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FORMING LIMIT DIAGRAMS

By

Jimmy Issa

A THESIS

Submitted to
Michigan State University
in partial fulfillment of the requirements
for the degree of

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ABSTRACT

FORMING LIMIT DIAGRAMS

By

Jimmy Issa

In this work, the Marciniak and Kuczynski model (MK) will be reviewed and will be generalized to a more general approach for the calculation of forming limit diagrams. In the new model we will account for all directions of the imperfection. This new approach has been used to calculate the theoretical FLDs (forming limit diagrams) for Vonmises, Barlat Yield96 and Barlat Yield2000 for aluminum alloy Al2008-T4 and compared with an experimental one. All the derivations needed for the implementation of the code are derived in the appendix for all yield functions. Finally the stretch stamping test will be modeled using LS-DYNA. A very good agreement has been achieved between the experimental and theoretical results.

To my family

[illegible]

ACKNOWLEDGMENTS

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LIST OF ABBREVIATIONS

σ - Cauchy stress

$\bar{\sigma}, \bar{\sigma}_{YF}$ - Yield stress from yield function

$\bar{\sigma}_{HR}$ - Yield stress from hardening rule

$\varepsilon, \bar{\varepsilon}$ - Cauchy and effective strain

e - Engineering Strain

n, k, ε_0 - Swift constant

A, B, C - Voce constants

f - Imperfection parameter

ρ - Strain ratio

α - Stress ratio

Φ - Yield ratio

t - Thickness

R - Ratio of plastic strain in the width and thickness direction

$\bar{R}$ - Average of R values

a - Exponent to account for crystallographically textured fcc and bcc metal

J - Jacobien

θ - Imperfection orientation

$C_1, C_2, C_3, C_6, \alpha_x, \alpha_y, \alpha_{z0}, \alpha_{z1}$ - Barlat yield96 material constants

$\alpha_1, \alpha_2, \alpha_3, \alpha_4, \alpha_5, \alpha_6, \alpha_7, \alpha_8$ - Barlat yield2000 material constants

Introduction

Forming limit diagrams (FLDs) have been valuable tools for analyzing sheet metal forming after the introduction of the idea in 1965. Stretching operations of sheet metals will normally lead to a development of a sharp localized neck on the surface where the plastic flow will localize and lead to failure. Forming limit diagrams have been first introduced by Keeler [1-2] and are still used as a failure criterion to assess the formability of sheet metals. Many tried to model the localized necking analytically after the introduction of the idea, starting with the pioneer work of Marciniak and Kuczynski [3] who first presented a mathematical model to help understand sheet metal deformation during forming. They modeled the imperfection as a long groove, perpendicular to the direction of the largest stress for positive major strains (RHS) or along Hill's direction of zero-extension (the strain increment parallel to the groove inside and outside of it is equal to zero) for the negative major strains (LHS). Using this approach theoretical FLDs have been calculated for linear strain path using different yield criteria, von Mises, Hill's 1948 [4-5-6], Hill's 1979 [5-6] and many other yield functions. Graf and Hosford [6] used an anisotropic yield function (Hill's 1979) and investigated the effect of the hardening exponent n , the strain rate sensitivity m , the imperfection f , the R-value and the yield function exponent a . F. Barlat [7] calculated the FLD using a yield function based on von Mises but with a high exponent. The effect of nonlinear strain paths has been

investigated on forming limits where experimental and numerical FLD have been predicted for changing strain paths for different materials [8-9-10-11]. M.C. Butuc [12-13] used a more general MK model and calculated the FLDs for von Mises, Hill's 1948, Hill's 1979 and Barlat Yld96 for Al6061.

In this work the MK model and a more general approach for the calculation of forming limit diagrams are explained in detail. The general approach is based on the MK model but is more general since there is no constraint on the groove direction and the limiting strain is calculated for all possible angles and minimized versus the angle of the groove. This new approach uses the Newton-Raphson method to solve for the transverse stress and effective strain in the groove. For this reason the second derivative of the yield function is needed for the evaluation of the Jacobian. I calculated the forming limit diagram for Al2008-T4 aluminum alloy using the modified MK model where von Mises, Yld96 and Yld2000 were used to describe the yield locus and modeled the hardening using both Voce and Swift models. The first and second derivatives for the three yield functions are evaluated in the appendix to make future implementation easier. F. Barlat [14] calculated the FLD by incorporating microstructural parameters, internal damage (voids) and crystallographic textures in his model and it has given very accurate results. Using a macroscopic approach, a very good agreement has been achieved between the calculated and the experimental FLD when the yield locus is described with Yld96 and the material hardening with Swift model.

CHAPTER 1: Experimental Forming Limits

Over the years experimental forming limit diagrams have been determined [4-15] for different linear strain paths and even for material pre-strained in different directions (biaxial tension, uniaxial tension and plane strain).

The most widely used technique involves printing or etching a grid of small circles of diameter d_0 on the sheet metal before deforming it. After stretching or forming the sheet, the circles will deform into a shape which resembles an ellipse. The forming process will stop when the first perceptible neck is observed. To find the limiting strains for different strain paths, lubrication and specimen widths are varied. To obtain the limit for negative minor strains, very narrow specimens are used and by increasing the width to full width specimen, we can find the limits up to balanced biaxial. After deformation the ellipses wholly or partially in the neck are considered failed, while the circles one or more diameters away from the neck are considered safe. The principal diameters of the failed ellipses are measured (major d_1 and minor d_2). The limiting strain can then be represented as engineering or true strains using the two formulas written below.

True Limiting Strains

$$\varepsilon_1 = \ln\left(\frac{d_1}{d_0}\right)$$

$$\varepsilon_2 = \ln\left(\frac{d_2}{d_0}\right)$$

Engineering Limiting Strains

$$e_1 = \frac{d_1 - d_0}{d_0}$$

$$e_2 = \frac{d_2 - d_0}{d_0}$$

Finally, Forming limit diagrams determined in different laboratories may be somewhat different. This is because the determination of the first perceptible neck is subjective. Many techniques have been used to determine the onset of the neck; visual inspection and feeling with the fingers. Also, there is no restriction placed on how far one should go from the neck before considering a circle to be safe. It is recommended that safe readings should be made at a distance at least equal to 1.5 circle diameter from the center of the neck. Below are two pictures of two different failed specimens.

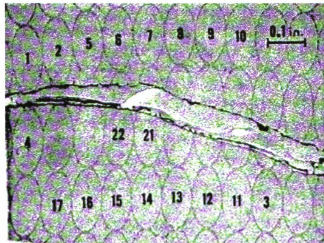


Fig1: Fractured specimen

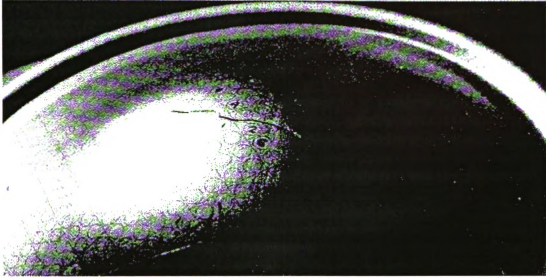


Fig2: Failure under biaxial tension

CHAPTER 2: MK Model

2.1 Introduction

The MK model is the first mathematical model presented by Marciniak and Kuczynski [3] to help understand localized necking in sheet metals. They assumed a long groove, a locally thinned region within which the strain will concentrate and will lead to failure due to the onset of localized necking. The origin of this imperfection is a variation in the sheet thickness [3], difference of the material strength [16] or a combination of both. The failure is assumed to occur when the ratio of the strain increment within the groove and outside of it reaches a critical value. There are some constraints on the orientation of the imperfection where it has been taken to be perpendicular to the axis of the largest principal stress and strain for positive major strains (RHS of the FLD), and in the direction of zero-extension for negative major strains (LHS of the FLD). In this section we will explain the algorithm for the calculation of FLDs based on the MK model. We will investigate the effect of the hardening exponent n , the strain rate sensitivity exponent m , the imperfection parameter f , the R -value and the yield function exponent a , where the yield locus will be described by Hill's 1979 yield function. We will investigate yield function differences in predicting localized necking. The corresponding algorithms are all written in Matlab.

2.2 Right hand side of forming limit diagrams

For positive minor strain the groove is assumed to be perpendicular to the axis of the largest principal stress and strain. The strain increments parallel to the groove within and outside of it are equal. The algorithm can be summarized by assuming, for a known strain path, the largest principal strain increment in the imperfection (region b) and then calculating the one in the homogeneous region (region a). The failure is assumed to occur when their ratio reaches a critical value (in the calculation below the critical value is assumed to be equal to 10). The algorithm below is written for von Mises yield function. The changes that will need to be implemented for other yield functions are described in the definition of each yield function. The Matlab programs are written for von Mises (Appendix A), Hill's quadratic (Appendix B), and non-quadratic yield functions (Appendix C). In the algorithm below indices a and b refer to the homogeneous and the defected region respectively. Indices 1 and 2 refer to the principal values. The figure below describes well the model.

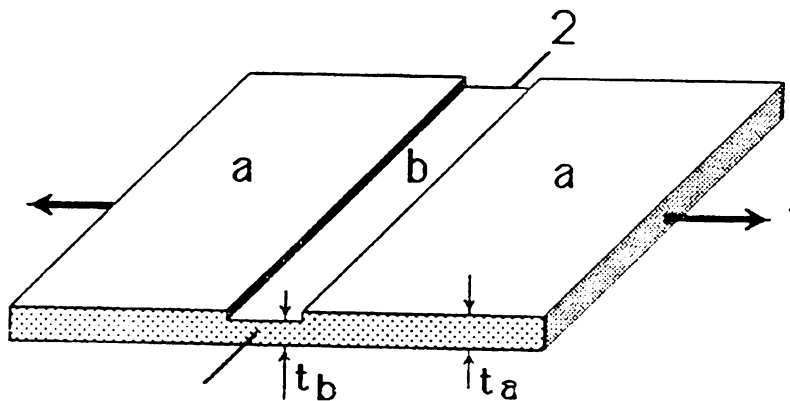


Fig3: RHS model

2.2.1: Algorithm

Under plane stress condition ($\sigma_3 = 0$), for a known strain ratio in region a:

$$\rho_a = \frac{d\varepsilon_{a2}}{d\varepsilon_{a1}}$$

The stress ratio is calculated:

$$\alpha_a = \frac{\sigma_{a2}}{\sigma_{a1}} = (2\rho_a + 1) / (\rho_a + 2)$$

The yield function is then evaluated:

$$\Phi_a = \frac{\overline{\sigma_a}}{\sigma_{a1}} = \sqrt{\alpha_a^2 - \alpha_a + 1}$$

Impose a value for the strain increment in the groove:

$$d\varepsilon_{b1} = 0.001$$

Assume a value for the strain increment in region a:

$$d\varepsilon_{a1}$$

Calculate the rest of the variables needed for region a:

$$\begin{aligned} d\varepsilon_{a2} &= \rho_a d\varepsilon_{a1} \\ d\varepsilon_{a3} &= -d\varepsilon_{a1} - d\varepsilon_{a2} \\ \overline{d\varepsilon_a} &= d\varepsilon_{a1} (1 + \alpha_a \rho_a) / \Phi_a \\ \beta_a &= \frac{\overline{d\varepsilon_a}}{d\varepsilon_{a1}} \end{aligned}$$

The equation below holds due to the compatibility requirement:

$$d\varepsilon_{b2} = d\varepsilon_{a2}$$

The rest of the variables in region b are evaluated:

$$\begin{aligned}
 d\varepsilon_{b3} &= -d\varepsilon_{b1} - d\varepsilon_{b2} \\
 \overline{d\varepsilon_b} &= d\varepsilon_{b1}(1 + \alpha_b \rho_b) / \Phi_b \\
 \beta_b &= \frac{\overline{d\varepsilon_b}}{d\varepsilon_{b1}} \\
 \rho_b &= \frac{d\varepsilon_{b2}}{d\varepsilon_{b1}} \\
 \alpha_b &= (2\rho_b + 1)(\rho_b + 2) \\
 \Phi_b &= \sqrt{\alpha_b^2 - \alpha_b + 1}
 \end{aligned}$$

From the force balance equation between region a and b the largest strain increment in the region a is recalculated. Then it is correspondingly adjusted and the process is repeated until the force balance is satisfied. The force balance equation is derived below:

$$F_{a1} = F_{b1}$$

For a unit width and thickness t , the force in the 1-direction is:

$$F_1 = \sigma_1 t = \overline{\sigma} \left(\frac{\sigma_1}{\overline{\sigma}} \right) t = \left(\frac{\overline{\sigma}}{\Phi} \right) t$$

The hardening rule with strain rate dependence can be written:

$$\overline{\sigma} = k \overline{\varepsilon}^n \dot{\overline{\varepsilon}}^m$$

Using the variables defined above the force can be expressed by:

$$F_1 = kt(\bar{\varepsilon} + d\bar{\varepsilon})^n \dot{\bar{\varepsilon}}^m / \Phi = kt(\bar{\varepsilon} + d\bar{\varepsilon})^n (\beta/\rho) \dot{\bar{\varepsilon}}_2^m / \Phi$$

By applying the above force expression to regions a and b, and after some adjustments, the force balance can be reduced to:

$$\frac{1}{\Phi_a} (\bar{\varepsilon}_a + d\bar{\varepsilon}_a)^n \left(\frac{\beta_a}{\rho_a} \right)^m \dot{\bar{\varepsilon}}_{a2}^m = \frac{f}{\Phi_b} \exp(\varepsilon_{3b} - \varepsilon_{3a}) (\bar{\varepsilon}_b + d\bar{\varepsilon}_b)^n \left(\frac{\beta_b}{\rho_b} \right)^m \dot{\bar{\varepsilon}}_{b2}^m$$

Where the imperfection parameter f and the current thickness are:

$$f = k_b t_{b0} / k_a t_{a0}$$

$$t = t_0 \exp \varepsilon_3$$

To solve the force balance equation, the Newton-Raphson or the Mid-point method can be used. Both methods were programmed in Matlab, but the Newton-Raphson proved to be faster. The same algorithm can be used for any yield function provided that the equations that involve the use of the yield function are changed correspondingly. Below, the algorithm will be applied to several different yield functions.

2.2.2 von Mises

The von Mises yield function is the most basic yield function describing the plastic behavior of isotropic materials. An isotropic material responds in the same manner when strained or stressed in any direction. All the derivations needed for the implementation of the algorithm for von Mises yield function are shown above. In plane stress applications the yield function is:

$$2\bar{\sigma}^2 = (\sigma_1 - \sigma_2)^2 + \sigma_1^2 + \sigma_2^2$$

The calculated forming limit diagrams below are for von Mises yield function where the imperfection parameter f , the hardening exponent n and the strain rate exponent m are variables. The FLD curves shown below have been plotted using the MK model. The groove does not change direction when the deformation occurs. So it is assumed to be perpendicular to the direction of the largest stress and strain and does not rotate during the straining of the sheet. The plots shown below indicate that formability increases and failure occurs at higher strains, with increased strain rate sensitivity and strain-hardening exponent, and decreased imperfection parameter.

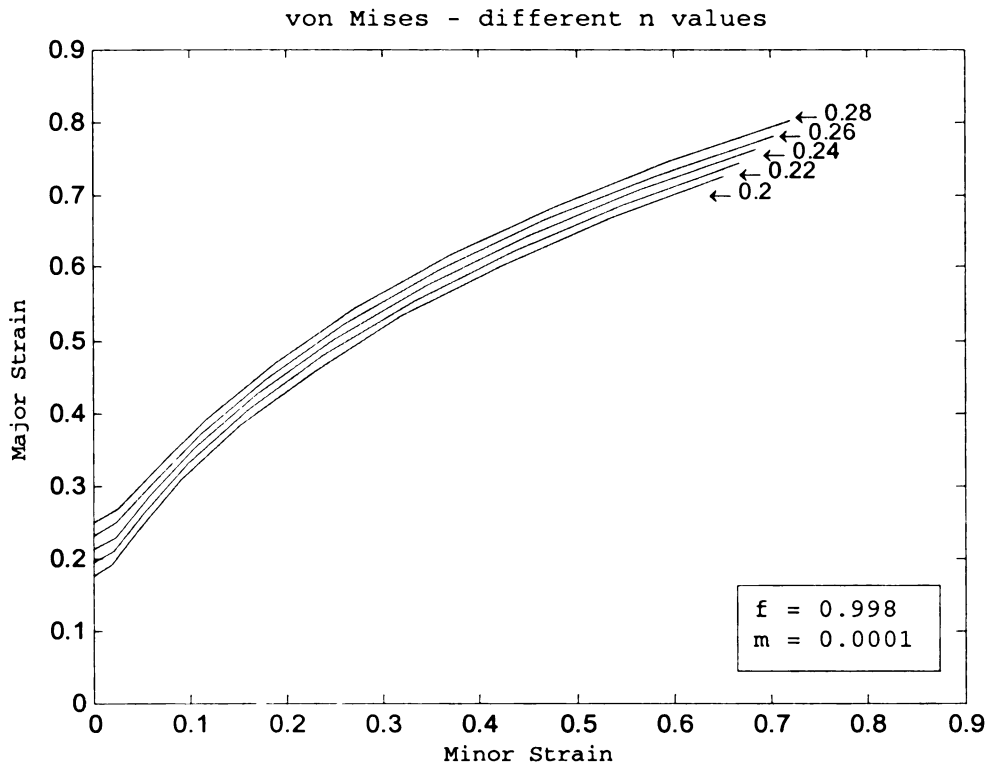


Fig4: Different n values

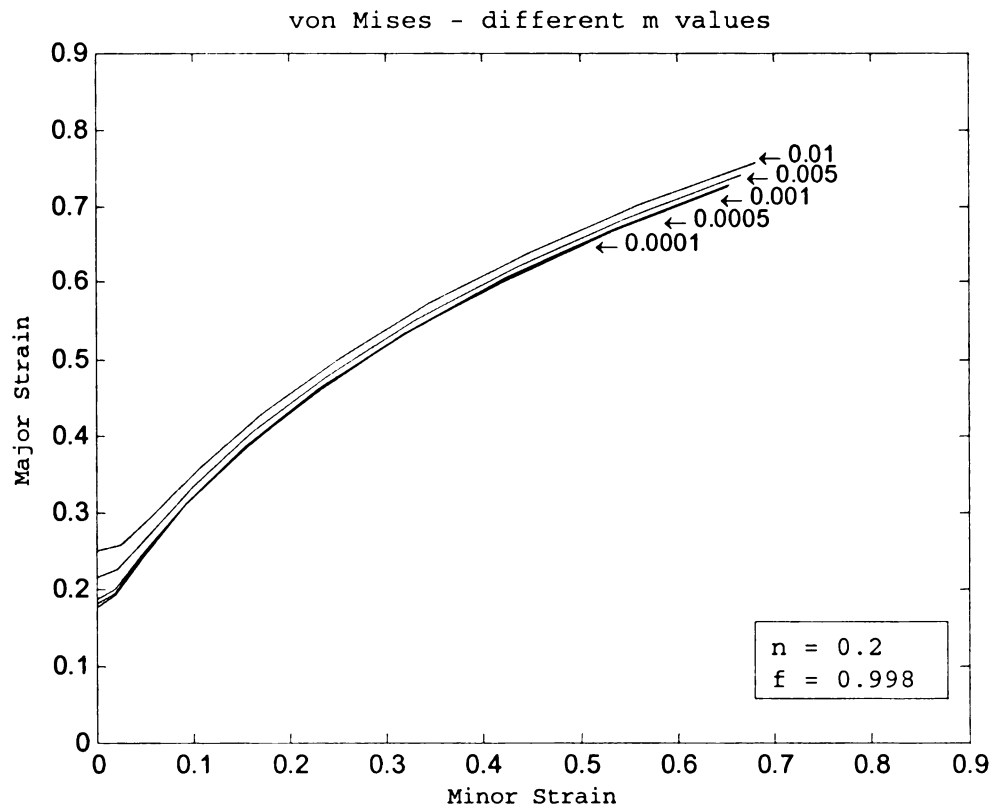


Fig5: Different m values

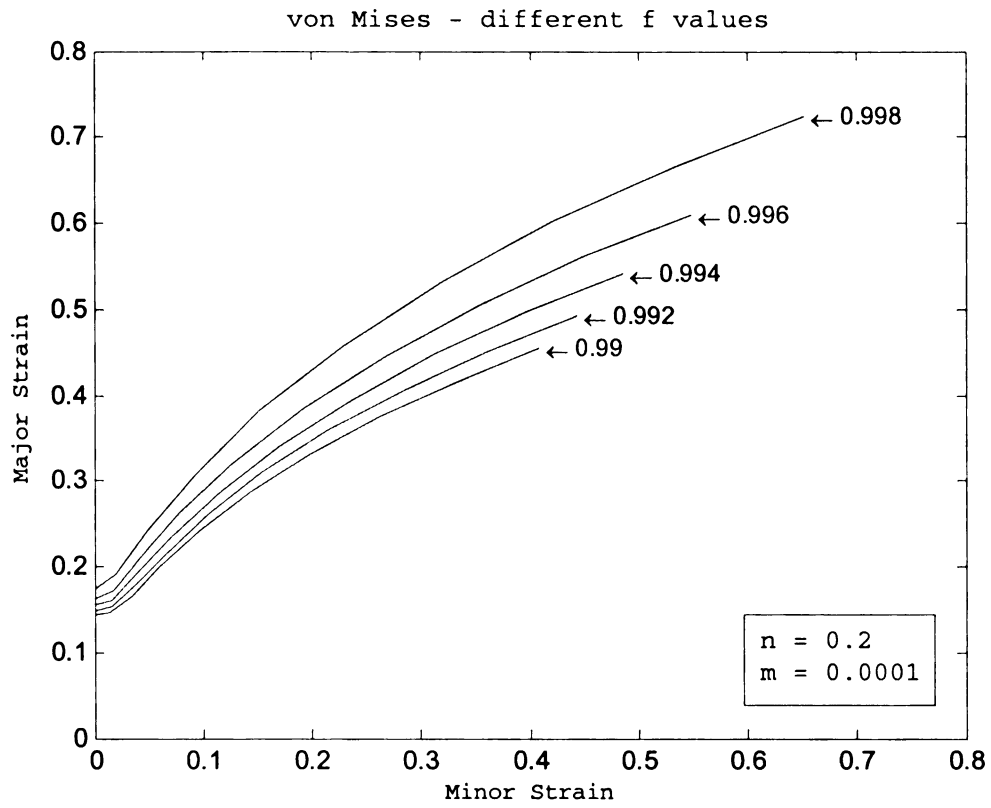


Fig6: Different f values

2.2.3: Hill's Quadratic 1948

The Hill's quadratic yield function was one of the first yield functions to describe the plastic anisotropy of metals. Hill has formulated a quantitative treatment of plastic anisotropy without regard to its crystallographic origin. A new parameter R has been used which is the ratio of plastic strain in the width and thickness direction for a tensile specimen cut from a sheet. The R-values usually vary with the test direction and it is common to characterize a material by an average R-value written below.

$$\bar{R} = \frac{R_0 + 2R_{45} + R_{90}}{4}$$

Where the indices represent the angles (with the rolling direction) to which the tensile specimen has been cut from the sheet. For R equal to 1 the quadratic hill's yield function will simply be reduced to von Mises. For plane stress loading the yield function simplify to the equation below:

$$\bar{\sigma}^2 = \sigma_1^2 + \sigma_2^2 - \frac{2R}{1+R} \sigma_1 \sigma_2$$

The only two expressions in the algorithm that should be changed in order to get the forming limit diagrams using Hill's 1948 are the stress ratio and the yield ratio. The stress ratio is a function of the strain ratio and the yield ratio is a function of the stress ratio. Those two functions are written below.

$$\alpha = (R + \rho(1 + R)) / (1 + R(\rho + 1))$$

$$\Phi = \sqrt{\frac{1}{1+R}(\alpha^2 + 1 + R(1-\alpha)^2)}$$

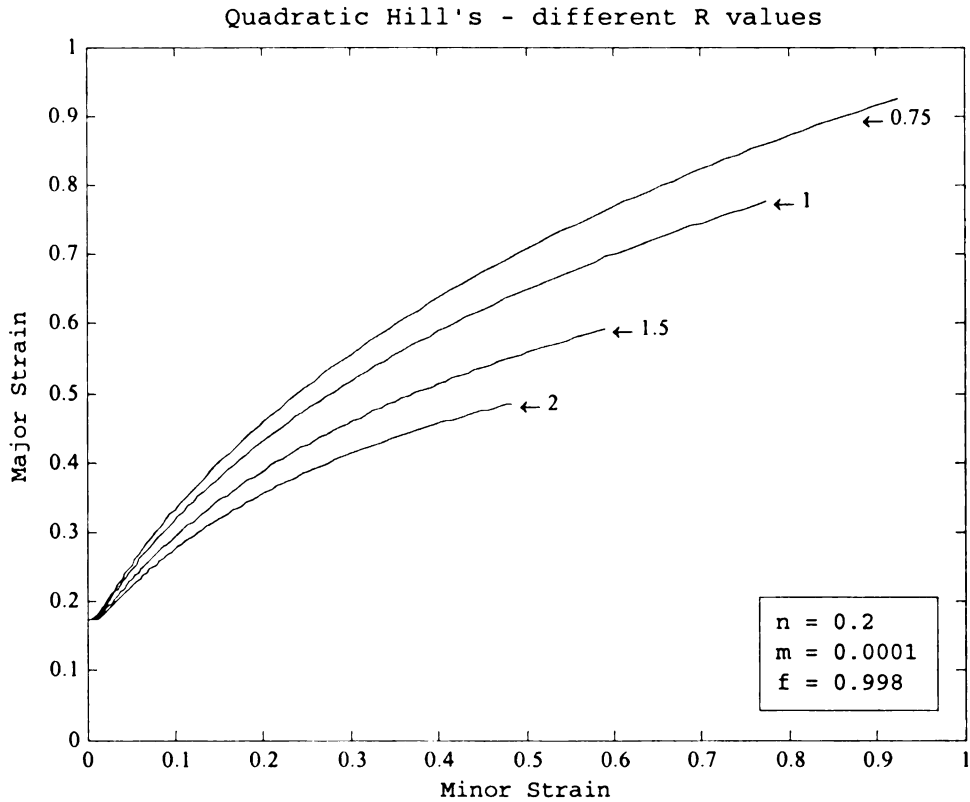


Fig7: Different R values

The figure above shows that the forming limit is very sensitive to the value of R and it decrease with increasing R. The forming limit for this case is calculated with the largest strain assumed to occur in the RD direction.

2.2.4: Hill's Non-Quadratic 1979

Hill's quadratic yield function overestimated the effect of the R-value on the shape of the yield loci. A new yield function has been developed to overcome this weakness where a higher exponent "a" has been used to account for crystallographically textured fcc and bcc metals. With a equal to 2 this criterion simplifies to Hill's quadratic. However, crystallographic calculations are much better approximated by a being equal to 6 for bcc and 8 for fcc metals. For plane stress loading this criterion simplifies to:

$$\bar{\sigma}^a (1 + R) = \sigma_1^a + \sigma_2^a + R(\sigma_1 - \sigma_2)^a$$

The yield ratio for this yield function can be expressed explicitly as a function of the stress ratio but the stress ratio cannot be expressed explicitly as a function of the strain ratio. A nonlinear equation should be solved in order to first get the stress ratio and then evaluate the yield ratio. The Newton Raphson method or the Midpoint method can be used to solve this equation. Below the non-linear equation and the yield ratio are written:

$$\rho(1 + R(1 - \alpha)^{a-1}) - \alpha^{a-1} + R(1 - \alpha)^{a-1} = 0$$

$$\Phi = \left(\frac{1}{1 + R} (\alpha^a + 1 + R(1 - \alpha)^a) \right)^{\frac{1}{a}}$$

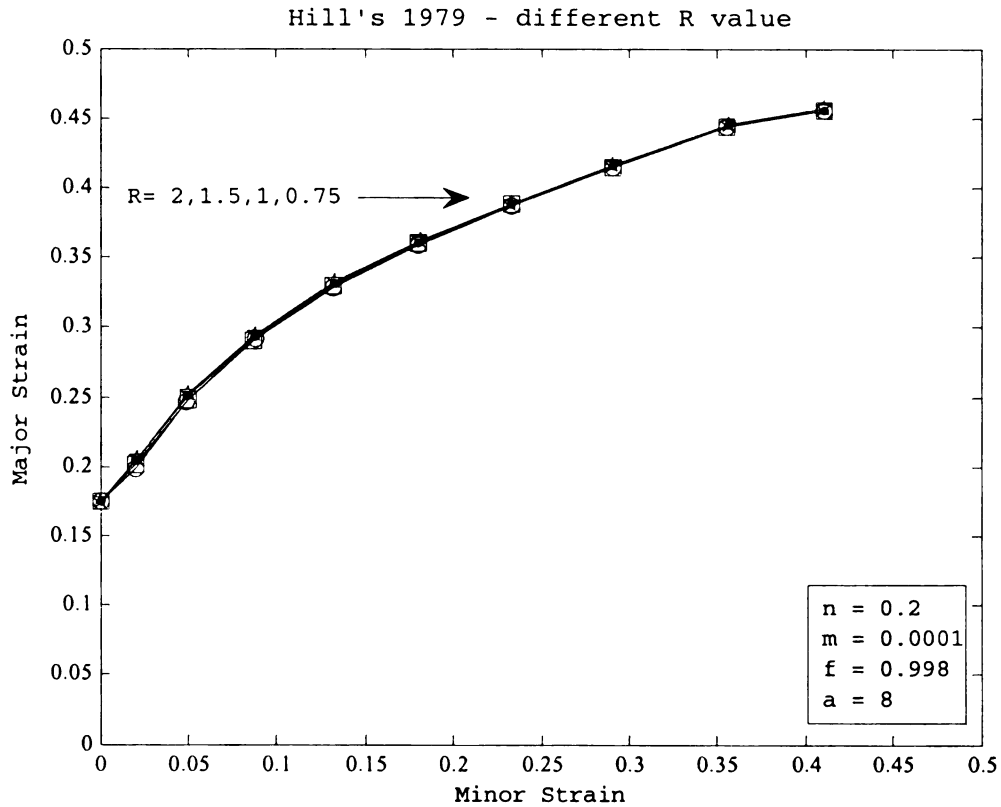


Fig8: Different R values

The plot above proves that the forming limit diagram is insensitive to the level of R, where it was very dependant when the quadratic hill's yield function was used. This independence of the FLD of the R-value gave the non-quadratic function a great importance because it matched with the experimental FLD where it has been shown to not depend on R values.

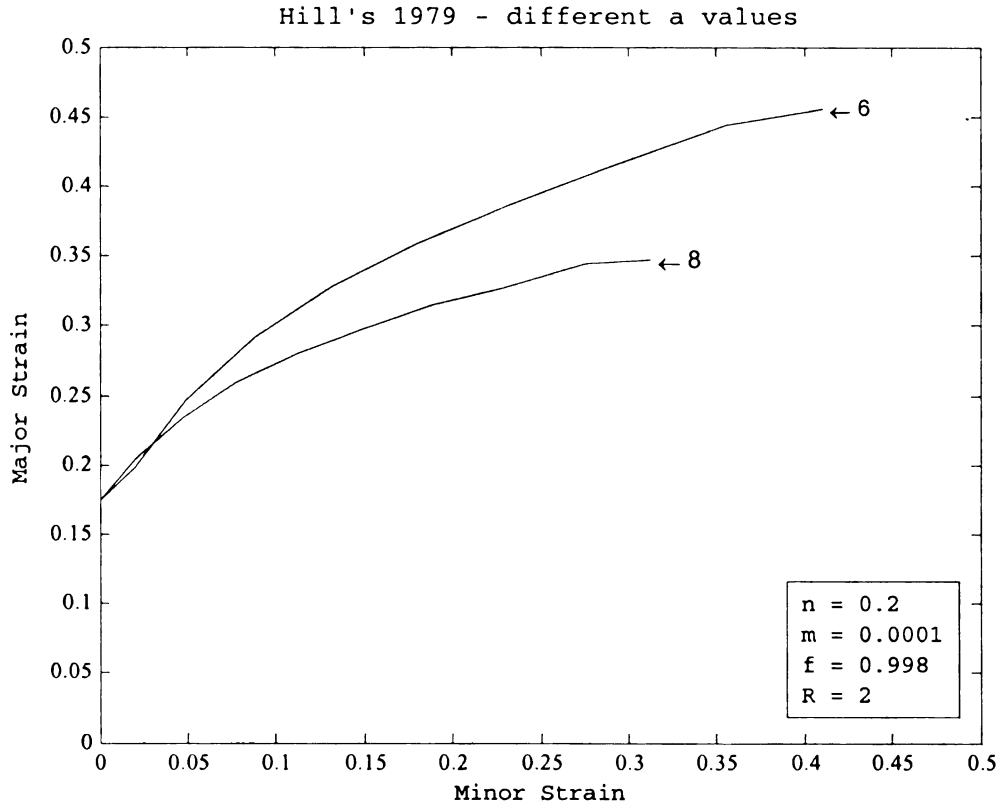


Fig9: Different a values

As mentioned before the exponent a , connected to the crystal structure, is equal to 6 or 8 for bcc and fcc alloys respectively. For the special case where R is equal to 1 and a equal to 2, Hill's 1979 will simply reduce to von Mises. The plot above shows a dependence of the FLD on the value of a .

2.3: Left hand side of forming limit diagrams

For the negative minor strain regime the groove direction is assumed to line up with Hill's direction of zero-extension such that the plane strain condition is satisfied. The existence of a critical thickness strain criterion for localized necking of sheets in the negative minor strain regime has been demonstrated. The effective limit strain in region a increases as the strain ratio becomes more negative and tends towards uniaxial tension. The limit thickness strain is a constant regardless of the imposed strain ratio, thus indicating the existence of a critical thickness strain for the onset of localized necking. The limiting thickness strain is dependant only on n , m and f . It is independent of the plastic anisotropy, the yield function and the plasticity theories (flow or deformation theory).

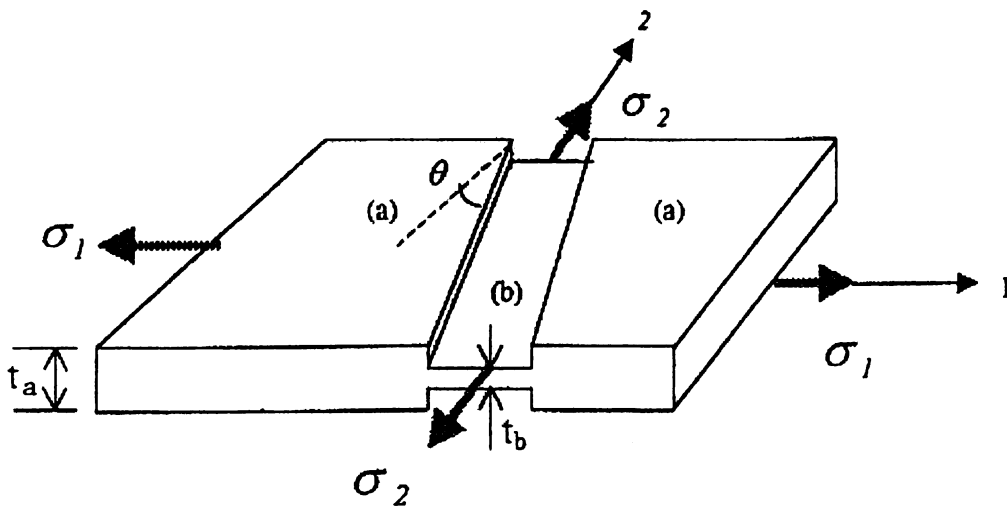


Fig10: LHS model

2.3.1: Algorithm

The algorithm is simple here because it involves the solution of a nonlinear equation with the thickness strain. A thickness strain increment is assumed in the imperfection and from the nonlinear equation the thickness strain in the homogeneous region is calculated until the thickness strain in the homogeneous region reaches a limiting value. This equation is solved using either Newton Raphson method or the Mid-Point method. After lengthy mathematical manipulation where the force balance equation coupled with the condition of Hill's zero extension have been used, the nonlinear equation simplifies to the equation below. The full derivation of this equation is in [17].

$$(d\varepsilon_3^a)^m (\varepsilon_{3o}^a + \varepsilon_3^a)^n \exp(\varepsilon_3^a) = f(d\varepsilon_3^b)^m (\varepsilon_{3o}^b + \varepsilon_3^b)^n \exp(\varepsilon_3^b)$$

2.3.2: Thickness limit

The limiting thickness strain in the homogeneous region is independent from the strain ratio. For each value of n , m and f the limiting strain in region a is constant. The variation of the limiting strain with n , m and f is shown in the plots below. By increasing the values of n and m , the limiting thickness strain increases by keeping a constant ratio with the thickness strain in the imperfection. By increasing the f values, the limiting thickness strain will increase with a small increase in its ratio with the limiting strain in the imperfection.

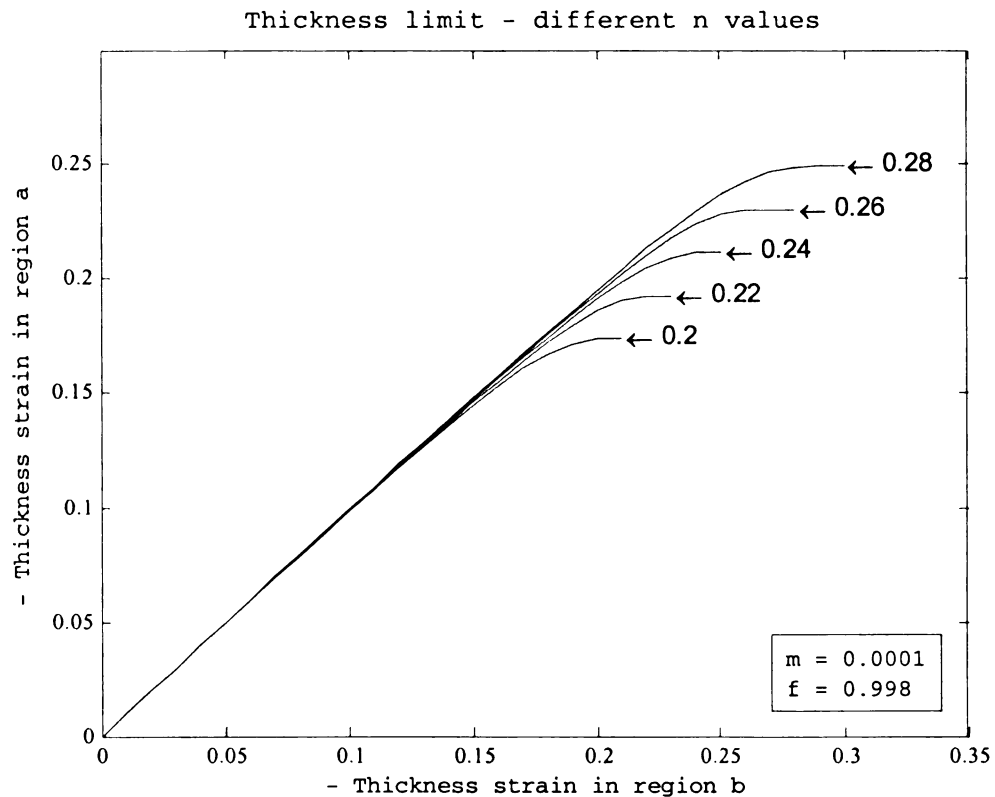


Fig11: Thickness limit Different n values

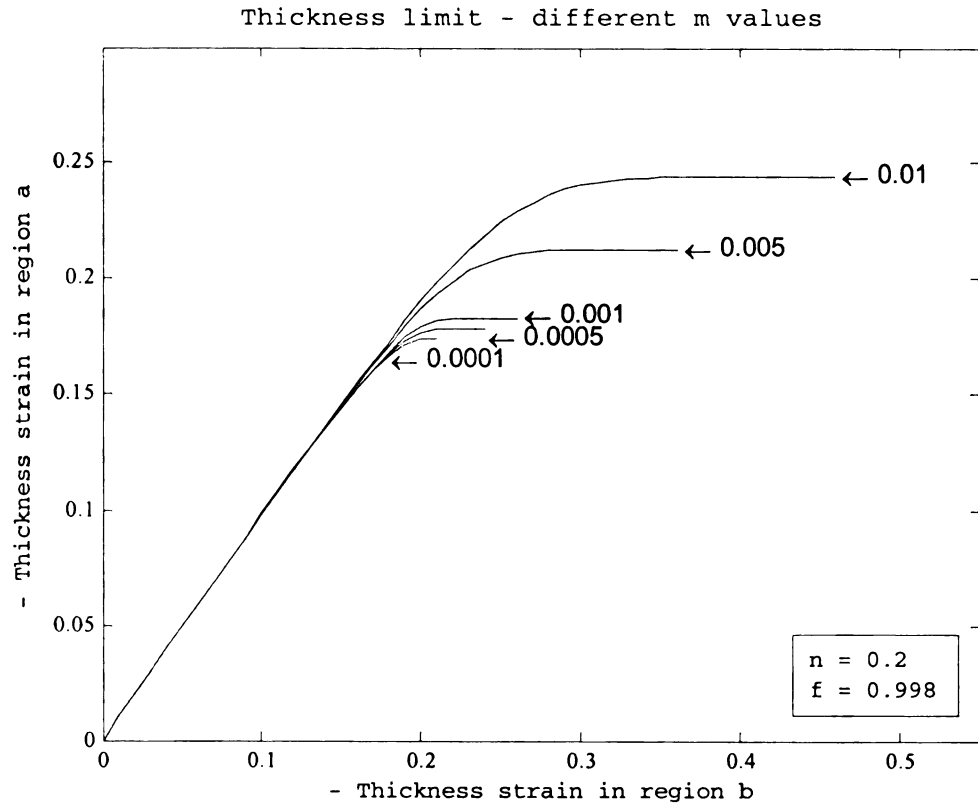


Fig12: Thickness limit Different m values

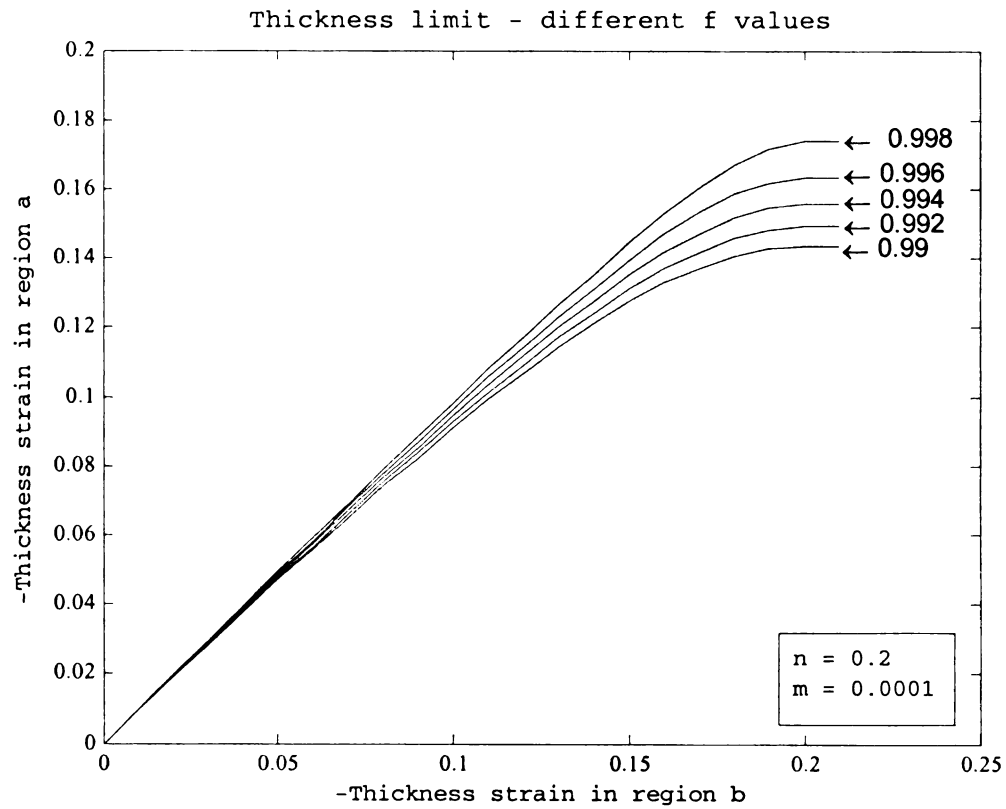


Fig13: Thickness limit Different f values

2.3.3: Results

The plots below show the effect of the hardening exponent n , the strain rate sensitivity parameter m and the imperfect parameter f . The FLD for the LHS increases with the increase in the values of n , m , and decrease with the increasing value of f . This is reasonable because an increase in the imperfection parameter means a deeper defect, which will lead to an earlier failure.

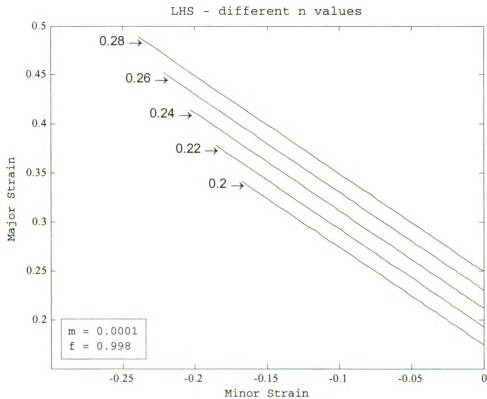


Fig14: LHS Different n values

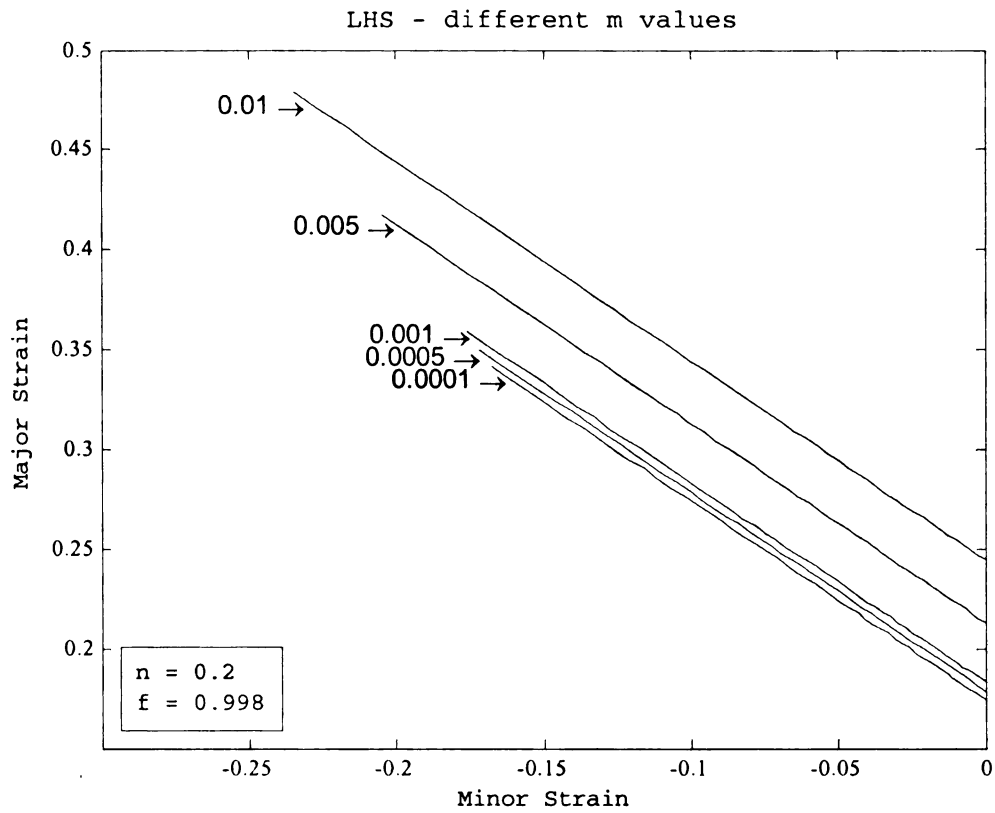


Fig15: LHS Different m values

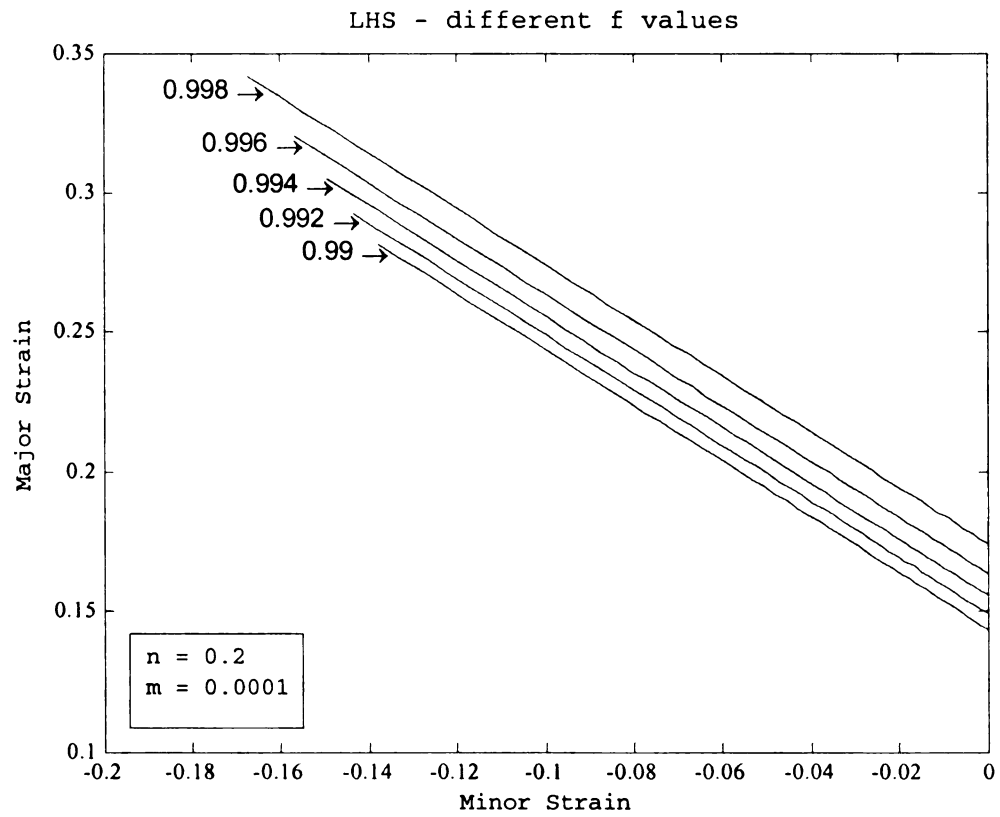


Fig16: LHS Different f values

CHAPTER 3: General MK Model

3.1: Introduction

In this chapter, we will explain in detail a general approach for the calculation of forming limit diagrams. This present model is an extension of a previous analysis by Marciniak and Kaczynski (MK-model), where it is generalized to account for all possible directions of the imperfection. The limiting strain is calculated for all possible angles and minimized versus the angle of the groove. We will use this approach to compare different yield functions starting with the most basic one for isotropic materials von Mises and two anisotropic yield functions Barlat Yld96 and Barlat Yld2000 proven to describe well the plastic behavior of aluminum sheet metals. Two work-hardening laws, Swift and Voce, will be used to model the effective stress-strain behavior of the material. This new approach uses the Newton-Raphson method to solve for the transverse stress and effective strain in the groove. For this reason, the second derivative of the yield function is needed for the evaluation of the Jacobian. The first and second derivatives of the three yield functions are evaluated in the appendix for implementation purposes. Forming limit diagrams will be calculated for aluminum alloy Al2008-T4 and will be compared with experimental data. Barlat calculated the FLD for Al2008-T4 by incorporating micro structural parameters, internal damage (voids) and crystallographic textures in his model and showed very accurate results. All computed FLDs assume that the largest strain is in the y-direction

(TD direction) in order to be on the safe side. A very good agreement between the calculated and the experimental FLD was obtained, when the Yld2000 and the voce hardening law were used. M.C. Butuc [12-13] already developed FLD for the Al6061 aluminum alloy based on von Mises, Hill's 1948, Hill's 1979 and Barlat Yld96yield function.

3.2: Modified M-K model

This model uses the same assumption proposed by Marciniak and Kuczynski [3] who assumed a long groove, a locally thinned region within which the strain will concentrate and will lead to failure due to the onset of localized necking. The origin of this imperfection is a variation in the sheet thickness [3], difference of the material strength [16], or a combination of both. The failure is assumed to occur when the ratio of the strain increment within the groove and outside of it reaches a critical value. As we mentioned earlier there is some constraints on the orientation of the imperfection. The modified M-K model will consider all possible orientations for the groove and consider the failure to occur when the ratio of the total strain increment inside and outside of the imperfection reach the value of 10.

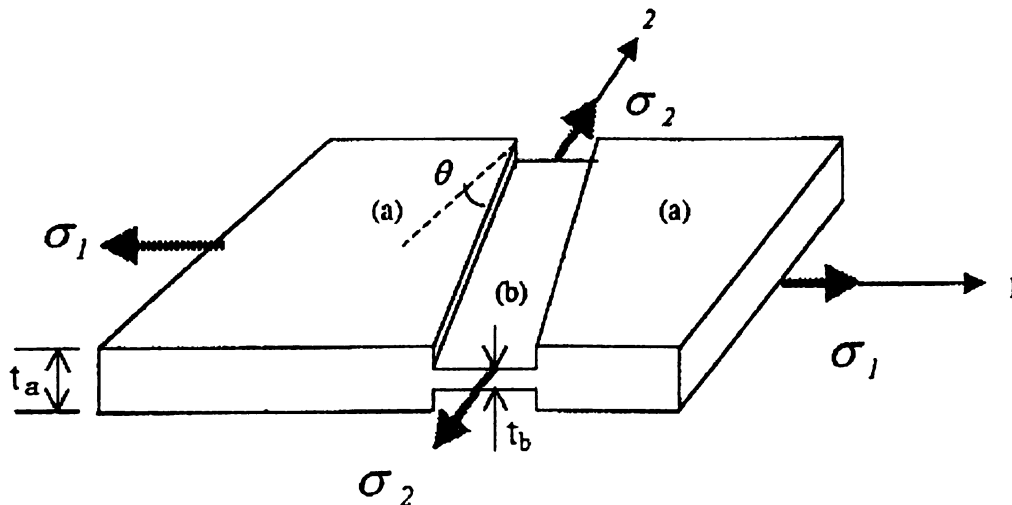


Fig17: Modified MK model

The model above consist of two regions, the homogeneous region "a" where the axis 1, 2 and 3 represent the principal directions of stress and strain and the groove region "b" where the axis n, t and 3 are the normal to the direction of the groove, the longitudinal direction, and the thickness direction. The M-K imperfection parameter is defined as the initial thickness ratio of the defected and the homogeneous area .The deformation within the groove occurs at a faster rate than the one in the homogeneous region and the plastic flow will localize in the imperfection area leading to eventual failure. Under deformation, the strains parallel to the imperfection direction within and outside of the groove are equal and forces perpendicular to the groove in both regions are the same. Those two constraints are called compatibility and equilibrium conditions. The groove is assumed to rotate while the deformation occurs. Let e and e_0 be the current and the initial sheet thickness. The value of the imperfection parameter, the compatibility and the equilibrium conditions can then be written as

Imperfection parameter: $f = e^b / e^a$

Compatibility condition: $d\varepsilon_u^a = d\varepsilon_n^b$

Equilibrium condition : $\sigma_{nn}^a e^a = \sigma_{nn}^b e^b$, $\sigma_{nt}^a e^a = \sigma_{nt}^b e^b$

3.2.1 Algorithm

Given the stress state in region “a”, the plastic strain is evaluated in region “b” for a known orientation of the groove. In the figure above we will assume the x-, y- and z-axis to be the orthotropic axes of the material (rolling, transverse and normal) that line up with axes 1, 2 and 3 respectively. For one increment we will explain clearly how to calculate the plastic strain in region “b”. First, assume a small increment of the effective strain in region “a” and then follow the steps below. The details below show how this algorithm should be used in order to calculate the FLD for proportional loading, by assuming that no shear stress exists in region “a”. This algorithm could be used to find the critical strain for an unknown strain path where the strain state will be an input from an FEM analysis.

3.2.1.1 Region “a”

- *Strain ratio*

Usually in the calculation of forming limit diagrams for linear strain path the strain ratio in region “a” will be an input.

$$\rho = \frac{d\varepsilon_{xx}^a}{d\varepsilon_{xx}^b}$$

- *Stress ratio*

The stress ratio is then evaluated by assuming that it is a variable equal to the stress in the y-direction, if the stress in the x-direction is set equal to 1 and shear stress equal to zero.

$$\alpha = \frac{\sigma_{yy}^a}{\sigma_{xx}^a}$$

For most yield functions, to get the stress ratio one will need to solve the following nonlinear equation:

$$d\varepsilon_{yy}^a - \rho d\varepsilon_{xx}^a = 0 \Rightarrow \frac{\partial \sigma_{yF}}{\partial \sigma_{yy}} - \rho \frac{\partial \sigma_{yF}}{\partial \sigma_{xx}} = 0 \quad \text{For } \sigma = \begin{bmatrix} 1 \\ \alpha \\ 0 \end{bmatrix}$$

The Newton Raphson or Mid-Point method could be used for this purpose. For the Mid-Point method the initial interval where the solution is located can be taken as [-1; 2].

- *Yield ratio*

The yield ratio is the ratio of the yield function to the stress in the x-direction. The yield ratio becomes the yield stress by assuming the stress in the x-direction to be equal to 1, the stress in the y-direction to be equal to the stress ratio, and the shear stress to be equal to zero.

$$\Phi = \frac{\bar{\sigma}}{\sigma_{xx}^a}, \quad \Phi = \bar{\sigma}_{yF} \quad \text{For } \sigma = \begin{bmatrix} 1 \\ \alpha \\ 0 \end{bmatrix}$$

- *Stress state*

Having assumed an increment for the effective strain, the yield stress can be evaluated using the hardening rule (swift or voce). Coupled with the yield ratio, the stress in the x-direction can be calculated, knowing the stress in the y-direction, and by setting the shear stress equal to zero (no shear stress in region "a").

$$\begin{aligned}\bar{\sigma}_{HR} &= \text{hardening}(\bar{\varepsilon}_a + d\bar{\varepsilon}_a) \\ \sigma_{xx}^a &= \bar{\sigma}_{HR} / \Phi \\ \sigma_{yy}^a &= \alpha \sigma_{xx}^a \\ \sigma_{xy}^a &= 0\end{aligned}$$

- *Strain state*

The strain state is calculated from the Flow Rule knowing the stress state in the orthotropic referential frame.

$$\begin{aligned}d\varepsilon_{xx}^a &= d\bar{\varepsilon}_a \frac{\partial \sigma_{YF}}{\partial \sigma_{xx}^a} \Rightarrow \varepsilon_{xx}^a = \varepsilon_{xx}^a + d\varepsilon_{xx}^a \\ d\varepsilon_{yy}^a &= d\bar{\varepsilon}_a \frac{\partial \sigma_{YF}}{\partial \sigma_{yy}^a} \Rightarrow \varepsilon_{yy}^a = \varepsilon_{yy}^a + d\varepsilon_{yy}^a \\ d\varepsilon_{xy}^a &= d\bar{\varepsilon}_a \frac{1}{2} \frac{\partial \sigma_{YF}}{\partial \sigma_{xy}^a} \Rightarrow \varepsilon_{xy}^a = \varepsilon_{xy}^a + d\varepsilon_{xy}^a\end{aligned}$$

- *Output needed*

To solve for the effective strain increment in region “b” four values are needed; the strain increment parallel to the groove, the strain in the thickness direction, the stress normal to the groove and the shear stress in the groove coordinate axis. Those four values are calculated as following:

$$\begin{aligned}
 d\varepsilon_{tt}^a &= d\varepsilon_{xx}^a \sin^2 \theta + d\varepsilon_{yy}^a \cos^2 \theta - d\varepsilon_{xy}^a \sin 2\theta \\
 \sigma_{nn}^a &= \sigma_{xx}^a \cos^2 \theta + \sigma_{yy}^a \sin^2 \theta + \sigma_{xy}^a \sin 2\theta \\
 \sigma_{nt}^a &= \sigma_{xy}^a (\cos^2 \theta - \sin^2 \theta) + (\sigma_{yy}^a - \sigma_{xx}^a) \sin \theta \cos \theta \\
 \varepsilon_3^a &= -(\varepsilon_{xx}^a + \varepsilon_{yy}^a)
 \end{aligned}$$

3.2.1.2 Region “b”

- *Nonlinear Equations*

For this region to complete our study we have to compute the effective strain increment and compare it with the one in the homogeneous region. Two nonlinear equations in the effective strain increment and the stress in the longitudinal direction must be satisfied. The first state that the yield stress computed from the yield function must equal the one computed from the hardening rule. The second is the compatibility equation. The two equations are written below.

$$G_1(d\bar{\epsilon}^b, \sigma_u^b) = \bar{\sigma}_{HR}^b - \bar{\sigma}_{YF}^b = 0$$

$$G_2(d\bar{\epsilon}^b, \sigma_u^b) = d\bar{\epsilon}_u^a - d\bar{\epsilon}_u^b = 0$$

To solve these equations the Newton-Raphson method is used. This method involves the computation of the Jacobian needed in the iterative evaluation of the solution. The solution is reached by iteration using the formula below:

$$\begin{bmatrix} d\bar{\epsilon}_{i+1}^b \\ \sigma_{u,i+1}^b \end{bmatrix} = \begin{bmatrix} d\bar{\epsilon}_i^b \\ \sigma_{u,i}^b \end{bmatrix} - J^{-1} \begin{bmatrix} G_1(d\bar{\epsilon}_i^b, \sigma_{u,i}^b) \\ G_2(d\bar{\epsilon}_i^b, \sigma_{u,i}^b) \end{bmatrix}$$

Where J is the Jacobian of the nonlinear equations. The Jacobian is the matrix of the derivatives of G_1 and G_2 with respect to the effective strain increment and the stress in the longitudinal direction. For this case the Jacobian is symmetric and its components are described below:

$$J = \begin{bmatrix} \frac{\partial G_1}{\partial d\bar{\epsilon}^b} & \frac{\partial G_1}{\partial \sigma_u^b} \\ \frac{\partial G_2}{\partial d\bar{\epsilon}^b} & \frac{\partial G_2}{\partial \sigma_u^b} \end{bmatrix}$$

The first component of the Jacobian is straightforward and is obtained by deriving either hardening rules (Swift or Voce)

$$\frac{\partial G_1}{\partial d\bar{\epsilon}^b} = \begin{cases} nK(\bar{\epsilon}_0 + \bar{\epsilon}^b + d\bar{\epsilon}^b)^{n-1} & \text{Swift hardening} \\ C \times B \exp[-C(\bar{\epsilon}^b + d\bar{\epsilon}^b)] & \text{Voce hardening} \end{cases}$$

As we mentioned above the Jacobian is symmetric and the non-diagonal component involve the first derivative of the yield function, which is derived in the Appendix.

$$\frac{\partial G_1}{\partial \sigma_{tt}^b} = \frac{\partial G_2}{\partial d\bar{\varepsilon}^b} = - \begin{bmatrix} \frac{\partial \bar{\sigma}_{YF}^b}{\partial \sigma_{xx}^b} & \frac{\partial \bar{\sigma}_{YF}^b}{\partial \sigma_{yy}^b} & \frac{\partial \bar{\sigma}_{YF}^b}{\partial \sigma_{xy}^b} \end{bmatrix} \begin{bmatrix} \sin^2 \theta \\ \cos^2 \theta \\ -\sin \theta \cos \theta \end{bmatrix}$$

The derivative of G_2 with respect to σ_{tt} involves the second derivative of the yield function, which is derived in the appendix.

$$\frac{\partial G_2}{\partial \sigma_{tt}^b} = -d\bar{\varepsilon}^b \begin{bmatrix} \sin^2 \theta & \cos^2 \theta & -\sin 2\theta \end{bmatrix} \begin{bmatrix} \frac{\partial^2 \bar{\sigma}_{YF}^b}{\partial \sigma_{xx}^2} & \frac{\partial^2 \bar{\sigma}_{YF}^b}{\partial \sigma_{xx} \partial \sigma_{yy}} & \frac{\partial^2 \bar{\sigma}_{YF}^b}{\partial \sigma_{xx} \partial \sigma_{xy}} \\ \frac{\partial^2 \bar{\sigma}_{YF}^b}{\partial \sigma_{yy} \partial \sigma_{xx}} & \frac{\partial^2 \bar{\sigma}_{YF}^b}{\partial \sigma_{yy}^2} & \frac{\partial^2 \bar{\sigma}_{YF}^b}{\partial \sigma_{yy} \partial \sigma_{xy}} \\ \frac{1}{2} \frac{\partial^2 \bar{\sigma}_{YF}^b}{\partial \sigma_{xy} \partial \sigma_{xx}} & \frac{1}{2} \frac{\partial^2 \bar{\sigma}_{YF}^b}{\partial \sigma_{xy} \partial \sigma_{yy}} & \frac{1}{2} \frac{\partial^2 \bar{\sigma}_{YF}^b}{\partial \sigma_{xy}^2} \end{bmatrix} \begin{bmatrix} \sin^2 \theta \\ \cos^2 \theta \\ -\sin \theta \cos \theta \end{bmatrix}$$

• System Solution

To solve the system of the two nonlinear equations we will use the Newton-Raphson method. The initial values are assumed to be the one of the homogeneous region. Using the equilibrium condition we are able to compute the normal stress to the groove and the shear stress in the groove. We assume the initial imperfection parameter to be the current for the first iteration so we can proceed with the calculations and provided that the variation of the imperfection parameter is too small at the beginning of the

deformation. Using the rotation rule below and the flow rule the stresses and strains in the groove in the orthotropic referential frame are determined and finally we calculate the strain increment in the groove in the longitudinal direction using the rotation formula:

$$d\bar{\varepsilon}^a, \sigma_{tt}^a \quad \begin{array}{c} \text{INITIAL} \\ \text{VALUES} \end{array} \rightarrow \begin{bmatrix} d\bar{\varepsilon}_0^b \\ \sigma_{tt0}^b \end{bmatrix} = \begin{bmatrix} d\bar{\varepsilon}^a \\ \sigma_{tt}^a \end{bmatrix}$$

$$\sigma_{nn}^a, \sigma_{nt}^a, f \quad \text{EQUILIBRIUM} \rightarrow \begin{array}{l} \sigma_{nn}^b = \sigma_{nn}^a / f \\ \sigma_{nt}^b = \sigma_{nt}^a / f \end{array}$$

$$\begin{bmatrix} \sigma_{nn}^b \\ \sigma_{tt}^b \\ \sigma_{nt}^b \end{bmatrix} \quad \text{ROTATION} \rightarrow \begin{array}{l} \sigma_{xx}^b = \sigma_{nn}^b \cos^2 \theta + \sigma_{tt}^b \sin^2 \theta - \sigma_{nt}^b \sin 2\theta \\ \sigma_{yy}^b = \sigma_{nn}^b \sin^2 \theta + \sigma_{tt}^b \cos^2 \theta + \sigma_{nt}^b \sin 2\theta \\ \sigma_{xy}^b = \sigma_{nt}^b (\cos^2 \theta - \sin^2 \theta) - (\sigma_{tt}^b - \sigma_{nn}^b) \sin \theta \cos \theta \end{array}$$

$$\begin{bmatrix} \sigma_{xx}^b \\ \sigma_{yy}^b \\ \sigma_{xy}^b \end{bmatrix} \quad \text{FLOW RULE} \rightarrow \begin{bmatrix} \varepsilon_{xx}^b \\ \varepsilon_{yy}^b \\ \varepsilon_{xy}^b \end{bmatrix}$$

$$\begin{bmatrix} \varepsilon_{xx}^b \\ \varepsilon_{yy}^b \\ \varepsilon_{xy}^b \end{bmatrix} \quad \text{ROTATION} \rightarrow d\varepsilon_{tt}^b = d\varepsilon_{xx}^b \sin^2 \theta + d\varepsilon_{yy}^b \cos^2 \theta - d\varepsilon_{xy}^b \sin 2\theta$$

Now having all the values needed to solve the nonlinear equation, the strain increment in the groove and the stress in the longitudinal direction of the groove are computed by iterating until a convergence value is reached. After that the current imperfection parameter and the angle of the band are updated. The process continues by assuming an effective strain increment in region a and calculating the one in the groove until the ratio of these two approaches a critical value corresponding to local instability. It is usually assumed that failure occurs when the strain increment in the homogeneous region reaches 10 percent or less than the one in the groove.

Imperfection parameter: $f = f_0 \exp(\varepsilon_3^b - \varepsilon_3^a)$

Groove orientation : $\tan(\theta + d\theta) = \tan \theta \frac{1 + d\varepsilon_{xx}^a}{1 + d\varepsilon_{yy}^a}$

The study above is done assuming the major strain to occur along the rolling direction (x-axis), but to obtain the full FLD we have to assume the case where the major strain occurs along the transverse direction where it will give the lowest values for the strain and will be used as the FLD for material. To calculate the FLD where the major strain occurs along the (y-axis), the only thing to do is to switch the x and y material coefficient and use the same algorithm. For Vonmises both FLDs are the same due to the symmetry of the yield function with the rolling and transverse directions, but for Barlat Yield96 and Barlat 2000 the full FLD is not symmetric with the first bisector.

3.3: Yield Functions

In this work three yield functions are used to calculate the theoretical FLDs for Al2008-T4, von Mises, Barlat Yield96 and Barlat Yield2000. A description of the yield functions and the derivation needed for the implementation of the codes are presented in the appendix.

3.4: Hardening rules

The coefficients of an effective stress-strain curve that defines the strain hardening of a material is found by fitting the experimental data obtained from a uni-axial tensile test to one of the following equations:.

Swift $\bar{\sigma} = K(\bar{\epsilon}_0 + \bar{\epsilon})^n$

Voce $\bar{\sigma} = A - B \exp(-C\bar{\epsilon})$

Where $\bar{\sigma}$ and $\bar{\epsilon}$ are the effective stress and effective strain respectively, and the rest of the variables are constants that are obtained from the by fitting the above equations of to experimental results from a tensile test.

3.5: Material constants

The material constants, needed for the implementation of the code for aluminum alloy Al2008-T4, describes for both two hardening rules and yield functions are presented below. The imperfection parameter is assumed to be 0.996.

Swift		Voce	
$\bar{\epsilon}_0$	0.002	A	447 Mpa
n	0.1882	B	248 Mpa
K	456.82 Mpa	C	4.3 Mpa

Table 1: Constants for voce and swift equation – Al2008T4

Yield96

a	c ₁	c ₂	c ₃	c ₆	α_x	α_y	α_{z0}	α_{z1}
8	1.1227	0.9154	1.0137	1.0578	1.55	1.75	1.0	0.6

Table 2: Constants for Barlat Yield96 – Al2008T4

Yield2000

a	α_1	α_2	α_3	α_4	α_5	α_6	α_7	α_8
8	0.91777	1.14937	1.17360	1.14556	1.05262	1.19593	1.01494	1.06188

Table 3: Constants for Barlat Yield2000 – Al2008T4

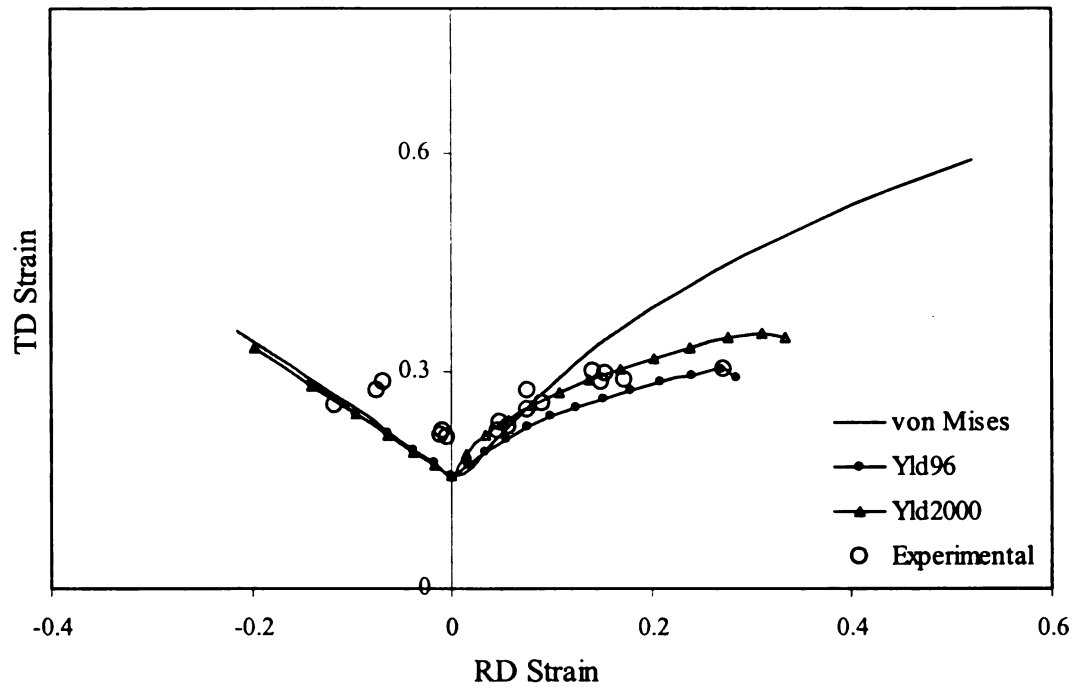


Fig18: Flds Al2008-T4-Swift

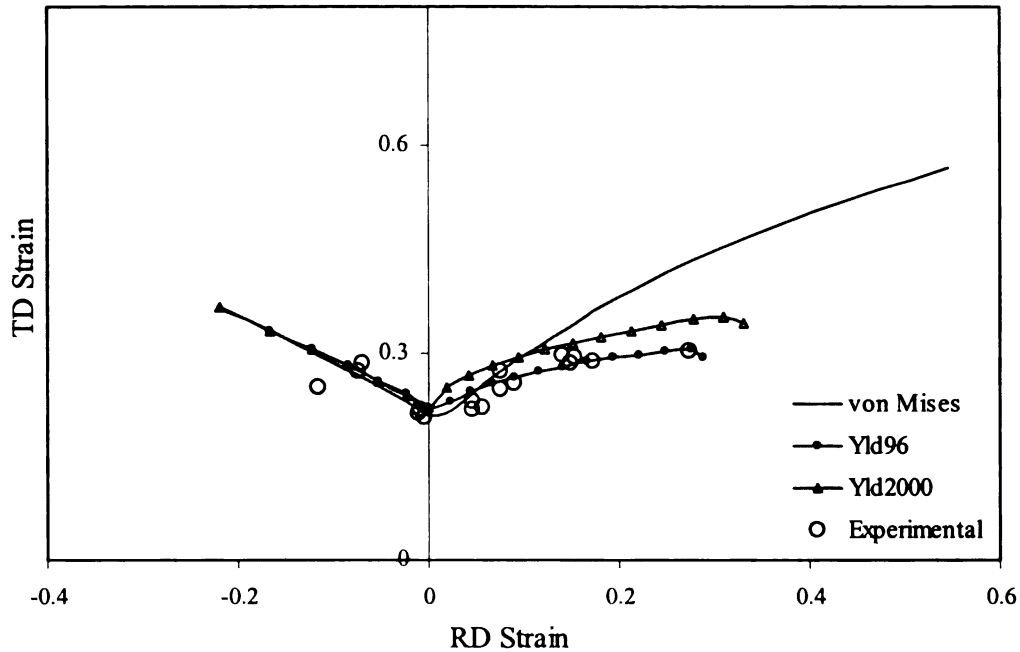


Fig19: Flds Al2008-T4-Voce

3.6: Results and discussions

The experimental and the computed FLDs for von Mises, Barlat Yld96 and Barlat Yld2000 yield function for Al2008-T4 using Swift hardening law are shown in Fig-18. It can be seen that von Mises over predicted the experimentally measured failure points due to the anisotropy of the metal and Yld96 and Yld2000 have given good result with Yld96 being more conservative. In Fig-18 the FLDs are calculated using Swift hardening rule where it is shown that failure points are well predicted when Yield96 describes the yield locus. For Al2008-T4 it is best to use Yield96 to describe the yield locus and Swift equation to model the hardening of the material to be on the safe side.

CHAPTER 4: Simulation of Stretch Stamping

4.1: Introduction

In many sheet metal forming simulations a forming limit diagram is used to predict failure. In sheet metal stamping simulations, the minor and major strains for all elements are plotted against each other and compared with the FLD curve to check whether or not strains at any element exceed the limiting curve. Usually the FLD is generated using both Voce and Swift hardening rule, and the most conservative one is used to check for the possibility of the development of a localized neck in the sheet. In this chapter stretch stamping of a sheet metal with a hemispherical punch is simulated using LS-DYNA and the major and minor strains of all the elements are tracked through the simulation to see how and when they exceed its the FLD limit. When one element reaches the limiting curve the simulation is stopped and the limiting punch depth is recorded. Although strain paths in this type of simulations are not linear, FLDs are still used to check for necking and has been shown to provide qualitatively accurate and failure valid prediction. The pure-stretch stamping test of a Al2008-T4 sheet using a hemispherical punch has been done experimentally and initial failure occurred was recorded for at a punch depth of 27mm at room temperature. The finite element simulation result with LS-DYNA also matched the experimental results, where it was showed predicted that the first time the strains in a first element exceeded reached the FLD, occurred when the punch depth was at 27mm.

4.2: Description of the model

In pure-stretch stamping, a sheet is clamped between a die and the blank holder, and a punch is used to deform the sheet. A full model has been used in this simulation to capture the anisotropic behavior of the AA3003-H111 sheet. The dimensions of the part, the material properties, and all the data needed to model this experiment is described below.

4.2.1: Part dimensions

The punch diameter is 4 in. The diameter of the die and the blank holder is 7.5 in. The diameter of the sheet is 3.54 in. The height of the die is 1.81 in, and 0.4 in for the blank holder. The radius of the surface of the die and blank holder parallel to the sheet is 2.3 in. The sheet is assumed to have a uniform thickness of 1mm. The initial distance between the die and the sheet was taken to be zero, and the one between the blank holder and the sheet was taken to be 0.5 mm to prevent any initial penetration. The initial distance between the punch and the sheet was assumed to be 3 mm. The radial distance between the punch and the blank holder was assumed to be 0.7mm.

4.2.2: Element type and material models

For all parts the "Belytschko-Tsay" (Shell element number 2 in LS-DYNA) was used, using 5 integration points for the sheet and 2 for the rest of the

parts. The punch and the blank holder were modeled as rigid materials "Steel" (material number 33 in LS-DYNA) and constrained to move only normal to the initial plane of the sheet. The die was modeled as rigid "Steel" but constrained to remain fixed during the forming process. The sheet was aluminum AA3003-H111 and modeled to follow BARLAT-YLD2000 (user material option has been used) and assumed to follow Voce and Swift hardening rule.

4.2.3: Material properties

The material constants for the aluminum and steel are shown below.

Steel	
Young's modulus	210 GPA
Poisson ratio	0.3
Density	7850 kg/m ³

Table 4: Properties of Steel

AA3003-H111	
Young's modulus	70 GPA
Poisson ratio	0.33
Density	2710 kg/m ³

Table 5: Properties of AA3003-H111

Voce	
A	144.015 MPA
B	76.217 MPA
C	13.058

Table 6: Constants for voce equation

Swift	
$\bar{\epsilon}_0$	5.7E-4
n	0.2157
k	199.82 MPa

Table 7: Constants for swift equation

Barlat YLD2000	
a	8
L_{111}	0.620325624
L_{112}	-0.310162812
L_{121}	-0.365965106
L_{122}	0.731930213
L_{166}	1.069533131
L_{211}	0.681653114
L_{212}	-0.331290171
L_{221}	-0.350798498
L_{222}	0.699980075
L_{266}	1.06096938

Table 8: Constants for Barlat Yield2000

See Appendix C for the definition of the parameters above.

4.2.4: Constraints and prescribed motion

The die was assumed to follow a prescribed motion (displacement), with the corresponding curve shown below. A force was applied on the blank holder to clamp the sheet to the die. This force started from zero and increased until it reaches 450 kN after nine seconds. A plot that describes this motion is shown below. A picture of the full model is also shown below.

AA3003-H111, RT

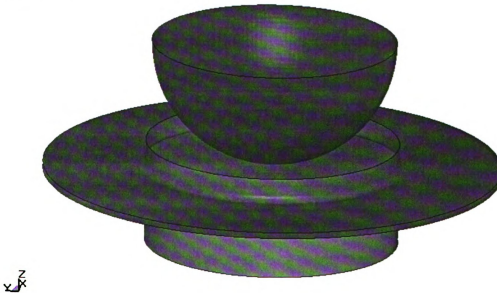


Fig20: Model stretch stamping

The plots show the prescribed motion for the punch and the blank holding force.

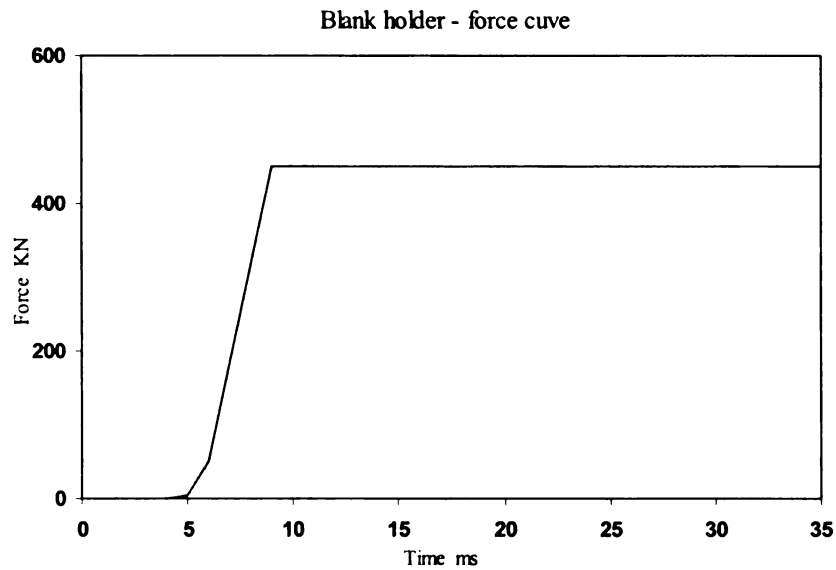


Fig21: Blank holder force

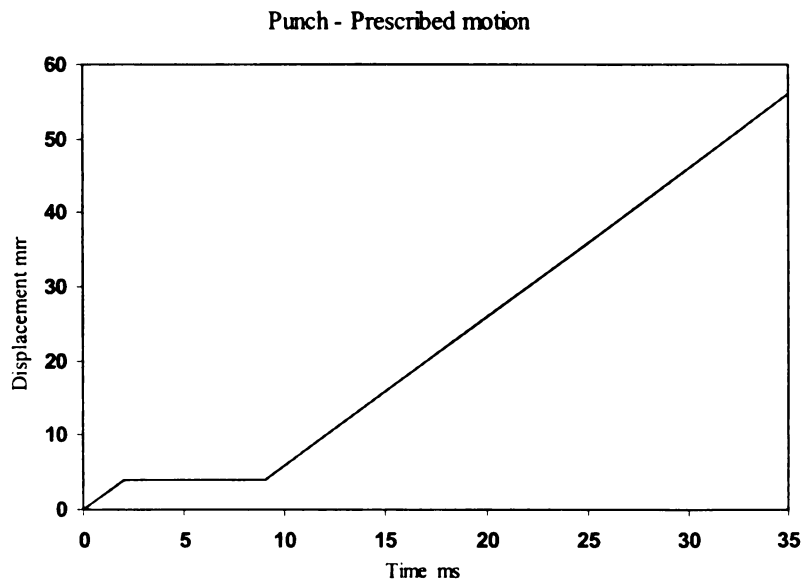


Fig22: Punch prescribed motion

4.3: LS-DYNA

After calculating the theoretical FLDs of the material using the program described in the previous chapter for both voce and swift hardening rules, the more conservative curve was used to check for failure.

4.3.1: Swift and Voce FLDs

For this case the FLD curve based on voce hardening rule turned out to give the more conservative FLD. Both FLDs were calculated with the larger principal strain assumed to occur in the TD direction to be on the safe side. The material coefficients for swift and voce hardening rules are written in the previous paragraph. Both voce and swift FLDs are plotted below.

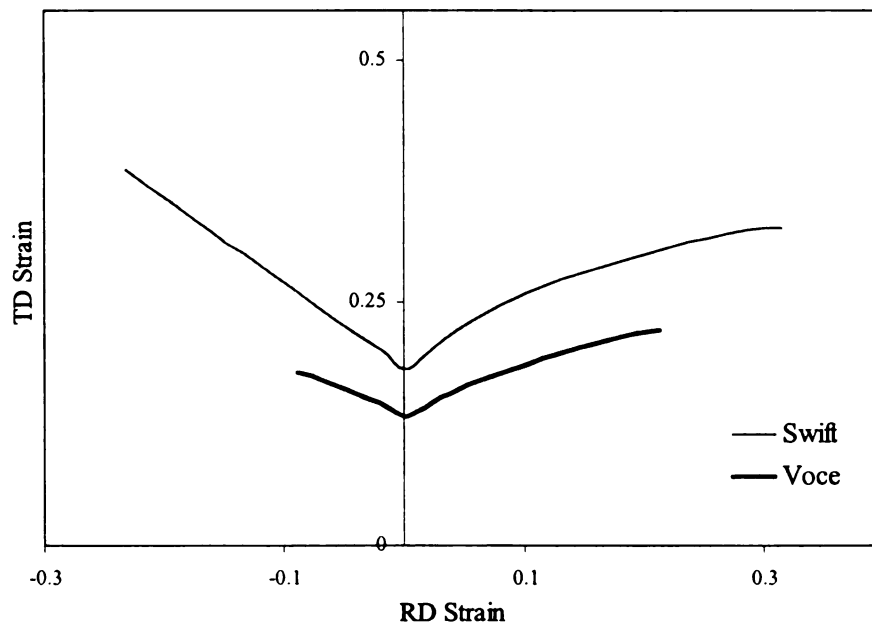


Fig23: FLD AA3003-H111

4.3.2: LS-DYNA FLDs

In LS-DYNA, to open a FLD curve, the points should be copied to a txt file starting with "fld_user" and followed by double the number of points. For AA3003-H111 the input FLD file is written below. In LS-DYNA the curve is read and plotted. The FLD curve used in this simulation is plotted below. Usually many curves are generated from one data input file. The most useful ones are the original FLD (Cracks curve) and a more conservative (Risk of Cracks curve) curve to keep one on the safe side. Materials are considered to be safe if they do not cross the crack curve. But it is better to be on the safer side where one should not cross the risk of crack curve.

fld_user	
36	
0.178426	-0.08921
0.168248	-0.0673
0.159279	-0.04778
0.150106	-0.03002
0.141445	-0.01414
0.132489	0.000002
0.137678	0.011016
0.147395	0.023585
0.15628	0.03751
0.1647	0.052713
0.172611	0.069068
0.180267	0.086552
0.187824	0.105236
0.195239	0.124963
0.202804	0.146084
0.210229	0.168262
0.217641	0.191588
0.221089	0.212251

Table9: FLD user

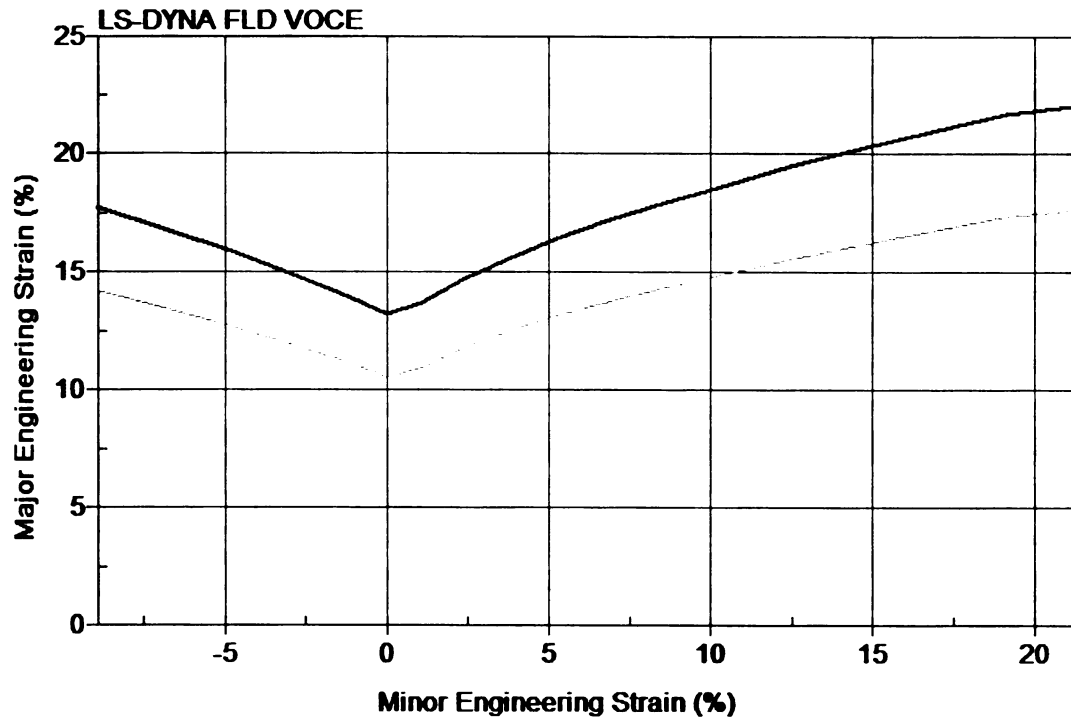


Fig24: LS-DYNA FLDs

4.4: Experimental Findings

The pure-stretch stamping test has been performed in the lab at room temperature. The picture below shows the fractured sheet at the depth of 27 mm.

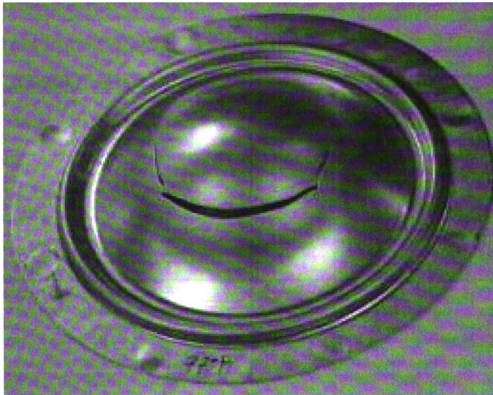


Fig25: Failed sheet

4.5: Numerical results

The computer simulation was stopped when the strains in an element crossed the forming limit diagram. The plots below show the strain distribution of all elements after 10 ms and at the failure time (23 ms). The strain paths range from plane strain for the element on the sides of the sheets to biaxial tension for the elements in the middle of the sheet. The displacement of the middle element is plotted too.

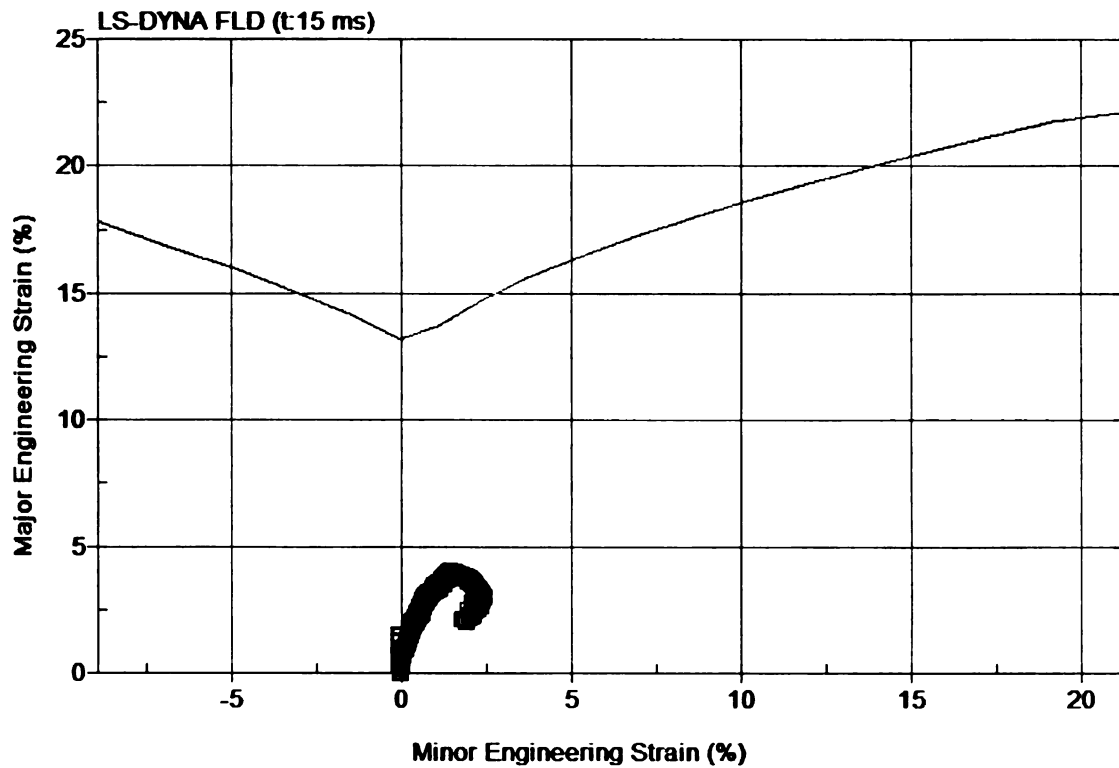


Fig26: FLD AA3003-H111 (t: 15 ms)

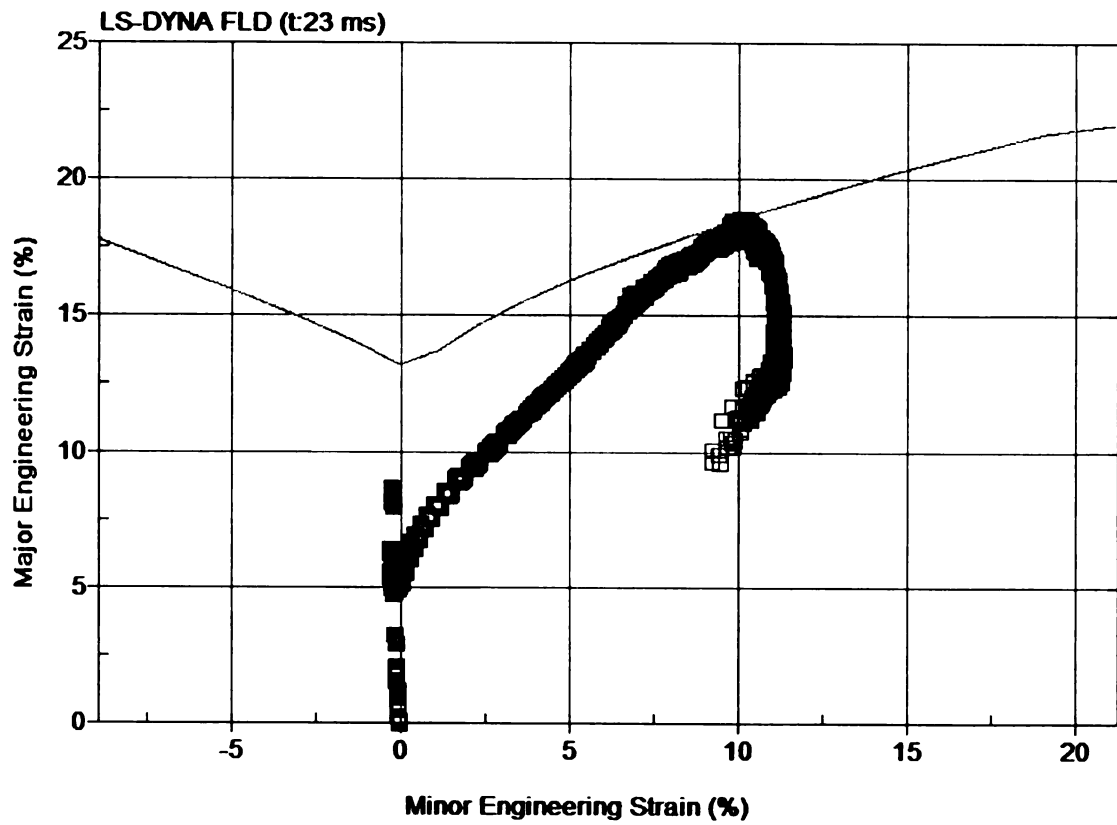


Fig27: FLD AA3003-H111 (t: 23 ms)

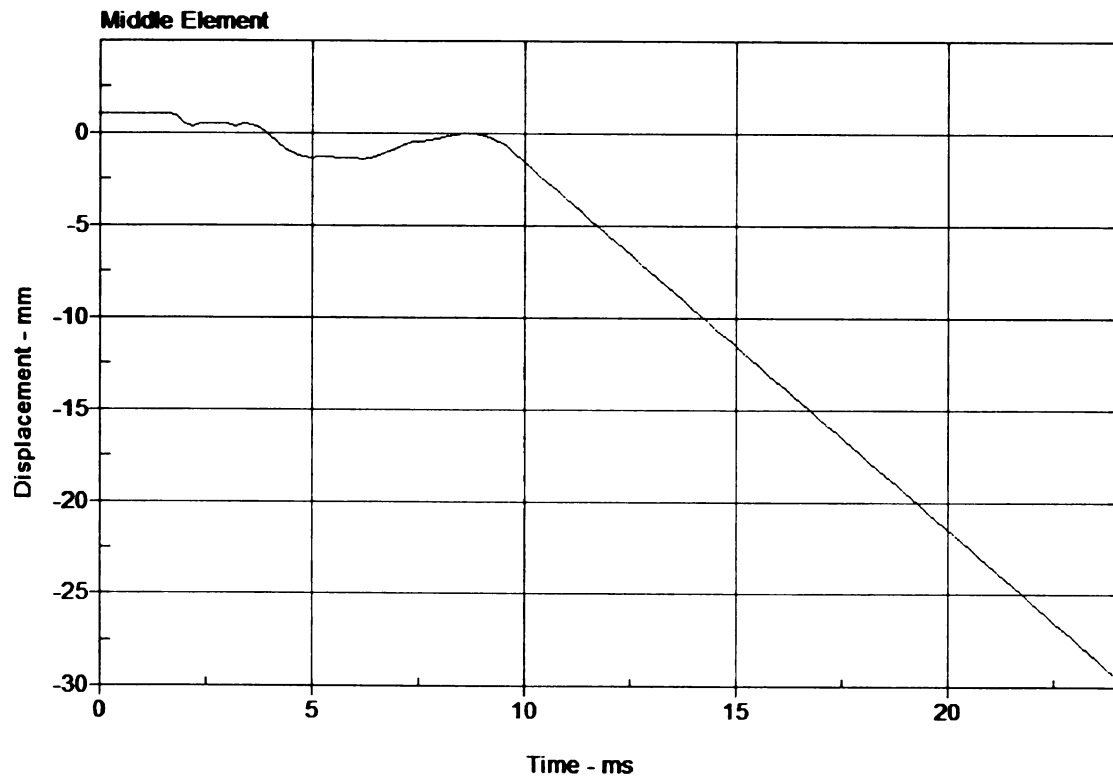
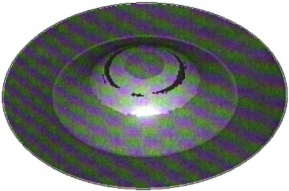


Fig28: Middle Element Displacement

The failure of the sheet has been found to take place in the cracks region in the picture below. In this region the strains in some of the elements have crossed the forming limit diagram and the sheet was considered to have failed due to the onset of localized necking. A comparison of the actual and predicted failed sheets indicates that a good qualitative match exists between experimental and numerical results.

Contours of Formability: Mid. Surface



Formability key

Cracks	
Risk of cracks	
Severe thinning	
Good	
Inadequate stretch	
Wrinkling tendency	
Wrinkles	



Fig29: FLD contour at failure

CHAPTER 5: Conclusions

The experimental and numerical results were very close. First the punch depth at failure was 27 mm for the experiment and 29 mm for the numerical simulation, which corresponds to a discrepancy of only 7.5%. More importantly, the failure occurred in the same location in both experimental and numerical simulation. The results below show that forming limits are a very useful tool to check for the failure of stamped sheet metals in numerical simulations. Experimental and numerical findings are compared in the picture below. The discrepancy between experimental and numerical findings can be a cause of some accuracy issues or even because the strain paths in the sheet are not linear for all elements. For elements on the boundary of the deformed sheet the deformation could be assumed to be linear, however it becomes non-linear as one moves toward the center of the sheet. For the pure-stretch stamping it has been shown [10,11] that the FLD lowers, in comparison with linear path deformations. This might explain the difference between the actual and predicted punch depth at the time of failure. And finally in all the numerical simulations we have assumed failure occurs when the strain increment in the imperfection region is ten times the value of the strain in the homogeneous region, and by assuming an imperfection parameter. All these assumptions could be partially responsible for the discrepancy between the experimental and numerical failure punch height.

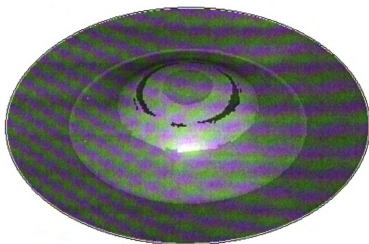


Fig30: Experimental and numerical findings

Appendix A – von Mises

It is the yield function that describes the yield locus of isotropic material.

A1 - Yield function-2d

$$\phi = \bar{\sigma}^2 = \sigma_{xx}^2 + \sigma_{yy}^2 + 3\sigma_{xy}^2 - \sigma_{xx}\sigma_{yy}$$

Where $\bar{\sigma}$ is the yield stress and the variables are the Cauchy stresses in the x-direction, y direction and the shear stress respectively.

A2 - First derivative

$$2\bar{\sigma} \frac{\partial \bar{\sigma}}{\partial \sigma_n} = \frac{\partial \phi}{\partial \sigma_n}$$

Where n refers to xx, yy and xy.

$$\frac{\partial \phi}{\partial \sigma_{xx}} = 2\sigma_{xx} - \sigma_{yy}$$

$$\frac{\partial \phi}{\partial \sigma_{yy}} = 2\sigma_{yy} - \sigma_{xx}$$

$$\frac{\partial \phi}{\partial \sigma_{xy}} = 6\sigma_{xy}$$

A3 - Second derivative

$$\frac{\partial^2 \bar{\sigma}}{\partial \sigma_n \partial \sigma_p} = \frac{1}{\bar{\sigma}} \left(\frac{1}{2} \frac{\partial^2 \phi}{\partial \sigma_n \partial \sigma_p} - \frac{\partial \bar{\sigma}}{\partial \sigma_n} \otimes \frac{\partial \bar{\sigma}}{\partial \sigma_p} \right)$$

Where n and p refer to xx, yy and xy.

$$\frac{\partial^2 \phi}{\partial \sigma_{xx}^2} = 2$$

$$\frac{\partial^2 \phi}{\partial \sigma_{yy}^2} = 2$$

$$\frac{\partial^2 \phi}{\partial \sigma_{xy}^2} = 6$$

$$\frac{\partial^2 \phi}{\partial \sigma_{xx} \partial \sigma_{yy}} = \frac{\partial^2 \phi}{\partial \sigma_{yy} \partial \sigma_{xx}} = -1$$

$$\frac{\partial^2 \phi}{\partial \sigma_{xx} \partial \sigma_{xy}} = \frac{\partial^2 \phi}{\partial \sigma_{xy} \partial \sigma_{xx}} = 0$$

$$\frac{\partial^2 \phi}{\partial \sigma_{yy} \partial \sigma_{xy}} = \frac{\partial^2 \phi}{\partial \sigma_{xy} \partial \sigma_{yy}} = 0$$

Appendix B – Barlat Yld96

Many yield functions have been developed to describe more accurately the plastic anisotropy of sheet metals. Barlat Yld96 [18] has proven to describe well the plastic behavior of aluminum alloys. Although its convexity cannot be proven and has complex derivatives, which make it a difficult yield function to implement, it describes the behavior of aluminum alloys very accurately.

B1 - Yield function-2d

$$\phi = 2\bar{\sigma}^a = \alpha_1 |S_2 - S_3|^a + \alpha_2 |S_3 - S_1|^a + \alpha_3 |S_1 - S_2|^a$$

Where S_1 , S_2 and S_3 are the eigen values of a stress tensor modified by a linear operator to obtain anisotropic properties, $\bar{\sigma}$ is the measure of the yield surface size, the coefficients α_1 , α_2 and α_3 are three functions of the angle β defined below and the exponent a , connected to the crystal structure, is equal to 6 or 8 for BCC and FCC alloys respectively. For plane stress the eight material coefficients needed to describe the anisotropy are

$$C_1, C_2, C_3, C_6, \alpha_x, \alpha_y, \alpha_{z0}, \alpha_{z1}$$

The stress tensor modified by a linear transformation is:

$$\begin{bmatrix} S_{xx} \\ S_{yy} \\ S_{xy} \end{bmatrix} = \begin{bmatrix} (C_2 + C_3)/3 & -C_3/3 & 0 \\ -C_3/3 & (C_1 + C_3)/3 & 0 \\ 0 & 0 & C_6 \end{bmatrix} \begin{bmatrix} \sigma_{xx} \\ \sigma_{yy} \\ \sigma_{xy} \end{bmatrix}$$

The eigen values of the modified stress tensor are:

$$\begin{aligned} S_1 &= \frac{S_{xx} + S_{yy}}{2} + \sqrt{\left(\frac{S_{xx} - S_{yy}}{2}\right)^2 + S_{xy}^2} \\ S_2 &= \frac{S_{xx} + S_{yy}}{2} - \sqrt{\left(\frac{S_{xx} - S_{yy}}{2}\right)^2 + S_{xy}^2} \\ S_3 &= -(S_1 + S_2) \end{aligned}$$

The coefficients α_k are found using the transformation rule:

$$\begin{aligned} \alpha_1 &= \alpha_x \cos^2 \beta + \alpha_y \sin^2 \beta \\ \alpha_2 &= \alpha_x \sin^2 \beta + \alpha_y \cos^2 \beta \\ \alpha_3 &= \alpha_{z0} \cos^2 2\beta + \alpha_{z1} \sin^2 2\beta \end{aligned}$$

Where β is the angle between the x direction (rolling direction) and the direction associated with principal stress S_1 and is found easily by:

$$2\beta = \tan^{-1} \left(\frac{2S_{xy}}{S_{xx} - S_{yy}} \right)$$

B2 - First derivative

$$\frac{\partial \phi}{\partial \sigma_n} = \frac{\partial \phi}{\partial S_i} \times \frac{\partial S_i}{\partial S_p} \times \frac{\partial S_p}{\partial \sigma_n} + \frac{\partial \phi}{\partial \alpha_i} \times \frac{\partial \alpha_i}{\partial \beta} \times \frac{\partial \beta}{\partial S_p} \times \frac{\partial S_p}{\partial \sigma_n}$$

$$2a\bar{\sigma}^{a-1} \frac{\partial \bar{\sigma}}{\partial \sigma_n} = \frac{\partial \phi}{\partial \sigma_n}$$

Where n and p denote xx, yy and xy and i for 1, 2 and 3.

$$\frac{\partial \phi}{\partial S_1} = -a\alpha_2 |S_3 - S_1|^{a-1} \text{sign}(S_3 - S_1) + a\alpha_3 |S_1 - S_2|^{a-1} \text{sign}(S_1 - S_2)$$

$$\frac{\partial \phi}{\partial S_2} = a\alpha_1 |S_2 - S_3|^{a-1} \text{sign}(S_2 - S_3) - a\alpha_3 |S_1 - S_2|^{a-1} \text{sign}(S_1 - S_2)$$

$$\frac{\partial \phi}{\partial S_3} = -a\alpha_1 |S_2 - S_3|^{a-1} \text{sign}(S_2 - S_3) + a\alpha_2 |S_3 - S_1|^{a-1} \text{sign}(S_3 - S_1)$$

If Δ is not equal to zero the derivative below can be calculated:

$$\Delta = (S_{xx} - S_{yy})^2 + 4S_{xy}^2$$

$$\frac{\partial S_1}{\partial S_{xx}} = \frac{1}{2} \left(1 + \frac{S_{xx} - S_{yy}}{\sqrt{\Delta}} \right) \quad \frac{\partial S_2}{\partial S_{xx}} = \frac{1}{2} \left(1 - \frac{S_{xx} - S_{yy}}{\sqrt{\Delta}} \right) \quad \frac{\partial S_3}{\partial S_{xx}} = - \left(\frac{\partial S_1}{\partial S_{xx}} + \frac{\partial S_2}{\partial S_{xx}} \right) = -1$$

$$\frac{\partial S_1}{\partial S_{yy}} = \frac{1}{2} \left(1 - \frac{S_{xx} - S_{yy}}{\sqrt{\Delta}} \right) \quad \frac{\partial S_2}{\partial S_{yy}} = \frac{1}{2} \left(1 + \frac{S_{xx} - S_{yy}}{\sqrt{\Delta}} \right) \quad \frac{\partial S_3}{\partial S_{yy}} = - \left(\frac{\partial S_1}{\partial S_{yy}} + \frac{\partial S_2}{\partial S_{yy}} \right) = -1$$

$$\frac{\partial S_1}{\partial S_{xy}} = \frac{2S_{xy}}{\sqrt{\Delta}} \quad \frac{\partial S_2}{\partial S_{xy}} = -\frac{2S_{xy}}{\sqrt{\Delta}} \quad \frac{\partial S_3}{\partial S_{xy}} = - \left(\frac{\partial S_1}{\partial S_{xy}} + \frac{\partial S_2}{\partial S_{xy}} \right) = 0$$

$$\frac{\partial S_n}{\partial \sigma_p} = L$$

Where L is the linear transformation matrix used to find the modified stress tensor.

$$L = \begin{bmatrix} (C_2 + C_3)/3 & -C_3/3 & 0 \\ -C_3/3 & (C_1 + C_3)/3 & 0 \\ 0 & 0 & C_6 \end{bmatrix}$$

$$\begin{array}{lll} \frac{\partial \phi}{\partial \alpha_1} = |S_2 - S_3|^a & \frac{\partial \alpha_1}{\partial \beta} = (\alpha_y - \alpha_x) \sin 2\beta & \frac{\partial \beta}{\partial S_{xx}} = -\frac{S_{xy}}{\Delta} \\ \frac{\partial \phi}{\partial \alpha_2} = |S_3 - S_1|^a & \frac{\partial \alpha_2}{\partial \beta} = (\alpha_x - \alpha_y) \sin 2\beta & \frac{\partial \beta}{\partial S_{yy}} = \frac{S_{xy}}{\Delta} \\ \frac{\partial \phi}{\partial \alpha_3} = |S_1 - S_2|^a & \frac{\partial \alpha_3}{\partial \beta} = 2(\alpha_{z1} - \alpha_{z0}) \sin 4\beta & \frac{\partial \beta}{\partial S_{xy}} = \frac{S_{xx} - S_{yy}}{\Delta} \end{array}$$

B3 - Second derivative

$$\begin{aligned} \frac{\partial^2 \phi}{\partial \sigma_n \partial \sigma_q} &= \frac{\partial^2 \phi}{\partial S_i \partial \sigma_q} \times \frac{\partial S_i}{\partial S_p} \times \frac{\partial S_p}{\partial \sigma_n} + \frac{\partial \phi}{\partial S_i} \times \frac{\partial^2 S_i}{\partial S_p \partial \sigma_q} \times \frac{\partial S_p}{\partial \sigma_n} + \frac{\partial^2 \phi}{\partial \alpha_i \partial \sigma_q} \times \frac{\partial \alpha_i}{\partial \beta} \times \frac{\partial \beta}{\partial S_p} \times \frac{\partial S_p}{\partial \sigma_n} \\ &+ \frac{\partial \phi}{\partial \alpha_i} \times \frac{\partial^2 \alpha_i}{\partial \beta \partial \sigma_q} \times \frac{\partial \beta}{\partial S_p} \times \frac{\partial S_p}{\partial \sigma_n} + \frac{\partial \phi}{\partial \alpha_i} \times \frac{\partial \alpha_i}{\partial \beta} \times \frac{\partial^2 \beta}{\partial S_p \partial \sigma_q} \times \frac{\partial S_p}{\partial \sigma_n} \end{aligned}$$

$$\frac{\partial^2 \bar{\sigma}}{\partial \sigma_n \partial \sigma_p} = \bar{\sigma}^{1-a} \left(\frac{1}{2a} \frac{\partial^2 \phi}{\partial \sigma_n \partial \sigma_p} - (a-1) \frac{\partial \bar{\sigma}}{\partial \sigma_n} \otimes \frac{\partial \bar{\sigma}}{\partial \sigma_p} \bar{\sigma}^{a-2} \right)$$

Indices n, p and q denote for xx, yy and xy and i for 1, 2 and 3.

Where:

$$\frac{\partial^2 \phi}{\partial S_i \partial \sigma_q} = \frac{\partial^2 \phi}{\partial S_i^2} \times \frac{\partial S_i}{\partial S_p} \times \frac{\partial S_p}{\partial \sigma_q} + \frac{\partial^2 \phi}{\partial S_i \partial \alpha_j} \times \frac{\partial \alpha_j}{\partial \beta} \times \frac{\partial \beta}{\partial S_p} \times \frac{\partial S_p}{\partial \sigma_q}$$

$$\frac{\partial^2 S_i}{\partial S_p \partial \sigma_q} = \frac{\partial^2 S_i}{\partial S_p \partial S_n} \times \frac{\partial S_n}{\partial \sigma_q}$$

$$\frac{\partial^2 \phi}{\partial \alpha_i \partial \sigma_q} = \frac{\partial^2 \phi}{\partial \alpha_i \partial S_j} \times \frac{\partial S_j}{\partial S_p} \times \frac{\partial S_p}{\partial \sigma_q}$$

$$\frac{\partial^2 \alpha_i}{\partial \beta \partial \sigma_q} = \frac{\partial^2 \alpha_i}{\partial \beta^2} \times \frac{\partial \beta}{\partial S_p} \times \frac{\partial S_p}{\partial \sigma_q}$$

$$\frac{\partial^2 \beta}{\partial S_p \partial \sigma_q} = \frac{\partial^2 \beta}{\partial S_p \partial S_n} \times \frac{\partial S_n}{\partial \sigma_q}$$

- The second derivative of function ϕ with respect to S_i is a symmetric second order tensor (3-3). The elements of this tensor are written below.

$$\begin{aligned} \frac{\partial^2 \phi}{\partial S_1^2} &= a(a-1)(\alpha_2 |S_3 - S_1|^{a-2} + \alpha_3 |S_1 - S_2|^{a-2}) & \frac{\partial^2 \phi}{\partial S_1 \partial S_2} &= \frac{\partial^2 \phi}{\partial S_2 \partial S_1} = -a(a-1)\alpha_3 |S_1 - S_2|^{a-2} \\ \frac{\partial^2 \phi}{\partial S_2^2} &= a(a-1)(\alpha_1 |S_2 - S_3|^{a-2} + \alpha_3 |S_1 - S_2|^{a-2}) & \frac{\partial^2 \phi}{\partial S_1 \partial S_3} &= \frac{\partial^2 \phi}{\partial S_3 \partial S_1} = -a(a-1)\alpha_2 |S_3 - S_1|^{a-2} \\ \frac{\partial^2 \phi}{\partial S_3^2} &= a(a-1)(\alpha_1 |S_2 - S_3|^{a-2} + \alpha_2 |S_3 - S_1|^{a-2}) & \frac{\partial^2 \phi}{\partial S_2 \partial S_3} &= \frac{\partial^2 \phi}{\partial S_3 \partial S_2} = -a(a-1)\alpha_1 |S_2 - S_3|^{a-2} \end{aligned}$$

- The second derivative of the stress vector S_i with respect to the stress tensor S_n is a third order tensor (3-3-3). The non-zero elements of this matrix are derived below.

$$\begin{aligned}\frac{\partial^2 S_1}{\partial S_{xx}^2} &= -\frac{\partial^2 S_1}{\partial S_{xx} \partial S_{yy}} = -\frac{\partial^2 S_1}{\partial S_{yy} \partial S_{xx}} = \frac{\partial^2 S_1}{\partial S_{yy}^2} = \frac{1}{2} \left(\left(\sqrt{\Delta} - \frac{(S_{xx} - S_{yy})^2}{\sqrt{\Delta}} \right) / \Delta \right) \\ \frac{\partial^2 S_1}{\partial S_{xx} \partial S_{xy}} &= -\frac{\partial^2 S_1}{\partial S_{yy} \partial S_{xy}} = \frac{\partial^2 S_1}{\partial S_{xy} \partial S_{xx}} = -\frac{\partial^2 S_1}{\partial S_{xy} \partial S_{yy}} = -2 \left(\left(\frac{(S_{xx} - S_{yy}) S_{xy}}{\sqrt{\Delta}} \right) / \Delta \right) \\ \frac{\partial^2 S_1}{\partial S_{xy}^2} &= 2 \left(\left(\sqrt{\Delta} - \frac{4S_{xy}^2}{\sqrt{\Delta}} \right) / \Delta \right)\end{aligned}$$

$$\begin{aligned}\frac{\partial^2 S_2}{\partial S_{xx}^2} &= -\frac{\partial^2 S_2}{\partial S_{xx} \partial S_{yy}} = -\frac{\partial^2 S_2}{\partial S_{yy} \partial S_{xx}} = \frac{\partial^2 S_2}{\partial S_{yy}^2} = -\frac{1}{2} \left(\left(\sqrt{\Delta} - \frac{(S_{xx} - S_{yy})^2}{\sqrt{\Delta}} \right) / \Delta \right) \\ \frac{\partial^2 S_2}{\partial S_{xx} \partial S_{xy}} &= -\frac{\partial^2 S_2}{\partial S_{yy} \partial S_{xy}} = \frac{\partial^2 S_2}{\partial S_{xy} \partial S_{xx}} = -\frac{\partial^2 S_2}{\partial S_{xy} \partial S_{yy}} = 2 \left(\left(\frac{(S_{xx} - S_{yy}) S_{xy}}{\sqrt{\Delta}} \right) / \Delta \right) \\ \frac{\partial^2 S_2}{\partial S_{xy}^2} &= -2 \left(\left(\sqrt{\Delta} - \frac{4S_{xy}^2}{\sqrt{\Delta}} \right) / \Delta \right)\end{aligned}$$

$$\begin{aligned}\frac{\partial^2 S_3}{\partial S_i \partial S_j} &= - \left(\frac{\partial^2 S_1}{\partial S_i \partial S_j} + \frac{\partial^2 S_2}{\partial S_i \partial S_j} \right) \\ \frac{\partial^2 S_3}{\partial S_i \partial S_j} &= 0\end{aligned}$$

- The second derivative of function ϕ with respect to α_i and S_i are second order tensor (3-3). The elements of this tensor are written below.

$$\begin{aligned}\frac{\partial^2 \phi}{\partial \alpha_1 \partial S_1} &= 0 & \frac{\partial^2 \phi}{\partial \alpha_1 \partial S_2} &= -\frac{\partial^2 \phi}{\partial \alpha_1 \partial S_3} = a|S_2 - S_3|^{a-1} \text{sign}(S_2 - S_3) \\ \frac{\partial^2 \phi}{\partial \alpha_2 \partial S_2} &= 0 & \frac{\partial^2 \phi}{\partial \alpha_2 \partial S_3} &= -\frac{\partial^2 \phi}{\partial \alpha_2 \partial S_1} = a|S_3 - S_1|^{a-1} \text{sign}(S_3 - S_1) \\ \frac{\partial^2 \phi}{\partial \alpha_3 \partial S_3} &= 0 & \frac{\partial^2 \phi}{\partial \alpha_3 \partial S_1} &= -\frac{\partial^2 \phi}{\partial \alpha_3 \partial S_2} = a|S_1 - S_2|^{a-1} \text{sign}(S_1 - S_2)\end{aligned}$$

- The second derivative of function α_i with respect to β is a vector of three elements derived below.

$$\begin{aligned}\frac{\partial^2 \alpha_1}{\partial \beta^2} &= 2(\alpha_y - \alpha_x) \cos 2\beta \\ \frac{\partial^2 \alpha_2}{\partial \beta^2} &= 2(\alpha_x - \alpha_y) \cos 2\beta \\ \frac{\partial^2 \alpha_3}{\partial \beta^2} &= 8(\alpha_{z1} - \alpha_{z0}) \cos 4\beta\end{aligned}$$

- The second derivative of the angle β with respect to S_p is a second order tensor (3-3).

$$\begin{aligned}\frac{\partial^2 \beta}{\partial S_{xx}^2} &= \frac{\partial^2 \beta}{\partial S_{yy}^2} = 2 \frac{S_{xy}(S_{xx} - S_{yy})}{\Delta^2} \\ \frac{\partial^2 \beta}{\partial S_{xx} \partial S_{yy}} &= \frac{\partial^2 \beta}{\partial S_{yy} \partial S_{xx}} = -2 \frac{S_{xy}(S_{xx} - S_{yy})}{\Delta^2} \\ \frac{\partial^2 \beta}{\partial S_{xx} \partial S_{xy}} &= -\frac{\partial^2 \beta}{\partial S_{yy} \partial S_{xy}} = \frac{-(S_{xx} - S_{yy})^2 + 4S_{xy}}{\Delta^2} \\ \frac{\partial^2 \beta}{\partial S_{xy} \partial S_{xx}} &= -\frac{\partial^2 \beta}{\partial S_{xy} \partial S_{yy}} = \frac{-(S_{xx} - S_{yy})^2 + 4S_{xy}}{\Delta^2} \\ \frac{\partial^2 \beta}{\partial S_{xy}^2} &= -8 \frac{S_{xy}(S_{xx} - S_{yy})}{\Delta^2}\end{aligned}$$

Appendix C - Barlat Yld2000

This is a new yield function that has been proposed by F. Barlat [19] to overcome the weakness of yield96 where it is proven to be convex and its mathematical formulation is less complicated. This yield function was developed for aluminum alloys and has been shown to describe well the anisotropic behavior of aluminum sheets.

C1 - yield function-2d

$$\phi = 2\bar{\sigma}^a = |X_1 - X_2|^a + |2Y_1 + Y_2|^a + |2Y_2 + Y_1|^a$$

Where X_1 , X_2 , Y_1 and Y_2 are the eigenvalues of a stress tensor $\mathbf{X}$ modified by a linear operator $\mathbf{L}$ to obtain anisotropic properties, the exponent a , connected to the crystal structure, is equal to 6 or 8 for BCC and FCC alloys respectively, $\bar{\sigma}$ is the measure of the yield surface size. For plane stress the eight material coefficients needed to describe the anisotropy and the linear operator $\mathbf{L}$ are written below.

$$\alpha_1, \alpha_2, \alpha_3, \alpha_4, \alpha_5, \alpha_6, \alpha_7, \alpha_8$$

$$\begin{bmatrix} L_{111} \\ L_{112} \\ L_{121} \\ L_{122} \\ L_{166} \end{bmatrix} = \begin{bmatrix} 2/3 & 0 & 0 \\ -1/3 & 0 & 0 \\ 0 & -1/3 & 0 \\ 0 & 2/3 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \alpha_1 \\ \alpha_2 \\ \alpha_7 \end{bmatrix} \quad \begin{bmatrix} L_{211} \\ L_{212} \\ L_{221} \\ L_{222} \\ L_{266} \end{bmatrix} = \frac{1}{9} \begin{bmatrix} -2 & 2 & 8 & -2 & 0 \\ 1 & -4 & -4 & 4 & 0 \\ 4 & -4 & -4 & 1 & 0 \\ -2 & 8 & 2 & -2 & 0 \\ 0 & 0 & 0 & 0 & 9 \end{bmatrix} \begin{bmatrix} \alpha_3 \\ \alpha_4 \\ \alpha_5 \\ \alpha_6 \\ \alpha_8 \end{bmatrix}$$

The linear operator **L** is found by a linear transformation from the anisotropy coefficients. The modified stress tensor **X** can be found by a linear transformation with **L**:

$$\mathbf{X} = \mathbf{L}\boldsymbol{\sigma}$$

$$\begin{bmatrix} X_{xx} \\ X_{yy} \\ X_{xy} \\ Y_{xx} \\ Y_{yy} \\ Y_{xy} \end{bmatrix} = \mathbf{L} \begin{bmatrix} \sigma_{xx} \\ \sigma_{yy} \\ \sigma_{xy} \end{bmatrix}$$

The eigenvalues of the modified stress tensor **X** are easily found:

$$X_1 = \frac{X_{xx} + X_{yy}}{2} + \sqrt{\left(\frac{X_{xx} - X_{yy}}{2}\right)^2 + X_{xy}^2} \quad Y_1 = \frac{Y_{xx} + Y_{yy}}{2} + \sqrt{\left(\frac{Y_{xx} - Y_{yy}}{2}\right)^2 + Y_{xy}^2}$$

$$X_2 = \frac{X_{xx} + X_{yy}}{2} - \sqrt{\left(\frac{X_{xx} - X_{yy}}{2}\right)^2 + X_{xy}^2} \quad Y_2 = \frac{Y_{xx} + Y_{yy}}{2} - \sqrt{\left(\frac{Y_{xx} - Y_{yy}}{2}\right)^2 + Y_{xy}^2}$$

C2 - First derivative

$$\frac{\partial \phi}{\partial \sigma_n} = \frac{\partial \phi}{\partial X_i} \times \frac{\partial X_i}{\partial X_p} \times \frac{\partial X_p}{\partial \sigma_n}$$

$$2\bar{\sigma}^{a-1} \frac{\partial \bar{\sigma}}{\partial \sigma_n} = \frac{\partial \phi}{\partial \sigma_n}$$

Where i denote indices for 1, 2 and n, p for xx, yy, xy.

$$\frac{\partial \phi}{\partial X_1} = a|X_1 - X_2|^{a-1} \text{sign}(X_1 - X_2)$$

$$\frac{\partial \phi}{\partial X_2} = -a|X_1 - X_2|^{a-1} \text{sign}(X_1 - X_2)$$

$$\frac{\partial \phi}{\partial Y_1} = 2a|2Y_1 + Y_2|^{a-1} \text{sign}(2Y_1 + Y_2) + a|2Y_2 + Y_1|^{a-1} \text{sign}(2Y_2 + Y_1)$$

$$\frac{\partial \phi}{\partial Y_2} = 2a|2Y_2 + Y_1|^{a-1} \text{sign}(2Y_2 + Y_1) + a|2Y_1 + Y_2|^{a-1} \text{sign}(2Y_1 + Y_2)$$

If Δ_x and Δ_y are not equal to zero the derivative below can be calculated:

$$\Delta_x = (X_{xx} - X_{yy})^2 + 4X_{xy}^2$$

$$\Delta_y = (Y_{xx} - Y_{yy})^2 + 4Y_{xy}^2$$

$$\frac{\partial X_1}{\partial X_{xx}} = \frac{1}{2} \left(1 + \frac{X_{xx} - X_{yy}}{\sqrt{\Delta_x}} \right)$$

$$\frac{\partial X_2}{\partial X_{xx}} = \frac{1}{2} \left(1 - \frac{X_{xx} - X_{yy}}{\sqrt{\Delta_x}} \right)$$

$$\frac{\partial X_1}{\partial X_{yy}} = \frac{1}{2} \left(1 - \frac{X_{xx} - X_{yy}}{\sqrt{\Delta_x}} \right)$$

$$\frac{\partial X_2}{\partial X_{yy}} = \frac{1}{2} \left(1 + \frac{X_{xx} - X_{yy}}{\sqrt{\Delta_x}} \right)$$

$$\frac{\partial X_1}{\partial X_{xy}} = \frac{2X_{xy}}{\sqrt{\Delta_x}}$$

$$\frac{\partial X_2}{\partial X_{xy}} = -\frac{2X_{xy}}{\sqrt{\Delta_x}}$$

$$\frac{\partial Y_1}{\partial Y_{xx}} = \frac{1}{2} \left(1 + \frac{Y_{xx} - Y_{yy}}{\sqrt{\Delta_y}} \right)$$

$$\frac{\partial Y_2}{\partial Y_{xx}} = \frac{1}{2} \left(1 - \frac{Y_{xx} - Y_{yy}}{\sqrt{\Delta_y}} \right)$$

$$\frac{\partial Y_1}{\partial Y_{yy}} = \frac{1}{2} \left(1 - \frac{Y_{xx} - Y_{yy}}{\sqrt{\Delta_y}} \right)$$

$$\frac{\partial Y_2}{\partial Y_{yy}} = \frac{1}{2} \left(1 + \frac{Y_{xx} - Y_{yy}}{\sqrt{\Delta_y}} \right)$$

$$\frac{\partial Y_1}{\partial Y_{xy}} = \frac{2Y_{xy}}{\sqrt{\Delta_y}}$$

$$\frac{\partial Y_2}{\partial Y_{xy}} = -\frac{2Y_{xy}}{\sqrt{\Delta_y}}$$

The first derivative of $\mathbf{X}$ with the stress tensor is equal to $\mathbf{L}$.

$$\frac{\partial X_n}{\partial \sigma_p} = L$$

C3 - Second derivative

$$\frac{\partial^2 \phi}{\partial \sigma_n \partial \sigma_p} = \frac{\partial^2 \phi}{\partial X_i \partial \sigma_p} \times \frac{\partial X_i}{\partial X_q} \times \frac{\partial X_q}{\partial \sigma_n} + \frac{\partial \phi}{\partial X_i} \times \frac{\partial^2 X_i}{\partial X_q \partial \sigma_p} \times \frac{\partial X_q}{\partial \sigma_n}$$

$$\frac{\partial^2 \bar{\sigma}}{\partial \sigma_n \partial \sigma_p} = \bar{\sigma}^{1-a} \left(\frac{1}{2a} \frac{\partial^2 \phi}{\partial \sigma_n \partial \sigma_p} - (a-1) \frac{\partial \bar{\sigma}}{\partial \sigma_n} \otimes \frac{\partial \bar{\sigma}}{\partial \sigma_p} \bar{\sigma}^{a-1} \right)$$

Where:

$$\frac{\partial^2 \phi}{\partial X_i \partial \sigma_p} = \frac{\partial^2 \phi}{\partial X_i \partial X_j} \times \frac{\partial X_j}{\partial X_n} \times \frac{\partial X_n}{\partial \sigma_p}$$

$$\frac{\partial^2 X_i}{\partial X_q \partial \sigma_p} = \frac{\partial^2 X_i}{\partial X_q \partial X_n} \times \frac{\partial X_n}{\partial \sigma_p}$$

- The second derivative of function ϕ with respect to X_i is a symmetric second order tensor (4-4). The non- zero elements are written below.

$$\frac{\partial^2 \phi}{\partial X_1^2} = \frac{\partial^2 \phi}{\partial X_2^2} = a(a-1) |X_1 - X_2|^{a-2}$$

$$\frac{\partial^2 \phi}{\partial X_1 \partial X_2} = \frac{\partial^2 \phi}{\partial X_2 \partial X_1} = -a(a-1) |X_1 - X_2|^{a-2}$$

$$\frac{\partial^2 \phi}{\partial Y_1^2} = a(a-1) \left(4|2Y_1 + Y_2|^{a-2} + |2Y_2 + Y_1|^{a-2} \right)$$

$$\frac{\partial^2 \phi}{\partial Y_2^2} = a(a-1) \left(4|2Y_2 + Y_1|^{a-2} + |2Y_1 + Y_2|^{a-2} \right)$$

$$\frac{\partial^2 \phi}{\partial Y_1 \partial Y_2} = \frac{\partial^2 \phi}{\partial Y_2 \partial Y_1} = 2a(a-1) \left(|2Y_1 + Y_2|^{a-2} + |2Y_2 + Y_1|^{a-2} \right)$$

- The second derivative of the stress vector X_i with respect to X_n is a third order tensor (4-6-6). The non-zero elements are written below.

$$\begin{aligned}
\frac{\partial^2 X_1}{\partial X_{xx}^2} &= -\frac{\partial^2 X_1}{\partial X_{xx} \partial X_{yy}} = -\frac{\partial^2 X_1}{\partial X_{yy} \partial X_{xx}} = \frac{\partial^2 X_1}{\partial X_{yy}^2} = \frac{1}{2} \left(\left(\sqrt{\Delta_x} - \frac{(X_{xx} - X_{yy})^2}{\sqrt{\Delta_x}} \right) / \Delta_x \right) \\
\frac{\partial^2 X_1}{\partial X_{xx} \partial X_{xy}} &= -\frac{\partial^2 X_1}{\partial X_{yy} \partial X_{xy}} = \frac{\partial^2 X_1}{\partial X_{xy} \partial X_{xx}} = -\frac{\partial^2 X_1}{\partial X_{xy} \partial X_{yy}} = -2 \left(\left(\frac{(X_{xx} - X_{yy}) X_{xy}}{\sqrt{\Delta_x}} \right) / \Delta_x \right) \\
\frac{\partial^2 X_1}{\partial X_{xy}^2} &= 2 \left(\left(\sqrt{\Delta_x} - \frac{4X_{xy}^2}{\sqrt{\Delta_x}} \right) / \Delta_x \right) \\
\frac{\partial^2 X_2}{\partial X_{xx}^2} &= -\frac{\partial^2 X_2}{\partial X_{xx} \partial X_{yy}} = -\frac{\partial^2 X_2}{\partial X_{yy} \partial X_{xx}} = \frac{\partial^2 X_2}{\partial X_{yy}^2} = -\frac{1}{2} \left(\left(\sqrt{\Delta_x} - \frac{(X_{xx} - X_{yy})^2}{\sqrt{\Delta_x}} \right) / \Delta_x \right) \\
\frac{\partial^2 X_2}{\partial X_{xx} \partial X_{xy}} &= -\frac{\partial^2 X_2}{\partial X_{yy} \partial X_{xy}} = \frac{\partial^2 X_2}{\partial X_{xy} \partial X_{xx}} = -\frac{\partial^2 X_2}{\partial X_{xy} \partial X_{yy}} = 2 \left(\left(\frac{(X_{xx} - X_{yy}) X_{xy}}{\sqrt{\Delta_x}} \right) / \Delta_x \right) \\
\frac{\partial^2 X_2}{\partial X_{xy}^2} &= -2 \left(\left(\sqrt{\Delta_x} - \frac{4X_{xy}^2}{\sqrt{\Delta_x}} \right) / \Delta_x \right) \\
\frac{\partial^2 Y_1}{\partial Y_{xx}^2} &= -\frac{\partial^2 Y_1}{\partial Y_{xx} \partial Y_{yy}} = -\frac{\partial^2 Y_1}{\partial Y_{yy} \partial Y_{xx}} = \frac{\partial^2 Y_1}{\partial Y_{yy}^2} = \frac{1}{2} \left(\left(\sqrt{\Delta_y} - \frac{(Y_{xx} - Y_{yy})^2}{\sqrt{\Delta_y}} \right) / \Delta_y \right) \\
\frac{\partial^2 Y_1}{\partial Y_{xx} \partial Y_{xy}} &= -\frac{\partial^2 Y_1}{\partial Y_{yy} \partial Y_{xy}} = \frac{\partial^2 Y_1}{\partial Y_{xy} \partial Y_{xx}} = -\frac{\partial^2 Y_1}{\partial Y_{xy} \partial Y_{yy}} = -2 \left(\left(\frac{(Y_{xx} - Y_{yy}) Y_{xy}}{\sqrt{\Delta_y}} \right) / \Delta_y \right) \\
\frac{\partial^2 Y_1}{\partial Y_{xy}^2} &= 2 \left(\left(\sqrt{\Delta_y} - \frac{4Y_{xy}^2}{\sqrt{\Delta_y}} \right) / \Delta_y \right) \\
\frac{\partial^2 Y_2}{\partial Y_{xx}^2} &= -\frac{\partial^2 Y_2}{\partial Y_{xx} \partial Y_{yy}} = -\frac{\partial^2 Y_2}{\partial Y_{yy} \partial Y_{xx}} = \frac{\partial^2 Y_2}{\partial Y_{yy}^2} = -\frac{1}{2} \left(\left(\sqrt{\Delta_y} - \frac{(Y_{xx} - Y_{yy})^2}{\sqrt{\Delta_y}} \right) / \Delta_y \right) \\
\frac{\partial^2 Y_2}{\partial Y_{xx} \partial Y_{xy}} &= -\frac{\partial^2 Y_2}{\partial Y_{yy} \partial Y_{xy}} = \frac{\partial^2 Y_2}{\partial Y_{xy} \partial Y_{xx}} = -\frac{\partial^2 Y_2}{\partial Y_{xy} \partial Y_{yy}} = 2 \left(\left(\frac{(Y_{xx} - Y_{yy}) Y_{xy}}{\sqrt{\Delta_y}} \right) / \Delta_y \right) \\
\frac{\partial^2 Y_2}{\partial Y_{xy}^2} &= -2 \left(\left(\sqrt{\Delta_y} - \frac{4Y_{xy}^2}{\sqrt{\Delta_y}} \right) / \Delta_y \right)
\end{aligned}$$

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