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HEAT TRANSFER PROBLEM IN CORE-ANNULAR FLOWS

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SOLUTIONS TO THE FULLY DEVELOPED CONVECTION HEAT TRANSFER PROBLEM IN CORE ANNULAR FLOWS

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ABSTRACT

SOLUTIONS TO THE FULLY DEVELOPED CONVECTION HEAT TRANSFER PROBLEM IN CORE ANNULAR FLOWS

By

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Heat transfer in circular pipes with two concentric regions (core annular flows) is a problem that arises in materials processing and the chemical industries. Examples of this problem include the flow of oil/water for a certain range of Reynolds numbers and flow through packed bed chemical reactors. In this work, the convection heat transfer problem is solved for a core-annular flow that is both thermally and hydrodynamically fully developed. Solutions are obtained for an outer surface of the pipe that is subjected to three different boundary conditions: uniform heat flux, constant wall temperature, and convective heat flux. The velocity and the temperature profiles are calculated analytically and Nusselt numbers are evaluated for the three cases. The analytical results are validated by comparison with the solutions for the single-phase fluid.

Dedicated to all my friends and professors at Michigan State University

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KEY TO SYMBOLS

C_p Heat Transfer Coefficient

D Diameter
H Pressure gradient
Pr Prandlt number
T Temperature
Nu Nusselt number
g Non Dimensional pressure gradient
h Heat transfer coefficient
k Thermal Conductivity
q Heat transfer
r Radius
u Velocity
v Non-Dimensional Velocity

GREEK ALPHABETS
αThermal diffusivity
γ Ratio of Thermal Conductivities
η Non-Dimensional Radius
λ Ratio of Dynamic Viscosities
μ Dynamic Viscosity
ν Kinematic Viscosity
θ Non-Dimensional Temperature
ρ Density
ω Ratio of Heat Capacities
Subscripts
i Inner Fluid
o Outer Fluid
m Mean
0 outer wall
m,1 Mean Property of the inner fluid
m,2 Mean Property of the outer fluid

INTRODUCTION

Important engineering problems in the materials processing and chemical industries involve heat transfer in a two-region channel flow. Examples include the flow of two immiscible fluids and the flow through a packed bed chemical reactor. One of the most important engineering applications of two-region flow is the flow of water and oil in a pipe. Pipeline transport of highly viscous oil needs enormous pumping pressures to overcome the high viscosity and the corresponding wall shear stress. It can be affected by heating the oil and insulating the pipeline [1]. However these operations involve considerable capital investments and operating expenditures. As discussed by Joseph et al. [2], oil companies have had an intermittent interest in the technology of water-lubricated transport of heavy oil since 1904.

In the past, experiments have been carried out to examine the possibility of water-lubricated transportation of oil as discussed by Chaves et al. [1]. A number of flow patterns were observed, such as water drops in oil, concentric oil-water flow and oil drops in water. It was found out that of all the observed flow patterns, the flow of the highly viscous oil as a core, with the water flowing only in the annular space between the pipe and the core walls was the most desirable for simultaneous flow [1]. The pressure drop measured over the pipe indicated that the addition of water greatly reduced the pressure gradient. Ooms et al. [3] discussed theoretical models for core annular flows of a very viscous oil core and a water annulus through a horizontal pipe. As long as oil core was supplied at a velocity above a certain critical value, a water film remained between the oil-core and the pipe wall.

However the amount of water used to transport the oil is determined by various stability analyses. If the water used is large, there is always a problem of dewatering and if the water used is less, there is a problem of fouling. Using cement-lined pipes as discussed by Arney et al. [4] can reduce fouling to a considerable extent. Joseph et al. [5] discussed the stability of annular flows. The annular flow is stable or can be stabilized when the fluid having the higher effective viscosity occupies the core region and lower viscosity fluid is in the annulus. Joseph et al. [6] also study the instability of the core annular flow as the thickness of the lubricating fluid in the system increases. Thus, volume ratio is a crucial factor in determining the stability. There is also a limitation on the velocities of the fluids. A minimum velocity exists below which the core-annular flow is unstable and results in oil slugs in water and a maximum velocity above which the flow is replaced by water emulsions in oil as discussed by Prezioski et al. [7]. Extent of the annular flow is determined by the location of the break-up point. Hason et al. [8] discovered that the wall-film broke up at low flow rates and thus destroys annular flow abruptly.

A practical aspect of this technology is the question of how to guarantee that a certain oil-water system will operate in core-flow pattern. In practical situations, this flow mode is brought about by using special inlet nozzles in combination with physiochemical agents to facilitate wetting of the pipe wall by water and to stabilize the oil-water interface [9]. Shut down and restart procedures must also be known. For the technique to be practical, it should be possible to restart a pipeline from the stratified oil-water situation to the core flow pattern in a reasonable amount of time without the requirement of excessive pumping power [9]. In the late nineteenth century, development of Moody's chart involving Reynolds number and Fanning's Friction factor facilitated the design and development of Single-fluid pipeline systems. Arnes et al. [10] extended this analysis to lubricated pipelining. Thus core-annular flow has been a topic of research for quite some time. However most of these works focus on the "fluid" part of the analysis. Most of the literature neglects the thermal analysis of the core-annular flow. The possibility of using these systems in cold regions such as on the ocean floors or in the arctic regions require a detailed heat transfer analysis for a variety of boundary conditions.

Although the uniform wall heat flux problem is rather trivial, the constant wall-temperature problem is a challenging one in the case of a single-fluid system. The case of a convection boundary conditions applied on the outside of the pipe is also non-trivial.

Solution to the heat transfer problem of a single fluid flowing down a pipe has been solved long ago by Graetz in 1885 [11]. Graetz neglected the variations of the properties of the fluid with the temperature and found a mathematical solution for the problem that was rediscovered by Nusselt in 1910[11]. In this work, a heat transfer analysis of the core-annular flow is performed using an approach different than the one used by Graetz to compute the temperature profiles and Nusselt numbers of the system for three boundary conditions on the outer wall- constant wall heat flux, constant wall temperature and convective heat flux

CHAPTER 1

PHYSICAL MODELS

The physical model for the two-region pipe flow and heat transfer problem is shown in Figure 1. Each region 1 or 2 contains a fluid that is immiscible with the fluid in the other region.

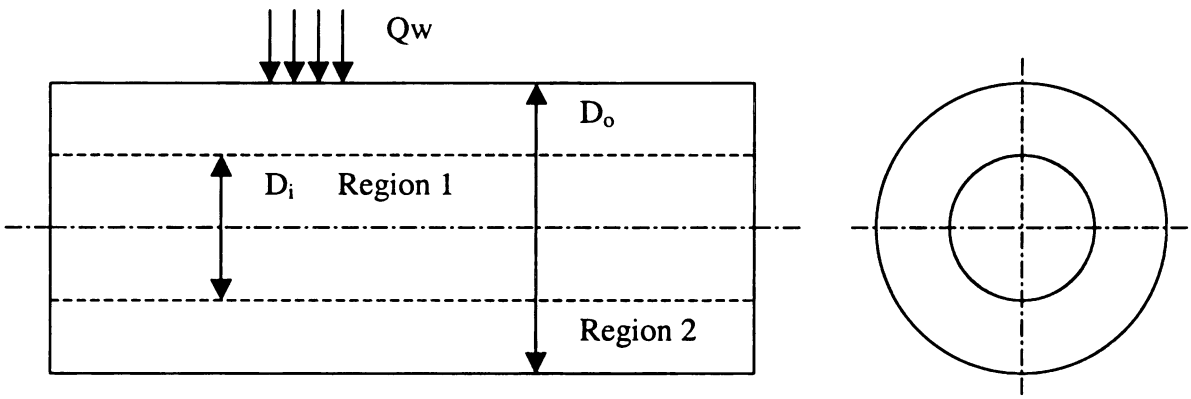


Figure 1.1: Physical model for two region flow

The following assumptions will be employed in the development of the solution to the heat transfer problem

(a) The flow is steady and hydrodynamically fully developed. This will lead to the only non-zero velocity component being that in the streamwise direction and a constant pressure gradient (- H) in the streamwise direction.

(b) The flow is thermally full developed. This will lead to a constant convective heat transfer coefficient on the outer surface and a constant temperature derivative in the streamwise direction.

(c) Three types of boundary conditions have been analyzed- constant heat flux on the outer wall, constant wall temperature and convective boundary condition.

The Continuity equation in the Cylindrical coordinates for steady and incompressible flow is [12]:

$$\frac{1}{r}\frac{\partial}{\partial r}(ru_r)+\frac{1}{r}\frac{\partial}{\partial \theta}(u_\theta)+\frac{\partial}{\partial z}(u_z)=0 \tag{1.1}$$

The components of the momentum equation are as follows:

r -momentum:

$$\frac{\partial u_r}{\partial t}+(U.\nabla)u_r-\frac{1}{r}u_\theta^2=-\frac{1}{\rho}\frac{\partial p}{dr}+g_r+\upsilon(\nabla^2 u_r-\frac{u_r}{r^2}-\frac{2}{r^2}\frac{\partial u_\theta}{\partial \theta}) \tag{1.2}$$

θ -momentum:

$$\frac{\partial u_\theta}{\partial t}+(U.\nabla)u_\theta+\frac{u_r.u_\theta}{r}=-\frac{1}{\rho r}\frac{\partial p}{d\theta}+g_\theta+\upsilon(\nabla^2 u_\theta+\frac{2}{r^2}\frac{\partial u_r}{\partial \theta}-\frac{u_\theta}{r^2}) \tag{1.3}$$

z -momentum:

$$\frac{\partial u_z}{\partial t}+(U.\nabla)u_z=-\frac{1}{\rho}\frac{\partial p}{dz}+g_z+\upsilon\nabla^2 u_z \tag{1.4}$$

where, the convective derivative is:

$$U.\nabla=u_r\frac{\partial}{\partial r}+\frac{1}{r}u_\theta\frac{\partial}{\partial \theta}+u_z\frac{\partial}{\partial z} \tag{1.5}$$

The Laplacian operator is:

$$\nabla^2=\frac{1}{r}\frac{\partial}{\partial r}(r\frac{\partial}{\partial r})+\frac{1}{r^2}\frac{\partial^2}{\partial \theta^2}+\frac{\partial^2}{\partial z^2} \tag{1.6}$$

Since a symmetric flow is assumed, u_θ=0. The flow is assumed to be hydrodynamically fully developed, u_r=0. Using this value of u_r in Eq (1.1), $\frac{\partial}{\partial z}(u_z)=0$. Since the flow is symmetric, there is no θ dependency. So u_z is a function of r only.

Consequently, when we use these above deductions in Eq (1.2), the r-momentum equation reduces to:

$$0 = -\frac{1}{\rho}\frac{\partial p}{dr} \tag{1.7}$$

which means that the pressure $p = f(\theta, z)$. The θ momentum equation (Eq 1.3) reduces to the following form:

$$0 = -\frac{1}{\rho}\frac{\partial p}{d\theta} \tag{1.8}$$

which implies that $p = f(z)$ only, in other words the pressure in the pipe flow changes only in the axial direction. The z-momentum equation is

$$\frac{\partial u_z}{\partial t} + (U.\nabla)u_z = -\frac{1}{\rho}\frac{\partial p}{dz} + g_z + \upsilon\nabla^2 u_z \tag{1.9}$$

Using the established facts that the flow is steady, $p = f(z)$ only, $g_z = 0$ and that u_z is a function of r only, Eq (1.9) can be reduced to the following form:

$$0 = -\frac{1}{r}\frac{\partial p}{\partial z} + \upsilon\nabla^2 u_z \tag{1.10}$$

Equation (1.10) further simplifies to,

$$\frac{1}{r}\frac{d}{dr}\left(r\frac{du_z}{dr}\right) = -\frac{H}{\mu} \tag{1.11}$$

where, H= $-\frac{dp}{dx}$ (the pressure gradient)

The energy equation [12] is analyzed next.

$$\rho c_p\left[\frac{\partial T}{\partial t} + (U.\nabla)T\right] = k\nabla^2 T + \mu\left[2\left(\varepsilon_{rr}^2 + \varepsilon_{\theta\theta}^2 + \varepsilon_{zz}^2\right) + \varepsilon_{\theta z}^2 + \varepsilon_{rz}^2 + \varepsilon_{r\theta}^2\right]$$

(1.12)

where,

$$\varepsilon_{rr} = \frac{\partial u_r}{\partial r}, \quad \varepsilon_{\theta\theta} = \frac{1}{r}\frac{\partial u_\theta}{\partial \theta} + \frac{u_r}{r}, \ \varepsilon_{zz} = \frac{\partial u_z}{\partial z}$$

$$\varepsilon_{\theta z} = \frac{1}{r}\frac{\partial u_z}{\partial \theta} - \frac{\partial u_\theta}{dz}, \varepsilon_{rz} = \frac{\partial u_r}{\partial z} + \frac{\partial u_z}{\partial r}, \varepsilon_{r\theta} = \frac{1}{r}\frac{\partial u_r}{\partial \theta} + \frac{\partial u_\theta}{\partial r} - \frac{u_\theta}{r} \tag{1.13}$$

Equation (1.12) simplifies to,

$$u_z \rho c_p \frac{\partial T}{\partial z} + u_r \rho c_p \frac{\partial T}{\partial r} - \left[\frac{k}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right) + \frac{k}{r^2}\frac{\partial^2 T}{\partial \theta^2} + k\frac{\partial^2 T}{\partial z^2}\right] = 0 \tag{1.14}$$

Since heat transfer is symmetric in the pipe, $\frac{\partial^2 T}{\partial \theta^2} = 0$. The flow is hydrodynamically fully developed which results in the radial component of the velocity to be zero ($u_r = 0$), Equation (1.12) becomes, after transposition,

$$u_z \left(\frac{\rho c_p}{k}\right)\frac{\partial T}{\partial z} - \left[\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right) + \frac{\partial^2 T}{\partial z^2}\right] = 0 \tag{1.15}$$

Letting $\alpha = \frac{k}{\rho c_p}$, the thermal diffusivity of the fluid,

$$\frac{u_z}{\alpha}\frac{\partial T}{\partial z} = \left[\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right) + \frac{\partial^2 T}{\partial z^2}\right] \tag{1.16}$$

If we neglect axial conduction relative to the radial conduction then, $\frac{\partial^2 T}{\partial z^2} = 0$, we obtain the following energy equation for laminar flow in a circular tube.

$$\frac{1}{r}\frac{\partial}{\partial r}\left(\frac{\partial T}{\partial r} r\right) = \frac{u_z}{\alpha}\frac{\partial T}{\partial z} \tag{1.17}$$

Since the flow is assumed to be thermally fully developed, the non-dimensional temperature profile is invariant with z [13]. This can be expressed in the following equation:

$$\frac{\partial}{\partial z}\left(\frac{T_w - T}{T_w - T_m}\right) = 0 \tag{1.18}$$

Differentiating and solving for $\partial T/\partial z$,

$$\frac{\partial T}{\partial z} = \frac{dT_w}{dz} - \frac{T_w - T}{T_w - T_m}\frac{dT_w}{dz} + \frac{T_w - T}{T_w - T_m}\frac{dT_m}{dz} \tag{1.19}$$

The next chapter deals with the first boundary condition, which is the constant heat flux on the outer wall.

CHAPTER 2

CONSTANT HEAT FLUX PROBLEM

The boundary condition of constant heat flux on the outer wall is considered below.

Writing the convection rate equation,

$$q''_w = h(T_w - T_m) = \text{Constant} \tag{2.1}$$

where T_w and T_m are the temperature of the outer wall and the mean temperature of the fluid respectively. Since the non-dimensional temperature is invariant in the flow direction, the boundary condition at the wall is then:

$$\frac{\partial}{\partial r}\left(\frac{\partial}{\partial z}\left(\frac{T_w - T}{T_w - T_m}\right)\right) = 0 \text{ At } r = R_o \tag{2.2}$$

which can be written as:

$$\frac{\partial}{\partial z}\left(\frac{\partial}{\partial r}\left(\frac{T_w - T}{T_w - T_m}\right)\right) = 0 \text{ At } r = R_o \tag{2.3}$$

This means that,

$$\left[\frac{\partial}{\partial r}\left(\frac{T_w - T}{T_w - T_m}\right)\right]_{r=R_o} = constant = -\frac{\left(\frac{\partial T}{\partial r}\right)_{r=R_o}}{T_w - T_m} \tag{2.4}$$

The wall heat flux can be defined as

$$q''_w = -k\left(\frac{\partial T}{\partial r}\right)_{r=R_o} \tag{2.5}$$

Comparing equations (2.3), (2.4) and (2.5), it can be deduced that h/k=constant, or in other words, h=constant for a constant property fluid [14].

Thus, it can be concluded from Eq (2.1) that $T_w - T_m$ = constant.

In other words, $\frac{dT_w}{dz} = \frac{dT_m}{dz}$ (2.6)

Using Eq (2.6) in Eq (2.4) we get,

$$\frac{\partial T}{\partial z} = \frac{dT_w}{dz} \tag{2.7}$$

Combining Equations (2.6) and (2.7)

$$\frac{\partial T}{\partial z} = \frac{dT_w}{dz} = \frac{dT_m}{dz} \tag{2.8}$$

Using Eq (2.8) in Eq (1.17), for constant heat flux on the outer wall, we get

$$u\frac{dT_m}{dz} = \frac{\alpha}{r}\frac{d}{dr}\left(r\frac{dT}{dr}\right) \tag{2.9}$$

Thus, the following differential equations are used to represent the conversation of momentum and energy within a region. Since there are 2 fluids in our problem, subscript i may be 1 or 2 to denote the region, 1 for the inner fluid and 2 for the outer fluid respectively.

$$\frac{1}{r}\frac{d}{dr}\left(r\frac{du_i}{dr}\right) = \frac{-H}{\mu_i} \tag{2.10}$$

And,

$$\frac{\alpha_i}{r}\frac{d}{dr}\left(r\frac{dT_i}{dr}\right) = u_i\frac{dT_m}{dz} \tag{2.11}$$

The boundary and matching conditions are as follows: -

At $r = 0$ (the centerline of the channel)

$$\frac{du_1}{dr} = 0 \tag{2.12}$$

$$\frac{dT_1}{dr} = 0 \tag{2.13}$$

At $r = R_i$ (the interface of the two regions)

$$u_1 = u_2 \tag{2.14}$$

$$\mu_1 \frac{du_1}{dr} = \mu_2 \frac{du_2}{dr} \tag{2.15}$$

$$P_1 = P_2 \tag{2.16}$$

$$T_1 = T_2 \tag{2.17}$$

$$k_1 \frac{dT_1}{dr} = k_2 \frac{dT_2}{dr} \tag{2.18}$$

At $r = R_o$ (the outer surface of the channel)

$$u_2 = 0 \tag{2.19}$$

$$k_2 \frac{dT_2}{dr} = q_w \tag{2.20}$$

The matching conditions on the pressure and the temperature along with the fully developed assumptions lead to the observation that both regions are subjected to the same pressure gradient (-H) and have the same axially temperature gradient $\frac{dT_m}{dz}$. An energy balance on a differential element of length Δz and radius R_o is carried out below.

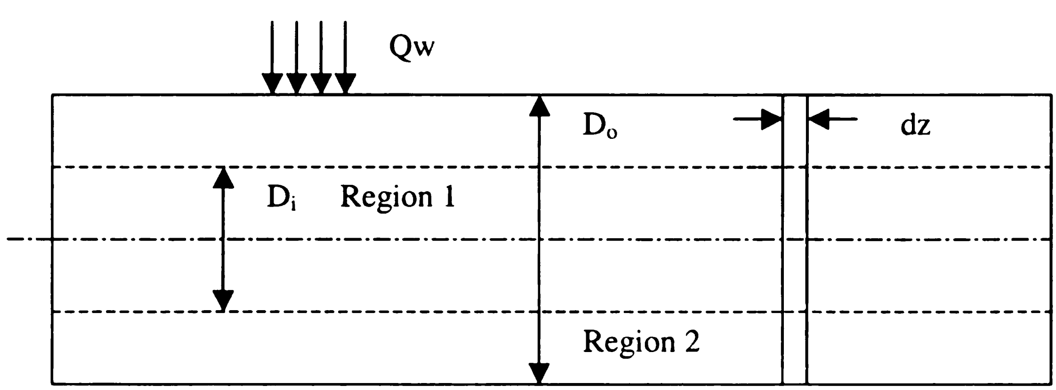


Figure 2.1: Energy balance on an element of length dz

$$2\pi R_o q_w dz = \left(\pi R_i^2 u_{m,1}\right)\rho_{f,1} c_{p,1} dT_m + \pi (R_o^2 - R_i^2) u_{m,2} \rho_{f,2} c_{p,2} dT_m \tag{2.21}$$

which can be written as: -

$$\frac{dT_m}{dz} = \frac{2R_o q_w}{(\rho c_p)_1 R_i^2 u_{m,1} + (\rho c_p)_2 (R_o^2 - R_i^2) u_{m,2}} \tag{2.22}$$

where

$$u_{m,1} = \frac{1}{\pi R_i^2} \int_0^{R_i} u_1(r) 2\pi r dr \tag{2.23}$$

$$u_{m,2} = \frac{1}{\pi (R_o^2 - R_i^2)} \int_{R_i}^{R_o} u_2(r) 2\pi r dr \tag{2.24}$$

NON-DIMENSIONALIZATION

Prior to solving the equations, the following non-dimensionalization is used.

$$\begin{aligned} \eta &= \frac{r}{R_o} \\ v_i &= \frac{u_i}{u_m} \\ G_i &= \frac{HR_o^2}{u_m \mu_i} \\ \theta_i &= \frac{T_i - T_c}{2R_o q_w / k_i} \end{aligned} \tag{2.25}$$

Then the momentum equation becomes:

$$\frac{1}{\eta}\frac{d}{d\eta}\left(\eta \frac{dv_i}{d\eta}\right) = -G_i \tag{2.26}$$

And, the energy equation becomes

$$\frac{1}{\eta}\frac{d}{d\eta}\left(\eta\frac{d\theta_i}{d\eta}\right)=\frac{1}{\Omega_{I,i}\eta_i^2 v_{m,I}+\Omega_{II,i}(1-\eta_i^2)v_{m,II}}v_i \tag{2.27}$$

In dimensionless form, the boundary and matching conditions become:

At $\eta=0$ (the centerline of the channel)

$$\frac{dv_1}{d\eta}=0 \tag{2.28}$$

$$\frac{d\theta_1}{d\eta}=0 \tag{2.29}$$

$$\theta_1=0 \tag{2.30}$$

At $\eta=\eta_i$ (the interface between the two regions)

$$v_1=v_2 \tag{2.31}$$

$$\lambda\frac{dv_1}{d\eta}=\frac{dv_2}{d\eta} \tag{2.32}$$

$$\theta_1=\theta_2 \tag{2.33}$$

$$\gamma\frac{d\theta_1}{d\eta}=\frac{d\theta_2}{d\eta} \tag{2.34}$$

At $\eta=1$ (at the outer surface of the channel)

$$v_2=0 \tag{2.35}$$

Several non-dimensional parameters appear from this scaling. We define them as

Thermal Conductivity Ratio: $\gamma=\frac{k_1}{k_2}$

Dynamic Viscosity Ratio: $\lambda=\frac{\mu_1}{\mu_2}$

Radius Ratio: $\eta_i = \dfrac{R_i}{R_o}$

Heat Capacity Ratio Function: $\Omega_{m,n} = \left\{\begin{array}{l} 1, m = n \\ \omega, m > n \\ \frac{1}{\omega}, m < n \end{array}\right\}$

Heat Capacity Ratio: $\omega = \dfrac{(\rho c_p)_1}{(\rho c_p)_2}$

Consider the momentum equation (2.26) $\dfrac{1}{\eta}\dfrac{d}{d\eta}\left(\eta\dfrac{dv_i}{d\eta}\right) = -G_i$

Solving the momentum equation separately for the 2 fluids, we get,

$$v_1 = A - B\eta^2 \tag{2.36}$$

$$v_2 = C - D\eta^2 \tag{2.37}$$

where, $A = \dfrac{g_2}{4} + \dfrac{1}{4}g_1\eta_i^2 - \dfrac{1}{4}g_2\eta_i^2 + Log[\eta_i]\eta_i\left(-\dfrac{1}{2}\lambda g_1\eta_i + \dfrac{g_2\eta_i}{2}\right)$

$$\text{B} = \frac{g_1}{4}$$

$$\text{C} = \frac{1}{4}g_2 + Log[\eta]\eta_i\left(-\frac{1}{2}\lambda g_1\eta_i + \frac{g_2\eta_i}{2}\right)$$

$$\text{D} = \frac{g_2}{4}$$

Thus, the actual velocity profiles are:

$$u_1 = \left(\frac{1}{4}hR_i^2\left(\frac{1}{\mu_1} - \frac{1}{\mu_2}\right) + \frac{hR_o^2}{4\mu_2}\right) - \frac{h}{4\mu_1}r^2 \tag{2.38}$$

$$u_2 = \frac{hR_o^2}{4\mu_2} - \frac{h}{4\mu_2}r^2 \tag{2.39}$$

The mean velocity profiles are:

For the inner fluid,

$$u_{m,1} = u_f = \frac{1}{\pi R_i^2} \int_0^{R_i} u_1(r) 2\pi r dr \tag{2.40}$$

For the outer fluid,

$$u_{m,2} = u_s = \frac{1}{\pi(R_o^2 - R_i^2)} \int_{R_i}^{R_o} u_2(r) 2\pi r dr \tag{2.41}$$

The mean velocity of the 2-fluid system as a whole is evaluated as follows:

$$u_m = \frac{1}{\pi R_o^2} \left(\int_{R_i}^{R_o} u_2 2\pi r dr + \int_0^{R_i} u_1 2\pi r dr \right) \tag{2.42}$$

The mean velocity profiles are then evaluated using the above equations:

$$u_{m,1} = \frac{h(2R_o^2 \mu_1 + R_i^2(-2\mu_1 + \mu_2))}{8\mu_1 \mu_2} \tag{2.43}$$

$$u_{m,2} = \frac{h(R_o^2 - R_i^2)}{8\mu_2} \tag{2.44}$$

$$u_m = \frac{h(R_o^4 \mu_1 + R_i^4(\mu_2 - \mu_1))}{8R_0^2 \mu_1 \mu_2} \tag{2.45}$$

The next step involves the calculation of the temperature profiles:

So considering the energy Equation, Eq (2.27),

$$\frac{1}{\eta}\frac{d}{d\eta}\left(\eta \frac{d\theta_i}{d\eta}\right) = \frac{1}{\Omega_{I,i}\eta_i^2 v_{m,I} + \Omega_{II,i}(1-\eta_i^2)v_{m,II}} v_i \tag{2.46}$$

INNER FLUID CALCULATIONS:

The non-dimensional temperature profile of the first fluid is calculated first using the above equation:

$$\theta_1 = E\eta^4 - F\eta^2 \tag{2.47}$$

where, $E = -\dfrac{g_1}{64(\Omega_{I,1}\eta_i^2 v_{m,I} + \Omega_{II,1}(1-\eta_i^2)v_{m,II}}$,

$$F = \frac{(-g_1\eta_i^2 - g_2 + g_2\eta_i^2)}{16(\Omega_{I,1}\eta_i^2 v_{m,I} + \Omega_{II,1}(1-\eta_i^2)v_{m,II})}$$

This is converted to the dimensional form to get the following expression for the temperature of the first fluid:

$$\text{Let m}=\Omega_{I,1}\eta_i^2 v_{m,I} + \Omega_{II,1}(1-\eta_i^2)v_{m,II} \tag{2.48}$$

$$T_1 = \frac{4Hqr^2(R_o^2 - R_i^2)\mu_1 - (Hqr^2(r^2 - 4R_i^2) - 32mk_1R_oT_cu_m\mu_1)\mu_2}{32mk_1R_ou_m\mu_1\mu_2} \tag{2.49}$$

Once the temperature profile of the inner fluid is known, its mean temperature is calculated.

$$T_{m,1} = \frac{1}{\pi R_i^2 u_{m,1}} \int_0^{R_i} T_1 u_1 2\pi r dr \tag{2.50}$$

The mean velocity of the inner fluid is:

$$u_{m,1} = \frac{1}{\pi R_i^2} \int_0^{R_i} u_1 2\pi r dr = \frac{H(2R_o^2\mu_1 + R_i^2(-2\mu_1 + \mu_2))}{8\mu_1\mu_2} \tag{2.51}$$

The mean temperature of the first fluid is:

$$T_{m,1} = \frac{\begin{array}{c}24HqR_o^4R_i^2\mu_1^2 + 384mk_1R_o^3T_cu_m\mu_1^2\mu_2 - 192mk_1R_oR_i^2T_cu_m\mu_1(2\mu_1 - \mu_2)\mu_2 + \\ 4HqR_o^2R_{i1}^4\mu_1(-12\mu_1 + 7\mu_2) + HqR_i^6(24\mu_1^2 - 28\mu_1\mu_2 + 7\mu_2^2)\end{array}}{192mk_1R_ou_m\mu_1\mu_2(2R_o^2\mu_1 + R_i^2(-2\mu_1 + \mu_2))} \tag{2.52}$$

The temperature at the interface of the two fluids is calculated as:

$T_i = T_1$ At $r = R_i$

$$T_i = \frac{4HqR_o^2R_i^2\mu_1 + 32mk_1R_oT_cu_m\mu_1\mu_2 + HqR_i^4(-4\mu_1 + 3\mu_2)}{32mk_1R_ou_m\mu_1\mu_2} \tag{2.53}$$

The heat transfer at the interface is given by:

$q_i = -k_1\dfrac{dT_1}{dr}$ At $r = R_i$

$$q_i = \frac{8HqR_o^2R_i\mu_1 - 2HqR_i^3\mu_2 - 2HqR_i(4R_i^2(\mu_1 - \mu_2) + R_i^2\mu_2)}{32mR_ou_m\mu_1\mu_2} \tag{2.54}$$

Finally, the Nusselt Number at the interface of the inner fluid of the inner fluid is given by:

$$Nu_{i,1} = \frac{q_i}{(T_i - T_{m,1})} * \frac{2R_i}{k_1} \tag{2.55}$$

with the values taken from Eq (2.52), (2.53), (2.54)

OUTER FLUID CALCULATIONS:

The next step involves calculating the temperature profile of the second fluid. The velocity and the mean velocity of the second fluid (outer fluid) are as follows:

$$u_2 = \frac{H(R_o^2 - r^2)}{4\mu_2} \tag{2.56}$$

$$u_{m,2} = \frac{1}{\pi(R_o^2 - R_i^2)} \int_{R_i}^{R_o} u_2 2\pi r dr = \frac{H(R_o^2 - R_i^2)}{4\mu_2} \tag{2.57}$$

Equation (2.46) is solved first to get the non-dimensionalzed temperature profile and then converted into the dimensional form to get the following result:

$$T_2 = \frac{\begin{aligned}&(Hnqk_2R_i^2(4R_o^2\mu_1 + R_i^2(-4\mu_1 + 3\mu_2)) + k_1(4Hnq(Log[\frac{r}{R_o}] - Log[\frac{R_i}{R_o}])R_i^4\mu_2 \\ &+ \mu_1(4HqR_o^2(mr^2 - (m + 2(m-n)Log[\frac{r}{R_o}] - 2(m-n)Log[\frac{R_i}{R_o}])R_i^2) + \\ &Hq(-mr^4 + (m + 4(m-2n)Log[\frac{r}{R_o}] - 4(m-2n)Log[\frac{R_i}{R_o}])R_i^4) + 32mnk_2R_oT_cu_m\mu_2)\end{aligned}}{32mnk_1k_2R_ou_m\mu_1\,\mu_2}$$

(2.58)

where $n = \Omega_{I,2}\eta_i^2 v_{m,I} + \Omega_{II,2}(1-\eta_i^2)v_{m,II}$

$$m = \Omega_{I,1}\eta_i^2 v_{m,I} + \Omega_{II,1}(1-\eta_i^2)v_{m,II} \tag{2.59}$$

The mean temperature of the second fluid is given by:

$$T_{m,2} = \frac{1}{\pi(R_0^2 - R_1^2)u_{m,2}} \int_{R_1}^{R_0} T_2 u_2 2\pi r dr =$$

$$\frac{\begin{aligned}&(7h^2mqk_1R_o^8\mu_1 + 12h^2mqk_1R_o^6R_i^2\mu_1 - 36h^2nqk_1R_o^6R_i{}^2\mu_1 + \\ &48h^2mqLog(R_i/R_o)k_1R_o^6R_i^2\mu_1 - 48h^2nqLog(R_i/R_o)k_1R_o^6R_i^2\mu_1 + \\ &24h^2nqk_2R_o^6R_1^2\mu_1 - 36h^2mqk_1R_o^4R_1^4\mu_1 + 84h^2nqk_1R_o^6R_i^4\mu_1 - \\ &24h^2mqLog(R_i/R_o)k_1R_o^4R_i^4\mu_1 + 48h^2nqLog(R_i/R_o)k_1R_o^4R_i^4\mu_1 - \\ &72h^2nqk_2R_o^4R_i^4\mu_1 + 20h^2mqk_1R_o^2R_i^6\mu_1 - 60h^2nqk_1R_o^2R_i^6\mu_1 + \\ &72h^2nqk_2R_o^2R_i^6\mu_1 - 3h^2mqk_1R_i^8\mu_1 + 12h^2nqk_1R_i^8\mu_1 - 24h^2nqk_2R_i^8\mu_1 - \\ &18h^2nqk_1R_o^4R_i^4\mu_2 + 24h^2nqLog(R_i/R_o)k_1R_o^4R_i^4\mu_2 + 18h^2nqk_2R_o^4R_i^4\mu_2 + \\ &24h^2nqk_1R_o^2R_i^6\mu_2 - 36h^2nqk_2R_o^2R_i^6\mu_2 - 6h^2nqk_1R_i^8\mu_2 + 18h^2nqk_2R_i^8\mu_2 + \\ &192hmnk_1k_2R_o^5T_c\mu_1\mu_2u_m + 384hmnk_1k_2R_o^3R_i^2T_c\mu_1\mu_2u_m + 192hmnk_1k_2R_oR_i^4T_c\mu_1\mu_2u_m\end{aligned}}{192hmnk_1k_2R_o(R_o^2 - R_i^2)^2\mu_1\mu_2u_m}$$

(2.60)

THE NUSSELT NUMBER AT THE OUTER WALL:

To find the Nusselt number at the outer wall, we need to compute the mean temperature of the system in total:

$$T_m = \frac{1}{\pi R_o^2 u_m}\left[\left(\int_0^{R_i} u_1 T_1 2\pi r dr\right) + \left(\int_{R_i}^{R_o} u_2 T_2 2\pi r dr\right)\right] \tag{2.61}$$

The temperature of the outer wall is given by:

$$T_{wall} = T_2 \text{ At } r = R_o \tag{2.62}$$

The heat transfer at the interface is given by:

$$q_w = -k_2 \frac{dT_2}{dr} \text{ at } r = R_o \tag{2.63}$$

The Nusselt number at the outer wall is given by:

$$Nu_o = \frac{q_w}{(T_m - T_{wall})} * \frac{2R_o}{k_2} \tag{2.64}$$

Where q_w, T_m, T_{wall} are given by Eq (2.63), (2.61) and (2.62)

RESULTS AND VALIDATION:

Since there is no literature with which we can validate the Nusselt numbers for the 2 fluids case, we validate it using the standard one-fluid case.

If we set $\eta_i = 1$. It means that the inner radius is equal to the outer radius, or in other words, there is only one fluid in the system and it's the inner fluid. And the Nusselt numbers should converge to the Nusselt number of the one-fluid case.

The two Nusselt numbers, the Nusselt number at the interface of the first fluid, $Nu_{i,1}$ and the Nusselt number at the outer wall, Nu_o were evaluated for radius ratio=1. The Nusselt number at the outer wall yielded a value of exactly $\frac{48}{11}\frac{k_1}{k_2}$. We get an extra factor of the ratio of the 2 thermal conductivities due to the way we define the Nusselt number. It has been defined with respect to the second fluid thermal conductivity. However there is no second fluid if the radius ratio is exactly equal to 1. Now, if it is scaled with respect to the first fluid (which is the only fluid present in the system), the Nusselt number simplifies to a value of exactly 4.36 that is the Nusselt number of a single fluid flowing in a pipe with constant heat flux on the outer wall [12].

The Nusselt number at the interface for the first fluid also yields a value of exactly $\frac{48}{11}$ if the radius ratio is equal to 1, which is to be expected because the interface is at the wall in that condition.

Another method in which the Nusselt number expression was validated was by assuming that both the fluids 1 and 2 are the same. In that case, the Nusselt number at the outer wall gave $\frac{48}{11}$ irrespective of the value of the radius ratio.

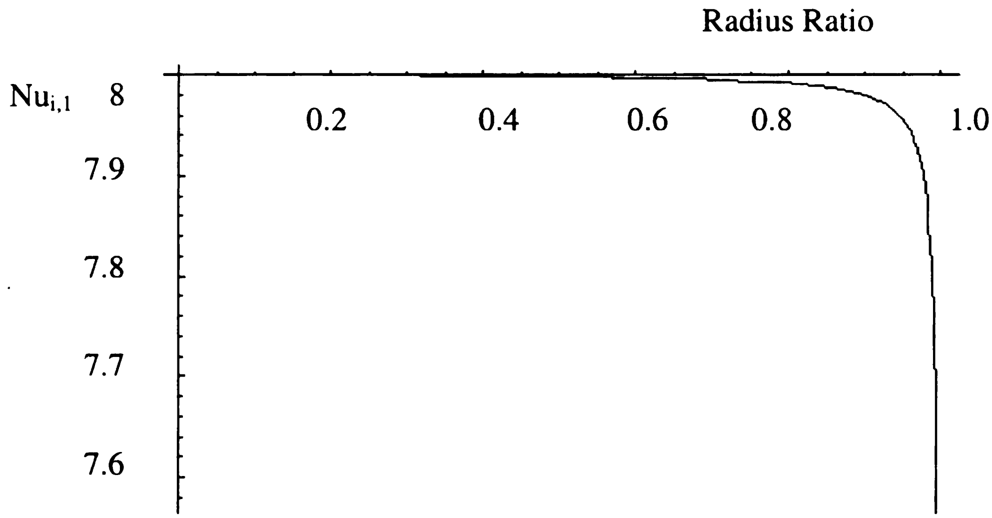


Figure 2.2: Nusselt number at the interface as a function of the radius ratio

As can be seen from Figure 2.2, the Nusselt number at the interface approaches the value of 8 when the radius ratio approaches zero. This is because the inner fluid flow becomes a plug flow as the radius ratio becomes very very small. And the Nusselt number of a plug flow is 8.

Although not exactly visible in the graph, when the radius ratio approaches 1, the Nusselt number is equal to 4.36.

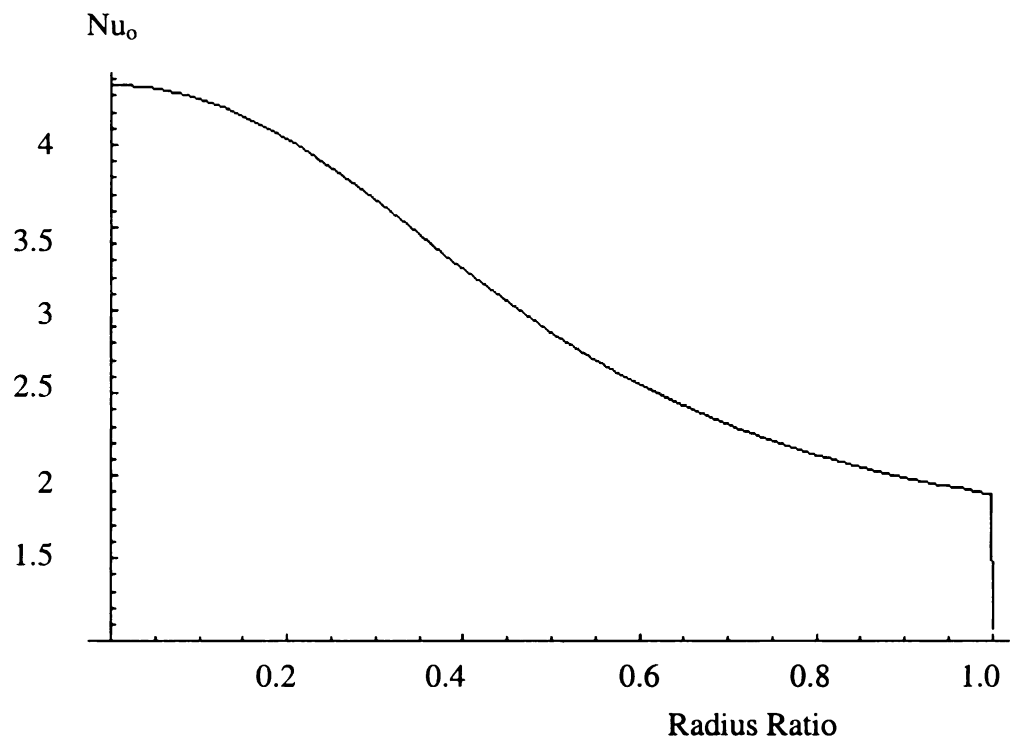


Figure 2.3: Nusselt number at the wall as a function of radius ratio

From Figure 2.3, we can see that the Nusselt number at the outer wall becomes 4.36 when the radius ratio equals zero, in which case the only fluid present in the system is the inner fluid. When the radius ratio equals 1, the Nusselt number at the outer wall converges to a value of 1.029. From the program, it has been observed that the Nusselt number at the outer wall converges to 1.029 at a value of radius ratio of 0.9999999999. However when the radius ratio becomes 1, the only fluid present in the system is the inner one. Thus, the Nusselt number at the outer wall must be scaled with respect to the inner fluid by multiplying 1.029 by the ratio of thermal conductivities. In this case, the ratio of thermal conductivities is:

$$\frac{k_1}{k_2} = \frac{0.145}{0.613} = 0.236$$

The Nusselt number at the outer wall becomes:

$$Nu_0 = \frac{1.029}{0.236} = 4.360$$

Thus, the results have been validated

Following are the plots of Temperature as a function of the radial position for different values of radius ratios. An entry temperature of $T_c = 100^o c$ was assumed with the following properties of the fluids:

Table 2.1:Properties of the two fluids

	Oil	Water
Density (ρ) kg/m^3	884.1	997
Heat Capacity (c_p) J/KgK	1909	4179
Viscosity (μ) Ns/m^2	0.486	0.855*10e-6
Thermal conductivity (k) W/mK	0.145	0.613
Thermal Diffusivity (α) m^2/s	0.859*10e-7	1.471*10e-7
Prandtl Number (Pr)	6400	5.83

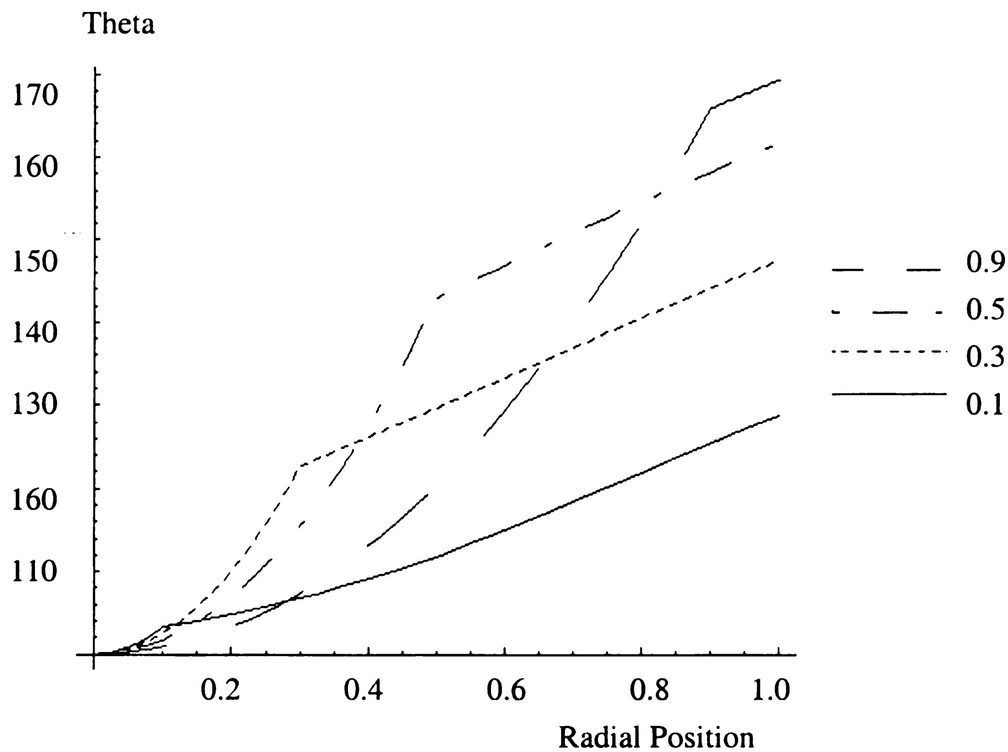


Figure 2.4: Temperature of the system as a function of radial position for different radius ratios.

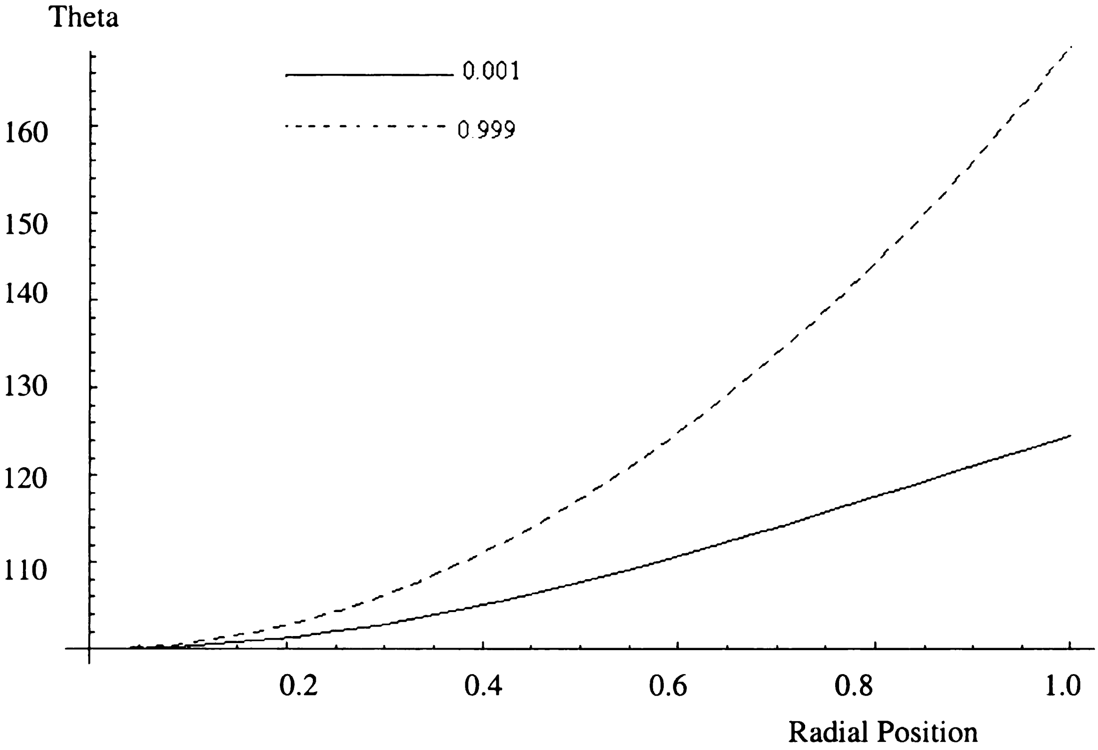


Figure 2.5: Temperature of the system as a function of radial position for extreme radius ratios.

CHAPTER 3

CONSTANT TEMPERATURE ANALYSIS

Consider Eq (1.18) and Eq (1.19)

$$\frac{\partial}{\partial z}\left(\frac{T_w - T}{T_w - T_m}\right) = 0 \tag{3.1}$$

$$\frac{\partial T}{\partial z} = \frac{dT_w}{dz} - \frac{T_w - T}{T_w - T_m}\frac{dT_w}{dz} + \frac{T_w - T}{T_w - T_m}\frac{dT_m}{dz} \tag{3.2}$$

The boundary condition of constant wall temperature reduces the above Eq (3.1) and Eq (3.2) to

$$\frac{\partial T}{\partial z} = \frac{T_w - T}{T_w - T_m}\frac{dT_m}{dz} \tag{3.3}$$

Figure 3.1 gives the energy balance on an element of length dz.

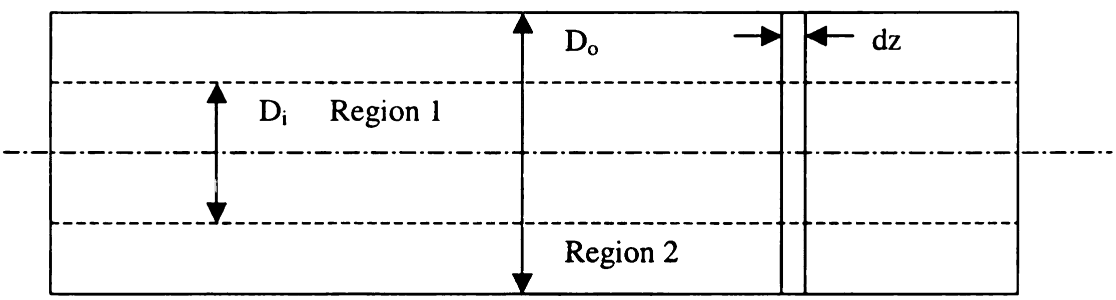


Figure 3.1: Energy balance on an element of length dz

$$2\Pi R_0 q_w(z)dz = \left(\Pi R_i{}^2 u_{m,1}\right)\rho_{f,1}c_{p,1}dT_m + \Pi(R_0{}^2 - R_i{}^2)u_{m,2}\rho_{f,2}c_{p,2}dT_m \tag{3.4}$$

$$\frac{dT_m}{dz} = \frac{2q_w{}'' R_o}{(\rho c_p)_1 R_1^2 u_{m,1} + \left(\rho c_p\right)_2 (R_o^2 - R_i^2)u_{m,2}} \tag{3.5}$$

$$\frac{dT_m}{dz} = \frac{2q_w''(x)R_o}{\{(\rho c_p)_2 R_o^2[\frac{(\rho c_p)_1}{(\rho c_p)_2}\frac{R_i^2}{R_o^2}u_{m,1} + (1-\frac{R_i^2}{R_o^2})u_{m,2}]\}} \tag{3.6}$$

Substituting the above expression from Eq (3.6) into Eq (3.3), we get:

$$\frac{\partial T}{\partial z} = \frac{(T_w - T)}{(T_w - T_m)} \frac{2q_w''(x)R_o}{\{(\rho c_p)_2 R_o^2[\frac{(\rho c_p)_1}{(\rho c_p)_2}\frac{R_i^2}{R_o^2}u_{m,1} + (1-\frac{R_i^2}{R_o^2})u_{m,2}]\}} \tag{3.7}$$

$$\frac{\partial T}{\partial z} = \frac{q_w''(x)}{(T_w - T_m)} \frac{2(T_w - T)}{\{(\rho c_p)_2 R_o^2[\frac{(\rho c_p)_1}{(\rho c_p)_2}\frac{R_i^2}{R_o^2}u_{m,1} + (1-\frac{R_i^2}{R_o^2})u_{m,2}]\}} \tag{3.8}$$

But, $$\frac{q_w''(x)}{(T_w - T_m)} = \text{h.} \tag{3.9}$$

From the definition of Nusselt number (on the outer wall),

$$Nu_o = \frac{h.(2R_o)}{k_2} \tag{3.10}$$

Using Eq (3.9) in the above definition of the Nusselt number, Eq (3.8) reduces to:

$$\frac{\partial T}{\partial z} = \frac{k_2 Nu_o}{2R_o} \frac{2(T_w - T)}{\{(\rho c_p)_2 R_o^2[\frac{(\rho c_p)_1}{(\rho c_p)_2}\frac{R_i^2}{R_o^2}u_{m,1} + (1-\frac{R_i^2}{R_o^2})u_{m,2}]\}} \tag{3.11}$$

$$\frac{\partial T}{\partial z} = \frac{\alpha_2 (T_w - T) Nu_o}{\{R_o^2[\frac{(\rho c_p)_1}{(\rho c_p)_2}\frac{R_i^2}{R_o^2}u_{m,1} + (1-\frac{R_i^2}{R_o^2})u_{m,2}]\}} \tag{3.12}$$

The energy equation Eq (1.17) $\frac{1}{r}\frac{\partial}{\partial r}\left(\frac{\partial T}{\partial r}r\right) = \frac{u_z}{\alpha}\frac{\partial T}{\partial z}$ reduces to the following form:

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right)=\frac{u_z}{\alpha}\frac{\alpha_2(T_w-T)Nu_o}{\{R_o^2[\frac{(\rho c_p)_1}{(\rho c_p)_2}\frac{R_i^2}{R_o^2}u_{m,1}+(1-\frac{R_i^2}{R_o^2})u_{m,2}]\}} \tag{3.13}$$

Since there is only one velocity component, the z-component, u_z is replaced by u for simplicity.

The Boundary and the matching conditions are as follows:

At $r = 0$ (at the centerline of the channel)

$$\frac{dT_1}{dr}=0 \tag{3.14}$$

At $r = R_i$ (at the interface of the two regions)

$$T_1 = T_2 \tag{3.15}$$

$$k_1\frac{dT_1}{dr}=k_2\frac{dT_2}{dr} \tag{3.16}$$

At $r = R_0$ (at the outer surface of the channel)

$$T_2 = T_{wall} \tag{3.17}$$

INNER FLUID CALCULATIONS:

Thus for the first fluid, the energy equation (3.14) becomes:

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T_1}{\partial r}\right)=\frac{u_1}{\alpha_1}\frac{\alpha_2(T_w-T)Nu_o}{\{R_o^2[\frac{(\rho c_p)_1}{(\rho c_p)_2}\frac{R_i^2}{R_o^2}u_{m,1}+(1-\frac{R_i^2}{R_o^2})u_{m,2}]\}} \tag{3.18}$$

$$u_1=\left(\frac{1}{4}hr_i^2\left(\frac{1}{\mu_1}-\frac{1}{\mu_2}\right)+\frac{hr_o^2}{4\mu_2}\right)-\frac{h}{4\mu_1}r^2 \text{ (From Eq 2.38)}$$

Defining new parameters:

$\theta_i = T_1 - T_w$ And $\eta = \frac{r}{R_o}$,

Equation (3.18) simplifies to:

$$\frac{1}{\eta}\frac{d}{d\eta}\left[\eta\frac{d\theta_1}{d\eta}\right] + [M - N\eta^2]Nu_o\theta_1 = 0 \tag{3.19}$$

where
$$M = \frac{\frac{\alpha_i}{4}(g_1 - g_2)\eta_i^2 + \frac{\alpha_i g_2}{4}}{\left(\eta_i^2 v_{m,1}\omega + (1-\eta_i^2)v_{m,2}\right)} \tag{3.20}$$

And
$$N = \frac{\frac{\alpha_i}{4}g_1}{\left(\eta_i^2 v_{m,1}\omega + (1-\eta_i^2)v_{m,2}\right)} \tag{3.21}$$

$$\alpha_i = \frac{\alpha_2}{\alpha_1} \tag{3.22}$$

And the other variables are same as defined in the constant heat flux problem.

Equation (3.19) is solved by the Method of Frobenius [15]:

Let
$$\theta_1 = \eta^s \sum_{n=0}^{\infty} a_n \eta^n \tag{3.23}$$

Substituting the value of θ_1 in Eq (3.19) we get:

$$\sum_{n=0}^{\infty}(n+s)(n+s-1)a_n\eta^{s+n-2} + \sum_{n=0}^{\infty}(n+s)a_n\eta^{n+s-2} + (M - N\eta^2)Nu_o\sum_{n=0}^{\infty}a_n\eta^{n+s} = 0 \tag{3.24}$$

Equating the coefficient of the lowest power of η which is the coefficient of η^{s-2}:

$$s(s-1)a_o + sa_o = 0 \tag{3.25}$$

which leads to s=0,0. (Double root)

If the indicial equation has 2 equal roots, as in Eq (3.25), at $y = m_1$ and $y = m_2$, the solution is given by [16] :

$$y = c_1 (y)_{m_1} + c_2 \left(\frac{dy}{dm} \right)_{m_2} \tag{3.26}$$

Thus, the solution in our case is given by:

$$\theta_1 = c_1 (\theta_1)_{s=0} + c_2 \left(\frac{d\theta_1}{ds} \right)_{s=0}$$

$$\theta_1 = c_1 \sum_{n=0}^{\infty} a_n \eta^n + c_2 \ln \eta \sum_{n=0}^{\infty} a_n \eta^n \tag{3.27}$$

From the boundary condition (3.14), $\frac{d\theta_1}{d\eta} = 0$ at $\eta = 0$, we get $c_2 = 0$. Eq (3.27) reduces to the following form:

$$\theta_1 = c_1 \sum_{n=0}^{\infty} a_n \eta^n \text{ or } \theta_1 = \sum_{n=0}^{\infty} A_n \eta^n \tag{3.28}$$

Substituting in Eq (3.24), we get,

$$\sum_{n=2}^{\infty} (n)(n-1) A_n \eta^{n-2} + \sum_{n=1}^{\infty} (n) A_n \eta^{n-2} + (M - N\eta^2) Nu_o \sum_{n=0}^{\infty} A_n \eta^n = 0 \tag{3.29}$$

Equating the corresponding coefficients of η powers to zero:

η^{-1}: $A_1 = 0$

η^0 : $2A_2 + 2A_2 + MNu_o A_o = 0 \Rightarrow A_2 = -\frac{MNu_o}{4} A_o$

η^1 : $6A_3 + 3A_3 + MNu_o A_1 = 0 \Rightarrow A_3 = 0$

$$\eta^2: \; 12A_4 + 4A_4 + Nu_o[MA_2 - NA_o] = 0 \Rightarrow A_4 = -\frac{[MA_2 - NA_o]}{16} Nu_o$$

$$\eta^3: 20A_5 + 5A_5 + Nu_o[MA_3 - NA_1] = 0 \;\Rightarrow\; A_5 = 0$$

$$\eta^4: \; 30A_6 + 6A_6 + Nu_o[MA_4 - NA_2] = 0 \Rightarrow A_6 = -\frac{[MA_4 - NA_2]}{36} Nu_o$$

In General:

$$A_n = 0 \text{ for n = odd,}$$

$$= \frac{Nu_o}{n^2}[-MA_{n-2} + NA_{n-4}] \text{ For n= even and n>2}$$

and, $$A_2 = -\frac{MNu_o}{4} A_o \qquad (3.30)$$

OUTER FLUID CALCULATIONS:

For the second fluid (outer fluid), the energy equation (3.13) becomes:

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T_2}{\partial r}\right) = \frac{u_2}{\alpha_2} \frac{\alpha_2 (T_w - T) Nu_o}{\{(\rho c_p)_2 R_o^2 [\frac{(\rho c_p)_1}{(\rho c_p)_2}\frac{R_i^2}{R_o^2} u_{m,1} + (1 - \frac{R_i^2}{R_o^2}) u_{m,2}]\}} \qquad (3.31)$$

$$u_2 = \frac{hr_o^2}{4\mu_2} - \frac{h}{4\mu_2} r^2 \text{ (From Eq 3.39)}$$

Equation (3.31) simplifies to:

$$\frac{1}{\eta}\frac{d}{d\eta}\left[\eta\frac{d\theta_2}{d\eta}\right] + [P - Q\eta^2] Nu_o \theta_2 = 0 \qquad (3.32)$$

$$P = \frac{\frac{g_2}{4}}{\left(\eta_i^2 v_{m,1} + \omega(1 - \eta_i^2) v_{m,2}\right)} \text{ And } Q = \frac{\frac{g_2}{4}}{\left(\eta_i^2 v_{m,1} + \omega(1 - \eta_i^2) v_{m,2}\right)}$$

Equation (3.32) is solved by the Method of Frobenius:

Let $\theta_2 = \eta^s \sum_{n=0}^{\infty} b_n \eta^n$

Substituting the value of θ_2 in Eq (3.32) we get:

$$\sum_{n=0}^{\infty}(n+s)(n+s-1)b_n\eta^{s+n-2} + \sum_{n=0}^{\infty}(n+s)b_n\eta^{n+s-2} + (M - N\eta^2)Nu_o \sum_{n=0}^{\infty} b_n\eta^{n+s} = 0 \tag{3.33}$$

Equating the coefficient of the lowest power of η which is the coefficient of η^{s-2} in Eq (3.33) to zero, we arrive at the following indicial equation:

$$s(s-1)b_o + sb_o = 0 \tag{3.34}$$

which leads to s=0. (Double root)

Thus, the solution in our case is given by:

$$\theta_2 = c_3(\theta_2)_{s=0} + c_4\left(\frac{d\theta_2}{ds}\right)_{s=0} \tag{3.35}$$

$$\theta_2 = c_3 \sum_{n=0}^{\infty} b_n\eta^n + c_4 \ln\eta \sum_{n=0}^{\infty} b_n\eta^n \tag{3.36}$$

From the boundary condition (3.15), (3.16), (3.17), we get,

$$\theta_2(\eta = 1) = 0 \tag{3.37}$$

$$\theta_1(\eta = \eta_i) = \theta_2(\eta = \eta_i) \tag{3.38}$$

$$k_1 \frac{d\theta_1}{d\eta} = k_2 \frac{d\theta_2}{d\eta} \text{ At } \eta = \eta_i \tag{3.39}$$

Equation (3.36) can be rewritten as:

$$\theta_2 = \sum_{n=0}^{\infty} B_n\eta^n + \sum_{n=0}^{\infty} D_n \ln\eta\eta^n \tag{3.40}$$

Substituting in Eq (3.33), we get,

$$\sum_{n=2}^{\infty}(n)(n-1)B_n\eta^{n-2} + \sum_{n=2}^{\infty}(n)(n-1)D_n \ln\eta\eta^{n-2} + \sum_{n=1}^{\infty}(n)D_n\eta^{n-2} + \sum_{n=0}^{\infty}D_n\eta^{n-2}(n-1) +$$

$$\sum_{n=1}^{\infty}(n)B_n\eta^{n-2} + \sum_{n=0}^{\infty}D_n\eta^{n-2} + \sum_{n=1}^{\infty}(n)D_n \ln\eta\eta^{n-2} + \sum_{n=o}^{\infty}(P)B_n\eta^n Nu_o + \sum_{n=0}^{\infty}(D_n)PNu_o\eta^n \ln\eta +$$

$$-\sum_{n=0}^{\infty}(B_n)QNu_o\eta^{n+2} - \sum_{n=0}^{\infty}(D_n)QNu_o\eta^{n+2}\ln\eta = 0 \tag{3.41}$$

Equating the corresponding coefficients of η powers to zero:

$$\eta^{-1} : 2D_1 + B_1 = 0$$

$$\eta^0 : 4B_2 + 4D_2 + PB_oNu_o = 0 \tag{3.42}$$

$$\eta^1 : 9B_3 + 6D_3 + PB_1Nu_o = 0$$

$$\eta^2 : 16B_4 + 8D_4 + Nu_o[PB_2 - QB_o] = 0$$

....

....

Equating the coefficients of Logarithmic terms:

$$\eta^{-1}\ln\eta : D_1 = 0$$

$$\eta^0 \ln\eta : 4D_2 + PD_oNu_o = 0 \tag{3.43}$$

$$\eta^1 \ln\eta : 9D_3 + PD_1Nu_o = 0$$

$$\eta^2 \ln\eta : 16D_4 + (PD_2 - QD_o)Nu_o = 0$$

From Eq (3.42) and Eq (3.43), we can deduce that all the odd D_i's are equal to zero.

In General, the values of D_i's are:

$$D_{2n+1} = 0,$$

$$D_2 = -\frac{PNu_0}{4} D_0$$

$$D_{2n} = \frac{Nu_0}{(2n)^2}\left[QD_{2n-4} - PD_{2n-2}\right] \qquad (3.44)$$

where n varies from 0 to ∞ and n is an integer.

When the use the fact that all the odd D_i's are equal to zero from Eq (4.44) in the set of Equation (3.42), we can deduce that all the odd B_i's are equal to zero.

In General:

$$B_{2n+1} = 0$$

$$B_{2n} = \frac{Nu_0}{(2n)^2}\left[QB_{2n-4} - PB_{2n-2}\right] - \frac{1}{n} D_{2n} \qquad (3.45)$$

$$B_2 = \frac{PNu_0}{4}\left(D_0 - B_0\right)$$

where n varies from 0 to ∞ and n is an integer.

Using the Boundary Condition (3.15)

$$\theta_1(\eta = \eta_i) = \theta_2(\eta = \eta_i)$$

$$\sum_{n=0}^{\infty} A_{2n}\eta_i^{2n} = \sum_{n-0}^{\infty} B_{2n}\eta_i^{2n} + \sum_{n-0}^{\infty} D_{2n}\eta_i^{2n}\ln\eta_i \qquad (3.46)$$

Using the Boundary Condition (3.16)

$$k_1\frac{d\theta_1}{d\eta} = k_2\frac{d\theta_2}{d\eta}$$

$$\gamma\sum_{n=1}^{\infty}(2n)A_{2n}\eta_i^{2n-1} = \sum_{n=1}^{\infty}(2n)B_{2n}\eta_i^{2n-1} + \sum_{n=1}^{\infty}(2n)D_{2n}\eta_i^{2n-1}\ln\eta_i + \sum_{n=0}^{\infty} D_{2n}\eta_i^{2n-1}$$

(3.47)

From Eq (3.30), Eq (3.44), Eq (3.45), we realize that all $A_{2i}'s$ are a function of A_0 only, all $D_{2i}'s$ are a function of D_0 only and all $B_{2i}'s$ are a function of D_0 and B_0 only. Equation (3.46) and Eq (3.47) are now solved to get D_0 and B_0 in terms of the third unknown which is A_0.

Now, the wall boundary condition $\theta_2(\eta = 1) = 0$ to get the following equation (and arrive at the following equation:

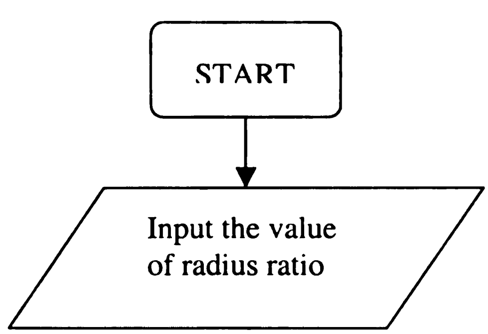
$$0 = \sum_{n=0}^{\infty} B_{2n}\eta^{2n} + \sum_{n=0}^{\infty} D_{2n}\eta^{2n} \ln\eta \text{ where } \eta = 1$$

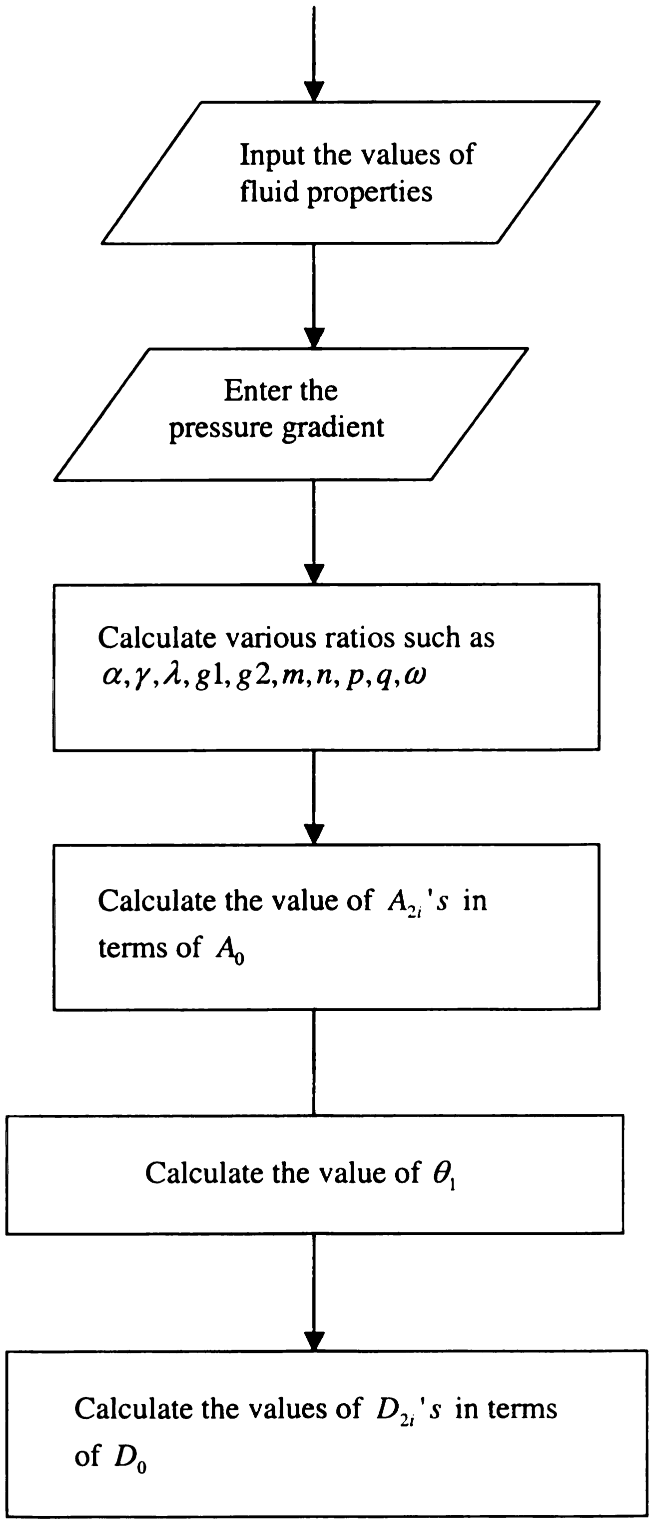
(3.48)

The values of D_0 and B_0 are known in terms of A_0. Thus Eq (3.48) takes the form:

$$A_0[equation\,*] = 0 \tag{3.49}$$

The equation* in Eq (3.49) is solved to get the Nusselt number in terms of all the two-fluids parameters. The value of the Nusselt number varies depends on the number of terms in the summation. However, as we later observe, the value converges after 10 terms and the value doesn't change appreciably upon increasing the number of terms than that. Following is the flowchart to solve the Nusselt number of a two-fluid system which is thermally and hydrodynamically fully developed and assuming that both the fluids are Newtonian.




Input the values of fluid properties
Enter the pressure gradient
Calculate various ratios such as $\alpha, \gamma, \lambda, g1, g2, m, n, p, q, \omega$
Calculate the value of A_{2i}'s in terms of A_0
Calculate the value of θ_1
Calculate the values of D_{2i}'s in terms of D_0

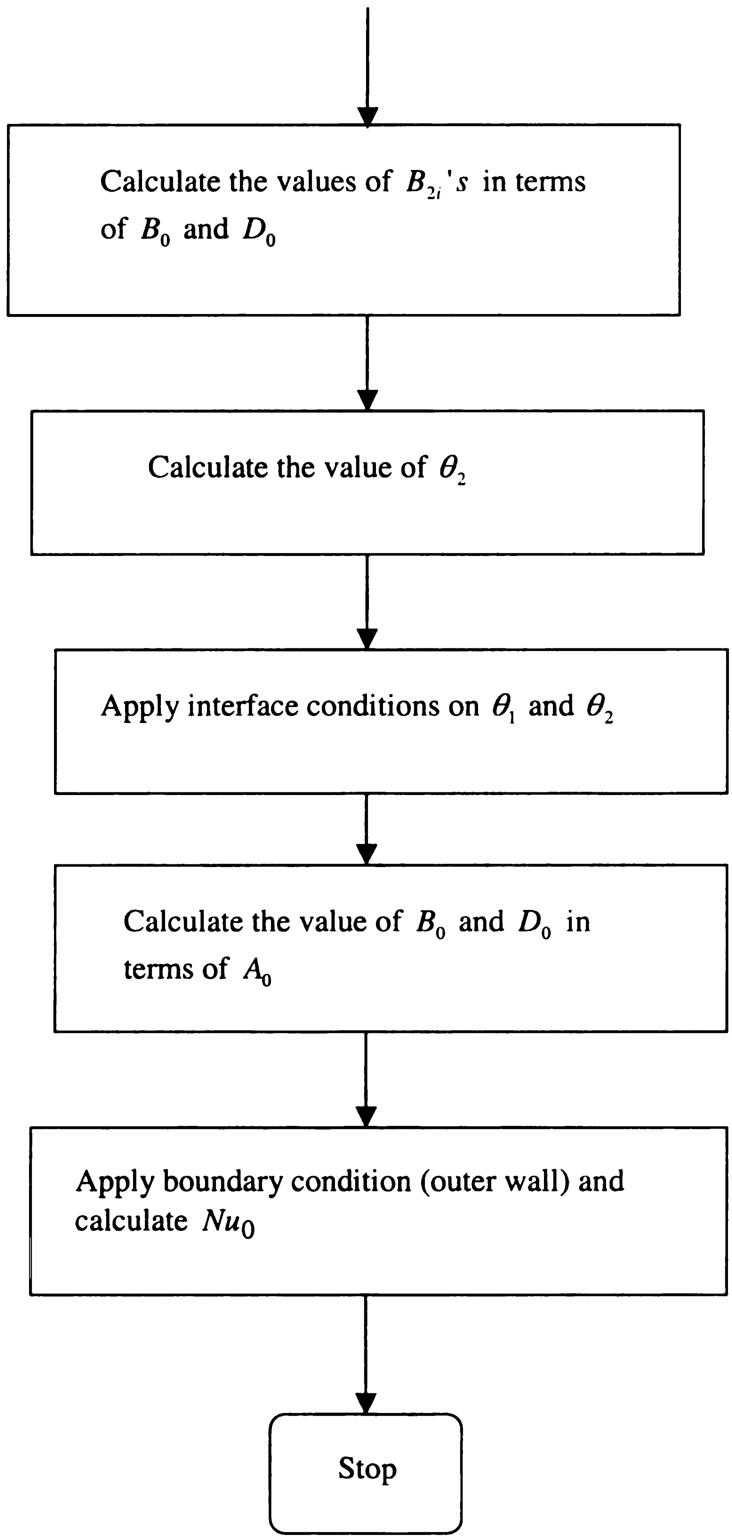


Figure 3.2: Flowchart to solve the Nusselt number for the two-fluid case

The point to be noted here is that the solution is not complete as yet. This is because we still have to find the temperature profiles of the two-fluids. And for that, we need to compute the values of A_0, B_0 and D_0. Since B_0 and D_0 are functions of A_0, the two temperature profiles can be evaluated once A_0 is known.

Consider Eq (3.12)

$$\frac{\partial T}{\partial z} = \frac{\alpha_2 (T_w - T) Nu_o}{\{R_o^2 [\frac{(\rho c_p)_1}{(\rho c_p)_2} \frac{R_i^2}{R_o^2} u_{m,1} + (1 - \frac{R_i^2}{R_o^2}) u_{m,2}]\}}$$

Equation (3.12) can be reduced to the following form:

$$\frac{\partial T}{\partial z} = \frac{\alpha_2 (T_w - T) Nu_o}{\{R_o^2 u_m [A]\}} \tag{3.50}$$

where $$A = \left(\frac{\rho c_{p,1}}{\rho c_{p,2}} \frac{u_{m,1}}{u_m} \frac{R_i^2}{R_o^2} + \left(1 - \frac{R_i^2}{R_o^2}\right) \frac{u_{m,2}}{u_m} \right) = \omega \eta_i^2 v_{m,1} + (1 - \eta_i^2) v_{m,2} \tag{3.51}$$

Let $\theta = T - T_w$ and $\zeta = \frac{z}{R_o}$

Equation (3.50) reduces to the following form:

$$\frac{\partial \theta}{R_o \partial \zeta} + \frac{Nu_0 \theta \alpha_2}{u_m R_o^2 A} = 0 \tag{3.52}$$

From Eq (3.28), $\theta_1 = \sum_{n=0}^{\infty} A_n \eta^n$

$$\theta_1 = A_0 + A_2 \eta^2 + A_4 \eta^4 + A_6 \eta^6 + \dots\dots\dots\dots$$

$$\theta_1 = A_0 (1 + \frac{A_2}{A_0} \eta^2 + \frac{A_4}{A_0} \eta^4 + \dots\dots\dots) \tag{3.53}$$

Equation (3.53) can also be written as:

$$\theta_1 = A_0 Y \text{ Where } Y = (1 + \frac{A_2}{A_0}\eta^2 + \frac{A_4}{A_0}\eta^4 + ...) \tag{3.54}$$

Substituting Eq (3.54) in Eq (3.52), we get the following equation:

$$\frac{Y}{R_o}\frac{\partial A_0}{\partial \zeta} + \frac{Nu_0 A_0 Y \alpha_2}{u_m R_o^2 A} = 0 \tag{3.55}$$

Or,

$$\frac{\partial A_0}{\partial \zeta} + \frac{Nu_0 A_0 \alpha_2}{u_m R_o A} = 0 \tag{3.56}$$

Define a new non-dimensional quantity called the Peclet number,

$$Pe = \frac{u_m . 2R_o}{\alpha_2} \tag{3.57}$$

Equation (3.56) reduces to the following form:

$$\frac{\partial A_0}{\partial \zeta} + \frac{2Nu_0 A_0}{PeA} = 0 \tag{3.58}$$

$$A_0 = C * \exp\left(\frac{-2Nu_0 \zeta}{PeA}\right) \tag{3.59}$$

Assume an inlet mean temperature of $\theta_{m,0}$ at the point where the flow gets hydrodynamically fully developed (z=0). The mean temperature at the point $\zeta = 0$ is

$$\theta_{m,0} = \frac{1}{\pi \eta_i^2 v_{m,1}} \int_0^{\eta_i} \theta_1 v_1 2\pi \eta d\eta \tag{3.60}$$

Using the value of $A_0 = C * \exp\left(\frac{-2Nu_0 \zeta}{PeA}\right)$ from Eq (3.59) in Eq (3.60),

$$C = \frac{\theta_{m,0}}{\frac{2}{\eta_i^2 v_{m,1}} \int_0^{\eta_i} \left(1 + \frac{A_2}{A_0}\eta^2 + \frac{A_4}{A_0}\eta^4 + ...\right)\left(a - b\eta^2\right) d\eta} \tag{3.61}$$

where, the velocity of the first fluid (inner fluid) is given:

$$v_1 = a - b\eta^2, \quad a = \frac{g_2}{4} + \frac{1}{4} g_1 \eta_i^2 - \frac{1}{4} g_2 \eta_i^2, b = \frac{g_1}{4}$$

Thus, the value of C is calculated from Eq (3.61) and the value of θ_1 is:

$$\theta_1 = A_0 + A_2\eta^2 + A_4\eta^4 + A_6\eta^6 + \dots\dots\dots \tag{3.62}$$

$A_n = 0$ For n = odd,

$= \frac{Nu_o}{n^2}[-MA_{n-2} + NA_{n-4}]$ For n= even and n>2

and, $$A_2 = -\frac{MNu_o}{4} A_o$$

$$M = \frac{\frac{\alpha_i}{4}(g_1 - g_2)\eta_i^2 + \frac{\alpha_i g_2}{4}}{\left(\eta_i^2 v_{m,1}\omega + (1-\eta_i^2)v_{m,2}\right)}, N = \frac{\frac{\alpha_i}{4} g_1}{\left(\eta_i^2 v_{m,1}\omega + (1-\eta_i^2)v_{m,2}\right)}$$

And $A_0 = C * \exp\left(\frac{-2Nu_0\zeta}{PeA}\right)$ where,

$$C = \frac{\theta_{m,0}}{\frac{2}{\eta_i^2 v_{m,1}} \int_0^{\eta_i} \left(1 + \frac{A_2}{A_0}\eta^2 + \frac{A_4}{A_0}\eta^4 + \dots\dots\dots\dots\right)\left(a - b\eta^2\right) d\eta}$$

RESULTS AND VALIDATION

The Nusselt numbers were calculated at three locations, at the outer wall, at the interface for the first fluid and at the interface for the second fluid. The two fluids considered for the sake of analysis were engine oil at the inside and water at the outside. The properties of oil and water are taken from Table 2.1.

$$\upsilon = \frac{k_1}{k_2} = 0.236$$

$$\lambda = \frac{\mu_1}{\mu_2} = 568421$$

$$\omega = \frac{(\rho c_p)_1}{(\rho c_p)_2} = 0.4051 \tag{3.63}$$

$$\alpha_i = \frac{\alpha_2}{\alpha_1} = 1.712$$

The Nusselt number at the outer wall (Nu_0) was calculated for different values of radius ratios varying from 0 to 1 following the steps outlined in the flowchart in Figure 3.1. Because there was no existing solution for Nusselt number in a two-fluid case, the results were compared with the Nusselt number at the outer wall for a single fluid and constant temperature boundary condition. The value of Nu_0 was evaluated at various values of radius ratio ($\eta_i = R_i / R_o$) close to zero. As the radius ratio approaches zero, it implies that the only major fluid in the system is the second fluid. Thus, the Nusselt number at the outer wall must approach the Nusselt number's value of the single fluid case. The Nusselt number at the outer wall of a tube with a single fluid flowing in it and constant wall temperature boundary condition is 3.65679 [12]. The value of Nusselt number Nu_0 in the two-fluid case was 3.6575 for $\eta_i = 0.01$ and 3.6568 for $\eta_i = 0.001$. Thus, these results show that the Nusselt number expression for the two-fluid system is right as it approaches the value of that of a single fluid system with similar boundary conditions.

The system also approaches the single-fluid system, if the radius ratio approaches 1. If η_i is close to 1, it would imply that the only component present in the pipe is the inner fluid (oil). Thus, the Nusselt number at the outer wall must converge to 3.65679. Nu_0 Was evaluated for radius ratios close to 1. The value of Nu_0 was 0.8648 for η_i =1. However it must be noted that Nu_0 is evaluated with respect to the outer fluid.

$$Nu_0 = \frac{q_w''}{(T_w - T_m)} \cdot \frac{2R_o}{k_2}.$$

Nu_0 must now be calculated with the inner fluid because if η_i =1, there is no outer fluid in the system. Thus 0.8648 is now scaled with respect to the inner fluid (the only fluid in the system).

$$Nu_0 = \frac{q_w''}{(T_w - T_m)} \cdot \frac{2R_o}{k_1}$$

Thus, $$Nu_0 = 0.8648 * \frac{k_2}{k_1}$$

$$Nu_0 = 0.8648 * \frac{0.613}{0.145} = 3.656$$

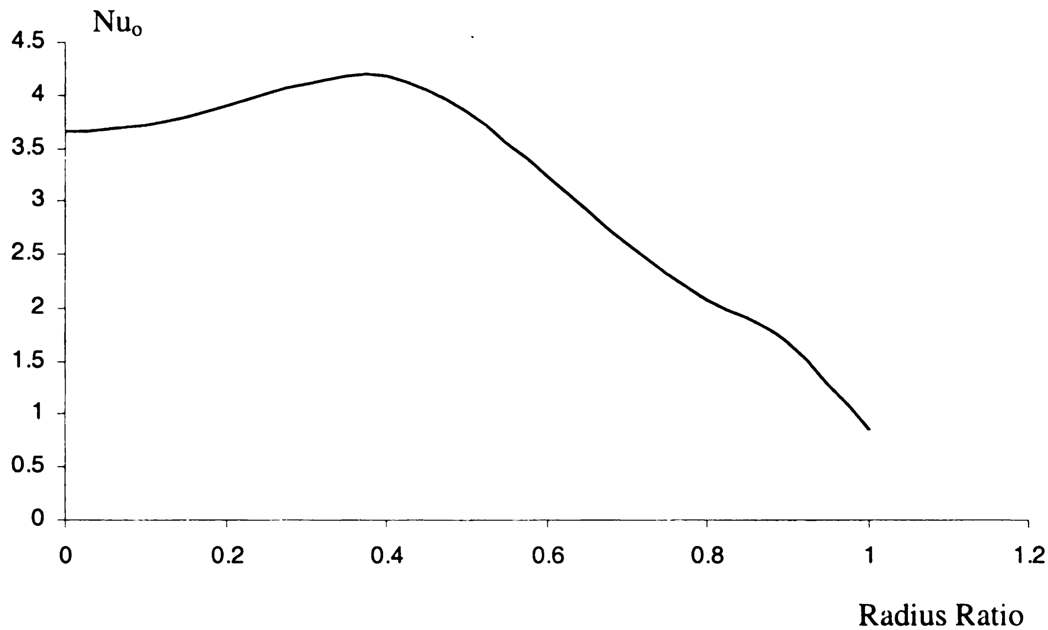


Figure 3.3: Nu_0 vs. η_i

From Figure 3.3, it can be observed that Nu_o increases as radius ratio increases to a value of 0.4 and then drops. To understand this phenomenon, the total energy in the fluid was plotted as a function of the radius ratio. Figure 3.4 shows the energy of the inner fluid, energy in the outer fluid and the total energy in the fluid as a function of the radius ratio. Total e denotes the energy in the entire fluid and e1 and e2 denote the energy in the first and the second fluid respectively. It can be seen that the total energy in the fluid increases till the radius ratio increases to a value of 0.4. The nusselt number at the outer wall is dependent on the heat transfer that in turn is dependent on the total energy in the fluid. Thus, the Nusselt number at the outer wall increases till the radius ratio of 0.4 and then drops, showing the same pattern as the total energy in the fluid.

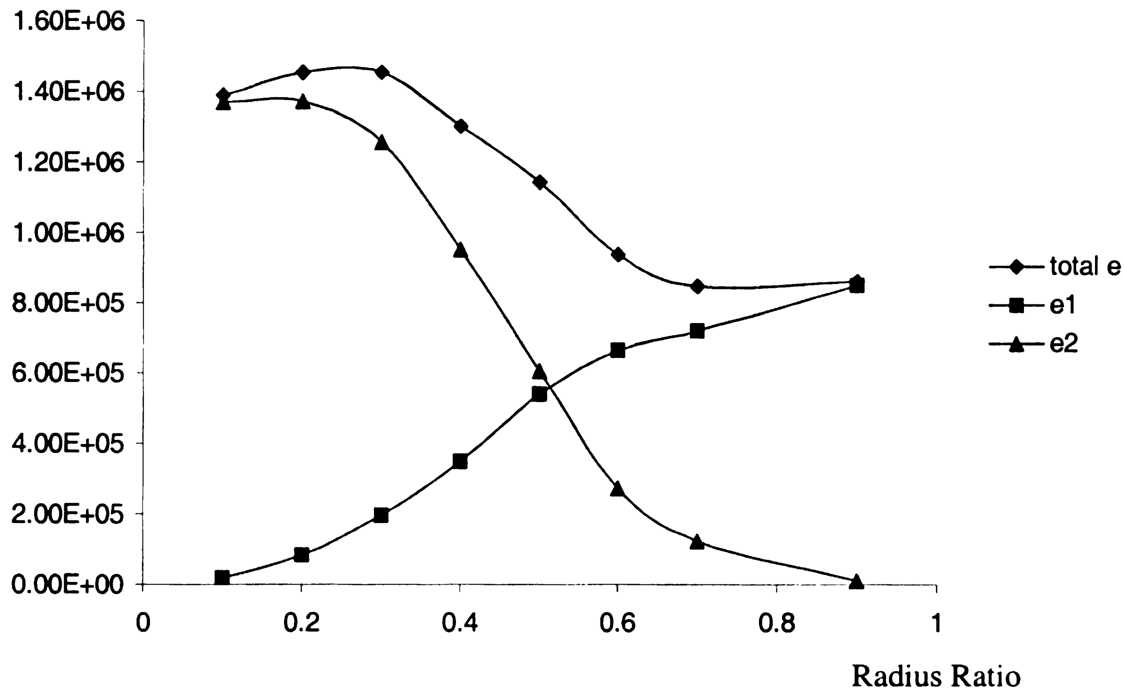


Figure 3.4: Energy in the fluids vs. η_i

The next step was calculating the Nusselt number at the interface for the inner fluid.

$$Nu_{i,1} = \frac{q_i}{(T_i - T_{m,1})} \frac{2r_i}{k_1} \tag{3.64}$$

$$q_i = k \frac{dT_1}{dr} |_{r=r_i}$$

T_i =Temperature of the interface

$T_{m,1}$ = Mean Temperature of the inner fluid

The Nusselt number at the interface is now verified again with the Nusselt number at the outer wall for the single fluid case. If the radius ratio goes to 1, $\eta_i = 1$, the interface is now at the wall, and thus $Nu_{i,1}$ should be same as that of the wall Nusselt number for the single fluid. The following chart shows how the radius ratio affects the Nusselt number at the interface.

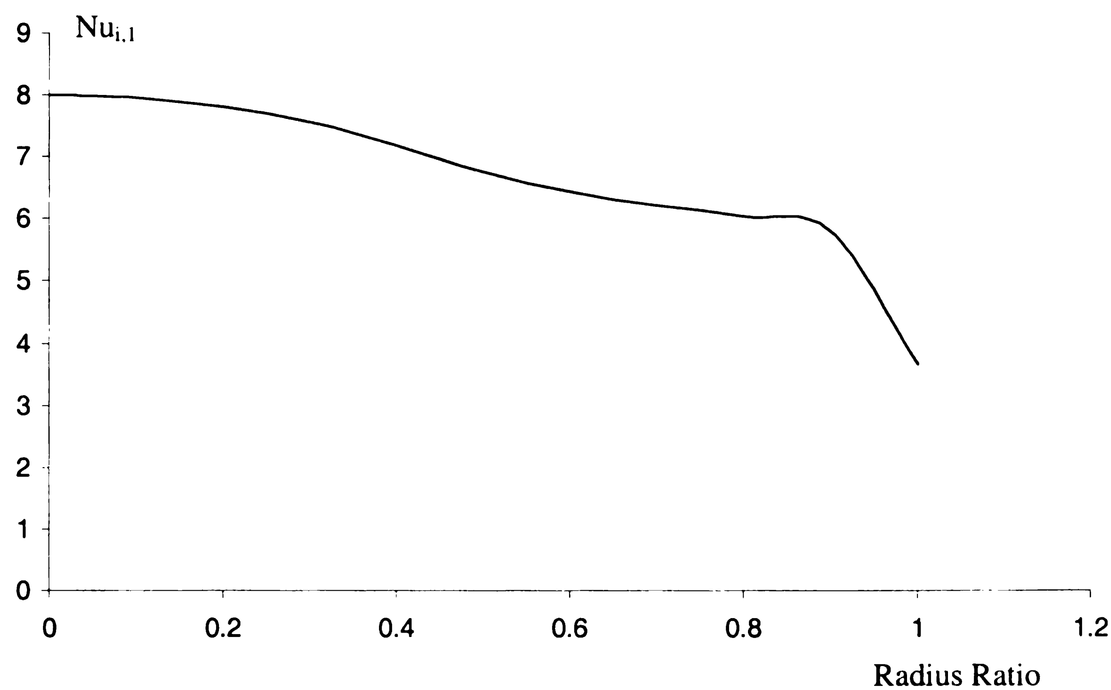


Figure 3.5: $Nu_{i,1}$ vs. η_i

As can be observed from Figure 3.5, the value of $Nu_{i,1}$ is 8 as the radius ratio approaches 0. This is because as the radius ratio is close to 0, it would mean that the inner fluid flow is same as that of a plug flow. As can be seen in literature, the Nusselt number for a plug flow type problem is 8. And as the radius ratio approached 1, the value of $Nu_{i,1}$ is exactly 3.657 that again validate our expression for the Nusselt number at the interface.

The next step that was done was to make the two fluids the same. If both the inner and the outer fluid are the same, the Nusselt number at the outer wall must be equal to 3.6567 for all values of radius ratio. The value of Nu_0 approached 3.6567 irrespective of the value of radius ratio thus validating the results. The following plots show the temperature variance with the Radial position.

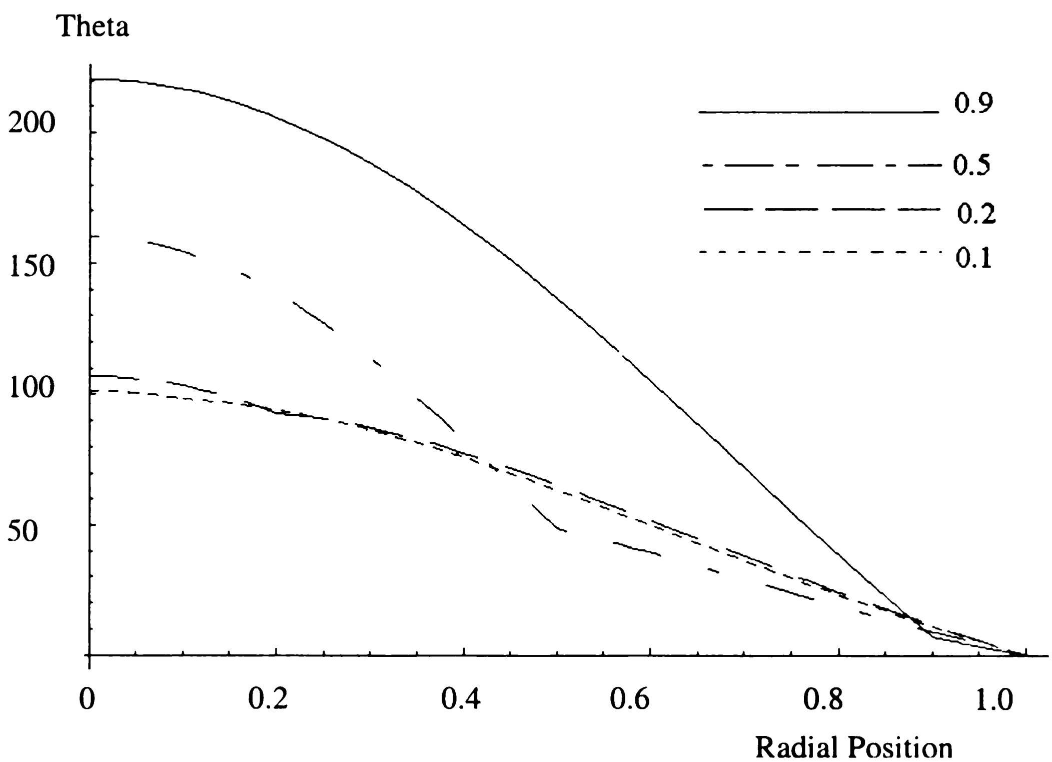


Figure 3.6: Temperature vs. radial position for different values of η_i

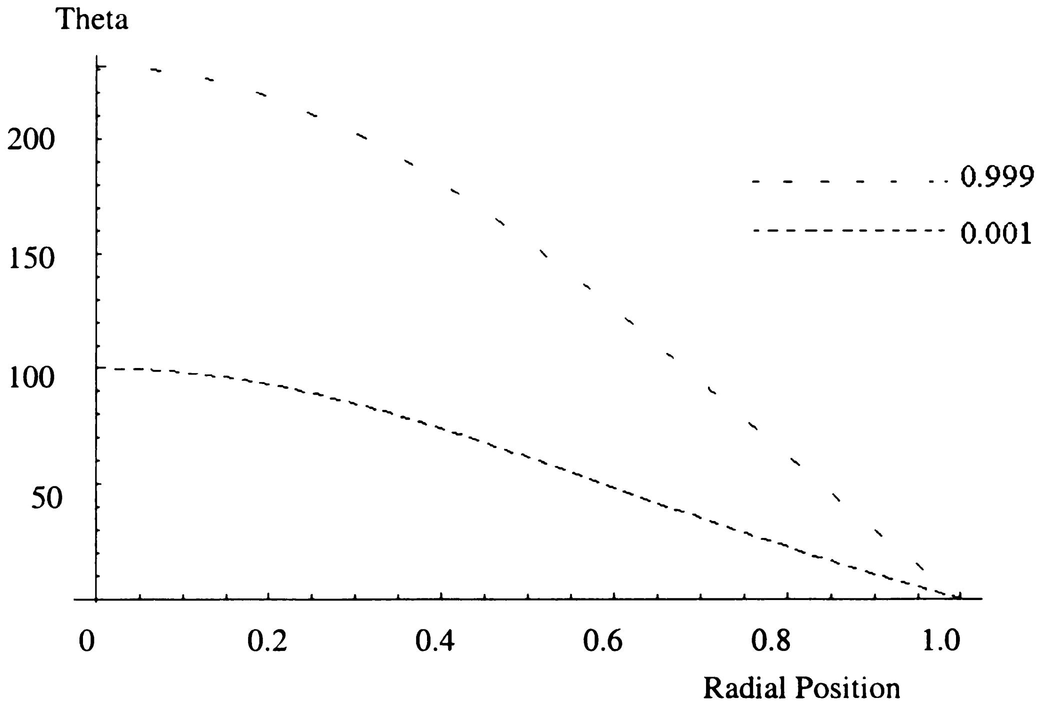


Figure 3.7: Temperature vs. radial position for extreme values of η_i

CHAPTER 4

CONVECTIVE BOUNDARY CONDITION

One of our initial assumptions in solving this problem was that the flow is thermally fully developed.

$$\frac{\partial}{\partial z}\left(\frac{T_w - T}{T_w - T_m}\right) = 0 \tag{4.1}$$

Equation (4.1) reduces to the following form:

$$\frac{\partial T}{\partial z} = \frac{dT_w}{dz} - \frac{T_w - T}{T_w - T_m}\left(\frac{dT_w}{dz} - \frac{dT_m}{dz}\right) \tag{4.2}$$

From Eq. (3.4) and Eq (3.5), the energy balance on a element of length dz gives,

$$2\Pi R_o q_w(x)dz = \left(\Pi R_i{}^2 u_{m,1}\right)\rho_{f,1}c_{p,1}dT_m + \Pi(R_o{}^2 - R_i{}^2)u_{m,2}\rho_{f,2}c_{p,2}dT_m \tag{4.3}$$

$$\frac{dT_m}{dz} = \frac{2q_w{}'' R_o}{(\rho c_p)_1 R_i^2 u_{m,1} + \left(\rho c_p\right)_2 (R_o^2 - R_i^2)u_{m,2}} \tag{4.4}$$

$$\frac{dT_m}{dz} = \frac{2q_w{}''(x)R_o}{\{(\rho c_p)_2 R_o^2[\frac{(\rho c_p)_1}{(\rho c_p)_2}\frac{R_i^2}{R_o^2}u_{m,1} + (1 - \frac{R_i^2}{R_o^2})u_{m,2}]\}} \tag{4.5}$$

The convective boundary condition implies that there is a fluid outside the pipe whose convective heat transfer coefficient is denoted by h_∞ [17]. Therefore, the boundary condition is,

$$q_w{}'' = h_\infty(T_\infty - T_w) = h_0(T_w - T_m) \tag{4.6}$$

Equation (4.6) reduces to

$$Nu_\infty(T_\infty - T_w) = Nu_0(T_w - T_m)$$

where,

$$Nu_\infty = \frac{h_\infty . 2R_o}{k_2} \text{ and } Nu_0 = \frac{h_0 . 2R_o}{k_2} \tag{4.7}$$

From Eq. (4.7) we can arrive at the value of the wall-temperature gradient in terms of the mean temperature and the two Nusselt numbers.

$$\frac{dT_w}{dz} = \frac{Nu_0}{Nu_0 + Nu_\infty} \frac{dT_m}{dz} \tag{4.8}$$

Substituting the above value from Eq (4.8) in Eq (4.2), we get,

$$\frac{\partial T}{\partial z} = \frac{Nu_0}{Nu_0 + Nu_\infty} \frac{dT_m}{dz} + \frac{T_w - T}{T_w - T_m} \frac{Nu_\infty}{Nu_0 + Nu_\infty} \frac{dT_m}{dz} \tag{4.9}$$

From the definition of the heat transfer at the wall,

$$q_w'' = h_\infty (T_\infty - T_w) \Rightarrow T_w = T_\infty - \frac{q_w''}{h_\infty} \tag{4.10}$$

Substituting this value in Eq (4.10)

$$\frac{\partial T}{\partial z} = \frac{Nu_0}{Nu_0 + Nu_\infty} \frac{dT_m}{dz} + \frac{T_\infty - \frac{q_w''}{h_\infty} - T}{T_w - T_m} \frac{Nu_\infty}{Nu_0 + Nu_\infty} \frac{dT_m}{dz} \tag{4.11}$$

$$\frac{q_w''}{h_\infty (T_w - T_m)} = \frac{h_0}{h_\infty} = \frac{Nu_0}{Nu_\infty} \tag{4.12}$$

$$\frac{\partial T}{\partial z} = \frac{Nu_0}{Nu_0 + Nu_\infty} \frac{dT_m}{dz} - \frac{Nu_0}{Nu_\infty} \frac{Nu_\infty}{Nu_0 + Nu_\infty} \frac{dT_m}{dz} + \frac{T_\infty - T}{T_w - T_m} \frac{Nu_\infty}{Nu_0 + Nu_\infty} \frac{dT_m}{dz} \tag{4.13}$$

$$\frac{\partial T}{\partial z} = \frac{T_\infty - T}{T_w - T_m} \frac{Nu_\infty}{Nu_0 + Nu_\infty} \frac{dT_m}{dz} \tag{4.14}$$

From Eq (4.5),

$$\frac{dT_m}{dz}=\frac{2q_w''(x)R_o}{\{(\rho c_p)_2R_o^2[\frac{(\rho c_p)_1}{(\rho c_p)_2}\frac{R_i^2}{R_o^2}u_{m,1}+(1-\frac{R_i^2}{R_o^2})u_{m,2}]\}}=\frac{2q_w''}{(\rho c_p)_2R_ou_mA}$$

where, $A=\frac{(\rho c_p)_2}{(\rho c_p)_1}\frac{R_1^2}{R_o^2}\frac{u_{m,1}}{u_m}+(1-\frac{R_1^2}{R_o^2})\frac{u_{m,2}}{u_m}$

$$\frac{\partial T}{\partial z}=\frac{T_\infty-T}{T_w-T_m}\frac{Nu_\infty}{Nu_0+Nu_\infty}\frac{2q_w''}{(\rho c_p)_{II}R_ou_mA} \tag{4.15}$$

From the definition of heat transfer at the wall,

$$q_w''=h_0(T_w-T_m) \tag{4.16}$$

Using the expression from Eq (4.16) in Eq (4.15), we get,

$$\frac{\partial T}{\partial z}=(T_\infty-T)\frac{Nu_\infty}{Nu_0+Nu_\infty}\frac{2h_0}{(\rho c_p)_{II}R_ou_mA} \tag{4.17}$$

$$h_0=Nu_0.k_2/2R_o \tag{4.18}$$

So, from Eq (4.17) and Eq (4.18), we get,

$$\frac{\partial T}{\partial z}=(T_\infty-T)\frac{Nu_\infty Nu_0}{Nu_0+Nu_\infty}\frac{\alpha_2}{R_o{}^2u_mA} \tag{4.19}$$

The energy equation for the flow is,

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right)=\frac{u}{\alpha}\frac{\partial T}{\partial z} \tag{4.20}$$

Using the value of $\frac{\partial T}{\partial z}$ from Eq (4.19) in Eq (4.20),

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right)=\frac{u}{u_m}\frac{\alpha_2}{\alpha}(T_\infty-T)\frac{Nu_0Nu_\infty}{Nu_0+Nu_\infty}\frac{1}{R_o^2A} \tag{4.21}$$

Non-dimensionalising Eq (4.21), $\theta = T - T_\infty$ and $\eta = \dfrac{r}{R_o}$, we get the following Eq (4.22) for the inner fluid.

$$\frac{1}{\eta}\frac{d}{d\eta}\left[\eta\frac{d\theta_1}{d\eta}\right] + [M - N\eta^2]Nu\ \theta_1 = 0 \tag{4.22}$$

where
$$M = \frac{\frac{\alpha_i}{4}(g_1 - g_2)\eta_i^2 + \frac{\alpha_i g_2}{4}}{\left(\eta_i^2 v_{m,1}\omega + (1-\eta_i^2)v_{m,2}\right)} \tag{4.23}$$

And
$$N = \frac{\frac{\alpha_i}{4}g_1}{\left(\eta_i^2 v_{m,1}\omega + (1-\eta_i^2)v_{m,2}\right)} \tag{4.24}$$

$$Nu = \frac{Nu_0 Nu_\infty}{Nu_0 + Nu_\infty}$$

The above set of equations (4.22)-(4.24) are same as that in the constant temperature case except that Nu_0 in Eq (3.19)-(3.22) is replaced with Nu.

Similarly, for the outer fluid, we get the following Eq (4.25)

$$\frac{1}{\eta}\frac{d}{d\eta}\left[\eta\frac{d\theta_2}{d\eta}\right] + [P - Q\eta^2]Nu\ \theta_2 = 0 \tag{4.25}$$

$$P = \frac{\frac{g_2}{4}}{\left(\eta_i^2 v_{m,1} + \omega(1-\eta_i^2)v_{m,2}\right)} \tag{4.26}$$

$$Q = \frac{\frac{g_2}{4}}{\left(\eta_i^2 v_{m,1} + \omega(1-\eta_i^2)v_{m,2}\right)} \tag{4.27}$$

Since the equations are of the same structure as the constant temperature case, the temperature profiles will also be similar. The boundary and interface conditions are given in Eq (4.28)-Eq (4.30)

At $\eta = 0$,

$$\frac{d\theta_1}{d\eta} = 0 \tag{4.28}$$

At $\eta = \eta_i$,

$$\lambda \frac{d\theta_1}{d\eta} = \frac{d\theta_2}{d\eta}$$

$$\theta_1 = \theta_2 \tag{4.29}$$

At $\eta = 1$,

$$\frac{d\theta_2}{d\eta} + \frac{Nu_\infty}{2}\theta_2 = 0 \tag{4.30}$$

Using the method of Frobenius, the expressions for theta1 and theta2 are given by Eq (4.31) and Eq (4.32).

$$\theta_1 = \sum_{n=0}^{\infty} A_n \eta^n \tag{4.31}$$

$$A_{2n+1} = 0 \text{ For n = odd,}$$

$$A_2 = -\frac{MNu}{4} A_o$$

$$A_{2n} = \frac{Nu_0}{(2n)^2}\left[NA_{2n-4} - MA_{2n-2}\right]$$

$$\theta_2 = \sum_{n=0}^{\infty} B_n \eta^n + \sum_{n=0}^{\infty} D_n \ln \eta\eta^n \tag{4.32}$$

$$D_{2n+1} = 0\,, B_{2n+1} = 0$$

$$D_2 = -\frac{PNu}{4}D_0\,,\quad B_2 = \frac{PNu}{4}(D_0 - B_0)$$

$$D_{2n} = \frac{Nu}{(2n)^2}[QD_{2n-4} - PD_{2n-2}] \quad , B_{2n} = \frac{Nu}{(2n)^2}[QB_{2n-4} - PB_{2n-2}] - \frac{1}{n}D_{2n}$$

or,

$$\theta_1 = \sum_{n=0}^{\infty} A_{2n}\eta^n \text{ And } \theta_2 = \sum_{n=0}^{\infty} B_{2n}\eta^n + \sum_{n=0}^{\infty} D_{2n}\ln\eta\eta^n$$

$$A_2 = -\frac{MNu}{4}A_o\,, D_2 = -\frac{PNu}{4}D_0\,,\quad B_2 = \frac{PNu}{4}(D_0 - B_0)$$

$$A_{2n} = \frac{Nu_0}{(2n)^2}[NA_{2n-4} - MA_{2n-2}]$$

$$B_{2n} = \frac{Nu}{(2n)^2}[QB_{2n-4} - PB_{2n-2}] - \frac{1}{n}D_{2n} \tag{4.33}$$

$$D_{2n} = \frac{Nu}{(2n)^2}[QD_{2n-4} - PD_{2n-2}]$$

Using the Boundary Condition (4.29)

$$\theta_1(\eta = \eta_i) = \theta_2(\eta = \eta_i)$$

$$\sum_{n=0}^{\infty} A_{2n}\eta_i^{2n} = \sum_{n-0}^{\infty} B_{2n}\eta_i^{2n} + \sum_{n-0}^{\infty} D_{2n}\eta_i^{2n}\ln\eta_i \tag{4.34}$$

Using the Boundary Condition (4.29)

$$k_1\frac{d\theta_1}{d\eta} = k_2\frac{d\theta_2}{d\eta}$$

$$\gamma\sum_{n=1}^{\infty}(2n)A_{2n}\eta_i^{2n-1} = \sum_{n=1}^{\infty}(2n)B_{2n}\eta_i^{2n-1} + \sum_{n=1}^{\infty}(2n)D_{2n}\eta_i^{2n-1}\ln\eta_i + \sum_{n=0}^{\infty}D_{2n}\eta_i^{2n-1}$$

(4.35)

From Eq (4.33) we realize that all $A_{2i}'s$ are a function of A_0 only, all $D_{2i}'s$ are a function of D_0 only and all $B_{2i}'s$ are a function of D_0 and B_0 only. Eq (4.34) and Eq (4.35) are now solved to get D_0 and B_0 in terms of the third unknown which is A_0.

Now, the wall boundary condition is solved to get the following equation

$$0=\sum_{n=1}^{\infty} 2n.B_{2n} + \sum_{n=0}^{\infty} D_{2n} + \sum_{n=0}^{\infty} \frac{Nu_\infty}{2} B_{2n} \tag{4.36}$$

The values of D_0 and B_0 are known in terms of A_0. Thus Eq (4.36) takes the form:

$$A_0[equation*] = 0 \tag{4.37}$$

The equation* in Eq (4.37) is solved to get the Nusselt number in terms of all the two-fluids parameters and Nusselt number of the fluid outside the tube.

RESULTS AND VALIDATION

The Nusselt numbers were calculated at the outer wall and at the interface. From Eq. (4.36), it can be seen that the Nusselt number at the outer wall, Nu_0, a function of the Nusselt number of the fluid outside the pipe, Nu_∞. The following table 4.1 shows the dependency of Nu_0 on Nu_∞ for a radius ratio of 0.001 and 1. As can be seen from Table 4.1, the Nusselt number at the outer wall converges to the constant temperature case in the case of very high Nu_∞ and to the constant heat flux case in the case of very low Nu_∞ [17]. The same dependency was observed for other radius ratios also. Figure. 4.1 shows Nusselt number at the outer wall for 3 different radius ratios as a function of Nu_∞.

Table 4.1: Dependency of Nu_0 on Nu_∞

η_i=0.0001		η_i=1	
Nu_∞	Nu_0	Nu_∞	Nu_0
10^6	3.65	10^6	0.86(3.65)
10^3	3.65	10^3	0.86(3.66)
10^2	3.66	10^2	0.87(3.66)
10	3.68	10	0.87(3.68)
1	4.26	1	1.00(3.98)
0.1	4.30	0.1	1.01(4.27)
0.01	4.35	0.01	1.02(4.33)
0.0001	4.36	0.0001	1.03(4.36)
10^{-6}	4.36	10^{-6}	1.03(4.36)

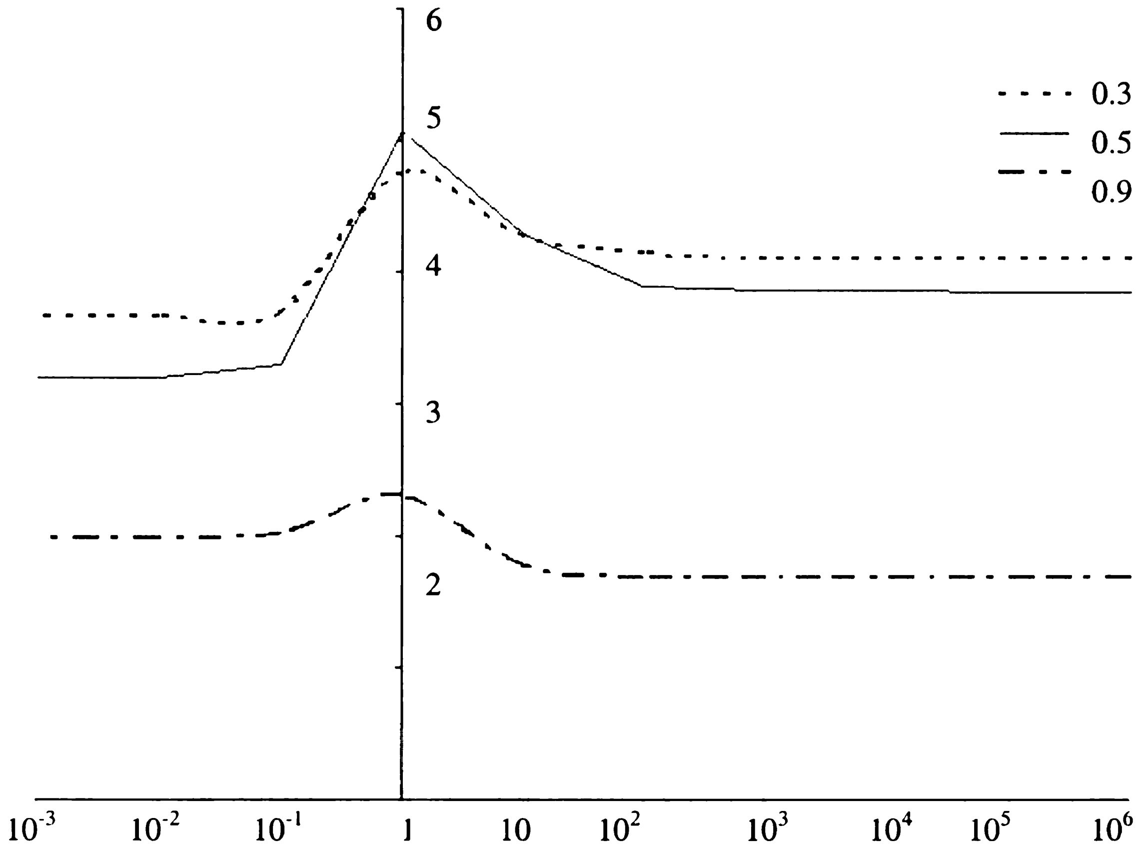


Figure 4.1: Nu_0 vs. Nu_∞ for different radius ratios

The values of A_0, B_0 and D_0 need to be computed. Since B_0 and D_0 are functions of A_0, the two temperature profiles can be evaluated once A_0 is known.

Consider Eq (4.17)

$$\frac{\partial T}{\partial z} = (T_\infty - T)\frac{Nu_\infty}{Nu_0 + Nu_\infty}\frac{2h_0}{(\rho c_p)_{II} R_0 u_m A} = \frac{\alpha_2 (T_\infty - T) Nu}{\{R_o^2 u_m A\}} \qquad (4.38)$$

where $$A=\left(\frac{\rho c_{p,1}}{\rho c_{p,2}}\frac{u_{m,1}}{u_m}\frac{R_i^2}{R_0^2}+\left(1-\frac{R_i^2}{R_0^2}\right)\frac{u_{m,2}}{u_m}\right)=\omega\eta_i^2 v_{m,1}+(1-\eta_i^2)v_{m,2}$$

Let $\theta = T - T_\infty$ and $\zeta = \dfrac{z}{R_0}$

Equation (4.38) reduces to the following form:

$$\frac{\partial\theta}{R_0\partial\zeta}+\frac{Nu\theta\alpha_2}{u_m R_0^2 A}=0 \tag{4.39}$$

From Eq (4.31), $\theta_1 = \sum_{n=0}^{\infty} A_n \eta^n$

$$\theta_1 = A_0 + A_2\eta^2 + A_4\eta^4 + A_6\eta^6 + \dots\dots\dots\dots$$

$$\theta_1 = A_0(1+\frac{A_2}{A_0}\eta^2+\frac{A_4}{A_0}\eta^4+\dots\dots\dots) \tag{4.40}$$

Equation (4.40) can also be written as:

$$\theta_1 = A_0 Y \text{ Where } Y=(1+\frac{A_2}{A_0}\eta^2+\frac{A_4}{A_0}\eta^4+\dots\dots\dots) \tag{4.41}$$

Substituting Eq (4.41) in Eq (4.39), we get the following equation:

$$\frac{Y}{R_0}\frac{\partial A_0}{\partial\zeta}+\frac{NuA_0Y\alpha_2}{u_m R_0^2 A}=0 \tag{4.42}$$

Or,

$$\frac{\partial A_0}{\partial\zeta}+\frac{NuA_0\alpha_2}{u_m R_0 A}=0 \tag{4.43}$$

Define a new non-dimensional quantity called the Peclet number,

$$Pe=\frac{u_m.2R_0}{\alpha_2} \tag{4.45}$$

Equation (4.43) reduces to the following form:

$$\frac{\partial A_0}{\partial \zeta} + \frac{2NuA_0}{PeA} = 0 \tag{4.46}$$

$$A_0 = C * \exp\left(\frac{-2Nu\zeta}{PeA}\right) \tag{4.47}$$

Assume an inlet mean temperature of $\theta_{m,0}$ at the point where the flow gets hydrodynamically fully developed (z=0). The mean temperature at the point $\zeta = 0$ is

$$\theta_{m,0} = \frac{1}{\pi\eta_i^2 v_{m,1}} \int_0^{\eta_i} \theta_1 v_1 2\pi\eta d\eta \tag{4.48}$$

Using the value of $A_0 = C * \exp\left(\frac{-2Nu_0\zeta}{PeA}\right)$ from Eq (4.47) in Eq (4.48),

$$C = \frac{\theta_{m,0}}{\frac{2}{\eta_i^2 v_{m,1}} \int_0^{\eta_i} \left(1 + \frac{A_2}{A_0}\eta^2 + \frac{A_4}{A_0}\eta^4 + \ldots\ldots\ldots\ldots\right)\left(a - b\eta^2\right) d\eta} \tag{4.49}$$

where, the velocity of the first fluid (inner fluid) is given:

$$v_1 = a - b\eta^2, \quad a = \frac{g_2}{4} + \frac{1}{4} g_1 \eta_i^2 - \frac{1}{4} g_2 \eta_i^2, b = \frac{g_1}{4}$$

Thus, the value of C is calculated from Eq (4.49) and the value of θ_1 is:

$$\theta_1 = A_0 + A_2\eta^2 + A_4\eta^4 + A_6\eta^6 + \ldots\ldots\ldots\ldots \tag{4.50}$$

$A_n = 0$ For n = odd,

$= \frac{Nu}{n^2}[-MA_{n-2} + NA_{n-4}]$ For n= even and n>2

and, $A_2 = -\frac{MNu}{4} A_0$

$$M = \frac{\frac{\alpha_i}{4}(g_1 - g_2)\eta_i^2 + \frac{\alpha_i g_2}{4}}{\left(\eta_i^2 v_{m,1}\omega + (1-\eta_i^2)v_{m,2}\right)} \qquad , N = \frac{\frac{\alpha_i}{4} g_1}{\left(\eta_i^2 v_{m,1}\omega + (1-\eta_i^2)v_{m,2}\right)}$$

and $$A_0 = C\exp\left(\frac{-2Nu\zeta}{PeA}\right) \text{ where,}$$

$$C = \frac{\theta_{m,0}}{\frac{2}{\eta_i^2 v_{m,1}} \int_0^{\eta_i} \left(1 + \frac{A_2}{A_0}\eta^2 + \frac{A_4}{A_0}\eta^4 + \ldots\ldots\ldots\ldots\right)\left(a - b\eta^2\right) d\eta}$$

The temperature profiles as shown below have been plotted with the Nusselt number Nu_∞ taken as 10.

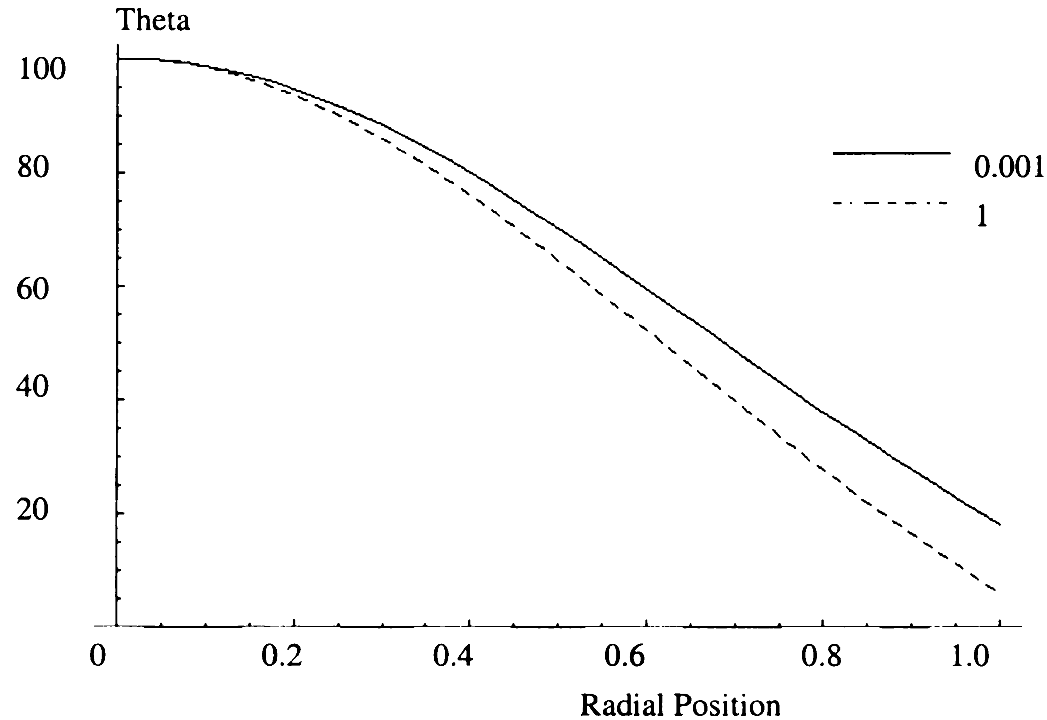


Figure 4.2: Temperature profiles for extreme radius ratios

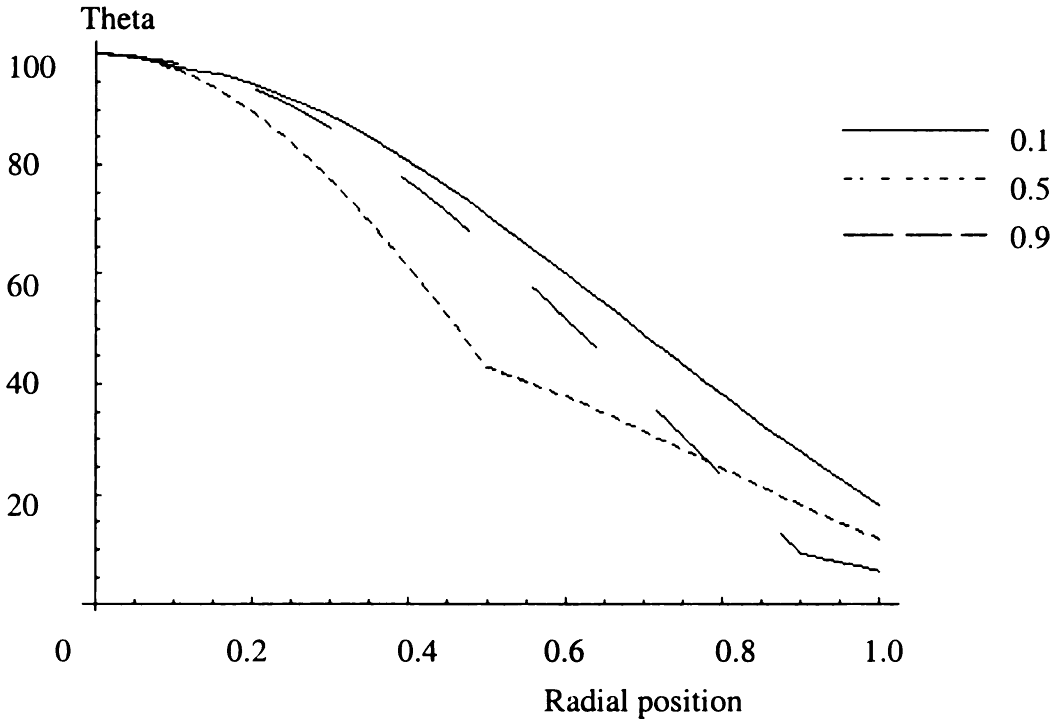


Figure 4.3:Temperature profiles for different radius ratios

CHAPTER 5

SUMMARY AND CONCLUSIONS

Core-annular flow, which has a very important role in oil industry, has been analyzed so far in this work. The oil-water flow has been analyzed for three boundary conditions- constant wall heat flux, constant wall temperature and convective boundary condition. In all the cases analytical solutions have been derived for velocity profiles, temperature profiles and Nusselt numbers. The Nusselt numbers thus evaluated analytically were validated by comparison with the solutions for the single fluid case.

The usefulness of water-lubricated oil flow can be found from the result that the mean velocity of the fluid system increased almost by a factor of 2000 for the same pressure gradient when the water in the system was increased from zero to 10%. Thus, the pumping pressure that is needed to pump the two-fluid flow, which has 10% water, is very less compared to the pumping pressure that would be needed to pump oil alone. The pumping pressure needed further decreases if the water in the pipe is increased. However the problem of dewatering limits the amount of water that can be used.

In the constant heat flux case, the Nusselt number at the outer wall converged to 4.36 in three cases-

(a) the radius ratio approached zero

(b) the radius ratio approached one

(c) the two fluid properties were made equal.

The Nusselt number at the interface for the inner fluid converged to 4.36 when the radius ratio approached one and converged to 8 when the radius ratio converged to 0. This implies that the inner fluid behaves like a slug flow when the inner radius becomes very small.

Similar analysis was performed in constant temperature case. The method of Frobenius was used to get the analytical solutions for temperatures and hence the Nusselt numbers. Similar to the first boundary condition, the Nusselt number at the outer wall approached 3.65 in three cases

(a) the radius ratio approached zero

(b) the radius ratio approached one

(c) the two fluid properties were made equal.

Similar to the constant heat flux case, the Nusselt number at the interface approached 8 when the radius ratio approached 0 and 3.65 when the radius ratio approached 1. Figure 3.3 shows that Nusselt number at the outer wall increases with radius ratio till a value of 0.4 and then keeps decreasing. As can be seen from Figure 3.4, the energy in the two-fluid system rises till the radius ratio reaches a value of 0.4 and then falls. The nusselt number at the outer wall is dependent on the heat transfer that in turn is dependent on the total energy in the fluid. Thus, the oil-water system will have maximum heat transfer at the wall at a radius ratio of 0.4.

The convective boundary condition was different from the above two conditions in that the Nusselt numbers are a function of the Nusselt number of the fluid outside the outer pipe (Nu_∞). As expected, the Nusselt number at the outer wall approached that of the constant heat flux case when the Nusselt number of the fluid outside the pipe (Nu_∞) was made zero (in other words, it was the zero wall heat flux case). The Nusselt number at the outer wall approached the constant temperature case when Nu_{∞} was made very large.

APPENDIX

The software "Mathematica" was used to solve for various parameters. Following are some of the important statements.

The following equations solve for the velocity profiles.

$$\frac{1}{\eta}\, D[\eta\, D[v_1[\eta],\ \eta],\ \eta]$$

$$\text{DSolve}[\% == -g_1,\ v_1[\eta],\ \eta]$$

$$\frac{1}{\eta}\, D[\eta\, D[v_2[\eta],\ \eta],\ \eta]$$

$$\text{DSolve}[\% == -g_2,\ v_2[\eta],\ \eta]$$

$$v1 = -\frac{1}{4}\eta^2 g_1 + \frac{g_2}{4} + \frac{1}{4} g_1 \eta_i^2 - \frac{1}{4} g_2 \eta_i^2 + \text{Log}[\eta_i]\, \eta_i \left(-\frac{1}{2}\lambda g_1 \eta_i + \frac{g_2 \eta_i}{2}\right)$$

$$v2 = \frac{g_2}{4} - \frac{\eta^2 g_2}{4} + \text{Log}[\eta]\, \eta_i \left(-\frac{1}{2}\lambda g_1 \eta_i + \frac{g_2 \eta_i}{2}\right)$$

$$u1 = -\frac{h r^2}{4\mu_1} + \frac{1}{4} h r_i^2 \left(\frac{1}{\mu_1} - \frac{1}{\mu_2}\right) + \frac{h r_o^2}{4\mu_2};$$

$$u2 = -\frac{h r^2}{4\mu_2} + \frac{h r_o^2}{4\mu_2};$$

$$\text{umean1} = \text{Integrate}[u_1\, 2\, \pi\, r,\ \{r,\ 0,\ r_i\}] \,/\, (\pi\, r_i^2)$$

$$\text{umean2} = \text{Integrate}[u_2 * 2 * \pi * r,\ \{r,\ r_i,\ r_o\}] \,/\, (\pi\, (r_o^2 - r_i^2))$$

$$u_m = \frac{1}{\pi\, r_0^2} * \left(\int_0^{r_i} u_1 * 2 * \pi * r\, dr + \int_{r_i}^{r_o} u_2 * 2 * \pi * r\, dr\right)$$

The following set of equations solve for the temperature profiles in the case of constant heat flux boundary condition. The variable I denotes the radius ratio.

$$\frac{1}{\eta}\, D[\eta\, D[\theta_1[\eta],\ \eta],\ \eta]$$

$$\text{DSolve}\left[\% == \frac{v_1}{m},\ \theta_1[\eta],\ \eta\right]$$

$$\frac{1}{\eta}\,\text{D}[\eta\,\text{D}[\theta_2[\eta],\ \eta],\ \eta]$$

$$\text{DSolve}\left[\% == \frac{v_2}{n},\ \theta_2[\eta],\ \eta\right]$$

$$\theta_1 = -\frac{\eta^4\ g_1}{64\ m} - \frac{\eta^2\ (-i^2\ g_1 + 2\,i^2\,\lambda\ \text{Log}[i]\ g_1 - g_2 + i^2 g_2 - 2\,i^2\ \text{Log}[i]\ g_2)}{16\,m}$$

$$T_1[r\_]\ := \theta_1 * 2 * r_o * q / k_1 + T_c$$

$$T_2[r\_]\ := \theta_2 * 2 * r_o * q / k_2 + T_c$$

$$\text{Temp}[r\_]\ := u_1[r]\ * T_1[r] *\ r * 2 * \pi$$

$$\text{Integrate}[\text{Temp}[r],\ \{r,\ 0,\ r_1\}]$$

$$\text{Tmean1}[r\_] = \% / (\pi * r_1{}^2)$$

$$\text{Qinterface} = -k\ (\text{D}[T_1[r],\ r];\ r = i$$

$$r = i;\ \text{Tinterface} = T_1[r\_]$$

$$\text{Nuinterface1} = \frac{\text{Qinterface}}{\text{Tinterface} - \text{Tmean1}[r]}$$

$$\text{Temp2}[r\_]\ := u_2[r]\ * T_2[r] *\ r * 2 * \pi$$

$$\text{Integrate}[\text{Temp2}[r],\ \{r,\ r1,\ ro\}]$$

$$\text{Tmean2}[r\_] = \% / (\pi * (ro^2 - r1^2))$$

$$\text{Tmean} = \frac{\text{Tmean1} * r_1{}^2 * \text{umean1} + \text{Tmean2} * (r_o{}^2 - r_1{}^2) * \text{umean2}}{r_o{}^2 * u_m};$$

$$r = r0;$$

$$\text{Twall} = T_2[r]$$

$$\text{Nuo} = \frac{q_w}{\text{Twall} - \text{Tmean}};$$

The following statements evaluate the nusselt number and the temperatures of the two fluids in the constant temperature boundary condition.

```
A[2] = -m*A[0] *nu0 / 4;

theta1 = A[0] + A[2] * (η^2);

Do[A[2 * i] = (nu0 / (4 * i * i)) * (n * A[2 * i - 4] -
            m* A[2 * i - 2]), {i, 2, 15, 1}];

Do[theta1 = theta1 + A[2 * i] * (η^(2 * i)), {i, 2, 15, 1}];

Dee[2] = -p * Dee[0] * nu0 / 4;

B[2] = -p* (B[0] - Dee[0]) * nu0 / 4;

theta2 = B[0] + B[2] * η * η + Dee[0] * Log[η] +
        Dee[2] * Log[η] * (η^2);

Do[Dee[2 * i] = (nu0 / (4 * i * i)) * (q * Dee[2 * i - 4] -
             p * Dee[2 * i - 2]), {i, 2, 15, 1}];

Do[B[2 * i] = (nu0 / (4 * i * i)) * (q * B[2 * i - 4] - p * B[2 * i - 2]) -
            (1 / i) * Dee[2 * i], {i, 2, 15, 1}];

Do[theta2 = theta2 + B[2 * i] * η^(2 * i) +
            Dee[2 * i] * Log[η] * (η^(2 * i)), {i, 2, 15, 1}];

dertheta1 = D[theta1, η];

dertheta2 = D[theta2, η];

η = 0.5; ni = 0.5;

ω = (884.1 * 1909) / (997 * 4179);

ni = 0.5;

α = 1.471 / 0.859;

γ = 0.145 / 0.613;

λ = 0.486 / (0.855 * (10^-6));
```

```
g1 = 1 * 0.2 * 0.2 / (2 * 2 * 0.486);

g2 = 1 * 0.2 * 0.2 / (2 * 2 * 0.855 * 10^-6);


vm2 = g2 / 4 - (g2 / 8) * (1 + (ni^2));

vm1 = (-g1 / 8 - g2 / 4 + g1 / 4) * (ni^2) + g2 / 4;

m = ((g1 - g2) * ni^2 + g2) * α / (4 * (ni^2 * vm1 * ω + (1 - ni^2) * vm2));

n = α * g1 / (4 * (ni^2 * vm1 * ω + (1 - ni^2) * vm2));

p = g2 / (4 * (ni^2 * vm1 * ω + (1 - ni^2) * vm2));

q = p;

A[0] = 1.0;

eqn = {theta1 - theta2 == 0, γ * dertheta1 - dertheta2 == 0};

Solve[eqn, {B[0], Dee[0]}];

Dee[2] = -p * Dee[0] * nu0 / 4;

B[2] = -p * (B[0] - Dee[0]) * nu0 / 4;

theta2 = B[0] + B[2] * η * η + Dee[0] * Log[η] + Dee[2] * Log[η] * (η^2);

Do[Dee[2 * i] = (nu0 / (4 * i * i)) * (q * Dee[2 * i - 4] -
        p * Dee[2 * i - 2]), {i, 2, 15, 1}];

Do[B[2 * i] = (nu0 / (4 * i * i)) * (q * B[2 * i - 4] - p * B[2 * i - 2]) -
      (1 / i) * Dee[2 * i], {i, 2, 15, 1}];

Do[theta2 = theta2 + B[2 * i] * η^(2 * i) +
     Dee[2 * i] * Log[η] * (η^(2 * i)), {i, 2, 15, 1}];


nu0 = 3.85; theta2

nu0 = 3.86; theta2
```

```
Plot[theta2, {nu0, 3.85, 3.86}]

thetaminlet = 100;

η = .; A[2] = -m * A[0] * nu0 / 4;
theta1 = A[0] + A[2] * (η^2);
Do[A[2 * i] = (nu0 / (4 * i * i)) * (n * A[2 * i - 4] - m * A[2 * i - 2]), {i, 2, 15, 1}];
Do[theta1 = theta1 + A[2 * i] * (η^(2 * i)), {i, 2, 15, 1}]
```

$$v1 = -\frac{1}{4}\eta^2 g1 + \frac{g2}{4} + \frac{1}{4} g1\, ni^2 - \frac{1}{4} g2\, ni^2$$

```
Integrate[(theta1 * v1 * η), {η, 0, ni}]

c = thetaminlet * ni * ni * vm1 / (2 * %)

a = ω * vm1 * ni * ni + (1 - ni * ni) * vm2; A[0] = c * Exp[-2 * nu0 * ζ / (pe * a)]
A[2] = -m * A[0] * nu0 / 4;

theta1 = A[0] + A[2] * (η^2);

Do[A[2 * i] = (nu0 / (4 * i * i)) * (n * A[2 * i - 4] - m * A[2 * i - 2]), {i, 2, 15, 1}];

Do[theta1 = theta1 + A[2 * i] * (η^(2 * i)), {i, 2, 15, 1}];

pe = 200; theta1

B[0] = B[0] * A[0]; Dee[0] = Dee[0] * A[0];

Dee[2] = -p * Dee[0] * nu0 / 4;

B[2] = -p * (B[0] - Dee[0]) * nu0 / 4;

theta2 = B[0] + B[2] * η * η + Dee[0] * Log[η] +
        Dee[2] * Log[η] * (η^2);

Do[Dee[2 * i] = (nu0 / (4 * i * i)) * (q * Dee[2 * i - 4] -
                p * Dee[2 * i - 2]), {i, 2, 15, 1}];

Do[B[2 * i] = (nu0 / (4 * i * i)) * (q * B[2 * i - 4] - p * B[2 * i - 2])
            - (1 / i) * Dee[2 * i], {i, 2, 15, 1}];
```

```
Do[theta2 = theta2 + B[2 * i] * η^(2 * i) +
        Dee[2 * i] * Log[η] * (η^(2 * i)), {i, 2, 15, 1}];
```

The same procedure as above is repeated for convective boundary condition except that the term Nu_0 in the constant temperature condition is replaced by Nu .

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