This is to certify that the
dissertation entitled

NONLINEARITY IN CENTRAL BANK INTERVENTION:
EVIDENCE FROM DM/USD MARKET

presented by

JONGBYUNG JUN

has been accepted towards fulfillment
of the requirements for the

Ph.D. degree in Economics

Major Professor's Signature

05.19.2004

Date

PLACE IN RETURN BOX to remove this checkout from your record.
TO AVOID FINES return on or before date due.
MAY BE RECALLED with earlier due date if requested.

DATE DUE	DATE DUE	DATE DUE
OCT 3 1 2006 0 8 2 9 0 6		

NONLINEARITY IN CENTRAL BANK INTERVENTION: EVIDENCE FROM DM/USD MARKET

By

Jongbyung Jun

A DISSERTATION

Submitted to
Michigan State University
in partial fulfillment of the requirements
for the degree of

DOCTOR OF PHILOSOPHY

Department of Economics

2004

ABSTRACT

NONLINEARITY IN CENTRAL BANK INTERVENTION: EVIDENCE FROM DM/USD MARKET

By

Jongbyung Jun

This dissertation consists of three empirical studies on the potential nonlinearity in a central bank's foreign exchange intervention and in the effects of intervention on the exchange rate. One interesting result is that despite general acceptance in the literature, a friction model which assumes a specific type of nonlinearity may not be better than a linear model in explaining intervention behavior. However, a more flexible nonlinear model, i.e. a threshold model, explains intervention better than a linear model. A threshold model is also found to be useful in characterizing the conditions on which intervention becomes effective in countering excessive exchange rate movements.

Under a floating exchange rate system, central banks do not intervene most of the time although exchange rates are fluctuating continuously. A friction model for intervention is based on a hypothesis that intervention occurs if the exchange rate is highly unstable and does not occur otherwise. While previous studies accept this hypothesis, without testing, as an appropriate explanation for the infrequency of intervention, the hypothesis is tested in this study with official daily data on intervention by US and German central banks. If the underlying hypothesis is true, then an economic model built on it (a friction model) must explain the actual

intervention better than a model without such information (a linear model). As reported in Chapter 1, however, the explanatory power of the friction model is lower than that of a linear model in terms of the degree of correlation (R^2) between the observed amount of intervention and the fitted values. This result implies that the core assumption of the friction model may not be true at least on a daily basis.

In Chapter 2, the test is about a similar hypothesis that a central bank's reaction to a low degree of instability is different from its reaction to a high degree of instability. This assumption is weaker than the friction hypothesis in that a central bank is allowed to react to small values, as well as large values, of the explanatory variables. The result is that a model allowing this type of regime-switching (a threshold model) explains intervention significantly better than a linear model. The relative frequency and average size of intervention are both larger on average in a high instability regime than in a low instability regime.

The empirical results in Chapter 3 indicate that intervention can be effective under certain conditions although not effective on average. First, when the size of intervention is large enough to exceed a threshold, such intervention tends to be effective. Secondly, intervention is effective when the short-run upward or downward trend is not too strong to lean against. Finally, if central banks wait for the right timing to break a trend driven by chartists or noise-traders, who make short-term trading decisions based on technical trading rules rather than economic fundamentals, such strategic intervention becomes effective.

TABLE OF CONTENTS

List of Tables

List of Figures

Chapter 1

The Friction Model And Foreign Exchange Intervention

1.1 INTRODUCTION

A central bank does not intervene frequently under a floating exchange rate system. In explaining intervention behavior of a central bank, therefore, it may sound reasonable to say that an approach reflecting this infrequency of intervention must be better than one ignoring such information. The purpose of this chapter is to test a claim in the previous literature that a friction model, which can accommodate the infrequency of intervention in a specific way, is better than a simple linear model in explaining intervention under a floating exchange rate system.

Under a floating exchange rate system, an exchange rate is supposed to be determined by market forces. However, many central banks occasionally have intervened in foreign exchange markets by buying or selling one currency against another. For the monetary authorities of the United States, i.e. the Department of the Treasury and the Federal Reserve, the purpose of foreign exchange intervention during the post-Bretton-Woods era has been "to slow rapid exchange rate moves and to signal the U.S. monetary authorities' view that the exchange rate did not reflect fundamental economic conditions," or in a simpler expression, "to counter disorderly market conditions."[1]

In the light of the purpose of intervention, foreign exchange intervention can be interpreted as a reaction by a central bank to disorderly market conditions. This implies that there can be a stable relationship between a central bank's intervention and some measures of market conditions, which is sometimes called a central bank reaction function. One interesting question about this relationship is whether it has sufficient regularity to be called a function. Unlike the exchange rate, which is determined by interactions among numerous buyers and sellers, intervention in one foreign exchange market is determined by one or two central banks. While the aggregated behavior of buyers and sellers in the market is likely to reveal some regularity or stability, a central bank's discretionary decisions on intervention may be quite arbitrary. Hence the reaction function may not be clearly defined. One reason for modelling and estimating a reaction function, therefore, is to investigate if a central bank's intervention behavior is consistent with the announced purpose of intervention so that a reaction function exists.

One of the main challenges in specifying a reaction function is that it is very difficult to find relevant explanatory variables. The announced purpose of intervention seems to be reasonable as a general description of the U.S. monetary authorities' foreign exchange policy. As a practical matter, however, it is not spe-

[1]For more details, visit www.newyorkfed.org/aboutthefed/fedpoint/fed44.html.

cific enough to guide an empirical researcher who is interested in explaining past intervention in terms of some measurable market conditions, or to guide a foreign exchange dealer who wants to make a prediction about future intervention. Economic theories also fail to provide useful guidelines for empirical researchers who want to find specific measures of 'the rapidity of exchange rate movements' or 'the discrepancy between the market exchange rate and a certain equilibrium rate based on fundamental economic conditions'. Not surprisingly, different empirical studies rely on different measures of the disorderly market conditions.

Another challenge in specifying a reaction function is to find an appropriate way of reflecting the infrequency of intervention. The variable measuring the amount of intervention takes zero values for the majority of the observations, while the explanatory variables are not zero, under a floating exchange rate system. This implies a nonlinear relationship because the amount of intervention does not increase or decrease approximately in proportion to the explanatory variables. One way to proceed is to approximate this potential nonlinear relationship with a linear model. This is the approach in Eijffinger, S. C. W. and A. P. D. Gruijters (1991). Another way is to model the probability of intervention rather than the quantity of intervention using a probit approach as in Baillie and Osterberg (1997) or a logit approach as in Frenkel and Stadtmann (2001). If the interest lies in the quantity of intervention rather than the probability of intervention, an appropriate model may be a Tobit model. Humpage (1999) and Almekinders and Eijffinger (1994) follow this approach.

However, a Tobit model takes either buying intervention or selling intervention, one at a time but not collectively, as the dependent variable. If one wants to explain both types of intervention simultaneously, then the friction model of Rosett (1959) is an available specification for the nonlinear reaction function. This approach allows us to use a data set where the two types of intervention are combined by recording the buying amounts as positive numbers and the selling amounts as negative numbers.

The friction model is an extension of a Tobit model, which is an appropriate model if, among other things, the dependent variable can be regarded as a continuous random variable that is restricted to be nonnegative and has positive probability at zero. A friction model also assumes nonzero probability at zero but the dependent variable is not restricted to be nonnegative. While a Tobit model is closely related to a probit model,[2] a friction model is closely related to an ordered-probit model with two cut-points.[3]

Alternatively, a friction model with y as the dependent variable can be interpreted as a three-regime switching model with $y > 0$, $y = 0$ and $y < 0$ in each regime. In this context, a Tobit model for time-series data is a regime-switching model between a regime of $y = 0$ and the other regime of $y > 0$, while a linear model is a one-regime model.

As an example, Rosett explains that the changes in the asset holdings (y)

[2]In a Tobit model, the dependent variable y is either zero or positive ($y = 0$ or $y > 0$). In a probit model, the dependent variable is either zero or one ($y = 0$ or $y = 1$).

[3]In a friction model, there are three possible outcomes for the dependent variable, i.e. $y < 0$, $y = 0$ or $y > 0$. If each outcome is recorded as $y = -1$, $y = 0$ and $y = 1$, then an appropriate model is an ordered-probit model with two unknown thresholds.

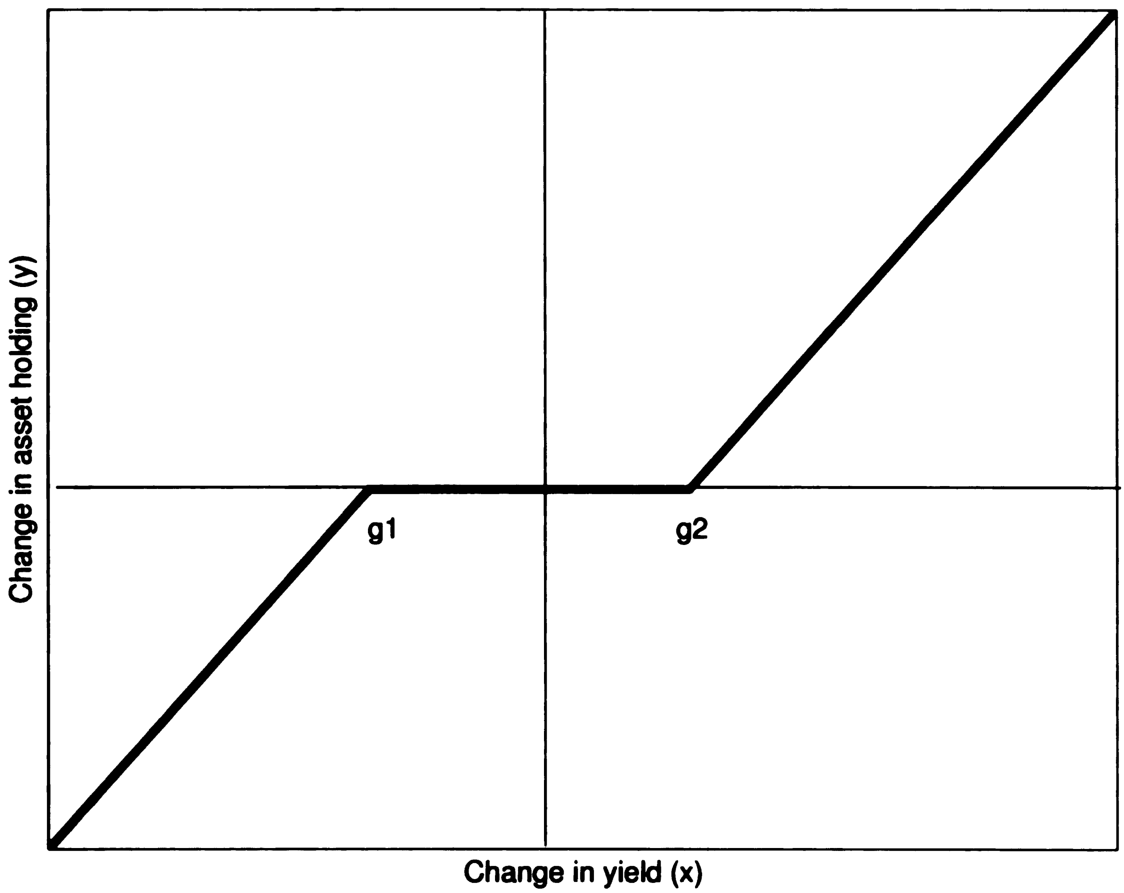


Figure 1.1: Friction model of Rosett

held by a certain class of investors may not respond to small changes in the yield (x) if the transaction cost exceeds the potential gains. The proposed relationship between the two variables is illustrated in Figure 1.1. In this picture, y is insensitive to x when x is relatively small in absolute value, i.e. between $g1$ and $g2$. This insensitivity is called friction. Outside the friction area, the relationship is assumed to be linear.

In the literature of central bank intervention, the friction model is used for the first time by Almekinders and Eijffinger (1996). Their approach is followed by two more recent papers of Kim and Sheen (2002), and Neely (2002). In Rosett's example, the source of friction is the transaction cost. In Almekinders and Eijffinger (1996), the main source of friction is "small realizations of the explanatory variables". The implication of the latter is that the central banks let the exchange rate float freely most of the time and intervene only if the exchange market is in seriously disorderly conditions, which will be signaled by 'large' realizations of the explanatory variables. Based on this assumption, they further claim that ordinary least squares (OLS) estimator for a linear reaction function is biased and inconsistent while a maximum likelihood estimator (MLE) based on a friction model suitably accounts for the infrequency and the resulting nonlinearity of intervention.

These claims of Almekinders et al. would be justified if there was an appropriate size of friction area such as the interval between g1 and g2 in Figure 1.1. However, if the friction area is too small or too large for a given sample, the advantage of the friction model would be insignificant. These possibilities are illustrated in Figure 1.2.[4] When the source of friction is the transaction cost as in Rosett's example, the friction area may be too small as in panel (a) in Figure 1.2. In the case of daily intervention reaction function, because the amount of intervention is zero for 70% or more of the observations,[5] the friction area may be too large as illustrated in panel (b) in Figure 1.2.

Given that the preferability of the friction model depends on the width of the

[4]While Figure 1.1 illustrates a positive relationship, the relationship in Figure 1.2 is negative. For example, when the currency is appreciating rapidly, the central bank will intervene and sell the currency.

[5]The Federal Reserve intervened on 137 days out of 651 (21%) between February 1987 and October 1989, during which the central bank was relatively active in intervention.

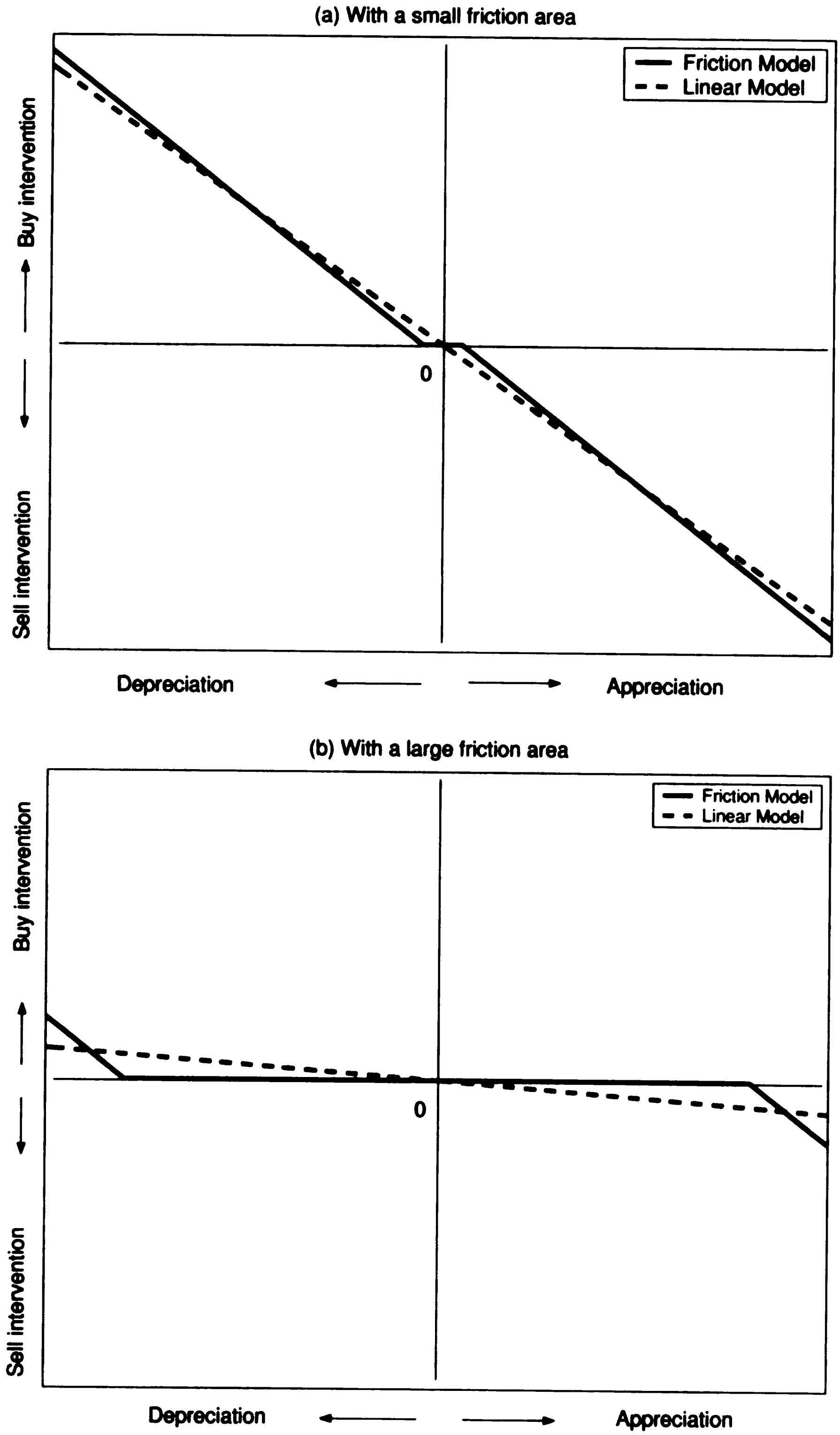


Figure 1.2: Friction models with different thresholds

friction area, it is necessary to test whether the friction model is substantially better than a linear model as a central bank's reaction function. However, the previous literature has failed to provide such a test.

This paper, by contrast, employs a procedure for testing nonlinearity in which the explanatory power of the friction model is compared with that of a simple linear model. The explanatory power is measured as the squared correlation coefficient of the actual quantities of intervention and their fitted values (R^2).[6]

A simulation result reveals that R^2 is a valid criterion for the comparison of explanatory powers in that the friction model tends to have higher R^2 than the linear model when the true data generating process is a friction model with an appropriate degree of friction. However, when the friction model is estimated with real data, with the same explanatory variables and a similar set of data as in Almekinders and Eijffinger (1996), R^2 of the model turns out to be smaller than the R^2 of the linear model, implying that the friction model is not necessarily better than a linear approximation of the reaction function in terms of explanatory power. This result is robust to some, although not exhaustive, variations in sample period and model specification.

To help understand this surprising result, Figure 1.3 depicts the official daily intervention by the Federal Reserve against one of the measures of the disorderly market conditions used in this paper and also in Almekinders and Eijffinger (1996). The sample period is between February 23, 1987 and October 31, 1989. The

[6]This approach is explained in Wooldridge (2002, Chapter 16) with an example of a Tobit model. The fitted values from the friction model are computed using the formula in Rosett (1959), which is re-derived in APPENDIX B of this dissertation in a clearer form.

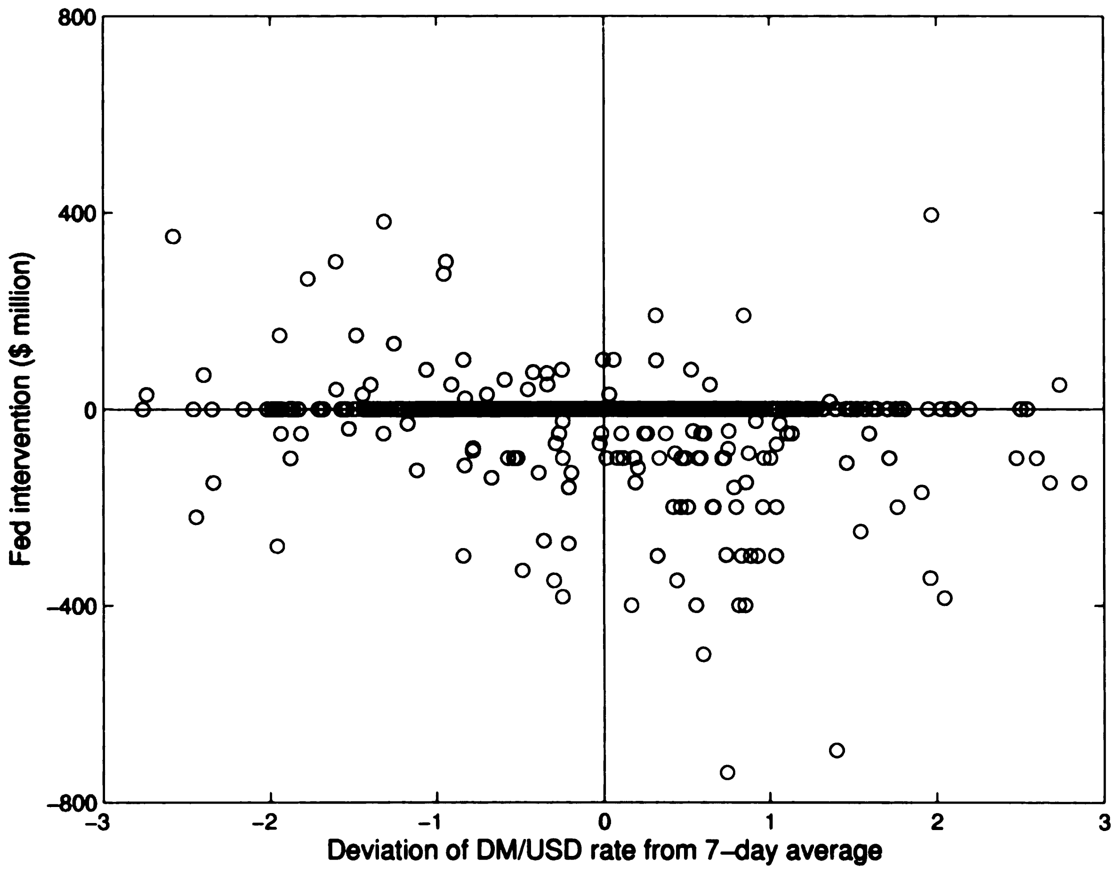


Figure 1.3: Federal Reserve daily intervention (Feb. 1987 - Oct. 1989)

amount of intervention is measured along the vertical axis. The horizontal axis measures how much the Deutsche mark per U.S. dollar exchange rate (hereinafter referred to as "DM/USD rate") is away from its 7-day moving average in percentage. Each small circles corresponds to one observation and the multitude of circles on the horizontal axis, i.e. zero amount of intervention, indicates that intervention is relatively infrequent in the sample period.

Figure 1.3 is similar to panel (b) of Figure 1.2 so that the friction area, if any, is large. In fact, more careful comparison reveals that the friction area is not clearly identified with the given data. On the one hand, large realizations of the explanatory variable do not warrant intervention. On the other hand, there are many occasions of intervention on days with small realizations of the explanatory variable. In addition, the figure shows that the Federal Reserve frequently sold U.S. dollars ($y < 0$) in response to the depreciation of the currency ($x < 0$), while it is required to buy rather than sell U.S. dollars to counter the currency's depreciation. These facts imply either that this particular explanatory variable is not a very good measure of the market conditions, or that the type of nonlinearity in the data is quite different from the nonlinearity underlying the friction model.

The problem may lie in the inappropriate data frequency. When the data frequency is daily as in this study, an underlying assumption is that the decisions on intervention are made on a daily basis. This seems to be too strong an assumption because it does not seem to be the responsibility of a central bank to maintain market stability on a daily basis under a floating exchange rate system. Lower frequency data, such as monthly or quarterly data, may reveal a clearer friction area with the expected polarizing tendency of buying intervention to the left and selling intervention to the right. The cost of lower frequency, however, is the significant reduction of the sample size, which can be detrimental to the empirical test for nonlinearity because each regime must have at least some observations. Besides, information on daily or weekly variations in the amount of intervention is lost in monthly or quarterly data.

Adding more observations over a longer sample period is not likely to be a perfect solution either, due to potential structural breaks. For example, a certain degree of disorderliness may be regarded as relatively small in one time period but large in another time period. Also, the tolerance level of the monetary authorities may depend on who the incumbent officials are. The inconsistency of the U.S. intervention policy is demonstrated by the dramatic change in the frequency of intervention around the Plaza Agreement in September 1985. While the U.S. monetary authorities abstained from intervening despite the rapid rise of the U.S. dollar sustained for about four years in early 1980's, they actively intervened in the last half of the decade. Since 1996, with another policy shift, the U.S. monetary authorities have not intervened in the foreign exchange markets.[7]

From a long-run perspective, one may say that the underlying assumption of friction model is consistent with the observed intervention behavior in that a central bank tends to intervene when the market is in highly disorderly conditions and does not intervene otherwise. However, from a shorter-run perspective, in particular on a daily basis, there is not enough evidence that the friction model is better than a simple linear approximation. After all, it seems to be quite difficult to find appropriate measures of the market conditions so that the friction area is clearly identified while at the same time decent number of observations are available in each regime of the nonlinear model.

The rest of this chapter is organized as follows: In section 2, the two models of central bank intervention, i.e. the linear model and the friction model used in Almekinders and Eijffinger (1996), are explained. While only a brief description

[7] Humpage and Osterberg (2000) claims that one reason for the dwindled intervention by the Federal Reserve since early or mid- 1990's may be that the officials are unconvinced of the effectiveness of intervention, particularly sterilized intervention.

of the friction model is given in the previous literature, the model is described in detail in this section, including a comparison with a Tobit model, derivation of the conditional mean function and interpretation of the parameters. In section 3, some issues on checking potential misspecification of the model are discussed. Section 4 describes the set of data used in the empirical study and section 5 provides estimation and test results. Section 6 offers a brief conclusion.

1.2 CENTRAL BANK REACTION FUNCTION

1.2.1 Linear Reaction Function

A linear model of daily central bank reaction function can be written as

$$y_t = \beta_0 + x_t\beta_\ell + u_t \tag{1.1}$$

where y_t is the amount of intervention on day t by a central bank, x_t is the $1 \times k$ vector of explanatory variables, β_ℓ is the $k \times 1$ vector of parameters, and u_t is the error term. The amount of intervention is positive when a central bank purchases the numeraire currency, which is the U.S. dollar in this paper, and negative when the central bank sells the currency.

In order to make the estimation results comparable, x_t consists of the same explanatory variables as in Almekinders and Eijffinger (1996). The first explanatory variable is the percentage deviation of the exchange rate from m-day moving average. Denoting S_t as the spot exchange rate on day t, which is the DM/USD rate, the percentage deviation is calculated as

$$dev_t = 100 \left[log(S_t) - log \left(\frac{1}{m} \sum_{i=1}^{m} S_{t-i} \right) \right]. \tag{1.2}$$

The moving average is a proxy for the central bank's target level of the exchange rate, which is assumed to exist. Therefore, dev_t is a measure of how far the exchange rate is away from the target level. The length of the moving average is set to be seven ($m = 7$) in Almekinders et al., which is arbitrary but there is no reliable guide in selecting the value of m. The empirical results with $m = 50$ as well as with $m = 7$ are reported in Section 5 in case the central banks have a longer time horizon.[8]

The second explanatory variable, vol_t, is a measure of the volatility of the exchange rate. This is the conditional variance of the log return of the exchange rate estimated with a GARCH (1,1) model. Note that the estimated variance is always positive while the dependent variable y_t may be either positive or negative. To correct this inconsistency, a sign is assigned to each of the estimated conditional variance. The sign is positive on the days when the exchange rate is greater than or equal to the market opening rate in New York on February 23, 1987, i.e. $DM/USD = 1.8255$. Almekinders et al. use this specific exchange rate as the proxy for the equilibrium level of the Louvre Accord on February 22, 1987.

vol_t is formally defined as

$$vol_t = D_t h_t, \tag{1.3}$$

[8]Kim and Sheen (2002) use $m = 150$. This choice of moving average length is not followed in this study because the corresponding series of dev_t does not pass a unit root test. See Chapter 2 (Section 2.2.1) for the unit root test results.

where D_t is a dummy variable defined as

$$D_t = \begin{cases} 1 & \text{if } S_t \geq 1.8255, \\ -1 & \text{if } S_t < 1.8255, \end{cases}$$

and the conditional variance h_t is estimated with the following model of the exchange rate return series.

$$100\,[log(S_t) - log(S_{t-1})] = c + \nu_t, \tag{1.4a}$$

$$\nu_t = h_t z_t, \qquad z_t \sim N(0,1), \tag{1.4b}$$

$$h_t = \omega + \alpha_1 \nu_{t-1}^2 + \alpha_2 h_{t-1}. \tag{1.4c}$$

Including an intercept, the generic form of the reaction function (1.1) can be now rewritten as

$$y_t = \beta_{\ell_0} + \beta_{\ell_1} dev_{t-1} + \beta_{\ell_2} vol_{t-1} + u_t, \tag{1.5}$$

which can be estimated by ordinary least squares (OLS). Note that the explanatory variables precede the dependent variable so that they are not correlated with the error. Equation (1.5) is not necessarily a correct specification for a central bank reaction function. It is likely that important variables such as lags of y_t are omitted, or the functional form is incorrect.[9] However, in the absence of an agreed-upon reaction function in the literature, this linear model serves as a benchmark in the evaluation of the friction model.

[9]Since vol_{t-1} in (1.5) is a generated regressor, in principle inference must reflect this. However, this issue is ignored in this paper with the assumption that the central banks respond to the estimated volatility rather than the true volatility because the latter is not observable to the central banks.

1.2.2 Friction Model

Tobit Models

A friction model can be interpreted as a combination of two Tobit models. Consider a Tobit model as

$$y_t^* = x_t\beta + \varepsilon_t, \qquad \varepsilon_t \mid x_t \sim N(0,\sigma^2), \tag{1.6a}$$

$$y_t = y_t^* - \delta^+ \qquad \text{if } y_t^* > \delta^+, \tag{1.6b}$$

$$y_t = 0 \qquad \text{if } y_t^* \leq \delta^+, \tag{1.6c}$$

where y_t and x_t are observable but y_t^* is an unobservable latent variable. With the assumption that x_t is a scalar, a set of data generated by this model is depicted in panel (a) of Figure 1.4. In this simulation $\delta^+ = 1.5$, $\beta = -1$ and x_t is randomly drawn from a uniform distribution between -5 and +5. ε_t is randomly drawn from a standard normal distribution. The sample size is 300. Note that y_t is limited to be nonnegative by (1.6b) and (1.6c).

In panel (b) of Figure 1.4, another Tobit model is depicted where y_t is limited to be nonpositive. This model can be written as

$$y_t^* = x_t\beta + \varepsilon_t, \qquad \varepsilon_t \mid x_t \sim N(0,\sigma^2), \tag{1.7a}$$

$$y_t = y_t^* + \delta^- \qquad \text{if } y_t^* < -\delta^-, \tag{1.7b}$$

$$y_t = 0 \qquad \text{if } y_t^* \geq -\delta^-. \tag{1.7c}$$

The diagram (b) is generated from the same simulated data on x_t and ε_t as in diagram (a). The parameter β maintains the same value of -1 but δ^- is set to be

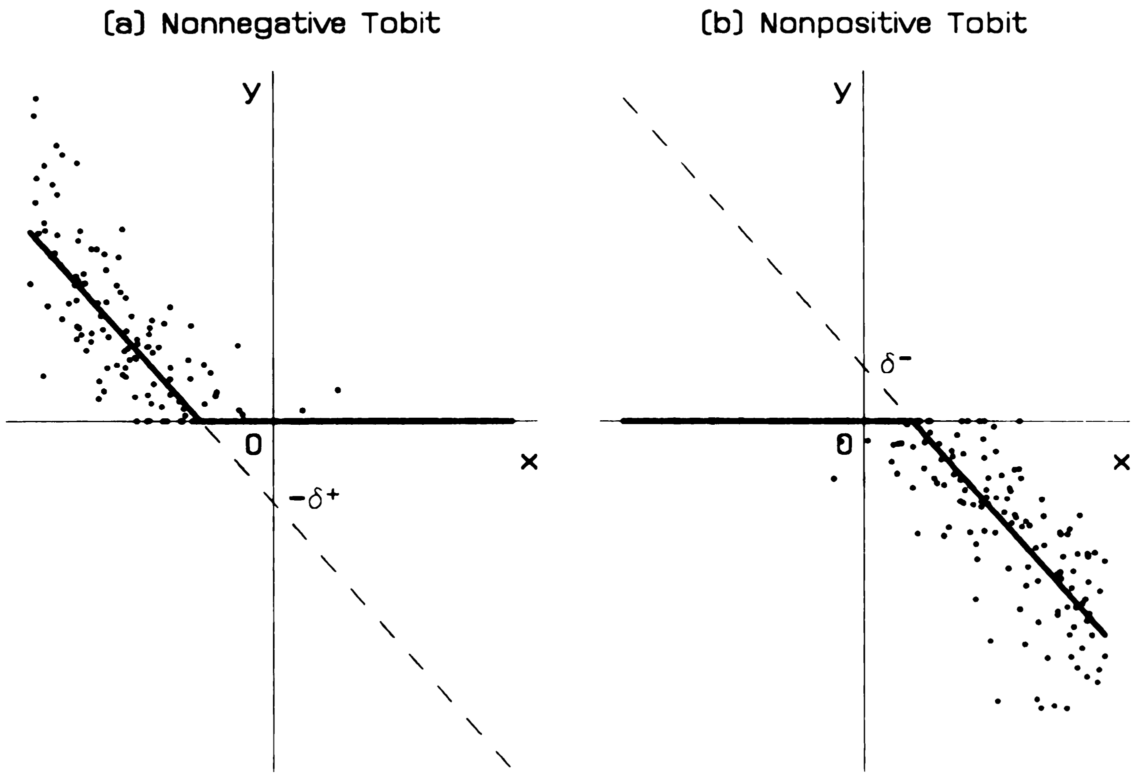


Figure 1.4: Simulation of Tobit approach

1. These two Tobit models are not necessarily symmetric around the origin since the intercepts may be different[10] although the slope coefficients are assumed to be the same as β.

[10]To obtain a friction model by combining the two Tobit models, it is required that $\delta^+ \geq -\delta^-$.

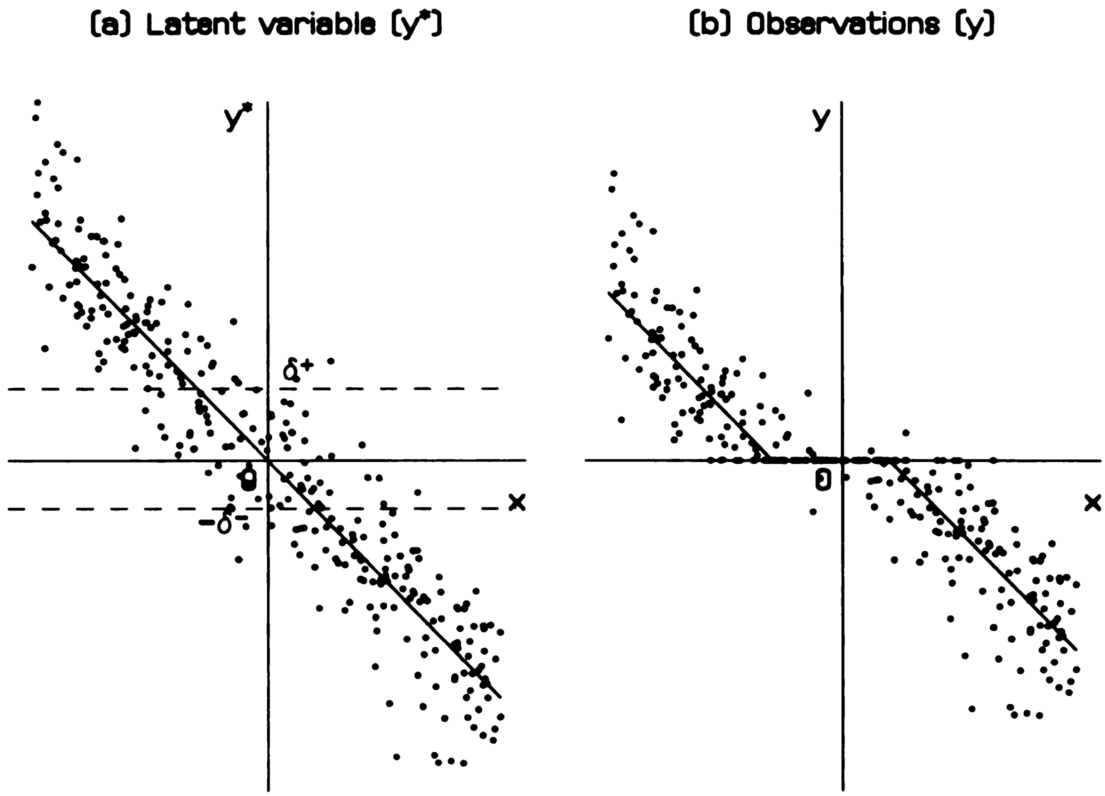


Figure 1.5: Simulation of friction approach

<u>A Friction Model</u>

By combining the two Tobit models[11], a friction model is obtained as

$$y_t^* = x_t\beta + \varepsilon_t, \qquad \varepsilon_t|x_t \sim N(0,\sigma^2), \tag{1.8a}$$

$$y_t = y_t^* - \delta^+ \qquad \text{if } y_t^* > \delta^+, \tag{1.8b}$$

$$y_t = 0 \qquad \text{if } -\delta^- \leq y_t^* \leq \delta^+, \tag{1.8c}$$

$$y_t = y_t^* + \delta^- \qquad \text{if } y_t^* < -\delta^-. \tag{1.8d}$$

Figure 1.5 illustrates this model using the same data set that has been used for the two Tobit models in Figure 1.4. Note that the thresholds $-\delta^- \leq y_t^* \leq \delta^+$ in the

[11]It is not possible to combine the two models if β's are not the same in (1.6a) and (1.7a) because $P(y > 0|x) + P(y < 0|x)$ may exceed 1. See APPENDIX C for details.

friction model are certain values of y_t^* rather than some values of the explanatory variable x_t. If there is only one explanatory variable, defining the thresholds in terms of y_t^* is equivalent to defining them in terms of x_t. However, if there are two or more explanatory variables, these two definitions will be different from each other.

If $\delta^+ = -\delta^- = \delta$, then $y_t = y_t^* - \delta$ for any y_t^* and the model becomes a linear model. In this respect, the friction model nests a linear model. Then it is tempting to test for nonlinearity with the null hypothesis $H_0 : \delta^+ = -\delta^-$ against an alternative $H_1 : \delta^+ > -\delta^-$. If interest lies in testing for linearity versus nonlinearity, then this is a valid test in that the test will reject the null hypothesis of linearity when the true model is nonlinear.

However, as for the foreign exchange intervention, the infrequency of intervention implies that the true model is not linear.[12] A linear model of intervention is just an approximation of the true nonlinear model. Given the nonlinearity of the reaction function, a relevant question is whether the friction model, based on a particular form of nonlinearity, is close to the true nonlinear model so that the friction model is better than a linear approximation in explaining observed data on intervention.

[12]If the true model is linear, then it is quite unlikely to observe $y = 0$ for 70% or 80% of the sample.

An Ordered Probit Model

If interest lies in the probability of buying or selling intervention rather than in the amount of intervention, then the friction model (1.8) can be transformed into a two-threshold ordered probit model.

$$y_t^o = x_t\beta_o + \varepsilon_t^o, \qquad \varepsilon_t^o|x_t \sim N(0,1), \tag{1.9a}$$

$$y_t = 1 \qquad \text{if } y_t^o > \delta_2, \tag{1.9b}$$

$$y_t = 0 \qquad \text{if } -\delta_1 \leq y_t^o \leq \delta_2, \tag{1.9c}$$

$$y_t = -1 \qquad \text{if } y_t^o < -\delta_1. \tag{1.9d}$$

An interesting property of a Tobit model is that its parameters are closely related to the parameters of a probit model as explained in Wooldridge (2002, Chapter 16). Likewise, the parameters of a friction model are closely related to the parameters of an ordered probit model. By comparing (1.9) with (1.8), it can be seen that

$$y_t^o = y_t^*/\sigma, \qquad \varepsilon_t^o = \varepsilon_t/\sigma, \tag{1.10a}$$

$$\beta_o = \beta/\sigma, \qquad \delta_2 = \delta^+/\sigma, \qquad \delta_1 = \delta^-/\sigma. \tag{1.10b}$$

Therefore, it is possible to check roughly if the friction model is an appropriate model by comparing the signs and sizes of the estimates with those of an ordered probit model.

1.2.3 Estimation and Hypothesis Tests

Estimation of a Friction Model

The parameter vector $\theta \equiv (\beta, \delta^+, \delta^-, \sigma)$ in the friction model of (1.8) can be estimated by the method of maximum likelihood. The log-likelihood for observation t is

$$\begin{aligned} \ell_t(\theta; y_t \mid x_t) = &\ 1(y_t > 0) \cdot log \left[\phi \left(\frac{y_t - x_t\beta + \delta^+}{\sigma} \right) / \sigma \right] \\ &+ 1(y_t < 0) \cdot log \left[\phi \left(\frac{y_t - x_t\beta - \delta^-}{\sigma} \right) / \sigma \right] \\ &+ 1(y_t = 0) \cdot log \left[\Phi \left(\frac{-x_t\beta + \delta^+}{\sigma} \right) - \Phi \left(\frac{-x_t\beta - \delta^-}{\sigma} \right) \right] \end{aligned} \quad (1.11)$$

where $1(\cdot)$ is an indicator function, which is 1 if the expression inside the parentheses is true and 0 otherwise.[13] Assuming that $y_t \mid x_t$ is independent for all $t = 1, \ldots, T$, the MLE of θ can be obtained by maximizing $\ell(\theta) = \sum_{t=1}^{T} \ell_t(\theta; y_t \mid x_t)$. With time series data, however, the assumption of independence may not be appropriate.[14] Then, MLE with $\ell(\theta) = \sum_{t=1}^{T} \ell_t(\theta; y_t \mid x_t)$ should be interpreted as the partial MLE (PMLE).[15]

[13]See APPENDIX A for the derivation of the log-likelihood.

[14]Since x_t does not include lags of y_t, the latent variable model (1.8a) is not dynamically complete. To compare the results with those in Almekinders and Eijffinger (1996), the specification is maintained in the initial estimation. Then, results with additional explanatory variables are reported later.

[15]More details about PMLE can be found in Wooldridge (2002, Chapter 13).

For the ordered probit model (1.9), the log-likelihood of observation t is

$$\ell_t(\beta_o, \delta_1, \delta_2; y_t \mid x_t) = 1(y_t = 1) \cdot log\,[1 - \Phi(-x_t\beta_o + \delta_2)]$$

$$+ 1(y_t = 0) \cdot log\,[\Phi(-x_t\beta_o + \delta_2) - \Phi(-xt\beta_o - \delta_1)]$$

$$+ 1(y_t = -1) \cdot log\,[\Phi(-x_t\beta_o - \delta_1)] \tag{1.12}$$

Goodness of Fit

The usual formula of R^2 for the linear model (1.5) is given as

$$R_\ell^2 = 1 - \frac{\sum_{t=1}^T (y_t - \hat{y}_t)^2}{\sum_{t=1}^T (y_t - \bar{y})^2} \tag{1.13}$$

where $\hat{y}_t$ is the fitted value of $E(y_t|x_t)$, and $\bar{y}$ is the sample average of y_t. Since the friction model is a nonlinear model and it is estimated by the method of maximum likelihood, (1.13) is not necessarily between zero and one. As an alternative, the R^2 for the friction model (1.8) is computed as the squared correlation of y_t and $\hat{y}_t$.[16]

$$R_f^2 = \frac{\left[\sum_{t=1}^T (y_t - \bar{y})\hat{y}_t\right]^2}{\left[\sum_{t=1}^T (y_t - \bar{y})^2\right]\left[\sum_{t=1}^T (\hat{y}_t - \bar{\hat{y}})^2\right]} \tag{1.14}$$

Dropping the time subscript, $E(y|x)$ for the friction model[17] is given as

$$\begin{aligned} E(y|x) &= P(y > 0|x) \cdot E(y|y > 0, x) + P(y < 0|x) \cdot E(y|y < 0, x) \\ &= \left[\Phi\left(\frac{x\beta - \delta^+}{\sigma}\right)(x\beta - \delta^+) + \sigma\,\phi\left(\frac{x\beta - \delta^+}{\sigma}\right)\right] \\ &- \left[\Phi\left(\frac{-x\beta - \delta^-}{\sigma}\right)(-x\beta - \delta^-) + \sigma\,\phi\left(\frac{-x\beta - \delta^-}{\sigma}\right)\right]. \end{aligned} \tag{1.15}$$

[16]Wooldridge (2002, Chapter 16) provides an example with a Tobit model.
[17]See APPENDIX B for the derivation.

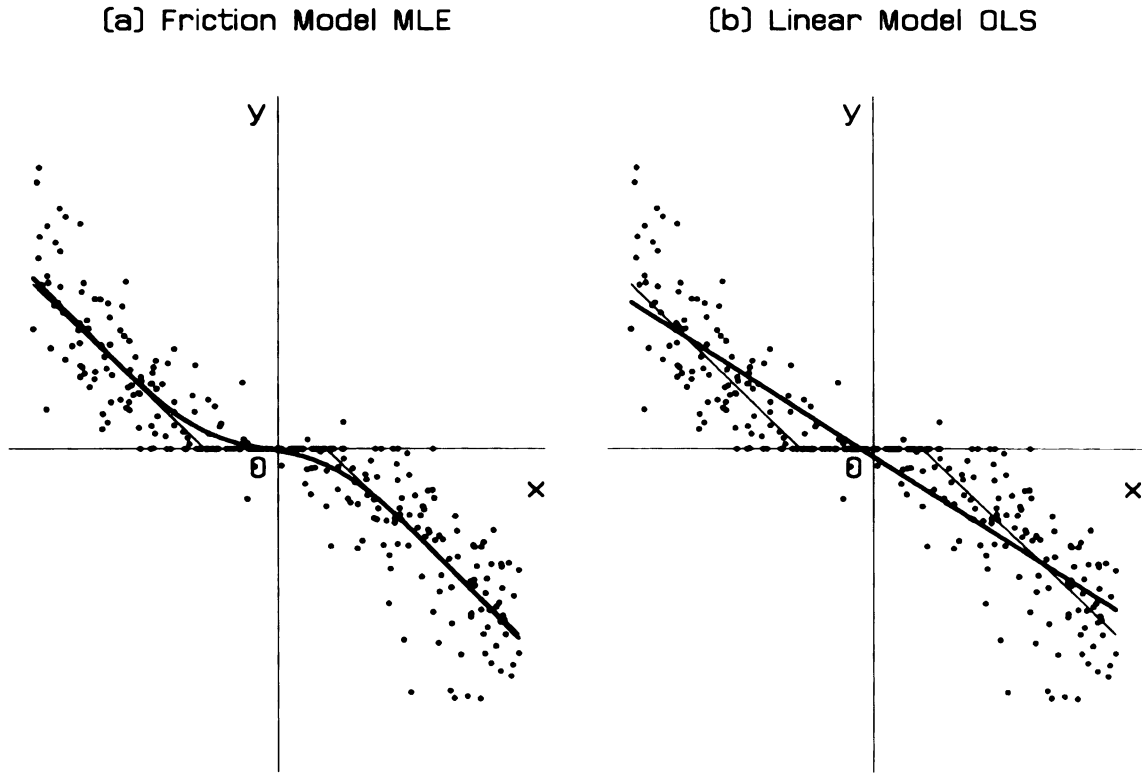


Figure 1.6: Fitted lines by friction model and linear model

Therefore,

$$\hat{y} = \left[\Phi\left(\frac{x\hat{\beta}_F - \hat{\delta}^+}{\hat{\sigma}}\right)(x\hat{\beta}_F - \hat{\delta}^+) + \hat{\sigma}\,\phi\left(\frac{x\hat{\beta}_F - \hat{\delta}^+}{\hat{\sigma}}\right)\right] - \left[\Phi\left(\frac{-x\hat{\beta}_F - \hat{\delta}^-}{\hat{\sigma}}\right)(-x\hat{\beta}_F - \hat{\delta}^-) + \hat{\sigma}\,\phi\left(\frac{-x\hat{\beta}_F - \hat{\delta}^-}{\hat{\sigma}}\right)\right] \tag{1.16}$$

where $\hat{\beta}_F, \hat{\delta}^+, \hat{\delta}^-$ and $\hat{\sigma}$ are the maximum likelihood estimates.

OLS estimates maximize the R^2 by definition while the MLEs do not necessarily. Hence it is not necessarily true to say that the friction model is worse than the linear model, even if the squared correlation coefficient of y and $\hat{y}$ from the MLE (R_f^2) is smaller than the R^2 from the OLS (R_ℓ^2). Nevertheless, if the friction model, including the normality and homoscedasticity assumption for the errors, is a significantly better specification than a linear model, then the MLE should beat the OLS in terms of the R^2 measure despite the intrinsic disadvantage. This is illustrated in Figure 1.6, where the fitted lines are computed using the same data in Figure 1.5. The diagram shows that the fitted line of the friction model MLE is closer to the true lines than that of the linear model OLS.

As a more formal justification, a simulation result is reported in Table 1.1. The simulation consists of 1,000 replications with the friction model of (1.8) as the data generating process. In each replication, 651 observations on y in (1.8) are generated using the same explanatory variables as in Almekinders et al. (1996). The true parameter values of the simulation are the estimates for the Federal Reserve's reaction function in the same paper (assuming symmetric response to depreciation and appreciation). In summary, the model of the simulations is

$$y_t^* = -107dev_{t-1} - 384vol_{t-1} + \varepsilon_t, \qquad \varepsilon_t \sim N(0, 218^2), \tag{1.17a}$$

$$y_t = y_t^* - 510 \qquad \text{if } y_t^* > 510, \tag{1.17b}$$

$$y_t = 0 \qquad \text{if } -315 \leq y_t^* \leq 510, \tag{1.17c}$$

$$y_t = y_t^* + 315 \qquad \text{if } y_t^* < -315. \tag{1.17d}$$

The mean values of the maximum likelihood estimates from the 1,000 replications, as reported in the third column of Table 1.1, are close to the true values in the second column. For a comparison of the explanatory powers of the friction model and the linear model, the ratio of the R^2 from the linear model OLS and the R^2 from the friction model MLE is also computed in each replication of the simulation. As reported in the lower part of the table, the mean value of the ratios (R_ℓ^2/R_f^2) is less than 1 implying that the friction model gives greater R^2 on average. Although there are cases in which the ratio exceeds 1, these cases are only 7 out of the 1,000 replications.

Table 1.1: Simulation results of the friction model

Variable	True Parameter	Mean MLE	RMSE	Bias
dev_{t-1}	-107	-106.53	14.28	0.47
vol_{t-1}	-384	-385.40	36.03	-1.40
δ^+	510	512.62	43.70	0.62
δ^-	315	315.58	30.89	0.58
σ	218	216.40	14.53	-1.60

Number of replications: 1,000

Mean of R_ℓ^2/R_f^2: 0.250/0.323 = 0.782

Range of R_ℓ^2/R_f^2: 0.584 $\sim$ 1.069

Cases of $R_\ell^2/R_f^2 > 1$: 7 out of 1,000

Overall, the simulation result confirms that the R^2 measures are reliable criteria for comparing explanatory powers of the two models in that a friction model has a substantially higher R^2 than a linear approximation when the true model is a friction model.

However, it is possible that the relative advantage of the friction model depends on the parameter values. As a sensitivity test, Table 1.2 reports additional simulation results with different values of the thresholds δ^+, δ^- and the error variance σ^2. These parameters in the friction model play an important role in determining whether $y = 0$ or not.[18] From the left panel of Figure 1.5, it can be seen that the

[18]The slope parameters (β) also may affect the relative explanatory power of the friction model.

number of observations with $y = 0$ will increase as δ^+ and δ^- get bigger because more values of y^* will lie between the two thresholds. Also, given δ^+ and δ^-, the smaller the error variance the more observations will have $y = 0$ as more and more values of y^* will be closer to the straight line of $x\beta$ and fall on the friction area of $(-\delta^-, \delta^+)$.[19]

Table 1.2: Average of R^2's with different parameter sets

		$\sigma = 100$	$\sigma = 150$	$\sigma = 218$	$\sigma = 250$	$\sigma = 300$
$\delta^+ = 700$	R^2_ℓ	0.149	0.158	0.162	0.163	0.158
$\delta^- = 400$	R^2_f	0.635	0.433	0.278	0.239	0.198
Average of ratio		0.238	0.377	0.604	0.705	0.822
		(0)	(1)	(2)	(22)	(68)
$\delta^+ = 510$	R^2_ℓ	0.270	0.266	0.250	0.242	0.223
$\delta^- = 315$	R^2_f	0.650	0.468	0.323	0.287	0.242
Average of ratio		0.417	0.575	0.782	0.851	0.926
		(0)	(0)	(7)	(24)	(105)
$\delta^+ = 300$	R^2_ℓ	0.533	0.474	0.394	0.360	0.307
$\delta^- = 200$	R^2_f	0.706	0.552	0.417	0.373	0.312
Average of ratio		0.756	0.860	0.947	0.968	0.985
		(0)	(0)	(31)	(104)	(214)

1) Number of replications: 1,000
2) Average of ratio $= \sum (R^2_\ell / R^2_f)/1000$
3) In parentheses are the number of replications in which $R^2_\ell > R^2_f$.

In this sensitivity test, additional sets of parameter values are selected so that both the case with the parameters larger than those in Table 1.1 and the case with

For the sake of simplicity, however, the slope parameters in the sensitivity test remain the same as in Table 1.1. Also note that the explanatory variables are fixed in each simulation.

[19]The relationship between $P(y = 0 \mid x)$ and $P(y \neq 0 \mid x)$ is illustrated in Figure A.1 of APPENDIX A.

smaller parameters are considered.

With the simulation results in Table 1.2, Figure 1.7 depicts the R_f^2 of the friction model MLE and the R_ℓ^2 of the linear model OLS for three different sets of the thresholds. In each panel, the three lines from top to bottom correspond to the pairs of thresholds (δ^+, $-\delta^-$)=(-200, 300), (-315, 510), and (-400, 700), respectively.

In the upper panel, R^2's of the friction model are depicted against the five values of the standard deviation of the error ($\sigma = 100, 150, 218, 250, 300$) on the horizontal axis. Regardless of the threshold levels, R_f^2 decreases as σ increases. Given σ, R_f^2 also decreases as the distance between the two thresholds, $\delta^+ - (-\delta^-)$, increases.[20]

In the lower panel, R^2's of the linear model show a similar pattern. The top two lines show that R_ℓ^2 tends to be negatively related to both σ and the distance between the two thresholds. In the case of (δ^+, $-\delta^-$)=(-400, 700), R_ℓ^2 is almost insensitive to changes in σ.[21]

Compared to the three lines in the upper panel, the three lines in the lower panel are farther apart from one another, which means that R_ℓ^2 is more sensitive to the changes in the thresholds than R_f^2 is. As the interval ($-\delta^-$, δ^+) increases, both decrease but R_ℓ^2 falls faster than R_f^2 does. On the other hand, the sensitivity

[20]From Figure 1.5, it can be seen that as the distance between δ^+ and δ^- gets bigger, more values of y^* fall on this interval. As a result, less observations have $y \neq 0$ and it becomes more difficult to predict y.

[21]In this case, R_ℓ^2 actually increases as σ increases from 100 to 250 presumably because not enough observations have $y \neq 0$ when σ is small relative to the distance between the thresholds. However, once sufficient observations have $y \neq 0$, R_ℓ^2 begins to decrease with σ.

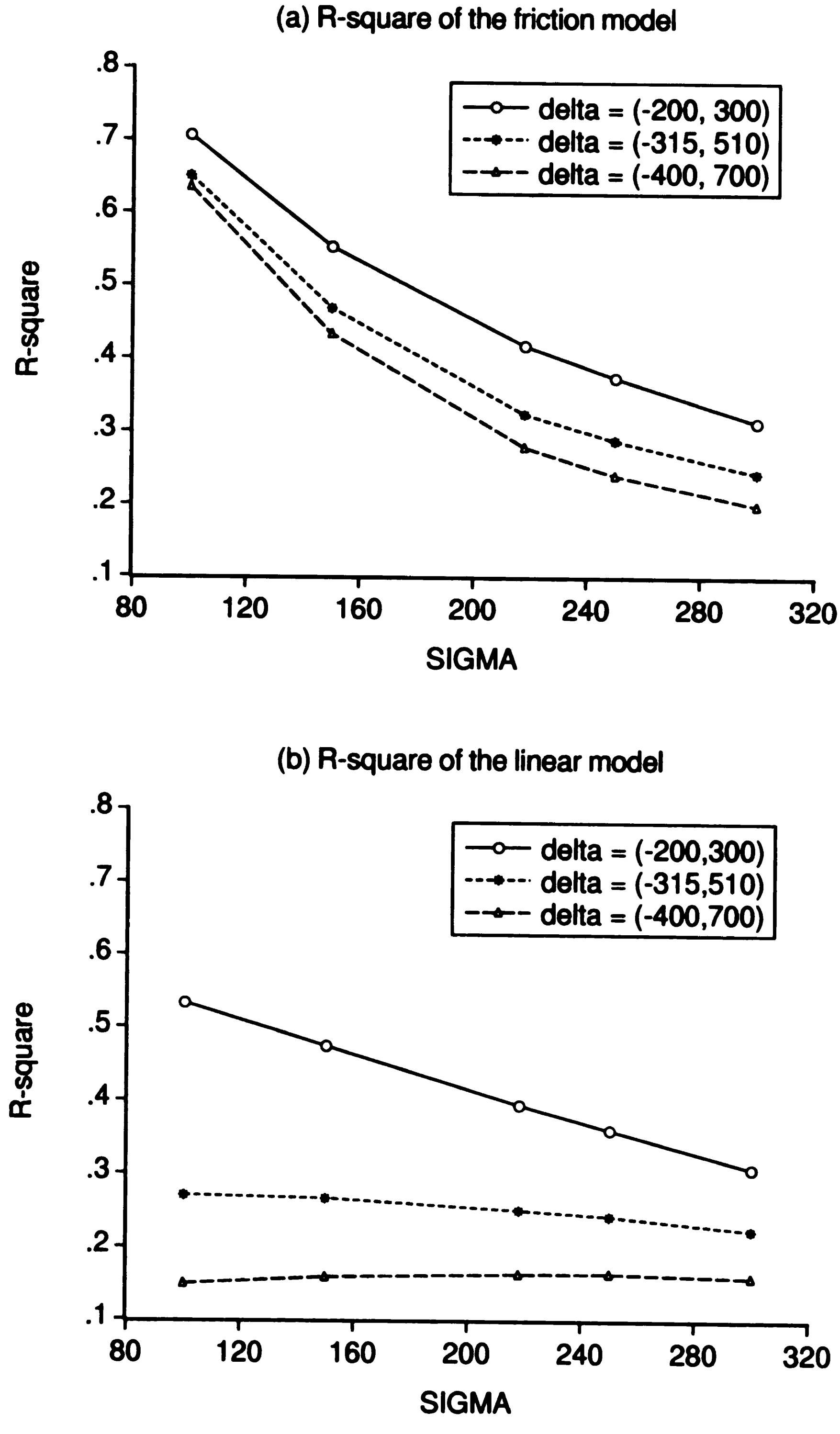


Figure 1.7: Sensitivity of R-squares

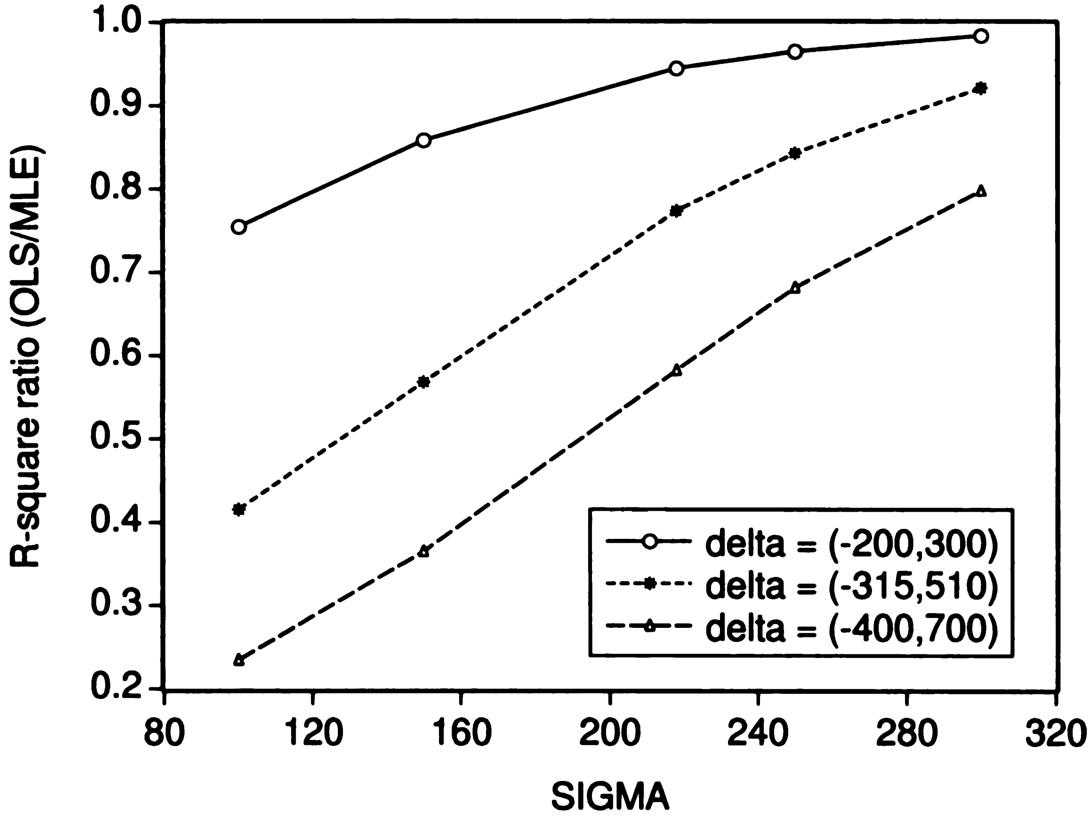


Figure 1.8: Sensitivity of R-squares

to changes in error variance is larger in R_f^2 than in R_ℓ^2. That is, R_f^2 falls faster than R_ℓ^2 as σ increases.

Consequently, the larger the error variance and the smaller the interval, the smaller gets the friction model's relative explanatory power. This is illustrated in Figure 1.8, where the average ratio of R_ℓ^2/R_f^2 from the 1000 replications are depicted. As represented by the tope line, the ratios are higher for smaller size of the interval. Along the line, the ratio gets higher as σ increases. In the extreme case where the error variance is large and the thresholds are small (σ = 300,

$\delta^+ = 300$, $\delta^- = 200$), the average ratio of R_ℓ^2/R_f^2 is close to one (0.985). Not surprisingly, the OLS frequently gives a higher R^2 than the MLE in this case (214 out of 1000).

Testing for Relative Explanatory Power

The simulation results indicate that R^2 of the friction model may not be significantly larger than the R^2 of a linear model if the error variance is large relative to the degree of friction (the distance between δ^+ and $-\delta^-$). Note that this is true even if δ^+ is significantly different from $-\delta^-$, e.g. $\delta^+ = 300$ and $-\delta^- = -200$. This finding is consistent with our earlier claim that testing $\delta^+ = -\delta^-$ with a likelihood ratio statistic is not a valid approach if what one wants to know is whether the friction model explains the data significantly better than a linear model.

Alternatively, the strategy in this study is to directly test the null hypothesis of $R_\ell^2 \geq R_f^2$ against the alternative hypothesis of $R_\ell^2 < R_f^2$.

$$H_0 : R_\ell^2 \geq R_f^2,$$

$$H_1 : R_\ell^2 < R_f^2.$$

Note that the null hypothesis is that the linear approximation of the reaction function is as good as the friction model in terms of R^2, i.e. in terms of the correlation between y and $\hat{y}$. The alternative hypothesis is that the friction model is significantly better than the linear approximation.

The test statistic F_r is defined as

$$F_r = \frac{R_f^2 - R_\ell^2}{1 - R_f^2}. \tag{1.18}$$

Since $F_r \leq 0$ under H_0, H_0 will be rejected for a large value of F_r. If H_0 is not rejected for a given sample, the implication is either that the friction model is true but the nonlinearity from friction is blurred by a large error variance, or that the nonlinearity in the reaction function is quite different from the nonlinearity described by a friction model.

This statistic, if the numerator and the denominator are divided by the respective degree of freedom, is similar to the usual F statistic for a linear model. Note, however, that F_r can be negative while usual F statistic is nonnegative, which indicates that the distribution of F_r is likely to be nonstandard. Therefore, the p-value will be computed by a parametric bootstrap with 1,000 replications. In each replication, a sample of 651 observations will be generated in the same way as in the previous simulation except that the parameter values for data generation in each replication are set to be the maximum likelihood estimates. The p-value is the number of replications where the F_r statistic from the replication is greater than the F_r statistic from the estimation.

Interpretation of the Parameters

In the friction model of (1.8), if $\delta^+ = -\delta^- = \delta$ so that there is no friction and the model is linear, then $y = y^* - \delta$. Moreover, if the intercept is zero ($\delta = 0$), then $y = y^*$. In the intervention context, assuming $\delta = 0$, y^* can be interpreted as the amount of intervention when the central bank attempts to counter any market instability no matter how small it may be, which should be the case under a fixed exchange rate system.

Under a floating exchange rate system, the infrequency of intervention requires $\delta^+ > 0$ and $-\delta^- < 0$, and actual intervention takes place only if y^* is above δ^+ or below $-\delta^-$. Since the actual amount of intervention y always falls short of the frictionless amount of intervention y^*, instability remains in the market even after an intervention. The limits of remaining instability are closely related to the two thresholds δ^+ and $-\delta^-$. Therefore, the values of these two thresholds reflect the tolerance levels of the central bank. Note that these tolerance levels are not in terms of the degree of instability, which is represented by the explanatory variables x, but rather in terms of the amount of intervention (y^*) as the central bank estimates it to be required if the bank decides to eliminate that much of instability.

In the linear model of (1.1) or (1.5), each element of β_ℓ has the interpretation of the partial effect of the corresponding explanatory variable on the conditional mean of y, i.e.,

$$\frac{\partial E(y|x)}{\partial x_j} = \beta_{\ell_j} \qquad \text{for } j = 1, \ldots, k. \tag{1.19}$$

In the case of the friction model, from (1.15),

$$\frac{\partial E(y|x)}{\partial x_j} = \beta_j \left[\Phi\left(\frac{x\beta - \delta^+}{\sigma}\right) + \Phi\left(\frac{-x\beta - \delta^-}{\sigma}\right) \right]. \tag{1.20}$$

β_j on the right hand side of this equation can be interpreted as the partial effect of x_j on the latent variable y^* (not on y) because from (1.8a),

$$\frac{\partial E(y^*|x)}{\partial x_j} = \beta_j. \tag{1.21}$$

In other words, β_j is the amount of intervention if the central bank attempts to completely offset the effects of a unit increase in x_j.

Since $P(y_t > 0 \mid x_t) = 1 - \Phi\left(\frac{-x_t\beta+\delta^+}{\sigma}\right)$ and $P(y_t < 0 \mid x_t) = \Phi\left(\frac{-x_t\beta-\delta^-}{\sigma}\right)$, the term inside the bracket in (1.20) is the probability of intervention given x.[22]

$$\Phi\left(\frac{x\beta-\delta^+}{\sigma}\right) + \Phi\left(\frac{-x\beta-\delta^-}{\sigma}\right) = P(y > 0 \mid x) + P(y < 0 \mid x)$$
$$= P(y \neq 0 \mid x). \tag{1.22}$$

By plugging (1.22) into (1.20),

$$\frac{\partial E(y|x)}{\partial x_j} = \beta_j \cdot P(y \neq 0 \mid x)$$
$$= \beta_j \cdot [1 - P(y = 0 \mid x)]. \tag{1.23}$$

The expected increase in actual intervention in response to a unit increase in an explanatory variable is the same as the discounted level of frictionless partial effect β_j, where the discount factor is the probability of intervention which in turn depends on the tolerance levels δ^+ and δ^-. Since $P(y \neq 0 \mid x) < 1$, the partial effect on y is smaller than β_j in modulus.[23]

$$\left|\frac{\partial E(y|x)}{\partial x_j}\right| < |\beta_j| = \left|\frac{\partial E(y^*|x)}{\partial x_j}\right|. \tag{1.24}$$

In the linear model, the partial effect of an explanatory variable is constant. In the friction model, however, the partial effect is not constant but depends on the level of all the explanatory variables, which is also the case in a Tobit model.[24] As a corollary, it is not possible to compare the linear versus nonlinear partial effects

[22]See (A.6) and (A.7) in APPENDIX A for the derivation of $P(y_t > 0 \mid x_t)$ and $P(y_t < 0 \mid x_t)$.
[23]See Figure A.1 in APPENDIX A.
[24]This implies that interpreting the estimated coefficient of dev_{t-1} as the expected amount of intervention in response to a one percentage point appreciation of U.S. dollar, as in Almekinders et al. (1996), is not valid.

unless x is given. On the other hand, it does not make much sense to directly compare $\hat{\beta}_{\ell_j}$ with $\hat{\beta}_j$ because β_{ℓ_j} measures the effect on y while β_j measures the effect on y^*. β_j will be always larger than β_{ℓ_j} in absolute value as illustrated by flatter OLS fitted line in panel (b) of Figure 1.6.

1.3 ON DIAGNOSTIC TESTS

One of the critical assumptions of the Rosett's friction model is that the conditional distribution of the error is normal and homoscedastic. If this assumption is violated, the MLE's of the parameters can be biased. Therefore, it is important to check normality and homoscedasticity. However, usual residual-based diagnostic tests are not available for the friction model. In the linear model of (1.1), the error term is

$$u_t = y_t - E(y_t|x_t) \tag{1.25}$$

where $E(y_t|x_t) = \beta_0 + x_t\beta_\ell$. In the case of the friction model, from (1.8) and (1.15),

$$\begin{aligned}\varepsilon_t &= y_t^* - x_t\beta \\ &\neq y_t - E(y_t|x_t).\end{aligned}$$

Therefore, the residual as an estimate of the error for observation t is not $y_t - \hat{y}_t$, where $\hat{y}_t$ is given in (1.16).

However, from (1.8),

$$\varepsilon_t = y_t - x_t\beta + \delta^+ \qquad \text{if } y_t > 0, \tag{1.26a}$$

$$\varepsilon_t = y_t - x_t\beta - \delta^- \qquad \text{if } y_t < 0. \tag{1.26b}$$

By substituting the maximum likelihood estimates for the parameters in (1.26), we can get the residual $\hat{\varepsilon}_t$ if $y_t \neq 0$. If $y_t = 0$, from (1.8a) and (1.8c),

$$-x_t\beta - \delta^- \leq \varepsilon_t \leq -x_t\beta + \delta^+. \tag{1.27}$$

So, residuals are not obtainable if the amount of intervention is zero, although interval estimates of the errors are obtainable by replacing the parameters in (1.27) with their maximum likelihood estimates. We may try testing normality of the errors only for the case of $y_t \neq 0$. However, nothing guarantees the normality of this part of the errors. Testing for serial correlation or heteroscedasticity based on residuals is not possible either.

1.3.1 Conditional Moment Tests

The problem related to unobservable residuals $\hat{\varepsilon}_t = y_t^* - x_t\hat{\beta}$ for those observations with $y_t = 0$ in the friction model is similar to the problem in a Tobit model where $\hat{\varepsilon}_t$ is not observable if $y_t = 0$. However, some diagnostic tests based on generalized residuals or conditional moments are available for a Tobit model as in Pagan and Vella (1989).

A similar approach may be applicable to the friction model. For example, if the error truly has the standard normal distribution, then

$$E(\varepsilon_t^3) = 0. \tag{1.28}$$

For the linear model, the sample analog of this population moment condition is

$$T^{-1}\sum \hat{\varepsilon}_t^3 \approx 0. \tag{1.29}$$

Since $\hat{\varepsilon}_t$ is not obtainable in the friction model if $y_t = 0$, (1.29) is replaced by

$$T^{-1}\sum \left[\hat{\varepsilon}_t^3 \cdot 1(y_t \neq 0) + \widehat{E(\varepsilon_t^3 \mid y_t = 0)} \cdot 1(y_t = 0)\right] \approx 0. \tag{1.30}$$

Note that (1.28) implies $E\left[E(\varepsilon_t^3 \mid y_t)\right] = 0$, and

$$\begin{aligned} E(\varepsilon_t^3 \mid y_t = 0) &= E(\varepsilon_t^3 \mid -\delta^- < y_t^* < \delta^+) \\ &= \sigma^3 \frac{(2 + a_t^2)\phi(a_t) - (2 + b_t^2)\phi(b_t)}{\Phi(b_t) - \Phi(a_t)} \end{aligned} \tag{1.31}$$

where $a_t \equiv \frac{-\delta^- - x_t\beta}{\sigma}$ and $b_t \equiv \frac{\delta^+ - x_t\beta}{\sigma}$.[25] The estimate $\widehat{E(\varepsilon_t^3 \mid y_t = 0)}$ is obtained by plugging the MLE estimates of σ, δ^-, δ^+ and β in (1.31).

Pagan and Vella (1989) provides a simple regression-based test procedure for the moment conditions of a Tobit model with a random sample. When the assumption of independent observations is violated, a robust estimator of the asymptotic variance of the test statistic is available, albeit more complicated. It seems interesting to design a test procedure for a friction model by extending the approaches developed for a Tobit model. However, the main focus of this study is to compare the relative explanatory power of the friction model against a linear model and the extension is left for future research.

1.3.2 Friction Model with Student's t-density

Another way to test for normality of the errors is to try using some other density function as the likelihood function. By comparing the R^2 measure of the maximum

[25]See APPENDIX D.

likelihood estimation with normal density and R^2 with some other density, it is possible to indirectly test whether the normality assumption is appropriate.

The alternative distribution for the error considered in this study is the Student's t-distribution. This distribution allows fatter tails than normal distribution which seems appealing in that observations violating the "leaning against the wind hypothesis"[26] seem to be more frequent than implied by a normal density. In Figure 1.9, which is a duplicate of Figure 1.3, many observations of selling U.S. dollar intervention (negative values of intervention) are on the days when the U.S. dollar is undervalued (negative values of deviation), i.e. leaning *with* the wind rather than *against* it. With lower frequency, the Federal Reserve buys U.S. dollars on the days when the dollar is overvalued.

The frequent observations of these counterintuitive types of intervention, where the signs of explanatory variables are inconsistent with the signs of observed intervention, indicate either that the error variance is quite large, or that the error distribution is severely leptokurtic.[27] In this respect, it will be interesting to see whether the observed data can be better explained by a t-density with a low degree of freedom than by a normal density with a very large variance.

The probability density of a random variable t with n degrees of freedom[28] is

$$f(t,n) = \frac{\Gamma((n+1)/2)}{\sqrt{\pi n}\,\Gamma(n/2)} \frac{1}{(1+t^2/n)^{(n+1)/2}}, \quad -\infty < t < \infty \tag{1.32}$$

[26]This hypothesis implies that $P(y > 0 \mid x < 0)$ and $P(y < 0 \mid x > 0)$ are high while $P(y < 0 \mid x < 0)$ and $P(y > 0 \mid x > 0)$ are low. See Figure A.1 in APPENDIX A.

[27]When the estimated $P(y < 0 \mid x < 0)$ or $P(y > 0 \mid x > 0)$ are unexpectedly high, it may also imply that the explanatory variables are inappropriate.

[28]This density function is from Hogg and Tanis (1997 page 600).

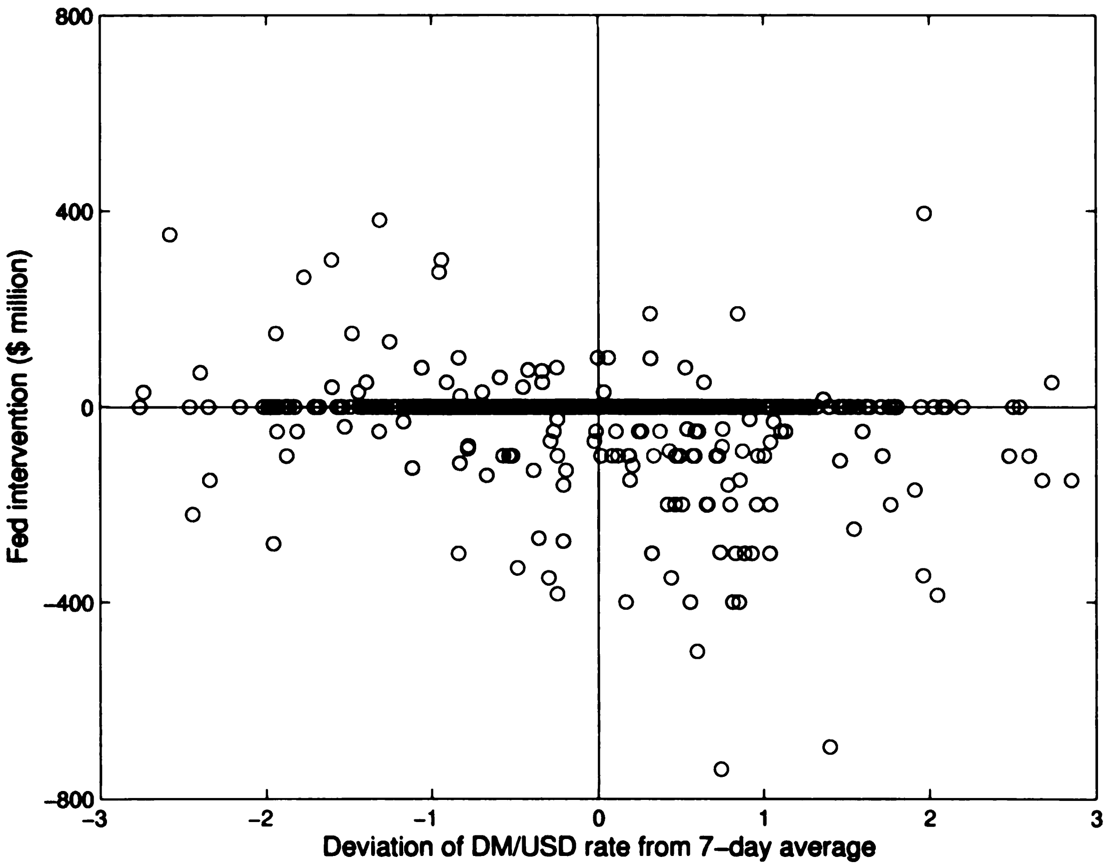


Figure 1.9: Intervention and deviation of DM/USD rate from moving average

where $\Gamma(\cdot)$ is the gamma function given as

$$\Gamma(\alpha) = \int_0^\infty x^{\alpha-1} e^{-x} dx, \quad 0 < t. \tag{1.33}$$

For the maximum likelihood estimation of (1.8) with $\varepsilon_t | x_t$ having a Student's t-distribution instead of a normal distribution, the log likelihood of (1.11) should be replaced by

$$\begin{aligned}
\ell_t(\theta) &\equiv log[L_t(\theta)] \\
&= 1(y_t > 0) \cdot log\left[f(y_t - x_t\beta + \delta^+)\right] \\
&\quad + 1(y_t < 0) \cdot log\left[f(y_t - x_t\beta - \delta^-\right] \\
&\quad + 1(y_t = 0) \cdot log\left[F(-x_t\beta + \delta^+) - F(-x_t\beta - \delta^-)\right] \qquad (1.34)
\end{aligned}$$

where $F(\cdot)$ is the t-distribution CDF.

For the computation of the R^2 measure, replace the conditional expectation of (1.15) with

$$\begin{aligned}
E(y|x) = F(x\beta - \delta^+)(x\beta - \delta^+) + \frac{n + (x\beta - \delta^+)^2}{n-1} f(x\beta - \delta^+) + \\
F(-x\beta - \delta^-)(x\beta + \delta^-) - \frac{n + (x\beta + \delta^-)^2}{n-1} f(x\beta + \delta^-). \qquad (1.35)
\end{aligned}$$

This conditional expectation exists if $n > 1$. Then, by plugging in the maximum likelihood estimates, it is possible to get fitted values of y and compute the squared correlation to compare it with the R^2 of OLS or R^2 of MLE with normal density. If $n \leq 1$, this comparison is not possible because the conditional mean does not exist.

1.4 THE DATA

The reaction functions of the two central banks are estimated separately assuming that each bank makes decisions independently.[29] Therefore, the dependent variable y_t can be either the Federal Reserve intervention or the Bundesbank intervention.

[29]This assumption does not necessarily rule out cases of joint intervention.

Official daily data on the sterilized intervention by the Federal Reserve and the Bundesbank are used for y_t. The net amount of US dollars (in millions) purchased on day t is recorded as a positive number and the amount sold as a negative number. The DM/USD exchange rate (S_t) is recorded at 9:30 in Paris on day t. The data set is the same as in Baillie and Osterberg (2000) covering the period between 1/5/87 $\sim$ 1/22/93.[30]

The sample period is matched with that of the Almekinders and Eijffinger (1996), which is from February 23, 1987 to October 31, 1989. Although their sample on the exchange rate consists of three intra-day observations in Frankfurt and four observations in New York while our sample consists of single observation per day in Paris, the estimation results are turned out to be similar to each other.

One of the explanatory variable, i.e. the signed conditional variance of the exchange rate vol_t as defined in (1.3) and (1.4), requires preliminary estimation of (1.4). The MLE estimates of this GARCH(1,1) model are reported in the second column of Table 1.3. They are quite similar to the two sets of estimates in Almekinders et al., which are duplicated in the third and fourth column in Table 1.3.

In Figure 1.10, the Federal Reserve's intervention is depicted against the signed volatility vol_{t-1}. Compared with the scatter plot against the deviation measure dev_{t-1} in Figure 1.9, the major difference is that there is no observation around the origin. However, this is not due to friction but due to the fact that the minimum

[30]The author thanks Dr. Baillie for providing the data.

Table 1.3: Estimated GARCH model for the DM/USD returns

Parameters	Estimates	Results in Almekinders et al.	
		Frankfurt rate	New York rate
c	0.005 (0.187)	0.015 (0.650)	0.013 (0.540)
ω	0.017 (2.349)	0.021 (2.770)	0.020 (2.830)
α	0.076 (4.124)	0.073 (4.470)	0.064 (4.130)
β	0.890 (42.128)	0.874 (31.510)	0.890 (34.830)
log likelihood	-667.62	-649.34	-684.79
skewness	0.11	0.04	-0.09
kurtosis	3.99	4.29	4.65
Q(12)	10.39	10.29	7.47
Q2(12)	10.30	20.63	6.88

* Number of observations: 651
* t-statistics are in parentheses (heteroskedasticity consistent).
* 5% critical vlaue for Ljung-Box Q-statistics is 21.03.

positive and the maximum negative values of the volatility measure are away from zero. In other respects, both diagrams are quite similar to each other. These diagrams suggest that the core assumption of the friction model may not be true at least in its strict form. Above all, zero values of intervention are observed not just over a subset of each explanatory variable around zero but rather spread over the entire range. Also note that nonzero values of intervention are not clustering around large realizations of the explanatory variables but rather many of them are observed when the explanatory variables take small values. Furthermore, there is no clear tendency of proportionate change between y and x, which implies that the adopted explanatory variables may not be relevant measures of the disorderly market conditions.[31]

[31] Unlike dev_{t-1} in Figure 1.9, the sign of vol_{t-1} is mostly consistent with the "leaning against the wind hypothesis" implying that the proxy of the equilibrium exchange rate of the Louvre

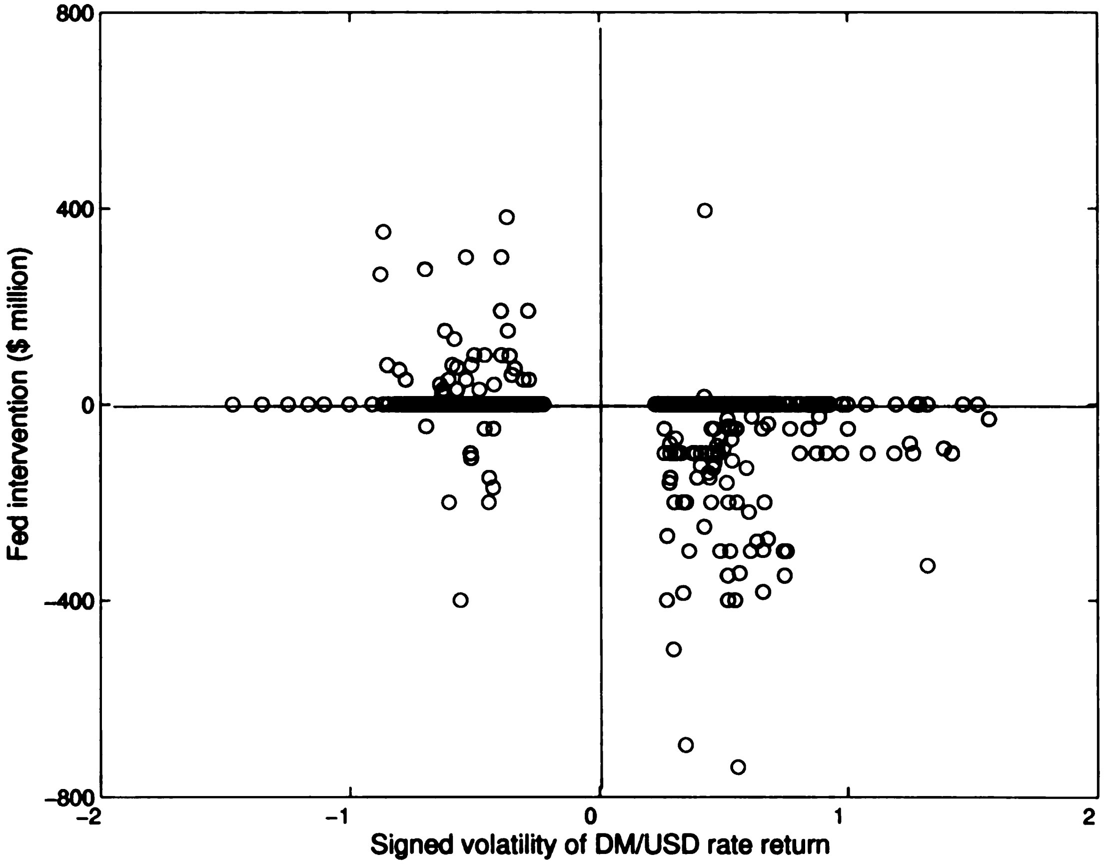


Figure 1.10: Intervention and volatility of DM/USD rate

1.5 ESTIMATION RESULTS

1.5.1 Federal Reserve Reaction Function

Table 1.4 reports the estimation results for the reaction function of the Federal Reserve. The OLS estimates of the linear model (1.5) are in the second column, while the maximum likelihood estimates of the friction model (1.8) are in the third column. The maximum likelihood estimates of Almekinders and Eijffinger (1996, column (3) in Table 2) are reproduced here in the last column for comparison.

Accord (1.8255 deutsche mark per U.S. dollar) may be a better target rate than the 7-day moving average.

Table 1.4: Federal Reserve reaction function

Variable	Linear-OLS		Friction-MLE		Almekinders et al.[2)]	
constant	-16.94***	(-5.22)				
dev_{t-1}	-18.16***	(-3.96)	-77.62***	(-4.22)	-106.91***	(-8.76)
vol_{t-1}	-45.89***	(-7.68)	-285.13***	(-9.00)	-383.92***	(-8.46)
δ^+			506.59***	(10.82)	509.97***	(10.61)
δ^-			342.33***	(10.09)	315.47***	(9.58)
σ	85.83		271.11***	(12.61)	218.51***	(14.51)
log L			-1171.98 (-3820.75)[3)]		-1168.40	
R^2	0.114		0.092			

1) t-statistics in parentheses (*** Significant at 1% level).
2) From Almekinders et al (1996, Column (3) in Table 2).
3) With the restriction of $\delta^+ = -\delta^-$.

All estimates of the linear model and the friction model are significant at 1% level[32] and have expected signs. The estimates and t-statistics in the third column are quite similar to those in the last column. This similarity implies that the data and estimation procedure of this study are similar to those in Almekinders and Eijffinger (1996).

The OLS estimates in the second column are much smaller in size than the maximum likelihood estimates. As discussed in section 1.2.3, this is because the OLS estimates are the partial effects of the explanatory variables on the expected amount of intervention $E(y|x)$, while the ML estimates are the partial effects on $E(y^*|x)$ which is the expected amount of intervention when the central bank wants to completely remove market disruptions.

[32] The t-statistics are based on a robust variance estimate.

Table 1.5: Estimated partial effects of Federal Reserve's intervention

Variables	OLS	MLE ($dev_{t-1} \leq 0$)	MLE ($dev_{t-1} > 0$)
dev_{t-1}	-18.16	-24.08	-11.31
vol_{t-1}	-45.89	-88.46	-41.55

In Table 1.5 are the estimated partial effects of the two explanatory variables. By comparing the second column and the third column, it can be seen that the partial effects for the friction model, which are evaluated at the sample means of the explanatory variables given $dev_{t-1} \leq 0$, are larger than the OLS estimates.[33] The partial effects at the sample mean given $dev_{t-1} > 0$ are in the last column, which are smaller than the OLS estimates.

If the friction model of (1.8) is correctly specified, a test for linearity is equivalent to testing if $\delta^+ = -\delta^-$. From the log likelihoods reported in Table 1.4 with or without the restriction, i.e. -1171.98 and -3820.75 respectively, it can be seen that the test statistic is huge ($LR = 2(-1171.98 + 3820.75) = 5297.54$). The null hypothesis of linearity is rejected in favor of nonlinearity, with the p-value close to zero.

Nevertheless, as reported on the last row of the table, R^2 of the linear model (0.114) is larger than the R^2 of the friction model (0.092). With $H_0 : R_\ell^2 \geq R_f^2$ against $H_1 : R_\ell^2 < R_f^2$, the test statistic and the p-value are

$$F_r = \frac{R_f^2 - R_\ell^2}{1 - R_f^2} = \frac{0.092 - 0.114}{1 - 0.092} = -0.024$$

$$Prob(F_r > -0.024) = 0.998.$$

[33]Note that the sample means of dev_{t-1} and vol_{t-1} are both close to zero so that the partial effects evaluated at sample means of the two variables are not interesting.

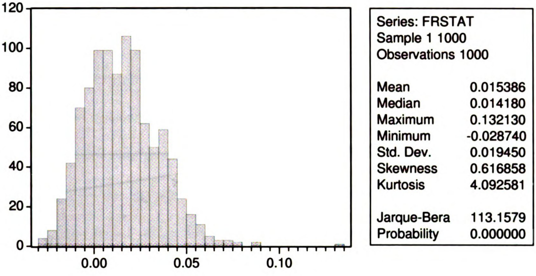


Figure 1.11: A histogram of F_r statistics

The test fails to reject H_0 with p-value close to 1, which indicates that there is not enough evidence that the friction model explains the intervention behavior of the Federal Reserve better than a linear model. $Prob(F_r > -0.024)$ is from a parametric bootstrap of 1,000 replications. In each replication, 561 random errors are drawn from $N(0, 271.1)$ distribution and values of y in the friction model (1.8) are computed with the estimated parameters and the fixed x.[34] The histogram in Figure 1.11 shows the frequencies of the 1,000 F_r statistics from the bootstrap.

In the upper panel of Figure 1.12, the fitted values of the friction model ($\hat{y}_f$ denoted by circles) are compared with those of the linear model ($\hat{y}_\ell$ denoted by the 45° line). The horizontal axis measures $\hat{y}_\ell$, i.e. the fitted values of the linear model OLS. Most of the fitted values of the friction model are around the 45°

[34] R_ℓ^2 is larger than R_f^2 in 230 replications out of the 1,000.

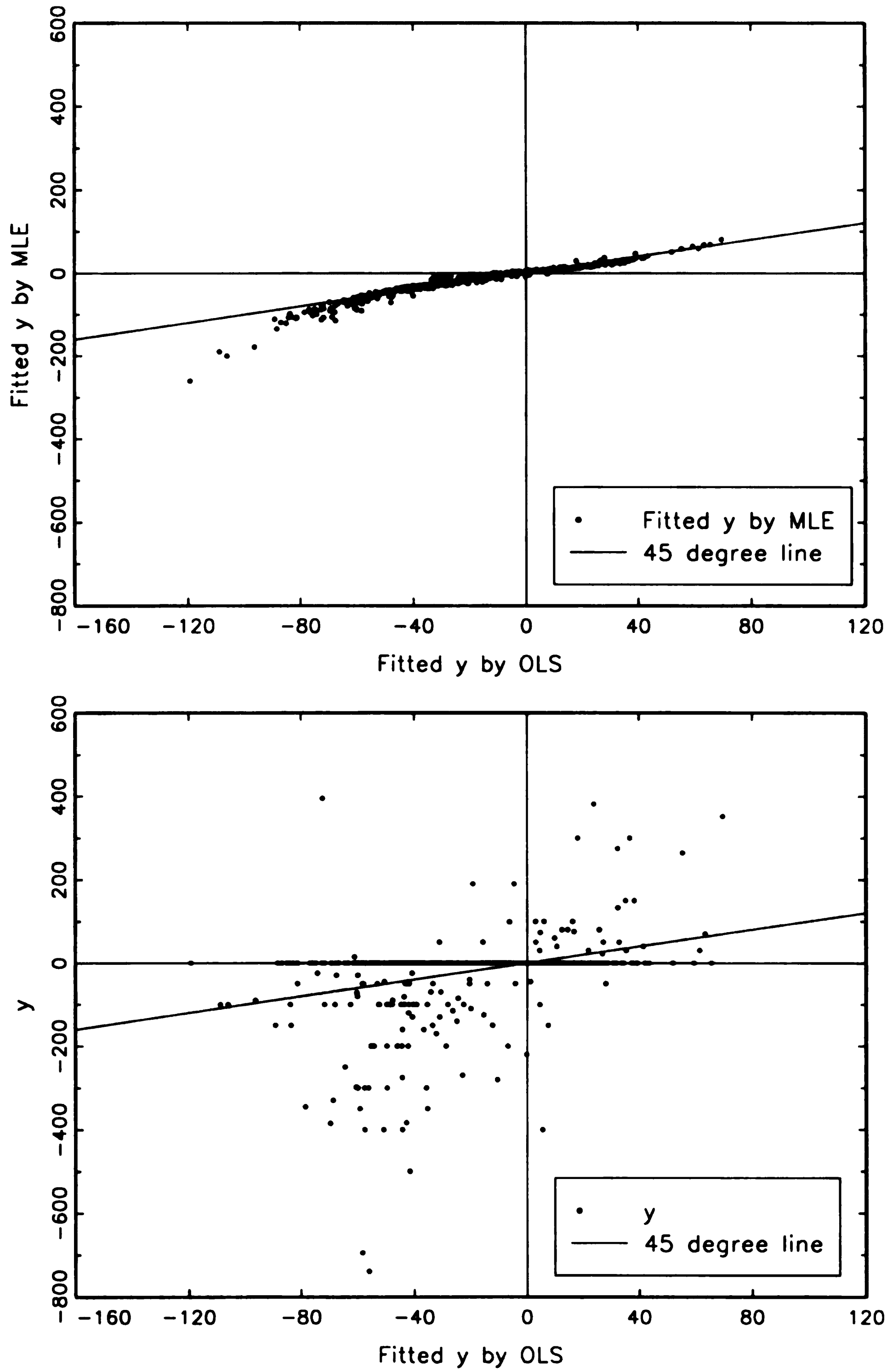


Figure 1.12: Comparison of fitted values

line, meaning that the two models' fittings are quite similar. The main difference between the two sets of fitted values occur when $\hat{y}_\ell \leq -80$, i.e. for large selling operations of \$80 million or more.

For these observations, the fitted values of the friction model are below the 45° line (in the upper panel) but the actual amounts of intervention, y, are above the 45° line (in the lower panel). As a result, when the fitted y by OLS is below -80, the linear fitted values are closer to the actual amounts of intervention than the fitted values of the friction model. This seems to be the main reason why the friction model has lower R^2 than the linear model.

Estimation with Ordered Probit Model

After transforming the data on intervention so that $y_t = 1$ for buying intervention and $y_t = -1$ for selling intervention, the ordered probit model (1.9) is estimated with the likelihood in (1.12).

The estimates are reported in the second column of Table 1.6. All parameters are significantly different from zero at 1% level. In the second column, the estimates

Table 1.6: Ordered Probit Model (Federal Reserve)

Variable	Ordered Probit-MLE		Estimates $\times\ \hat{\sigma}$ 2)	Friction-MLE
dev_{t-1}	-0.286***	(-4.02)	-77.55	-77.62***
vol_{t-1}	-1.139***	(-9.47)	-308.88	-285.13***
δ^+	1.884***	(18.12)	510.86	506.59***
δ^-	1.291***	(17.06)	349.88	342.33***

1) t-statistics in parentheses (*** Significant at 1% level).
2) $\hat{\sigma} = 271.11$ from Table 1.4.

of the ordered probit model are multiplied by the estimated standard deviation of the error of the friction model ($\hat{\sigma}$). As shown in (1.10b), these values must be similar to the corresponding estimates of the friction model, which is reproduced in the last column. Comparing the last two columns shows that this is indeed the case.

The next table, Table 1.7, summarizes the predictive power of the ordered probit model. Out of the three possible outcome, i.e. buying, no intervention, or selling, the type of predicted intervention is the one with the highest estimated probability. As shown in the third column, the model predicts 0 days of buying operation and 18 days of selling operation. The rest 633 days are predicted to go without intervention. The fourth column shows how many times the predictions are correct for each possible outcome. The last column reports the number of days on which the predicted outcome is different from actually observed outcome. None of the buying operations are correctly predicted and only 8 out of 101 selling operations are correctly predicted.[35]

Table 1.7: Prediction by Ordered Probit Model (Federal Reserve)

y	Observations (A)	Predicted[1)]	Correctly Predicted (B)	Error[2)] (C)
1 (buy)	36	0	0	36
0	514	633	504	10
-1 (sell)	101	18	8	93

1) $y_t = i$, $i = 1, 0, -1$, is predicted if $y_t = i$ maximizes $P(y_t \mid x_t)$.
2) $C = A - B$.

[35]The pseudo R^2 is 0.15.

This pattern of prediction is similar to the one implied by the estimated friction model. In the case of the friction model, among the three types of intervention, the estimated probability of buying, $P(y > 0 \mid x)$, is the highest on none of the days in the sample. The probability of selling intervention, $P(y < 0 \mid x)$, is estimated to be the highest on 12 days while $P(y = 0 \mid x)$ is estimated to be the highest on the rest 639 days.

Overall, both the friction model and the ordered probit model do not have substantial power in predicting intervention. This result helps us understand why the friction model is not significantly better than a linear model. The friction model specifies both probability and quantity for each type of intervention. However, the predicted direction of intervention (buy, sell, or do not intervene) rarely matches the observed direction. Consequently, the friction model does not explain the data better than a linear approximation.

Partially Asymmetric Reaction Function

One of the explanatory variable, dev_{t-1}, represents the distance in percentage between the exchange rate and its average during the previous 7 days. The exchange rate is moving upward (appreciation of U.S. dollar against Deutsche mark) when $dev_{t-1} > 0$, and moving downward (depreciation of U.S. dollar against Deutsche mark) when $dev_{t-1} < 0$.

One implicit assumption underlying the estimation results in Table 1.4 is that the Federal Reserve's reaction is the same for appreciation and depreciation of its currency. However, as shown in Figure 1.9, the central bank's reaction may

Table 1.8: Federal Reserve reaction function (asymmetric)

Variable	Linear-OLS		Friction-MLE		Almekinders et al.[2]	
Constant	-17.66***	(-3.52)				
pos.dev	-17.13*	(-1.80)	-60.01*	(-1.77)	-121.55***	(-6.07)
neg.dev	-19.14**	(-2.47)	-95.05***	(-3.03)	-92.75***	(-4.74)
vol_{t-1}	-45.97***	(-7.65)	-286.64***	(-9.03)	-381.99***	(-8.37)
δ^+			520.89***	(10.11)	493.70***	(9.56)
δ^-			329.46***	(9.14)	327.10***	(9.00)
σ	85.90		271.01***	(12.64)	217.77***	(14.39)
log L			-1171.70 (-3820.73)[3]		-1168.03	
R^2	0.114		0.096			

1) t-statistics in parentheses (*, **, *** significant at 10%, 5% and 1% level, respectively).
2) From Almekinders et al (1996, Column (4) in Table 2).
3) With the restriction of $\delta^+ = -\delta^-$.

be different depending on whether the U.S. dollar is appreciating or depreciating. In order to take care of this potential asymmetry of reaction, Almekinders and Eijffinger (1996) re-estimate the friction model with positive deviation and negative deviation as two separate explanatory variables. Following this approach, the estimation results for the asymmetric reaction function of the Federal Reserve are reported in Table 1.8.

On the whole, the results are similar to the symmetric case in Table 1.4 except that the estimated coefficient of positive deviation is slightly smaller than the coefficient estimate of the negative deviation and not significant at 5% level (but significant at 10% level) for both the linear model and the friction model. This result is consistent with Figure 1.9 where selling as well as buying observations are frequently observed when the deviation is negative.

Fully Asymmetric Response

To see if other parameters as well as the slope coefficients for deviations are different between appreciation regime and depreciation regime of the U.S. dollar, fully asymmetric versions of the linear model and the friction model are estimated.[36] The results are in Table 1.9. In the second and fourth column are the estimated parameters when the daily exchange rate is at or below its 7-day moving average, i.e. when the U.S. dollar is depreciating against Deutsche mark, for the linear model and friction model, respectively.

Table 1.9: Federal Reserve reaction function (fully asymmetric)

Variables	OLS		MLE	
	β_2	$\beta_1 - \beta_2$	β_2	$\beta_1 - \beta_2$
Constant	-20.02* (-2.78)	4.02 (0.40)		
$dev7_{t-1}$	-21.27* (-2.19)	2.03 (0.14)	-79.74* (-2.31)	19.50 (0.37)
vol_{t-1}	-52.02* (-6.10)	11.98 (1.00)	-352.77* (-7.48)	123.98 (1.96)
δ^+			471.80* (7.56)	83.07 (0.86)
δ^-			322.67* (6.69)	14.07 (0.20)
σ	85.96		224.68* (10.33)	73.99 (1.93)
log likelihood			-1164.76	
R^2	0.116		0.102	

1) t-statistics in parentheses (* Significant at 5% level).
2) Number of observations: 651.
3) Parameters are β_2 if $dev_{t-1} \leq 0$, β_1 otherwise.

Instead of the corresponding parameters when the exchange rate is above the moving average, the differences of the parameters between depreciating regime and

[36] See APPENDIX C for details of the fully asymmetric model and the estimation strategy.

appreciating regime are estimated and reported in the third and fifth column. This method makes it easier to test whether the differences are statistically significant or not. Both the linear model OLS and the friction model MLE indicate that the differences are not significant at 5% level. As a result, there is not much improvement in R^2 compared to the symmetric or the partially asymmetric model. Once again, however, OLS gives higher R^2 than the MLE of the nonlinear model (0.116 versus 0.102).

1.5.2 Bundesbank Reaction Function

The estimation results of both the symmetric and partially asymmetric cases for the Bundesbank intervention are given in Table 1.10. The estimates are similar to those for the Federal Reserve intervention.

Again, the likelihood ratio test for linearity rejects the null hypothesis of $\delta^+ = -\delta^-$ with the test statistic $LR = 2 * (-1475.73 + 3997.40) = 5043.34$ for the symmetric model and $LR = 2 * (-1475.30 + 3997.39) = 5044.19$ for the partially asymmetric model. The p-values in both cases are virtually zero.

However, the R^2 measures of the ML estimation (0.088 and 0.091) are smaller than those of the OLS (0.097, 0,097). Consequently, the $H_0 : R^2_\ell \geq R^2_f$ is not rejected with test statistic $F_r = (0.088 - 0.097)/(1 - 0.088) = -0.01$ and p-value $= 0.945$. (303 out of 1000 replications in the bootstrap have $R^2_\ell > R^2_f$.) Also note that the R^2's are somewhat smaller than those of the Federal Reserve reaction functions.

The estimation results for a fully asymmetric model in Table 1.11 tell a similar

Table 1.10: Bundesbank reaction function

Variable	Symmetric reaction			Asymmetric reaction		
	OLS	MLE	A-E1	OLS	MLE	A-E2
Constant	-18.39*			-19.06*		
	(-4.21)			(-2.57)		
dev_{t-1}	-30.61*	-111.51*	-79.77*			
	(-5.20)	(-5.70)	(-9.27)			
pos.dev				-29.66*	-88.91*	-55.07*
				(-2.17)	(-2.53)	(-3.44)
neg.dev				-31.52*	-134.00*	-103.56*
				(-3.33)	(-4.09)	(-6.56)
vol_{t-1}	-40.92*	-207.83*	-221.07*	-41.00*	-209.50*	-224.96*
	(-5.03)	(-7.34)	(-8.30)	(-5.02)	(-7.38)	(-8.27)
δ^+		500.67*	381.79*		519.22*	406.91*
		(10.90)	(12.13)		(10.29)	(10.94)
δ^-		303.20*	226.79*		286.78*	205.32*
		(9.89)	(11.65)		(8.28)	(8.99)
σ	112.59	297.08*	196.32*	112.68	297.08*	195.69*
		(12.53)	(18.65)		(12.52)	(18.72)
log L		-1475.73	-1300.18		-1475.30	-1298.70
		(-3997.40)			(-3997.39)	
R^2	0.097	0.088		0.097	0.091	

1) t-statistics in parentheses (* Significant at 5% level).

2) Columns (4) and (7) from Table 2 in Almekinders et al.

story as that of the Federal Reserve intervention. One exception is that the estimated difference for the slope of the conditional volatility is significant at 5% level. On the whole, the reactions of the two central banks do not change significantly between depreciating and appreciating regime. Therefore, only the empirical results of the symmetric models are reported in the rest of this paper.

Table 1.11: Bundesbank reaction function (fully asymmetric)

Variables	OLS		MLE	
	β_2	$\beta_1 - \beta_2$	β_2	$\beta_1 - \beta_2$
Constant	-14.09 (-1.59)	-9.28 (-0.64)		
$dev7_{t-1}$	-27.66* (-2.56)	0.57 (0.03)	-87.34* (-2.48)	25.64 (0.46)
vol_{t-1}	-52.61* (-5.27)	23.46 (1.43)	-281.96* (-6.46)	142.85* (2.42)
δ^+			439.81* (7.46)	159.94 (1.66)
δ^-			323.54* (5.80)	-66.57 (0.93)
σ	112.64		269.18* (8.69)	41.91 (0.94)
log likelihood			-1467.89	
R^2	0.101		0.099	

1) t-statistics in parentheses (* Significant at 5% level).

2) Number of observations: 651.

3) Parameters are β_2 if $dev_{t-1} \leq 0$, β_1 otherwise.

1.5.3 Friction Model with Student's t-density

As an indirect test for normality of the errors, Table 1.12 reports the results of the friction model MLE with Student's t-density. The estimates are much smaller

Table 1.12: Reaction functions with t-density

Variables	Federal Reserve		Bundesbank	
	Estimates	(t-statistics)	Estimates	(t-statistics)
dev_{t-1}	-6.28*	(-5.69)	-1.49	(-1.66)
vol_{t-1}	-32.22*	(-8.01)	-4.61*	(-3.62)
δ^+	48.72*	(9.27)	42.14*	(5.10)
δ^-	41.49*	(8.36)	10.30*	(7.00)
degree of freedom	0.39*	(31.07)	0.39*	(29.09)

than the normal-density MLEs. They are even smaller than the OLS estimates.

The estimated degree of freedom (0.39) is less than unity. Therefore, the conditional mean specified in equation (1.35) does not exist and we cannot rely on the squared correlation measure to evaluate the explanatory power of this approach. On the other hand, the small estimated degree of freedom indicates that the error distribution has fat tails and the normality assumption of the friction model is likely to be inappropriate.

1.5.4 With Additional Explanatory Variables

In Table 1.13, the estimation results for both central banks using other explanatory variables are reported so as to see if the lower R^2 of the friction model is due to omitted variables problem.

Table 1.13: Reaction functions with other variables

Variables	Federal Reserve		Bundesbank	
	OLS	MLE	OLS	MLE
Constant	-12.59* (-3.85)		-13.62* (-3.43)	
$dev50_{t-1}$	-7.37* (-4.59)	-38.93* (-5.92)	-12.20* (-6.06)	-48.81* (-7.47)
vol_{t-1}	-20.70* (-3.84)	-168.06* (-6.23)	-7.48 (-0.93)	-85.05* (-3.63)
int_{t-1}	0.27* (5.03)	0.78* (7.18)	0.27* (4.40)	0.62* (5.52)
ret_{t-1}	3.94 (0.91)	13.70 (0.82)	-1.30 (-0.21)	-7.92 (-0.46)
δ^+		478.64* (9.88)		461.22* (11.19)
δ^-		346.60* (9.96)		301.40* (10.20)
σ	80.95	234.13* (11.22)	103.53	245.29* (12.98)
log L		-1105.32		-1393.87
R^2	0.234	0.211	0.258	0.234

1) t-statistics in parentheses (* Significant at 5% level).
2) Number of observations: 634.

Now the deviation variable is based on 50-day moving average of the daily exchange rate. The underlying assumption is that the central banks look at about two-month average of the exchange rate rather than 7-day average when they make decisions about whether to intervene, or about how much to buy or sell.

In addition, two more variables are added. The lagged dependent variable (lag order = 1) is added to make the latent variable model closer to a dynamically complete model, hence reducing serial correlation in the errors. The log return of the exchange rate on day $t-1$ is also added to see if amount of intervention reflects daily fluctuation in the exchange rate as well as deviation from longer-run average.

Unlike Neely (2002), the exchange rate returns are not statistically significant in any of the four cases reported in Table 1.13. In contrast, the lagged intervention is significant and has the correct sign in all cases. All other variables are also significant at 5% level except for the OLS estimate of the volatility variable in the Bundesbank reaction function. The R^2 measures are much higher now with the additional variables, and OLS still beats the MLE for both banks.

1.5.5 With Extended Sample Period

To see whether the results change with different sample period, the sample period is extended from that of Almekinders et al., i.e. 2/23/87 ~ 10/31/89, to 1/5/87 ~ 1/22/93. The sample size is more than doubled. The results are summarized in Table 1.14.

In terms of included variables, there are three different models. In Model A, the variables are the deviation of the exchange rate from its 7-day moving average, and the signed conditional variance of the exchange rate, which are the same as

Table 1.14: R^2 with extended sample (1/5/87 $\sim$ 1/22/93)

	Federal Reserve		Bundesbank	
	OLS	MLE	OLS	MLE
Model A	0.081	0.065	0.062	0.056
Model B	0.227	0.186	0.213	0.171
Model C	0.226	0.186	0.213	0.169

1) Included variables are $dev7_{t-1}$, vol_{t-1} for model A, $dev50_{t-1}$, vol_{t-1}, y_{t-1} and ret_{t-1} for model B, $dev50_{t-1}$, vol_{t-1} and y_{t-1} for model C.

2) Number of observations: 1456 for A, 1413 for B & C.

in Almekinders et al. In Model B, the variables are the same as in Table 1.13 where the deviation is from 50-day moving average and two more variables (lagged intervention and exchange rate return) are added. In Model C, the insignificant return series are dropped from Model B.

OLS of the linear model still outperforms the friction model MLE in all three cases and for both central banks.

1.6 CONCLUDING REMARKS

The core assumption of the friction model is that the dependent variable is insensitive to the changes in the explanatory variables over some range of the explanatory variables. This assumption, albeit plausible, does not seem to be consistent with the daily sterilized intervention data of the Federal Reserve and Bundesbank in the DM/USD market. It is shown in this chapter that a friction model, as adopted in Almekinders and Eijffinger (1996), is not better than a simple linear model in terms of in-sample explanatory power measured by the squared correlation coefficient of the actual and fitted values of intervention.

Although it is yet to be seen whether the results of this study are robust to further variations in sample periods and also in currencies, it seems doubtful that the friction model can explain the intervention behavior substantially better than a linear model as long as similar explanatory variables, particularly in terms of data frequency, are used as in Almekinders and Eijffinger (1996). This is not surprising in that it is not the responsibility of the central banks to secure an orderly market on a daily basis under the free floating exchange rate system.

As pointed out in Neely (2000), intervention may be a highly political process. Ito (2002) provides an example where the change of a key official dramatically changes the intervention behavior of the Japanese monetary authorities. Furthermore, central banks may rely on more powerful policy measures such as monetary policy rather than sterilized intervention, the effectiveness of which is at most unclear, when the instability in the foreign exchange market is too high to counter with limited foreign reserves. In short, there is not enough evidence that supports the friction hypothesis for foreign exchange intervention under a floating exchange rate system, at least on a daily basis.

Chapter 2

Threshold Nonlinearity In Central Bank Reaction Function

2.1 INTRODUCTION

Although infrequency of foreign exchange intervention indicates a nonlinear reaction function, it is shown in the previous chapter that the form of nonlinearity may not be consistent with the one implied in the friction model of Rosett (1959). If the friction model is a correct specification, then days of intervention ($y \neq 0$) must cluster around large positive or negative values of the explanatory variables while days without intervention ($y = 0$) must be coupled with small values of the explanatory variables. On the contrary, as seen in Figure 1.9 and Figure 1.10, observations with intervention and observations without intervention are both scattered over the entire range of the explanatory variables.

Decisions on intervention involve two stages. In the first stage a central bank has to decide whether to intervene or not. If the bank decides to intervene, then the next decision is about how much to buy or sell a currency against another. As a reaction function, a friction model attempts to explain the two stages of decision-making simultaneously. The first decision depends on whether the degree of instability in the market exceeds a certain threshold levels or not. Once a central bank decides to intervene, the amount of intervention (amount of U.S. dollars purchased against Deutsche marks) is in proportion to the degree of instability. Albeit attractive, this model seems to be too specific about the form of nonlinearity to be true.

As an alternative, a more flexible nonlinear model is considered in this chapter, which is a threshold model à la Hansen (2000). As a modelling tool of a central bank reaction function, a threshold model assumes that the expected amount of intervention is in proportion to the explanatory variables but the parameters in the conditional mean function, $E(y \mid x) = x\beta$, may change depending on the level of a threshold variable.

Assuming that the threshold variable is the same as the explanatory variable, a three-regime threshold model is illustrated in Figure 2.1. Note that the slope of the reaction function changes at the two thresholds in the figure, $g1$ and $g2$. The underlying assumption of this threshold model is that a central bank's reaction changes depending on the level of the disorderliness measured by x. It is further assumed that the central bank becomes more sensitive to disorderly market conditions when x is below $g1$ or above $g2$.

This univariate threshold model in Figure 2.1 is similar to its friction model counterpart in Figure 1.5 in that both models are nonlinear but piecewise linear. However, there are substantial differences between the two models. First, the

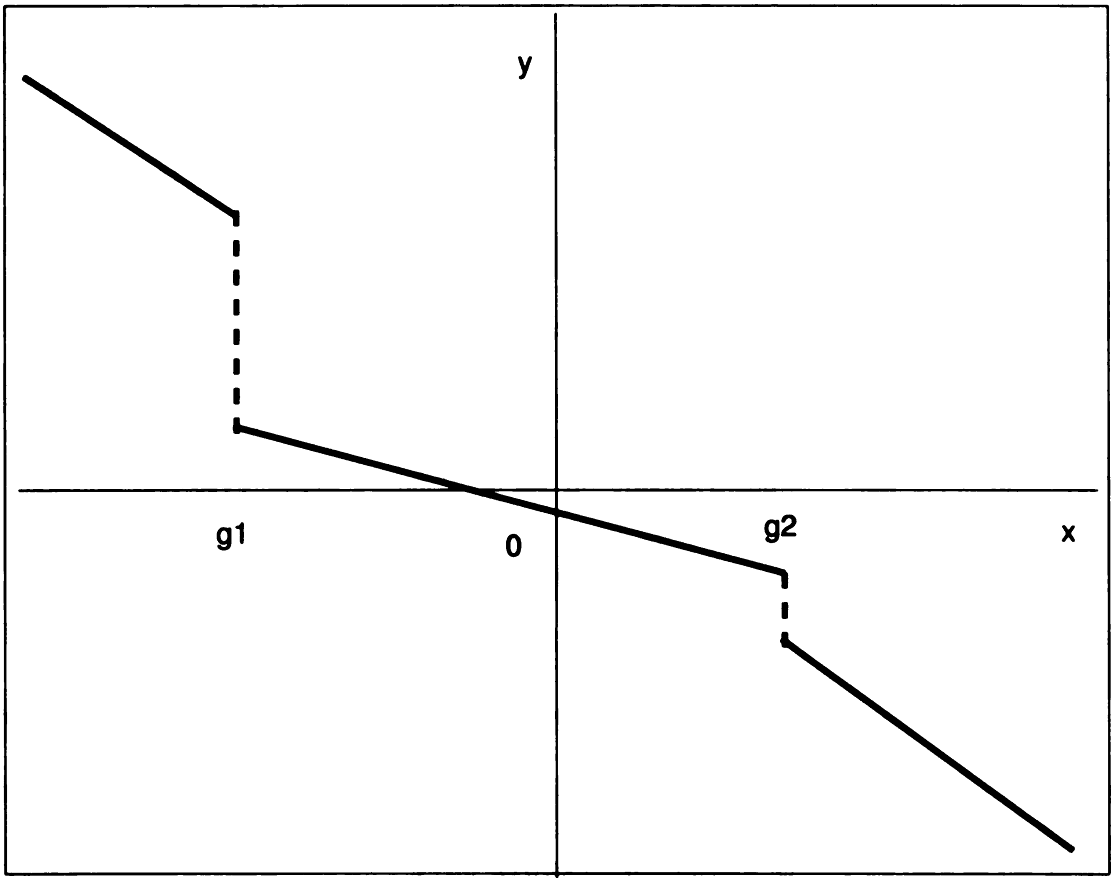


Figure 2.1: Threshold nonlinearity in a reaction function

threshold variables that determine the regimes are different. In the friction model (1.8), the threshold variable is the latent variable y^*, which is the amount of intervention if the central bank were to intervene no matter how small the level of disorderliness may be. In the threshold model, the threshold variable q is one of the measures of the degree of disorderly market conditions. While y^* is not observable in the friction model, q in the threshold model is observable.

Secondly, the slope of the middle regime is restricted to be zero in the friction model. There is no such restriction in the threshold model so that the slope may be nonzero. Therefore, the existence of friction becomes a testable hypothesis with a threshold model while it is embedded in the friction model.

Finally, the friction model requires explicit assumptions on the density function of the errors and it is estimated by the method of maximum likelihood. As explained in the previous chapter, it is not easy to test for specification errors since residuals are not obtainable. A threshold model, on the contrary, can be estimated by the method of least squares without assuming normality of the regression errors. Residual based tests for misspecification are also applicable with this modelling approach.

In the following section, the specification, estimation and test strategies of a threshold model are described in the context of foreign exchange intervention, which relies heavily on Hansen (1997, 1999, and 2000). Section 3 explains the data set, and section 4 presents the results of estimation and inference where some evidence is found in favor of a two-regime or three-regime threshold model against a linear model. As expected, intervention is more frequent and larger in size in the outer regimes than in the middle regime. Section 5 concludes with a summary.

2.2 THE MODELS

2.2.1 Measures of Disorderly Market Conditions

Rewrite the linear model of central bank reaction function in (1.1) as

$$y_t = x_t \beta + u_t \tag{2.1}$$

where the $(k+1) \times 1$ parameter vector β now includes the intercept β_{ℓ_0} as well as the slope coefficients $\beta_{\ell_1}, \ldots, \beta_{\ell_k}$. One important question in specifying this linear model is what measures of disorderly market conditions should be included in the vector of explanatory variables x_t. We simply followed Almekinders and Eijffinger (1996) in the previous chapter so that the estimation results were comparable. In this chapter, we want to be more careful about choosing the variables.

Although most empirical studies include deviation of exchange rate from some target levels and a measure of volatility of the exchange rate, there is no clear answer to the appropriate measures of the disorderly market conditions. Consequently, different studies include different variables in x_t. Almekinders and Eijffinger (1996) include the deviation of DM/USD rate from 7-day moving average and the conditional volatility estimated by GARCH(1,1). Frenkel and Stadtmann (2001) use 25-day moving average as a short-run target level of exchange rate, and purchasing power parity as a long-run target. For the volatility measure, they also rely on a GARCH model but they include lagged interventions of the central bank and its foreign counterpart as additional explanatory variables. Humpage (1999 Appendix) uses 10-day moving average, daily exchange rate return, a dummy variable representing the relative importance of the exchange market conditions among the policy objectives of the Federal Open Market Committee, and a 10-day rolling standard deviation of the exchange rate as a measure of its volatility. Kim and Sheen (2002) include interest rate differentials, profitability of intervention, and a ratio of foreign reserves to imports as a proxy of the budget constraint, as well as the usual deviation and volatility measures of the exchange rate. In calculating the deviation, they use a 150-day moving average rule following Neely (1998) and LeBaron (1999) who claims that it is the common choice among the market traders.

We follow these previous literature to include a deviation variable and a measure of exchange rate volatility. The deviation measure (dev_t) is defined in (1.2), which is

$$dev(m)_t = 100\left[log(S_t) - log\left(\frac{1}{m}\sum_{i=1}^{m} S_{t-i}\right)\right]. \quad (2.2)$$

The order of moving average (m) ranges from $m = 7$ to $m = 150$ in the above mentioned literature. We choose $m = 7$ for a short term target exchange rate and $m = 25$ for a longer term target. We do not choose the value of $m = 10$ as in Humpage (1999 Appendix) since it is similar to $m = 7$. The value of $m = 150$ in Kim et al. is also excluded because the corresponding series of dev_t is close to a unit root series. The Augmented Dickey-Fuller unit root test results are in Table 2.1.

Table 2.1: ADF unit root test statistics for dev_t series

Sample	*dev7*	*dev25*	*dev150*	Obs.
02/23/87 ~ 10/31/89	-8.62***	-4.03***	-2.09	651
01/05/87 ~ 01/22/93	-12.36***	-6.07***	-2.39	1459

1) 4 lags and an intercept are included in the tests.
2) *** significant at 1%, ** at 5%, * at 10% level.
3) Critical values for both samples: -3.4 at 1%, -2.9 at 5%, -2.6 at 10% level.

As for the volatility measure, the dominant choice seems to be the GARCH estimates. However, we need an appropriate signing method for these estimates since our combined intervention data can be either positive or negative while the estimated conditional variance is always positive. One possibility is to sign them with the sign of the exchange rate return as in Kim and Sheen (2002). One drawback with this approach is that the sign of exchange rate return changes almost every day while the direction of intervention is quite persistent. Almekinders et al.'s approach described in the previous chapter has the merit that the sign does not change too often. Therefore, we will continue to use their volatility measure as defined in (1.3) and (1.4).

Following Kim and Sheen (2002) and Neely (2002), we also include lags of intervention variable as the explanatory variables. The order of autoregressive terms p is chosen so as to eliminate the serial correlation in the residuals. Other variables such as daily exchange rate return, intervention by the foreign central bank and the interest rate differential are not included since they are in general not statistically significant in the previous literature or in our own preliminary estimation results. The rest of the variables mentioned above are also not included since daily observations on such variables are not available.

Consequently, the vector x_t is

$$x_t = \begin{pmatrix} 1 & dev7_{t-1} & dev25_{t-1} & vol_{t-1} & y_{t-1} & y_{t-2} & \cdots & y_{t-p} \end{pmatrix} \tag{2.3}$$

where $dev7_{t-1}$ and $dev25_{t-1}$ denote $dev(m)_{t-1}$ in (2.2) with $m = 7$ and $m = 25$, respectively. Although this seems to be a relatively parsimonious specification, it will help us reduce the time required for bootstrap in the test procedure of the threshold model considered below. With a three-regime model, one more variable means three more parameters to estimate which significantly increase the running time of the bootstrap procedure that is required for testing linearity.

2.2.2 Threshold Models

An ℓ-regime threshold model allows the parameter vector β in the linear model (2.1) to change ℓ times based on the values of a threshold variable q_t. A three-regime threshold model can be written as

$$y_t = x_t\beta_1 \cdot 1(q_t \leq \gamma_1) + x_t\beta_2 \cdot 1(\gamma_1 < q_t \leq \gamma_2) + x_t\beta_3 \cdot 1(q_t > \gamma_2) + \varepsilon_t \qquad (2.4)$$

where $1(\cdot)$ is the indicator function, and $\beta_i = (\beta_{i0}\,\beta_{i1}\,\ldots\,\beta_{ik})'$ for $i = 1, 2, 3$. Figure 2.1 in the previous section is a special case of (2.4) with one regressor ($k = 1$).

If x_t consists of lags of y_t only, it is called a TAR model. In addition, if q_t is one of the lagged y_t, the model becomes a SETAR model. We use some exogenous variables such as deviation and volatility measures as explanatory variables in addition to the autoregressive terms of y_t. Thus we call the model a threshold model, following Hansen (2000), rather than a TAR model. In our analysis, the threshold variable q_t is one of the exogenous variables. Specifically, we try all three candidates, $dev7_{t-1}$, $dev25_{t-1}$ and vol_{t-1}, one by one as the threshold variable and then choose the one that minimizes the sum of squared residuals.

The parameters (β_1, β_2, β_3, γ_1, γ_2) can be estimated by the method of sequential conditional least squares. If the thresholds γ_1 and γ_2 are given, the model becomes linear in the rest of the parameters and can be rewritten as

$$y_t = \xi_t \theta + \varepsilon_t \tag{2.5}$$

where $\theta = (\beta_1,\ \beta_2,\ \beta_3)$ and the $3(k+1)$ row vector ξ_t is

$$\xi_t = (x_t \cdot 1(q_t \le \gamma_1) \quad x_t \cdot 1(\gamma_1 < q_t \le \gamma_2) \quad x_t \cdot 1(q_t > \gamma_2))\,. \tag{2.6}$$

Now, θ in (2.5) can be estimated by OLS.

Define the set of threshold observations Γ as

$$\Gamma = \{q_t \mid t = 1, \ldots, T\}. \tag{2.7}$$

For each pair of $(q_i,\ q_j) \in \Gamma^2$ where $i,\ j = 1, \ldots, T$ and $q_i < q_j$, substitute $(q_i,\ q_j)$ for $(\gamma_1,\ \gamma_2)$ in (2.6). Estimate (2.5) by OLS and obtain the sum of squared residuals $S(q_i,\ q_j)$ which is defined as

$$S(q_i,\ q_j) = \sum_{t=1}^{T} (y_t - \xi_t \theta)^2. \tag{2.8}$$

Then the pair of $(q_i,\ q_j)$ that minimizes the sum of squared residuals will be the threshold estimates $(\hat{\gamma}_1,\ \hat{\gamma}_2)$. The estimates of the other parameters, $(\hat{\beta}_1,\ \hat{\beta}_2,\ \hat{\beta}_3)$, are obtained as the OLS estimates of equation (2.4) with $\gamma_1 = \hat{\gamma}_1$ and $\gamma_2 = \hat{\gamma}_2$. Note that this procedure requires up to $T(T-1)/2$ OLS regressions. With T=648, this means about 210,000 regressions.[1] Since we have three candidates for the threshold variable, the whole estimation process requires about 630,000 regression. Although this is not a problem in the estimation stage, it becomes a critical problem in the

[1] Actual number of regressions is smaller than this because we have to allow minimum number of observations in each regime.

hypothesis testing where the computation of bootstrap p-value requires thousands of replications of this procedure. In order to reduce this computational burden, Hansen (1999) proposes a two-step estimation procedure where the two thresholds are estimated sequentially one by one. Details of this two-step procedure will be described later.

In order to estimate the multi-regime model of (2.4), some restrictions must be imposed on the range of γ_1 and γ_2 so that each regime has at least the minimum required number of observations. One obvious requirement is that the number of observation of each regime, T_i for $i = 1, 2, 3$, must be greater than or equal to the number of explanatory variables, $k + 1$. Another more important requirement is concerned with the asymptotic properties of the estimators and test statistics. For a linear model, we can rely on consistency of the OLS estimator if T is large. Similarly, with the three regime model of (2.4), we need to have large T_i for each regime because the conditional least square procedure is equivalent to splitting the sample into three sub-samples and then estimate β_i using only T_i observations.[2] Hansen (1999) explains that it is necessary to have $T_i/T \geq \tau$ for some $\tau > 0$ as $T \to \infty$. In practice, however, it is inevitable to choose τ somewhat arbitrarily. Hansen (1999) suggests $\tau = 0.10$. We start with $\tau = 0.15$ so that each regime has approximately 100 observations or more, and then see how the results change with lower or higher values of τ.

[2]This involves three smaller regressions and it is faster than running one large regression with equation (2.5).

2.2.3 Hypothesis Testing

If there exists a friction area in the reaction function, i.e. the slope is zero between $g1$ and $g2$ in Figure 2.1, then the parameters in (2.4) must meet two conditions. First, all the parameters in the middle regime must be zero ($\beta_2 = 0$), while the slope coefficients in the two outer regimes are not zero ($\beta_{i1}, \ldots, \beta_{ik} \neq 0$ for $i = 1, 3$). Secondly, since the friction is assumed to be around zero values (i.e. over small realizations) of the threshold variable, we must have $\gamma_1 < 0$, $\gamma_2 > 0$. These conditions form a testable hypothesis as

$$H_0 : \gamma_1 < 0, \ \gamma_2 > 0, \ \beta_1 \neq 0, \ \beta_2 = 0, \text{and} \beta_3 \neq 0, \tag{2.9}$$

where each parameter vector β_i for $i = 1, \ldots, 3$ is zero if all of its elements are zero.

However, this hypothesis is meaningful only if there exist three distinct regimes, or a significant threshold effect, in the central bank reaction function. Thus, it is necessary to test if the nonlinear model in (2.4) is a better specification than the linear model in equation (2.1). This is equivalent to testing the null hypothesis of

$$L_0 : \beta_1 = \beta_2 = \beta_3 = \beta. \tag{2.10}$$

Hansen (1999) suggest a test statistic of

$$F = T \left(\frac{S_L - S_T}{S_T} \right) \tag{2.11}$$

where S_L is the sum of squared residuals from LS estimation of the linear model (2.1) and S_T is the sum of squared residuals from the three-regime model (2.4).

When the thresholds γ_1 and γ_2 are known, F is asymptotically equivalent to the usual F statistic. Since the thresholds are unknown and not identified under (2.10), however, F follows an unknown asymptotic distribution. We rely on bootstrapping methods to compute the p-values with and without the conditional heteroscedasticity assumption.

Under homoscedastic error assumption, we get a set of bootstrap errors, $\tilde{e} = \{\tilde{e}_t \mid t = 1, \ldots, T\}$ by randomly drawing T times with replacement from the OLS residuals $\hat{e} = \{\hat{e}_t \mid t = 1, \ldots, T\}$ of the linear model (2.1). Then a set of data on the dependent variable is generated by

$$\tilde{y}_t = \tilde{x}_t\hat{\beta} + \tilde{e}_t \tag{2.12}$$

where $\tilde{x}_t = (1, dev7_{t-1}, dev25_{t-1}, vol_{t-1}, \tilde{y}_{t-1}, \tilde{y}_{t-2}, \cdots, \tilde{y}_{t-p})'$ and $\hat{\beta}$ is the OLS estimate of equation (2.1). Substituting $\tilde{y}_t$ for y_t in the linear model and the threshold models, the models are re-estimated to get one value of $\tilde{F}$ which is

$$\tilde{F} = T\left(\frac{\tilde{S}_0 - \tilde{S}_1}{\tilde{S}_1}\right) \tag{2.13}$$

where $\tilde{S}_0$ and $\tilde{S}_1$ are the sum of squared residuals from the linear model and the threshold model, respectively, with the bootstrap data. Out of 2,000 replications, the proportion of $\tilde{F}$ greater than F is the approximate p-value.

Under heteroscedastic error assumption, the procedure is a bit more complicated because we have to impose heteroscedasticity on the bootstrap errors $\tilde{e}$. See APPENDIX E for a detailed description of the bootstrap procedure.

If the linearity hypothesis is rejected in favor of the threshold nonlinearity, we can move on to test H_0 in (2.9). Hansen (1997) explains that since the threshold estimates are super-consistent for the true thresholds due to the discrete nature of the parameter space, inference on the slope parameters can be done in the usual way, i.e. relying on the asymptotic normality of the slope estimators $(\hat{\beta}_1, \hat{\beta}_2, \hat{\beta}_3)$.

Unfortunately, the asymptotic distribution of the threshold estimators $(\hat{\gamma}_1, \hat{\gamma}_2)$ are not developed so far for three or higher order models. However, Bai (1997) and Bai and Perron (1998) show that the estimate of the threshold in a two-regime model is consistent for one of the two thresholds in the three-regime model.

Hansen (1997, 2000) in turn provides a method to construct an asymptotically valid confidence interval for the threshold of the two-regime model. Noting that the likelihood ratio statistic $LR(\gamma)$ is a function of the threshold γ, he recommends inverting the $LR(\gamma)$ statistic to get the confidence interval for γ. See APPENDIX F for details.

Therefore, it is possible to test either $\gamma_1 < 0$ or $\gamma_2 > 0$ if some regularity conditions are met. For this purpose we also estimate a two-regime model as

$$y_t = x_t\alpha_1 \cdot 1(q_t \leq \gamma) + x_t\alpha_2 \cdot 1(\gamma > q_t) + \nu_t. \tag{2.14}$$

The fact that the threshold estimate $\hat{\gamma}$ for the two-regime model is the same with either $\hat{\gamma}_1$ or $\hat{\gamma}_2$ is very useful in reducing the running time for the estimation of the three-regime model of (2.4).[3] The two-step approach proposed by Hansen (1999) consists of the following steps:

- Estimate the two-regime model and obtain $\hat{\gamma}$ which is the element of Γ that

[3]It is also true that $\hat{\alpha}_1 = \hat{\beta}_1$ if $\hat{\gamma} = \hat{\gamma}_1$, or $\hat{\alpha}_2 = \hat{\beta}_3$ if $\hat{\gamma} = \hat{\gamma}_2$.

minimizes the sum of squared residuals.

- Estimate the three-regime model (2.5) after setting γ_1 and γ_2 in (2.6) as either $(\hat{\gamma},\ q_s)$ or $(q_s,\ \hat{\gamma})$ for each $q_s \in \Gamma$. Since $\gamma_1 < \gamma_2$ by assumption, the rule is

$$(\gamma_1,\ \gamma_2) = (q_s,\ \hat{\gamma}) \quad \text{if } q_s \leq \hat{\gamma},$$

$$(\gamma_1,\ \gamma_2) = (\hat{\gamma},\ q_s) \quad \text{if } q_s > \hat{\gamma}.$$

If there are N distinct elements in Γ for each of the M candidates of the threshold variable, then the two-step approach requires up to $(2N-1)M$ regressions.[4] With $N = 648$ and $M = 3$, it is only 3,885 OLS regressions which is far less than the over 0.6 million regressions of the joint estimation approach.[5]

2.3 THE DATA

We use the official data on Federal Reserve and Bundesbank intervention in the DM/USD market during 2/23/87 ~ 10/31/89. This is the same data set used in chapter 1, which in turn is a subset of the data set used in Baillie and Osterberg (2000) covering the period between 1/5/87 ~ 1/22/93. We focus on the subsample period to minimize the potential effects of structural breaks over time.[6]

[4] For each candidate of the threshold variable, N regressions are required in the first stage and $N-1$ regressions in the second stage.

[5] After allowing for the minimum number of observations in each regime, the required number of regressions is smaller than this. With the minimum requirement of sample $\tau = 0.15$, the joint estimation requires up to $0.55N(0.55N-1)/2*3 \approx 190{,}000$ regressions while the two-step estimation involves maximum $(0.7N+0.4N)*3 = 2{,}138$ regressions. In computing the bootstrap p-values, we use the estimated threshold, hence $M = 1$. For 2,000 bootstrap replications, it takes about 1.5 hours with the two-step approach on a personal computer with a Pentium IV 2.4Gh CPU. With the joint estimation approach, it will take more than a week.

[6] It is possible theoretically to allow both regime switching and structural breaks in a nested threshold model with two threshold variables, e.g. *dev*25 and time. However, it seems difficult to

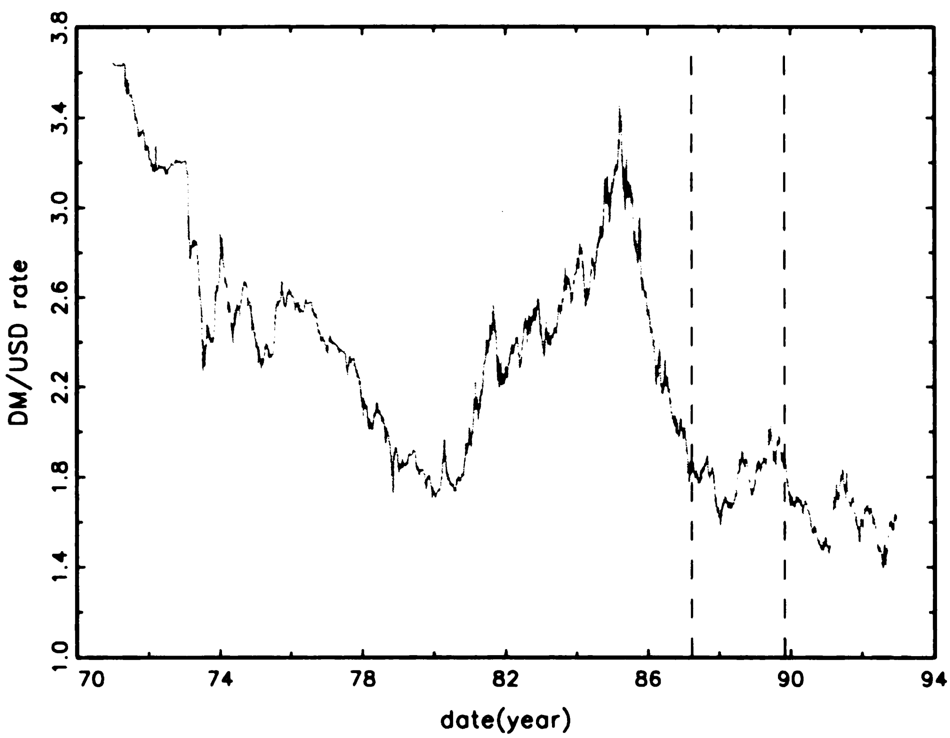


Figure 2.2: DM/USD exchange rate (1/4/1971 ~ 1/22/1993).

Excluding holidays and weekends, our sample has total 651 daily observations. Federal Reserve intervenes 137 times, of which 36 are buying operations and the rest 101 are selling operations. Bundesbank intervenes 173 times with 42 buying and 131 selling operations.

The asymmetry between buying and selling operations seems to reflect the fact that during the sample period the value of the US dollar against deutsche mark had been around the historic minimum level of the floating exchange rate system. The exchange rate movement between 1/4/1971 and 1/22/1993 is shown in Figure 2.2 where our sample period is between the two dashed lines. Since the exchange

find appropriate inference methodology for such a case. Also, the noticeable drop in the frequency of intervention in 1990 and thereafter makes it difficult to rely on asymptotics.

rate is low in our sample period due to the sharp decrease in the previous two years, the agencies seem to have been more worried about the possibility of rapid upward movement of the exchange rate than rapid downward movement.

The amount of intervention at time t is the amount of US dollars bought by Federal Reserve against Deutsche marks between the market closing time on day $t-1$ and the market closing time on day t. The exchange rate at time t is the bid rate at 9 : 30 AM in Paris, which is 3 : 30 AM in New York time. Therefore, the deviation variable dev_{t-1} precedes y_t by about 13 hours.

2.4 ESTIMATION AND TEST RESULTS

We estimate the reaction functions for the Federal Reserve and the Bundesbank. For each bank, the two-regime and three-regime threshold reaction functions are estimated with GAUSS programs, which are revised versions of those available at Bruce Hansen's web site.[7]

2.4.1 Federal Reserve Reaction Function

Tests for nonlinearity

Although estimation precedes hypothesis tests, we begin with the results of the tests whether two-regime or three-regime threshold nonlinearity exists in the reaction function.

As reported in Table 2.2, the test statistic F_{12} for one-regime linear model against two-regime model is 62.37. The p-values are computed as the proportion of

[7] www.ssc.wisc.edu/ bhansen/progs/progs_threshold.html

Table 2.2: Test for nonlinearity (Federal Reserve, $\tau = 0.15$)

	F-statistic	Bootstrap p-values		
		Homoscedastic	Heteroscedastic1*	Heterosecdastic2**
F_{12}	62.37	0.0000	0.0755	0.0160
F_{13}	67.68	0.0000	0.1355	0.0470
F_{23}	4.84	1.0000	0.9400	0.9580

* h_t is from a regression of $\hat{e}_t^2$ on x_t^2 keeping all elements of x_t.

** Insignificant elements of x_t are dropped in estimating h_t.

those bootstrap simulations out of 2,000 replications that have the F-statistic larger than 62.37. When the errors in the linear model is assumed to be homoscedastic, the p-value is zero. Therefore, we reject the null hypothesis of linearity in favor of a two-regime threshold nonlinearity. With correction for heteroscedasticity, the p-value is about 0.08 and we reject the null of linear reaction function at 10% level but not at 5% level. However, when the conditional variance h_t is re-estimated with only significant regressors in equation (E.2), the bootstrap p-value drops to about 0.02.

Against three-regime alternative, the F_{13} statistic from the estimation is 67.68, which is not much different from the statistic of 62.37 for the two-regime alternative. The bootstrap p-value is still close to zero with homoscedasticity. With heteroscedasticity, the p-value is 0.1355 and we fail to reject the linearity at 10% level. Again using the re-estimated h_t with only significant elements of x_t^2, we get a p-value below 0.05.

On the other hand, the F_{23} statistic is clearly insignificant with or without the heteroscedasticity assumption for the regression errors.

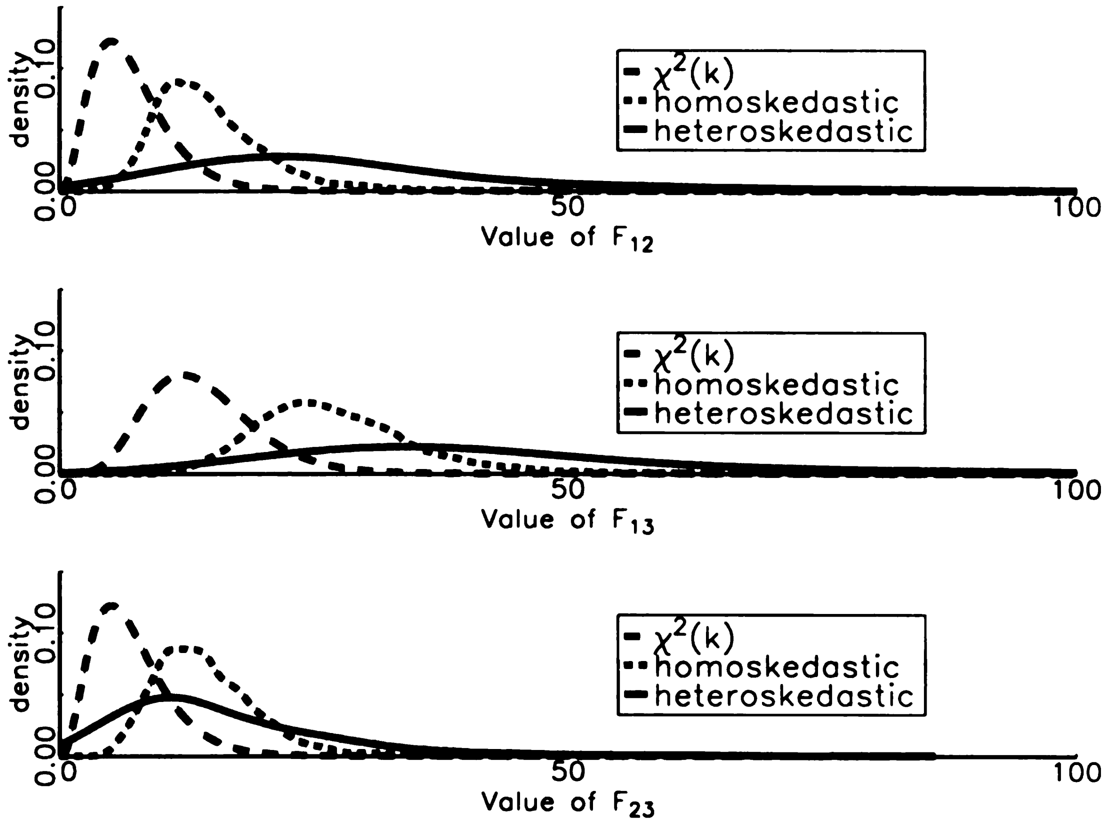


Figure 2.3: Distribution of F-statistics (Federal Reserve 87 $\sim$ 89).

The clear difference between the homoscedastic p-values and heteroscedastic p-values are depicted in Figure 2.3. The three graphs in the upper panel are, respectively, the true density of χ^2 distribution with $k = 7$ degrees of freedom, the kernel density estimated with the 2,000 bootstrap data with homoscedastic error assumption, and the kernel density estimated with heteroscedasticity (keeping all regressors). The kernel density graphs are to the right of the χ^2 density, reflecting the sampling errors in the threshold estimates. Of the two kernel densities, the one with heteroscedasticity has much higher dispersion.

In the middle panel, the three graphs for F_{13} statistic reveal a similar pattern.

Comparing the two panels, we can also see that F_{13} statistics tend to have fatter tails on the right than F_{12} statistics implying that a lot more improvement in goodness-of-fit is required for a three-regime model to beat a linear model, in particular when there is heteroscedasticity. In the lower panel are the density graphs of the test statistics for the null of two regimes against three regimes. The three density graphs for F_{23} statistics are closer to one another compared to the cases for F_{12} and F_{13} statistics.

<u>Linear reaction function</u>

The estimation results are in Table 2.3. The second column is for the linear model. The next two columns are for the two-regime threshold model, and the last three columns contain the estimation results with the three-regime model.

As for the linear model, all three measures of the disorderly market conditions ($dev7_{t-1}$, $dev25_{t-1}$, vol_{t-1}) are significant at 5% or lower level and have the expected negative signs implying that the Federal Reserve's intervention tends to be against the wind and in proportion with the degree of market instability.

Out of the three lags of the dependent variables, y_{t-1} and y_{t-3} are significant at 1% and 10% level, respectively, while y_{t-2} is insignificant. Overall, the implication is that recent interventions increase the expected amount intervention in the near future. These three lags are enough to eliminate serial correlation in the residuals. Ljung-Box test statistics indicate that there are no strong serial correlations in the residuals up to the order of 12 (Q(12) = 13.07, 5% critical value 21.03).

However, there is very strong evidence for heteroscedasticity (Q2(12)=80.83).

Table 2.3: Federal Reserve reaction function (2/23/87 ~ 10/31/89, q=dev25)

Variable	Linear	Two-regime		Three-regime		
		Reg1	Reg2	Reg1	Reg2	Reg3
Constant	-10.36***	-4.39*	97.55*	-4.44	-2.79	97.55*
	(2.83)	(2.40)	(58.66)	(14.03)	(2.22)	(58.66)
$dev7_{t-1}$	-11.14**	-8.25**	3.31	-3.80	-11.19***	3.31
	(4.70)	(3.38)	(18.75)	(6.26)	(3.85)	(18.75)
$dev25_{t-1}$	-5.84**	-1.78	-58.24**	-3.00	-3.97	-58.24**
	(2.41)	(2.14)	(24.52)	(6.75)	(2.92)	(24.52)
vol_{t-1}	-22.05***	-17.65***	-47.90***	-24.34***	-12.62***	-47.90***
y_{t-1}	0.25***	0.26***	0.12	0.21*	0.27***	0.12
y_{t-2}	-0.03	0.12	-0.23***	0.16	0.08	-0.23***
y_{t-3}	0.17*	0.05	0.18	0.10	0.05	0.18
Obs.	648	542	106	194	348	106
Buy	36	36	0	28	8	0
Sell	101	55	46	13	42	46
$\hat{\sigma}$	77.63	57.60	132.90	66.06	52.30	132.90
R^2	0.28	0.24	0.27	0.25	0.22	0.27
$F_{all=0}$	42.22	28.92	5.97	10.66	15.87	5.97
(*Pvalue*)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)
$F_{b_2 b_3 b_4=0}$	21.11	9.83	4.50	3.13	5.81	4.50
(*Pvalue*)	(0.00)	(0.00)	(0.01)	(0.03)	(0.00)	(0.01)
$\hat{\gamma}_1/\hat{\gamma}_2$		2.09		-1.08 / 2.09		
R^2(all)	0.28	0.35		0.35		
Skewness	-1.96	-1.14		-1.15		
Kurtosis	17.23	14.07		14.07		
Q(12)	13.07	14.50		15.50		
Q2(12)	80.83	76.43		77.43		

1) Heteroskedasticity consistent standard errors in parenthesis.
2) *** significant at 1%, ** at 5%, * at 10% level.
3) $F_{all=0}$ tests overall significance except the constant.
4) $F_{b_2 b_3 b_4=0}$ tests significance of the three exogenous variables.

A separate regression of the squared residuals on squared regressors, as reported below, also indicates that the errors are heteroscedastic. Therefore, White's heteroscedasticity consistent standard errors are reported in parentheses for all three models in Table 2.3.

$$\begin{array}{rcllll} \hat{e}_t^2 & = & 1091.58 + 291.68\, dev7_{t-1}^2 & + 885.82\, dev25_{t-1}^2 & -1662.80\, vol_{t-1}^2 \\ & & (1236.88) \;\; (716.34) & (297.01) & (2351.78) \\ & & -0.01\, y_{t-1}^2 \; -0.04\, y_{t-2}^2 \; + 0.16\, y_{t-3}^2 & & \\ & & (0.02) \quad\quad (0.02) \quad\quad (0.10) & & \end{array}$$

· $R^2 = 0.13$
· F-statistic (p-value)= 100.37 (0.00)
· Standard errors in parentheses.

Despite the strong evidence for heteroscedasticity, we cannot claim that the functional form of the heteroscedasticity is correct. Note that the R^2 is relatively low at 0.13 and the squared explanatory variables are insignificant except $dev25_{t-1}^2$. We have already seen that dropping the insignificant regressors (keeping intercept even if insignificant) changes the nonlinearity test results in 2.3. Also, some of the coefficients are negative so that the fitted conditional variances are non-positive for 20 out of 648 observations (3.1 %). In the bootstrap procedure, we drop these 20 observations.

The standardized errors $\breve{e}$ obtained by equation (E.1) are depicted in Figure 2.4 together with the autocorrelograms of $\breve{e}$ and $\breve{e}^2$. The Ljung-Box test statistics are Q(12)=18.23 for the standardized errors and Q2(12)=1.06 for the squared errors. Both are below the 95% critical value of the $\chi^2(12) = 21.03$, justifying the random draws from these standardized errors in the bootstrap procedure.

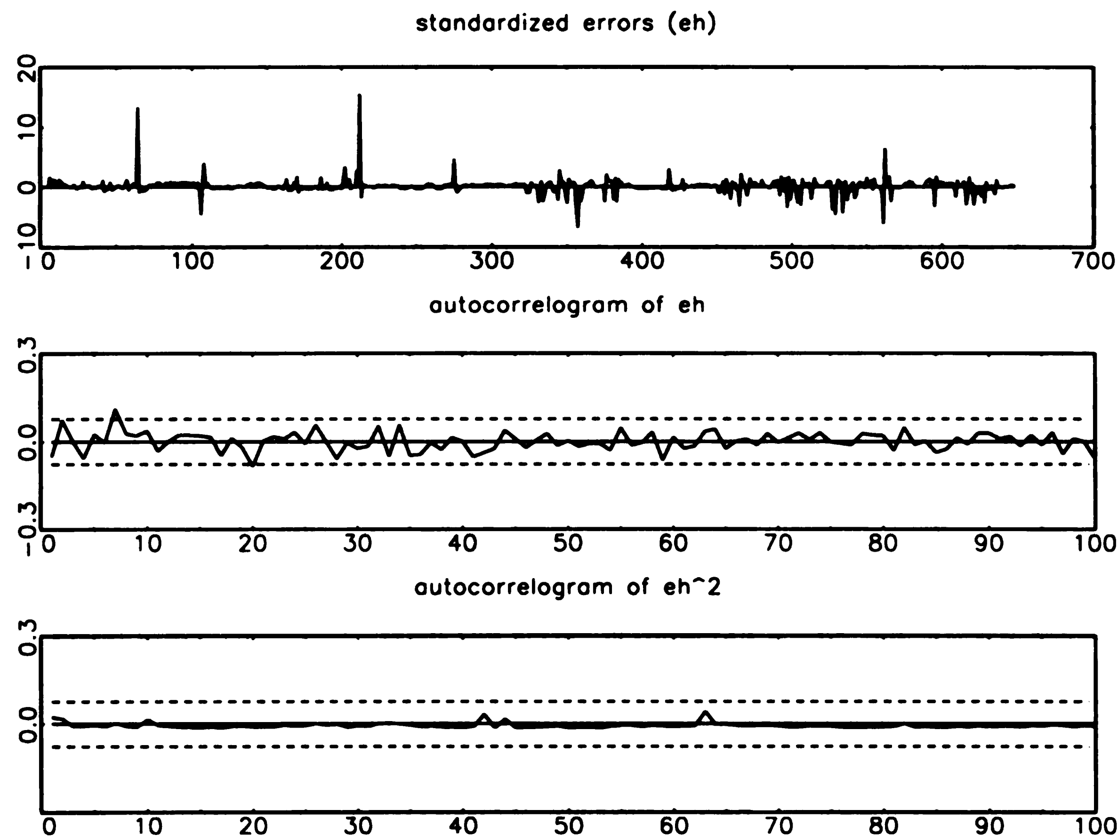


Figure 2.4: Standardized errors (Federal Reserve 87 $\sim$ 89).

Two-regime model

With the two-regime model, the optimal threshold variable is estimated to be $dev25_{t-1}$ and the point estimate of the threshold γ is 2.09. This positive threshold indicates that the strongest nonlinearity in the Federal Reserve's reaction function exists when the dollar appreciates rapidly. This supports our earlier claim that the two central banks are more sensitive to appreciation than depreciation of the dollar in light of the history of the exchange rate fluctuations under the floating rate system.

In order to see if the threshold is indeed positive, we estimate its 95% confidence interval using the LR statistic defined in equation (F.1) and the criterion in equation (F.7) in APPENDIX F. When $\hat{\eta}^2$ in (F.1) is estimated by a polynomial regression, the confidence interval is [-2.36, 2.20] while it is [1.42, 2.20] with kernel regression (Epanechnikov). In Figure 2.5 $LR(\gamma)$ is depicted against γ. With the polynomial regression, the entire graph is way below the 95% critical value for any values of γ. Therefore, there is no meaningful interval estimate for the threshold. Since the linearity test statistic F_{12} was significant at least at 10% level, we believe the $\hat{\eta}^2$ is overestimated by the polynomial regression so that the $LR(\gamma)$ statistic is scaled down too much. With the kernel estimate of $\hat{\eta}^2$, on the other hand, the confidence interval exists and clearly positive.

From Table 2.3, we see that about 84% of the observations belong to regime 1 and the rest 16% belong to regime 2. Since most of the observations are in regime 1, the slope estimates of this regime are similar to those of the linear model. In regime 2, the slope estimates for $dev25_{t-1}$ and vol_{t-1} are much bigger in modulus than in regime 1 but the shorter term deviation measure ($dev7_{t-1}$) is insignificant. All 46 occasions of intervention in regime 2 are selling operations, which means the Federal Reserve leans against the wind when the exchange rates are above the target of 25-day moving average by 2.1% or more.

The R^2 of the two-regime model (0.35) is much larger than the linear model R^2 (0.28), which is consistent with the significance of the F_{12} statistic in Table 2.3.

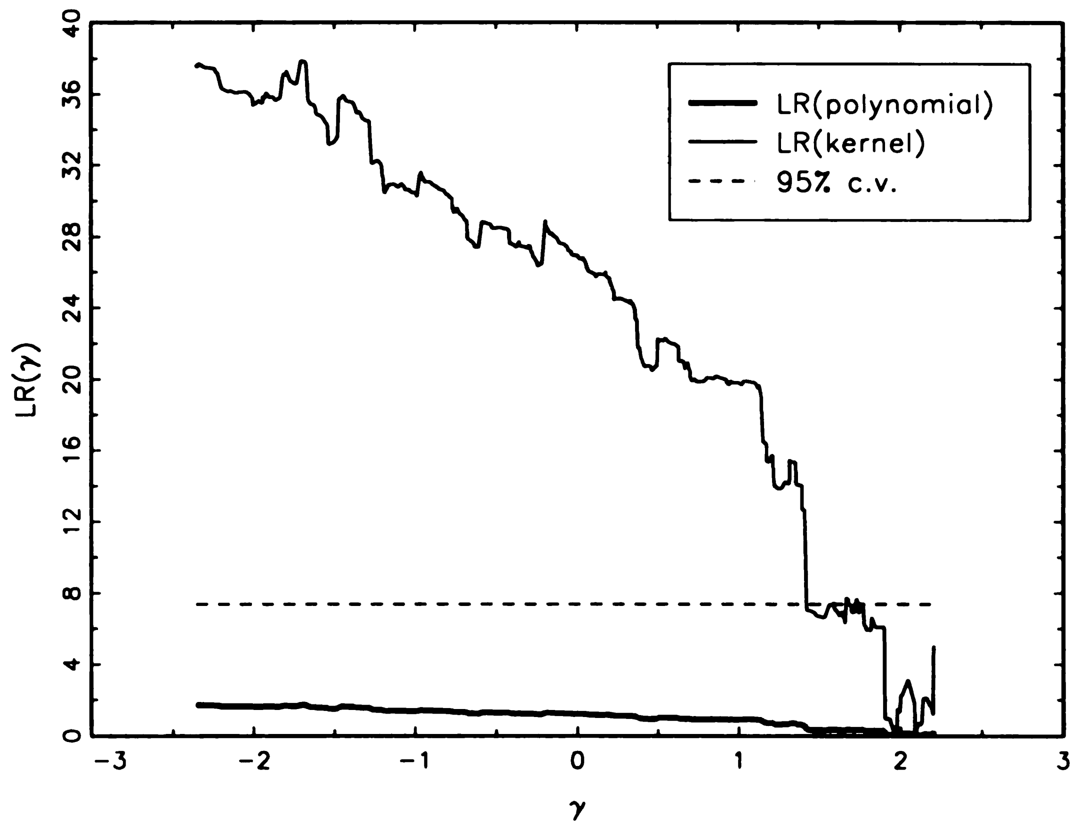


Figure 2.5: Confidence interval of threshold (Federal Reserve 87 ~ 89).

<u>**Three-regime model**</u>

In the case of the three-regime model, the estimate of the second threshold is $\hat{\gamma}_1 = -1.08$. It has negative sign. Note that the other threshold estimate $\hat{\gamma}_2 = 2.09$ is the same as $\hat{\gamma}$ of the two-regime model. Consequently, the third regime is the same as the second regime of the two-regime model. The opposite signs of the two threshold estimates are consistent with the friction hypothesis.

As we have seen in the nonlinearity tests, however, this model is no better than the two-regime model. We find some evidence confirming this earlier finding. There is no gain in terms of R^2 and the slope estimates of the mid-regime are closer

Table 2.4: Descriptive statistics (Federal Reserve, $\tau = 0.15$)

	Regime 1	Regime 2	Regime 3
Frequency of buying	0.14	0.02	0.00
Frequency of selling	0.07	0.12	0.43
Mean buying ($Million)	18.47	2.60	0.00
Mean selling ($Million)	9.10	15.62	91.10

to the estimates of regime 1 while they are quite different from the estimates of regime 3 which is already identified by the two-regime model.

The explanatory variables are jointly significant in all three regimes. The three measures of market instability are also jointly significant in all three regimes. The significance of the explanatory variables in the middle regime is noteworthy since they must be jointly insignificant if there is friction in the reaction function.

With this three-regime model, we don't see the zero values of intervention clustering around "small realizations" of the threshold variable as the friction hypothesis suggests. In regime 1, there is no intervention for 83% of the observations. This no-intervention ratio is 86% in regime 2 and 57% in regime 3.

However, there is a tendency for the Federal Reserve to buy dollars more often and in larger amounts in regime 1, where the dollar is depreciating, than in other regimes. Likewise, the central bank sells more frequently and in larger quantities in regime 3, where the dollar is appreciating, than in other regimes. This finding is summarized in Table 2.4.

Table 2.5: Sensitivity to τ (Federal Reserve)

	$\tau = 0.05$			$\tau = 0.10$			$\tau = 0.20$			$\tau = 0.30$		
	R1	R2	R3	R1	R2	R3	R1	R2	R3	R1	R2	R3
q_t	dev7			dev7			dev25			dev25		
$\hat{\gamma}$	1.35			1.33			1.82			1.27		
$\hat{\gamma}_1$	0.99			-1.41			-1.54			-1.08		
Obs.	533	55	60	65	519	64	156	361	131	194	261	193
Buy	35	0	1	17	18	1	25	11	0	28	7	1
Sell	60	14	27	9	63	29	12	36	53	13	23	65
F_{12}	.178 (0.083)			.126 (.034)			.064 (0.017)			.049 (0.013)		
F_{13}	.337 (0.180)			.177 (.063)			.117 (0.029)			.053 (0.010)		
F_{23}	.785 (0.795)			.571 (.597)			.887 (0.899)			.334 (0.293)		
F_{slope}	.00	.28	.33	.01	.00	.33	.01	.00	.00	.03	.01	.00

1) R1, R2, R3 are the regimes in the three regime model.
2) q_t is the estimated threshold variable.
3) P-values of F_{ij} statistics are reported with heteroscedasticity assumption. In the parentheses are the p-values based on conditional volatilities re-estimated without insignificant variables.
4) F_{slope} is the p-value for the F-statistic testing joint significance of the exogenous variables in each regime.

Sensitivity of results

The above empirical results are obtained by setting the minimum sample size τ in each regime as 15% of the whole sample. In order to see if this choice is critical for the results, we re-estimate the models with different values of τ as summarized in Table 2.5.

The estimated threshold variable is $dev7_{t-1}$ when $\tau < 0.15$ while it is $dev25_{t-1}$ otherwise. This implies that when the market is extremely unstable, the Federal Reserve's intervention is more sensitive to shorter run movements of the exchange rate.

The first threshold estimate is positive in all cases. Except for $\tau = 0.05$, the second threshold is estimated to be negative. The positive second threshold for $\tau = 0.05$ suggests that further nonlinearity may exist in the outer regimes of the three-regime model.

For the cases where the two thresholds are opposite in signs, buying operations are relatively more frequent in the depreciating regime and selling operations are more frequent in the appreciating regime.

The test statistics for linearity against two- or three-regime alternative are insignificant at 5% level except for F_{12} statistic with $\tau = 0.30$. However, when the parameters of the conditional variance equation (E.2) were re-estimated after dropping insignificant explanatory variables, the p-values of the statistics are much smaller as reported in the parentheses of the table. With this change, two- and three-regime models are significantly better than a linear model at 5% level for most of the cases with $\tau \geq 0.10$. One exception is for the F_{13} statistic with $\tau = 0.10$.

Again, for the cases with positive and negative thresholds, the variables that measure market instability ($dev7_{t-1}$, $dev25_{t-1}$, vol_{t-1}) are jointly significant in all sub-regimes except in regime 3 with $\tau = 0.10$.

Overall, the results are quite similar with different values of τ and do not provide sufficient evidence for the existence of friction.

Table 2.6: Test for nonlinearity (Bundesbank, $\tau = 0.15$)

	F-statistic	p-values	
		Homoscedastic	Heteroscedastic
F_{12}	81.32	0.0000	0.0190
F_{13}	98.35	0.0000	0.0295
F_{23}	15.12	0.4690	0.4485

2.4.2 Bundesbank Reaction Function

Tests for nonlinearity

The test results in Table 2.6 indicate that the two-regime or three-regime model is better than a linear model in explaining the Bundesbank's reaction during the period covered by Almekinders et al.'s sample. The F_{12} and F_{13} statistics are significant at lower than 5% level even without dropping insignificant regressors in estimating the conditional variance h_t. The F_{23} statistic is still insignificant although the p-values are much smaller than those for the Federal Reserve. The simulated distribution of the statistics are in Figure 2.6, where the graphs have similar patterns as the Federal Reserve's case in Figure 2.3.

Linear reaction function

The estimation results for the Bundesbank reaction function are reported in Table 2.7. We add four lags of y_t instead of three to eliminate serial correlation in the errors. There are signs of heteroscedasticity and the estimated conditional variance equation (not reported) is similar to the Federal Reserve's case in that only $dev25_{t-1}$ is significant at 10% or lower level. The standardized residuals and

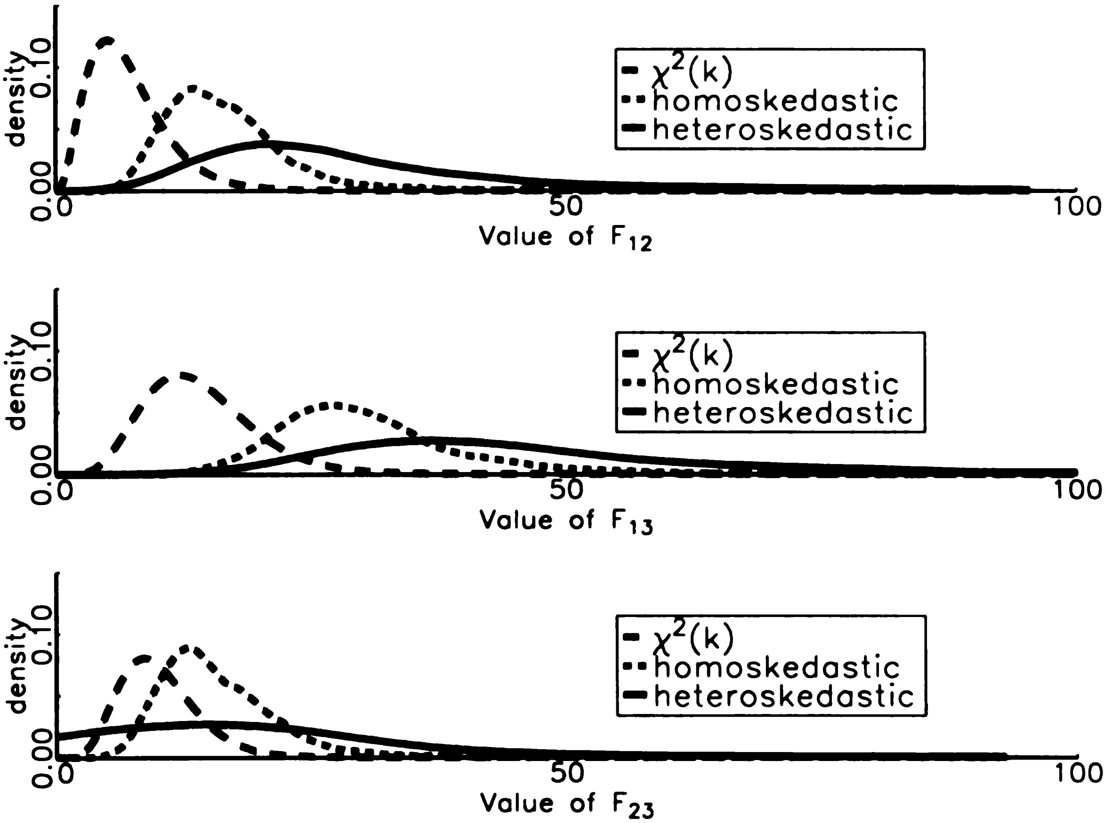


Figure 2.6: Distribution of F-statistics (Bundesbank 87 $\sim$ 89).

the autocorrelograms are depicted in Figure 2.7. Ljung-Box statistics are less than 95% critical values with Q(12)=16.15, Q2(12)=6.11.

From the second column in Table 2.7, we see that the short-run deviation measure $dev7_{t-1}$ and the volatility measure vol_{t-1} are not significant even at 10% level while they were significant at 5% level for the Federal Reserve's case. In fact, $deve7_{t-1}$ is not significant in any of the regimes of the multi-regime models, either. This implies that the Bundesbank is less sensitive to short-run fluctuations of the exchange rate than the Federal Reserve is. The R^2 of the linear model is 0.29 which is close to 0.28 for the Federal Reserve's reaction function.

Table 2.7: Bundesbank reaction function (2/23/87 ~ 10/31/89, q=dev25)

Variable	Linear	Two-regime		Three-regime		
		Regl	Reg2	Regl	Reg2	Reg3
Constant	-9.26**	-2.87	54.01	21.15	-3.12	54.01
	(3.60)	(2.94)	(91.37)	(23.92)	(3.23)	(91.37)
$dev7_{t-1}$	-9.03	-2.76	-2.33	-14.11	-0.98	-2.33
	(5.75)	(4.42)	(28.30)	(13.55)	(4.75)	(28.30)
$dev25_{t-1}$	-9.00***	-6.06***	-40.38	6.10	-6.93**	-40.38
	(2.99)	(2.26)	(38.29)	(8.01)	(3.01)	(38.29)
vol_{t-1}	-12.19	-19.69***	-6.30	-49.42***	-10.95	-6.30
y_{t-1}	0.24***	0.34***	0.00	0.53***	0.26***	0.00
y_{t-2}	0.03	0.04	-0.03	-0.19**	0.12	-0.03
y_{t-3}	0.11	0.05	0.20*	0.06	0.07	0.20*
y_{t-4}	0.13**	-0.03	0.40***	-0.05	-0.06	0.40***
Obs.	647	549	98	119	430	98
Buy	42	42	0	29	13	0
Sell	131	75	56	10	65	56
$\hat{\sigma}$	100.21	72.79	177.44	87.49	66.87	177.44
R^2	0.29	0.26	0.26	0.36	0.20	0.26
$F_{all=0}$	38.12	27.09	4.42	9.04	14.95	4.42
(*Pvalue*)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)
$F_{b2b3b4=0}$	14.24	8.10	1.16	3.06	2.87	1.16
(*Pvalue*)	(0.00)	(0.00)	(0.33)	(0.03)	(0.04)	(0.33)
$\hat{\gamma}_1/\hat{\gamma}_2$		2.20		-2.01 / 2.20		
R^2(all)	0.29	0.37		0.39		
Skewness	-1.60	-0.91		-1.09		
Kurtosis	15.38	14.88		15.17		
Q(12)	14.65	14.37		12.76		
Q2(12)	94.48	63.30		63.84		

1) Heteroskedasticity consistent standard errors in parenthesis.
2) *** significant at 1%, ** at 5%, * at 10% level.
3) $F_{all=0}$ tests overall significance except the constant.
4) $F_{b_2b_3b_4=0}$ tests significance of the three exogenous variables.

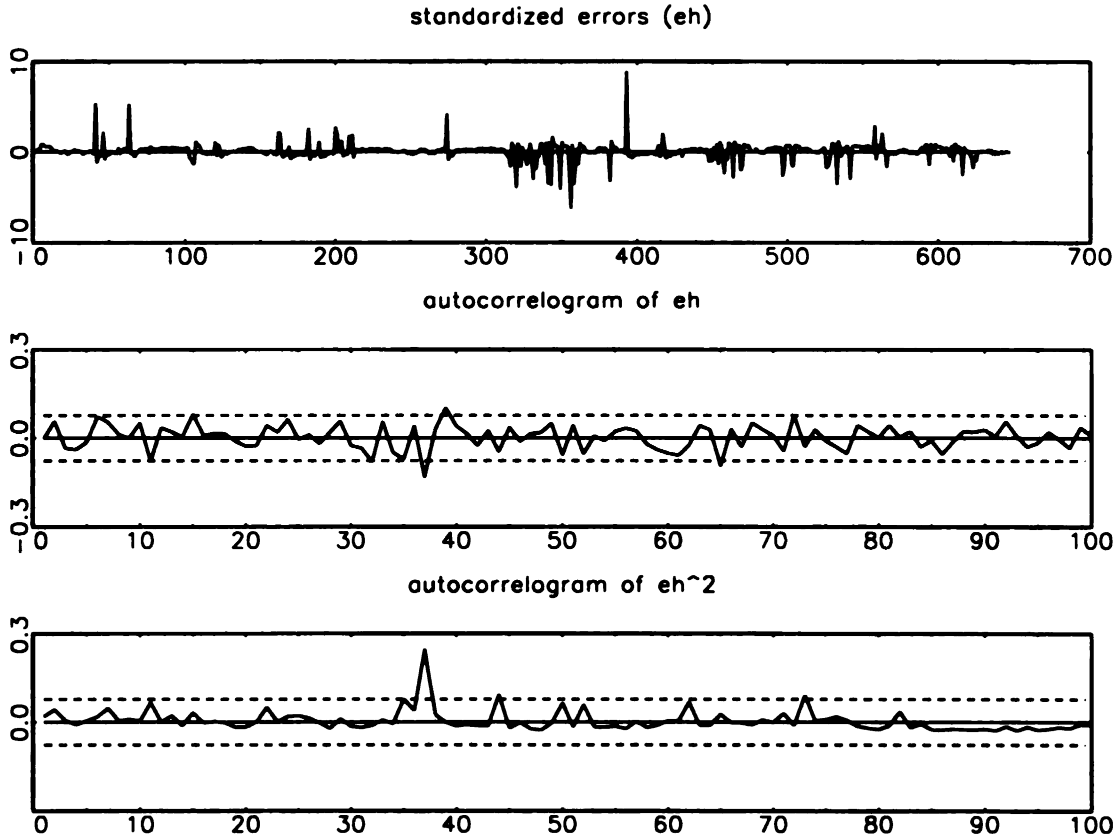


Figure 2.7: Standardized errors (Bundesbank 87 $\sim$ 89).

Two-regime model

The estimated threshold variable is the same with that of the Federal Reserve's reaction function, i.e. the deviation from 25-day moving average. The first threshold is 2.20 which is similar to 2.09 for the Federal Reserve. Unlike the Federal Reserve's case, the 95 % confidence interval with polynomial regression is positive (0.21, 2.20) as well as the interval with kernel regression (1.70, 2.20). The test statistics are depicted in Figure 2.8. The upper limits of these intervals are the maximum value of the threshold variable indicating that the minimum sample restriction of τ = 0.15 is binding. This binding restriction implies that there is a

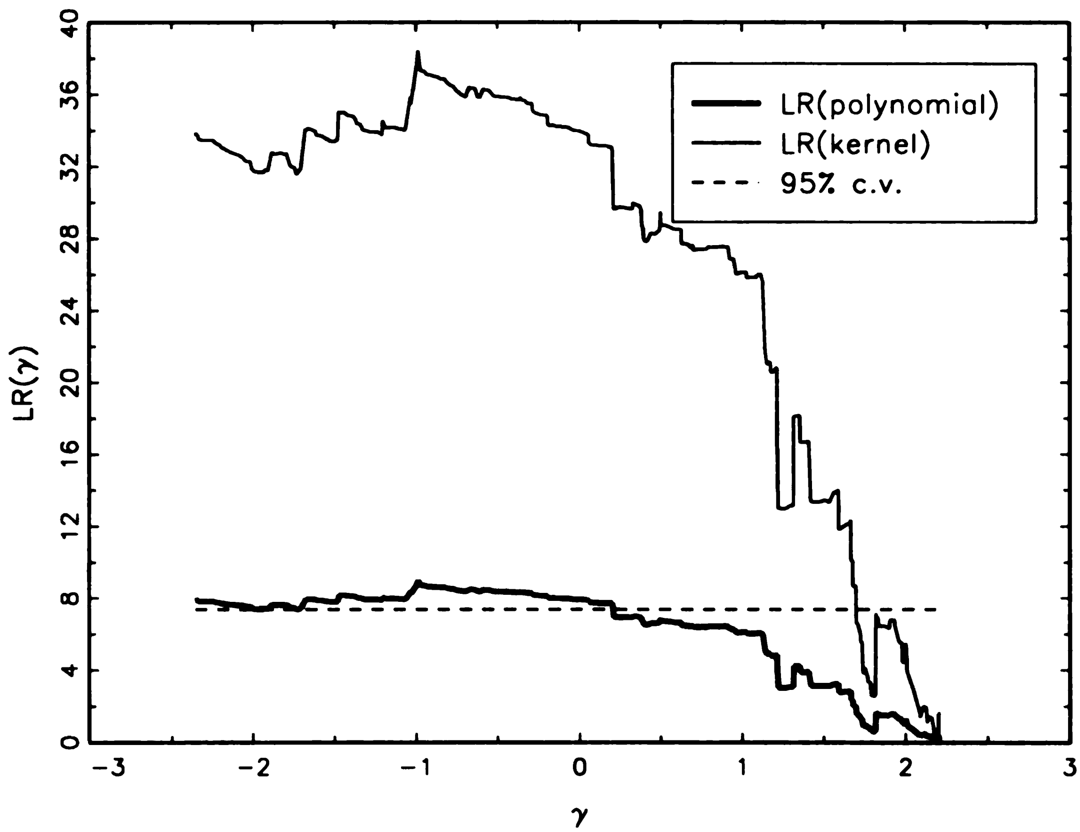


Figure 2.8: Confidence interval of threshold (Bundesbank 87 ~ 89).

strong nonlinearity in the region where the deviation above 25-day moving average is large.

Three-regime model

The second threshold is -2.01. The additional regime increases the R^2 to 0.39 from 0.37 of the two-regime model while there was no such improvement for the Federal Reserve's reaction function. In regime 1, the volatility measure is significant but the two deviation measures are insignificant at 10% level. In regime 3, none of the measures of market instability are significant. This suggests that once the

Table 2.8: Descriptive statistics (Bundesbank, $\tau = 0.15$)

	Regime 1	Regime 2	Regime 3
Frequency of buying	0.24	0.03	0.00
Frequency of selling	0.08	0.15	0.57
Mean buying ($Million)	41.42	5.12	0.00
Mean selling ($Million)	7.39	15.36	131.23

exchange rate is away from the target by more than 2% or so, then the amount of intervention does not grow in proportion to the deviation. On the other hand, in regime 2 where the deviation is small, the deviation from 25-day moving average is significant at 5% level. This result contradicts with the friction hypothesis.

Like the Federal Reserve's intervention, however, both the frequency and average amount of buying intervention are apparently higher than those of selling intervention in regime 1 where the U.S. dollar is depreciating. Similarly, the frequency and average amount of selling is higher than those of buying intervention in regime 3, where the U.S. dollar is appreciating, as reported in Table 2.8.

Sensitivity of results

Table 2.9 reports some of the estimates and statistics for different values of the minimum sample ratio τ. They show similar patterns as those for the Federal Reserve intervention in Table 2.5.

However, the estimated threshold variable is the deviation measure from monthly target $dev25_{t-1}$ for all cases while it was $dev7_{t-1}$ for the Federal Reserve's intervention when $\tau < 0.15$. The Bundesbank seems to be less sensitive to short-run

Table 2.9: Sensitivity to τ (Bundesbank)

	$\tau = 0.05$			$\tau = 0.10$			$\tau = 0.20$			$\tau = 0.30$		
	R1	R2	R3	R1	R2	R3	R1	R2	R3	R1	R2	R3
q_t	dev25			dev25			dev25			dev25		
$\hat{\gamma}$	2.46			2.46			1.81			1.24		
$\hat{\gamma}_1$	3.20			-2.55			-1.92			-0.53		
Obs.	569	39	39	84	485	78	129	386	132	248	205	194
Buy	42	0	0	22	20	0	29	13	0	37	5	0
Sell	84	17	30	8	76	47	11	49	71	19	29	83
F_{12}	0.078			0.024			.005			0.001		
F_{13}	0.104			0.030			.017			0.013		
F_{23}	0.597			0.513			.347			0.498		
F_{slope}	.00	.45	.83	.06	.00	.43	.04	.01	.02	.06	.02	.00

1) R1, R2, R3 are the regimes in the three regime model.
2) q_t is the estimated threshold variable.
3) P-values of F_{ij} statistics are reported with heteroscedasticity assumption. In the parentheses are the p-values based on conditional volatilities re-estimated without insignificant variables.
4) F_{slope} is the p-value for the F-statistic testing joint significance of the exogenous variables in each regime.

movements of the exchange rate even when the deviation is large in size.

Another noticeable difference is that the test statistics for linearity against multiple regimes are significant at 5% level even without dropping the insignificant variables in the conditional variance estimation. Therefore, the evidence for nonlinearity is stronger for Bundesbank's reaction function than for Federal Reserve's.

The measures of disorderly market conditions are highly jointly significant in the middle regime as shown by small p-values for the F_{slope} statistic. In the outer regimes the variables are insignificant for some values of τ. Therefore, the rejection of the friction hypothesis in its strict form does not depend on the values of τ.

2.5 CONCLUSION

The Federal Reserve and the Bundesbank's reaction functions are estimated with a linear and two threshold type nonlinear models using the official intervention data in the DM/USD exchange market during 2/23/87 $\sim$ 10/31/89. This sample period is the same as in Almekinders et al. (1996) and enables us to compare the results with those of the friction model and to reduce potential effects of structural breaks that may exist in a longer sample period.

Unlike the friction model considered in Chapter 1, the multi-regime threshold models tend to have significantly higher R^2 than a linear model. As a result, we could reject linearity of the reaction function in favor of two or three-regime piece-wise nonlinearity at about 10% or lower level.

However, the implied nonlinearity of the intervention reaction functions is not consistent with the friction hypothesis which claims that observations on intervention are mostly zero when the exchange rate is around the target level while they are mostly nonzero otherwise. We tested this hypothesis by two criteria. First, we checked whether the three-regime model is better than a two-regime model as well as a linear model. Secondly, we tested whether those variables measuring the degree of disorderly market conditions are jointly insignificant in the mid-regime while significant in the two outer-regimes. Neither of the two central bank reaction functions met these conditions.

Strictly speaking, therefore, we failed to find evidence for the friction hypothesis in that substantial amount of interventions are observed together with small realizations of the threshold variable and also the central banks do not intervene on many of the days on which the exchange rate deviates from a target level by large amount.

However, we found that the central banks tend to intervene more frequently and in large amount when the deviation is large. We also found that this tendency was stronger in dollar appreciation regime than in depreciation regime during our sample period.

Chapter 3

Conditions For Effective Intervention

3.1 INTRODUCTION

In the previous two chapters, the common question is how the amount of intervention is determined. In contrast, this chapter is concerned with another interesting question about foreign exchange intervention, that is whether such intervention is effective or not.

So far, empirical studies have generally failed to find substantial evidence in favor of the stabilizing effect of infrequent intervention. This is particularly true for sterilized intervention, the impact of which on money supply is offset by open market operations in the opposite direction. For example, Rosenberg (1996), after surveying 18 widely cited empirical studies, published between 1983 and 1994, concludes that "while episodes of successful intervention can be found, no systematic relationship between intervention and exchange rates has been uncovered".[1] More recently, Humpage and Osterberg (2000) reasserts the ineffectiveness of foreign

[1] One well-known exception is Dominguez and Frankel (1993).

exchange intervention.

Notwithstanding the general consensus in the academic literature on the inefficacy of intervention, Neely (2001) reports a survey result of 22 central banks which is in sharp contrast. Most of the central banks (21 banks) relied on intervention between 1990 and 2000, although with varying amounts and frequencies. Why do central banks continue to intervene if it is indeed ineffective? This incongruity is partially resolved in the previous literature where it is also suggested that intervention may sometimes, but not always, influence exchange rates through a mechanism known as the signalling channel or expectations channel.

If intervention is effective sometimes but mostly ineffective, then a natural question is what the conditions for effective intervention are. One answer to this question is that the effectiveness may depend on the way intervention is implemented. In this context, Hung (1997) points out that the monetary authorities should take a more strategic approach to increase the probability of success with limited resources for intervention. In fact, the survey in Neely (2001) also reveals that most central banks (19 out of 20) reflect the market reaction to previous intervention when they make decisions on subsequent rounds of intervention. This implies that central banks have been already exerting some deliberate efforts to increase the probability of success. Even though this conjecture on strategic intervention is quite interesting, it is unclear so far, as noted in Baillie, Humpage and Osterberg (2000), how closely the success is related to intervention strategies.

As an attempt to characterize the conditions for effective intervention, this paper investigates how the efficacy of intervention varies depending on three factors, i.e. the size of intervention, the strength of the exchange rate movement being countered, and the timing of intervention. It is often suggested or implied in the previous literature that these factors may play an important role for the success or failure of intervention. However, few studies thus far have actually tested this possibility in a systematic way.

In the empirical tests of this study, each of the three factors is considered separately for the sake of simplicity.[2] The common hypothesis in each test is that intervention becomes effective when the level of the factor considered is in a certain range, while it is ineffective otherwise. The threshold model described in the previous chapter is an appropriate model for this type of nonlinear effects of intervention. The threshold variable in each test is one of the three factors above. This threshold-effect approach differs from the linear-effect approach of the previous literature in that the latter tests the overall effectiveness of intervention while the former attempts to identify the conditions under which intervention becomes effective.

The empirical results for the Federal Reserve System (hereinafter referred to as the "Fed") and the Bundesbank intervention indicate that all three factors do matter for effective intervention.[3] Specifically, the first condition is that the size of intervention should be larger than a certain threshold to be effective.[4] This

[2] A joint test, if any, is obviously more desirable. However, it seems very difficult to consider the three factors simultaneously in a tractable model.

[3] The Fed and the Bundesbank are known to have routinely sterilized their intervention.

[4] In the case of the Federal Reserve intervention, the estimated thresholds for buying and selling operations are \$128 million and \$61 million, respectively.

condition is related to the familiar proposition that intervention is ineffective in general because usual amount of intervention is too small relative to the volume of trading in the foreign exchange market.

Secondly, intervention becomes effective when the short-term trend in the exchange rate that the central banks attempt to counter is fairly weak. If the short-term trend being countered is too strong, then intervention fails to reverse the trend.

Note that the above two conditions do not rely on any specific channels through which sterilized intervention take effects. The effect may come through changes in relative supply of foreign and domestic assets (portfolio-balance channel), or through changes in market expectations rendered by the signal or information contents of intervention (signalling channel).

On the contrary, the third condition depends upon a specific transmission mechanism, the "noise-trading channel", which is originally proposed by Hung in her unpublished paper and cited in Rosenberg (1996) and Hung (1997). Noise-traders, or chartists, are the traders in the foreign exchange markets who make trading decisions based on technical trading rules rather than economic fundamentals. One common feature of various technical trading rules is to forecast future movements of an exchange rate by extrapolating its recent movements.[5] Frankel and Froot (1990) note that, in response to the poor short-term forecasting performance of exchange

[5]For example, a popular trading rule recommends to buy a currency when its value is above the recent lowest level (a filter rule) or above the moving average of a given length (a moving average rule) by more than a certain percentage. Rosenberg (1996, Chapter 12) provides a good summary of popular trading rules.

rate models based on economic fundamentals, the majority of the foreign exchange forecasting firms, surveyed annually by *Euromoney* magazine between 1978 and 1988, have switched from fundamental analysis to technical analysis around mid 1980s. This trend is reconfirmed by Taylor and Allen (1992) who report a survey result that about 90 percent of the traders in London use some form of technical analysis.

On this background, Hung claims that, since noise traders may be the main source of short-term exchange rate instability, central banks may improve the effectiveness of foreign exchange intervention by identifying the right timing implied by the trend-following trading rule. She suggests, for example, to wait until noise-traders drive the exchange rate sufficiently up or down. At this point, central banks can break the trend with ease since noise-traders themselves suspect that the exchange rate has overshot some appropriate level and consequently the momentum toward further deviation is relatively weak.

More recently, Sarno and Taylor (2001) suggest a similar channel of effects, which is termed the "coordination channel". They complement Hung's proposition by pointing out that the central bank may serve as a coordinator for those traders who are aware of severe misalignment in an exchange rate but reluctant to bet individually against a sustained trend. The estimation and test results in this chapter strongly support these claims.

In testing for efficacy of intervention, the effect of intervention is directly measured by percentage change in the exchange rate for one day after intervention.

However, the amount of intervention is measured by a 3-day average (moving average) rather than the daily amount of intervention with the assumption that intervention on consecutive days contains more information than an isolated single day intervention.[6] As shown in Figure 3.1, the 3-day average dominates all other

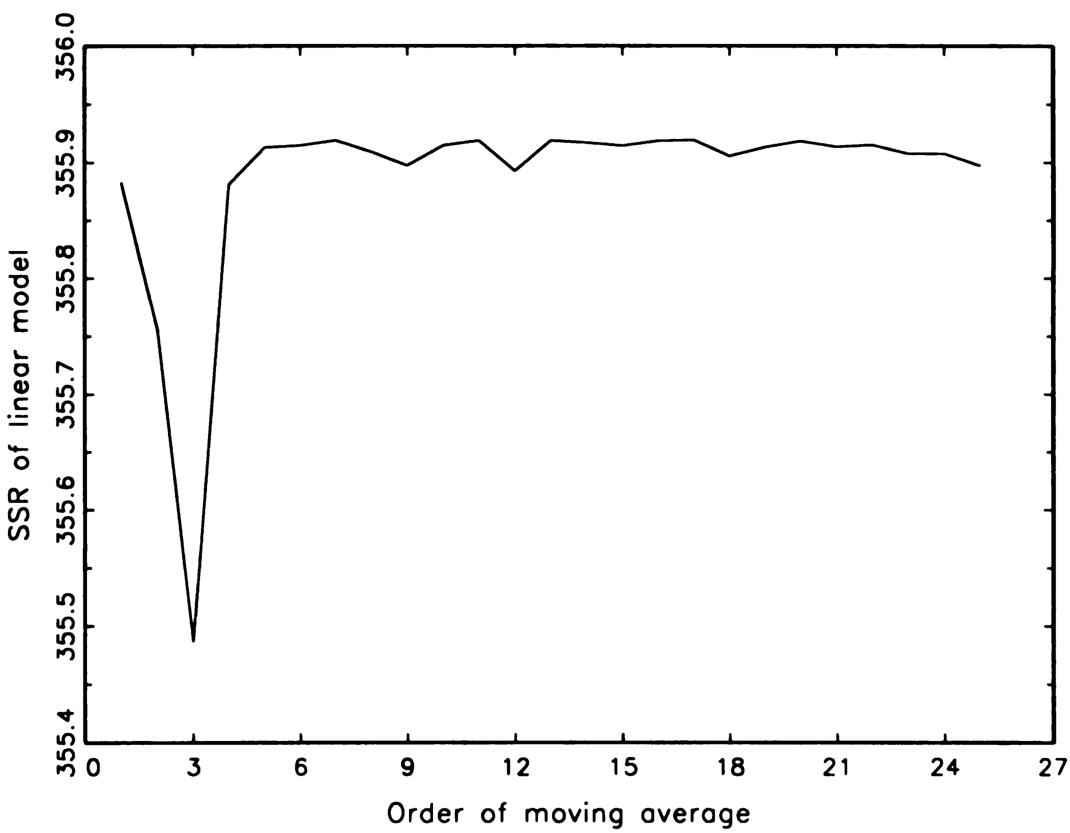


Figure 3.1: Choice of the moving average of intervention

choices of time span, including the single day, in minimizing the sum of squared residuals (SSR) for a linear effect model. Each SSR in this figure is obtained by regressing the daily change in DM/USD rate (1987 - 1989) on an intercept and an

[6]This approach is in line with recent tendency in the literature that favors signaling or expectations channel over portfolio-balance channel with emphasis on the role of intervention as a medium of information transmission, as in Baillie, Humpage and Osterberg (2000).

n-day moving average of intervention for $n = 1, \dots, 25$.

As an illustration of the potential nonlinear effects of intervention, Figure 3.2 compares the fitted line of a linear model with that of a nonlinear model. In the upper panel, the estimated line is from a linear regression of the percentage change in daily DM/USD rate on the 3-day average amount of intervention. In the lower panel, the fitted line is given by Loess regression in order to identify potential nonlinearity. While the OLS line is almost flat, the slope of the Loess line becomes noticeably positive when the average amount of purchase is greater than about 200 million US dollars (large buying interventions).

The rest of this chapter is organized as follows. The next section discusses some issues specific to modelling the effects of intervention within a threshold model framework. Section 3 describes the data. Estimation and test results are presented in section 4 and a brief conclusion is in section 5.

3.2 THE MODELS

3.2.1 Models of Linear Effects

In order to test if intervention is effective, it is necessary to look at whether the exchange rate moves in the desired direction in response to recent intervention. The usual measure of daily exchange rate fluctuation is the log return. Let S_t be the DM/USD spot exchange rate. Then the log return y_t is defined as

$$y_t \equiv 100 \times (log(S_t) - log(S_{t-1})). \tag{3.1}$$

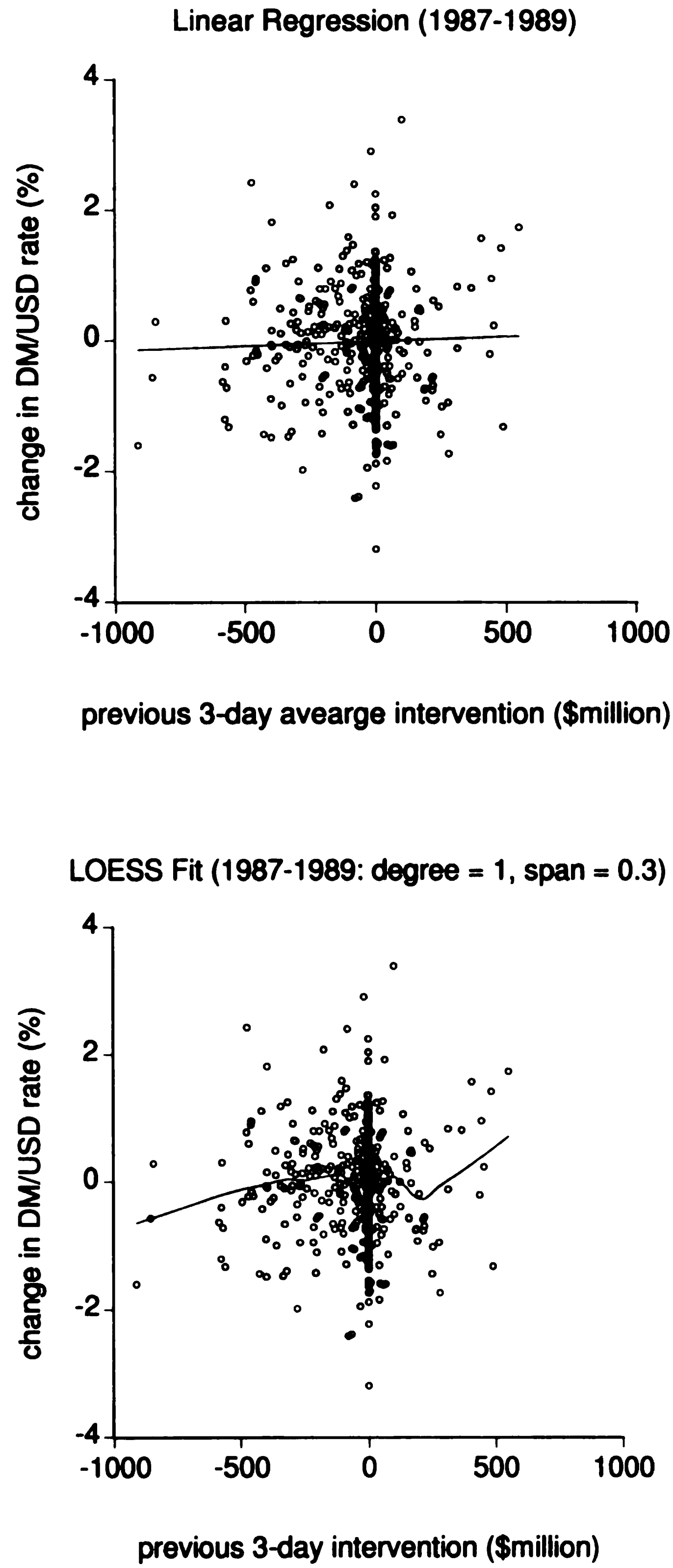


Figure 3.2: Linear vs. nonlinear effect of intervention

With the assumption that intervention has a linear effect on returns, the model can be written as

$$y_t = c + \theta x_t + u_t \tag{3.2}$$

where x_t is the average amount of intervention for previous three days. The log return of the exchange rate may be affected by intervention either by the Fed or the Bundesbank, or both. Therefore, the amount of daily intervention is measured as the sum of U.S. dollars purchased by the two banks. As in the previous chapters, the amounts are recorded as negative numbers when the banks sell U.S. dollars so that x_t is a real number. This explanatory variable is defined more formally as

$$x_t = intv3_{t-2} = fed3_{t-2} + bun3_{t-2} \tag{3.3}$$

$$fed3_{t-2} = \frac{1}{3}\sum_{i=1}^{3} fed1_{t-i-1} \tag{3.4}$$

$$bun3_{t-2} = \frac{1}{3}\sum_{i=1}^{3} bun1_{t-i-1} \tag{3.5}$$

where $fed1_t$ is the amount of intervention by the Fed on day t, and $bun1_t$ is the corresponding intervention by the Bundesbank. Note that x_t is the average amount of intervention between $t-2$ and $t-4$ and thus precedes y_t, which is the percentage change of the exchange rate between t and $t-1$.

The exchange return series y_t is known to have *GARCH* type conditional heteroscedasticity. Therefore, the error term of the linear model can be specified as

$$u_t = z_t \sigma_t \tag{3.6}$$

$$\sigma_t^2 = \omega + \beta\sigma_{t-1}^2 + \alpha u_{t-1}^2, \tag{3.7}$$

where $z_t \sim i.i.d.\ N(0,1)$.

3.2.2 Definition of Effectiveness

Suppose the Fed believes that the DM/USD exchange rate is above the appropriate level, i.e. the US dollar is overvalued against the deutsche mark. Then the Fed will sell US dollars, $x_t < 0$, to increase the relative supply of dollar-denominated assets in the economy. If the DM/USD rate falls subsequently, $y_t < 0$, then such intervention is successful. Similarly, buying intervention, $x_t > 0$, is successful if $y_t > 0$. To sum, the conditions for successful intervention are

$$x_t < 0 \Rightarrow y_t < 0, and \tag{3.8}$$

$$x_t > 0 \Rightarrow y_t > 0. \tag{3.9}$$

Note, however, that these conditions for sucess, (3.8) or (3.9), are sufficient but not necessary for effectiveness of intervention. For one thing, intervention can be regarded as effective so long as the trend of appreciation becomes weaker due to intervention, even if the trend is not reversed. Furthermore, depending on the basis of comparison, this 'weaker trend' criterion may mean different things. By comparing y_t with y_{t-1}, Humpage (1999) counts observations with $x_t < 0 \Rightarrow 0 < y_t < y_{t-1}$[7] and $x_t > 0 \Rightarrow y_{t-1} < y_t < 0$[8] as success.

On the other hand, the so-called "leaning against the wind effect" requires comparison between the observed return y_t and the unobservable return that might

[7] After a selling U.S. dollar operation seeking to counter rapid appreciation, the currency keeps to appreciate but the degree of appreciation is smaller than the degree in the previous day.

[8] After a buying operation seeking to counter rapid depreciation of U.S. dollar, the currency still depreciates rather than appreciates. However, the degree of depreciation is smaller than the degree in the previous day.

have been realized if there had not been any intervention. The law of demand and supply says that increase in supply, whether it is a flow or a stock, should lower the price while decrease in supply should increase the price, other things being equal. Therefore, selling U.S. dollar intervention should decrease the DM/USD rate, and buying intervention should increase the exchange rate. That is, intervention should be effective. However, many empirical studies report perverse signs of θ with the linear model (3.2). The culprit is generally believed to be the policy endogeneity or the simultaneity bias. That is, the effect of intervention is often dominated by the other stronger determinants of the exchange rate, which are the very things that are being countered. Of course, this counterfactual effect is not testable.

For simplicity, the focus in this study is on the slope coefficient θ in (3.2), which is expected to be positive if intervention is effective. This coefficient measures the partial effect of intervention on the exchange rate return, i.e. $\theta = \partial E(y \mid x)/\partial x$. A positive slope, $\theta > 0$, does not necessarily mean that the conditions for success in (3.8) and (3.9), or the secondary conditions of Humpage (1999), are satisfied.[9] However, it does mean that additional amount of intervention tends to move the exchange rate to the desired direction.

3.2.3 Nonlinearity and Threshold Models

Theoretically speaking, a threshold model may have any finite number of regimes. This study, however, considers the model with two or three regimes only. Although

[9] For example, even with $\theta > 0$ and $x_t > 0$, the predicted value of exchange return may be negative ($\hat{y}_t = \hat{c} + \hat{\theta}_x t < 0$) if $\hat{c}$ is negative large in absolute value.

a complete description of the nonlinearity may require higher-order threshold models, a three-regime model will be sufficient to ascertain nonlinearity, if any, in the effects of sterilized intervention.[10] The two-regime model can detect the strongest nonlinearity while the three-regime model enables us to see, for example, whether both large selling and buying operations are more effective than medium-sized intervention when the amount of intervention is used as the threshold variable.

The two-regime model is specified as

$$y_t = (c_1 + \theta_1 x_t) \cdot 1(q_t \leq \gamma) + (c_2 + \theta_2 x_t) \cdot 1(q_t > \gamma) + u_t \tag{3.10}$$

and the three-regime model as

$$\begin{aligned} y_t = {} & (c_1 + \theta_1 x_t) \cdot 1(q_t \leq \gamma_1) + (c_2 + \theta_2 x_t) \cdot 1(\gamma_1 < q_t \leq \gamma_2) \\ & + (c_3 + \theta_3 x_t) \cdot 1(q_t > \gamma_2) + u_t \end{aligned} \tag{3.11}$$

where $1(\cdot)$ is the indicator function. The minimum sample size of each regime is, to begin with, 5% of the number of unique observations on the threshold variable $\tau = 5\%$. Then, the result will be compared with the cases of τ=10%, 15% and 20%.

[10]One obstacle to specifying the model with four or more regimes is the infrequency of interventions. The data set used in this paper has 220 days of intervention which is only about 30% of the 723 total observations.

The threshold variable q_t is defined as one of the following three.

$$\text{Case 1:} \quad q_t = intv3_{t-2} \tag{3.12}$$

$$\text{Case 2:} \quad q_t = dev20_{t-3} \equiv 100\left(log\left(S_{t-3}\right) - log\left(\frac{1}{20}\sum_{i=1}^{20} S_{t-2-i}\right)\right) \tag{3.13}$$

$$\text{Case 3:} \quad q_t = dev50_{t-3} \equiv 100\left(log(S_{t-3}) - log\left(\frac{1}{50}\sum_{j=1}^{50} S_{t-2-j}\right)\right). \tag{3.14}$$

In Case 1, the threshold variable, which is the 3-day average amount of intervention, is the same as the explanatory variable, i.e. $q_t = x_t$. The hypothesis in this case is that intervention tends to be effective when the size of intervention is below γ_1 or above γ_2, as illustrated in Figure 3.3.[11]

In Case 2, the threshold variable, $dev20_{t-3}$, is the deviation of the DM/USD rate from its 20-day moving average.[12] It is the proxy for the strength of the short-term trend of the exchange rate being countered. Admittedly, this measure of deviation may not be the best proxy. However, this measure seems to be a reasonable proxy in that the distance between the exchange rate and the 20-day moving average gets bigger when there is a rapid upward or downward trend, as shown in the upper panel of Figure 3.4.[13] The lower panel of this figure illustrates the hypothesis that intervention is not effective when the upward or downward trend in the exchange rate is too strong (the far-left or far-right graph) but effective

[11] Note that in this figure y is allowed to be discontinuous.

[12] Since the explanatory variable x_t is an average of daily purchases of U.S. dollars from $t-2$ to $t-4$, the deviation is measured on day $t-3$ which is in the middle of the three days. This also holds for $dev50_{t-3}$ in Case 3.

[13] $dev20_{t-3}$ is measured in percentage. As a result, the percentage deviation becomes smaller as the 20-day moving average increases even if the distance between the exchange rate and the 20-day moving average remains the same.

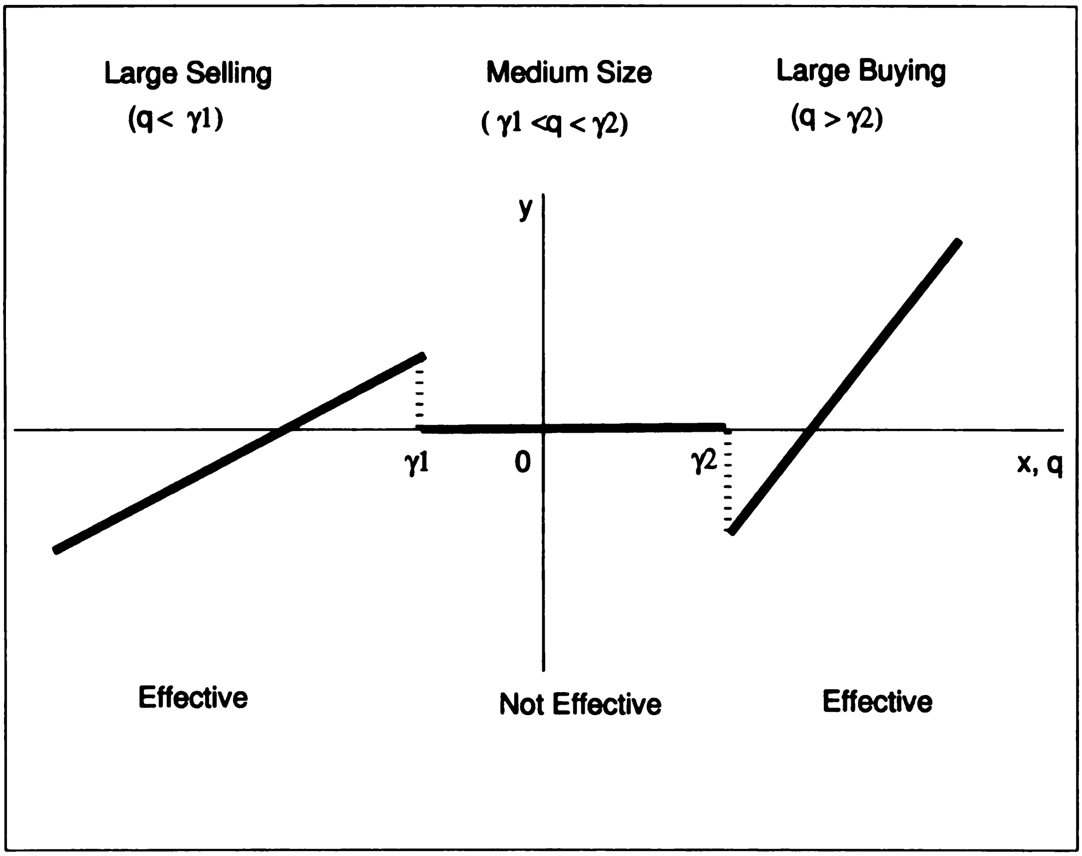
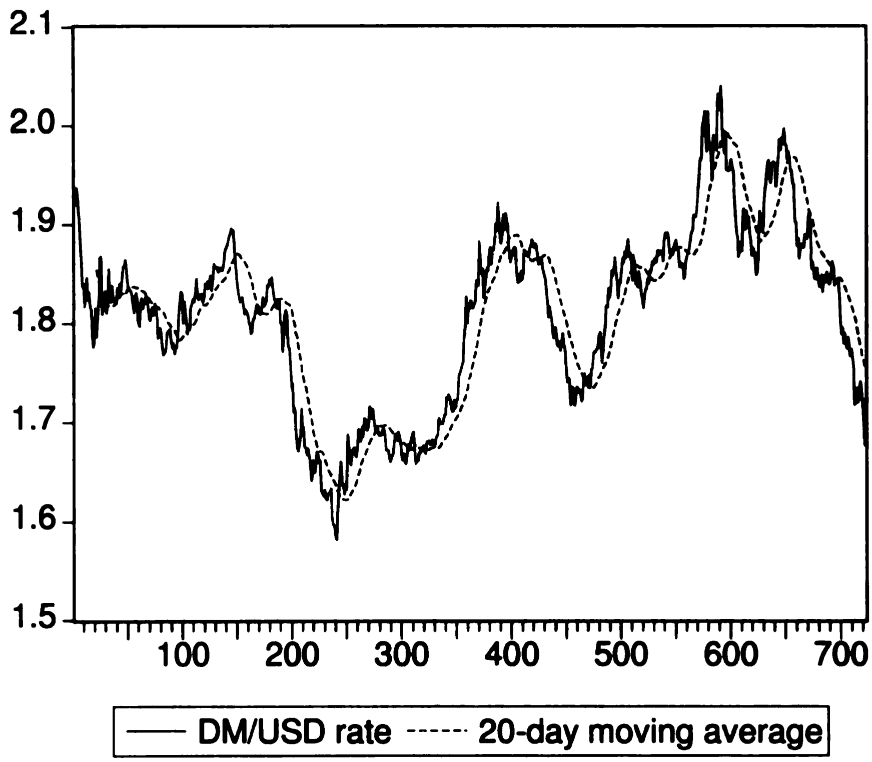


Figure 3.3: Threshold effect by size of intervention.

when the short-term trend is relatively weak (the middle graph).

In case 3, the threshold variable is the deviation from a 50-day moving average of the exchange rate. The 50-day moving average, which is depicted as the dotted line in the upper panel of Figure 3.5, serves as the proxy for a longer-run trend of the exchange rate. The hypothesis in this case is illustrated by the three graphs in the lower panel of Figure 3.5. One of the popular technical trading rule is to sell a currency when it is depreciating and buy when it is appreciating. If noise traders have already sold enough of a currency following a depreciating short-term



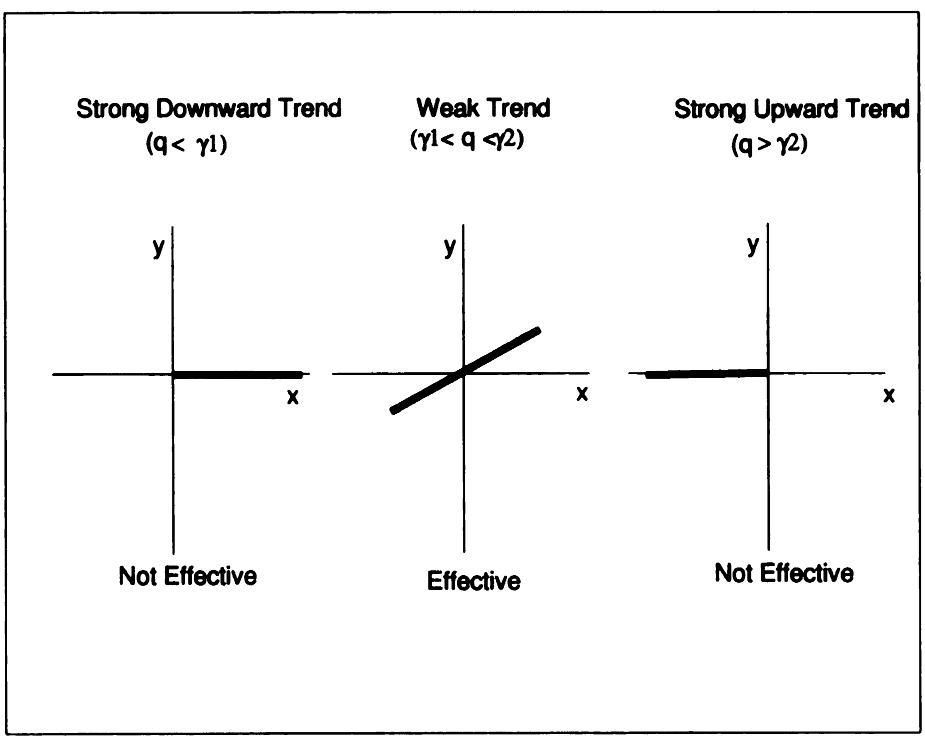


Figure 3.4: Threshold effect by strength of short-term trend.

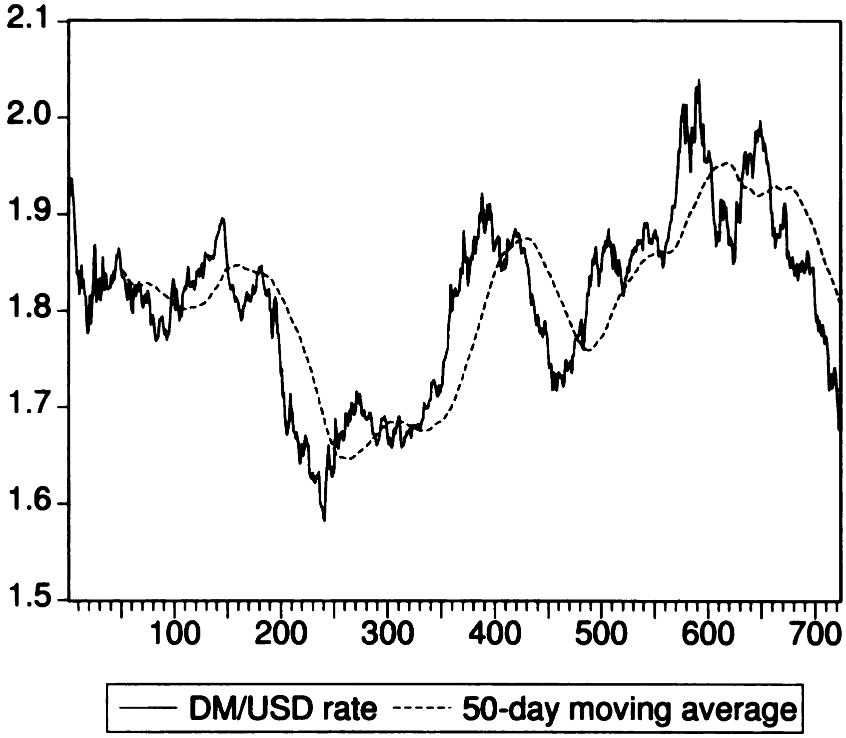
trend so that the exchange rate is sufficiently below the longer-run trend ($q < \gamma_1$), then buying intervention is likely to be effective. This situation corresponds to the far-left graph in the lower panel of Figure 3.5. In the opposite situation described in the far-right graph of the figure, where the noise traders are in heavy overbought positions ($q > \gamma_2$), selling intervention is also hypothesized to be effective.[14]

In Case 2 and Case 3, the order of moving average is $m = 20$ and $m = 50$, respectively. These numbers are chosen based on an interesting pattern, as illustrated in Figure 3.6, in the response of the sum of squared residuals of the two-regime threshold model (3.10) to different values of m.[15] In the figure, the global minimum is at $m = 80$ while there is a local minimum at $m = 20$. Note that the $SSR(m)$ is increasing in m for $20 < m < 35$ while it is decreasing in m for $35 < m < 80$. As it turns out, this non-monotonic behavior of $SSR(m)$ is closely related to the clear difference between the pattern of nonlinearity in Case 2 and the pattern of nonlinearity in Case 3. In case 2, intervention becomes effective when the threshold variable $dev20_{t-3}$ is relatively small, i.e. when the exchange rate is *closer* to the short-run moving average (with m around 20).[16] In sharp contrast, intervention becomes effective in Case 3 when the exchange rate is *away* from the

[14]The graphs in Figure 3.5 are drawn under the assumption that the central banks buy U.S. dollars when the DM/USD rate is below the longer-run trend and sell when the exchange rate is above the longer-run trend.

[15]In this experiment, the minimum sample size is set to be 5% of the unique values of the threshold variable ($\tau = 5\%$).

[16]This is consistent with the moving average rule or the filter rule of the noise-traders. For instance, the moving average rule recommends to buy a currency if its value goes up more than $\alpha\%$ above a moving average. From a central bank's point of view, this implies that the appropriate timing of intervention is before the value of the currency reaches the threshold level of $\alpha\%$. Once the value of the currency goes beyond the threshold level, it will be much difficult to counter the trend because the trend will be enhanced by quite a large number of traders.



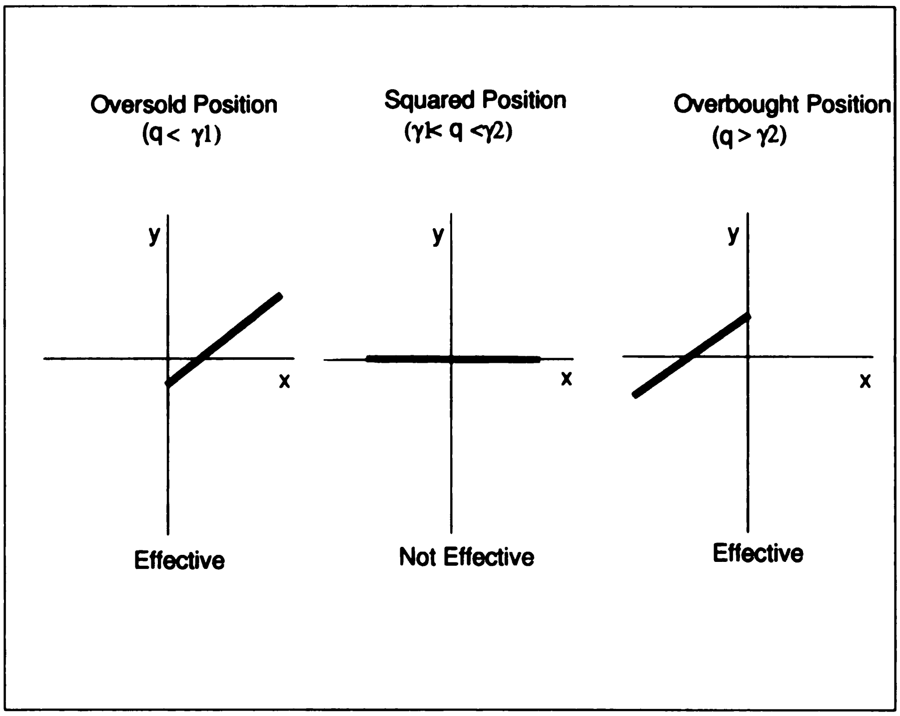


Figure 3.5: Threshold effect by noise trading channel.

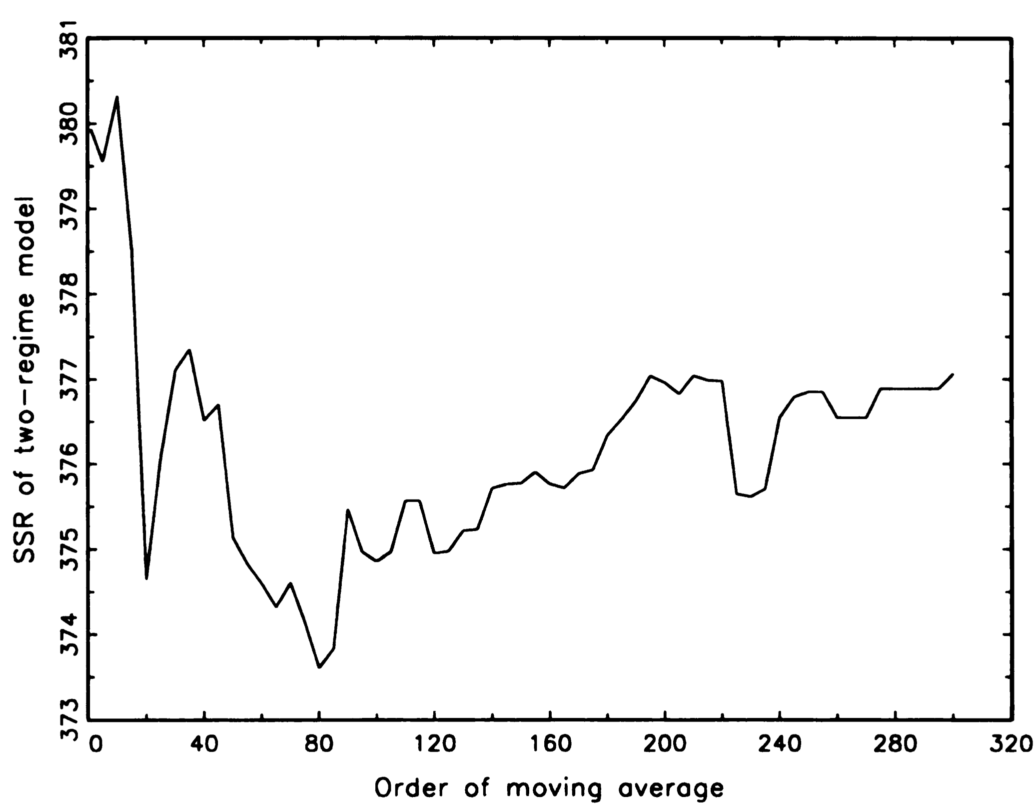


Figure 3.6: Choice of the moving average of exchange rate

longer-run moving average (with m around 80) as claimed by Hung (1997) and Sarno et al. (2001). Therefore, $m = 20$ is a natural choice in Case 2. In Case 3, we choose $m = 50$ rather than $m = 80$ because the deviation of exchange rate from 80-day moving average is quite close to a unit root series.

3.2.4 Estimation and Test Strategies

The linear effect model of (3.2) can be estimated by ordinary least squares. The threshold models of (3.10) and (3.11) can be estimated by the method of sequential conditional least squares as explained in Hansen (2000).

To see whether there is a significant threshold effect in each of the three cases, it is necessary to test the null hypothesis of $c_1 = c_2$ and $\theta_1 = \theta_2$ against $c_1 \neq c_2$ or $\theta_1 \neq \theta_2$ in the two-regime model (3.10).

The test strategy is to compare the explanatory powers of two competing models. When the test is about the linearity against two-regime nonlinearity, the test statistic is

$$F_{12} = T \times \frac{SSR_1 - SSR_2}{SSR_2} \tag{3.15}$$

where SSR_1 is the sum of squared residuals of the linear model (3.2), SSR_2 is the sum of squared residuals of the threshold model (3.10) and T is the sample size. Similarly, test statistics for linearity against three regimes and for two regimes against three regimes are

$$F_{13} = T \times \frac{SSR_1 - SSR_3}{SSR_3} \tag{3.16}$$

$$F_{23} = T \times \frac{SSR_2 - SSR_3}{SSR_3} \tag{3.17}$$

where SSR_3 is the sum of squared residuals from the three-regime model (3.11).

When the threshold is known, the asymptotic distribution of F_{ij} statistic is χ^2 with k or $2k$ degrees of freedom, where k is the number of regressors in the linear model, including the intercept. Since γ is not known and it is not identified under the null hypothesis of no threshold effect while SSR_2 and SSR_3 depend on the value of the threshold(s), the asymptotic distribution is not χ^2. Hansen (1996) provides the asymptotic distribution of these statistics but the distribution does not allow the critical values to be tabulated. As an alternative, the same paper provides a simulation scheme to approximate the p-values. For homoscedastic errors, it is suggested to replace y_t with $\varepsilon_t \sim N(0,1)$ and compute the test statistics with sufficiently large number of replications. For heteroscedastic errors, y_t is replaced by $\hat{u}_t \cdot \varepsilon_t$. Some examples of application are in Hansen (2000) for a two-regime threshold model and Hansen (1999) for a TAR model of up to three regimes.

However, this approximation of asymptotic p-values is not directly applicable to our analysis. One condition for the approximation procedure is that the errors are independent over time. This condition is violated in our model due to the GARCH effect in the conditional variance. As an alternative, the tests in this chapter rely on a bootstrap procedure as in Hansen (1999).

The bootstrap data in each application will be generated based on the GARCH specification in (3.7). Specifically, standardized residuals for z_t in (3.6) are obtained by MLE of a linear-GARCH(1,1) model (or a two-regime-GARCH(1,1) model for F_{23} statistic). A set of bootstrap errors are drawn from these standardized residuals with replacement.[17] Then data on σ_t, u_t and y_t will be generated sequentially with the ML estimates of the parameters $(\hat{\omega}\ \hat{\beta}\ \hat{\alpha})$ in (3.7), and $\hat{\theta}$ in (3.2) (or $\hat{\theta}_1$ and $\hat{\theta}_2$ in (3.10) for F_{23} statistic).

However, note that the test statistics in (3.15) - (3.17) are to be computed based on least squares estimation without explicit consideration on the GARCH property of the errors. One reason for this is it is computationally much easier to implement. The OLS estimators are not efficient but still consistent with a large sample.[18] Another reason is that once the GARCH specification is included in

[17] If the time dependent heteroscedasticity is not removed, so that the residuals are not independent, random drawing from the residuals does not make much sense.

[18] It is possible to estimate a threshold model by sequential MLE, that is by comparing the fitted likelihoods instead of R^2 from OLS for each value of the observed threshold variable. However, the properties of such slope estimates are unknown while LS estimates are known to

the multi-regime model, the regimes may be determined largely by the GARCH parameters ($\hat{\omega}$ $\hat{\beta}$ $\hat{\alpha}$ in (3.7)) rather than the coefficients of the explanatory variables ($\hat{\theta}$). Although this possibility itself is interesting, the focus in this paper is the effect of intervention on the level of the exchange rate.[19]

3.3 THE DATA

Like in the previous chapters, the main data set for the empirical part of this chapter is the one in Baillie and Osterberg (2000), which contains daily data on the DM/USD exchange rate and official interventions by the Fed and the Bundesbank during 01/03/1987 - 01/22/1993. The exchange rates are observed at 9:30 AM Paris time and originally provided by Olsen and Associates of Zurich, Switzerland.

Figure 3.7 depicts the exchange rate movements and sum of interventions by the two central banks during the original sample period. To minimize potential effect of structural breaks, the sample period is limited to 01/03/1987 - 12/29/1989, which is the left side of the vertical line in Figure 3.7.[20] From Figure 3.7, it can be seen that the frequency of interventions has significantly dropped in the early 1990s implying potential structural breaks.[21]

---

be consistent and asymptotically normal.

[19]By adopting a threshold model with four or more regimes, at least theoretically, it is possible to consider both nonlinearity in the volatility and the nonlinearity in the exchange return. However, such a higher-order threshold model seems to be very demanding in terms of minimum sample size in each regime as well as computational burden.

[20]During this sample period, the two central banks are known to have intervened intensively. The first observation falls on the first business day in 1987 and the last observation on the last business day in 1989. The period roughly covers the so-called 'post-Louvre era', which has been a popular object of analysis in the previous literature.

[21]It is at least theoretically possible to consider threshold effects and structural breaks simul-

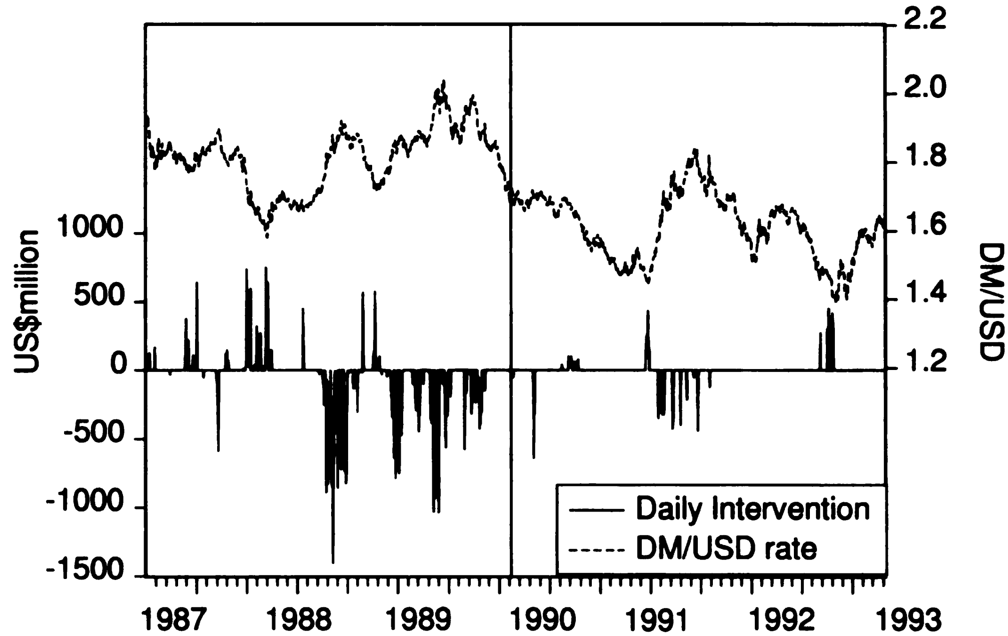


Figure 3.7: Exchange rate and intervention.

Excluding holidays and weekends, the sub-sample has 723 daily observations instead of 1464 in the whole sample. Although the total number of observations has decreased by more than 50%, the decrease in number of days with nonzero net intervention is much smaller (from 272 to 220). Detailed numbers of interventions by bank and type of intervention are given in Table 3.1.

Table 3.1: Number of interventions (1987 - 1989)

	Fed	Bun	Sum	Joint
Buy	36	48	59	25
Sell	100	132	161	71
Total	136	180	220	96

* *Sum* is the number of days of intervention by either central bank.
** *Joint* is the number of days of intervention by both central banks.

taneously in a nested threshold model where the regimes are defined by two or more threshold variables. This approach, however, is beyond the scope of this paper.

y_t is the percentage change in DM/USD rate from 9:30 AM on day $t-1$ in Paris to 9:30 AM on day t. To avoid correlation between the errors and the explanatory variable, y_t is matched with interventions before 9:30 AM on day $t-1$. Thus, x_t is the average amount of US dollars purchased per day between day $t-4$ and $t-2$.[22] With this transformation of the data, four observations are lost and the final size of the sample in our regressions is 719. The 3-day moving average of interventions, $intv3_{t-2}$, has 344 nonzero observations with 105 positive values and 239 negative values.

In Case 2 and Case 3, the threshold variables are the deviations of exchange rates from 20- and 50-day moving averages. One critical requirement of the threshold model is that the threshold variable should be (strictly) stationary. If the threshold variable is non-stationary, hence no tendency for mean-reverting, the idea of switching over a limited number of regimes depending on the value of the threshold variable does not make much sense.[23] Table 3.2 reports the results of ADF unit root tests for our threshold variables. The null hypothesis of unit root is rejected in all three cases.

[22]The amount of intervention on day $t-i$ for $i = 2, 3, 4$ is the amount of US dollars purchased between the market closing time on day $t-i-1$ and the market closing time on day $t-i$.

[23]For the two-regime model (3.10), for example, it is required to have some observations with $q_t \leq \gamma$ and other observations with $q_t > \gamma$. If q_t is a unit-root time series, it may be the case that $q_t > \gamma$ always after some time point, hence no more observations for regime 1.

Table 3.2: ADF unit root tests for threshold variables

Sample	*intv3*1)	*dev20*2)	*dev50*2)
ADF statistic	-6.20***	-5.23***	-2.85***
Obs.	716	722	722

1) 4 lags and an intercept are included in the test.
2) Insignificant lags and intercept are dropped in the test.
*** significant at 1%, ** at 5%, * at 10% level.

3.4 RESULTS

3.4.1 Threshold Effect by Amount of Intervention

To see if effectiveness of intervention depends on the size of intervention, two and three regime threshold models are estimated together with a linear effect model. This is Case 1 in (3.12) where the threshold variable is the same with the explanatory variable $intv3_{t-2}$. The minimum number of observations in each regime of the two- or three-regime threshold model is restricted to be 5% (13 observations) of the unique observations on the threshold variable (267 observations). The estimation result is in Table 3.3.

From the second column, it can be seen that the intervention variable $intv3_{t-2}$ is insignificant in the linear model, which is consistent with the overall results in the previous literature, except that the sign is correctly positive here while previous studies have often reported a result with the negative sign. With two-regime model, effect of intervention gets significant when the 3-day average intervention exceeds a threshold of $128 million. The small number of buying interventions that belong to

Table 3.3: Threshold effect by amount of intervention ($q_t = intv3_{t-2}$)

Variable	Linear	Two-regime		Three-regime		
		Reg1	Reg2	Reg1	Reg2	Reg3
Constant	-0.014	-0.007	-1.622***	0.329**	-0.035	-1.622***
	(0.028)	(0.029)	(0.449)	(0.135)	(0.029)	(0.449)
$intv3_{t-2}$	0.019	0.020	0.641***	0.136**	0.173	0.641***
	(0.032)	(0.034)	(0.216)	(0.060)	(0.141)	(0.216)
Obs.	719	695	24	129	566	24
Buy	105	81	24	0	81	24
Sell	239	239	0	129	110	0
$\hat{\gamma}$		1.281		-0.613 / 1.281		
R^2	0.001	0.020		0.033		
Q(20)	19.533	20.823		20.604		
Q2(20)	111.309	106.057		103.725		

1) 95% critical value for $\chi^2(20)$: 31.41
2) Minimum sample size in each regime is 5% of unique observations on q_t.
3) Heteroscedasticity consistent standard errors are in parentheses.
4) **, *** significant at 10% and 5% level, respectively.

regime 2 (24 out of the 105 observations that have positive 3-day average), implies that buying interventions must be exceptionally large to be effective. When this condition is satisfied, for each additional $100 million purchased, the exchange rate is estimated to increase by about 0.64% the next day. Table 3.4 shows that 0.64% is slightly less than the sample standard deviation of the dependent variable while $100 million is about one standard deviation of the intervention variable.

The results for the three-regime model reported in the last three columns of the table, indicate that selling intervention greater than $61 million is also effective. Unlike the buying interventions, more than half (129 out of 239) of selling operations satisfy this condition. Note that the Ljung-Box tests with the residuals and

Table 3.4: Descriptive statistics of the variables

Variable	Mean	Std Dev	Variance	Minimum	Maximum
y_t (%)	-0.019	0.731	0.534	-3.198	3.386
$intv3_{t-2}$ ($100 million)	-0.261	1.079	1.165	-6.841	4.133

squared residuals find little evidence for significant serial correlation but strong evidence for heteroscedasticity.

The test results for threshold nonlinearity are reported in Table 3.5. The p-value in each test is the ratio of the number of bootstrap replications out of 2,000 that give larger values of the test statistics than the one based on the observed data. The bootstrap data are generated with the following estimated GARCH models to take care of heteroscedasticity. When the null model is the linear model (F_{12} and F_{13}), the data generating process is approximated by the following Linear-GARCH(1,1) model, which is separately estimated with the data.

$$\underset{}{y_t} = \underset{(0.025)}{-\,0.010} \; \underset{(0.033)}{+\,0.008\, intv3_{t-2}}$$

$$\sigma_t^2 = \underset{(0.007)}{0.019} \; \underset{(0.019)}{+\,0.088\, u_{t-1}^2} \; \underset{(0.021)}{+\,0.879\, \sigma_{t-1}^2}.$$

When the null hypothesis is a two-regime model (F_{23}), the data are generated with

$$
\begin{aligned}
y_t =&(-0.001 \; + 0.017 \; intv3_{t-2}) \cdot (q_t \le 1.281) \\
&\;(0.025) \quad (0.033) \\
&+(-1.210 \; + 0.480 \; intv3_{t-2}) \cdot (q_t > 1.281) \\
&\;(0.729) \quad (0.373) \\
\sigma_t^2 =& \; 0.018 \; + 0.081 \; u_{t-1}^2 \; + 0.886 \; \sigma_{t-1}^2. \\
&\;(0.008) \quad (0.020) \qquad (0.021)
\end{aligned}
$$

The Ljung-Box test statistics with the standardized residuals are $Q(20) = 18.32$ for the residuals and $Q2(20) = 24.83$ for the squared residuals of the linear-GARCH model, and $Q(20) = 18.75$ and $Q2(20) = 24.94$ for the two-regime-GARCH model. None of these statistics are significantly different from zero at a conventional significance level. Thus random drawings from these standardized residuals are justified for the bootstrap procedures.

The bootstrap p-values in Table 3.5 indicate that both two-regime and three-regime models are significantly better than a linear model at less than 10% level. However, the evidence for the three-regime model is weaker in the sense that the test fails to reject the two-regime model against the three-regime model at 10% or lower level although the F_{23} statistic becomes significant at a more generous level of 15%.

On the other hand, when it is assumed that the thresholds are given as the estimates, hence sampling errors on the thresholds are ignored, the p-values from

Table 3.5: P-values for testing number of regimes

	F_{12}	F_{13}	F_{23}
F-statistic	13.942	23.832	9.701
Bootstrap P-value	0.056	0.063	0.150
$\chi^2(2)$ P-value	0.001		0.008
$\chi^2(4)$ P-value		0.000	

F_{ij} is the test statistic for i-regime(s) against j-regimes.

χ^2 distribution indicate that the difference in the coefficients are very significant among the three regimes.

The above test results are based on the restriction that each regime in the threshold models must have at least 5% of the unique observations on the threshold variable ($\tau = 5\%$). Table 3.6 shows how the test results change depending on the

Table 3.6: P-values for different restrictions on minimum sample size

τ	F_{12}	F_{13}	F_{23}
0.05	0.056	0.063	0.150
0.10	0.168	0.180	0.346
0.15	0.141	0.216	0.563
0.20	0.129	0.164	0.397

F_{ij} is the test statistic for i-regime(s) against j-regimes.

value of τ. When the minimum sample size increases to 10% or higher level, none of the test statistics are significant at 10% significance level. This phenomenon is related to our earlier finding that only exceptionally large size buying interventions are effective. When this type of intervention is allowed to form a separate regime by a small value of τ, the multi-regime models are significantly better than a linear model. For a larger value of τ, however, the threshold effect is diluted since heterogeneous observations are forced to be included in the same regime.

3.4.2 Threshold Effect by Strength of Wind

In Case 2, the test is about whether the effectiveness of intervention depends on how strong the short-run trend is. The degree of short-run momentum (q_t) is measured by the deviation of DM/USD rate from its previous 20-day moving average as discussed earlier.

Table 3.7: Threshold effect by strength of wind ($q_t = dev20_{t-3}$)

Variable	Linear	Two-regime		Three-regime		
		Reg1	Reg2	Reg1	Reg2	Reg3
Constant	-0.014	-0.122***	0.089**	-0.055	-0.312***	0.089**
	(0.028)	(0.040)	(0.040)	(0.044)	(0.083)	(0.040)
$intv3_{t-2}$	0.019	0.100	0.038	0.055	0.989***	0.038
	(0.032)	(0.067)	(0.038)	(0.066)	(0.259)	(0.038)
Obs.	719	347	372	269	78	372
Buy	105	93	12	85	8	12
Sell	239	48	191	38	10	191
$\hat{\gamma}$		-0.083		-0.670 / -0.083		
R^2	0.001	0.021		0.045		
Q(20)	19.533	20.717		20.708		
Q2(20)	111.309	109.326		104.165		

1) 95% critical value for $\chi^2(20)$: 31.41
2) Minimum sample size in each regime is 5% of unique observations on q_t.
3) Heteroscedasticity consistent standard errors are in parentheses.
4) *, **, *** significant at 10%, 5% and 1% level.

In Table 3.7 are the estimation results with minimum sample size of 5%. As shown in the last three columns of the Table, intervention is significantly effective

Table 3.8: Bootstrap p-values for testing number of regimes

	F_{12}	F_{13}	F_{23}
$F-statistic$	15.027	33.617	18.210
$P-value$	0.024	0.005	0.014

F_{ij} is the test statistic for i-regime(s) against j-regimes.

Table 3.9: P-values for different restrictions on minimum sample size

τ	F_{12}	F_{13}	F_{23}
0.05	0.024	0.005	0.014
0.10	0.018	0.003	0.007
0.15	0.011	0.023	0.320
0.20	0.008	0.028	0.589

F_{ij} is the test statistic for i-regime(s) against j-regimes.

in the middle regime where the wind is weak. The estimated thresholds are -0.670 and -0.083 implying that if central banks intervene when the exchange is below the 20-day moving average by more than 0.08% but less than 0.67%, such intervention tends to be effective. Like in Case 1, this condition for effectiveness is very tight since only 8 out of 105 buying interventions and 10 out of 239 selling interventions meet this condition.

The tests for linearity reported in Table 3.8 show that both of the two multi-regime models are significantly better than the linear model. Also, the three-regime model is significantly better than the two-regime model.

When the minimum sample size (τ) is increased to 10% of the whole sample,[24]

[24] All observations on the threshold variable are unique in Case 2 and Case 3.

the results remain the same. For larger values of τ, the tests still reject the linear model against the multi-regime models. However, the three-regime model is no longer better than the two-regime model.

3.4.3 Threshold Effect by Noise Trading Channel

The question in Case 3 is whether intervention tends to be effective if the central banks wait for the right timing rather than lean against a very strong upward or downward momentum developed by noise traders or chartists. We approximate the

Table 3.10: Threshold effect by noise trading channel ($q_t = dev50_{t-3}$)

Variable	Linear	Two-regime		Three-regime		
		Reg1	Reg2	Reg1	Reg2	Reg3
Constant	-0.014	-0.236***	0.002	-0.236***	-0.020	0.392**
	(0.028)	(0.090)	(0.030)	(0.090)	(0.030)	(0.187)
$intv3_{t-2}$	0.019	0.260***	0.002	0.260***	-0.093*	0.162**
	(0.032)	(0.078)	(0.035)	(0.078)	(0.051)	(0.066)
Obs.	719	119	600	119	525	75
Buy	105	54	51	54	51	0
Sell	239	2	237	2	166	71
$\hat{\gamma}$		-3.959		-3.959 / 3.634		
R^2	0.001	0.020		0.038		
Q(20)	19.533	17.617		21.708		
Q2(20)	111.309	108.937		100.044		

1) 95% critical value for $\chi^2(20)$: 31.41
2) Minimum sample size in each regime is 5% of unique observations on q_t.
3) Heteroscedasticity consistent standard errors are in parentheses.
4) *, **, *** significant at 10%, 5% and 1% level.

right timing with the deviation of exchange rate from its previous 50-day (about two months) moving average. When a newly developed trend by the chartists is not supported by the corresponding change in the economic fundamentals, it will be much easier for the central banks to counter such a trend or instability.

With $\tau = 5\%$, the estimation results are reported in Table 3.10. The estimated two-regime model indicates that when the exchange rate is below the longer-run equilibrium by more than 4%, buying intervention has significant effect on the exchange rate movement. More than half of the observed buying operations (54 out of 105) meet this condition. The three-regime model shows that when the deviation is positive and larger than 3.6%, selling intervention is also effective. About 30% of the selling operations meet this second condition.

Table 3.11: Bootstrap p-values for testing number of regimes ($\tau = 0.1$)

	F_{12}	F_{13}	F_{23}
$F - statistic$	14.286	27.741	13.193
$P - value$	0.043	0.020	0.065

F_{ij} is the test statistic for i-regime(s) against j-regimes.

The test statistics for threshold effects are reported in Table 3.11. Both F_{12} and F_{13} statistics are statistically different from zero implying that the multi-regime models are better than the linear model. F_{23} statistic is insignificant at 5% level but significant at 10% level.

On the other hand, the evidence for these threshold effects by noise-trading channel is stronger with $\tau = 10\%$ as reported in Table 3.12. For higher values of τ, the threshold effects become weaker. In comparison to Case 1 and 2, however, the threshold effect in Case 3 spans a fairly large portion of the observed data.

Table 3.12: P-values for different restrictions on minimum sample size (τ)

τ	F_{12}	F_{13}	F_{23}
0.05	0.043	0.020	0.065
0.10	0.024	0.005	0.030
0.15	0.021	0.012	0.111
0.20	0.129	0.152	0.326

F_{ij} is the test statistic for i-regime(s) against j-regimes.

3.5 CONCLUSION

Using threshold models and official data on the Fed and Bundesbank intervention in the DM/USD market during 1987 - 1989, it is tested whether nonlinearity exists in the effects of sterilized intervention on the DM/USD exchange rate. This is the first empirical study testing explicitly the potential conditions for effective intervention with nonlinear specifications, given that sterilized intervention has been ineffective on average in stabilizing the exchange rates.

First, I find that intervention is effective if the size is exceptionally large, particularly when the interventions is a buying US dollar operation so as to support the value of this currency against deutsche mark. Secondly, intervention tends to be effective if a short-run trend in the exchange rate movement is not too strong to lean against. Finally, intervention can be effective if the central banks take a strategic approach to beat the noise-traders. While only a few observations in the sample meet the first two conditions, a large share of the sample (about 50% of buying and 30% of selling operations) meet the third condition.

For the third test, based on a proposition in Hung (1997)and Sarno et al. (2001), it is assumed that short-run fluctuation of the exchange rate is mainly driven by noise-traders or chartists who rely heavily on technical analysis as well as fundamental analysis. Our estimation and test results indicate that the central banks may increase the effectiveness of intervention by waiting, instead of attempting to counter newly developed strong momentum, until the daily exchange rate deviates from a bimonthly or longer-run moving average beyond the estimated thresholds.

It is yet to be seen whether these results are robust across different pairs of currencies or over different time periods. Nevertheless, I believe that this study demonstrates a new approach in evaluating the performance of intervention under the floating exchange rate system, which can provide some useful guidelines for future studies as well as for many central banks in designing future intervention strategies.

Appendix A

Log Likelihood of the Friction Model

The friction model is

$$y_t^* = x_t\beta + \varepsilon_t, \qquad \varepsilon_t | x_t \sim N(0, \sigma^2), \tag{A.1a}$$

$$y_t = y_t^* - \delta^+ \qquad \text{if } y_t^* > \delta^+, \tag{A.1b}$$

$$y_t = 0 \qquad \text{if } -\delta^- \leq y_t^* \leq \delta^+, \tag{A.1c}$$

$$y_t = y_t^* + \delta^- \qquad \text{if } y_t^* < -\delta^-. \tag{A.1d}$$

By substituting y_t^* with (A.1a) in equations (A.1b) and (A.1d), we get

$$y_t = x_t\beta - \delta^+ + \varepsilon_t \qquad \text{if } y_t > 0, \tag{A.2a}$$

$$y_t = x_t\beta + \delta^- + \varepsilon_t \qquad \text{if } y_t < 0. \tag{A.2b}$$

Note that $y_t > 0$ if and only if $y_t^* > \delta^+$ and $y_t < 0$ if and only if $y_t^* < -\delta^-$. With the normality assumption for the errors, $\varepsilon_t | x_t \sim N(0, \sigma^2)$, the conditional distribution of y_t is given as

$$y_t | x_t \sim N(x_t\beta - \delta^+, \sigma^2) \qquad \text{if } y_t > 0, \tag{A.3a}$$

$$y_t | x_t \sim N(x_t\beta + \delta^-, \sigma^2) \qquad \text{if } y_t < 0. \tag{A.3b}$$

Let $L_t(\theta) \equiv L(\theta; y_t \mid x_t)$ be the likelihood function for observation t. From (A.3), we see that

$$L_t(\theta) = \frac{1}{\sigma} \cdot \phi\left(\frac{y_t - x_t\beta + \delta^+}{\sigma}\right) \qquad \text{if } y_t > 0, \qquad \text{(A.4a)}$$

$$L_t(\theta) = \frac{1}{\sigma} \cdot \phi\left(\frac{y_t - x_t\beta - \delta^-}{\sigma}\right) \qquad \text{if } y_t < 0. \qquad \text{(A.4b)}$$

where $\phi(\cdot)$ is the standard normal probability density function (PDF).

In the case of $y_t = 0$,

$$\begin{aligned} L_t(\theta) &= P(y_t = 0 \mid x_t) \\ &= 1 - P(y_t > 0 \mid x_t) - P(y_t < 0 \mid x_t). \end{aligned} \qquad \text{(A.5)}$$

In Figure A.1, the relationship among $P(y_t = 0 \mid x_t)$, $P(y_t > 0 \mid x_t)$ and $P(y_t < 0 \mid x_t)$ is illustrated, assuming $\beta < 0$. $P(y_t = 0 \mid x_t)$ is the shaded area under the PDF of $N(x_t\beta, \sigma^2)$. Note that $P(y_t > 0 \mid x_t)$ is larger than $P(y_t < 0 \mid x_t)$ when $x_t < 0$ but $P(y_t < 0 \mid x_t) \neq 0$. Likewise, $P(y_t > 0 \mid x_t) \neq 0$ when $x_t > 0$, which is not illustrated in the figure.

From (A.2a),

$$\begin{aligned} P(y_t > 0 \mid x_t) &= P(x_t\beta - \delta^+ + \varepsilon_t > 0 \mid x_t) \\ &= P(\varepsilon_t > -x_t\beta + \delta^+ \mid x_t) \\ &= 1 - P(\varepsilon_t \leq -x_t\beta + \delta^+ \mid x_t) \\ &= 1 - \Phi\left(\frac{-x_t\beta + \delta^+}{\sigma}\right) \end{aligned} \qquad \text{(A.6)}$$

where $\Phi(\cdot)$ is the standard normal cumulative density function (CDF). Also from

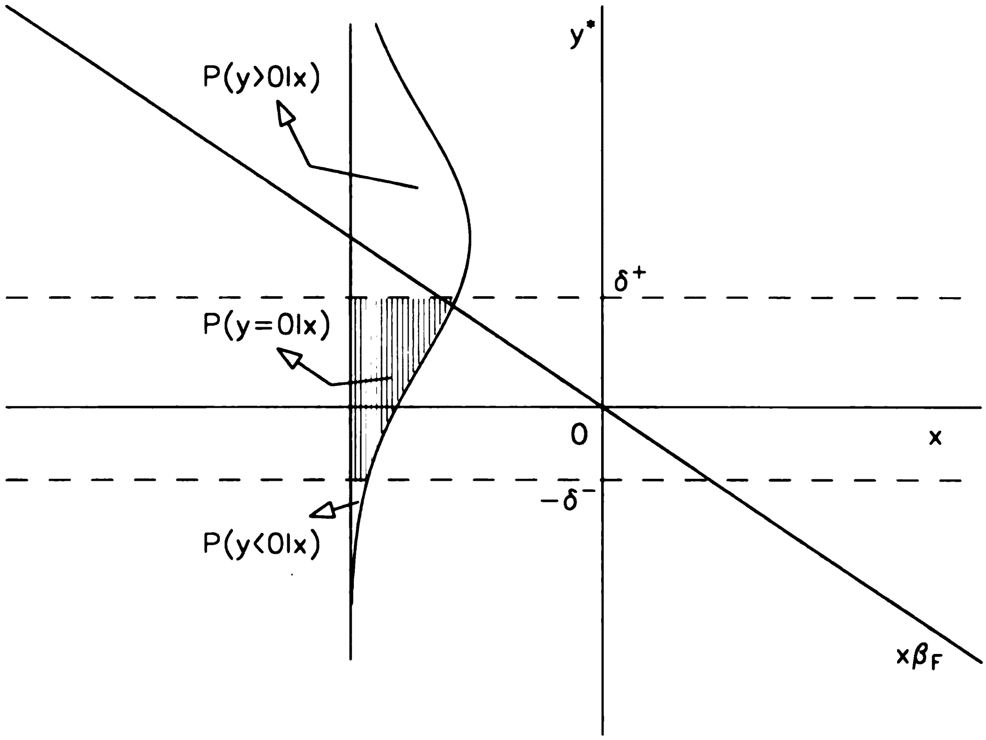


Figure A.1: Conditional density of y*

(A.2b),

$$P(y_t < 0 \mid x_t) = P(x_t\beta + \delta^- + \varepsilon_t < 0 \mid x_t)$$
$$= P(\varepsilon_t < -x_t\beta - \delta^- \mid x_t)$$
$$= \Phi\left(\frac{-x_t\beta - \delta^-}{\sigma}\right). \qquad (A.7)$$

By plugging (A.6) and (A.7) into (A.5) and rearranging,

$$L_t(\theta) = \Phi\left(\frac{-x_t\beta + \delta^+}{\sigma}\right) - \Phi\left(\frac{-x_t\beta - \delta^-}{\sigma}\right) \quad \text{if } y_t = 0. \qquad (A.8)$$

From (A.4) and (A.8), the log-likelihood for observation t is obtained as

$$\begin{aligned}
\ell_t(\theta) &\equiv log[L_t(\theta)] \\
&= 1(y_t > 0) \cdot log\left[\phi\left(\frac{y_t - x_t\beta + \delta^+}{\sigma}\right)/\sigma\right] \\
&+ 1(y_t < 0) \cdot log\left[\phi\left(\frac{y_t - x_t\beta - \delta^-}{\sigma}\right)/\sigma\right] \\
&+ 1(y_t = 0) \cdot log\left[\Phi\left(\frac{-x_t\beta + \delta^+}{\sigma}\right) - \Phi\left(\frac{-x_t\beta - \delta^-}{\sigma}\right)\right] \qquad \text{(A.9)}
\end{aligned}$$

where $1(\cdot)$ is the indicator function.

Appendix B

Conditional Mean of the Friction Model

Dropping the time subscript from the friction model of (A.1), $E(y|x)$ is given as

$$E(y|x) = P(y > 0|x) \cdot E(y|y > 0, x) + P(y < 0|x) \cdot E(y|y < 0, x), \qquad \text{(B.1)}$$

where the two probability terms are given in (A.6) and (A.7). The two expectation terms can be derived from (A.2) with the normality assumption for the errors. From (A.2a),

$$\begin{aligned}
E(y|y > 0, x) &= E(x\beta - \delta^+ + \varepsilon \mid \varepsilon > -x\beta + \delta^+, x) \\
&= x\beta - \delta^+ + E(\varepsilon \mid \varepsilon > -x\beta + \delta^+, x) \\
&= x\beta - \delta^+ + \sigma E\left(\frac{\varepsilon}{\sigma} \mid \frac{\varepsilon}{\sigma} > \frac{-x\beta + \delta^+}{\sigma}, x\right) \\
&= x\beta - \delta^+ + \sigma \frac{\phi\left((-x\beta + \delta^+)/\sigma\right)}{1 - \Phi\left((-x\beta + \delta^+)/\sigma\right)} \qquad \text{(B.2)}
\end{aligned}$$

where the last equality follows from the fact that $(\varepsilon/\sigma)|x \sim N(0,1)$, and $E(z|z > c) = \phi(c)/[1 - \Phi(c)]$ if $z \sim N(0,1)$. See Wooldridge (2000 chapter 17) for details of this approach applied to the case of a Tobit model. Similarly, from (A.2b),

$$
\begin{aligned}
E(y|y<0,x) &= E(x\beta+\delta^- + \varepsilon \mid \varepsilon < -x\beta-\delta^-, x) \\
&= x\beta+\delta^- + E(\varepsilon \mid \varepsilon < -x\beta-\delta^-, x) \\
&= x\beta+\delta^- + \sigma E\left(\frac{\varepsilon}{\sigma} \mid \frac{\varepsilon}{\sigma} < \frac{-x\beta-\delta^-}{\sigma}, x\right) \\
&= x\beta+\delta^- - \sigma E\left(-\frac{\varepsilon}{\sigma} \mid -\frac{\varepsilon}{\sigma} > \frac{x\beta+\delta^-}{\sigma}, x\right) \\
&= x\beta+\delta^- - \sigma\frac{\phi\left((x\beta+\delta^-)/\sigma\right)}{1-\Phi\left((x\beta+\delta^-)/\sigma\right)}.
\end{aligned}
\tag{B.3}
$$

After plugging equations (A.6), (A.7), (B.2) and (B.3) into (B.1) and rearranging, we get

$$
\begin{aligned}
E(y|x) = &\left[\Phi\left(\frac{x\beta-\delta^+}{\sigma}\right)(x\beta-\delta^+) + \sigma\,\phi\left(\frac{x\beta-\delta^+}{\sigma}\right)\right] \\
&- \left[\Phi\left(\frac{-x\beta-\delta^-}{\sigma}\right)(-x\beta-\delta^-) + \sigma\,\phi\left(\frac{-x\beta-\delta^-}{\sigma}\right)\right]
\end{aligned}
\tag{B.4}
$$

where we rely on the properties of $\phi(c) = \phi(-c)$ and $\Phi(c) = 1 - \Phi(c)$ to simplify the notation.

Appendix C

Friction Model of a Fully Asymmetric Reaction Function

In the friction model of (A.1), it is assumed that y_t^* is a linear function of x_t. One implication of this assumption is that the central bank's reaction to abrupt appreciation is the same as its reaction to depreciation. Consequently, β remains the same whether $y > 0$ or $y < 0$ although the intercepts ($-\delta^+$ and δ^-) may be different in size. If the central bank responds differently to appreciation and depreciation of its currency so that the slope parameter β changes, this symmetric reaction function becomes invalid. To account for potential asymmetry, separate Tobit models may be estimated for buying intervention and selling intervention, respectively. Alternatively, the friction model may be modified to allow asymmetric responses.

It seems almost impossible, without additional arbitrary assumptions, to allow different slope coefficients between buying interventions ($y > 0$) and selling interventions ($y < 0$) in the friction model. This is because the distribution of $y|x$ is not defined for some x if the slope parameters are not constant. To see this, note that for the normal cdf $\Phi(\cdot)$,

$$\begin{aligned} \Phi(a) + [1 - \Phi(b)] &< 1 \quad \text{if } a < b, \\ &= 1 \quad \text{if } a = b, \\ &> 1 \quad \text{if } a > b. \end{aligned} \tag{C.1}$$

From (A.7) and (A.6),

$$P(y < 0|x) + P(y > 0|x) = \Phi\left(\frac{-x\beta - \delta^-}{\sigma}\right) + \left[1 - \Phi\left(\frac{-x\beta + \delta^+}{\sigma}\right)\right]. \tag{C.2}$$

Since $\delta^+ > 0$ and $\delta^- > 0$ by assumption, $-x\beta - \delta^- < -x\beta + \delta^+$, which is the first case in (C.1). Therefore, $P(y < 0|x) + P(y > 0|x) < 1$ as it should be so that the distribution of $y|x$ is well defined. However, if the slope parameters are different as β_1 for $y > 0$ and β_2 for $y < 0$,

$$P(y < 0|x) + P(y > 0|x) = \Phi\left(\frac{-x\beta_2 - \delta^-}{\sigma}\right) + \left[1 - \Phi\left(\frac{-x\beta_1 + \delta^+}{\sigma}\right)\right]. \tag{C.3}$$

Note that for some x, $-x\beta_2 - \delta^- > -x\beta_1 + \delta^+$ so that $P(y < 0|x) + P(y > 0|x) > 1$, which is the third case in (C.1). Therefore, the distribution of $y|x$ is not defined for some x if $\beta_1 \neq \beta_2$.

Although it is not possible, or intractably complicated, to allow asymmetry based on the values of y, it is possible to allow asymmetry based on the values of an explanatory variable or other exogenous variable. Let s_t be an observable threshold variable so that the parameters in (A.1) are different between the two cases of $s_t > 0$ and $s_t \leq 0$. Daily log return or deviation from m-day moving average of the exchange rate may be used as s_t.

If $s_t > 0$,

$$y_{1t}^* = x_t\beta_1 + \varepsilon_{1t}, \qquad \varepsilon_{1t}|x_t \sim N(0, \sigma_1^2), \tag{C.4a}$$

$$y_t = x_t\beta_1 - \delta_1^+ + \varepsilon_{1t} \qquad \text{if } y_{1t}^* > \delta_1^+ \quad (y_t > 0), \tag{C.4b}$$

$$y_t = 0 \qquad \text{if } -\delta_1^- \leq y_{1t}^* \leq \delta_1^+, \tag{C.4c}$$

$$y_t = x_t\beta_1 + \delta_1^- + \varepsilon_{1t} \qquad \text{if } y_{1t}^* < -\delta_1^- \quad (y_t < 0) \tag{C.4d}$$

and if $s_t \leq 0$,

$$y_{2t}^* = x_t\beta_2 + \varepsilon_{2t}, \qquad \varepsilon_{2t}|x_t \sim N(0, \sigma_2^2), \tag{C.5a}$$

$$y_t = x_t\beta_2 - \delta_2^+ + \varepsilon_{2t} \qquad \text{if } y_{2t}^* > \delta_2^+ \quad (y_t > 0), \tag{C.5b}$$

$$y_t = 0 \qquad \text{if } -\delta_2^- \leq y_{2t}^* \leq \delta_2^+, \tag{C.5c}$$

$$y_t = x_t\beta_2 + \delta_2^- + \varepsilon_{2t} \qquad \text{if } y_{2t}^* < -\delta_2^- \quad (y_t < 0). \tag{C.5d}$$

Since (C.4) and (C.5) are of the same form as the symmetric friction model of (A.1), the log-likelihood for observation t must be the same as (1.11) with appropriate replacement of the parameters. Therefore,

$$\ell_t(\theta) = 1(s_t > 0)\ell_{1t}(\theta) + 1(s_t \leq 0)\ell_{2t}(\theta), \tag{C.6}$$

where

$$\begin{aligned}\ell_{1t}(\theta) = {} & 1(y_t > 0) \cdot log\left[\phi\left(\frac{y_t - x_t\beta_1 + \delta_1^+}{\sigma_1}\right)/\sigma_1\right] \\ & + 1(y_t < 0) \cdot log\left[\phi\left(\frac{y_t - x_t\beta_1 - \delta_1^-}{\sigma_1}\right)/\sigma_1\right] \\ & + 1(y_t = 0) \cdot log\left[\Phi\left(\frac{-x_t\beta_1 + \delta_1^+}{\sigma_1}\right) - \Phi\left(\frac{-x_t\beta_1 - \delta_1^-}{\sigma_1}\right)\right]\end{aligned} \tag{C.7}$$

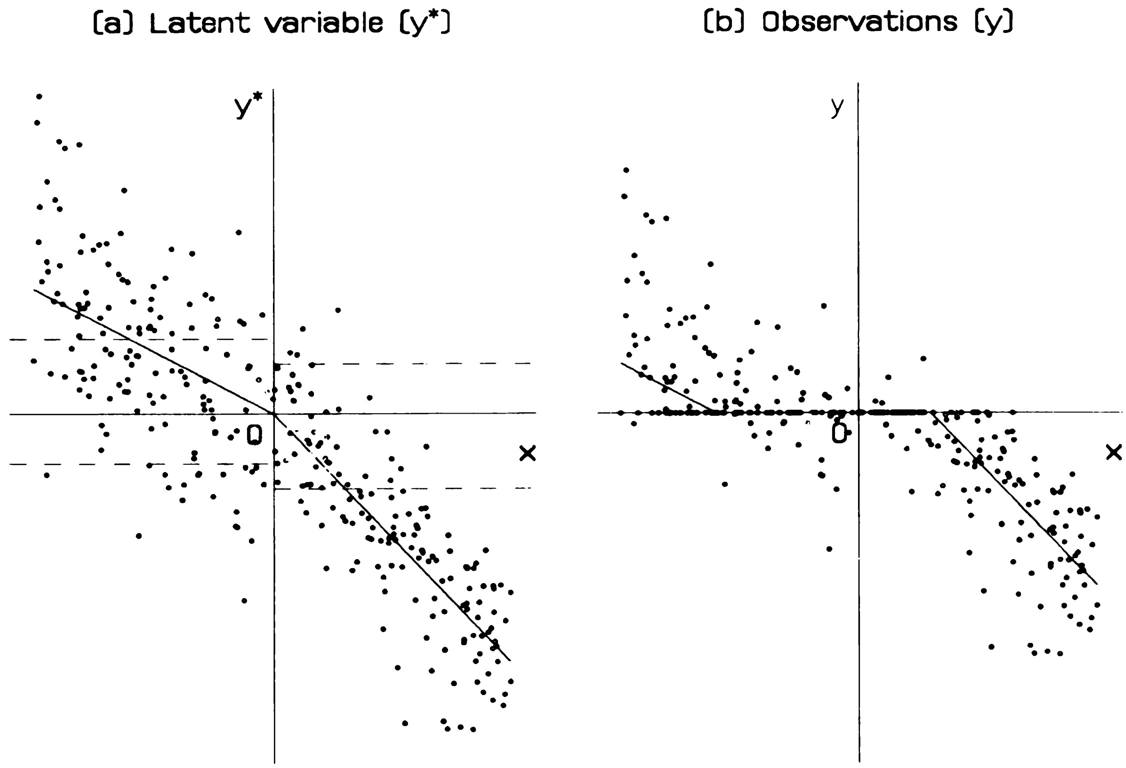


Figure C.1: Simulation of asymmetric friction approach

and

$$
\begin{aligned}
\ell_{2t}(\theta) = {} & 1(y_t > 0) \cdot log \left[\phi \left(\frac{y_t - x_t\beta_2 + \delta_2^+}{\sigma_2} \right) / \sigma_2 \right] \\
& + 1(y_t < 0) \cdot log \left[\phi \left(\frac{y_t - x_t\beta_2 - \delta_2^-}{\sigma_2} \right) / \sigma_2 \right] \\
& + 1(y_t = 0) \cdot log \left[\Phi \left(\frac{-x_t\beta_2 + \delta_2^+}{\sigma_2} \right) - \Phi \left(\frac{-x_t\beta_2 - \delta_2^-}{\sigma_2} \right) \right]. \qquad \text{(C.8)}
\end{aligned}
$$

This asymmetric version of the friction model is illustrated in Figure C.1 for the simple case where x_t is a scalar and $s_t = x_t$. The values of parameters are $\theta_1 \equiv (\beta_1, \delta_1^+, \delta_1^-, \sigma_1) = (-1, 1, 1.5, 1)$ and $\theta_2 \equiv (\beta_2, \delta_2^+, \delta_2^-, \sigma_2) = (-0.5, 1.5, 1, 1.5)$. The asymmetric model considered in Almekinders et al., where the asymmetry is allowed only for the slope coefficient of the deviation variable dev_{t-1}, is a restricted version of this fully asymmetric friction model.

In order to make the comparison of the estimation results of this asymmetric model with those of the symmetric model in (A.1), we can rewrite the model as

$$y_t^* = x_t\beta_t + \varepsilon_t, \qquad \varepsilon_t|x_t \sim N(0, \sigma_t^2), \tag{C.9a}$$

$$y_t = x_t\beta_t - \delta_t^+ + \varepsilon_t \qquad \text{if } y_t > 0, \tag{C.9b}$$

$$y_t = x_t\beta_t + \delta_t^- + \varepsilon_t \qquad \text{if } y_t < 0, \tag{C.9c}$$

$$y_t = 0 \qquad \text{otherwise}, \tag{C.9d}$$

where $\beta_t \equiv \beta_2 + (\beta_1 - \beta_2)(s_t > 0)$ and δ_t^+, δ_t^- and σ_t are defined in the same way. We estimate $\beta_a \equiv \beta_2$ and $\beta_b \equiv \beta_1 - \beta_2$. By looking at the t-statistics of $\beta_1 - \beta_2$, we can see whether this difference in parameters is significantly different from zero or not.

The corresponding asymmetric linear model is

$$y_t = [\beta_{01} + x_t\beta_{\ell_1} + u_{1t}](s_t > 0) + [\beta_{02} + x_t\beta_{\ell_2} + u_{2t}](s_t \le 0) \tag{C.10}$$

$$= \beta_{0t} + x_t\beta_{\ell_t} + u_t, \tag{C.11}$$

where $\beta_{0t} = \beta_{02} + (\beta_{01} - \beta_{02})(s_t > 0)$, $\beta_{\ell_t} = \beta_{\ell_2} + (\beta_{\ell_1} - \beta_{\ell_2})(s_t > 0)$ and $u_t = u_{1t}(s_t > 0) + u_{2t}(s_t \le 0)$.

The conditional expectation $E(y_t|x_t, s_t)$ for the asymmetric friction model of (C.9) is easily obtained as in (B.4) by replacing β with β_t, δ^+ with δ_t^+ and so on. Therefore, we can obtain and compare the R^2 measures from these asymmetric linear model and nonlinear model.

Appendix D

Generalized Residuals of the Friction Model

For a Tobit model, Pagan and Vella (1989) show that a generalized residual can be computed and used for their proposed diagnostic tests, which are based on moment conditions. For the friction model (A.1), it is also possible to define generalized residuals which may be useful when the usual residuals are not obtainable from the estimated nonlinear model.

Define η_t as

$$\eta_t = \begin{cases} y_t - x_t\beta + \delta^+ & \text{if } y_t > 0 \\ y_t - x_t\beta - \delta^- & \text{if } y_t < 0 \\ E(y_t^* \mid y_t = 0) - x_t\beta & \text{if } y_t = 0. \end{cases} \tag{D.1}$$

Note that when $y_t = 0$,

$$\begin{aligned}
E(y_t^* \mid y_t = 0) - x_t\beta &= E(\varepsilon_t \mid y_t = 0) \\
&= E(\varepsilon_t \mid -\delta^- < y_t^* < \delta^+) \\
&= E(\varepsilon_t \mid -\delta^- < x_t\beta + \varepsilon_t < \delta^+) \\
&= E(\varepsilon_t \mid -\delta^- x_t\beta < \varepsilon_t < \delta^+ - x_t\beta) \\
&= E\left(\frac{\varepsilon_t}{\sigma} \mid \frac{-\delta^- x_t\beta}{\sigma} < \frac{\varepsilon_t}{\sigma} < \frac{\delta^+ - x_t\beta}{\sigma}\right).
\end{aligned}$$

Let

$$\begin{aligned}
a_t &\equiv \frac{-\delta^- - x_t\beta}{\sigma} \\
b_t &\equiv \frac{\delta^+ - x_t\beta}{\sigma}.
\end{aligned}$$

Then,

$$\begin{aligned}
E(\varepsilon_t \mid y_t = 0) &= \sigma E\left(\frac{\varepsilon_t}{\sigma} \mid a_t < \frac{\varepsilon_t}{\sigma} < b_t\right) \\
&= \sigma\frac{\phi(a_t) - \phi(b_t)}{\Phi(b_t) - \Phi(a_t)} \qquad \text{(D.2)}
\end{aligned}$$

where the last equality is from the fact that for a random variable $z \sim N(0, 1)$,

$$\begin{aligned}
E(z \mid a < z < b) &= \frac{1}{\Phi(b) - \Phi(a)} \int_a^b z\phi(z)dz \\
&= \frac{1}{\Phi(b) - \Phi(a)} [-\phi(z)]_a^b \\
&= \frac{\phi(a) - \phi(b)}{\Phi(b) - \Phi(a)}.
\end{aligned}$$

The generalized residual $\hat{\eta}_t$ is defined as

$$\hat{\eta}_t = \begin{cases} y_t - x_t\hat{\beta} + \hat{\delta}^+ & \text{if } y_t > 0 \\ y_t - x_t\hat{\beta} - \hat{\delta}^- & \text{if } y_t < 0 \\ \widehat{E(\varepsilon_t \mid y_t = 0)} & \text{if } y_t = 0. \end{cases} \tag{D.3}$$

where $\widehat{E(\varepsilon_t \mid y_t = 0)}$ is obtained by plugging the maximum likelihood estimates of the parameters in (D.2).

Although this generalized residual cannot be treated in the same manner as the usual residual from a linear model, it can be used to test conditional moment restrictions. For example, a test for first-order serial correlation may be based on $T^{-1}\sum \hat{\eta}_t\hat{\eta}_{t-1}$, which can be justified by noting that

$$\begin{aligned} E(\varepsilon_t\varepsilon_{t-1}) &= E[E(\varepsilon_t\varepsilon_{t-1} \mid y_t, y_{t-1})] \\ &= E(\varepsilon_t \mid y_t)E(\varepsilon_{t-1} \mid y_{t-1}) \end{aligned}$$

under the assumption that ε_t is independent of ε_{t-1}.

If a test for specification error involves a higher moment of the error, it can be computed in a similar way. For example, to test whether $E(\varepsilon_t^3) = 0$, a generalized sample analog can be obtained from the fact $E(\varepsilon_t^3) = E[E(\varepsilon_t^3 \mid y_t)]$ as

$$\begin{aligned} &T^{-1}\sum \hat{\varepsilon}_t^3 && \text{if } y_t \neq 0, \\ &T^{-1}\sum \widehat{E(\varepsilon_t^3 \mid y_t = 0)} && \text{if } y_t = 0 \end{aligned}$$

where $\widehat{E(\varepsilon_t^3 \mid y_t = 0)}$ is the conditional moment $E(\varepsilon_t^3 \mid y_t = 0)$ evaluated at the maximum likelihood estimates.

$$
\begin{aligned}
E(\varepsilon_t^3 \mid y_t = 0) &= E(\varepsilon_t^3 \mid -\delta^- < y_t^* < \delta^+) \\
&= E(\varepsilon_t^3 \mid -\delta^- < x_t\beta + \varepsilon_t < \delta^+) \\
&= E(\varepsilon_t^3 \mid -\delta^- x_t\beta < \varepsilon_t < \delta^+ - x_t\beta) \\
&= E\left(\frac{\varepsilon_t^3}{\sigma} \mid \frac{-\delta^- x_t\beta}{\sigma} < \frac{\varepsilon_t}{\sigma} < \frac{\delta^+ - x_t\beta}{\sigma}\right) \\
&= \sigma^3 \frac{(2 + a_t^2)\phi(a_t) - (2 + b_t^2)\phi(b_t)}{\Phi(b_t) - \Phi(a_t)}
\end{aligned}
$$

where a_t and b_t are as defined above, and the last equality is from the fact that for a random variable $z \sim N(0,1)$,

$$
\begin{aligned}
E(z^3 \mid a < z < b) &= \frac{1}{\Phi(b) - \Phi(a)} \int_a^b z^3 \phi(z) dz \\
&= \frac{1}{\Phi(b) - \Phi(a)} \left[-(2 + z^2)\phi(z) \right]_a^b \\
&= \frac{(2 + a^2)\phi(a) - (2 + b^2)\phi(b)}{\Phi(b) - \Phi(a)}.
\end{aligned}
$$

Appendix E

Bootstrap Procedure with Heteroscedasticity

First, each element of the OLS residual vector $\hat{e}$ is divided by an estimate of the conditional standard deviation $(\sqrt{\hat{h}_t})$ to obtain a set of homoscedastic errors of

$$\breve{e} = \{\breve{e}_1, \ldots, \breve{e}_T \mid \breve{e}_t = \hat{e}_t/\sqrt{\hat{h}_t}, \quad t = 1, \ldots, T\}. \tag{E.1}$$

The conditional variance estimate, $\hat{h}_t$, is obtained as the fitted value from an auxiliary regression of $\hat{e}_t^2$ on $x_t^2 = (1, dev7_{t-1}^2, dev25_{t-1}^2, vol_{t-1}^2, y_{t-1}^2, y_{t-2}^2, \cdots, y_{t-p}^2)'$, i.e.

$$\hat{e}_t^2 = x_t^2\delta + v_t \tag{E.2}$$

$$\hat{h}_t = x_t^2\hat{\delta} \tag{E.3}$$

where v_t is an error term and $\hat{\delta}$ is the vector of OLS estimates in that auxiliary regression.

Now the random draws are from these standardized errors of $\breve{e}$. Then, the t-th bootstrap error $\tilde{e}_t$ is

$$\tilde{e}_t = \breve{e}_t\sqrt{\tilde{h}_t} \tag{E.4}$$

where $\tilde{h}_t = \tilde{x}_t^2 \hat{\delta}$ and $\tilde{x}_t = (1, dev_{t-1}, dev25_{t-1}, vol_{t-1}, \tilde{y}_{t-1}, \tilde{y}_{t-2}, \cdots, \tilde{y}_{t-p})$. Once $\tilde{e}_t$ is given, the value of the dependent variable $\tilde{y}_t$ is computed by (2.12). Note that $\tilde{h}_t \neq \hat{h}_t$ because $\tilde{x}_t \neq x_t$. Since $\tilde{x}_t$ contains lags of $\tilde{y}_t$, in each replication the bootstrap data on $\tilde{h}_t$, $\tilde{e}_t$ and $\tilde{y}_t$ must be computed recursively. The rest of the bootstrap procedure is the same as the homoscedastic case.

Appendix F

Confidence Interval for a Threshold

Consider an LR statistic for $H_0 : \gamma = \gamma_0$ as

$$LR(\gamma) = T\left(\frac{\hat{\sigma}^2(\gamma) - \hat{\sigma}^2(\hat{\gamma})}{\hat{\eta}^2(\hat{\gamma})}\right) \tag{F.1}$$

where

$$\hat{\sigma}^2(\gamma) = \frac{1}{T}\sum_{t=1}^{n}\hat{e}_t(\gamma)^2, \tag{F.2}$$

$$\eta^2(\gamma) = \frac{(\beta_1 - \beta_2)'V(\beta_1 - \beta 2)}{(\beta_1 - \beta_2)'D(\beta_1 - \beta 2)}, \tag{F.3}$$

$$V = E(x_t x_t' e_t^2 | q_t = \gamma),$$

$$D = E(x_t x_t' | q_t = \gamma).$$

and $\hat{e}_t(\gamma)$ is the residual from two-regime threshold regression. The numerator and denominator of $\eta^2(\gamma)$ can be estimated either by polynomial regression on ($1\ q_t\ q_t^2$), or by kernel regression. See Hansen(1997, 2000) for details.

Note that the LR statistic is a measure of the change in error variance as the threshold moves away from $\hat{\gamma}$. It is standardized by a scale factor $\hat{\eta}^2$ which becomes $\hat{\sigma}^2(\hat{\gamma})$ if the regression error of the two-regime model is homoscedastic. Asymptotic distribution (cdf) of the LR statistic is

$$P(LR \leq c) = \left(1 - e^{-c/2}\right)^2. \tag{F.4}$$

Let c_β be the β level critical value (e.g. $\beta = 0.95$). Then

$$P(LR \leq c_\beta) = \beta. \tag{F.5}$$

Using (F.4), solve for c_β.

$$c_\beta = -2ln(1 - \sqrt{\beta}). \tag{F.6}$$

Then the β level confidence interval for γ consists of those elements of q_t for $t = 1, \ldots, T$ such that

$$LR(q_t) \leq -2ln(1 - \sqrt{\beta}). \tag{F.7}$$

BIBLIOGRAPHY

Almekinders, G. J. and S. C. W. Eijffinger (1996): "A friction model of daily Bundesbank and Federal Reserve intervention," Journal of Banking and Finance, 20, 1365-1380.

Almekinders, G. J. and S. C. W. Eijffinger (1994): "The ineffectiveness of central bank intervention," in S. C. W. Eijffinger ed. (1998), Foreign Exchange Intervention: Objectives and Effectiveness. Cheltenham and Northampton: Edward Elgar Publishing Limited.

Bai, J. (1997): "Estimating multiple breaks one at a time," Econometric Theory, 13, 315-352.

Bai, J. and P. Perron(1998): "Estimating and testing linear models with multiple structural changes," Econometrica, 66, 47-78.

Baillie, R. T. and W. P. Osterberg (1997a): "Central bank intervention and risk in the forward market," Journal of International Economics, 43, 483-497.

Baillie, R. T. and W. P. Osterberg (1997b): "Why do central banks intervene?" Journal of International Money and Finance, 16, 909-919.

Baillie, R. T. and W. P. Osterberg (2000): "Deviations from daily uncovered interest rate parity and the role of intervention" Journal of International Financial Markets, Institutions and Money, 10, 363-379.

Baillie, R. T., O. F. Humpage and W. P. Osterberg (2000): "Intervention from an information perspective," Journal of International Financial Markets, Institutions and Money, 10, 407-421.

Cox, D. R. and E. J. Snell (1968): "A General definition of residuals," Journal of the Royal Statistical Society. Series B (Methodological), 30, 248-275.

Dominguez, K. M. and J. A. Frankel (1993): "Does foreign-exchange intervention matter? The portfolio effect," The American Economic Review, 83, 1356-1369.

Dominguez, K. M.(2003): "The market microstructure of central bank intervention," Journal of International Economics, 59, 25-45.

Eijffinger, S. C. W. and A. P. D. Gruijters (1991): "On the short term objectives of daily intervention by the Deutsche Bundesbank and the Federal Reserve System in the U.S. dollar - Deutsche mark exchange market," in S. C. W. Eijffinger ed. (1998), Foreign exchange intervention: objectives and effectiveness. Cheltenham and Northampton: Edward Elgar Publishing Limited.

Frankel, J. A. and K. A. Froot (1990): "Chartists, Fundamentalists, and Trading in the Foreign Exchange Market," The American Economic Review, 80, No.2, Papers and Proceedings of the Hundred and Second Annual Meetings of the American Economic Association (May, 1990), 181-185.

Franses, P. H. and D. van Dijk (2000): Non-linear time series models in empirical finance. Cambridge: Cambridge University Press.

Frenkel, M. and G. Stadtmann (2001): "Intervention reaction functions in the dollar-deutsche mark market," Financial Markets and Portfolio Management, 15, 328-343.

Hansen, B. E. (1996): "Inference when a nuisance parameter is not identified under the null hypothesis," Econometrica, 64, 413-430.

Hansen, B. E. (1997): "Inference in TAR Models," Studies in Nonlinear Dynamics and Econometrics, 2, 1- 14.

Hansen, B. E. (1999): "Testing for linearity," Journal of Economic Surveys, 13, 551-576.

Hansen, B. E. (2000): "Sample splitting and threshold estimation," Econometrica, 68, 575-603.

Hogg, R. V. and E. A. Tanis (1997): Probability and statistical inference. 5th edition. Upper Saddle River: Prentice-Hall, Inc.

Humpage, O. F. (1999): "U.S. intervention: Assessing the probability of success," Journal of Money, Credit, and Banking, 31, 731-746.

Humpage, O. F. (1988): "Intervention and the dollar's decline," Economic Review, Federal Reserve Bank of Cleveland, 24(2), 2-16.

Humpage, O. F. and W. P. Osterberg (2000): "Why intervention rarely works," Federal Reserve Bank of Cleveland Economic Commentary, February 1.

Hung, J. H. (1997): "Intervention strategies and exchange rate volatility: a noise trading perspective," Journal of International Money and Finance, 16, 779-793.

Ito T. (2002): "Is foreign exchange intervention effective?: The Japanese experience in the 1990S," NBER Working Paper 8914.

Kearns J. and R. Rigobon (2002): "Identifying the efficacy of central bank interventions: The Australian case," NBER Working Paper 9062.

Kim S. K. and J. Sheen (2002): "The determinants of foreign exchange intervention by central banks: evidence from Australia," Journal of International Money and Finance, 21, 619-649.

LeBaron, B. (1999): "Technical trading rule profitability and foreign exchange intervention," Journal of International Economics, 49, 125-143.

Neely C. J. (1998): "Technical analysis and the profitability of U.S. foreign exchange intervention," Review, Federal Reserve Bank of St. Louis, July/August, 3-17.

Neely C. J. (2000): "Are changes in foreign exchange reserves well correlated with official intervention?," Review, Federal Reserve Bank of St. Louis, September/October 2000, 17-31.

Neely C. J. (2001): "The practice of central bank intervention: Looking under the hood," Review, Federal Reserve Bank of St. Louis, May/June 2001, 1-10.

Neely C. J. (2002): "The temporal pattern of trading rule returns and exchange rate intervention: intervention does not generate technical trading profits," Journal of International Economics, 58, 211-232.

Pagan A. and F. Vella (1989): "Diagnostic tests for models based on individual data: A survey," Journal of Applied Econometrics, 4(S), S29-S59.

Potter S. M. (1999): "Nonlinear time series modelling: An introduction," Journal of Economic Surveys, 13, 505-524.

Rosenberg M. R. (1996): Currency forecasting: A guide to fundamental and technical models of exchange rate determination. Chicago: Irwin Professional Publishing, Chapter Eleven, 276-323.

Rosett R. N. (1959): "A Statistical model of friction in economics," Econometrica, 27, 263-267.

Taylor M. P. and H. Allen (1992): "The use of technical analysis in the foreign exchange market," Journal of International Money and Finance, 11, 304-314.

Tersvirta, T. (1998): "Modelling economic relationships with smooth transition regressions," in A. Ullah and D.E.A. Giles (eds.), Handbook of Applied Economic Statistics, New York: Marcel Dekker, 507-552.

Wooldridge J. M. (2000): Introductory econometrics: A modern approach. Cincinnati, OH: South-Western College Publishing.

Wooldridge J. M. (2002): Econometric analysis of cross section and panel data. Cambridge: The MIT Press.