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UNDERSTANDING ALGEBRA AND FUNCTIONS: AN EXPLORATION OF THE LEARNING EXPERIENCES OF PREVIOUSLY UNSUCCESSFUL STUDENTS IN CORE-PLUS COURSE 1A

By

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ABSTRACT

UNDERSTANDING ALGEBRA AND FUNCTIONS:
AN EXPLORATION OF THE LEARNING EXPERIENCES
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A major focus of mathematics reform at the secondary level has been directed at the teaching and learning of algebra, driven by the belief that mathematics instruction—and algebra instruction, in particular—must effectively reach all students. It is widely agreed upon in mathematics reform literature that algebra is a gatekeeping course whereby students who do not succeed are denied access to equal participation in a technologically oriented society; moreover, algebra is accessible to all students only when it can be understood conceptually, explored within contexts that are meaningful to students.

In this study, the author explores the learning experiences—focusing on the algebra and functions content strand—of six students, each of whom had previously experienced non-success in school mathematics, in the first part of the first course of Core-Plus, a reform high-school mathematics curriculum. Through an analysis of classroom observations and in-depth interviews conducted at three different moments in time of the course, the author investigates the nature of the students' previous non-successes, their experiences of the Core-Plus curriculum, and their understandings of algebra and functions as presented in Core-Plus Course 1A. After developing individual portraits of each student, the author reads across all six participants for common features

and patterns of their school-mathematics histories, their experiences of the Core-Plus curriculum, and their understandings of algebra and functions.

The findings coalesce around issues attendant to representations of mathematical phenomena, the role of contextualized situations in providing access to abstract mathematics, and a construct called "mathematics community membership" that captures a number of qualities that the author claims to be closely associated with navigating a reform curriculum successfully. The implications call for both high school mathematics teachers and mathematics teacher educators to devote serious attention to students' alienation from the subject and to explicitly socialize them into expressly mathematical modes of thinking, speaking, and behaving.

To Kirsten
who challenged me, as a teacher,
to make Algebra a meaningful learning experience
for previously unsuccessful students

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To me, this is the most important part of my dissertation. I have waited to compose it until the very end; it is the finishing touch and the culminating piece of this work. I recognize that this project is not the work of just the one person whose name appears in the byline; it is the collaboration of many people to whom I am deeply indebted. Without them, this volume would never have come together. More importantly, without them, I would not have the benefit of knowing and understanding all that I have learned throughout this course of study and this research project.

To the young women who were the focus of my study, I am very grateful. I learned a great deal through my interactions with and observations of them. Through their participation, they have contributed a great deal to the betterment of mathematics education and, because of that, I credit each one of them with being highly successful!

To the math teachers at the high school where I conducted my study, I am also very grateful. They welcomed me as one of their own, and I learned a great deal from them as well, visiting their classes and listening to their lunchtime conversations. Their openness and collegial spirit allowed me to see that, through their collective commitment to effectively reaching all students through their mathematics instruction, they too have contributed a great deal to the betterment of mathematics education.

To (Mr) Bill Rosenthal, my dissertation director, I don't have the words to express the depth of my appreciation. We began and ended our time at Michigan State University simultaneously (1992-1998), yet our greatest work of collaboration had not yet begun. If it weren't for Bill, I am convinced this project would never have commenced; and I am absolutely certain that I would not have finished what was started

so many years earlier were it not for him. He believed in me more than I did myself. Bill encouraged, supported, empathized with, listened to, understood, challenged, and pushed me all along the way, from the time I enrolled in his *Mathematical Ways of Knowing* course in 1994 to the final editing of this dissertation. I can only hope to be able to contribute as much to my own students' pedagogical betterment as Bill has contributed to mine.

To the members of my dissertation committee—Sandy Wilcox, Susan Melnick, Jack Smith, and Dan Chazan—I also extend my appreciation. In addition to their valuable contributions as members of my committee, they have contributed a great deal to my growth as a learner, a teacher, and a scholar. I had the privilege of learning most of what I know about qualitative research methods from Sandy. My first foray into teaching elementary math methods was in collaboration with her; and I finished my graduate assistant career at MSU working on a research project that Sandy was directing. I appreciate Sandy most for her mentorship and for the serious approach she takes when considering the fledgling research of her students. I had the privilege of getting to know Susan during my first year at MSU in the professional seminar course she co-taught. I appreciate Susan most of all for her frankness, her tough-mindedness, and her sense of humor. Susan has "kept me honest" when it comes to addressing issues of diverse learners and equity in education. While I did not have the privilege of taking a course from either Jack or Dan, I have learned a tremendous amount from them about mathematics and mathematics education. I have benefited, not only from the research about which they have written, but most of all from the many hours they have spent sharing their research and contributing ideas to the research of others in Math Education

Seminars and in the Math Learning Research Group. I greatly appreciate Jack and Dan for the encouragement and support they give to graduate students who are still trying to find their way doing math education research.

To Taylor University where I am currently employed, I am greatly beholden. Through their Doctoral Studies Assistance program, I was granted a year's leave with funding in order to complete my degree. I was provided with a research office on campus where I was able to work without distraction while I collected and analyzed my data. My colleagues at Taylor have been tremendously encouraging and supportive, especially during this second year of work on my dissertation as I have split my time between my teaching responsibilities and writing. My good friend and mentor, Pam Medows deserves special credit for standing in the gap for me—doing things such as advising my students and grading papers—so that I could devote as much time as possible to finishing my dissertation. I have been tremendously blessed to be a part of the Taylor community whose support has been a large contributing factor to my completing this project.

To my mentors and friends at Michigan State University, I also owe a debt of gratitude. So many contributed to my education there, in ways too numerous to mention, yet I would be remiss if I did not acknowledge Glenda Lappan, Deborah Ball, Maggie Lampert, and Deb Smith for the ways in which they helped me grow as a learner, as a teacher, and as a scholar. I also wish to acknowledge fellow graduate students Kara Suzuka, Sarah Theule Lubienski, Ruth Heaton, and Duane Castanier for the friendship they shared with me and for the significant ways in which they, too, helped me grow as a learner, a teacher, and a scholar.

To my parents who have always loved me and supported me, I am so very indebted. They have always wanted the best for me and have done whatever they could to provide that for me. From the time they saw me off to kindergarten to the completion of this "paper," they have encouraged me to fulfill my potential through education. I could not have reached this point without them.

To my husband Phil, I am indebted most of all. Mere words cannot adequately express my love and appreciation for all that he has done to help me finish my degree. Phil has not known me apart from having a dissertation to write, and he has patiently endured the process without complaint as I have devoted much more time and energy to the completion of my dissertation than I have to him. He has done far more than his share of laundry, cooking, cleaning, doing dishes, running errands, and getting us moved into our "new" house during Spring Break of this year! And throughout this long journey, Phil has not wavered in providing unselfish support, offering encouragement, sharing laughter, lending a listening ear, and showing unconditional love. God has blessed me in indescribable ways by bringing Phil into my life. Without him, I never would have become "Dr. Macomber."

TABLE OF CONTENTS

LIST OF TABLES

LIST OF FIGURES

CHAPTER 1

INTRODUCTION

My Teaching Experiences

In Fall 1989, just months after the publication of NCTM's original *Standards* document, I enrolled in a secondary mathematics methods course at the University of Kansas under the instruction of a soon-to-be member of the NCTM Board of Directors, Dr. Beverly Nichols. Having caught the vision of math reform in my methods class, I began my first job as a high school math teacher determined to make mathematics learning a meaningful experience for my students. I taught in a rural school district in northeast Kansas. The average graduating class was 50; the average class size was no more than 20. I taught Pre-Algebra, Algebra Two, and Geometry the first year, Algebra One, Algebra Two, and Geometry the second year. The algebra texts I used were the classic "Dolciani" textbooks (Dolciani, brown, & Cole, 1984; Dolciani, Sorgenfrey, Brown, & Kane, 1986).

Not having had a student-teaching experience in mathematics,[1] my first year as a math teacher was eye opening. My algebra classes posed the greatest challenges. My Pre-Algebra students (placed in this course, rather than Algebra One, because they were thought to need remediation of basic math skills in order to prepare them to succeed in Algebra One) were sick and tired of doing the same things year after year (Flanders, 1987). They complained that they had done "all that stuff" in the Pre-Algebra book before. It was clear they felt stigmatized by having been placed in Pre-Algebra in the

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[1] Mathematics is an endorsement area that I added to my original teaching certificate. I student taught in 1987 when I first obtained my teaching certificate in French, but I was not required to do another student teaching experience when adding the endorsement in math.

first place, and many lacked the belief that they were capable of learning math at all. Many did not aspire to go to college, nor did their parents encourage them to do so. Math was a necessary evil they had to endure to graduate high school. Many times they asked, "Why do we need to know this?" And I struggled to help them, spending hours upon hours trying to find real-life applications for the material they were learning, trying to create projects or unique activities that would allow them to use their math skills in a context that was interesting and perhaps meaningful to them. But this work was difficult, time-consuming, and not often successful.

Then there were my Algebra Two classes. They really struggled, too! It was as if all of their algebra learning had completely evaporated during the previous year when they had taken Geometry. We limped along through the text, until I finally decided—in January—that most of students' difficulties stemmed from not being able to remember the basic algorithms for algebraic/symbolic manipulation. Without these, the further we progressed through the text (topics), the more difficult it became for them to be successful. After spending a few weeks on rudimentary review of the rules for solving algebraic equations, the highest-achieving students seemed to have been helped and enjoyed more success than before. Yet the majority still struggled with, actually resisted, the things we continued to work on, lamenting that they could memorize the steps well enough, but *Why* did they do them? What sense did it make? How could they *understand* what they were doing, instead of just blindly following the algorithms I made them write down? Again, I struggled to help them, spending hours upon hours trying to find real-life applications (from other textbooks) for the rules and procedures I was teaching, trying to create projects or unique activities that might help them make connections across mathematics topics. But this was difficult, time-consuming, and not often successful. It soon became clear to me that, without a reform-minded curriculum, teaching in reform-consistent ways proved extremely difficult indeed.

Graduate School

After two years of teaching high school in Kansas, I began my doctoral program at Michigan State. My first assistantship involved working for Glenda Lappan, Bill Fitzgerald, and Betty Phillips on the Connected Mathematics Project (CMP) middle school curriculum. As I saw it, the major goal of this curriculum was to engage students in investigations of math concepts in contexts that were meaningful, as opposed to the "naked numbers" problems that were so common in traditional math curricula. I was especially intrigued to make the acquaintance of Jim Fey, one of the five collaborating authors of the CMP curriculum. Fey's area of expertise was algebra and functions. He was in charge of finding ways to introduce algebra and functions concepts via meaningful contexts. At times he admitted he was truly challenged. He was determined nonetheless.

I also learned that Fey was a member of a high school mathematics curriculum-writing team, the Core-Plus Mathematics Project (CPMP).[2] The name Core-Plus, as I understood it, was an outgrowth of the *Standards* concept of a core curriculum for high school mathematics, which advocated that *all* students learn math in ways that connected many topics each year (algebra, geometry, data analysis, probability, discrete math, trigonometry, and so on) and delved more deeply into these interrelated topics for each of three years of high school (Meiring et al., 1992; NCTM, 1989).

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[2]Not coincidentally, the Connected Mathematics Project and the Core-Plus Mathematics Project coordinated their efforts so that the Core-Plus (high school) curriculum would essentially pick up where the Connected Math (middle school) curriculum leaves off.

Through what I knew of both these curricula, I held high hopes that therein might lie some of the answers that I had sought as a novice high school math teacher trying to help my students experience math in ways that didn't simply leave them bored and unengaged doing "naked number" problems for which numerous algorithms—understood procedurally (at best)—must be memorized and performed in order to be successful.

Another significant part of my graduate studies at Michigan State was my work on the Mathematics and Teaching through Hypermedia (M.A.T.H.) Project with Deborah Ball and Maggie Lampert from 1993-1995. My work involved videotaping classes and workshops taught by Ball and Lampert, organizing and preparing data in electronic format for entry into an interactive database, as well as using their hypermedia materials (Lampert & Ball, 1998) to teach several elementary math methods courses.

From these experiences, I came to think in a whole new way about what it means for students to understand mathematics. I saw how fragile students' seemingly "correct" understandings can be. I witnessed firsthand the fact that, although students may give a "right" answer, this does not necessarily indicate that their understandings are correct (Erlwanger, 1975); I also came to appreciate the fact that, although students may give a "wrong" answer, it does not necessarily indicate a complete lack of understanding or mathematical insight (Ball, 1997; Duckworth, 1996; Heaton, 2000; Rosenthal, 1999). Indeed, I grew more and more intrigued and interested in those students who did not always have the right answers, those who seemed to struggle with their mathematical understandings, those who were really trying to make sense of the concepts for themselves (rather than just memorizing a set of steps to follow blindly). It was these students whose mathematical insights had to be uncovered by eliciting their thinking about problem-solving tasks so as to help them pursue the promising leads that, often, they themselves didn't even know they had stumbled onto (Duckworth, 1996; Labinowicz, 1987; NCTM, 1991).

As I made my way through my doctoral program at Michigan State, I knew that I wanted to make the Core-Plus curriculum the focus of my dissertation because, as a high school mathematics curriculum, it was one that I knew truly made an effort to facilitate students' learning of mathematics in ways that also deepened their relational, rather than just their instrumental, understandings of mathematics (Skemp, 1978). However, the curriculum was very new (first published for nationwide release in 1998), and I had barely had the opportunity to look it over, let alone see how it played out in the classroom. In the summer of 1997, I was able to attend a week-long conference for Michigan math education leaders who were working on the effort to help implement the Core-Plus curriculum throughout the state, which gave me my first actual glimpse at some of the actual investigations and lessons. At the time, I figured that, for my dissertation, I would undertake some kind of comparative (theoretical) curricular analysis. Little did I know that just a year later I would have the opportunity to actually try my own hand at teaching the curriculum.

Back to Teaching

During the 1998-99 school year, I found myself back in the high school math classroom at a laboratory school on the Ball State University campus. Because of the academic freedom of the laboratory-school environment, teachers were allowed to exercise the prerogative to supplement or replace the selected textbooks for their courses.

I knew that my Algebra One class was considered a remedial section—a collection of students who were placed in the course as ninth-graders (a year behind the majority of the students at the school) because each had struggled with math in their previous school years. They had been deemed not-yet-ready for algebra in eighth grade, or they had not yet passed Algebra One in previous attempts. Indeed, one third of the students in the class would be taking it for (at least) the second time. Eleven of 18 students in the class were at risk of failing, or seriously struggling in, the course because of at least one (and in most cases, several) factors such as absenteeism, repeated suspensions from school, previous academic failure, learning disabilities, and so on.

The standard textbook for the course (Schultz, Hollowell, & Ellis, 1997), which was on the state-approved list, struck me as an updated version (i.e., lots of color pictures related to teen life in the late 1990s) of the text I had used nearly a decade earlier in my Pre-Algebra classes in Kansas (Willcutt, Fraze, & Gardella, 1984). I wondered: If these students had not previously been successful with this kind of approach to learning mathematics, then why would repeating the same curriculum be of help? So I decided to use the Core-Plus curriculum in this class. The second and third units of the first-year course (Coxford et al., 1998b) are primarily focused on algebra concepts; and the first unit lays the groundwork for students to be able to do the data analysis (using graphing calculators to interpret various representations of data in tabular and graphical forms) that was essential to Units 2 and 3. So we set out to work on the first three units.

Although Course 1 in its entirety includes eight units of study covering algebra and functions, statistics and probability, geometry and trigonometry, as well as discrete math, my students and I stuck to the algebra-focused units—not only because the students were enrolled in an algebra course, but also because, in keeping a pace that afforded students time to master the concepts, we never got farther than Unit 3 by the end of the school year. And, although it seemed unsatisfactory that the students had covered just three of the four units of Part A of Core-Plus Course 1 for the school year, I witnessed some truly satisfying results. I observed these "remedial" students' success in making sense of algebraic concepts (Skemp's relational understanding), discussing them with a mathematical sophistication that I had not seen even from more advanced students in traditionally taught second-year algebra courses.[3]

My Consequent Research Interests

From this experience grew my research interests. Initially, this study was designed to help answer the question, *What kinds of understandings of algebra and functions do previously unsuccessful students possess after studying Units 1, 2, and 3 of Core-Plus Course 1?* Understandings of algebra and functions was the figure being studied; previously unsuccessful students was the ground:

---

[3]The entirety of Core-Plus Course 1 contains eight units—four units in Part A and four units in Part B. While three units of Part A amount to just 40% of the entire Course 1 curriculum, the curriculum developers have acknowledged that there will be some students who need two school years to complete both parts of the Course 1 curriculum (Coxford et al., 1998d, p. 4). Assuming that this pace was appropriate for my Algebra One students, the three units of Part A that they completed amount to 80% of the Course 1A curriculum, which we supplemented with six weeks of traditional algebra skills work from the school's pre-selected text for Algebra One (Schultz et al., 1997).

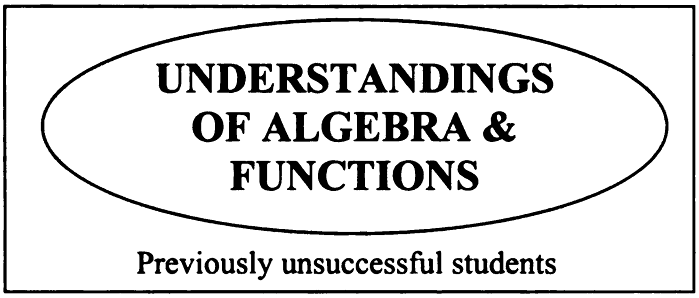

Figure 1.1
First iteration of research focus

As described in Chapter 4, my data collection began with a survey given to a large group of students as a means of selecting a small number of study participants. The survey had been carefully designed to help discern which students were previously unsuccessful in school mathematics (according to criteria described in Chapter 3) based on their entering attitudes toward the subject and its study. As soon as interviewing commenced with the selected study participants, I began to inquire about their previous non-successes in math. Indeed, every participant had something in her personal math history that was qualitatively unsuccessful. The variety among these previous non-successes was quite interesting and insightful, and my research focus shifted slightly. I became as interested in the participants' previous non-successes in school mathematics as I was in their understandings of algebra and functions:

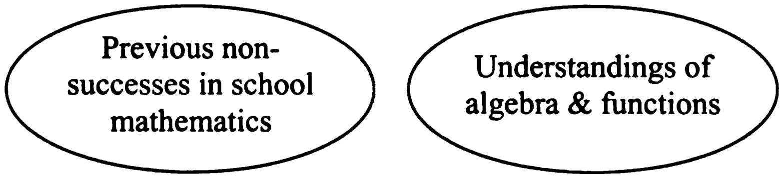

Figure 1.2
Second iteration of research focus

As a part of my interview protocol, I had included questions about students' learning experiences in Core-Plus Course 1. These questions were meant to help detect whether participants' entering attitudes toward school mathematics were changing over time. Each time I asked these questions of a participant, I took more and more interest in her learning experiences in Core-Plus Course 1. As I collected these data, the observations I was making about students' learning experiences seemed, in some ways, to be the bridge that I used to connect students' previous non-successes in school mathematics with what students were coming to understand about algebra and functions during their studies in Core-Plus Course 1. In other words, *students' experiences of the Core-Plus Course 1 curriculum* became the vehicle through which I was beginning to make sense of their understandings of algebra and functions (Erickson & Shultz, 1992). Yet, at that time, students' learning experiences played a secondary role to the two main components of my inquiry.

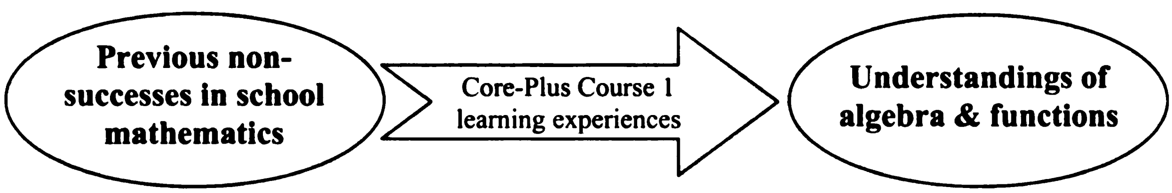

Figure 1.3
Third iteration of research focus

During the final phase of my research, in the course of trying to make sense of all my data, I came to fully appreciate the importance of students' learning experiences in Core-Plus Course 1. It has shown itself to be a third focal point, meriting equal attention with the other two:

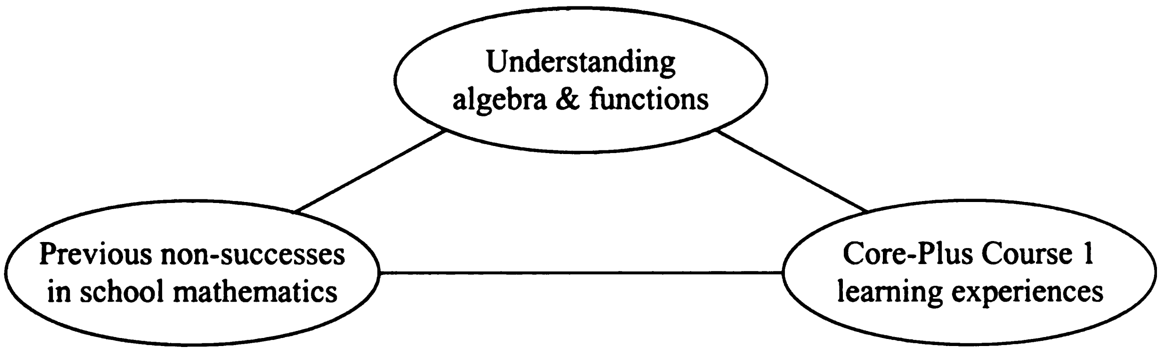

Figure 1.4
Final iteration of research focus

The three prongs of this inquiry are interrelated. Each informs the other two and cannot be fully explained or understood without them.

In this final iteration of my research focus, there has also been a subtle but important change in the framing of the exploration of students' understandings of algebra and functions. In the diagram above, note that the focus formerly worded "Understandings of algebra and functions," is now worded "Understanding algebra and functions." "Understandings" as a *noun* has been replaced by "understanding" as a *verb*. This is intentional. It reflects the ongoing nature of coming-to-understand things. Understandings, as any constructivist teacher of mathematics knows, are fragile and ever changing—especially in the beginning stages of learning new ideas, skills, and concepts. Understanding is a learning *process*. This study is but one small snapshot of that process.

My Questions

Stated as questions, the three prongs of inquiry that compose this study are:

- What is the nature of these students' previous non-successes in school mathematics?

- What are these students' experiences of the Core-Plus curriculum?
- What do these students understand about algebra and functions (at three different moments in time)?

CHAPTER 2

OVERVIEW OF MATHEMATICS-EDUCATION AND ALGEBRA REFORM

Everybody counts

The last two decades of the 20th Century have proven to be a significant period of reform in school mathematics in the United States. This movement was driven by a number of factors including revolutionary changes in the work world brought about by the advent of technology (Johnston & Packers, 1987; Secretary's Commission on Achieving Necessary Skills, 1990) and reports about the miserable state of mathematical knowledge of schoolchildren in the U.S. compared to other nations worldwide (National Commission for Excellence in Education, 1983). Led by the National Council for the Teachers of Mathematics (NCTM), math educators across the country began to reconsider what mathematics is important for schoolchildren to learn (NCTM, 1980) as well as what teaching approaches are most effective in helping students achieve both computational and conceptual understandings of the mathematics they study in school.

One of the fundamental issues underlying this reform movement has been the concern for those students who, for a variety of reasons, have been historically disadvantaged in or, worse yet, filtered out of mathematics as a field of study, as a career option, as a subject in school in which they can succeed—females, students of color, students from lower socioeconomic backgrounds, special-needs students. "Opportunity for all" was a defining goal of the original *Standards* document, which declared,

> The social injustices of past schooling practices can no longer be tolerated. Current statistics indicate that those who study advanced mathematics are most often white males. Women and most minorities study less mathematics and are seriously underrepresented in careers using science and technology. Creating a

> just society in which women and various ethnic groups enjoy equal opportunities and equitable treatment is no longer an issue. Mathematics has become a critical filter for employment and full participation in our society. We cannot afford to have the majority of our population mathematically illiterate: Equity has become an economic filter. (NCTM, 1989, p. 4)

At the same time that NCTM published its original *Standards* document, the National Research Council released its landmark publication *Everybody Counts: A Report to the Nation on the Future of Mathematics Education* (1989). The key theme for which this document is best known is its declaration, "Mathematics must become a pump rather than a filter in the pipeline of American education" (p. 7). That is to say that, rather than weeding out and thus preventing all but the most gifted or advantaged students from pursuing mathematics as a field of study and as a career choice, the way in which mathematics is taught and learned in our society must become such that it enables and empowers all students to succeed as mathematically literate participants in society (NCTM, 1989; NRC, 1989).

While *Everybody Counts* served as an outline of the philosophical underpinnings of the burgeoning mathematics reform movement, the first NCTM *Standards* document provided the practical recommendations for changes needed in the content and emphasis of school mathematics. These recommendations called for decreased the attention paid to complex and/or tedious paper-and-pencil computations; rote practice and memorization of rules, formulas, and facts without understanding; isolated treatment of arithmetic procedures and mathematical topics; and developing skills out of context. Recommendations were also included to increase emphasis on developing thinking, reasoning, and problem-solving strategies; understanding of the meanings of numbers, operations, and the interconnectedness of math topics; recognizing patterns and relationships; being able to communicate mathematically and work collaboratively with peers; and using a variety of methods, including technology and mathematical models, to solve problems (1989, pp. 20-21, 70-73).

The second of NCTM's *Standards* documents (1991)[1] summarized the changes in instructional practice needed to bring about these results:

> Woven into the fabric of the *Professional Standards for Teaching Mathematics* are five major shifts in the environment of mathematics classrooms that are needed to move from current practice to mathematics teaching for the empowerment of students. We need to shift —
>
> - toward classrooms as mathematical communities—away from classrooms as simply a collection of individuals;
> - toward logic and mathematical evidence as verification—away from the teacher as sole authority for right answers;
> - toward mathematical reasoning—away from merely memorizing procedures;
> - toward conjecturing, inventing, and problem solving—away from an emphasis on mechanistic answer-finding;
> - toward connecting mathematics, its ideas, and its applications—away from treating mathematics as a body of isolated concepts and procedures. (p. 3)

Algebra for all

A major focus of reform at the secondary level has been directed at the teaching and learning of algebra. For decades, the traditional ninth-grade algebra course has been the critical filter for high school mathematics students (Chazan, 1996; Moses, 1994; Moses & Cobb, 2001; Usiskin, 1993, 1995). The prototypical algebra course—in which the skills practice focuses heavily on symbolic manipulation and the use of algorithms to solve problems out of context, equations and functions must be graphed by hand (using only paper and pencil), and classroom instruction is exclusively characterized by an observational-learning model whereby the teacher lectures and demonstrates problem-

[1]Two additional volumes (NCTM, 1995, 2000) round out the set of *Standards* documents.

solving routines that individual students imitate—is a realistic description of the high school mathematics experience most adults in the United States have known.

The Algebra Working Group, convened by the NCTM in the mid-1990s to rethink the algebra curriculum, summarized the need for algebra reform:

> The algebra experience for all students in grades K-12 must be greatly improved. All students need the tools and mathematical thinking fostered by the study of algebra to function effectively in a dynamic and challenging world. Currently many students are being disenfranchised by lack of access to algebra as well as an impoverished curriculum. Algebra needs to change. (Burrill et al., 1994, p. 3)

Among the primary tenets of its work, the Algebra Working Group asserts that "[t]echnology changes the way algebra should be taught, learned, and used" and that "[t]he world presents problems and challenges that demand a different kind of algebraic knowledge" (p. 4). The recommendations made by the Algebra Working Group built directly upon the reform vision of algebra put forth by the NCTM in 1989 (and further refined in 2000):

> Instructional programs from prekindergarten through grade 12 should enable all students to —
> - Understand patterns, relations, and functions
> - Represent and analyze mathematical situations and structures using algebraic symbols
> - Use mathematical models to represent and understand quantitative relationships
> - Analyze change in various contexts. (NCTM, 2000, p. 296)

Specific recommendations for changes in the content and emphasis of the study of algebra and functions at the high school level appeared in the original NCTM *Standards* document:

Topics to receive increased attention	Topics to receive decreased attention
ALGEBRA • The use of real-world problems to motivate and apply theory • The use of computer utilities to develop conceptual understanding • Computer-based methods such as successive approximations and graphing utilities for solving equations and inequalities • The structure of number systems • Matrices and their applications	ALGEBRA • Word problems by type, such as coin, digit, and work • The simplification of radical expressions • The use of factoring to solve equations and to simplify rational expressions • Operations with rational expressions • Paper-and-pencil graphing of equations by point plotting • Logarithm calculations using tables and interpolation • The solution of systems of equations using determinants • Conic sections
FUNCTIONS • Integration across topics at all grade levels • The connections among a problem situation, its model as a function in symbolic form, and the graph of that function • Function equations expressed in standardized form as checks on the reasonableness of graphs produced by graphing utilities • Functions that are considered as models of real-world problems	FUNCTIONS • Paper-and-pencil evaluation • The graphing of functions by hand using tables of values • Formulas given as models of real-world problems • The expression of function equations in standardized form in order to graph them • Treatment as a separate course

(1989, pp. 126-127)

Reform-based secondary mathematics curriculum and the Core-Plus Mathematics Project

In addition to the recommended curricular changes in content and emphasis, the 1989 Standards document also introduced a new paradigm for the structure of high school mathematics: the core curriculum (NCTM, 1989, pp. 123-136). The "core topics" include not only the traditional topics of algebra, geometry, trigonometry, and functions, but also statistics, probability, and discrete mathematics. It is emphasized that "the core curriculum is intended to provide a common body of mathematics ideas accessible to all students" (p. 123). Moreover, the idea of a core curriculum was proposed with the "expectation that mathematical ideas will grow and deepen as students progress through the curriculum" (p. 124).

While the 1989 *Standards* document offers guidelines for the content of a core curriculum in the form of fourteen curricular standards for Grades 9-12 (pp. 137-186), it does not provide many practical suggestions. However, in 1992, as part of its Addenda Series for Grades 9-12 (Meiring, Rubenstein, Schultz, de Lange, & Chambers, 1992), NCTM did provide "several possible curriculum models for organizing the mathematics content recommended in the *Curriculum and Evaluation Standards*" (p. vi). Still, only nine sample lessons for each of two curriculum models are provided in the *Core Curriculum* Addenda booklet. Thus, it has been left to curriculum developers to design instructional materials and texts that exemplify the core curriculum envisioned by the NCTM:

> In 1990, the National Science Foundation (NSF) responded to the call for new materials by funding curriculum development projects at the elementary school, middle school, and secondary school levels. At the secondary level, five projects were funded....All five secondary school mathematics projects are currently available as multiyear textbook series. (Martin et al., 2001, pp. 540-541)

The Core-Plus Mathematics Project (CPMP), which developed the textbook series *Contemporary Mathematics in Context*, is one of the five secondary school projects that the NSF funded. The Core-Plus curriculum is a four-year integrated mathematics program (Coxford et al., 1998d, p. 1). The first three courses are intended to be studied by all students, the fourth course by college-intending students. Each of the first three courses comprises seven units of study followed by a "culminating capstone experience" that

> ...is a thematic, two-week project-oriented activity that enables students to pull

> together and apply the important modeling concepts and methods developed in the entire course. (Coxford et al., 1998d, p. 4)

Throughout each of the seven units of Courses 1-3, four content strands are interwoven, which were designed to address the curriculum standards for Grades 9-12 (NCTM, 1989). Some or all of the content strands—algebra and functions, geometry and trigonometry, statistics and probability, and discrete math—may be incorporated into any given unit with various degrees of emphasis (Coxford et al., 1998d).

Also in keeping with the recommendations of the NCTM *Standards* (1989), the instructional approach of the Core-Plus program differs from the traditional model that has been known to characterize high school math classes for decades prior to the reforms that have taken place in the 1980s and 1990s:

- Students focus on real-life problem situations from which new mathematical skills emerge—in contrast with the traditional skills focus on symbolic manipulation and algorithms to solve problems that lack any context;
- Students employ graphing calculators to examine multiple representations of data and functions—in contrast with the traditional approach which prioritizes solving and graphing equations using only paper and pencil;
- Students tackle mathematical investigations in small groups followed by a teacher-guided whole-group discussion of the mathematical underpinnings—in contrast with the traditional approach that emphasizes an observational-learning instructional model where the teacher lectures and demonstrates algorithmic problem-solving routines that individuals imitate.

The "math wars"

Reformed math curricula have not been embraced with open arms by all, however. The Core-Plus developers themselves are well aware of the backlash:

> The spirited debates about the reform of school and undergraduate mathematics have led some proponents and opponents of change to indulge in such angry rhetoric that the controversy has come to be referred to as the "math wars." (Schoen, Fey, Hirsch, & Coxford, 1999, p. 445)

These debates date as far back as 1995, when a number of concerned parents organized to oppose the math curriculum being used in California schools. The middle school math curriculum at issue was adopted by the Palo Alto school district because it conformed to the state's math frameworks, adopted in 1992, which were based upon the 1989 NCTM *Standards* recommendations for reforming school math instruction. Shortly thereafter, a rival group of parents who favored the new math curriculum rose up to show their support (Sommerfeld, 1996; West, 1995a, 1995b). Not quite a year later, "similar battles [were] being fought in other states, from Iowa to Texas" (Sommerfeld, 1996, p. 1).

> [J]ust as the new curricula, teaching methods, and assessment strategies are beginning to be tested in schools and universities across the country and are beginning to show promise of reaching the objectives of reform, critics have challenged the content goals, the pedagogical principles, and the assessment practices that are at the heart of the reform agenda. What seemed to be an overwhelming national consensus on directions for change in mathematics education is now facing passionate resistance from some dissenting mathematicians, teachers, and other citizens. Wide dissemination of the criticisms—through reports in the media, through Internet mailings, and through debates in the meetings and journals of mathematics professional societies—has shaken public confidence in the reform process. (Schoen et al., 1999, pp. 444-445)

The critics—including everyone from parents and teachers to national politicians—charge that the reform curricula have done away with the learning of "basic skills."[2] They also oppose the student-centered nature of the reform approach, and deplore the deviation from standard-looking textbook fare of many reform curricula (Loveless, 1997).

The battle rages on, and opponents gained new fervor when the federal Department of Education released a list in late 1999 naming ten reform curricula as "exemplary" or "promising" (Viadero, 1999a, 1999b). Even national legislators were drawn into the dispute after "nearly 200 mathematicians, physicists, and other scholars took out a full-page advertisement in *The Washington Post* to air their opposition" to the list of endorsed math curricula (Klein, 2000; Viadero, 2000).

The Core-Plus curriculum has not been exempt from the "math wars" controversy. For instance, in one of the schools that piloted Core-Plus, there were widely contrasting opinions as to whether high school students who studied the Core-Plus curriculum were adequately prepared for their college mathematics courses (Clayton, 2000a).

The criticism leveled at Core-Plus increased when it appeared on the Department of Education's list of "exemplary" curricula:

> The Department of Education's expert panel, assembled to find the best math programs in the United States, had a key problem, critics say. It relied heavily on studies of student achievement that were authored or co-authored by the directors of the programs themselves—or by people with close institutional or other ties to the program. (Clayton, 2000b, p. 15)

The chief complaint of critics was that results of field tests of the curricula—Core-Plus and Connected Math (Lappan, Fey, Fitzgerald, Friel, & Phillips, 1998), in particular—had not been subject to independent evaluation such as that required for

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[2]Indeed, the 2000 NCTM *Principles and Standards* was written in large part for the purpose of a détente in the "math wars" (Hoff, 2000).

publication in peer-reviewed journals (Clayton, 2000b).

Further research needed

Despite the fact that the lack of independent evaluation does not render invalid the studies that were done by program-affiliated researchers (Clayton, 2000b), the Core-Plus curriculum developers have acknowledged the need for further research on the effectiveness of reform curricula in general (Huntley, Rasmussen, Villarubi, Sangtong, & Fey, 2000). They continue to carry out their own studies on the effects of Core-Plus (Schoen, Cebulla, & Winsor, 2001; Schoen, Finn, Griffin, & Fi, 2001; Schoen, Hirsch, & Ziebarth, 1998; Schoen & Pritchett, 1998; Ziebarth, Slezak, Lagrange, & Kleinfelter, 1997) while inviting others to join them in the process of evaluating the comparative effects of a reformed high school math curriculum to that of a traditional curriculum:

> The broad purpose of this study was to test the vision of reform proposals in recent advisory documents like the NCTM *Standards* by comparing effects of a curriculum designed to implement the *Standards* to those of more conventional curricula. We collected and analyzed extensive data on student learning of algebra from both kinds of curricula and found considerable support for main themes of the reform. However, no single study will provide complete or conclusive evidence.
>
> Our study suggests some important patterns of consequences from curricular, instructional, and assessment practices in high school mathematics. Those patterns suggest areas in which both reform and traditional curricula need to be improved if they are to reach widely agreed-upon goals. But they also leave open the fundamental questions about what understanding and skill in algebra is most important for students to acquire from their school mathematics experience. Furthermore, they suggest some aspects of both reform and traditional curricula that need to be studied in more depth with methods other than those used in this study. (Huntley et al., 2000, pp. 359-360)

Occupying a unique spot in the existing literature

In my search to find other studies on the Core-Plus curriculum that have been shared with and reviewed by the public to date, I have found almost one-and-a-half dozen[3]—two studies published in peer-reviewed journals (Huntley et al., 2000; Wilson & Lloyd, 2000), eight Ph.D. dissertations (Hetherington, 2000; Kahan, 1999; Kett, 1997; Latterell, 2000; Lloyd, 1996; Truitt, 1998; Tyson, 1996; Walker, 1999), six papers presented at major conferences (Schoen, Cebulla, & Winsor, 2001; Schoen, Finn, et al., 2001; Schoen et al., 1998; Schoen & Pritchett, 1998; Van Zoest & Bohl, 2000; Ziebarth et al., 1997), and one master's thesis (Kohmetscher, 2001). Ten of the 17 study reports have at least one author who is directly affiliated with the CPMP, either as a member of the curriculum project team or as an instructor who helped pilot the curriculum.[4] Of the seven reports whose authors are not directly affiliated with the CPMP (Hetherington, 2000; Kahan, 1999; Kohmetscher, 2001; Lloyd, 1996; Tyson, 1996; Van Zoest & Bohl, 2000; Wilson & Lloyd, 2000), all but Hetherington and Kohmetscher could certainly be considered indirectly linked to the project.[5]

Of these 17 Core-Plus studies, 8 focus the lens of research on students, 8 focus on teachers, and 1 (the master's thesis) focuses on the parents' role in the Core-Plus

[3]This number represents publicly reviewed presentations of *distinct* studies. A "publicly reviewed" presentation includes an article published in a peer-reviewed journal, a paper presented at a major conference, or a dissertation/thesis. Multiple presentation modes of the same study are not included.
[4]As mentioned in the previous section, one of the most pointed criticisms that was leveled at the Core-Plus curriculum when it received its "exemplary" rating from the federal Department of Education in Fall 2000 was precisely this: that evaluations of the program's effectiveness were co-authored by those directly or indirectly affiliated with the CPMP (Clayton, 2000b).
[5]Melvin (Skip) Wilson was a longstanding colleague of the late Art Coxford at the University of Michigan, who served as one of the co-directors of the CPMP, and Wilson served as the major advisor for Lloyd's doctoral dissertation. Laura Van Zoest is a colleague of co-director Christian Hirsch at Western Michigan University and served as the major advisor for Rebecca Walker's doctoral dissertation (a CPMP team member mentioned above). Tyson completed her doctorate at the University of Iowa, where co-director Henry Schoen is on the faculty, and Kahan completed his doctorate at the University of Maryland where co-director Jim Fey is based.

curriculum. The eight studies that address students' issues (Huntley et al., 2000; Kahan, 1999; Latterell, 2000; Schoen, Cebulla, & Winsor, 2001; Schoen et al., 1998; Schoen & Pritchett, 1998; Tyson, 1996; Walker, 1999) are all quantitative in nature, each involving several hundred students as study subjects (taking tests, completing surveys, or providing college placement exam scores). Two of these student-focused studies examine students' preparation for, or transition to, college mathematics (Schoen, Cebulla, & Winsor, 2001; Walker, 1999). Walker (1999) also examines students' (epistemological) conceptions of mathematics; while Schoen and Pritchett (1998) examine students' (epistemological) perceptions of and (affective) attitudes towards mathematics. Tyson (1996) explores gender differences on performance tests. Schoen et al. examine students' quantitative reasoning, Latterell (2000) examines students' general problem-solving ability, Kahan (1999) examines students' competence with proof, and Huntley et al. (2000) are unique in their focus on students' "understanding, skill, and problem-solving ability in *algebra*" (p. 328, emphasis added).

Given this map of the current Core-Plus research literature, my research occupies a unique spot. It joins the ranks of the eight studies that focus on students' issues, yet I am the only student-focused researcher who is neither directly nor indirectly affiliated with the CPMP. Mine is the only study to focus exclusively on those students who have been previously unsuccessful in school mathematics.[6] Moreover, to my knowledge, I am the first to examine students' learning using qualitative methods. This has allowed me to explore issues that other researchers have undertaken—students' understandings of algebra and functions; and students' attitudes towards mathematics (which is part of the

[6] A complete explication of what is meant by "previously unsuccessful" students is included in Chapter 3.

exploration of students' previous non-successes in school mathematics)—yet paint them in different hues by focusing on just six students.

Perhaps most important, however, is the new perspective that I bring. My study is novel in its exploration of students' experiences of the curriculum.

> [V]irtually no research has been done that places student experience at the center of attention...If the student is visible at all in a research study he is usually viewed from the perspective of adult educators' interests and ways of seeing, that is, as failing, succeeding, motivated, mastering, unmotivated, responding, or having a misconception. Rarely is the perspective of the student herself explored. Classroom research typically does not ask what the student is up to, nor does it take a critical stance toward its own categories and assumptions so as to question whether "failing" or "mastering" or being "unmotivated" or "misconceiving" adequately captures what the student might be about in daily classroom encounters with curriculum. (Erickson & Shultz, 1992, pp. 467-468)

Holt et al. (2001) have taken a first step in presenting first-hand accounts of students' experiences with a reform high school mathematics curriculum. My approach is to present third-person reports of students' experiences with such a curriculum, incorporating the added information of students' past experiences in school mathematics, while focusing on a particular strand of content learning—algebra and functions. While I do not claim to present a study that examines both the social participation and subject matter task structures in depth (Erickson & Shultz, 1992), I believe I do take a first step toward offering the kind of investigation that Erickson and Shultz advocate:

> Presently it is not at all clear how social relations with the teacher and with fellow students become part of the math problem the student encounters, nor do we know how the student's "archaeology" of past experience with mathematics in both its social and intellectual aspects enters into the experience of the academic task structure of the moment at hand. (1992, p. 475)

CHAPTER 3

CONCEPTUAL FRAMEWORK

Before venturing any further to describe this study, it is important to offer a synopsis of my own ideological stance on the issues that play a major role in shaping the direction(s) of my research in order for the reader to better understand the terrain of mathematics education reform that I plan to explore.

Issues related to math and algebra reform

Perhaps the most significant of these issues is the belief that mathematics instruction (and algebra instruction, in particular) must effectively reach all students (Chazan, 1996; Hirsch & Coxford, 1997; NCTM, 1989, 2000; Secada, 1992; Usiskin, 1993, 1995). "All students" includes those typically and historically filtered out of their study of mathematics – those students who are tracked into a lower level of mathematics instruction in school, including (especially) girls, students of color, and, very often, students who do not enjoy the socioeconomic and educational advantages that are common to middle- and upper-class students (National Research Council, 1989).

To "effectively reach" all students, many believe and have argued that, mathematics instruction (and algebra instruction, in particular) must take on new forms, different from the traditional approach (Chazan, 2000; Moses & Cobb, 2001; NCTM, 1991). Chazan argues that non-traditional approaches to first-year algebra instruction are imperative because the traditional curriculum drives out lower-track students and deprives them of conceptual understandings. Moses makes a similar case: Algebra is a gatekeeping course whereby students, who do not succeed, are denied access to equal participation in a technologically oriented society (which he considers a civil right as significant as the right to vote); moreover, algebra is accessible to all students *only* when it can be understood conceptually, learned experientially, explored within contexts that are meaningful to students.

Another key to *effectively* reaching all students lies in the *type* of understanding that has been chosen as the goal for mathematics education (Skemp, 1978). Skemp makes a strong argument in favor of relational understanding[1] as a better goal for mathematics education than instrumental understanding. It is students' relational understanding (knowing what to do *and why*) that I am investigating in this study rather than their instrumental understanding (merely knowing what to do).

Issues pertaining to "previously unsuccessful" students

The students whom I have chosen to make the focus of my study are a subset of those whom I have dubbed "previously unsuccessful." This is a term I have selected to describe students who have been alienated in their earlier study of mathematics. A student's feelings of alienation might be linked to her/his gender, ethnicity, socioeconomic status, language background, special needs, learning style, work ethic, or a poor experience in a previous math class; the reason behind students' alienation was not of particular importance to me, however. Instead, it was those characteristics that unsuccessful students commonly exhibit that held the key to identifying those who would make the best candidates for study participants.

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[1]Buxton (1978) calls this "insightful" understanding, and Hiebert et al. in Hiebert (1986) refer to this as "conceptual" understanding.

House (1988) describes seven components of success for students in math and science that were particularly useful in helping me pinpoint identifying characteristics of (previously) unsuccessful students. Of seven characteristics discussed by House, I chose four—two of which were broken into subcategories—that I felt could be detected through students' self-reports on a written questionnaire. The other three characteristics are ones that I felt students would not be able to report themselves, but could only be discerned through my first-hand observation of and personal interviews with the students.

Characteristics to be determined from students' self-reports

SELF-CONFIDENCE: The student believes she can learn. She doesn't feel helpless, and she feels math is for her.

Public: This is the confidence a student possesses about her own skills when those skills can be viewed by others.

Personal: This is a student's private knowledge of how well she can do.

ENTHUSIASM: The student possesses a sense of ownership, curiosity, drive to learn. She possesses a respect for math and technology; a willingness to think creatively, to take risks, to subject her ideas to the scrutiny of others; and an excitement about learning math.

SIGNIFICANCE: The significance of math is not lost on the student. She sees applications of math in her everyday life. She views the world through math-colored glasses and possesses an awareness of the interrelatedness of math, science, and technology.

STICK-TO-IT-IVENESS:

Long-range: The student plans to take more math (and math-related) courses and those that are more challenging than what is required.

Short-range: The student possesses perseverance. She stays on task and isn't afraid to grapple with important ideas or hard problems. The student doesn't necessarily need answers quickly, doesn't give up easily, and is not easily distracted. She is willing to think and struggle, to trust her own judgments, and to risk being wrong.

Because House describes components of *success* for students in mathematics and science, the lack of the above-named characteristics were taken as indicators of non-success.

Characteristics to be determined from personal interviews and observations

Although House (1988) discusses another three characteristics of success (understanding, competence, and communication), I have compiled a set of my own indicators of success – characteristics to be determined from my personal interviews with and classroom observations of the study participants. These indicators were born early in my data analysis and were used throughout the data analysis process. While I do not take a formal inventory of these characteristics for each participant, the presence or absence of these indicators was helpful in discerning each participant's overall success throughout Core-Plus Course 1A.[2]

These indicators of success fall into two general categories, which are related to two of my research questions.

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[2]Here, and throughout the dissertation, I use "1A" when referring specifically to the first half of the curriculum only (as in "My study is about Course 1A"); otherwise, when referring in general to the first course in the Core-Plus curriculum (as in "Joy was enrolled in Course 1 as a ninth-grader"), I use just "1."

ATTITUDES TOWARDS MATHEMATICS: These are related to participants' learning experiences in Core-Plus Course 1. The component parts of this category that proved to be of particular interest include

Satisfaction: The student feels positively about her mathematics learning experience. She derives a certain amount of fulfillment and/or enjoyment from her math studies.

Change in attitude: The student's dispositions towards math, over time, become more like those expected of successful students.

MATHEMATICS ACHIEVEMENT: This is related to participants' understanding algebra and functions throughout Core-Plus Course 1A. The component parts of this category that proved to be of particular interest include

Text interpretation: The student understands the nature of a posed problem – what it is asking the problem-solver to find out and what is involved in solving it.

Explanation of thinking: The student is able to verbally articulate her mathematical thinking about a problem. She can explain her problem-solving process to another person in such a way that he can understand her strategies and insights.

Mathematical abstraction: The student is able to move beyond the problem context to comprehend the mathematical principle(s) involved in the problem.

Retention: The student retains (or is able to recall without too much difficulty) and is able to utilize mathematical concepts and skills learned previously.

Invention: The student makes successful use of alternate means of problem solution.

The characteristics described above are indicators of *success* for students in mathematics; therefore, the lack of these characteristics are taken as indicators of non-success.

Finally, it is important to note that, according to these criteria, "previously unsuccessful" does not necessarily refer to the grades a student has received in her mathematics classes, nor does it necessarily refer to a student's conceptual grasp of the subject matter. A student may receive good grades, yet still feel estranged from her math studies if she is bored with or uninterested in the subject, if she sees no relevance of her math studies to her everyday life, or if she has come to believe that mathematics is simply not for her. Likewise, a student may also possess some propensity for mathematics learning and demonstrate keen insight about mathematical concepts, yet she may nonetheless feel disaffected by her math studies if she believes her math skills are inferior to others', if she sees no practical reason for having to pass the required math classes in order to graduate high school, or if she sees no value in pursuing a difficult problem to its resolution.

A map of the algebra terrain

In order to answer the question of what these students understand about algebra and functions, it is also necessary to map out the terrain that comprises algebra, to stake out the territory that is the mathematical focus of this research. The map included here represents a high school math teacher's view of algebra, drawn from the canon of various 1980s and 1990s U.S. high school math curricula, from traditional to reformed, as well as the NCTM *Standards* documents (Coxford et al., 1998b; Dolciani et al., 1984; Dolciani et al., 1986; Larson, Kanold, & Stiff, 1997; Murdock, Kamischke, & Kamischke, 1998;

NCTM, 1989, 2000; Schultz et al., 1997).

Six fields of study within algebra are identified: multiple representations, functions, levels of abstraction, number relationships, means of solution, and number groups. No hierarchy is implied in the listing of these fields; indeed, they are inter-connected and are typically studied as such, with various elements from each field introduced and revisited at various times throughout the course of one's studies of high school algebra. It should be noted, however, that more traditional high school curricula tend to introduce elements, and even some fields, in a more linear and less integrated fashion than do the more reformed curricula.

As a means of constructing this map of the algebra terrain, I began by listing all the topics I had taught as a high school math teacher of Algebra One, Algebra Two, and Core-Plus Course 1A. I also considered what algebra topics are necessary for subsequent mathematics courses (trigonometry, precalculus, calculus, statistics) as well as for other "client courses" such as chemistry and physics. Finally, I also considered what sorts of mathematical skills were included on college entrance exams (SAT and ACT) and on college math placement tests. The resulting set of topics is the algebra map that you see here, after being organized and arranged into the fields of study listed above.

This algebra map is offered with specific purposes in mind and is, admittedly, limited in its utility. First of all, this map is presented as a means of broadly sketching out the territory of high school algebra studies in the United States over the last two decades. A macroscopic view is presented; as such, it is meant as an aid in orienting yourself, while exploring the question of what previously unsuccessful students understand about algebra and functions, in order to be able to locate the region in which a student's algebraic understandings lie.

Secondly, it is *not* to be inferred that a mapping of this type is meant to convey the entire picture of what algebra learning is all about. A listing of topics cannot address the sum of what students should know and be able to do from their study of high school algebra (Schoenfeld, 1994). Certainly, there are concepts and skills, mathematical ways of knowing and doing, as well as connections between topics, included in the study of high school algebra, that are not (and could not be) portrayed here.

Thirdly, this algebra map is not intended to represent a vision for what school algebra could or should be (a project that lies far beyond the scope of this dissertation!). This map is not intended to reflect any particular vision of algebra that has been put forth (e.g., Chazan, 1996, 2000; Edwards, Jr., 1990; Lacampagne, Blair, & Kaput, 1995; NCTM, 1989; Usiskin, 1988, 1995); nor does it represent my own vision for what school algebra could or should be. The map presented here is simply meant as a tool for analysis, a means to an end, and not as an end in itself.

Issues not addressed in this study

There are several issues—related to gender, social class, and immigrant students—that I have deliberately chosen not to address in this study. Because of who the study participants turned out to be, any one (or all) of these issues could have been addressed yet, in the interest of limiting the scope of my analysis, I have set these issues aside and have focused solely on those issues that are raised in my research questions.

First, all participants were girls, and there are significant concerns about girls and mathematics learning discussed in the reform literature (E.g., Clewell, Anderson &

Table 3.1 – Map of the Algebra Terrain

VARIOUS REPRESENTATIONS

- graph
- table of data
- rule (equation)
    - explicit
    - iterative/recursive

MEANS OF SOLUTION

- algorithms
    - solving single equations
    - systems of equations
        - substitution
        - linear combination
    - factoring
    - division of polynomials
        - synthetic
        - long division
    - completing the square
- graphical
    - points and lines (sketched)
    - linear programming
    - systems of quadratics
- The Quadratic Formula
- matrices
- technology-assisted
    - graphing (scatterplots, lines, both, etc.)
    - rule-generated tables
    - conjecture & investigation
- student-invented means

LEVELS OF ABSTRACTION

- using math to make sense of a situation
- mathematizing a situation *(mathematical modeling; Now-Next; translating words to symbols)*
- symbolic abstraction *(naked numbers, unknowns, variables, symbols)*
- symbolic generalization *(properties, theorems, patterns)*

FUNCTIONS

- continuous functions
    - linear functions
    - non-linear functions
        - exponential
        - quadratic
            - quadratic relations (conics)
- discontinuous functions
    - discrete
    - stepwise
    - rational algebraic functions
- inverse functions
- composition of functions

NUMBER RELATIONSHIPS

- inequalities
- ratio and proportion
- variation/rate of change
    - constant (slope)
    - exponential
    - inverse
- absolute value & distance
- trigonometric ratios

NUMBER GROUPS

- negative numbers
- real numbers
- irrational numbers
- imaginary numbers
- complex numbers

Thorpe, 1992; Fennema & Leder, 1990; Secada, 2000; Secada, Fennema & Adajian, 1995). However, much more investigation of these particular issues would have to have been planned into the study and, as it was, such issues did not present themselves and are therefore not included in the final analysis of the data.

Second, among the very limited data that I gathered about participants' personal backgrounds, there are indications that several of them were likely related to factors having to do with social class, and there are significant concerns about the role of "cultural confusion" (Thuele-Lubienski, 1996) concerning, in particular, students of lower-SES standing in the reformed mathematics classroom. However, given the nature of my inquiries, I did not have access to data related to participants' socioeconomic statuses. Thus, these issues do not play a role in my analysis of the data.

Lastly, one participant (Monica) was, in fact, a recent immigrant to the United States, and there were several indications that issues pertinent to immigrant students played a role in her school experience. However, exploring these issues was well beyond the scope of this study. Therefore, these issues are not mentioned in my analysis of the data.

CHAPTER 4

METHODS OF DATA COLLECTION AND ANALYSIS

Research Context

Selecting a site for my research

I sought, as a site for my research, an Indiana school district that uses the Core-Plus curriculum in its high school(s). Because of my interest in studying previously unsuccessful students' understandings of first-year algebra concepts, it was necessary to find a district that did not track its lower-achieving students into course offerings other than Core-Plus. Through the use of a massive on-line database maintained by the Indiana Department of Education (2002), I was able to identify a single qualifying school district in the state of Indiana whose district included six public high schools with an average enrollment of just over 1400 students per school.

Another parameter for locating a suitable research site was a school where the math teachers had "bought into" the Core-Plus curriculum and who had experience teaching Core-Plus *in the spirit in which it was intended* by the curriculum developers. This parameter served to narrow the choices of potential school sites to one or two. By time my request to conduct research within the school district was approved by the teachers' union discussion committee, I was given permission to work at only one school, which I will call Oliver High School. At Oliver, there were two teachers teaching the math classes in which students studied the first three units of Core-Plus Course 1 during the first half of the school year.[1] Between these two teachers, there were eight class

[1]Oliver High School observed a Block-4 schedule during the 2001-02 school year. Under the Block-4 plan, students took four full-year courses during the fall semesters which ran from mid-August to late October

sections with a combined enrollment of 150 students.

The Community

The community where Oliver High School is located, which I will call River City, is a pleasant mid-size midwestern city.[2] River City has garnered several national awards recognizing its desirable characteristics such as a cost of living that is 7% below the national average, a property crime rate that is 20% less than the national average, and a violent crime rate that is about half the national average (Find Your Spot, 1999). River City's major employers include three hospitals (employing 7,000), two automotive divisions (employing a combined 5,500), the River City school corporation (which employs 3,800), a national insurance company headquarters (employing 3,400), a telephone company (employing 2,500), and an electric motor manufacturer (1,800 employees). The city is home to four minor league sports teams, twelve institutions of higher education (the largest of which is a public university enrolling nearly 14,000 students), several museums, at least three civic theaters, and a children's zoo, as well as 2,200 acres of public parks and playgrounds.

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and from late October to early January. Similarly, four full-year courses were taken during the spring semesters which ran from early January to mid-March and from mid-March to the end of May. It was arranged, by the Oliver High School Math Department, for the Core-Plus Course 1 curriculum to be taught for three of the four semesters that comprise a nine-month school year: Students study three units of the Core-Plus Course 1 curriculum for half the school year, attending class every other day (which equals one semester), combined with the study of the other five units which are studied for half the school year, attending class every day (which equals two semesters). Half of the students taking Core-Plus Course 1 in a given school year attend math class every day in the fall, studying Units 1-5; these students attend math class every other day in the spring, studying Units 6-8. The other half of students taking Core-Plus Course 1 in a given school year attend math class every other day in the fall, studying Units 1-3; these students attend math class every day in the spring, studying Units 4-8. Because I was interested in only the first three (algebra-focused) units of Core-Plus Course 1, I conducted my research with the latter group of students.

[2] "Mid-size city" as defined by the U.S. Census Bureau: the central city of a CMSA (Consolidated Metropolitan Statistical Area) or MSA (Metropolitan Statistical Area) with population less than 250,000 (Indiana Department of Education, 2002).

The School

Oliver High School, I believe, is representative of high schools with graduating classes of at least 300 students located in mid-size midwestern cities. To give the reader a more elaborate yet general description of the type of school being described here, suffice it to say that this was neither an urban nor a suburban school. That is, the River City school corporation was characterized by neither extreme in terms of racial demographics, academic achievement, socioeconomic standing, nor issues of safety.

For instance, the student population at Oliver was comprised of 32.5% non-white students.[3] The graduation rate was 84.2%, and the college attendance rate (those seniors who continued their education at two- or four-year colleges or universities after high school) was 68.0%. For college-bound seniors, the average composite SAT score was 928 (compared to the Indiana average of 1000 and the national average of 1020) with 40% of all 12th grade students at Oliver taking the SAT exam. During the year that I collected my data, 29.4% of Oliver High School students qualified for free lunch and another 7.3% qualified for reduced lunch. As for issues of school violence, in my experience, there was a safe atmosphere within Oliver High School—there was a security guard or two whose presence was nominal, school faculty and staff wore ID badges (yet students did not), and the comings and goings of school visitors was monitored very loosely at best. Statistically speaking, the percent of students who were suspended at Oliver during the 2001-02 school year was 18.1%, while 7 students out of 1323 were expelled (Indiana Department of Education, 2002).

[3]All statistical figures reflect data for the years 2000-02.

The Curriculum

The year when I collected my data at Oliver was the fourth year during which Core-Plus had been used in the River City schools. From talking with several different people within the district,[4] I learned that the decision to adopt the Core-Plus curriculum district-wide was one that was reached after much debate and difference of opinion. The River City district superintendent had announced that reform-based mathematics curricula would be adopted at both the middle school and high school levels, in keeping with the statewide mathematics initiative that was underway. The superintendent selected Oliver's math department chairperson and two other math teachers from the district to work for an entire school year with half-day released time to prepare for the curricular change. Three curricula, pre-selected by the superintendent, were carefully examined and piloted by the selection committee, and town meetings were held to allow the public an opportunity to give input.

After the selection of Core-Plus as the new high school math curriculum, it became district-wide policy that all 9th graders were to be automatically enrolled in Core-Plus Course 1 if they had not already passed the course by the time they began high school. Students first have the opportunity to enroll in Core-Plus Course 1 (for which they receive high school credit) in 8th grade. However, those students deemed not yet ready for this course (i.e., those who would be considered "lower track") are enrolled in Eighth Grade Math instead (equivalent to taking Pre-Algebra rather than Algebra I).

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[4]These informants included the River City district's Director of Planning, Assessment & Learning Technologies; one of the district's high school principals (who was a middle school principal in River City schools at the time of the Core-Plus curriculum adoption decision); an assistant principal at Oliver H.S. (who was also a middle school principal in River City schools at the time of the Core-Plus curriculum adoption decision); as well the math department chair and one of the math department faculty members from Oliver High School.

Among all the high schools in River City, it was the math department members at Oliver who embraced the Core-Plus curriculum most readily at the time of its adoption. Three years after its implementation, the math faculty at Oliver seemed united in their commitment to teaching the curriculum in the spirit in which the curriculum developers intended (which cannot be said of all other high schools district-wide). Oliver's math teachers enjoyed a common lunch hour that allowed ample opportunity for collegial support and collaboration of planning. Moreover, Oliver's math department chair provided teachers with unit tests and a listing of selected homework problems for use with all students, helping ensure that everyone got a uniform curricular experience regardless of the instructor s/he had.

The Students

The six students in my interview cohort were representative of previously unsuccessful, white female students who come from middle- and working-class families. Although a much more elaborate description of what is meant by "previously unsuccessful" was given in Chapter 3, suffice it to say that these students were representative of those girls who are one or more years behind the norm[5] in taking their first-year high school math class and whose mean overall-attitude scores fall approximately within the middle 50% of all scores.[6] All of the students in my interview cohort took Eighth Grade Math followed by Core-Plus Course 1 in 9th grade. Moreover,

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[5]The norm to which I refer is that which is advocated by Usiskin (1987), whereby average students take first-year algebra in the eighth grade. This, however, is not the norm at Oliver High School, where a majority of students take Core-Plus Course 1 as ninth-graders.

[6]Mean overall-attitude scores can range from 1.0 (low) through 5.0 (high). Scores for students in the interview cohort ranged from 2.33 and 3.28. Of the 31 survey-taking students, 5 had scores lower than 2.33 and 8 had scores higher than 3.28. For a detailed description of how these scores were derived, refer to the section "Analysis of Initial Survey Data during the Data Collection Phase" later in this chapter.

two of the six students in the interview cohort were repeating Core-Plus Course 1 as tenth-graders.

Data Collection

Initial Survey

In order to have a means of selecting a small cohort of students whom I could interview throughout the course of their studies in Core-Plus Course 1A, my plan was to administer a survey to all students enrolled in the every-other-day fall block of first-year Core-Plus about their attitudes and behaviors related to their math studies. In addition to attitude questions, the survey also included two problem-solving tasks: one abstract ("naked numbers") problem and one contextualized problem which yielded the same solution set as the traditional problem. (This Initial Survey can be found in Appendix A.) The classroom teachers distributed take-home letters with permission forms for parents to sign and return in a postage-paid envelope. Of the 150 students who received these in class, 34 returned the forms.[7] From the group of students taking the survey, I planned to select 8-12 interviewees who seemed to be previously unsuccessful in mathematics as determined by self-reported attitudes and behaviors typically associated with non-success in school mathematics (House, 1988; Prawat, Lanier, Byers, & Anderson, 1983) and based on their performance on the problem-solving tasks.

On the three school days following the administration of the surveys, I observed survey-taking students to gather supporting evidence for my preliminary categorization of

[7]Of the 34 students who returned a permission form, however, two of them never ended up taking the survey, and the survey of a third student was eliminated from the pool after I learned that this student was diagnosed as profoundly learning disabled and would not qualify as an interview candidate for the purposes of the study.

students as previously unsuccessful or successful based on their overall attitude scores. I took care to note students' modes of participation in class (verbal, written, and in collaboration within small groups) as well as any evidence of how well they seemed to be understanding the mathematics being discussed. I was also able to collect information on students' race, sex, year in high school, and family structure. The latter set of information was based in part on my classroom observations, in part on data I was able to obtain from the school (name of guardian, race, and year in high school), and in part from the parents' signatures on permission forms (which offered some insight as to whether a student lived with one or two parents or step-parents). Using all of this data, I separated the survey-taking students into three groups: a group of 13 who I thought would be the best interview candidates (Target Group 1); a group of 10 who I thought would be satisfactory interview candidates (Target Group 2); and a group of 8 whom I would not contact about the possibility of participating in the Interview Phase of my study.

The best interview candidates, for the purposes of my study, were going to be those who seemed to have had some previous non-success in school mathematics. Thus, those students in the group of 8 whom I did not contact for participation in my study were those whose attitude scores on the initial survey revealed attitudes toward math that were highly consistent with attitudes/responses expected of successful students. (An explanation of how these scores were determined was given in Chapter 3.) Students placed in Target Group 1 were those whose attitude scores on the initial survey revealed attitudes toward math that were most consistent with attitudes/responses expected of unsuccessful students. Students placed in Target Group 2 were those whose attitude scores were not high enough to be ruled out as non-candidates for the interview phase of the study, but not as low as those placed in Target Group 1. Indeed, Target Group 2 students' scores on the attitude survey and the problem tasks revealed mixed results: some signs of previous non-success but also some glimmers of successful traits as well.

Students' scores on the problem tasks of the initial survey were also taken into consideration. I was most interested in those students whose scores on the contextualized task were especially high at the same time that their scores on the abstracted problem were especially low. This was important to me because I had observed, when teaching Core-Plus Course 1A myself a few years earlier, that the Core-Plus curriculum seemed to provide previously unsuccessful Algebra One students an entrée into the subject matter—perhaps, I hypothesized, because of the contextualized nature of the mathematics.

From among the 23 students contacted about possible participation in the study as interviewees, 9 female students (4 from Target Group 1 and 5 from Target Group 2) consented to participate. It is perhaps not unusual that the interview cohort consisted of all girls, given the fact that 21 of the 31 survey-takers were girls, and 18 of the 23 students contacted about possible participation in the study were girls. It was also not intended that gender be an analytical category of research, although, once the cohort of interviewees was determined, gender issues were certainly considered in the interpretation of the research results. Nevertheless, issues particular to girls' experiences in school mathematics did not present themselves and are therefore not included in the final analysis of the data.

Table 4.1
Summary data on interview participants as derived from initial survey[8]

Pseudonym	Target Group	Grade Level	Taking Course 1 for the ... time	# of Interviews with this student	Initial Survey -- Attitude Score Means[9]							Initial Survey -- Problem Solving Scores	
					Overall Attitude Score Average	Public Self-Confidence	Personal Self-Confidence	Enthusiasm	Significance of Math	Long-term Stick-to-it-iveness	Short-term Stick-to-it-iveness	Abstract Problem	Contextualized Problem
Terri	1	10	2nd	3	2.33	3.0	1.7	1.7	2.7	2.0	3.0	1	3.5
Andrea	1	10	2nd	1[10]	2.44	2.3	2.7	1.3	2.7	2.3	3.3	0	4
Monica	1	9	1st	3	2.83	2.7	3.0	2.0	3.0	3.0	3.3	1	4
Amy	2	9	1st	3	2.94	2.7	2.7	1.7	3.3	4.3	3.0	1	4
Joy	2	9	1st	3	3.00	4.0	2.0	2.0	3.3	2.7	4.0	0	4
Adrienne	2	10	2nd	3	3.11	3.7	3.7	2.7	2.0	2.7	4.0	3	4
Carissa	1	11	2nd	1[10]	3.17	4.3	3.3	2.3	1.3	2.7	3.3	0	3.5
Jamie	1	12	3rd	1[11]	3.19	3.3	3.3	3.7	2.5	2.0	3.7	0	2.5
Sara	2	9	1st	3	3.28	4.0	3.3	2.7	2.7	3.0	4.0	4	4
MEAN SCORES (n=31) :					**3.01**	**3.2**	**3.0**	**2.5**	**2.8**	**3.0**	**3.4**	**0.8**	**3.1**

Classroom Observations

Not only did I make classroom observations immediately following the initial survey, but I also continued to observe interview participants in the math classroom

[8]Shaded boxes indicate scores that both (a) suggest previous non-success and (b) also fall below the average score for all survey participants.
[9]A score of 3.0 is neutral. Scores below 3.0 indicate responses like those expected from unsuccessful students; scores above 3.0 indicate responses like those expected from successful students.
[10]Due to extenuating circumstances, this student was not able to meet more than one time for an interview.
[11] After the first interview, this student was no longer enrolled in Core-Plus Course 1 and therefore did not continue to be part of the interview cohort.

setting throughout the time that interviews were being conducted.

Table 4.2
Summary of classroom observations of interview cohort

Dates	Timing of observations relative to interviews	Terri	Monica	Amy	Joy	Adrienne	Sara
October 4-8	Immediately following initial survey	2	2	2	1	1	1
October 23-25	Beginning of Interview Round 1	1	1	1	1	1	0
January 24-25	Middle of Interview Round 2	1	1	1	1	1	1
April 9-12	End of Interview Round 3	1	1	1	1	1	1
	TOTAL # OBSERVATIONS:	**5**	**5**	**5**	**4**	**4**	**3**

This technique allowed me to continue collecting data on the interviewees' modes of participation in class and on their apparent understandings of the mathematics being discussed. It also provided me insight into the routines and procedures of each math class, which helped me to more accurately understand and interpret students' reports about their affective experiences in the classrooms of various teachers.

Table 4.3
Summary of interviewees' math class enrollment

	Fall (August to January)		**Spring (January through May)**		
	Mr. Harper	**Mr. Larson**	**Mr. Barnes**	**Mr. Everett**	**Mr. Harper**
Terri	Section "A"			Section "F"	
Monica	Section "B"				Section "G"
Amy	Section "C"		Section "H"		
Joy	Section "D"		Section "H"		
Adrienne		Section "E"			Section "G"
Sara		Section "E"			Section "J"

At the time that students were first made aware of my conducting research in the Core-Plus Course 1A classes, they were informed that I would like to make a few classroom visits to observe the students I would be interviewing as they participate in class and that I might also like to note interactions that other students in class had with interviewees. It was explained in the original information letter, which all students received, that I would only take notes on the participation of students whose parents had provided written consent. On the permission forms where parents gave consent for their child to participate in the initial survey, they also had the option of giving consent for me to take notes on their child's participation in class.

Although all students and teachers in the classes that I visited had been made aware that I was conducting research about the Core-Plus curriculum, the particular identities of the participating students were kept confidential. Classroom teachers were informed that I would not reveal to them which of their students I was observing during my classroom visits. This was done, in part, to protect the confidentiality of participating students. More importantly, it was done to guard against any stigmatization of participating students, due to the fact that my study focused on students with prior histories of non-success in school mathematics. Therefore, at no time were any students made aware that my study focused on previously unsuccessful students. Great care was taken to let students know only that they were selected for participation as interviewees on the basis of being able to "contribute valuable additional information to my research."

My relationship with the classroom teachers throughout all of my visits to the school was very open and supportive. It was made clear to the classroom teachers that I was not there to evaluate in any way their pedagogy, but that I was there to observe interviewees' participation in class. Teachers readily accepted this at face value, welcomed any questions that I had for them, and provided any supporting materials that I requested (copies of tests or homework templates, seating charts, grading criteria, etc.). I was made to feel a part of the group during the dozen or so lunch hours that I spent eating and talking with the math department faculty on the days I was observing. Indeed, several of the math teachers, including some whose classes were not a part of my study in any way, bid me a fond farewell when my data collection at Oliver drew to a close.

Interviews

The second phase of my data collection was the interview phase. Between late October and mid-April, I conducted audiotaped interviews with each of six participants three times: once initially to obtain baseline data, and again at the conclusions of the second and the third (algebra-focused) units of study in Core-Plus Course 1. According to the curriculum developers, of the Course 1 units of study, the algebra content strand is a primary emphasis in Units 2, 3, 5, 6, and 8; the functions strand is a primary emphasis in Units 2, 3, 6, and 8. Throughout the other units of the course, connections to both the algebra and functions content strands are included but not emphasized (Coxford et al., 1998c, p. x). Therefore, my study focused particularly on Units 2 and 3 of the Course 1 curriculum—both of which were studied by my interview participants during the first half of the academic year—because my interests were limited to students' understandings of algebraic concepts and functions that would typically be learned in a traditional first-year algebra course. As the last lesson of Unit 1 serves as a segue between the study of statistics and data analysis (the primary focus of Unit 1) and the study of algebra and functions (the primary focus of Unit 2), I also chose to investigate students' earliest conceptions of algebra and functions by including a problem from Lesson 4 of Unit 1 in the Post-Unit-2 Interview.

Each interview lasted between 30 and 60 minutes and was conducted in a small, private room at Oliver High School at the end of the school day.[12] The purpose of the interviews was to obtain information about students' affective experiences in Core-Plus Course 1 as well as their developing understandings of the algebra and functions concepts being studied. An additional purpose of the first interview was to obtain information about students' personal math histories. (A Summary of Instruments chart, which provides a snapshot of the data collected, is included in Appendix A.)

At each interview session, students were asked to complete a written checklist about their familiarity with pertinent math topics being explored in that interview's problem-solving tasks and a written attitude questionnaire (similar to the one given initially). (See Familiarity Checklist in Appendix A.) After students completed the written surveys (which typically took about 5 minutes total), they were then asked to verbally respond to a set of questions that pertained to their experiences in math class (which typically took 10 minutes). (See Interview Protocol A2 in Appendix A.) The majority of the interview session was devoted to the students' solving of three problem tasks. The tasks were taken directly from an assessment resource booklet written by Core-Plus curriculum authors (Coxford et al., 1998a). Each task was presented to the student on a worksheet on which she wrote out her problem solution. After work on each problem-solving task, the student was asked the same basic set of questions about her understandings of the mathematical concepts involved in the task. (See Interview Protocol B in Appendix A.)

[12]This room was used during the school day by the school psychologist for testing and interviewing of student-clients.

Data Analysis

Although in-depth analysis of the data was not undertaken until after completing data collection, I did make a point to conduct some preliminary and ongoing analyses throughout both phases of data collection, acknowledging that, when one collects qualitative data, the "focus [of the phenomena under study] may shift as analytical categories and theory 'emerge' from the data" (Lancy, 1993, p. 2). Indeed, such constant comparison methodology (Bogdan & Biklen, 1998) helped me in subsequent data collection to tweak questions or (re-)direct the focus of my queries in response to interesting leads that were uncovered along the way.

<u>Analysis of Initial Survey Data during the Data Collection Phase</u>

The survey instrument was designed to be the primary means of identifying, among the students who completed the questionnaire, those who possessed school attitudes and behaviors that are typically associated with previous non-success in math. Survey items were written to elicit students' attitudes in several categories (House, 1988), as shown below.

<u>PUBLIC SELF-CONFIDENCE IN ONE'S OWN MATHEMATICAL ABILITIES</u>

1. I often volunteer to answer questions in math class.
2. I understand math well enough to help a classmate who is struggling.
3. I do <u>not</u> like to go to the board to show my answer to a math homework problem.

<u>PERSONAL SELF-CONFIDENCE IN ONE'S OWN MATHEMATICAL ABILITIES</u>

4. I am able to tell for myself whether an answer on my math homework is correct (or at least reasonable) <u>without</u> looking up the answer.
5. Math often does <u>not</u> make sense to me.
6. I am naturally good at math.

ENTHUSIASM ABOUT MATHEMATICS

7. Math was one of my favorite subjects in middle school.
8. I love playing games in math where you have to be the fastest one to answer in order to win.
9. I can imagine myself completing a math-related major in college.

PERCEIVED SIGNIFICANCE OF MATHEMATICS

10. I want to get a job that uses as little math as possible.
11. A student doesn't need more than 2 years of math in high school if he (or she) is not going to go to college.
12. A student doesn't really need to take 4 years of math in high school unless she (or he) is going to be a math major in college.

LONG-TERM STICK-TO-IT-IVENESS IN THE STUDY OF MATHEMATICS

13. I would be happy to get a C as my grade in Core-Plus Course 1.
14. I do not plan to take more math courses than the two that are required to graduate from high school (Core-Plus Course 1 and Core-Plus Course 2).
15. I want to graduate from high school with an Academic Honors Diploma.

SHORT-TERM STICK-TO-IT-IVENESS ON MATHEMATICAL TASKS

16. When a math problem is challenging, I usually keep working at it until I get it figured out.
17. After finishing a math test, I usually check over my answers before turning it in.
18. When I get back my graded math tests, I usually go over the problems I missed so I understand why I got them wrong.

In response to each item, students were asked to indicate whether they strongly agreed (SA), agreed (A), were undecided (U), disagreed (D), or strongly disagreed (SD) with the statement. As survey data came in, I scored the responses to the attitude questions (1, for the response most likely from an unsuccessful student, through 5, for the response most likely from a successful student), as shown in the example below. (The complete Scoring Rubric for the Attitude Questions can be found in Appendix B.)

Table 4.4
Example of scoring procedure for survey items

	SA	A	U	D	SD
4. I am able to tell for myself whether an answer on my math homework is correct (or at least reasonable) without looking up the answer.	5	4	3	2	1
5. Math often does not make sense to me.	1	2	3	4	5

This scoring procedure was used to calculate an overall attitude score (mean of the 18 responses), as well as specific attitude scores (mean of the 3 responses) in each of the six subcategories: Public Self-Confidence, Personal Self-Confidence, Enthusiasm, Significance, Long-Term Stick-to-it-iveness, and Short-Term Stick-to-it-iveness. I also developed my own rubrics (similar to that used by Thompson & Senk, 1993) to score students' work on the problem-solving tasks. (See Appendix B.)

This analysis of survey data during the data collection phase was essential to being able to proceed with the next phase of data collection, as it provided the means for selecting those students who best qualified as interviewees according to the purposes of the study.

Analysis of Interview Data during the Data Collection Phase

Ongoing analyses of interview data proved helpful as well. After each interview with a participant, I wrote an interview memo. The memo served several purposes. First, it allowed me to record data from the interview that could not be recorded on audiotape—for example, a student's counting on her fingers, or numbers she punches into the calculator. These data would have been quickly forgotten and forever lost if not recorded soon after the interview.

Second, the interview memo became a sort of abstract for the interview, or a catalog of noteworthy events that took place during the interview. I anticipated there being—at some point while doing in-depth analysis and/or writing—particular instances from interviews that I wished to go back and examine. It would be very time-consuming to have to listen to three hours of interviews on audiotape to locate such an instance (assuming I was able to remember which informant was involved in the instance I seek), yet it would be fairly simple to look back through the interview memos to locate the desired episode.

Lastly, and perhaps most importantly, the interview memos served as a forum where I kept track of issues that presented themselves as potentially salient. Because I was not able, due to time constraints, to transcribe the interviews as they occurred, the interview memos were most useful as a growing set of research notes. I made a special effort to note unexpected happenings—such as misinterpretations of a problem task that was posed, or delightful insights on the part of the interviewe that I hadn't anticipated—especially if such insights were common to more than one participant. For example,

> Joy isn't the first one who seemed not to be able to grasp the abstract idea of a y=x line going through any (unplotted) points where y=x; others also seemed able to focus/conceive of such points only in terms of the already-plotted points (74, 74) and (80, 80).
> (Memo of Interview Two with Joy, 1/17/02)

I also made note of particular verbal exchanges and student explanations that I wanted to go back and examine using the transcript, once it was available.

> We were pursuing the question of whether the graphs of the two trips' expenses were ever going to meet or cross. *There is a fascinating explanation on the*

transcript for the reasoning she came up with by looking at the hand-drawn graph and examining & conjecturing about its downward slope!!
(Memo of Interview Two with Sara, 1/09/02)

Likewise, I noted hunches, hypotheses, and interesting leads as they began to present themselves.

> Amy surprised me by not remembering a lot of this stuff from Unit 3 – calculating linear regression models, writing equivalent equations, distributive property, and so on. Both Terri and Adrienne – whom Amy has outperformed mathematically in previous interviews – remembered these things from Unit 3 quite a bit better. *This made me wonder whether the fact that Terri and Adrienne are both taking the course for the second time has anything to do with their remembering the stuff better than Amy, who is taking the course for the very first time?*
> (Memo of Interview Three with Amy, 3/12/02)

Further Analysis of Initial Survey and Interview Data

Once all interviews were complete, the first task I undertook was to transcribe the interview tapes. Audiotapes of 14 of the 15 interviews conducted with the six participants were available.[13] I personally transcribed all three interviews with Adrienne and then sought assistance in transcribing the remaining interviews, choosing instead to spend my time working on other means of data analysis. The transcriptionists I hired typed up only the problem-solving portions of the interviews. Meanwhile, I recorded students' responses to the protocol questions in charts for easy comparison of various interviewees' answers to similar questions.

I also scored interviewees' responses to the attitude questionnaire items from each interview in the same way that the attitude questionnaire items were scored from the initial survey. Then I studied these scores, both for the initial survey and for the attitude questionnaires given at each interview, and analyzed them for patterns—not only patterns

[13]The audiotape of Interview One with Amy was blank. It is suspected that the Play button was pushed without also pushing Record on the cassette recorder.

among the responses of each interviewee but also patterns among the responses of all interviewees as a group. In the responses to the initial survey questions, I discovered a fascinating trend: The interviewees' responses revealed consistently low levels of enthusiasm for math at the same time that their responses revealed consistently high levels of short-term stick-to-it-iveness. That is, despite their not liking math as a subject, the interviewees nevertheless demonstrated productive learning habits in their math studies. (Whole Group – Initial Survey Results are included in Appendix B.)

Perhaps the most significant component of data analysis was the evaluation of students' problem-solving work on the interview tasks. Prior to undertaking the grading of students' work, I carefully examined each task that was used in the interviews and noted each mathematical concept or problem-solving skill that interested me. I then categorized these evaluation items according to type.

Table 4.5
Performance Skills and their Types for Interview 1 Task 3

PERFORMANCE SKILL	TYPE
Understanding that the question implies *Which option is more economical?*	U
Recognizing that the answer to the question has more than one case	U
Using paired data to compare two functions	TS
Using an explicit formula to compare two functions	RS
Deriving NOW-NEXT equations for calculating values	R
Using the Answer Key on the TI-83 to quickly calculate values iteratively (Any Task)	C

Table 4.6
Definitions for Performance Skill Types

U	Comprehension of what the problem was asking; Understanding the nature of the problem
TS	This item notes whether a table aided the student in obtaining the solution
RS	This item notes whether a rule (equation) aided the student in obtaining the solution
R	This is a skill that supports the use of a rule (equation) to solve the problem
C	Calculator skill

In order to score each student's performance on the evaluation items for each task of each interview, I not only found it necessary to consult the answers students had written on their task worksheets, but I also had to consult the interview transcripts for students' explanations of their thinking while completing the tasks, and I consulted the interview memos where I had written notes on each student's work on each task during the interview. Once the scoring of each participant's problem-solving work was complete, I then began to analyze the results of the evaluations both by task and by skill type, looking for patterns in each student's performance as well as patterns among the various students' problem-solving work.

Writing as Data Analysis

The data analysis described above was undertaken as a direct means of making sense of the data. Without this kind of data analysis, one has little to write about. Nevertheless, writing can be used as a tool of data analysis itself (Richardson, 1994), as an indirect means of making sense of the data. This was the case in my research. In two instances—writing portraits of the six participants in Chapter 5 and creating a map of the algebra terrain in Chapter 3—writing/creating was used for the express purpose of further analyzing data. In the third instance I describe below, additional insights about the data came as an unintended, but welcome, result of writing up field notes.

Portraits of the Six Participants

The portraits of the six interview participants presented in Chapter 5 were written with the primary purpose of providing a purely descriptive introduction to the students whose experiences of the Core-Plus curriculum and understandings of algebra and functions are being explored in this study. However, a secondary purpose in writing these portraits was to gain further insight and information about patterns that might exist across participants described in Chapter 6. Indeed, as I looked at each student's set of data, with an eye toward answering each of the three questions that drive this study (What is the nature of these students' previous non-successes in school mathematics? What are these students' experiences of the Core-Plus curriculum? What do these students understand about algebra and functions?), I made new discoveries and came to better (and sometimes differently) understand the data I had.

Use of the Algebra Map

Similarly, the algebra map presented in Chapter 3 was created with the initial purpose of providing a description of the algebra terrain that was used as a conceptual framework in this research. The algebra map was also created with the intent to use it as an analytic framework for helping identify what participants understood about algebra and functions (presented in Chapter 6). However, the process of creating the map of the algebra terrain was also an exercise in preliminary data analysis. As I collected and organized the various fields and topics within fields that appear on the map, I was also mentally forming new categories of analysis for later data exploration. This, in turn, helped inform my ultimate interpretations of what participants understood about algebra and functions; it also gave me a head start in identifying what participants did not understand about algebra by virtue of the fact that certain topics appearing on the map were not part of the Core-Plus Course 1A curriculum. While the algebra map served as a tool for directly analyzing the data; the writing/creating of the algebra map also served as an indirect means of making sense of the data.

The Emergence of Findings

As mentioned in a previous section of this chapter, one of my methods of keeping track of data during the data collection process—writing a memo following each interview—turned out, quite unintentionally, to also be a method of preliminary data analysis. It was in the course of recording unexpected happenings (misinterpretations or interesting insights of students), as well as hunches, hypotheses, and interesting leads that I first began to take note of early patterns beginning to present themselves.

It did not become apparent, however, that findings were perhaps beginning to emerge until I began to write about my work-in-progress. The sequence of my collecting data and drafting a dissertation proposal were a bit unconventional and, as it turned out, the writing of my dissertation proposal took place between the second and third rounds of interviews that I conducted with participants. As I worked to make some sense of the data I had collected during the first two rounds of interviews in order to present the study-in-progress to my dissertation committee, I began to see that the early patterns that had begun to take shape seemed to hold some promise and deserved further investigation. During the third round of interviews, then, I paid close attention to the learning issues that seemed to underlie these patterns I was noticing, taking care to note whether additional information would help clarify the fuzzy picture that I thought I might be seeing. Indeed it did.

It is worth noting that, of these three categories of findings described in the second part of Chapter 6, two were quite unexpected (mathematical community membership and language issues); the third (contextualized versus abstracted mathematics) ran parallel to—but did not confirm—a conjecture that I had in mind as I began the study: that the contextualized nature of the Core-Plus curriculum provided students an entrée into the subject matter that previously unsuccessful students did not experience with traditional curricula (which typically deal far more, and often exclusively, with abstracted mathematics). It was *not* my aim to confirm this conjecture. Instead, I set it aside to allow the data to speak to me, not wanting to have any part in trying to make the data say something I was hoping it would.

In the following two chapters, then, I present my full collection of findings. Chapter 5 is a descriptive overview of each of the six participants individually. This first level of analysis lays the groundwork for the broader analysis and interpretation in Chapter 6. These second-level findings include patterns that did (or did not) exist across participants in each of the three inquiry areas (previous non-success in math, experience of the Core-Plus curriculum, and understanding of algebra and functions) as well as the two categories of findings related to learning issues: mathematical community membership and contextualized versus abstracted mathematics. These findings, in turn, lead to the implications for high school math teachers and mathematics teacher educators in Chapter 7.

CHAPTER 5

PORTRAITS OF THE SIX PARTICIPANTS

Portraits of the participants are intended to provide the reader with vivid images of each of the six students whose learning experiences are explored in this study. As I conducted my research with these students, the more I learned about who they were as people, what made them unique individuals, the better I was able to understand their learning experiences. Their stories vary quite a bit—they come from different personal backgrounds, their previous and current school experiences differ—yet they also have a number of elements in common. What makes these students distinct is described here in Chapter 5. That which that they share in common is described in Chapter 6.

As I have reflected on all that I've come to know about these six students and their learning experiences in high school math, I have been reminded many times of other students—those that I myself have taught in high school math classes in the past. As I gained insight about the things that make learning mathematics, and algebra in particular, more or less challenging for these students, I found myself looking for ways that I might apply these findings to my own teaching of math, and algebra in particular, should I ever find myself back in a high school classroom again. Additionally, I sought particular meanings for members of the community of mathematics teacher educators. These are the implications that I share in Chapter 7.

Organization of the Portraits

These portraits are organized around the three questions that guide this study: What is the nature of these students' previous non-successes in school mathematics? What are these students' experiences of the Core-Plus Course 1A curriculum? What do these students understand about algebra and functions (at three different moments in time)? In addressing each question for each participant, an attempt at uniformity was made, yet the priority was to offer data that would allow the reader to best understand each participant as an individual. Data that did not offer salient information about a participant is not included in the portrait. So, varying sets of tables (and, in some cases, merely *excerpts* of data tables) appear among the portraits, for instance. Complete versions of the data tables can be found in Appendix B. Furthermore, data that proved to be important in the second level of analysis—looking for patterns across participants, or that which was relevant to the three categories of findings pertaining to learning issues—are presented in Chapter 6. The portraits are purposefully meant to be descriptive, and *not* interpretive, in nature. Interpretation of findings appears in Chapter 6. Thus this chapter and the next are meant to complement one another.

There were many ways in which the portraits could have been ordered. Because this is a study of the ways in which students understand algebra and functions, and because all findings ultimately have something to say about students' mathematics learning, I ordered the portraits according to the participants' overall success on the series of interview problem-solving tasks. I begin with Adrienne, whose performance was consistently the most successful of all six participants; I conclude with Monica who experienced the greatest difficulty in successfully completing the interview tasks.

Adrienne

Adrienne was a 16-year-old tenth-grader, taking Core-Plus Course 1 for the second time. Adrienne's parents were divorced, and she lived with her mother, and at least one sibling (a brother a year older than she).

<u>Distinguishing Characteristics of Adrienne</u>

My overall, lasting impression of Adrienne was of her extremely shy but well-mannered nature. During my interactions with and observations of Adrienne, she almost never spoke unless spoken to, and she didn't make a lot of eye contact. In the classroom, it seemed almost as if she made an effort not to be noticed among the group. It truly seemed painful for Adrienne when all eyes were on her, as if she might not survive the heat of the spotlight. On the other hand, Adrienne could do much better if and when the teacher helped her one on one.

Yet at the same time, Adrienne did participate in class verbally. This was a fascinating phenomenon that I noticed during my classroom observations: The teacher would pose a question to the class and, in response, a number of students would voice an answer—Adrienne's being a regular participant. The teacher (this was actually true of both math teachers that Adrienne had) would rarely call on an individual student; instead, he listened to the chorus of answers, identified the answer he was seeking, and continued with the lesson.

My initial reaction to this classroom practice—as a former high school math teacher and now a student teacher supervisor at the university—was to think that the discussion would be better managed if the teacher called on students who raised their hands. I was concerned about those students who gave wrong answers that seemed largely overlooked by the teacher: How would these students arrive at correct understandings if their misunderstandings were not addressed? I wondered.

Adrienne offered a new spin on this, however. Having the freedom to speak up without raising her hand actually provided a safe way for Adrienne to participate verbally—something she would never do if she were required to raise her hand in order to be individually acknowledged by the teacher. Indeed, in gathering information on Adrienne's previous experiences in math classes, I asked her to describe one of her "least successful experiences in a math class, like a negative memory that may really stand out."

> Adrienne: I don't like being called on in class and giving the wrong answer.
> Interviewer: OK. Has it happened sometimes?
> Adrienne: M Hm.
> Interviewer: and then you just feel mortified?
> Adrienne: Yeah.
> (Interview One, 10/19/01)

Interestingly, Adrienne didn't view this calling out of answers as bona fide participation. During the first interview, I asked Adrienne whether she ever volunteered in class in response to a question the teacher asked. Adrienne's answer was "I don't volunteer—I just yell[1] out the answer" (Interview One, 10/19/01). During the last interview, I again questioned Adrienne about her classroom participation:

> Interviewer: Did you participate in the class discussion today?
> Adrienne: No, well, I do. We don't raise our hands.
> (Interview Three, 3/11/02)

What's more, Adrienne seemed empowered to participate because of this non-formalized approach to classroom discussion:

> Interviewer: Think back to a time when the answer you gave in class happened to be wrong. What happened?

[1]It should be clarified that what Adrienne refers to as "yelling," I considered to be speaking at a normal conversational volume.

Adrienne: There's always people yelling out answers. He just takes the right one and writes it down.
Interviewer: So it doesn't keep you from calling out an answer, even if you're not that sure.
Adrienne: Nh nh. [No.]
(Interview Three, 3/11/02)

Not only did the freedom to call out answers at will allow Adrienne the opportunity to participate verbally in class, it also helped her understand things better. Of Mr. Larson, Adrienne said, "When I say something wrong, he just goes through and explains it better. I like that. He explains it better" (Interview One, 10/19/01).

In contrast to the informal and unofficial way that Adrienne participated verbally in class, she approached her individual written work with meticulous and methodical work habits. She worked in a very careful manner, keeping good records—either written or through verbal commentary—of everything she did. Adrienne often had good mathematical insights, demonstrated strong commonsense reasoning, and displayed high levels of perseverance and curiosity in pursuing interesting questions beyond the extent of the question asked in the given problem task. Adrienne was also quite skilled in the use of her TI-83 calculator, more so than any of the other participants. Despite all this, Adrienne seemed to have little awareness of how good her mathematical skills and understandings actually were.

Adrienne's Previous Non-Success in School Mathematics

Personal math history. Adrienne had failed Core-Plus Course 1 as a freshman. In fact, going into her second year of high school, Adrienne did not yet have enough credits to officially qualify for sophomore standing. One of the only pieces of information I gathered about Adrienne's unsuccessful first experience in Core-Plus Course 1 surfaced when I asked her opinion of her current Core-Plus Course 1 class. She said, "It's a small class so we get more attention" (Interview One, 10/29/01). Adrienne said there were 20 in her class last year compared to 10 in her current class. She also reported that her current teacher explained things better than her teacher the previous year. Yet even Adrienne was rather surprised that she was doing so much better this second time through the course.

I had the distinct impression that Adrienne seemed to believe that success in math classes was due, at least in part, to chance or luck of the draw (getting a good teacher or a small class). For example, in her third interview Adrienne had expressed ambivalence about whether she was looking forward to taking Core-Plus Course 2 in the coming school year (she checked No Opinion/Undecided). When I asked her about this, she said her brother—who was a grade ahead of her in school—hadn't yet passed Core-Plus Course 1, which made her a bit hesitant/apprehensive about Core-Plus Course 2. And this, despite the fact that she was getting an A in Mr. Harper's class—a class that had over 30 rambunctious students in it. While it seemed quite clear to me that she had come a very long way in her ability to succeed in a math class, Adrienne seemed uncertain of her own chances for success in the future.

Entering attitudes towards mathematics. Adrienne's general feelings toward math as a subject were surprisingly quite positive. In the initial interview, she reported that she thought it to be "pretty easy" as well as "fun and interesting." She even said she had "always liked math," that it was "fun working with the numbers." She also reported that she used math outside of school when trying to anticipate biweekly paycheck totals for her friend, taking into account hours, overtime, taxes, etc. (Interview One, 10/29/01).

On the attitude questions of the initial survey, Adrienne's mean response score was 3.11—only slightly above a neutral score of 3.0—which indicates that her responses, on average, were only slightly more like those responses expected of successful students than those expected of unsuccessful students. This score placed Adrienne squarely in the middle of the group of 31 survey-takers—one of four whose score was the median score (z = +0.13)—yet she had the second highest score among the six participants.

Adrienne's survey responses identified short-term stick-to-it-iveness and both personal and public self-confidence as her strongest suits. In each of these categories, Adrienne's responses were above neutral (indicating they were responses expected from successful students) and above average for the entire group of survey-takers. Adrienne's personal self-confidence score was also the highest among the group of six participants.

Table 5.1
Adrienne's scores on initial survey attitude questions by category[2]

	Response Mean	Means within categories					
		Self-confidence, Public	Self-confidence, Personal	Enthusiasm	Significance	Stick-to-it-ive-ness, Long-term	Stick-to-it-ive-ness, Short-term
ADRIENNE	**3.11** z = +0.13	**3.7** z = +0.64	**3.7** z = +0.65	**2.7** z = +0.20	**2.0** z = -0.78	**2.7** z = -0.28	**4.0** z = +0.60
MEANS (n=31):	3.01	3.20	2.99	2.46	2.80	2.96	3.43
StDevP:	0.752	0.722	1.042	1.050	1.033	1.057	0.951

There were three areas in which Adrienne's responses revealed less-than-neutral attitudes[3]. The first was enthusiasm for math as a subject. (It should be noted, however, that Adrienne's enthusiasm was above average for the group of survey-takers as a whole, z = +0.19, and tied for the highest Enthusiasm subscore among the six interview participants.) Second, Adrienne was slightly less-than-neutral and slightly below average (z = -0.27) in her long-term stick-to-it-iveness (i.e., the desire to pursue math studies

[2]A complete table of initial survey response data can be found in Appendix B.
[3]On the attitude survey response scale, the middle score of 3 is neutral or undecided.

throughout high school and college). Third, in terms of seeing math as an important area of study, Adrienne's responses revealed the lowest score of all six interview participants, which was also fairly low for the group of survey-takers as a whole (z = -0.76).

Adrienne's Experiences of the Core-Plus Course 1A Curriculum

Adrienne's opinion of Core-Plus Course 1 was that it was easy, and she reported that it got easier for her as the year progressed. Indeed, throughout the school year, she received continually improving above-average math grades.

> Adrienne reported that, since the last interview, the Core-Plus Course 1 class had gotten "a lot easier" because she understands it so much better. She said it just made a lot of sense, like it had not before. This definitely was borne out in the interview/tasks. She didn't really have an explanation for why things have come so easily to her this year, especially in light of her having failed last year. Even her familiarity survey shows that she didn't understand most of the Unit 2 topics prior to studying them this semester in Mr. Larson's class.
> (Memo of Interview Two, 1/17/02)

Table 5.2
Adrienne's Familiarity Checklists

Below is a list of the topics in Chapter N of your math book. For each one, mark an X in one of the boxes to show which statement best describes how familiar you were with the topic before this semester.	I already knew how to do these very well.	I already knew how to do these somewhat.	I had seen these before, but I knew very little about them.	I had never seen these before, and I did not know anything about them.	I don't remember studying this in this chapter.
CHAPTER 1.					
Stem-and-leaf plots		X			
Number line plots		X			
Histograms			X		
Mean, median, mode		X			
Five-number summary			X		
Box plots			X		
Scatterplots		X			
Plots over time			X		

Table 5.2 (cont'd)

CHAPTER 2.					
Using a y=x line		X			
Graphing (x,y) data			X		
Writing a NOW-NEXT expression		X			
Using the ANS key to calculate numbers repeatedly (TI-83)		X			
Writing a rule using letters				X	
Using a rule to produce a table (TI-83)				X	
Choosing Xmin, Xmax, Ymin, Ymax to set the viewing window (TI-83)				X	
Using a rule to produce a graph (TI-83)				X	
Using rules to produce non-linear graphs				X	
CHAPTER 3.					
Draw a line to fit the pattern in a plot		X			
Find the linear regression model for a set of data (TI-83)		X			
Make predictions using a linear model	X				
Writing equivalent equations by rearranging, combining, and expanding terms		X			
Solve an equation such as 3x + 12 = 45 without the use of a table or graph	X				
Using a table to find values of variables that satisfy the conditions of two equations (TI-83)			X		
Finding the rate of change (the slope) and y-intercept of a linear graph				X	
Use the equation of a linear model to make a quick sketch of the graph by hand				X	

Adrienne mentioned in every interview that her teachers took the time to explain things and were very helpful when she had questions. From Adrienne's point of view, this was crucial to her success in class, especially because her experiences with work in small groups (of students) did not prove to be as helpful to her as did raising her hand and receiving help from the teacher.

During the first interview, Adrienne said that the people in her group worked separately (side-by-side) for the most part, but that they would attempt to help one another when someone didn't understand; yet if they couldn't figure it out, they would then ask the teacher for assistance. By the second interview, Adrienne reported that she raised her hand and asked the teacher "a lot of questions" and that the girl she was usually paired with for small-group work could only help her somewhat. Then, in Adrienne's spring-term class, the teacher had ceased to allow students to work in small groups because they were messing around more than they were working. Even so, the teacher permitted students to ask a neighbor for help when needed, and he was readily available to assist students as well.

On the written questionnaires administered at each of the three interviews, Adrienne's responses were consistently very positive (almost all "agree" responses and a few "strongly agree" responses, which correspond to responses expected from successful students). There were just six instances, in fact, where her answers changed over the course of the semester. In the first case, Adrienne's answer changed from negative (Disagree) to positive (Agree) from the first to the second interview in response to the following two statements:

- I take part in whole-group class discussions on a regular basis.
- When a math problem is challenging, I usually keep working at it until I get it figured out.

In the second case, Adrienne's answer to three of the questions was more strongly positive at the end of the first term (second interview) than during the middle of either the fall or the spring terms (first and third interviews):

- Most of the things we're learning in Core-Plus Course 1 make sense to me.
- If I work at it hard enough, I know I can succeed in this class.
- I find it helpful that we use graphing calculators in this class.

In the third case, Adrienne's answer declined from positive (Agree) to neutral (Undecided) from the second to the third interview in response to the following question:

- Based on my experience in Core-Plus Course 1, I am looking forward to taking Core-Plus Course 2.

When I asked Adrienne about her ambivalence about taking the second Core-Plus course, she told me that her older brother was still trying to pass it as a junior, and this caused her to question her own chances of being able to pass the course.

Table 5.3
Adrienne's responses to attitude questionnaires from interview sessions

ATTITUDE SUB-CATEGORIES	Adrienne			Interviews – Attitude Questions
	29-Oct	17-Jan	11-Mar	
	Int 1	Int 2	Int 3	
Public Self-Confidence	2	4	4	1. I take part in whole-group class discussions on a regular basis.
	4	4	NA	2. I contribute my fair share of ideas during small-group investigations.
Personal Self-Confidence	4	5	4	3. Most of the things we're learning in Core-Plus Course 1 make sense to me.
	4	5	4	4. If I work at it hard enough, I know I can succeed in this class.
Enthusiasm	4	4	4	5. I am interested in the things we're learning in Core-Plus Course 1.
	4	4	3	6. Based on my experience in Core-Plus Course 1, I am looking forward to taking Core-Plus Course 2.
Significance of Math	4	4	4	7. The problems we work on help me see the usefulness of math in everyday life.
	4	5	4	8. I find it helpful that we use graphing calculators in this class.
Short-term Stick-to-it-iveness	2	4	4	9. When a math problem is challenging, I usually keep working at it until I get it figured out.
	4	4	4	10. I am putting forth my best effort in my Core-Plus Course 1 class.
RESPONSE MEAN:	3.6	4.3	3.9	

Adrienne's Performances on Algebra and Functions Tasks

Adrienne stood out among the six participants because of her consistently strong performance on the problem-solving tasks throughout the entire study. Her score for the contextualized problem task on the initial survey was perfect (z = +0.73) and that for the abstracted problem task was third highest (z = +1.89) among the group of 31 survey-takers. Adrienne's performance on the problem tasks posed during the interviews also revealed good understandings of algebra and functions. For all nine tasks, Adrienne scored fours, based on a four-point rubric used to evaluate a student's holistic solution for (i.e., ultimate success on) each problem task.

Table 5.4
Adrienne's scores on the interview problem-solving tasks

	Overall Solutions			Mean Score for all Individual Skills used	Z-score for Group Stats	Group Stats
	T1	T2	T3			
Interview 1	4	4	4	3.46	+0.20	Mean = 3.263 StDevP = 1.009
Interview 2	4	4	4	3.90	+0.64	Mean = 3.320 StDevP = 0.912
Interview 3	4	4	4	3.48	+0.44	Mean = 2.928 StDevP = 1.269

Table 5.5
Scoring Rubric for interview problem-solving tasks

4	Student did/understood this satisfactorily
3	Student eventually did/understood this with support from the interviewer
2	Student did/understood this somewhat/partially
1	Student did not do/understand this satisfactorily
0	Student did not attempt this method/approach

A closer examination of the individual skills on each task also showed that

Adrienne possessed good algebra skills and understandings of algebra and functions. Her task-skills performance means on all tasks and sub-tasks ranged from 3.00 to 4.00, where 3 indicates that the student satisfactorily completed a skill or understood a concept with scaffolding from the interviewer, and 4 indicates that the student satisfactorily completed a skill or understood a concept all on her own. (For a complete listing of all the task-skills that were evaluated, see Appendix B.)

Adrienne equaled or scored higher than the mean for all six participants on all tasks and subtasks as well. In the majority of instances, her own mean score exceeded the group's mean score by a noticeable amount. This is not to say that Adrienne's performances on these tasks was flawless, but her overall performance was the best of the interview cohort.

Table 5.6
Adrienne's performance means for evaluation items

Performance Mean	Adrienne	MEAN for all 6 participants
Int1 Task-Skills Combined	**3.46**	**3.25**
Int1 - Task 1	**3.29**	2.91
Int1 - Task 2	**4.00**	3.75
Int1 - Task 3	**3.50**	3.51
Int2 Task-Skills Combined	**3.90**	**3.46**
Int2 - Task 1	**3.88**	3.14
Int2 - Task 2	**4.00**	3.65
Int2 - Task 3a-3d	**4.00**	3.85
Int2 - Task 3e-3f	**3.78**	3.08
Int3 Task-Skills Combined	**3.48**	**2.91**
Int3 -Task 1	**3.00**	2.90
Int3 -Task 2	**3.82**	3.12
Int3 -Task 3, concrete	**3.83**	3.14
Int3 -Task 3, abstracted	**3.00**	1.88
MEAN for 3 INTERVIEWS	**3.62**	**3.21**
StDevP		0.486
Z-score for 3-Interview Mean	**0.84**	

Most remarkable was how well Adrienne did on the abstracted items of the third interview's Task 3, on which all other participants struggled (Adrienne's mean score was 3.00 while other participants' scores ranged from 1.00 to 2.50, with a mean of 1.65 for those five participants). What's more, according to Adrienne's self-assessment of her entering knowledge of these particular topics, these were not skills that she possessed prior to her experience in Mr. Larson's Core-Plus Course 1 class (see Table 5.2 above). Of all six participants, Adrienne seemed to have the best recall of a variety of things studied several months prior with just a bit of support on my part.

Sara

Sara was a 14-year-old ninth-grader, taking Core-Plus Course 1 for the first time. Sara's parents were divorced, and she lived—seemingly as an only child—with her mother.

Distinguishing Characteristics of Sara

My overall, lasting impression of Sara was of her emotional detachment with me. Her primary loyalties seemed to lie with a particular peer group, with whom she "clowned" both in and outside of class. She seemed to have no concern for what her teachers (and I) thought of her and her actions, but she did seem highly conscious of maintaining the approval of her friends.

Sara was generally soft-spoken but very matter-of-fact with her interview responses; it was usually hard to get her to elaborate. In fact, I would frequently offer a summary of what I perceived Sara to be saying, and would ask if it was correct—hoping she would say more herself—but most often she would just nod in the affirmative. (Even so, when she disagreed with my paraphrasing, she spoke up definitively.)

Sara approached her class work in much the same way: She put forth effort on that which came easily, but had no interest in pressing farther on that which took extra thought, hard work, or sustained effort (such as proving an answer or double-checking it for accuracy). Sara was quite confident in her own abilities—sometimes too much so for her successful completion of mathematics problems—and seemed satisfied that her first try would get her by well enough. Indeed, she could do better on most math tasks with much less effort than many other students.

Nevertheless, in my first interview with Sara, she seemed reliant on the calculator for certainty about her calculations. Her confidence in her own mental arithmetic was not great, and she did make some minor errors at times. It took my suggestion that she do so to get her to use the calculator initially. When she did use it, there seemed to be a boost in her self-confidence regarding the accuracy of her answers. Still, she didn't persevere with the calculator, similar to the tendency noted above; it was a tool to assist her in her first try at an answer, but it was not seen as a tool for exploring a problem-solving situation more deeply. Nonetheless, over time, Sara became quite adept at using the calculator; her recall for procedures and techniques involving the TI-83 was greater than that of most interviewees, yet this potential often went untapped because Sara simply lacked the interest in pursuing most problems beyond the minimum required.

Sara's minimal-effort approach got her into trouble at times when it didn't fit with the teachers' expectations for her work ethic. If Sara understood the key concept early in the lesson, she might sleep for the rest of the class period; or if Sara knew points weren't going to be given for written work, Sara might refuse to do it. And, when pressed to conform to the teachers' expectations, Sara acted out with stubbornness and rebellion. Nonetheless, Sara's academic work was good enough to earn her passing grades, which was satisfactory to her (Question 13, Initial Survey, 10/04/01).

Sara's Previous Non-Success in School Mathematics

Personal math history. Sara reported that, in sixth grade, she was the top math student in her class. Consequently, as a seventh-grader, she was placed in an Eighth Grade Math class along with other sixth- and seventh-grade students. However, she did not experience success.

Sara struggled to understand the topics being studied in the Eighth Grade Math class and asked to be placed into Seventh Grade Math instead, although her request was not granted. Sara did poorly enough in Eighth Grade Math during her seventh-grade year that she had to retake the course during her eighth-grade year. Thus, she began the Core-Plus curriculum as a ninth-grader.

Entering attitudes towards mathematics. Sara's general opinion of math was that it was "okay" but "not [her] favorite subject" yet she felt she did "pretty good" in it. When asked to describe her most successful experience in a math class, or a positive memory that stood out, Sara explained that her knowledge of math helped her in other classes, such as when she has to measure stock in shop class. She also said she used math outside of school when shopping to add up the prices before paying or to figure sale discounts. (Interview One, 10/31/01)

Compared to the entire group of 31 students who took the initial survey, Sara's response mean on the attitude survey placed her above the group's response mean (z-score = +0.35). This score ranked Sarah ninth (tied with one other student) among the 31 survey-takers when ranked from highest response mean (rank of 1) to lowest (rank of 31), placing her in the upper part of the third quartile (although her response mean was nevertheless only slightly above neutral at 3.28).

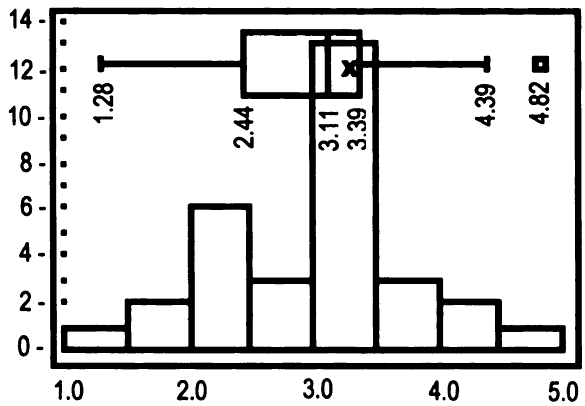

Figure 5.1
Histogram & Overlaid Boxplot of mean scores on initial survey attitude questions, showing Sara's placement

Sara's survey responses identified public self-confidence, short-term stick-to-it-iveness, and personal self-confidence as her strongest suits. In each of these categories, Sara's responses were above neutral (indicating they were responses expected from successful students) and above average for the entire group of survey-takers. The areas in which Sara's responses revealed below-neutral attitudes (indicating they were responses expected from unsuccessful students) were enthusiasm for math as a subject and significance (seeing math as an important area of study). While Sara's score in the significance category was below the mean for the group of survey-takers (z = -0.13), her enthusiasm score was actually above average for the group (z = +0.20). Sara's mean score in the category of long-term stick-to-it-iveness was exactly neutral and almost exactly the same as the mean score for the group (z = +0.04).

Table 5.7
Sara's scores on initial survey attitude questions by category[4]

	Response Mean	Means within categories: Self-confidence, Public	Self-confidence, Personal	Enthusiasm	Significance	Stick-to-it-ive-ness, Long-term	Stick-to-it-ive-ness, Short-term
SARA	**3.28** z = +0.36	**4.0** z = +1.10	**3.3** z = +0.33	**2.7** z = +0.20	**2.7** z = -0.13	**3.0** z = +0.04	**4.0** z = +0.60
MEANS (n=31):	3.01	3.20	2.99	2.46	2.80	2.96	3.43
StDevP:	0.752	0.722	1.042	1.050	1.033	1.057	.951

Sara's Experiences of the Core-Plus Course 1A Curriculum

Sara's initial opinion of Core-Plus Course 1 was that it was "easy" but that it repeated a lot of the same mathematics that she had done in previous math courses. The familiarity checklists that Sara completed at each interview seemed to support the assertion that she was already familiar with most topics of Core-Plus Course 1A prior to her study of them in ninth grade. Indeed, with what seemed to be minimal effort on Sara's part, she was receiving above-average grades in Core-Plus Course 1.

[4]Complete tables of initial survey response data can be found in Appendix B.

Table 5.8
Sara's Familiarity Checklists

Below is a list of the topics in Chapter N of your math book. For each one, mark an X in one of the boxes to show which statement best describes how familiar you were with the topic before this semester.	I already knew how to do these very well.	I already knew how to do these somewhat.	I had seen these before, but I knew very little about them.	I had never seen these before, and I did not know anything about them.	I don't remember studying this in this chapter.
CHAPTER 1.					
Stem-and-leaf plots	X				
Number line plots	X				
Histograms		X			
Mean, median, mode	X				
Five-number summary		X			
Box plots		X			
Scatterplots		X			
Plots over time			X		
CHAPTER 2.					
Using a y=x line		X			
Graphing (x,y) data		X			
Writing a NOW-NEXT expression			X		
Using the ANS key to calculate numbers repeatedly (TI-83)	X				
Writing a rule using letters		X			
Using a rule to produce a table (TI-83)		X			
Choosing Xmin, Xmax, Ymin, Ymax to set the viewing window (TI-83)		X			
Using a rule to produce a graph (TI-83)		X			
Using rules to produce non-linear graphs		X			
CHAPTER 3.					
Draw a line to fit the pattern in a plot		X			
Find the linear regression model for a set of data (TI-83)			X		
Make predictions using a linear model		X			
Writing equivalent equations by rearranging, combining, and expanding terms		X			
Solve an equation such as 3x + 12 = 45 without the use of a table or graph	X				
Using a table to find values of variables that satisfy the conditions of two equations (TI-83)		X			
Finding the rate of change (the slope) and y-intercept of a linear graph	X				
Use the equation of a linear model to make a quick sketch of the graph by hand	X				

By the third interview, Sara's general opinion of Core-Plus Course 1 was that it was "boring." The aspect she enjoyed least about the class was "all the work that we have to do." What she enjoyed most was the fact that her friends were in the same class and that they clowned around together. Still, Sara continued to receive above-average grades in the class.

On the written questionnaires administered at each of the three interviews, Sara's responses were largely positive (almost all "agree" responses and a few "strongly agree" responses, which correspond to responses expected from successful students). However, responses to six of the ten questions were lower on the third interview questionnaire (completed in April 2002) than they were on the first (completed in late October 2001). Indeed, Sara's questionnaire response mean dropped from 4.3 to 3.5 from Interview One to Interview Three.

Table 5.9
Sara's responses to attitude questionnaires from interview sessions

ATTITUDE SUB-CATEGORIES	Sara			Interviews -- Attitude Questions
	31-Oct	8-Jan	9-Apr	
	Int 1	Int 2	Int 3	
Public Self-Confidence	4	4	4	1. I take part in whole-group class discussions on a regular basis.
	4	4	4	2. I contribute my fair share of ideas during small-group investigations.
Personal Self-Confidence	4	4	4	3. Most of the things we're learning in Core-Plus Course 1 make sense to me.
	5	4	3	4. If I work at it hard enough, I know I can succeed in this class.
Enthusiasm	4	3	3	5. I am interested in the things we're learning in Core-Plus Course 1.
	4	4	3	6. Based on my experience in Core-Plus Course 1, I am looking forward to taking Core-Plus Course 2.
Significance of Math	5	4	3	7. The problems we work on help me see the usefulness of math in everyday life.
	5	4	4	8. I find it helpful that we use graphing calculators in this class.
Short-term Stick-to-it-iveness	4	4	4	9. When a math problem is challenging, I usually keep working at it until I get it figured out.
	4	3	3	10. I am putting forth my best effort in my Core-Plus Course 1 class.
RESPONSE MEAN:	4.3	3.8	3.5	

While one cannot conclude solely on the basis of the questionnaire data that Sara's attitude toward her math studies declined during the course of the study, there was a notable decline in attitude reported by Sara herself across interviews, which was corroborated by my occasional classroom observations from October to April. For instance, Sara was initially not included in the target group of interview participants due to seemingly productive levels of in-class participation. Yet, in Interview Two, Sara reported that she regularly slept through her math classes and got two disciplinary referrals in two weeks' time from her math teacher. Moreover, in Interview Three, Sara indicated that she was not in the habit of ever opening her math book and stated that the presence of her friends and their clowning around in class was what she enjoyed most about her math class during the second half of the school year.

Sara's Performance on Algebra and Functions Tasks

Despite the fact that Sara's attitude toward her math studies declined throughout the study, she consistently demonstrated above-average achievement on the algebra and functions tasks. Indeed, Sara's scores for the two problem tasks on the initial survey were perfect and above-average in comparison to the group of 31 survey-takers, equaled by only one other student (contextual problem score of 4, group mean score of 3.15; abstract problem score of 4, group mean score of 0.77). (See complete chart of Initial Survey results on the problem tasks in Appendix B.)

Sara's performance throughout the study on the problem tasks posed during the interviews also revealed good understandings of algebra and functions. For eight of the nine tasks, Sara scored fours, based on a four-point rubric used to evaluate a student's holistic solution for (i.e., ultimate success on) each problem task.

Table 5.10
Sara's Scores on the interview problem-solving tasks

	Overall Solutions			Mean Score for all Individual Skills used	Z-score for Group Stats	Group Stats
	T1	T2	T3			
Interview 1	4	4	4	3.71	+0.45	Mean = 3.263 StDevP = 1.009
Interview 2	4	4	4	3.68	+0.39	Mean= 3.320 StDevP = 0.912
Interview 3	4	4	2	3.37	+0.35	Mean = 2.928 StDevP = 1.269

A closer examination of the individual skills on each task also shows that Sara possessed good algebra skills and understandings of algebra and functions. With the exception of the abstract portion of Task Three on Interview Three, Sara's task-skills performance means (for the 10 other tasks and sub-tasks) ranged from 3.25 to 4.00, where 3 indicates that the student satisfactorily completed a skill or understood a concept with support from me, and 4 indicates that the student satisfactorily completed a skill or understood a concept all on her own. (For a complete listing of all the task-skills that were evaluated, see Appendix B.)

Table 5.11
Sara's performance means for evaluation items

Performance Mean	Sara	MEAN for all 6 participants
Int1 Task-Skills Combined	**3.71**	**3.25**
Int1 - Task 1	**3.75**	2.91
Int1 - Task 2	**4.00**	3.75
Int1 - Task 3	**3.50**	3.51
Int2 Task-Skills Combined	**3.68**	**3.46**
Int2 - Task 1	**3.43**	3.14
Int2 - Task 2	**3.80**	3.65
Int2 - Task 3a-3d	**3.90**	3.85
Int2 - Task 3e-3f	**3.56**	3.08
Int3 Task-Skills Combined	**3.37**	**2.91**
Int3 -Task 1	**3.25**	2.90
Int3 -Task 2	**3.83**	3.12
Int3 -Task 3, concrete	**4.00**	3.14
Int3 -Task 3, abstracted	**1.25**	1.88
MEAN for 3 INTERVIEWS	**3.59**	**3.21**
StDevP		0.486
Z-score for 3-Interview Mean	**0.78**	

Amy

Amy was a 15-year-old ninth-grader taking Core-Plus Course 1 for the first time. Amy lived with both her parents and a couple of siblings.

Distinguishing Characteristics of Amy

My overall, lasting impression of Amy was of her boredom with the subject of math.

> Amy's responses to the personal math history questions confirmed what I had already observed: She does very well in math (has always gotten As or Bs), but she does not like it as a school subject. She says she finds the subject really "boring." Indeed, from my in-class observations, she appears very bored. (Memo from Interview One, 11/07/01)

Amy's typical classroom demeanor was quiet and compliant. Indeed, one of her teachers referred to her as the "girl in the back who doesn't say anything" (Observation Notes, 1/24/02). Despite her silence, one could tell that Amy participated in the class; she smiled at the teacher's jokes and responded when spoken to. She listened when the teacher demonstrated something at the board and took notes, asked questions when necessary, but for the most part seemed to prefer to be a silent participant in class. She appeared to do her work in class and get it done quickly, and then have nothing else to do that interested her.

Amy's Previous Non-Success in School Mathematics

Personal math history. Of the six participants, Amy is one whose previous school math experiences do not reveal obvious instances of non-success. Amy reported that throughout middle school, she had gotten As and Bs in math. Still, she was not placed in Core-Plus Course 1 in the eighth grade as many students were.

It was in her 8th-grade math class, however, that Amy reported having had her least successful experience (a negative memory that stood out). The teacher of the class, Amy reported, "didn't teach us anything." Instead, the students worked on their own from the book, and the teacher would interact if the students initiated questions. Otherwise, he just let them work without explaining anything to them. Indeed, Amy's familiarity checklists differed noticeably from the other six participants' in that she seemed far less familiar with many topics in Chapters 1 and 2 than other participants taking Core-Plus Course 1 for the first time. Yet, by contrast, her familiarity with Chapter 3 topics was actually noticeably better than others'.

Table 5.12
Amy's Familiarity Checklists

Below is a list of the topics in Chapter N of your math book. For each one, mark an X in one of the boxes to show which statement best describes how familiar you were with the topic <u>before</u> <u>this</u> <u>semester</u>.	I already knew how to do these very well.	I already knew how to do these somewhat.	I had seen these before, but I knew very little about them.	I had never seen these before, and I did not know anything about them.	I don't remember studying this in this chapter.
CHAPTER 1.					
Stem-and-leaf plots	X				
Number line plots		X			
Histograms				X	
Mean, median, mode	X				
Five-number summary				X	
Box plots				X	
Scatterplots			X		
Plots over time				X	

Table 5.12 (cont'd)

CHAPTER 2.					
Using a y=x line		X			
Graphing (x,y) data		X			
Writing a NOW-NEXT expression			X		
Using the ANS key to calculate numbers repeatedly (TI-83)				X	
Writing a rule using letters		X			
Using a rule to produce a table (TI-83)				X	
Choosing Xmin, Xmax, Ymin, Ymax to set the viewing window (TI-83)				X	
Using a rule to produce a graph (TI-83)			X		
Using rules to produce non-linear graphs				X	
CHAPTER 3.					
Draw a line to fit the pattern in a plot	X				
Find the linear regression model for a set of data (TI-83)					X
Make predictions using a linear model	X				
Writing equivalent equations by rearranging, combining, and expanding terms		X			
Solve an equation such as 3x + 12 = 45 <u>without</u> the use of a table or graph		X			
Using a table to find values of variables that satisfy the conditions of two equations (TI-83)		X			
Finding the rate of change (the slope) and y-intercept of a linear graph		X			
Use the equation of a linear model to make a quick sketch of the graph by hand	X				

Entering attitudes towards mathematics. Despite Amy's above-average grades in past mathematics classes, her responses to the attitude questions on the initial survey revealed some surprisingly below-average scores when compared to the entire group of survey-takers or to the group of six participants. Amy scored below neutral (indicating that her responses were like those expected from unsuccessful students) and below average for the group of 31 survey-takers in three of the six categories (enthusiasm for math, public and personal self-confidence). Amy also scored below average for the group of 31 survey-takers in the short-term stick-to-it-iveness category ($z = -0.46$), although her mean score was neutral. Amy's scores in three of these aforementioned categories (enthusiasm for math, public self-confidence, and short-term stick-to-it-iveness) were the lowest of all six participants as well.

Table 5.13
Amy's scores on initial survey attitude questions by category

	Response Mean	Means within categories					
		Self-confidence, Public	Self-confidence, Personal	Enthusiasm	Significance	Stick-to-it-ive-ness, Long-term	Stick-to-it-ive-ness, Short-term
AMY	**2.94** z = -0.09	**2.7** z = -0.74	**2.7** z = -0.31	**1.7** z = -0.76	**3.3** z = +0.51	**4.3** z = +1.30	**3.0** z = -0.46
MEANS (n=31):	3.01	3.20	2.99	2.46	2.80	2.96	3.43
StDevP:	0.752	0.722	1.042	1.050	1.033	1.057	0.951

By contrast, Amy scored above neutral (indicating that her responses were like those expected from successful students) and above average in the other two categories (long-term stick-to-it-iveness and seeing math as an important area of study). It must be noted, however, that Amy's actual responses to the questions in the significance category were, on average, only slightly above neutral. Amy's mean scores in both of these categories were the highest of the six participants' scores, yet it is Amy's mean score in the long-term stick-to-it-iveness category that sharply stands out.

Amy was the only one of the six participants with a score in the long-term stick-to-it-iveness category (pursuing the study of math throughout high school and college) that was above neutral (indicating her responses were like those expected from successful students). Moreover, hers is the third highest score of all 31 survey-takers in this category. Were it not for Amy's successful disposition toward the pursuit of math throughout high school and college, Amy's overall response mean on the initial survey attitude questions would not have been seemingly so "average" among the 31 survey-takers (z = -0.09). As it was, since the majority of Amy's responses pertaining to her current school math experience were neutral or negative (i.e., consistent with responses expected from unsuccessful students), she placed 20th among the group of 31, in the middle of the second quartile.

Table 5.14
Amy's responses to the initial survey attitude questions

Questions pertaining to current in-school experience

Responses like those expected from successful students:

I would be happy to get a C as my grade in Core-Plus Course 1.					SD
I often volunteer to answer questions in math class.		A			

Neutral responses:

I understand math well enough to help a classmate who is struggling.			U		
I am able to tell for myself whether an answer on my math homework is correct (or at least reasonable) without looking up the answer.			U		
When a math problem is challenging, I usually keep working at it until I get it figured out.			U		
I usually check over my answers on a math test before turning it in.			U		
When I get back my graded math tests, I usually go over the problems I missed so I understand why I got them wrong.			U		
Math often does not make sense to me.			U		

Responses like those expected from unsuccessful students:

I am naturally good at math.				D	
I love playing math games in class where you have to be the fastest one to answer in order to win.				D	
I do not like to go to the board to show my answer to a math homework problem.	SA				
Math was one of my favorite subjects in middle school.					SD

Table 5.14 (cont'd)

Questions pertaining to plans beyond high school

Responses like those expected from successful students:

I want to graduate from high school with an Academic Honors Diploma.	SA				
A student doesn't really need to take 4 years of math in high school unless she (or he) is going to be a math major in college.				D	
A student doesn't need more than 2 years of math in high school if he (or she) is not going to go to college.				D	

Neutral response:

I do not plan to take more math courses beyond the two that are required to graduate from high school (Core-Plus Courses 1 and 2).			U		

Responses like those expected from unsuccessful students:

I can imagine myself completing a math-related major in college.				D	
I want to get a job that uses as little math as possible.		A			

Amy's Experiences of the Core-Plus Course 1A Curriculum

Amy's initial opinion of Core-Plus Course 1 was that it was "boring." At the end of the fall term, Amy reported feeling the very same way, although her feelings apparently were not particular to Core-Plus Course 1, but pertained to math classes in general:

Amy: I don't like math. (laugh) I never have! (laugh) ... I hate math. I've never had very good, I've never had teachers that I liked very well.
Interviewer: You still do really well in it, though, it seems like.
Amy: It amazes me 'cause I didn't do very well in it last year, oh, well, I got like a B. I've never had very good teachers so I just don't like math. Plus it's boring.
(Interview Two, 12/18/01)

Interestingly, however, during her third interview, in the middle of the spring term, Amy reported that it was the first time in her life that she had enjoyed a math class. She attributed this to the teacher who, she said, made the class fun by joking around. "It's more interesting in [his] class, so you pay more attention, but he's a little bit harder on you. He, like, pushes you more" (Interview Three, 3/12/02).

Still, Amy's attitude questionnaires from the three interviews didn't reveal any real changes over time. Her enthusiasm for math continued to be the category in which her responses were lowest (most like responses expected from unsuccessful students). She also remained quite neutral in her short-term stick-to-it-iveness, despite her agreement with the statement "If I work hard enough at it, I know I can succeed in this class."

Table 5.15
Amy's responses to attitude questionnaires from interview sessions

ATTITUDE SUB-CATEGORIES	Amy			Interviews -- Attitude Questions
	5-Nov	17-Dec	12-Mar	
	Int 1	Int 2	Int 3	
Public Self-Confidence	4	4	3	1. I take part in whole-group class discussions on a regular basis.
	4	3	4	2. I contribute my fair share of ideas during small-group investigations.
Personal Self-Confidence	3	4	4	3. Most of the things we're learning in Core-Plus Course 1 make sense to me.
	4	4	4	4. If I work at it hard enough, I know I can succeed in this class.
Enthusiasm	3	3	3	5. I am interested in the things we're learning in Core-Plus Course 1.
	3	2	3	6. Based on my experience in Core-Plus Course 1, I am looking forward to taking Core-Plus Course 2.
Significance of Math	4	3	3	7. The problems we work on help me see the usefulness of math in everyday life.
	5	5	4	8. I find it helpful that we use graphing calculators in this class.
Short-term Stick-to-it-iveness	3	3	3	9. When a math problem is challenging, I usually keep working at it until I get it figured out.
	4	3	3	10. I am putting forth my best effort in my Core-Plus Course 1 class.
RESPONSE MEAN:	3.7	3.4	3.4	

Amy's Performance on Algebra and Functions Tasks

To sum up my assessment of Amy's performance on the problem-solving tasks, I would say that she has very strong intuitions about mathematics. Amy used shortcuts in her first attempt at a problem (e.g., using an explicit formula rather than working iteratively to develop a table of numbers) where others struggled to envision such a shortcut, even when prompted. She often seemed to make her way easily through problem tasks that other participants found difficult or challenging. And in several instances, despite insisting she did not remember ever doing such problems in class, Amy outperformed other participants after receiving a minimal amount of coaching.

On five of the nine problem-solving tasks that were posed, Amy was successful in completing these on her own (those with scores of 4). For the other four tasks (those with scores of 3), Amy successfully completed them with my assistance.

Table 5.16
Amy's scores on the interview problem-solving tasks

	Overall Solutions			**Mean Score for all Individual Skills used**	**Z-score for Group Stats**	**Group Stats**
	T1	**T2**	**T3**			
Interview 1	3	4	3	3.20	-0.06	Mean = 3.263 StDevP = 1.009
Interview 2	4	4	4	3.54	+0.24	Mean = 3.320 StDevP = 0.912
Interview 3	4	3	3	3.36	+0.34	Mean = 2.928 StDevP = 1.269

In terms of individual skills used to solve each of the problem tasks, Amy's mean scores hovered near average or a bit above. Amy's biggest challenges were in working with the mathematical ideas in abstraction, apart from the context of the problem:

> Interestingly, for both Parts A and C [of Item 2 on Task 2], Amy went directly to wanting/trying to write the equation out by immediately inserting a die-value. I asked her to stick with the X first, which she did. Yet, her first inclination was to go straight to the application rather than stick with the abstraction. She seemed far more at ease when working with the application (calculating a value using the number from the die) than when working with the equation in the abstract (writing out the equation leaving it in terms of the variable X).
> (Memo of Interview Three, 3/12/02)

The same was true for Items e and f on Task 3 of Interview 2: Amy struggled to think beyond the immediate context of the problem to answer theoretically, based on the pattern that could be seen in the numbers being examined.

Table 5.17
Amy's performance means for evaluation items

Performance Mean	Amy	MEAN for all 6 participants
Int1 Task-Skills Combined	**3.20**	**3.25**
Int1 - Task 1	**2.78**	2.91
Int1 - Task 2	**4.00**	3.75
Int1 - Task 3	**3.75**	3.51
Int2 Task-Skills Combined	**3.54**	**3.46**
Int2 - Task 1	**3.38**	3.14
Int2 - Task 2	**4.00**	3.65
Int2 - Task 3a-3d	**4.00**	3.85
Int2 - Task 3e-3f	**2.40**	3.08
Int3 Task-Skills Combined	**3.36**	**2.91**
Int3 -Task 1	**3.25**	2.90
Int3 -Task 2	**3.40**	3.12
Int3 -Task 3, concrete	**4.00**	3.14
Int3 -Task 3, abstracted	**2.50**	1.88
MEAN for 3 INTERVIEWS	**3.36**	**3.21**
StDevP		0.486
Z-score for 3-Interview Mean	**0.31**	

Joy

Joy was a 15-year-old ninth-grader, taking Core-Plus Course 1 for the first time. Joy lived with her father, stepmother, and four siblings (ranging in age from 5 to mid-20s) in a two-bedroom house.

Distinguishing Characteristics of Joy

I am left with two overall, lasting impressions of Joy: her amazing ability to rise above her circumstances and her brilliantly disguised contempt for mathematics. Joy had very difficult family relationships, yet she was a wonderfully cheerful and optimistic person. Although she seemed to receive no familial support for her endeavors, Joy participated in the school orchestra and was very actively involved in her church youth group.

When I observed Joy in class, she was engaged, worked diligently, frequently asked questions of the teacher, and displayed a clever sense of humor. She always spoke in complete sentences and her speech was peppered with a sophisticated vocabulary. Other students seemed to regard Joy as the one to turn to if they had questions about their assignments in class. Indeed, she was earning above-average grades in Core-Plus Course 1. Yet, in my interviews with Joy, I was astonished to learn that she absolutely despised mathematics and saw no use for it whatsoever.

When I pointed out to Joy that her productive classroom behaviors and her good grades in math seemed very atypical given her strong insistence of dislike for the subject, she attributed it to her "work ethic." It was also Joy's desire to attend a private college that drove her to do her very best in school. She explained that she had a couple of opportunities for college scholarships that were contingent on her continuing to get good grades in high school. One of these was Indiana's Twenty-first Century Scholars program in which Joy was enrolled. The program grants tuition awards for up to four years of college at in-state (public or private) institutions of higher education. In order to qualify, students must enroll in the program as a seventh- or eighth-grader, must meet income eligibility (i.e., qualify for free or reduced lunch at school), and must make a commitment to complete high school with a GPA of 2.0 or better and not use illegal drugs or alcohol nor commit a crime.

Joy's Previous Non-Success in School Mathematics

Personal math history. It had not always been true that Joy had gotten good grades in her math classes. She reported that in the sixth grade her math grades ranged from Ds to C-plusses, and in seventh grade from Ds to As. It was not until the eightth grade that Joy's math grades were consistently high Bs or As. Perhaps it was Joy's enrolling in the Twenty-first Century Scholars program that caused the change, although I never had the occasion to inquire. It might also have been Joy's low grades in sixth and seventh grade that kept her from starting Core-Plus Course 1 until the ninth grade, rather than in the eighth grade as some students did.

Entering attitudes towards mathematics. Joy's general opinion of math was one of strong dislike for the subject:

> I can't stand it 'cause I see no reason why, how it's gonna help me when I am older. Like if I'm standing at the end of the line, and I get my groceries done, and like they say, "You can't have this! unless you tell me the area of this cylinder!" (Interview One, 11/08/01)

When asked if she ever used math outside of school, Joy said, "I'm sure I do, but not that I can think of" (Interview One, 11/08/01).

Joy's responses to the attitude questions on the initial survey also revealed a very curious mix of attitudes towards math. On the one hand, Joy was adamant in her survey responses that math often does not make sense to her, that she is not naturally good at math, and that math was not one of her favorite subjects in middle school. Yet, on the other hand, she said she was able to tell for herself whether an answer on her math homework was correct (or at least reasonable) without looking up the answer and that she understands math well enough to help a classmate who is struggling. She strongly disagreed that she wanted to get a job that uses as little math as possible and said she could imagine herself completing a math-related major in college.

True to what I observed of Joy in class, all of her responses to questions about classroom behaviors were consistent with those expected of successful students. However, for most questions pertaining to plans beyond high school, Joy's responses were much more typical of unsuccessful students.

Table 5.18
Joy's responses to the initial survey attitude questions

Questions pertaining to current in-school experience
Responses like those expected from successful students:

I understand math well enough to help a classmate who is struggling.		A			
I am able to tell for myself whether an answer on my math homework is correct (or at least reasonable) without looking up the answer.		A			
When a math problem is challenging, I usually keep working at it until I get it figured out.		A			
I usually check over my answers on a math test before turning it in.		A			
When I get back my graded math tests, I usually go over the problems I missed so I understand why I got them wrong.		A			
I often volunteer to answer questions in math class.		A			
I do not like to go to the board to show my answer to a math homework problem.				D	
I would be happy to get a C as my grade in Core-Plus Course 1.				D	

Responses like those expected from unsuccessful students:

Math often does not make sense to me.	SA				
I am naturally good at math.					SD
Math was one of my favorite subjects in middle school.					SD
I love playing math games in class where you have to be the fastest one to answer in order to win.					SD

Table 5.18 (cont'd)

<u>Questions pertaining to plans beyond high school</u>

Responses like those expected from successful students:

I want to get a job that uses as little math as possible.					SD
I can imagine myself completing a math-related major in college.		A			

Neutral response:

A student doesn't really need to take 4 years of math in high school unless she (or he) is going to be a math major in college.			U		

Responses like those expected from unsuccessful students:

I do <u>not</u> plan to take more math courses beyond the two that are required to graduate from high school (Core-Plus Courses 1 and 2).		A			
A student doesn't need more than 2 years of math in high school if he (or she) is not going to go to college.		A			
I want to graduate from high school with an Academic Honors Diploma.				D	

Not surprisingly, Joy's overall mean score for the initial attitude survey yielded a perfectly-neutral score of 3.00. Compared to the entire group of 31 students who took the initial survey, Joy's score was almost exactly the same as the group's response mean (z = -0.01).

Table 5.19
Joy's scores on initial survey attitude questions by category

		Means within categories					
	Response Mean	Self-confidence, Public	Self-confidence, Personal	Enthusiasm	Significance	Stick-to-it-ive-ness, Long-term	Stick-to-it-ive-ness, Short-term
JOY	**3.00** z = -0.01	**4.0** z = +1.10	**2.0** z = -0.95	**2.0** z = -0.44	**3.3** z = +0.51	**2.7** z = -0.28	**4.0** z = +0.60
MEANS (n=31):	3.01	3.20	2.99	2.46	2.80	2.96	3.43
StDevP:	0.752	0.722	1.042	1.050	1.033	1.057	.951

Joy's survey responses identified public self-confidence and short-term stick-to-it-iveness as her strongest suits. In each of these categories, Joy's responses were above neutral (indicating they were responses expected from successful students) and above average for the entire group of survey-takers. In fact, in both of these categories, Joy also scored highest out of all six participants. While Joy's response mean in the category of significance of math (seeing it as an important area of study) was closer to neutral, it also was above neutral and above average for the group of 31 survey-takers as well as for the group of six participants.

The areas in which Joy's responses revealed below-neutral attitudes (indicating they were responses expected from unsuccessful students) were personal self-confidence, enthusiasm for math as a subject and long-term stick-to-it-iveness (pursuing the study of mathematics in high school and college). These last two items are not at all surprising; however, Joy's lowest score comparatively, in the area of personal self-confidence ($z = -0.95$), seemed quite puzzling given her recent track record in math classes.

Joy's Experiences of the Core-Plus Course 1A Curriculum

Joy's initial opinion of Core-Plus Course 1 was that it was a lot easier than she thought it would be: "I don't know, I just thought it'd be harder than my eighth-grade class, and it wasn't" (Interview One, 11/08/01). Indeed, the topics in Chapter 1 were nothing new to Joy for the most part, according to her familiarity checklist:

Table 5.20
Joy's familiarity checklist for Chapter 1 topics

Below is a list of the topics in Chapter N of your math book. For each one, mark an X in one of the boxes to show which statement best describes how familiar you were with the topic before this semester.	I already knew how to do these very well.	I already knew how to do these somewhat.	I had seen these before, but I knew very little about them.	I had never seen these before, and I did not know anything about them.	I don't remember studying this in this chapter.
CHAPTER 1.					
Stem-and-leaf plots	X				
Number line plots	X				
Histograms		X			
Mean, median, mode	X				
Five-number summary				X	
Box plots	X				
Scatterplots	X				
Plots over time		X			

Yet this pleasant surprise didn't seem to have any influence on Joy's enjoyment of the class:

> Interviewer: What do you enjoy least about Core-Plus Course 1?
> Joy: I don't know, the fact that it's a math class. (laugh)
> (Interview One, 11/08/01)

Joy's feelings didn't change any throughout the school year, either. At each interview, when asked her general opinion of the class, Joy reiterated her dislike for the subject.

On the written questionnaires administered at each of the three interviews, Joy's responses were generally consistent over time, exhibiting the same curious mix of responses (fairly successful class behaviors and fairly negative attitudes towards the subject) with the mean overall scores hovering right around neutral. There was one question on the surveys, asked both initially as well as at each of the three interviews, however, where Joy's answers were not consistent over time: the question of whether or not math makes sense to the student. On the initial survey, Joy strongly agreed with "Math often does not make sense to me," yet in two of the three interviews, she agreed with "Most of the things we're learning in Core-Plus Course 1 make sense to me." (At the second interview, Joy was undecided.)

The only truly noticeable change over time was in Joy's involvement in small-group work. Initially, she reported contributing her fair share despite the fact that others in the group didn't do much work, other than to copy down answers (without understanding the problems) when the teacher went over them. By the end of the fall-term, the "group" work had become a formality where students sat in their assigned groupings but worked independently (if at all). Then, during the spring term, Joy's new teacher did not utilize small-group work in class.

Table 5.21
Joy's responses to attitude questionnaires from interview sessions

ATTITUDE SUB-CATEGORIES	Joy 8-Nov Int 1	Joy 14-Jan Int 2	Joy 12-Apr Int 3	Interviews – Attitude Questions
Public Self-Confidence	4	4	4	1. I take part in whole-group class discussions on a regular basis.
	5	1	NA	2. I contribute my fair share of ideas during small-group investigations.
Personal Self-Confidence	4	3	4	3. Most of the things we're learning in Core-Plus Course 1 make sense to me.
	5	5	4	4. If I work at it hard enough, I know I can succeed in this class.
Enthusiasm	1	1	1	5. I am interested in the things we're learning in Core-Plus Course 1.
	3	1	2	6. Based on my experience in Core-Plus Course 1, I am looking forward to taking Core-Plus Course 2.
Significance of Math	1	1	1	7. The problems we work on help me see the usefulness of math in everyday life.
	3	4	4	8. I find it helpful that we use graphing calculators in this class.
Short-term Stick-to-it-iveness	2	3	3	9. When a math problem is challenging, I usually keep working at it until I get it figured out.
	5	4	4	10. I am putting forth my best effort in my Core-Plus Course 1 class.
RESPONSE MEAN:	3.3	2.7	3.0	

Joy's Performance on Algebra and Functions Tasks

Perhaps my overall assessment of Joy's performance on the problem-solving tasks could be summed up by the following comment written in my first interview memo:

> I would have to say that Joy has the strongest constructivist skills of anyone! Basically, it comes down to the fact that Joy may not seem to start out very strong, but by the time she is done, she has discovered/revised/constructed a solution that is mathematically superior to the rest!
> (Memo of Interview One, 11/13/01)

Joy successfully completed all but two of the nine problem-solving tasks that were posed. Four of these seven tasks were done with my aid (those with scores of 3), while she satisfactorily understood the other three (those with scores of 4) on her own.

Table 5.22
Joy's scores on the interview problem-solving tasks

	Overall Solutions			Mean Score for all Individual Skills used	Z-score for Group Stats	Group Stats
	T1	T2	T3			
Interview 1	3	4	3	3.46	+0.20	Mean = 3.263 StDevP = 1.009
Interview 2	2	4	4	3.47	+0.16	Mean = 3.320 StDevP = 0.912
Interview 3	3	1	3	2.85	-0.06	Mean = 2.928 StDevP = 1.269

On the two tasks where Joy did not successfully complete the problems, there was a common theme among these tasks: Their mathematical content dealt with the most traditional of all algebra topics—making sense of the coordinate plane and symbolically manipulating equations to solve for an unknown. Even Joy herself, on several different occasions, declared her struggles in these areas: During the second interview she said she "never understood why Mr. Harper kept adding an X onto the end of problems like Y = 2 + 5X" (1/14/02). During the third interview she mentioned her unfamiliarity with the coordinate plane: "I understand <u>most</u> of the stuff I do, but some of it I don't get. Like that grid with the Y and the X" (4/12/02). Indeed, many of the topics having to do with these types of traditional algebra were completely new to Joy:

Table 5.23
Joy's familiarity checklists for Chapters 2 and 3

Below is a list of the topics in Chapter N of your math book. For each one, mark an X in one of the boxes to show which statement best describes how familiar you were with the topic <u>before this semester</u>.	I already knew how to do these very well.	I already knew how to do these somewhat.	I had seen these before, but I knew very little about them.	I had never seen these before, and I did not know anything about them.	I don't remember studying this in this chapter.
CHAPTER 2.					
Using a y=x line					X
Graphing (x,y) data			X		
Writing a NOW-NEXT expression	X				
Using the ANS key to calculate numbers repeatedly (TI-83)		X			
Writing a rule using letters	X				
Using a rule to produce a table (TI-83)	X				
Choosing Xmin, Xmax, Ymin, Ymax to set the viewing window (TI-83)	X				
Using a rule to produce a graph (TI-83)					X
Using rules to produce non-linear graphs			X		
CHAPTER 3.					
Draw a line to fit the pattern in a plot			X		
Find the linear regression model for a set of data (TI-83)				X	
Make predictions using a linear model			X		
Writing equivalent equations by rearranging, combining, and expanding terms		X			
Solve an equation such as 3x + 12 = 45 <u>without</u> the use of a table or graph			X		
Using a table to find values of variables that satisfy the conditions of two equations (TI-83)				X	
Finding the rate of change (the slope) and y-intercept of a linear graph				X	
Use the equation of a linear model to make a quick sketch of the graph by hand				X	

On the whole, Joy's performance on the problem tasks was average for the group of six participants. Her mean combined-skills scores for the three interviews placed her just above the mean for the entire group of six participants (z = +0.10).

Table 5.24
Joy's performance means for evaluation items

Performance Mean	**Joy**	**MEAN for all 6 participants**
Int1 Task-Skills Combined	**3.46**	**3.25**
Int1 - Task 1	**3.25**	2.91
Int1 - Task 2	**4.00**	3.75
Int1 - Task 3	**3.67**	3.51
Int2 Task-Skills Combined	**3.47**	**3.46**
Int2 - Task 1	**2.57**	3.14
Int2 - Task 2	**3.60**	3.65
Int2 - Task 3a-3d	**4.00**	3.85
Int2 - Task 3e-3f	**4.00**	3.08
Int3 Task-Skills Combined	**2.85**	**2.91**
Int3 -Task 1	**2.88**	2.90
Int3 -Task 2	**2.50**	3.12
Int3 -Task 3, concrete	**3.67**	3.14
Int3 -Task 3, abstracted	**2.25**	1.88
MEAN for 3 INTERVIEWS	**3.26**	**3.21**
StDevP		0.486
Z-score for 3-Interview Mean	**0.10**	

Terri

Terri was a 15-year-old sophomore, taking Core-Plus Course 1 for the second time. She lived with her grandmother and her younger brother.

Distinguishing Characteristics of Terri

My overall, lasting impression of Terri was of her resilience and the knack she had for chatting, almost non-stop, through our interviews. Terri was open and friendly and exhibited a happy-go-lucky attitude about school and about life in general, although it was clear that she was not free of struggles in either of these areas.

From the very first interview, Terri shared many details of her life with me as if we had been close friends for a long time. She loved to share the exciting news of her day, interesting anecdotes about friends and family members, and updates on her activities both in and out of school. From this, I gained tremendous insight into Terri's background, which served to underscore her amazing recovery from a tremendously unsuccessful school year she had had as a freshman. Most fascinating was the mature outlook she possessed as a result of this experience—full of healthy regret over the year of school she squandered yet making the most of the new school year, doing things the right way the second time around.

<u>Terri's Previous Non-Success in School Mathematics</u>

Personal math history. Terri had failed Core-Plus Course 1 as a freshman, along with English, biology, and physical education, all of which she was repeating as a sophomore. She reported that her chronic absenteeism as a freshman was to blame for her not passing these classes. Although she was doing rather well (receiving above-average grades) her second time through Core-Plus Course 1, Terri described math, in general, as her "worst subject," saying she had never done well in it.

Entering attitudes towards mathematics. Compared to the entire group of 31 students who took the initial survey, Terri's response mean on the attitude survey placed her in the middle of the first quartile ($z = -0.90$) with the sixth lowest overall response mean. Similarly, Terri's mean scores in every category were below the group mean. In four of these categories—personal self-confidence, enthusiasm for math, long-term stick-to-it-iveness, and seeing math as an important area of study—the scores were also below neutral (indicating they were responses expected from unsuccessful students). In two categories—public self-confidence and short-term stick-to-it-iveness—the response scores averaged out to neutral, even though they were below the mean for the group of 31 survey-takers. In comparison to the six participants, Terri's mean scores were lowest in four categories and next to lowest in two of them (public self-confidence and significance).

Table 5.25
Terri's scores on initial survey attitude questions by category

	Response Mean	Means within categories					
		Self-confidence, Public	Self-confidence, Personal	Enthusiasm	Significance	Stick-to-it-ive-ness, Long-term	Stick-to-it-ive-ness, Short-term
TERRI	**2.33** z = –0.90	**3.0** z = –0.28	**1.7** z = –1.27	**1.7** z = –0.76	**2.7** z = –0.13	**2.0** z = –0.91	**3.0** z = –0.46
MEANS (n=31):	3.01	3.20	2.99	2.46	2.80	2.96	3.43
StDevP:	0.752	0.722	1.042	1.050	1.033	1.057	0.951

Terri's Experiences of the Core-Plus Course 1A Curriculum

In stunning contrast to her responses to the initial survey attitude questions, Terri's responses to the attitude questionnaires at each of the three interviews were overwhelmingly positive (i.e., consistent with responses expected from successful students) and consistently the highest of all six participants. The attitude statements used for the interview questionnaires were written so that, in all cases, positive responses ("Agree" or "Strongly Agree") would be the ones expected from successful students. As can be seen in Table 5.27, every single one of Terri's responses across the three interviews was positive, with the vast majority of them very positive.

TABLE 5.26
Terri's responses to attitude questionnaires from interview sessions

ATTITUDE SUB-CATEGORIES	Terri			Interviews – Attitude Questions
	25-Oct	17-Dec	6-Mar	
	Int 1	Int 2	Int 3	
Public Self-Confidence	5	5	5	1. I take part in whole-group class discussions on a regular basis.
	5	5	5	2. I contribute my fair share of ideas during small-group investigations.
Personal Self-Confidence	4	4	5	3. Most of the things we're learning in Core-Plus Course 1 make sense to me.
	5	5	5	4. If I work at it hard enough, I know I can succeed in this class.
Enthusiasm	4	4	5	5. I am interested in the things we're learning in Core-Plus Course 1.
	5	5	5	6. Based on my experience in Core-Plus Course 1, I am looking forward to taking Core-Plus Course 2.
Significance of Math	4	4	5	7. The problems we work on help me see the usefulness of math in everyday life.
	5	5	5	8. I find it helpful that we use graphing calculators in this class.
Short-term Stick-to-it-iveness	4	4	5	9. When a math problem is challenging, I usually keep working at it until I get it figured out.
	5	5	5	10. I am putting forth my best effort in my Core-Plus Course 1 class.
RESPONSE MEAN:	4.6	4.6	5.0	

Terri herself marveled at her newfound success the second time through Core-Plus Course 1, and wasn't quite sure how to account for the dramatic turnaround. (Certainly her consistent attendance and applying herself to her studies made a difference, yet she was still struggling to pass English for the second time.) Across the three interviews, Terri attributed her success in Core-Plus Course 1 to various factors. In the first interview, Terri cited the small class size (13 students), the teacher's helpful explanations of the material, and the functional nature of her small group ("They're not the kind of people that just want to work by themselves; they're willing to help us") as factors that contributed to her success. In the second interview, Terri again mentioned the small class size and the help that she obtained from the people in her small group as factors that contributed to her success. However, the change of classes for the spring term brought changes in Terri's class size (25 students) and in the classroom teacher she had; she also reported that the small group (now five students instead of three) was not nearly as helpful. Still, she continued to receive above-average grades, and her attitude scores were the highest possible at the third interview.

Terri's Performance on Algebra and Functions Tasks

On most of the algebra and functions tasks, Terri's performance revealed good understandings. Six of the nine tasks were completed successfully. Terri completed two of these tasks satisfactorily on her own (those with scores of 4), and she completed four of them with my support from the interviewer (those with scores of 3).

Table 5.27
Terri's scores on the interview problem-solving tasks

	Overall Solutions			Mean Score for all Individual Skills used	Z-score for Group Stats	Group Stats
	T1	T2	T3			
Interview 1	1	3	4	3.00	-0.26	Mean = 3.263 StDevP = 1.009
Interview 2	3	4	3	3.33	+0.01	Mean = 3.320 StDevP = 0.912
Interview 3	3	2	1	2.54	-0.31	Mean = 2.928 StDevP = 1.269

On two of the three tasks that Terri did not complete successfully—namely Tasks 2 and 3 of Interview Three—the mathematical content involved topics that Terri reported not having remembered studying in class. (Terri's classroom teacher nevertheless reported that students did, in fact, study these topics in class.)

Table 5.28
Terri's familiarity checklist for Chapter 3

Below is a list of the topics in Chapter N of your math book. For each one, mark an X in one of the boxes to show which statement best describes how familiar you were with the topic before this semester.	I already knew how to do these very well.	I already knew how to do these somewhat.	I had seen these before, but I knew very little about them.	I had never seen these before, and I did not know anything about them.	I don't remember studying this in this chapter.
CHAPTER 3.					
Draw a line to fit the pattern in a plot	X				
Find the linear regression model for a set of data (TI-83)	X				
Make predictions using a linear model	X				
Writing equivalent equations by rearranging, combining, and expanding terms					X
Solve an equation such as 3x + 12 = 45 without the use of a table or graph		X			
Using a table to find values of variables that satisfy the conditions of two equations (TI-83)		X			
Finding the rate of change (the slope) and y-intercept of a linear graph		X			
Use the equation of a linear model to make a quick sketch of the graph by hand					X

These tasks also involved the most traditional of all algebra topics—symbolically manipulating equations to solve for an unknown and making sense of the coordinate plane. Terri had expressed in her very first interview that algebra was hard for her. In fact, when asked what she enjoyed least about math, "algebra" was Terri's answer. "It's not an easy subject," she said (Interview One, 10/25/01).

As for Task 1 of Interview One: Although this proved to be one of the most challenging tasks across all interviews (based on the mean skills score across all six participants), Terri struggled with this task more than anyone. Terri's difficulties stemmed primarily from the wording of the question. (See Task Sheet in Appendix A.) Because the problem in this task did not say, "*At what time* are the temperatures the same?" Terri understood the question ("When do the temperatures meet?") to be referring to various temperatures that were the same regardless of the time, allowing for multiple answers. Terri had similar difficulties with the wording of the problem questions on each of the three tasks of Interview One, but none that interfered with her success on the problem in such a major way as it did for the first task.

Table 5.29
Terri's performance means for evaluation items

Performance Mean	**Terri**	**MEAN for all 6 participants**
Int1 Task-Skills Combined	**3.00**	**3.25**
Int1 - Task 1	**1.75**	2.91
Int1 - Task 2	**3.50**	3.75
Int1 - Task 3	**4.00**	3.51
Int2 Task-Skills Combined	**3.33**	**3.46**
Int2 - Task 1	**3.29**	3.14
Int2 - Task 2	**3.50**	3.65
Int2 - Task 3a-3d	**3.60**	3.85
Int2 - Task 3e-3f	**2.33**	3.08
Int3 Task-Skills Combined	**2.54**	**2.91**
Int3 -Task 1	**3.13**	2.90
Int3 -Task 2	**2.60**	3.12
Int3 -Task 3, concrete	**2.17**	3.14
Int3 -Task 3, abstracted	**1.00**	1.88
MEAN for 3 INTERVIEWS	**2.96**	**3.21**
StDevP		0.486
Z-score for 3-Interview Mean	**-0.58**	

Monica

Monica was a 14-year-old freshman, taking Core-Plus Course 1 for the first time. She lived with her parents, all of whom immigrated from war-torn Eastern Europe when Monica was in fifth grade. Monica lived in a neighboring midwestern state for two years before moving to River City for her seventh-grade year. She spoke English without an accent. The only hint of her being a non-native English speaker was that her reading level seemed to be below grade level—perhaps fifth or sixth grade:

> [S]he had trouble pronouncing "economical" as she read the DVD problem aloud. But I said it for her, and she repeated it perfectly. I asked her if she knew what it meant, and she said No. I said it means "it's a better deal" to which she said that's what she thought, but she wasn't sure.
> (Memo from Interview One, 10/23/01)

During her freshman year, English as a New Language was one of the classes in Monica's daily schedule (seeming to take the place of the English class other students would typically take as freshmen); otherwise, she attended classes with everyone else. Besides Monica's having told me during her first interview that she immigrated to the United States, there was very little indication of this during my interactions with her.

<u>Distinguishing Characteristics of Monica</u>

My overall lasting impression of Monica was of her silly-hearted nature. She was talkative, always in a hurry, seemingly carefree, and a bit less mature than most ninth-graders I've known. The traits she exhibited caused me to regard her more as a junior high school student than a high-schooler. Monica also seemed hyperconscious about wanting to fit in socially among her peers, and even though this was consistent with her junior-high-school-like behavior, I always wondered how much influence her having immigrated in the middle of her schooling experience had on her drive to assimilate and be accepted by her friends.

Monica worked through the problem-solving tasks I gave her as quickly as she could. It seemed far more important to her to get through as much as possible, as fast as possible, than it was to ensure quality or accuracy of her work. In fact, I'm not sure that Monica perceived a connection between the speed of her work and its quality/accuracy.

Nevertheless, Monica also seemed to want to please me and was interested in knowing how I evaluated her performance on the problem tasks. Monica thanked me wholeheartedly at the end of each interview. She told me, on more than one occasion, how much she liked meeting with me to work on math. She begged me at the end of the study to continue to work with her—more that school year as well as during the following school year—because she felt that the interview sessions actually helped improve her math performance in class.

Monica's Previous Non-Success in School

Personal math history. Monica reported that in seventh grade she was enrolled in "ESL remedial math" and that she received a letter grade of A, both in that class and in her eighth-grade math class. There was nothing in her personal math history that pointed to obvious non-success in school math. However, Monica was among those who did not take Core-Plus Course 1 until the ninth grade, although many students in the River City school district take the course in the eighth grade for high school credit.

Entering attitudes toward mathematics. Monica's self-described opinion of math was fairly positive. "I think it will really help me in the future," she said (Interview One, 10/22/01). She very much enjoyed using a calculator in class, but disliked taking tests and quizzes. When asked to describe one of her most successful experiences in a math class, Monica recalled having passed a very hard test in the eighth grade; when was asked to describe a least successful experience, she replied, "I don't know."

Based on her responses to the attitude questions on the initial survey, Monica seemed more like a previously unsuccessful student, however. Compared to the group of 31 survey-takers, Monica's overall response mean was at the lower end of the second quartile; she placed 22nd of 31 when ranking overall response means from high to low. Across the six categories of responses, Monica's scores were below the group mean in three categories (public self-confidence, enthusiasm for math, and short-term stick-to-it-iveness). In two of those categories (public self-confidence and enthusiasm for math), Monica's scores were below neutral as well (indicating that her responses were like those expected from unsuccessful students). In the three categories where Monica scored above the group mean, it was only slightly so, and her mean scores were exactly neutral in all three cases (significance of math as a field of study, long-term stick-to-it-iveness, and personal self-confidence).

Table 5.30
Monica's scores on initial survey attitude questions by category

	Response Mean	Means within categories					
		Self-confidence, Public	Self-confidence, Personal	Enthusiasm	Significance	Stick-to-it-ive-ness, Long-term	Stick-to-it-ive-ness, Short-term
MONICA	**2.83** z = −0.23	**2.7** z = −0.74	**3.0** z = +0.01	**2.0** z = −0.44	**3.0** z = +0.19	**3.0** z = +0.04	**3.3** z = −0.11
MEANS (n=31):	3.01	3.20	2.99	2.46	2.80	2.96	3.43
StDevP:	0.752	0.722	1.042	1.050	1.033	1.057	0.951

<u>Monica's Experiences of the Core-Plus Course 1A Curriculum</u>

Monica's attitude questionnaires from each of the interviews were quite positive. Her responses to half the attitude questions became more positive over time; all but one of the rest of her responses remained steadily positive across interviews. The one response that became neutral, rather than positive, related to the use of calculators—perhaps because, at the time of the third interview, Monica reported that she and her classmates were not allowed to use calculators for most of their work (on the geometry-focused Chapter 5).

Table 5.31
Monica's responses to attitude questionnaires from interview sessions

ATTITUDE SUB-CATEGORIES	Monica			Interviews -- Attitude Questions
	22-Oct	24-Jan	18-Mar	
	Int 1	Int 2	Int 3	
Public Self-Confidence	4	4	4	1. I take part in whole-group class discussions on a regular basis.
	4	4	4	2. I contribute my fair share of ideas during small-group investigations.
Personal Self-Confidence	4	5	5	3. Most of the things we're learning in Core-Plus Course 1 make sense to me.
	4	5	5	4. If I work at it hard enough, I know I can succeed in this class.
Enthusiasm	3	4	4	5. I am interested in the things we're learning in Core-Plus Course 1.
	4	5	5	6. Based on my experience in Core-Plus Course 1, I am looking forward to taking Core-Plus Course 2.
Significance of Math	4	4	4	7. The problems we work on help me see the usefulness of math in everyday life.
	4	5	3	8. I find it helpful that we use graphing calculators in this class.
Short-term Stick-to-it-iveness	3	4	4	9. When a math problem is challenging, I usually keep working at it until I get it figured out.
	4	4	4	10. I am putting forth my best effort in my Core-Plus Course 1 class.
RESPONSE MEAN:	3.8	4.4	4.2	

Monica's experiences working in small groups with other students did not seem, from an outsider's (especially a teacher's) viewpoint, to be as successful as those of the other five participants; however, Monica did not seem negatively affected. In the first interview, Monica described the nature of her small-group work to be four students working individually on problems, regularly comparing answers with each other. Monica admitted she did not always understand the answers she put down; if the majority of the group had an answer that differed from hers, then she would "copy off of theirs." She reported making an attempt to get group members to explain such a problem to her, but that this wasn't usually successful. She also reported that her attempts to explain her differing answers to others was usually not met with good reception. Still, Monica seemed largely unfazed. She was pleased to be getting an above-average grade in the class and enjoyed her self-perceived success.

Monica's Performance on Algebra and Functions Tasks

Monica struggled the most of all of the participants with the algebra and functions tasks. Of the nine tasks, Monica was able to complete only one of them (Task 2 of Interview Three) successfully, and that was done with my support. Among the eight tasks with which Monica had difficulty, she understood three of them partially successfully when judging performance on each task holistically.

Table 5.32
Monica's scores on the interview problem-solving tasks

	Overall Solutions			Mean Score for all Individual Skills used	Z-score for Group Stats	Group Stats
	T1	T2	T3			
Interview 1	1	3	1	2.69	-1.52	Mean = 3.263 StDevP = 1.009
Interview 2	2	2	2	2.86	-1.69	Mean = 3.320 StDevP = .912
Interview 3	1	1	1	1.85	-1.56	Mean = 2.928 StDevP = 1.269

Monica's scores on the individual evaluation items help show a more complete picture of her performance on the interview tasks. These numbers allude to the fact that Monica was able to successfully complete parts of problems within each task. In a couple of instances (Task 1 of Interview One and the first part of Interview Two Task 3), her subscores were not too far below the mean for all participants.

Nevertheless, Monica lacked conceptual understanding of the mathematics of the problem tasks. Her procedural knowledge was good in many instances—and perhaps this was the reason behind her successful grades in class—but the interview problem-solving tasks focused largely on probing for conceptual understanding, and Monica turned up lacking deep and flexible understandings of the mathematics being explored.

Table 5.33
Monica's performance means for evaluation items

Performance Mean	Monica	MEAN for all 6 participants
Int1 Task-Skills Combined	**2.69**	**3.25**
Int1 - Task 1	**2.63**	2.91
Int1 - Task 2	**3.00**	3.75
Int1 - Task 3	**2.67**	3.51
Int2 Task-Skills Combined	**2.86**	**3.46**
Int2 - Task 1	**2.33**	3.14
Int2 - Task 2	**3.00**	3.65
Int2 - Task 3a-3d	**3.57**	3.85
Int2 - Task 3e-3f	**2.40**	3.08
Int3 Task-Skills Combined	**1.85**	**2.91**
Int3 -Task 1	**1.88**	2.90
Int3 -Task 2	**2.56**	3.12
Int3 -Task 3, concrete	**1.17**	3.14
Int3 -Task 3, abstracted	**1.25**	1.88
MEAN for 3 INTERVIEWS	**2.47**	**3.21**
StDevP		0.486
Z-score for 3-Interview Mean	**-1.70**	

CHAPTER 6

BROADER DATA ANALYSIS AND INTERPRETATIONS

In the previous chapter, I discussed my findings about the six participants as individuals. These portraits were descriptive in nature and focused on characteristics that made each participant unique unto herself. The collection of information presented in Chapter 5 constitutes the first level of data analysis.

This chapter, then, represents the second, more in-depth level of data analysis. I discuss my findings about the participants as a group. I take a step back, and with a wider focus attempt to identify patterns across participants. In some instances, however, patterns do not exist and generalizations cannot be made. These are findings as well.

My primary goal in this chapter, though, is to highlight the commonalities among participants. Where there are commonalities to be found, I look for ways to interpret them. Nevertheless, I take great care not to attribute causality in my interpretations. It is far beyond the scope of this study to be able to identify causes and effects of students' learning. Instead, I look to the data to help me make sense of what these students understand about algebra and functions and of the nature of their learning experiences in Core-Plus Course 1A.

I begin this chapter by discussing trends that exist in each of the categories that define this study: students' previous non-successes in school mathematics, students' learning experiences in Core-Plus Course 1A, and students' understanding algebra and functions. In the second part of this chapter, I discuss three categories that emerged as I conducted my study.

Patterns across Participants

<u>Characteristics of these students' previous non-successes in school mathematics</u>

The most basic characteristic common to all six participants was the fact that they did not take Core-Plus Course 1 until their freshman year in high school, despite the fact that students in River City schools were allowed to take the course as eighth graders with a teacher's recommendation. Four of the participants had, in the previous year or two, received grades of D or F in math. This was not true of Amy and Monica. Yet, in Amy's case, she cited her previous year's math class as her least successful, and her self-assessments of familiarity with topics in the first couple chapters of the Core-Plus Course 1 curriculum was noticeably lower than those of other participants taking the course for the first time. In Monica's case, she had taken "remedial math" for ESL students two years previous, as she assimilated to a country, culture, and language that were brand new to her as a fifth grader.

Another general characteristic that the participants shared was their entering attitudes towards mathematics; in response to questions about their dispositions toward math, these six participants' answers were, on average, *unlike* those expected from successful students. All six participants' mean responses in the category of Enthusiasm for mathematics were below neutral[1]; and four of the six scored below the group mean of the other 25 students who completed the same attitude survey. Five of the six students disagreed—four of them adamantly so—with the statement, "Math was one of my favorite subjects in middle school." (Adrienne was the exception.)

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[1] A score of 3.0 is neutral. Scores below 3.0 indicate responses like those expected from unsuccessful students; scores above 3.0 indicate responses like those expected from successful students.

Table 6.1
Initial Survey Attitude Questions: Analysis of Participants' Responses

Category Mean for n=25	Category Mean for 6 participants	CATEGORY	Category Means for each participant						Response Scores[1] for Questions						Mean for Q	Q #
			Sara	Adm	Joy	Amy	Mon	Terri	Sara	Adm	Joy	Amy	Mon	Terri		
3.17	3.33	Public Self-Confidence	4.0	3.7	4.0	2.7	2.7	3.0	4	2	4	4	4	4	3.0	1
									4	4	4	3	3	2	2.7	2
									4	5	4	1	1	3	2.3	3
3.05	2.72	Personal Self-Confidence	3.3	3.7	2.0	2.7	3.0	1.7	3	4	4	3	3	3	2.8	4
									3	4	1	3	2	1	1.8	5
									4	3	1	2	4	1	1.8	6
2.54	2.11	Enthusiasm	2.7	2.7	2.0	1.7	2.0	1.7	2	4	1	1	1	1	1.3	7
									3	2	1	2	4	3	2.0	8
									3	2	4	2	1	1	1.7	9
2.79	2.83	Significance of Math	2.7	2.0	3.3	3.3	3.0	2.7	3	2	5	2	2	1	2.0	10
									3	2	2	4	4	3	2.5	11
									2	2	3	4	3	4	2.7	12
2.96	2.94	Stick-to-it-iveness (Long-term)	3.0	2.7	2.7	4.3	3.0	2.0	2	4	4	5	2	2	2.8	13
									3	2	2	3	2	2	1.8	14
									4	2	2	5	5	2	2.7	15
3.40	3.56	Stick-to-it-iveness (Short-term)	4.0	4.0	4.0	3.0	3.3	3.0	4	4	4	3	4	3	3.0	16
									4	4	4	3	3	2	2.7	17
									4	4	4	3	3	4	3.0	18
									2.61	2.44	2.33	2.44	2.28	1.83		
									Response Mean for each participant							

Table 6.1 (cont'd)

Q #	QUESTIONS
1	I often volunteer to answer questions in math class.
2	I understand math well enough to help a classmate who is struggling.
3	I do <u>not</u> like to go to the board to show my answer to a math homework problem.
4	I am able to tell for myself whether an answer on my math homework is correct (or at least reasonable) <u>without</u> looking up the answer.
5	Math often does <u>not</u> make sense to me.
6	I am naturally good at math.
7	Math was one of my favorite subjects in middle school.
8	I love playing math games in class where you have to be the fastest one to answer in order to win.
9	I can imagine myself completing a math-related major in college.
10	I want to get a job that uses as little math as possible.
11	A student doesn't need more than 2 years of math in high school if he (or she) is not going to go to college.
12	A student doesn't really need to take 4 years of math in high school unless she (or he) is going to be a math major in college.
13	I would be happy to get a C as my grade in Core-Plus Course 1.
14	I do <u>not</u> plan to take more math courses beyond the two that are required to graduate from high school (Core-Plus Courses 1 and 2).
15	I want to graduate from high school with an Academic Honors Diploma.
16	When a math problem is challenging, I usually keep working at it until I get it figured out.
17	I usually check over my answers on a math test before turning it in.
18	When I get back my graded math tests, I usually go over the problems I missed so I understand why I got them wrong.

In the category of Long-term Stick-to-it-iveness (pursuing the study of math throughout high school and college), five of the six participants' response means were at or below neutral as well as at or below the group mean of the other 25 students who completed the same attitude survey. (Amy was obviously an outlier with a score of 4.3, which was tied for third highest score among the entire group of 31 survey-takers.)

Another attitude that the participants shared was, by contrast, one that was associated with being successful: Short-term Stick-to-it-iveness. In response to the statement, "When a math problem is challenging, I usually keep working at it until I get it figured out," four of the six participants agreed while two (Amy and Terri) were undecided. In response to the statement, "When I get back my graded math tests, I usually go over the problems I missed so I understand why I got them wrong," four of the participants agreed while two (Amy and Monica) were undecided. Half of the participants also agreed that "I usually check over my answers on a math test before turning it in." Only Terri disagreed; Amy and Monica were undecided.[2]

Characteristics of these students' learning experiences in school mathematics

All six participants, as a group and individually, were deemed "previously unsuccessful" in mathematics at the outset of the study on the basis of possessing attitudes towards mathematics that were typical of unsuccessful students and/or having had experiences in previous math classes that were unsuccessful. On the other hand, all six participants reported receiving above-average grades in Core-Plus Course 1 throughout the study. So it is natural to ask whether these *previously* unsuccessful students were realizing success.

[2] Perhaps what Amy lacks in Stick-to-it-iveness in the short run she makes up for over the long term.

The answer is not as simple as it seems. In explicating what was meant by the term "previously unsuccessful" in Chapter 3, I was careful to point out that success is not defined solely by the grades that a student receives nor by her conceptual grasp of the mathematics. As in Joy's case, a student might receive grades of A or B in math, and she might possess good understandings of the mathematics, yet she would not be considered truly successful—according to the conceptual framework of this study—as long as she is estranged from her mathematics studies because she experiences these as neither relevant nor important to her life. In fact, Joy provides the best example of just how elusive the answer can be to the question of whether a student is successful in mathematics, as defined in this study. Only after thoroughly analyzing Joy's learning experiences in Core-Plus Course 1 did I realize that Joy did <u>not</u> enjoy overall success because she did not experience satisfaction.

Interviewer:	It seems like you've consistently, on questions like [Attitude Questionnaire #5, 6, 7], you always check in the negative area, 'I strongly disagree' or 'I disagree.'
Joy:	Yeah.
Interviewer:	And yet you seem to do very well in class, like the questions that you ask, and the things that you answer. And I walked by in the hallway upstairs and your name is on the Honors List.
Joy:	Math was the only class I got a B in ... [or] I would've been on [the list for] Highest Honors.
Interviewer:	It just is really fascinating to me that somebody who feels fairly uninterested, or that kind of thing about math, still does so well. Do you know why that is?
Joy:	I mean I think it's retarded, and I hate it all, and it's really stupid.
Interviewer:	Uh huh, and you see lots of people, I would say <u>most</u> people, that causes them just to not try; they don't turn in work, 'cause they don't want to, and they wouldn't be getting a B.
Joy:	Well, maybe I just have a strong work ethic or something because I realize I <u>have</u> to do it. It's just (pause) realizing, not in denial about it or anything. I realize I have to do it to pass [ninth] grade. I know it's important to some people.

(Interview Three, 4/12/02)

In the same way, Amy and Sara cannot be categorized as having fundamentally successful learning experiences in Core-Plus Course 1 because of their lack of satisfaction with their math studies. For both these girls, their grades were above average and they possessed fairly good understandings of the mathematics, yet they expressed boredom with and a sense of alienation from the subject.

Interviewer:	How well are you doing in the class right now? Do you know roughly what your grade is?
Sara:	B.
Interviewer:	Is that what you expect to get based on what you know you're putting into it and how much you understand?
Sara:	Actually, I figured I would have a better grade 'cause I do all my work and all of it gets given back to me with a good grade on it.
Interviewer:	So it's really lower than you expect?
Sara:	[Nods.]
Interviewer:	Do you have an idea why that might be?
Sara:	Probably because the way I act in his class. I act like I don't care. I sleep almost every day. I get mouthy with him. (laugh)

(Interview Two, 1/08/02)

In Sara's case it seemed that, if anything, this estrangement was growing over the course of the study. Sara's change in attitude seemed to be declining, and she reported less and less productive in-class behaviors over time.

Interviewer:	I saw you kind of chuckling to yourself about [Attitude Questionnaire Item] number 6.
Amy:	I don't like math. (laugh) I never have! (laugh) ... I hate math. I've never had very good, I've never had teachers that I liked very well.
Interviewer:	You still do really well in it, though, it seems like.
Amy:	It amazes me 'cause I didn't do very well in it last year, oh, well, I got like a B. .... I've never had very good teachers so I just don't like math. Plus it's boring.
Interviewer:	Did your opinion of Core-Plus Course 1 as a class change any since we last talked?
Amy:	It's still boring.

(Interview Two, 12/18/01)

In Amy's case, however, there was a glimmer of hope that she might be on a more successful trajectory than before, as she reported during the third interview in mid-March that her satisfaction was growing: She was enjoying a math class for the first time in her life.

Monica also cannot be categorized as having fundamentally successful learning experiences in Core-Plus Course 1; however, the reasons for this are quite different than for the students mentioned above. If Monica's success were judged on the basis of her self-reports, she would seem quite successful: Not only did her attitude questionnaire responses suggest consistently successful characteristics, but she also reported receiving slightly above average grades. However, Monica's understandings of the mathematics were good on the surface only; that is, most of Monica's understandings of the mathematics were procedural at best. The evaluation of her problem-solving tasks showed that she did not fully or deeply—i.e., conceptually (Hiebert, 1986)—understand the mathematics.

2. **Sara has a weekend job as a waitress at her family's restaurant. She works 8 hours on Saturdays and 4 hours on Sundays. Her pay is $2.50 per hour plus tips.**

   a) **Choose letters to represent the variables *daily income* and *daily tips*. Write a rule that gives Sara's income on Saturday as a function of her tips for that day.**

   Letter for daily income L ***Letter for daily tips*** D ***Rule*** = L + D

   $20

   b) **Write a rule that gives Sara's income on Sunday as a function of her tips for that day.**

   Rule 10$

   c) **One Saturday, Sara earned $70.00 in tips. Use your rule to calculate her income for that day.**

   Saturday income $90

   d) **The next day was Mother's Day, which is usually a good day for tips. If Sara earned $80.00 in tips on Mother's Day, how much did she earn that day?**

   Mother's Day income $90

Figure 6.1
Monica's work on Task 2 of Interview Two

a.	Using I for income and T for tips, $I = T + (2.50)(8)$ or $I = T + 20$
b.	On Sunday, [Sara] works 4 hours so her income I is given by the following rule: $I = T + (2.50)(4)$ or $I = T + 10$
c.	Set $T = 70$ in the rule in part a. $I = 70 + 20 = 90$. [Sara]'s income that Saturday was $90.
d.	Mother's Day is a Sunday so set $T = 80$ in the rule in part b: $I = 80 + 10 = 90$.

Figure 6.2
Suggested solutions from Coxford et al. (1998a, p. 66)

Compared to the list of indicators of success in the category of mathematics achievement described in Chapter 3, Monica was lacking in all five areas (text interpretation, explanation of thinking, mathematical abstraction, retention, and invention). Interestingly, of these five areas in which Monica was less than successful, text interpretation and explanation of thinking stand out because of her status as a recent immigrant and non-native English speaker. I certainly wondered about the effects of her ESL status on her ability to communicate effectively in the language of mathematics. In turn, I wondered about the effects of Monica's less-than-successful math communication ability on her understanding of and her achievement in mathematics. (These language issues are taken up in more depth in a later section of this chapter.)

Terri can be described very similarly to Monica. According to Terri's self-reports, she seemed rather successful. Her attitude questionnaire responses were the highest of any participant across all three interviews, and Terri reported receiving slightly above average grades throughout the school year. Yet, Terri's understandings of the mathematics, like Monica's, were not well-rooted conceptually. Terri performed quite a bit better than Monica on the problem-solving tasks, yet the limits of her understandings revealed themselves more quickly during interviews than for other participants, especially with respect to mathematical abstraction:

> Terri did the same thing as Adrienne did: RE-LABELED the y-axis differently for the second graph.
>
> On one hand, I can see why: They chose their range for the y-axis based on the Washington DC numbers (20 students, $400); but this isn't a large enough range for the Hawaii trip (20 students, $10,000). …
>
> On the other hand, even when I challenged Terri's choice to re-number (because she was struggling with that a bit) and asked whether she could use the same numbers, she said No. And then, after she got her Hawaii points plotted, I asked about the comparative costs of the two trips: Which one is higher? (Hawaii) then why are the points lower than the others? (because the numbers on

> the y-axis are different). Terri didn't seem to see the difficulty that I perceived with the Hawaii points looking lower/smaller on the graph when they are actually higher. Another math-community-member thing to which Terri hasn't been initiated.
>
> Other than these things, I'd say Terri's comprehension of the mathematics of the problem was good.
>
> (Memo for Interview Two, 1/08/02)

I have found it very difficult to reach a definitive conclusion about Terri's fundamental success in Core-Plus Course 1. Terri showed fantastic improvement in many areas, not only compared to her performance from her previous experience in Core-Plus Course 1, but also compared to her performances at the beginning of the study. Terri showed a dramatic change in attitude over the course of the study: Her attitude scores went from lowest of the six participants on the initial survey to highest of all participants at all three interviews. Terri also improved in her mathematics performances as time went on. For instance, at Interview One, Terri abandoned her attempt to graph any part of the solution for Task 1 (which called for the student to graph two functions simultaneously to find their point of intersection), yet she graphed one function with considerable success at Interview Two (Task 3). Nevertheless, even in this instance, Terri was not completely successful; given another opportunity at Interview Three (Task 2), Terri was unable to graph two functions simultaneously (this time on the calculator). Nonetheless, Terri was able to compare these functions using a dual table on the TI-83, and thus succeeded with the less abstract representation of the two.

So, my answer to the question of whether Terri enjoyed fundamentally successful learning experiences in Core-Plus Course 1 is a qualified yes. Terri is on a trajectory of improvement, although she still has quite a ways to go—especially with problems at higher levels of abstraction (which is an issue area that is taken up in more depth in a later section of this chapter, "Contextualized versus Abstracted Mathematics"). Terri is not enjoying the kind of unqualified success that one would wish for her to ultimately enjoy; yet, considering where she came from (having failed Core-Plus Course 1 the previous year) and where she is headed (having shown dramatic improvement over the course of the study), Terri is well on her way to enjoying fundamentally successful learning experiences in mathematics.

Adrienne is the one participant who seemed to enjoy unquestioned success in all areas. This is not to say she never had difficulty or showed no indications of fragile or incomplete understandings of mathematics. Nonetheless, Adrienne's understandings of the mathematics were strong across all three interviews (overall, the best of all six participants),[3] her attitude questionnaire responses consistently indicated success, and the grades she received were well above average throughout the school year. In addition, the methodical and meticulous ways in which Adrienne worked on the problem-solving tasks drew my attention to a set of behaviors, common to mathematicians, that contribute to a person's success in mathematics when exercised. (These issues are taken up in more depth in a later section of this chapter called "Mathematics Community Membership.") Especially in light of the fact that Adrienne was taking Core-Plus Course 1 for the second time, Adrienne is the best example of a previously unsuccessful student who was subsequently enjoying fundamentally successful learning experiences in mathematics.

Interviewer: Tell me your opinion of Core-Plus Course 1 as a class.
Adrienne: It's a small class so we get more attention. I like that.
Interviewer: What do you enjoy most about the class?

[3] See Table B.6 in Appendix B. Appendix C contains my interview memo of Interview 2 with Adrienne; the reader can compare Adrienne's performance on Tasks 2 and 3 to Monica's and Terri's performance on these same tasks described above.

Adrienne: It's easier, not easier, but easier to learn now. He goes through stuff, explains it better.

(Interview One, 10/29/01)

Interviewer: Did your opinion of Core-Plus Course 1 as a class change any since the first interview?

Adrienne: I actually got easier, I think. The stuff we worked on, I understood it a lot better than the earlier stuff.

Interviewer: Do you think it was because you already had the class once before?

Adrienne: I don't remember doing that stuff.

Interviewer: What grade were you getting in class towards the end of the semester?

Adrienne: A-minus.

Interviewer: Is that the grade you expected to get?

Adrienne: Not really. I [expected to get] probably a B.

Interviewer: Do you think your grade is a good representation of how well you understood the material?

Adrienne: Yeah.

Interviewer: Do you have any ideas as to why your grade turned out higher than what you thought it might be?

Adrienne: I understood the stuff better than I thought I would.

(Interview Two, 1/17/02)

Characteristics of these students' understandings of algebra and functions

It has been no small task to try to answer the question, What do these students understand about algebra and functions? It is important that the reader keep in mind that my study provides a very limited amount of data to use in answering this question. Three snapshots, in the form of interviews, were taken of each of the six participants. These snapshots provide a glimpse of what a student knew about algebra and functions at a very early stage in her learning; they focus only on the content of Lesson 4 of Chapter 1, Chapter 2, and Chapter 3 of Core-Plus Course 1.

Core-Plus is an integrated, spiraling curriculum that incorporates various mathematical strands (algebra and functions, geometry and trigonometry, statistics and probability, and discrete mathematics) throughout all three courses of the high school curriculum. So a student's opportunities to learn about algebra and functions are spread across the 21 chapters that comprise the three-year Core-Plus curriculum.[4] Therefore, only a brief introduction to the many topics that appear on the map of the algebra terrain (Table 3.1) are offered in the first three chapters of Core-Plus Course 1.

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[4] Each course in the Core-Plus curriculum is comprised of seven units (chapters) that introduce the mathematics content as well as 1 unit (chapter) at the end of each course that is described as a "capstone" to the seven preceding units. This capstone unit is "a thematic, two-week, project-oriented activity that enables students to pull together and apply the important modeling concepts and methods developed in the entire course" (Coxford et al., 1998d, p. 7).

Table 6.2

Core-Plus Course 1 Chapters 1, 2, 3 and their algebra content [5]

ALGEBRA MAP		Ch 1.4	Ch 2.1	Ch 2.2	Ch 2.3	Ch 2.4	Ch 3.1	Ch 3.2	Ch 3.3
Various Represen-tations	graph	c	c		c	a	a,b	a,f,g	e
	table of data		c		c	a	a	a,f,g	e
	rule (equation)—explicit				a	a	a	f,g	a
	rule (equation)—iterative/recursive			b,d					
Functions	continuous functions—linear			(a)		(b)		(e)	
	continuous functions—non-linear			(a)		(b)		(e)	
	discontinuous functions—discrete	(d)							
	inverse functions								
	composition functions								
Levels of Abstraction	using math to make sense of a situation	d	b,d		c		a	b,g	a,b,c
	mathematizing a situation (math'l modeling, Now-Next; translating words into symbols)		a	c			b,c	d,g	a
	symbolic abstraction (variables, unknowns, naked #s)					b		b,c	c,d
	symbolic generalization					b		e	
Number Relationships	inequalities								a
	ratio and proportion								
	variation/rate of change—constant (slope)			a			c	b	
	variation/rate of change—exponential			a					
	variation/rate of change—inverse								
	absolute value & distance								
	trigonometric ratios								

[5] The letters in the table refer to objectives for Chapters 1, 2, and 3 that appear in the Teacher's Guide for Core-Plus Course 1 (Coxford et. al., 1998c, pp. T1B, T97B, T157B). Parentheses around letters indicate use without addressing any related concepts. A complete listing of these objectives and their corresponding letter codes appears in Appendix B.

TABLE 6.2 (cont'd)

ALGEBRA MAP		Ch 1.4	Ch 2.1	Ch 2.2	Ch 2.3	Ch 2.4	Ch 3.1	Ch 3.2	Ch 3.3
Means of Solution	algorithms—solving a single equation							b,c	b,c
	algorithms—systems of equations								
	factoring								
	division of polynomials								
	completing the square								
	graphical—lines and/or points (sketched)	a,b	c				b,c		
	graphical—linear programming								
	graphical—systems of quadratics								
	The Quadratic Formula								
	matrices								
	technology-assisted—graphing (scatterplots, lines, linear models, etc.)	a,b	c			a	b,c	d	b,e
	technology-assisted—rule-generated tables		c		b,c	a			b,e
	technology-assisted—conjecture & investigation		d		c,d		c		
	student-invented means								
Number Groups	negative numbers					(a)			
	real numbers					(a)			
	irrational numbers								
	imaginary numbers								
	complex numbers								

Table 6.3

Interview Tasks and their algebra content[6]

ALGEBRA MAP		Int 1 Task 1	Int 1 Task 2	Int 1 Task 3	Int 2 Task 1	Int 2 Task 2	Int 2 Task 3	Int 3 Task 1	Int 3 Task 2	Int 3 Task 3
Various Represen-tations	graph	5,6,7,8,9			1		2,3,4,5,12,15	4	9	1,2,3,4,5,6
	table of data	3	1	3	6		1,13,14,16		6,7	
	rule (equation)—explicit			4			1,8,11,13		6,9	
	rule (equation)—iterative/recursive			5,6						
Func-tions	continuous functions—linear	3,4						2,3,5,8	6,7,9	
	continuous functions—non-linear (inverse variation)						11,12,13, 14, 15,18,19			
Levels of Abstraction	using math to make sense of a situation		2	2	5,7,8	5	7,10,16, 17,21	1,3	8,11	
	mathematizing a situation (math'l modeling, Now-Next; translating words to symbols)	1,2,4,5		1	3	1,2	9,12	2,6,7	4	
	symbolic abstraction (variables, unknowns, naked #s)				4	3			1,2,3, 5,12	7,8
	symbolic generalization				9		18,19	5,8		9,10
Number Relation-ships	variation/rate of change—constant (slope)				3					
	variation/rate of change—exponential									
	variation/rate of change—inverse						7,10,21			
Means of Solution	algorithms—solving single equations					4			3,5,12	
	graphical—points and lines (sketched)	4,5			2		11,12			
	technology-assisted—graphing (scatterplots, lines, linear models, etc.)						6,14,15	6,7	9,10	
	technology-assisted—rule-generated tables						13		6,7	
	technology-assisted—conjecture & investigation						19,20			
	student-invented means	3		3,4						

[6] The numbers in the table refer to evaluation item numbers that appear in the Listings of Evaluation Items included in Appendix B.

Another limitation to the snapshots of data that I have is the fact that only a brief sampling of skills, topics, and concepts could be explored in the set of three 60-minute interviews conducted with each participant. Approximately 45 minutes per interview was spent working on the problem-solving tasks; so an understandably limited amount of information could be gathered in 2 hours 15 minutes on what each participant understood about algebra and functions.

Moreover, these snapshots were taken over time – the first in October or November; the second in December or January; the third in March or April. Two caveats must be noted about this. First, as I mentioned in Chapter 1, my focus is on students' understanding (a verb in the progressive tense) algebra and functions: The emphasis is on learning and understanding as a process over time. Because the learning is ongoing and certainly extended beyond the time frame of my study—in fact, students were just beginning the algebra-focused Chapter 6, not included in my study, during the time when Interview Three was being conducted—the answer to what students understand does, in fact, change over time.

Second, it is also true that the answer to this question changes over time because the learning of new concepts is often fragile and easily dislodged. This was seen during the first couple interviews in the form of negative transfer, when students would attempt to (mis-)apply a calculator skill they had just learned in class, for instance, to a problem on an interview that was related to material they had learned in a previous chapter. It also was clear that the amount of time that had passed between students' having worked on certain topics and skills in class and their having to revisit them in an interview one to three months later resulted in some deterioration of their facility with those skills.[7]

Given these qualifications, I will now attempt to summarize what the six participants understood about algebra and functions. I will begin with a finer-grained analysis for each area of the algebra map, although there are very few patterns to be found that hold for all six participants. So, I will follow up with a broader set of findings about students' understandings of algebra and functions.

Specific areas of the algebra terrain. The map of the algebra terrain (Table 3.1) is comprised of six broad fields of study: various representations, functions, levels of abstraction, number relationships, means of solution, and number groups. Each of these fields consists of several elements. In this section, I offer a fine-grained, composite analysis of what the six participants understood about algebra in the categories of these fields and their elements, based on the problem-solving work they did on the interview tasks.

<u>Various Representations</u>. The representations that students were asked to use in the interview problem-solving tasks were graphs (both those sketched by hand and those derived on the TI-83 graphing calculator), tables of data (both written and those derived on the TI-83 calculator), and explicit rules (the term used in Core-Plus for equations). Iterative rules appeared only through students' electing to use them on their own. Of the nine tasks, there were only three (Interview Two, Task 3; Interview Three, Tasks 1 and 3)

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[7] This phenomenon was seen most dramatically in Interview Three, conducted in March and April. The problem-solving tasks were taken from Chapter 3 material, which the students had begun studying in late November and had finished studying in early January (with the break over Christmas taking place as well). Most participants remarked during Interview Three that they did not remember doing certain things in class, even though a survey of their teachers indicated that they had, in fact, spent class time working on those very things.

where students were actually directed in the problem to utilize more than one representation when solving a problem. In the other six tasks, there was only one kind of representation used (additional ones, if used, were incorporated by a student's choice). Thus, I deliberately chose to use the term *various* representations here, so as not to be confused with *multiple* representations, which often implies the use of several, equivalent representations used flexibly together (e.g., Brenner et al., 1997). The evaluation of students' facility with various representations on these problem-solving tasks does <u>not</u> reflect their facility with multiple representations used in connection with one another.

Upon examining students' scores on evaluation items related to each of these various representations, it would be fair to say that most students for the most part were able to use graphs, tables, and explicit rules effectively. (And, although none of the interview problem-solving tasks overtly called for the use of iterative rules, there were four students whose recent use of them in class resulted in positive transfer to Task 3 of Interview One.) In most cases where participants' performances were weak initially, they showed improvement over time with each of these representations.

There were a couple of exceptions to the above analysis. The first is Terri and her use of graphs. Terri was unable to draw a graph representation at Interview One, but did fairly well drawing a graph representation at Interview Two. She was unable, however, to make use of the calculator's graphing capabilities during all three interviews; and she struggled with understanding a graph representation in the abstract (Interview Three, Task 1). As for her use of tables, Terri showed proficiency on the calculator at Interview Three; she also made use of tables when they were provided for her in written form, but she lacked the invention needed to create her own when it might have been helpful. Terri also showed proficiency using rules at Interview Three in conjunction with calculator-generated tables, but not for calculator-generated graphs:

> Part E [of Task 2, Interview Two] was not as difficult as Parts A-D, since its completion doesn't depend on previously-derived answers. With some prompting, Terri did fairly well with the Table solution method. And she did not hesitate in the slightest when she interpreted what the answer "4" meant.
>
> For the Graph solution method, however, Terri used lists. She entered the values she had from the Table into lists and then proceeded to set up two Stat Plots. I helped with getting the lists into the right spots for Xlist and Ylist (to move us along so we could get to the parts where I was really interested in her understanding, of the interpretation of the solution method), although there was evidence that she understood this herself. At some point, she entered the two expressions into Y1 and Y2, so that they appeared in the window with her two Stat Plots, but this was merely coincidental (that she got the graphed functions along with her plotted pairs).
>
> (Memo for Interview Three, 3/11/02)

The second exception to the above analysis is Monica and her use of graphs, tables, and rules. Monica did all right when it came to graphing data on a scatterplot, but was completely unable to employ mathematical abstraction in making sense of the graph representation. She also was unable to utilize the calculator's graphing capabilities at Interview Two, but demonstrated fairly good retention of such skills at Interview Three with my support.

> To begin with, Monica really didn't recall having done this. At some point during work on this task, she stated how much she forgot over Christmas Break, and how long it had been since they worked on this type of stuff given that it's now March and they did this before mid-January.
>
> So I walked Monica through Part B, telling her at each step what to do. When we ultimately arrived at a prediction of 363.8 meters, Monica was convinced that this was the best answer, giving a lot of credit to the authority of the calculator. She was also impressed with herself that the answer was so close to the one that she had given (369 m).
>
> I should also note that, when Monica wanted to view the scatterplot/graph of the data she had entered into Lists 1 and 2 (stories, meters), I told her she would have to set the StatPlot. Yet, she seemed completely unfamiliar with how to do this. Since Plot 1 was already set up for (Type) scatterplot and for Xlist:L1, Ylist:L2 -- which was coincidentally just what she needed -- I just told her all she

> needed to do was turn the plot On, choosing not to linger on the mechanics of this with her.
>
> I did much less guiding on Part C; Monica seemed to remember, for the most part, what to do after having done Part B. She did show some insight that others prior to her had not -- realizing that she only needed to re-enter List 1 data and could reuse List 2 data. (Perhaps this was aided by my having starred the columns in which we were interested for each part of the problem.) She also realized that she didn't need to change the Window setting for Ymin, Ymax, and Yscl (although I hadn't realized it).
> (Memo for Task 1 of Interview Three, 3/19/02)

Monica also struggled with tables, unless they were provided for her in written form (indicating little, if any, invention on her part); again, she did all right with tables at Interview Three with support from me.

> I paraphrased a great deal in explaining Part E and proceeded to prompt Monica through the Table Method [using the graphing calculator], although she really didn't seem to recall having done this.[8] Still, once she had Jerome's and Lin's equations entered as Y1 and Y2, Monica was able to comfortably find the answer of X=4 as the value that would cause Jerome and Lin to both score 48 points.
> (Memo for Task 2 of Interview Three, 3/20/02)

Lastly, when it came to using rules, Monica struggled as well, unless the equations were provided for her.

<u>Means of Solution</u>. The means of solution that students were asked to use in the interview problem-solving tasks were algorithmic (solving single equations of several types), graphical (sketching points and/or lines), and technology-assisted means of solution in several forms (graphing scatterplots, lines, and using linear models; rules-generated tables; and conjecturing and investigating using the TI-83). There were also several instances where students demonstrated invention, making successful use of alternate means of solution.

[8] The transcript for this portion of Interview Three with Monica is included in Appendix C.

On problems that called for solution by means of solving single equations, there were four types of equations involved.

Table 6.4
Four types of equations appearing in the interview problem-solving tasks

Type I.	Y = 2.50(8) + X	Given X, solve for Y.
Type II.	Y = 1(2X) + 4(X+4) + 4X	Given X, solve for Y.
Type III.	Y = 11X + 12	Given Y, solve for X.
Type IV.	8 + 10X = 12 + 9X	Solve for X.

Type I appeared in Interview Two Task 2, and the other three types were part of Interview Three Task 2. The equations of Type I and II were presented within context, while the equations of Type III and IV were presented as naked numbers. Overall, all six participants were able to solve equations of Types I, II, and III rather successfully; only Sara was able to solve an equation of Type IV, which she did without hesitation or error.

On problems which called for solution by sketching points and/or lines on a graph, all six participants were ultimately successful, showing improvement over time from Interview One to Interview Two. Three of the six participants (Amy, Terri, Monica) struggled at both Interviews One and Two, however, when asked to sketch a graph to find a point of intersection of two functions; yet Adrienne, who didn't even attempt the sketched-graph method at Interview One, was successful in sketching two sets of points of the same set of axes at Interview Two with my assistance.[9]

[9] The transcript for this portion of Interview Two with Adrienne is included in Appendix C.

Problem Task 3, Parts B & E

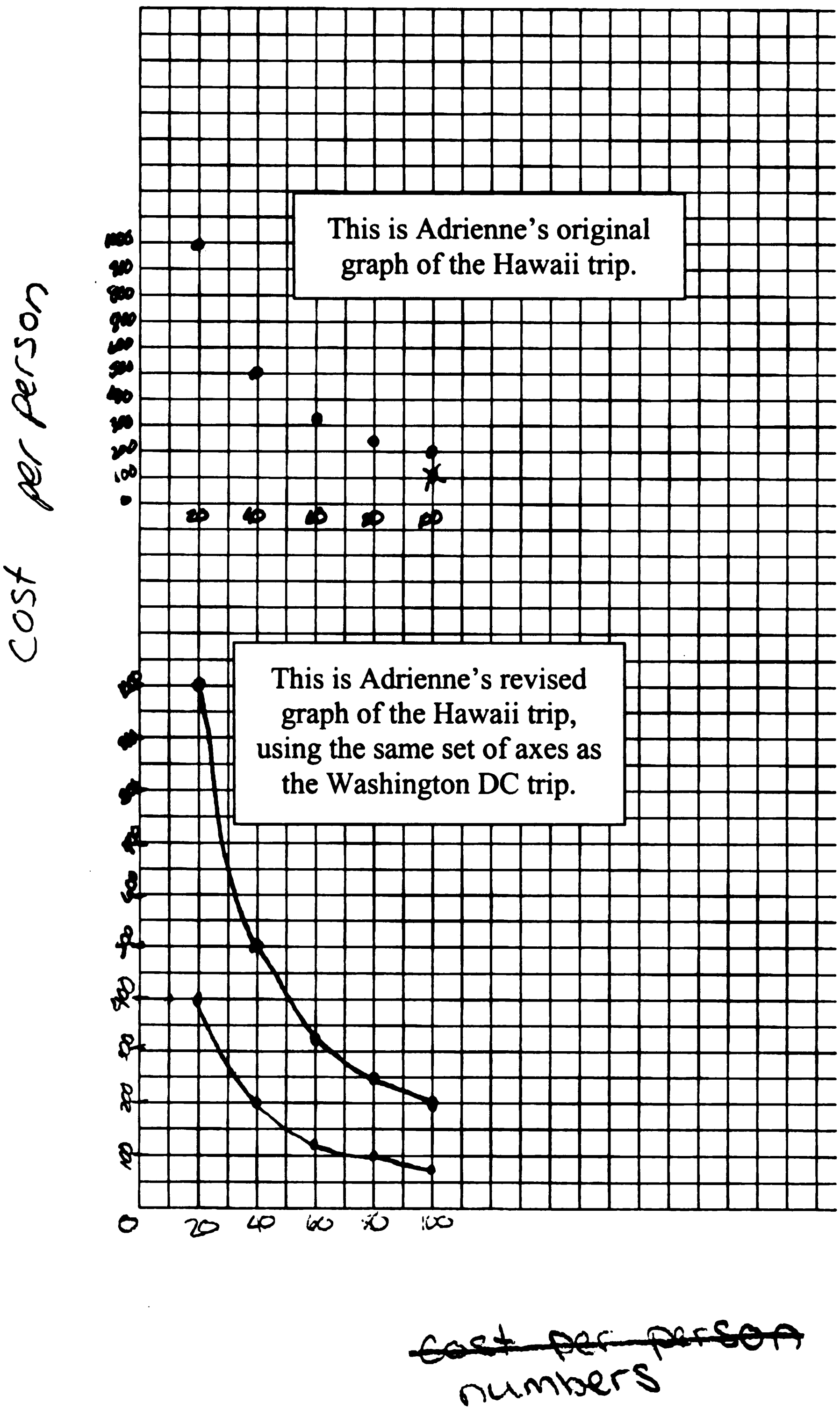


Figure 6.3
Adrienne's work on Task 3 of Interview Two

On problems that called for solution by using the TI-83 calculator to generate graphical representations, all six participants were able to successfully make a scatterplot from paired data (Interviews Two and Three). All six participants were also able to successfully use the graphing calculator to find a linear regression line for sets of paired data (Interview Three). However, on those tasks which called for graphing functions (linear and non-linear), all participants except Adrienne struggled initially (Interview Two); ultimately, Sara and Amy were able to do so as well, but Joy, Terri, and Monica were not successful in using the calculator to graph functions (Interview Three).

Table 6.5
Performance scores for participants' using rules to graph functions[10]

		Adrn	Sara	Amy	Joy	Terri	Mon
Interview 2 Task 3e	Graphed one function on calculator by entering rule as Y=	4	0	1	0	0	0
Interview 2 Task 3e	Graphed two functions on calculator by entering rules as Y=	0	0	0	0	0	0
Interview 3 Task 2.e2	Graphed two functions on calculator by entering rules as Y=	4	3	4	0	0	1

On problems which called for solution by using the TI-83 calculator to generate tables, all six participants were ultimately successful (Interview Three). This is very interesting when one considers that the generation of a table and the generation of a function on the TI-83 calculator both require the entry of a "Y=" rule; the only difference between obtaining a table and obtaining a graph is in the window that one views. All six

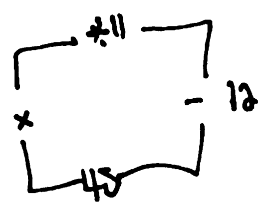
[10] Score interpretation is as follows: "4" indicates that the student did or understood the item satisfactorily; "3" indicates that the student eventually did or understood this with support from the interviewer; "2" indicates that the student did or understood this somewhat or partially; "1" indicates that the student did not do or understand this satisfactorily; and "0" indicates that the student did not attempt this method or approach.

participants were far more adept with the rule-generated table on the TI-83 than they were with graphing functions, especially when it came to finding a point of intersection of two functions (Interview One). It is interesting to note that, similarly, four of six participants were more adept with the hand-written table than they were with a sketched graph when it came to finding a point of intersection of two functions at Interview One.

Although technology-assisted conjecture and investigation as a means of solution were not included as part of any of the tasks, there was one instance where I improvised and extended my line of questioning on Interview Two Task 3. This occurred during the third participant's interview (that is, two participants had already completed their interviews and were not asked this set of questions). However, of the four participants (Adrienne, Sara, Joy, Monica) whom I pressed to explore the problem further, three (Adrienne, Sara, Joy) were able to make a reasonable conjecture and two of those (Adrienne, Joy) were able to successfully pursue the answer using the TI-83 calculator and its rule-generated table, comparing the Y-values of the two inverse-variation functions as X approached positive infinity.

As for student-invented means of solution, or student-selected alternate means of solution, the results were varied. Because no clear patterns are evident, here it is difficult to generalize across participants. There are two interesting things of which to take note, however. First, Joy stood out as one who was rather gifted in the area of invention. There was at least one instance during every interview where I was surprised by the method Joy selected and by her success in obtaining a solution utilizing that method. For instance, Joy was the only participant who successfully (and perfectly) completed Interview One Task 1 by graphing the two functions simultaneously on one graph to find their point of intersection; she did not even find it necessary to write out a table of paired data to do so! On the other hand, there also seemed to be at least one instance during each interview where I was surprised to see Joy struggle with concepts that other participants handled quite easily. In at least one case—solving a Type III equation at Interview Three—Joy compensated for her inability to successfully employ the algorithmic method of solution by using her own method of guess and check with complete success.

Joy:	Eleven X plus twelve, ummm, well, yeah. (pause) Mmm, how about (pause) one (long pause while student thinks)
Interviewer:	In Mr. Harper's class when you guys worked on these kinds of problems
Joy:	M hm.
Interviewer:	you made this little diagram that went like this.[11]



Joy:	M hm.
Interviewer:	Do you remember that? Does that help you?
Joy:	Yeah, I
Interviewer:	Started with the X here and said what's the first thing that happens to X?
Joy:	Umm, it's multiplied
Interviewer:	Multiplied by, uh huh, so multiplied by eleven.
Joy:	Yeah, but don't you do that opposite?
Interviewer:	And, oh yeah, I was goin', you're right. (laughs) Yes, divide by eleven, then what happens?
Joy:	Minus twelve.
Interviewer:	Yeah. Oops, wrote the twelve way over there. And that equals, do you write the forty-five here, I think? (Interviewer is not sure how to draw the circle diagram.)
Joy:	(laughs) Don't know.
Interviewer:	Does that help at all?
Joy:	It would if I knew ~ (laughs)

[11] This is the interviewer's (incorrect) attempt to recreate the "circle diagram" that students were taught to use to solve equations of Type III. See Figure B.1 in Appendix B or the correct circle diagram.

Interviewer: (laughs) Okay.
Joy: Um. But how do I divide this? (laughs)
Interviewer: Then I think how it worked, you go backwards.
Joy: Oh.
Interviewer: You worked it this other way.
Joy: So you take forty-five minus twelve.
Interviewer: That's right.
Joy: Okay. (laughs)
Interviewer: I think that's how that worked. Anyway, it's not helping you any more than it's helping me apparently. (laughs)
Joy: (pause) It's like thirty-six, isn't it?
Interviewer: Thirty-three.
Joy: Thirty-three?
Interviewer: Right. Five minus two, is a three.
Joy: Oh yeah.
Interviewer: Four minus one is a three.
Joy: Okay. Then thirty-three divided by eleven is three.
Interviewer: Right.
Joy: So X equals three.
Interviewer: Okay, so write that down.
Joy: *I'd actually come up with that ~*
Interviewer: Yeah.
Joy: *in my head*
Interviewer: How'd ya do it?
Joy: *Um, I was thinking, I knew that, like, eleven times whatever the number equals, you know, twenty-two or thirty three*
Interviewer: Okay, okay.
Joy: *and I was adding twelve to it.*
Interviewer: So you were just kind of guessing and checking
Joy: Yeah.
Interviewer: for something here that would make that whole thing work. Okay.
(Interview Three, 4/12/02)

The second item of interest to note is that, across interviews, tables became the solution method of choice, when participants were left to their own choosing, and their skills in using tables improved over time. Initially (Interview One), just two of the four participants (Adrienne and Sara) recorded data in a format resembling a table, and that was with some aid from me. At Interview Two, four of the participants (Adrienne, Sara, Amy, Joy) initiated the use of a table when no solution method was suggested (demonstrating invention); Adrienne used the calculator to create a rule-generated table for herself, while the other three wrote out their own tables of data on paper (Task 3). At Interview Three, participants were asked in Task 2 to solve the problem utilizing a variety of methods and then indicate the method they preferred. All six participants utilized the table as a means of solution most successfully, and all six participants named the table as their preferred method of solution.

Levels of Abstraction. There were four levels of abstraction that students encountered on the interview problem-solving tasks. These are listed and briefly described in Table 6.5.

Table 6.6
Levels of Abstraction encountered in the interview problem-solving tasks

Level I.	**Using math to make sense of a situation** *Organizing information, describing patterns or relationships, predicting outcomes*
Level II.	**Mathematizing a situation** *Using iterative Now-Next equations, translating words into symbols, modeling a situation mathematically*
Level III.	**Symbolic abstraction** *Working with naked numbers, unknowns, variables, symbols*
Level IV.	**Symbolic generalization** *Identifying mathematical patterns*

In looking for patterns across all participants, two general conclusions can be drawn from the evaluation data. First and most striking is the fact that, the lower the level of abstraction, the more successfully the participants performed. One can see this by examining Table 6.7 below.[12] To aid analysis, the scores were shaded in such a way that the darker the shading, the more successful the participant's score.[13] As one scans the

[12] A complete set of evaluation inventories—one for each of the six fields on the algebra map—can be found in Appendix B.
[13] Entries labeled "NA" or "0" do not apply. These indicate, respectively, that a participant did not attempt to utilize the named skill or concept, or that the interviewer did not ask/pose/present that particular question to the participant.

table from top to bottom, one notices that the lighter shades (less successful) are more dominant lower on the chart, where the more abstract levels are shown.

Second and nearly as striking is the fact that, the stronger the participant performed mathematically overall (across all interviews and tasks), the stronger she is at the higher levels of abstraction. This pattern can also be seen in the shading of the table. Participants were listed on the table from left to right in order of most successful (Adrienne) to least successful overall (Monica) based on participants' performance means for evaluation items (included in Appendix B). As one scans the table from upper left to lower right, a pattern can be seen in the table: The upper left half of the chart is dominated by dark shades but gives way to light shade in the lower right half of the chart. In other words, in Adrienne's and Sara's columns (the students who performed most successfully overall), the dark shades (indicating success) extend the farthest down into Level IV. In Amy's and Joy's columns (less successful overall than Adrienne and Sara), the dark shades begin to mix with the lighter shades (indicating some lack of success) at Level III. In Terri's and Monica's columns (the students who performed least successfully overall), the lighter shades begin to appear at Level II.

Table 6.7
Evaluation Inventory for Levels of Abstraction

LEVELS OF ABSTRACTION -- Using math to make sense of a situation			**Adr**	**Sara**	**Amy**	**Joy**	**Terri**	**Mon**
2	Int1 - T2	Recognizing that the answer has more than one case	4	4	4	4	3	3
2	Int1 - T3	Recognizing that the answer to the question has more than one case	3	3	3	3	4	1
5	Int2 - T1b	Using points where y>x to answer something about the overall data	4	4	4	4	3	1
7	Int2 - T1c	Using points where y<x to answer something about the overall data	4	4	4	4	0	0
8	Int2 - T1c	Using points where y=x to answer something about the overall data	4	0	4	0	3	0
5	Int2 - T2e	Being able to explain why two equation-values are the same if their "constants" differ	4	4	4	4	4	4
7	Int2 - T3c	Being able to describe the relationship between the function variables	4	4	4	4	4	4
10	Int2 - T3d	Predicting the shape of the graph for a function similar to one already graphed	4	4	4	4	4	NA
16	Int2 - T3e	Made her own written table	4	4	4	4	0	0
17	Int2 - T3e	Predicting a y-value for an x-value not in the data set	4	4	2	4	NA	4
21	Int2 - T3f	Being able to describe how the two functions compare	4	4	4	4	4	3
1	Int3 - T1a	Finding a linear model that is a good fit for the trend in the data by eyeing with a clear ruler	4	4	3	4	4	3
3	Int3 - T1a	Using the estimated linear model to predict a y-value for an x-value not in the data set	4	4	4	2	4	1
8	Int3 - T2e1	Understanding what the x- and y-values represent from the problem context	4	4	4	2	4	4
11	Int3 - T2e2	Understanding what the x- and y-values represent from the problem context	4	4	4	0	3	0
LEVELS OF ABSTRACTION -- Mathematizing a situation			**Adr**	**Sara**	**Amy**	**Joy**	**Terri**	**Mon**
1	Int1 - T1	Vocabulary -- "At a rate of ... per ..."	4	4	4	4	4	3
2	Int1 - T1	Understanding that the question implies ***At what time*** *are the temps the same?*	2	4	4	4	1	1
4	Int1 - T1	Using graph to find a point of intersection of two functions	0	3	2	4	1	1
5	Int1 - T1	Graphing Time versus Temperature	0	3	4	4	0	4
1	Int1 - T3	Understanding that the question implies *Which option is more economical?*	4	4	4	4	4	3
3	Int2 - T1a	Understanding what the y=x line represents	3	2	3	1	3	1
1	Int2 - T2a/b	Translating words into an algebraic representation/expression	4	4	4	4	2	2
2	Int2 - T2a	Assigning letters to variables	4	4	4	4	4	4
9	Int2 - T3d	Used Washington rule as a model for Hawaii rule	?	4	4	?	3	0
12	Int2 - T3e	Graphed <u>two functions</u> on paper (using same set of axes appropriately)	3	4	1	4	1	2
2	Int3 - T1a	Understanding which line is the best fit and knowing why (when eyeing with a clear ruler)	2	4	4	2	4	2
6	Int3 - T1b/c	Using the TI-83 to find the Linear Regression line (y = a + bx)	4	4	3	3	3	3
7	Int3 - T1b/c	Using the linear regression model to predict a y-value for an x-value not in the data set	4	4	3	3	4	3
4	Int3 - T2c	Translating words into algebraic represent'n/expression (using a similar, worked-out example)	4	4	4	1	1	2

Table 6.7 (cont'd)

			Adr	Sara	Amy	Joy	Terri	Mon
LEVELS OF ABSTRACTION -- Symbolic Abstraction								
4	Int2 - T1a	Recognizing that there are many places on graph (other than data points) where y=x	0	0	0	1	0	0
3	Int2 - T2a	Writing equivalent expressions/equations	4	3	4	2	0	0
1	Int3 - T2a	Manipulating expressions/equations -- applying the distributive property	4	4	1	1	1	1
2	Int3 - T2a	Manipulating expressions/equations -- collecting & combining like terms	4	4	1	1	1	1
3	Int3 - T2b	Using a given y-value to solve for X --> y = 11x + 12	4	4	NA	4	4	4
5	Int3 - T2d	Using a given x-value to solve for Y --> y = 1(2x) + 4(x+4) + 4x	4	4	4	4	2	4
12	Int3 - T2e3	Solving for X --> 8 + 10x = 12 + 9x	0	4	0	0	0	0
7	Int3 - T3c	Using knowledge about the y-intercept value appearing y= -2+3x to plot a point at (0,-2)	4	1	4	3	0	2
8	Int3 - T3c	Using knowledge about the slope value in y= -2+3x to sketch an "uphill" line thro y-int	3	2	4	3	0	1
LEVELS OF ABSTRACTION -- Symbolic Generalization			**Adr**	**Sara**	**Amy**	**Joy**	**Terri**	**Mon**
9	Int2 - T1d	Knowing what the data do/not suggest	4	2	1	1	2	1
18	Int2 - T3e	Understanding the infinite extent of a function	4	4	NA	4	NA	2
19	Int2 - T3e	Making reasonable conjecture abt numerical patterns as fxns extend twd x-approach-+-infinity	3	4	NA	4	NA	1
5	Int3 - T1a	Understanding what makes good estimate & why (cf interpolating betw 80- & 110-story bldgs)	1	1	1	1	1	1
8	Int3 - T1d	Understanding of what makes a good estimator and why (lines where r=.87 and r=.999998)	1	1	4	4	1	1
9	Int3 - T3a	Understanding what C and D represent in an abstracted linear equation --> y = C + Dx	1	1	1	1	1	1
10	Int3 - T3a	Knowing which of C or D is slope and which is y-intercept	4	1	1	2	1	1

Functions. The types of functions that appeared in the interview problem-solving tasks were linear (Interview One, Task 1; Interview Three, Tasks 1 and 2) and non-linear—namely, a function involving inverse variation (Interview Two, Task 3). When I initially chose these tasks for use at the various interviews, I had not analyzed their content to be sure that I had problems which would provide data for all areas of the algebra terrain; so I was a bit chagrined that I ended up with what seemed like data that was inadequate for helping me gauge what participants knew about functions from their study of Core-Plus Course 1A.

The criteria that I used when selecting problems for the interview tasks, however, involved finding a set of problems that was a good representation of the mathematics in Core-Plus Course 1, Chapters 1, 2, and 3. After reconfirming for myself that the interview tasks were, in fact, a good representation of the mathematics in these chapters, I was again puzzled at the small amount of evaluation items among the interview data that pertained to functions. The next step I took was to revisit the mapping of the mathematical content from Chapters 1, 2, and 3 (Table 6.2). What I began to realize was that, although students are exposed to and work with various types of functions in several places across these three chapters of the curriculum, the focus is not on learning about concepts related to functions, their characteristics, and their behavior; rather, the focus is on the *number relationship* aspects of two variables. For instance, both a "linear model for loss" and an "exponential model for loss" are used in Lesson 2 of Chapter 2, but they are investigated as "iterative concept[s]" using Now-Next expressions only (Coxford et al., 1998c, pp. T110-T121). Indeed, the Teacher's Guide points out that this was an intentional aspect of the design of the curriculum:

> It is important that students be encouraged to think about variables as quantities that change – either over time or in relation to other variables. The most important notion of variable is not simply that of a letter or an unknown. We eventually will develop the familiar concept of function carefully. However, at this time we want only to lay the foundation for that concept by using the phrase *is a function of* to indicate a relation between variables. (Coxford et al., 1998c, p. T101; emphasis in the original)

Nevertheless, do the data tell us anything about participants' understanding of functions—even if it has not been explicitly included in Chapters 1, 2, and 3? I think so. First, students worked on three tasks involving linear functions. On Interview One Task 1, there were mixed results. (See first two rows of Table 6.8.) Only Sara was successful on both items related to functions, although Adrienne, Amy, and Joy did all right on one item or the other.[14] On Tasks 1 and 3 of Interview Three, however, the success is more widespread—especially when one considers and arranges the data according to the level of abstraction of the evaluation items. Evaluation items at the lower levels of abstraction (other than Level IV) reveal fairly good success on the part of most participants, with Monica the exception; although Monica, along with all but Adrienne, does show marked improvement at Level II over time. Certainly, the participants possess previous knowledge to build upon—possibly the "foundation" mentioned above?—when, in fact, functions are introduced explicitly in the curriculum (Chapter 6 of Course 1).

[14] See Appendix C for work on Task 1 of Interview One by Sara, Adrienne, Amy, and Joy.

Table 6.8
Levels of Abstraction for Evaluation Items related to Linear Functions

FUNCTIONS -- Linear			Adrienne	Sara	Amy	Joy	Terri	Monica	Level of Abstraction
4	Int1 - T1	Using graph to find a point of intersection of two functions	0	3	2	4	1	1	
3	Int1 - T1	Using paired data to find a point of intersection of two functions	4	4	3	0	1	2	II
9	Int3 - T2e2 (fxn)	Graphed two functions on calculator by entering rules as Y=	4	3	4	0	0	1	
3	Int3 - T1a	Using the estimated linear model to predict a y-value for an x-value not in the data set	4	4	4	2	4	1	I
2	Int3 - T1a	Understanding which line is the best fit and knowing why (when eyeing with a clear ruler)	2	4	4	2	4	2	II
6	Int3 - T2e1 (fxn)	Entered two rules as Y= and examined table values for comparison	3	4	4	4	4	3	II
7	Int3 - T2e1 (fxn)	Finding a point of intersection of two functions (TI-83 table)	4	4	4	3	4	3	II
5	Int3 - T1a	Understanding what makes a good estimate and why (cf. interpolating betw 80- and 110-story bldgs)	1	1	1	1	1	1	IV
8	Int3 - T1d	Understanding of what makes a good estimator and why (lines where r=.87 and r=.999998)	1	1	4	4	1	1	IV

Something similar could certainly be said for participants' understandings of non-linear functions as well. The participants had not encountered a function of inverse variation in their classroom studies, yet three of them (Adrienne, Sara, and Joy) had some good, albeit incomplete, understandings of concepts related to the behavior of the functions $c = 20000/n$ and $c = 8000/n$ in Task 3 of Interview Two.[15]

[15] See Appendix C for transcript excerpts from Interview Two of Task 3 for Adrienne, Sara, and Joy.

Table 6.9
Evaluation Items related to Non-Linear Functions

		FUNCTIONS -- Non-Linear (Inverse variation)	Adrienne	Sara	Amy	Joy	Terri	Monica	Level of Abstraction
11	Int2 - T3e (fxn)	Graphed one function on calculator by entering rule as Y=	4	0	1	0	0	0	
12	Int2 - T3e (fxn)	Graphed two functions on paper (using same set of axes appropriately) … Complementary Strategy	3	4	1	4	1	2	II
13	Int2 - T3e (fxn)	Entered two rules as Y= and examined table values for comparison … Complementary Strategy	4	2	0	4	NA	0	
14	Int2 - T3e (fxn)	Entered two sets of data into lists to make dual function representation	0	4	0	0	2	0	
15	Int2 - T3e (fxn)	Graphed two functions on calculator by entering rules as Y=	0	0	0	0	0	0	
18	Int2 - T3e	Understanding the infinite extent of a function	4	4	NA	4	NA	2	IV
19	Int2 - T3e	Making a reasonable conjecture about the two numerical patterns as the functions extend toward x-approaching-positive-infinity	3	4	NA	4	NA	1	IV

Number Relationships. The interview problem-solving tasks, unfortunately, included only a few items that probed students' understandings of number relationships, all of them appearing in the problem-solving tasks of Interview Two (not coincidentally, because Chapter 2 was the key place in the text where number relationships were a focus of mathematical investigation). The number relationships that were explored in Interview Two were constant and inverse variation. Upon examining the evaluation data for participants' performances on questions about these two types of variation, it is striking how different the results are for constant variation and for inverse variation.

Table 6.10
Evaluation Items related to Number Relationships

NUMBER RELATIONSHIPS -- Constant variation			**Adrienne**	**Sara**	**Amy**	**Joy**	**Terri**	**Monica**
3	Int2 - T1a	Understanding what the y=x line represents		2		1		1
NUMBER RELATIONSHIPS -- Inverse variation			**Adrienne**	**Sara**	**Amy**	**Joy**	**Terri**	**Monica**
7	Int2 - T3c	Being able to describe the relationship between the function variables	4	4	4	4	4	4
10	Int2 - T3d	Predicting the shape of the graph for a function similar to one already graphed	4	4	4	4	4	NA
21	Int2 - T3f	Being able to describe how the two functions compare	4	4	4	4	4	

However, I would venture to say that the difference relates not to the type of number relationship at issue, but rather to the context (or lack thereof) in which the number relationship questions were posed. In the case of Task 1 (constant variation), the question about number relationships involves the rather abstract notion of a "Y=X line," whereas in Task 3 (inverse variation), the questions are closely related to the context of the problem. This difference in participants' performance on tasks involving contextualized versus abstracted mathematics is among the set of categories that emerged during the course of my data collection and ongoing analysis; it will be discussed in a subsequent section.

Number Groups. The field of number groups, as represented on the map of the algebra terrain, involves the study of various types of numbers—negative, real, irrational, imaginary, and complex—and their properties. While there were a couple of interview problem-solving tasks that made use of negative and real numbers, there were no evaluation items that probed participants' understandings of these number groups, nor was there any evidence among the interview data that students possessed understandings or awareness of number groups per se.

Commonalities of participants' understandings of algebra and functions. Having worked my way through the algebra terrain, analyzing what participants understood about algebra and functions, I now turn to a couple of common characteristics that I found as I looked for patterns in participants' understandings of algebra and functions.

Initiating and Connecting Representations. A common theme across all three sets of interview tasks was the idea of various representations—a graph, a rule, and a table that were different, but obviously related, representations of the same problem situation.

(Obvious to some, that is, but not to the participants.) This is another pattern I noticed: that participants were not able, at this stage in their learning, to move freely between and among representations.

Interview Three Task 1 is but one of the tasks that brought this pattern to light. It presented a table of paired data to be graphed on a scatterplot, then asked the participant to draw a line of best fit on the graph and predict a Y-value for an X-value not found among the data. The second part of the problem asked participants to calculate a linear regression line for the data, and then use the same X-value, together with the equation for the linear regression line, to predict a Y-value. In essence, the task involved using two representations—a graph and an equation—for the same situation, making a prediction using each of the representations, and then comparing the results. Although students were able to create the representations and make the predictions successfully, their understanding of the connections between the two representations seemed rather tenuous.

A second, closely related pattern I noticed was that without some support—whether from me during the interview or from the design of the problem itself—participants often could not initiate a representation on their own. For instance, the three problem-solving tasks of Interview One were mathematically very similar to one another. For each task, there were two equations to be examined for their point of intersection. Task 2 provided a table with the rows and columns already labeled; the participant needed only calculate the numerical amounts for the table cells. Task 3, however, provided only a large blank space for the participant to solve the problem in any way she saw fit.

The results were the same across all participants: Task 2 was far easier for participants to solve. Task 3 probably would not have been solved by any of the participants had it not been for a great deal of assistance on my part. Without a suggestion for how to solve the problem, the participants were at a loss as to how to proceed. However, the more structure that was provided for solving a problem, the more successful the participants. (Terri's performance on Task 3 is the lone exception.)

Table 6.11
Participants' holistic performance scores on tasks with varying degrees of structure for problem solving

Interview 1	**Adrn**	**Sara**	**Amy**	**Joy**	**Terri**	**Mon**
Most Structured: Task 2	4	4	4	4	3	3
Partly Structured: Task 1	4	4	3	3	1	1
Least Structured: Task 3	4	4	3	3	4	1

Over time, several students showed improvement in initiating representations (Interview Two, Task 3), but seeing the connections among multiple representations continued to be challenging for participants, especially at the conceptual (as opposed to the procedural) level of understanding. The two examples below from Interview Three, Task 2 illustrate this:

> Adrienne did well on this problem. Her understanding of it, up through Part E, the Table Method was great!
>
> When we got to Part E, the Graph Method, Adrienne showed the limit of her conceptual understanding. This was fascinating too. Adrienne got the equations entered, and the window set, but the lines were so close together that it was tremendously difficult to tell where they crossed (same problem I ran into when working this problem myself! In fact, I wish the problem had been crafted differently so there wasn't this problem!). So, with a little prompting from me, Adrienne began to zoom in on the area of intersection by adjusting the window bit by bit.
>
> Eventually, Adrienne got to the point where she was using the Trace key to jump back and forth between the two lines, very close to where they intersected. She was looking at their X- and Y-values, seeing that they were very close. She

> concluded that Y=43 and X=3.5 was as close as we could approximate. Then I asked her about what X=3.5 would mean in terms of the die. She wondered aloud whether she should round that up to 4? Then she decided to zoom in a bit more, and very quickly, after zooming in for the last time, the Trace key landed her on the point Y=48 and X=4. Still, I say that this showed the edge of her knowledge/understanding because she was OK with the two methods (Table and Graph) yielding different answers to this problem.
> (Memo for Interview Three Task 2, 3/11/02)
>
> Unlike any of the previous interviewees, Sara didn't hesitate in completing any of these problems – not even the three methods of Part E – and did them correctly, after basic getting-started prompts from me. She wasn't able to articulate that the x=4 stood for the 4 that was rolled on the die, but she didn't seem to flinch at attempting any of the methods.
> (Memo for Interview Three Task 2, 4/09/02)

While I found it natural to move back and forth between the various representations and to select an appropriate representation to use to solve a problem task, the same was not true for the participants. The connections between the representations seemed especially opaque to participants, often occluding their use of various representations. I was mistaken to have considered these connections to be self-evident.

Fragility of Understanding. According to the original design of my study, I planned to conduct each of the three interviews with participants immediately following the conclusion of their study in class of Chapters 1, 2, and 3, respectively. As it turned out, however, I was unable to begin the third round of interviews until the beginning of March, which was nearly three months after participants had concluded their study of Chapter 3 in class. (Although the fall term extended into the second week of January, most, if not all, of participants' work on Chapter 3 was completed in December.) While such a gap between participants' work in class and their work in an interview on the same material did not seem ideal, I realized that it would provide me with an opportunity to see how participants' understandings fared over time.

To my dismay, there were two areas in which participants' knowledge had noticeably faded—in the use of the TI-83 graphing calculator and in their facility with certain types of algebraic-manipulation skills. There were differences, however, in students' recall of what they had learned after being reminded. With the calculator, I coached the participants along, reminding them of the steps to take to figure a linear regression line, for instance. Although all of the participants needed help getting started with the calculator, once they did get started, they were able to continue without much additional assistance. The skills they had learned a couple of months back resurfaced without all that much difficulty.

> For the linear regression line calculations in Parts B and C, Amy insisted she did not recall ever having done this in Mr. Harper's class. *I must check with him on this!* So, I led her through what to do for Part B so that we could discuss the prediction that the calculator/data yielded and compare it to the predictions discussed in Part A. Then, Amy was able to complete Part C pretty much on her own. I even commented to her that she was "either a very quick study," or that she actually had done this in the past, because she picked up on it so quickly after I showed her how to do the first one.
> (Memo for Interview Three Task 2, 3/12/02)

> Even though she didn't recall having done the linear regression stuff in class, Sara took right to it once I got her going. She even said "I understand now" as if, like a toddler does, she was making it clear that she didn't want my help because she could do it on her own. She didn't hesitate, when looking at the LinReg results screen, in writing down the proper equation $y = a + bx$. Neither did she hesitate in substituting the given x-value into the equation to come up with a prediction.
> (Memo for Interview Three Task 2, 4/09/02)

On the other hand, some of the participants' previous work on algebraic manipulation did not seem to resurface, even after being shown what to do. Most striking was their inability to apply the distributive property and to collect and combine like terms in a problem such as $Y = 2(2X) + 3(X + 4) + 4X$ (Task 2, Part A). Only Adrienne and Sara were able to successfully transform the given equation to $Y = 11X + 12$; the others didn't even know where to begin. Once the equation was in this form, however, all participants (with the exception of Amy, whom I neglected to ask) were able to successfully solve for X when given a Y value (with all but Joy using the technique they were taught in class).

Although students had not made use of the TI-83 calculators in the interim between their work on Chapter 3 and Interview 3 (according to reports from both participants and teachers), their latent knowledge of how to use the calculator for some nontrivial tasks returned surprisingly quickly. On the other hand, although students probably had not made use of the their algebraic-manipulation skills in the interim between their work on Chapter 3 and Interview 3 (during which they studied the discrete-math-focused Chapter 4 and the geometry-focused Chapter 5), their knowledge of how to apply the distributive property, how to collect and combine like terms—skills they very likely had practiced in middle school math classes—did not resurface at all for most participants.

Interpretations

The first part of this chapter has been devoted to broader data analysis: looking for patterns in the data that shed light on characteristics of students' previous non-successes in school mathematics, characteristics of students' learning experiences in Core-Plus Course 1A, and characteristics of students' understandings of algebra and functions. These findings, as well as the findings I presented in Chapter 5, have been primarily descriptive. In this second part of Chapter 6, then, I offer some interpretations of my findings.

These interpretations share a theme. They can be used to answer the question, What facilitated or inhibited participants' mathematics learning? Each of these explanations emerged from the data itself. In the midst of my data collection, I began to formulate conjectures about what I was finding. As I proceeded, I looked closely to see if additional data would confirm and flesh out these hunches. Over time, these three categories began to take shape very clearly; and, as I have proceeded with my data analysis through writing, these interpretations have been underscored and confirmed numerous times.

Contextualized Versus Abstracted Mathematics

I have alluded several times heretofore to the differences in participants' successes on problem-solving tasks when those tasks involve math concepts that are presented in context versus math concepts that are presented in the abstract. The results of participants' successes across the different levels of abstraction—described in a preceding section—is by far the best illustration of this finding, yet it is but one example of many. These same differences—students' successes with problems in context versus students' struggles with problems in the abstract—appeared among the types of equations that students were able to solve as well as among the data on students' work with number relationships in context and in the abstract, students' ability to use rule-generated tables of data far better than rule-generated graphs (both on the TI-83 calculator) to find the intersection of two functions, and students' ability to work with scatterplots of data points much more easily than with graphs of the functions that generated those data. It was quite evident that, for these previously unsuccessful students, their performances on problem-solving tasks were much stronger when the mathematics was contextualized, but the more removed the mathematics was from a context, the more the participants' skills seemed to deteriorate.

This is a fascinating finding, but a very difficult one to interpret. On first reflection, this seems to affirm the choice of the Core-Plus authors to present "Contemporary Mathematics in Context" (the title of the textbook series):

> The curriculum builds upon the theme of *mathematics as sense-making*. Through investigations of real-life contexts, students develop a rich understanding of important mathematics that makes sense to them and which, in turn, enables them to makes sense out of new situations and problems. (Coxford et al., 1998c, p. 1, emphasis in the original)

Participants were indeed able to make sense of important mathematics that were couched in the contexts of real-life situations.

Yet, I can only be partly enthusiastic about this finding. First of all, I must reserve judgment as to whether the participants were enabled to make sense out of new situations and problems. From my study, I am only willing to agree that participants were enabled to make sense out of new situations and problems *when they, too, were couched in a real-life context.* Yet one very important and powerful use of the language of mathematics is to express mathematical ideas in the abstract, to generalize principles and theories that can be applied in many ways across numerous contexts (NCTM, 2000). Given the limited data that I collected on my participants, I have not seen evidence that they are (yet) able to engage in making sense out of new situations and problems *through the use of mathematics in the abstract.* Students' mathematics education is incomplete without this skill.

Secondly, I have concerns about students getting to the place where they will be able to engage in making sense out of new situations and problems through the use of mathematics in the abstract. I worry about Theule-Lubienski's (1996) finding that students often become stuck in the contexts of problems, losing a handle on making sense of important mathematics couched in the contexts of real-life situations. Although Theule-Lubienski found that the differences fell along SES lines and socioeconomic factors were not considered in my study, I share her concern that some students are less able to step back from the context of a problem and see the mathematical point because of the way in which they approach contextualized problems. There were participants in my own study who got caught up in the contexts of certain problems (Interview One, Task 3, for instance) and consequently missed the mathematical objectives of the tasks at times.

This leaves me with questions that my study cannot answer. Is it simply too early in their study of the Core-Plus curriculum for participants to have reached the stage in their mathematical development where they are facile with mathematical ideas in the abstract? Will the participants achieve the same degree of success with abstracted problems that they now enjoy with contextualized problems as they gain more experience in the Core-Plus curriculum? Does the Core-Plus curriculum provide students the learning opportunities they need to make a successful transition from using mathematics in context to making use of mathematics in the abstract? What facilitates or hinders students' success in making this transition? Needless to say, more research is warranted in order to answer these questions. Moreover, it is essential to find answers, as students' mathematics education is incomplete without the ability to use abstracted mathematics:

> ... it sounds promising to allow students to explore math through familiar contexts. But in learning mathematics, the goal is not to stay tied to the contexts, but to abstract the mathematical principles so they can be utilized elsewhere. (Theule-Lubienski, 1996, p. 212)

Language Issues

A second category that emerged during the course of my data collection and analysis had to do with language issues—repeated occasions where participants had difficulty with the text that accompanied a problem-solving task. It became evident that one of the major factors that inhibited participants' mathematics learning had to do with the language in which the problems were written. At first, I presumed that it was a matter of the text's being written at a reading level that was too advanced for the participants, yet a Raygor Readability Estimate placed the reading level of the texts of the interview tasks at about Grade 7.

What I then began to realize was that participants possessed certain limitations in their proficiency with the more formal language that is used by the mathematics community to convey ideas among its members (used in math textbooks and by math teachers, for example)—i.e., the mathematics register (Pimm, 1987). Because the nature of the interviews was interactive, it was quite easy for me, as the interviewer, to assist the participant with the language she found difficult to navigate and to get her back on track. Yet I wondered: In the day-to-day work of completing homework assignments or written assessments for math class, how would a student fare if she found the text of the problem or task difficult to interpret appropriately? I suspect that, all too often, what is in actuality a lack of mathematics-language comprehension is mistaken for a lack of mathematics-concept comprehension.

Consider the following problem posed in Interview One Task 3:

> You are considering joining a karate club. At Champion's Corner, there is no membership fee and lessons cost $6.00 per hour. At Bowles Academy, there is a

> yearly membership fee of $24, and lessons cost $4.50 per hour. Which club would you join? Show or write a short explanation of how you decided.

For her answer, Adrienne chose Champion's Corner, explaining, "If you go for only a couple hours its [sic] cheaper." What Adrienne wrote is correct; however, in evaluating her response, I had to judge it as having missed part of the mathematical point of the problem because Adrienne did not interpret the question appropriately. To one who is acquainted with the mathematics register, it is clear that the question is asking, "Which option is more economical?" and that the answer consists of three cases: One club is less expensive over the short term, the other club costs less in the long run, and the key to the answer is to find out the number of hours for which they cost the same amount. Were this a problem on a homework assignment or a test, Adrienne would get partial credit at best, but would not be given credit for a correct answer. Yet, during the interview, with prompting from me, Adrienne was able to successfully arrive at the three-part answer. Adrienne's only error was in failing to interpret the text of the problem according to the mathematics register.

> Part of learning mathematics is learning to speak like a mathematician, that is, acquiring control over the mathematics register. ...
>
> Are pupils sufficiently aware of the existence of a mathematics register which is employed within the school? ...
>
> In particular, how do pupils keep such meanings and uses separate and how do they acquire knowledge of when to invoke one meaning rather than another? How aware are teachers themselves when they are using various registers? Do teachers explain these differences to their pupils, or do they assume that their pupils know which items are occurring in the mathematics register, and when using this particular register is appropriate? (Pimm, 1987, pp. 76-77)

It was clear from my study that participants were not sufficiently aware of the existence of a mathematics register; in fact, I myself was only beginning to recognize my own use of the mathematics register. Given the limited data that I collected on my participants and their study of the Core-Plus curriculum, I did not see evidence that such language issues were addressed or explained to students in overt ways. At best, when participants struggled during an interview to understand the mathematics register, I provided a paraphrase for them, stating the problem in everyday terms. In retrospect, this maneuver enabled participants to successfully answer the problem in question, but did little, if anything, to help them develop their own skills in decoding the mathematics register for themselves. Thus, although the question of how students acquire knowledge of how to appropriately interpret the mathematical register is left to further research, it is irrefutable that the answer to this question is the key to helping students overcome the obstacle to success in school mathematics that such language issues pose.

"Mathematics Community Membership"

Issues pertaining to "mathematics community membership" are closely related to the language issues discussed above: I came up with the term "mathematical community membership" to refer to the collection of characteristic behaviors that mathematicians possess, similar to the collection of characteristic behaviors of scientists that science educators aim to instill in even the youngest of their students (National Research Council, 1996). Facility with the mathematics register is a prime example of the types of behaviors that belong to this collection. Other examples of characteristic behaviors of mathematicians and other members of the mathematics community include using a systematic method when looking for all possible solutions to a problem, organizing data into a table, recording one's work in such a way that others can interpret one's solution process, using an alternate method of solution to verify an answer, and so on.

Figures 6.4 and 6.5 below are provided in order to give the reader an idea of the impact on problem-solving success that even these benign mathematician behaviors can have. Figure 6.4 is an example from the initial survey of one student's use of a systematic method when looking for all possible solutions to the following problem:

> I have a bunch of dimes and nickels in my pocket. I need 50 cents to buy a candy bar. List the different combinations of dimes and nickels I could use to equal 50 cents.

Figure 6.5 is an example from a student who did not utilize this mathematician behavior.

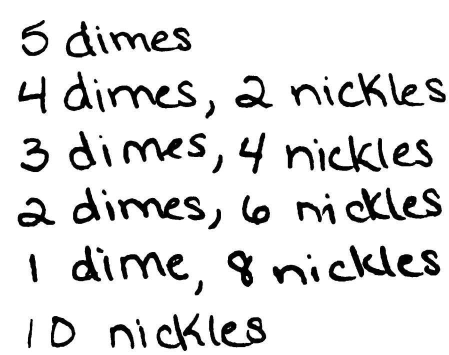


Figure 6.4
One student's use of a systematic method to find all possible solutions to the contextualized problem from the initial survey

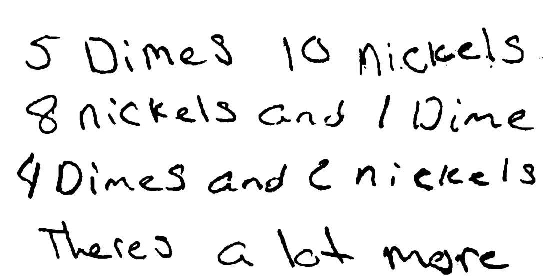


Figure 6.5
Another student's less successful approach to the contextualized problem from the initial survey

To one who already possesses membership in the mathematics community (such as myself) by virtue of having learned these behaviors and employing them habitually, these actions are taken for granted. Thus, it is easy to miss their significance: For those students who do not employ the behaviors commonly used by mathematicians, completing a mathematical task is an endeavor that is confusing and chaotic, mysterious and discouraging. By contrast, completing a mathematical task successfully can be facilitated by the use of these common mathematician behaviors.

Indeed, one of the major things that facilitated participants' mathematics learning was the degree to which they had been initiated into the mathematics community—i.e., the degree to which they had experienced mathematical enculturation (Bishop, 1988). Given the limited data that I collected on my participants and their study of the Core-Plus curriculum, however, I did not see evidence that mathematician behaviors were addressed or explained to students in overt ways. Unfortunately, as is the case with the mathematics register, students are left to their own devices to learn these traits that are essential to success in school mathematics. Further research is certainly warranted to help develop ways in which these traits can be intentionally taught to students.

CHAPTER 7

IMPLICATIONS

A Letter to High School Math Teachers

Dear High School Math Teachers,

As I have carried out my study of previously unsuccessful students, of their learning experiences in Core-Plus Course 1A, and of their understandings of algebra and functions, my thoughts have regularly turned to the question of What implications do my findings suggest for the high school mathematics classroom? I've often reflected, as a former high school math teacher myself, upon what I would do differently in my teaching as a result of the things I have learned through this research if I were to find myself back in the high school mathematics classroom. I would like to share my thoughts with you, my colleagues.

I have learned a lot about this whole notion of success and non-success. My own understanding of what it means for a student to be successful in mathematics has changed a great deal during the course of my research. I was guilty of simplistically categorizing those students who received above-average grades in my classes as successful and those students who received below-average grades in my classes as unsuccessful. I had barely taken into consideration what students' own experiences of the curriculum were like. I'm afraid in doing so, I let many slip through the cracks.

I think of students like Joy or Amy, for example. Joy and Amy were practically straight-A students and exhibited every observable characteristic of highly successful students—attentive and engaged in class, asking and answering questions, completing their assignments correctly, working productively with classmates—yet they loathed mathematics. Because of their disdain for the subject, it's quite probable that they won't choose careers related to mathematics, because they don't see it as something relevant or interesting. Yet, were there a way to draw them into the fold, some means of turning them on to the excitement of studying mathematics, the possibilities are quite enticing—to think that we'd have these young women, full of potential and the drive to succeed, pursuing careers in mathematics and blazing a trail for other young women to follow. Yet this will never happen if their teachers know nothing of their feelings of alienation in school mathematics.

> Schools have an obligation to ensure that all students participate in a strong instructional program that supports their mathematics learning. High expectations can be achieved in part with instructional programs that are *interesting for students and help them see the importance and utility of continued mathematical study for their own futures.* (NCTM, 2000, p. 13, emphasis added)

I don't claim to have an answer to the question of *how* to bring these young women around. I wish I did. However, I think the place to start is by getting to know our students and working to understand what their experiences in our classes are like.

Getting to know what my students' experiences in mathematics are like can also help me reach those students who are not as seemingly successful as Joy and Amy. Who's to say, for instance, what the reason was for Adrienne's or Terri's failure to pass Core-Plus Course 1 the first time they took it?

> Certain students may not be "good at math," not because of inherent lack of aptness in cognizing mathematical operations but because they experience as unfamiliar or as alienating the participation structures by which mathematics is customarily engaged in the classroom. (Erickson & Shultz, 1992, p. 477)

In fact, it was true for Sara, by her own reports, that she found the participation structures by which math was engaged in the Core-Plus classroom alienating. Could this have also been the reason behind her failure of Eighth Grade Math as a seventh-grader? It's impossible for me to say. Yet I certainly believe that working to understand how students are experiencing mathematics in the classroom is the first line of defense in the struggle to keep them from being filtered out of the mathematics pipeline (National Research Council, 1989).

I have also learned a great deal about the fragile nature of students' understanding, both in getting it to germinate and in getting it to take root. As a result of my research, I have developed a brand new appreciation for the difficult work that students face in learning the stuff of algebra. I'm sorry to say that I had gained too much distance from my own first days of encountering the complex ideas of functions, multiple representations, and various levels of abstraction. I had begun to think that those understandings and connections, which now seem quite clear and obvious to me, were similarly quite clear and obvious to students.

As I look back now at my data collection experiences, I'm surprised at the things that surprised me. Why was I puzzled that Joy did not recognize the infinitely many places where Y=X on a graph that had just two such points (Interview Two Task 1)? Why did I marvel at the fact that Adrienne did not realize that a graph of (*Atlanta temperature, Fort Wayne temperature*) did not actually help solve the problem of when two temperatures would meet (Interview One, Task 1)? Why did I find it baffling that Sara could not sketch the graph of $Y= -2 + 3X$ correctly even though she could correctly explain a negative y-intercept and a positive slope (Interview Three Task 3)? Why was I so "stunned" that Amy so thoroughly confused the rules for applying the distributive property? (Or did she?)

> I was quite stunned at how little Amy knew/remembered about simplifying equations and the rudiments of algebra. For instance, for Part A, Amy ultimately ended up with Y = 9X • 2X • X + 4 and explained that she got the 9X from these boldfaced/underlined elements in the given equation: Y = **2**(2X) + **3**(X+4) + **4**X , the 2X came from the first parentheses, and the X+4 came from the second set of parentheses. (To her credit, she wasn't at all confident about this answer!) When I proposed that someone else had gotten Y = 11X + 12 for an answer, Amy admitted she had "no idea" how they got that. Even when I showed her how it was derived, it didn't really seem to make sense to her.
> (Interview Two Task 3, 12/17/01)

If I allow myself to lose sight of just how challenging it is to wrap one's mind around (let alone master) such concepts, then, as a teacher, I will only be able to reach those students for whom the learning of mathematics comes most easily. I will leave these other students behind in the dust; and if they weren't previously unsuccessful when they reached my class, they surely will be unsuccessful in my class. Forgetting how complex and confusing these ideas are for the novice will prevent me from seeing the need to offer a variety of learning opportunities for students to nurture their budding knowledge and to allow it to grow and take root.

I think that the best way to help students' knowledge to mature is to probe their thinking, challenge them to understand their own ways of understanding. When a student gives a right answer, I should not make the mistake of equating that with correct understanding; when a student gives a wrong answer, I should not make the mistake of equating that with incorrect understanding. Only by finding out what my students think, how they've reasoned through things, how they've interpreted the questions I've asked or the problems I've posed, will I be able to prune the incomplete understandings and foster new, more hardy knowledge growth.

Finally, I have become aware, for the first time, of how essential it is to our students' success that they be explicitly taught to "walk the walk" and "talk the talk" of mathematicians. Prior to my conducting this study, I was unaware that there was a language or a set of behaviors unique to those who have been enculturated into the mathematics community. The need for intentionally helping socialize students according to the ways of the mathematicians was transparent to me. Yet I have come to see how vital this kind of instruction is for our students' success.

Some students, like Adrienne, will pick up mathematical habits through skillful imitation of their teachers – working methodically through a problem one step at a time, keeping good record of their steps. Others, like Joy, will only see madness in the method and will not follow suit – losing her way in resorting to her own devices.

> Joy right away asked whether she had to do it the way she "was supposed to do it" or the way she wanted to do it. I said she could choose. So she did this:

$$x - 8 = 10$$

$$\begin{array}{r} 10 \\ -8 \\ \hline \end{array}$$

$$x = 2$$

> (Interview Two, 1/14/02)

Some students, like Joy, will seem to naturally speak the language of the mathematics classroom with tremendous fluency. Others, like Monica, will struggle with this language in very unexpected ways (especially if the English language is not completely familiar):

> I said something about "draw[ing] a line through the points" and she questioned me about what I meant by motioning to see if I meant to connect the points dot-to-dot. (Interview Three Task 1, 3/18/02)

If I assume that students will learn to speak the language and imitate the behaviors that I exhibit in the mathematics classroom without my intentional help, I will be shortchanging many students for whom deliberate instruction in these areas could be of significant benefit to their mathematics learning success. Those unable to crack the code themselves will remain outsiders wondering at the mysterious ways of mathematics learning, possibly believing in the myth that they are simply unable to learn mathematics.

> Some teachers can be reluctant to provide such unashamedly personal images [of the way they metaphorically understand concepts], claiming that it is not at all clear that they will be usable by anyone else, and may well confuse. However, by not talking about such things at all, the existence of rich inner mental realms in which mathematics properly takes place will remain undiscovered by many pupils, who will see only the external mathematical world of symbols on paper and operations on them. Whether or not a particular offered image is successful in illuminating a concept, it at least serves the purpose of indicating that image-making is an appropriate activity for pupils to be engaged in, and that the teacher has personal images of the mathematics in question. (Pimm, 1987, p. 97)

In the same way that a master trains an apprentice, showing him how to use the tools of the trade, how to think and act like a craftsman, I believe we, as math teachers, ought to train our students. Through our own careful reflection on the mathematical ways of knowing and doing that we possess, and through overt means of sharing these with our students so that they can learn how to think and act like mathematicians, I believe we will see greater learning success among the students in our classrooms.

And I trust that greater learning success is the goal that you, like I, seek for your students!

Sincerely, Angie

A Letter to Math Teacher Educators

Dear Math Teacher Educators,

Although the research interests that motivated this study were born from my past experiences as a high school math teacher and I have carefully considered the implications of my findings for those who teach high school math, I have been challenged to consider the implications of my findings for mathematics teacher educators as well because these days I play a role in educating prospective mathematics teachers. These are my thoughts:

As I think back on my interactions with the previously unsuccessful students who participated in my study and on what I have come to understand about their mathematics learning experiences prior to as well as during the year that I conducted my study, it strikes me that the mathematics reform literature addresses our obligation to effectively reach all students without addressing the question of *how* to accomplish that goal. Granted, one of the mantras of mathematics reform is "mathematics [success] for all," and there has been quite a bit written about how to accomplish the goal of implementing mathematics reform. However, in my fourteen years of participating in mathematics reform both as a math teacher and as a math teacher educator, I have not encountered resources that address the complexities of effectively meeting the learning needs of those students who struggle.

I am thinking of students such as Adrienne and Terri who struggle once through a course to no avail before experiencing success; students such as Sara and Amy and Joy who struggle to find any satisfaction, meaning, or relevance in their math studies;

students such as Monica who struggle to understand the language of mathematics so that the essence of what they're supposed to be learning eludes them. These types of students struggle with their math classes while their teachers hardly notice. (I know this is true, based on my own experience as one such teacher.) Not only do I still puzzle over how to effectively reach these students, I puzzle still more over how to prepare prospective mathematics teachers to effectively reach these students.

☙

One insight I have garnered from my research is that teachers must make it their business to find out what their students' mathematics learning experiences are like *from those students' perspectives.* This is a particular challenge when I realize that, based on my own school math experiences, my perspective of mathematics learning is quite different from that of students like my study participants; somehow I just succeeded in math with very little effort, always enjoyed my math studies, and ...developed fairly well-rooted conceptual understandings of the mathematics on my own. Yet this cannot be expected for students, like my study participants, for whom math achievement does not come easily, for whom the "real-life" contexts of math problems are not meaningful, or for whom the mathematics register is not a familiar language.

Another insight I have garnered from my research is that teachers must accept it as their responsibility to intentionally socialize their students into mathematical ways of thinking, speaking, and behaving. This, too, is a particular challenge when I realize that, in my own school math experiences, I was never explicitly socialized to become a mathematics community member; somehow I just acquired a large repertoire of mathematician behaviors on my own. Yet it is not enough to hope that this will just happen for students who, like my study participants, could be that much more successful if they were explicitly taught mathematical modes of thinking, speaking, and behaving. Just as Lisa Delpit (1988) maintains that the rules of "the culture of power" must be explicitly taught to children who grow up outside of it, as a result of this study, I maintain that the same is true about the culture of *mathematical* power.

☙

So how do we, as mathematics teacher educators, teach prospective (and practicing) math teachers to attend to their future students' learning experiences? How do we, as mathematics teacher educators, teach teachers to attend to their future students' mathematical enculturation? Both of these are questions to which I do not yet have complete answers. Nevertheless, it is imperative that expertise in each of these areas be acknowledged as essential components of pedagogical content knowledge of mathematics.

I believe that the place to start is by attending to our own students' learning experiences in, and of, the math-teacher education classroom. As a math teacher educator, I must model for and with prospective math teachers the process of attending to students' (i.e., again, their own) learning experiences and to students' (i.e., their own) mathematical enculturation. Math teacher educators can learn by doing; similarly, prospective-math-teacher students can learn what it is like to learn by doing as they see their instructor learning by doing. This work is a bit like navigating a hall of mirrors, which is admittedly tricky, but important nonetheless (Lampert & Eshelman, 1995). Without creating ways for prospective teachers to learn to understand their students' math-learning experiences and to intentionally socialize their students into mathematical ways of thinking and behaving, I believe many more school math students will fall through the cracks of mathematics reform.

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And I trust that finding answers to the question of how to keep school math students from falling through the cracks is the goal that you, like I, seek for your prospective-math-teacher students!

Sincerely, Angie

APPENDICES

APPENDIX A

DATA COLLECTION INSTRUMENTS

Table A.1
Summary of Instruments

	SURVEY PHASE	*INTERVIEW PHASE*		
INSTRUMENT: (DATE): *# STUDENTS:*	**Initial Survey** (Oct. 2-5) ***n=31***	**Base-Line Interview** (Oct. 22-Nov. 26) ***n=9***	**Post-Unit-2 Interview** (Dec. 14-Jan. 31) ***n=6***	**Post-Unit-3 Interview** (Mar. 6-Apr. 12) ***n=6***
Affective Questions	20 agree/disagree items	10 agree/disagree items	Same as Base-Line agree/disagree items	Same as Base-Line agree/disagree items
Algebra/Functions Tasks ***Contextualized***	• Find (*d*,*n*) pairs (Open-ended)	• Compare two functions (Open-ended) • Complete table • Make graph	• Scatterplot problem with *y*=*x* line: Plot points and interpret • Write and use function rule • Make table and graph, then interpret	• Find linear regression of two scatterplots • Make table and graph each comparing two equations, then interpret
Abstract	• Complete table of values	N/A	N/A	• Slope and *y*-intercept interpretation
Symbolic Manipulation	N/A	N/A	N/A	• Solve three types of equations
Interview Protocols	N/A	• **Protocol A1:** Questions about the student's personal math history • **Protocol A2:** Questions about the student's recent experiences in the Integrated Math course • **Protocol B:** Questions about the student's understandings of the problem-solving task	• **Protocol A2:** Questions about the student's recent experiences in the Integrated Math course • **Protocol B:** Questions about the student's understandings of the problem-solving tasks	• **Protocol A2:** Questions about the student's recent experiences in the Integrated Math course • **Protocol B:** Questions about the student's understandings of the problem-solving tasks

Initial Survey Instrument

Please respond to the 18 statements below. For each one, mark an X in one of the boxes to show whether you **Strongly Agree**, **Agree**, have **No Opinion (or** are **Undecided)**, **Disagree**, or **Strongly Disagree** with the statement. Please complete the two math problems on the back as well.

Thank you very much for your time!

	Strongly Agree	Agree	No Opinion (or Undecided)	Disagree	Strongly Disagree
1. I often volunteer to answer questions in math class.					
2. I understand math well enough to help a classmate who is struggling.					
3. I do not like to go to the board to show my answer to a math homework problem.					
4. I am able to tell for myself whether an answer on my math homework is correct (or at least reasonable) without looking up the answer.					
5. Math often does not make sense to me.					
6. I am naturally good at math.					
7. Math was one of my favorite subjects in middle school.					
8. I love playing math games in class where you have to be the fastest one to answer in order to win.					
9. I can imagine myself completing a math-related major in college.					
10. I want to get a job that uses as little math as possible.					
11. A student doesn't need more than 2 years of math in high school if he (or she) is not going to go to college.					
12. A student doesn't really need to take 4 years of math in high school unless she (or he) is going to be a math major in college.					
13. I would be happy to get a C as my grade in Integrated Math 1-2.					
14. I do not plan to take more math courses beyond the two that are required to graduate from high school (Integrated Math 1-2 and Integrated Math 3-4).					
15. I want to graduate from high school with an Academic Honors Diploma.					
16. When a math problem is challenging, I usually keep working at it until I get it figured out.					
17. I usually check over my answers on a math test before turning it in.					
18. When I get back my graded math tests, I usually go over the problems I missed so I understand why I got them wrong.					

Initial Survey Instrument Scoring Rubrics (cont'd)

1. I have a bunch of dimes and nickels in my pocket.
   I need 50 cents to buy a candy bar.
   List the different combinations of dimes and nickels I could use to equal 50 cents:

2. Complete the table using the equation $5y = 50 - 10x$

x	1	4	5
y			

Interview Protocols

These are sample questions for the individual interviews with students. Specific questions will depend upon each student's responses to agree/disagree statements and problem-solving tasks that have been completed.

PROTOCOL A1:
Questions about the student's personal math history

- Tell me your opinion of math as a school subject.
  What do you enjoy most about math?
  What do you enjoy least about it?
  How well would you say you do in math?
- Describe to me one of your most successful experiences in a math class.
- Describe to me one of your least successful experiences in a math class.
- Do your parents ever help you with your math homework?
  If yes: Give me an example.
  If no: Why not?
- Do you ever use math outside of school?
  If yes: Give me an example.
  If no: Why not?

PROTOCOL A2:
Questions about the student's recent experiences in the Integrated Math course

- Tell me your opinion of Integrated Math 1-2 as a class.
  What do you enjoy most about the class?
  What do you enjoy least about it?
  How well would you say you are doing in the class?
- What did you study in class this past week?
- Did you participate in the class discussion this past week?
  If no: Why not?
  If yes: Give me an example.
    Did your answer happen to be wrong?
    If yes: Give me an example.
      How did you know your answer was wrong?
      How did you feel?
      How did the teacher respond?
- Did you contribute any ideas to a small-group investigation this past week?
  If no: Why not?
  If yes: Give me an example.
    Did anyone disagree with your idea(s)?
    If yes: Give me an example.
      How did you know that someone disagreed with your idea(s)?
      How did you feel?
      How did your classmate(s) respond?

Interview Protocols (cont'd)

<u>PROTOCOL B:</u>
Questions about the student's understandings of the problem-solving task(s)

- How confident are you that your answer to this problem is correct?
  Tell me why you feel this way.
- Explain to me what you were thinking when you solved this problem.
- Is there any other way to solve that problem?
  If yes: Explain it to me.
- Is there any other answer that would (also) be correct?
  If yes: Explain it to me.
- *In the event that the student's answer is (partially) incorrect:*
  I know of someone else who worked on this problem. She came up with <u>(correct answer)</u> for an answer. How is that answer different from yours?
  Can you imagine how she might have gotten this answer instead of the one you got?
  If yes: Explain your thinking to me.

  Would you like to revise your answer?
  If yes: Explain your revised answer to me.
- Imagine that you are showing this problem to an 8th-grade friend who is not taking Integrated Math right now. Tell me what you would say to her/him so that s/he understands what the problem is asking.
  How would you explain your method of solving the problem so that your friend is convinced that your answer is correct?

Interview Attitude Questionnaire

Please respond to the 10 statements below about your experiences in your Core-Plus Course 1 math class.

For each one, mark an X in one of the boxes to show whether you **Strongly Agree**, **Agree**, have **No Opinion (or** are **Undecided)**, **Disagree**, or **Strongly Disagree** with the statement.

	Strongly Agree	Agree	Undecided or No Opinion	Disagree	Strongly Disagree
1. I take part in whole-class discussions on a regular basis.					
2. I contribute my fair share of ideas during small-group time.					
3. Most of the things we're learning in Core-Plus Course 1 make sense to me.					
4. If I work at it hard enough, I know I can succeed in this class.					
5. I am interested in the things we're learning in Core-Plus Course 1.					
6. Based on my experience in Core-Plus Course 1, I am looking forward to taking Core-Plus Course 2.					
7. The problems we work on help me see the usefulness of math in everyday life.					
8. I find it helpful that we use graphing calculators in this class.					
9. When a math problem is challenging, I usually keep working at it until I get it figured out.					
10. I am putting forth my best effort in my Core-Plus Course 1 class.					

Interview One Familiarity Checklist

Below is a list of the topics in Unit 1 of your math book.

For each one, mark an X in one of the boxes to show which statement best describes how familiar you were with the topic before this semester.

	I already knew how to do these very well.	I already knew how to do these somewhat.	I had seen these before, but I knew very little about them.	I had never seen these before, and I did not know anything about them.	I don't remember studying these in Unit 1.
Stem-and-leaf plots					
Number line plots					
Histograms					
Mean, median, mode					
Five-number summary					
Box plots					
Scatterplots					
Plots over time					

Interview One Task 1

1. At 3:00 P.M., the temperature is 86° F in Atlanta and is decreasing at a rate of 3 degrees per hour.

   At the same time, the temperature is 56° F in Fort Wayne and is increasing at a rate of 2 degrees per hour.

   When will the temperatures be the same?

   Show how to use a graph to solve the problem:

Interview One Task 2

2. You want to join a DVD club to buy movies on DVD.
   Your friend says that it is more economical to buy the DVDs at the mall.

   You decide to consider your options:
   The DVD club has a membership fee of \$40 and each movie costs \$20.
   The store in the mall charges \$30 for each DVD.

   a) Complete the table.

Number of DVDs	1	2	3	4	5	6	7	8
DVD Club Cost (\$)								
Mall Cost (\$)								

   b) What would you say to your friend about which option is more economical?

Interview One Task 3

3. You are considering joining a karate club:

   At Champion's Corner, there is no membership fee and lessons cost $6.00 per hour.
   At Bowles Academy, there is a yearly membership fee of $24, and lessons cost $4.50 per hour.

   Which club would you join? ______________________________

   Show or write a short explanation of how you decided:

Interview Two Familiarity Checklist

Below is a list of the topics in Unit 2 of your math book.

For each one, mark an X in one of the boxes to show which statement best describes how familiar you were with the topic before this semester.

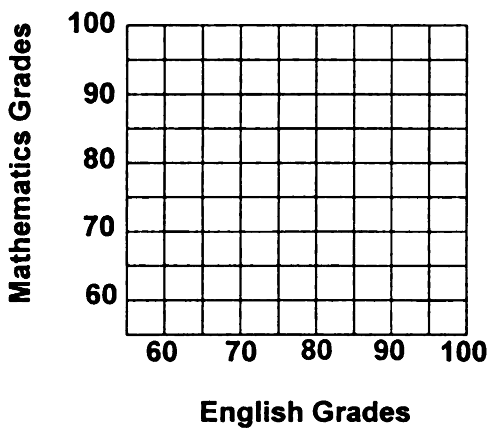
TOPIC	I already knew how to do these **very well.**	I already knew how to do these **somewhat.**	I had seen these before, but I **knew very little** about them.	I had never seen these before, and I **did not know anything** about them.	I **don't remember** studying this in Unit 2.
Using a $y = x$ line					
Graphing (x, y) data					
Writing a NOW-NEXT expression					
▪ Using the ANS key to calculate numbers repeatedly					
Writing a rule using letters					
▪ Using a rule to produce a table					
▪ Choosing Xmin, Xmax, Ymin, Ymax to set the viewing window					
▪ Using a rule to produce a graph					
Using rules that produce non-linear graphs					

Interview Two Task 1

1. Some students are better at some school subjects than they are at others. The end-of-semester English and mathematics grades of 12 students are given below.

English	95	73	84	90	70	64	80	74	90	88	80	76
Mathematics	85	75	80	66	87	83	91	74	94	74	80	63

a) Make a scatterplot of these grades on the axes below. Draw the line $y = x$ on your scatterplot.



b) How many students had a higher grade in mathematics than in English? ________

c) How many students had a higher grade in English than in mathematics? ________

d) Some people say that mathematics is more difficult than English.
Do these data support that statement? ________

Interview Two Task 2

2. Sara has a weekend job as a waitress at her family's restaurant. She works 8 hours on Saturdays and 4 hours on Sundays. Her pay is $2.50 per hour plus tips.

   a) Choose letters to represent the variables *daily income* and *daily tips*. Write a rule that gives Sara's income on Saturday as a function of her tips for that day.

      Letter for daily income _______ *Letter for daily tips* _______ *Rule -* ____________________

   b) Write a rule that gives Sara's income on Sunday as a function of her tips for that day.

      Rule _______________________

   c) One Saturday, Sara earned $70.00 in tips. Use your rule to calculate her income for that day.

      Saturday income _______________________

   d) The next day was Mother's Day, which is usually a good day for tips. If Sara earned $80.00 in tips on Mother's Day, how much did she earn that day?

      Mother's Day income _______________________

   e) How does the amount that Sara earned on Sunday (in part d) compare to the amount she earned on Saturday in part c?

Interview Two Task 3

3. Oliver High School's seniors would like to take a class trip to Washington, DC. The tour company offers to arrange the entire trip for a set fee of $8,000. This cost is to be shared annually by the seniors who go on the trip. If n seniors go on the trip, you can calculate the cost c to each one by the rule $c = \frac{8000}{n}$

   a) Complete the table below. Show (or explain) your work.

Number (n)	20	40	60	80	100
Cost per Person (c)					

   Show work:

   b) On a separate sheet of paper, make a graph of the (*number, cost*) relation.
   Explain to me, as you make your graph, what you are doing and what you are thinking.

   c) As accurately as possible, complete this sentence describing the pattern relating the number of seniors going on the trip and the cost per person.

   As the number of seniors going on the trip increases …

   Explain how the shape of the graph for part b matches the description you wrote above.

Interview Two Task 3 (cont'd)

d) Some seniors were trying to get the others to agree to a senior trip to Hawaii instead of to Washington, DC. The tour company could arrange that, too, but the total cost would be \$20,000. Write a rule that gives the cost *c* per senior going on a trip to Hawaii for various numbers *n* who go.

Rule ______________________

Predict how the shape of the graph for the Hawaii trip will look compared to that for the Washington, DC trip.

e) On the graph that you made for part b, make a graph of the (*number, cost*) relation for the Hawaii trip.

Explain to me, as you work on the problem, what you are doing and what you are thinking.

f) Explain how the cost for the Hawaii trip compares to the cost of the trip to Washington, DC.

Interview Two Task 3 (cont'd)

Problem Task 3, Parts B & E

Interview Three Familiarity Checklist

Below is a list of the topics in Unit 3 of your math book.

For each one, mark an X in one of the boxes to show which statement best describes how familiar you were with the topic before you studied Unit 3 this school year.

TOPIC	I already knew how to do these **very well.**	I already knew how to do these **somewhat.**	I had seen these before, but I **knew very little** about them.	I had never seen these before, and I **did not know anything** about them.	I **don't remember** studying this in Unit 3.
Draw a line to fit the pattern in a plot					
Make predictions using a linear model					
Finding the *rate of change*, the *slope*, and the *y-intercept* of a linear graph					
▪ Find the linear regression model for a set of data					
Use the equation of a linear model to make a quick sketch of the graph by hand					
▪ Using a table to find values of variables that satisfy the conditions of two equations or inequalities					
Solve an equation such as $3x + 12 = 45$ without the use of a table or graph					
Writing equivalent equations by rearranging, combining, and expanding terms					

Interview Three Task 1

1. The following table gives heights of eight tall buildings in the United States in stories, in feet, and in meters.

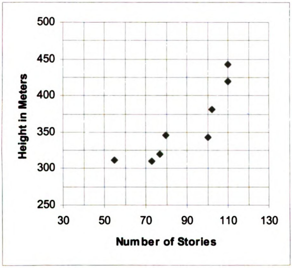
City	Building	Stories	Height ft	Height m
Chicago	Sears Tower	110	1,454	443
New York	World Trade Center	110	1,377	419
New York	Empire State	102	1,250	381
Chicago	AMOCO	80	1,136	346
Chicago	John Hancock Center	100	1,127	343
New York	Chrysler	77	1,046	319
Atlanta	Nations Plaza	55	1,025	312
Los Angeles	First Interstate World Center	73	1,017	310

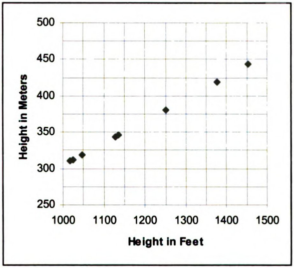
Source: *The 1993 Information Please Almanac.* Boston: Houghton Mifflin Company, 1992.

Below are two scatterplots, one of (*number of stories, height in meters*) and the other of (*height in feet, height in meters*).

a) For each scatterplot, draw a linear model that you believe is a good fit for the trend in the data.

Explain why you drew the lines where you did.





For the scatterplot on the left, use your linear model to predict the height in meters of a 90-story building:

____

For the scatterplot on the right, use your linear model to predict the height in meters of a 1400-foot building:

____

Interview Three Task 1 (cont'd)

b) Write an equation for the linear regression line for the (*number of stories, height in meters*) data. Use it to predict the height in meters of a 90-story building.

Linear regression line: y = ________________________

Predicted height of 90-story building: ________________

c) Write an equation for the linear regression line for the (*height in feet, height in meters*) data. Use it to predict the height in meters of a 1400-foot building.

Linear regression line: y = ________________________

Predicted height of 1400-foot building: ________________

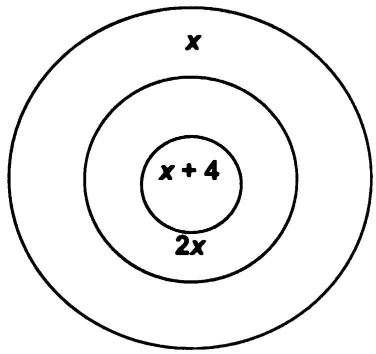
d) Compare the two scatterplots in part a. Based on your comparison, in which estimate would you have more confidence, the one in part b or the one in part c? Explain why.

More confidence in part b _____ *part c* _____

Explanation:

Interview Three Task 2

2. Jerome and Lin invented a game. They take turns throwing 10 darts (one dart at a time, of course) at a target like the one shown at the right. After each has a turn, one rolls a regular die (with 1 to 6 dots on each face). The result of the roll of the die is x, which they then use with the equations corresponding to where their darts landed to compute the number of points each player earned for that turn.



   a) In his first try, Jerome had two darts in the bull's-eye, worth $2x$ points each; three in the second ring, worth $x + 4$ points each; and four in the outer ring, worth x points. One dart was outside the outer ring. On his first turn then, Jerome earned y points where $y = 2(2x) + 3(x + 4) + 4x$.
   Write an equivalent equation in the form $y = a + bx$. Show how you got your answer.

   Equivalent equation: $y =$ ____________

   b) After rolling the die, Jerome found that he had earned 45 points. Write and solve an equation whose solution is the number that Jerome rolled on the die.

   Equation: ____________

   Solution: $x =$ ________

   c) On her first turn, Lin had one dart in the inner bull's-eye, four in the second ring, and four in the outer ring. Write an equation that gives y, the number of points that Lin made on this turn, in terms of x.

   $y =$ _________________________

Interview Three Task 2 (cont'd)

d) Use the results of parts b and c to find the number of points Lin earned on her first turn. (Use the same number Jerome rolled for x.) Show and explain your work.

Lin's points: ________

Work and explanation:

e) On their second turns, the equation for Jerome's points was equivalent to $y = 8 + 10x$, and the equation for Lin's points was equivalent to $y = 12 + 9x$. Before rolling the die to get x, they wondered what value of x would give them the same number of points. Find the answer to their question, and show how it can be found using a table, a graph, and an equation.

Value of x that gives Jerome and Lin the same number of points: $x =$ ________

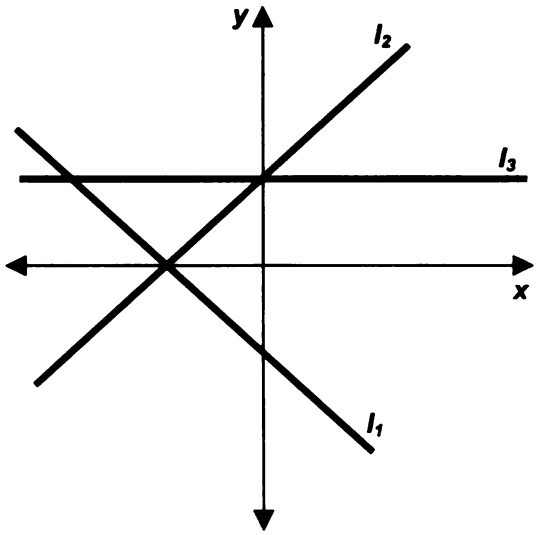
How to use a table:

How to use a graph:

How to write and solve an equation:

Interview Three Task 3

3. Three lines l_1, l_2, and l_3 are graphed below. As you learned in Unit 3, each line has an equation of the form $y = c + dx$.



a) Describe c and d for each of the three lines as completely as you can from the information given in the graph. For example, indicate which values are positive, negative, or zero, and why you think so.

Description for line l_1:

Description for line l_2:

Description for line l_3:

b) Are the values of c equal in the equations of any two of the three lines l_1, l_2, and l_3? Are the values of d equal? Explain your answer.

Values of c: *Yes* _____ *No* _____

Values of d: *Yes* _____ *No* _____

Explanation:

APPENDIX B

INSTRUMENTS OF ANALYSIS, DATA SUMMARIES, AND LONGER DATA EXAMPLES

Initial Survey Instrument Scoring Rubrics

Please respond to the 18 statements below. For each one, mark an X in one of the boxes to show whether you **Strongly Agree**, **Agree**, have **No Opinion (or are Undecided)**, **Disagree**, or **Strongly Disagree** with the statement. Please complete the two math problems on the back as well.

Thank you very much for your time!

	Strongly Agree	Agree	No Opinion (or Undecided)	Disagree	Strongly Disagree
1. I often volunteer to answer questions in math class.	**5**	**4**	**3**	**2**	**1**
2. I understand math well enough to help a classmate who is struggling.	**5**	**4**	**3**	**2**	**1**
3. I do <u>not</u> like to go to the board to show my answer to a math homework problem.	**1**	**2**	**3**	**4**	**5**
4. I am able to tell for myself whether an answer on my math homework is correct (or at least reasonable) <u>without</u> looking up the answer.	**5**	**4**	**3**	**2**	**1**
5. Math often does <u>not</u> make sense to me.	**1**	**2**	**3**	**4**	**5**
6. I am naturally good at math.	**5**	**4**	**3**	**2**	**1**
7. Math was one of my favorite subjects in middle school.	**5**	**4**	**3**	**2**	**1**
8. I love playing math games in class where you have to be the fastest one to answer in order to win.	**5**	**4**	**3**	**2**	**1**
9. I can imagine myself completing a math-related major in college.	**5**	**4**	**3**	**2**	**1**
10. I want to get a job that uses as little math as possible.	**1**	**2**	**3**	**4**	**5**
11. A student doesn't need more than 2 years of math in high school if he (or she) is not going to go to college.	**1**	**2**	**3**	**4**	**5**
12. A student doesn't really need to take 4 years of math in high school unless she (or he) is going to be a math major in college.	**1**	**2**	**3**	**4**	**5**
13. I would be happy to get a C as my grade in Integrated Math 1-2.	**1**	**2**	**3**	**4**	**5**
14. I do <u>not</u> plan to take more math courses beyond the two that are required to graduate from high school (Integrated Math 1-2 and Integrated Math 3-4).	**1**	**2**	**3**	**4**	**5**
15. I want to graduate from high school with an Academic Honors Diploma.	**5**	**4**	**3**	**2**	**1**
16. When a math problem is challenging, I usually keep working at it until I get it figured out.	**5**	**4**	**3**	**2**	**1**
17. I usually check over my answers on a math test before turning it in.	**5**	**4**	**3**	**2**	**1**
18. When I get back my graded math tests, I usually go over the problems I missed so I understand why I got them wrong.	**5**	**4**	**3**	**2**	**1**

Initial Survey Instrument Scoring Rubrics (cont'd)

1. I have a bunch of dimes and nickels in my pocket.
   I need 50 cents to buy a candy bar.
   List the different combinations of dimes and nickels I could use to equal 50 cents:

SOLUTION:

Dimes	0	1	2	3	4	5
Nickels	10	8	6	4	2	0

SCORING RUBRIC:

0 no attempt

1 one correct pair

1.5 two correct pairs

2 six correct (listed values/not # of d,n)
or three incorrect/unreasonable

2.5 three correct pairs

3 four correct pairs

3.5 five correct pairs

4 six correct pairs

2. Complete the table using the equation $5y = 50 - 10x$

SOLUTION:

x	1	4	5
y	8	2	0

SCORING RUBRIC:

0 no attempt

1 major errors

2 decreasing values, proportionally-spaced; no work shown

3 all but last step OK; nearly correct work shown

4 correct, or correct with very minor error

Table B.1
Initial Survey Data Summary

Rank from High to Low	Response Mean	Ranking of Response Mean	Z-Score for Response Mean	Category Means												Problem-Solving Tasks			
				Self-confidence, Public	Z-Score for Self-Conf, Public	Self-confidence, Personal	Z-Score for Self-Conf, Personal	Enthusiasm	Z-Score for Enthusiasm	Significance	Z-Score for Significance	Stick-to-it-ive-ness, Long-range	Z-Score for Long-range Stick-to-it	Stick-to-it-ive-ness, Short-range	Z-Score for Short-range Stick-to-it	Score on Abstract Problem	Z-Score for Abstract Problem	Score on Contextual Problem	Z-Score for Context Problem
31	1.28	Min	-2.30	2.3		1.0		1.0		1.0		1.3		1.0		1		3	
30	1.50	Q1	-2.01	2.3		1.3		1.0		1.0		1.7		1.7		0		3.5	
29	1.89	Q1	-1.49	1.7		1.3		2.0		2.7		1.7		2.0		1		4	
28	2.22	Q1	-1.05	3.0		1.3		1.0		2.7		2.7		2.7		0		0	
27	2.28	Q1	-0.97	3.3		3.0		2.0		1.0		1.7		2.7		0		3	
26. Terri	**2.33**	**Q1**	-0.90	**3.0**	**-0.28**	**1.7**	**-1.27**	**1.7**	**-0.76**	**2.7**	**-0.13**	**2.0**	**-0.91**	**3.0**	**-0.46**	**1**	**0.20**	**3.5**	**0.31**
24T	2.44	Q1	-0.75	3.0		1.7		1.0		2.0		2.0		5.0		0		4	
24T	2.44	Q1	-0.75	2.3		2.7		1.3		2.7		2.3		3.3		0		4	
23	2.47	Q2	-0.72	2.3		1.3		4.5		2.3		2.7		2.3		0		1.5	
22. Monica	**2.83**	**Q2**	-0.23	**2.7**	**-0.74**	**3.0**	**0.01**	**2.0**	**-0.44**	**3.0**	**0.19**	**3.0**	**0.04**	**3.3**	**-0.11**	**1**	**0.20**	**4**	**0.75**
20T	2.94	Q2	-0.09	3.0		3.7		3.0		2.0		2.0		4.0		0		2.5	
20T. Amy	**2.94**	**Q2**	-0.09	**2.7**	**-0.74**	**2.7**	**-0.31**	**1.7**	**-0.76**	**3.3**	**0.51**	**4.3**	**1.30**	**3.0**	**-0.46**	**1**	**0.20**	**4**	**0.75**
18T	3.00	Q2	-0.01	3.3		4.3		2.0		3.7		2.0		2.7		0		4	
18T. Joy	**3.00**	**Q2**	-0.01	**4.0**	**1.10**	**2.0**	**-0.95**	**2.0**	**-0.44**	**3.3**	**0.51**	**2.7**	**-0.28**	**4.0**	**0.60**	**0**	**-0.67**	**4**	**0.75**
14T	3.11	Median	0.13	3.0		2.3		2.7		2.0		4.3		4.3		1		1	
14T. Adrn	**3.11**	**Median**	0.13	**3.7**	**0.64**	**3.7**	**0.65**	**2.7**	**0.20**	**2.0**	**-0.78**	**2.7**	**-0.28**	**4.0**	**0.60**	**3**	**1.93**	**4**	**0.75**
14T	3.11	Median	0.13	3.7		3.0		2.7		3.7		3.0		2.7		0		3	
14T	3.11	Median	0.13	3.3		3.3		1.7		3.0		3.3		4.0		2		3	

Table B.1 (cont'd)

12T	3.17	Q3	0.21	4.3		3.3		2.3		1.3		2.7		3.3		0		3.5	
12T	3.17	Q3	0.21	2.0		3.3		2.7		4.0		3.0		4.0		0		2.5	
11	3.19	Q3	0.24	3.3		3.3		3.7		2.5		2.0		3.7		0		2.5	
9T. Sara	**3.28**	**Q3**	0.36	**4.0**	**1.10**	**3.3**	**0.33**	**2.7**	**0.20**	**2.7**	**-0.13**	**3.0**	**0.04**	**4.0**	**0.60**	**4**	**2.79**	**4**	**0.75**
9T	3.28	Q3	0.36	3.0		4.0		1.8		1.7		1.8		2.4		0		0	
8	3.39	Q3	0.50	3.0		4.3		3.3		2.3		4.0		3.3		0		3.5	
7	3.44	Q4	0.58	3.7		4.0		2.7		3.7		3.0		3.7		0		2	
6	3.50	Q4	0.65	4.0		3.3		1.7		4.0		4.3		3.7		2		4	
5	3.72	Q4	0.95	2.7		4.3		3.3		3.0		5.0		4.0		2		4	
4	3.83	Q4	1.09	3.7		3.7		3.0		4.0		4.3		4.3		1		3.5	
3	4.11	Q4	1.46	4.0		3.7		4.0		4.3		4.0		4.7		0		4	
2	4.39	Q4	1.83	4.3		4.3		4.3		4.3		4.3		4.7		4		4	
1	4.82	Max	2.41	4.7		4.3		5.0		5.0		5.0		5.0		0		4	
Mean of 31 Means:	**3.01**			**3.20**		**2.99**		**2.46**		**2.80**		**2.96**		**3.43**		**0.77**		**3.15**	
StDevP	**0.752**			**0.722**		**1.042**		**1.050**		**1.033**		**1.057**		**0.951**		**1.156**		**1.144**	
Mean of 25 Means:	3.03			3.17		3.05		2.54		2.79		2.96		3.40		0.56		2.96	
StDevP	0.841			0.764		1.098		1.135		1.127		1.125		1.033		0.983		1.199	
Mean of 6 Means:	2.92			3.33		2.72		2.11		2.83		2.94		3.56		1.67		3.92	
StDevP	0.262			0.255		0.938		0.674		0.828		0.905		0.974		0.373		1.484	

Table B.2
Explanation of Evaluation Item Types

U	Comprehension of what the problem was asking; Understanding the nature of the problem
E	Extension; Going beyond what the problem is asking to explore ideas about the mathematics
T	Skills that support using a table for solving
G	Skills that support using a graph for solving
R	Skills that support using a rule (equation) for solving
P	Skills that support using the coordinate plane for solving
M	Skills that support using symbolic manipulation for solving
J	Conjecturing; Using what you already know to form a conjecture
TS	Items where the table aided the student in obtaining the solution
GS	Items where the graph aided the student in obtaining the solution
RS	Items where the rule (equation) aided the student in obtaining the solution
PS	Items where the coordinate plane aided the student in obtaining the solution
MS	Items where symbolic manipulation aided the student in obtaining the solution
JS	Items where the conjecture aided the student in obtaining the solution
C	Calculator skill

Table B.3
Interview One Tasks: Listing of Evaluation Items and Item Types

Interview & Task	Item Type	Evaluation Item (skill or concept)
Int1 - T1	U	Vocabulary -- "At a rate of ... per ..."
Int1 - T1	U	Understanding that the question implies ***At what time*** *are the temperatures the same?*
Int1 - T1	TS	Using paired data to find a point of intersection of two functions
Int1 - T1	GS	Using graph to find a point of intersection of two functions
Int1 - T1	G	Graphing Time versus Temperature
Int1 - T1	G	Choosing appropriate ranges for axes
Int1 - T1	G	Choosing appropriate scales for axes
Int1 - T1	G	Interpolating for points that do not fall on the grid lines
Int1 - T1	G	Understanding where the origin is on the coordinate plane
Int1 - T2	T	Correctly completing a dual table of data
Int1 - T2	U	Recognizing that the answer has more than one case
Int1 - T3	U	Understanding that the question implies *Which option is more economical?*
Int1 - T3	U	Recognizing that the answer to the question has more than one case
Int1 - T3	TS	Using paired data to compare two functions
Int1 - T3	RS	Using an explicit formula to compare two functions
Int1 - T3	R	Deriving NOW-NEXT equations for calculating values
Int1 - T*	C	Using the Answer Key on the TI-83 to quickly calculate values iteratively (Any Task)

Table B.4
Interview Two Tasks: Listing of Evaluation Items and Item Types

Interview & Task	Item Type	Evaluation Item (skill or concept)
Int2 - T1a	G	Interpolating for points that do not fall on the grid lines
Int2 - T1a	G	Drawing the y=x line accurately on the graph
Int2 - T1a	U	Understanding what the y=x line represents
Int2 - T1a	U	Recognizing that there are many places on graph (other than data points) where y=x
Int2 - T1b	GS	Using points where y>x to answer something about the overall data
Int2 - T1b	TS	Identifying points on the TABLE where y>x (or y<x or y=x)
Int2 - T1c	GS	Using points where y<x to answer something about the overall data
Int2 - T1c	GS	Using points where y=x to answer something about the overall data
Int2 - T1d	E	Knowing what the data do/not suggest
Int2 - T2a/b	R	Translating words into an algebraic representation/expression
Int2 - T2a	R	Assigning letters to variables
Int2 - T2a	R	Writing equivalent expressions/equations
Int2 - T2c/d	RS	Using the equations (rules) to find total earnings
Int2 - T2e	U	Being able to explain why two equation-values are the same if their "constants" differ
Int2 - T3a	RS	Using a rule to find the y-values for a table of data [Washington DC trip]
Int2 - T3b	G	Using appropriate axes (x-axis for independent variable)
Int2 - T3b	G	Choosing appropriate ranges for axes
Int2 - T3b	G	Choosing appropriate scales for axes
Int2 - T3b	G	Interpolating for points that do not fall on the grid lines
Int2 - T3b	C	Entered one set of data into lists to make scatterplot representation
Int2 - T3c	U	Being able to describe the relationship between the function variables
Int2 - T3d	R	Writing a rule (formula) for a given situation [Hawaii trip]
Int2 - T3d	RS	Used Washington rule as a model for Hawaii rule
Int2 - T3d	J	Predicting the shape of the graph for a function similar to one already graphed
Int2 - T3e (fxn)	C	Graphed one function on calculator by entering rule as Y=
Int2 - T3e (fxn)	GS	Graphed <u>two functions</u> on paper (using same set of axes appropriately) ... Complementary Strategy
Int2 - T3e (fxn)	TS	Entered <u>two rules</u> as Y= and examined table values for comparison ... Complementary Strategy
Int2 - T3e (fxn)	C	Entered <u>two sets</u> of data into lists to make dual function representation
Int2 - T3e (fxn)	C	Graphed <u>two functions</u> on calculator by entering rules as Y=
Int2 - T3e	T	Made her own written table
Int2 - T3e	J	Predicting a y-value for an x-value not in the data set
Int2 - T3e	E	Understanding the infinite extent of a function
Int2 - T3e	J	Making a reasonable conjecture about the two numerical patterns as the functions extend toward x-approaching-positive-infinity
Int2 - T3e	JS	Using the TI-83 to check out conjectures
Int2 - T3f	U	Being able to describe how the two functions compare
Int2 - T3e (fxn)	C	Graphed one function on calculator by entering rule as Y=

Table B.5
Interview Three Tasks: Listing of Evaluation Items and Item Types

Interview & Task	Item Type	Evaluation Item (skill or concept)
Int3 - T1a	G	Finding a linear model that is a good fit for the trend in the data by eyeing with a clear ruler
Int3 - T1a	U	Understanding which line is the best fit and knowing why (when eyeing with a clear ruler)
Int3 - T1a	GS	Using the estimated linear model to predict a y-value for an x-value not in the data set
Int3 - T1a	U	Understanding the connection between the scatterplot representation and the context situation
Int3 - T1a	E	Understanding what makes a good estimate and why (cf. interpolating betw 80- and 110-story bldgs)
Int3 - T1b/c	C	Using the TI-83 to find the Linear Regression line (y = a + bx)
Int3 - T1b/c	RS	Using the linear regression model to predict a y-value for an x-value not in the data set
Int3 - T1d	E	Understanding of what makes a good estimator and why (lines where r=.87 and r=.999998)
Int3 - T2a	M	Manipulating expressions/equations -- applying the distributive property
Int3 - T2a	M	Manipulating expressions/equations -- collecting & combining like terms
Int3 - T2b	M	Using a given y-value to solve for X --> y = 11x + 12 [Type Beta]
Int3 - T2c	R	Translating words into an algebraic representation/expression (using a similar, worked-out example)
Int3 - T2d	RS	Using a given x-value to solve for Y --> y = 1(2x) + 4(x+4) + 4x [Type Alpha]
Int3 - T2e1 (fxn)	T	Entered two rules as Y= and examined table values for comparison
Int3 - T2e1 (fxn)	TS	Finding a point of intersection of two functions (TI-83 table)
Int3 - T2e1	U	Understanding what the x- and y-values represent from the problem context
Int3 - T2e2 (fxn)	C	Graphed two functions on calculator by entering rules as Y=
Int3 - T2e2	GS	Finding a point of intersection of two functions (TI-83 graph)
Int3 - T2e2	U	Understanding what the x- and y-values represent from the problem context
Int3 - T2e3	MS	Solving for X --> 8 + 10x = 12 + 9x [Type Gamma]
Int3 - T3a	U	Knowing that "y-intercept" describes the place where a line crosses the y-axis
Int3 - T3a	P	Identifying positive and negative y-intercepts, given 3 lines drawn on the coord plane
Int3 - T3a	U	Knowing that "slope" describes the uphill or downhill characteristic of a line
Int3 - T3a	P	Identifying positive, negative, and zero slopes, given 3 lines drawn on the coord plane
Int3 - T3b	PS	Correctly identifying lines with same y-intercept
Int3 - T3b	PS	Correctly identifying lines with same slope
Int3 - T3c	U	Using knowledge about the y-intercept value appearing in y = -2 + 3x to plot a point at (0,-2)
Int3 - T3c	U	Using knowledge about the slope value appearing in y = -2 + 3x to sketch an "uphill" line thro y-int
Int3 - T3a	U	Understanding what C and D represent in an abstracted linear equation --> y = C + Dx
Int3 - T3a	U	Knowing which of C or D is slope and which is y-intercept

Table B.6
Task Evaluation Mean Score Summary

Performance Mean	Adrienne	Sara	Amy	Joy	Terri	Monica	MEAN for all 6 participants
Int1 Task-Skills Combined	**3.46**	**3.71**	**3.20**	**3.46**	**3.00**	**2.69**	**3.25**
Int1 - Task 1	3.29	3.75	2.78	3.25	1.75	2.63	2.91
Int1 - Task 2	4.00	4.00	4.00	4.00	3.50	3.00	3.75
Int1 - Task 3	3.50	3.50	3.75	3.67	4.00	2.67	3.51
Int2 Task-Skills Combined	**3.90**	**3.68**	**3.54**	**3.47**	**3.33**	**2.86**	**3.46**
Int2 - Task 1	3.88	3.43	3.38	2.57	3.29	2.33	3.14
Int2 - Task 2	4.00	3.80	4.00	3.60	3.50	3.00	3.65
Int2 - Task 3a-3d	4.00	3.90	4.00	4.00	3.60	3.57	3.85
Int2 - Task 3e-3f	3.78	3.56	2.40	4.00	2.33	2.40	3.08
Int3 Task-Skills Combined	**3.48**	**3.37**	**3.36**	**2.85**	**2.54**	**1.85**	**2.91**
Int3 -Task 1	3.00	3.25	3.25	2.88	3.13	1.88	2.90
Int3 -Task 2	3.82	3.83	3.40	2.50	2.60	2.56	3.12
Int3 -Task 3, concrete	3.83	4.00	4.00	3.67	2.17	1.17	3.14
Int3 -Task 3, abstracted	3.00	1.25	2.50	2.25	1.00	1.25	1.88
MEAN for 3 INTERVIEWS	**3.62**	**3.59**	**3.36**	**3.26**	**2.96**	**2.47**	**3.21**
Z-score for 3-Interview Mean	0.84	0.77	0.32	0.10	-0.52	-1.52	**StDevP= 0.486**

List of Objectives

I. Unit 1 Objectives:

C. To compare sets of data using scatterplots and the line y=x, and to interpret these comparisons for the real-world contexts that gave rise to the data

4. **Lesson 4 Objectives:**
    - **a.** To use scatterplots to find an association between two variables
    - **b.** To use the line y=x to analyze data on a scatterplot
    - **c.** To compare the information provided by various types of plots
    - **d.** To analyze plots over time

II. Unit 2 Objectives:

A. To begin developing students' sensitivity to the rich variety of situations in which quantities vary in relation to each other

B. To develop students' ability to represent relations among variables in several ways – using tables of numerical data, coordinate graphs, symbolic rules, and verbal descriptions – and to interpret data presented in any one of those forms

C. To develop students' ability to recognize important patterns of change in single variables and related variables

1. **Lesson 1 Objectives:**
    - **a.** To identify key variables in the situation to be modeled
    - **b.** To collect data that will suggest the pattern relating those variables
    - **c.** To make a table and graph the data to look for patterns
    - **d.** To make predictions that go beyond the data (extrapolations and interpolations)
2. **Lesson 2 Objectives:**
    - **a.** To focus students' attention on patterns of change in variables
    - **b.** To explore iterative and recursive change
    - **c.** To examine equations relating NOW and NEXT
    - **d.** To introduce and utilize the iteration capabilities of a graphing calculator or computer software
3. **Lesson 3 Objectives:**
    - **a.** To develop students' ability to summarize patterns relating variables using rules
    - **b.** To utilize rules along with a graphing calculator or computer software
    - **c.** To illustrate the power of a rule and its capabilities when used with a graphing calculator or computer software
    - **d.** To explore difficult questions relating variables by using rules and technology
4. **Lesson 4 Objectives:**
    - **a.** To begin to understand and develop an appreciation for the power of technology-generated tables and graphs and the essential role of algebraic rules in providing directions to a calculator or computer software
    - **b.** To recognize the connection between two-variable relationships and their graphs

List of Objectives (cont'd)

III. Unit 3 Objectives:

A. To recognize patterns in tables and graphs of data that are modeled well by linear equations

B. To write equations in the form y = a + bx to model linear patterns in graphs or numerical data

C. To use table, graph, or symbolic representations of linear models to answer questions about the modeled situation: (1) Find y for a given x; (2) Find x for a given y (i.e., solve equations and inequalities); and (3) Describe the rate at which y changes as x changes.

1. **Lesson 1 Objectives:**
   - **a.** To organize and interpret sets of data from real-world situations using a table, graph, and equation
   - **b.** To use a variety of methods to estimate the graph and equation of a line that fits a given set of data
   - **c.** To draw and use a modeling line to predict the value of one variable given the value of the other and to describe the rate at which one variable changes as the other changes

2. **Lesson 2 Objectives:**
   - **a.** To construct a table of values from a given graph and examine these two representations for common patterns
   - **b.** To write an equation of a line given its slope and y-intercept and interpret the meaning of the equation, slope, and y-intercept in real-world problem situations
   - **c.** To write an equation of a line given two points on the line, explain the process for doing so, and interpret the meaning of the equation in a real-world problem situation
   - **d.** To produce a linear regression model using technology, and interpret the meaning
   - **e.** To describe how to use the numbers a and b to sketch the graphs of equations in the form y = a + bx and write equations in this form given a graph or appropriate points of a graph
   - **f.** To explain the relationships among the graph, equation (y = a + bx), and table of values for a linear model
   - **g.** To use and interpret the meanings of linear graphs, tables, rules, and related procedures in real-world contexts to which they apply

3. **Lesson 3 Objectives:**
   - **a.** To write linear equations and inequalities that match important questions about linear models or real-world situations
   - **b.** To solve linear equations and inequalities by inspection of appropriate graphs and tables of (x,y) values, explain the methods of solution, and interpret the meaning of the solution in the real-world context
   - **c.** To use the "undoing" and "balancing" methods to solve simple linear equations, explain how to do so, and interpret the meaning of the solution in real-world contexts
   - **d.** To use the commutative property of addition and the distributive property of multiplication over addition to rewrite linear models in equivalent forms
   - **e.** To test the equivalence of equations by comparing their tables and graphs

Table B.7
Evaluation Inventory for Various Representations

VARIOUS REPRESENTATIONS -- Graph			Adrn	Sara	Amy	Joy	Terri	Mon
5	Int1 - T1	Graphing Time versus Temperature	0	3	4	4	0	4
6	Int1 - T1	Choosing appropriate ranges for axes	4	4	2	2	0	4
7	Int1 - T1	Choosing appropriate scales for axes	4	4	2	2	0	4
8	Int1 - T1	Interpolating for points that do not fall on the grid lines	4	0	2	4	0	0
9	Int1 - T1	Understanding where the origin is on the coordinate plane	1	4	2	2	0	2
1	Int2 - T1a	Interpolating for points that do not fall on the grid lines	4	4	4	4	4	4
2	Int2 - T3b	Using appropriate axes (x-axis for independent variable)	4	3	4	4	4	4
3	Int2 - T3b	Choosing appropriate ranges for axes	4	4	4	4	4	4
4	Int2 - T3b	Choosing appropriate scales for axes	4	4	4	4	2	2
5	Int2 - T3b	Interpolating for points that do not fall on the grid lines	4	4	4	4	4	4
12	Int2 - T3e (fxn)	Graphed <u>two functions</u> on paper (using same set of axes appropriately)	3	4	1	4	1	2
15	Int2 - T3e (fxn)	Graphed <u>two functions</u> on calculator by entering rules as Y=	0	0	0	0	0	0
9	Int3 - T2e2 (fxn)	Graphed <u>two functions</u> on calculator by entering rules as Y=	4	3	4	0	0	1
1	Int3 - T3a	Knowing that "y-intercept" describes the place where a line crosses the y-axis	4	4	4	4	2	1
2	Int3 - T3a	Identifying positive and negative y-intercepts, given 3 lines drawn on the coord plane	4	4	4	4	1	1
5	Int3 - T3b	Correctly identifying lines with same y-intercept	4	4	4	4	1	1
3	Int3 - T3a	Knowing that "slope" describes the uphill or downhill characteristic of a line	4	4	4	3	3	1
4	Int3 - T3a	Identifying positive, negative, and zero slopes, given 3 lines drawn on the coord plane	3	4	4	3	2	1
6	Int3 - T3b	Correctly identifying lines with same slope	4	4	4	4	4	2
4	Int3 - T1a	Understanding the connection between the scatterplot representation and the context situation	4	4	4	4	4	1
VARIOUS REPRESENTATIONS -- Table			**Adrn**	**Sara**	**Amy**	**Joy**	**Terri**	**Mon**
1	Int1 - T2	Correctly completing a dual table of data	4	4	4	4	4	3
3	Int1 - T1	Using paired data to find a point of intersection of two functions	4	4	3	0	1	2
3	Int1 - T3	Using paired data to compare two functions	3	3	0	0	0	0
6	Int2 - T1b	Identifying points on the TABLE where y>x (or y<x or y=x)	4	4	4	0	4	4

Table B.7 (con'td)

1	Int2 - T3a	Using a rule to find the y-values for a table of data [Washington DC trip]	4	4	4	4	4	4
13	Int2 - T3e (fxn)	Entered two rules as Y= and examined table values for comparison	4	2	0	4	NA	0
14	Int2 - T3e (fxn)	Entered two sets of data into lists to make dual function representation	0	4	0	0	2	0
16	Int2 - T3e	Made her own written table	4	4	4	4	0	0
6	Int3 - T2e1 (fxn)	Entered two rules as Y= and examined table values for comparison	3	4	4	4	4	3
7	Int3 - T2e1 (fxn)	Finding a point of intersection of two functions (TI-83 table)	4	4	4	3	4	3
VARIOUS REPRESENTATIONS -- Rule (equation), explicit			**Adrn**	**Sara**	**Amy**	**Joy**	**Terri**	**Mon**
4	Int1 - T3	Using an explicit formula to compare two functions	0	0	4	4	4	4
1	Int2 - T3a	Using a rule to find the y-values for a table of data [Washington DC trip]	4	4	4	4	4	4
8	Int2 - T3d	Writing a rule (formula) for a given situation [Hawaii trip]	4	4	4	4	3	0
11	Int2 - T3e (fxn)	Graphed one function on calculator by entering rule as Y=	4	0	1	0	0	0
13	Int2 - T3e (fxn)	Entered two rules as Y= and examined table values for comparison	4	2	0	4	NA	0
6	Int3 - T2e1 (fxn)	Entered two rules as Y= and examined table values for comparison	3	4	4	4	4	3
9	Int3 - T2e2 (fxn)	Graphed two functions on calculator by entering rules as Y=	4	3	4	0	0	1
VARIOUS REPRESENTATIONS -- Rule (equation), iterative			**Adrn**	**Sara**	**Amy**	**Joy**	**Terri**	**Mon**
5	Int1 - T3	Deriving NOW-NEXT equations for calculating values	0	4	0	0	0	0
6	Int1 - T*	Using the Answer Key on the TI-83 to quickly calculate values iteratively (Any Task)	4	0	4	0	4	0

Table B.8
Evaluation Inventory for Means of Solution

MEANS OF SOLUTION -- Algorithms, Solving single equations			**Adrn**	**Sara**	**Amy**	**Joy**	**Terri**	**Mon**
4	Int2 - T2c/d	Using the equations (rules) to find total earnings	4	4	4	4	4	2
5	Int3 - T2d	Using a given x-value to solve for Y --> y = 1(2x) + 4(x+4) + 4x	4	4	4	4	2	4
3	Int3 - T2b	Using a given y-value to solve for X --> y = 11x + 12	4	4	NA	4	4	4
12	Int3 - T2e3	Solving for X --> 8 + 10x = 12 + 9x	0	4	0	0	0	0
MEANS OF SOLUTION -- Graphical, Points & Lines (sketched)			**Adrn**	**Sara**	**Amy**	**Joy**	**Terri**	**Mon**
5	Int1 - T1	Graphing Time versus Temperature	0	3	4	4	0	4
4	Int1 - T1	Using graph to find a point of intersection of two functions	0	3	2	4	1	1
2	Int2 - T1a	Drawing the y=x line accurately on the graph	4	4	3	3	4	3
2	Int2 - T3b	Graphing Number versus Cost, using appropriate axes	4	3	4	4	4	4
12	Int2 - T3e (fxn)	Graphed two functions on paper (using same set of axes appropriately)	3	4	1	4	1	2
MEANS OF SOLUTION -- Technology-assisted, graphing			**Adrn**	**Sara**	**Amy**	**Joy**	**Terri**	**Mon**
6	Int2 - T3b	Entered one set of data into lists to make scatterplot representation	4	4	4	4	4	3
11	Int2 - T3e (fxn)	Graphed one function on calculator by entering rule as Y=	4	0	1	0	0	0
14	Int2 - T3e (fxn)	Entered two sets of data into lists to make dual function representation	0	4	0	0	2	0
15	Int2 - T3e (fxn)	Graphed two functions on calculator by entering rules as Y=	0	0	0	0	0	0
6	Int3 - T1b/c	Using the TI-83 to find the Linear Regression line (y = a + bx)	4	4	3	3	3	3
7	Int3 - T1b/c	Using the linear regression model to predict a y-value for an x-value not in the data set	4	4	3	3	4	3
9	Int3 - T2e2 (fxn)	Graphed two functions on calculator by entering rules as Y=	4	3	4	0	0	1
10	Int3 - T2e2	Finding a point of intersection of two functions (TI-83 graph)	3	3	4	0	2	0
MEANS OF SOLUTION -- Technology-assisted, rule-generated t			**Adrn**	**Sara**	**Amy**	**Joy**	**Terri**	**Mon**
13	Int2 - T3e (fxn)	Entered two rules as Y= and examined table values for comparison	4	2	0	4	NA	0
6	Int3 - T2e1 (fxn)	Entered two rules as Y= and examined table values for comparison	3	4	4	4	4	3
7	Int3 - T2e1 (fxn)	Finding a point of intersection of two functions (TI-83 table)	4	4	4	3	4	3

Table B.8 (cont'd)

MEANS OF SOLUTION – Technology-assisted, conjecture & investigation			**Adrn**	**Sara**	**Amy**	**Joy**	**Terri**	**Mon**
19	Int2 - T3e	patterns as the functions extend twd x-approach-+-infinity	3	4	NA	4	NA	1
20	Int2 - T3e	Using the TI-83 to check out conjectures	4	2	NA	4	NA	NA
MEANS OF SOLUTION – Technology-assisted, student-invented means			**Adrn**	**Sara**	**Amy**	**Joy**	**Terri**	**Mon**
3	Int1 - T1	Using paired data to find a point of intersection of two functions	4	4	3	0	1	2
3	Int1 - T3	Using paired data to compare two functions	3	3	0	0	0	0
4	Int1 - T3	Using an explicit formula to compare two functions	0	0	4	4	4	4
5	Int1 - T3	Deriving NOW-NEXT equations for calculating values	0	4	0	0	0	0
6	Int1 - T*	Using the Answer Key on the TI-83 to quickly calculate values iteratively (Any Task)	4	0	4	0	4	0
16	Int2 - T3e	Made her own written table	4	4	4	4	0	0
MEANS OF SOLUTION – Alternate means of solution			**Adrn**	**Sara**	**Amy**	**Joy**	**Terri**	**Mon**
1	Int2 - T3a	Using a rule to find the y-values for a table of data [Washington DC trip]				4		
13	Int2 - T3e (fxn)	Entered <u>two rules</u> as Y= and examined table values for comparison … Complementary		2				
16	Int2 - T3e	Made her own written table	4					
3	Int3 - T1a	Using the estimated linear model to predict a y-value for an x-value not in the data set				2		
7	Int3 - T1b/c	Using the linear regression model to predict a y-value for an x-value not in the data set					4	
3	Int3 - T2b	Using a given y-value to solve for X –> y = 11x + 12				4		
10	Int3 - T2e2	Finding a point of intersection of two functions (TI-83 graph)					2	

Table B.9 Evaluation Inventory for Levels of Abstraction

LEVELS OF ABSTRACTION -- **Using math to make sense of a situation**			Adr	Sara	Amy	Joy	Terri	Mon
2	Int1 - T2	Recognizing that the answer has more than one case	4	4	4	4	3	3
2	Int1 - T3	Recognizing that the answer to the question has more than one case	3	3	3	3	4	1
5	Int2 - T1b	Using points where y>x to answer something about the overall data	4	4	4	4	3	1
7	Int2 - T1c	Using points where y<x to answer something about the overall data	4	4	4	4	0	0
8	Int2 - T1c	Using points where y=x to answer something about the overall data	4	0	4	0	3	0
5	Int2 - T2e	Being able to explain why two equation-values are the same if their "constants" differ	4	4	4	4	4	4
7	Int2 - T3c	Being able to describe the relationship between the function variables	4	4	4	4	4	4
10	Int2 - T3d	Predicting the shape of the graph for a function similar to one already graphed	4	4	4	4	4	NA
16	Int2 - T3e	Made her own written table	4	4	4	4	0	0
17	Int2 - T3e	Predicting a y-value for an x-value not in the data set	4	4	2	4	NA	4
21	Int2 - T3f	Being able to describe how the two functions compare	4	4	4	4	4	3
1	Int3 - T1a	Finding a linear model that is a good fit for the trend in the data by eyeing with a clear ruler	4	4	3	4	4	3
3	Int3 - T1a	Using the estimated linear model to predict a y-value for an x-value not in the data set	4	4	4	2	4	1
8	Int3 - T2e1	Understanding what the x- and y-values represent from the problem context	4	4	4	2	4	4
11	Int3 - T2e2	Understanding what the x- and y-values represent from the problem context	4	4	4	0	3	0
LEVELS OF ABSTRACTION -- **Mathematizing a situation**			Adr	Sara	Amy	Joy	Terri	Mon
1	Int1 - T1	Vocabulary -- "At a rate of ... per ..."	4	4	4	4	4	3
2	Int1 - T1	Understanding that the question implies ***At what time are the temps the same?***	2	4	4	4	1	1
4	Int1 - T1	Using graph to find a point of intersection of two functions	0	3	2	4	1	1
5	Int1 - T1	Graphing Time versus Temperature	0	3	4	4	0	4
1	Int1 - T3	Understanding that the question implies *Which option is more economical?*	4	4	4	4	4	3
3	Int2 - T1a	Understanding what the y=x line represents	3	2	3	1	3	1
1	Int2 - T2a/b	Translating words into an algebraic representation/expression	4	4	4	4	2	2

Table B.9 (cont'd)

2	Int2 - T2a	Assigning letters to variables	4	4	4	4	4	4
9	Int2 - T3d	Used Washington rule as a model for Hawaii rule	?	4	4	?	3	0
12	Int2 - T3e (fxn)	Graphed two functions on paper (using same set of axes appropriately)	3	4	1	4	1	2
2	Int3 - T1a	Understanding which line is the best fit and knowing why (when eyeing with a clear ruler)	2	4	4	2	4	2
6	Int3 - T1b/c	Using the TI-83 to find the Linear Regression line (y = a + bx)	4	4	3	3	3	3
7	Int3 - T1b/c	Using the linear regression model to predict a y-value for an x-value not in the data set	4	4	3	3	4	3
4	Int3 - T2c	Translating words into an algebraic representation/expression (using a similar, worked-out example)	4	4	4	1	1	2
LEVELS OF ABSTRACTION -- Symbolic Abstraction			**Adr**	**Sara**	**Amy**	**Joy**	**Terri**	**Mon**
4	Int2 - T1a	Recognizing that there are many places on graph (other than data points) where y=x	0	0	0	1	0	0
3	Int2 - T2a	Writing equivalent expressions/equations	4	3	4	2	0	0
1	Int3 - T2a	Manipulating expressions/equations -- applying the distributive property	4	4	1	1	1	1
2	Int3 - T2a	Manipulating expressions/equations -- collecting & combining like terms	4	4	1	1	1	1
3	Int3 - T2b	Using a given y-value to solve for X --> y = 11x + 12	4	4	NA	4	4	4
5	Int3 - T2d	Using a given x-value to solve for Y --> y = 1(2x) + 4(x+4) + 4x	4	4	4	4	2	4
12	Int3 - T2e3	Solving for X --> 8 + 10x = 12 + 9x	0	4	0	0	0	0
7	Int3 - T3c	Using knowledge about the y-intercept value appearing in y = -2 + 3x to plot a point at (0,-2)	4	1	4	3	0	2
8	Int3 - T3c	Using knowledge about the slope value appearing in y = -2 + 3x to sketch an "uphill" line thro y-int	3	2	4	3	0	1
LEVELS OF ABSTRACTION -- Symbolic Generalization			**Adr**	**Sara**	**Amy**	**Joy**	**Terri**	**Mon**
9	Int2 - T1d	Knowing what the data do/not suggest	4	2	1	1	2	1
18	Int2 - T3e	Understanding the infinite extent of a function	4	4	NA	4	NA	2
19	Int2 - T3e	Making reasonable conjecture abt numerical patterns as fxns extend twd x-approach-+-infinity	3	4	NA	4	NA	1
5	Int3 - T1a	Understanding what makes a good estimate and why (cf. interpolating betw 80- and 110-story bldgs)	1	1	1	1	1	1
8	Int3 - T1d	Understanding of what makes a good estimator and why (lines where r=.87 and r=.999998)	1	1	4	4	1	1
9	Int3 - T3a	Understanding what C and D represent in an abstracted linear equation --> y = C + Dx	1	1	1	1	1	1
10	Int3 - T3a	Knowing which of C or D is slope and which is y-intercept	4	1	1	2	1	1

Table B.10 Evaluation Inventory for Functions

FUNCTIONS -- Linear			Adr	Sara	Amy	Joy	Terri	Mon
4	Int1 - T1	Using graph to find a point of intersection of two functions	0	3	2	4	1	1
3	Int1 - T1	Using paired data to find a point of intersection of two functions	4	4	3	0	1	2
2	Int3 - T1a	Understanding which line is the best fit and knowing why (when eyeing with a clear ruler)	2	4	4	2	4	2
3	Int3 - T1a	Using the estimated linear model to predict a y-value for an x-value not in the data set	4	4	4	2	4	1
5	Int3 - T1a	Understanding what makes a good estimate and why (cf. interpolating betw 80- and 110-story bldgs)	1	1	1	1	1	1
8	Int3 - T1d	Understanding of what makes a good estimator and why (lines where r=.87 and r=.999998)	1	1	4	4	1	1
6	Int3 - T2e1 (fxn)	Entered <u>two rules</u> as Y= and examined table values for comparison	3	4	4	4	4	3
7	Int3 - T2e1 (fxn)	Finding a point of intersection of two functions (TI-83 table)	4	4	4	3	4	3
9	Int3 - T2e2 (fxn)	Graphed <u>two functions</u> on calculator by entering rules as Y=	4	3	4	0	0	1
FUNCTIONS -- Non-Linear (Inverse variation)			Adr	Sara	Amy	Joy	Terri	Mon
11	Int2 - T3e (fxn)	Graphed one function on calculator by entering rule as Y=	4	0	1	0	0	0
12	Int2 - T3e (fxn)	Graphed two functions on paper (using same set of axes appropriately) ... Complementary Strategy	3	4	1	4	1	2
13	Int2 - T3e (fxn)	Entered two rules as Y= and examined table values for comparison ... Complementary Strategy	4	2	0	4	NA	0
14	Int2 - T3e (fxn)	Entered two sets of data into lists to make dual function representation	0	4	0	0	2	0
15	Int2 - T3e (fxn)	Graphed two functions on calculator by entering rules as Y=	0	0	0	0	0	0
18	Int2 - T3e	Understanding the infinite extent of a function	4	4	NA	4	NA	2
19	Int2 - T3e	Making a reasonable conjecture about the two numerical patterns as the functions extend toward x-approaching-positive-infinity	3	4	NA	4	NA	1

Table B.11 Evaluation Inventory for Number Relationships

NUMBER RELATIONSHIPS -- Constant variation			Adrn	Sara	Amy	Joy	Terri	Mon
3	Int2 - T1a	Understanding what the y=x line represents	3	2	3	1	3	1
NUMBER RELATIONSHIPS -- Inverse variation			Adrn	Sara	Amy	Joy	Terri	Mon
7	Int2 - T3c	Being able to describe the relationship between the function variables	4	4	4	4	4	4
10	Int2 - T3d	Predicting the shape of the graph for a function similar to one already graphed	4	4	4	4	4	NA
21	Int2 - T3f	Being able to describe how the two functions compare	4	4	4	4	4	3

Interview 2 with Adrienne—Interview Memo, 1/22/02

Questions about the Core-Plus Course 1 class

Adrienne has been the most pleasantly-surprising interviewee this round with her positive attitude and her successful performance on the math tasks; she truly has had the best conceptual understanding of [the five] interviewees to date. (It was difficult to find the edge of her understanding!) Her attitude survey shows all Agree or Strongly Agree responses.

Adrienne reported that, since the last interview, the Core-Plus Course 1 class had gotten "a lot easier" because she understands it so much better. She said it just made a lot of sense, like it had not before. This definitely was borne out in the interview/tasks. She didn't really have an explanation for why things have come so easily to her this year, especially in light of her having failed last year. Even her familiarity survey shows that she didn't understand most of the Unit 2 topics prior to studying them this semester in Mr. Larson's class. ...

Problem Solving Tasks

Task 1: Grades in English vs. grades in Math.

Adrienne showed excellent plotting and interpolating skills. She worked very carefully, neatly, precisely -- definite math-community-membership on those counts. She had an answer of "6" down originally for Part C because her (74, 74) point looks like it falls below her y=x line; but, in discussing an alternate way of finding points for higher grades in one subject than the other (using the table), Adrienne changed her answer to "5" because she found the two points for which the math & English grades were the same. ...

Task 2: Sara the waitress.

Adrienne is the first one who did not use I and T or D (alliterative variables) for their rules; instead, she chose y and x. This seemed to indicate loudly & clearly to me Adrienne's math-community-membership on this count as well!

The same thing about the hours seeming like they should be a variable but being a given number in the Rule was again puzzling to Adrienne. After I helped out a tiny bit, she figured it right out.

Her first attempt at the Rule was Y = 2.50 X, but in explaining it to me, she changed it to Y = 2.50 + X. But she realized that still didn't fit exactly with her explanation. So then – all without my prompting – she finally settled on Y = 2.50 (8) + X. She was going to put in a multiplication sign at first between the 2.50 and the 8, but she chose the parentheses saying "This is what I usually use." Another indicator of math-community-membership (MCM)! *especially because* it has not occurred to any other interviewees to use parentheses to symbolize the multiplication operation.

One final indicator of math-community-membership was what Adrienne decided to write down for Part E. I verbally asked the question stated there ("How does the amount that Sara earned on Sunday compare to the amount she earned on Saturday?"), and Adrienne (had actually already) said they were the same. Then I queried her about that with my probe "Someone else said Saturday's income should be less because the tips were less" but Adrienne immediately explained it and then said, "Is that what you want me to put down?" I said yes, so she wrote "She worked less hours on Sunday than Saturday." The reason I thought of this as a MCM indicator was because Adrienne seemed conscientious about filling out all the answers on the sheet. No other interviewees that I recall have shown this; many seem not to care about writing stuff down, or they view it as an obligation "if they have to" (Joy, for instance); but Adrienne seems to have the disposition that this is for the important recording of known information, being sure that all the answers are kept in the record … kind of like a good science lab student who will follow the protocol of recording all observations carefully and precisely *because she wants to behave properly as a scientist* and not just because (/when) she will get points for her grade for doing so.

Task 3: Senior class trip.

There were many fascinating aspects to watching Adrienne work on this task! Adrienne exhibited MCM, I thought, by drawing the separating line between her entry of 133.33 and 100 on her table for Part A. Just one other interviewee (Sara?), so far, has done this.

Also, Adrienne found the entries for the table in Part A by punching in 8000/n on the calculator. So, the fact that she wrote 133.33 for 60 people also indicates MCM, in my opinion. *I think she even said "133 dollars and 33 cents" as she wrote it!* (Check transcript.)

Adrienne was very adept in her use of the graphing calculator for this task. The only place she faltered was in remembering to set up the STATPLOT so that the points on the table would show. I helped her just a little with this.

As Adrienne read the text for Part D, it seemed fairly clear that she also possessed adequate MCM to understand the use of the "c" and the "n" in the last sentence. Other interviewees, just in their reading-aloud, have stumbled on these. Adrienne's inflection was just how a MC member would read it!

On the graph for Part B, Adrienne labeled the X-axis "cost per person" at first. Then she corrected herself and simply crossed that out and wrote "numbers" instead. MCM? *(I'm not sure if this was why or not.)*

Her numbering both axes from zero and LABELING the origin with a zero showed MCM to me.

Finally, it was most interesting when it came time to draw the graph for Part E. I was feeling certain that Adrienne would extend the numbering of her y-axis from Part B, because there was plenty of room. However, she instead drew a completely new graph in an unused area of the graph paper. This was perhaps the one place where Adrienne did not exhibit MCM *(I would expect a MC member to automatically assume that you would use the same set of axes to graph the second function.)*.

I waited for her to finish without interrupting, but then I probed her about comparing the two graphs. Then it occurred to her that she might put them on the same set of axes for comparison. So she redid the graph and we pursued the rest of the questions I've been using for all interviewees (but which don't appear on my interview protocol). I remember that Adrienne did well on these, but I don't recall exactly how she answered (about whether the functions would ever eventually cross or meet up). CHECK THE TRANSCRIPT FOR THIS.

I do see that, on the graphing calculator, Adrienne went as far as

X = 11400 Y1 = 1.7544 Y2=.70175

and that she was skipping by 100s for the X-values (more comfortable than Joy, for instance, who thought she might miss it by skipping by more than 1s -- ?).

Other settings Adrienne used on the calculator:

WINDOW	STAT-Edit
Xmin = 0	L1 = 20, 40, 60, 80, 100
Xmax = 100	L2 = 400, 200, 133.33, 100, 80
Xscl = 10	
Ymin = 0	
Ymax = 400	
Yscl = 50	

She also had both functions entered into the Y = window. Y1 = 20000/X and Y2 = 8000/X. That reminds me of how Adrienne *explained to me*, when she entered Y1, that she would use "an X instead of n" even though she had written c = 20,000/n for the Rule for Part D. Also a sign of MCM – being conscious of (and reporting that she was) using an X for some other variable-letter.

Also, the fact that she wrote her Rule for Part D as it would be entered into the calculator (using the slash for a division sign and writing the rule left to right instead of writing it like the text showed it for Part A with 8000 *OVER* n) seems to indicate mathematical/conceptual thinking-ahead-to-solving-the-problem and some *problem-solving-independent-thinking*, instead of just following (more blindly?) after the model of how the text did it in a previous part of the task.

Interview Three with Monica—Excerpt from Transcript of Task 2, 3/20/02

Interviewer: Alrighty. Now, second time around, they each took one turn. Now they take a second turn. On their second turn, the equation for Jerome's points was equivalent, or the same as ...

Interviewer: Jerome's points was equivalent to Y equals eight, eight plus ten X.

Monica: Okay.

Interviewer: And the equation for Lin's points was equivalent to Y equals twelve plus nine X. So they're saving us the trouble of doing all this. Okay? They're just gonna give us a short cut. Before they roll the dice they wondered whether there was some number on the dice that would make them tie. Okay? So you're supposed to find out the answer to their question and there's three, at least three different ways you can useo this. Okay? To solve this problem, and two of these I'm pretty sure you did in class. You might have also done [the Equation Method] and so we're gonna try that. Okay?

Monica: Uh huh.

Interviewer: Do you remember using your calculator to compare the values of two equations in Mr. Harper's class?

Monica: Like Stat and Enter, is that ~ ?

Interviewer: Yes. (pause) Well, yeah, you could use the Stat and Enter or you could also use the table over here. Do you remember using that table like that?

Monica: Whoa.

Interviewer: If Stat and Enter is more familiar to you, use that. Okay?

Monica: Uh, yeah.

Interviewer: Alright.

Monica: No numbers. [The student is referring to the use of a rule with a variable to generate a table as opposed to entering numbers into a lists.]

Interviewer: Okay, yeah, [the List Method (using Stat & Enter keys) is] the one where you have to punch in the numbers.

Monica: Oh. I don't know how to use that.

Interviewer: Okay. Let me help. Um, I think the table's actually a little easier.

Monica: Yeah.

Interviewer: And so those numbers right there, one, two, three, four, five, six. Since they tell us in the problem that he has, his points equal Y, are the same as Y equals eight plus ten X.

(Monica enters 8 + 10X into Y1 in the Y= window.)

	Yeah, you can punch that into there. And on the next line [in the Y= window] is [Lin's equation].
	(Monica enters 12 + 9X for Y2.)
	Oh sorry. Use [the 2nd button for] the table.
Monica:	Oh.
Interviewer:	And it'll actually tell you the points. So, if they roll a six, how much does she get for points?
Monica:	Sixty-eight.
Interviewer:	And what does he get?
Monica:	Sixty-six.
Interviewer:	Okay. The question was, is there a number on the dice that would make them tie?
Monica:	(pause) Yep.
Interviewer:	Which?
Monica:	Four.

Adrienne's work on Task 3 of Interview Two—Comparing two functions

Interviewer: OK. Now. When you had the graph over here [on the calculator]

Adrienne: M Hm.

Interviewer: THAT one. (pause) Yeah. It's got your dots which represented the Washington, DC stuff, and then it put this line on here that represented the Hawaii stuff. And it looks to me like, eh, eh, like you had said, y'know, it would be bigger.

Adrienne: M Hm.

Interviewer: But HERE [on the graphs drawn on paper], the two of these look the very same to me. (pause) Do you know what I mean?

Adrienne: Because of the amount skipped between the twenty and forty? Or is the

Interviewer: Yeah, 'cause if I cut this out [the Hawaii graph drawn at the top] and laid it on top, I'm guessing those would be almost identical.

Adrienne: M Hm.

Interviewer: Y'know, like held 'em up to the light and I'd have [points on the Hawaii graph] right over the [points on the Washington DC graph].

Adrienne: M Hm.

Interviewer: So what do you think about that?

Adrienne: (pause) Mm, pretty much the same

Interviewer: OK.

Adrienne: if you look.

Interviewer: OK, so, can you think of why these [on paper] look like they would be identical if you laid one on top of the other? But here [on the calculator], when they're on the very same graph, this one looks bigger?

Adrienne: (pause) I'm not sure.

Interviewer: OK. (pause) One of the things I wondered about was, I was surprised when you started way up here, but then you numbered these from zero to a thousand.

Adrienne: Oh you

Interviewer: And I was wondering about starting with the zero down here. Is that possible to do it that way?

Adrienne: M Hm.

Interviewer: OK, is there any reason why you decided to do it that way?

Adrienne: (laugh) No.

Adrienne's work on Task 3 of Interview Two—Comparing two functions (cont'd)

Interviewer:	OK, that's fine! (laugh) 'Cause I've had people do it both ways, but you're saying you could've used the same zero, four hundred,
Adrienne:	Yeah.
Interviewer:	and gone up to a thousand? (pause) Would that make the two of those, then, look like it does here [on the calculator]?
Adrienne:	(pause) Yyeh
Interviewer:	OK.
Adrienne:	probably.
Interviewer:	OK.
Adrienne:	It'd be closer together and easier to compare
Interviewer:	OK, alright.
Adrienne:	that way.

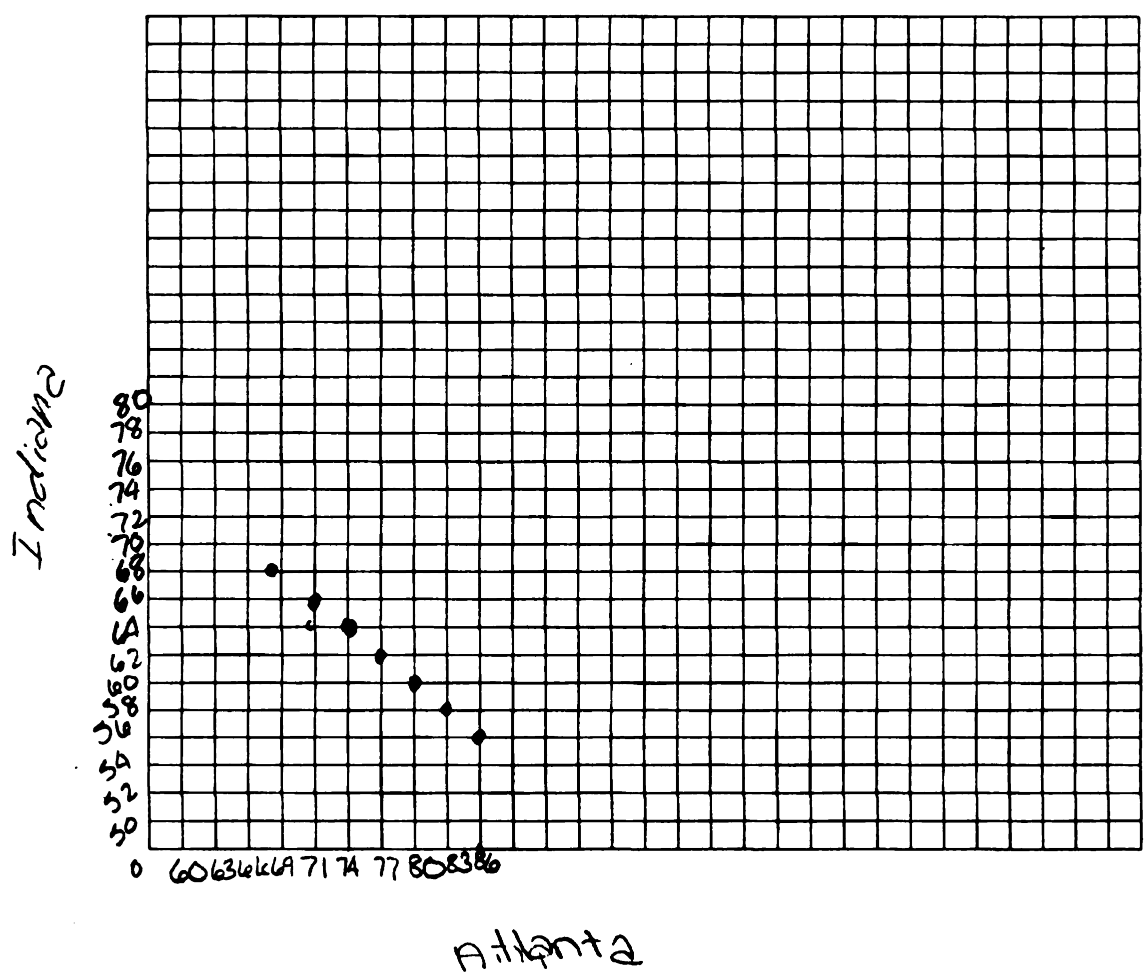
Sara's work on Task 1 of Interview One—Comparing two functions

1. At 3:00 P.M., the temperature is 86° F in Atlanta and is decreasing at a rate of 3 degrees per hour.
   At the same time, the temperature is 56° F in Fort Wayne and is increasing at a rate of 2 degrees per hour.

   When will the temperatures be the same?

86	56	74	64
− 3	+ 2	71	66
83	58	68	68
− 3	+ 2	65	70
80	60	62	72
− 3	2	59	74
77	62		

Show how to use a graph to solve the problem:



Adrienne's work on Task 1 of Interview One—Comparing two functions

1. At 3:00 P.M., the temperature is 86° F in Atlanta and is decreasing at a rate of 3 degrees per hour.
   At the same time, the temperature is 56° F in Fort Wayne and is increasing at a rate of 2 degrees per hour.

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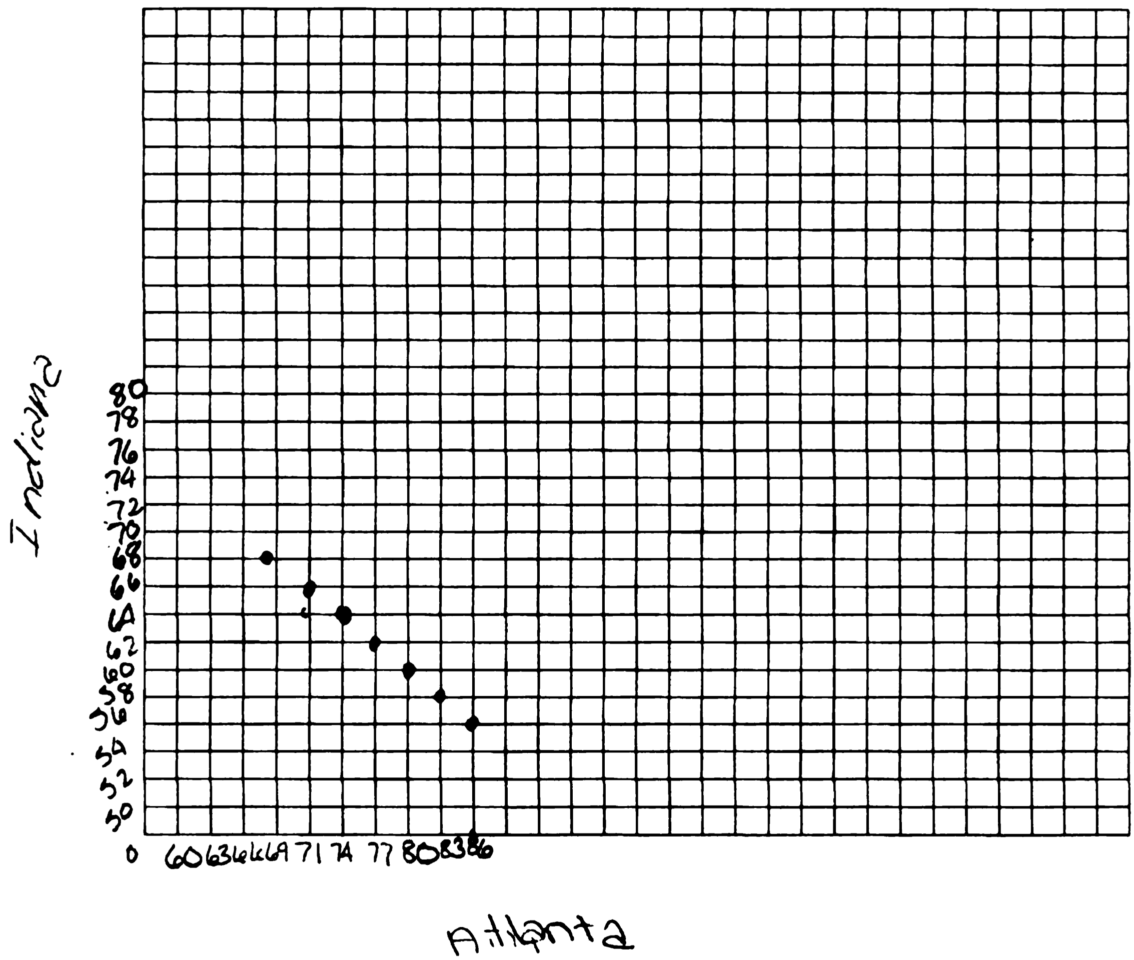


Amy's work on Task 1 of Interview One—Comparing two functions

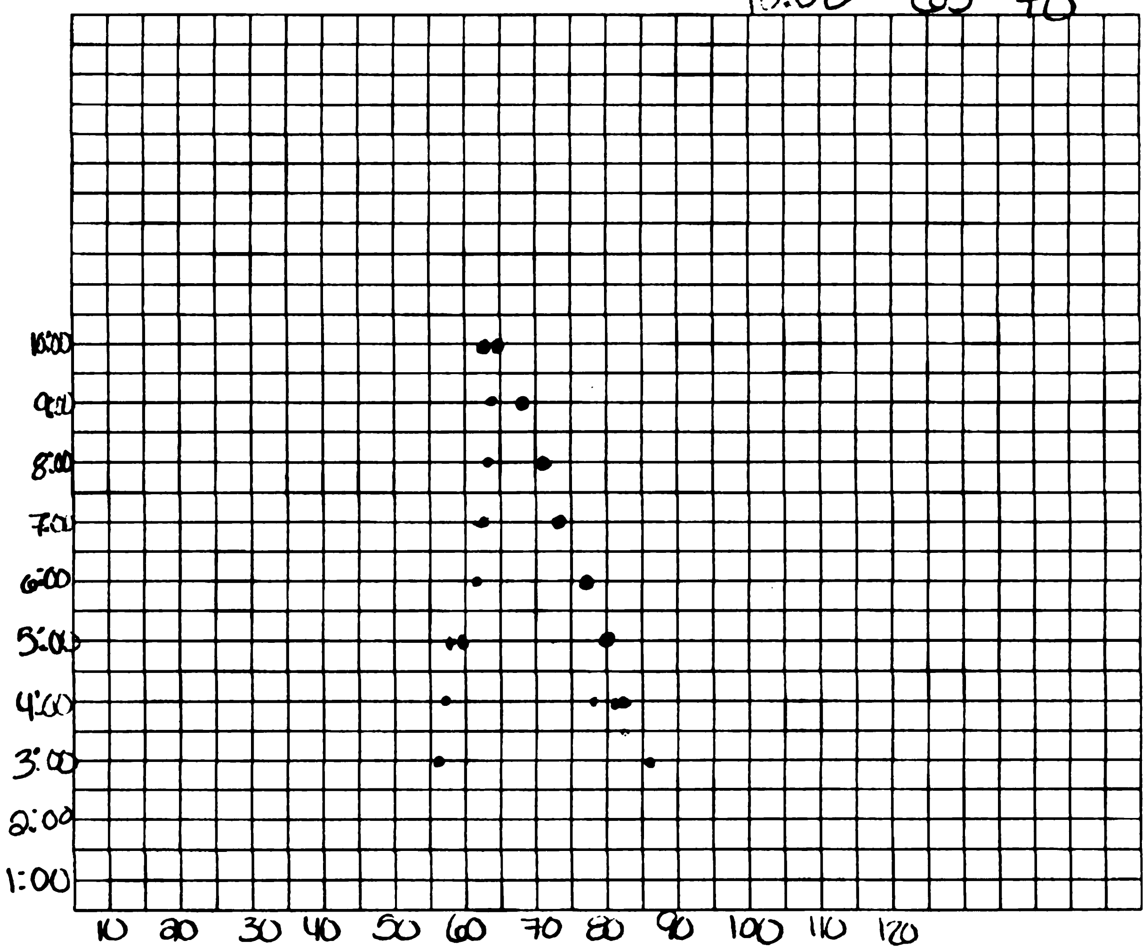
1. At 3:00 P.M., the temperature is 86° F in Atlanta and is decreasing at a rate of 3 degrees per hour.
   At the same time, the temperature is 56° F in Fort Wayne and is increasing at a rate of 2 degrees per hour.

   When will the temperatures be the same?

   9:00

   3:00-86, 56
   4:00-83 58
   5:00-80 60
   6:00-77 62
   7:00-74 64
   8:00 71 66
   *9:00 68 68
   10:00 65 70

   Show how to use a graph to solve the problem:



Joy's work on Task 1 of Interview One—Comparing two functions

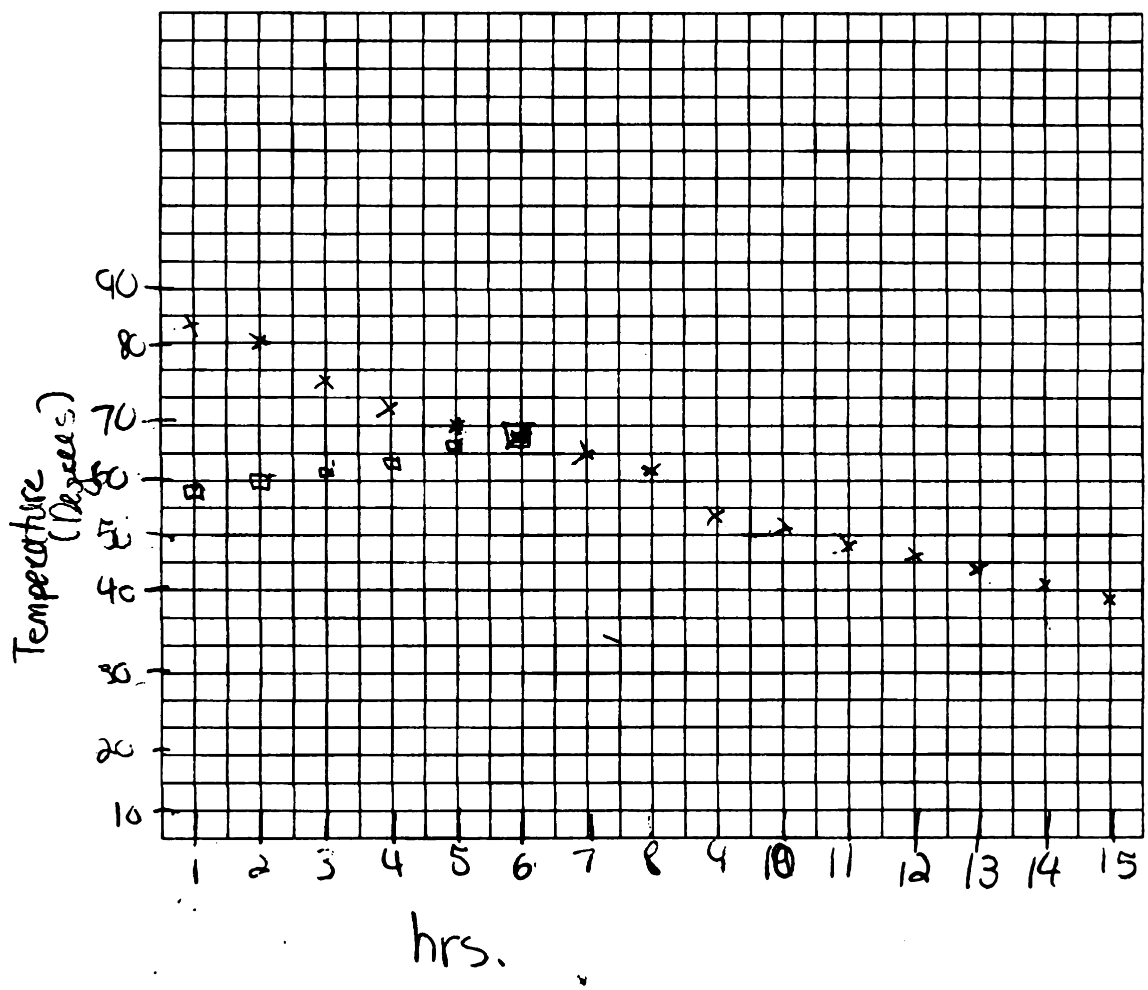
1. At 3:00 P.M., the temperature is 86° F in Atlanta and is decreasing at a rate of 3 degrees per hour.
   At the same time, the temperature is 56° F in Fort Wayne and is increasing at a rate of 2 degrees per hour.

   When will the temperatures be the same?

15hr

$$\begin{array}{r} 86 \\ -56 \\ \hline 30 \end{array}$$

Show how to use a graph to solve the problem:

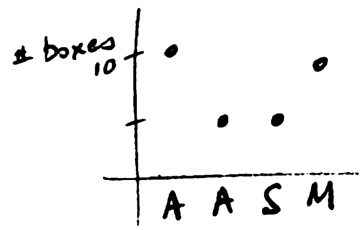


Adrienne's work on Task 3 of Interview Two
Conjecturing about the functions $c = 20000/n$ and $c = 8000/n$

Interviewer: OK. Here's my next question. I drew this little sketch here.

Adrienne: M Hm.

Interviewer: This, sometimes it makes sense to connect dots for graphs and sometimes it doesn't; and here's one where it doesn't. Pretend this is a graph of people selling Girl Scout cookies.



Adrienne: M Hm.

Interviewer: And, so, here's you, and you sold ten. Here's me, I sold five. Somebody else sold five and somebody else sold eight. It wouldn't really make sense for us to connect those dots because they're comPLETEly different people.

Adrienne: Yeah.

Interviewer: In the case of THESE [senior trips data], would it make sense to connect those dots, do you think?

Adrienne: (pause) It doesn't really matter, you could because it's at a downward slope

Interviewer: OK.

Adrienne: and not up and down, up and down.

Interviewer: OK, OK. Umm, I'm also wondering about, you had talked at one point, y'know, whether we would use the same numbers: twenty, forty, sixty, eighty, a hundred?

Adrienne: M Hm.

Interviewer: Umm, and I asked, "Well, would it make a difference if you went twenty, thirty, fifty, seventy, ninety?" Umm, is there a way, using this graph, to tell what those numbers would be? Like if I wanted to know, I've got fifty students who want to go on the class trip, and we want to know the price for Washington compared to Hawaii. (pause) Can you tell us how much that would be?

Adrienne: Not exactly, but, uh (pause) precisely [sic] four hundred?

Interviewer: OK, and how did you figure

Adrienne: About four hundred.

Interviewer: that out? You're estimating?

Adrienne: The space in between the forty and the sixty

Interviewer: OK.

Adrienne's work on Task 3 of Interview Two
Conjecturing about the functions $c = 20000/n$ and $c = 8000/n$ (cont'd)

Adrienne:	Go up [i.e., interpolating] and it, may be in between the forty and the sixty dots.
Interviewer:	OK.
Adrienne:	And right there should be about
Interviewer:	OK, and what do you think it would be for the Washington trip?
Adrienne:	For what?
Interviewer:	The Washington trip
Adrienne:	Five hundred?
Interviewer:	with the pencil dots.
Adrienne:	Oh.
Interviewer:	For fifty students, yeah.
Adrienne:	(pause) About one hundred.
Interviewer:	OK. And is there a way to check that?
Adrienne:	On the calculator?
Interviewer:	OK. So go ahead and see how close you came with your estimates.
Adrienne:	(pause) Go by (pause) tens.
Interviewer:	OK.
Adrienne:	Starting at
Interviewer:	So you're just gonna change the table? And have it come up with?
Adrienne:	Uh huh.
Interviewer:	Alright.
Adrienne:	(pause) Four hundred.
Interviewer:	OK.
Adrienne:	For the
Interviewer:	That's exactly what you said, isn't it?
Adrienne:	M Hm.
Interviewer:	OK.
Adrienne:	And I'm not sure how to check the, unless I put this rule in.
Interviewer:	OK.
Adrienne:	Forget what that was.
Interviewer:	It's on the other side.
Adrienne:	Oh. (pause) Mm. Eighty
Interviewer:	Could you figure it the same way that you figured these? 'Cause these you came up with without putting that table in. (pause) Remember that?
Adrienne:	Yeah. Prob, yeah.
Interviewer:	OK, how would you do that then?

Adrienne's work on Task 3 of Interview Two
Conjecturing about the functions $c = 20000/n$ and $c = 8000/n$ (cont'd)

Adrienne:	The same way it said to do, I wrote my rule.
Interviewer:	OK.
Adrienne:	I'd just type it in
Interviewer:	OK. Alright.
Adrienne:	the number.
Interviewer:	And you can do it either way you wish.
Adrienne:	(pause) Eight thousand divided by X. (pause) For fifty. (pause) That'd be four hundred. Oh! (pause) One sixty
Interviewer:	OK. (pause) Alrighty, so four hundred for the Hawaii trip, a hundred and sixty
Adrienne:	M Hm.
Interviewer:	for the Washington trip. (pause) Alright. OK, now my next question brings those two together: If you were to draw the connecting lines for these dots, would that help you with those values, like if I wanted to know fifty students? If I wanted to know ninety students? Something like that.
Adrienne:	(pause) Not, kinda, but not really 'cause you're still gonna have to go up and find where the line hits
Interviewer:	OK.
Adrienne:	that.
Interviewer:	OK, alright.
Adrienne:	But yeah.
Interviewer:	Alright. And another question about connecting those. If you were to connect those dots, would you connect, would it make sense to connect them with straight lines, y'know, like to use a ruler like this, y'know [demonstrates what she means with ruler, miming drawing the lines], and then, y'know, connect 'em like this [more demo], etc.?
Adrienne:	No.
Interviewer:	What would you do instead?
Adrienne:	Just draw openly, for it, with your hand
Interviewer:	OK, why don't you
Adrienne:	use a curve
Interviewer:	OK, why don't you pick one of those and see how well you can do
Adrienne:	I'll do ~ ~
Interviewer:	OK. (laugh)
Adrienne:	(pause while student sketches curve through points) Dot that's wrong.
Interviewer:	OK, alright, so you're just trying to draw a smooth curve
Adrienne:	M Hm.
Interviewer:	that connects all of those. Alright.

Adrienne's work on Task 3 of Interview Two
Conjecturing about the functions $c = 20000/n$ and $c = 8000/n$ (cont'd)

Adrienne:	This one [the point (100, 80)] doesn't really look
Interviewer:	OK, it doesn't follow that same
Adrienne:	Nh nh.
Interviewer:	curve, you're thinking? And it may be because, y'know, there's such a tiny amount in there
Adrienne:	Yeah.
Interviewer:	Um, y'know, that this might not be just exactly right, y'know and, which is always a problem (laugh), y'know when you're trying to graph something with a lot of numbers by hand.
	OK! One more set of questions for you! Um, well, lemme have you go ahead and -- I told you I was gonna have you only do one – but I'll make you do the other one
Adrienne:	Both of them?
Interviewer:	Yeah, 'cause that's what my next question relates to.
Adrienne:	(pause while student sketches curve) I'm not that good at it.
Interviewer:	(laugh) It looks you're pretty good at it. (laugh) I think that looks good.
	Alright. If we continued to go on, let's say we went to two hundred students, three hundred students, four hundred students, how would those two curves continue on? Can you predict?
Adrienne:	They'd probably both come down at the same, mm, m mm (pause) I'm not sure. Probably about the same amount? Maybe, umm, after a while (pause) it'd be the, like (pause) (laugh) I'm not sure.
Interviewer:	OK, by the same amount you mean, like, they're about one finger width apart
Adrienne:	Yeah.
Interviewer:	And you think they would continue to be about one finger width apart as they go?
Adrienne:	(pause) M (longer pause) Nnno. I'm not sure. (laugh)
Interviewer:	OK. (pause) Is there a way that you could use the calculator to check it out?
Adrienne:	Probably.
Interviewer:	OK.
Adrienne:	For, if, see if they'll put two [punches some buttons] Wait, lemme go back. [more punching]
Interviewer:	So you'll go back to that table?
Adrienne:	M Hm.
Interviewer:	And have it
Adrienne:	~ ~ for two hundred, and they're a hundred apart.

Adrienne's work on Task 3 of Interview Two
Conjecturing about the functions *c = 20000/n* and *c = 8000/n* (cont'd)

Interviewer: OK.

Adrienne: (pause) For the Hawaii it'd be a hundred, and for Washington forty. And

Interviewer: OK.

Adrienne: About forty apart there, when they were sixty apart there. Then for four hundred, there's only thirty

Interviewer: OK.

Adrienne: Five hundred there's twenty, twenty-four.

Interviewer: OK.

Adrienne: And, then there's twenty. And (pause) seventeen,

Interviewer: OK.

Adrienne: just about, not counting that.

Interviewer: OK. Alright.

Adrienne: And then fifteen ~

Interviewer: OK, so how would those look on the graph then? Does that help you?

Adrienne: Coming closer. They come closer together.

Interviewer: OK, so as they, they go they get closer and closer.

Adrienne: M Hm.

Interviewer: Do you think there would be a point – y'know, if we had all kinds of paper, just kept going – umm, where those two would ever meet up?

Adrienne: (pause) Yeah.

Interviewer: OK.

Adrienne: I don't know where, but they probably would,

Interviewer: OK.

Adrienne: 'cause this one gets shorter, by more

Interviewer: OK.

Adrienne: The Hawaii ones get shorter by more than the Washington one.

Interviewer: OK. Now is there a way you could use the calculator to try and see where they might meet up?

Adrienne: Yeah, you can just use the, um (pause)

Interviewer: And then how are you gonna know when they meet up?

Adrienne: It'll be the same number.

Interviewer: Where?

Adrienne: Going straight across [on the calculator's table display].

Interviewer: In those two, the Y1 and the Y2 column?

Adrienne: M Hm.

Interviewer: OK.

Adrienne's work on Task 3 of Interview Two
Conjecturing about the functions $c = 20000/n$ and $c = 8000/n$ (cont'd)

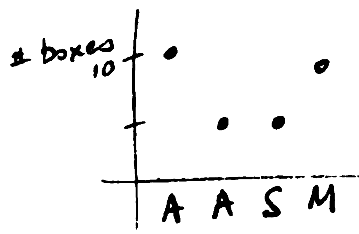
Adrienne:	(pause while working on calculator)
Interviewer:	I promise this is my last question. (laugh) But you were doing so well, I thought I'd go further. (laugh) Plus we had some time. (pause) How close are they getting? I can't hardly ~
Adrienne:	They're like three apart
Interviewer:	OK.
Adrienne:	for each one.
Interviewer:	(pause while student continues with calculator) That's really cheap, isn't it? (laugh) I'm thinking like a DOLLAR or something to go on the trip, of course you're putting up with fifty-nine hundred other people! (both laugh)
Adrienne:	(pause) I don't know!
Interviewer:	Are they not coming together?
Adrienne:	Nh nh.
Interviewer:	How close are they getting?
Adrienne:	There's like two dollars
Interviewer:	OK.
Adrienne:	About one-something in between.
Interviewer:	OK. (pause) Do you think it's possible they'll NEVER MEET?
Adrienne:	Maybe! 'Cause no matter what, that one's still gonna keep going down.
Interviewer:	OK.
Adrienne:	I don't know.
Interviewer:	OK. (pause) Well you did very well. (pause) The answer is they aren't ever gonna meet.
Adrienne:	They aren't?
Interviewer:	(laugh) No!
Adrienne:	Yeah, they ~ ~
Interviewer:	And you're exactly right because this one keeps going down and THIS one keeps going down. They may get CLOser,
Adrienne:	Yeah.
Interviewer:	buuut, y'know, you could take a microscope to that graph, y'know, and there'd still be just a little tiny space between
Adrienne:	Yeah.
Interviewer:	And, the other way of kind of thinking or reasoning through that is that this is now down to like seventy cents
Adrienne:	M Hm.
Interviewer:	per person to go, but, y'know, the question that one of my interviewees and I were talking about was whether it would ever go below the zero.

Adrienne's work on Task 3 of Interview Two
Conjecturing about the functions $c = 20000/n$ and $c = 8000/n$ (cont'd)

	Well, as long as you have a certain amount of cost for the trip, and a certain number of people going, you can't take the cost divided by the people and end up with a cost, y'know, per person
Adrienne:	(laugh) Where they ~ ~
Interviewer:	is zero.
Adrienne:	Yeah.
Interviewer:	Yeah. And so that's how we were thinking about that, too, that it could get smaller and smaller and maybe get to pennies, or tenths of pennies, or something like that
Adrienne:	M Hm.
Interviewer:	but it's NEVer gonna get to below zero, and no matter how many, like y'know, if you have a twenty-thousand-dollar trip and an eight-thousand-dollar trip, there's still gonna be a difference in price
Adrienne:	Yeah.
Interviewer:	Y'know, even if I said let's take a MILLION people. (laugh) Y'know, well, the total cost is a little more, so the cost per person is gonna be a little more
Adrienne:	OK.
Interviewer:	Y'know, for even the Hawaii travelers. (pause)
	You did VERY well on that! (laugh)
Adrienne:	(laugh) Thank you.

Sara's work on Task 3 of Interview Two
Conjecturing about the functions $c = 20000/n$ and $c = 8000/n$

Interviewer: Okay. Is it correct, would you say, to connect the dots? I mean is that valid? Because I asked somebody else this, there's some graphs where it just doesn't make any sense at all. Let's say we're selling Girl Scout cookies, and these are the number of boxes so five and ten and here's me and I've sold ten and here's you over here and you sold five and a couple of other students. Now it wouldn't make any sense at all, you know what I mean



Sara: Yeah.

Interviewer: to connect those dots? Does it make sense [for the senior class trips] to connect the dots or is it more like [the Girl Scout cookie graph]?

Sara: Actually it probably wouldn't matter whether you connect the dots or not

Interviewer: Okay.

Sara: cause that just helps you see the dots better, I think.

Interviewer: Okay. Okay. Alright. Umm, what if, one of your first questions you asked me was if we were going to use the same numbers here. Could you still get your results and stuff and be able to tell me how the two trips compared and stuff ? Um, or even do them on the calculator at the same time if we had used different numbers like ten and thirty, fifty, you know that kind of thing?

Sara: That would have worked

Interviewer: Okay.

Sara: too,

Interviewer: Okay.

Sara: also as well.

Interviewer: Okay. So I'm wondering if the line helps tell you anything about those other points on the graph.

Sara: Yes it does. I didn't realize that before, but yes it does.

Interviewer: Okay. So, go ahead.

Sara: It would go through all the numbers like say ~ ~ I'll just put a fake one in. Like it goes through that or something.

Interviewer: Okay.

S; It would go through the points of all the other numbers.

Interviewer: Okay. So based on that you, what would you predict, um, it would cost for fifty students?

Sara: For fifty students it'd cost about a hundred and sixty-five

Interviewer: Okay.

Sara: to a hundred and seventy.

Sara's work on Task 3 of Interview Two
Conjecturing about the functions $c = 20000/n$ and $c = 8000/n$ (cont'd)

Interviewer:	And that's, uh, for which trip? (pause) 'Cause
Sara:	For the New York, or the Washington DC.
Interviewer:	Okay. And how much would it cost them to go to Hawaii if there were fifty kids going?
Sara:	(pause, while Sara sketches a line on the graph connecting the points) About four hundred and twenty.
Interviewer:	Okay. Alright. Now when you started sketching the line you started all the way up here [from the highest point on the graph] rather than just draw a straight line between the two why did you do that?
Sara:	I don't know.
Interviewer:	Okay. Now are those suppose to be straight lines between the two? or are they curving? because one of the things you said
Sara:	They're kind of curving.
Interviewer:	umm, you described this as non-linear decreasing. (laughs) Which was correct. Umm, and so I am wondering about that word, non-linear. Do you know what that refers to?
Sara:	It means not a straight line.
Interviewer:	Okay.
Sara:	That should be kind of curved.
Interviewer:	So that would be curvy?
Sara:	Yeah,
Interviewer:	Okay.
Sara:	Just a little bit.
Interviewer:	Okay.
Sara:	So that would be about four hundred,
Interviewer:	Okay. Alright.
Sara:	a little over maybe
Interviewer:	Alright. Well and I was thinking it made sense, you know, that I was, I do the very same thing you know if I'm gonna try to connect these dots with a curve I, you know, start at the beginning so I can kind of get a feel for, y'know, how that thing is gonna curve. So I think that made sense. I was just wondering if that was
Sara:	Yeah.
Interviewer:	in your thinking at all. (laugh) Alright. I think we have one more question to go. Umm, explain how the cost for the Hawaii trip compares to the cost of the trip to Washington.
Sara:	It would be quite a bit more.
Interviewer:	Okay. In all cases?
Sara:	In all cases because there's a difference in the overall price
Interviewer:	Okay.
Sara:	and the one to DC would be less because the overall all price is less
Interviewer:	Okay.
Sara:	than the Hawaii one.

Sara's work on Task 3 of Interview Two
Conjecturing about the functions $c = 20000/n$ and $c = 8000/n$ (cont'd)

Interviewer:	Okay. But I noticed how you commented about there being a huge jump between here and here and when you said that, I noticed how there also is a big jump between the difference, like for these twenty people it's REALLY different, y'know, in cost, like a thousand compared to four hundred but when you get down to here (laugh) the difference isn't so much.
Sara:	Yeah.
Interviewer:	Like I was thinking, y'know, I might not as hard a time talking the parents into going to Hawaii instead of Washington, DC if I we had a hundred kids going because the amounts aren't as different but here there's no way I could make that case.
Sara:	Yeah.
Interviewer:	And why do you think that is that, uh, when you've got fewer students its REALLY a big difference?
Sara:	Because there's a lot, there's a big difference in the cost, the overall cost
Interviewer:	Okay. Okay.
Sara:	there's twelve thousand dollars' difference.
Interviewer:	Okay.
Sara:	and it would be more for each student.
Interviewer:	Okay. Alright. Do you think there is ever a point, like of we just continued to go on and on and on, um, would those cost ever meet up, do you think?
Sara:	I don't think so.
Interviewer:	Okay. Why not?
Sara:	Because there would al (hesitation) Well actually I think there might, because these ones, cause there's, they're bigger then smaller and smaller.
Interviewer:	Okay.
Sara:	and I'm sure they would meet somewhere way along the line.
Interviewer:	Okay. Alright. Do you have just a ballpark guess how many people you might have to take to get those to meet up?
Sara:	(pause, writing)
Interviewer:	Okay tell me what you did there and why you did that?
Sara:	I seen how many over it was
Interviewer:	Okay.
Sara:	and how many down it was
Interviewer:	Okay.
Sara:	And I'm gonna go here
Interviewer:	Okay.
Sara:	and see it again [Sara is approximating the slope between different sets of two consecutive points.]
Interviewer:	Okay.
Sara:	Umm, (pause) over the same amount, then down there's one, two, three and a half

Sara's work on Task 3 of Interview Two
Conjecturing about the functions $c = 20000/n$ and $c = 8000/n$ (cont'd)

Interviewer: Okay.

Sara: and we go over and then down two

Interviewer: Okay.

Sara: and probably over and down so it would be point five

Interviewer: Okay.

Sara: so it would be point five down (pause) and that would be (whispering, counting) one, two, three, four. (pause) It would be right here.

Interviewer: Okay.

Sara: (pause) Umm, I don't know. I don't think they would

Interviewer: Okay.

Sara: because you can't get any smaller than zero. (pause) Because the next one would be right here.

Interviewer: Oh I see, the difference that you're decreasing it by.

Sara: M Hm, because that one went down three point five

Interviewer: Okay.

Sara: and then take away three point five minus one point five was two

Interviewer: Okay.

Sara: and that one was exactly two

Interviewer: Okay.

Sara: Then you take another one point five away

Interviewer: Uh hu. I see.

Sara: ~~~~

Interviewer: ~~~~ (talking over each other) and so if you took the point five here minus the one point five it's gonna, alright.

Sara: be a negative.

Interviewer: Okay. That's excellent thinking. Alright, umm, is there, do you know of a way for the calculator to show you those numbers or the graphed non, non line (laugh) the curve, going way out without actually having to sit there and punch in all the numbers on your list?

Sara: No.

Interviewer: Okay. I wanna show you. See if this sounds familiar. Oops, if you, let's turn the plots off. Oop, too far. (pause) There we go. If you go to this Y equals window (pause) and you use your

Sara: Okay.

Interviewer: rule. Does that ring a bell?

Sara: (pause) You can't put *n* in the calculator so you have to make it like Y.

Interviewer: Okay. Okay.

Sara: That's a Y ~

Interviewer: And that makes sense doesn't it because you've got an X here and you've got a Y here?

Sara: M Hm.

Interviewer: Okay.

Sara: Okay, then you would go to your tables, no Table Set, and you start it at

	zero and go up by one. (pause) I'll go by five.
Interviewer:	Okay.
Sara:	And, and you would go to your table. (pause) And zero says error because there is no cost for zero students.
Interviewer:	(laugh) Okay, uh huh.
Sara:	And then you would, (pause) now what am I looking for?
Interviewer:	So okay, so the line that you are on what does that tell me there?
Sara:	That for ninety students it costs two hundred and twenty-two dollars.
Interviewer:	Okay. Now can you, what we were trying to figure out is if they're ever going to meet up.
Sara:	Okay.
Interviewer:	So we put in this one. Now we want that one. And is there a way to get all of those showing so that we could, like, compare 'em?
Sara:	That would be the closest one right there. I don't think that they would ever meet up because there's, they would never meet up, because those two right there are the closest.
Interviewer:	Okay. Where you got, what is that point?
Sara:	One forty and Y was one hundred and forty two point eight six [Here, the student is erroneously comparing the X-value and the Y1-value of the same function to determine whether the two functions, Y1 and Y2, will ever intersect.]
Interviewer:	Okay. And what does that stand for? The one forty and the one forty two point six?
Sara:	The one forty is how many students would go, and the one forty two point eighty-six is the cost, is how much it would be.
Interviewer:	Okay. For what?
Sara:	The Hawaii trip.
Interviewer:	Okay. How much would it be for the Washington, DC trip? For one hundred and forty kids?
Sara:	(talking as she punches into the calculator) Y equals, no, clear this, Y equals eight thousand divided by next, (pause) one go up by five
Interviewer:	Okay.
Sara:	and (pause), that's the closest one, ninety one students at the cost of eighty seven dollars and ninety hun,ninety-two cents. [Again, the student is erroneously comparing the X-value and the Y2-value of the same function to determine whether the two functions, Y1 and Y2, will ever intersect.]
Interviewer:	Okay. So when you say the closest one where that number and that number are pretty similar.
Sara:	M Hm
Interviewer:	Alrighty.
Sara:	But they will never meet up.
Interviewer:	Okay.

Joy's work on Task 3 of Interview Two
Conjecturing about the functions $c = 20000/n$ and $c = 8000/n$

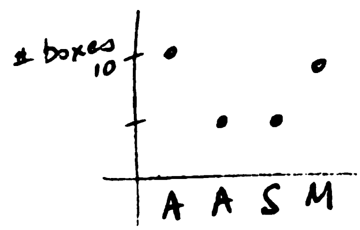
I: Okay. Alright, um, I have a question about connecting those dots.

S: Yeah.

I: Some graphs, it makes sense to connect the dots and others it doesn't.

S: Yes.

I: Okay. For example, let's say I have this graph and it's people selling Girl Scout cookies.



S: Uh huh.

I: And the first person sold ten boxes, the next one sold five. That one sold five, that sold about eight. It wouldn't really make sense to connect those dots.

S: Yeah, I know.

I: Would it make sense to connect the dots on those graphs?

S: Mmm. (pause) I guess, if you wanted it to. It can but (pause) mm, yeah. It would make sense.

I: Okay. Would it help for any reason?

S: (pause) Mmm, help for any reason?

I: Yeah.

S: (pause) I don't think so.

I: Okay. What if you wanted to find out how much it would cost for thirty seniors to go either to Hawaii or to Washington, DC?

S: Then you could see if you made a line through it . (laughs)

I: Okay. So, would the line help in that case?

S: Yeah.

I: Alright. And how would you use the line to determine what it would cost?

S: You'd look, you'd, like go where you, from where thirty would be

I: Okay.

S: then go up and see where it meets the line.

I: Okay. Alright. What would you estimate for the pencil ones?

S: I accidentally put that forty,

I: Okay.

S: oh well.

I: So what would you estimate the cost would be for thirty students to go to

Washington, DC?

S: Umm. (pause) I'm gonna make a line here.

I: Okay.

S: Does it have to follow the dots? Or does it

I: That's up to you. What do you think?

S: I don't know, I guess. How 'bout yeah?

I: Okay.

S: Okay. Thirty it would cost about two hundred and sixty.

I: Okay. Is there a way to check that to see if that's right?

S: Umm, yes there is.

I: Okay how?

S: You could put (pauses to pick up the calculator) eight thousand divided by thirty.

I: What'd you get?

S: Two hundred and sixty-six point six.

I: Ooh, pretty good. Okay. Umm, let's see. Almost out of questions. I bet you're happy.

S: Oh joy.

I: (laugh) Oh yeah, explain how the, it's right here on the thing, explain how the cost of the Hawaii trip compares to the cost of the Washington, DC trip.

S: They follow the same pattern as far as, like, the numbers decreasing but the Hawaii trip costs more.

I: Okay. Umm, you see how these lines are, like, decreasing as they go?

S: Yeah.

I: And if you connected both of those,

S: Yeah.

I: Do you think there would ever be a point at which they came to, they crossed? (pause) You know what I'm saying?

S: They might meet up at the same point.

I: Okay. You think they would, or they wouldn't, or you don't know?

S: Probably.

I: Alright. Where would you guess that to be?

S: I dunno. I think ~ ~ ~

I: Like fairly near by or way out?

S: Mmm, (pause) Mmm, near by?

I: Okay. Go ahead and show me how you'd find out.

S: Can put it in Y equals, I think. (pause) Hmm, where is that? Then you go to second graph, and they're both on here.

Joy's work on Task 3 of Interview Two
Conjecturing about the functions $c = 20000/n$ and $c = 8000/n$ (cont'd)

I: Mmmmm.

S: Then you see where they equal the same. I think.

I: Okay.

S: Maybe I did that wrong. But that's okay.

I: Looks good to me. (pause) So you're looking for which two to equal the same?

S: Yup.

I: The, these two? The Y1 and the Y2? Or these two?

S: Hold on. (laugh) I'm not sure.

I: Okay.

S: Oh, the Y1 and the Y2.

I: Okay.

S: (pause) Okay that's the top of the other thing. Maybe I do not want to go by twenties there.

I: Okay.

S: Start at two hundred and forty. Why on earth would I do that?

I: I think its just because you scrolled down the list a ways and that's were it was.

S: Aye. (pause) What do I want to start at? How about, well we know it's not there by one hundred (pause) so

I: Okay.

S: How 'bout ones? Why the heck not? (long pause as Joy scrolls down the table on the calculator)

I: Are they getting closer?

S: Mmm, not really. They're getting close ~

I: Okay. (laugh)

S: (pause) Mmm, I'm not sure they will.

I: Okay. How far have you gone there? (pause) What if you jumped way way up?

S: Then I would get confused. [Student does not want to scroll through the table by X-value increments greater than 1.]

I: Oh, okay. (laugh)

S: (pause) Because sometimes if you do it way up then you get, like, you see where it, above where it matches.

I: After it's crossed.

S: And miss it.

I: Okay. Okay.

S: (pause) like the same thing when you're doing things that when they turn positive

I: Uh huh.

S: like when it hits bottom

Joy's work on Task 3 of Interview Two
Conjecturing about the functions *c = 20000/n* and *c = 8000/n* (cont'd)

I: Uh huh.

S: you can't tell, but it's already at a negative [describing to a critical value of a function]

I: Okay.

S: so you know whatever it did, you just don't know when.

I: Okay.

S: I don't think its going to happen.

I: No. Where are you at now?

S: Seventy.

I: Are you getting any closer than you have been?

S: Ahh, sixty-five and twenty-six, twenty-five and sixty-four, mmm, maybe. Perhaps. (pause) Yeah, they're getting closer.

I: Okay. (long pause) What'd you think?

S: I'm gonna change this real quick.

I: What'd you get to, four fourteen?

S: Yeah.

I: Okay.

S: So I'm gonna put, five, fives.

I: Okay.

S: Okay that would be ~ ~? I don't know. (pause, while scrolling farther)

I: That's getting' cheap isn't it? (laugh) Fifteen dollars to go then?

S: Yeah.

I: to Washington DC. It'd be a deal.

S: Yeah probably. (pause) Hm, I'm just thinking it's not gonna cross. (pause) Although it probably did somewhere.

I: You think this is ever gonna get to zero?

S: Yeah.

I: So it would be free to go?

S: Eventually (pause) it'll get to zero.

I: Okay.

S: But will it get to zero before it crosses (pause) ~ trying to tell. (long pause) I don't think they're gonna cross.

I: Okay. Where are you at now?

S: Eighteen and seven. [These are the Y-values for Y2 and Y1, respectively.]

I: Okay. (laugh) That's a lot closer than it was. That's with [X-values of] what, eleven hundred fifteen, sixteen?

S: They might cross at, like, negative something.

Joy's work on Task 3 of Interview Two
Conjecturing about the functions $c = 20000/n$ and $c = 8000/n$ (cont'd)

I: Okay. Alright, so is that the point at which you give up?

S: Yeah, perhaps.

I: (laugh) Alright. That was the most persistent of anybody I've asked. Very good. (laugh)

S: Does it cross? [Joy is asking the interviewer for the answer.]

I: No, it doesn't. It actually never reaches zero, either. But it gets soooooo tiny you can hardly tell ~ ~ ~.

S: It never reaches zero, it goes by ~ ?

I: It would never reach zero.

S: Never?

I: Nh nh. I mean it would just, it'd go to fractions of cents and stuff. Because if you took

S: Point zero?

I: Well, yeah it would go to, m hm, zero point something, but it would never get to the zero or below the zero because you would always have a certain amount of money divided by a certain number of people, and you're never gonna get zero. You know what I'm saying?

S: M hm.

I: Like if you had twenty thousand divided by twenty thousand it'd be down to a dollar apiece; or if you had forty thousand people it'd be down to fifty cents a piece. (pause) It's a, it's a looong ways out there to find. (laughs) But it actually gets closer and closer to zero and never reaches. Is that what you expected?

S: Mmm. I dunno.

I: (laugh) Well you did very well.

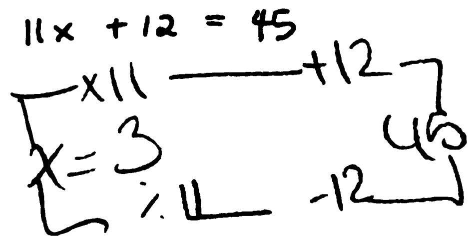


Figure B.1
Correct circle diagram

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