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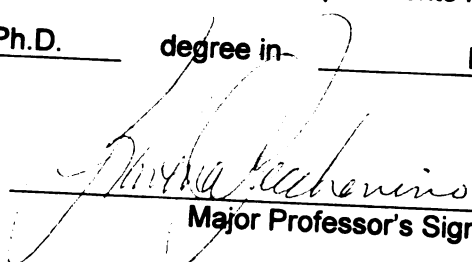
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ESSAYS ON INFLATION STABILIZATION  
IN A SMALL OPEN ECONOMY

By

Hyuk-jae Rhee

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Submitted to  
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# **ABSTRACT**

## **ESSAYS ON INFLATION STABILIZATION IN A SMALL OPEN ECONOMY**

By

Hyuk-jae Rhee

The main objective of this dissertation is to develop models to explain the stylized facts of various stabilization programs observed in developing countries in a single unified framework. The basic model is Obstfeld and Rogoff's dynamic general equilibrium Mundell-Fleming model with cash-in-advance constraints. I extend the model to allow for sticky inflation. With this framework I simulate the disinflation experiments in high inflation environments.

Chapter 1: "Exchange-Rate-Based Stabilization" examines inflation stabilization using the rate of devaluation as a nominal anchor under a fixed or pre-determined exchange rate regime. This study shows that the credible reduction in the devaluation rate induces initial expansion of non-tradable output and produces sustained expansion. The inflation rate, however, shows substantial persistence even if the policy is fully credible. This is because of backward-looking wage contracts. This result is quite consistent with the stylized fact of successful exchange rate-based stabilization programs. An incredible reduction of the devaluation rate results in a "boom-recession cycle" and inflation inertia, which has been observed in stabilization programs. Therefore, I conclude that as long as the indexation mechanism has a backward-looking component, credible disinflation can

not reduce the rate of inflation rapidly and inflation shows inertia. I also point out that the credible disinflation can be welfare-improving regardless of inflation persistence.

Chapter 2: “Money-based stabilization” analyzes the disinflation attempt in which the growth rate of the money supply is used as the nominal anchor under the flexible exchange rate regime. This study shows the initial contraction in the non-tradable sector when the growth rate of the money supply is reduced. This is because the initial appreciation of the nominal and real exchange rates. When the supply-side effect, however, dominates, we observe long-run expansion of the non-tradable sector. In the case of an incredible policy, we can replicate the “recession-boom cycle” that we observed in money-based stabilization programs. In both cases, inflation persistence is obtained. This is mainly due to the backward-looking indexation in wage contracts. This study also suggests that the credible money-based disinflation can be welfare-enhancing regardless of initial recession and inflation persistence.

Chapter 3: “Interest rate rule for inflation targeting” analyzes the nominal interest rate policy for reducing high inflation. I use the same model and same approach as in the previous chapters. However, I look at the effect of an inflation targeting interest rate rule. The interest rate policy generates a severe recession with inflation inertia throughout the inflation stabilization program is observed even if the policy is fully credible. Therefore I argue that the nominal interest rate is not the appropriate policy instrument for fighting high inflation.

## ***To My Parents***

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I forever owe a special debt of gratitude to the chair of my dissertation Committee, Rowena Pecchenino, who taught me macroeconomics, for her suggestions, encouragements, and comments on this dissertation. The same debt I have towards Steven Matusz Andrei Shevchenko, Tomas Jeitschko for having offered me support and advice. I am honored to call myself their student.

Throughout all the hours of work my wife provided an emotional and philosophical support. Of course, I am grateful to my parents for their patience and *love*. Without them this work would never have come into existence (literally).

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# Chapter 1

## Exchange rate-based stabilization

### 1.1 Introduction

Chronic inflation has been one of the distinguishing macroeconomic phenomena in developing countries since World War II.<sup>1</sup> Particularly, some Latin American countries have suffered from high (relative to developed countries) and persistent rates of inflation, which in some cases have lasted up to the present day. Those countries have engaged in repeated inflation stabilization. For example, in the late 1970's, the Southern-Cone countries, Argentina, Chile and Uruguay, fought inflation by implementing exchange-rate based stabilization programs. Unfortunately, most of these stabilization attempts have been unsuccessful in eliminating inflation. In the last ten years, however, countries such as Argentina, Israel, and Mexico have succeeded in reducing inflation close to international levels for substantial periods.

These episodes of stabilization policy in chronic inflation countries have gained much attention from macroeconomists. Especially, the stabilization programs of

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<sup>1</sup>The term was first coined by Pazos (1972). According to his analysis, in chronic inflationary environment, inflation is relatively high compared to industrial countries and is persistent. Moreover, inflation does not have an inherent propensity to accelerate.

the Southern-Cone countries gave new challenges to macroeconomic theory. The common stylized fact we observed in the stabilizations, both exchange rate-based and money-based programs, is inflation persistence. Contrary to the expectation of both policy makers and economists, inflation converged slowly to the target rate, and exhibited considerable inertia. The most intriguing and puzzling phenomena is the business cycle associated with inflation inertia. In exchange rate-based programs, real economic activities, such as consumption and output, expanded in the early stage of program. Later in the program, even before the program was abandoned, consumption and output shrank and the trade balance went into deficit, so recession set in.<sup>2</sup> This boom-recession cycle of exchange-based stabilization programs was a puzzle to conventional macroeconomists who believed the stabilization should have resulted in an immediate contraction.

A lot of theoretical research has sought to explain these intriguing phenomena observed in the stabilization programs, especially in exchange-based programs. Early work by Rodriguez (1982) and Dornbusch (1982) offers a simple description of the Southern-Cone exchange rate-based programs of the late 1970s. They emphasized the presence of sticky inflation. Rodriguez assumes adaptive expectations to point out the backward-looking price and wage behavior. Dornbusch assumes rational expectations and stickiness of inflation. According to this hypothesis, under perfect capital mobility, if aggregate demand depends negatively on the real

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<sup>2</sup>In Argentina, for instance, the stabilization program was launched in 1978. By the 1980s, the annual inflation rate was 88 per cent, but the devaluation rate was 23 per cent. Moreover, the growth rate of consumption was 1.9 per cent in 1978, but in the first year of program, 1979, it increased to 12.3 per cent.

interest rate and positively on the real exchange rate (i.e., the relative price of tradable goods in terms of non-tradable goods), reduction of the devaluation rate decreases the nominal interest rate, which leads to lower real interest rates because of sticky inflation. The reduction of the real interest rate induces an initial expansion of output. After that, the domestic currency begins to appreciate since domestic inflation shows inertia and remains above the devaluation rate. Eventually, the real appreciation shrinks domestic consumption and so output falls and recession sets in.

Those sticky inflation models give a lucid explanation of the phenomena observed in the stabilization programs, especially inflation persistence and boom-recession cycles. The inflation inertia results from sticky inflation in both models. However, these early models have critical defect. Both models specify reduced-form behavioral equations rather than derive them. We find this very ad-hoc. In the case of Rodriguez(1982), the model relies on non-rational behavior and inflation stickiness is assumed, not derived as in Dornbusch(1982).

An alternative explanation to the major effects of stabilization is the “temporariness hypothesis” by Calvo and Végh (1993, 1994a). This hypothesis considers the case in which prices or wages are sticky due to staggered contracts, but inflation is fully flexible because of forward-looking contracts. These models are based on the optimizing behavior of individuals. Under these circumstances, this hypothesis argues that slow convergence of inflation and the initial boom in real economic activities arise from lack of credibility, not from sticky inflation. Rapid convergence

of inflation, therefore, can be achieved under the fully credible stabilization policy even if the price level exhibits stickiness.

This hypothesis seems to be successful in capturing the stylized facts associated with the recent stabilization programs, especially exchange rate based programs. However, these models are not successful in pointing out the exact cause of inflation inertia. In these models, since prices are set in a purely forward-looking manner, the inflation rate is independent of past inflation and depends only on future(expected) inflation. Thus, inflation is quite flexible while the price level is sticky. Therefore, credible disinflation policies can reduce inflation dramatically without any output cost and no inflation inertia is found, which is not consistent with reality.<sup>3</sup> Therefore, purely forward-looking staggered contract models are not useful in explaining the inflation dynamics when inflation shows stickiness.

To overcome the flaw, Calvo and Végh (1994b) incorporate backward-looking indexation into an intertemporal optimizing model and show a fully credible reduction in the devaluation rate can result in low inflation convergence. However, Calvo and Végh start from the Arrow-Debreu style model with sticky prices and assume that output is demand-determined. Thus, it lacks a solid rationale for the demand-determined output. And also to obtain the boom-recession cycle of the stabilization, their model depends on the relative size of the intertemporal and intratemporal elasticities of substitution.

All the explanations examined so far are based on demand-side considerations.

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<sup>3</sup>Agénor and Montiel (1999) pointed out that countries with chronic inflation derived various indexation mechanisms which are quite backward looking.

This is due to the fact that most literature was inspired by the Southern-Cone tabilitas of the late 1970s. In the recent programs, such as Mexico's 1987 and Argentina's 1991 convertibility plan, it has been argued that inflation stabilization may have played an important role in unleashing supply-side responses in labor and investment. This literature suggests that credible stabilization induces sustained expansion of real economic activities. In Lahiri (2001) and Roldos (1997), the nominal interest rate introduces a distortion between consumption and leisure. When inflation falls, labor supply increases. This, in turn, leads to a rise in the desired capital stock and, hence, in investment. These supply-side considerations can reproduce the main stylized facts observed in the successful stabilizations (Argentina (1991-1994) and Mexico (1988-1992) in which the initial boom was not followed by a recession.

In this paper, we consider the supply-side effects of disinflation with inflation } inertia in exchange rate-based programs within a New-Keynesian framework. To } consider the supply-side response, we introduce imperfect competition in the non-tradable sector with endogenous labor supply. The reason we adopt imperfect competition in the model is that imperfect competition is the main characteristic of the industrial structure of developing countries.<sup>4</sup> Our model is based on Obstfeld and Rogoff's "Exchange Rate Dynamic Redux" in 1995. We modified Obstfeld and Rogoff's model in two ways. First, we assume a small-open economy with a cash-in-advance constraint in which the change of monetary policy including

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<sup>4</sup>See Agénor and Montiel (1999)

the change of money growth rate has a long-run effect on the economy as in the two-country model in Obstfeld and Rogoff. Second, we adopt a Calvo-type contract model to obtain the inflation dynamics in a way that nominal wage contracts are staggered. Our model is a modified version of Calvo's (1983) forward-looking model including backward-looking components by Ghezzi (2001). We combine the staggered wage contract with price setting of imperfectly competitive firms to obtain the inflation persistence. Incorporating the backward-looking wage contracts into the model is quite consistent with the wage indexation of most chronic inflation circumstances.<sup>5</sup>

We apply this model to investigate the effects of inflation stabilization in a small open economy. We have two possible experiments, one is exchange-based stabilization which is perfectly credible and the other is temporary stabilization. From the first experiment we are able to replicate qualitatively the stylized facts of the exchange-based stabilization programs in which inflation persistence coexists with the sustained expansion of domestic sector. Sticky inflation combined with supply side effects produces inflation persistence and a sustained expansion of the non-tradable sector. This result is quite consistent with the stylized fact of successful stabilization programs, Argentina (1991-1994) and Mexico (1988-1992), in which the initial boom was not followed by a recession. In this regard, the model we present here is quite successful. It should be noticed that the resulting inflation persistence and associated business cycle have very important policy implications.

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<sup>5</sup>For instance, Edwards (1991) pointed out that the primary cause of inflation inertia in the Chilean stabilization is backward-looking wage indexation.

First, inflation persistence cannot be avoided even if policy is fully credible as long as there is a backward-looking indexation in price or wage setting. To achieve rapid stabilization, the indexation mechanism should be considered. The second implication is that the economy can enjoy a sustained expansion during the stabilization. Therefore slow adjustment of inflation does not weaken the credibility of policy. The results are not consistent with the arguments of the “temporariness hypothesis” by Calvo and Végh (1993, 1994a) in which inflation persistence and associated business cycles are merely due to the lack of credibility. For the temporary disinflation scenario, we see that the inflation rate accelerates after the initial decrease, and the “boom-recession again cycle” is observed.

The plan of this paper is as follows. We consider the stylized facts of exchange rate-based stabilization in Section 1.2. We present the basic model and the equilibrium in Section 1.3. The impact effects and transitional dynamics of a fully credible and temporary disinflation policy are discussed in Section 1.4. In Section 1.5, we draw the main conclusions. Technical derivations are relegated to Appendices.

## **1.2 Evidence on the real effects of exchange rate-based stabilization in chronic inflation countries**

During the past 40 years there have been 13 major exchange rate-based stabilization in Argentina, Brazil, Chile, Israel, Mexico, and Uruguay. More often than not, stabilization attempts have failed and inflation come back. During the 1980s, however, some countries - most notably, Chile, Israel, Mexico, and, more recently,



Table 1.1: Major exchange-rate based inflation stabilization plans

Program	Begin/end date	Succeed?
Brazil 1964	March 1964 - August 1968	Yes
Argentina 1967	March 1967 - May 1970	No
Uruguay 1968	June 1968 - December 1971	No
Chilean tablita	February 1978 - June 1982	Yes
Uruguayan tablita	October 1978 - November 1982	No
Argentine tablita	December 1978 - February 1981	No
Israel 1985	July 1985 - December 1991	Yes
Austral (Argentina)	June 1985 - September 1986	No
Crudo (Brazil)	February 1986 - November 1986	No
Mexico 1987	December 1987 - November 1986	Yes
Uruguay 1990	December 1990 - present	Yes
Convertibility (Argentina)	April 1991 - present	Yes

Major source: Reinhart and Végh (1995) and Calvo and Végh (1999)

Argentina - have succeeded in drastically reducing their rates of inflation. Table 1.1 depicts twelve major exchange-rate based stabilization in chronic inflation countries in the last 40 years.

Based on these episodes, the literature has identified the following main stylized facts associated with exchange rate-based stabilizations, which were not successful:

- (1) Slow convergence of the inflation rate to the rate of devaluation.
- (2) Initial increase in real activities, particularly, real GDP and private consumption, followed by a later contraction.
- (3) Real appreciation of the domestic currency.
- (4) Deterioration of the trade balance and current account balance.

To take a closer look at the main stylized facts, we need to investigate the time-series data for inflation stabilization plans. Calvo and Végh (1999) constructed the panel of annual observations for four countries (Argentina, Chile, Israel, and

Uruguay) for 16 years from 1978 to 1993. Figure 1.1 and 1.2 show the dynamics of the rate of devaluation and inflation, real exchange rate, real GDP, and real private consumption observed during the seven exchange rate-based stabilization plans listed in table 1.1, which includes tablita implemented in Argentina, Chile, and Uruguay and the Israeli 1985 plan, the Argentine Austral plan, the Uruguayan 1990 plan, and the Argentine 1991 plan.<sup>6</sup>

Panel A in figure 1.1 illustrates the behavior of the rate of devaluation and inflation. Even if inflation is highly responsive to the reduction in the rate of devaluation, it remains above the rate of devaluation and then lags it as the rate of devaluation increases. Panel B shows that the real exchange rate appreciates at the initial stage of the stabilizations.

Calvo and Végh (1999) constructed the panel of annual observations for four countries (Argentina, Chile, Israel, and Uruguay) for 16 years from 1978 to 1993. Figure 1.1 and 1.2 show the dynamics of the rate of devaluation and inflation, real exchange rate, real GDP, and real private consumption observed during the seven exchange rate-based stabilization plans listed in table 1.1, which includes tablita implemented in Argentina, Chile, and Uruguay and the Israeli 1985 plan, the Argentine Austral plan, the Uruguayan 1990 plan, and the Argentine 1991 plan.

Figure 1.2 presents the evidence of the “boom-recession cycle” in the growth of per capita GDP, consumption. Panel A shows that real GDP growth increases in the initial periods of stabilization and decreases sharply thereafter. The same

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<sup>6</sup>In the figure 1.1 and 1.2, time profiles are based on the stabilization time. Stabilization time is denoted by  $T + j$ , where  $T$  is the year in which the stabilization program was implemented and  $j$  is the number of years preceding or following the year of stabilization

Table 1.2: Major stabilization plans with crisis

Program	Main features
Argentina 1967	An 82% decline in reserves
Uruguay 1968	An 81% decline in reserves
Chilean tablita	A 65% decline in reserves
Uruguayan tablita	A 90% decline in reserves by March 1983
Argentine tablita	A 71% decline in reserves by April 1982
Austral (Argentina)	A 75% decline in reserves by September 1987
Crudo (Brazil)	A 58% decline in reserves by March 1987
Mexico 1987	An 85% decline in reserves between Feb 1994 and Jan 1995

Major source: Reinhart and Végh (1995) and Calvo and Végh (1999)

pattern is observed for private consumption. This stabilization time profiles point to the presence of a boom-recession cycle associated with exchange rate-based stabilization.

A notable aspect of exchange rate-based stabilization programs is that a vast majority have ended in balance-of-payments crises. Table 1.2 lists all the major programs, which were ended in crises with large losses of international reserves.

All the major programs, which were unsuccessful to eliminate the chronic inflation have ended in full-blown crises with large losses of international reserves except the Mexico's plan. This evidence suggests that there is a link between the dynamics of the exchange rate-based stabilization and the eventual collapse of the programs.

## 1.3 Basic model

In this section we introduce the aggregate demand side through the household's maximization problem and then the aggregate supply side in which the non-tradable good sector is the locus of the imperfect competition. Next, we solve for a long-run symmetric steady state where all prices are fully flexible. We, then, introduce sticky prices through staggered wage contracts. In the last part of this section, we derive the short-run dynamic system for the next section.

### 1.3.1 The aggregate demand side

Our model is based on Obstfeld and Rogoff's (1995) perfect-foresight general equilibrium Mundell-Fleming model. However, while they specify the models in discrete time, we assume that time is continuous. Instead of a two-country setting, we consider the small-country model in which non-tradable sector is the locus of imperfect competition, and also a cash-in-advance economy is assumed.

Consider a two sector small open economy which is perfectly integrated with the rest of the world in goods and capital markets. The economy is inhabited by a continuum of identical households which reside on the interval  $[0,1]$ . The representative household derives utility from the consumption of tradable goods, non-tradable goods and leisure. This economy contains a continuum of differentiated non-tradable goods which are indexed by  $z$  and distributed uniformly on  $[0,1]$ .

The lifetime utility of the representative household is

$$\int_0^\infty U(C_t^T, C_t^N, 1 - l_t) \exp(-\beta t) dt, \quad (1.1)$$

where  $U(\cdot)$ , the instantaneous utility function, is increasing, twice-continuously differentiable, and strictly concave;  $C_t^T$  is consumption of the tradable good and  $C_t^N$  is a non-tradable consumption index defined by

$$C_t^N = \left[ \int_0^1 c_t^N(z)^{\frac{\theta-1}{\theta}} dz \right]^{\frac{\theta}{\theta-1}}, \quad (1.2)$$

where  $c^N(z)$  is the household's consumption of good  $z$ , and  $\theta > 1$  is the elasticity of substitution between non-tradable goods. The last term in the instantaneous utility function in Eq.(1.2.1) captures the utility from leisure or disutility from labor supply (  $l_t$  represents total hours worked by the household), and  $\beta$  is the positive and constant subjective discount rate.

We employ the convention of letting  $E$  represent the nominal exchange rate in units of domestic currency per unit of foreign currency, while  $P^T$  denotes the foreign currency price of the tradable good.<sup>7</sup> Here,  $P^N$  denote the aggregate domestic currency price index of the non-tradable goods, defined as

$$P_t^N = \left[ \int_0^1 p_t^N(z)^{1-\theta} dz \right]^{\frac{1}{1-\theta}}. \quad (1.3)$$

The real exchange rate (the relative price of tradable goods in terms of non-tradable goods) can be defined as  $e = \frac{EP^T}{P^N}$ . For simplicity, we assume that  $P^T = 1$  and is constant. By this assumption, we can suppress the unnecessary foreign inflation

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<sup>7</sup>We assume that the law of one price holds for the tradable good. Therefore, the domestic currency price of the tradable good is  $E_t P_t^T$ .

rate from the model.

To complete the household's maximization problem, let's look at the individual household's budget constraint. We assume that the representative household can hold two types of assets as their financial wealth: domestic non-interest bearing currency and an internationally tradable bond with a constant real interest rate (in terms of tradable goods). Since the domestic currency and the international bond are the only two assets held by individual consumers, we have

$$a_t = m_t + b_t, \quad (1.4)$$

where  $a, m, b$  stand for real financial wealth, real money balance and the stock of real bonds in terms of domestic price of tradable goods, respectively.<sup>8</sup>

We assume that the household has a constant endowment flow of tradable goods,  $y_t^T$ , while it derives human wealth from supplying labor to firm  $z$  for the nominal wage  $W_t$ . The household also owns firm  $z$ . Therefore, it can earn the profit of the firm,  $\Pi_t(z)$ . Then the household's dynamic budget constraint is governed by the following differential equation:

$$\dot{a}_t = ra_t - i_t m_t + \frac{\Pi_t(z)}{E_t P^T} + \frac{W_t l_t}{E_t P^T} + y_t^T + \tau_t - \frac{C_t^N}{e_t} - C_t^T, \quad (1.5)$$

where  $\tau$  is real transfers from the government in terms of tradable goods;  $i$  is the instantaneous nominal interest rate in terms of domestic currency;  $r$  is the constant real interest rate in terms of tradable goods.<sup>9</sup>;  $W_t$  the is money wage rate. Notice that in Eq.(1.2.5), household's expenditure includes the opportunity

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<sup>8</sup>If we define nominal financial wealth, money balances and the stock of bonds as  $A, M$  and  $B$ , respectively, then real variables can be defined as follows;  $a = \frac{A}{E P^T}$ ,  $m = \frac{M}{E P^T}$ , and  $b = \frac{B}{E P^T}$ .

<sup>9</sup>Since we assume that  $P^T = 1$  and is constant,  $r$  is also the world nominal interest rate.

cost of holding real money balances,  $im$ .

In order to carry out consumption expenditures, the consumer is required to hold sufficient domestic money. Following Feenstra (1985), the cash-in-advance constraint which the consumer faces is thus

$$\alpha\left(\frac{C_t^N}{e_t} + C_t^T\right) \leq m_t, \quad \alpha > 0, \quad (1.6)$$

where  $\alpha$  represents the length of time that money has to be held to finance consumption expenditure.<sup>10</sup> Eq.(1.2.6) implies that minimum required money balances are proportional to the value of consumption expenditures. If the nominal interest rate,  $i$ , is positive, the consumer will hold the minimum required money balances. Then the cash-in-advance constraint (1.2.6) will be binding.

Using Equations (1.2.4), (1.2.5) and (1.2.6), and imposing the transversality conditions, we can derive the household's intertemporal budget constraint which is given by

$$a_0 + \int_0^\infty \left( \frac{W_t l_t}{E_t P^T} + y_t^T + \frac{\Pi_t(z)}{E_t P^T} + \tau_t \right) \exp(-rt) dt = \int_0^\infty \left( \frac{C_t^N}{e_t} + C_t^T \right) (1 + \alpha i_t) \exp(-rt) dt, \quad (1.7)$$

where  $a_0$  stands for the initial level of financial wealth. This equation says that the household's lifetime expenditure equals the present discounted value of its lifetime income. At each point in time  $t$ , the representative consumer's expenditure consists of the cost of consumption,  $\frac{C_t^N}{e_t} + C_t^T$ , plus the opportunity cost of holding real balances,  $i_t m_t$ .

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<sup>10</sup>The cash-in-advance constraint can be written as  $m_t \geq F(\alpha) \equiv \int_t^{t+\alpha} \left( \frac{C_s^N}{e_s} + C_s^T \right) ds$ . A Taylor-series expansion gives  $F(\alpha) = \alpha \left( \frac{C_t^N}{e_t} + C_t^T \right) + \frac{1}{2} \alpha^2 \frac{d}{dt} \left( \frac{C_t^N}{e_t} + C_t^T \right) + \dots$ , so (1.2.6) can be interpreted as a first-order approximation

The decisions the consumer has to make at each point in time are how many tradable and nontradable goods to consume and how much to work. Therefore, the consumer's optimization problem is to choose the paths of  $C_t^T$ ,  $C_t^N$  and  $l_t$  to maximize lifetime utility,(1.2.1), subject to initial wealth,  $a_0$ , and the intertemporal budget constraint,(1.2.7).

The first-order conditions for this optimization problem are<sup>11</sup>

$$U_{C_t^T}(C_t^T, C_t^N, l_t) = \lambda(1 + \alpha i_t) \quad (1.8)$$

$$U_{C_t^N}(C_t^T, C_t^N, l_t) = \lambda\left(\frac{1 + \alpha i_t}{e_t}\right) \quad (1.9)$$

$$\frac{U_{C_t^T}(C_t^T, C_t^N, l_t)}{U_{C_t^N}(C_t^T, C_t^N, l_t)} = e_t \quad (1.10)$$

$$U_{l_t}(C_t^T, C_t^N, l_t) = \lambda\left(\frac{W_t}{E_t P^T}\right), \quad (1.11)$$

where  $\lambda$  is the (time-invariant) Lagrange multiplier associated with the household's intertemporal budget constraint (1.2.7). As usual, it can be interpreted as the marginal utility of wealth. Equations (1.2.8) and (1.2.9) indicate that at an optimum, the consumer equates the marginal utility of consumption of tradable and non-tradable goods to the marginal utility of wealth times the effective prices of goods. The effective prices of goods consist of the market prices, unity in the case of tradables and  $\frac{1}{e_t}$  in the case of non-tradables, plus the opportunity cost of holding the  $\alpha$  units of money that are needed to purchase both goods,  $\alpha i$  and  $\alpha \frac{i}{e_t}$ , respectively. Equation (1.2.10) is the familiar condition that at an optimum the

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<sup>11</sup>To eliminate inessential dynamics and ensure the existence of a steady state, we assume that  $\beta = r$ .



marginal rate of substitution between tradable and non-tradable goods is equal to the relative price of tradable goods in terms of home goods which is the real exchange rate. Equation (1.2.11) is the Euler equation for optimal labor supply. It ensures that the marginal disutility of labor (due to forgone leisure) equals the marginal utility from consuming the extra income from another unit of labor supplied. The term  $(1 + \alpha i_t)$  generated by cash-in-advance constraint must be treated with special attention in this model. This is the usual monetary wedge due to the fact that any form of wealth has to be liquidated before goods can be purchased. Even if this monetary wedge increases the effective price of consumption, it does not affect the price of leisure.

In order to obtain a closed form solution, we assume the following instantaneous utility function

$$U(C_t^T, C_t^N, l_t) = \gamma \log(C_t^T) + (1 - \gamma) \log(C_t^N) + \rho \log(1 - l_t), \quad (1.12)$$

where  $\rho > 0$ . Then, first order conditions are

$$\frac{\gamma}{C_t^T} = \lambda(1 + \alpha i_t) \quad (1.13)$$

$$\frac{1 - \gamma}{C_t^N} = \lambda \left( \frac{1 + \alpha i_t}{e_t} \right) \quad (1.14)$$

$$\frac{C_t^T e_t}{C_t^N} = \frac{\gamma}{1 - \gamma} \quad (1.15)$$

$$\frac{\rho}{1 - l_t} = \lambda \left( \frac{W_t}{E_t P_t^N} \right) \quad (1.16)$$

. Using (1.2.14) and (1.2.16), we can get

$$\frac{\rho}{1 - l_t} = \frac{1 - \gamma}{C_t^N} \frac{W_t}{P_t^N} (1 + \alpha i_t)^{-1}. \quad (1.17)$$

From Eq.(1.2.17) we can see that individual's optimal labor supply is dependent on the nominal interest rate. Since the nominal interest rate introduces a distortion between consumption and leisure, a change in the interest rate makes the consumer substitute leisure for consumption due to the change in the effective price of consumption. This, in turn, leads to a change in the long-run level of non-tradable output.

### 1.3.2 The government

The other participant in this economy is the government. Since Ricardian equivalence holds in this model, we can assume that the government runs a balanced budget in every period. We also assume no government spending. With these assumptions, we can simplify government behavior. It is assumed that the government holds internationally tradable bonds (international reserves) which pay the world real interest rate in terms of tradable goods and issues non-interest bearing debt (domestic currency).

The evolution of the government's stock of net foreign bonds is governed by the following differential equation

$$\dot{h}_t = rh_t - \tau_t + \frac{\dot{M}_t}{P_t^T E_t}, \quad (1.18)$$

where  $h_t$  is the government's stock of internationally tradable bonds (international reserves). Notice that  $\frac{\dot{M}_t}{P_t^T E_t}$  is the government's seignorage profits. Integrating equation (1.2.18) imposing the transversality condition,<sup>12</sup> we can derive the gov-

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<sup>12</sup>The appropriate transversality condition is  $\lim_{t \rightarrow \infty} h_t \exp(-rt) = 0$ .

ernment's intertemporal budget constraint

$$\int_0^\infty \tau_t \exp(-rt) dt = h_0 + \int_0^\infty (\dot{m}_t + \epsilon_t m_t) \exp(-rt) dt, \quad (1.19)$$

where  $\epsilon_t = \frac{\dot{E}_t}{E_t}$  is the nominal rate of depreciation and  $h_0$  is the initial level of the government's stock of foreign bonds.<sup>13</sup> The government's intertemporal budget constraint indicates that the present value of transfer expenditure has to be equal to the initial stock of government-held international bonds (i.e., international reserves) and revenues from seignorage. By equation (1.2.19), we implicitly assume that the government returns to the consumer all of its revenues.

### 1.3.3 The aggregate supply side

We now turn to the supply side of the economy. For simplicity, we assume that the supply of tradable goods is exogenous and fixed at the constant level  $y^T$  (i.e.,  $y_t^T = y^T$  for all  $t$ ) and its domestic price will be determined by the law of one price.

In this model, it is assumed that the non-tradable goods sector is imperfectly competitive and there is a continuum of imperfectly competitive firms indexed by  $z \in [0, 1]$ . Each producer produces a differentiated good and acts as a monopolistic competitor, choosing the nominal price and the level of production of the good.

Given the CES non-tradable consumption index, Eq.(1.2.2), an individual's

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<sup>13</sup>The government's seignorage revenue  $\frac{\dot{M}_t}{P_t^T E_t}$  is equal to  $\dot{m}_t + \epsilon_t m_t$  in real term (in terms of tradable goods).

demand for non-tradable good  $z$  in period  $t$  is

$$C_t^N(z) = \left[ \frac{p_t^N(z)}{P_t^N} \right]^{-\theta} C_t^N, \quad (1.20)$$

where  $\theta$  is the price elasticity of demand.

According to Eq.(1.2.20), a monopolistically competitive firm that produces a non-tradable good faces the downward-sloping curve

$$y_t^{Nd}(z) = \left[ \frac{p_t^N(z)}{P_t^N} \right]^{-\theta} C_t^{NA}, \quad (1.21)$$

where  $C_t^{NA} dz = \int_0^1 C_t^N dz = C_t^N$  is aggregate per capita non-tradable goods consumption.

Labor is the only input into production. The technology available to firms is identical and is linear in labor:

$$y_t^N(z) = l_t. \quad (1.22)$$

Firm- $z$ 's profit is

$$\Pi_t(z) = p_t(z) y_t^d(z) - W_t l_t. \quad (1.23)$$

At any time  $t$ , a monopolistically competitive firm maximizes profit, Eq.(1.2.23), subject to the demand function, Eq.(1.2.21), and the production function, Eq.(1.2.22).

Solving the firm's problem:

$$\frac{\theta - 1}{\theta} y_t^{-\frac{1}{\theta}}(z) P_t^N C_t^{\frac{1}{\theta}} = W_t. \quad (1.24)$$

Since the elasticity of demand,  $1/\theta$ , of all non-tradable goods is identical and every producer has the same technology, they produce the same level of output

and set the same price. Therefore, for any two producers,  $0 < z < z' < 1$

$$\begin{aligned} y_t^N(z) &= y_t^N(z') = y_t^N \\ p_t^N(z) &= p_t^N(z') = p_t^N. \end{aligned} \quad (1.25)$$

It follows that non-tradable good's aggregate supply and price index simplify to

$$\begin{aligned} y_t^{AN} &= \int_0^1 y_t^N(z) dz = y_t^N \\ P_t^N &= \left[ \int_0^1 p_t^{N1-\theta}(z) dz \right]^{\frac{1}{1-\theta}} = p_t^N. \end{aligned} \quad (1.26)$$

The equilibrium of non-tradable goods can be obtained by Eqs.(1.2.21) and (1.2.25)

$$y_t^N = C_t^N. \quad (1.27)$$

Therefore, from Eq.(1.2.24)

$$p_t^N(z) = \frac{\theta}{\theta - 1} W_t, \forall z. \quad (1.28)$$

Equation(1.2.28) shows that producer with market power sets prices above marginal cost, which is the money wage rate. Thus if it cannot adjust its price it is willing to produce to satisfy demand in the face of fluctuations in demand. Therefore, in the short-run, if the price is sticky, output will be demand-determined.

### 1.3.4 A symmetric steady state

In the model, perfect capital mobility is assumed. It implies that

$$i_t = r + \epsilon_t. \quad (1.29)$$

In the steady state, all prices are fully flexible. Thus the symmetric equilibrium implies

$$C_t^N = y_t^N = C_t^{AN}, \forall z. \quad (1.30)$$

In the steady state where all prices are fully flexible, the supply condition determines the level of non-tradable output. It means that the long-run level of output is totally dependent on labor supply. Using Eqs. (1.2.17), (1.2.28) and (1.2.30), we get the symmetric steady state level of output of non-tradables

$$y_{ss}^N = [1 + (\frac{\rho}{1-\gamma})(\frac{\theta}{\theta-1})(1 + \alpha i_{ss})]^{-1}, \quad (1.31)$$

where  $i_{ss} = r + \epsilon_{ss}$  is the steady state level of nominal interest rate. In the steady state, all prices are fully flexible and all exogenous variables are constant. Therefore, the long-run level of non-tradable output is determined by the supply side. According to Eq.(1.2.31), however, we can see that non-tradable output depends on the nominal interest rate, which is also a function of the monetary policy variable, the devaluation rate. The reason is as follows. Since the nominal interest rate introduces a distortion between consumption and leisure, if there is a permanent decrease in the nominal interest rate, the household will substitute leisure for consumption due to a lower effective price of consumption. This, in turn, leads to a rise in the long-run level of output as in Lahiri(2001) and Roldos(1997).<sup>14</sup> This result is consistent with the disinflation scenario of Cooley and Hansen (1989, 1995) in a closed economy model in which variations in the rate of inflation can

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<sup>14</sup>Their models, however, are based on Arrow-Debreu economy. Thus, the welfare implication of disinflation policy implied our model will be different from their models.

affect the steady state labor supply and consumption (or output).

In this model, each producer has monopoly power, so the long-run level of non-tradable output is lower than the socially optimal level. To see this, let's look at the social planner's problem. The social planner tries to maximize the utility of non-tradable consumption considering the disutility from labor supply.<sup>15</sup>

$$\max[(1 - \gamma) \log y^N - \rho \log(1 - y^N)]. \quad (1.32)$$

The solution is

$$y_s^N = \left( \frac{1 - \gamma}{1 - \gamma + \rho} \right) > y_{ss}^N. \quad (1.33)$$

From Eq.(1.2.33), we can see that the equilibrium output of non-tradables is inefficiently low under imperfect competition. This fact has important implications for economic fluctuation as well as monetary policy to stabilize it, as long as prices are not perfectly flexible because of menu costs or staggered price or wage contracts. Since the market equilibrium is lower than the socially optimal level, recessions and booms have asymmetric effects on welfare (Mankiw, 1985). Since equilibrium output is less than socially optimal level, a boom brings output closer to the social optimum, whereas a recession in the non-tradable sector pushes it farther away. In this model, however, the long-run equilibrium level of output is subject to a monetary policy variable, the devaluation rate. Therefore, a change in monetary policy can lead the economy closer to the optimum.

In order to investigate the dynamic behavior of the current account triggered by the policy change, consider the economy's dynamic resource constraint. To

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<sup>15</sup>Remember that by the production condition  $y^N = l$ .

obtain an economy's dynamic resource constraint, combine the consumer's dynamic budget constraint with the profit of the firm and the market clearing condition for the non-tradable goods sector, then

$$\dot{k}_t = rk_t + y^T - c_t^T, \quad (1.34)$$

where  $k_t = b_t + h_t$  which is the total amount of foreign bonds held by the economy. Equation (1.2.34) represents the economy's current account balance. It indicates that the current account balance is the difference between tradable goods income and consumption of tradable goods.

Combining Equations (1.2.7), (1.2.22), (1.2.14) and (1.2.24) yields the economy's intertemporal constraint

$$k_0 + \int_0^\infty y^T \exp(-rt) dt = \int_0^\infty c_t^T \exp(-rt) dt, \quad (1.35)$$

where  $k_0$  denotes the economy's initial stock of foreign bonds. Equation (1.2.35) states that the initial stock of foreign bonds plus the present value of all future tradable output must equal the present value of tradable consumption. At the steady state,  $C_t^T = C_{ss}^T$ . Therefore,  $C_{ss}^T = y^T + rk_0$  and the current account is balanced at the steady state.

### 1.3.5 Staggered wage contract and short-run dynamics

So far, we have assumed flexible prices and solved for the long-run equilibrium. We now introduce the assumption that the price level of the non-tradable good is sticky because of staggered wage contracts and are ready to consider the short-run dynamics of the economy. In the short-run, the supply of non-tradable goods



will be demand determined and prices will be given by a modified version of the staggered-price model of Calvo (1983) and Ghezzi (2001), including backward-looking indexation consistent with our discussion in Section 1.1. We assume nominal wages are sticky as a result of staggered wage contracts.

Following Calvo (1983), we assume that each individual can change wages only when a wage signal is received. If individuals do not change the wage or is setting a new wage at time  $t$ , the probability (density) that the wage will last for  $s$  more periods is given by the geometric distribution

$$\delta \exp(-\delta s) \tag{1.36}$$

and is, therefore, independent of  $t$  and of the amount of time the wage has lasted at  $t$ . And it is also stochastically independent across the individual wage setters. The expected length of a wage duration is  $1/\delta$ . Therefore, nominal wages specified by the contract will be fixed for the duration of the contract and the contract itself is staggered.

The aggregate (log of) newly posted wage level is

$$w_t = \delta \int_{-\infty}^t x_s \exp(-\delta(t-s)) ds, \tag{1.37}$$

where wage set at time  $s$ ,  $x_s$ , is weighted by the probability that it continues in effect at  $t$ ,  $\exp(-\delta(t-s))$ .<sup>16</sup>

For analytical simplicity, we assume that at any time  $t$ , there are two types of wage contractors: half of the individual contractors set their nominal wage in a

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<sup>16</sup>Since price is a fixed markup over wage, there is no distinction between wage and price in this model.

purely forward-looking manner and half of them set in a backward-looking manner. By this mechanism we can easily incorporate the backward-looking indexation, which is proposed by Taylor (1980), into the Calvo model.<sup>17</sup>

The forward-looking wage contractor will set wages according to the standard Calvo model

$$x_t^F = \delta \int_t^\infty [w_s + \phi(\log c_s^N - \log y_{ss}^N)] \exp(-\delta(s-t)) ds, \quad (1.38)$$

where  $\phi$  reflects sensitivity of contract wages to future excess demand conditions ( $\log c_s^N - \log y_{ss}^N$ ), where  $y_{ss}^N$  is the steady state level of non-tradable output. According to Eq.(1.2.38), we can see that when setting a contract, forward-looking agents will take into account the future average wage level and excess demand during the length of contract.

For the other half of the contractors, wages are assumed to be set according to a backward-looking rule. Specifically, we assume that the wage contract at  $t$  indexes money wages to the current aggregate wage level plus a weighted average of past inflation and current excess demand conditions.

$$x_t^B = w_t + \frac{1}{\delta} \eta_t + \varphi(\log c_t^N - \log y_{ss}^N), \quad (1.39)$$

where  $\varphi(> 0)$  reflects the sensitivity of the current money wage to the current level of excess demand, and

$$\eta_t = \delta \int_{-\infty}^t \pi_s \exp(-\delta(t-s)) ds \quad (1.40)$$

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<sup>17</sup>In original Taylor models, current wage is set according to the past wage level and expected future wage rate including past and future excess demand condition. Thus, Taylor models can generate wage level inertia.

$$\dot{\eta}_t = \delta(\pi_t - \eta_t). \quad (1.41)$$

Equation (1.2.40) indicates that backward-looking wage contracts at  $t$  index current nominal wages to the current aggregate wage level, a weighted average of past inflation  $\eta_t$ , adjusted by  $1/\delta$ , which is the expected length of the price duration, and the current level of excess demand.<sup>18</sup>

Therefore, the log of wage set at time  $t$  is

$$x_t = \frac{1}{2}x_t^F + \frac{1}{2}x_t^B. \quad (1.42)$$

At any point in time at which every variable is continuous, we can differentiate Eq.(1.2.37) with respect to time to get<sup>19</sup>

$$\pi_t = \dot{w}_t = \delta[x_t - w_t]. \quad (1.43)$$

Combining (1.2.38), (1.2.39), and (1.2.42)

$$x_t = \frac{1}{2}[w_t + \frac{1}{\delta}\eta_t + \varphi(\log c_t^N - \log y_{ss}^N)] + \frac{1}{2}[\delta \int_t^\infty (w_s + \phi(\log c_s^N - \log y_{ss}^N)) \exp(-\delta(s-t)) ds]. \quad (1.44)$$

Differentiating (1.2.44), we obtain the changes in the newly set wage rate

$$\dot{x}_t = 2\pi_t - \eta_t - \frac{\delta}{2}(\varphi + \phi)(\log c_t^N - \log y_{ss}^N) + \frac{1}{2}\varphi \log \dot{C}_t^N. \quad (1.45)$$

In order to obtain non-tradable inflation dynamics, differentiate (1.2.44) with respect to time and substitute it into (1.2.45)

$$\dot{\pi}_t = \delta(\pi_t - \eta_t) - \frac{\delta^2}{2}(\varphi + \phi)(\log c_t^N - \log y_{ss}^N) + \frac{1}{2}\delta\varphi \log \dot{C}_t^N. \quad (1.46)$$

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<sup>18</sup>In equation (1.2.41), the adjustment parameter is not necessarily equal to  $\delta$ . It could be  $\varsigma \neq \delta$  and, therefore  $\eta_t = \varsigma \int_{-\infty}^t \pi_s \exp(-\delta(t-s)) ds$ . For simplicity, we just assume  $\varsigma = \delta$ . See Ghezzi(2001)

<sup>19</sup>  $\pi_t = \log \dot{P}_t^N = \log \dot{W}_t$

This is the equation for the inflation dynamics. Eq.(1.2.46) indicates that change in the rate of inflation is dependent on two sources. First, it depends on the excess demand conditions: the change of inflation is negatively related to excess demand. The second term of (1.2.46) shows this relation. This is the same result as Calvo (1983) and Calvo and Végh (1993, 1994a). The intuition for this higher order inverse Phillips curve is that if there is excess demand at  $t$ , individuals who revise their wages at time  $t$  set higher wages. Therefore, the higher excess demand at time  $t$ , the higher the rate of inflation will be. However, since agents set their nominal wages in a forward-looking manner, wage setters at  $s > t$  do not take excess demand at time  $t$  into account. Hence, the higher the excess demand at  $t$ , the greater the reduction in the inflation rate for  $s > t$  will be.

In equation (1.2.46), we can see that there is another source for the change of the rate of inflation. It shows that current inflation depends on a weighted average of past inflation. Especially, when averaging past inflation, the inflation rate in the recent past receives more weight which is the same formulation as adaptive expectations. If current inflation is lower than  $\eta_t$ , an average of past inflation, the newly contracted wage has the lower premium over the current price level, which will be translated into lower inflation. This makes inflation sticky. The first term on the right-hand side of (1.2.46) represents this relation. The change of aggregate demand in the non-tradable good sector affects inflation positively. This is the original Phillips curve mechanism. In this regards, equation (1.2.46) can be interpreted as a traditional Expectations-Augmented Phillips curve.

In this model, there is another source for inflation dynamics. In contrast to a constant steady state level of non-tradable output as in Calvo and Calvo and Végh models, the long-run level of non-tradable output is subject to nominal variables, the devaluation rate, since the nominal interest rate introduces a wedge between consumption and labor supply. Therefore, non-tradable output shows different dynamics with respect to the policy change in this model. The wage contract implies that the nominal wage has its own persistence in addition to the inertia in the driving term, which is the change in the level of excess demand. By this specification, we can overcome the inability of the standard-new Keynesian contract model to generate significant inflation persistence.

In order to analyze the dynamic system of the economy, we will need to obtain the equilibrium path of consumption of tradables and non-tradables. From Eqs (1.2.14), (1.2.16) and (1.2.31), we can see that

$$C_t^T = \frac{\gamma}{\lambda}(1 + \alpha i_t)^{-1} \quad (1.47)$$

$$C_t^N = \frac{1 - \gamma}{\gamma} e_t C_t^T, \quad (1.48)$$

and

$$\lambda = \frac{\gamma \int_0^\infty (1 + \alpha i_t)^{-1} \exp(-rt) dt}{k_0 + \frac{1}{r} y^T} \quad (1.49)$$

is the shadow value of wealth, which is constant as long as there is no policy shocks to the economy. From Eqs.(1.2.47) and (1.2.48), we can obtain that

$$\log C_t^T = \zeta - \alpha i_t \quad (1.50)$$

$$\log C_t^N = \log \frac{1 - \gamma}{\gamma} + \log C_t^T + \log e_t, \quad (1.51)$$

where  $\zeta = \frac{\gamma}{\lambda}$  is constant.

Differentiating (1.2.51) with (1.2.50) yields

$$\log \dot{C}_t^N = \log \dot{e}_t - \alpha \dot{i}_t. \quad (1.52)$$

By definition,  $e_t = \frac{E_t P^T}{P_t^N}$ . Therefore,

$$\log \dot{e}_t = \epsilon_t - \pi_t. \quad (1.53)$$

Using the fact that  $\dot{i}_t = \dot{\pi}_t$ , we can obtain

$$\log \dot{C}_t^N = (\epsilon_t - \pi_t) - \alpha \dot{\pi}_t. \quad (1.54)$$

In the short-run where all prices are sticky, the equilibrium level of employment is demand-determined. Therefore, non-tradable output is also demand-determined. It implies

$$\log y_t^N = \log C_t^N. \quad (1.55)$$

Combining (1.2.54) and (1.2.46) with the equilibrium condition of non-tradable goods, we can obtain the differential equation governing the change of rate of inflation

$$\dot{\pi}_t = \frac{2\delta}{2 + \alpha\delta\varphi}(\pi_t - \eta_t) + \frac{\delta\varphi}{2 + \alpha\delta\varphi}(\epsilon_t - \pi_t) - \frac{\delta^2(\varphi + \phi)}{2 + \alpha\delta\varphi}(\log y_t^N - \log y_{ss}^N). \quad (1.56)$$

Then, the change of non-tradable good can be obtained as

$$\log \dot{y}_t^N = -\frac{2\alpha\delta}{2 + \alpha\delta\varphi}(\pi_t - \eta_t) + \frac{2}{2 + \alpha\delta\varphi}(\epsilon_t - \pi_t) + \frac{\alpha\delta^2(\varphi + \phi)}{2 + \alpha\delta\varphi}(\log y_t^N - \log y_{ss}^N). \quad (1.57)$$

Therefore, dynamics of the economy consist of three-equation system [Eqs. (1.2.41), (1.2.56), and (1.2.57)], in output of non-tradables,  $y_t^N$ , inflation of non-tradables,  $\pi_t$ , and  $\eta_t$ , a weighted average of past inflation.

## 1.4 Exchange rate-based stabilization

This section studies the effects of stabilization policies. We will examine the effects of an exchange rate-based stabilization. First, we consider the credible stabilization plan in which the reduction of the devaluation rate is believed to be permanent. Next, the effect of the temporary reduction in the devaluation rate is considered.

Suppose that, prior to any stabilization (for  $t < 0$ ), the devaluation rate is  $\epsilon^H$ , and is expected to remain at that level forever. For a given devaluation rate  $\epsilon^H$ , the economy is at a steady state characterized by

$$\begin{aligned}
i_{ss} &= r + \epsilon^H \\
\pi_{ss} &= \eta_{ss} = \epsilon^H \\
C_{ss}^N &= y_{ss}^N((i_{ss})) = [1 + (\frac{\rho}{1-\gamma})(\frac{\theta}{\theta-1})(1 + \alpha i_{ss})]^{-1} \\
C_{ss}^T &= y^T + rk_0 \\
e_{ss} &= (\frac{\gamma}{1-\gamma}) \frac{C_{ss}^N}{C_{ss}^T}.
\end{aligned} \tag{1.58}$$

At a steady state, consumption of tradable goods is equal to its permanent income level, while consumption of non-tradables equals its full-employment level. The full employment level of non-tradables is less than the socially efficient level due to the imperfection in the non-tradable sector. However, this full-employment level of output is not constant. Monetary policy which affects the nominal interest rate can change the long-run level of non-tradable output. Nominal variables (the interest rate and the rate of inflation) are growing at the devaluation rate,  $\epsilon^H$ .

The real exchange rate is equal to the ratio of the steady state consumption of non-tradables to tradables.

To examine the transitional adjustment of the economy, we have to consider the dynamic system. The system is characterized by the differential equations (1.2.41), (1.2.56), and (1.2.57) with three variables  $\log y$ ,  $\pi$  and  $\eta$ . Since the system is linear, we do not need any approximations.

The dynamic system for  $\log y$ ,  $\pi$  and  $\eta$  is

$$\begin{aligned} & \begin{bmatrix} \log \dot{y}_t^N \\ \dot{\pi}_t \\ \dot{\eta}_t \end{bmatrix} \\ &= \begin{bmatrix} \frac{\alpha\delta^2(\phi+\varphi)}{\delta^2(\phi+\varphi)} & -\frac{2(1+\alpha\delta)}{\delta(\varphi-2)} & \frac{2\alpha\delta}{\delta} \\ \frac{\mu}{\mu} & \frac{\mu}{\mu} & -\frac{2\delta}{\delta} \\ 0 & -\delta & \delta \end{bmatrix} \times \begin{bmatrix} \log y_t^N - \log y_{ss}^N \\ \pi_t - \epsilon_t \\ \eta_t - \epsilon_t \end{bmatrix}, \end{aligned} \quad (1.59)$$

where  $\mu = 2 + \alpha\delta\varphi > 0$

Now, we need to investigate the stability of the system. It exhibits two convergent (non-positive) roots and one explosive (non-negative) root (see Appendix A.1.1). At time  $t$ , current inflation,  $\pi_t$ , is given by an average of past inflation, current excess aggregate demand and future economic conditions, including future inflation. Therefore,  $\pi_t$  becomes a jump variable at time  $t$  even if it cannot adjust immediately to a new equilibrium value. The partial forward-looking indexation (half of households set the new wage in a forward-looking manner) makes inflation jump initially to the point between the new steady state and the initial steady state after a shock. And  $\log y_t^N$  is also a jump variable (notice that by the equilibrium condition,  $\log y_t^N = \log c_t^N$ ). However, as one can see in (1.2.35),  $\eta_t$  is a predetermined variable. Thus, there is one predetermined variable ( $\eta$ ) and two jump ( $\pi$  and  $\log y^N$ ) variables. However, since the jump of  $\log y^N$  is exogenous to



the solution of (1.3.2), the system exhibits saddle path stability.<sup>20</sup> Also the roots may be complex conjugates, and the system may show cyclical dynamics.<sup>21</sup>

Since the system has three variables, it will be difficult to analyze the equilibrium paths. However, it is possible to see the exact transitional dynamics of the system by resorting to the methods of dominant eigenvalue proposed by Calvo (1987). The system has two non-positive roots and one non-negative root. Let  $\nu_i, i = 1, 2$ , be the non-positive roots and  $\nu_3$  be the non-negative root, with  $\nu_1 > \nu_2$  ( $\nu_1$  is the dominant eigenvalue). Setting to zero the constant corresponding to the unstable root ( $\nu_3$ ),<sup>22</sup> the solution to dynamic system (1.3.2) can be expressed as:

$$\begin{aligned}\log y_t^N - \log y_{ss}^N &= A_1 \omega_{11} \exp(\nu_1 t) + A_2 \omega_{21} \exp(\nu_2 t) \\ \pi_t - \epsilon_t &= A_1 \omega_{12} \exp(\nu_1 t) + A_2 \omega_{22} \exp(\nu_2 t) \\ \eta_t - \epsilon_t &= A_1 \omega_{13} \exp(\nu_1 t) + A_2 \omega_{23} \exp(\nu_2 t),\end{aligned}\tag{1.60}$$

where  $A_i, i = 1, 2$ , and  $\omega_{ij}, j = 1, 2, 3$ , denote the constants and the element of the eigenvector associated with root  $\nu_i$ .

From the assumption  $\nu_1 > \nu_2$ , it follows that (see Appendix A.1.2):

$$\lim_{t \rightarrow \infty} \frac{\log y_t^N - \log y_{ss}^N}{\pi_t - \epsilon_t} = \frac{\omega_{11}}{\omega_{12}} > 0.\tag{1.61}$$

This implies that as  $t$  becomes large, if the initial value of  $\log y^N$  and  $\pi$  are not on the ray corresponding to solution,  $\nu_2$ , then the ratio of  $\frac{\log y_t^N - \log y_{ss}^N}{\pi_t - \epsilon_t}$  will converge to the slope of the dominant eigenvector ray ( $\frac{\omega_{11}}{\omega_{12}}$ ), which is positive. It implies

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<sup>20</sup>See, Ghezzi (2001).

<sup>21</sup>For more detail, see Blanchard and Khan (1980) and Turnovsky (1995).

<sup>22</sup>It is clear from the transversality condition that a necessary condition for convergence of the system is that  $\nu_3 = 0$ .

that, graphically, the system will converge asymptotically to the dominant eigenvector ray. Figure 1 shows the phase diagram for  $\log y_t^N$  and  $\pi_t$  with the dominant eigenvector ray.

### 1.4.1 Permanent reduction in the devaluation rate

Consider the impact of a once-and-for-all reduction in the rate of devaluation. The case corresponds to the credible disinflation that the public believes the devaluation rate to remain at the lower level for the future.

At the initial steady state,  $\epsilon = \epsilon^H$ . At  $t = 0$ , the authorities announce the following policy that is unanticipated:

$$\begin{aligned}\epsilon_t &= \epsilon^H, \text{ for } t \leq 0 \\ \epsilon_t &= \epsilon^L, \text{ for } t > 0,\end{aligned}$$

where  $\epsilon^H > \epsilon^L$

According to Equation (1.2.29), on impact, the nominal interest rate jumps to a new level,  $r + \epsilon^L$ , falls by the same amount as the rate of devaluation. Since the reduction in the rate of devaluation is fully credible, the public believes that nominal interest rate remains at the lower level,  $r + \epsilon^L$ , forever.

From Eq.(1.2.49), if the nominal interest rate is constant over time,

$$\lambda = \frac{\gamma}{rk_0 + y^T}(1 + \alpha i)^{-1}. \quad (1.62)$$

Hence, Eq.(1.2.47) with (1.3.5) implies

$$C^T = rk_0 + y^T. \quad (1.63)$$

The consumption of tradable goods is constant over time. By (1.3.6),  $C^T$  is equal to its permanent income level. The reason is as follows. Even if the nominal interest rate is reduced, it is constant over time at the lower level, which implies the constant effective price of tradable consumption at the lower level. Hence, the consumer does not have any incentives to engage in intertemporal consumption substitution. This implies that the current account does not change at all during the transition to the new steady state. As implied by (1.2.48), the consumption of non-tradables does not change on impact as well because the real exchange rate is predetermined. However, this result depends on the specific form of utility function and is not a general case.<sup>23</sup>

Inflation also falls on impact. The forward-looking contracts make the inflation rate jump on impact. The inflation rate, however, will jump to a point between  $\epsilon^H$  and  $\epsilon^L$ . This is due to the backward-looking indexation of wage contracts.<sup>24</sup> The initial impact effect of the reduction in the rate of devaluation on inflation and non-tradable output is shown in Fig 1-3. The initial impact implies a movement from the point of  $SS_0$  to  $A$ .

However, this shows the partial effect of the reduction in the nominal interest rate on consumption and output. The reduction in the nominal interest rate implies from (1.2.17) and (1.2.31) that non-tradable output does increase after the initial

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<sup>23</sup>If the intertemporal elasticity of substitution is greater than the intratemporal elasticity of substitution, tradable consumption will increase on impact and there will be the initial jump of the non-tradable consumption. Therefore, the change of consumption on impact is dependent on the relative magnitudes of the elasticities of intertemporal and intratemporal substitution. In our model, both elasticities are equal to 1. Hence, there is no jump of the tradable and non-tradable consumption; see Calvo and Végh (1994b)

<sup>24</sup>According to Ghezzi (2001), the size of jump of the inflation rate depends negatively on the degree of backward-looking indexation

impact. The reduction in the nominal interest rate that constitutes reduction in the monetary wedge generated by the cash-in-advance constraint. This increases consumer's labor supply, which induces output expansion in the non-tradable sector. Hence, by the market-clearing condition of non-tradable goods, consumption also increases on impact. Since the greater labor supply implies more labor income, the consumer will increase demand for non-tradables. This permanent increase in labor supply triggers a positive wealth effect. Therefore, the supply-side effects of disinflation come in through the change in the labor supply.<sup>25</sup>

The effect of the reduction in the rate of devaluation on inflation and non-tradable output is shown in Fig 1-1. The steady state moves from  $SS_0$  to  $SS_1$ , the steady state rate of inflation falls from  $\epsilon^H$  to  $\epsilon^L$ , and steady state non-tradable output is higher than it was before the reduction in the devaluation rate. At point  $A$ , inflation starts to fall. Non-tradable output (consumption) also begins to increase. The economy will move asymptotically to the dominant eigenvector ray, and proceeds towards the new steady state,  $SS_1$ .

During the transition, inflation converges to  $\epsilon^L$  slowly and shows substantial persistence. This is due to the backward-looking indexation of wage contracts. This result is broadly consistent with the scenario of inflation inertia described by Rodriguez (1982), Dornbusch (1982), and Calvo and Végh (1994b). In their models inflation exhibited considerable inertia even if the disinflation program was fully credible. However, these sticky inflation models can not reproduce the styl-

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<sup>25</sup>In Argentina (1991-1994) and Mexico (1988-1992), the labor participation rates seems to rise after the stabilization; see Roldos (1997).

ized fact of an initial boom after exchange rate-based stabilization. Specifically, in Calvo and Végh, the initial consumption expansion depends on the elasticities of intertemporal and intratemporal substitution. In our model sticky inflation combined with supply side effect can mimic one of most important stylized fact of stabilizations, the initial boom. After the initial boom, non-tradable output (or consumption) expands up to the new steady state. The economy experiences a sustained expansion in the domestic sector through the labor supply effect. These results are quite consistent with the evidence of the successful stabilizations with a high degree of credibility in Mexico(1988-1992) and Argentina (1991-1994) [See Roldos (1997)]. The distinguishing feature of this model compared to the previous ones is that disinflation produces a sustained expansion even if inflation shows persistence.

The dynamic adjustment of the main variables is illustrated in Figures 1.4 to 1.9. The first thing we have to notice in this simulation is the movement of the real exchange rate during adjustment. Due to the sticky inflation, the rate of inflation remains above the nominal devaluation rate for some time. From (1.2.53), we can see that the real exchange rate appreciates, which is one of the main stylized facts in exchange-rate based stabilization. In this model, real appreciation co-exists with the expansion of the domestic sector. Another interesting result is that inflation is below the nominal anchor's growth rate (the rate of devaluation) for some period of time: inflation shows undershooting. The reason is as follows After disinflation is implemented, the real exchange rate appreciates due to the sticky inflation. To

achieve a high level of non-tradable output (or consumption), however, the real exchange rate should be higher (real depreciation) in the new steady state than in the initial steady state since

$$e_{ss1} = \frac{y_{ss1}^N}{rk_0 + y^T}. \quad (1.64)$$

For a subsequent real depreciation, some period of inflation below  $\epsilon_L$  is required.

This result follows from the fact that

$$\dot{e}_t = e_t(\epsilon_t - \pi_t). \quad (1.65)$$

The path of the real exchange rate is depicted in Figure 1.7.

It should be noted that in this model inflation stabilization proves to be welfare-improving. Since the reducing inflation is beneficial in cash-in-advance models with a labor-leisure choice and imperfect competition in goods market. The lower inflation reduces the transaction cost of holding money. Hence it allows the individuals more free time for productive activities: more consumption and production. In a broad sense, disinflation is welfare improving since socially unproductive efforts to conserve on money balances can be eliminated.

#### 1.4.2 Temporary reduction in the devaluation rate

Consider now the implication of a temporary reduction in the rate of devaluation. This corresponds to the case of lack of credibility: the authority announces a permanent reduction in the rate of devaluation, but the public believes that the rate of devaluation will go back to its original level after a certain period of time. The initial steady state corresponds to a rate of devaluation  $\epsilon_t = \epsilon^H$ . At  $t = 0$ , the

devaluation rate is set to a lower level,  $\epsilon^L$ , but at time  $T$ , it is increased back to its original level.

More specifically, for some  $T > 0$ ,

$$\begin{aligned}\epsilon_t &= \epsilon^L, & 0 \leq t < T \\ \epsilon_t &= \epsilon^H, & t \geq T,\end{aligned}$$

where  $\epsilon^H > \epsilon^L$ .

From Eq.(1.2.29), we can see that the nominal interest rate will be low from 0 to  $T$ , and higher after  $T$ . Thus, (1.2.47) implies that consumption of tradables,  $C_t^T$ , will be high between 0 and  $T$ , and low afterwards.<sup>26</sup> The reason is as follows. Since the consumer expects the effective price of tradables (market price and the opportunity cost of the holding money) to be lower between 0 and  $T$  than after  $T$ , he or she will engage in intertemporal consumption substitution. Hence, consumption will be increased during the interval  $[0, T)$  and lowered after  $T$ .

The economy's intertemporal budget constraint implies that before the stabilization policy is reversed, consumption of tradable goods is larger than permanent income, while consumption of tradables is lower than the initial permanent income after  $T$ . This implies

$$\begin{aligned}C_H^T &> y^T + rk_0, & 0 \leq t < T \\ C_L^T &< y^T + rk_0, & t \geq T.\end{aligned}\tag{1.66}$$

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<sup>26</sup>By Eq (1.2.49), we can see that  $\lambda$  will be constant during the transition.

The time path of the current account is given by Eq.(1.2.34). The increase in the consumption of tradables the permanent income induces the current account to jump into deficit. During the transition, the current account deficit will be larger steadily since the interest rate income on net foreign asset declines. When the stabilization is abandoned,  $C_t^T$  goes down below permanent income and brings the current account into balance. The time path of the consumption of tradables and the current account are illustrated in Figure 1.15, and 1.16 respectively. This is quite consistent with the deterioration of the current account balance observed in the early stage of the Southern-Cone stabilization.

We now turn to examine the behavior of non-tradable output and inflation. On impact, non-tradable consumption jumps, unlike the case of permanent disinflation. We can see this from Eq.(1.2.48). Since the real exchange rate is pre-determined, the increase in the consumption of tradable goods induces the non-tradables to jump on impact. This is consistent with the stylized fact of Southern-Cone exchange rate-based stabilizations in which consumption and output expand at the beginning of the programs. Inflation also falls. However, the size of the initial jump of inflation will be smaller than the permanent case. This is due to the lack of credibility. The public believes that the money growth rate will revert to the original level which is higher than it is now. The initial effect will be a move from  $SS_0$  to  $A$  in Fig.1.8.

After the initial impact, non-tradable output (or consumption) starts to increase. For  $0 < t < T$ , the nominal interest rate will be lower than after  $t \geq T$ .



The households supply more labor to the non-tradable sector during the transition than after the policy reverts to the original level. Therefore, non-tradable output will be higher during the transition. This is due to the intertemporal substitution effects of labor supply. Since the inflation is above the devaluation rate, the real exchange rate appreciates. In spite of real appreciation, we observe that consumption and output of non-tradables expand during the transition. This result is different from the previous literature [Calvo and Végh (1993) and Calvo and Végh (1994a)]. In their model, there is a long-lived recession after the initial boom in non-tradable output until the policy is reversed. This is mainly due to the fact that the supply-side effect of disinflation is not considered. Inflation also begins to increase as the public expects higher inflation in the future. The expected inflation rate will be incorporated into the wage contracts, especially into forward-looking contracts. This induces the rate of inflation to keep increasing.

During the transition, for  $0 < t < T$ , the system will be governed by the dominant divergent path (See Appendix A.1.3).

$$\lim_{t \rightarrow \infty} \frac{\log y_t^N - \log y_{ss1}^N}{\pi_t - \mu_H} = \frac{\omega_{31}}{\omega_{32}}. \quad (1.67)$$

In this model, the sign of  $\frac{\omega_{31}}{\omega_{32}}$ , the slope of divergent path, is ambiguous [see (A.1.11)]: it can be positive or negative. However, even if the slope is positive, it is flatter than the dominant eigenvector ray (in new steady state). Therefore, the transitional dynamic of the system can not be affected by the indeterminacy. Figure 1.10 depicts the transitional adjustment of non-tradables and inflation. The

direction of motion is indicated by the arrows.<sup>27</sup>

At  $T$ , the rate of devaluation reverts to  $\epsilon^H$ . The economy now converges to the dominant eigenvector ray and moves to the original steady state. As the nominal interest rate goes back to the original level, the households decrease their labor supply. Thus, non-tradable output (or consumption) also shrinks and the recession sets in. Since the policy is reversed at  $T$ , expectations adjust and inflation starts to fall, converging to the original level. This result highlights the importance of the policymaker's credibility. The credibility of policy will affect people's expectations, and produce unpredicted results.

The dynamic path of the main variables is depicted in Figure 1.11 to 1.16. One of distinguishing features of this policy relative to the credible one is that the temporary policy induces the current account deterioration. The movement of tradable consumption and current account dynamics are illustrated in Figure 1.15 and 1.16.

## 1.5 Concluding remarks

This paper has extended Obstfeld and Rogoff's dynamic general equilibrium Mundell-Fleming model to allow for sticky inflation in a way that can be used to analyze the effect of exchange rate-based stabilization.

The large theoretical literature has provided useful insights into different aspects of both failed and successful exchange-rate based stabilizations. The model

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<sup>27</sup>In Figure 1.10 we assume that the divergent path is negatively sloped. However, as is mentioned in the main text, we observe the same transitional path under a positively sloped divergent path.

presented in this paper can replicate the stylized facts observed in both failed and successful disinflation programs within a single analytical framework. In that respect, this study is quite successful.

This paper also has a very important policy implication for inflation stabilization. As illustrated in the main text, stabilization, even if it is fully credible, is always accompanied with inflation persistence. Therefore, the inflation inertia during the program does not necessarily mean the possibility of failure if backward-looking indexation in price or wage setting exists. Therefore, as long as the policy maker continues to pursue its goal, he or she can reduce inflation without any output cost: he can improve welfare. Another piece of advice for the policy maker implied in this paper is that the economy's indexation mechanism should be altered to accommodate rapid disinflation before the stabilization is launched.

This paper can be extended in several ways. First, it can be applied to the money-based stabilization under the flexible exchange rate. And another promising way would be to adapt the model to analyze the effect of interest rate policy on the economy including capital flows. For this, the model must explicitly incorporate capital accumulation.

In this paper, we assume the degree of backward looking indexation for analytical simplicity. However, the degree of indexation has a significant implication for the economic welfare during the stabilization. As the degree of indexation declines, the model predicts the rapid convergence of inflation. This will enhance the welfare effects of inflation stabilization. To study the relationship between the degree

of indexation and economic welfare of inflation stabilization is also promising.

## 1.6 Appendix

### 1.6.1 The stability of Equation (1.2.59)

The stability of the system requires that among the eigenvalues of the matrix associated with dynamic system (1.2.59), one has a positive real part and two eigenvalues have negative real parts.

The eigenvalues  $\lambda_i, i = 1, 2, 3$ . of the system

$$\begin{aligned} & \begin{bmatrix} \log y_t^N \\ \pi_t \\ \eta_t \end{bmatrix} \\ &= \begin{bmatrix} \frac{\alpha\delta^2(\phi+\varphi)}{\mu} & -\frac{2(1+\alpha\delta)}{\mu} & \frac{2\alpha\delta}{\mu} \\ \frac{\delta^2(\phi+\varphi)}{\mu} & \frac{\delta(\varphi-2)}{\mu} & -\frac{2\delta}{\mu} \\ 0 & -\delta & \delta \end{bmatrix} \times \begin{bmatrix} \log y_t^N - \log y_{ss}^N \\ \pi_t - \epsilon_t \\ \eta_t - \epsilon_t \end{bmatrix} \end{aligned} \quad (1.68)$$

have the following properties

$$\det(A) = \lambda_1 \lambda_2 \lambda_3 = \frac{\delta^3}{\mu} > 0 \quad (1.69)$$

$$\lambda_1 \lambda_2 + \lambda_2 \lambda_3 + \lambda_1 \lambda_3 = -\frac{\delta^2}{\mu}(\phi + \alpha\delta) < 0 \quad (1.70)$$

$$Tr(A)^2 - 4\det(A) > 0. \quad (1.71)$$

From (A.1.2), we can see that the three eigenvalues are positive or one is positive and two are negative. From (A.1.3), the case of three positive roots can be ruled out. As a result, the system has two negative roots. Furthermore, according to (A.1.4), the eigenvalues may be complex conjugates and the system may exhibit cyclical behavior.

### 1.6.2 The dominant eigenvalue ray

We assume that  $\nu_i, i = 1, 2$  is the non-positive roots, with  $\nu_1 > \nu_2$ . Then, for  $i = 1, 2$ , it follows that

$$\begin{bmatrix} \frac{\alpha\delta^2(\phi+\varphi)}{\mu} - \nu_i & -\frac{2(1+\alpha\delta)}{\mu} & \frac{2\alpha\delta}{\mu} \\ \frac{\delta^2(\phi+\varphi)}{\mu} & \frac{\delta(\varphi-2)}{\mu} - \nu_i & -\frac{2\delta}{\mu} \\ 0 & -\delta & \delta - \nu_i \end{bmatrix} \times \begin{bmatrix} \omega_{i1} \\ \omega_{i2} \\ \omega_{i3} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}, \quad (1.72)$$

where  $\omega_{ij}, j = 1, 2, 3$ , are the element of the eigenvector associated with root  $\nu_i$ .

Therefore,

$$\frac{\omega_{i1}}{\omega_{i2}} = \frac{[2\delta - 2\nu_i - 2\alpha\delta\nu_i]/\mu(\delta - \nu_i)}{[\alpha\delta^2(\phi + \varphi) - \nu_i\mu]/\mu} > 0. \quad (1.73)$$

Now, let's look at the dominant eigenvector ray. Setting to zero constant corresponding to the unstable root ( $\nu_3$ ), the solution to this system takes the form

$$\begin{aligned} \log y_t^N - \log y_{ss}^N &= A_1\omega_{11} \exp(\nu_1 t) + A_2\omega_{21} \exp(\nu_2 t) \\ \pi_t - \epsilon_t &= A_1\omega_{12} \exp(\nu_1 t) + A_2\omega_{22} \exp(\nu_2 t) \\ \eta_t - \epsilon_t &= A_1\omega_{13} \exp(\nu_1 t) + A_2\omega_{23} \exp(\nu_2 t), \end{aligned} \quad (1.74)$$

where  $A_i, i = 1, 2$ , denote the constants associated with root  $\nu_i$ . Since  $\nu_1 > \nu_2$ , it follows that:

$$\begin{aligned} \lim_{t \rightarrow \infty} \frac{\log y_t^N - \log y_{ss}^N}{\pi_t - \epsilon} &= \lim_{t \rightarrow \infty} \frac{A_1\omega_{11} \exp(\nu_1 t) + A_2\omega_{21} \exp(\nu_2 t)}{A_1\omega_{12} \exp(\nu_1 t) + A_2\omega_{22} \exp(\nu_2 t)} \\ &= \lim_{t \rightarrow \infty} \frac{A_1\omega_{11} + A_2\omega_{21} \exp((\nu_2 - \nu_1)t)}{A_1\omega_{12} + A_2\omega_{22} \exp((\nu_2 - \nu_1)t)} \\ &= \frac{\omega_{11}}{\omega_{12}} > 0. \end{aligned} \quad (1.75)$$

### 1.6.3 Properties of system for temporary stabilization

We assume that  $\nu_i, i = 1, 2$  are non-positive roots and  $\nu_3$  is the non-negative one.

Then, for  $0 < t < T$ , the solution to the system is

$$\begin{aligned}\log y_t^N - \log y_{ss1}^N &= B_1 \omega_{11} \exp(\nu_1 t) + B_2 \omega_{21} \exp(\nu_2 t) + B_3 \omega_{31} \exp(\nu_3 t) \\ \pi_t - \epsilon^H &= B_1 \omega_{12} \exp(\nu_1 t) + B_2 \omega_{22} \exp(\nu_2 t) + B_3 \omega_{32} \exp(\nu_3 t) \\ \eta_t - \epsilon^H &= B_1 \omega_{13} \exp(\nu_1 t) + B_2 \omega_{23} \exp(\nu_2 t) + B_3 \omega_{33} \exp(\nu_3 t),\end{aligned}\quad (1.76)$$

and for  $t \geq T$

$$\begin{aligned}\log y_t^N - \log y_{ss0}^N &= C_1 \omega_{11} \exp(\nu_1 t) + C_2 \omega_{21} \exp(\nu_2 t) \\ \pi_t - \epsilon^L &= C_1 \omega_{12} \exp(\nu_1 t) + C_2 \omega_{22} \exp(\nu_2 t) \\ \eta_t - \epsilon^L &= C_1 \omega_{13} \exp(\nu_1 t) + C_2 \omega_{23} \exp(\nu_2 t),\end{aligned}\quad (1.77)$$

where  $B_i$  and  $C_i, i = 1, 2, 3$  are constants associated with root  $\nu_i$ , and  $\omega_{ij}, i, j = 1, 2, 3$  are the elements of eigenvector for roots  $\nu_i$ .

For  $0 < t < T$ , the system will be governed by the unstable root,  $\nu_3$ . This means that the system converges asymptotically to the divergent path, which is given by

$$\begin{aligned}\lim_{t \rightarrow \infty} \frac{\log y_t^N - \log y_{ss1}^N}{\pi_t - \epsilon^L} &= \lim_{t \rightarrow \infty} \frac{B_1 \omega_{11} \exp(\nu_1 t) + B_2 \omega_{21} \exp(\nu_2 t) + B_3 \omega_{31} \exp(\nu_3 t)}{B_1 \omega_{12} \exp(\nu_1 t) + B_2 \omega_{22} \exp(\nu_2 t) + B_3 \omega_{32} \exp(\nu_3 t)} \\ &= \frac{\omega_{31}}{\omega_{32}}.\end{aligned}\quad (1.78)$$

From (A.1.11) with  $\nu_3 > 0$ , we can see that the sign of  $\frac{\omega_{31}}{\omega_{32}}$  cannot be determined.

Even if, however, it is positive, the slope is flatter than the dominant eigenvector ray. For  $t \geq T$ , the dominant eigenvector ray is same as (A.1.8).

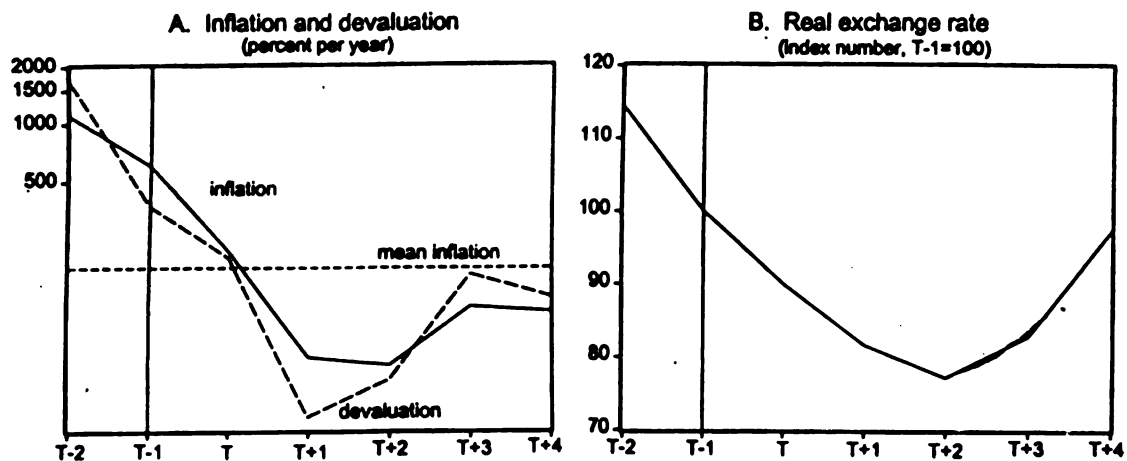


Figure 1.1: Exchange rate-based stabilization

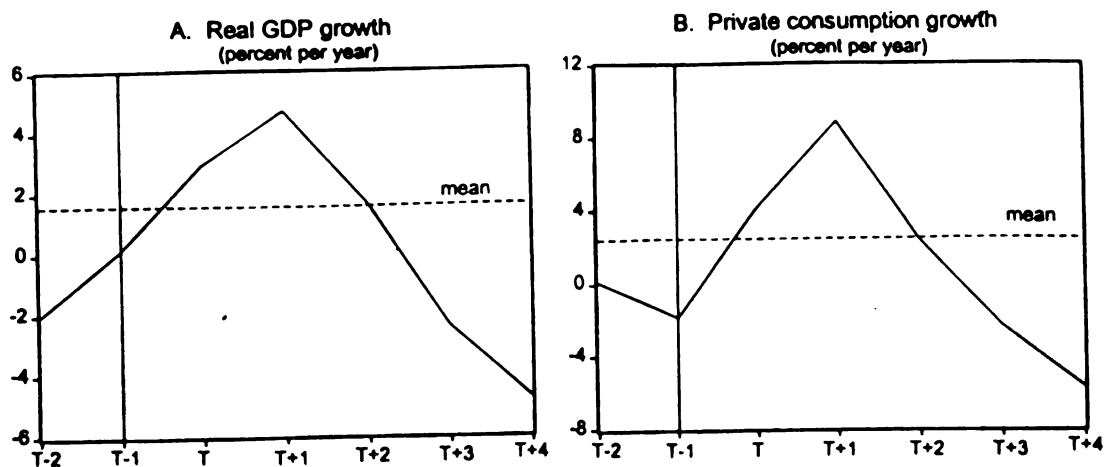


Figure 1.2: GDP and consumption in exchange-rate based stabilization



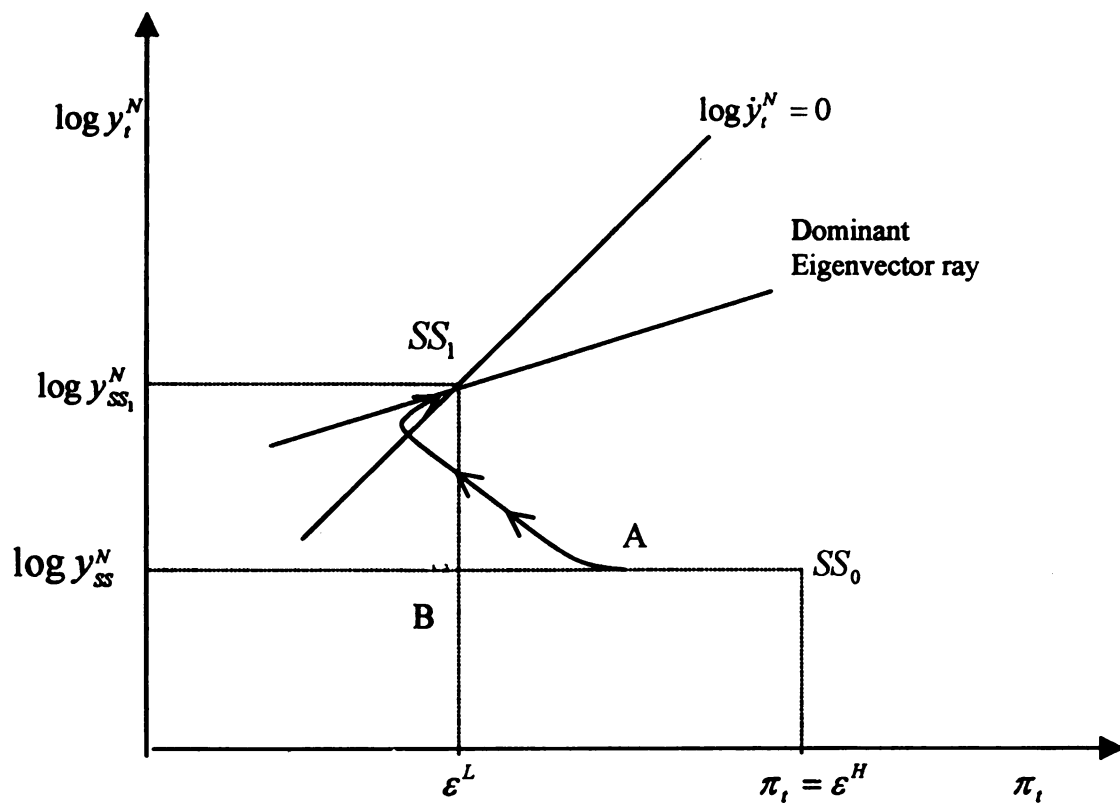


Figure 1.3: Dynamic system of exchange rate-based stabilization

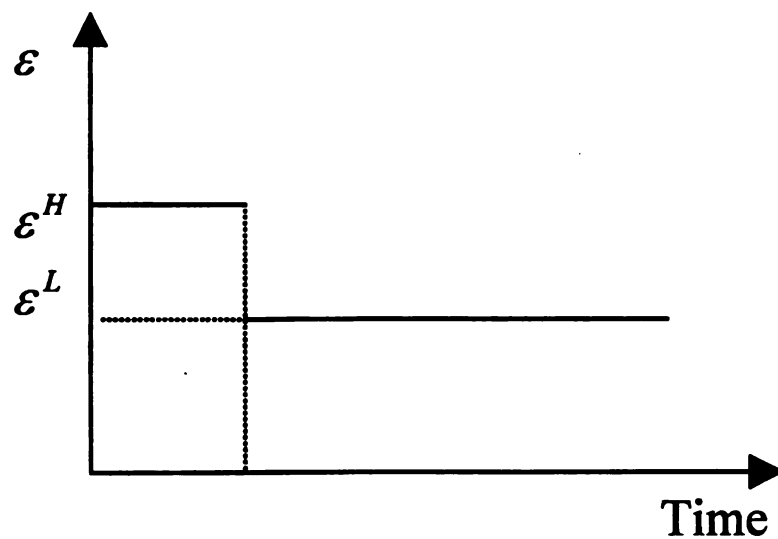


Figure 1.4: Time path: rate of devaluation

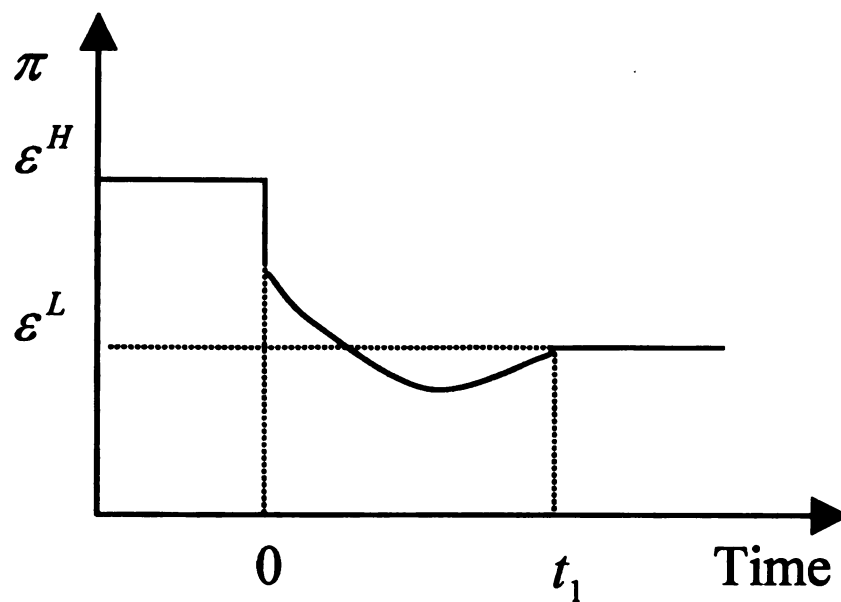


Figure 1.5: Time path: inflation

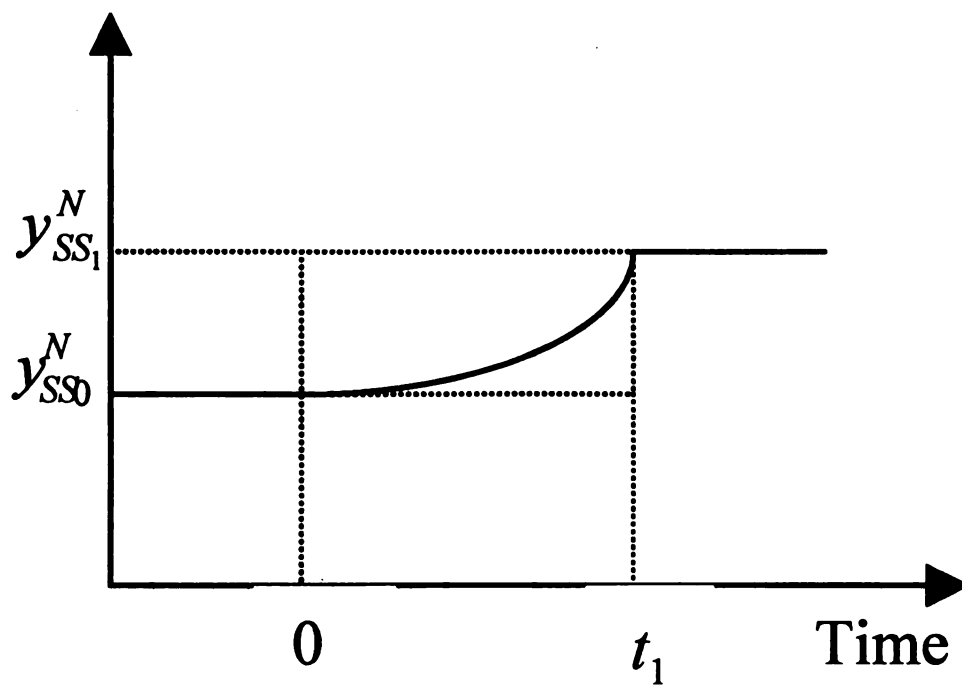


Figure 1.6: Time path: output of non-tradables

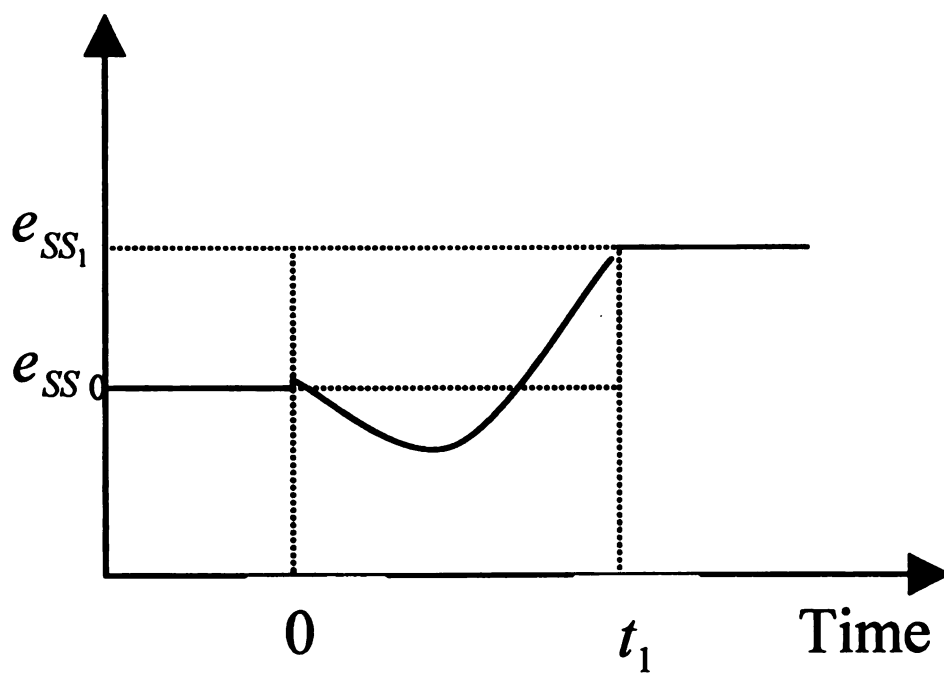


Figure 1.7: Time path: real exchange rate

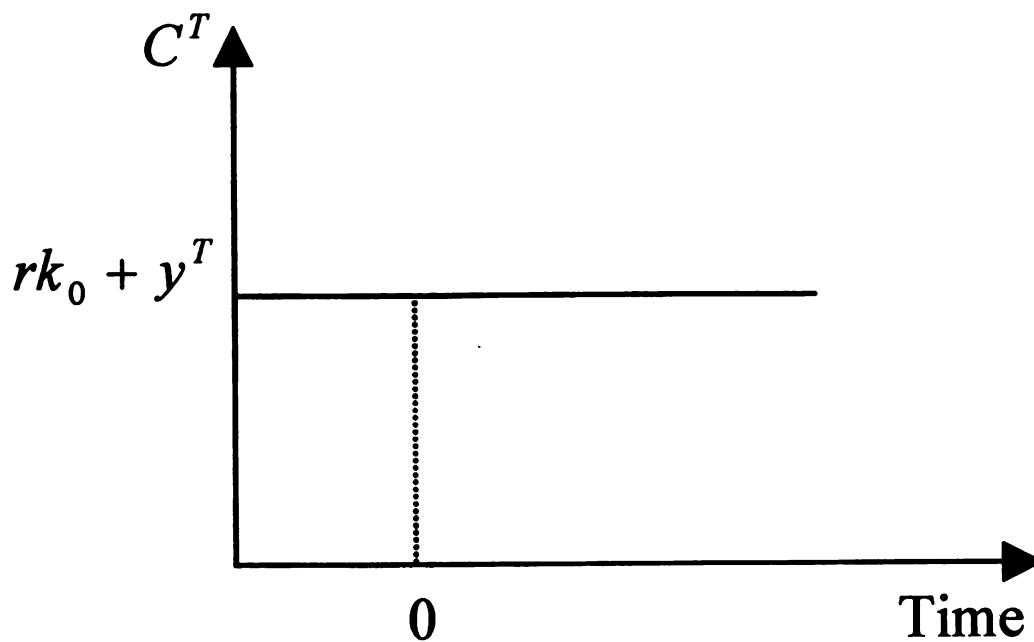


Figure 1.8: Time path: consumption of tradables

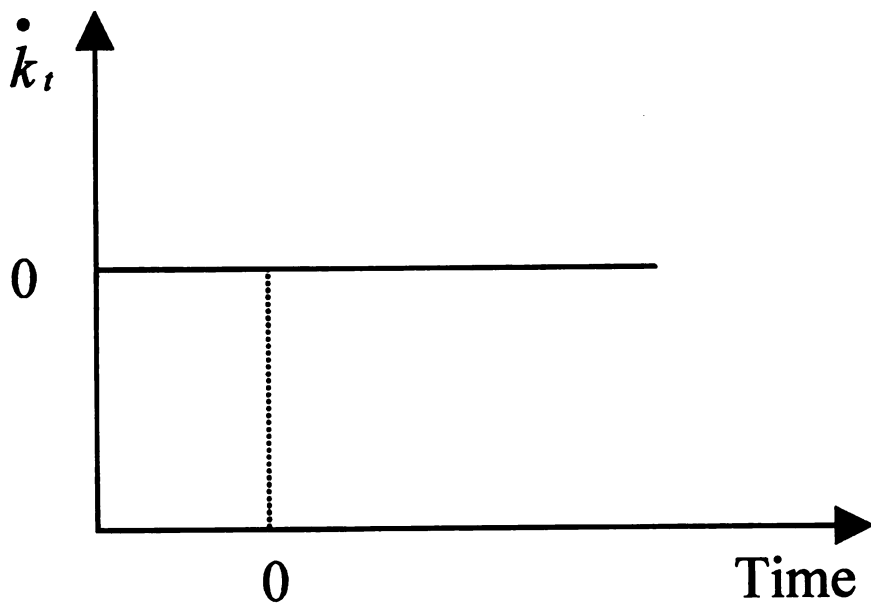


Figure 1.9: Time path: current account

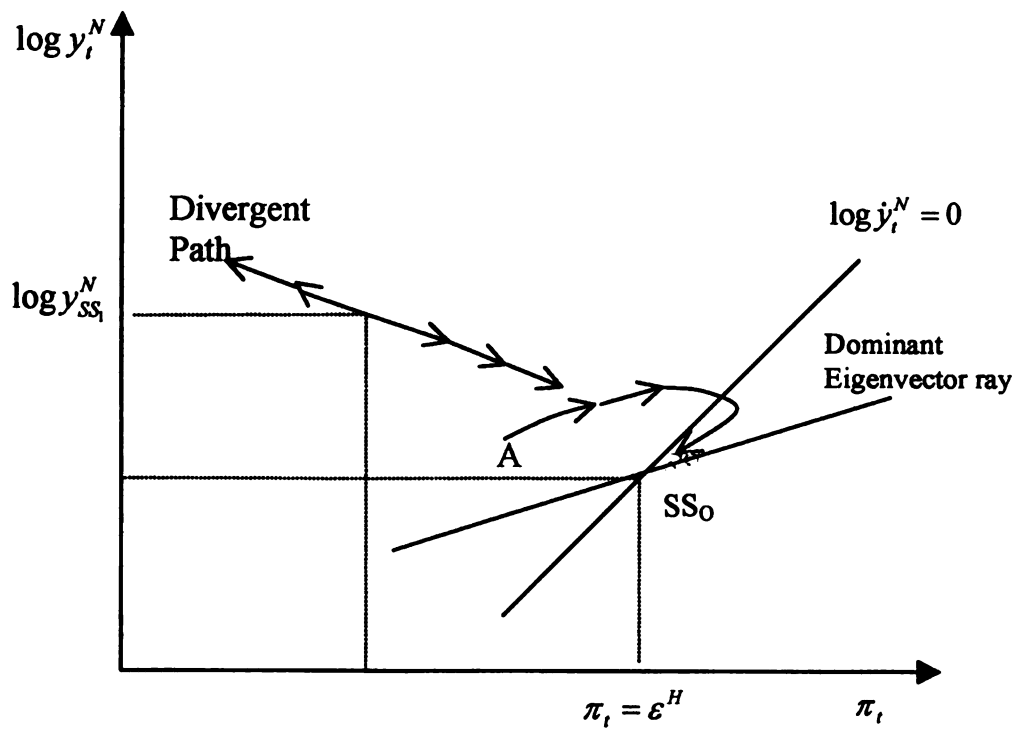


Figure 1.10: Dynamic system of temporary stabilization

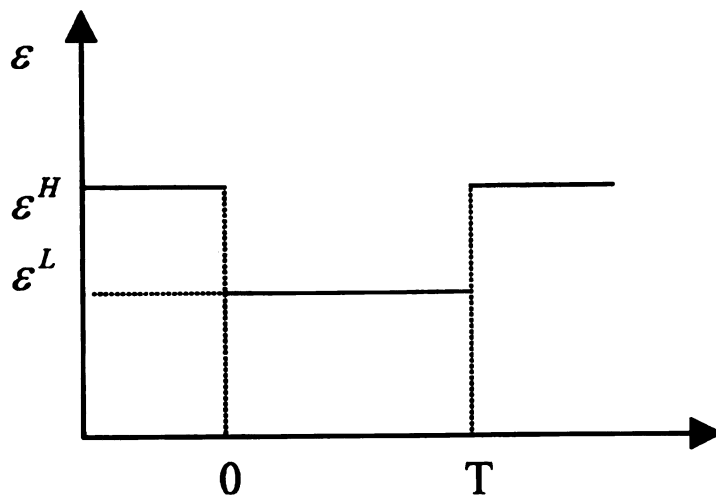


Figure 1.11: Time path: rate of devaluation

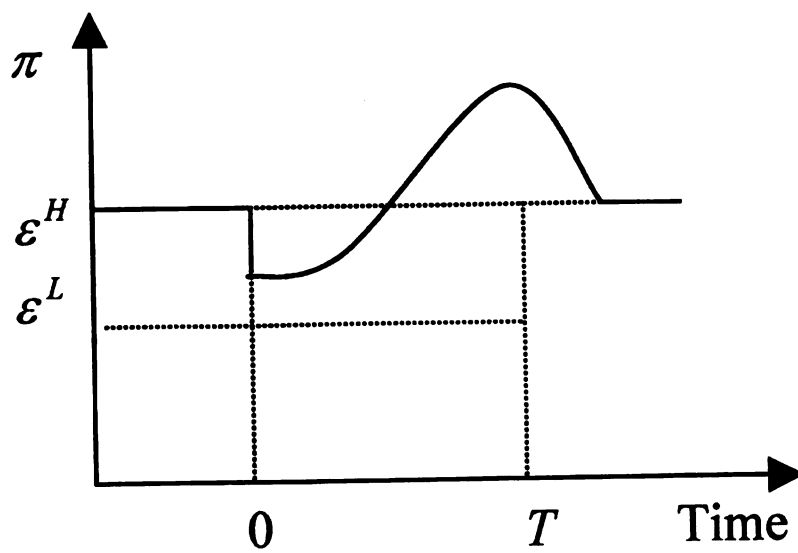


Figure 1.12: Time path: inflation

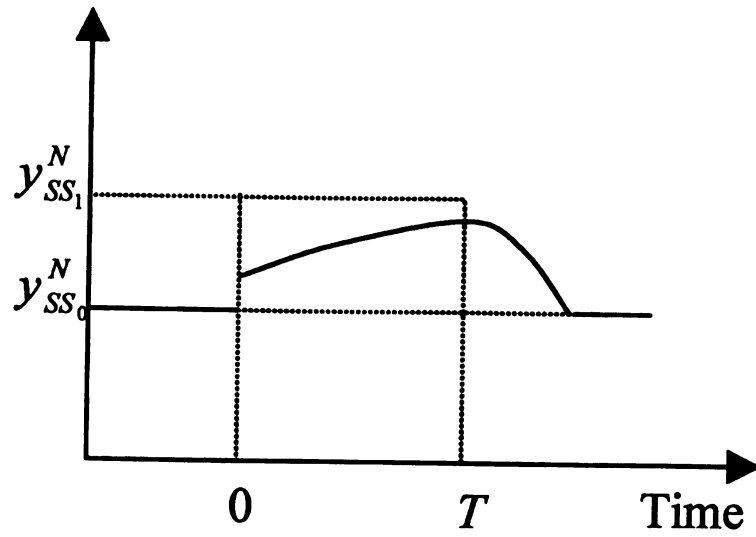


Figure 1.13: Time path: output of non-tradables

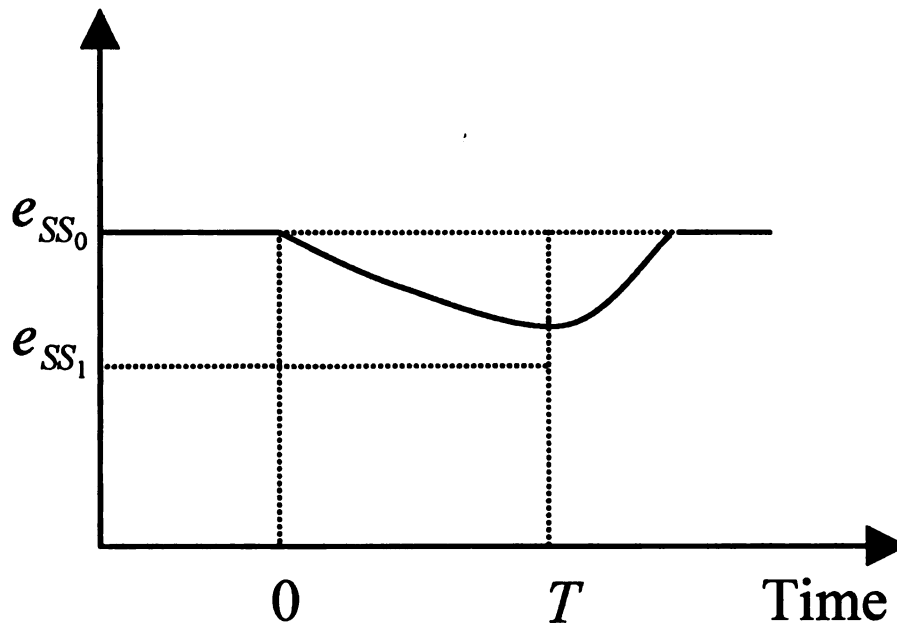


Figure 1.14: Time path: real exchange rate

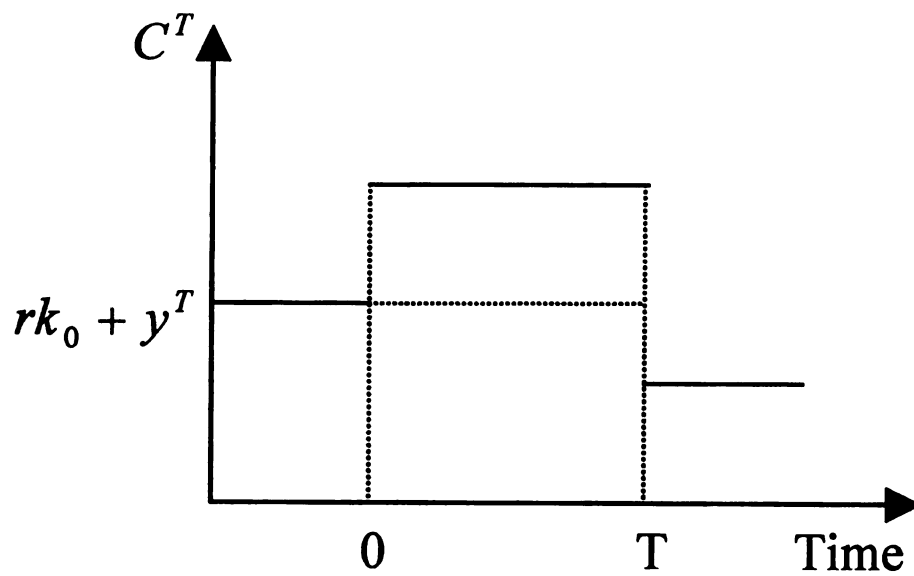


Figure 1.15: Time path: consumption of tradables

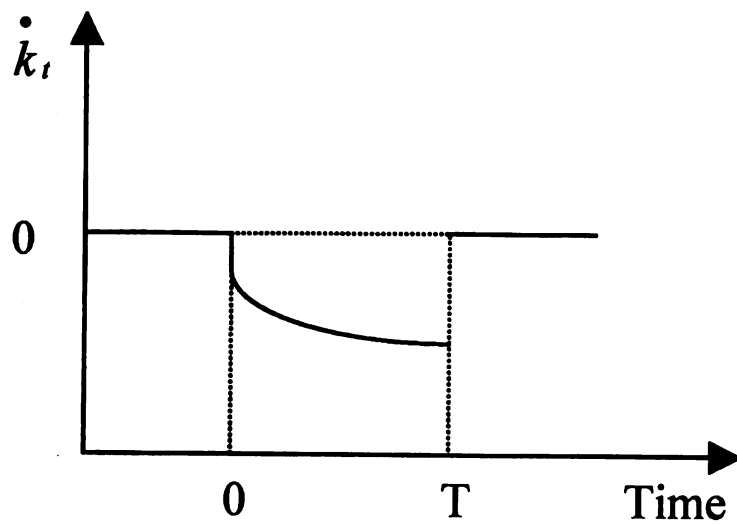


Figure 1.16: Time path: current account

# Chapter 2

## Money-based stabilization

### 2.1 Introduction

Since the end of World War II, many developing countries have experienced high and persistent inflation<sup>1</sup>, which in some cases has lasted until today. Chronic inflation is one of the distinguishing macroeconomic characteristics of developing countries. For the last four decades, countries afflicted by chronic inflation have engaged in repeated inflation stabilization programs. For example, in the late 1970's, the Southern-Cone countries, Argentina, Chile and Uruguay, implemented exchange-rate based inflation stabilization, and Brazil and Peru in the early 1990's fought chronic inflation with money growth rates. Unfortunately, most of these stabilization attempts have failed in the sense that they did not reduce the inflation rate to the international level. In the last ten years, however, countries such as Argentina, Israel, and Mexico have succeeded in reducing inflation close to international levels for substantial periods.

The stabilization plans implemented in the last four decades, whether they were

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<sup>1</sup>Pazos(1972) refers to this phenomenon as "chronic inflation". According to his analysis, chronic inflation is relatively high compared to industrial countries and persistent. Moreover, chronic inflation does not have an inherent propensity to accelerate.



successful or not, have provided a lot of issues and puzzles to macroeconomists. The common stylized fact we observed in the stabilizations, both exchange rate-based and money-based programs, is inflation persistence. Contrary to the expectation of both policy makers and economists, inflation converged slowly to the target rate and exhibited considerable inertia. The most intriguing and puzzling phenomena is the business cycle associated with inflation inertia. In exchange rate-based programs, real economic activity, such as consumption and output, expands in the early stage of the program. Later in the program, even before the program is abandoned, consumption and output shrink and the trade balance goes into deficit, so recession sets in.<sup>2</sup> This boom-recession cycle of exchange-based stabilization programs was a puzzle to conventional macroeconomists who believed the stabilization should have resulted in an immediate contraction. In the case of money-based stabilization, the cost of disinflation comes in the early stage of the program.

A lot of theoretical research has sought to explain these intriguing phenomena observed in the stabilization programs, especially in exchange-based programs. Early work by Rodriguez (1982) and Dornbusch (1982) offers a simple description of the Southern-Cone exchange rate-based programs of the late 1970s. They emphasized the presence of sticky inflation. Rodriguez assumes adaptive expectations to point out the backward-looking price and wage behavior. Dornbusch assumes

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<sup>2</sup>In Argentina, for instance, the stabilization program was launched in 1978. By the 1980s, the annual inflation rate was 88 per cent, but the devaluation rate was 23 per cent. Moreover, the growth rate of consumption was 1.9 per cent in 1978, but in the first year of the program, 1979, it increased to 12.3 per cent.

rational expectations and stickiness of inflation. According to this hypothesis, under perfect capital mobility, if aggregate demand depends negatively on the real interest rate and positively on the real exchange rate (i.e., the relative price of tradable goods in terms of non-tradable goods), reduction of the devaluation rate decreases the nominal interest rate, which leads to lower real interest rates because of sticky inflation. The reduction of the real interest rate induces an initial expansion of output. After that, the domestic currency begins to appreciate since domestic inflation shows inertia and remains above the devaluation rate. Eventually, the real appreciation shrinks domestic consumption and so output falls and recession sets in.

Those sticky inflation models give a lucid explanation of the phenomena observed in the stabilization programs, especially inflation persistence and the boom-recession cycle. Inflation inertia results from sticky inflation in both models. However, these early models have a critical defect. Both models specify reduced-form behavioral equations rather than derive them. We find this very ad-hoc. In the case of Rodriguez (1982), the model relies on non-rational behavior and inflation stickiness is assumed, not derived as in Dornbusch(1982).

An alternative explanation to the major effects of stabilization is the “Temporariness hypothesis” by Calvo and Végh (1993, 1994). This hypothesis considers the case in which prices or wages are sticky due to staggered contracts, but inflation is fully flexible because of forward-looking contracts. These models are based on the optimizing behavior of individuals. Under these circumstances, this hypothe-

sis argues that slow convergence of inflation and the initial boom in real economic activity arise from lack of credibility, not from sticky inflation. Rapid convergence of inflation, therefore, can be achieved under a fully credible stabilization policy even if the price level exhibits stickiness. With the same assumptions, Calvo and Végh (1994, 1999) replicate the recession-boom cycle in a money-based stabilization with no inflation inertia.

This hypothesis seems to be successful in capturing the stylized facts associated with the recent stabilization programs, especially exchange rate based programs. However, these models are not successful in establishing the exact cause of inflation inertia. In these models, since prices are set in a purely forward-looking manner, the inflation rate is independent of past inflation and depends only on future (expected) inflation. Thus, inflation is quite flexible while the price level is sticky. Therefore, credible disinflation policies can reduce inflation dramatically without any output cost and no inflation inertia is found, which is not consistent with reality.<sup>3</sup> Also, this hypothesis cannot replicate both the initial recession and inflation inertia simultaneously in a money-based program. In Calvo and Végh (1999), money-based stabilizations replicate the initial contraction without inflation persistence under the credible policy. Therefore, purely forward-looking staggered contract models are not useful in explaining inflation dynamics when inflation shows stickiness.

All the explanations examined so far are based on demand-side considerations.

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<sup>3</sup>Agénor and Montiel (1999) point out that countries with chronic inflation derived various indexation mechanisms which are quite backward looking.

This is due to the fact that most literature was inspired by the Southern-Cone tabilitas of the late 1970s. In the recent programs, such as Mexico's 1987 and Argentina's 1991 convertibility plan, it has been argued that inflation stabilization may have played an important role in unleashing supply-side responses in labor and investment. This literature suggests that credible stabilization induces sustained expansion of real economic activity. In Lahiri (2001) and Roldos (1997), the nominal interest rate introduces a distortion between consumption and leisure. When inflation falls, labor supply increases. This, in turn, leads to a rise in the desired capital stock and, hence, in investment. These supply-side considerations can reproduce the main stylized facts observed in the successful stabilizations (Argentina (1991-1994) and Mexico (1988-1992) in which the initial boom was not followed by a recession.

Most literature focused on the exchange rate-based stabilizations since money-based programs in chronic inflation countries have been much less common than programs based on the exchange rate.<sup>4</sup> Several authors provide a number of reasons why policymakers prefer an exchange rate-based program to a money-based one [Robero and Végh (1994), and Calvo and Végh (1999)]. First, since the velocity of money may be quite unstable during the stabilization, it is difficult for a policymaker to determine the appropriate growth rate of the money supply in practice. Second, in the transition from high to low inflation, the economy may experience a period of deflation. Thus, policymakers will engineer a one time in-

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<sup>4</sup>According to Reinhart and Végh, among 17 stabilization programs from 1964 to early 1990's, there are only 5 money-based programs.

crease in the money supply to avoid deflation, which weakens the policymaker's credibility. Based on arguments mentioned above, it is not surprising to find that money-based stabilization does not enjoy a high reputation in the academic literature or in practice. However, there are several reasons to analyze money-based stabilizations here. First, stabilization programs supported by the use of IMF resources have usually been money-based programs.<sup>5</sup> Second, since most exchange rate-based stabilizations end up in balance-of-payment crises,<sup>6</sup> there is a need to find possible policy instruments to bring down high inflation without triggering crises. It should be noted that a lot of exchange rate-based stabilization programs have failed. Therefore, for a country which experienced a series of failed exchange rate-based programs, it could be wise to switch the anchor. Lastly, there is a possibility that successful disinflation in a money-based program might lead to the supply side responses we observed in the exchange-rate programs.

In this paper, we consider the supply-side effects of disinflation with inflation inertia in money-based programs within a New-Keynesian framework. To consider the supply-side response, we introduce imperfect competition in the non-tradable sector with endogenous labor supply. The reason we adopt imperfect competition in the model is that imperfect competition is the main characteristic of the industrial structure of developing countries.<sup>7</sup> Our model is based on Obstfeld and Rogoff's (1995) "Exchange Rate Dynamic Redux". We modify Obstfeld and Rogoff's model in two ways. First, we assume a small-open economy with a cash-

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<sup>5</sup>See Agénor and Montiel (1999).

<sup>6</sup>See Calvo and Végh (1999).

<sup>7</sup>See Agénor and Montiel (1999)

in-advance constraint in which a change of monetary policy, including a change of the money growth rate, has a long-run effect on the economy as in the two-country model in Obstfeld and Rogoff. Second, we adopt a Calvo-type contracts model to obtain the inflation dynamics. Our model is a modified version of Calvo's (1983) forward-looking, staggered wage contracts model. However, we incorporate backward-looking indexation into wage contracts. We combine the staggered wage contract with price setting of imperfectly competitive firms to obtain inflation persistence. Incorporating backward-looking wage contracts into the model is quite consistent with the wage indexation of most chronic inflation circumstances.<sup>8</sup>

We apply this model to investigate the effects of inflation stabilization in a small open economy. We have two possible experiments, one is a money-based stabilization which is perfectly credible and the other is temporary stabilization. From the first experiment we are able to replicate qualitatively the stylized facts of the money-based stabilization programs in which inflation persistence coexists with the "recession-boom cycle". Sticky inflation combined with supply side effects produces inflation persistence and a sustained expansion of the non-tradable sector. In this regard, the model we present here is quite successful. It should be noted that the resulting inflation persistence and associated business cycle have very important policy implications. First, inflation persistence cannot be avoided even if the policy is fully credible as long as there is backward-looking indexation in price or wage setting. To achieve rapid stabilization, the indexation mechanism

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<sup>8</sup>For instance, Edwards(1991) pointed out that the primary cause of inflation inertia in the Chilean stabilization is backward-looking wage indexation.

should be considered. The second implication is that after the initial cost of disinflation, the economy can enjoy a sustained expansion during the stabilization. Therefore slow adjustment of inflation does not weaken the credibility of policy. These results are not consistent with the arguments of Calvo and Végh's (1993, 1994) "temporariness hypothesis" in which inflation persistence and associated business cycles are merely due to the lack of credibility. For the temporary disinflation scenario, we see that the inflation rate accelerates after an initial decrease, and the "recession-boom-recession again cycle" is observed.

The plan of this paper is as follows. We consider the stylized facts of exchange rate-based stabilization in Section 2.2. We present the basic model and the equilibrium in Section 2.3. The impact effects and transitional dynamics of a fully credible and temporary disinflation policy are discussed in Section 2.4. In Section 2.5, we draw the main conclusions. Technical derivations are relegated to Appendices.

## **2.2 Evidence on the real effects of money-based stabilization in chronic inflation countries**

Money-based programs in chronic inflation countries have been much less common than programs based on the exchange rate. According to Reinhart and Végh, among 17 stabilization programs from 1964 to early 1990's, there are only 5 money-based programs. Table 2.1 shows presents the main features of five major money-based programs undertaken in the last 25 years. The following stylized facts observed in the money-based programs listed above have been identified.

Table 2.1: Money-based inflation stabilization plans

Program	begin/end date	Initial/Lowest inflation
Chile 1975	April 1975 - December 1977	393.4 / 63.4
Bonex (Argentina)	December 1989 - February 1991	4923.3 / 287.3
Collar (Brazil)	March 1990 - January 1991	5747.3 / 1119.5
Dominican Republic	August 1990 - 1999	60.0 / 2.5
Peru 1990	August 1990 - 1999	12377.8 / 10.2

Major source: Kiguel and Liviatan (1996), and Calvo and Végh (1999)

(1) Slow convergence of the inflation rate to the rate of growth of the money supply.

(2) Initial contraction in economic activities. A sharp, though short-lived, contraction in real GDP, consumption, and investment seems to follow the implementation of money-based programs.

(3) Real appreciation of the domestic currency.

These stylized facts are less surprising in that they seem to broadly conform with available evidence for industrial countries. To illustrate some of these stylized facts, Figure 2.1 shows the panel of annual observations for five countries (Argentina, Brazil, Chile, Dominican republic, and Peru) for 25 years from 1971 to 1995 constructed by Calvo and Végh (1999).<sup>9</sup> Panel A shows that the inflation rate falls dramatically in the first year after stabilization but begins to rise again soon afterwards. and inflation rate does remain above the rate of money growth rate. This evidence shows the existence of inflation persistence. Real GDP growth falls in the year of stabilization from -3.8 to -7.3 percent, but recover soon afterward. The evidence is thus consistent with a short-lived but sharp contraction in

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<sup>9</sup>In the figure 1.1 and 1.2 , time profiles are based on the stabilization time. Stabilization time is denoted by  $T + j$ , where  $T$  is the year in which the stabilization program was implemented and  $j$  is the number of years preceding or following the year of stabilization



economic activity (Panel B). The real exchange rate appreciates throughout the program (Panel C), while the current account shows no clear pattern (Panel D).

## 2.3 Basic model

In this section we consider a two sector small open economy, which is perfectly integrated with the rest of the world in goods and capital markets. First, we introduce the demand side through the households' maximization problem and then the aggregate supply side in which the non-tradable sector is the locus of imperfect competition. The next step is to solve for a symmetric steady state where all prices are fully flexible. We, then, introduce sticky prices through staggered wage contracts. In the last part of this section, we derive the short-run dynamic system for the next section.

### 2.3.1 The aggregate demand side

Our model is based on Obstfeld and Rogoff's (1995) perfect-foresight general equilibrium Mundell-Fleming model. The economy is inhabited by a continuum of identical households which reside on the interval  $[0,1]$ . The representative household derives utility from the consumption of tradable and non-tradable goods and leisure. This economy contains a continuum of differentiated non-tradable goods which are indexed by  $z$  and distributed uniformly on  $[0,1]$ .

The lifetime utility of typical households is given by

$$U_t = \int_0^\infty [\gamma \log(C_t^T) + (1 - \gamma) \log(C_t^N) + \rho \log(1 - l_t)] \exp(-\beta t) dt, \quad (2.1)$$

where  $C_t^T$  is the consumption of the tradable good and  $C_t^N$  is a non-tradable consumption index, defined by

$$C_t^N = \int_0^1 [c_t^N(z)^{\frac{\theta-1}{\theta}}]^{\frac{\theta}{\theta-1}} dz, \quad (2.2)$$

where  $c^N(z)$  is the household's consumption of good  $z$ ,  $\beta$  is the positive and constant subjective discount rate, and  $\theta > 1$  is the elasticity of substitution between non-tradable goods. The last term in the lifetime utility function in Eq.(2.2.1) captures the utility from leisure or disutility from labor supply ( $l_t$  represents total hours worked by the household), and we assume  $\rho > 0$ .

We employ the convention of letting  $E$  represent the nominal exchange rate in units of domestic currency per unit of foreign currency, while  $P^T$  represents the foreign currency price of the tradable good.<sup>10</sup> Here,  $P^N$  denotes the aggregate domestic currency price index of the non-tradable goods, defined as

$$P_t^N = \int_0^1 [p_t^N(z)^{1-\theta}]^{\frac{1}{1-\theta}} dz. \quad (2.3)$$

The real exchange rate (the relative price of tradable goods in terms of non-tradable goods) can be defined as  $e = \frac{EP^T}{P^N}$ . For simplicity, we assume that  $P^T = 1$  and is constant. By this assumption we can suppress the unnecessary foreign inflation rate from the model.

The typical household can hold two types of assets as their financial wealth: domestic non-interest bearing currency and an internationally tradable bond with a constant real interest rate (in terms of tradable goods). Since the domestic

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<sup>10</sup>We assume that the law of one price holds for the tradable good. Therefore, the domestic currency price of the tradable good is  $E_t P_t^T$ .

currency and the international bond are the only two assets held by individual consumers, we have

$$a_t = m_t + b_t, \quad (2.4)$$

where  $a$ ,  $m$ , and  $b$  stand for real financial wealth, real money balances and the stock of real bonds in terms of domestic price of tradable goods, respectively.<sup>11</sup>

We assume that the household has a constant endowment flow of tradable goods,  $y_t^T$ , while it derives human wealth from supplying labor to firm  $z$  for the nominal wage  $W_t$ . Households also own firm  $z$ . Therefore, they can earn the profit of the firm,  $\Pi_t(z)$ . Then the household's dynamic budget constraint is governed by the following differential equation

$$\dot{a}_t = ra_t - i_t m_t + \frac{\Pi_t(z)}{E_t P^T} + \frac{W_t l_t}{E_t P^T} + y_t^T + \tau_t - \frac{C_t^N}{e_t} - C_t^T, \quad (2.5)$$

where  $\tau$  is real transfers from the government in terms of tradable goods;  $i$  is the instantaneous nominal interest rate in terms of domestic currency;  $r$  is the constant real interest rate in terms of tradable goods;<sup>12</sup>  $W_t$  is the money wage rate. Notice that in Eq.(2.2.5) the household's expenditure includes the opportunity cost of holding real money balances,  $im$ .

In order to carry out consumption expenditures, the household is required to hold sufficient domestic money. Following Feenstra(1985), the cash-in-advance constraint which the household faces is thus

$$\alpha \left[ \frac{C_t^N}{e_t} + C_t^T \right] \leq m_t, \quad \alpha > 0, \quad (2.6)$$

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<sup>11</sup>If we define nominal financial wealth, money balances and the stock of bonds as  $A$ ,  $M$  and  $B$ , respectively, then real variables can be defined as follows:  $a = \frac{A}{E P^T}$ ,  $m = \frac{M}{E P^T}$ , and  $b = \frac{B}{E P^T}$ .

<sup>12</sup>Since we assume that  $P^T = 1$  and is constant,  $r$  is also the world nominal interest rate.

where  $\alpha$  represents the length of time that money has to be held to finance consumption expenditure.<sup>13</sup> Eq.(2.2.6) implies that the minimum required money balances are proportional to the value of consumption expenditures. If the nominal interest rate,  $i$ , is positive, the household will hold the minimum required money. Then the cash-in-advance constraint (2.2.6) will be binding.

Using Equations (2.2.4), (2.2.5) and (2.2.6), and imposing the transversality conditions, we can derive the household's intertemporal budget constraint which is given by

$$a_0 + \int_0^\infty \left( \frac{W_t l_t}{E_t P^T} + y_t^T + \frac{\Pi_t(z)}{E_t P^T} + \tau_t \right) \exp(-rt) dt = \int_0^\infty \left( \frac{C_t^N}{e_t} + C_t^T \right) (1 + \alpha i_t) \exp(-rt) dt, \quad (2.7)$$

where  $a_0$  stands for the initial level of financial wealth. This equation says that the household's lifetime expenditure equals the present discounted value of its lifetime income. At each point in time  $t$ , the representative household's expenditure consists of the cost of consumption,  $\frac{C_t^N}{e_t} + C_t^T$ , plus the opportunity cost of holding real balances,  $i_t m_t$ .

The decisions the household has to make at each point in time are how many tradable and non-tradable goods to consume and how much to work. Therefore, the household's optimization problem is to choose the path of  $C_t^T$ ,  $C_t^N$  and  $l_t$  to maximize lifetime utility, (2.2.1), subject to initial wealth,  $a_0$ , and the intertemporal budget constraint, (2.2.7).

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<sup>13</sup>The cash-in-advance constraint can be written as  $m_t \geq F(\alpha) \equiv \int_t^{t+\alpha} \left( \frac{C_s^N}{e_s} + C_s^T \right) ds$ . A Taylor-series expansion gives  $F(\alpha) = \alpha \left( \frac{C_t^N}{e_t} + C_t^T \right) + \frac{1}{2} \alpha^2 \frac{d}{dt} \left( \frac{C_t^N}{e_t} + C_t^T \right) + \dots$ , so (2.2.6) can be interpreted as a first-order approximation.

The first-order conditions for this optimization problem are<sup>14</sup>

$$\frac{\gamma}{C_t^T} = \lambda(1 + \alpha i_t) \quad (2.8)$$

$$\frac{1 - \gamma}{C_t^N} = \lambda\left(\frac{1 + \alpha i_t}{e_t}\right) \quad (2.9)$$

$$\frac{\rho}{1 - l_t} = \lambda\left(\frac{W_t}{E_t P_t^N}\right), \quad (2.10)$$

where  $\lambda$  is the (time-invariant) Lagrange multiplier associated with the household's intertemporal budget constraint (2.2.7). As usual, it can be interpreted as the marginal utility of wealth. Equations (2.2.8) and (2.2.9) indicate that at an optimum, the household equates the marginal utility of consumption of tradable and non-tradable goods to the marginal utility of wealth times the effective prices of goods. The effective prices of goods consist of the market prices, unity in the case of tradables and  $\frac{1}{e_t}$  in the case of non-tradables, plus the opportunity cost of holding the  $\alpha$  units of money that are needed to purchase both goods,  $\alpha i$  and  $\alpha \frac{i}{e_t}$ , respectively. Equation (2.2.10) is the Euler equation for optimal labor supply. It ensures that the marginal disutility of labor (due to forgone leisure) equals the marginal utility from consuming the extra goods from another unit of labor supplied. The term  $(1 + \alpha i_t)$  generated by the cash-in-advance constraint must be treated with special attention in this model. This is the usual monetary wedge due to the fact that any form of wealth has to be liquidated before goods can be purchased. Even if this monetary wedge increases the effective price of consumption, it does not affect the price of leisure.

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<sup>14</sup>To eliminate inessential dynamics and ensure the existence of a steady state, we assume that  $\beta = r$ .

Using (2.2.8) and (2.2.9), we can get

$$\frac{C_t^T e_t}{C_t^N} = \frac{\gamma}{1 - \gamma}. \quad (2.11)$$

And combine (2.2.9) and (2.2.10)

$$\frac{\rho}{1 - l_t} = \frac{1 - \gamma}{C_t^N} \frac{W_t}{P_t^N} (1 + \alpha i_t)^{-1}. \quad (2.12)$$

Equation (2.2.11) is the familiar condition that at an optimum, the marginal rate of substitution between tradable and non-tradable goods is equal to the relative price of tradable goods in terms of non-tradable goods, which is the real exchange rate. From Eq.(2.2.12) we can see that the individual's optimal labor supply is dependent on the nominal interest rate. Since the nominal interest rate introduces a distortion between consumption and leisure, a change in the interest rate makes the household substitute leisure for consumption due to the change in the effective price of consumption. This, in turn, leads to a change in the long-run level of non-tradable output.

### 2.3.2 The government

The other participant in this economy is the government. Since Ricardian equivalence holds in this model, we can assume that the government runs a balanced budget in every period. We also assume no government spending. With these assumptions, we can simplify government behavior. It is assumed that the government holds internationally tradable bonds (international reserves) which pay the world real interest rate in terms of tradable goods and issues non-interest bearing

debt (domestic currency).

The evolution of the government's stock of net foreign bonds is governed by the following differential equation:

$$\dot{h}_t = rh_t - \tau_t + \frac{\dot{M}_t}{P_t^T E_t}, \quad (2.13)$$

where  $h_t$  is the government's stock of internationally tradable bonds (international reserves). Notice that  $\frac{\dot{M}_t}{P_t^T E_t}$  is the government's seignorage profits. Integrating equation (2.2.13) and imposing the transversality condition, we can derive the government's intertemporal budget constraint

$$\int_0^\infty \tau_t \exp(-rt) dt = h_0 + \int_0^\infty (\dot{m}_t + \epsilon_t m_t) \exp(-rt) dt, \quad (2.14)$$

where  $\epsilon_t = \frac{\dot{E}_t}{E_t}$  is the nominal rate of depreciation and  $h_0$  is the initial level of the government's stock of foreign bonds.<sup>15, 16</sup> The government's intertemporal budget constraint indicates that the present value of transfer expenditure has to be equal to the initial stock of government-held international bonds (i.e., international reserves) and revenues from seignorage. By equation (2.2.14), we implicitly assume that the government returns to the consumer all of its revenues.

### 2.3.3 The aggregate supply side

We now turn to the supply side of the economy. For simplicity, we assume that the supply of tradable goods is exogenous and fixed at the constant level  $y^T$  (i.e.,  $y_t^T = y^T$  for all  $t$ ) and its domestic price will be determined by the law

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<sup>15</sup>The government's seignorage revenue  $\frac{\dot{M}_t}{P_t^T E_t}$  is equal to  $\dot{m}_t + \epsilon_t m_t$  in real term (in terms of tradable goods).

<sup>16</sup>The appropriate transversality condition is  $\lim_{t \rightarrow \infty} h_t \exp(-rt) = 0$ .

of one price.

In this model, it is assumed that the non-tradable good sector is imperfectly competitive and there is a continuum of imperfectly competitive firms indexed by  $z \in [0, 1]$ . Each producer produces a differentiated good and acts as a monopolistic competitor, choosing the nominal price and the level of production of the good.

Given the CES non-tradable consumption index, Eq.(2.2.2), an individual's demand for non-tradable good  $z$  in period  $t$  is

$$C_t^N(z) = \left( \frac{p_t^N(z)}{P_t^N} \right)^{-\theta} C_t^N, \quad (2.15)$$

where  $\theta$  is the price elasticity of demand.

According to Eq.(2.2.15), a monopolistically competitive firm that produces a non-tradable goods faces the downward-sloping curve

$$y_t^{Nd}(z) = \left[ \frac{p_t^N(z)}{P_t^N} \right]^{-\theta} C_t^{NA}, \quad (2.16)$$

where  $C_t^{NA} dz = \int_0^1 C_t^N dz = C_t^N$  is aggregate per capita non-tradable good consumption.

Labor is the only input into production. The technology available to firms is identical and is linear in labor:

$$y_t^N(z) = l_t. \quad (2.17)$$

Firm  $z$ 's profit is

$$\Pi_t(z) = p_t(z) y_t^d(z) - W_t l_t. \quad (2.18)$$



At any time  $t$ , a monopolistically competitive firm maximizes profit, Eq.(2.2.18), subject to the demand function, Eq.(2.2.16), and the production function, Eq.(2.2.17). Solving the firm's problem:

$$\frac{\theta - 1}{\theta} y_t^{-\frac{1}{\theta}}(z) P_t^N C_t^{\frac{1}{\theta}} = W_t. \quad (2.19)$$

Since the elasticity of demand,  $1/\theta$ , of all non-tradable goods is identical and every producer has the same technology, they produce the same level of output and set the same price. Therefore, for any two producers,  $0 < z < z' < 1$

$$\begin{aligned} y_t^N(z) &= y_t^N(z') = y_t^N \\ p_t^N(z) &= p_t^N(z') = p_t^N. \end{aligned} \quad (2.20)$$

It follows that the non-tradable good's aggregate supply and price index simplify to

$$\begin{aligned} y_t^{AN} &= \int_0^1 y_t^N(z) dz = y_t^N \\ P_t^N &= \left[ \int_0^1 (p_t^N(z))^{1-\theta} dz \right]^{\frac{1}{1-\theta}} = p_t^N. \end{aligned} \quad (2.21)$$

The equilibrium in a non-tradable good's market can be obtained by Eq.(2.2.16) and Eq.(2.2.21)

$$y_t^N = C_t^N. \quad (2.22)$$

Therefore, from Eq.(2.2.19)

$$p_t^N(z) = \frac{\theta}{\theta - 1} W_t, \forall z. \quad (2.23)$$

Equation (2.2.23) shows that a producer with market power sets prices above marginal cost, which is the money wage rate. Thus, if it cannot adjust its price, it is

willing to produce to satisfy demand in the face of fluctuations in demand. Therefore, in the short-run, if the price is sticky, output will be demand-determined.

### 2.3.4 Equilibrium conditions and a symmetric steady state

In this paper, perfect capital mobility is assumed. It implies that

$$i_t = r + \epsilon_t. \quad (2.24)$$

Equilibrium in the non-tradable goods market implied by Eq.(2.2.22) is

$$y_t^N = C_t^N.$$

In order to investigate the dynamic behavior of the current account triggered by a policy change, consider the economy's dynamic resource constraint. Combining the household's dynamic budget constraint with the profit of the firm and the market clearing condition for the non-tradable goods sector yields the economy's intertemporal budget constraint:

$$\dot{k}_t = rk_t + y^T - c_t^T, \quad (2.25)$$

where  $k_t = b_t + h_t$  is the total amount of foreign bonds held by the economy. Equation (2.2.25) represents the economy's current account balance. It indicates that the current account balance is the difference between tradable goods income and consumption of tradable goods.

Combining Equations (2.2.7), (2.2.22), (2.2.14) and (2.2.24) yields the economy's intertemporal constraint

$$k_0 + \int_0^\infty y^T \exp(-rt) dt = \int_0^\infty C_t^T \exp(-rt) dt, \quad (2.26)$$

where  $k_0$  denotes the economy's initial stock of foreign bonds. Equation (2.2.26) states that the initial stock of foreign bonds plus the present value of all future tradable output must equal the present value of tradable consumption. At the steady state,  $C_t^T = C_{ss}^T$ . Therefore,  $C_{ss}^T = y^T + rk_0$  and the current account is balanced at the steady state.

In the steady state, all prices are fully flexible. Thus the symmetric equilibrium implies

$$C_t^N = y_t^N = C_t^{AN}, \forall z. \quad (2.27)$$

In the steady state where all prices are fully flexible, the supply condition determines the level of non-tradable output. It means that the long-run level of output is totally dependent on labor supply. Using Equations (2.2.12), (2.2.23) and (2.2.27), we get the symmetric steady state level of output of non-tradables

$$y_{ss}^N = [1 + (\frac{\rho}{1-\gamma})(\frac{\theta}{\theta-1})(1 + \alpha i_{ss})]^{-1}, \quad (2.28)$$

where  $i_{ss} = r + \epsilon_{ss}$  is the steady state level of the nominal interest rate. In the steady state, all prices are fully flexible and all exogenous variables are constant. Therefore, the long-run level of non-tradable output is determined by the supply side. According to Eq.(2.2.26), however, we can see that non-tradable output depends on the nominal interest rate, which is also a function of the monetary policy variable. The reason is as follows. Since the nominal interest rate introduces a distortion between consumption and leisure, if there is a permanent decrease in the nominal interest rate, the household will substitute leisure for consumption due to a lower effective price of consumption. This, in turn, leads to a rise in the

long-run level of non-tradable output. This result is consistent with the disinflation scenario of Cooley and Hansen (1989, 1995) in a closed economy model in which variations in the rate of inflation can affect the steady state labor supply and consumption (or output).

In this model, each producer has monopoly power, so the long-run level of non-tradable output is lower than the socially optimal level. To see this, let's look at the social planner's problem. The social planner tries to maximize the utility of non-tradable consumption considering the disutility from labor supply.<sup>17</sup>

$$\max[(1 - \gamma) \log y^N - \rho \log(1 - y^N)]. \quad (2.29)$$

The solution is

$$y_s^N = \left( \frac{1 - \gamma}{1 - \gamma + \rho} \right) > y_{ss}^N. \quad (2.30)$$

From Eq.(2.2.30), we can see that the equilibrium output of non-tradables is inefficiently low under imperfect competition. This fact has important implications for economic fluctuations as well as monetary policy to stabilize it, as long as prices are not perfectly flexible because of menu costs or staggered price or wage contracts. Since the market equilibrium is lower than the socially optimal level, recessions and booms have asymmetric effects on welfare (Mankiw, 1985). Since equilibrium output is less than the socially optimal level, a boom brings output closer to the social optimum, whereas a recession in the non-tradable sector pushes it farther away. In this model, however, the long-run equilibrium level of output is subject to a monetary policy variable. Therefore, a change in monetary policy

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<sup>17</sup>Remember that by the production condition  $y^N = l$ .

can lead the economy closer to the optimum level.

### 2.3.5 Staggered wage contract and short-run dynamics

So far, we have assumed flexible prices and solved for the long-run equilibrium. We now introduce the assumption that the price level of the non-tradable good is sticky because of staggered wage contracts, and we are ready to consider the short-run dynamics of the economy. In the short-run, the supply of non-tradable goods will be demand determined and prices will be given by a modified version of the staggered-price model of Calvo (1983) and Ghezzi (2001), including backward-looking indexation consistent with our discussion in Section 2.1. We assume nominal wages are sticky as a result of staggered wage contracts.

Following Calvo (1983), we assume that each individual can change wages only when a wage signal is received. If individuals do not change the wage or is setting a new wage at time  $t$ , the probability (density) that the wage will last for  $s$  more periods is given by the geometric distribution

$$\delta \exp(-\delta s) \tag{2.31}$$

and is, therefore, independent of  $t$  and of the amount of time the wage has lasted at  $t$ . It is also stochastically independent across the individual wage setters. The expected length of wage duration is  $1/\delta$ . Therefore, nominal wages specified by the contract will be fixed for the duration of the contract and the contract itself is staggered.

Then the aggregate (log of) newly posted wage level follows

$$w_t = \delta \int_{-\infty}^t x_s \exp(-\delta(t-s)) ds, \quad (2.32)$$

where the wage set at time  $s$ ,  $x_s$ , is weighted by the probability that it continues in effect at  $t$ ,  $\exp(-\delta(t-s))$ .

For analytical simplicity, we assume that at any time  $t$  there are two types of wage contractors: half of the individual contractors set their nominal wage in a purely forward looking manner and half of them set in a backward-looking manner. By this mechanism, we can easily incorporate backward-looking indexation, which is proposed by Taylor (1980), into the Calvo model.<sup>18</sup>

The forward-looking wage contractor will set wages according to the standard Calvo model

$$x_t^F = \delta \int_t^{\infty} [w_s + \phi(\log c_s^N - \log y_{ss}^N)] \exp(-\delta(s-t)) ds, \quad (2.33)$$

where  $\phi$  reflects sensitivity of contract wages to future excess demand conditions ( $\log c_s^N - \log y_{ss}^N$ ), where  $y_{ss}^N$  is the steady state level of non-tradable output. According to Eq.(2.2.33), we can see that when setting a contract, forward-looking agents will take into account future average wage level and excess demand during the length of contract.

For the other half of the contractors, wages are assumed to be set according to a backward-looking rule. Specifically, we assume that the wage contract at  $t$

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<sup>18</sup>In the original Taylor models, current wage is set according to the past wage level and the expected future wage rate including past and future excess demand condition. Thus, Taylor models can generate wage level inertia.

indexes money wages to the current aggregate wage level plus a weighted average of past inflation and current excess demand conditions

$$x_t^B = w_t + \frac{1}{\delta}\eta_t + \varphi(\log c_t^N - \log y_{ss}^N), \quad (2.34)$$

where  $\varphi > 0$  reflects the sensitivity of the current money wages to the current level of excess demand, and

$$\eta_t = \delta \int_{-\infty}^t \pi_s \exp(-\delta(t-s)) ds \quad (2.35)$$

$$\dot{\eta}_t = \delta(\pi_t - \eta_t). \quad (2.36)$$

Equation (2.2.34) indicates that backward-looking wage contracts at  $t$  index current nominal wages to the current aggregate wage level, a weighted average of past inflation  $\eta_t$ , adjusted by the expected length of the price duration,  $1/\delta$ , and the current level of excess demand.<sup>19</sup>

Therefore, the log of wage set at time  $t$  is

$$x_t = \frac{1}{2}x_t^F + \frac{1}{2}x_t^B. \quad (2.37)$$

Since price is a fixed markup over wage, there is no distinction between wage and price inflation in this model. Therefore, we can differentiate Eq.(2.2.32) with respect to time to get<sup>20</sup>

$$\pi_t = \dot{w}_t = \delta[x_t - w_t]. \quad (2.38)$$

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<sup>19</sup>In equation (2.2.36), the adjustment parameter is not necessarily equal to  $\delta$ . It could be  $\varsigma \neq \delta$  and, therefore  $\eta_t = \varsigma \int_{-\infty}^t \pi_s \exp(-\delta(t-s)) ds$ . For simplicity, we just assume  $\varsigma = \delta$ . See Ghezzi (2001).

<sup>20</sup>  $\pi_t = \log \dot{P}_t^N = \log \dot{W}_t$ .

Combining Equations (2.2.33), (2.2.34) and (2.2.37), we obtain

$$x_t = \frac{1}{2}[w_t + \frac{1}{\delta}\eta_t + \varphi(\log c_t^N - \log y_{ss}^N)] + \frac{1}{2}[\delta \int_t^\infty (w_s + \phi(\log c_s^N - \log y_{ss}^N)) \exp(-\delta(s-t)) ds]. \quad (2.39)$$

Differentiating (2.2.39), we obtain the changes in newly set wage rate

$$\dot{x}_t = 2\pi_t - \eta_t - \frac{\delta}{2}(\varphi + \phi)(\log c_t^N - \log y_{ss}^N) + \frac{1}{2}\varphi \log \dot{C}_t^N. \quad (2.40)$$

In order to obtain non-tradable inflation dynamics, differentiate (2.2.39) with respect to time and substitute it into (2.2.40)

$$\dot{\pi}_t = \delta(\pi_t - \eta_t) - \frac{\delta^2}{2}(\varphi + \phi)(\log c_t^N - \log y_{ss}^N) + \frac{1}{2}\delta\varphi \log \dot{C}_t^N. \quad (2.41)$$

This is the equation for the inflation dynamics. Eq.(2.2.41) indicates that a change in the rate of inflation is dependent on two sources. First, it depends on excess demand conditions: the change of inflation is negatively related to excess demand. The second term of (2.2.41) shows this relation. This is the same result as Calvo and Végh (1993,1994). The intuition for this higher order inverse Phillips curve is that if there is excess demand at  $t$ , individuals who revise their wages at time  $t$  set higher wages. Therefore, the higher excess demand at time  $t$ , the higher the rate of inflation will be. However, since agents set their nominal wages in a forward-looking manner, wage setters at  $s > t$  do not take excess demand at time  $t$  into account. Hence, the higher the excess demand at  $t$ , the greater the reduction in the inflation rate for  $s > t$  will be.

In equation (2.2.41), we can see that there is another source for the change in the rate of inflation. It shows that current inflation depends on a weighted average



of past inflation. Especially, when averaging past inflation, the inflation rate in the recent past receives more weight, which is the same formulation as adaptive expectations. If current inflation is lower than  $\eta_t$ , an average of past inflation, the newly contracted wage has a lower premium over the current price level, which will be translated into lower inflation. This makes inflation sticky. The first term on the right-hand side of (2.2.41) represents this relation. The backward-looking wage contract mechanism implies that inflation has its own persistence in addition to the inertia in the driving term, which is the change in the level of excess demand. By this specification, we can overcome the inability of the standard new-Keynesian contract model to generate significant persistence of inflation. The change of aggregate demand in the non-tradable good sector affects inflation positively. This is the original Phillips curve mechanism. In this regard, equation (2.2.41) can be interpreted as a traditional Expectations-Augmented Phillips curve.

In this model, there is another source for inflation dynamics. In contrast to a constant steady-state level of non-tradable output as in Calvo and Végh models, the long-run level of non-tradable output is subject to a nominal variable, since the nominal interest rate introduces a wedge between consumption and labor supply. Therefore, non-tradable output shows different dynamics with respect to the policy change in this model.

In order to analyze the dynamic system of the economy, we will need to obtain the equilibrium path of consumption of tradables and non-tradables. From

Equations (2.2.8), (2.2.11) and (2.2.30), we can see that

$$C_t^T = \frac{\gamma}{\lambda}(1 + \alpha i_t)^{-1} \quad (2.42)$$

$$C_t^N = \frac{1 - \gamma}{\gamma} e_t C_t^T \quad (2.43)$$

and

$$\lambda = \frac{\gamma \int_0^\infty (1 + \alpha i_t)^{-1} \exp(-rt) dt}{k_0 + \frac{1}{r} y^T} \quad (2.44)$$

is the shadow value of wealth, which is constant as long as there are no policy shocks to the economy. From Equations (2.2.42) and (2.2.43), we can obtain that

$$\log C_t^T = \zeta - \alpha i_t \quad (2.45)$$

$$\log C_t^N = \log \frac{1 - \gamma}{\gamma} + \log C_t^T + \log e_t, \quad (2.46)$$

where  $\zeta = \frac{\gamma}{\lambda}$  is constant.

Differentiating (2.2.45) with (2.2.46) yields

$$\log \dot{C}_t^N = \log \dot{e}_t - \alpha \dot{i}_t, \quad (2.47)$$

By definition,  $e_t = \frac{E_t P^T}{P_t^N}$ . Therefore,

$$\log \dot{e}_t = \epsilon_t - \pi_t, \quad (2.48)$$

Using the fact that  $\dot{i}_t = \dot{\pi}_t$ , we can obtain

$$\log \dot{C}_t^N = (\epsilon_t - \pi_t) - \alpha \dot{\pi}_t. \quad (2.49)$$

In the short-run where all prices are sticky, the equilibrium level of employment is demand-determined. Therefore, non-tradable output is also demand-determined.

It implies

$$\log y_t^N = \log C_t^N. \quad (2.50)$$

Combining Equations (2.2.22), (2.2.41) and (2.2.44), we can obtain the differential equation governing the change of the rate of inflation

$$\dot{\pi}_t = \frac{2\delta}{2 + \alpha\delta\varphi}(\pi_t - \eta_t) + \frac{\delta\varphi}{2 + \alpha\delta\varphi}(\epsilon_t - \pi_t) - \frac{\delta^2(\varphi + \phi)}{2 + \alpha\delta\varphi}(\log y_t^N - \log y_{ss}^N). \quad (2.51)$$

Then, the change of non-tradable good can be obtained as

$$\log \dot{y}_t^N = -\frac{2\alpha\delta}{2 + \alpha\delta\varphi}(\pi_t - \eta_t) + \frac{2}{2 + \alpha\delta\varphi}(\epsilon_t - \pi_t) + \frac{\alpha\delta^2(\varphi + \phi)}{2 + \alpha\delta\varphi}(\log y_t^N - \log y_{ss}^N). \quad (2.52)$$

Therefore, the dynamics of the economy consist of a three-equation system (Equations. (2.2.36), (2.2.51), and (2.2.52)), in output of non-tradables,  $y_t^N$ , inflation of non-tradables,  $\pi_t$ , and  $\eta_t$ , a weighted average of past inflation.

## 2.4 Money-based stabilization

This section studies the effects of stabilization policies. We will examine the effects of money-based stabilization in which the government lets the exchange rate float and uses the money supply as the nominal anchor. First, we consider a credible stabilization plan in which the reduction of the growth rate of money supply is believed to be permanent. Next, the effect of a temporary reduction in the money growth rate is considered.

Before we proceed, we need to investigate the relationship between the growth rate of the money supply and the devaluation rate. According to the definition of real money balances ( $m_t = M_t/E_t P^T$ ), the path of real money balances is governed by

$$\frac{\dot{m}_t}{m_t} = \mu_t - \epsilon_t \quad (2.53)$$

where  $\mu_t (\equiv \dot{M}_t/M_t)$  is the growth rate of the money supply. Eq.(2.3.1) is an unstable differential equation. Unless  $\mu_t = \epsilon_t$ , real money balances are ever increasing or decreasing. To ensure stability, the devaluation rate,  $\epsilon_t$ , needs to be equal to the growth rate of the money supply. It implies that under a flexible exchange rate regime, the devaluation rate,  $\epsilon_t$ , will adjust instantaneously to its new value when the growth rate of money supply,  $\mu_t$ , changes.

Now we will introduce a money-based stabilization policy. Suppose that, prior to any stabilization (for  $t < 0$ ), the growth rate of the money supply is  $\mu^H$ , and is expected to remain at that level forever. For a given money supply growth rate, the economy is at a steady state characterized by

$$\begin{aligned}
 i_{ss} &= r + \mu^H \\
 \pi_{ss} &= \eta_{ss} = \mu^H \\
 C_{ss}^N &= y_{ss}^N((i_{ss})) = [1 + (\frac{\rho}{1-\gamma})(\frac{\theta}{\theta-1})(1 + \alpha i_{ss})]^{-1} \\
 C_{ss}^T &= y^T + rk_0 \\
 e_{ss} &= (\frac{\gamma}{1-\gamma}) \frac{C_{ss}^N}{C_{ss}^T}.
 \end{aligned} \tag{2.54}$$

At a steady state, consumption of tradable goods is equal to its permanent income level, while consumption of non-tradables equals its full-employment level. The full employment level of non-tradables is less than the socially efficient level due to the imperfection in the non-tradable sector. However, this full-employment level of output is not constant. Monetary policy which affects the nominal interest rate can change the long-run level of non-tradable output. Nominal variables (the interest rate and the rate of inflation) are growing at the money supply's growth rate,  $\mu^H$ . The real exchange rate is equal to the ratio of the steady state consumption of non-tradables to tradables.

To examine the transitional adjustment of the economy, we have to consider

the dynamic system. The system is characterized by the differential equations (2.2.41), (2.2.51), and (2.2.52), with  $\mu_t = \epsilon_t$ . Since the system is linear, we do not need any approximations.

The dynamic system for  $\log y$ ,  $\pi$  and  $\eta$  is

$$\begin{bmatrix} \log \dot{y}_t^N \\ \dot{\pi}_t \\ \dot{\eta}_t \end{bmatrix} = \begin{bmatrix} \frac{\alpha\delta^2(\phi+\varphi)}{\chi} & -\frac{2(1+\alpha\delta)}{\chi} & \frac{2\alpha\delta}{\chi} \\ \frac{\delta^2(\phi+\varphi)}{\chi} & \frac{\delta(\varphi-2)}{\chi} & -\frac{2\delta}{\chi} \\ 0 & -\delta & \delta \end{bmatrix} \times \begin{bmatrix} \log y_t^N - \log y_{ss}^N \\ \pi_t - \mu_t \\ \eta_t - \mu_t \end{bmatrix}, \quad (2.55)$$

where  $\chi = 2 + \alpha\delta\varphi > 0$ .

Now, we need to investigate the stability of the system. It exhibits two convergent (non-positive) roots and one explosive (non-negative) root (see Appendix A.2.1). At time  $t$ , current inflation,  $\pi_t$ , is given by an average of past inflation, current excess aggregate demand and future economic conditions, including future inflation. Therefore,  $\pi_t$  becomes a jump variable at time  $t$  even if it cannot adjust immediately to a new value. The partial forward-looking indexation (half of households set the new wage in a forward-looking manner) makes inflation jump initially to the point between the new steady state and the initial steady state after a shock.  $\log y_t^N$  is also a jump variable (notice that by the equilibrium condition,  $\log y_t^N = \log c_t^N$ ). However, as one can see in (2.2.35),  $\eta_t$  is a predetermined variable. Thus, there is one predetermined variable ( $\eta$ ) and two jump ( $\pi$  and  $\log y^N$ ) variables. However, since the jump of  $\log y^N$  is exogenous to the solution of (2.3.3), the system exhibits saddle path stability.<sup>21</sup> Also the roots may

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<sup>21</sup>See, Ghezzi (2001).

be complex conjugates, and the system may show cyclical dynamics.<sup>22</sup>

Since the system has three variables, it will be difficult to analyze the equilibrium paths. However, it is possible to see the exact transitional dynamics of the system by resorting to the methods of dominant eigenvalue proposed by Calvo (1987). The system has two non-positive roots and one non-negative root. Let  $\nu_i, i = 1, 2$ , be the non-positive roots and  $\nu_3$  be the non-negative root, with  $\nu_1 > \nu_2$  ( $\nu_1$  is the dominant eigenvalue). Setting to zero the constant corresponding to the unstable root ( $\nu_3$ ),<sup>23</sup> the solution to dynamic system (2.3.3) can be expressed as

$$\begin{aligned}\log y_t^N - \log y_{ss}^N &= A_1 \omega_{11} \exp(\nu_1 t) + A_2 \omega_{21} \exp(\nu_2 t) \\ \pi_t - \mu_t &= A_1 \omega_{12} \exp(\nu_1 t) + A_2 \omega_{22} \exp(\nu_2 t) \\ \eta_t - \mu_t &= A_1 \omega_{13} \exp(\nu_1 t) + A_2 \omega_{23} \exp(\nu_2 t),\end{aligned}\tag{2.56}$$

where  $A_i, i = 1, 2$ , and  $\omega_{ij}, j = 1, 2, 3$ , denote the constants and the element of the eigenvector associated with root  $\nu_i$ .

From the assumption  $\nu_1 > \nu_2$ , it follows that (see Appendix A.2.2):

$$\lim_{t \rightarrow \infty} \frac{\log y_t^N - \log y_{ss}^N}{\pi_t - \mu} = \frac{\omega_{11}}{\omega_{12}} > 0.\tag{2.57}$$

This implies that as  $t$  becomes large, if the initial value of  $\log y^N$  and  $\pi$  are not on the ray corresponding to solution,  $\nu_2$ , then the ratio of  $\frac{\log y_t^N - \log y_{ss}^N}{\pi_t - \mu}$  will converge to the slope of the dominant eigenvector ray ( $\frac{\omega_{11}}{\omega_{12}}$ ), which is positive. It implies that, graphically, the system will converge asymptotically to the dominant

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<sup>22</sup>For more detail, see Blanchard and Khan (1980) and Turnovsky (1995).

<sup>23</sup>It is clear from the transversality condition that a necessary condition for convergence of the system is that  $\nu_3 = 0$ .

eigenvector ray. Figure 2.1 shows the phase diagram for  $\log y_t^N$  and  $\pi_t$  with the dominant eigenvector ray.

### 2.4.1 Permanent reduction in the growth rate of money supply

We, now, consider the impact of a once-and-for-all reduction in the growth rate of the money supply. This case corresponds to the credible disinflation policy. Suppose that at time  $t < 0$ , the economy is at the initial steady state. In terms of Figure 2.1, the initial steady state is at point  $SS_0$ . At time 0, the policy-maker announces that he or she will reduce the growth rate of the money supply permanently, which is unanticipated. Formally, the policy is

$$\mu_t = \mu^H, \text{ for } t \leq 0$$

$$\mu_t = \mu^L, \text{ for } t > 0.$$

On impact, the nominal exchange rate adjusts instantaneously to its lower value.<sup>24</sup> The nominal appreciation leads to an appreciation of the real exchange rate since the price level of non-tradables is sticky. The nominal interest rate also jumps to a new level,  $(r + \mu^L)$  by the same amount as the money growth rate. Now it is easy to see the equilibrium path of the main variables. Since the nominal interest rate remains at the new level, we can see from Eq.(2.2.44) that

$$\lambda = \frac{\gamma}{rk_0 + y^T} (1 + \alpha i)^{-1}. \quad (2.58)$$

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<sup>24</sup>On impact, real money balances must increase to equilibrate money market. This can only be achieved with instantaneous appreciation of nominal exchange rate.

Combining (2.2.42) and (2.3.6), we obtain

$$C^T = rk_0 + y^T. \quad (2.59)$$

The consumption of tradable goods is constant over time and equal to its permanent income level. The reason is as follows: even if the nominal interest rate is reduced, it is constant over time at the lower level, which implies a constant effective price of tradable consumption at the lower level. Hence, the consumer does not have any incentives to engage in intertemporal consumption substitution. This implies that the current account does not change at all during the transition to the new steady state. The path of consumption of non-tradables can be derived from Eq.(2.2.43) and the path of the real exchange rate. Since the consumption of the tradable good continues to be equal to permanent income, consumption (or output) of non-tradable goods falls on impact as the real exchange rate appreciates. Inflation also falls on impact. However, the inflation rate will jump to a point between  $\mu_H$  and  $\mu_L$ . This is due to the backward-looking indexation of wage contracts even if the forward-looking contracts make it possible that the inflation rate jumps on impact.<sup>25</sup> The impact effects of the reduction in the money growth rate on inflation and non-tradable output is illustrated as the movement from  $SS_0$  to  $A$  in Fig 2.2.

After the initial impact, the supply-side effects of disinflation come in through the change in the labor supply. The reduction in the nominal interest rate implies a reduction in the monetary wedge (or distortion), generated by the cash-in-advance

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<sup>25</sup> According to Ghezzi (2001), the size of jump of the inflation rate depends negatively on the degree of backward-looking indexation.



constraint, between consumption and leisure. Thus, the individual household increases labor supply, which leads non-tradable output to expand. Since the greater labor supply implies more labor income, the household will increase demand for non-tradables. This permanent increase in labor supply triggers a positive wealth effect on non-tradable output. At  $A$ , non-tradable output starts to increase and inflation keeps falling slowly. Finally, they converge to their new steady state ( $SS_1$ ).

The dynamic path of the main variables we are concerned with is depicted in Figure 2.3 to 2.8. The first thing we have to notice in this experiment is the movement of inflation. We observe that inflation shows inertia. This result is quite consistent with a stylized fact in money-based stabilizations described by Agénor and Montiel (1999), and Calvo and Végh (1999). In this model, however, inflation persistence can be observed even if the disinflation policy is fully credible. This result implies that backward-looking indexation is an important factor in explaining inflation inertia in stabilization programs. Another intriguing phenomenon is that inflation shows undershooting during the transition. Before it reaches the new level, inflation is below the growth rate of money supply for some period of time. The reason is as follows: on impact, the real exchange rate appreciates. To achieve a high level of non-tradable output (or consumption), the real exchange rate should be higher (real depreciation) in the new steady state than in the initial steady state as

$$e_{ss1} = \frac{y_{ss1}^N}{rk_0 + y^T}. \quad (2.60)$$

For a subsequent real depreciation, some period of inflation below  $\mu_L$  is required.

This result follows from the fact that

$$\dot{e}_t = e_t(\mu_t - \pi_t). \quad (2.61)$$

The path of the real exchange rate is depicted in Figure 2.5. Inflation undershooting can be understood more easily if the money market equilibrium condition is investigated. Let  $n_t = M_t/P_t^N$  be the real money balance in terms of non-tradables. Then, the dynamic path of real money balances is

$$\dot{n}_t = n_t(\mu_t - \pi_t). \quad (2.62)$$

After the reduction in the money growth rate, real money balances (in terms of non-tradables) start to decrease as the money growth rate is lower than the inflation rate. This result is consistent with the fact of lower non-tradable output. However, the rise in real money balances is necessary to accommodate the increase in non-tradable consumption. This is achieved by inflation undershooting. The time path of real money balances is illustrated in Figure 2.8.

As pointed out in Section 2.2, exchange rate based programs are preferred to money based programs since a period of deflation may appear in money-based stabilizations. Thus, policymakers will engineer a one time increase in the money supply to avoid deflation, which weakens the credibility of the policy maker. The resulting inflation undershooting effect in this model eliminates the possibility of an increase in the money supply during the disinflation program.

In this paper, we have succeeded in producing another stylized fact of money-

based stabilizations. From the above simulation, non-tradable output (or consumption) shows a sharp and short-lived contraction after the implementation of a money-based program. This result is broadly consistent with empirical and theoretical regularities documented by Calvo and Végh (1999) and Fischer (1986). It also fares well with the notion that disinflation is contractionary in industrial countries (Gordon, 1982). The distinguishing feature of this model compared to the previous literature is that disinflation produces a sustained expansion through the labor supply effect even if inflation shows persistence. This result is also consistent with the evidence of the successful exchange rate-based stabilizations.

It should be noted that in this model, inflation stabilization proves to be welfare-improving: the reduction in inflation is beneficial in cash-in-advance models with a labor-leisure choice and imperfect competition in the goods market. Lower inflation reduces the transaction cost of holding money. Hence it allows individuals more free time for productive activities: more consumption and production. In a broad sense, disinflation is welfare improving since socially unproductive efforts to conserve on money balances and the inefficiency of the imperfect competition can be eliminated.

#### **2.4.2 Temporary reduction in the growth rate of money supply**

We now turn to investigate a temporary stabilization experiment. This corresponds to the case of lack of credibility: the authority announces a permanent reduction in the growth rate of money supply, but the public believes that the money growth

rate will go back to its original level after a certain period of time. The initial steady state corresponds to a money growth rate  $\mu_t = \mu^H$ . At  $t = 0$ , the money growth rate is set to a lower level,  $\mu^L$ , but at time  $T$ , it is increased back to its original level. More specifically, for some  $T > 0$ ,

$$\begin{aligned}\mu_t &= \mu^L, & 0 \leq t < T \\ \mu_t &= \mu^H, & t \geq T\end{aligned}$$

where  $\mu^H > \mu^L$ .

From Eq.(2.3.1), we can see that during the transition (the interval  $[0, T)$ ), the rate of devaluation is lower than after  $T$ . Perfect capital mobility implies that the nominal interest rate is also relatively low during the transition. Therefore, on impact consumption of the tradable jumps up and remains constant during the transition. At  $T$ , tradable consumption decreases. We can see this from (2.2.42). The reason is as follows: since the consumer expects that the effective price of non-tradables (market price and the opportunity cost of holding money) will be lower between 0 and  $T$  than after  $T$ , he or she will engage in intertemporal consumption substitution. Hence, non-tradable consumption will be higher during the interval  $[0, T)$  and lower after  $T$ .

The economy's intertemporal budget constraint implies that before the stabilization policy is reversed, consumption of tradable goods is higher than permanent income, while consumption of non-tradables is lower than initial permanent income after  $T$ .

This implies

$$\begin{aligned} C_H^T &> y^T + rk_0, & 0 \leq t < T \\ C_L^T &< y^T + rk_0, & t \geq T. \end{aligned} \quad (2.63)$$

The time path of the current account is given by Eq.(2.2.25). The increase in the consumption of tradables over permanent income induces the current account to jump into deficit. During the transition, the current account deficit will grow steadily since the interest income on net foreign asset declines. When the stabilization is abandoned,  $C_t^T$  falls below the permanent income level and brings the current account into balance.

We now investigate the behavior of non-tradable output and inflation. Since the real exchange rate appreciates on impact, it is difficult to determine the initial effect of the temporary disinflation on non-tradable consumption. We can see this from Eq.(2.2.43): the movement of the real exchange rate and tradable output (induced by the change in the nominal interest rate) go in opposite directions. However, by investigating the money market, we can establish that non-tradable output jumps down on impact. From the definition of real money balances (in terms of tradables and non-tradables), we can see<sup>26</sup>

$$n_t = e_t m_t. \quad (2.64)$$

On impact, real appreciation reduces real money balances in terms of non-tradable goods.

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<sup>26</sup>From the definition, real money balances in terms of tradable and non-tradable goods can be expressed as  $m_t = \frac{M_t}{E_t P_t^T}$ ,  $n_t = \frac{M_t}{P_t^N}$ . Therefore,  $n_t = e_t m_t$ .

Substituting (2.3.10) with (2.2.45) into the cash-in-advance constraint (2.2.6), we obtain

$$n_t = \frac{\alpha}{1 - \gamma} C_t^N. \quad (2.65)$$

Therefore, initially, non-tradable consumption jumps down as in the permanent disinflation case. Inflation also falls on impact. However, the size of the initial jump of inflation will be smaller than the permanent case. This result is due to the lack of credibility. The public believes that the money growth rate will revert to the original level which is higher than it is now. The initial effect will be a move from  $SS_0$  to  $A$  in Fig 2.9.

After the initial impact, non-tradable output (or consumption) starts to increase. For  $0 < t < T$ , the nominal interest rate will be lower than after  $t \geq T$ . Households supply more labor to the non-tradable sector during the transition than after the policy reverts to the original level. Therefore, non-tradable output will be higher during the transition. This is due to the intertemporal substitution effects of labor supply. Inflation also begins to increase as the public expects higher inflation in the future. The expected inflation rate will be incorporated into wage contracts, especially into forward-looking contracts. This induces the rate of inflation to keep increasing.

During the transition, for  $0 < t < T$ , the system will be governed by the dominant divergent path (See Appendix A.2.3).

$$\lim_{t \rightarrow \infty} \frac{\log y_t^N - \log y_{ss1}^N}{\pi_t - \mu_H} = \frac{\omega_{31}}{\omega_{32}}. \quad (2.66)$$

In this model, the sign of  $\frac{\omega_{31}}{\omega_{32}}$ , the slope of the divergent path, is ambiguous [see (A.2.11)]: it can be positive or negative. However, even if the slope is positive, it is flatter than the dominant eigenvector ray (in the new steady state). Therefore, the transitional dynamic of the system will not be affected by the indeterminacy. Figure 2.11 and 2.12 depict the transitional adjustment of inflation and non-tradables. The direction of motion is indicated by the arrows.<sup>27</sup>

At  $T$ , the growth rate of the money supply increases back to  $\mu^H$ . The economy now converges to the dominant eigenvector ray and moves to the original steady state. As the nominal interest rate goes back to the original level, the households decrease their labor supply. Thus, non-tradable output (or consumption) also shrinks and recession sets in. Since the policy is reversed at  $T$ , expectations adjust and inflation starts to fall, converging to its original rate. This result highlights the importance of policymaker credibility. The credibility of policy will affect people's expectations, and produce unwanted results.

The dynamic path of the main variables is depicted in Figure 2.10 to 2.15. One of the distinguishing features of this policy relative to the credible one is that the temporary policy induces a deterioration of the current account. The movement of tradable consumption and current account dynamics are illustrated in Figures 2.14 and 2.15.<sup>28</sup>

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<sup>27</sup>In Figure 2.3 we assume that the divergent path is negatively sloped. However, as is mentioned in the main text, we observe the same transitional path under a positively sloped divergent path.

<sup>28</sup>Figures do not include the transitional adjustment of the nominal exchange rate and real money balances. However, it is not difficult to see if we consider the behavior of inflation and the real exchange rate.

The behavior of non-tradables and inflation are similar to Calvo and Végh's (1993) under the incredible policy scenario. In their model, a temporary reduction in the rate of growth of the money supply induces an initial decrease in consumption of non-tradable and inflation. After the initial impacts, inflation starts to increase and converges slowly to its original level. The consumption of non-tradables also shows a similar dynamic adjustment. The initial recession is followed by an expansion of the non-tradable sector to its original level. In this simulation, however, the transitional dynamics show a little difference: for non-tradable output, we observe "boom-recession" cycle after initial recession, and inflation shows overshooting. This is mainly due to the supply-side effect and backward-looking indexation, which are not considered in Calvo and Végh (1993).

## 2.5 Concluding remarks

The main objective of this study is to explain the stylized facts of the recent money-based stabilization programs within a New-Keynesian framework. We extend Obstfeld and Rogoff's (1995) dynamic general equilibrium Mundell-Fleming model to allow for sticky inflation. This is achieved by Ghezzi's (2001) staggered contract model including backward-looking wage indexation.

We used the model to study money-based stabilizations. Our model produces inflation inertia, which is the main stylized fact of inflation stabilization (both money and exchange rate based). Backward-looking indexation prevents inflation



from converging rapidly to the new nominal anchor. This result implies that slow convergence of the rate of inflation cannot be avoided unless backward-looking indexation is eliminated.

Another important result is that the model successfully replicates the “recession-boom cycle” in the credible disinflation scenario. We achieve this result by incorporating imperfect competition and endogenous labor supply into a small open economy, high inflation environment. The initial contraction after the stabilization attempt is typical of disinflation programs when the money growth rate is used as a nominal anchor [Fischer (1986) and Gordon (1982)]. In this exercise, an initial recession is observed after disinflation is implemented. Moreover, the credible reduction in the money growth rate is capable of replicating the sustained expansion of the domestic sector, which is observed in the successful exchange rate based disinflation. This result is quite consistent with the disinflation scenario in the closed economy model in Cooley and Hansen (1989,1995). In this respect, our study is quite successful.

In this paper, the resulting dynamic behavior of inflation and non-tradable output under the permanent disinflation programs provides a counter argument to the “temporariness hypothesis” of Calvo and Végh (1993, 1994). Our result suggests that the main causes of inflation inertia and the associated business cycle may not be an incredible policy, which is claimed by many authors. As long as the indexation mechanism has a backward-looking component, credible disinflation cannot reduce the rate of inflation rapidly. Combined with the supply-side effect, inflation

persistence co-exists with sustained expansion of domestic output.

The model has clear welfare implications for disinflation policy. Reducing inflation is beneficial in a cash-in-advance model with labor-leisure choice and imperfection in the goods market. Lower inflation reduces transaction costs. Hence it allows the households more free time for productive activities: more consumption and production. In a broad sense, permanent disinflation can eliminate existing inefficiencies in the economy: high inflation and low levels of production. Therefore, it is definitely welfare enhancing to implement credible disinflation. In this paper, we assume the degree of backward looking indexation for analytical simplicity. However, the degree of indexation has a significant implication for the economic welfare during the stabilization. As the degree of indexation declines, the model predicts the rapid convergence of inflation. According to the model, rapid convergence of inflation implies the less severe recession during the early stage of stabilization. This will enhance the welfare effects of inflation stabilization. To study the relationship between the the degree of indexation and economic welfare of inflation stabilization is also promising.

## 2.6 Appendix

### 2.6.1 The stability of Equation (2.3.3)

The stability of the system requires that among the eigenvalues of the matrix associated with dynamic system (2.3.3), one has a positive real part and two eigenvalues have negative real parts.

The eigenvalues  $\nu_i, i = 1, 2, 3$ . of the system

$$\begin{aligned} & \begin{bmatrix} \log \dot{y}_t^N \\ \dot{\pi}_t \\ \dot{\eta}_t \end{bmatrix} \\ &= \begin{bmatrix} \frac{\alpha\delta^2(\phi+\varphi)}{\chi} & -\frac{2(1+\alpha\delta)}{\chi} & \frac{2\alpha\delta}{\chi} \\ \frac{\delta^2(\phi+\varphi)}{\chi} & \frac{\delta(\varphi-2)}{\chi} & -\frac{2\delta}{\chi} \\ 0 & -\delta & \delta \end{bmatrix} \times \begin{bmatrix} \log y_t^N - \log y_{ss}^N \\ \pi_t - \mu_t \\ \eta_t - \mu_t \end{bmatrix} \end{aligned} \quad (2.67)$$

have the following properties

$$\det(A) = \nu_1\nu_2\nu_3 = \frac{\delta^3}{\mu} > 0 \quad (2.68)$$

$$\nu_1\nu_2 + \nu_2\nu_3 + \nu_1\nu_3 = -\frac{\delta^2}{\mu}(\phi + \alpha\delta) < 0 \quad (2.69)$$

$$Tr(A)^2 - 4\det(A) > 0. \quad (2.70)$$

From (A.2.2), we can see that the three eigenvalues are positive or one is positive and two are negative. From (A.2.3), the case of three positive roots can be ruled out. As a result, the system has two negative roots. Furthermore, according to (A.2.4), the eigenvalues may be complex conjugates and the system may exhibit cyclical behavior.

### 2.6.2 The dominant eigenvalue ray

We assume that  $\nu_i, i = 1, 2$  is the non-positive roots, with  $\nu_1 > \nu_2$ . Then, for  $i = 1, 2$ , it follows that

$$\begin{bmatrix} \frac{\alpha\delta^2(\phi+\varphi)}{\chi} - \nu_i & -\frac{2(1+\alpha\delta)}{\chi} & \frac{2\alpha\delta}{\chi} \\ \frac{\delta^2(\phi+\varphi)}{\chi} & \frac{\delta(\varphi-2)}{\chi} - \nu_i & -\frac{2\delta}{\chi} \\ 0 & -\delta & \delta - \nu_i \end{bmatrix} \times \begin{bmatrix} \omega_{i1} \\ \omega_{i2} \\ \omega_{i3} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}, \quad (2.71)$$

where  $\omega_{ij}, j = 1, 2, 3$ , are the element of the eigenvector associated with root  $\nu_i$ .

Therefore,

$$\frac{\omega_{i1}}{\omega_{i2}} = \frac{[2\delta - 2\nu_i - 2\alpha\delta\nu_i]/\chi(\delta - \nu_i)}{[\alpha\delta^2(\phi + \varphi) - \nu_i\chi]/\chi} > 0 \quad (2.72)$$

Now, let's look at the dominant eigenvector ray. Setting to zero the constant corresponding to the unstable root ( $\nu_3$ ), the solution to this system takes the form

$$\begin{aligned} \log y_t^N - \log y_{ss}^N &= A_1\omega_{11} \exp(\nu_1 t) + A_2\omega_{21} \exp(\nu_2 t) \\ \pi_t - \mu_t &= A_1\omega_{12} \exp(\nu_1 t) + A_2\omega_{22} \exp(\nu_2 t) \\ \eta_t - \mu_t &= A_1\omega_{13} \exp(\nu_1 t) + A_2\omega_{23} \exp(\nu_2 t), \end{aligned} \quad (2.73)$$

where  $A_i, i = 1, 2$ , denote the constants associated with root  $\nu_i$ . Since  $\nu_1 > \nu_2$ , it follows that

$$\begin{aligned} \lim_{t \rightarrow \infty} \frac{\log y_t^N - \log y_{ss}^N}{\pi_t - \mu} &= \lim_{t \rightarrow \infty} \frac{A_1\omega_{11} \exp(\nu_1 t) + A_2\omega_{21} \exp(\nu_2 t)}{A_1\omega_{12} \exp(\nu_1 t) + A_2\omega_{22} \exp(\nu_2 t)} \\ &= \lim_{t \rightarrow \infty} \frac{A_1\omega_{11} + A_2\omega_{21} \exp((\nu_2 - \nu_1)t)}{A_1\omega_{12} + A_2\omega_{22} \exp((\nu_2 - \nu_1)t)} \\ &= \frac{\omega_{11}}{\omega_{12}} > 0. \end{aligned} \quad (2.74)$$

### 2.6.3 Properties of system for temporary stabilization

We assume that  $\nu_i, i = 1, 2$  are non-positive roots and  $\nu_3$  is the non-negative one.

Then, for  $0 < t < T$ , the solution to the system is

$$\begin{aligned}\log y_t^N - \log y_{ss1}^N &= B_1 \omega_{11} \exp(\nu_1 t) + B_2 \omega_{21} \exp(\nu_2 t) + B_3 \omega_{31} \exp(\nu_3 t) \\ \pi_t - \mu^H &= B_1 \omega_{12} \exp(\nu_1 t) + B_2 \omega_{22} \exp(\nu_2 t) + B_3 \omega_{32} \exp(\nu_3 t) \\ \eta_t - \mu^H &= B_1 \omega_{13} \exp(\nu_1 t) + B_2 \omega_{23} \exp(\nu_2 t) + B_3 \omega_{33} \exp(\nu_3 t),\end{aligned}\quad (2.75)$$

and for  $t \geq T$

$$\begin{aligned}\log y_t^N - \log y_{ss0}^N &= C_1 \omega_{11} \exp(\nu_1 t) + C_2 \omega_{21} \exp(\nu_2 t) \\ \pi_t - \mu^L &= C_1 \omega_{12} \exp(\nu_1 t) + C_2 \omega_{22} \exp(\nu_2 t) \\ \eta_t - \mu^L &= C_1 \omega_{13} \exp(\nu_1 t) + C_2 \omega_{23} \exp(\nu_2 t),\end{aligned}\quad (2.76)$$

where  $B_i$  and  $C_i, i = 1, 2, 3$  are constants associated with root  $\nu_i$ , and  $\omega_{ij}, i, j = 1, 2, 3$  are the elements of eigenvector for roots  $\nu_i$ .

For  $0 < t < T$ , the system will be governed by the unstable root,  $\nu_3$ . This means that the system converges asymptotically to the divergent path, which is given by

$$\begin{aligned}\lim_{t \rightarrow \infty} \frac{\log y_t^N - \log y_{ss1}^N}{\pi_t - \mu^L} &= \lim_{t \rightarrow \infty} \frac{B_1 \omega_{11} \exp(\nu_1 t) + B_2 \omega_{21} \exp(\nu_2 t) + B_3 \omega_{31} \exp(\nu_3 t)}{B_1 \omega_{12} \exp(\nu_1 t) + B_2 \omega_{22} \exp(\nu_2 t) + B_3 \omega_{32} \exp(\nu_3 t)} \\ &= \frac{\omega_{31}}{\omega_{32}}.\end{aligned}\quad (2.77)$$

From (A.2.11) with  $\nu_3 > 0$ , we can see that the sign of  $\frac{\omega_{31}}{\omega_{32}}$  cannot be determined.

Even if, however, it is positive, the slope is flatter than the dominant eigenvector ray. For  $t \geq T$ , the dominant eigenvector ray is same as (A.2.8).

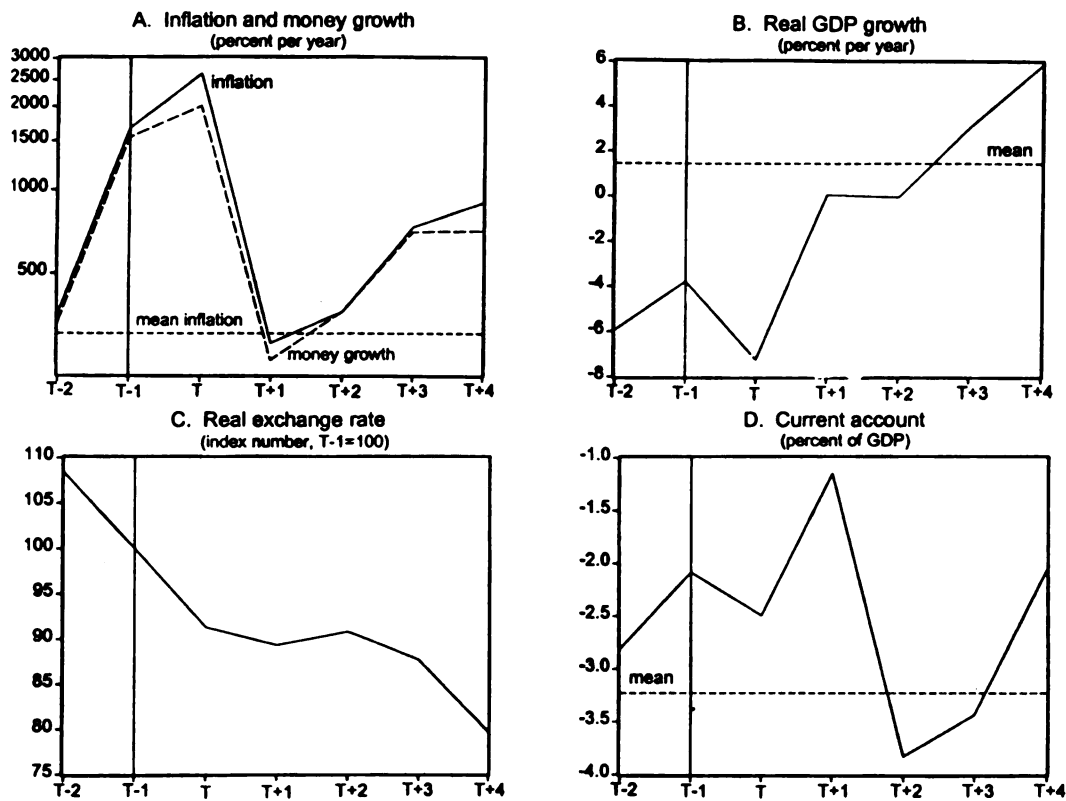


Figure 2.1: Money-based stabilization

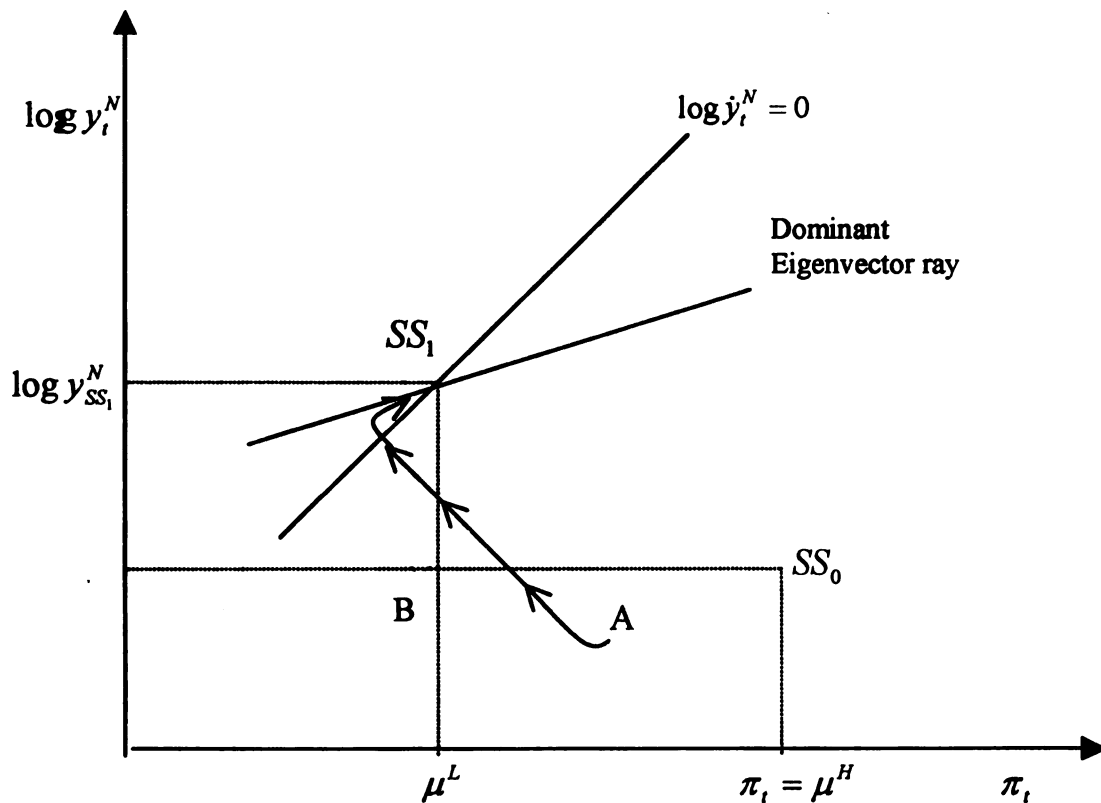


Figure 2.2: Dynamic system of money-based stabilization

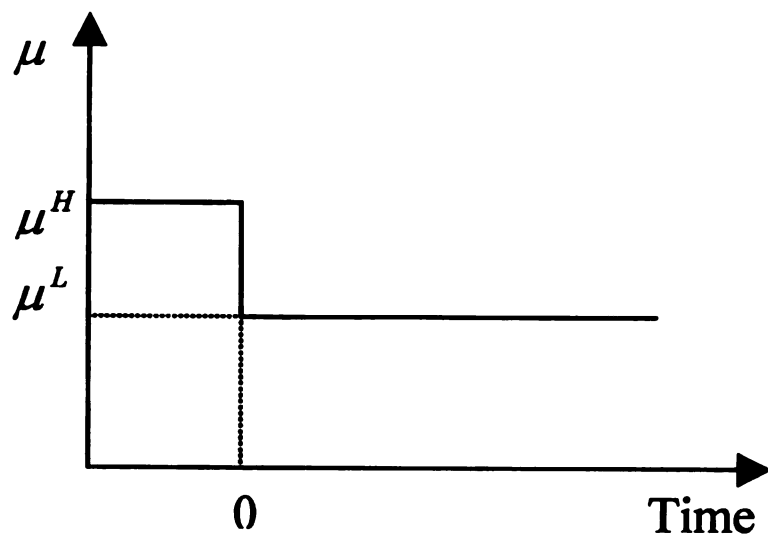


Figure 2.3: Time path: rate of money growth

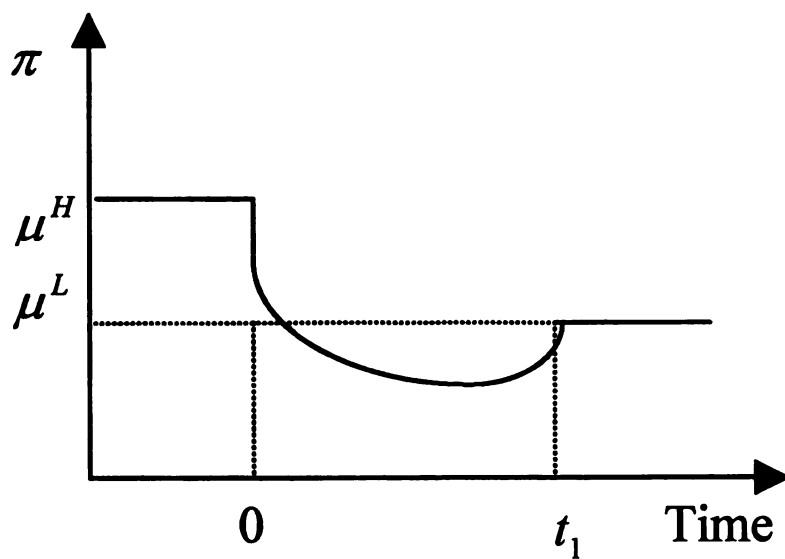


Figure 2.4: Time path: inflation

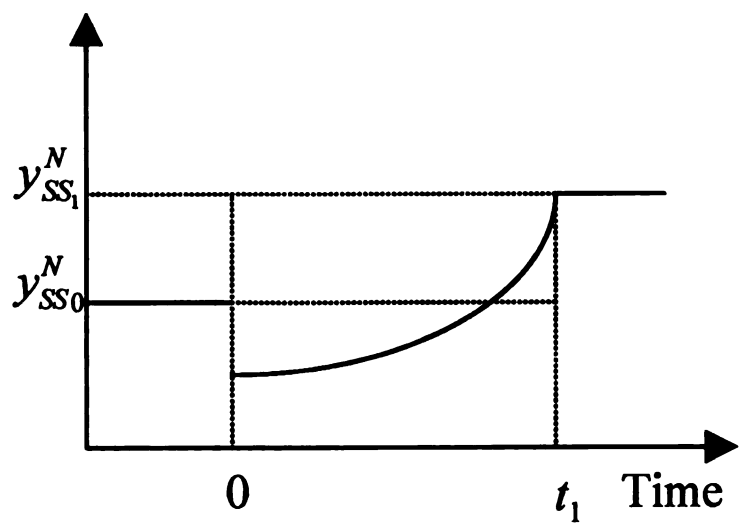


Figure 2.5: Time path: output of non-tradable

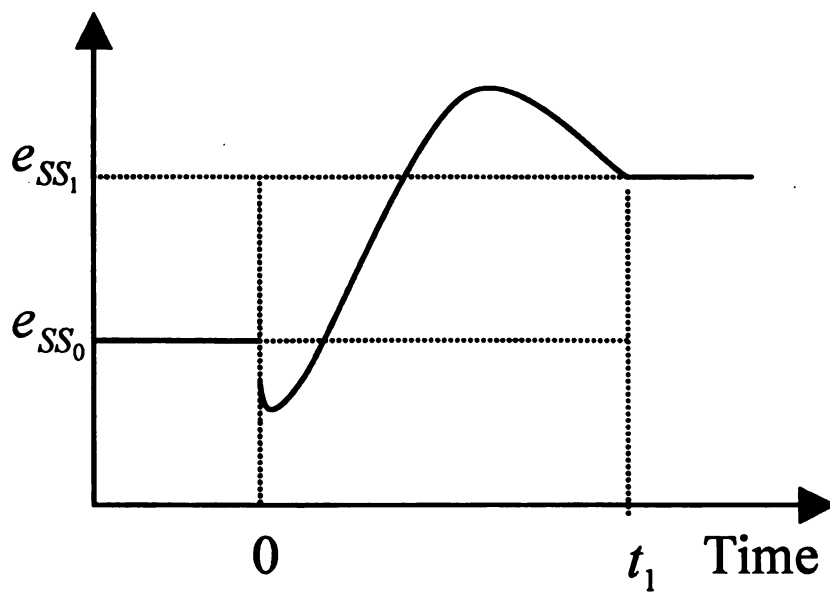


Figure 2.6: Time path: real exchange rate



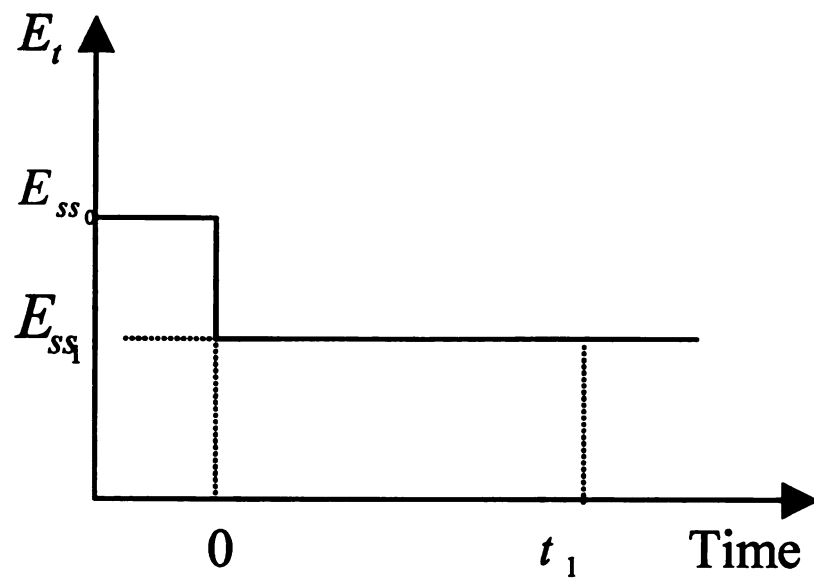


Figure 2.7: Time path: nominal exchange rate

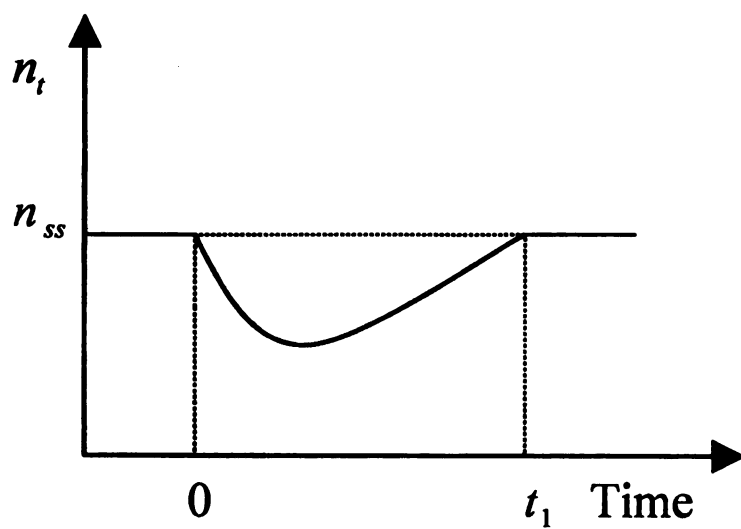


Figure 2.8: Time path: real money balance

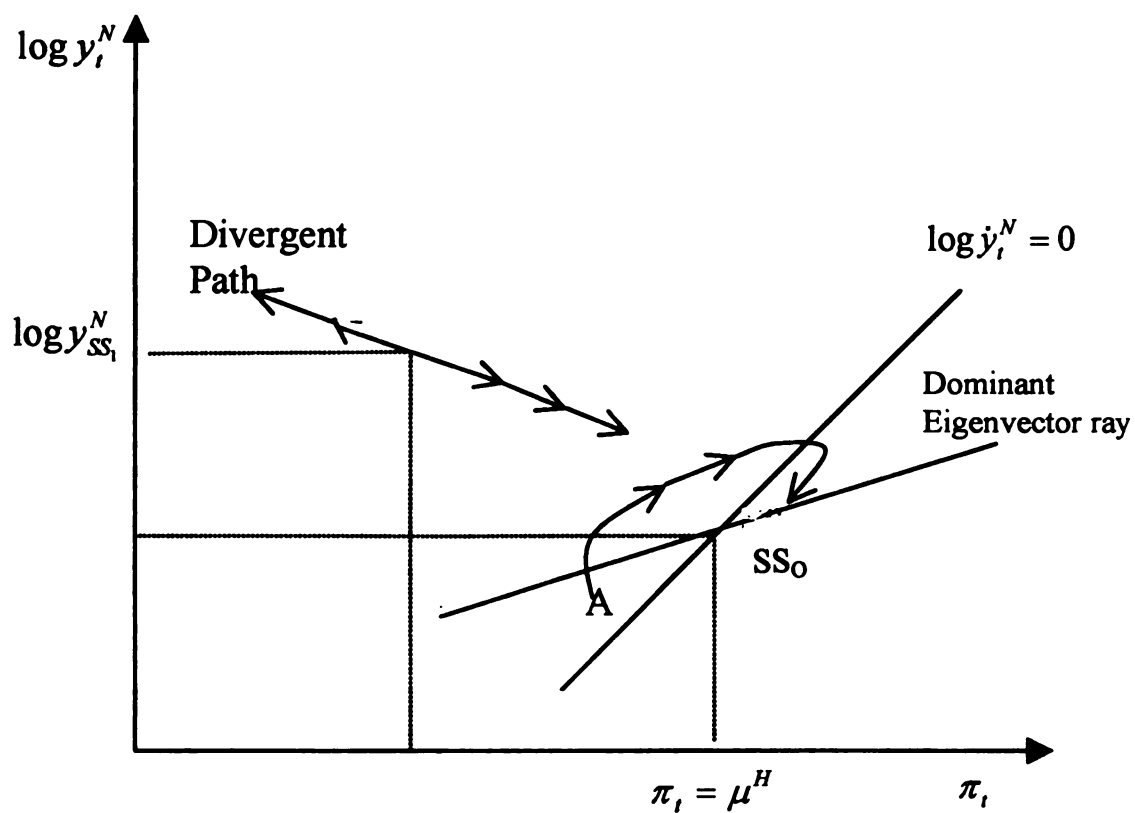


Figure 2.9: Dynamic system of temporary stabilization

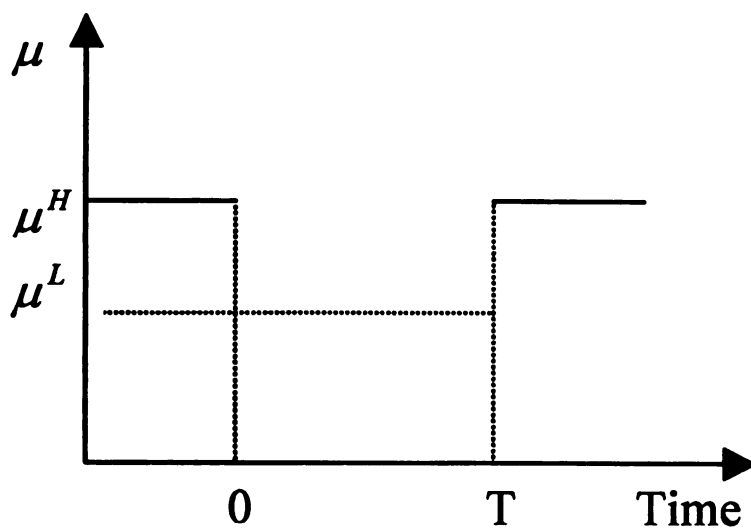


Figure 2.10: Time path: Money growth rate

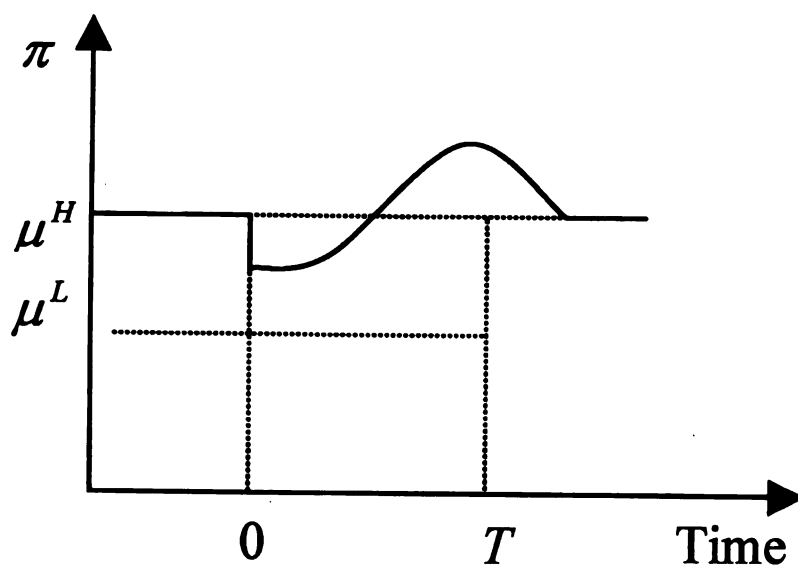


Figure 2.11: Time path: inflation

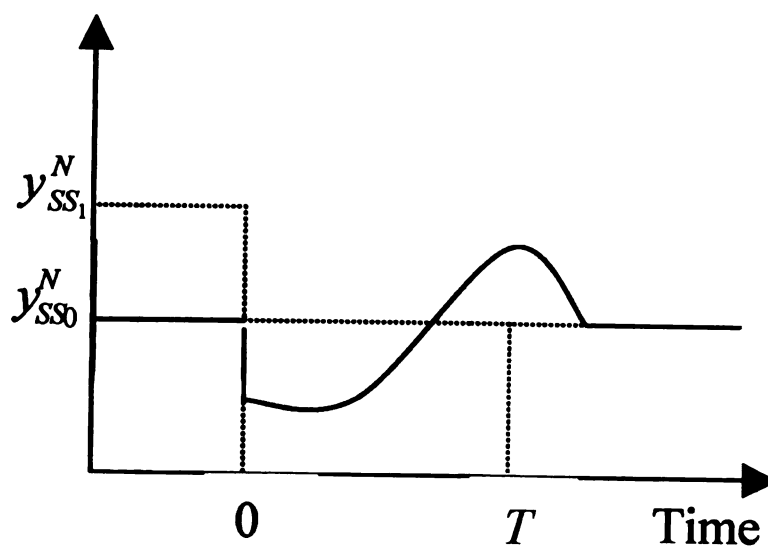


Figure 2.12: Time path: output of non-tradables

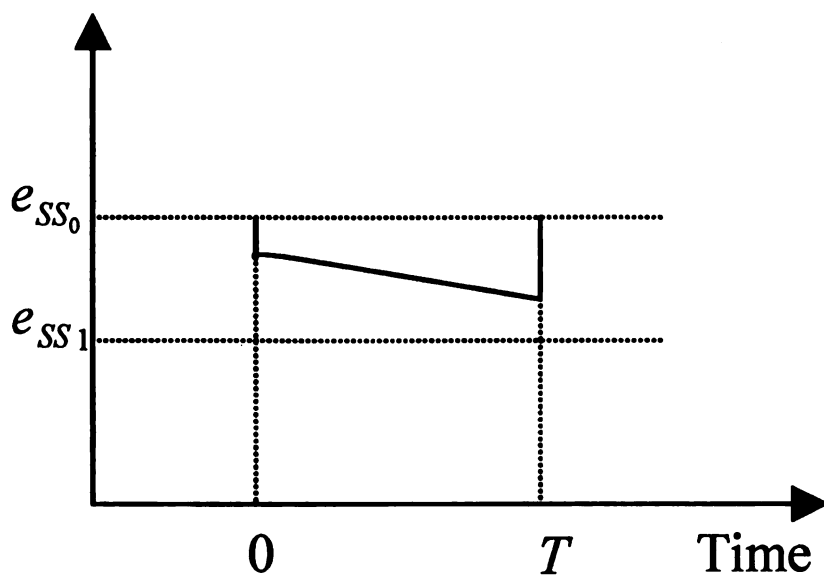


Figure 2.13: Time path: real exchange rate

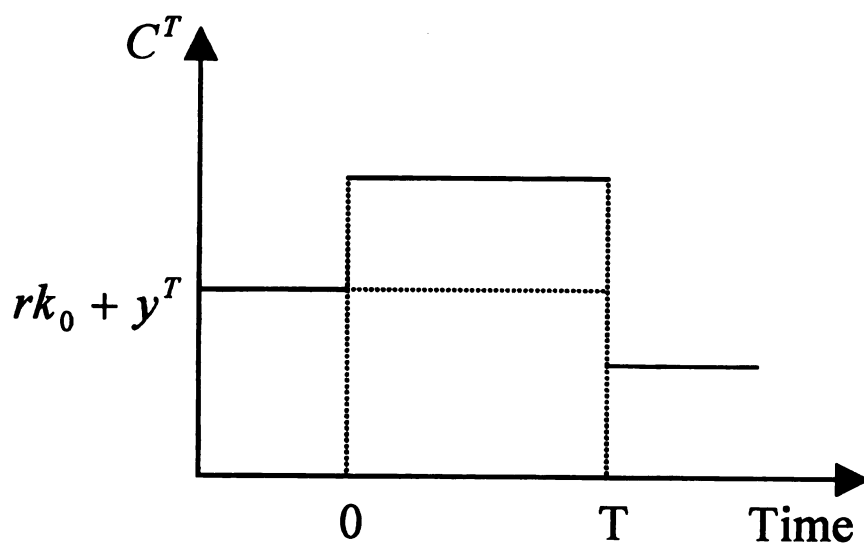


Figure 2.14: Time path: consumption of tradables

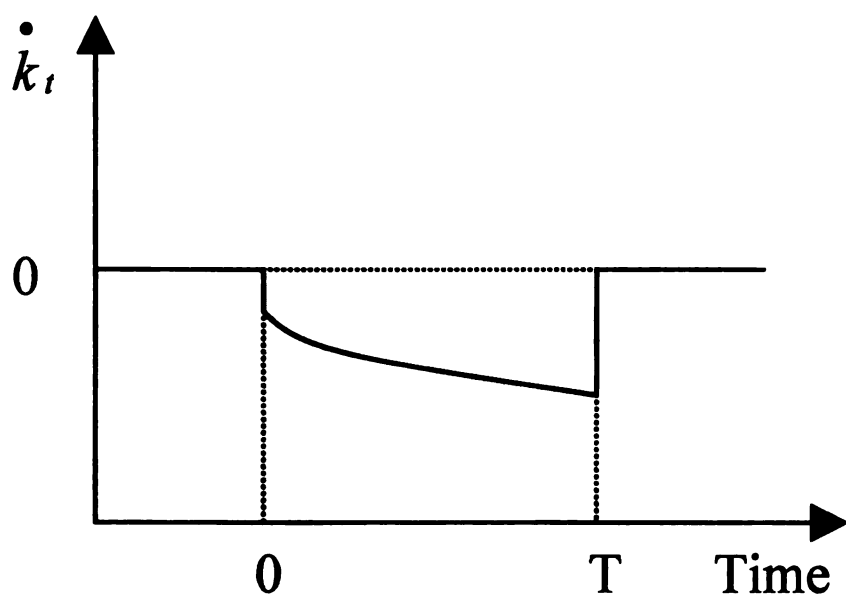


Figure 2.15: Time path: current account

# Chapter 3

## Interest rate policy

### 3.1 Introduction

In industrialized countries, short-term nominal interest rates are the most common operating instrument for the monetary policy,<sup>1</sup> and are most often used to achieve the target inflation rate. Many industrialized countries have implemented inflation targeting using the short-term interest rate as a nominal anchor. These countries include New Zealand, Canada, United Kingdom, Sweden, Finland, Australia, Spain, and, of course, United States. In the United States, the Federal Reserve raises the federal funds rate when the inflation rate is above the its target level.

Since World War II, developing countries, especially Latin American countries, have suffered from chronic inflation. Policymakers in those countries have engaged in repeated inflation stabilization attempts. Monetary policy for inflation stabilization in those countries is typically cast as choice between an exchange rate-based stabilization (using an exchange rate as a nominal anchor under a fixed or prede-

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<sup>1</sup>See, for instance, Batten et al (1990), and Bernanke and Mishikin (1992)

terminated exchange rate), or money-based stabilization (using the growth rate of the money supply under the flexible exchange rate). Most stabilization plans, however, have failed. Those which rely on the rate of devaluation have often generated dramatic balance-of-payment crises. The unsustainable increase in devaluation finally induces a speculative attack against the domestic currency. The Mexican crisis of December 1994 and the recent Argentine crisis are perfect examples.

In countries inflicted by chronic inflation, short-term interest rates have also played a key role in inflation stabilization programs during the 1980s. Even in the exchange rate-based stabilization program, policymakers raise interest rates to ensure a rapid fall in inflation as well as to prevent a speculative attack against the domestic currency. In the 1980's, Argentina, Brazil and Israel supported stabilization efforts by hiking up the interest rate (Calvo and Végh (1995), and Végh (1998)). In the case of Mexico, analysts blame the monetary authority's reluctance to raise interest rates as a main factor in triggering the December 1994 crisis.

Considering the high popularity of interest rate policy in practice, it is surprising to find that it does not enjoy high reputation in the inflation stabilization literature. Calvo and Végh (1995) analyzed the effectiveness of interest rate policy in inflation stabilization in the context of the small open economy, flexible price model. The key assumption of the model is that the representative consumer is subject to a liquidity-in-advance constraint under which the consumer must hold money and interest bearing liquid bonds (highly liquid bank deposits). The policymaker controls the deposit interest rate. In this set-up, a permanent increase in

the deposit rate produces an increase in money demand. Therefore, with a constant nominal money supply, the price level and the inflation fall to increase real money balances. When, an increase in the deposit rate is temporary, the inflation rate rises continuously after the initial fall in the inflation rate. This result thus suggests that, in the presence of flexible prices, the increasing interest rate is not an effective tools for reducing inflation. Calvo and Végh (1995) provided useful insights into the role of interest rate policy inflation stabilization. However, it also has some defects. In this model, the nominal interest rate is divided into two separate rates: a deposit rate and a money rate. Even if policymakers increase the deposit rate, the money rate is constant by assumption. If the deposit rate is increased by policy, we would expect that the money rate would also increase. So the assumption of a constant money rate is quite inapplicable to the real world. In the real world, if the deposit rate increases the money rate also goes up. When this is the case, it cannot produce the increase in money demand and the fall in the price level that they suggest.

In this paper, we analyze the effect of interest rate policy on inflation stabilization. Specifically, we consider the nominal interest rate rule for inflation stabilization in a small open economy. We depart from Calvo and Végh's model by assuming there is only one nominal interest rate, which is common in the literature. An analytical problem of using the interest rate as a policy instrument is that interest rate targeting leads to a nominal indeterminacy in equilibrium. In their famous paper, Sargent and Wallace (1975) pointed out that the price level



will be made indeterminate by pure interest rate pegging under a flexible price model. With sticky prices, nominal interest pegging leads to indeterminate inflation (Calvo, 1983). The indeterminacy problem can be eliminated by designing appropriate money supply targets (MaCallum (1981), Canzoneri, Henderson, and Rogoff (1985)), or controlling the interest rate on deposits (Calvo and Végh (1995, 1996)).

To ensure determinacy, we adopt an inflation targeting rule. In this case, the policymaker announces the inflation target and sets the interest rate according to the deviation of the actual inflation rate from the target inflation rate. In this paper, we do not derive the inflation targeting policy rule from the policymaker's optimization problem. The policy rule we consider is a variant of the Taylor rule in which the nominal interest rate responds positively to inflation. We simulate the model to analyze the effect of the interest rate rule on inflation and other macro-variables in a small open economy.

Our model is based on Obstfeld and Rogoff's (1995) "Exchange Rate Dynamic Redux". We modify Obstfeld and Rogoff's model in two ways. First, we assume a small-open economy with a cash-in-advance constraint in which a change of monetary policy, including a change of the money growth rate, has a long-run effect on the economy as in the two-country model in Obstfeld and Rogoff. Second, we adopt a Calvo-type contracts model to obtain the inflation dynamics. Our model is a modified version of Calvo's (1983) forward-looking, staggered wage contracts model. However, we incorporate backward-looking indexation into wage contracts.

We combine the staggered wage contract with price setting of imperfectly competitive firms to obtain inflation persistence. Incorporating backward-looking wage contracts into the model is quite consistent with the wage indexation of most chronic inflation countries.<sup>2</sup>

We apply this model to investigate the effects of inflation targeting interest rate policy. In this experiment, we find that interest rate policy triggers a severe recession during stabilization. The recession arises from both demand and supply side effects. When policymakers use the growth rate of the money supply for inflation stabilization (money-based stabilization), only an aggregate demand-oriented recession was observed. In this regard, the model we present here has very important policy implications in terms of output stabilization. The initial cost of disinflation is much higher if the nominal interest rate rather than the growth rate of money supply is used. Inflation persistence is also implied. Inflation inertia can not be avoided in disinflation as long as there is backward-looking indexation in price or wage setting regardless of policy instruments. Hence to achieve rapid stabilization, the indexation mechanism should be considered. Also the slow convergence of inflation deepens the initial recession through supply side effects. This result distinguishes the interest rate rule from a money-based stabilization. In case of money-based stabilizations, backward-looking indexation matters in terms of inflation stabilization. Indexation, however, plays a key role in inflation stabilization as well as output stabilization in interest rate policy.

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<sup>2</sup>For instance, Edwards(1991) pointed out that the primary cause of inflation inertia in the Chilean stabilization is backward-looking wage indexation.

Another feature of interest rate policy is that it causes real and nominal appreciation during the initial stabilization period. This result ensures that increasing the interest rate during the stabilization process prevents the domestic currency from depreciating. Hence it is possible that a policymaker can avoid a possible speculative attack on the domestic currency, such as those observed in Latin American exchange rate-based inflation stabilization, when he or she uses the nominal interest rate as an anchor.

This paper proceeds as follows. Section 3.2 develops the basic model and an interpretation of model is presented. Interest rate policy with inflation targeting is examined in section 3.3. Section 3.4 closes the paper.

## **3.2 Basic model**

In this section we consider a two sector small open economy, which is perfectly integrated with the rest of the world in goods and capital markets. First, we introduce the demand side through the households' maximization problem and then the aggregate supply side in which the non-tradable sector is the locus of imperfect competition. The next step is to solve for a symmetric steady state where all prices are fully flexible. We, then, introduce sticky prices through staggered wage contracts. In the last part of this section, we derive the short-run dynamic system for the next section.

### 3.2.1 The aggregate demand side

Our model is based on Obstfeld and Rogoff's (1995) perfect-foresight general equilibrium Mundell-Fleming model. The economy is inhabited by a continuum of identical households which reside on the interval  $[0,1]$ . The representative household derives utility from the consumption of tradable and non-tradable goods and leisure. This economy contains a continuum of differentiated non-tradable goods which are indexed by  $z$  and distributed uniformly on  $[0,1]$ .

The lifetime utility of typical households is given by

$$U_t = \int_0^\infty [\gamma \log(C_t^T) + (1 - \gamma) \log(C_t^N) + \rho \log(1 - l_t)] \exp(-\beta t) dt, \quad (3.1)$$

where  $C_t^T$  is the consumption of the tradable good and  $C_t^N$  is a non-tradable consumption index, defined by

$$C_t^N = \int_0^1 [c_t^N(z)^{\frac{\theta-1}{\theta}}]^{\frac{\theta}{\theta-1}} dz, \quad (3.2)$$

where  $c^N(z)$  is the household's consumption of good  $z$ ,  $\beta$  is the positive and constant subjective discount rate, and  $\theta > 1$  is the elasticity of substitution between non-tradable goods. The last term in the lifetime utility function in Eq.(3.2.1) captures the utility from leisure or disutility from labor supply ( $l_t$  represents total hours worked by the household), and we assume  $\rho > 0$ .

We employ the convention of letting  $E$  represent the nominal exchange rate in units of domestic currency per unit of foreign currency, while  $P^T$  represents the foreign currency price of the tradable good.<sup>3</sup> Here,  $P^N$  denotes the aggregate

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<sup>3</sup>We assume that the law of one price holds for the tradable good. Therefore, the domestic currency price of the tradable good is  $E_t P_t^T$ .

domestic currency price index of the non-tradable goods, defined as

$$P_t^N = \int_0^1 [p_t^N(z)^{1-\theta}]^{\frac{1}{1-\theta}} dz. \quad (3.3)$$

The real exchange rate (the relative price of tradable goods in terms of non-tradable goods) can be defined as  $e = \frac{EP^T}{P^N}$ . For simplicity, we assume that  $P^T = 1$  and is constant. By this assumption, we can suppress the unnecessary foreign inflation rate from the model.

The typical household can hold two types of assets as their financial wealth: domestic non-interest bearing currency and an internationally tradable bond with a constant real interest rate (in terms of tradable goods). Since the domestic currency and the international bond are the only two assets held by individual consumers, we have

$$a_t = m_t + b_t, \quad (3.4)$$

where  $a, m$ , and  $b$  stand for real financial wealth, real money balances and the stock of real bonds in terms of domestic price of tradable goods, respectively.<sup>4</sup>

We assume that the household has a constant endowment flow of tradable goods,  $y_t^T$ , while it derives human wealth from supplying labor to firm  $z$  for the nominal wage  $W_t$ . Households also own firm  $z$ . Therefore, they earn the profit of the firm,  $\Pi_t(z)$ . Then the household's dynamic budget constraint is governed by the following differential equation:

$$\dot{a}_t = ra_t - i_t m_t + \frac{\Pi_t(z)}{E_t P^T} + \frac{W_t l_t}{E_t P^T} + y_t^T + \tau_t - \frac{C_t^N}{e_t} - C_t^T, \quad (3.5)$$

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<sup>4</sup>If we define nominal financial wealth, money balances and the stock of bonds as  $A, M$  and  $B$ , respectively, then real variables can be defined as follows;  $a = \frac{A}{EP^T}$ ,  $m = \frac{M}{EP^T}$ , and  $b = \frac{B}{EP^T}$ .

where  $\tau$  is real transfers from the government in terms of tradable goods;  $i$  is the instantaneous nominal interest rate in terms of domestic currency;  $r$  is the constant real interest rate in terms of tradable goods;<sup>5</sup>  $W_t$  is the money wage rate. Notice that in Eq.(3.2.5) household's expenditure includes the opportunity cost of holding real money balances,  $im$ .

In order to carry out consumption expenditures, the household is required to hold sufficient domestic money. Following Feenstra(1985), the cash-in-advance constraint which the household faces is thus

$$\alpha \left[ \frac{C_t^N}{e_t} + C_t^T \right] \leq m_t, \quad \alpha > 0, \quad (3.6)$$

where  $\alpha$  represents the length of time that money has to be held to finance consumption expenditure.<sup>6</sup> Eq.(3.2.6) implies that the minimum required money balances are proportional to the value of consumption expenditures. If the nominal interest rate,  $i$ , is positive, the household will hold the minimum required money. Then the cash-in-advance constraint (3.2.6) will be binding.

Using Equations (3.2.4), (3.2.5) and (3.2.6), and imposing the transversality conditions, we can derive the household's intertemporal budget constraint which is given by

$$a_0 + \int_0^\infty \left( \frac{W_t l_t}{E_t P^T} + y_t^T + \frac{\Pi_t(z)}{E_t P^T} + \tau_t \right) \exp(-rt) dt = \int_0^\infty \left( \frac{C_t^N}{e_t} + C_t^T \right) (1 + \alpha i_t) \exp(-rt) dt, \quad (3.7)$$

---

<sup>5</sup>Since we assume that  $P^T = 1$  and is constant,  $r$  is also the world nominal interest rate.

<sup>6</sup>The cash-in-advance constraint can be written as  $m_t \geq F(\alpha) \equiv \int_t^{t+\alpha} \left( \frac{C_s^N}{e_s} + C_s^T \right) ds$ . A Taylor-series expansion gives  $F(\alpha) = \alpha \left( \frac{C_t^N}{e_t} + C_t^T \right) + \frac{1}{2} \alpha^2 \frac{d}{dt} \left( \frac{C_t^N}{e_t} + C_t^T \right) + \dots$ , so (3.2.6) can be interpreted as a first-order approximation

where  $a_0$  stands for the initial level of financial wealth. This equation says that the household's lifetime expenditure equals the present discounted value of its lifetime income. At each point in time  $t$ , the representative household's expenditure consists of the cost of consumption,  $\frac{C_t^N}{e_t} + C_t^T$ , plus the opportunity cost of holding real balances,  $i_t m_t$ .

The decisions the household has to make at each point in time are how many tradable and non-tradable goods to consume and how much to work. Therefore, the household's optimization problem is to choose the path of  $C_t^T$ ,  $C_t^N$  and  $l_t$  to maximize lifetime utility, (3.2.1), subject to initial wealth,  $a_0$ , and the intertemporal budget constraint, (3.2.7).

The first order conditions for this optimization problem are:<sup>7</sup>

$$\frac{\gamma}{C_t^T} = \lambda(1 + \alpha i_t) \quad (3.8)$$

$$\frac{1 - \gamma}{C_t^N} = \lambda\left(\frac{1 + \alpha i_t}{e_t}\right) \quad (3.9)$$

$$\frac{\rho}{1 - l_t} = \lambda\left(\frac{W_t}{E_t P_t^N}\right), \quad (3.10)$$

where  $\lambda$  is the (time-invariant) Lagrange multiplier associated with the household's intertemporal budget constraint (3.2.7). As usual, it can be interpreted as the marginal utility of wealth. Equations (3.2.8) and (3.2.9) indicate that at an optimum, the household equates the marginal utility of consumption of tradable and non-tradable goods to the marginal utility of wealth times the effective prices of goods. The effective prices of goods consist of the market prices, unity in the

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<sup>7</sup>To eliminate inessential dynamics and ensure the existence of a steady state, we assume that  $\beta = r$ .

case of tradables and  $\frac{1}{e_t}$  in the case of non-tradables, plus the opportunity cost of holding the  $\alpha$  units of money that are needed to purchase both goods,  $\alpha i$  and  $\alpha \frac{i}{e_t}$ , respectively. Equation (3.2.10) is the Euler equation for optimal labor supply. It ensures that the marginal disutility of labor (due to forgone leisure) equals the marginal utility from consuming the extra goods from another unit of labor supplied. The term  $(1 + \alpha i_t)$  generated by the cash-in-advance constraint must be treated with special attention in this model. This is the usual monetary wedge due to the fact that any form of wealth has to be liquidated before goods can be purchased. Even if this monetary wedge increases the effective price of consumption, it does not affect the price of leisure.

Using (3.2.8) and (3.2.9), we can get

$$\frac{C_t^T e_t}{C_t^N} = \frac{\gamma}{1 - \gamma}. \quad (3.11)$$

And combine (3.2.9) and (3.2.10)

$$\frac{\rho}{1 - l_t} = \frac{1 - \gamma}{C_t^N} \frac{W_t}{P_t^N} (1 + \alpha i_t)^{-1}. \quad (3.12)$$

Equation (3.2.11) is the familiar condition that at an optimum the marginal rate of substitution between tradable and non-tradable goods is equal to the relative price of tradable goods in terms of non-tradable goods which is the real exchange rate. From Eq.(3.2.12) we can see that the individual's optimal labor supply is dependent on the nominal interest rate. Since the nominal interest rate introduces a distortion between consumption and leisure, a change in the interest rate makes



the household substitute leisure for consumption due to the change in the effective price of consumption. This, in turn, leads to a change in the long-run level of non-tradable output.

### 3.2.2 The government

The other participant in this economy is the government. Since Ricardian equivalence holds in this model, we can assume that the government runs a balanced budget in every period. We also assume no government spending. With these assumptions, we can simplify government behavior. It is assumed that the government holds internationally tradable bonds (international reserves) which pay the world real interest rate in terms of tradable goods and issues non-interest bearing debt (domestic currency).

The evolution of the government's stock of net foreign bonds is governed by the following differential equation:

$$\dot{h}_t = rh_t - \tau_t + \frac{\dot{M}_t}{P_t^T E_t}, \quad (3.13)$$

where  $h_t$  is the government's stock of internationally tradable bonds (international reserves). Notice that  $\frac{\dot{M}_t}{P_t^T E_t}$  is the government's seignorage profits. Integrating equation (3.2.13) and imposing the transversality condition, we can derive the government's intertemporal budget constraint

$$\int_0^\infty \tau_t \exp(-rt) dt = h_0 + \int_0^\infty (\dot{m}_t + \epsilon_t m_t) \exp(-rt) dt, \quad (3.14)$$

where  $\epsilon_t = \frac{\dot{E}_t}{E_t}$  is the nominal rate of depreciation and  $h_0$  is the initial level of the government's stock of foreign bonds.<sup>8,9</sup> The government's intertemporal budget constraint indicates that the present value of transfer expenditure has to be equal to the initial stock of government-held international bonds (i.e., international reserves) and revenues from seignorage. By equation (3.2.14), we implicitly assume that the government returns to the consumer all of its revenues.

### 3.2.3 The aggregate supply side

We now turn to the supply side of the economy. For simplicity, we assume that the supply of tradable goods is exogenous and fixed at the constant level  $y^T$  (i.e.,  $y_t^T = y^T$  for all  $t$ ) and its domestic price will be determined by the law of one price.

In this model, it is assumed that the non-tradable good sector is imperfectly competitive and there is a continuum of imperfectly competitive firms indexed by  $z \in [0, 1]$ . Each producer produces a differentiated good and acts as a monopolistic competitor, choosing the nominal price and the level of production of the good.

Given the CES non-tradable consumption index, Eq.(3.2.2), an individual's demand for non-tradable good  $z$  in period  $t$  is

$$C_t^N(z) = \left( \frac{p_t^N(z)}{p_t^N} \right)^{-\theta} C_t^N, \quad (3.15)$$

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<sup>8</sup>The government's seignorage revenue  $\frac{\dot{M}_t}{P_t^T E_t}$  is equal to  $\dot{m}_t + \epsilon_t m_t$  in real term (in terms of tradable goods).

<sup>9</sup>The appropriate transversality condition is  $\lim_{t \rightarrow \infty} h_t \exp(-rt) = 0$ .

where  $\theta$  is the price elasticity of demand.

According to Eq.(3.2.15), a monopolistically competitive firm that produces a non-tradable goods faces the downward-sloping curve

$$y_t^{Nd}(z) = \left[ \frac{p_t^N(z)}{P_t^N} \right]^{-\theta} C_t^{NA}, \quad (3.16)$$

where  $C_t^{NA} dz = \int_0^1 C_t^N dz = C_t^N$  is aggregate per capita non-tradable good consumption.

Labor is the only input into production. The technology available to firms is identical and is linear in labor

$$y_t^N(z) = l_t. \quad (3.17)$$

Firm  $z$ 's profit is

$$\Pi_t(z) = p_t(z) y_t^d(z) - W_t l_t. \quad (3.18)$$

At any time  $t$ , a monopolistically competitive firm maximizes profit, Eq.(3.2.18), subject to the demand function, Eq.(3.2.16), and the production function, Eq.(3.2.17).

Solving the firm's problem:

$$\frac{\theta - 1}{\theta} y_t^{-\frac{1}{\theta}}(z) P_t^N C_t^{\frac{1}{\theta}} = W_t. \quad (3.19)$$

Since the elasticity of demand,  $1/\theta$ , of all non-tradable goods is identical and every producer has the same technology, they produce the same level of output and set the same price. Therefore, for any two producers,  $0 < z < z' < 1$

$$\begin{aligned} y_t^N(z) &= y_t^N(z') = y_t^N \\ p_t^N(z) &= p_t^N(z') = p_t^N. \end{aligned} \quad (3.20)$$

It follows that the non-tradable good's aggregate supply and price index simplify to

$$\begin{aligned} y_t^{AN} &= \int_0^1 y_t^N(z) dz = y_t^N \\ P_t^N &= [\int_0^1 (p_t^N(z))^{1-\theta} dz]^{\frac{1}{1-\theta}} = p_t^N. \end{aligned} \quad (3.21)$$

The equilibrium in a non-tradable good's market can be obtained by Eq.(3.2.16) and Eq.(3.2.21)

$$y_t^N = C_t^N. \quad (3.22)$$

Therefore, from Eq.(3.2.19)

$$p_t^N(z) = \frac{\theta}{\theta - 1} W_t, \forall z. \quad (3.23)$$

Equation(3.2.23) shows that a producer with market power sets price above marginal cost, which is the money wage rate. Thus, if it cannot adjust its price, it is willing to produce to satisfy demand in the face of fluctuations in demand. Therefore, in the short-run, if the price is sticky, output will be demand-determined.

### 3.2.4 Equilibrium conditions and a symmetric steady state

In this paper, perfect capital mobility is assumed. It implies that

$$i_t = r + \epsilon_t. \quad (3.24)$$

Equilibrium in the non-tradable goods market implied by Eq. (3.2.22) is

$$y_t^N = C_t^N.$$

In order to investigate the dynamic behavior of the current account triggered by a policy change, consider the economy's dynamic resource constraint. Combining the household's dynamic budget constraint with the profit of the firm and the market clearing condition for the non-tradable goods sector yields the economy's intertemporal budget constraint

$$\dot{k}_t = rk_t + y^T - c_t^T, \quad (3.25)$$

where  $k_t = b_t + h_t$  is the total amount of foreign bonds held by the economy. Equation (3.2.25) represents the economy's current account balance. It indicates that the current account balance is the difference between tradable goods income and consumption of tradable goods.

Combining Equations (3.2.7), (3.2.22), (3.2.14) and (3.2.24) yields the economy's intertemporal constraint

$$k_0 + \int_0^\infty y^T \exp(-rt) dt = \int_0^\infty C_t^T \exp(-rt) dt, \quad (3.26)$$

where  $k_0$  denotes the economy's initial stock of foreign bonds. Equation (3.2.26) states that the initial stock of foreign bonds plus the present value of all future tradable output must equal the present value of tradable consumption. At the steady state,  $C_t^T = C_{ss}^T$ . Therefore,  $C_{ss}^T = y^T + rk_0$  and the current account is balanced at the steady state.

In the steady state, all prices are fully flexible. Thus the symmetric equilibrium implies

$$C_t^N = y_t^N = C_t^{AN}, \forall z. \quad (3.27)$$

In the steady state where all prices are fully flexible, the supply condition determines the level of non-tradable output. It means that the long-run level of output is totally dependent on labor supply. Using Eqs.(3.2.12), (3.2.23) and (3.3.27), we get the symmetric steady state level of output of non-tradables

$$y_{ss}^N = [1 + (\frac{\rho}{1-\gamma})(\frac{\theta}{\theta-1})(1 + \alpha i_{ss})]^{-1}, \quad (3.28)$$

where  $i_{ss} = r + \epsilon_{ss}$  is the steady state level of the nominal interest rate. In the steady state, all prices are fully flexible and all exogenous variables are constant. Therefore, the long-run level of non-tradable output is determined by the supply side. According to Eq.(3.2.26), however, we can see that non-tradable output depends on the nominal interest rate, which is also a function of the monetary policy variable. The reason is as follows. Since the nominal interest rate introduces a distortion between consumption and leisure, if there is a permanent decrease in the nominal interest rate, the household will substitute leisure for consumption due to a lower effective price of consumption. This, in turn, leads to a rise in the long-run level of non-tradable output. This result is consistent with the disinflation scenario of Cooley and Hansen (1989, 1995) in a closed economy model in which variations in the rate of inflation can affect the steady state labor supply and consumption (or output).

In this model, each producer has monopoly power, so the long-run level of non-tradable output is lower than the socially optimal level. To see this, let's look at the social planner's problem. The social planner tries to maximize the utility of

non-tradable consumption considering the disutility from labor supply.<sup>10</sup>

$$\max[(1 - \gamma) \log y^N - \rho \log(1 - y^N)]. \quad (3.29)$$

The solution is

$$y_s^N = \left( \frac{1 - \gamma}{1 - \gamma + \rho} \right) > y_{ss}^N, \quad (3.30)$$

From Eq.(3.2.30), we can see that the equilibrium output of non-tradables is inefficiently low under imperfect competition. This fact has important implications for economic fluctuations as well as monetary policy to stabilize it, as long as prices are not perfectly flexible because of menu costs or staggered price or wage contracts. Since the market equilibrium is lower than the socially optimal level, recessions and booms have asymmetric effects on welfare (Mankiw, 1985). Since equilibrium output is less than the socially optimal level, a boom brings output closer to the social optimum, whereas a recession in the non-tradable sector pushes it farther away. In this model, however, the long-run equilibrium level of output is subject to a monetary policy variable. Therefore, a change in monetary policy can lead the economy closer to the optimum level.

### 3.2.5 Staggered wage contract and short-run dynamics

So far, we have assumed flexible prices and solved for the long-run equilibrium. We now introduce the assumption that the price level of the non-tradable good is sticky because of staggered wage contracts, and we are ready to consider the short-run dynamics of the economy. In the short-run, the supply of non-tradable

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<sup>10</sup>Remember that by the production condition  $y^N = l$ .

goods will be demand determined and prices will be given by a modified version of the staggered-price model of Calvo (1983) and Ghezzi (2001), including backward-looking indexation consistent with our discussion in Section 3.1. We assume nominal wages are sticky as a result of staggered wage contracts.

Following Calvo (1983), we assume that each individual can change wages only when a wage signal is received. If individuals do not change the wage or is setting a new wage at time  $t$ , the probability (density) that the wage will last for  $s$  more periods is given by the geometric distribution

$$\delta \exp(-\delta s) \tag{3.31}$$

and is, therefore, independent of  $t$  and of the amount of time the wage has lasted at  $t$ . And it is also stochastically independent across the individual wage setters. The expected length of wage duration is  $1/\delta$ . Therefore, nominal wages specified by the contract will be fixed for the duration of the contract and the contract itself is staggered.

Then the aggregate(log of) newly posted wage level follows

$$w_t = \delta \int_{-\infty}^t x_s \exp(-\delta(t-s)) ds, \tag{3.32}$$

where the wage set at time  $s$ ,  $x_s$ , is weighted by the probability that it continues in effect at  $t$ ,  $\exp(-\delta(t-s))$ .

For analytical simplicity, we assume that at any time  $t$ , there are two types of wage contractors: half of the individual contractors set their nominal wage in purely forward looking manner and half of them set in a backward-looking manner.



By this mechanism, we can easily incorporate backward-looking indexation, which is proposed by Taylor (1980), into the Calvo model.<sup>11</sup>

The forward-looking wage contractor will set wages according to the standard Calvo model

$$x_t^F = \delta \int_t^\infty [w_s + \phi(\log c_s^N - \log y_{ss}^N)] \exp(-\delta(s - t)) ds, \quad (3.33)$$

where  $\phi$  reflects sensitivity of contract wages to future excess demand conditions ( $\log c_s^N - \log y_{ss}^N$ ), where  $y_{ss}^N$  is the steady state level of non-tradable output. According to Eq.(3.2.33), we can see that when setting a contract, forward-looking agents will take into account the future average wage level and excess demand during the length of contract.

For the other half of the contractors, wages are assumed to be set according to a backward-looking rule. Specifically, we assume that the wage contract at  $t$  indexes money wages to the current aggregate wage level plus a weighted average of past inflation and current excess demand conditions.

$$x_t^B = w_t + \frac{1}{\delta} \eta_t + \varphi(\log c_t^N - \log y_{ss}^N), \quad (3.34)$$

where  $\varphi(> 0)$  reflects the sensitivity of the current money wages to the current level of excess demand, and

$$\eta_t = \delta \int_{-\infty}^t \pi_s \exp(-\delta(t - s)) ds \quad (3.35)$$

$$\dot{\eta}_t = \delta(\pi_t - \eta_t). \quad (3.36)$$

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<sup>11</sup>In the original Taylor models, current wage is set according to the past wage level and the expected future wage rate including past and future excess demand condition. Thus, Taylor models can generate wage level inertia.

Equation (3.2.34) indicates that backward-looking wage contracts at  $t$  index current nominal wages to the current aggregate wage level, a weighted average of past inflation  $\eta_t$ , adjusted by the expected length of the price duration,  $1/\delta$ , and the current level of excess demand.<sup>12</sup>

Therefore, the log of wage set at time  $t$  is

$$x_t = \frac{1}{2}x_t^F + \frac{1}{2}x_t^B. \quad (3.37)$$

Since price is a fixed markup over wage, there is no distinction between wage and price inflation in this model. Therefore, we can differentiate Eq.(3.3.32) with respect to time to get<sup>13</sup>

$$\pi_t = \dot{w}_t = \delta[x_t - w_t]. \quad (3.38)$$

Combining Equations (3.2.33), (3.2.34) and (3.2.37), we obtain

$$x_t = \frac{1}{2}[w_t + \frac{1}{\delta}\eta_t + \varphi(\log c_t^N - \log y_{ss}^N)] + \frac{1}{2}[\delta \int_t^\infty (w_s + \phi(\log c_s^N - \log y_{ss}^N)) \exp(-\delta(s-t)) ds]. \quad (3.39)$$

Differentiating (3.2.39), we obtain the changes in newly set wage rate

$$\dot{x}_t = 2\pi_t - \eta_t - \frac{\delta}{2}(\varphi + \phi)(\log c_t^N - \log y_{ss}^N) + \frac{1}{2}\varphi \log \dot{C}_t^N. \quad (3.40)$$

In order to obtain non-tradable inflation dynamics, differentiate (3.2.39) with respect to time and substitute it into (3.2.40)

$$\dot{\pi}_t = \delta(\pi_t - \eta_t) - \frac{\delta^2}{2}(\varphi + \phi)(\log c_t^N - \log y_{ss}^N) + \frac{1}{2}\delta\varphi \log \dot{C}_t^N. \quad (3.41)$$

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<sup>12</sup>In equation(3.2.36), the adjustment parameter is not necessarily equal to  $\delta$ . It could be  $\varsigma \neq \delta$  and, therefore  $\eta_t = \varsigma \int_{-\infty}^t \pi_s \exp(-\delta(t-s)) ds$ . For simplicity, we just assume  $\varsigma = \delta$ . See Ghezzi(2001)

<sup>13</sup>  $\pi_t = \log \dot{P}_t^N = \log \dot{W}_t$

This is the equation for the inflation dynamics. Eq.(3.2.41) indicates that a change in the rate of inflation is dependent on two sources. First, it depends on excess demand conditions: the change of inflation is negatively related to excess demand. The second term of (3.2.41) shows this relation. This is the same result as Calvo and Végh (1993, 1994). The intuition for this higher order inverse Phillips curve is that if there is excess demand at  $t$ , individuals who revise their wages at time  $t$  set higher wages. Therefore, the higher excess demand at time  $t$ , the higher the rate of inflation will be. However, since agents set their nominal wages in a forward-looking manner, wage setters at  $s > t$  do not take excess demand at time  $t$  into account. Hence, the higher the excess demand at  $t$ , the greater the reduction in the inflation rate for  $s > t$  will be.

In equation (3.2.41), we can see that there is another source for the change in the rate of inflation. It shows that current inflation depends on a weighted average of past inflation. Especially, when averaging past inflation, the inflation rate in the recent past receives more weight which is the same formulation as adaptive expectations. If current inflation is lower than  $\eta_t$ , an average of past inflation, the newly contracted wage has a lower premium over the current price level, which will be translated into lower inflation. This makes inflation sticky. The first term on the right-hand side of (3.2.41) represents this relation. The backward-looking wage contract mechanism implies that inflation has its own persistence in addition to the inertia in the driving term, which is the change in the level of excess demand. By this specification, we can overcome the inability of the standard new-Keynesian

contract model to generate significant persistence of inflation. The change of aggregate demand in the non-tradable good sector affects inflation positively. This is the original Phillips curve mechanism. In this regard, equation (3.2.41) can be interpreted as a traditional Expectations-Augmented Phillips curve.

In this model, there is another source for inflation dynamics. In contrast to a constant steady-state level of non-tradable output as in Calvo and Végh models, the long-run level of non-tradable output is subject to a nominal variable, since the nominal interest rate introduces a wedge between consumption and labor supply. Therefore, non-tradable output shows different dynamics with respect to the policy change in this model.

In order to analyze the dynamic system of the economy, we will need to obtain the equilibrium path of consumption of tradables and non-tradables. From Eqs (3.2.8), (3.2.11) and (3.2.30), we can see that

$$C_t^T = \frac{\gamma}{\lambda} (1 + \alpha i_t)^{-1} \quad (3.42)$$

$$C_t^N = \frac{1 - \gamma}{\gamma} e_t C_t^T \quad (3.43)$$

and

$$\lambda = \frac{\gamma \int_0^\infty (1 + \alpha i_t)^{-1} \exp(-rt) dt}{k_0 + \frac{1}{r} y^T} \quad (3.44)$$

is the shadow value of wealth, which is constant as long as there are no policy shocks to the economy. From Eqs.(3.2.42) and (3.2.43), we can obtain that

$$\log C_t^T = \zeta - \alpha i_t \quad (3.45)$$

$$\log C_t^N = \log \frac{1 - \gamma}{\gamma} + \log C_t^T + \log e_t, \quad (3.46)$$

where  $\zeta = \frac{\gamma}{\lambda}$  is constant.

Differentiating (3.2.45) with (3.2.46) yields

$$\log \dot{C}_t^N = \log \dot{e}_t - \alpha \dot{i}_t. \quad (3.47)$$

By definition,  $e_t = \frac{E_t P^T}{P_t^N}$ . Therefore,

$$\log \dot{e}_t = \epsilon_t - \pi_t. \quad (3.48)$$

Then, we can obtain

$$\log \dot{C}_t^N = (\epsilon_t - \pi_t) - \alpha \dot{i}_t. \quad (3.49)$$

This is the equation for the aggregate demand dynamics of non-tradables. Equation (3.2.49) indicates that a change in demand for non-tradable goods arises from two sources: real appreciation and the change in the nominal interest rate. The increase in the nominal interest rate reduces the demand for non-tradables by increasing the effective price of non-tradable goods. Real appreciation induces the intratemporal substitution between non-tradable and tradable goods.

In the short-run where all prices are sticky, the equilibrium level of employment is demand-determined. Therefore, non-tradable output is also demand-determined. It implies

$$\log y_t^N = \log C_t^N. \quad (3.50)$$

Combining equations (3.2.49) and (3.2.50), we can obtain the differential equation governing the change of the aggregate demands for non-tradable outputs

$$\log \dot{y}_t^N = (\epsilon_t - \pi_t) - \alpha \dot{i}_t. \quad (3.51)$$

### 3.3 An interest rate rule with an inflation target

This section studies the effects of inflation stabilization policy. We consider the case that policymakers announce an inflation target, and let the nominal interest rate vary according to the change in inflation rate.

#### 3.3.1 Dynamic System

Suppose that policy makers have an inflation target and adjust the nominal interest rate gradually to achieve the target rate. Formally

$$\dot{i}_t = \chi(\pi_t - \bar{\pi}), \quad (3.52)$$

where  $\chi > 0$ . This policy regime implies that the nominal interest rate is increased when the inflation rate is above its target ( $\bar{\pi}$ ).

From equation (3.2.51), it follows that

$$\log y_t^N = (\epsilon_t - \pi_t) - \alpha\chi(\pi_t - \bar{\pi}). \quad (3.53)$$

Combining Equations (3.2.41), (3.2.50), (3.2.51) and (3.3.1), we are able to obtain

$$\dot{\pi}_t = \delta(\pi_t - \eta_t) - \frac{\delta^2}{2}(\varphi + \phi)(\log y_t^N - \log y_{ss}^N) + \frac{1}{2}\delta\varphi[(\epsilon_t - \pi_t) - \alpha\chi(\pi_t - \bar{\pi})]. \quad (3.54)$$

Equations (3.2.36), (3.3.1), (3.3.2), and (3.3.3) constitute a four-equation differential equations system in  $i_t, y_t^N, \pi_t$ , and  $\eta_t$ .

Now we will introduce a stabilization policy. Suppose that, prior to any stabilization (for  $t < 0$ ), the target rate of inflation is  $\pi^H$ , and is expected to remain

at that level forever. For a given inflation rate, the economy is at a steady state characterized by

$$\begin{aligned}
\epsilon_{ss} &= \pi^H \\
i_{ss} &= r + \pi^H \\
\pi_{ss} &= \eta_{ss} = \pi^H \\
C_{ss}^N &= y_{ss}^N(i_{ss}) = [1 + (\frac{\rho}{1-\gamma})(\frac{\theta}{\theta-1})(1 + \alpha i_{ss})]^{-1} \\
C_{ss}^T &= y^T + rk_0 \\
e_{ss} &= (\frac{\gamma}{1-\gamma}) \frac{C_{ss}^N}{C_{ss}^T}.
\end{aligned} \tag{3.55}$$

At a steady state, consumption of tradable goods is equal to its permanent income level, while consumption of non-tradables equals its full-employment level. The full employment level of non-tradables is less than the socially efficient level due to the imperfect competition in the non-tradable sector. However, this full-employment level of output is not constant. Monetary policy which affects the nominal interest rate can change the long-run level of non-tradable output. Nominal variables (the interest rate and the rate of depreciation) are growing at the inflation rate,  $\pi^H$ . The real exchange rate is equal to the ratio of the steady state consumption of non-tradables to tradables.

To examine the transitional adjustment of the economy, we have to consider the dynamic system. The linear approximation of the system around the steady-state

is given by

$$\begin{bmatrix} \dot{i}_t \\ \log \dot{y}_t^N \\ \dot{\pi}_t \\ \dot{\eta}_t \end{bmatrix} = \begin{bmatrix} \chi & 0 & 0 & 0 \\ -\alpha\chi & 0 & -1 & 0 \\ -\frac{1}{2}\alpha\phi\varphi\chi & -\frac{\delta^2}{2}(\varphi + \phi) & -\delta(\frac{1-\chi}{2}) & -\delta \\ 0 & 0 & \delta & -\delta \end{bmatrix} \times \begin{bmatrix} i_t - i_s s \\ \log y_t^N - \log y_{ss}^N \\ \pi_t - \pi_H \\ \eta_t - \pi_H \end{bmatrix}. \quad (3.56)$$

Now, we need to investigate the stability of the system. It exhibits two convergent (non-positive) roots and two explosive (non-negative) roots (see Appendix A.3.1). At time  $t$ , current inflation,  $\pi_t$ , is given by an average of past inflation, current excess aggregate demand and future economic conditions, including future inflation. Therefore,  $\pi_t$  becomes a jump variable at time  $t$  even if it cannot adjust immediately to a new value. The partial forward-looking indexation (half of households set the new wage in a forward-looking manner) makes inflation jump initially to the point between the new steady state and the initial steady state after a shock.  $\log y_t^N$  is also a jump variable (notice that by the equilibrium condition,  $\log y_t^N = \log c_t^N$ ). However, as one can see in (3.2.35),  $\eta_t$  is a predetermined variable. Also since the interest rate rule is specified in terms of rates of change, the nominal interest rate is a predetermined variable at each instant in time. Thus, there are two predetermined variables ( $i$  and  $\eta$ ) and two jump ( $\pi$  and  $\log y^N$ ) variables. Therefore, the system exhibits saddle path stability. Also the roots may be complex conjugates, and the system may show cyclical dynamics.<sup>14</sup>

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<sup>14</sup>For more detail, see Blanchard and Khan (1980) and Turnovsky (1995).



Since the system has four variables, it will be difficult to analyze the equilibrium paths. However, it is possible to see the exact transitional dynamics of the system by resorting to the methods of dominant eigenvalue proposed by Calvo (1987). The system has two non-positive roots and two non-negative roots. Let  $\nu_i, i = 1, 2$ , be the non-positive roots and  $\nu_j, j = 3, 4$  be the non-negative roots, with  $\nu_1 > \nu_2$  ( $\nu_1$  is the dominant eigenvalue). Setting to zero the constant corresponding to the unstable root ( $\nu_3$  and  $\nu_4$ ),<sup>15</sup> the solution to dynamic system (3.3.5) can be expressed as

$$\begin{aligned}
i_t - i_{ss} &= A_1 \omega_{11} \exp(\nu_1 t) + A_2 \omega_{21} \exp(\nu_2 t) \\
\log y_t^N - \log y_{ss}^N &= A_1 \omega_{12} \exp(\nu_1 t) + A_2 \omega_{22} \exp(\nu_2 t) \\
\pi_t - \pi_H &= A_1 \omega_{13} \exp(\nu_1 t) + A_2 \omega_{23} \exp(\nu_2 t) \\
\eta_t - \pi_H &= A_1 \omega_{14} \exp(\nu_1 t) + A_2 \omega_{24} \exp(\nu_2 t),
\end{aligned} \tag{3.57}$$

where  $A_i, i = 1, 2$ , and  $\omega_{ij}, j = 1, 2, 3, 4$ , denote the constants and the element of the eigenvector associated with root  $\nu_i$ .

From the assumption  $\nu_1 > \nu_2$ , it follows that (see Appendix A.3.2)

$$\lim_{t \rightarrow \infty} \frac{\log y_t^N - \log y_{ss}^N}{\pi_t - \pi_H} = \frac{\omega_{12}}{\omega_{13}} > 0. \tag{3.58}$$

This implies that as  $t$  becomes large, if the initial value of  $\log y^N$  and  $\pi$  are not on the ray corresponding to solution,  $\nu_2$ , then the ratio of  $\frac{\log y_t^N - \log y_{ss}^N}{\pi_t - \pi_H}$  will converge to the slope of the dominant eigenvector ray ( $\frac{\omega_{12}}{\omega_{13}}$ ), which is positive. It

implies that, graphically, the system will converge asymptotically to the dominant

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<sup>15</sup>It is clear from the transversality condition that a necessary condition for convergence of the system is that  $\nu_3 = \nu_4 = 0$ .

eigenvector ray. Figure 3.1 shows the phase diagram for  $\log y_t^N$  and  $\pi_t$  with the dominant eigenvector ray.

### 3.3.2 An interest rate rule with an inflation target

We, now, consider the impact of an inflation targeting policy which is a once-and-for-all reduction in the target rate of inflation. Suppose that at time  $t < 0$ , the economy is at the initial steady state. In terms of Figure 3.1, the initial steady state is at point  $SS_0$ . At time 0, the policymaker announces that he or she will reduce the target rate of inflation permanently, which is unanticipated. Formally, the policy is

$$\begin{aligned}\pi_t &= \pi^H, \text{ for } t \leq 0 \\ \pi_t &= \pi^L, \text{ for } t > 0.\end{aligned}$$

On impact, inflation jumps down as the policy maker reduces the target rate of inflation. However, the inflation rate will jump to a point between  $\pi_H$  and  $\pi_L$ . This is due to the backward-looking indexation of wage contracts even if the forward-looking contracts make it possible that the inflation rate jumps on impact. Thus the inflation rate is higher than the target rate on impact. After initial impact, the inflation rate declines to the target rate slowly. The backward-looking indexation of wage contracts also play a key role in slow convergence of inflation rate to the target rate, inflation inertia.

From Eq(3.3.1), we can see that on impact the nominal interest rate begins to

increase as current inflation is higher than the target rate. The increase in interest rate continues until inflation reaches the target rate. During this transition, the economy experiences a higher nominal interest rate. A higher nominal interest rate induces gradual nominal appreciation. And the nominal appreciation leads to an appreciation of the real exchange rate since the domestic inflation is sticky.

Now consider the supply-side effects of disinflation. The increase in the nominal interest rate implies a higher monetary wedge (or distortion), generated by the cash-in-advance constraint, between consumption and leisure. Thus, the individual household decreases labor supply, which leads non-tradable output to contract. Since the lower labor supply implies less labor income, the household will reduce demand for consumption of non-tradables. This decrease in labor supply triggers a negative wealth effect on non-tradable output.

Now look at the policy effect on consumption. It is not obvious whether consumers would reduce their consumption of the tradable good. First, consumer would reduce the consumption of the tradable as the nominal interest rate increases. From Eq(3.2.45), we can see

$$\log \dot{C}_t^T = -\alpha \dot{i}_t. \quad (3.59)$$

The reason is as follows. Since the consumer finds that the effective price of tradables (market price and the opportunity cost of holding money) will be higher, he or she will engage in intertemporal consumption substitution. Hence, tradable consumption will be reduced after impact. However, the real appreciation triggered by the increase in the nominal interest rate reduces the relative price of tradable

goods in terms of non-tradables. Thus consumers will increase consumption of tradables. The total effect depends on the size of intertemporal and intratemporal elasticities of substitution. In this paper, we assume the both elasticities of substitutions are equal to 1, which implies that those effects cancel each other out. As a result consumption of tradable goods remains unchanged. Consumption of non-tradables, however, jumps down on impact after the target rate is lowered. The initial increase in the nominal interest rate sparks the nominal appreciation. The nominal appreciation leads to an appreciation of the real exchange rate which in turn reduces consumption of non-tradables. The initial jump of non-tradables is induced by intratemporal substitution. After initial impact, due to the increase in the nominal interest rate, the consumer finds that the effective price of tradables (market price and the opportunity cost of holding money) will be higher, he or she reduces consumption of non-tradable goods. This is the intertemporal consumption substitution effect. Also supply-side effects play a key role in reducing non-tradable consumption. Hence, tradable consumption will be reduced after impact. The overall effect means that the economy jumps to the point A in Figure 3.1 and converges to the point B. So initial disinflation policy triggers a severe recession.

After initial impact, the nominal interest rate is still increasing as the inflation rate is higher than the target rate. So consumption and output of non-tradables decreases until inflation converges to the target rate. The recession observed under the interest rate rule is more severe and longer than in the money-based stabi-

lization case. In the money-based stabilization, the initial recession results from decreases in demand. After the initial recession, the economy starts to recover immediately. However, interest rate policy induces a long-lived, severe recession since it is triggered by a decrease in demand as well as in supply (decrease in labor supply). After initial impact, the economy moves to the point B in Figure 3.1.

An intriguing phenomenon observed in this exercise is that inflation shows undershooting during the transition. Before it reaches the new level, inflation remains below the target rate for substantial periods. The intuition behind the inflation undershooting result just discussed is as follows. The permanent reduction in the target inflation rate implies that, in the new steady state, inflation, and thus the nominal interest rate, will be lower, and output higher. Hence real money balances in the new steady state will be higher. How will the increase in real money balances come about? Since nominal money balances do not change at all, the only way for the economy to generate higher real money balances is for the inflation to undershoot. Thus tight monetary policy forces the economy to undergo a deflationary period.

As inflation undershoots its long run value, the target rate (or new steady state value), the nominal interest rate starts decreasing. Therefore, the supply-side effects of disinflation come in through the change in the labor supply. The reduction in the nominal interest rate implies a reduction in the monetary wedge (or distortion), generated by the cash-in-advance constraint, between consumption and leisure. Thus, the individual household increases labor supply, which leads non-

tradable output to expand. Since the greater labor supply implies more labor income, the household will increase demand for non-tradables. This permanent increase in labor supply triggers a positive wealth effect on non-tradable output. At  $B$ , non-tradable output starts to increase and inflation keeps increasing slowly. The economy begins to recover from recession. Inflation also converges to the new steady state level. Finally, they converge to their new steady state ( $SS_1$ ). The dynamic path of non-tradable goods and inflation is depicted in Figure 3.1.

The dynamic paths of the other main variables we are concerned with are depicted in Figure 3.2 to 3.7. The first thing we have to notice in this experiment is the movement of inflation. We observe that inflation shows inertia. This result is mainly due to backward-looking indexation in wage the contract. As long as the indexation mechanism has a backward-looking component, the policymaker can not expect rapid disinflation. Inflation persistence in this model also is the main cause for the disinflationary recession. The slow convergence of inflation triggers a higher nominal interest rate, which significantly reduces consumption and output of non-tradables. If inflation converges to the target rate rapidly, the economy avoids a recession during the transition period. This result assures a well established notion that disinflation is contractionary. The contractionary effect of disinflation, however, usually arises from contraction in aggregate demand. In this paper, the demand side as well as the supply side contracted during disinflation, which makes the recession more severe. In a high inflation environment, high nominal interest rates significantly reduce labor supply since the public tries to

reduce the high opportunity cost of money. Hence it allows individuals less time for productive activities: less consumption and production. Therefore, increasing the nominal interest rate for disinflation results in a recession.

The movement of real and nominal exchange rates are presented in Figure 3.6 and 3.7. We can see that for substantial periods of time, both real and nominal exchange rates appreciate due to the high nominal interest rate.

### **3.4 Concluding remarks**

This paper examines the inflation stabilization when policymakers use the nominal interest rate as main anchor. Recently, policymakers increasingly view short-term nominal interest rates as the main instruments of monetary policy, often in conjunction with an inflation target. Inflation targeting has been implemented in many industrial countries. Short-term interest rates have also played a key role in the exchange rate-based inflation stabilization programs in chronic inflation countries to ensure a rapid fall in inflation as well as to prevent a speculative attack against the domestic currency.

Policymakers prefer a higher interest rate for two reasons: first, exchange-rate based stabilizations often end up with balance of payment crises. Therefore, interest rate policy if it is used under the exchange rate-based stabilization has better chance to bring down inflation without crisis. Second, under interest rate policy, it is much easier for the public to identify the government's disinflation policy than under money based stabilization in which the growth rate of money supply is used.

This paper has simulated an inflation targeting rule in the context of a sticky-inflation, small-open economy model. Especially, we focus on the inflation stabilization effect of nominal interest rate policy in high inflation environment. In this case, the nominal interest rate is the main anchor.

The main lesson from the analysis is that using the nominal interest rate may be not a good policy option to fight high inflation. A high interest rate succeeds in reducing inflation in the long-run, but results in severe recession during the transition period. This recession arises from both demand and supply sides effects. In the present paper, the individual's optimal labor supply is dependent on the nominal interest rate. Since the nominal interest rate introduces a distortion between consumption and leisure, a change in the interest rate makes the household substitute leisure for consumption due to the change in the effective price of consumption. Hence high and increasing interest rates substantially reduce labor supply. From this perspective, an inflation targeting rule is not appropriate in high inflation environment under which the cost of holding money is high even if the high interest rate due to the inflation inertia prevents speculative attacks against the domestic currency during the stabilization. The speed of convergence of inflation we observed in this experiment is not significantly different from exchange rate and money-based stabilizations with a backward-looking wage contract.

In this paper, however, we do not consider the case that the policymaker raises the nominal interest rate under the exchange-rate based policy. Applying interest policy to the exchange rate-based stabilization would be an promising extension.



Another possible extension is to incorporate the target inflation rate into a backward-looking indexation. In that case, we can expect that inflation converges to the target rate rapidly, and no recession results.

## 3.5 Appendix

### 3.5.1 The stability of Equation (3.3.5)

The stability of the system requires that among the eigenvalues of the matrix associated with dynamic system (3.3.5), two eigenvalues have positive real parts and the other two eigenvalues have negative real parts.

The eigenvalues  $\lambda_i, i = 1, 2, 3, 4$ . of the system

$$\begin{bmatrix} \dot{i}_t \\ \log \dot{y}_t^N \\ \dot{\pi}_t \\ \dot{\eta}_t \end{bmatrix} = \begin{bmatrix} \chi & 0 & 0 & 0 \\ -\alpha\chi & 0 & -1 & 0 \\ -\frac{1}{2}\alpha\phi\varphi\chi & -\frac{\delta^2}{2}(\varphi + \phi) & -\delta(\frac{1-\chi}{2}) & -\delta \\ 0 & 0 & \delta & -\delta \end{bmatrix} \times \begin{bmatrix} i_t - i_{ss} \\ \log y_t^N - \log y_{ss}^N \\ \pi_t - \pi_H \\ \eta_t - \pi_H \end{bmatrix} \quad (3.60)$$

have the following properties

$$\det(A) = \lambda_1\lambda_2\lambda_3\lambda_4 = \frac{\delta^3\chi}{2}(\chi + \phi) > 0 \quad (3.61)$$

$$\lambda_1\lambda_2 + \lambda_1\lambda_3 + \lambda_1\lambda_4 + \lambda_2\lambda_3 + \lambda_2\lambda_4 + \lambda_3\lambda_4 = -\frac{\delta^2}{\mu}(\phi + \alpha\delta) < 0 \quad (3.62)$$

$$Tr(A)^2 - 4\det(A) > 0. \quad (3.63)$$

From (A.3.2), we can see that the four eigenvalues are positive or two are positive and two are negative. From (A.3.3), the case of four positive roots can be ruled out. As a result, the system has two negative roots. Furthermore, according to (A.3.4), the eigenvalues may be complex conjugates and the system may exhibit cyclical behavior.

### 3.5.2 The dominant eigenvalue ray

We assume that  $\nu_i, i = 1, 2$  is the non-positive roots, with  $\nu_1 > \nu_2$ . Then, for  $i = 1, 2$ , it follows that

$$\begin{bmatrix} \chi - \nu_i & 0 & 0 & 0 \\ -\alpha\chi & 0 - \nu_i & -1 & 0 \\ -\frac{1}{2}\alpha\phi\varphi\chi & -\frac{\delta^2}{2}(\varphi + \phi) & -\delta(\frac{1-\chi}{2}) - \nu_i & -\delta \\ 0 & 0 & \delta & -\delta - \nu_i \end{bmatrix} \times \begin{bmatrix} \omega_{i1} \\ \omega_{i2} \\ \omega_{i3} \\ \omega_{i4} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \quad (3.64)$$

where  $\omega_{ij}, j = 1, 2, 3, 4$ , are the element of the eigenvector associated with root  $\nu_i$ .

Therefore,

$$\frac{\omega_{i2}}{\omega_{i3}} = -\frac{1}{\nu_i} > 0. \quad (3.65)$$

Now, let's look at the dominant eigenvector ray. Setting to zero constant corresponding to the unstable roots ( $\nu_3, \nu_4$ ), the solution to this system takes the form

$$\begin{aligned} i_t - i_{ss} &= A_1\omega_{11} \exp(\nu_1 t) + A_2\omega_{21} \exp(\nu_2 t) \\ \log y_t^N - \log y_{ss}^N &= A_1\omega_{12} \exp(\nu_1 t) + A_2\omega_{22} \exp(\nu_2 t) \\ \pi_t - \pi_H &= A_1\omega_{13} \exp(\nu_1 t) + A_2\omega_{23} \exp(\nu_2 t) \\ \eta_t - \pi_H &= A_1\omega_{14} \exp(\nu_1 t) + A_2\omega_{24} \exp(\nu_2 t), \end{aligned} \quad (3.66)$$

where  $A_i, i = 1, 2$ , denote the constants associated with root  $\nu_i$ . Since  $\nu_1 > \nu_2$ , it follows that:

$$\begin{aligned} \lim_{t \rightarrow \infty} \frac{\log y_t^N - \log y_{ss}^N}{\pi_t - \pi_H} &= \lim_{t \rightarrow \infty} \frac{A_1\omega_{12} \exp(\nu_1 t) + A_2\omega_{22} \exp(\nu_2 t)}{A_1\omega_{13} \exp(\nu_1 t) + A_2\omega_{23} \exp(\nu_2 t)} \\ &= \lim_{t \rightarrow \infty} \frac{A_1\omega_{12} + A_2\omega_{22} \exp((\nu_2 - \nu_1)t)}{A_1\omega_{13} + A_2\omega_{23} \exp((\nu_2 - \nu_1)t)} \\ &= \frac{\omega_{12}}{\omega_{13}} > 0. \end{aligned} \quad (3.67)$$

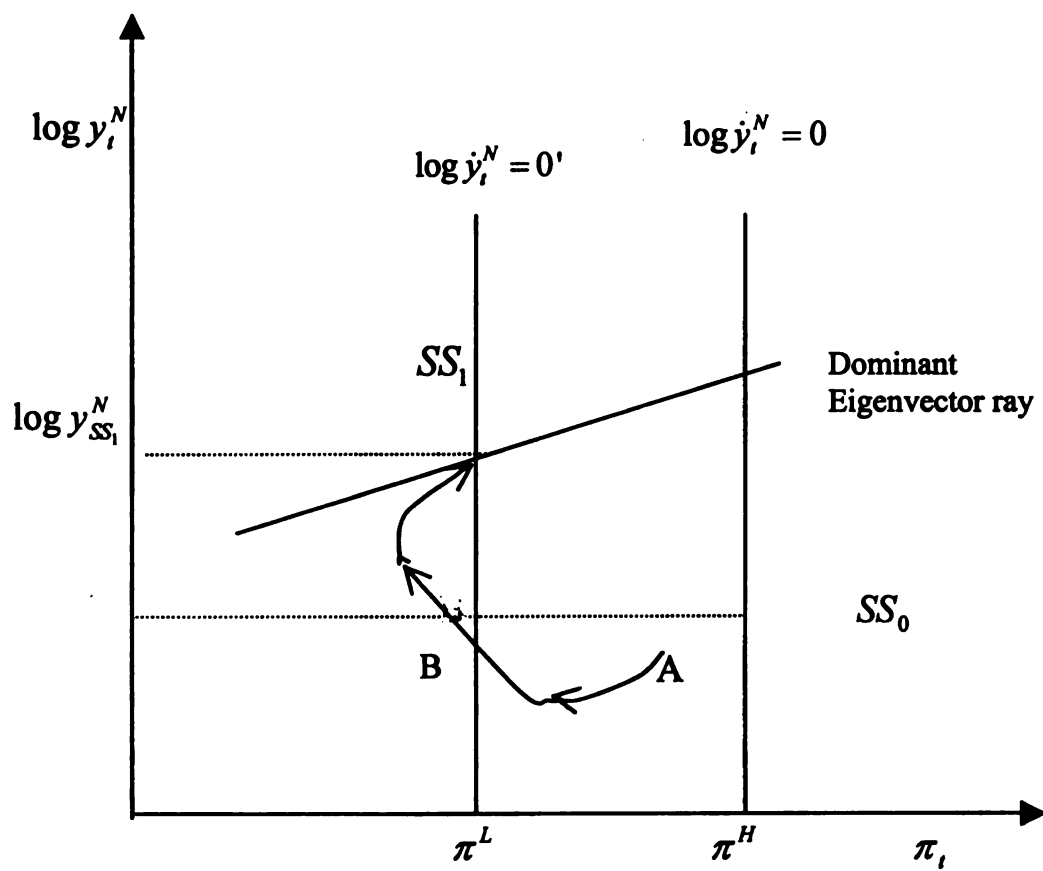


Figure 3.1: Dynamic system

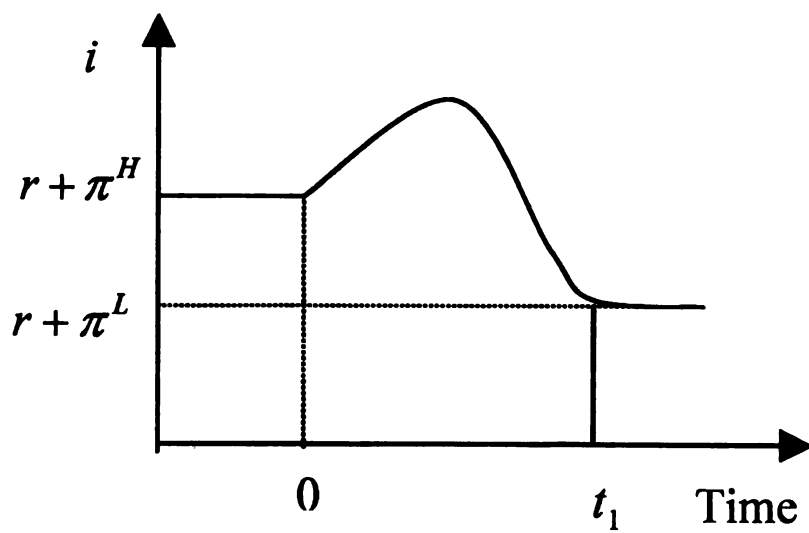


Figure 3.2: Time path: nominal interest rate

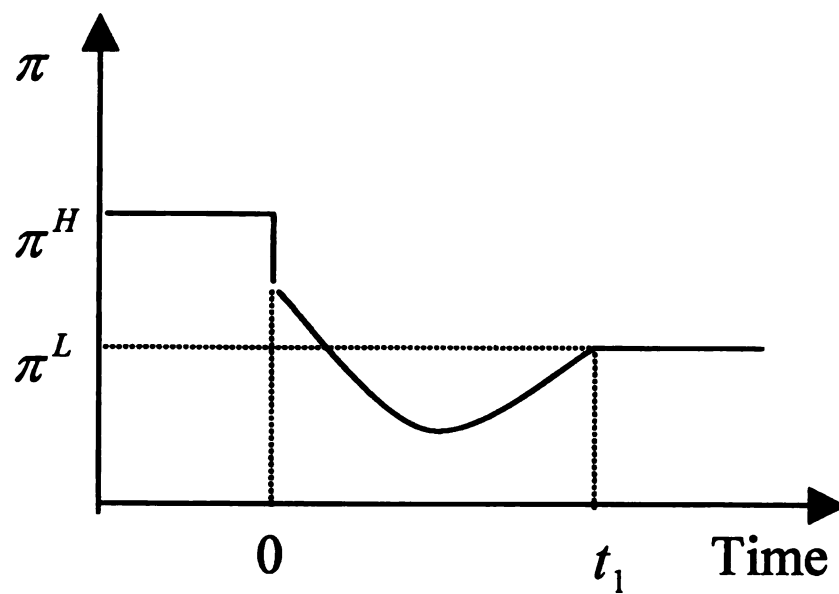


Figure 3.3: Time path: inflation

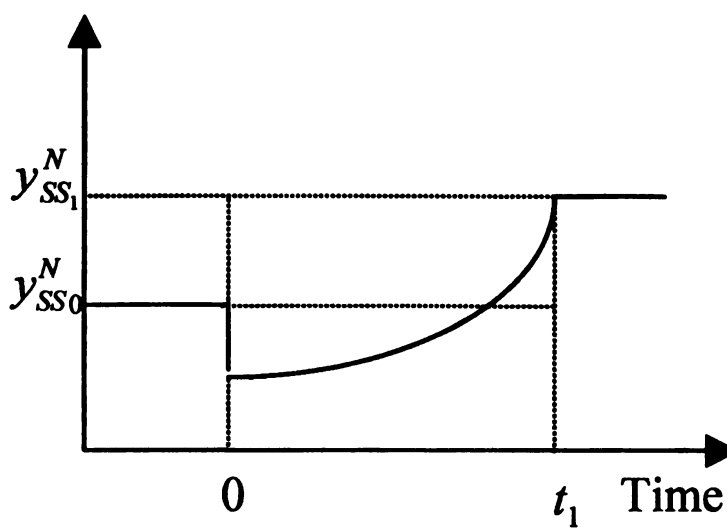


Figure 3.4: Time path: output of non-tradables

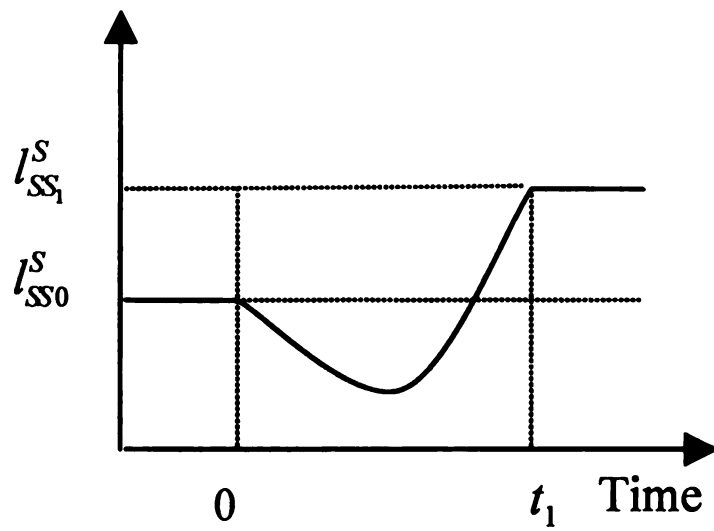


Figure 3.5: Time path: labor supply

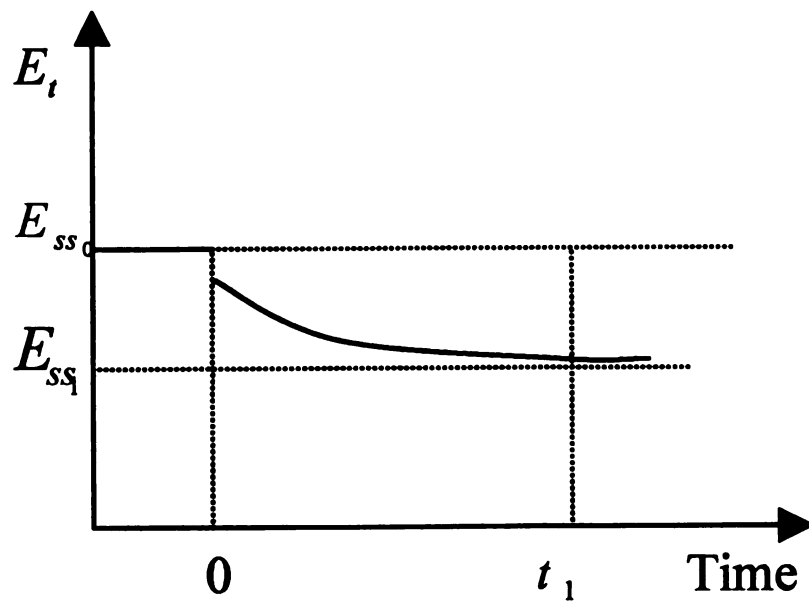


Figure 3.6: Time path: nominal exchange rate

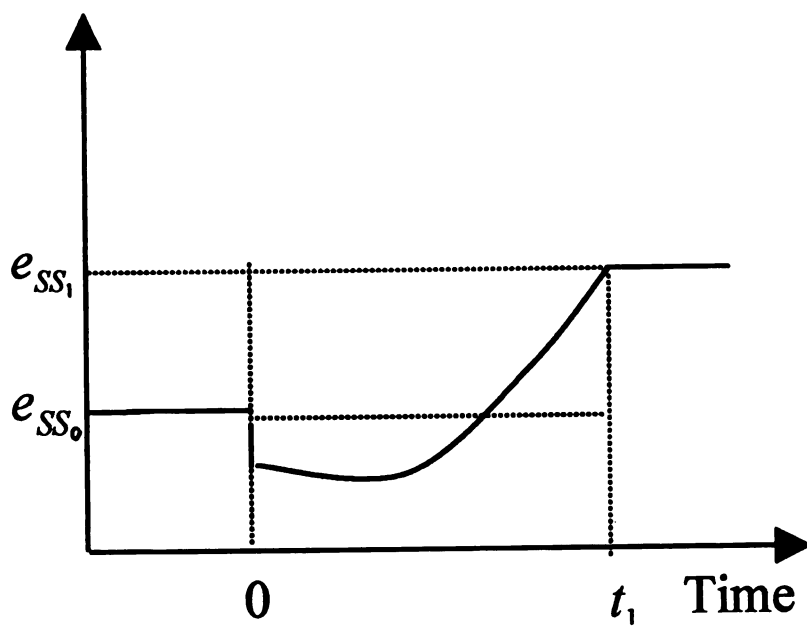


Figure 3.7: Time path: real exchange rate

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