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VALIDATION OF TWO GROWTH AND YIELD MODELS ON
RED PINE PLANTATIONS IN MICHIGAN

presented by

ERIN E. SMITH-MATEJA

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of the requirements for the

M.S. degree in Forestry

Karen Potter Witter
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VALIDATION OF TWO GROWTH AND YIELD MODELS ON RED PINE PLANTATIONS IN MICHIGAN

By

Erin E. Smith-Mateja

A THESIS

Submitted to
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ABSTRACT

VALIDATION OF TWO GROWTH AND YIELD MODELS ON RED PINE PLANTATIONS IN MICHIGAN

By

Erin E. Smith-Mateja

Two red pine thinning study sites were used to validate two different forest growth and yield computer models, the Forest Vegetation Simulator (FVS) Lakes States variant, an individual tree model and Red Pine Al Lundgren (RPAL), a stand level model. Both study sites had been established as red pine plantations in the 1930's and became thinning study sites in the 1960's. Approximately every five to ten years (between 1964, 1965 through 1991, 1992) the stands were thinned and individual tree measurements were taken on every tree in the study. Both growth models "grew" the data from the second inventory date through the last inventory. The simulated estimates of diameter at breast height (dbh) for FVS, trees per acre, and basal area were compared to the actual measurements. Two types of simulations were projected using FVS; with diameter growth calibration and without diameter growth calibration. One type of RPAL simulation was run.

FVS predicted more accurate results than RPAL for trees per acre but not necessarily for basal area per acre. FVS predicted more accurately dbh and basal area per acre with dbh growth calibration turned on, it had little to no effect on more accurately estimating mortality on either site. With calibration turned on FVS predicted up to twenty- seven years of growth with an absolute mean error less than 1.0 inch.

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INTRODUCTION

Red pine or Norway pine, *Pinus resinosa*, is commonly regarded as Michigan's most significant commercial softwood species, and with good reason. In 1992 red pine production was valued at $25.3 million (Potter-Witter 1995). Red pine covers 897,200 acres of Michigan timberland, 641,200 acres in the Lower Peninsula, and 256,000 acres in the Upper Peninsula (Leatherberry and Spencer 1996). Average net annual growth (1980-1992) was 78,310 thousand ft^3 and removals were 15,980 thousand ft^3 (Leatherberry and Spencer 1996). In 1994 timber industries in Michigan produced over 93,261 MBF of sawlogs, approximately 56,040 cords of pulpwood and almost 3,053 MCF of industrial fuelwood from red pine (Hackett and Pilon, 1997). Red pine has been managed since the turn of the 20^{th} century (Eyre and Zehngraff 1948). Numerous research articles have been written on the best way to manage red pine in the Lake States. Research results (or advice) vary depending on the desired timber products, site quality and location, and initial densities.

Considering the value of the resource and the body of work on red pine growth and yield, resource managers need some way to compare alternate management regimes which will allow them to pick the best management scenario to meet their objective for their site. Growth models are an excellent way to do this. However how does a resource manager evaluate what model is best to use? In the Lake States there are presently two main PC based computer models that can be used to predict red pine growth and yield; the Forest Vegetation Simulator- Lakes States variant (FVS-LS) or RPAL (REDPINE – Allen Lundgren). Validating a model's performance is an essential part of model development and revision. Previously, the only sub-regional (using only Michigan tree data) validation of FVS-LS was done on hardwood stands on 5 and 10-year growth (Canavan and Ramm 2000, Guertin and Ramm 1996). RPAL was based on unpublished equations and no validation study has been published. Given the importance of red pine in the Lake States it is necessary that land managers have a model that correctly predicts growth to compare alternative management treatments, whether the objective is timber volume, wildlife habitat or recreational area.

This paper explores FVS-LS prediction of individual tree and stand level attributes and RPAL prediction of stand level attributes. The validation project uses two long-term thinning-study sites in lower and upper Michigan. These study sites have had repeated measurements taken approximately every five to ten years for the past 27 years, which make them an excellent resource to test growth models. In addition to testing the models prediction accuracy over time, at different sites and densities, the study also tests how much better FVS predicts using diameter calibration. Diameter calibration is a unique function of the FVS program. It allows the users to include dbh increment cores or past diameter measurement (at least five measurements for each species), which FVS uses to scale its own growth equations to more closely represent past growth on the site.

Previous validation studies of Lakes States growth and yield models use at least three variables to test the model performance, diameter at breast height, trees per acre and basal area per acre (Holdaway and Brand 1983, Kowalski and Gertner 1989, Guertin and Ramm 1996, Canavan and Ramm 2000). These three variables are good descriptors of a stand's growth and mortality. "An analysis of dbh predictions provides an indication of the predictive ability of individual tree growth equations functions. The accuracy of predicted TPA reflects the effectiveness of the mortality model at simulating mortality due to stand composition. BA predictions, which can be viewed as a combination of the growth and mortality models, provide a measure of overall model performance" (Canavan and Ramm 2000). Once a model can be "trusted" to give accurate results of diameter or basal area growth and mortality, then it can used to predict more specific and management oriented variables, such as volume or percent canopy cover.

The models performance was measured by how well it predicted stand growth and mortality to the stand's actual growth and mortality. This is called bias, the difference between the observed measurement and the predicted measurement. This difference is used to calculate two statistics which give a description of the error; the mean error (the average difference between the predicted and observed values) and the standard deviation (the variability of the error estimate). These statistics are often referred to as accuracy and precision. "Accuracy is an expression of how close something is to the correct answer....Precision, on the other hand, is just a consequence of repeatability" (Iles 2003). These two concepts are essential when testing a model. It is imperative to know how close the prediction is to the correct answer and how much variability there is in the predictions. The classic text book example of this is throwing a dart at a bullseye. Accuracy is how close the darts land to the bullseye, while precision describes how close the darts are to each other. The objective of this study is to test how close the predicted answer is to the observed and how reliable that estimate is under varying conditions. Standards for accuracy and precision can vary greatly in natural resource planning and projects. Large scale forest projects covering millions of acres, over a long time frame may not require a highly accurate model. On the other hand a manager setting up a 10-year thinning project for a private landowner's woodlot would want a model with very high accuracy and low variability. When considering the information most land managers are interested in they tend not to be just trees per acre and basal area, but other variables which are calculated from these base stand structure components, such as percent canopy cover, down woody debris and volume estimates. Validation should first occur on this base component. In FVS these equations are diameter growth and mortality. In RPAL these equations are basal area growth and mortality.

Objectives

The objectives of this research are to:

1. Compare the ability of the Forest Vegetation Simulator Lakes States variant (FVS-LS) to accurately predict red pine diameter growth, basal area growth, and mortality over varying time intervals up to 27 years for two Michigan plantation sites.
2. Compare the ability of REDPINE -Allen Lundgren (RPAL) to accurately predict red pine basal area growth and mortality over varying time intervals up to 27 years for two Michigan plantation sites.
3. Compare the effects of including past diameter measurements in the FVS-LS inputs on the accuracy and precisions of diameter growth, basal area growth, and mortality.
4. Compare the ability of FVS-LS and RPAL to accurately predict growth under varying thinning densities.

LITERATURE REVIEW

Red pine has been managed in the Lake States since the start of the 20th century (Eyre and Zehngraff 1948). Many of the early management studies are from Minnesota and deal with different methods of regenerating or thinning natural stands. Buckman's (1962) study of permanent plots in Minnesota initiated the move from normal yield tables to stand-level models for red pine. Some of the models developed by Buckman, including a three dimensional response surface, are still in use today (Ramm 1997). Buckman and Lundgren (1962) looked at growth response to removing hardwood competition from three red pine plantations planted by the Civilian Conservation Corps (CCC) in northern Minnesota. Lundgren (1963) used a modified soil expectation value to analyze the economic returns from releasing these plantations. Lundgren (1965) also examined land expectation values and return rates for a range of thinning intensities and rotation ages and recommended regular thinnings to 90 ft/acre. This work was later expanded to a series of tables and worksheets for estimating income, costs, and rate of return from growing red pine for pulp and sawtimber (Lundgren 1966). Lundgren assembled a number of published and unpublished stand-level growth and yield models into a mainframe computer program called REDPINE. The program was used to evaluate the effect of initial density and different levels of thinning intensity on yield (Lundgren

1981) and prepare graphical guidelines. Using REDPINE, Lundgren (1983) found that normal yield tables underestimated cubic foot yield for both thinned and unthinned stands on average sites. REDPINE was adopted from the mainframe to the personal computer in 1984 and named RPAL for REDPINE -Allen Lundgren (Ramm 1990).

In Michigan, early work focused on quantifying the response to different thinning intensities. Day and Rudolph (1971) used residual spacing or percent height as a thinning guide in a red pine plantation at the Dunbar Forest in Michigan's Eastern Upper Peninsula (UP). They found significant differences in basal area (BA) and cubic foot volume production between thinning intensities, and used stumpage prices and costs to determine total value production over 15 years. Rudolph *et al.* (1982) collected data from a red pine plantation in Michigan's Upper Peninsula and from a plantation in the Lower Peninsula to evaluate competition quotient as a guide for thinning. Rudolph *et al.* (1984) summarized results from three thinning studies in Michigan and compared growth, yield, changes in stand structure, and value of residual stands for a wide variety of thinning treatments. Marty (1988) analyzed the results of a 18 year thinning study in northern lower Michigan and recommended that managers interested in red pine sawtimber start at lower densities (200 - 500 TPA) than generally used.

DeNaurois and Buongiorno (1986) compared red pine pulpwood production to pulp and sawtimber production across a range of sites in Wisconsin using soil expectation values and internal rate of return. Dual management was deemed superior, and the authors found that plantation density of 680 TPA with heavy thinning every 15 years until rotation at 45 years maximized returns[1]. Harms *et al.* (1990) evaluated the potential

[1] Grossman and Potter-Witter (1991) later found that the TWIGS economic algorithm that DeNaurois and Buongiorno used incorrectly calculated the rate of returns.

market for red pine utility poles in Wisconsin, where pole production was minor, along with the regional market. Grossman and Potter-Witter (1991) used the TWIGS projection system (Miner *et al.* 1988) and data from the Kellogg Experimental Forest in southwest Michigan and Petawawa Forest in Ontario to determine that management strategies including utility poles maximized returns. They recommended plantation densities of 890 trees per acre and thinning intensities above 110 ft^2/acre basal area. Marty and Potter-Witter (1992) used Forest Survey data from recent Michigan inventories, along with empirical yield tables developed for Michigan (Hahn and Stelman 1984) to predict that current rates of red pine production could not be maintained in the future without significant increases in planting.

There have been a number of studies on red pine growth and yield in the Lake States. Martin and Ek (1984) discovered that simple empirical models provided the best fit for diameter and height growth for 20-58 year old plantations in Wisconsin. . Reed *et al.* (1986) developed compatible equations for red pine, allowing estimation of cubic foot volume to any upper stem diameter. Zarnoch *et al.* (1982) used the Weibull probability density function to model changes in stand structure over time. In the 1990's, red pine growth and yield information was compiled into an expert system (Rauscher *et al.* 1990), survival was modeled using an artificial neural network (Guan and Gertner 1991), and an interactive program was developed to produce optimal thinning schedules through dynamic programming (Rose and Chen 1995). Erickson (1996) summarized height and diameter growth over 59 years for one plot in a 1.4 acre site in northern Minnesota. In contrast, Fowler (1997) measured over 3,500 tree from 27 red pine stands across Michigan to develop new individual tree volume equations.

Given the number of studies on red pine, do resource managers have an easy way to compare alternate thinning treatments? Resource managers can choose among three projection systems for red pine in the Lake States; all of which originated with the USDA Forest Service. The Woodsman's Ideal Growth Projection System (TWIGS) is an individual tree, distance-independent growth and yield simulator developed by the USDA Forest Service's North Central Forest Experiment Station to simulate growth of mixed species stands (Miner *et al.* 1988). The regional models in Lake States TWIGS were calibrated with 80,000 trees from over 1,500 plots in Wisconsin, Michigan, and Minnesota (Miner *et al.* 1988). Lake States TWIGS has been superseded by the Forest Vegetation Simulator (FVS), also developed by the USDA Forest Service (Bush and Brand 1995). FVS began as PROGNOSIS, an individual tree, distance-independent growth and yield projection system developed for the Inland Empire area of Idaho and Montana. The file input and output format used in PROGNOSIS was adopted as the standard for all growth and yield projection systems used by the USDA Forest Service. The Lake States TWIGS variant of the Forest Vegetation Simulator (FVS-LS) uses this standard file input and output and uses the growth and mortality functions based on Lake States TWIGS and can be run stochastically or deterministically. One important difference is that Lake States TWIGS uses annual increments, while FVS-LS calculates 10 year growth by multiplying the annual increment by 10 and then scaling back if the growth period is less than 10 years (Bush and Brand 1995). Predicted individual tree and stand-level attributes, including both stand tables and volume tables, can be obtained for any year in a projection. FVS-LS can simulate a wide range of silvicultural prescriptions and provide economic analyses of the prescriptions.

There have been two sub-regional validations of TWIGS and later FVS-LS, both using hardwood stands in the northern Lower Peninsula. The TWIGS study (Guertin and Ramm 1996) showed that the precision was quite variable, five-year diameter growth was predicted within $\pm$ 0.3 inches for the five species studied. Mean errors for basal area projections were within $\pm$ 5 ft^2/acre for all species and mean error for trees per acre (TPA) was within $\pm$ 20 TPA for all species but other red oak. A follow-up study five years later found that FVS-LS consistently overestimated 10-year diameter growth across seven hardwood species (Canavan 1997).

RPAL (Ramm 1990) is a deterministic, interactive program based on stand-level models of growth and yield for red pine plantations. The user provides information on stand site index, stand age, density in basal area (BA) and trees per acre (TPA), desired residual BA, and the thinning interval. The residual BA and the projection period may be unique for each thinning interval. The stand is described before and after each thinning; through stand age, quadratic mean diameter at breast height (dbh), average height of dominant and codominant trees, basal area per acre and trees per acre. Total volume in cunits per acre and merchantable volume in standard cords per acre or thousand board feet (MBF) per acre, (International 1/4-inch rule), are shown for the stand before and after thinning. The volume removed during thinning is also given. RPAL does not provide any financial analysis. Hyldahl and Grossman (1994) filled this gap with a financial spreadsheet RPGROW$ that incorporated the models used in RPAL.

RPAL was initially based on row-thinning but was extended to other thinning regimes. Stand growth is driven by two different basal area growth models. The growth model for stands less than 25 years old was developed by Lundgren using data from unthinned red pine plantations in Michigan, Minnesota and Wisconsin (Wambach 1967). The growth model for stands over 25 years old predicts average annual BA growth (in square feet per acre) as a simple quadratic function of BA, stand age, and site index (Buckman 1962). Total cubic foot volume per acre is estimated using a stand level model with stand BA and average stand height (Buckman 1962). Volume is then allocated between size classes using a ratio based on quadratic stand diameter (QSD).

METHODS

Data Description

The validation data were collected from two-long term study sites in Michigan, the Hiawatha National Forest in the eastern Upper Peninsula (Hiawatha site) and the W. K. Kellogg Experimental Forest (Kellogg site) in the southwestern Michigan. The Kellogg site is owned and managed by Michigan State University's Department of Forestry.

The Hiawatha site was planted in 1938 with approximately six by six foot spacing. It is located in the Sault Ste. Marie district near Trout Lake on level area where East Lake, Rubicon and Rousseau fine sands are present. In 1962 it became a thinning study established as a randomized complete block design (RCBD) with four blocks (Rudolph *et al.*1984). Each block contained 16 treatments, with each being a 0.10 acre in size. This study examined 12 of the treatments (Table 1), four of the treatments were taken out of this study because the initial thinning method was the percent of height method and the researchers felt this method was no longer commonly used to manage red pine. The percent of height thinning method is set up so the average spacing between residual trees is a specified percent of the average dominant and codominant stand height (Rudolph *et al.*1984). On each 0.10 acre treatment plot diameter at breast height for each tree was measured along with the height of three to five trees on the plot. These measurements were taken in 1962, 1965, 1969, 1976, 1982, and 1992 and the blocks were thinned in 1962, 1969, 1976, 1982 and 1992, except for a few plots for which basal areas were too low to thin in 1982 and 1992. The researchers then calculated the basal area per acre and trees per acre for the years that measurement or thinning occurred.

Table 1. Hiawatha National Forest thinning study treatments

Treatment:	Description of treatment
BAr30:	Initial thinning to a residual BA of 30 ft2/acre
BAr45:	Initial thinning to a residual BA of 45 ft2/acre
BAr60:	Initial thinning to a residual BA of 60 ft2/acre
BAr80:	Initial thinning to a residual BA of 80 ft2/acre
BAr100:	Initial thinning to a residual BA of 100 ft2/acre
BAr130:	Initial thinning to a residual BA of 130 ft2/acre
BAr160:	Initial thinning to a residual BA of 160 ft2/acre
Row2nd:	Initial thinning removed every other row
Row3rd:	Initial thinning removed every third row
Row4_2:	Initial thinning removed every fourth row; at second thinning removed center row
Row4th:	Initial thinning removed every fourth row
Control:	Control - no thinning

The Kellogg study site was planted in 1936 and 1937 with spacing of approximately seven by eight feet. This site was planted on rolling hills with Oshtemo loamy sand. In 1960 it became a thinning study with nine thinning treatments applied in a randomized complete block design with four replications of each treatment (Rudolph et al 1984). These treatments (Table 2) were also 0.10 acres in size. On each 0.10 acre treatment plot diameter at breath height for each tree was measured along with the height on three to five trees on the plot. Measurement and thinning occurred in 1960, 1964, 1967, 1972, 1980, 1985 and 1991. The author then calculated the basal area per acre and trees per acre for these years.

Table 2. W.K. Kellogg Forest Thinning Treatments

Treatment:	Description of treatment
Row2nd:	Initial thinning removed every 2nd row
Row3rd:	Initial thinning removed every 3rd row
Row4th:	Initial thinning removed very 4th row
Row4_2:	Initial thinning removed every 4th row, at second thinning removed center row
BAr90:	Initial thinning to a residual BA of 90 ft2/acre
BAr70:	Initial thinning to a residual BA of 70 ft2/acre
BAr110:	Initial thinning to a residual BA of 110 ft2/acre
BAr130:	Initial thinning to a residual BA of 130 ft2/acre
Control:	Control – no thinning

The researchers on this project did not set up the thinning studies or take the measurements, and thus did not have a choice on what measurements were taken, the size of the plots or how and when the stands were thinned. However a data set of this type is not common in forestry, because of the completeness of the experimental design (RCBD) and the span in years of the repeated measurements. In forestry it is more common to find data sets which cover a large spatial distribution (across a state or National Forest), however rarely do they have permanent plots with repeated measurements over a decade. The uniqueness of this data and foresight of the designers of the experiment, make this data an excellent choice to test the basic equations in a forest growth and yield model.

Growth Simulations

Three types of growth simulations were run for validation. The first used RPAL, which is a stand level program. The data needed for input were stand age, basal area, trees per acre, desired residual basal area after thinning and the length of the projection. For the Hiawatha site, stand age (28 years in 1965), BA at age 28, TPA at age 28, site index at a base age of 50 years, the desired residual BA, and the specified projection interval ending in 1969, 1976, 1982, 1992 were used. For the Kellogg site, stand age (27 years in 1964), BA at age 27, TPA at age 27, site index at a base age of 50 years, the desired residual BA and the projection lengths ending in 1964, 1967, 1972, 1980, 1985, 1991. The residual basal area of the plots was entered at the beginning of each projection cycle for Hiawatha and Kellogg sites to match the actual residual basal area, while trees per acre was only entered at the start of the program, either 1965 or 1964.

Two types of FVS-LS simulations were run for each block of each treatment; with FVS diameter calibration turned off, the other with diameter calibration turned on. This was to compare how well FVS predicted tree growth without past diameter information. The simulations that did not include past diameter measurements were abbreviated 'NA' for no ancillary data. The simulations that included past diameter measurements were abbreviated 'ΔD'. All runs included plot name, age, site index (base age 50 years), dbh (1965: Hiawatha, 1964: Kellogg), height if measured, and the specific year the tree was cut. The ΔD simulations also included diameter growth, which was calculated from the diameter from the simulation start date minus the previous diameter measurement. The Hiawatha growth measurements were from age 25 to 28, and Kellogg growth measurements were from age 27 to 31. The Hiawatha site had a three year growth measurement and the Kellogg site had a four year growth measurement period that FVS used to scale or calibrate the growth equations to more closely match the growth that was occurring in the plantation at that time. In the FVS tree list all trees were coded with the species code "RP" indicating red pine plantation trees and the model was run deterministically.

Validation

The validation examined twelve treatments on the Hiawatha National Forest (Table 1) and nine treatments on the Kellogg Forest (Table 2). Basal area and trees per acre error were calculated for the RPAL simulations. Basal area, trees per acre, and diameter at breast height error were calculated for FVS-LS simulations. Other studies of Lakes States growth and yield models use at least three variables to test the model performance, diameter at breast height, trees per acre and basal area per acre (Holdaway and Brand 1983, Kowalski and Gertner 1989, Guertin and Ramm 1996, Canavan and Ramm 2000). Testing trees per acre evaluates the mortality equations in both models. Since RPAL is a stand level model evaluating basal area estimates tests the models base growth equations and overall estimates of growth and mortality. In FVS, testing the diameter growth equations evaluates individual tree growth equations and evaluating basal area provides an over all estimate of how FVS is predicting growth and mortality. Mean error and standard deviations were the statistics chosen to display the error because this is what other Lake States validation studies have used and most professionals understands these statistics with little to no explanation needed for interpreting the results.

Mean error and standard deviation were calculated from the bias; where the bias is defined as the error between actual and predicted measurement for each treatment. Error was calculated as predicted value minus observed value, to be consistent with the other Lake States validation studies (Holdaway and Brand 1983, Holdaway and Brand 1986, Guertin and Ramm 1996, Canavan and Ramm 2000). Overestimates therefore were positive numbers.

The formula used to calculate mean error and standard deviation were:

$$\text{Mean Error } (\bar{e}) = \frac{\sum_{i=1}^{n} (\hat{y}_i - y_i)}{n}$$

$$\text{Standard Deviation of the error} = \sqrt{\sum_{i=1}^{n} \frac{(e_i - \bar{e})^2}{n-1}}$$

where $\hat{y}_i$ is the predicted TPA, BA, DBH
y_i is the actual TPA, BA, DBH
$e_i = (\hat{y}_i - y_i)$
n = sample size

The Hiawatha simulations projected growth from 1965 through 1992, with cycle boundaries at 1969, 1976, 1982 and 1992. The Kellogg simulations projected growth from 1964 through 1991, with cycle boundaries at 1967, 1972, 1980, 1985 and 1991. For example a 0.10 acre treatment plot on Hiawatha site would be "grown" from 1965 to 1969 (4 years). The simulated tree or stand conditions were checked against the actual tree or stand conditions for 1969 and bias was calculated. The stand then continued to "grow" another seven years and the simulated tree or stand conditions were compared to the actual conditions in 1976 and bias was calculated. This process continued so that at every cycle boundary the simulated tree or stand conditions were compared to the actual conditions. From the bias at each cycle boundary, mean error and standard deviations were calculated. The mean errors and standard deviation calculated at each cycle boundary for stand level measurements (TPA and BA) had a sample size of four (for each treatment). The sample size for DBH error decreased over time as trees were cut and was dependant on the type of treatment. Treatments that had a high residual basal area (less trees cut per acre) had a larger sample size then those with low residual basal area treatments.

Figure 1. Diagram of the cycle boundaries at which bias was calculated for the Hiawatha and Kellogg sites.

Hiawatha site

simulation length (years)	0	4	11	17	27
cycle boundaries	1965	1969	1976	1982	1992

Kellogg site

simulation length (years)	0	3	8	16	21	27
cycle boundaries	1964	1967	1972	1980	1985	1991

RESULTS

Diameter at Breast Height Error

Only FVS-LS runs were used to predict dbh error because RPAL, a stand level model, does not predict individual tree diameters.

Hiawatha - FVS-LS, on average predicted diameter growth with better accuracy for the ΔD (calibrated) simulations than for the NA (not calibrated) simulations (Table 3). For both types of simulations FVS-LS over-predicted diameter growth, except in treatments with low residual basal areas. In the last cycle (27 years) five of the twelve treatments had errors greater than 1.0 inches in the NA simulation and only one of the twelve treatments in the ΔD simulations had an error greater than 1.0 inches (Table 4). When absolute mean error was calculated, averaged over the treatments the absolute mean error over the 27 year growth projection was 0.4 inches (ΔD) and 0.8 inches (NA) (Table 3). No treatment had an error greater than 1.78 inches in dbh. Absolute mean error increased as the length of the projection increased.

Kellogg - Unlike the Hiawatha results, FVS did not necessarily predict diameter growth with better accuracy for the ΔD simulations than for the NA simulations (Table 5). FVS-LS over-predicted growth for all trees in the NA projections except for the thin every 4^{th} row treatment (Row 4^{th}) where growth was under-predicted. In both runs as projection length increased so did the error. Almost all of the simulations had an error of less than one inch, except for the longer projection lengths in the control, thin every 4th row, and 70 ft^2/acre thinnings. Only treatments thin every 2nd row and thin to a basal area of 90 ft^2/acre of the NA simulation had error of more than 2.0 inches. These errors occurred in the 27-year interval. Absolute mean error for the ΔD simulation showed an error less than 1.0 inches up to the 27 years of growth, while the NA simulation only showed an absolute mean error of less than 1.0 inch up to eight years of growth (Table3).

Table 3 . Hiawatha site and Kellogg site absolute mean error for all treatments by cycle length.

Hiawatha site		NA	ΔD	RPAL
DBH	4 years	0.3	0.2	
	11 years	0.5	0.3	
	17 years	0.7	0.4	
	27 years	0.8	0.4	
BA	4 years	12.3	5.8	2.0
	11 years	18.9	6.7	5.9
	17 years	19.1	8.5	5.4
	27 years	24.2	11.4	17.9
TPA	4 years	3.2	2.8	3.8
	11 years	5.6	4.7	36.4
	17 years	8.1	7.3	40.6
	27 years	24.1	23.9	37.5
Kellogg site		NA	ΔD	RPAL
DBH	3 years	0.1	0.1	
	8 years	0.4	0.3	
	16 years	1.0	0.5	
	21 years	1.3	0.6	
	27 years	1.5	0.8	
BA	3 years	2.4	7.4	3.2
	8 years	8.3	10.4	6.2
	16 years	18.1	13.2	18.9
	21 years	19.6	16.0	7.6
	27 years	30.0	19.5	18.9
TPA	3 years	10.0	9.4	10.1
	8 years	21.6	20.5	57.3
	16 years	37.9	36.3	84.2
	21 years	44.1	42.4	58.2
	27 years	52.3	50.5	59.3

Table 4. **Hiawatha mean error ($\bar{e}$) and standard deviation (s) of mean error for estimated diameter at breast height by treatment and projection length for the two types of simulation. (Error expressed as predicted value minus observed value.)**

					-- NA Simulation --		-- ΔD Simulation --	
treatment	projection cycle	age	QMD*	n	$\bar{e}$	s	$\bar{e}$	s
BAr30	65-69 = 4yrs	32	8.7	59	-0.52	0.23	-0.47	0.27
	65-76 = 11yrs	39	10.9	30	-0.77	0.30	-0.78	0.51
	65-82 = 17yrs	45	12.9	21	-0.99	0.45	-0.97	0.74
	65-92 = 27yrs	55	15.0	21	-0.59	0.62	-0.58	1.02
BAr45	65-69 = 4yrs	32	8.2	85	-0.28	0.25	-0.44	0.73
	65-76 = 11yrs	39	10.4	46	-0.48	0.50	-0.37	0.49
	65-82 = 17yrs	45	12.6	29	-0.91	0.62	-0.80	0.61
	65-92 = 27yrs	55	14.8	29	-0.26	2.59	-0.11	2.56
BAr60	65-69 = 4yrs	32	7.8	121	-0.05	0.57	-0.04	0.56
	65-76 = 11yrs	39	9.9	71	-0.26	0.97	-0.29	0.96
	65-82 = 17yrs	45	11.8	47	-0.54	1.40	-0.60	1.42
	65-92 = 27yrs	55	13.7	47	-0.30	1.72	-0.41	1.76
BAr80	65-69 = 4yrs	32	7.1	173	0.13	0.22	0.05	0.21
	65-76 = 11yrs	39	8.9	110	0.07	0.40	-0.15	0.36
	65-82 = 17yrs	45	10.5	73	-0.26	0.47	-0.56	0.45
	65-92 = 27yrs	55	12.5	58	-0.24	0.63	-0.65	0.59
BAr100	65-69 = 4yrs	32	6.9	220	0.19	0.17	0.12	0.17
	65-76 = 11yrs	39	8.3	146	0.28	0.31	0.08	0.34
	65-82 = 17yrs	45	9.5	105	0.33	0.77	0.04	0.82
	65-92 = 27yrs	55	11.2	84	0.62	1.51	0.22	1.55
BAr130	65-69 = 4yrs	32	6.4	302	0.29	0.38	0.17	0.38
	65-76 = 11yrs	39	7.6	218	0.51	0.58	0.18	0.58
	65-82 = 17yrs	45	8.7	162	0.53	0.97	0.08	0.98
	65-92 = 27yrs	55	10.1	130	0.68	1.25	0.08	1.27
BAr160	65-69 = 4yrs	32	6.0	395	0.38	0.54	0.20	0.54
	65-76 = 11yrs	39	6.9	305	0.73	0.71	0.31	0.72
	65-82 = 17yrs	45	7.7	241	0.89	1.00	0.32	1.03
	65-92 = 27yrs	55	8.7	200	1.27	1.37	0.53	1.41
Row2nd	65-69 = 4yrs	32	6.5	231	0.19	0.29	0.08	0.29
	65-76 = 11yrs	39	8.3	138	0.26	0.53	-0.03	0.53
	65-82 = 17yrs	45	9.6	96	0.21	0.73	-0.19	0.73
	65-92 = 27yrs	55	11.4	79	0.56	1.54	0.00	1.53
Row3rd	65-69 = 4yrs	32	6.0	332	0.31	0.23	0.13	0.23
	65-76 = 11yrs	39	7.5	212	0.56	0.47	0.10	0.49
	65-82 = 17yrs	45	8.6	158	0.61	0.67	-0.03	0.72
	65-92 = 27yrs	55	10.0	129	1.16	2.12	0.32	2.13
Row4_2	65-69 = 4yrs	32	6.1	331	0.36	0.46	0.19	0.45
	65-76 = 11yrs	39	7.0	224	0.71	1.09	0.28	1.10
	65-82 = 17yrs	45	8.4	159	0.97	1.75	0.36	1.80
	65-92 = 27yrs	55	10.2	129	1.55	2.60	0.73	2.69
Row4th	65-69 = 4yrs	32	5.9	366	0.35	0.40	0.16	0.41
	65-76 = 11yrs	39	7.4	218	0.69	0.82	0.17	0.87
	65-82 = 17yrs	45	8.6	161	0.71	1.15	0.00	1.23
	65-92 = 27yrs	55	10.3	118	1.06	1.92	0.10	2.03
Control	65-69 = 4yrs	32	5.9	466	0.36	0.33	0.18	0.34
(no thin)	65-76 = 11yrs	39	6.3	463	0.76	0.49	0.38	0.52
	65-82 = 17yrs	45	6.6	454	0.95	0.77	0.54	0.81
	65-92 = 27yrs	55	7.3	378	1.78	2.25	1.51	2.26

* QMD: The plots actual quadratic mean diameter at the end of the measurement cycle (start of the next cycle, before thinning.)

Table 5. Kellogg mean error (ē) and standard deviation (s) of mean error for estimated diameter at breast height by treatment and projection length for the two types of simulation. (Error expressed as predicted value minus observed value.)

				- - NA Simulation - -			- - ΔD Simulation - -	
treatment	projection cycle	age	QMD*	n	ē	s	ē	s
BAr70	64-67 = 3yrs	31	7.3	150	0.16	0.76	-0.35	0.77
	64-72 = 8yrs	36	7.9	150	0.31	1.64	-0.52	1.87
	64-80 = 16yrs	44	10.8	77	1.01	3.18	-1.29	3.31
	64-85 = 21yrs	49	11.9	69	1.13	3.68	-1.15	3.48
	64-91 = 27yrs	55	12.9	69	1.63	4.20	-1.67	4.39
BAr90	64-67 = 3yrs	31	7.5	168	0.21	1.46	0.00	1.48
	64-72 = 8yrs	36	9.0	115	0.94	2.98	0.41	2.72
	64-80 = 16yrs	44	11.0	91	1.77	3.86	0.85	3.92
	64-85 = 21yrs	49	12.1	83	1.92	4.40	0.84	4.48
	64-91 = 27yrs	55	12.7	83	2.17	4.68	0.97	4.77
BAr110	64-67 = 3yrs	31	7.3	205	0.08	1.01	-0.10	1.02
	64-72 = 8yrs	36	8.8	135	0.26	1.68	-0.23	1.71
	64-80 = 16yrs	44	10.7	104	1.23	3.45	0.36	3.49
	64-85 = 21yrs	49	12.0	93	1.59	4.19	0.59	4.24
	64-91 = 27yrs	55	12.5	93	1.98	4.56	0.85	4.61
BAr130	64-67 = 3yrs	31	7.1	255	0.18	1.14	0.01	1.14
	64-72 = 8yrs	36	8.2	184	0.61	2.10	0.18	2.09
	64-80 = 16yrs	44	9.7	147	1.61	3.54	0.85	3.53
	64-85 = 21yrs	49	11.0	131	1.81	4.05	0.92	4.06
	64-91 = 27yrs	55	11.6	131	1.94	4.32	0.96	4.34
Row2nd	64-67 = 3yrs	31	7.2	159	0.18	1.35	-0.08	1.35
	64-72 = 8yrs	36	7.8	159	0.70	2.34	0.23	2.35
	64-80 = 16yrs	44	10.2	106	1.38	3.55	0.27	3.72
	64-85 = 21yrs	49	11.5	85	1.64	4.37	0.35	4.48
	64-91 = 27yrs	55	12.3	85	2.08	4.62	0.63	4.87
Row3rd	64-67 = 3yrs	31	7.2	194	0.02	0.64	-0.25	0.64
	64-72 = 8yrs	36	8.6	122	0.07	0.97	-0.65	0.97
	64-80 = 16yrs	44	10.4	98	0.83	2.85	-0.47	2.90
	64-85 = 21yrs	49	11.7	81	1.34	4.02	-0.22	4.05
	64-91 = 27yrs	55	12.6	81	1.49	4.30	-0.30	4.34
Row4_2	64-67 = 3yrs	31	6.8	250	0.06	0.69	-0.18	0.69
	64-72 = 8yrs	36	7.8	171	0.62	1.93	0.03	1.94
	64-80 = 16yrs	44	9.9	123	1.09	3.02	0.00	3.09
	64-85 = 21yrs	49	11.2	109	1.22	3.51	-0.07	3.61
	64-91 = 27yrs	55	12.0	109	1.40	4.12	-0.04	3.86
Row4th	64-67 = 3yrs	31	7.0	232	-0.10	0.27	-0.21	0.19
	64-72 = 8yrs	36	8.1	158	-0.18	0.74	-0.47	0.56
	64-80 = 16yrs	44	10.0	110	-0.42	1.21	-0.97	0.74
	64-85 = 21yrs	49	11.3	86	-0.75	1.45	-1.39	0.73
	64-91 = 27yrs	55	12.0	86	-0.53	2.27	-1.23	1.72
Control (no thin)	64-67 = 3yrs	31	6.8	289	0.18	0.69	-0.06	0.70
	64-72 = 8yrs	36	7.3	288	0.45	0.80	-0.10	0.82
	64-80 = 16yrs	44	8.0	269	1.09	1.57	0.25	1.64
	64-85 = 21yrs	49	8.5	253	1.51	2.19	0.60	2.27
	64-91 = 27yrs	55	9.0	234	1.96	2.76	1.04	2.83

* QMD: The plots actual quadratic mean diameter at the end of the measurement cycle (start of the next cycle, before thinning.)

Basal Area Error

Basal area mean error and standard deviation were calculated for the four plots for each treatment and projection length (Table 6 and Table 7). Error was calculated for all three simulations types, FVS-LS (NA and ΔD) and RPAL.

Hiawatha-ΔD simulations projections were more accurate than the NA simulations. The absolute mean error was two to three times as great is the NA as in the ΔD simulations. As with the dbh mean error projections, the ΔD simulations were more likely to under-predict in treatments with low residual basal areas, and tended to over predict in treatments with high residual basal areas. RPAL predicted stands with high residual basal areas with lower mean errors. In all but the two lightest thinning treatments the mean error was less than 10 ft^2/acre up to a 17 year projection cycle. In treatments with low residual basal area by the last cycle (27 years), the model over-predicted basal area up to 40 ft^2/acre (Table 6). With all three projection types, as time increased so did the error (Table 3).

Table 6. Hiawatha mean error (ē) and standard deviation (s) of mean error for estimated basal area by treatment and projection length for three types of simulation. (Error expressed as predicted value minus observed value.)

				-----BA NA-----		-----BA AD-----		-----BA RPAL----	
treatment	projection length	age	BA**	ē	s	ē	s	ē	s
BAr30	65-69 = 4yrs	32	60	-7.06	2.44	-6.46	3.19	-0.67	2.88
	65-76 = 11yrs	39	48	-6.75	1.92	-6.69	4.51	16.18	2.10
	65-82 = 17yrs	45	47	-7.20	1.76	-6.97	4.52	13.20	2.19
	65-92 = 27yrs	55	65	-5.48	3.38	-5.22	8.14	38.70	6.63
BAr45	65-69 = 4yrs	32	78	-5.32	1.71	-4.02	1.14	-1.35	2.54
	65-76 = 11yrs	39	67	-6.32	1.73	-5.03	0.62	12.55	2.97
	65-82 = 17yrs	45	63	-9.17	2.83	-8.12	1.57	10.90	1.65
	65-92 = 27yrs	55	83	-6.50	5.90	-4.85	4.48	40.23	4.85
BAr60	65-69 = 4yrs	32	99	-2.25	2.62	-2.05	1.65	-1.35	2.20
	65-76 = 11yrs	39	91	-6.38	4.92	-6.91	4.12	5.70	3.07
	65-82 = 17yrs	45	86	-10.01	3.83	-10.83	5.23	6.50	2.74
	65-92 = 27yrs	55	116	-8.57	7.15	-10.32	10.21	26.33	6.18
BAr80	65-69 = 4yrs	32	120	4.45	2.25	1.96	0.71	0.53	2.47
	65-76 = 11yrs	39	116	1.22	5.18	-4.38	1.29	5.88	3.20
	65-82 = 17yrs	45	109	-5.87	3.55	-11.87	1.40	0.88	2.19
	65-92 = 27yrs	55	124	-5.68	5.29	-13.36	2.29	14.90	6.08
BAr100	65-69 = 4yrs	32	143	7.01	3.98	3.09	1.56	-0.25	2.33
	65-76 = 11yrs	39	138	8.92	4.51	2.46	6.43	0.55	6.68
	65-82 = 17yrs	45	128	6.94	3.81	-2.11	7.99	4.65	7.42
	65-92 = 27yrs	55	141	11.63	7.34	1.75	14.33	14.55	6.61
BAr130	65-69 = 4yrs	32	172	14.33	2.06	7.55	1.66	2.38	1.21
	65-76 = 11yrs	39	169	21.75	4.71	6.67	2.42	2.18	3.46
	65-82 = 17yrs	45	164	17.55	4.07	-0.05	3.63	-0.70	2.06
	65-92 = 27yrs	55	179	20.09	4.79	-1.74	7.34	4.90	2.87
BAr160	65-69 = 4yrs	32	193	22.74	4.87	10.83	3.55	5.43	3.49
	65-76 = 11yrs	39	194	40.34	5.60	14.23	9.06	2.78	4.85
	65-82 = 17yrs	45	191	41.40	10.75	10.65	15.08	-0.27	4.42
	65-92 = 27yrs	55	201	53.92	18.01	16.45	22.81	6.05	8.49
Row2nd	65-69 = 4yrs	32	130	6.70	1.36	2.08	0.87	-1.48	2.73
	65-76 = 11yrs	39	126	6.80	6.07	-2.15	5.18	6.05	4.16
	65-82 = 17yrs	45	119	3.64	9.68	-6.47	8.45	3.88	4.40
	65-92 = 27yrs	55	136	9.25	13.25	-4.45	10.91	15.63	2.58
Row3rd	65-69 = 4yrs	32	160	15.91	2.03	5.93	1.01	1.83	1.64
	65-76 = 11yrs	39	157	21.86	4.09	1.37	1.47	3.75	1.92
	65-82 = 17yrs	45	153	18.78	5.80	-5.12	3.22	-0.10	2.85
	65-92 = 27yrs	55	163	28.68	15.58	-1.19	11.69	9.93	2.32
Row4_2	65-69 = 4yrs	32	163	17.58	3.03	7.59	2.28	3.70	1.53
	65-76 = 11yrs	39	144	24.27	1.25	4.92	2.75	4.98	4.51
	65-82 = 17yrs	45	139	19.98	5.89	-1.23	6.59	-0.27	4.61
	65-92 = 27yrs	55	151	30.73	9.90	1.55	12.46	10.00	4.03
Row4th	65-69 = 4yrs	32	171	19.18	2.32	6.90	2.55	3.60	1.47
	65-76 = 11yrs	39	155	24.73	3.79	1.46	6.08	3.88	4.25
	65-82 = 17yrs	45	153	18.32	5.32	-8.47	6.46	-1.30	2.67
	65-92 = 27yrs	55	154	20.64	3.44	-10.62	8.82	9.93	3.60
Control	65-69 = 4yrs	32	215	25.44	0.58	11.15	5.11	1.58	1.60
(no thin)	65-76 = 11yrs	39	246	57.49	1.85	23.65	9.55	-6.75	2.41
	65-82 = 17yrs	45	270	69.89	2.38	30.33	12.50	-21.95	2.74
	65-92 = 27yrs	55	275	89.13	9.46	65.13	18.05	-23.13	10.00

** BA: The plots actual basal area per acre at the end of the measurement cycle (start of the next cycle, before thinning.)

Kellogg-Unlike the Hiawatha results, in most cases the FVS-LS NA simulations predicted basal area more accurately than the ΔD simulations (Table 7). The ΔD projections were more likely to under predict basal area, while the only NA projections that under-predicted was the thin every 4th row treatment. The difference in the absolute mean error between the two simulation types was not as large as that in the Hiawatha simulations. RPAL mean basal area error was less than 10 ft^2/acre up to eight years in all treatments. In stands with heavier thinning treatments, the mean error dramatically increased in the 16 year cycle. In most cases the 27 year growth showed an over prediction of 11 to 28 ft^2/acre, except in the control case where RPAL under-predicted growth by 10 ft^2/acre.

Table 7. Kellogg mean error (ē) and standard deviation (s) of mean error for estimated basal area by treatment and projection length for three types of simulation. (Error expressed as predicted value minus observed value.)

				-----BA NA-----		-----BA AD-----		-----BA RPAL-----	
	projection length	BA**	age	ē	s	ē	s	ē	s
BAr70	64-67 = 3yrs	100	31	0.70	6.04	-5.45	9.29	-0.22	7.24
	64-72 = 8yrs	115	36	4.49	6.04	-8.06	9.29	3.03	7.24
	64-80 = 16yrs	106	44	7.47	6.00	-12.88	10.40	20.08	5.38
	64-85 = 21yrs	115	49	6.73	6.91	-16.82	11.25	9.55	5.69
	64-91 = 27yrs	128	55	14.85	11.35	-13.76	15.34	26.05	2.24
BAr90	64-67 = 3yrs	121	31	1.95	13.88	-5.11	13.95	-0.25	16.07
	64-72 = 8yrs	112	36	10.29	13.88	-3.93	13.95	17.50	16.07
	64-80 = 16yrs	119	44	18.24	21.85	-4.95	20.09	31.20	15.06
	64-85 = 21yrs	128	49	16.37	22.02	-9.99	20.38	6.78	4.90
	64-91 = 27yrs	146	55	21.76	20.77	-9.84	19.26	19.58	3.50
BAr110	64-67 = 3yrs	151	31	-1.64	9.32	-9.39	9.52	-5.87	5.22
	64-72 = 8yrs	135	36	0.54	9.32	-14.59	9.52	4.58	5.22
	64-80 = 16yrs	137	44	12.21	25.25	-12.06	23.58	34.73	4.61
	64-85 = 21yrs	140	49	13.94	30.61	-13.20	26.91	7.30	7.48
	64-91 = 27yrs	155	55	22.03	35.32	-10.39	31.29	20.78	12.71
BAr130	64-67 = 3yrs	168	31	3.14	19.88	-5.20	15.85	-3.42	9.11
	64-72 = 8yrs	156	36	11.13	19.88	-6.26	15.85	6.48	9.11
	64-80 = 16yrs	150	44	27.11	38.89	-0.86	32.35	31.90	24.87
	64-85 = 21yrs	163	49	25.00	42.41	-6.27	33.62	8.88	18.22
	64-91 = 27yrs	186	55	71.86	122.48	35.42	116.64	28.03	47.26
Row2nd	64-67 = 3yrs	106	31	0.81	22.97	-6.81	24.12	-1.25	24.51
	64-72 = 8yrs	116	36	9.47	22.97	-5.67	24.12	6.08	24.51
	64-80 = 16yrs	124	44	16.02	35.61	-13.83	31.92	15.05	3.64
	64-85 = 21yrs	122	49	13.37	37.69	-16.82	31.93	15.70	19.09
	64-91 = 27yrs	136	55	22.24	44.20	-14.57	38.96	20.13	6.61
Row3rd	64-67 = 3yrs	137	31	-0.57	5.28	-8.67	4.73	-5.75	4.97
	64-72 = 8yrs	123	36	0.10	5.28	-15.17	4.73	7.05	4.97
	64-80 = 16yrs	130	44	9.77	12.20	-15.83	13.21	21.60	19.15
	64-85 = 21yrs	126	49	12.49	17.68	-14.71	20.56	9.28	7.41
	64-91 = 27yrs	146	55	15.41	18.63	-17.87	23.59	18.78	8.88
Row4_2	64-67 = 3yrs	153	31	1.23	12.66	-9.57	11.35	-5.67	10.17
	64-72 = 8yrs	128	36	10.00	12.66	-10.78	11.35	9.08	10.17
	64-80 = 16yrs	138	44	10.91	18.50	-21.93	15.38	7.73	7.17
	64-85 = 21yrs	150	49	8.45	18.75	-28.76	15.73	5.40	8.04
	64-91 = 27yrs	171	55	12.66	20.42	-32.41	17.42	11.63	8.25
Row4th	64-67 = 3yrs	153	31	-5.12	19.59	-9.87	4.44	-6.02	4.42
	64-72 = 8yrs	142	36	-7.49	19.59	-17.45	4.44	2.10	4.42
	64-80 = 16yrs	149	44	-13.12	30.82	-28.77	6.74	7.33	3.61
	64-85 = 21yrs	147	49	-19.19	33.07	-35.04	5.99	2.80	2.33
	64-91 = 27yrs	164	55	-16.36	40.27	-35.04	8.06	14.63	6.71
Control	64-67 = 3yrs	179	31	6.79	3.73	-6.22	1.94	-0.37	1.00
(no thin)	64-72 = 8yrs	206	36	21.12	3.73	-11.68	1.94	0.33	1.00
	64-80 = 16yrs	234	44	48.39	9.64	-7.80	5.51	-0.30	4.51
	64-85 = 21yrs	246	49	60.88	16.26	-2.45	11.17	-2.85	12.09
	64-91 = 27yrs	258	55	72.89	13.61	6.56	9.13	-10.20	11.49

** BA: The plots actual basal area per acre at the end of the measurement cycle (start of the next cycle, before thinning.)

Trees per acre Error

As with the basal area calculations of mean error, the trees per acre mean error was based on a sample size of four for each treatment (Table 8 and Table 9). Error was calculated for all three simulation types.

Hiawatha-The FVS simulations had a mean error of ± 10 TPA except for the high residual basal area treatments, BAr160, Row4_2, Row4th, and the Control. The worst prediction accuracy for FVS was the control treatment (no thinning). The control treatment, with a 27-year prediction interval, over-predicted by more than 175 trees per acre. There was very little difference in the error between the NA and ΔD simulations. RPAL also did a poor job at predicting treatments with higher residual basal area. In the row thinning treatments and treatments with a residual basal over 100 ft^2/acre RPAL over-predicted by 21 to 95 TPA after the first cycle. In the control treatment, RPAL under-predicted between 34 and 103 TPA over the 27 year period (Table 8).

Table 8. **Hiawatha mean error (ē) and standard deviation (s) of mean error for estimated trees per acre by treatment and projection length for three types of simulation. (Error expressed as predicted value minus observed value.)**

				-----TPA NA-----		-----TPA ΔD-----		-----TPA RPAL -----	
treatment	projection length	age	TPA***	ē	s	ē	s	ē	s
BAr30	65-69 = 4yrs	32	148	-0.18	0.02	-0.18	0.02	0.00	0.00
	65-76 = 11yrs	39	75	-0.25	0.02	-0.25	0.02	1.75	3.77
	65-82 = 17yrs	45	53	-0.27	0.03	-0.27	0.03	-10.75	3.77
	65-92 = 27yrs	55	53	-0.43	0.04	-0.43	0.04	-10.75	3.77
BAr45	65-69 = 4yrs	32	213	-0.26	0.04	-0.26	0.05	0.00	0.00
	65-76 = 11yrs	39	115	-0.38	0.06	-0.38	0.06	9.75	3.95
	65-82 = 17yrs	45	73	-0.37	0.05	-0.37	0.05	-2.00	7.16
	65-92 = 27yrs	55	70	1.91	4.96	1.91	4.96	0.50	6.56
BAr60	65-69 = 4yrs	32	300	2.14	4.97	2.14	4.97	2.50	5.00
	65-76 = 11yrs	39	175	1.91	4.93	1.91	4.93	11.50	4.20
	65-82 = 17yrs	45	115	1.90	4.92	1.90	4.92	5.50	8.85
	65-92 = 27yrs	55	115	1.55	4.88	1.55	4.88	5.50	8.85
BAr80	65-69 = 4yrs	32	433	-0.52	0.02	-0.52	0.02	0.00	0.00
	65-76 = 11yrs	39	275	-0.91	0.04	-0.91	0.04	13.00	13.61
	65-82 = 17yrs	45	183	-0.93	0.03	-0.93	0.03	7.00	12.75
	65-92 = 27yrs	55	145	-1.17	0.05	-1.18	0.05	7.25	9.91
BAr100	65-69 = 4yrs	32	550	-0.66	0.08	-1.10	0.85	0.00	0.00
	65-76 = 11yrs	39	365	-1.21	0.14	-1.22	0.16	24.50	6.95
	65-82 = 17yrs	45	260	1.16	4.94	1.14	4.92	23.25	6.85
	65-92 = 27yrs	55	205	3.30	5.68	3.28	5.66	21.25	10.59
BAr130	65-69 = 4yrs	32	755	-0.94	0.06	-1.05	0.12	0.00	0.00
	65-76 = 11yrs	39	543	0.66	4.98	0.53	4.95	30.75	14.86
	65-82 = 17yrs	45	400	2.90	5.79	2.80	5.81	40.25	17.06
	65-92 = 27yrs	55	328	2.33	5.82	2.22	5.83	46.75	21.03
BAr160	65-69 = 4yrs	32	978	8.66	19.89	7.86	19.74	10.00	20.00
	65-76 = 11yrs	39	753	7.28	19.84	6.26	19.70	50.75	20.19
	65-82 = 17yrs	45	590	9.17	24.81	8.16	24.68	61.50	23.27
	65-92 = 27yrs	55	485	10.62	23.40	9.63	23.05	69.50	27.33
Row2nd	65-69 = 4yrs	32	575	1.67	4.90	1.58	4.84	2.50	5.00
	65-76 = 11yrs	39	343	1.34	4.97	1.31	4.94	62.00	17.80
	65-82 = 17yrs	45	238	1.23	4.93	1.20	4.89	45.00	22.91
	65-92 = 27yrs	55	193	3.33	5.58	3.30	5.54	42.50	23.85
Row3rd	65-69 = 4yrs	32	828	1.14	4.76	0.52	4.43	2.50	5.00
	65-76 = 11yrs	39	520	7.61	7.62	6.97	7.18	91.00	18.20
	65-82 = 17yrs	45	385	6.92	7.27	6.15	6.72	80.00	16.43
	65-92 = 27yrs	55	300	18.17	15.14	17.23	15.31	76.00	19.54
Row4_2	65-69 = 4yrs	32	818	8.66	19.84	7.99	19.55	10.00	20.00
	65-76 = 11yrs	39	540	17.65	17.88	16.68	17.60	-7.25	14.52
	65-82 = 17yrs	45	363	28.08	34.29	31.14	30.62	20.25	20.12
	65-92 = 27yrs	55	275	44.24	37.51	43.47	36.87	47.25	31.70
Row4th	65-69 = 4yrs	32	908	6.04	9.56	4.91	9.39	7.50	9.57
	65-76 = 11yrs	39	525	17.86	11.42	17.08	11.49	95.00	13.74
	65-82 = 17yrs	45	380	19.90	14.78	19.02	14.87	88.50	25.53
	65-92 = 27yrs	55	318	24.25	16.49	23.26	16.39	89.25	26.81
Control	65-69 = 4yrs	32	1165	7.77	14.36	5.25	14.97	-10.75	15.22
(no thin)	65-76 = 11yrs	39	1158	10.16	22.68	2.75	25.55	-39.75	37.54
	65-82 = 17yrs	45	1135	24.20	28.10	14.00	29.35	-103.25	30.19
	65-92 = 27yrs	55	945	177.55	51.76	179.50	63.05	-34.00	64.47

***** TPA: The plots actual trees per acre at the end of the measurement cycle (start of the next cycle, before thinning.)**

Kellogg- Error was much larger for all three types of simulations on the Kellogg site than for the Hiawatha. The FVS-LS, NA and ΔD simulations showed an increase in error as the projection intervals increased, however this was not necessarily true for the RPAL projections. The worst prediction accuracy for FVS-LS was the control treatment (no thinning). The control treatment with a 27-year prediction interval was over-predicted by more than 130 TPA (NA simulation) and 120 TPA (ΔD simulation). RPAL over-predicted TPA after the first cycle by 12 to 162 TPA (Table 9).

Table 9. **Kellogg mean error (ē) and standard deviation (s) of mean error for estimated trees per acre by treatment and projection length for three types of simulation. (Error expressed as predicted value minus observed value.)**

				-----TPA NA-----		-----TPA AD-----		-----TPA RPAL-----	
treatment	projection length	age	TPA***	ē	s	ē	s	ē	s
BAr70	64-67 = 3yrs	31	360	14.59	22.38	14.31	22.93	15.00	22.17
	64-72 = 8yrs	36	353	21.64	22.38	21.04	22.93	22.50	22.17
	64-80 = 16yrs	44	168	21.49	18.90	21.33	18.99	55.00	31.80
	64-85 = 21yrs	49	148	23.79	19.09	23.60	19.17	28.50	11.96
	64-91 = 27yrs	55	145	25.92	23.58	25.68	23.68	31.00	12.83
BAr90	64-67 = 3yrs	31	405	14.54	33.12	14.29	33.23	15.00	62.41
	64-72 = 8yrs	36	250	36.64	33.12	36.39	33.23	93.75	62.41
	64-80 = 16yrs	44	180	46.03	44.37	45.64	44.58	98.75	60.64
	64-85 = 21yrs	49	160	45.67	44.44	45.22	44.67	42.25	20.11
	64-91 = 27yrs	55	160	45.11	44.51	44.60	44.76	42.25	20.11
BAr110	64-67 = 3yrs	31	518	14.44	23.65	14.14	23.77	15.00	18.49
	64-72 = 8yrs	36	320	16.51	23.65	16.09	23.77	89.00	18.49
	64-80 = 16yrs	44	218	41.12	43.78	40.47	43.73	148.25	26.06
	64-85 = 21yrs	49	180	45.59	48.59	44.98	48.25	78.25	15.00
	64-91 = 27yrs	55	183	47.46	51.33	46.75	50.88	80.75	18.66
BAr130	64-67 = 3yrs	31	620	16.73	40.22	16.20	39.99	17.50	19.54
	64-72 = 8yrs	36	425	33.72	40.22	33.18	39.99	86.00	19.54
	64-80 = 16yrs	44	300	65.65	62.44	65.21	62.25	162.25	91.47
	64-85 = 21yrs	49	258	67.79	64.00	67.28	63.72	102.00	20.61
	64-91 = 27yrs	55	258	99.33	118.96	98.72	118.77	115.25	70.28
Row2nd	64-67 = 3yrs	31	383	14.63	62.45	14.53	62.58	15.00	62.38
	64-72 = 8yrs	36	360	36.76	62.45	36.51	62.58	37.50	62.38
	64-80 = 16yrs	44	223	41.22	71.95	41.05	71.88	49.00	22.21
	64-85 = 21yrs	49	170	41.15	71.77	41.00	71.65	46.00	31.70
	64-91 = 27yrs	55	168	43.25	76.55	43.06	76.39	32.25	13.33
Row3rd	64-67 = 3yrs	31	483	2.00	5.11	1.72	5.30	-1.00	9.59
	64-72 = 8yrs	36	303	1.65	5.11	1.34	5.30	63.00	9.59
	64-80 = 16yrs	44	223	21.06	32.54	20.62	32.09	76.00	51.22
	64-85 = 21yrs	49	173	28.37	37.59	27.96	37.06	46.75	22.11
	64-91 = 27yrs	55	173	27.87	37.33	27.41	36.73	46.75	22.11
Row4_2	64-67 = 3yrs	31	623	1.68	49.73	0.79	48.74	2.50	39.50
	64-72 = 8yrs	36	393	33.36	49.73	31.88	48.74	59.50	39.50
	64-80 = 16yrs	44	258	47.30	47.66	45.56	46.12	75.00	18.57
	64-85 = 21yrs	49	220	49.20	51.86	47.47	50.47	74.75	35.07
	64-91 = 27yrs	55	220	48.17	51.12	46.25	49.65	74.75	35.07
Row4th	64-67 = 3yrs	31	580	-2.39	4.43	-1.61	2.47	0.00	18.41
	64-72 = 8yrs	36	395	-4.05	4.43	-2.92	2.47	53.50	18.41
	64-80 = 16yrs	44	275	-2.72	2.70	-1.64	0.22	70.75	8.34
	64-85 = 21yrs	49	215	-2.04	1.25	-1.51	0.20	61.75	6.65
	64-91 = 27yrs	55	213	-0.01	5.73	0.58	5.11	64.25	8.81
Control	64-67 = 3yrs	31	723	9.14	9.09	7.43	8.36	10.00	11.81
(no thin)	64-72 = 8yrs	36	720	10.01	9.09	5.40	8.36	11.25	11.81
	64-80 = 16yrs	44	673	54.26	24.50	45.45	22.54	22.50	25.51
	64-85 = 21yrs	49	633	93.61	46.19	82.89	43.23	43.75	40.48
	64-91 = 27yrs	55	585	133.44	15.76	121.49	14.45	46.00	10.36

*** TPA: The plots actual trees per acre at the end of the measurement cycle (start of the next cycle, before thinning.)

DISCUSSION

FVS predicted more accurate results than RPAL for trees per acre but not necessarily for basal area per acre. FVS predict more accurately diameter at breast height and basal area per acre with diameter growth calibration turned on, diameter growth calibration had little to no effect on more accurately estimating mortality on either site.

FVS Options and Defaults

The FVS validation runs used version 6.2 of the FVS-LS variant with a revision date of 12/01/1995. FVS variants are continuously being updated and improved. These same data run through a current version of the model would yield different results. There are many ways users can make adjustments in an FVS simulation to produce more accurate results. One example of this involves using serial correlation of diameter growth. This feature improves estimates from cycle to cycle by assuming the error terms from a previous cycle are correlated with error terms of the next cycle. In other words, the error term is not randomly distributed at each cycle. Trees that were growing well previously continue to grow well and those that are not growing well continue to grow poorly (Dixon 2002). This feature improves the overall distribution of diameter growth in a stand. This feature was turned off by default in the version used for this validation project. It could have been turned on by the use of a keyword, however the user did not know of this feature at that time. This feature is now on by default in more current version of FVS-LS.

RPAL Options and Defaults

The RPAL program had a date of 9/2/1999, with no known revisions to date. RPAL is comprised of two basal area growth equations, one for stands less than 25 years old and another for stands greater than or equal 25 years old. Stands that are less than 25 years old (at dbh) use an equation developed by Lundgren (1981). Stands greater than or equal to 25 years old (at dbh) use an equation developed by Buckman (1962) that is based on basal area per acre, age, and site index (base age 50 years). Therefore the only the basal area growth equation for trees greater then 25 years in age was tested in this study. Unlike FVS, there are no parameters that can be used to alter the growth equations in RPAL except the site index set at the beginning of each run. The only changes that can be made from cycle to cycle are the desired residual basal area after treatment and the cycle length. These make the model easier to use and lead to more consistency among users, yet also limits the capability of the user to make the model more site or location specific.

Model Performance

In both models as time increased error also tended to increase. This is in part due to error propagation; over time the error builds upon its self. So that bias results are used to predict into the future more bias results. "Outputs or estimates from the previous period become inputs or initial values for the next. In this network of calculations, uncertainty estimations can be a complex and tedious process at best" (Mowrer and Haas 1991). FVS-LS also did a better job at predicting the Hiawatha site compared to the Kellogg site. This is probably in part due to the fact that the FVS-LS equations were not created with data from southern lower Michigan (Miner *et al.* 1988).

Overall, the dbh mean errors were better predicted for FVS runs that used past growth (ΔD) information to scale the equations than those that did not (NA). The absolute dbh mean errors for the ΔD simulations were less than 1.0 inch. The scale factors for Kellogg ranged from 0.42 to 0.75 with a mean of 0.56. The Hiawatha scale factors ranged from 0.47 to 1.16 with a mean of 0.78. On average, these were biased low. This is due to how the diameter growth equations were applied. The first thinning occurred (1962: Hiawatha, 1960: Kellogg) at the start of the growth measurement period. The growth measurement period used to calibrate the dbh growth was three years long (1962 to 1965) for the Hiawatha site and four years long (1960 to 1964) for the Kellogg site. If the increase in growth was not immediate after the thin, in other words there was a year or more delay before the trees took full advantage of the thin, then FVS would not have captured the impact of the thinning in the short growth measurement period. Therefore, the scale factor applied to the thinned stand may have scaled down growth too much for the rest of the projection. A longer measurement period may have given more accurate results. A second reason that the scale factors were biased low was because mortality trees were recorded as "dead" trees in the input tree list instead of "recent mortality" trees. If the trees had been recorded as recent mortality trees then FVS would have included them in the stand density calculations which then would have affected how the scale factors were calculated. It is important that users realize the impact that the scale factor has on future tree simulated growth.

There were three main problems with how FVS calibrated the runs in this study. First, the measurement periods in the simulations were too short and reflected the pre-thinned growth, therefore scaling down (slowing) the original growth equations. Second, trees were marked as "dead" instead of "recently dead," making FVS incorrectly predict past density. A third problem is that the growth measurements were not adjusted for bark thickness. This could have been corrected by indicating the correct parameters on the FVS GROWTH keyword. Since diameter increments included bark growth this added another bias to the results.

For the FVS runs, mean error for trees per acre were very similar between the scaled and non-scaled simulations for both sites. Typically denser red pine stands have little mortality and will stagnate until released. Therefore, it would have been expected that the models might over predict mortality (under-predict TPA). FVS and RPAL, however, in most cases under-predicted mortality. The TWIGS mortality (Buchman 1983, Buchman et al 1983, Buchman and Lentz 1984) equations imbedded in the FVS framework actually calculate the survival probability for each tree. These survival rates are annual rates and are converted to mortality rates and scaled to the number of years in the run (Dixon 2002, Bush and Brand 1995). This estimate of mortality is then applied to each tree record (or sampled tree). RPAL predicts stand level mortality in terms of basal area mortality. This is then converted to TPA mortality, using the assumption that the mean dbh of dead trees is equal to the stand's QMD minus one standard deviation (Lundgren, 1981). Both models may have under-predicted mortality because neither assumed any type of stochastic mortality. The inventory history indicates that some trees died by snow or ice storms, but no description was given for other trees. From the previous inventory, however, most trees appeared to put on good growth prior to the cycle in which death occurred. From this it is assumed that many of the trees died from stochastic events such as wind, ice, snow, insect or disease. This suggests that when running simulations, in order to get reasonable estimates of mortality, it is important to include stochastic mortality events that are typical for the region and the species modeled. In FVS a mortality multiplier could be included to model these events. RPAL does not have this functionality and therefore stochastic mortality would have to be applied outside of the computer program.

For the typical user of FVS, who may not have a complete understanding of the system, it is important to realize there are many adjustments that can be made to FVS in order improve accuracy. FVS uses common stand exam data but is a complicated growth and yield program. In order appropriately use the model a user must read the user's manual (Dixon 2002), variant guide (Bush and Brand 1995) and probably take an FVS course. The more the user understands the capabilities of FVS, and the process it uses to compute growth, the better estimates the user will obtain. At the beginning of this study the author was a novice FVS user and found that even with out making many adjustments to improve estimates of growth, FVS-LS did perform well in the prediction of individual tree diameters, stand density, and mortality in most cases. RPAL on the other hand was very easy to use. No user's manual was needed to learn how to use the model and it needed few input values (all stand level information). Over all it did a good job predicting basal area growth, however it did not do a good job predicting trees per acre.

CONCLUSION

Land managers are under increasing pressure to meet their own objectives or those set by the company which they work, or the public they serve. Whether it is short or long term timber revenues, recreation areas, or wildlife habitat, RPAL and FVS can both be used to estimate future red pine stand structure to help land managers make more informed decisions. Land managers can apply knowledge from any number of red pine management studies and test to see what management strategy will best meet the objectives for their site. When comparing the two models FVS is more robust than RPAL. That it is able to simulate almost any type of thinning, it allows for mixed species stands and with past tree growth information (increment core data) the model will calibrate growth to the specific site. However with the heterogeneity of red pine plantations, managers may not need such a robust model, as red pine plantation thinning regimes tend to be uncomplicated and growth is heterogeneous. One of the most common quotes in modeling is, "All models are wrong, but some are useful" (George E. P. Box). None the less, users of models want the predicted results to be truth. Model results are used to make decisions. In forestry with 30 to 200 year timber rotations and uncountable site variations, perfecting a model is virtually impossible. Both of these models, however, perform well if the objective is to take standard forest inventory data to compare future management options. For example, holding all things constant, will one thinning regime versus another produce more red pine utility poles at final harvest. These models may not predict exactly the number of poles produced in the future but they will estimate whether one management plan will significantly provide more than another. FVS was a good predictor of diameter growth and was even better when diameter calibration was used. Absolute mean error at both sites when diameter calibration was included was less 1.0 inch and in many cases less then 0.5 inches. In many cases RPAL's absolute mean error for basal area was less then FVS. FVS and RPAL did poorly in predicting mortality past the first cycle. At the Kellogg site (Lower Michigan) FVS frequently underestimated mortality by 50 to 180 trees per acre. The large bias and variability that both models showed in predicting trees per acre could certainly be a problem in estimating volume production for the site. FVS and RPAL also lost accuracy and precision as cycle length increased.

Foresters and natural resource professionals are under increasing pressure to make the "right" forest management decision. A quality growth model can be one more tool in their toolbox to help them make more informed decisions.

Future validation studies need to continue on growth and yield models in the Lake States. This study only examined the red pine large tree growth and mortality equations for RPAL and FVS-LS. Validation of the basal area growth equations for trees less than 25 years old should be done on RPAL and validation should be done on seedling/sapling growth in FVS-LS for both hardwood and softwood species. This study covered a small spatial distribution but was longitudinal in design. Follow-up studies should also consider both longitudinal and spatial data to give a more complete understanding of the model's performance through out Michigan and over time. When consistent biases are discovered old equations should be corrected or new equations should be developed. In modeling, validation and revision should be an integral part of the model development and maintenance process.

Figure 2. Hiawatha site mean error for estimated diameter at breast height by treatment and prjection length for the FVS with out DBH growth calibration (NA). (Error expressed as predicted value minus observed value.)

Image presented in color.

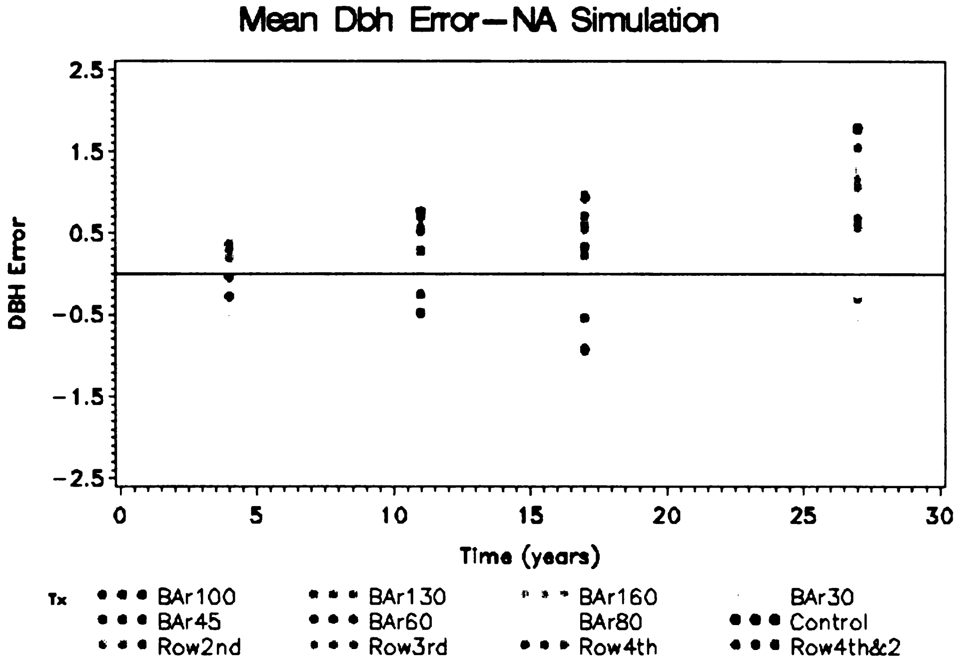


Figure 3. Hiawatha site mean error for estimated diameter at breast height by treatment and projection length for the FVS with growth calibration (ΔD). (Error expressed as predicted value minus observed value.)

Image presented in color.

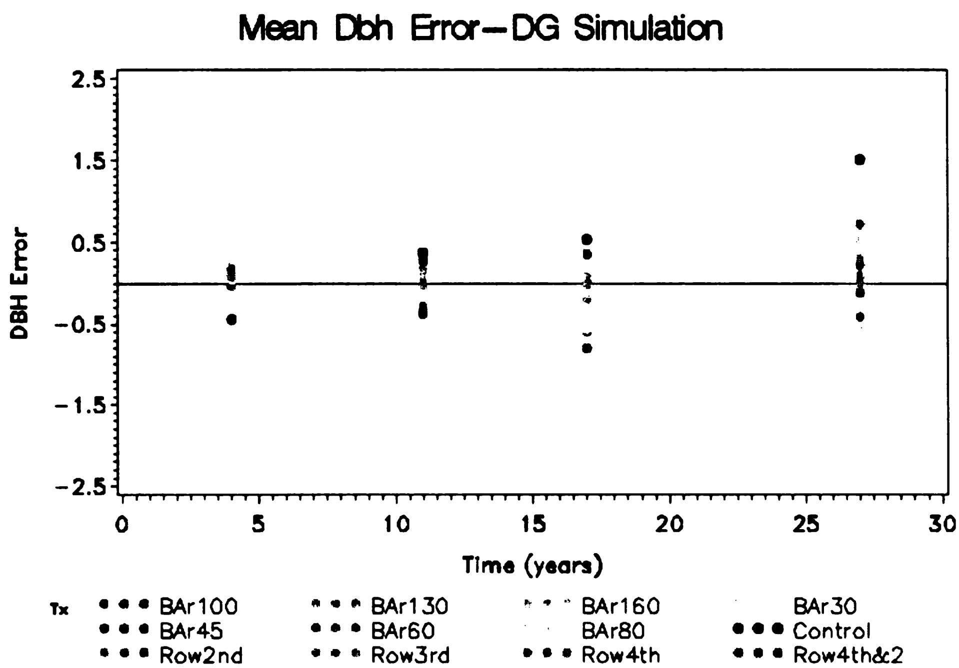


Figure 4. Kellogg site mean error for estimated diameter at breast height by treatment and projection length for the FVS with out DBH growth calibration (NA). (Error expressed as predicted value minus observed value.)
Image presented in color.

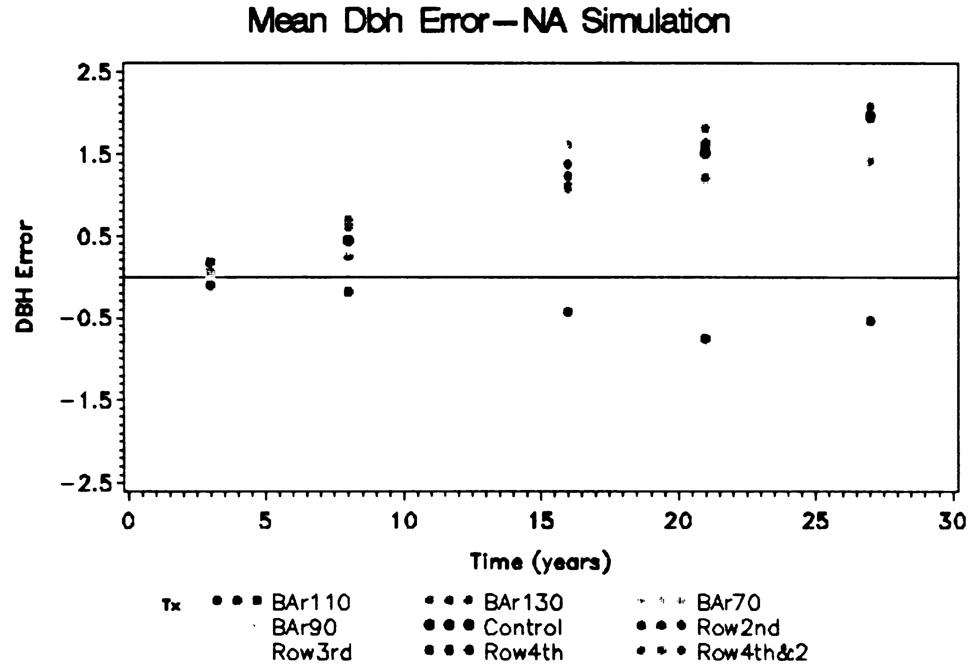


Figure 5. Kellogg site mean error for estimated diameter at breast height by treatment and projection length for the FVS with growth calibration (ΔD). (Error expressed as predicted value minus observed value.)
Image presented in color.

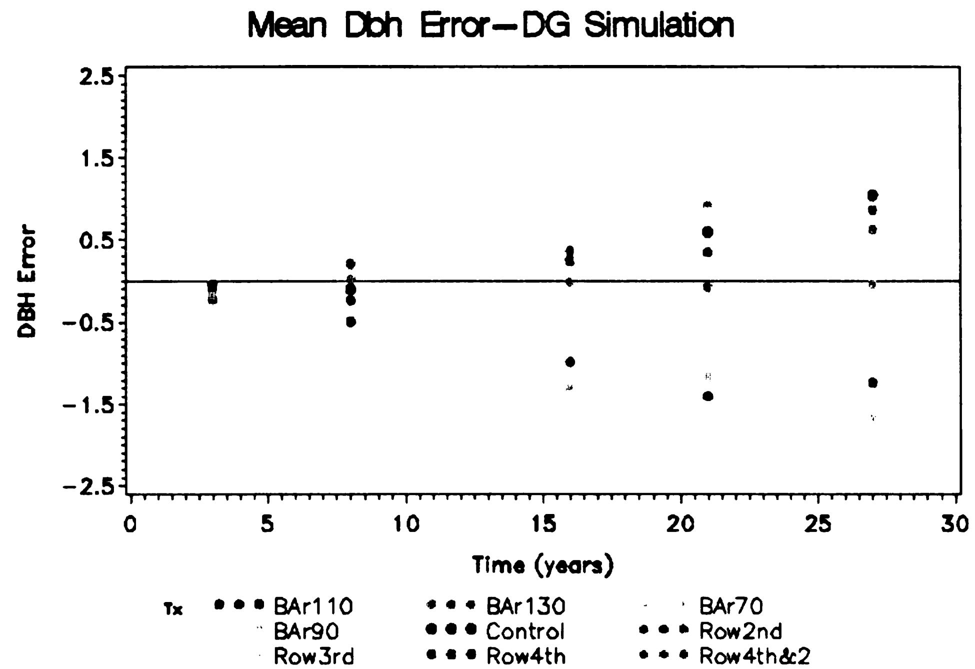


Figure 6. Hiawatha site mean error for estimated basal area per acre by treatment and projection length for the FVS with out DBH growth calibration (NA). (Error expressed as predicted value minus observed value.)
Image presented in color.

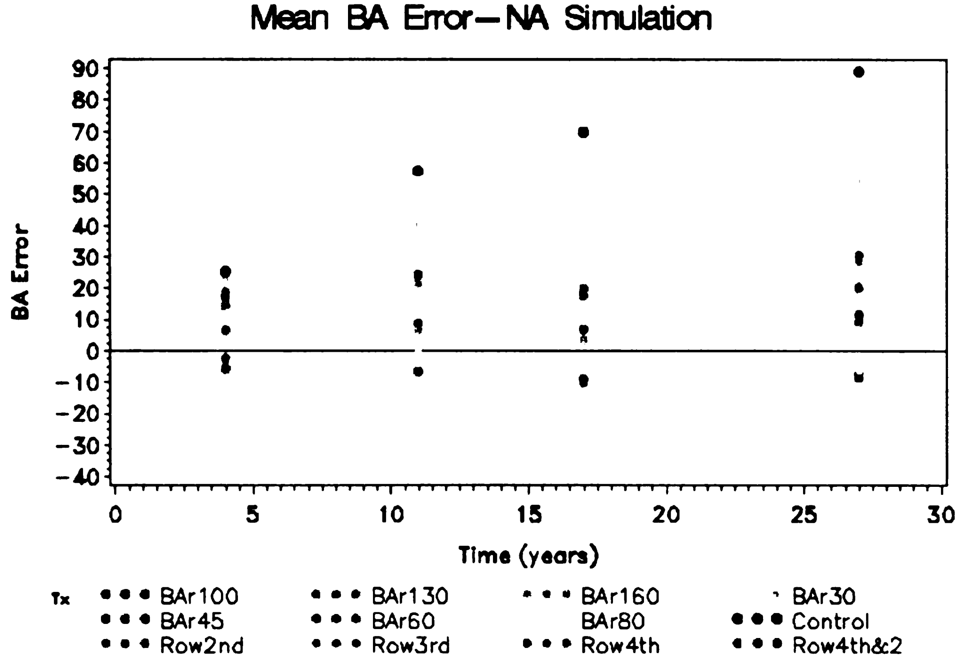


Figure 7. Hiawatha site mean error for estimated basal area per acre by treatment and projection length for the FVS with growth calibration (ΔD). (Error expressed as predicted value minus observed value.)
Image presented in color.

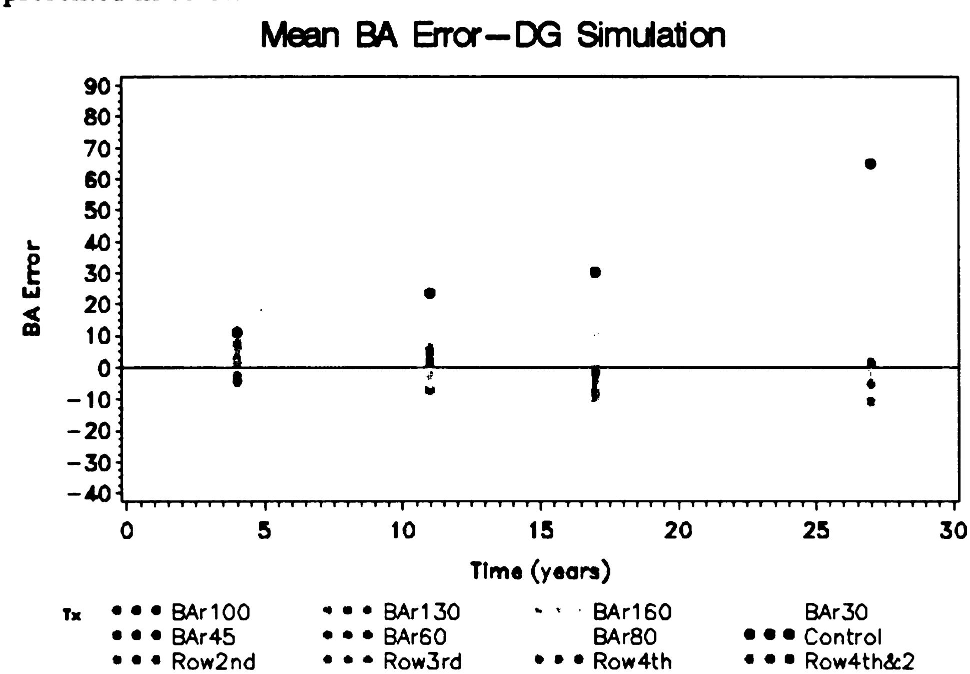


Figure 8. Hiawatha site error for estimated basal area per acre by treatment and projection length for RPAL. (Error expressed as predicted value minus observed value.)
Image presented in color.

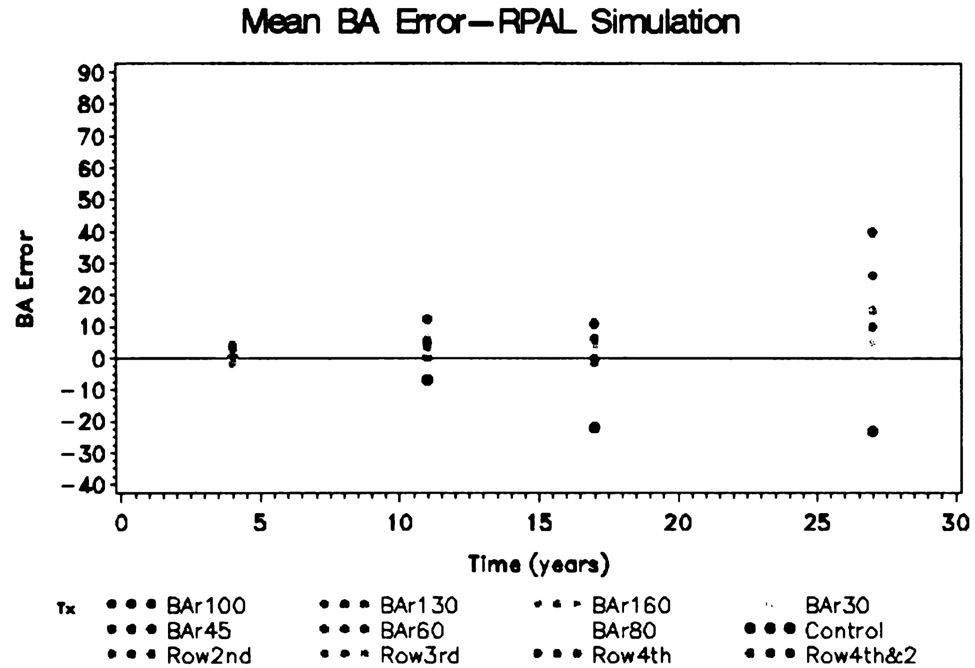


Figure 9. Kellogg site mean error for estimated basal area per acre by treatment and projection length for the FVS with out DBH growth calibration (NA). (Error expressed as predicted value minus observed value.)
Image presented in color.

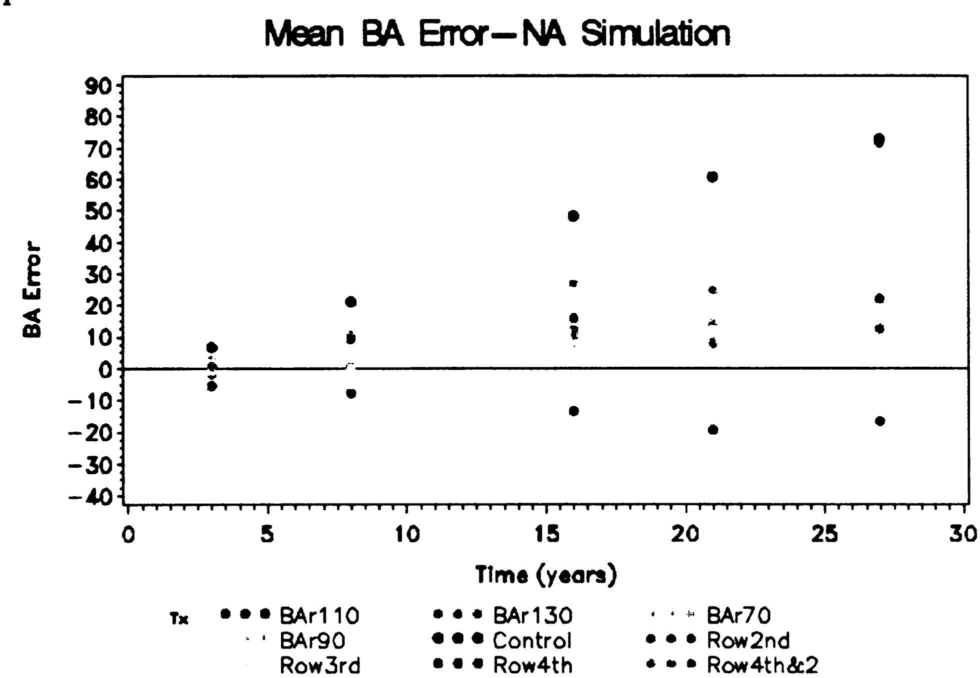


Figure 10. Kellogg site mean error for estimated basal area per acre by treatment and projection length for the FVS with growth calibration (ΔD). Error expressed as predicted value minus observed value.
Image presented in color.

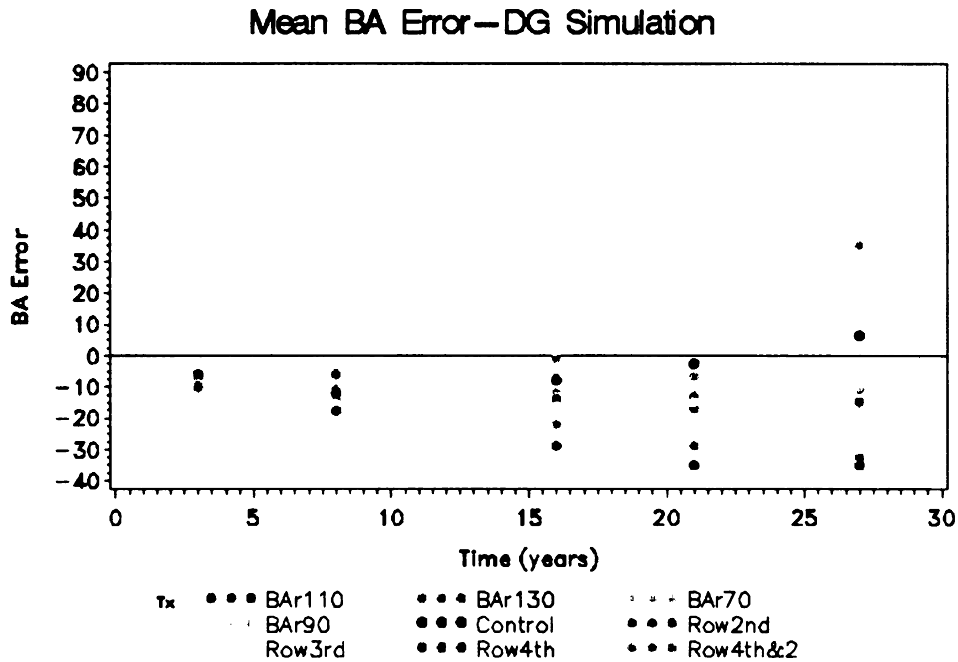


Figure 11. Kellogg site mean error for estimated basal area per acre by treatment and projection length for RPAL. (Error expressed as predicted value minus observed value.)
Image presented in color.

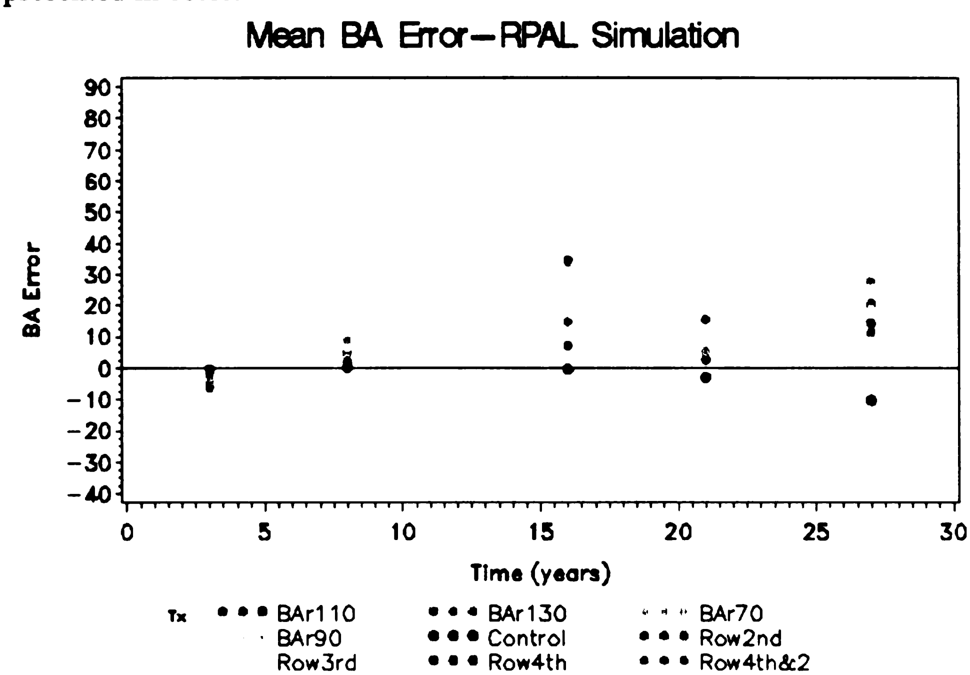


Figure 12. Hiawatha site mean error for estimated trees per acre by treatment and projection length for the FVS with out DBH growth calibration (NA). (Error expressed as predicted value minus observed value.)
Image presented in color.

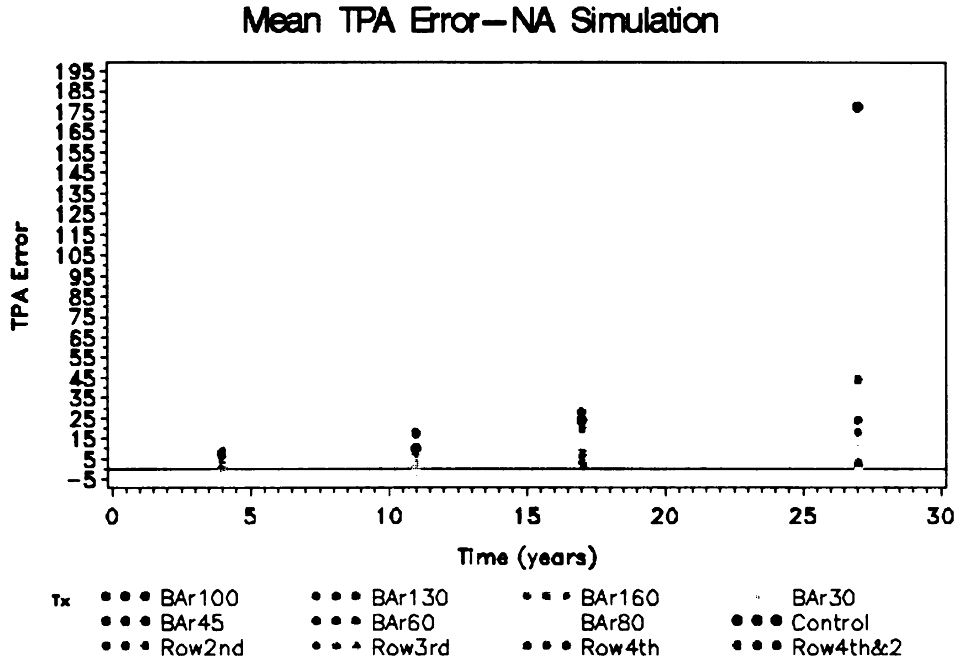


Figure 13. Hiawatha site mean error for estimated trees per acre by treatment and projection length for the FVS with growth calibration (ΔD). (Error expressed as predicted value minus observed value.)
Image presented in color.

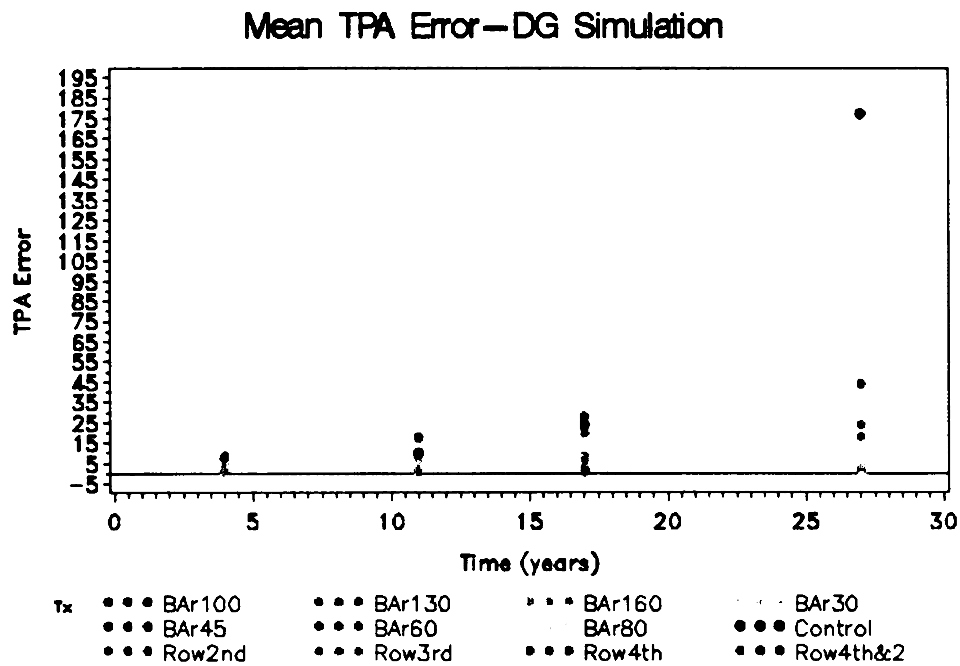


Figure 14. Hiawatha site mean error for estimated trees per acre by treatment and projection length for RPAL. (Error expressed as predicted value minus observed value.)
Image presented in color.

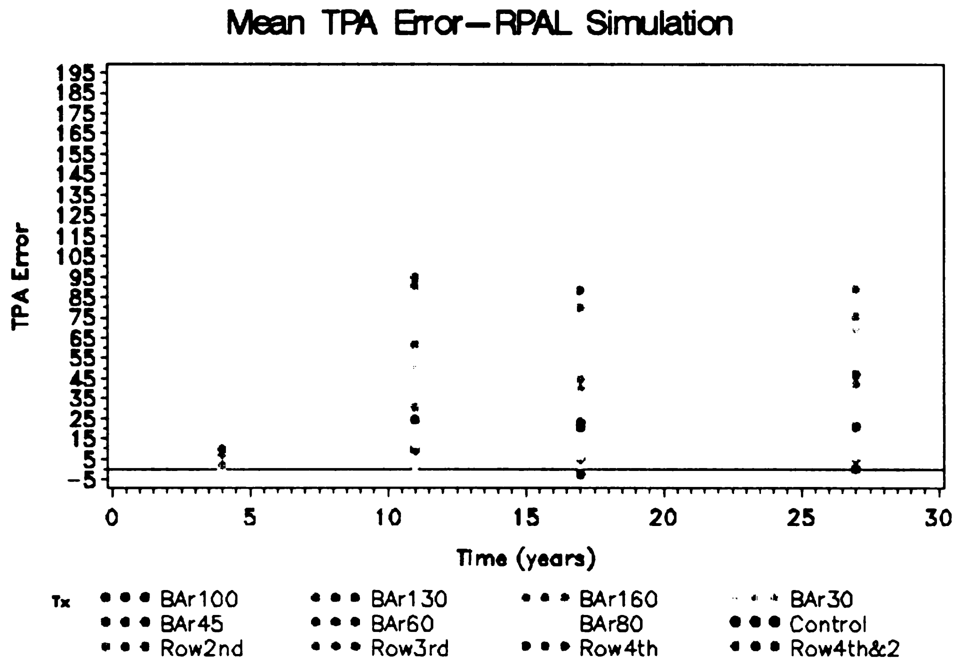


Figure 15. Kellogg site mean error for estimated trees per acre by treatment and projection length for the FVS with out DBH growth calibration (NA). (Error expressed as predicted value minus observed value.)
Image presented in color.

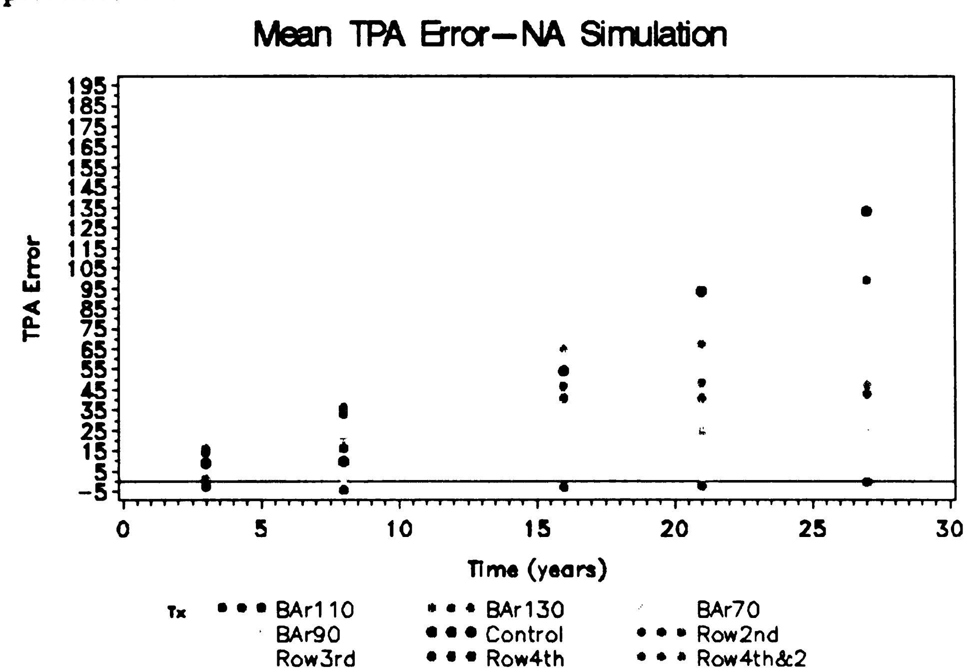


Figure 16. Kellogg site mean error for estimated trees per acre by treatment and projection length for the FVS with growth calibration (ΔD). (Error expressed as predicted value minus observed value.)
Image presented in color.

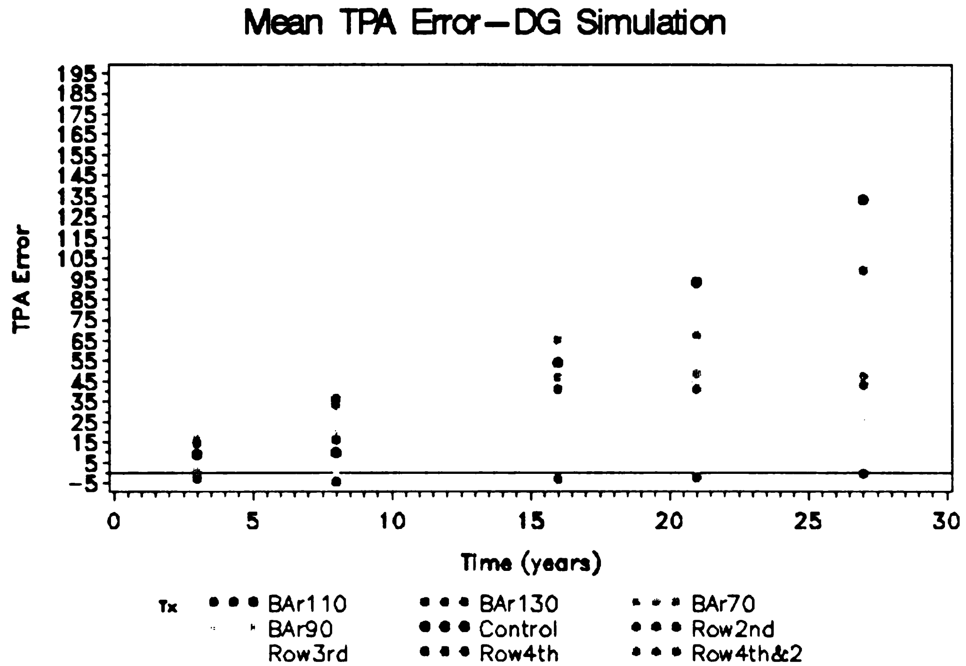
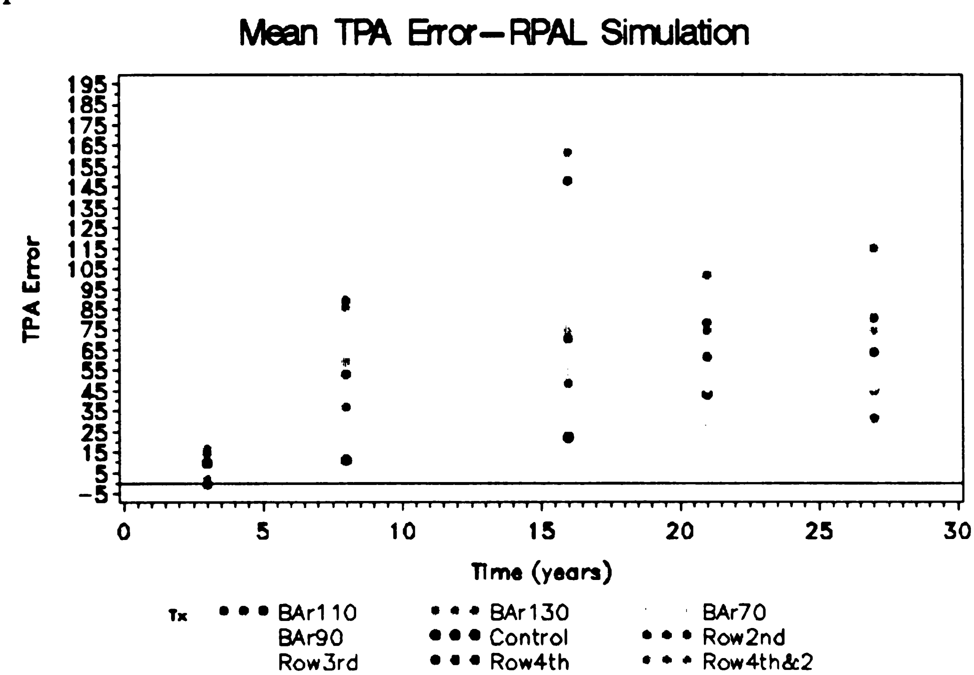


Figure 17. Kellogg site mean error for estimated trees per acre by treatment and projection length for RPAL. (Error expressed as predicted value minus observed value.)
Image presented in color.



APPENDICES

Appendix A. Thinnings at the Kellogg and Hiawatha site. (Adapted from Rudolph *et al.* 1984)

Hiawatha site				
Treatment name	Initial thinning	1969	1976	1982
BAr30:	30 ft2/acre	30 ft2/acre	30 ft2/acre	none
BAr45:	45 ft2/acre	45 ft2/acre	45 ft2/acre	none
BAr60:	60 ft2/acre	60 ft2/acre	60 ft2/acre	none
BAr80:	80 ft2/acre	80 ft2/acre	80 ft2/acre	85 ft2/acre
BAr100:	100 ft2/acre	100 ft2/acre	100 ft2/acre	105 ft2/acre
BAr130:	130 ft2/acre	130 ft2/acre	130 ft2/acre	135 ft2/acre
BAr160:	160 ft2/acre	160 ft2/acre	160 ft2/acre	165 ft2/acre
Row2nd:	every other row	90 ft2/acre	95 ft2/acre	100 ft2/acre
Row3rd:	every third row	120 ft2/acre	120 ft2/acre	125 ft2/acre
Row4_2:	every fourth row	every center row	110 ft2/acre	115 ft2/acre
Row4th:	every fourth row	22% height	22% height	22% height
Control:	none	none	none	none

Kellogg site						
Treatment name	Initial thinning	1967	1970	1974	1980	1985
BAr70:	70 ft2/acre	none	70 ft2/acre	none	95 ft2/acre	none
BAr90:	90 ft2/acre	90 ft2/acre	none	95 ft2/acre	105 ft2/acre	none
BAr110:	110 ft2/acre	110 ft2/acre	none	115ft2/acre	120 ft2/acre	none
BAr130:	130 ft2/acre	130 ft2/acre	none	135 ft2/acre	140 ft2/acre	none
Row2nd:	every other row	none	85 ft2/acre	none	100 ft2/acre	none
Row3rd:	every third row	100 ft2/acre	none	105 ft2/acre	105 ft2/acre	none
Row4_2:	every fourth row	every center row	none	110 ft2/acre	125 ft2/acre	none
Row4th:	every fourth row	115 ft2/acre	none	120 ft2/acre	125 ft2/acre	none
Control:	none	none	none	none	none	none

Appendix B. FVS-LS growth and mortality equations (Bush and Brand 1995)

FVS-LS Diameter Growth:

The LS-TWIGS diameter growth equation is comprised of two parts: a growth equation which predicts growth as if there were no competition (Hahn and Leary 1979) and a modifier equation which reduces potential tree growth to reflect stand competition based on stand basal area and the size of each tree in relation to the tree of average stand diameter (Holdaway 1984) (Miner et al. 1988).

The diameter growth equation is:

$$PG = A1 - A2*DA3 + A4*SI*CR*DA5$$

where:

PG = potential annual diameter growth (inches/year)
D = current tree diameter at breast height
SI = site index (base age 50)
CR = crown ratio code
A1 = 0.09446
A2 = 0.00012
A3 = 2.0596
A4 = 0.00035
A5 = 0.2423

The modifier equation is:

$$CM = 1 - \exp\{-f(D/AD)*g(AD)*[(BAMAX - BA)/BA]1/2\}$$

where:

CM = competition modifier
BAMAX = maximum basal area expected for the species (RP=350)
BA = current basal area
AD = average stand diameter
f(R) = a function characterizing the individual tree's relative diameter effect on the average stand diameter
= $B1*[1 - \exp(B2*D/AD)]B3 + B4$
g(AD) = a function characterizing the average stand diameter effect on the modifier
= $C1*(AD + 1)C2$
B1 = 2.310
B2 = -1.670
B3 = 3.94
B4 = 0.00
C1 = 0.441
C2 = 0.173

A diameter adjustment factor is then added onto the product of the growth and modifier equations. The equation, by Holdaway (1985), is as follows:

$$DAF = E1*D + E2*D2 + E3$$

where:

DAF = diameter adjustment factor
D = diameter breast height
E1 = 0.00
E2 = -0.00017
E3 = 0.018

LS-TWIGS calculates diameter growth on a yearly basis, FVS calculates diameter growth on a 10 year interval. Therefore, the final diameter growth is multiplied by 10. Since diameter, basal area, average stand diameter, and crown ratio change on a yearly basis, diameter growth predicted by the LS-TWIGS variant of FVS is slightly different than LS-TWIGS.

FVS- LS Mortality:

The individual-tree mortality model is that which is discussed in Buchman et al. (1983) with additional species coefficients in Buchman (1983) and Buchman and Lentz (1984). The equation is as follows:

$$M = 1 - B0 - [1/(1 + \exp(n))]$$

where:

$$n = B1 + B2*(DGR/10)B3 + B4*(D-1)B5 * \exp[-B6*(D-1)]$$

and:

M = tree's annual probability of mortality
D = diameter at breast height
DGR = diameter growth
B0 = 0.9997
B1 = 1.9953
B2 = 57.97
B3 = 1.012
B4 = 0.26480
B5 = 1.6260
B6 = 0.1273

LS-TWIGS calculates mortality on a yearly basis, FVS calculates it every 10 years. Hence, an interest rate approach is used for mortality periods longer than one year. Since D and DGR change annually in LS-TWIGS, the LS-TWIGS variant of FVS results in slightly different mortality estimates.

Appendix C. RPAL basal area growth equations for stands greater than 25 years in age.

RPAL Basal Area Growth (25 < age):

For stands over 25 years old at dbh, the average annual basal area growth in square feet per acre is estimated from Buckman (1962):

where:

$$ABAG = 1.6889 + 0.041066*B - 0.00016303*B^2 - 0.0769*A + .0002274*A^2 + 0.064415*S$$

All basal area increments are constrained by the maximum annual diameter growth defined from Lundgren (1981):

$$DMAX = 0.007 * S * e^{(-0.01*BHA)}$$

where:

DMAX = maximum annual diameter growth (inches)
S = site index
BHA = tree age at Dbh

This maximum constraint is applied to a tree of mean stand diameter.

RPAL Mortality:

If a stand has over 40 sq.ft./acre of basal area, mortality is estimated in terms of basal area per acre:

$$BAM = B*exp(-20*S/B)$$

where:

BAM = annual basal area mortality (ft^2 per acre)
S = site index at age 50
B = basal area in sq.ft. per acre

It is assumed that the mean dbh of the dead trees will be equal to the stand's quadratic mean dbh minus one standard deviation (Lundgren 1981), or MDBH - SD, where:

$$SD = 0.37628 * MDBH * exp(-0.093346*MDBH)$$

Mortality is then estimated in number of trees per acre (MNOT):

$$MNOT = BAM/((MDBH - SD)^2 * \pi/576)$$

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