between the two is that the source of the competitive threat in the pure duopoly case is another, identically specified private firm, whereas in the mixed duopoly case it is the welfare-maximizing public sector firm.

The public sector firm in our model is a welfare maximizer to whom the value of the prize embodies the social welfare. This means, first, that the "W's" in the payoff function are greater for the public sector firm than they will be for the private (more discussion on the assumption and relative values of the prize follows). Second, the public sector firm, as a social welfare maximizer, takes into account the flow cost of research incurred by all firms in the economy. In the duopoly case where one firm is public and the other private, the public sector firm's payoff function is written as:

$$V_S=\int_0^{\infty} e^{-(h[x_P]+h[x_S]+r)t}(h[x_S]W_{WS}[x_S]/r+h[x_P]W_{LS}[x_P]/r+W_S-x_S-x_P)dt \quad \text{(III.5a)}$$

$$=\frac{(1/r)\left(h[x_S]W_{WS}[x_S]+h[x_P]W_{LS}[x_P]\right)+W_P-x_P-x_S}{h[x_S]+h[x_P]+r} \quad \text{(III.5b)}$$

The maximization of the equation III.5 yields the public sector firm's reaction curve which is defined by the condition $\frac{\partial V_S}{\partial x_S} = 0$, which is true, if and only if, $R_S[x_S, x_P]=0$

$$R_S[x_S, x_P] \equiv \begin{bmatrix} h'[x_S](W_{WS}[x_S] - W_S) + \\ (1/r)h'[x_P]h[x_S](W_{WS}[x_S] - W_{LS}[x_P]) - \\ r - h[x_P] - h[x_S] + h'[x_P](x_P + x_S) + \\ h[x_S]W'_{WS}[x_S]\left(1 + (1/r)\left(h[x_P] + h[x_S]\right)\right) \end{bmatrix} = 0 \qquad \text{(III.6)}$$

The reaction curve of the public sector firm in the mixed oligopoly also reveals that the public sector firm faces the 'profit incentive' and the 'rivalry threat' from the opposing firm. For the public sector firm the 'profit incentive' is a misnomer (since the value of the prize to it is total welfare, and not private profits as the name implies), although it is still the incentive to innovate in the absence of a rival. As with the earlier case, the smaller the rivalry and profit incentives the later is the date of innovation

In summary, equations (III.2), (III.3), (III.4) and (III.6) are, respectively, the best response functions for a monopolist, duopolist (in a pure duopoly), private firm in the mixed duopoly and the public sector firm (in a mixed duopoly). In all these cases we see that each firm faces a profit incentive and a competitive threat (except in the case of the monopolist, where it does not face a rival). The profit incentive is a function of how large the difference is between current profits and profits if the firm wins, and similarly the competitive threat is a function of how big the difference in profits is between winning and losing. Clearly, if the differences are small, then the firms will be conducting less research, which will delay the expected date of innovation. For the public sector firm in the mixed duopoly, the profit incentive and the competitive threat also matter (only that it is not profits that the public-sector firm is after but welfare). But since it takes into account the total R&D cost, the effect of R&D cost on the public sector firm $(x_P + x_S)$, relative to private firm (x_P), is to bring closer the expected date of innovation.

Lastly we note that in each of the four best response functions the presence of the term the term $\frac{\partial W_{Wi}}{\partial x_i}$. This term, which reflects the marginal change in the prize value due to a change in own research, is a direct consequence of our assumptions regarding the endogeneity of the value of the prize. If we assume that profits are concave with respect to own research then the endogenous nature of the prize value brings closer the expected date of innovation. This implies all firms carry out a greater amount of research relative to the case where the prize value is exogenous. To put it differently, when R&D complements the innovation at a later stage, firms have an incentive to increase their research effort. High amounts of research, or the so-called over-investment problem (Dasgupta and Stiglitz, 1980), could therefore be explained by the presence of such a complementary effect.

III.4. Comparative Analysis

Having established the nature of the game and market structure we now turn our attention to the relative ranking of the equilibrium research effort by individual firms (x_M, x_D, x_P, and x_S) as well the industry (x_M, $2x_D$, and x_P+x_S). First, however, we need to make certain assumptions about the relative value of the prize in the different markets and for the different firms. Since the relative values of W will determine the equilibrium values of research effort for all the firms, we need to make our assumptions regarding them explicit.

Assumption 1: The winning firm profits (for the private firms) or welfare (for the public sector firm) are greater than current profits/welfare. That is $W_{Wi}[x_i] > W_i$. This assumption ensures that the 'profit incentive' to innovate is always positive.

Assumption 2: In the pure duopoly the profits in the winning state are greater than or equal to in the losing state. Moreover, profits are increasing in R&D for winning firm and decreasing for the losing firm. Assume that firm 1 emerges as the winner and acquires the rights to a superior cost reducing technology. In the second stage, the two firms play a Cournot game. If we assume increasing costs of production in the second stage, then it can be shown that in equilibrium the winning firm will produce more than the losing firm. Consider, for example, a second stage product market where $P = a - bQ$ is the inverse demand function (for $Q = q_{WD} + q_{LD}$). Assume that prior to the race, the cost of production for both firms was $C[\gamma, q] = \gamma q^2 / 2$ where the parameter γrepresents the technological opportunity due to the successful research, such that $\partial C / \partial \gamma < 0$. At the completion of the race, the losing firm will continue to produce at the pre-innovation cost, but the winner obtains a better technology such that it lowers its costs to $C[\gamma, q] = \overline{\gamma} q^2 / 2(1 + x_D)$ where $\overline{\gamma} < \gamma$. The $\left(1 + x_D\right)$ term reflects the fact that the winning firm also gains from its research effort of the first stage. The post-innovation maximization problems for the winning and losing firm are therefore, respectively

$$\max_{q_{WD}} W_{WD} = P q_{WD} - C[\overline{\gamma}, q_{WD}, x_D^*] \tag{III.7}$$

$$\max_{q_{LD}} W_{LD} = P q_{LD} - C[\gamma, q_{LD}] \tag{III.8}$$

The two firms acting simultaneously and non-cooperatively solve their maximization problem. From the first order condition for a maximum it can be easily shown (see appendix) that the standard Cournot equilibrium when costs are asymmetric will prevail, that is[13]

1. $q^*_{WD}[x^*_D] \geq q^*_{LD}[x^*_D]$

2. $W^*_{WD}[x^*_D] \geq W^*_{LD}[x^*_D]$

3. $\partial W^*_{WD}[x^*_D] / \partial x^*_D > 0$

4. $\partial W^*_{LD}[x^*_D] / \partial x^*_D < 0$

Assumption 3: In the mixed duopoly, no public production or further research takes place in the second stage. Should the public sector firm win the race, it licenses its technology to the private firm. Alternatively, if the private firm wins the race in the mixed case, it does not face a rival in the second stage and assumes the role of a monopolist. As we are assuming the same demand and technology characteristics across the different market structures, it follows that the profits of the monopolist, should it succeed in innovating (in the monopoly case), equal the profits of the winning firm in the mixed duopoly, if and only if, the research effort by the two firms was equal in the first stage[14]. On the other hand if the public sector firm wins the first stage race, and in the absence of further research or production by the public sector firm in the second stage, it licenses the technology to the private firm. While the private firm is still a monopolist even with the license (by virtue of the fact that no rival exists), the terms of the license are such that it is not allowed to produce at the profit maximizing level (where marginal

[13] The asterisks represent equilibrium values from the solution of the optimization problem

[14] The claim here is that if $x_M = x_P = x$ then $W_{WM}[x] = W_{WP}[x]$

cost (MC) equal marginal revenue (MR)) but rather at a level where the welfare losses associated with monopoly production are minimized, though not necessarily eliminated. This is because the terms of the license also have to be incentive compatible in the sense that the profits for the private firm from production using the new licensed technology are greater than, or equal to, the profits associated with the older technology.

Therefore, it is reasonable to think that the welfare attained with a public sector firm innovating is higher than the welfare attained when the innovator is a private firm, simply because a public sector firm would license and contract its innovation to the private firm which would have to price its innovation in order to maximize social welfare taking into account consumer surplus. With $W_{WS}^{*}[x_S^{*}]$, as the total welfare generated by the innovation when the innovator is the public sector firm, and $W_{LS}^{*}[x_P^{*}]$ as the total welfare generated when the innovator is the private firm. Finally, $W_{WP}^{*}[x_P^{*}]$ is the value of the private benefits when the private firm innovates, and $W_{LP}^{*}[x_S^{*}]$ when it does not. Given these definitions, one has $W_{WS}^{*}[x_S^{*}] > W_{LS}^{*}[x_P^{*}] \geq W_{WP}^{*}[x_P^{*}] \geq W_{LP}^{*}[x_S^{*}]$ and $q_{LP}^{*}[x_S^{*}] \geq q_{WP}^{*}[x_P^{*}]$[15].

The generalized functional form of the hazard rate and the prize value does not permit us to solve explicitly for equilibrium research effort in each case that could be compared across the three different market structure. However, if we assume that the equilibrium research conditions x_M^{*}, x_D^{*}, x_P^{*}, and x_S^{*} solve their respective best response function, we can derive a set of sufficient conditions to evaluate the relative magnitude of

[15] For a more formal treatment of this result, refer to the appendix.

research among the firms. To illustrate this approach, assume that, in a two firm strategic game, x_i^* and x_j^* solve the best response function $R_i[x_i, x_j]$ and $R_j[x_i, x_j]$ for firms i and j, respectively (that is, $R_i[x_i^*, x_j^*] = R_j[x_i^*, x_j^*] = 0$). Assume also that there is an asymmetric relationship between the two firms such that, *a priori*, we are unable to determine the relative levels of research i.e. $x_i^* \gtreqless x_j^*$. In such cases it is possible to derive a set of sufficient conditions that will satisfy $x_i^* > x_j^*$, $x_i^* = x_j^*$ and $x_i^* < x_j^*$. We do so by taking the difference of the best response function of one firm evaluated at the other firm's optimal research levels (i.e., $R_j[x_i^*, x_i^*]$) and the best response of the other firm evaluated at its optimal level (i.e., $R_i[x_i^*, x_j^*] = 0$). If we assume that the underlying value function for both firms is concave with a relative maximum at x_i^* and x_j^*, and that the second order condition is satisfied ($\partial^2 V_i[x_i]/\partial x_i^2 < 0$), then we can claim that the conditions under which $R_j[x_i^*, x_i^*] - R_i[x_i^*, x_j^*] \gtreqless 0$ imply $x_i^* \gtreqless x_j^*$.

We apply this strategy separately to compare and derive the sufficient conditions to evaluate the equilibrium research effort between a) a monopolist and firms in a pure duopoly, b) a monopolist and firms in the mixed duopoly, and c) firms in a duopoly and firms in the mixed duopoly.

III.4.1. Monopoly vs. Pure Duopoly

The first order condition of a monopolist evaluated at the equilibrium value for the duopolist is expressed as:

$$R_M[\overset{*}{x_D}] = \begin{bmatrix} h'[\overset{*}{x_D}](W_M[\overset{*}{x_D}] - W_M) - r - h[\overset{*}{x_D}] + \overset{*}{x_D} h'[\overset{*}{x_D}] + \\ h[\overset{*}{x_D}] W_{WM}'[\overset{*}{x_D}](1 + h[\overset{*}{x_D}]/r) \end{bmatrix} \tag{III.9}$$

From this we subtract the best response function of a duopolist (equation (III.3)) evaluated at the equilibrium value of the R&D expenditure, $\overset{*}{x_D}$:

$$\overset{*}{R_D}[\overset{*}{x_D}] = \begin{bmatrix} h'[\overset{*}{x_D}](W_{WD}[\overset{*}{x_D}] - W_D) + (1/r)h'[\overset{*}{x_D}]h[\overset{*}{x_D}](W_{WD}[\overset{*}{x_D}] - W_{LD}[\overset{*}{x_D}]) \\ -r - 2h[\overset{*}{x_D}] + \overset{*}{x_D} h'[\overset{*}{x_D}] + h[\overset{*}{x_D}] W_{WD}'[\overset{*}{x_D}](2 + (1/r)h[\overset{*}{x_D}]) \end{bmatrix} = 0$$

(III.10)

Evaluating and collecting terms in the expression $R_M[\overset{*}{x_D}] - \overset{*}{R_D}[\overset{*}{x_D}]$, we get

$$\begin{aligned} R_M[\overset{*}{x_D}] - \overset{*}{R_D}[\overset{*}{x_D}] = {} & h[\overset{*}{x_D}]h'[\overset{*}{x_D}](1/r)(W_{LD}[\overset{*}{x_D}] - W_{WD}[\overset{*}{x_D}]) + \\ & h[\overset{*}{x_D}](W_M'[\overset{*}{x_D}] - W_{WD}'[\overset{*}{x_D}]) + \\ & h'[\overset{*}{x_D}]\Big((W_D - W_{WD}[\overset{*}{x_D}]) + (W_M[\overset{*}{x_D}] - W_M)\Big) + \\ & h[\overset{*}{x_D}](1 + (1/r)h[\overset{*}{x_D}](W_M'[\overset{*}{x_D}] - 2W_{WD}'[\overset{*}{x_D}]) \end{aligned} \tag{III.11}$$

We consider first the conditions under which the optimal research effort of the duopolist in a (pure) duopoly is greater than that of the monopolist (i.e. $\overset{*}{x_D} > \overset{*}{x_M}$), which is implied by $\overset{*}{R_M}[\overset{*}{x_D}] - \overset{*}{R_D}[\overset{*}{x_D}] < 0$. For this inequality to be satisfied it suffices that each of the four terms (lines) in equation (III.11) be less than zero (the sufficient conditions). Assumption 2 unambiguously implies that the first term is negative; the profits of the winner are greater than that of the loser in a duopoly. The second term will be negative ($h[\overset{*}{x_D}](W_M'[\overset{*}{x_D}] - W_{DW}'[\overset{*}{x_D}])$ <0) iff $W_M'[\overset{*}{x_D}] < W_{DW}'[\overset{*}{x_D}]$; the marginal changes in second period profits (at the duopolist's optimal level) are greater for the winning duopoly firm than it is for the monopolist. For the third term to be negative requires that the condition $|W_D - W_{DW}[x_D^*]|>|W_M - W_M[x_D^*]|$ be met[16]. For the final term in equation (III.11) to be negative requires that $\frac{r}{h[x_D^*]} < 2W_{DW}'[x_D^*] - W_M'[x_D^*]$.[17]

The implication and interpretation of these conditions are as follows. Prior to the innovation the firms in a duopoly make equal but lower profits than the sole firm in a monopoly. If the innovation in question results in significantly higher profits for the winning firm (relative to the losing) in a duopoly, then these conditions are sufficient to ensure that the optimal research effort by each firm in a duopoly will be greater than that of a monopolist. It follows than that if $x_D^* > x_M^*$, than the industry wide research effort in a duopoly will be greater than in a monopoly as well (i.e. $2x_D^* > x_M^*$).

Consider now the reverse case where a monopolist undertakes more research than both firms in a duopoly (i.e., $2x_D^* < x_M^*$)[18]. The conditions that would imply this result would require that $R_M[2x_D^*] - R_D^*[x_D^*]>0$, which is expressed as:

$$\begin{aligned} R_M[2x_D^*] - R_D^*[x_D^*] = {} & (2h[x_D^*] - h[2x_D^*] + 2x_D^* h'[2x_D^*] - x_D^* h'[x_D^*]) + \\ & (h'[x_D^*](W_{D1} - W_{WD}[x_D^*]) + h'[2x_D^*](W_M[2x_D^*] - W_{M1}) + \\ & (1/r)h[x_D^*]h'[x_D^*](W_{LD}[x_D^*] - W_{WD}[x_D^*]) + \\ & (h[2x_D^*]W_M'[2x_D^*] - h[x_D^*]W_{WD}'[x_D^*] + \\ & (1/r)(h[2x_D^*]^2 W_M'[2x_D^*] - 2h[x_D^*]^2 W_{WD}'[x_D^*]) \end{aligned} \tag{III.12}$$

---

[16] Consider the extreme case where the innovation is drastic allowing the winning firm in the duopoly to serve the whole market then $W_{DW}[x_D^*] > W_{DL}[x_D^*] = 0$. Having driven off the other firm, the duopoly winner will act as a monopolist, such that $W_{DW}[x_D^*] = W_M[x_D^*]$.

[17] This being a consistent, but stronger condition to $W_{WD}'[x_D^*] > W_M'[x_D^*]$

[18] Which implies that $x_D^* < x_M^*$

A sufficient condition for (III.12)>0 is that each term in (III.12) be non-negative, which we consider next. The first term is always positive due to the assumptions on the hazard rate.[19] For the second term to be greater than zero, we first note that it can be expressed as the following inequality:

$$\begin{gathered}(h'[x_D^*](W_{D1} - W_{DW}[x_D^*]) + h'[2x_D^*](W_M[2x_D^*] - W_{M1}) \\ > \\ h'[x_D^*]\Big((W_{D1} - W_{DW}[x_D^*]) + (1/2)(W_M[2x_D^*] - W_{M1})\Big)\end{gathered} \tag{III.13}$$

If the right hand side of the inequality in (III.13) is non-negative then it follows that left hand side is also non-negative. Thus, for

$h'[x_D^*]((W_{D1} - W_{DW}[x_D^*]) + (1/2)(W_M[2x_D^*] - W_{M1}))>0$ implies that

$\left|W_{D1} - W_{DW}[x_D^*]\right| < \left|(1/2)(W_M[2x_D^*] - W_{M1})\right|$. That is, the change in profits for the winning duopoly firm relative to pre-innovation profits is less than half the change for the monopolist.

The third term will be less than or equal to zero since from assumption 2, we know that $W_{LD}[x_D^*] \leq W_{WD}[x_D^*]$. Therefore, a sufficient condition for (III.12)>0 would necessarily require that $W_{LD}[x_D^*] - W_{WD}[x_D^*] = 0$, implying that the gains from winning and losing are the same in the duopoly. Such a situation would arise if there are perfect spillovers allowing the losing duopolist to appropriate all the returns of the winning firm's prize. A weaker condition would require that the winning duopolist prize in the second stage allow it only slightly higher profits than the losing firm's profits

[19] Note that $2h[x_D^*] > h[2x_D^*]$ and $2x_D^* h'[2x_D^*] > x_D^* h'[x_D^*]$

$W_{WL}[x_D^*] - W_{WD}[x_D^*] \approx 0$. One implication of this condition would be that the innovation is non-drastic and "small".

As with the second term, the fourth term in equation (III.12) can be expressed as an inequality such that

$$h[2x_D^*]W_M^{'}[2x_D^*] - h[x_D^*]W_{DW}^{'}[x_D^*] > h[x_D^*]\left((1/2)W_M^{'}[2x_D^*] - W_{DW}^{'}[x_D^*]\right) \geq 0.$$

Thus, for the fourth term to be non-negative requires that following condition be met:

$$\frac{\partial W_M[2x_D^*]}{\partial x_D^*} \geq 2\frac{\partial W_{DW}[x_D^*]}{\partial x_D^*}.$$

This condition implies the slope of the profit function for the monopolist is relatively constant (relative to the duopoly case). This sufficient condition is consistent with the others in that innovation for the monopolist is significant but insignificant for the duopoly winner.

The last sufficient condition for (III.12)>0 requires that the following inequality be satisfied: $(1/r)(h[2x_D^*]^2 W_M^{'}[2x_D^*] - 2h[x_D^*]^2 W_{WD}^{'}[2x_D^*])>0$. Assume that the change in profits for the monopolist due to first stage research is constant, such that $\partial W_i\,[\alpha x^*]/\partial x^* = \alpha c$ for all $\alpha > 0$. We have established that for the fourth term in (III.12) to be non-negative requires $\frac{\partial W_M[2x_D^*]}{\partial x_D^*} \geq 2\frac{\partial W_{DW}[x_D^*]}{\partial x_D^*}$. If we assume that

$\frac{\partial W_M[2x_D^*]}{\partial x_D^*} = 2\frac{\partial W_M[x_D^*]}{\partial x_D^*} = 2\frac{\partial W_{DW}[x_D^*]}{\partial x_D^*}$ than the last term in equation (III.12) can be reduced to $(1/r)(h[2x_D^*]^2 - h[x_D^*]^2)W_M^{'}[x_D^*]$. This term will be non-negative if and only if $h[2x_D^*]^2 > h[x_D^*]^2$, which is always true. Table AIII.1 provides a summary of the sufficient conditions for ranking the research effort of a monopolist and firms in a duopoly.

III.4.2. Monopoly vs. Mixed Duopoly

We next derive the sufficient conditions under which $x_P^* \gtrless x_M^*$ and $x_S^* \gtrless x_M^*$ (which taken together would imply that $x_P^* + x_S^* \gtrless x_M^*$). We first derive the sufficient conditions for the equilibrium research of the private firm (in the mixed duopoly) to be greater than that of the monopolist. This condition, $x_P^* > x_M^*$, is implied by $R_P[x_M^*, x_S^*] - R_M^*[x_M^*] > 0$ where $R_P[x_M^*, x_S^*]$ is the best response of the private firm in the mixed duopoly (III.4) evaluated at the equilibrium research level of the monopolist and the public sector firm, and $R_M^*[x_M^*]$ is the monopolist best response (equation (III.9)). Evaluating and collecting the terms in the expression $R_P[x_M^*, x_S^*] - R_M^*[x_M^*]$, we get

$$\begin{aligned} R_P[x_M^*, x_S^*] - R_M^*[x_M^*] = {} & h'[x_M^*](W_{M1} - W_{P1}) + \\ & h'[x_M^*](W_{WP}[x_M^*] - W_M[x_M^*]) + \\ & h[x_M^*] + (1/r)h[x_M^*]^2)(W'_{WP}[x_M^*] - W'_M[x_M^*] + \\ & (1/r)h'[x_M^*]h[x_S^*](W_{WP}[x_M^*] - W_{LP}[x_M^*]) + \\ & h[x_S^*]\left(h[x_M^*]W'_{WP}[x_M^*] - (1/r)\right) \end{aligned} \quad \text{(III.14)}$$

A sufficient condition for $R_P[x_M^*, x_S^*] - R_M^*[x_M^*] > 0$ is that each term in equation (III.14) be non-negative. From our assumptions on the prize value, we note that $W_{M1} = W_{P1}$ (pre-innovation profits for the both the monopolist and the private firm in the mixed duopoly are equal) and that $W_{WP}[x_M^*] = W_M[x_M^*]$ (post-innovation profits, when both monopolist and the private firm are successful in research, are equal as well). These assumptions imply that the first three terms in (III.14) all converge to zero. The fourth term is non-negative and follows from assumption 1 $(W_{WP}[x] \geq W_{LP}[x])$. Thus the sufficient condition for $R_P[x_M^*, x_S^*] - R_M^*[x_M^*] > 0$ is reduced to $rh[x_M^*]W_{WP}^{'}[x_M^*] > 1$.

For equilibrium research of the monopolist to be greater than the private firm in the mixed duopoly simply requires that we reverse the previous condition. That is, for $R_P[x_M^*, x_S^*] - R_M^*[x_M^*] < 0$ (which implies $x_P^* < x_M^*$) suffices that the sufficient condition $rh[x_M^*]W_{WP}^{'}[x_M^*] \leq 1$ be met[20]. Further, since the profits from winning for the private firm in the mixed duopoly will never be less than the profits from losing (i.e., $W_{WP}[x] \geq W_{LP}[x]$), another sufficient condition that would guarantee $x_P^* < x_M^*$ is $W_{WP}[x] = W_{LP}[x]$. Recall that the when the private firm losses the race to the public sector firm, it can either continue using the pre-innovation technology in the second stage or license the technology from the public sector firm (which does not produce in the second stage), and produce at level determined from the maximization of the public sector firm's objective (see appendix). The implication of the condition $W_{PW}[x] = W_{PL}[x]$ is that the public sector firm essentially allows the private firm to earn rents that the private firm would earn under the scenario where private firm actually had won the race.

[20] Note that the first three terms in (III.14) equal to zero

Next we derive the conditions for which the equilibrium research by the public sector is greater than the monopolist (i.e. $x_S^* > x_M^*$) which is now implied by $R_S[x_P^*, x_M^*] - R_M^*[x_M^*] > 0$. Evaluating and collecting the terms in this expression, we get

$$
\begin{aligned}
R_S[x_P^*, x_M^*] - R_M^*[x_M^*] = {} & h'[x_M^*]\Big((W_{M1} - W_M[x_M^*]) + (W_{WS}[x_M^*] - W_{S1})\Big) + \\
& h[x_M^*](W_{WS}'[x_M^*] - W_M'[x_M^*]) + \\
& (1/r)\Big(h[x_M^*]^2(W_{WS}'[x_M^*] - W_M'[x_M^*]) + h[x_M^*]h[x_P^*]W_{WS}'[x_M^*]\Big) + \\
& x_P^* h'[x_M^*] - h[x_P^*] + \\
& (1/r)h[x_P^*]h'[x_M^*](W_{WS}[x_M^*] - W_{LS}[x_P^*])
\end{aligned}
\tag{III.15}
$$

As before each line in equation (III.15) is a term, and it suffices that it be non-negative to satisfy the inequality $x_S^* > x_M^*$. The first term will be non-negative if and only if $\left|W_{M1} - W_M[x_M^*]\right| \le \left|W_{WS}[x_M^*] - W_{S1}\right|$, that is the absolute gains in profits for the monopolist are less than the welfare gains to the public sector firm due to the innovation. This sufficient condition is consistent with $W_{WS}'[x_M^*] \ge W_M'[x_M^*]$ which ensures that the second and third terms are non-negative.

The fourth term will be non-negative if and only if $x_P^* h'[x_M^*] \ge h[x_P^*]$. To guarantee that the fourth term is non-negative it suffices that curvature of the hazard rate be constant (i.e., a linear hazard rate $h[x] = x$). The last term in equation (III.15) is non-negative for $W_{WS}[x_M^*] \ge W_{LS}[x_P^*]$. While welfare will be greater when the public sector wins as opposed to when it losses (assumption 3), this assumption requires that research effort, *x*, across the two states be equal. Clearly, when $x_M^* \ge x_P^*$, then

$W_{WS}[x_M^*] \geq W_{LS}[x_P^*]$ will hold. However, if $x_M^* < x_P^*$, then the comparison between the welfare in the two states remains ambiguous therefore one has to assume that $W_{WS}[x_M^*] \geq W_{LS}[x_P^*]$ or that the public sector firm is always larger when the public sector firm wins as opposed to when it losses.

Lastly, we state the sufficient conditions for the case where the equilibrium research by the monopolist is greater than the public sector firm (i.e., $x_S^* < x_M^*$)[21]. Thus, relative to public's welfare, the gains to the monopolist are now larger such that $W'_{WS}[x_M^*] < W'_M[x_M^*]$ and $\left|W_{M1} - W_M[x_M^*]\right| > \left|W_{WS}[x_M^*] - W_{S1}\right|$ (which would now ensure that the first three terms in equation (III.15)). For the last two terms in (III.15) to be negative simply suffices that $x_P^* < x_M^*$, and the sufficient conditions that are implied by it. Table AIII.2 summarizes these results for the comparison between the monopolist and the firms in the duopolist. Note that while $x_P^* > x_M^*$ and $x_S^* > x_M^*$ implies $x_P^* + x_S^* > x_M^*$, we are unable to establish whether $x_P^* < x_M^*$ and $x_S^* < x_M^*$ implies $x_P^* + x_S^* < x_M^*$. The best we can do here is to say that sufficient conditions for $x_P^* < x_M^*$ and $x_S^* < x_M^*$ are the necessary conditions for $x_P^* + x_S^* < x_M^*$.

III.4.3 Mixed Duopoly vs. Pure Duopoly

Our final comparative analysis is between the firms in the mixed duopoly and firms in a pure duopoly. Here we seek to derive the sufficient conditions under which

[21] As one would expect this implies reversing the sufficient conditions for $x_S^* > x_M^*$.

$x_D^* \gtrless x_P^*$ and $x_D^* \gtrless x_S^*$. If the sufficient conditions for $x_D^* > x_P^*$ are not inconsistent for those that would satisfy $x_D^* > x_S^*$, then we can claim to have found the sufficient condition for $2x_D^* > x_S^* + x_P^*$. Similarly, if the sufficient conditions for $x_D^* < x_P^*$ and $x_D^* < x_S^*$ are not inconsistent, then those conditions would imply that the aggregate research effort in the mixed market is greater than that in the duopoly market (i.e., $2x_D^* < x_S^* + x_P^*$).

We first examine the sufficient conditions that would satisfy $x_P^* > x_D^*$ implied by $R_P[x_D^*, x_S^*] - R_D^*[x_D^*] > 0$, where

$$\begin{aligned}
R_P[x_D^*, x_S^*] - R_D^*[x_D^*] = h[x_D^*] - h[x_S^*] + \\
&(1/r)h'[x_D^*]h[x_D^*]\Big(W_{LD}[x_D^*] - W_{WD}[x_D^*]\Big) + \\
&(1/r)h'[x_D^*]h[x_S^*]\Big(W_{WP}[x_D^*] - W_{LP}[x_S^*]\Big) + \\
&h'[x_D^*]\Big((W_D - W_{WD}[x_D^*]) + (W_{WP}[x_D^*] - W_P)\Big) + \\
&h[x_D^*]\Big(W_{WP}'[x_D^*] - W_{WD}'[x_D^*]\Big) + \\
&(1/r)h[x_D^*]\Big(W_{WP}'[x_D^*](h[x_D^*] + h[x_S^*]) - W_{WD}'[x_D^*](2h[x_D^*])\Big)
\end{aligned} \quad \text{(III.16)}$$

As before the sufficient conditions for $R_P[x_D^*, x_S^*] - R_D^*[x_D^*] > 0$ require that each term in (III.16) be non-negative. For the first term in (III.16) to be non-negative would require that the equilibrium research effort by a firm in the duopoly be greater or equal than that of public sector firm ($x_D^* \geq x_S^*$). The second term will be non-negative if we assume that profits for firms in the duopoly are equal, win or lose (i.e. $W_{LD}[x_D^*] = W_{WD}[x_D^*]$). The third term is always non-negative and follows from assumption 2 and the earlier sufficient condition $x_D^* \geq x_S^*$ (hence, $W_{WP}[x_D^*] \geq W_{LP}[x_S^*]$).

For the fourth and fifth terms to be non-negative implies that the profits from winning in the mixed duopoly case increases faster than in the pure duopoly case, such that $W'_{WP}[x_D^*] \geq W'_{WD}[x_D^*]$ and $\left|W_D - W_{WD}[x_D^*]\right| \leq \left|W_{WP}[x_D^*] - W_P\right|$. The last term will be non-negative if and only if $W'_{WP}[x_D^*](h[x_D^*] + h[x_S^*]) \geq W'_{WP}[x_D^*](2h[x_D^*])$.

A key sufficient condition in establishing that $x_D^* < x_P^*$ has been the assumption that $x_D^* \geq x_S^*$. Are the sufficient conditions that would satisfy $x_D^* \geq x_S^*$ consistent with those for $x_D^* < x_P^*$? We examine this next. For $x_D^* \geq x_S^*$ to hold implies that $R_S[x_P^*, x_D^*] - R_D^*[x_D^*] \leq 0$ also needs to hold, where

$$
\begin{aligned}
R_S[x_P^*, x_D^*] - R_D^*[x_D^*] = {} & (1/r)h'[x_D^*]h[x_D^*]\left(W_{LD}[x_D^*] - W_{WD}[x_D^*]\right) + \\
& (1/r)h'[x_D^*]h[x_P^*]\left(W_{WS}[x_D^*] - W_{LS}[x_P^*]\right) + \\
& h'[x_D^*]\left((W_D - W_{WD}[x_D^*]) + (W_{WS}[x_D^*] - W_S)\right) + \\
& h[x_D^*](W'_{WS}[x_D^*] - W'_{WD}[x_D^*]) + \\
& (1/r)h[x_D^*]\left(1 + W'_{WS}[x_D^*](h[x_D^*] + h[x_P^*]) - W'_{WD}[x_D^*](2h[x_D^*])\right) \\
& x_P^* h'[x_D^*] - h[x_P^*]
\end{aligned}
$$

(III.17)

For $R_S[x_P^*, x_D^*] - R_D^*[x_D^*] \leq 0$, a sufficient condition is that each term in the equation be less than or equal to zero. The first term will be less than or equal to zero by assumption 2, $W_{LD}[x_D^*] \leq W_{WD}[x_D^*]$. However, to be consistent with the sufficient conditions that satisfy $x_D^* < x_P^*$ requires the stronger condition $W_{LD}[x_D^*] = W_{WD}[x_D^*]$, which is maintained here as well. The sign on the second term is ambiguous, as assumption 2 no longer holds due to asymmetric research effort. That is, since $x_D^* < x_P^*$, the term $[W_{WS}[x_D^*] - W_{LS}[x_P^*]]$ cannot be signed without further assumptions on the value of the welfare in the losing and winning states. Therefore, we require the stronger condition than the public's welfare in the winning state (evaluated at the research effort of the duopolist) be equal to or less than the losing state (evaluated at the research effort of the private firm in the mixed duopoly). The third and fourth term, taken together imply that relative to the pre-innovation profits, the gains from innovation are greater to the winning duopolist than they are for the public sector firm should it win. That is $W_{WS}'[x_D^*] \le W_{WD}'[x_D^*]$ and $\left|W_D - W_{WD}[x_D^*]\right| \ge \left|W_{WS}[x_D^*] - W_S\right|$. The fifth term will be less than or equal to zero if and only if $W_{WS}'[x_D^*](h[x_D^*] + h[x_P^*]) < W_{WD}'[x_D^*](2h[x_D^*]) + 1$. Note that this sufficient condition is consistent with our assumptions that $x_P^* > x_D^*$ and $W_{WS}'[x_D^*] > W_{WD}'[x_D^*]$ which implies $W_{WS}'[x_D^*](h[x_D^*] + h[x_P^*]) > W_{WD}'[x_D^*](2h[x_D^*])$. The last term will be less than or equal to zero if and only if $x_P^* h'[x_D^*] < h[x_P^*]$.

To derive the sufficient conditions that would guarantee the reverse relationship, i.e. $x_P^* < x_D^* < x_S^*$, simply implies that we reverse the above condition. For the sake of brevity we present those results and a summary of the sufficient conditions in Table AIII.3a. The conditions summarized in Table AIII.3a cannot, however, be used to make statements on the aggregate research relationship between the mixed and pure duopoly. To do so requires that we derive conditions for $x_D \gtrless x_S, x_P$, which implies $2x_D \gtrless x_S + x_P$. Table AIII.3b summarize these sufficient conditions and follows from equation (III.17). That is to derive the sufficient conditions that would simultaneously satisfy $x_D^* > x_S^*$ and $x_D^* > x_P^*$ requires that both (III.16) and (III.17) be non-negative. These sufficient conditions are summarized in the first column of Table III.3b. The second column lists those sufficient conditions that would reverse this relationship, such that they satisfy $x_D^* < x_S^*$ and $x_D^* < x_P^*$

III.5. Discussion

The sufficient conditions that have been derived, which allow us to rank the research effort in the three markets, are primarily a function of three properties. First, how the profits/welfare are distributed, post-innovation, in the pure and mixed duopoly. Second, how the profits/welfare are increasing for the winning firm in research effort (x). Lastly how hazard rate changes in x (or the curvature properties of the hazard rate). In all the comparison we note that our assumptions about these three properties allow us to derive the sufficient conditions and hence a particular ranking. What do the sufficient conditions mean and what are the implications for genomics research?

Consider first our comparison of mixed and pure duopoly where our interest was to explore the conditions under which a mixed duopoly would perform more research than a pure duopoly (or vice versa; refer to Table AIII.3b). For the research intensity of the mixed market to be greater than that of the pure duopoly it suffices that four conditions be satisfied. The first condition, that profits be distributed equally across both the winning and losing states in the pure duopoly, will be satisfied if one assumes that the losing firm easily imitates the innovation. Since the current market structure resembles mixed duopoly, this would suggest that one reason why the pure duopoly has not taken hold is because intellectual property rights are ill defined. Indeed, the U.S. Patent and Trademark Office has had difficulties in reliably defining the scope and scale of patents in genomics and biotech in general (Padron and Uranga, 2001). Such uncertainty may lead to greater likelihood of patent infringement and lowering of expected profits. Where property rights protection is weak the winning firm will have difficulty appropriating the returns of its innovation[22]. Another scenario where the first condition would also arise is if the innovation is non-drastic such that the winning firm does not have a significant advantage over the losing firm or the losing firm can invent around the innovation to effectively achieve technological parity.

The second and third conditions relate to the profits/welfare function in the winning state. The second condition is satisfied if we assume that the first-stage R&D enters into the firm's second stage profit function as a autonomous cost reduction. For example, if we assume that the cost of production, post innovation, for the winning firm is $C[\underline{\gamma}, q_{WD}, x_D^*] = (\underline{\gamma})q_{WD} - x_D^*$, and for the losing firm $C[\gamma, q_{LD}] = (\gamma)q_{LD}$ then it can be shown that $W_{Li}[x_j^*] < W_{Wi}[x_g^*]$ for $i = S, P, D$ and $\forall x_g, x_j > 0$ (see appendix). Again, given the presence of a mixed duopoly in genomics research, this implies that the knowledge gained from the sequencing acts as "lump-sum" transfer from the first stage to the second stage for the winning firm. This case would arise in genomics if the effect of the sequencing stage has negligible or no effect on further research stages (such as

[22] A simple example illustrates this point. Consider that post innovation, the winning duopolist is able to lower its cost to $C[\underline{\gamma}, q_{WD}, x_D^*]$. If the losing firm is able to imitate the technology than it too will have the same technology and cost function, that would result in equilibrium conditions where both firms produce the same amount and make equal profits (i.e. with all firms having the same cost function, the symmetric result would hold).

breeding for better varieties) for firms in the pure duopoly but a positive effect for firms in the mixed duopoly.

The third condition relates to the curvature properties of the profit (for the private firm) and welfare (for the public) functions. That is if the profit function of the winning private firm (in the mixed duopoly) and the welfare function of the public sector firm are "more" concave in x than the profit function of the winning duopolist, then this condition is satisfied. To the extent that private and public sector firms have taken a different research strategies in genomics, the research production of firms in both stages in a pure douopoly may indeed differ from that of the firms in the mixed. As stated in the introduction to this chapter, the public sector firm has essentially pursued a strategy where it expends more research resources in the initial stages so that it does not have to in the latter. The private firms have proceeded to invest less now in the hopes to be the first to sequence, even though it might entail more investments later to understand significance of the sequenced genome. This implies that the public sector firm would benefit more from the knowledge gained from the sequencing stage, and hence raising the marginal productivity of successive stages.

The last sufficient condition for the mixed duopoly to perform more R&D than that in the pure duopoly can be restated as such:

$$1+W_{WP}'[x_D^*](h[x_D^*]+h[x_S^*])-W_{WD}'[x_D^*](2h[x_D^*])>\frac{h[x_S^*]}{h[x_D^*]}>1 \quad \text{for } x_S^*>x_D^*>0.$$

A case where this condition is satisfied is when we assume that the hazard rate is bounded such that $h[x]\rightarrow 1$ as $x\rightarrow\infty$ (e.g. a logarithmic function) and that equilibrium R&D is "high" for both the firm in the private duopoly and the public sector firm (i.e.

$\frac{h[x_S^*]}{h[x_D^*]}$ approaches 1 from above). Since $W_{WP}'[x_D^*](h[x_D^*]+h[x_S^*]) > W_{WD}'[x_D^*](2h[x_D^*])$,

it follows that the left hand side will be greater than $\frac{h[x_S^*]}{h[x_D^*]}$, iff, $W_{WP}'[x_D^*] \gg W_{WD}'[x_D^*]$.

An interpretation of this sufficient condition is that the innovation is drastic for the winning private firm in the mixed duopoly and non-drastic for the winning duopolist (in the pure duopoly).

We next turn to the interpretation of the conditions under which a mixed duopoly would perform more R&D than a monopolist (refer to Table AIII.2). Three of the four conditions are also related to the properties of the profit/welfare function and hazard rate. If for example, the public sector firm's welfare function is more concave in R&D than the monopolist than the first condition is satisfied. The second sufficient condition is satisfied if we assume that the hazard rate is relatively constant, for example a linear hazard rate. The third condition requires that public welfare be strictly greater in the winning state than in the losing state, an example of which was given earlier.

The fourth sufficient condition ($rh[x_M^*]W_{WP}'[x_M^*] > 1$) follows from the earlier sufficient conditions and is met when those conditions suffice. Recall that when the winning firm in the mixed duopoly wins it becomes a monopolist in the second stage. Hence $W_{WP}[x_M^*] = W_M[x_M^*]$. Since for the first sufficient condition to be met, it suffices that the public sector firm's welfare function be more concave than the monopolist, it follows that it should be also more concave than the winning private firm's profit function in the mixed duopoly.

In our comparison of the mixed duopoly with that of pure duopoly we also derived a set of sufficient conditions, which, if met, ranked individual research effort at the firm level (Table AIII.3a). The rankings, however, do not allow for an unambiguous statement on whether at the industry level one market performs more R&D than the other. Focusing on the sufficient conditions which ranks the amount of research performed by the public sector firm above that of the private firms (either in the mixed duopoly or the pure duopoly), we note that all the sufficient conditions relate to the losing and winning profits of the private firms (which need to be equal) and the concavity properties of the public sector firm's welfare function in the winning state (the public sector firm's welfare function has to be more concave in R&D than the profit function of the private firms).

Our analysis also presents a comparison of the pure duopoly market with that of a monopoly market (Table AIII.1). Here, too, the sufficient conditions for ranking the industry R&D in the two markets is function of curvature properties of the profit function and the hazard rate as well as the distribution of profits in the duopoly. For example, when the profits after the race are distributed evenly among winners and losers and the profits for the monopolist are larger than it is for the winning duopolist, then it suffices that the sole firm in the monopoly market performs more research than the firms in the duopoly.

III.6. Conclusions

The analysis of this chapter was motivated by the observation that in genomics research, the amount of R&D expended to win an R&D race affects not only the probability of success but also downstream profits. Further, we observed that the public research sector is engaged in fierce competition with private firms in a variety of genomics projects. Since it was not clear, a priori, the reasons for the public sector firm to undertake genomics research (especially in light of the fact that activities are similar to those of the private firms), we set out in this chapter to derive a set of sufficient conditions under which R&D effort across different plausible and observed markets in genomics research could be ranked. It was found that the sufficient conditions relate to

- the concavity properties of the profit/welfare function with respect to first stage research,
- the distribution of profits across firms in the duopoly markets,
- the magnitude of the gains in the second (profit/welfare) from the innovation relative to other firms as well as pre-innovation profits/welfare,
- the curvature properties of the hazard rate.

One interpretation of the concavity of profit/welfare is how useful first stage R&D (or the knowledge gained in the racing stage) complements the innovation that is eventually employed in applied research or production. For the firm for which the complementarity effect is the greatest, the incentive to conduct more research will also be larger. Given the disparate research strategies of the public and private firms in genomics, it was suggested that the public sector firm by pursuing a different research strategy is able to exploit more effectively the complementarities between the sequencing stage and research stages that follow.

When comparing with the duopoly market, how the profits are distributed across the two firms affects has implication for the sufficient conditions. Distribution of profits across firms can be affected by the nature of the innovation (whether drastic or non-drastic) and/or the ease to which the winning innovation can be imitated by the losing firm. For example, if the innovation is minor such that the distribution of profits of the two firms remains relatively unchanged then the incentives to innovate are less. Similarly if the innovation is easily copied, due to weak property rights perhaps, then the incentives remain weak. In such cases, we find that the mixed market undertakes more R&D then the pure duopoly and a role for the public sector exists. Given the uncertain nature of intellectual property rights in genomics, it can be argued that expected profits, and hence incentives for research, for a private firm are low.

For firms in one market to perform more research relative to firms in another market (e.g., firms in mixed relative to firms in pure duopoly), it also suffices that the absolute profit/welfare gains (relative to the pre-innovation case) be greater than the other market. For example, in the comparison between the mixed duopoly with that of the monopoly market, a sufficient condition under which the public sector performs more research than the monopolist is if the gains to public sector firm's welfare are larger than they are to the monopolist's profits. It is not difficult to imagine cases where this would be true. Consider a case where, prior to innovation, a firm holds a monopoly in the breeding of a particular crop. The public sector firm, in competition with the monopolist, is able to sequence the genome of the crop before the monopolist (i.e., the public sector firm wins the race), and makes available to the monopolist the sequence for use in its breeding program. Assuming that the public sector firm sets the terms for the use of the genome such that the monopolist has to produce at the welfare maximizing level, it is easy to envision in this case that the gains to public sector firm will be greater relative to the gains to the monopolist.

In summary, this chapter has outlined the conditions under which a mixed duopoly, and by extension a public sector firm, would conduct more research than other plausible market structures. That a number of those conditions may resemble the attributes of the genomics research may help explain why a mixed market has taken hold as opposed other market structures. It is important to note that early research on genomics was based solely on the efforts of the public sector, suggesting that the conditions derived ranking the mixed market above others may have been stronger. As patent rights become more defined and the research focus shifts from sequencing to more applied uses of the genetic information, one can expect that the conditions that rank pure duopoly and monopoly over mixed duopoly would prevail, and hence the role of the public sector would diminish.

CHAPTER IV

COOPERATIVE AND NONCOOPERATIVE R&D IN A MIXED DUOPOLY WITH SPILLOVERS

IV.1. Introduction

In the 70s and 80s, there was well-known concern about the decline in the growth of productivity in the U.S. economy as well as the loss of international competitiveness amongst U.S. firms. It was felt that a lack of institutions to foster technology transfer and research sharing among the various agents, both private and public, was a key factor for the lack of vitality in the economy (Hall, Link and Scott, 2001). In response to these concerns, government intervention, in the form of legislation, was required to encourage the transfer of technology between public and private sector. As Day-Rubenstien and Fuglie (2001) state, "these laws affected ownership rights to new technology developed with federal funds and established new mechanisms for collaboration between public and private researchers." Research partnerships were an important element in the policy response to global competition, with the explicit recognition that industry had to rely heavily on U.S. universities and publicly funded institutions to ensure the success of the research being undertaken.

To encourage research joint ventures among different research enterprises, the 1984 National Cooperative Research Act (NCRA) encouraged joint research among private firms by providing for anti-trust exemption for private participants in such research consortia. Further, to specifically encourage collaborative research amongst private and public sectors, Cooperative Research and Development Agreements

(CRADAs) were implemented with the goal of developing and commercializing technology. CRADAs are very specific arrangements defining how research responsibilities are shared and results disseminated among the collaborating partners (Day-Rubenstien and Fuglie, 2000). CRADAs have been credited for developing the anti-cancer drug Taxol and commercializing useful agricultural technologies (Day-Rubenstien and Fuglie, 2000).

By strengthening the linkages between publicly financed research with that of a private firm, the primary policy aim was to increase the economic returns of public R&D by disseminating those innovations and R&D outputs that have prospects for commercialization. From the perspective of the private firm, the benefits of partnership with a public 'firm' (such as a university), stems from the perception of the private firm that the public institutions are engaged in "new" science. The public sector therefore provides its private collaborators with research insights that anticipate future research problems, especially with regard to the use of basic knowledge (Hall, Link and Scott, 2001).

While the benefits for both public and private firms may seem apparent, Hall, Link and Scott (2001) report that of all research joint ventures registered under the NCRA, only 15% involve universities. This observation leads them to ask whether university research participation in such projects was not warranted because of the characteristics of the research or whether certain institutional barriers prevented such collaboration from taking place. And even though the number of joint ventures between public and private institutions is not widespread (relative to partnerships amongst private firms) concern has been expressed that such collaborations divert the public research agenda from its central mission and may create an unfair competitive advantages for specific firms (Cohen and Noll, 1995).

In this essay we develop a theoretical model of mixed duopoly to study the effects of public-private partnerships. Specifically our objectives are to provide a better characterization of the incentive for a private firm to collaborate with the public sector, which could explain the low levels of partnerships between private firms and universities reported by Hall, Link and Scott (2001). Second we examine whether public sector objectives—to further social welfare through research—is compromised from partnering with the private firm[1]. Third by introducing a welfare maximizing public sector firm in the analysis of cooperative and non-cooperative R&D, we examine the robustness of earlier results that examined cooperative research between only private firms. Lastly we take issue with the standard assumption regarding the motivation for a public sector firm. Most partial-equilibrium models of mixed oligopoly have assumed that the public sector firm maximizes a social welfare function composed of the unweighted sum of consumer surplus and producer surplus.[2] Such an assumption is sufficient for a normative policy analysis, not to mention that it is also practical and simplifies the analysis considerably. However since it cannot be guaranteed that the public sector firm actually does or can maximize the unweighted social welfare function, which would suggest that the analysis merits an extentsion to allow for positivistic analysis. A weighted social welfare function allows for a political economy perspective of the solution as well as simplifying our analysis of cooperation of public and private firms.

---

[1] Social welfare (of the public sector firm's welfare) is defined as a the summation of producer profits and consumer surplus.
[2] Where producer surplus is simply the summation of individual profits.

Our approach and model are similar to the one found in the inspiring analysis of d'Aspremont and Jacquemin (1988) (d'A&J) who studied the behavior of a R&D performing firms in a (pure) duopoly. Their analysis consisted of comparing the consequences of cooperative and noncooperative R&D in terms of a two-stage model of a duopoly with R&D spillovers. They find that R&D agreements between otherwise competing firms increases the R&D expenditure level at sufficiently large spillovers, although the R&D is still lower then what would be achieved at the socially efficient level. The significance of this result is that it is contrary to what one would expect, i.e. a reduction of R&D due to less wasteful duplication.

In our model we introduce a welfare maximizing public sector firm that competes with a profit maximizing private firm. As in d'A&J we study the affects of cooperation and non-cooperation between the two firms in the presence of spillovers and ask whether the d'A&J results would also hold in the mixed duopoly case. We analyze two types of markets, one where the public sector firm is a participant in both R&D and production stages and another where it only engages in R&D. Our simulation results show that in all instances, the public sector firm's objective is advanced through partnerships although that is not always the case for the partnering private firm. That, for certain ranges of spillovers, private profits declines due to partnering with the public sector firm implies that private firms may not have strong incentives to collaborate with the public sector firm. When there is no participation by the public in the production stage we find that our results are consistent with the observation of Hall, Link and Scott who suggest that partnerships are more likely when private firm's research is difficult to appropriate (i.e. when spillovers are high).

The plan of this paper is as follows. We develop the general model in section II, introducing the structure of the two markets, the nature of the game and some general solution concepts. We specialize the model in section III employing the example of d'A&J for comparison. Simulation of the equilibrium values is provided in this section. Section IV develops the social welfare criterion for the purposes of comparing the result to an efficiency standard. Section V concludes the paper.

IV.2. The Model

Consider a duopoly consisting of a private firm (*P*) and a public sector firm (*S*) producing a homogenous product. They face a negatively sloped demand $P[Q]$, where Q is the aggregate output of this product ($Q = q_P + q_S$). The two firms engage in Cournot competition that proceeds in two stages. In the first stage the expenditure on cost-reducing R&D is determined, which lowers the cost of production in the second stage. In the second stage equilibrium levels of output are determined endogenously, taking as given the R&D completed in the first stage. As in previous multi-stage models of this nature, the solution is obtained recursively, where the equilibrium of the entire game is a subgame perfect equilibrium. Each firm has a cost of production $c_i[x_i, x_j]$, which is a function the amount of cost reducing research it undertakes (x_i), and the amount of R&D conducted by the other firm x_j which enters via 'spillovers' and where the nature of the spillover is embodied in $c_i[x_i, x_j]$. The cost of undertaking R&D is specified as $R_i[x_i]$, and it is assumed that there are diminishing returns to R&D. $(\partial R_i[x_i]/\partial x_i > 0$ and $\partial^2 R_i[x_i]/\partial(x_i)^2 > 0)$.

The spillovers signify that there are externalities to R&D such that some benefits of each firm's R&D effort flows, without payment, to the other firm. What proportion of those benefits are ultimately captured by the receiving firm is a function of how much knowledge (embodied in an innovation) its competitor allows to flow to the other firm as well as the usefulness and adaptability of that knowledge in the receiving firm's own production process[3]. If one assumes that the nature of the R&D discovery process and innovation is similar across firms (i.e. the two firms have the same learning paths), then the proportion of research that spills over is more a function of the former rather than the latter. By this assumption then, the firm from which the spillovers originate has control over how much knowledge is "shared" with its rivals via patents, trade secrets and other intellectual property protection mechanisms[4]. However complete control of one's research may not always be possible especially if the research develops characteristics of a public good or if IP protection is not a possibility. If the appropriability of research returns is difficulty under existing intellectual property institutions the public sector will usually dominate such research (as it occurs in basic research).

We describe two possible market regimes in the presence of a public sector firm. The first regime is one where the public sector firm competes in both the R&D and product markets. This market is akin to the one discussed in d'A&J, only that here we replace one of the profit maximizing profit firms with a welfare maximizing public sector firm. The second regime is more reflective of the true nature of the public-private

---

[3] As in Chapter II, we differentiate here between 'output' spillovers and spillovers that occur during the discovery process. Spillovers in this chapter refer to output spillovers as they reduce the cost of production in the second stage without affecting the cost or productivity of the innovation process itself. They are not, however, to be confused with appropriability, which relates to how much the returns of the innovation are captured by the innovation.

[4] That is, since its spillovers effect the production stage, the firm has control as to how much spillovers are allowed.

interaction in R&D, in that the public sector firm is not engaged in any second stage production, but the outputs of its first stage R&D effort spillover and reduces the production costs of competing private firms in the second stage. For the second regime we shall assume that there are two profit maximizing private firms that compete in the second stage. The two private firms, join the public sector firm in first stage, and engage in R&D competition. Whereas the incentive to conduct R&D for the private firms in this market is due to their profit maximizing behavior, the public sector firm's incentive are due to its desire to increase welfare which occurs when the benefits of its research spillover to other firms.

Following d'A&J and Suzumura (1992), for both these markers we examine two equilibrium concepts. The first is one where firms act independently and noncooperatively throughout the game, so that the equilibrium of the second stage game is Cournot-Nash equilibrium, and that of the entire game is a subgame perfect equilibrium. The second equilibrium concept originates from collaborative R&D in the first stage between the private and public sector firms; the public and private firms coordinate their R&D in the first stage with the understanding that no further cooperation will occur in the product market stage. We first provide some general equilibrium concepts to the model. Later we specialize the model to correspond to the market assumptions of d'A&J and compare the comparative statics and equilibrium concepts of the different markets/games. Table IV.1 provides an overview of the two market regimes that we study in this chapter and the sections in which they appear.

Table IV.1: Overview of the Cooperative and Noncooperative Game

	Regime 1	Regime 2
Non-Cooperative Behavior	• 2 firms (1 private, 1 public) • 2 stages (R&D and output mkt.) • Both firms compete non-cooperatively in the two stages GM:§IV.2.1.1 SM:§IV.3.1.1	• 3 firms (2 private, 1 public) • 2 stages (R&D and output mkt.) • Public sector firm not a participant in the output mkt (second stage) • All firms compete non-cooperatively in the two stages GM:§IV.2.2.1 SM:§IV.3.3.1
Cooperative Behavior	• 2 firms (1 private, 1 public) • 2 stages (R&D and output mkt.) • Firms cooperate in the R&D stage but compete non-cooperatively in the output stage GM:§IV.2.1.2 SM:§IV.3.1.1	• 3 firms (2 private, 1 public) • 2 stages (R&D and output mkt.) • Public sector firm not a participant in the output mkt. (second stage) • Public sector firm cooperates with one of the private firms in the R&D stage GM:§IV.2.2.2 SM:§IV.3.3.2

GM refers to the generalized model and SM refers to the specialized model.

IV.2.1. Regime 1: Mixed Duopoly in Both Stages

In this market, private and public sector firms compete in both the R&D and the production markets (or stages). Whereas the public institutions are known to actively participate in the R&D stage, the notion that the public sector firm also competes at the production level may seem less compelling. Our justification for explicitly involving the public sector firm in the production stage is the observation that in many instances the public sector firm contracts out its innovation (through a license for example) to a private firm on terms that would be welfare maximizing and not necessarily profit maximizing. The private firm's incentive to participate in such a contractual arrangement would stem from it gaining access to a superior technology or any transfers from the public sector firm to fill the shortfall in profits from producing at the welfare maximizing level. A second justification for having the public sector firm compete in both stages is to think of the second stage as another R&D stage, where firms compete to produce a final innovation that is homogenous across firms and sold non-exclusively to the consumers of innovation in the second stage. For example in genomics research, the sequencing and identification phase provides better tools for breeding and transformation of plants. The final innovation, a transgenic plant, is then marketed for mass dissemination.

IV.2.1.1. Non-Cooperative Game

To characterize the second stage equilibrium, we begin with the classical model of Cournot equilibrium. The objective of the private firm is to choose its output q_P to maximize its second-stage profits:

$$\max_{q_p} \pi_p = \left(P[Q] - c_P[x_p, x_S]\right) q_p - R_P[x_p] \tag{IV.1}$$

The public sector firm maximizes social welfare where social welfare is represented by the summation of consumer and producer surplus. The producer rent is simply the sum of the profits of each firm in the output market $(\pi_P + \pi_S)$. The consumer surplus reflects the utility of the individuals and is calculated by taking the integral over the demand function less the consumer payments. That is if $P[Q]$ is the inverse demand function and Q^* is the equilibrium output, the consumer rent is given by

$V[Q] = \int_0^{Q^*} P[Q^*]\, dQ - P^* Q^*$. The public sector firm's maximization is thus

$$\max_{q_S} W = w_C V[Q] + w_S\left[\left(P[Q]-c_S[x_S,x_P]\right)q_S - R_S[x_S]\right] + w_P\left[\left(P[Q]-c_P[x_P,x_S]\right)q_P - R_P[x_P]\right] \tag{IV.2}$$

where w_C, w_S and w_P are the weights assigned by the public sector firm to consumer surplus, public sector firm profits and private firm profits, respectively. Assuming an interior optimum for each firm and that the second order condition is satisfied, this implies that the Nash-Cournot equilibrium must satisfy the following first order conditions:

$$\frac{\partial \pi_P[q_P,q_S]}{\partial q_P} = P[Q^*] + P'[Q^*]q_p^* - c_P[x_p,x_S] = 0 \tag{IV.3}$$

$$\frac{\partial W[q_P,q_S]}{\partial q_S} = w_C V'[Q^*] + P'[Q^*](w_P q_P^* + w_S q_S^*) + w_S P[Q^*] - w_S c_S[x_S,x_P] = 0 \tag{IV.4}$$

It is convenient to rearrange the first order conditions so that they have an elasticity interpretation. Letting $s_i^* = q_i^* / Q^*$ denote each firm's share of aggregate output, we can write the first order conditions for the private and public sector firm's as

$$P[Q^*]\left(1+\frac{s_P^*}{\varepsilon}\right) = c_P[x_p,x_S] \tag{IV.5}$$

$$P[Q^*]\left(w_S + \frac{w_P s_P^* + w_S s_S^*}{\varepsilon}\right) - w_S c_S[x_S,x_P] = -w_C V'[Q^*] \tag{IV.6}$$

where ε is the elasticity of market demand. A common interpretation of the traditional Cournot model is that it is in some sense the "in between" case of monopoly and that of pure competition (Varian (1996), pp. 288). That interpretation however does not hold in the present context. Consider the "competitive" case such that s_i^* approaches zero so that each firm has insignificant share of the market. Assuming equal weights

($w_C = w_S = w_P = 1$), equal marginal costs across firms, and constant market elasticity, the public sector firm will choose an output such that the marginal (increase) in consumer surplus equals the marginal (decrease in) total domestic profits. de Fraja and Delbono (1989) show that when the market is competitive, all firms, public and private, produce an output such that their own marginal cost equals the price.

On the other hand if the product market is characterized as a mixed *duopoly* where only a public sector firm competes with a private firm, then the private firm will produce where the price is a constant markup on marginal cost in which case $P[Q^*] \geq c_P[x_P, x_S]$. The production decision of the public sector firm is also a function of the markup on marginal cost, where the markup by the public sector firm is always greater than that of the private firm (i.e. $\left(1+\frac{s_P^*}{\varepsilon}\right) < \left(1+\frac{1}{\varepsilon}\right)$). In the absence of any consumer concerns ($w_C = 0$) and assuming that the marginal cost of production is the same across the firms, then the private firm will underbid the price of the public sector firm, which can lead to corner solutions. If, however, the marginal change in consumer surplus due to public sector firm production is large such that $P[Q^*]\left(1+\frac{s_P^*}{\varepsilon}\right) > P[Q^*]\left(1+\frac{1}{\varepsilon}\right) - V'[Q^*]$, then the public sector firm will underbid the price of the private firm, which too can result in corner solutions.

For the maximization problem of the private and public sector firms, we also have the second order conditions, which respectively take the form:

$$\frac{\partial^2 \pi_P}{\partial (q_P)^2} = 2P'[Q^*] + P''[Q^*]q_p^* \leq 0 \tag{IV.7}$$

$$\frac{\partial^2 W}{\partial (q_S)^2} = w_C V''[Q^*] + 2w_S P'[Q^*] + P''[Q^*](w_P q_P^* + w_S q_S^*) \leq 0 \qquad \text{(IV.8)}$$

If we assume that the inverse demand and consumer surplus are twice differentiable functions such that $P'[Q] < 0$, $P''[Q] \leq 0$, $V'[Q] > 0$, $V''[Q] \geq 0$, then the SOC for the private firm is always satisfied. However the SOC of the public sector firm is satisfied only if appropriate restrictions are placed on the weights (i.e. cannot have weights on consumer surplus, w_C, too large relative to the profits of the two firms).

The existence and uniqueness of the Cournot equilibrium are well known and have been studied by De Fraja and Delbono (1989, 1987), the issue of whether the Cournot equilibrium is stable has not be adequately addressed[5]. It turns out that under the assumption of equal weights for producer profits and consumer surplus in the public sector firm's objective, the model is indeed unstable. A sufficient condition for the equilibrium to be stable is if one assumes that the weight of the public sector firm is larger than that of the private firm in the public sector firm's welfare (objective) function.

Lemma IV.1: A sufficient condition for the Cournot equlibrium in the output market of a mixed duopoly to be stable is $w_S > w_P$.

Proof: For the stability condition in the Cournot equilibrium to be met requires:

[5] Stability in the Cournot equilibrium, or the Cournot adjustment process, is generally regarded as unsound because the underlying assumption that rivals do not change their output during the course of the game is often invalid. al-Nowaihi and Levine (1985) however point out that this only true if firms have full knowledge of their own and rivals cost functions along with the demand curve. Under the more plausible assumption of partial information, the Cournot adjustment process is an attempt at a solution and a "reasonable approximation to a richer kind of consistent-expectiations formation." (al-Nowaihi and Levine, 1985).

$$\left|\frac{\dfrac{\partial^2 \pi_P}{\partial q_P \partial q_S}}{\dfrac{\partial^2 \pi_P}{(\partial q_P)^2}}\right| = \left|\frac{\dfrac{\partial P[Q]}{\partial q_S} + q_P \dfrac{\partial^2 P[Q]}{\partial q_P \partial q_S}}{2\dfrac{\partial P[Q]}{\partial q_P} + q_P \dfrac{\partial^2 P[Q]}{(\partial q_P)^2}}\right| < 1 \tag{IV.9}$$

$$\left|\frac{\dfrac{\partial^2 W}{\partial q_S \partial q_P}}{\dfrac{\partial^2 W}{(\partial q_S)^2}}\right| = \left|\frac{(w_S + w_P)\dfrac{\partial P[Q]}{\partial q_P} + (w_P q_P + w_S q_S)\dfrac{\partial^2 P[Q]}{(\partial q_S \partial q_P)} + w_C \dfrac{\partial V[Q]}{(\partial q_P)^2}}{2 w_S \dfrac{\partial P[Q]}{\partial q_S} + (w_P q_P + w_S q_S)\dfrac{\partial^2 P[Q]}{(\partial q_S)^2} + w_C \dfrac{\partial V[Q]}{(\partial q_S)^2}}\right| < 1 \tag{IV.10}$$

If we assume symmetry, we have $\dfrac{\partial P[Q]}{\partial q_S} = \dfrac{\partial P[Q]}{\partial q_P}$,

$\dfrac{\partial^2 P[Q]}{(\partial q_S \partial q_P)} = \dfrac{\partial^2 P[Q]}{(\partial q_P \partial q_S)} = \dfrac{\partial^2 P[Q]}{(\partial q_S)^2} = \dfrac{\partial^2 P[Q]}{(\partial q_P)^2}$ and $\dfrac{\partial V[Q]}{(\partial q_S)^2} = \dfrac{\partial V[Q]}{(\partial q_P)^2}$. It follows than that the denominator in equations (IV.9) and (IV.10) has to be larger than their respective numerators for the stability condition to suffice. While the private firm's reaction function is always stable, the public sector firm's reaction function is stable only for $w_S > w_P$ but stability cannot be guaranteed for $w_S \leq w_P$.

Consider next the first stage of this game. The private firm, having chosen its output that maximizes profits in the second stage (conditional on research level x_i), now will choose the research level that maximizes profits given its equilibrium output function of x_i. That is in the second stage it maximizes

$$\max_{x_P} \pi_P^* = \left(P[Q^*[x_P, x_S]] - c[x_P, x_S]\right) q_P^*[x_P, x_S] - R_P[x_P] \tag{IV.11}$$

Similarly the public sector firm maximizes its welfare in the first stage given its output choice in the second stage:

$$\max_{x_S} W^* = w_C V[Q^*[x_S, x_P]] + \\ w_S\left(\left(P[Q^*[x_S, x_P]] - c_S[x_S, x_P]\right) q_S^*[x_S, x_P] - R_S[x_S]\right) + \\ w_P\left(\left(P[Q^*[x_P, x_S]] - c_P[x_P, x_S]\right) q_P^*[x_P, x_S] - R_P[x_P]\right) \tag{IV.12}$$

Applying the properties of the envelope theorem, the first order condition for the maximization of the private and public objectives in the first stage simplify to:

$$\left(-\frac{\partial c[x_P^*, x_S^*]}{\partial x_P}\right) q_P^* + \left(P[Q^*] - c[x_P^*, x_S^*]\right) \frac{\partial q_P^*[x_P^*, x_S^*]}{\partial x_P} = R_P'[x_P^*] \tag{IV.13}$$

$$-w_S\left(\frac{\partial c_S[x_S^*, x_P^*]}{\partial x_S} q_S^* + \left(P[Q^*] - c_S[x_S^*, x_P^*]\right) \frac{\partial q_S^*}{\partial x_S}\right) - \\ w_P\left(\frac{\partial c_P[x_P^*, x_S^*]}{\partial x_S} q_P^* + \left(P[Q^*] - c_P[x_P^*, x_S^*]\right) \frac{\partial q_P^*}{\partial x_S}\right) = w_S R_S'[x_S] \tag{IV.14}$$

We also have the second order conditions for each firm which take the form:

$$\left(-\frac{\partial c_P}{\partial x_P}\right)\left(\frac{\partial q_P^*}{\partial x_P} - 1\right) - q_P^* \frac{\partial^2 c_P}{\partial (x_P)^2} + P[Q^*] \frac{\partial^2 q_P^*}{\partial (x_P)^2} - R_P''[x_P^*] \le 0 \tag{IV.15}$$

$$w_S\left(\left(-\frac{\partial c_S}{\partial x_S}\right)\left(\frac{\partial q_S^*}{\partial x_S} - 1\right) - q_S^* \frac{\partial^2 c_S}{\partial (x_S)^2} + P[Q^*] \frac{\partial^2 q_S^*}{\partial (x_S)^2}\right) - \\ w_P\left(\left(-\frac{\partial c_P}{\partial x_S}\right)\left(\frac{\partial q_P^*}{\partial x_S} - 1\right) - q_P^* \frac{\partial^2 c_P}{\partial (x_S)^2} + P[Q^*] \frac{\partial^2 q_P^*}{\partial (x_S)^2}\right) - R_S''[x_S^*] \le 0 \tag{IV.16}$$

The first order conditions (equations (IV.13) and (IV.14)) simply state that the two firms conduct research up to that point where the marginal cost of doing research ($R_i'[x_i]$) is equal to the marginal benefits that accrue to the each firm. In the case of private firm these benefits are its cost saving due to research and its increased second stage profits brought about from a change in the quantity supplied as a result of the

R&D.[6] The marginal benefits to the public sector firm, however is the aggregation of marginal benefits due to its research in the second stage on both itself (i.e. the public sector firm) and the private firm. This is a natural outcome of our assumption that the public sector firm is a welfare maximizer that takes into account the quantity and research output of all firms in maximizing its objectives.

Stability is also required in the second stage (Henriques, 1990) and takes the form

$$\left| \frac{\dfrac{\partial^2 \pi_P^*}{\partial x_P \partial x_S}}{\dfrac{\partial^2 \pi_P^*}{(\partial x_P)^2}} \right| = \left| \frac{N_P}{SOC_P} \right| < 1 \tag{IV.17}$$

$$\left| \frac{\dfrac{\partial^2 W^*}{\partial x_S \partial x_P}}{\dfrac{\partial^2 W^*}{(\partial x_S)^2}} \right| = \left| \frac{w_S N_S + w_P N_P}{SOC_S} \right| < 1 \tag{IV.18}$$

where $N_i = \left(-\dfrac{\partial c_i}{\partial x_j} \right) \left(\dfrac{\partial q_i^*}{\partial x_j} - 1 \right) - q_i^* \dfrac{\partial^2 c_i}{\partial x_i \partial x_j} + P[Q^*] \dfrac{\partial^2 q_i^*}{\partial x_i \partial x_j}$ for $i = P, S \;\; i \neq j$ and SOC_i is the second order conditions stated in equations (IV.15) and (IV.16). Without knowledge of the nature of the Nash-equilibrium in the second (output) stage, we cannot claim that the stability condition is satisfied. However we note that if the Nash-equilibrium is symmetric such that $q_P^*[x_P, x_S] = q_S^*[x_P, x_S]$, then the reaction functions of the private and public sector firm are stable in the R&D space as $R_i^{*}[x_i] > 0$ for $i = P, S$.

---

[6] Notice that if the second stage were a competitive market such that $P[Q^*] = c_S[x_S^*, x_P^*]$, the marginal cost of research would simply equal the cost saving due to research.

IV.2.1.2. Cooperative Game

Next we consider the case where the two firms coordinate their research activities in the first stage but remain competitors in the second stage[7]. When all firms have the same profit-maximizing objective, the firms simply choose that level of research that maximizes their joint profits. In the context of mixed duopoly, however, the notion of joint *profits* is inappropriate as the two firms have divergent objectives; while the private firm still seeks to maximize profits, the public sector firm maximizes welfare. However, if one assumes that firms maximize their joint *objectives*, where joint objectives are simply the summation of their individual objectives (i.e. profit for the private and welfare for the public sector firm), then we can define joint objectives of the two firms, as a function of x_P and x_S, in the first stage as:

$$\begin{aligned} J^* &= W^*[x_P, x_S] + w_J \pi_P^*[x_P, x_S] \\ &= w_C V[Q^*[x_P, x_S]] + w_S \pi_S^*[x_P, x_S] + (w_P + w_J)\pi_P^*[x_P, x_S] \\ &= w_C V[Q^*[x_P, x_S]] + w_S \pi_S^*[x_P, x_S] + w_{JP} \pi_P^*[x_P, x_S] \end{aligned} \qquad \text{(IV.19)}$$

where w_J is the weight of private firm's profits in joining the cooperative agreement. We see now that having parameterizing weights allows us to analyze different forms of cooperative arrangements. One could consider, for example, the case where the private firms profits are weighted more than that of the consumer surplus or the profits of the public sector firm (the weight will certainly be greater than if the private firm did not collaborate). At the other extreme would be the case where the weights are skewed towards the profits of the two firms, in which case the joint objectives would be in line

---

[7] Since the second (output) stage of the market game is non-cooperative and therefore unchanged, we restrict our analysis to the first stage which is now characterized by cooperation.

with the concept of joint profits considered by d'A&J and Suzumura (for example, $w_C = 0$ and $w_{JP} \geq w_S$ in equation (IV.19)).

The private firm chooses x_P and the public sector firm chooses x_S in maximizing J^*. Assuming an interior optimum for each firm, this means that a Nash-Cournot equilibrium must satisfy the two first order conditions in the R&D space:

$$\begin{bmatrix} -w_S\left(\frac{\partial c_S[x_S^*, x_P^*]}{\partial x_S} q_S^* + \left(P[Q^*] - c_S[x_S^*, x_P^*]\right)\frac{\partial q_S^*}{\partial x_S}\right) - \\ w_{JP}\left(\frac{\partial c_P[x_P^*, x_S^*]}{\partial x_S} q_P^* + \left(P[Q^*] - c_P[x_P^*, x_S^*]\right)\frac{\partial q_P^*}{\partial x_S}\right) \end{bmatrix} = w_S R_S'[x_S] \qquad \text{(IV.20)}$$

$$\begin{bmatrix} -w_S\left(\frac{\partial c_S[x_S^*, x_P^*]}{\partial x_P} q_S^* + \left(P[Q^*] - c_S[x_S^*, x_P^*]\right)\frac{\partial q_S^*}{\partial x_P}\right) - \\ w_{JP}\left(\frac{\partial c_P[x_P^*, x_S^*]}{\partial x_P} q_P^* + \left(P[Q^*] - c_P[x_P^*, x_S^*]\right)\frac{\partial q_P^*}{\partial x_P}\right) \end{bmatrix} = w_{JP} R_P'[x_P] \qquad \text{(IV.21)}$$

The F.O.C. are similar to public sector firm's F.O.C. in the non-cooperative game and imply that research occurs at that level where marginal costs of research are equal to its marginal benefits. The key difference resulting from the joint maximization of first stage objectives is that both firms take into account the benefits of its research on that of the other firm. Whereas the public sector firm was previously internalizing those benefits in the non-cooperative case, the fact that R&D in the first stage is now combined, the private is forced to internalize the benefits of its research on the public sector firm as well.

IV.2.2. Regime 2: Mixed Duopoly in R&D Stage Only

In this section we consider a two-stage model of mixed oligopolistic competition but in contrast to the previous section, we assume here that the public sector firm does not engage in any production (i.e. $q_S = 0$). Instead we allow for quantity competition to occur between two private firms in the product market (second) stage. In the first stage, the two private firms, along with the public sector firm, decide on their cost-reducing R&D either cooperatively or noncooperatively. The incentive for the public sector firm to perform cost reducing R&D in the absence of any production, results from the spillovers of its research to the private firms. To the extent that private firm's benefit from public sector firm's research spillover, public welfare is increased and hence the public sector firm's welfare is advanced.

IV.2.2.1. Non-Cooperative Game

In the non-cooperative case, no firm cooperates with any other in either of the stages. Since only private firms are engaged in quantity competition in the second stage, the standard Cournot result follows. That is, in the second stage, the two private firms (denoted $P1, P2$) maximize their profits:

$$\max_{q_i} \ \pi_i = \left(P[Q] - c_i[x_i, x_j, x_S]\right)q_i - R_i[x_i] \tag{IV.22}$$

$$\text{for } i, j = P1, P2 \text{ and } i \neq j$$

where, as before, cost is a function of own research x_i, the research of the other private firm x_j, and the research of the public sector firm x_S. Assuming an interior optimum for each firm, the Nash-Cournot equilibrium must satisfy the two first order conditions:

$$\frac{\partial \pi_i[q_i, q_j]}{\partial q_i} = P[Q^*] + P'[Q^*]q_i^* - c_i[x_i, x_j, x_S] = 0 \tag{IV.23}$$

Writing in terms of market elasticity of demand,

$$P[Q^*]\left(1 + \frac{s_i^*}{\varepsilon}\right) = c_i[x_i, x_j, x_S] \tag{IV.24}$$

In the previous stage, the two private (and symmetric) firms join the public sector firm in choosing R&D levels. The payoff to the two private firms and public sector firm at this stage can be written as

$$\pi_i^* = \left(P[Q^*[x_i, x_j, x_S]] - c_i[x_i, x_j, x_S]\right) q_i^*[x_i, x_j, x_S] - R_i[x_i] \tag{IV.25}$$

$$W^* = w_C V[Q^*[x_i, x_j, x_S]] + \sum_i^{P1,P2} w_i \pi_i^* - R_S[x_S] \tag{IV.26}$$

The first order conditions are

$$\left(-\frac{\partial c_i[x_i^*, x_j^*, x_S^*]}{\partial x_i}\right) q_i^* + \left(P[Q^*] - c[x_i^*, x_j^*, x_S^*]\right) \frac{\partial q_i^*[x_i^*, x_j^*, x_S^*]}{\partial x_i} = R_i'[x_i^*] \tag{IV.27}$$

$$\sum_i^{P1,P2} -w_i \left(\frac{\partial c_i[x_i^*, x_j^*, x_S^*]}{\partial x_S} q_i^* + \left(P[Q^*] - c_i[x_i^*, x_j^*, x_S^*]\right) \frac{\partial q_i^*}{\partial x_S} \right) = R_S'[x_S] \tag{IV.28}$$

As before, the first condition (IV.27) indicates that for each private firm the marginal benefit from a unit of research should equal the marginal research cost, where marginal benefit consists of the marginal cost saving due to research and the (commensurate) increase in second stage profits. Equation (IV.28) indicates that the public sector firm chooses a research level such than the weighted marginal (decrease in) total benefits from research equals the marginal (increase in) the cost of doing research to itself. If one compares these condition with the corresponding one under the non-cooperative case

(IV.13) and (IV.14), one observes that the quantity supplied by each firm is symmetric when no production by the public sector firm takes place and that how much the research the public sector firm undertakes is a function of how much public research is "used" by the private firms. In the extreme case if R&D by the public sector firm has no effect on the unit cost of the private firms such that $\partial c_i / \partial x_S = \partial q_i / \partial x_S = 0$ then $x_S^* = 0$.

IV.2.2.2. Cooperative Game

In Regime 2, cooperation implies that the public sector firm jointly undertakes R&D with one of the private firms. Because of symmetry it does not matter as to which private firm the public sector firm collaborates with, and henceforth we simply assume that the collaboration is with $P1$. The joint objective maximization is therefore much like (IV.19)with the profits of the collaborating private firm receiving a higher weight than the case were it not to collaborate (for $w_J > w_{P1}$). The joint objective maximization problem of the public and firm $P1$ in the first stage is

$$\begin{aligned} \max_{x_S, x_{P1}} W^* &= w_C V[Q^*[x_i, x_j, x_S]] + (w_{P1} + w_J)\pi^*_{P1} + w_{P2}\pi^*_{P2} - R_S[x_S] \\ &= w_C V[Q^*[x_i, x_j, x_S]] + w_{JP1}\pi^*_{P1} + w_{P2}\pi^*_{P2} - R_S[x_S] \end{aligned} \qquad \text{(IV.29)}$$

The payoff to the non-collaborating private firm ($P2$) remains unchanged from its payoff in the non-cooperative case (i.e. equation (IV.25)) and is retained for $i = P2$. The first order conditions from the maximization of (IV.29) and (IV.25) (for $i = P2$) are:

$$(-w_{JP1} - w_{P2})\sum_i \left(\frac{\partial c_i}{\partial x_{P1}} q_i^* + \left(P[Q^*] - c_i \right) \frac{\partial q_i^*}{\partial x_{P1}} \right) = w_{JP1} R'_{P1}[x^*_{P1}] \qquad \text{(IV.30)}$$

$$(-w_{JP1} - w_{P2})\sum_i \left(\frac{\partial c_i}{\partial x_S} q_i^* + \left(P[Q^*] - c_i\right) \frac{\partial q_i^*}{\partial x_S} \right) = R_S^{'}[x_S^*] \qquad \text{(IV.31)}$$

$$\left(-\frac{\partial c_{P2}}{\partial x_{P2}} \right) q_{P2}^* + \left(P[Q^*] - c_{P2}\right) \frac{\partial q_{P2}^*}{\partial x_2} = R_{P2}^{'}[x_{P2}^*] \qquad \text{(IV.32)}$$

Equations (IV.30) to (IV.32) determine, respectively, the choice of research level for the cooperating private firm, public sector firm, and the non-cooperating private firm. The private firm that collaborates with the public sector firm is now takes into account the benefits of its research on all firm and chooses R&D where the total (weighted) marginal benefits equal the (weighted) marginal costs. The private that does not collaborate does not have to take into account the externalities of its research on others and chooses R&D where marginal benefits from doing R&D equal its marginal cost of that R&D. While the marginal benefits of doing R&D for the cooperating firm are higher, as it takes into account the marginal benefits of its rival), the cooperating firm may also incur a higher marginal cost of R&D (due to the weight). Therefore, it remains ambiguous whether the cooperating private firm performs more R&D than the other (or vice versa), as the equilibrium R&D level is indeterminate. Table IV.2 provides a summary of the key results of this section.

Table IV.2: Summary of Results for §IV.3.2.

	Regime 1	Regime 2
Output Mkt. Non-cooperative solution	• $w_S > w_P$ • Pvt. firm produces where $MR = MC$ • Public sector firm produces where marginal consumer surplus equals total marginal profits	• No production by public sector firm • Pvt. firms produce where $MR = MC$
R&D Mkt. Non-cooperative solution (NC)	• Pvt. firm does R&D where marginal benefits (MB) equal MC • Public sector firm does R&D where weighted total MB equals MC	Same as Regime 1 but with two private firms
Cooperative Solution (C)	• Public and private firms do R&D where weighted total MB equals MC	• Public and C-private firms do R&D where weighted total MB equals MC • NC-Pvt. firm does R&D where $MB = MC$

IV.3. Specializing the Model

In this section we shall specialize the games of section 2 to provide us with a sharper charecterization of the equilibrium conditions. In so doing, it will develop a systematic methodology for analyzing the properties of a two-stage model of mixed duopoly.

IV.3.1. Regime 1: Mixed Duopoly in Both Stages

The public and private firms face and inverse demand $P[Q] = a - bQ$ where $Q = q_P + q_S$ is the total quantity produced and $a, b > 0$. Let the cost of production of each firm be $c_i[q_i, x_i, x_j] = (A - x_i - \beta_j x_j)q_i$ for $i = P, S \quad i \neq j$. We assume that $0 < A < a$ and $x_i + \beta_j x_j \leq A$. The β_j parameter signifies that there are externalities to R&D, and by specifying a different spillover parameter for the two firms we are allowing for asymmetries in spillovers to exist. The returns to research are assumed to be diminishing, and hence we specify the cost of R&D as quadratic. As stated earlier, each firm's strategy is to choose a level of R&D (x_i) and a subsequent production choice based on their R&D choice. We examine the cooperative and non-cooperative games next.

IV.3.1.1. Non-Cooperative Game

The profit of firm P and the welfare of firm S at the second stage, conditional on R&D levels (x_P, x_S) are, respectively:

$$\pi_{\mathrm{P}} = P[Q]q_P - c_P q_P - \gamma \frac{x_P^2}{2} \tag{IV.33}$$

The public sector firm maximizes social welfare where social welfare is represented by the summation of consumer and producer surplus. For a linear demand function and constant per unit costs c_i , $i = P, S$, the public sector firm maximizes welfare choosing q_S

$$W = w_C\left[\frac{bQ^2}{2}\right] + w_P \pi_P + w_S \pi_S \tag{IV.34}$$

where π_P is the private firm's profits as specified in equation (IV.33),

$\pi_{\mathrm{S}} = P[Q]q_S - c_S q_S - \lambda \frac{x_S^2}{2}$ is the public sector firm's profits, and $bQ^2/2$ is the consumer surplus. From the first-order conditions for a maximum for the private and public sector firms, we derive the following reaction functions in output space, taking R&D levels as given:

$$q_P[q_S] = \frac{a - q_S - c_P}{2b} \tag{IV.35}$$

$$q_S[q_p] = \frac{w_S(a - c_S) - (w_P + w_S - w_C)bq_P}{(2w_S - w_C)b} \tag{IV.36}$$

Whereas the second order condition for the private firm is always satisfied ($\partial^2 \pi_P / \partial q_P^2 = -2b \le 0$), the second order condition for the public sector firm requires that $\partial^2 W / \partial q_S^2 = b(w_C - 2w_S) \le 0$ or $w_C \le 2w_S$. Further, for the equilibrium to be stable it suffices that $\left|\partial q_i / \partial q_j\right| < 1$ (Henriques, 1990; Seade, 1980). The private firm's reaction curve in the output space will always be stable in this example, as $\left|\partial q_P / \partial q_S\right| = 1/2$. The stability condition for the public sector firm is different and requires that $\left|\frac{\partial q_S}{\partial q_P}\right| = \left|-\frac{w_P + w_S - w_C}{w_C - 2w_S}\right| < 1$ which is true for $w_S > w_P$ and $\forall w_C$ or $2w_C > 3w_S + w_P$.

Taken together, the second order condition and the stability conditions imply that $2w_S \ge w_C > \frac{3w_S + w_P}{2}$. Therefore for a stable equilibrium to exist, the public sector firm's *profits* must be given a higher weight than the private firm's profits in the its welfare maximization problem ($w_S > w_P$).

From the equations (IV.35) and (IV.36), the Nash-Cournot equilibrium is computed to be:

$$q_P^*[x_P, x_S] = \frac{w_C(a - c_P) - w_S(a - 2c_P + c_S)}{(w_C + w_P - 3w_S)b} \tag{IV.37}$$

$$q_S^*[x_P, x_S] = \frac{(w_P - w_C)(a - c_P) - w_S(a - 2c_S + c_P)}{(w_C + w_P - 3w_S)b} \tag{IV.38}$$

Notice that if the public sector firm maximized profits instead of welfare, such that $w_C = w_P = 0$, then the Nash Cournot equilibrium is completely symmetric.

At the preceding stage, define the first-stage payoff function of the private and public sector firm's as

$$\pi_P^*[x_P, x_S] = b(q_P^*[x_P, x_S])^2 - \gamma \frac{x_P^2}{2} \tag{IV.39}$$

$$W^*[x_P, x_S] = F_1[x_P, x_S] - \gamma w_P \frac{x_P^2}{2} - \gamma w_S \frac{x_S^2}{2} \tag{IV.40}$$

where $F_1[x_P, x_S]$, defined in the appendix, is the net benefits that the public sector firm receives from the second stage, and q_P^* and q_S^* are the Cournot-Nash equilibrium quantities. Then, the Nash equilibrium of the first-stage game, denoted by $[x_P^*, x_S^*]$ is characterized under the assumption of interior optimum and second-order conditions by $\partial\pi_P^*[x_P^*, x_S^*]/\partial x_P = 0$ and $\partial W^*[x_P^*, x_S^*]/\partial x_S = 0$. As was the case in the output space, stability is also required in the R&D space such that $\left|\partial x_i / \partial x_j\right| < 1$ (Henriques, 1990). To explore these stability conditions we follow Henriques (1990) and employ reduced form reaction functions in the R&D space. Matters are considerably simplified if we assume that in the maximization of its objective function, the public sector firm places a higher weight to the profits of its firm such that $w_S = 2$, and $w_P = w_S = 1$. Recall that these weights will satisfy the stability conditions in the quantity space, and will be used, from this point forward.

From the maximization of the first stage profits (for the private firm, eqn (IV.39)) and welfare (for the public sector firm, eqn (IV.40)), we have the following reduced form reaction functions[8]:

$$x_P[x_S] = \frac{(2\beta_P - 3)\left((a-A) + x_S(3\beta_S - 2)\right)/8b}{\partial^2 \pi_P^* / \partial (x_P)^2} \tag{IV.41}$$

$$x_S[x_P] = -\frac{\left((a-A)(18+\beta_S) - x_P(26 - 44\beta_P - 27\beta_S + 26\beta_P\beta_S)\right)/16b}{\partial^2 W^* / \partial (x_S)^2} \tag{IV.42}$$

where the denominator in the above two equations are the second order conditions that must be less than 0 for an equilibrium. Differentiating (IV.41) and (IV.42) with respect to x_P and x_S yields, respectively:

$$\frac{\partial x_P}{\partial x_S} = -\frac{(3-2\beta_P)(3\beta_S - 2)}{(3-2\beta_P)^2 - 8b\gamma} \tag{IV.43}$$

$$\frac{\partial x_S}{\partial x_P} = \frac{26 - 27\beta_S + (26\beta_S - 44)\beta_P}{44 + (27\beta_S - 52)\beta_S - 32b\gamma} \tag{IV.44}$$

The absolute value of these functions must be less than 1 for the stability condition to suffice. Notice that for sufficiently large $b\gamma$ the second order and the stability conditions for both firms is satisfied for all values of the spillover parameters. For smaller values of $b\gamma$, appropriate restrictions need to be placed on the spillovers that would satisfy the equilibrium concepts. Henriques (1990), for example, shows for the pure duopoly case that when $b\gamma = 1$, the non-cooperative model is unstable for low levels of spillovers,

---

[8] With $w_S = 2$, and $w_P = w_S = 1$, we note that the second order condition for maximization of the each firm's objective requires that $(3-2\beta_P)^2/8 < b\gamma$ (for the private firm) and $44 + \beta_S(27\beta_S - 52)/32 < b\gamma$ (for the public sector firm).

which results in corner solutions.[9] However if one were to scale up the research cost to, say $\gamma = 5$, then the stability condition is satisfied $\forall \beta_i \in [0,1]$ for $i = P, S$. Since our interest in this paper is to explore how the equilibrium values are affected by the presence of a public sector firm as well as the consequences of cooperative and non-cooperative R&D with such a firm, we shall assume that $b\gamma$ are large enough to satisfy all the conditions for an unique and stable equilibrium.[10]

Table AIV.3 gives the Nash-equilibrium values for the R&D (x_i^*), output ($q_i^*[x_S^*, x_P^*]$), private firm's profits $\pi_P^*[q_P^*[x_S^*, x_P^*], q_S^*[x_S^*, x_P^*]]$ and public sector firm's welfare $W^*[q_P^*[x_S^*, x_P^*], q_S^*[x_S^*, x_P^*]]$ for different combinations of spillovers. For the simulation we assumed $a = 10$, $A = 7$, $b = 1$, and $\gamma = 5$, but the comparative static results that follow can be generalized for large range of values for $a > A > 0$ and $b\gamma > 5$[11].

From the inspection of the simulation results we have the following comparative static relationships:

1. $\forall \beta_i \in |0,1|$, $x_S^* > x_P^*$ and $q_S^* > q_P^*$. For $w_C = w_P = 1$ and $w_S = 2$, it can be verified from equations (IV.37) and (IV.38), that $q_S^* > q_P^*$ $\forall \beta_i \in |0,1|$.

---

[9] In the pure duopoly, non-cooperative case of d'A&J and Henriques, $\dfrac{\partial x_i}{\partial x_j} = \dfrac{-2(2-\beta)(2\beta-1)/9b}{(2(2-\beta)^2/9b) - \gamma}$, which is unstable for $\beta < 0.17$ and $b = \gamma = 1$.

[10] In the simulations that follow, we have assumed that $\gamma = 5$.

[11] For $0 < b\gamma < 5$, restrictions on the spillovers need to be placed so that an interior solution is obtained.

2. Private firm R&D (x_P^*), output (q_P^*), and profits (π_P^*) are all decreasing in own spillovers. That is $\frac{\partial x_P^*}{\partial \beta_P}, \frac{\partial q_P^*}{\partial \beta_P}, \frac{\partial \pi_P^*}{\partial \beta_P} < 0$, which implies that as appropriability of the returns to private R&D declines, the private firm decreases its research. Public sector firm output (q_P^*) is increasing in private firm spillovers $\frac{\partial q_S^*}{\partial \beta_P} > 0$. With higher levels of spillover, the public sector firm faces a lower unit cost of production allowing it increase the quantity supplied.
3. For low levels of public sector firm spillover ($\beta_S \leq 0.75$), public research is increasing in private firm spillovers. For high levels of public sector firm research *and* high levels of private research spillovers, public research is decreasing in private firm spillovers. For example when $\beta_S = 0.95$, then an increase in private spillover from $\beta_P = 0.75$ to $\beta_P = 0.95$ results in a decline in public R&D (from 0.416 to 0.415).
4. Private firm R&D (x_P^*), output (q_P^*), and profits (π_P^*) are all increasing in public sector firm spillovers $\frac{\partial x_P^*}{\partial \beta_S}, \frac{\partial q_P^*}{\partial \beta_S}, \frac{\partial \pi_P^*}{\partial \beta_S} > 0$.
5. The affect of public sector firm spillovers (β_S) on public sector firm's welfare is ambiguous. We note however that with increasing public R&D spillover total output (and by extension, consumer surplus) is increasing, whereas the equilibrium price is declining.

Our interpretation of these results is as follows. Profit-maximizing firms react to a loss of appropriability by decreasing their research effort. Our result is consistent with this behavior; the easier it becomes for the public sector firm to employ the private firm's research (i.e. increasing β_P), the incentive to conduct R&D declines for the private firm and hence a decline in equilibrium private firm R&D. The reaction of the public sector firm to an increase in spillover is ambiguous; public R&D declines with increasing β_S but increases for very high β_S. This result is similar to the one observed in Chapter II where, for specification 1, the decrease in appropriability of public research resulted, initially, in a decline in public R&D but for very low appropriability of public research (α>0.89), equilibrium public research increased public R&D increased with increasing inability to appropriate the returns.

A decrease in the appropriability of public research (increasing β_S), has a positive affect on private firms R&D as well as total R&D. This implies than any decline in public R&D is more than compensated for by increase in R&D by the private firm. In the output market, we note that the quantity supplied by the public sector firm is declining unambiguously in β_S, while the quantity supplied by the private firm is increasing in β_S. Since total output is also increasing in β_S, the decrease in the output of the public sector firm is also more than compensated by the output of the private firm.

That the public sector firm's output and profits (π_S) unambiguously declines with β_S while its research does not may seem surprising. One would expect that an increase in public sector firm's R&D due to increasing β_S, would imply a proportionate increase in output as the marginal cost of production is decreasing. For example, when β_S increases from 0.75 to 0.95 in Table AIV.3, the marginal cost for the public sector firm declines but its output (q_S) and profits (π_S) are not increasing. The reason for this is that with such high amounts of leakage of public research occurring, the private firm benefits more and is able to reduce its marginal cost much more than the public sector firm can. With a bigger decline in its marginal cost, the private firm is able to increase its output, taking away some of the market share of the public sector firm. The equilibrium results being that the private firm sees an increase in its output whereas the public sector firm sees a decline. Equilibrium profits of the public sector firm (π_S^*) decline since it incurs a higher research cost, the benefits of which it is unable to appropriate due to higher spillovers.

IV.3.1.2. Cooperative Game

In the second game, under the regime where the public sector firm participates in the second stage, the public and private firms cooperate in R&D, while remaining competitors in the output market. The firms maximize their joint objectives, defined earlier, as a function of R&D. In the context of the present market structure, joint objectives are defined as:

$$\begin{aligned} J^* &= W^* + \pi_P^* \\ &= w_C V[q_P^*[x_P^*, x_P^*], q_S^*[x_P^*, x_P^*]] + w_S \pi_S^*[q_P^*[x_P^*, x_P^*], q_S^*[x_P^*, x_P^*]] + \\ &\quad w_P \pi_P^*[q_P^*[x_P^*, x_P^*], q_S^*[x_P^*, x_P^*]] + \pi_P^*[q_P^*[x_P^*, x_P^*], q_S^*[x_P^*, x_P^*]] \\ &= w_C V[q_P^*[x_P^*, x_P^*], q_S^*[x_P^*, x_P^*]] + w_S \pi_S^*[q_P^*[x_P^*, x_P^*], q_S^*[x_P^*, x_P^*]] + \\ &\quad w_J \pi_P^*[q_P^*[x_P^*, x_P^*], q_S^*[x_P^*, x_P^*]] \end{aligned} \tag{IV.45}$$

where $w_J = w_P + 1$ is the weight now given to the private firm in the joint objective maximization. Since the second stage remains non-cooperative, we still require that the stability condition be satisfied and hence retain the assumption that $w_S > w_P$. However in the first stage under cooperation a different set of stability and second order conditions need to be satisfied for a Nash-Cournot equilibrium. The stability and second order condition in this case are, respectively:

$$\left| \frac{\dfrac{\partial^2 J^*}{\partial x_i \partial x_j}}{\dfrac{\partial^2 J^*}{(\partial x_i)^2}} \right| < 1 \text{ for } i, j = P, S \text{ and } i \neq j$$

$$\frac{\partial^2 J^*}{(\partial x_i)^2} \leq 0 \text{ for } i, j = P, S$$

Both these conditions are satisfied for $w_C = 1$ and $w_J = w_S = 2$ and for sufficiently large value of $b\gamma$ (see appendix). As before, the following reduced form reaction functions are obtained from the maximization of the first stage profits in this game[12]:

$$x_P[x_S] = \frac{\left(x_S(38 - 52\beta_P - 45\beta_S + 38\beta_P\beta_S) - 7(a - A)(1 + 2\beta_P)\right)/16b}{\partial^2 J^* / \partial(x_P)^2} \quad \text{(IV.46)}$$

$$x_S[x_P] = \frac{\left(x_P(38 - 52\beta_P - 45\beta_S + 38\beta_P\beta_S) - 7(a - A)(2 + \beta_S)\right)/16b}{\partial^2 J^* / \partial(x_S)^2} \quad \text{(IV.47)}$$

Table AIV.4 provides the simulation results for the Nash-Cournot equlibrium values in the cooperative case for Regime 1. From the table, we observe that

1. $\forall \beta_i \in |0,1|$ $q_S^* > q_P^*$. Firm output is increasing in public sector firm spillovers.

$\dfrac{\partial q_P^*}{\partial \beta_S}, \dfrac{\partial q_S^*}{\partial \beta_S} > 0$. Generally speaking public sector firm output is increasing in

---

[12] Where we have assumed $w_C = w_P = 1$ and $w_J = w_S = 2$.

private spillovers, except when public sector firm spillovers are low (in which case it is decreasing). Industry output is increasing in spillovers $\dfrac{\partial Q^*}{\partial \beta_i} > 0$

2. $\forall \beta_i \in |0,1|$ private research is increasing in spillovers (i.e. $\dfrac{\partial x_P^*}{\partial \beta_P}, \dfrac{\partial x_P^*}{\partial \beta_S} > 0$).
3. Private firm profits are increasing in public sector firm spillovers and generally decreasing in own spillovers.[13] As such, the highest private profits are obtained when there are low private spillovers and high public spillovers.[14] Notice that at high levels of private spillovers and low public spillovers private equilibrium profits are actually negative. This occurs because the firm is not generating enough revenue (due to lower output) to compensate the higher cost of R&D and in spite of the lower unit cost of production. That is to say that the cost of R&D has increased at a faster rate than the cost saving from production along with a loss in market share
4. The affect of private and public research spillovers is ambiguous for public welfare, although it is the highest at high levels of both private and public spillovers.

The fact that public welfare is maximized when spillovers are high and the private firm's profits are maximized when *private* spillovers are low reflects the divergent objectives of the two firms. When spillovers are high, both firms are not only able to share each others research outputs, they also conduct more R&D. This lowers their marginal cost of production allowing them to increase their output. The industry supply

---

[13] Except when public spillovers are low in which case private research is increasing in private spillovers.
[14] For example, $\beta_P = 0, \beta_S = 0.99$ we get $\pi_P^* = 0.92706$

curve shifts to the inelastic portion of the demand curve where the consumer surplus is greater and the profits to the private firm lower. Since the gains to the consumers are higher than the losses to the private firm, public sector firm welfare is higher.

How do the cooperative and non-cooperative cases compare? The key difference is in the way x_P changes with respect to β_P; whereas it is decreasing in the non-cooperative case it is increasing in the cooperative case. The joint objectives that arise from cooperation imply that the private firm is now forced to internalize the consumer benefits that arise from research. In some sense, the R&D stage is characterized by one public sector firm, but with the greater weight being given to the profits of the producers then the welfare of consumers. Whether it is advantageous for the private and public sector firm to cooperate is function of how easy it is for the firms to imitate each other's R&D, which is captured by the spillover parameter. Table AIV.5 provides a matrix for comparing key equilibrium values for total R&D (X^*), private firm profits (π_P^*), consumer surplus V^*, and public sector firm's welfare between the two games and at different level of spillovers. From Table AIV.5 we see that cooperation always increases public sector firm welfare, regardless of the spillovers between firms. This is not the case for the profits of the private firm, as profits (π_P^*) are higher only if the spillovers are low. That is when the research of the public and private firm is easy to appropriate (low β_i) then the private firm will obtain higher profits from cooperating than from not cooperating. For most other levels of appropriability, however the private does not gain much from collaborating. The intuition behind this result is that by cooperating with a public sector firm, the private firm acts less like a profit maximizing firm and more like a welfare maximizing 'public' firm. Since we have established that a public sector firm (generally speaking) increases its R&D with higher rates of spillovers, the private firm also will increase its R&D as β_P increases. The increased R&D it conducts in advancing the joint objectives, however come at a cost of decreased private profits. That is the private firm, by collaborating with the public sector firm, over-invests and beyond the point where a profit maximizing firm normally would when spillovers are high. If spillovers are low, the private and public sector firm gains from cooperation because they are able to lower their R&D expenditures ($x_P^C < x_P^{NC}$ for low spillover) mitigating the over-investment problem when research is appropriable.

IV.3.2. Regime 2: Mixed Duopoly in R&D Stage Only

In this section we specialize the model for the case where the public sector firm does not compete in the second (output) stage and is only a producer of research in the first (R&D) stage. Two symmetric private firms that compete in quantities in the second stage undertake production. The two private firms also conduct (production) cost reducing R&D alongside the public sector firm. Spillovers from research exist between all three firms (two private and one public), such that all firms benefit from each other's research. As in the earlier case we analyze two types of games, first where all firms conduct research non-cooperatively, and in the second we allow for cooperative behavior between one of the private firms and the public sector firm. To allow for comparisons between this market structure and the one discussed earlier, we assume the same cost function and inverse demand of the earlier market.

IV.3.2.1. Non-Cooperative Game

The second stage of the non-cooperative game is exactly as the one that appears in d'A&J, except that in the present case the private firms also benefit from the spillovers of a public sector firm. The profit of firm i at the second stage, conditional on first stage R&D (x_i for $i = P1, P2, S$) is expressed as

$$\pi_i = P[Q]q_i - c_i q_i - \gamma \frac{x_i^2}{2} \tag{IV.48}$$

for $i = P1, P2$ and $i \neq j$.

where $P[Q] = a - b(q_i + q_j)$ is the inverse demand, $c_i = A - (x_i + \beta_P x_j + \beta_S x_S)$ is the unit cost of production and $(\gamma / 2)x_i^2$ is the cost of R&D. We assume that spillovers between the two private firms are symmetric but allow for difference in spillovers between the public sector firm and the private firm. The reaction functions in the output space, taking R&D levels as given is

$$q_i[q_j] = \frac{a - bq_j - c_i}{2b} \tag{IV.49}$$

for $i = P1, P2$ and $i \neq j$.

As indicated by Henriques (1990), these output reaction functions meet the stability conditions because of the assumptions on the cost, market demand and that R&D levels are given. The Nash-Cournot equilibrium is computed to be

$$q_i^* = \frac{a - 2c_i + c_j}{3b} \tag{IV.50}$$

for $i = P1, P2$ and $i \neq j$.

In the first stage, we introduce a public sector firm that along with the two private firms chooses R&D levels. The private firms choose that level of R&D that maximizes profits, whereas the public sector firm maximizes welfare. Profits and public sector firm welfare in the first stage are written as:

$$\pi_i^*[x_i, x_j, x_S] = b(q_i^*[x_i, x_j, x_S])^2 - \gamma \frac{x_i^2}{2} \tag{IV.51}$$

$$\begin{aligned} W_2^*[x_P, x_S] &= V[q_{P1}^*[x_{P1}, x_{P2}, x_S], q_{P1}^*[x_{P1}, x_{P2}, x_S]] + \\ &\quad \sum_i^{P1,P2} \pi_i^*[q_{P1}^*[x_{P1}, x_{P2}, x_S], q_{P1}^*[x_{P1}, x_{P2}, x_S]] - \gamma \frac{x_S^2}{2} \\ &= F_2\left[x_{P1}, x_{P2}, x_S\right] - \gamma \frac{x_{P1}^2}{2} - \gamma \frac{x_{P2}^2}{2} - \gamma \frac{x_S^2}{2} \end{aligned} \tag{IV.52}$$

where $F_2\left[x_{P1}, x_{P2}, x_S\right]$ is defined in the appendix. In the R&D space, the following reduced form reaction functions are derived from the first order conditions for a maximum of equations (IV.51) and (IV.52):

$$x_i[x_j, x_S] = -\frac{2(2-\beta_P)(a - A + x_j(2\beta_P + 1) + \beta_S x_S)/9b}{\partial^2 \pi_i^* /(\partial x_i)^2} \tag{IV.53}$$

for $i = P1, P2$ and $i \neq j$.

$$x_S[x_{P1}, x_{P2}] = -\frac{4\beta_S(2(a-A) + (x_{P1} + x_{P2})(1+\beta_P))/9b}{\partial^2 W_2^* /(\partial x_S)^2} \tag{IV.54}$$

Assuming that the second order conditions are satisfied and that model is stable[15], there exists a unique solution satisfying $\partial\pi_i^* / \partial x_i = 0$ for $i = P1, P2$ and $i \neq j$ and $\partial W^* / \partial x_S = 0$ for which:

$$x_i^* = \frac{(a-A)(2-\beta_P)}{4.5b\gamma - (2-\beta_P)(1+\beta_P) - 4(\beta_S)^2} \qquad \text{(IV.55)}$$

for $i = P1, P2$

$$x_S^* = \frac{8\beta_S(a-A)}{4.5b\gamma - (2-\beta_P)(1+\beta_P) - 4(\beta_S)^2} \qquad \text{(IV.56)}$$

From the equilibrium values of R&D, we note that private research is increasing in public spillovers and decreasing in private spillovers ($\partial x_i / \partial\beta_P < 0$ and $\partial x_i / \partial\beta_S > 0$). Public research also is increasing in public spillovers $\partial x_S / \partial\beta_S > 0$; public R&D is however decreasing in private spillovers greater that half and increasing for spillovers less than half ($\partial x_S / \partial\beta_P < 0$ for $\beta_P > 0.5$ and $\partial x_S / \partial\beta_P > 0$ for $\beta_P < 0.5$)[16]. Our simulation results (Table AIV.6) confirm these comparative statics, and we also observe that firm output profits and public welfare are also increasing in public spillovers. Note that firm output declines for $\beta_P > 0.5$, but firm profits are unambiguously increasing in

---

[15] The second order conditions are $\frac{\partial^2\pi_i^*}{\partial(x_i)^2} = \frac{2(2-\beta_P)^2}{9b} - \gamma \leq 0$ for $i = P1, P2$ and $\frac{\partial^2 W_2^*}{\partial(x_S)^2} = \frac{8(\beta_S)^2}{9b} - \gamma \leq 0$. The stability conditions require that $\frac{\partial x_S}{\partial x_i} = \left|\frac{4\beta_S(1+\beta_P)}{-8(\beta_S)^2 + 9b\gamma}\right| < 1$, $\frac{\partial x_i}{\partial x_S} = \left|-\frac{2\beta_S(2-\beta_P)}{2(2-\beta_P)^2 - 9b\gamma}\right| < 1$ and $\frac{\partial x_i}{\partial x_j} = \left|\frac{2(2-\beta_P)(1-2\beta_P)}{2(2-\beta_P)^2 - 9b\gamma}\right| < 1$

[16] These comparative statics are given in the appendix.

the private spillovers. The increase in producer surplus (industry profits) are however not large enough to offset the losses in consumer surplus (due to declining output) which occurs at higher levels of spillovers and is the reason as to why public sector firm welfare (W_S^*) declines at high spillovers.

When $\beta_S = 0$, the non-cooperative game under Regime 2 corresponds to the non-cooperative game of the d'A&J. It is clear that the presence of a spillovers from a public sector firm increases private firm research and output (i.e. $x_i^{\beta_S=0} < x_i^{\beta_S>0}$ and $q_i^{\beta_S=0} < q_i^{\beta_S>0}$ for $i = P1, P2$).

IV.3.2.2. Cooperative Game

Assume now that the public sector firm and one of the private firms join to maximize their joint objectives by choosing R&D levels in the first.[17] The second stage remains unchanged, and the two private firms continue to compete in quantities, given their first stage R&D. As was the case with Regime 1, 'joint objectives' imply that the profits for the collaborating private firm are given a higher weight than the case where it did not collaborate. Since we have implicitly defined the public's welfare function to be equally weighted (and equal to one) across consumers and producers (see equation (IV.52)), the implication of cooperation is that the weight on the profits of the cooperating firm (P1 in this case) is larger. The joint objectives of the cooperating firms as function of R&D is than

[17] Because of symmetry, it does not matter here as to which private firm the public sector firm cooperates with. We shall assume that the cooperating private firm is *P1*.

$$\begin{aligned}J^*[x_P,x_S] &= V[q^*_{P1}[x_{P1},x_{P2},x_S],q^*_{P2}[x_{P1},x_{P2},x_S]]+\\&\quad w_{P1}\pi^*_{P1}[q^*_{P1}[x_{P1},x_{P2},x_S],q^*_{P2}[x_{P1},x_{P2},x_S]]+\\&\quad \pi^*_{P2}[q^*_{P1}[x_{P1},x_{P2},x_S],q^*_{P2}[x_{P1},x_{P2},x_S]]\\&\quad -\gamma\frac{x_S^2}{2}\\&= F_3[x_{P1},x_{P2},x_S]-\gamma\frac{x_{P1}^2}{2}-\gamma x_{P2}^2-\gamma\frac{x_S^2}{2}\end{aligned} \tag{IV.57}$$

where $w_{P1}=2$ and F_3 is defined in the appendix. The first stage profits of the firm that does not cooperate with the public sector firm remains the same and are given by equation (IV.51), only here it is just for $P2$. As in the earlier cases, we derive the following reduced form reaction functions in the R&D space:

$$x_{P1}[x^*_{P2},x^*_S]=-\frac{((4+\beta_P)(a-A)+x_{P2}(11-\beta_P(32-11\beta_P)-2\beta_S x_S(4+\beta_P))/9b}{\partial^2 J^*_2/\partial(x_{P1})^2} \tag{IV.58}$$

$$x_{P2}[x_{P1},x_S]=-\frac{2(2-\beta_P)(a-A+x_{P1}(2\beta_P+1)+\beta_S x_S)/9b}{\partial^2\pi_{P2}/(\partial x_{P2})^2} \tag{IV.59}$$

$$x_S[x_{P1},x_{P2}]=-\frac{2\beta_S(5(a-A)+x_{P2}(1+4\beta_P)+x_{P1}(4+\beta_P))/9b}{\partial^2 J^*_2/(\partial x_S)^2} \tag{IV.60}$$

Assuming that the second order conditions are satisfied and that the reaction are stable in the R&D space, we have the following unique solution for the equilibrium with cooperation in this case:

$$x^*_{P1}=\frac{2(a-A)(6-\beta_P(11-\beta_P(11-\beta_P(6-\beta_P)+b\gamma)-4b\gamma}{D} \tag{IV.61}$$

$$x^*_{P2}=\frac{2(a-A)(2-\beta_P)(3-\beta_P(4-\beta_P)-2b\gamma)}{D} \tag{IV.62}$$

$$x^*_S=\frac{2(a-A)\beta_S(\beta_P(26-11\beta_P)-15+10b\gamma)}{D} \tag{IV.63}$$

where D is given in the appendix. Table AIV.7 presents the simulation results for the equilibrium values for different levels of spillovers. Comparing across equilibrium research levels for the three firms (the above equations and Table AIV.7), it is not clear as to which firms conduct the most R&D or whether there are unambiguous benefits for the private (in terms of profits) for cooperating with the public sector firm. From our simulation we observe that:

1. $\forall \beta_i \in |0,1|$ (for $i = P, S$), $q_{P1}^* > q_{P2}^*$. Further firm output is increasing in public spillovers. $\partial q_i^* / \partial \beta_S$ for $i = P1, P2, S$. By cooperating with the public sector firm, the private firm produces above the profit maximizing level as it internalizes the gains to consumers. As such the quantity produced by the private firm if it acted non-cooperatively is lower than that produced when it acts cooperatively (i.e. $q_{P1}^{*C} > q_{P1}^{*NC}$)
2. For all three firms, firm R&D is increasing in public spillovers ($\partial x_i^* / \partial \beta_S > 0$). Whereas the R&D of the cooperating private firm and the public sector firm are increasing with increasing private spillovers, it is decreasing for the non-cooperative firm ($\partial x_i^* / \partial \beta_P > 0$ for $i = P1, S$ and $\partial x_{P2}^* / \partial \beta_P < 0$). The industry unit costs of production are lowered when more R&D is done and shared by as many firms as possible. The cooperating firms would prefer this state as it increases output and their joint objectives (even though it comes at a cost of lower profits for the firm that cooperates). The non-cooperating firm however will reduce its research level for higher private spillovers not only because it is able to gain from the R&D of the other private firm (which is increasing its R&D) but

also because it gains (free-ride) of its rivals research. Interestingly the profits of the non-cooperating firm are higher for higher private spillovers implying that there are gains to cutting back its own in-house research and free riding off others.

3. $\forall \beta_i \in |0,1|$ (for $i = P, S$), public welfare and the profits for the non-cooperating firm are increasing in spillovers. For $P1$, profits are increasing in public spillovers $\forall \beta_S \in |0,1|$. $P1$ profits are however increasing for $\beta_P < 0.5$ and increasing for $\beta_P > 0.5$. Hence the reduction in profits for the cooperating firm (for the $\beta_P > 0.5$ cases) are offset by the increase in consumer surplus and the profits of the non-cooperating, resulting in higher overall public sector firm's welfare.

In comparing the equilibrium outcome of the non-cooperative with that of the cooperative case, we note that industry research and output are higher at all levels of spillovers in the cooperative case (see comparative summary in Table AIV.8). The affect of cooperation on profits is less clear. If both public and private spillovers are low, the cooperating firm gains, as its profits are higher relative to the non-cooperative case. When private spillovers are high, than the profits from cooperating are lower than they are from non-cooperating. Notice however that a firm would make higher profits if it did not cooperate than when it did at high levels of spillovers (i.e. for $\beta_P > 0.5$ $\pi_{P2}^C > \pi_{P1}^C$). Generally speaking public welfare is higher in the cooperative case than it is in the non-cooperative case. An exception appears to be for "low" private spillover and "medium" public spillover whence the welfare in the non-cooperative case is higher. This may seem somewhat counterintuitive considering that both consumer surplus and profits are higher in the cooperative case than they are in the non-cooperative case. However since public research is increasing in β_S, so is the cost doing research. If the increase in the cost of undertaking R&D is higher than the gains to producers and consumers, than public welfare will be lower.

IV.4. Welfare

To put our results in perspective, we define in this section an efficiency criterion to which the results are compared. We employ the d'A&J definition of social welfare $W[Q]$ as the sum of consumer's surplus and the producer's surplus[18].

$$SW[Q] = V[Q] - CQ - \gamma X^2 \tag{IV.64}$$

where $C = A - (1+\beta)X$ is the unit cost of production. Given R&D, the socially optimal output is given by

$$Q^* = \frac{a-C}{b} \tag{IV.65}$$

In the preceding stage, social welfare is now expressed as

$$SW^* = b\left(Q^*[X]\right)^2 - \gamma X^2 \tag{IV.66}$$

The social planner maximizes the social welfare function choosing R&D in the first. From the first order conditions for a maximum[19], the efficient level of R&D is

$$X_{SW}^* = \frac{(a-A)(1+\beta)}{2b\gamma - (\beta+1)^2} \tag{IV.67}$$

---

[18] The social welfare here corresponds to the public sector firm's welfare when no profit maximizing firms are present. In this case the public sector firm's maximization is equivalent to the social planner's problem.

[19] The second order condition require that $(1+\beta)^2 / 4 < b\gamma$

Table AIV.9 provides the simulation results for the socially efficient values of R&D and output. It is easily verified that higher levels of spillovers result in more production but also higher R&D[20]. Note as would be expected in the first best solution, our simulation show that price equals marginal cost ($P[Q^*] = C$) when the social planner allocates resources.

How do the cooperative and non-cooperative games of the two regimes compare with the first-best solution? Table AIV.10 provides a comparative summary of the equilibrium output and welfare across the regimes, where, for the comparison to be meaningful, we have assumed that private and public spillovers are equal ($\beta_P = \beta_S = \beta$).

Our results confirm the earlier findings of d'A&J but also provide a new perspective on how those results are affected by the presence and cooperation between public and private firms[21]. Confirmation of the d'A&J result comes in the observation that for "low" (symmetric) spillovers, noncooperation increases output quantities but at high spillovers cooperation increases output quantities. Distinguishing between the two regimes, the results reveal that the second best for output is due to Regime 1 in most cases, except when spillovers are very high.

The comparison of industry research across the two regimes shows that for low spillovers, the industry in the two regimes will over-invest relative to the first-best. Note however that in all cases, the second best appears to be either the non-cooperative or

---

[20] $\partial Q^* / \partial \beta = x / b > 0$ and $\partial X^*_{SW} / \partial \beta = \dfrac{(a-A)((1+\beta)^2 + 2b\gamma)}{((1+\beta)^2 - 2b\gamma)^2} > 0$

[21] Recall that the d'A&J analysis focused on the cooperative and noncooperative outcome between two profits maximizing firms (i.e. a pure duopoly).

cooperative equilibrium under the first regime. This result is in accordance with that of the Chapter II, where we saw that the presence of public sector firm in an R&D race tends to move the industry towards the socially optimal level of research.

IV.5. Conclusions

This chapter adapted the d'A&J framework to study the cooperative and noncooperative equilibrium in the presence of a public sector firm that seeks to maximize welfare. Our analysis was motivated by two issues, one positivistic and the other more normative, concerning public and private sector partnerships The first issue is why more partnerships among public and private sector R&D firms are not observed? A second issue is whether the partnering of a public sector firm with that of a private one, will work against the public sector firm's ultimate objective of raising welfare?

We conducted our analysis first in a general framework where we found that a sufficient condition for a stable equilibrium in mixed oligopoly is obtained only if the public sector firm's profits are weighted higher than that of the private firm in the objective function of the public sector firm. The general model provides us with decision criterion on how much output and R&D the public and private firms perform. A key result of this section is that the private firm by jointly maximizing its objectives with a public sector firm, conducts R&D at the point where weighted *total* marginal benefits equals its marginal cost. As such, by internalizing the gains to its research that accrue to itself and its collaborator, the private firm will conduct more R&D than it's profit maximizing level, assuming that there is perfect sharing of the innovation in the second stage of the game which remains noncooperative.

To provide more clearer and definitive results, we specialize the general model using the framework of d'A&J. Specifically, we address the issue that while research partnerships were encouraged as important element in the policy response to global competition, the number of public-private partnerships has remained low, relative to partnerships among private firms. The results of the model simulation provides why this may be the case. If we assume that the public sector firm conducts both R&D and production, as was the case in Regime 1, we find that profits from cooperating for the private firm is not always unambiguously a Pareto improvement. Specifically we find that for high spillovers, profits of the private when it collaborates with the public sector firm are actually lower than the non-cooperative case (in Regime 1) or lower than the firm that does not cooperate (in Regime 2). Hence the incentive to participate in a joint collaborative effort will not be all that compelling for the private firm, when spillovers are high. On the other hand when spillovers are low, then the profits for the private firm from cooperating are higher than from not cooperating, suggesting that at very low levels of R&D cooperation will take place.

Our finding that low levels of cooperation is due to high spillovers suggest that the level of spillovers between private and public sector firms is an important ingredient in the formation of research partnerships. That high level of spillovers may discourage public-private partnerships, especially in agricultural research, is consistent with evidence that agricultural research is characterized by the presence of large spillovers (Evenson, 2000; Fuglie et al., 1996).

Lastly we observed that under Regime 1, cooperation for the public sector firm's welfare was always a Pareto improvement. This result is also true under Regime 2, but for a higher level of private spillover. This implies that collaboration between a private and public sector firm does not compromise the objective of the public sector firm, and if anything tends to increase it. Given that cooperation is a Pareto improvement for the public sector firm and may not be for the private firm, the results suggest that private firm be given incentives to undertake cooperative research. Such incentives could take the form of subsidization of private research or that the profits of the private firm that collaborates be weighted more heavily relative to consumer welfare. As long as the decline in consumer welfare is offset by the increase in producer welfare, the public sector firm's welfare will be higher and the incentive for the private firm to cooperate will exist.

CHAPTER V

CONCLUSIONS

The structural changes that have occurred in the U.S. agricultural R&D market over the last two decades have served as the basic research motivation for this thesis. In particular, we focused attention on the following observed facts: that a) there has been a shift in the balance between public and private research activities, b) there has been an increasing resort to intellectual property rights by both sectors and c) the phenomenon of public and private research partnerships is on the rise, although not as common as one would expect. The objective of this research was not to present a comprehensive explanatory model of the agricultural R&D industry, but to provide a limited framework for analyzing the effects and implications of the changing market structure through the modeling of strategic micro-level interactions between firms in the two sectors. Employing a mixed oligopoly framework, which aptly characterizes the nature of the market structure in agriculture R&D, we presented theoretical models to investigate the relationship of public and private research in the presence of appropriability and spillovers (chapter II); a comparative analysis of non-competitive markets and where the research has the attributes of genomics R&D (chapter III); and finally, an analysis of cooperative and noncooperative R&D in the presence of the public sector firm (chapter IV). Collectively, the results of this dissertation address the research questions that were posed in the context of the recent structural changes in the agricultural R&D sector noted in the introductory chapter. Next we summarize the key results and show how they relate to the issues that we sought to explore.

In chapter II, we show that the differential amounts of research being performed by the public and private sectors can be understood through the modeling of appropriability and spillover of public research. Our results demonstrate that if the appropriability of public research is low, such that the private firm can easily imitate the innovation of the public sector firm, the private sector loses the incentive to conduct its own research. However, since the public sector firm is a social welfare maximizer, the appropriability condition does not affect its behavior. As a welfare maximizer, the public sector firm is only concerned with total welfare and not its distribution among the producers. With total welfare remaining unchanged, the public sector firm's response function is therefore unaffected. This is in contrast to a private firm which will cutback its R&D effort if the returns to its research are not appropriable (Poyago Theotoky, 1998).

The shift in the reaction curve of the private firm due to appropriability of the public sector firm's research will, of course, change the equilibrium level of R&D. While the equilibrium research of the private firm is always lower as a result of lower appropriability of public research, the effect on the public sector firm's equilibrium R&D is ambiguous. The ambiguity, however, provides us with another interpretation and extension of the D&D and P-T results. A key finding of D&D was that a mixed duopoly is welfare superior to the pure duopoly in spite of the distortion that prevails due to over-investment in R&D. The comparable result in the P-T model is more ambiguous as social welfare can be higher or lower than the private duopoly. In our analysis, the parameterization of the appropriability condition allows for calculation of the first best solution, whereby the mixed duopoly is not only welfare superior to the pure duopoly but is equal to the welfare obtained by a social planner. This implies that regardless of the value of the innovation, and as long as the public sector firm can choose the level of appropriability of its innovation, the public sector firm can restore the correct incentives for R&D.

We show that if we extend our analysis to account for private firm research capacity, the D&D result can be confirmed and made more robust under a set of sufficiently strong assumptions. Under these assumptions the private firm's reaction is unambiguously upward sloping and rises with increasing appropriability of the public sector firm's research. The D&D result is also confirmed if we assume that R&D spillovers occur during the course of the race (as opposed to the appropriability condition which are essentially spillovers after the conclusion of the race).

The implication of this result is that, in the presence of spillovers (both ex-post innovation and during the course of the R&D race) an increase in public research can result in an increase in private research. The effect of private research on public R&D is a bit more ambiguous, but the results suggest that for 'low' ('high') levels of private R&D an increase in private research reduces (increases) public research. An empirical test of this hypothesis by Fuglie and Walker (2001) found that an increase in private breeding on a commodity led to a small but statistically significant reduction in public research on that commodity. Note that from a social welfare perspective, if higher spillovers lead to an equilibrium condition in which both private and public R&D is higher, it may lead to a reduction in welfare as one is moving away from the first best solution. That is, spillovers may exacerbate over-investment, in which case the pure duopoly may perform better[1].

[1] The conditions under which this would occur is if the pure duopoly not spillovers occur, but they do in the mixed as modeled for specification II and III in chapter II.

Relating the approbiablity and spillover condition in our models to the use of intellectual property protection by the public sector, our results suggest that the effect of IP protection of public sector innovation needs to be evaluated within the context of the response that the private firm elicits. The extreme cases of perfect protection and no protection of public sector firm's innovation are not welfare maximizing. Another example showed that spillovers during the course of the R&D race may result in higher private and public R&D in equilibrium, although the welfare may be lower because of over-investment due to increased research activity.

While the primary focus of the analysis in chapter II was to investigate how public research appropriability and spillover impacted the research activity of the private firm, it also highlighted the regulatory role of a public sector firm in a research market. Considering that a standard theme in the approach to market structure and innovation is whether there is over- or under-investment in research, the results show the usefulness of public R&D as part of technology policy to correct such distortions[2].

In chapter III, we extended the analysis to consider, not the desirability of a particular market structure *per se*, but the conditions under which a particular market structure would imply higher levels of R&D in genomics. The modeling of chapter III took into consideration some of the key elements of research in genomics, which is characterized as a race and where the resources expended on the race impact the value of the innovation or downstream profits/welfare. The results of the analysis suggest that the distribution of profits among competing firms, the complementarity of the first stage research with second stage output, and the curvature properties of the research function

[2] Subsidies, taxes, patent laws and other legal regulation being some of the other technology policy tools available to the government.

can, in large part, explain which market structure among monopoly, pure duopoly, and mixed duopoly undertakes more research. In particular, we found that the R&D in the mixed duopoly will be greater than the other markets if a) property rights are weak such that firms can easily copy each other's innovation, b) first stage research is not does not complement well the downstream stages for firms in the pure duopoly, and c) the gains from R&D activity are larger for firms in the mixed market relative to firms in the other market. These sufficient conditions would generally be met for R&D on commodities for which there is weak IP protection (such as non-hybrid crops), research effort of the R&D race is limited to the race itself and limited in scope, and the market is not large enough for the innovator to get a significant fraction of the total welfare. It was suggested that these conditions resemble the current state of genomics R&D, as patent rights in genomics are ill-defined leading to lower expected profits for firms, and private firms pursue a research strategy in which the current sequencing effort does not significantly impact research downstream. In contrast, the public sector firm, by pursuing a different research strategy and expending more effort on the sequencing stage, may increase the marginal productivity of its research effort.

The last essay explored the issue of cooperative and noncoopertive behavior in the presence of a public sector firm. We showed in the essay that under certain conditions a private firm may not have incentives to collaborate with a public sector firm as such collaboration subverts its profit maximizing objectives as (under cooperation) it produces where total benefits are maximized and not simply its private benefits. From the perspective of the public sector firm, the cooperation with the private firm is welfare improving so long as cooperation implies the maximization of joint objectives, which includes producer and consumer welfare. This result suggests that private firms be given incentives to undertake cooperative research. Such incentives could take the form of subsidization of private research or that the profits of the collaborating private firm be weighted more heavily relative to consumer welfare.

The recent evolution of agriculture R&D industry structure is consistent with theoretical models of R&D races in mixed oligopoly. The driving forces in these models are spillovers, appropriability, cooperation, and competition. Although these models show that the role of the public sector has shifted away from that of provider of a public good (R&D) to that of a regulator of non-competitive effects, there remains an important role for the public sector as provider of R&D.

The models explored in this study, while largely serving its purpose of examining the issues raised in the introduction, have limitations leaving room for future work. All the models of this study have been cast under the assumption of an oligopolistic market, as our interest was to examine the nature of the behavioral response of the private and public sectors. This assumption does not allow for an examination of how the intensity of competition amongst several private and public sector firms affects innovation. The issue of market performance was address in chapter III, but only in the context of monopolistic and oligopolistic markets. We argued that high fixed costs would not allow for a competitive market. However, a more thorough treatment of this topic would allow for competitive markets, one where both private and public sector firms are generalized to n firms. Generalization to several firms also allows one to consider heterogeneity of firms with respect to size and on how the welfare of such firms are weighted (a concept introduced in chapter IV) in the public sector firm's objective function.

A second limitation of our analysis relates to our implicit assumption that in the mixed oligopoly the private and public sector firms produce a homogenous research output and hence are in direct competition with each other. This is certainly true for the genomics example, but may not be appropriate for research where public and private sectors take on distinct and separate roles. That is, the public sector is generally considered to be engaged in pre-technology and basic research whereas the private sector conducts more applied research.

The purpose of the dissertation was to provide within the chosen game theoretic analytical framework the range of choices of available market structures wherein public and private firms can collaborate to produce the optimum level of R&D agricultural research. These choices need to be narrowed down by further case-specific parameterization of the model structure. Even within the broad framework chosen, there is admittedly considerable scope for further extension and choice of more realistic assumptions.

Lastly, the question of empirically implementing the model for a number of real life situations also needs to be explored at some length. The main testable hypothesis that have been generated by the models of this dissertation are

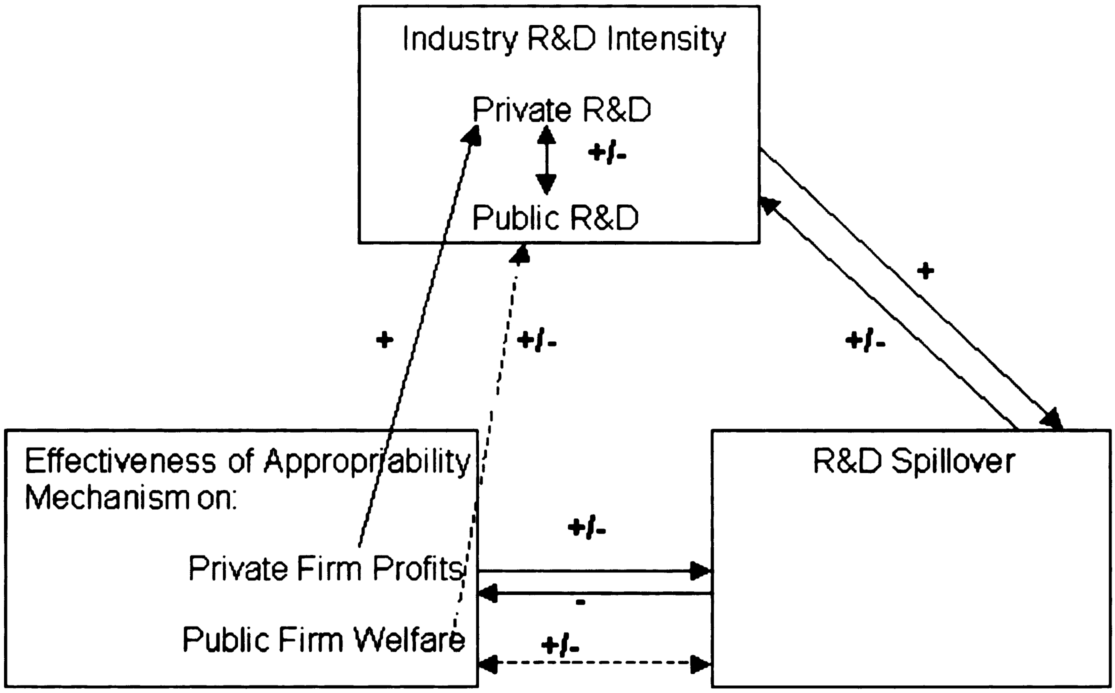
1) Greater ability by the public sector to appropriate its innovations will lead to a decline in the private research. Greater research spillovers from public sector research will lead to an increase in private sector research.
2) The rate of innovation in genomics for private and public sector firms differs. The private firm employs a strategy in which it sequences the genome quickly but at the expense of long-term profits. The public sector firm maximizes

long-term welfare gains by sequencing the genome more accurately and thoroughly.

3) The presence of public research in agriculture, and especially biotechnology research, reduces welfare losses associated with an imperfectly competitive research market.
4) Private firms collaborate with public sector firms when research spillovers are low.

APPENDICES

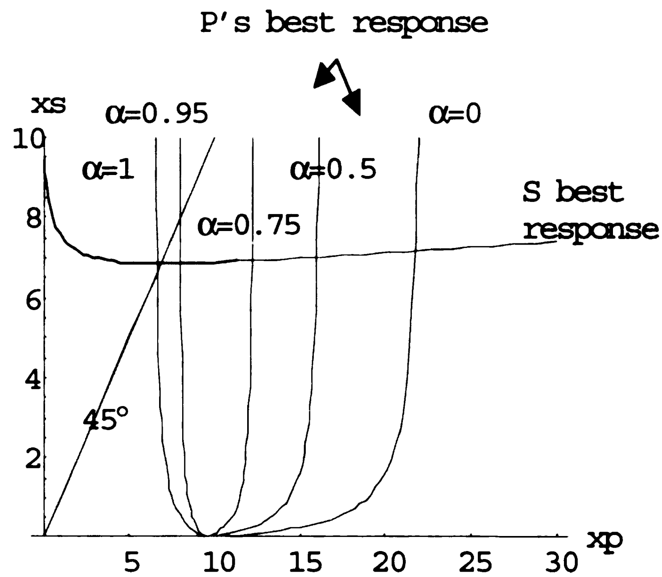
Figure AII.2: Relationship Between R&D Intensity, Appropriability and Spillovers



Note: Dashed lines from the appropriability box relate to public R&D and solid lines relate to private R&D.

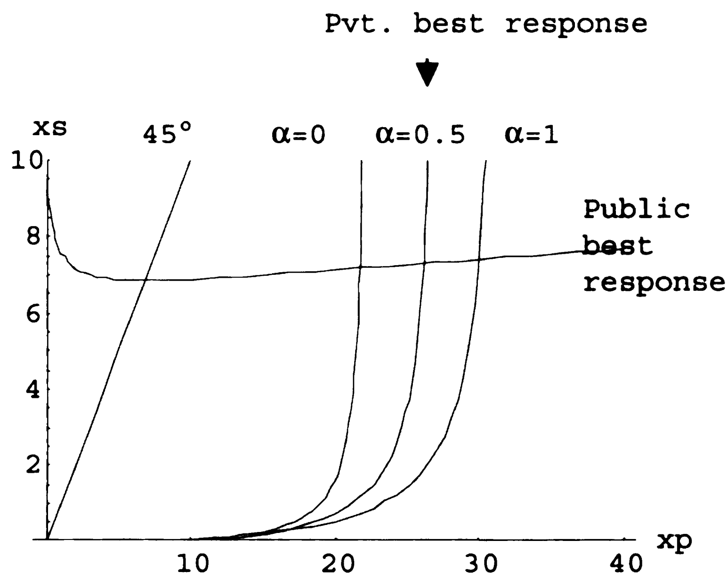
Source: Adapted from Cohen and Walsh (2000).

Figure AII.3: Public and Private Best Response Under Varying Levels of α for Specification 1.



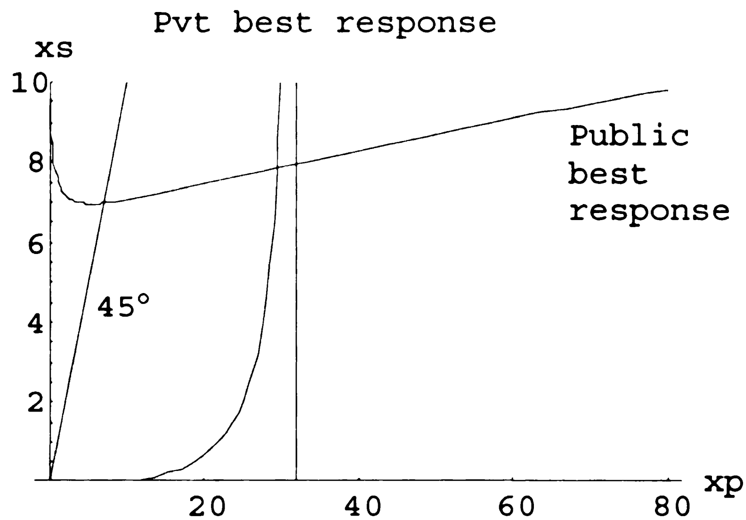
Note: Simulation using $h[x] = x/(1+x)$, $W = 100$ and $r = 0.1$

Figure AII.4: Public and Private Best Response Under Varying Levels of α for Specification 2.



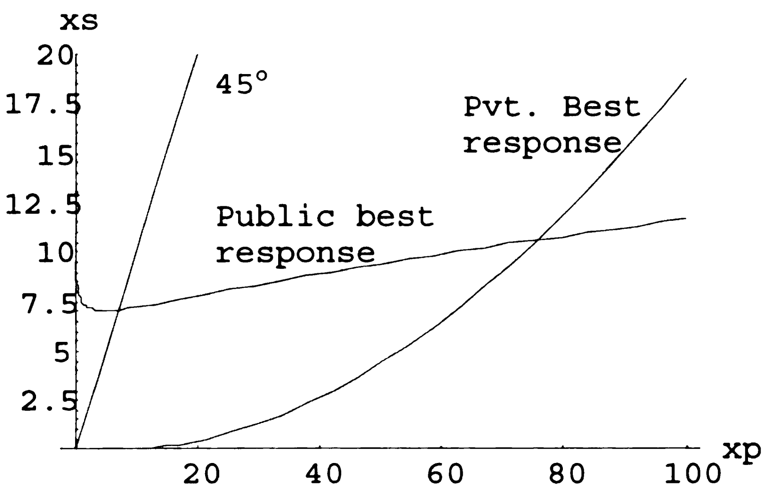
Note: Simulation using $h[x] = x/(1+x)$, $W = 100$ and $r = 0.1$

Figure AII.5a: Public and Private Best Response Under Varying Levels of β for Specification 3 (β=0.5).



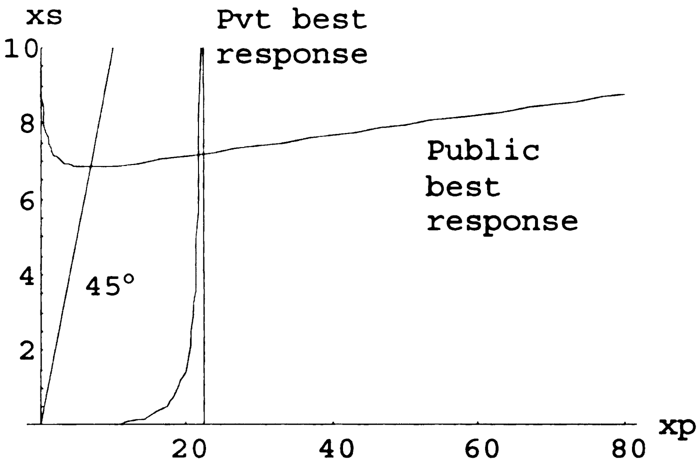
Note: Simulation using $h[x] = x/(1+x)$, $W = 100$ and $r = 0.1$

Figure AII.5b: Public and Private Best Response Under Varying Levels of β for Specification 3 ($\beta = 1$)



Note: Simulation using $h[x] = x/(1+x)$, $W = 100$ and $r = 0.1$

Figure AII.5c: Public and Private Best Response Under Varying Levels of β for Specification 3 (β=0)



Note: Simulation using $h[x] = x/(1+x)$, $W = 100$ and $r = 0.1$

Chapter III

More on Assumption 2:

The second stage post innovation profits of the winning and losing doupolist is given by equations III.7 and III.8:

$$\max_{q_{WD}} W_{WD} = P[Q]q_{WD} - C[\bar{\gamma}, q_{WD}, x_D^*] \tag{A.1}$$

$$\max_{q_{LD}} W_{LD} = P[Q]q_{LD} - C[\gamma, q_{LD}] \tag{A.2}$$

where $P[Q] = a - (q_{WD} + q_{LD})$ is the inverse demand, $C[\underline{\gamma}, q_{WD}, x_D^*]$ is the cost of production for the winning firm and $C[\gamma, q_{LD}]$ is the cost of production for the losing firm. Consider three cost regimes, one where marginal cost is increasing (decreasing returns to scale), and two others where marginal cost is constant but differ in how first stage R&D affects production cost (a lump sump reduction in one case and a unit reduction in another). Mathematically the three cost regimes are specified as:

1. Increasing costs: $C[\underline{\gamma}, q_{WD}, x_D^*] = \frac{\underline{\gamma}}{(1+x_D)} \frac{(q_{WD})^2}{2}$, $C[\gamma, q_{LD}] = \gamma \frac{(q_{LD})^2}{2}$, where

   $\underline{\gamma} < \gamma$, $x_D^* \geq 0$

2. Constant costs with per unit cost decreasing with R&D:

   $C[\underline{\gamma}, q_{WD}, x_D^*] = (\underline{\gamma} - x_D^*)q_{WD}$, $C[\gamma, q_{LD}] = (\gamma)q_{LD}$. Assume, for the constant cost cases, that $a > \gamma > \underline{\gamma} > 0$ and $\underline{\gamma} > x_D$

3. Constant costs with lump sump reduction in total cost:

   $C[\underline{\gamma}, q_{WD}, x_D^*] = (\underline{\gamma})q_{WD} - x_D^*$, $C[\gamma, q_{LD}] = (\gamma)q_{LD}$

Substituting the different cost specifications into equations (III.7) and (III.8), and from the first order condition for a maximum of the profit expression, we can solve for the equilibrium quantities produced by the two firms in the second stage given x_D^* and that a winner to the R&D race has been determined.

1. If increasing costs then

$$q_{WD}^*[x_D^*] = \frac{a(1+x_D^*)(1+\gamma)}{(1+x_D^*)(3+2\gamma)+\underline{\gamma}(2+\gamma)} \tag{A.3}$$

$$q_{LD}^*[x_D^*] = \frac{a(1+x_D^*+\underline{\gamma})}{(1+x_D^*)(3+2\gamma)+\underline{\gamma}(2+\gamma)} \tag{A.4}$$

2. If constant costs than

$$q_{WD}^*[x_D^*] = \frac{a-2\underline{\gamma}+\gamma+2x_D^*}{3} \tag{A.5}$$

$$q_{LD}^*[x_D^*] = \frac{a+\underline{\gamma}-2\gamma-x_D^*}{3} \tag{A.6}$$

3. If constant costs with lump sump reduction due to R&D

$$q_{WD}^*[x_D^*] = \frac{a-2\underline{\gamma}+\gamma}{3} \tag{A.7}$$

$$q_{LD}^*[x_D^*] = \frac{a+\underline{\gamma}-2\gamma}{3} \tag{A.8}$$

It is easily verified that under the three regimes, $q_{WD}^*[x_D^*] \geq q_{LD}^*[x_D^*]$, $W_{WD}^*[x_D^*] \geq W_{LD}^*[x_D^*]$, $\partial W_{WD}^*[x_D^*]/\partial x_D^* > 0$ and $\partial W_{LD}^*[x_D^*]/\partial x_D^* < 0$.

More on Assumption 3:

When the mixed duopoly private firm wins, it faces no rival and its maximization problem is simply the maximization of a monopolist. That is it maximizes the following profit expression

$$\max_{q_{WP}} W_{WP} = Pq_{WP} - C[\overline{y}, q_{WP}, \overset{*}{x_P}] \tag{A.9}$$

From the first order condition, the private firm chooses that output level for which $MR = MC$. At that output level, profits and the public welfare can be calculated[1]. On the other hand if the public sector firm emerges as the winner, since it does not participate in the second stage it necessarily must introduce the technology through a licensing arrangement with the private firm. The licensing arrangement is such that it maximizes the public welfare subject to the incentive compatibility condition of the private firm. The output level that comes from solving this maximization is then used to calculate the equilibrium profits for the private firm and welfare for the public sector firm. The maximization problem for the public sector firm then is written as

$$\max_{q_{LP}} \ W_{WS} = \int_0^{Q^*} P(Q)dQ - P^*Q^* + W_{LP}[x_S] + F \quad \text{s.t. } W_{LP}[x_S] \geq W_P \tag{A.10}$$

where

$\int_0^{Q^*} P[Q]dQ - P^*Q^*$ is the consumer surplus

F is the licensing fee charged by the public sector firm

$W_{LP}[x_D] = Pq_{LP} - C[\overline{y}, q_{LP}, \overset{*}{x_M}] - F$ is the private firm's second stage profits when it licenses the technology from the public sector firm (the winner)

[1] Public sector firm's welfare in the second stage is defined as the summation of consumer and producer surplus

$W_P = Pq_P - C[\gamma, q_P] = W_M$ is the private firms pre-innovation profits, where we assume that pre-innovation profits of a private firm is equivalent to that of the monopolist, pre-innovation.

From the first order conditions for a maximum of the public sector firm's objective function eqn. (A.10), the equilibrium levels of quantity produced and profits can be calculated. It can be shown (see appendix) that in equilibrium

$W^*_{WS}[x^*_S] > W^*_{LS}[x^*_P] \geq W^*_{WP}[x^*_P] \geq W^*_{LP}[x^*_S]$ and $q^*_{LP}[x^*_S] \geq q^*_{WP}[x^*_P]$.

To show that $W^*_{WS}[x^*_S] > W^*_{LS}[x^*_P] \geq W^*_{WP}[x^*_P] \geq W^*_{LP}[x^*_S]$, consider Figures A.1 and A.2 below which assumes a linear inverse demand curve and increasing marginal cost. If the winner of the R&D race is the private firm than it takes on a monopoly position (in the absence of another firm). Monopoly profits are maximized where $MR = MC$ (figure A.1), in which case monopoly profits ($W^*_{WP}[x^*_P]$) are given by area C and consumer surplus is given by area A. The public sector firm's welfare is simply the aggregation of monopolist's profits and consumer surplus, i.e. $W^*_{LS}[x^*_P] = A + C$.

Next consider the case where the winner of the R&D race is the public sector firm that must license the technology to the private firm. However to prevent monopoly pricing, and the associated welfare loss, the public sector firm dictates the terms of the licenses such that welfare is maximized (the first best welfare outcome) which occurs where $P = MC$. In this case, the licensees' profits ($W^*_{LP}[x^*_P]$) are given by area E and the consumer surplus is given by area D in Figure A.2. Comparing the two figures, it is clear that $W^*_{WS}[x^*_P] = D + E > W^*_{LS}[x^*_P] = A + C$. Moreover $W^*_{LP}[x^*_P] = E < W^*_{WP}[x^*_P] = E$.

Figure A.1: Second stage mixed market if private firm is the winner of the R&D race

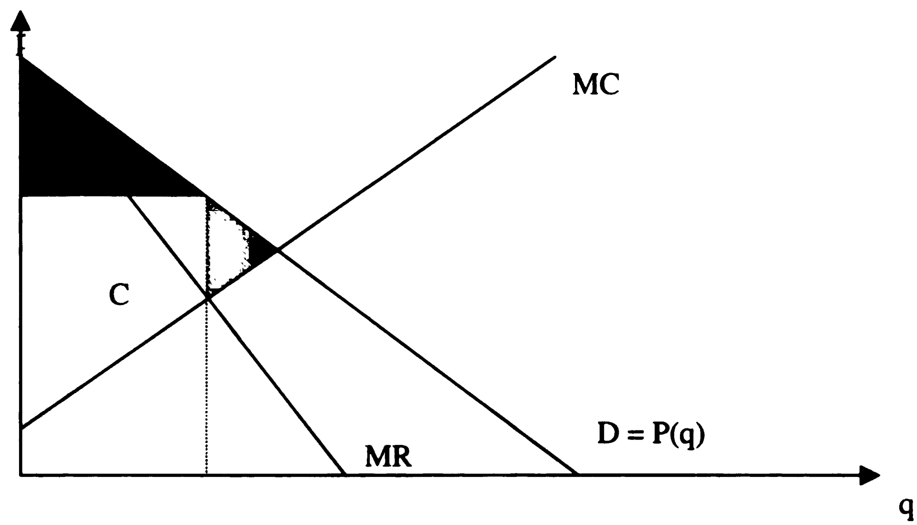


Figure A.2: Second stage mixed market if public firm is the winner of the R&D race

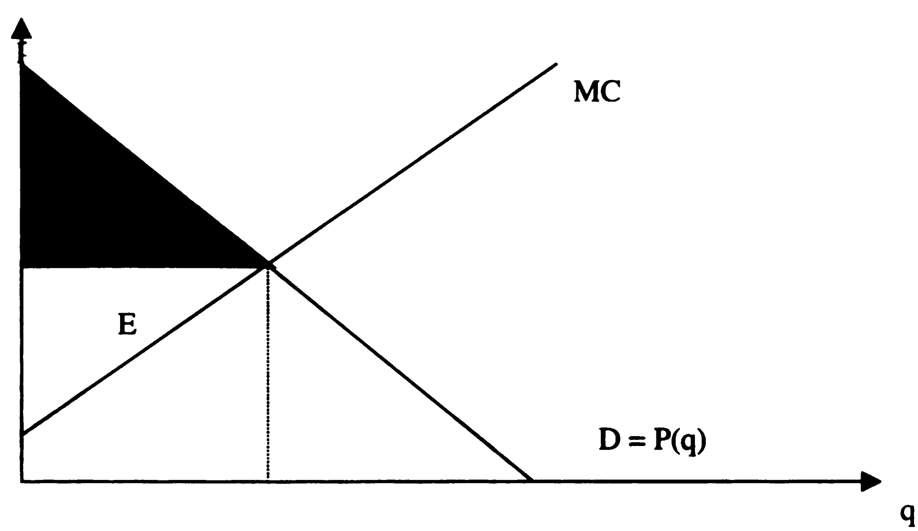


Table AIII.1. Sufficient Conditions for Ranking Monopoly and Pure Duopoly Research Effort

$x_M^* < x_D^*$ (implies $x_M^* < 2x_D^*$)	$x_M^* > 2x_D^*$
1. The profit function of the winning doupolist is increasing at a faster rate in x than the monopolist's profit function (i.e $\frac{\partial W_M[x_D^*]}{\partial x} < \frac{\partial W_{WD}[x_D^*]}{\partial x}$) 2. Relative to their respective pre-innovation profits, the duopoly winner makes more profits than a monopolist. $\lvert W_D - W_{WD}[x_D^*]\rvert > \lvert W_M - W_M[x_D^*]\rvert$ 3. $\frac{r}{h[x_D^*]} < 2W_{WD}'[x_D^*] - W_M'[x_D^*]$	1. The difference between the duopoly winner and loser is not too big (i.e. $W_{LD}[x_D^*] - W_{WD}[x_D^*] \approx 0$) 2. Relative to their respective pre-innovation profits, the duopoly winner makes lower profits than a monopolist. $\lvert W_D - W_{WD}[x_D^*]\rvert < \lvert W_M - W_M[x_D^*]\rvert$ 3. The slope of the profit function is relatively constant (with respect to x such that $\frac{\partial W_M[2x_D^*]}{\partial x} \geq 2\frac{\partial W_{WD}[x_D^*]}{\partial x}$ holds

Table AIII.2. Sufficient Conditions for Ranking Monopoly and Mixed Duopoly Research Effort

$x_P^* > x_M^*$ and $x_S^* > x_M^*$ (implies $x_P^* + x_S^* > x_M^*$)	$x_P^* < x_M^*$ and $x_S^* < x_M^*$ (implies $x_P^* + x_S^* < x_M^*$)
1. Post-innovation, public welfare increases faster than monopoly profits (i.e. $W_{WS}'[x_M^*] \geq W_M'[x_M^*]$ and $\left\|W_{M1} - W_M[x_M^*]\right\| < \left\|W_{WS}[x_M^*] - W_{S1}\right\|$)	1. Post-innovation, monopoly profits increases faster than public welfare (i.e. $W_{WS}'[x_M^*] \leq W_M'[x_M^*]$ and $\left\|W_{M1} - W_M[x_M^*]\right\| > \left\|W_{WS}[x_M^*] - W_{S1}\right\|$)
2. The hazard rate is relatively constant such that $x_P^* h'[x_M^*] \geq h[x_P^*]$.	2. Post innovation profits for the private firm is the same regardless of whether it wins or losses (i.e. $W_{WP}[x] = W_{LP}[x]$)
3. Public welfare is (strictly) larger when the public sector firm wins relative to when it loses. $W_{WS}[x_M^*] > W_{LS}[x_P^*]$	3. $rh[x_M^*]W_{WP}'[x_M^*] < 1$
4. $rh[x_M^*]W_{WP}'[x_M^*] > 1$	

Table AIII.3a. Sufficient Conditions for Ranking Pure Duopoly and Mixed Duopoly Research Effort

$x_P^* > x_D^* > x_S^*$	$x_P^* < x_D^* < x_S^*$
1. Post innovation profits in the duopoly are equal across states ($W_{LD}[x_D^*] = W_{WD}[x_D^*]$).	1. Post innovation profits in the duopoly are equal across states ($W_{LD}[x_D^*] = W_{WD}[x_D^*]$).
2. Post innovation welfare for the public sector firm in the mixed duopoly is equal across states ($W_{LS}[x_D^*] = W_{WS}[x_P^*]$).	2. Post innovation profits for the private firm in the mixed duopoly are equal across states ($W_{LP}[x_D^*] = W_{WP}[x_S^*]$).
3. Private firm's profits in the mixed duopoly are always greater in the winning state than in the losing state	3. Public welfare is always greater in the winning state than in the losing state.
4. Relative to the pre-innovation profits the winning firm in a pure duopoly gains less than the winning private firm in the mixed duopoly, such that $W'_{WP}[x_D^*] > W'_{WD}[x_D^*]$ and $\lvert W_D - W_{WD}[x_D^*]\rvert < \lvert W_{WP}[x_D^*] - W_P\rvert$.	4. Relative to the pre-innovation profits the winning firm in a pure duopoly gains more than the winning private firm in the mixed duopoly, such that $W'_{WP}[x_D^*] < W'_{WD}[x_D^*]$ and $\lvert W_D - W_{WD}[x_D^*]\rvert > \lvert W_{WP}[x_D^*] - W_P\rvert$.
5. Relative to the pre-innovation profits the winning firm in a pure duopoly gains more than the welfare gains of the winning public sector firm in the mixed duopoly, such that $W'_{WS}[x_D^*] < W'_{WD}[x_D^*]$ and $\lvert W_D - W_{WD}[x_D^*]\rvert < \lvert W_{WS}[x_D^*] - W_S\rvert$.	5. Relative to the pre-innovation profits the winning firm in a pure duopoly gains less than the welfare gains of the winning public sector firm in the mixed duopoly, such that $W'_{WS}[x_D^*] > W'_{WD}[x_D^*]$ and $\lvert W_D - W_{WD}[x_D^*]\rvert > \lvert W_{WS}[x_D^*] - W_S\rvert$.
6. $W'_{WP}[x_D^*](h[x_D^*] + h[x_S^*]) > W'_{WD}[x_D^*](2h[x_D^*])$	6. $W'_{WP}[x_D^*](h[x_D^*] + h[x_S^*]) < W'_{WD}[x_D^*](2h[x_D^*])$
7. $W'_{WS}[x_D^*](h[x_D^*] + h[x_P^*]) < W'_{WD}[x_D^*](2h[x_D^*]) + 1$	7. $W'_{WS}[x_D^*](h[x_D^*] + h[x_P^*]) > W'_{WD}[x_D^*](2h[x_D^*])$
8. $x_P^* h'[x_D^*] < h[x_P^*]$	

Table AIII.3b. Sufficient Conditions for Ranking Pure Duopoly and Mixed Duopoly Research Effort

$x_D > x_S, x_P$ (implies $2x_D < x_S + x_P$)	$x_D < x_S, x_P$ (implies $2x_D < x_S + x_P$)
1. The profits from winning are strickly greater than that from losing in the duopoly case. ($W_{LD}[x_D^*] < W_{WD}[x_D^*]$)	1. The profits from winning or losing in the duopoly case are equal ($W_{LD}[x_D^*] = W_{WD}[x_D^*]$)
2. The profits/welfare from winning or losing in a mixed duopoly are equal. $W_{LS}[x_P^*] = W_{WS}[x_D^*]$ and $W_{LP}[x_S^*] = W_{WP}[x_D^*]$	2. The profits/welfare from winning are strickly greater than losing in a mixed duopoly. $W_{LS}[x_P^*] < W_{WS}[x_D^*]$ and $W_{LP}[x_S^*] < W_{WP}[x_D^*]$
3. The change in profits (welfare) for the private (public) firm in the mixed duopoly is less than that for the doupolist. That is $W'_{WS}[x_D^*] < W'_{WD}[x_D^*]$ and $W'_{WP}[x_D^*] < W'_{WD}[x_D^*]$.	3. The change in profits (welfare) for the private (public) firm in the mixed duopoly is greater than that for the doupolist. That is $W'_{WS}[x_D^*] > W'_{WD}[x_D^*]$ and $W'_{WP}[x_D^*] > W'_{WD}[x_D^*]$.
4. $\begin{bmatrix} h[x_D^*](1 + W'_{WP}[x_D^*](h[x_D^*] + h[x_S^*]) - \\ W'_{WD}[x_D^*](2h[x_D^*])) \end{bmatrix} < h[x_S^*]$	4. $\begin{bmatrix} h[x_D^*](1 + W'_{WP}[x_D^*](h[x_D^*] + h[x_S^*]) - \\ W'_{WD}[x_D^*](2h[x_D^*])) \end{bmatrix} > h[x_S^*]$
5. $\begin{bmatrix} 1 + W'_{WP}[x_D^*](h[x_D^*] + h[x_P^*]) - \\ W'_{WD}[x_D^*](2h[x_D^*]) \end{bmatrix} > \dfrac{h[x_P^*] - x_P^* h'[x_D^*]}{h[x_D^*]}$	

Chapter IV

1. Under Regime II, comparative statics in the noncooperative case are:

$$\frac{\partial x_i}{\partial \beta_P} = \frac{2(a-A)\left(2\left((\beta_P-2)^2+4\beta_S^2\right)-9b\gamma\right)}{\left(4+2\beta_P(1-\beta_P)+8\beta_S^2-9b\gamma\right)^2} < 0 \text{ for “large” } b\gamma$$

$$\frac{\partial x_i}{\partial \beta_S} = -\frac{32(a-A)(\beta_P-2)\beta_S}{\left(4+2\beta_P(1-\beta_P)+8\beta_S^2-9b\gamma\right)^2} > 0$$

$$\frac{\partial x_S}{\partial \beta_S} = \frac{8(a-A)\left(2\beta_P(\beta_P-1)+8\beta_S^2+9b\gamma-4\right)}{\left(4+2\beta_P(1-\beta_P)+8\beta_S^2-9b\gamma\right)^2} > 0 \text{ for “large” } b\gamma$$

$$\frac{\partial x_S}{\partial \beta_S} = -\frac{16\beta_S(a-A)(2\beta_P-1)}{\left(4+2\beta_P(1-\beta_P)+8\beta_S^2-9b\gamma\right)^2} \le 0 \text{ for } \beta_P \le 0.5 \text{ (and } > 0 \text{ for } \beta_P > 0.5\text{)}$$

2. Definition of F_1, F_2, F_3 and D

$$F_1[x_P, x_S] \equiv \frac{1}{2b(w_C+w_P-3w_S)^2} \times \begin{pmatrix} (a-c_P)^2 w_C w_P (2w_C+w_P) - \\ 2w_S(a-c_P)\left(w_C^2(a-c_S)+w_C w_P(3a-4c_P+c_S)+w_P^2(c_S-c_P)\right)+ \\ w_S^2\left(w_C\left(4a^2-c_P^2+8c_Pc_S-3c_S^2-2a(3c_P+c_S)\right)-2w_P(5a-4c_P-c_S)(c_P-c_S)\right)+ \\ 2w_S^3(a+c_P-2c_S)^2 \end{pmatrix}$$

$$F_2[x_{P1},x_{P2},x_S] \equiv \frac{1}{18b} \times \begin{pmatrix} 8(a-A)^2 + 8(a-A)\big((1+\beta_P)(x_{P1}+x_{P2}) + 2x_S\beta_S\big) - \\ 2x_{P1}x_{P1}(7+\beta_P(7\beta_P - 22)) + (x_{P1}^2 + x_{P2}^2)(11+\beta_P(11\beta_P - 14)) + \\ 8x_S(1+\beta_P)(x_{P1}+x_{P2}) + 8(x_S\beta_S)^2 \end{pmatrix}$$

$$F_3[x_{P1},x_{P2},x_S] \equiv \frac{1}{18b} \times \begin{pmatrix} 10(a-A)^2 + 4(a-A)\Big((x_{P1}(1+4\beta_P) + x_{P2}(4+\beta) + 5x_S\beta_S\Big) - \\ 2x_{P1}x_{P1}(11+\beta_P(11\beta_P - 32)) + x_{P2}^2(19+\beta_P(13\beta_P - 22)) + \\ x_{P1}^2(13+\beta_P(19\beta_P - 22)) + 4x_{P2}x_S(4+\beta_P)\beta_S + \\ 4x_{P1}x_S(1+4\beta_P)\beta_S + 10x_S^2\beta_S^2 \end{pmatrix}$$

$$D \equiv 2(\beta_P - 1)\Big(6 + 15\beta_S^2 + \beta_P(1+\beta_P(\beta_P - 4) - 11\beta_S^2)\Big) + b\gamma(35 - 38\beta_P + 17\beta_S^2 + 20\beta_S^2) - 18(b\gamma)^2$$

3. Tables[1]:

Table AIV.3: Nash Equilibrium Values for the Noncooperative Case in Regime 1.

$\beta_S \rightarrow$ / $\beta_P \downarrow$		0.05	0.25	0.50	0.75	0.95
0.05	x_P^*	0.204412	0.229652	0.257525	0.284426	0.306962
	x_S^*	0.418009	0.398702	0.389502	0.393119	0.404437
	X^*	0.622421	0.628354	0.647027	0.677545	0.711399
	q_P^*	0.70487	0.791903	0.888018	0.980779	1.05849
	q_S^*	1.81557	1.74552	1.67624	1.61771	1.5742
	Q^*	2.52044	2.53742	2.56426	2.59849	2.63269
	c_S^*	6.57177	6.58982	6.59762	6.59266	6.58021
	c_P^*	6.77469	6.67067	6.54772	6.42073	6.30882
	π_P^*	0.39238	0.495261	0.622778	0.759682	0.884838
	$V[Q^*]$	3.17632	3.21926	3.28771	3.37607	3.46552
	π_S^*	1.21132	1.12601	1.02561	0.922133	0.830124
	W^*	5.99135	5.96655	5.96171	5.98001	6.01061
	$P[Q^*]$	7.47956	7.46258	7.43574	7.40151	7.36731
0.25	x_P^*	0.16224	0.183723	0.207294	0.229684	0.248098
	x_S^*	0.437709	0.415794	0.403071	0.402941	0.410971
	X^*	0.599949	0.599517	0.610365	0.632625	0.659069
	q_P^*	0.64896	0.734891	0.829175	0.918736	0.992393
	q_S^*	1.88621	1.81789	1.75048	1.69442	1.65374
	Q^*	2.53517	2.55278	2.57965	2.61315	2.64613
	c_S^*	6.52173	6.53828	6.54511	6.53964	6.527
	c_P^*	6.81587	6.71233	6.59117	6.46811	6.36148
	π_P^*	0.355344	0.45568	0.580104	0.71219	0.830961
	$V[Q^*]$	3.21353	3.25834	3.32731	3.41428	3.501
	π_S^*	1.29991	1.22015	1.12592	1.02962	0.945177
	W^*	6.16871	6.15432	6.15926	6.18572	6.22231
	$P[Q^*]$	7.46483	7.44722	7.42035	7.38685	7.35387
0.50	x_P^*	0.120058	0.13684	0.155172	0.172366	0.186282

[1] Tables IV.3 to IV.10 are all based on calculating the equilibrium using the following parameter values $a=10$, $A=7$, $b=1$, and $\gamma=5$. Note that $X=\sum x_i$, $Q=\sum q_i$ and all other values are defined in the text.

	x_S^*	0.453494	0.429507	0.413793	0.410338	0.415382
	X^*	0.573552	0.566347	0.568966	0.582704	0.601664
	q_P^*	0.600288	0.684199	0.775862	0.861829	0.93141
	q_S^*	1.94216	1.87582	1.81034	1.75646	1.71808
	Q^*	2.54244	2.56002	2.58621	2.61829	2.64949
	c_S^*	6.48648	6.50207	6.50862	6.50348	6.49148
	c_P^*	6.85727	6.75578	6.63793	6.51988	6.4191
	π_P^*	0.324311	0.421315	0.541766	0.668474	0.780772
	$V[Q^*]$	3.23201	3.27684	3.34423	3.42772	3.50989
	π_S^*	1.37184	1.29816	1.21061	1.12163	1.04454
	W^*	6.30001	6.29447	6.30722	6.33947	6.37973
	$P[Q^*]$	7.45756	7.43998	7.41379	7.38171	7.35051
0.75	x_P^*	0.0852334	0.0975079	0.110878	0.123313	0.133271
	x_S^*	0.46214	0.436841	0.419164	0.41346	0.416446
	X^*	0.547373	0.534349	0.530042	0.536772	0.549717
	q_P^*	0.568223	0.650052	0.739184	0.822083	0.888471
	q_S^*	1.97189	1.90661	1.84209	1.78924	1.75195
	Q^*	2.54012	2.55667	2.58128	2.61132	2.64042
	c_S^*	6.47394	6.49003	6.49768	6.49406	6.4836
	c_P^*	6.89166	6.79328	6.67954	6.56659	6.47111
	π_P^*	0.304715	0.398799	0.515658	0.637806	0.744979
	$V[Q^*]$	3.2261	3.26827	3.33149	3.40951	3.48592
	π_S^*	1.41025	1.34051	1.25741	1.17332	1.1011
	W^*	6.35132	6.34809	6.36196	6.39395	6.43309
	$P[Q^*]$	7.45988	7.44333	7.41872	7.38868	7.35958
0.95	x_P^*	0.0606795	0.0695127	0.0791146	0.0880039	0.0950821
	x_S^*	0.464984	0.438985	0.420266	0.413356	0.415224
	X^*	0.525663	0.508498	0.499381	0.501359	0.510306
	q_P^*	0.551632	0.631933	0.719223	0.800036	0.864383
	q_S^*	1.98067	1.91539	1.8508	1.79795	1.76078
	Q^*	2.5323	2.54733	2.57002	2.59798	2.62516
	c_S^*	6.47737	6.49498	6.50458	6.50304	6.49445
	c_P^*	6.91607	6.82074	6.71075	6.60198	6.51046
	π_P^*	0.295093	0.38726	0.501634	0.620696	0.724556
	$V[Q^*]$	3.20626	3.24443	3.30251	3.37476	3.44574

	π_S^*	1.42099	1.35259	1.27117	1.18915	1.11915
	W^*	6.34334	6.33688	6.34649	6.37376	6.40858
	$P[Q^*]$	7.4677	7.45267	7.42998	7.40202	7.37484

Table AIV.4. Nash Equilibrium Values for the Cooperative Case in Regime 1.

$\beta_S \rightarrow$ / $\beta_P \downarrow$		0.05	0.25	0.50	0.75	0.95
0.05	x_P^*	0.09456	0.120671	0.150997	0.18405	0.215795
	x_S^*	0.357304	0.356545	0.37409	0.409371	0.45238
	X^*	0.451864	0.477217	0.525088	0.593421	0.668175
	q_P^*	0.653303	0.726066	0.812712	0.909022	1.00258
	q_S^*	1.80582	1.75768	1.71262	1.67303	1.64039
	Q^*	2.45912	2.48374	2.52533	2.58206	2.64297
	c_S^*	6.63797	6.63742	6.61836	6.58143	6.53683
	c_P^*	6.88757	6.79019	6.66196	6.50892	6.35444
	π_P^*	0.404451	0.490768	0.6035	0.741635	0.888752
	$V[Q^*]$	3.02364	3.08449	3.18865	3.33351	3.49266
	π_S^*	1.31133	1.2269	1.11667	0.98056	0.833823
	W^*	6.05074	6.02905	6.02549	6.03626	6.04905
	$P[Q^*]$	7.54088	7.51626	7.47467	7.41794	7.35703
0.25	x_P^*	0.179065	0.197152	0.21997	0.246092	0.271666
	x_S^*	0.348215	0.354874	0.377775	0.416286	0.460788
	X^*	0.527279	0.552026	0.597746	0.662377	0.732455
	q_P^*	0.700866	0.762322	0.84026	0.929825	1.01771
	q_S^*	1.79474	1.76123	1.72834	1.69866	1.674
	Q^*	2.49561	2.52355	2.5686	2.62848	2.69171
	c_S^*	6.60702	6.59584	6.56723	6.52219	6.4713
	c_P^*	6.80352	6.71413	6.59114	6.44169	6.29058
	π_P^*	0.411053	0.483962	0.585069	0.713172	0.851225
	$V[Q^*]$	3.11403	3.18415	3.29885	3.45446	3.62264
	π_S^*	1.30742	1.23612	1.13679	1.00948	0.870319
	W^*	6.13992	6.14035	6.1575	6.18659	6.2145
	$P[Q^*]$	7.50439	7.47645	7.4314	7.37152	7.30829
0.50	x_P^*	0.272536	0.28555	0.302799	0.323077	0.343091
	x_S^*	0.359341	0.367827	0.391858	0.430769	0.475023

	X^*	0.631877	0.653376	0.694656	0.753846	0.818114
	q_P^*	0.720073	0.777829	0.852417	0.938462	1.02249
	q_S^*	1.85036	1.82185	1.79389	1.76923	1.74939
	Q^*	2.57043	2.59968	2.64631	2.70769	2.77188
	c_S^*	6.50439	6.4894	6.45674	6.40769	6.35343
	c_P^*	6.7095	6.62249	6.50127	6.35385	6.20564
	π_P^*	0.332815	0.401171	0.497397	0.619763	0.751203
	$V[Q^*]$	3.30356	3.37916	3.50148	3.6658	3.84165
	π_S^*	1.3891	1.32132	1.22515	1.10118	0.966061
	W^*	6.41456	6.42298	6.44917	6.48793	6.52497
	$P[Q^*]$	7.42957	7.40032	7.35369	7.29231	7.22812
0.75	x_P^*	0.372782	0.382002	0.394778	0.410192	0.425557
	x_S^*	0.391542	0.396238	0.41671	0.452698	0.494644
	X^*	0.764323	0.778239	0.811488	0.86289	0.920201
	q_P^*	0.708705	0.769426	0.845953	0.932115	1.0147
	q_S^*	1.97495	1.94221	1.91123	1.88548	1.86608
	Q^*	2.68365	2.71163	2.75718	2.8176	2.88077
	c_S^*	6.32887	6.31726	6.28721	6.23966	6.18619
	c_P^*	6.60764	6.51894	6.39687	6.25028	6.10453
	π_P^*	0.154848	0.227204	0.326012	0.448197	0.576861
	$V[Q^*]$	3.601	3.67648	3.80102	3.96943	4.14943
	π_S^*	1.56695	1.49358	1.39228	1.26519	1.12944
	W^*	6.88974	6.89084	6.91159	6.948	6.98517
	$P[Q^*]$	7.31635	7.28837	7.24282	7.1824	7.11923
0.95	x_P^*	0.470929	0.476194	0.484629	0.495701	0.507195
	x_S^*	0.435395	0.432718	0.446615	0.477565	0.515951
	X^*	0.906324	0.908912	0.931243	0.973266	1.02315
	q_P^*	0.678135	0.745729	0.827446	0.916166	0.999118
	q_S^*	2.13643	2.09292	2.05304	2.02154	1.99911
	Q^*	2.81456	2.83864	2.88049	2.93771	2.99823
	c_S^*	6.11722	6.1149	6.09299	6.05152	6.00221
	c_P^*	6.5073	6.41563	6.29206	6.14613	6.00265
	π_P^*	-0.0945683	-0.0107907	0.0975048	0.225061	0.355121
	$V[Q^*]$	3.96088	4.02895	4.14861	4.31507	4.49469
	π_S^*	1.80824	1.72204	1.60883	1.47315	1.33271
	W^*					

	$P[Q^*]$	7.4828 7.18544	7.46223 7.16136	7.46378 7.11951	7.48643 7.06229	7.51523 7.00177

Table AIV.5: Comparing Equilibrium Values Between Cooperative and Noncooperative Cases Under Regime 1

$\beta_S \rightarrow$ / $\beta_P \downarrow$	Low (0.05)	Medium (0.5)	High (0.95)
Low (0.05)	$X^C < X^{NC}$ $V^C < V^{NC}$ $\pi_P^C > \pi_P^{NC}$ $W^C > W^{NC}$	$X^C < X^{NC}$ $V^C < V^{NC}$ $\pi_P^C < \pi_P^{NC}$ $W^C > W^{NC}$	$X^C < X^{NC}$ $V^C > V^{NC}$ $\pi_P^C > \pi_P^{NC}$ $W^C > W^{NC}$
Medium (0.5)	$X^C > X^{NC}$ $V^C > V^{NC}$ $\pi_P^C > \pi_P^{NC}$ $W^C > W^{NC}$	$X^C > X^{NC}$ $V^C > V^{NC}$ $\pi_P^C < \pi_P^{NC}$ $W^C > W^{NC}$	$X^C > X^{NC}$ $V^C > V^{NC}$ $\pi_P^C < \pi_P^{NC}$ $W^C > W^{NC}$
High (0.95)	$X^C > X^{NC}$ $V^C > V^{NC}$ $\pi_P^C < \pi_P^{NC}$ $W^C > W^{NC}$	$X^C > X^{NC}$ $V^C > V^{NC}$ $\pi_P^C < \pi_P^{NC}$ $W^C > W^{NC}$	$X^C > X^{NC}$ $V^C > V^{NC}$ $\pi_P^C < \pi_P^{NC}$ $W^C > W^{NC}$

Table AIV.6. Nash Equilibrium Values for the Noncooperative Case in Regime 2.

$\beta_S \rightarrow$ / $\beta_P \downarrow$		0.05	0.25	0.50	0.75	0.95
0.05	x_{Pi}^*	0.286169	0.289568	0.300733	0.321384	0.347336
	x_S^*	0.0293506	0.148496	0.308444	0.494438	0.676859
	X^*	0.601688	0.727633	0.909909	1.13721	1.37153
	q_{Pi}^*	1.10065	1.11372	1.15666	1.23609	1.33591
	Q^*	2.2013	2.22745	2.31333	2.47219	2.67181
	c_{Pi}^*	6.69806	6.65883	6.53001	6.29172	5.99228
	π_{Pi}^*	1.0067	1.03076	1.11177	1.26971	1.48304
	$V[Q^*]$	2.42285	2.48076	2.67574	3.05586	3.56929
	W^*	4.43409	4.48714	4.66144	4.9841	5.39003
	$P[Q^*]$	7.7987	7.77255	7.68667	7.52781	7.32819
0.25	x_{Pi}^*	0.258589	0.261682	0.271845	0.290657	0.314324
	x_S^*	0.029553	0.149533	0.31068	0.49827	0.682533
	X^*	0.546731	0.672897	0.854369	1.07958	1.31118
	q_{Pi}^*	1.10824	1.1215	1.16505	1.24567	1.3471
	Q^*	2.21648	2.24299	2.3301	2.49135	2.69421
	c_{Pi}^*	6.67529	6.63551	6.50485	6.26298	5.95869
	π_{Pi}^*	1.06102	1.08656	1.17259	1.3405	1.56769
	$V[Q^*]$	2.45638	2.5155	2.71468	3.10341	3.62938
	W^*	4.57624	4.63272	4.81855	5.16373	5.60013
	$P[Q^*]$	7.78352	7.75701	7.6699	7.50865	7.30579
0.50	x_{Pi}^*	0.222332	0.225	0.233766	0.25	0.270433
	x_S^*	0.0296443	0.15	0.311688	0.5	0.685096
	X^*	0.474308	0.6	0.779221	1.	1.22596
	q_{Pi}^*	1.11166	1.125	1.16883	1.25	1.35216
	Q^*	2.22332	2.25	2.33766	2.5	2.70433
	c_{Pi}^*	6.66502	6.625	6.49351	6.25	5.94351
	π_{Pi}^*	1.11221	1.13906	1.22955	1.40625	1.64551
	$V[Q^*]$	2.47158	2.53125	2.73233	3.125	3.65669
	W^*	4.6938	4.75313	4.94856	5.3125	5.77432
	$P[Q^*]$	7.77668	7.75	7.66234	7.5	7.29567
0.75	x_{Pi}^*	0.184706	0.186916	0.194175	0.207612	0.224517
	x_S^*	0.029553	0.149533	0.31068	0.49827	0.682533
	X^*	0.398966	0.523364	0.699029	0.913495	1.13157

	q_{Pi}^*	1.10824	1.1215	1.16505	1.24567	1.3471
	Q^*	2.21648	2.24299	2.3301	2.49135	2.69421
	c_{Pi}^*	6.67529	6.63551	6.50485	6.26298	5.95869
	π_{Pi}^*	1.1429	1.17041	1.26308	1.44395	1.68867
	$V[Q^*]$	2.45638	2.5155	2.71468	3.10341	3.62938
	W^*	4.74	4.80042	4.99953	5.37063	5.84209
	$P[Q^*]$	7.78352	7.75701	7.6699	7.50865	7.30579
0.95	x_{Pi}^*	0.154091	0.155921	0.161933	0.173053	0.187027
	x_S^*	0.0293506	0.148496	0.308444	0.494438	0.676859
	X^*	0.337532	0.460339	0.632309	0.840544	1.05091
	q_{Pi}^*	1.10065	1.11372	1.15666	1.23609	1.33591
	Q^*	2.2013	2.22745	2.31333	2.47219	2.67181
	c_{Pi}^*	6.69806	6.65883	6.53001	6.29172	5.99228
	π_{Pi}^*	1.15207	1.1796	1.27232	1.45306	1.6972
	$V[Q^*]$	2.42285	2.48076	2.67574	3.05586	3.56929
	W^*	4.72483	4.78484	4.98253	5.3508	5.81834
	$P[Q^*]$	7.7987	7.77255	7.68667	7.52781	7.32819

Table AIV.7: Nash Equilibrium Values for the Cooperative Case in Regime 2.

$\beta_S \rightarrow$ / $\beta_P \downarrow$		0.05	0.25	0.50	0.75	0.95
0.05	x_{P1}^*	0.300122	0.304608	0.319532	0.347944	0.38521
	x_{P2}^*	0.284898	0.289156	0.303323	0.330294	0.36567
	x_S^*	0.0368146	0.186824	0.391955	0.64021	0.897788
	X^*	0.621834	0.780588	1.01481	1.31845	1.64867
	q_{P1}^*	1.11022	1.12682	1.18202	1.28713	1.42499
	q_{P2}^*	1.09576	1.11214	1.16663	1.27036	1.40642
	Q^*	2.20598	2.23895	2.34865	2.55749	2.83141
	c_{P1}^*	6.68379	6.63423	6.46932	6.15538	5.74361
	c_{P2}^*	6.69826	6.64891	6.48472	6.17215	5.76217
	π_{P1}^*	1.00741	1.03775	1.14193	1.35404	1.65962
	π_{P2}^*	0.997774	1.02782	1.131	1.34108	1.64374
	$V[Q^*]$	2.43318	2.50646	2.75808	3.27037	4.00843
	W^*	4.43498	4.48477	4.64694	4.94082	5.29673
	$P[Q^*]$	7.79402	7.76105	7.65135	7.44251	7.16859
0.25	x_{P1}^*	0.324667	0.329619	0.346116	0.377613	0.41911
	x_{P2}^*	0.25565	0.259549	0.272539	0.297341	0.330016
	x_S^*	0.0375567	0.190648	0.400378	0.65522	0.92115
	X^*	0.617874	0.779816	1.01903	1.33017	1.67028
	q_{P1}^*	1.14741	1.16491	1.22321	1.33452	1.48118
	q_{P2}^*	1.09564	1.11235	1.16802	1.27432	1.41436
	Q^*	2.24305	2.27726	2.39123	2.60884	2.89553
	c_{P1}^*	6.60954	6.55783	6.38556	6.05664	5.62329
	c_{P2}^*	6.66131	6.61038	6.44074	6.11684	5.69011
	π_{P1}^*	1.05302	1.08539	1.19675	1.42447	1.75475
	π_{P2}^*	1.03704	1.06892	1.17859	1.40286	1.72812
	$V[Q^*]$	2.51564	2.59296	2.859	3.40303	4.19205
	W^*	4.60218	4.6564	4.83357	5.15707	5.55363
	$P[Q^*]$	7.75695	7.72274	7.60877	7.39116	7.10447
0.50	x_{P1}^*	0.349429	0.354871	0.373022	0.407785	0.453797
	x_{P2}^*	0.222364	0.225827	0.237378	0.2595	0.28878
	x_S^*	0.0383314	0.194641	0.409194	0.670992	0.945824
	X^*	0.610125	0.775338	1.01959	1.33828	1.6884

	q_{P1}^*	1.17535	1.19366	1.25471	1.37164	1.52641
	q_{P2}^*	1.11182	1.12913	1.18689	1.2975	1.4439
	Q^*	2.28717	2.32279	2.4416	2.66914	2.97031
	c_{P1}^*	6.53747	6.48356	6.30369	5.95922	5.50328
	c_{P2}^*	6.601	6.54808	6.37151	6.03336	5.58579
	π_{P1}^*	1.0762	1.10998	1.22643	1.46568	1.8151
	π_{P2}^*	1.11253	1.14745	1.26783	1.51515	1.87637
	$V[Q^*]$	2.61558	2.69767	2.9807	3.56215	4.41137
	W^*	4.80065	4.86039	5.05637	5.4174	5.86638
	$P[Q^*]$	7.71283	7.67721	7.5584	7.33086	7.02969
0.75	x_{P1}^*	0.371817	0.377705	0.397369	0.435125	0.485292
	x_{P2}^*	0.19032	0.193334	0.203399	0.222725	0.248404
	x_S^*	0.0389714	0.197943	0.416496	0.684104	0.966439
	X^*	0.601108	0.768981	1.01726	1.34195	1.70013
	q_{P1}^*	1.18729	1.20609	1.26889	1.38945	1.54964
	q_{P2}^*	1.14192	1.16	1.22039	1.33635	1.49042
	Q^*	2.32921	2.3661	2.48928	2.7258	3.04007
	c_{P1}^*	6.48349	6.42781	6.24183	5.88475	5.41029
	c_{P2}^*	6.52887	6.4739	6.29033	5.93785	5.46951
	π_{P1}^*	1.06405	1.09801	1.21532	1.45723	1.81263
	π_{P2}^*	1.21342	1.25216	1.38593	1.66181	2.0671
	$V[Q^*]$	2.71261	2.79921	3.09826	3.71499	4.621
	W^*	4.98629	5.05142	5.26583	5.66404	6.16572
	$P[Q^*]$	7.67079	7.6339	7.51072	7.2742	6.95993
0.95	x_{P1}^*	0.39016	0.396407	0.417289	0.457451	0.510956
	x_{P2}^*	0.164541	0.167176	0.175982	0.19292	0.215484
	x_S^*	0.0394021	0.200165	0.421419	0.692968	0.980424
	X^*	0.594103	0.763749	1.01469	1.34334	1.70686
	q_{P1}^*	1.18657	1.20558	1.26908	1.39123	1.55395
	q_{P2}^*	1.17529	1.19411	1.25702	1.378	1.53917
	Q^*	2.36187	2.39969	2.5261	2.76922	3.09312
	c_{P1}^*	6.45156	6.39473	6.20482	5.83955	5.35293
	c_{P2}^*	6.46284	6.4062	6.21688	5.85278	5.3677
	π_{P1}^*	1.0274	1.06057	1.17524	1.41236	1.76206
	π_{P2}^*	1.31363	1.35604	1.50267	1.80584	2.25297

	$V[Q^*]$	2.78921	2.87926	3.19059	3.8343	4.7837
	W^*	5.12636	5.1957	5.42452	5.85199	6.39566
	$P[Q^*]$	7.63813	7.60031	7.4739	7.23078	6.90688

Table AIV.8: Comparing Equilibrium Values Between Cooperative and Noncooperative Case Under Regime 2

$\beta_S \rightarrow$ / $\beta_P \downarrow$	Low (0.05)	Medium (0.5)	High (0.95)
Low (0.05)	$X^C > X^{NC}$ $V^C > V^{NC}$ $\pi_{P1}^C > \pi_{P1}^{NC} = \pi_{P2}^{NC} > \pi_{P2}^C$ $W^C > W^{NC}$	$X^C > X^{NC}$ $V^C > V^{NC}$ $\pi_{P1}^C > \pi_{P2}^C > \pi_{P1}^{NC} = \pi_{P2}^{NC}$ $W^C < W^{NC}$	$X^C > X^{NC}$ $V^C > V^{NC}$ $\pi_{P1}^C > \pi_{P2}^C > \pi_{P1}^{NC} = \pi_{P2}^{NC}$ $W^C > W^{NC}$
Medium (0.5)	$X^C > X^{NC}$ $V^C > V^{NC}$ $\pi_{P1}^C < \pi_{P1}^{NC} = \pi_{P2}^{NC} < \pi_{P2}^C$ $W^C > W^{NC}$	$X^C > X^{NC}$ $V^C > V^{NC}$ $\pi_{P1}^C < \pi_{P1}^{NC} = \pi_{P2}^{NC} < \pi_{P2}^C$ $W^C > W^{NC}$	$X^C > X^{NC}$ $V^C > V^{NC}$ $\pi_{P2}^C > \pi_{P1}^C > \pi_{P1}^{NC} = \pi_{P2}^{NC}$ $W^C > W^{NC}$
High (0.95)	$X^C > X^{NC}$ $V^C > V^{NC}$ $\pi_{P1}^C < \pi_{P1}^{NC} = \pi_{P2}^{NC} < \pi_{P2}^C$ $W^C > W^{NC}$	$X^C > X^{NC}$ $V^C > V^{NC}$ $\pi_{P1}^C < \pi_{P1}^{NC} = \pi_{P2}^{NC} < \pi_{P2}^C$ $W^C > W^{NC}$	$X^C > X^{NC}$ $V^C > V^{NC}$ $\pi_{P2}^C > \pi_{P1}^C > \pi_{P1}^{NC} = \pi_{P2}^{NC}$ $W^C > W^{NC}$

Table AIV.9: Socially Efficient Values at Different β Values.

	0.05	0.25	0.5	0.75	0.95
X_{SW}^*	0.354032	0.444444	0.580645	0.756757	0.943929
Q^*	3.37173	3.55556	3.87097	4.32432	4.84066
SW	5.0576	5.33333	5.80645	6.48649	7.26099
C	6.62827	6.44444	6.12903	5.67568	5.15934
$P[Q^*]$	6.62827	6.44444	6.12903	5.67568	5.15934

Table AIV.10: Comparison of Nash-Equilibrium Output and Research Across Models

0.05	0.25
$X_1^{NC} > X_2^C > X_2^{NC} > X_1^C > X^{SW}$ $Q^{SW} > Q_1^{NC} > Q_1^C > Q_2^{NC} > Q_2^C$	$X_2^C > X_2^{NC} > X_1^C > X_1^{NC} > X^{SW}$ $Q^{SW} > Q_1^{NC} > Q_1^C > Q_2^{NC} > Q_2^C$

Table AIV.10 cont'd

0.5	0.75
$X_2^C > X_2^{NC} > X_1^C > X^{SW} > X_1^{NC}$ $Q^{SW} > Q_1^C > Q_1^{NC} > Q_2^C > Q_2^{NC}$	$X_2^C > X_2^{NC} > X_1^C > X^{SW} > X_1^{NC}$ $Q^{SW} > Q_1^C > Q_2^C > Q_1^{NC} > Q_2^{NC}$

Table AIV.10 cont'd

0.95
$X_2^C > X_2^{NC} > X_1^C > X^{SW} > X_1^{NC}$ $Q^{SW} > Q_2^C > Q_1^C > Q_2^{NC} > Q_1^{NC}$

BIBLIOGRAPHY

Al-Nowaihi, A. and P.L. Levine (1985), "The Stability of the Cournot Oligopoly Model: A Reassessment" *Journal of Economic Theory* 35:307-321.

Alston, J.M., B.J. Craig, and P.G. Pardey eds., (1997). *Paying for Agricultural Productivity.* Washington, DC: International Food Policy Research Institute.

Alston, J.M., P.G. Pardey, and V.H. Smith (1998). "Financing Agricultural R&D in Rich Countries: What's Happening and Why." *Australian Journal of Agricultural and Resource Economics* 42(1): 51-82.

Arrow, K.J. (1962). "Economic Welfare and the Allocation of Resources for Invention." In R. Nelson (ed.) *The Rate and Direction of Inventive Activity: Economic and Social Factors*, Princeton, NJ: Princeton University Press.

Azzam, M.A. and H. Andersson, (2001). "Market Power and Cost-Efficiency Effects of Concentration in a Mixed Oligopoly." Mimeo.

Bennetzen, J. (2002). "Opening the Door to Comparative Plant Biology" *Science* 296(5565):60-63.

Bernstein, J. I. and M. I. Nadiri (1989). "Research and Development and Intra-industry Spillovers: An Empirical Application of Dynamic Duality." *Review of Economic Studies* 56(2): 249-67

Brennan, M.F. C.E. Pray, and A. Courtmanche (2000), "Impact of Industry Concentration on Innovation in the U.S. Plant Biotech Industry." In W.H. Lesser (ed). *Transitions in Agbiotech: Economics of Strategy and Policy.* Storrs, CT: Food Marketing and Policy Center, University of Connecticut.

Butler, L.J. and M.W. Marion (1985), "Importance of Public Plant Breeders and Competition in the Seed Industry." *Journal of Agronomic Education.* 14:11-15.

Cohen, W. and J.P. Walsh (2000), "R&D Spillovers, Appropriability and R&D Intensity: A Survey Based Approach." Mimeo.

Cohen, L., and R.G. Noll (1995). "The Future of the National Research Labs." Paper presented at the National Academy of Sciences Colloquium on Science, Technology and the Economy, Irvine, CA, October 20-24.

d'Aspremont, C., and A. Jacquemin (1988). "Cooperative and Noncooperative R&D in Duopoly with Spillovers" *American Economic Review*, 78(5):1133-37.

Dasgupta, A. and J. Stiglitz (1980). "Industrial Structure and the Nature of Innovative Activity" *Economic Journal*, 90(358):266-93.

Day-Rubenstein, K. and K.O. Fuglie (2000), "The CRADA Model for Public-Private Research Technology Transfer in Agriculture" In K.O. Fuglie and D.E. Schimmelpfenning (eds.) *Public-Private Collaboration in Agricultural Research.* Ames, IA: Iowa State University Press.

De Bondt, R. (1997). "Spillovers and Innovative Activities." *International Journal of Industrial Organization* 15(1): 1-28

Delbono, F. and V. Denicolo (1993). "Regulating Innovative Activity: The Role of a Public Firm." *International Journal of Industrial Organization* 11(1): 35-48.

Delbono, F and V. Denicolo (1991). "Races of Research and Development between Firms with Different Incentives to Innovate" *Recherches Economiques de Louvain* 57(2):103-23

de Fraja, G. and F. Delbono, (1987). "Oligopoly, Public Firm and Welfare Maximization: A Game-Theoretic Analysis," *Giornale degli Economisti e Annali di Economia* 46(7-8):416-35.

de Fraja, G. and F. Delbono, (1989). "Alternative Strategies of a Public Enterprise in Oligopoly" *Oxford Economic Papers* 41(2):302-11.

de Fraja, G. and F. Delbono, (1990). "Game Theoretic Models of Mixed Oligopoly," *Journal of Economic Surveys*, 4(1):1-17.

Economic Research Serive (2002), "Agricultural Research Funding in the Public and Private Sectors." Data downloaded from www.ers.usda.gov/data/agresearchfunding/ on May 11, 2002.

Evenson, R.E. (2000), "Agricultural Technology Spillovers." In K.O. Fuglie and D.E. Schimmelpfenning (eds.) *Public-Private Collaboration in Agricultural Research.* Ames, IA: Iowa State University Press.

Fulton, M. and K. Giannakas (2001), "Agricultural Biotechnology and Industry Structure." *AgBioForum* 4(2):137-151.

Fuglie, K.O. (2000). "Trends in Agricultural Research Expenditure in the United States." In K.O. Fuglie and D.E. Schimmelpfenning (eds.) *Public-Private Collaboration in Agricultural Research.* Ames, IA: Iowa State University Press.

Fuglie, K.O., N. Ballenger, K. Day, C. Klotz, M. Ollinger, J. Reilly, U. Vasavada and J. Yee (1996). *Agricultural Research and Development: Public and Private Investments Under Alternative Markets and Institutions.* USDA-ERS-NRED Agricultural Economic Report No. 735. Washington, DC: U.S. Department of Agriculture.

Griliches, Z (1992), "The Search for R&D Spillovers." *Scandinavian Journal of Economics* 94(3):529-547.

Geroski, P.A. (1995), "Do Spillovers Undermine the Incentive to Innovate?" In S. Dowrick (ed) *Economic Approaches to Innovation.* Aldershot (U.K.): Edward Algar.

Henriques, I. (1990), "Cooperative and Noncooperative R&D in Duopoly with Spillovers: Comment" *American Economic Review*, 80(3):638-40.

Hall, B.H., A.N. Link and J.T. Scott (2001), "Barriers Inhibiting Industry from Partnering with Universities: Evidence from the Advanced Technology Program" *Journal of Technology Transfer* 26:87-98.

Huffman, W. and R.E. Evenson (1992). "Contributions of Public and Private Science and Technology to U.S. Agricultural Productivity" *American Journal of Agricultural Economics*. 74(3): 751-56.

Kalaitzandonakes, N. and M. Hayenga (2000). "Structural Change in the Biotechnology and Seed Industrial Complex: Theory and Evidence." In W.H. Lesser (ed). *Transitions in Agbiotech: Economics of Strategy and Policy.* Storrs, CT: Food Marketing and Policy Center, University of Connecticut.

Kamien, M. and N. Schwartz (1982), *Market Structure and Innovation*, New York, NY: Cambridge University Press.

Lee, T. and L. L. Wilde (1980). "Market Structure and Innovation: A Reformulation." *Quarterly Journal of Economics* 94(2): 429-36.

Levin, R. A. Klevorick, R. Nelson and S. Winter (1987) "Appropriating the Returns from Industry Research and Development." *Brookings Papers on Economic Activity*,pp 783-820

Loury, G. (1979). "Market Structure and Innovation" *Quarterly Journal of Economics*, 93(3):395-410.

Lieberman, M.B. (1984) "The Learning Curve and Pricing in the Chemical Industries," *Rand Journal of Economics* 34:217-233.

Maredia, M. F. Erbisch, A. Naseem, A. Hightower, J. Oehmke, D. Weatherspoon, C. Wolf (1999) "Public Agricultural Research and the Protection of Intellectual Property Rights: Issues and Options. *AgBioForum*, 2(3&4).

McMillan, G.S., F. Narin and D.L. Deeds (2000) "An Analysis of the Critical Role of Public Science in Innovation: The Case of Biotechnology" *Research Policy* 29(1):1-8.

Monsanto Corporation, (1998) *Annual Report*. St. Louis: Monsanto.

National Science Foundation (1996), National Report of R&D Resources, Washington DC, NSF.

Nett, L. (1991). "Mixed Oligopoly—A Survey" Department of Economics Discussion Paper No. A-358. Bonn: University of Bonn.

Nett, L. (1994). "Why Private Firms Are More Innovative Than Public Firms." *European Journal of Political Economy* 10(4): 639-53.

Nti, K.O. (1999). "Stability in the Patent Race Contests of Lee and Wilde." *Economic Theory*, 14:237-245.

Oehmke, J., D. Weatherspoon, C. Wolf, A. Naseem, M. Maredia, and A. Hightower (2000) "Is Agriculture Still a Public Good?" *Agribusiness* 16(1):68-81.

Padron, M.S. and M.G. Uranga (2001), "Protection of Biotechnological Inventions: A Burden Too Heavy for the Patent System." *Journal of Economic Issues*, 35(2):315-22.

Poyago-Theotoky, J. (1998). "R&D Competition in a Mixed Duopoly under Uncertainty and Easy Imitation." *Journal of Comparative Economics* 26(3): 415-28.

Reinganum, J. F. (1989). "The Timing of Innovation: Research, Development, and Diffusion." In R. Schmalensee and R. D. Willig. *Handbook of Industrial Organization*. Amsterdam, Elsevier Science Publications.

Ronald, P. and H. Leung (2002), "The Most Precious Things Are Not Jade and Pearls...," *Science* 296:58-59.

Sabido, A.C. (1994). "R&D Investment in Strategic Settings: A Survey of Patent Races" *Estudios Economicos* 9(1):61-118.

Scherer, F.M. (1980) *Industrial Market Structure and Economic Performance*. Boston, MA: Houghton Mifflin Company.

Schumpeter, J.A. (1934). *The Theory of Economic Development*. Cambridge, MA: Harvard University Press

Seade, J. (1980), "The Stability of the Cournot Revisited" *Journal of Economic Theory* 23:15-27.

Spence, M. (1984). "Cost Reduction, Competition, and Industry Performance." *Econometrica* 52(1): 101-21

Suzumura, K. (1992). "Cooperative and Noncooperative R&D in an Oligopoly with Spillovers" *American Economic Review*, 82(5):1307-20.

Tennbekk, B. "Marketing Cooperatives in Mixed Duopolies." *Journal of Agricultural Economics*, 46:33-45.

Thursby, J.G. and M.C. Thursby (2000) "Who is selling the Ivory Tower: Sources of Growth in University Licensing." NBER Working Paper 7718. Cambridge: National Bureau of Economic Research.

van Cayseele, P.J.G. (1998) "Market Structure and Innovation: A Survey of the Last Twenty Years" *De Economist* 146(3):391-417.