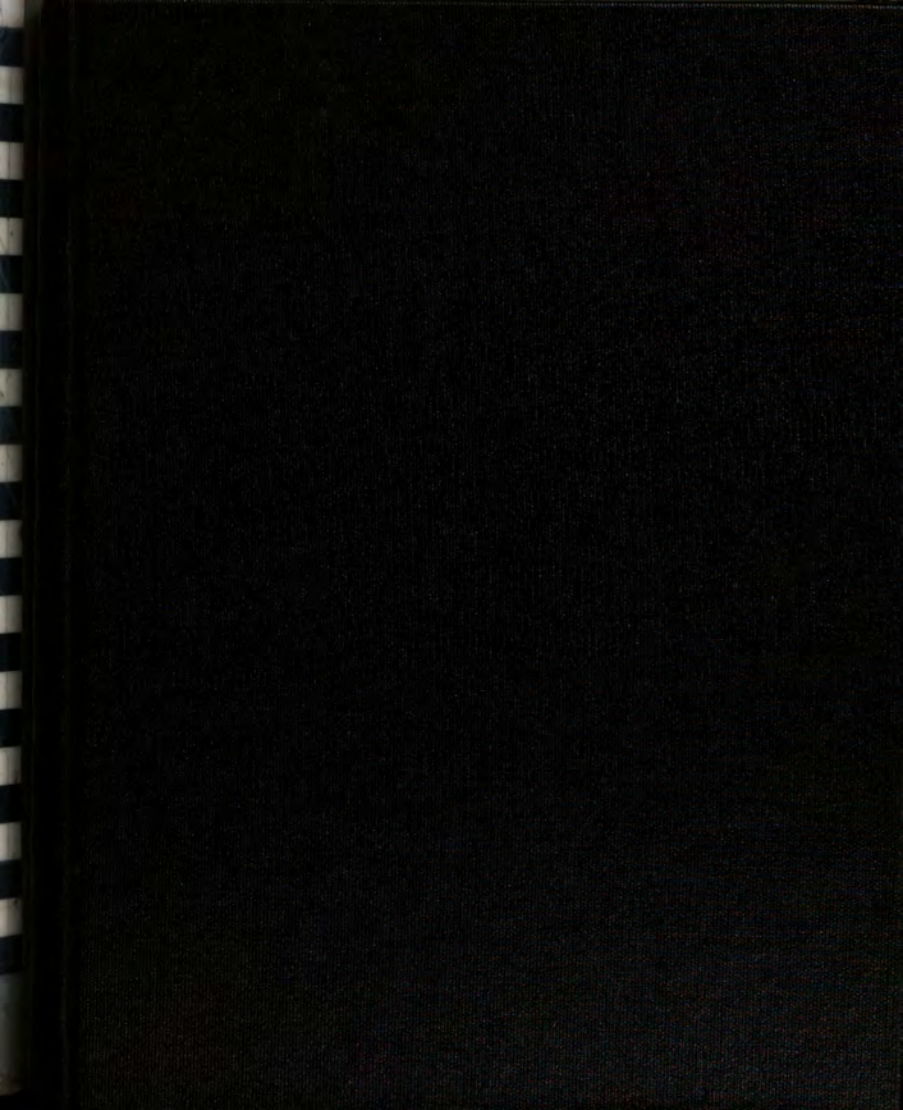






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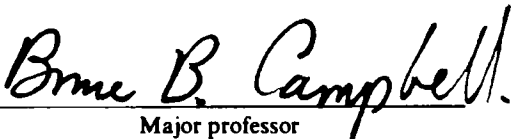
REPRESENTATIONS OF TIME AND SPACE
IN THE PLAYER PIANO STUDIES
OF CONLON NANCARROW

presented by

Julie Anne Scrivener

has been accepted towards fulfillment
of the requirements for

Ph.D. degree in Music


Major professor

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**REPRESENTATIONS OF TIME AND SPACE
IN THE PLAYER PIANO STUDIES
OF CONLON NANCARROW**

By

Julie Anne Scrivener

A DISSERTATION

**Submitted to
Michigan State University
in partial fulfillment of the requirements
for the degree of**

DOCTOR OF PHILOSOPHY

School of Music

2002

ABSTRACT

REPRESENTATIONS OF TIME AND SPACE
IN THE PLAYER PIANO STUDIES
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By

Julie Anne Scrivener

The Studies for Player Piano by Conlon Nancarrow (1912–1997) have been studied for their innovations in rhythm and tempo, including the use of isorhythm, different techniques of acceleration and deceleration, and most particularly, simultaneous use of different tempos in sometimes very complex mathematical relationships and textures. The present study examines specific structural features native to the Studies, including the pervasive use of mathematical ratios, the structural use of convergence points in the canonic Studies, and the presence of fractal characteristics (especially forms).

The first chapter contains a brief biography of Nancarrow, a review of the literature, and introduces the reader to general characteristics of the Studies. In Chapter 2, the use of ratios to control various musical parameters are identified, including: tempo relationships, rhythmic motives, melodic and harmonic materials, and structure. The use of ratios is the focus of a comprehensive analysis of Study No. 34.

Chapters 3 and 4 are concerned with features of “tempo canons”: a new form created by Nancarrow in which individual voices present canonic material at different speeds. Chapter 3 examines the structural use of convergence points in the canonic Studies. The structural impact of the convergence points, and techniques for emphasizing and de-emphasizing them, are the focus for this chapter. The chapter concludes with an analysis of Study No. 27.

Chapter 4 identifies fractal features in Nancarrow’s tempo canons. The defining fractal characteristics of self-iteration, scaling, and space-filling are identified as they

correspond to formal features in which the same musical material is presented at different tempos to create large-scale “fractal forms.” Study No. 32, an example of a fractal form, is analyzed in detail.

The fifth chapter is devoted to analysis of two of Nancarrow’s later works that contain some of his more mature compositional techniques: Studies No. 25 and 45. The analysis of No. 25 focuses on the use of the layering technique, while the analysis of No. 45 draws comparisons with a much earlier piece (Study No. 3) and particularly examines the presence of multidimensionality.

The final chapter summarizes the findings of the study and offers suggestions for future study of this compelling twentieth-century music.

ACKNOWLEDGMENTS

During these past seven years spent pursuing this degree program, I have come to the inescapable recognition that an effort of this magnitude can only be accomplished through the encouragement, kindnesses, and forbearances of many people. I have been truly fortunate to be surrounded by people who have kept me steady and given me what I needed, and I would like to acknowledge a few of them here.

My guidance committee and dissertation chair, Dr. Bruce Campbell, provided excellent guidance and was always a font of encouragement and sound advice, both in terms of guiding my academic program and fostering my career. I appreciate the efforts of the members of my dissertation committee, Dr. Charles Ruggiero, Dr. Gordon Sly, and Dr. Dale Bonge, who were responsive when I needed feedback and offered valuable insights.

I thank the Dean's office of the College and Arts and Letters at Michigan State University, particularly Dr. Patrick McConeghy, for financial assistance in the form of a Dissertation Completion Fellowship. The award came at a crucial time and was instrumental in allowing me to complete the writing. Dr. Frederick Tims, the School of Music graduate advisor, provided critical last-minute assistance in completing the application and getting it to the college on time.

My research on Nancarrow would never have gotten off the ground without timely assistance and guidance from two pioneers in the field, Dr. Kyle Gann and Dr. Margaret Thomas, both of whom were helpful and encouraging correspondents and generous in their offers of assistance. I am also grateful for the assistance I received from James Tenney in locating unpublished scores.

I could never have completed this degree program or this dissertation while working full-time at Western Michigan University without the forbearance and flexibility

of my co-workers and, most especially, my supervisors. Special thanks are extended to Dr. Rollin Douma and Dr. Shirley Scott of The Graduate College, and Dr. Thomas Seiler and Dr. Paul Szarmach of The Medieval Institute.

The people of Gull Lake United Methodist Church in Richland, where I have spent the past 16 years as choir and worship accompanist, organist, choir director, and now music director, have been my rock. I especially owe a debt of gratitude to my friends in the chancel choir, some of whom I am sure are convinced that references in this work to “spastic rhythm” and “temporal dissonance” refer to them! My other musical friends at Gull Lake UMC—Betty LeRoy, John and Martha Kuch, John Hill, Allison Hendrix, and Craig and Joan Schroeder—have brightened my life with their beautiful music, and I have been blessed in my many musical collaborations with them.

A great gift that I have received as a result of this degree program has been the friendship of fellow doctoral students Diane deVries and Andrea Dykstra. We shared the trials of being “non-traditional” commuter students, trying to balance work and home responsibilities with the rigors of a doctoral program. My journey was enriched by the treasure of their friendship. Thank you, Diane and Andrea!

Last, but *certainly* not least, my family—my parents, Leslie and Sharon Gummert and my brothers, Randy Gummert and Eric Gummert—were always interested in my progress and proud of my endeavors. My children, Garrett and Taryn Scrivener, who were only 5 and 2 when I began working on this degree, did not always understand why I had to work so much or even wanted to, but have now seen the fruits of this intense labor and will hopefully realize in their own lives that hard work really does pay off. Now that this is finished, they can call me “Dr. Mom.”

Julie Anne Scrivener

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CHAPTER 1

INTRODUCTION AND REVIEW OF THE LITERATURE

Conlon Nancarrow (1912–1997) considered time “the last frontier in music” (Garland 1982, 185). Like a pioneer forging his way across untested territory, Nancarrow faced numerous obstacles in his struggle to find a performing medium that would faithfully reproduce his bold musical concepts without the limitations of human performers. And, like most pioneers, Nancarrow seemed guided by two old proverbs—“where there’s a will, there’s a way” and “necessity is the mother of invention”—which speak to the resourcefulness that ultimately led him to the seemingly odd combination of the player piano with an esoteric experimental music rich in rhythmic complexity. The results have irrevocably stretched the boundaries of human perception and musical thinking, and forever altered the landscape of time in music.

Nancarrow’s almost fifty Studies for player piano represent an extraordinary body of work in their focus on a single performing medium (the player piano), in their exploitation of that medium’s unique expressive abilities, and in their tremendous diversity that belies a long evolutionary development over time. I can think of no other body of work, musical or otherwise, that consumed an artist for so long in so thoroughly exploring the expressive limits of that medium, and so absorbed his attention that he completely turned away from writing for other instruments until very late in life. Close to fifty years were devoted to these almost fifty pieces, involving hundreds of thousands of holes punched laboriously by hand on large rolls of paper to create works that fly by in a matter of minutes—all to realize the composer’s intentions in the purest way possible, directly from his hands to the player piano with no performer in between. The

achievement is even more extraordinary considering the almost complete isolation in which the Studies were produced; after moving to Mexico in 1940, Nancarrow remained (by choice) very much an “underground” composer until his reputation reached the mainstream in the 1980s.

In some ways it is understandable that Nancarrow’s work has not received the attention that it deserves. It is perceived by some as being melodically and harmonically unsophisticated (for example, by Pierre Boulez, who found Nancarrow’s pitch vocabulary disappointing compared to his innovative rhythmic structures [Gann 1995, 10–11]). Also, the unusual nature of its performance, in which no live performers are involved, has limited the programming of this music on contemporary music concerts. In comparison to other “performerless” music such as electronic music, music for player piano has been even more rarely programmed. Few people have heard a “live” performance of a Nancarrow player-piano piece, and his music has been mostly disseminated through recordings.

James Tenney, writing in 1977, found the continuing obscurity of Nancarrow’s work to be “nothing short of scandalous” (Tenney 1977, 41). In 1986, Carlsen observed (in the first dissertation devoted to Nancarrow’s music) that the critical literature on the Studies for player piano remained very small. Although Nancarrow’s work has become increasingly well-known (due partially to his selection in 1982 as one of the first class of recipients of the MacArthur Foundation “genius grants,” and the release in 1988 and 1989 of most of the Studies on recordings by Wergo), this deficit in the critical literature has been only partially addressed with the publication of a major analytical book by Kyle Gann (1995), a 1996 dissertation by Margaret Thomas, and a small number of articles and reviews.

The present dissertation is intended to expand the critical literature by examining specific structural features native to the Studies. These features include the pervasive use of mathematical ratios, the structural use of convergence points in the canonic Studies,

and the presence of “fractal forms” in certain of the canonic Studies. In Chapter 2, the examination of the use of ratios in the Studies identifies Nancarrow’s favored ratios and their relationship to the Fibonacci series and pitch ratios in the justly-tuned chromatic scale. Uses of ratios to control various parameters are identified, including: tempo relationships, rhythmic motives, melodic and harmonic materials, and structure. The presence of ratios is the focus of a comprehensive analysis of Study No. 34.

The next two chapters are concerned with features of Nancarrow’s “tempo canons”: canonic pieces in which the various voices present canonic material at proportionally-related speeds. Chapter 3 examines the structural use of convergence points in the canonic Studies; convergence points are a prominent feature of these Studies, because the canonic voices are either converging or diverging most of the time. The structural impact of the convergence points, and techniques for emphasizing and de-emphasizing them, are the focus for this chapter. The chapter concludes with an analysis of Study No. 27.

Chapter 4 identifies fractal features in Nancarrow’s tempo canons, particularly formal features in which the same musical material is presented at different tempos to create large-scale “fractal forms.” Perhaps the most compelling canonic fractal forms are created in the canons whose overall form is either converging (where just one convergence point occurs at the very end of the piece) or converging-diverging (an arch form where a single convergence point occurs somewhere in the middle of the piece). Study No. 32, an example of the former, is analyzed in detail.

Chapter 5 is devoted to the analysis of two of Nancarrow’s later works that contain some of his more mature compositional techniques: Studies No. 25 and 45. The analysis of No. 25 focuses on the use of the layering technique, while the analysis of No. 45 draws comparisons with a much earlier piece (Study No. 3) and particularly examines the presence of multidimensionality.

Chapter 6, the final chapter, presents conclusions and offers suggestions for further research into the Studies.

The remainder of Chapter 1 continues with a brief biographical summary and an examination of the four major analytical sources on Nancarrow's Studies: Carlsen, Gann, Thomas, and CD liner notes and an article written by James Tenney. The chapter then introduces the reader to general characteristics of the Studies, including rhythm, pitch, harmony, structure, and texture; notational features unique to Nancarrow's Studies are also examined. Finally, the analytical treatments of four Studies (Nos. 8, 19, 23, and 35) found in each of the four major sources are compared.

Biographical Summary and Review of Existing Literature

Conlon Nancarrow was born in Texarkana, Arkansas on October 27, 1912, and died August 10, 1997 at his home in Mexico City. His childhood was musically unexceptional, although it is noteworthy that the home contained a player piano (about which Nancarrow said, "I was fascinated by this thing that would play all of these fantastic things by itself" [Gagne and Caras 1982, 292]). Young Conlon took trumpet lessons, and by the time he completed high school he was good enough to get steady work as a jazz trumpet player.

Nancarrow's advanced music education consisted of a single semester (not four years as is sometimes reported) at Cincinnati College-Conservatory of Music, where he ostensibly studied composition but neither made an exceptional mark nor was profoundly influenced. It was here, however, that he first heard Stravinsky's *Rite of Spring*, which made a deep and lasting impression on him. Years later he pointed to this experience as life-altering and identified Stravinsky and Bach as his favorite composers (Gagne and Caras 1982, 282–83). Nancarrow's exposure to Bach apparently came about through his private study in counterpoint with Roger Sessions in Boston, where he moved after leaving Cincinnati. This was unquestionably his most rigorous compositional training. While in Boston, Nancarrow also became acquainted with Walter Piston and Nicolas Slonimsky. These relationships began under the guise of studying composition, but according to Nancarrow neither man taught him much about composition. As Nancarrow

said about Slonimsky,

Well, with him it wasn't really studying. I went to see him a few times, showed him a few pieces of music, and he commented on them. It wasn't studying because he didn't teach; he was just saying, "Well, this is good, this is bad." . . .

This "studying" didn't go on for very long. I finally got to be friendly with him, and I would go to his house for dinner, and the studying sort of drifted off into the background. (Gagne and Caras 1982, 283-84)

Nancarrow's experience in working with Piston was similar in that he shared a few of his pieces with him but they eventually became just occasional dining companions.

It was in Boston that Nancarrow became affiliated with and joined the Communist Party. In an extraordinary turn of events, Nancarrow (as one of few musicians in the Party) was asked to organize a Lenin Memorial Concert; what is truly extraordinary is that the Party was able to book Boston Symphony Hall for this event *and* Nancarrow managed to convince Piston to not only program his "Sonata for Oboe and Piano" but to perform the piano part. (This gesture is quite a testament to Piston's regard for Nancarrow; the administrative powers at Harvard let Piston know in no uncertain terms that he should never do such a thing again.)

In 1936 Nancarrow pulled out his trumpet and embarked upon a two-month playing tour in Europe. He returned briefly to the United States before signing up with the Abraham Lincoln Brigade to spend two years fighting the Fascist Franco regime in the Spanish Civil War. Upon his return to the United States in 1939, Nancarrow began to experience governmental harassment for his political affiliation, and as a result of this he moved to Mexico City in 1940. He became a Mexican citizen in 1956. In interviews in his later years he continued to express exasperation with the U.S. government; for a number of years after Nancarrow became a Mexican citizen he was considered an "undesirable alien" and unable to get a U.S. visa. He was finally able to travel to the U.S. in 1981, and at last began hearing his music performed at new music festivals and receiving commissions and requests to serve as composer-in-residence. The receipt of the

MacArthur grant in 1982 solidified his new-found fame (as well as easing his financial worries), and during the last fifteen years of his life he travelled quite frequently to the States.

Even before he moved to Mexico, Nancarrow had become personally acquainted with or known to a number of important twentieth-century musical figures. Besides Sessions, Piston, and Slonimsky, Nancarrow's music was known as early as the 1930s by Elliott Carter, Aaron Copland, and Henry Cowell. It was Slonimsky who was responsible for introducing Nancarrow's earlier music to a larger audience. While Nancarrow was away fighting in Spain, Slonimsky—without Nancarrow's knowledge—submitted several of his pieces (*Toccata* for violin and piano and two pieces for piano: *Prelude for Piano*, and *Blues*) to *New Music*, which published them in 1938. Nancarrow's first player piano piece, *Rhythm Study No. 1*, was also published without his knowledge. At some point he had given a copy to Carter, and in 1951 Carter arranged for the piece to be professionally copied and submitted it to *New Music*. Nancarrow learned of this by chance some five years later!

The Player Piano Studies

Nancarrow's player piano output includes forty-nine numbered studies.¹ The most comprehensive source to date on Nancarrow's music is Kyle Gann's *The Music of Conlon Nancarrow* (1995). The other major sources of analytic information on Nancarrow's Studies are the liner notes by James Tenney for the five volumes of recordings produced by Wergo, and dissertations by Philip Carlsen (1986) and Margaret Elida Thomas (1996). These are the main sources that will be reviewed and compared in this discussion of the literature.

Kyle Gann, *The Music of Conlon Nancarrow*. Gann's book includes information

¹The Studies are actually numbered through 51, but Nos. 38 and 39 were re-numbered by Nancarrow as Nos. 43 and 48, respectively.

based on personal interviews with Conlon Nancarrow in Mexico City from the late 1980s until 1993, where he was also given access to Nancarrow's studio with the original punching scores² and the piano rolls. Gann also was able to interview or contact a number of other people in Nancarrow's life, including his first and second wives and his brother. With this information he is able to paint a vivid and illuminating portrait of the composer's early life and influences. In addition, examination of both the punching scores and the piano rolls provided Gann with highly useful information about the construction of each Study, and allowed him to analyze each in considerable detail without relying completely on Nancarrow's hand-drawn scores.

Gann looks at each piece in Nancarrow's entire output (including his smaller output of "live performer" pieces) and analyzes each in brief analytical synopses that range from somewhat superficial to fairly comprehensive. Prior to this, however, the book contains useful chapters discussing the general characteristics of Nancarrow's music (a general discussion of these characteristics will be given in the second part of this chapter) and a well-informed biography. Gann's discussion of musical characteristics gives the reader a thorough understanding of the relationships in Nancarrow's music between tempo, rhythmic ratios, pitch ratios, and the harmonic series, and the terminology associated with tempo canon. A particularly useful feature of the first chapter is the "Morphology of Nancarrow's canons" on pp. 22–27, which graphically illustrates the formal structures of most of the canons (except for No. 43, which apparently was inadvertently omitted). He also discusses melodic and harmonic tendencies and the special kinds of notation used in Nancarrow's scores.

Because "there is no wholly satisfactory order in which to discuss the Studies for Player Piano," Gann discusses the works by creating the following taxonomy, which is "based on the primary compositional technique of each study" (p. 69):

²The "punching score" is a graphically drawn score on which Nancarrow plotted out a piece before punching the piano roll. He described it as "pretty much illegible for anyone else" (Duckworth 1995, 47).

1. "Blues years: the ostinato studies" covers the early Studies No. 1, 2a, 2b, 3 (a-e), 5, and 9. Gann notes that:

When Nancarrow began experimenting with polytempo, he turned first to a simple device that would show it to striking effect: ostinato, a phrase persistently repeated without variation. It was a natural choice, for ostinato was much in the air in modernist music of the 1920s and 30s. (pp. 69-70)

Gann points to Stravinsky (especially *Rite of Spring*) and Bartók as being particularly influential on these early pieces, which are also heavily influenced by Nancarrow's background as a jazz trumpeter. The I-IV-V harmonic and phrase structure is pervasive here, and Nancarrow builds several of these pieces by transposing large sections of musical material to the subdominant and dominant levels in an obvious blues pattern. He also already begins exploiting the ability of the player piano to perform polytempo by building in underlying rhythms that are in ratios such as 4:7 (Study No. 1), 4:5, 4:7, 5:7, 5:9, and 10:12:15:20 (Study No. 2a), and 2:3:5 (Study No. 3). Further, Nancarrow begins creating series of events that use arithmetical acceleration or deceleration: for instance, a series of notes with durations of 3, 4, 5, 6, 7, 8, and 9 beats to effect a *ritardando*. This is a technique that will recur in many later studies.

2. "Isorhythm: the numbers game" examines Studies No. 6, 7, 10, 11, 12, and 20. Gann points out that in the 1940s and 50s Nancarrow was collecting Indian music, and "analyses of the isorhythmic works will make Nancarrow's affinity for Indian rhythmic thinking obvious, whether he drew on the influence consciously or not" (p. 85). In advancing from ostinato to isorhythm, it is important to note, as Gann says, that:

There is a fine line between ostinato and isorhythm, since an ostinato reiterates the same rhythm as well as the same pitch, and the studies discussed here form an admittedly heterogeneous group. What they do have in common is some *internal dissociation of rhythm and pitch* [emphasis mine], a use of *talea* independent of *color*.³ (pp. 86-87)

³In the terminology associated with isorhythm, *talea* refers to the isorhythm's repeating rhythmic pattern, while *color* is the repeating melodic pattern. They are often of different lengths and consequently out-of-phase with each other.

Gann's discussions of the internal workings of these pieces are richer in detail than those in the first section—the longest discussion of the blues-ostinato pieces is that concerning Study No. 3, which covers the five movements of this study in just over five pages; the discussions of the isorhythm studies range from $2\frac{1}{2}$ to 7 pages for each. Gann focuses much attention on the isorhythmic structures of the studies and their applications to themes and structures. In several places, he acknowledges the medieval origins of isorhythm by comparing Nancarrow's technique to that of composers of that time period, as he does for Study No. 11, and uses terminology more consistent with that time period, such as the word "chant" in the discussion of Study No. 12 to refer to modally-inflected themes. Interestingly (and somewhat disappointingly), no further mention is made of Indian music or its relation to the studies discussed in this section.

3. "Canon: phase I" concerns itself with Studies No. 4, 13, 14, 15, 16, 17, 18, 19, 26, 31, 34, 44, 49, and 50. Gann points to these studies as being "the ones in which contrast of perceptibly different tempos is the primary focus" (p. 111). The studies discussed in this section combine the use of isorhythm with another old and familiar device: canon. Continuing the development of polytempo, Nancarrow creates a new kind of "tempo canon" by establishing increasingly complex tempo ratios between the canonic voices. Gann continues from the previous chapter the analogy to medieval and Renaissance music, tracing Nancarrow's tempo canons back to the prolation canons of composers such as Josquin des Prez, but identifies Nancarrow as "the first explorer of the true tempo canon" (p. 111).

In this section, Gann begins a practice—continued throughout the book—of using pitch analogies to relate tempos to one another, such as referring to a tempo that is twice as fast as another tempo as being an "octave" or multiple of the first tempo.⁴ Gann thoroughly documents here the rich diversity of tempo ratios created by Nancarrow in

⁴Readers who are not thoroughly conversant with the nature of pitch ratios in the just intonation system will find much helpful information on Kyle Gann's Web site at <<http://home.earthlink.net/~kgann/tuning.html>>.

these Studies.

The pieces discussed in this section abound with numerical series such as $2 + 3 + 4 + 5 + 4 + 3$ (Study No. 4) and the formula $\{n - 1, n, n + 1, n\}$, which is expressed in various manifestations in Studies No. 13–19. The series are used rhythmically in two ways: to construct isorhythms in individual voices, and the shorter patterns such as $\{n - 1, n, n + 1, n\}$ are used to create larger rhythmic resultants, such as the 336-note resultant used in most of Studies No. 13–19. Gann's identification of occurrences of numerical series in these pieces is quite thorough.

As is to be expected, Gann also devotes much attention in this section to identifying canons, their echo distances and pitch intervals of imitation. He also notes the location of convergence points and summarizes the canonic forms, which range from the standard types identified by Thomas (such as converging, diverging, converging-diverging [arch], etc.—see Figure 1-17, below) to the more unusual forms that result from tempo switches, such as that in Study No. 17.

4. "Stretching time: the acceleration studies" deals with Studies No. 8, 21, 22, 23, 27, 28, 29, and 30 and begins with this wonderfully perceptive comment: "None of Nancarrow's achievements is more original, more uniquely his own, than the *sense of curved time* [my emphasis] he has created by means of long, slow, smooth acceleration and *ritardando*" (p. 146). Gann begins this section by introducing and comparing the two different types of acceleration/deceleration: arithmetical and geometric.⁵

Gann identifies the presence of acceleration/deceleration in two ways, depending on whether the process is arithmetical or geometric. For arithmetical changes, either the note value changes incrementally (e.g., effecting an acceleration by decreasing each note value by the same note value—say a sixteenth note—from the previous value), or, in the cases

⁵Nancarrow did not have geometric acceleration available to him until Study No. 21. Until that point, his punching machine had a notched mechanism that allowed only certain increments to be punched on the roll; Nancarrow had the machine rebuilt to allow any increment to be punched. See Carlsen (1988) p. 4 and Duckworth (1995) pp. 47–48.

where Nancarrow uses proportional notation rather than note values, either the same invariable measured unit is added or subtracted, or the change is created through a numerical series such as 10, 9, 8, . . . , 2, 1 (a “countdown”). Geometric acceleration/deceleration is *always* notated with proportional notation, because it cannot be represented with conventional note values. Whenever proportional notation is involved, Gann identifies the rates of change by carefully measuring distances on the score(s) and/or piano rolls, usually in millimeters. Geometric acceleration in Nancarrow’s Studies is often indicated by a percentage value; for instance, the subtitle to Study No. 27 is “Canon 5%/6%/8%/11%.”

Although all of the Studies in this section feature acceleration/deceleration in some way, many continue to exhibit features discussed in earlier sections—such as isorhythm and canon—and Gann continues to point out these features. But the focus here is on rates of change.

5. “Beyond counterpoint: the sound-mass canons” concentrates on Nancarrow’s later canons: Studies No. 24, 32, 33, 36, 37, 40, 41, 43, and 48. As the chapter name implies, these canons are notable for achieving large sound effects, and Gann suggests that one is more likely to hear mass effects and structural points in these canons than individual voices. In this chapter he notes several differences between the earlier and later canons:

(a) The later canons exhibit considerable concern for structural placement of convergence points (CPs) and their climactic effects, whereas in the earlier canons CPs more often amounted to just a “collision of voices” (p. 173). Gann points out the tendency in the later canons for Nancarrow to use longer phrases further from the CPs and progressively shorter gestures as the CP approaches, and for the placement of CPs to affect a greater variety of details throughout the canon.

(b) In the later canons, Nancarrow seems less concerned about clear audible distinctions between extremely close ratios. Whereas the early canons primarily were restricted

to ratios involving 3, 4, and 5, these later canons use ratios as close as 24:25 (No. 43) and 60:61 (No. 48) and even irrational numbers such as $\sqrt{2}:2$ (No. 33) and $e:\pi$ (No. 40). Gann suggests that it was no longer necessary to create audible tempo distinctions because “here Nancarrow is not illustrating tempo differences, he is using subtleties of tempo to create forms and textures that had never been heard before, and which could have been created no other way” (p. 175).

(c) In the later canons (specifically Nos. 40, 41, 43, and 48), Nancarrow favors a major tenth level of imitation in two-voice canons, that—coupled with his preference for the melodic minor third—creates a “bittersweet and ambiguous major/minor harmony” (p. 203).

This chapter contains some of Gann’s most lengthy and involved analyses; he devotes seven pages each to Nos. 24, 37 and 40, eleven pages to No. 41 (the piece that is also the most extensively analyzed by Thomas), and—in his most extensive review—sixteen pages to No. 48 (a 3-movement piece in which the third movement is the first two movements performed together). In his analyses of these canons, Gann continues to identify occurrences of ratios between voices and pays special attention to the placement of CPs. About Study No. 37, he notes that “Convergence points are brilliantly de-emphasized in this study, for this is the work in which Nancarrow learned how to create beautiful effects with convergence points by *omitting* them” (p. 195). Later, about Study No. 41a, he points out the presence of a false CP, something highly unusual for Nancarrow.

6. “Synthesizing a language,” the final section, covers Studies No. 25, 35, 42, 45, 46, 47, and 51. These are the pieces in which Gann believes Nancarrow is “synthesizing a heterogeneous language, reaping the benefits of decades of experimentation, and starting to combine structural ideas into new hybrids” (p. 240). Of the first piece discussed, Study No. 25, Gann says “no other composition is such a treasure chest of every type of idea Nancarrow has worked with” (p. 241).

Although Nancarrow continued to make new discoveries and use new techniques in these pieces (e.g., the use of a tone row and the sustain pedal in Study No. 25, the “spastic” irrationally-derived ostinato rhythm in Studies No. 45–47), he relies heavily on the techniques of canon, isorhythm, tempo ratios, and acceleration that have served so well. Gann points out that, while none of the works in this chapter *are* canons, most of them *contain* canons (p. 240).

In these works Nancarrow is at the height of his creative powers and in complete command of his compositional tools—Gann remarks that “By now he is so expert at his favorite devices that isorhythmic canons, accelerating canons, isorhythmic ostinatos, even isorhythmic canons of arpeggiated ostinatos, appear almost effortlessly” (p. 240). These devices reach unprecedented levels of complexity, with tempo ratios as complex as the 144:182:351:468:585:624:819:936:1638 ratio that results in Study No. 46, and isorhythms reaching extreme lengths, such as the 99-note recurring isorhythm built entirely of eighth notes and dotted eighths (values of 2 and 3 in mathematical terms) in Study No. 47.

Philip Carlsen, *The Player-Piano Music of Conlon Nancarrow: An Analysis of Selected Studies*. Prior to Gann, the most thorough treatments of any of Nancarrow’s Studies were to be found in Carlsen’s dissertation, which was completed in 1986 with a monograph of the same title published in 1988. Unless otherwise noted, all discussion in this study of Carlsen’s contributions will refer to the 1988 monograph.

Carlsen analyzes five Studies selected from the first published collection: *Conlon Nancarrow: Selected Studies for Player Piano*, published by Soundings Press in Spring–Summer 1977 as *Soundings, Book 4*. The Studies he chooses to examine are Nos. 8, 19, 23, 35, and 36. His treatment of four of these (Nos. 8, 19, 23, and 35) will be reviewed more thoroughly in the last section in this chapter, where the four major sources are compared and contrasted. Thus, unlike Gann and Thomas, Carlsen establishes no taxonomy

for the review of the Studies because the five works selected comprise a “convenience sample.”

In Carlsen’s preface and introductory chapter he includes much useful background information. He characterizes Nancarrow’s scores as “extremely accurate in their spatial representation of rhythm and tempo” (p. ix), and points out that even the scores that are in metered rather than proportional notation use spatial representation of elapsed time: i.e., note attacks are carefully placed on the staff to represent the exact time at which the attack occurs. He comments on Nancarrow’s view toward performers and how he actively sought to eliminate the performer after enduring several disastrous performances of his music. The player piano itself is examined in terms of how it was popularly perceived—particularly by Stravinsky and Hindemith, who wrote for it, and by Cowell, who saw great potential in it—and how the mechanical characteristics of this instrument affected Nancarrow’s compositions for it. For instance, the paper roll uptake mechanism of the player piano introduces a naturally gradual *accelerando* as a piece progresses and the amount of paper on the uptake roll increases. Nancarrow actually did not mind this effect and likened it to the effect of “long, rhythmic African drum performances” (p. 69, note 15).

Carlsen structures his dissertation to focus most thoroughly on Study No. 19:

The present study does not attempt to survey Nancarrow’s entire output or, for that matter, to deal comprehensively with more than a handful of his compositions. A large portion of the analysis will be devoted to a single short work, Study No. 19. One may argue that it is not among the greatest of Nancarrow’s compositions, but it provides many insights into his overall compositional style. Analysis of Study No. 19 will set the stage for examination, in slightly lesser detail, of several other works, concentrating on their unique features but relating them to other studies. (p. 5)

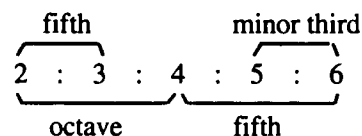
Carlsen’s analyses are more comprehensive than most of Gann’s and focus on a number of key areas. He calls canon “the predominant structural device in [Nancarrow’s] music” (p. 18), yet only one of the Studies that Carlsen selects (No. 19) is truly canonic, so this is necessarily a limited topic of discussion in his analyses. He expresses surprise

that the literature to that point had virtually ignored pitch elements in Nancarrow's music; it is not unexpected, then, that Carlsen himself examines this aspect of each Study. In the process he identifies recurrent motives (with the two most prominent involving the ascending fourth and the "partitioned" minor third, i.e., a minor third partitioned into a whole step and a half step) and even melodic segments that appear in more than one work. Long-range harmonic goals are also identified in some pieces, and Carlsen points out that the pitch levels representing I, IV, and V are frequently present.

Symmetry of design on different levels is a frequent topic for Carlsen's observations. In two of the Studies (19 and 36), Carlsen observes that the player piano's central note, E₄, was the axis of symmetry for the deployment of the voices. Small-scale symmetry is observed in mirrored rhythms (No. 35) and in mirrored pitch patterns (No. 8). In Study No. 35 he found formal symmetry among the sections of the piece. Carlsen also identifies correlations between the partitioning of musical "space" and ratios found in common intervals. In Study No. 36, he observes the prevalence of the major seventh chord as a structural device—its presence in determining the levels of imitation calls attention to the major seventh chord itself as a symmetrical structure, consisting of major thirds on the top and bottom separated by a minor third in the middle. Carlsen also refers in this same Study to a theme whose "second appearance is twice as fast as the first; it occurs at a point . . . exactly halfway between the beginning and the canonic midpoint, providing an obvious parallel with the well-known acoustical fact that the halves of a string vibrate twice as fast as the whole" (p. 30). In his analysis of No. 8, Carlsen makes the following observation:

When the second voice enters, it does so at a point exactly halfway through the first voice's isorhythmic pattern; the third voice enters exactly three-quarters of the way through the pattern The positioning of these entrances is specifically related to the divisions of successive intervals in the overtone series. Thus, the entrance of the second voice exactly halfway through the pattern parallels, in the overtone series, the division of the octave (2:4) into fifth (2:3) and fourth (3:4); by the same token, the further subdivision of that half into quarters parallels the subdivision of the fifth (4:6)

into major and minor thirds (4:5 and 5:6). Obviously, it is no coincidence that the bass voice is characterized by octaves, the next by fifths, and the highest by minor thirds:



(p. 43)

Carlsen obviously had great confidence in the intentionality with which these structures were created, and the arguments he puts forth in support of this view are both intriguing and persuasive.

In Carlsen's final chapter, he reflects in broad terms on what he calls "the motivating principles behind the music" (p. 66). The player-piano medium itself is one of these. Carlsen observes a fortuitous compatibility between the instrument and Nancarrow's interest in matters numerical and spatial:

The [piano] roll is highly conducive to the use of exact measurements, templates, replications, and geometric constructions; thus, to a certain extent, it exerts an influence that is purely visual and mathematical. Nancarrow, with his mathematical and structural turn of mind, is quite receptive to such influences. (p. 66)

Margaret Elida Thomas, *Conlon Nancarrow's "Temporal Dissonance": Rhythmic and Textural Stratification in the Studies for Player Piano*. Thomas completed her dissertation in 1996, for which she had access to both Carlsen's 1986 dissertation and the 1988 monograph based on it, and unpublished drafts of portions of Gann's book (Thomas 1996, ii). Thomas sets out to explore "the rhythmic and textural techniques" of the Studies, and "temporal dissonance in particular" (abstract). The term "temporal dissonance" is traced back to Nancarrow himself, who said in an interview with Roger Reynolds that he had an "interest in temporally dissonant relationships" (Reynolds 1984, 5). In the course of this dissertation, Thomas makes at least brief mention of all the Studies except Nos. 13, 22, 26, 28, 29, 30, 34, 43, 44, 46, 47, 48, 49, and 50 (see Appendix C for a complete index to Studies referenced in Thomas).

A key part of Thomas's discussion is the development of a concept she calls "multi-dimensionality." This is a somewhat elusive concept that is presented in terms of its relationship to the techniques of accumulation (progressive addition of musical layers), canon, and heterophony that she identifies in Nancarrow's works. Thomas's definition of heterophony includes not only line doubling at the octave, but other varieties of parallel melodic movement (including triads); even lines which are only loosely coordinated melodically are considered a heterophonic texture if "the heterophonic pitch relationship causes the voices to lose their individual identities" (p. 90). (See Figure 1-26 for such a texture.) Thomas further states that multi-dimensionality is neutralized in the presence of heterophony. Finally, Thomas examines multidimensionality in relationship to temporal dissonance, for which she states multidimensionality is a prerequisite.

Thomas's development of the concept of multidimensionality and its relationship to temporal dissonance is somewhat compromised by repeated references to a lack of coordination or relationship between the temporally dissonant layers, yet an insistence later on that in order for a temporal conflict to exist, there must be a discernible relationship between the musical layers—and indeed, throughout the dissertation there is a somewhat uneasy balance between these conflicting views. In the abstract, Thomas refers to "Nancarrow's aesthetic goal of 'temporal dissonance': two or more *seemingly uncoordinated* streams of music . . . presented simultaneously" (emphasis mine). Later, the qualification of *seeming* uncoordination between the layers is not so carefully observed and Thomas begins instead to equate asynchronous with unrelated. On p. 3 she states that:

My particular interest in Nancarrow's studies centers on the extreme form of polyphony created by his simultaneous use of *uncoordinated, independent* streams of music, and the philosophy of musical rhythm and time embodied therein. . . . the multidimensionality of the works [consists of] the stratification of their musical time into a number of *asynchronous* layers. (emphasis mine).

Further, she notes that "the works feature several discrete, concurrent temporal processes,

so that at any given moment a study has a relatively expansive spatial quality owing to the *irreconcilability* of its layers” (pp. 3–4, emphasis mine). Finally, on p. 9 she allows that “it is not mere independence between layers for which Nancarrow strives with his rhythmic techniques, but real conflict, which is crucial to his overriding aesthetic goal of creating an effect he calls temporal dissonance.” Yet it still seems that for a conflict to exist, there must be some discernible relationship. This matter is not finally settled until Thomas states on p. 114 that “the clearer the conflicting relationship between [the] voices the more easily we can perceive them as [temporally] dissonant.” And,

Textural density does not translate into temporal dissonance. Instead, dissonance seems to rely heavily on the perceptibility of the relationship of simultaneous voices, whether in terms of their tempo proportions, their metrically significant simultaneous articulations (or lack thereof), or their act of converging. (p. 122)

Thomas begins the dissertation with a brief survey of Nancarrow’s compositional techniques and characteristics of the Studies; her observations are incorporated into the section on “Musical Characteristics of Nancarrow’s Studies” presented later in this chapter.

In a very useful section on musical influences, Thomas examines two important influences on Nancarrow’s music: Henry Cowell, and jazz. Cowell’s *New Musical Resources* of 1930, which sets forth his groundbreaking theories on rhythm and metric relationships, had a profound influence on Nancarrow. Cowell espoused the use of proportional rhythmic relationships that relate directly to interval ratios. Thomas notes that such a relationship could be expressed in three different ways (see Figure 1-1): rhythmically, metrically, and temporally. By far the most common expression of this concept in Nancarrow’s Studies is the tempo ratio.

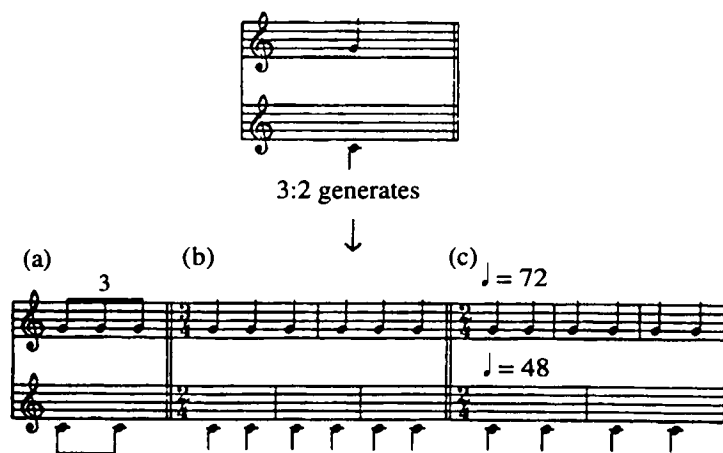


Figure 1-1. Figure in Thomas (1996, p. 189) illustrating the 3:2 (perfect fifth) ratio applied to: (a) rhythm, (b) meter, and (c) tempo.

Thomas notes that, whereas Nancarrow applied Cowell's ideas on rhythmic proportions in the *Studies for Player Piano*, his musical goals were different than Cowell's:

The contrasting simultaneous rhythms, meters, and tempos he [Cowell] discusses are related by proportions. The parallel to Nancarrow's asynchronous layers is unmistakable, of course, particularly in his use of proportionally-related tempos. There is a fundamental difference in the intended effect of those proportions, however: whereas Nancarrow strives for temporal dissonance, Cowell uses ratios derived from the overtone series to create what he called "rhythmic harmony." (Thomas 1996, 21)

Cowell's ideas were also influential on the form of Nancarrow's *Studies*. In Cowell's system, large groupings equivalent to formal sections could be articulated by the arrival of voices in conflicting tempos at a simultaneous downbeat. These "convergence points" are indeed of great interest in the *Studies*, particularly in the tempo canons, and will be the subject of Chapter 3 in this dissertation.

The influence of jazz on Nancarrow is just as pervasive. For a time, Nancarrow performed as a jazz trumpeter, and his lifelong love of jazz emerges in his music in many ways. Thomas identifies the influence of boogie-woogie in Nancarrow's early *Studies*, and standard blues progressions, "walking" bass lines, and jazz accompaniment patterns are used to great effect in a number of Nancarrow's pieces. Rhythmic influences of jazz

include swung (long-short) rhythmic divisions and the evocation of improvisational practice by placing rhythmic attacks “around the beat.” Thomas also points out that poly-rhythm and polymeter are typical features of jazz. Nancarrow also used a great many jazz harmonic idioms, including seventh, ninth, and substitution chords, as well as blues scales.

Thomas’s second chapter focuses on specific works by four other twentieth-century composers who, like Nancarrow, had an interest in “exploring the possibility of presenting simultaneous layers of music that are related temporally in unconventional ways” (p. 29; note the assertion here that the layers are temporally related). Thomas looks at these works “in order to see how differently temporal stratification can function” (p. 29), and identifies in each a different genesis for temporal conflict. In the first work examined, Charles Ives’s *Washington’s Birthday* (1909), the genesis of the temporal conflict is Ives’s desire to recreate his actual musical experience of simultaneous performing forces. Thomas begins to examine here the concept of a “perceptibility threshold” (p. 34, note 12) at which individual layers of sound coalesce into a composite mass.

The next piece examined is the first movement of Henry Cowell’s *Quartet Romantic*. Here the genesis of temporal conflict is, of course, Cowell’s theory of temporal ratios. Thomas explains Cowell’s compositional method, in which he converts the pitch ratios of simple four-voice chorales to rhythmic ratios (a “harmony of rhythm”) with the result being that “the effect of the movement is correspondingly progression-like, in that the rhythms regularly proceed from one set of ratios to the next” (p. 41).

The second movement of Elliott Carter’s “String Quartet No. 2” features a constant pulsation in the second violin, which functions as the time keeper while the other instruments develop unique rhythmic personalities. As in Cowell, the genesis of temporal conflict in this piece is a variety of tempo ratios related by simple integers. Here, however, the ratios were not arrived at by conversion from a harmonic system, but through a series of ratios all relating to the basic tempo of $\text{♩} = 140$ and expressed through Carter’s

technique of “metric modulation.”

Thomas’s final example is György Ligeti’s “Automne à Varsovie” from *Piano Etudes*, Book One. Thomas notes that Ligeti faced a greater challenge in “articulating a stratified texture with just one instrument” (p. 47). Ligeti creates temporal conflict between the pianist’s two hands by articulating almost continuous sixteenth notes throughout the piece and eventually requiring the performer to articulate every fifth pulse with the right hand and every third with the left to form a 5:3 ratio.⁶

Chapter III continues the development of the concept of multidimensionality, and concentrates on canon and textural accumulation as means of achieving it. Heterophony is also examined as a non-multidimensional texture that is created by coordination of pitch-class content among musical layers. Thomas further identifies factors such as register, pitch class, and rhythmic, metric, and temporal organization as being influential on the individuality of layers.

In examining processes of textural accumulation, Thomas identifies some Studies that clearly cross “the perceptibility threshold for multidimensionality” (p. 63) due to extreme textural density. She classifies canon in Nancarrow’s Studies as a special kind of accumulation, and comments that “Indeed, as we will see, the technique of canon does not necessarily produce a clarity of layers. Sometimes canon seems not to be the aesthetic point of a study at all, but rather a tool for creating a texture in which voices that are theoretically independent are not perceptually distinct” (p. 73). In one instance, while discussing Study No. 31, she makes a point regarding difficulties in perception when canonic entrances are widely separated, but mistakenly calculates the gap between the opening of the study and the second voice as $38\frac{1}{2}$ seconds when the gap is really only

⁶There is a slight miscalculation here regarding the tempo of the 5-grouping in the right hand: the quarter-note pulse at the opening is MM 144; when the 5-grouping begins, creating a 5:4 ratio against the pulse (one pulse every 5 sixteenth notes rather than every 4), Thomas calculates this new pulse at MM 113 when it is actually closer to MM 115 ($144 \cdot 4/5 = 115.2$).

about $12\frac{1}{2}$ seconds.⁷ In the course of this discussion, and later in regard to Study No. 20, Thomas performs some motivic analysis, which is unusual for this dissertation.

The fourth chapter is devoted to an exploration of temporal dissonance in the Studies, particularly ways in which Nancarrow controls levels of temporal dissonance. Thomas notes (p. 93) that this quality is pervasive in Nancarrow's music, most noticeably in the Studies in which tempo proportions are used, but it is present even in those Studies that do not contain multiple simultaneous tempos. There are, of course, differing levels of temporal dissonance, and Thomas observes that moments of relative consonance often serve a formal function in the Studies (p. 96). Absent other mitigating factors, the most extreme levels of temporal dissonance can be found in the Studies which use irrational number proportions, since there is a total lack of coordination between the voices, and the potential for points of simultaneity is virtually non-existent.

Thomas suggests that "one of the clearest compositional strategies in which to hear proportions is canon" (p. 97), and then proceeds to examine levels of temporal dissonance in individual Studies. In this section, she makes many observations about the perceptual levels of dissonance attendant to techniques such as ostinato/isorhythm and canon in an environment where proportional tempos are operating. She identifies many instances in which the levels of temporal dissonance are exaggerated when Nancarrow subverts potential points of simultaneity by one of two techniques: (1) shifting meters (i.e., moving the downbeat around), and (2) avoiding the placement of note attacks on potential points of simultaneity by using notes longer than the basic pulse or using rests. In illustrating the second technique, Thomas cites an example from Study No. 9 (see Figure 1-2) in which two voices share an eighth-note pulse at a ratio of 5:3; this, then, sets up the possibility of a simultaneity every fifth note in the faster voice and every third

⁷This is easily verified on the recording. The tempo is $\text{♩} = 105$ for 22 measures of $\frac{2}{4}$ time. There is a simple math error here: Thomas multiplied the 22 measures by $105/60$ rather than $60/105$ (each measure elapses in .57 seconds [$60/105$] and $60/105 \times 22 = 12.57$ seconds).

note in the slower. However, due to the use of rests in the lower voice, five of nine possible simultaneities are avoided (significantly, all three of the simultaneities that occur on mutual downbeats—the first, fifth, and ninth—are retained).



Figure 1-2. Segment from Study No. 9 (as renoted by Thomas, 1996, p. 260) exhibiting a 5:3 rhythmic ratio in which potential simultaneities are avoided. Actual simultaneities are indicated by a vertical broken line, while potential simultaneities are marked with an asterisk.

Thomas points to one additional factor that contributes to temporal dissonance, and that is the mixing together in the Studies of three “time types”: (1) material that uses regular meters and non-changing tempos; (2) non-metered, proportionally notated material; and (3) material that gradually changes speeds. She comments that “These different time types raise new issues with regard to an examination of temporal dissonance: tempo ratios and simultaneous articulations generally no longer serve as markers for dissonance” (p. 125).

For Thomas’s fifth chapter she selected Study No. 41 for a comprehensive analysis. This Study is one of Nancarrow’s longest and most complex. It is in three “movements”: both movements 41A and 41B are two-voice converging-diverging (arch) canons based on irrational ratios, and movement 41C is the two previous movements played together. The subtitle for 41A is $\sqrt[1]{\pi}/\sqrt[2]{2/3}$, while 41B’s subtitle is $\sqrt[3]{\pi}/\sqrt[3]{13/16}$. Thomas’s analysis treats A, B, and C separately. For A she identifies six “families” of motives or

melodic gestures and the implied rhythmic patterns that they represent. She notes also several pc sets that are prominent in some of the motives, and observes that the single convergence point that occurs about 70% of the way through the movement is the only "clear formal division" (p. 147), although she points to a prominent moment of silence that occurs a short time later as being significant. In B she notes the similar formal arch shape, although in B the prominent moment of silence is placed directly after the convergence point and Thomas emphasizes the support given to the arch shape by respective processes of *accelerando* and *ritardando*. Five motivic families are identified, and their implied rhythms analyzed, but this time no pitch analysis in the form of pc sets takes place. Only one pitch is identified as structural, and that is B, which ends both movements (and consequently 41C) and begins 41B, forming a bridge from 41A. The analysis of 41C focuses on the "truly staggering complex" (p. 160) that is created when the two movements come together; Thomas examines the complementarity of the formal shapes of the first two movements, and certain similarities that exist among the motivic gestures of 41A and 41B.

In her conclusion, Thomas summarizes the types of textures and textural devices that have been discussed, and acknowledges that "Because my analysis of temporal dissonance has (necessarily) been so contextually based, issues of perception have been prominent in my discourse" (p. 166). She then focuses on the canonic technique, questioning why Nancarrow uses it when

the canonic process is highly obscured by the complexity of the canonic line, the number of voices, the speed at which it progresses, and so on. This raises the question of why canon is used in these cases: if we cannot hear it, what is the point of the canon? (p. 167)

Thomas's final comments concern the importance of the player piano in realizing Nancarrow's goals of temporal dissonance and in imparting a "superhuman" quality to his music.

James Tenney. Gann, Carlsen, and Thomas all make occasional reference to the liner notes prepared for the Wergo recordings by James Tenney and the 1977 article in *Soundings*, Book 4. The 1977 article covers all the Studies through No. 41, and the Wergo liner notes through No. 41 are almost exactly reproduced from this article (with the exception of No. 30, which is not included in the Wergo recordings). Tenney's comments about Studies No. 42–50 were apparently added in 1988 and 1989. Those Studies about which Tenney wrote for both the 1977 article and the Wergo liner notes (i.e., Nos. 1 through 41) are sometimes treated in more detail in the liner notes. In a few unusual cases, there is considerably more detail—usually about formal matters—in the liner notes; two good examples are the analyses for Nos. 7 and 9. Yet other Studies receive only very cursory treatment in both writings.

Tenney laments in his introduction that “the continuing obscurity of Nancarrow’s work is nothing short of scandalous” (1977, p. 41). Still, he confidently predicts that

21st-century historians will rank Conlon Nancarrow’s *Studies for Player Piano* with the most innovative works of Ives, Schoenberg, Stravinsky, Webern, Varèse, Partch, Cage, Xenakis—and perhaps a very few others—as the most significant works composed since 1900. This prediction may seem extravagant to some, but I am convinced that, when Nancarrow’s music is as accessible and widely known as that of his contemporaries and immediate predecessors, its importance will be just as widely recognized, and there will remain no room for doubt. (Wergo Vols. I/II, liner notes, p. 1)

Tenney identifies several terms that he finds critical to a discussion of Nancarrow’s music, and these include: (a) aggregate, “complex sounds and sound-configurations which are perceived as singular textural elements” (1977, p. 44); (b) resultant, “a complex but perceptually singular ‘layer’ or stratum in the polyphonic texture” (1977, p. 44); and (c) compound-polyphonic, a term used frequently by Tenney to refer to the texture created when aggregates and/or resultants are used (but also used in a way as to be easily confused with heterophonic: “each voice is compound-monophonic, its melodic lines generally doubled in octaves or filled out by triads” [Wergo Vols. III/IV, liner notes, p. 20]).

In writing about the Studies, Tenney creates the following roughly chronological taxonomy:

1. The “early group,” consisting of Studies 1 through 12. Tenney characterizes these works as clearly tonal (or modal) and notated metrically except for No. 8. Many of them have origins in blues, ragtime, or other jazz styles, and while ostinatos are frequently present, strict canonic writing is rarely used. In the individual analyses, Tenney observes numerous tempo ratios in effect and the frequent use of rhythmic series, either separately or as part of an isorhythm. Different versions of the trochaic rhythm pattern (long-short) are observed, sometimes in the same piece (e.g., No. 2), and these include 3:1, 2:1, 3:2, and 5:3. Tenney also mentions several times a “rubato-effect” which he mentioned earlier (1977, p. 44) as one of six rhythmic procedures but never really defines. This effect is observed by Tenney in Studies No. 6, 10, and 12. In No. 6, it refers to an ostinato that regularly alternates between measures of the same length that are divided into four and five eighth notes; in No. 10, a “freely evolving melodic line” is created from numerous arithmetical accelerations (“countdowns”) to form a “kind of ‘written out’ rubato” (1977, pp. 50–51); and in No. 12, several countdown devices are evident that impart a feeling of rubato.

2. The next group is Studies No. 14 through 19, which were part of a set of Seven Canonic Studies (including No. 13, for which Nancarrow never released the score) all using tempo ratios based on the numbers 3, 4, and 5. Tenney characterizes these as having a more abstract style, with no hints of blues or ragtime; using canon as a formal organizing procedure; and showing Nancarrow’s increasing interest in tempo as a structural parameter. Tenney’s analyses include a structural diagram for each piece showing the number of canonic voices and how they relate to one another registrally and temporally in terms of tempos used, entrance points for the voices, and any convergence points that occur. Melodic elements are analyzed with respect to tonal centers, presence of mixed meters, and use of various doublings. Tempo switches are noted in Nos. 15 and

17, and in No. 16 four duration series are identified that contribute to the two resultant strata.

3. Tenney's next group is called "The Middle Period," and includes Studies No. 20 through 32. This is by no means a homogeneous group, and in his "Overview" section Tenney actually identifies three groups among these works: Nos. 20–27, characterized by "precisely controlled, very gradual changes of tempo, and a fairly consistent use of a non-metrical rhythmic notation for the scores" (1977, p. 45); Nos. 28–30, similar to 20–27 but distinguished from them by being originally planned for prepared player piano; and Nos. 31–32, which return to metrical notation and fixed rational tempo ratios. The analyses in this section are at times highly detailed. Tenney observes that Study No. 21 could be perceived not as the beginning of a new group, but the end of an earlier period. This Study ("Canon X") features a gradual acceleration in one voice against a gradual *ritardando* in the other, and it is the last piece Nancarrow punched with a roll-punching machine that punched in fixed incremental units rather than on a continuous time scale. Tenney also comments that "Beginning with Study #21, Nancarrow's harmonic-melodic language becomes more and more difficult to relate to traditional tonality" (1977, p. 55). Other noteworthy features of these Studies include the two pieces (Nos. 22 and 27) that Nancarrow punched with the new roll-punching machine, which have percentages in their subtitles to indicate the rates of tempo change; the "exploded" notation technique Nancarrow began using in No. 25 to accommodate very fast note fragments; the difficulties Nancarrow encountered in writing pieces for "prepared" player piano (Nos. 28–30); and the seeming return in Nos. 31–32 to the simpler textures and rhythmic procedures as seen in Nos. 14–19.

4. Last in Tenney's 1977 taxonomy are Studies No. 33–41, which are marked by the use of higher-order (but still rational) tempo relations; the introduction of *irrational* tempo ratios (e.g., Nos. 33 and 40); the use of two player pianos simultaneously; and richer textures and a greater variety of aggregate structures. Study No. 34 is the one piece

in this group in which Tenney finds simple textures and a minimum of aggregates being used, and he surmises that this is to allow the rather complex tempo relationships to be clearly heard. Study No. 36 prominently features linear aggregates (grace-note appoggiaturas, triadic arpeggios, and chromatic glissandos), while in the remaining Studies vertical structures predominate. In Studies No. 40 and 41, Tenney finds that the texture is almost completely composed of aggregates:

each stratum consists of a succession of glissandos, chords, repeated-notes, trills and tremolos—nearly every basic type of aggregate ever used in earlier Studies, and more *different* types than in any one previous Study. These aggregates are varied (in duration and/or pitch-register)—and juxtaposed in time—in a way which suggests the way in which the *object sonore* is treated in *musique concrète*. (1977, p. 64)

5. After Tenney's 1977 article appeared he wrote of Studies No. 42–50 for the Wergo liner notes. For these he classified the most recent group as Nos. 40–50, creating a slight overlap with the previous group. He notes that these Studies primarily use proportional notation, that references to jazz and Spanish music appear again, and that some new elements appear, such as the use of aleatoric procedures (No. 44) and an introduction of live performers in the *Concerto for Pianola* that was the basis for No. 49. The analyses in this group are, by far, the most detailed that Tenney produced. Several of the pieces (Nos. 40, 41, 45, 48, and 49) are in multiple movements, and three of these (40, 41, and 48, along with No. 44) involve simultaneous use of two pianos. Tenney continues to focus on the presence of aggregates, which ones are used in a piece, and how the collections of aggregates might change in the course of a piece. For example, about Study No. 41 he identifies “glissandos, arpeggios, and other, miscellaneous ‘running’ figures, sometimes alone, but often leading to, passing ‘through’, or seeming to arise out of, sustained tones and chords” (Wergo Vols. I/II, pp. 8–9); and about No. 48 he observes “the great variety of linear aggregate types used, including trills, tremolos, arpeggios, glissandos, and miscellaneous melodic figures, all at dizzyingly high speed” (Wergo Vol. 5, liner notes, p. 10). His analysis of the three movements of Study No. 45 is perhaps the

most thorough of all. He reviews each movement separately in terms of repeating harmonic-melodic progressions and ostinato bass-lines (largely based on blues), the presence of canons, and textural changes. (He does not recognize, as Gann does, an irrational number derivation of the “peculiarly uneven rhythm” [Wergo Vol. V, liner notes, p. 7] that is the basis of the bass lines in the first and third movements). No. 45 is one of several jazz-inflected works in this group that prompt Tenney to observe, “In fact, it now seems to me quite possible that much of what I have called Nancarrow’s ‘abstract’ style might better be understood as simply a new form of jazz” (Wergo Vol. V, liner notes, p. 11).

Musical Characteristics of Nancarrow’s Studies

General observations about characteristics and features of Nancarrow’s Studies are given specific attention in Gann (pp. 1–35), Thomas (pp. 7–19), in various parts of Carlsen’s discussion of individual Studies, and in a brief overview in Tenney (*Soundings*, Book 4, pp. 43–46). Particular attention is paid by these sources to matters of notation, which is an important element of Nancarrow’s work. This summary of the literature will make note of notational features as they relate to other features.

Rhythmic Features

Tenney points to rhythmic procedures as one of the two most important characteristics of Nancarrow’s works (along with polyphonic texture), and makes the following observation:

Rhythmic independence is essential to real polyphony. Conversely, the most thorough exploration of rhythmic possibilities must include simultaneous as well as successive rhythmic relations. So it is not surprising that a music inspired primarily by an interest in rhythm would be polyphonic in its very essence. (1977, p. 43)

Nancarrow himself commented on the primacy of rhythm to his compositional method by

remarking that “the thematic notions [in my music] are usually rhythmical rather than melodic” (Reynolds 1984, 10).

Nancarrow uses two primary types of notation to convey rhythm in his Studies: conventional metered notation and proportional notation. Conventional rhythmic notation usually involves the use of meter and bar-lines, even when each voice is in a different tempo (such as in Figure 1-2). In proportional notation, the passage of time is carefully measured on the scores; sometimes Nancarrow will indicate this with a notation such as:



while at other times no such notation is given. As Carlsen is careful to point out, *all* of Nancarrow’s scores use spatial representation of time, even when meters and regular note values are used:

throughout a study, a certain amount of horizontal space on the score page represents a specific amount of time, and the spaces between notes in the score are the same—or proportionally the same—as those on the roll. . . . This exact relationship between space and time is true even of the studies notated with time signatures, barlines, and the standard symbols for durations and rests. (Carlsen 1988, 5)

The length of a sustained note in proportional notation is indicated with a thick line extending to the right of the note for its duration. Thomas notes (1996, pp. 8, 19) that in his proportional notation Nancarrow generally used quarter notes for sustained notes and eighth notes for staccato articulations (see Figure 1-3). Notes longer than a quarter note are unusual in the Studies, primarily because a sustained note on the player piano necessitates that a long row of holes be laboriously punched on the roll. Also, like the perforations on a sheet of postage stamps, such long rows of holes made the roll paper vulnerable to tearing, so Nancarrow took considerable care in placement of sustained notes.



Figure 1-3. Proportional notation in "Study No. 8," p. 18.⁸

Perhaps the most characteristic hallmark of Nancarrow's Studies is the simultaneous use of different rhythms, meters, or beat divisions, sometimes in addition to different tempos. This is evident already in the first Study, where an ostinato in which the same measure length is divided into both $\frac{7}{8}$ and $\frac{4}{4}$ time is pitted against a variety of different tempos and meters (see Figure 1-4).



Figure 1-4. $\frac{7}{8}$ against $\frac{4}{4}$ ostinato with conflicting meters in Study No. 1 (p. 9). The symbol J is a five-division note, in this case a five-thirty-second note.

⁸In this example, two other characteristics of Nancarrow's score notation are evident: (1) the entire length of the staff, from beginning to end (about 165 millimeters), is used for purposes of notating elapsed time, and (2) to accommodate this, the beginning clefs are always written to the left of the staff, as are any accidentals that apply to the first note or chord. Nancarrow also tended to avoid writing meter changes on the staff and usually wrote them above the staff instead.

Jazz rhythmic features abound in Nancarrow's Studies. There are many variations on the trochaic (long-short) "swung" rhythm, sometimes even within the same piece. For example, in Study No. 3b both the rhythmic divisions 3:2 and 5:3 are used. As Gann remarks,

Nancarrow has from the very beginning used the player piano to *recreate* rhythmic liberties taken in performance that no notation could convey. In the studies based on [stride piano] (Nos. 3, 4, 10, 45), he has implicitly acknowledged that jazz pianists hardly ever play a dotted rhythm in a 3:1 ratio; instead, Nancarrow often divides his beats into ratios of 3:2, 5:3, or 8:5, all divisions based on the Fibonacci series, related to the intuitively pleasing Golden Section as well as closer to live performance practice. (p. 9)

The 5-division notational symbol [] shown in Figure 1-4 above also found frequent usage in beat divisions of 5:3, in which it complements a 3-division note such as dotted eighth or sixteenth notes. See Figure 1-5.



Figure 1-5. The 5-division symbol used to express a 5:3 division of a half note in Study No. 3b, p. 16.

Earlier, in the discussion by Thomas on the influence of jazz on Nancarrow, Nancarrow's tendency to place notes "around the beat" was noted, and this is another characteristic rhythmic feature in his music (see Figure 1-6). Other features that have already been mentioned include the "countdown" accelerative technique (progressively smaller units between attacks; see Figure 1-7 for a metric countdown used to particularly dramatic effect to close Study No. 12), and other types of numeric series that often involve adjacent numbers. (See Figure 1-23 for a series that alternately decelerates and accelerates.)

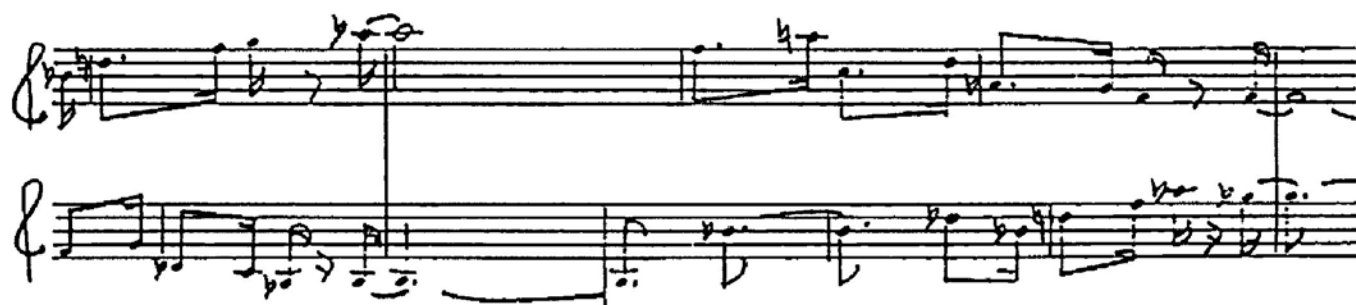


Figure 1-6. Melodic writing “around the beat” in Study No. 2 (p. 4).



Figure 1-7. The “countdown” technique at the closing of Study No. 12.

Rhythmic ostinati and isorhythms are a significant component of Nancarrow’s rhythmic landscape. In the literature reviewed here the terms ostinato and isorhythm are sometimes used interchangeably, but a distinction should be made between them. At times the authors seem to insist that an isorhythm involves both repeating rhythmic and pitch elements which are often of different lengths and out of phase with each other:

There is a fine line between ostinato and isorhythm, since an ostinato reiterates the same rhythm as well as the same pitch, . . . [while isorhythm involves] some internal dissociation of rhythm and pitch, a use of *talea* independent of *color*. (Gann 1995, 85–86)

. . . duration series are often paired with pitch series, which may be of different lengths, creating isorhythm. (Thomas 1996, 11–12)

[Nancarrow] revived the medieval technique of isorhythm (though inspired by a strong interest in the *tāla* structure of Indian music), employing multiple repetition of the same rhythm against different pitch sequences. (Gann 2001, 606)

Gann, however, often refers to rhythmic patterns alone as isorhythms. A slightly different phrase is used by Carlsen, who discusses the four closely-related repeating rhythmic patterns in Study No. 19 based on the series $\{n - 1, n, n + 1, n\}$ (the same patterns are shown in Figure 1-11 below) as “isorhythmic patterns,” which he later calls “*tales*” (Carlsen 1988, 7). Thus, for Gann and Carlsen, at least, an isorhythm does not necessarily require a repeating pitch element.

The term *ostinato* is just as ambiguous. Gann’s definition of *ostinato* as “a phrase persistently repeated without variation” (p. 69) does not distinguish between a rhythmic *ostinato* or one that combines melody with rhythm. I think it is safe to say, however, that the authors reviewed here always choose the term *ostinato* over *isorhythm* when the rhythm involves the same rhythmic unit repeated over and over, rather than a variety of rhythmic values—e.g., a steady repetition of eighth notes or eighth notes separated by eighth rests would be considered an *ostinato*, even if a repeating pitch element is present. Thus the term *isorhythm* should be held to the more exacting standard of involving varying rhythmic values. The repeating pattern in an *ostinato* also tends to be of brief length whereas *isorhythmic* patterns are generally longer and more complex. In Figure 1-8, for instance, Thomas states that the lower voice in the opening of Study No. 2 is “a small-scale example of isorhythm” (p. 12); she refers to the two lines operating together as an *ostinato* (as does Tenney [Wergo Vols. III/IV, p. 7]). Gann refers to each of the



Figure 1-8. The opening of Study No. 2.

lines in this segment as ostinati, while the bottom line is additionally an “isorhythm-within-an-ostinato” (p. 73).

Nancarrow sometimes creates resultant rhythms out of constituent numerical series and other components. An early and transparent example of a rhythmic ostinato, in Study No. 6, involves the continuous alternation between measures of $\frac{5}{8}$ and $\frac{4}{8}$ time in which all measures elapse at the same rate (1 measure per second); see Figure 1-9.



Figure 1-9. First system of Study No. 6, showing ostinato created by alternating different eighth-note divisions of the same measure length.

Several other Studies provide clearer examples of what the writers consider isorhythm. In Study No. 7, Gann (p. 88) identifies the following three isorhythms with lengths of 18, 24, and 30 eighth notes (a 3:4:5 ratio):

18 isorhythm = 5 + 4 + 2 + 3 + 4 (= 5 notes)

24 isorhythm = 5 + 5 + 2 + 4 + 3 + 2 + 3 (= 7 notes)

30 isorhythm = 3 + 2 + 2 + 3 + 2 + 3 + 3 + 2 + 3 + 2 + 2 + 3 (= 12 notes, palindromic)

Thomas refers to these patterns merely as “durational series” (p. 91) and Tenney calls them “duration series” (Wergo Vols. III/IV, p. 9). In Study No. 11, Gann and Thomas both identify the following accelerating eighth note pattern of 60 beats divided into three segments:

5 5 6 4 | 5 5 3 4 3 | 5 4 3 3 3 2
(4 notes) (5 notes) (6 notes)

Thomas calls this a “duration series” (p. 115) while Gann notes that this isorhythm is part of a 120-note “isorhythmic melody” (p. 97). Note the gentle acceleration built into the

series.

In an example using a numerical series, Gann identified the additive formula $\{n - 1, n, n + 1, n\}$ as the basis for all the rhythms in Studies No. 13–19. In Study No. 14, the series is applied to four voices that create the resultant rhythm shown in Figure 1-10. Later, in Studies No. 45, 46, and 47, Nancarrow creates a rhythm “not susceptible to rational notation” (Gann 1995, 257) by taking a collage of templates representing different tempo relations, and putting them all together.⁹

eighth-note beats	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17
voice #1 (3+4+5+4)	♪			♪				♪	♪				♪				♪
voice #2 (4+5+6+5)	♪				♪	♪				♪	♪					♪	♪
voice #3 (5+6+7+6)	♪	♪				♪	♪					♪	♪				
voice #4 (6+7+8+7)	♪	♪					♪	♪							♩		
resultant rhythm:	3/8 ♩				4/8 ♩ ♩ ♩ ♩				6/8 ♩ ♩ ♩ ♩				3/8 ♩		2/8 ♩		

Figure 1-10. Derivation of resultant rhythm in Study No. 14 (Scrivener 2000, 189).

Finally, Gann mentions (pp. 9–10) Nancarrow’s somewhat unique ability to combine both additive and divisive rhythms in his Studies, paying homage to Stravinsky’s legacy in the use of additive rhythms and Cowell’s ideals in the use of divisive rhythm—particularly the different division of the same unit of time in different voices. Manifestations of additive rhythm include the use of short repeating rhythmic patterns that are often grouped into larger “hypermeasures.”

Pitch and Melodic Features

Carlsen observes that:

the pitch organization in much of Nancarrow’s music . . . is based on traditional tonality. This is true even in the later studies, which are generally more chromatic or freely atonal than the earlier ones. Tonal centers, even if not

⁹The precise tempo relations used are unknown because Nancarrow could not recall what templates he had used to construct this rhythm. See Chapter 5 for a conjectural explanation for the derivation of this rhythm.

explicit, are frequently implied. Close-spaced, root-position major triads are ubiquitous, and nearly all of the studies end with some sort of V-I cadence (usually in the form of an ascending fourth). (pp. 19–20)

The real significance of this is, according to Carlsen, that “the prevailing diatonic context makes a perfect foil for the rhythmic complexities” (p. 20).

There are several prevailing melodic figures that have been identified in Nancarrow's Studies, including the ascending perfect fourth; the minor third, particularly the “partitioned minor third” (minor third divided into a whole step and half step; see Figure 1-11); and the melodic pattern of a whole step followed by a minor third ($\hat{5} - \hat{6} - \hat{1}$) which is variously described as pentatonic and derived from blues (an example can be seen in the lower voice of Figure 1-6). (This melodic pattern is also prevalent in nineteenth-century hymnody from the southeastern United States, although this was not likely a significant influence on Nancarrow.)



Figure 1-11. Opening of Study No. 31 showing prevalence of melodic “partitioned minor third.”

The influence of blues on Nancarrow's melodic writing can be seen in passages such as the one in Figure 1-12 showing blues scales used in Study No. 45b.

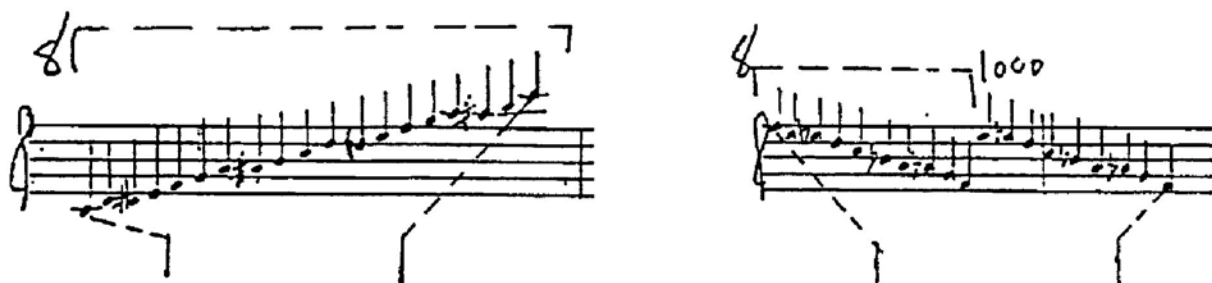


Figure 1-12. Blues scale patterns in Study No. 45b (p. 7). The notation Nancarrow uses is “exploded,” which is used when the space on the staff is insufficient for the number of notes; the given notes take place on the staff below in the time frame indicated by the dotted lines.

An especially characteristic melodic tendency is scalar patterns that are in the major form ascending and minor form descending. See Figure 1-13 (see also the top two lines

of Figure 2-6 [p. 71]).



Figure 1-13. Major ascending/minor descending scalar passages from: (top) Study No. 6 (p. 5) and (bottom) Study No. 7 (p. 1—major/minor passage is combined with 5 + 4 + 2 + 3 + 4 duration series).

Harmonic Features

This discussion has already noted Nancarrow's preference for the elements of traditional tonality, including harmony consisting of prevalent thirds, sixths, octaves, and root-position triads; implied tonal centers; traditional cadence structures; and major and minor scale structures. He also frequently used jazz harmonies such as seventh and ninth chords (including those with raised fifths and flatted ninths) and major/minor triads, and blues scales with flatted sevenths and simultaneous raised and lowered thirds. Despite his rigorous counterpoint training with Sessions, however, Nancarrow claimed to have little melodic or harmonic imagination.¹⁰ Figure 1-14 shows a fairly typical triadic texture in Nancarrow. In this example, the third canon in Study No. 37, the top five of twelve voices enter with a level of imitation each time of a fourth higher.

Parallel harmonies like those in Figures 1-13 and 1-14 are indeed quite common in Nancarrow's Studies. The presence of parallelism and other non-functional harmonic structures is what prompts Thomas to comment that "Coordination of pitch class content among layers, in fact, creates a non-multidimensional texture type called heterophony" (p. 53). Gann points out that "chords appear not as products of voice-leading or tonal function, but almost always in parallel, as textural extensions of a single line" (p. 12). He

¹⁰"One of the main reasons I suppose that I concentrate more on tempo relationships is that I have little melodic or harmonic invention" (Reynolds 1984, 6).



Figure 1-14. Entrance of top five voices (of twelve) in third canon of Study No. 37 (p. 9).

goes on to suggest that Nancarrow uses the following harmonic structures “in increasing order of emphasis” to fortify a melodic line:

- 1 thirds
- 2 octaves
- 3 parallel major triads
- 4 parallel seventh or ninth chords
- 5 larger chords generated from a single interval, or
- 6 chords which, when condensed within an octave, fill out a contiguous diatonic or chromatic scale segment. (Gann 1995, 12–13)

These parallel structures are often built up through the accumulation process. For instance, in Study No. 27 Nancarrow begins with an unharmonized ostinato and progressively harmonizes it with thicker harmonic structures. Figure 1-15 shows the four notes of the original ostinato and the harmonic manipulations to which it is subjected.

As already noted by Carlsen, many of Nancarrow’s Studies end with a V–I final cadence, and interior V–I cadences are also plentiful. In the case of canons in which imitations are at intervals other than the octave (the vast majority), adjustments would be

needed at the end to effect a V–I ending; Study No. 19 includes such a final cadence (perhaps because it is the concluding piece in the set of pieces Nos. 13–19). However, many endings have no harmonic adjustment and conclude straightforwardly in multiple key areas. Study No. 12 includes a 3-voice canon which establishes three key areas related by fifths: D, A, and E. The voices cadence at the end on complete triads in these same three key areas (see Figure 1-7). There is also a number of interior V–I cadences in this canon that are characterized by extended harmony (see Figure 1-16).

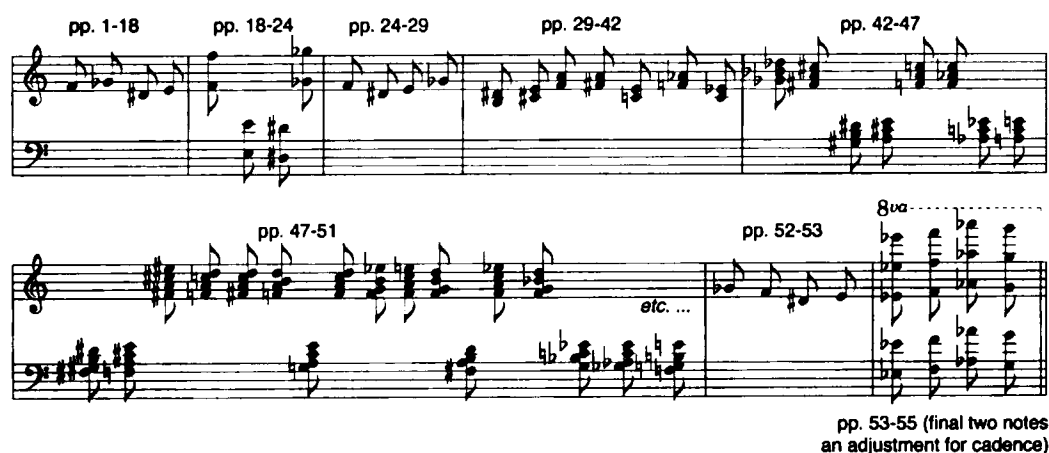


Figure 1-15. Ostinato line of four chromatically contiguous pitches in Study No. 27 and progression of its harmonic manipulations.

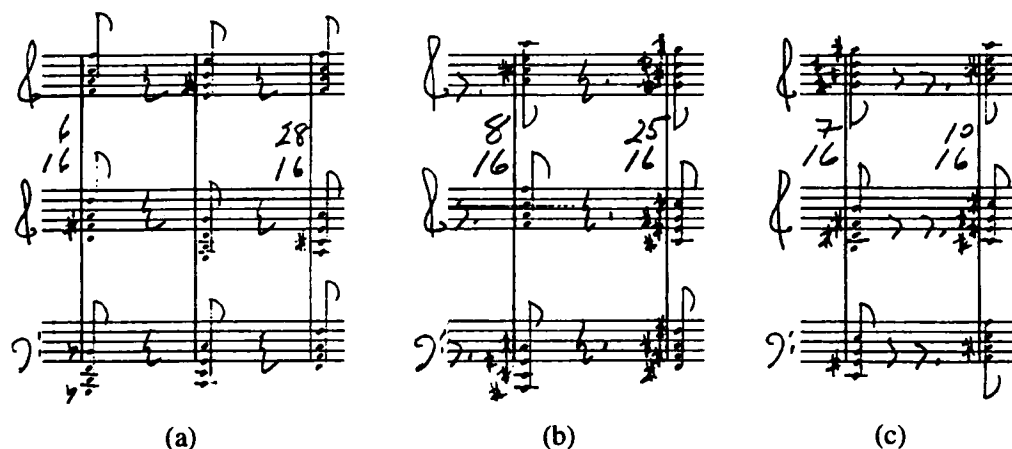


Figure 1-16. Interior V–I cadences in Study No. 12: (a) p. 28, first system; (b) p. 28, second system; (c) p. 29, second system.

The influence of blues extended to Nancarrow's use in several pieces of blues harmonic progressions (e.g., I-IV-I-V-I) as the basis for ostinati on which an entire piece is built and also for levels of transposition of large sections in some cases. This was particularly common in his earlier pieces, such as Nos. 2, 3 (the "Boogie-Woogie Suite"), and 11, but he returns to this harmonic pattern in some later pieces such as No. 45.

Finally, the levels of imitation used in Nancarrow's tempo canons have a significant impact on the harmonic environment. In the earlier tempo canons and other canonic pieces the level of imitation tended to be at the octave or fourth/fifth; gradually Nancarrow began to move to major thirds/tenths to establish what Gann called a "bittersweet and ambiguous major/minor harmony" (p. 203) in two-voice canons such as No. 43. Table 1-1 provides a summary of levels of imitation in some of Nancarrow's Studies.

Table 1-1

Levels of Imitation in Selected Studies

Study No.	No. of voices	Levels of imitation
4	3	2 octaves, 2 octaves
8	3	2 octaves, 2 octaves
12	3	perfect fifth, perfect fifth
14	2	2 octaves + perfect fifth
15	2	3 octaves
18	2	octave
19	3	perfect eleventh, perfect eleventh
24	3	contains series of canons with levels of imitation ranging from root position triads to first inversion triads to open fifths and octaves
32	4	perfect fifth between all voices
36	4	major tenth, minor tenth, and major tenth (creating a widely-spaced major seventh chord)
43	2	major third
49a	3	2 octaves + perfect fourth, 3 octaves + perfect fifth
49b	3	perfect twelfth, perfect twelfth

Structural Features

Canon is such a pervasive technique in Nancarrow's Studies that it is treated at considerable length in the four sources. Tenney speaks quite eloquently of the importance of this device in Nancarrow's Studies:

In his frequent use of *canon* as an organizing device, Nancarrow has revived an ancient tradition that perhaps began with Machaut in the 14th century, and reached a point of culmination in the *Musikalisches Opfer* of Johann Sebastian Bach before lapsing into a period of neglect as a formal procedure. But no survey of the past history of the canon could have given us any idea of the incredible wealth of utterly *new* possibilities hidden within this humble device. The word itself has had an intriguing history: deriving from the Greek *kanna* meaning simply a length of reed or cane, it became *kanon*—model, measure, law, or rule. But like some elegant mathematical equation, what it “models” is an infinite variety of particular manifestations, becoming the very “measure” of musical/experiential *time*. (Tenney, liner notes to Wergo Vols. III/IV, p. 19)

Thomas creates a taxonomy of canons showing the conventional “1:1” canon plus four possibilities involving voices moving at different speeds (Figure 1-17). Nancarrow wrote tempo canons representing each of the four basic types except diverging, as well as numerous canons that combine various aspects of the basic types.

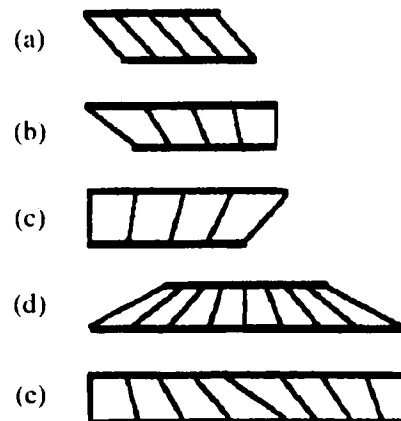


Figure 1-17. Thomas's (1996, p. 186) taxonomy of canons: (a) conventional; (b) converging; (c) diverging; (d) converging-diverging (arch); and (e) diverging-converging.

Gann's section on tempo canon terminology (pp. 19–28) introduces several

important terms: convergence point, convergence period, tempo switch, and echo distance. The **convergence point** is “the infinitesimal moment at which all lines have reached identical points in the material they are playing” (p. 21). Convergence points (CPs) are so critical to the structure of tempo canons that Chapter 3 of this work will be devoted to them. The **convergence period** is “the hypermeasure that exists between (potential) simultaneous attacks in voices moving at different tempos” (p. 21), which Gann specifies as being important to keep track of in canons in which meters change frequently. Not all canons contain a **tempo switch**, “a device in which Nancarrow switches the [faster] line to the [slower] tempo and vice versa” (p. 21), but when they appear they always occur halfway between CPs involving the affected voices. Finally, the **echo distance** is “the temporal gap between an event in one voice and its corresponding recurrence in another” (p. 21); as voices approach a CP the echo distance decreases, and as voices move away from a CP the echo distances increases. Gann’s illustration of these terms is shown in Figure 1-18.

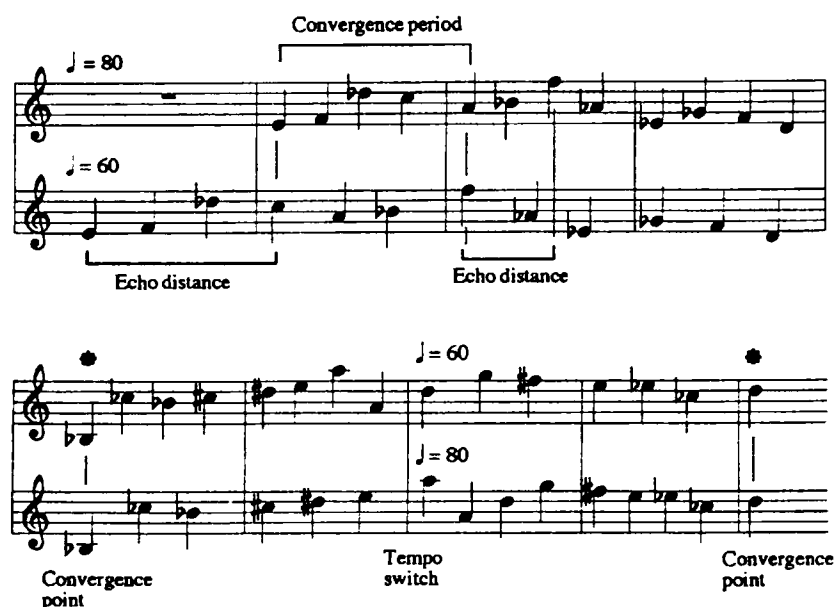


Figure 1-18. Gann’s tempo canon terminology: convergence point, convergence period, tempo switch, and echo distance (Gann 1995, 20).

Symmetry of various kinds is also structurally significant in Nancarrow's Studies. The arch-shaped converging-diverging tempo canons exhibit symmetry about the central CP. Other non-canonic pieces are virtually palindromic, including Studies No. 1 and 43. Palindromic isorhythms, such as the $3 + 2 + 2 + 3 + 2 + 3 + 3 + 2 + 3 + 2 + 2 + 3$ isorhythm found in Study No. 7, abound in the Studies; several other lengthy examples from Study No. 8 are shown in Figure 1-23. Also, Figure 1-24 shows a complex series of isorhythms that is symmetrical. An example of a mirrored rhythm is located by Carlsen in Study No. 35 (see Figure 1-19).



Figure 1-19. Mirrored rhythm in Study No. 35 (Carlsen 1988, 56).

In Study No. 19, a 3-voice tempo canon subtitled "Canon 12/15/20," Nancarrow symmetrically partitioned the keyboard into overlapping 4-octave segments about the player piano's middle note, and he employed a similar procedure in Study No. 36 (see Figure 1-20).

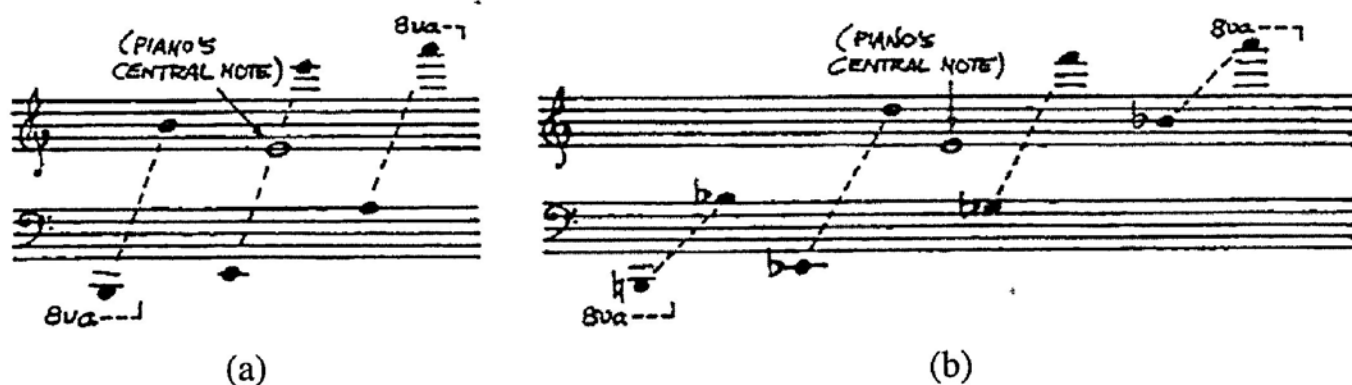


Figure 1-20. Symmetrical deployment of canonic voices in (a) Study No. 19 and (b) Study No. 36 (Carlsen 1988, 6, 24).

Symmetries also occur in some of the Studies' harmonic structures. The major seventh chord—a symmetrical structure with two major thirds separated by a minor third—defines the levels of imitation in Study No. 36 and pervades that piece's harmonic

environment (including the final chord). In Study No. 27, a series of eleven 4-voice canons, the levels of imitation are symmetrical in a number of the canons (see Table 1-2).

Table 1-2

Symmetrical Levels of Imitation in Selected Canons of Study No. 27

Canon #	Levels of imitation (highest to lowest voices)
1	octave + major sixth octave + minor sixth octave + major sixth
2	octave + perfect fifth 2 octaves + major second octave + perfect fifth
3	octave + minor sixth 2 8ves + major second octave + minor sixth
4	octave + minor sixth 2 octaves octave + minor sixth
8	octave + major third 2 octaves + major seventh octave + major third

Melodic and pitch inversions are also fairly frequent, such as the second ostinato in Study No. 9 and mirrored glissandi in Study No. 25 (see Figure 1-21).

Mathematical ratios also play a significant structural role in Nancarrow's Studies; this topic will be explored more thoroughly in Chapter 2. In particular, ratios based on pitch intervals of the justly-tuned scale are prominently used. Besides being used deliberately in establishing relative speeds between voices in tempo canons, mathematical ratios are used by Nancarrow in a number of ways, such as creating relationships between pitch ratios and melodic and harmonic features; and to control such factors as the levels of imitation between voices, positioning of voice entrances, and other structural concerns.

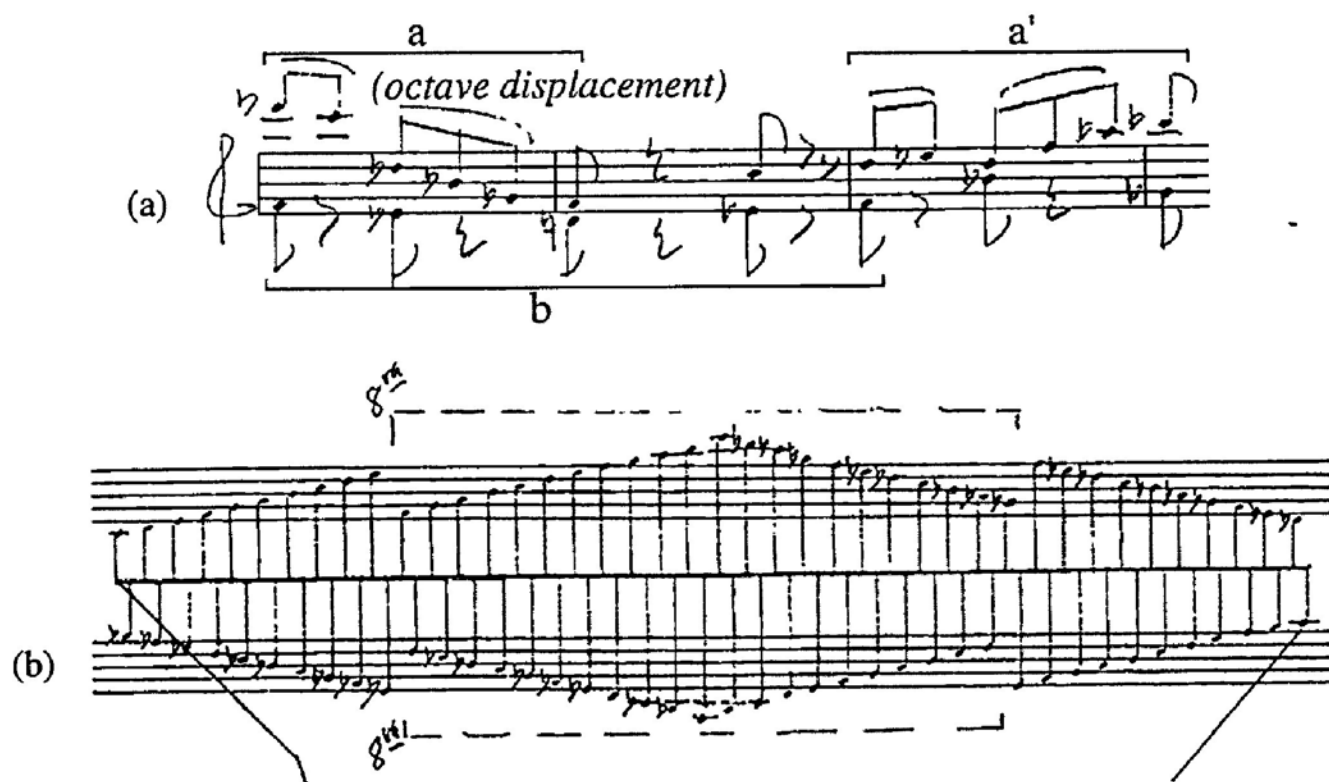


Figure 1-21. Symmetrical melodic structures in (a) Study No. 9 (p. 5) and (b) Study No. 25 (p. 60).

Texture, Timbre, and Dynamics

Several textural features of the Studies can be traced to limitations of the player piano medium. The first is the prevalence of staccato notes—Nancarrow reported to Reynolds (1984, p. 20) that this was because punching sustained notes was simply more work, as a line of holes must be punched for sustained notes whereas a single punch produced a staccato note. (Eventually Nancarrow had his punching machine altered so that he could punch a line of four holes at once, and this alleviated the problem somewhat.) The other textural feature affected by the player piano was the number of simultaneous notes that could be sounded—due to potential loss of air pressure from the sounding mechanism and the possibility that too many holes punched too close together could cause the roll to perforate, the player piano is limited to playing about sixteen notes at a time (Carlsen 1988, 71 note 7). The player piano has a sustain pedal, but Nancarrow used it in only one piece (No. 25), so the overall texture in the Studies tends to be somewhat dry and spare.

Carlsen notes Nancarrow's tendency to assign faster tempos to higher voices, adding that:

Compelling acoustical reasons have of course also made this true of much other music; one could cite, as Cowell does, the model of the overtone series, where the higher notes vibrate at faster frequencies than the lower ones. But it is interesting that, in most cases, the only concession Nancarrow makes to the differences in tone quality between high and low registers is to assign them different tempos. In other respects, high and low are treated equally; Nancarrow does not usually compensate for the quick decay and light sound of notes in the upper registers. (Carlsen 1988, 23)

Finally, despite the fact that gradual crescendos and decrescendos are possible on the player piano, Nancarrow has a decided preference for terraced dynamics. Gann explains the mechanics behind the process this way:

At one edge of the piano roll is a place for up to three holes that determine how much air pressure the machine uses, thus how much volume it produces. For whatever reason, these holes are designated 2, 4, and 6. Eight dynamic levels are possible, ranging from no holes to all three. In Nancarrow's punching scores, he notates those levels as (-0-), (-2-), (-4-), (-6-), (4-2), (6-2), (6-4), and (6-4-2). He rarely seems to need all eight levels in one work, however. Study No. 48a contains six dynamic levels, and in the final score Nancarrow notates (6-4-2) and (6-4) as *ff*, (-6-) as *f*, (-4-) as *mf*, (-2-) as *p*, and (-0-) as *pp*. (Gann 1995, 29)

Comparison of Analyses of Selected Studies

It is instructive at this point to compare the analytical treatment of the Studies in the various sources. The most comprehensive sources on the individual Studies are Gann and Tenney. As we have seen, Gann reviews each of the Studies, and Tenney's liner notes for the Wergo recordings and 1977 article in *Soundings*, Book 4 cover all of them, as well, although in considerably less detail. However, Carlsen's monograph and Thomas's dissertation offer more detailed analyses of fewer, selected Studies. There are four Studies—Nos. 8, 19, 23, and 35—that are reviewed at least to some extent by all four sources, and these provide a useful sample for comparison of the four major analytical sources.

Study No. 8 is discussed by Gann on pp. 148–58, by Carlsen on pp. 34–46, by Thomas on pp. 89–91, and in Tenney’s liner notes to *Wergo Vols. III/IV* on p. 11. There is general agreement that non-strict canonic imitation occurs in much of the piece, which is in four voices, and both Carlsen and Gann identify three sections to the study and agree on their locations (neither Thomas nor Tenney comments on the form). Gann and Tenney observe that this is the first of the Studies to use proportional notation, and Gann calls this Nancarrow’s first experiment in acceleration; Tenney is characteristically succinct in commenting only on the notation and the “gradual fluctuations in tempo, alternately accelerating and decelerating, in a way that appears to be quite independent of the other voices, and unrelated to any sense of meter” (p. 11). The acceleration is arithmetical and accomplished through using multiples of an invariant background unit, although Gann credits Nancarrow for developing some numerical series in the work that approximate geometric acceleration. The accelerating rhythm of the opening line is discussed by both Gann and Carlsen, but Gann’s “conjectural notation” portrays the acceleration of the melody while Carlsen’s re-notation portrays only its triple time feel (see Figure 1-22).



Figure 1-22. (a) Carlsen’s (p. 35) and (b) Gann’s (p. 149) re-notations of the opening rhythm in Study No. 8. (Carlsen’s example begins with the score’s second system and corresponds to the pick-up to Gann’s ¹⁹32 measure.)

The accelerative process illustrated in Figure 1-22 is actually isorhythmic, and both Gann and Carlsen devote the majority of their attention to identifying the rhythmic series used and their various manifestations. Gann identifies three accelerating/decelerating isorhythmic series used in the Study while Carlsen refers to “accelerating succession[s] of durations” (p. 35). Because of the non-metric proportional notation, identifying the rhythmic series requires that the length of time between notes be measured on the score in millimeters; however, Carlsen and Gann convert these measurements for purposes of analysis in different ways. Carlsen identifies the number of separate elements in the series and then assigns each value an ordinal number. Gann identifies an “invariant background durational unit” (p. 148) in the rhythmic series, and expresses each duration in the first series as a multiple of that background unit. For instance, both Carlsen and Gann describe the first rhythmic series as consisting of nineteen durations, with 41 millimeters being the longest and 10 being the shortest: Carlsen expresses that series in values from 1 (longest) to 19 (shortest); Gann divides each measurement by the background unit (ca. 1.7 mm.) to arrive at a series that progresses from 24 (longest) down to 6 (shortest). Note that Carlsen’s system creates an inverse relationship with respect to length of the note while Gann’s does not.

Gann’s three series correspond to the three sections of the work, although he notes that the first series appears in all three sections. Strangely, he does not follow through with division by the background unit in expressing the second series. Figure 1-23 shows Gann’s three series and Carlsen’s graphic representation of the first series; the values expressed in Gann’s three series are not directly comparable, because the second series is not expressed in multiples of the background unit.

Carlsen, too, devotes a considerable portion of his analysis to examining all the duration series, particularly their symmetrical applications. The symmetrical structures of the series are readily evident in Figure 1-23. Other symmetrical structures identified by Carlsen include the canonic subject of the second section, a 35-note palindromic line

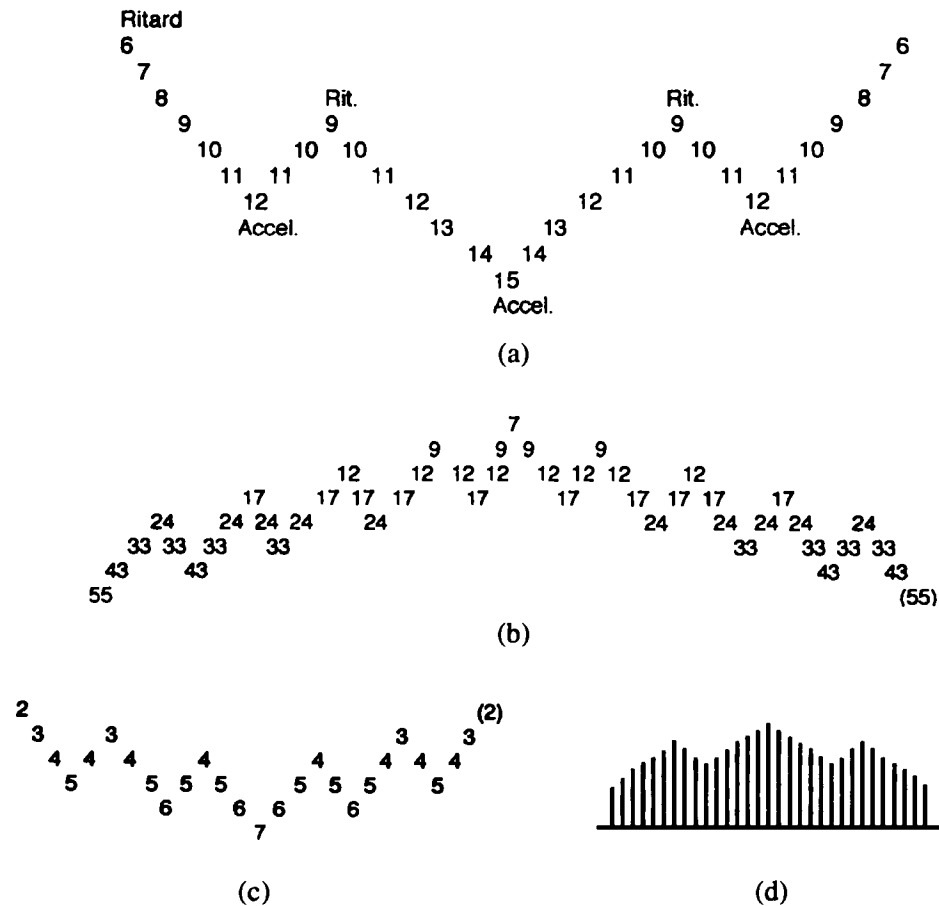


Figure 1-23. Gann's three isorhythmic series identified in Study No. 8: (a) section 1, with the series expressed as multiples of the background durational unit (p. 150); (b) section 2, expressed in actual measurements in millimeters (p. 151); and (c) section 3, expressed as multiples of the background unit (p. 152); plus (d) Carlsen's graphic representation of the series in section 1 (p. 37).

consisting of only three pitches; and the pitch sequence of the top voice in this same section, an ascending scalar movement in the treble from F# to B and back that is mirrored in a descending scalar movement in the bass from F# to C#. When these materials are combined with the symmetrical rhythmic series that abound in this piece, it is clear that the work is as much about symmetry as it is about acceleration.

The work's third section introduces a different idea—that of heterophony among three canonic voices. As shown in Figure 1-24 from Carlsen, the canon is duplicated at alternate notes first at the octave above in the bottom voice, at the fifth above in the

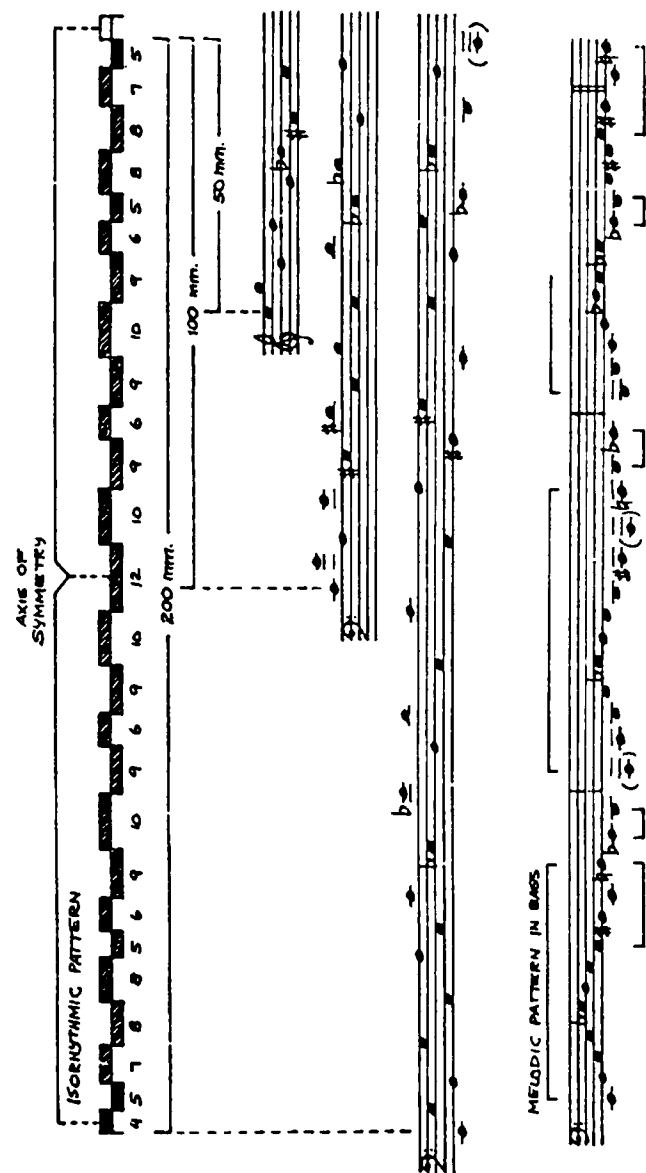


Figure 1-24. Carlsen's summary of the canonic trio in the third and final section in Study No. 8 (p. 45). (The designation "mm." in the diagram refers to millimeters and not to measures.)

middle voice, and at the minor third in the top voice. The result is a consistent texture of major triads expressing the symmetrical rhythmic series of this section.

Thomas chooses to focus on this last section in her discussion on heterophony as an example of heterophony involving parallel movement of voices in intervals other than octaves. She disagrees with Gann and Carlsen by stating that "this is a rhythmic but not pitch canon" (p. 90); it is not clear why she would not comment on the canon, except that she seems to be much more focused on the creation of vertical triadic sonorities than on the linear element. She believes that there is so much coordination among the voices that "the heterophonic pitch relationship causes the voices to lose their individual identities" (p. 90). At the point that the third voice comes in, she analyzes on the score the triadic structures that result, as shown in Figure 1-25 (Gann performs a similar analysis on p. 152).

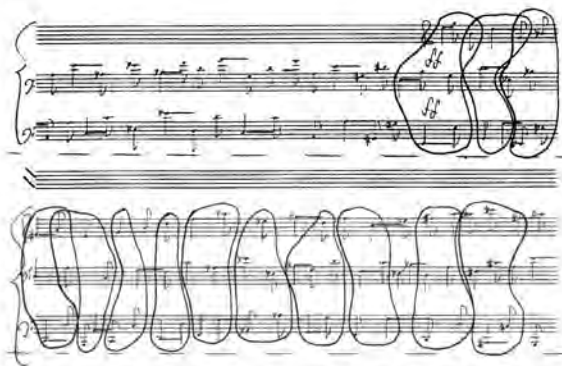


Figure 1-25. Thomas's identification of triadic structures in the trio canon of the third section of Study No. 8 (p. 250). (This point corresponds to the entrance of the third voice in Figure 1-24).

Study No. 19 (Canon 12/15/20) is discussed by Gann on pp. 114–15 and 119–20, by Carlsen on pp. 11–28, by Thomas on pp. 87–88, and by Tenney in the liner notes to *Wergo Vols. III/IV* on p. 24. The form of this piece is rather transparent: as the title indicates, it is a 3-voice canon with the tempo ratio of 12:15:20. It is also a converging canon, with the three voices converging on the very last note of the piece. As is typical in Nan-carrow, the slowest/lowest voice enters first, followed by a higher/faster voice and finally the highest/fastest voice.

Tenney refers to the voices as “strata,” which is an apt term since each is a composite of four rhythmic layers. Much attention is focused on the rhythmic resultant, with Carlsen, Gann, and Thomas all detailing the following information: (a) the additive formula $\{n - 1, n, n + 1, n\}$ is used to establish the following rhythmic patterns (in eighth-note beats) for the highest to lowest voices: $3 + 4 + 5 + 4$, $4 + 5 + 6 + 5$, $5 + 6 + 7 + 6$, and $6 + 7 + 8 + 7$; (b) the same melody is projected in all four voices, with the lower voices occasionally dropping notes along the way in order to keep up. The result is, as Thomas notes, an “incipient form of heterophony” (p. 87), which can be seen in Figure 1-26.

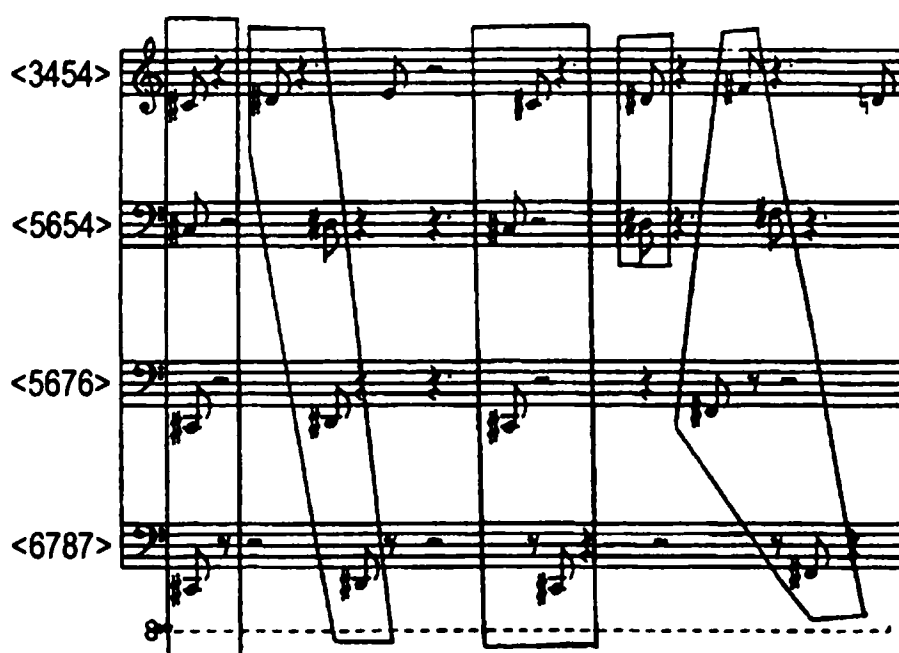


Figure 1-26. Heterophony as projected through four rhythmic layers in Study No. 19 (Thomas 1996, 247). Note the omission of some notes in the lower and slower layers (E_4 , for example, is projected only in the top voice). The number series at the left of each staff represents the note durations in eighth notes according to the formula $\{n - 1, n, n + 1, n\}$.

The registral placement of the voices is of interest to Carlsen and Thomas. Carlsen notes the symmetrical placement of the voices about the player piano's middle note (E_4) into overlapping four-octave segments, with the interval of imitation between the voices being an eleventh. To Thomas, this—in tandem with the heterophony displayed in each canonic layer—results in a lack of registral separation: “the potential for multidimensionality inherent in the registral polyphonization of the single line is denied by its heterophony” (p. 88).

Carlsen devotes the most attention and speculation to the use of tonality in the work and attendant harmonic planning and motion. Tenney, Carlsen, and Gann all note the adjustment in the canon at the very end so that all voices end on $F\#$ and effect essentially a V–I cadence, and Gann calls this “the most traditionally tonal of Nancarrow’s tempo canons” (p. 120). Carlsen asserts that Nancarrow’s compositional method was to first plot out the rhythmic relationships, then carefully select melodic pitches that would create the desired harmonies since “most of the harmonies in Study No. 19 are major and minor triads or other traditional chords, connected moreover by quite elegant voice-leading” (p. 10). Figure 1-27 is representative of the strong tonal implications that result when all three voices are part of the texture.

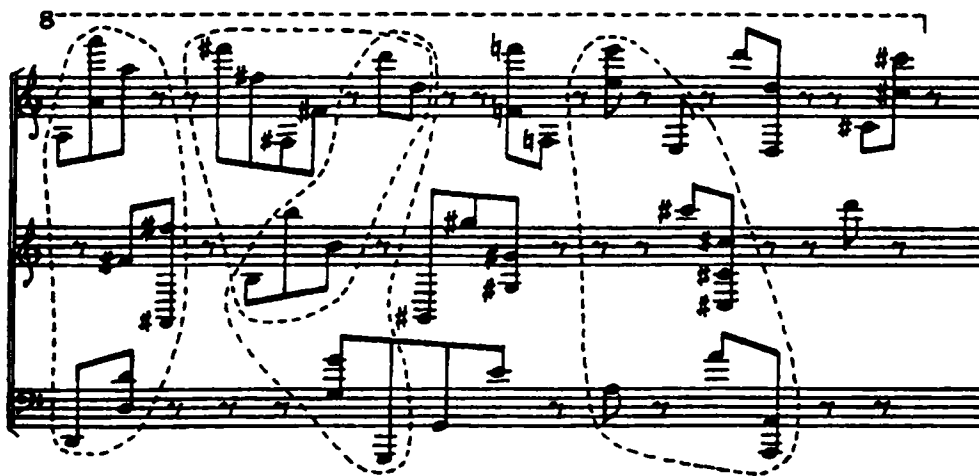


Figure 1-27. Triadic structures in Study No. 19, systems 7–8 (Carlsen 1988, 14).

Carlsen then turns his attention to debating the specific tonality of the piece: he makes a strong case for E being the most prominent tonality, but finds support for C# and F# (and a long-range tonal motion between the two) as well.

With all of the above information in mind, Carlsen includes in his analysis an inferred “underlying set of precompositional rules for rhythmic organization” (p. 9), which I include here in its entirety:

- (1) Establish four closely related isorhythmic patterns (*taleae*).
- (2) Give a different pattern to each of four octaves constituting the range of a voice, assigning the fastest pattern to the highest octave, the next-fastest to the second octave down, and so on.
- (3) Determine the length of the study by finding that point after the beginning at which notes in all four patterns will coincide.
- (4) Replicate the rhythmic patterns for two other voices after deciding on tempo ratios. Assign the fastest tempo to the highest voice. Carefully measure and draw these patterns on a piano roll or on music manuscript paper, working backwards from a simultaneous ending. At this point, the specific number of notes in the study, the rhythms, and the relationships between voices are set—the only thing that remains is to choose pitches. (It is perhaps worth reemphasizing that there are twelve lines of rhythmic activity—four for each voice.)
- (5) Determine the interval of imitation by symmetrically partitioning the piano into overlapping four-octave segments, one for each voice.
- (6) Write a melody for the soprano octave of the lowest voice. Fill in the pitches for the alto, tenor, and bass octaves by referring to the soprano line. (Notes in a lower octave must be of the same pitch-class as simultaneous or proximate soprano notes.)
- (7) Now that the pitches are all set for the lowest voice, use the interval of imitation (an octave plus a fourth) to derive all the pitches for the other two voices. Make a slight adjustment in notes at the very end to achieve a sort of V–I cadence (C# to F#) and the piece is finished. (p. 9)

Study No. 23 is discussed by Gann on pp. 156–59, by Carlsen on pp. 47–54, by Thomas on pp. 128–32, and by Tenney on p. 14 of the liner notes to *Wergo Vols. III/IV*. Carlsen, Gann, and Thomas all agree that the piece is divided into four primary sections, and they are in agreement as to where the sections divide. Carlsen and Gann both comment on the total lack of canonic imitation in this piece. The sources are in general consensus that there is little relationship between the two voices and, as Carlsen (1988) says,

the “two voices remain quite independent of one another from beginning to end, and it appears that one of Nancarrow’s purposes here is simply to exploit the differences between them” (p. 47).

Thomas points to this piece as one involving different “time types,” with one of the two voices being metered most of the way through and the other voice being proportionally notated and highly variable in speed. The metered voice frequently changes meters, but a steady eighth-note pulse is maintained. One of the more interesting effects in the second section occurs in the unmetered voice: a countdown device is combined with a duration series in which durations are tied directly to pitch level. The 71 notes from B_0 to A_6 are assigned a gradually decreasing duration ranging in measurement from 59 millimeters to $\frac{3}{4}$ millimeter. Carlsen points out that durations that are related by the ratio of 2:1 are a major ninth apart (see Figure 1-28). This same rhythmic series was actually established in the first section as part of a 71-note accelerando in which the pitches are not stated in chromatic order. Both Carlsen and Thomas correctly note that in

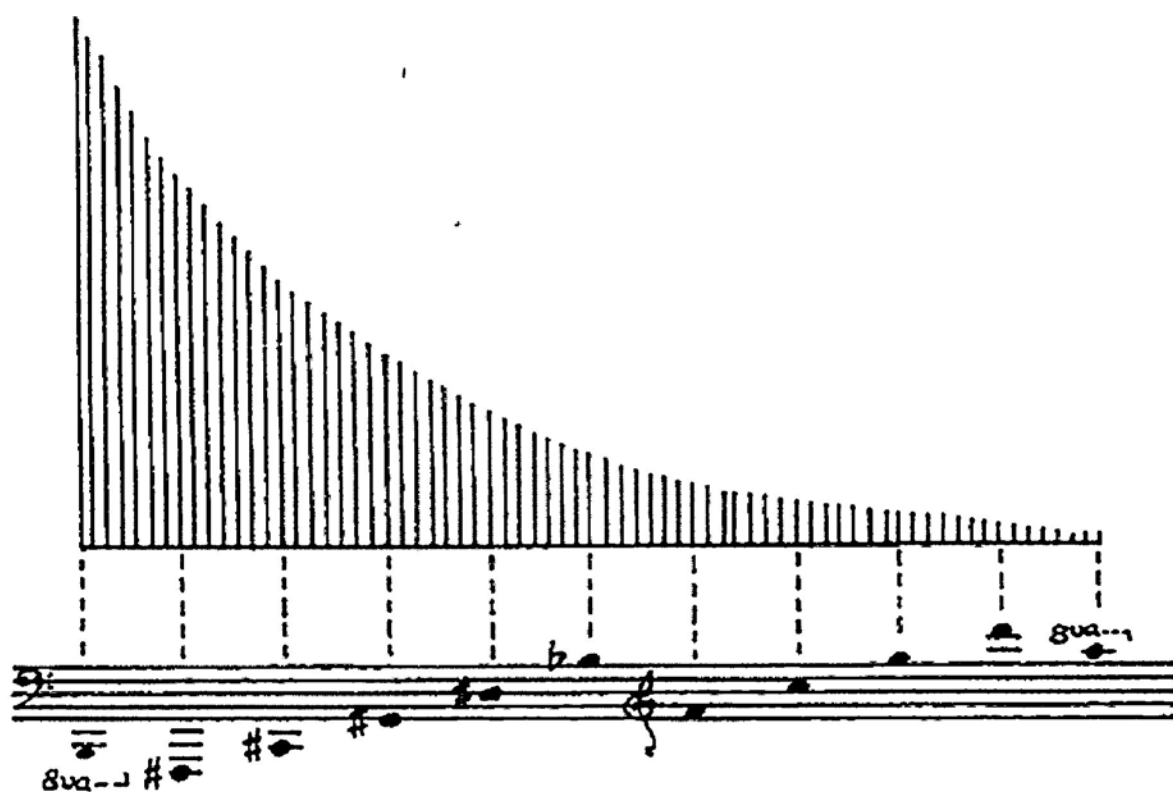


Figure 1-28. Duration series in Study No. 23 (Carlsen 1988, 49).

the second section the durations are ordered from lowest to highest, while in the opening accelerando they are not.

Thomas focuses on the second section's countdown device in which the lower voice states pitches first in quintuplet groups, then quadruplets, triplets, and finally duplets (Carlsen enumerates 23 quintuplets, 28 quadruplets, 44 triplets, and 71 duplets). For each grouping, the duration of each note is the same and is determined by the duration assigned to the first pitch of the grouping. Additionally, at each point where the size of the groupings decreases, the tempo of the upper voice increases. Thomas identifies several passages in this section where there are near convergences in attacks between the lower voice and the metered upper voice; one such example is shown in Figure 1-29. In the first

The figure displays three systems of musical notation for a piano study. Each system is a grand staff with a treble clef on the upper staff and a bass clef on the lower staff. The first system begins with a mezzo-forte (mf) dynamic marking. The second system is marked with a 3/4 time signature. The third system is marked with a 4/4 time signature. In each system, the lower staff contains complex rhythmic groupings (quintuplets, quadruplets, triplets, and duplets) while the upper staff has a more regular metered pattern. Horizontal lines with breaks (dashed lines) are drawn across the lower staves to highlight specific points of near convergence between the attacks of the two voices. These broken underlines appear in the first system at two points, in the second system at one point, and in the third system at two points.

Figure 1-29. Moments of near convergence of attacks in second section of Study No. 23 (Thomas, p. 287). Broken underlines beneath second staff indicate near convergences.

case (from the first to the second systems) the bottom voice nearly converges with the quarter note pulse in the upper voice, and in the second case (second to third systems) it nearly converges with the eighth note.

Tenney does not delve into the underlying system of this section, but comments on the effect of the faster, higher figures that are intermittently stated: “Occasionally interjected into this texture we hear fast, high-pitched four- or five-note figures. These are notated on the same staff as the lower voice, but they suggest to the ear the presence of an intermittent *third voice* in the texture” (p. 14).

Carlsen, Gann, and Thomas all comment on the structure of the metered voice, particularly its pitch content (its rhythm is generally agreed to consist of somewhat random meter changes with a steady eighth-note pulse.) In the opening section and through the statement of quintuplets in the lower voice, the upper voice consists of only combinations of dyads and single notes in which Thomas identifies the prevalence of set classes 3-1 [012] and 3-2 [013]. Carlsen astutely observes that in the second section, as the groupings decrease in size, the chords in the upper voice at first become thicker: where the lower voice has quadruplets, the dyads in the upper voice have been replaced by trichords; and with triplets in the lower voice, the upper voice states four-note chords; the upper voice returns to dyads where the lower voice has duplets.

The third and fourth sections are given the fullest treatment by Carlsen. The third section does not include the metered voice and features an upper voice with a gradually accelerating sequence and a lower voice of seemingly random sustained pitches that use the same 71-note duration series already seen. Carlsen makes an extraordinary case that Nancarrow arrived at the pitches in the lower voice:

by drawing diagonal lines on the roll through the beginnings or ends of notes that have already been plotted, using the intersections of those diagonal lines with various other lines to determine the pitches of later notes. He thereby establishes a long chain of interrelationships between the pitches and temporal placements of almost all the notes in the passage. (p. 52)

Figure 1-30 shows his attempt to verify this method. Gann was quite skeptical that Nancarrow would have worked this way, and indeed says that “Nancarrow denies that he ever did such a thing, or that he ever let the visual appearance of the roll influence a musical decision. Indeed, it strikes me that such a non-musical strategy would be foreign to his thinking” (p. 293, note 8).

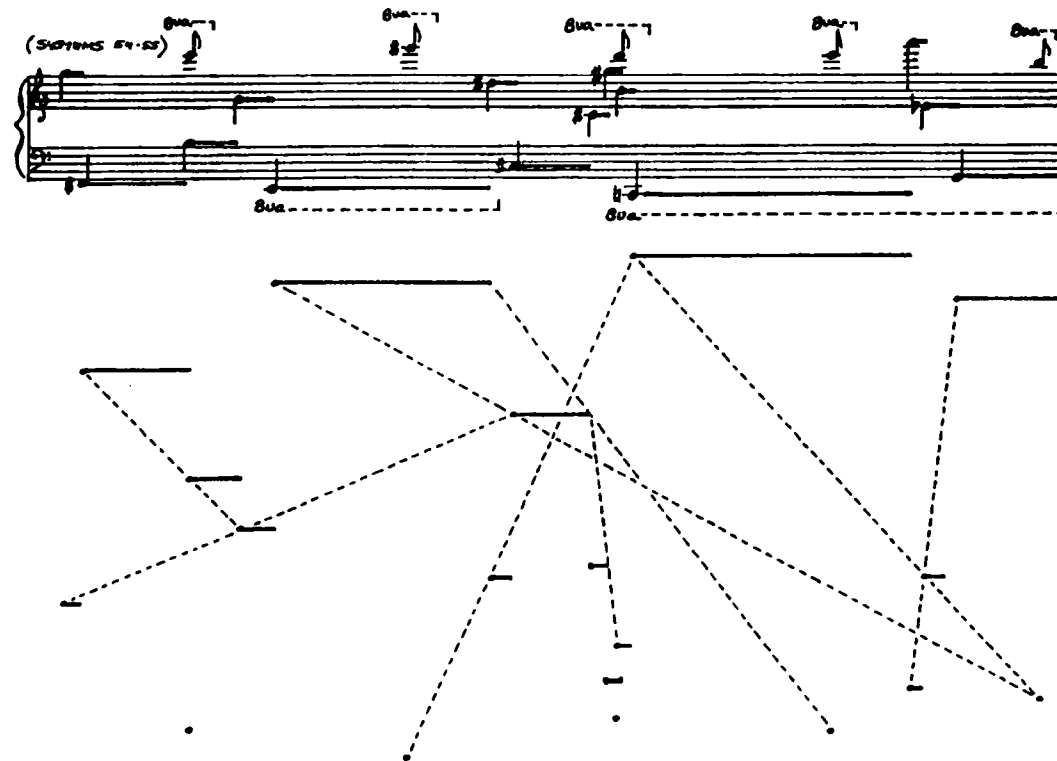


Figure 1-30. Carlsen’s (1988, p. 53) diagram illustrating the plotting of pitches in systems 54–55 of Study No. 23. The diagram below the staff represents the piano roll, on which pitches are ordered lowest to highest from top to bottom.

Study No. 35 is covered by Gann on pp. 248–53; Tenney very briefly on p. 16 of the notes to Vols. I–II and on p. 62 of *Soundings*, Book 4; Thomas on pp. 63–65; and Carlsen (1988) on pp. 55–65. All four sources note that this piece has canonic features but that exact canonic imitation is only hinted at and rarely used. They also make note of the large number of different tempos used (eleven), and Carlsen points out how unusual it is for Nancarrow to use more discrete tempo markings than the maximum number of

sounding voices (five in this case). Gann makes an interesting observation in pointing out that the fastest five tempos are “octave multiples” of slower tempos, “leaving only six real tempos” (Gann 1995, 249).

Tenney’s description is, as usual, very cursory—it does not identify the number of formal sections, nor much about the pitch elements except to say that there are “different aggregate-types used, which include octaves, 3rds, triads, and miscellaneous chords.” There is also no mention of the jazz elements (both rhythmic and melodic) that are identified by the other sources as being quite prominent. Carlsen notes, for instance, the trochaic (“swung”) rhythm that characterizes the main melodies, and points to the constant evolution of the melodies as suggesting improvisation; he also characterizes some accompanimental patterns as “boom-chick” and notes a call-and-response pattern between voices that mimics jazz performance. Gann also finds similarities to jazz in the $\frac{6}{8}$ syncopated melody and the call-and-response pattern.

There is slight disagreement between Gann and Carlsen on the form of the piece (Thomas discusses only the first nine pages of the work and does not comment on the form). Gann identifies eight sections in the study and discusses each in turn. He goes into considerable detail for each section about the progression of two similar “basic melodic patterns” that he labels A and B, and notes for each section the tempo ratios that result with the various combinations of the eleven different tempos. Gann identifies the fifth section as a “brief bridge,” but in Carlsen’s formal diagram this section belongs to the previous section, giving a total of seven sections in his analysis (see Figure 1-31, in which the unbroken vertical lines delineate the seven sections). Carlsen’s formal diagram corroborates his assertion that there is a sectional “acceleration” where each section is shorter than the previous section, and he astutely identifies a “grand substructure” in which various combinations of overlapping sections (A, B, C, and D in the example) are exactly the same length (measured in millimeters since the score is notated in “proportional notation” in which elapsed time is conveyed on the horizontal axis).

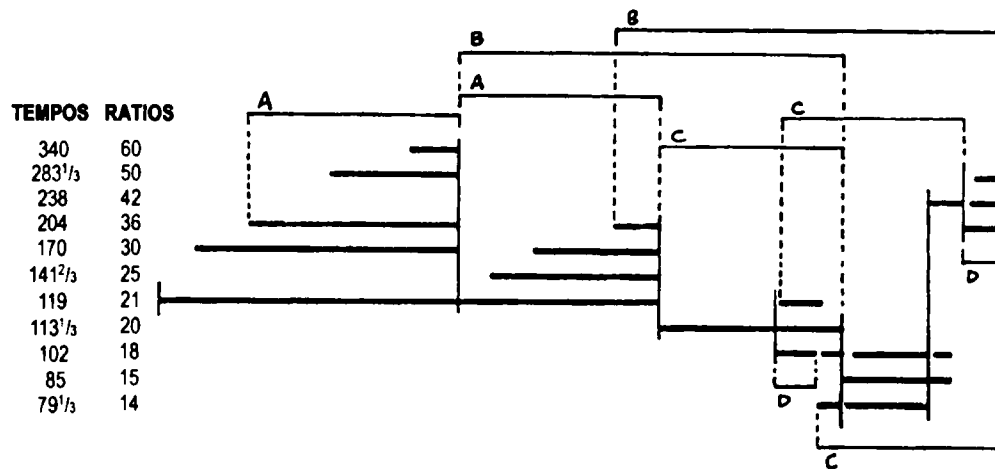


Figure 1-31. Carlsen's formal diagram for Study No. 35 (1988, p. 65) showing seven sections and proportional relationships (A, B, etc.).

Gann, Carlsen, and Thomas all characterize the melodic material in the first section as going through a three-phase evolutionary process, and Gann and Carlsen use similar diagrams to present the progression of the five voices through this process. This section is the only one discussed by Thomas, and it is interesting that she chooses this quasi-canonic piece to lead into her discussion on canon. Thomas discusses this piece as an example of accumulating layers that maintain their textural identity, and points to the progressively higher pitch and faster tempo of each successive voice as contributing to this perception: "There is no pitch imitation, but this broader form of imitation is vivid, and, in fact, is partially responsible for the perceptibility of the individual layers in the accumulation, since we can follow a layer's functional progression" (Thomas 1996, 65). Like Gann (and, to some extent, Carlsen), Thomas comprehensively follows the progression of the melody and accompaniment patterns through each of the five voices.

Gann and Tenney both note the presence in this study of isorhythms—or, more appropriately, recurring rhythmic patterns. The main melody incorporates a rhythmic pattern (in eighth note durations) of $2 + 3 + 3 + 2 + 4 + 3 + 4 + 2$ that is stated three times, the final time in retrograde. This pattern recurs in later sections, and other isorhythmic patterns are used as well with several of them used together near the end of the piece.

Little mention is made by any of the writers on the pitch content of the piece, despite the overwhelming presence of [025] in the main melody and its progressive versions. Gann, Carlsen, and Thomas do note the importance of the perfect fourth and minor third as motivic material, but structural pitches are rarely identified. The formal analyses by Gann and Carlsen include no mention of structural pitches or large-scale movement around pitch centers, although such assertions could certainly be made.

Summary of the Literature

The analyses by Gann, Thomas, Carlsen, and Tenney offer varying viewpoints on Nancarrow's Studies and focus on different aspects of them. Thomas focuses her discussion most intently on the layering technique and the factors that contribute to or detract from multidimensionality; she is less likely than some of the other sources to comment on form or perform melodic analysis.

The two authors who create a classification system for the Studies, Gann and Tenney, use completely different approaches. Gann classifies all the Studies into six groups based on the compositional techniques used, along with formal features; the categories are well-defined and provide for useful discussion of compositional techniques such as isorhythm, canon, acceleration, and use of ratios, and the forms that result. Tenney's system is chronological, which works fairly well for some periods in Nancarrow's career but not as well in his middle period; Tenney's remarks tend to focus on the textures created by the interaction of the musical layers.

Carlsen analyzes only five Studies selected from book 4 of the *Soundings* series, focusing especially on symmetry of design at different levels and the use of ratios in Nancarrow's forms. He chooses for his most thorough analysis Study No. 19, a 3-voice converging tempo canon, as an example of a work especially providing insights into Nancarrow's "overall compositional style."

My intent in the present work is to build on the work of these four authors by

presenting chapters focused on specific structural attributes of Nancarrow's Studies: the use of ratios, the presence and placement of convergence points, and fractal formal features. Each of these three chapters will include an analysis of a work representative of the chapter topic. A final analytical chapter (Chapter 5) will be devoted to analyses of two works—Nos. 25 and 45—that are representative of Nancarrow's mature compositional style.

Our look at these structural features begins in Chapter 2 with the use of ratios in Nancarrow's Studies.

CHAPTER 2

THE USE OF RATIOS IN NANCARROW'S STUDIES

Nancarrow's Studies are pervaded by the deliberate use of ratios, particularly ratios based on intervals found in just intonation and the overtone series. Nancarrow has deployed ratios in his compositions in remarkably inventive ways to control many of his musical materials. Besides the most obvious—the deliberate use of ratios in the establishment of tempo proportions in the tempo canons—Nancarrow used ratios to create isorhythmic and melodic patterns, to establish relationships between pitch materials and other details, and in certain structural features. Sometimes the same ratio is used within a piece in a number of different ways to create clever but subtle connections between musical features.

Ratios have been classified as far back as the ancient Greek mathematicians into the following five types (Barbera 1984, 194–95; all examples given here are whole-integer ratios representative of pitch intervals):

- (1) Multiple: the greater term is some multiple of the lesser term, e.g., 3:1.¹
- (2) Superparticular: the greater term is exactly one larger than the lesser (with the lesser term greater than one in order to exclude the ratio 2:1, which is defined as a multiple), e.g., 4:3, 25:24.

¹Pitch interval ratios can be presented both with the lesser term first (2:3) or the reverse (3:2), but in both cases the same interval (in this case, the perfect fifth) is represented. The latter system is more predominantly used today and will generally be preferred in this study to refer to pitch ratios, although Cowell used the first system and Nancarrow followed suit. It is useful to be conversant in both systems. The first system (e.g., 2:3) relates the pitch interval to the portion of a string length that will create the purely-tuned interval—thus, for the perfect fifth, the interval is created between a string's full length and the string shortened to 2/3 of its original length. However, the second system (e.g., 3:2) has the advantage of placing all the intervals between the unison and the octave on a logarhythmic scale between 1 and 2, with $\sqrt{2}$ representing the octave's halfway point (the tritone).

(3) Superpartient: both terms are greater than one and the greater term contains the lesser plus more than one part (but less than the whole) of the lesser—i.e., the greater term is something less than twice the lesser. The mathematical expression is $(x + m):x$, $x > m > 1$; examples include 5:3, 8:5, 16:9.

(4) Multiple superparticular: the greater term is one greater than some multiple of the lesser term, e.g., 7:2, 7:3, 10:3.

(5) Multiple superpartient: the greater term contains the lesser more than once plus more than one part (but less than the whole) of the lesser, e.g., 8:3, 24:7.

The most common types of ratios representing pitch intervals, and thus used in Nancarrow's music, are multiple and superparticular ratios. Additionally, it is important to note that ratios consisting of adjacent pairs in the Fibonacci series 1, 1, 2, 3, 5, 8, . . ., in which the greater term is at least 5 (e.g., 5:3, 8:5, 13:8) are superpartient ratios; ratios consisting of pairs of Fibonacci numbers in which a number is skipped in between and the greater term is at least 5 are multiple superpartients, e.g., 5:2, 8:3, 13:5. Since a number of Fibonacci-related ratios describe pitch intervals (e.g., 5:3 = major sixth, 8:5 = minor sixth, 8:3 = perfect eleventh), these ratios show up with some frequency in Nancarrow's work. Multiple superparticular ratios are probably the least commonly represented.

In this chapter, references to interval equivalents of ratios will follow either Cowell (1930) or Abraham and von Hornbostel (1994); see Figure 2-1. Figure 2-1a identifies the four different sizes of half-step available in Cowell's chromatic scale, ranging in size from a 4.2% increment (25:24) to a 7.1% increment (15:14); Figure 2-1b relates intervals in the just and Pythagorean systems and shows some less common interval ratios.

The relationship of pure interval ratios to the harmonics of the overtone series should not be overlooked. Figure 2-2 illustrates the overtone series based on C.

C	1:1	
C#	15:14	(15:14)
D	9:8	(21:20)
E _b	6:5	(16:15)
E	5:4	(25:24)
F	4:3	(16:15)
F#	7:5	(21:20)
G	3:2	(15:14)
A _b	8:5	(16:15)
A	5:3	(25:24)
B _b	7:4	(21:20)
B	15:8	(15:14)
C	2:1	(16:15)

(a)

	12-tone tempera- ment	Just	Pythago- rean	Ratios
Minor second (semitone)	100	70 112	90	24:25 243:256 15:16
Major second (whole tone)	200	182 204 231	204	9:10 8:9 7:8
Minor third	300	267 316		6:7 5:6
Major third	400	386	408	4:5 64:81
Fourth	500	498	498	3:4
Tritone	600	583 590		5:7 32:45
Fifth	700	702	702	2:3
Minor sixth	800	814		5:8
Major sixth	900	884	906	3:5 16:27
Minor seventh	1000	969 996	996	4:7 9:16
Major seventh	1100	1088		8:15
Octave	1200	1200	1110 1200	128:243 1:2

(b)

Figure 2-1. (a) Cowell's justly-tuned chromatic scale, with all interval ratios in relation to C (Cowell 1930, 107); the right column's parenthetical figures indicate size of half-steps between adjacent pitches. (b) Interval ratios and cent sizes in just and Pythagorean intonation (Abraham and von Hornbostel 1994, 451).

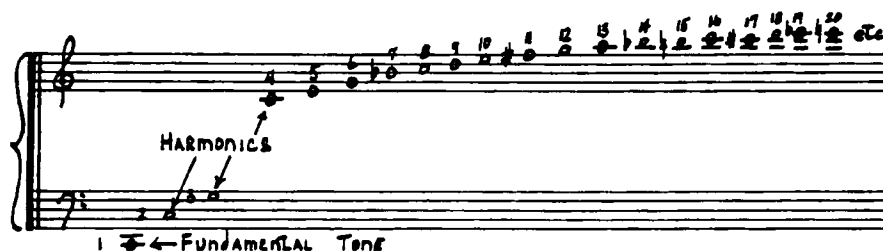


Figure 2-2. The overtone series and its numbered harmonics (Jorgensen 1977, 16).

Based on the overtone series, Jorgensen makes the following observations about interval ratios and harmonics:

Intervals taken from consecutive harmonics [superparticular ratios]—

Ratio ²	Name	Size in Cents
1/2	octave	1200
2/3	fifth	702
3/4	fourth	498
4/5	major third	386
5/6	minor third	316
6/7	small minor third	267
7/8	maximum tone	231
8/9	major tone	204
9/10	minor tone	182
10/11	minimum tone	165
11/12	3/4 tone	151
⋮	⋮	⋮
15/16	just diatonic semitone	112

Intervals taken from every second harmonic [superpartient ratios]—

Ratio	Name	Size in Cents
3/5	major sixth	884
5/7	small tritone	583
7/9	large major third	435
9/11	neutral third	347

Intervals taken from every third harmonic [superpartient ratios]—

Ratio	Name	Size in Cents
4/7	harmonic minor seventh	969
5/8	minor sixth	814
7/10	large tritone	617

(Jorgensen 1977, 17)

²Notice the use of the virgule, rather than the colon, in Jorgensen's discussion to separate the terms of the pitch ratio. Nancarrow also uses the virgule in describing his tempo ratios.

The importance of the overtone series in musical materials other than pitch was a major theme in Cowell's *New Musical Resources*. In its "Introduction" he writes:

The purpose of *New Musical Resources* is not to attempt to explain the materials of contemporary music, many of which are not included in its discussion, but to point out the influence the overtone series has exerted on music throughout its history, how many musical materials of all ages are related to it, and how, by various means of applying its principles in many different manners, a large palette of musical materials can be assembled. . . . The very fact that such materials are built on the overtone series, which is the greatest factor in musical relationship, shows that they probably have potential musical use and value. . . .

The result of a study of overtones is to find the importance of relationships in music and to find the measure by which every interval and chord may be related. . . . It is also discovered that rhythm and tone, which have been thought to be entirely separate musical fundamentals (and still may be considered so in many ways) are definitely related through overtone ratios. (Cowell 1930, x–xi)

Ratios in Other Twentieth-Century Music

One of Cowell's primary ideas in *New Musical Resources* that greatly intrigued and motivated Nancarrow was the creation of rhythmic complexes based on pure interval ratios. Cowell's book contains a number of diagrams, like that in Figure 2-3, showing equal measure lengths divided according to a common pitch ratio such as the 3:4:5 second-inversion major triad.

As has been frequently noted in the literature, Cowell remarked on the potential utility of the player piano in realizing his complex tempo relationships as well as the general under-utilization of the capabilities of the player piano:

Some of the rhythms developed through the present acoustical investigation could not be played by any living performer; but these highly engrossing rhythmical complexes could easily be cut on a player-piano roll. This would give a real reason for writing music specially for player-piano, such as music written for it at present does not seem to have, because almost any of it could be played instead by two good pianists at the keyboard. (Cowell 1930, 64–65)

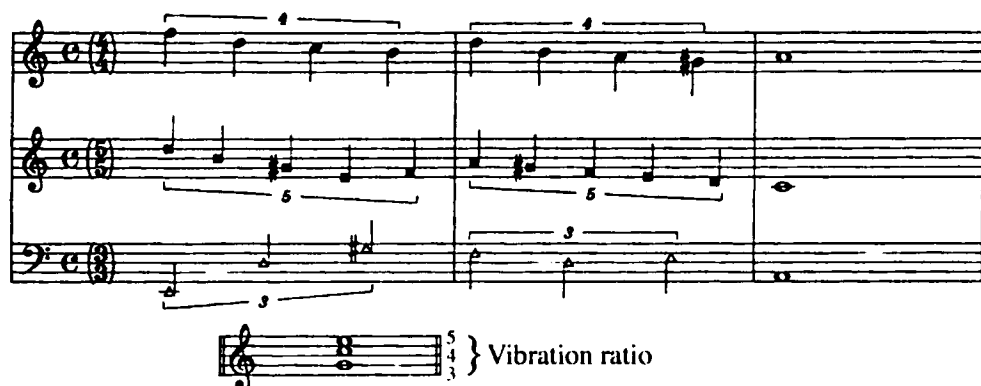


Figure 2-3. Diagram showing 3:4:5 temporal relationship from Cowell's *New Musical Resources* (cited in Gann 1995, 6). Note how Cowell uses different shaped noteheads for different divisions of the measure.

Cowell himself applied these ideas to the composition of live performance pieces such as *Quartet Romantic*, one of the pieces discussed by Thomas (1996). Cowell developed the rhythmic framework for the first movement of this piece in several stages: (1) he composed several chorale-like themes in four-part harmony (see Figure 2-4a); (2) the pitch ratios of the chorale chords were converted to rhythmic ratios, which then provided the rhythm for the movement; and (3) the pitch content was changed to virtually eliminate the correlations between the chorale's pitches and the final rhythm (see Figure 2-4b).

Thomas comments perceptively about Cowell's motivation for changing the pitch content in order to dissociate it from the original chorale themes:

It may seem odd that once Cowell has derived the rhythmic ratios from his themes, he completely changes the pitch content, so that there is virtually no correlation between the ratios of the actual pitches in the music and their rhythms. Is not such a correlation the basis of his entire theory of rhythm, after all? But what Cowell has actually done is taken the theory one step further: by negating the explicit association of pitch with rhythm, he forces the 'harmony' of rhythm to stand on its own. Because he has composed the rhythm with the aid of traditional four-voice chorales, he has ensured that the contrasting simultaneous rhythms, as well as juxtaposed rhythms, as they change, will comply to what he considers a 'natural' progression, just as the chords of the chorales comply to tonal progressions. Furthermore, the rhythms change on a fairly periodic basis, since they are governed by the original harmonic rhythm of the chorales, and the effect of the movement is correspondingly progression-like, in that the rhythms regularly proceed from one set of ratios to the next. (Thomas 1996, 41)

(a)

chorale theme

$J = 100$

etc.

ratios derived from overtone series

1	1	1
6	$5/3$	$7/2$
5	$5/2$	6
4	4	$4/2$
2	$2/3$	$2/3$

rhythms derived

(b)

$J = 100$

Flute 1

Flute 2

Violin

Viola

Figure 2-4. Stages of composition of the first movement of Cowell's *Quartet Romantic*: (a) chorale theme and its resultant pitch ratios; (b) first page of the score, showing the conversion of the chorale theme's pitch ratios to rhythmic ratios (Thomas 1996, 203–04).

Elliott Carter was another composer who created complexes of rhythmic relationships using Cowell’s ideas about interval ratios. Carter’s technique of metric modulation is based on setting up a system of tempos in which simple ratio relationships are prevalent and speeds are changed by changing the division of the beat. Thomas examined the second movement of Carter’s String Quartet No. 2 for its tempo relationships, which are shown in Figure 2-5. The majority of the paired tempo relationships identified in Figure 2-5a involve superparticular ratios.

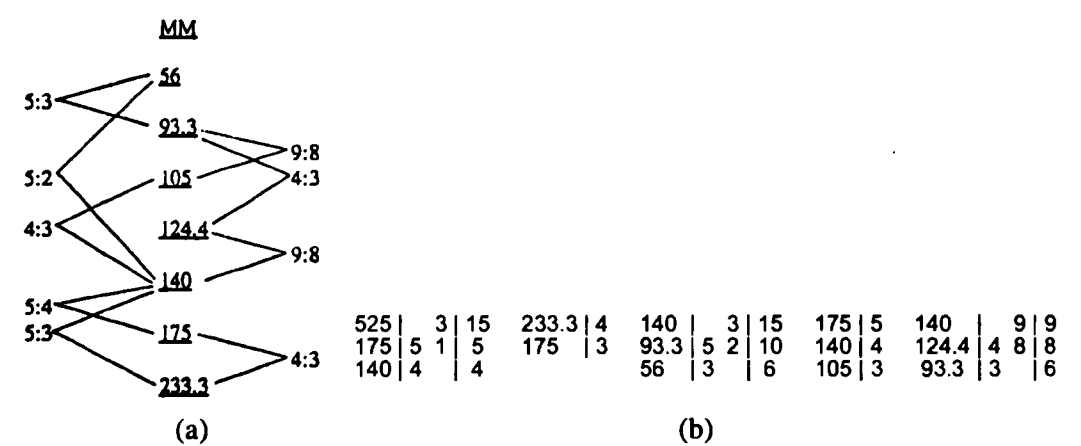


Figure 2-5. Tempos and relative ratios in Elliott Carter’s String Quartet No. 2, movement 2 (Thomas 1996, 45): (a) MM speeds used (MM=525 is not represented) and ratios of selected pairs of tempos; (b) combinations of speeds and ratios.

Claims have been made for other composers that they may have used ratios, particularly for structural proportions, but there is some question as to how much a role intuition played in the presence of the proportions that have been identified and how much was due to deliberate application of ratios. Kramer (1988) has examined structural proportion in Stravinsky’s music and claims that in many of his compositions, Stravinsky used the same “proportional ratio consistently throughout a piece” (p. 303). For instance, Kramer identifies the ratio 3:2 as describing many of the structural proportions in *Symphonies of Wind Instruments*, although he allows that “Stravinsky’s proportional consistencies are never exact, which implies that he did not consciously calculate sectional

durations” (p. 289); indeed, in his analysis Kramer allows deviation from 3:2 within a range of 7%, or from 1.40 to 1.61 as being “equivalent” to 3:2. In Stravinsky’s late ballet *Agon*, Kramer’s analysis identifies the piece’s structural ratio as being 1.19:1, which he says is “not as strange as it may at first seem, as it is really $\sqrt[4]{2}:1$ ” (p. 298)!

Kramer also treats the topic of golden-section proportions in music, noting that

[t]heorists and analysts have been studying the use of the ratio 1.62:1 by composers as diverse as Barber, Hindemith, Schoenberg, Bartók, Webern, Berg, Prokofiev, Debussy, Ravel, Delius, Rachmaninoff, Fauré, Scriabin, Saint-Saëns, MacDowell, Dvorák, Wolf, Brahms, Wagner, Tchaikovsky, Schumann, Mendelssohn, Chopin, Schubert, Beethoven, Mozart, Haydn, Bach, Handel, Sermisy, Jannequin, Gibbons, Binchois, Dunstable, Ockeghem, Obrecht, Dufay, and Machaut. (Kramer 1988, 303–04)

Thus, just as Kramer may have been “reaching” in asserting a structural ratio of $\sqrt[4]{2}:1$ in a piece of Stravinsky’s, there is evidence that many analyses concerning the presence of the golden section in the music of the composers listed above use approximations or even ranges of deviation (as Kramer has done) to prove the point. Still, the quest to find golden-section proportions in music is strongly rooted in its ubiquitous presence all around us, as Kramer notes quite convincingly:

It should not be surprising that golden-mean proportions and Fibonacci numbers appear in music. Numerous mathematical properties of the Fibonacci series have appealed to artists and scientists for centuries, and golden-section proportions are frequently found in nature, human or otherwise. There is experimental evidence, for example, that the golden mean determines the ratio of people’s positive to negative value judgments. . . . There is also experimental evidence that rectangles (cards, mirrors, pictures, etc.) proportioned according to the golden section (ratio of the longer to the shorter side is 1.62) appeal to our sense of symmetry. . . . Fibonacci numbers are used in certain electrical networks, and they are approximated in the structures of atomic and subatomic particles. The ratios of the distances of the satellites of Jupiter, Saturn, and Uranus from their parent planets approximate 1.62, as do (in a weaker approximation) the distances of the planets (including the asteroid belt) from the sun. . . . Fibonacci numbers relate to the numbers of years in cycles between peaks and peaks, peaks and lows, and lows and lows of the stock market. It has even been suggested that Fibonacci numbers determine the lengths of cycles of grasshopper abundance, automobile factory sales, the ratio of male to female conceptions, advertising effectiveness, sunspots, tree ring size, rainfall in India, Nile floods, financial panics, and furniture production! (Kramer 1988, 305)

Uses of Ratios in Nancarrow's Music

While Henry Cowell led the way in showing how pitch ratios and the overtone series could be used to create new musical textures, no other composer ventured further than Nancarrow did in using ratios to govern so many different musical parameters. Nancarrow extended Cowell's ideas to go beyond the simpler pitch ratios of just intonation to ratios as close as 60:61 and even ratios involving irrational numbers such as $\sqrt{2}$, and he further extended Cowell's work by using ratios not only to control rhythmic parameters but to establish relationships among features such as pitch, duration, and structural elements.

The player piano medium itself, of course, played a crucial role in the realization of these ideas on proportions. Nancarrow shared with Reynolds the following insights about his working process:

I mark out on a blank roll of paper all of the proportional relationships of tempo, using what I think is going to be the smallest (fastest) note value as the unit of measure. Of course, occasionally, if I have to use something even faster, I just go over the roll and put in the smaller values, showing the relations to the basic scale in the score. I mark the whole thing out from beginning to end on the blank player piano roll. . . . Then I take the marked proportions from the roll onto the music paper. It is not as exact as the roll, but it's fairly accurate so that the vertical relationships of tempo units will be more or less what I see graphically on the paper. (Reynolds 1984, 13)

The graphical and visual nature of the process of creating the rolls lent itself extremely well to the types of calculations and alignment procedures that were a necessary part of Nancarrow's composition process. Carlsen acknowledged this when he said:

Some of the most characteristic and innovative elements of Nancarrow's music were born right on the blank surface of the unpunched paper roll. . . . The roll is highly conducive to the use of exact measurements, templates, replications, and geometric constructions; thus, to a certain extent, it exerts an influence that is purely visual and mathematical. (Carlsen 1988, 66)

Tempo Ratios

The most obvious and deliberate use of ratios in Nancarrow's Studies occurs in the pieces known as "tempo canons," or those pieces in which canonic material is played simultaneously at proportionally-related tempos. Nancarrow's tempo canons are representative of large-scale polytempo as envisioned by Cowell.

The ratios given in the subtitles of Nancarrow's tempo canons relate directly to the tempo markings for the individual voices; the number of components in the ratio indicates the number of canonic voices in the study (it should be noted that a canonic "voice" is not necessarily a monophonic [unharmonized] musical line, but is more likely a polyphonic musical stratum—sometimes quite dense—that is repeated at different pitch levels in successive voices). Table 2-1 describes most of Nancarrow's tempo canons, including the tempo ratios and the tempo markings used in the scores.

Nancarrow's stated purpose in writing these canons

was [to explore] my interest in temporally dissonant relationships. Temporal dissonance is as hard to define as tonal dissonance. I certainly would not define a temporal relation of 1 to 2 as dissonant, but I would call a 2 to 3 relation mildly dissonant, and more and more so up to the extreme of the irrational ones (Reynolds 1984, 5).

Gann recognizes that, in the earlier Studies through No. 19, Nancarrow restricted his focus primarily to ratios involving 3, 4, and 5 (No. 19's ratio, 12:15:20, is actually a combination of a 3:4 ratio and 4:5 ratio). Starting with No. 24 Nancarrow began to climb higher in the overtone series for his ratios, and in Study No. 33, with its ratio of $\sqrt{2}/2$, he used his first irrational ratio to reach a new extreme of temporal dissonance. The irrational number $\sqrt{2}$ (1.4142136...) represents the equal-tempered tritone (among the most dissonant and unstable intervals in tonal music), which is commonly given the ratio 7:5. Nancarrow casts attention in this piece on this irrational number by using a variety of ratios that are rational approximations of $\sqrt{2}$. Gann (p. 187) identifies the ratios 7:5 and 10:7 as being used in the final section, where in one voice only note values of five and

Table 2-1

Relationship of Canon Ratios and Tempo Markings

Study	Tempo Markings
No. 14, "Canon 4/5"	$\text{♩} = 88, \text{♩} = 110$
No. 15, "Canon 3/4"	$\text{♩} = 165, \text{♩} = 220$
No. 17, "Canon 12/15/20"	$\text{♩} = 138, \text{♩} = 172.5, \text{♩} = 230$
No. 18, "Canon 3/4"	$\text{♩} = 168, \text{♩} = 224$
No. 19, "Canon 12/15/20"	$\text{♩} = 144, \text{♩} = 180, \text{♩} = 240$
No. 24, "Canon 14/15/16"	$\text{♩} = 149 \frac{1}{3}, \text{♩} = 160, \text{♩} = 170 \frac{2}{3};$ $\text{♩} = 224, \text{♩} = 240, \text{♩} = 256$
No. 31, "Canon 21/24/25"	$\text{♩} = 105, \text{♩} = 120, \text{♩} = 125$
No. 32, "Canon 5/6/7/8"	$\text{♩} = 85, \text{♩} = 102, \text{♩} = 119, \text{♩} = 136$
No. 33, "Canon $\sqrt{2}/2$ "	$\text{♩} = 140\sqrt{2}, \text{♩} = 280$
No. 34, "Canon $\frac{9}{4/5/6} / \frac{10}{4/5/6} / \frac{11}{4/5/6}$ "	the three voices generally exhibit a 9:10:11 tempo ratio—the opening markings are $\text{♩} = 72, \text{♩} = 80$, and $\text{♩} = 88$; the application of the 4:5:6 ratio results in a succession of progressively faster tempos in each voice, peaking at $\text{♩} = 264$ in the fastest voice
No. 36, "Canon 17/18/19/20"	$\circ = 85, \circ = 90, \circ = 95, \circ = 100$
No. 37, "Canon 150/160 $\frac{5}{7}$ / 168 $\frac{3}{4}$ / 180 / 187 $\frac{1}{2}$ / 200 / 210 / 225 / 240 / 250 / 262 $\frac{1}{2}$ / 281 $\frac{1}{4}$ "	$\text{♩} = 150, \text{♩} = 160 \frac{5}{7}, \text{♩} = 168 \frac{3}{4}, \text{♩} = 180,$ $\text{♩} = 187 \frac{1}{2}, \text{♩} = 200, \text{♩} = 210, \text{♩} = 225,$ $\text{♩} = 240, \text{♩} = 250, \text{♩} = 262 \frac{1}{2}, \text{♩} = 281 \frac{1}{4}$
No. 40, "Canon $\bullet/\pi$ "	tempo markings not used—Nancarrow notates a simple diagram which gives timings that establish a relationship of <i>roughly</i> $\bullet$ to π
No. 41A, "Canon $\frac{1}{\sqrt{\pi}}/\sqrt{2/3}$ " No. 41B, "Canon $\frac{1}{3\sqrt{\pi}}/3\sqrt{13/16}$ "	Nancarrow does not give tempo markings; the canons are intended to be played together, and he establishes the following relationships using total playing times: $\frac{1}{\sqrt{\pi}} = 5'00''; \sqrt{2/3} = 7'40''; \frac{1}{3\sqrt{\pi}} = 4'35''; 3\sqrt{13/16} = 6'10''$
No. 43, "Canon 24/25"	$\circ = 120, \circ = 125$
No. 48, "Canon 60/61"	tempo markings not used—uses proportional notation

seven sixteenth notes are used while the other voice consists only of values of seven and ten sixteenth notes (recall, from earlier in this chapter, Jorgensen's description of 7:5 as the "small tritone" and 10:7 as the "large tritone").

After reaching extremes of irrational tempo ratios in Studies No. 40 and 41, Nancarrow returned to the use of simpler ratios. His later pieces Nos. 45, 46, and 47, although not nominally tempo canons, contain some tempo canon sections that have ratios using lower numbers such as 3, 4, 5, and 6.

Despite the concentration on smaller numbers in many of the Studies' tempo relationships, changing beat divisions and the progression of tempo canons can and do result in fairly large "tempo complexes," or multiple tempo ratios. Gann identifies tempo complexes of 2:3:5:8:14 in Study No. 1, 21:30:36:50:60 in Study No. 35, 27:32:36:42 and 450:525:588:700 in Study No. 37, and 144:182:351:468:585:624:819:936:1638 in Study No. 46.

Although Nancarrow's first named tempo canon is No. 14, many of the pieces prior to No. 14 use tempo ratios in various ways. Nancarrow began exploring different simultaneous tempos based on interval ratios in his first player piano piece, "Rhythm Study No. 1," in which the two tempos—120 and 210—are related by a ratio of 4:7 (minor seventh). Within this tempo relationship Nancarrow uses at least twelve durations that create a duration range from 2 to 35 on what Gann calls a "quasi-logarithmic scale" (p. 70). A portion of this Study is shown in Figure 1-4 (p. 31).

Tempo ratios are often expressed, especially in the earlier pieces, on a smaller scale as ostinati or isorhythms. Study No. 2 (Figure 1-8, p. 34) has already been shown as an example of an ostinato combining divisions of 5 and 6. In Study No. 7, Nancarrow creates his major themes out of three isorhythms consisting of 18, 24, and 30 beats, respectively (a 3:4:5 ratio). The ratio 3:4:5 (representing the second-inversion major triad as shown in Figure 2-3) seems to be a favorite of Nancarrow's. He uses it in numerous other studies, including No. 9 where the rhythmic relationships among the three main

themes are permeated by this ratio, and in Study No. 46, where a “polytempo ostinato” projects the ratio 3:4:5 (see Figure 2-6 for Gann’s renotation of this proportionally-notated study).



Figure 2-6. Study No. 46, 3:4:5 polytempo ostinato as renoted by Gann (p. 264).

In Study No. 11, Nancarrow creates a 60-beat duration series divided into three 20-beat segments:

5 5 6 4 | 5 5 3 4 3 | 5 4 3 3 3 2
 (4 notes) (5 notes) (6 notes)

The progressively smaller note values create an acceleration effect, and the three segments exhibit a kind of 4:5:6 ratio in the numbers of division of the 20-beat segments.

A different type of ostinato is shown in Figure 1-9 (p. 35), where, in Study No. 6, a single line alternates between 4- and 5-divisions of a measure.

Another matter for consideration in Nancarrow’s tempo canons is the relative duration of the sections resulting from the application of the tempo ratio. Tempo and duration have an inverse relationship, and the “duration ratio” is the inverse of the tempo ratio. In certain of Nancarrow’s tempo canons with three or more voices this inverse proportion has a significant effect on the relative proportions of the piece. Study No. 17 offers an apt example. As shown in Figure 2-7a, each voice states each of the three sections once, in direct succession. The duration lengths of the sections do not relate by 12:15:20, however, but by the inverse of this ratio. As shown in Figure 2-7b, the inverse of the ratio is

determined by maintaining the relationship of the extreme terms and reversing the relationship of the interior ratios (12:15 [4:5] and 15:20 [3:4]). The result is the ratio 12:16:20, or 3:4:5, and this is the ratio by which sections C, A, and B, respectively, relate to each other in duration.

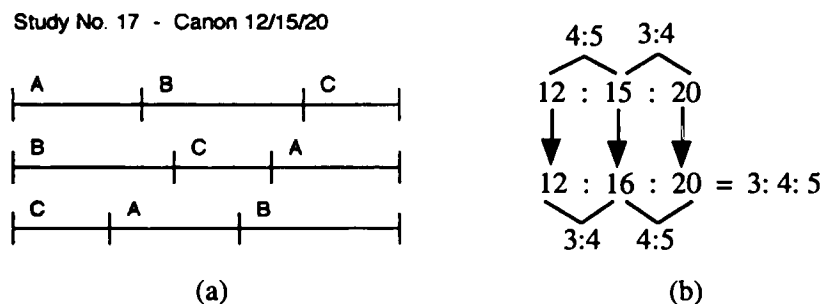


Figure 2-7. (a) Formal diagram of Study No. 17, “Canon 12/15/20” (Gann 1995, 22). Section “A” states the 15-tempo, section “B” the 12-tempo, and section “C” the 20-tempo. (b) The “duration ratio” resulting from a 12:15:20 tempo ratio is 3:4:5. This is the ratio which describes the relative durations of sections C, A, and B.

In a two-voice canon, then, say “Canon 4/5,” the ratio inverts upon itself so that the duration of the 4-voice is 5 and of the 5-voice it is 4.

“Scales” of Tempos

In addition to the numerous tempo canons in which the same tempo ratios are heard between the voices, Nancarrow composed several pieces in which a succession of tempos—often based on interval ratios—is used.

Technically, Study No. 37 does not meet this definition as, rather than being a scalar succession of tempos, the simultaneous relative speeds of its twelve voices are defined by the justly-tuned scale. However, since its tempos exactly relate to the interval ratios of Cowell’s chromatic scale (see Figure 2-1a) it seems best to discuss it in this section. One of the fascinating aspects of Study No. 37 is the way in which the “scale of tempos” aurally illustrates the slightly varying sizes of half steps in Cowell’s scale (see Figure 2-8). Note, for instance, the relative smallness of intervals between the $262\frac{1}{2}$ -

Handwritten musical score for Study No. 37, showing 11 staves. Each staff has a tempo marking at the beginning and a dynamic marking at the end. The tempo markings are: $\text{♩} = 281\frac{1}{4}$, $\text{♩} = 262\frac{1}{2}$, $\text{♩} = 250$, $\text{♩} = 240$, $\text{♩} = 225$, $\text{♩} = 210$, $\text{♩} = 200$, $\text{♩} = 187\frac{1}{2}$, $\text{♩} = 180$, $\text{♩} = 168\frac{3}{4}$, and $\text{♩} = 160\frac{5}{8}$. The dynamic markings are: ff , ff , ff , ff , ff , ff , ff , ff , ff , ff , and ff . The score is labeled (37) at the bottom.

Figure 2-8. First page of score to Study No. 37, showing rhythmic expression of Cowell's justly-tuned scale. The rhythmic "interval" between each pair of voices represents a half-step.

voice and the 250-voice (a 21:20 ratio) and between the 250-voice and the 240-voice (a 25:24 ratio), especially as compared to a 15:14 interval such as that between the 225-voice and the 210-voice.

Study No. 28, a non-canonic piece based on large-scale acceleration, finds the composer again using the ratios of the justly-tuned chromatic scale to establish tempo relationships, this time to establish a “scale” of acceleration. Gann (p. 164) describes how an incremental acceleration in this study from $\text{♩} = 252$ to $\text{♩} = 864$ adheres to a “scalar” progression as shown in Table 2-2.

Table 2-2

Scalar Acceleration in Study No. 28

Tempo (♩ =)	Ratio*	Corresponding note	Tempo (♩ =)	Ratio*	Corresponding note
252	7:10	F#	480	4:3	F
270	3:4	G	504	7:5	F#
288	4:5	A $\flat$	540	3:2	G
300	5:6	A	576	8:5	A $\flat$
315	7:8	B $\flat$	600	5:3	A
337.5	15:16	B	630	7:4	B $\flat$
			675	15:8	B
384	16:15	C#	720	2:1	C
405	9:8	D	765	17:8	C#
432	6:5	E $\flat$	810	9:4	D
450	5:4	E	864	12:5	E $\flat$

*Ratios are in relationship to C, representing 1:1 and the “tonic tempo” at $\text{♩} = 360$

As has already been observed about Cowell’s chromatic scale (Figure 2-1a), there are four different interval ratios for half steps in this scale and they are represented in Study No. 28 as follows: the largest increment, 15:14 (increment of 7.1%), occurs between F#–G and B–B; the ratio 16:15 (6.7%) occurs between G–A $\flat$, B–C, C–C#, D–E $\flat$, and E–F; the ratio 21:20 (5%) occurs between A–B $\flat$ and F–F#; and the smallest

increment, 25:24 (4.2%) occurs between $A\flat$ -A and B-E.³ The resulting overall acceleration ($\downarrow = 252$ to $\downarrow = 864$) is equivalent to the pitch interval of an octave plus a diminished seventh, or the pitch ratio 24:7 (10:7 [interval from F# to C] + 12:5 [interval from C to $E\flat$ the octave above] = 24:7).

The final section of this chapter will analyze in detail Study No. 34, a piece in which the two ratios 9:10:11 and 4:5:6 combine to create a long succession of tempos.

Rhythm/Pitch Analogues

A number of Nancarrow's Studies contain ratios simultaneously expressing both pitch and rhythm or duration. Gann credits Nancarrow's work in associating pitch with duration as placing him among

the great innovators of the 1950s, at least for those whose conception of that period assumes a European model. For Study No. 23 is a pioneering work—Nos. 28 and 29 are others—in the dependent association of frequency with duration, an idea that was very much “in the air” between 1949 and 1960. (Gann 1995, 156)

In the first work cited above by Gann, Study No. 23, Nancarrow uses the pitch ratio 9:4 (major ninth) to determine the relative length of 71 consecutive pitches from B_0 to A_6 . For each major ninth higher the pitch progresses in this series, the note duration decreases by half (Carlsen 1988, 49—refer again to Figure 1-28, p. 56). In Figure 2-9 from this Study, pitches are arranged in five-note groupings, with the duration of all notes (written in proportional notation) in each group determined by the duration of its first note. Note the progressively longer notes as the first pitch of the group becomes lower.

³Inexplicably, Nancarrow uses two different ratios for the two occurrences of C#, and neither of them is the 15:14 ratio used by Cowell (Figure 2-1). The 16:15 ratio used for the first occurrence of C# is actually what Cowell specifies for $D\flat$ (Cowell 1930, 99). That does not explain, however, why Nancarrow's second occurrence of C# (17:8) is not an octave of the first (and nowhere does Cowell use the lower octave of this ratio, 17:16, for either C# or $D\flat$); the second ratio would have to be 32:15 rather than 17:8 to be an octave duplication of 16:15. As a result, the two intervals C#-D differ: the first C#-D interval is an increment of 5.5%, or the rather unwieldy ratio of 135:128, while the second (“higher”) C#-D interval is a 5.9% increment, ratio 18:17—the only two occurrences of these intervals in this Study.

of three pitches a half-step apart that are repeatedly articulated but in an irregular fashion. Gann claims that these clusters of pitches, which can be represented by a pitch ratio of 15:16:17 (two adjacent half-steps), also exhibit a 15:16:17 speed ratio, with the highest note in each cluster being rearticulated 17:16 as fast as the middle note and the lower note rearticulated 15:16 the speed of the middle note. This is difficult to verify due to the nature of measuring duration in Nancarrow's proportional notation, and arriving at these numbers involves working with approximations and averages. What Gann has done is to measure in millimeters the distance between rearticulated notes and determine an average. In the note cluster shown in Figure 2-11, my own measurements show that the average distance between successive articulations of the note A is about 25.8 mm in the score (measurements in Figure 2-11 are reduced). To effect a 15:16:17 ratio with this measurement as the middle figure, the measurements for the faster (upper) voice would need to be about 24.3 mm and the slower (lower) voice about 27.5 mm. The actual average measurements are 24.0 mm for the upper voice and 27.5 mm for the lower voice, which is remarkably close to the ratio in question. One must keep in mind that Nancarrow's hand-written scores represent only a rough approximation of the piano rolls, which are more precise.



Figure 2-11. Three-note cluster from Study No. 28 (p. 3), exhibiting 15:16:17 tempo and pitch ratio. Rearticulations in the middle note are regular but some rearticulations of the upper and lower notes are omitted by Nancarrow.

The next example involves metric notation and is even more overt. In Study No. 5, two rhythmic ostinatos are established which are related by the ratio 7:5 (the “small tritone”), and the opening interval between these two voices is the tritone C–F# (see Figure 2-12). The two lines share a $\frac{35}{16}$ meter signature, but the distance between notes in the top line is multiples of seven sixteenth notes and in the bottom line multiples of five sixteenth notes (thus the least common multiple, 35, in the meter signature). Although the tonal

implications of these two lines are actually C major and B pentatonic, the harmonic use of the tritone at the very beginning of this rhythmic context is a striking parallel to the rhythmic relationship.

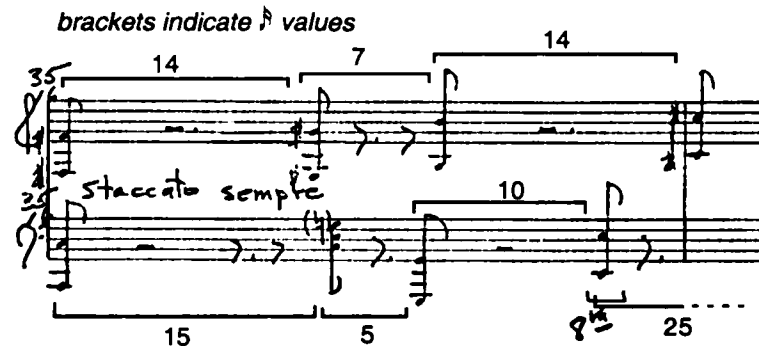


Figure 2-12. Opening of Study No. 5, showing 7:5 rhythmic ratio and opening tritone interval between voices.

Like the example cited earlier in this section where Gann identifies the ratio 15:16:17 in both pitch and rhythm, Study No. 31 (“Canon 21/24/25”) exhibits a relationship between the tempo ratio of its title and its opening pitches (see Figure 1-11, p. 37). The opening pitches, G–A–B $\flat$, relate to 21:24:25 in this way: 8:7 (24:21) is a large whole tone in just intonation (231 cents—see Figure 2-1b); 25:24 is the smallest minor second in just intonation (just 70 cents); and 25:21 is one of five possible minor thirds in Cowell’s chromatic scale (Gann 1995, 199—although the actual ratio for the minor third from G to B $\flat$ in Cowell’s scale is 7:6). Of course, the pitch associations with just intonation (here and elsewhere) are merely academic as Nancarrow’s pieces are all performed on pianos tuned in equal temperament.

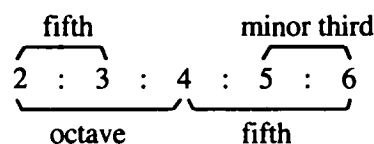
Structural Uses of Ratios

Occasionally ratios appear in Nancarrow’s Studies in structural roles. Levels of imitation (see Table 1-1, p. 41) would seem an area ripe for exploitation in this manner, but there is little use of ratios to connect levels of imitation in canonic pieces with other

elements. One significant exception to this is Study No. 34, where the ratio 4:5:6 both determines the succession of tempos and describes the level of imitation, which is a major triad. This Study will be examined more fully at the end of this chapter.

Carlsen uncovers an extraordinary case of proportional structural integrity in Study No. 8. The final section is a 3-voice canon in which the lowest voice first states the canon heterophonically in octaves, the middle voice in fifths, and the upper voice in minor thirds, creating a texture of root-position triads. The positioning of the entrances in this canon is carefully worked out so that:

When the second voice enters, it does so at a point exactly halfway through the first voice's isorhythmic pattern; the third voice enters exactly three-quarters of the way through the pattern. . . . The positioning of these entrances is specifically related to the divisions of successive intervals in the overtone series. Thus, the entrance of the second voice exactly halfway through the pattern parallels, in the overtone series, the division of the octave (2:4) into fifth (2:3) and fourth (3:4); by the same token, the further subdivision of that half into quarters parallels the subdivision of the fifth (4:6) into major and minor thirds (4:5 and 5:6). Obviously, it is no coincidence that the bass voice is characterized by octaves, the next by fifths, and the highest by minor thirds:



(Carlsen 1988, 43)

The structural application of these ratios is vividly illustrated in Figure 1-24 (p. 51).

In another particularly fine example of a ratio being used structurally in different ways, Study No. 3a (from the "Boogie-Woogie Suite") uses a 3:5 ratio on two structural levels. On a smaller level Nancarrow creates a 2:3:5 rhythmic ratio between three ostinatos; see Figure 2-13, where the ostinatos are in lines 2, 4, and 7 (recall that the note symbol $\sqcap$ in line 2 is Nancarrow's own symbol for a 5-division note, in this case in the meter $\frac{20}{16}$). The piece concludes with an eight-voice texture (also Figure 2-13) in which five voices are expressing irregularly-spaced staccato chords against the three ostinato lines, thus creating a textural expression of the 3:5 ratio.

The musical score for Study No. 3a, concluding texture, is presented across eight staves. The notation is as follows:

- Staff 1:** Treble clef, key signature of one sharp (F#). It begins with a whole note chord, followed by a half rest, and ends with a half note chord.
- Staff 2:** Treble clef, key signature of one sharp. It contains a continuous ostinato pattern of eighth notes.
- Staff 3:** Treble clef, key signature of one sharp. It contains a continuous ostinato pattern of eighth notes.
- Staff 4:** Treble clef, key signature of one sharp. It contains a continuous ostinato pattern of eighth notes.
- Staff 5a:** Treble clef, key signature of one sharp. It contains a continuous ostinato pattern of eighth notes.
- Staff 5b:** Bass clef, key signature of one sharp. It contains a continuous ostinato pattern of eighth notes.
- Staff 6:** Bass clef, key signature of one sharp. It contains a continuous ostinato pattern of eighth notes.
- Staff 7:** Bass clef, key signature of one sharp. It contains a continuous ostinato pattern of eighth notes.
- Staff 8:** Bass clef, key signature of one sharp. It contains a continuous ostinato pattern of eighth notes.

Figure 2-13. Concluding texture of Study No. 3a (score, p. 35). Ostinato lines 2, 4, and 7 express a 2:3:5 rhythmic ratio, while the entire texture expresses a 3:5 ratio (the three ostinato lines against five lines of irregularly-spaced chords [lines 1, 3, 5a/b, 6, and 8]).

Analysis of Study No. 34: “Canon $\frac{9}{4/5/6}/\frac{10}{4/5/6}/\frac{11}{4/5/6}$ ”

This chapter concludes with an analysis of Study No. 34, focusing particularly on the use of ratios in its composition. The copy of the score to No. 34 that I was able to obtain was, unfortunately, missing page 9. When multiple attempts to obtain the missing page were unsuccessful, I ultimately resorted to transcribing the missing page myself by extrapolating themes from elsewhere in the work and, as needed, determining transposition levels and rests between themes from the taped performance. The transcribed page emulates Nancarrow’s proportional notation and the 177-millimeter staff length of my copy to facilitate accurate measurements of elapsed time. The transcribed page is shown in Appendix D.

Overview

Study No. 34 is a three-voice converging tempo canon with a single convergence point on the last attack. Its melody is multi-thematic, with a rondo-like A theme separated by themes B, C, . . . , G, although Tenney describes it as “a fugue, with thematic material in each voice alternating between ‘expositions’ of a fairly long ‘subject’ (heard seven times in each of the three voices) and six ‘episodes’ involving related but distinctly different melodic material” (Wergo Vols. III/IV, liner notes, p. 26). Figure 2-14 is a proportional representation of the progression of themes (A1, B, A2, . . . , A7) in each voice and shows the levels of imitation between the voices.

The 4:5:6 ratio of the title describes the major triad, and as shown in Figure 2-14 the levels of imitation for all A statements happen to spell out major triads (except in A7, which is imitated at the double octave), creating a structural application of the ratio. In contrast, none of the non-A sections imitates voices at the major triad; the actual levels of imitation in these sections (ignoring octave displacements) are:

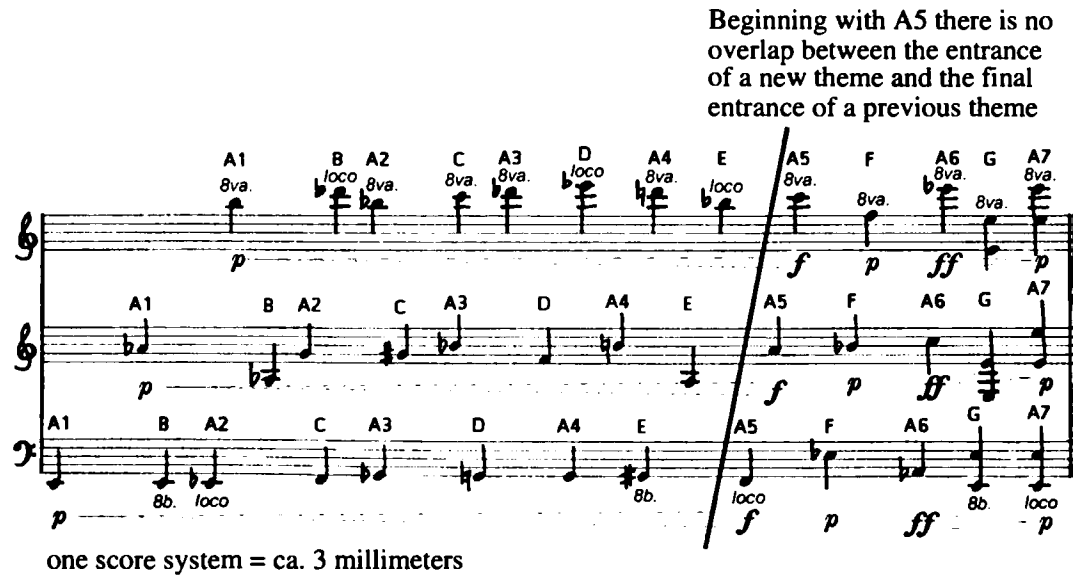


Figure 2-14. Proportional diagram of entrance of themes and levels of imitation in Study No. 34.

section B = ascending (enharmonically spelled) first inversion minor triad

section C = ascending (enharmonically spelled) root position minor triad

section D = descending cluster of major seconds

section E = ascending cluster of minor seconds

section F = ascending perfect fifths

section G = ascending octaves

In effect, the levels of imitation are contracting through section E before expanding dramatically in sections F and G. The cluster of major seconds in section D could be construed as another representation of the 9:10:11 ratio. The major second is usually expressed by the ratios 9:8 or 10:9; however, 11:10 could be a very small major second, and Nancarrow used a similar technique in Study No. 28 when he used a 15:16:17 ratio to express both a rhythm and a melodic figure of contiguous half-steps (see Figure 2-11).

An important structural point occurs at the entrance of A5 (see Figure 2-14). This is the first thematic section in which the first entrance of the theme (always in the lowest voice) follows the last entrance of the previous theme (in the highest voice). From this

point on there is no longer overlapping of entrances of new themes with entrances of old themes. The significance of this point in the piece is emphasized by the arrival at the loudest dynamic in the piece thus far (*f*), and by the division of the piece at A5 into the first eight sections and the last five sections, or the golden section (8:5).⁴

Besides defining the levels of imitation (and thus, to some extent, the harmonic content) of the A sections in the piece, the 4:5:6 ratio is represented in No. 34's tempo scheme. The entire meaning of the enigmatic title refers to a basic 9:10:11 tempo ratio between the three voices, with the A sections of each voice then subjected to a series of 4:5:6 operations to create an accelerating "scale of tempos" in the A sections of each voice. For instance, the progression of tempos in the "9-voice" (lowest voice)⁵ begins with the following tempos in the A sections (Gann 1995, 130):

tempo:	72		90		108		144		180		216
ratio:	4	:	5	:	6	:	8	:	10	:	12

In other words, the ratio 9:10:11 can be thought of as applying to the vertical axis (between the voices) and 4:5:6 to the horizontal axis (time) to create an accelerating series of tempos. Gann calls the result an acceleration canon, but "the acceleration is by steps rather than gradual" (Gann 1995, 130).⁶ Figure 2-15 shows the succession of tempos that results from the application of the two ratios in all three voices.

About the tempo structure, Tenney remarks that "Study #34 is in some ways the most elaborate of all the *Studies* in its use of rational tempo relations as an integral structural parameter" (Tenney, Wergo Vols. III/IV, liner notes, p. 26) while Gann says "With its ratios within ratios, this is one of Nancarrow's most experimental studies, and it is his last canon in which no strong connection is made between form and content" (Gann

⁴I make this assertion while keenly aware of the tendency—as expressed by Kramer (1988)—to "find" the golden section to fit one's momentary purpose.

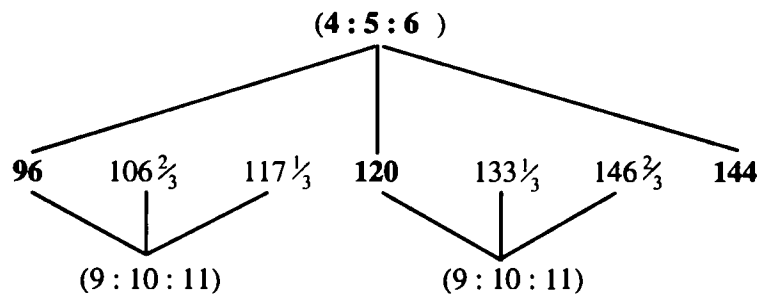
⁵As is usual in Nancarrow's tempo canons, in No. 34 the lowest voice is the slowest and the fastest voice the highest.

⁶Gann uses the term "acceleration canon" to refer to canons containing either or both accelerating and decelerating processes. This canon accelerates only to section A6 and then decelerates to the end of the piece.

Section:	A1	B	A2	C	A3	D	A4	E	A5	F
11-voice:	88	$106\frac{2}{3}$	110	$117\frac{1}{3}$	132	$133\frac{1}{3}$	176	-- $146\frac{2}{3}$	220	176
10-voice:	80	96	100	$117\frac{1}{3}$	120	--120	160	-- $146\frac{2}{3}$	200	160
9-voice:	72	96	90	$106\frac{2}{3}$	108	120	144	-- $133\frac{1}{3}$	180	144
Section:	A6	G (deceleration begins) -----					A7 (incomplete statement) -			
11-voice:	264	$234\frac{2}{3}$	220	176	$146\frac{2}{3}$	132	$117\frac{1}{3}$	110	88	$73\frac{1}{3}$
10-voice:	240	$213\frac{1}{3}$	200	160	$133\frac{1}{3}$	120	$106\frac{2}{3}$	100	80	$66\frac{2}{3}$
9-voice:	216	192	180	144	120	108	96	90	72	60
(returns to original tempos)										

Figure 2-15. Succession of tempos in Study No. 34 (based on Gann 1995, 131). Shaded rectangles indicate 9:10:11 tempo complexes; recurring 9:10:11 tempo complexes are indicated by ovals. Dotted lines between adjacent tempos in the top row indicate breaks in acceleration. Note that: (a) tempos in all A statements adhere to the 9:10:11 ratio; (b) tempos peak in all voices in section A6, with steady deceleration beginning in section G; and (c) the piece ends slower than it began.

1995, 132). As shown in Figure 2-15, the 9:10:11 tempo ratio between the voices is consistent across all the A sections and in sections F and G, but a different process takes place in sections B, C, D, and E. As a result, the acceleration is not constant through section A6 and sections E and F are actually decelerations. The 9:10:11 ratio is still operational in these sections, however, but it has effectively been doubled. If one traces from the lowest voice of section B, up to the 10-voice and 11-voice, then from the bottom voice of section C and up that column, and so on in sections D and E, one ends up with the following series of tempos: 96 96 $106\frac{2}{3}$ $106\frac{2}{3}$ $117\frac{1}{3}$ $117\frac{1}{3}$ 120 120 $133\frac{1}{3}$ $133\frac{1}{3}$ $146\frac{2}{3}$ $146\frac{2}{3}$. The starting point, 96, creates a 9:10:11:12 ratio with the three tempos in section A1. Omitting the duplications, the series is revealed as a succession of 9:10:11-related numbers as shown below:



The next number in the series, the tempo 144 in the bottom voice of section F, does not seem to fit the steadily accelerating pattern until one applies the ratio 4:5:6 to the numbers 96 and 120 (the first terms of the two 9:10:11 series) to yield 96:120:144. Thus 144 becomes the lowest term in section F, where a consistent 9:10:11 relationship between the voices is established and maintained to the end of the piece. The result of all these procedures is nineteen tempo changes in each voice, with a general acceleration through section A6 and continuous deceleration in sections G and A7.

Changes in dynamic generally parallel changes in relative speed. The dynamic increases steadily from *p* at the beginning to *f* at section A5 (unaffected by the deceleration in section E). At section F, where the tempo slows again, the dynamic drops suddenly to *p* again for another crescendo to *ff* at A6 (the piece's fastest and loudest section), and from there the dynamic grows softer to the end of the piece. The result is a most unusual ending for Nancarrow as the decrescendo is accompanied by a gradual ritardando that winds the piece down to slower than it began. Gann describes this ending as "coy" and notes that no other of Nancarrow's tempo canons "ends simultaneously on a ritard" (Gann 1995, 132); the effect is appreciably heightened by the simultaneous decrescendo and is quaintly reminiscent of the winding down of a child's music box.

Themes

Themes A through G of Study No. 34 are shown in Figures 2-16 through 2-22. (All themes shown are from the lowest voice.)

The seven themes share three important motives: motive **a**, a pitch motive, is the same pitch repeated three or four times with rests in between (motive **a'** is a two-note pitch repetition); motive **b**, a rhythmic motive, is two consecutive eighth notes; and motive **c**, a pitch motive, is a broken major triad (another manifestation of the ratio 4:5:6). Motive **b** is often combined with one of the pitch motives, and its pitch content is dominated by the interval of a minor second (34 occurrences out of 58 total), followed by the P4/P5 interval which occurs twelve times. The only other intervals stated are the minor third (seven times) and major second (five times).

As is typical for Nancarrow, in this piece there are very few long notes: the longest is a quarter note, and the majority of notes are staccato eighth notes. Nancarrow frequently changes the meter; the time signature always has an eighth-note denominator and meters range from $\frac{4}{8}$ to $\frac{9}{8}$. Only three measures do not begin with an attack. Isorhythms or other obvious rhythmic patterns are difficult to ascertain, although there are isolated numeric sequences affiliated with motive **a** such as the 2 + 3 + 2 + 1 rhythmic pattern in mm. 7–8 of theme C (Figure 2-18).

Theme A (120 ♩s) -- first statement in lowest voice

1

(8b. throughout)

8

piece ends here in statement A7

15

21

entire theme traces icl

Figure 2-16. Theme A of Study No. 34.

Theme B (40 ♩'s) -- first statement in lowest voice (system 6)

----- rhythmic sequence 1 -----

----- rhythmic sequence 2 -----

1 (8b. throughout)

----- rhythmic sequence 3 -----

4

Figure 2-17. Theme B of Study No. 34.

Theme C (67 ♩'s) -- first statement in lowest voice (system 13)

----- rhythmic sequence 1 -----

----- rhythmic sequence 2 -----

1 (8b. throughout)

5

2 + 3 + 2 + 1

entire theme traces icl

Figure 2-18. Theme C of Study No. 34.

Theme D (103 ♩'s) -- first statement in lowest voice (system 20)

----- rhythmic sequence 1 -----

----- rhythmic sequence 2 -----

1 (8b. throughout)

6

11

15

first two phrases trace icl

entire theme traces icl

Figure 2-19. Theme D of Study No. 34.

The A theme (Figure 2-16) bears a notable resemblance to the opening of No. 31 (Figure 1-11, p. 37). No. 31 states motives **a** and **b** (a half step) immediately in the first three measures, with the same articulations as No. 34's A theme and clearly outlining G minor (the "partitioned minor third" of which Nancarrow was so fond). No. 34's opening theme hints at E minor but it is not until mm. 5–6 that the first three scale degrees are clearly stated, and until then the tonal environment is quite ambiguous. The note E articulated at the beginning of theme A is an important tonal anchor, however.⁷ It reappears in the next statement of **a** at mm. 8–9 (which, in the abbreviated A7, is the ending of the piece with the E stated in octaves in all three voices). At m. 17, the motive **a** is stated on G a minor third higher than the opening E; this G will be another important tonal anchor, especially in the middle themes. Motive **c** has two isolated appearances in

Theme E (138 ♭'s) -- first statement in lowest voice (system 27)

-- sequence 1b --

(8b. throughout)

----- sequence 1a -----

----- sequence 2a -----

----- sequence 2b -----

3 + 4 + 5

----- sequence 3a -----

----- sequence 3b -----

Figure 2-20. Theme E of Study No. 34.

⁷ All references to specific pitches in themes will refer to the bottom voice, from which Figures 2-16 through 2-22 are taken. It is the bottom voice's E tonal center that is affirmed in the A7 statement when all voices are imitating in octaves.

Theme F (130 ♩s) -- first statement in lowest voice (system 35)

Figure 2-21. Theme F of Study No. 34.

Theme G (50 + 62 ♩s) -- first statement in lowest voice (system 41)

Figure 2-22. Theme G of Study No. 34.

theme A. The entire theme traverses the interval of a half step (ic1), and this interval class shapes several of the themes. The theme is metrically diverse but begins with nine measures of $\frac{4}{8}$ time.

Theme B (Figure 2-17), at 40 $\frac{1}{8}$ s in length, is the shortest of the themes; the predominant meter is $\frac{7}{8}$. The theme is dominated by motive **c**, while motive **a** is notably absent. Theme B is much more organic than theme A; it consists of three phrases, each beginning with two contiguous statements of **c** followed by eighth notes and rests. The three phrases are rhythmically identical for the first eight notes. The second phrase begins a major third higher than the first, with the third phrase beginning on the same E that began the first phrase. The theme concludes with its only statement of motive **b**, stating a half step.

Theme C (Figure 2-18) begins with two sequential phrases that combine all three motives, each ending with **a'**. The three statements of motive **a'** are related by thirds. The predominant meter of theme C is $\frac{8}{8}$, and there is an arithmetical rhythmic series (2 + 3 + 2 + 1) in the 4-note statement of motive **a** in mm. 7–8. Like theme A, theme C traverses ic1 from beginning to end.

Theme D (Figure 2-19) is dominated by $\frac{5}{8}$ meter and also traverses ic1 from beginning to end. Its first two phrases also trace ic1, and the predominant melodic movement in this theme is by half step; chromatic scale passages occur at mm. 12–13 and 15–17. There are also several examples of Nancarrow's tendency to use a major form of a scale ascending and a minor form descending, such as mm. 8–10 and 12–14. Except for one statement of **a'**, motive **a** is virtually absent, and there are no occurrences of motive **c**.

Theme E (Figure 2-20) is the longest theme (138 $\frac{1}{8}$ s) and is metrically diverse, although 15 of the 26 measures are in $\frac{5}{8}$. Only motives **a/a'** and **b** appear, and all but one statements of the **b** motive state a half step. This theme abounds with sequential phrases, labeled sequences 1a/b, 2a/b, and 3a/b; sequence 2 changes pitch direction slightly, but is

similar enough to sound sequential. The statement of motive **a** in mm. 12–15 marks the halfway point of the theme and is highlighted in importance by its four-note repetition and its arithmetical deceleration (3 + 4 + 5 eighth-note beats). Deceleration of the **a** motive is also a feature of sequence 1a/b (note that section E represented a deceleration within the general acceleration that occurs through section A6).

Theme F (Figure 2-21) is also metrically diverse but tends toward the longer measures such as $\frac{7}{8}$, $\frac{8}{8}$, and $\frac{9}{8}$; there are no $\frac{4}{8}$ measures. A new feature that appears here for the first time is pitch dyads of either perfect fifths or minor thirds. Only motives **a'** and **b** are present, although the first four iterations of the perfect fifth G–D (mm. 2, 7, 8, and 11) trace an expansion of motive **a**. The remaining three P5 dyads trace a chromatic descent from the G in m. 11 to F# in m. 15 and F at the very end; the descent concludes at the beginning of theme G with the note E.

Theme G (Figure 2-22) is metrically diverse, is stated entirely in octaves, and shares some features with Theme A: quarter notes appear in mm. 17, 18, and 19, and like theme A, statements of the **b** motive tend to be perfect fourths and fifths. There is only one statement of motive **a** but it is the same pitch (G) and rhythmically the same as the statement in mm. 12–15 of theme E; motive **c** returns here after a long absence. Theme G is divided into two parts with an unmeasured rest in between; the second half of the theme contains four tempo changes, all decelerations (making it the only theme with internal tempo changes with the exception of the last statement of A). The end of the theme elides with the beginning of A7, at the A theme's original pitch level plus the upper octave.

It is noteworthy that, in sections F and G/A7, the perfect fifth dyads in F and octave dyads in G/A7 are identical to the levels of imitation in these sections.

The features of the individual themes are summarized in Table 2-3.

Examination of the seven themes reveals that the most important structural pitches in Study No. 34's canonic subject are affiliated with appearances of motive **a**. The first such pitch is the opening note E expressed by the first statement of **a** in Theme A and

reiterated by the second statement at mm. 8–9. The next statement of **a**, at mm. 17–18, affirms the second structural pitch, G. The next statements of **a** are on E_b (theme A, m. 23) and F (theme C, mm. 7–8).

Table 2-3
Features of Themes in Study No. 34

Theme:	A	B	C	D	E	F	G
Contains pitch motive a/a'	•		•	•	•	•	•
Contains rhythmic motive b	•	•	•	•	•	•	•
Contains pitch motive c	•	•	•				•
Contains quarter notes	•						•
Traces ic1 from beginning to end	•		•	•			
Traces ic1 within phrases				•		•	
Contains dyads						•	•
Contains rhythmic or melodic sequences		•	•		•		
Contains arithmetical rhythmic series in motive a			•		•		•
Contains internal tempo changes							•

Although the marked absence of motive **a** in theme D would seem to indicate little of significance going on there structurally, themes C and D together trace a series of chromatic pitches about the main structural interval E–G as shown in Figure 2-23. The same six pitches shown in Figure 2-23—E_b, E, F, F[#], G, and G[#]—are also the starting pitches of themes A1 through A6 in the lowest voice (see Figure 2-14 [p. 88]).

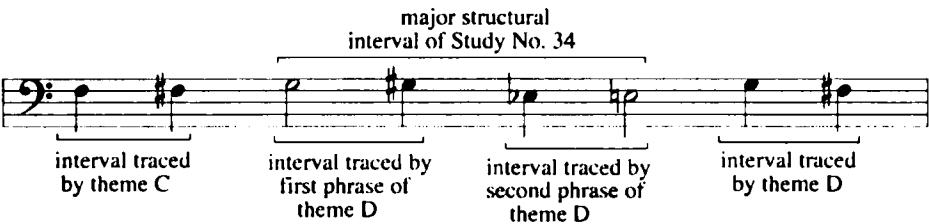


Figure 2-23. Chromatic movement about the main structural interval, E–G, in themes C and D.

Theme E has two statements of motive **a**, both stating the pitch G. The second statement, as noted earlier, marks the halfway point of this theme and its rhythm is an arithmetical deceleration. The pitch G continues to be dominant in theme F; although this theme contains no statements of motive **a**, the motive is traced through the perfect fifth dyads G–D which then chromatically trace the interval from G back down to the note E that begins theme G. The first half of theme G again traces the interval from E to G again, and the second half of the theme contains a 4-note version of motive **a** on G that is rhythmically the same as the statement in the middle of theme E; the second half of theme G concludes with a return to the opening pitch E, which becomes the beginning of section A7.

Rests

The themes are connected by a series of rests that are either metrically notated or unmeasured. Including the rests, the entire lengths of the canonic subjects are approximately 7912 millimeters in the lowest voice, 7103 millimeters in the middle voice, and 6440 millimeters in the highest voice, or roughly a 9:10:11 ratio. Because of this one might expect, perhaps, that Nancarrow would maintain the 9:10:11 relationship between the voices in the rest sections as well, but this is not the case. Only a few of the rests between themes maintain 9:10:11, and Nancarrow's system of determining the amount of rest between themes seems somewhat arbitrary at times. Certain relationships between voices are simple to ascertain while others are baffling.

The rest measurements between sections, as measured either in millimeters on the score or in measured rests, are shown in Table 2-4.

The liberties that seem to be taken with some of the rests and the resultant entrances of themes can, at least in some cases, possibly be attributed to Nancarrow placing theme entrances to coincide with events in other theme statements. Theme E provides a useful example. Each of its three entrances aligns almost perfectly with an attack in another

voice, as shown in Figure 2-24.

Table 2-4
Rest Measurements Between Theme Sections in Study No. 34

Rests between sections:											
	A1-B	B-A2	A2-C	C-A3	A3-D	D-A4	A4-E	E-A5	A5-F	F-A6	A6-G
High	{ @	21	{ @	39	54	*30	*76	15	186	115	51
Voice	$\text{♩}=88$	mm.	$\text{♩}=110$	mm.	mm.	mm.	mm.	mm.	mm.	mm.	mm.
Middle	{ @	20	{ @	78	62	25	*82	84	207	128	74
Voice	$\text{♩}=80$	mm.	$\text{♩}=100$	mm.	mm.	mm.	mm.	mm.	mm.	mm.	mm.
Low	{ @	52	{ @	94	63	94	83	*101	228	142	112
Voice	$\text{♩}=72$	mm.	$\text{♩}=90$	mm.	mm.	mm.	mm.	mm.	mm.	mm.	mm.
Approx. Ratio:	9:10:11	2:5	9:10:11	2:4:5	6:7	-	-	-	9:10:11	9:10:11	4:6:9

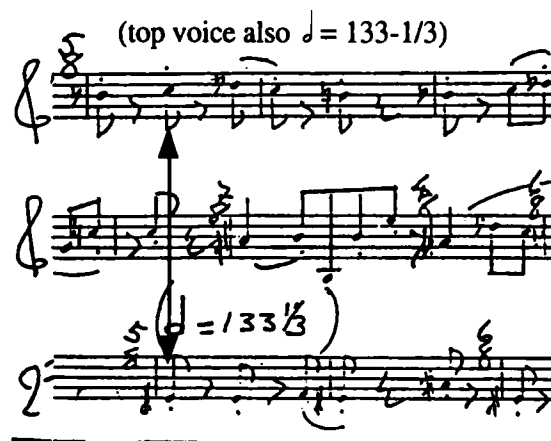
Note: Shaded areas denote measurements that are similar enough to be considered equivalent for the purpose of determining ratios.

* These measurements are at least partially taken on transcribed p. 9 and are more vulnerable to error.

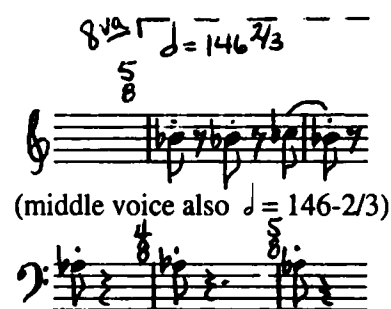
Summary and Conclusions

The net effect of the temporal procedures in this piece is gradually accelerating and then rapidly decelerating sets of closely related tempos that are usually slightly out of phase with each other. However, a few places of near unanimity between voices emerge from the texture in stark relief. Figure 2-25 shows several such passages. Although the constant shift in tonal areas within the seven themes makes coordination of tonal areas between voices quite haphazard, these three passages show a remarkable degree of tonal coherence in addition to near-rhythmic unity: in passage (a), a coordinated shift from D_b to C; in (b), coordination around G minor; and, in (c), arrival at a C major triad followed by movement to an octave E_b . The prevalence of motive a/a' in all three segments is worth noting. The limited rhythmic vocabulary of the themes increases the likelihood of rhythmic similarity as seen in these passages. Passages (a) and (b) show that, because of

Entrance of theme E in bottom voice:



Entrance of theme E in top voice:



Entrance of theme E in middle voice:



Figure 2-24. Entrance of theme E in each of the three voices (Study No. 34, p. 8 and transcribed p. 9).

the progression of the tempo canon, ratios larger than the piece's basic 9:10:11 ratio are possible. In addition, passages (b) and (c) are taken from two-voice textures [and passage (a) is opposite a significant rest in the other voice], adding greater emphasis to the sense of coordination in those passages.

Overall, there are three ways in which the three voices of Study No. 34 can move in and out of phase with each other. We have already looked at two: tonal, in which frequent changes in tonal center left few opportunities for coherence; and rhythmic/temporal, in which the simultaneous combination of similar rhythms and closely-related tempos also occurred infrequently. The other feature that changes among the voices is which of the

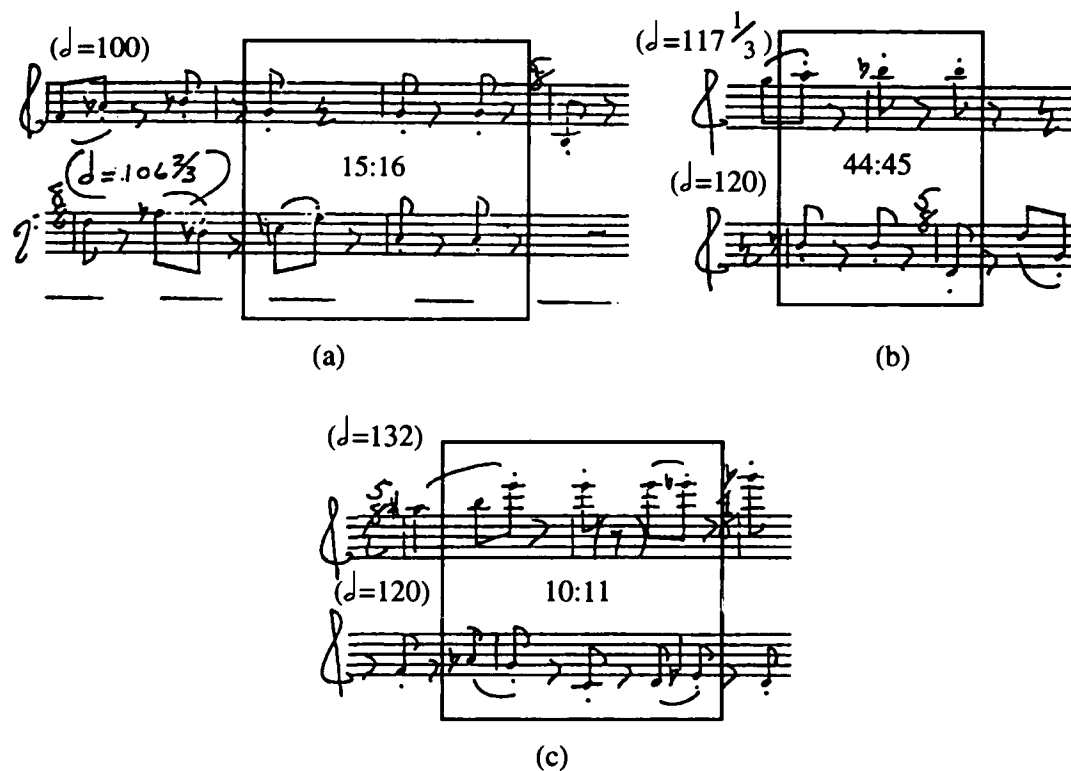


Figure 2-25. Passages of near-rhythmic and temporal unanimity between voices: (a) p. 4, top system (similar rhythm, 15:16 tempo ratio); (b) p. 6, second system (identical rhythm, 44:45 tempo ratio); and (c) p. 7, second system (identical rhythm, 10:11 tempo ratio).

seven themes (A through G) is being stated in each voice. Although the same sequence of themes occurs in each voice, because of the converging canon there is a fairly limited number of places where all three voices are stating the same theme. Figure 2-26 graphically represents areas in Study No. 34 where all three voices are stating the same thematic material.

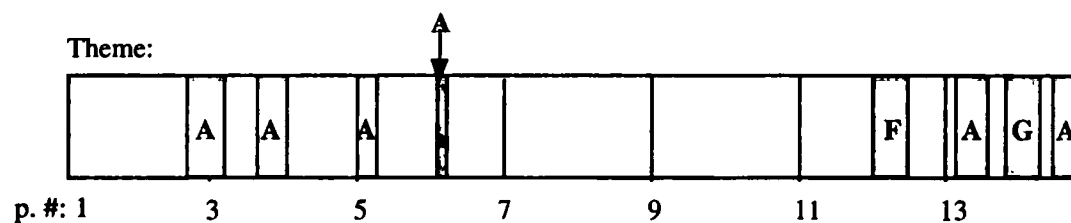


Figure 2-26. Graph depicting areas of Study No. 34 where all three voices are stating the same thematic material.

As shown in the graph, at the beginning of the piece only theme A is stated simultaneously in all three voices, and this is only because of the rondo-like recurrences of theme A. For instance, in the first shaded A section on the graph, section A2 is being stated in the bottom voice while the middle and top voices are stating section A1; the second shaded section includes A2 in the bottom and middle voices and A1 in the top voice; and so on (compare to Figure 2-14). The converging nature of the canon, with its significant delays in the entrance of the middle and top voices, creates enough distance between the B, C, D, and E sections that they are never occurring at the same time in all three voices (the brevity of themes B and C also contributes to their separation between voices). Once section F arrives, however, the canon has converged enough so that sections F and G and the last two A sections, A6 and A7, have more significant portions of simultaneity taking place.

Several processes occur in this Study that create points of structural significance. The first such point is the beginning of section A5, which is the arrival point of a crescendo that began at the beginning of the piece and is also the point at which emerging themes no longer overlap with beginnings of prior themes; this section is also the ninth theme section of 13 overall, dividing the piece into parts of eight and five (the golden section). Furthermore, A5 is the point at which levels of imitation in non-A sections stop contracting (reaching minor seconds in section E) and begin expanding (to perfect fifths in F and octaves in G).

At section F there is a marked lull in the action and extreme textural break. Besides the abrupt change in dynamic, this theme is preceded by the longest rest in the entire piece. Like section E, section F's tempos decelerate from the previous section, and this is where the two tempo schemes shown in Figure 2-15 "collide"; from this point on all tempo complexes are related by the 9:10:11 ratio. Section F introduces another new textural component: dyads of perfect fifths and minor thirds; the P5 dyads match the level of imitation in this section, and this will continue into section G–A7 where the dyads

change to octaves. The roots of the P5 dyads trace an expansion of motive **a** on G, the upper pitch of the major structural interval E–G. Also beginning in section F, simultaneous statements of the same theme in all voices can be heard again (see Figure 2-26).

The fastest and loudest point in the piece is reached in section A6. From here on, tempos rapidly decelerate; as sections G and A7 are the only sections in which there are multiple tempo changes (there are five tempos in section G and four in section A7—see Figure 2-15), the deceleration to the end is much more rapid (and steeper, since the piece ends slower than it began) than the somewhat halting acceleration at the beginning of the piece; see Figure 2-27, which illustrates this feature using the three extreme tempos from the bottom voice.



Figure 2-27. Dynamics and relative scale of acceleration and deceleration in bottom voice of Study No. 34.

As shown in the example, the study reaches its fastest (and loudest) point almost 6/7 of the way through, with the tempo increasing to three times its beginning level. During the last roughly 1/10 of the piece, the tempo decelerates to 5/18 of its fastest with tempo changes occurring at the very end of the piece with increasing rapidity: the final seven tempo changes, from the middle of section G to the end, occur every 14, 21, 15, 12, 10, 11, and 7 eighth-note beats, respectively. The final tempo, $\text{♩} = 60$, applies to only the final three notes, which are a statement of motive **a** on the original pitch, E. Figure 2-28 shows the final system of the Study, which includes the convergence to the final E in triple octaves. The piece's overall structural interval, E–G, is the final interval in all three voices.



Figure 2-28. Concluding system of Study No. 34.

CHAPTER 3

CONVERGENCE POINTS IN NANCARROW'S TEMPO CANONS

Chapters 3 and 4 will concentrate on two aspects of Nancarrow's tempo canons: convergence points, discussed in this chapter; and fractal formal features, discussed in Chapter 4. The most important structural feature of the tempo canons is the convergence point, or "the infinitesimal moment at which all lines have reached identical points in the material they are playing" (Gann 1995, 21). Convergence points (CPs) serve varying purposes in Nancarrow's Studies: sometimes they seem to be the *raison d'être* for an entire piece (particularly in his earlier tempo canons with just one CP, such as Nos. 14, 18, and 19), sometimes they vary considerably in significance within the same piece (such as the thirteen CPs in No. 24), and sometimes they serve to mark the most structurally significant points in a piece (such as the three CPs in No. 27).

Unlike conventional canons in which the voices proceed at the same tempo, tempo canons present the possibility for one or more CPs between the canonic voices because the voices are either converging or diverging most of the time. Recall, from Chapter 1, Margaret Thomas's classification of tempo canons into four basic types (Figure 1-17, p. 42): (1) the converging canon, with one CP at the end; (2) the diverging canon, which begins with a CP; (3) the converging-diverging (arch) canon, in which a single CP is somewhere in the middle; and (4) the diverging-converging canon, which begins and ends with a CP. Nancarrow wrote examples of each basic type (although he did not write any complete diverging canons—there is, after all, not much aural interest in a piece that

diverges immediately away from a beginning point of convergence, never to return¹).

This chapter will examine the presence and placement of CPs in Nancarrow's tempo canons and their significance as related to the overall structure. CPs will be examined in their more conventional uses in the canons, and other devices examined will include tempo switches and overlaps (which can be used to create additional CPs) and techniques for emphasizing and de-emphasizing CPs. Several of Nancarrow's more complex works feature many CPs, and further complexity is possible through the addition of more voices to a canon, providing opportunities for interior CPs among smaller groups of voices. A special sub-type of Nancarrow's tempo canons is the "acceleration canon," in which voices are accelerating or decelerating, usually by established percentages. Nancarrow wrote several such pieces, and this chapter will conclude with an analysis of one of them: Study No. 27 ("Canon 5%/6%/8%/11%"), a four-voice acceleration canon with three CPs.

The effect of convergence in Nancarrow's tempo canons is perceptually heightened because it involves not only temporal convergence, but convergence of canonic material. The choice of canon as a means of portraying temporal proportions is quite natural. Thomas notes the aptness of this technique when she says "One of the clearest compositional strategies in which to hear proportions is canon" (Thomas 1996, 97); she adds further that in Nancarrow's tempo canons "there is a palpable sense of the voices being in different places at the same time, of gradually moving closer together, of a brief moment of coordination, and then a departure" (p. 66). This "brief moment of coordination" is the CP, or Gann's "infinitesimal moment" at which temporally divergent voices converge.

A CP is an excellent example of what Jonathan Kramer refers to as a "timepoint,"

¹Perhaps a good real-life analogy to the perception of the diverging-converging process is airplane travel. A passenger on an airplane has a far more accurate perception of when the plane is going to meet the ground during the landing process (i.e., convergence) than of when the plane is going to reach cruising altitude after takeoff (i.e., divergence). Also, the passenger's perception of exactly when the landing will occur becomes more and more accurate as the plane approaches the point of "convergence" with the ground and is much more acute than the perception of when cruising altitude will be reached.

or “an instant, analogous to a geometrical point in space” (Kramer 1988, 454). A timepoint has no dimension, as explained by Kramer:

But what is a timepoint? Whereas a timespan is a specific duration (whether of a note, chord, silence, motive, or whatever), a timepoint really has no duration. *We hear events that start or stop at timepoints, but we cannot hear the timepoints themselves* [emphasis mine]. A timepoint is thus analogous to a point in geometric space. By definition, a point has no size: It is not a dot on the page, although a dot may be used to represent a point. Similarly, a *staccato* note or the attack of a longer note necessarily falls on and thus may represent a timepoint, but a timepoint in music is as inaudible as a geometric point is invisible. A point in space has zero dimensions; Space itself is three-dimensional; a plane has two dimensions; a line has one; and a point has none. (Kramer 1988, 82–83)

Like one-dimensional geometric lines in space, Nancarrow’s canonic lines converge at this dimensionless point in time, the timepoint. Although “we cannot hear the timepoints themselves,” the CP is an audible event that takes place at this point. Eytan Agmon, in his discussion of musical durations as mathematical intervals, noted that “every common musical duration is *uniquely associated* with a single moment in time, namely its attack” (Agmon 1997, 48). Indeed, most of Nancarrow’s CPs do occur on a coincident beat attack; in a number of cases, however, CPs occur on rests; and, in several cases (the end of Studies No. 32 and 37), a CP is at the end of a long held note. But, whether articulated or not, a CP always represents a timepoint.

The process of temporal convergence, and the CPs which result from it, is virtually undiscussed in the literature as it is a phenomenon that occurs in the music of very few composers (with Charles Ives being probably the other most significant practitioner of large-scale temporal divergence). A search for the term “convergence point” turned up nothing in the literature; looking for the term “convergence” itself turned up a small number of articles, but none dealt with temporal convergence. There is, of course, a considerable body of literature on rhythmic and metric divergence (e.g., Harald Krebs, Edward Cone, Kramer) but true temporal divergence seems to be a relatively unexplored area.

Tempo Canon Terminology and Features

At this point, review of a number of terms relevant to tempo canons will be useful. Recall from Chapter 1 (pp. 42–43) that Gann defined the terms *convergence point*, *convergence period*, *tempo switch*, and *echo distance*. These terms were all illustrated in Figure 1-18 (p. 43). An additional illustration directly from Nancarrow's scores will be given here to further elucidate these features and introduce some additional concepts. Figure 3-1 shows a portion of Study No. 14 ("Canon 4/5") at the point where the CP

At ⊗ the echo distance equals the convergence period (5 beats in top voice, 4 beats in bottom)

The figure shows a musical score for Study No. 14, Canon 4/5, by John Nancarrow. It consists of two systems, each with four staves. The top two staves of each system are in treble clef, and the bottom two are in bass clef. The score is written in 4/5 time. Arrows indicate echo distances between notes in different voices. Dotted lines show potential points of simultaneity. A convergence point (CP) is marked at the beginning of the second system. A circled X (⊗) marks a point where the echo distance equals the convergence period (5 beats in top voice, 4 beats in bottom).

Figure 3-1. Convergence point at middle of Study No. 14 (score, p. 3). Canon is in two voices (bottom voice is two staves in bass clef, top voice is two staves in treble clef); CP takes place at beginning of second system. Arrows illustrate selected echo distances. Dotted lines show potential points of simultaneity; the *convergence period* is the distance between potential simultaneities. Notice that in the fifth convergence period before and after the CP, the echo distance equals the length of the convergence period—a distance from the CP that is functionally related to the ratio 4:5.

occurs, and this example will illustrate the relationships among the terms convergence point, convergence period, echo distance, and potential points of simultaneity. Study No. 14 is a two-voice arch (converging-diverging) canon in which the CP occurs exactly in the middle of the piece.

Echo distances are shown in Figure 3-1 by arrows. Gann pointed out that the echo distance “will grow shorter and shorter as the convergence point is approached, reach zero at the convergence point, then grow progressively longer as it moves away” (Gann 1995, 21), and this is clearly seen in this example. Thomas alludes to the aural effect of the echo distance when she discusses the relative degree of temporal dissonance near what she calls the “point of synchrony”:

The notable and increasingly proportionately significant changes in the gaps, and the attendant modifications in temporal dissonance, are responsible for the intense perceptual focus a point of synchrony achieves in a tempo canon, whether that synchrony occupies the beginning of a diverging canon, the ending of a converging canon, or the middle of a converging-diverging canon. Although it is clear that voices at different tempos are gradually and continuously drawing near to or away from one another, their degree of dissonance can seem nearly uniform for a good portion of a canon. Only near the point of synchrony does the fast approaching/departing [temporal] consonance claim perceptual prominence. (Thomas 1996, 138)

Gann generalizes that as a CP approaches in a piece and the echo distance decreases that motives tend to splinter and become briefer, and that the opposite occurs as a CP recedes and the echo distance increases:

Typically, in the late canons, the following motion occurs: immediately following any convergence point, a quick echoing of brief figures creates excitement signalling the entrance to a new section of the piece. Usually voices lose their distinguishability in a bristling texture, then slowly separate. Short figures are gradually displaced by longer and longer motives which sound calmer (by virtue of their temporal stability) but also more complex, even developmental. As a new convergence point is approached, the echo-tempo picks up again, and figures splinter into ever briefer motives, creating a deliciously gradual feeling of cumulative climax. (Gann 1995, 173–74)

Another event (or series of events) that can signal the approach of a CP is what Gann calls “collective effects,” or a place in the music where the individual voices

coalesce to form a single gesture. Glissandos, arpeggios, and scalar passages are frequently used by Nancarrow in this way, as are other patterns that may at first be quite separated in the voices before merging together.

Recall that the convergence period is the distance between potential simultaneities. Gann points out that in canons involving superparticular ratios the echo distance “will approximate n beats at a point n convergence periods from a convergence point” (Gann 1995, 21). In Figure 3-1, this is obscured somewhat by rests but can be most clearly seen at the points marked $\otimes$, where the echo distance is equal to the convergence period of five beats in the top voice and four beats in the bottom. At these points it is easiest to see that the respective lengths of the echo distance (in beats) in each voice are proportional to the operational tempo ratio at that point; for example, at a point in this 4:5 canon where the echo distance is 2 beats in the top voice, it will be 1.6 beats in the bottom voice.

Gann defines the convergence period as “the hypermeasure that exists between (potential) simultaneous attacks in voices moving at different tempos” (1995, p. 21). Kramer defines *hypermeasure* as a “group of measures that functions on a deep hierarchic level much as does a measure on the surface” (Kramer 1988, 453). Gann’s use of the word hypermeasure, however, unfortunately implies a metrical hierarchy that may or may not be present, as the beginning of a convergence period does not necessarily coincide with a metric accent. Also, a convergence period in Nancarrow’s tempo canons could be as short as three or four beats in those canons with smaller ratios, certainly too brief to be considered a true hypermeasure.

In some of Nancarrow’s canons where the meters do not change, the location of some convergence periods is clearly marked by shared barlines. Also, for as long as the metric pulse does not change, the length of the convergence period remains the same. Shared barlines in Nancarrow’s scores are important markers of convergence and sometimes the only way one can accurately measure the location of events elsewhere in the score (such as the number of beats elapsed to the entrance of a later voice).

Figure 3-2 shows the beginning of a convergence period from Study No. 36 (“Canon 17/18/19/20”), where a convergence occurs every 17 half notes in the bottom voice, every 18 half notes in the second lowest voice, every 19 half notes in the second highest voice, and every 20 half notes in the highest voice. However, since the meter signature is $\frac{4}{4}$ there is a shared barline only every 17/18/19/20 measures, with the beginning of another convergence period occurring exactly halfway between the shared barlines.²

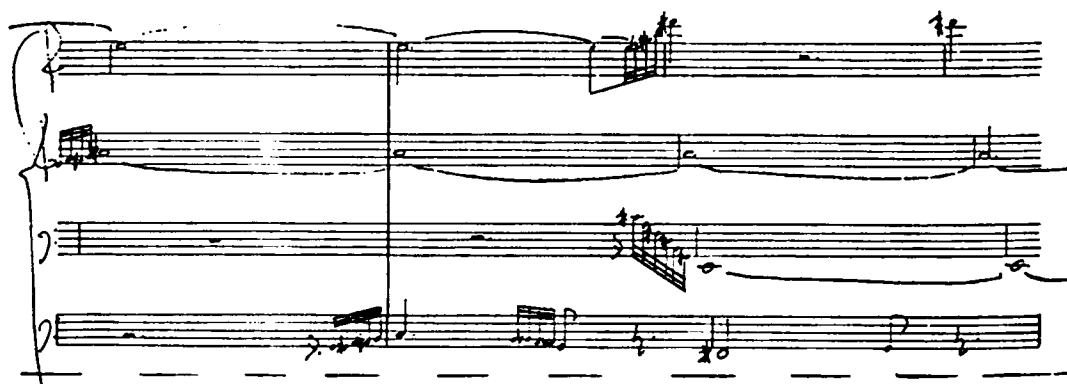


Figure 3-2. Beginning of convergence period (at shared barline) in Study No. 36 (p. 4 of score).

Two additional terms, *timespan* and *point of simultaneity*, are important in making determinations about the placement of CPs and the amount of delay that is needed in later-entering voices to make a CP occur at a certain point. Kramer defines the *timespan* as the “interval between two timepoints” (Kramer 1988, 454). For determining the timespan of Nancarrow’s canonic subjects, the critical question becomes the location of the first and last timepoints. The first timepoint is clearly the first beat attack. There are generally two possibilities, however, for the location of the final timepoint: coinciding with either the attack of the final note, or the release of the final note (which can be said to coincide with the attack of the hypothetical beat that would follow the end of the final note). The most important criteria for making the determination of which should be the final timepoint are usually how the canon ends metrically and its prevailing metric pulse.

²There are also convergence periods every 17/18/19/20 quarter notes, eighth notes, sixteenth notes, etc., but these are not viable metric pulses.

Study No. 14 (which was the subject of Figure 3-1) will serve as an example of a canonic subject in which the final timepoint coincides with the final attack. Gann notes that there are 337 total eighth-note beats in the subject of Study No. 14 (Gann 1995, 117). I concur with this assessment because of the concluding nature of the final attack (with the remainder of the final measure filled with rests) and because the final note matches the prevailing eighth-note metric pulse (see Figure 3-3). By measuring from the *attack* of beat 1 to the *attack* of beat 337, the distance between these two timepoints, or the time-span, is $337 - 1 = 336$ beats. From that measurement we can determine that the midpoint of the canon, where the CP occurs, is the *attack* of beat 169, with 168 beats occurring prior to this point and 168 beats after. Beat 169 is, indeed, the location of the CP.

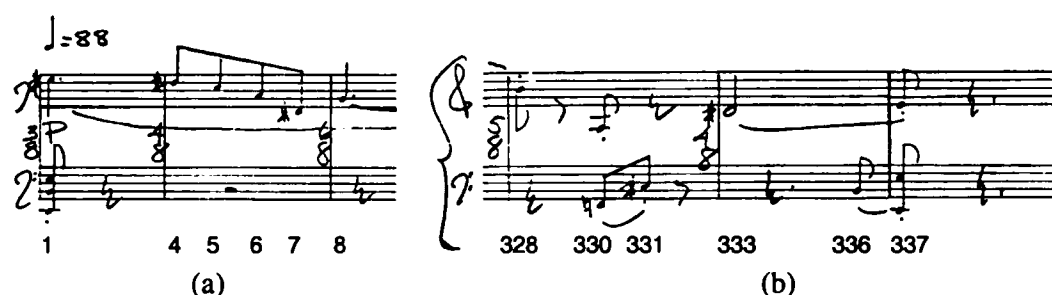


Figure 3-3. (a) Beginning and (b) ending (score, p. 5) of Study No. 14; eighth-note beats of note attacks are numbered. (See also Figure 1-10, p. 36, regarding derivation of Study No. 14's rhythm.)

A different situation occurs in Study No. 36 ("Canon 17/18/19/20"), a four-voice arch canon where the final timepoint is the release of the final note and not the final attack. Figure 3-4 shows the beginning and end of Study No. 36. Gann quite correctly identifies the half-note beat as the pulse,³ but incorrectly states that there is a total of 842 such beats. There are actually 843 beats, consisting of the following: 105 measures of $\frac{4}{4}$ (210 ds) + 1 measure of $\frac{6}{4}$ (3 ds) + 534 measures of $\frac{2}{4}$ (534 ds) + 48 measures of $\frac{4}{4}$ (96 ds) = 843 half-note beats (see also Figure 3-13).

³ Although the tempo markings state the whole-note beat, the half note is the smallest reasonable pulse because there is an extended section of $\frac{2}{4}$ time and also a measure of $\frac{6}{4}$.

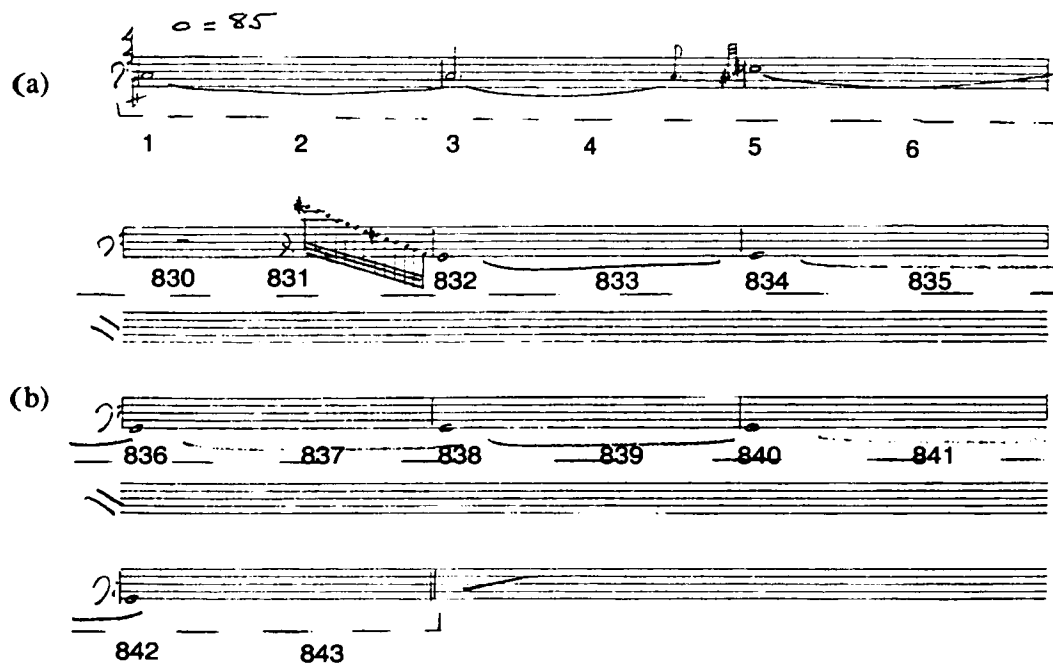


Figure 3-4. Beginning (a) and ending (b) of Study No. 36; half-note beats are marked.

Agmon considers the question of how the duration of a complete musical composition relates to its final attack, and posits as a basic principle that “the end of a piece of music is *never* its last attack. A composition’s last attack cannot be an end because it necessarily initiates a musical duration—specifically, the duration of the musical event that immediately follows” (Agmon 1997, 46). He concludes that “the duration of a piece of music constitutes a closed interval $[x,y]$, where x is (usually) the piece’s first attack, and y is (practically always) a mentally supplied terminal beat shortly following the last attack” (p. 51). Although determining the timespan of Nancarrow’s canonic subjects is similar to but not quite the same as determining the rhythmic close of a piece, Agmon’s observations offer some helpful ideas about measuring the timespan beyond a final attack. In order to calculate the timespan in cases such as these, the attack of the down-beat that follows the release of the final attack can serve as the final timepoint (in essence, the mentally supplied terminal beat). Thus, in our example of Study No. 36, the final attack (beat 832) is obviously not the final timepoint. Because the last note is held

for considerably longer than the metric pulse, and extends through the end of the final measure, the attack of hypothetical beat 844 serves as the final timepoint, confirming the timespan as 843 half-note beats.

Measuring the timespan is somewhat simpler when the CP occurs at the end of the canon (i.e., converging and diverging-converging canons) because, in these pieces, the final CP defines the concluding timepoint. Most of the time (e.g., Studies No. 18, 19, and 24) the CP occurs on the final attack and the timespan would be measured from the first attack to the final attack. Occasionally, however (e.g., Study No. 32), the CP occurs after the final attack and the timespan must be measured to a timepoint that follows the CP (usually a mentally-supplied terminal beat).

Potential points of simultaneity (i.e., the beginning of a convergence period) have tremendous importance in the structure of tempo canons. Obviously, a CP can occur only at a point of simultaneity in all sounding voices. A piece may have a large number of potential points of simultaneity, if the tempo ratio involves fairly small numbers, or it may theoretically have only one point of simultaneity (especially in those tempo canons whose proportions involve irrational numbers—although, practically speaking, there may still be a number of places in these pieces where virtual simultaneities occur that are beneath the threshold of perceptual discrimination, or “just noticeable difference”). At the same time, many potential points of simultaneity are de-emphasized through the use of rests or held notes (for instance, see Figure 1-2, p. 23, as well as Figure 3-1).

The importance of the location of potential points of simultaneity to the structure of tempo canons can be illustrated as shown in Figure 3-5. This example compares the two Studies just presented, Nos. 14 and 36, both arch canons with a single CP in the middle. Nancarrow composed Study No. 14 so that its CP would fall exactly in the middle of its 337-beat theme—that is, on beat 169. In Study No. 36, the midpoint of the canon’s 843-beat timespan falls between beats 421 and 422, and is obviously not an appropriate spot for an audible CP. Nancarrow thus moves the CP slightly later to beat 427; this slight

shift means that 426 half-note beats occur before the CP, and 417 occur after.

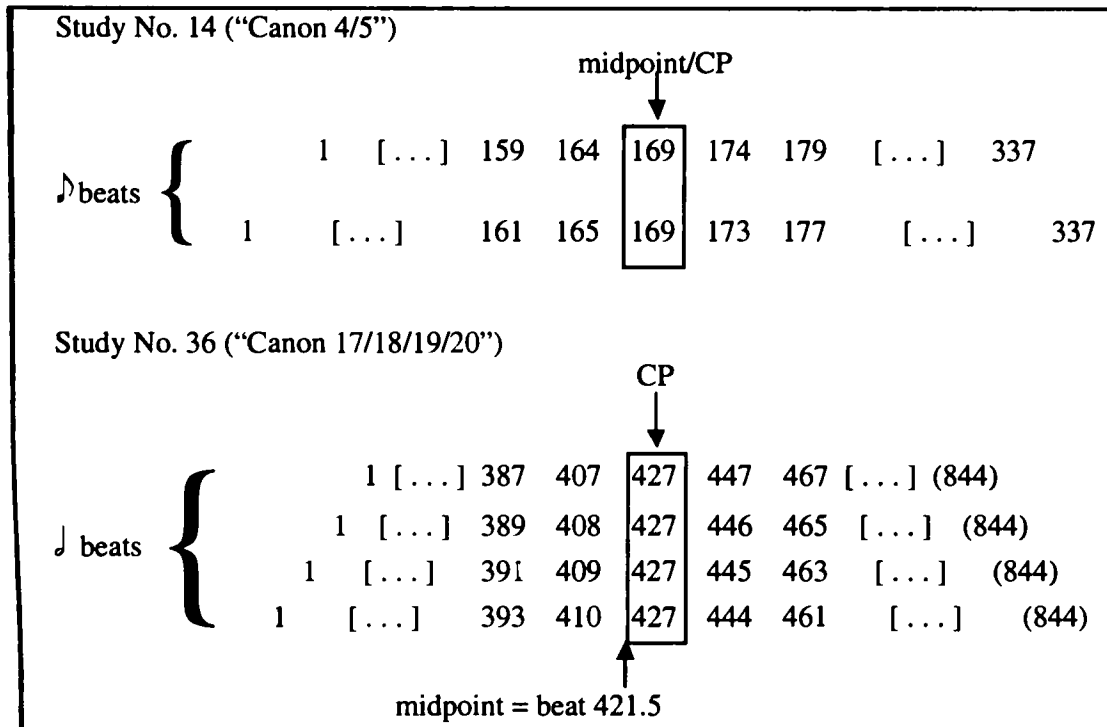


Figure 3-5. Placement of interior CPs in relation to canonic midpoint, and the role of potential points of simultaneity. In Study No. 14 (top), canonic midpoint coincides with simultaneous beat in both voices, allowing CP to coincide with midpoint; in Study No. 36 (bottom), canonic midpoint does not coincide with simultaneous beat in all voices and canon is constructed so that CP is delayed to a later beat. Simultaneities noted are *potential* points of simultaneity as there is not necessarily an attack in each voice at these points.

One can visualize, in Figure 3-5, the separate voices in each piece being represented by proportionally-sized blocks of wood in which a notch in the middle represents the midpoint. The shorter blocks (representing the faster, later-entering voices) are slid into position above the longest block. In Study No. 14, the notches representing the midpoints in each voice would be perfectly aligned; in Study No. 36, they would be slightly staggered.

If it does not matter to the composer whether or not the entrances of later-entering voices coincide with beat attacks in voices that are already sounding (i.e., with the beginning of a convergence period)—and for Nancarrow it often did not—then the CP can

essentially be placed wherever the composer chooses. Convergence periods will then proceed both backward and forward from the CP, depending on its location. In the two Studies shown in Figure 3-5, the entrance of the second voice in No. 14 is between beats 33 and 34 of the first voice, and none of the later voices in No. 36 enters on a beat attack in an earlier voice. In fact, in most of Nancarrow's canons later entrances do not coincide with beat attacks. One clear exception is Study No. 18 ("Canon 3/4"), a two-voice converging canon with the CP on the final attack. Since the 1,680 half-note-beat timespan of this canon is divisible evenly by both factors in the canon's ratio (3 and 4), the faster voice in this canon enters after 420 half-note beats have elapsed, coincident with the 421st beat attack of the slower voice. Thus, in this Study the second voice enters at the beginning of a convergence period, and this is confirmed in the score by a shared barline at the entrance of the second voice.

Finally, two tempo change devices are used with some frequency in Nancarrow's tempo canons: the *tempo switch* and the *tempo overlap*. The tempo switch is defined by Gann:

Another major, but less audible, event is the *tempo switch*, a device in which Nancarrow switches the fastest line to the slowest tempo and vice versa, so that the line that has been lagging catches up with the one that has been pulling ahead. By mathematical necessity, the tempo switch always occurs halfway between two convergence points. (Gann 1995, 21)

Tempo switches are useful in creating additional CPs. They are a significant part of Study No. 24 ("Canon 14/15/16"), where they are used to create thirteen CPs which delineate twelve sections.

Occasionally a tempo changes in only one voice to create a *tempo overlap*, where the same tempo is occurring in more than one voice. Figure 3-6 shows how two tempos can form a tempo overlap in a diverging-converging canon. This is, in fact, exactly how Study No. 15 ("Canon 3/4") is configured. The tempo overlap section in a two-voice canon will consist of a portion of the entire canon that is equal to $\frac{1}{m+n}$, where m and n

are the two elements of the tempo ratio. For instance, in Figure 3-6 the two tempos (and the sectional durations) are related by the ratio 3:2, so the tempo overlap section occupies one-fifth of the entire canon.

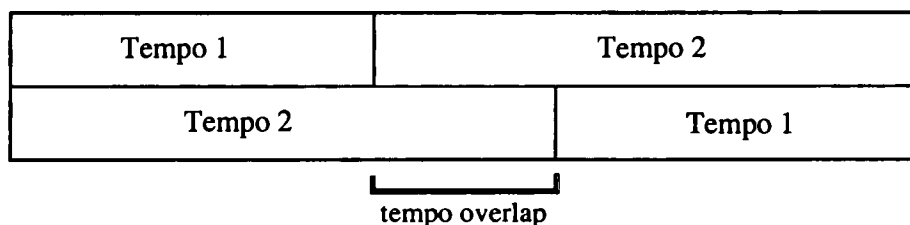


Figure 3-6. Diagram of a tempo overlap at a 3:2 ratio in a diverging-converging canon.

Another Study with tempo overlaps is No. 17, whose structural diagram was shown in Figure 2-7 (p. 78). Tempo overlaps occur between section A in the top and bottom voices, and between section B in the top and bottom voices (see Figure 3-7). Recall that the “duration ratio” of this 12:15:20 tempo canon is 3:4:5, which is how the durations of sections C, A, and B, respectively, relate to each other. In general, in a three-voice tempo canon where the elements of the duration ratio (*not* the tempo ratio) are $x:y:z$ (with x being the smallest element and z the largest), the difference between the durations of x and y related to the length of the entire piece is $\frac{y-x}{x+y+z}$, between y and z it is $\frac{z-y}{x+y+z}$,

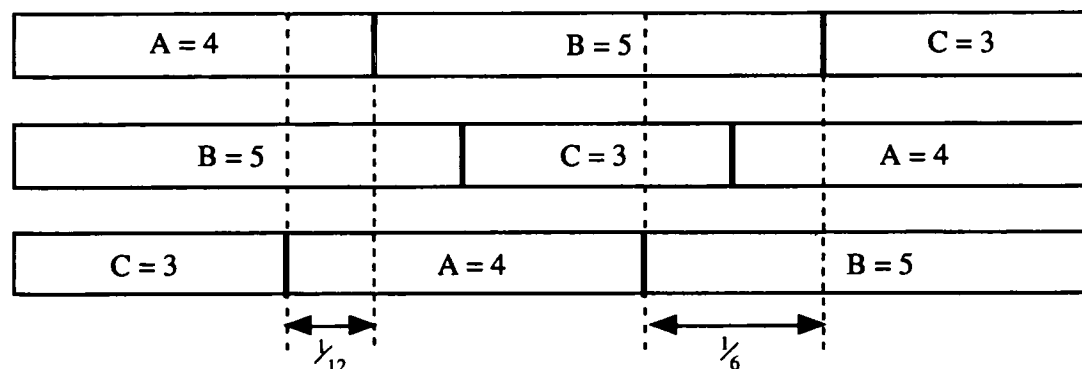


Figure 3-7. Structure of Study No. 17 (“Canon 12/15/20”) showing the relative duration of tempo overlap sections. The duration ratio is 3:4:5 between sections C, A, and B, respectively. The first overlap is between A sections in the top and bottom voices, and the second overlap is between B sections in the same voices. There is no overlap involving C, the shortest section.

and between x and z it is $\frac{z-x}{x+y+z}$. Thus, in No. 17 the sum of the duration ratio's elements, x:y:z, is 12, and the durational relationship between contiguous elements in the ratio (3:4 and 4:5) is one-twelfth of the whole piece while between the outer elements (3:5) it is one-sixth. As shown in Figure 3-7, the tempo overlap involving section A occupies one-twelfth of the piece, and that involving section B occupies one-sixth of the piece. The section B overlap is long enough to be bisected by a tempo change in the middle voice.

Rhythmic and Metric Convergence

Although the primary focus of this chapter is canonic convergence in tempo canons, the topic of rhythmic and metric convergence on a smaller scale in some of Nancarrow's other Studies should be briefly considered. Thomas discusses the metrically different ways in which contrasting tempos can be placed against each other (see Figure 3-8). She identifies three degrees of temporal dissonance based on the number of "metrically significant" downbeats shared by the two voices, which in this example exhibit a 4:5 ratio: (a) in this scenario, metric accents occur in both voices at all convergences for minimal temporal dissonance; (b) here, both voices share the same meter for a metric accent every fourth eighth note, but the number of *shared* metric accents is reduced to once every fifth downbeat in the faster voice and fourth downbeat in the slower voice—this is mild temporal dissonance; and (c) in this scenario, there are no metric accents in common for maximum temporal dissonance.

These rhythmically- and metrically-based periods of convergence are often the result of rhythmic ostinatos, and even in Studies that are not tempo canons these can reach extremes of complexity. In Nancarrow's first player piano Study, "Rhythm Study No. 1 for Player Piano," two lines in $\frac{7}{8}$ and $\frac{4}{4}$ meters create an ostinato that runs through the entire piece (see Figure 3-9). Each line declaims a chord at regular intervals, and the interval of iteration is generally twice as fast in the $\frac{7}{8}$ voice to create a 7:4 tempo ratio

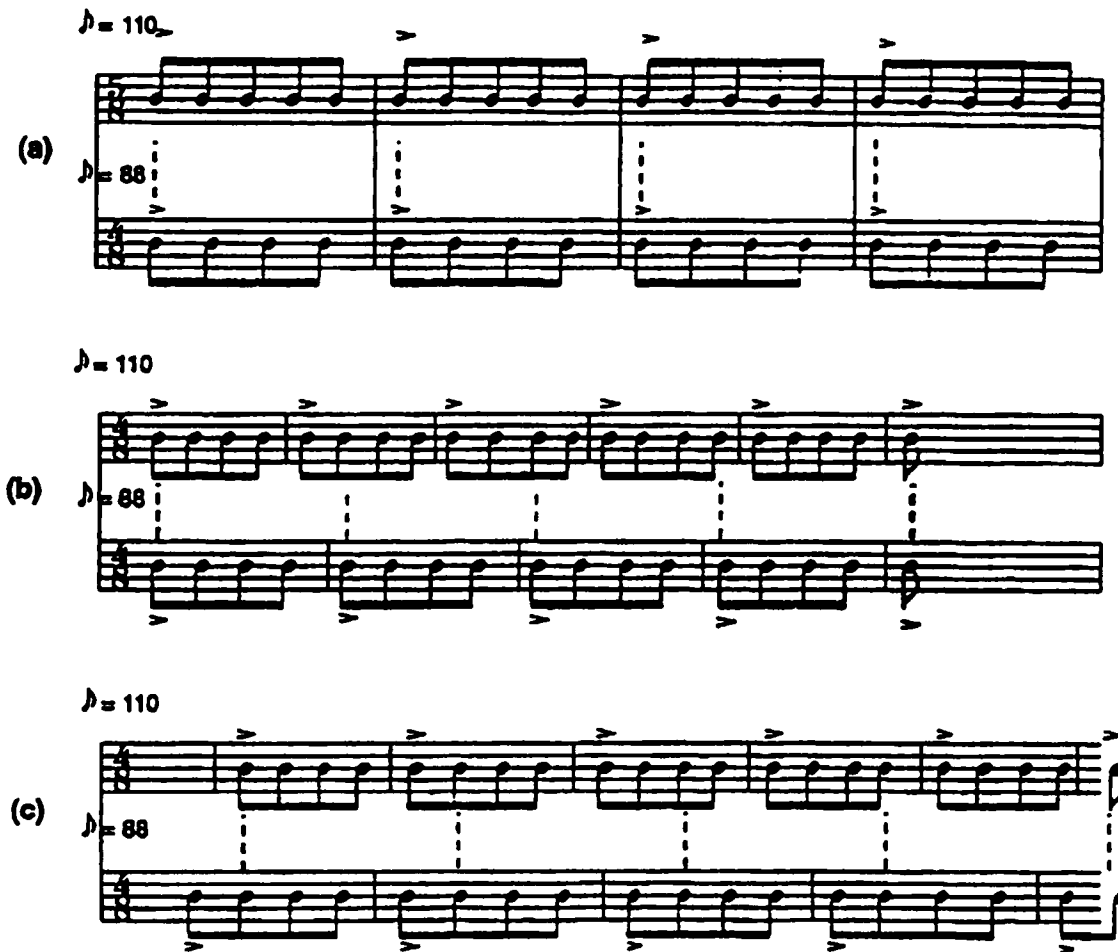


Figure 3-8. Three degrees of temporal dissonance at 4:5 tempo ratio as shown in Thomas (1996, p. 253): (a) all shared downbeats are metrically significant in both voices for minimal temporal dissonance; (b) only some shared downbeats are metrically significant in both voices for moderate temporal dissonance; and (c) no shared downbeats are metrically significant in both voices for maximum temporal dissonance.

between the voices most of the time. Nancarrow creates an elegantly balanced arch-like structure, with the first 14 measures and the last 13 measures containing no simultaneities (maximum temporal dissonance somewhat as illustrated in Figure 3-8c, although here the dissonance is even more extreme because there are no shared articulations—accented or otherwise—at all). The rate of iteration increases incrementally and proportionately to the beginning of measure 21, which begins a section of convergences on every other downbeat in both parts of the ostinato (closer to Thomas's minimal temporal dissonance as shown in Figure 3-8a). The 3-bar convergence periods of mm. 15–21 are then balanced

by the same in mm. 81–87, and then at m. 87 Nancarrow again creates increased temporal dissonance by thwarting all simultaneities. In this section, after the final simultaneity at m. 87, the rate of iteration now incrementally decreases to return to the original rates; this time, however, the changes in the rates of iteration are staggered in the two voices to create additional ratios of 7:3 and 7:5.

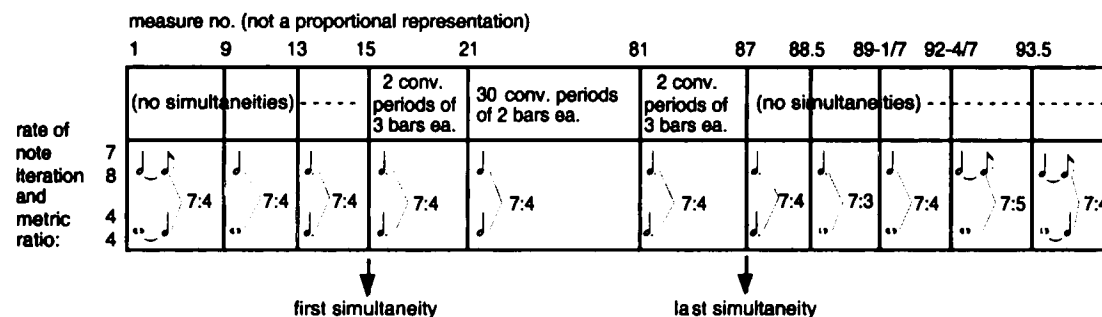


Figure 3-9. Relationship of simultaneous beats and convergence periods between two ostinato lines in "Rhythm Study No. 1."

Convergence Points in Nancarrow's Tempo Canons

Tempo Canons With One (or Almost One) CP

Two of the basic types of tempo canon classified by Thomas (Figure 1-17, p. 42) have just one CP: the converging-diverging (arch) canon, and the converging canon.⁴ In the arch canon the CP falls somewhere in the middle, and in the converging canon it falls at the end. Figure 3-10 shows line diagrams of Nancarrow's tempo canons with just one CP.

Some of Nancarrow's earlier pieces have interesting examples of CPs. Study No. 4 is a loosely canonic piece and an early precursor of the tempo canon technique; a crude variety of tempo canon is effected by notating the canon in different voices in different time values, e.g., five sixteenth notes tied together in one voice, dotted eighth notes in

⁴The diverging canon type also has a single CP, but Nancarrow did not write any complete examples of these.

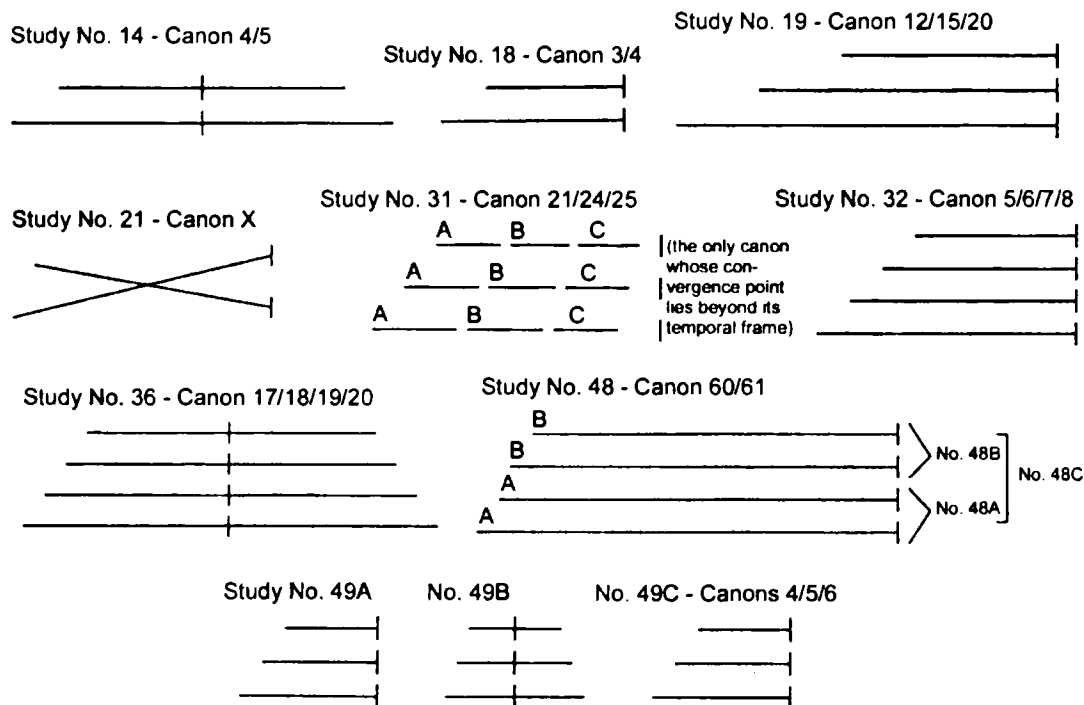


Figure 3-10. Line diagrams describing Nancarrow's tempo canons with only one CP (Gann 1995, 22, 23, 25, and 26).

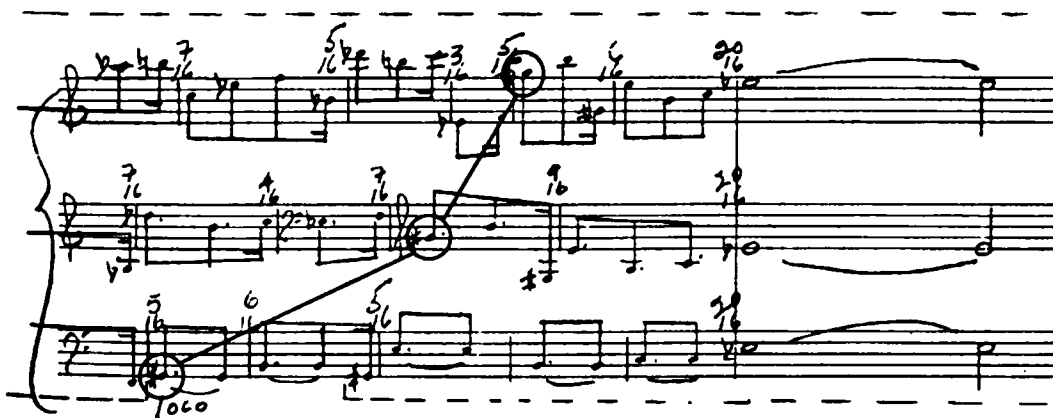


Figure 3-11. Convergence point from Study No. 4 (score, p. 10) in a passage exhibiting a loosely interpreted 2:3:5 rhythmic ratio.

another, and eighth notes in another (a 2:3:5 ratio). This is the ratio in effect in the passage shown in Figure 3-11, which shows the one CP in this piece involving all three voices (two systems prior to this point is a CP involving two of the voices). Note, however, the inconstancy with which the tempo/rhythmic ratio is applied in that pitches that are sixteenth notes are the same value in all voices; this procedure is consistent with

the free rhythmic imitation that takes place in the rest of the Study.

Arch Canons. Nancarrow's arch canons vary widely in ambition and scope. Study No. 14 has its CP exactly at its canonic midpoint (see Figure 3-1), and the realization of this structure seems to be the primary goal for the piece. Study No. 36 has one of the most spectacular CPs in all of Nancarrow's *oeuvre* in terms of its preparation and realization. Study No. 21 has one true canonic CP in addition to an area of "crossing tempos" and a loosely canonic CP on the last note. Study No. 49B returns to a simpler texture but also has a rather arresting CP.

As has already been noted, Study No. 14 (the first that Nancarrow subtitled "Canon") exhibits the basic arch canon form with the CP exactly in the middle of its 337-beat theme (see Figure 3-12). With a tempo ratio of 4:5, the faster voice occupies 80% of the duration of the slower voice, with the remaining 20% of the timespan split so that 33.6 beats elapse prior to that voice's entrance, and 33.6 beats after it concludes; thus, the second voice does not begin at the beginning of a convergence period with the lower voice, but rather three-fifths of the way between the lower voice's beats 33 and 34.

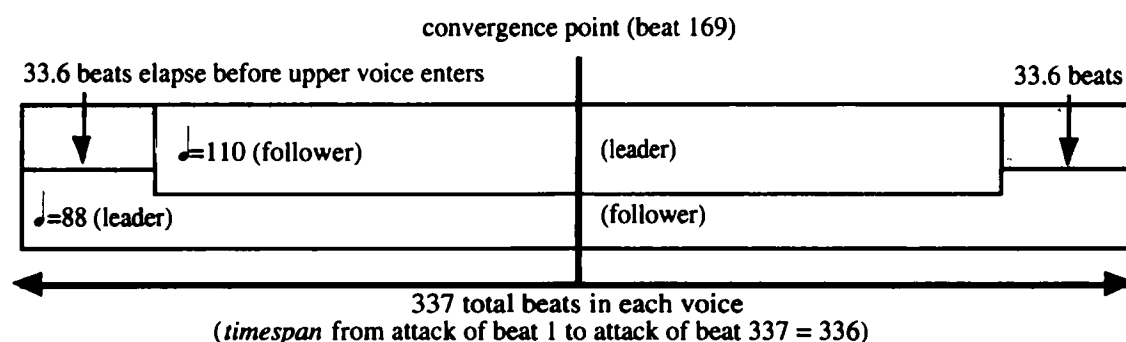


Figure 3-12. Structural diagram of Study No. 14 ("Canon 4/5"), an arch canon.

Another important feature of arch canons, and any tempo canons with interior CPs, is the changing leader-follower relationship between the voices that takes place at interior CPs (see the CP in Figure 3-1). In fact, in Study No. 14 there is so little else calling attention to the CP that the changing leader-follower relationship is thrust into relief and

is what most immediately captures the listener's attention. Gann points out (p. 118) that Nancarrow exerted so much rhythmic control in the creation of his canonic subject in No. 14 that melodic and harmonic coordination between the voices is virtually absent, and this lack of coordination does little to prepare and support the CP. There is also a fairly low incidence, in general, of rhythmic coordination between the voices due to the numerous rests; both Gann and Thomas note that the fairly small 4:5 ratio between the voices would potentially allow for convergence every four to five beats, but actual simultaneous attacks are avoided more often than realized. Thus, in Study No. 14 the CP's placement at the exact midpoint defines the structure, but Nancarrow's highly contrived rhythmic scheme precludes a high incidence of convergence at the metric level.

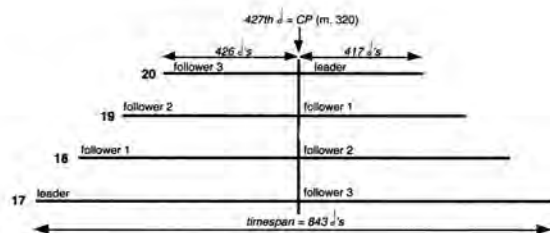
The formal sophistication of Study No. 36 ("Canon 17/18/19/20") makes it a stark contrast to the somewhat humble beginnings of Study No. 14. It is an arch canon in which the canonic midpoint falls between beats because the timespan is an odd number: 843 half-note beats, placing the canonic midpoint between beats 421 and 422. In this canon, then, it is not possible for the CP and midpoint to coincide on a beat attack as they did in Study No. 14 (see again Figure 3-5). Nancarrow instead delays the CP to beat 427, thus creating two sections consisting of 426 and 417 beats (see Figure 3-13).

Philip Carlsen notes that the placement of the CP on beat 427 creates a particular relationship of the elapsed time between statements of a recurring theme (which he calls a rondo theme) within the canon and the CP (see Figure 3-13, bottom diagram):

The rondo theme's second appearance [at m. 107] is twice as fast as the first; it occurs at a point (system 36 in the bass voice) exactly halfway between the beginning and the canonic midpoint, providing an obvious parallel with the well-known acoustical fact that the halves of a string vibrate twice as fast as the whole. (Carlsen 1988, 30)

Carlsen slightly misspeaks here by noting that the relationship is with the canonic midpoint when it is actually with the CP, as shown in Figure 3-13. Carlsen's observation concerning the acoustical parallel can be carried further by noting that the first three

statements of the theme occur on C_2 , $F\#_3$, and C_3 so that the second statement “bisects” not only the elapsed time to the CP but the octave between the first and third statements. Such connections are quite common in Nancarrow.



	a $\downarrow$ elapse needed before and after faster voices for CP 0 (occur at midpoint between beats 425 and 425)	Actual $\downarrow$ elapse before entrance of faster voices	Actual $\downarrow$ elapse after conclusion of faster voices
18-voice	23.4 (1/18 of 421.5)	23.7 (1/18 of 426)	23.2 (1/18 of 417)
19-voice	44.4 (2/19 of 421.5)	44.8 (2/19 of 426)	43.9 (2/19 of 417)
20-voice	63.2 (3/20 of 421.5)	63.9 (3/20 of 426)	62.6 (3/20 of 417)

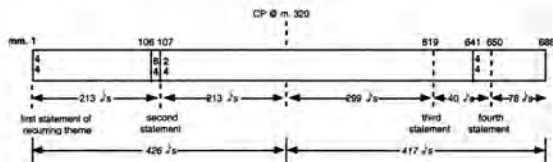


Figure 3-13. Diagram of Study No. 36 (“Canon 17/18/19/20”), an arch canon with the CP slightly past the midpoint (top); table of elapsed half-note beats to entrances of higher voices in comparison to the first (lowest) voice (middle); and structural diagram of the canon, showing entrance points of recurring theme and relationship of theme statements to CP (bottom).

The table of elapsed beats in Figure 3-13 describes the number of beats elapsed to

the entrance and from the drop-out point of the top three voices, in relation to the first (or lowest) voice. The first column shows the beat elapse needed to place the CP exactly at the canonic midpoint; the second and third columns show the beat elapses that actually occur prior to and after, respectively, the statements in the top three voices.

None of the voices enters at the beginning of a convergence period, as shown in Figure 3-14. In fact, even the first voice begins on the last beat of a convergence period, as half-note beat 2 marks the beginning of the first complete convergence period. The second voice's 23.7-beat elapse places its entrance within the second convergence period. Figure 3-14 also shows how the echo distance decreases by one beat with the passing of each convergence period; at the beginning of each period, the number of beats between the voices is equal to the number of convergence periods remaining to the CP.

	beat number of  beats at beginning of convergence period							
20-line					7	27	47	67
19-line				9	28	47	66	85
18-line			13	31	49	67	85	103
17-line	2	19	36	53	70	87	104	121
	25	24	23	22	21	20	19	18
	Number of convergence periods to CP							

Figure 3-14. Half-note beats associated with beginnings of convergence periods at the beginning of Study No. 36. The difference between beats in each voice at each point is equal to the number of convergence periods remaining to the CP—for instance, where the difference between beat values is 20, the distance to the CP is 20 convergence periods.

The stunning CP in Study No. 36 (Figure 3-15) is one of Nancarrow's finest moments. Even the composer was surprised, upon hearing the passage where the CP occurs, to hear the effect of what Reynolds refers to as "low-register difference tones" (Reynolds 1984, 16) generated by the extremely rapid iteration of notes in all the voices. Carlsen describes the passage this way:

In this passage, the four voices sweep up to their point of convergence with rapid chromatic glissandos, arriving simultaneously on their highest notes—Bb₃, D₃, F₆, and A₇.⁵ Each high note is then rapidly repeated at the speed of a quarter-note in its own voice's particular tempo. . . . Even in the slowest voice, the reiterated high notes fly by at a rate of more than 340 per minute (5 per second). But that is not all: the spaces between high notes are filled in with ascending thirty-second-note glissandos. At such speeds, the perception of individual pitches is impossible. The thirty-second-notes (which, in the bass voice, move at over 3000 per minute or 50 per second) are fast enough to theoretically start generating additional low pitches with frequencies in the range of approximately 50–60 Hz. (Carlsen 1988, 25)

The convergence point is preceded by what Gann identifies as a “mega-glissando” that begins 80 measures before the CP, creating a fine example of a “collective effect” that is an extraordinarily long preparation to the CP. Figure 3-16 shows a section in the middle of this passage. Exactly halfway between the beginning of the mega-glissando and the CP, the C₁ half note in the lowest voice begins a restatement of the opening transposition levels, which spell out a widely spaced major seventh chord. The last voice reaches its restatement at the beginning of the next convergence period; two measures prior to this, the lowest voice starts another restatement, this time an octave higher.

This section of rising glissandos creates a tremendous build-up to the CP. The glissandos emphasize to the listener the leader-follower relationship of the four voices, and Nancarrow very cleverly manipulates the decreasing echo distance, by using steadily decreasing gaps between the glissandos, to eventually create long, continuous glissandos through the four voices. After the CP, the glissandos descend through the voices and the listener is once again acutely aware of how the leader-follower relationship has changed at the CP.

An unusual and unique canon is Study No. 21 (“Canon X”), an acceleration canon whose subtitle is derived from the manner in which the tempos of the two voices “cross” in the course of the piece. Study No. 21 is unlike any of the other tempo canons, with its gradually changing tempos (accelerating in one voice while the other voice decelerates)

⁵The levels of imitation in Study No. 36 spell out a major seventh chord.

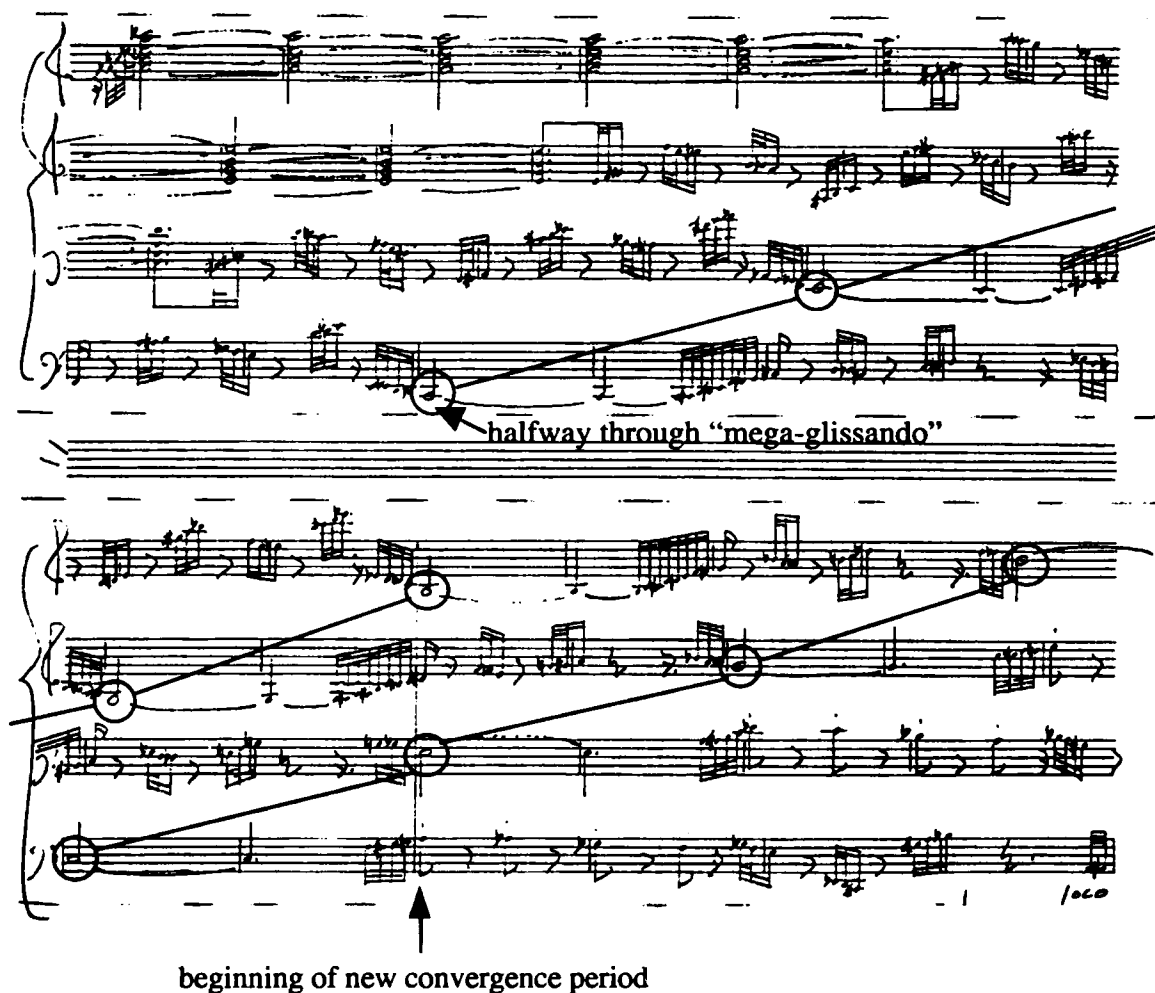


Figure 3-16. Section of Study No. 36 halfway between beginning of “mega-glissando” and the CP (score, p. 21) where original pitch levels are restated in all four voices.

that almost imperceptibly “cross” somewhere in the middle of the piece. Gann reports that in punching this piece, Nancarrow “went through a tedious process of placing the [roll punching] mechanism one half or one quarter of the way between notches” in order to effect geometric acceleration and deceleration; the process was so frustrating that Nancarrow “had the machine altered so that, instead of punching holes at points indicated by notches on the mechanism, it would move along a smooth continuum, capable of placing a hole at any point” (Gann 1995, 106).

In No. 21’s proportionally-notated score, there is no meter and no shared downbeats are notated, but it appears that there are three significant convergences. This Study thus

fits somewhat uneasily into this section on arch canons with one interior CP; however, only one of the convergences is a true canonic CP, and it does appear in the interior of the piece. The first convergence, just over one-third of the way through the piece where the two voices cross tempos, is not a canonic convergence point but more of a convergence area (see Figure 3-17). Thomas (1996, pp. 126, 281) identifies the bracketed area in the figure as a passage of “nearly simultaneous motion.” However, the tempos of the two lines actually coincide somewhere near the beginning of the next system, where the note articulations are nearly evenly staggered between the voices. The crossing of the tempos is thus not marked by a convergence point.

The image shows two systems of musical notation. The upper system consists of two staves, with the upper staff marked *mp*. A bracketed area in the middle of the system is labeled "area identified by Thomas (1996) as 'nearly simultaneous motion' (p. 126)". The lower system also consists of two staves. A bracketed area at the beginning of the system is labeled "tempos actually coincide near the beginning of this system".

Figure 3-17. Area of tempo crossing in Study No. 21 (score, p. 7). The lower voice is accelerating and the upper voice decelerating. Bracketed area is identified by Thomas as representing “nearly simultaneous motion” (Thomas 1996, 126).

At just over two-thirds of the way through the piece, an actual canonic CP does occur (see Figure 3-18). Its significance is enhanced by several factors: the 54-note row (B–C–A_♭ . . .) that comprises the canon begins again in each voice; there is a registral shift upward in the lower voice; and the texture of the upper voice changes from triple

octaves to quadruple octaves. Thus, this CP is both the conclusion of a converging canon and the beginning of a diverging canon. At this point in the piece there are about 16 notes in the bottom voice between successive articulations in the upper voice.

Study No. 21 concludes with a non-canonic CP on the final note; at this point there are about 48 notes in the lower voice between articulations in the upper voice. The canon is altered at the end so the piece ends on a V-I cadence with a quintuple C octave.



Figure 3-18. Convergence point in Study No. 21 (score, p. 14). The arrow indicates the CP, where the canon starts over in both voices, a registral shift occurs in the lower voice, and the texture in the upper voice (top two staves) changes from triple octaves to quadruple octaves.

The final arch canon shown in Figure 3-10 is Study No. 49B, the middle of three movements based on the tempo ratio 4:5:6. This author was not able to obtain a copy of the score, and according to Gann (1995, p. 136) there are considerable discrepancies between the score and the Wergo recording. Analysis of this piece here is thus somewhat limited.

No. 49B's three voices are monophonic throughout, and like all of Nancarrow's tempo canons the piece begins with the bottom voice, followed by the faster middle voice, and then the fastest top voice. The voices appear to be symmetrically arranged about the CP, which occurs just past the midpoint of the piece (at 1'23", or 52% of the way through the piece's total 2'40"⁶). The two most striking features about the CP are the

⁶The natural acceleration of the piano roll may account for the slightly longer duration of the first half.

way in which all three voices accelerate (not just become more rhythmically active, but actually accelerate the beat) in the 10 seconds leading to the CP; and the convergence of the three voices on a sustained major triad. Since the voices originally entered at a level of imitation at the twelfth between all voices, at some point prior to the CP the levels of imitation change.

After the CP, the voices are only loosely canonic and certain motives from earlier in the piece become more affiliated with specific voices. A second convergence occurs at about 2'04", where first the middle voice, then the lower and top voices state an ascending and descending melodic minor scale, with the three voices converging on octaves. Thus, through the course of the piece the levels of imitation have converged from twelfths, to spelling out a major triad, to octaves. Also, the expected leader-follower relationship of voices after the CP, or top-middle-bottom, is not strictly adhered to in stating canonic material.

Converging Canons. The next group of canons to be examined is the converging canons shown in Figure 3-10 (p. 122), which include Nos. 18, 19, 31, 32, 34, 48, 49A, and 49C.

One of the distinguishing characteristics of converging canons is that the leader-follower relationship of voices is maintained to the final CP. This is easily seen in a structural diagram for Study No. 18 ("Canon 3/4"), which was mentioned earlier as an example of a canon where the entrance of a later voice coincides with the beginning of a convergence period (see Figure 3-19). This is made possible because the canon's time-span (1,680 eighth-note beats) is divisible by both 3 and 4 and the first voice begins at the beginning of a convergence period.⁷

⁷It should further be noted that 1,680 is also divisible by 5; had Nancarrow wanted to make this a 3-voice canon with a 3:4:5 ratio, the entrance of the third voice would *also* have coincided with the beginning of a convergence period—specifically, after elapse of 672 beats in the bottom voice and 336 in the middle. In a way it is unfortunate that he did not write such a canon, as none of his converging canons is structured so

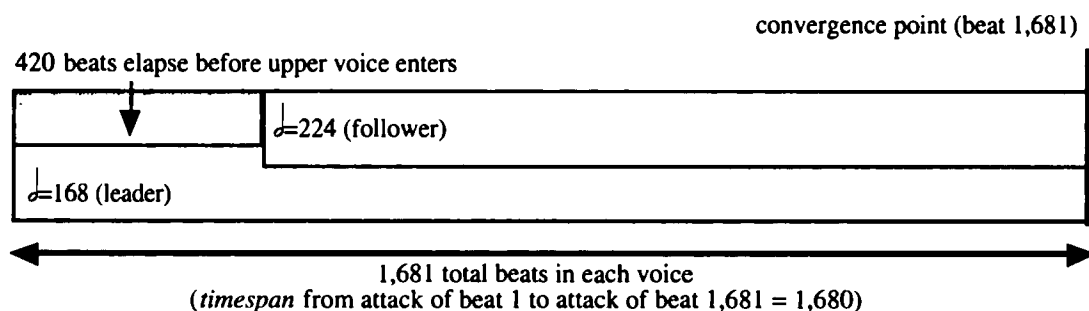


Figure 3-19. Structural diagram of Study No. 18 ("Canon 3/4"), a converging canon in which the entrance of the second voice coincides with the beginning of a convergence period.

In Study No. 19 ("Canon 12/15/20"), like most of Nancarrow's tempo canons, later-entering voices do *not* enter at the beginning of a convergence period (Figure 3-20). The duration ratio of the three sections is 3:4:5 (see Figure 2-7, p. 78), and the bottom voice relates to the middle voice by the ratio 4:5 and to the top voice by the ratio 3:5. The timespan of 336 eighth-note beats is divisible by 3 and 4, but not by 5; thus, since both the middle and top voices relate to the bottom voice in ratios containing the factor 5, neither of these voices enters at the beginning of a convergence period with the bottom voice. The middle voice enters after 20% of the bottom voice has elapsed, or one-fifth of the way between beats 67 and 68. In the top voice, the tempo relationship with the bottom

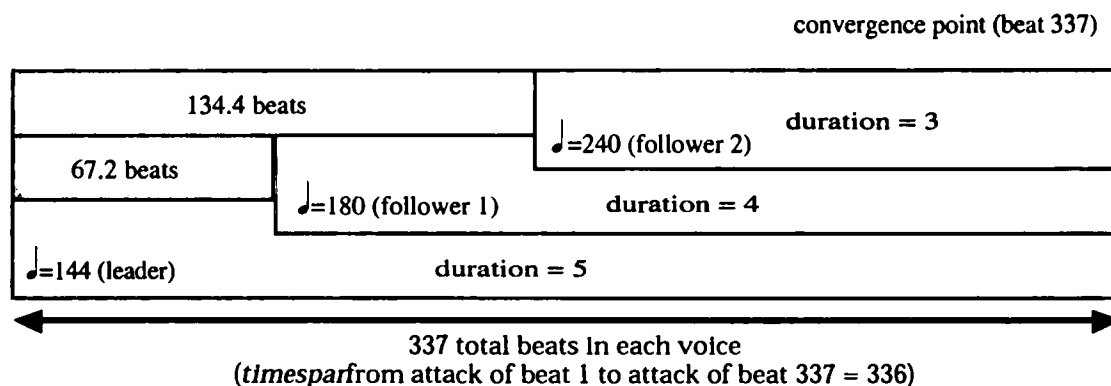


Figure 3-20. Structural diagram of Study No. 19 ("Canon 12/15/20").

that all voices enter at the beginning of a convergence period and this would have been an impressive effect.

voice is 3:5 and it enters after 40% of the bottom voice has elapsed, or two-fifths of the way between beats 134 and 135.

The CP at the end of Study No. 19 is noteworthy because Nancarrow somewhat uncharacteristically alters the interval of imitation between the voices (from elevenths to double octaves) to create a unison V-I cadence (see Figure 3-21). I surmise that he may have done this to convey an additional sense of finality because this canon concludes the six-canon set of Nos. 13–19, all of which are based on the rhythmic series $\{n - 1, n, n + 1, n\}$.

The image shows a handwritten musical score for the conclusion of Study No. 19. It consists of four staves, each with a different clef (soprano, alto, tenor, and bass). The notation includes various rhythmic values and accidentals. A diagonal line is drawn across the staves, indicating a change in the interval of imitation from elevenths to double octaves. The score concludes with a unison V-I cadence.

Figure 3-21. Conclusion of Study No. 19 (“Canon 12/15/20”; score, p. 7), showing the convergence point and alteration of final pitches to create V-I final cadence.

Study No. 32 (“Canon 5/6/7/8”) is a straightforward four-voice converging canon whose structure is shown in Figure 3-22. The duration ratio for the four voices is 105:120:140:168. No. 32’s final convergence point (see Figure 3-23) is unusual in that it occurs at the final cut-off—that is, at the hypothetical downbeat that follows the final beat. There are 431 $\downarrow$ beats in the canon, and the timespan is the same number because

the CP actually occurs on hypothetical beat 432. The second voice enters five-sixths of the way between beats 71 and 72 in the lowest voice; the third voice enters one-seventh of the way between beats 123 and 124; and the top voice enters five-eighths of the way between beats 161 and 162. This canon will be the subject of more thorough analysis in Chapter 4.

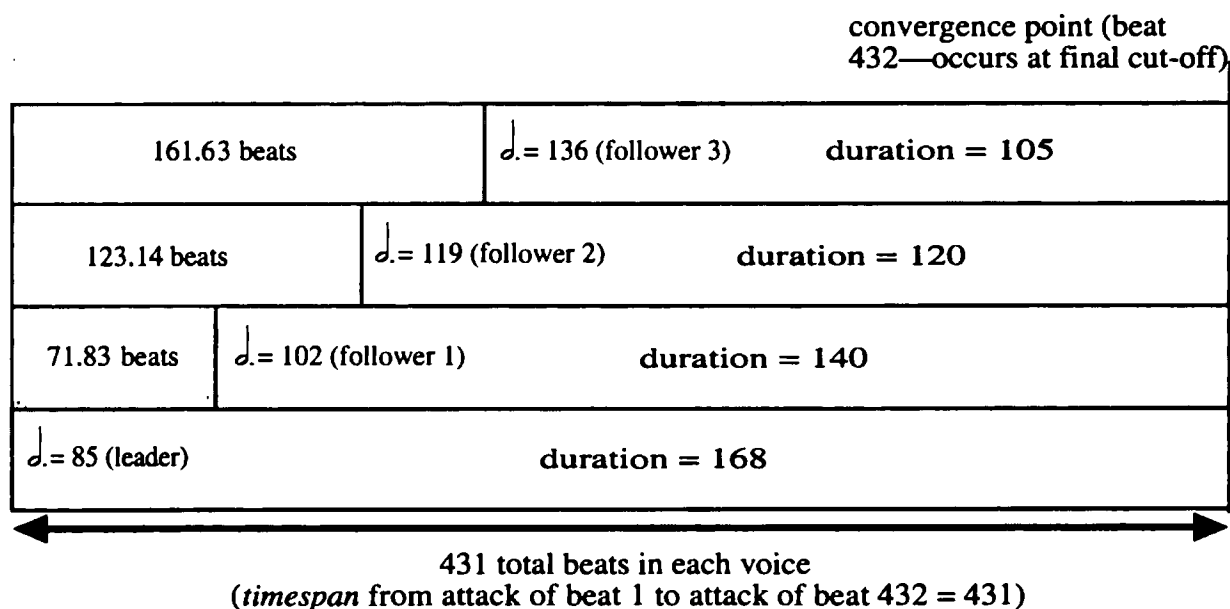


Figure 3-22. Structural diagram of Study No. 32 (“Canon 5/6/7/8”).



Figure 3-23. Convergence point of Study No. 32 (score, p. 11): the CP is the final cut-off.

In Study No. 34 (“Canon $\frac{9}{4/5/6}/\frac{10}{4/5/6}/\frac{11}{4/5/6}$ ”), which was the analytical subject of Chapter 2 (see, in particular, Figures 2-14 [p. 88] and 2-15 [p. 90]), measuring entrances to later-entering voices is complicated by the almost complete lack of noted shared downbeats in the score, the multiple sections of the canon, and the unmetered rests that occur between many of the sections (making it impossible to establish a length for the canon’s timespan; see Table 2-4, p. 100). There is even an unmetered rest within section G (although it appears to relate to the 9:10:11 ratio effective in that section). The CP (Figure 2-28, p. 105) occurs on the attack of the final beat (this is, in fact, the only downbeat in the entire score shared by all three voices).

Study No. 48 (“Canon 60/61”) is a remarkable achievement in convergence. It consists of three movements: movements A and B are each 60:61 canons, which are then played together to create movement C. The performance of C involves two carefully coordinated player pianos. Movement A has a major tenth level of imitation, with an echo distance that begins at about eight seconds (the score is proportionally notated and includes additive acceleration in all four voices, thus measurements are inexact), dwindling to zero over the course of some 127 pages. There is a minimal level of convergence between the two voices built into the canon; the most pervasive is regularly-recurring pairs of arpeggios in which the second of the pair in the bottom voice aligns with the first in the top voice (see Figure 3-24a).

Movement B is slightly shorter than movement A, as, during the performance of C, it should enter after movement A’s 61-voice (see Figure 3-24b). From Nancarrow’s score it is not clear what is the intended ratio between the two movements; it is not 60:61, but closer to 34:35 (based on page count measurements), but in the actual performance of C the B voices enter about a page earlier than indicated in the score, so this makes the actual ratio closer to approaching 60:61. Movement B’s interval of imitation is the perfect fifth; this allows the final chord of movement C to spell out a B₁ major seventh chord at the extraordinary moment of coordination where all four voices converge. Dynamics play

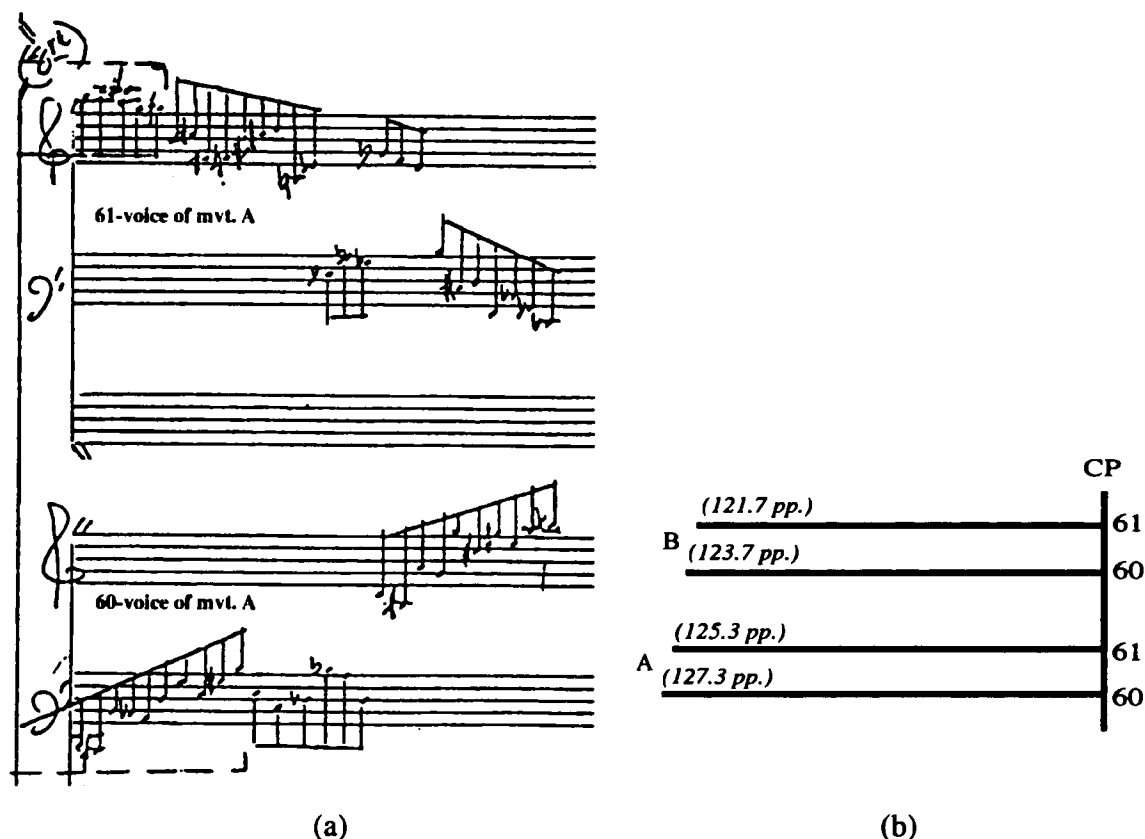


Figure 3-24. (a) Convergence of arpeggio figures in Study No. 48A (p. 43 of score); (b) diagram of proportions in Study No. 48 (movement C is movements A and B played together).

a critical role in the preparation for this convergence; Nancarrow brilliantly creates an almost complete coordination of rapidly changing and highly contrasting dynamics (mostly between *ff* and *pp*) for the final 28 pages of the piece, creating a convergence of dynamics long before the temporal convergence.

Studies Nos. 49A and 49C are both converging canons in the ratio 4:5:6 (which gives them a duration ratio of 10:12:15). Again, due to the lack of a score these remarks are somewhat limited. The entrance timings of later voices in both movements conform to the 4:5:6 tempo ratio. The CP in both movements is preceded by a section of glissandi that coalesce in all three voices into larger, sweeping glissandi. The section which concludes movement C is more extensive and the sweeping glissandi cover a wider range for a more satisfying effect. This movement also contains an earlier converging glissando section about one-third of the way through the piece.

Study No. 31 (“Canon 21/24/25”) is an unusual case. It is a converging canon with no CP, as the canon is truncated before the CP is reached. As shown in Figure 3-25, the canon’s actual length is 669 eighth-note beats, which is 36 beats shy of the 705 needed to reach convergence (the timespan is 704 beats) at a ratio of 21:24:25. The lack of a temporal convergence is seemingly underscored by the three key areas a fifth apart to create what Thomas (1996, p. 67) refers to as an “unresolved asynchronicity” (see the canon’s conclusion in Figure 3-26). The canon is in three distinct sections, each separated by an eight-measure rest, and the third section is both registrally enlarged (the level of imitation increases from a fifth between voices to an octave plus a fifth) and the canonic line reinforced by octaves and full of melodic leaps. Based on these features, Thomas offers a very perceptive and plausible reason for the lack of convergence at the end:

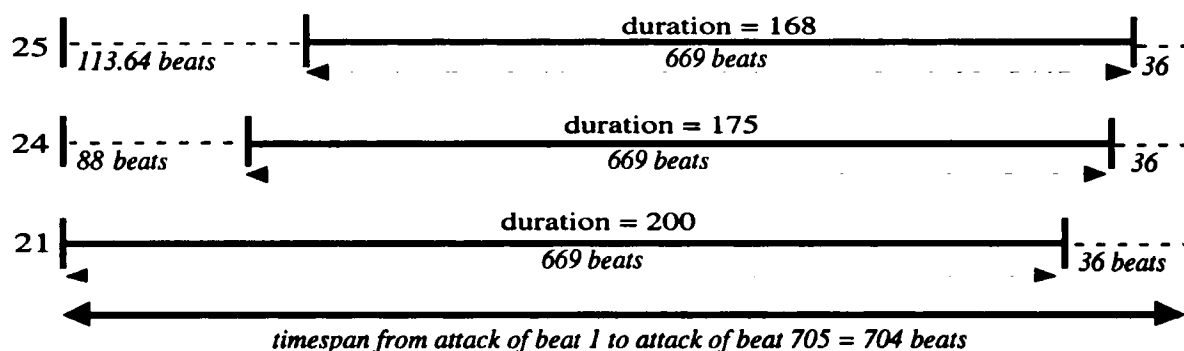


Figure 3-25. Structural diagram of Study No. 31 (“Canon 21/24/25”), a converging canon that ends before the CP is reached.

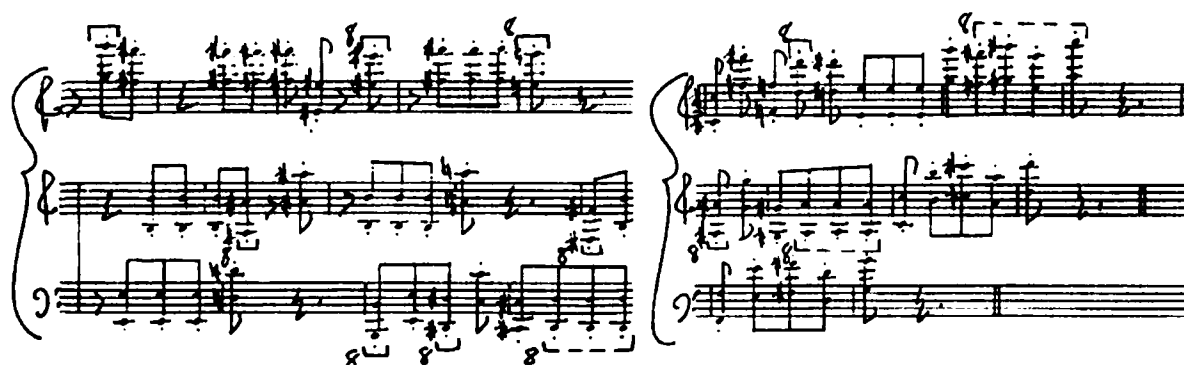


Figure 3-26. Conclusion of Study No. 31 (“Canon 21/24/25”; score, pp. 8–9).

It is in the final section of the study that the canon becomes most difficult to perceive, and in this lies what may have been a potential reason for ending the study before it can converge on a point of synchrony. Each single voice is difficult to follow in and of itself in this section because of its extreme registral leaps and because the articulation types are limited exclusively to staccato. . . . Since it is difficult to follow a single voice, it is nearly impossible to follow the canonic relationship among voices. The staggered ending thus reorients us to the canonic nature of the passage: as each voice drops from the texture in quick succession we are reminded retrospectively of them as linear entities, and we hear in their concluding gestures their shared material and imitative relationship. (Thomas 1996, 69–70)

As shown in Figure 3-25, the entrance of the middle voice in Study No. 31 converges with the eighty-ninth beat in the bottom voice, and the top voice enters $\frac{16}{25}$ of the way between the bottom voice's beats 113 and 114. Although the timespan of 704 is not evenly divisible by either 7 or 8 (the two factors to which 21 and 24 reduce), the entrance of the middle voice coincides with a beat attack (beat 89) in the bottom voice because the bottom voice does not start at the beginning of a convergence period; the first complete convergence period begins at beat 5 in the bottom voice, which *does* allow convergence at beat 89 when the second voice enters ($89 - 5 = 84$, a number evenly divisible by 7).

Tempo Canons With More Than One CP

Canons with more than one CP are of the following types: diverging-converging canons (which begin and end with a CP), canons with interior tempo switches, and special cases such as acceleration canons. Figure 3-27 shows the simpler canonic structures that contain more than one CP.

One of the basic types of tempo canon, the diverging-converging canon, has two CPs (one at the beginning and one at the end). Nancarrow wrote two examples of this type of canon: Studies No. 15 and 17 (see Figure 3-27). Canons of the diverging-converging type have at least one tempo overlap (the number depends on the number of voices and tempos in the canon). Study No. 15 contains a simple example of a tempo overlap. This two-voice Study has a tempo and duration ratio of 3:4 and a canonic subject

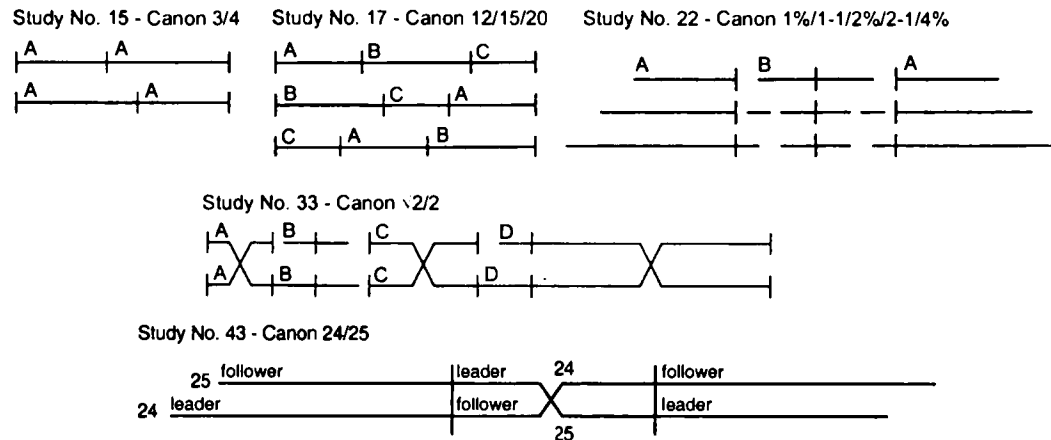


Figure 3-27. Line diagrams describing structures of Nancarrow's tempo canons with more than one CP (Gann 1995, 22, 23, and 25).

of 336 eighth-note beats; the canon is stated twice in each voice, once at the slower tempo and once at the faster tempo. When the faster voice repeats the canon at the slower tempo, both voices are stating the slower tempo for a period of 84 eighth notes (one-fourth of the canon's length), and this section comprises the middle one-seventh of the entire length of the piece (see Figure 3-28). An interesting by-product of this structure is that the top voice, which initially states the faster tempo, remains the leader in stating the canon throughout the piece. The piece is firmly entrenched in G major, and Nancarrow composes the canon so that a G cadence occurs three-fourths of the way through, allowing the two voices to coordinate in a cadence on G at the beginning of each statement; the canon concludes with a V7-I cadence (see Figure 3-29).

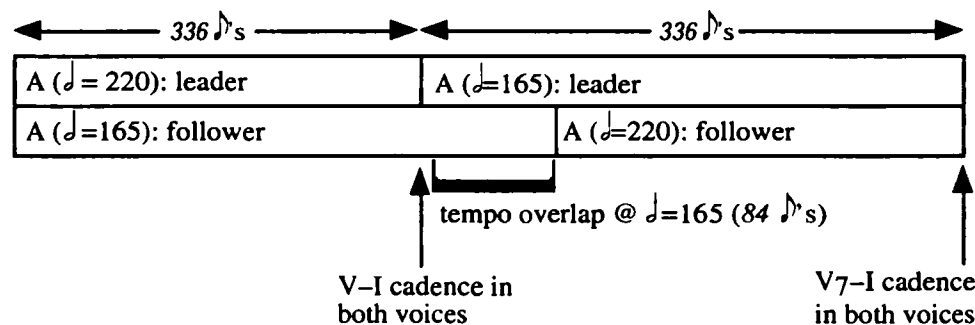


Figure 3-28. Structure of Study No. 15 ("Canon 3/4"), a diverging-converging canon in two voices.

Beginning:

Beginning of canon's repeat in top voice and of tempo overlap section:

V-I cadence in G built in 3/4 of way through canon

Beginning of canon's repeat in bottom voice and end of tempo overlap section:

Final convergence point:

Figure 3-29. Structural highlights of Study No. 15: cadential figures at beginning and end of canon statements in each voice.

Another important structural feature of diverging-converging canons is that each voice successively states every tempo, with each voice stating the tempos in a different order and each tempo uniquely associated with a canonic section that is the same length in each voice. This can be clearly seen in the structure of Study No. 17 ("Canon 12/15/20"), which was shown in Figure 3-7 (p. 118). The three sections are stated once in each voice, with the A section associated with the tempo $\text{♩} = 172.5$ and a duration of 4, the B section associated with the tempo $\text{♩} = 138$ and a duration of 5, and the C section associated with the tempo $\text{♩} = 230$ and a duration of 3. As shown in Figure 3-7, there are no tempo overlaps involving C, the shortest section. The longest section, B, has the longest overlap, with the A section having an overlap half that length.

The same principles are applied in Figure 3-30, which shows the structure of a conjectural four-voice diverging-converging canon and the associated tempo overlaps.

A = 3	B = 4	C = 5	D = 6
B = 4	A = 3	D = 6	C = 5
C = 5	D = 6	A = 3	B = 4
D = 6	C = 5	B = 4	A = 3

Figure 3-30. Conjectural example of a diverging-converging canon in four voices and tempos, where the tempo ratio is 10:12:15:20 and the duration ratio is 3:4:5:6. There are no tempo overlaps involving A (the fastest tempo), overlaps of $\frac{1}{18} + \frac{1}{18} = \frac{1}{9}$ of the total piece involving B, $\frac{4}{18} = \frac{2}{9}$ involving C, and $\frac{6}{18} = \frac{1}{3}$ involving D (the slowest tempo).

Study No. 17 does not exhibit as much harmonic convergence at section changes as No. 15 does, and, with almost continuous eighth-note motion in three different (and quite fast) tempos, it would be difficult to hear such coordination anyway. The CP at the beginning of No. 17 coordinates on a F major triad while the final CP features a cadence on A (see Figure 3-31). Throughout the piece, the A section always begins on the note C, the B section on A, and the C section on F. Nancarrow keeps to a limited number of frequently changing key areas (mostly F, A minor, C, G, and D, with their affiliated accidentals B $\flat$, F $\sharp$, C $\sharp$, and G $\sharp$), which allows some brief and random areas of harmonic convergence to occur. Two such areas are shown in Figure 3-32; the first example shows an area of F major in the bottom two voices where the middle voice begins section C; and the second example shows an area of A minor in all three voices at the point where section B starts in the bottom voice. The melodic construction is highly organic, and the majority of the melodic patterns are shown in these examples: the melodic movement is all in eighth notes in groupings of one to four notes, mostly monophonic with occasional single octaves, and a preponderance of movement by half steps.

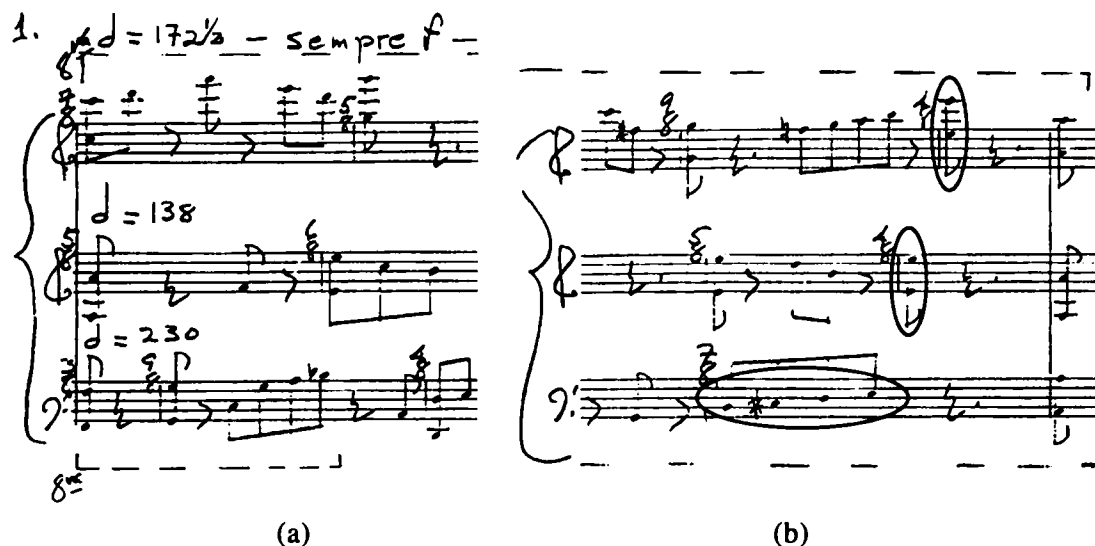


Figure 3-31. (a) Beginning and (b) ending (score, p. 12) of Study No. 17 ("Canon 12/15/20"). Circled notes in (b) are notes altered to create the cadence.

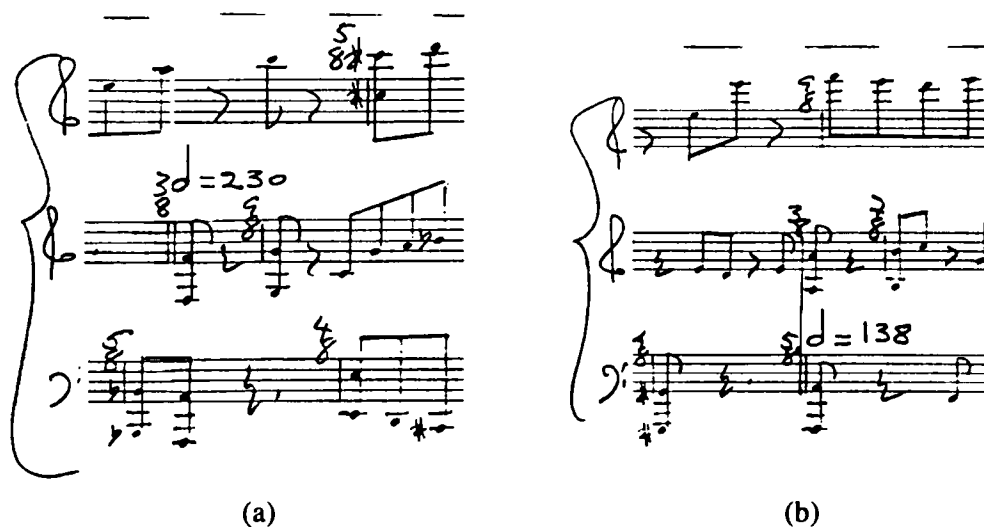


Figure 3-32. Brief areas of harmonic convergence in Study No. 17: (a) entrance of section C ($\text{♩} = 230$) in middle voice (p. 5); and (b) entrance of section B ($\text{♩} = 138$) in bottom voice (p. 7).

Several Studies (Nos. 24, 33, and 43) use internal tempo switches to create additional CPs. The structures of Nos. 33 and 43 are shown in Figure 3-27; No. 24's structure is shown in Figure 3-36 (p. 150). Study No. 43 ("Canon 24/25"), with its single tempo switch and two CPs, provides a simple structural framework for looking at the tempo

switch; its two CPs and the tempo switch are shown in Figure 3-33. The two voices of Study No. 43 are not only very close in tempo ratio (24:25) but also in their interval of imitation, a major third. Gann describes the proportions of the Study in relation to the CPs as follows (p. 219):

	Exposition	CP	CP	Dénouement
Convergence Periods*:	50	24	68.5	
Beats:	1198	588	1643	
*given in quarter notes				

Thus, the tempo switch occurs exactly halfway between the two CPs, after twelve convergence periods at a point where the echo distance is twelve quarter-note beats (this is shown by the arrow connecting the $\frac{6}{4}$ bars in the middle example of Figure 3-33). The timespan to the first CP is 1198 quarter-note beats, and the higher voice enters $\frac{23}{25}$ of the way between beats 47 and 48 in the lower voice. At the end of the piece, the lower voice drops out with 65.72 ($65\frac{18}{25}$) beats remaining in the higher voice.

As he did in Study No. 36, Nancarrow precedes the first CP with ascending movement. The second CP, which is actually a rest, is followed by descending movement in sixteenth notes (the most rhythmically active section of the piece). The CPs are used to set off a middle section in the palindromic structure of the piece; Gann describes that structure in this way:

This 24/25 canon is more nearly palindromic than No. 36; no extravagant post-CP event appears to throw the symmetry off balance. Instead, the textures and motives of the first half of the piece return in almost exact reverse order in the second half, though varied, some of them more elaborately developed, others shortened.

. . . No. 43 is palindromic in spirit, although there are no actual retrograde passages. (Gann 1995, 218)

The “palindromic spirit” and arch shape of the piece are also reinforced by increases and then decreases in note density and dynamics before and after the CP section. Gann’s generalization about the splintering of motives that become briefer as a CP approaches somewhat holds true here, as some of the motives prior to the first CP are

Approach to first CP (p. 14):

Tempo switch (p. 18):

Second CP (p. 21):

Figure 3-33. Score sections from Study No. 43 showing approach to first CP, tempo switch, and final CP.

subjected to rhythmic compression, and in the section leading up to the CP there is a countdown extending from 12 quarter notes down to 1.

The section between the two CPs consists entirely of a 3-note motive, expressing the first three degrees of the minor scale in broken octaves. This motive appears earlier, in measure 150 (all references to locations in the score will refer to the bottom voice) where the pitches are E_b -F- G_b . At the first CP (see Figure 3-33) both voices are stating the motive in their treble registers, with the bottom voice stating C-D- E_b while the top voice states E-F#-G. At measure 199, the bass register enters in the bottom voice on F-G- A_b and in the top voice on A-B-C, creating the hexachords F-G- A_b -C-D- E_b and A-B-C-E-F#-G which are suggestive of the Dorian mode. At the tempo switch, only the bottom tri-chord is being stated in each voice, and shortly thereafter tonal centers begin shifting, first every three measures to as often as every three beats. Five measures before the second CP, the original hexachords return (see second CP in Figure 3-33).

Due to the frequently changing meters in Study No. 43 there are few shared bar-lines, but there is one area about 40 measures before the first CP where “collective effects” are clearly illustrated within a single convergence period.⁸ As shown in Figure 3-34, a collective glissando begins in the bottom voice prior to the first shared barline, followed by a pattern of a chord, broken octave, and lengthy rest that are alternated between the voices. The second iteration of this pattern in the lower voice coincides with the beginning of the next convergence period.

Study No. 33, with its irrational tempo ratio of $\sqrt{2}:2$, presents special challenges in measuring and calculating distances and durations. The structure of the Study is shown in Figure 3-27. There are four distinct canons (which Gann labels A through D in the diagram, but refers to as Canons 1 through 4 in his narrative): Canon 1 is a diverging-converging canon with a tempo switch in the middle; Canon 2 is an arch canon with a CP

⁸The number of beats in each voice within this convergence period would seem to indicate a score error: there are 26 beats in the top voice to the bottom voice's 25, instead of the 25 and 24 that would be indicated by the tempo ratio.

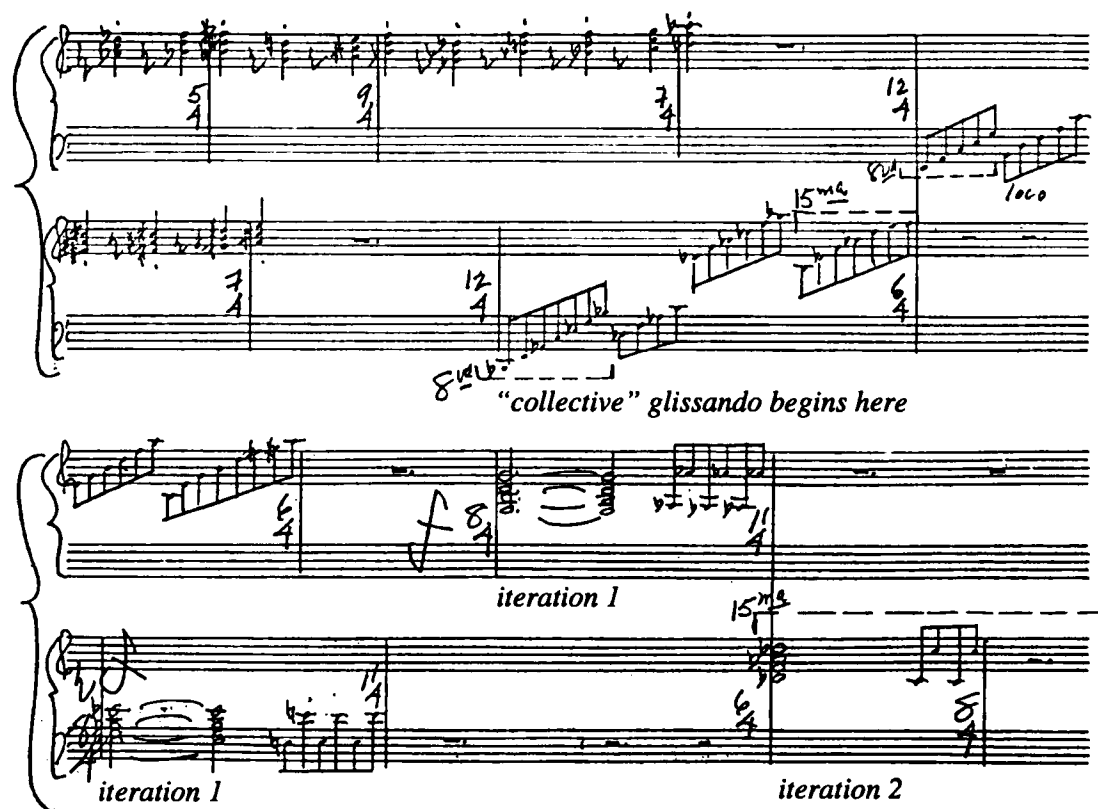


Figure 3-34. "Collective effects" in Study No. 43 (score, p. 11).

in the middle; Canon 3 is another diverging-converging canon; and Canon 4 is also a diverging-converging canon, but it begins with a short interlude prior to the canon's first CP. All told, there are seven CPs. Only the first one takes place on a note attack in both voices; most of the remainder occur either on held notes or rests. Even the final CP does not take place on a coincident beat attack. See Table 3-1 for a description of the canons and convergence points in Study No. 33.

Due to the irrational ratio, tempo switches do not occur at beat attacks. For instance, Canon 1 is 46 measures (184 quarter-note beats) in length and the tempo switch is notated as taking place near the beginning of measure 20 in the bottom (slower) voice and measure 28 in the top (faster) voice. A duration of 46 measures divided according to the ratio $\sqrt{2}:2$ results in a split of approximately 26.95 and 19.05 measures ($\sqrt{2}x + 2x = 46$; $x = 3.414$); thus the tempo switch actually takes place just prior to the completion of

measure 27 in the top voice and just after the beginning of measure 20 in the bottom voice.

Table 3-1

Description of Canons and Convergence Points in Study No. 33

Event		Description
Canon 1		diverging-converging canon, tempo switch in the middle
	CP 1	articulated, coincides with initial attack of Canon 1 in both voices
	CP 2	unarticulated, is the final cutoff of Canon 1
Canon 2		arch canon, no tempo switch
	CP 3	in middle of Canon 3; unarticulated, takes place in middle of measure on a held note—changing leader-follower relationship is clearly audible following the rest
Canon 3		diverging-converging canon, tempo switch in middle
	CP 4	at beginning of Canon 3; unarticulated, begins with a rest
	CP 5	closes Canon 3 and elides with the beginning of Canon 4 where it is articulated in bottom voice only
Canon 4		diverging-converging canon, begins with a brief interlude before CP 6; tempo switch between CPs 6 and 7
	CP 6	unarticulated, between two quarter note rests
	CP 7	unarticulated, occurs just after last note attack in lower voice

Figure 3-35 shows three of the unarticulated CPs in Study No. 33. CP 4 marks the beginning of Canon 3, and begins on a rest. CP 6 is technically the beginning of the diverging-converging canon in Canon 4, but it also occurs at a rest. CP 7 is an enigma—the final beat attacks in both voices are *almost* coincident, but not quite, as indicated by the lack of a shared barline at the final measure. The effect, then, is that the top voice finishes slightly ahead of the bottom voice to create a sort of V–I cadence in D_b. Unfortunately, it is impossible to calculate exactly where the final CP falls in the last measure. Due to a shared barline, the exact location of CP 6 is discernible, but because the exact measurement from CP 6 to the tempo switch between CP 6 and CP 7 is not known or calculable, it is not possible to use the measurement of that half of Canon 4 to

determine the full canon's length and project the exact ending point of the canon. If it were possible to calculate the exact measurement of the first half of the canon to the tempo switch, the exact location of CP 7 would be calculable. Thus its location can only be approximated.

Figure 3-35 shows three musical excerpts from Study No. 33, "Canon $\sqrt{2}/2$ ". Each excerpt consists of two staves. (a) CP 4: The top staff has a tempo marking $(\text{♩} = 2 \times 0)$ and a dynamic marking pp . The bottom staff has a tempo marking $(\text{♩} = 140 \times \sqrt{2})$ and a dynamic marking pp . An arrow points to the convergence point labeled CP. (b) CP 6: Similar notation with a pp marking. An arrow points to the convergence point labeled CP. (c) CP 7: Similar notation with a pp marking. An arrow points to the convergence point labeled CP. Handwritten annotations include $16x$, $14x$, and $8x$ in the bottom staff of (c).

Figure 3-35. Convergence points in Study No. 33 ("Canon $\sqrt{2}/2$ "): (a) CP 4, (b) CP 6, and (c) CP 7 (pp. 12, 25, and 52, respectively, of the score).

Study No. 24 ("Canon 14/15/15") has twelve canonic sections demarcated by thirteen CPs (see canon's structure in Figure 3-36). Each of the twelve canons is of the diverging-converging type, with a tempo switch between the outer voices (i.e., the fastest and slowest voices) in the middle of each canon; the middle voice carries the middle tempo throughout the piece. Canon 10, however, has a tempo overlap section in the middle before the tempos switch. Also, in addition to the tempo switches in each of the other eleven canons, an additional tempo switch is inserted at the beginning of Canon 9. Thus, there are twelve tempo switches and one tempo overlap in this Study.

Table 3-2 lists and describes the canons and CPs that occur in Study No. 24. As the table shows, the majority of the CPs elide the end of one canon with the beginning of

Study No. 24 - Canon 14/15/16

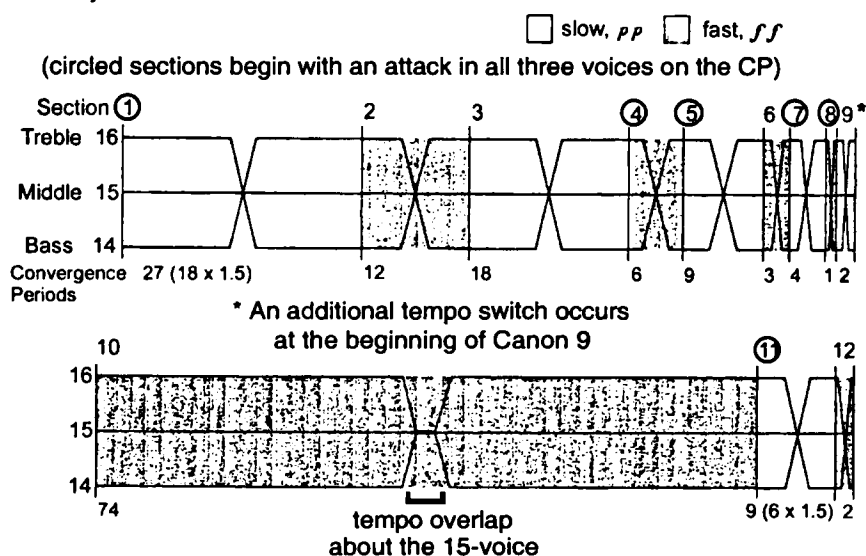


Figure 3-36. Structure of Study No. 24 ("Canon 14/15/16"; modified from Gann 1995, 23).

another; the exceptions are CPs 4, 7, and 10. The table also shows that CPs 1, 4, 5, 7, 8, 11, and 13 are articulated in all voices while the others are unarticulated (i.e., take place on rests or held notes).

CPs and tempo switches sometimes occur with startling rapidity in this Study, particularly in the area of Canons 6–9 where (as shown in Figure 3-36) CPs occur every 3, 4, 1, and 2 convergence periods, respectively. Figure 3-37 shows Canon 8 (the briefest canon, consisting of only one convergence period of 15 sixteenth notes) and the beginning of Canon 9. As mentioned earlier, an additional tempo switch is inserted at the beginning of Canon 9. Normally the leader-follower relationship between the top and bottom voices reverses at each CP; at Canon 9, however, the additional tempo switch allows the bottom voice to remain the leader for both canons.

At Canon 10 (by far the longest canon, comprising 74 convergence periods), Nan-carrow inserts a tempo overlap on the middle tempo ($\downarrow = 240$) that lasts for 60 sixteenth notes (four convergence periods). The tempo overlap allows for a rare rhythmic convergence to be created in all three parts by repeating the same rhythmic pattern three times, as

Table 3-2

Description of Canons and Convergence Points in Study No. 24

Event	Description
Canons 1–12	every canon is a diverging-converging canon; Canon 10 is the only canon which has a tempo overlap rather than a tempo switch in the middle
CP 1	articulated in all voices at beginning of Canon 1
CP 2	unarticulated (occurs at dotted quarter rest), elides with beginning of Canon 2
CP 3	unarticulated (occurs at half rest), elides with beginning of Canon 3
CP 4	articulated in all voices, Canon 3 extends one $\frac{9}{8}$ measure beyond this CP before Canon 4 begins
CP 5	articulated in all voices, elides with beginning of Canon 5
CP 6	unarticulated (occurs at eighth rest), elides with beginning of Canon 6
CP 7	articulated in all voices, occurs one measure <i>beyond</i> beginning of Canon 7
CP 8	articulated in all voices, elides with beginning of Canon 8
CP 9	unarticulated (occurs at quarter rest), elides with beginning of Canon 9 (there is also a tempo switch at this CP)
CP 10	unarticulated (occurs near end of long held note in all voices), precedes beginning of Canon 10 by five sixteenth notes
CP 11	articulated in all voices, elides with beginning of Canon 11
CP 12	unarticulated (occurs at seven eighth rests), elides with beginning of Canon 12
CP 13	articulated on last attack of Canon 12

shown in Figure 3-38. After this rhythmic convergence, the top and bottom voices switch to the tempo opposite the one they had before the convergence.

Study No. 37, whose structure is shown in Figure 3-39, is another canon with multiple CPs. Its twelve voices state the tempo ratio of the justly-tuned scale (see again Figure 2-1a, p. 66), which seems imposing but allows Nancarrow the ability to create widely-ranging textures and to combine the tempos in different groupings (comparable to

CP at beginning of section 8

tempo switch

"silent" CP at beginning of section 9

section 9 has tempo switch at beginning as well as in middle

Figure 3-37. Study No. 24 (portions of second and third systems of p. 12), showing CPs 8 and 9.

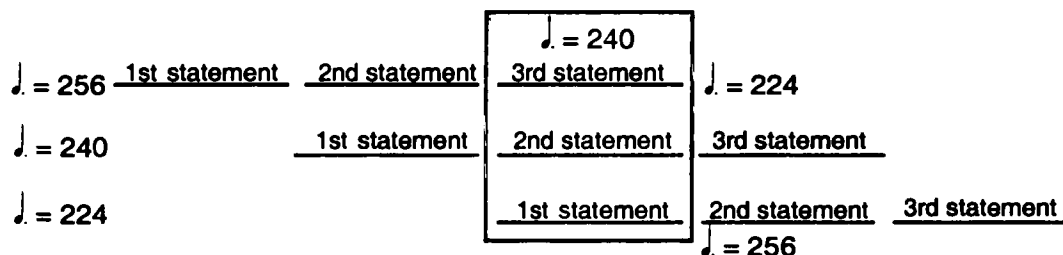


Figure 3-38. Structure of rhythmic convergence area in middle of Canon 10, Study No. 24.

chordal structures) that are no longer rigidly bound by his practice of assigning the slowest tempo to the lowest voice, etc. Gann identifies twelve canons and five CPs in this Study, and although Figure 3-39 does not very accurately portray this, different tempos are consistently applied to different voices. In Canon 1, the tempos are in order from fastest in the top voice to slowest in the bottom voice and expressing a rhythmic analogue of just intonation (see Figure 2-8, p. 79); in Canon 2, the order of tempos is reversed. In later sections of the Study, Nancarrow creates "chordal" groupings of tempos, such as in Canon 6 where the tempo order from bottom to top is 150 187½ 240 160⅔ 200 250

168 $\frac{3}{4}$ **210** 262 $\frac{1}{2}$ 180 **225** 281 $\frac{1}{4}$ (see Figure 3-40, left edge of score); as shown by the bold and italic type, the tempos are arranged into three interlocking “diminished seventh chord” groupings. Gann has plotted the tempo relationships of all of No. 37’s canons on a musical staff by converting each tempo to its pitch analogue, and compared this to the transposition levels of the voices (Gann 1995, 196), allowing one to see the considerable variety Nancarrow brings to the canonic process.

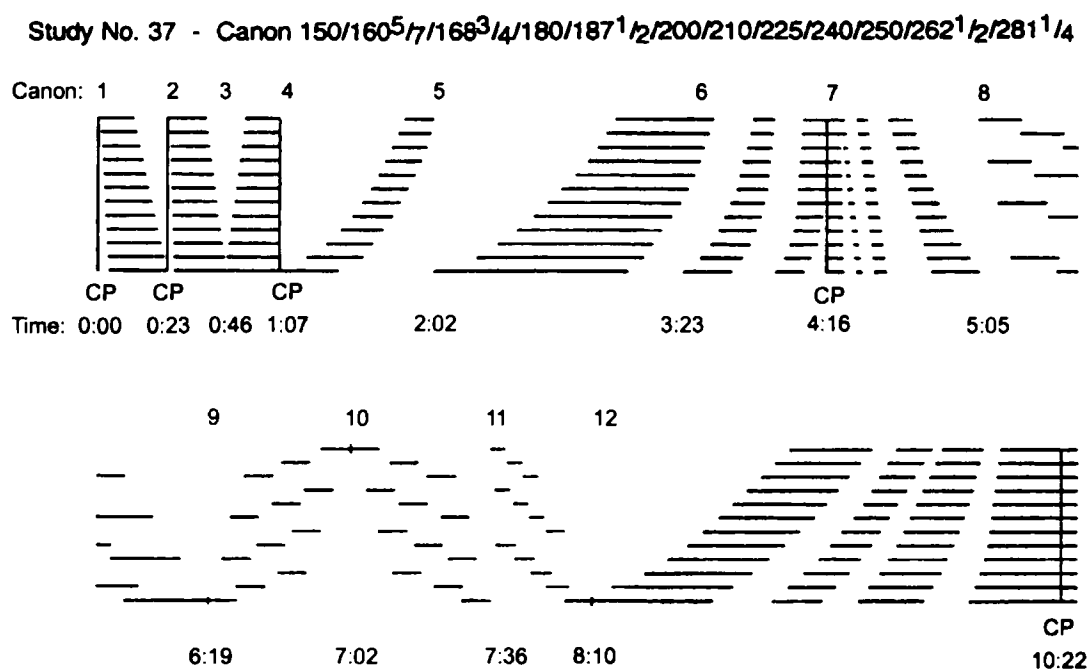


Figure 3-39. Structural diagram of Study No. 37 (modified from Gann 1995, 26), showing the twelve canons and five convergence points.

Convergences between voices are clearly important here as in many places Nancarrow has marked with vertical broken lines where entrances of new voices coincide with beats in other voices (something he could have marked in many other scores, but did not), even when the voices are widely separated in the score and the convergence is with an interior beat rather than a downbeat. These convergences between voices are more common than might be supposed from the unwieldy tempo ratios. In several of the later canon sections (e.g., canons 8, 9, and 10) where tempos are in “minor third” groupings, simpler tempo ratios such as 6:5 and 7:6 are common between voices. These middle and

later canonic sections, then, are more temporally consonant, which is a virtual necessity for perceptual purposes because the melodic material here is much more active than in the earlier sections.

Gann says about the convergence points in No. 37, “Convergence points are brilliantly de-emphasized in this study, for this is the work in which Nancarrow learned how to create beautiful effects with convergence points by *omitting* them” (p. 195). The CPs are not really omitted, of course, but unarticulated, as not one of the five CPs occurs at a note attack. CPs 1 and 2 occur on a rest, with the voices entering in a staggered fashion just after the CP. CP3 also takes place at a rest, just after the conclusion of canon 3. The most climactic CP is unquestionably CP4 at the beginning of canon 7 (Figures 3-39 and 3-40). This CP marks the most rhythmically active section of the piece and comes at a place where the texture thickens to all twelve voices after an extended section in canons 4, 5, and 6 where the voice saturation is considerably thinner. In comparison, the concluding CP5 is notably anti-climactic, occurring at a whole note in all twelve voices that is held over from twelve tied half notes and marked with a fermata.

The processes of acceleration and deceleration create special challenges for planning and placement of convergence points, and for this we look at Studies No. 8 and 22. Recall that Nancarrow used two varieties of acceleration: arithmetical and geometric (see pp. 10–11), where arithmetical tempo changes involve adding or subtracting the same invariable rhythmic unit to the previous value, and geometric changes involve changing note values by an invariable percentage. Arithmetical acceleration creates a constantly increasing or decreasing rate of change, but can be notated in conventional notation (see, for example, the “countdown” at the end of Study No. 12, Figure 1-7, p. 33); geometric acceleration creates a smooth continuum of changing speed that must be notated in proportional notation.

Although it predates the first tempo canons, Study No. 8, an early example of Nancarrow’s acceleration technique, makes considerable use of convergence points. Its

The image displays a handwritten musical score for CP4 from Study No. 37, page 40. The score is written on multiple staves, with a central vertical line indicating a change in the musical structure. The tempo markings are as follows:

- Top left: $J = 281\frac{1}{4}$
- Top right: $J = 281\frac{1}{4}$
- Second row left: $J = 225$
- Second row right: $J = 262\frac{1}{2}$
- Third row left: $J = 180$
- Third row right: $J = 250$
- Fourth row left: $J = 262\frac{1}{2}$
- Fourth row right: $J = 240$
- Fifth row left: $J = 210$
- Fifth row right: $J = 225$
- Sixth row left: $J = 168\frac{3}{4}$
- Sixth row right: $J = 210$
- Seventh row left: $J = 250$
- Seventh row right: $J = 200$
- Eighth row left: $J = 200$
- Eighth row right: $J = 187\frac{1}{2}$
- Ninth row left: $J = 160\frac{5}{8}$
- Ninth row right: $J = 180$
- Tenth row left: $J = 240$
- Tenth row right: $J = 168\frac{3}{4}$
- Eleventh row left: $J = 187\frac{1}{2}$
- Eleventh row right: $J = 160\frac{5}{8}$
- Twelfth row left: $J = 150$
- Twelfth row right: $J = 150$

Figure 3-40. CP4 from Study No. 37 (p. 40 of score). End of canon 6 is shown, where tempos are arranged in groupings analogous to interlocking diminished seventh chords. Canon 7 begins at CP4, which is a sixty-fourth rest at beginning of $\frac{4}{4}$ measure; a tempo switch in some voices also takes place at this CP.

first section, called by Carlsen the “trio” because it is in three voices, consists of canonic lines that alternately accelerate and decelerate within a series of nineteen durations, each of which consists of a pair of notes in a 2:1 rhythmic relationship (an eighth note followed by a quarter note with a sustain line—see Figure 1-3, p. 31); the 2:1 rhythmic pattern creates a loping effect that particularly elucidates the processes of acceleration and deceleration. Although the notation is proportional, the process of tempo change is arithmetical as an invariable background unit is continually added or subtracted to determine the next note values (see again pp. 48–51; note in Figure 1-22, p. 48, the two possibilities posed by Carlsen and Gann for metric notation in this section).

Nancarrow imposes strict order on the form of this section by bringing in each new voice at a convergence point as shown in Figure 3-41. As Carlsen notes, the main pitch in this section is G, and all CPs take place on G octaves or triadic intervals involving G. No two voices are ever stating the same rate of change at the same time, even if they are using the same tempo process. For instance, in the fourth phrase of this section, where both the first and second voices are in accelerating-decelerating patterns, the patterns begin and end at different places in the nineteen-duration series and also change speed direction at different places. One of the interesting effects of this procedure is that the echo distance between voices is constantly different, and not steadily increasing or decreasing as would normally be the case.

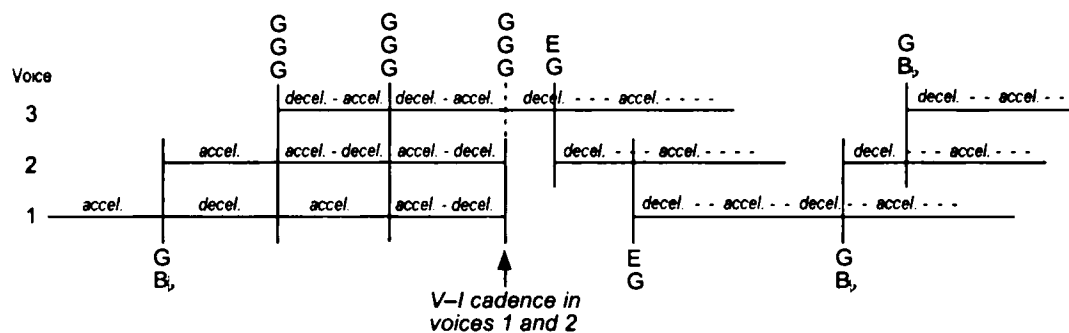


Figure 3-41. First section of Study No. 8, showing convergence points (vertical connecting lines), whether durational series are accelerating or decelerating, and pitches at CPs. (Diagram is not strictly proportional.)

Despite this variety in rate of change within the voices, Nancarrow is able to ensure convergences at regular time intervals by keeping the total number of background units the same in each voice within each phrase. There are also occasional near-convergences that pop out of the texture due to the use of the same limited number of durations within each voice; also, the short note in each durational pair casts such nearly-aligned attacks, when they do occur, in stark relief.

In contrast, Study No. 22 (“Canon 1%/1½%/2¼%”⁹), with its geometric acceleration, has only three widely-spaced convergence points (see Figure 3-27, p. 140): the first concludes Canon A, a converging canon; the second is in the middle of canon B, an arch canon; and the third begins Canon A’, a diverging canon. Study No. 22 is a strict palindrome in both rhythm and pitch; CP2, which takes place at a rest, is thus the rhythmic line of symmetry in the piece.

In Canon A, the first note of each voice is the same length (75 millimeters in the score’s proportional notation), but by the CP the different rates of acceleration (with the 2¼% line accelerating 125% as fast as the 1% line) have resulted in very different lengths for the last note before the CP: 7.5 mm. in the fastest voice, or a duration 10% as long as at the beginning; 17 mm. in the middle voice, or about 23% as long as at the beginning; and 30 mm. in the slowest voice, or about 40% as long as at the beginning. Thus, at the CP the fastest voice has sped up to four times the speed of the slowest voice. As a result, as the CP approaches one hears the exponential change in the echo distance between the voices.

Analysis of Study No. 27: “Canon 5%/6%/8%/11%”

This chapter concludes with an analysis of Study No. 27, one of two “acceleration canons” written by Nancarrow (the other being No. 22). Like Study No. 22, Study No. 27 has three widely-spaced convergence points. However, whereas No. 22 consists of three

⁹Each contiguous pair of percentages relates to the same ratio—3:2.

of the basic canon types (converging, converging-diverging, and diverging), Thomas notes that No. 27 is unlike any of the four basic canon types and instead “incorporate[s] the technique of gradually changing speeds, so that the relationship between the voices is constantly changing” (p. 13).

General Observations

The canonic material is primarily in four voices (sometimes doubled) representing the four acceleration/deceleration percentages of the subtitle. Due to the geometric acceleration technique, the Study is notated entirely in proportional notation. In the middle of the canonic texture is a line of non-varying tempo that is commonly referred to as the “clock line,” representing what Nancarrow referred to as the ticking of an “ontological clock” (Reynolds 1984, 9), or a time-keeping device against which one can detect the constantly changing speeds of the other voices. According to Jan Jarvlepp (1983/84, 219), Nancarrow considered the clock line to be an ostinato even though it seems to have no recurring pattern of pitch or rhythm. Thomas’s description of the clock line (refer back to Figure 1-15, p. 40) emphasizes its function as an ostinato:

It is presented in staccato articulations at intervals equivalent to either quarter or eighth notes. . . . The basic rhythmic and pitch material of this layer is limited, to be sure, but it is subjected to perpetual, random reorderings. As a result, it has no recurring patterns nor metrical implications. Nonetheless, the layer functions contextually as an ostinato because of its constant presence and its relatively steady character: although its articulations are not regular, its tempo is unchanging, while all the other layers in the study are continually subjected to *accelerandi* and *ritardandi*. (Thomas 1996, 14)

Figure 3-42 shows Gann’s formal diagram of Study No. 27. It identifies the relative positions of the clock line and the four canonic voices, with the percentage of acceleration or deceleration indicated for each voice. It also identifies Gann’s eight sections; these eight sections correspond to the changing textures of the clock line as shown in Figure 1-15 (p. 40). Section 1 contains four different canons (identified separately by Gann because the levels of imitation do not remain the same), and I have modified Figure

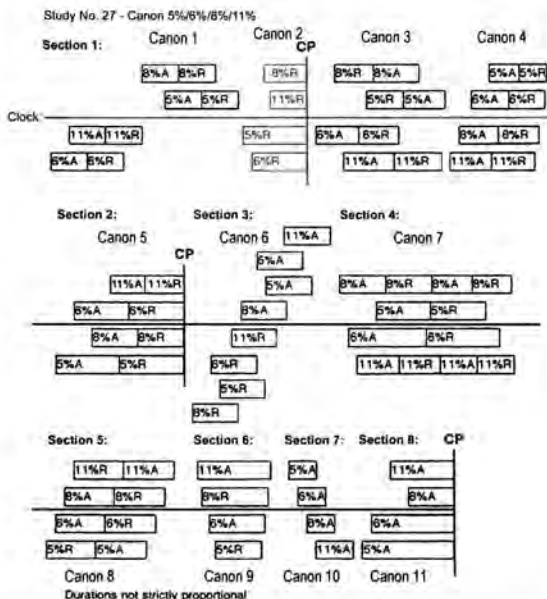


Figure 3-42. Formal diagram of Study No. 27 (modified from Gann 1995, 161). Percentages of acceleration (A) or deceleration (R, for *ritardando*) are given for each voice. Gann's diagram has been modified to show convergence points and the locations of the eleven canons; considerable adjustments have been made to voice entrances and endings to better reflect relative relationship of voices.

3-42 to identify these four canons; each of the other sections has one canon, for a total of eleven. CPs (at the end of Canons 2, 5, and 11) are also identified, and I have modified the diagram to more accurately reflect the relative relationships between canons and

where voices enter and drop out. As shown in the diagram, within each canon each percentage is uniquely affiliated with a specific voice that does not change within that canon; in some canons both acceleration and deceleration are used, and in some only one process is used.

Overall Structure

The importance of convergence points in this Study seems a matter for some disagreement. Gann asserts (p. 163) that “convergence point is not an important issue in this study.” Thomas, on the other hand, points to the first CP as a place where “the process of convergence is very plainly the point of this particular canon” (p. 135), and, indeed, it seems that with the kind of mathematical manipulations that were required to create this and the other CPs it could hardly be otherwise. If convergence is not the main point of this Study, it certainly is at least a significant one. The first two CPs involve the canonic voices but not the clock line, while the CP that concludes the piece involves all the voices.

Thomas characterizes this Study as “decidedly non-tonal” (p. 17), and, although like many of Nancarrow’s pieces it contains numerous tonal elements such as scales and triadic structures, there is little evidence of functional tonality or long-range tonal motion. These tonal elements tend to be used together in a manner suggestive of what Tenney refers to as “aggregates”—for instance, the seventh chords in the clock line in Canon 9 (see Figure 1-15, p. 40).

Carlsen points to this Study (along with Nos. 19 and 36) as one in which “It is significant that the exact midpoint on Nancarrow’s instruments is the note E_4 This has a direct bearing on the symmetrical structures Nancarrow sets up using every note on the instrument” (p. 5). The clock line itself, consisting of the pitches $D\#_4$, E_4 , F_4 , and $G\flat_4$, is almost evenly deployed about the piano’s central note, and thus it not only continuously marks time in the piece, but also constantly articulates the vertical axis about the

center of the player piano keyboard. Symmetries also exist in the levels of imitation within some canons, as was shown in Table 1-2 (p. 45). Figure 3-43 shows the levels of imitation in each of the canons; these intervals form vertically symmetrical structures in Canons 1, 2, 3, 4, 6, 8, 9, and 11, while Canons 5, 7, and 10 are not quite symmetrical.



Figure 3-43. The levels of imitation between adjacent canonic voices and order of entrances of the eleven canons of Study No. 27.

The first three lines of the ubiquitous clock line are shown in Figure 3-44. The

clock line consists of two rhythmic values related 2:1, with the longer value articulating the stated 220 tempo. Thomas points out that the clock line never approaches temporal consonance with the other voices (p. 134), and since the rhythmic values in the canonic voices are always changing there are only random, non-consecutive, cases where these values match the clock line's.

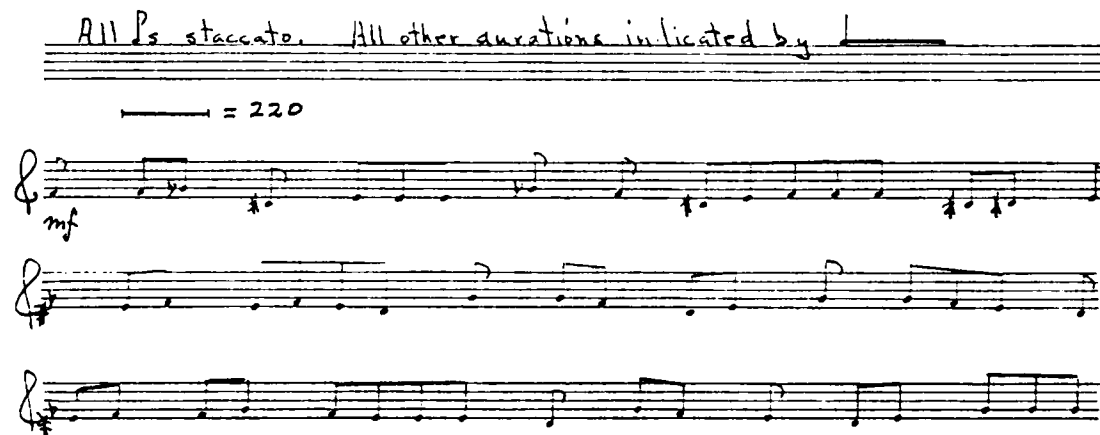


Figure 3-44. The first three lines of the clock line in Study No. 27, showing limited pitch content and proportional notation of the tempo.

All references to the clock line in the main analytical sources comment on its apparent total randomness of both pitch and rhythm throughout the piece, and any rhythmic or melodic patterns or non-musical associations that Nancarrow may have incorporated into this line are so well hidden as to be indecipherable. I even entertained the possibility that Nancarrow might have created the rhythm by using a system such as Morse code to encode a text in this line; however, in Morse code the duration of a dash relates to a dot by the ratio 3:1 and the system includes breaks equal to a dot between elements of a character, the length of a dash between characters, and the length of seven dots between words.¹⁰ Since the only relative durations in Nancarrow's clock lines are lengths of 2 and 1, this system could not have been used. Short repeated pitches appear fairly frequently in the clock line, with the first set of three repeated notes articulating E₄, the piano's central

¹⁰See www.soton.ac.uk/~scp93ch/morse/morse.html.

note. The longest series of short notes in the line is nine, and of long notes it is five. Despite the overall appearance of randomness in both pitch and rhythm in this ever-present line, it still seems likely that some type of patterning must have been involved in its creation. For one thing, as Pickover (1995) has noted, it is quite difficult for humans to generate truly random sequences:

Sit down at a computer keyboard . . . and randomly press the 1 and 0 keys. Try to make the string of numbers as “patternless” as possible. In other words, try to generate random numbers. *[Pickover then gives a 6-line example of 1’s and 0’s that he generated.]*

. . . I have tested dozens of supposedly random sequences typed by colleagues, and found out that it is curiously difficult for humans to type patternless sequences. To start with, we would expect about a 50 percent occurrence of each digit, and I did amazingly well. The [example I typed] had 49.3 percent 1’s and 50.7 percent 0’s. Next, we would expect 25 percent occurrences of the following pairs: 00, 11, 01, 10. In fact, doing my best to make a random sequence, I produced 15 percent, 13 percent, 36 percent, and 36 percent occurrences, respectively. Apparently my fingers preferred to oscillate rather than producing doublets such as 00 and 11. Perhaps I was trying to avoid clumpiness of digits, when in fact strings of identical digits should exist in a truly random sequence. (Pickover 1995, 233)

In creating the clock line, then, Nancarrow would not only have been faced with this difficulty in creating random rhythmic values, but randomness among the four pitches. Perhaps he created a 4 x 2 matrix (four pitches by two rhythmic values) and threw darts at it!

The percentages in the Study’s subtitle are used to create both accelerating and decelerating series of rhythmic values. Table 3-3 shows the multiplication factors needed to derive successive values of the series. The factors can be applied over a series of accelerating or decelerating values by using successive multiplication; for example, an accelerating series of twenty values (i.e., a series of nineteen value changes) at 6% that begins with a rhythmic value of 100 millimeters will end with a value of $100 \cdot 0.9434^{19}$, or 33.1 millimeters, while a decelerating series of twenty values at 6% that begins with a rhythmic value of 20 millimeters will end with a value of $20 \cdot 1.06^{19}$, or 60.5 millimeters.

Table 3-3

Multiplication Factors for Acceleration and Deceleration in Study No. 27

Percentage	Multiplication Factors	
	<u>Acceleration</u>	<u>Deceleration</u>
5%	0.9524	1.05
6%	0.9434	1.06
8%	0.9259	1.08
11%	0.9009	1.11

Sections of the Study

Canons 1 and 2. Study No. 27's first CP concludes the section consisting of these two canons. Canon 1 is one of only a few canons in the piece in which more than one basic rhythmic value is used, and in this way it forms a rhythmic parallel to the clock line. Canon 2 forms a pitch parallel to the clock line in that each of the four canonic lines, like the clock line, consists of four contiguous chromatic pitches. Both Canons 1 and 2 (as well as 3 and 4—see Figure 3-43) have symmetrical patterns of levels of imitation between the voices. In Canon 1, the first two voices (A6%/R6% and A11%/R11%) finish stating the canon before the other two voices enter; in Canon 2, the four voices enter in succession so that the canon is eventually being stated in four voices at once.

Thomas asserts that “the canonic and ostinato layers seem completely unrelated and unrelatable” (p. 133); however, in the first canon rhythmic values are sometimes halved and doubled to create a 4:2:1 rhythmic duration relationship, and this partially parallels the 2:1 relationship of the values in the clock line. Because both accelerative and decelerative processes are used in this canon, Jarvlepp (1983–84) astutely points out (p. 220) the simultaneous existence of three different kinds of tempo layers: steady tempo, accelerating, and decelerating.

The pitch content and relative rhythm of the lowest voice of Canon 1 are shown in

Figure 3-45. The first six notes—with the first three in the equivalent of quarter notes and the next three in eighth notes—along with the first rest, serve to set up a pattern that divides the line primarily into trichords. This trichordal division is maintained throughout much of the canon through rests and changing note values. The most prevalent pitch-class set represented is 3-4 [015], while sets 3-2 ([013], Carlsen’s “partitioned minor third”), 3-3 [014], and 3-9 [027] are presented three times each; melodic movement by half steps and P4/P5 prevails. The canon’s concluding pentachord, 5-Z18, is a subset of both the opening hexachord 6-Z10 and the hexachord that precedes it, 6-Z11.

Figure 3-45. Canon 1 of Study No. 27 in the lowest (A6%/R6%) voice, in rhythmic notation showing relative rhythmic values.

Figure 3-46 shows a section from Canon 1 where the canon is in two voices. This passage is a rare instance in Study No. 27 where the voices are close enough together in stating the canon that the echo distance is audible. There are two leader-follower switches on this page, and these are made more clearly audible because of the rests that accompany the switches. The passage is especially interesting because of the symmetry of the intervals between the voices at these points.

Canon 2’s pitch content is, like the clock line, limited to four contiguous chromatic pitches, as shown in Figure 3-47. The canon concludes with a CP on the attack of the

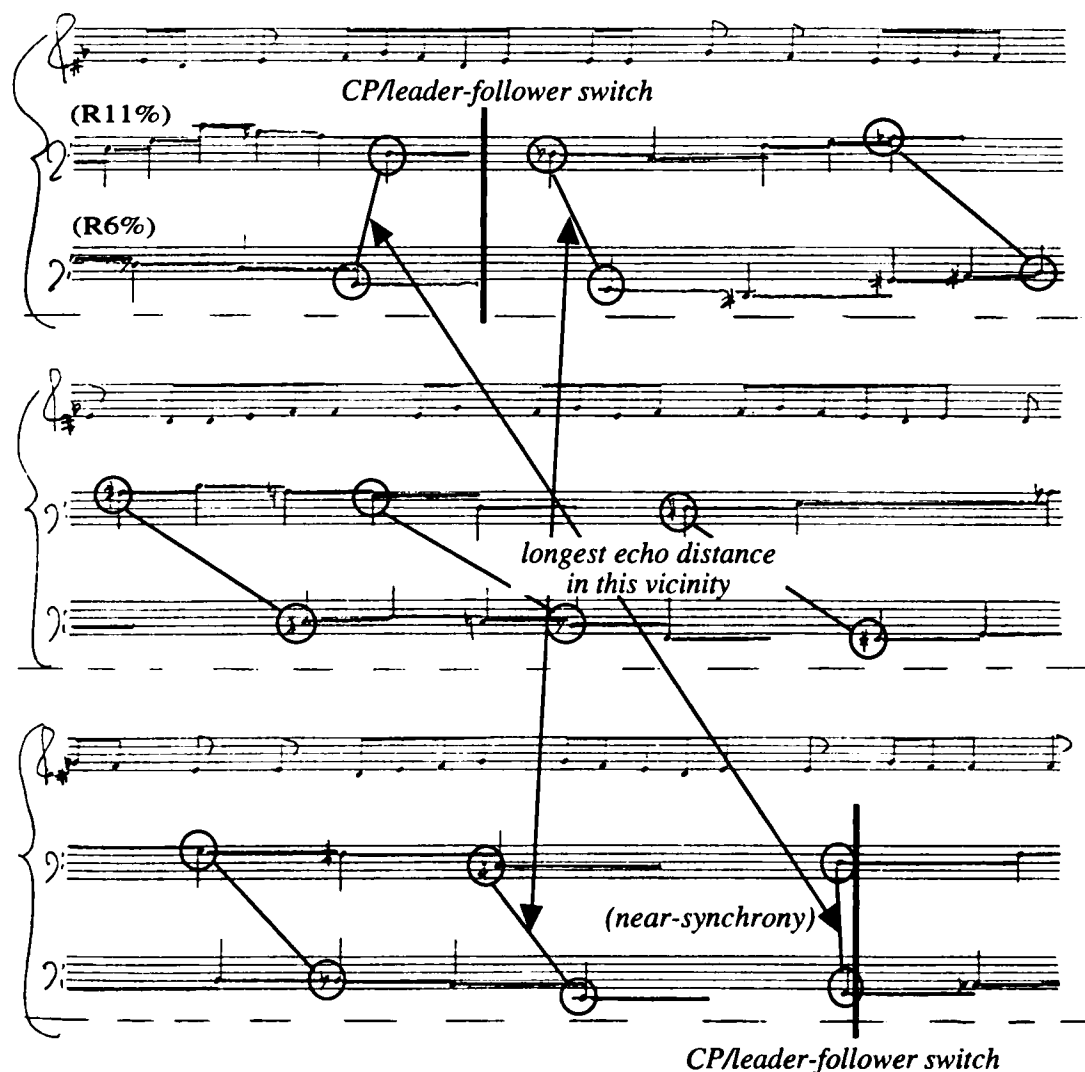


Figure 3-46. Section from Canon 1 of Study No. 27 (p. 3), showing two inaudible CPs and their leader-follower switches, pitch symmetry about these switches, and near-synchrony at second switch. (Rests in bottom staff of middle and bottom systems have been added after notes F_1 and E_1 as the omission of these rests were errors in the score.) The longest echo distance is reached halfway between the two leader-follower switches; selected echo distances are shown by circled notes connected with a line.

final pitch, which is the same as the first pitch of the canon's pitch cluster (a minor 7th chord on E); these pitches, as shown in the figure, create a symmetrical pitch structure about the $D\#$ in the clock line.

Thomas finds the CP at the end of this canon to be remarkable in its swift movement from extreme temporal dissonance to consonance:

What is so remarkable about this canon is its single-minded pursuit of convergence, and its rapid attainment of it. Within the span of approximately eighteen seconds the four related but temporally clashing canonic voices resolve their dissonance through their convergence upon a simultaneity, which represents consonance. Because the voices not only start at different speeds but also proceed at different rates of change, the convergence is dramatically faster than in a tempo proportion canon: the pace of the process itself is magnified through the use of gradually changing speeds. (Thomas 1996, 135–36)

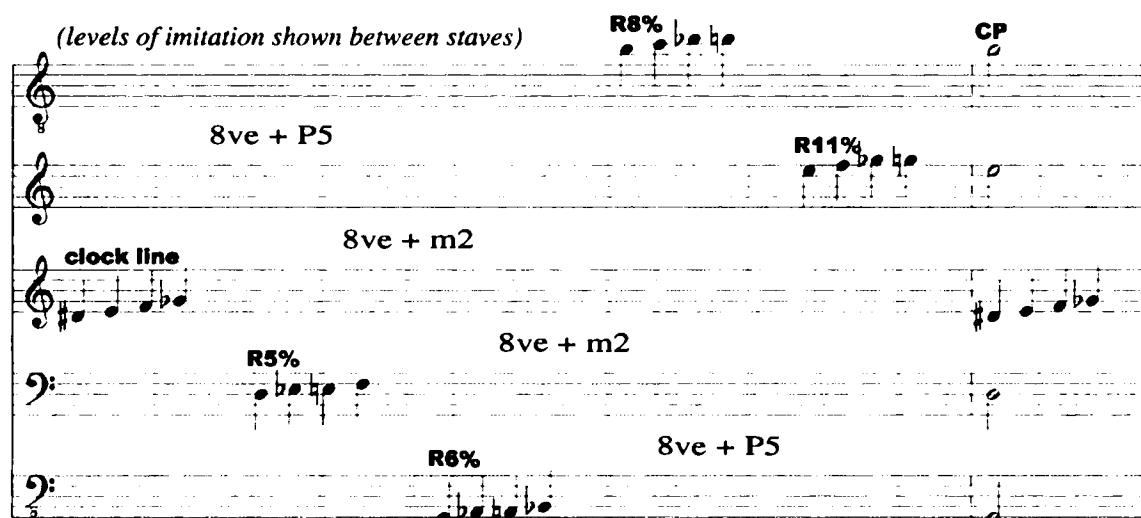


Figure 3-47. Pitch content of Canon 2, Study No. 27. Chord at CP is symmetrical about the D# in the clock line.

As Thomas notes, the four voices in the canon begin at “different speeds” (i.e., with different durations); however, the length of the final duration (the CP) is the same in all voices. Table 3-4 reports the actual and calculated measurements in millimeters of the twenty-three decelerating pitches in Canon 2; the final note in the series represents the CP, which occurs at the attack of the final note in the canon. Within a certain amount of tolerance, there is generally good agreement between the actual and calculated measurements. The canon’s final pitch, the CP, is 108 millimeters long in all four voices, and this value happens to relate to only one of the voices: it is roughly double the value in the series at that point in the 5% voice.

It is interesting to speculate how the CP at the end of Canon 2 might have represented a greater achievement in convergence had the opening values in *each* voice been

Table 3-4

Calculated and Actual Duration Measurements in Canon 2 of Study No. 27

Rit. 5%		Rit. 6%		Rit. 8%		Rit. 11%	
<u>Calc.</u>	<u>Actual</u>	<u>Calc.</u>	<u>Actual</u>	<u>Calc.</u>	<u>Actual</u>	<u>Calc.</u>	<u>Actual</u>
18.7	18.5	13.2	12	8.3	7.5	4.1	3
19.6	18.5	14.0	13	9.0	8	4.6	4
20.6	19.5	14.9	14	9.7	8.5	5.1	4
21.6	20.5	15.8	15	10.5	10	5.7	4.5
22.7	22	16.7	16	11.3	10.5	6.3	5
23.8	23	17.7	17	12.3	11	7.0	6
25.0	25	18.8	18	13.2	12	7.7	7
26.3	25.5	19.9	19	14.3	14	8.6	7
27.6	26.5	21.1	19.5	15.4	15	9.5	9.5
29.0	28	22.4	22	16.7	15.5	10.6	8.5
30.4	30	23.7	22.5	18.0	16.5	11.7	10.5
31.9	31	25.1	25	19.4	19.5	13.0	12.5
33.5	32.5	26.6	25.5	21.0	19	14.5	13
35.2	35	28.2	27	22.7	22	16.1	14.5
37.0	36	29.9	29.5	24.5	24	17.8	16.5
38.8	38.5	31.7	30.5	26.5	25.5	19.8	18.5
40.7	41.5	33.6	33	28.6	27.5	22.0	21
42.8	41.5	35.6	35	30.9	30.5	24.4	24
44.9	45	37.8	37	33.3	33	27.1	26.5
47.2	45.5	40.1	40	36.0	35.5	30.0	29.5
49.5	51	42.5	42.5	38.9	41	33.3	34.5
*52	52	*45	45	*42	42	*37	37
(54.6)	108	—	108	—	108	—	108

Calculation of opening values required in all voices to result in final value of 108:

18.7	—	15.2	—	10.0	—	5.5	—
—	108	—	108	—	108	—	108

Score measurements are to the nearest half millimeter.

Circled measurements are taken over a system break and more subject to inaccuracy.

*These are actual measurements from which the increasingly smaller values are calculated (5% values multiplied by 0.9524, 6% values by 0.9434, 8% values by 0.9259, and 11% values by 0.9009). The largest measurement was used to calculate smaller values because it would be less subject to inaccuracy.

such that 108 was double the value in the series at the CP. The bottom part of Table 3-4 shows the calculated values for opening durations that *would* have resulted in this “durational convergence” on the value 108 at the CP. The resulting values in the 6%, 8%, and 11% voices are closest to the values that occur in the third note of each series (shaded area in table). It would have been a simple matter for Nancarrow to begin the 6%, 8%, and 11% voices with these values in order to facilitate this durational convergence, and it is unclear why he did not do so.

Canons 3 and 4. These two canons conclude the first section, in which the clock line is still iterating single pitches (see Figure 1-15, p. 40). In Canon 3, the voices above the clock line (8% and 5%) first decelerate and then accelerate, while the voices below the clock line (6% and 11%) do the opposite. In Canon 4, each voice first accelerates and then decelerates, with the fastest-changing voice (11%) at the bottom and the slowest (5%) at the top.

Figure 3-48 shows Canon 3’s 23-note canon as it appears in the first voice (A6%/R6%). As Gann notes (p. 160), the canon has a narrow pitch focus of a major sixth. The canon’s opening contour is similar to that of Canon 1, and there is a fairly strong implication of first E major and then B minor in the last half of the canon.

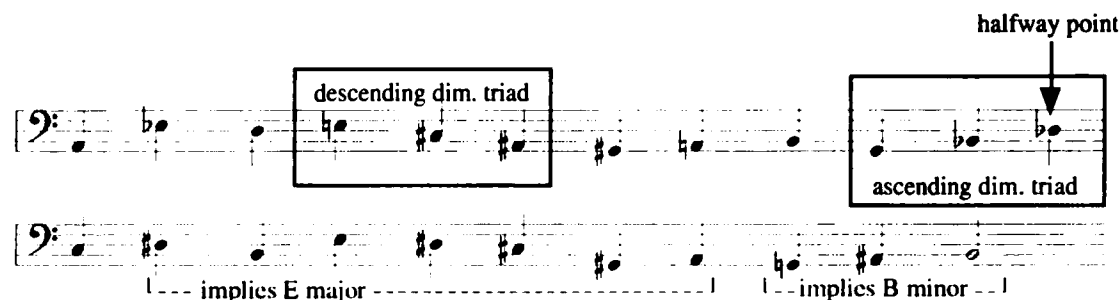


Figure 3-48. Canonic line of Canon 3, Study No. 27.

The halfway point of the canon, the twelfth note, is where the A6%/R6% voice is notated to switch from acceleration to deceleration, but in the score the R5%/A5% and

A11%/R11% voices are marked to switch on the note prior to this, and the R8%/A8% voice on the note following the halfway point. Besides the fact that this seems a violation of the spirit of the canonic process, there are other reasons to believe that all the voices should be marked in the score to switch at this halfway point. Based on measurements in the score, it appears that the tempo change process occurs as follows: ten duration changes take place in one direction (either accelerating or decelerating), with the duration of the eleventh note used again for the twelfth note, followed by ten duration changes in the opposite direction, with the final (23rd) note a longer duration which is apparently unrelated to the prevailing duration series. If the switch takes place on the middle note and the same number of duration changes is taking place on both sides of this point, the opening value and the value preceding the long concluding note should be roughly the same. Indeed, this does seem to be the case with all four voices, leading to the reasonable conclusion that the switch in each voice should be marked at the twelfth note of the canon.

Canon 4 unites an organic principle of organization in both pitch and rhythmic duration, organizing both around five-note groupings separated by barlines. The melody's five-note pattern is a sort of major/minor upper trill, including a total of 24 groupings and 16 iterations of the trill figure as shown in Figure 3-49; groupings of one to four of these "trills" are followed by a single short note. Fourths/fifths and triadic structures appear to

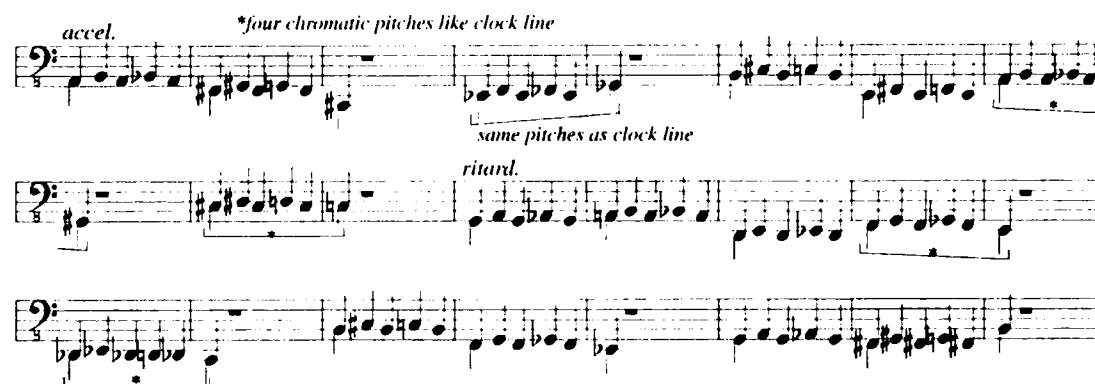


Figure 3-49. Canon 4 of Study No. 27, from lowest voice (A11%/R11%).

dominate the melodic movement between the first notes of each “measure”; the trill in the fourth and fifth measures spells out the clock line’s tetrachord, and several other groupings include the chromatic tetrachord. The deceleration portion of the canon is two groupings longer than the acceleration portion.

Like the pitch changes, the rhythmic durations of this canon change with each five-note grouping. Figure 3-50 shows a representative portion of the score and duration measurements from this section.

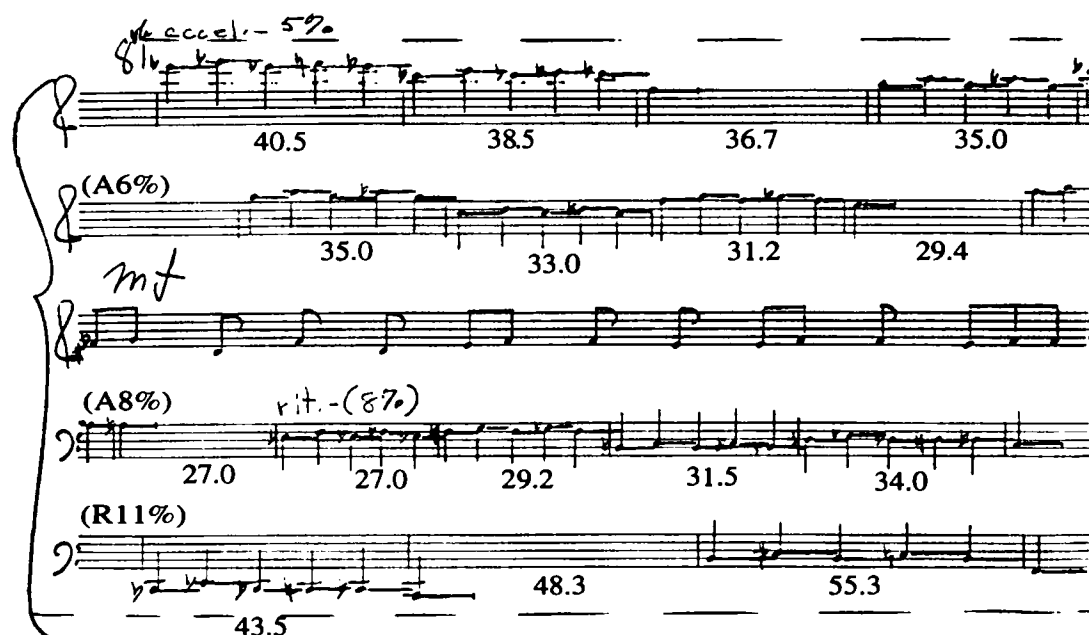


Figure 3-50. Segment of Canon 4, Study No. 27 (p. 16, second system), at entrance of 5% voice. Measurements from the score are given in millimeters for each five-note grouping (the example is reduced from the score).

Canon 5. Canon 5 introduces two related texture changes: the canon includes many half-step trills, and the ostinato line is reinforced by octaves and registrally divided with the bottom half-step pair (D#/E) in the bass register and the top pair (F/G_b) in the treble. Like Canon 4, this canon’s melody is arranged with variable groupings of trills followed by short notes.

Figure 3-51 shows a passage from Canon 5 where there is a near temporal

convergence among the 11% and 8% voices and the ostinato. The switch to “rit.” is almost simultaneous in these voices (although in the 11% voice the switch is marked one measure earlier than in any of the other voices). At the ritardando, the 11% voice’s tempo approximates 880 and the 8% voice’s 440, while the ostinato is maintaining its steady 220 tempo, for an overall tempo ratio of nearly 4:2:1.

The image shows a handwritten musical score for Canon 5 of Study No. 27. It consists of five staves. The top staff is marked with "rit. (11%)" and "R6%". The fifth staff is marked with "rit. (8%)" and "R5%". The score includes various musical notations such as notes, rests, and dynamic markings. The notation is dense and includes many trills and ornaments.

Figure 3-51. Passage of near temporal convergence in Canon 5 of Study No. 27 (p. 22, second system). At R11% (top line), tempo is ca. 880 while at R8% (fifth line), tempo is ca. 440.

Canon 5 concludes with the Study’s second CP, with the convergence taking place at the release of the final note. The convergence at the note release is made the more striking by the sudden change in dynamic from *f* to *p* and the change back to a single note texture in the clock line immediately following the CP. The final values in each voice at the CP are not equivalent and thus not a “durational convergence,” but in each voice the durations of the notes at the CP appear to be close to the next value in the deceleration series taking place at that point (although in some places, particularly in the 11%

voice, Nancarrow's score measurements do not match the series very well).

Canon 6. At Canon 6, the ostinato line returns to its original single-note texture, and the dynamic level drops from *f* to *p*. As shown in Figure 3-42, each voice first decelerates in the bass register below the clock line, and then, in tempo, leaps five octaves plus a minor sixth to the treble register above the clock line for its accelerative portion. This idea of registral separation is an extension of the texture used for the clock line in Canon 5.

Canon 6's melodic line is shown in Figure 3-52. Thanks to the registral separation within the canon, it covers the widest range in the Study—in fact, the lowest note of the lowest voice (8%) occurs at the last note of the ritardando portion and is the lowest note on the player piano keyboard (B_0), and the highest note of the highest voice (11%) is at the end of the acceleration and is the piano's highest note, A_7 . Nancarrow has thus included a rough parallel between frequency and speed—the slowing pitches sink to the lowest register and the accelerating pitches climb to the top of the keyboard. Combined with the very close intervals of imitation ($M2 - m2 - M2$), this texture is an extreme contrast to any heard in the Study thus far.

rit. (31 notes)

retrograde inversion of last 13 notes

accel. (32 notes)

8va-

lowest note on keyboard *

8va-

retrograde inversion of first 13 notes

* interval between bass and treble portions = 5 8ves + m6

highest note on keyboard (A_7) in highest canonic voice (11%)

Figure 3-52. Canon 6 of Study No. 27 in the lowest (R8%/A8%) voice.

The canon opens with a chromatic descending line in which the intervals progressively widen (five m2's, two M2's, three m3's, and a M3) in a manner reminiscent of the overtone series in retrograde. The first and last thirteen notes of the canon are the same melodic series in retrograde inversion; between these extremes, melodic movement again favors intervals of fourths and fifths. This canon's use of retrograde inversion at the beginning and end of the melody and the use of the entire range of the keyboard highlight the significance of Canon 6 as the midpoint of the Study.

The tempo change scheme in Canon 6 is very straightforward, and each voice begins and ends with the same durational value.

Canon 7. Canon 7's melody consists of sustained and staccato notes, all of which remain in tempo. The texture thickens dramatically in this canon. The clock line expands to major and minor thirds, and the canonic lines are doubled five octaves apart—a simultaneous (vertical) expression of the registral separation idea that was sequential (horizontal) in Canon 6. And, like Canon 6, which covered the entire range of the player piano keyboard, Canon 7 covers almost the entire range (minus the very top note). The extreme pitches, the B₀ in the lowest octave of the 11% voice and the G₇[#]/A₇_b in the highest octave of the 8% voice, are reiterated over and over again, particularly in a passage of compound melody near the middle of the canon. The compound melody consists of the extreme registral pitch plus a descending chromatic line; these two extreme lines of compound melody are shown in Figure 3-53. Note that the extreme repeated pitch is a long note and the other notes are staccato during the chromatic descent, while the repeated pitch is a staccato note (and its spelling changes) and the others sustained during the chromatic ascent section.

As shown in Figure 3-42, the tempo change pattern in the 8% and 11% voices is accel.—rit.—accel.—rit., while in the 5% and 6% voices it is just accel.—rit.; this is the only canon in this piece in which a different number of changes takes place in different voices.

persistent repetition of high A/G#

compound melody consisting of repeated high pitch and chromatic descent and ascent through a tritone

alignment

persistent repetition of lowest note on keyboard

chromatic tritone descent (all staccato notes)

compound melody with chromatic descent to lowest note

compound melody with chromatic descent to lowest note

Figure 3-53. Portions of 8% and 11% voices, Canon 7 of Study No. 27 (pp. 32–34 of score), showing compound melody consisting of repetition of extreme pitches and chromatic descent/ascent. (Both lines are doubled in the score—the figure shows the registral extremes: the top line of the 8% voice and the bottom line of the 11% voice.)

The 98 pitches of the canon are divided differently in each voice between acceleration and deceleration, with the pitches divided as follows:

8% voice: A = 36 R = 34 A = 17 R = 11 / = 98 total

5% voice: A = 46 R = 52 / = 98 total

6% voice: A = 49 R = 49 / = 98 total

11% voice: A = 24 R = 27 A = 25 R = 22 / = 98 total

Canon 8. From here to the end of the piece, the percentages stay in voice order: first

from slowest change (5%) in the bottom voice to fastest change (11%) in the top voice in Canons 8 and 9, reversing in Canon 10, and reversing back in Canon 11.

This canon's main textural idea is the minor/major triad. The clock line here expands to major and minor triads: the $D\#_4$ and E_4 are the fifths of the $G\#/A_b$ minor/major and A minor/major triads, while the F_4 and $F\#/G_b$ are the roots of F minor/major and $F\#/G_b$ minor/major triads, forming a complex of eight triads in the clock line. The minor/major triad is also spelled out by the levels of imitation (B , D , $D\#$, and $F\#$ —see Figure 3-43). Additionally, occasional half-step trills take place in the canon melody on the third of the triad to create alternate minor and major triads (except for the final trill, which alternates between the major third and the fourth scale degree).

Figure 3-54 shows the canon melody from the bottom line ($R5\%/A5\%$). Canon 8 contains two rhythmic values in the ratio 2:1, a feature that was previously used only in Canon 1. The lowest note on the keyboard, B_0 , that was prominent in both Canons 6 and 7 is just as much so here: the lowest voice enters first, on a B major triad built on this note that is repeated twice more. The melody in the lowest voice is tonally organized around B , with the first section (to the point where the tempo direction changes) concluding on a supertonic triad, which near the middle of the second half of the melody

Figure 3-54. Melody of Canon 8 from lowest voice, Study No. 27, rewritten in rhythmic notation.

becomes a minor/major triad and alternates in function between the supertonic and the V/V, finally becoming (enharmonically) V/V moving to V at the very end. In the highest voice (11%), the keyboard's highest note A₇ is stated four times.

Canon 9. Canon 9 is rhythmically the most complex canon, encompassing note values related by the ratio 4:3:2:1. Figure 3-55 shows the canon melody as it would appear in rhythmic notation in the lowest voice; the treble and bass registers in each voice are rhythmically coordinated but there is never a simultaneous note attack between them. The trill in some triads in the treble register involving the chord third and the fourth degree is retained from the end of Canon 8.



Figure 3-55. Melody of Canon 9 from lowest voice, Study No. 27, rewritten in rhythmic notation.

The clock line in Canon 9 states various kinds of seventh chords, again with D₄ and E₄ at the top of the chord structures containing those notes, and F₄ and F₄/G₄ at the bottom of chords containing them. The levels of imitation in Canon 9 are the same as the pattern in the clock line: m2 – m2 – m2 – m2, descending in the bass from the highest voice's D₁ to the keyboard's lowest note, B₀, in the lowest voice, thus filling out the chromatic tetrachord below the clock line. The most extreme high pitch in the treble of the

highest voice is A_{b7} , just below the top of the keyboard.

Like an arch canon, Canon 9's voices are placed almost symmetrically about the durational center of the canon, but, unlike the arch canon in the middle of Study No. 22, the symmetrical placement of the voices here does not allow for a convergence in the middle because of the constantly changing tempos.

Canons 10 and 11. The final two canons are each very brief and both use only acceleration. Canon 10's 21-note melody is an ascending chromatic scale extending an octave plus a minor sixth, against a clock line that has returned to its original form of four chromatic pitches. The intervals of imitation progressively increase by a minor second: the interval between the top two voices is a perfect fourth, between the second and third voices an augmented fourth, and between the third and fourth voices a perfect fifth (see Figure 3-43). The beginning pitches of the four voices— $C\#$, $G\#$, D , and G —add two notes above and below the chromatic space occupied by the clock line, and the A that concludes the first voice and the B_b that concludes the third voice add two more pitches to the chromatic space.

At the conclusion of Canon 10, both the clock line and the lowest voice of the canon conclude on the note E_b , which is then expanded to four octaves and repeated four times in the clock line; the clock line states only the major second E_b – F in Canon 11 (with the exception of the final two notes—see Figures 3-42 and 3-43).

Canon 11 is the most tonally-oriented canon in the Study; all the intervals of imitation are octaves, and the pitch content is entirely G major. The major second that is being expressed in the clock line is imitated in the canonic lines, where every note is expressing a major second trill (thus, no trills involving the intervals B – C and $F\#$ – G appear in the canon). The final CP (see Figure 3-56), unlike the first two, incorporates the clock line and occurs on the final staccato attack; thus there is no opportunity for a “durational convergence” as there was in the first CPs.

The image displays a handwritten musical score for the conclusion of Study No. 27, page 55. The score is organized into two main systems, each containing four staves. The first system's staves are labeled from top to bottom as: clock line (two staves), A11%, A8%, A6%, and A5%. The notation includes various musical symbols such as notes, rests, and dynamic markings. The second system follows a similar structure with four staves, also labeled from top to bottom as: clock line (two staves), A11%, A8%, A6%, and A5%. The handwriting is clear and legible, with some annotations above the staves.

Figure 3-56. Conclusion of Study No. 27 (p. 55). Order of voices from top to bottom of each system is: clock line (two staves), A11%, A8%, A6%, and A5%.

Summary and Conclusions

In Study No. 27, Nancarrow was able to express the same ideas in several different contexts, sometimes sequentially and sometimes simultaneously. The clock line's chromatic tetrachord, for instance, is manifested in the pitch content of individual voices in Canon 2, in the opening pitches of the canonic voices in Canon 6, and in the levels of imitation in Canon 9. As another example, the concept of registral expansion moves through three consecutive canons: first in the clock line in Canon 5, then in the canonic voices of Canon 6, and then a simultaneous (vertical) expression of this idea in Canon 7.

Thomas found Study No. 27 to be "decidedly non-tonal" (p. 17), but several tonal areas do achieve some significance. The final tonal goal is clearly G, the note immediately above the chromatic tetrachord of the clock line; G major is stated unequivocally in all the voices of Canon 11 and this canon is a shocking contrast to the tonal aimlessness and ambiguity of the rest of the Study. G is established almost immediately in Canon 1, being the first pitch in the lowest voice, and the lowest voice in Canon 2 begins on this same pitch. By the conclusion of Canon 4, however, with its F#–B final interval in the lowest voice, B plays a more prominent role—particularly with the emphasis on B₀ in the bass, which reaches a peak in the compound melody section of Canon 7. The filling out of the chromatic tetrachord below the clock line (D, C#, C, and B) in the bass register of the four canonic voices of Canon 9 provides a final emphasis on B before a G tonal center is reasserted in Canons 10 and 11.

Study No. 27's convergence points do little to support the establishment of these tonal centers. Each of the CPs is structured differently. The first two CPs involve only the canonic voices, take place at the end of a decelerating section, and end with sustained notes; the first CP occurs at the beginning of this note, and the second CP is at the end of the sustained note. The first CP has a durational convergence on a value related to only one of the voices while the second CP's final sustained note is of different durations and

represents the next series value in each voice. The Study's final CP, which concludes the piece, takes place in all the voices (including the clock line) at the end of an accelerating passage on the attack of the final staccato note; this CP also includes a pitch convergence on G.

It is somewhat surprising to note in Study No. 27 how little control Nancarrow seemed to exert in areas where he had typically done so in many of his other tempo canons: for instance, in the establishment of tonal centers, areas of harmonic convergence, and various kinds of symmetry. There is also very little in this Study that could be considered a "collective effect" other than the similar gestures in the canonic voices; these gestures are generally quite disjointed and rarely achieve a level of interdependence except at the CPs.

When considering both Studies No. 22 and 27 together, it is apparent that Nancarrow had far less success in these two geometric acceleration canons in creating collective effects, symmetry, and harmonic convergence than he did in a piece such as Study No. 8, which is based on arithmetical acceleration. The arithmetical acceleration technique allows for a common background unit, while geometric acceleration does not. Without the background unit, it is nearly impossible to overcome the independence of the voices to create any sense of interdependence. Nancarrow must have felt this, and one is left to wonder how satisfied he was with the results of the acceleration canon technique when considering Gann's observation (p. 163) that Nancarrow wrote no more acceleration canons after Study No. 27.

CHAPTER 4

FRACTAL FORMAL FEATURES IN THE TEMPO CANONS

In addition to convergence points, another interesting feature of some of Nancarrow's tempo canons is that their formal structures exhibit properties characteristic of fractals. The following sections will introduce the reader to fractals, recount the ways in which they have been observed in sound and music, and demonstrate how they are represented in Nancarrow's tempo canons.

Introduction to Fractals

Fractal geometry is a relatively newly-described branch of mathematics based on the 1977 work of Benoît Mandelbrot, in which elements of self-iteration, scaling, and space-filling are recognized in a variety of naturally-occurring objects as widely diverse as coastlines, plants, and bodily structures such as the brain and bronchial lobes. Manfred Schroeder (1991) describes the prevalence and nature of fractals, first by noting their close relationship to chaotic systems:

But no matter how chaotic life gets, with all regularity gone to bits, another fundamental bulwark often remains unshaken, rising above the turbulent chaos: *self-similarity*, an invariance with respect to *scaling*; in other words, invariance not with *additive* translations, but invariance with *multiplicative* changes of scale. In short, a self-similar object appears unchanged after increasing or shrinking its size. Indeed, in turbulent flows, large eddies beget smaller ones, and these spawn smaller ones still—and so on *ad infinitum* (almost). In general, one of the conspicuous consequences of self-similarity is the appearance of exceedingly fine-grained structures, now generally called *fractals* after Benoît B. Mandelbrot, the father of fractals. (Schroeder 1991, 1–2)

The term *fractal* escapes easy definition. Mandelbrot himself deliberately avoided

defining the term, but he did venture so far as to identify a "family of shapes I call *fractals*," further noting that "Some fractal sets are curves or surfaces, others are disconnected 'dusts,' and yet others are so oddly shaped that there are no good terms for them" (Mandelbrot 1977/1982, 1).

As the study of fractals has advanced, several properties have been acknowledged as characteristic. The first two such properties, as mentioned above by Schroeder, are related: the properties of *self-similarity* and *scaling*. According to Larry Solomon (1998), "Perhaps the most important defining property of fractals is self similarity on many different scales; i.e., they have self-iterating geometric structures that repeat in different sizes" (no page). These properties can be seen in both natural and representational objects. Solomon uses the example of a fern frond (Figure 4-1a), a naturally-occurring object in which the same characteristic shape is iterated on a number of different scales; Figure 4-1b shows fractal art that resembles the shape of the fern frond.



Figure 4-1. (a) A fern frond, a naturally-occurring fractal shape (Solomon 1998); three different scalings of the frond shape are highlighted, and even smaller iterations of the shape are present; (b) "fern-like" fractal art (Sprott 1996, 105).

Fractals can exist in one, two, or three dimensions; the Menger sponge, a three-dimensional fractal, is shown in Figure 4-2.

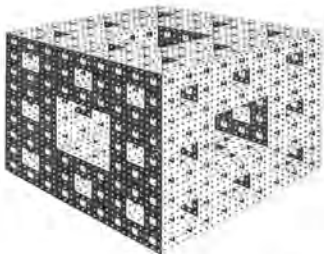


Figure 4-2. The Menger sponge: a fractal in three dimensions, or what Mandelbrot calls a “spatial universal curve” (Mandelbrot 1977/1982, 145).

Another important property of fractals is that of *space-filling*. Consider the line fractal known as a Peano curve (Figure 4-3). With each new iteration of the generating shape, the surface space is more tightly filled and the length of the line drawing the curve increases. The number of iterations and the length of the line can approach infinity, moving toward filling the space but never completely doing so (since in geometry a line, by definition, has no breadth). The iterative process that results in the generation of the fractal shape is known as an “iterated function system,” or IFS (see Schroeder 1991, Barnsley 1996, and Frame 1996). The IFS consists of an initiator shape and a generating shape, or the function that is repeatedly applied to create the figure.

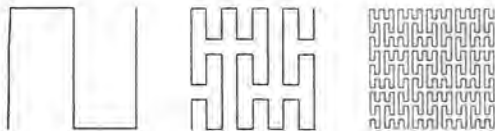


Figure 4-3. A Peano curve, which illustrates the space-filling property of fractals (Mason and Saffle 1994, 31). With each further iteration of the curve, the length of the line drawing the curve approaches infinity.

A spectacular example of a space-filling curve, or what Barnsley calls "a sequence of curves 'converging to' a space-filling curve," is shown in Barnsley (1993, p. 241). The figure is not reproduced here because the limits of resolution will not do it justice. Two similar examples from Barnsley are shown in Figure 4-4.

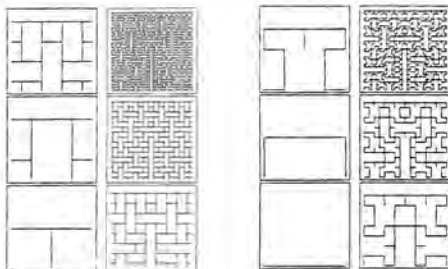


Figure 4-4. "Space-filling curves" from Barnsley (1993), p. 242.

Fractals in Sound and Music: A Review of the Literature

The close relationship between music and geometry goes back thousands of years to the Greek quadrivium. Mandelbrot's theories on fractals began to be applied to music and sound beginning in 1978, with fractal structures being identified in the nature of sound itself (Gardner 1978; Schroeder 1991), in melodies (Mason and Saffle 1994; Chesnut 1996), and in the phrase structures of binary and ternary forms (Solomon 1998). Fractals have also been explored as a means of composing music (Gardner 1978; Dodge and Bahn 1986) and creating new self-similar scales (Pierce, in Schroeder 1991). Among the fractal structures that have been related to musical structures are the Cantor comb, Sierpinski's triangle, Peano curves, and the Koch snowflake.

Mason and Saffle began their 1994 article with the following observations:

Symmetries and self-similarities abound in the world of geometry, and a great deal of music also shares these properties. Throughout the nineteenth and twentieth centuries, historians and theorists have discovered musical self-similarities of various kinds. These include the micro- and macrolevel “bar” forms that Alfred Lorenz discovered in Wagner’s music dramas; the recurrent melodic and harmonic background patterns in European art music of the eighteenth, nineteenth and early twentieth centuries discovered by Heinrich Schenker and other Schenkerian theorists; rhythmic patterns discovered by Leonard Meyer and his colleague Grosvenor Cooper; and generative organizational patterns in tonal music, discovered by Fred Lerdahl and Ray Jackendoff. (p. 31)

The nature of sound itself has been found to display fractal characteristics. In 1964, before fractals had even been described, Roger Shepard reported on a special class of sounds in which the property of scaling is actually present in the waveform itself. These sounds, which Mandelbrot later termed “scaling noises,” have the fascinating property that the pitch *decreases* as the frequency *increases* (Schroeder 1991, 95). Schroeder (1991) gives an example of another self-similar waveform in which frequency doubling results in no change in pitch (Figure 4-5). Synthetic waveforms have also been produced based on fractal properties, such as Mackenzie’s “fractalized” sine wave shown in Figure 4-6.



Figure 4-5. A self-similar waveform in which frequency doubling results in no change of pitch (Schroeder 1991, 96).

Richard Voss focused on how the fractal nature of sound relates to the construction of pleasing melodies. Gardner (1978) describes how Voss identifies the frequency spectra for three types of “noise”—white, $1/f$ (“pink”), and Brownian (see Figure 4-7)—and demonstrates how the properties of these different waveforms could be interpreted as

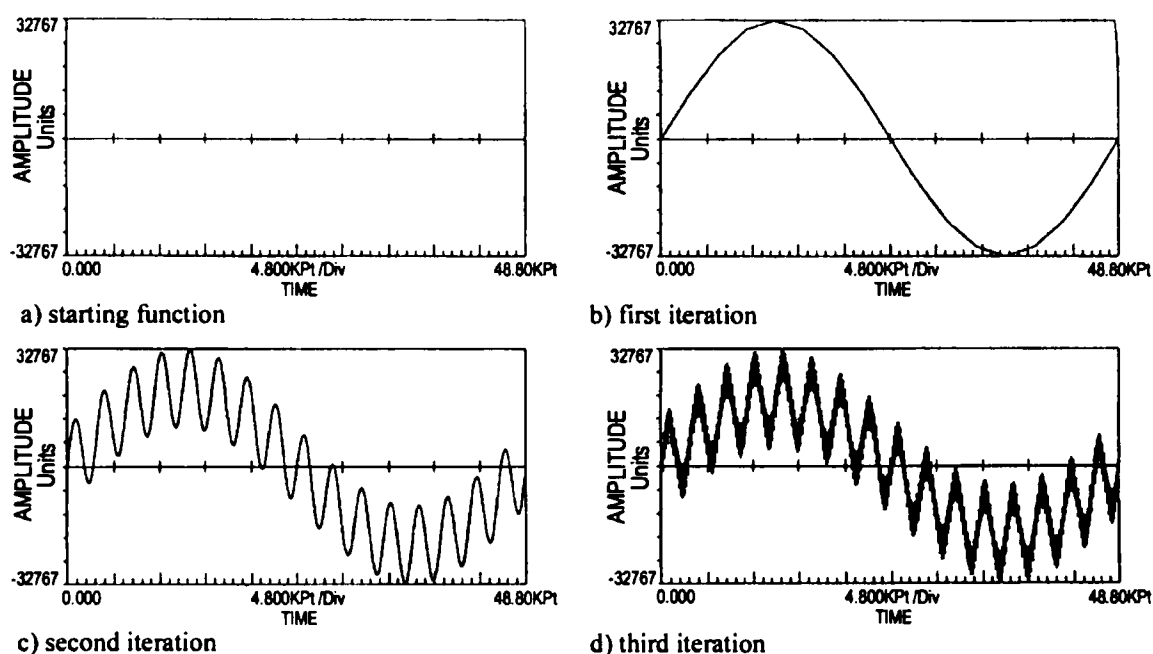


Figure 4-6. Three iterations of a “fractalized” sine wave (Mackenzie 1996, 237).

melodies incorporating elements of randomness (white noise), correlation (Brownian noise), or both (pink noise) as exhibited in the sound patterns. It turns out that $1/f$ (“pink”) noise exhibits fractal self-similarity whereas white and Brownian noise do not, and it is the melodies based on moderately correlated $1/f$ noise that most people in a test audience found most appealing, based on the melodies’ effective balance between complete randomness (surprise) and extreme correlation (expectation).

Fractal properties of melodic structures have been further studied by Mason and Saffle (1994), who showed how right-angle drawings known as Lindenmayer (L-system) curves could be used to create melodies—albeit of questionable musical value. Melodies are created from the curves by interpreting horizontal line segments as durations and vertical line segments as pitches (Figure 4-8). Mason and Saffle also assert that many existing melodies can be shown to have strong correlations with L-system curves, although their work in 1994 is very preliminary. They did, however, identify L-system curves that “generate tunes that are similar or even identical to hundreds of existing melodies by classical and popular composers” (p. 35), and they go on to discuss several

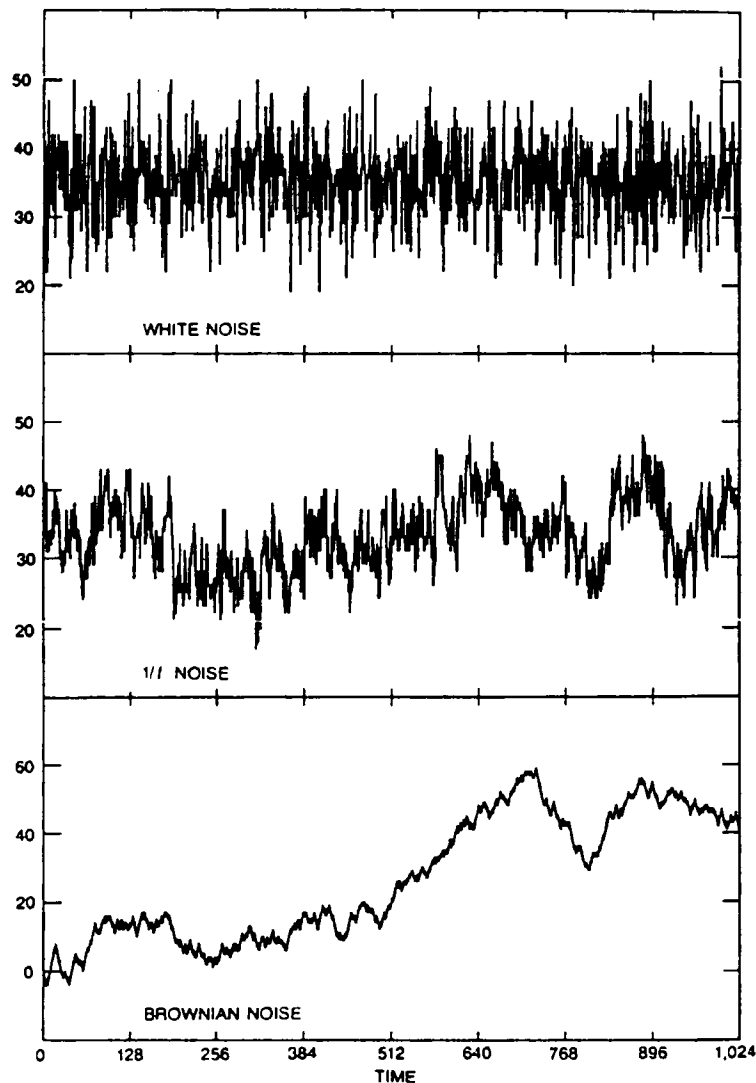


Figure 4-7. Typical patterns of white noise, “ $1/f$ [pink] noise,” and “Brownian noise” (Gardner 1978, 21). White noise (spectral density = $1/f^0$) exhibits properties of extreme randomness within a limited range, with no correlation between events. “Brownian noise” (spectral density = $1/f^2$) exhibits properties of extreme correlation between events and a tendency to “wander” over the spectrum. “ $1/f$ noise” (spectral density = $1/f$) is moderately correlated.

such tunes. Mason and Saffle’s work with the melodic associations of these curves prompted them to observe that:

Aspects of certain theories about the origins and fundamental structures of melodies suggest that much—perhaps all—beautiful music is, in some essential sense, fractal in its melodic material and internal self-similarity. (p. 35)

Schroeder (1991, pp. 99–101) described the self-similar structures in the equal-tempered scale, with the equal-tempered semi-tone functioning as the unit of self-

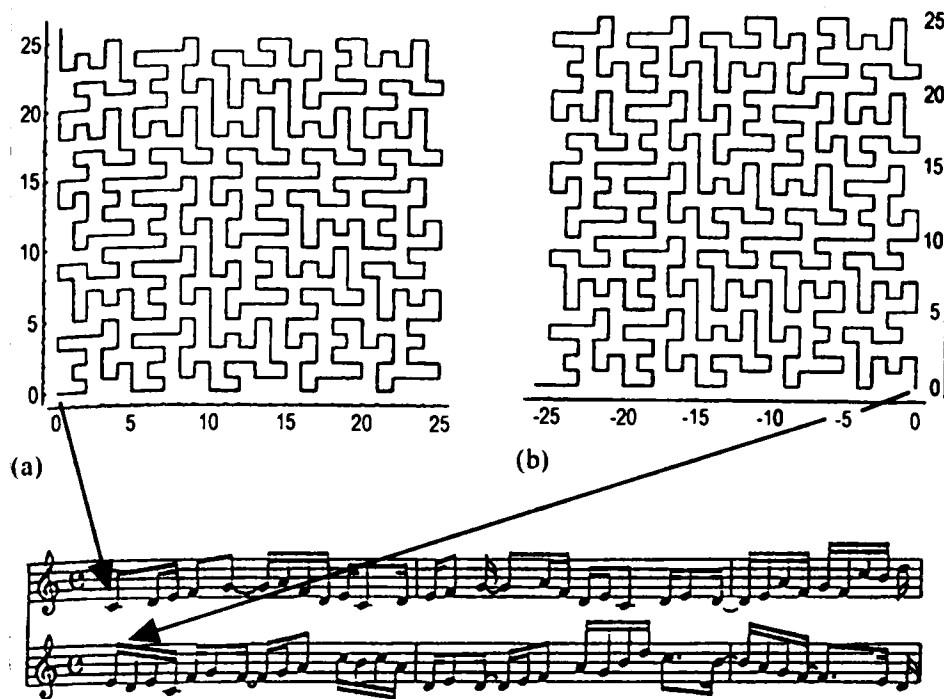
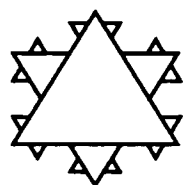


Figure 4-8. Construction of a “right-angle canon” from Lindenmayer curves (Mason and Saffle 1994, 32–33). The method involves reading the horizontal lines of curves as *durations* of notes and the vertical lines as *pitch intervals* between notes. In this example, the smallest line segment represents either one scale step (if vertical) or one sixteenth note (if horizontal). If the curve begins with a horizontal line, the pitch of the first note is assumed to be the first note of the chosen scale or mode (C major in this example); only the duration for the first note is taken from the curve. If the curve begins with a vertical line, the first note can be read as the number of forward moves up or down from the first note of the given scale or mode. Curve (b) is a 90° counterclockwise rotation of (a).

similarity in creating a musical scale. The near equality of $3:2^{12}$ (≈ 129.7 , or twelve perfect fifths) to 2^7 ($= 128$, or seven octaves) creates a nearly-closed system. Schroeder then described the “Pierce scale” (p. 102) in which John R. Pierce attempted to generate a self-similar scale like the equal-tempered scale in which the octave (2:1) is replaced by the “tritave” (3:1). The “semi-tone” that results is $3^{\frac{1}{13}}$, with the result that the Pierce system is as nearly well-closed as the equal-tempered scale, although Schroeder reports that the musical utility of this scale is “open to debate” (p. 102).

Dodge and Bahn (1996) applied the properties of self-iteration, scaling, and space-filling to the composition of a musical phrase and related the results to the fractal known as the Koch snowflake, as shown in Figure 4-9.



(a) The Koch snowflake.



(b) A "generating motif" of music composed of a few intervals and durations (analogous to the largest triangle in the Koch snowflake).



(c) The first and second layers of a polyphonic musical composition based on the generating motif. The first layer (bottom voice) is the original motif, while the second layer (upper voice) is merely a faster (and transposed) repetition of that motif added to each of the original motif's notes (analogous to the smaller triangles attached to the larger triangles).

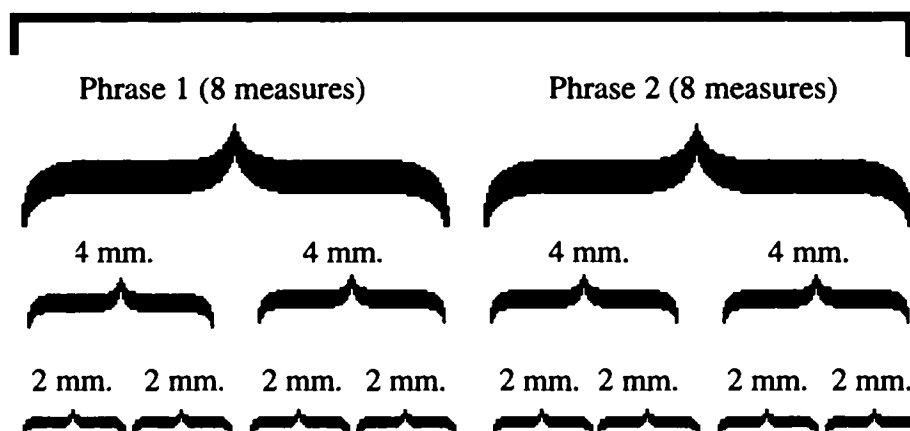
Figure 4-9. Properties of self-iteration, scaling, and space-filling in a musical segment (Dodge and Bahn 1996, 190).

Solomon (1998) demonstrated how fractal structures can be identified in musical phrase structures. A simple example is the Cantor comb (Figure 4-10a), which Solomon



(a)

Period (16 measures)



(b)

Figure 4-10. (a) A Cantor comb, and (b) a musical phrase division that corresponds to the Cantor comb (Solomon 1998).

correlated to phrase divisions in a typical binary form (Figure 4-10b). The Cantor comb itself is defined by subsequent iterations of ternary division (of which the middle third is blank); still, it is useful to see how its structure somewhat correlates with phrase structures of subsequent binary divisions.

Measuring Dimension in Fractals

Every fractal can be described in terms of its fractal dimension:

How big is a fractal? When are two fractals similar to one another in some sense? What experimental measurements might we make to tell if two different fractals may be metrically equivalent? . . .

There are various numbers, associated with fractals, which can be used to compare them. They are generally referred to as fractal dimensions. They are attempts to quantify a subjective feeling we have about how densely the fractal occupies the metric space in which it lies. Fractal dimensions provide an objective means for comparing fractals.

Fractal dimensions are important because they can be defined in connection with real-world data, and they can be measured approximately by means of experiments. For example, one can measure the “fractal dimension” of the coastline of Great Britain; its value is about 1.2. Fractal dimensions can be attached to clouds, trees, coastlines, feathers, networks of neurons in the body, dust in the air at an instant in time, the clothes you are wearing, the distribution of frequencies of light reflected by a flower, the colors emitted by the sun, and the wrinkled surface of the sea during a storm. (Barnsley 1993, 171)

As Pickover (1995) describes it, “the fractal dimension . . . characterizes the size-scaling behavior of the pattern. This value gives an indication of the degree to which the pattern fills the space” (p. 204). The fractal dimension further serves to describe whether the fractal is between the topological dimension of a point (0) and a line (1), whether it is two-dimensional (a dimension between 1 and 2), or three-dimensional (a dimension between 2 and 3); Frame (1996) notes that the dimensions of a line segment = 1, a filled-in square = 2, and a solid cube = 3 (p. 38).

The most commonly used fractal dimension is called the Hausdorff dimension (D_H ; Schroeder 1991, p. 15 ff.). The Hausdorff dimension describes the relationship between the number of self-similar objects that are created between successive iterations and the

size of these objects compared to the original. Figure 4-11 shows a simple derivation of this dimension based on self-similarity in lines, squares, and cubes. Connors (1994) gives the formula $N = S^D$, where N is the number of self-similar pieces in the divided figure, S is the scaling factor (the factor by which each smaller piece relates to the original—in each case here, $S = 4$)¹, and D (the power to which S is raised) is the dimension. Solving the equation for D yields:

$$S^D = N$$

$$\log S^D = \log N$$

$$D \log S = \log N$$

$$D_H = \log(N)/\log(S)$$

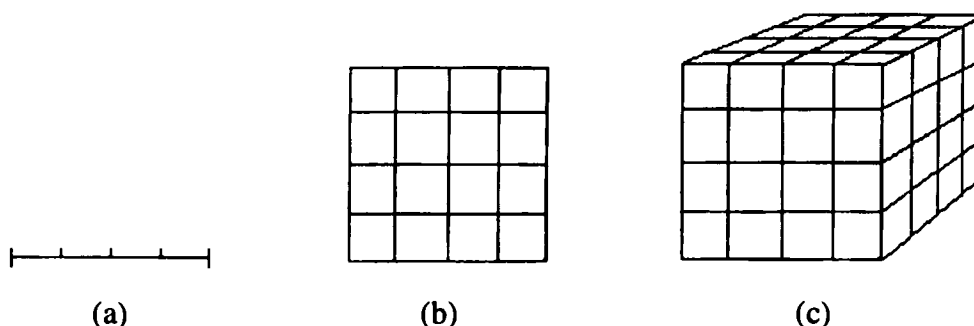


Figure 4-11. Illustration of derivation of dimension via self-similarity (Connors 1994). In the line segment in (a), four self-similar pieces are each $\frac{1}{4}$ the size of the original, i.e., $4 = 4^1$ pieces; the square shown in (b) consists of 16 self-similar pieces with sides $\frac{1}{4}$ the size of the original, i.e., $16 = 4^2$; and the cube in (c) consists of 64 self-similar pieces with sides $\frac{1}{4}$ the size of the original, i.e., $64 = 4^3$ pieces. In each of these simple cases, the exponent gives the dimension; thus, the dimension of (a) = 1, of (b) = 2, and of (c) = 3.

Some examples follow to illustrate the calculation of this fractal dimension using this formula, $D_H = \log(N)/\log(S)$. Let us first consider the Cantor comb shown in Figure 4-10a. The first iteration of the generating function—to remove the middle third of the line's length—results in two line segments one-third the size of the original, and this continues through successive iterations potentially *ad infinitum*. The dimension of the

¹The scaling factor refers to how the *side length* of the scaled object compares to the original, not its area or volume in the case of two- or three-dimensional figures.

Cantor comb is thus described:

$$D_H = \log(2)/\log(3) = 0.63$$

Thus, the Cantor comb is a one-dimensional fractal with its dimension being between that of a point and a line.

Now let us look at some two-dimensional fractals and their dimensions. The Sierpinski triangle (Figure 4-12) divides the initiating shape into three copies of itself, each of which has a line length one-half the size of the original (i.e., each of the three sides of the initiating triangle is bisected to form the generating shape, a blank inner triangle). The dimension is calculated as:

$$D_H = \log(3)/\log(2) = 1.58$$

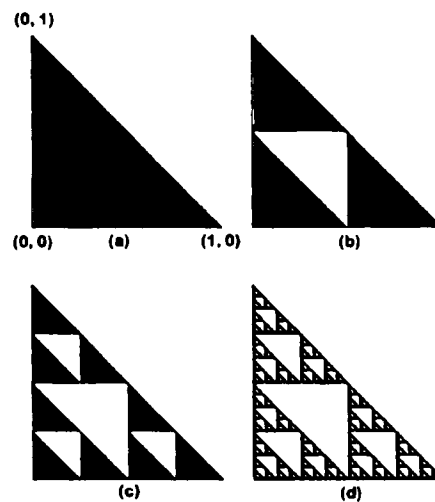


Figure 4-12. The Sierpinski triangle, $D_H = 1.58$ (Frame 1996, 39).

Another two-dimensional fractal is the Cantor gasket, shown in Figure 4-13. It divides the initiating shape into eight copies of itself, each with a line length one-third the size of the original. The dimension is thus:

$$D_H = \log(8)/\log(3) = 1.89$$

At this point, in comparing Figures 4-12 and 4-13, one will notice that as the fractal dimension approaches 2, the space is more solidly filled. Thus, in a sense, the fractal dimension describes how much of the metric space in which the fractal exists is “carved

out” to create the fractal figure, with a dimension closer to 1 indicating that more is carved out than in a figure with a dimension closer to 2.

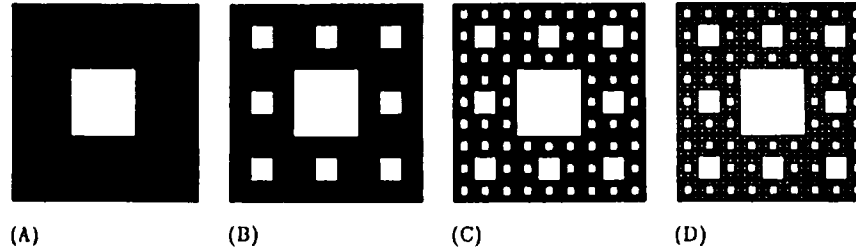


Figure 4-13. The Cantor gasket, $D_H = 1.89$ (Schroeder 1991, 179).

The Menger sponge that was shown in Figure 4-2 will illustrate dimensions in three-dimensional fractals. A comparison with Figure 4-13 reveals that this fractal is the three-dimensional equivalent of the Cantor gasket. Determining its dimension necessitates being able to determine how many cubes, of side-length one-third, are created by each iteration of the generating function. There are 20 such cubes (eight on the top layer, eight on the bottom layer, and the four corners of the middle layer). Thus the fractal dimension is:

$$D_H = \log(20)/\log(3) = 2.73$$

which confirms that the figure is three-dimensional.²

Fractals are also described in terms of whether the fractal is totally disconnected, “just-touching,” or overlapping. Fractals such as the Cantor comb (Figure 4-10a), or “dust” fractals, are totally disconnected; their successive iterative generations are not touching any previous generations. In “just-touching” fractals such as Sierpinski’s triangle (Figure 4-12) and the Cantor gasket (Figure 4-13), successive iterations are wholly

² A curiosity about the dimension relationship between the Cantor gasket and the Menger sponge is that the decimal part of their dimensions (.89 for the Cantor gasket and .73 for the Menger sponge) is not equivalent as one would expect them to be. Note that if the Menger sponge contained 24 self-similar cubes rather than 20, the decimal portion of its D_H would be exactly the same as the Cantor gasket’s [$\log(24)/\log(3) = 2.89$]. The reason for the discrepancy is that the cube of side length one-third at the very center of the Menger sponge appears in all six of the cuts from one side of the figure to the other; this overlapping cube can be subtracted only once from the possible total of 27, with the other six eliminated cubes coming from the center of each of the main cube’s faces to result in 20.

contained within the initiating shape and the points which define the extremes of the generating shape create smaller figures that are just touching. Fractal shapes can also overlap, and these are the kinds of fractals we will see in Nancarrow's tempo canons.

Fractal Characteristics in Nancarrow's Music

Although Nancarrow's tempo canons do exhibit the fractal characteristics of self-similarity, scaling, and space-filling, they exhibit as well some important departures from the fractals we have observed so far.

The first (and most important) difference is that the percentage of scaling between successive iterations in tempo canons involving more than two voices is not the same, as it would be in standard fractals based on iterated function systems (IFS). For instance, let us look at the procedure used in the construction of the Cantor comb from Figure 4-10a. Figure 4-14 shows the successive division of a line extending from 0 to 1 into one-third segments. Note that between any pair of successive iterations, the same ratio of division is in effect; the scaling factor is always 3.

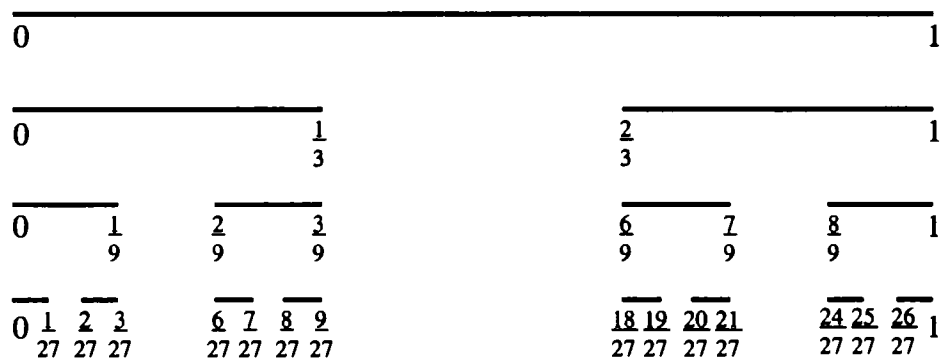


Figure 4-14. Construction of the Cantor comb (after Barnsley 1993, 44). The line in each previous iteration is divided into thirds, with the middle third left blank; the scaling factor is always 3.

Nancarrow's tempo canons, on the other hand, usually exhibit tempo (or duration) relationships based on superparticular ratios, which in the case of 3- or more element ratios ensures that the scaling between different pairs of adjacent voices will not be the

same. For example, in a 12:15:20 tempo canon, which has a duration ratio of 3:4:5, the scaling between 3:4 and 4:5 is not equivalent. In a duration ratio such as 4:6:9, on the other hand, both adjacent pairs are related by the same ratio; such a ratio is symmetrical in that the tempo ratio and its inversion, the duration ratio, are the same. Nancarrow did not normally use such symmetrical tempo ratios, however, with perhaps the only example being the ratio $1\%/1\frac{1}{2}\%/2\frac{1}{4}\%$ of Study No. 22, an acceleration canon in which the ratio between both pairs of adjacent voices is 3:2. Thomas cites Nancarrow's avoidance of geometric ratios when she says "The ratios are geared toward creating the effect of independence between the layers, and to that end Nancarrow avoids geometric ratios (such as 3:6:12) which would create tempos potentially subordinate to a common, broad rate of pulsation" (Thomas 1996, 9).

Another way in which Nancarrow's tempo canons differ from typical fractals is that the scaling compression is in only one dimension (time/tempo) as melodic intervals (and therefore pitch ranges) are not compressed. Figure 4-15 represents as an example the structure of Study No. 14, a two-voice arch canon, in terms of the pitch range of the two canonic voices and their tempo/duration relationship. The interval of imitation is two octaves plus a perfect fifth, and both voices have a total pitch range of 45 pitches. Thus the scaling of the second and faster voice involves only the tempo and not the pitch range. (Note that this was also the case in the example from Dodge and Bahn, Figure 4-9, which has a scaling factor of 2). This discrepancy will be ignored in this study when determining fractal dimensions of the tempo canons and dimensions will be based only on scaling relationships of the durations of the canonic voices.

Finally, a third difference between Nancarrow's tempo canons and typical fractals is their self-limiting nature. Most of the tempo canons discussed here contain only two or three scaled generations while the most complex (Study No. 36, "Canon 17/18/19/20") has four. Meanwhile, the number of generations in a geometric fractal is potentially infinite.

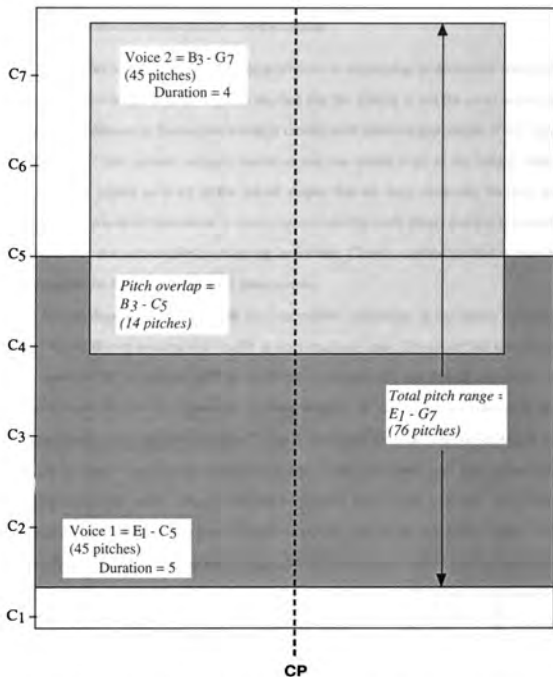


Figure 4-15. Diagram of Study No. 14 ("Canon 4/5"), showing pitch ranges of two canonic voices and tempo/duration relationship. Voice 2 is compressed in time but not in pitch.

Measuring Fractal Dimension in the Tempo Canons

These and other factors lead to complications in attempting to derive the fractal dimensions of the tempo canons. Besides the fact that the scaling is not the same across all iterative generations, in Nancarrow's tempo canons each iterative generation of the "generating shape" (the canonic subject) results in just one scaled copy of the subject rather than multiple copies as in all of the fractal shapes that we have observed. Further, the "fractalized" voices in Nancarrow's tempo canons overlap each other, and the Hausdorff equation does not account for overlapping in fractals. Clearly another method is needed to determine the fractal dimension of these canons.

In consideration of the fact that each successive generation in the tempo canons is scaled by a different amount and results in only one new copy, I propose that the dimension equation $D_H = \log(N)/\log(S)$ be modified for calculating the fractal dimension of tempo canons so that N represents the *total* number of scaled voices (including the original voice, or the "generating shape") and S represents the total relative scaling of the duration of these voices. (This equation will not, as already noted, take into account the overlapping of the scaled objects.) Figure 4-16a will serve as an example. The figure simultaneously illustrates two hypothetical tempo canons: on the left side a canon with the duration ratio 2:3:4:5:6, and on the right side a duration ratio of 3:4:5:6. For the canon with the duration ratio 2:3:4:5:6, $N = 5$ and S is the sum of the five scaled durations, $1 + 0.83 + 0.67 + 0.5 + 0.33$, more precisely summed up as $\frac{2 + 3 + 4 + 5 + 6}{6} = \frac{20}{6} = 3.33$. Thus, $D_H = \log(5)/\log(3.33) = 1.34$. For the canon with the duration ratio 3:4:5:6, $N = 4$ and $S = \frac{3 + 4 + 5 + 6}{6} = \frac{18}{6} = 3$; $D_H = \log(4)/\log(3) = 1.26$. [Log(4)/log(3) also happens to describe a fractal known as the Koch curve, in which the middle third of a line segment is repeatedly raised to the point of an equilateral triangle; see Figure 4-16b.]

This definition of fractal dimension will now be applied to several of Nancarrow's tempo canons. The diagrams in this section will show the relationships between canonic

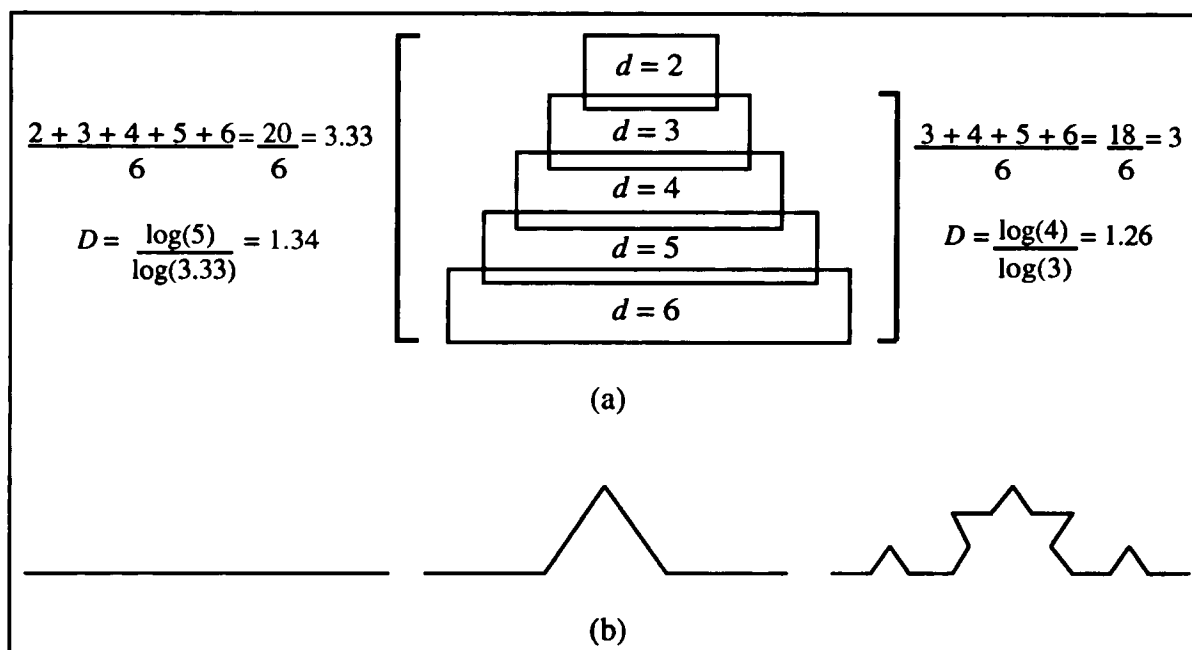


Figure 4-16. (a) Hypothetical tempo canons of duration ratio 2:3:4:5:6 (left side of diagram) and 3:4:5:6 (right side); fractal dimension (D) is calculated by dividing log of number of scaled objects (including the generator) by the log of the total of the relative dimensions. (b) $D = 1.26$ [$\log(4)/\log(3)$] also describes the Koch curve, in which each line segment is replaced by a segment $4/3$ the size of the original.

voices in terms of duration, range, and overlap of voices. The horizontal x axis in the diagrams will represent the duration of each Study (which will, of course, vary from Study to Study), while the vertical y axis will represent the player piano's entire pitch range from B_0 to A_7 .

Figure 4-15 shows the fractal formal structure for Study No. 14. This tempo canon consists of just two scaled voices, with relative scaled durations of 1 and 0.8; therefore, $D_H = \log(2)/\log(1.8) = 1.18$.

Figure 4-17 shows the structure for Study No. 18 ("Canon $3/4$ "). In this Study, the second voice of the tempo canon consists of only the treble melody. For these two scaled lines, the relative scaled durations are 1 and 0.75, and $D_H = \log(2)/\log(1.75) = 1.24$.

Figure 4-18 shows the structure for Study No. 19 ("Canon $12/15/20$ "). Its duration ratio is 3:4:5, and it consists of three scaled voices with a total scaled duration of $\frac{3+4+5}{5} = 2.4$; thus, $D_H = \log(3)/\log(2.4) = 1.25$.

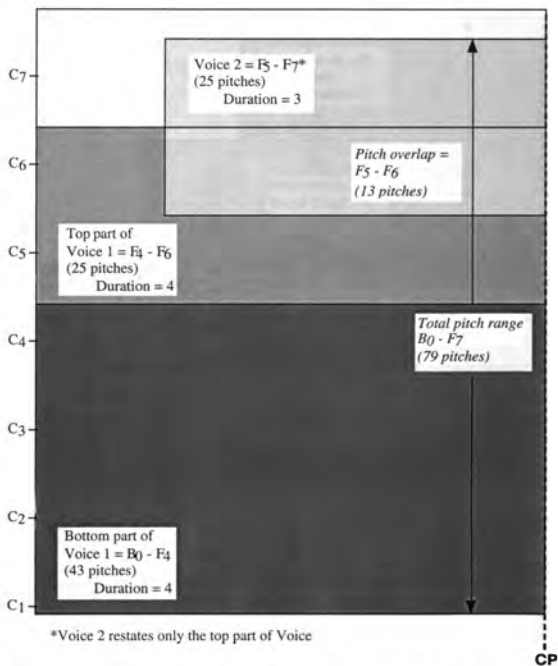


Figure 4-17. Diagram of Study No. 18 ("Canon 3/4"); $D_H = \log(2)/\log(1.75) = 1.24$.

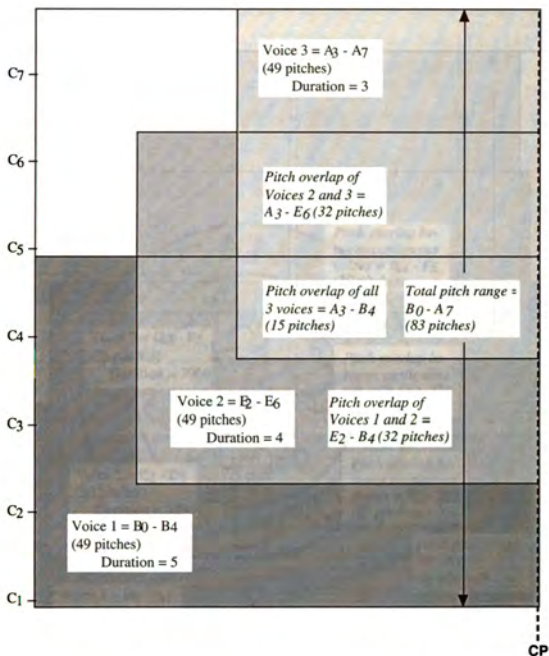


Figure 4-18. Diagram of Study No. 19 ("Canon 12/15/20"); $D_H = \log(3)/\log(2.4) = 1.25$.

Figure 4-19 presents the structure for Study No. 36 ("Canon 17/18/19/20"). This canon's duration ratio is 2907:3060:3230:3420; its four scaled voices have a total scaled duration of $\frac{2907 + 3060 + 3230 + 3420}{3420} = \frac{12617}{3420} = 3.69$. Thus, $D_H = \log(4)/\log(3.69) =$

1.06.

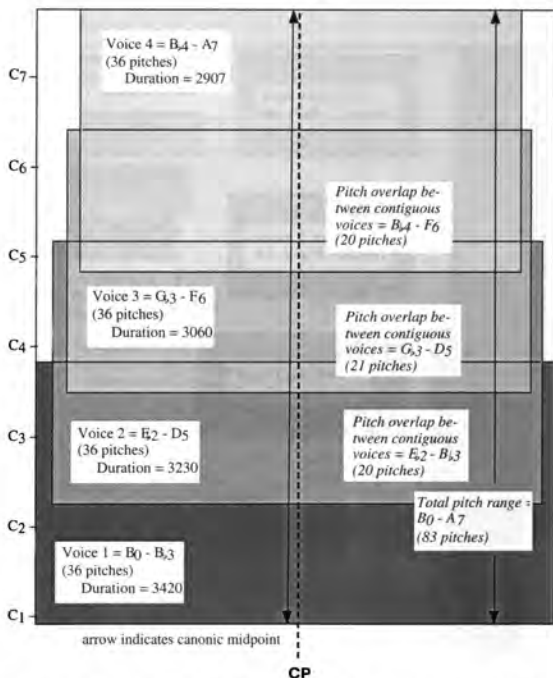


Figure 4-19. Diagram of Study No. 36 ("Canon 17/18/19/20"). Recall that in this canon the CP is slightly past the canonic midpoint (see Figures 3-5 and 3-13). $D_n = \log(4)/\log(3.69) = 1.06$.

Figure 4-20 presents the structural diagram for Study No. 31 ("Canon 21/24/25").

The duration ratio here is 168:175:200; the three scaled voices have a combined scaled

duration of $\frac{168 + 175 + 200}{200} = \frac{543}{200} = 2.72$. Therefore, $D_u = \log(3)/\log(2.72) = 1.10$.

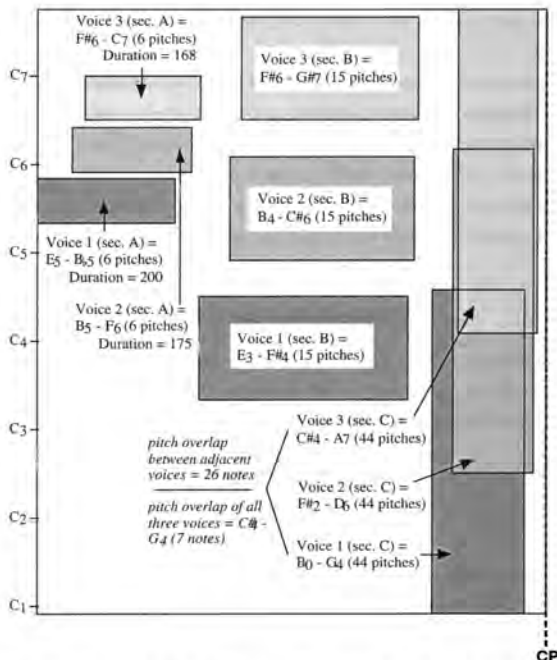


Figure 4-20. Diagram of Study No. 31 ("Canon 21/24/25"). The canon melody is divided into parts A, B, and C, each separated by eight measures; interval of imitation in section A is a perfect fifth, and in sections B and C a perfect twelfth. The duration ratio in each section is 168:175:200; $D_u = \log(3)/\log(2.72) = 1.10$.

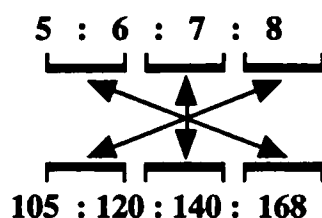
A review of the fractal dimensions determined for these tempo canons reveals the following tendencies:

1. The fractal dimension is generally higher in tempo canons with more voices (for example, the hypothetical 2:3:4:5:6 and 3:4:5:6 canons shown in Figure 4-16a), especially when the following is true:

2. The fractal dimension tends to be higher as the total scaled duration decreases; in other words, the smaller the duration of the scaled voices compared to the original, the higher the fractal dimension is. See, for example, Studies No. 18 (Figure 4-17) and 19 (Figure 4-18) and the two hypothetical canons in Figure 4-16(a); compare to the minimally-scaled voices in Studies No. 36 (Figure 4-19) and No. 31 (Figure 4-20). Consider also the example given by Dodge and Bahn (Figure 4-9) in which the scaled copy is half the duration of the original; in this case, $D_H = \log(2)/\log(1.5) = 1.71$.

Analysis of Study No. 32: "Canon 5/6/7/8"

This chapter now concludes with an analysis of Study No. 32, a four-voice converging canon in which the final cut-off is the CP (see Figure 3-23, p. 135). The derivation of Study No. 32's duration ratio from the tempo ratio 5:6:7:8 is shown below:



This duration ratio and the elapsed number of beats in the first voice to the entrances of later voices were shown in Figure 3-22 (p. 135). The canon's timespan is 431 dotted quarter-note beats (the canon is written entirely in $\frac{6}{8}$ and $\frac{9}{8}$ meters) and the CP occurs on the hypothetical downbeat of beat 432.

The fractal formal structure of Study No. 32 is shown in Figure 4-21. As shown in the figure, the interval of imitation between all voices is the perfect fifth, and the four

scaled voices have a total scaled duration of $\frac{105 + 120 + 140 + 168}{168} = \frac{533}{168} = 3.17$; thus

$$D_H = \log(4)/\log(3.17) = 1.20.$$

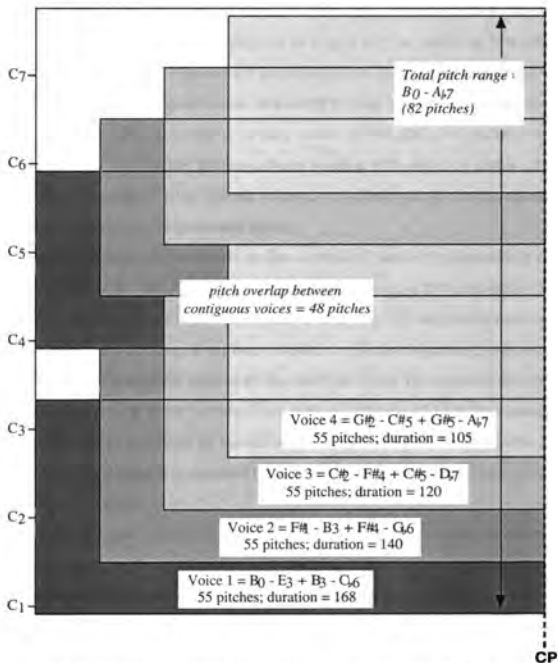


Figure 4-21. Structural diagram of Study No. 32 ("Canon 5/6/7/8"); $D_H = \log(4)/\log(3.17) = 1.20$.

Although our analysis of fractal dimension does not take into account pitch dimension, it is apparent—in comparing Figure 4-21 with Figures 4-15, 4-17, 4-18, 4-19, and 4-20—that Study No. 32 has the largest pitch overlap (in both absolute and relative terms) of any of these pieces. Study No. 14 (Figure 4-15) has a pitch overlap of 31% (14 of 45 pitches) between adjacent voices, Study No. 18 (Figure 4-17) an overlap of 52% (13 of 25 pitches), Study No. 19 (Figure 4-18) an overlap of 65% (32 of 49 pitches), Study No. 36 (Figure 4-19) an overlap of between 56% and 58% (20 or 21 of 36 pitches), and Study No. 31 (Figure 4-20) an overlap in its third section of 59% (26 of 44 pitches), while Study No. 32's pitch overlap between adjacent voices is 87% (48 of 55 pitches). This considerable overlap of pitch between voices is an intrinsic part of the characteristic sound of this piece, as will be revealed shortly.

Gann identifies three sections in the canonic melody. Section 1 is comprised of 164 beats, or 38% of the canon's total length; section 2 is 77 beats, or 18%; and section 3 is 190 beats, or 44%. The structural proportions of Study No. 32's sectional divisions are shown in Figure 4-22. There is one near-coincidence of section beginnings, as shown in the figure by the dashed line between the first and third voices. The beginning of section 2 in the third voice is within one beat of the beginning of section 3 in the first (bottom) voice: the first two sections of the bottom voice comprise $164 + 77 = 241$ beats, while the third voice's beat delay to its entrance ($123 \frac{1}{7}$) plus the duration of its first section ($164 \cdot \frac{5}{7} = 117 \frac{1}{7}$) adds up to $240 \frac{2}{7}$.

In another near-coincidence, the entrance of the fourth (top) voice is just over two beats prior to the beginning of the first voice's section 2. The nearness of this entrance to the beginning of the first voice's section 2 is indicated by comparing the number of elapsed beats to the entrance of the fourth voice (161.63 beats) and the number of beats in the canon's first section (164). Also, as indicated by the arrow in Figure 4-22, the beginning of section 2 in the fourth voice happens to coincide very neatly with the piece's golden section.

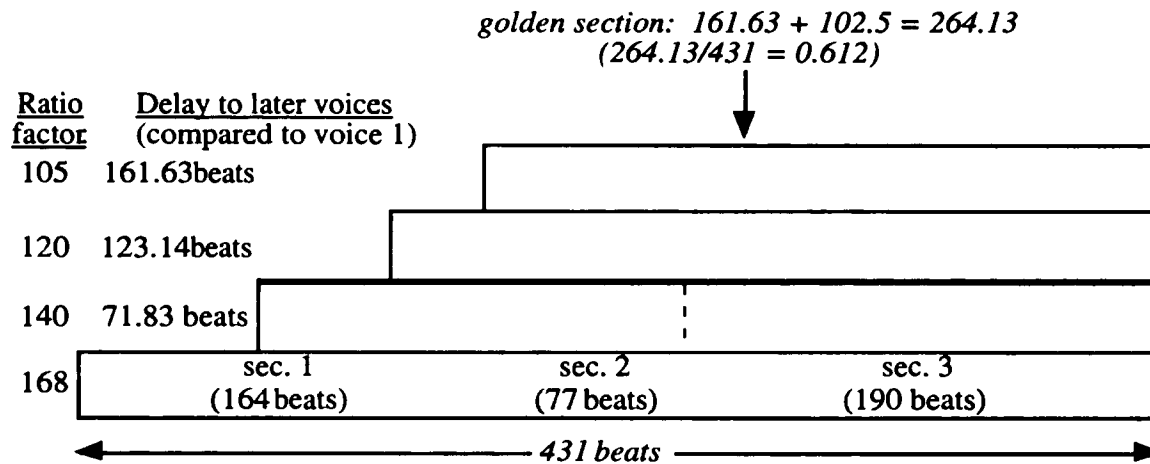


Figure 4-22. Proportional dimensions of Study No. 32 (duration ratio = 105:120:140:168) showing sections 1, 2, and 3 in each voice and beat delay to entrance of later voices. Entrance of section 2 in third voice nearly coincides with beginning of section 3 in first (bottom) voice (shown by dotted line).

Gann's diagram of the structure of No. 32 (Figure 4-23) is apparently based on the tempo ratio 5:6:7:8 rather than the duration ratio (although its proportions at the left seem to indicate the correct ratios—compare with Figure 4-22); it incorrectly implies a coincidence between the beginning of section 2 in the fourth voice and the beginning of section 3 in the second voice.³ In the score, this relationship is actually off by a whole score system: in the top voice the elapsed beat count is $161\frac{5}{8}$ beats to its entrance plus 102.5 beats ($164 \cdot \frac{5}{8}$) in section 1 for a total of $264\frac{1}{8}$; in the second voice, section 3 begins after $71\frac{5}{6}$ beats prior to its entrance, plus section 1's $136\frac{2}{3}$ beats ($164 \cdot \frac{5}{6}$) and section 2's

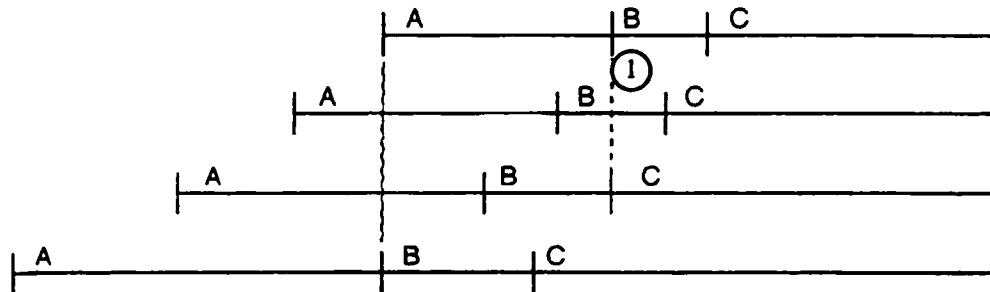


Figure 4-23. Structural diagram of Study No. 32 ("Canon 5/6/7/8") from Gann (1995, p. 185). The alignment indicated at ① does not really occur.

³Gann's diagram labels the sections A, B, and C although in his text he refers to them as 1, 2, and 3.

$64 \frac{1}{6}$ beats ($77 \cdot \frac{5}{6}$) for a total of $271 \frac{2}{3}$ beats. Gann's diagram does, however, correctly identify the near-coincidence of the entrance of the fourth voice with the beginning of section 2 in the bottom voice.

Study No. 32's highly organic canon melody is monophonic throughout and contains the seven motives identified in Table 4-1. The table shows the motives in their most common rhythmic forms as well as showing examples of rhythmic augmentations and diminutions that occur. Each motive is classified as having a primary beat division of either the quarter note or dotted quarter note; this classification is generally determined by the most common rhythmic relationship between the motive's first two notes. Motives *a*, *c*, and *e* are based on the quarter note division, and motives *b*, *b2*, *d*, and *f* on the dotted quarter note.

Table 4-1
Descriptions of Motives in Study No. 32's Canon Melody

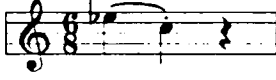
<u>Motive</u>	<u>Description and Comments</u>	<u>Primary Beat Division</u>
<i>a</i>	descending major triad; motive does not appear in sec. 2 <u>Rhythmic forms:</u>  most common form (m. 1)  augmented form (m. 156)	
<i>b</i>	ascending or descending minor third <u>Rhythmic forms:</u>  most common form (m. 3)  augmented form (m. 9)  diminished form (m. 115)	

Table 4-1 (cont'd.)

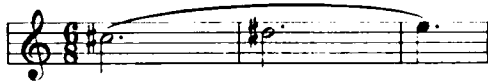
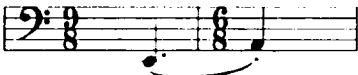
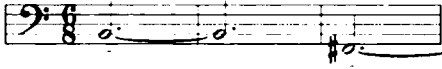
<i>b2</i>	two successive interlocking ascending minor thirds; motive does not appear in sec. 2	
	<u>Rhythmic forms:</u>	
		most common form (m. 24)
		diminished form (m. 66)
<i>c</i>	ascending or descending half step	
	<u>Rhythmic forms:</u>	
		most common form (m. 13)
		minimally augmented form (m. 6)
		maximally augmented form (m. 46)
<i>d</i>	ascending or descending octave (or multi-octave) leap; largest leap = 4 octaves (e.g., mm. 6-7)	
	<u>Rhythmic forms:</u>	
		most common form (m. 17)
		augmented form (m. 106)
		diminished form (m. 31)
<i>e</i>	ascending or descending "partitioned minor third" (3- note figure consisting of major second and minor second); motive does not appear in sec. 2	
	<u>Rhythmic forms:</u>	
		most common form (m. 50)
		augmented form (m. 182)

Table 4-1 (cont'd.)

<i>f</i> ascending or descending perfect fourth; most commonly in descending form in sec. 2 <u>Rhythmic forms:</u>  	♩ most common form (m. 26) augmented form (m. 79)

Canon's Section 1

Section 1 (164 beats, mm. 1–74) introduces all seven of the motives shown in Table 4-1. Figure 4-24 shows Study No. 32's melodic line as it appears first in the lowest voice. The canon is written in the figure on three staves to clarify its compound nature as noted by Gann (1995, p. 182): a high melodic line of staccato and sustained notes; a line of sustained notes in the bottom area of the bass clef; and mostly staccato notes in the extreme bass register.

Immediately, the canon establishes a conflict between the ♩ rhythmic division of *a* and the ♩ division of *b*, and by m. 10 the ♩ division is added. The last half of section 1 (after the second voice enters in m. 34) is less rhythmically active, contains numerous rests, and has few notes in the top register, all of which allow the beginning of the second voice to be heard more clearly.

The beginning of the canon is subdivided by three statements of motive *a*: first in A major, then F# major (mm. 18–19), and D major (mm. 27–28). The statements of the motive in A and D are the most similar, with each subsection stating the *a*, *b*, and *c* motives in that order. The dominant note, E, is prominent throughout section 1, and is traced in a slow chromatic descent (F#–F–E) in the deepest bass register prior to the statement of the F# major triad as well as being a melodic goal in mm. 38–52. The half-step motive *c* is twice used in this section in both ascending and descending forms to lead

Figure 4-24 (cont'd.)

The musical score is divided into four systems, each with three staves. The first system (measures 39-46) shows a melodic line in the treble staff and a bass line in the lower bass staff, with a middle bass staff. The second system (measures 47-54) continues the melodic and bass lines. The third system (measures 55-65) shows a more complex melodic line in the treble staff. The fourth system (measure 66) shows a final melodic line in the treble staff and a bass line in the lower bass staff. The score includes various musical notations such as notes, rests, and dynamic markings (p, f, b, d, e, b2, (d)).

to the same pitch in different registers: at mm. 13–14 and 16 to lead first up and then down to the pitch E, and at mm. 46–48 and 53–55 to lead first up and then down to the pitch C. The pitch E also represents the tonal center of an extended passage from measure 38 to measure 52.

Thomas (1996) identifies the clear registral separation in this canon as contributing to its extreme sense of multidimensionality: “internal registral gaps can make an ostensibly single line sound like more than one line, thereby extending and possibly also confounding a work’s multidimensionality” (p. 84). This extreme sense of registral separation

also occurred in Canon 7 of Study No. 27 (see Figure 3-53, p. 175), where canonic lines are separated by five octaves and compound melody is also established.

Section 2

Several motives (*a*, *b*2, and *e*) rather conspicuously drop out in the canon's second section (77 beats, mm. 76–107), which is shown in Figure 4-25. Section 1's upper register is also not present in this section, and the fastest rhythmic movement is by dotted half note; thus the quarter-note beat division is entirely absent. Gann describes this as the “get out of the way section” (p. 183), and that is an apt description of the effect of its condensed low register, elongated rhythmic movement, and motivic restriction. Section 2 is framed on both ends by motive *d* (the only occurrences of this motive in this section), and the section's five phrases each end with a staccato quarter note followed by a rest.

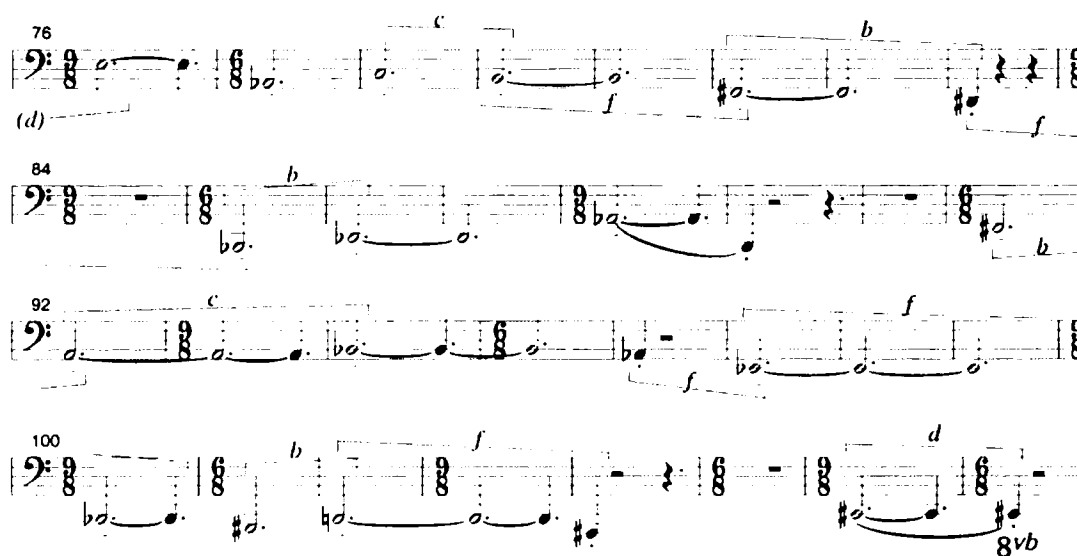


Figure 4-25. Canon melody in section 2 (mm. 76–107).

Figure 4-26 (cont'd.)

Section 3

Section 3, the longest of the sections (190 beats, mm. 109–95), is shown in Figure 4-26. The separation into three registers returns, and all seven motives are used again, and rhythmic augmentation is a significant feature of this section, particularly in those motives that in section 1 were based on the quarter note division. The restatement of motive *a* in dotted half notes at m. 156 establishes a particularly significant harmonic goal.

Section 3 concludes with two statements of *e*, one ascending and one descending, that appear in rhythmically augmented form a few measures before the stacks are stated. The two statements rotate about the note C#: first D–C#–B in mm. 176–77, followed by C#–D#–E in mm. 182–84. This section actually begins at m. 172 with the note B₀ initiating a 4-octave leap up to an interrupted ascending statement on *e* on B₄; there is then a

4-octave leap back down from D_3 to D_1 and an inverted statement of e . The $C\#_1$ in m. 181 initiates another ascending 4-octave leap into the ascending statement of e on $C\#$; the $C\#_3$ beginning in m. 184 bisects the 4-octave leap.

Effect of All Voices Together

Thomas (1996) points to this piece as exhibiting extreme multidimensionality in two ways: first, in the separation of the canonic voice into three registral layers which, when all four voices are sounding, obscures the canon; and second, the variety of rhythmic durations that, when combined with the tempo ratio 5:6:7:8, creates a variety of closely-related durations. Table 4-2 shows the rhythmic durations cited by Thomas (1996, p. 245); Thomas points out the duplication of the tempo value 204 (which is also related to tempos 102 and 306) and the closeness among values such as 119, 127.5, and 136, and between 170 and 178.5.

Table 4-2
Relative Tempos of Durations Occurring in Study No. 32
(after Thomas 1996, 245)

<u>Ratio of Voice Tempo</u>	<u>Rhythmic Duration</u>			
				
8	 = 136	 = 204	 = 272	 = 408
7	 = 119	 = 178.5	 = 238	 = 357
6	 = 102	 = 153	 = 204	 = 306
5	 = 85	 = 127.5	 = 170	 = 255

Nancarrow seems to have written the canon so that convergence of tonal areas occurred to varying degrees at entrances of later voices. Figure 4-27 shows the entrance of voices 2, 3, and 4. At the entrance of voice 2 (Figure 4-27a), an area of A major (of which the first triad in voice 2 is V) is maintained for several measures; at the entrance of

voice 3 (Figure 4-27b), a brief period of B major takes place; and at the entrance of voice 4 (Figure 4-27c), the key area is alternately B minor and major.

(a)

(b)

(c)

Figure 4-27. Tonal areas represented at entrances of new voices: (a) entrance of voice 2 (p. 1), A major; (b) entrance of voice 3 (p. 2), B major; and (c) entrance of voice 4 (p. 3), B minor/major.

There is another extended area of tonal convergence involving all four voices, and this occurs on pp. 8–9 of the score, where first a section of predominantly F major (first

two systems of p. 8) is followed by a section of G major which begins with a statement of motive *a* on a C major triad in the bottom voice, followed immediately by the same triad in rhythmically augmented form. There is a remarkable degree of tonal homogeneity in the G major section, broken only by an E_b in the bottom voice and a C# in the second voice. This section, placed so near the end of the piece, is quite prominent due to the length of time it stays in the same tonal area and because of the augmented statement of *a*. It is also significant, I believe, that it is approximately at the first augmented statement of *a* (measure 156) in the lowest voice that the listener really begins to ascertain the echo distance between the voices and anticipate the approaching convergence point. The profile of the augmented *a* statement is so prominent that it is recognized immediately as it is restated at progressively shorter time intervals in succeeding voices. The fact that the rhythmically augmented major triad is immediately preceded by the same triad in the original note values also serves to heighten the listener's awareness of the augmented statement.

The rhythmically-augmented statement of *a* in the passage shown in Figure 4-28 is the beginning of the arrival at Study No. 32's most significant tonal goal. Figure 4-29 shows the piece's large-scale movement between tonal centers, from the A major triad expressed in motive *a* at the opening to the rhythmically-augmented A major triad two octaves higher at m. 156 in the top voice. With this movement, the piece completes a fifth stack *below* A to complement the fifth stack above A represented by the interval of imitation between the voices. The figure identifies the two major key areas established by the statements of *a* in the entrance of the four voices (A major in the first two voices, and B major in the last two), and how these two key areas plus the C major triad at m. 156 in the bottom voice project motive *e*. The figure also shows how the movement in the opening two key areas from the tonic triad to the triad statement in the subdominant is answered in key areas G and A at the end of this large section by movement from the subdominant to the tonic.



Figure 4-28. Bottom system of p. 8 and top system of p. 9 from score of Study No. 32, showing extended section of G major beginning in bottom voice with restatement of *a*.

The piece concludes with two fifth stacks: one sustained for 23 dotted half notes and built on C $\sharp_3$ in the lowest voice (beginning at m. 184), and the other a dotted quarter note in m. 190 (six measures from the end) built on B $_0$. In all four voices, the two stacks together spell out the hexachord B–F $\sharp$ –C $\sharp$ –G $\sharp$ –D $\sharp$ –A $\sharp$, and the roots of these two stacks are actually based in the two rhythmically-augmented statements of *e* that conclude the canon.

Finally, the organicity of the motives in Study No. 32 (especially motives *b* and *e*, which both cover the interval of a minor third, along with *a*, which contains a minor third) and the fact that motives like *b*, *c*, and *f* involve inversion of *motion* (i.e., ascending for descending or vice versa) rather than inversion of interval, allow for effects such as

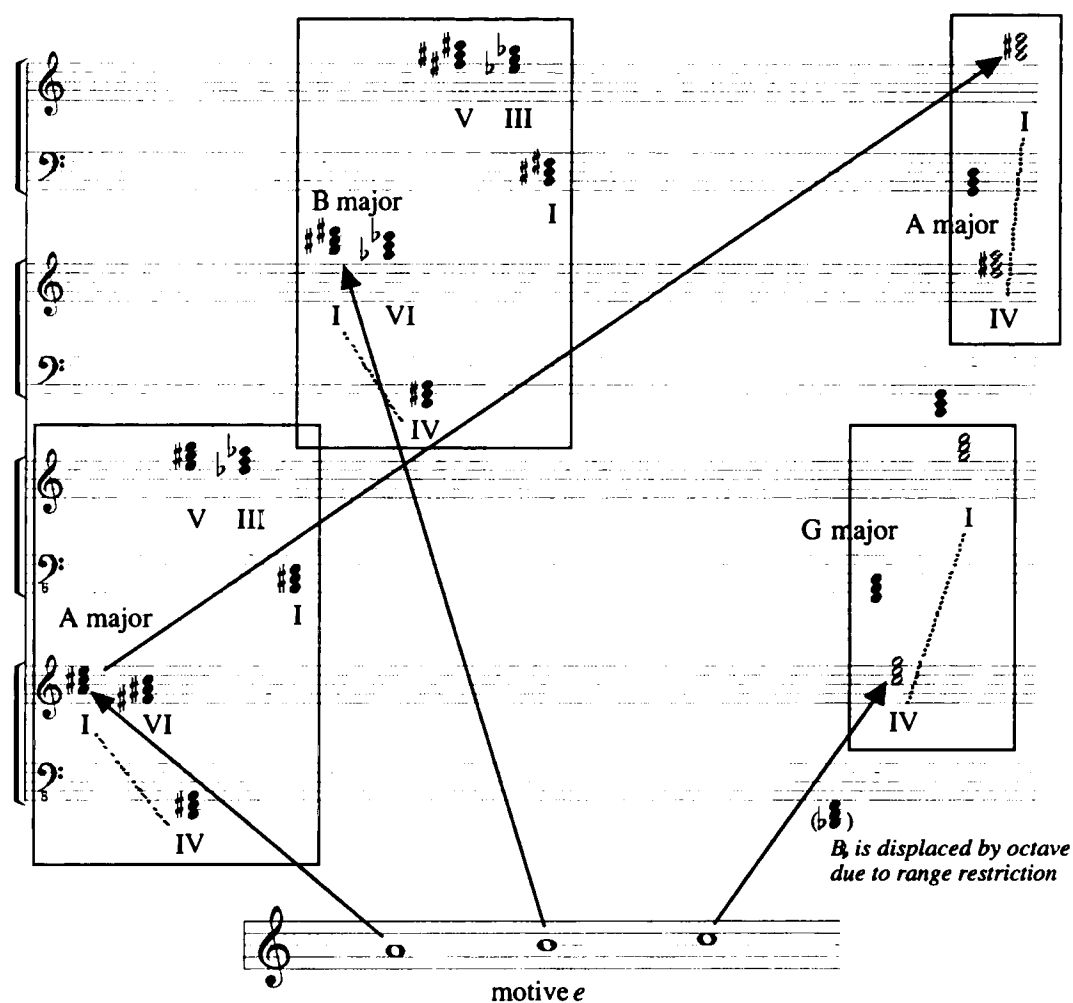


Figure 4-29. Tonal area movement in Study No. 32, from beginning of piece to bottom of p. 9. Occurrences of motive *a* (descending major triad) from sections 1 and 3 are plotted (open notes represent rhythmic augmentation) and key areas related to an enlarged expression of motive *e* (ascending "partitioned minor third"). (Proportions are approximate.)

voice exchanges to occur with some frequency. Figure 4-30 shows several examples: example (a) shows two instances of the same motive inverted and rotating about the same pitch, and examples (b) and (c) are voice exchanges involving the minor third interval.

With its monophonic canon, highly organic motivic content, highly regular metric notation, and consonant level of imitation between voices, Study No. 32 seems at first to be rather unadventurous. However, Nancarrow injects the piece with several elements that introduce different layers of complexity: the widely separated registral layers within

the canon, and the conflicting (and sometimes changing) beat division of quarter note and dotted quarter note, which—when combined with the tempo ratio of 5:6:7:8—create a carefully modulated texture where the distinctness of the individual elements is blurred to varying degrees. As Thomas (1996) notes, once all four voices are active the precise identity of any particular layer is extremely difficult to ascertain, and Tenney (notes to *Wergo Vols. I/II*) speaks of a “perceptual realignment between phrases” (p. 15).

Figure 4-30 consists of three musical diagrams labeled (a), (b), and (c). Diagram (a) shows three staves with musical notation. Arrows indicate the rotation of motive *b* around a central pitch and motive *e* around another. Diagram (b) shows two staves with musical notation, illustrating a voice exchange between motives *a* and *b*. Diagram (c) shows two staves with musical notation, illustrating a voice exchange involving motive *b* in contrary motion.

Figure 4-30. Motivic combinations using contrary motion: (a) motive *b* rotating about pitch D#/E₁ and motive *e* rotating about pitch A (p. 3, top system); (b) voice exchange involving motives *a* and *b* (p. 3, bottom system, top two voices); and (c) voice exchange involving motive *b* in contrary motion (p. 4, middle system, bottom two voices).

Ultimately, however, the strong profile of the *a* motive anchors the listener through this morass, and its reappearance in rhythmically-augmented form (particularly after its absence in section 2) at a point near the end of the canon where the echo distance begins to be clearly heard begins to impart a welcome clarity to the piece. The moment of clarity notwithstanding, once A major seems to be affirmed with the augmented statement of *a* in the top voice, the piece ends without tonal or rhythmic closure: the final two stacks of

open fifths a step apart shift the piece to a B major tonal area, and the last note attack occurs five measures before the inaudible CP. It is a curious twist, allowing the piece's tonal momentum to be overstepped while its rhythmic momentum stops short of completion.

CHAPTER 5

ANALYSIS OF STUDIES NO. 25 AND 45

This chapter will present analyses of two of Nancarrow's more substantial compositions that employ a variety of compositional techniques: Studies No. 25 and 45. This examination will particularly seek to reveal "connecting threads" between these and other Studies, with the analysis of Study No. 25 focusing on its use of a twelve-tone row, palindromic features, underlying rhythmic pulse, use of the layering technique, and connections to Study No. 27. The analysis of No. 45, meanwhile, will concentrate on drawing comparisons between it and a much earlier Study, No. 3.

Chronology

Before considering the analyses of Studies No. 25 and 45, it is helpful to make a few remarks about the chronology of the Studies. It is not possible to pinpoint even the year of composition for most of Nancarrow's Studies, especially the earlier ones. According to Gann (p. 68), Nancarrow refused to put dates on any of his Studies, and by the time he was interviewed about specific works it was often impossible for him to provide much detail about date of composition. That, plus Nancarrow's isolation in Mexico and the fact that many of his pieces were not published until years after their composition, makes the construction of a chronology for the Studies difficult to do with any accuracy or precision.

Gann constructs a chronology of the composition of Nancarrow's Studies based on the best information available to him, including interviews with the composer and inspection of piano rolls and punching scores. His chronology, in part, is as shown in Table 5-1

(compiled from Gann 1995, 69 and “Conlon Nancarrow: List of Works”).

Table 5-1

Gann’s Chronology of Composition of Nancarrow’s Works

Studies	Year completed	Year begun	Year published
3	1948		1983
2a	late 1940s		1984
2b	ca. 1950		
1	by 1951		1951 in <i>New Music</i>
4–30	1948–1960		No. 25 in 1975; Nos. 8, 19, 23, and 27 in 1977; Nos. 4, 5, 6, 7, 9, 10, 11, 12, 14, 15, 16, 17, 18, 20, 21, 24, and 26 in 1984
31, 32, 33, 37	1965–1969		No. 31 in 1977; No. 37 in 1982; No. 33 in 1984
34, 35, 36		1965–1969	—
34, 35, 36, 40, 41	1969–1977		Nos. 35, 36, and 40 in 1977; No. 41 in 1981
43, 48		1969–1977	—
42–50	1977–1988		

Note: Nos. 38 and 39 were renumbered as Nos. 43 and 48, respectively.

From this chronology, one can make the following observations. Study No. 1, the first of Nancarrow’s published Studies, was not the first to be composed; that distinction belongs to Study No. 3, the “Boogie-Woogie Suite.” Also, the chronology reflects a five-year hiatus from composing from about 1960 to 1965; Gann refers to this time period when he characterizes Study No. 31 as Nancarrow’s attempt to “get the old compositional pitching arm back in shape” (p. 129). Further, it can be seen that there is a considerable lag time between composition dates and publication, particularly for the Studies written in the late 1940s. Finally, a number of Studies remain unpublished, including 13 and 30 (both of which Nancarrow was unsatisfied with and “withdrew” the scores), 32, 34, and 42–50.

Analysis of Study No. 25

Study No. 25 conveniently marks the approximate halfway point in the body of Nancarrow's player piano compositions, and its description by Gann as a "treasure chest of every type of idea Nancarrow has worked with" (p. 241) highlights why it makes a suitable subject for study in this chapter. Duckworth (1995, p. 48) described the piece in an interview with Nancarrow as "the most spectacular of those early pieces," to which Nancarrow replied, "Well, it is sort of dramatic. But there's nothing special; it's just a lot of flurry." As shown in Table 5-1, Study No. 25 was only the second of the Studies to be published, appearing in print 24 years after the publication of "Rhythm Study No. 1."

There are several characteristics that distinguish Study No. 25. It is the only one of Nancarrow's Studies that uses a twelve-tone row in the traditional sense, and it is the only Study in which the player piano's sustain pedal is used. The row matrix, as identified by Gann (p. 242), is shown in Table 5-2; forms of the row that appear in this piece are shaded in the table.

Study No. 25 is written entirely in Nancarrow's typical style of proportional notation, with sustained notes written as quarter notes with thick extension lines and staccato notes as eighth notes. The piece's structure, which is shown in Figure 5-1, is based on five layers of distinct types of figures that drop in and out of the texture. Figure 5-1 shows the piece divided into nine sections¹ (proportionally represented in the figure) based on textural changes. The figure also describes the following activity taking place in Study No. 25: appearances of the five musical layers; appearances of the twelve-tone row beginning on D#/E_b and its transformations; and various tempo pulses that occur and the ratios between layers expressing different tempos.

¹ Gann divides No. 25 into eight sections, combining what I have identified as sections 7 and 8 into a single section; I felt that the introduction of the acceleration technique and changes in the pitch content of the chords layer at page 60 merited a separate section designation.

Table 5-2

12-tone Row Matrix for Study No. 25 (Gann 1995, 242)

	I0	I10	I3	I6	I11	I1	I2	I4	I9	I8	I5	I7
P0	D#	C#	F#	A	D	E	F	G	C	B	G#	A#
P2	F	D#	G#	B	E	F#	G	A	D	C#	B _b	C
P9	C	B	D#	F#	B	C#	D	E	A	G#	F	G
P6	A	G	C	D#	G#	A#	B	C#	F#	F	D	E
P1	E	D	G	B _b	D#	F	F#	G#	C#	C	A	B
P11	D	C	F	A _b	C#	D#	E	F#	B	B _b	G	A
P10	C#	B	E	G	C	D	D#	F	A#	A	F#	G#
P8	B	A	D	F	B _b	C	C#	D#	G#	G	E	F#
P3	F#	E	A	C	F	G	A _b	B _b	D#	D	B	C#
P4	G	F	B _b	D _b	G _b	A _b	A	B	E	D#	C	D
P7	A#	G#	C#	E	A	B	C	D	G	F#	D#	F
P5	G#	F#	B	D	G	A	B _b	C	F	E	C#	D#

Nancarrow builds Study No. 25 around forms of the row that begin or end with D#/E_b, most notably P0/R0 and P5, and this pitch is the goal reached at the end of sections 1, 2, 4, and 5. Also, by ending the row with A#/B_b, Nancarrow sets up a possibility of a V-I cadence when P0 is immediately restated, as it is a total of seven times in the first section (pp. 1-7). At the first restatement of P0 in section 1 this V-I formula is already quite evident. Figure 5-2 shows the second page of the score, where the first restatement of P0 occurs; V-I is clearly stated and reinforced by the bottom notes of the arpeggios in the top layer. Further, the appearance in the row of G#/A_b prior to A#/B_b adds a suggestion of IV, although this pitch is not reinforced by its accompanying arpeggio.

Gann points out the major second interval on both ends of the row, which allows Nancarrow to overlap some row statements in section 2 by two notes. The row's two opening pitches, D#-C#, later become the level of imitation stated in the two-voice

Sections:		1	2	3	4	5	6	7	8	9
Layers:	arpeggios/ glissandos	end of sustained chords always punctuated with arpeggios	regularly pulsing @ tempo = ca. 82, pulse changing to 36 and 66	pulsing in $\bullet$ and ♩ values at ca. $\text{♩} = 58$ P7, R8, R11, P0	tempo canon at M2 below 21:25 and 21:25 ratio, unarticulated CP	layers switch roles here from sec. 1: staccato chords punctuate end of arpeggios held with pedal	4-voice canon in 9:10:12:15 ratio, in ♩ and ♩ values		acceleration rate = approx. 2.25%	
	sustained single pitches	P0 started 3 times		←	near end of section R0/R10					
sustained chords		P0 started 4 times	pulse of 41 established at arpeggio's pulse of 66 (ca. 5:8 ratio) P5, R15, R0, 110	7:10 ratio				sustained and staccato chords are combined in one layer	acceleration rate = approx. 4.75% P0/R0 and P10/R10	
	staccato notes		in broken octaves, varying durations							
staccato chords				pulsing in ♩ and ♩ values at ca. $\text{♩} = 82$ P10, P9, P8		brief pulsed section near the end at tempo = ca. 201 P11, P3		P11, P3, P0, P8		

1.4" pause 0.5" pause 1.0" pause

Figure 5-1. Description of layers in Study No. 25 (section durations proportionate).

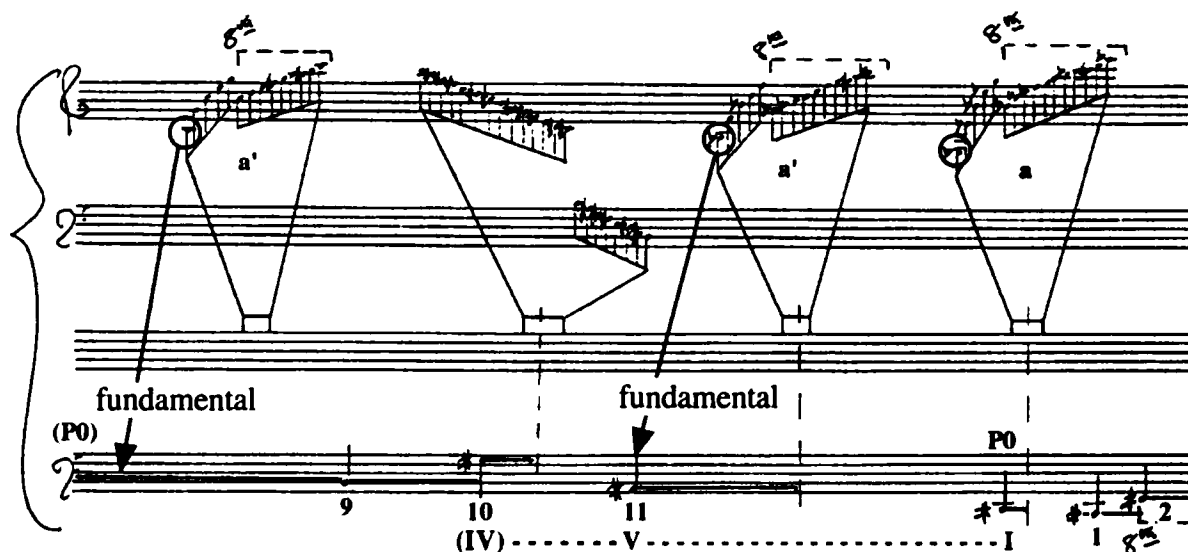


Figure 5-2. Study No. 25, top system of p. 2 from score, showing (IV)–V–I cadence at second statement of P0 in bottom voice. Vertical dotted lines indicate arpeggios articulating breaks in the sustained notes of bottom voice. Figure *a* is a 16-note arpeggio based on the overtone series, while figure *a'* is a 15-note arpeggio in which the fundamental of the overtone series is stated in the sustained bottom voice.

tempo canon of section 4, and row P10, which is used throughout section 8, begins with the second pitch of this important structural interval, C#.

The score segment shown in Figure 5-2 also illustrates several other features of this section of the work: an arpeggio figure *a/a'* (*a* being a 16-note arpeggio stating the first sixteen harmonics of the bottom pitch, while *a'* is a similar 15-note arpeggio in which the fundamental is stated in the sustained voice and harmonics 2 through 16 are stated in the arpeggio), which is prominent throughout the piece; the use of scalar glissandos (here a B major scale) within the arpeggio layer; and the use of arpeggios and glissandos in the top layer to punctuate breaks in the sustained line in the bottom layer. Later in the piece's first section, the 16-note overtone series arpeggio figure is divided so that the top note is in the sustained layer.

Nancarrow's most prominent uses of the layering technique prior to Study No. 25 were in Nos. 3a and 5, both of which were examples of cumulative layering where new layers are continually added to the musical texture and sustained to the end of the piece. Study No. 25 represents a different type of layering—one in which, as shown in Figure 5-

1, the layers intermittently drop in and out of the texture. Another difference is that, while the layers in previous Studies tended to be registrally distinct (a concept carried to the extreme in No. 5, in which the entire keyboard is distributed among thirteen layers with no pitch overlap), there is considerable registral overlap and changing of registers among the layers in No. 25.

All of the layers in No. 25 are established by the third section. As Figure 5-1 shows, the arpeggiated layer is the only one appearing throughout the Study, and in sections 6 and 9 it is the only layer (although in section 6 there are four different arpeggiated layers forming a 9:10:12:15 tempo canon). Tenney comments about the arpeggio/glissando layer (liner notes to Vols. III/IV, p. 15) that "Fast arpeggios and/or glissandos had previously been used in a similar way only in Studies #5 and 12."

The arpeggio/glissando layer contains the following kinds of figures: the 15- and 16-note harmonic overtone arpeggios (figures **a** and **a'** shown in Figure 5-2, and there is also a similar 15-note figure later in section 1 in which the fundamental is present in the arpeggio and the sixteenth harmonic appears in the sustained layer), always ascending through section 3, changing to descending figures in section 4, then a mixture of both ascending and descending; major scale glissandos, both ascending and descending, of up to $3\frac{1}{2}$ octaves in length; arpeggios of major triads and seventh chords, often in combinations; and what Gann calls a "simultaneous glissando" (p. 243) figure, a lengthy glissando combining a major scale going in one direction with the Locrian mode in the other direction (see Figure 5-3). This layer must be what Thomas was referring to when she spoke of this Study as having a high "concentration of tonal figures in a non-tonal setting" (Thomas 1996, 17).

The remaining non-arpeggio layers are nearly constantly stating the row (or portions of it) in one way or another, with the exception of the staccato notes layer in section 2. The layers of sustained single pitches and sustained chords are similar in function and presentation, with the layer of single pitches usually progressing to chords. In the

sustained and staccato chords layers, the row is embedded in a number of different chord structures, including triads and dominant seventh chords (section 1), 8-note stacks of thirds (section 2), 10-note chromatic chords (section 3), 7-note chords containing all the pitches of the Lydian mode (sections 5 and 7), and pairs of unrelated triads (section 8).

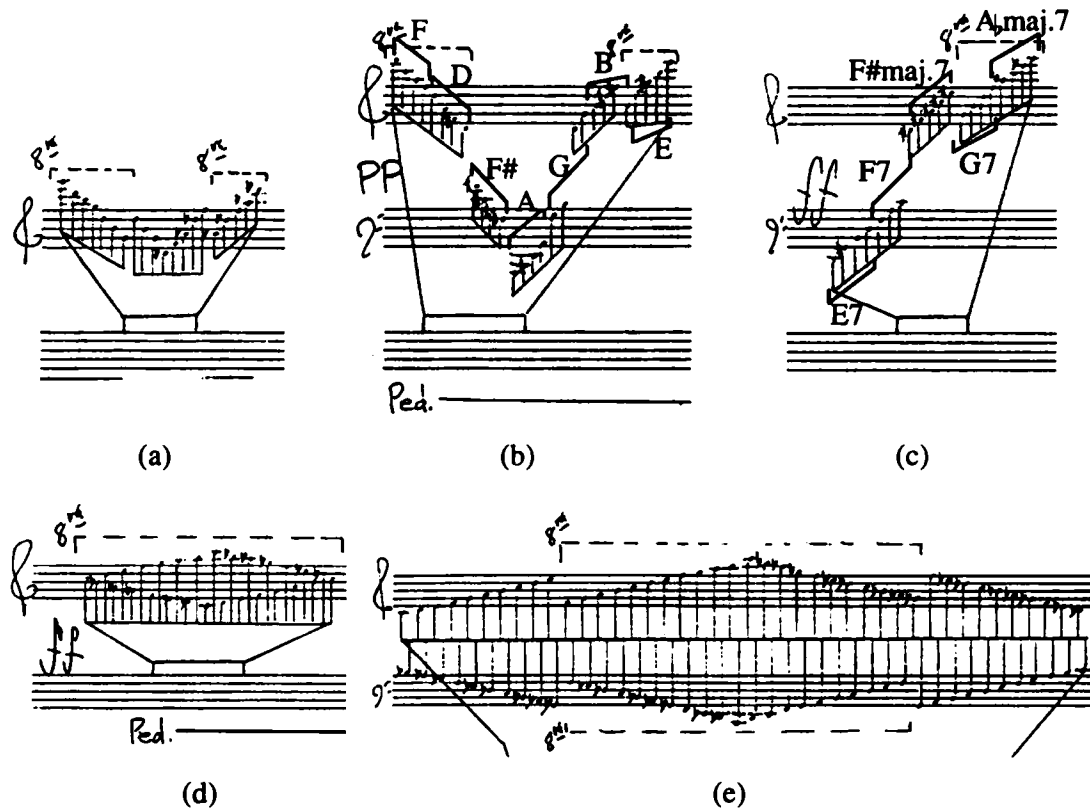


Figure 5-3. Figures found in the arpeggio/glissando layer, Study No. 25: (a) figure combining two different triads (score, p. 18, top system); (b) figure of seven different consecutive triads (p. 34, bottom system); (c) figure of alternating dominant and major seventh chords (p. 40, bottom system); (d) simultaneous ascending major and descending Locrian glissando (p. 36, bottom system); and (e) longer version of same figure (p. 60, top system). All of the examples use the "exploded drawing" technique where the bracket indicates where on the home staff the figure belongs (bracket and home staff not shown in [e]).

Structure

Gann comments that "Never before or since [No. 25] has Nancarrow used so many devices in a single movement, and yet the piece holds together like a rock through the

audacity of its timbral gesture" (p. 242). It is, however, more than timbral gesture that holds No. 25 together. Other characteristics that serve to unify the piece include the use of the tone row and a sense of an underlying pulse.

The nature of 12-tone composition, with its row that can be used forward and backwards, is conducive to the incorporation of palindromic elements, and indeed Study No. 25 contains significant palindromic organization as a result of the presence of the row. Thomas comments on the significance of palindromes and retrograde in Study No. 25:

Retrograde can also play an important part in the construction of the studies, as in Study No. 25, for example, which, as the only study using a 12-tone row, relies more heavily on serial techniques than most. The row accounts for some but not all of the material in the study, however. When it is present it is subjected to various transpositions and retrogrades, and the retrogrades of the row are generally rotated, as well. Some of the other material in the study is organized palindromically, with non-retrogradable pitch-class patterns. It is more characteristic to find, as in many of the studies, only small bits of material subjected to retrograde. (Thomas 1996, 15)

Thomas understates the prevalence of palindromic organization in Study No. 25. Section 4 is entirely a retrograde of section 1, with the exceptions that the sustained layer is reduced to single notes rather than the successively larger chord structures found in section 1, there are no breaks in the sustained layer in section 4 as there are in section 1, and the arpeggios are stated in section 4 at half the tempo of section 1; these characteristics can be seen in Figure 5-4, which shows the relationship of the beginning of section 1 to the end of section 4. The reduction of the sustained layer to single notes in section 4 is a practical matter, since this section is a tempo canon at the major second below in a very low register (the closest level of imitation ever used by Nancarrow, and the only one in which the echoing voice is lower than the original voice) and using section 1's full chord structures at that close interval would result in a very muddy texture. The relationship of arpeggios to sustained notes is maintained in section 4, however, so that arpeggio figures that were punctuating breaks in the sustained line are now stated during a sustained note; compare, for example, the placement of the F# arpeggio and G major scale figures in Figure 5-4. The other effect of the palindrome is that the

Figure 5-4. Palindromic organization of sections 1 and 4: (a) opening of Study No. 25 (p. 1, top system); (b) end of section 4 (pp. 32–33). Arpeggios in section 4 are stated at half the tempo of section 1. Location of convergence point in section 4's tempo canon is shown at the beginning of first system in (b). Notice that arpeggios in (b) are stated at half the tempo of those in (a).

presentation of row R0 in section 4 instead of P0 eliminates a cadence formula such as section 1's IV–V–I (G#–A#–D#) from the end of P0 to the beginning of its restatement.

Another significant example of palindromic structure occurs in section 3, which contains a layer of 10-note block chords and a layer of arpeggios that contain two

different major triads, first descending then ascending. The arpeggio layer contains a palindrome in the intervals between the two triads. Two forms of the row are presented across both the falling and rising triads, and the two different triads in each figure are determined according to the pattern shown in Table 5-3.

Table 5-3

Derivation of Palindromic Interval Pattern Between Arpeggio Triads in Section 3

Row: P7	0	1	2	3	4	5	6	7	8	9	10	11
Falling triads:	B _b	A _b	D _b	E	A	B	C	D	G	G _b	E _b	F
Rising triads:	D	A	G	B _b	B	F#	E	E _b	D _b	A _b	F	C
Row: R11	11	0	1	2	3	4	5	6	7	8	9	10
Row: R8	0	1	2	3	4	5	6	7	8	9	10	11
Falling triads:	G _b	E	G	A _b	E _b	D _b	C	B _b	F	D	A	B
Rising triads:	D _b	F#	A	D	E	F	G	C	B	A*	B _b	E _b
Row: P0	1	2	3	4	5	6	7	8	9	10	11	0

Notes: Dotted line indicates line of symmetry between first and second halves of palindrome; shaded area indicates where missing accidentals in the score (system 44) obscure the relationship between the triads.

*According to the row, this pitch should be A_b. Presumably this is a score error.

The reflexive use of both a prime form of the row and a retrograde in the two concurrent patterns (falling and rising triads) and the migration of both patterns up a half step in the second half of the palindrome yields the following progression of intervals between the triads for the first half of the progression, which is mirrored in the second half:

M3 m2 T T M2 P4/5 M3 m2 T M2 M2 P4/5

Note how Nancarrow rotates the rows for the rising triads (the last in each arpeggio) so that the E_b triad of row P0 ends the section.

Yet another palindromic structure occurs in section 6, where on p. 45 a fifth voice joins the four-voice tempo canon, stating ascending 20-note arpeggios of major triads. The triads, whose roots span the tritone from E_b to A, are stated in this order:

(E_b F# A) F E_b A_b F G F# E | E_b A | E F# G F A_b E_b F

The palindrome begins with the fourth triad and breaks off for two chords after the first seven triads of the palindrome, before restating the fourth through tenth triads in reverse order.

Rhythm and Pulse

Gann cites the “audacity of timbral gesture” as primarily holding Study No. 25 together, but the feeling of a strong underlying metric pulse is also a critical unifying element in a piece such as No. 25, where such seemingly unrelated and uneven rhythmic gestures are juxtaposed between and within sections. The most consistent rhythmic element throughout the piece is the speed of the notes in the arpeggios, which are stated at three related speeds: the fastest speed, a blistering tempo of about 10,560 (176 notes/second), is used in section 1; the arpeggios in section 2 are one-fourth this tempo, or about 2640; and the remainder of the piece is essentially at half the original tempo, or 5280. Even in section 8, where both the sustained and arpeggio/glissando layers are accelerating, the 5280 tempo is maintained within the arpeggios although the distance between their onsets is decreasing. There is one lengthy glissando in section 7 (p. 54) that returns to the fastest (10,560) tempo, but otherwise the 5280 tempo is maintained in the arpeggios from section 3 to the end, and this is the tempo of section 9’s monstrous final arpeggio/glissando figure.

Figure 5-1 identifies the various tempos that appear in Study No. 25 and in which sections they appear. Only sections 2, 3, 5, and 6 contain discernible tempos, with the approximate tempos shown in Table 5-4 represented. As can be seen in the table, there is one tempo continuation between sections 2 and 3 but otherwise little clear relationship

among tempos appearing in different sections; the common tempo, 82, is established at the beginning of section 2 and then changes to one-half that speed, 41, before both those values are used in a mixed context in section 3 (essentially ♩ and ♪ values at ♩ = 82). One other tempo related to tempos 41 and 82 is 123 in section 6.

Table 5-4

Approximate Tempos Represented in Study No. 25

Sec. 2	Sec. 3	Sec. 5	Sec. 6
36			
41			
	58		
66			69
82	82		92
			103
			123
			152
		201	

The variety of note durations in some sections of this piece almost defies any form of systematization, with durations indeed seeming to be random as Gann suggests (p. 242). In section 7, however, all the note durations tend to coalesce into a limited number of duration clusters as shown in Table 5-5; because of variability in score measurements, this leaves open the possibility of durations being derived from multiples of a common background unit. This possibility seems all the more likely since one of the duration clusters (96–98) appears in both layers, indicated by bold type in the table. In the sustained layer, durations are approximately divisible by 4, but in the arpeggio layer any background unit is indiscernible and likely very small (perhaps as small as a single notch on the punching mechanism?).

Table 5-5
Clusters of Durations in Section 7

	Duration measurements in millimeters	Possible background unit?
Sustained layer:		
9 durations:	[7/8 11/11.5/12 15/16/16.5 22.5/23 28/28.5/29 36/37 50/51/51.5/52 67/68/69 96/97/97.5/98	] 2 3 4 6? 7 9 13 17 24 division of 4?
Arpeggio layer:		
7 durations:	[21/21.5 30/30.5 38.5/39/39.5 63/63.5/64 81.5/82/82.5/83 96/97 120.5/121.5	] ???

Sectional Characteristics in Study No. 25

Section divisions in No. 25 are clearly demarcated by the diversity of the textural layers, and sometimes by cadences supported by the tone row. There are also three obvious pauses in the work, at the end of sections 1, 3, and 4 as shown in Figure 5-1. The diversity of the combinations of the layers allows each of the nine sections to be distinctly different. Each section will be briefly described below.

Section 1 (p. 1 through system 13, top of p. 7) establishes immediately the contrast of the arpeggio/glissando layer against the sustained notes layer (see Figure 5-4a). As Gann notes (p. 242), the sustained layer states P0 three times as sustained notes, twice as the root of root position triads gradually rising in register, and twice as the root of dominant seventh chords (although three of the chords at the bottom of p. 6 appear to be

missing the requisite notes—see Appendix A).

Through most of the section, the arpeggio/glissando figures punctuate a break in the sustained layer, although these figures appear elsewhere as well. The tone row does not appear to be operative in the arpeggio/glissando layer, which is only loosely related harmonically to the sustained layer. The punctuating arpeggios and glissandos are more likely to be harmonically related to the notes they reinforce in the sustained layer (see, for example, Figure 5-2). A number of the overtone series arpeggios are of the 15-note variety where either the fundamental or the highest harmonic is not stated in the arpeggio but in the sustained layer (these are marked *a'* in Figure 5-2 and in Figure 5-5, below). By the conclusion of section 1, a considerable level of harmonic coordination between the two layers is achieved and the section concludes on a strong V–I cadence to E_b , as shown in Figure 5-5. There is a 1.4" pause between sections 1 and 2.

Figure 5-5. End of section 1, Study No. 25 (score, p. 7, top system). Arpeggio/glissando and sustained notes layers achieve a level of harmonic coordination prior to and at the cadence.

Section 2 (system 14, bottom of p. 7 to middle of system 35, p. 16) presents an

immediate contrast to section 1 in its change of dynamic level from *ff* to *pp* and in its statement of arpeggios at one-fourth the speed of the first section. The sense of D#/E_b as a tonal goal is also completely dissipated. The first layer to enter is a 4-note major triad staccato arpeggio pulsing at the 82 tempo in the bass; its pitch pattern seems to be random and includes numerous repeated triads. This layer is soon joined by an 8-note chain of thirds that is sustained for random durations; Gann points out the presentation of row forms P5, RI5, R0 and I10 in the roots of these arpeggios. Shortly thereafter a layer of broken octaves, characterized by Gann as an “erratic stride bass” (p. 245) enters in the deep bass; the rhythm here appears to be truly random: the ascending order of the broken octaves never changes, but the distance between the notes of the broken octaves varies widely from about 2 to 63 millimeters. The section concludes with the sustained arpeggio layer stating the 41 tempo, while the staccato layer (now expanded to 7-note major triads) states the pulse of 66, a roughly 5:8 ratio. The statement of row P5 at the end of the sustained layer allows the section to end on a third stack built on E_b.

The two layers in section 3 (middle of system 35, p. 16 to middle of system 46, p. 21) form a rhythmic canon relating by the tempo ratio of 7:10. The top layer expresses two major triads in each 20-note arpeggio (see Figure 5-3a), stating the row according to the palindromic pattern shown in Table 5-3; this layer’s tempo is about 58, with the arpeggios expressing the equivalent of quarter and half note durations. The bottom layer of staccato chords also expresses quarter and half note durations, but in the tempo 82 that carries over from section 2. Gann analyzes the widely-spaced chords in this layer as containing “every note of the chromatic scale except for those a minor third and a minor tenth above the bass” (p. 245), although the missing pitches are actually a minor third and minor seventh above the bass. The three statements of the row in this layer are found first in the lowest pitch, then in the second lowest, and finally in the third lowest. About every two systems, the dynamic level increases by one marking, effecting a crescendo across the section from *pp* to *ff*.

There are two convergences between the two layers in this section: one at the bottom of p. 17, and the other at the bottom of p. 18 (see Figure 5-6). A representation of the entire rhythmic structure of section 3 is shown in Figure 5-7; as the figure shows, in addition to the two convergences noted, the final two possible convergences are subverted, as is the first one, which occurs at the third staccato chord.

The figure displays a musical score segment from section 3, page 18, bottom system. It consists of two staves. The upper staff features a series of arpeggiated chords, with notes labeled (P7), (R11), and (P9). The lower staff shows a series of staccato chords, with notes labeled 2, 3, 4, and 5. A vertical line with an upward-pointing arrow labeled 'convergence' indicates the point where the two layers meet. Below the staves, the text '(underlying rhythm @ ♩ = 82)' is present.

Figure 5-6. Score segment from section 3 (p. 18, bottom system), showing the second of two convergences between arpeggio and staccato chord layers.

Section 4 (middle of system 46, p. 21 to middle of system 59, p. 33) is a two-voice converging-diverging tempo canon at the ratio 21:25. The prime row's opening dyad, D#-C#, defines the pitch levels of the two voices of this tempo canon.

As noted earlier, the lower voice is a complete palindrome of section 1 except that its sustained layer remains in single notes rather than becoming progressively thicker chords (such as those shown in Figure 5-5) and this section's arpeggios are stated at half the speed of section 1's. There is an interesting moment in the canon where one falsely

senses a change in the leader-follower relationship between the voices; as shown in Figure 5-8, the same B $\flat$ major arpeggio appears first in the upper voice at a place where the lower voice is still leading.

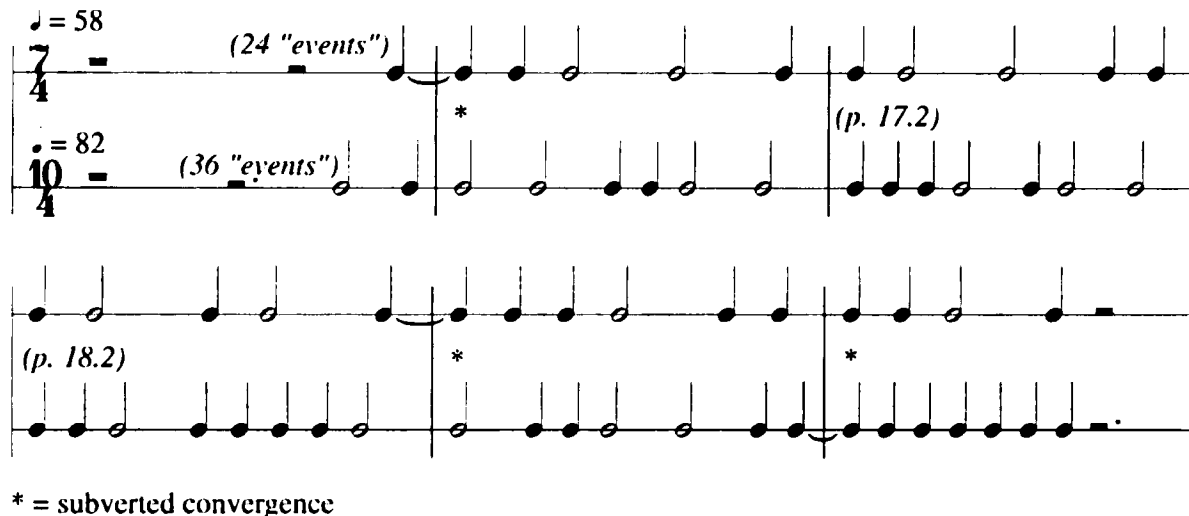


Figure 5-7. Rhythmic scheme underlying section 3; convergence period is 7 beats in top layer (arpeggios) against 10 beats in bottom layer (staccato chords). Only two of five possible convergences are articulated in both voices.

The canon's CP (shown in Figure 5-4b) barely misses being articulated in both voices and occurs very near the end of the section, just prior to the last four notes of the sustained layer. The restatement in reverse of the opening material from section 1, bringing the bass line back to the opening D $\sharp$, and the 1 second pause before section 5 establish the end of section 4 as the close of the first half of the piece.

Section 5 (middle of system 59, p. 33 through first third of system 77, p. 42) introduces a new texture: sustained arpeggio/glissando figures punctuated by staccato chords. The chords in this case are 7-note chords, spelling "a Lydian scale built above the root" (Gann 1995, 245). It should be noted that the Lydian mode bears a relationship to harmonics 8–16 of the overtone series figure.² As Gann notes, the chords spell out first P11

²Specifically, the top octave of this figure is what is known as the "overtone scale," and it contains both a raised fourth (characteristic of the Lydian mode) and a flatted seventh (characteristic of the Mixolydian mode).

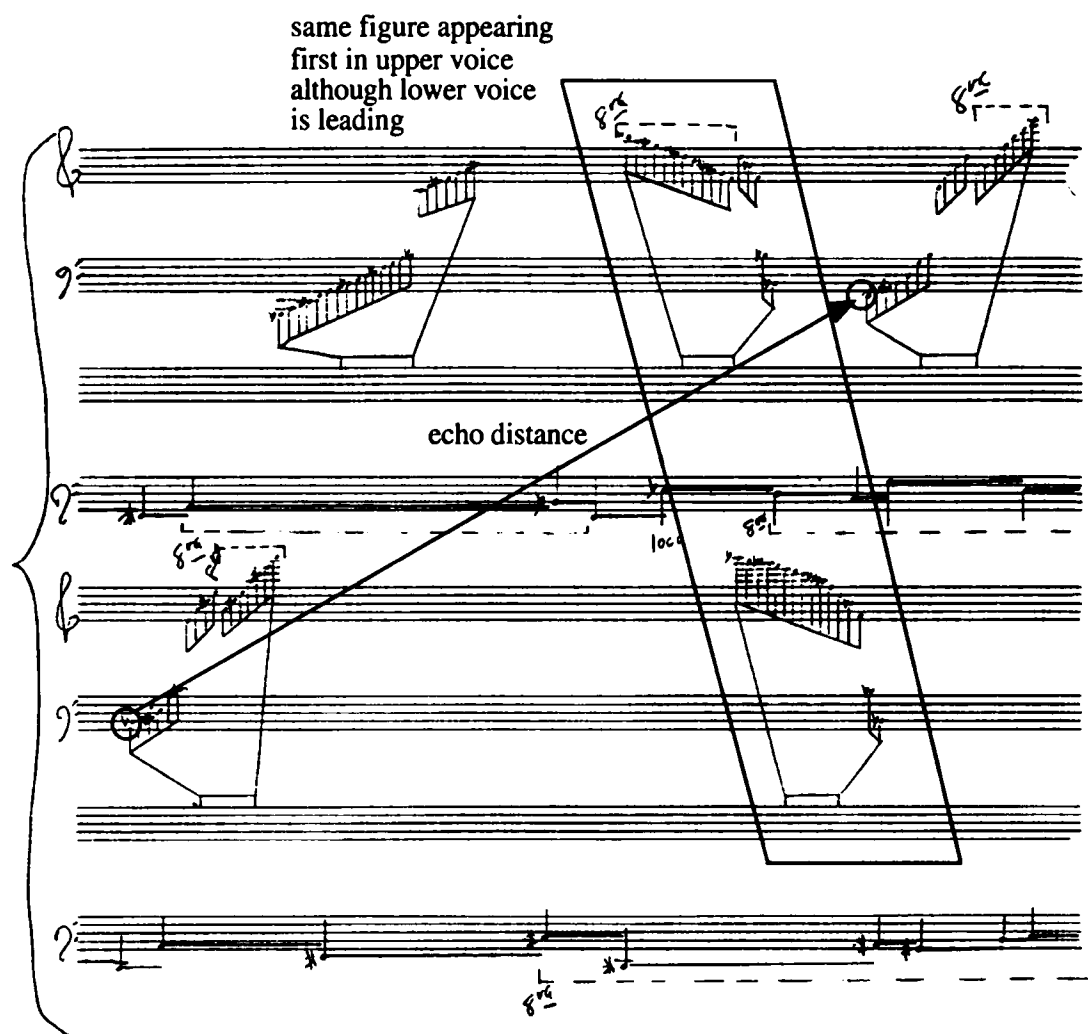
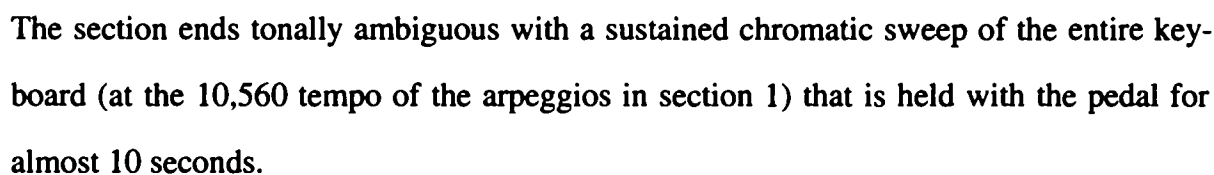


Figure 5-8. Segment from section 4 (p. 28) where same figure appears in both voices, giving impression that upper voice is leading although it is not.

in the bass notes, then P3 in the second-lowest notes. The arpeggio/glissando figures in this section include the most lengthy and elaborate so far, including figures of 210 notes (pp. 35–36), 99 notes (p. 37), 91 notes (pp. 38–39), and 153 notes (p. 39).

Gann finds that “beneath the [arpeggio/glissando] figures the chords seem to imply an underlying beat of about 208, a duration of about 13.7 millimeters in the score” (p. 246). This beat is actually established only very briefly near the end of the section (and my calculation is that it is closer to 201), stating the following rhythmic equivalent:



Section 6 (middle third of system 77, p. 42 to beginning of system 90, p. 49) is another tempo canon, this one in four voices at the ratio 9:10:12:15 with the interval of imitation between all voices being a perfect twelfth. The canonic voice consists of four-note major triad arpeggios (a texture similar to the beginning of section 2) stating a rhythm of durations related 2:1. The canon is converging but, as Gann points out, it ends before reaching its CP, an idea that Nancarrow would expand to an entire piece in Study No. 31.

Near the end of the section, the rhythm changes to just short durations, setting up more frequent possibilities of convergences among the voices. Figure 5-9 shows the last full page of the section, with convergences marked. Over the four-voice tempo canon a fifth voice enters stating 19-note arpeggios at a tempo of 69, creating an overall tempo complex of 27:30:36:40:45 (Gann, p. 246); the arpeggios in this voice consist of major triads spread over six octaves, and state the nearly palindromic series noted earlier.

The tone row does not appear in this section at all. In each of the five voices, the arpeggios stated traverse the interval of a tritone as shown in Table 5-6; each pair of voices in the tempo canon, then, is contiguous in terms of pitch. As shown in Figure 5-9, this results in quite a few dissonances at points of rhythmic convergence (and elsewhere): thus, the section ends with a high degree of rhythmic convergence, but not of tonal convergence. Four of the six rhythmic convergences on this page are at the interval of a major or minor second (reduced from the compound intervals), and there is little tonal connection between the 19-note arpeggio line and the tempo canon lines (with the exception of the A₁ arpeggio). The section ends on an F major arpeggio in the 19-note voice, providing closure with its beginning on an F arpeggio in the first tempo canon voice.

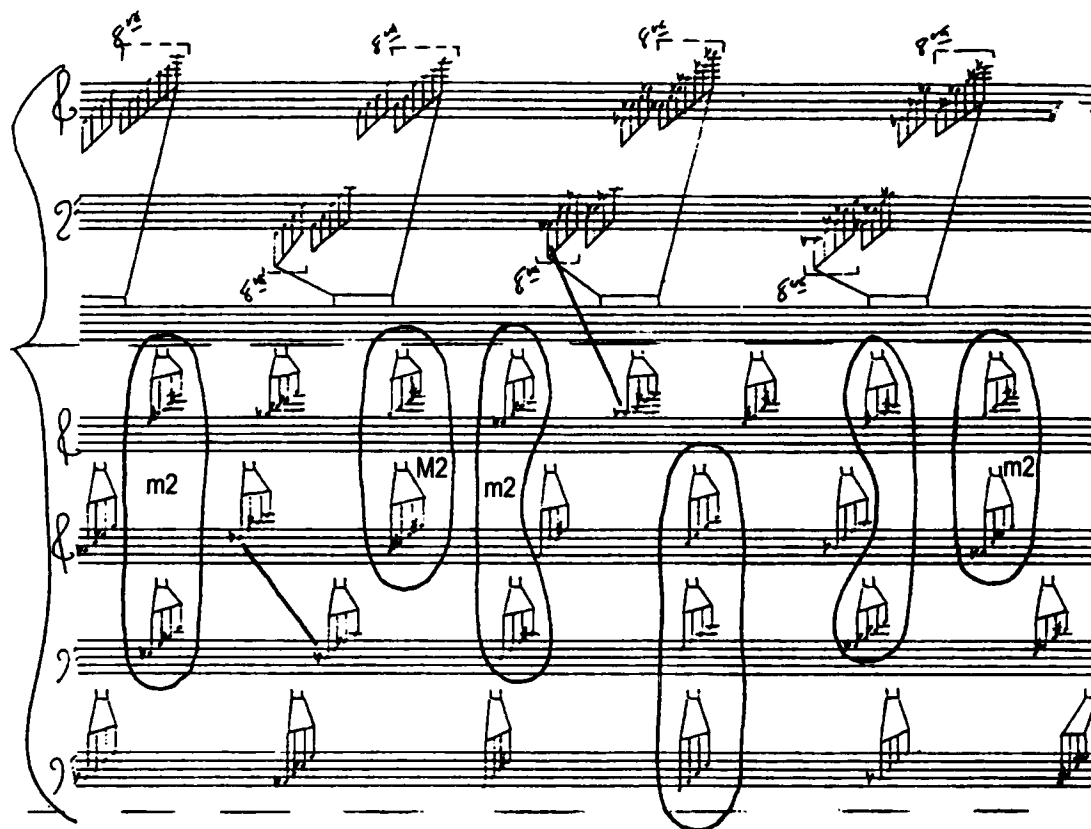


Figure 5-9. Study No. 25 (p. 48), showing multiple convergences in 9:10:12:15 tempo canon in section 6. Rhythmic convergences are circled, and dissonant intervals within rhythmic convergences identified. Lines indicate the same major triad stated in different voices.

Table 5-6

Range of Arpeggios in Each Voice of Section 6, Study No. 25

19-note arpeggios	E _b -A
tempo canon voice 4	D-G#
tempo canon voice 3	G-C#
tempo canon voice 2	C-F#
tempo canon voice 1	F-B

(voices in score order)

Section 7 (beginning of system 90, p. 49 through two-thirds of system 101, p. 60) is very similar to section 5's two layers in the elaborateness of its arpeggios/glissandos and because it restates the 7-note Lydian chords found in the earlier section. As noted earlier

in Table 5-5, the durations in the sustained layer condense into nine duration clusters, while the arpeggio layer has seven duration clusters. The sustained layer combines both sustained and staccato chords.

In addition to arpeggiated triads and major scale glissandos, many of the arpeggio figures in section 7 consist of chains of major seventh chords; see Figure 5-10.

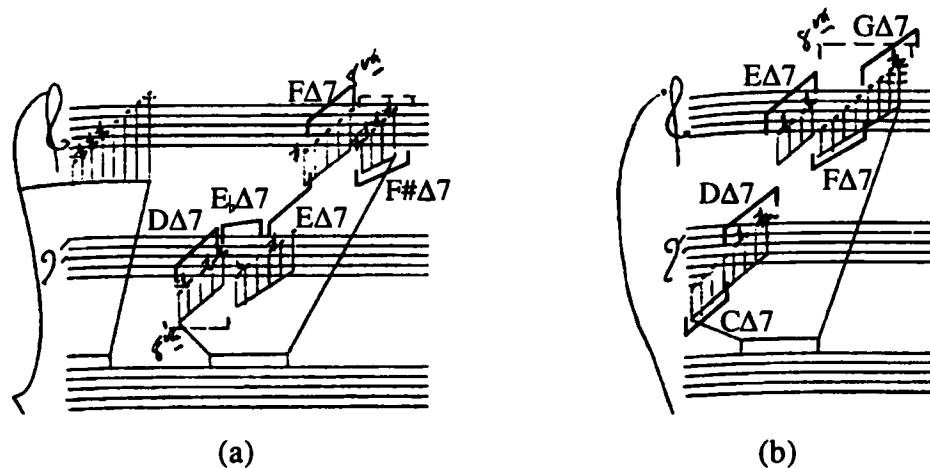


Figure 5-10. Arpeggio figures of chains of major seventh chords, section 7: (a) chromatically rising roots (p. 57); (b) diatonically rising roots (p. 59).

Section 8 (last third of system 101, p. 60 through first third of system 114, p. 73) is separated from section 7 by the rising C major/descending C Locrian simultaneous glissando figure, and its texture is very similar to section 7. Several changes do take place, however: staccato chords disappear from the sustained layer, which is no longer stating the 7-note Lydian chords but two different triads based on the prime/retrograde pairs P0/R0 and P10/R10 (another reference to the D#–C# dyad that dominated section 4); and the arpeggio layer is now stating 20-note rising arpeggios consisting of alternating major and dominant seventh chords, e.g., DΔ7–E7–FΔ7–A_b7–GΔ7, all of which begin on either D_b, D, or E_b (another projection of the interval D#–C#). The arpeggios alternate through the three beginning pitches, but each arpeggio contains a different mixture of seventh chords.

The most important change in this section is the introduction of acceleration. Both layers accelerate at different rates, and both have an underlying rhythm of long and short

durations. The layers are in a rhythmic canon, with the second duration of the arpeggio layer corresponding to the first duration of the sustained layer. Gann identifies the rates of acceleration as 6.44% in the chords and 3.22% in the arpeggios but does not supply any supporting measurements or calculations. My own measurements, taken over a variety of segments in this section, consistently indicate acceleration rates of approximately 4.75% in the chords and 2.25% in the arpeggios. Here are sample measurements in the chords layer (the multiplication factor for acceleration by 4.75% is $1/1.0475 = 0.9547$), where the first duration is 62.5 mm. (the opening duration is a short duration and all measurements for calculating acceleration are to short durations):

62.5 mm. to 43 mm. in 8 steps: $62.5 \times (0.9547)^8 = 43.1$

62.5 mm. to 29.5 mm. in 16 steps: $62.5 \times (0.9547)^{16} = 29.7$

62.5 mm. to 11 mm. in 37 steps: $62.5 \times (0.9547)^{37} = 11.2$

Measurements in the arpeggio layer are almost as consistent (the multiplication factor for acceleration by 2.25% is $1/1.0225 = 0.9780$). The first duration³ is 80.5 mm. (a long duration—all subsequent measurements are to long durations):

80.5 mm. to 62.5 in 11 steps: $80.5 \times (0.9780)^{11} = 63.0$

80.5 mm. to 46 mm. in 24 steps: $80.5 \times (0.9780)^{24} = 47$

80.5 mm. to 24 mm. in 53 steps: $80.5 \times (0.9780)^{53} = 24.8$

This section invites comparisons to two other Studies. The accelerative percentage of 2.25% was, of course, used in Study No. 22 (“Canon $1\%/1\frac{1}{2}\%/2\frac{1}{4}\%$ ”). The use of multiple rhythmic values in an accelerative context is an idea that reappears in Study No. 27 (“Canon $5\%/6\%/8\%/11\%$ ”), reaching a peak in Canon 9 of No. 27 where four different rhythmic values are used.

Section 9 (middle third of system 114, p. 73 to end), described by Gann as an “arpeggiated cacophony” of “continuous single notes” (p. 248), consists of many of the figures already seen: major scale passages, major triad arpeggios,⁴ major and dominant

³In the arpeggio layer, all “durations” given are measurements between the first notes of adjacent arpeggios. The tempo within arpeggios does not change.

⁴In this entire Study, I noticed only one minor triad arpeggio: an F# minor arpeggio presented in the first six notes of section 9. This seems likely to be an error in the score.

seventh chords, and broken octaves (a relic from section 2's staccato notes layer). The extremes of the player piano keyboard are rapidly juxtaposed in two places as shown in Figure 5-11.

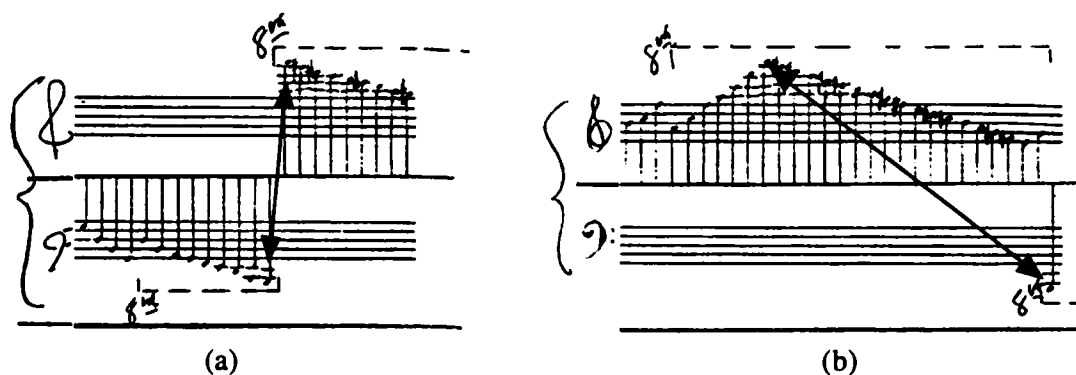
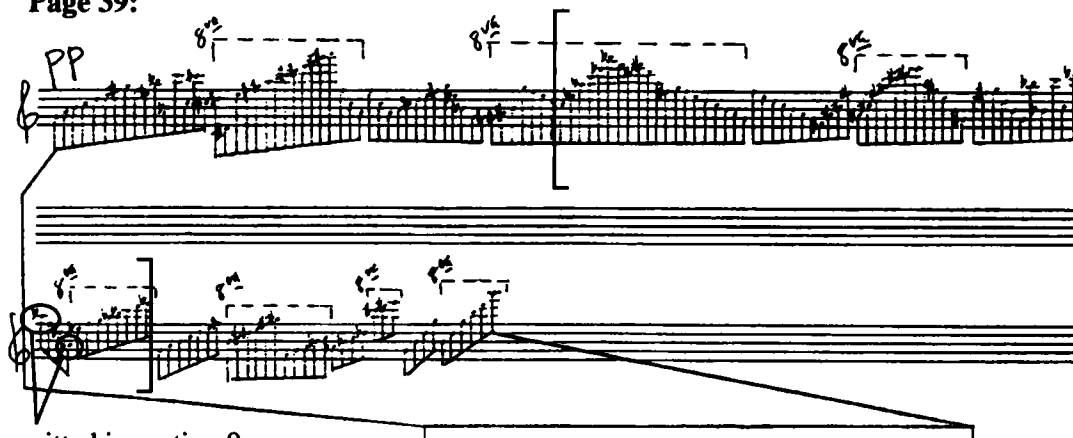


Figure 5-11. Two rapid juxtapositions of keyboard extremes in Study No. 25, section 9: (a) p. 74, top system; and (b) p. 74, bottom system.

Despite the seeming stream-of-consciousness derivation of the pitch material in this section, close examination of the pitch content reveals that several lengthy arpeggio/glissando passages from earlier in the piece are repeated verbatim in section 9. For example, on p. 35 of the score there is a 210-note arpeggio whose first 165 notes are repeated verbatim beginning on the top system of p. 76; and on p. 39 there is a 153-note arpeggio of which about the middle third is almost entirely repeated in the bottom system of p. 73 (see Figure 5-12).

The 15- and 16-note overtone series figure does not appear in section 9 (in fact, it last appears in section 5). The other most recognizable arpeggio/glissando figure from the first eight sections, the simultaneous glissando figure, does not reappear in section 9 until the very end. Section 9's final gesture bears a notable resemblance to this simultaneous glissando figure as it first appears in section 1. The figure from p. 6 (Figure 5-13a) combines the C major scale and the G₇ pentatonic scales to fill out the chromatic space; the scales in the first half of the figure are alternated and split between the bass and treble registers as marked in the figure. The gesture at the very end of the piece (Figure 5-13b)

Page 39:



omitted in section 9

Page 73:

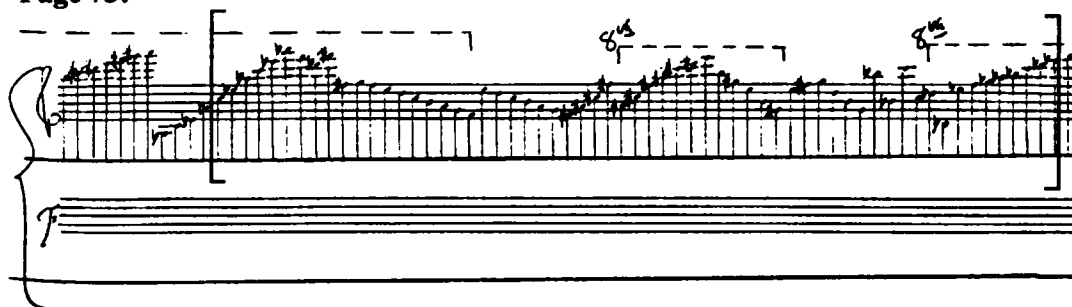


Figure 5-12. Lengthy arpeggio/glissando figure from section 5 (p. 39, top of figure) repeated almost verbatim in section 9 (p. 73, bottom of figure).

is a shorter simultaneous glissando that combines the F# major scale with the G pentatonic scale while maintaining a similar split between the registers; the chromatic space in this figure is not quite filled as B appears in both scales, and C does not appear in either. The gesture ends with an F# major triad arpeggio; the last high C# before the arpeggiated F# major triad creates a V–I tonal movement to the final high F#.

Progression of Pitch Centers in No. 25

Study No. 25's nine sections divide fairly clearly into two parts: the palindrome of section 1 in section 4 closes out the first part, and the second part consists of sections 5 through 9. The most important pitch centers in the piece trace the first three pitches in P0,

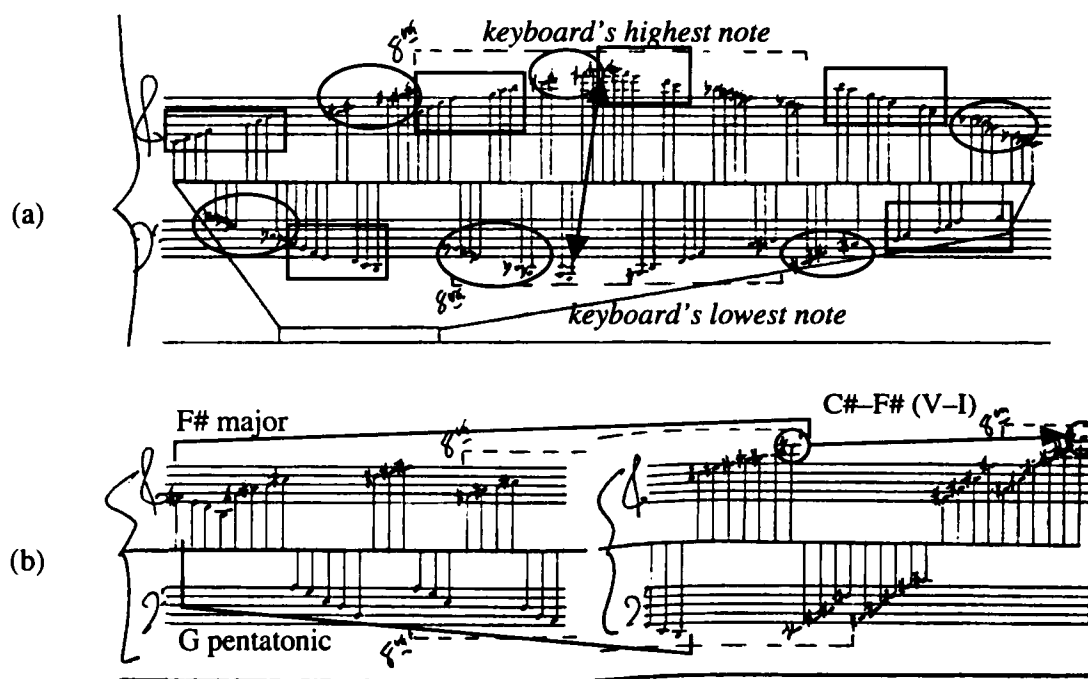


Figure 5-13. Comparison of alternating major (in rectangles) and pentatonic (in ovals) pitch patterns in simultaneous glissando figures from (a) p. 6 and (b) pp. 76–77 of Study No. 25.

D#–C#–F#, with pitch centers D# and C# dominating the piece through section 5, and a V–I movement from C# to F# taking place in sections 8 and 9. D#, then, assumes a submediant role, for which pitches 10 and 11 of the row, G# and A#, briefly create a secondary cadence formula in section 1. Figure 5-14 summarizes important tonal centers in the piece.

Connections Between Studies No. 25 and 27

Study No. 25 appears to have more in common with Study No. 27, which presumably was written at most a few years later, than with any of the other Studies. Several interesting parallels can be drawn. Two of the parallels involve No. 27's clock line (see Figure 3-44, p. 162). The first is a sense of rhythmic randomness that appears in the "erratic stride bass" of the staccato note layer in No. 25's section 2; No. 27's clock line consists of apparently random long and short durations relating by a 2:1 ratio. The

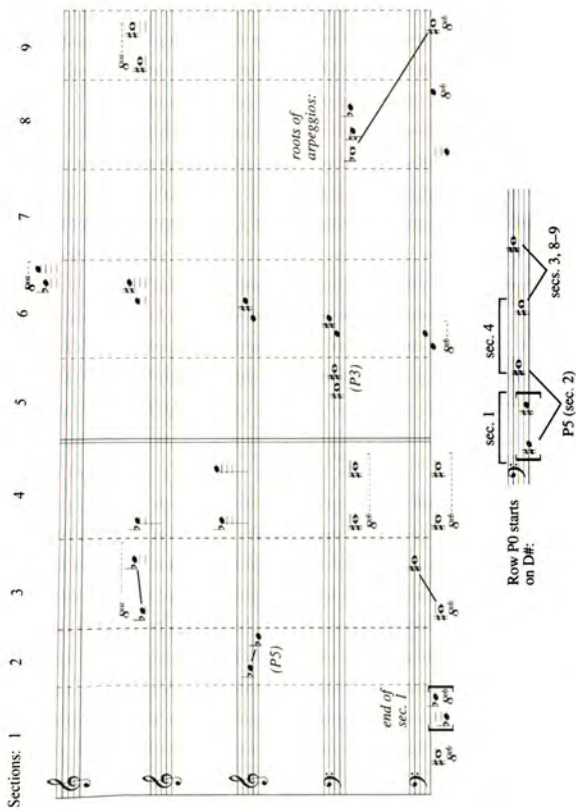


Figure 5-14. Diagram of pitch center progression in Study No. 25 (sections not proportional).

concept of randomness was already discussed in Chapter 3 (pp. 162–63), particularly as it applies to binary decisions, and whenever one encounters seemingly random elements in Nancarrow's work it seems presumptuous to dismiss these elements as truly random.

Another parallel with the clock line occurs in No. 25's sections 3, 5, and 7, where the row is embedded in different positions within different chord structures, much like the clock line's four pitches move to different positions within the structures shown in Figure 1-15 (p. 40). In Study No. 27, however, the clock line pitches occupy varying positions within the chord structures, whereas in No. 25 the row pitches usually maintain the same relative position.

The other parallel concerns the geometric acceleration technique employed in No. 25's section 8, where values in the series are sometimes either halved or doubled to form two different accelerating rhythmic values (such as ♩ and ♪). This same technique was used briefly in Study No. 22, but was brought to full fruition in No. 27 (see Figures 3-45 and 3-55, pp. 165 and 177).

Analysis of Study No. 45

Study No. 45 is a three-movement work that resembles Study No. 3, the "Boogie-Woogie Suite," in its use of a boogie-woogie bass line as the bottom layer of a multi-layered texture that moves through transpositions based on a I–IV–V harmonic pattern. Since these two "suites" are based on the same premise and they virtually frame Nancarrow's compositional career,⁵ the two works offer an almost irresistible opportunity to observe evolution of Nancarrow's technique and compositional goals over his career. Also, since there is not a great deal of substantive information that can be added to Kyle Gann's thorough documentation of the materials and formal procedures used in Study No. 45 (Gann 1995, pp. 256–63), the present analysis will be focused less on that task

⁵No. 3, the first piece Nancarrow composed, was written by 1948, and No. 45 was completed between 1977 and 1984, when it was performed in Los Angeles (Gann, p. 48); see Table 5-1.

than on elucidating relationships with No. 3 (although material supplementary to Gann's analysis will be provided when possible). A particular challenge in analyzing Study No. 45 is the inexactitude of the score, especially in the third movement (of which my copy is quite faded and at times difficult to read).

Much of this analysis of No. 45 will build upon Thomas's work on the concept of multidimensionality and temporal dissonance. The analysis will seek to compare degrees of multidimensionality within Nos. 3 and 45, particularly in the domains of rhythm (i.e., temporal dissonance) and tonality. The analysis will show that No. 45 exhibits a higher degree of both rhythmic (or temporal) and tonal dissonance. The analysis will further reveal that, while temporal dissonance is more evident in Study No. 45, multidimensionality is more pronounced in some of the textures of Study No. 3.

The comparative analysis will primarily involve only movements (a), (b), and (e) of No. 3, as these are the only movements involving a true boogie-woogie bass line. Table 5-7 shows the harmonic pattern and bass line used in each of the movements included in the analysis. As can be seen in the table, Nancarrow used different harmonic patterns for every movement, with actual blues patterns being used only in Study No. 3. The patterns in No. 45 also primarily involve I, IV, and V, but are not traditional 12-bar blues patterns and very short in comparison; No. 45a also incorporates a V/V tonality in the harmonic pattern. The only bass line pattern that is duplicated is that used for Nos. 3e and 45c (although in 45c the pattern is used only sporadically along with several other patterns).

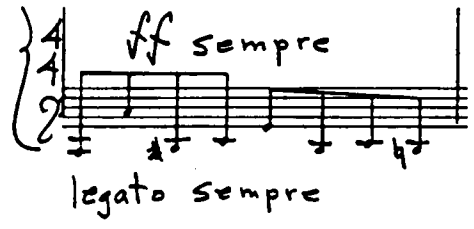
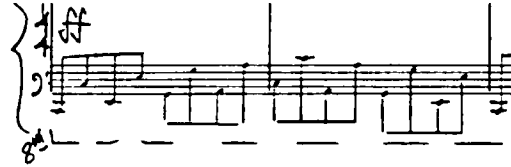
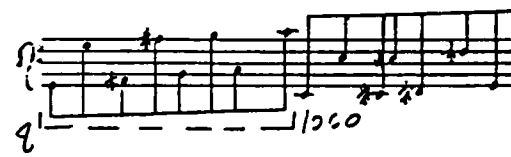
Multidimensionality and Temporal Dissonance in Study No. 3

Thomas's concept of multidimensionality is grounded in the typically multi-layered texture of Nancarrow's works:

The texture of Nancarrow's studies is best described as multilayered: a work typically contains a number of independent, contemporaneous layers that are irreconcilable to a single governing musical process, and that, as a consequence, create an extreme form of polyphony. (Thomas 1996, 52)

Table 5-7

Harmonic Patterns and Bass Lines in Studies No. 3 and 45

Study and Movement	Harmonic Pattern	Bass Line
3a	I-I-I-I IV-IV-I-I V-V-I-I	
3b	I-IV-I-I IV-IV-I-I V-IV-I-I	
3e	I-I-IV-I-V-I	
45a	I-IV-V/V-V (preceded by a 6-note dominant)	
45b	I-IV-V-I	
45c	(no continuing pattern)	

Thomas points to a number of musical features that may contribute to a work's multidimensionality, including "differences in melodic content, register, articulation, rhythm, tempo, and/or relative placement within shared melodic material (canon)" (p. 5). Factors that can increase a work's multidimensionality include "registral polyphonization" (melodic separation of a layer into several layers, similar to compound melody),

mixing together different "time types" (e.g., metered rhythm, proportionally-notated rhythm, and accelerating/decelerating rhythms), and introducing rhythmic variety and meter changes to reduce the incidence of possible rhythmic simultaneities.

Factors that can diminish a work's multidimensionality include melodic coordination of pitch-class content (what Thomas calls "heterophony"), registral overlap, a high frequency of simultaneous note articulations among layers, and coalescence of the layers into a sound mass or groups of layers (a phenomenon that occurs once the "perceptibility threshold" is exceeded and individual layers are no longer perceived as such; common causes of this are sheer density of sound and extreme note speed). Multidimensionality can also be somewhat negated by harmonic coordination among layers, a feature that will be particularly examined in Study No. 45.

For those works that are considered less multidimensional, Thomas says, "What sets [some] studies fundamentally apart from those that are multidimensional is the *reconcilability of the layers to a governing process*: metrically, temporally, in pitch content, and/or phrase structure" (p. 54, emphasis mine). In essence, then, Thomas's concept of multidimensionality depends on the manner in which two features are achieved in Nancarrow's music: *differentiation* of the layers, and *reconcilability* of the layers. Thomas points out further that "There is no absolute boundary between polyphonic and multidimensional textures, of course, but rather a continuum of the reconcilability of layers. This reconcilability is principally a matter of the degree of differentiation among simultaneous layers" (p. 58).

Thomas identifies multidimensionality as being a prerequisite for temporal dissonance, but not all multidimensional textures are temporally dissonant:

There is an intimate link between the concept of multidimensionality . . . and temporal dissonance. Multidimensionality is, in effect, a prerequisite for temporal dissonance: in order for two or more voices to have a dissonant relationship they must first be convincingly differentiated and independent. As we have seen, layers forming a multidimensional texture can be distinguished through differences in register, melodic material, position in a canon, placement of entrance, tempo, meter, and/or rhythmic design. If their differenti-

ation involves significant conflicts in the time parameter, then temporal dissonance is created. Not all multidimensional textures are temporally dissonant, however. (Thomas 1996, 94)

Temporal dissonance, as expressed through multidimensional musical textures, was identified by Nancarrow as his primary compositional goal and led to his development of the tempo canon.

Thomas points to Study No. 3a as having an “intensely multidimensional” texture (p. 165). No. 3a’s concluding eight-layer texture, built through the process of accumulation, was shown in Figure 2-13 (p. 86). Thomas describes the influence of polyrhythm and polymeter on this texture:

Polyrhythm and polymeter themselves are common in jazz In jazz, moreover, polyrhythm and polymeter are not often simple affairs, but rather they can involve quite a number of voices, creating an extended polyphonic texture. One need only look as far as Study No. 3(a), a boogie-woogie work with eight discrete layers, to find an analog. (pp. 26–27)

One of the ways in which Nancarrow increases multidimensionality in this texture is by expanding the original boogie-woogie bass line (line 7 in Figure 2-13) to two additional layers (lines 2 and 4) stating the same line in different registers and at different tempos; here the three layers form a 2:3:5 tempo ratio (essentially, a 3-voice tempo canon) and there are two octaves between each pair of layers. Another multidimensional technique used here to differentiate layers is registral partitioning: although there is some overlap among layers, each layer occupies a distinct registral space. Harmonically, multidimensionality is mostly expressed through layer 5, which states a F#–C# open fifth in four octaves (pitches not shown in Figure 2-13) against the prevailing C major of the other layers. Also, the transpositions to subdominant and dominant levels according to the blues harmonic pattern are slightly staggered among the different layers according to where they are in their rhythmic patterns, and this contributes to the differentiation of the layers and a greater sense of multidimensionality.

Temporal dissonance in No. 3a is also fairly pronounced, especially in the final texture. Gann identifies, in addition to the 2:3:5 rhythmic ratio between the three ostinato

line layers, five isorhythmic layers which are notated in the same meter as layer 7 but whose rhythmic patterns are 23, 29, 39, 43, and 47 sixteenth notes in length.⁶ Nowhere in this concluding texture is there a simultaneity involving all the layers—not even at the last note, leaving the temporal dissonance unresolved. Thomas finds overall that Study No. 3a, at its conclusion, reaches a state of such temporal and textural density that it perches precariously on the perceptibility threshold “beyond which it is difficult to perceive the individual layers and, by extension, the multiple processes and multidimensionality they create” (p. 60).

Thomas refers to No. 3b as having a more conventional texture that is not so extremely multidimensional as No. 3a's. The movement opens with two layers: the boogie-woogie bass line and a melodic layer. Both layers are rhythmically coordinated in a 5:3 swung division of the quarter note beat, and, although the melodic layer's note attacks often occur “around the beat,” they are coordinated with the bass layer's short notes. The entrance of a third layer, however, introduces an element of multidimensionality in an interesting guise: its straight quarter and eighth note rhythm sounds dissonant in the prevailing swung-note context. Thomas points specifically to the fourth layer, with its basic pulse of a dotted quarter note, as “setting up the multidimensional character of the remainder of the study” (p. 56). As in No. 3a, here Nancarrow uses numerous meter changes and isorhythmic patterns of varying lengths (including several arithmetical decelerations, or “countdowns”) to introduce further rhythmic variety (see Figure 5-15). At the thickest point of the piece there are five distinct layers (see Figure 5-16) but in this canonic section (this is a traditional canonic texture, not a tempo canon), rhythmic variety has been reduced to small rhythmic groupings that mostly match the bass line; the grouping of 3 + 2 + 3 sixteenth notes, for instance, merely subdivides the first value of the bass line's 5 + 3 pattern. Thus, as the textural density in this piece increases, the temporal dissonance decreases. In the section shown in Figure 5-16, how-

⁶These patterns are not, as Gann implies (p. 77), based all on prime numbers as 39 is not prime.

ever, there is a greater degree of tonal dissonance between the bass and upper layers with the introduction in the melodic layers of decorative minor seconds.



Figure 5-15. Temporal dissonance in Study No. 3b: use of $\frac{6}{8}$ and $\frac{12}{16}$ meters in upper three voices against $\frac{4}{2}$ meter in bass ostinato voice (score, p. 7).



Figure 5-16. Five-layer canonic texture from Study No. 3b (score, p. 21). Numeric patterns indicate sixteenth note rhythmic groupings.

Thomas says very little about multidimensionality in No. 3e although temporal dissonance is more intensely displayed here than in 3b. This movement is a good example of a piece in which multidimensionality is confined almost entirely to temporal dissonance; the temporal dissonance itself is expressed more successively than cumulatively. The straight eighth notes of the blistering fast bass line are never accompanied by more than three other layers at a time, but a continuous stream of rhythmic patterns that conflict with the bass line's $\frac{4}{4}$ meter challenge the listener's metric orientation. Some of these patterns, which share the eighth note pulse with the bass line, include a $\frac{12}{8}$ pattern, a $\frac{7}{4}$ pattern, a $\frac{9}{8}$ pattern, and patterns of 6 + 7 + 4, 4 + 3 + 3, and 7 + 3 sixteenth notes; other patterns align less frequently with the bass, such as a $\sim \text{♪}$. (5 + 3) pattern aligning with the half note pulse in the second statement of the harmonic pattern.

Multidimensionality in No. 3e is tempered by consistent tonal coordination (the major/minor tonic triad is particularly prominent) and repeated use of the same melodic motives among the layers (see Figure 5-17). Differentiation of the layers is denied by the consistent harmonic coordination among the layers; at the transpositions of the bass line to IV, V, and back to I, the melodic layers are always in harmonic accord with the bass.

The final section of the piece is characterized by a rhythmic disintegration in the bass layer, with first every ninth eighth note being replaced by a rest, then every eighth, seventh, and so on (Gann, p. 80). Concurrent with this process is a melody in the top layer that is progressively lengthened rhythmically, allowing the layers to further separate (especially when the $\frac{15}{16}$ meter is introduced in the second statement; see Figure 5-18).



Figure 5-17. Recurring melodic fragments in Study No. 3e, showing use of major/minor tonic: (a) score, p. 4; (b) score, p. 7 (this segment has a $\frac{9}{8}$ rhythmic pattern in the middle line).

Multidimensionality in No. 45

Turning now to Study No. 45, this examination of multidimensionality will focus primarily on degrees of accord among the musical layers in terms of rhythm and tonality. Tonally, No. 45 is somewhat of an anomaly among Nancarrow's late Studies. Once he began writing tempo canons, it was unusual for Nancarrow to use the same tonality in all musical layers at once; see, for instance, the variety of levels of imitation listed in Table 1-2 (p. 45). As Gann noted, in the later 2-voice tempo canons, beginning with No. 40,

Nancarrow's favored interval of imitation was a tenth, which imparted a "bittersweet and ambiguous major/minor harmony" (p. 203).

The figure consists of three musical staves, labeled (a), (b), and (c), each showing a piano score with a treble and bass clef. The time signature is 12/16. Staff (a) shows a melodic line in the treble clef with a series of notes and rests, and a bass line with a series of eighth notes. Staff (b) shows a similar melodic line in the treble clef, but with a more complex rhythmic pattern in the bass line. Staff (c) shows a melodic line in the treble clef with a series of notes and rests, and a bass line with a series of eighth notes. The notation is handwritten and includes various musical symbols such as notes, rests, and accidentals.

Figure 5-18. Rhythmic disintegration in bass line and progressively slower melodic statement at end of Study No. 3e: (a) p. 21; (b) p. 23; and (c) p. 25.

We begin by examining the background rhythmic scheme in the three movements. Gann characterizes No. 45 as exhibiting "chaotic rhythmic energy" (p. 10). The

difference that one most immediately notices between Nos. 45 and 3 is the change in the rhythm of the bass line from straight, rapidly pulsing (or consistently swung) eighth notes in No. 3 to a loping, lopsided rhythm in No. 45 (see Table 5-7). The unevenness of the bass rhythm in the slower middle movement (45b), in particular, could remind one of a weaving drunk. Gann points to this uneven rhythm as an “irrational ‘spastic rhythm’” (p. 256) of fifteen durations that runs through all three movements of No. 45 as well as Nos. 46 and 47 (all of which were originally part of a six-movement work).⁷ This rhythmic pattern is not subject to rational notation and is notated proportionally by Nancarrow (the fifteen-duration “spastic rhythm” can be seen in Table 5-7 in the entry for No. 45a). All Nancarrow could recall about the derivation of the rhythmic series was that he “took a bunch of [tempo] templates and started putting them together, purely intuitively, and finally came up with that proportion that I liked” (Gann 1995, 257). The exact derivation of the series has remained a mystery.

As it turns out, a good share of the durations in the series can be deduced in a systematic way and related to consistent divisions of the time continuum. Gann reports (p. 257) the measurements of the values in the rhythmic series at its slowest point, expressed as averages in millimeters on the score. Using Gann’s measurements, I multiplied each by 10 to convert all to whole numbers and added all the values together to determine a cumulative value for the series:

$$378 + 357 + 250 + 220 + 270 + 132 + 460 + 128 + 318 + 357 + 92 + 171 + 382 + 224 + 435 = 4174$$

Dividing each duration by the cumulative value allows each to be expressed as a percentage of the whole series. An examination of the resulting values reveals that in a number of cases the values could be represented quite closely by fractions with

⁷This point is later contradicted when Gann notes that 45b’s bass rhythm is a 24-note ostinato derived from “a measure divided simultaneously into three, four, five, and fourteen equal divisions” in which the bass line’s rhythmic values “skip freely back and forth between the 3-, 4-, and 5-tempos” (p. 258). The second movement does not, in fact, contain the 15-note “spastic rhythm.”

denominators of prime numbers, especially 11, 17, 19, 29, and 31. This characteristic begins with the first value, 378, which is extremely close to $\frac{1}{11}$ of 4174 (379.5).

Recognizing that Gann's measurements represent an average approximation from Nancarrow's imperfect score, I then created a grid dividing the value of 4174 equally into 11, 17, 19, and 31 parts and looked for close matches with the duration measurements. It turns out that all but four of the rhythmic values can be related to the resulting values; two of the remaining values can be related fairly closely to the common fractions $\frac{1}{2}$ and $\frac{3}{4}$, indicating that Nancarrow may have also used a 4-template; the remaining two values are not close enough to any of the values in the grid to be related to them with any certainty (although one is fairly close to $\frac{29}{29}$). See Table 5-8 for full results.

The ten values that closely match the prime-number-denominator fractions in Table 5-8 are close enough (particularly the values 378, 735, 1475, 1607, 2195, 2962, and 3515), within a reasonable margin of error to accommodate inexactitude of measurements from the score, to be compelling evidence that these divisions may have been the ones used by Nancarrow to create the rhythm. Table 5-8 also shows that two contiguous pairs having the same denominator appear in the table ($\frac{3}{17}$ and $\frac{4}{17}$, and $\frac{16}{19}$ and $\frac{17}{19}$), further supporting this conclusion. It is particularly interesting to note that Nancarrow settled on some divisions that have historically been used to create equal-tempered scales, most particularly the 19-division (proposed by Salinas in 1577) and the 31-division (proposed by Vicentino in 1555); see Jorgensen (1977, p. 376). Given Nancarrow's penchant for use of references to historical tuning systems in his rhythmic schemes, even though this may have been a serendipitous accident it is not entirely surprising to find such a connection.

Of course, Nancarrow was aided in the creation of this rhythm by having the benefit of the modified punching mechanism that allowed him to punch a hole on the piano roll virtually anywhere on the time continuum, an ability he didn't have until Study No. 22. The availability of this technique also created another difference between Nos. 3 and 45 in the constancy of the underlying rhythm. In No. 3 the rhythmic background unit stays

Table 5-8

Possible Derivation of Rhythmic Values in No. 45's "Spastic Rhythm"

Step 1: Rhythmic series is given as a series of relative values and running cumulative:

Values: 378 357 250 220 270 132 460 128 318 357 92 171 382 224 435

Running

cumulative: 378 735 985 1205 1475 1607 2067 2195 2513 2870 2962 3133 3515 3739 4174

Step 2: Values from grid dividing 4174 into equal parts of 11, 17, 19, and 31 (one value is derived from a 13-division) are compared to running cumulative values to identify closely matching note attacks (the derived value identifying the note attack defines the end of the previous note):

Individual Value	Cumulative Value	Closest Match
378	378	1/11 (379.5)
357	735	3/17 (736.6)
250	985	4/17 (982.1)
220	1205	9/31 (1211.8)
270	1475	6/17 (1473.2)
132	1607	5/13 (1605.4)
460	2067	none; value is 0.495 of cumulative total, suggesting relation to 1/2 (2087)
128	2195	10/19 (2196.8)
318	2513	none; value is 0.602
357	2870	none; value is 0.688 (20/29 = 0.690)
92	2962	22/31 (2962.2)
171	3133	none; value is 0.751 of cumulative total, suggesting relation to 3/4 (3130.5)
382	3515	16/19 (3514.9)
224*	3739	17/19 (3734.6)

*The final value, 435 ($435/4174 = 0.104$), approximates $3/29$ (0.103)

constant throughout a movement,⁸ while geometric acceleration takes place at the end of two movements in No. 45. In order to create an effect of tempo changes in No. 3, Nan-carrow relied on the technique of arithmetical deceleration near the end of 3b. The constancy of the eighth note is eroded at the end of No. 3e by the "dropping out" technique shown in Figure 5-18 that makes for a very effective winding down of this whirlwind-like

⁸With the exception of a curious notation "accel. poco a poco" at the beginning of No. 3b, which is unexplained except for the possibility that it refers to the natural slight acceleration that takes place as the piano roll dwindles during playback.

piece.

No. 45a

Interestingly, Tenney points to the unevenness of the bass octaves as contributing to a degree of registral polyphonization in the bass layer:

There is . . . one additional perceptual result of this rhythmic irregularity which I could not have predicted before hearing it: the upper and lower notes of the successive broken octaves seem to separate into two distinct “streams,” each of which is more internally cohesive—and more clearly segregated from the other—than seems to be the case with a line of broken octaves in an even, regular rhythm. Thus, what’s usually heard as a single voice is here split into two voices . . . (Tenney, Vol. V, p. 8)

In addition to the inherently uneven bass line rhythm described in the previous section, No. 45a is structurally uneven in another important way. The 192-note *color* (pitch pattern) of the bass line is not evenly distributed among the four harmonic areas of the harmonic pattern but is divided as follows:

$$\begin{array}{ccccccc} \text{I (C)} & \rightarrow & \text{IV (F)} & \rightarrow & \text{V/V (D)} & \rightarrow & \text{V (G)} \rightarrow \text{I (C)} \\ \text{♩ beats: [6]} & & 54 & & 46 & & 50 & & 42 & = & 192 \end{array}$$

Thus the pattern spends the most amount of time in I, and the least amount of time in V. The *color* is preceded by a 6-note dominant pitch pattern (as shown in Table 5-7), and each new transposition level is preceded for six notes by its dominant.

There are eight complete statements of the 192-note *color*. The smallest common denominator of the 15-note rhythmic pattern and the 192-note pitch pattern is 960 notes, or every five statements of the *color*. The beginning of the 15-note rhythmic pattern rarely aligns with a change in transposition level, doing so on only six occasions (numbers in parentheses indicate during which *color* the alignment occurs): IV₍₁₎, I₍₃₎, V₍₃₎, IV₍₆₎, I₍₇₎, and V₍₇₎.

Temporal dissonance in No. 45a is achieved in several ways. First, the piece mixes “time types,” with the result that there are virtually no (if any) simultaneities between the upper (melodic) layers and the bass. The first two time types are metered rhythm and

irregular (proportionally-notated) rhythm. Along with the proportionally-notated bass in No. 45a there is no actual metered rhythm, but there is a strong sense of meter in some of the melodic figures of the upper layers, especially earlier in the piece. Some examples are shown in Figure 5-19.

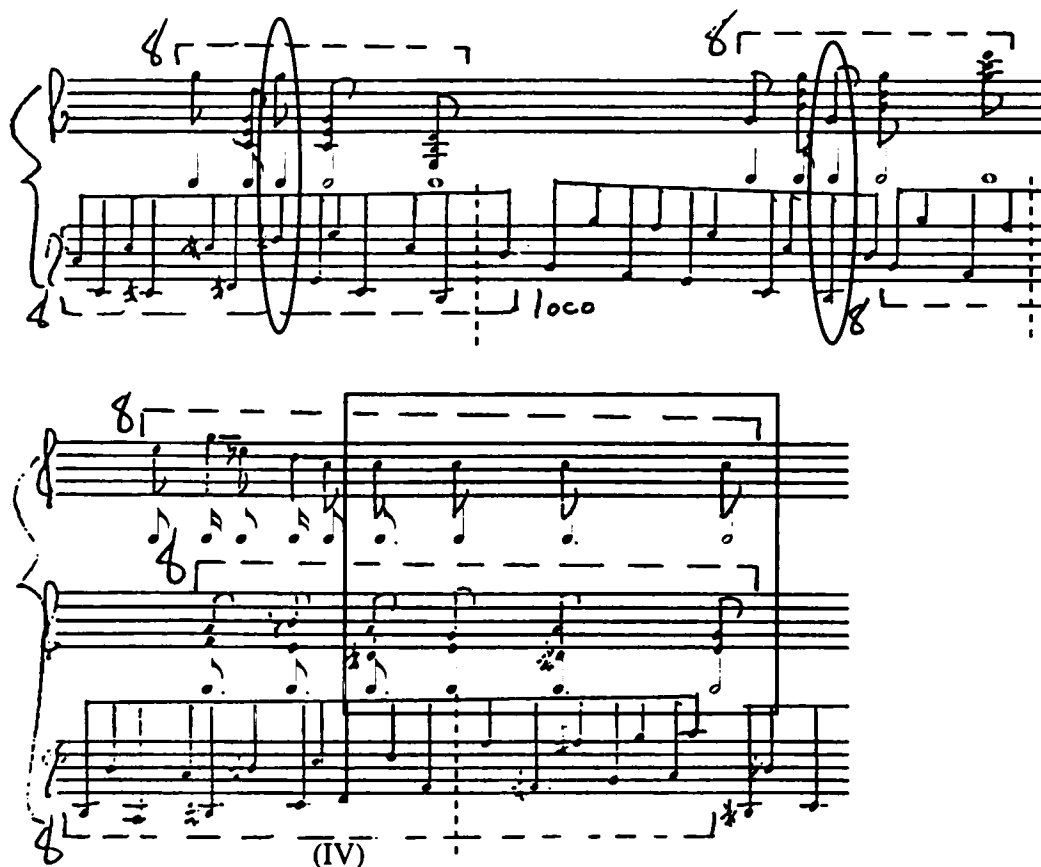


Figure 5-19. Passages in No. 45a's melodic layer that emulate metered rhythm: (top) p. 2, bottom system; (bottom) p. 3, top system (note the rhythmic coordination between the top two layers). Rhythmic approximations are shown between the staves; dotted lines in bass layer indicate divisions of the 15-note rhythm. In the top example there appears to be a rhythmic simultaneity between the layers on the third note of both statements in the top layer. In the bottom example, an area of additive deceleration in the top layers is indicated by the rectangle.

Further temporal dissonance is created by the use of acceleration, the other "time type" identified by Thomas. A case of additive deceleration is shown in Figure 5-19. The eighth and final section of No. 45a introduces geometric acceleration in the form of scalar glissandi inserted within a line of octaves (see Figure 5-20). The distance between the

first pair of glissandi is 77 millimeters; in 18 steps, this measurement decreases to 25 millimeters, an overall acceleration of about 6.5% (although the amount is variable—for the first 8–10 steps of the accelerative process the acceleration is closer to 4.75%). Nan-carrow used this technique in a very similar way once before, with arpeggio figures in section 8 of Study No. 25. In both cases, the figures themselves are not accelerating, but the space between their attacks steadily decreases.

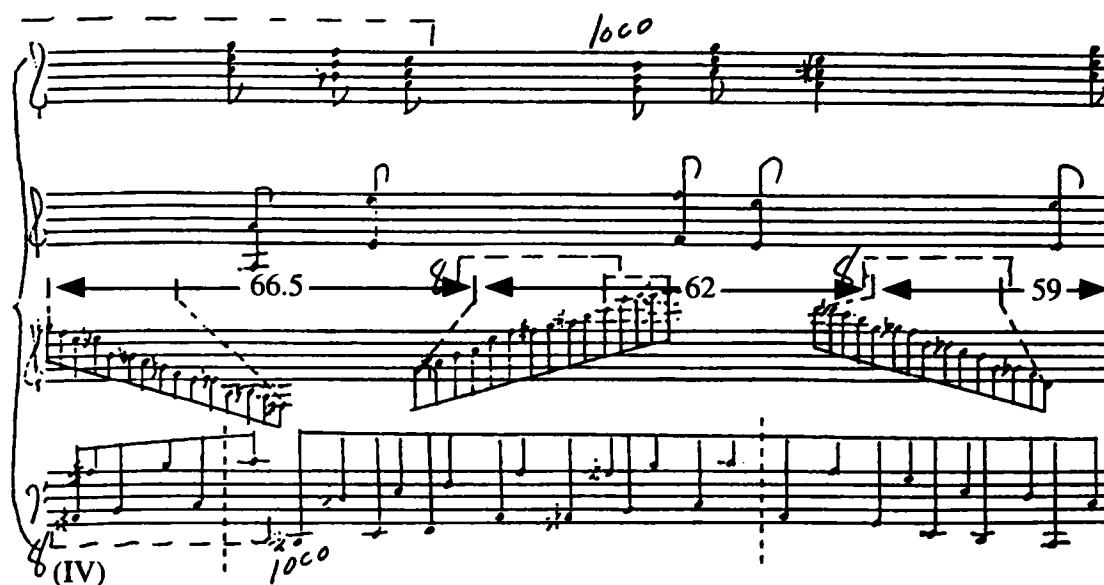


Figure 5-20. Acceleration between glissandi in final section of No. 45a (score, p. 19). Millimeter measurements between glissando attacks are given. Acceleration rate ranges from 4.75% to 6.5%.

The subversion of potential simultaneities also contributes to temporal dissonance, and, as Gann points out, the presence of converging canons that are cut off just prior to their convergence is a hallmark of Study No. 45. In 45a, there is one such canon, a two-voice canon in an approximate 5:7 tempo ratio. The ending of this canon, with its near-convergence, is shown in Figure 5-21. There appears, however, to be a rhythmic convergence between all three voices on the last note of the canon in the leading voice; also at this very spot, there is a break in the broken octave pattern of the bass line (the only such break in this movement). The canon's ending also neatly coincides with arrival at the 6-note dominant leading to the return to I.



Figure 5-21. End of tempo canon in No. 45a, stopping just short of convergence (score, p. 14). Circled area indicates break in broken octave pattern of bass line. The dotted lines between layers indicate an apparent rhythmic simultaneity, which happens to immediately precede the arrival at I's dominant. Arrows indicate echo distance.

Earlier in the canon, Nancarrow sets up the canon melody so that the two layers neatly switch between triads and single pitches; see Figure 5-22.

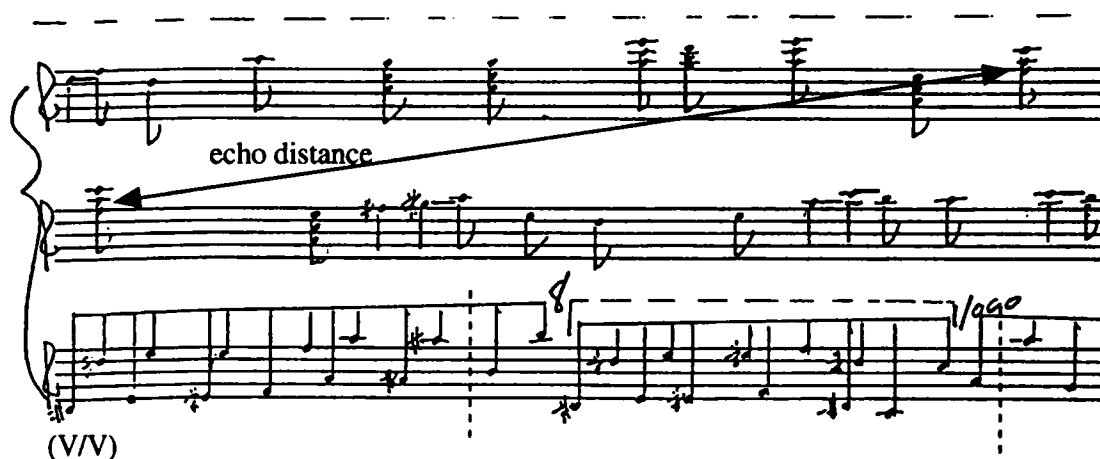


Figure 5-22. Earlier section of 5:7 tempo canon in No. 45a (score, p. 13). Arrow shows echo distance. Canon is effectively constructed so that layers switch between triads and single pitches.

In sum, then, temporal dissonance in No. 45a is achieved through the irregular bass rhythm, mixing of the three time types, and the subversion of potential simultaneities as most significantly expressed through the thwarting of the convergence of the tempo

canon. As shown in Figures 5-19, 5-20, 5-21, and 5-22, the melodic materials of No. 45a consist primarily of single pitches, triads (overwhelmingly parallel major triads), octaves, and glissando scalar figures. The glissando figures (see Figure 5-20) are generally either one or two octaves long and state the boogie-woogie bass line pattern (without the broken octaves) beginning on the dominant (i.e., the pattern shown in Table 5-7); the glissandi are almost always stated in a direction opposite to the direction of the bass (the last two figures in Figure 5-20 being notable exceptions).

The tonal environment is primarily major with jazz inflections, e.g., flatted third and seventh scale degrees appearing along with the same degrees unaltered, as well as occasional diminished seventh chords. There is considerable tonal coordination among the layers—with the melodic layers coordinating with the transposition levels in the bass—until the last few sections of the piece. For instance, for the first seven statements of the *color*, the glissandi are overwhelmingly tonally coordinated with the transposition level being stated, although occasionally a glissando will anticipate the arrival of a tonal area or extend a tonal area past the arrival of a new area. Table 5-9 identifies the tonics of the glissandi appearing in each tonal area. In section 8, as shown in the table, the attacks between glissandi are accelerating, and the increasing numbers of glissandi reach more remote key areas than have appeared before; note, however, the statement of I–IV–V/V–V in the last four glissandi, all being stated within the V area.

There are places where the melodic layers veer noticeably away from the bass layers. Such a place occurs just prior to the entrance of the 5:7 tempo canon in section 6. On the previous page, the melodic layer extends the V/V tonal area (D) for at least half a system beyond the movement to tonal area V; see Figure 5-23.

Table 5-9

Tonics of Glissandi Stated in No. 45a

Section	Transposition levels/Glissandi	Comments
2	I (C): C	double-octave glissandi
	IV (F): D	
	V/V (D): A	
	V (G): G	
3	I (C): C, C	single-octave glissandi
	IV (F): B _b , B _b , F	
	V/V (D): F, F, F, D	
	V (G): G, G	
4	I (C): none	double-octave glissandi
	IV (F): none	
	V/V (D): D, G	
	V (G): D, D	
5	I (C): C, F	double-octave glissandi
	IV (F): B _b	
	V/V (D): A	
	V (G): G, G	
8	I (C): D, E, A, B _b	double-octave glissandi
	IV (F): B, C, G, D	
	V/V (D): A _b , B, F, E, D	
	V (G): G, A, B _b , F, D, G, C	

Notes: There are no glissandi in sections 6 and 7. Arrows indicate anticipations of new tonal area or extensions of previous area. Shaded tonics in last section are stating the final four transposition levels (IV–V/V–V–I).

After the increased differentiation between the two melodic layers of the tempo canon in section 6, section 7's two melodic layers are, by contrast, extremely tonally coordinated with each other but only somewhat loosely with the bass layer; see Figure 5-24. The middle layer brings back a melody from earlier in the piece (p. 4) which has a recurring motive that clearly outlines the four transposition levels (the system immediately preceding this example is shown in Figure 5-21).

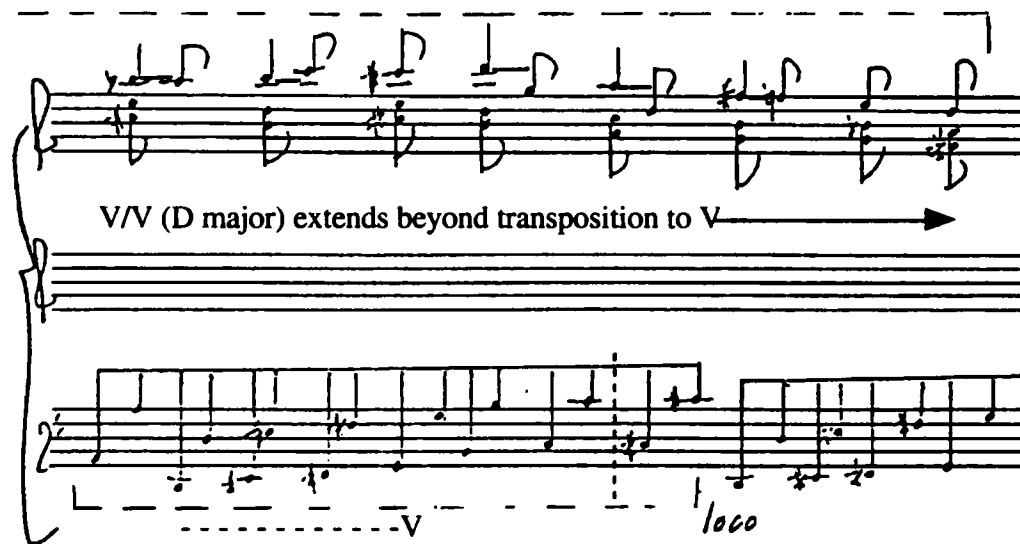


Figure 5-23. Extension of V/V tonal area beyond arrival at V area in No. 45a (score, p. 11).

Thus, overall No. 45a exhibits a variety of multidimensional textures in terms of its tonal cohesion. Thomas asserted that multidimensionality in a piece is rarely static, instead usually representing a continuum, and that is certainly the case in this movement. Both temporal dissonance and multidimensionality reach a peak in section 8 (the final section) where acceleration between the glissandi accentuates the temporal disparity and the more remote tonal areas expressed in the glissandi and the triad layer separate these layers tonally from the bass layer.

No. 45b

No. 45b has many of the same multidimensional features as No. 45a. Its rhythm is more noticeably based on metric configurations, however, and it has a higher frequency of simultaneous note attacks between layers. Acceleration does occur in places, but when it does it is arithmetical acceleration rather than the more multidimensional geometric acceleration that was used in 45a. The upper melodic layers, of which there are only two, are tightly coordinated both rhythmically and tonally with each other, but less so with the bass line. The upper layers do faithfully adhere to the tonal transpositions of the bass, but the changes are often staggered between the bass and upper layers.

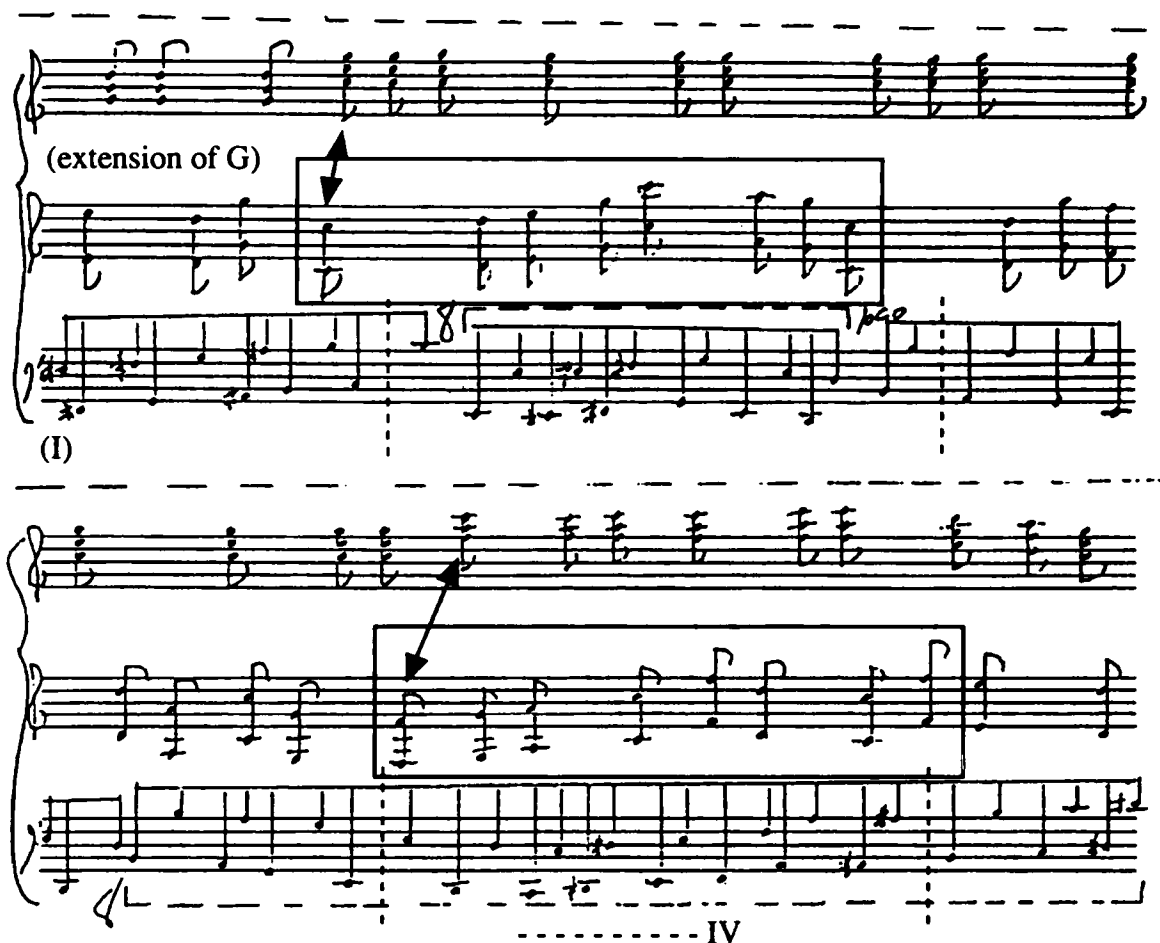


Figure 5-24. Section in No. 45a's section 7 (score, p. 15) where upper two melodic layers are closely coordinated tonally but not tightly coordinated with changes of transposition level in the bass layer. Rectangle identifies repeating melodic pattern.

As already noted, the bass line rhythm in this movement is based, according to Gann, on a background measure (not notated as such in the proportionally-notated score) simultaneously divided into 3, 4, and 5 parts. This same rhythmic idea, a “polytempo ostinato,” was used in Study No. 46 (shown in Figure 2-6, p. 77). Gann also asserts that the upper melodic layers relate to this same background measure as a 14-division of the measure. Presumably, then, the entire movement could have been written in metric notation, but in proportional notation the metric relationships among layers are somewhat less clear. If Gann’s assertions about the 3, 4, 5, and 14 divisions are correct (and I have no reason to believe they are not), then the top layers and bass layer share a common down-beat, which presumably would allow for frequent simultaneities among the layers.

As Gann further notes, the rhythm in the upper melodic layers consists of values that are multiples of the same background unit, and these multiples are most frequently 2, 3, 5, 8, or 10 units in duration (and thus heavily based on Fibonacci numbers; rhythmic groupings such as 2 + 3 + 3 and 3 + 5 + 5 also imply Fibonacci). Short ostinati and isorhythmic passages abound. The two passages shown in Figure 5-25 contain an isorhythmic pattern consisting of a four-note pitch pattern with a 2 + 3 + 3 rhythmic pattern. (In this section, figures showing score segments from No. 45b will have the relative values of the durations noted.)

Figure 5-25 consists of two musical examples, (a) and (b), each showing a piano accompaniment and a melodic line. The piano accompaniment is in a 2+3+3 rhythmic pattern, with the first two notes of each group being short and the third being a longer note. The melodic line is a 4-note ostinato. In (a), the melodic line is ascending, and in (b), it is descending. The notes are aligned with the third note of each piano group. The relative values of the durations are noted above the notes: 3, 2, 3, 3, 2, 3, 3, 2, 3, 3, 2, 3.

Figure 5-25. Two instances of a 4-note ostinato against thirds aligning with every third note (the short note value in each 2 + 3 + 3 rhythmic pattern): (a) ascending pattern in No. 45b, p. 3; (b) descending pattern in No. 45b, p. 4.

The Fibonacci-related rhythmic values also are used to create rhythmic augmentation. Figure 5-26 shows the same melody shown in Figure 5-25b rhythmically augmented as a 3 + 5 + 5 rhythmic pattern.

The score examples shown in Figures 5-25 and 5-26 illustrate complete rhythmic

agreement between the melodic layers. Later in the movement, a steady 5 rhythmic value is established in the lower layer, setting up the syncopation shown in Figure 5-27.

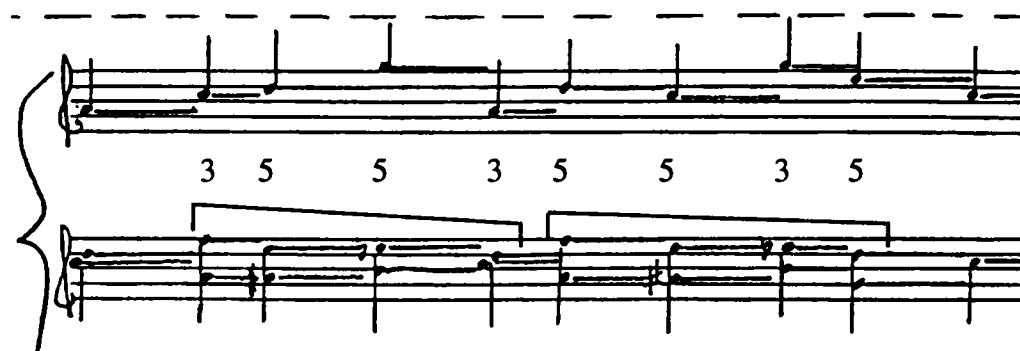


Figure 5-26. Same melodic pattern from Figure 5-25b rhythmically augmented to 3 + 5 + 5 (score, p. 6).

Multidimensionality is heightened in No. 45b by structural imbalances that drive a wedge between the bass and melodic layers, and deny the listener's expectations about regularity in the recurring harmonic pattern. Although the harmonic structure of No. 45b is based on the pattern I-IV-V-I, there are some inconsistencies in the pattern (occasionally an area appears out of sequence, and on p. 13 the area V/V appears twice). Also, the

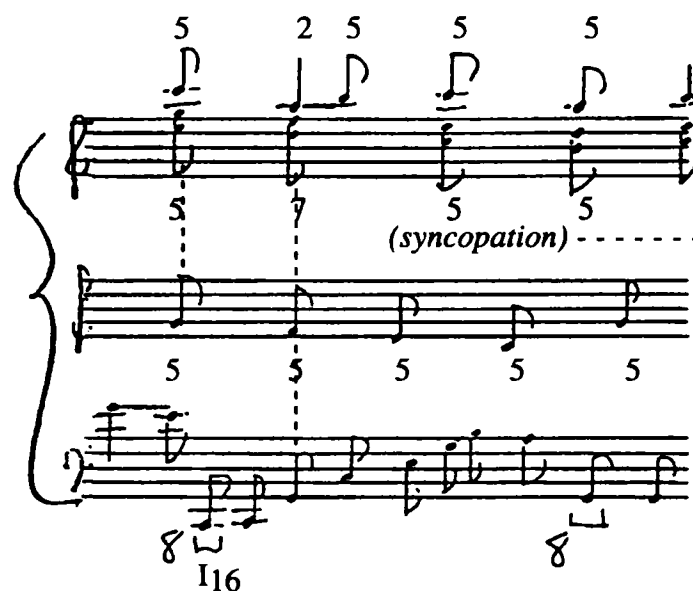


Figure 5-27. Syncopation between two melodic layers in 45b (score, p. 12). I_{16} refers to the sixteenth time through the I bass pattern. Note the low incidence of simultaneities between the bass layer and upper layers.

piece ends much the same way as No. 45a did, with a rapid cycling through remote key areas. Pages 14–18 of the score progress through the following keys:

F A_b B E A B C E_b A_b F D G B_b E_b A_b B G_b D_b A_b F C A D

The majority of tonal movement is by either minor third (up or down) or perfect fourth/fifth, and from the C highlighted in the line above, those are the only intervals of movement. Nancarrow moves to these areas by making adjustments in two parts of the bass pattern: the resolution of the seventh scale degree (the highest note of the pattern) by either minor second or major second; and by varying which scale degree the resulting resolution note is in the new key. Only two patterns are used prior to the cycling of keys beginning on p. 14; thereafter three new patterns (plus two irregular patterns that are each used only once) are introduced that allow Nancarrow to create the tonal movements shown in Table 5-10 and Figure 5-28.

Table 5-10

Five Patterns of Transposition in Bass Line at End of Study No. 45b

Transposition	Resolution of Opening Key's 7th	Resolution Note's Scale Degree in New Key
up a perfect fourth (Fig. 5-28a)	m2	third
up a major second (Fig. 5-28b)	m2	fifth
up a minor third (Fig. 5-28c)	M2	fourth
down a minor third (Fig. 5-28d)	m2	octave
down a perfect fourth (Fig. 5-28e)	m2	second

Like the inconsistent number of notes in No. 45a's harmonic pattern, the sense of regularity in No. 45b provided by the repeating harmonic pattern is compromised further by disparities in the durational length of the key areas. Because there is no systematic coordination of changes to new tonal areas with the downbeat of the background measure, there seems to be no rhyme or reason to the lengths of the areas. For instance, the first six key areas have lengths of 96, 107, 111, 121, 121, and 87 millimeters; the range of

lengths is from 48 (in the accelerating section on p. 10)⁹ to 147 millimeters, and adjoining areas are often of quite different lengths.

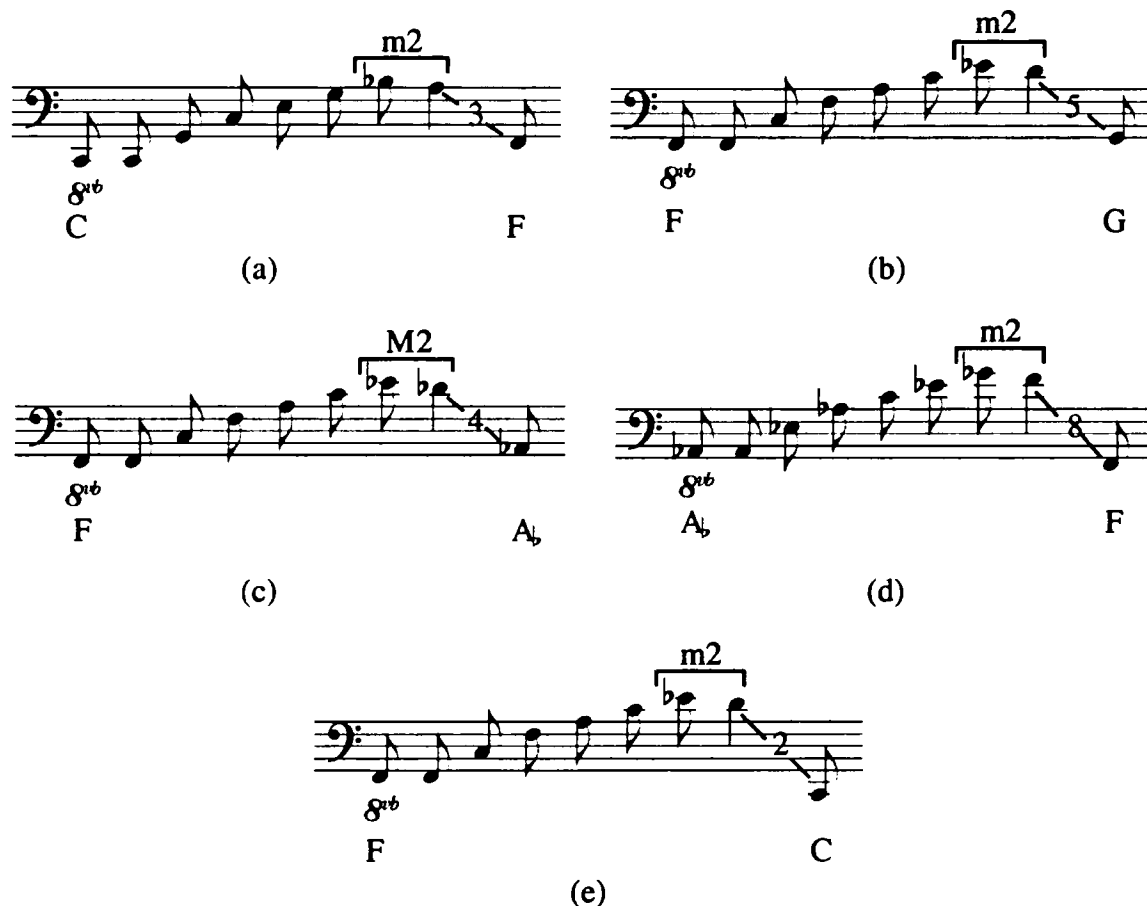


Figure 5-28. Alternate transpositions in No. 45b's bass line pattern: (a) movement to key a perfect fourth higher; (b) movement to key a major second higher; (c) movement to key a minor third higher; (d) movement to key a minor third lower; and (e) movement to key a perfect fourth lower.

Structure is further thrown off balance because the arrival of new sections in No. 45b only rarely coincides with a return to I (in No. 45a, by contrast, all new sections coincided with an arrival at I). Gann identifies nine sections in No. 45b according to textural changes; it is, in fact, difficult to ascertain clear section breaks because there is so little alignment with new textures and arrival at returning key areas.

As mentioned earlier, the upper two melodic layers are fairly coordinated tonally, but less so with the bass layer. This changes remarkably in the closing section during the

⁹There is a duration of 45 at the end of p. 11, but that appears to be due to a missing system.

rapid cycling through remote key areas. The last three systems of the piece are shown in Figure 5-29; as shown in the figure, all three layers are tonally coordinated during the last seven key areas. (Incidentally, the key areas G_b, D_b, A_b, and F are virtually identical in duration, perhaps indicating the length of a background measure or hypermeasure.) The last four areas (F, C, A, and D) include a strong melodic emphasis on the note establishing each as a dominant seventh, and the piece ends in strongly dominant fashion on a D9 chord. The curious ending on a D dominant chord functions essentially as a half cadence to lead the listener directly into No. 45c, which begins with a chromatic glissando starting on G, leading to an extended G dominant harmony. Not only does 45b end on a non-tonic harmony, there are no simultaneities between any of the layers at the end, further intensifying the lack of resolution.

No. 45c

The structural irregularities in Nos. 45a and 45b pale in comparison to those in 45c—the “spastic rhythm” returns in its full glory, appearing not only in the bass but in the melodic layers, and the boogie-woogie bass line and the implication of tonal areas I, IV, and V only loosely undergird the structure. Tenney says about 45c that “the process of abstraction . . . is carried still further, juxtaposing and superimposing fragmentary, altered elements of a boogie-woogie texture in highly unpredictable ways” (notes to Vol. V, p. 9).

The boogie-woogie pattern itself (Table 5-7) is used only occasionally, and it appears not only in the bass but in several melodic statements (see Figure 5-30).

One other melodic motive is prominent throughout the movement: a first 3-note, then 4-note pattern outlining the “partitioned minor third” so prevalent in Nancarrow’s work. The motive is presented in a variety of textures, including single notes, octaves, parallel major triads, and as part of more complex textures (see Figure 5-31). The motive is actually a “head motive” from a much longer theme (53–54 notes long) that recurs in

The image displays three systems of musical notation for Study No. 45b. Each system consists of a piano part (grand staff) and a violin part (single staff).
 - The first system includes fingerings (2 5, 3 2 5, 3 2 5, 3 2 5, 3, 2 3, 2 3, 2 5) above the violin staff. The piano part has a label (B) and a G chord indicated below. The violin part has a sfz dynamic marking.
 - The second system has fingerings (2 5, 2 3, 5...) above the violin staff. The piano part has labels for Ab, F, and C chords below. The violin part has sfz dynamics and a 7/1000 tempo marking at the end.
 - The third system has labels for A and D chords below the piano part. The violin part has a sfz dynamic and a (D9) chord label. Arrows connect specific notes between the piano and violin parts across the systems, indicating harmonic coordination.

Figure 5-29. The end of Study No. 45b (score, pp. 17–18). From the G₇ area to the end there is complete tonal coordination among all layers. In the last four key areas (F, C, A, D) there is a melodic emphasis on the dominant seventh, culminating in a D9 chord that becomes the dominant to the first harmony of No. 45c.

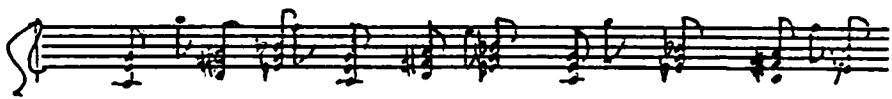


Figure 5-30. Melodic uses of the boogie-woogie bass pattern in No. 45c: (a) score, p. 2; (b) score, p. 6. [Note: the score copy to which I had access is quite badly faded in this movement. I have attempted to heighten certain details in these figures where possible.]

several sections, including being the theme for the last (and longest) of three tempo canons in the movement. The motive is further identifiable by its fairly consistent rhythmic shape: a long note, followed by a shorter note, and a long note (exact proportions vary).

Unlike movements (a) and (b), No. 45c does not really mix time types as the rhythms are mostly based on the “spastic rhythm” and everything is proportionally notated. In this movement more than the others, it seems that Nancarrow studiously avoided simultaneities among layers. To achieve this, Nancarrow introduced a new procedure for use of the 15-note spastic rhythm that guarantees no simultaneities among layers: in much of the piece, the rhythm is divided among the layers. The piece’s final section (what Gann identified as section 8) is perhaps the best example of this; see Figure 5-32.

Although the division of the spastic rhythm among layers does heighten the sense of temporal dissonance due to the lack of simultaneities, in a way this same procedure introduces a sense of registral polyphonization since all the layers are participating in stating the same rhythmic pattern. Besides the use of the spastic rhythm to avoid simultaneities and lack of a semblance of metered rhythm, temporal dissonance is also introduced into

(a) 

(b) 

(c) 

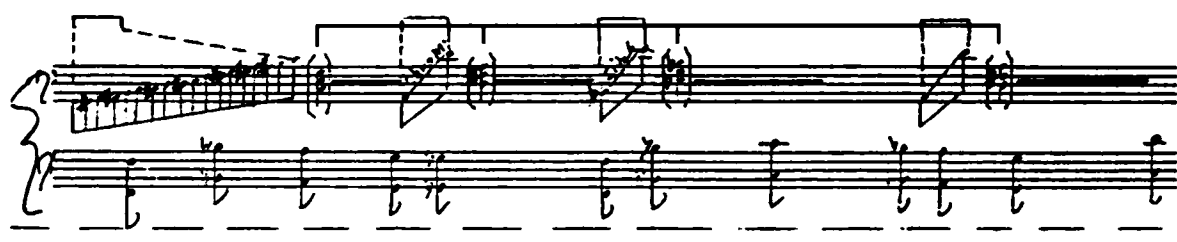
(d) 

Figure 5-31. Recurring melodic motive in No. 45c: (a) first appearance, as a 3-note pattern, in interlocking form (ascending triads below and descending single notes above), score, p. 1; (b) expansion to 4-note form, repeating the second note (as triads and octaves), score, p. 3; (c) as the highest pitch in series of glissandi followed by sustained polychord (score, pp. 5–6); (d) preceded by chromatic glissandi (score, p. 16).

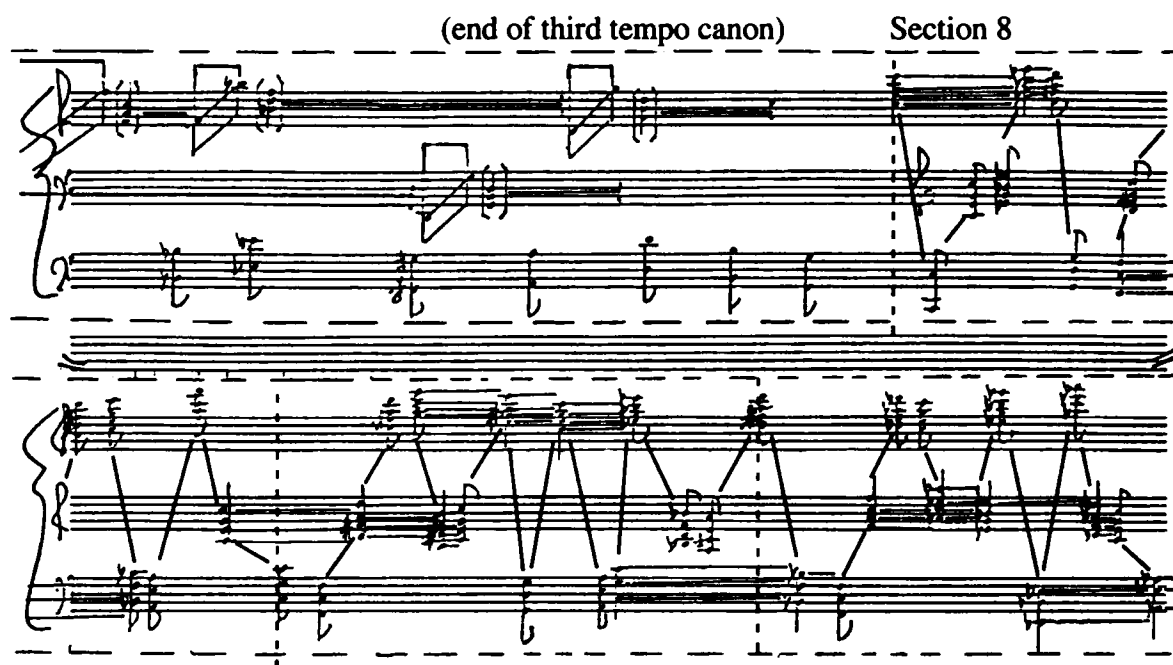


Figure 5-32. Division of the “spastic rhythm” among all layers in section 8 of No. 45c (score, p. 21). The 15-note segments demarcated by the vertical dotted lines are all the same length. This section is preceded by the last of three tempo canons.

No. 45c by three converging tempo canons, all of which stop just short of convergence and leave the temporal processes unresolved. All of the canons end on G—the first on G octaves, the second on G major triads, and the third (Figure 5-32) also on G triads. The tempo canons are, ironically enough, the only places in this movement where occasional simultaneities occur between layers.

Acceleration, the third time type, is not much of a presence in this movement. The only instance is one of sectional deceleration, created by an increase in the length of the 15-note rhythm statements on pp. 15 and 16 (between the second and third tempo canons) by a 1:2:3 ratio. The boogie-woogie bass pattern is first stated twice at a length of 75 millimeters per rhythmic statement, then twice at 150 millimeters, then—underneath the third tempo canon—a number of times at ca. 225 millimeters. The effect of the first deceleration is quite dramatic because it occurs at the highest point of this widely-ranging line and before the tempo canon voices enter.

As can also be seen in Figure 5-32, No. 45c has sections of thick parallel

harmonies, specifically parallel major triads that introduce a sense of polytonality to the texture. There are also thick polychords in places, such as the stacks of two minor seventh chords with roots a major third apart shown in Figure 5-31c. Another prominent polychordal structure involves stacked dominant seventh chords with roots a fifth apart, several of which appear at the beginning of the piece: each of the first three phrases begins with a long chromatic glissando followed by a sustained polychord (see Figure 5-33). As Figure 5-33 illustrates, the introduction of more remote keys happens much earlier in No. 45c than it did in 45a or 45b. No. 45c begins conventionally enough, with the first phrase forming the dominant to the boogie-woogie bass line in C and the second phrase to F, but the third statement resolves into a B_b tonality in the top voice (shown in Figure 5-30a) before the head motive enters in E_b.

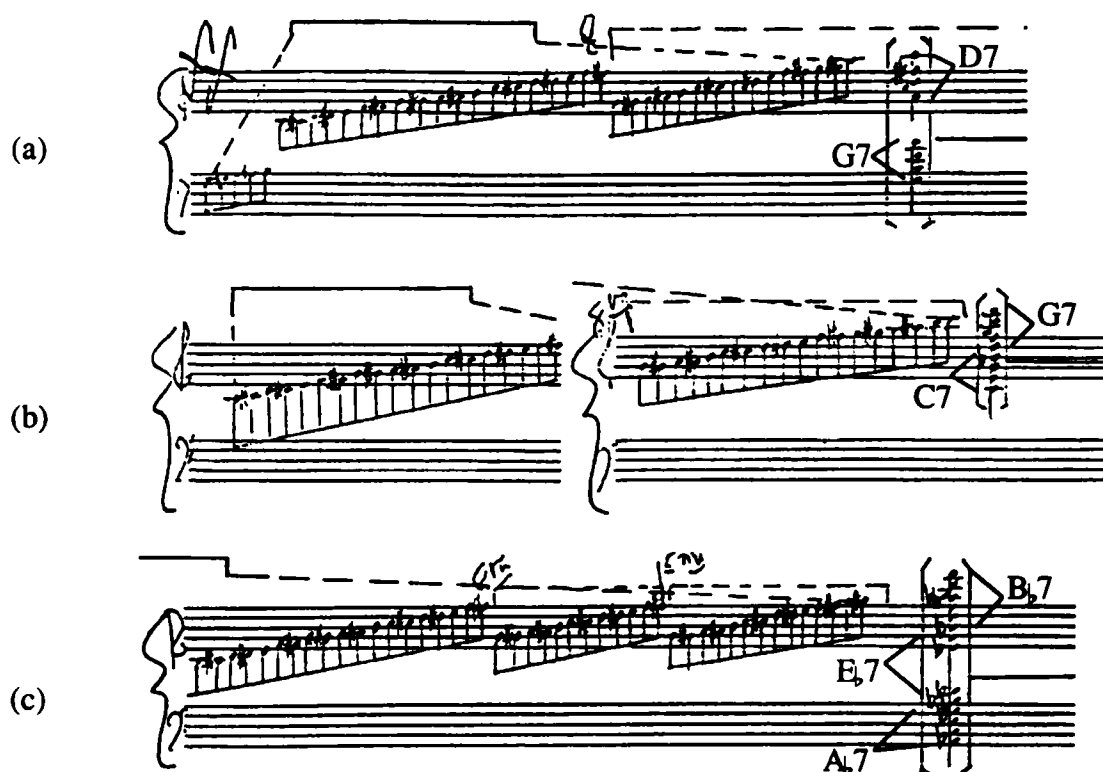


Figure 5-33. Dominant seventh polychords in each of No. 45c's first three phrases: (a) p. 1, first system; (b) p. 1, third and fourth systems; and (c) p. 2, first system.

It is this frequent use of parallel major triads in the theme (and elsewhere) and the

resulting subversion of tonal function within each layer that contributes most significantly to tonal multidimensionality in this piece. Figure 5-34 shows the very end of No. 45c. Here shifts in tonal center, as determined by the bottom pitch of each chord, are fairly coordinated among the three layers and the shifts in tonal center are in close proximity to the beginnings of the 15-note rhythmic pattern, as well as being reinforced by V-I cadences (although the attacks are staggered). However, the constant movement

The figure displays a musical score for the ending of No. 45c, consisting of three staves. The top staff is marked with a treble clef and a key signature of one sharp (F#). The middle and bottom staves are marked with bass clefs. The music features complex polyphonic textures with multiple voices. Vertical dotted lines indicate the beginning of 'spastic rhythm' statements. Circled chords indicate where tonal shifts occur, specifically to D, F, and G. The bottom staff is labeled 'G dominant polychord'. The score includes various musical notations such as notes, rests, and dynamic markings.

Figure 5-34. Ending of No. 45c (score, p. 23), showing near-coincidence of shifts in tonal centers with beginning of "spastic rhythm" statements (demarcated by vertical dotted lines—rhythmic values are still being divided among the layers) and parallel triadic harmony, culminating in a G dominant polychord (G major, B major, and D major triads). Circled chords indicate where tonal shifts occur. Staggered V-I cadences are shown in D, F, and G.

in parallel major triads (and open fifths in the bass), in opposition to functional tonality, adds significantly to multidimensionality. This is perhaps most striking at the G dominant polychord that immediately precedes the final phrase. The function is clearly a dominant to C, but the actual tonal center is not confirmed until the line of broken octaves continues in the bass. (Nancarrow used a similar polytonal technique in cadences in Study No. 12—see Figure 1-16, p. 40).

Structurally, the I–IV–V–I harmonic pattern on which 45a and 45b were based has been reduced in 45c to little more than a touchstone, and, as Tenney’s comment reveals, there is much fragmentation of the background harmonic pattern. Tonal areas I (C) and IV (F) do figure prominently in a number of sections (see Figures 5-31d and 5-32, for instance, for examples of I), but V (G) functions only in brief sections that are bridges to I. There are also numerous secondary tonal areas of significance, including E_b on page 2, A on page 3 (Figure 5-31c), and D on page 4.

Concluding Remarks

The three movements of Study No. 45, described by Gann as a “Boogie-woogie Suite No. 2” (p. 263), employ a number of different techniques that enhance both temporal and tonal multidimensionality. Some of these techniques were also used in the first “Boogie-woogie Suite,” Study No. 3, and some evolved as Nancarrow gained composing experience (particularly through the development of the tempo canon) and access to more precise punching mechanisms.

Multidimensionality is expressed as temporal dissonance in No. 45 in several ways. Mixing of time types (metered, proportional, and accelerative) occurs in movements (a) (Figure 5-19) and (b) (Figure 5-27), but almost not at all in (c). Still, temporal dissonance seems most intense in 45c, where the 15-note proportional “spastic rhythm” (Tables 5-7 and 5-8) is partitioned among layers (Figure 5-32) and simultaneous attacks are virtually non-existent. No. 45b is the least dissonant temporally; although its bass line is still

metrically irregular, the upper melodic layers are in a pseudo-metric rhythm with regularly-recurring duration values (2, 3, 5, 8, etc.) and are often in metric accord, as well as having a fairly high incidence of simultaneities with the bass line. The use of metric regularity allows Nancarrow to incorporate syncopation into 45b (Figure 5-27), a feature non-existent in the other two movements.

No. 45a's most striking temporal dissonance is achieved through geometric acceleration (Figure 5-20), and it is the only movement to use this form of acceleration. Arithmetical tempo change also occurs in this movement (Figure 5-19), as it does in No. 45b (Figures 5-25 and 5-26). Further, there is sectional acceleration in 45b and deceleration in 45c.

Temporal dissonance is also imparted to these pieces by the subverting of the temporal convergence process in several tempo canons that abruptly end just prior to convergence. Such canons occur in 45a (Figure 5-21) and three times in 45c (the end of the last canon is shown in Figure 5-32).

The potential for tonal multidimensionality in these works is primarily dependent on two factors: (1) the number of layers operating at any given time, and (2) the harmonic expectations established by the background harmonic pattern in each movement. Multidimensionality is increased as more layers introduce different tonal areas and as tonal areas fail to conform to harmonic expectations according to the pattern. In No. 45a, tonal multidimensionality is heightened where tonal areas in the upper layers either anticipate or lag behind new areas in the bass (Figure 5-23); where the upper layers are tonally coordinated with each other but not the bass (Figure 5-24); and where key areas remote to the original I-IV-V/V-V harmonic pattern are introduced, particularly the rapid cycling of remote key areas (combined with geometric acceleration) at the piece's end (Table 5-9).

Tonal dissonance in No. 45b is more moderate. Its two upper layers are highly coordinated rhythmically and tonally (Figures 5-25 and 5-26) but less so with the bass

layer (Figure 5-27). Like No. 45a, 45b ends by rapidly cycling through a number of keys remote to the original harmonic pattern (Table 5-10 and Figure 5-28).

No. 45c has the densest texture of all three movements (Figures 5-32 and 5-34), and tonal dissonance in this movement is represented in the use of parallel major harmonies and dense polychords (Figure 5-33). As a result of the parallel harmonies, even passages in which there is tonal agreement among layers (Figure 5-34) have a high incidence of extra-tonal pitches. Also, tonal areas remote to the I-IV-V-I harmonic pattern that subtly underlies this movement are introduced much earlier than in the first two movements.

Because of the frequency with which it strays from the basic harmonic framework, No. 45c is the most extreme and dramatic example of the sort of structural multidimensionality that exists in Study No. 45. No. 45a is the most structurally regular movement; although its tonal areas are of different lengths, its sections clearly coincide with returns to the I tonal area and sections often begin with a simultaneity among layers. However, its structure begins to disintegrate at the end with the rapid cycling through remote key areas and the introduction of geometric acceleration. No. 45b introduces further structural disintegration with its highly variable length of tonal areas (varying by a factor of about 3) and in how its sections so rarely coincide with I or even the beginning of a new tonal area. It, too, undergoes harmonic disintegration at the end (Figure 5-29).

In comparing Study No. 45 to its early ancestor, Study No. 3, it is the structural irregularities of No. 45, especially the inherently uneven rhythms of the bass layer, where this "second boogie-woogie suite" differs most strikingly from Study No. 3. Nonetheless, the structural disintegration that is so much a part of No. 45 has a predecessor in the rhythmic disintegration that takes place at the end of No. 3e (Figure 5-18). From there, this process of disintegration advances to tonal disintegration at the end of 45a and, to a lesser extent, 45b, and culminates in the structural disintegration of the I-IV-V-I pattern throughout 45c. It also finds expression in structural irregularities such as the uneven lengths of tonal areas in movements (a) and (b).

Beyond this, multidimensionality is expressed in different ways in Nos. 3 and 45. In the three movements of No. 3 examined here ([a], [b], and [e]), textural accumulation, or the gradual addition of differentiated new layers to the texture, is much more in evidence than in No. 45, which generally has no more than two melodic layers and a bass layer. Within No. 45's layers, however, there is more likely to be registral polyphonization than there is in No. 3.

Another basic difference between the multidimensional textures of the two suites is in the potential for simultaneities among the layers. Even though portions of No. 3 are thickly layered and the layers are effectively differentiated, the use of short, recurring rhythmic patterns and steady ostinati—couched in a variety of tempo ratios—results in a higher incidence of simultaneities than in No. 45, with its skewed rhythms in the bass that rarely find accord with the other layers.

Further multidimensionality is introduced into No. 45 by the incorporation of converging tempo canons that don't quite converge, thick parallel major harmonies (particularly in movement 45c), and a greater likelihood of anticipation or delay of new tonal areas between the upper layers and the bass (especially in movements [a] and [b] where the harmonic patterns are still functionally present in the bass line). A greater mixing of time types than in No. 3, and the use of geometric acceleration in No. 45a, add further to the potential for multidimensionality in No. 45. Thus, while Nancarrow achieved a greater variety of differentiated layers in Study No. 3, in No. 45 he achieved multidimensionality in a greater variety of ways.

CHAPTER 6

SUMMARY, CONCLUSIONS, AND SUGGESTIONS FOR FUTURE STUDY

The music of Conlon Nancarrow has proven increasingly fascinating to students of twentieth-century music, but has continued to suffer from a lack of serious attention. Some of the reasons for this include reluctance by some to take seriously Nancarrow's performing medium of choice, the player piano, while others (Boulez, for example) have cited the simplicity of Nancarrow's tonal materials in dismissing his music. Nancarrow's isolation in Mexico and his seeming lack of interest in advocating his own music also contributed to a continuing obscurity.

At its heart, Nancarrow's music is characterized by layers and the widely varying ways in which these layers interact with each other, particularly rhythmically. As Thomas (1996) showed in her work on multidimensionality and temporal dissonance in Nancarrow's *Studies*, the continuum of multidimensional complexity in the music is indeed very broad, extending from pieces that could be described as non-multidimensional (No. 37) to the simpler two-layer pieces with frequent rhythmic simultaneities (such as Nos. 14, 15, and 18) to those that teeter on the brink of the "perceptibility threshold," such as Nos. 3a, 41, and 48.

Nancarrow's favored compositional device of the tempo canon has two features—the use of tempo ratios, and placement of convergence points—that are intrinsically related and directly impact the formal interaction of the musical layers. Chapter 2 showed that ratios in Nancarrow's music extend beyond just the use of ratios to relate different tempos to each other; ratios also governed and created connections between pitch patterns, intervals of imitation between voices, and structure. Nancarrow's use of ratios was

heavily influenced by those found in just intonation, but he also sought to use more complex ratios, up to and including irrational numbers such as π and $\sqrt{2}$. Chapter 2 also introduced the concept of the duration ratio, the ratio that describes the relative duration of sections in a tempo canon; the duration ratio becomes an important concept in Chapter 3's discussion of convergence points.

Chapter 3 discussed the four basic types of tempo canons, of which the most common in Nancarrow's music are the converging canon and the converging-diverging (arch) canon. Considerations for measurement of the canon melody's timespan and the concept of potential points of simultaneity were discussed as they related to the placement of CPs. By introducing the technique of the tempo switch, Nancarrow was able to create more complex structures with greater numbers of convergence points (CPs), perhaps best represented by Studies No. 24 and 43. Several of his works also include multiple sections of shorter tempo canons (e.g., Study No. 37), allowing further structural freedom in the placement of CPs.

The tempo canon device also gave birth to a new kind of musical form, one in which characteristics of fractals are represented. Chapter 4 introduced the characteristics of fractals (self-iteration, scaling, and space-filling) as defined in fractal geometry, and reviewed the ways in which fractals have been identified in music and sound. The concept of fractal dimension was introduced and reviewed as it relates to traditional fractals, and a modification of the equation for finding the Hausdorff dimension, $D_H = \log(N)/\log(S)$, was proposed for finding fractal dimension in Nancarrow's tempo canons, which, unlike traditional theoretical fractals, are self-limiting in the number of iterations and do not have consistent scaling. The modified equation produces the fractal dimension based on the number of scaled objects (e.g., canonic voices) and the average of their scaling relationships. It was concluded, based on the application of the modified equation to Studies No. 14, 18, 19, 31, and 36 that:

1. The fractal dimension is generally higher in tempo canons with more voices,

especially when the following is true:

2. The fractal dimension tends to be higher as the total scaled duration decreases; in other words, the smaller the duration of the scaled voices compared to the original, the higher the fractal dimension is. See, for example, Studies No. 18 (Figure 4-17) and 19 (Figure 4-18) and the two hypothetical canons in Figure 4-16a; compare to the minimally-scaled voices in Studies No. 36 (Figure 4-19) and No. 31 (Figure 4-20).

Finally, Chapter 5 was devoted to the analysis of two pieces: Studies No. 25 and No. 45. Study No. 25 is the only one of Nancarrow's works to use a 12-tone row, which it does in conventional ways. Retrograde is found not only in the way the tone row is used, but in entire sections that are retrogrades of other sections. The work's five layers include a layer of arpeggios and glissandos that extends throughout the piece, culminating in the blistering final section where Nancarrow applies the sustain pedal for the only time in the Studies. Study No. 25 is found to have interesting parallels with Study No. 27 in the use of geometric acceleration, rhythmic randomness, and the embedding of a line (No. 25's tone row or No. 27's "clock line") within different chord structures.

Study No. 45, a three-movement piece based on boogie-woogie bass lines and short blues harmonies, is analyzed in view of how it achieves multidimensionality as described by Thomas (1996), and the results are compared to Study No. 3, the "Boogie-woogie Suite." The pieces achieve multidimensionality in different ways. No. 3 relies more heavily on textural accumulation, or the gradual addition of temporally diverse layers, reaching a peak in No. 3a where multidimensionality teeters on the brink of the "perceptibility threshold." No. 45 does not reach such extremes of multidimensionality since it uses fewer layers, but it achieves a wider variety of multidimensional textures by incorporating techniques of registral polyphonization, less tonal coordination among layers, more mixing of time types (including acceleration in some layers against steady tempo in other layers), and tonal and structural disintegration.

Research Issues

Score Inaccuracies

Future research on Nancarrow's Studies will have to address the problem of occasional inaccuracy in his scores. Although Nancarrow's scores are carefully drawn and a very helpful aid to the analyst, they contain numerous errors and omissions. Appendix A of this dissertation identifies a number of score inaccuracies found by this author, but the list is by no means complete. The final authority on Nancarrow's true intentions in the Studies is the player piano rolls themselves (and by extension the recordings made from the rolls), of which the scores represent Nancarrow's best—but imperfect—representation.

There are some score problems that will only be resolved by analysis of either the piano rolls themselves or extremely slow playback. Study No. 25 provides some excellent examples of apparent score errors that defy verification from listening to full-speed playback of the recordings. Section 8, the final section of this Study, blitzes by the listener at a speed of about 88 notes per second, or as Gann describes, "too fast for the ear to register as individual events" (p. 248). Difficulties in analyzing this section are compounded by the apparent errors in the score. The section contains numerous major key scalar passages, some several octaves long; the same key area is maintained throughout each passage. Figure 6-1 identifies two areas in this section where scalar passages appear to be intended but all the necessary accidentals are not there. In example (a), the passage at first appears to be a G_b major scale, but only the first of four C's in the passage is flatted, leading one to question whether the passage is really in G_b or D_b ; and, in example (b), a 2-octave-plus segment in B major is missing four sharps in the middle. (It turns out that this latter omission is confirmed by comparison with p. 35, where this passage earlier occurs.)

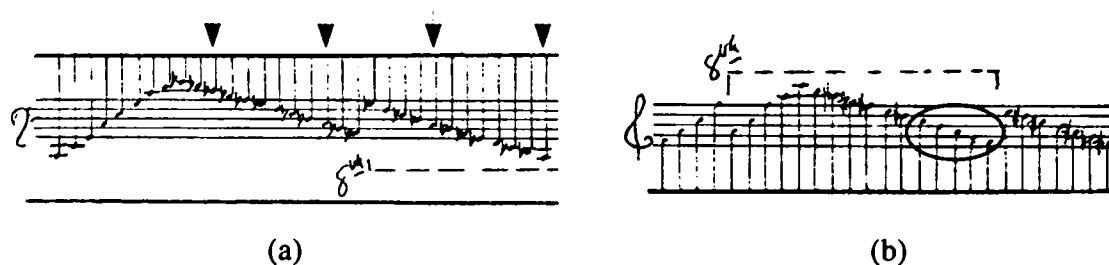


Figure 6-1. Questions about score accuracy in Study No. 25: (a) first C in scalar passage is flatted while three others are not (from bottom system, p. 75); (b) scalar passage apparently in B major in which sharps appear to be missing in circled area (from top system, p. 76).

The average listener will not be able to verify such errors by listening to a full-speed playing of this piece—only very slow playback or examination of the piano rolls will allow such errors to be corroborated with complete assurance. And, in the case of this section from Study No. 25, the use of the sustain pedal through this entire passage exacerbates the difficulty of making this assessment from the recording alone.

In general, when analyzing the Studies from the scores, the analyst must be constantly aware of the compositional processes Nancarrow has used. Maintaining this awareness will occasionally lead to fruitful questions about score accuracy. Many of the errors—particularly rhythmic and metric errors, such as missing rests and time signatures—are obvious at once. The plethora of repeating patterns (canons, ostinati, isorhythms, tone rows, repeating chord patterns, blues harmonic patterns, etc.) in Nancarrow's music allows one to arrive at all sorts of other questions about score accuracy. Some of these types of errors include chord spellings, rhythmic and pitch patterns (such as ostinati and countdown patterns), and other variations to established patterns. Here are some examples (all are chronicled in Appendix A):

1. Chord spellings. In Study No. 1 (which is not in Nancarrow's hand but was professionally copied for publication in Henry Cowell's *New Music* in 1952 [Gann 1995, 70]), the opening section has a repeating series of three-note open fifth chords on D_b, G_b, and A_b; on p. 2 at the end of the second system there is a three-note chord spelled D_b–G_b–D_b.

Another example is in Study No. 11, which is a series of changing textures based on a blues harmonic pattern and a fifteen-note isorhythm. In the fifth texture (begins p. 7), there is a major seventh chord (with a missing fifth) in the first and third voices from the top at the beginning of each $\frac{5}{8}$ measure, with the exception of the last measure in the second system of p. 7; my belief is that the two notes in the top staff in this measure are one ledger line too high, and should be A and D rather than C and F, making the chord B $\flat$ -A-D.

2. Rhythmic and pitch patterns. In Study No. 12, the meter signature in the partial measure at the end of the first system on p. 9 should be $\frac{9}{16}$ rather than $\frac{6}{16}$; this not only reflects the actual rhythmic content of the measure, but the metric pattern 5-4-3-4-5-9, of which the $\frac{9}{16}$ measure is a part, has been stated several times earlier in the piece.

In Study No. 3a (first movement of the "Boogie-woogie Suite"), the entire movement is built over a 12-bar blues harmonic pattern. On p. 16, during the thirteenth statement of the pattern, the F pattern from the end of the second system should only be repeated once instead of four times. The third and fourth systems should have the following harmonic patterns: F-C-C and G-G-C.

Some errors in patterns are more difficult to detect. Study No. 5 contains an elaborate system of rhythmic countdowns in which the rests between short ostinato patterns progressively decrease in length by ten sixteenth notes at a time; eventually there are eight different patterns taking place. Careful monitoring of the countdowns (which can somewhat easily be done because each system is exactly 70 sixteenth notes long) reveals that on p. 13 there is an ostinato statement missing in the tenth system from the top. On p. 19, in the second system the $\frac{9}{16}$ ostinato pattern is placed one sixteenth note too soon. There are also some inconsistencies in the length of the rests, with some of them being from one to ten sixteenth notes off according to the established pattern.

All of the examples listed above are from Studies that are metrically notated. Score problems are more difficult to detect, of course, in scores that are proportionally notated,

although awareness of patterns and systems such as canon and ostinati continue to be helpful in spotting score errors in these Studies. In Study No. 27, for instance, it is evident from observing the canon on p. 3 that rests are missing in the bottom voice after F in the second system and after E in the third system (in each case, half the value of the sustained note should be a rest). In other scores such as that to No. 21 ("Canon X"), one has to wonder how accurate the proportional notation is throughout such a complex score, especially in the more densely notated sections.

In the Studies where geometric acceleration/deceleration is used, it is sometimes perilously difficult to determine the percentage of change being used when Nancarrow has not specified it. Recall, for instance, my disagreement with Gann on the rates of acceleration in Study No. 25 (Chapter 5) and elsewhere.

Final Thoughts

Conlon Nancarrow's Studies for Player Piano represent a remarkable compositional achievement, one in which the composer spent some fifty years exploring the possibilities of the player piano as a performing medium that afforded him the compositional freedom he craved. While not every one of the Studies is a masterpiece, each makes a contribution in addressing interesting compositional problems set forth by the composer, and most of these problems involve creating different rhythm and temporal relationships.

Nancarrow's greatest compositional achievement, I believe, is in the creation of the genre of the tempo canon, of which Studies No. 36, 40, 41, and 48 represent the most spectacular examples. It is in these pieces that Nancarrow most successfully creates structures that capitalize on the dynamic relationship between ratios and placement of convergence points. The fractal forms created in many of the tempo canons also represent a unique musical achievement.

Nancarrow's achievements in the use of accelerative techniques must also not be overlooked. In the ability to create "curved time," as Gann calls it, Nancarrow found an

exciting new way to constantly alter the dynamic relationship among layers, either gradually through geometrical acceleration or exponentially through additive acceleration.

Where does research on Nancarrow's *Studies for Player Piano* go from here? From my vantage point in 2002, five years after Nancarrow's death, the only thing that is clear is that the field is still emerging. A very good start has been made in introducing Nancarrow's music to a wider audience, with Carlsen's insightful 1986 monograph, Gann's comprehensive book of 1995, and Thomas's groundbreaking analysis of multidimensionality and temporal dissonance in 1996. There are many more contributions waiting to be made, and much more to be found in this music. It is my sincere hope that interest in the music of this reclusive and innovative composer will continue to build among twenty-first-century musicians, and that it will increasingly receive the wider critical attention it so richly deserves.

APPENDIX A: ERRATA TO SCORES OF THE PLAYER PIANO STUDIES

Conlon Nancarrow's hand-written scores, while generally quite precise, contain numerous errors. Most commonly these are errors of omission (e.g., missing rests, accidentals, dots, ties, meter markings, clef signs, and octave displacement lines); more occasionally, notes are misplaced on the staff by one line or space. The errors can often be deduced due to the use of transparent compositional methods such as canon, isorhythm, and other clearly recurring patterns.

In order to be as profitable as possible to scholars studying Nancarrow's scores, the following list of score errors includes those that are already noted in the major sources in the literature as well as those noted by this author during the course of this study. In all cases, the final authority on the accuracy of the scores is the recordings made from the piano rolls; all errors listed here have been verified on the recordings whenever possible.

•noted in Margaret Thomas, *Conlon Nancarrow's 'Temporal Dissonance': Rhythmic and Textural Stratification in the Studies for Player Piano* (1996)
+noted in Kyle Gann, *The Music of Conlon Nancarrow* (1995) or in Gann's score copies
§noted in Philip Carlsen, *The Player Piano Music of Conlon Nancarrow: An Analysis of Selected Studies* (1988)

Study #1 (This is the only published score not in Nancarrow's hand. Apparently Elliott Carter had the score recopied before publication in *New Music*. See Gagne and Caras, 1982, p. 291).

E^b
A^b

p. 2, m. 4, bass clef: chord should be spelled D^b

p. 9, third system, bottom staff: bar line is missing after first two notes; sixth note (D₁) should be a dotted eighth note (dot is missing)

p. 10, second system, second staff: fifth note should be C#₆ (sharp sign is missing)

p. 14, first system, bottom staff: dotted-sixteenth rest missing at end of first complete measure

Study #3a

p. 4, second system, middle (second) staff: eighth rest missing on seventh eighth-note beat

p. 8, third system, middle staff: quarter rest missing after each sixteenth note

p. 16, third system, bass staff: ostinato should switch to C pattern in second measure; also, in last system ostinato should be G pattern for first two measures and become C pattern in last measure

Study #3b

- p. 1, first system, bass staff: second note in first measure (D_{b3}) should be dotted eighth note (dot is missing)
- p. 5: all initial clefs are missing from both systems
- p. 5, first system, bottom staff: second note in first measure should be a dotted eighth note (dot is missing)
- p. 5, second system, bottom staff: first note should be B_{b1} (flat sign is missing)
- p. 8, second system, bottom staff: repeat sign is missing in second measure
- p. 12, bottom system, top staff: in second measure, first and sixth chords should contain $C\#$ octaves (both C_s s are missing the sharp sign)
- p. 21, first system, third staff: in second measure, fourth chord should be a dotted eighth note (dots are missing)
- p. 24, last system: initial clefs missing (treble-treble-bass)

Study #3c

- p. 3, third system, top staff: first note in second measure (A_{b4}) should be dotted quarter note (dot is missing)

Study #3d

- p. 1, fourth system, top staff: second chord of second measure should be $D\#$ octave (sharp sign is missing on top note)
- p. 1, fourth system, bottom staff: dotted-half note chord on second beat should have flat signs repeated for notes B_{b3} and D_{b4}

Study #4

- p. 1, second system, treble staff: tie is missing from first note (E_{b6})
- p. 1, third system, bass staff: F_2 should be tied over for 6 bars rather than change to G_2
- p. 7, second system, middle staff: first measure contains 8 sixteenth notes and does not fit $\frac{6}{16}$ meter; perhaps dot should be removed from dotted quarter?; also, in the same measure, the staff above should contain 5 sixteenth notes—flag should be added to first note. It is difficult to determine what is intended here since each line of the system might contain a different (and constantly varying) number of sixteenth-note beats.
- p. 8, first system, bass staff: third note from end of that line should be A_{b1} (flat sign is missing)
- p. 8, second system, top staff: penultimate note should be B_4 and not G_4
- p. 8, second system, bass staff: $\frac{5}{16}$ meter signature is missing after $\frac{13}{16}$ measure
- p. 8, last system, bass staff: in the three $\frac{5}{16}$ measures that follow the $\frac{10}{16}$ measure, the rhythm should be ♪.♪ (flag is missing from first note in each of those measures)
- p. 10, second system, bass staff: meter should not change to $\frac{5}{16}$ until third measure
- p. 10, last system, bass staff: meter signature for first complete measure should be $\frac{5}{16}$ and for next measure $\frac{6}{16}$
- p. 12, first system, bass staff: last note of that line should be E_{b2} (flat sign is missing)

Study #5

- p. 13, tenth staff: ⁷16 ostinato (see ostinato figure on p. 11) should be stated beginning with 47th sixteenth note of that line—figure should be preceded by blank ³⁵16 measure and ²¹16 measure
- p. 19, second staff: ostinato figure should be notated one sixteenth note later than it is
- p. 20, third staff: time signature ³⁵16 should precede last note (C₇)
- p. 28, fifth staff: three time signatures missing (²¹16, ³⁰16, ²¹16)
- p. 32, first staff: sharp sign missing for F#₆ across whole line
- p. 32, fourth staff: flat signs missing from chord B_{b5}–D_{b7} on last note

Study #6

- p. 11: all initial clef signs are missing (treble-treble-bass-bass)

Study #7

- p. 1, third system, treble staff: tie missing between sixth and seventh measures
- p. 1, last system, treble staff: tie missing between fourth and fifth measures

Study #8

- p. 1, third system, bottom (bass) staff: first note (G₂) should be an eighth note

Study #9

- p. 3, first system, top staff: m. 4 should be ^{D6}~~E~~₅ eighth note on beat 1, and present mm. 4–5 should become mm. 5–6 (in other words, present mm. 4–5 should be notated one measure later)
- p. 14, third system, bottom staff: sharp sign is missing for bottom note of second chord (should be F# octave)

Study #10

- +p. 4, second system, bottom staff: in third measure, chord should change to ^{F3}^{D3}G₂ and be tied over to fourth measure

Study #11

- p. 7, second system, top staff, last measure: chord should be ^{D7}A₆ for correct voicing of M7 chord (no 5th) that occurs consistently on each ⁵8 bar that opens a 20 eighth-note segment

Study #12

- p. 4, second system, top staff: dotted eighth rest needed at beginning of line to fill up ¹⁵16 measure

- p. 6, first system, bottom staff, third measure: flag is missing on second note (should be dotted eighth note)
- p. 9, first and second systems: time signature at end of first system should be $\frac{9}{16}$ (not $\frac{6}{16}$) to be consistent with metric pattern of opening canon the first three times, and following measure should have $\frac{6}{16}$ signature
- p. 12, third system: last meter change ($\frac{27}{16}$) is notated one bar too early

Study #16

- p. 7, staff "e": first note of second measure should be $\downarrow$ rather than $\downarrow$

Study #17

- p. 4, first system, top staff: first chord on that line should be an octave G
- p. 4, third system, top staff: tempo change ($\downarrow = 138$) not notated at double bar ($\frac{5}{8}$ measure)
- p. 8, first system, bottom staff: clef should be bass and not treble

Study #20

- +(Gann notes existence [p. 291, n4] of errors in the score, particularly in rhythmic series)

Study #22

- p. 5, second system, middle staff: this staff should begin with a continuation of the rest that ended the previous line

Study #23

- p. 1, third system, bottom staff: 8va. b. line is missing below this staff (without this, Gann's assertion [p. 156] that the same note is not used twice in this line is not true)

Study #24

- p. 12, third system, bottom staff: chord in first $\frac{6}{4}$ measure should be open fifth stack, and F_2 should not be flatted (see Gann, p. 176, transposition levels for Canon 9)

Study #25

- p. 1, first system, top staff: third "exploded" arpeggio is based on overtone series, so fifth note should be A# rather than A
- p. 2, first system, bottom staff: extension line after B_2 is missing (note should be sustained)
- p. 5, first system, fourth staff: top note of second chord should be $C\#_7$ (leger line is missing)
- p. 6, second system, bottom staff: first, sixth, and seventh chords are missing notes in this context of root position dominant seventh chords
- p. 7, first system, bottom two staves: note E_b appears to be missing (context is root position dominant seventh chords)
- p. 8, second system, second staff: extension lines to indicate sustained notes are missing from final chord in the system

- p. 20, second system, arpeggio layer: accidentals are missing in second and third arpeggios; second arpeggio should state first a B₁ and then a C triad, while third arpeggio should state first an F and then a B triad
- p. 21, first system, arpeggio layer: accidentals are incorrect in first arpeggio, which should state first a D and then an A₁ (not A) triad
- p. 21, second system: left brace is missing
- p. 22, first system, arpeggio layer: in second arpeggio, final note should probably be G₃ rather than D₄ to fit pattern of 16-note arpeggios stating first 16 notes of the overtone series
- p. 23, first system, arpeggio layer: first arpeggio appears to be missing note E_{b6} (to complete the overtone series)
- p. 24, sixth staff: in third (last) arpeggio 8 b. marking is missing under last five notes
- p. 24, bottom staff: stems missing from all notes
- p. 26, bottom staff: bass clef sign missing between sixth and seventh notes
- p. 32, fourth staff: initial bass clef sign is missing
- p. 41, first system, arpeggio layer: final 16-note arpeggio is based on G overtone series, and last two notes should be G₃ and G₂
- +p. 44, second system, top two staves: C₃ in first arpeggio is probably an error as it does not fit E_b major arpeggio (see Gann p. 294 n. 5)
- p. 50, bottom staff: to fit the row statement (pitch 6 of P11), bottom note of second chord should be E and not D#
- p. 53, fourth staff: extension lines for sustained notes are missing after first chord
- p. 59: left brace is missing, and bottom three clefs from top down should be treble, treble, bass
- p. 59, bottom three staves: extension lines from previous page should extend to beginning of first staccato chord
- p. 74, bottom system: left brace is missing, and clefs should be treble, bass
- p. 76, top system: under the second 8va. marking, it appears that last five notes are missing sharps to fit B major scale

Study #26

- p. 5, second system: all initial clefs are missing; the order, from top, should be: treble, treble, treble, treble, bass, bass

Study #27

- p. 3, second system, bottom staff: there should be a brief rest notated after F₁, occupying half the note value
- p. 3, third system, bottom staff: there should be a brief rest notated after E₁, occupying half the note value

Study #28

- p. 2, first system, third staff (first "occupied" staff): octave higher designation is missing from third note through bass clef
- p. 3, second system, third staff (first "occupied" staff): line denoting octave lower is missing

Study #31

- p. 2, third system: eighth note rests are missing in entire line (meter is $\frac{4}{8}$)

- p. 3, second system, middle (second) staff: rhythm in sixth complete measure should be 

Study #32

- p. 1, fourth system: $\frac{9}{8}$ meter marking is missing in third measure and $\frac{6}{8}$ meter marking in fourth measure
- p. 2, fourth system, middle staff: *8va b.* continuation line is missing under first two measures
- p. 3, first system, middle staff: first note of fourth full measure should be $F\sharp$ rather than F_b
- p. 3, second system, bottom staff: $\frac{6}{8}$ meter marking is missing in last measure of the line
- p. 5, second system, bottom staff: all meter changes are missing in this line: $\frac{9}{8}$ in second complete measure, $\frac{6}{8}$ in next measure, and $\frac{9}{8}$ in final measure
- p. 5, third system, second staff from top: octave higher designation in mm. 4 and 5 is erroneous
- p. 5, third system, bottom staff: *8va b.* marking should continue through first note
- p. 6, first system, second staff from top: note in last complete measure should be F_2 rather than F_3
- p. 6, second system, top staff: $\frac{9}{8}$ meter marking is missing in last (incomplete) measure
- p. 6, third system, top staff: $C\sharp$, $\downarrow$ is missing (tied over from previous system) in incomplete measure at beginning of line
- p. 6, third system, top staff: $\frac{6}{8}$ meter marking is missing in first complete measure
- p. 6, third system, top staff: note in seventh complete measure should be C_4 rather than C_3
- p. 8, first system, bottom staff: $\frac{9}{8}$ meter marking is missing in first full measure
- p. 8, second system, bottom staff: $\frac{6}{8}$ meter marking is missing in first full measure
- p. 9, first system, bottom staff: all meter changes are missing in this line: $\frac{9}{8}$ in second complete measure, $\frac{6}{8}$ two measures later
- p. 10, third system, top staff: change to treble clef missing in third full measure

Study #33

- p. 30, second system, bottom staff: first complete measure should have a $\frac{5}{16}$ meter signature

Study #34

- p. 5, last system, middle staff: $\frac{8}{8}$ measure is missing three eighth rests (beats 2, 4, and 7)
- p. 7, second system, top staff, fifth full measure: eighth rest missing on second beat
- p. 11, second system, bottom staff, third measure: rest on second beat should be eighth rest rather than quarter rest

- p. 11, third system, middle staff, third measure: eighth rests missing on beats 2 and 4
- p. 11, third system, bottom staff: articulations missing in entire line (all eighth notes staccato except those beamed to a following eighth note; beamed eighth notes slurred together)

Study #35

- +p. 27, last system: tempo designation in bottom two staves should be $\text{♩} = 283\frac{1}{3}$ rather than 204

Study #36

- p. 2, third system, top staff: $\frac{4}{4}$ meter signature should appear prior to the entrance of this voice
- p. 18, third system, bottom staff: in last measure, top note of half-note chord should be D_{b3} (flat sign missing)
- p. 47, first system, third staff: $\frac{4}{4}$ meter change should be inserted at beginning of third full measure
- p. 48, first system, bottom staff: $\frac{4}{4}$ meter change should be inserted at beginning of third measure

Study #37

- p. 60, seventh staff from top: $\frac{4}{8}$ measure is missing an eighth rest at end of measure
- p. 67, seventh staff from top: tempo marking ($\text{♩} = 200$) is missing at beginning of this staff

Study #41a

- p. 1, first system, top notated staff: change to bass clef is needed before final four notes of system
- p. 3, third (last) system, third staff: line indicating drone note should be on B_2 rather than G_2
- p. 5, first system, top staff: 23-note run at beginning of system should be marked *pp* rather than *ff*

Study #41b

- p. 25, second system: left brace is missing in this system

Study #43

- p. 1, second system, bottom staff: rests are missing after first quarter note in $\frac{11}{4}$ measure: dotted half rest followed by whole rest
- p. 1, third system, third staff: initial treble clef is missing
- +(marked in Gann's copy of the score) p. 1, third system, fourth (lowest) staff: first note of $\frac{7}{4}$ measure should be $E\#_3$, not $G\#_3$
- p. 6, second system, bottom staff: first $\frac{4}{4}$ measure is missing a quarter rest on second beat

- p. 6, third system, second staff from top: first $\frac{4}{4}$ measure is missing a quarter rest on second beat
- p. 9, first system, third staff from top: quarter rest is missing at end of $\frac{11}{4}$ measure
- p. 21, second system, second and fourth staves: in measure that begins with shared barline, clef should change from treble to bass
- p. 35, third system, bottom two staves, and p. 36, first system, bottom two staves: $\frac{8}{4}$ measure that begins on bottom of p. 35 concludes after first half note at top of p. 36; barline and $\frac{10}{4}$ meter signature should be inserted after this half note and the present $\frac{10}{4}$ barline deleted

Study #45a

- p. 9, top system, middle staff: glissando at beginning of system apparently is missing a concluding A_4
- p. 11, top system, middle staff: glissando is missing a concluding E_4
- +p. 12, bottom system, top staff: tenth note should be a C major triad rather than note G_6
- p. 21: piece concludes in the bass layer with a C octave, which is not noted

Study #45b

- +p. 2, first system, bass staff: the eighth note (F_2) occurs earlier in the recording than indicated on the score—it occurs before, not after, the grace notes and chord in top voice
- p. 8, bottom system, middle staff: after fourth eighth note, a B_4 eighth note is missing (in the same rhythm as previous four notes)
- +p. 9: Gann notes that an entire system (system 23) is apparently missing after top system on this page
- p. 13, bottom system, top staff: the glissando at the end of the line appears to be missing the note D_5
- p. 14, second system, middle staff: *8b.* indication missing under first four notes
- p. 14, second system, top staff: *8va.* indication missing over chord $F\#-A\#-C\#$
- p. 15, middle system, top staff: sustain line from top staff of previous system is missing
- p. 15, bottom system, bottom staff: after second C_4 (near middle of system), note E_{b4} is missing
- p. 17, top system, bottom staff: fourth note (written as $F\#_2$) should be one octave higher ($F\#_3$)
- p. 18, top system, bottom staff: eighth note (written as C_4) should be one octave lower (C_3)

Study #45c

- p. 2, third system, bottom staff: eleventh and twelfth eighth notes should be A_b , rather than A natural
- p. 8, fourth system, top staff: A major triad is sustained after third glissando, but is missing in score
- p. 14, first system, second staff: line should end with a major triad eighth note built on F_4

Study #48

pp. 3–6: some of the initial clefs are missing on the bottom system (p. 3, treble clef on third line from bottom, bass clef on second line from bottom; p. 4, same as p. 3; p. 5, treble clef on third line from bottom; and p. 6, bass clef on second line from bottom)

APPENDIX B: AVAILABLE SCORES, RECORDINGS,
AND WRITINGS FOR EACH STUDY

Key:
Study #
<u>Score</u>
<u>Recording</u>
<u>Writings</u>

N.B. Each of the Studies is discussed by Kyle Gann in *The Music of Conlon Nancarrow*. (Cambridge: Cambridge University Press, 1995) so that information is not repeated in the listings below.

#1

Score: Nancarrow, Conlon. *Rhythm Study No. 1 for Player Piano*. 1952.

Recordings: Nancarrow, Conlon. *Studies for Player Piano*, Vols. III/IV [Studies No. 1, 2a, 2b, 7, 8, 9, 10, 11, 12, 13, 15, 16, 17, 18, 19, 21, 23, 24, 25, 27, 28, 29, 33, 34, 36, 43, 46, 47, and 50]. Compact discs. Wergo: WER 60166/67-50, 1987.

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#2

Score: Nancarrow, Conlon. *Collected Studies for Player Piano, Vol. 5: Studies No. 2, 6, 7, 14, 20, 21, 24, 26, and 33 for Player Piano*. Santa Fe: Soundings Press, 1984.

Recordings: Nancarrow, Conlon. *Studies for Player Piano*, Vols. III/IV [Studies No. 1, 2a, 2b, 7, 8, 9, 10, 11, 12, 13, 15, 16, 17, 18, 19, 21, 23, 24, 25, 27, 28, 29, 33, 34, 36, 43, 46, 47, and 50]. Compact discs. Wergo: WER 60166/67-50, 1987.

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#4

Score: Nancarrow, Conlon. *Studies No. 4, 5, 9, 10, 11, 12, 15, 16, 17, and 18 for Player Piano (Volume 6)*. Santa Fe: Soundings Press, 1985.

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#5

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Recordings: Nancarrow, Conlon. *Studies for Player Piano*, Vols. I/II [Studies No. 3a, 3b, 3c, 3d, 3e, 4, 5, 6, 14, 20, 22, 26, 31, 32, 35, 37, 40a, 40b, 41a, 41b, 41c, 44; Tango?]. Compact discs. Wergo: WER 6168-2/6169-2, 1987.

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#12

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#13

Score: never released by Nancarrow (see Gann, p. 68)

Recordings: Nancarrow, Conlon. *Studies for Player Piano*, Vols. III/IV [Studies No. 1, 2a, 2b, 7, 8, 9, 10, 11, 12, 13, 15, 16, 17, 18, 19, 21, 23, 24, 25, 27, 28, 29, 33, 34, 36, 43, 46, 47, and 50]. Compact discs. Wergo: WER 60166/67-50, 1987.

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#15 (Canon 3/4)

Score: Nancarrow, Conlon. *Studies No. 4, 5, 9, 10, 11, 12, 15, 16, 17, and 18 for Player Piano (Volume 6)*. Santa Fe: Soundings Press, 1985.

Recordings: Nancarrow, Conlon. *Studies for Player Piano*, Vols. III/IV [Studies No. 1, 2a, 2b, 7, 8, 9, 10, 11, 12, 13, 15, 16, 17, 18, 19, 21, 23, 24, 25, 27, 28, 29, 33, 34, 36, 43, 46, 47, and 50]. Compact discs. Wergo: WER 60166/67-50, 1987.

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#16

Score: Nancarrow, Conlon. *Studies No. 4, 5, 9, 10, 11, 12, 15, 16, 17, and 18 for Player Piano (Volume 6)*. Santa Fe: Soundings Press, 1985.

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#17 (Canon 12/15/20)

Score: Nancarrow, Conlon. *Studies No. 4, 5, 9, 10, 11, 12, 15, 16, 17, and 18 for*

Player Piano (Volume 6). Santa Fe: Soundings Press, 1985.

Recordings: Nancarrow, Conlon. *Studies for Player Piano*, Vols. III/IV [Studies No. 1, 2a, 2b, 7, 8, 9, 10, 11, 12, 13, 15, 16, 17, 18, 19, 21, 23, 24, 25, 27, 28, 29, 33, 34, 36, 43, 46, 47, and 50]. Compact discs. Wergo: WER 60166/67-50, 1987.

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#18 (Canon 3/4)

Score: Nancarrow, Conlon. *Studies No. 4, 5, 9, 10, 11, 12, 15, 16, 17, and 18 for Player Piano (Volume 6)*. Santa Fe: Soundings Press, 1985.

Recordings: Nancarrow, Conlon. *Studies for Player Piano*, Vols. III/IV [Studies No. 1, 2a, 2b, 7, 8, 9, 10, 11, 12, 13, 15, 16, 17, 18, 19, 21, 23, 24, 25, 27, 28, 29, 33, 34, 36, 43, 46, 47, and 50]. Compact discs. Wergo: WER 60166/67-50, 1987.

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#19 (Canon 12/15/20)

Score: Nancarrow, Conlon. *Conlon Nancarrow: Selected Studies for Player Piano* [With Critical Material by Gordon Mumma, Charles Amirkhanian, John Cage, Roger Reynolds, and James Tenney], edited by Peter Garland. Soundings, Book 4. Berkeley, CA: Soundings Press, Spring-Summer 1977.

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- Writings: Tenney, James. "Studies for Player Piano Vol. IV: Notes on *Studies #9, 11, 12, 13, 16, 17, 18, 19, 27, 28, 29, 34, 36, 46, 47*." In *Conlon Nancarrow: Studies for Player Piano*, Vols. III/IV [compact discs], pp. 20-30. Wergo: WER 60166/67-50, 1987.
- Carlsen, Philip. *The Player Piano Music of Conlon Nancarrow: An Analysis of Selected Studies*. I.S.A.M. Monographs, no. 26. Brooklyn, N.Y.: Institute for Studies in American Music, 1988.
- Thomas, Margaret Elida. *Conlon Nancarrow's 'Temporal Dissonance': Rhythmic and Textural Stratification in the Studies for Player Piano* [Ph.D. dissertation, Yale University]. Ann Arbor, Mich.: University Microfilms, 1996 [see Appendix C of this dissertation for page numbers].
- Wierzbicki, James. "Nancarrow: Studies for Player Piano, Nos. 9, 11-13, 16-19, 27-29, 34, 36" [review]. *High Fidelity/Musical America* 34 (November 1984): 45-46.

#20

- Score: Nancarrow, Conlon. *Collected Studies for Player Piano, Vol. 5: Studies No. 2, 6, 7, 14, 20, 21, 24, 26, and 33 for Player Piano*. Santa Fe: Soundings Press, 1984.
- Recordings: Nancarrow, Conlon. *Studies for Player Piano*, Vols. I/II [Studies No. 3a, 3b, 3c, 3d, 3e, 4, 5, 6, 14, 20, 22, 26, 31, 32, 35, 37, 40a, 40b, 41a, 41b, 41c, 44; Tango?]. Compact discs. Wergo: WER 6168-2/6169-2, 1987.
- Nancarrow, Conlon. *Complete Studies for Player Piano*, Vol. 1 [Studies No. 3, 20, 41]. 1750 Arch: S-1768, 1977.
- Writings: Tenney, James. "Studies for Player Piano Vol. I: Notes on *Studies #3 (a-e), 20, 41 (a-c), 44*." In *Conlon Nancarrow: Studies for Player Piano*, Vols. I/II [compact discs], pp. 7-11. Wergo: WER 6168-2/6169-2, 1987.
- Thomas, Margaret Elida. *Conlon Nancarrow's 'Temporal Dissonance': Rhythmic and Textural Stratification in the Studies for Player Piano* [Ph.D. dissertation, Yale University]. Ann Arbor, Mich.: University Microfilms, 1996 [see Appendix C of this dissertation for page numbers].

#21 (Canon X)

- Score: Nancarrow, Conlon. *Collected Studies for Player Piano, Vol. 5: Studies No. 2, 6, 7, 14, 20, 21, 24, 26, and 33 for Player Piano*. Santa Fe: Soundings Press, 1984.
- Recordings: Nancarrow, Conlon. *Studies for Player Piano*, Vols. III/IV [Studies No. 1, 2a, 2b, 7, 8, 9, 10, 11, 12, 13, 15, 16, 17, 18, 19, 21, 23, 24, 25, 27, 28, 29, 33, 34, 36, 43, 46, 47, and 50]. Compact discs. Wergo: WER 60166/67-50, 1987.
- Nancarrow, Conlon. *Complete Studies for Player Piano*, Vol. 3 [Sonatina, Studies Nos. 1, 2, 7, 8, 10, 15, 21, 23, 24, 25, 33]. 1750 Arch: S-1786, 1981.
- Writings: Tenney, James. "Studies for Player Piano Vol. III: Notes on *Studies #1, 2a, 2b, 7, 8, 10, 15, 21, 23, 24, 25, 33, 43, 50*." In *Conlon Nancarrow: Studies for Player Piano*, Vols. III/IV [compact discs], pp. 7-19. Wergo: WER 60166/67-50,

1987.

Thomas, Margaret Elida. *Conlon Nancarrow's 'Temporal Dissonance': Rhythmic and Textural Stratification in the Studies for Player Piano* [Ph.D. dissertation, Yale University]. Ann Arbor, Mich.: University Microfilms, 1996 [see Appendix C of this dissertation for page numbers].

#22 (Canon 1%/1-1/2%/2-1/4%)

Score: unpublished

Recordings: Nancarrow, Conlon. *Studies for Player Piano*, Vols. I/II [Studies No. 3a, 3b, 3c, 3d, 3e, 4, 5, 6, 14, 20, 22, 26, 31, 32, 35, 37, 40a, 40b, 41a, 41b, 41c, 44; Tango?]. Compact discs. Wergo: WER 6168-2/6169-2, 1987.

Nancarrow, Conlon. *Complete Studies for Player Piano*, Vol. 2 [Studies No. 4, 5, 6, 14, 22, 26, 31, 32, 35, 37, 40]. 1750 Arch: S-1777, 1979.

Writings: Tenney, James. "Studies for Player Piano Vol. II: Notes on *Studies #4, 5, 6, 14, 22, 26, 31, 32, 35, 37, Tango?, 40 (a, b).*" In *Conlon Nancarrow: Studies for Player Piano*, Vols. I/II [compact discs], pp. 7-11. Wergo: WER 6168-2/6169-2, 1987.

#23

Score: Nancarrow, Conlon. *Conlon Nancarrow: Selected Studies for Player Piano* [With Critical Material by Gordon Mumma, Charles Amirkhanian, John Cage, Roger Reynolds, and James Tenney], edited by Peter Garland. Soundings, Book 4. Berkeley, CA: Soundings Press, Spring-Summer 1977.

Recordings: Nancarrow, Conlon. *Studies for Player Piano*, Vols. III/IV [Studies No. 1, 2a, 2b, 7, 8, 9, 10, 11, 12, 13, 15, 16, 17, 18, 19, 21, 23, 24, 25, 27, 28, 29, 33, 34, 36, 43, 46, 47, and 50]. Compact discs. Wergo: WER 60166/67-50, 1987.

Nancarrow, Conlon. *Complete Studies for Player Piano*, Vol. 3 [Sonatina, Studies Nos. 1, 2, 7, 8, 10, 15, 21, 23, 24, 25, 33]. 1750 Arch: S-1786, 1981.

Writings: Tenney, James. "Studies for Player Piano Vol. III: Notes on *Studies #1, 2a, 2b, 7, 8, 10, 15, 21, 23, 24, 25, 33, 43, 50.*" In *Conlon Nancarrow: Studies for Player Piano*, Vols. III/IV [compact discs], pp. 7-19. Wergo: WER 60166/67-50, 1987.

Carlsen, Philip. *The Player Piano Music of Conlon Nancarrow: An Analysis of Selected Studies*. I.S.A.M. Monographs, no. 26. Brooklyn, N.Y.: Institute for Studies in American Music, 1988.

Thomas, Margaret Elida. *Conlon Nancarrow's 'Temporal Dissonance': Rhythmic and Textural Stratification in the Studies for Player Piano* [Ph.D. dissertation, Yale University]. Ann Arbor, Mich.: University Microfilms, 1996 [see Appendix C of this dissertation for page numbers].

#24 (Canon 14/15/16)

Score: Nancarrow, Conlon. *Collected Studies for Player Piano, Vol. 5: Studies No. 2, 6, 7, 14, 20, 21, 24, 26, and 33 for Player Piano*. Santa Fe: Soundings Press, 1984.

Recordings: Nancarrow, Conlon. *Studies for Player Piano*, Vols. III/IV [Studies No. 1, 2a, 2b, 7, 8, 9, 10, 11, 12, 13, 15, 16, 17, 18, 19, 21, 23, 24, 25, 27, 28, 29, 33,

34, 36, 43, 46, 47, and 50]. Compact discs. Wergo: WER 60166/67-50, 1987.
 Nancarrow, Conlon. *Complete Studies for Player Piano*, Vol. 3 [Sonatina, Studies Nos. 1, 2, 7, 8, 10, 15, 21, 23, 24, 25, 33]. 1750 Arch: S-1786, 1981.
Writings: Tenney, James. "Studies for Player Piano Vol. III: Notes on *Studies #1, 2a, 2b, 7, 8, 10, 15, 21, 23, 24, 25, 33, 43, 50*." In *Conlon Nancarrow: Studies for Player Piano*, Vols. III/IV [compact discs], pp. 7-19. Wergo: WER 60166/67-50, 1987.
 Thomas, Margaret Elida. *Conlon Nancarrow's 'Temporal Dissonance': Rhythmic and Textural Stratification in the Studies for Player Piano* [Ph.D. dissertation, Yale University]. Ann Arbor, Mich.: University Microfilms, 1996 [see Appendix C of this dissertation for page numbers].

#25

Score: Nancarrow, Conlon. "Study No. 25 for Player Piano." *Soundings* 9 (Summer 1975).
Recordings: Nancarrow, Conlon. *Studies for Player Piano*, Vols. III/IV [Studies No. 1, 2a, 2b, 7, 8, 9, 10, 11, 12, 13, 15, 16, 17, 18, 19, 21, 23, 24, 25, 27, 28, 29, 33, 34, 36, 43, 46, 47, and 50]. Compact discs. Wergo: WER 60166/67-50, 1987.
 Nancarrow, Conlon. *Complete Studies for Player Piano*, Vol. 3 [Sonatina, Studies Nos. 1, 2, 7, 8, 10, 15, 21, 23, 24, 25, 33]. 1750 Arch: S-1786, 1981.
Writings: Tenney, James. "Studies for Player Piano Vol. III: Notes on *Studies #1, 2a, 2b, 7, 8, 10, 15, 21, 23, 24, 25, 33, 43, 50*." In *Conlon Nancarrow: Studies for Player Piano*, Vols. III/IV [compact discs], pp. 7-19. Wergo: WER 60166/67-50, 1987.
 Thomas, Margaret Elida. *Conlon Nancarrow's 'Temporal Dissonance': Rhythmic and Textural Stratification in the Studies for Player Piano* [Ph.D. dissertation, Yale University]. Ann Arbor, Mich.: University Microfilms, 1996 [see Appendix C of this dissertation for page numbers].

#26 (Canon 1/1 [/1/1/1/1])

Score: Nancarrow, Conlon. *Collected Studies for Player Piano*, Vol. 5: *Studies No. 2, 6, 7, 14, 20, 21, 24, 26, and 33 for Player Piano*. Santa Fe: Soundings Press, 1984.
Recordings: Nancarrow, Conlon. *Studies for Player Piano*, Vols. I/II [Studies No. 3a, 3b, 3c, 3d, 3e, 4, 5, 6, 14, 20, 22, 26, 31, 32, 35, 37, 40a, 40b, 41a, 41b, 41c, 44; Tango?]. Compact discs. Wergo: WER 6168-2/6169-2, 1987.
 Nancarrow, Conlon. *Complete Studies for Player Piano*, Vol. 2 [Studies No. 4, 5, 6, 14, 22, 26, 31, 32, 35, 37, 40]. 1750 Arch: S-1777, 1979.
Writings: Tenney, James. "Studies for Player Piano Vol. II: Notes on *Studies #4, 5, 6, 14, 22, 26, 31, 32, 35, 37, Tango?, 40 (a, b)*." In *Conlon Nancarrow: Studies for Player Piano*, Vols. I/II [compact discs], pp. 7-11. Wergo: WER 6168-2/6169-2, 1987.
 La Barbara, Joan. "The Remarkable Art of Conlon Nancarrow." *High Fidelity/Musical America* 34 (May 1984): 12-13.

#27 (Canon 5%/6%/8%/11%)

Score: Nancarrow, Conlon. *Conlon Nancarrow: Selected Studies for Player Piano* [With Critical Material by Gordon Mumma, Charles Amirkhanian, John Cage, Roger Reynolds, and James Tenney], edited by Peter Garland. Soundings, Book 4. Berkeley, CA: Soundings Press, Spring-Summer 1977.

Recordings: Nancarrow, Conlon. *Studies for Player Piano*, Vols. III/IV [Studies No. 1, 2a, 2b, 7, 8, 9, 10, 11, 12, 13, 15, 16, 17, 18, 19, 21, 23, 24, 25, 27, 28, 29, 33, 34, 36, 43, 46, 47, and 50]. Compact discs. Wergo: WER 60166/67-50, 1987.

Nancarrow, Conlon. *Complete Studies for Player Piano*, Vol. 4 [Studies Nos. 9, 11, 12, 13, 16, 17, 18, 19, 27, 28, 29, 34, 36]. 1750 Arch: S-1798, 1984.

Writings: Tenney, James. "Studies for Player Piano Vol. IV: Notes on *Studies #9, 11, 12, 13, 16, 17, 18, 19, 27, 28, 29, 34, 36, 46, 47.*" In *Conlon Nancarrow: Studies for Player Piano*, Vols. III/IV [compact discs], pp. 20-30. Wergo: WER 60166/67-50, 1987.

Jarvlepp, Jan. "Conlon Nancarrow's *Study #27 for Player Piano* Viewed Analytically." *Perspectives of New Music* 23/1-2 (Fall-Winter 1983/Spring-Summer 1984), 218-22.

Thomas, Margaret Elida. *Conlon Nancarrow's 'Temporal Dissonance': Rhythmic and Textural Stratification in the Studies for Player Piano* [Ph.D. dissertation, Yale University]. Ann Arbor, Mich.: University Microfilms, 1996 [see Appendix C of this dissertation for page numbers].

Wierzbicki, James. "Nancarrow: Studies for Player Piano, Nos. 9, 11-13, 16-19, 27-29, 34, 36" [review]. *High Fidelity/Musical America* 34 (November 1984): 45-46.

#28

Score: unpublished

Recordings: Nancarrow, Conlon. *Studies for Player Piano*, Vols. III/IV [Studies No. 1, 2a, 2b, 7, 8, 9, 10, 11, 12, 13, 15, 16, 17, 18, 19, 21, 23, 24, 25, 27, 28, 29, 33, 34, 36, 43, 46, 47, and 50]. Compact discs. Wergo: WER 60166/67-50, 1987.

Nancarrow, Conlon. *Complete Studies for Player Piano*, Vol. 4 [Studies Nos. 9, 11, 12, 13, 16, 17, 18, 19, 27, 28, 29, 34, 36]. 1750 Arch: S-1798, 1984.

Writings: Tenney, James. "Studies for Player Piano Vol. IV: Notes on *Studies #9, 11, 12, 13, 16, 17, 18, 19, 27, 28, 29, 34, 36, 46, 47.*" In *Conlon Nancarrow: Studies for Player Piano*, Vols. III/IV [compact discs], pp. 20-30. Wergo: WER 60166/67-50, 1987.

Wierzbicki, James. "Nancarrow: Studies for Player Piano, Nos. 9, 11-13, 16-19, 27-29, 34, 36" [review]. *High Fidelity/Musical America* 34 (November 1984): 45-46.

#29

Score: unpublished

Recordings: Nancarrow, Conlon. *Studies for Player Piano*, Vols. III/IV [Studies No. 1, 2a, 2b, 7, 8, 9, 10, 11, 12, 13, 15, 16, 17, 18, 19, 21, 23, 24, 25, 27, 28, 29, 33, 34, 36, 43, 46, 47, and 50]. Compact discs. Wergo: WER 60166/67-50, 1987.

Nancarrow, Conlon. *Complete Studies for Player Piano*, Vol. 4 [Studies Nos. 9, 11,

- 12, 13, 16, 17, 18, 19, 27, 28, 29, 34, 36]. 1750 Arch: S-1798, 1984.
- Writings: Tenney, James. "Studies for Player Piano Vol. IV: Notes on *Studies* #9, 11, 12, 13, 16, 17, 18, 19, 27, 28, 29, 34, 36, 46, 47." In *Conlon Nancarrow: Studies for Player Piano*, Vols. III/IV [compact discs], pp. 20–30. Wergo: WER 60166/67-50, 1987.
- Wierzbicki, James. "Nancarrow: Studies for Player Piano, Nos. 9, 11–13, 16–19, 27–29, 34, 36" [review]. *High Fidelity/Musical America* 34 (November 1984): 45–46.

#30

- Score: Nancarrow never produced a score (see Gann, p. 169)
- Recordings: No commercially available recordings (see Gann, p. 169)

#31 (Canon 21/24/25)

- Score: Nancarrow, Conlon. *Conlon Nancarrow: Selected Studies for Player Piano* [With Critical Material by Gordon Mumma, Charles Amirkhanian, John Cage, Roger Reynolds, and James Tenney], edited by Peter Garland. Soundings, Book 4. Berkeley, CA: Soundings Press, Spring–Summer 1977.
- Recordings: Nancarrow, Conlon. *Studies for Player Piano*, Vols. I/II [Studies No. 3a, 3b, 3c, 3d, 3e, 4, 5, 6, 14, 20, 22, 26, 31, 32, 35, 37, 40a, 40b, 41a, 41b, 41c, 44; Tango?]. Compact discs. Wergo: WER 6168-2/6169-2, 1987.
- Nancarrow, Conlon. *Complete Studies for Player Piano*, Vol. 2 [Studies No. 4, 5, 6, 14, 22, 26, 31, 32, 35, 37, 40]. 1750 Arch: S-1777, 1979.
- Writings: Tenney, James. "Studies for Player Piano Vol. II: Notes on *Studies* #4, 5, 6, 14, 22, 26, 31, 32, 35, 37, *Tango?*, 40 (a, b)." In *Conlon Nancarrow: Studies for Player Piano*, Vols. I/II [compact discs], pp. 7–11. Wergo: WER 6168-2/6169-2, 1987.
- Thomas, Margaret Elida. *Conlon Nancarrow's 'Temporal Dissonance': Rhythmic and Textural Stratification in the Studies for Player Piano* [Ph.D. dissertation, Yale University]. Ann Arbor, Mich.: University Microfilms, 1996 [see Appendix C of this dissertation for page numbers].

#32 (Canon 5/6/7/8)

- Score: unpublished
- Recordings: Nancarrow, Conlon. *Studies for Player Piano*, Vols. I/II [Studies No. 3a, 3b, 3c, 3d, 3e, 4, 5, 6, 14, 20, 22, 26, 31, 32, 35, 37, 40a, 40b, 41a, 41b, 41c, 44; Tango?]. Compact discs. Wergo: WER 6168-2/6169-2, 1987.
- Nancarrow, Conlon. *Complete Studies for Player Piano*, Vol. 2 [Studies No. 4, 5, 6, 14, 22, 26, 31, 32, 35, 37, 40]. 1750 Arch: S-1777, 1979.
- Writings: Tenney, James. "Studies for Player Piano Vol. II: Notes on *Studies* #4, 5, 6, 14, 22, 26, 31, 32, 35, 37, *Tango?*, 40 (a, b)." In *Conlon Nancarrow: Studies for Player Piano*, Vols. I/II [compact discs], pp. 7–11. Wergo: WER 6168-2/6169-2, 1987.
- La Barbara, Joan. "The Remarkable Art of Conlon Nancarrow." *High Fidelity/Musical America* 34 (May 1984): 12–13.
- Thomas, Margaret Elida. *Conlon Nancarrow's 'Temporal Dissonance': Rhythmic*

and Textural Stratification in the Studies for Player Piano [Ph.D. dissertation, Yale University]. Ann Arbor, Mich.: University Microfilms, 1996 [see Appendix C of this dissertation for page numbers].

#33 (Canon $\sqrt{2}/2$)

Score: Nancarrow, Conlon. *Collected Studies for Player Piano, Vol. 5: Studies No. 2, 6, 7, 14, 20, 21, 24, 26, and 33 for Player Piano*. Santa Fe: Soundings Press, 1984.

Recordings: Nancarrow, Conlon. *Studies for Player Piano, Vols. III/IV* [Studies No. 1, 2a, 2b, 7, 8, 9, 10, 11, 12, 13, 15, 16, 17, 18, 19, 21, 23, 24, 25, 27, 28, 29, 33, 34, 36, 43, 46, 47, and 50]. Compact discs. Wergo: WER 60166/67-50, 1987.

Nancarrow, Conlon. *Complete Studies for Player Piano, Vol. 3* [Sonatina, Studies Nos. 1, 2, 7, 8, 10, 15, 21, 23, 24, 25, 33]. 1750 Arch: S-1786, 1981.

Writings: Tenney, James. "Studies for Player Piano Vol. III: Notes on *Studies #1, 2a, 2b, 7, 8, 10, 15, 21, 23, 24, 25, 33, 43, 50*." In *Conlon Nancarrow: Studies for Player Piano, Vols. III/IV* [compact discs], pp. 7-19. Wergo: WER 60166/67-50, 1987.

Thomas, Margaret Elida. *Conlon Nancarrow's 'Temporal Dissonance': Rhythmic and Textural Stratification in the Studies for Player Piano* [Ph.D. dissertation, Yale University]. Ann Arbor, Mich.: University Microfilms, 1996 [see Appendix C of this dissertation for page numbers].

#34 (Canon $\frac{9}{4/5/6} / \frac{10}{4/5/6} / \frac{11}{4/5/6}$)

Score: unpublished; string trio version exists (see Gann, p. 132)

Recordings: Nancarrow, Conlon. *Studies for Player Piano, Vols. III/IV* [Studies No. 1, 2a, 2b, 7, 8, 9, 10, 11, 12, 13, 15, 16, 17, 18, 19, 21, 23, 24, 25, 27, 28, 29, 33, 34, 36, 43, 46, 47, and 50]. Compact discs. Wergo: WER 60166/67-50, 1987.

Nancarrow, Conlon. *Complete Studies for Player Piano, Vol. 4* [Studies Nos. 9, 11, 12, 13, 16, 17, 18, 19, 27, 28, 29, 34, 36]. 1750 Arch: S-1798, 1984.

Writings: Tenney, James. "Studies for Player Piano Vol. IV: Notes on *Studies #9, 11, 12, 13, 16, 17, 18, 19, 27, 28, 29, 34, 36, 46, 47*." In *Conlon Nancarrow: Studies for Player Piano, Vols. III/IV* [compact discs], pp. 20-30. Wergo: WER 60166/67-50, 1987.

Wierzbicki, James. "Nancarrow: Studies for Player Piano, Nos. 9, 11-13, 16-19, 27-29, 34, 36" [review]. *High Fidelity/Musical America* 34 (November 1984): 45-46.

#35

Score: Nancarrow, Conlon. *Conlon Nancarrow: Selected Studies for Player Piano* [With Critical Material by Gordon Mumma, Charles Amirkhanian, John Cage, Roger Reynolds, and James Tenney], edited by Peter Garland. Soundings, Book 4. Berkeley, CA: Soundings Press, Spring-Summer 1977.

Recordings: Nancarrow, Conlon. *Studies for Player Piano, Vols. I/II* [Studies No. 3a, 3b, 3c, 3d, 3e, 4, 5, 6, 14, 20, 22, 26, 31, 32, 35, 37, 40a, 40b, 41a, 41b, 41c, 44; Tango?]. Compact discs. Wergo: WER 6168-2/6169-2, 1987.

Nancarrow, Conlon. *Complete Studies for Player Piano*, Vol. 2 [Studies No. 4, 5, 6, 14, 22, 26, 31, 32, 35, 37, 40]. 1750 Arch: S-1777, 1979.

Writings: Tenney, James. "Studies for Player Piano Vol. II: Notes on *Studies #4, 5, 6, 14, 22, 26, 31, 32, 35, 37, Tango?, 40 (a, b)*." In *Conlon Nancarrow: Studies for Player Piano*, Vols. I/II [compact discs], pp. 7–11. Wergo: WER 6168-2/6169-2, 1987.

Carlsen, Philip. *The Player Piano Music of Conlon Nancarrow: An Analysis of Selected Studies*. I.S.A.M. Monographs, no. 26. Brooklyn, N.Y.: Institute for Studies in American Music, 1988.

Thomas, Margaret Elida. *Conlon Nancarrow's 'Temporal Dissonance': Rhythmic and Textural Stratification in the Studies for Player Piano* [Ph.D. dissertation, Yale University]. Ann Arbor, Mich.: University Microfilms, 1996 [see Appendix C of this dissertation for page numbers].

#36 (Canon 17/18/19/20)

Score: Nancarrow, Conlon. *Conlon Nancarrow: Selected Studies for Player Piano* [With Critical Material by Gordon Mumma, Charles Amirkhanian, John Cage, Roger Reynolds, and James Tenney], edited by Peter Garland. Soundings, Book 4. Berkeley, CA: Soundings Press, Spring–Summer 1977.

Recordings: Nancarrow, Conlon. *Studies for Player Piano*, Vols. III/IV [Studies No. 1, 2a, 2b, 7, 8, 9, 10, 11, 12, 13, 15, 16, 17, 18, 19, 21, 23, 24, 25, 27, 28, 29, 33, 34, 36, 43, 46, 47, and 50]. Compact discs. Wergo: WER 60166/67-50, 1987.

Nancarrow, Conlon. *Complete Studies for Player Piano*, Vol. 4 [Studies Nos. 9, 11, 12, 13, 16, 17, 18, 19, 27, 28, 29, 34, 36]. 1750 Arch: S-1798, 1984.

Writings: Tenney, James. "Studies for Player Piano Vol. IV: Notes on *Studies #9, 11, 12, 13, 16, 17, 18, 19, 27, 28, 29, 34, 36, 46, 47*." In *Conlon Nancarrow: Studies for Player Piano*, Vols. III/IV [compact discs], pp. 20–30. Wergo: WER 60166/67-50, 1987.

Carlsen, Philip. *The Player Piano Music of Conlon Nancarrow: An Analysis of Selected Studies*. I.S.A.M. Monographs, no. 26. Brooklyn, N.Y.: Institute for Studies in American Music, 1988.

Thomas, Margaret Elida. *Conlon Nancarrow's 'Temporal Dissonance': Rhythmic and Textural Stratification in the Studies for Player Piano* [Ph.D. dissertation, Yale University]. Ann Arbor, Mich.: University Microfilms, 1996 [see Appendix C of this dissertation for page numbers].

Wierzbicki, James. "Nancarrow: Studies for Player Piano, Nos. 9, 11–13, 16–19, 27–29, 34, 36" [review]. *High Fidelity/Musical America* 34 (November 1984): 45–46.

#37 (Canon 150/160⁵/₇/168³/₄/180/187¹/₂/200/210/225/240/250/262¹/₂/281¹/₄)

Score: Nancarrow, Conlon. *Study No. 37 for Player Piano*. Santa Fe: Soundings Press, 1982.

Recordings: Nancarrow, Conlon. *Studies for Player Piano*, Vols. I/II [Studies No. 3a, 3b, 3c, 3d, 3e, 4, 5, 6, 14, 20, 22, 26, 31, 32, 35, 37, 40a, 40b, 41a, 41b, 41c, 44; Tango?]. Compact discs. Wergo: WER 6168-2/6169-2, 1987.

Nancarrow, Conlon. *Complete Studies for Player Piano*, Vol. 2 [Studies No. 4, 5, 6,

14, 22, 26, 31, 32, 35, 37, 40]. 1750 Arch: S-1777, 1979.

Writings: Tenney, James. "Studies for Player Piano Vol. II: Notes on *Studies #4, 5, 6, 14, 22, 26, 31, 32, 35, 37, Tango?, 40 (a, b)*." In *Conlon Nancarrow: Studies for Player Piano*, Vols. I/II [compact discs], pp. 7–11. Wergo: WER 6168-2/6169-2, 1987.

Thomas, Margaret Elida. *Conlon Nancarrow's 'Temporal Dissonance': Rhythmic and Textural Stratification in the Studies for Player Piano* [Ph.D. dissertation, Yale University]. Ann Arbor, Mich.: University Microfilms, 1996 [see Appendix C of this dissertation for page numbers].

#38

renumbered by Nancarrow as Study No. 43

#39

renumbered by Nancarrow as Study No. 48

#40

Score: Nancarrow, Conlon. *Conlon Nancarrow: Selected Studies for Player Piano* [With Critical Material by Gordon Mumma, Charles Amirkhanian, John Cage, Roger Reynolds, and James Tenney], edited by Peter Garland. Soundings, Book 4. Berkeley, CA: Soundings Press, Spring–Summer 1977.

Recordings: Nancarrow, Conlon. *Studies for Player Piano*, Vols. I/II [Studies No. 3a, 3b, 3c, 3d, 3e, 4, 5, 6, 14, 20, 22, 26, 31, 32, 35, 37, 40a, 40b, 41a, 41b, 41c, 44; Tango?]. Compact discs. Wergo: WER 6168-2/6169-2, 1987.

Nancarrow, Conlon. *Complete Studies for Player Piano*, Vol. 2 [Studies No. 4, 5, 6, 14, 22, 26, 31, 32, 35, 37, 40]. 1750 Arch: S-1777, 1979.

Writings: Tenney, James. "Studies for Player Piano Vol. II: Notes on *Studies #4, 5, 6, 14, 22, 26, 31, 32, 35, 37, Tango?, 40 (a, b)*." In *Conlon Nancarrow: Studies for Player Piano*, Vols. I/II [compact discs], pp. 7–11. Wergo: WER 6168-2/6169-2, 1987.

La Barbara, Joan. "The Remarkable Art of Conlon Nancarrow." *High Fidelity/Musical America* 34 (May 1984): 12–13.

Thomas, Margaret Elida. *Conlon Nancarrow's 'Temporal Dissonance': Rhythmic and Textural Stratification in the Studies for Player Piano* [Ph.D. dissertation, Yale University]. Ann Arbor, Mich.: University Microfilms, 1996 [see Appendix C of this dissertation for page numbers].

#41

Score: Nancarrow, Conlon. *Study No. 41 for Player Piano*. Santa Fe: Soundings Press, 1981.

Recordings: Nancarrow, Conlon. *Studies for Player Piano*, Vols. I/II [Studies No. 3a, 3b, 3c, 3d, 3e, 4, 5, 6, 14, 20, 22, 26, 31, 32, 35, 37, 40a, 40b, 41a, 41b, 41c, 44; Tango?]. Compact discs. Wergo: WER 6168-2/6169-2, 1987.

Nancarrow, Conlon. *Complete Studies for Player Piano*, Vol. 1 [Studies No. 3, 20, 41]. 1750 Arch: S-1768, 1977.

Writings: Tenney, James. "Studies for Player Piano Vol. I: Notes on *Studies #3 (a–e)*,"

20, 41 (a–c), 44.” In *Conlon Nancarrow: Studies for Player Piano*, Vols. I/II [compact discs], pp. 7–11. Wergo: WER 6168-2/6169-2, 1987.

Thomas, Margaret Elida. *Conlon Nancarrow’s ‘Temporal Dissonance’: Rhythmic and Textural Stratification in the Studies for Player Piano* [Ph.D. dissertation, Yale University]. Ann Arbor, Mich.: University Microfilms, 1996 [see Appendix C of this dissertation for page numbers].

#42

Score: unpublished; Gann makes reference to viewing the punching score (p. 34) and the “original manuscript” (p. 254); no information available on a published score

Recordings: Nancarrow, Conlon. *Studies for Player Piano*, Vol. V [Studies No. 42, 45a, 45b, 45c, 48a, 48b, 48c, 49a, 49b, 49c]. Compact disc. Wergo: WER 60165-50, 1988.

Writings: Tenney, James. “Studies for Player Piano Vol. V: Notes on *Studies* #42, 45, 48, and 49.” In *Conlon Nancarrow: Studies for Player Piano*, Vol. V [compact disc], pp. 7–12. Wergo: WER 60165-50, 1988.

Thomas, Margaret Elida. *Conlon Nancarrow’s ‘Temporal Dissonance’: Rhythmic and Textural Stratification in the Studies for Player Piano* [Ph.D. dissertation, Yale University]. Ann Arbor, Mich.: University Microfilms, 1996 [see Appendix C of this dissertation for page numbers].

#43

Score: unpublished

Recordings: Nancarrow, Conlon. *Studies for Player Piano*, Vols. III/IV [Studies No. 1, 2a, 2b, 7, 8, 9, 10, 11, 12, 13, 15, 16, 17, 18, 19, 21, 23, 24, 25, 27, 28, 29, 33, 34, 36, 43, 46, 47, and 50]. Compact discs. Wergo: WER 60166/67-50, 1987.

Writings: Tenney, James. “Studies for Player Piano Vol. III: Notes on *Studies* #1, 2a, 2b, 7, 8, 10, 15, 21, 23, 24, 25, 33, 43, 50.” In *Conlon Nancarrow: Studies for Player Piano*, Vols. III/IV [compact discs], pp. 7–19. Wergo: WER 60166/67-50, 1987.

#44

Score: aleatoric in nature, no real score?

Recordings: Nancarrow, Conlon. *Studies for Player Piano*, Vols. I/II [Studies No. 3a, 3b, 3c, 3d, 3e, 4, 5, 6, 14, 20, 22, 26, 31, 32, 35, 37, 40a, 40b, 41a, 41b, 41c, 44; Tango?]. Compact discs. Wergo: WER 6168-2/6169-2, 1987.

Writings: Tenney, James. “Studies for Player Piano Vol. I: Notes on *Studies* #3 (a–e), 20, 41 (a–c), 44.” In *Conlon Nancarrow: Studies for Player Piano*, Vols. I/II [compact discs], pp. 7–11. Wergo: WER 6168-2/6169-2, 1987.

#45

Score: unpublished; Gann makes reference to viewing the punching score (p. 34) and his Xerox score (p. 257); no information available on a published score

Recordings: Nancarrow, Conlon. *Studies for Player Piano*, Vol. V [Studies No. 42, 45a, 45b, 45c, 48a, 48b, 48c, 49a, 49b, 49c]. Compact disc. Wergo: WER 60165-50, 1988.

Writings: Tenney, James. "Studies for Player Piano Vol. V: Notes on *Studies* #42, 45, 48, and 49." In *Conlon Nancarrow: Studies for Player Piano*, Vol. V [compact disc], pp. 7–12. Wergo: WER 60165-50, 1988.

La Barbara, Joan. "The Remarkable Art of Conlon Nancarrow." *High Fidelity/Musical America* 34 (May 1984): 12–13.

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#46

Score: unpublished; Gann makes reference to viewing the punching score (p. 34) and a final score (p. 263) which lacks barlines (p. 265)

Recordings: Nancarrow, Conlon. *Studies for Player Piano*, Vols. III/IV [Studies No. 1, 2a, 2b, 7, 8, 9, 10, 11, 12, 13, 15, 16, 17, 18, 19, 21, 23, 24, 25, 27, 28, 29, 33, 34, 36, 43, 46, 47, and 50]. Compact discs. Wergo: WER 60166/67-50, 1987.

Writings: Tenney, James. "Studies for Player Piano Vol. IV: Notes on *Studies* #9, 11, 12, 13, 16, 17, 18, 19, 27, 28, 29, 34, 36, 46, 47." In *Conlon Nancarrow: Studies for Player Piano*, Vols. III/IV [compact discs], pp. 20–30. Wergo: WER 60166/67-50, 1987.

#47

Score: Gann notes the score was lost and he worked from the roll (pp. 31, 266)

Recordings: Nancarrow, Conlon. *Studies for Player Piano*, Vols. III/IV [Studies No. 1, 2a, 2b, 7, 8, 9, 10, 11, 12, 13, 15, 16, 17, 18, 19, 21, 23, 24, 25, 27, 28, 29, 33, 34, 36, 43, 46, 47, and 50]. Compact discs. Wergo: WER 60166/67-50, 1987.

Writings: Tenney, James. "Studies for Player Piano Vol. IV: Notes on *Studies* #9, 11, 12, 13, 16, 17, 18, 19, 27, 28, 29, 34, 36, 46, 47." In *Conlon Nancarrow: Studies for Player Piano*, Vols. III/IV [compact discs], pp. 20–30. Wergo: WER 60166/67-50, 1987.

#48 (Canon 60/61)

Score: unpublished; Gann notes that it is not as accurate as the punching score

Recordings: Nancarrow, Conlon. *Studies for Player Piano*, Vol. V [Studies No. 42, 45a, 45b, 45c, 48a, 48b, 48c, 49a, 49b, 49c]. Compact disc. Wergo: WER 60165-50, 1988.

Writings: Tenney, James. "Studies for Player Piano Vol. V: Notes on *Studies* #42, 45, 48, and 49." In *Conlon Nancarrow: Studies for Player Piano*, Vol. V [compact disc], pp. 7–12. Wergo: WER 60165-50, 1988.

#49 (Three Canons, 4/5/6)

Score: unpublished

Recordings: Nancarrow, Conlon. *Studies for Player Piano*, Vol. V [Studies No. 42, 45a, 45b, 45c, 48a, 48b, 48c, 49a, 49b, 49c]. Compact disc. Wergo: WER 60165-50, 1988.

Writings: Tenney, James. "Studies for Player Piano Vol. V: Notes on *Studies* #42, 45,

48, and 49." In *Conlon Nancarrow: Studies for Player Piano*, Vol. V [compact disc], pp. 7–12. Wergo: WER 60165-50, 1988.

#50 (Canon $\frac{5}{3}$)

Score: unpublished

Recordings: Nancarrow, Conlon. *Studies for Player Piano*, Vols. III/IV [Studies No. 1, 2a, 2b, 7, 8, 9, 10, 11, 12, 13, 15, 16, 17, 18, 19, 21, 23, 24, 25, 27, 28, 29, 33, 34, 36, 43, 46, 47, and 50]. Compact discs. Wergo: WER 60166/67-50, 1987.

Writings: Tenney, James. "Studies for Player Piano Vol. III: Notes on *Studies #1, 2a, 2b, 7, 8, 10, 15, 21, 23, 24, 25, 33, 43, 50*." In *Conlon Nancarrow: Studies for Player Piano*, Vols. III/IV [compact discs], pp. 7–19. Wergo: WER 60166/67-50, 1987.

APPENDIX C: INDEX OF REFERENCES TO SPECIFIC STUDIES IN THOMAS (1996)

Study No. 1— 18, 31, 168n3

Study No. 2 — 8, 12, 14, 15, 18, 22, 24, 26, 27, 101–04, 168n3

Study No. 3 — 12, 14, 15, 18, 24, 25, 26, 27, 34n12, 53, 54–60, 62, 64, 165, 168n3

Study No. 4 — 22, 24, 86n10

Study No. 5 — 10, 14, 17, 34n12, 60–63, 64, 168n3

Study No. 6 — 16, 24, 25, 53, 168n3

Study No. 7 — 12, 16, 17, 18, 89, 91–92, 118–21, 168n3

Study No. 8 — 8, 10, 11, 12, 13, 19, 24, 86n10, 89–91, 167

Study No. 9 — 7, 8, 13, 104–08, 168n3

Study No. 10 — 15, 18, 24, 27, 53

Study No. 11 — 6, 7, 10, 15, 24, 27, 53, 115–18, 136, 138–41, 151

Study No. 12 — 8, 17, 24, 53, 168n3

Study No. 14 — 13, 17, 66–67, 97–99, 136–38, 167, 168n3

Study No. 15 — 13, 16, 17, 94, 96, 97, 99–101, 167, 168n3

Study No. 16 — 122–24

Study No. 17 — 13, 17

Study No. 18 — 13, 96, 168n3

Study No. 19 — 13, 16, 17, 87–88, 168n3

Study No. 20 — 8, 17, 18, 81–84, 86n10, 88–89, 165

Study No. 21 — 8, 13, 15, 17, 125–26, 127

Study No. 23 — 8, 10, 17, 128–32, 134, 151

Study No. 24 — 13, 16, 17, 26, 70–74, 76, 84, 167

Study No. 25 — 13, 15, 17, 19, 127–28

Study No. 27 — 8, 10, 13, 14, 17, 132–36

Study No. 31 — 8, 13, 17, 66–70, 72, 12–22

Study No. 32 — 7, 13, 84–86

Study No. 33 — 13, 17, 34n12, 74–77, 97, 108–10, 114, 151, 167

Study No. 35 — 63–65

Study No. 36 — 13, 19, 96

Study No. 37 — 13, 18, 23, 77–81, 84, 86, 96, 165

Study No. 40 — 19, 97, 110–14

Study No. 41 — 8, 19, 28, 97, 114, 141, 142–64, 167

Study No. 42 — 24

Study No. 45 — 24, 27

APPENDIX D: TRANSCRIBED P. 9 OF STUDY NO. 34

9.

$\text{♩} = 176$

8ve

$\text{♩} = 146 \frac{2}{3}$

$\text{♩} = 146 \frac{2}{3}$

(Transcribed and notated by Julie Scrivener - February 2001)

(34)

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