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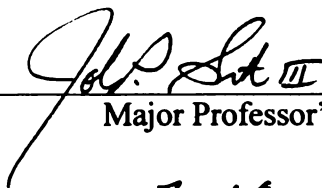
MIDDLE SCHOOL MATHEMATICS STUDENTS'
MOTIVATIONS FOR PARTICIPATING IN
WHOLE-CLASS DISCUSSIONS: THEIR BELIEFS,
GOALS, AND INVOLVEMENT

presented by

AMANDA JANSEN HOFFMANN

has been accepted towards fulfillment
of the requirements for the

Doctoral degree in Counseling, Educational
Psychology, and Special Education



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**MIDDLE SCHOOL MATHEMATICS STUDENTS' MOTIVATIONS
FOR PARTICIPATING IN WHOLE-CLASS DISCUSSIONS:
THEIR BELIEFS, GOALS, AND INVOLVEMENT**

By

Amanda Jansen Hoffmann

A DISSERTATION

**Submitted to
Michigan State University
in partial fulfillment of the requirements
for the degree of**

DOCTOR OF PHILOSOPHY

Department of Counseling, Educational Psychology, and Special Education

2004

ABSTRACT

MIDDLE SCHOOL MATHEMATICS STUDENTS' MOTIVATIONS FOR PARTICIPATING IN WHOLE-CLASS DISCUSSIONS: THEIR BELIEFS, GOALS, AND INVOLVEMENT

By

Amanda Jansen Hoffmann

Whole-class discussions in mathematics classrooms are considered to foster active sense-making and intellectual autonomy among students. Through participating in these discussions, students have the opportunity to develop skills of mathematical communication, reasoning, and justification. However, middle school students may resist participating in whole-class discussions if they perceive social consequences resulting from this activity.

Research on mathematics classroom discourse typically focuses on the role of the teacher in discourse, examining student variables as outcomes to measure the effectiveness of the teachers' strategies. Alternatively, in this study, students' beliefs and goals are examined for how they influence students' participation in classroom discourse rather than as outcomes.

I assessed beliefs and goals of 15 target students from two seventh grade mathematics classrooms through one-on-one interviews and a Likert-scale survey instrument. Students' talk in interviews was analyzed through the use of a framework that included imperative verbs to capture idealized states, repetition to capture emphasis, and connections to affect to capture relative importance to the student. This framework

allowed for a more rigorous analysis of students' beliefs in contrast to reporting any and all of their responses to interview questions.

Students' involvement in classroom discourse was described based on an analyses of videotaped classroom discussions about four investigation problems from the *Connected Mathematics Project* Standards-based mathematics curriculum.

Results from this study indicate that students' involvement in classroom discussions is influenced by their social goals and epistemological beliefs. Students who believed they learned mathematics through a process of negotiation and associated a low level of risk with participating in discussion were more likely to extend their participation during an interaction, critique the thinking of their classmates, and talk about mathematics at a high level of explicit meaning. There were also differences in students' involvement between the target students based on their classrooms.

This study illustrates how adolescence intersects with the mathematics reform movement by taking into account students' perspectives. Future research investigating how beliefs and goals relate to students' involvement in discussions may explain how a classroom of students together supports the development of effective classroom discussions.

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ACKNOWLEDGEMENTS

I would like to express my appreciation for those who played a role in supporting the work of my dissertation.

I would first like to express my gratitude to my dissertation chair and academic advisor, Dr. Jack Smith. Since the beginning of my academic career at Michigan State, he has been helpful, concerned, and thoughtful. I am particularly grateful for his careful reading of multiple editions of each of these chapters, even while away on sabbatical overseas. I consider myself fortunate to have Jack as my mentor.

Additionally, my committee members, Dr. Ralph Putnam, Dr. Sandra Crespo, and Dr. Ellen Altermatt each played an important role in my work. Ralph and I had many engaging discussions about analyzing the classroom discourse, in no small part supported by his knowledge of studying classroom processes. Sandra consistently helped me feel like a professional researcher when I was feeling small, and she saw possibilities in my ideas even when they were first developing. Ellen's analytic perspective on issues such as units of analysis helped me see this study in new ways. They all asked challenging and helpful questions throughout my journey. For each of them, I am very thankful.

Two funding sources – the MSU / Spencer Research Training Grant and the Graduate School's Dissertation Completion Fellowship – provided me with the opportunity to develop and execute a complex study.

I am honored to have shared in the classroom experiences of the teachers and students of these two classrooms in the 2002-03 school year. They have taught me much more than what I was able to capture in these pages.

I have been blessed with a supportive group of friends and family. My mom, dad, and sister have encouraged me to pursue my dreams and have been my biggest fans. Thank you for being so proud of me. Many of my friends during graduate school have helped me maintain my sanity and thoughtfulness during the past two years, and for that I am grateful. I am especially appreciative of Jane Pizzolato's continual encouragement and for her critical eyes as my best first reader.

My husband, Eric, has been unconditionally loving and supportive throughout graduate school. He gave me space when I needed it, reminded me to stop working when I needed balance, made me laugh when I took myself too seriously, and believed in me when I did not believe in myself. Words cannot thank him enough for being such a wonderful partner.

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CHAPTER ONE

INTRODUCTION

Familiar images of middle school mathematics classrooms have not commonly included whole-class discussions. A more typical image includes a teacher describing procedures for students to learn and practice. The teacher delivers a body of knowledge that students take in. Current reforms in mathematics education challenge this image, suggesting alternative images of teaching and learning mathematics that promote, among other things, shifts in the roles of students and teachers (NCTM, 1989, 1991, 2000). These reforms suggest teachers should involve students in the process of socially (re-) constructing mathematical knowledge. In this construction process, whole-class discussions of problems and potential solutions play a prominent role.

Creating whole-class discussions that involve students in the local construction of knowledge may be difficult in mathematics because of the need for students to acquire knowledge of algorithms, understandings, and relationships established by the mathematical community, and because of the expectations students may have about their role as the receiver of knowledge in the classroom. Although these seem like logical assumptions, there is more to learn about whether and how students experience these difficulties. Insights from students in mathematics classrooms where whole-class discussions consistently take place may illuminate opportunities that teachers have to increase depth and breadth of student engagement in mathematics classroom discussions.

By whole-class discussions, I mean large group interactions during mathematics class in which students present their solutions to a problem and alternative solutions are

contrasted to one another. In these discussions, students are invited to voice arguments for or against the solutions. The National Council of Teachers of Mathematics [NCTM] advocates discussion in their Standards documents (NCTM, 1989, 1991, 2000). They encourage mathematics teachers to create environments for students to communicate with one another, listen carefully, and critique each others' ideas, in order to promote autonomous thinking among students and to provide students with opportunities to learn how to justify their thinking mathematically. However, the *Principles and Standards of School Mathematics* (NCTM, 2000) also mentions challenges involved with holding such discussions in the middle school setting:

For some students, participation in class discussions is a challenge. For example, some students in the middle grades are often reluctant to stand out in any way during group interactions. (NCTM, 2000, p. 61)

Students' reluctance may be due to their perceived social repercussions of participation. In other words, if a student participates and is incorrect, he may fear that their classmates will pass negative judgment upon him. If a student participates and is either correct or thoughtful about mathematics, she may be concerned that their peers will think she is altogether too dedicated to school. During adolescence, the development of metacognition leads to a greater awareness of what others might think, and a heightened sensitivity to the possibility of being judged. Discussion-rich environments may pose challenges not only for students who are reluctant to participate, but also for those who lack the linguistic and social competencies to engage fully in the classroom discourse. Although the risks associated with publicly sharing one's thinking may be a hindrance for early adolescents' participation in classroom discussions, it is possible that sharing

strategies for solving problems in mathematics class may benefit the middle school student and meet some of their needs, overriding the associated risks.

As teachers engage students in talking about mathematics, they may assume that their students are familiar with the ways classrooms function, yet lack awareness of the complexity of students' roles in the construction of the classroom micro-culture and the larger school culture (Sarason, 1982). Teachers may not know how the history of their students' relationships with one another over preceding years, or even from the cafeteria at lunchtime that day, influences their social interactions about the subject matter during class. Research characterizing students' perspectives on the social dimension of the classroom, among other issues, can provide insight into the development of classroom micro-cultures. Lubienski (2000a, 2000b), for example, has shown that working class middle school students prefer more direction from the teacher than their middle class peers in discussion-intensive mathematics classrooms.

In this study, I examined two seventh-grade classrooms where whole-class discussion took place on a regular basis. In these classrooms, I examined and analyzed the range of students' perspectives, in terms of their beliefs and goals, and whether and how these beliefs and goals related to their involvement in whole-class discussions. Students' involvement in whole-class discussion may relate to their beliefs and goals in ways that help teachers and mathematics educators gain an understanding of why some students get involved in discussion and others do not, or why some students adopt productive ways of talking during class discussion and others do not. For example, students who have beliefs about the nature of mathematics such that they view the domain as flexible, pursuing multiple solution paths to a problem, may be more likely to

participate in an extended discussion around alternative methods for solving a problem. Students without this belief may resist getting involved in such a discussion. The primary goal of this study is to capture how junior high students experience Standards-based mathematics settings, in terms of their involvement in whole-class discussions, their beliefs and goals in these settings, and how their actions and beliefs interrelate.

In this chapter, I first describe the theoretical perspective orienting this study. Then I define the constructs in this study – students’ beliefs, goals, and their involvement in whole-class discussions – and describe my orientation to studying relations between them. Next, I share some of my personal history to illustrate my own journey toward this study. Finally, I describe the goals of this study and the structure of this dissertation.

Theoretical Perspective

This project is designed to situate research on students’ motivation in the activity of whole-class discussion and the subject matter of mathematics. Mathematics educators have called for analyses of students’ motivation that take into account subject matter and issues related to the mathematics reform movement (e.g., Middleton & Spanias, 1999). Analyzing beliefs and goals in light of a specific activity and subject matter allows for the potential of developing new frameworks of beliefs and goals to study with respect to students’ experiences as learners. When the study of motivation is situated in a context, the beliefs and goals students express may be more specific to the activity and subject matter, and less general in comparison to constructs such as learning goals (Ames, 1992; Dweck, 1986).

Following symbolic interactionism (Blumer, 1969, as cited in Cobb, Yackel, & Wood, 1993), in this study, I assume that beliefs and goals have a reflexive relationship

with classroom discourse practices. On the one hand, beliefs shape students' involvement and participation. On the other hand, beliefs change as discourse practices change. For students to participate in a particular way, such as critique the solution of a classmate, their beliefs and goals would need to support this behavior. They may hold the belief that a student has the authority to evaluate, or the goal to help a classmate. If students are encouraged to change how they participate, their beliefs and goals may shift after trying on new behavior. However, students' beliefs and goals could relate to their discourse practices in a range of ways.

We therefore conjectured that students develop specifically mathematical beliefs and values that enable them to act as increasingly autonomous members of the classroom mathematical community as they participate in the negotiation of sociomathematical norms... Once again, this conjecture is open to empirical investigation. (Cobb, Stephan, McClain, & Gravemeijer, 2001, p. 124)

Empirical investigation of how students' beliefs and goals are related to their involvement in classroom discourse practices would allow for verifying, enriching, or problematizing current theories in mathematics education about relations between students' beliefs, goals, and discourse practices.

Mathematics education researchers have begun to examine links between students' beliefs, goals, and their classroom discourse practices, but coordinating analyses of individual students' perspectives with analyses of the social setting of the classroom is a complex endeavor. While some of this work has been taken up (e.g., Bowers & Nickerson, 2001; Cobb et al., 2001; Lo, Wheatley, & Smith, 1994; Stephan,

Cobb, & Gravemeijer, 2003), these studies have placed more emphasis on the analysis of the classroom practices, complementing these analyses with case studies of small numbers of students (between two and five). In this study, I coordinated analyses of individual students' perspectives with the analysis of social settings by shifting the emphasis onto individual students and including more cases ($N = 15$) of students' involvement, beliefs, and goals, with complementary analyses of classroom practices.

My emphasis on individual cases of students focuses more on a cognitive analysis of a plurality of individuals than on the activity of a collective (Stephan et al., 2003). A primary focus on individual students allows for mapping out the diversity of students' perspectives in their social context. An examination of a wide range of students' views is limited with an analysis of only two to five students. Another difference between this study and others that have attempted to coordinate analyses of the individual and social setting is that other studies have focused on ways of thinking and talking about mathematics specifically, rather than my focus on beliefs and goals related to students' roles as learners. The practices of each local classroom community are also described briefly in this study, in contrast to studies that emphasize descriptions of classroom practices over individual perspectives.

Students' Motivations: Beliefs and Goals

Research on students' experiences in Standards-based mathematics settings have tended to focus on student achievement (e.g., Huntley, Rasmussen, Villarubi, Sangtong, & Fey, 2000; Reys, Reys, Lapan, & Holliday, 2003; Riordan & Noyce, 2001; Senk & Thompson, 2002). Notable exceptions include studies of students' attitudes, beliefs, and achievement and how they change in the context of new curricula or pedagogical

approaches (e.g., Wood & Sellers, 1997). There exists a variety of methods for exploring students' experience of Standards-based curricula beyond achievement, including capturing their voices and telling their stories (e.g., Boaler, 1998; Holt et al., 2001) or interpreting their dispositions, beliefs, attitudes, or motivational goals from self-reported survey data or interview data. The field of mathematics education has more to understand about the students' experiences with Standards-based curricula, since these curricula are relatively new. There may be more diversity existing among students' perspectives than is currently known, as each student may interpret the same activity in a classroom from their own unique viewpoints.

...each individual person in the classroom creates his own unique construction of the rest of the participants, of their goals, of the interactions between herself and the others and of all the events, tasks, mathematical contents which occur in the classroom. (Bishop, 1985, p. 26)

Examining how classrooms that use Standards-based curricula are seen and felt by students is important work, as students' experiences have not received sufficient attention from educators (Erickson & Shultz, 1992).

One goal of the NCTM Standards and the National Research Council (Kilpatrick, Swafford, & Findell, 2001) is to promote "productive" dispositions among students.

...the tendency to see sense in mathematics, to perceive it as both useful and worthwhile, to believe that steady effort in learning mathematics pays off, and to see oneself as an effective learner and doer of mathematics.(Kilpatrick et al., 2001, p. 131)

Helping students develop particular dispositions would support students' motivations for engaging in learning or doing mathematics. We do not know, however, which particular dispositions support students' involvement in class discussions during mathematics class.

From an educational psychologist's perspective, people's motivation to engage in particular behaviors may be explained through expectancy-value models (Eccles & Wigfield, 2002). Expectancies include beliefs about one's capabilities, addressing the question, "Can I do this?" Additionally, expectancies may include epistemological beliefs about the nature of knowledge or subject matter. Since implementing whole-class discussion in mathematics classrooms can promote shifts in expected roles of teacher and students, this study focuses on students' epistemological beliefs with respect to the process of learning mathematics: the process of learning and students' and teachers' roles. Values may include reasons for doing the activity, addressing the question, "Why do I want to do this?" Studying students' beliefs and goals in mathematics classrooms implementing whole-class discussion could provide insight into their dispositions and motivations for involvement in Standards-based settings.

Defining beliefs. Beliefs are considered to be an ill-defined construct (Schoenfeld, 1992; Thompson, 1992). One struggle with this construct involves parsing the definition of beliefs as exclusive from other phenomenon, such as knowledge, attitudes, or values. While it might be tempting to assume that beliefs is a term for which there exists a shared understanding, Furinghetti and Pehkonen (2001, 2002) have shown that researchers' definitions are not consistent, one to another.

For the purpose of this study, beliefs are defined as students' personal knowledge (Polanyi, 1958) about the process of learning mathematics that serve as the "assumptions

from which individuals make decisions about the actions they will undertake”

(Kloosterman, Raymond, & Emenaker, 1996, p. 39). This personal knowledge is generally not subject to objective scrutiny and/or justification, so there is little sense in asking if or evaluating whether beliefs are “right” or “wrong.” Beliefs are an important element in students’ learning, as they are a part of their schema through which they make sense out of the world.

...the beliefs students construct, the overall goals they establish, and the contexts in which they do mathematics are their attempts to find a viable way of operating in the classroom. (Cobb, 1986, p. 8)

Students’ attempts to make sense out of the learning process are cognitive interpretations, dependent upon what an individual knows (knowledge) or assumes to be true (beliefs), and this sense-making activity is provoked in the social setting of the classroom. Beliefs may also have an emotional response tied to them (Mandler, 1989).

Some researchers refer to beliefs as “generally stable” (McLeod, 1992), while others (cf., diSessa, Elby, & Hammer, 2002; Furinghetti & Pehkonen, 2002; Pehkonen & Furinghetti, 2001) describe beliefs as more contextually dependent. This study examines students’ beliefs during a relatively short period of time, and I suggest that there is substantial stability of these students’ beliefs during the time period of the study, due to the short time period and the level of commitment the students appear to express.¹ Beliefs may be held at varying degrees of commitment. However, for the purpose of this study, I examined beliefs for which students express at a strong level of commitment, as these beliefs may also be those that they act upon.

¹ Whether the same beliefs are stable over much longer periods of time is not addressed in the study and therefore an open question.

Range of students' goals. I operationally define students' goals as the specific reasons they cite for their actions, which are more specific than the broader psychological assumptions through which students interpret their world (beliefs). Goals are the desired end-states that orient students' actions—what actions are intended to achieve. Students may have multiple goals for participating in whole-class discussions during mathematics class, not all of which may appear to be obviously related to their beliefs about learning mathematics, but can directly affect how students choose to participate when talking about mathematics. Some potential goals students might act on include: (1) *learning goals*, such as whether students are motivated by a need to perform well or develop a deep understanding (Ames, 1992; Dweck, 1986); (2) *social goals*, e.g., whether students are motivated by gaining the attention of a peer or looking good in front of others and presenting one's self in a particular way (Goffman, 1959); (3) *efficiency goals*, such as finishing the problems as soon as possible so as not to have homework; and (4) *academic but non-mathematical goals*, as students may have a goal of focusing on their mathematics, but a stronger goal toward something else, such as memorizing their vocabulary for another class. Students holding these goals may also hold goals more closely related to developing a deeper understanding of mathematical content.

Students' beliefs and goals may exist in clusters, supporting one another to create motivational paths (Pintrich, 2003) for students' involvement in whole-class discussions. These motivational paths may be the set of beliefs that provide the psychological context for decision-making and the goals providing an internal stimulus for action. Additionally, multiple beliefs and goals held in isolation may set up competing priorities for the student.

Students' Involvement in Whole-Class Discussion

Students may act on their beliefs and goals during their involvement in whole-class discussion. Reforms in mathematics education, both national and international, call for improving students' proficiencies in mathematical communication. In the United States, the National Council of Teachers of Mathematics has addressed the significance of discourse in mathematics classrooms (NCTM, 1991):

Discourse refers to the ways of representing, thinking, talking, agreeing, and disagreeing that teachers and students use to engage... The discourse embeds fundamental values about knowledge and authority. Its nature is reflected in what makes an answer right and what counts as legitimate mathematical activity, argument, and thinking. (p. 20)

Through their involvement in mathematics classroom discourse, students attempt to communicate about mathematics, which makes their thought process more public, allowing teachers to assess their students informally and allowing students to practice reasoning about mathematics. Making one's thinking public is a practice that some mathematics teachers are attempting to engage in with their students (e.g., Lampert, 2001). Since discussions are a fairly new practice in mathematics classrooms, recent research has examined them.

In the mathematics education research literature, researchers have attempted to document what these whole-class discussions should or could look like in classrooms (e.g., Chazan, 2000; Heaton, 2000; Lampert, 2001). Julianne Turner and colleagues (Turner et al., 1998) determined that upper-elementary mathematics classroom teachers who fostered high-involvement classrooms scaffolded instruction through techniques

such as transferring responsibility. Knuth and Peressini (2001) contrasted the differences between univocal and dialogic discourse (following Wertsch, 1991) – an attempt to convey a specific message versus more two-way, give-and-take communication, respectively – in their writing for mathematics classroom teachers.

In this study, rather than focusing on how teachers can foster productive whole-class discussions about mathematics, I examine students' contributions during these discussions, along with an analysis of their beliefs and goals. This analysis of students' contributions during whole-class discussion allows for an investigation of what students do in Standards-based mathematics classrooms, providing another look at their experiences with the implementation of standards-based mathematics curricula. My analysis of the range of students' contributions during whole-class discussion included an examination of their levels of reasoning about mathematics as they communicate in this curricular setting (cf., Stein & Lane, 1996). Additionally, I analyzed students' contributions in terms of indications that they are taking on an active, autonomous role in their classroom, such as a willingness to assert themselves or to critique their classmates' solutions (Walen, 1994).

Relations Between Beliefs, Goals, and Involvement in Whole-Class Discussion

Certainly, there is no reason to expect any simple relationship (or any set of relationships) between students' beliefs or goals and their participation in whole-class discussions. On the one hand, classroom practices may moderate relations between beliefs and actions, as classroom norms set up expectations for what constitutes appropriate behavior, and beliefs are aligned due to the process of negotiating the norms (Cobb et al., 1993). On the other hand, individual variation among students, such as

socio-economic status (Lubienski, 2000a, 2000b) or previous history as a student and home influences (Ridlon, 2001), may moderate the relation between students' beliefs and their participation in mathematics classroom discussion. My students' histories prior to their seventh grade mathematics classroom may result in wide variation among their beliefs and goals. Additionally, students who express a particular belief may involve themselves in discussion in a variety of ways. For example, students who express the belief that mathematics problems have more than one solution may participate by offering an alternative solution, or they may listen carefully to the discussion to make sure they do not miss hearing one of the several solution methods being discussed.

Attempts to study links between students' beliefs and their actions (or their teachers') are common among researchers who adopt a cognitive perspective on learning. In mathematics education, students' beliefs have been studied in relation to their problem solving behaviors (Cobb, 1985; Schoenfeld, 1985, 1988) and group work (Kloosterman et al., 1996). Schoenfeld (1985, 1988) found that high school students who held a belief that they should be able to solve mathematics problems quickly, in 12 minutes or less, also exhibited a lack of persistence when working on challenging problems. Cobb (1985) demonstrated that a first-grade student who had an ego-involvement learning goal orientation also had an instrumental view of mathematical knowledge, while another student with a task-involvement learning goal orientation had a more relational understanding of mathematics. Kloosterman and colleagues (1996) determined that fourth through sixth grade students developed preferences over time for group work in classrooms where the teacher implemented group work, suggesting that students' beliefs about group work were related to teachers' practices. This dissertation study focused on

examining relations between students' beliefs and goals and their involvement in classroom discussions rather than engagement in problem solving activity or group work.

Author's Brief History Toward This Dissertation

My intellectual evolution toward this study began even before I started my graduate studies. Some of the curiosities leading to this study began from my time as a classroom teacher, and even my time as a mathematics student. Below, I briefly discuss some of my personal history as it relates to the origin of this dissertation. My initial motivation, which came out of my teaching experience, helps to explain how and why I see the results of this sort of research as relating to classroom practice.

Students' beliefs and goals. The origin of this dissertation lies in my curiosity about students' relationships with the school subject of mathematics. These curiosities stemmed from my experiences as both a junior high public school teacher and a mathematics education researcher.

I taught junior high mathematics for three years. I sometimes heard my students express extreme reactions to mathematics and my class. They had reactions such as looking forward to coming to class, expressing that it was their favorite class, as well as resisting the activity by not doing homework or participating. Some students started off disinterested in learning and left with a better attitude. Others challenged me for giving them too much homework when they would have rather been playing sports after school, perhaps growing disinterested in mathematics. I became particularly interested in how a teacher could help students overcome mathematics anxiety, as I seemed to experience a few successes with some of my students, and perhaps helped to foster new cases of mathematics aversion among other students. As a teacher, it was tempting to assume my

students would all develop the beliefs and goals I wanted them to hold, but every student appeared to interpret their experience in my classroom differently. I would teach similarly in different class periods, but each class period had a slightly different atmosphere. Depending on the make-up of students in the classroom and the time of day, some class periods felt alive and engaged, others felt easily distracted and scattered, and others felt quieter and perhaps even dull at times. Individual variation between students' experiences in mathematics classrooms has fascinated me from my time as a classroom teacher.

As a researcher, early in graduate school I interviewed high school students about their experiences moving out of Standards-based mathematics curricula and into a more traditional mathematics setting (Smith & Berk, 2001; Smith, Herbel-Eisenmann, Jansen, & Star, 2000; Smith et al., 2001). I did not expect consistent responses from all of the students, but I was taken aback at the wide range of reactions students had to the curricular shift. Some students were relieved they were in a setting where the teacher was more prescriptive, since their performance on high school mathematics was recorded on their transcripts, and direct instruction from the teacher helped these students understand their teacher's expectations. Other students missed the opportunity to voice their thinking during class, and seemed to be annoyed the teacher was telling them so much rather than letting the students try to do the mathematics, so they felt as if some intellectual power was taken from them. Again, there was a great deal of diversity among the students' perspectives, but as my role with students shifted from teacher to researcher, I was given privileged access to more of these students' thinking.

Central to both of these experiences was my interest in taking students' perspectives seriously. I believe students have valuable insights into their learning experiences. While I do not believe teachers should always act to satisfy students' felt needs or address their emotional reactions, gaining a sense of what students think about, and mapping the range of students' perspectives, could inform important instructional thinking and decisions.

Students' involvement in whole-class discussions. I have not had extensive exposure to whole-class discussions during mathematics class prior to doing this study. As a mathematics student, I did not experience learning through discussion. The mathematics I studied in high school was structured by the Saxon textbook series (Saxon, 2003). In these classrooms, the teacher demonstrated a new skill each day, and in the homework we practiced a few exercises of the new skills and reviewed all previously taught skills. I was successful in mathematics in high school. As a mathematics major in college, I chose to retake calculus in order to provide myself with a solid foundation in the subject, even though I had already taken two years of calculus in high school and earned excellent grades. At the university, I experienced reform-oriented mathematic courses. I took Harvard Consortium calculus (Hughes-Hallett, Gleason, & McCallum, 1994) through multivariable calculus, and had a similar textbook series for differential equations, but whole-class discussion did not appear to me as an important focus in these courses. Instead, as a student, I noticed an increased emphasis on story problems, graphs, and the use of graphing calculators, and during class I noticed that it seemed to take a longer time and many chapters to get to what I thought were the "real" ideas in calculus, such as the chain rule, as I recalled from my first pass through calculus in high school.

As a teacher of general mathematics and Algebra I at the junior high level, I was introduced to teaching through the use of whole-class discussion through the district's mathematics specialist, who would come visit my classroom and model demonstration lessons with my students. She would teach two of my three seventh grade class periods about once a month. I would participate in the first one as a student, observe the second time, and I would teach the lesson and she would observe me during the third class period. When I did not have these model lessons to follow, I struggled to implement whole-class discussions in my mathematics classroom. I had a hard time coming up with good questions to pose to the students. I found it challenging to select mathematical tasks that were rich enough to provoke thoughtful discussions. At times, I believed it was a good idea to engage students in talking about mathematics, but, as a novice teacher, I sought to maintain control of my classroom, and I viewed opening up the floor to their ideas as a somewhat chaotic experience that I would rather avoid, especially with the pressure I was feeling to raise my students' test scores. It was the general sense among my colleagues that it was more efficient to raise test scores through teaching mathematics by direct instruction.

As a researcher, I have come to believe there is a great deal to understand about students' experiences in whole-class discussions during mathematics class. On the one hand, I read reform documents and research articles suggesting how discussions could help students become autonomous learners of mathematics. On the other hand, I recall from teaching that what is helpful for one student can be threatening for another. I now believe more empirical evidence is required for making sense out of the range of mathematics students' experiences with whole-class discussion. University teacher

educators and reform documents encourage teachers to foster discussions in their mathematics classrooms, but not all whole-class discussions are created equal, nor do all students experience these discussions about mathematics in the same way. I have read many research reports in which students' beliefs are "read off" of their discourse practices in mathematics classrooms. I designed this study to examine what more we might have to learn about students' beliefs in discussion-oriented mathematics classrooms from talking with students about their experiences as well as observing and documenting their participation in them.

Goals of This Study

From these prior experiences and relevant research, I designed this study to examine the following questions: How do students in early adolescence experience classrooms that use Standards-based mathematics curricula? In particular, what motivates these students to participate in whole-class discussion? My analysis has three primary outcomes: (a) A framework for students' beliefs and goals that describes the diversity of students' beliefs and goals emerging from work with Standards-based mathematics curricula; (b) a description of the range of ways that these students involved themselves in whole-class discussions, including their mathematical reasoning and communication strategies that may reveal students' sense of autonomy with respect to learning and doing mathematics; and (c) an examination of the relations between students' beliefs and goals and their involvement in whole-class discussion.

This study also has a series of secondary goals. I explored whether students' involvement was related to their classroom settings and local classroom practices. I wanted to develop an analytical framework for assessing students' beliefs based on their

talk in interviews. I wanted to design the study as a case of a coordinated analysis between individual and social levels of learning and doing mathematics in school, with the awareness that analyses at each level, individual and social, would present challenges. I wanted the results of this study to allow me to revisit the conceptualizations of motivational and socio-emotional constructs, such as epistemological beliefs and mathematics anxiety, by studying them and similar constructs in Standards-based mathematics settings.

The body of the dissertation is made up of six additional chapters. Chapter Two, that follows, relates this study to previous research and unanswered questions in the fields of mathematics education and educational psychology. In Chapter Three, I describe my methods of data collection and analysis. While collecting the data was relatively straightforward, scoring and analyzing the results, particularly those that concern how beliefs and goals relate to involvement in whole-class discussion, was a more involved process. Chapters Four through Six present the results. I describe the results of the analyses of students' beliefs and goals in Chapter Four – primarily from interview data and secondarily from survey data. In Chapter Five, I present the results of the analyses of students' involvement in whole class discussion, preceded by an introduction to the discourse practices in each classroom, and ending with analyses of whether target students participated differently depending on their classrooms. The results of the analyses of relations between students' beliefs and goals and their patterns of involvement are described in Chapter Six. Finally, I discuss my interpretations of the results, suggest future research, and present implications of the results in Chapter Seven.

CHAPTER TWO

LITERATURE REVIEW

This study was designed to examine seventh grade students' beliefs and goals about learning mathematics as related to their involvement in whole-class discussions. The goals of the study lie at the intersection of at least five significant areas of educational research: (a) the mathematics reform movement in the US, including the development of NCTM Standards-based curricula and teaching practices; (b) adolescent development; (c) beliefs about learning mathematics; (d) students' motivations, such as their academic and social goals; and (e) discourse in mathematics classrooms.

To bound the task of reviewing the research literature relevant to this study, I chose to discuss studies at the intersection of at least two of the five areas listed. Each of the five topics listed above would merit its own literature review, so examining work at the intersection of these topics approximates the boundaries of this study. In a few instances I also included foundational pieces that were relevant to only one of the five themes. An example of a foundational piece would be a frequently cited reference (e.g., Mehan's (1979) book in which he describes the Initiate-Respond-Evaluate interaction sequence in classroom discourse). Generally, I placed an emphasis on peer reviewed empirical research articles. However, I additionally included some chapters from edited books.

There are bodies of research that I could discuss in this chapter, but I have chosen not to, at least in great detail. I could include a discussion of research on teachers' beliefs (e.g., Aguirre & Speer, 2000; Good, Grouws, & Mason, 1990; Stipek, Givvin, Salmon, &

MacGyvers, 2001; Thompson, 1984; Thompson, Phillip, Thompson, & Boyd, 1994), because they could influence how teachers structure whole-class discussions, but I will not, since this line of work has been pursued to some detail already.² Likewise, I could discuss the growing body of research assessing the impact of the NCTM Standards, such as whether students studying mathematics using Standards-based curricula perform better than, or at least equal to, students studying mathematics in traditional curricular settings (e.g., Huntley, Rasmussen, Villarubi, Sangtong, & Fey, 2000; Reys, Reys, Lapan, & Holliday, 2003; Riordan & Noyce, 2001; Senk & Thompson, 2002; Wood & Sellers, 1997), but I will not, since this study is not designed to be an evaluation of Standards-based mathematics curricula. Additionally, I could discuss research that examines students' beliefs in terms of their diversity of conceptions about a particular concept in mathematics (e.g., Szydlik, 2000; Thompson, 1994), but this study is designed to capture the socio-emotional context of learning mathematics in a discussion-oriented setting rather than students' understandings of a mathematical concept.

This study focuses on individual variation in adolescent students' beliefs and goals related to learning and doing mathematics, and how these beliefs and goals relate to their involvement in discussion-oriented mathematics classrooms. Rather than examine what is normative, or most common, among the students in the two mathematics classrooms that I visited, I wanted to map the diversity of students' perspectives. This is in contrast to researchers who choose a perspective on students such that they a set of beliefs that correspond with patterns in classroom talk. The danger of focusing only on the most common patterns across students is the possibility of minimizing some students perspectives in the discussion of improving mathematics classrooms – when those

² For literature reviews, see Pajares (1992) or Thompson (1992).

students' experiences cut against the grain of what is normative. If teachers and researchers hope to promote mathematics learning among all students, it seems appropriate to examine the range of students' perspectives rather than only those that are the most common.

One of the challenges with studying students' perspectives is interpreting their beliefs and goals through a deficit model, and discussing some beliefs and goals as more productive or beneficial than others. So, another way of talking about normative, rather than what is most common in a classroom, is relative to an external view of what is good and productive. I strive to break away from this sense of normative as well and honor the students' perspectives as a worthy starting point rather than a potentially unproductive one. Rather than critique the students' perspectives, I would like to rest the critique on the goals of the mathematics reform, allowing the students' experiences in Standards-based curricular settings provide a lens for revisiting NCTM's recommendations for increased public classroom discourse.

My purpose in this chapter is to review literature at the intersection of research on Standards-based mathematics classrooms, adolescents' learning in school settings, students' beliefs about learning mathematics, students' goals in mathematics classrooms, and discourse in mathematics classrooms. In order to discuss this wide range of literature, I will first review research literature relevant to *students' beliefs and goals*, as it intersects with either Standards-based mathematics settings or adolescents' learning. Then, I will review research literature relevant to the study of *classroom discourse*, as it intersects with teaching and learning in Standards-based mathematics settings, adolescence, or students' beliefs and goals.

Students' Beliefs and Goals in Mathematics Classrooms

A psychological perspective on student learning involves an examination of mental structures, such as beliefs and goals. Beliefs are students' expectations and assumptions about the process of learning and doing mathematics that provide a psychological context for decision making and behaving in classrooms. Goals are what students want to accomplish in their actions – the states of affairs that actions are designed to achieve.

Studying students' beliefs involves an analysis of the assumptions and expectations students hold and act upon related to the process of learning and doing mathematics. Studying students' goals differs from studying students' beliefs, as goals are more specific objectives students' hold in relation to particular activities rather than learning and doing mathematics more generally. Beliefs provide a psychological backdrop for learning, while goals orient actions more directly. Beliefs may be held at a more unconscious level than goals, as students may not think much about what they believe about mathematics, and as a result are not very aware of their own beliefs (Lester, 2002). They may not be cognizant that others could believe differently. Regardless of their level of awareness, students hold multiple beliefs and goals at varying levels of commitment, acting on different goals depending on the situation. I will first discuss research on students' beliefs about learning mathematics, and then I will discuss research on students' goals for learning in mathematics classrooms.

Students' Beliefs about Learning Mathematics

Beliefs have been studied extensively in mathematics education; a literature review discussing the range of research on students' beliefs about mathematics was

published almost 20 years ago (Underhill, 1988), while a book discussing the role of beliefs in mathematics education was published recently (Leder, Pehkonen, & Torner, 2002). Students' beliefs have been studied in mathematics education for two primary purposes: (a) to understand how student thinking moderates the process of learning mathematics and (b) to assess teaching interventions, such as Standards-based curricula and teaching practices.

Beliefs moderate the process of learning mathematics. The study of students' beliefs as an important phenomenon in the learning process began as an effort to move away from process-product orientations in educational research. A large body of research accumulated in the 1970's and early 1980's, the results of which demonstrated that certain teaching practices led to improved student outcomes, such as achievement or time on task (e.g., Clark et al., 1979). Process-product research was complexified as researchers began to study how students' thinking played a role in the learning process (e.g., Peterson & Swing, 1982; Winne & Marx, 1982).

Researchers have examined mathematics students' beliefs in terms of how they moderate the process of learning and motivation to learn. For example, with respect to motivation, Stodolsky, Salk, and Glaessner (1991) suggested students' beliefs about the nature of the school subject are related to their learning goals. Cobb (1985) demonstrated this relationship through two case studies of first grade students, illustrating that a student with an ego-involvement learning goal, such as a focus on performance, expressed that mathematics problems and procedures were unrelated to one another. Another student with a task-involvement learning goal, or a persistence in learning the material, expressed

beliefs that mathematics procedures were related. Beliefs about the nature of the school subject co-occurred with particular learning goals for these students.

Based on the assumption that students' operational definitions of the school subject would be a hidden factor in students' confidence and perceived usefulness toward mathematics, Kouba and McDonald (1991) studied over 1,200 students across elementary grades (K-6) to determine whether they saw different situations as involving mathematics (or not). They also conducted classroom interviews to identify the characteristics that distinguished the different ways in which students defined mathematics. Their results indicated that K-6 students believed mathematics is a narrow domain, primarily about numbers and operations, an exclusive domain, such that it only occurs in school and is isolated from other school subjects, and is an upwardly shifting domain, such that once the students consider the situation to be automatic and easily known, the students no longer perceive the situation as mathematical. One concern these researchers expressed was that if students believed that mathematics is always difficult, they may discount a portion of their understandings about mathematics, and then their confidence would decrease.

Relations have been found between students' beliefs and their problem solving behaviors. Schoenfeld (1985, 1988, 1989) found that high school students who held a belief they should be able to solve mathematics problems in 12 minutes or less also exhibited a lack of persistence when working on challenging problems. This belief has been referred to as "quick learning." It has also been found to be a strong predictor of high school GPA; the less students believed in quick learning, the higher GPA they earned (Schommer, Calvert, Gariglietti, & Bajaj, 1997). With regards to undergraduates'

problem solving behaviors, Pajares and Miller (1994) found that self-efficacy was more predictive of success in problem solving than math self-concept, perceived usefulness of mathematics, prior experience with mathematics, or gender. In another study of undergraduates, Lerch (2004) found that lack of confidence and previous lack of success in mathematics led to a lack of persistence with solving a problem. If they did persist, students continued with unsuccessful strategies when working on unfamiliar problems, suggesting a dependency upon particular solution strategies for specific problem types. Thus, beliefs have been shown to provide a psychological foundation for problem solving that leads to persistence, or lack thereof, and higher performance in school.

Most of the studies of whether and how students' thinking during instruction, including their beliefs, influence mathematics learning have been conducted prior to the current U. S. mathematics education reform movement. A recent exception is the work of the QUASAR project [Quantitative Understanding: Amplifying Student Achievement and Reasoning] (Henningsen & Stein, 1997; Stein & Lane, 1996). Although QUASAR did not explicitly focus on the analysis of students' beliefs, their research acknowledges that students' thinking can mediate their experiences with mathematical tasks. They proposed that a mathematical task exists at three levels: as represented in the curricular materials, as set up by the teacher in the classroom, and as implemented by students in the classroom. Further, students' dispositions toward learning mathematics (which include their beliefs) are conjectured in their framework to be a factor that influences students' implementation of a task. Additional work is needed that explores whether and how students' beliefs influence students' learning in reform-oriented mathematics classrooms, particularly with respect to specific activities such as participation in whole-

class discussion, in order to build upon the research previously conducted on individual students' problem solving.

Different kinds of beliefs can affect the learning of mathematics. Beliefs about the nature of mathematics may provide a psychological foundation for holding additional beliefs, such as confidence or interconnectedness, breadth, isolation, and difficulty of the school subject, in clusters with learning goals. Beliefs about the process of learning mathematics, such as whether mathematics can be learned quickly, and beliefs about the self as a learner of mathematics may affect students' performance and problem solving behaviors. Epistemological beliefs, such as those about the nature of knowledge and the process of learning, may moderate students' learning processes.

Assessing classroom interventions: Developing productive beliefs. Since the mathematics reform movement in the US strives to promote certain productive beliefs among students, mathematics education researchers have also studied students' beliefs as an outcome for evaluating the effectiveness of a teaching intervention. If some students' beliefs are considered to be problematic, e.g., that mathematics problems can always be solved quickly or that the school subject is narrow and made up of isolated facts and procedures, then changing the culture of mathematics classrooms in schools may promote more productive beliefs. Researchers have analyzed whether implementing Standards-based teaching or curricula has resulted in changes in students' beliefs. While some of this research has addressed the development of pre-service teachers' beliefs (Ambrose, Clement, Philipp, & Chauvot, 2004; Cooney, Shealy, & Arvold, 1998), this section will address research on students' beliefs in the settings of interventions related to mathematics reform. Belief change may be considered among one of the outcomes of

teaching interventions related to mathematics reform, across age groups: elementary school (Kloosterman, Raymond, & Emenaker, 1996; Nicholls, Cobb, Wood, Yackel, & Patashinck, 1990), high school (Boaler, 1997; 1998), and post-secondary school (Hofer, 1999).

Nicholls et al. (1990) conducted a study of six second grade classrooms, one taught according to constructivist views of mathematics and five that were taught more traditionally, to examine the dimensions of second grade students' theories about success in school mathematics. Students in the non-traditional classroom expressed a higher task-orientation, where the goal is to gain understanding rather than be superior to others (ego-orientation), in comparison to the students in the traditional classrooms. Also, students in the non-traditional classroom were more likely to express the belief that success depends on effort, attempts to understand, and cooperation with peers, in comparison to students in the more traditional classrooms.

Kloosterman et al. (1996) studied elementary students' beliefs for three years, starting when they were in first through fourth grades, and found that beliefs about learning and doing mathematics were relatively stable over time. These students were at a school that taught mathematics through problem solving. Results of this study showed students' beliefs were strongly tied to their classroom environments. Students approved of cooperative learning, for example, only when their teachers were using it. These students also expressed a narrow belief about the usefulness of mathematics and tended to like mathematics more as it became more difficult, appreciating the challenge of mathematics.

Boaler (1997; 1998) compared high school students' experiences over three years from two schools with similar demographic profiles, but with different curricular settings: one school with open-ended mathematics activities and the other with a more traditional textbook approach. She found that students in the open-ended setting were more likely to express enjoyment in doing mathematics and to appreciate thinking for themselves over memorizing. Students in the traditional setting were more passive about their learning, were more likely to have a set view of mathematics as a disconnected collection of exercises, rules, and equations, and viewed mathematics as relating less to the world than other school subjects.

Hofer (1999) also contrasted students in two different curricular settings, but she compared college undergraduates at one university who experienced different forms of Calculus: one that emphasized active and collaborative learning both in and out of class and primarily focused on word problems, and the other a more traditional approach of lecture and demonstration. The students in the non-traditional calculus course were found to have more sophisticated beliefs about mathematics; they were particularly less likely to believe that doing mathematics involves getting a right answer quickly. Achievement was positively correlated with sophistication in mathematical beliefs (as in Schommer et al. (1997), and students with sophisticated beliefs about mathematics were more likely to have mastery orientations toward learning mathematics.

Some studies reporting students' reactions to teaching interventions have assessed their perspectives more broadly than in terms of their beliefs alone. Bay, Beem, Reys, Papick, and Barnes (1999) studied sixth and seventh grade students' reactions to

Standards-based mathematics curricula.³ In letters students wrote about their experiences with these curricular series, students reported positive experiences with hands-on activities, group work, and the new content, an increase in application problems, and improvement in attitudes toward mathematics. However, the diversity across classrooms, in terms of students' perspectives on the difficulty of the curricula, the use of technology, and their assessments of their own progress, suggests implementation varied, and success of the curricula with students is dependent upon the teacher. Reactions from older students were reported in their own voices (Holt et al., 2001), as high school students commented on their experiences with the *Interactive Mathematics Project* [IMP] – one of the NSF-funded Standards-based high school curricula. Five female students said that they appreciated IMP's emphasis on problem solving, group work, and communication skills, and expressed a need for more practice doing algebraic manipulation. They generally believed the program did prepare them for college. In both cases, students were able to identify salient features of new mathematics curricula and mentioned ways in which they benefited from their experiences learning from it.

Since these six studies focus on assessing curricular or teaching interventions designed to improve students' learning of mathematics, the results emphasized the commonalities in students' experience, whether the intervention was the classroom or school. In each case, the interventions had some impact on students' beliefs that was assumed to be positive. It appears that students develop beliefs somewhat consistent with their teachers' implementation of a curriculum or other reform-oriented teaching practice. Additionally, these studies all took place in settings where non-traditional mathematics teaching and curricula were novel, as these approaches are relatively new. One question

³ Connected Mathematics Project and Sixth Through Eighth Mathematics, now titled MATH Themes.

they raise is what shape students' experiences and beliefs will take in non-traditional mathematics settings after they have been in them for extended periods of time, as their perspectives may shift after novelty wears off.

Development of students' beliefs about learning mathematics. Researchers who study students' beliefs about mathematics have been curious about the factors that influence belief development. The goals of the mathematics reform are based on the assumption that teaching and curricular interventions can make a difference in students' beliefs and learning. Beginning with the scholarship of Doyle (1983, 1988) and moving into the work of the QUASAR project (Henningsen & Stein, 1997; Silver & Stein, 1996; Stein, Grover, & Henningsen, 1996; Stein & Lane, 1996), one premise is as follows:

...the work students do, which is defined in large measure by the tasks teachers assign, determines how they think about a curriculum domain and come to understand its meaning. (Doyle, 1988, p. 167)

Thus, continued exposure to certain types of mathematics tasks, perhaps those at a higher cognitive demand, those that afford flexible solutions, or those that are more related to the world outside of school, may promote productive beliefs among mathematics students. Also, teachers' implementation of mathematics tasks may influence students' beliefs. However, there are a number of issues related to individual diversity among students that also support the development of beliefs, in addition to classroom influences, such as socio-economic class, race, the culture of students' home environments, and gender differences.

Franke and Carey (1997) found that demographics (particularly race and socio-economic status [SES]) may moderate students' belief development, as they studied

elementary school students' perspectives on what it means to learn mathematics with a curriculum that has a problem solving emphasis (Cognitively Guided Instruction [CGI] (Carpenter, Fennema, & Franke, 1996)). Contrasting students who were predominantly White and middle class with students who were predominantly African American and low SES, they found that the White, middle class students were more likely to say that doing mathematics involved spending a long time on a problem and that mathematical success was determined by the strategy one used. In contrast, African American, lower SES students said that mathematical success was determined through obtaining correct answers and by speed and accuracy. These findings imply a need for further study on the development of students' beliefs in Standards-based settings, as curricula alone does not appear to predict the development of particular beliefs.

Ridlon's (2001) case study of a seventh grade student revealed the influence of family on students' beliefs and resistance to participate in learning mathematics from a problem centered mathematics program. The case focused on a student from a rural farming community. His parents were third generation soybean farmers. His mother was involved in school, calling the teacher four times in a nine week period and writing two notes, on her own initiative. Mark did not volunteer in class initially, and became more withdrawn throughout the nine-week period. He refused to write in a math journal, and resisted working in groups. The researcher determined that "the idea of speaking your mind and multiple correct answers was confusing for him" (p. 59), and that Mark's "well-established habit of bowing to authority and accepted procedures without making sense of them" was strengthened by his southern, rural upbringing. In this case, the curriculum

and teaching style were unsettling to the students' beliefs, which the researcher attributed to his cultural background.

Lubienski (2000a; 2000b) taught a socio-economically diverse group of 18 seventh grade students in one class using the *Connected Mathematics Project* curricular series, and determined that their views about whole-class discussions differed based upon their socio-economic backgrounds. Higher-SES students considered the discussions to be a helpful forum for exchanging ideas and were conceptually oriented during discussions. Lower-SES students preferred more teacher direction and were more often focused on giving correct answers to specific problems. Her research suggests that students' socio-economic status moderates students' beliefs about learning in discussion-oriented mathematics classrooms.

There is conflicting evidence on gender differences in students' beliefs about learning mathematics. Studies from the 1970's and 1980's (e.g., Fennema & Peterson, 1985; Fennema & Sherman, 1976) indicate differences between males and females in terms of confidence in mathematics and perceived usefulness. However, gender differences in beliefs may be disappearing, as more recent studies investigating gender differences in beliefs do not show gender differences. For example, Vanayan, White, Yuen, and Teper (1997) only found gender differences in terms of more boys saying that they were good at mathematics, but both boys and girls said that they liked mathematics, and no other gender differences in beliefs were found, such as relevance of mathematics. A study of a girls-only middle school mathematics class revealed that the students' experience enhanced their ability to learn math and their view of themselves as

mathematicians (Streitmatter, 1997), suggesting that girls' beliefs may improve in single gender classrooms.

This growing body of research has begun to explore the ways in which students' demographic backgrounds may shape the development of students' beliefs about learning mathematics. But it is also possible that the age and developmental challenges of students may influence the character of their beliefs. The age of the student can affect what they are cognitively capable of understanding, as well as the students' heightened sense of how their classmates perceive them (Elkind, 1978). Edwards and Ruthven's (2003) study is an example of how the age of the student can influence their beliefs. They replicated an approach taken by Kouba and McDonald (1991), asking students whether certain everyday situations involved mathematics, with junior high and high school students rather than elementary school students. These older students' beliefs about the domain of mathematics were broader than the elementary students from the previous study. The developmental trajectory of the student is another factor that contributes to the diversity of students' beliefs.

Categories of mathematics-related beliefs. A wide range of beliefs related to learning mathematics can be studied. DeCorte, Op'tEynde, and Verschaffel (2002) and McLeod (1992) proposed two somewhat similar, but slightly different, frameworks for studying students' beliefs in mathematics classrooms. DeCorte et al. (2002) distinguished between three categories of mathematics-related beliefs: beliefs about mathematics education, beliefs about the self in relation to mathematics, and beliefs about the social context. Beliefs about mathematics education include beliefs about the nature of the school subject, about the process of learning and problem solving, and beliefs about

mathematics teaching. Beliefs about the self in relation to mathematics learning include motivational beliefs, such as self-efficacy beliefs, control beliefs, task value beliefs, and goal orientation beliefs. Beliefs about the social context include students' beliefs about their classroom norms, their expectations for their teacher's role, their role, and their classmates' roles in their current classroom, and beliefs about aspects of their classroom culture that are specific to mathematical activity, such as what counts as a different solution. Alternatively, McLeod (1992) proposed categorizing beliefs in terms of those that are about the nature of mathematics, about the self, about mathematics teaching, and about the social context, separating those that are about the nature of mathematics from those about the process of teaching.

Epistemological beliefs are assumptions about the nature of knowledge and the process of knowing. Hofer and Pintrich's (1997) review of research of epistemological beliefs research, synthesized epistemological beliefs into four categories, two that refer to the nature of knowledge and two that address the nature of knowing. Beliefs referring to the nature of knowledge include *certainty of knowledge* (whether knowledge is fixed or fluid) and *simplicity of knowledge* (whether knowledge is an accumulation of isolated facts or highly interrelated concepts). Beliefs addressing the nature of knowing include *source of knowledge* (whether knowledge resides in an external authority or within the student's ability to construct knowledge) and *justification for knowing* (how individuals evaluate knowledge claims). These beliefs could be stated in domain-specific terms, using *mathematical* as a modifier for knowledge, such as beliefs of the simplicity of *mathematical* knowledge – whether mathematics is a coherent system or a collection of isolated pieces.

Whether beliefs are domain-specific (localized to the school subject of mathematics) or general (pertaining to a range of subject matters) is currently an active and open question. In support of the latter view, Schommer and Walker (1995) found that students held a range of conceptions at consistently sophisticated levels across the domains of both mathematics and social studies, including conceptions of knowledge as less certain or simple, learning as a not quick process, and one's ability not being fixed. However, Buehl, Alexander, and Murphy (2002) present challenges to this work. They found evidence supporting the domain-specificity of certain beliefs (e.g., knowledge utility or value – mathematics was more related to other areas than history) when survey items were worded in reference to disciplines.

Another way of describing epistemological beliefs about the process of learning would be in terms of women's ways of knowing (Belenky, Clinchy, Goldberger, & Tarule, 1986). Three of these ways of knowing may be particularly relevant to students who participate in whole-class discussions: received, procedural, and constructed. *Received knowers* conceive of themselves as capable of receiving, even reproducing, knowledge from the all-knowing external authorities but not capable of creating knowledge on their own. *Procedural knowers* are invested in learning and applying objective procedures for obtaining and communicating knowledge. *Constructed knowers* view all knowledge as contextual, experience themselves as creators of knowledge, and value both subjective and objective strategies for knowing. Mathematics students may be less likely to express evidence of subjective knowing, as the nature of mathematics is rarely perceived as entirely subjective. Additionally, if they participate in whole-class discussion, they are not exhibiting silence, or experiencing themselves as mindless and

voiceless and subject to the whims of external authority. While these may be considered *women's* ways of knowing, they may have characteristics that apply to both young men and young women. Epistemological beliefs have typically been studied in college students and adults, but if younger students have moved beyond relying on receiving knowledge from an authority, this may be significant enough to document. Researchers have documented students' epistemological beliefs as early as fourth grade (Johnston, Woodside-Jiron, & Day, 2001).

Beliefs about the self as a learner of mathematics have already been explored in great detail. One example is the breadth of research on self-efficacy. Self-efficacy in mathematics is whether or not students believe they can control or regulate their own learning and master academic tasks in mathematics (Bandura, 1993, 1997), and these self-beliefs of efficacy play a role in students' motivation. Students with a strong sense of self-efficacy with respect to a particular domain or task may be more likely to attribute their success to their own efforts. Additionally, students may be more likely to put effort into activities that they believe they can succeed in. Efficacy beliefs may be unrealistic, over-estimates or under-estimates of one's capabilities. They have been found to determine goal setting, activity choice, willingness to expend effort, and persistence (Bandura, 1997).

Even though self-efficacy in mathematics is challenging to assess (Pajares & Miller, 1995), self-efficacy is considered to moderate students' mathematics problem solving more effectively than other variables, such as self-concept, usefulness of mathematics, or gender (Pajares & Miller, 1994). This relationship may be developmental, as Kloosterman et al. (1996) found a lack of relation between self-

confidence and achievement for first grade students, but a strong relation between the variables for third grade students. Additionally, Pietsch, Walker, and Chapman (2003) found self-efficacy in mathematics to be more highly related to mathematics performance than self-concept among secondary school students. As self-efficacy has been found to be a strong predictor of engagement in problem solving and achievement in mathematics, it may also relate to students' participation in whole-class discussion during mathematics class.

Overall, while a wealth of research on students' beliefs has been taken up in mathematics education, new questions still remain. While research has suggested that students' epistemological beliefs play a role in mathematics learning, such as relations to performance goals and problem solving behaviors, it is not clear how epistemological beliefs would be related to students' experiences of learning in Standards-based mathematics classrooms. While there is some evidence that students develop more sophisticated epistemological beliefs in Standards-based settings, how do these beliefs then influence students' future experiences with learning mathematics?

Students' Goals in Mathematics Classrooms

Recall that goals, in contrast to beliefs, are what students want to achieve through their actions, rather than broader assumptions students use for interpreting a situation (beliefs). Students may be more cognizant of, and perhaps more able to articulate, their goals than their beliefs. The study of whether and how goals influence students' behaviors and achievement in school has been taken up more by researchers in the field of educational psychology than those in mathematics education. Many educational psychologists have either situated their studies in mathematics classrooms, or suggested

that their studies of adolescent motivation could be relevant to mathematics learning. They have primarily studied students' goals in relation to achievement. As with the research on beliefs, these researchers have assumed that some goals are more productive than others and that classrooms can be structured in order to help students develop more productive goals. Research on students' goals in school has been dominated by a focus on learning goals, but recently has begun to expand the focus onto students' social goals.

Learning goals. Two primary distinctions have been made between students' goals for learning academic content: the difference between (a) seeking positive evaluations of one's competence and avoiding negative evaluations of one's competence or (b) a focus on mastering tasks and increasing one's understanding of the content. These goals have fallen under a variety of labels, including ego-involved and task-involved goals (Nicholls et al., 1990), performance and learning goals (Dweck, 1986), and performance and mastery goals (Ames, 1992). For the sake of clarity, I will refer to the goals related to evaluation of performance as *performance* goals and the goals related to understanding content as *mastery* goals.

An additional distinction has been made between two types of performance goals: *performance-approach* and *performance-avoidance* goals. Students who hold performance-approach goals want to be recognized positively for their competence, and students who hold performance-avoidance goals try to avoid looking incompetent (Midgley, Kaplan, & Middleton, 2001).

The development of learning goals has been studied among middle school students (e.g., Middleton, Kaplan, & Midgley, 2003). Mastery and performance goals appear to be fairly stable over time during adolescence. However, students who expressed

high self-efficacy and performance-approach early in middle school shifted toward performance avoidance goals later in middle school. These students may have a need to protect their self-efficacy, and if it is threatened, they may avoid engaging in increased competition or unfamiliar material.

Meyer and Turner (1997) found that fifth and sixth grade students in problem solving environments fell primarily along two dimensions: challenge-seekers and challenge-avoiders. Challenge-seekers expressed mastery goals, higher self-efficacy than challenge-avoiders, and a tolerance for failure, while challenge-avoiders expressed a high negative affect after failure, performance goals, lower self-efficacy, and used surface-level strategies when solving mathematics problems. The researchers suggested following up these studies of students' motivations in mathematics classrooms with additional research in order to examine whether the patterns hold in other settings, as their studies focused on a small number of classrooms.

In another study, Summers, Schallert, and Ritter (2003) determined that middle school mathematics students expressing a low level of mastery goals were more influenced by comparisons to close friends than to other students in the class, suggesting issues of relatedness are integral to students who do not have high mastery goals.

School-related goals: Beyond learning goals. Adolescents may also strive to attain a range of school-related goals, not only learning goals. These include task-related goals, cognitive goals, or social relationship goals (Wentzel, 1999). Task related goals are focused on learning content. Cognitive goals are related to seeking challenges. Social relationship goals are pursued in order to feel connected to others. Sharing of mathematical strategies in discussion could address task-related goals of mastering

subject matter or meeting a standard of achievement. Coming up with an alternative solution to share may address cognitive goals of engaging in creative thinking or satisfying an intellectual challenge. Sharing one's perspective during mathematics class may also support social relationship goals of gaining approval from others, establishing personal relationships with teachers or peers, or cooperating with classmates. Desiring the opportunity to succeed in the task of discussing mathematics problem strategies may be related to more than one school related goal, and goals other than mastery or performance goals.

Additionally, students may orient their classroom behavior to meet personal needs. In self-determination theory (Deci & Ryan, 1985), these include the need for a sense of relatedness to others, for autonomy, and for competence. Of course, pursuit of these needs is only possible a range of suitable classroom environments. Discussing strategies during mathematics class can foster a sense of relatedness, as students may offer their thinking in support of the thinking of another student in order to help each other. The opportunity to share strategies can foster a sense of autonomy, as students are encouraged to share their own thinking in these mathematics classroom discussions. Mathematics classroom discussions are also an opportunity for students to demonstrate competence, as students can show what they know through these discussions. Researchers have begun to study students' autonomy in learning mathematics (e.g., Kamii, 1985; Yackel & Cobb, 1996), and their self-efficacy and confidence (e.g., Pajares & Miller, 1994; Pietsch et al., 2003). However, mathematics education research could do more to take up issues of relatedness, particularly in light of NCTM's recommendations

for increased interactions in classroom discourse, since increase in discussion may lead to increased opportunities for public performances and evaluations.

Social goals. Although it may make intuitive sense that students' social goals and sense of relatedness are significant elements of students' experiences in school settings, researchers have only recently begun to address their role in the learning process. For example, in students from 3rd – 6th grade, their sense of relatedness to parents, teachers, and peers was found to contribute to classroom engagement (Furrer & Skinner, 2003). This suggests that the quality of relationships experienced by students in the classroom has implications for their academic success.

Additionally, students' goals are often assessed via surveys, and social goals have not usually been included on survey instruments. When studying middle school students' motivations inductively through listening to students' talk in interviews, Dowson and McNerney (2003) discuss five social goals upon which students operate in social contexts: social affiliation, social approval, social responsibility, social status, and social concern. The authors suggest students' motivation be conceptualized as a process of managing multiple goals, both academic and social goals, as has Wentzel (1999). Listening to how students talk about their motivations may reveal a broader range of goals beyond the traditionally studied learning goals, and students may discuss multiple goals specific to the activity they are participating in, such as whole-class discussions about mathematics.

Patterns Among Beliefs, Goals, and Academic Risk

Doyle's (1983) review of the role of academic work on students' learning states that since academic tasks are embedded in an evaluation system, students work on these

tasks under conditions of ambiguity and risk. It is possible that risk is heightened not only when the task is designed to assess understanding and is less opinion oriented (Doyle, 1983), but also when it takes place in a public setting. Whether students experience a classroom to be a risky environment may be related to their beliefs and goals as well as their opportunities to participate during class.

If students perceive a task to have a high level of risk, they may act in such a way that reduces the risk of the task. Risk-reducing actions may include as performing only the minimum requested problems, restricting the amount of input given to the teacher to minimize the risk of exposing errors or in order to elicit assistance, getting a teacher or another student to answer on their behalf (Doyle, 1983). Perception of risk involved in publicly discussing a task may be related to whether and how students choose to participate in whole-class discussions.

This sense of risk can be considered in relation to students' beliefs and behaviors in school settings. For example, students' perceptions of risk may be related to their experiences with teachers in the classroom, as Ryan, Gheen, and Midgley (1998) found that sixth grade mathematics students with lower self-efficacy who may otherwise avoid seeking help were more likely to do so in a classroom with a teacher who attended to students' socio-emotional needs. This implies that teachers who attend to socio-emotional aspects of the classroom environment may support students' learning.

In order to assess students' experiences in discussion, it seems important to also consider students' social goals, in addition to their learning goals. Students' learning goals seem particularly important given both the activity of discussion and the developmental time frame of adolescence. During adolescence, students may have a

heightened sense of how others perceive them, or a sense of an imaginary audience (Elkind, 1978). As a participant in dialogue with others, people are naturally attuned to issues of presentation of self.

Let us turn now from the others to the point of view of the individual who presents himself before them. He may wish them to think highly of him, or to think that he thinks highly of them, or to perceive how in fact he feels toward them, or to obtain no clear-cut impression; he may wish to ensure harmony so that the interaction can be sustained, or to defraud, get rid of, confuse, mislead, antagonize, or insult them (Goffman, 1959, p. 3).

Individuals engaging in discussion with others, then, are not only thinking about the content they are discussing, but also about how they are viewed by the other participants in the dialogue and their relationships with them. Considering students' learning goals alone does not effectively capture the range of possibilities of students' social goals during whole-class discussion about mathematics during middle school, such as a sense of harmony among the group.

But it can be challenging to hypothesize which patterns of student involvement may relate to students' beliefs and goals. This is partially because what may seem like a significant connection between beliefs, goals, and involvement to a researcher may not be a significant connection to the student. For example, recent research on 11-13 year-olds' prosocial behaviors indicated that students perceive a wider range of behaviors than are normally studied to be part of their prosocial activity, such as standing up for others, encouraging others, helping others develop skills, including others who are left out, or being humorous (Bergin, Talley, & Hamer, 2003). Traditional research on prosocial

development in youth has focused on behaviors such as sharing, helping, volunteering. Since researchers may not make the same connections between beliefs, goals, and involvement that students make, including students' perspectives on their involvement in classrooms may allow for a more thorough description of these potential relations than would be possible through observation alone.

Discourse in Mathematics Classrooms

As I mentioned in Chapter One, the National Council of Teachers of Mathematics has made recommendations for mathematics teaching in order to improve student learning. The image of mathematics classrooms promoted by NCTM looks quite different from asking students to imitate and mimic procedures as presented by the teacher. In an often quoted passage, the authors of the *Professional Standards for Teaching Mathematics* state:

We need to shift toward classrooms as mathematical communities – away from classrooms as simple collections of individuals; toward logic and mathematical evidence as verification – away from the teacher as the sole authority for right answers; toward mathematical reasoning – away from merely memorizing procedures; toward conjecturing, inventing, and problem solving – away from an emphasis on mechanistic answer-finding; toward connecting mathematics, its ideas and its applications – away from treating mathematics as a body of isolated concepts and procedures... (NCTM, 1991, p. 3)

NCTM's image sees the classroom as a community of learners who reason and communicate about mathematics together to verify their ideas. Recommendations for

creating such mathematics classrooms include orchestrating discourse between the teacher and students.

The teacher of mathematics should promote classroom discourse in which students: listen to, respond to, and question the teacher and one another; use a variety of tools to reason, make connections, solve problems, and communicate; initiate problems and question; make conjectures and present solutions; explore examples and counterexamples to investigate a conjecture; try to convince themselves and one another of the validity of particular representations, solutions, conjectures, and answers; rely on mathematical evidence and argument to determine validity. (NCTM, 1991, p. 45)

So, in these classrooms imagined by the NCTM, students are expected to talk about mathematics in particular ways, including initiating problems, presenting solutions, and questioning the thinking of others and trying to verify the ideas of their classmates and teacher with evidence and reasoning. Classrooms fostering these sorts of interactions are considered to be an advance over classrooms that emphasize imitating procedures, because students may develop autonomous thinking such that they are able to communicate, reason, and justify their ideas about mathematics to themselves and others.

Classroom discussion is considered to be a valuable practice in many subject matters, not mathematics alone. John Bruer (as cited in Cazden, 2001) suggests five hypotheses as to why promoting classroom discourse would be an effective teaching strategy across the curriculum: (a) thinking is made public, and skilled thinkers can model their thinking to others; (b) the task of thinking can be distributed among the classroom community; (c) dialogue involves both language comprehension and language

production, and production is considered to be more demanding, so participating in discussions may result in greater depth of processing; (d) classroom discourse sends the message that thinking and intelligence are valued; (e) discourse does not simply make thought visible, but thinking is internalized discourse. These hypotheses were developed from information processing and social psychology.

If discourse is thought about in terms of students' opportunities to try on new ideas about mathematics, talking about mathematics is an opportunity to learn through the process of internalization, as described by Vygotsky (1978). Appropriation, a term introduced by Leontev (1981), is also sometimes used to describe this process of the transformation of knowledge, and at times this term is preferred because it acknowledges the two directions – students can appropriate the ideas of their teachers and teachers can appropriate the ideas of their students. However, discussing learning in terms of appropriation or internalization does not effectively explain why or how individuals construct knowledge differently through discourse. Studies of classroom discourse could benefit from addressing the cognitive and socio-emotional diversity among learners who participate in classroom discourse.

Research on classroom discourse in mathematics classrooms has begun to acknowledge and describe the role that individual students have in shaping mutually constituted classroom norms (e.g., Stephan, Cobb, & Gravemeijer, 2003), but it is still common among researchers who study mathematics classroom discourse to focus on describing classroom discourse in normative terms or the role of the teacher in orchestrating effective classroom discourse.

Describing Mathematics Classroom Discourse

As teachers and researchers attempt to foster classroom productive and powerful discourse about mathematics, careful descriptions have emerged in the literature. Lampert (1990), Yackel and Cobb (1996), and Kazemi and Stipek (2001) each describe characteristics of discourse in mathematics classrooms designed to alter the roles of the teacher and the student and promote autonomous thinking about mathematics concepts rather than procedures. Lampert (1990) illustrated how she and her fifth grade students talked about exponents, including the need to talk about talking as well as mathematics content, the challenge of negotiating the meaning of exponents, and determining that finding the answer is not the signal to stop thinking. Yackel and Cobb (1996) described classroom norms that influenced first grade students' learning of place value.⁴ These norms were termed "sociomathematical," since they were at the intersection between expectations for social interactions and appropriate ways of doing mathematics.

Sociomathematical norms described by Yackel and Cobb (1996) include using mathematics to justify explanations rather than social status and authority, conceptual rather than procedural explanations, and reflecting on the explanations of students. Kazemi and Stipek (2001) extended this work by describing sociomathematical norms in fourth and fifth grade classrooms where teachers pressed for conceptual learning by: (a) moving beyond procedural descriptions to mathematical arguments, (b) seeking to understand relations among multiple strategies, (c) discussing errors in order to reconceptualize a problem, explore contradictions in solutions, and pursue alternative strategies; and (d) collaborative work also involves individual accountability and seeking consensus through mathematical argumentation. In these studies, the researchers agree on

⁴ "Norm" refers to regular patterns of interaction during a discussion.

the importance of talking about talking with students, such as continuing the conversation once the answer has been found and talking about more than the procedures for solving the problem. Additionally, discussing students' strategies themselves, and looking for relations between them, is considered fruitful for developing mathematical understandings.

Describing classroom discourse, however, has been a part of the practice of educational researchers outside of mathematics and before the NCTM reforms. There has been an interest in understanding the nature of talk in classrooms to helping students become a part of the culture of their classroom, as practices of talking in school may be different than those at home (Shultz, Florio, & Erickson, 1982), and educators may be concerned with helping students adopt the discourse of the discipline (Lemke, 1990). One challenge of studying talk in classrooms has consistently been finding the balance between studying the structure of the talk and accounting for the subject matter in the discourse. The tension lies between an analysis of the form and function and the content in the talk.

Form and function. Talk in classrooms may be examined in terms of its form and function. One common way of describing the form of classroom talk is in terms of whether or not the interactions follow a traditional form of teacher-initiated questions, student responses, and teacher evaluations (I-R-E) (Mehan, 1979). Talk that follows this form is considered to function as a message of the teacher as authority, in terms of who controls what is discussed and who determines the accuracy and validity of students' responses. While I will not describe an exhaustive review of the ways talk in classrooms can be studied in terms of form and function, I will present some of the forms and

functions of talk that have been studied with adolescents, or in mathematics classrooms, or both.

The form of talk can be studied in terms of the settings in which talk takes place, who is speaking to whom, and the ways talk is structured in particular interactions or utterances. Classroom discourse has been studied in different settings, such as public talk in front of the class or private talk between the teacher and the student (e.g., Hart, 1989) as well as either large group (e.g., Hamm & Perry, 2002; Nathan & Knuth, 2003) or small group discussions (e.g., Nussbaum, 2002). The form of talk is sometimes described in terms of whether the interactions are monologic, between a teacher and one student, or dialogic, involving multiple students such that they address one another directly (Nystrand, 1997). Specific utterances within interactions have also been examined for form, such as whether and how teachers revoice, or repeat, students' utterances in order to call attention to them or appropriate them in discussion (O'Connor, 1998). The form, or structure, of talk can be analyzed at many levels down to the level of word choice.

Analyzing the function of talk is more interpretive than analyzing the form of talk, and analyzing the form of talk is often the way researchers operationalize analysis of the function of talk. One example was mentioned above, in the case of the I-R-E form indicating the function of communicating teacher's authority. Monologic and dialogic forms of talk have been utilized by researchers studying the function of talk in mathematics classrooms in terms of whether autonomous thinking is promoted among students. Hamm and Perry (2002) analyzed whether the function of talk in six first grade mathematics classrooms was at a high level of "mathematical discourse" – talk not entirely scripted by the teacher where ideas were exchanged – by looking at the degree to

which the dialogue was reciprocal. Nathan and Knuth (2003) examined one sixth grade teacher's classroom for monologic and dialogic talk over two years to document changes over time. In both studies, teachers were challenged to move beyond monologic talk.

Content. As an alternative to describing the form and function of classroom talk, researchers have also described the content of the talk in classrooms, which allows researchers to capture more of the subject matter in the discourse. Hamm and Perry (2002) also analyzed the discourse in the six first grade classrooms in terms of the degree of mathematical analysis, or the level at which students engaged in higher order thinking about mathematics, such as searching for mathematical patterns, making mathematical conjectures, evaluating, arguing, and inventing original procedures. Lubienski (2000a; 2000b) studied her own teaching of a seventh grade mathematics classroom using the *Connected Mathematics Project* textbook series, and analyzed students' involvement in whole-class discussions in terms of the content of their talk, including whether the students discussed answers to problems, in terms of how to get an answer, with a focus on answer over method, or whether the student talked about patterns, such as pointing out the existence of a pattern or explaining a pattern. Describing the content of classroom talk captures subject matter, but sometimes at the expense of describing how the opportunities to talk about the content may arise in the interactions, which may be afforded by an analysis of form and function.

Whether the analysis is on the form and function of classroom discourse or the content of the talk in classroom discourse, there are challenges with studying classroom discourse in mathematics classrooms. Analyzing and describing the form and function of talk may compete with a focus on accounting for the subject matter in the talk.

Additionally, since teachers play a prominent role in classroom discussions, even those who attempt to abide by the recommendations of the mathematics reform movement, analyses of classroom discourse often illuminate more about the teacher's talk than students'. Examining the nature of talk in classrooms does not necessarily provide insight for why and how some students are more involved than others, as it does not account for the agency of individual students.

Best Practices: Mathematics Classroom Discourse

Research on mathematics classroom discourse has also moved beyond to describing the nature of discussions to assess the effectiveness of discussions in mathematics classrooms. Effectiveness has been operationalized in terms of a range of outcomes: (a) the quality of the mathematical talk, such as whether the talk maintained cognitive demand or mathematical precision, (b) whether students were highly involved, (c) the reported motivation of the students, and (d) students' achievement. If NCTM recommends implementing discussion in classrooms, the assumption that discussions are beneficial should be evaluated according to a variety of outcomes.

Quality of mathematical talk as outcome. The effectiveness of a discussion may be assessed with respect to the quality or level of mathematical talk in the discussion itself. Henningsen and Stein (1997) examined four middle school mathematics classrooms and determined factors that assisted students' engagement at high levels of cognitive demand, such as scaffolding and consistently pressing students to provide meaningful explanations or make meaningful connections, and factors that led to the decline of cognitive demand, such as the removal of challenging aspects of the problem, lack of time, or inappropriateness of the task. Removal of challenging aspects of the

problem and scaffolding or pressing students occurred as part of classroom discussion. Nathan and Knuth's (2003) study of a sixth grade teacher's attempt to include more dialogic talk in her classroom resulted in a decrease in the mathematical precision of the talk among the class. Certain features of classroom discussions may support students' attempts to talk about mathematics more than others, such as scaffolding, that may be challenging to incorporate when the teacher is initially trying to shift her role in classroom discussions.

High involvement as outcome. Discussions in mathematics classrooms have also been evaluated with respect to whether or not students are highly involved. Turner et al. (1998) examined at least four classroom discussions from each of seven fifth and sixth grade mathematics classrooms, and analyzed the self-reports of students' involvement from six target students in each classroom. She determined that three of the seven classrooms had higher involvement from the target students. Two of the high-involvement teachers demonstrated scaffolding in terms of negotiating understanding, adjusting instruction in response to students, and transferring responsibility. They also held students accountable for their own understanding provided intrinsic supports for students' motivation, such as evoking students' curiosity, providing encouragement, and advocating risk-taking. The third high involvement teacher did not focus on negotiating understanding, but did focus on transferring responsibility and intrinsic supports. The students in the high involvement classrooms expressed that their skills were suited for working at the level of challenge in their classrooms. The students in the low involvement classrooms reported their skills as exceeding the classroom challenges, and these classrooms were characterized as adhering to I-R-E sequences, emphasizing

procedures and extrinsic supports for motivation. These results from the field of educational psychology are somewhat consistent with the results from mathematics education: effective classroom discussions move away from IRE and procedural talk toward a focus on students' ideas, but Turner's work contributes that the role of affect and motivational support plays a significant role in classroom discourse when assessing students' involvement.

Students' motivation as outcome. Students' self-reports of their motivation are another outcome that may be used to assess the effectiveness of discussion. Turner and colleagues (Turner, Meyer, Midgley, & Patrick, 2003; Turner et al., 2002) examined sixth grade mathematics classrooms and found that, while instructional discourse similarly focused on students' understanding of mathematics, the classrooms differed in terms of the support for students' autonomy and intrinsic motivation. The students in the classroom with more supportive motivational discourse reported less negative affect and self-handicapping, or avoidance behaviors, than did students in the classroom with less supportive motivational discourse. Students reported using avoidance strategies less often in classrooms that they perceived as emphasizing learning, understanding, effort, and enjoyment. These findings suggest that environments that support mastery over performance combine cognitive and affective components of teaching and learning.

Turner and colleagues have made an effort to integrate an individual viewpoint with a social perspective on studying involvement, with their use of self-reports of students' motivation and involvement as outcomes. However, their perspective on students' motivation does not go beyond students' learning goals, and students' motivation may be broader than these goals.

Student achievement as outcome. Classroom discussions about mathematics may also be assessed in terms of their impact on student learning or achievement. Hiebert and Wearne (1993) assessed the relationships between teaching and learning of mathematics in six second-grade classrooms in one school. Two of these classrooms focused on constructing relationships over practicing procedures. These two classrooms had higher gains on the achievement measures of place value understanding and both routine and novel computation. The researchers determined similarities in the instructional features of these two classrooms that appear to have supported the students' learning in comparison to the four more traditional classrooms. The students in the "learning" classrooms received fewer problems, spent more time with each problem, were asked more questions asking them to describe and explain alternative strategies, and both talked more often and used longer responses. The researchers concluded that both instructional tasks and classroom discourse support students' learning, and they were not surprised that particular instructional tasks occurred together with forms of discourse.

Through examining a range of outcomes, such as student achievement, motivation, involvement, and the quality of talk about mathematics during classroom discussion, researchers have determined characteristics of effective classroom discussions, such as explaining alternative strategies and looking for relationships between them, scaffolding students' efforts to explain their thinking about mathematics, and scaffolding intrinsic motivation. Additionally, the nature of the task supports the quality of discourse. Characteristics of ineffective discussions about mathematics include removing challenging aspects of the task, not spending enough time or too much time on a task, emphasizing procedures, following an I-R-E sequence, and scaffolding extrinsic

motivation. However, these characteristics of classroom discussions emphasize what the teacher could do, or could avoid doing, but say little about whether and how students' characteristics relate to their involvement in classroom discussions about mathematics.

Student Characteristics and Involvement in Mathematics Classroom Discourse

Teachers may not be aware of the challenges students face with discussions during mathematics class. Lampert (1990) reported the range of ways her fifth grade students were challenged by learning in a discussion-oriented setting. Some students preferred to look to either the teacher or a more knowledgeable classmate as an authority. Some did not realize that using a rule is different from explaining why it works. Students kept silent for a variety of reasons, such as not having the words to express their thinking out loud, lacking the courage to share their thinking, or because they copied another student's work. Social power and deciding by majority was an effective means of justifying mathematical ideas, for some students. Finally, some students appeared to believe that if something was wrong with their reasoning, then something was wrong with them as an individual. Some of these issues decreased over the year in her classroom, but the challenges that her students experienced provide insight for why some mathematics students may hesitate to get involved in whole-class discussion, or not benefit from their involvement. Not all students experience whole-class discussions about mathematics in the same way. A range of student characteristics may contribute to how students experience and participate in these discussions, such as personality traits, dispositions, race, socio-economic status, or gender.

Students' personality traits and dispositions. To date, mathematics educators have not explicitly analyzed whether and how students' personality traits or dispositions,

including their beliefs or attitudes, shape students' involvement in whole-class discussions. However, educational psychologists have conducted studies of whether and how students' personality traits or dispositions shape students' involvement in discussion more generally (Nussbaum, 2002; Nussbaum & Bendixen, 2003).

Nussbaum (2002) examined whether extroverted students in small group discussions with other extroverts participated differently than a small group of introverted students. Four extroverted sixth grade students and four introverted sixth grade students discussed issues about urban planning. These students were sorted into these two groups based on their responses to a personality questionnaire. The extroverted students' involvement had significantly more contradictions and counterexamples and a greater tendency to use conflictual discourse during the small group discussions. In contrast, the introverted students worked with one another more collaboratively. The study was replicated with 16 undergraduate pre-service teachers taking an educational psychology course, and the results were similar: introverted students were more likely to participate in collaborative discourse while extroverted students were more likely to challenge one another.

Nussbaum and Bendixen (2003) used self-reported data (rather than observations of students' involvement in discussion) to analyze whether undergraduates' epistemological beliefs, desire for warmth in relationships, and need for cognitive challenge predicted whether they were more likely to avoid or approach arguments. Their results demonstrated that epistemological beliefs, such as certain and simple knowledge, and desire for warmth were paired with avoiding arguments, while the need for cognitive

challenge occurred with approaching arguments. Additionally, assertiveness predicted both approaching and avoiding arguments.

These studies are two of the few examining whether and how students' personality traits or dispositions relate to their involvement in discussions. However, neither of these studies took into account the subject matter involved in the discussions. Although the results of these studies suggest introverted students are less likely to challenge one another during discussion, as they prefer collaborative talk, and that epistemological beliefs, need for warmth, and cognitive challenge affect students' willingness to participate or avoid arguments, it is not clear which aspects of students' disposition may relate to what forms of talk during students' involvement in mathematics class.

Socio-economic status and race. Classroom discussions are only effective when they benefit students from all racial and socio-economic backgrounds. White (2003) documented how two third grade teachers used classroom discourse to promote the mathematical learning of their diverse students, including over a third African-American and Hispanic students. Over the school year, the teachers came to value students' ideas in the discussions, explored students' answers, incorporated students' background knowledge, and encouraged student-to-student communication. According to White, "these teachers' practices help dispel the myth that African American and Hispanic students must be told how to think about and solve mathematics problems" (p. 51), and this study serves as an existence proof of the possibility of effective classroom discussions with diverse groups of students.

However, Lubienski (2000a; 2000b) studied students from diverse socio-economic backgrounds, looking at individual students' involvement rather than the collective discussions about mathematics, and determined that students from lower socio-economic backgrounds participated differently from the higher SES students. Lower SES students required more external direction from their teacher, and the higher SES students participated in the discussions as a forum for exchanging ideas. Lubienski's work suggests that discussions in mathematics classrooms may be more aligned with middle class cultures. There is more to understand about how students' backgrounds relate to how they interpret and experience discussions in mathematics classrooms.

Gender. Some researchers have found that male and females may participate differently in mathematics classroom discussions. For example, Hart (1989) studied seventh graders and determined that more boys than girls participated in public interactions with the teacher, as opposed to private interactions; were more likely to volunteer during discussions; were more likely to call out; and were more likely to share their thinking when it was incorrect. Her results suggested that if certain forms of participation are more valued and privileged in mathematics classrooms, males may benefit more from the experience of discussions. However, more recent literature on differences between how different genders participate in discussions more generally suggests that the differences are often small (Goldsmith & Fulfs, 1999). Although this is not an extensive representation of the research on gender differences in involvement in whole-class discussion, I mean to suggest that there is more to understand about how the young men and women involve themselves in discussion during mathematics class.

In summary, there is a need to coordinate studies of individual students' beliefs and goals with their involvement during mathematics class discussions. Such analyses can illuminate students' role in creating classroom norms of interaction during mathematics class. Research attempting to coordinate the analyses of the individual and social aspects of experiencing class discussion have focused only on a few cases, such as Lo, Wheatley, and Smith's (1994) case study of one third-grade student. This student developed increasingly sophisticated understandings of arithmetic over time while he actively assisted classmates with sense-making during discussion by indicating when he did not understand or when he disagreed with a classmate. With respect to beliefs about participating in discussions, he expressed that sharing ineffectively was a form of dishonesty. He was comfortable with the chaos that went along with disagreement in discussion, instead finding it to be intellectually stimulating. Looking beyond a case of one student can allow for describing a broader range of the beliefs relevant to learning mathematics in discussion-oriented settings.

A range of student characteristics may influence how students involve themselves in discussion during mathematics class. Mathematics educators have begun to study whether and how differences in race, socio-economic backgrounds, and gender influence students' involvement, while the influence of students' personality traits and dispositions on their involvement in mathematics classroom discussions remains largely unexplored. Additionally, accounting for subject matter in the analysis of students' involvement continues to be a challenge. Research on students' motivation in learning mathematics needs to be reconceived in light of reform-oriented mathematics classrooms (Middleton & Spanias, 1999).

CHAPTER THREE

METHODS

This study is a snapshot in time of 15 target students' beliefs and goals and their involvement in whole-class discussions in two different mathematics classrooms during the Spring of 2003. I studied students' beliefs and goals through my interpretation of their self-reported data (one-on-one interviews and Likert-scale surveys), and I studied students' involvement and participation in whole-class discussion through videotaped records of students' behaviors during mathematics class. These data were analyzed both at the individual student level and at the aggregate levels, grouped by both classroom and groups based on shared beliefs and goals, in order to assess relations between students' beliefs and goals and their involvement in whole-class discussion. As stated in Chapter One, I was most interested in examining students' motivations for participating in discussions during mathematics class.

I did not have specific conjectures as to the nature of the relations between students' beliefs and goals and their involvement in whole-class discussion. Thus, the data collection focused on open-ended approaches to gathering self-reported data for studying beliefs and goals: interviews rather than forced-choice survey instruments. This follows the suggestion of Aikenhead, Fleming, and Ryan (1987) who critique the use of Likert-scale instruments in studying students' perspectives. Considering students' perspectives is important because teachers could benefit from an increased awareness of the complexity of students' roles in the classroom micro-culture and the larger school culture (Sarason, 1982), and the social-emotional consequences of engaging students in

discussion in mathematics class remain largely unexplored (Chazan & Ball, 1995).

Research suggests that teachers could improve upon their assessment of their students' motivations (Givvin, Stipek, Salmon, & MacGyvers, 2001).

Research Questions

In order to study adolescents' motivations for participating in Standards-based mathematics classrooms, I designed my methods of inquiry to address the following primary research questions:

1. What are students' beliefs and goals?
2. What are students' patterns of involvement in their whole-class discussions about mathematics?
3. What are the relations between students' beliefs, goals, and their patterns of involvement?

Each of the primary research questions was supported by a series of secondary questions, which were as follows:

1. What are students' beliefs and goals with respect to learning mathematics in discussion-oriented classrooms?
 - a. What are the relations between students' beliefs and goals?
 - b. To what extent are the target students' beliefs representative of the populations of their classrooms?
2. What are students' patterns of involvement during whole-class discussions about mathematics?
 - a. What are the similarities and differences between the discussions in each of two mathematics classrooms?

- b. How do students' patterns of involvement differ by classroom?
- 3. What are the relations between students' patterns of involvement and their beliefs and goals?
 - a. Do students who express particular beliefs involve themselves differently in whole-class discussions?
 - b. Do students who express particular goals involve themselves differently in whole class discussions?
 - c. Do students who express particular clusters of beliefs and goals involve themselves differently in whole-class discussions?

The purpose of this chapter is to discuss the context and approach to my inquiry. First off, I will describe the setting of the study, including the two classrooms and the target students. Then, I will discuss methods of data collection and analysis as related to each of the three primary research questions.

Setting: School, Curriculum, and Classrooms

This study took place at Two Rivers⁵ Middle School (grades 6-8), the single middle school in a school district serving a rural community in Mid-Michigan. I spent approximately 100 total hours in two mathematics classrooms at Two Rivers Middle School during the 2002-03 school year as a participant-observer for the purpose of this study.

The school is located in a *small town*, according to the National Center for Education Statistics. The occupations of the parents and adult community members of this town are a mix between farmers and commuters to the more urban settings, which are located approximately 30 miles from their town. The school enrolls approximately 440

⁵ All names are pseudonyms.

students each year, approximately 150 students per grade. In 2001, 98.4% of the student body was White, 0.7% Native American, 0.7% Hispanic, and 0.2% Black. 12.4% of the student body received free or reduced-price lunch under the National School Lunch Program as a result of low family income.⁶

Standards-based mathematics curriculum. I selected this school because the students' experience with Standards-based curricula and teaching was not novel. The mathematics curricular experience for students in this school district starts with the Investigations textbook series (TERC, 1998) in elementary school and moves into the *Connected Mathematics Project* [CMP] textbook (Lappan, Fey, Fitzgerald, Friel, & Phillips, 1997) in middle school. These two curricular series are complementary in that their development was funded by the National Science Foundation in response to the calls of the NCTM Standards (1989, 2000). Most studies of students' experiences with Standards-based texts are conducted during early stages of implementation. In contrast, these students only know Standards-based mathematics curricula.

As there is a low attrition rate in this school,⁷ most of the students at Two Rivers Middle School have studied mathematics with a Standards-based text from early in elementary school until the present. The implications of this include students experiencing problems in real-world contexts and experiencing teaching practices that included, to some degree, small group work and whole-class discussions for most of their time in school.

Another reason for selecting this school was the teachers' experiences with this textbook series. The school was one of the initial sites to implement CMP, and has used

⁶ Statistics from: <http://www.ses.standardandpoors.com/> (04/05/03)

⁷ Participants reported attending school with their classmates since kindergarten, generally.

the textbook series since it was written. The mathematics teachers at this school across the grade levels have participated in professional development with the authors of CMP, worked with the authors as consultants, and have presented about their teaching with CMP to other teachers across the country. The administration and the teachers demonstrated overarching support for this mathematics curriculum.

In 2002-03, the school was piloting the revision of CMP texts. As a result, the students had less colorful texts and the teachers did not have teachers' editions. The teachers mentioned not having teachers' editions affecting them in terms of not always being told the larger mathematical concept of the investigation and not having specific guiding questions to use in whole-class discussion. Since they would normally have this information in a teachers' edition, sometimes they would email or call the textbook authors for insights.

Two of the three seventh grade teachers at this school participated in this study during the 2002-03 school year. I selected two teachers because more than one classroom would afford a broader range of students' beliefs and goals, as well as patterns of involvement in whole-class discussions, that would not be solely shaped by the teacher. I did not choose to work with the third teacher in order to keep the sample size manageable for an exploratory study, and because, based on communication with these two teachers, other teachers at this school, and members of the textbook authors who were familiar with this site, the third teacher was relatively less faithful to the curriculum and did not attempt to coordinate with the other two teachers. Below I describe the two teachers and their classrooms that participated in this study, Mrs. Evans's fifth-hour class and Ms. Carson's third-hour class. These two teachers' class periods were selected because they

were both seventh grade sections and they were relatively close to one another in time of day, and also near each teacher's planning period or lunch hour, in case I needed time to interact with the teachers.

Mrs. Evans' fifth-hour class. Mrs. Evans participated in this study during her 16th year of her teaching career. She was certified to teach elementary school and was in her 9th year of teaching middle school mathematics at Two Rivers Middle School and of teaching with the CMP texts, primarily seventh and eighth grade. Mrs. Evans was one of the original teachers invited to pilot the revised 7th grade CMP curriculum. She worked with the curriculum developers during the school year and summertime to design and carry out professional development for CMP teachers as well as evaluate new problems and construct assessment items. In her experience at this middle school, she had also taught some sections of science. I informally observed two of her classes, a seventh grade and eighth grade section, on an average of one day a week during the 2001-02 school year prior to this study. The purpose of this pilot work was to characterize features of whole-class discussion about mathematics. During 2002-03, all of her classes were mathematics, three eighth grade sections and two seventh grade sections.

Her fifth-hour seventh grade CMP students participated in this study. The class periods were 58 minutes long, beginning at 12:21 p.m., and it was the second class period after the seventh graders' lunch and the second to last period before the end of the day. In the fall, this class consisted of 24 students, 10 males and 14 females. In the spring, 27 students were in the class, 15 females and 12 males. As students' schedules changed between fall and spring semesters, there were 10 new students in the class and 7 students were no longer in the class. Mrs. Evans mentioned to me on more than one occasion that

her 5th hour was her biggest challenge to manage in terms of classroom behavior after the students' schedules changed for spring semester, potentially due to the combination of the time of day and the mix of personalities in the class.

Ms. Carson's third-hour class. Ms. Carson participated in this study during her 4th year of teaching, and she taught with the CMP text for all of these years. She is certified to teach at the secondary level. 2002-03 was her second year of teaching at Two Rivers Middle School, as she changed schools after her first two years of teaching. She learned how to teach with the CMP curriculum through coaching from another experienced CMP teacher during her first two years of teaching at her previous school, and through collaborations with her experienced colleague at Two Rivers Middle School, Mrs. Evans. They talked almost daily at lunch, before school, after school, or between classes about how their students were performing on similar units and about their approaches to teaching different units. They appeared to collaborate with each other more than with the other seventh and eighth grade mathematics teacher. Mrs. Carson has also participated in multiple professional development programs sponsored by the CMP textbook developers. During 2002-03, Ms. Carson also taught three sections of eighth grade math and two sections of seventh grade.

Her third-hour seventh grade class participated in this study. This class period was also 58 minutes long, beginning at 9:41 a.m. In the fall, the class consisted of 21 students, 11 females and 11 males. In the spring, 18 students were in the class, 10 males and 7 females. As students' schedules changed between fall and spring semesters, there were 2 new students in the class and 5 students were no longer in the class. Mrs. Carson mentioned to me on more than one occasion that upon the schedule change for the spring

semester, her 3rd hour happened to be her lowest performing class academically. This class was also her smallest enrollment section.

Fidelity to CMP. Both teachers were committed to following the CMP texts, utilizing the range of materials and teaching suggestions made by the authors, such as the text's reflection questions, and having their students keep a notebook that included their responses to these reflection questions and also their work on investigation problems, homework, and vocabulary words. The teachers did not supplement the textbook with worksheets of practice problems. The teachers did implement partner quizzes, as suggested by the textbook authors, about once a chapter.

Participants: Target Students

The analyses in this study involve data from 15 target students during the Spring of 2003. I selected seven students (out of 21) in Ms. Carson's classroom and eight (out of 24) students in Mrs. Evans's classroom to be target students. I initially selected 20 target students in the Fall of 2002, 10 from each classroom through purposeful sampling (Miles & Huberman, 1994; Patton, 1990) along the dimensions of gender and frequency of participation in classroom discussion, as judged from my fall observations. I shared the selection of participants with the teachers to inquire as to whether I also would have diversity along the dimension of achievement. The teachers both agreed that I had a broad representation of the range of students in the classrooms, based on achievement (performance on the first quiz) and participation. Table 3.1 presents a list of participants by teacher, gender, and level of observed participation in the fall.

Table 3.1: Participant Information

Pseudonym	Teacher	Gender	Fall Involvement
Abby*	Carson	F	Low
Allen	Carson	M	Moderate
Allison	Carson	F	Moderate
Colleen	Carson	F	Low
Hannah	Carson	F	Low
Laura*	Carson	F	High
Max	Carson	M	Moderate
Pete	Carson	M	Low
Shawn*	Carson	M	High
Tim	Carson	M	High
Alex	Evans	M	High
Alyssa	Evans	F	Low
Becky	Evans	F	Moderate
Bill	Evans	M	High
Jon*	Evans	M	Low
Marissa	Evans	F	Low
Molly	Evans	F	High
Robert*	Evans	M	Moderate
Steve	Evans	M	Moderate
Tricia	Evans	F	Moderate

*Fall participant only.

There were approximately three students at each level of involvement from both classrooms in the fall, with one additional low involvement student from Ms. Carson's class and one additional moderately involved student from Mrs. Evans's class. These students were representative of the demographics of their school in terms of race: all of the target students were White, as were most of the students at this school. I did not obtain data as to whether any of the participating target students received free or reduced lunch. Due to the changes in classroom populations at the beginning of the Spring semester, as described above, three of the fall target students from Ms. Carson's class and

two of the fall target students from Mrs. Evans's class were not included in the analyses of this study from the spring.

This study focused on the data during the Spring semester primarily due to the surprisingly large (to me) change in classroom population. Initially, I intended to study change over time in students' beliefs and goals and their involvement in whole-class discussion. The high turn over at semester was an additional variable I had not accounted for in the design of this study, and changes in target students' participation may have been influenced by the change in classroom population. In this study, then, I did not examine the change over time in students' beliefs and goals or their involvement in whole-class discussion. Rather, this is a descriptive study of how students' express beliefs and goals related to learning mathematics at one point in time. Results describe whether and how these beliefs and goals relate to students' involvement in whole-class discussions during this point in time: late February through March, 2003, while the students were studying an algebra CMP unit: *Moving Straight Ahead*.

Recall that I designed the study in order to address three primary research questions. The following table illustrates how the data sources correspond with my three research questions:

Table 3.2: Data Sources in relation to Research Questions

Research Question	Data Sources
1. What are students' beliefs and goals with respect to learning mathematics in discussion-oriented classrooms?	1. a. Interviews with 15 target students. 1. b. Likert-scale survey instruments given to population of each classroom.
2. What are students' patterns of involvement during whole-class discussions about mathematics?	2. Videotaped observations of whole-class discussions from each classroom.
3. What are the relations between students' patterns of involvement and their beliefs and goals?	3. Analyses of above data sources

Data Collection: Students' Beliefs and Goals

I had two data sources for data on students' beliefs and goals: interviews and surveys. I will first discuss the interviews, and then I will discuss the surveys.

Interviews. Interviews were conducted in late March or early April with the 15 target students, and they were one-on-one, between the target student and myself. Each interview lasted between 30 and 45 minutes, and was conducted during the student's math class in a small room either in the school's library or the front office, depending on availability.

The interview was designed to elicit data on students' beliefs about learning mathematics. I began the interview with open-ended questions to assess students' beliefs about learning mathematics, such as describe a successful mathematics student or a good mathematics teacher. One of the open-ended questions, about describing a successful mathematics student, was inspired by Spangler (1992). As the interview continued, I asked the students to elaborate on some of their responses to items from a survey instrument (5=True, 1 = Not At All True) that students completed in early March, following the technique of Aikenhead et al. (1987).

Some interview questions were more conducive to eliciting data on students' beliefs and goals about learning mathematics in discussion-oriented settings. Examples of interview questions that consistently elicited data relating to students' motivations to participate in whole-class discussion about mathematics included:

- Let's say that your school needed to hire a new 7th grade math teacher, and they wanted to get some students' ideas about what would make a good 7th grade math teacher. What would you tell them?
- Let's say a student moved into your classroom, and they wanted to know what they had to do to be successful in math, what would you tell them?
- Elaborate on your response to this survey item: When my teacher asks a question in math class, it is important that I explain how I did the problem, not give my answer.
- Elaborate on your response to this survey item: To work on math problems, I have to be taught the rules and steps, or else I can't solve them.

- Do you think about yourself as somebody that likes to talk about their ideas in class or somebody that would rather listen to other people's ideas or in between?

A complete list of interview questions can be found in the appendix.

Survey instrument. The survey instrument, *Students' Mathematical Views* [SMV], was a forced choice Likert-scale instrument with 63 items (see Appendix B for items and scales). The SMV survey was administered after a quiz during the students' mathematics class. I selected these scales based upon themes in previous research on students' beliefs and goals associated with learning and doing mathematics, such as research on students' confidence in mathematics (Kloosterman, 1991), students' task and ability orientations (Ames, 1992; Dweck, 1986), students' autonomy in learning and doing mathematics (Yackel & Cobb, 1996), the usefulness of mathematics (Boaler, 1997), the interconnectedness of mathematics (Gfeller, 1999), whether students view mathematics as conceptual or calculational (Thompson, Phillip, Thompson, & Boyd, 1994) or procedural or relational (Skemp, 1978), and whether students considered the use of multiple methods when solving a problem to be an important part of doing mathematics (Weinstein, 2000).

The scales on the survey drew from previously published instruments. I designed the survey from scales used in a previously constructed survey or based upon single items from instruments used in previous studies. The nine scales were confidence (Fennema & Sherman, 1976), task orientation (Midgley et al., 1996), ability orientation (Midgley et al., 1996), process versus product (Schoenfeld, 1989), autonomy and authority (Grouws, 1994; Schoenfeld, 1989), usefulness (Fennema & Sherman, 1976; Grouws, 1994),

structure (Grouws, 1994), conceptual versus procedural (Grouws, 1994; Schoenfeld, 1989), and multiple methods (Grouws, 1994; Schoenfeld, 1989). The confidence, task orientation, ability orientation, structure, and usefulness scales were imported intact from previously established instruments, and the process versus product, autonomy and authority, conceptual versus procedural, and multiple methods scales were written by myself based upon items used in previous research. Table 3.3 presents the name of each scale (with its eventual reliability scores), its definition, and a sample item.

Table 3.3: Survey Scales

Title of scale (α)	Definition	Sample Item
Confidence (0.91) 10 Items	I am good in math and successful in math class vs. I am not able to learn or be successful in math class.	27. I am sure that I can learn math.
Task Orientation (0.83) 6 Items	I value developing a deeper understanding of mathematics and working hard.	2. I like doing problems in math class that I'll learn from even if I make a lot of mistakes.
Ability Orientation (0.87) 6 Items	I value getting good grades and being recognized for my successes when learning.	3. I want to do better than other students in my math class.
Process vs. product (0.39) 6 Items	The process (or the product) is most important to focus on when learning mathematics.	16. When my teacher asks a question in math class, it is important that I explain how I did the problem, not just give my answer.
Autonomy & authority (0.43) 7 Items	Students are capable of discovering mathematical patterns vs. the teacher should provide the students for clear rules for doing the math problems.	40. To work on math problems, I have to be taught the rules & steps, or else I can't solve them.
Usefulness (0.67) 8 Items	The extent the mathematics we do in school is related to life outside of school	22. Math is a worthwhile subject for me.

Table 3.3, continued

Structure (0.56) 8 Items	Mathematics is an interrelated set of ideas vs. made up of isolated ideas	21. Finding answers to one type of math problem cannot help you solve other types of problems.
Conceptual vs. procedural (0.33) 7 Items	Mathematics is primarily about ideas vs. mathematics is primarily about procedures	8. The math that I learn in school is mostly a set of rules to memorize
Multiple methods (0.37) 6 Items	Mathematics problems have one right solution vs. multiple solutions are possible for solving the same problem.	35. It is possible to approach the same math problem in more than one way

Reliability scores were high for the confidence and ability orientation scales (above 0.80), moderate for task orientation, usefulness, and multiple methods (above 0.60, but below 0.80), and low for the rest of the scales. The entire survey instrument can be found in the appendix.

Data Analysis: Students' Beliefs and Goals

In order to analyze students' beliefs and goals, I analyzed both the interviews with students and their survey responses. Interviews were the leading data source, since I was not certain I was aware of the range of students' beliefs and goals prior to this study in such a way that would allow me to design a survey instrument. Therefore, the survey was generally insufficient for addressing Research Question #1. I transcribed the audiotapes of the interviews myself. Rather than analyzing the interview data solely for content in students' talk, I applied an additional level of rigor through the development of an analytic framework for examining cues in talk that suggests the existence of beliefs.

Analytic framework: Analysis of students' interviews. In complement to studies of students' beliefs and goals through the use of forced choice instruments,

students' talk in their interview statements was analyzed through an examination of three types of language cues. Through a close analysis of students' talk, the following language cues appeared to provide evidence of beliefs and goals: (1) verbs expressed in terms of idealized states or preferences, (2) affective statements, and (3) repetition.

The subjective knowledge of beliefs is considered to exist in the form of idealized states. Abelson (1979) suggests that beliefs do not exist only at the level of personal truths (Polanyi, 1958), but additionally manifest in the form of idealized states.

Belief systems often include representations of 'alternative worlds,' typically the world as it is and the world as it should be. ... The world must be changed in order to achieve an idealized state... (Abelson, 1979, p. 357)

Beliefs held in the form of idealized states focus on how the world *should* operate rather than how it does operate. These idealized states may be recognized through a participant's use of stating the imperative (i.e., "I need to..." "I have to..." "I should...") to express their perception that their world should operate in particular ways. The conceptualization of beliefs as idealized states may lead to a sufficient (but not necessary) criterion of imperative statements in the participant's responses for indication of a belief. Bills (1999) utilized similar linguistic cues, modal auxiliaries (may, might, can, could, shall, should, will, would, ought, need), in order to study students' attitudes when talking about math.

An example of the use of imperative statements to express a belief can be seen in this interview with a 7th grade student, Molly,⁸ who was asked to discuss her responses on a beliefs survey instrument. She used a series of imperatives to describe the importance of understanding in mathematics.

⁸ pseudonym

AJH: OK. The math I learn in school is mostly a set of rules to memorize. You put 1, not at all true, what do you think?

MH: It's not just to memorize, you have to understand it.

AJH: OK.

MH: They're not just, there are rules, but you have to know, like, why it's a rule and why it's so important, so I don't think it's true that you just have to memorize it. You also have to know its meaning and understand it.

AJH: Yeah, so memorizing's not enough.

MH: No. Not in math. (10/10/02, emphasis added)

Her repeated use of "...you have to..." suggests a belief in understanding as an important element in learning mathematics beyond memorizing procedures. The relationship between the idealized states of beliefs and imperative statements may assist in analyzing verbal data, such as interviews and questionnaire responses, for evidence of beliefs. Similarly, goals may be expressed in terms of desires or preferences, such as "I want to..." or "I hope to..." An examination of verb usage could reveal some evidence of a belief or goal.

Carefully attending to participants' expression of affect can also help the researcher to establish whether the evidence lends itself to claims about beliefs, values, goals, or attitudes. Hannula (2002) focused on students' affective statements as evidence of students' attitudes, and beliefs are closely related to affect (McLeod, 1992). If students discuss an emotional reaction, the psychological backdrop to their reaction is evidence of their beliefs.

Another excerpt from Molly's interview illustrates how a student may demonstrate attitudes and beliefs in conjunction through her affective statements.

MH: I like the way she teaches in that class.

AJH: What do you like about it?

MH: I like that she put, she makes it funny, and she gets the whole class involved, so, I don't know, it makes it a little bit more fun to learn to do. (10/10/02)

Molly's attitude toward learning mathematics was a positive reaction, as she enjoys her teacher's approach and her experience of learning. This enjoyment appears to be connected to her belief that the teacher should express her sense of humor and get the students involved in the class. Again, these statements alone are not sufficient evidence for a belief, but are selected to illustrate how looking at affective statements can reveal relations between beliefs and attitudes. Similarly, affective statements could provide further evidence for a goal, as students may emphasize reasons for their behavior with a sense of affect. Stronger evidence for a belief or goal occurs when evidence of affect converge with additional evidence of verbs indicating idealized states or preferences.

Tannen (1989) suggests that one of the functions of repetition in discourse is to aid comprehension through providing redundancy. Repetition can also be thought of as a form of emphasis as seen in songs, poetry, or oratorical discourse. Instances of repetition in the students' interview statements, especially when triangulated with affective statements and statements of idealized or preference verbs, may also suggest a belief or goal, as the repetition may indicate a form of emphasis. For example, we see above in Molly's affective statement the repetition of "she makes it funny," and "makes it a little

bit more fun.” Also, in the example of her idealized statement, she repeats that memorizing is not important, echoing the survey statement.

In order to identify themes in the content of students’ beliefs and goals, data analysis was conducted through constant comparative analysis of interview transcripts (Glasser & Strauss, 1967), with the use of the analytic framework of idealized states, affect, and repetition to guide the process. Students’ talk in interviews had to meet two of the three criteria in the analytic framework in order to be considered as expressing a particular belief or goal in their interview talk. This process allowed for building codes and allowing themes to emerge from the data, as an alternative to forced choice survey instruments. I used the qualitative software NVivo to help me track my process and organize my data. Codes were verified through the use of peer debriefing. Cobb & Whitenack (1996) suggest two types of peer debriefers, those who are familiar with the participants and the study and those who are not. I tested the credibility and resonance of my analysis by sharing data with colleagues who were more and less familiar with this study for feedback on the development of codes and relations between them.

The analyses of student’s interviews went through multiple cycles of analysis. Initial code development was based on interviews conducted in the fall. I developed general codes motivated by research on students’ motivation, such as self-efficacy, learning goals, and codes for students’ beliefs about their own role and their teacher’s role in learning mathematics. The spring interview questions focused more directly on students’ participation in classroom discussions, which resulted in the revision of the initial set of codes to include students’ social goals and perceptions of risk associated with participating in whole-class discussions. Students’ beliefs about the teacher and

students' roles in learning were collapsed into codes for students' epistemological beliefs about learning mathematics. Students' perception of risk seemed to capture students' motivations for participating more so than self-efficacy, based on the students' talk in interviews, so the self-efficacy code was eliminated. Specific codes used in this study, and the results from these analyses, are discussed in Chapter Four (Results).

Survey analyses. I collected survey responses from all students in both classrooms ($N = 42$ in the Spring), and I imported the numerical data into SPSS for statistical analyses. I wanted to address two sub-questions with the analysis of survey data. How did the target students' beliefs compare to the populations of their classrooms? Do students in different groups (classrooms, groups by shared beliefs or goals) express different beliefs? I used unpaired t-tests to compare groups of students for each of the scales presented earlier. I used a significance level of $p < 0.05$ to determine differences between groups.

In order to compare target students to the populations of their classrooms, I compared the target student's mean on each scale to their class mean on that scale in terms of standard deviations. If the student's mean was within his or her class's standard deviation, I considered the target student's mean to be similar to the classroom population. If the target student's mean was more than one standard deviation above or below the mean, I considered the target student to be different from the class population. This identification of target students into three categories relative to the views of their entire class (above, within, or below the class mean) was useful to see the extent to which target students expressed similar beliefs to their classmates.

Data Collection: Students' Involvement in Whole-Class Discussion

I visited each classroom and videotaped observations of all whole-class discussions between February 10 and March 7, 2003. I videotaped whole-class discussions during the CMP unit *Moving Straight Ahead*. These days encompassed work on the series of problems from the first two Investigations from the unit. An Investigation is a set of two, three, or four exploratory problems along a theme, both a thematic context and a mathematical theme. I did not videotape after March 7, because the teachers did not want to be videotaped during the third and fourth investigations, because these investigations were being pilot tested as part of the revisions of the CMP texts.

From this corpus of video records, I analyzed four consecutive instances of whole-class discussion from each classroom. In addition to choosing these four days based on the consecutive coverage of material, I asked each target student to watch three to five minutes of video from their classroom's discussion of Problem 1.3, and I asked whether this was a typical whole-class discussion for their class. They all agreed that the video captured a typical class discussion. So, based on their reactions, I claim that these days were representative of the whole-class discussions in these two classrooms. I did not analyze the first few days of videotaping in order to give the students time to get used to the camera being around, as these days involved some initial attention-seeking moves from the students that faded somewhat over time. Table 3.4 describes which days of whole-class discussions that I analyzed.

Table 3.4: Analyzed Discussions

Ms. Carson	Mrs. Evans
2/20/03 (Problem 1.3, 42 min discussion)	2/20/03 (Problem 1.3, 45 min)
2/21/03 (ACE day, 7 min)	2/21/03 (Problem 1.3 cont., 33 min)
2/25/03 (Problem 2.1 & 2.2, 15 min)	2/25/03 (Problem 2.1 & 2.2, 30 min)
2/26/03 (Problem 2.3, 20 min)	3/03/03 (Problem 2.3, 20 min)

There are evident lapses in time between videotaped discussions of problems. February 20 and 21 were a Thursday and Friday, respectively. On Monday, February 24, both classes used the time for seatwork and other class business. The days between Tuesday, February 25, and Monday, March 3, for Mrs. Evans' class included an "A.C.E. day"⁹ on February 26, a teacher in-service day on February 27 and a day off from school on February 28 for the end of the marking period.

The problems discussed over these four days included problems 1.3, 2.1 & 2.2 (as a joint problem), and 2.3. The mathematical content of these problems involved linear relationships. This content was significant to the research, because students' transitioning into algebra during middle school may provide new challenges for participating in whole-class discussion. I will describe the problems in more detail below.

Problem 1.3 was about pledge plans for participating in a walkathon. In the problem, the students are encouraged to compare three different pledge plans.

Leanne says that each sponsor should pay \$10 regardless of how far a person walks. Gilberto says that \$2 per kilometer would be better because it would bring in more money. Alana points out that if they ask for too much money, not as

⁹ An A.C.E. day is a class period spent working on problems similar to the investigation problems as seatwork.

many people will want to be sponsors. She suggests that they ask each sponsor for \$5 plus 50 cents per kilometer.

In the text, students are asked to create a table, graph, and an equation to represent how much money would be taken in by each of these pledge plans. Additionally, the text asked students to locate particular values, such as how much a sponsor would owe for 8 kilometers, and the students are requested to write new equations, such as a pledge plan with a larger or smaller rate than those shown, and to describe a pledge plan that is not linear. In both teachers' implementations of this problem, students were asked to describe how they knew the pledge plans were linear based on the table, graph, or equation, but this question was not a part of the problem in the text.

Problems 2.1 and 2.2 were about two brothers, Henri and Emile, who were going to race each other. The older brother, Emile, learned at school that his walking rate was 2.5 meters per second, and he timed his little brother Henri and determined his walking rate to be 1 meter per second. Henri challenged Emile to a race, and Emile wanted to design a race that Henri would win, but would still be close. In problem 2.1, students were asked to come up with a suggested distance for the race such that Henry would still win, but in which Emile would come close to winning. In problem 2.2, students were asked to justify their answer with a table and a graph. They were also asked to create equations and relate the y-intercepts in the equations to the y-intercepts in the graphs.

In problem 2.3, students were asked to compare price plans for either a membership to a movie theater (a \$49 membership fee and \$1 per movie) or to pay \$4.50 for each movie. Students were asked to compare the plans for a certain number of movies (20), a certain amount of money (\$120), to determine the number of movies in which the

cost of both plans would be equal, and to explain why the relationships in the plans are linear.

Across these problems, a range of concepts and skills were discussed, including graphing a line, constructing an equation for a linear relationship, creating a table for a situation, comparing relationships to determine which one was smaller or lower for a particular portion of the domain, determining whether a relationship was linear, and moving between representations.

I videotaped whole-class discussions over these problems with one digital camera on a tripod. I placed the camera in one spot in the classroom, either in the back of the classroom or the front corner of the classroom, moving the camera to catch the image of the speaker who had the floor. I zoomed in on a speaker if necessary for identification, such as if it wasn't clear who was speaking because the student's back was to the camera. I also drew a floor plan of the classroom to identify the speakers when I would watch the videotape. The microphone on the camera was sufficient for picking up the dialogue during whole class discussion.

Data Analysis: Students' Involvement in Whole-Class Discussion

In order to prepare the classroom data for analysis, I transferred the digital videotapes to .MPEG video files using Dazzle hardware and Movie Star software. I downloaded these .MPEG files to compact disks for portability. I imported these files into Transana software for transcribing and qualitatively analyzing videotape data. This software allowed me to use the computer's keyboard to stop and start the video while transcribing, and allowed me to timestamp the transcript to match up with the video. I

transcribed the clips of whole-class discussion myself, making notes along the way about target students.

My unit of analysis for coding classroom discourse was an *interaction segment*. I define an interaction segment as either a student or teacher initiated series of turns in an interaction around a consistent topic. Interaction segments were a useful unit for the analysis of the content of students' talk during discussion because they encompassed both questions and replies. I segmented all transcripts into interaction segments. Then I coded all of the interaction segments that involved target students' vocal participation. Target students participated in 72% of the overall interaction segments from Mrs. Evans's class and 65% of the overall interaction segments from Ms. Carson's class. More than one student may have participated in an interaction segment.

I was interested in developing codes for capturing students' autonomy, initiative, or confidence in their talk, as well as instances of mathematical justification. I selected these themes before analyzing the data because the goals of conducting discussions during mathematics class are often to promote active sense making and intellectual autonomy among students as well as promoting mathematical reasoning and communication. To examine these themes, I developed codes through a constant comparative process (Glasser & Strauss, 1967). Specific codes, and the results from these analyses will be presented in Chapter Five (Results), as they were indeed results of intensive work with the video data.

Data Analysis: Relations Between Students' Beliefs and Goals and Their Involvement

My primary goal in this study is to analyze whether students who express similar beliefs and goals also participate in similar ways during whole-class discussion. In order to pursue this question, I created groups of students based on their shared beliefs or shared goals. For each belief or goal, there were two groups of students. Usually these groups were constructed by putting the students who did express a particular belief or goal in one group and those who did not express the belief or goal into another group. Then I counted the number of times the student participated in a particular type of interaction segment, calculated the percentage of participation according to that type of interaction segment for each target student, and then calculated the mean and standard deviations of percentages of participation in each type of interaction segment for the belief or goal groupings of students. To determine whether the “belief groups” of students participated differently, I calculated unpaired t-tests. Results from these analyses are presented in Chapter Six (Results).

I conducted similar analyses, grouping students by teacher rather than beliefs or goals, in order to explore whether or not target students participated differently by classroom rather than belief group. These results are presented in Chapter Five.

My Role in the Research Process

Since I acted as both a participant-observer in these classrooms and an interviewer of these students, I would like to be explicit about my role in these classrooms and my relationship with the teachers and their students.

Initially, when I arrived in the fall, students in both classrooms labeled me as the “camera lady.” Their exposure to me was as a person who stood behind a video camera for two to three weeks each semester during their math class. I had the opportunity to introduce myself to each classroom in the fall and invite the students to participate in the study, but other than that, I had no interaction with the class as a large group. Mrs. Evans referred to me by first name to the students, while Ms. Carson referred to me as Mrs. Hoffmann to the students. Also, the school librarian made me a badge with my name on it, similar to those worn by the staff at the school, but the students who didn’t work with me directly still only referred to me as the “camera lady.”

My videotaped observations were focused on whole-class discussions, so I moved out from behind the camera during other parts of the class period. When students were working independently or in small groups during seatwork, I would wander from table to table, as both classrooms had tables with three or four students at a table. Sometimes I took on the role of observer of these groups, saying nothing, but more often I would interact with the students. I let the teachers know that they could consider me a resource during seatwork time, because I am a former junior high mathematics teacher, I have a bachelor’s degree in mathematics, and I am familiar with the textbook series. When I was interacting with the students, I would ask questions such as, “How do you know you’re right?” whether or not the student was correct. I would also ask them how they solved the problems. Sometimes students would thank me for “helping” them, which I found interesting, because I never gave the students explicit instructions for how to solve the problems. I interacted with the students by asking questions rather than telling them how to work out problems.

Students appeared to value the attention they perceived to receive from interacting with me. Students in the class periods I visited would declare themselves as “special” to the students in the preceding and following class periods during the passing period as I was bringing my equipment in and out of the classroom. Students in class periods that I did not visit would ask me when I was going to videotape them. I do not see this “specialness” as interfering with the videotaped discussions, as I did not observe differences between the discussions on the days I videotaped (after the initial day or two of attention-seeking behaviors) and the days I was observing without a camera before or after conducting an interview. When completing survey instruments, students worked quietly and intently, sometimes putting comments in the margins about their opinions about the items. When I was interviewing students, they would ask me who was getting interviewed next. I would tell them that students who asked to be interviewed would not be getting interviewed. However, I had already selected target students, and I sometimes interviewed students who had asked to be interviewed. It seemed as though my extended time spent in the classroom helped the target students open up with me during the interviews, since I was not a stranger and had invested in getting to know them and their classmates.

I spent time talking with the teachers before and after class. Ms. Carson had a preparation period immediately after my observation of her class, during fourth hour, which intersected with Mrs. Evans’s lunch period. I would often debrief with Ms. Carson about the day as I took down my camera equipment, and she would share with me what she thought did and did not go well, in her opinion, during that class period. I responded to these comments in an effort to empathize with her concerns, but I did not offer her

advice. Then, I would set up my camera in Mrs. Evans's classroom during her lunch hour. Sometimes these two teachers would chat with each other during this time period, and other times I would talk with Mrs. Evans. Our conversations were mostly about what was going on in Mrs. Evans's life outside of the class period I was observing, such as with her children or with the graduate level course she was taking that semester. These conversations also focused on procedural aspects of the study, such as whether the next day would be appropriate for videotaping and for scheduling times to administer the survey or scheduling times to interview target students.

Some of my conversations with the teachers included me reassuring them that this study was about their students rather than an evaluation of their teaching. These sorts of reassuring conversations would occur usually after administering a survey, and the teachers would ask whether their students did well on the survey. This question surprised me at first, because I did not look at the students' responses on the survey items as an evaluation of the teachers. However, it appeared as though they wanted their students to respond in particular ways to the survey, and they would have considered some responses to be better than others, while I did not have the same value associated with these responses, nor did I have the same stakes associated with the students' responses on the survey that these teachers may have. Similar conversations also occurred after interviewing some of the students. I responded to their questions by assuring them that no one particular response was "better" than another, based on my orientation to the data. I did not share individual students' responses with the teacher, nor did they ask to see them. The teachers may have been concerned with how my dissertation results would be perceived by those who were not supportive of the CMP textbook series.

Results

The following three chapters present the results from these analyses. In Chapter Four, I present the results of interviews with 15 target students and their expressions of their beliefs and goals related to learning mathematics and participating in whole-class discussion. Additionally, survey analyses are presented in Chapter Four. In Chapter Five, I present the results of the analyses of four days of whole-class discussion in each of the two classrooms. Common patterns of involvement among target students are presented, as well as results as to whether or not target students participate similarly or differently in each classroom. In Chapter Six, I present the results of the analyses of students' involvement in discussion, and whether students involved themselves differently depending on the beliefs or goals they expressed.

CHAPTER FOUR

RESULTS: STUDENTS' BELIEFS AND GOALS IN DISCUSSION-ORIENTED MATHEMATICS CLASSROOMS

Students' motivations in general, and their beliefs and goals in particular, are not commonly studied in a situated manner. If the study takes place in a particular subject matter, it is usually to isolate a variable rather than to study the learning of that particular content. This study focused on mathematics learning because students may associate particular epistemological beliefs with this content that differ from their beliefs about other subject matters, such as social studies or English. This study is situated in discussions-oriented mathematics classroom because of NCTM's recommendations for promoting classroom discourse. What were these seventh grade students' beliefs and goals for participating in mathematics classroom discussions?

I structured this study of students' motivations to include their beliefs about learning mathematics, as these beliefs create the psychological context of what is possible for students in their classroom activity, and their academic and social goals related to their involvement in whole-class discussion during mathematics class. Although I collected both interview and survey data from my participants, the majority of the data presented in this chapter are from the interviews with the 15 target students. In this chapter, I will present the target students' beliefs and goals first from my analysis of their talk in interviews, followed by analyses of students' responses on survey instruments. Findings from students' interview data will be depicted by (a) describing students' beliefs and goals (b) presenting how beliefs and goals co-occurred, or clustered, for individual

students, and (c) briefly addressing potential relations between beliefs and students' performance in their mathematics classes. Then, I will present the following survey results from the population of students in these two classrooms: (a) relations between beliefs, (b) differences between classrooms, and (c) whether the target students were representative of their class.

Students' Talk During Interviews: Beliefs and Goals

The principal goal for this study was to pursue an understanding of the nature of students' motivations to participate in discussions about mathematics. I wanted to focus my analysis on a bottom-up examination of students' talk in interviews, because whether and how students' motivations may support and constrain students' involvement in discussions during mathematics class remains largely unexplored. However, I had some of my own conjectures that these students might express based on prior research and my own experiences working with middle school students.

My selection of survey scales reveals my conjectures for the beliefs and goals that these students would express. For example, I conjectured that students would be motivated to participate through their learning goals, as assessed by the task orientation and the ability orientation scales on the survey instrument. I also expected that students who were mastery-oriented might participate in order to learn material, while those who had more performance-oriented goals may be participating for recognition or to avoid appearing unintelligent. Additionally, I conjectured that students would be motivated to participate through their belief that mathematics was a flexible discipline, as assessed by the multiple methods scale on the survey instrument. I conjectured that students who believed that mathematics was a flexible discipline would participate in order to share

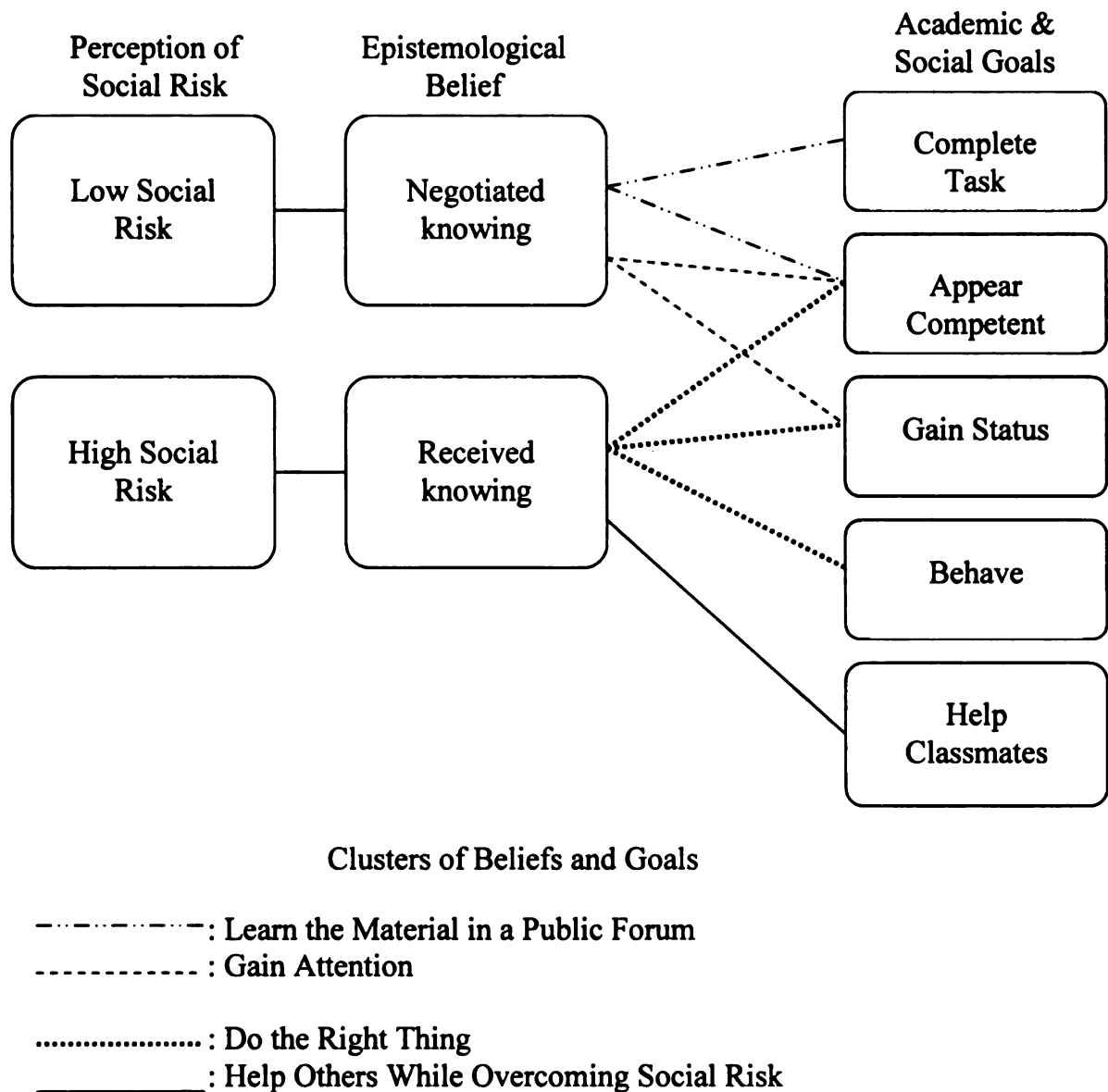
alternative solutions. While I had these conjectures, I did not assume that I knew enough about students' beliefs and goals in these settings to test hypotheses, so I relied more on open-ended interview data to assess students' beliefs and goals. The interviews analyzed in this chapter were conducted in mid-March through early-April 2003.

I hypothesized that students' beliefs and goals would vary as much within classrooms as they did between them, as students have histories as learners and doers of mathematics prior to their experiences in these classrooms, but I wanted to investigate this assumption empirically in both survey and interview analyses. The surveys analyzed in this chapter were collected in mid-March 2003.

The 15 target students expressed four clusters of beliefs and goals in their talk during interviews. Each cluster consists of (a) the students' perception of the level of risk associated with participating in whole class discussion (*social risk*), (b) students' beliefs about learning mathematics (*epistemological beliefs*), and (c) students' academic and social goals, such as their task goals (their investment in *completing the task* efficiently), their goal to *appear competent* publicly, goal to *gain status* among their classmates, their goal to *behave* appropriately, and their goal to *help their classmates*. These four clusters were far from evident at the conclusion of the interviews. They only emerged after many cycles of analysis and complementary reading of relevant research literature.

I describe these four clusters in terms of thematic titles: *learn the material in a public forum*, *gain attention*, *do the right thing*, and *help others while overcoming social risk*. Figure 4.1 represents the clusters of students' perceptions of social risk, epistemological beliefs, and academic and social goals.

Figure 4.1: Seventh Grade Students' Motivations



Initially, two clusters of students were formed based on relations between students' perception of social risk and epistemological beliefs. *Learn the material in a public forum* and *gain attention*, are clusters in which students perceived a low level of risk associated with sharing their thinking publicly during a whole-class discussion about mathematics. These students also did not believe learning mathematics was a process of receiving

knowledge – that is, of taking in the knowledge presented by an authority, such as the teacher or a knowledgeable peer. Rather, they believed that that learning mathematics was a process of negotiating ideas. Differences between these two clusters lay in students' academic and social goals, as the students in *learn the material in a public forum* expressed goals of completing the task, while the students in *gain attention* were less task oriented and more interested in appearing competent and gaining status for that competence. Students in the other two clusters, *do the right thing* and *help others while overcoming social risk*, are clusters in which students expressed a higher level of risk associated with involvement mathematics class discussions. They also expressed the epistemological belief that learning mathematics was a process of receiving knowledge from an external source. But there were differences between these two clusters in the students' social goals. The students in *help others while overcoming social risk* shared a social goal of helping their classmates, while students in *do the right thing* shared social goals of behaving appropriately, appearing competent, and gaining status among their classmates. The latter group did not take on the goal to help others; they focused on their own learning.

These four cluster categories were not distributed equally across the 15 target students. Overall, as Table 4.1 shows, there were fewer *gain attention* and *do the right thing* students across the two classes.

Table 4.1: Target Students in Each Belief Cluster

	Ms. Carson's students	Mrs. Evans's students
Learn the material in a public forum	Tim, Allison	Alex, Steve, Becky
Gain attention		Molly, Bill
Do the right thing	Max, Pete	Marissa
Help others while overcoming social risk	Allen, Colleen, Hannah	Alyssa, Tricia

The more common clusters among these target students were *learn the material in a public forum* ($N = 5$) and *help others while overcoming social risk* ($N = 5$). Only one cluster, *gain attention*, was limited to target students in one of the classrooms – though given the small sample size, little can be concluded from this empty cell.

Before discussing the four clusters in greater detail, I will discuss the nature of the students' talk during interviews with respect to each of the beliefs and goals that make up these clusters: perception of social risk, epistemological beliefs, and their academic and social goals. I will also address the analytic process for claiming the student held a belief or a goal in order to provide some justification, beyond the general methods pursued, for the clusters.

Perceptions of Social Risk

During my interviews with target students, I asked them questions about their experiences during classroom discussions, such as whether they would rather listen to others talk or participate in the whole-class discussion. When students described their

preferences for involvement in whole-class discussion, a number of them ($N = 6$) mentioned that participating in discussion felt threatening, because they could be incorrect in front of others. I concluded that these students perceived participating in whole-class discussion to be a high-risk activity. Other students did not view participation as a threatening experience. Instead, they perceived participating in classroom discussion to be a moderate to low risk activity. Table 4.2 summarizes the definition for these beliefs about social risk and presents a quote to illustrate the nature of students' talk for each belief.

Table 4.2: Definitions for Perceptions of Social Risk

Perception of Social Risk	Definition	Illustrative Quote
High	Sharing thinking publicly is threatening, because of the chance of being incorrect in front of classmates.	"...when I'm <u>put on the spot</u> , I kind of go off track. I don't know how. Every time I'm <u>put on the spot</u> in front of an audience, I just <u>panic</u> and I can't really think straight." (Allen, 3/19/04, #70, Ms. Carson)
Moderate	Students were somewhat concerned about being incorrect, but not consistently.	Talk from interview demonstrated some threat, but also some indifference.
Low	A lack of concern for being incorrect in front of their classmates.	"...if I get something <u>wrong</u> , then I can see what I did <u>wrong</u> , and they'll, like, they'll <u>help me</u> and <u>show me how to do it</u> ." (Molly, 3/13/03, #29, Mrs. Evans)

As discussed in Chapter Three (Methods), one utterance or one turn in an interview was not enough evidence for determining whether a student held a belief. I used an analytic framework consisting of three criteria: repetition within the interview, imperative verbs, and relation to affect. Allen and Molly's quotes above illustrate some of these frames in the underlined phrases (emphasis added).

Allen expressed the idea of being “put on the spot” repeatedly and a sense of “panic” associated with this experience. He was speaking about his experiences participating in class discussions. The high sense of affect, expressed through use of “panic,” indicates a high sense of risk associated with sharing his thinking during class. He also repeated this feeling of being “put on the spot” in other turns during the interview, with comments such as, “...going up in front of everybody, that just makes me nervous” (3/19/04, #90). This increased affect around sharing one’s thinking publicly was a common indicator of a high sense of risk in students’ talk.

Molly talked about being “wrong” in front of classmates by emphasizing the benefits rather than a sense of negative affect. She talked about getting help and being shown how to do an incorrect problem. Additionally, when asked directly about whether she minded being incorrect in front of her classmates, she spoke as follows:

I’d like to see if I, because I could be right, and even if I’m not, I really don’t feel bad, because other people aren’t always right, either, so it, just, it doesn’t really bother me. (3/13/03, #232, Mrs. Evans).

She repeated this sense of not feeling “bad” and that it “doesn’t really bother” her to be incorrect. Since she mentioned benefits with being publicly corrected, mentioned not associating negative affect with being incorrect in front of classmates, and spoke consistently about this issue more than once in the interview, she appears to have a low sense of risk associated with sharing her thinking in front of her classmates.

Students from both classrooms expressed varying levels of perceived risk. Table 4.3 illustrates the level of risk reported by the target students in each classroom.

Table 4.3: Perceived Social Risk by Classroom

	Ms. Carson's class	Mrs. Evans's class
High Perceived Risk	Allen, Colleen, Hannah,	Alyssa, Marissa, Tricia
Moderate Perceived Risk	Max, Pete	
Low Perceived Risk	Allison, Tim	Alex, Becky, Bill, Molly, Steve

The six students who spoke about perceiving a high level of risk associated with sharing their thinking during class discussion all talked similarly to Allen. They expressed feeling threatened when they were not sure that they were correct. For example, Colleen talked about only sharing her thinking when she was certain she was correct.

Colleen: It depends on the problem. Usually I just like to sit and listen.

AJH: OK.

Colleen: If I know, I know the correct answer, I know how I got it, then I would kind of say it. (3/26/03, #61-63, Mrs. Carson)

The repetition of “I know,” suggests a level of confidence she must have before joining the large group conversation. Otherwise, she expressed a preference for observing and taking it all in, as she said “usually I just like to sit and listen.” Another target student, Tricia, spoke about how it was important for successful mathematics students to volunteer their thinking during class. When I asked her whether she volunteered her thinking during class, she spoke as follows:

Tricia: Sometimes. I'm kind of really shy, so I'm like super-conscious about when it comes to answering in front of people. I get, like, all nervous and stuff.

AJH: So, what do you mean, can you say a little more about that?

Tricia: Well, like, in math, I used to be, like, if a teacher called on me or

something, and I had my hand raised, my face would turn all red, and I'd get really nervous and I'd start sweating, until I got the answer out, and, like, it, I was, like, always nervous that it would be wrong. And, I'm not so much nervous any more, but I'm still kind of worried about if the answer's wrong, like, I'll get, like, messed up or something. (3/18/03, #52-54)

Tricia talked about her experience of being incorrect in front of the class with many affective terms ("really shy," "super-conscious," "worried") with repetition ("nervous"), suggesting that these experiences of sharing her thinking have an impact on her and are threatening. Also, the repetition of "wrong" in her talk helps illustrate her fear: she does not want to be incorrect in front of the class.

The seven students who spoke about perceiving a low level of risk associated with sharing their thinking during class discussion spoke about the benefits associated with sharing their thinking and did not mention threat associated with sharing their thinking during class discussion. Becky described a successful mathematics student in the following manner:

And when you have to do problems, don't just sit there. You have to get into the conversation in order to actually get it yourself and make you understand it, don't just understand it like how other kids do it. (3/27/03, #42, Mrs. Evans)

For Becky, to understand (or to "actually get it") means having her own ideas, as indicated by the repetition of "understand." Putting her ideas on the floor during the large group discussion is a way for her to realize these ideas, as indicated by the use of imperative verbs such as "have to" and "make you." Another target student, Steve, spoke about the importance not just paying attention, but also participating.

Both, really, because, like, you can't just sit back the whole time and just let everybody else teach you. You'll know how to do it, but you wouldn't have tried it, you wouldn't know if it would work or not. So, if you're out there, trying to throw out your ideas, you could actually find for all, a new way of doing a math problem. (3/26/03, #92, Mrs. Evans)

Steve's repeated use of "try" suggested that even some effort on the students' part to participate was necessary, because otherwise, "you wouldn't know if it would work or not." Also, the use of the strong verb of "can't" in reference to the process of receiving knowledge showed the importance he placed on participation.

The two students whose talk suggested a moderate level of perceived risk expressed both feeling threatened and seeing the benefits of sharing their thinking publicly. For example, when I asked Max directly whether he was comfortable sharing his thinking if he was not certain that he was correct, he spoke as follows:

Uh, no. I'm not right, I'm not going to raise my hand, if I don't know if I'm right or something. If I get it wrong and then she says it's wrong, I don't care, but if I think I got it right, then I'll raise my hand, but if I don't know, then I probably won't. (3/26/03, #259, Ms. Carson)

While he didn't have a strong sense of affect associated with being publicly incorrect ("I don't care..."), Max did consider whether or not he might be correct, by his repetition of whether or not he is "right" or whether he "knows." Sometimes he thought he would be correct when he was not, and this did not bother him. Max did not choose to participate if he did not think his answer was correct enough to share.

The difference between students with a moderate sense of risk and a high sense of

risk is that, while both groups hesitate to participate unless they are correct, those with a moderate sense of risk do not have a sense of negative affect or threat connected with being wrong in front of their classmates or teacher. Another target student, Pete, demonstrated this when I asked him what it was like to volunteer to share his thinking.

Pete: Kind of, like, weird. You're not sure whose is right. Theirs is right, or yours is right, or you're going to get wrong. That's what.

AJH: OK. And do you mind that?

Pete: Not really. (3/25/03, #82-84, Ms. Carson)

While he recognized that he could be publicly incorrect, this did not appear to bother him, other than it being "kind of, like, weird."

The difference between students with a low and moderate sense of risk was that the students with a low sense of risk explicitly mentioned benefits to sharing their thinking during class discussion. Students with a moderate sense of risk did not talk about these benefits in their interviews. Students with a high sense of risk mentioned the perceived threat associated with involvement in whole-class discussion repeatedly throughout the interview and often in response to an open ended question rather than a question specifically about participation, while those with a moderate sense of risk did not mention feeling threatened repeatedly. I also found that these differing levels of social risk correlated with different views on what it takes to learn mathematics.

Epistemological Beliefs

As discussed in Chapter Three (Methods), my interview questions were also designed to reveal evidence of students' beliefs about learning mathematics. I asked the students several open-ended questions to assess these beliefs, including what a successful

mathematics student should do and what a good teacher does. As a result, students' talk was rich with data about the process of learning mathematics.

Students talked about the process of learning mathematics either as receiving knowledge from an authority (*received knowing*) or as negotiating between their thinking and the ideas of others (*negotiated knowing*). *Received knowing* is similar to the epistemological belief in Belenky, Clinchy, Goldberger, & Tarule (1986). Students with this belief expressed that the work or wisdom of others was central to the process of learning mathematics, and they did not talk about themselves as capable of creating knowledge. *Negotiated knowing* is a blend between Belenky et al.'s procedural and constructed knowledge. Any departure from received knowing was considered an advance, and the nature of mathematics as a school subject may not lead as naturally to constructed knowing. Students with this belief had less reliance on authority. Instead, they looked for feedback on their own thinking. Table 4.4 summarizes the definition for these beliefs and presents a quote to illustrate the nature of students' talk for each belief.

Table 4.4: Definitions of Epistemological Beliefs

Epistemological Belief	Definition	Illustrative Quote
Received knowing: External source of knowledge	Knowing involves obtaining knowledge from an outside authority.	"...you <u>gotta pay attention</u> so you know what's going on, and if you don't, then you're pretty much lost, and you won't be able to really <u>catch up</u> real fast, it might take you a while to <u>catch up</u> , so you <u>gotta really pay attention</u> , and you <u>gotta</u> listen a lot." (Allen, 3/19/03, #46, Ms. Carson) ¹⁰

¹⁰ (Pseudonym of student, date of interview, line number of turn, teacher's pseudonym)

Table 4.4, continued

Negotiated source of knowledge	Knowing involves getting feedback on one's own ideas and comparing the ideas of others with your own.	"...if you <u>talk through it</u> , and you, like, <u>talk about it</u> , you <u>might realize</u> something you did wrong, if you <u>talk about it</u> , you <u>might say, oops</u> , I timesed when I was supposed to divide or something." (Molly, 3/13/03, #58, Mrs. Evans)
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Allen's quote contains some repetition and imperative verbs. His use of "pay attention" and "catch up" repeatedly suggests not only the importance of staying focused, but also a sense of difficulty associated with not staying focused. He said that if he did not pay attention, he would be lost, which suggests that he needed to hear what is going on around him to receive the knowledge. Additionally, the importance placed on paying attention is indicated by his use of "gotta" (or "have to"). From this quote, we can conjecture that Allen holds a belief of knowledge coming from an external source. At least one other turn during the interview that met two of the three criteria was necessary for claiming that a student held a belief.

Additional evidence for Allen's belief in knowledge residing in an external source was in how he talked about the teacher's role in learning mathematics: "...you've [the teacher] gotta keep them [the students] under control, and you've gotta, like, kind of, well, you've gotta kind of get their attention." (3/19/03, #114) His repeated use of "gotta" in reference to how the teacher needs to help the students stay focused was another example of an imperative verb, emphasizing the importance of helping the students pay attention. Though he focused on the teacher's role, the implications of that role were consistent with a student's role of primarily paying attention to the teacher. This quote demonstrates repetition within in the interview of the importance of students paying attention. This idea repeated in other instances throughout his interview as well. This

sense of a need to pay attention over other activities during mathematics class suggested a reliance on an external source of knowledge.

In contrast, Molly's talk indicated that knowledge did not come exclusively from an external source, as seen through the repetition and affect in her quote in the table above. She repeated "talk through it" and "talk about it [the problem]" in the quote, indicating a commitment to share her own thinking about the mathematics problem. She also repeated "might," as in "you might realize" and "you might say, oops." While this verb is more hesitant than an imperative verb, the content of her talk suggests that talking about the problem can have a benefit, potentially learning more about the problem. Her use of "might" could be because sometimes she was correct, and did not need to realize what she did wrong. Her use of "oops" suggests affect associated with being incorrect, and that she may not have had the opportunity to get her thinking corrected if she had not shared it. Her knowledge of mathematics appears to come both from her own thinking and from others, as she shares her thinking for feedback.

Additional evidence for Molly's belief in knowing mathematics through negotiation rather than an external source came from quotes in her interview, such as when she was asked about how often students should be asked to share their thinking in mathematics class.

In, like, I think every situation, because different people, like, learn different ways and different problems might have different troubles. Like, I might find one problem really easy or another problem really hard, so you need to explain it, even if you do get it, even if it is hard. (3/13/04, #97, Mrs. Evans)

While she acknowledged that explaining is not easy (“even if it is hard”) and that sometimes she did not need to be corrected when she explained (“even if you do get it”), she also recognized that others could benefit from her explanations. Her repetition of “different” indicates a sense of the range of learners’ understandings, and how some students will understand one explanation over another. Thus, she said with an imperative verb, “you need to explain it.” If students explain, others can learn if the problem was hard, so students should try to figure it out for themselves and share, both for their own benefit, and the benefit of others. For Molly, knowing was a process of negotiation of ideas between the students and the teacher.

Students from both classrooms expressed evidence of both epistemological beliefs in about equal numbers (Table 4.5).

Table 4.5: Epistemological Beliefs by Classroom

	Ms. Carson’s class	Mrs. Evans’s class
Received knowing	Allen, Colleen, Hannah, Max, Pete	Alyssa, Marissa, Tricia
Negotiated knowing	Allison, Tim	Alex, Becky, Bill, Molly, Steve

Although relatively more students in Ms. Carson’s class expressed knowing to be a received process and more students in Mrs. Evans’s class expressed knowing to be a negotiated process, these samples are small and represent less than half of each class. In addition, my sampling within classes was not necessarily random. It is not reasonable to claim that students from Ms. Carson’s class were more likely to believe in received knowing.

The eight students who spoke about knowing as a process of receiving knowledge gave similar accounts to Allen’s. For example, Marissa also spoke about the importance

of paying attention. When asked whether she would choose to participate in the discussion or listen during class, she responded as follows:

I think paying attention. I like, just, watching people more than I like jumping right in, arguing, getting in the fight, all that good stuff. I think I'd rather just look and pay attention and just, yeah. (3/31/03, #80)

Marissa expressed affect and a preference for watching, looking, and paying attention repeatedly. Additionally, when asked what a good student needed to do, her first response was, "I think, um, they have to pay attention." Then she talked about not being intimidated by the teacher's jokes and the need to take notes, but then she revisited the importance of paying attention. "I think you just have to pay attention. Keep yourself on the ball, keep watching everybody." (3/31/03, #40). She repeated the imperative verb of "have to" as she repeated her statements about paying attention. Attention was an important issue for about half of the target students when they talked about the process of learning. These students did not talk about the importance of having their own ideas and sharing them with others, often devaluing their own ideas in their talk. I interpret talk about paying attention to the teacher as indicating beliefs of received knowing. Such talk indicated that students were listening rather than speaking and focusing on the authority in the classroom as the teacher rather than the students.

The seven students for whom learning was a process of negotiation likewise spoke similarly to Molly. They did not emphasize paying attention in their talk about learning mathematics. Rather, they talked about getting feedback on their own thinking. Tim described how successful students participate in class.

I'd say, listen carefully to the teacher and other people when they share ideas and participate by, just participate, because sometimes when you do answer something, it just clicks in your head and then you know what you're talking about after you think about it a little bit, so just listen more in class, and then you'll be able to come up with more ideas about how easy to be able to do problems. (3/13/03, #35, Ms. Carson)

He repeated the verb "participate," and spoke in terms of commanding students to participate in his verb tense. Also, he mentioned that in the activity of participating, the student could come up with his or her own ideas, as they could "just click in your head." Another instance when Tim mentioned a preference for sharing his thinking in class was in the following quote:

Well, a lot of times I want to know if my ideas are wrong or right, and if they're wrong, I want to know, so I can correct the next time that I do it, so I like to be picked on, just to, like, raise my hand, then she picks on me. I like to be picked on just as much as everybody else does. Maybe more. (3/13/03, #49)

He spoke of wanting to know and preferring to be called on by the teacher. This repeated preference suggests a commitment to the practice of sharing his thinking for the sake of being corrected. The preference suggests affect supporting his belief in knowing as a negotiated process. He even claims that he wants his ideas to be critiqued in order to determine if his thinking is correct or not. This was representative of many of the negotiated knowers.

Relation Between Epistemological Beliefs and Level of Social Risk

Students' epistemological beliefs about learning mathematics were coherently

related to their beliefs about the risk associated with sharing their thinking publicly during large group discussion in mathematics class. Students who expressed that the potential of being wrong in front of their classmates was somewhat threatening to them also believed that learning mathematics was a process of receiving knowledge. Students who expressed that involvement in class discussions was a low risk activity also believed that learning mathematics was a process of negotiating knowledge. Table 4.6 displays the target students by perceived levels of risk and epistemological beliefs.

Table 4.6: Students' Perceived Levels of Risk by Epistemological Beliefs

	High level of risk	Moderate level of risk	Low level of risk
Received knowing	allen, colleen, hannah, ALYSSA, MARISSA, TRICIA	max, pete	
Negotiated knowing			tim, allison, ALEX, BECKY, BILL, MOLLY, STEVE

*Lower case: Ms. Carson's class. Upper case: Mrs. Evans's class

This relation between students' epistemological beliefs and perceived level of risk associated with classroom participation held true for students in both classrooms. This consistency of a shared level of risk within the groups of students who expressed a similar epistemological belief grouped the target students into two categories: (a) those who expressed a belief in received knowing and expressed a high or moderate level of risk associated with classroom participation and (b) those who expressed a belief in negotiated knowing and expressed a low level of risk associated with classroom participation. The relation between students' perceived level of social risk and epistemological beliefs establishes an initial set of two co-occurring beliefs, for two

clusters of beliefs.

Academic and Social Goals

Target students varied within these two clusters of beliefs in terms of their academic and social goals. During the interviews, students expressed five different goals: (a) an academic goal of completing the task efficiently (b) a social goal of appearing competent in front of their classmates, (c) a social goal of gaining status for improvements in their competence or (d) their behavior, and (e) a social goal of behaving appropriately, and helping their classmates. Table 4.7 summarizes the definitions for these goals and presents a quote to illustrate the nature of students' talk for each goal.

Table 4.7: Definitions of Academic and Social Goals

Goal	Definition	Illustrative quote
Complete task	Focus on own learning over classmates'. Emphasized a high priority of wanting to get own work finished efficiently rather than supporting their classmates' understandings.	Becky: ... I <u>like</u> having to <u>do</u> them [math problems], like writing them down, reading them, instead of, like, having to, like, Mrs. Evans says it, like, she reads it when you do it as a class. I guess I like that, like, sometimes, but not all the time. <u>I'd rather sit down and do it myself.</u> (3/27/03, #124, Mrs. Evans)
Appear competent	Show others that you understand or can do the mathematics. Not necessarily about demonstrating improvement in one's competence.	Hannah: Like, if I understand something, and <u>someone else in my class doesn't</u> , or <u>nobody else in my class doesn't</u> , I'm <u>confident</u> that I can do better and maybe I can get more done. (3/25/03, #90, Ms. Carson)
Gain status	Focus on receiving recognition. Gain attention for having important contributions about mathematics, earn increased status from demonstrating improvement in class, or gain recognition for appropriate behavior.	Max: ... I <u>want to prove it that I know it better</u> than I was. ... <u>last time I didn't really know a lot</u> and this time I'm doing pretty good. (3/26/03, #90-92, Ms. Carson)

Table 4.7, continued

Behave	Follow teacher's expectations for good behavior. Not necessarily about gaining recognition for this choice to behave appropriately.	Marissa: Yeah, another thing with the class is that people <u>disrupt</u> , and that kind of throws you off, about train of thought, and during the problem, Mrs. Evans has to stop, <u>and it's like, ah, you just want to yell out something</u> . Yeah, that's another thing, is people, I think that's kind of, not really intimidating, but it's just <u>disrupting and rude. Rude</u> . (3/31/03, #62, Mrs. Evans)
Help classmates	Focus on classmates' learning. Teach classmates if you know more than they might.	Tricia: Um, you <u>should</u> try to <u>help</u> , like, explain it to people, and give out your ideas, and, like, it's OK if they're not right, because people in the class will <u>help</u> you understand them. They'll explain it to you, they'll <u>help</u> you, like, figure it out and stuff. So I think you <u>should</u> try to volunteer as much as you can. (3/18/03, #50, Mrs. Evans)

Among these five goals, one was more academic than the others: *complete the task*. This goal was explicitly about their mathematical work. The other goals addressed concern for themselves in the social context of the classroom (*appear competent, gain status*) and concern for others (*behave, help classmates*). Some social goals were more prominent in target students from one classroom over another, while the academic goal of complete the task was not found among target students in Ms. Carson's class. Table 4.8 displays which target students expressed particular academic and social goals.

Table 4.8: Academic and Social Goals of Target Students by Classroom

	Complete Task	Appear Competent	Gain Status	Behave	Help Classmates
Ms. Carson's class		Allen Hannah Max Pete Tim	Max Pete	Colleen Hannah Max Pete	Allen Allison Colleen Hannah Max Pete Tim
Mrs. Evans's class	Alex Alyssa Becky Molly Steve	Alex Becky Bill Marissa Molly Tricia	Bill Marissa Molly	Alyssa Becky Marissa Steve	Alyssa Bill Tricia

In Ms. Carson's class, all of the students expressed a desire to help classmates, and almost all expressed a goal of demonstrating their competence. None of them demonstrated a goal for completing the task in their talk. In Mrs. Evans's class, more than half of the students demonstrated this goal in their talk. Many of her students also expressed the goal of appearing competent as well. The other social goals were demonstrated by at least some of the students in Mrs. Evans' class. It is not necessarily the case that these are the social goals that are the most normative among Mrs. Evans or Ms. Carson's classrooms. Rather these are the prominent social goals among the target students. The extent to which the target students are representative of the population of their respective classrooms will be addressed later in the chapter.

In the proceeding paragraphs, I will discuss the evidence necessary for attributing each of the academic and social goals: complete the task, appear competent, gain status, behave, and help classmates.

Complete the task. Students' goal of completing the task stood out in contrast to their social goals, not only because it was the one academic goal students expressed, but because of the students' focus on their own individual learning rather than the collective learning of the class. Students with a goal of completing the task usually did not mention wanting to help classmates during their interview. They emphasized a high priority of wanting to get own work finished rather than supporting their classmates' understandings. These students appeared to be interested in completing their work, and did not mention attaining a deep level of understanding.

Becky's quote in Table 4.7 above expressed a preference that she "liked" and "would rather" spend time doing problems than talking about them and reading about them as a class. She also repeated this idea within her interview. Both the repetition and connection to affect reveal a level of commitment to wanting to spend time working on the problem on her own.

Additionally, Alex talked about wanting to get his work done in class because he did not want to take homework home. This meant that he would prioritize his own work over helping classmates.

Alex: Um, I like to get my homework done before, so then I won't have anything to do at, like, home or something, but, like, I usually try and get that done, and then I'll help 'em. So, like, if somebody asks me a question, I'll be, like, uh, hold on, just a second. I gotta finish this. (04/03/04, #50, Mrs. Evans)

He demonstrated his commitment to this idea through repetition of getting the work "done" or "finished," and through his use of an imperative verb, "gotta." Alex wanted to finish his work to have time for outside activities, while Becky would rather be given the

time and space to work alone. She believed working alone allowed her to complete her work.

Students with this goal talked more about completing the tasks than gaining a deeper understanding of the content. This talk had evidence of an individual focus toward the task than the perspectives of other students.

Appear competent. Students' whose talk suggested a goal of demonstrating competence wanted to let others know that they understood or could do the mathematics. This goal was inherently social because it was centrally concerned with communication with others, as opposed to the focus above on mathematical tasks.

In her illustrative quote from Table 4.7, Hannah expressed feeling more confident when she understood a math problem and other students did not. She spoke of other students not understanding with repetition. Her talk about confidence was evidence of affect. Having the opportunity to look better in comparison to the class was important to her. Her interest in comparing herself to classmates approaches a concern for appearances.

Similarly, Marissa spoke about doing better than her classmates and feeling more confident as a result.

Marissa: I think when you feel like you've done better than the class, I like it when teachers have on their progress report the class average, and then you're like, yay, I did better than the whole class did from the class average. Because then it can just boost your confidence a little bit and feel, oh, yeah. Go me.

(3/31/03, #133, Mrs. Evans)

In both classrooms, the teachers would periodically publicly post students' grades, with student-selected nicknames rather than their own names, and students could examine how they were doing in relation to one another. Marissa's repetition of "doing better than the class" and her use of affective phrases both indicate the importance of this goal for her. Her concern for appearance is revealed in the public reporting of these grades, as all students received these grade printouts, and their grades were posted regularly on a bulletin board under student-selected nicknames.

Students with the goal of demonstrating competence spoke about their own performance in relation to others, and the opportunity to appear as if they are doing better than others, but not necessary demonstrating that they have improved in mathematics.

Gain status. Students whose talk revealed a concern for gaining status were focused on recognition from others. This recognition took several forms, including attention for contributing important ideas about mathematics during the class discussing, earning increased status from demonstrating improvement in class, or gaining recognition for appropriate behavior. They want to do mathematical work, but they also want to be recognized for it.

Max's quote in Table 4.7 demonstrated his commitment to gaining status by contrasting how he "knows it better" now with how "last time I didn't really know a lot," repeating this idea in additional turns in the interview. He also expressed a desire to "want to prove that I know it." This desire is a preference, indicating a sense of affect supporting the goal.

Molly explained that she liked sharing her thinking in class because she enjoyed getting attention for her thinking about mathematics.

Molly: ... I like to talk in front of people, so people, like, pay attention to me and everything... I really like to talk to people and tell them what I think of it.

(3/13/03, #52, Mrs. Evans)

She repeated how much he “likes to talk in front of people,” because of the attention she receives when she has the opportunity to express herself. Both this repetition and connection to affect demonstrate a level of commitment to the goal of gaining status.

Students’ gaining status talk was focused on how others might view them, in terms of their improvement in their mathematical competence, behavior, or the opportunity to gain their attention. The difference between this social goal and that of demonstrating competence or appropriate behavior more generally is how students’ talked about wanting to be noticed for *improvement* in their behavior or competence in mathematics class if they had the goal of gaining status.

Behave. Some students’ talk particularly emphasized the goal of satisfying their teacher’s expectations for good behavior. These students may have differed from those who were interested in gaining status for this behavior in that they were not concerned with recognition for their own choice to behave appropriately, but more concerned that their peers to choose to behave more appropriately.

For example, in Marissa’s quote in Table 4.7, she repeats the words “disrupt” and “rude.” Additionally, she expressed affect in how someone might want to yell out, in frustration with those who are rude. It was not so much her own behavior she was concerned with, but of the ideal student who should be behaving appropriately, herself and others included.

When asked about being a successful mathematics student, another student, Pete emphasized good behavior, not because it would help him understand, but because it would help him raise the participation portion of his grade (in Ms. Carson's class).

Pete: Uh, participate a lot. Pay attention. And don't talk while the teacher's talking... because if you don't participate, then she'll just call on you, and if you weren't paying attention, then you might not get it right or might have no clue what she's talking about. (3/25/03, #26-32, Ms. Carson)

He repeated "pay attention," emphasizing this as an activity of what a good student should do, but accompanied with "don't talk while the teacher's talking." This suggests a sense of taking one's turn to talk when it is most appropriate. He did not always talk about paying attention in light of assisting his learning.

Help classmates. Finally, other target students were concerned not just for their own learning, but that of their classmates. They spoke of assisting others who did not understand. It was as if they did not view the teacher as the only one in the class who could instruct others about mathematics, but that students had a role in this process as well.

In Table 4.7, for example, Tricia spoke of how students ideally "should try to help." This helping behavior was not one-way. She intended to give help in recognition of others' help to her, past and future. Most of the other students with this goal also expressed this symmetric social relationship. Not only did she use an imperative verb, but she repeated these words, suggesting a commitment to this goal. She spoke about this idea in other turns during the interview as well.

Another student, Colleen, also spoke similarly about how she tries to help her classmates when they do not understand.

Colleen: Well, like, if people are struggling, then I like to raise my hand, like, give them a boost.

AJH: What do you mean?

Colleen: Like, if they're, like, struggling, like, they know the answer, but they can't say how they got it or how they got it, but they don't know the answer, I'll raise my hand to, like, kind of jump in a little bit. (3/26/03, #71-73, Ms. Carson)

She repeated that students might be “struggling” and that her response would be to “raise her hand” and “jump in” or “give them a boost.” She also expressed this with affect, saying that she “liked to” do this. Students expressing the goal of helping classmates spoke of whether others understood and if they could have a role in helping them understand.

Students' goals: Summary. Students' talk in interviews demonstrated evidence for five goals, one academic and four social goals: complete the task, appear competent, gain status, behave, and help classmates. When I examined relations between these goals, students' epistemological beliefs, and perceptions of social risk, I determined four clusters of beliefs and goals for these target students, as illustrated in Figure 4.1. Clusters were first determined through differences in combinations of epistemological beliefs and social risk. Then, students were grouped again based on shared goals. Below, I describe in further detail the variations between and within these clusters.

Four Clusters of Beliefs and Goals: Variations Between and Within

As described initially at the beginning of this chapter, these 15 students held four different clusters of beliefs and goals: learn material in public forum, gain attention, do the right thing, and help others while overcoming social risk. Each of these clusters is a set of co-occurring beliefs and goals as expressed by two or more target students. Students within each cluster also diverged, not sharing other goals, creating diversity within clusters as well as between them.

Learn material in public forum. Students in this cluster shared the perception that involvement in whole-class discussion was a low risk activity and the belief that learning mathematics involved negotiating knowledge. Additionally, all of the students who expressed the academic goal of completing the task are in this cluster. These beliefs and goals appear to focus on a concern for learning the content, suggesting that the risk of sharing one's thinking is beneficial. Alex, a member of Mrs. Evans' class, demonstrates this perspective.

AJH: How important is to you to be asked to explain what you think? If you were never asked to explain what you think, would that...

Alex: Uh, you'd just, I don't think you'd learn as much. Other kids wouldn't learn as much, you wouldn't learn as much.

AJH: How wouldn't you learn as much by not being asked to explain?

Alex: Because you can learn more once you have other people's opinions sometimes. And, like, because if you're not getting something, and then if you say something, it might just spark something in your mind. (4/03/03, #225-228, Mrs. Evans)

His use of “you,” including “I don’t think you’d learn as much,” and “if you’re not getting something,” suggests that he’s focused on whether or not he understands the material. Additionally, his talk about both learning from “other people’s opinions” and his own ideas, such as a “spark” in his mind, suggests a belief in a negotiated source of knowledge that rests in both internal understandings and those of others. Other students who shared this perspective talked about getting their questions addressed by their teacher, such as Steve, who mentioned little to no concern about putting his question in front of his classmates.

AJH: If a new student moved into your class and they had to know what they needed to do to be successful, what would you say?

Steve: Pay attention. Do all your work. And, uh, like, don’t be afraid to ask questions. Don’t be afraid to, you know, jump right in to a question that she asked, you know. Always try to be in the group, you know? In the big group discussion.

AJH: Oh, yeah?

Steve: Yeah. Because then you’ll understand it more and you’ll interact with the question more. (3/26/03, #69-72, Mrs. Evans)

He recognized that some students might “be afraid” to jump in to the discussion, but he encourages them to think otherwise, as he has found benefits to his own learning from doing so.

Target students in this particular cluster appeared to value input from their teacher over their classmates. Their lower sense of risk when sharing thinking publicly in class

appeared to support a willingness to trust their own thinking, but still value the input their teacher in developing their thinking about mathematics.

The salient beliefs among students in this cluster is the low level of social risk combined with a belief in the process of knowing as negotiation, as the students expressed that it was worth participating in order to learn. The students in this cluster from Mrs. Evans' class also shared a task orientation goal, or a focus on their own understanding over the need to help others. All of the students in this cluster from Ms. Carson's class valued helping others. These students did not end up in the cluster of *help others while overcoming social risk* because they expressed a lower level of risk. Additional variation among students within this cluster included the goal of helping others and demonstrating competence. With respect to competence, Allison and Steve specifically said they were not concerned with demonstrating competence or appearing as one of the smartest students in the class.

Gain attention. In addition to believing in knowing as a process of negotiation and associating a lower social risk with participation, some students also expressed a social goal of gaining status. This cluster is similar to the previous cluster, *learn material in a public forum*, but the students expressed an additional goal for receiving attention. This goal was expressed with a level of commitment so strong that it distinguished these two students, Molly and Bill, as appearing as slightly different than the others in previously described cluster. These students were also consistently concerned with demonstrating their competence.

Molly wanted to get attention for her ideas about mathematics, as she said, "...I like to talk in front of people, so people, like, pay attention to me and everything"

(3/13/03, #52, Mrs. Evans), while Bill, another student in Mrs. Evans's class, seemed to want to demonstrate that he was more than the class clown. At the end of the interview, in order to follow up on some of his comments, I asked him directly about whether he participated in order to receive attention. He spoke as follows:

Bill: Uh, sometimes I do, and, like, [mentions students' names], and there's a couple people, three or four people who do it because they want attention. But, uh, sometimes I do it because, like, I want people to know that I know how to do the problem, and I'm just not, like, fooling around and stuff all the time, like, some people think. (3/25/03, #129, Mrs. Evans)

He wanted not only to get people's attention because of his sense of humor, but also specifically so they would think of him as more than someone who was "fooling around and stuff all the time."

Molly and Bill expressed consistent epistemological beliefs of negotiating knowledge, perceptions of low social risk, and social goals of gaining status and demonstrating competence. They differed with respect to the goal of helping classmates, as Molly did not express this goal to the extent that Bill did. No target students from Ms. Carson's class were in this cluster.

Do the right thing. The second two clusters both include target students with the epistemological belief of knowing as a process of receiving knowledge and a moderate to high perception of risk associated with sharing their thinking during class discussion. In this cluster, *do the right thing*, students also consistently held the social goal of gaining status, demonstrating competence, and demonstrating appropriate behavior. These students especially expressed a concern with behaving appropriately in their mathematics

class in order to seek a level of social status. Further evidence from Marissa included talking about how others might view her.

I don't want to be, to put down or to make people uncomfortable. Because I wouldn't, I don't know, I try to be friendly, I don't know if I am all the time, but, um, it's not nice. The world doesn't respect you if you put people down, it doesn't help your social career or your business career, either. (3/31/03, 60, Mrs. Evans)

Her consideration for whether her classmates are uncomfortable was directly related to how her behavior would reflect upon herself, whether it would help her "social career," which indicates a concern for status.

Target students in this cluster were from the group of students who believed in received knowing, perceived a higher sense of social risk, and consistently held social goals for gaining status, demonstrating competence, and demonstrating appropriate behavior. Among the students within this cluster, there was diversity in whether they would help others, paralleling the diversity in the *gain attention* cluster. Those from Ms. Carson's class expressed the goal of helping others, while the student in this cluster from Mrs. Evans's class did not.

Help others while overcoming social risk. Most of the students who expressed a belief in knowing as a process of receiving knowledge, or obtaining knowledge from an external source, were in this cluster. All of these students expressed a high level of social risk. Additionally, students in this cluster consistently expressed a social goal of wanting to help their classmates, while students in the previous cluster did not, and they did not consistently express a goal of wanting to gain status or behave appropriately. They talked

about competence more in terms of avoiding appearing incompetent than appearing competent.

Students' sense of social risk and their goal of helping one another appeared to interact. It appeared as though these target students identified with what it is like not to understand, and were interested in helping others who feel this way. This excerpt from Tricia's interview combined two illustrative quotes presented earlier in this chapter. They occurred together as part of a longer exchange in her interview, demonstrating how intertwined the goal of helping was for students with a high perception of social risk.

Tricia: Um, you should try to help, like, explain it to people, and give out your ideas, and, like, it's OK if they're not right, because people in the class will help you understand them. They'll explain it to you, they'll help you, like, figure it out and stuff. So I think you should try to volunteer as much as you can.

AJH: OK. Um, do you find that you ever volunteer?

Tricia: Sometimes. I'm kind of really shy, so I'm like super-conscious about when it comes to answering in front of people. I get, like, all nervous and stuff.

AJH: So, what do you mean, can you say a little more about that?

Tricia: Well, like, in math, I used to be, like, if a teacher called on me or something, and I had my hand raised, my face would turn all red, and I'd get really nervous and I'd start sweating, until I got the answer out, and, like, it, I was, like, always nervous that it would be wrong. And, I'm not so much nervous any more, but I'm still kind of worried about if the answer's wrong, like, I'll get, like, messed up or something. (3/18/03, #50-54, Mrs. Evans)

She spoke of understanding that it was valuable to give help, that she appreciated when

others helped her, and that she is shy of being incorrect in front of her peers, although less shy now than she used to be.

Students in this cluster also had beliefs in received knowing. When asked whether she was someone who participated in the discussions or not, she said, “I’m more towards the listening” (3/18/03, #80). She said that, in order to be successful in mathematics, students should pay attention.

Pay attention, because sometimes some of the problems are really hard, and you don’t get, like, parts of it, like yesterday in the worksheet we were working on, um, Mrs. Dole said the hardest part was on the tables, to figure out the equation. And, like, if you pay attention in class, like I got the equation right away...

(3/18/03, #44)

This focus on taking in knowledge, such as how to figure out the hardest equations, indicated evidence of received knowing.

Other students spoke similarly to Tricia, such as Hannah from Ms. Carson’s class. Hannah said that helping others was important, when she said, “if people don’t understand it, then you should give how you do it.” (3/25/03, #102, Ms. Carson) However, she also revealed a high level of social risk in her talk with statements such as, ‘Sometimes I’ll share, but sometimes I just like to listen to what other people think, I’ll think what I did was wrong’ (3/25/03, #58, Ms. Carson). Her lack of faith in her own thinking combined with the phrase “sometimes I’ll share” suggested a hesitancy to participate due to a heightened sense of risk associated with the activity.

Hannah was also a received knower. When I asked her how she would face a brand new math problem, she said that she could not work on it without being told how to

do it.

Well, if you just give me the problem, and I don't know how to do it, like, if you gave me this book and told me to do it, I wouldn't be able to do it, because I wouldn't understand how to. But if you told, if you taught me how to do it, and how to get through it, then I'd be able to probably do it. (3/25/03, #142)

She emphasized that she would not know how, would not be able to, and would not understand how to do a new problem through her repetition of these similar phrases. She said that if she was “told” then “taught” how to do the problem, then she could probably do it. Her use of probably indicates some lack of faith in her own skills even after being taught. She appears to be more reliant on an authority to learn mathematics than her own ideas.

Students in this belief cluster appeared to be concerned primarily with avoiding the appearance of being incompetent, with their shared perception of a high social risk and a social goal of helping their classmates. They expressed that they were generally not interested in participating because of the potential threat of being incorrect in front of peers and their belief that learning occurred in the process of receiving knowledge rather than negotiating. However, these target students consistently said that if they were to participate, it would be to help their classmates, perhaps because they identified with not being able to understand. These students also were received knowers. Target students in this cluster did not consistently share social goals of demonstrating competence or appropriate behavior.

Diversity within Clusters: Third Quarter Performance

While this study was not designed to explore whether students' beliefs and goals related to their achievement in mathematics class, it is reasonable to examine briefly whether students' beliefs and goals related to their achievement. I collected students' third quarter grades from each classroom as evidence of their achievement, as the rest of my data was also gathered during this academic quarter.¹¹ It may be reasonable to assume that achievement moderates students' beliefs and goals, thus explaining their participation, if there is a relation between students' beliefs and goals and their participation. However, within each cluster of beliefs and goals, there was diversity among students' third quarter performance.

Relations between students' grades and their beliefs are presented below. Are the students who express higher levels of perceived social risk and epistemological beliefs of received knowing also the students who are less successful in mathematics? Are the students who are the most successful in mathematics also the students who believed in negotiated knowing and lower levels of perceived social risk?

Since the relation between students' epistemological beliefs and perception of social risk primarily determined the clusters, Table 4.9 illustrates students' third quarter performance in relation to these beliefs.

¹¹ I acknowledge that there are potential problems with using class grades as achievement data, as they include indices of effort, such as participation grades, as well as performance on mathematical tasks. However, since some students look at course grades as feedback on their performance, they may provide a useful distinction between participants.

**Table 4.9: Target Students' Performance (Letter Grade in Mathematics)
By Beliefs (Epistemological Beliefs and Perception of Risk)**

	A	B	C	D	E
Negotiated & Low Risk	tim, ALEX, BECKY, MOLLY	BILL	allison, STEVE		
Received & Moderate Risk			max	pete	
Received & High Risk		MARISSA, TRICIA	allen, ALYSSA	colleen, hannah	

*lower case: Ms. Carson's class. UPPER CASE: Mrs. Evans's class

Within belief clusters, there was diversity among students' performance. Generally, higher performing students were in the clusters of those who believed in knowing as negotiation and a lower perception of risk, but students with these beliefs also earned grades as low as C's. Two students who earned B's during third quarter expressed a high perception of risk and a belief in knowing as a received process, but otherwise the students with these beliefs were the lower performing students.

Survey Analyses

In addition to interviewing 15 target students, I surveyed all students in both classrooms ($N = 42$) using a Likert-scale instrument that assessed a set of beliefs about and orientations toward learning mathematics. Through my survey analyses, I addressed the following secondary questions: (a) Are there relations between students' beliefs at the group level? (b) Do students beliefs differ by classroom? (c) To what extent are the target students representative of their classrooms?

Recall from Methods (Chapter Three), that I used scales from Fennema-Sherman (1976) in order to assess their *confidence* in mathematics and whether they believe mathematics is *useful*; I used scales from the Conceptions of Mathematics Inventory in

order to assess the *structure* and interconnectedness of mathematics; scales from the Patterns of Adaptive Learning Survey (Midgley et al., 1996) were used to assess students' *task* and *ability orientations*; and I designed four scales based on single items from Schoenfeld (1989) to assess whether students focused on the *process* or product, whether they focused on *concepts* while learning mathematics, whether they believed they could learn *autonomously* or had to rely on an authority, and whether they believed mathematics problems could be solved using *multiple methods* or solution paths. The reliability of three of these nine scales was ≥ 0.70 (Cronbach's α): confidence, task orientation, and ability orientation. However, I will report survey results for all scales. Means, standard deviations, and reliabilities (Cronbach's α) are shown in Table 4.10.

Table 4.10: Survey Results Across the Two Classrooms (N = 42)
Scale Means, Standard Deviations, and Reliabilities

Scale	Mean	S.D.	Cronbach's α
(1) Confidence	3.16	0.34	0.91
(2) Task Orientation	3.53	0.79	0.83
(3) Ability Orientation	3.03	1.07	0.87
(4) Process over Product	3.15	0.36	0.39
(5) Autonomy	3.48	0.54	0.43
(6) Usefulness	3.00	0.33	0.67
(7) Structure	3.09	0.49	0.56
(8) Conceptual	3.33	0.63	0.33
(9) Multiple Methods	3.16	0.43	0.37

Relations Between Beliefs.

I analyzed whether there were relations between beliefs across both classrooms in order to determine whether any of the beliefs assessed through survey scales would cluster together. Intercorrelations could reveal alternative belief clusters not assessed through the analysis of students' talk during interviews. Table 4.11 illustrates the intercorrelations for the nine survey scales.

Table 4.11: Intercorrelations Among Survey Scales

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
(1) Confidence	-								
(2) Task Orientation	-0.05	-							
(3) Ability Orientation	-0.11	0.24	-						
(4) Process over Product	0.06	0.27	0.59**	-					
(5) Autonomy	0.11	0.09	0.29*	0.34**	-				
(6) Usefulness	-0.06	0.08	-0.03	-0.04	0.22	-			
(7) Structure	0.31**	0.17	0.09	0.29*	0.44**	0.09	-		
(8) Conceptual	0.05	0.16	0.23	0.46**	0.28*	0.00	0.50**	-	
(9) Multiple Methods	-0.07	0.11	0.30**	0.26*	0.10	0.09	0.17	0.32**	-

* $p \leq 0.10$

** $p \leq 0.05$

While a number of inter-scale correlations were significant, if scales with reliabilities below 0.50 are excluded, only one of the correlations was significant. There was a significant positive correlation (0.31) between confidence and structure. In other words, the students with a higher sense of self-confidence in mathematics were more likely to believe that concepts in mathematics were interconnected, and students with lower levels of self-confidence were more likely to believe that ideas in mathematics are isolated from one another. It is unclear if the beliefs as measured on the survey are generally unrelated, or if low reliability scores led to lack of relations.

Differences Between Classrooms

In order to determine whether the students in the two classrooms expressed similar or different beliefs in their survey responses, I calculated means and standard deviations for each classroom, and calculated two-tailed t-test comparisons between the class means, as seen in Table 4.12.

Table 4.12: Survey Results: Comparisons Between Classrooms
Means (Standard Deviations)

	(1)*	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
Carson N=16	3.21 (0.44)	3.44 (0.57)	2.92 (1.19)	3.07 (0.36)	3.53 (0.55)	3.05 (0.42)	2.99 (0.51)	3.19 (0.57)	3.07 (0.39)
Evans N=26	3.13 (0.27)	3.59 (0.90)	3.11 (1.01)	3.21 (0.36)	3.46 (0.54)	2.97 (0.27)	3.15 (0.48)	3.41 (0.66)	3.21 (0.46)

*p<0.05

The classrooms were significantly different on one scale: *confidence*. Otherwise the classrooms were not significantly different on any of the other scales. However, I see this significant difference as occurring more due to the standard deviations than a large difference between scale means. Differences between the confidence scale means were

less than 0.1, which does not appear to be a practically significant difference. As a group, the students in the two classrooms did not appear to hold different beliefs about learning mathematics, other than their confidence in mathematics.

Additionally, I examined differences between classrooms in terms of students' beliefs as expressed in interviews. I assessed whether the target students from each classroom were distributed between the two epistemological belief categories differently than expected by chance. Due to the low numbers, expected values less than ten (2×2 matrix), I used Fisher's Exact Test.

Table 4.13: Epistemological Belief by Classroom

	Received Knowing	Negotiated Knowing
Ms. Carson's Class	Allen, Colleen, Hannah, Max, Pete (5)	Tim, Allison (2)
Mrs. Evans's Class	Alyssa, Marissa, Tricia (3)	Alex, Becky, Bill, Molly, Steve (5)

The two-sided Fisher's exact test did not confirm that the beliefs in these two classrooms occurred more frequently than we would expect due to chance ($p = 0.315$). It may be equally likely that students with epistemological beliefs of knowing as a process of receiving knowledge would be in Ms. Carson's classroom as in Mrs. Evans's classroom.

Target Students Relative to their Classmates

In order to determine whether the target students were similar to their classmates in terms of their responses on the survey, I compared each target student's mean to their class means for each survey scale. Recall that Ms. Carson's class had seven target students, while Mrs. Evans's class had eight target students. In Table 4.14 below, students' means were categorized as "within" if they were within one standard deviation above or below the class mean; otherwise, the student was counted as either above or

below the mean. Table 4.14 illustrates the number of students in each classroom who were within the class mean, above the mean, and below the mean.

Table 4.14: Survey Results: Target Students in Relation to Their Respective Populations

Frequency Within, Above, and Below their Class Mean

	Ms. Carson's class			Mrs. Evans's class		
	Within	Above	Below	Within	Above	Below
(1) Confidence	4	3	0	5	1	2
(2) Task Orientation	6	0	1	6	1	1
(3) Ability Orientation	3	1	3	5	2	1
(4) Process over Product	4	2	1	6	2	0
(5) Autonomy	5	1	1	5	1	2
(6) Usefulness	4	2	1	7	1	0
(7) Structure	6	0	1	7	0	1
(8) Conceptual	6	1	0	7	1	0
(9) Multiple Methods	3	2	2	6	1	1

In Mrs. Evans's class, for every survey scale, five or more target students were within one standard deviation of their class mean. In Ms. Carson's class, five or more target students were within one standard deviation of their class mean for four out of nine scales: task orientation, autonomy, structure, and conceptual. Four or three of the target students were within the mean on the other five survey scales. More of the target students in Mrs. Evans's class were within the mean survey responses than those in Ms. Carson's class. In each class, most target students were within the mean. The 15 target students appear to be reasonably representative of each classroom of students. Table 4.15 shows which target students responded above and below their class mean.

**Table 4.15: Survey Results: Target Students in Relation to their Class Scale Mean
Survey Scale by Students**

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
Ms. Carson's class									
Allen			B				B		
Allison			B			B			B
Colleen			B	B	A	A			
Hannah	A								
Max	A	B		A		A			A
Pete	A				B				B
Tim			A	A				A	A
Mrs. Evans's class									
Alex	B		A		A				B
Alyssa	A	B	B	A	B	A			
Becky			A						
Bill									
Marissa								A	A
Molly					B		B		
Steve	B			A					
Tricia		A							

A=Above, B=Below

In Ms. Carson's class, three target students were either above or below the class mean on four or more of the survey scales: Colleen, Max, and Pete. In Mrs. Evans's class, two target students were either above or below the class mean on four or more survey scales: Alex and Alyssa. The rest of the target students (four in Ms. Carson's classroom, six in Mrs. Evans's classroom) were within the class mean on four or more survey scales. Generally, target students were within their class's mean scores on the survey.

Summary

Analyses of students' talk during their interviews revealed that their epistemological beliefs interacted with their perceptions of social risk, and that social goals factored strongly into their motivations to participate during whole-class discussion. Students expressed that participating in whole-class discussion has benefits, such as opportunities to learn content through being corrected or hearing new ways of

solving problems, opportunities to demonstrate competencies in both their mathematical understandings and behavior, as well as opportunities to gain recognition for these competencies, and opportunities to be supportive and helpful to classmates. However, some students found whole-class discussions to be threatening due to the potential of being publicly incorrect in front of their classmates and teacher.

The 15 target students could be separated into four clusters of beliefs and goals based on their perceptions of social risk, epistemological beliefs, and academic and social goals. I refer to these clusters as: *learn content in a public forum*, *gain attention*, *do the right thing*, and *help others while overcoming social risk*. Two of these clusters, *learn the material in a public forum* and *gain attention*, appear to be more self-focused while the other two appear to be more community-focused. The students in the self-focused clusters emphasized goals that benefit the student, such as learning content, gaining status, or appearing competent. The goals of the students in the community focused-clusters included behaving and helping classmates. On the one hand, the students with beliefs of negotiated knowledge and low social risk may have productive beliefs that allow them to take advantage of opportunities to learn in discussion-oriented mathematics classrooms. On the other hand, students with social goals of helping and behaving may have productive goals related to fostering a cohesive classroom community.

Students' beliefs did not appear to be bound by classrooms, though there were some differences between classrooms among students' goals. Both classrooms included target students who expressed three out of the four clusters of beliefs and goals. Fisher's Exact Test did not reveal significant differences between the classrooms based upon epistemological beliefs. Eight of the nine survey scales were not significantly different.

All of the target students in Ms. Carson's classroom expressed a goal for helping classmates, while this was not the case among Mrs. Evans's target students. None of Ms. Carson's target students expressed a goal for completing the task, while this goal was mentioned by more than half of the target students in Mrs. Evans's class.

Third quarter grades did not appear to completely explain differences in students' beliefs. While the higher performing students expressed beliefs about negotiated knowing and low social risk, there were also some students who earned C's in this category. Students with beliefs of received knowing and high social risk earned grades ranging from B's to D's, with more of the lower performing students holding these beliefs. Students who earned B's and C's expressed both types of beliefs, which suggests that students with moderate grades could hold either type of belief.

In the proceeding chapter, I will situate these students' participation in context by describing the nature of discussions in each classroom, and then I will present the results that demonstrate how the target students participated in these whole-class discussions.

CHAPTER FIVE

RESULTS: THE NATURE OF WHOLE-CLASS DISCUSSIONS AND TARGET STUDENTS' PATTERNS OF INVOLVEMENT

In the previous chapter, data suggested that target students' beliefs and goals were not completely shaped by their current classroom experiences. Students' beliefs and goals varied within each classroom, and some students with similar beliefs and goals were in different classrooms. As the overarching goal of this study is to examine whether and how students' beliefs and goals relate to their participation in whole-class discussions, the next logical step is to examine the patterns of participation that took place in each classroom. The results presented in this chapter address the following question: How do the target students participate in whole-class discussions? I chose to examine students' involvement in whole-class discussion rather than engagement in other activities, such as small group discussions and individual problem solving, because of the particular push to encourage large group discussion in the NCTM reform movement and the potential threat adolescents might experience when expected to speak about their thinking in front of their peers.

In order to examine students' involvement in whole-class discussions, I first compare and contrast the nature of discussions in the two classrooms in the beginning sections of this chapter. Similarities and differences in classroom discussions, including the amount of structure or flexibility in each classroom, reveal students' opportunities to participate in each setting. Then, I compare and contrast the target students' participation

in the latter sections of the chapter in terms of their level of involvement and the nature of their involvement.

The Nature of Whole-Class Discussions in Two Mathematics Classrooms

Before presenting the target students' patterns of involvement, I illustrate the nature of whole-class discussions in these two mathematics classrooms. Since these two teachers shared a commitment to faithfully implementing their Standards-based mathematics curricula, they may have conducted their whole-class discussions somewhat similarly. These similarities may have provided a common experience for structuring students' opportunities to engage in whole-class discussion. Each teacher may vary in how they implemented these discussions as well, and these variations may have created differential opportunities for students to involve themselves in particular forms of classroom talk.

Similarities

Whole-class discussions in these two classrooms had some similarities that potentially shaped students' involvement. These similarities included the teachers' commitment to whole-class discussions, the teachers' commitment to following the textbook series, using the CMP instructional model, and pursuing multiple solution methods for solving or interpreting a mathematics problem.

Commitment to Textbook Series

The teachers expressed similar levels of commitment to implementing discussions and involving students in the construction of meaning during mathematics class (personal communication, 08/02). One way they both put this level of commitment into practice was by dedicating a portion of time to whole-class discussion for every lesson.

Essentially, whole-class discussions were occurring in both classrooms. At the minimum, if students wanted to talk about mathematics in whole-class discussion, they had the opportunity in both settings. However, I will present later in this chapter that the teachers dedicated different amounts of time to whole-class discussion. These two classrooms were both settings in which students were invited to talk about mathematics.

Also, the teachers demonstrated commitment to the CMP textbook series. They communicated with each other in order to stay on a similar schedule, and each used the investigation problems as the foundation for their lessons. Classroom discussions, within a day or two of each other, were about similar mathematics problems, since students were working on the same mathematics tasks in small groups or individually before the discussions. Additionally, both teachers utilized features of the textbook series including reflection questions at the end of each investigation, assigned A.C.E. problems for homework, and did not supplement the textbook series with additional practice problems. Lessons in both classrooms were based on the same mathematical tasks, those directly from the textbook series.

Instructional Model

The teachers both implemented CMP's recommended Launch, Explore, and Summarize instructional model (Lappan, Fey, Fitzgerald, Friel, & Phillips, 1996), which is another example of their commitment to the textbook series. According to this model, *Launch* involves introducing an investigation problem to students, *Explore* involves class discussion of students' ideas from working on the investigation in small groups or individually, and *Summarize* involves wrapping up the discussion, helping students take away the main ideas. This *Launch, Explore, Summary* model takes the place of the

teacher's presentation of content in a more traditional mathematics setting, in which the teacher models steps for solving a problems and provides the students with opportunities for guided practice. Instead, during the *Launch*, students are introduced to a problem to work on. During *Explore*, students spend time working on these problems either individually or in small groups, and they also talk about their thinking during whole-class discussion. This is different from guided practice time during a traditional lesson, since students have not been given explicit instruction for how to solve the problems, but instead are invited to try the problems on their own first. During whole-class discussion, students' ideas about how to solve the problems serve as a replacement for the teacher's prescriptions of the steps for solving the problems. Then, during *Summarize*, with the teacher's guidance, the class determines which new concepts and approaches to problem solving came out of working on that investigation problem, similar to a closure of a traditional lesson, but perhaps with more student involvement.

Along with the shared Launch, Explore, Summarize instructional model, both teachers structured the mathematics class periods with similar activities each day. Typical activities in each of these classrooms included: warm-up problems at the beginning of the class period, grading homework, introducing a new problem for investigation (*Launch*), seatwork over new problems (*Explore*) or homework problems, and large group discussion about the seatwork (*Explore and Summarize*). Although this structure was adhered to in both classrooms, later in this chapter I will demonstrate differences in the implementation in this structure, including the time spent on each activity in the class period and the nature of the talk during whole-class discussion.

Pursuit of Multiple Solution Methods

One similarity in the nature of classroom discussions was that both teachers solicited multiple methods for solving or interpreting the mathematics problems during whole-class discussion. In Problem 2.3 from the second investigation in *Moving Straight Ahead*, students examined whether or not it was a better deal to get a membership to a movie theater or to pay per movie. Both classrooms discussed multiple solution methods, but there were differences in what counted as an alternative solution in each classroom, such as whether the focus was on calculation paths or various representations. For example, Ms. Carson solicited additional responses from students in the following manner:

T: OK, did anybody else get 69 dollars for a member to see 20 movies?

{Ss raise their hands.}

T: Did anybody do it differently than what Tyler did? Holly, what did you do?

(interaction segment #2, 2/26/03)

Ms. Carson consistently solicited an alternative perspective from students by asking, “Did anybody do it differently from [student’s name]?” Ms. Carson did not request alternative perspectives through any other types of questions. Her consistent use of the verb “do” in these questions when she solicited multiple solution methods indicates a focus on different calculation paths over other conceptions of multiple solution methods.

In contrast, Mrs. Evans’s requests for alternative perspectives on the problems involved requesting more than alternative calculation methods, and also involved reflecting back students’ perspectives for feedback. In the following example from the

whole-class discussion of Problem 2.3, Mrs. Evans's request for an alternative method involved a request for a different perspective other than an additional calculation path.

T: So I could have used that calculator and table. It happened at 14, they were both 63 dollars. Did anybody find that 14 in a different way than table?

Jim: Um, yeah, I did. I did it on a graph. (Interaction segment #20, 3/03/03, Evans)

Mrs. Evans' use of "a different way than table," and her listing of what they could have used (calculator and table) indicates that alternative forms of representations also counted as multiple solution methods. These representations included the use of the calculator for calculating values or using an equation, finding the values in a table, using the graph, as mentioned by Jim. Bringing in alternative representations in addition to multiple calculation paths indicates that Mrs. Evans's whole-class discussions encompassed a broader perspective for what sounds as an alternative solution. Using a table for solving a problem rather than a graph is a way of taking a new lens on the relationship, perhaps seeing something about the relationship differently as a result of the alternative representation, while an alternative calculation path may not shed new light on the relationship, but rather allows students to become more proficient in their calculation skills.

Another way Mrs. Evans requested multiple solution methods was in her requests for students to evaluate the solutions of others. In the following example, Mrs. Evans asked Molly to discuss her critique of another student's solution and invited others to share their viewpoints as well.

T: Molly is saying she doesn't like this one, say it one more time, 'cause I'm not sure it's all been heard.

Molly: because, 10, it's saying that for every kilometer you go, you're adding 10 more, so if you go 2, you're gonna have 20, and it's really only 10.

T: so you would disagree with that one.

Molly: Yeah.

T: How about other people? (Interaction segment #32, 2/21/03, Evans)

Mrs. Evans's use of "like," in reference to whether Molly "liked" the problem she was critiquing was one way that she invited students to share alternative perspectives on the problem. Then, once one student put her ideas on the floor, rather than evaluating her response, Mrs. Evans asked what others thought. While this is not the same as asking whether anyone did the problem differently, it is a request for alternative perspectives on the problem. In the act of critiquing one another, students may suggest an alternative solution method to contrast with the method they are critiquing.

Both classrooms explored multiple methods for examining linear relationships, with differences in what counted as an alternative solution. Ms. Carson explicitly solicited alternative calculation paths, while Mrs. Evans additionally requested alternative representations, as well as asking students to evaluate one another's solutions.

The similarities between these two classrooms may provide common experiences for these seventh graders around participating in whole-class discussion about mathematics. Both classrooms had discussions about mathematics over the same mathematical tasks. Both teachers implemented a similar instructional model, as advocated by the textbook series, and a similar series of typical activities during a class

period. Additionally, multiple solution methods were pursued in both classrooms. It is unclear whether the similarities between the two classrooms, relative to more traditional settings, or the differences between the whole-class discussions play a stronger role in considering the development of students' beliefs and goals and their participation practices. Compared to the population of mathematics teachers across the nation, these two teachers may be relatively similar, but there were also some differences between them, beyond what counted as an alternative solution in each classroom.

Differences

As mentioned above, while these two teachers implemented the *Launch, Explore, Summarize* instructional model, the teachers varied in their implementation of the instructional model. For example, during the *Launch* in Ms. Carson's class, the teacher read the problem to the students from the textbook, while Mrs. Evans's class spent slightly more time on this event in the model, and this time was spent in terms of the teacher retelling the problem in an elaborated form, connecting back to previous investigation problems, with the students interjecting jokes or comments. The *Explore* and *Summarize* lesson elements included the whole-class discussions, and based on variations in the discussions, Ms. Carson's class appeared to be more structured, while Mrs. Evans's class appeared to be more flexible.

In the *Explore* and *Summarize* portions of Ms. Carson's lessons, the whole-class discussions appeared to be able to be predicted by the pages of the textbook, as the content of the interaction segments consistently followed the order of each of the sub-problems in the investigation problem (part a, b, c, etc.). Exploratory work around the investigations appeared to happen less during whole-class discussion and more during

seatwork time, when the teacher would go around from table to table (with three to four students at a table, and students were invited to work together, but they also had the option to work alone) and consult with students. More time was spent on seatwork during Explore in Ms. Carson's class than in Mrs. Evans's class. The purpose of the whole-class discussion appeared to be having students check their answers from their work on the investigations, to get feedback as to whether they were right or wrong. The teacher evaluated whether the solutions were correct rather than other students, and this was seen in the adherence to the Initiate-Respond-Evaluate (Mehan, 1979) model of discourse during whole-class discussion in Ms. Carson's class. During these discussions, students rarely contributed off-topic comments, and the teacher was the initiator of new topics of discussion rather than students. Based on how the content of the talk followed the structure of the sub-problems of the investigation problems, the adherence to the I-R-E-model, the role of the teacher as the primary evaluator, and the lack of student initiated or off-topic talk, Ms. Carson's whole-class discussions appeared to be highly structured.

In Mrs. Evans's class, the whole-class discussions also covered all of the sub-problem in the investigation, but the events in the discussion were less predictable. At times, students initiated new topics of conversation, such as asking for clarification about a particular idea, going back to a previous problem, or revising their vocabulary. The teacher took up these suggestions to varying degrees rather than moving to the next sub-problem. It appeared that the teacher would follow up on the student-initiated topics if these aligned with her mathematical agenda for the day, or if they seemed particularly mathematically poignant. Students had not completed or clarified their thinking by the time the classroom discussion took place; these discussions appeared more exploratory in

nature than those in Ms. Carson's class, as students sometimes revised their own contributions during discussion. Sometimes the class did not finish discussing an investigation in one class period, and they would continue talking about it in the next class period. This did not generally happen in Ms. Carson's classroom. Mrs. Evans's class spent more time on whole-class discussion than Ms. Carson's class, and relatively less time on seatwork. Additionally, students in Mrs. Evans's class would not only initiate mathematical topics during the discussion, but would also make off-topic comments. Since the discussion did not always follow the order of the sub-problems in the investigation, students initiated discussion topics, both mathematical and off-topic, and the whole-class discussions sometimes extended beyond the class period, if necessary, the discussions in Mrs. Evans's class appeared to be more flexible.

In order to systematically examine the apparent differences between the two classrooms, I pursued two lines of analysis: (a) I identified segments of activity in the two classrooms and compared and contrasted the amount of time spent on each activity, and (b) I identified elements of classroom discourse that would allow me to assess the structure or flexibility of the whole-class discussions. The class periods analyzed below included the days spanning five consecutive investigation problems from *Moving Straight Ahead*.

Amount of Time Spent on Typical Activities

As mentioned above, Mrs. Evans's class spent more time on classroom discussion, while Ms. Carson's class spent more time on seatwork. In order to examine the amount of time spent on discussion or seatwork in these two classrooms in more detail, I measured the time spent on each of the typical activities: warm-up problems at

the beginning of the class period, grading homework, introducing a new problem for investigation (*Launch*), seatwork over new problems (*Explore*) or homework problems, and large group discussion about the seatwork (*Explore and Summarize*). I compared and contrasted the time spent at the level of the class period and at the level of each lesson, since the *Launch, Explore, Summarize* sequence sometimes occurred over multiple days.

Time spent over the average class period. In order to examine the time spent on each activity at the level of a particular day, I averaged the time spent on each activity for ten observed days spanning five consecutive investigation problems. The only activity that happened with regularity in each classroom every day was the warm-up problems on the overhead at the beginning of each classroom. Otherwise, homework problems may or may not have been graded in class. Some days, students turned in the homework problems and the teacher would grade them. Also, on some days the entire class period was taken up by seatwork with little to no classroom discussion, while other days the class period mostly consisted of whole-class discussion, with little to no seatwork. Given this variability day to day, I pursued the contrast between classrooms by examining the average time spent on each activity across ten days of observation. The following table represents a typical day in each classroom, as calculated by taking the average time spent on an activity across the observed days.¹²

¹² Days included in this analysis include 2/11, 2/13, 2/18-2/21, 2/24-2/26, & 3/03.

Table 5.1: Average Minutes Spent on Typical Activities during a Class Period

	Ms. Carson's Class	Mrs. Evans's Class
Warm-up problems / class business	10.6	15.4
Grade homework	3.3	5.4
Introduce new problem (<i>Launch</i>)	3.1	3.3
Seatwork (<i>Explore</i>)	24.5	8.9
Whole-class discussion (<i>Explore & Summary</i>)	15.4	21.4
TOTAL*	~57	~54

*Totals do not add up to 58 minutes due to transition times between activities.

Similarities in the ways the two classrooms used time included the warm-up problems (or “openers”), grading homework, and introducing the new problems (launch). Each of the classrooms opened the period by working on a set of 1-5 problems that as students were getting settled. The teacher would then go over these problems with the class and talk about any class business. During the four weeks of classroom observation in the spring semester, Mrs. Evans’ class spent about five minutes more on the warm-up problems and class business than Ms. Carson’s class. Neither class spent much of the period grading homework or discussing answers to homework problems. Instead, the teachers would more often grade students’ homework themselves and hand them back to the students with feedback.

The principal differences between the two classrooms were in the time spent on seatwork and whole-class discussion activities. Ms. Carson’s class spent about three

times as much time on seatwork. During seatwork in Ms. Carson's class, students often worked alone, but they also had the option to ask questions to the other two to three students sitting at the table or of the teacher. Mrs. Evans' class did not spend as much time on seatwork. When students worked on investigation problems during seatwork time in Mrs. Evans's class, they usually worked with a partner or in small groups, in contrast to the students in Ms. Carson's class who worked more independently. Mrs. Evans spent extended periods of time sitting down and talking about the investigation problem with one group or another during this time, spending time with one or two of the five or six small groups, in contrast to Ms. Carson who walked around the room taking questions from students with raised hands from almost every small group. Whole-class discussion time also appeared to be different in each classroom, with Mrs. Evans' class typically spending more time on discussion than Ms. Carson's class – approximately 50% more time.

Time spent over the average lesson. As mentioned earlier, each day varied, so characterizing each classroom in terms of a typical day could be misleading, as sometimes no time was spent on an activity, such as seatwork or discussion, on a particular day. The unit of a "lesson" encompassed the *Launch, Explore, Summarize* sequence for an investigation problem, and this sequence usually took place over the span of two or three days. The following table illustrates the average time each class spent on introducing new problems, doing seatwork, and in discussion over problems 1.2 to 2.3 in the Moving Straight Ahead textbook, a total of five problems.

Table 5.2: Average Time Spent on Typical Activities during a Lesson

	Ms. Carson's Class	Mrs. Evans's Class
Introduce new problem (<i>Launch</i>)	6.5	9
Seatwork (<i>Explore</i>)	33.25	18
Whole-class discussion (<i>Explore & Summary</i>)	30.75	45.25

At the level of a lesson, similar patterns follow as those seen at the level of a class period: each class spent a relatively small amount of time introducing the problem, with Ms. Carson's class spending less time on this activity. Ms. Carson's class spent more time on seatwork (almost twice as much as Mrs. Evans' class) and Mrs. Evans class spent more time on large group discussions (approximately 1.5 times as many minutes).

Interaction segments and time spent on discussion per day. The additional time Mrs. Evans's class spent on discussion provided students in her class with more opportunities to talk about mathematics in front of the whole class. Another way to demonstrate that the students had increased opportunities to participate in Mrs. Evans's class would be to compare the number of interaction segments in the whole class discussions. Table 5.3 illustrates the number of interaction segments in each classroom across four days of classroom discussion.

Table 5.3: Number of Interaction Segments per Day

	2/20/03	2/21/03	2/25/03	2/26/03	3/03/03	Average
Ms. Carson's class	81	16	41	24		41
Mrs. Evans's class	68	83	53		28	58

Mrs. Evans's class, on average, had the opportunity to participate in almost 50% more interaction segments, as Mrs. Evans's class had an average of 58 interaction segments over these four days and Ms. Carson's class had 41. Three of the four days in Mrs. Evans's classroom had 50 or more interaction segments, while only one of the days in Ms. Carson's classroom had 50 or more interaction segments. Looking across the four days, there was also variability in the amount of interaction segments for both classrooms. Ms. Carson's class had greater variability, with a range between 16 and 81 interaction segments. This variability in interaction segments was generally paralleled in the amount of time spent on discussion during these four days.

Table 5.4: Time Spent (in minutes) on Whole-Class Discussion per Day

	2/20/03	2/21/03	2/25/03	2/26/03	3/03/03
Ms. Carson's class	42	7	30	13	
Mrs. Evans's class	45	33	36		20

Generally, more interaction segments occurred during a longer duration of class discussion. Exceptions, when increased interaction segments were not accompanied by increased in time spent in whole-class discussion, were related to the pace of the discussion, as more interaction segments could occur over a similar time period with more overlapping talk and less time between turns. The highest number of interaction segments, 83, occurred during a 33 minutes discussion in Mrs. Evans's class rather than the longest times spent on classroom discussion 42 or 45 minutes. Again, there was more variability in the amount of time spent on class discussion in Ms. Carson's class, with a range between 13 and 42 minutes. Three of Mrs. Evans's whole-class discussions were at or above 30 minutes on these four days, while two of Ms. Carson's fell into this category.

An examination of the number of interaction segments per day and the time spent on whole-class discussion per day also illustrates that Mrs. Evans's students had more opportunities to participate in class discussion.

The increased time spent on whole-class discussion and the decreased time spent on seatwork in Mrs. Evans's class appears to be related to the flexibility of the nature of the discussions. A teacher who is willing to dedicate more time to discussion may also be more willing to follow up on students' initiation of discussion topics, either mathematical or off-topic. Such follow up on student-initiated topics did not occur in Ms. Carson's class, as students did not initiate topics for discussion. It is not clear whether more time was spent on discussion because Mrs. Evans was willing to follow up on students' contributions or if Mrs. Evans followed up on students' contributions because she was willing to spend more time on whole-class discussion. Additionally, if Ms. Carson's class spent more time on seatwork, it is possible that the students had the opportunity for more flexible discussions with the teacher during this activity, so the purpose of the whole-class discussions in Ms. Carson's class may have instead focused on making sure the students were correcting their answers for the investigation problem. The amount of time spent on discussion appears to be related to whether the whole-class discussion was more structured or flexible.

Contrasting the Nature of Classroom Talk

In addition to the differences in the allocation of time, I selected three elements of classroom talk to analyze in order to assess whether Mrs. Evans's classroom was more flexible and Ms. Carson's was more structured. These characteristics were created from a bottom-up comparison of my observations of the two classrooms, as well as an analysis

of the literature on classroom discourse: (a) the opportunities for off-topic discussion, (b) how students would obtain the floor, and (c) the extent the discussions maintained or deviated from the IRE model (Mehan, 1979).

Variability in amount of off-topic discussion. In order for an interaction segment to be coded as “off topic talk,” the entire interaction had to consist of content separate from the mathematics under discussion; it was not coded as off topic if only part of the segment addressed alternative content.

In Ms. Carson’s class, the topic under discussion in the large group was always the mathematics content, during the four days analyzed, as well as during my informal observations throughout the study. Only one-percent of the exchanges on 2/20/03 were off the mathematical topic in Ms. Carson’s class from the four days of analyzed whole-class discussion, and it was two students mentioning that they were missing the vocabulary sheet.

In Mrs. Evans’ class, by contrast, while much of the large group discussion focused on mathematics, some of the interaction segments also consisted of off-topic talk, such as whether the students were required to take notes, whether they would have homework, or generally teasing one another.

Table 5.5 illustrates the percentage of off-topic interaction segments for both classrooms across four days of whole-class discussion.

Table 5.5: Percentage of Off-Topic Interaction Segments

	2/20/03	2/21/03	2/25/03	2/26/03	3/03/03	Average
Ms. Carson’s class	1%	0%	0%	0%		0.25%
Mrs. Evans’s class	7%	17%	13%		7%	11%

Generally, in Ms. Carson's class, students do not engage in off-topic talk, while in Mrs. Evans's classroom, students engage in off-topic talk in approximately 10% of the interaction segments. While this may seem like a significant chunk of interactions to spend during the classroom discussion on off-topic talk, it is possible that the Mrs. Evans's willingness to allow off-topic talk creates a comfortable space for student's to initiate talk about mathematics as well.

The following examples illustrate that students primarily initiated the off-topic talk in Mrs. Evans's class, but the teacher also would engage in the off-topic talk as well. Some of these interaction segments addressed classroom management, such as the example below.

T: Seth.

Steve: Can you tell Alyssa to stop clicking her pens and everything?

T: Please stop clicking your pens.

(Interaction segment #6, 2/20/03)

In other cases, students were attempting to negotiate the amount of work they were expected to do.

Alyssa: Are we going to have homework tonight?

T: Yes.

Alyssa: No!

Gary: Yes! (Interaction segment #10, 2/21/03, Evans)

Others were about interpreting the text, such as a series of interaction segments discussing how to pronounce a character's name from a CMP investigation, Gilberto, which was an unusual name for White students in a rural setting to encounter.

T: Yes?

Sally: Um, I told my dad, the Gilberto thing.

T: Uh, huh.

Sally: And, um, he says that it's Gilberto {emphasis on the G sound}.

T: We can pronounce it that way if you want. (Interaction segment #8, 2/21/03, Evans)

Some of the off topic interactions were initiated by the teacher as well, some related to classroom management, and others were joking around with students. The example below is a case of the students and teacher making jokes together.

T: I lost my opener. {in a funny voice}

Bill: I lost my brain a long time ago.

Alyssa: You never had a brain.

Albert: We lost our can opener.

{Ss laugh.} (Interaction segment #65, 2/21/03, Evans)

Approximately one-third of the off topic interaction segments were initiated by Mrs. Evans and the rest were initiated by students, suggesting that the teacher may have accepted, and perhaps implicitly encouraged, slipping back and forth between off-topic and on-topic talk. This focus on both mathematical and non-mathematical talk suggests a more informal tone in Mrs. Evans' whole-class discussions than in Ms. Carson's whole-class discussions. It is possible that the informal tone led to students also being more likely to initiate talk about mathematics as well as talk about off-topic issues.

Variability in amount of students' attempts to obtain the floor. Students initiated some interaction segments; to qualify as "student initiated" segments, students

either had to call out to get the floor, or brought up a topic for discussion, when called upon by the teacher, that diverged from the question asked by the teacher. Additionally, the segment did not qualify as student-initiated if the solicitation was not picked up on by the teacher or other students; instead, it was considered to be an attempt at student-initiation, also coded as assertive talk in terms of individual students' patterns of involvement. Some of the segments that were coded as "student initiated" were about mathematics and others were off-topic.

In Ms. Carson's class, students raised their hand to be called on, and generally did not speak until recognized formally by the teacher. Out of the four analyzed whole class discussions, 4% of the interaction segments were student initiated. Since there were so few student initiated interaction segments, there was not a typical segment. In the following example, Tim initiated an interaction segment correcting a calculation.

Tim: There's negative 3 up there.

Max: It should be negative 1 point 5.

T: Because we're trying to get a negative 3 up here?

Tim: Yeah.

T: So I should have a negative 1 point 5?

Ss: Yeah.

T: Then, plus a negative three, I get... (Interaction segment #13, 2/21/03, Carson)

In this case, the students put the idea on the floor, and Ms. Carson took the idea up by asking follow-up questions to clarify what the students said. Then she took the floor back from the students, redirecting the discussion, as she asked, "and if I look at my table, when X is 2, what does it say for Y?"

In Mrs. Evans' class, student initiated interaction segments were substantially more frequent. Table 5.6 illustrates the percentage of interaction segments that were initiated by students across the four analyzed days of classroom discussion.

Table 5.6: Mrs. Evans's Class's Student Initiated Interaction Segments

	2/20/03	2/21/03	2/25/03	3/03/03	Average
Student-initiated segments	49%	36%	17%	14%	29%

21% of the total amount of student initiated interaction segments was also off-topic, while the rest of the segments were about mathematics. Since examples of off-topic segments were presented above, the following are examples of student initiated interaction segments about mathematics. In the first example, Becky initiates an interaction segment by asking a question about the format of a proposed equation, asking about the order of the symbols, specifically whether the equal sign could go closer to the end rather than the beginning of the equation.

Becky: Why don't you just do 2 times K equals M?

T: Could I do 2 times K equals M?

Jim: No.

T: Can I write it in the other direction?

Ss: No. Yes.

T: What tells me that that's OK?

Alex: Uh, because it's Y equals and that equals Y. (Interaction segment #22, 2/21/03, Evans)

At times, student initiated segments were similar to Becky's segment above: A student posed a question to the teacher, and then the teacher reflected the question back to the rest of the class. Other instances of student-initiated segments were students putting their ideas on the floor, when the teacher was collecting a range of students' perspectives. In the example below, the teacher initially asks the class, "What about this one?" The teacher calls the class's attention to a particular students' suggested equation, $y = 10$, for expressing a relationship such that a participant in a walkathon would collect a flat rate of \$10 from those who are interested in donating rather than a fee per kilometer. Mrs. Evans is asking the class to evaluate this equation. Alex presents his opinion, then Jamie also offers her opinion, without being solicited explicitly.

T: What about this one?

... [Omitted: joke from Bill.]

Andrew: I don't get it. I don't get how, like, how would you write that. Like, it's not mathematical. It doesn't seem, like, mathematical.

T: It doesn't seem mathematical to say money equals 10.

Andrew: Yeah.

Josh: Yeah, it does.

T: Yes.

Jamie: I don't think it shows enough information about the kilometers, like if someone had that equation, they could just, they would be, like, what? Huh? It wouldn't show enough information.

T: So you like this one because it shows what Bill was saying, that she ain't getting anything for those kilometers?

Jamie: It shows, like, the table. Like, you could make a table out of it.

(Interaction segment #36-38, 2/21/03, Evans)

Since Jamie presented her thinking about the problem right after Alex's, not in response to an explicit solicitation for alternative methods or perspectives, Jamie's contribution is more student initiated than teacher initiated. Mrs. Evans at times took on the role of a moderator, collecting a range of students' perspectives, such as Alex's and Jamie's above and reflecting them back to the class. Mrs. Evans reflected back Jamie's response by saying, "So you like this one because it shows what Bill was saying, that she ain't getting anything for those kilometers?" These questions called attention to the student initiated responses and also provided the student the opportunity to elaborate on his or her response. Mrs. Evans's role was less that of an explicit evaluator, and more of an implicit evaluator, as she would call attention to particular student contributions through her questions. More student initiated interaction segments occurred in Mrs. Evans's class, perhaps in part due to the openings for off topic talk, and also in part due to the teacher's acceptance for collecting several student responses to one question.

Variability in adherence to I-R-E discourse structure. A common interaction pattern in classrooms is the Initiate-Respond-Evaluate structure for an interaction segment, where the teacher initiates a question, the student responds, and the teacher evaluates (Mehan, 1979). Mrs. Carson's class used the I-R-E structure with only slight deviations, while Mrs. Evans's class rarely used the I-R-E structure, and deviated more dramatically from this model. I will present examples of the ways in which each teacher deviated from this model to demonstrate that Mrs. Evans was further away from the I-R-E structure.

Interaction segments in Ms. Carson's classroom deviated slightly from the I-R-E structure, in formats such as extended I-R-E's, or initiate-respond-initiate-respond-evaluate. In the example below, Ms. Carson asks how to find a dependent variable if they know a particular value for an independent variable.

T: Um, 8 kilometers, and we want to know how much each person would make if they walked 8 kilometers. For Leanne, how much?

Allen: 10 bucks.

T: How do you know?

Allen: Because she's, because, like, she doesn't even have to walk. She, like, a sponsor has to pay 10 bucks no matter how far she walks.

T: OK. ... (Interaction segment #46, 2/20/03, Carson)

This example is very close to the I-R-E structure. The interaction segment begins with the teacher asking a question. A student responds. The teacher then asks a follow-up question. The student responds with an elaborated answer, and the teacher evaluates ("OK," meaning "correct"). These interactions were typical in Ms. Carson's class. After responding to a question with an answer, the teacher replied with a follow up question about the process (e.g., "How do you know?"), and the student would then explain their process for obtaining the answer. This I-R-I-R-E structure is not exactly like the I-R-E structure, but is similar, and was the primary deviation from the I-R-E structure in Ms. Carson's classroom.

Interaction segments in Mrs. Evans's classroom deviated more dramatically from the I-R-E structure in that the teacher did not evaluate students' responses as directly, and more than one student would respond. A typical deviation from the I-R-E structure in

Mrs. Evans's class involved the teacher initiating a question, then collecting a range of students' responses. The teacher then reflected the responses back to the students, asking them to evaluate. The teachers' role was one of revoicing (O'Connor, 1998) and asking for clarification. In the following example, Mrs. Evans asks the students for the equation for a relationship, the relationship mentioned above about the participant in the walkathon who wants to collect pledges at a flat rate of \$10.

T: Let's talk about Leanne. With Leanne, we have the same variables of money and kilometers. How do you get the money for Leanne?

Jim: Oh! I know!

Becky: You get 10 dollars the whole time.

T: I get 10 dollars all the time. How do I write that as an equation?

Becky: 10.

Jim: You go money equals 10 dot K, or also known as time.

T: Becky says 10. Jim says 10 dot K.

Alex: Um, OK, I wrote Y equals zero times X plus 10.

Bill: That's what I put.

Molly: Um, the 10, Jim's... wouldn't work, because that's saying for every that kilometer that you go, you add 10 more. That's not going to work.

T: So, you don't like this one? I'm not sure that everyone at that back table is listening.

As this example illustrates, Mrs. Evans would ask a question (e.g., "How do you get the money for Leanne?"), and a number of students would reply, as if it was understood that questions were posed to the group rather than one student at a time. Additionally, the

teacher would intervene to reflect back some of the collected responses (“Becky says... Jim says...”) and focus the class’s attention on another student’s critique (“So, you don’t like this one?”). This participation structure, collecting students’ responses and critiques, deviated highly from the I-R-E structure, and did not occur in Ms. Carson’s classroom.

Since students would interject and initiate interaction segments, joking would occur periodically, more than one student would respond to a teacher initiated question, and the teacher would not directly evaluate student solutions, Mrs. Evans’s class’s interaction segments rarely aligned with the I-R-E structure. It was challenging to label any of the interaction segments in Mrs. Evans’s class as following the I-R-E structure, particularly due to the lack of explicit evaluation by the teacher. In the following example, Mrs. Evans inquires as to how to set up a table.

T: ...So where did you put the 45 head start on here?

S: At zero.

T: At the zero, because it wasn't at zero, zero, like the other ones. (Interaction segment #17, 2/25/03)

Mrs. Evans asked a question, a student answered, and she revoiced, elaborating slightly. This was the way Mrs. Evans presented her evaluation of students’ answers. These interaction segments, teacher initiates a question, student responds, teacher revoices, was the closest approximation to the I-R-E structure in Mrs. Evans’s classroom, although the specific evaluation portion is implicit in the last turn. This form of interaction segment occurred less than 5% of the time in each class period, as more than one student usually participated in an interaction segment.

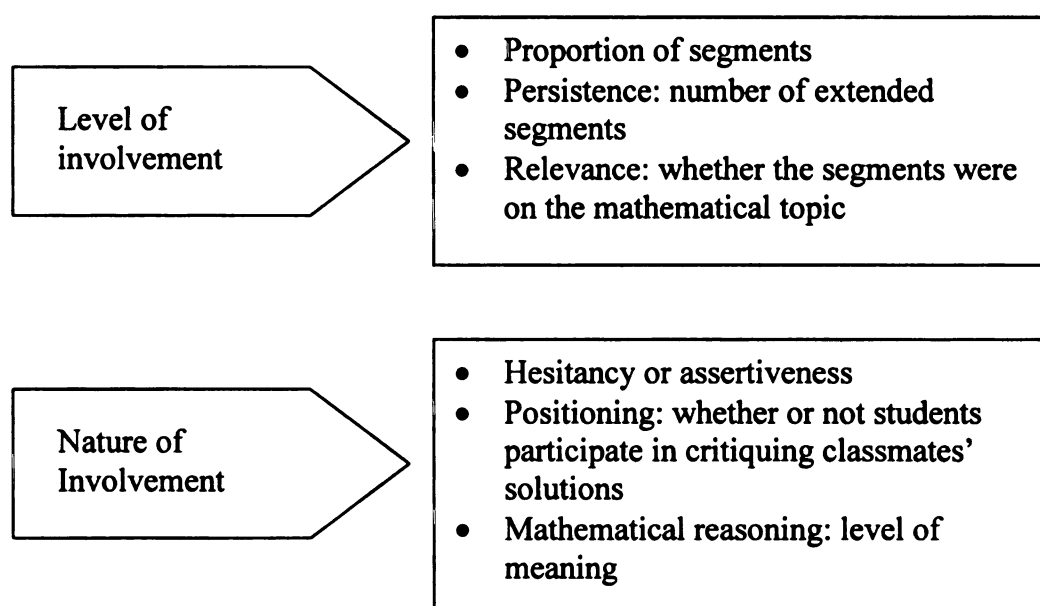
Summary: Differences between whole-class discussions. Mrs. Evans and Ms. Carson's classrooms differed in the amount of off-topic talk, student-initiated talk, and deviation from the I-R-E model, suggesting that Mrs. Evans's class was more flexible than Ms. Carson's, as perceived in my classroom observations. Varying degrees of student initiation and teacher control in the classroom talk may affect whether students have the opportunity to act on their beliefs and goals. For example, in classrooms where it is more common for students to initiate an interaction segment, it may be less risky to ask a question during whole-class discussion, while in a classroom in which students rarely initiate interaction segments, it may be more of a challenge to ask a question. A student may have beliefs or goals that support the importance of asking questions during whole-class discussion, but certain settings may make this act more possible than others. However, students could experience each of these settings differently; a classroom that may be inviting and comfortable to one student may be threatening to another, which suggests no immediately obvious mapping between students' beliefs and goals and each of these classroom settings. The relations between the target students' motivations and their involvement in classroom discourse will be discussed in more detail in Chapter Six.

Target Students' Patterns of Involvement

Although there are similarities and differences between the duration and nature of classroom discussions, in order to examine whether and how beliefs and goals influenced classroom talk, I also analyzed whether and how the target students each involved themselves in the whole-class discussions. I selected and analyzed four patterns of student involvement that were potentially comparable among the target students across both classrooms. These were: (a) their level of involvement; (b) their hesitancy and

assertiveness in their talk; (c) their positioning in terms of how they critiqued others' ideas; and (d) their mathematical communication in terms of their explanations and justifications. These patterns of involvement became the focus of this analysis because they each occurred in varying degrees across both classrooms, and because they bring the analysis closer to the central issue of how beliefs and goals correlate with patterns of participation, in these 15 students. Figure 5.1 summarizes the ways I assessed students' involvement in whole-class discussion.

Figure 5.1: Seventh Grade Students' Involvement in Whole Class Discussions about Mathematics



In order to assess the degree to which students participated at all, I assessed their level of involvement in terms of three factors: (a) the percentage of interaction segments they participated in, (b) whether they persisted in their talk to be involved in more extended interaction segments, and (c) whether their talk was about a relevant mathematical topic. I assessed the nature of students' involvement in terms of their hesitancy and

assertiveness since these particular kinds of involvement could be evidence of students' confidence while talking during class. I also assessed the nature of their involvement in terms of positioning, or whether or not the students participated in critiquing a solution or mathematical idea on the floor. Positioning and critiquing is a way of interacting in mathematics classrooms that may promote autonomous sense-making and reasoning. Finally, I assessed the nature of students' involvement in terms of their mathematical reasoning, characterizing their explanations and justifications in terms of whether and how students focused their talk on procedures and calculations or concepts and meaning. Results for these four patterns of involvement – level of involvement and three factors in the nature of students' involvement – across the four days of whole-class discussion are presented below for each target student in both classrooms.

Level of Involvement

As discussed in Chapter Three (Methods), one dimension I considered in selecting the target students was their frequency of participation. So, it was not surprising when these same students were involved at varying degrees when I observed in the spring. However, I selected the target students in September, so their level of participation in the spring may have changed since the fall.

In order to analyze students' level of involvement in the spring, I focused on three factors: (a) the proportion of interaction segments in which they participated, (b) whether their participation in an interaction was extended in those segments (amounting to taking more than two turns in an interaction), and (c) whether their talk was about mathematics or off-topic in those segments. Table 5.7 below summarizes how determinations of low,

moderate, and high were made for assessing students' level of involvement along these criteria.

Table 5.7: Criteria for Students' Levels of Involvement

	Percentage of Segments	Extended Participation ≥ 2 turns	Non-Mathematical Talk
High	$\geq 10\%$	No	Low
	5 – 10%	Yes	Low
Moderate	5 – 10%	No	Low
	10% +	No	High
	$\leq 5\%$	Yes	Low
Limited	$\leq 5\%$	No	Low
	5 – 10%	No	High

In order to assess the level of target students' participation as limited, moderate, or high, I first determined the percentage of segments they participated in, then determined whether they participated in any extended segments, and finally determined whether they participated in a high or low amount of mathematical talk. Target students were considered to participate at a *high* level involvement amounted to participating in either a high percentage (above 10%) of segments or a moderate percentage (5-10%) of segments with extended participation. *Limited* involvement amounted to participating in either a low percentage of segments (less than 5%) or a moderate percentage of segments (5-10%) with a high amount of non-mathematical talk. Target students who participated at a *moderate* involvement either participated for a moderate percentage of interaction segments, for a high percentage of segments but with a high amount of non-mathematical

talk, or for a low percentage of segments with extended participation. Below I will discuss target students' participation along each of the criteria for level of involvement: percentage of segments, extended participation, and non-mathematical talk.

Proportion of Interaction Segments

In order to determine the percentage of interaction segments that the target students participated in, I counted the number of interaction segments in which a target student participated for each of the four analyzed whole-class discussions and divided that number by the total interaction segments for that discussion. There may have been more than one target student participating in each interaction segment. Then, I found the average percentage of interaction segments that the target student participated in across the four days. Table 5.8 illustrates the percentage of interaction segments that the target students participated in across the four analyzed whole-class discussions.

Table 5.8: Target Students' Percentages of Participating Interaction Segments
Class Day by Target Student

	2/20/03	2/21/03	2/25/03	2/26/03	3/03/03	Average
Ms. Carson's class						
Allen	8.6%	12.5%	9.8%	0%		7.3%
Allison	9.9%	6.3%	9.8%	0%		6.5%
Colleen	3.7%	0%	2.4%	4.2%		3.4%
Hannah	7.4%	6.3%	4.9%	4.2%		5.7%
Max	21.0%	31.3%	14.6%	8.3%		18.8%
Pete	9.9%	12.5%	0%	8.3%		7.7%
Tim	11.1%	25%	7.3%	12.5%		14%
Mrs. Evans's class						
Alex	4.4%	12%	1.9%		35.7%	13.5%
Alyssa	4.4%	9.6%	1.9%		10.7%	6.7%
Becky	2.9%	9.6%	7.5%		3.6%	5.2%
Bill	22.1%	25.3%	34%		14.3%	23.9%
Marissa	0%	0%	0%		0%	0%
Molly	7.4%	4.8%	7.5%		10.7%	7.6%
Steve	20.1%	21.7%	11.3%		7.1%	15.1%
Tricia	0%	0%	1.9%		3.6%	1.4%

There were students in both classrooms that participated in every percentage range. Five students participated in a high percentage ($\geq 10\%$) of interaction segments (Max and Tim, in Ms. Carson's class, and Alex, Bill, and Steve, in Mrs. Evans' class). Seven students participated in a moderate percentage (5-10%) of interaction segments (Allen, Allison, Hannah, and Pete, in Ms. Carson's class, and Alyssa, Becky, and Molly, in Mrs. Evans' class). Three students participated in a limited proportion ($\leq 5\%$) of interaction segments (Colleen in Ms. Carson's class and Marissa and Tricia in Mrs. Evans' class). On this basis, the classes, as represented in the focal students, look approximately equivalent. However, frequency of participation was not considered sufficient for calculating students' level of involvement, particularly because frequency of participation may reveal only how often the teacher called on the student and does not take into account whether the talk was on task. I also considered whether and how students participated within the interaction segments, including whether they participated for an extended time or whether the talk was about mathematics.

Extended Interaction Segments

Some of these students participated in more extended segments than others. Students who participated in an interaction segment by taking more than one turn, and did so over more than one large group discussion (at least two days out of four), were considered students with more extended participation in their level of involvement. The following example, from Mrs. Evans' class on 2/21/03, demonstrates a case of Becky's extended participation.

T: I owe them 2 dollars. So if I show up at the walkathon, and I don't walk at all, what happens?

Becky: You get 5 dollars.

T: Where does it say I get 5 dollars if I don't walk at all?

Becky: It's starting at 5.

T: It's starting out at 5 dollars. What happens if I walk a kilometer?

Jim: You lose two of them!

Becky: You have 3.

T: I walk another kilometer?

Ss shout answer. (Interaction segment #75, 2/21/03, Evans)

In this interaction segment, we see Becky continuing her participation, even when not called on directly, when other students are also taking up the floor, such as Jim. This extended participation indicates a persistence on Becky's part to continue to be a part of the conversation about this mathematical topic. This persistence is indicative of engagement on the student's part. If a student is willing to persist in talking about the topic, they are likely also engaged in thinking about it, although there may be students who are engaged in thinking about the topic who are not talking.

Additionally, the following example, from 2/21/03, demonstrates extended participation in an interaction segment from Ms. Carson's class.

T: OK. Doesn't count by anything. Good. So, which tables did you decide would represent linear relationships? Max?

Max: Uh, 1 and 4.

T: Why 1?

Max: Because, um, on the y-axis it's always 3, so it's always at a constant rate.

T: What is the constant rate?

Max: Uh, 3.

T: It's always at 3? But the constant isn't three, what is the rate?

Max: Oh.

T: Meaning what is it changing by, Max?

Max: Nothing.

T: So the number is?

Max: Zero.

T: The rate, it does have a constant rate of zero, it's always three, so that one's linear. What's my equation for that one? Table one. Albert? (Interaction segment #6, 2/21/03, Carson)

In both classrooms, the teacher's follow-up questions appeared to be posed in such a way that either the student who was most recently speaking was invited to continue, or any other student was welcome to participate. On the one hand, answering a teacher's follow-up question may not appear like persistence, but more like responding directly to a question, students did not always respond to these follow-up questions, sometimes waiting for other students to respond instead.

Students who participated in two or more extended interaction segments were those who persisted in their talk, according to this analysis. Table 5.9 demonstrates how many total extended interaction segments that the target students participated in across all four analyzed whole-class discussions.

Table 5.9: Number of Extended Interaction Segments for each Target Student

	Percentage Extended Interaction Segments	Extended Interaction Segments
Ms. Carson's class		
Allen	6.7%	1
Allison	7.7%	1
Colleen	0%	0
Hannah	7.7%	1
Max	16.7%	5
Pete	9.1%	1
Tim	27.8%	5
Mrs. Evans's class		
Alex	26.9%	7
Alyssa	6.3%	1
Becky	23.1%	3
Bill	24.1%	14
Marissa	0%	0
Molly	6.3%	1
Steve	30.3%	10
Tricia	0%	0

In Ms. Carson's class, two students persisted in their participation: Max and Tim. Both of these students participated at a high percentage. In Mrs. Evans's class, four students persisted in their participation: Alex, Becky, Bill, and Steve. Alex, Bill, and Steve participated at a high percentage, but Becky participated for a moderate percentage of interaction segments, and persistence in her participation changed her level of participation from moderate to high. No other students' level of participation changed as a result of considering persistence, or participating in extended interaction segments.

Off-Topic Talk

In addition to percentage of interaction segments and extended participation, or persistence, students' talk was assessed in terms of relevance, or participation in mathematical or non-mathematical talk. Target students' interaction segments were

examined in terms of whether they participated in non-mathematical, or off-topic, talk during the whole class discussion.¹³ Students with a high amount of non-mathematical talk participated in off-topic interaction segments for more than half of their total interaction segments. Students with a low amount of mathematical talk participated in off-topic interaction segments for less than half of their total interaction segments.

Most students did not participate at a high level of non-mathematical talk. No target students in Mrs. Carson's class participated in non-mathematical talk. Only Max participated in any non-mathematical talk across the four analyzed days of whole-class discussion in Mrs. Carson's class, and he only participated in one non-mathematical interaction segment. Max also participated in the most interaction segments out of all of the target students in Mrs. Carson's class. In Mrs. Evans's class, two students participated in a high amount of non-mathematical talk: Bill and Alyssa. Bill also participated in the most interaction segments out of all of the target students in his class. Since Bill and Alyssa both participated in non-mathematical talk for half or more of their interaction segments, their level of participation changed from the percentage range to the next lowest level. While Bill participated in a high percentage of interaction segments, his high level of non-mathematical talk shifted his level of participation down from high to moderate. Alyssa participated in a moderate percentage of interaction segments, but her high level of non-mathematical talk shifted her level of participation down from moderate to low.

The following table presents the results for the target students' levels of involvement by classroom. The three factors were taken into consideration: proportion of

¹³ Off-topic interaction segments were discussed previously in the chapter, when the classrooms were contrasted.

interaction segments, persistence (extended interaction segments), and relevance (lack of non-mathematical talk).

Table 5.10: Classes by Target Students' Levels of Involvement

	Ms. Carson's class	Mrs. Evans's class
High Involvement	Max, Tim	Alex, Steve
Moderate Involvement	Allen, Allison, Hannah, Pete	Becky, Bill, Molly
Limited Involvement	Colleen	Alyssa, Marissa, Tricia

Not surprisingly, the distribution of students across levels of participation is somewhat of a bell-curve distribution, with more students participating at a moderate level of participation, and less at high and limited levels. In addition to the fact that this variation is to some degree naturally occurring in most classrooms, I selected target students in part to achieve this sort of distribution.

Nature of Involvement

Looking beyond the level of students' involvement, or the frequency of their participation, I also examined the nature of students' involvement in terms of three factors: hesitancy or assertiveness, positioning, and mathematical reasoning. Each of these factors, and evidence for students' participation according to these factors, will be presented below.

Hesitancy and Assertiveness

One of the ways I assessed the nature of students' talk during whole-class discussions was by examining whether they were hesitant or assertive during interaction

segments. I conjectured that students' assertiveness could indicate a high degree of confidence or self-efficacy in mathematics, while students' hesitancy could indicate a lower degree of confidence or self-efficacy in mathematics. Alternatively, assertive or hesitant interaction segments may be fostered more in one classroom than another.

Students' participation in an interaction segment was coded as *hesitant* if they exhibited some hedging or backing down. For example, Allison's hesitancy in the interaction segment below from Ms. Carson's class is an example of a student backing down from an interaction:

T: OK, any other estimates? Allison?

Allison: Um, I think it's 3, never mind.

T: Nick? (interaction segment #38, 2/20/03)

Allison backed down from the interaction by starting to provide an answer, but then stopped, ending her turn with "never mind." Backing down from an interaction was considered a form of hesitancy because the student did not stand by her initial answer.

Students also demonstrated hesitancy by hedging in their talk when expressing solutions. The example below is a case of hedging from Ms. Carson's class.

T: Let's do this. Look at Alana's first. Where do we see the y-intercept in her equation? Allen?

Allen: You see it at, uh, like, in between like, um, 3 point, uh, I don't know how to explain it. Hold on. It's like, we see it around, like, 5x and 10y. You see it.

T: OK, what I'm talking about, Allen, is this point right here. OK? This point right here. Where do I see that in her equation?

Allen: OH! OK. Um.

T: Or maybe I can't? Max? (Interaction segment #25, 2/20/03, Carson)

In this example, Allen's expression of "I don't know how to explain it. Hold on," indicates hedging, or a sense of buying time to figure out what he thinks. Hedging was considered evidence of a lack of certainty on the student's part about their answer, since the student did not respond as if he immediately knew the answer.

Participation in an interaction exchange was coded as *assertive* differently in each classroom, as the differences in structure and teacher control in the classrooms created different possibilities for exhibiting assertive behavior. In Ms. Carson's classroom, assertive talk looked like soliciting the floor or speaking without being called upon. In the example below, Allen comments aloud in response to the teacher's statement and is not called upon.

T: one thing that I did not see anybody in this class do, but I did see in 2nd hour, was the graph.

Allen: I know, I saw that up there.

T: If I were to make a graph, what would go up here? Mara? (Interaction segment #1, 2/25/03, Carson)

When students commented aloud without being called upon in Ms. Carson's class, the teacher rarely pursued their comments with a follow-up question. Since students' comments were not pursued by the teacher, students may have been less likely to initiate interaction segments.

Behaviors such as soliciting the floor or speaking without being called upon were more normative for all students in Mrs. Evans' class, so assertive talk instead looked like initiating a new topic of conversation. Some of these instances of assertive talk were also

coded as off-topic talk. In the example below, Alyssa initiates an off-topic interaction segment.

Alyssa: Do you want us to write this down?

Ss: Yes, she just said that. She just said yes.

Gary: Oh, my God. Tells how much our group pays attention.

Bill: It's not our group, it's just.

Gary: Those girls. (Interaction segment #16, 2/25/03, Evans)

Alyssa's initiation of a question about whether they had to take notes was a shift from the mathematical topic on the floor, as prior to Alyssa's question, the class was discussing whether it would help to represent a particular relationship from a story problem in tabular form. In this example, the students followed up on Alyssa's question rather than the teacher. The teacher or classmates may not have followed up additional instances of assertive talk, such as jokes. In the example below, Bill makes a joke.

T: So, I'm feeling like yesterday's discussion, even though it got a little weird, it might have been helpful for more people besides Steve.

Bill: I want a bronto burger. (Interaction segment #5, 2/21/03, Evans)

This comment, and other similar jokes, was spoken loud enough to make it to the floor, so it was considered to be an attempt at a student initiated interaction segment. Attempted student interaction segments were coded as assertive talk because the student was taking the initiative to begin an interaction. The turn did not have to be followed up on in order to be coded as assertive talk.

Other instances of assertive talk were more about mathematics, in order to pursue a question or clear up one's own confusion. In the example below, Mrs. Evans asks the

students for their perspective on an equation, one student evaluated the equation , and the teacher revoiced his response. Alex then initiates a question about the form of the equation itself.

T: What about this one?

Bill: That's a loser one, too.

T: That's a loser one, too.

Ss: No...

Alex: I don't get it. I don't get how, like, how would you write that. Like, it's not mathematical? It doesn't seem, like, mathematical.

T: It doesn't seem mathematical to say money equals 10.

Alex: Yeah.

Jim: Yeah, it does. (Interaction segments #36-37, 2/21/03, Evans)

In this example, the teacher followed up on Alex's statement with revoicing. The next series of interaction segments pursued Alex's concern. In Mrs. Evans's class, students sometimes refocused the talk to pursue their own questions about the mathematics under discussion.

The following table presents the percentages of the target students' interaction segments that were coded as hesitant or assertive out of their individual totals of interaction segments.

Table 5.11: Percentages of Hesitancy and Assertiveness by Target Student

	Hesitancy	Assertiveness
Ms. Carson's class		
Allen	27%	33%
Allison	31%	0%
Colleen	20%	0%
Hannah	31%	0%
Max	3%	27%
Pete	0%	9%
Tim	6%	22%
Mrs. Evans's class		
Alex	0%	46%
Alyssa	0%	38%
Becky	0%	15%
Bill	0%	62%
Marissa	0%	0%
Molly	0%	13%
Steve	0%	39%
Tricia	0%	50% ¹⁴

Only target students in Ms. Carson's classroom expressed hesitancy, and all but one of the target students in Ms. Carson's class exhibited some expression of hesitancy in their talk. All of the male target students in Ms. Carson's class exhibited assertive talk. All but one of the target students in Mrs. Evans's class exhibited assertive talk, and Mrs. Evans's target students expressed higher percentages of assertive talk than those in Ms. Carson's class. Some of the students who exhibited hesitancy also exhibited assertiveness. For example, Allen sometimes spoke up without being called upon. These striking differences between the two classrooms in terms of assertive and hesitant patterns of involvement suggests that classroom practices may also play a strong role in shaping this form of involvement, but the next chapter will also examine whether beliefs and goals also shape this form of involvement.

¹⁴ 1 out of 2 contributions

Positioning

The nature of students' involvement was also examined in terms of their positioning. In these classrooms, students were invited to varying degrees to critique their classmates' solutions. I refer to this form of talk as positioning, as through the act of critique, they agree and disagree with the thinking of their classmates, aligning themselves for and against each other. In the following example from Ms. Carson's class, Tim positions himself against Albert's solution.

T: The rate, it does have a constant rate of zero, it's always three, so that one's linear. What's my equation for that one? Table one. Albert?

Albert: Wouldn't it be y equals zero x ?

T: You guys agree?

Ss: mm, hmm

Tim: No, Y equals 3.

T: Allen?

Allen: That's what I put, I put y equals 3.

Pete: Yeah, I had that, too.

T: Any other thoughts?

Tim: If you had y equals zero x , it'd be all zero.

T: It's be all zero. And it's not all zero, is it?

Tim: No.

T: It's always 3. Y is always 3, there's your equation for table 1. Max, you also said 4, right? (Interaction segments #7-8, 2/21/03, Carson)

In this segment, Tim's involvement was coded in terms of "disagree" for his form of positioning, because he was the first one to disagree, and he offered a justification for his response. Ms. Carson's question, "You guys agree?" may have invited students to evaluate the solution on the floor. Such questions were not commonly posed in Ms. Carson's class, as interaction segments in this class were more often in the form of the I-R-E structure, with the teacher evaluating directly.

Positioning appeared to be more normative in Mrs. Evans' class, as students offered critique without being solicited by the teacher, and the teacher explicitly encouraged critique with which comments she highlighted in the talk and the nature of her questions. For example, students were discussing whether or not a particular relationship was linear, and the equations for representing these relationships. The problem was about pledge plans for a walkathon. One person, Leanne, did not collect money per kilometer, but instead asked people to donate a flat fee of \$10. Three forms of equations were proposed: $y=10$. $y=0x+10$. $y=10x$.

Molly: Um, the 10, Jim's... wouldn't work, because that's saying for every that kilometer that you go, you add 10 more. That's not going to work.

T: So, you don't like this one? I'm not sure that everyone at that back table is listening.

... [classroom management talk omitted]

T: Molly is saying she doesn't like this one, say it one more time, 'cause I'm not sure it's all been heard.

Molly: because, 10, it's saying that for every kilometer you go, you're adding 10 more, so if you go 2, you're gonna have 20, and it's really only 10.

T: so you would disagree with that one.

Molly: Yeah.

T: How about other people? (Interaction segments #30, 32, 2/21/03, Evans)

In this case, Molly is providing her critique of the $y = 10x$ solution. She began positioning herself against that solution, and the teacher noticed that the class was not attending to her talk, so through classroom management talk, the teacher attempted to focus the class on her disagreement.

Students also agreed with the solutions of their classmates. For example, in the same discussion over Leanne's pledge plan, Bill positioned himself to be for, rather than against, a solution presented by Alex ($y = 0x + 10$):

T: Bill.

Bill: I agree, with Alex, because, uh, he ain't getting paid per kilometer.

...[jokes and clarification question by student omitted]

T: And Bill was saying he liked that one, but I missed why. Why did you like that one?

Bill: Because, uh, she ain't gonna run, uh, she ain't getting money whenever she walks, but she is getting 10 dollars.

T: So you like saying it's zero times the number of kilometers, and then she just gets 10 bucks.

Bill: Yeah, she gets 10 bucks for the fun of it. (Interaction segments #33, 36, 2/21/03, Evans)

Again, Mrs. Evans focused the talk back to a student's move to critique, encouraging a justification for the position that was not initially presented.

The following table illustrates students' positioning in the two classrooms, with a percentage of times they positioned themselves out of their total interaction segments.

Table 5.12: Percentages of Interaction Segments Involving Positioning

	Positioning
Ms. Carson's class	
Allen	20%
Allison	0%
Colleen	0%
Hannah	0%
Max	0%
Pete	9%
Tim	11%
Mrs. Evans's class	
Alex	19%
Alyssa	6%
Becky	8%
Bill	5%
Marissa	0%
Molly	38%
Steve	12%
Tricia	0%

Positioning occurred among more target students in Mrs. Evans's class than those in Ms. Carson's class. Only three students, all boys, participated in positioning in Ms. Carson's class. The only students in Mrs. Evans's class who did not participate in positioning rarely participated at all over the four analyzed whole-class discussions. Marissa and Tricia did not participate in positioning, and each participated in only two interaction segments over the four days.

Students' Mathematical Reasoning

The content of students' contributions in their talk also varied in how they reasoned about mathematics. For this phase of analysis, interaction segments were only coded if they contained mathematical reasoning. Students' mathematical reasoning in an interaction segment was coded in terms of three levels: low, moderate, and high levels of

explicit meaning. These levels were inspired by QUASAR's (Henningsen & Stein, 1997; Silver & Stein, 1996; Stein, Grover, & Henningsen, 1996; Stein & Lane, 1996) analyses of problem solving in Standards-based mathematics classrooms, but modified given the nature of the content in these discussions. The conceptual talk in these mathematics lessons was around the idea of how one could know whether a relationship was linear.

Table 5.13 illustrates the coding of students' mathematical reasoning:

Table 5.13: Definitions of Students' Mathematical Reasoning

Level of Mathematical Reasoning	Code	Definition	Illustrative Example
Low Explicit Meaning	Facts	Student answers a question such as naming a feature on a graph or definition previously discussed.	T: Say it loud. The y-intercept. What definition do we have for the y-intercept? Hannah? Hannah: Where the line crosses the y-axis. (Interaction segment #31, 2/25/03, Carson)
	Procedures without meaning	Student describes how he or she solved a problem by talking about calculations.	T: What about Gilberto? For him to make 10 dollars? Allison? Allison: OK, what I did is that I did 10 divided by 2, and I got 5. T: Super... (Interaction segment #54, 2/20/03, Carson)
Moderate Explicit Meaning	Procedures with meaning	Student describes how he or she solved a problem by talking about why the calculations were appropriate or why and how they work.	T: ... what about Alana? Colleen? Colleen: 5 plus point 50 K equals C. T: What does that mean? Colleen: 5 plus 50 cents every kilometer, you get 5 dollars, times every kilometer, you get 50 cents. T: Good. ... (Interaction segment #16, 2/20/03, Carson)

Table 5.13, continued

	Interpreting representations	Student describes how to read a representation such as a graph or table. (Reading an equation is coded as “procedures with meaning.”)	T: ...where do I, if I have my table of time, here’s my distance, for Emile and Henri, how do I see this point of intersection on the table, because some of you did make the table, where do I see that? Tim? Tim: When it’s on the graph, or, the table, when it says 75 and 75 on the distance. T: Yeah, so they’re both the same... (Interaction segment #29, 2/25/03, Carson)
High Explicit Meaning	Concepts	Student talks about a solution in terms of concepts such as constant rate or in light of the problem context in a way that is not as closely tied to the procedures.	T: Steve. Steve: OK, thank you. See, didn’t she, didn’t someone give her a pledge for 5 dollars? T: That’s what she’s saying, yeah. Steve: If someone gave her a pledge for 5 dollars, that’s just like the zero, you know? (Interaction segment #20, 2/20/03, Evans)
	Justifying linearity	Student presents an argument for why a relationship is linear or not.	T: OK, Molly, other thoughts. Molly: For Alana, the reason why I thought it was, um, linear, I don’t know if it is, because a unit rate is when it’s at 1, and you really shouldn’t count the zero, because it’s not really at that, you know what I mean, because, like, one kilometer is her unit rate, that’s what she gets per kilometer, so that’s why. (Interaction segment #9, 2/20/03, Evans)

Although there were two codes for each level, results are presented for each student in each classroom according to each of the three levels. Percentages of student talk at each

level were calculated for number of coded segments out of his or her total segments that were coded as mathematical talk. Table 5.14 presents the results for levels of mathematical talk for each student and the average percentages for each class. Students were assigned an overall level of explicit meaning based upon which category they had a higher percentage in. If the student was within 10% between two categories, he or she was assigned both categories overall.

Table 5.14: The Level of Mathematical Reasoning for Target Students

	Low Explicit Meaning	Moderate Explicit Meaning	High Explicit Meaning	Overall
Ms. Carson's class				
Allen	58%	34%	8%	Low
Allison	54%	46%	0%	Low / Moderate
Colleen	20%	60%	20%	Moderate
Hannah	73%	27%	0%	Low
Max	50%	50%	0%	Low / Moderate
Pete	64%	27%	9%	Low
Tim	41%	59%	0%	Moderate
Class average	51%	43%	6%	Low / Moderate
Mrs. Evans's class				
Alex	24%	47%	30%	Moderate
Alyssa	40%	40%	20%	Low / Moderate
Becky	36%	45%	18%	Low / Moderate
Bill	42%	29%	20%	Low
Marissa	0%	0%	0%	None
Molly	31%	15%	54%	High
Steve	19%	50%	32%	Moderate
Tricia	0%	0%	0%	None
Class average	24%	58%	23%	Moderate

In Ms. Carson's class, *moderate* was the highest level of explicit meaning among the target students' mathematical reasoning. Three of the students reasoned at a *low* level of explicit meaning, while two others reasoned between *low* and *moderate*. No students in Ms. Carson's class reasoned at a *high* level of explicit meaning. Only three of Ms.

Carson's seven students participated in any high explicit meaning interaction segments, and two of them participated high explicit meaning interaction segments for less than 10% of their total interaction segments that included mathematical reasoning. These two students, Allen and Pete, were also the only two students who positioned themselves in Ms. Carson's class. The third student, Colleen, also reasoned at a higher explicit level of meaning than her classmates, but did not position herself. *Procedures without meaning* was the most frequent form of mathematical reasoning among the target students in Ms. Carson's class.

More target students in Mrs. Evans's classroom reasoned at a higher level of meaning than those in Ms. Carson's class. *Justifying linearity*, reasoning at the highest level of explicit meaning, occurred among the target students in Mrs. Evans's class. *Procedures with meaning* was the most frequent form of mathematical reasoning for the target students in Mrs. Evans's class, which was higher than the most frequent form of reasoning among the target students in Ms. Carson's class, *procedures without meaning*. One of the students in Mrs. Evans's class reasoned at a *high* level of explicit meaning, while no students in Ms. Carson's class reasoned overall at a *high* level of explicit meaning. Only one of the students reasoned at a *low* level of explicit meaning, in contrast to three in Ms. Carson's class. Two of the students reasoned between *low* and *moderate* and two also reasoned at a *moderate* level, similarly to the students in Ms. Carson's class. Molly, the student who reasoned at the highest level, also positioned most often. The two students who reasoned at a moderate level also positioned at levels above 10%, which were the next two most frequent participators in positioning after Molly.

Statistical Analyses

In order to test these analyses of whether target students' patterns of involvement differed statistically between the two classrooms, I conducted unpaired t-tests to compare the target students' mean percentages in each classroom on the different patterns of involvement. Results from these analyses can be seen in Table 5.15.

**Table 5.15: Comparisons Between the Two Classrooms:
Target Students' Patterns of Involvement**

	<i>M %</i> (<i>SD</i>)		<i>t</i> (13)	<i>p</i>	<i>d</i>
	Carson	Evans			
Level of Involvement					
Frequency	9.26 (4.77)	8.94 (7.93)	9.13	0.93	0.05
Extended Interaction Segments	10.94 (8.86)	14.61 (12.69)	0.64	0.53	0.36
Nature of Involvement					
Assertive	13.00 (14.14)	32.88 (21.29)	2.09	0.06	1.16
Hesitant	16.73 (13.47)	0.00 (0.00)	3.53	0.01**	1.96
Positioning	5.71 (7.89)	11.00 (12.57)	0.96	0.36	0.53
Explicit Mathematical Meaning					
Low	51.43 (17.18)	24.00 (16.69)	3.13	0.01**	1.73
Moderate	43.29 (14.12)	28.38 (20.67)	1.61	0.13	0.89
High	5.29 (7.63)	21.75 (17.65)	2.28	0.04*	1.27

Note: Percentages out of each target students' totals.

* $p < .05$. ** $p < .01$.

These analyses provide another lens for determining whether target students participated differently in each classroom. Target students' involvement in terms of frequency, extended interaction segments, and positioning did not appear to differ between the two classrooms. Participation in assertive interaction segments and segments involving moderate level of explicit mathematical meaning did not differ much, but perhaps slightly, between classrooms. Interaction segments involving hesitancy and high and low levels of explicit mathematical meaning were significantly different between the groups of target students in each classroom.

Summary: Nature of Whole-Class Discussion & Target Students' Involvement

Before discussing whether and how target students participation practices related to their beliefs and goals in Chapter Six, I will revisit and summarize the main findings of this chapter. I analyzed both similarities and differences between the whole-class discussions in these two classrooms as well as patterns of involvement for the target students.

These two classrooms shared a model of instruction, *Launch, Explore, Summarize*, used the same mathematical tasks from the CMP textbooks, and each devoted time to whole-class discussion. However, Ms. Carson's class spent more time on seatwork, while Mrs. Evans's class spent more time in whole-class discussion. Both classrooms pursued multiple solution methods, but Ms. Carson's class emphasized multiple ways of calculating an answer, while Mrs. Evans's class also discussed alternative representations. What counted as an alternative or different solution varied between the two classrooms.

Ms. Carson's whole class discussions were more structured than Mrs. Evans's. The discussions in Ms. Carson's classroom followed the I-R-E structure closely, while Mrs. Evans's classroom deviated more dramatically from the I-R-E structure, alternatively collecting a range of student responses, less evaluation directly from the teacher, and some student initiated questions. In Mrs. Evans's classroom, there were student initiated interaction segments and off-topic talk, which rarely happened, if ever, in Ms. Carson's classroom.

The target students' level of involvement was similar in both classrooms, with most participating at a moderate level, and a few in each classroom participating at a high and limited level.

The nature of the target students' involvement was assessed in terms of their hesitancy and assertiveness, positioning, and mathematical reasoning. In terms of their hesitancy and assertiveness, only the target students in Ms. Carson's class expressed hesitancy, while all but one of Mrs. Evans's target students expressed assertiveness. Four out of seven target students, all boys, in Ms. Carson's class expressed assertiveness. The differences in hesitancy were supported by statistical analyses.

With respect to positioning, three out of seven target students engaged in positioning in Ms. Carson's class, while six out of eight target students engaged in positioning in Mrs. Evans's class. The three target students who were involved in positioning in Ms Carson's class were either at moderate or high levels of involvement, but one of the students at a high level of involvement did not participate in positioning. All three of these students engaged in assertive talk. The two target students in Mrs. Evans's class who did not engage in positioning were at low levels of involvement. One

the one hand, positioning appears to relate to level of involvement, but on the other, positioning may be moderated by teacher, as Mrs. Evans appeared to encourage more positioning than Ms. Carson. Statistical tests did not support differences in positioning between the groups of target students in each classroom.

The target students' mathematical reasoning differed between the two classrooms, with more target students engaging in higher explicit levels of meaning in Mrs. Evans's classroom. Mathematical reasoning generally appeared to be related to positioning, as the students with higher levels of mathematical reasoning also participated in higher percentages of positioning. One exception is Colleen, from Ms. Carson's class, who reasoned at a moderate level of explicit meaning, but did not position herself. However, she also participated at a limited level of involvement, the only student in Ms. Carson's class to do so. On the one hand, target students' levels of mathematical reasoning appear to relate to their engagement in positioning. On the other, mathematical reasoning could also be shaped by the teacher's practices, since the target students in Mrs. Evans's class reasoned at higher levels. This difference could also be due to sampling. The differences in students' levels of mathematical reasoning were supported by statistical analyses.

The differences in students' patterns of involvement between these two classrooms suggests that classroom practices may play a role in shaping these target students' patterns of involvement, particularly in terms of hesitant talk, and low and high explicit mathematical reasoning. However, there were other patterns of involvement that did not appear to be different between classrooms. Students' individual agency may lead to some patterns of involvement to be moderated by factors such as their beliefs and

goals. In the next chapter, I will explore whether students' beliefs and goals relate to these participation practices.

CHAPTER SIX

RESULTS: RELATIONS BETWEEN STUDENTS' BELIEFS, GOALS, AND THEIR INVOLVEMENT IN WHOLE-CLASS DISCUSSIONS

The overarching goal of this study was to examine whether and how students' beliefs and goals influence students' participation in whole-class discussions about mathematics. In this chapter, I will examine whether groups of students who shared similar beliefs participated similarly or differently in class discussions. It may be reasonable to expect that students' participation in whole-class discussions about mathematics would be shaped by classroom discourse practices or the ways of participating promoted by the teacher rather than by the students' beliefs about learning mathematics. From this perspective, it is more the participation in these different classrooms that shaped students' beliefs rather than the reverse. In Chapter Five, I explored students' patterns of participation from this perspective. In this chapter, I explore relations between students' beliefs, goals, and their involvement. Such analyses could provide insights about students' roles in the development of classroom discourse practices.

The analyses in Chapter Four showed that the students in the two classrooms did not differ dramatically in their beliefs and goals. The 15 target students express four different clusters of beliefs and goals in their interviews. The first distinction between these four clusters is that two of them, *learn the material in a public forum* and *gain attention*, contained negotiated knowers and low social risk students, while the other two clusters, *do the right thing* and *help others while overcoming social risk*, contained

received knowers and high social risk students. Students' academic and social goals then distinguished between each pair of clusters. *Learn the material in a public forum* included a completing the task goal, and *gain attention* had a gain status goal, while they both shared an appear competent goal. In addition to goals to appear competent and gain status, students in *do the right thing* had a goal to behave. The students in *help others while overcoming social risk* held a social goal of helping others. Out of the four clusters, *learn the material in a public forum* is focused on mastering mathematics, while the other three are predominantly focused on social concerns. *Gain attention* is more self-focused, while *do the right thing* and *help others while overcoming social risk* are more community-focused.

Students' beliefs and goals were not distributed differently than we would expect due to chance between classrooms. In their survey responses, the population of students from both classrooms differed on only one of the nine scales (confidence). These data suggested either that students could hold beliefs and goals independent from their classroom experiences, or that both classrooms supported similar beliefs and goals among the students.

In Chapter Five, I examined the extent to which the target students' patterns of involvement varied between the two classrooms. Students' involvement varied in terms of the nature of their involvement, particularly in terms of the hesitancy expressed in their discourse, and low and high levels of explicit meaning expressed when talking about mathematics during whole-class discussion. In Ms. Carson's class, more target students expressed hesitancy in their talk and a low level of explicit meaning. In Mrs. Evans's classroom, more target students talked about mathematics at high level of explicit

meaning. Many patterns of involvement were more similar than different between the two classrooms, including indicators of the level of students' involvement, such as frequency and participation in either extended or off-topic interaction segments. Additionally, the nature of students' involvement did not vary between classrooms in terms of their assertive talk, positioning, and moderate level of explicit meaning expressed when talking about mathematics.

In this chapter, I examine whether target students participated differently in terms of the beliefs and goals that they expressed in interviews – that is, I directly explore the issue of whether individual students' expressed beliefs and goals influence their participation in discussion. Rather than grouping the students by classroom, in this chapter, I group the target students by the beliefs and goals they expressed in the interviews. I first examine whether patterns of involvement differed with respect to epistemological beliefs and the perceived level of social risk associated with participating in whole-class discussion.¹⁵ Then, I examine whether students who express particular academic and social goals participate differently than those who do not. Finally, I examine whether involvement in whole-class discussion about mathematics appears to vary based on the four clusters of beliefs and goals discussed in Chapter Four.

Epistemological Beliefs, Social Risk, and Involvement

In Chapter Four, the target students' interview responses led to two belief clusters depending on their expression of (a) epistemological beliefs and (b) perception of risk associated with participating in whole class discussion, when moderate and high risk students were collapsed into one cluster. Eight target students talked about learning as a

¹⁵ As social risk and epistemological beliefs co-varied in this sample almost perfectly, there was little to gain from looking how each related to students' participation.

process of receiving knowledge and associated a high level of risk with participating, while seven target students talked about learning as a process of negotiation and associated a lower level of risk with participating.

In order to analyze whether these two groups of students participated differently, I conducted unpaired t-tests to assess for differences in the mean percentages for each pattern of involvement. I set the level of significance to be a two-tailed p value $< .05$. I calculated each student's percentages for frequency differently than I calculated his or her percentages for the various patterns of involvement. Frequency percentages were calculated by dividing the target student's total interaction segments by their classroom's overall total of interaction segments across the four analyzed discussions. Means and standard deviations of these frequency percentages were calculated for the two belief groups. Percentages for the other patterns of involvement were calculated by dividing the number of interaction segments in which the target student exhibited a particular pattern of involvement (e.g., hesitant talk, positioning) by the target student's total interaction segments. Using each student's individual total for these percentages approximates what would be normative for that student. Means and standard deviations were calculated for the two belief groups for each pattern of involvement. Table 6.1 displays the mean percentages of the two groups for the nine patterns of involvement from Chapter Five, the t-value of the difference between the means, p-value, and effect size of the difference.

Table 6.1: Comparisons Between Belief Groups: Target Students' Patterns of Involvement, Version A

	M % (SD)		<i>t</i> (13)	<i>p</i>	<i>d</i>
	Received Knowing & High Risk	Negotiated Knowing & Low Risk			
Level of Involvement					
Frequency	6.79 (5.72)	11.72 (6.55)	1.56	0.14	0.86
Extended	5.80 (5.79)	20.88 (9.80)	3.69	0.01**	2.05
Off Topic	14.48 (26.02)	18.09 (18.03)	0.31	0.76	0.17
Nature of Involvement					
Assertive	19.57 (19.81)	28.24 (21.74)	0.81	0.43	0.45
Hesitant	10.10 (13.38)	5.19 (11.47)	0.76	0.46	0.42
Positioning	4.42 (7.23)	13.62 (12.26)	1.73	0.11	0.96
Explicit Meaning					
Low	38.09 (28.37)	36.58 (13.34)	0.13	0.90	0.07
Moderate	29.73 (21.47)	41.68 (14.62)	1.24	0.24	0.69
High	7.18 (8.78)	21.72 (18.90)	1.96	0.07	1.09

Note: Percentages out of each target students' totals.

p < .05. ** *p* < .01.

Table 6.1 demonstrates that negotiated knowers and low social risk students were more likely to participate in extended interaction segments. The *p*-values for positioning and talking about mathematics at a high level of explicit meaning suggest that there may be some differences between the belief groups in these patterns of involvement, as the *p*-values are near the significance cutoff.

To follow up on these potential differences between belief groups, I calculated the target students' percentages for patterns of involvement using different totals. Instead of examining the expected percentage for the individual student, by dividing out of the individuals' totals, I divided out of their classroom total interactions. Calculating the percentages out of the classroom total interactions could approximate how often the target student would participate in that pattern of involvement during a class period. Table 6.2 displays the new mean percentages for the nine patterns of involvement (frequency remains unchanged), the *t*-value of the difference between these means, *p*-value, and effect size of the difference.

Table 6.2: Comparisons Between Belief Groups: Target Students' Patterns of Involvement, Version B

	M % (SD)		<i>t</i> (13)	<i>p</i>	<i>d</i>
	Received Knowing & High Risk	Negotiated Knowing & Low Risk			
Level of Involvement					
Frequency	6.79 (5.72)	11.72 (6.55)	1.56	0.14	0.86
Extended	0.67 (1.02)	2.69 (2.06)	2.45	0.03*	1.36
Off Topic	0.67 (1.49)	2.95 (4.51)	1.36	0.20	0.75
Nature of Involvement					
Assertive	1.46 (1.86)	4.35 (5.38)	1.43	0.18	0.80
Hesitant	0.77 (1.08)	0.44 (0.92)	0.63	0.54	0.35
Positioning	0.36 (0.65)	1.35 (0.91)	2.43	0.03*	1.35

Table 6.2, continued**Explicit Meaning**

Low	2.88 (2.94)	3.14 (2.10)	0.19	0.85	0.11
Moderate	2.11 (2.56)	3.38 (1.63)	1.13	0.28	0.62
High	0.29 (0.31)	1.54 (1.24)	2.77	0.02*	1.54

Note. Percentages out of total interaction segments across the four days in each class.
(Carson: 162. Evans = 232.)

* $p < .05$. ** $p < .01$.

The results in Table 6.2 show that negotiated knowers with a low perception of risk were more likely to extend their interaction segments, engage in positioning, and use a high level of explicit meaning in their talk. The differences between the means for these patterns of involvement in Table 6.2 may not be impressive when viewed in terms of the sizes of the percentages, but if we examine how the proportional relationships between the percentages, we see more dramatic differences. The negotiated knowers and low risk target students participated in extended and positioning segments approximately four times as often as the received knowers and high risk target students, and participated in talking about mathematics at high levels of explicit meaning over five times as often.

The analyses presented in Tables 6.1 and 6.2 demonstrate that participation in extended interaction segments occurred more often among negotiated knowers and low social risk students. Involvement in positioning and talking about mathematics at high levels of explicit mathematical meaning was also somewhat more likely to occur among these students. These three patterns of involvement are more related to target students' beliefs rather than their experiences in their classrooms.

There was some diversity in these patterns within belief groups. Table 6.3 presents the individual students' percentages in each belief group for extended, positioning, and high explicit meaning.

Table 6.3: Diversity Within Belief Groups
Patterns of Involvement by Target Students, as Grouped by Beliefs

	% Out of Individual Totals		
	Extended	Positioning	High Explicit Meaning
Received Knowing & High Risk			
allen	6.67	20.00	8.33
ALYSSA	6.25	6.25	20.00
colleen	0.00	0.00	20.00
hannah	7.69	0.00	0.00
MARISSA	0.00	0.00	0.00
max	16.67	0.00	0.00
pete	9.09	9.09	9.09
TRICIA	0.00	0.00	0.00
Mean	5.80	4.42	7.18
(SD)	(5.79)	(7.23)	(8.78)
Negotiated Knowing & Low Risk			
ALEX	26.92	19.23	29.41
allison	7.69	0.00	0.00
BECKY	23.08	7.69	18.18
BILL	24.14	5.17	19.35
MOLLY	6.25	37.50	53.85
STEVE	30.30	12.12	31.25
tim	27.78	11.11	0.00
Mean	20.88	13.26	21.72
(SD)	(9.80)	(12.26)	(18.90)

*Lower case: Ms. Carson's class. Upper case: Mrs. Evans's class

For example, Max extended his interaction segments at a more similar level of frequency to the negotiated knowers and low social risk students. Molly and Allison did not extend their interaction segments as often as the other students in their belief group. Allen

participated in positioning more similarly to negotiated knowers and low social risk students in contrast to the students in his own belief group. Allison was the only student among the negotiated knowers and low social risk students who did not participate in positioning at all. Allison and Tim, both from Ms. Carson's class, did not engage in talking about mathematics at as high levels of explicit mathematical meaning as the other students in their belief group, the rest of whom were in Mrs. Evans's class. Additionally, Alyssa and Colleen talked about mathematics at higher levels of explicit mathematical meaning. Alyssa was from Mrs. Evans's class, while Colleen was in Ms. Carson's class. Colleen was the only student in Ms. Carson's class who talked about mathematics at a high level of explicit mathematical meaning. While the differences between belief groups are statistically significant, individual anomalies are worth noting. In particular, the differences between belief groups for high levels of explicit mathematical meaning appears to be explained somewhat more by classrooms than beliefs, when diversity within the groups are examined.

Looking back at Tables 6.1 and 6.2, it is noteworthy that the belief groups did not differ in other patterns of involvement. Target students' frequency of participation, off-topic talk, assertive talk, hesitant talk, and talk about mathematics at low and moderate levels of explicit meaning did not differ according to belief group. The patterns of involvement that did differ by classroom – students' assertive talk, hesitant talk, and talk about mathematics at a low level of explicit meaning – did not differ according to the belief groups. However, students' talk about mathematics at a high level of explicit mathematical meaning differed by both belief groups and classrooms.

Academic and Social Goals and Involvement

Target students also were divided into groups based on their academic or social goals. In the interviews, target students expressed academic goals about completing their tasks social goals about appearing competent, gaining status, appropriate behavior, and helping their classmates. In this section of the chapter, I examine whether target students' involvement differed in terms of whether or not they expressed any of these five academic or social goals. I conducted unpaired t-tests between the group of students who expressed each goal and the group that did not for all nine involvement categories.

Complete task. Five students of the 15 students were focused on completing their tasks as a goal during whole-class discussions. All were in Mrs. Evans's class: Alex, Alyssa, Becky, Molly, and Steve. Students who expressed this goal participated significantly differently from those who did not with respect to positioning and high levels of explicit meaning. ($p < 0.05$). These students positioned more often (16.56% in comparison to 4.54%) and used higher levels of explicit meaning more frequently (30.54% compared to 5.68%) when talking about mathematics than those who did not express a goal of completing the task.

Appear competent. Eleven students discussed the goal of appearing competent during whole-class discussions. From Ms. Carson's class, Allen, Hannah, Max, Pete, and Tim expressed this goal. From Mrs. Evans's class, Alex, Becky, Bill, Marissa, Molly, and Tricia expressed this goal. Students with this goal did not participate significantly different from those who did not express this goal on any of the forms of involvement.

Gain Status. Five students were concerned with gaining some form of status, either in terms of their mathematical ideas, their improved behavior, or their improved

academic performance. From Ms. Evans's class, Max and Pete were concerned with status, while Marissa, Molly, and Bill were concerned with status in Mrs. Evans's class. Students who expressed this goal did not participate significantly differently from those who did not express this goal on any of the forms of involvement.

Behave. Eight target students were concerned with behaving appropriately during whole-class discussion. Alyssa, Becky, Marissa, and Tricia expressed this goal in Mrs. Evans's class. From Ms. Carson's class, Colleen, Hannah, Max, and Pete, from Ms. Carson's class, expressed this goal. Students with this goal participated significantly differently ($p < .05$) with respect to extended segments (7.85% to 18.54%) and positioning (2.88% to 15.02%). The students who did not express the goal of behaving appropriately involved themselves more often in these forms of involvement. This is consistent with the received knowers and high social risk students. Becky was the only student expressing this goal outside of this belief group.

Help classmates. Ten target students wanted to participate in whole-class discussion in order to help their classmates. From Mrs. Evans's class, Alyssa, Bill, and Tricia expressed this goal, and all of the students in Ms. Carson's class expressed this goal. Students with this goal participated significantly differently ($p < .05$) from those who did not with respect to high levels of explicit meaning. Those expressed the goal of helping classmates involved themselves less often in talking about mathematics with high levels of explicit meaning (7.68% as compared to 26.54%).

This is the opposite of the results from the group of students with the goal of completing the task. The students who did not express the goal to help others were Alex, Becky, Marissa, Molly, and Steve, from Mrs. Evans's class. This is almost the same

group as the students with the goal of completing the task. Marissa is in the group with the helping goal, but Alyssa was in the group with the goal of completing the task. It may be the case that students who were more self-focused and academic focused were more likely to talk about mathematics at high levels of explicit mathematical meaning.

Out of these five social goals, three appeared to be related to students' involvement in whole-class discussion about mathematics: Complete the task, behave, and help classmates. The goals of appearing competent and gaining status did not appear to be related to students' patterns of involvement. Students with the goal of completing the task participated more often in critiquing or positioning during discussion and talked about mathematics at a high level of explicit mathematical meaning. Alternatively, students positioned less and engaged in less extended segments if they had the goal of behaving, and they were less likely to talk about mathematics at a high level of mathematical meaning if they had the goal of helping their classmates.

Clusters and Involvement

Students expressed four clusters of beliefs and goals in their interviews: Learn the material in a public forum, gain attention, do the right thing, and help others while overcoming social risk. Table 6.4 displays which students fell into which cluster.

Table 6.4: Students' Belief Clusters
Individual Students by Belief Cluster and Classroom

	Ms. Carson's students	Mrs. Evans's students
<i>(1) Learn the material in a public forum</i>	Tim, Allison	Alex, Steve, Becky
<i>(2) Gain attention</i>		Molly, Bill
<i>(3) Do the right thing</i>	Max, Pete	Marissa
<i>(4) Help others while overcoming social risk</i>	Allen, Colleen, Hannah	Alyssa, Tricia

To examine whether expressing a particular cluster of beliefs and goals was related to how students' involved themselves in whole-class discussion about mathematics, I explored the percentages of average involvement for each cluster. Table 6.5 presents these average percentages across students in each cluster for the nine forms of involvement.

Table 6.5: Differences in Involvement by Belief/Goal Clusters
Mean Percentages of Involvement by Forms of Involvement and Cluster

	(1)	(2)	(3)	(4)
Level of Involvement				
Frequency	10.03	15.55	8.72	5.46
Extended	23.15	15.20	8.59	4.12
Off Topic	12.82	31.25	0.00	22.5
Nature of Involvement				
Assertive	24.63	37.29	11.92	24.17
Hesitant	7.27	0.00	1.11	15.49
Positioning	10.03	21.34	3.03	5.25
Explicit Meaning				
Low	34.73	41.19	37.88	38.21
Moderate	49.50	22.21	25.76	32.12
High	15.77	36.60	3.03	9.67

Ideally, I would run an ANOVA to determine differences within and between groups. However, one of the clusters, *gain attention* (2), has only two members, which does not allow me to run such an analysis.

Based on percentages alone, these students apparently involved themselves differently depending on which cluster they expressed. Students in cluster (2), *gain attention*, differed the most dramatically from the other clusters, participating more frequently, in more off-topic interactions, more assertive interactions, utilized positioning more often, as well as both low and high levels of explicit meaning in their mathematical talk. Findings related to cluster (2) should be interpreted with hesitancy, because only two students are included in this group. Cluster (1), *learn the material in a public forum*, included a group of students who were more likely to participate in extended interactions and talk about mathematics at moderate levels of explicit mathematical meaning. Students who participated in hesitant talk were more likely to be in cluster (4), *help others while overcoming social risk*. Students in cluster (3), *do the right thing*, never participated in off-topic talk.

Summary

Through comparing a series of groups of students, involvement in discourse appears to be related to students' beliefs with respect to the following forms of involvement: extended interaction segments, positioning, and the use of high levels of explicit meaning. Diversity within belief groups suggests that students' talk about mathematics at high levels of explicit levels of mathematical meaning may be more different by classroom.

When students were grouped by their goals, differences between the groups were along those same three patterns of involvement: extended interaction segments, positioning, and high levels of explicit mathematical meaning. The students with a more self-focused goal of completing the task were more likely to position themselves during

discussion and talk about mathematics at a high level of explicit mathematical meaning. The patterns of involvement of students with other self-focused goals, appearing competent and gaining status, did not differ. The involvement of students with community focused goals, behaving and helping, did differ. Students with a goal of behaving were not as likely to position or participate in extended segments. Students with a goal of helping were less likely to talk about mathematics at a high level of explicit mathematical meaning.

There were also differences in patterns of involvement when students were grouped by clusters. The involvement of the students in cluster (2), *gain attention*, differed most markedly, but there were only two students in this cluster. Cluster (1), *learn the material in a public forum*, contained students who were more likely to extend their interactions. Students in cluster (3), *do the right thing*, never participated in off-topic talk. More hesitant talk took place by the students in cluster (4), *help others while overcoming social risk*.

CHAPTER SEVEN

DISCUSSION

This study was designed to explore whether and how middle school students' beliefs and goals influenced their involvement in whole-class discussions, in contrast to research that focuses on the collective aspects of whole-class discussion (e.g., Stephan, Cobb, & Gravemeijer, 2003) and the teacher's efforts to structure the classroom discourse (e.g., Nathan & Knuth, 2003). This study's primary contribution is a set of relationships between students' beliefs and goals and their patterns of involvement. In particular, students who believed in negotiated knowing and low social risk participated in more extended interaction segments, increased positioning, and at a higher level of explicit mathematical meaning. Students who believed in received knowing and high risk were more likely to participate in these shorter interaction segments and to talk about mathematics at lower levels of explicit meaning. Such results provide an explanation of how students assist in establishing or maintaining classroom norms. Students have developed beliefs and goals, and associated expectations for behavior, as a result of previous educational experiences. Then, they may enter a new classroom and expect to act on these beliefs and expectations for behavior. Teachers play a prominent role in structuring the whole-class discussions, but students also shift the nature of whole-class discussion by initiating their own questions or evaluating a classmates' solution even if not explicitly requested by the teacher.

Some of the additional contributions of this study include a framework of middle school students' beliefs and goals in discussion-oriented mathematics classrooms;

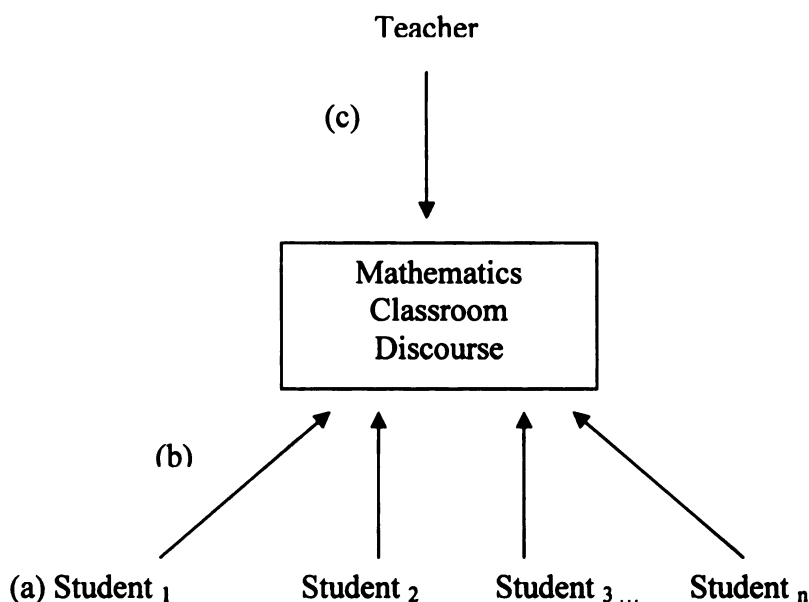
revisiting constructs such as mathematics anxiety in light of mathematics reforms; and an analytic framework for the analysis of students' beliefs. Based on students' interview data, a framework of students' beliefs and goals in discussion-oriented classrooms should include their social goals as well as their beliefs about learning mathematics. Previous studies of mathematics anxiety have not taken place in reform-oriented mathematics classrooms, and these results suggest that discussion-oriented classrooms may foster a performance anxiety in mathematics classrooms for some students. When analyzing students' beliefs from interview data, perspectives from discourse analysis, such as the use of repetition (Tannen, 1989), can lead to stronger claims.

In this chapter, I will first review and interpret the results from this dissertation. Then I will discuss the contributions of this study, the limitations of the study, the implications of these results for mathematics classroom practice, and future research suggested by these results.

Review and Interpretations of Results

Figure 7.1 illustrates the relations between the results that I will discuss in this chapter: (a) the range of students' beliefs and goals that they bring into their mathematics classroom, (b) whether and how students' beliefs and goals influenced their involvement in whole-class discussions, and (c) whether and how the classroom discourse practices or the teachers' discourse practices influenced students' involvement in whole-class discussions.

Figure 7.1: Interactions in a Mathematics Classroom



Range of Students' Beliefs and Goals

One of the goals of teaching mathematics is to foster productive dispositions for learning and doing mathematics. Dispositions include students' beliefs, their attitudes, and their emotions. Beliefs provide the psychological backdrop for students' goals. Many studies of students' motivation and beliefs in mathematics classrooms were conducted before the NCTM reforms. Research on students' motivation in mathematics classrooms has not taken into account students' beliefs and goals in relation to participating in classroom discussions—a key element of classroom practice proposed in these reforms. The range of students' beliefs and goals in discussion-intensive mathematics classrooms should be mapped and described in order to understand the motivations of all students.

In this study, an examination of students' beliefs and goals about learning mathematics in discussion-intensive mathematics classrooms, the following patterns were observed among the 15 target students' beliefs and goals: (a) epistemological beliefs

correlated with their perception of risk associated with involvement in classroom discussion, and (b) target students expressed both academic and social goals that supported their involvement in classroom discussion.

Epistemological beliefs and perception of risk. The 15 target students in this study expressed beliefs and goals in four different clusters. Two of the clusters, *learn the material in a public forum* and *gain attention*, included the epistemological belief of learning as a process of negotiating knowledge as well as lower perceived level of risk associated with participating in whole-class discussions about mathematics. Students in the other two clusters, *do the right thing* and *help others while overcoming social risk*, expressed the belief that learning mathematics involved receiving knowledge from an authority and that participating in discussion involved a higher level of risk.

So epistemological beliefs and perception of risk correlated for the target students in sensible ways. Students who saw benefits to sharing one's ideas during mathematics class discussion, the negotiated knowers, were not threatened by the idea of participating. For these students, the benefits of involvement outweighed the risk of being incorrect in front of their peers. Alternatively, some students did feel somewhat threatened by the idea of participating in class discussions. These students were the received knowers. They may have found safety in the belief that the process of learning mathematics involves receiving knowledge from an authority. This belief may protect them from sharing ideas in front of classmates and allow them to avoid being publicly incorrect. If the student did not view himself or herself as one of the authorities in the classroom about the current mathematics problem under discussion, they may not interact during class in particular

ways. Future research could examine the development of students' epistemological beliefs in relation to their perceptions of social risk.

Table 7.1: Epistemological Beliefs by Perception of Social Risk

Epistemological Beliefs	Perception of Social Risk	
	High	Low
Received	<i>Avoid Involvement</i> (<i>N</i> = 7)	<i>Devalue Involvement</i> (<i>N</i> = 0)
Negotiated	<i>Face-Saving Involvement</i> (<i>N</i> = 0)	<i>Approach Involvement</i> (<i>N</i> = 8)

Table 7.1 indicates that the target students' expression of beliefs in this study did not fall along all possible relations between epistemological beliefs and perception of social risk. While the participants' interview data from this study demonstrated relations between epistemological beliefs and perceptions of social risk, additional relations may be revealed among future participants. For example, students who believed that learning mathematics involved negotiation could have expressed a higher sense of social risk, even though none of the negotiated knowers in this study expressed perceptions of higher social risk. Negotiated knowers could believe that knowing mathematics requires the sharing and interchange of ideas, but might still see this practice as somewhat threatening. I conjecture that the involvement of students who believe in negotiated knowing and experience high social risk may be observed as *face-saving* behaviors, such as not always participating when incorrect or backing out of an interaction. Also, future participants may believe that learning mathematics involves receiving knowledge and also express a low level of social risk. These students might not be intimidated by the idea of participating in discussion, but do not see the purpose of doing so, given their

epistemological beliefs. I conjecture that these students may *devalue involvement*. While none of the participants in this study expressed the other two possible combinations between epistemological beliefs and perceptions of social risk, other middle school mathematics students may hold them.

Future research could examine students' epistemological beliefs and perceptions of social risk in a variety of mathematics classrooms. Expanding the analysis into a wider range of classrooms from this study would provide the opportunity to expand this dissertation's framework of students' beliefs and goals.

No previous study has examined relations between students' epistemological beliefs and perception of risk. One previous study examined relations between two second grade students' beliefs about the nature of mathematics, a domain specific epistemological belief, and their learning goals (Cobb, 1985). If learning goals are related to students' epistemological beliefs, they may also be related to students' perception of risk. However, Cobb's study is different from this dissertation, as the students were younger, there were only two cases analyzed in the article, and the constructs were different. Researchers could examine relations between students' perception of risk and their learning goals, as well as relations between students' beliefs about the process of learning mathematics and their beliefs about the nature of mathematics.

Students' goals: Academic and social. The four clusters differed due to students' goals, both academic goals and learning goals. Students in *learn the material in a public forum* ($N = 5$) expressed the academic goal of wanting to participate in discussion in order to complete the task as well as a goal of appearing competent in front of classmates. Students in the other three clusters did not cite goals related to completing

their mathematical tasks when they shared their reasons for participating in whole-class discussion. In the cluster *gain attention*, students ($N = 2$) expressed goals of appearing competent and gaining status for their thinking. These two clusters of students both expressed competence goals, but one cluster was more focused on completing content, while the other was attending to gaining status. The students in the group *learn the material in a public forum* were the only ones whose principal focus was learning the content; the others were focused on social goals.

The two other clusters, *do the right thing* and *help others while overcoming social risk*, also were made up of students with similar epistemological beliefs and perceptions of social risk, and these clusters differed in terms of the goals that students expressed. Students in the cluster of *help others while overcoming social risk* ($N = 5$) were focused on the goal of helping their classmates, while students in the cluster of *do the right thing* ($N = 3$) were concerned with issues of appropriateness and appearance, such as appearing competent, gaining status, and behaving. The focus in the former group was clearly social, where the latter group was more focused on themselves as individuals.

Students' may particularly attend to social issues in discussion-intensive mathematics classrooms during middle school. Both the activity of whole-class discussion and the developmental challenges of adolescence could alert students to attend to social goals. Students' social goals reflected some element of performance or awareness of how others perceive them, whether it was in terms of appearing competent intellectually, getting recognized by others for the content of their thinking, appearing socially appropriate, or helping their classmates learn.

For students who held it, the academic goal of completing the task seemed to relate to their view of learning as a negotiated process. If students wanted to figure out what they needed to know in order to get their work done, they could focus their participation in order to get the information they need to help them accomplish this goal. The goal of completing the task may have appealed to some students' sense of efficiency. They may have valued efficiently finishing the task over any risks involved in participation.

Target students' goals could also be considered either self-focused or community-focused. The goals of learning content, appearing competent, and gaining status were more self-focused. Students' efforts to achieve these goals are not out of concern for their classmates, except if they indirectly ask questions or share mathematical thinking that could potentially provide insights. These actions seem secondary to the goal of appearing competent, perhaps more important for gaining status, and most likely for learning the content. Though it is somewhat counter-intuitive, the class may benefit indirectly from students who act on their self-focused goals. For example, if a student asks a question in order to help himself or herself complete their assignment, another student may have had that same question and therefore benefit from the response, from the teacher or from peers. In contrast, behaving and helping classmates were community-focused goals. Good behavior allows more students to focus on paying attention to the discussion rather than other distractions. Helping their classmates is a supportive action of one's classmates, as it is an effort to help them learn mathematics.

It is noteworthy that the students who expressed what might be considered the more "productive" epistemological beliefs and perceptions of social risk—at least as

viewed from the reformer's perspective—(negotiated knowing and low risk) appear to hold more self-focused goals, such as completing the task, demonstrating competence, and gaining status. Compare this focus on oneself to the call by NCTM, "We need to shift toward classrooms as mathematical communities" (NCTM, 1991, p. 3). The social nature of a community involves considering the needs of others and interacting out of consideration of these needs. The students who may have expressed the more productive beliefs toward learning mathematics were less focused on considering the needs of the others in their classroom community.

In contrast, the target students in this study who spoke of learning mathematics as a process of receiving knowledge and of participating in discussion as a high-risk activity may not have expressed the more "productive" beliefs about learning mathematics, but they may have expressed more productive social goals for fostering a sense of community in their classrooms. These students were more likely to express social goals of helping classmates and behaving. These goals suggest a focus on the needs of others: helping others learn and establishing an atmosphere conducive to learning through their good behavior. Even though these students do not express a belief in the negotiation of knowledge, they may still participate in the activity of classroom discussion due to an interest in supporting their community.

A range of beliefs and goals support the emergence of norms in mathematics classrooms, from epistemological beliefs about negotiating mathematical knowledge to social goals about helping classmates and behaving. Social goals are important factors in the development of a mathematical community. Attending to social processes in the classroom can allow for more harmonious discussions. If no students were focused on

behaving, then the discussions may flow less effectively. However, there are elements of risk taking associated with sharing one's thinking in a public space, so getting any initial ideas on the floor from students during discussion requires at least some of the students to be less concerned with the social consequences associated with taking these risks.

"Productive" beliefs and goals in a mathematical community may include beliefs about learning mathematics, such as negotiated knowing and low social risk, as well as less self-focused goals, such as helping classmates and behaving.

Teachers could consider how to honor productive goals of helping others and behaving while considering the learning beliefs of these students (received knowing and high social risk). For example, teachers who make an effort to balance out the time spent in whole-class discussion with time spent in small group discussion may be considering the level of risk involved in whole-class discussion. Students can meet their goal of helping their classmates by working together in smaller groups, and they may find small group work to be less risky.

I did not explicitly design this study to assess social goals. Data on social goals arose from questions such as those inquiring why students chose to get involved in class discussions. If I asked more general questions about whether and why the student wanted to learn mathematics, I could have learned about a wider range of social goals, such as *social affiliation*, *social approval*, *social responsibility*, *social status*, and *social concern* (Dowson & McInerney, 2003). Target students' goal to help others could be considered a *social concern* goal, a concern for assisting others in their academic development. Their goal to gain status is similar to the goal of *social status*, or to achieve academically to attain social position. Additionally, students' goal to appear competent may be aligned

with either performance goals or *social approval* goals, depending on how the student talked about his or her goal to appear competent. Finally, target students' goal to behave is similar to the social goal of acting out of *social responsibility*, to meet social role obligations or to follow social and moral rules. Target students did not extensively mention social goals of *social affiliation*, to enhance a sense of belonging, or *social approval*, to gain the approval of peers, teachers, or parents.

However, one student, Marissa, mentioned defining herself against a group of students, which would be the antithesis of a *social affiliation* goal. She mentioned to me that she did not participate in class often because she didn't want to be like one of those "preppy people." I asked her what she meant by this:

...to me preppy people are people who can brag about what they wear, brag about their grades, brag just brag. I don't know. I don't like that, to brag about things. Maybe I do, I don't know, maybe I don't catch it. I don't think I do (3/31/03, #66).

She did not want to come across as a certain type of person. Sometimes participating in mathematics class felt like bragging to her, because she would have been showing off about what she knew. She made the effort to not affiliate herself with students that she saw doing this. Marissa was the only student whose interview talk hinted at social affiliation goals, or anti-affiliation, in this case. Adolescents may be particularly focused on issues of affiliation and anti-affiliation, as they wrestle with the question, "who am I in relation to others?" However, I interpreted this data as indicating a goal of behaving, or, in Dowson & McInerney's terms, social responsibility, since not bragging may also be refraining from an action that may be offensive to others.

In the Principles and Standards for School Mathematics, NCTM suggests “...some students in the middle grades are often reluctant to stand out in any way during group interactions,” (NCTM, 2000, p. 61) This study sheds light on some adolescents’ goals that could explain this potential reluctance. They may hold self-focused goals, such as those related to gaining status and appearing competent, due to their concern with an imaginary audience during adolescence (Elkind, 1978). Additionally, students may define appropriate behavior in ways that would cause them to hesitate to participate, as the students from working class backgrounds in Lubienski’s research (2000a, 2000b) or students from rural farming communities, as the student in Ridlon’s case study (2001). More research is needed to explore whether and why middle school students may be reluctant to participate in mathematics classroom discussions, and to determine characteristics of classrooms in which middle school students become less reluctant to participate.

Students’ Beliefs and Goals Influencing their Involvement

Students’ involvement in whole-class discussion appeared to be influenced by their beliefs and goals, particularly their participation in extended interaction segments, positioning, and high levels of explicit mathematical meaning. Students who expressed epistemological beliefs of negotiating knowledge and a perception of lower social risk were more likely to participate in these forms of involvement. Target students who expressed social goals of completing the task, gaining status, and behaving appropriately were more likely to participate in interactions with high levels of explicit mathematical meaning. Positioning occurred more frequently among target students who expressed the goal of completing the task and less frequently among target students who expressed the

goal of behaving appropriately. Extended interaction segments occurred less frequently among these target students as well. These three patterns of involvement – extended segments, positioning, and high levels of explicit mathematical meaning – appear to be influenced by students' beliefs and goals.

Extended segments and positioning. Students who participated in these interaction segments held the floor for multiple turns and critiqued the solutions of others. These ways of talking could suggest that these students have developed mathematical autonomy.

...autonomy is defined with respect to students' participation in the practices of the classroom community. In particular, students who are intellectually autonomous in mathematics are aware of, and draw on, their own intellectual capabilities when making mathematical decisions and judgments as they participate in these practices (Kamii, 1985). These students can be contrasted with those who are intellectually heteronomous and who rely on the pronouncements of an authority to know how to act appropriately (Yackel & Cobb, 1996, p. 43). If autonomy can be read off of students' involvement in whole-class discussions, those who participating in extended interaction segments and positioning may appear autonomous, since they expressed a stronger commitment to the ideas on the floor. But I would argue that an examination of behavior alone is not sufficient for determining whether students are mathematically autonomous. A range of beliefs or goals may support the same action. However, out of the target students in this study, more students who participated in extended and positioning interaction segments also expressed beliefs and goals that could suggest intellectual autonomy.

Students with epistemological beliefs about negotiated knowing, perception of low social risk, and a goal of completing the task were more likely to be involved in extended interaction segments and positioning. These beliefs and goal may indicate autonomy, particularly in light of students' epistemological beliefs. Negotiated knowing was partially determined by whether students suggested that their ideas were important to examine and compare to others' ideas in the process of learning mathematics. In the act of examining their own ideas, they drew on their own intellectual capabilities.

Students who hold beliefs and goals that lead to positioning and extended interaction segments (negotiated knowing, low risk, complete the task) may not be threatened by the idea of others critiquing them. Instead, they see this activity as a part of the learning process. They may not mind when they are critiqued, and they are less hesitant to critique the thinking of others.

Those less likely to position or participate in extended interaction segments were students with beliefs about received knowing, perceptions of high social risk, and those with the goal of behaving. They may be treating their classmates as they would prefer to be treated. They may prefer not to be critiqued themselves, so they do not participate in the activity of critiquing others. Additionally, their belief in received knowing suggests that they depend on an authority for learning, such as their teacher. While they may acknowledge that they can learn from, and even help, their classmates, they may not act as a knowledgeable authority through positioning.

High levels of explicit mathematical meaning. Students were more likely to involve themselves in mathematical talk with higher levels of explicit meaning if they expressed beliefs about negotiated knowing and low social risk and goals of completing

the task, gaining status, and behaving appropriately. Students with beliefs about negotiated knowing and low social risk may attain a depth of mathematical meaning in their talk through the process of relating their solutions to others. Students with a goal of completing the task may be invested in attaining a deeper understanding than students without this goal. A goal of gaining status may lead students to want to prove that they know mathematics well, resulting in higher explicit meaning. A goal of appropriate behavior may lead students to talk about mathematics with high levels of explicit meaning if they perceive that such talk is expected in their classroom.

Attending to whether and how students' beliefs and goals are related to their involvement in whole-class discussions provides insight into why some students may pick up on the discourse practices encouraged by particular teachers. Their beliefs and goals may fit with those being promoted by the teacher, and this fit can result in ease of involvement. Additionally, some students may shape the discourse practices of the classroom by initiating some patterns of involvement not explicitly encouraged by the teacher. For example, Allen and Tim initiated some efforts to position and critique in Ms. Carson's classroom. Ms. Carson did not invite this type of participation as purposefully as Mrs. Evans did. Instead, Ms. Carson tolerated the student's contribution, and then took back the roles of initiating and evaluating. Mrs. Evans used questions directed to students such as, "What do we think?" and these questions may be likely to evoke more critique and positioning. The teacher may not encourage positioning or extended segments, but students may assert their involvement in such a manner due to their expectations they developed from previous experiences in mathematics classrooms where teachers requested critique.

Teachers' Discourse Practices Influencing Students' Involvement

Students' involvement may also be influenced by the classroom discourse practices, as the teacher may take on a particular role in structuring these practices. Target students' involvement in each of these two classrooms differed in terms of their hesitancy and low and high levels of explicit mathematical meaning. The teachers may have influenced students' participation by structuring the classroom talk.

Levels of explicit mathematical meaning. Mrs. Evans and Ms. Carson each may have influenced the level of explicit mathematical meaning in their target students' interactions. Target students from Mrs. Evans's class were more likely to participate at high levels of explicit meaning, while target students from Ms. Carson's class were more likely to participate at low levels of explicit meaning. These teachers differed in terms of their use of the I-R-E participation structure as well as the nature of the questions they posed to their students.

Interaction segments in Ms. Carson's classroom were more aligned with the I-R-E structure than those in Mrs. Evans's class. Ms. Carson would take on the tasks of posing questions to the students and the role of evaluating students' responses. Rather than evaluating students' responses, Ms. Evans would ask the students, "What do *we* think?" Encouraging students to evaluate one another's thinking may have helped students' talk move to a higher level of explicit meaning.

Others questions that the teachers posed to students could have influenced the level of explicit mathematical meaning in students' talk. Ms. Carson would ask her students question such as "How did you solve this problem?" or "How do you know if you're correct?" Students would often respond by describing procedures, or lower levels

of explicit meaning. In contrast, Mrs. Evans would ask her students questions such as, “How do we know this is a linear relationship?” or “If Molly solved this with a graph, how would someone else solve this with a table?” These questions may be more likely to promote conceptual or relational responses from students, which would be at a higher level of explicit meaning.

Hesitant talk. While some of the teacher’s discourse may have influenced the level of explicit mathematical meaning in the students’ talk, other features of their talk may have influenced the less mathematical features of students’ talk, such as their hesitancy. Students who expressed hesitancy were those who either hedged in their talk or backed out of an interaction. Teachers’ discourse practices may have encouraged students to not be hesitant, such as affirming their participation through revoicing (O’Connor & Michaels, 1998) and following up on students’ attempts to direct the classroom talk. Alternatively, teachers’ discourse practices may have been implicitly accepting of students’ hesitancy, through maintaining control of the talk through I-R-E interaction segments.

Mrs. Evans was more likely to revoice students’ contributions and follow up on the students’ questions and comments during large group discussion. These discourse practices appeared to be well established by the spring. If students’ ideas are implicitly made valuable through the teacher’s appropriation of them, students may feel as if their thinking is affirmed. If students’ thinking is affirmed, they may not have a reason to express hesitancy in their interactions.

As mentioned previously, Ms. Carson was more likely to use the I-R-E format in her interaction segments. This teacher took on a more authoritative role during whole-

class discussions. Students in this classroom may have felt less empowered to participate in this classroom, and backed out of interactions or hedged during interaction when they were unsure.

These results suggest that the differences in how teachers structure classroom talk could influence their students' involvement in whole-class discussion. Teachers may not be cognizant of the ways in which their talk could have supported higher levels of explicit mathematical meaning in students' talk. Supporting the development of students' mathematical communication and reasoning would involve assisting students to communicate with high levels of explicit mathematical meaning. Teachers who ask questions that promote high levels of explicit meaning and structure discussions that support conceptual depth are not always easy to find when studying mathematics classrooms (e.g., Hamm & Perry, 2002).

Contributions from this Study

The results of this study contribute to and extend the research literature in mathematics education and educational psychology. For mathematics educators, these results provide new content for (a) discussing how issues of adolescent development intersect with the mathematics reform movement, as well as (b) revisiting the construct of mathematics anxiety. For educational psychologists, these results suggest the importance of revisiting the nature of constructs, such as (c) students' learning goals and (d) academic risk, within the context of subject matter and specific activities. Additionally, researchers interested in discourse analysis or analyses of students' thinking may engage in dialogue around (e) the analytic framework I presented for the analyses of students' beliefs. I will discuss each of these five contributions below.

Adolescent Development Intersecting with Mathematics Reform

Through the analyses of students' beliefs and goals, it appears as though students operate on academic and social goals simultaneously, such as students who are interested in completing their task and appearing competent to others. The goal of appearing competent may be heightened during adolescence due to students' development of metacognition and increased awareness that others may be thinking about them and potentially judging them. Social goals may have increased importance in the middle school years over the elementary school years. During adolescence, a significant task for young people is to develop an identity that fits them. As they attempt to work on their senses of self, they define themselves as alike and apart from others, and this activity is largely at work among students in middle school classrooms. Additional research could explore the change over time in students' social goals in mathematics classrooms.

Students' social goals may provide the opportunity to move beyond their beliefs about learning mathematics. Seven of the eight students who expressed beliefs that knowing mathematics involves receiving knowledge also expressed the social goal of helping their classmates. It may be that students could begin to move away from received knowing as they make attempts to help their classmates. If their efforts to help others through sharing their ideas in whole-class discussion, they could receive feedback on their thinking through this involvement and begin to relate their ideas to those of others. Then students may begin to experience, appreciate, and come to believe in the process of negotiating knowledge in mathematics through attempts to help. Future research could explore the development of adolescents' epistemological beliefs in mathematics

classrooms, including whether students' social goals act as a mechanism to support the development of their epistemological beliefs.

It is common for mathematics educators to briefly acknowledge that socio-emotional factors influence students' learning of mathematics. However, socio-emotional aspects of adolescence are more than a footnote in seventh graders' experiences of learning mathematics. Rather, they are central to the process of learning. If students' perception of risk in the mathematics classroom is aligned with their beliefs about the process of learning, and epistemological beliefs may be also related to their achievement (Schommer et al., 1997) or their learning goals (Cobb, 1985), then attending to socio-emotional factors in the mathematics classroom could promote more productive learning goals or perhaps promote increased achievement.

The *Principles and Standards of School Mathematics* (NCTM, 2000) discusses both content and process standards for teaching and learning mathematics. The content standards (e.g., algebra, geometry, measurement) include detailed recommendations for what teachers should address with their students across grade bands. In contrast, the process standards are less detailed in their recommendations across grade bands. For example, the appendix of the PSSM does not break down process standards by grade bands, but it does discuss content standards by grade bands.

In the larger body of the text, the process standard of *communication* for the 6-8 grade band begins to address some of the developmental concerns of adolescence.

During adolescence, students are often reluctant to do anything that causes them to stand out from the group, and many middle-grades students are self-conscious and hesitant to expose their thinking to others. Peer pressure is powerful, and a

desire to fit in is paramount. Teachers should build a sense of community in middle-grades classrooms so students feel free to express their ideas honestly and openly, without fear of ridicule (NCTM, 2000, p. 267)

There is a need for more detailed recommendations for teachers to consider when they are working to encourage students to “feel free to express their ideas honestly and openly, without fear of ridicule.” One of these recommendations could include considering whether to refer to students’ solutions by attributing them to a specific student (e.g., “Becky said...” versus “one student said...”), as some students may appreciate being singled out, while others might not. Also, sometimes teachers are encouraged to develop student-to-student dialogue in their classrooms (Nathan & Knuth, 2003), but teachers should also reflect on potential social implications of such a practice, as students may call on their friends or call on people in order to single them out unfavorably. The mathematics education community could benefit from addressing social implications of whole-class discussion.

Educational psychologists have begun to explore whether and how upper elementary school teachers’ motivational discourse supports students’ motivation (Turner et al, 1998, 2003). Supportive motivational discourse in the classrooms from these studies involved evoking students’ curiosity, providing encouragement, commenting on progress, responding positively to errors, advocating risk taking, and talking about challenge as desirable. Classrooms with this supportive motivational discourse were more likely to have increased involvement from students, students were more likely to perceive tasks as challenging rather than threatening, and students were less likely to report self-handicapping behaviors. Attending to relations between teachers’ motivational discourse

in the analysis of classroom talk is one way that researchers have acknowledged the role of affect in learning. Future research in mathematics education should do more to account for socio-emotional factors in the process of students' learning, particularly the socio-emotional factors most relevant to students in their respective stages of development, such as issues of competence in adolescence.

Revisiting Mathematics Anxiety

Mathematics anxiety as a psychological construct received much attention in mathematics education in the 1970's and 1980's. Sheila Tobias wrote an article in *Ms.* in the late 1970's (Tobias, 1976) that called public attention to mathematics anxiety. Her article stated that the use of the term "anxiety" cast women's feelings about mathematics as pathological, since more women suffered from mathematics anxiety. After this article it appeared, educators considered it advisable to focus on math anxiety as a psychological state rather than as a defect (Tobias and Weissbrod, 1980). Mathematics anxiety has since been studied in relation to achievement and performance (Hembree, 1990; Ma, 1999). However, the construct may be under-conceptualized, as mathematics anxiety has similarities with mathematics avoidance (Dew, 1984), low confidence in mathematics, (Fennema & Sherman, 1976) or low self-efficacy in mathematics (Cooper, 1991).

Mathematics anxiety appears to be related to students' test anxiety (Hendel, 1980), such that the two are difficult to separate (McLoed, 1992). The anxiety may be rooted in performance, or a fear that one may not perform as well as he or she would like on tests in general and mathematics in particular. Opportunities to perform would be more threatening for a student who is experiencing mathematics anxiety.

Given the performance component of mathematics anxiety, the results of this study suggest that whole-class discussions about mathematics may foster another form of mathematics anxiety among students, social mathematics anxiety. Social mathematics anxiety would then be the fear of being incorrect in front of peers. It may occur among students who perceive an extremely high sense of social risk in discussion-intensive mathematics settings. It is possible that teachers' supportive motivational discourse can serve to reduce the development of this anxiety.

Both the performance dimension of mathematics anxiety and the increased opportunities for student performance in Standards-based mathematics settings suggests future research on the nature and development of mathematics anxiety. Anxiety about mathematics may be socially constructed through interactions in the mathematics classroom. Students who express anxiety may be reacting to a classroom that emphasizes performance or competition, either explicitly or implicitly. If students experience mathematics anxiety as a perceived personal deficit (e.g., I am afraid I won't be able to do this, because I don't think I am good at it.), then that deficit may be most worrisome and painful in the adolescent years when students are actively comparing themselves to others.

Revisiting Learning Goals

The results from this study may appear to parallel previous research in educational psychology on students' learning goals. The target students revealed competence goals of avoiding the appearance of incompetence (performance-avoidance goals) as well as being gaining status for their ideas and appearing competent (performance-approach goals) in their interview talk. The students did not express goals

related to understanding the material (mastery goals) as clearly. Rather, they talked about completing the task before them, which may be more of an efficiency goal.

While performance and mastery goals are common constructs in the study of student learning, these constructs may be too generally conceived to be explanatory. These goals may have specific components that, once broken down, may provide insights that are helpful for considering learning process in specific subject matters, such as mathematics, and in relation to specific activities, such as involvement in whole-class discussions. Dowson & McInerney (2003) determined that students' goals were multi-dimensional constructs, including affective and behavioral components as well as cognitive components, after interviewing 86 middle school students, ages 12-15. For example, students may demonstrate performance goals¹⁶ through behaviors that relate to measuring performance relative to classmates, such as working harder than necessary to secure a higher grade. Additionally, affective reactions could indicate performance goals, particularly those that may be maladaptive for engaging in academic work, such as debilitating anxiety about schoolwork. Finally, Dowson and McInerney described cognitive components of performance goals, indicated by a variety of relatively shallower cognitive processes, such as copying notes from the blackboard without thinking about them. Future research could examine components of performance, mastery goals, and social goals in various subject matters, as Dowson and McInerney did not account for subject matter in their study.

Components of performance goals that target students expressed were along the affective dimension, manifested in students' expressions of their perceptions of social

¹⁶ Dowson & McInerney (2003) did not distinguish between performance-approach and performance-avoidance goals.

risk and overlapping with social goals. Target students also expressed a goal of wanting to appear competent. This goal may be related to performance goals, and it goal also has features of a social goal, of being interested in gaining approval for one's ability, based on how students talked about wanting to appear competent. Examining students' goals through their talk may reveal more of the components of the goals that relate to specific activities or subject matters.

Revisiting Risk in Academic Work

Students' perceived level of risk associated with participating in whole-class discussions about mathematics could be related to the risk inherently involved in academic work (Doyle, 1983). Academic risk is has been discussed in light of the evaluative nature of classrooms. Teachers evaluate student performance constantly, and their parents and classmates may be made aware of these evaluations.

Doyle (1983) writes about academic tasks being associated with dimensions of ambiguity and risk. Less ambiguous tasks are those for which a precise answer may be pre-defined, or for which a method of generating an answer is readily available. The level of ambiguity is considered to be inherent in the task. Risk is a feature that Doyle attributes to the teacher, with respect to the stringency of the evaluative criteria and the possibility of meeting these criteria.

Results from this study suggest that students may not only see risk in the academic work that is graded, but also in the academic work that is evaluated more informally, such as their interactions about mathematics in the public forum of whole-class discussions. Students may not only be concerned with informal evaluations from the teacher, but also those from their classmates. Future analyses of the risk involved with

academic work should address both graded and ungraded academic work, and evaluations from both teachers and classmates.

Methods: Analytical Framework for Analysis of Students' Beliefs

It is a challenge to claim with confidence that a student holds a "belief." Whether or not a student is aware of his or her beliefs, students' level of commitment toward the beliefs, the stability of the beliefs, and the inter-relations and overlapping of beliefs all complicate the process of analyzing students' beliefs. Through designing this study, I developed an analytic framework for analyzing students' talk in interviews in order to claim with greater confidence that students were expressing a belief, inspired by previous analyses of students' talk in order to infer their attitudes (Bills, 1999). According to the framework developed for this dissertation, repetition (Tannen, 1989) was indicative of commitment. Use of imperative verbs, such as "need to" or "have to," suggested the idealized state that beliefs may hold (Abelson, 1979). Affective statements suggested beliefs, due to the strong relations between beliefs and affect (Hannula, 2002). Recall that before claiming that a student held a belief, I analyzed the student's interview talk in order to determine whether their talk met any two of the three criteria explicated above.

This framework for inferring students' beliefs from their talk may not be the only relevant framework for its purpose. However, being explicit about one's methods for analyzing and inferring beliefs from students' talk adds a level of rigor in support of claims. This framework afforded a distinction between talk students used when answering my questions for the sake of being polite, and the talk that revealed stronger evidence of their beliefs. Additionally, being explicit about methods supports the opportunity for replication of studies. Future research on students' beliefs should

continue to be explicit about methods for inferring beliefs from students' (or teachers') talk.

Limitations

While this study contributes to the research literature, there are limitations to the generalizability of these results. Future research is needed to determine whether these results about middle school students' involvement in whole-class discussions about mathematics hold only for White seventh graders in rural communities in a Midwestern state, or only among students in schools or classrooms that use Standards-based mathematics curricula.

Certain background data may have been reasonable to collect, but was not collected, in this study. Lubienski's (2000a, 2000b) research on students' participation in a seventh grade Connected Mathematics Project classroom explicitly examined how social class appeared to influence students' involvement. SES data was not collected for the participants in this dissertation study, which eliminated the possibility of analyzing for patterns among students from particular social classes. While it is not realistic to expect to analyze for every background dimension in one study, the potential parallels between Lubienski's study and this dissertation did not become clear to me until I was nearing the end of my data analysis. Future research could extend these two studies in order to determine whether social class appeared to shape students' epistemological beliefs about the process of learning as well as perception of social risk.

Additionally, the analyses of students' beliefs and goals relied on my interpretations of students' interview data. While I have argued earlier in this manuscript that this is a strength, due to the openness to students' perspectives that interviews can

provide, as well as the analytic framework I can employ to analyze students' talk, I want to acknowledge the complexities associated with interviewing students. For one, these seventh grade students had the opportunity to observe me talking with their teachers informally before and after class. While I told students that whatever they shared with me would be kept confidential, it is possible that they would remain skeptical about this, and censor themselves in light of their awareness of my relationship with their teacher. Also, they had a relationship with me outside of the interview setting, since I was in their classrooms regularly in both the spring and previous fall. To a certain extent, their interview responses could have been about their attempts to develop a positive relationship with me as the researcher and classroom visitor. For example, if students are concerned with good behavior, they may have images of what good behavior during an interview or appropriate interview responses should be. While I believe that my analytic framework assists in sorting some of this "good student" talk out of the analysis, if they repeated the talk enough during the interview, their responses mentioned out of appropriateness concerns may be included in these results. In spite of some of the drawbacks of interview analyses, they can also afford access to students' thinking that Likert-scale surveys do not, as can open-ended questionnaires (Aikenhead, 1987).

Implications

While I intend to share the results of this study with the teachers I worked with, at the time of drafting this manuscript, I have not yet done so. My current thinking about how the results of this study inform mathematics classroom practice are based upon my previous interactions with these two classroom teachers during data collection. I recall two questions from them that influence how I view the implications of this study. I will

share these questions, and explain how this dissertation helped me think about these questions, below. Implications from this dissertation address knowledge for teachers about students' beliefs as well as classroom interventions for potentially influencing students' beliefs and goals.

Teachers' Knowledge of Students' Beliefs and Goals and Their Development

Frequently after I administered surveys, and sometimes after I interviewed students, the teachers would ask me, "How did they do?" I was initially surprised by this question. I did not have strong preferences for how I would prefer the students to respond on either the survey or the interviews. These teachers had a different sort of investment in the students' responses. They teach a middle school Standards-based mathematics curriculum that is under scrutiny by parents and high school teachers. Their students' responses in surveys and interviews could be another outcome that to be used by someone, perhaps even me, as ammunition either for or against CMP, or for or against their teaching strategies. I see their question as reasonable, given the nature of the data I am collecting. In order to respond to their question, I intend to share my results with these teachers, and ask them what they think. How do *they* think the students did? What do they make out of my interpretations of the data? Would they interpret it differently?

Research has demonstrated that teachers and students make different judgments of students' motivations (Givvin et al., 2001), so it would not be a surprise if these teachers had not identified the same beliefs and goals for these particular students. If these beliefs and goals are important to the target students, it would be interesting to learn whether they are important to their teachers. If they are not, perhaps teachers could benefit from knowledge of students' beliefs and goals, in terms of which beliefs and goals students

may act upon when involving themselves in whole-class discussion. With this knowledge, teachers may interpret their students' involvement differently than they did before, and perhaps structure their classroom conversations differently as a result.

Teachers could also benefit from knowledge about the development of students' beliefs and goals. While this study did not explicitly address development of students' beliefs and goals, such knowledge could assist teachers by releasing a burden of feeling responsible for completely shaping students' dispositions. Students' beliefs and goals develop over long periods of time as a result of their cultural experiences and home lives outside of school in addition to their classroom experiences. During interviews, many students cited experiences from previous years when justifying their perspectives. For example, when students talked about the importance of being able to solve problems with more than one method, they shared experiences from elementary school, particularly multi-digit multiplication or addition. Students' histories in classrooms over time influence their beliefs. Current teachers may be planting seeds of influence that may take time to manifest. Teachers have a great deal to be responsible for, so results from research on the development of students' beliefs could help them see both the possibilities for influencing students' beliefs as well as the limited influence they may have. Such insights, while making them feel less powerful and effectual, may also release a bit of the burden of wanting to influence dispositions in addition to teaching content.

I would not interpret the results of this dissertation as an evaluation of the effectiveness of CMP or of these teachers' practices. Several of students' patterns of involvement appear to be influenced by their beliefs and goals, and these beliefs and

goals could have developed outside of school as well as in school, in previous classrooms as well as their current classroom.

Classroom Interventions

There may be ways in which teachers can structure classroom discussions that influence students' involvement in them, which may, in turn, influence students' beliefs and goals related to learning mathematics. I remember an instance when one of the teachers asked me about what I thought she could do to help her students get involved in classroom discussion. The teachers' second question, then, was "What can I do?" Rather than answering her, I asked her what *she* thought she should do, as I didn't have my thoughts formulated in that moment, and I was concerned about interfering with the study itself.

The results of this study suggest classroom interventions for supporting teachers' efforts to structure their classroom discourse in order to influence students' talk about mathematics. Recall that target students' involvement differed by classroom in terms of hesitancy and level of explicit mathematical meaning. Hesitancy may be considered a social element of talk, while level of explicit meaning is about the mathematics in the talk.

In order to encourage and foster students' autonomous thinking about mathematics, it may be worthwhile to attempt to reduce their hesitancy in their involvement. There was less hesitancy in Mrs. Evans' classroom. In Mrs. Evans' classroom, there was off-topic or non-mathematical talk, some humor, and also students took the opportunity to shift the discussion topics. It may have been possible that the off-topic and humor allowed the students to feel comfortable to initiate discussion topics.

Additionally, fostering supportive motivational discourse (Turner, 1998; 2003), fostering courage and humility among the classroom community (Lampert, 1990), and talking about expectations for talking (Yackel & Cobb, 1996) may assist in creating a classroom community in which students feel safe to contribute their own ideas and speak with less hesitancy.

Many researchers share the goal of wanting to understand how to promote class discussions in which students participate at a high level of explicit meaning (e.g., QUASAR, 1996; Hamm & Perry, 2002). Results from this study suggest that teachers should ask students questions that are relational and conceptual rather than procedural and to minimize the focus on correct answers. Additionally, teachers could shift their role such that students are asked to evaluate the solutions of their classmates. However, previous research has shown that initial attempts to shift the authority of the teacher may decrease the level of explicit meaning in the talk (Nathan & Knuth, 2003). Teachers may need to learn skills for scaffolding students' critique and positioning in order to maintain higher levels of explicit mathematical meaning in the talk.

A challenge in structuring classroom discussions may be the need for decisions of which to focus on first: creating a welcoming community that may reduce hesitancy or foster high explicit meaning in students' mathematical talk. With adolescent students' focus on social goals, it appears to be important to teach students how to be in community with one another as well as teach them academic content. There is more to learn about the delicate balance between teaching students how to participate in a mathematics learning community and teaching students mathematics, as the two are interrelated and inseparable in discussion-intensive classrooms. Given the middle school

student population, it may be fruitful to invest heavily in the development of classroom community earlier in the school year, which could provide a foundation that then opens up the opportunity to take risks related to sharing their ideas about mathematics.

It is not clear which comes first, involvement or changes (even minor adjustments) in beliefs. Students may develop new patterns of involvement and trying on ways of thinking about mathematics through talking about mathematics, and then their beliefs may shift in line with these ways of talking and thinking about mathematics. Alternatively, students may develop new beliefs and goals given life outside of the classroom, or even new events within the classroom, and shift their involvement accordingly.

Future Research

While this dissertation provides new insights on students' beliefs, goals, and involvement in mathematics classroom discussions, more questions remain to be explored. There are many open questions for the fields of mathematics education and educational psychology in general—some of which are implicit in the previous sections of this chapter. Additionally, there are some specific next steps I intend to undertake in my own research program in order to extend these results. Below I will first present the range of open questions, and then I will discuss my own next steps as a researcher of these issues.

Open Questions for the Field

While this study may contribute to the research literature, the results also suggest unanswered questions for the field of mathematics education and educational psychology. Mathematics educators could continue to take up questions related to socio-emotional

factors in the learning process. While this study was a descriptive study of relations between students' beliefs and goals and their involvement in whole-class discussions, future research is needed to take up questions related to the development of these beliefs and goals and the classroom processes that influence students' involvement.

Importance of socio-emotional factors in mathematics learning. This study described some of the socio-emotional factors among seventh graders as they learn mathematics in discussion-intensive settings. Socio-emotional issues have not been taken up as frequently as research on student cognition in mathematics education (Lubienski & Bowen, 2000). What are the additional socio-emotional factors that are a part of students' learning experiences in Standards-based mathematics settings? How might these socio-emotional processes facilitate or inhibit learning? In particular, do Standards-based classrooms foster or curb mathematics anxiety or academic risk? If so, how? If social goals are an important factor in students' experiences in Standards-based mathematics classrooms, what are the behavior, cognitive, and affective components of these goals? How do the answers to these questions change as students develop and age?

Development of students' beliefs and goals. The relations among students' beliefs and goals as described in this study suggest that even though these beliefs and goals may co-occur at one point in time, they may not have all developed at once. Which came first, and how did the development of an earlier belief or goal support the development of future beliefs and goals? Do students' epistemological beliefs predict the development of social risk, or vice versa? If so, which epistemological beliefs, those about the nature of the subject matter or the process of learning the content, predict the development of social risk? Do students' social goals act as a mechanism to promote the

development of epistemological beliefs? As students act on the social goal of helping their classmates, do they start to value the process of negotiating knowledge?

Examining students' beliefs and goals in a variety of mathematics classrooms – those with both Standards-based and non-Standards-based curricula, those in a range of geographic locations – would allow for contrasts. These contrasts would allow for an examination of which classroom processes play a stronger role in belief and goal development in mathematics classrooms. Which classroom processes foster particular beliefs and goals?

Age contrasts across elementary, middle, and high school would allow for an examination of whether adolescence is a time when students are reluctant to participate and why. Do students' social goals indeed become heightened in adolescence? If so, are any social goals heightened more than others? Do some of these social goals become less important as students age?

Furthering My Own Research Agenda

As I advance my own research agenda, I will build upon this study. My immediate next steps will extend the results from my dissertation by examining students' beliefs, goals, and involvement in additional middle school classrooms in different settings and in multiple subject matters. Not only will I diversify my analyses, but I will also incorporate contrasts that allow me to compare students' beliefs, goals, and involvement in different academic domains. Further steps include the following: (a) extending these analyses to focus on pre-service teachers' beliefs, goals, and involvement in their mathematics education courses, (b) studying the range of adolescent young women's ways of learning mathematics, (c) studying teachers' assessments of students'

beliefs and goals compared to their own assessments, and (d) the development of students' beliefs and goals.

Extending these results: Generality & domain specificity. As mentioned previously, this study was conducted with two classrooms in the same school, with 15 target students, all of whom were White and from a small Midwestern town. These results are not necessarily generalizable to all populations. A small-scale descriptive study allows for developing models and new hypotheses to test in larger samples. It is possible that the results from this dissertation will hold in other settings, but examinations of other classrooms and communities would allow me to expand the framework of beliefs and goals in light of new data. Additionally, new relations between students' beliefs and goals and their involvement may manifest in with more students in more classrooms.

I would also like to extend this study by conducting similar analyses in other academic domains, such as English, social studies, or science, to contrast with students' experiences in mathematics classrooms. This would allow for analyses of the domain specificity of students' beliefs and goals and their relation to students' involvement. The issue of domain specificity of epistemological beliefs is currently under debate in the field of educational psychology. Also, the examination of other academic domains could shed light on the specific nature of mathematics anxiety.

Pre-service mathematics teachers' beliefs, goals, and involvement. Taking the analyses into other populations could also extend these results. The field of mathematics education is interested in improving teacher education. Research on whether and how pre-service teachers' beliefs and goals influence their learning, particularly their involvement in discussions, could contribute to understanding pre-service teacher

learning. Would results differ for pre-service elementary teachers in contrast to pre-service secondary teachers?

Adolescent young women's ways of learning mathematics. An issue left unexamined thus far in this dissertation is whether and how sex differences factor into relations between students' beliefs and goals and their involvement. Target students' involvement appeared to be influenced by their beliefs and goals, but were these beliefs and goals explained by sex differences? Certainly there were more young women who expressed beliefs about received knowing and high social risk, but there were also young men in this category. While there were more young men who expressed beliefs about negotiated knowing and low social risk, there were some young women who also expressed these beliefs.

Rather than generating claims that describe what the majority of adolescent young women appear to believe, it may be beneficial to examine the diversity within young women's beliefs. Future research could develop a framework of adolescent young women's ways of learning mathematics. Such a framework would be useful for breaking down gender stereotypes of young women in mathematics classrooms.

Teachers' assessments of students' beliefs and goals. Students' beliefs and goals are influenced by what teachers expect from them—though teachers' influence may change as students grow older. Teachers may implicitly make assumptions about the beliefs and goals that their students hold and treat them accordingly. I am interested examining teachers' assessments of students' beliefs and goals and comparing their assessments with students' assessments of themselves, similar to Givvin et al. (2001). Such a study could also serve as an intervention for teachers, helping them develop

knowledge of students' beliefs and goals in mathematics classrooms. Teachers could benefit from being more aware of their students' beliefs and goals, or at least how their assessments may not be similar to students' assessments of themselves, so that teachers' implicit expectations of students do not serve as self-fulfilling prophecies in unproductive ways.

Development of students' beliefs and goals. Why do some students develop beliefs about negotiated knowing and others develop beliefs about received knowing while simultaneously experiencing mathematics learning in reform settings? There may be important variations between teachers' practices in reform settings that promote the development of certain beliefs and goals over others. But other developmental factors may be situated more in the lives of students than in the beliefs and practices of their teachers. For example, how do factors in students' lives outside of school contribute to the development of their beliefs and goals, such as socio-economic class (Lubienski, 2000a, 2000b)? And how might dimensions of personality, e.g., introversion vs. extroversion, interact with the developmental challenges of adolescence in the very public setting of the classroom—especially those where whole classroom discussion is an important activity?

This dissertation study serves as a foundation for a breadth of future research in my research program. While I have learned ways in which seventh grade students' beliefs and goals influence their involvement, I am left with many new questions as a result of conducting this study.

Summary

The perspectives of the target students in this study expressed beliefs and goals related to seeking and avoiding opportunities to involve themselves in whole-class discussion. Students' motivation in mathematics classrooms has been typically studied in terms of performance and mastery goals, self-efficacy, or self-handicapping. These students spoke about their experiences in ways that suggested other constructs. Their talk in interviews illustrated that epistemological beliefs and social goals were pursued simultaneously in their attempts to get involved with or resist participation in mathematics classroom discussions. Mathematics educators could benefit from attending to students' social goals, as creating an effective classroom learning community involves attention to social harmony as well as conceptual depth in the subject-matter. Students' motivations for avoiding or approaching involvement were related to whether they were involved in positioning, or critiquing their classmates' solutions, extended interaction segments, and talk at a high level of explicit mathematical meaning. While their teachers' practices also supported a high level of explicit mathematical meaning, students' beliefs may predict who is more likely to involve themselves in these interactions. Continued efforts to examine relations between students' beliefs and goals and their involvement in discussions may explain how a distribution of diverse students can work together to support the development of classroom norms.

Closing

The perspectives of the target students in this study expressed beliefs and goals related to seeking and avoiding opportunities to involve themselves in whole-class discussion. Students' motivation in mathematics classrooms has been typically studied in

terms of performance and mastery goals, self-efficacy, or self-handicapping. These students spoke about their experiences in light of other constructs. Their talk in interviews illustrated that epistemological beliefs and social goals were pursued simultaneously in their attempts to get involved with or resist participation in mathematics classroom discussions. Mathematics educators could benefit from attending to students' social goals, as creating a classroom community is both about social harmony as well as conceptual depth. Students' motivations for avoiding or approaching involvement related to whether they were involved in positioning, or critiquing their classmates' solutions, extended interaction segments, and talk at a high level of explicit mathematical meaning. Although their teachers' practices also supported a high level of explicit mathematical meaning, students' beliefs may predict who is more likely to involve themselves in these interactions. Continuing to examine relations between students' beliefs and goals and their involvement may explain the how a distribution of diverse students can work together to support the development of classroom norms.

Students' experiences in mathematics classrooms should not be reduced to achievement measures alone. The socio-emotional factors that support and constrain students' involvement are inextricably related to the learning of mathematics content.

Students do not just learn methods and processes in mathematics classrooms, they learn to *be* mathematics learners and their learning of content knowledge cannot be separated from their interactional engagement in the classroom, as the two mutually constitute one another at the time of learning. (Boaler, 1999, p. 380, italics in original)

After the school year ends, how students felt about themselves during their mathematics class may remain long after the formulas fade.

APPENDIX A

Interview Protocol, Spring 2003

Introduction:

I have been spending time in your classroom recently and back in September and October because I am trying to better understand how kids learn math in order to improve math textbooks and help teachers. So, I am trying to understand very carefully, as a researcher, how YOU think. Let's pretend I'm trying to create a robot who thinks just like you, who thinks and learns just like you learn. When you answer my questions today, try to give me as many details as you can in order to help me make that thinking machine. OK?

How do you feel math has been going for you since the last time I saw you?

- What has been going well? What do you think has been helpful about the class lately? What has helped you learn?
- Can anything about your class be improved to make your experience in class better? If you wanted to make the situation a better learning experience, what could your teacher do? What could your peers do? What could YOU do?

Say a new student moved into your school and they wanted to know what they had to do to be successful in your math class. What would you say?

- What exactly are you paying attention to? Remember that we want to make a robot that thinks like you!
- People talk about paying attention to a lot of different parts of the class. Some people say that they pay attention to the teacher's explanations of the problem's directions, some say that they listen carefully to a classmate's explanation of a problem, others say that they listen carefully to the teacher explaining the steps for how to calculate the answer to a problem, and there still more things to pay attention to. Do you pay attention to any of those things? Are any of them more important than the others? Can you try to specifically describe what you are paying attention to?
- Do you do anything else in class besides PAY ATTENTION? Do you ever talk or share your thinking about a problem? How often?
- What is more important for your learning – to talk through your thinking with someone or to listen to the thinking of another? Why would you say so?

Say that your school needed to hire a new 7th grade math teacher and they were asking students for advice on what would make a good 7th grade math teacher. What would you say?

- Does your current teacher do any of these things? Do you wish she would?
- Which one of your teachers did something like this, if anyone?

Would you rather listen to the ideas of others or share your own ideas? Why would you say so?

Elaboration on responses to survey:

3. I want to do better than other students in my math class.

- Why? (or why not?)

- Do you compare yourself to other people in the class? Does this affect how you learn?
 - How important is doing better than other students to you?
 - How important is it to you to make sure no one in the class is left behind when moving on to a new problem? How about moving on to a new book?
16. When my teacher asks a question in math class, it is important that I explain how I did the problem, not just give my answer.
- (If it is important to them to explain...) How important is it to explain your thinking? Does it help you learn or not?
 - When do you think you should be asked to explain and share your thinking? In what ways?
 - When are you asked by your teacher to explain your thinking? Does this help you learn?
35. It is possible to approach the same math problem in more than one way.
- Is it important to you to be able to solve a math problem in more than one way? Why or why not?
 - Does being able to solve a math problem in more than one way help you learn anything or does it really matter?
40. To work on math problems, I have to be taught the rules & steps, or else I can't solve them.
- Can you try a new problem without help and be able to solve it?
 - If you need some help to get started, what kind of help do you need? Please be specific as you can. Again, remember that we need to try to create a robot to think just like you think!

APPENDIX B

Students' Mathematical Views Instrument: 9 Scales, 63 items

Not at all True	Not Very True	Somewhat True	Mostly True	True
1	2	3	4	5

Confidence scale:

- 1. I feel secure about attempting math problems.
- 27. I am sure that I can learn math.
- 36. I think I could handle more difficult math.
- 46. I can get good grades in math.
- 63. I have a lot of self-confidence when it comes to math.
- 41. I'm no good in math.
- 14. I'm not the type to do well in math.
- 28. Even though I study, math is unusually hard for me.
- 55. Most subjects I can handle OK, but I have a knack for messing up math.
- 61. Math is my worst subject.

Task Orientation scale:

- 2. I like doing problems in math class that I'll learn from even if I make a lot of mistakes.
- 15. An important reason why I do my work for math class is because I like to learn new things.
- 26. I like work in math class best when it really makes me think.
- 37. An important reason why I do my work for math class is because I want to understand it.
- 47. An important reason I do my work for math class is because I enjoy it.
- 56. I do my work for math class because I'm interested in it.

Ability Orientation scale:

- 3. I want to do better than other students in my math class.
- 10. I would feel successful if I did better than most of the other students in my math class.
- 25. I'd like to show my math teacher that I'm smarter than the other kids in this class.
- 38. Doing better than other students in this math class is important to me.
- 48. I would feel really good if I were the only one who could answer the teacher's questions in (math) class.
- 57. It's important to me that the other students in this math class think that I am good at my work.

Process vs. Product scale:

- 4. When my teacher asks a question in math class, I have to say the right answer to answer it correctly.
- 16. When my teacher asks a question in math class, it is important that I explain how I did the problem, not just give my answer.
- 24. Just because I got the wrong answer doesn't mean I don't know how to do the problem.
- 39. If I get the wrong answer, I do not know how to do the problem.
- 49. I want to hear how other students did the math problem, not just what they got for an answer.
- 58. I do not think it is important for students to share how they did a math problem, only the answers they got.

Autonomy & Authority scale:

- 5. In math, it is possible to discover things by myself.
- 11. Math problems can be solved by thinking carefully, not just by the math rules I learn in school.
- 23. I cannot discover things about numbers in math class without being taught.
- 40. To work on math problems, I have to be taught the rules & steps, or else I can't solve them.
- 42. In math, the teacher has the answer and it is the student's job to figure it out.
- 50. When a classmate and I don't agree on an answer, we can usually think through the problem together until we have a reason for what is correct.
- 29. When a classmate and I don't agree on an answer in math, we need to ask the teacher or check the book to see who is correct.

Usefulness scale:

- 6. I'll need math for my future work.
- 17. Math has very little to do with my life.
- 22. Math is a worthwhile subject for me.
- 30. Taking math classes is a waste of time.
- 32. Knowing math will help me earn a living.
- 43. Math will not be important to me in my life's work.
- 51. I will use math in many ways as an adult.
- 59. I expect to have little use for math when I get out of school

Structure scale:

- 7. Tables and graphs have little to do with other things in math like formulas and equations.
- 12. Often a single mathematical idea will explain many equations or steps for doing a problem.
- 21. Finding answers to one type of math problem cannot help you solve other types of problems.
- 31. Math is mostly thinking about relationships among things (for example: numbers, points, and lines).

- 33. There are few connections between the different mathematical topics I have studied (for example: measurement and fractions).
- 44. Ideas learned in one math class can help you understand material in the next math class.
- 52. Math consists of many unrelated topics.
- 60. Most mathematical ideas are related to one another.

Conceptual vs. Procedural scale:

- 8. The math that I learn in school is mostly a set of rules to memorize
- 18. The math that I learn in school is a way to think about the world around me
- 20. The math that I learn in school is mostly about computation, like addition, subtraction, multiplying, or dividing.
- 34. The math that I learn in school is mostly a set of steps to remember
- 53. The math that I learn in school is not just a set of rules.
- 45. The math that I learn in school is about seeing new relationships in data
- 64. The math that I learn in school is mostly about ideas

Multiple Methods scale:

- 9. A math problem can only be done correctly in one way
- 19. A math problem can be solved correctly in more than one way
- 35. It is possible to approach the same math problem in more than one way
- 54. It is not possible to approach the same math problem in more than one way.
- 13. Once I get an answer I know is correct, looking at a second way to do the problem is pointless
- 62. When your method of solving a problem is different from your teacher's method, your method can be as correct as your teacher's.

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