This is to certify that the
dissertation entitled

Homework as a Boundary Tool: A Case of Teacher Wang's
Homework Activities in Middle Grades Mathematics Teaching
in Shanghai

presented by

Yanping Fang

has been accepted towards fulfillment
of the requirements for the

Ph.D. degree in Curriculum, Teaching and Educational Policy

Major Professor's Signature

6/27/05

Date

HOMEWORK AS A BOUNDARY TOOL: A CASE OF TEACHER WANG'S HOMEWORK ACTIVITIES IN MIDDLE GRADES MATHEMATICS TEACHING IN SHANGHAI

By

Yanping Fang

A DISSERTATION

Submitted to
Michigan State University
in partial fulfillment of the requirements
for the degree of

DOCTOR OF PHILOSOPHY

Department of Teacher Education

2005

ABSTRACT

HOMEWORK AS A BOUNDARY TOOL: A CASE OF TEACHER WANG'S HOMEWORK ACTIVITIES IN MIDDLE GRADES MATHEMATICS TEACHING IN SHANGHAI

By

Yanping Fang

Homework is a tool for school learning around the world. It is devised mainly for students to practice what is taught. In China, homework is also made for teachers; it is a teacher's job responsibility to mark student work and provide feedback to students in a timely manner. But teachers also analyze homework to inform their teaching. My observations of one teacher, Teacher (Tr.) Wang and her colleagues, a group of 8th grade mathematics teachers in a middle school in Shanghai, reveal the prominence of homework in her daily work.

In this study, I investigate four major homework-related activities of Tr. Wang and describe and analyze what they each entailed and made possible for her teaching and student learning. These four activities are: marking homework, explaining selected errors to the whole class, tutoring individual students on their errors and talking with colleagues about homework-related issues. Drawing on theory and methods from cultural-historic activity theory, cognitive psychology, ethnography and classroom discourse analysis, I collected and analyzed telephone interviews with mathematics teachers in Shanghai, focused observations of Tr. Wang's homework activities in her office and classrooms, audiotaped classroom and office observations, field notes, marked student work samples, and curriculum materials.

Analyses of the above data suggest several findings about how homework was used by Tr. Wang. (1) In marking homework, she created a communicative system of symbols and signs on student work to communicate teacher feedback; while marking, she was making sense

of student problems of learning, selecting typical and important errors to explain and tutor. (2) In explaining selected homework errors to the class, Tr. Wang offered structured and detailed explanations of the important mathematics behind the errors from multiple perspectives. (3) In tutoring, she summoned students she identified to her office for individual assistance in which she diagnosed their learning issues and offered guidance for how to make corrections. (4) While marking homework together with her deskmate colleague, they shared student problems in homework and their stress from the challenges of helping all students learn. She and her math colleagues also informally deliberated on the problems arising from teaching and homework and figured out specific ways to resolve the problems. Such collective problem solving provided valuable learning opportunities for Tr. Wang and her colleagues.

This study offers a window into a community of practice and the systematic use of homework as a tool to advance a teacher's pedagogical reasoning and action. In this process, Tr. Wang polished raw errors and turned them into valuable teaching opportunities. Homework was used as a boundary object (Wenger, 1998) to enable the teacher to cross the boundary between teaching and learning, to coordinate the goals of her different activities and center them around student learning, and with her colleagues, to collectively inquire into ambiguities in the curriculum and student learning. Embedded in these activities are long traditions of a Confucian culture that emphasize good performance in examinations. This study has implications for rethinking the pedagogical role of homework. It suggests organizing teachers' work to enable teachers to develop their content and teaching knowledge as well as knowledge about student learning through their daily practice.

ACKNOWLEDGEMENTS

This dissertation is not possible without numerous support from professors, colleagues and friends. I am most indebted to my advisor, Lynn Paine's boundless support, nurture, and encouragement throughout my doctoral program at Teacher Education Program, Michigan State University. It was through her research grant that I was able to secure assistantships to support my study, gain research apprenticeship in her NSF-sponsored grant from which the research questions were raised and developed. Over the years, Lynn not only patiently guided my research assistantship work, course taking but also provided huge support and friendship to my family to make it possible for us to brave together the entire arduous journey of pursuing an advanced degree in a new culture together. Professionally, Lynn served as a role model to me in terms of how to become a serious scholar and a responsible teacher educator who cares very much about her students.

Next, my deep appreciation goes to my Dissertation Committee, Helen Featherstone, Ralph Putnam, Sandy Wilcox, and Gary Sykes. They offered sustained support and encouraging words at critical times, waited patiently to read my chapters and gave thoughtful feedback to my writing. Revision of the dissertation would not see remarkable improvement without their concerted effort in putting their insights together to come to consensus on major revision suggestions. Their successful collaboration has made the writing process a much more enjoyable experience.

Third, I want to thank Tr. Wang and her colleagues who opened the door to my observation of their daily work, accepted my numerous informal interviewing, and allowed me to follow them to meetings and have lunch conversations. I also would like to thank Professor Gu Linyuan of East China Normal University and Head of National Elementary and Secondary Teacher Professional Development Research Center, Shanghai Branch for his making time for interviews and conversations. His ideas have widened my view of the middle school mathematics teaching in Shanghai and the embedded Chinese cultural heritage.

I also appreciate the wonderful editing assistance provided by friends and colleagues. Among them, Tina Urbain, Marcy Wood, and Kristen Millar offered additional editing support that improved the reading of the language.

Last but not least, my endless love and thankfulness for my husband, Jianjiang Wu and my daughter, Xiaotong Wu. Without their patience, dedication, and understanding, this dissertation and degree would be far from reach.

TABLE OF CONTENTS

LIST OF TABLES

LIST OF FIGURES

A De

Nove

teach

home

left

the ca

buildi

two hi

Wang

all cla

classro

floor

reside

studen

[1] The na

[2] This is

workday

[3] I use th

embody

whether

"Teache

her.

[4] "Class

students

progres

class dir

simply t

[5] Startin

year of

that the

grade at

class siz

Chapter One

The Prominence of Homework in Tr. Wang's Daily Work

A Depiction of Teacher Wang's[1] Typical Workday[2]

At 7:40, early in the morning on a chilly late autumn day in the middle of November, Teacher Wang, a middle-aged and experienced 8th grade mathematics teacher at Forest Land Middle School in Shanghai, was busy marking student homework in her third-floor teachers' office. Visible from a half-open window to the left of her desk, the golden and brownish leaves of the sycamore trees shuddered in the early winter chill. Located at the right end of an L-shaped, simple, 5-story school building, the small grade-level office was shared by six teachers: two math teachers, two history teachers, one Chinese teacher and one English teacher. Except for Tr. Wang[3] and another younger female history teacher, the other four officemates were all class directors[4] (banzhuren), head teachers each responsible for one of the classrooms they were teaching. Another 8th grade office was located on the fourth floor

In this relatively academically strong school in a heavily populated residential area, for the past decade or so, all 8th grade classes were made up of 60 students each[5]. Tr. Wang and three other colleagues shared the teaching of six 8th

---

[1] The names for the teachers and the students are all pseudonyms.
[2] This is written based on what took place on the first of my full-day observations of Tr. Wang's workdays in November and December of 2002.
[3] I use the abbreviation "Tr." to refer to "Teacher," as Liping Ma (1999) did in her work. I use it to embody the respectful title that Chinese society has traditionally assigned to a teacher. No matter whether it is in the school or out in the neighborhood community, a teacher is always addressed as "Teacher Wang," "Teacher Li," etc., by those who are acquainted with or even familiar with him or her.
[4] "Class director" is a verbatim translation of *banzhuren,* who acts as a head teacher of a classroom of students, taking full responsibility for monitoring their physical safety, disciplinary conduct, academic progress, and emotional health inside and outside school, keeping in direct contact with parents. A class director, in this sense, is different from a *homeroom* teacher and has more responsibilities than simply being a subject teacher. In any school, a portion of subject teachers also serve as *banzhuren.*
[5] Starting in 2002, with the passing of the baby boom era, the number of students enrolled in the first year of junior secondary schools started to drop, and the new Shanghai municipal policy stipulated that the average secondary school class size be no more than 48. The average class size for the sixth grade at Forest Land School was 43 in 2002, which means that when this cohort entered 8th grade, the class size would be considerably smaller than it was in 2002.

grade cl

a weaker

sitting a

teacher

taught (

whose

(

periods,

Group

and org

school

colleag

one per

wide po

took up

don't ha

offices

homewo

consider

H

busy ge

she was

workbo

they all

assignm

and, as

putting

The diff
weaker st
in both c
Since ther
and comp
usually w
adjusted

grade classrooms in the school. She taught Class 4 and Class 2, a stronger class and a weaker one (which was called the "parallel" class[6]*). Tr. Zhao, a male teacher sitting across from her in the same office, taught Class 5, and Tr. Li, a young female teacher who was also the school's vice principal and had an office on the 2nd floor, taught Class 6. Class 1 and Class 3 were taught by Tr. Hu, a young male teacher whose office was on the 4th floor.*

On average, Tr. Wang's official workload included three 40-minute-long periods, about two hours per day. As head of the school's Math Teaching Research Group (MTRG), she attended a one-period weekly school-wide TRG Heads Meeting and organized a biweekly one-period Math TRG Meeting for all math teachers in the school. As an 8th grade math teacher, biweekly she also joined the other three colleagues in the 8th grade Math Lesson Preparation Group (MLPG) Meeting for one period. Besides these teaching-related meetings, she participated in the school-wide political study for one hour every Friday afternoon. The meetings altogether took up about 2 hours of her time per week, on average. Since teachers in China don't have desks in the classrooms, they spend their non-teaching hours in their offices. Tr. Wang said she often spent more than 3 hours daily in her office marking homework and dealing with issues related to homework, which exceeded considerably her daily teaching time.

Having arrived half an hour before the school day started, Tr. Wang was busy getting a portion of one of her classes' homework marked. On being asked why she was doing so, she responded with a sense of urgency, "I have to use them [the workbooks being marked] for today's class!" "Students all did differently; I mean, they all made mistakes of different types," she added. For these geometry proof assignments, she ticked, crossed items off, or wrote comments on each step of a proof and, as usual, wrote the date at the end of the day's assignments (11.19.02) before putting it in the finished pile. "I've got to train their logical thinking habits from

[6] The difference between the two classes was that there was a greater proportion of mathematically weaker students in Class 2 compared to that of Class 4, but there were mathematically strong students in both classes. Because of this, Class 2 was not called a weak class but a class "parallel" to Class 4. Since there is no tracking in middle schools in Shanghai, both classes need to learn the same content and complete the same basic homework assignments assigned to all eighth-graders; but Class 4 usually was assigned a few more additional problems. As discussed in later chapters, Tr. Wang adjusted her teaching of the same lessons to meet the different needs of the two classes.

early on

mind. sh

pen. On

the error

confused

appear a

homewo

"desk m

they ma

other th

on colle

entered

she and

Zhao we

briefly a

discusse

animate

given in

W

fourth fl

She bega

in three

tutored

Five, on

perpend

teaching

the same

teachers

this class

early on!" she explained about her meticulous homework-marking. With this goal in mind, she frowned at errors, paused, then added symbols or comments with her red pen. Once in a while, she turned to the cover to read the name and let out a sigh at the errors and confusions.

During break, she tutored a boy on a homework assignment that he had felt confused about and had come to her for help with. (Details of this tutoring session appear at the beginning of Chapter Six, on tutoring students on homework.) Her homework-marking continued into the second period, when she was joined by her "desk mate," Tr. Zhao. He sat down at his desk, which was adjoined to hers, and they marked homework together, facing each other. They discussed and showed each other their students' problems. (These conversations are discussed in Chapter Seven, on collegial homework conversations.) In the second half of the same period, Tr. Li entered to share her teaching concerns. It was the week after the mid-term exam, and she and Tr. Hu had just begun teaching the unit on functions while Tr. Wang and Tr. Zhao were in the final week of teaching geometry proofs. The three colleagues talked briefly about their Lesson Preparation Meeting to be held in the afternoon and then discussed at length the ambiguity of a functions-related homework assignment and animatedly shared their students' homework errors. (Details of this conversation are given in Chapter Seven's opening vignette.)

When the music sounded to start the third period, Tr. Wang walked to the fourth floor to teach Class 4, carrying the marked workbooks and her teaching tools. She began teaching by spending the first 10 minutes explaining the issues she found in three of the homework assignments she had marked, including the one she had tutored the boy over during break. (Details of her explanation appear in Chapter Five, on explaining homework errors.) By explaining errors in the homework on perpendicular bisectors assigned the previous day, she made a transition to the new teaching topic, the angle bisector theorem. During the break, she went to Class 2 on the same floor. After getting things ready, she walked into the other 8th grade teachers' office next to Class 2, a place she regularly visited before or after teaching this class to talk to Tr. Hu. She told him about their group meeting in the afternoon.

When th
way in w
perpetu
first of th
F
portion
room on
to them
lunch co
classroo
homewo
forgotte
homewo
blackbo
who did
brief tut
teachers
third bo
and sho
explaine
first two
his care
frustrate
figures?
improve
H
each oth
to appea
the hom
students
aimed at

When the fourth period began, she started the lesson somewhat differently from the way in which she had started in Class 4, spending 6 minutes reviewing the perpendicular bisector theorem and its converse and about 4 minutes explaining the first of the three problems she had just explained to Class 4.

Returning to her office after the 4th period, Tr. Wang marked the remaining portion of Class 4's homework. At 11:45, she went to lunch in the teachers' dining room on the first floor. (Teachers' lunches are usually well-prepared and provided to them for free in schools in Shanghai, and student lunches are usually prepared by lunch companies and delivered to the schools, where students eat in their own classrooms.) After lunch, at about 12:35, she resumed marking her students' homework. The student math monitor came in and apologized for something he'd forgotten to do that morning: record the names of those who did not submit their homework. It was his responsibility to write those names on one corner of the blackboard for the class director teacher. She asked him to call two of his classmates who did not do their homework very well. Very soon, the two boys arrived. After a brief tutorial, they were asked to sit in two teachers' empty seats (since the head teachers were in their classrooms monitoring lunch) and correct their mistakes. A third boy came in with his Volume B book in hand. He'd been sick the previous day and showed her his make-up homework for that day. Finding an error, Tr. Wang explained it to him and he also took a seat to complete the correction. One of the first two boys, tall and strong, who was often called to Tr. Wang's office because of his careless homework, approached her several times to ask for help. Obviously frustrated, she said to the boy, "Liu Long, could you use pencil and ruler to draw the figures? Look at this mess! Could you make it out?" The boy returned to the seat to improve his drawing.

For the first period in the afternoon, Tr. Wang, Tr. Zhao, and Tr. Hu joined each other in the library for their meeting, waiting for Tr. Li, the busy vice principal, to appear. While waiting, they started sharing student learning problems found in the homework. A frustrated Tr. Hu drew for his colleagues a figure of a problem his students had badly bungled. The teachers then discussed the school policy that aimed at higher average class scores during final exams, which meant that they had to spend

to achie

where ca

for the d

shown a

different

focusing

students

scoring

scores

how sc

those st

attempt

those sc

approac

strategy

the fill-

difficult

concepts

who wou

student

amount

similar

were to

students

reference

R

resource

This is or
contain br
experiment

to spend more time improving the poor performance of the weaker students in order to achieve this higher average score for their own classrooms. Then they checked where each of them was in their teaching and agreed on the schedules for preparing for the district final exam. Ten minutes later, with the vice principal still not having shown up, the three members started to develop a strategic plan aiming at "treating different levels of students with due measures," or working to raise test scores by focusing their teaching on different goals for different groups of students. They broke students into three groups, according to their scores on the midterm exam. For those scoring below 30 percent, the teachers' strategy was "to eliminate the single digit scores" by consolidating their algebra concepts they are taught and helping them to know some basic concepts of geometry, such as how to use the sum of a triangle. For those students scoring between 40 and 50 percent, the teachers' strategy was to attempt to increase their scores above 60 percent by offering remedial classes. For those scoring above 60 percent, considered the achieving students, the teachers' approach was to focus them on the more challenging optional exam problems. Their strategy was to transform the problems they had used before, for instance, changing the fill-in-the-blank exercises into proof problems. Their objective was not to pursue difficult problems but to make the students solidly understand the fundamental concepts and procedures. Finally, they divided up the work among themselves as to who would be responsible for constructing worksheets for the various levels of student understanding. They agreed that even though this would pose a daunting amount of extra work, the expectations for these additional worksheets would be similar to those for both regular homework assignments and exam sheets: all errors were to be tracked, explained by the teacher, and corrected by students, with students' corrections written on Post-it® Notes stuck right beside the error for later reference.

Returning to her office, Tr. Wang started flipping through one of the two resource books, Tests, Comments and Analysis[7] *(ce ping), thinking. "I told Class*

[7] This is one of the two widely circulated resource books (the other is called The Same Step) that contain brief quizzes, analyses of results, and key points that were collected from teachers and experimented with in schools for quite a few years in the mid-1980s. It was written in strict

Two th...

she said

saying

period

called

on their

disagree

prove

we think

Wang

called t

no. 2 an

briefly

day's h

he was

came to

and how

homewo

classes

other of

During

father's

The sad

and he

complian
sometime
[1] In China
in their ow
either do
class mor
drop in to
are able to
very often

Two that I would give them a quiz this afternoon [during their self-study period[8]]," she said to herself. "I am wondering whether they really are able to do these." So saying, she stood up, went upstairs to give the quiz, and did not return until the third period started at 2:50. She continued marking the homework books for Class 2 and called 8 students (3 girls and 5 boys) from Class 2 for additional individual tutorials on their errors. At 3:45, two boys from Class 4 came to seek help because they disagreed with each other on a proof. "He thinks that this problem expects you to prove congruence twice [the congruence of two pairs of triangles] and I disagree. So we think we should ask you [about this]," said one of the boys. After following Tr. Wang's explanation, they headed to the door. "Li Dongping," Tr. Wang suddenly called to one of them, "Your homework today was not done very well and problems no. 2 and 3 were both proved in a wrong way." She asked him to come over and briefly tutored him on both problems. Noticing that Li had not corrected the previous day's homework mistakes, she asked him to make the corrections and show her when he was done.

Around 4:00 PM, a female math teacher who had retired the previous year came for a visit. She discussed her current teaching assignment at a private school and how the years of having a heavy load of marking and dealing with student homework had made her work seem much easier now that she had two smaller classes to teach. Much of the later part of her conversation with Tr. Wang and two other office mates focused on a female student who was in Tr. Wang's Class 4. During most of the previous year, the girl had copied homework and faked her father's signature on her test sheets. Nobody found her out until later in the year. The sad father blamed the teacher for not finding out early enough to take action, and he talked directly with the principal. Tr. Wang, still marking homework, shared

---

compliance with the municipal textbooks. Tr. Wang chose quizzes, additional teaching examples, and sometimes homework assignments from it.

[8] In China, there are several self-study periods in a week, usually in the afternoon, when students stay in their own classrooms for one or two periods at a time without the supervision of a teacher. Students either do their homework or prepare for the next day's lessons. The class is left to the charge of the class monitors, although the class director, the teacher who takes responsibility for this class, will drop in to check briefly on order and safety. Teachers of major subjects (math, Chinese, and English) are able to borrow these periods to give feedback on homework or a quiz, which Teacher Wang did very often.

that the

proofs

with Tr

until 5:3

girl wh

The Pro

homew

ended th

issues th

assistanc

talked w

exchang

Group M

indirectl

T

Wang's

that the girl did not improve much and currently was struggling with geometric proofs.

Tr. Wang ended the day still marking students' homework and discussing with Tr. Zhao ways to improve their students' final exam scores. She did not go home until 5:30. Tr. Zhao left for home ten minutes later after tutoring on her homework a girl who was regarded as one of the weakest math students in his class.

The Prominence of Homework in Tr. Wang's Workday

As the opening vignette depicts, on this typical, intense workday for Tr. Wang, homework figures prominently in almost every major piece of her practice. She started and ended the day by marking and commenting on homework; she explained a few problematic issues that she identified in homework to students during her teaching; she offered tutoring assistance to individual students on their homework during breaks in her office; and she talked with colleagues about problems related to homework. In addition to informal exchanges with other teachers, formal meetings such as the biweekly Lesson Planning Group Meeting in the afternoon created occasions in which homework was directly or indirectly discussed in planning for the approaching review for the district final exam.

The chart below gives a visual representation of the prominence of homework in Tr. Wang's work activities on this day.

T

comme

minutes

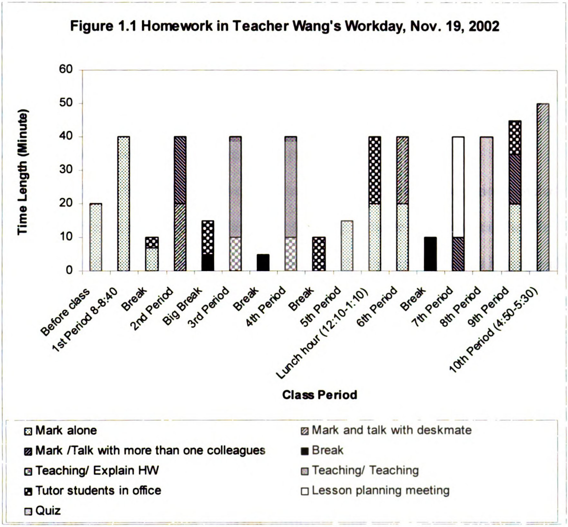
of 60 stu

indicate

Zhao, tal

about 90

marking.



The bars marked with tiny dots indicate Tr. Wang's time spent marking and commenting on homework by herself. This took up a remarkable amount of time, 142 minutes (2.22 hours) for a load of 120 copies of workbooks to mark for her two classrooms of 60 students each. The bars in thin stripes, accompanying most of the dotted ones, indicate that she was marking homework together with her desk mate and colleague, Tr. Zhao, talking while marking. Throughout the day they spent a considerable amount of time, about 90 minutes (1.5 hours), marking homework together and sharing ideas while marking. The thicker stripes of opposite direction represent the time when more than two math co

about 5

Zhao, an

Preparat

teachers

In

during s

Wang ev

white ar

These br

proporti

as percen

Work act

Marking

Marking
with desk

Sharing w
closely re

Explainin
class and
students o

Total tim

Other*

Teaching

Note: * The
unable to f

math colleagues were discussing issues related to homework. The day's discussions were about 50 minutes in total, which included a 20-minute conversation among Tr. Wang, Tr. Zhao, and Tr. Li in the office, the conversation in the first 10 minutes of the Lesson Preparation Meeting among Tr. Wang, Tr. Zhao, and Tr. Hu, and the chat among other teachers in the office with the visiting retired teacher.

In addition, the smaller checkered patterns located between class periods, including during short breaks, after lunch, or during her teaching, indicate the moments when Tr. Wang explained homework errors to the whole class (the checkered patterns in gray and white) and tutored individual students on their homework errors (those in black and white). These brief segments accounted for 55 minutes of the day. Table 1.1 below shows the proportions of time spent on homework-related work activities and the activity of teaching as percentages of her total work hours on that day.

Table 1.1 Homework-Related Activities as Percentages of Tr. Wang's Total Work Time on November 19, 2002

Work activity	Time involved / total work time in the day [expressed in minutes (hours)]	As % of total work time of the day
Marking homework alone	142 (2.22)/505 (8.5)	28%
Marking homework and/or sharing with desk-mate colleague	90 (1.50)/505 (8.5)	18%
Sharing with colleagues on issues closely related to homework	50 (.83)/505 (8.5)	10%
Explaining homework to whole class and tutoring individual students on homework errors	55 (.92)/505 (8.5)	11%
Total time related to homework	340 (5.67)/505 (8.5)	67%
Other*	45 (.75)/505 (8.5)	9%
Teaching and administering quiz	120 (2.00)/505 (8.5)	24%

Note: * There are 45 minutes unaccounted for, when Tr. Wang went to attend to other business to which I was unable to follow and observe her.

activitie

40-minu

totaling

official

one perio

typical e

needed t

necessar

algebraic

(

shadowe

consiste

Wang's

workday

students

in and ac

homewo

given tea

explanat

Furtherm

issues re

On this particular day, almost 70 percent (67%) of the workday was spent on activities directly related to student homework. In contrast, the bars in gray indicate the two 40-minute periods of classroom teaching plus another period spent administering a quiz, totaling 120 minutes (2 hours) of teaching time, which was Tr. Wang's average weekly official teaching load. But note that she had only two periods to teach on Tuesdays and the one period that she used for a quiz for Class 2 was a student self-study period. This was a typical expenditure of time when the course content was geometric proofs and Tr. Wang needed to pay particular attention to each step of a proof in marking homework. The time necessary to mark the homework shortened when the topic shifted to functions involving algebraic calculations.

Of the 18 teaching days that I observed, fifteen were full-day observations. I shadowed Tr. Wang from the beginning until the end of the day. The data gathered show consistently that homework was highly visible in and across the routine activities of Tr. Wang's practice. Homework is not only important because it consumed much of her workday. More importantly, it was prominent in her teaching and daily interactions with students and colleagues. When focusing on the trajectory of homework, on how it was used in and across her teaching activities and during those dynamic interactions, we can see that homework was a powerful tool she used to collect information about student learning of the given teaching topics and to help her make decisions about what topics needed further explanation in her teaching and which students needed more tutoring assistance. Furthermore, homework was also a tool Tr. Wang and her colleagues used to deliberate on issues related to student learning and teaching. Therefore, homework not only organized but also

teaching

systems

students

degrees

grade m

while sa

segments

(p. 31).

math teac

E

math teac

Greeno's

novice an

performed

componen

texts) can

completed

adapt the

math teach

plays a ver

helping to

but also shaped her work of teaching, weaving in and out of the activities that made up her teaching practice.

As a tool for teaching and learning in schools used across differing education systems around the world, homework is typically viewed as a task primarily concerning students. It involves students, teachers, and curriculum in different ways and to varying degrees in different countries. For instance, consider the TIMSS videotape study of 8th grade math lessons in Japan, the U.S., and Germany. Stigler and Hiebert (1999) noted that, while samples of math lessons in the U.S. and Germany often "begin with relatively long segments of checking homework, Japan begins with a quick review of yesterday's lesson" (p. 31). TIMSS-R (2000, p. 205) surveys of 28 jurisdictions in the U.S. confirmed that math teachers spent 15 percent of a lesson checking on homework.

Even with their cursory check of homework at the beginning of a lesson, American math teachers are able to know a lot about what students might know. Leinhardt's and Greeno's study (1991) of the knowledge gap between the homework-checking practices of novice and expert teachers in U.S schools found that "homework correction [checking]" performed by an expert teacher "is an ideal example of how one rather small lesson component (it lasts 2 to 5 minutes and is rarely mentioned by teachers, student teachers, or texts) can help achieve multiple goals" such as taking attendance, knowing who has not completed the day's assignment, finding out what mistakes there are, and deciding how to adapt the lesson to overcome existing problems (p. 238-241). That a Chinese middle school math teacher's day revolved around student homework attests to the fact that homework plays a very significant role in the teacher's learning and assessment of student learning, helping to inform her teaching in many profound ways. In a culture that attaches importa

right" in

Researc

experie

research

2

3

Overview

D

consecut

and learn

Viewing

this study

interacti

dimensio

develop

Th

introduce

importance to exams, homework is also a tool that Tr. Wang used to help students "get it right" in the exams.

Research Questions

In this dissertation, I study the role of homework in the practice of Tr. Wang, an experienced 8^{th} grade mathematics teacher in Shanghai. I aim to answer three major research questions:

1. What is entailed in the homework activities of a Chinese math teacher's daily practice?
2. What did Tr. Wang's homework activities make possible for her teaching and student learning?
3. What kind of teaching practice does Tr. Wang's homework activities make possible?

Overview of the Chapters

Drawing on data about and analysis of Tr. Wang's daily work activities across three consecutive teaching weeks, this study provides a rich description of the kind of teaching and learning practices that are supported by a teacher's homework-related activities. Viewing this through the lenses of cultural-historic activity theory and cognitive psyhology, this study offers a thorough examination of the knowing and learning of Tr. Wang in her interactions with homework. It also investigates the infrastructure – the social and cultural dimensions of the work in enabling and supporting such practice – which allows teachers to develop this kind of daily practice that uses assessment for ongoing decision making.

These research questions are considered over seven chapters. This initial chapter introduces the centrality of homework in Tr. Wang's practice. The second chapter

intr
rela
subs
and t
and h
serve
chapte
explain
during
begins
typical
findings
practices

introduces and develops the conceptual and analytical lenses of activity theory and other related theories of teaching and learning that help to frame the study and guide the subsequent analysis. It also presents the research methodology, including the study design and the methods of data collection and analysis. Chapter Three describes the curriculum and how homework, closely aligned to the curriculum, is designed both to organize and to serve as a vital resource for each day's teaching and learning. In each of the following four chapters, one major homework-related activity is examined: marking homework, explaining selected errors to a whole class, tutoring individual students on homework during breaks, and discussing homework-related issues with colleagues. Each chapter begins with a short vignette, which further details an aspect of the role of homework in the typical teaching day presented in this chapter. Finally, Chapter 8 presents the major findings and highlights the main implications that this study provides for policies and practices in curriculum development, teacher learning, and teacher development.

(

work in

the ma

importa

the task

studying

Three. I

construc

of a com

Wang's

work to

informa

the data

this study

The miss

Th

learning

Chapter Two

Theoretical Framework and Research Methodology

Introduction

Chapter One brought the prominence of student homework in Tr. Wang's daily work into the focus of attention and established her homework-related work activities as the major research subject of this study. In this chapter, I attempt to achieve two important tasks in four separate sections. In Section One, I provide a brief introduction of the tasks that this chapter attempts to achieve. In Section Two, I locate the importance of studying Tr. Wang's homework activities in the relevant research literature. In Section Three, I examine Tr. Wang's homework activities by viewing them from a set of major constructs from cultural-historic activity theory and cognitive psychology. With the help of a composite theoretical framework, I attempt to form a better understanding of how Tr. Wang's homework activities enabled her to generate and use information from student work to support her teaching and student learning. In Section Four, I provide background information about how the study has come into being and its research design as well as the data collection and analysis techniques used. I also provide an explanation of what this study can and cannot do as a piece of research.

A Review of Literature

The missing middle level: teachers' work practice, a site of current research.

There is increasing attention to research on teacher knowledge and teacher learning in the area of knowledge use in teachers' work and their daily practice. There are numer

(2001)

theory

knowle

and hel

"lookin

student

Mewbo

"bundle

ready" (

S

teachers

teachers'

One exp

Study) d

affect th

2002). Th

provide g

have tim

inquire c

T

and struc

(Engestr

numerous reasons for this increased attention. Here, I name just a few. First, Ball & Bass (2001) among others, are concerned about the seemingly unbridgeable divides between theory and practice, between expert teachers' knowledge theorized from research and knowledge used in real practice, and therefore, between such codified expert knowledge and helping teachers learn it. Researchers in this group have come to see the need for "looking closely at teaching" to observe how teachers "puzzle about mathematics" and student learning "in order to uncover knowledge in practice" (Ball, Lubienski, & Mewborn, 2000, p. 452-453). They believe that such uncovered knowledge is not "bundled in advance with learning and pedagogy" and thus, is "pedagogically useful and ready" (p. 453) for teachers to learn and use for teaching.

Second, researchers of international comparative education also found that what teachers do, what roles they undertake, and how the public perceives the importance of teachers' work vary considerably from culture to culture (LeTendre et al, 2001, 2002). One explanation from analysis of TIMSS (Third International Mathematics and Science Study) data offers that "local and national organizations can make important rules that affect the way teachers structure their work day" (LeTendre et al, 2002, p. 22 ; Levitt, 2002). Therefore, a study on Tr. Wang's workday and how it is structured is able to provide good insights into how a teacher's work can be organized to allow him or her to have time to use tools, such as curriculum and homework, and interact with colleagues to inquire collectively into problems arising from teaching and student learning.

Third, researchers from outside education note the "dichotomy between systems and structures, on the one hand, and the daily practices, on the other hand …" (Engestrom, 1998, p. 76). Engestrom (1998) explained that between the two levels, there is a mi

taken-f

and ide

student

middle

homew

Homew
world a

little ab

(Kalove

decade.

opposed

Despite

in Chin

teachers

related

frequent

attempte

improvi

are imp

"Quality
its contra
commitm
person (a

is a missing middle level, which "consists of relatively inconspicuous, recurrent, and taken-for granted aspects of school life" (p. 76, cited in Cobb et al., 2003) that "are sense and identity building processes which determine what it means to be a teacher or a student in an institutional setting" (Cobb et al., 2003). In the school settings in China, this middle level is dynamic and filled with teachers' mathematical sense making through homework activities in which teachers and students participate.

Homework remains invisible in the research literature both in the English-speaking world and in China.

In spite of much public and media attention to homework, we actually know very little about it. Homework has been one of the "most entrenched institutional practices" (Kalovec and Buell, 2000) in educational systems around the world. In China, in the past decade, the reform of "quality" or "disposition-oriented" education *(sushi jiaoyu*[1]*)* (as opposed to examination-oriented education) called for reducing student workload. Despite the fact that homework has been highly visible in teaching and student learning in China, little research has been done on the role of homework in student learning or teachers' work. In the Western world, Cooper (1994, 1984) reviewed 100 or so studies related to homework done in English. He lamented that these studies "are more frequently used to fuel debates rather than to resolve them" (1994, p. xiii). Most studies attempted to answer one of two major questions: first, whether homework is effective in improving academic learning and/or learning attitudes, and second, which assignments are important. He also found 6 studies that compared the effects of giving different

[1] "Quality education" challenges traditional, examination-oriented education and is best defined in terms of its contrast with academic, promotion-driven schooling. The two hallmarks of quality education are a commitment to all children (not just those heading on to further education) and to educating the whole person (and not just advancing the cognitive abilities needed for university admission).

instruct

signific

strategi

45). In t

how it c

address

seldom

the orga

classes a

basis in

brief hor

(TIMSS,

brief hor

wrong ar

U.S. do r

studied t

perspect

student

tool for

Paine, 2

been m

instructional feedback or of different methods of grading homework assignments. No significant effect was found because of the small sample sizes and because the "different strategies for providing feedback differ little in their influence on homework" (1994, p. 45). In these research findings, homework was studied as a given. Instead of studying how it could be altered to support teaching and learning, the studies were done mainly to address the debates over whether and how much homework should be given.

It could be because of these many debates in the Western world that homework is seldom regarded as a pedagogical tool for teachers in the research literature. In the U.S., the organization of a math teacher's work usually means that a teacher teaches five classes a day and has no time to grade or mark student homework carefully on a regular basis in order to gather information about student learning issues to inform teaching. The brief homework checking in the beginning few minutes of a lesson is a routine practice (TIMSS, 1998; Leinhardt and Greeno, 1991; Leinhardt, 1990). Study of expert teachers' brief homework checking found that it was able to help the teacher know who got things wrong and make adjustment in teaching (Leinhardt, 1990). Maybe because teachers in the U.S. do not have time to use homework in a more systemic way to inform teaching, those studied by Leinhardt and her colleagues "only mention of [homework from the perspective of] managing student behavior or lesson content rather than in the context of student learning" (p. 22).

Unlike homework, the curriculum material has received research attention as a tool for teacher learning in daily practice or in reforms in China (Ma, 1999; Wang & Paine, 2003) and in the U.S (Ball & Cohen, 1996; Collopy, 2003). Recently, there has been more research advocating using student work as a site of or a tool for teacher learnir

2003;

Repres

can bec

examin

through

researcl

commu

practice

teacher

work" (p

N

and their

students

learning

own. It p

and assis

I

Tr. Wan

homewo

of the pr

learning and development in professional communities in the U.S. (for example, Little, 2003; Lieberman & Miller, 2000). In her article, "Inside Teacher Community, Representations of Classroom Practice, " Little (2003) found that teachers' workplaces can become resources for developing their teaching practice when they come together to examine and support each others' teaching and respond to differences and conflict through channels such as studying student work. As she pointed out, "[...] relatively little research examines the specific interactions and dynamics by which professional community constitutes a resource for teacher learning and innovations in teaching practice. In particular, few studies go 'inside teacher community' to focus closely on the teacher development opportunities and possibilities that reside within ordinary daily work" (p.23).

My dissertation study provides an example of going inside teachers' daily work and their communities of practice. It probes into how Tr. Wang, together with her students and colleagues, used homework as a tool to inform teaching and assist student learning. This study focuses on the invisible middle level which has a dynamic life of its own. It produces a portrait of how a teacher used the information in her work to support and assist student learning.

Framing Practice, Work and Community of Practice

How do I examine and conceptualize homework and the related work activities in Tr. Wang's practice? I turn to cultural historic activity theory to study Tr. Wang's homework practice in part because it helps offer major constructs that make visible parts of the practice that are important but easily missed. Although I draw heavily on major ideas of a

psycholo

Wang's p

and tools

rules bot

expert te

illustrate

An activ

I

of interc

compon

which c

(1990).

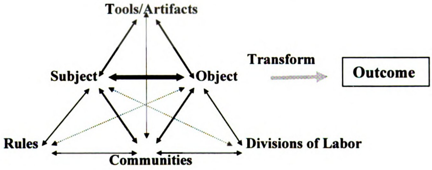
Engestr

ideas of activity theory, I also consider relevant theoretical frames in cognitive psychology to bring more light onto the questions that I attempt to answer. I view Tr. Wang's practice as constituting an activity system that involves the subject, the object and tools; as occurring within a community of practice with shared knowledge, skills and rules both in the use of tools and the discourse of use; as structured routine actions in expert teaching; as tool-mediated pedagogical reasoning and action. In this section, I illustrate these concepts as the major constructs of my theoretical framework.

An activity system mediated by tools and communities of practice

I view Tr. Wang's homework activities as an activity system, comprising of a set of interconnected activities. This notion of an activity system helps me to map out the components of Tr. Wang's activities and their interrelationships in a comprehensive way, which can be visually represented in the triangular diagram below devised by Engestrom (1990).

Figure 2.1. The Human Activity System (Engestrom, 1990)



In studying human labor activity as developmental work in social institutions, Engestrom (1990) brought Vygotsky's (1978) idea of the tool- and sign-mediated nature of huma

nature o

first I la

carefull

by one.

I

the acti

her pers

or *cons*

(for ins

transfor

material

to a pro

to her a

learning

subject

internal

directio

Marx's

labor a

in the p

For ins

of human activity together with Leont'ev's (1977) view of the community-mediated nature of collective human activity. In the following, for convenience of presentation, first I lay out in more details the major components of the system and then examine more carefully the tool-mediated and the community-mediated dimensions of the system one by one.

In this diagram of the activity system, Tr. Wang is the human *subject* engaged in the activities. Taking her as the subject primarily, I view homework and its actions from her perspective. Student homework constituted the central *object* that she interacts with or *constructs* in these activities. The object refers to the raw material or problem space (for instance, errors and learning issues in homework) that the subject constructs and transforms into desired outcome. It is Tr. Wang's motivation to *transform* the raw material, those errors and misconceptions, into a value-added *outcome,* such as a solution to a problem or the understanding needed in order to make corrections, that gave meaning to her activities and drove them toward the desired outcome of improving student learning of the mathematical content (Kuunti 1996, p. 25).

Transformation is also a mutual process. The object is transformed, but so are the subject and the mediating tools. This dynamic mutual transformation represents the internal relationships of the activity system, which is visually represented by the bi-directional arrows in the triangular diagram of the human activity system. It follows Marx's theory of praxis, as Wertch (1981) describes it: "in carrying out continually the labor activity, humans do not simply transform nature: they themselves are transformed in the process" (p. 134-135). This transformation of human subjects happens over time. For instance, Tr. Wang's and her colleagues' knowledge about their subject matter, teaching, and student learning, as well as their skills in helping to improve student learning, can change and grow into more sophisticated forms as they accumulate experience from their daily homework-related practice.

In this homework activity system, the interaction between Tr. Wang and student homework (the subject and the object) was the center of attention, hence their being highlighted in bold in the diagram. Using activity theory helps me see that this interaction was both individual and social, that is, not only mediated by tools but also by the communities of practice with their norms, rules and division of labor. In this sense, "the individual and the social levels are interlinked at the same time" (Kuunti, 1996, p. 25). That is to say that Tr. Wang's actions of marking homework, gathering information from it, and using that knowledge to make sense of student learning were all part of her everyday practice and were firmly embedded in the social matrix of students, colleagues, tools, and artifacts. Such an assumption means that I have to understand and examine homework in this matrix.

Object of the activity system. Object is a central construct in activity theory. In the following, I illustrate three major points regarding the nature of the object from the activity theory point of view. First, not every object can become the object of an activity. The capacity for a thing or phenomenon to become the object of an activity is determined by whether it meets a human need (Leont'ev, 1977, p. 54). In school learning activities, homework is an object that students interact with as an everyday learning task. It is mainly homework's capacity to meet the need students have to practice in order to retain the knowledge and skills conveyed through teaching that allow me to see homework as an object of school learning activities.

to an ob

(p. 27).

an und

major p

as Enge

the driv

requires

languag

Homew

evolutio

develop

Confuci

educatio

the orde

transforn

perspect

reconstr

underst

Second, as Kuunti (1996) summarized, "[a]n activity is a form of doing directed to an object, and activities are distinguished from each other according to their objects" (p. 27). By understanding activities as distinguished by objects, I take homework not as an undifferentiated process but rather as a set of homework activities. In this way, a major purpose will be to unpack what that set includes and what each activity entails. Just as Engestrom (1990) argued, construction of the object based on human needs becomes the driving force that gives shape and direction to activity.

Third, the "principle of object relatedness of activity" (Engestrom, 1999, p. 22) requires that objects be distinguished from goals as the terms are used in everyday language. As Engestrom stated,

> Objects are not to be confused with goals. Goals are attached to specific actions. Actions have clear points of beginning and termination and relatively short half-lives. Activity systems (shaped by objects) evolve through long historical cycles in which clear beginnings and ends are difficult to determine (p. 381).

Homework, a key object for school learning around the world, has undergone an evolution since schooling was invented. The contemporary thinking that guides the development of Shanghai's curriculum and homework evolved from a deep-rooted Confucian heritage of learning. For instance, in Chapter Three, I discuss those traditional education principles that emphasized the prevention of evil, the timeliness of teaching, the order in which content was arranged (for example, from simple to difficult), and transforming weaknesses and failures into strengths and means of pursuing success.

Construction, reconstruction, and transformation of the object. From the perspective of activity theory, it is important to understand how the object is constructed, reconstructed and transformed. This helps me focus on Tr. Wang's actions and understand how she constructed, reconstructed and transformed the object into desired outcome

through

contents

Wang's

individu

I

directio

the char

redefin

made up

be furth

of stude

subject

technica

Chapter

theory a

compris

symbols

her feed

pedago

used he

and tut

outcomes. Put it in another way, "[A]n activity system constantly generates actions through which the object of the activity is enacted and reconstructed in specific forms and contents" (Engestrom 1999, p. 381). These specific forms and contents can include Tr. Wang's explanations about the errors given to students and the tutoring dialogues with individual students.

Engestrom (1999) referred to these actions as *search actions* always in the direction of identifying problems and seeking to define or resolve them. Thus, they bear the characteristics of "the creative potential of activity in object construction and redefinition" (p. 381). From this viewpoint, I need to examine Tr. Wang's activities as made up of a sequence of such search actions, such as identifying errors and problems to be further addressed in explaining and tutoring, that aimed at gaining insight about issues of student learning.

Activities mediated by tools. Human labor activities are tool-mediated. That is, the subject never interacts directly with the environment/object without the mediation of technical, social or psychological tools (Vygotsky, 1978; Wertsch, 1985). As described in Chapter One, Tr. Wang's homework activities were mediated by a set of tools. Activity theory as a frame prompts me to focus on and inquire about the range of tools that comprise this set. Certainly, they included the red pen she used to mark homework, the symbols and signs she assigned as she evaluated student work that were to communicate her feedback to students, and her own knowledge and understanding of the content, pedagogy and student learning that she deployed as she assessed student learning. As she used homework information to create further assistance to students' learning, explaining and tutoring also became additional pedagogical tools.

A

mediatec

commun

construc

homewo

into lear

colleagu

S

commun

commun

other w

by *rules*

respons

assistan

complet

respons

of labo

activit

among

all 8th

comm

Activities mediated by communities of practice. Tr. Wang's activities were also mediated by the participation of students and colleagues, who made up the two distinctive communities with which she worked and to which she belonged. In other words, this construction of the communicative system surrounding homework (via the marking of homework and the transformation of the errors and learning issues in student homework into learning outcomes) was collectively produced by Tr. Wang and her students and colleagues.

As the subject of the activity system, Tr. Wang's relationship with these communities was mediated by rules; the relationship between the object and the communities was mediated by division of labor (Kuutti, 1996, p. 29; Leont'ev, 1977). In other words, her interactions with students and homework were constrained and regulated by *rules,* such as the norms and conventions of teaching in China and her official work responsibilities (as marking homework and providing timely feedback and additional assistance to students were officially part of Tr. Wang's work). In the meantime, completion of homework and marking and giving feedback on homework were a set of responsibilities shared between the teacher and her students, representing a clear division of labor between them. At the same grade level, the math teaching and homework activities of all six 8th grade classrooms were divided among four teachers, Tr. Wang among them. When these teachers came together to share issues of learning reflected in all 8th grade student homework, the object became a shared, common property of the community of teaching.

Locatin

(1998)

of my tr

the thre

the syst

cultural

commu

relation

(p. 12-

summar

enterpr

mathem

to share

students

account

"pedag

commu

and art

charact

how th

and ma

Locating homework-related activities in communities of practice.

Using major concepts of communities of practice developed mainly by Wenger (1998) and Lave & Wenger (1991), I am able to bring new light onto the major constructs of my theoretical framework. I illustrate four areas that this new light helps illuminate: the three essential properties of a community of practice, homework as boundary object, the system of homework use as technology of the community of practice; and the shared cultural heritage.

Three essential properties of a community of practice. Wenger (1998) defined a community of practice as having three essential properties: *a joint enterprise, mutual relationships, and a well-honed repertoire of ways of reasoning with tools and artifacts* (p. 12-13). Cobb and colleagues (2003) in studying a community of mathematics teaching summarized the constituents of the three properties of such a community. To them, a joint enterprise comprise of shared activities aimed to ensure that students mastered the central mathematics ideas and perform well in the high-stakes exams. Mutual relationships refer to shared norms, such as curriculum standards, and the responsibility for teaching that students to learn mathematics well. Their mutual relationships hold each other accountable when they justify their interpretations of the problems as well as "pedagogical decisions and judgments" in their collective deliberations. In the community, the teachers share "a well-honed repertoire of ways of reasoning with tools and artifacts" (Cobb et al., 2003, p. 14-15). These three properties offer more specific characteristics that help me view and describe Tr. Wang's community of teaching and how they come together to reason with problems arising from teaching, student learning and managing with dilemmas of teaching.

objects

own pu

organi

a signi

activiti

commu

the ins

custom

tool for

differen

homew

teachin

grade le

mathem

systema

student

Such sy

commu

and inf

commu

Homework as a boundary object. Star (1989) developed the idea of boundary objects – the shared informational objects that can be used by different groups for their own purposes. Boundary object is widely used in studies of artificial intelligence and organizational memory (such as Bannon and Kuutti, 1996). A boundary object "can play a significant role in enabling the members of different communities to coordinate their activities even when they are used differently and [have] different meanings in each community" (Cobb et al., 2003, p. 19). In Wenger's study of the insurance claims agency, the insurance claims form was regarded by him as a boundary object that connected the customers, the insurance claims agents, and the medical institutions. It was also used as a tool for novice agents to make sense of the business when they learned to deal with the different communities. When I view homework as a boundary object, I talk about homework as crossing the boundaries on three levels in Tr. Wang's community of teaching: the boundary between the teacher and students, between teachers of the same grade level, and between and among her own different work activities.

System of homework use as technology of community of practice. In the mathematics teaching community, Tr. Wang and her mathematics colleagues systematically and routinely use student homework as tools to continuously assess student learning and inform their decision making in and for teaching on a daily basis. Such systematic use of tools is called by Lave and Wenger (1991) as the "technology of a community of practice" (p. 103). The technology also consists of the access to knowledge and information as a primary condition for participation and the discourse of the community, what the members talk and how they talk. Hence, taking this construct serious

homew

practice

idea of

of-class

Educat

al., 19

well in

learning

with ac

homew

member

students

homew

and reas

back in

I

taken as

compon

be "acc

directed

Engestr

seriously means that I need to examine discourse and technology that are part of homework activity.

Homework as a shared cultural heritage. Homework, in the form of "out-of-class practice/exercise" *(kewai lianxi)* was a tool for Confucius's teaching as well. Confucius's idea of how to balance the complementary relationship between in-class learning and out-of-class practice, which was called *zangxi xiangfu*, was summarized in *Record of Education*, one of the earliest education essays, written two thousand years ago (Wang et al., 1994, p. 16). In a cultural tradition that orients school-based learning to performing well in examinations, homework has been used as a resource that organizes teaching and learning. The institutional setting also has been designed to provide teachers and students with access to using homework as a tool for teaching and learning. The systematic use of homework enabled information flow and access to the information and use for the members of the community. Tr. Wang's discourse used in explaining and tutoring students on homework errors as well as the informal collegial conversations around homework issues are all part of this technology, with which the gathering of information and reasoning of student learning and subject and curricular knowledge are all integrated back into their daily practice.

In activity theory, internal tensions and contradictions in an activity system are taken as "motivating force of change and development" (Engestrom, 1999, p. 9). As the components of the activity system continually transition and transform, such tensions can be "accentuated," for instance, between the collective motive of the activity and goal-directed individual actions and between tools and actions (Leont'ev, 1978 cited in Engestrom, 1999). This perspective is useful in examining the societal and cultural contex

instanc

but it a

encoun

teacher

the ma

transfo

actions

resultin

process

mathem

activity

process

assume

Cognit

studyin

draw on

and the

have lo

contexts of and the role of artifacts in the work of Tr. Wang and her colleagues. For instance, the curriculum served as a source of information guiding teaching and learning, but it also created problems, ambiguities, and difficulties that teachers and students encountered daily. Errors in student homework both created frustrations and stress for teachers and generated opportunities for the teachers to probe into the errors' causes and the mathematics entailed in them.

Since activity theory studies the processes of object construction and transformation and the development of practices, it requires that human interaction and actions be analyzed in the context of development. Activity theory views all practices as resulting from specific historical developments and as continuously re-forming as processes. Tr. Wang's work activities reflected an institutionalized tradition of mathematics teaching and learning in China. In studying her daily practice, I viewed her activity system and practice as a continuously unfolding and on-going development process, worked out daily and across time. This developmental point of view makes me assume learning from practice and daily work as continuous improvement over time.

Cognitive research on expert teachers' instructional actions.

In my study, I also take advantage of the approaches of cognitive research in studying the expert teachers' structure of knowledge and instructional actions. I mainly draw on the related research carried out by Leinhardt (1991, 1990) and Berliner(1986) and the Shulman's (1987) pedagogical reasoning and action.

Expert teaching as schemata of organized routine actions. Cognitive psychologies have long studied expert classroom instruction. Among them, the research carried out by

Leinhardt (1991, 1990) and Berliner (1986) are very helpful in my examination of Tr. Wang's practice. These researchers provide a conceptualization that is particularly useful in delineating "how information is processed, how memory is organized, how demands are handled, how planning systems are constructed, and how schemas of actions are structured" (Leinhardt, 1990, p. 19). Taking advantage of these tools of cognitive psychology, I aim to reveal Tr. Wang's routine actions and processes of gathering, making sense of, and using information in her homework-related activities.

Shulman's model of pedagogical reasoning and actions. Shulman (1987)'s classic model comprises "a cycle through the activities of comprehension, transformation, instruction, evaluation, and reflection" with comprehension as the "starting point and terminus for the process" (p.14). This model may well be used to characterize Tr. Wang's practice as an expert teacher. From the perspective of her homework activities, homework can also be viewed as an import object that she constructs and reconstructs every day in pursuit of understanding about student learning and how to help them improve learning. The comprehension she acquired could be in the form of the information she gathered and used in adjusting her lessons by adding and enacting her "on-line" (p. 18) decisions through explaining selected errors at the beginning of a lesson.

The "transformation stage" occurs before "instruction" in which the teacher "interpret text", figuring out the best modes of "representation", selecting suitable instructional approaches and considering how to adapt and tailor the teaching to meet the needs of students as a group or individually. This stage captures Tr. Wang's transformation of the material of learning in the form of student homework into content and forms (such as explanations and tutoring dialogues) accessible to student underst

homew

and st

IRE pa

"teache

a topic

event is

respons

analytic

choral r

in her sh

homewo

Unit of a

dynamic

Tr. War

into use

actions

its own

within t

understanding about their errors and how to make corrections. Viewed in this way, homework becomes a boundary object that connects these different instructional actions and stages and interweaves pedagogical reasoning and actions.

IRE patterns characterizing traditional classroom teaching discourse

Cazden (1988) regards that traditional classroom teaching is characterized by "teacher-led speech event" (p. 99) "in which the teacher controls both the development of a topic (and what counts as relevant to it) and who gets a turn to talk." (p. 30) This speech event is a classroom discourse featured by an IRE pattern – teacher initiation, student response and teacher evaluation of student response. This pattern provides a very useful analytical tool in interpreting the teacher-student interaction routines (such as students' choral responses). It also helps to explain how Tr. Wang leads students in big classrooms in her sharing with them her feedback to homework and making explanations to homework detailed and structured.

Unit of analysis.

Although teacher actions are important sites of analysis in interpreting Tr. Wang's dynamic practice, they cannot be treated as unit of analysis in studying work activities. In Tr. Wang's homework activities, teacher actions were directed at transforming homework into useful resources for teaching and learning for both students and teachers. These actions make sense only when they are examined as part of an activity system guided by its own cultural and social norms and ideologies and put in the context of its long history within the examination system. Just as Tickhomirov (1988, p. 113, cited by Engestrom, 1999, p

attainm

and m

practic

1999, p

activity

homew

actions

subject

within

local, s

Therefo

the acti

activitie

practice

analysis

actors w

below is

1999, p.22) pointed out, "focusing exclusively on the level of actions highlights goal attainment and problem solving but makes it very difficult to analyze the socio-cultural and motivational basis of goal formation and problem finding." Individuals' collective practices "are not reducible to sums of individual actions [...]" (Engestrom & Miettinen, 1999, p. 11).

Throughout this study, I use the activity system as the unit of analysis. Tr. Wang's activity system consisted of multiple object-oriented activities, that is, multiple homework-oriented activities, each of which consisted of a distinctive set of goal-directed actions aimed at forming and transforming the object. On the one hand, actions reveal the subject's goal-attainment and problem-solving processes. Viewing actions and activities within an activity system "makes it possible to include both historical continuity and local, situated contingency in the analysis" (Engestrom & Mietinnen, 1999, p. 9). Therefore, using the activity system as the unit of analysis allows me both to probe into the actions that Tr. Wang took in transforming the objects of the homework-related activities and to study her activity system as an embodiment of a social and cultural practice of teaching with a long tradition and cultural heritage. As a result, the unit of analysis in studying human-mediated activity is an activity system with a community of actors who have a common object of activity (Cole and Engestrom, 1993). Figure 2.2 below is a sketch of the structure of this unit of analysis.

Signific

work ac

in an exp

daily, ro

Wang's

that took

centerin

better un

learning

unpack t

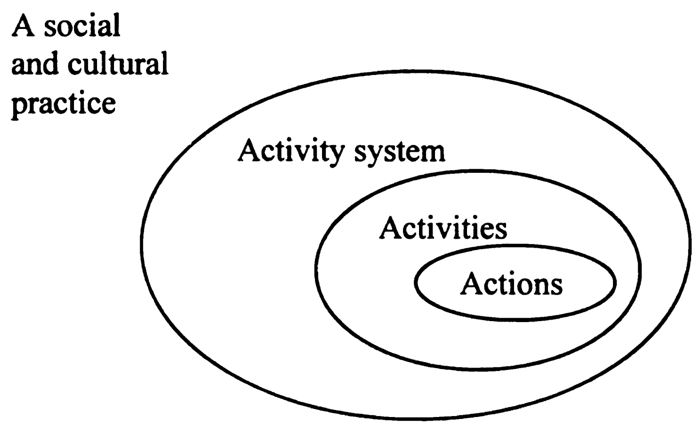
order to

lies in th

homewo

mathem

Figure 2.2 Activity System as Unit of Analysis



Significance of the Study

This study is a close examination of a middle school mathematics teacher's daily work activities centering on student homework. In tracing the trajectory of homework use in an experienced 8th grade math teacher's daily work, I offer a rich description of the daily, routine activities surrounding student homework. In doing so, I uncover Tr. Wang's ongoing sense making of student learning, mathematics content, and teaching that took place while she marked homework. I probe into how the collegial conversations centering on homework provided opportunities for teachers to inquire into and develop a better understanding of the ambiguity in the curriculum and in problems of student learning. These homework-related processes enabled Tr. Wang and her colleagues to unpack the mathematics entailed in the homework errors and the curricular ambiguities in order to make them accessible for students' understanding. The significance of the study lies in the rich description of a teacher's practice and how she and her colleagues used homework as tools to inform their decision making and to adjust to students' developing mathematics understanding. This study also offers a comprehensive analysis of how homew

1998)

cultura

impact

Overvi

activitie

chapter

establis

the ma

that w

homew

her inter

student

and cul

homewo

practice

marking

on error

chapters

chapter

Wang's

homework is designed and used to serve as an important "boundary object" (Wenger, 1998) of teaching. In addition, it also examines this practice in the context of Confucian cultural and educational traditions to understand its relation with the traditions and their impact on the practice.

Overview of Chapters

The organization of the dissertation reflects the conceptualization of homework activities as a dynamic activity system located in a community of practice. The first chapter documented the prominence of homework in Tr. Wang's work day and established the homework activities as focus of the study. In Chapter Two, I introduced the major constructs from activity theory, communities of practice and cognitive theories that will guide my considering of her homework-related activities and her use of homework across these activities to create teaching and learning opportunities as well as her interactions with her colleagues to collectively deliberate on issues of teaching and student learning. Chapter Three focuses on the curriculum. It introduces the rationale for and cultural dimensions present in the design of the curriculum, how the use of homework is closely aligned to it, and how Tr. Wang enacted the curriculum in her practice. Chapters Four through Seven each deal with a different homework activity: marking homework, explaining homework to a whole class, tutoring individual students on errors, and conversing with colleagues on homework-related issues. In each of these chapters, I continue to narrate the day first introduced in Chapter One, beginning each chapter with a different but relevant real teaching and homework scenarios from Tr. Wang's work day. I organize the chapters by presenting the specific object of the activity (marking, explaining, tutoring, and conversing) and then focusing on the actions that Tr. Wang took to construct and transform the object into desired outcome. Chapter Eight pulls these together to recap key aspects of Tr. Wang's practice. It also discusses the limitations of such practice as well as the implications of the study for curriculum, teaching, and professional development. Taken together, these eight chapters serve as a window onto Tr. Wang's mathematics teaching practice, which is informed and shaped by her use of student homework to capitalize on errors.

Research Methodology

Background of the Study

This is a study of how an eighth grade math teacher uses homework as a tool for creating pedagogical and learning opportunities for students, herself, and her colleagues. Although the data comes from a case of middle school mathematics education in China, the nature of the study is inherently comparative. As a young scholar from China receiving advanced training in the U.S. in the field of education and research, my experiences in taking courses, performing comparative research on teacher induction in China and the U.S, and observing the U.S. school system made it possible for me to ask the questions that this study attempts to answer. This rewarding experience equipped me with new perspectives with which to make meaning by comparing and contrasting, and learning by "making the familiar strange." It is because of this experience that I take a renewed interest in studying the "familiar" system in which I was brought up and introducing it to others. All of these experiences factored into my choices data collection, analysis and writing and reinforced my sometimes implicit comparative stance.

This a dissertation has an international comparative stance. While it focuses on data from a Chinese mathematics teacher's work practice, it makes an effort to draw on research literature from both the U.S. and China. Given the remarkably larger volume of research in the field of teaching, teacher learning, and professional development in the U.S. and my own background of advanced training in the U.S., literature produced by U.S. researchers plays a prominent role in shaping my approach.

An interpretive approach. In this study I aim to understand how a middle school math teacher, Tr. Wang, used homework as a tool to create learning opportunities for students to learn and how it was used to generate inquiry and problem solving in teachers' work. Because of this, data need to be collected from the major artifacts of work that the teacher used and produced in her daily practice and "the flow of work actions" (Engestrom, 1996, p. 5) in her work setting. This determined that the nature of the study be qualitative and primarily based on observation. I appear as an outsider to the teacher's work and to the mathematics and pedagogical problems from student homework that Tr. Wang and her colleagues deliberated over and addressed on a daily basis. So I approached this study using an interpretive method of inquiry (LeCompte & Preissle, 1993).

Research Design

An emerging research problem. In many ways, the emergence of the research problem and the initial design of this study resembles the "theoretical sampling" in grounded theory as developed by Glaser and Strauss (1967) and Strauss and Corbin (1990). This refers to the process in which the problem and the theory emerge along with the data collection and the analysis. My initial interest was driven by the dynamic interac

teache

same g

me to t

markin

work.

collabo

teacher

broadly

is used

learning

allows

charact

constrai

interact

helps th

nodes ir

underst

(1999)

investig

school

interactions in the middle school teachers' grade-level offices in Shanghai, where teachers shared information and collaborated informally in educating students of the same grade in various subjects. Analysis of the videotaped data and interviews all pointed me to the unusual amount of time that teachers, particularly the math teachers, spent marking student homework and talking about the problems they encountered in student work. As I was seeking to understand how teachers learn from the information flow and collaboration, homework emerged as a center piece of my inquiry. This emerging area – teachers learning from their work by using homework as tools – guided me to read more broadly and do more interviews to further develop my conceptual framework.

Case study approach. To better understand an activity system in which homework is used to generate new activities, interactions, and opportunities for teaching and learning, this study looked closely at one teacher's work. Using a case study approach allows for intensive, in-depth examination of "the interaction of significant factors characteristic of the phenomenon" (Merriam, 1988, p. 10), such as the curriculum constraints, pedagogical values of errors, teacher actions, collegial interactions, and their interactions with the sociocultural context. Using the activity system as a unit of analysis helps the study pay attention to all the components of the activity system (i.e., all of the nodes in the diagram) and their interplay in order to form a comprehensive and systemic understanding of the inner workings of the system. At the same time, as Engestrom (1999) noted, I have to enter "a dialogical relationship" between "the local activity under investigation" and my own constructed view of the system (p. 10).

Selection of the school and participants. Tr. Wang's school is a popular middle school in the neighborhood of a busy residential area in the southwest portion of

Shang

projec

offices

conten

proofs

activit

2001.

Resear

in the s

dozen

intervi

smooth

intense

term ex

spent in

that I s

consecu

Mather
project,
induction
This st
course p
The on
guided le
The obse
the work.

Shanghai Municipality, China. Her school was one of the schools visited by our MGM project[2] in May, 2001. I stayed longer to do a videotape study of the 6th grade teachers' offices[3]. For my data collection in November 2002[4], I chose the 8th grade because the content being taught during the month of my fieldwork happened to cover both geometry proofs and functions. This allowed content comparison in relation to homework activities. I chose to observe Tr. Wang mainly because during my videotape study in June 2001, I was impressed by her thoughtful words at the school MTRG (Math Teaching Research Group) meeting during which colleagues commented on a new teacher's lesson in the school's yearly teaching competition for young teachers. She was also one of the dozen teachers I interviewed long distance on the phone prior to my field work.

These phone interviews helped to clarify and specify field observation and interview protocols, and familiarity with the school made my negotiation of entry smoother than they otherwise would have been. Yet the time constraints made the trip intense. I arrived to find that schools across Shanghai were having their week-long mid-term exams, and no classes were in session for me to observe. The first week was then spent in interviewing teachers and researchers and in collecting written documents. After that I shadowed Tr. Wang and followed her from her classrooms to her office for three consecutive weeks.

---

[2] Mathematics and Science Middle Grades Teacher Induction in Selected Countries, NSF-sponsored project, 1998-2001. The findings of the study were published in the book entitled *Comprehensive teacher induction – Systems for early career learning* (Britton, Paine, Pimm and Raizen, 2003).

[3] This study was sponsored by the Spencer Research Training Grant and the findings were written up in course papers and presented at AERA and CIES.

[4] The one-month field trip in November 2002 was made possible by my student interns having a one-month guided lead teaching in their internship schools, during which they did not attend course work on campus. The observation site was familiar but I had no choice about the timing of the fieldwork as it depended on the work commitments I had in the U.S.

possib

comm

In tern

averag

contai

and th

pingxi

occasi

Five a

chose

Teach

Lesson

teachi

consis

freque

inform

topic c

inform

part o

a hub

becar

Although Tr. Wang is the subject of the activity system, her work would not be possible without the participation of her students, on whose homework she marked and commented. She taught math for two of the six 8th grade classrooms, Class 4 and Class 2. In terms of student composition, there was a larger proportion of students with above average and average mathematics achievement in Class 4 than in Class 2, but both classes contained some mathematically weak students. The former was called a stronger class and the latter was called a parallel class, instead of a weak class (in Chinese translation, *pingxin*). She taught the two classes the same content at the same pace, while occasionally assigning more homework for the stronger class. As illustrated in Chapters Five and Six, she treated the two classes differently, such as in how many errors she chose to explain during a lesson and whom she selected to receive tutoring.

Tr. Wang belonged to two formal, subject-matter-related groups at the school, the Teaching Research Group for all mathematics teachers, which she chaired, and the Lesson Planning Group for 8th grade math teachers. The groups met biweekly to discuss teaching-related issues and school plans and work. The lesson preparation group consisted of Tr. Wang and three other colleagues, Tr. Zhao, Tr. Li, and Tr. Hu. They frequently met informally, in addition to their formal meetings, to share teaching information and problems. Since they shared the same object, homework was also the topic of most of their informal conversations. Tr. Wang's homework activities were often informed by this informal collegial sharing, and these discussions became an important part of the data. Besides these subject-matter-related informal interactions, her office was a hub of information about students shared by teachers of other subject areas. These also became sites of observation and data.

Data Collection

I designed the study to collect data from multiple sources about the use of homework in different activities in order to help shed light on how the activity system worked. These data include: direct and focused observations, audio-taped natural flow of office and classroom activities, interviews, marked student work samples, and curriculum documentation. The following describes the rationale for and methods of data collection.

Observation. As an outside researcher, observation of the daily activities of Tr. Wang's work was the major vehicle for gathering information and "record[ing] the behavior as it [was] happening" (Merriam, 1988, p. 88). Sitting at the deskside of Tr. Wang (and also Tr. Zhao, as his desk adjoined hers), I was able to watch all of that transpired in the routine dynamics of the office and the classrooms: what she was marking and writing on the student work, her facial expressions, her verbalized thoughts, what she said and showed to Tr. Zhao, sometimes her interactions with other colleagues in the office, and her tutoring of students on homework during the noisy breaks. I observed every period that she taught to identify links between her marking of homework and her classroom instruction. These natural events were recorded on audiotapes while in my field notes I carefully kept detailed notes of the time sequences of each shift of events, the facial expressions, the gestures, direct quotations, and which students were tutored and on what material. My observation write-ups, storing the living moments of Tr. Wang's work events side-by-side with my personal thoughts and comments, fill two thick, sturdy notebooks.

While observing, I was aware of my role as an outside researcher, not a participant in the events. Sometimes, however, participation was required, as people around me wanted to have a little conversation or situations developed that expected a specific reaction from me (verbal or otherwise) in order to maintain politeness. At times, I was tempted to join the flow of a conversation and forgot my role as an observer, thus yielding a segment of data partially invalid. But these incidents did not in themselves make me a participant.

Tape recording. Audiotaped observations yielded a reliable source of data from which useful segments were played back, sometimes repeatedly, and then transcribed and compared with field notes to be coded. The three weeks of observation produced 32 cassette tapes that were duplicated for use, the originals being stored for safekeeping. The transcribing, however, proved extremely challenging, given the nature of the research questions that I aimed to answer. First of all, four major homework activities occurred in the natural flow of the work day. They often followed the expected sequence but many times there were unexpected and intervening events in the process. At the backstage of teaching organized classroom teaching continued, involving all participants in the game. Teasing these four activities out of the flow of a certain day often involved playing back and listening repeatedly to all tapes of the day. Tutoring dialogues with students during breaks were extremely hard to make out given the noisy background and students' low voices. The challenges of untangling the messy dynamics of a work day are part of the reality fieldwork of this kind of encounters.

Interviews. The most interesting part of the field work turned out to be interviews. Initially, I conducted several structured interviews according to preplanned protocols designed to learn about what the teacher was thinking while marking homework. These interviews produced responses that were repetitive and very brief. Leinhardt's (1990) warnings rang clearly: "[...] teachers are somehow less able than others to identify important features of their skilled performance [...] because it is inherently problematic for anyone both to engage in an act skillfully and to accurately interpret it" (p. 20, citing Ericsson & Simon, 1984).

I quickly shifted gears and performed such interviews only once a week as a formal weekly summary by the teacher of her work. I replaced the interviews with occasional random questions following the flow of her work, trying not to interrupt it. The busy, quick-paced, on-going activities often did not allow for asking any questions. This in fact made observations the primary tool of data collection.

Besides interviews with teachers, I also conducted formal interviews with teacher researchers and educational research experts to learn about their attitudes towards teaching and homework and their expectations of teachers in the new round of reforms. Among them, interviews with Professor Gu[5] provided important insights into how they studied but did not publish their findings about teaching and teachers' use of homework in practice.

Marked student work samples and teacher analysis of mid-term exams. In order to read and understand the signs, symbols, and comments that Tr. Wang marked and wrote with her red pen in each student's work every day, I regularly read through the marked student workbooks on the teacher's desk. I recorded the unusual symbols and asked the

---

[5] Professor Gu is well known for his 20-year stint of research and experimentation in mathematics education in rural Shanghai that considerably improved the overall academic performance of the local schools in the 1970s-90s.

teacher about the meaning of certain signs or comments. I also made copies of a few days' marked work that reflected those errors selected by the teacher to explain to and tutor students. I noticed that both Tr. Wang and her colleague, Tr. Zhao marked the student corrections of errors before they marked the newly completed work. I also noticed there were a set of signs frequently used to mark the incorrect writing or procedures. Collected student work samples provided an important site where the meanings and purposes of marking were revealed and which showed how the marked work informed the teacher's choice of candidate problems for further teaching to students. Reading the limited local literature on marking homework in general also indicated that Tr. Wang's system of marking signs and symbols was broadly shared in mathematics teaching in China.

In addition to marked student work, I also collected and analyzed the mid-term exam sheets along with the teacher's required analysis of the results, calculation of the errors distribution, and suggestions for improving the grade situation. This analysis was compared with the marking of homework to see how each stood in the cycle of student learning. This revealed that errors are always tracked and provided with feedback and analysis, whether present in homework or tests and exams.

Curriculum documents. I collected those curricular materials used as routine tools of teaching and learning and read them closely to see their connection to how selected student work problems appearing in Tr. Wang's tutoring, in-class explanations, and conversations with other teachers. These included homework assignment sources that included the textbook; Volume A and Volume B *(A ce and B ce)*, the two main student workbooks, used on alternating days; Same Step *(tongbu,* an abbreviation of *Same Step*

Practice and Improvement), an additional resource book with extra practice and tests; and Tests & Commentary Analysis *(ceshi yu pingxi*), a book containing homework practice and tests, accompanied by comments on and analysis of student learning problems and errors. The last two extra resource books were officially approved in Shanghai and used by schools according to the levels and needs of their students. I also obtained a copy of Teaching Research Material *(jiaoxue cankao ziliao)*, a manual containing teaching schedules, content analysis, teaching suggestions, and a key to assignments at the end of the small book. These constitute the important and highly visible artifacts of the daily teaching work in Shanghai.

A careful analysis of the design schemes of these documents is required to provide clues for an understanding of how they are designed in ways that regulate and control teachers' work and how they were also able to serve as important tools to raise questions and generate information for teachers in problem-solving situations. Such an analysis is also required to illustrate the properties of the curriculum as an artifact of a cultural practice that embodies the heritage and "wisdom of practice" (Shulman, 1987). This analysis is provided in Chapter Three.

Summary of data sources. Taken together, these data sources provide a detailed picture of Tr. Wang's homework-related work activities. Table 2.1 below summarizes the variety of data collected during the field trip from November to December of 2002.

Table 2.1 Summary of Data Collected in the Field

Activities/Events/Artifacts	**Type of Data**	**Dates Collected or Duration**
Homework activities in the office	Researcher field notes from observations, audiotapes.	Ongoing, November 11-December 3, 2002.
Homework activities in teaching	Classroom observation field notes, audiotapes.	Ongoing, November 12-December 3, 2002.
Structured and random interviews with teachers and math education researchers	Researcher field notes, audiotapes.	Random: ongoing, November 14 – December 3, 2002. Structured: weekly for four weeks.
Marked student work samples	Copies of work samples.	On-going, twice weekly during fieldwork; and collected workbooks sent by mail after field work
Curriculum materials	Textbook; workbooks Vol. A and Vol. B; Same Step; Test & Commentary Analysis; Teaching Research Material; teacher analysis of mid-term exams and unit tests.	Some were collected during prior research work and others were purchased during field trip.

Methods of Data Analysis

As a study adopting an interpretive approach, several key methods of data analysis have to be used. To infer meaning and allow for triangulation, the study employed the following methods for data analysis: the grounded theory approach; draftsmanship; data triangulation; data reduction; and discourse analysis. In the following, I provide a brief description of each method.

Developing grounded theory: As a participant observer of collective teacher learning in the workplace over the years, the focus of my research problem underwent a sharpening process as I observed and read more (and took more related coursework) to deepen my understanding of the problem. I started by fervently studying Dewey's philosophical stance toward the interplay among the curriculum, the child, and the educative experience to rethink the curriculum and teachers' study of the curriculum on the job. As Dewey's view became inadequate to explain the cultural mediation of such learning in different education systems, I came to appreciate Marx's theory of human labor activity and praxis as an insight into how humans use tools to transform nature and are themselves transformed in the process. Using Marx's theory of social practice to help develop the notion of legitimate peripheral participation of apprentices learning from conventional work practices, Lave and Wenger (1991) helped me conceptualize teachers' use of curriculum and homework as developing the technology of practice in communities of practice and the notion of the importance of the transparency of such technology in enabling learning. But the way they treated such learning as a unidirectional movement from peripheral to more central roles in their work leaves out the contradictions, tensions, and problem-solving situations of real work situations.

Vygotsky's theory of social and cultural mediation in human cognition and development enabled me to group teacher workplace learning and their tools of learning into two big categories: social mediation by the communities of practice and the cultural mediation of values, tools, and resources. But this theory is unable to offer a more concrete framework to connect the bits and pieces of such a learning web into a clear structure. While analyzing the data, I re-read some activity theory work, which I originally had not appreciated because of my initial superficial understanding, and the structure and constructs finally came together.

Coming to really appreciate the usefulness of this theory has been another long and gradual process which included a thorough reading of Engestrom's works and those of his predecessors. His triangular configuration of the activity system was the structural map that I had sought for a long time. His developmental work research has offered a wide and deep enough lens to allow in both more light and deeper view on the workplace learning of teachers that I examine. His study of team meetings in industrial plants also provided categories and coding schemes for my discourse analysis.

Drafting work. As I get inside this activity system to analyze and interpret my data, I constantly find myself drawing circles, charts, and tables, driven by a need to present visually the interconnected web of data. The following words from Engestrom and Middleton (1996) summarized very well the nature of and need for studying work practices from the point of view of an activity system.

> *Transcripts of talk are complemented and enriched by visual representations of work settings and specific sequences of interaction. This complementarity of textual and graphic modes of representation is a distinctive expansion of the more traditional models of discourse and conversation analysis. [...] Visual representation serves as a reflexive function in that they break down the tight flow of written argument, forcing both the writer and reader to stop and look, and then to realign the two modalities (p. 5).*

Triangulating data. I used triangulation of data from observation field notes, audiotapes, interviews, and document analysis to cross check my interpretations and add to the completeness of the descriptions. This combination of data sources allowed "data gathered in one way to cross check the accuracy of data gathered in another way" (LeCompte & Preissle, 1993, p. 48). In addition, the multiple sources of data allowed me to substantiate my arguments and strengthen the constructs in the developing process.

Seeking patterns and reducing data. Homework emerged as the research problem from prior research on teachers' interactions in their offices. Viewing and transcribing the videotaped data allowed me to pay attention to those collegial interactions involved in marking homework and tutoring students on homework during breaks. These visible homework activities became important areas of observation during my field work. In the field, data collected was guided implicitly by interrelated goals: to search for the invisible or embedded relationships between and among homework-related activities; to identify their connections to the teacher's efforts to assist student learning; to understand the teachers' own meaning making based on homework. With these goals in mind, I had to observe as much as possible in the field to form a comprehensive picture of Tr. Wang's practice. This required data-reducing in the cyclical analyzing and selecting of data for focal analysis.

I approached data reduction in several stages that were not at all linear or sequential. The first stage involved writing thematic memos and vignettes of the mathematics problem surfaced in the homework related activities to strengthen major themes and their inner connections. I wrote long analytical memos on individual broader but recurring themes as one way to pull together potential data that supported the themes and generated new themes, such as how homework was used as a tool in Tr. Wang's practice and how I was able to tell how Tr. Wang used homework.

I drafted problem vignettes depicting activities surrounding homework on a number of individual days as another way to draw the inner relationships of the activities and to look more thoroughly into the object of the activities. The vignettes yielded detailed descriptions of the dynamic office events and classroom teaching scenarios that served as important backgrounding for the homework activities. In these vignettes, I highlighted the homework errors in two ways. First, I focused on the errors that the teacher explained during her teaching and compared and contrasted with those that she tutored to individual students and discussed with colleagues. In doing so, I was able to trace which errors were dealt with in which activities and whether they had connections between and among them. Second, I focused on the mathematics behind the errors that the teacher explained and tutored to students and discussed with colleagues to examine their importance in teaching and learning the related topics. Together, these vignettes allowed me to pull out a set of themes to organize and select data for more detailed analysis along the line of the object of an activity, its construction, and its process of transformation.

The second stage began with a more thorough examination of the object, the homework errors and learning problems selected by Tr. Wang for more teaching and tutoring. I started by carefully reading collected student workbooks and making a list of those errors used by the teacher in different homework-related activities. I recorded the different types of symbols and comments that she assigned to and wrote on these errors, from which I tried to make sense of the teacher's feedback and what she wanted to communicate to her students through it. (See more details about this analysis in Chapter Three.)

Based on the above two stages, in the third stage, I singled out those marked errors that were used in other homework activities. I did a thorough inventory of all explaining and tutoring segments, breaking them down by their content, procedures, and major characteristics. The inventory helped to identify major patterns across those segments, which were then used to select representative segments for detailed transcription and coding. I also did the same for teachers' conversations about homework and pared them down to a small set of conversations for detailed transcription and coding. Details about the transcription and coding are provided in chapters four through seven.

Knowing the subject matter of errors. The data analysis came to a long "halt" when I searched and read more about the mathematics related to those errors and the research on their teaching and learning. This process was made arduous particularly by the lack of readings on geometry proofs published in recent decades in the U.S. Most readings found are about debates on whether to teach this content area and when and how to teach it, rather than containing a thorough discussion of the content itself. In terms of content on functions, there are big differences in the mathematics terms used in the two countries' curricular materials, the topics chosen to teach, and where those topics are located in the curriculum. Eventually, this process familiarized me with the content and enabled me to talk about the importance of the errors in terms of the mathematics entailed and their roles in pedagogy and learning. The cultural differences in the designing, framing, and sequencing of curricular topics are an intriguing area for further study.

Discourse analysis. I drew on Cazden's (1988) approach to classroom discourse patterns, mainly the IRE (Initiation-Response-Evaluation) patterns, to code and make sense of Tr. Wang's explanations of homework at the beginning of lessons and her tutoring of students on homework as an extension of classroom teaching. The similarities and differences reflected in the discourse patterns not only help to illustrate the teacher-student interaction patterns in a traditional Chinese mathematics classroom but also revealed the cultural features of the discourse of explaining errors. I also used other coding approaches, including Engestrom's coding of phases and categories of artifacts in team meetings. Descriptions of the coding procedures are found in subsequent chapters.

Limitations of the Study

As a piece of interpretive research, this study has a number of limitations. The most prominent ones include, first of all, my lack of a deep and thorough understanding of the mathematics entailed in the errors chosen by the teacher for additional teaching, despite efforts to educate myself through substantial reading and talking with mathematics colleagues. With a more solid subject matter training and pedagogical knowledge, I would have been able both to articulate and to present the analysis and findings in a clearer, more sensible way. Second, as both an insider and outsider in the educational system under study, I sometimes felt it hard to gauge my position and felt uncertain or confused about what constituted a neutral position from which to present data and analysis truthfully. In other words, I may have interpreted certain pieces of data with a cultural bias of which I am unaware or lacked the sensitivity to interpret them to fit readers' cultural frames in order to help them understand better. This limitation may be hard for a novice researcher to improve, and it certainly may render my descriptions and arguments ambiguous at points.

Third, my study relies heavily on observation data. On the one hand, this is required by interpretive research; on the other hand, it prevented me from getting insiders' perspectives more systematically (for instance, how they felt students' understandings had changed after explaining of or tutoring on homework errors, that is, whether these interactions made any difference and how they could tell). Such perspectives would strengthen my argument that Tr. Wang's additional assistance pertaining to homework had brought greater understanding to students.

Fourth, in addition to the above limitations, the generalizability of this study is limited by its size. Drawing on one case study has allowed a richer and deeper view of one teacher's homework practices (LeCompte & Preissle, 1993), yet there always lurks a certain danger that the case may be inappropriately generalized onto work practices surrounding homework held by other teachers or schools in Shanghai or broader China.

Despite its lack of generalizability, this study can raise important conceptual questions in research on the roles that homework can play in mathematics teaching and learning. My study also helps in rethinking homework practices and demonstrating how homework can be used as a pedagogical tool to provide learning opportunities for teachers based on their practice.

Chapter Three:
Locating Tr. Wang's Homework
in Shanghai's Mathematics Teaching and Learning System

Introduction

Chapter One gave prominence to the homework-related activities in Tr. Wang's daily work – marking and commenting on students' homework, explaining selected problems at the beginning of a lesson, tutoring individual students on homework errors during breaks, and talking and sharing with colleagues about homework issues. Chapter Two provided an introduction to how concepts of activity theory and cognitive research allow us to investigate how homework has the capacity to become the object of her major activities that orient her work to identifying student learning problems and responding to and using them to create additional teaching and learning opportunities. Both Tr. Wang and homework are part of communities of practice in which social and cultural traditions and values give shape and meaning to the curriculum in which homework takes its shape. In this chapter, I explore the major traditions and rationale beneath the design of the curriculum, homework's place in the curriculum, and how curriculum (and homework) shape the teaching and learning systems and are shaped through their use.

The curriculum that I refer to include the Curriculum Standards for Nine-Year Compulsory Education, the 8^{th} grade mathematics textbook for semester one and the Teaching Reference Material (the official teachers' guide) for the same semester. By homework, I refer to the assignments designed in the textbook and in the two workbooks called *Volume A* and *Volume B* for the semester which were assigned on alternating days. Homework also included the assignments from another two resource books used by most secondary schools in Shanghai, *Same Step Practice and Improvement* (abbreviated as Same Step) (*tongbu xionglian yu tigao*) and *Tests and Commentary Analysis* (*ceshi yu pingxi*). Tr. Wang and her colleagues assigned homework (also used quizzes) from these two resources, usually for weekends, when students finished the other assignments from the textbooks and the two workbooks. In my study, homework is regarded as a component in the curriculum designed for use by both teachers and students. At times, if I sound like talking about the curriculum and homework are two separate things, I do so to emphasize homework.

The municipality-wide mathematics curriculum used in Shanghai *creates* a system of teaching and learning, and Tr. Wang's daily work practice centering on homework reinforces and in some ways *recreates* that system. In this chapter, I focus on the role of the curriculum in conjunction with homework in creating the system of teaching and learning. I discuss three overarching features of the curriculum, homework's place within it, and how the system in each instance is recreated by the homework activities of Tr. Wang. First of all, the curriculum is characterized by clearly defined goals and objectives, and homework as a component is aligned closely with the curriculum and its goals. It organizes the content structure, defines the work responsibilities of a teacher, and guides the curriculum's actual enactment in teaching and learning.

Second, the spiral and sequential organization of the curriculum (Bruner, 1960/1967) creates "meaningful verbal learning" as discussed by Ausubel (1963, 1968) which is characterized by longitudinal coherence across grades and connectedness between and among ideas. Homework, as a major means for consolidation in this teaching and learning system, is reconstructed and transformed by Tr. Wang in collaboration with her students and colleagues through her daily homework activities.

Third, variation of forms *(bianshi)* and "practice by variation of forms" *(bianshi xunlian)* is a major theme and method in curriculum and homework design. Used in Tr. Wang's practice, this design method has a direct impact on her homework activities. This method is generally conceived as "designing and using one substantive problem in teaching a lesson by changing it multiple times in its forms and along different dimensions" (Cai, 1995, p. 23.; Gu, 1994, p. 191). Cai and others have observed that this "one-problem-multiple-changes" (OPMC) instructional approach has been used regularly in Chinese and Japanese classrooms. Careful analysis of the curriculum, homework practice exercises, and relevant literature shows that there are other versions of the so-called "practice by variation of forms" besides OPMC, however. Observation of Tr. Wang's teaching and daily work helps illustrate how such a method is enacted in the practice of teaching and learning and how her homework activities play a crucial role in making this system work. In addition, such design of practice purposefully induces students to err and makes them learn from discerning the errors and lets teachers utilize the errors as resources for helping students learn (Interview with Gu[1], November 15, 2002).

These features of the curriculum and homework also reflect the Chinese cultural traditions of teaching and learning dating back thousands of years, which emphasize prevention *(yu)*, timeliness *(shi)*, sequence *(xu)* and transformation of weaknesses into

[1] Gu, Lingyuan is a leading mathematics researcher and mathematics education professor in China (located in Shanghai). He is also mentioned later in this chapter.

stre

feat

und

of T

Th

Dej

sel

cur

"cei

cur

Ma

elen

(zu

wor

call

curr

syst

---

[2] Th
year
[3] M
pro
and
Lyn
Ya

strengths by learning about the weaknesses *(chang shan jiu shi)*[2]. By illustrating these features, I make an effort to locate homework in this teaching and learning system and to understand how it works both as a component of the curriculum and as an important part of Tr. Wang's daily practice.

The Context of Homework: A Teaching and Learning System Guided by Clearly Defined Content Goals and Objectives

In a prior study I participated in, we examined mathematics teacher induction in selected countries[3] and found that the municipality-wide lower secondary mathematics curriculum in Shanghai serves for novice teachers as "a teacher of teachers" and is "central to the work of teachers and schools." In teaching and learning, the "official curriculum goals" have "tremendous power" (Paine, Fang, Wilson, 2003, p. 36, 37, 54). Ma (1999) in her comparative study of Chinese and American teachers' understanding of elementary mathematics also found that "studying teaching materials intensively" *(zuanyan jiaocan)* (p.130) not only "occupies a significant status in Chinese teachers' work" (p.135) but is also a major way those whom she studied came to acquire what she called "profound understanding of fundamental mathematics" (PUFM). Obviously, the curriculum guides math teachers' work and creates a certain type of teaching and learning system. One feature of this system is that the curriculum standards and benchmarks

---

[2] These are education principles found in Record of Education (xue ji), written more than two thousand years ago. It is regarded as one of the earliest essays on principles of teaching and learning in the world..

[3] Middle Grades Mathematics and Science Teacher Induction Study in Selected Countries, NSF-sponsored project, 1998-2001. Shanghai, China, and France were chosen as sites for mathematics teacher induction and New Zealand and Switzerland as sites for science teacher induction. The Shanghai team, chaired by Lynn Paine, included Dan Chazan, David Pimm, Suzanne Wilson and two graduate research assistants, Yanping Fang and Violeta Lazarovici.

consistently and clearly define content goals and requirements and impose a rigorous pacing guide.

Content Goals and Bloom's Taxonomy. Tr. Wang, like all middle school teachers in Shanghai, has been using a set of teaching materials written by the Shanghai Municipal Curriculum and Teaching Materials Committee. These were issued by the Shanghai Education Press in 1991 after several years' pilot experimentation[4]. For a long time, the curricular standards throughout China, including those for Shanghai, have been in terms similar to Bloom's taxonomy[5]. These standards are written as a set of four ascending categories in terms of teaching content and requirements: to know, to understand, to master, and to apply. The two units that I observed, on geometry proofs and on direct and inverse proportion functions, are covered in both the Curriculum Standards (1998, p. 50-52) and the Teaching Reference Materials (1996, p. 35, 40, 46) in the following table form (the portions in bold type represent content areas for which I observed teaching):

[4] A new set of curriculum materials piloted for quite a number of years was expected to be produced for circulation among schools in Shanghai by 2004.
[5] Bloom broke down competences and the skills demonstrated into 5 categories: Knowledge, comprehension, application, analysis and synthesis. See Bloom, B.S. (Ed.) (1956) Taxonomy of educational objectives: The classification of educational goals: Handbook I, cognitive domain. New York ; Toronto: Longmans, Green

Table 3.1 Teaching Content, Objectives, and Teaching Hours for the Unit on Functions and Geometry Proofs[6]

Unit & Section[7]	Content	Objectives				Tchg Hrs.
		To Know	To Under-stand	To Master	To Apply	
Unit 21: Functions						
21.1-2 Meaning and basic properties of proportions			√	√		2
21.3 Direct proportion functions	**Meaning of DPFs**			√		**1**
	Analytic expression of DPFs			√		**1**
21.4 Graphs and properties of direct proportion functions	**Graphs and properties of DPFs**			√		**2**
21.5 Inverse proportion functions, graphs and properties	**Inverse proportion functions, graphs & properties**			√		**1**
21.6 Functions	**Definition and symbols**		√			**1**
	Domain range and value of a function			√		**2**
21.7 Ways to express a function	**Ways to express a function**			√		**1**
Practice and exercises						1
Unit 22: Geometry Proofs						
22.1 *Original Book of Geometry*	Origin of Euclidean geometry and value of learning deduction and inference.	√				
22.2 Propositions, axioms, and theorems	Concepts of a proposition, an axiom and a theorem		√			1
22.3 Steps of a proof	Steps and format for writing a proof	√				1
22.4 Demonstration examples of writing proofs	13 Demonstration examples			√		7
	Proof of a key theorem			√		
22.5 Converse propositions and theorems	**Concept**		√			**3**
	Take a perpendicular bisector and angle bisector respectively as sets of points			√		
*22.6 Unequal relations of sides and angles in a triangle	Two key theorems					

[6] This table is a direct translation of the official curriculum both in content and form.

[7] Although the standard sequence normally schedules geometry as content for the later half of every semester, Tr. Wang and her math colleague in her office continued from before the midterm exam teaching geometry and did not start functions until after they had finished with geometry. For the past three years, schools have been given freedom as to which one (algebra or geometry) they want to teach first. Some schools choose to teach two weeks of algebra and then two weeks of geometry, or vice versa.

Practice and exercises	3

Note: * The asterisked item indicates optional content. Tr. Wang did not teach it. This topic is emphasized in the content designated for the second semester of 8th grade.

These unified content goals for teaching, together with a guiding timeframe for implementing them, create a highly uniform teaching and learning system: every math teacher is teaching the same basic content required of virtually every student at almost the same time in schools across Shanghai. For a Western expert visiting classrooms there for the first time, such uniformity is especially impressive. "Because of the striking continuity of content across the Shanghai area (and only set of texts approved), when we went to the same grade lesson the next day, we saw the next lesson," wrote David Pimm in his observation notes when he visited with the project team in the fall of 1999. Needless to say, the mid-term and final exam weeks are also fixed as the same two weeks across all schools. Since there is no tracking in nine-year compulsory schools[8], every student is required to learn and master the same basic knowledge and skills in order to complete middle school. In alignment with the content goals and with the unit and section sequences, every student receives the same set of official workbooks for homework, *Volume A and Volume B* assigned on alternate days. Although teachers receive another two resource books in the curriculum material set from which they can choose supplementary practice problems or quizzes according to their needs, Volumes A and B of homework constitute the fundamental requirements for each middle school student.

[8] Tr. Wang taught two classes, one composed of students who were stronger in math and one having a larger proportion of mathematically weaker students. Both classes were assigned and required to complete homework in Volume A and Volume B. She also gave additional homework assignments from other resource books to both classes, and when she did so, she gave the weaker class a smaller amount of such homework than she did the stronger one.

In alignment with the content standards, in designing homework, a topic that deserves more teaching attention is also matched with a greater amount of homework. For instance, if a concept's objective is listed only as "to know," some introduction is to be given in teaching, but no homework is designed for the concept or topic. For concepts having objectives of "to understand" (e.g., the concept of converse propositions and theorems), relatively fewer hours for teaching and fewer assignments are given. The majority of homework is found in those concepts or topics having the objective of "to master." The Curriculum Standards (1998) state that "to master" means to understand the knowledge, to form skills through practice, and to be able to perform simple applications. (p. 17) Thus, the formation of skills through practice is expected to be achieved mainly through homework. Of the 50 topics which appeared in the 8th grade standards, 7 of them have objectives of "to apply," and all of them are geometry topics. In the Teaching Standards, "'To Apply' means to be able to use knowledge to solve problems in a synthetic and flexible way" (p. 17). Hence "mastery" already includes "application." This is different from what mathematics reformers in the U.S refer to as application by connecting to real world situations[9].

Immediately following each topic in the Teaching Reference Material or TRM (the teaching guide that will be discussed later in the chapter) are more detailed teaching objectives, requirements, content analysis and teaching suggestions. For instance, the teaching of geometry proofs in the first semester of 8th grade has three broader goals:

> *First, to master accurate geometry concepts and geometry language; second, to learn the procedures (steps) of deductive reasoning; and third, through appropriate practice in solving problems, to master the fundamental thinking methods (observation, comparison and contrast, analysis, synthesis, abstraction,*

[9] More recently, reformers in China have argued for the same notion of "to apply" as a curricular goal.

summarization, induction and deduction) such as to form the habit of logical thinking as a basis for life-long learning in the future." (TRM, 1996: 53)

The TRM reminds teachers that since this marks the beginning of learning geometry proofs, strong emphasis must be laid on "the norms of writing, or expression format" and that "every claim be supported with evidence" (p. 54). It also reminds teachers that they should use an appropriate amount of homework practice and adopt a cautious attitude to prevent students from developing bad habits, confused thinking, and missing details or writing casually. (p. 54)

In the classroom teaching and homework activities on geometry proofs, Tr. Wang worked painstakingly towards these goals and requirements. Most of her attention was focused on dealing with students' sloppiness in writing and logical thinking. One of the most common errors in geometry homework at this time is called "missing conditions" *(lou tiaojian)*, in which students would tend to construct claims without using sufficient conditions or by ignoring a given condition so that the logic is not strict *(bu yanmi)*. "At heart, they understood it," Tr. Wang explained; "it's just that they assumed that it is too obvious to write." For example, a girl she tutored told her that she did not write down the given condition because she wanted to reduce the number of steps. This appears to be a rather procedural problem, but Tr. Wang rigorously marked out those missing conditions with specific symbols whenever she saw one, tutored individual students to call their attention to such problems, and emphasized them in her teaching.

Analysis of the sampled marked homework shows that about two-thirds of the marked places in geometry homework are directly or indirectly related to this "missing c

w

r

a

g

a

b

d

e

th

ab

m

ad

ov

m

tea

pro

pe

pro

car

T

on f

condition" problem[10]. This problem was frequently covered in the conversations Tr. Wang had with her officemate, Tr. Zhao, when they marked homework together. "Problems will arise later on," asserted Tr. Wang as she explained her strenuous effort. "I need to train their habits of strict logical deduction from early on!" This is how she accounted for the kind of importance she attached to this goal within her teaching of geometry proofs. She also explained that, in the 9th grade, when students would be allowed to write proofs in a simplified way, they would easily make logical mistakes because the more simplified the method, the more confused the proofs would be if they did not start with sufficient conditions and support each step with good evidence/conditions.

Using Homework to Work Within and Beyond Curriculum Guidance. Although the number of hours allotted for teaching each topic is clearly paced (as shown in the above table of standards), as an experienced teacher who had taught secondary school for more than 20 years, Tr. Wang generally followed the given time frame but made visible adjustments according to her pedagogical needs. Such needs are found to be largely based on the performance of students as seen in the number and kind of errors that students made in homework and her sense of what the errors meant in terms of their impact on teaching and learning. For example, on the day when she started teaching inverse proportion functions, she was very happy that the chemistry teacher offered her one period of his classes that day, since he could not teach that period. "There are still lots of problems in the student homework," she explained; "if I stop to explain the problems, I cannot teach the new content as planned; but "if I do not, the problems pile up and that

[10] Tr. Wang also found the term "jumping steps" used in talking about homework errors in teaching the unit on functions.

becomes a bigger problem" (observation notes, November 26, 2002). Most of the time she used in dealing with student homework was outside of the official instructional hours. She believed that, without taking time to deal with the problems reflected in student homework, moving on to the next lesson would not be as fruitful. In the meantime, she did not want to delay instruction.

In teaching about direct and inverse proportion functions, Tr. Wang made the adjustments shown in the table below:

Table 3.2 Tr. Wang's Adjusted Teaching Schedule in Teaching Direct and Inverse Proportion Functions

Official Curriculum				Enacted		
Content sequence	Content	Content goals (master)	Hours	Hours taught of req. curric.	Hours added	Added content
21.3 Direct proportion functions (DPF)	Meaning of DPFs	√	1	1		
	Analytic expression of DPFs	√	1			
21.4 Graphs and properties of direct proportion functions	Graphs and properties of DPFs	√	2	1	1	*Practice lesson on DPFs*
21.5 Inverse proportion functions (IPF), graphs and properties	Inverse proportion functions (IPF), graphs and properties	√	1	1	1	*Practice lesson comparing DPFs and IPFs*
Total hours			5	5		

She finished teaching the meaning/definition of a direct proportion function and its analytical expressions in one period instead of the two recommended by the TRM. Then she continued teaching and finished the graphs and properties of a direct proportion function in one period instead of the given two periods. In so doing, she used this saved period as a so called "practice lesson" *(xiti ke)* to review and consolidate all of the content on direct proportion functions by presenting more practice exercises that were based on the property just taught but had to be solved by using the knowledge about indexes and inequalities that had been taught previously. This lesson also tacitly prepared students for her upcoming teaching of the range and domain of a function. After teaching inverse proportion functions and their graphs and properties in one period, she used the other saved period for another "practice lesson" in which she put the two types of functions together for comparison and contrast and provided practice exercises to help students see that the two functions are mutually transformable. The homework assignments she chose for that day from one of the two additional resource books matched the teaching content she was supplementing that day.

Obviously, two things supported her, allowing her to teach successfully at a quicker and tightened pace. First, because of her familiarity with the teaching content, how students learn the topics, and the trajectory of the curriculum, she was able to build connections between the sections to tighten the content without losing the students. She also knew where to stop to consolidate the learned content before moving on to new content. Second, she marked and provided feedback on student homework in a prompt and rigorous manner, and she engaged in conversations with colleagues about problems encountered in student homework, incorporating ideas she gained from the conversations into her feedback to students. In this way, she cleared obstacles in the way of students' learning and prepared them to continue learning about a topic.

Role and Organization of the Curriculum for Teaching – Analysis of the Geometry Proof Unit

As mentioned earlier, recent studies on mathematics education in China point to the importance of the curriculum in supporting teachers as they learn the content and pedagogy necessary for their practice (Paine, Fang & Wilson, 2003; Wang & Paine, 2003; Ma, 1999). This triggers the question of why the curriculum is able to play such a role and how this role is enacted in the real practice of teaching and learning. Below, I examine the role and the features of the curriculum, in particular noting the sequential organization of the curriculum and its ability to shape teaching and learning.

The role of the curriculum in teachers' work. Both Ma and our mathematics teacher induction study in Shanghai found that teachers in Shanghai, particularly novice teachers, study the curricular materials to acquire "wisdom of practice" (Shulman, 1987). The materials help them to connect what they will teach with how to teach it. The novice teachers we studied started to learn the lower secondary curriculum through what are framed in pre-service pedagogical courses as the important points, difficult points and hinges of teaching. They continued studying the curriculum, the Teaching Reference Material (TRM), and the textbooks through these frames in their planning, teaching, and formal and informal conversations with colleagues even after beginning to teach. Our analysis of the first semester, 7th grade TRM showed that each chapter and section lays out the important points to study, what constitute the difficulties for children in learning the important points, and what pedagogical considerations a teacher should make in order to help students overcome the difficulties and get at the important points (p. 51).

Ma (1999) characterized what she called "profound understanding of fundamental mathematics" that the experienced teachers in her study possessed in terms of their knowledge's depth, breadth, and thoroughness. "Depth" refers to the teacher's ability to connect the ideas at hand to the most important mathematical ideas, even across the material taught in different grades; "breadth" refers to the ability to connect ideas of similar conceptual power; and "thoroughness" is the ability to weave the various ideas

into a coherent whole (p. 121). She also found that when these teachers talked about teaching of the subject matter, their talk tended to turn such connections into four similar "properties" that are more concretely connected with mathematics learning: "connectedness," or their tendency to draw connections between both superficial and underlying concepts, procedures and operations; "multiple perspectives," or their appreciation of different facets of an idea and of using multiple approaches to solve a problem; "basic ideas," or knowing well those "simple but powerful basic concepts and principles of mathematics"; and "longitudinal coherence," or their creation of opportunities to revisit crucial concepts students had learned before and to pave the way for their learning of what would come next (p. 123).

In our earlier interviews with novice teachers and their mentors and in our observations of the novices' teaching, they very frequently talked about "familiarizing themselves with the teaching materials" *(shuxi jiaocai)* in terms of the important, difficult and hinge points of teaching. It is interesting that in my observations of and interviews with Tr. Wang and her colleagues and other experienced teachers, these terms were rarely heard. Tr. Wang said that she had used the TRM everyday when she had just started to teach, but she no longer did so. She did think of using it, however, to check the standard key to a student homework problem given at the back of the little book every time she or her colleagues were confronted with multiple student answers or multiple solutions to a problem. It seems that for novice teachers seeking to access the basic content and organization of the subject matter, the three points of teaching offer convenient assistance. Experienced teachers who have begun by familiarizing themselves with the field using those frames and have continued studying the teaching material through teaching "do not invent connections between and among mathematical ideas, but reveal and represent them in terms of mathematics learning" (Ma, 1999, p.122). My study suggests that teachers' homework activities allow them to connect their knowledge about the content, the curriculum, and teaching with student learning. They make such connections in their daily work by marking homework and by identifying and making sense of student learning difficulties in order to provide timely assistance through in-class explanations and tutoring. They also make these connections by discussing curricular and homework issues with colleagues.

The rationale of the curriculum design. In recent decades, the Chinese mathematics curriculum has been written in the spirit of Bruner's "the structure of the subject" and Ausubel's "meaningful verbal learning." These theories have obviously had sufficient impact on the design discourse and methods of school mathematics education curriculums in China (Zhang, Tang & Liu, 1991; Gu, 1994; Wang, 1995; Zhu & Wang, 1998). Gu (1994, p.168-169), a leading Chinese mathematics education researcher located in Shanghai, has summarized such impact in the following way: Bruner's theory about the structure of knowledge and sequence of materials to be learned has aided us in understanding how we should organize teaching content. His theory, however, is too general to guide classroom teaching. In comparison, Ausubel's theory of assimilation in concept learning, such as is found in superordinate and subordinate learning and the relationship between the two, has practical implications for classroom teaching, especially in teaching concepts. Both Bruner and Ausubel were regarded as concerned with building meaningful connections in the "structure of a subject" (1960 and 1966) and in a learner's "cognitive structure" (1963) to increase fruitful transfer and retention of knowledge learned. In Shanghai's current math curriculum, Bruner's direct impact is reflected in its spiral structure, while Ausubel's influence is found in a common discourse pattern in the TRM (Teaching Reference Material) that specifically states the connections between a new concept and the previously learned one(s).

While in North America Ausubel has been considered more of a behaviorist, in the Chinese research literature on mathematics teaching and learning, he is viewed as more of a key figure in cognitive psychology, together with Bruner. Ausubel is mostly known for his identification of principal variables in the cognitive structure and using them to manipulate the organization of learning material and thereby to modify the cognitive structure a learner developed in the learning and retention of meaningful material. As he put it, "...new ideas and information can be effectively learned and retained only to the extent that more inclusive and appropriately relevant concepts are already available in cognitive structure to serve a subsuming role or to furnish ideational anchorage" (1963, p.79; 1968, p.153). Therefore, the two related variables essential for facilitating subsequent learning are the availability of anchoring ideas in the cognitive structure and their stability, clarity, and discriminability from the learning material (1968). Ausubel recognized Bruner's and others' contribution to "the substantive organizational problem" which refers to "identifying the basic organizing concepts in a given discipline" (p. 152). He proposed using his principal variables in "the presentation and sequential arrangement of component units" (1968, p.152) and their topics and even sub-topics. His key construct of anchoring ideas refers to the goal to bridge the gap between what the learner already knows and what he needs to know before he can successfully learn the task at hand.

a

ar

di

hi

pa

ke

cr

ma

ca

an

lea

"s

[d

ne

enl

lea

de

org

geo

obs

Anchoring ideas or *organizers* should be "the more general and inclusive ideas of a discipline" because they have "greater inherent stability," "greater explanatory power," and "integrative capacity" (p.148). In the meantime, the new learning material should be discriminable from previously learned concepts so as to enable transferability. We see hints of this in Ma's (1999) conception of teachers' knowledge package – within the package, different pieces are not only connected but also carry different weight, with the key pieces playing similar roles as the anchoring ideas (p. 53).

For students, such sequential organization of units (among units and within a unit) creates "sequential school learning" (p.158) in which knowledge presented in the earlier material can act as an organizer (introductory) for learning the subsequent material. Such carefully sequenced learning material ensures "gradation of difficulty" to secure anchoring posts for learning and retention of later knowledge items. For such sequential learning to be most effective, Ausubel suggested that each unit should also have a "separate organizer" (p.158). By using "comparative organizers" (p.143) that "explicitly [delineate] similarities and differences between the two sets of ideas" to make the old and new knowledge more discriminable and familiar, as mentioned above, sequencing will enhance "the organizational strength of cognitive structure" of both the material and the learner. These ideas of anchoring, sequencing, and organizing run throughout the development of Shanghai's curriculum and of the TRM.

Below I present an analysis of how the Shanghai Teaching Reference Material organizes the content of the first semester of 8th grade and organizes its analysis of geometry proofs and direct and inverse proportion functions, the two units I happened to observe being taught during my field work. This analysis demonstrates how the middle school mathematics curriculum is designed based on Bruner's spiral structure and Ausubel's sequential organization of units and topics. For a culture that gives authority to texts (Paine, 1990) and attaches importance to order and sequence in designing learning materials and teaching sequentially, Bruner and Ausubel provided much guidance in terms of principles and methods of curriculum design.

Analysis of the Unit on Geometry Proofs. The design of the unit on geometry proofs is a representative example; the unit, its topics, and its sub-topics are carefully sequenced to take advantage of the available anchor points, and they use organizers purposefully to scaffold learning of current and future concepts. The content analysis of the unit in the TRM starts by locating the position and role of geometry proofs in the spiral structure of the middle grades' geometry to identify what is already available in the learner's cognitive structure. This unit, as it states, marks a transition from experimental geometry in 6^{th} and 7^{th} grade to proof geometry in 8^{th} grade. It reminds the teacher that in experimental geometry, students learned a set of 40 geometry properties through hands-on experimentation with concrete shapes and by simply knowing the simple reasoning behind certain properties. It explains that the teaching of those properties at the early stages of geometry learning provided simple reasoning without proofs. This unit makes the transition by supplementing or restoring the necessary proofs to the 40 properties. Through proofs, these learned properties can now become theorems to support deductions made in doing proof problems. In this way, the 40 properties are utilized as anchor posts to scaffold the learning of geometry proofs.

To facilitate smooth transition and transfer, the congruence of triangles taught in the previous semester (the second semester of 7^{th} grade) is chosen as "the optimal entry point" (TRM, 54) into learning proofs. In the previous semester, simple reasoning was used as a rudimentary version of proofs: for example, if two given triangles are congruent, a student is asked to tell why they are congruent based on the given conditions and figures. In fact and in format, such reasoning is virtually the same as a proof. The purpose of such organization is to create a sense of familiarity:

> it looks as though hardly any new theorems or content is involved; yet it is an indispensable section in helping to deepen the understanding of propositions, axioms and theorems. While doing so, it focuses students' attention on learning the procedures of writing a proof, laws of solving problems and significance of the subsequent necessity of auxiliary lines (TRM, p.54).

Such an organization uses the congruence of triangles as an organizer, benefiting not only from this available anchor idea in the learner's cognitive structure but also from its considerable ability to be discriminated from previously learned concepts in order to facilitate transfer to the learning of the new and related concepts.

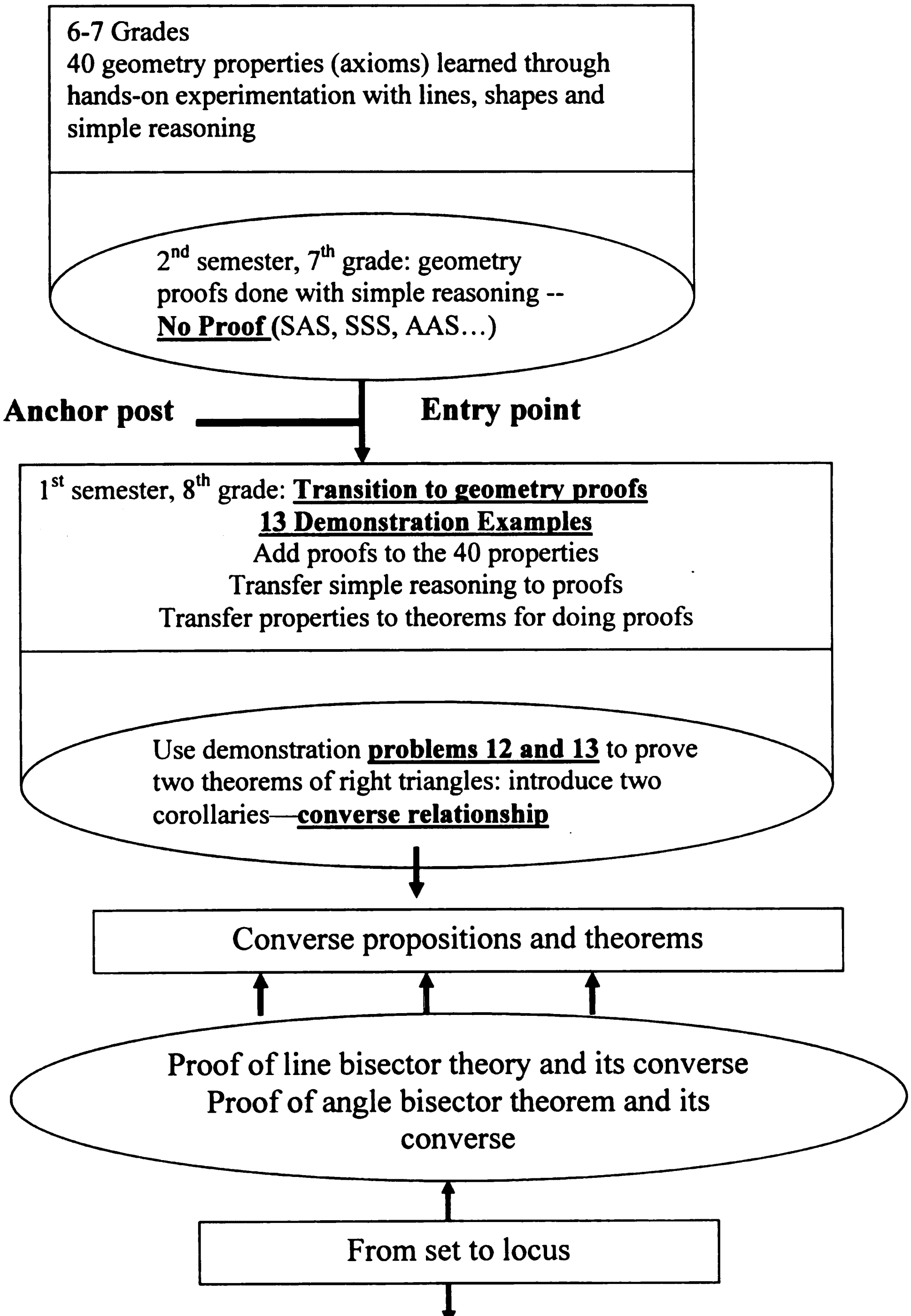
In terms of content, this unit is broken down into two sections, 22.4 and 22.5, as indicated in the above table of contents. Starting with the congruence of triangles, the first section is made up of 13 carefully designed proof examples, arranged in an ascending order of difficulty, each playing a distinctive role. This section introduces and practices the basic procedures of writing a proof, the rules and format to follow to make a proof logically clear and strict. The second section, relatively brief, is about converse propositions and theorems. After a short introduction of the concept, it sets out to prove and practice two related theorems and their converse theorems: specifically, it works with line and angle bisectors with the help of proofs of the congruence of triangles and by adding auxiliary lines, as learned in the previous section. Such an arrangement facilitates sequential learning by organizing a subsequent section on the basis of the preceding one. Using a comparative organizer to introduce two seemingly new concepts, line and angle bisectors, enables comparison and contrast based on the common attribute of "bisector" that is used in the differing contexts of lines and angles. Furthermore, as the final section of geometry proofs in this semester, it builds connections that lead to continued learning of proofs in the next semester. Here, line and angle bisectors are expressed in terms of set; for instance, the line bisector theorem works with the idea that any point on a bisector of a line segment is of equal distance to the ends of the segment. In the second semester of 8^{th} grade, in teaching the concept of locus, the above example is used to compare the similarities and differences between set and locus and what each theory can do. In order to support teachers in teaching the two concepts, in the 8^{th} grade TRM of semester one, teachers are not only reminded in detail of the similarity between set and locus but also how to state the theorems in terms of locus. In so doing, the TRM prepares the teachers in advance to teach the concept of locus that immediately follows in the second semester.

Within section one, the 13 proof examples are also carefully sequenced to build a gradation of difficulty. The first two examples move from a simple two-step deduction to produce a proof to a two-step proof of the congruence of triangles. The three examples following (3-5), all requiring more than two deduction steps to construct a proof, are designed to train students how to analyze a geometry problem. Starting with Example 6, the level of difficulty is raised; two major geometry properties are used in order to construct a proof. Example 7 introduces for the first time a case of proving two pairs of triangles to be congruent. Examples 8 and 9 bring in the idea of producing a proof with the help of adding an auxiliary line. They are followed by Examples 10 and 11, which require adding two auxiliary lines to work out a proof. (Example 10 is regarded as adding one auxiliary because the two auxiliary lines are the same auxiliary added in two locations.)

Built on the basis of the proof of the congruence of triangles and of students' experience in adding auxiliaries, Examples 12 and 13 are designed to have students use these prior experiences to prove two important theorems about right triangles. More specifically, students will prove that the two acute angles of a right triangle added together equal 90 degrees, and they will prove that, if one angle of a right triangle is 30 degrees, then the side that angle faces is equal to half of the hypotenuse. In Example 12, which seeks a proof for the latter theorem, two corollaries of this theorem are introduced: that, in a right triangle, if an acute angle is 30 degrees, then the side it faces is equal to half of the hypotenuse; and that, in a right triangle, if a side is equal to half of the hypotenuse, then the angle it faces is 30 degrees. Example 13 uses this second corollary to solve a given problem. In fact, these two corollaries are of converse relationship. Although the TRM does not suggest this, Tr. Wang made use of these two corollaries' converse relationship to introduce the line bisector theorem and its converse theorem in the lesson that immediately followed. The Curriculum Standards allocate 7 teaching periods altogether for teaching the 13 examples. Because of this carefully designed sequence, the teacher is advised to follow this given sequence and to avoid any increase in the level of difficulty. But Tr. Wang often made adjustment to the official curricular sequence by making teaching of the similar topic more compact so that she could use more time for additional practice lessons for consolidating the new content and add more variety of practice problems by changing their forms so that students could see the same concept from different perspectives, an approach to be discussed further later in this chapter.

The following chart is a visual representation of the sequential organization of the unit on geometry proofs, as illustrated above. The rectangles indicate content or topic areas while the ovals indicate the anchor points.

Figure 3.1 Content Analysis and Teaching Suggestions for Geometry Proof Unit



Legend: the ovals represent the content designed as anchoring posts, as a basis for moving on to the new content, represented by the rectangles. However, these two types of content are not separate; they are complementary to each other.

As will be discussed later in Chapters Five and Seven, this kind of sequential organization was carried out forcefully and effectively in Tr. Wang's teaching of geometry proofs, given that the nature of geometry proofs facilitates sequential learning. A similar kind of organization in the unit on functions, however, created problems in teaching. The unit starts with the concept of proportionality and uses it to serve as an anchor point for introducing the concepts of direct proportion functions and inverse proportion functions. Built on these two concepts as specific cases of functions (as well as anchor points for functions in general), the unit moves to the general concept of a function and its properties. The major problem in this organization is the weak and insufficient handling of the beginning section on the concept of proportionality. Instead of focusing on the substance of direct and inverse proportionality, this section barely introduces proportionality, and then goes on to introduce its properties, without even mentioning what constitute the concepts of direct and inverse proportions. In doing so, the section fails to utilize the concepts of direct and inverse proportionality as anchor points for the launching of the upcoming concepts of direct and inverse proportion functions.

The beginning of the section on direct proportion functions introduces the idea of direct proportions by using the relationship between the distance (s) and time (t) traveled by a car at constant speed (v) and calls the proportionality of distance and time (s/t) equaling a constant value (v) a direct proportion. It then names the relationship between t and s ($s = vt$) as a direct proportion function. Later it calls the multiplication of two variables (st) equaling a constant value (v) an inverse proportion and the relationship between distance and time (t = v/s) an inverse proportion function. Such an organization is likely to make students memorize the example given instead of understanding the concepts of direct and inverse proportion functions, confusing students with this set of relationships that seems to be arbitrary.

Furthermore, in the old version of the curriculum, the concepts of a function and its properties were introduced before the specific cases of direct and inverse functions were taught. Tr. Wang and her colleagues, especially Tr. Zhao, the novice teacher, found this change very difficult to handle in terms of when to teach and how far they should go in teaching the general concept of functions. They believed this change, as well as the weak handling of the concepts of direct and inverse proportionality, had created problems for both teaching and learning. The subsequent chapters show that this problem was further discussed by Tr. Wang and her colleagues on quite a few occasions when they began teaching the unit on functions. When the teachers came together to deliberate on the embedded controversies and to seek solutions or ways to manage the uncertainties of teaching, the weaknesses in the curricular design turned out to be opportunities for teacher learning.

In summary, in spite of its flaws, the way that the Teaching Reference Material is written bears teachers in mind and treats them as learners by offering a thorough guide to them as they travel through and familiarize themselves with the structure of the curriculum. The analysis of the content and detailed teaching suggestions help the teachers connect the teaching material with how to teach it by giving each section of the curriculum its rationales so that the teachers are able to use the rationale as explicit analytic tools in learning and teaching the content. The stumbling blocks in the curricular design also prompted teachers to come together to make sense of the ambiguities and figure out ways to manage these difficult situations. The capacity of the curriculum to serve as "tools for teacher learning" also reveals another aspect of the connected and coherent nature of the curricular materials: it is written with "the wisdom of practice." To cite what we find in our mathematics teacher induction study in Shanghai,

> *... the materials support communication about teaching, and are themselves the product of a long-term conversation among Shanghai's mathematics educators, written iteratively, with authors self-consciously reflecting on what has and has not worked in the past. The materials thus represent shared knowledge about a specific and specified curriculum that is located in and among teachers. They are, to paraphrase Shulman (1987), the wisdom of practice, the products of teachers' and administrators' individual and collective inquiries into how and what instructional materials best support pupil learning...* (Paine, Fang and Wilson, 2003, p.54).

Consolidation and homework. Ausubel (1968) emphasized that for effective sequential learning to occur, "the antecedent step is always consolidated" (p.158). He insisted on not introducing new material in the sequence "until all previous steps are thoroughly mastered" (p.159). Consolidation being a key condition for ensuring successful sequential learning, Ausubel suggested "confirmation, correction, clarification, differential practice, and review in the course of repeated exposure, with feedback, to learning material" (p.159) as major avenues for achieving it: Tr. Wang had a pacing guide to follow; her regular lesson lasted only 40 minutes, and her official teaching hours for each of her two classes were no more than 1.5 hours per day. She used these different approaches in her lessons and relied on homework as a major avenue to consolidate learning and make up for the lack of class time for more thorough teaching.

In the sequential organization of the curriculum, homework is carefully designed to fit in the flow of the sequence, serving the purposes of practice and consolidation. Strict monitoring rules were set up to make sure all students completed and handed in their homework books every morning. After the workbooks arrived at her desk every morning, Tr. Wang spent most of her non-teaching hours marking homework, tutoring students on their homework mistakes, and talking and sharing with colleagues about homework issues. In fact, as the upcoming chapters illustrate, Tr. Wang, together with her students and colleagues, reconstructed and transformed the homework errors and learning obstacles into more teaching and learning opportunities that could lead to solid understanding of the concepts taught.

Differential Practice or One Problem with Multiple Changes (OPMC) in Curriculum, Homework, and Teaching

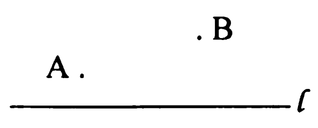
The Confucian tradition of education emphasizes the importance of practice in assisting learning. Confucius considered "learning" or "study" (in Chinese *xuexi*) as made up of two parts: study or learning *(xue)* and practice *(xi).* To him, practice was an important auxiliary tool for studying. This was best illustrated by his famous saying: "studying combined with timely and appropriate practice provides a happy learning experience that produces good results" *(xue er shi xi zhi, bu yi yue hu).* The Chinese curriculum developers and researchers brought up in Confucius tradition are inclined to look for more concrete theories that are able to guide them in designing teaching materials capable of enabling and carry on such learning. Contemporary cognitive psychology has much to offer.

In terms of cognitive science, practice is a major factor that has an impact on cognitive structure. It does many things, including increasing the "stability and clarity of the newly-learned meanings in cognitive structure" and facilitating "the learning and retention of related new learning tasks" (Ausubel 1968, p. 274). One of the methods of practice commonly used is referred to by Ausubel as "differential practice of the more difficult components of a task" (p. 160). Even though he did not say much about what this kind of practice means and how it is designed or how it works, mathematics educators in China, who studied Bruner and Ausubel's ideas seriously, developed a set of such practices to guide the curriculum design and pedagogical recommendations. Over the years, the term *bianshi,* meaning to change or modify forms or formats in the design of pedagogical or practice problems, has become highly visible in research and teaching practice in Shanghai. This term is often seen or heard in the literature on mathematics education, in the curriculum resource books, such as the TRM, in conversations about teaching, and in real classroom practice, including Tr. Wang's teaching,.

A mathematics teachers' professional development course book on the psychology of secondary mathematics teaching, published by Shanghai Education Press in 1995, introduces Bruner's and Ausubel's cognitive theories and paraphrases the following definition for differentiation or variation by changing forms *(bianshi)*: "it is a form or method achieved through modifying the nonessential attributes of a similar learning target. By changing the angles and methods that students use in observing such a learning target, it makes the essential properties stand out for students to view so that they can abandon the non-essential ones to learn more accurate and stable concepts for future transfer" (Wang, 1995, p.73). According to Gu, this kind of practice is "a major characteristic of student homework in China." He compared mathematics homework in China with that in Western countries, mainly in the U.S.: "While it [homework in China] features the changes made in forms based on related key disciplinary features of mathematics, the homework in Western countries highlights the changes made in a problem's contexts of use [where the knowledge learned is to be applied], which does not often reflect the features of change in mathematics" (personal interview with Gu, November 22, 2002).

In teaching, such variation is often referred to as the one-problem-with-multiple-changes (OPMC) instructional approach. As Cai (1987) and others (Hashimoto, 1987; Zhong, 1988) observed, this theme or method is generally conceived as designing and using one substantive problem in teaching a lesson by changing it multiple times in its forms and along different dimensions. It has been regularly used in Chinese and Japanese classrooms to achieve coherence in instruction.

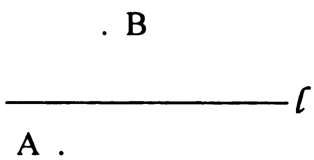
In Tr. Wang's teaching, she often employed such an approach in her teaching. For example, on the morning of November 18, 2002, she taught her stronger class (Class 4) the "line bisector theorem" and its converse theorem[11]. After working on two practice problems with the whole class to consolidate the theorems, she gave a "practical problem"[12]: A telephone booth is going to be built on the side of a road and has to be of equal distance to point A and point B which are also on the side of the road. Where should this phone booth be built? She drew the following figure and led the students to analyze the problem:



[11] Line bisector theorem: A (Any) point on a bisector of a line is of equal distance to both ends of the line/segment. Converse: A (Any) point that is of equal distance to both ends of a line segment is on the bisector of this line segment.

[12] A practical problem is usually considered either a word problem or a problem applicable to real life situations.

After that, she continued with a similar problem: we want the booth to be located where the sum of its distance from point A and point B is smallest. She suggested taking A and B as two residential quarters:

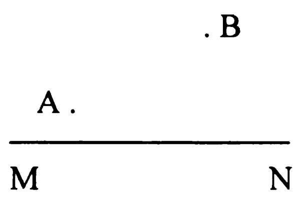


After she led the class to analyze this problem, she added: "If the two residential quarters are not on the same side of the road, you all seemed to know. What if they are on the same side of the road?" She heard some students suggest that they move one to the same side of the road, causing all students to laugh and sparking an enthusiastic discussion. Although I do not know whether students all figured out these problems, the point here is not to argue they do but to illustrate how Tr. Wang used as a practice approach to prepare students for doing the homework she was going to assign after demonstrating and leading students through these examples.

Of the homework assigned at the end of the lesson, two of the problems -- one in the textbook (Exercise 1, 1st semester, 8th grade, p. 79) and the other in Volume A (Exercises 1, 22.5 (2), p. 37) -- are versions of OPMC for the above two types of examples. By teaching the above versions of the problems, Tr. Wang obviously had intended to prepare students to do the homework of the day.

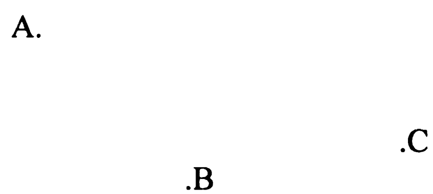
The textbook assignment was:

Say how to find a point P on line MN in the given figure to make PA=PB.



Meanwhile, Exercise 1 in Volume A looked like this:

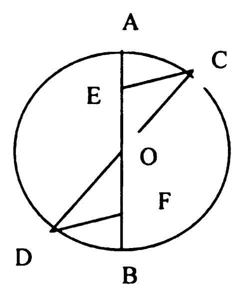
> *How to find a point, which is of equal distance to the three residential quarters, A, B and* C.



There are many similar examples in Tr. Wang's teaching where she purposefully built connections between her teaching examples and the day's homework to facilitate students' completion of their homework. In a number of cases, a given homework problem was beyond the scope of the examples given in the textbook and taught in class. She would design or choose a similar problem to teach in a lesson before she assigned the homework.

Most of the semester's homework assignments are designed to provide students with more opportunities to practice and reinforce a concept that they have just learned, yet without making the problems very different and, thus, difficult. For instance, the TRM of 2nd semester, 7th grade (47), near the end of the "Teaching Suggestions" on the congruence of triangles, proposes the following OPMC practice, using exercise 2 of 18.3

(3)[13] in the textbook (105-106) to illustrate the two typical change methods in geometry: "stretching the head" and "stretching the foot."



Exercise 2

Exercise 2: As shown in the figure, it is already known that AB and CD are two diameters of circle O; E and F are two points on AB; ∠ECO=∠FDO. Are ΔCEO and ΔDFO congruent? Why?

"Stretching the head": (1) Change ∠ECO=∠FDO into CE//DF to practice from the perspective of the properties of two parallel line segments; (2) Change ∠ECO=∠FDO into "Point E and F are midpoints of OA and OB, respectively"; (3) Change ∠ECO=∠FDO into AE=BF, etc.

"Stretching the foot": (1) Change ΔCEO ≌ ΔDFO into CE=DF; (2) Change ΔCEO ≌ ΔDFO into OE=OF; (3) Change ΔCEO ≌ ΔDFO into AE=BF, etc.

"Stretching the head" is a method used to diversify a problem by changing among given conditions, while "stretching the foot" diversifies by changing between and among

[13] 18.3 (3): Chapter 18: Congruence of Triangles; Section 3: Judging congruence of two triangles; and (3): Sub-section 3: S.S.A cannot determine congruence of two triangles.

what is to be proven and what is given. The TRM suggests that using these kinds of practice should reduce the difficulty faced by students in learning geometry proofs.

Gu (1994) called such changes in practice exercises or problems a method to manipulate the problems and promote as much as possible their "pedagogical functions or values" (p. 138). He found that the great majority of the basic exercises in secondary mathematics (in the textbook and the homework books, Volume A and B) can be modified in appropriate ways to serve a greater number of pedagogical purposes. Such modifications would build a gradual slope of difficulty, disperse the level of difficulty, and "provide different levels of students effective practice and develop their independent thinking ability" (138). Since one major purpose of practice by variation in forms, such as introduced above, is to help the students learn the fundamental properties of a math topic, I find such variation and modification to be able to develop students' ability to understand a fundamental mathematical idea from multiple perspectives.

In Tr. Wang's real practice, however, such a design of practice problems would not be able to achieve its desired effect without her intensive work activities related to student homework. These activities, to be presented in the following four chapters, include Tr. Wang's time and energy spent marking homework, selecting difficult or important points (in the form of student errors) to explain in teaching, providing additional tutoring to individual students on their errors, and constantly engaging in informal problem-solving conversations with her colleagues. These activities were able to achieve two pedagogical functions as Tr. Wang enacted the intended curriculum through her daily work.

First, diversifying a standard or essential problem by changing and utilizing related or familiar transferable attributes of the problem requires that students have a firmer understanding or memory of both the previously learned concept or procedure and the essential attribute(s) involved in the problem. Because of this, each change tends to pose a certain level of difficulty (or become a completely new problem) for different students, which requires them to "use their minds", as she often put it, when doing their homework. Tr. Wang's weaker students and even quite a number of her average students encountered such difficulties, for example, in the above homework from Volume A. Students not only needed to understand the line bisector theorem and its converse theory in order to draw the line bisectors, they also needed to see the relationship between this point and the bisectors to understand that they only need to draw the bisectors of two line segments. The standard way of constructing the drawing required for the assignment is to use a ruler and compass to draw the line bisectors, which is a skill that students had learned previously. When all this knowledge and these skills are required for one problem, it creates difficulty for a considerable number of students, even for those in Tr. Wang's stronger class. Because of this, Tr. Wang chose to explain in her teaching the major errors that she found in the student homework of that day, demonstrating, for instance, how to write a complete construction procedure and why only two perpendicular bisectors are needed. Such examples show that the pedagogical functions of such practice in Tr. Wang's teaching are carried out mainly through her daily homework activities, without which, successful consolidation of the topic taught would not happen easily or in a timely manner.

Second, in teaching and learning, this OPMC approach reflects one of the four important traditional methods stressed in the *Record of Education*: teaching and learning by analogy (the other three being questions and answers, explanation, and practice). Teaching and learning by analogy means to infer or deduce attributes or meaning from one thing and apply it to the knowing and learning of things of the same category. This way of teaching, as was used by Tr. Wang, raises the efficiency of learning and assists students in developing their ability to think and reason (Wang et al, 1994, p. 68-69).

Summary

This chapter opens with a look at how the curriculum organizes teaching, with the help of its clearly stated goals and a rigorous pacing guide, and how, in the case of Tr. Wang, homework is used to adjust to and meet her students' needs in her teaching. This chapter then argues that the curriculum, so designed, is a tool for teacher learning, allowing teachers to improve their teaching knowledge. To explain this capacity of the curriculum, I explore the curricular design rationale and methods in light of the curriculum and cognitive theories of Bruner and Ausubel. I delineate their influence on the curriculum development research and discourse in China and the ways in which their popularity in Chinese research and teaching reflects a deep rooting in the traditional beliefs regarding teaching and learning.

The effects of such curricular design is further illustrated with a specific analysis of the unit on geometry proofs. The chapter ends with a look at a special feature of the design -- aiming to increase the pedagogical and cognitive values of practice problems by changing their formats, as in the OPMC model. An example of one of Tr. Wang's lessons is provided to show how she used this design approach in her teaching to take advantage of the pedagogical values such a change in formats was able to bring to teaching. Again, this kind of teaching practice has much to do with a Confucian teaching method that aims at promoting comprehension by analogy.

Chapter Four:
The Role of Marking Homework in Teaching and Learning Geometry Proofs

Introduction

Homework-marking scenarios on November 19, 2002

It was Tuesday morning. Tr. Wang was going to teach Class 4 in the third period and Class 2 in the fourth period, but now she was behind her desk, marking student homework. Her desk was a busy sight. Two piles of different types of homework books were stacked against the wall by her desk. One was a pile of thin and larger (12 inches x 8 inches) workbooks called Volume A, with officially printed math exercises[1]*, figures, and tables that were used on alternating days in tandem with Volume B across all secondary schools in Shanghai. The other was a pile of thin and much smaller (6.5 inches x 8.5 inches) blank exercise-books in which students did their additional homework assignments for the weekend, which had been collected the day before. Tr. Wang required her students to use two sets of such blank exercise-books: one for geometry and one for algebra homework. She had taught the perpendicular bisector theorem and its converse theorem*[2] *on Monday, and at the end of the lesson, she assigned all four problems from the textbook and all three from Volume A as homework. The Volume A workbooks from both classes had been collected and delivered by the math monitors to her desk just before the first period started. After marking two dozen students' homework assigned over the weekend, she spent most of the first period marking Class 4's Volume A workbooks because she was going to use the marked homework to give feedback to students at the beginning of their lesson. Her homework marking continued in the second period and she returned to it after she finished teaching (at the end of the fourth period). She did not stop until 11:45 AM when she went to lunch.*

---

[1] I do not distinguish purposefully between the terms of math exercises and math problems in this study; I use them to refer to the assignments for practice and consolidation purposes both given during class and for work at home.

[2] The perpendicular bisector theorem states that any point on the perpendicular bisector of a line segment is equidistant to both ends of the segment. Its converse states that any point that is equidistant to both ends of a line segment is on its perpendicular bisector.

After lunch, at around 12:35, she resumed her marking of homework. She frowned at a workbook and turning to the cover, she broke out: "Wang Bing has become more out of shape now! Such careless work!" "Yes. I think so, too," agreed the young female English teacher. Sitting back to back with Tr. Wang, she was the boy's class director. "This boy is good at everything but his study," she told Tr. Wang, "but he's strong at using his hands. He won a prize for the science model he made recently." Liu Long, a tall boy weak in math, was again seen sitting at an empty teacher's desk making up an incomplete assignment after Tr. Wang's brief tutoring and instruction. She asked Liu Long to get Wang Bing for her from the classroom. On hearing that the Chinese teacher was using the after-lunch hour, which seemed to be always monopolized by her math tutorial, to dictate new Chinese idioms to the students, she sighed, "Ai, these students are very busy, too!" Liu Long came over to ask Tr. Wang to check his corrections. Noticing that he had missed the logic by confusing the sequence of the steps in his proof, she compared his error with a "failure to follow traffic regulations." She provided further explanation and gave him another problem to do on an extra piece of paper. The next workbook that she began marking had two small Post-it® notes stuck on top of the incorrectly done problems, on which the student had carefully written the corrections. She scanned over and checked over the two corrections first before marking the day's assignments.

As introduced earlier, much of Tr. Wang's daily work was dedicated to marking student homework in her grade-level teachers' office. As we revisit the day of November 19, 2002, from the above vignette, homework marking stands out again. With two different types of workbooks submitted by students on alternate days to be marked and returned by the teacher, Tr. Wang's desk was always a site of much activity. She and her colleagues jokingly described such sites, at which they marked countless pages of student work, as being "buried in homework piles" (which did not always suggest a positive phenomenon[3]). As described in the opening vignette, marking homework was also interspersed with other activities, such as tutoring students on homework and talking with colleagues sitting nearby. These accompanying activities were largely triggered by the ongoing homework-marking work, as Tr. Wang put into use the information she obtained from homework, such as deciding to tutor a problematic student or exchanging with her colleagues certain information about the student(s) in question.

Chapter overview

In this chapter, I investigate Tr. Wang's homework-marking activity by examining the major outcomes that it created for the teacher and her students – a communicative system of symbols, sense made of student learning, and a basis for shaping and being shaped by other homework activities (as illustrated in Figure 4.1 and the above paragraph). Here I lay out my research questions, data, and data analysis. After this, the chapter breaks down into three sections. The first brief section examines the meaning of routine homework-marking actions that create a system of symbols and signs. In the second section, I illustrate this system as an error-centered and process-oriented two-way communicative system that provides the teacher with assistance in error correction. The third section looks into the process of homework marking as the teacher's gathering of information about student learning and making sense of student learning problems, which leads to her making decisions about how to respond to the errors in order to improve student learning. Appendix 1 contains marked student work samples

[3] Saying that a teacher is always "buried in homework piles" could suggest that the teacher was either incompetent in handling homework efficiently or returning student work on time or even could mean that the teacher was ignoring other important teaching work.

that have been translated into English. These work samples will be discussed in the second section of this chapter.

Research questions

In this chapter, I aim to answer the following two central research questions: First, what did Tr. Wang's activity of marking homework entail? Second, what did this activity make possible for Tr. Wang's (and her colleagues') work? These questions will be answered through the more specific questions listed below:

What did Tr. Wang's activity of marking homework entail?

- What were the routine actions of homework marking? What did these actions reflect about the nature of homework marking as an institutional norm and responsibility?
- What were the symbols and signs Tr. Wang marked on the pages? What kinds of meanings did they convey to students?

What did this activity make possible for Tr. Wang's work?

- What were the characteristics of the communicative system?
- How did such a system help with teaching and learning?
- What were the goals of homework marking?
- How did Tr. Wang gather and use information to inform her teaching practice?
- How did this process reflect her pedagogical reasoning and action?

Together, these questions provide a detailed examination of the activity of homework marking in terms of what it involves, what it produces, what it affords for teaching and learning, and what it reveals about the nature of Tr. Wang's teaching practice.

Data and data analysis

Data. I draw on multiple sources of data to describe Tr. Wang's homework-marking activity and what happened in the process of and as a result of marking homework. I use observation data and interviews to illustrate her routine actions and how she collected information and used that information to help her think about teaching, student learning, and appropriate actions to take next. I rely on marked homework samples to find out what symbols, signs, and comments the teacher assigned to student work. I make sense of these symbols to determine what feedback the teacher wanted to provide to students and the nature of the feedback in helping with student corrections.

Data Analysis: I used a basic document analysis approach, sorting, categorizing, and comparing the marked symbols and signs in order to determine what each category in general stands for in the system. I then compared my own analysis with informal interviews with Tr. Wang to determine whether my own interpretation was accurate and to see what I might have missed. I also paid attention to those symbols in the assignments that she chose to explain and tutor to students, so that I could compare them with and deliberate on their actual use in teaching. By triangulating these different sources of information, I was able to form a picture of the homework-marking system and trace the trajectory and cycles of how these symbols were used and how the activity of marking informed and was informed by other homework activities.

More specifically, I also used observations of the teacher's visible actions to make sense of what was going on in her mind, that is, those invisible actions that took place while she was marking homework. For instance, I paid attention to the teacher's facial expressions and inadvertent utterances during her homework-marking actions. I also take note of those tutorial actions and actions of talking with colleagues that occurred while she was marking homework in order to look at how these concurrent actions revealed her thinking in the middle of her homework marking. Analysis of these visible acts offered important insights about her mental activity. For instance, by interpreting the utterances that she made (such as complaints and comments on student work) while marking and commenting on homework, I was able to pick out what information she was collecting and using. By looking at the relationship between homework marking and the subsequent activities she engaged in as an immediate result of marking, I was able to map out the system of the activity of marking homework.

Understanding the Meaning of Routine Actions and Interactions

To an outside observer, a teacher sitting at a desk covered with large piles of homework, would appear on first impression to be doing nothing but repetitively making check marks and X's, marking and flipping of the pages of student workbooks one by one. The following excerpt of the field notes from my first day of observation does create such an impression. Even though I had already begun observing her teaching, this was the first time that I had sat down at a desk a little distance away from her[4] and observed Tr. Wang marking homework in her teachers' office:

> *She has just made a check mark by an item in the first problem of an open student workbook,[5] and now she is **frowning** at the next problem, which she <u>assigns an x</u>.*

---

[4] I quietly sat some distance away during my first observation, as I was feeling cautious about making her uncomfortable or being intrusive to her flow of work as an imposition from outside.

[5] Every morning, as soon as students arrived at school, the row leaders (each of which is responsible for the 7 or 8 students in his or her row) collected the workbooks in their rows. To make the teacher's homework-marking work easier, they opened every workbook to the relevant page, stacked them on top of one another, and then folded them all together into a small pile before handing the pile to the classroom math monitor. The math monitor was responsible for collecting all of the workbooks and delivering them to the teacher. He or she also kept a record of who failed to turn in homework and wrote the names of these

*She appears **annoyed** when coming to the next workbook and turns to its cover to see whose work it is. She underlines a certain section, marks it with a big question mark and then **lets out a sigh**. She has just made another check mark and then signed something as she closes this finished workbook and places it in a separate pile. It has taken her about two minutes to finish this workbook. Now she starts with another open workbook. She scans the first problem, writes something in it, and then makes a check mark next to it. She does the same with the second problem, and then the third and fourth ones. She writes the date of the day she marked this homework [later I realize that she always ends her homework marking by dating it, which is a norm for marking homework in China] and puts it away. This one just took her about one minute. ... She's just finished with another one - putting it away, taking it back again to write something in it, and then putting it back on the finished pile. She continues marking homework and the same marking actions repeat. Now, it's almost 35 minutes into her homework marking. She's finished 3 folds, about two dozen workbooks. She is holding them in her hands and tapping them against the desktop to make a straight and neat pile (field notes, November 13, 2002).*

This little excerpt captures the following chain of actions repeatedly performed by Tr. Wang as she marked and commented on student homework with her red pen (represented by the underlined verbs above): underlining; making check marks, X's, or question marks; occasionally turning to the cover to see the name; signing the date; closing the workbook; placing it on top of a (separate) pile; and again scanning, correcting, and writing comments on the next. She repeated the same set of actions for 120 student workbooks for about 3 hours that day and would follow the same routine on every work day during the days when she taught geometric proofs. As my observation went on, this was found to be a set of routine homework-marking actions that she performed for hours each day as part of her work responsibilities.

On the surface level, these actions do seem to paint the mechanical and redundant operational part of her work, the gray background of most institutional work life. The teachers' work responsibilities laid out at the end of the Curricular Standards for

---

students on one corner of the blackboard for the class director teacher or *banzhuren*, who held the responsibility to discipline those who failed to submit homework.

Mathematics for Nine-Year Compulsory Education (1998) stipulate that marking homework and providing students with timely feedback and tutoring constitute an important part of a math teacher's job responsibilities that are to be regularly evaluated as part of his or her work performance (p.13). Tr. Wang shared that student workbooks would be randomly collected by the administrators to assess how well the teachers were doing their job (Interview, November 20, 2002). As marking homework often took a large proportion of a math teacher's work time, teachers' complaints about the time-consuming nature of the work could often be heard, and the time constraints produced tension for the teachers.

Observing Tr. Wang's homework marking also allowed me to dig into its deeper level: performing those repetitive actions for hours daily created detailed feedback for students via homework about their daily performance. Tr. Wang communicated her feedback and requirements through those red-inked symbols and signs. At the same time, as she made check marks and X's, frowned and verbalized her frustrations, she was found examining student learning, making sense of their problems and collecting information about her past lessons to help her develop future lessons.

Just as she put it, "I always think while I am marking student work, 'Why did the student make such a mistake? What caused this mistake? What exactly was the student thinking?'" (casual conversation with Tr. Wang, November 19, 2002). The wrinkling of her forehead and expressions of frustration on her face (as in the bold-face verbs in the above field note excerpt) all suggest such inner wondering while she was marking homework. Just as Philip Jackson (1990) made meaningful those routine actions and interactions recurring from moment to moment in the elementary classrooms, in this

section, I dig for the meaning and significance in and of Tr. Wang's routine, repetitive actions and interactions with student homework. I agree with Jackson that "the taken-for-granted and common and ordinary aspects of our lives" require "renewed understanding" so that the "significance of the trivia" would "help reveal the sublime" (xviii-xix).

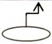
What of the sublime could be revealed in the "trivia" of Tr. Wang's marking of geometric proof homework? In the following section, I examine Tr. Wang's activity of marking homework as creating a communicative system and as gathering and using information to help students consolidate learning and make corrections.

Marking Homework as Creating a Communicative System

Symbols, signs and written comments as a tacit communicative language. As illustrated in the observation field note excerpt above, the underlined action verbs, such as making check marks, X's, and other symbols or signs; writing comments; and dating the marked work are a set of basic actions that Tr. Wang performed with her red pen. Each of the actions produced a red symbol or a sign that conveyed feedback to her students. Below is a list of such actions matched side by side with the most commonly resultant symbols, signs and written comments found in the marked geometric proof homework. I also list the locations where these were assigned or written and give brief explanations of the meanings they generally have.

Table 4.1 Common Symbols, Signs and Written Comments Used by Tr. Wang in Marking Geometric Proof Homework

Action	Symbols and comments	How and where is it assigned?	Meaning and implications
Making a check	√	At the end of a short proof and at the end of each step of a long proof.	Correct and acceptable.

mark			
Making an x	×	Usually at the end of a wrong step or steps.	Wrong; please correct or redo.
Assigning other symbols	(...)	At the end of a statement or condition in a proof.	Failure to back up with reason or explain the source.
	……	At the end of a proof-writing or drawing-construction exercise.	Writing is incomplete.
	? or ___? Or …? (?)	Question mark may be assigned by itself to a step or steps of a proof; or a sentence or two may be underlined, circled, or have an ellipsis written underneath with a question mark written at the end of these other markings. Usually assigned to an inappropriate step that has a missing statement or condition, or that lacks clarity or logic.	How does this come about? Or, a condition is missing. Please correct.
	_____?× _____× (×)	A sentence is underlined or circled, with a question mark and/or x at the end of these markings. Usually assigned to a step.	This statement is wrong or logically unacceptable. Please correct.
	∨ or ∧ <··or ··>	Insertion signs. Usually in the middle of a step or in between two steps.	A condition or words are missing here. Please correct.
		A sentence or condition is circled, and an arrow marks the way to its appropriate location.	This condition or step should be moved to a more appropriate location (as indicated).
	⇒ or ⇐	Usually at the transition from one step to another.	Logically incoherent or requiring more attention to logic here.
	☆	At the end of marking the day's work.	Good or excellent work.
	Δ	At the end of marking the day's work.	Please correct your mistakes!
Written comments	"Why?" "Please try the other way." "Please make up corrections." "Please write clearly." "Missed doing one exercise!" "Late work!"	"Why?" or "Please try the other way" appear at the end of a step or an exercise. Reminders about corrections and clear writing always appear at the end of marking the day's work.	Emphatically reminding, using words to indicate some of the above symbols for emphasis. Serious reminders to discipline students.
Signing the date	e.g., 11.14 (November 14)	At the end of the day's marked work.	Ending the marking; signaling that the work has been marked; and providing a record for future reference.

As shown in the above table, besides a check mark or Xx to indicate a correct or incorrect answer in proof writing, numerous other symbols or signs were widely used in her marking of geometric proof homework. Appendix 1[6] displays such symbols and signs in a few student homework samples marked by Tr. Wang. For example, the ellipsis sign indicated incomplete writing or missed information (such as in student work sample 3) while an ellipsis or a question mark inside parentheses specifically referred to a failure to back up a statement with reason (such as in student work sample 4). A question mark usually signaled the teacher's puzzlement about how the student reached a step or suggested that there was something missing in getting to this statement or step. In this case, a question mark more often than not followed an underlined or circled sentence or step (such as in student work sample 1).

Insertion signs opening at all four directions were also frequently used to indicate where there was missing information or steps. Sometimes the teacher added her suggestions for correction as well. Circling a sentence or step and using an arrow to lead it to its designated location was often given as a reminder that the proof writing suffered from a problem of sequence (such as in student work samples 2 and 6). A double-lined arrow was sometimes found to indicate a problem related to logical coherence between steps (such as in student work sample 6). When written comments were used, they voiced the teacher's extra emphasis on a certain idea (as in those listed in the table above), such as the need for more explanation ("Why?") and the need to produce clear and tidy work

[6] These student work samples were translated verbatim with the notation to the teacher's marking provided. Given limited space, they were selected for two reasons: first as typical examples to represent their particular category of errors, and also because they reflect how the errors are used in teachers' full-class explanations and individual tutoring.

as well as timely completion and corrections ("You have not corrected this exercise!"). Very often her symbols were also found to mend the format of proof writing; for example, she rewrote "Solution" as "Proof," since students had not yet gotten used to using this term, and she added format symbols to proof steps (student work sample 4, "$\because$ & $\therefore$") and drawings (such as in student work sample 3) .

In aggregate, these symbols and signs highlighted the common errors that students made when they began to learn how to write a geometry proof: incomplete writing, missing conditions, incoherent structures or wrong sequences, and improper format and use of language. In geometry proofs, these symbols and signs had become a set of tacit language that the teacher used to communicate the norms and expectations of learning via homework as a mutually binding responsibility to reinforce timely work and correction (see student work samples 2 and 4 with corrections marked again by the teacher).

A Euclidean geometry proof is a special form of mathematical language that has a "restricted format" and its own "linguistic conventions" and "rituals" that appeal to "abstraction, formalization, and justification" (Davis and Hersh, 1981, p. 148; Shanghai Curriculum Standards, 1998; Gina, 1990; Fowcett, 1938). Therefore, in terms of functions, these symbols and signs resemble those commonly used in editing a piece of writing, and the mechanics and criteria of a sound proof share those of a piece of good writing: complete sentences, coherent ideas and steps (paragraphs), and convincing arguments backed up with evidence as well as sound logic in the argumentation. Tr. Wang used such marking symbols to edit her students' proof writing to encourage them to develop sufficient evidence to support their arguments, clarity and logical coherence in their sequencing, and concise writing, those essential habits of good deductive reasoning that she strived to develop in her students. However, these symbols and signs are not unique to Tr. Wang's practice; rather, they are widely shared across the mathematics teaching and learning system in China, particularly in teacher-student homework communication (Ren and Zhang, 2000). Many of these marking symbols are unique to geometry proofs, particularly those that are process driven, such as arrows, insertion signs, and underlines and circles with question marks or X's as indications of different causes for logical problems. They are not part of the marking of homework on functions.

Error-centered. In terms of distribution, most of the items in the above list of marking symbols and signs were used to mark unacceptable or problematic proof writing. A correctly done exercise received a check mark and usually was passed by without further attention, while an unacceptable or problematic exercise received all the other possible signs as the teacher marked the proof step by step. Errors become the center of this communication in which it was tacitly understood that an exercise which received a marking symbol other than a check mark required correction. Error correction was closely monitored by the teacher, who would check on and mark the correction or redo of an exercise before marking the new assignments.

As discussed later in this chapter, additional measures would usually be taken, such as explaining to the whole class selected errors and tutoring individual students in order to assist them, supervise them, and ensure that they were able to make corrections. As we follow the trajectory of an error from its being marked as such to the way in which Tr. Wang would use it to guide additional pedagogical attention and tutoring, we see that this error-centered nature of homework marking is also characterized by its use as a resource for teaching and learning to offer students the understanding they needed in order to make corrections and to prevent future errors. Just as Ren Yong (2000), a special rank teacher[7] put it, "A teacher's purpose in marking homework is to examine the teaching effect, learn about the level of mastery of the knowledge and skills and help students to correct mistakes" (p.160).

Process-oriented. In terms of location, as demonstrated in the above table and in the student work samples in Appendix 1, almost all these signs and symbols are found marking some aspect of a certain step or group of steps within a proof, which suggests that Tr. Wang was not only paying attention to the correctness or incorrectness of the conclusions made or results reached, but also taking time to read through the procedures of the proofs written by students. In other words, her reading and marking of student work is process-oriented. In her marked geometric proof homework, students did not just receive a check mark or x to indicate a whole proof that was correct or faulty; these marks could often be found used to indicate a correct or faulty step (see student work samples 1, 2 and 6). Some examples of process-based markings include the underlining or circling of a sentence or a step and the marking of a question mark or x at the end, indicating an inappropriate step or faulty logic missing a condition or conditions (student work sample 2); using a double-lined arrow to indicate a lack of logical or structural coherence in the transition between steps; using an insertion sign to indicate missing information or conditions at a particular point (see student work sample 6); and circling a

[7] This is an honorable rank conferred to a very small number of school teachers around the entire country for their special contribution to education and teaching. It is not a professional rank, as the professional rank of teachers usually has three levels: a preliminary rank for young teachers with five or fewer years of experience; a middle-level rank for teachers with more than five years of experience, after taking enough in-service education credits at a district-level college of education and certain qualifying examinations. A senior rank teacher is one who not only has passed in-service classes with certain research requirements designed for experienced teachers, but also has established a successful teaching record and has had certain publications in journals for teachers and teaching. Tr. Wang is a senior rank teacher.

sentence or condition to move it with an arrow to the appropriate location (student work sample 6)[8].

This attention given to process is required by the nature of geometry proofs, which are structured as chains of logically connected arguments (Davis and Hersh, 1981) It also shows that Tr. Wang exerted effort to develop her students' logical, deductive reasoning habits in the early stage of learning to write proofs. Therefore, given the context of teaching and learning geometry proofs, this type of patching and stitching work in giving student homework feedback in helping students learn to write coherent and complete proofs should not be very surprising.

Two-way communication targeting correction of errors. Yet, the teacher's intentions embodied in the marking of homework and in how she used errors in teaching and assisting students' learning is worthy of careful attention. It reveals a communicative system that was not satisfied with simply giving careful feedback indicating what students had done wrong and where and reminding them of how they might possibly correct it. Tr. Wang's expectation and tacit rule of homework was that corrections be made, and be made promptly. This was a rule often followed conscientiously by most of Tr. Wang's students. Students' corrections were made in a number of ways: written side by side with the original work if there is enough space (as in student work samples 3 and 4 in the appendix); written on a Post-it® note and stuck on top of the original work (student work sample 3 and 5[9]); or simply adding corrections to the wrong steps (see the student work sample 5), often using a differently colored pen. As Tr. Wang's deskmate

[8] Some of the student work samples included in Appendix 1 are those that were also selected by the teacher to explain at the beginning of a subsequent lesson or to tutor an individual student over during a break or other non-teaching hours, as will be discussed in section 2.
[9] I removed the Post-it notes for scanning purposes.

pointed out, these corrections are made and displayed in ways that would allow students to use them for comparison and contrast for later reference. No matter which type of correction it was, Tr. Wang marked it again with care to show that it is central to her daily work to make sure that every error is corrected. Viewing students' work samples, one could be impressed with the students' carefully displayed corrections, as well as the carefully written check marks and dates on each day's assignments.

Assisted error correction. Tr. Wang thought correcting mistakes would be more difficult than just completing the homework and handing it in. Yet, students were not left alone in their struggle to make corrections. To reinforce the correction of homework, Tr. Wang practiced a rigorous regime. First of all, as mentioned earlier, she began marking a new day's submitted homework by checking whether the student had made corrections to the previous mistakes, marking them carefully before moving on to the newly completed assignments. If she found that correction had not been made, she would remind the student again by writing at the end of the day's work with a heavy tone of emphasis, "Please make up your correction!" Second, more importantly, she took other actions to assist students in correcting their errors. One of the pedagogical actions that Tr. Wang immediately executed following the homework-marking activity was explaining at the beginning of an immediately subsequent lesson one or two major exercises or a major dimension of an exercise that students did incorrectly[10]. Her other major action was tutoring individual students on their particular mistakes during breaks or the after-lunch hour.

[10] It is worth noting that Tr. Wang gave priority to the homework of the class that she would teach first, so that she could give the class feedback with the information about errors and misconceptions present in their work, and so that she could pick up one or two examples to explain. This seems to be very different from many U.S teachers, who check homework only to make sure every student gets the correct answer; that is, their focus is on correct answers instead of wrong answers.

As discussed in detail in the upcoming chapters, through these two complementary activities, Tr. Wang explains to students both in a group and individually why the mistakes she chooses to explain are important in learning geometry proofs, how they should correct them, and how they could prevent them from happening again. She used the two activities to supplement her homework marking, allowing her to do what she could not do simply by communicating her feedback to students through marked homework, such as making the mathematical ideas behind the errors visible and providing students with the necessary understanding for performing the corrections.

By ensuring that corrections are made, homework marking goes beyond a one-way communication; it encourages, monitors, and assists students in seeing why they are mistaken and what the mathematics behind the errors is, enabling them to correct errors based on understanding. Not until the correction was marked "confirmed" by the teacher would a full cycle of communication via a homework exercise come to a stop.

Similar cycles also took place with quizzes, tests, and exams, as the teacher used most of these symbols to mark test sheets and exam papers. While marking these, she recorded on a separate, blank exam or test paper the names of her students under each of the questions they got wrong, then calculating the percentage of students getting each question wrong. She used a weekly after-school hour to provide explanations for mid-term exam and geometric proof unit test questions to her students one by one. She made use of the statistical information she collected on the blank test paper to stress those areas that more students got wrong during explanations in class. She also required students to correct the errors they made on their exam and test papers by redoing those incorrectly done items on a Post-it® note to be affixed aside the incorrectly done problem. In the same way that she checked student corrections in the homework, she also checked and marked their corrections on the tests by collecting and marking them again.

In addition, for midterm and final exams, every teacher had to fill out several pages of item analysis developed by the municipality. Such analysis included calculating the distribution of scores for each classroom by students and by sections, computing the percentage of correct answers, and producing a record of which part of the exam still required more of her and her students' attention because of a low rate of correct answers in these sections or to these problems. This was followed by a brief written analysis of potential causes of such low rates of correct answers and a plan of possible strategies to use for improvement in the given areas. The last section of the form requested the teacher's comments on the how well the test was written and suggestions for the test writers as to what improvements were needed. The school and the school districts that administered the exams collected such formal analyses of mid-term and final exams as serious feedback to inform their future test producing. Hence, the summative evaluation was also treated in the same ways as the formative evaluations: errors were studied and used as resources for teaching and learning. They were carefully marked, analyzed and explained; student corrections were strictly monitored and marked, and parent signatures were required. The final analysis of the high-stakes exams (the mid-term and the final exams) treated the teacher as a researcher gathering information about errors, analyzing this information, and providing feedback that helped improve future district and municipality test construction. The cycle finally ended by ensuring that measures were taken to help students "get it right" on the exams. This response to exams is also a highly public system, in which Tr. Wang and her colleagues came together in her office to informally compare and discuss the forms they had filled out and how well their classes had performed.

Goal-directed actions of homework marking and assisting correction of errors. Actions are goal-directed (Vygotsky, 1978; Wertch, 1985; Engestrom, 1996, 1999). Tr. Wang's homework-marking actions and methods of following up on the errors with more teaching and tutoring assistance were directed by the official curricular goals as well as her own pedagogical purposes. First of all, the official curriculum defines the goals of teaching geometry proofs as "mastering the precise geometry concepts and language; learning the deductive reasoning procedures and steps; and mastering the commonly used thinking methods to form logical thinking habits for future life-long learning" (Teaching Reference Material, 1st semester, 8th grade, p. 53). Against such ambitious goals, homework is used as a major vehicle of teaching and learning. As the TRM states, "Practice and consolidation has to be ensured through an appropriate amount of homework; the teacher needs to adopt careful approaches to prevent the bad habits of confused thinking and writing without needed details or careful thinking" (p. 54). Tr. Wang's meticulous effort in marking homework communicated such clear expectations, and her persistence in making sure that errors were corrected aimed at developing student habits of good deductive thinking. It also reflected that the goals of the curriculum and of her teaching were in compliance with her job responsibilities stipulated in the curriculum standards, requiring her to carefully mark homework and provide timely feedback to students.

Second, Tr. Wang's actions were guided by clear pedagogical purposes. In her own words, she uses these symbols and comments as tools for analyzing student work, helping students with correction of their errors and encouraging them to learn. "I cannot just make a check mark or an x; I have to do analysis to show where it is mistaken and remind them of how to correct the mistake," she pointed out, going on to say that "Even for the weak students whom I know sometimes copy from other students' work, I have to do such analysis to encourage them to try to work on their proofs; simply giving them big X's will make them give up totally" (observation notes, November 15, 2002). Her words conveyed the value that she attached to "analysis" (instead of just marking the errors) in the nurturing of students' good attitudes and motivation to learn geometric proof writing via homework. Third, in her teaching, she believes that without making sure the errors are corrected and understood, "problems will accumulate and teaching will have difficulty in moving on smoothly, particularly in learning geometry proofs, where each concept is built upon and related to a previous one" (personal interview, November 19, 2002).

Gathering and Using Information for Student Learning.

In this section, I discuss Tr. Wang's marking homework as a process of sorting and gathering information, using information to make sense of student learning and make decisions for further teaching. I also illustrate this process as one of stress intertwining with sense making as Tr. Wang assessed student learning against how well her previous class had gone. Viewing it in her activity system, marking homework can be viewed as her engagement in a process of pedagogical reasoning and action.

A process of sorting and gathering information. Marking homework is a process of gathering information about student learning. There are several observable actions that

Tr. Wang performed that suggested what kind of information she was collecting. First, she always sorted the most problematic workbooks into a separate pile and placed it at the top of the finished pile but facing the opposite direction to distinguish them from the unsorted workbooks below. This top pile usually contained students singled out for additional tutoring assistance. While marking homework, she tended to turn to the cover to see to whom the workbook belonged. When encountering a poorly finished assignment, she would read aloud the name of the student and let out a sigh to vent her frustration. Hearing the student name, her colleagues sitting closer by, who taught other subjects to this student, would fill her in with what they knew about the student, thus enriching her knowledge about the student. For instance, in the opening vignette, Wang Bing's class director teacher informed Tr. Wang of the strong suits of this boy who did poorly in his mathematics homework, his strength in non-academic areas and his family circumstances that might be causes of the poor work. Therefore, this kind of information is specific to an individual student, not only about him or her as a learner of mathematics but also of other subjects, and about the student's personal traits and family situations.

The second most visible set of actions came in the form of her inadvertent utterances with which she blurted out what was on her mind. She made these utterances often, as she found many students made mistakes with the same assignment. One example of this took place on November 20, 2002, when she marked homework on angle bisectors; many students missed the condition of perpendicularity in using the theorem of DCCC (distances from the center of a circle to two equal chords of the same circle are equal -- see student work sample 2 in the appendix). She made many statements about these frequent, similar errors: "DCCC again!" and "Another with DCCC!" She chose this assignment to explain to the whole class in the subsequent lesson. Such verbalized thoughts are important clues to the kind of information she was gathering and, as discussed in later chapters, how she would use such information.

As discussed below, Tr. Wang was not only gathering information from her marking of homework, but also using the information to make sense of student learning and to guide her in making decisions for teaching. Such information is learner-, content-, and situation-specific, oriented towards teaching and learning.

Using information to make sense of student learning and to make decisions for teaching. As mentioned above, accompanying Tr. Wang's homework-marking actions were certain habitual actions, such as sorting workbooks, checking the student names on the workbooks, and verbalizing her frustrations with students or completed problems ridden with errors, as well as those non-verbal acts (such as making facial expressions) mentioned earlier. These verbal or non-verbal behaviors also could suggest that she was making sense of student learning. In the early stage of my observations, one of my attempts at a formal interview with Tr. Wang about what she was thinking while marking homework yielded the following brief response:

> *While I am marking homework, I am checking on whether student work today is up to my expectations. If I feel that the effect of yesterday's lesson should be [for students] to have no problems, but students display many problems in their work, then I start to wonder why students all made mistakes on such a simple exercise. In the process of marking, I am thinking why this student made a mistake, why he [or she] made such a kind of mistake. Then, when I turn over to the cover to look at the name, my state of mind returns to normal. That's all that is on my mind while I am marking* (personal interview, November 19, 2002).

Her words confirmed that she was pondering the types of errors students committed and trying to figure out their causes in terms of whether the errors were made by a group of students or an individual student. Such ponderings often tied the specific and particular instances of an error to the learner's characteristics as she associated a certain error with who the student is. However, it should be noted that she was not simply judging the errors according to who the learner happened to be, but rather according to what he or she was thinking while doing the homework. For example, she said in an interview, "I have to sometimes pause and make myself think about what kind of perspectives a certain student is taking in reading and thinking about the same exercise." (Interview, November 15, 2002)

However, such interviews did not go far in learning about how the teacher reasoned about student thinking and what kind of decisions she was making for her teaching based on this reasoning. Yet, observations of her as she used the information gathered for further teaching shed light on the importance of such reasoning. As Chapters Five and Six reveal, when she explained selected assignments in her teaching, she tended to explain more than one error she found in an assignment and offer multiple perspectives in her explanations. When she tutored students, her wonderings about a student's error(s) led her to focus on the student's particular learning problem and further diagnose the causes. These pedagogical actions and what they helped make possible for her teaching do demonstrate that homework marking was an activity that engaged Tr. Wang in gathering information and making sense of student learning. This was often done together with her deskmate colleague, Tr. Zhao, in their homework-marking conversations, discussed in detail in Chapter Seven. This was a process that not only enabled her to make decisions about what additional teaching was needed but also readied her for turning such reasoning into immediate pedagogical actions. This is what I regard as a valuable way of using homework as an object to inform practice from moment to moment and to allow her to base her decision making on the central issues of student learning and her students' specific learning needs.

The information gathering and using processes are similar to what Leinhardt & Greeno (1991) called information schemata. The way schemata work, as she noted by citing the studies of teachers' planning networks[11], resembles recording information generated by interactions in planning on a kind of "cognitive blackboard" that allows information "generated in actions to be saved, revised, and used in later actions" (p. 235). In this sense, marking homework was also a planning process. In her work hours in the office, Tr. Wang seldom planned for her lessons. She usually took out her lesson plans that she had constructed at home to read quickly but carefully during the break before she went to teach her lesson(s). In some ways, the homework marking was a form of active planning. As she was gathering information and wondering about student learning issues, she was also making decisions about ways to adjust her planned lesson so that she could make space to clean up the messy processes occurring in those errors, to remove the demonstrated misconceptions and provide help in completing the necessary procedures.

Figure 4.1 below sketches one dimension of the reasoning and decision-making flow in the activity of marking homework. In the system of Tr. Wang's homework activities, this flow was immediately channeled into two other activities as it fed back into her teaching. The information she collected was able to support her subsequent pedagogical actions in these two activities: explaining and tutoring students on homework errors. To some extent, this process resembled Shulman's (1987) model of pedagogical reasoning and action (p. 15).

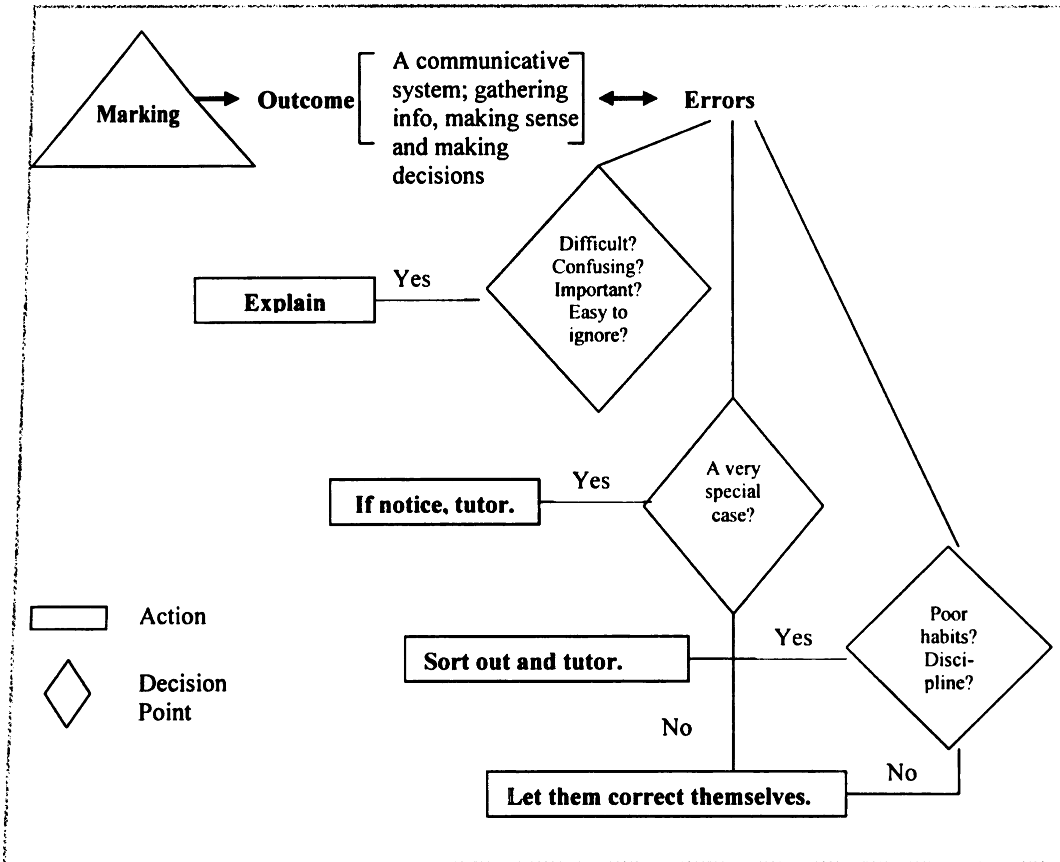
---

[11] The two studies cited by Leinhardt are: Heyes-Roth, B. & Heyes-Roth, H. (1978) *Cognitive processes in planning* (Report R-2366-ONR). Santa Monica, CA: Rand Corporation. Stefik, M. (1981). Planning with constraints (MOLGEN: Part 1) *Artificial Intelligence*, 16, 111-140.

While his model describes the entire process of teaching from planning and instruction to evaluation and reflection, Tr. Wang's model started with marking homework and reasoning about student learning in relation to her teaching. It ended with further actions for teaching and assisting student learning as a result of applying her informed decisions into practice. However, her reasoning about student thinking behind their errors still continued during explaining and tutoring students on homework. In her practice, her reasoning was almost always followed by immediate pedagogical actions; and in performing those actions, she continued her reasoning, which would prompt further immediate actions, for instance, adjusting her questions she asked during explaining an error or asking them from different perspectives. Such reasoning was supported by her information from student work.

In many ways, Tr. Wang's actions to transform the object, the errors, into student understanding resembles the transformation in Shulman's model that proceeds instruction: the teacher undertakes planning for materials to use for teaching, ways to best represent them, methods to teach them and strategies for how to adapt to the needs of different students. The difference between the two kinds of transformation lies in how it is achieved: Tr. Wang achieved it through marking homework, making sense of students' learning problems and taking actions to address the problems in teaching. In this entire process, homework served as a boundary object (Wenger, 1998, p. 58) that allowed her to save and use information, which "enables (a) skilled teacher(s) to deal with interactions between disparate goals and activities" (Leinhardt & Greeno, 1991, p. 235)

Figure 4.1 A System of Information Gathering and Use in Tr. Wang's Homework Activities



Internal tension in the activity of marking homework. As mentioned earlier, those verbal and non-verbal actions accompanying Tr. Wang's marking of homework exposed the psychological dimension of her work: stress and sense-making intertwines. According to what she shared, while marking, she was measuring the effectiveness of her previous lesson against how well students performed in the homework. It was frustrating when she found "students all made mistakes in different ways in the assignments" (observation, November 19, 2002). As all activities are driven by certain internal tensions, while a source of frustration, this process also propelled her to wonder about the causes of student errors at the same time (for instance, by thinking about who made the error(s) and why), as well as to take prompt action to handle the errors. As Chapter Seven indicates, such stress was often shared and alleviated when Tr. Wang was marking homework and talking with her deskmate colleague, as carefully discussed in chapter seven on collegial homework conversations

Discussion

From the point of view of assessment, homework marking is a form of continuous daily informal assessment of student academic performance. By informal, I mean that homework was not counted in the final composition of a student's grade. It was dealt with mostly during the teacher's non-teaching hours. In this regard, it has every appeal of what reformers currently look for as an ideal way of assessing student learning. Today's reform in assessment echoes the goals set up two decades ago: assessment should be a means of setting more appropriate targets for students, focusing staff development efforts for teachers, encouraging curriculum reform, and improving instruction and instructional materials (Darling-Hammond & Wise, 1985).

Tr. Wang used student homework as a tool to assess student learning on a daily basis. She was able to use what she learned to continuously gauge her teaching's ability to meet students' learning needs and to create more opportunities to assist students in improving their learning. She was also able to accumulate knowledge about the content in relation to curricular goals and means of helping students better learn their subject matter. Her knowledge about student learning and her skills in navigating their learning were developed in and from her assessment practice through homework. The primary aim of assessment is to foster learning of worthwhile academic content for all students (Wolf, Bixby, Glenn, & Gardner, 1991). Tr. Wang's work was designed to achieve such an aim.

Summary

In conclusion, the trajectory of collecting and using homework information shows that marking homework helped Tr. Wang to do two things. First, her homework marking created a teacher-student communicative system of symbols and signs. Second, the process-oriented attribute of homework marking also enabled Tr. Wang to make sense of student errors and misconceptions and to gather the information necessary for her to make decisions about how to deal with errors further. In this sense, she is seen as collecting, saving, and using the information to connect and coordinate the goals of her different work activities surrounding homework and to concentrate her efforts on how to "eradicate the errors" and assist understanding, the actions that I will explore in the next section.

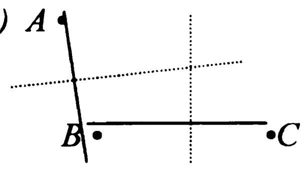
I argue that marking homework is a process in which Tr. Wang gathered, reasoned about, and saved information about student learning difficulties and misconceptions encountered in the form of homework errors; transformed them into teachable moments; and used these to coordinate her teaching goals with student learning. In the meantime, she also used homework as a daily assessment tool to inform her about student learning and about means of better helping them to learn. Homework is a tool that generated information for her practice and guided her knowing in and from her practice.

Chapter Five: Explaining Homework Errors to Whole Class in Teaching Geometry Proof and Function

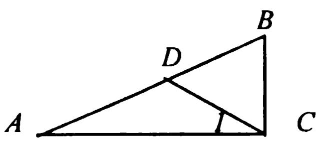
Introduction

Explaining Homework at the Beginning of the Third Period, November 19, 2002

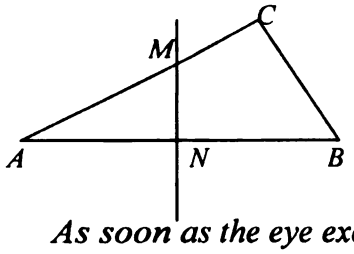
The music bell sounded to announce the start of the third period of the morning. Tr. Wang stopped reading her lesson plan. With about a dozen marked homework books under her arm, teaching materials in one hand, and a big wooden triangle and a protractor in the other, she walked briskly upstairs to Class 4 on the fourth floor where both of her two classes were located. The melodious music for the "eye protection exercises"[1] was being aired in the loud speaker to the accompaniment of which students were massaging the major facial points with their eyes closed. This regular sized classroom (about 300 square feet) was fully packed with 60 students seated in four rows. Each row had eight desks and each desk seated two students, leaving a narrow passage in between the rows that allowed one person to pass at a time. In front of the crowded classroom well-lit by the three big windows was a small desk for the teacher on which to place her textbooks and teaching tools. Before the music bell signaled the start of the third period, Tr. Wang finished drawing on the blackboard the following three geometry figures selected from the marked homework problems.



(Find a point which is equidistant to the three residential sites, point A, B, and C.) –***Vol. A***, [Exercises 22.5 (2): 1.], p. 37



(Given: See drawing, in ΔABC, $\angle ACB=90°$, $\angle 1=\angle B$. To prove: D is on the perpendicular bisector of AC. – ***Vol. A***, [Exercises 22.5 (2): 3.], p. 37



(Given: See drawing, $\angle C=90°$, the bisector of AC intersects with AC and AB at point M and N respectively, and AM = 2CM.
To prove: $\angle A=30°$. ---***Textbook*** [Exercises 22.5 (2): 4.], p. 79

As soon as the eye exercises ended, she approached the boy sitting closest to the loor. He had not done one of the homework assignments and had left it blank. She told im to make it up and let her know if he could not do it. The monitor returned the marked omework to students and Tr. Wang talked to some of them about their mistakes. When

---

There are two longer breaks in most school days in China, one in the middle of the morning and the other the middle of the afternoon, each lasting 20 minutes. The first ten minutes is dedicated to routine eye otection exercises in which students massage their eyes.

the music bell started again, the loud and noisy sixty 14-year-olds in school sport-suits uniforms (girls in dark pink and boys in gray) quieted down quickly.

"Class begins!" announced Tr. Wang. "Stand up," commanded the class monitor. The whole class (including me) stood up and said in unison, "Teacher, good morning." "Sit down, please," said the teacher, and the class was summoned. "I started marking your homework as soon as I arrived this morning," said Tr. Wang. "In doing proofs," she continued, "our classmates either missed an arm or leg." She read the names of the students who did well and then directed student attention to the first figure on the blackboard. She used students' wrong methods and procedures as examples in guiding them to work together with her on the three proofs again. In the first problem, she focused on how to construct the figure and adequately write the procedures of such a construction. She began by reviewing the theorem of perpendicular bisector. She then made use of the "two points" – the end points of a line segment to "three points" – the three residential areas. After leading students to see that the point, P, is where the perpendicular bisector of AB and BC meet, she asked them, "Do we need to connect A and C and draw the perpendicular bisector of AC too?" Students answered, "No." "But some students did," she added. "If the construction is not accurate enough," she continued, "you cannot make this bisector meet with those two above at the same point". (See student work sample 3 in Appendix 1.)They she probed, "Why it's enough to just get the perpendicular bisectors of AB and BC?" "Because AP=BP and BP=BC," many students answered together. Then she emphasized that it is important to write the construction procedures very clearly and asked them how they should write it. She let students say it slowly and while repeating what they said, she wrote the steps carefully on the blackboard.

In the second problem, she led students to see that there is no need to add an auxiliary line segment and why they need to prove AD=DC. By making use of the given relationships between the angles ($\angle ACB=90^\circ$, $\angle 1=\angle B$), they can get that the two sides, AD and DC are equal, which proves that Point D is on the perpendicular bisector of a given side. For every step, she asked students to give reasons for a conclusion they made. For the third problem, the one on which she tutored the boy a while ago, she asked students to compare and contrast it with the second one to see that this exercise needs an auxiliary line. "Look, this picture looks incomplete; it now requires you to think of an auxiliary. What could we get if we connect B and M? Could you notice the picture now is complete and could you see its connection with the bisector? How about AM and MB?" "(They are) Equal," many students answered together.

Then she helped students see that with the given relationship, AM=2CM, they get BM=2CM. "When it tells you the relationship between segments, how should you use the relationship?" she probed, "Don't we use it to get the size of angles?" "Then here you can only rely on this right triangle (MBC), right? Here this (BM) is twice of this (CM), Conversely, how much is this (CM) of this (BM) (pointing the drawing on the board)?" "Then, we know the right side is half of the hypotenuse, how many degrees is angle MBC?" "Yes, 30 degrees. Some students got stuck here." "Then, how about angle CMB?" "Yes, it is 60 degrees." "Then which angle can you get next?" Pause. "Angle A *is in which triangle?" "Oh yes, angle AMB, how do you get it?" " A few other students got stuck here" When she finally led the students to prove ∠ACB=30°, she summarized, "So later on, if a perpendicular bisector misses the other segment, you need to connect it in order to make use of this theorem. Some students did not finish this proof. Please finish it or correct your mistakes. Don't lose points (here she referred to tests and exams; homework was rarely graded for credit or counted as points in tests or exams) at such small places." Ten minutes later, she started the new lesson by announcing in a new and different tone, "Today, we're going to study another bisector theorem: an angle bisector and its converse theorem." She made a transition to angle bisector theorem from reviewing the perpendicular bisector theorem.*

During the ten-minute break, she walked to Class 2 (two doors away from Class 4, with Class 3 in between) to get ready for teaching the next period. She drew two figures on the blackboard and after that, as she often did, walked into the other 8th grade teachers' office next to Class 2 to talk with Tr. Hu, the young male math teacher who taught Class 3 and Class 5, about their Lesson Preparation meeting in the first period after lunch. Instead of starting Class 2 with homework, Tr. Wang began with reviewing the concept of perpendicular bisector theorem and its converse and then she explained the first of the three exercises that she explained to Class 4 before moving to the new content of the day.

The above vignette captures Tr. Wang explaining selected homework assignments in the beginning of the third period to a crowded but orderly classroom of 60 8th graders. There are several noticeable features about her explaining. For instance, she drew the figures on the board and referred to them in the course of her explaining. Her explanation was formal and detailed. She used the information she collected from marking in her explanations – what was wrong and the proportion of students who got something wrong- and used two contrasting assignments to illustrate a point. She also tended to ask students questions and led the explaining by questioning. Such features will be discussed in detail as part of Tr. Wang's pedagogy of explaining homework errors to the whole class.

In the tradition of Chinese education, explaining is a routine pedagogical approach. In Chinese, "explaining homework" means speaking about and commenting on homework *(jiangping zuoye)*. "Explaining" in Chinese literarily means "speaking and giving meaning or solving." *Record on the Subject of Education* (translated by James

Legge, 1885) says that a teacher should explain "if pupils are not able to put questions" and then the teacher "should put subjects before them (p. 90)." In explaining homework, teachers put before their students the subject of errors, the knowledge and skills required for students to know and understand the errors in order to correct them.

As a homework-related activity, the object for explaining homework to whole class is the selected assignments or errors that Tr. Wang chose. As discussed in Chapter Four, these errors were chosen while she was marking homework and her decision for choosing them was also informed by other concurrent activities, such as tutoring students on errors during the break and talking with her deskmate colleague, Tr. Zhao. The face-to-face nature of explaining requires the active participation and engagement of students, who are subgroups of the community of teaching. Her goal of explaining is to repair students' flawed or incomplete understanding promptly. The immediate outcome of the activity is in the form of her detailed explanations that help students see what is wrong and how to do corrections.

Chapter Overview

This chapter consists of four sections. In the Introduction, I began with a small vignette of what Tr. Wang's activity of explaining looks like. It continues from the initial introduction of Tr. Wang's workday (11-19-02) that opened the first chapter. I also introduce the research questions, data and approach to data analysis for this chapter. To provide a storyline, the second section summarizes the settings (the who, what, when, where and how) of the activities of explaining homework to whole class in Tr. Wang's two teaching sections that I observed. I focus the third section on two dimensions of the object of the activity: the subject matter of errors – what constituted the errors and the explanations provided by Tr. Wang; and the construction of errors – the identification and selecting of the errors, that is, what informed Tr. Wang's selecting of the errors. In the third section, I also examine the pedagogical actions in explaining errors – how the explanations were conducted or how the object was being transformed into the subject matter that students need to know and understand to do correction. In the last section, I summarize the chapter and highlight a few major findings. Together, the four sections offer a detailed analysis of Tr. Wang's activity of explaining homework errors.

Research questions

There are three central constructs considered in framing the research questions. First, the object of the activity, which refers to the selected errors/assignments. Second, the process of identifying and selecting the errors to be explained, which I refer to as construction of the object. Third, the process of making the mathematics entailed in the errors accessible to students, which, in terms of activity theory, is the transforming of the object into an immediate outcome – which is referred to as Tr. Wang's explanations provided to the errors.

To offer a description of Tr. Wang's activity of explaining, I will focus on the object and the immediate outcome; the selecting/construction of the object; and transforming of the object. To understand the subject matter and student learning-related implications of the errors, I put the object and outcome, that is, the errors and their explanations side by side for examination. More specifically, my questions examined in this chapter become:

Object and outcome:

- What are the errors Tr. Wang chose to explain?
- What are the explanations she gave to students?
- What are the mathematical, curricular and student learning implications entailed in the targeted errors?

Selecting as construction of the object:

- What might be the major factors that bore upon her selecting of errors to explain to whole class?
- What are the sources that might have informed her decision making in selecting the errors to explain?

On the transformation level:

- How did she explain the errors to transform them into forms and content accessible for student to understand?
- What pedagogical actions did she take to conduct explaining?
- What are some major characteristics of her explaining?
- In what ways did her pedagogy of explaining reflect the tradition of mathematics teaching and discourse in China?

Taken together, answers to the above questions aim to provide a deep analysis of Tr. Wang's explaining homework errors.

Data and Data Analysis

Data. In this chapter, I draw on two sets of data from the two units I observed in November 2002: Geometry Proofs and Functions. More specifically, I choose to focus data analysis on two sections in the two units: the section on *Converse Propositions and Theorems* and the section on *Direct Proportion and Inverse Proportion Functions.* I focus on these because of their timing in my field work and because they enable comparison and contrast of data in two different content domains.

In terms of the location of the two sections in their respective units, the section in the Geometry Proofs Unit was the last section and I was able to do thorough observation which started three days after my formal entry into the field school. The section in the

Functions Unit was taught immediately following the above geometry section. Using these two sections allows me to have a connected view of the time sequence and the routines of the teacher's daily work. Second, the two sections, when put side by side, offer opportunities for comparison and contrast of the teacher's use of homework in explaining (and also later, tutoring) across two different content domains: geometry and algebra (taking function as a special domain of algebra).

The section on *Converse Propositions and Theorems* was taught on four consecutive teaching days, from 11-15-02 to 11-20-02 (Tr. Wang did not explain homework errors on 11-18). The section on *Direct Proportion and Inverse Proportion Functions* was taught on five consecutive teaching days from 11-21-02 to 11-27-02. The full day observations make it possible for me to weave the data together to form a picture that shows how the teacher's use of homework mediated her teaching and learning. The following is a table of the two sections and the exercises selected for explanation in teaching the two sections.

Table 5.1 Basic Information of the Exercises Explained in the Sections of Geometry Proof and Function

Date/	Homework topic	No. of Assignments	No. of assignments chosen to explain
Geometry Proofs: Section on Converse Propositions and Theorems			
Friday, Nov. 15, 2002	*Two theorems of right triangles (11-13) and two corollaries (11-14)*	4 exercises assigned	1 for Class 2 (Ex. 2) 3 for Class 4: (Ex. 1 and 3/Vol. A and Ex. 4/ textbook)
Tuesday, Nov.19, 2002	*Perpendicular bisectors*	Both classes: Ex. 1, 2 & 3 in Volume A (p. 37). Extra for Class 4: Ex. 2, 3 & 4, Textbook (p. 79)	2 for Class 2: Ex. 1/Vol and Ex. 2/Vol. B:
Wednesday, Nov. 20, 2002	*Review of different theorems taught from 11/13-19*	For both classes: Ex. 1, 2, & 3, Vol. B (p. 42) Extra for Class 4: Ex. 1, 2 &3, Textbook (p. 81-82)	Ex. 2 for both classes
Functions: Section on Direct and Inverse Proportion Functions			
Thursday, Nov. 21, 2002	*The concept of direct proportion functions (DPFs)*		Ex. 1 (6); Ex. 4, & Ex. 5 Vol. B
Friday, Nov. 22,2002	*How to get the analytical expression (formula) of a DPF*		3 given to Class 2
Monday, Nov. 25, 2002	*Using the properties of the graphs to get the analytical expression of a DPF*		2 for Class 2 (Ex. 2 and 5); 2 for Class 4 (Ex. 5 and 6)
Tuesday, Nov. 26, 2002	*End-of-unit geometry proof exercises* *Format of exercises related to DPF*		1 for Class 2 (Ex. 1, Vol. B)
Wednesday, Nov. 27, 2002	*Inverse proportion functions (IPFs)*		2 for both Class 2 and Class 4 (Ex. 2 and 3, Vol. B)

The errors chosen and explained during the brief teaching segments at the beginning of a lesson and sometimes during the after lunch hours make up the bulk of the data for this chapter. I draw on these data and their analysis to see what these errors are, their pedagogical and learning values and how they are used to create teachable moments for students.

Data analysis. From the lens of activity theory, understanding the object requires understanding how the errors are constructed (selected) and transformed (explained). The analysis was conducted on three levels to form a general picture of the object and the changing processes through selecting and explaining.

On the first level, the selection of errors to explain, my analysis is aimed at understanding the nature and role of the errors. Since my data collecting is observation-based, it did not yield substantive data about the exact considerations in terms of pedagogy, content and learning considerations that Tr. Wang made in selecting to explain these errors. Therefore, I interpreted the value of the errors and their importance towards student learning of the content by looking carefully at both object and the immediate outcome, which is, the errors and their given explanations. By focusing on the errors and her explanations at the same time, I was able to understand the mathematics, curricular location, student learning and pedagogical characteristics entailed in the errors. I examined the errors and the exercises from which they were derived in relation to their location in the curricular sequence and other errors students made in the two sections. I look at the mathematics entailed, the purpose of the exercises, and the types and importance of errors in relation to student learning of the topic or concept.

To understand the mathematics and student learning entailed in the errors, I coded her explanation of the errors from the transcribed classroom teaching segments in which she explained the exercises and errors. The coding categories include the content knowledge and skills, their location in the curriculum, research on student learning of the content, and the importance of the assignment to teaching and learning.

Since the homework activities Tr. Wang engaged in were interrelated and each could inform the other(s), on this first level, I also looked for likely sources that might have informed her decision making in selecting the errors. To do so, I juxtaposed several data sources for triangulation purposes. I examined the teachers' remarks in the detailed observation notes taken in her office on several occasions: when she marked homework alone; when she interacted with students coming for tutoring; and when she talked with colleagues while marking homework, to form a more concrete picture of what informs her decision making in error selection.

On the second level, errors and explaining, my analysis is intended to understand the teacher's pedagogical actions in explaining, that is, how she explained the errors in ways to make the mathematics entailed accessible to students and how she used the information she collected from student errors to assist her explaining process. The analysis on this dimension was done on both the structure and discourse of explaining. I examined the activity (or event) structure and routine actions of explanation. I analyzed the discourse of explaining to reveal the interaction patterns of the explanations.

A lesson is viewed as made up of major segments called activity structures, such as checking homework, presenting new material, having independent seatwork etc. (Stoldolsky 1983; Leinhardt & Greeno, 1991:236). Explaining homework errors at the beginning of a lesson is such an activity structure that has a life of its own propelled by the distinctive error-driven motif of the teacher.

In skilled teaching, teachers are seen performing a set of activities with facility and ease. This is because "skilled teachers have a large repertoire of activities that they perform fluently" which are referred to as "routines" (Leinhardt & Greeno, 1991: 235).

When routines are established, students as well as the teacher have developed organized actions (or schema of actions) that they perform regularly. Routines "allow relatively low-level activities to be carried out efficiently, without diverting significant mental resources from the more general substantive activities or goals of teaching" (p. 235).

The activity of explaining homework is itself a visible and brief routine lesson activity. For it to be carried out efficiently in a short time frame, the activity structure also contains recognized patterns and routines to engage students and help them focus on the substantive content of explanation. Such routine actions give shape to structured explanations and allow the teacher to orchestrate the details of explanations and convey her messages clearly in short time durations to bring about the desired outcome of understanding.

However, studying expertise as well-performed schemata of actions alone is not able to account for the pedagogical features of Tr. Wang's explaining. As a social practice immersed in a culture rooted in Confucian educational thinking and values, the patterns and purposes of explaining need also to be related to this cultural context for fuller understanding. As Engestrom (1996) pointed out, expertise should be "understood as formulable as part of ordered social interactions rather than preexisting cognitive schema and as containing the basis for the creative generation of new ways of doing things rather than depending upon the orthodoxy of received wisdom." His words ring true particularly for the activity of explaining and tutoring students on errors. They occur and exist only when the teacher is in the accompaniment of students—in the moments of interaction with students. Her explanations were not static but formulated and adjusted by information about student learning of the content and her own teaching of the content.

They were continued to be readjusted in the process of her giving the explanations and tailoring her assistance to individual students, as in tutoring.

Cazden (1988) approached classroom discourse analysis by looking at the IRE (initiation-response-evaluation) structure of the teacher-student interaction in traditional classroom teaching. Such an approach is not only able to reveal the content (what happens or is communicated) of interaction but also expose the respective roles played by the teacher and students in making meaning of the errors. This approach guided my examination of the explanations occurring during the teacher's work day.

Explaining homework errors to students is a "teacher-led speech event" (Cazden, 1988, p. 99) "in which the teacher controls both the development of a topic (and what counts as relevant to it) and who gets a turn to talk" (p. 30). It is also different from a regular lesson segment in that in the activity of explaining both the teacher and the students share the information, albeit to different levels of understanding and sophistication, about the context of a homework assignment and what possibly has led to the error or errors. Therefore, the coding of the teacher initiation, student response and teacher evaluation sequences in every explaining segment is less about how a student response or interaction occurred. It is more about how the teacher initiated the response and how explanation was enriched with the help of the teacher's information and her probing for more understanding about student problems of learning.

The codes are also directed to teacher-student interaction routines (such as students' choral responses) to open up windows onto how she manipulated the information she gathered from marking homework to orchestrate the explanation, engage the students and move the explanation forward. They reflect some of the cultural dimensions of Tr. Wang's teaching practice given the large class size and teacher's leading role that requires a different kinds of student participation in the form of active mental participation (Briggs, 1996).

On both levels, I drew on interviews and other observation data wherever necessary to help cross-reference the coding and better understand the construction of the object as well as its transforming process. Although I do not report it here, I also examined the relationship between Tr. Wang's homework activities and her teaching in an effort to make out how her homework activities mediate her classroom teaching. Such examinations are attempted in order to shed light on what roles these homework activities play in the teaching and learning of geometry proof and the concept of function.

Explaining Homework across the Two Teaching Sections

Settings of explaining homework

As mentioned in the beginning of this chapter, explaining homework errors is an activity that Tr. Wang undertook, following the activity of marking homework, to communicate her feedback on issues, often errors, to her students in the classrooms. To describe what these activities are like, the five W's (who, what, when, where and how) that Tharp and Gallimore (1988) used are able to help make sense of the "interlocked dimensions of the activity settings" (p. 74). Through the five W's I summarize the participants, place, time and motivating force of this activity in Table 5.2 below.

Table 5.2 Settings of Explaining Homework Errors in Sections on Geometry Proofs and Functions

Day	Who/Where, what and when?	For how long?	(Threads to) Why?
Fri 11-15-02	**Class 2**: One exercise explained at the beginning of the first period	2.8 minutes (proceeded by a 3-minute **review**)	--Arrived at office half an hour before work to mark a proportion of homework for immediate use (*Did not observe Class 4.)
Tue 11-19-02	**Class 4:** Three exercises explained at the beginning of the third period **Class 2:** One exercise explained at beginning of 4th period	**Class 4:** 9 minutes (proceeded by a 2-minute review) **Class 2:** 4 minutes (proceeded by a 8.8-minute **review**)	Wondered aloud while marking, "It looks that writing about this construction is difficult for the students…" *(Observation notes.)*
Wed 11-20-02	**Class 4:** One exercise explained at the beginning of the 4th period—***Segment 4*** **Class 2:** One exercise explained during the later half of the after lunch hour—***Segment 5***	**Class 4:** 9 minutes **Class 2:** 10-minute explaining and 10-minute face-to-face marking of corrections and answering questions	--Arrived at office half an hour before work to mark a proportion of homework for immediate use
Thu 11-21-02	**Class 4:** Three exercises explained at the beginning of the weekly after-school hour, 4:30-5:30 PM.—***Segment 6***	**Class 4:** 10 minutes	The collegial conversation in the morning discussed the controversial key to one homework assignment that she shared with students.
Fri 11-22-02	**Class 2:** Three exercises explained during student self-study hour at around 3:15 PM—***Segment 7***	**Class 2:** 15 minutes	Finishing homework of Class 2 at around 3:15 PM, she said, "I've got to go to Class 2 to explain for 10 minutes. They did a really poor job."
Mon 11-25-02	**Class 4:** One exercise explained at the beginning of the first class in AM. **After lunch:** **Class 2:** One exercise explained at 12:55 **Class 4:** One exercise explained at 1:00PM	**Class 4:** 3 minutes (proceeded by a 5-minute **review**) **After lunch:** **Class 2:** 5 minutes and more time monitoring correction **Class 4:** 8 minutes	--Arrived at office half an hour before work to mark a proportion of homework for immediate use --When she marked homework and tutored students at her office in the morning, she found that she needed to give some prompt to students regarding one assignment.
Tue 11-26-02	**Class 2:** One function exercise explained during the last ten minutes of the 5th period, the last morning period—***Segment 11***	**Class 2:** 10 minutes	--She disagreed with the key to an assignment given in the TRM; she found students had given

	After lunch hour **Class 4:** One function and 2 geometry exercises explained at 12:45 PM—***Segment 12*** **Class 2:** No explaining: face-face marking of corrections only	**After lunch hour:** **Class 4:** 10 minutes **Class 2:** 15 minutes	different answers too. --She just finished marking the geometry proof assignments for Class 4 and wanted to comment on it.
Wed 11-27-02	**Class 2:** Two exercises explained at beginning of 1st period AM—***Segment 13*** **Class 4:** Two exercises explained at beginning of 2nd period AM—***Segment 14***	**Class 2:** 8 minutes **Class 4:** 7 minutes	--Arrived at office half an hour before work to mark a proportion of homework for immediate use; --She found students approached the two assignments in different ways.

The above outline shows that explaining homework errors is a routine activity that Tr. Wang conducted nearly everyday and sometimes more than once a day (11-25 & 26). Always taking place in the classrooms, the explanations are brief (usually ranging from 3-12 minutes in length) and often occurred at the beginning of a lesson or after lunch[2] except for 11-21-02, which occurred during the weekly after-school hour and 11-22-02, during the student self-study hour. For those occurring at the beginning of a lesson, they appeared to be well planned, such as her coming to office half an hour early to mark a certain amount of homework and carrying them to her first class to share with and explain to students. Meanwhile, for those that occurred during the non-teaching hours, they seemed quite spontaneous, as she was prompted to react immediately to issues she encountered in the process of marking homework. In all cases, whether they were planned or spontaneous, these activities were driven by a strong motivation to address quickly the problems in student homework she found and shared with colleagues.

[2] It was between 12:30-1:15 for all middle schools. In fact, this period of time was used by many math teachers in the middle schools that I observed.

The settings of her explaining activity also point readers to those days and segments in which she apparently treated her two classrooms differently, Class 4, the stronger one and Class 2, the parallel one (a relatively weaker one)[3]. For instance, on Nov. 19, more exercises in Class 4 were explained than in Class 2 (3 versus 1) while more time for reviewing was given to Class 2 (for more preparation for the explaining) than in class 4 (2 minutes versus 8.8 minutes). On both Nov. 25 and Nov. 26, after lunch, she explained the same assignment to both classes. She spent more time explaining to Class 4 while more time on monitoring and marking student corrections face-to-face in Class 2. Even when she explained the same exercises in similar amounts of time to both classes, for instance, on Nov. 27, analysis of her explanations found that she gave more sophisticated and thorough explanation to her stronger class which was more able to follow the fast-paced and rich explanations. Analysis also shows that she made adjustments to her explanations, for instance, to meet the needs and learning levels of her weaker students, something which will be talked about in more detail later in this section. As mentioned earlier, it appeared that her decisions to explain certain errors included both planned ones and spontaneous ones reflecting the situated nature of her decision making processes during marking homework.

Errors, Explanations and an Informed Decision Making Process

In explaining homework, the selected errors are taken as the object of the activities and explanations as the immediate outcome of the activity. Construction of the

[3] Explained in a note in an early chapter, the two classes both have mathematically strong and weak students; but compared with the stronger class, the weaker one has relatively a smaller proportion of stronger students and a larger proportion of weaker ones.

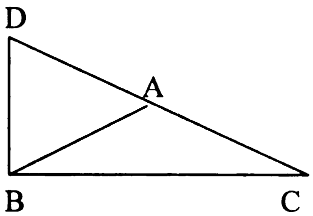
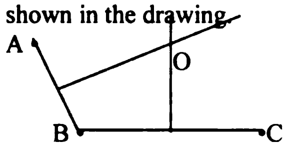
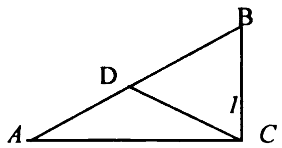
object refers to the teachers' selecting and meaning-making process. In a more narrow sense, the human subject molds the object into a value-added product. Viewed more broadly, to quote Lektorsky (1984), it means that the subject "singles out the useful properties of the object to develop social practice." While marking homework, Tr. Wang chose the errors that would help students understand better the difficult and important points embodied in the errors. Such a selection process not only informs her about student learning and what she needed to do to help students make better sense of the errors but also enabled the information to be used and fed back into the system of her activity for her eventual purposes of teaching. In addition, her explanations expanded the raw errors with new meanings – what is wrong, why it is wrong, what constitutes a correct solution or procedure and how to make corrections.

In this section, I examine the nature and the role of the errors selected for explaining, the mathematics and student learning implications entailed in the errors as well as the selection process of the errors.

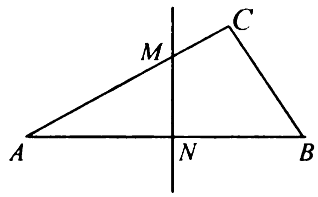
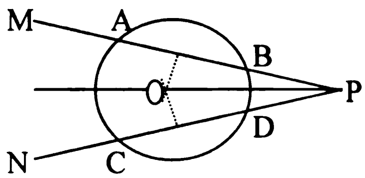
Selected errors in the two teaching sections

Locating the errors in the curricular sequence. Chapter Three demonstrated that teaching and learning a new concept successfully is determined by the mastery of prior concepts. That is a major reason for Tr. Wang to choose to explain the errors in the homework promptly and get students ready for learning a new topic. The learning of new topics or concepts depends on the mastery of previously learned ones. As the last section of *Geometry Proof*, teaching of *Converse Propositions and Theorems* builds on the previous concepts, such as construction of figures, definition of lines and shapes, adding of auxiliaries and procedures of proof writing. As shown in the table below, the curriculum also prepares for making a transition to this section through the converse relation of the two corollaries of the right triangle theorem taught on 11-14-02. In *Function*, as the beginning chapter, *Direct and Inverse Proportion Function* depends on knowledge and skills learned in earlier and the current grades, such as laying out algebraic expressions, equation, equation system and the correspondence of points on a plane with real numbers in coordinates and direct and inverse proportionality. As the errors unfold, one can notice the intricate relation between the errors and these previous knowledge and skills. As the teacher explained the errors, one can see how she assisted students in brushing up on and reinforcing the understanding of the previous concepts and building connection to extend the current learning.

Table 5.3 Location of the Errors Chosen to Explain in the Daily Homework Assignments in the Section, *Converse Proposition and Theorems*

Date/ Topic of homework	**Context of the day's assignments**	**Exercise chosen to explain**	**Error chosen to explain**
Friday, Nov. 15, 2002 Two theorems (11-13) and two corollaries of right triangles[4] (11-14)	Ex. 1-4, Vol. A/p.35-36: One filling blank and three proof writing exercises. Apply the theorems (see footnote) in Rt Δ created by altitudes in other shapes: altitude on the hypotenuse of a Rt Δ ; altitude on the side and on the base of an isosceles Δ ; and a right trapezoid with an angle of 60 degrees—the need to draw an auxiliary altitude to make a Rt Δ .	**Ex. 2.** An isosceles triangle with a base angle equal to 15°. To prove: the altitude on one side is half of the side. (Students are to do the drawing) [Vol. A/p. 35. Ex 22.4 (8) 2.] 	**<u>Error 1</u>** **Ex. 2.** Opening the proof writing by citing the congruent base angles ($\angle$C=$\angle$ABC= 15°)
Tuesday, Nov. 19, 2002 Perpendicular bisector	Both classes: Ex. 1, 2 & 3 in Volume A (p. 37). Extra for Class 4: Ex. 2, 3 & 4, Textbook (p. 79) One construction, one filling blanks (with calculated lengths of sides and degrees of angles), and 4 proof writing. Apply the perpendicular bisector (PB) theorem and its converse in different types of exercises: construction---finding a point of equidistance to a given segment or points; calculation of angles and proportion of sides in given right triangle; and proof writing.	**Ex.1.** Say how to find a point of equidistance to the three residential sites shown in the drawing. (The drawing is the completed version of the construction) [Vol. A/p. 37, Ex. 22.5 (2) 1.] **Ex. 2.** The given: See drawing, in ΔABC, $\angle ACB = 90°$, $\angle$1=$\angle$B. Prove: D is on the perpendicular bisector of AC. 	**<u>Error 2</u>** **Ex.1.** (1) In this construction, two perpendicular bisectors (PB) suffice but many students drew a third one for AC; (2) Failure to write the complete construction methods and conclusion in standard construction language. **<u>Error 3</u>** **Ex.2** Drawing an unnecessary auxiliary, the PB of AC and used it as median with D not given as midpoint. Creates the givens and mixing up

[4] Theorem : In a right triangle, the median on the hypotenuse is half of the hypotenuse. Corollary 1: In a right triangle, if an acute angle is 30 degrees, the right side it faces is half of the hypotenuse. Corollary 2: In a right triangle, if a right side is half of the hypotenuse, the angle it faces is 30 degrees.

	In proof writing, apply the theorem in a quadrilateral with perpendicular diagonals as well as in triangles: given relationships between sides or those between angles to prove that a point is on a PB; or given the PB and sides, to get the degree of an angel.	[Vol. A/p. 37, Ex. 22.5 (2) 3.] **Ex. 3.** The given: See drawing, $\angle C = 90°$, the perpendicular bisector of AC intersects with AC and AB at point M an dN, and AM=2CM. To prove: $\angle A = 30°$.  [Textbook/p. 79. Ex. 22.5 (2): 4]	what is the given with what is to prove. **<u>Error 4</u>** **Ex. 3.** A few students failed to use the concept of PB to draw the auxiliary by connecting M and B.
Wed. Nov. 20, 2002 Angle bisector * Note the day's two errors are arranged in the order that was explained.	For both classes: Ex. 1, 2, & 3, Vol. B (p. 42) Extra for Class 4: Ex. 1, 2 &3, Textbook (p. 81-82) Apply the angle bisector theorem and its converse by finding the distances from a point to two intersecting rays and to the sides of an angle. Finding the distances from the intersecting point of a vertex of a triangle	**Ex. 2.**[5] The given: See the drawing, Circle O intersects $\angle MPN$ to get AB=CD. To prove: PO is bisector of $\angle MPN$. [Vol. B/p. 42. Ex. 22.5 (3) 2.] 	**<u>Error 5</u>** **Ex. 2** 1) Drew the distances from O to AB and CD but failed to write the construction in accurate language 2) Failing to indicate them as perpendiculars and use them as sufficient conditions to justify that the two distances are equal;

[5] The TRM gives a rationale for choosing to represent the content in the concept of set versus locus (to be taught in the second semester of 8th grade) and compares their similarities. Note that Schoenfeld (1991)'s chapter in *Informal reasoning and education (1991)* in Perkins and J.W.Segal (Eds) p.311-343, used this similar construction on some college students and Fawcett's (1938) use of it on high school students. The later was both based on the concept of locus. They use it to let students construct by drawing on the theorems while this exercise was used in Shanghai's 8th grade as a situation to practice on the theorem of an angle bisector.

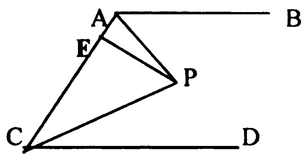
	intersected in an angle to the sides of the angle; Proving that the center of a circle that intersects an angle (or two intersecting rays) is on the angle's bisector: given either that the intersected arcs are equal and or that the intersected chords (which are on the two sides of the angle) are equal. Two parallel lines intersected by a third line forming two angles. Find the distance from the intersecting point of the angle's bisector to the two parallel lines.	<u>Ex. 1.</u> As in the figure below, AB//CD, AP and CP respectively bisect $\angle$ BAC and $\angle$ DCA. If the altitude of Δ PAC, PE=8cm, then the distances from AB and CD are respectively ___________.  (Take advantage of a shared side to allow substitution of equal distances.)	3) Failing to use the concept of distance indicated as perpendicularity to justify that O is on the bisector of angle MPN. **<u>Error 6</u>** The exercise requires filling the length of two distances. A number of students only filled in one. They were misled by being given only one blank. They did not pay attention to the word, "respectively".

Table 5.3 shows that homework is designed to consolidate learning by changing a problem in multiple forms to increase the pedagogical value. One can notice such a pattern in the table above, as well. In the second column, the design of one problem (or concept) with multiple changes in format (such as the OPMC talked about in chapter three) is clearly reflected. For instance, the assignments for Nov. 15, 2002 aim to practice the two theorems of right triangles and their converses by using the concept (approach) of an altitude (perpendicularity) located in different geometric shapes: a right triangle, isosceles triangle and a trapezoid. Assignments for Nov. 19, 2002 apply the perpendicular bisector in different triangles and the relationships of their sides. Those for Nov. 20, 2002 are designed to apply the angle bisector theorem and its converse in the context of angles formed by two rays, intersected by circles or in between two parallel lines. The design also used different types of exercises, such as construction, filling blanks and proof writing.

However, one noticeable pattern in the explanations and errors emerges from this context: although the designed changes provide difficulty for students, what they got wrong was not typically with the newly taught concepts or topics that they needed to practice through these changes. Instead, the errors, especially those chosen by Tr. Wang for further attention, were more fundamental to the learning of geometric proofs and functions in general. For instance, Error 1 and 2 are both concerned with the fundamental knowledge and skills for axiomatic structure and constructions in geometry.

Major factors impacting Tr. Wang's selecting of errors to explain. Tr. Wang selected the errors to explain to all students either based on their being typical or difficult or both. The time factor was also important in selecting the number of assignments and which assignment to explain. For instance, on Nov. 15, 2002, during the first period, she needed more time to teach the new concept, converse propositions and theorems. So she chose to explain the second assignment as a brief example (2.8 minutes) to illustrate what she found across the homework of the day – students' incomplete writing – which she figuratively called, "missing an arm or leg". On that day, a small number of students also made mistakes in the first fill-in-the-blank exercise that practiced on the proportion between the segments of the given sides of a right triangle and the last proof writing in the context of a trapezoid. She tutored a girl on both of these errors during a morning break. From this tutoring (to be discussed in more detail in Chapter Six), she found the girl was able to know the theorem quite well, but had numerous errors in applying them in writing her proofs.

On the next day, Nov. 19, 2002, as depicted in the beginning vignette of this chapter, when the content for teaching can allow more time, she chose to explain three of the marked assignments to illustrate two major points both of which are fundamental but difficult and important for students to learn. The first was how to construct perpendicular lines and how to write the construction procedures in standard geometric language. The second was how to know when and whether an auxiliary line is needed or not. She said, "Because of time, I cannot explain all the problematic ones; I choose the most typical ones, the ones not necessarily just typical or difficult; they are the ones I believe that need to be emphasized again" (interview, 11-19-02). Her beliefs about what needed to be emphasized in student learning were both determined by availability of teaching time and whether the error(s) represented typical student learning difficulty or important points that the curriculum stipulated.

Unrelated and trivial at first sight. A first glance at the above selected errors given in Table 5.3 shows that they do not bear directly upon the learning of the newly taught theorems or concepts (for instance, Error 1 is unrelated to the corollaries of right triangle theorems taught on the day when the homework was assigned, 11-15-02 or Error 5 , unrelated to angle bisector theorem and its converse on 11-20-02). Instead, they appeared to be trivial. For example, Error 1 does not have to do with the context of the assignment, applying the theorem in an isosceles triangle: students wrote in the first sentence of the proof that the base angles are equal instead of the isosceles sides are equal[6]. Error 5 has to do with the auxiliary distances (see the dotted segments on the figure in the table) students drew from the center of the circle to the chords (AB and CD). After drawing the two distances, many students could not write how the auxiliaries were drawn; they started proof writing with the reason that the two chords are equal and

[6] In Shanghai mathematics textbooks, an isosceles triangle is defined based on its sides and the equal base angles are deduced from the definition based on equal sides.

therefore, the two distances (auxiliaries) are equal without indicating (in perpendicularity) that the two auxiliary are distances.

Beneath the errors and explanations lie fundamentals of the deductive system. Careful reading of the explanations that Tr. Wang gave to her students on these errors reveals important mathematics behind them that the students lacked. Both Error 1 and Error 5 have important implications for student learning of the nature of the deductive system: the need to understand that the axiomatic proof is established on the basis of definitions or some "rock bottom self-evident facts upon which the whole structure is to rest" (Davis & Hersh, 1981, 149). As she questioned her students, "What's the reason for $\angle C = \angle ABC$ (the two base angles)?" She wanted to let her students understand that the two equal base angles are derived from the two equal sides that define an isosceles triangle and "there is a logical syllogism in this step," as she said, that cannot be missed. Included in Error 5 are two key requirements for learning to write a good and rigorous proof, to justify a statement with sufficient evidence and to put the conditions and steps in logical sequence. The mathematics entailed in the errors and the explanations given by the teacher and research related to student learning of related topics are summarized in Table 5.4 below, which illustrates that these seemingly trivial errors entail important mathematics and considerations about student learning of the mathematics involved.

Table 5.4 Mathematics and Student Learning Entailed in the Errors in Geometric Proof.

Types of error Error/Date	Mathematics entailed	Role of the error in student learning
Nature of axiomatic system Error 1 (11-15-02) Error 5	*"There is a logical reasoning segment (syllogism) in this step (that you cannot miss)."* Deductive and axiomatic system starts with a definition or axiom ; justification based on sufficient	Students transitioning from van Hiele Level 2-3. "...do not grasp the meaning of deduction in an axiomatic sense, e.g., do not see the need for definitions and basic assumptions" (Fuys et al, 1988).

(11-20-02)	conditions	
Deductive reasoning is needed in a construction problem. Error 2 (11-19-02) Error 3 & 4 (11-19-02)	**Ex. 1** *"Do we need to draw a third perpendicular bisector?"* Deduction is needed to reason and justify that it is sufficient to connect two segments (instead of three) and draw their perpendicular bisectors intersecting at one point Procedures of construction Language of construction Writing of the procedures of construction in clear construction language	"...students do not often see the connection between construction and proof problem when a construction problem is given before a proof." (Schoenfeld, 1991:319) "Have we found the point?" "Many students drew 3 PBs. Is it Necessary?" questioned Tr. Wang repeatedly. "... Does this exercise need an auxiliary?" Tr. Wang questioned students.
How to decide where and what auxiliary is needed. Error 3 & 4	**Ex.2** "When is an auxiliary needed?"—use of counter examples Knowledge and skills to find auxiliary lines to assist finding a proof; write the construction in geometric language.	**Ex. 2 & 3.** "(F)inding the lines is part of finding a proof, and this may be no easy matter" (Davis and Hersh, 1981, p.150)[7] According to the Teaching Reference Material (Manual), knowing when and how to tell if an auxiliary is necessary is both an important and difficult point (p. 55-59).
Logical sequence in proof writing. Error 5 (11-20-02) Error 1 (11-15-02)	*"Distance has to be used twice in constructing this proof"* The rigor of deductive proof demands justification of a statement with sufficient conditions and put them in a logical sequence.	The place of a concept in the axiomatic chain or the "chains of deductive proof" (Brumfiel, 1973:102).: how it becomes part of the deductive chain and how it is used to extend the chain. (Farrell, 1987:239) "Some students, after drawing the perpendiculars, wrote, because AB=CD, (so) OE=OF (E on AB and F on CD). This because statement, is this right or wrong?"—asked Tr. Wang.
Understanding of language Error 6	"Respectively" Language use in assisting understanding of mathematics	

The rest of the selected errors above are also emblematic of other different dimensions of geometry proof, such as the need for deduction in doing construction (Error 2), the ability to see where an auxiliary is needed and how to draw them (Error 4, 5

[7] Davis, P.J. & Hersh, R. (1981). *The mathematical experience.* Boston: Houghton and Mifflin.

& 6), use of geometric language in both construction and proof (Error 2, 4 & 5) and use of language in mathematics (Error 6). These errors also represent issues related to student learning of geometric proof widely identified and documented in important research on mathematic education. In Fuys and colleagues study (1988), for example, students, like those of Tr. Wang's who are at the transition from van Hiele Level 2 to Level 3, typically do not recognize the need for definitions in a proof. Schoenfeld (1991, p. 149) found that college and high school students do not see that constructions and proofs are connected when they are given a construction before a proof.

Error Selecting as Construction

Errors in the section on functions. When we look at the errors that Tr. Wang chose to explain in the section on *Functions, Direct and Inverse Functions,* a different picture emerges. While the errors in geometry proof indicate a definitive incorrect or inadequate response or writing based on the rules and logic of deductive system, many of the errors in functions do not necessarily have a clear cut basis for whether they constitute errors. In other words, the criteria as to whether an answer is wrong or which answers, given multiple ones, are better, was context-dependent. Such contexts can be ambiguous regarding multiple answers and approaches to solving the same problem. Table 5.5 below presents these errors in the contexts of the daily assignments, the selected assignments and the errors chosen to explain in the section on *Converse Propositions and Theorems.*

Table 5.5 Location of the Chosen Errors in the Daily Homework Assignments in the Section, *Converse Propositions and Theorems.*

Date	Design context of the assignments	Exercise chosen to explain	Context of Errors chosen to explain
Thursday, Nov. 21, 2002 **Topic of teaching:** *Direct Proportion Function (DPF)* **Topic of explained homework** *Direct Proportion Function (DPF)*	Ex.1-5 (22.3), Vol. B/p.25-26: Based on the concept of DPF, judge whether the given representations are DPFs and write the analytical expression of relationship: symbols (equations and proportions), situations (radius and perimeter in the formula of a circle's perimeter of and orbiting of a satellite around the earth), and a data table.	**Ex. 1** In the following functions, please tick on those that are DPFs. **(6).** y : x=1 : 4 **Ex. 4** Filling gasoline into the tank at even speed, the quantify (q)of filled gas and the time (t) it takes at different time intervals: t 1 2 3 4 5 6 q 1.5 3 4.5 6 7.5 9 (1) How to say that q and t are direct proportion. (2) Find its coefficient (3) Write the analytical expression of relationship[8] ("formula") of the function **Ex. 5** It takes 10 3/5 hour for a manmade scientific experiment satellite to orbit the earth 6 rounds. How many hours it takes to orbit 14 rounds?	**<u>Error 1</u>** Some students answered yes and others, no. The key in the Teaching Reference Manual says yes, but other references say no. **<u>Error 2</u>** Error 2: Students could interpret data from the table but were unable to state why the data are of direct proportionality; confusion b/w q is function of t and t is function of q **<u>Error 3</u>** Many students used arithmetic approach. She compared arithmetic and function approach.
Friday, Nov. 22, 2002 **Topic of teaching** *DPF--Graph and its property* **Topic of homework explained:** *DPF: its concept, analytic expression of relationship and its use*	Assigned from another reference resource, "Same Step", Ex. 3-7/p. 50 Practice on the concept of DPF by giving other forms of symbols: algebraic expressions of direct proportionality; and given the values of the variables, ask to get the value of the coefficient and then write the analytic expression of relationship of a function. Assigned three	From "Same Step" Ex. 3, 4, 7. Ex. 3: Known: y-3 and 5x are in direct proportion and when x=2, y=8. Get (1) the expression of relationship –y as function of x; (2) y's value when x=-4 Ex. 4. Known: y+1 and 2x-1 are of direct proportionality and when $x=\sqrt{2}$, $y=\sqrt{2}-\frac{3}{2}$, find the analytical expression of relationship of this function. **Ex. 7** Known: y+a and x+b are of direct proportionality; when x=1, y=-2 and when x=-1, y=1. Find	**<u>Error 4</u>** (Similar type of error for the explained exercises) Students failed to follow the mathematical format the teacher laid out to do such exercises: use "Let" Sentence: Let the analytical expression of the function be ...and plug the obtained value of the coefficient into the supposed function to get the expression of relationship of the function or the desired

[8] In the Shanghai textbook (p. 46), the expression used to represent the relationship between the dependent and independent variables are called solving analytical expression *(jie xi shi).* Many textbooks in the U.S. call it "formula" or "rule". But in Chicago school textbook, it is called, "expression of direct proportionality". It is also called, "defining expression of relationship" (Conversation with Dr. Danielle Chazan, 1-24-05). Tr. Wang often called it both "analytical expression" and "expression of relationship." I translate it here into a composite one, "analytical expression of relationship".

	geometry proof exercises: Ex. 16, 17, &18, Vol. A/p. 45 Practice, on angle bisector in a right triangle, perpendicular bisector in more complicated intercepted shapes.	the analytical expression of this function.	value of a point Where to plug it in and which variable and which variable are of direct proportionality.
Monday, Nov. 25, 2002 **Topic of teaching:** *Mixed and extended practice on DPF.* **Topic of homework explained:** *DPF--Graph and Its Property*	Ex. 1-3, Vol. A/p.22 Practice on the property of a DPF by given the graph and the value of a point on the graph and ask to write the analytic expression of the function; or given the graph and a point on it with one value unknown, get the value of this unknown. Assigned on the same day more practice of similar exercises to make use of the properties of a direct proportion function.	**Ex. 2** As in the graph [a line passing the I and III quadrants with two points A (3, 2) and] point M (-4, b) is on the graph of this function, get the value of b. Ex. 6, Vol. B/p. 27—assigned the same day: Known: the function, y=-3x+(n+4) and its graph passes the origin, get the value of n.	**<u>Error 5</u>** Error 5: Similar to Error 4 but summarized again the type of exercises: a point on the graph of a function; and reiterated the format for doing such exercises. **<u>Difficulty:</u>** Thinking that student might have difficulty with this exercise, she went to both classes after lunch to explain what it means for a function's graph to pass the origin of the coordinate.
Tuesday, Nov. 26, 2002 **Topic of teaching**: *Inverse Proportion Function and Graph and Its Property* **Topic of explained homework:** DPF and its graph and property	Ex. 1-6, Vol. B/p. 26-27 More exercises in the form of a given point on the graph of a function or the location of the graph line, and ask to write the analytical expression of the function or to draw the graph, or get an unknown value of the given point.	Ex. 1. Known: the graph of function, y=(a-3)x+(b+2) passes the origin, then: (A)b=-2; (B)a=3; (C) a$\neq$3; (D) a$\neq$3 and b=-2 (Assigned on Mon. 11-25) Ex. 4. The graph of a direct proportion function passes $(-3\sqrt{5}, 5)$. Find the analytical expression of the function and tell which quadrants it passes. Explained the three geometry exercises assigned on No. 22.	**<u>Error 6</u>** The key given in TRM says (D) but Tr. Wang thought it should not necessarily be (D) because it does not say that it is a direct proportion function. **<u>Error 7</u>** Failure to plug in the obtained value of *k*, the coefficient in the supposed analytical expression to get the function. **<u>Error 8</u>** Reiterated the accuracy in drawing the graph: even and big enough unit on a coordinate, indicating the direction and origin on the graph. **<u>Error 9</u>** She was happy to share

			that most students improved in giving sufficient conditions in their proof writing, but there were several students who still missed conditions.
Wednesday, Nov. 27, 2002 **Topic of teaching:** *Mixing and extending of DPF and IPF, their graphs and properties.* **Topic of homework explained:**	Ex. 1-8, Vol. A/p. 22-24 Given an IPF, draw its graph; given two points on a IPF's graph with one unknown value, get the value; given a set of value for x and y, get the expression of relationship; given that a DPF and a IPF both pass the same point, get the expressions of both functions and draw graphs; word problems.	**Ex. 7.** The "gear plate" of a bicycle has 46 cogs and does 100 spins per minute; a "flywheel" has 20 cogs, how many spins does it make per minute? **Ex. 8** A pile of coal---originally it was planned to be consumed at 3 tons a day and was able to last 95 days. When it was consumed for 24 days, the furnace was renovated and it saved 0.6 ton of coal a day. How many days could the pile of coal still last?	**<u>Error 10</u>** Many students used arithmetic approaches. Tr. Wang compared the use of arithmetic, equation, and function approach and their advantage and disadvantage and led students to see the usefulness of function in daily life and production work...

Similar to geometry proof, the assignments given in functions are also designed to practice on the concept taught on the previous day by applying it in different contexts. The domain specific contexts in functions are the different representations of functions: algebraic expressions, tables, graphs, real situations, which require students to interpret the data about the relationships and translate between these representations. However, as shown in the above table, the context of the errors in functions is not clear cut. They can be grouped into three different categories according to the types of errors: those that are incorrect or inadequate mathematically; those that induce ambiguous perspectives or approaches; and those that do not seem to belong either of the two categories. From the following table, a considerable proportion of them do not constitute mathematically wrong or partially wrong responses.

Table 5.6 Types of Errors in the Section on Functions and their Attributes

Types of Errors	Incorrect or inadequate	Ambiguous	Cannot be classified
	Error 2; Error 4; Error 5; Error 7 & Error 8	Error 1, Error 3, Error 6 & Error 10	Regarded as difficult; Error 9: geometry proof
Attributes	Pertaining to inadequate steps and procedures; failure to comply with format and rules regarding expression of proportionality and the functions and translation between different representations of function as well as drawing accurate graphs.	Different answers from student work and different keys from teachers' and other references make sense from their own light; student use of arithmetic approach from elementary school are correct but inadequate compared to function approach; and the teacher disagreed with the key given in the Teachers' Reference.	A student coming to ask a question about the day's homework often could suggest to her that students could have difficulty in doing a certain problem; so she went to the classroom to explain or tutor. Assigned and explained homework on geometry proof during teaching function.

Each of the three categories of errors deserves attention in its own right. Among them, however, the errors that stand out are those that are producing ambiguity and multiple perspectives or have multiple ways of solution For example, she explained, y:x=1:4 or y/x=1/4 is only of direct proportionality and cannot be regarded as a direct proportion function because given the range of x, x cannot be 0 while its analytical expression of direct proportionality, y=4/x, the range of x could be all real numbers. Similarly, in Error 6, she disagreed with the key given in the Teachers' Reference Material. She explained: if the function, y=(a-3)x+(b+2) is indicated as a direct proportion function, then when its graph passes the origin, answer (D) $a \neq 3$ and b=-2 is correct; but since it is not indicated as a direct proportion function, all the other answers could also be correct in other respective contexts.

Error 3 and Error 10 are interesting cases as well in teaching direct and inverse proportion functions. In both cases, most students, including the stronger ones, reacted to the familiar situations of word problems in elementary grades and used arithmetic

operations to get the answers correctly without using algebraic expressions, such as equations. The teacher did not mark their answers as wrong but, through explanation, assisted them in understanding the differences between these approaches by comparing and contrasting the limitations and strengths of each of the approaches. She assisted students to see that the one-to-one correspondence of the two variables of the functions has wide applications in life (graphs of weather forecast and stock exchange) and work (mechanical design, such as designing bicycles of different sizes and functions).

In choosing to explain this set of errors, the teacher opened up ambiguity and controversies to students, treating them as learners that have their own thinking albeit needing assistance. As commonly assumed, under centralized education systems, teachers have to strictly follow the given curriculum and treat it as the source of authority. The above situations help illustrate how, in implementing the pace and goals of the official curriculum, the source of authority was also open to question, debate and examined from multiple perspectives. This helps reveal a dimension of mathematics teaching in Tr. Wang's practice which suggests that mathematics is not viewed as cut and dried facts that often have only one correct answer; but as open ended and encouraging viewing an answer and making meaning from different perspectives.

Borasi (1996), in proposing a pedagogy of "using errors as springboard for inquiry" (p. 30), viewed errors from a radical constructivist view: "What constitute as errors are no longer clear cut" and "... most often the decision of whether something constitutes an error may depend both on the context and the person who making that decision" (p. 30). This is also reflected in the above group of errors selected by Tr. Wang to explain to her students. Here, the teacher made decisions about what constitute errors based on student responses in their homework and her explanations given to these errors are targeted at student problems of learning. Borasi argues for using error to introduce a sense of inquiry and to combat reductive views of mathematics as being concerned with rigid, formulaic correctness, rather than problem-solving. Tr. Wang was guiding her students to do inquiry by introducing to the ambiguity embedded in the official curriculum.

Even though the constructed nature of errors is more prevalent in functions, the mathematics and student learning entailed in them are important. The explanations all reflect the careful direction on the part of the teacher based on student problems and efforts that she made to help see the mathematics lying beneath the errors so that they could form the understanding needed both conceptually and in terms of procedure to correct the errors. The table below helps illustrate such implications.

Table 5.7 Mathematics and Student Learning Entailed in the Errors in Functions

Types of error Error/Date	Mathematics entailed	Role of the error in student learning
Concept of direct proportion and direct proportion function Error 1 and Error 2	Direct proportion and direct proportion function can be distinguished by their difference in format, range and domain. Translating data in a table representing direct proportionality into natural language and into algebraic symbols.	Learning of the function concept is intimately linked to study of prototypes (families of functions), multiple representations, and transformations. (Confrey and Smith, 1991) Students are found to have difficulty translating the different representations of functions. (Callaghan, 1998: 23)
Concept of DPF*($k \neq 0$, the constant item is zero)* in conjunction with its graph and property Error 6	Algebraic representation of a direct proportion function and its relation to its graphic representation and the properties of the graph.	"Students need to be able to understand information presented in different formats as well as perform transitions among the various representations" (Callaghan, 1998: 23)
Interference of strongly retained early knowledge and skills Error 3 and Error 10	Arithmetic, algebraic, proportional approaches and understanding are bases for understanding functions but they are different concepts in nature.	Transfer skills are prerequisite to the integration of information about functions into a single, unified conceptual image. (Schwartz and Dreyfus, 1999)
Error 4, Error 5, Error 7, and Error 8	Mathematical rules, format and methods; algebraic procedures; relationships of variables represented in coordinate and graphs and the properties of the graphs.	Students need for a strong operational base before being introduced to the structural conception of concept. (Sfard, 1989, 1992). A certain amount of algebraic knowledge and methods is needed for studying functions (Sierpinska, 1992)
Error 9	Geometric proof writing	

Selection as reasoning and decision making informed by multiple interactions. As the above analysis illustrates, the errors explained by Tr. Wang imply important mathematics and student learning of it. In geometry proof, she was assisting students to learn from the errors how the axiomatic system works and in functions, she was guiding them to understand the concept of functions, specifically in terms of direct proportion function, as dependency of variables and how to interpret and translate between different representations of such relationships. Through the errors in functions, she also helped them see that there is not one single answer for a problem or one way to solve a problem. These important implications reveal the basis of her selecting these errors and her goals of making students understand the concepts and procedures entailed.

Nonetheless, the process it took to determine which errors to pick for further attention is not so straightforward. The organization of the teacher's work and her work space allow her to use student homework to make sense of student learning through meticulous marking of each workbook, to interact with students through additional tutoring on homework, and to interact with colleagues through conversations over making homework. These interactions all informed her in one way or another on her decision to choose errors to explain. While observing her marking homework in her office, as presented in Chapter Four, I was able to capture some of her on-going thinking that was revealed by the words, comments, or complaints she uttered to herself or to me about the assignments she was marking. The patterns of student errors were identified through long hours of marking. The students coming in during break time to ask questions about homework or who were called to the office for tutoring also helped her know more about the problems students encountered and helped her decide whether to chose to explain a certain problem she just tutored. For instance, on 11-19, she learned from tutoring a boy student coming to ask a question about Exercise 4 in the textbook (p. 79) that students might have difficulty in seeing the need to add the auxiliary, so she shared that she chose to explain by comparing and contrast this exercise with another exercise that does not require adding auxiliary. On 11-20, tutoring a weak student with

Exercise 1 helped her see the need to emphasize the meaning of the word "respectively" during her explanation in the following period.

Her conversations with colleagues about homework not only informed her selection of the errors but also how she explained them. This happened very frequently, especially during teaching the functions when they encountered controversies about the standard keys and different ways students approached an exercise. When Chapter Seven digs further into such collegial interactions, it not only will showcase how mathematics and student learning are richly embedded in their conversations but also how the teachers negotiated meanings and came to decisions as to how to deal with such situations. As discussed in Chapter 3, the curriculum and the different nature of the content domains bear directly upon the teachers' conversations around homework in both content and the questions that triggered the conversations.

Section Summary

Error selection is a kind of a construction process. I note three dimensions: first, the context-dependent nature of the errors, mostly in functions that induce multiple views and approaches; second, the selection is informed by different interactions between the teacher with the participating material and persons surrounding her. Third, such interactions mediate the teacher's making sense of student thinking entailed in the errors. Such a process was not finished with selection; it continued over the course of her explanation. As demonstrated in the next section, it is quite evident that, during her explanation, as she probed and questioned students, she was able to realize other possible routes student might have taken that led to the errors. As a result, she was able to add other perspectives and produce new layers of explanation to the errors. Therefore, if what went into this process of selecting and explaining errors was how the teacher tried to understand student thinking, then construction weaves selection into explanation during which errors are being transformed into forms that are more accessible to the student understanding that she aimed at to create.

Explaining and Transforming Errors

The above section illustrated that the errors Tr. Wang selected to explain to the whole class entail important mathematics and implications for student learning. In this section, I attempt to describe the pedagogical actions that Tr. Wang took to make the entailment accessible to students' understanding. I also aim to describe the pedagogical features of Tr. Wang's explaining and relate them to the traditions of teaching and education embedded in Chinese culture.

Established Activity Structure and General Routine Actions

As the opening vignette of a real teaching scenario portrays, Tr. Wang's explaining of homework errors has a certain structure. Coding analysis of the brief explaining segments in the observed lessons demonstrate established structure which consists of three, sometimes four routine steps. Within each step, there are a number of routine actions frequently performed by the teacher and her students. These steps included most often a review of previously taught content; introducing the problem context (the task assigned) and the error(s) the teacher found in the homework; giving explanations through interaction with students; and summarizing and generalizing in the end. Each step has its own goal(s) and is performed by a set of goal-directed actions.

It is such a “clear and consistent event structure” that “allow(s) participants to attend to content rather than procedures” (Cazden, 1988, p. 47) This structure made it possible for the teacher to convey to the students the important ideas behind the errors and/or problems within a constrained time frame. The routine actions that they performed regularly “allow relatively low-level activities to be carried out efficiently, without diverting significant mental resources from the more general substantive activities or goals of teaching” (Leinhardt & Greeno, 1991, p. 235). Therefore, clearly mapping out the activity or event structure and those routine actions is essential to understand what an explanation looks like in terms of its general procedures and actions.

Reviewing to build connections between old and new knowledge. Tr. Wang often started a lesson with a brief review of the knowledge taught in the previous day(s) or semesters, which seems to be commonly seen in math lessons in East Asian classrooms, such as those found in Japan (Stigler & Hiebert, 1999) and the famous secondary school teachers’ math lessons recommended to teachers in China (Committee of Record of Famous Teachers’ Lessons—Secondary Math, 1992 & 1999). Although many of her reviews directly transitioned into presenting new content, 6 out of 14 segments across 3 of the 8 teaching days, segments 1-5 & 8 on 11-15, 11-19 and 11-25, were followed by explaining homework, accounting for about 43 percent of all reviews. All the reviews shared quite a visible and consistent goal of building a bridge between related prior knowledge and the new knowledge to be taught. These are identifiable by her transitional remarks she made to connect the review with the new content to be taught at the end of each review.

For instance, depicted in the opening vignette of this chapter, on Nov. 19, 2002, Tr. Wang started to explain homework after she reviewed the perpendicular bisector theorem and its converse taught on the previous day: all points of equidistance to the endpoints of a line segment are on the perpendicular bisector of the line segment. She used the review to move from the "two points" (she referred to the endpoints of a given or constructed segment) to the three points---"the three residential areas" in the assignment (see Table 5.3 for the details and of this assignment and its drawing) that she started to explain to class:

> *Then, the homework we did yesterday. There are three points given here (pointing to the board at the already drawn points), now we need find the point, so that its distances to these three points are all equal. Now here, there are three points (emphatically), how should we solve this problem?*

To use a much briefer explaining segment she gave to Class 4 on Nov. 15, 2002, when she taught the new concept of converse propositions and theorems. She began the lesson with a 3-minute review of the two corollaries of the right triangle theorem she taught in the previous two days. She called on two students, a girl and a boy, to recall each corollary. To end the review, she made the following transition (the left column of the transcribed segment indicate the I (initiation) – R (response) – E (Evaluation) (Cazden, 1988) :

I	T:	Then, next, we move on to our new lesson. As in the two corollaries, *(in slower and softer tone)* what kind of **relationship** there exists between their statements and conclusions?
R	S:	Exactly converse. Starting to discuss noisily…
I	T:	What kind of relationship? Like this statement…

With another question after students' response, she called on more students to think and respond by comparing the relationship between the "if… then…" statements of the two corollaries. She then guided them to why they are of converse relationships. In doing so, she took advantage of the review and made a smooth transition to the new topic of the day.

In teaching the section on *Function,* Tr. Wang's reviews are mostly independent from explaining homework except for Nov. 25, 2005. One major reason is that she often took time to explain homework during her non-teaching hours and analysis of her explanations shows that the explanations to the errors themselves already encompass brushing up old content to help understand the current new content. This is because although function is a new content, it is built on the basis of previous algebraic content, such as equation, coordinate, and proportion.

Introduce the problem contexts and error(s). With or without a review proceeding her explaining homework, Tr. Wang would offer a complete introduction to major assignments that she was going to explain. For instance, to continue with the segments of Nov. 15, 2002, she started explaining and commenting on student homework by sharing her feedback:

T: Our classmates in proof, I marked homework of 3 groups as soon as I arrived in the morning, in proof, like these two corollaries, you all used very well. But, some, the majority of classmates, in proof, either missed an arm or a leg. For example, yesterday, we wrote the "Given" and 'To prove' on our own. This, AB=AC, $\angle C = 15°$, and the altitude on the side, BD. It requires us to prove that BD is half of the side, right? Some classmates (wrote their) proof in this way: the first sentence, because angle C=angle ABC=15 degrees…

S: *(A few students)* Wrong. *(More students joined in noisily.)*

In this brief introduction, she did several things: first, she shared with students the status of her marking homework (attaching a sense of importance to homework as she always did). To relate to the review, she commented that students used the two corollaries quite well in their homework and then focused directly on the problem of proof writing (missing of conditions) that she found in their homework. She then provided an example to illustrate her point. She went through the assignment virtually line by line and then zoomed in on the first sentence where the error lay. She also drew the sketch on the board and referred to it while explaining step by step. Generally, in introducing the errors, she often performed the following actions: shared feedback and commented on what students did well and what they did poorly; then she gave an example or two to illustrate her point; she drew a sketch or graph on the board and referred to it while going over the requirements of the assignment.

In the key assignments she focused on in explaining the problems in function, such as "the filling gas tank" (see Table 5.3, Ex. 4, 11-21-02) and the "gear plate and flying wheel of a bicycle" (Ex. 7, 11-27-02) (see Table 5.4 for the two exercises) among others. This sequence of actions was more apparent in the introduction she gave at the beginning of her lesson on Nov. 20, 2002 given below:

> T: *(Tr. Wang went to teaching with carrying a big pile of marked homework.) (Holding the bigger pile in her hand)* These many students have all got wrong. Wang XXX *(She read the names of the smaller pile one by one—about 11 names)* These students all paid attention to when to use perpendicularity. Those classmates who got it wrong, please make corrections during the rest time after lunch. *(She start drawing the figure on board while the monitor was handing the workbooks to the students.)* Those who have got your workbooks, please take a look at where you got wrong. (*Sound of the drawing tools and traveling chalk on board was audible while students were talking about their workbook.)* Where *(have you got)* wrong? In general, mistakes were found in Problem No. 2. Do you know what

you got wrong? Some students after drawing the perpendiculars *(she drew them on the blackboard, pause a while),* and then said, because, AB = CD, (so) OE = OF. This so statement, is it right or wrong?

S: *(A boy first and more students joined him.)* Wrong.

In terms of interaction with students, introducing is different than reviewing and developing an explanation (the subsequent step) in which the teacher engaged students by initiating their responses with questions and then she evaluated them to move on. In introducing the problem context, the teacher most often shared, commented and led the talk by herself. As a small but formal proportion of the entire explanation, it achieved several important purposes: it communicated clearly to students how the teacher viewed where they stand in learning; it roused students to recall from memory their previous day's thinking in performing those tasks and get their attention onto the same page through visually presenting them on board; it launched a leading line that pointed to what they lacked, missed, or erred; in other words, it got the students ready for learning about their problems or errors.

Developing explanations. This is the essential step or component in the activity structure of explaining homework errors. I call it "develop explanations" for two main reasons: first, it involves interactions between the teacher and her students to come up with the explanations even though the responses given by students are often brief and obvious. In introducing the problem context on Nov. 15, 2002 (which was given above), she focused on the error:

			… …
1	I	T:	Some classmates *(wrote their)* proof in this way: **the first**
2			**sentence, because angle C=angle ABC=15 degrees…**
3	R	S:	*(A few students)* Wrong. *(More students joined in noisily.)*
4	I	T:	This first sentence, is it right or not? *(With emphatic tone)*
5	R	S:	*(More students)* No, not right.

I	T:	Why not?
R	S:	*(Noisily)* You can't immediately say it *(it's not immediately known)...*
E/I	T:	$\angle C=\angle ABC$, what's its reason?
R	S:	*(A few students)* AB=AC.
Ev/ Ex	T:	**AB=AC. Then angle C equals to angle ABC. Right? Included in this step there is a logical deductive segment** (syllogism). Right? There is a syllogism (segment) like this.
	S:	*(A student repeating the teacher)* A logical deductive segment.
I/ Ev/ Ex	T:	Then how can you start by stating that the two angles are equal? **This is not a given condition, is it? They are deducted from AB=AC.** So we should say...
R	S:	*(Trying to answer at the same time)* Because AB=AC...
Summarize & generalize		*(Picking up)* So we should say: **because AB equals AC, $\angle C$ equals $\angle ABC$; and because $\angle C$ equals 15°, then $\angle C$ equals $\angle ABC$ equals 15°.** Is it like this? So our class all tends to err at such small issues... A small problem like this in the first sentence, the entire proof is not valid. But in using the theorems like these *(pointing at board),* our classmates did generally well. So we really need to pay attention to the small details in writing our proofs. Please correct your errors after lunch.
Transition to new lesson		[Then, next, we proceed to our new lesson. Like the two theorems, *(slower and quieter)* what kind of relationship exists between their statements and conclusions?]

In this brief segment of explanation, there is a clear pattern of IRE (teacher initiation-student response-teacher evaluation). Even though student responses are very brief, such as Line 3, a one word response while Line 5, a three-word response to a yes-or-no question, each time, the teacher made use of their responses as a form of contribution for her to continue initiation or evaluation. For instance, in the quick IR exchange from Line 1-11, in Line 4, the teacher questioned with a more emphatic tone about whether the students thought the step was wrong after she heard only a proportion of them answered (Line 3) until more students answered "No" (Line 5). In Line 7, when students answered why, "You cannot immediately say it...", the teacher followed with "$\angle C=\angle ABC$" echoing the "it" in student response and initiated another question (Line

8). When students responded why in Line 10, "AB=AC", the teacher confirmed their answer by repeating it in Line 11.

This brief and quick exchange helps illustrate the face-to-face nature of explaining homework: the teacher held the key but instead of just telling students what went wrong and how to correct it, her goal was obviously to engage students and create an interaction maintained and enlivened by a flow of her questioning on student wrong step(s). This forces them to respond and recall from their solution steps and push them to think over again step by step and sentence by sentence. Therefore, the explaining was not able to be accomplished by the teacher alone—she needed the part of students' involvement, their being there bringing with them their attention and memory of how they performed the homework tasks explained,

Students' involvement in the explaining is also evident in terms of the evaluations given by the teacher to student responses in the interaction. In each evaluation (Line 11 & Line 17-18), where a topic or subtopic ends (Cazden, 1998:36), is an explanation generated. That is, her summing up of a topic led to the explanation she wanted to draw on. But notice that the two teacher evaluations occurred in different ways: the former was led up to by the IR patterns (AB=AC, then angle C equals to angle ABC. Included in this step there is a logical deductive) while the later was created by a teacher-initiated and teacher-responded format where the teacher seemed to be doing some sort of thinking or pushing for her thinking in her mind. Such a "monologue" gave rise to a different interpretation of the error—it is not a given condition, so cannot be treated as such. This little pattern will be further discussed in the coming subsection as I try to make sense of how the teacher continued to make sense of student thinking in the process of working out explanations to error situations.

In explaining geometry homework, the teacher-student interaction also took two other familiar forms: teacher initiated questioning and student choral responses and teacher-student responding together to a teacher-initiated questioning, such as in her explaining given to Class 4 on Nov. 19, 2002 (see the transcripts in full in Appendix 1, Samples of Marked Homework) when she explained the second and third assignment. Although these appear to be variant forms of interaction, they all conform to the IRE patterns in general. Compared with the interactions in explaining homework in the section on function, there are more IRE patterns in the geometry and more teacher explaining in the later. In terms of location, the section investigated here is at the end of geometry proof unit, so students had acquired familiarity of the content. The sentence by sentence structure of proof writing also seems to be easier to launch quick and short questioning and clear and brief responses. The lack of student response or less frequent responses was largely due to the fact that the teacher did not create initiations and she tended to provide longer explanations on her own. Given that the concept of function is a new topic, more teacher explanation was needed. Quite a number of assignments incur controversial answers that required the teacher to offer analysis while explaining them. The lack of response did not mean that they were not participating. When the teacher paused with a question inviting disagreements or discussion, a noisy discussion usually broke out all of a sudden, which shows that students were mentally engaged while the teacher did her explaining.

Developing explanations to the errors is achieved mainly through teacher-student collaborative interactions. The teacher engaged the students by initiating brief responses from them through questioning and orchestrated the details of explanation by making direct or indirect uses of student responses. A development thread is visible when she evaluated student responses and offered her expanded explanations. The interactions sometimes took place within the teacher herself when she responded to her own questions. As further discussed later, this teacher-initiation and teacher-response pattern would be seen as a kind of "inevitable improvisation" (Cazden, 1988, p. 43) in which the teacher pushed herself for different interpretations of the errors (problems) being explained.

Summarize and generalize. In developing the explanations, Tr. Wang led her students to open up different ways of viewing an error or multiple dimensions of the error, such as in the segment presented above. Most of her explanations did not end here; instead, in the way she introduced the problem context and the error, she would close up by summarizing the key ideas (such as she did from Line 20-22) and highlighting the important role that the concerned error would play for determining a successful proof writing (Line 23-24) or solution in function. She would always end by reminding students to make corrections and ask them to show her their corrections which required their understanding to make and it was this understanding that her error explaining tried to achieve.

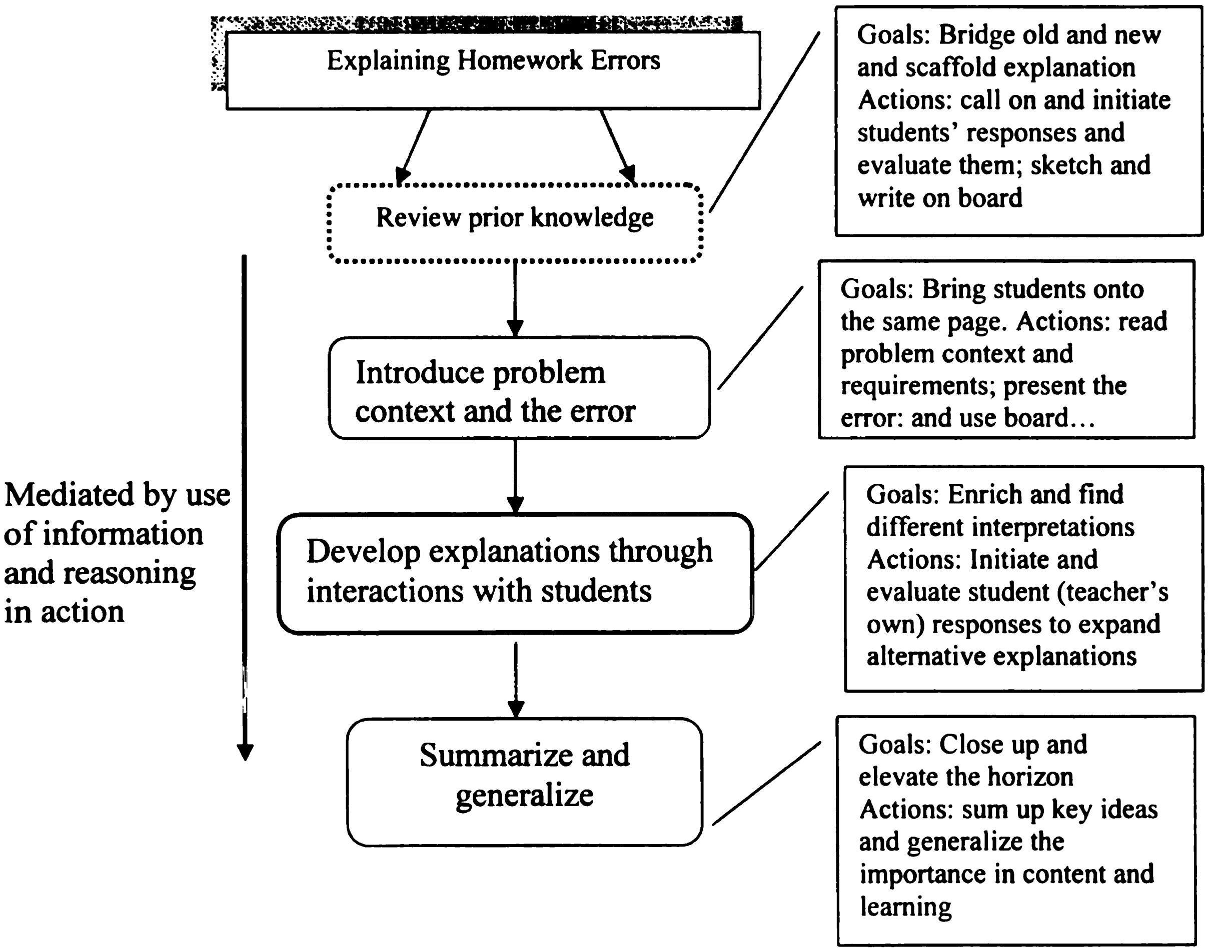
This final step of explaining is found at the end of all her explaining to geometry proof homework, which could always give students a sense of structure and completion—the nature of a proof writing and model for students how to achieve such structure and completion. In explaining homework on function, not all explanations ended with a summary. For instance, when she explained the controversial keys, her explanations would end when she finished explaining what were the controversies, what caused them and why. However, for those that did, the summary played a similar role. For instance, on 11-27-02, after she finished sharing three different approaches—the arithmetic, the algebraic, and the function approach she found students used in their homework to get the solution of the "gear plate" and "flying wheel" problem (See Table 5.3 for details), she summarized in the following way:

1 T: Right? First, you need to find the relationship between the
number of cogs and the number of rotations and then, write the
2 analytical expression according to this relationship. Good.
3 That's all for this problem of ours. When you come across a
4 problem in the future, *(you need to)* deal with it according to
5 the circumstances. We have learned so many methods:
6 arithmetic, learned at elementary school; equation, learned at
7 junior one; and now we've just learned function. We have so
8 many methods to solve a practical problem. But now, which
9 method do you think is the most advanced?
10 S: *(Several boy students)* Arithmetic! *(Others started laughing*
11 *and discussing)*
12 T: This is the most rudimentary one, right?
13 S: *(Students still discussing and laughing)*
14 T: Function is the highest level method *(among them).* Right? It is
15 able to deal with the changing relationships *(which she already*
16 *explained in detail before she summarized here.)*

The above summary to the given problem not only summarizes the major ideas already explained (Line 1-2) and briefly generalizes the approach—handling a future problem "according to circumstances" (Line 3-4), but also adds another new piece of knowledge—"which is most advanced approaches" (Line 8-9)—by calling on students (Line 10) to answer and then evaluating their answer (Line 12), giving the right one (Line 14) and telling students why function is the most advanced (Line 14-15). (In the phase of developing the explanations, she already talked in detail about what a function approach can do in situations where the other two approaches do not work, so here she ended by just summarizing it briefly.)

In summary, explaining homework errors generally consists of the above 3 (or 4) steps and each step achieves its certain goals through a number of generally conceived actions. The following diagram summarizes this activity structure.

Figure 5.1 Structure and Pedagogical Actions in Tr. Wang's Explaining Activity



Discussion

Good explanations shared some general characteristics which features the teacher's knowledge of the "routes to understanding that student had experienced" by "getting inside the head of the learner" (Beveridge and Rimmershaw, 1991, p. 289). These brief but well structured explanations given by Tr. Wang clearly demonstrate her use of the information and knowledge about student issues of learning she collected and saved in her working memory while marking student work. In the meantime, good explanations share cultural characteristics of what explaining is. Leading a review and also a formal lesson by numerous questioning and student choral responses was a cultural approach of mathematics teaching well documented by Gu and his colleagues (1999) in their observation of middle school mathematics classroom practices in Shanghai. While this approach appeared to highly engage students to follow the teacher's orchestrating, they felt it a barrier to more active student participation with the teacher as the leader of the scene most of the time. In the case of explaining homework errors in the classroom, given the brevity of time but rich and detailed information shared with a large room of 60 students, I argue that such questioning, repetition and choral responses enabled the teacher to reach most of the students in ways that drew their participation. With students' being able to recollect their solution or proof writing process they did the night before in doing their homework, such engagement was made possible to a great extent. The shared repertoire of homework was the bounding and boundary object for this homework activity.

Section Summary

In the above subsection, I have attempted to offer a general description of what an activity of error explaining looks like by depicting its internal structures. Pedagogically each of the explaining steps diagramed above also represents a teacher strategy in constructing the explanations: reviewing to connect with new content or through homework to new content; introducing the problem to help students to recall task performance; giving students explanations with analysis by engaging them to participate and navigating explaining through questioning; and summarizing key points and highlighting the significance of the problem or error in the content and its learning. These strategies were deployed by Tr. Wang to lead students towards structured and principled ways of knowing mathematics and make sense of errors by not only knowing what is wrong, how to correct them, but why it is wrong and important for learning.

By analysis, I refer to the approaches opposite to direct showing and telling of what and how of a mathematic algorithm, concept or procure; I refer to those that give students the analysis as to what and why behind an algorithm, concept or procedure. More specifically, I refer to those strategies used by Tr. Wang in transforming the errors into multilayered and multidimensional interpretations entailed in the error situations. Such strategies include, among others, adjusting her explanations to meet the learning needs of her two different classrooms; strategically repeating to reach all students; analyzing problem situations, comparing and contrasting problems and approaches (such as the one just mentioned), using questioning to push students to recall and think (such as in the IRE patterns of her discourse discussed in the above subsection), improvising and generalizing. All these strategies, in one way or another, push for what goes beneath the problem situation mathematically and in terms of learning. Although I have discussed briefly some of the strategies, such as questioning and improvising, I choose to elaborate them in a different light: how the teacher used them and the information to mediate opportunities to learn for students and herself, which is symbolized by the downward line and arrow in the diagram above.

Summary

This chapter has addressed three tasks. First, it shows the reader what errors that Tr. Wang chose to explain in teaching the two units on geometric proof and functions, nature of the errors, the mathematics and student learning of the mathematics laying beneath the errors. Review of literature on teaching and student learning of the concerned topics reveals that the mathematics and student learning entailed in the errors are both important in the structure of the content and have rich implications for learning it. Second, it reviews the process of how she identified and selected these errors to explain. This process is shown as one that she made sense of student learning problems informed by other concurrent homework activities, marking, tutoring student and conversations with colleagues. Third, it unfolds that process in which the errors were explained in structured and principled ways and expanded into concrete elements constitutive of mathematics and learning. Together, in accomplishing these tasks, this chapter gives a thorough description of the subject matter and pedagogy of Tr. Wang's explaining of errors in homework.

Chapter Six: Tutoring Individual Students on Homework Errors in Teaching Geometry Proof and Function

Introduction

Tutoring students during break and after lunch on November19, 2002

When the first period ended, a boy from Class 4 came in with his textbook in hand to seek help with a previous day's homework problem, Exercise 4 in the textbook (p. 79). His submitted homework was included in the pile that the math monitor just delivered to Tr. Wang. Amidst the huge racket outside, she guided the boy to see the need of drawing an auxiliary line segment to make use of the perpendicular bisector theorem. She then probed by asking what he could get by knowing that one segment of the longer side is twice the other segment. She pointed at the figure, paused, and waited for the boy to think. While the boy was answering her questions, he realized that he was able to work out the rest of the problem on his own, so he left happily. After tutoring the boy, she also chose to explain it as soon as she taught the third period in Class 4.

After lunch, at about 12:30, she resumed marking homework. The math monitor came in and asked to make up one thing that he forgot to do this morning: to record the names of those who did not submit their homework since he had to write them down on a corner of the blackboard as usual for the class director teacher to see. She asked him to call two of his classmates who did not do their homework very well. Very soon, the two boys arrived. After brief tutoring, they were asked to correct their mistakes on the two empty teachers' seats (since the class director teachers were at their classrooms monitoring lunch). A third boy came in with his Volume B in hand. He said he was sick the other day and showed her the homework he made up for that day. Finding that there was one problem wrong, she explained it to him and he took another seat to complete the correction. One of the first two boys, tall and strong, who was often called to see Tr. Wang at her office because of his careless homework habit, approached her several times to ask for help. "Liu Long," she said to him, frustrated, "could you use pencil and ruler to draw the figures? Look at this mess! Could you make it out?" The boy went back to the seat to draw more clearly.

In contrast to explaining homework errors to all students, which happened in the orderly and typically quiet formal classroom settings, tutoring individual students on their homework errors occurred in the hustle and bustle of the noisy 10-minute breaks in between the lesson periods or after lunch hours in the teachers' office. It was a time when students were all suddenly released from classrooms, running around the open building, making lots of noise. Teachers returned to the office from their teaching, often with students brought from classroom or sent by other teachers to the office to discipline. The quiet office turned into a busy sight.

At such times, it was a common scene that Tr. Wang (often her desk mate colleague as well) was surrounded by students who she summoned to her desk to get quick teacher feedback or make prompt corrections on their homework. Students also came voluntarily to seek help with homework or extracurricular practice exercises they chose to do on their own. For both "compulsory" and "voluntary" tutoring, she always worked with them one on one to give differentiated individual assistance. This kind of tutorial practice on homework is commonly called "coaching and assisting and guiding homework," or in Chinese, *fudao zuoye.* Because it is given one on one, it is also called "face to face marking (homework)" *(mian pi zuoye).*

Errors and problematic student work were singled out for more pedagogical attention in explaining and tutoring while Tr. Wang was marking homework. "Students all made errors but their errors were all different!" remarked Tr. Wang broodingly one morning when she saw me entering the office. The tutorial assistance conducted on a one-on-one basis (even though students often came in a group) allowed her to address the idiosyncratic nature of the remaining individual problems case by case. Such short and quick tutorial sessions enabled the teacher to probe and diagnose student errors in a more focused manner.

In some ways, I would argue, tutoring and explaining run parallel to each other like two rail tracks. One helps address typical and important learning problems for whole classrooms of students, while the other secured her attention to care for individual students' errors. The two activities worked together to create possibilities for her assistance to reach students of different levels. Her colleague's metaphor compared the intensity of and their own tenacity to holding themselves accountable for the homework errors to a "battle field" scene, in which they fought with errors and other problematic issues as if they were "eradicating the bandits—wherever they are hiding, we track them down and wipe them out" (observation, 11-18-02).

Chapter Overview

This chapter attempts to offer a rich description of Tr. Wang's tutoring activities in four sections. The first section introduces the activity through a brief vignette of Tr. Wang's tutoring activities on Nov. 19, 2002. It provides a list of the research questions, data and data analysis conducted to answer the questions. In the second section, I outline the settings of the tutoring activities – the who, what, when, where and how of the activities. In the third section, I examine how the object of tutoring (the individual students' homework errors and needs for assistance with homework) is transformed and the characteristics. In the fourth section, I use examples of tutoring dialogues to illustrate the dynamics of Tr. Wang's tutoring discourse as she diagnosed student learning problems to offer needed guidance. Finally, the summary in the last section brings the chapter together in its examination of Tr. Wang's tutorial assistance to her students.

Research Questions

Drawing on multiple sources of data, as given below, this chapter provides a detailed description of Tr. Wang's tutoring activity. To form a complete picture of this activity, I develop the research questions on three levels: the general settings of the activities, which characterize what these activities look like from my observation; the object, or the individual students' errors that the teacher identified and students' self-identified problems and questions; the construction of the object, which refers to the object and processes through which these errors and needs were made available for the teacher to address; and the transforming of the object, which includes the pedagogical actions that Tr. Wang took to diagnose the causes of the errors and offered advice to do corrections and provide assistance that addressed students' needs .

On the activity level of settings:

- What are the general characteristics of the tutoring activities occurred during my observation?

On the construction level:

- Who were the students tutored?
- What errors were they tutored on and what homework needs did she assist with?
- How did the teacher identify students to be tutored?
- How did the teacher get students to tutor?

On the transformation level:

- How did Tr. Wang initiate and lead the tutoring?
- In what ways did students participate in the tutoring?
- What patterns characterize the tutorial dialogues in terms of the teacher's goals and actions?
- What opportunities to learn were made possible for students and the teacher through tutoring?

Responses to these questions offer a basic view of the activity settings of tutoring, present the goals and actions of the teacher in identifying and getting students to come to the tutoring and describe how the teacher tutored in ways to achieve her goals.

Data and Data Analysis

Data. The data came from my observations and interviews across the days during which Tr. Wang taught the two sections on *Converse Propositions and Theorems* and

Direct and Inverse Proportion Functions. To render a more complete view of the homework activities in the days that I observed, I choose to examine the same teaching days from which data on the activities of explaining were drawn. This includes the days from November 13 to 28 with a focus on observations during November 15-27, 2002.

I draw on analysis of three main types of data to respond to the research questions. Data for the object level come from student work and the observation record of tutoring events. Data for identifying problems and students to tutor came from observation field notes and the tape recorded observations of Tr. Wang marking homework and tutoring students during the breaks and after lunch in her grade-level teachers' office. Field notes mainly record the time and sequence of events, background information of the marked homework including brief teacher utterances, which students (their learning level) and what problem were tutored on. The tape recorder was used to capture the details of the dynamics of tutoring, such as teacher utterances during marking and during her identifying students to tutor as well as the informal interviews with the teacher.

Data analysis. Data analysis was carried out in a number of ways. I looked at what constitutes tutoring activities in Tr. Wang's teaching of the two sections to look for patterns that characterized the student participants and their participation in tutoring. I did document analysis on student work being tutored and the learning levels of the students to make sense of the errors. I also coded the tutorial dialogues between the teacher and the students to identify patterns.

Settings of Tutoring: Diversified Learners and Their Needs

In tutoring, there were students that the teacher identified and summoned for tutoring who are referred to here as compulsory tutoring students and there were also those who came to the teacher for help with their homework who are referred to as voluntary tutoring students. The voluntary group was made up of students who were generally considered above average in mathematics learning. Within the former group of compulsory tutoring students, however, there were a wide range of students in terms of their mathematics learning levels. Their errors and learning levels were therefore varied greatly and so was the teacher's ways of tutoring. Therefore, these contextual issues about tutoring will reveal the "interlocked" activity settings of tutoring, the "small recurrent dramas of everyday life" of Tr. Wang and her students "played on the stages of" of the workplace and schools (Tharp & Galimore, 1988, p.72). In this section, I will examine carefully who these students were and what problems and issues that the teacher assisted them with to address through tutoring.

Identifying and Getting Students to Come for Tutoring ---Small Recurrent Dramas of Daily Work Life

The following table lists the settings of tutoring across the days of Tr. Wang teaching the two sections on *Converse Propositions and Theorems* and *Direct and Inverse Proportion Functions* with a focus on November 15-27. The information is organized in Tharp & Galimore's (1988)[1] terms of the five Ws: who (participants—Tr. Wang and her students tutored), when and where (time and place where tutoring was conducted, including its duration), how (types of tutoring students received including

---

[1] Tharp, R.G. & Gallimore, R. (1988). *Rousing minds to life: Teaching , learning, and schooling in social context.* Cambridge University Press.

how they came for tutoring and the general features of tutoring) and what (the errors or problematic issues that students were tutored on). Again, my effort in trying to sort through the messy and fluid web of a teacher's busy workday produces a dense table that demands careful reading into the information provided. This table provides a basis for answering a series of questions concerning what is tutoring and the goals or purposes of tutoring.

Table 6.1 Settings of Tutoring Activity in Tr. Wang's Daily Practice

Date/ Phase of Day	**Who & When** * *Where –referring to the teachers' office except for otherwise noted.*	**How & What** (Types of tutoring /Content & amount tutored)	**For how long?**	**Person-times (p-t)/ Total time**
Friday,11/15/02				
AM	**A girl/Above-average Break b/w 4th -5th period (11:10-11:20)**	Spotted by the teacher in the office, she was called to teacher's desk for tutoring/(Ex. 2, 1 & 4/Vol. A)	5 minutes	1 person-time 5 minutes *Half day observation
Tuesday, 11/19/02				
AM	A boy/Above average Break b/t 1st-2nd period (8:45-50)	Sought help with a question about Ex.4/Textbook/p. 79	3 minutes	15 person-times (Minutes) 54 minutes
After lunch	2 boys/Below average (12:30-50)	Called to tutor on homework on Vol. B, and had them correct in the office	20 minutes	
	A boy/above average (12:40-50)	Sought help with homework for his absence the day before	(10 minutes)	
PM	8 students from C2[2] (3 girls and 5 boys)/ Average or below average in math Break b/w (2:50-3:06)	Called to office to tutor on errors in Vol. A	16 minutes	
	Two boys, C4/Above average Break b/w 8th-9th period (3:45-48)	Seeking teacher opinions about their disagreements on a homework assignment	2minutes	
	One of the above two boys (3: 49-3:52)	Tr. W remembered one of the boy's homework errors and	3 minutes	

[2] On 11-19-2002, she could not use the after-lunch hour for Class 2 when she learned the Chinese teacher was there to dictate new words. So she called 8 students from Class 2 for individual tutoring when she found them during their self-study hour in the afternoon.

		asked him to stay for immediate tutoring		
Wednesday, 11/20/02				
AM	Two boys from Class 4/Below average Break b/t 3rd and 4th period (10:30-40 AM)	Called to the office by the math monitor for tutoring	5 minutes	3 person-times 12 minutes
After lunch	*Explained an assignment to Class 2 and stayed longer walking in b/w seat columns to answer questions and mark corrections			
PM	A girl/Above average 5:00-5:10 PM, after school	Came to ask 2 homework questions (T & CA[3]/p.83, Test 1, Ex. 3 and...)	7 minutes	
	* In PM, Tr. W was invited to observe a young male 9th grade math teacher's trial lesson for the following day's Competition and had discussions about the lesson.			
Thursday, 11/21/02				
	* No tutoring. She adjusted her three periods in the morning to observe three lessons in the school's Yearly New Star Young Teacher Teaching Competition.[4].			
Friday, 11/22/02				
AM				7 person-times 22 minutes
After lunch	Two girls/ Strong in math After lunch: 12:45-12:50 PM	Sought help with two problems from an extra resource book	5 minutes	
	A girl/Average After lunch: 12:50-53 PM	Spotted in the office and called for tutoring (T&CA/homework)	3 minutes	
	Two girls/Above average After Lunch 12:53-58 PM	Came to ask two problems/Homework on textbook	5 minutes	
PM	A boy/Below average (Break b/t 6th -7th period, 2:00-2:05 PM)	Called to office for tutoring/ T&CA/homework	5 minutes	
	A girl/Below average 2:05-2:13	Called to office for tutoring/ T&CA/homework	4 minutes	
Monday, 11/25/02				
AM	A boy/Below average Break b/t 2nd-3rd period (9:30-9:35 AM)	Spotted in the office and called for tutoring/ask redo an assignment	5 minutes	13 person-times 40 minutes
	4 girls/average Break b/t 4th and 5th period (11:10-11:20 AM)	Called to office for tutoring/being reminded of errors and corrections	9 minutes	
After lunch (12:25-12:55)	Two girls/Strong in math (12:25-12:29)	Sought help with a problem on function in an extra resource book	4 minutes	
	Boy 1/Below average (12:29-12:37)	Called to office for tutoring	4 m (8 m in all)	

[3] Assessments and Commentary Analysis *(Ceshi yu Pinxi)*

[4] All math teachers spent a special lunch gathering in the school dining room with the District Teaching Research Expert hearing him debrief his observation feedback and his views about the new reform and his recent 3-month Canadian visit and observations.

	Boy 2/above average	Called to office for tutoring on function homework	3 m	
	Boy 3/Above average	Called to office for tutoring on geometry and function homework	5 minutes	
	Boy 4/Below average	Called to office for tutoring on geometry and function homework	4 minutes	
	Same two Girls/Strong in math	Sought help with another function problem in the same resource book	3 minutes	
PM	A boy/Above average Break b/w 6th-7th period (2:02-05)	Sought help with a geometry homework	3 minutes	
	* Tr. Wang finished marking all homework after the 7th period and went to a movie for all teachers of the school.			
Tuesday, 11/26/02				
AM	A girl/average Break b/w 2nd-3rd period (9:32-9:35)	Called to office for tutoring/Geometry proof homework Ex. 2	3 minutes	7 person-times 21 minutes
	A boy/Below average Break b/w 2nd-3rd period (9:36-9:40)	Called to office for tutoring/Geometry proof homework Ex. 3	4 minutes	
After lunch	Explained to Class 4 one function and 2 geometry exercises for 10 minutes (12:45-55)			
	Class 2: Answering questions and face-face marking of corrections and getting students ready for drawing hyperbola for next day's lesson,15 minutes (12:56-1:10)			
PM	3 students, Break b/t 6th-7th period (2:00-2:10)	Sought advice on how to draw hyperbola	3 minutes	
	3 students/above average After the test, break b/w 8th-9th period, 3:42-47	Followed the teacher to inquire about uncertain places in the test		*Test, 7th-8th period (2:10-3:40)
	A girl/average After test, 3:47-3:50	Called to tutor on a geometry proof assignment	3 minutes	
	A boy/Above average After test, 3:51-4:02	Called to tutor on a geometry proof assignment	8 (lasted 11m)	
Wednesday, 11/27/02				
AM	A girl/Average, C4 Break b/w 2nd-3rd period (9:32-37)	Sought help with a wrong corrections on a geometry assignment and another function assignment	5 minutes	4 person-times 12 minutes
	A girl/Strong in math, C4 Break b/w 3rd-4th period (10:32-35)	Sough help with a function problem from an extra resource book	3 minutes	
After lunch(12:30-47)	The two girls/Strong in math	Came to see their test scores and watched grading		
	Two boys/strong	Asked teacher to mark their test sheets and watched grading		
	Two boys/average (12:43-12:47)	Called to office to show their error corrections to teacher and received tutoring	4 minutes	
PM	* Math Teaching Research Group Meeting * Marking test papers			

Total Person-times/Time	50 person-times 165 minutes

A busy routine practice in the teachers' office. As shown in the above table, 32 out of the 50 person-times of tutoring took place in the brief 10-minute breaks across the day, accounting for 66 percent of the total. On average, every person-time would take 3.3 minutes; but on some occasions, particularly after lunch, a few students would be required to make corrections right after tutoring at one of the empty teacher's desks, which could take more than 10 minutes. The pressure for time and the number of students to tutor added to the busy and noisy atmosphere of the routine tutoring practice taking place in the small busy teachers' office in between the lesson periods.

The focus group. From the column in the table listed as "How and What", there were two different groups coming for tutoring, the larger group (62%), which I list under "compulsory" group, were summoned by the teacher to her desk for tutoring on homework while the smaller group (38%), listed as "voluntary" came for tutoring without being asked, of which a smaller percentage (14%) asked for help with questions in addition to the teacher-assigned homework. These statistics are given in the table below.

Table 6.2 How Students Came for Tutoring?

	How students came for tutoring?	Number of person-times	As % of the total 50 person-times
Compulsory	**Spotted in or called to the office to be tutored on homework**	**31**	**62%**
Voluntary	Came voluntarily for help with homework	12	24%
	Came voluntarily for help with questions extra to homework	7	14%

There are two major reasons why the study of tutorial dialogues should mainly focus on this teacher identified "compulsory" group. First, this is determined by the purpose of the study. Aimed at understanding Tr. Wang's homework-related practice, her knowledge use and decision making in the process of her daily practice, this group represents those students she sent for tutoring based on her knowledge and information acquired from student homework directly. Second, the students in the voluntary group came to the teacher with their own self-identified questions which were different in nature from errors already committed and identified by the teacher and therefore the tutoring offered by the teacher would tend to differ in content and strategies as well. Therefore, the compulsory group represents this focal population. The other group will be mentioned to show the diversity of students being tutored. When focusing on this group, the central question of concern is how the teacher made decisions to call these students for tutoring and what kind of processes is involved in getting them to her desk side. Behind such processes was a dynamic information or communication system at work. It is an important part of the tutoring activity that needs to be introduced.

How students were brought in for tutoring—An information circulatory network. The teachers' office is a dynamic workplace where teachers mark student work, talk with students and interact with colleagues (Ma, 1999). Seating the teachers responsible for teaching the same grade level of students together to share the same office space is to facilitate information exchange about the common group of students. An earlier study on teachers' lives in their offices in Shanghai[5] found that teachers studied compared their offices as a "beehive" to highlight the information exchange about students (pollen) and processing of shared information and knowledge for helping students learn (processing pollen to make honey). For Tr. Wang, gathering information from and dealing with

[5] Study entitled, "Teachers' Offices in Shanghai as a Site for Teacher Learning in School Settings" sponsored by Spencer Research Training Grant (2000-2002), which led to the present study of homework activities.

homework in the office could be compared to collecting pollen as well. Similar to bees sharing information about direction and location of pollen, colleagues as well as students supported her with her effort to assist student learning in three major dimensions.

First of all, her office colleagues, especially the *banzhuren*, the class director teacher, kept her informed about other aspects of the students whose homework was unsatisfactory and the possible causes which were unknown to her as their math teacher. For instance, in the opening vignette on 11-19-02, when she uttered a student's name with frustration when she found that his homework repeatedly contained more errors than others even in his corrections, his *banzhuren* teacher sitting behind her added, "This student is not doing well academically; but he likes sports and science and is very strong at hands-on activities. One of his science model has recently won the school competition!" Days later, on 11-22-02, when she complained about this same student, "Lin," whose homework was getting worse for several days in a row, this *banzhuren* teacher shared that the student's father had to take frequent business trips out of Shanghai and the child would not listen to his mother while the father was not home. Such bits and pieces of information acquired about her students informally would help clear up her "blind spots" and got more rounded views about her "headache" students.

Second, as mentioned in Chapter 2, the math monitors and the *banzhuren* teachers supported her to ensure all students submit their homework and deliver them to her desk so that her central attention could be focused on marking and dealing with completed homework. The monitors helped collect student work every morning and kept a record of who failed to submit homework on one side of the blackboard so that the *banzhuren* teacher could follow up with them by finding out why they did not turn it in. Third, during break or after lunch, the teacher asked the math monitor or other students who happened to be in the office to help call those students from classroom for tutoring *(qu jiao ren)* because "their homework has problems" *(zuoye you wenti)*. Often, the students who just finished tutoring were sent to get more students to come for tutoring. In this way, students formed a quick and efficient chain of support to get the students requested by the teacher to her desk side.

As a community of practice, colleagues and students, as the members of the community, not only shared knowledge, information and responsibilities but also supported each other in circulating and accomplishing them. It was such a network of relationships served as the infrastructure that enabled Tr. Wang's quick tutorials to be conducted promptly.

Meeting of different levels of learners and diversified needs.

As Table 6.1 and 6.2 indicate, tutored students range from different academic backgrounds and they came for tutoring as two distinctive groups, the voluntary and the compulsory. To help the reader to get a fuller picture of the diversified needs, I would like to look at the composition of both groups and the two classrooms before narrowing down to focus on the needs of the focus group. For convenience of observation and analysis, I divide the tutored students roughly into four levels: strong, above average, average, and below average[6]. Different levels of learners brought with them different problems and issues with their homework. Those seeking tutoring voluntarily were either

[6] According to students' academic performance in the mid-term mathematics exam that they took a week before I started my observation, the average was around 70-75 on a 100 scale and the above average was anywhere from 75 to below 90 and the below average would be those that below 70, sometimes at the passing level (60) and more often with a failing grade of lower than 60. The strong ones usually achieved 90-100. And there were also several mathematically very weak students in each class that achieved bottom scores in the range of below 20.

mathematically strong or above average students. They often came with questions that require more analyzing and doing of the mathematics. The two strong girl students came regularly (during teaching of functions) to seek help with questions from extra resource books they purchased outside of school containing additional practice problems related to the course content. According to Tr. Wang, since the reform movement of "reducing students' academic load" *(jianqin xueye fudan)* started in 1999, schools and teachers were not allowed to purchase or recommend students to purchase extracurricular practice resource books widely available in the market. But the assignments from schools were not enough for those strong students, so they chose, often at the request of their parents, to do more similar assignments from additional resource books. In the case of these two girls bringing extracurricular materials, she would often work together with them on those "new" problems on separate pieces of paper that she collected from used student workbooks. Here she was more like an experienced peer, working out the problem step by step, drawing and showing the given conditions on the graphs, pushing them to analyze the situation before explaining the steps to them.

Those who come without being asked also sought different teacher assistance with homework, including making up missed lessons and assignments and seeking teacher opinion when they had disagreements (11-19-02). Those who raised questions about homework often mirrored back to Tr. Wang what students still did not understand well and what kind of support she still needed to provide. For instance, after lunch on 11-22-02, two girls (above average) came for help with the homework on the textbook that asks students to judge whether the given situations constitute direct proportion. Some of these exercises were completed during the previous day's class led by the teacher. She provided them with thorough explanation and learned that students still had problems understanding the basic concept of direct proportion, the basis for understand direct proportion function. On several occasions, after tutoring such students, she might have thought that other students could have the same difficulty, so she included the problem she had tutored to explain during a lesson to the whole class (such as on 11-19-02 after tutoring a boy who came to ask a question after the first period in the morning and on 11-25-02 after a boy asked for help with a geometry proof problem in the afternoon.)

Her two classrooms were differentiated by the proportion of mathematically poor achieving students in each class. Class 2 has a larger number of below average learners (more than one-third but less than half), which she often said gave her a lot of headaches in marking their homework. Therefore, to make good use of her time, she often started by marking the homework of her stronger class, Class 4, because it was much easier to plough through their work and then she could call students with problems to tutor immediately. With most of the students coming for voluntary tutoring being also from Class 4, 80 percent of the total person-times of tutoring were from Class 4.

For Class 2, instead of calling large groups of students to tutor during breaks, she approached them mainly from two ways: first, she often used the after-lunch hour for explaining and tutoring to Class 2 and stayed much longer to answer student questions and monitor and mark their corrections by walking in between the rows in their room. Second, marking and tutoring for Class 4 would often remind her of possible difficulties for Class 2, so she often went to offer prompt explaining or tutoring during their after lunch or self-study hours in the middle of marking or immediately after tutoring a student. But once in a while when she could not find time to go to Class 2 for tutoring, she would call those, usually in a large group, who needed help to her office for tutoring. For instance, eight students were called to her office for tutoring from Class 2 in the afternoon of 11-19-02. In this way, she was able to manage to attend to homework problems promptly in both classes in her busy days. Obviously, Tr. Wang gave more attention to those weak students in the stronger class given that they had greater "malleability." On the other hand, because students were not strictly tracked into two streams, the strong and the weaker, it was unlikely for her to give similar one-on-one tutorial to all weak students.

However, the "compulsory" group were not just students who were poor achieving or below average and were generally regarded as needy of help. According to my detailed record of all students who received tutoring, among this group, there were often average and above average students as well. The composition of this group in terms of levels of learning is given in the table below:

Table 6.3 Composition of the Compulsory Group by Academic Level and Gender

Above Average / % of Total person-times (31)	Average / % of Total person-times (31)	Below Average / % of Total person-times (31)	Girls/% of Total person-times (31)	How did they come for tutoring?
6 (20)	14 (45)	11 (35)	12 (40)	4 (spotted in the office): 2 female-above average

Of those called by the teacher for homework tutoring, 80 percent were average or below average students. This indicates that Tr. Wang's attention was concentrated on the students who required more effort to monitor and improve or whom she believed were more likely to go downhill if her attention lapsed. Usually the below average learners were a small number of boys who were located in between a passing and failing grade and could improve if they showed more care or effort with homework. Her attention to this small group in tutoring was remarkable. Among them was one boy, Little Dragon (to be discussed more in the next section), who was called to come to the office or spotted in office for tutoring almost on a daily basis (11-19, 11-20, 11-25 and 11-26) and twice on 11-25. He was often, among a few other boys, kept for redos or correction after lunch, during which Tr. Wang could answer their questions while she was marking homework. Most of the time, Little Dragon was found with sloppy work in his handwriting, used ball pen to do the drawing or chart, and very incomplete format. One redo sometimes took several rounds to make it accepted by the teacher.

Most of the above average or average students were expected to make corrections on their own without being tutored. When they received tutoring, it was mainly because their homework was exceptionally unsatisfactory on a certain day that caught the teacher's attention. She always found girls generally produced neat and more carefully done homework than boys but she found that they did not do as well as the boys on exams. One main reason, according to her observation, was that girls liked to do homework in groups rather than independently. Girls were also too shy to ask questions if they did not understand. Most of these girls were either called to office or spotted in the office for tutoring. For instance, of the 4 students who were spotted in the office and then were called to her desk for tutoring, two were girls and above average students in math.

Students who were identified for tutoring not only included a wide range of learning ability but also were often tutored on different problem areas or issues of learning. In general, below average students and some of the average students were considered by the teacher as having "poor learning or homework habits". She believed that it was their poor habit rather than "poor learning ability" that dragged them behind.

She closely kept track of them by urging them to draw accurate pictures and graphs, write neatly and pay attention to whether they followed the required format. At the same time, she alerted them when they ignored correcting the errors. For the above average learners and some of the average learners, her attention was focused on helping them find out what was wrong, why and how to correct the wrong. In summary, her approaches to and responses in tutoring students varied according to their different academic background and also the nature of the problems they faced. However, her goal to help them to improve and achieve the desired understanding was always the same. This directed her effort in helping students figure out what and why a step or solution was wrong and offering assistance as to how to correct them across different levels of learners.

Transformation of the Object of Tutoring – Reaching Different Learners and Diversified Needs: An Examination of Tutorial Dialogues

In this section, I take the reader into the dynamic tutoring dialogues taking place in the teachers' office. Compared with explaining homework to the whole class in the form of classroom discourse, tutoring dialogues between the teacher and the student are an extended form of instructional discourse taking place outside the classroom. Both forms of discourse were targeted to student homework problems and errors. Whether her goal was to bring the desired understanding accessible to all students or to individual students was also informed by her knowledge of individual students performance and learning levels.

Tutoring, like explaining, is a routinely performed activity in Tr. Wang's daily practice. As an experienced teacher, she displayed "an organized set of actions that are performed fluently as routines" (Leinhardt & Greeno, 1991; Littman, 1991) in her assisting individual learners to know and understand their errors. By examining the dynamic teacher-student interactions in the tutoring dialogues, I aim to answer two interrelated questions: what actions and strategies did the teacher take to fulfill her tutoring goals and what did her tutoring make possible for helping her and her students learn?

Capturing the structural features of an "unstructured" activity.

In activity theory, constraints and internal tensions are often regarded as the driving force that propels an activity to move forward. As discussed in Chapter Five, explaining often appeared as a formal segment of a lesson and within the confines of a formal lesson, it has a very structured discourse that consists of four distinctive steps or phases (reviewing, introducing, explaining, and summarizing) with clear goals for each step and well performed actions to reach them. In comparison, while free from constraints of a formal lesson, tutoring was situated in the rushed 10-minute breaks and after lunch hours in the teacher's office. In this busy atmosphere, Tr. Wang managed to get those students sorted out for tutoring to come to her desk side for quick tutorials. Meanwhile, the content and patterns of the tutoring dialogues were dependent on who the learners were and what problems they faced. Since she aimed at finding out why a student made certain errors in order to provide assistance that met the needs of this particular learner, the discourse often influenced teacher-initiated diagnosis of what might be the causes of the wrong step or solution, which was accompanied by her assistance with repair actions. The word "repair" means not merely correcting but also offering explaining to causes of a wrong answer and suggesting measures to correct it. It is a word often used in the research literature in computer assisted learning and tutoring. (Goodyear, 1991)

A discourse aimed to diagnose and repair the error situations.

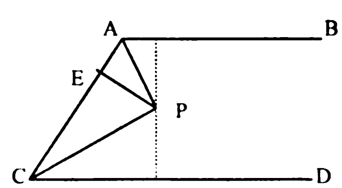
Most of the tutoring dialogues were characterized by teacher-led questioning to initiate student responses. Similar to explaining homework to whole class, the teacher navigated the process through questioning and then built on student responses; but different from explaining, student responses were not just used as a way to maintain their attention as the teacher moved on, they were used for the teacher to base her judgment or diagnosis, a precondition for offering needed repair. In this sense, it is very similar to a physician's diagnosis at a clinic, which often consists of inquiring into the symptoms in detail to help arrive at a decision about what goes wrong with the patient. Since "correction is best studied within the sequential organization of the interaction themselves (Fox, 1991:153)", in the following, I choose to illustrate a few major patterns of the teacher's diagnostic and repairing actions and strategies. In initiating and maintaining the tutoring interaction, her assistance helped the students find and understand error situations and know how to make corrections.

Tutoring interactions led by questions. One of the major patterns of Tr. Wang's tutoring actions is that she led both the diagnostic and repair interactions with the students by questions. Below, I examine such interactions under the key questions she asked of the boy student in a 3-minute tutoring session. For convenience in presenting the analysis, after a brief overview of the background, I provide the problem and drawing that the teacher and student worked on as well as a full transcript of the tutoring.

On Wednesday, November 20, 2002, Tr. Wang finished teaching her first period in Class 4. In the first ten minutes, she explained two homework assignments (see Chapter Five for more details), one of which she explained was the first assignment that requires students to fill the data of distances from a provided drawing. The assignment provides one blank space but it says the distances should "respectively be ____." Many students only filled in one distance, 8, so she reminded them of this cue word that requires two pieces of data, that is, two 8s, and asked them to be cautious and careful in reading the meaning of a given problem. After teaching, she continued marking homework of Class 4. When the third period ended, she finished marking most of the workbooks for this class and asked a student she saw in the office to send for two boys from Class 4, including Little Dragon. After quickly reminding the first boy of his careless writing and asked him to make corrections at an empty desk, she turned to Little Dragon, who had produced an unexpected answer to the first assignment: 16.

Little Dragon, as mentioned earlier in this chapter, was a one of those who were on the verge of getting a failing grade because of what Tr. Wang described as careless work habits and not making enough effort in studying. Because he was called to the office for tutoring nearly every day, he was becoming a little confrontational and the teacher was becoming impatient with him. The following transcript of their dialogue indicates such an atmosphere to a certain extent.

Problem 1: As shown in the figure below, AB//CD, AP and CP bisects ∠BAC and ∠DCA; if the altitude of ΔPAC, PE = 8 cm, then the distance from point P to AB and CD are respectively ________.



(Note: the dotted segments are added auxiliaries drawn by students.)

1 T: From where do you get 16? *(She questioned frustratingly.)*
2 Boy: *(He said something very indistinctively)*
3 T: The distance from this point (P) to here, how did you get it?
4 S: 8
5 T: How did you get it? *(A little impatient.)*
6 S: I got the distance of AP and CP. *(He said in a matter-of-fact way.)*
7 T: Can you get AP?! The distance from P to which segment?
8 S: To AB.
9 T: To AB. Which segment is it?
10 S: This segment *(pointing on the drawing on the page of his workbook)*.
11 T: Eh (yes), draw a perpendicular segment, this segment! What are AP
12 and CP? What is distance from a point to a line segment?
13 S: A perpendicular line from the point to the line.
14 T: The length of the perpendicular segment from this point to this straight
15 line. Pass this point draw a perpendicular line, and the length of the
16 perpendicular line. *(She pointed at the drawing while explaining)* How
17 can it be CP? The distance from a point to a segment, you cannot call it
distance by casually connecting it to a line. *(She said with a rising tone to suggest questioning.)*

---

18 You said you got AP and then you got 8? How did you get 8? It says
19 this (EP) is 8, how can AP be 8? This is a right triangle (AEP), and AP
20 is hypotenuse, how can AP be 8?
21 S: This is 8, this is half of that. *(Pointing at the drawing)*
22 T: Which is which one's half?
23 S: This (EP) is half of this (AP).
24 T: Why is this half of this *(Pointing at the same drawing)?*
25 S: It is a right triangle.
26 T: Because it is right triangle, the hypotenuse is twice of a right side?
27 S: And also, there… *(He wanted to continue but interrupted by the teacher.)*
28 T: Is there a 30 degree angle? *(She asked importantly.)*
29 S: Oh, no. There isn't. *(Suddenly realizing there is not such an angle, his voice became more hushed compared to when he began the dialogue..)*
30 T: If there is no 30 degree, how can there be a relation of double? Then,
31 how should you do it? *(In a seriously questioning tone.)*
32 S: *(No answer from student)*.

---

33 T: This is bisector of an angle, and this is perpendicular segment, this
34 means that this point is on the bisector, right? *(Pointing at the drawing)*
35 S: Yes, it is.
36 T: This is perpendicular segment and the distance from P to AC is 8, then
37 how about its distance to AB?
38 S: 8.
39 T: Why?

40	S:	The distance from this point to **the ends** are equal.
41	T:	To the **two sides** of the angle. The distances from any point on the
42		bisector to both sides of the angle are equal. This is bisector of this
43		angle and this (P) is a point on the bisector. What special character does
44		this point have? Distance to the two sides of the angle? Then, which are
45		the two sides of this angle?
46	S:	AC and AB.
47	T:	Then, what relationship is there between the distance to AB and that to
48		AC?
49	S:	Equal.
50	T:	Equal. Then, what is the distance to AC?
51	S:	8.
52	T:	Then, the distance to AB?
53	S:	Also 8.
54	T:	OK. Here CP bisects this angle, then which two sides are sides of this
55		angle?
56	S:	AC and CD.
57	T:	AC, CD. (Point P's) distance to both sides are equal. This one is 8, and
58		the other?
59	S:	8.

The tutoring dialogue consists of three distinctive segments. The first segment (Line 1-17) starts with the teacher directly questioning Little Dragon (LD) on how he got the wrong answer. It ended after she did a little repair and went back to her original question. Below I offer a short interpretation of what was going on in the interactions in each of the three segments.

Brief as it is, the first segment is quite a dense chain of questioning and answering to interpret. When Tr. Wang did not quite get what LD answered (Line 2), she pointed at the drawing to show the distance is from P to AB (Line 3), and LD's response "8" was correct (Line 4) (which might be as a result of the teacher's explaining the assignment in the first period). Without confirming that this answer was correct, she pushed again with the same question (Line 5). This time, LD's response provided a clue to her—he confused the two given angle bisectors to be the distances to AB and CD. In a surprised tone, she asked him again to make sure that they were both referring to the same distance, "The distance from P to which segment?" (Line 7) When LD answered it is to AB, she asked him to point it her on the drawing (Line 9) to confirm what he said. He pointed at the right place. Sounding confused at LD's contradictory responses, she asked him for the definition of distance from a point to a line segment (Line 12) and he answered the definition correctly. At this point, she managed to do a little repair (Line 14-17) by pointing out how this distance is drawn and led to where so as to show him that the distance is not CP.

In this 17-line segment, the teacher asked 19 questions. Although the student responded with no more than one sentence each time, they offered the teacher clues to his routes of thinking. Her questions tended to repeat themselves, which shows her effort to find out how he got his answer as well as her frustration. In the meantime, the questions directed LD to answer, to point, and to define, but his contradictory responses (saying the wrong segments as distances but pointing at the correct ones and knowing the correct definition) seemed to make the teacher still confused and frustrated. Instead of giving up by simply telling LD that he was wrong and telling him the right way to approach it, Tr. Wang seemed to try to push further and give LD the opportunity to articulate how he got his answer so that she could "see" it and repair it.

To do so, following her telling and showing LD that CP is not the distance, she resumed her questioning on AP wondering aloud with questions: AP is the hypotenuse (of right triangle, AEP), how can it be 8? (Line 19-20) The brief exchange immediately followed was another chain of brief questions aligned with brief responses in which the teacher led LD to verbalize his thinking: This (EP, given) is 8 and this (EP) is half of that

(AP). Her questions directed LD to confirm his answer by asking him to point at these segments on the drawing (line 22, 24) and pushed him for his reasoning (that it is a right triangle) (Line 25) and she then completed his reasoning for him (hypotenuse is half of the right angle side) (Line 26). Her success in entrapping LD to share his approach encouraged him to share his wrong step (Line 27): he used the right triangle theorem that can get EP, a right angle side, as half of the hypotenuse, AP if the angle opposite to EP is 30 degrees. LD was stopped abruptly in the middle by the teacher's question, "Is there a 30 degree (angle) here?" He immediately realized all that he reasoned was invalid because there was no 30 degree angle known in this right triangle and then he fell silent (Line 32) when the teacher questioned his logic again (Line 30-31).

In this quick and short interaction, the teacher was mainly seen probing and diagnosing what might have caused LD's wrong answer. Even though she might have known already what led him to his wrong thinking, it appears that she wanted to see it and make the student see it too by pushing him to revisit his own reasoning in reaching the answer. Particularly, when this assignment is a filling blank exercise, there was no proof writing or procedures for the teacher to refer to in finding out the confused thinking that LD had had in reaching his answer. Tr. Wang's assistance helped her confirm what thinking process LD went through and make him recognize it. In trying to answer the teacher's questions, the boy was given an opportunity to verbalize this thinking, clarifying and sometimes defending his ideas even though he might not feel like doing so.

Strategic diagnosis and thorough repair. The power of the above brief diagnosis also lies in the teacher's strategic use of "*entrapping*" and "*tracing consequences to a*

contradiction" that Socrates often used in his dialogue with the slave boy, Meno (Collins & Stevens, 1991: 222-225)[7]. Follow the chain of consequences until the slave boy (here Little Dragon) recognized the contradiction (no 30 degree angle). By entrapping the student, the teacher created a kind of conflict in the student that could "expose and challenge the student's misconceptions" (Borasi, 1996, p. 41). In fact, by explaining and tutoring homework errors, the teacher often had students face their errors and challenge them step by step with her probing questions. This aspect of the interaction with LD was very typical of her tutoring of the other students.

With this strategic diagnosis taking effect, Tr. Wang got ready for the next repair step in the third segment which is composed of three sub-segments. In the first sub-segment (Line 33-40), the teacher virtually repaired by reteaching the definition and property of an angle bisector. She pointed at the drawing to locate the essential elements of the definition for the students: AP is the angle bisector and EP is the perpendicular segment and so P is on this segment (Line 33-34). She ended her sentence with a question to invite LD to confirm what she said (Line 34). In this way, she got things prepared for him to use the property of angle bisector to tell that distance from point P to AB is also 8 (Line 36-38). To make sure that LD really understood, she made him to answer why it is also 8, to which LD gave an almost accurate response (Line 39-40). This initial part of repair was done mainly by teacher's showing, telling and confirming.

Although the student response was correct, she did not let it go. She heard from his response to her questioning of "Why?" (Line 39) "the ends" instead of "two sides" of the angle, she was not assured whether he understood it or this was simply a slip of

[7] Collins, A. & Stevens, A.L. (1991). A cognitive theory of inquiry teaching. In Peter Goodyear (ed.) *Teaching knowledge and intelligent tutoring.* New Jersey: Ablex Publishing Corporation, 1991

tongue or he mixed it up with the perpendicular bisector theorem (point on the bisector to the two ends of the segment) that she had just taught two days before. She picked up from his response and fixed it carefully by replacing "the ends" with "two sides" (Line 41) and said the property of the angle bisector verbatim. She then repeated what she did in the initial repair by pointing at the drawing to locate for him the bisector and the point P on it. To double check, she asked him to tell her which were the two sides of the angle they were talking about before she moved on (Line 44-45). This second repair was rendered very thorough with a few repetitions to reinforce. The success of this repair was evident from the boy's collaborating in finishing her questions in both the second and the third sub-segment which was a quick and smooth procedural repair achieved on the basis of the thorough scaffold built in the previous repairs.

The above tutoring sessions demonstrate that Tr. Wang's diagnostic and repair actions were navigated by her questioning to initiate and push the student to verbalize his thinking. Student responses usually became part of the links in the chain of questioning and responding that circulated the tutoring dialogue. These questions, whether high level or not, wove the interactions together and moved the tutorial forward to create opportunities for the teacher to "enter" the mind of the learner and for the learner to "see" his own thinking and mistakes.

Summary

In summary, there are a few important points that this chapter has helped to make. First, we see in tutoring individual students the different roles played by a mathematics teacher. She was not just responsible for planning and teaching lessons, even though explaining and tutoring as well as marking could be important part of planning, but also for strictly holding themselves and students accountable for "getting things correct" and making sure their "net" caught all problems, errors and the students who committed them. In tutoring those strong students coming with additional work they chose for themselves, she acted like an experienced peer working together with them through those "new" practice problems step by step. In tutoring the average and below average students, she not only tutored and challenged their thinking but also acted as a disciplinarian strictly cultivating their basic habit of doing math and producing good and acceptable work. She communicated with colleagues to learn about other dimensions of the students that were unknown to a subject (math) teacher and she sought alliance with students and let them play a role in monitoring and assisting her work with their peers. "Students (their good and bad points/characters) are in the palm of our hands" (phone interview with a retired middle school math teacher, 9-24-02). This suggests strict control over students, but what the teacher really meant was that through homework activities, problems would never slip out of their hands. Homework is one key tool they used to ensure this.

Second, both explaining and tutoring were concerned with student errors and problems of learning. They were conducted in ways orienting to the results. At the same time, however, the teacher seemed to be always driven more by her own pedagogical needs and a sense of completion of her work—in which students "getting it" is a necessary and natural part. In other words, she always took students' faced problems upon herself and found any available opportunities inside and outside of her teaching to offer additional assistance. Understanding to what extent this is the product of a "examination-driven" system will be examined more carefully in the next chapter when investigating the teachers at work in their office.

Third, examining tutoring offers another window into which to view how errors are used constructively as pedagogical and learning resources. Again, it shows that tutoring was able to create ample opportunities to make the teacher and the students benefit from collaboratively learning about the error processes and making effort to remove misconceptions and correct the wrong. The power of errors, when they are used in such dynamic interactive inquiries, are utilized and expanded with their disappearance. According to Piagetian constructivist viewpoints, teaching errors in such ways would induce cognitive conflict or disequilibrium (Piaget, 1970) and cause conflict to be resolved. Such a view usually advocates creating environment that students can come into such conflict on their own with the teacher in the facilitating role. We see Tr. Wang was also creating such environment for students to recognize and probe into errors and conflict, yet not so much by facilitating as by carefully leading. The teacher also led students to monitor their own error committing and eliminating activities in their homework created by structured explaining and challenging tutoring. This created a process more like one of "metacognition" (Schoenfeld, 1992)[8] in which students, in the long run, could learn to monitor their cognitive processes in the case of following how an error was made, how to recognize, interpret it and find out the routes and causes so as to correct it.

Finally, to give a sense of ending for the story of Tr. Wang's tutoring (and explaining and marking), I would like to share that in a phone conversation during

[8] Schoenfeld, A.H. (1992). Learning to think mathematically: Problem solving, metacognition and sense making in mathematics. In D.A. Grouws (Ed.), *Handbook of research on mathematics teaching and learning* (o. 334-370). New York: Macmillan.

summer of 2003, Tr. Wang told me that Little Dragon and a small proportion of below average students were accepted by junior technical schools. She also shared that their academic performance were not up to entering into senior high schools as their parents all wished their only child to go to but they have enough basic fundamental math to pursue a successful technical school program. She seemed to suggest that her persistent tutoring effort with Little Dragon was paid off.

Chapter Seven: Collegial Homework Interaction and Conversations in Teaching Geometry Proofs and Functions

Introduction

Homework-related collegial conversations, November 19, 2002

She continued marking when the second period started. Tr. Zhao, the male math teacher in his early forties whose desk adjoined hers, returned from his teaching and sat down, facing Tr. Wang, to mark his homework. He had taught advanced math in a college for more than ten years and it was his third year teaching in this middle school where his son was in the sixth grade. He taught Class 6 of the 8th grade and computer for the 6th grade. While marking homework, Tr. Wang said, "Students all seemed to have difficulty in constructing the figure (in Exercise 1, Volume A which asks students to find the point of equidistance to three given points) and writing the construction procedures."[1] Tr. Zhao agreed. "Look," he showed her a student's work, "do you have such students in your class? This figure just rotates a little. They couldn't recognize it any more!" As they hit upon this frequent issue about "figures," they both felt that although in class they drew figures on the blackboard and both Volume A and B have figures printed ready for homework, students did not get enough practical experience in reading and constructing figures on their own. To them, being able to construct figures is vitally important to learning geometry, so they both liked the opportunity that the extra blank exercise-books created for students to copy the problems and/or draw figures on their own when they do the proofs or calculations. They discussed the possibility of using quizzes and tests that require students to construct figures. Even with such moment-to-moment informal exchanges going on, Tr. Wang never seemed to stop marking homework.

At 9:10, twenty minutes into their homework marking and talking, the screen door opened with a cracking sound and in came Tr. Li, a young female math teacher who looked to be in her mid-thirties. She taught Class 1, and also was the school's vice principal. Her office was located in the administration wing on the second floor, so she visited the two colleagues' office whenever she had a teaching- or homework-related concern. On seeing her, Tr. Wang said to her, "Eh, I am thinking about having our Lesson Preparation Group Meeting this afternoon to plan for the District Final Exam, now that we are finishing up with geometry proofs." "Sure," Li responded, "but I have been teaching the unit on functions after Midterm instead of continuing with geometry. Tr. Hu is doing the same as me. I simply could not find the time to mark geometry proof homework if I continued to teach geometry all the way through!" she exclaimed. Then

[1] Students were already taught how to construct bisectors with a ruler and a pair of compasses in second semester of 7^{th} grade. Here it requires students to do the construction by using the Perpendicular Bisector Theorem taught the previous day.

she added that this shift of focus would not create a problem since she did not have much geometry left from Midterm to teach after finishing with functions.

"Eh, I have to ask," she continued, "for direct proportion functions, do you also teach the analytical expressions, the graphs, and the domains for those practical problems?[2] *" "Yes, I think so," answered Tr. Wang, "you teach them when you come across them, but not with particular emphasis though. Isn't that the focus of the last section on functions?" Tr. Li's question reminded Tr. Wang of another exercise in Volume B (p. 25) that Tr. Hu, the other young male teacher, had discussed with her in his office the other day. She opened to that page and read: "Judge whether the following given functions are direct proportion functions: No. (6) -- y: x=1:4." They started a heated discussion around this problem. Tr. Wang said that the key to the Teaching Reference Material*[3] *states that the function is a direct proportion function, but Tr. Li said that the key to Same Step (one of the two official resource books for teachers and students) states that it is not, and she stressed several times that she also had asked people (experts, in a sense) in other schools, who had said that it is not.*

The three teachers tried to examine the given problem, y:x=1:4, from several perspectives: first, its domain is not "any real number" since it is not a line that passes the original point on a coordinator. Second, although it can be written as y=1/4 x, the domain range is different when written in this way. Tr. Wang said, "Every year at this time, this is an issue much debated among teachers." "It is simply very difficult for students to accept the concept of one-to-one corresponding relationships when they just start with functions!" added Tr. Zhao. "Well, I think it very confusing to give the definition of a direct proportion function in two different forms, first proportional and then linear, y in terms of x (y=1/4 x)!" said Tr. Li grumpily. "Still," reminded Tr. Wang, "when dealing with a practical problem (such as the relationship between the perimeter and number of sides of a square: y=4 x), the domain should be considered according to the meaning of the problem (here x>0)." After several rounds of debates, about ten minutes later, they all agreed that since in giving formal exam questions, such ambiguous and confusing questions were often purposefully avoided, they should not be spending too much time hair splitting.

When Tr. Wang reminded them again of the afternoon meeting, they started throwing out ideas about how they needed to give different levels and types (i.e, geometry or algebra) of worksheets to different levels of students after finishing with all teaching content and how they should share the work among themselves. Since Tr. Zhao had been quite concerned about the school's policy on higher average class scores, he suggested that the school should be ranking student final exam results by how well students performed individually instead of by how a class performed on average. Tr. Li, the vice

---

[2] What was meant here as "practical problems" were those that have a practical situation to them, say for instance, the relation between the perimeter and number of sides of a square, relation between distance that a vehicle runs at a constant speed and the time it takes, and so on.

[3] Teaching Reference Material is the direct translation of *Jiaoxue Cankao Ziliao*, a major curricular document for teachers in which it provides detailed unit, lesson, and section analysis on content and pedagogy, and student learning issues. See more detailed discussion in Chapter Three.

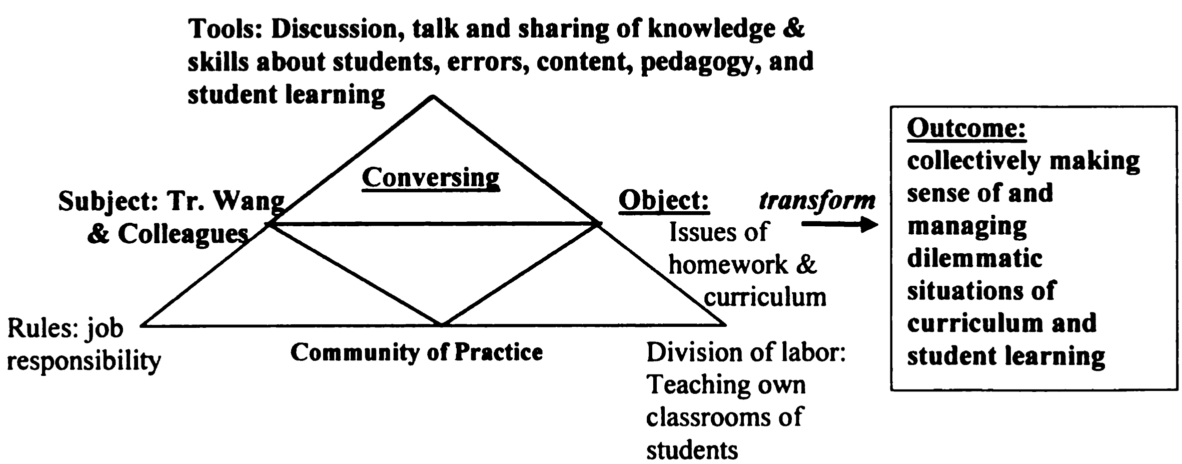
principal, worried that Tr. Zhao was suggesting not to put more effort into teaching weaker students. She explained that in the 9th grade, the graduating grade, more attention was given to those planning to take the senior high entrance examination, but she felt that all the other grades should still pay attention to every student. Tr. Zhao said he was misunderstood and what he meant was not giving up on weaker students but requiring them to master more fundamental or rudimentary requirements because of their learning capacity. Tr. Li agreed with him. Tr. Wang shared her idea about what more specifically they should do to put Tr. Zhao's idea into action and suggested that they work out a more detailed action plan at their meeting in the afternoon.

Their conversation naturally moved to those weaker students who both frustrated them and made them laugh until they cried with the dramatic examples of their poor attitudes and learning habits. Near the end of their conversation, Tr. Wang stopped participating and began to read her lesson plan sheets, inserted in her textbook. When I told Tr. Zhao that it was amazing that their conversation was all about homework, content, and student learning, he responded seriously, "How can you learn math without doing homework and it almost equals no homework if we don't mark their homework!" "If you don't push," Tr. Wang echoed, "he (the student) will not use his mind to think while doing homework!"

Surrounded by students and colleagues at her grade-level office, Tr. Wang's activities surrounding homework were never solitary. As she was marking student workbooks at her desk and selecting errors to explain and students to tutor, her math colleague, Tr. Zhao, sitting across from her, was often doing the same work for his class. As in the opening vignette, performing similar tasks and sharing similar concerns during their non-teaching hours frequently triggered their informal conversations about the tasks at hand.

Such conversations were often joined and enriched by her other two math colleagues who had offices on different floors and came to visit whenever they had problems and concerns. Most often, these problems and concerns voiced in their conversations were related directly or indirectly to issues arising from student homework. On this particular day, Tr. Li's visit to her two homework-marking colleagues gave rise to a rich conversation that lasted for 20 minutes. They talked about teaching, student learning, how to deal with weaker learners and errors in students' homework, focusing half of their conversation on an ambiguous homework assignment. The fact that these homework-related issues often brought the colleagues together makes them a shared object. (See diagram below). Such conversations opened up avenues for Tr. Wang to share her knowledge and thinking about student learning with her colleagues and for them to examine problems in teaching and student learning together.

Figure 7.1 Collegial Interaction and Homework Discussion Activity



As the figure indicates, the subject of this activity is Tr. Wang, accompanied by her colleagues. Sharing the work responsibility for the same group of students and using the same homework and curriculum, these colleagues had similar concerns about teaching and student learning that brought them together for informal interactions on a daily basis. Driven by the same desire to manage the uncertainties of teaching and the issues of student learning, their interaction created collegial deliberations on the real problems arising from their work. It was through such frequent and informal interactions that the teachers found opportunities to talk about those problems and put their minds together to better understand and deal with them.

There were generally two types of such informal talk. The first type, represented by the beginning portion of the above vignette, are casual conversations between Tr. Wang and Tr. Zhao while marking homework. In this type of conversation, they randomly shared issues they encountered in student work. The second type are collegial conversations about a certain problem deriving from teachers' use of homework. In this type of talk, the teachers were driven by a desire to work out the problem. Both types of conversations provided opportunities for the teachers to share problems and concerns about their teaching, the curriculum and student learning, albeit in different ways.

Chapter overview

In this chapter, I choose to focus on three homework-related teacher conversations and build each into a case that reflects a different dimension of what such conversations entailed and made possible for teachers and their work. The first case comes from a homework-marking conversation between Tr. Wang and Tr. Zhao and illustrates the content and dynamics of their dialogue. It is a case of how such conversation mirrors two aspects of marking homework: it creates opportunities for colleagues to manage the stress and to make sense of student learning through sharing and verbalizing their frustrations with student errors and learning issues.

The second and third cases are two problem-solving scenarios in which the colleagues' use of homework in teaching triggered ambiguities and controversies for them to discuss and ultimately understand math and math teaching and learning better.

The third case is based on the conversation among the three teachers in the above vignette. Both cases shed light on the collaborative nature of the processes through which colleagues came together to uncover the mathematics and curricular complications embodied in the uncertainties of improving student learning. Conversation 2 highlights the ways such conversations are deeply involved in subject matter knowledge and pedagogical content knowledge. Case 3, also a problem-solving scenario, highlights the collective use of curriculum materials as tools to unpack the causes of ambiguity in a homework assignment.

Research questions

Using the three cases, I aim to answer two major questions: 1) what do these conversations look like and entail, and 2) what did homework conversations make possible for teachers and their work. More specifically, I intend to answer these questions: what were the content and dynamics of those conversations? What opportunities did the conversations create for teachers to make better sense of the content of teaching, their pedagogy, and student learning? How were the tools and artifacts of teaching used in the process of creating those opportunities?

Data and data analysis

Data. I draw on informal collegial conversations that occurred in Tr. Wang's office on the days in which she taught the two sections on geometry proofs *(Converse Propositions and Theorems)* and functions *(Direct and Inverse Proportion Functions).* More specifically, I choose the following three conversations for detailed analysis:

Table 7.1. Data Sources for Collegial Homework-Related Conversations

	Date	Type	Topic	Length
Conversation 1	11/18/02	Sharing by two colleagues while marking homework	Student errors, teaching, curriculum, education in general	2 hours 9:30-11:30 AM
Conversation 2	11/15/02	Problem-solving as deepening subject matter knowledge and pedagogical content knowledge	Turning a proposition or theorem into its converse	6 minutes
Conversation 3	11/19/02	Problem-solving in collective use of tools	Different answers to judging if a certain given proportion is a direct proportion function	20 minutes (with first half on the problem Tr. Li brought)

As indicated in the above table, there were roughly two major types of conversations: first, those homework-marking conversations, such as the one briefly described in the beginning of the opening vignette, which feature the sharing of incidental topics as colleagues marked homework, and second, those conversations that brought the colleagues together because of a homework-related issue that drew disagreements among themselves and student answers or between external teaching references. I choose the three conversations as representative of the two major types of conversations.

Conversation 1 (November 18, 2002) represents those "marking conversations" occurring on a daily basis for different lengths of time when Tr. Wang and Tr. Zhao both marked homework during their non-teaching hours. It is a typical example of how the things that they happened to find in student homework, which often were unexpected errors and poor quality work, led them to talk about other related issues, such as their teaching, the curriculum, societal influence on students, and so on. In some ways, these talks were rambling conversations with no serious intention to dig deep into an issue, although once in a while they went more in-depth as they dwelled on a topic longer. One of the major roles played by such sharing was that it helped reduce the degree of stress involved in facilitating student learning and reflected in marking homework. I choose to describe this long homework-marking conversation a case of homework-mediated sharing as stress management.

Both conversations 2 and 3 represent those "problem-solving" scenarios in which there was a long and focused discussion on a homework-related problem that a teacher brought to consult with the other colleagues. Conversation 2 took place immediately after Tr. Wang taught her first period on the concept of "converse propositions and theorems" on the morning of November 15, 2002. She was wondering how to explain what was going on when students had different but reasonable answers to several problems within the same set of assignments. This prompted her to start a fairly long discussion with Tr. Zhao as soon as she returned to her office. I choose this as an exemplary case to illustrate how the two colleagues, with the help of their subject matter knowledge, tried to unpack the concrete mathematics buried underneath the concise form of the concepts in the homework assignments.

Conversation 3 happened four days later, on the morning of November 19, 2002, when Tr. Li came to Tr. Wang and Tr. Zhao, who were marking homework in their office. This twenty-minute conversation was described in detail in the opening vignette to give the reader a sense of what they really talked about and how their conversation developed. Both conversations 2 and 3 were mainly about how the teachers used their subject matter knowledge to make sense of ambiguous homework assignments. But each conversation reflects a different aspect of such use. The first one reflected how teachers used their content knowledge as a major device in exploring the problem regarding converse propositions and theorems, while the second conversation was a case of how they used curricular materials as tools to help them uncover different dimensions of the concept of direct proportion functions.

Data analysis. All three conversations were transcribed and translated verbatim. Careful comparisons were made with observation notes in order to add in the cues of verbal and non-verbal expressions, such as silence when reading from a book, tones and intonation, facial expressions (showing eagerness, thinking, wrinkling eyebrows to indicate thought, or expressing frustration). These cues enliven the conversational dynamics and provide the reader and researcher with other contextual information needed for further coding and interpretation. In terms of coding, I conducted two fundamental levels of analysis across the conversations to understand what the conversations were about and how they were related to the nature and purposes of the teachers' work. First, at the descriptive level, I coded the content (**phases** of sharing or discussion and the **topics** shared or discussed) and the dynamics (the **turns**[4], actions, verbal and non-verbal cues). Second, at the interpretive level, I looked for themes by identifying patterns and categories of shared topics.

Given the differences between the two types of conversations, coding was done differently for each conversation. In coding the first two conversations, I adopted an approach similar to developing conceptual categories and relationships in grounded theory (Glaster & Strauss, 1967).[5] First, I coded a conversation sequentially, labeling each type by category. I repeated such coding a few times until no "new" or unfamiliar categories emerged. I then combined the similar categories into clusters and gave each

[4] The categories in bold type indicate those that will be presented and discussed later in this chapter.
[5] Glaser, B. G., and Strauss, A. L. (1967). *The discovery of grounded theory.* Chicago: Aldine.

cluster a more stable name that was able to reflect the properties of the cluster. For instance, such categories in conversation 1 included "sharing problems from homework" or "interpreting/understanding the problems," and in conversation 2, whether the topic or progression of discussion of the topic belonged to a category that involved "subject matter knowledge," "pedagogy," or both, "pedagogical content knowledge."

Second, since all the categories were extracted from natural conversations with interweaving topics, their interrelations were important sites to infer how meanings were developed by different actions and instantiations. Therefore, I looked for whether there were connections between and among the categories or concepts and how such connections were displayed. Third, I inferred meaning by restoring life to the categories. I returned to each category to code the action verbs, verbal and non-verbal cues, what or who initiated or shifted the topic, what was brought in to extend or use as a reference to elaborate a topic. Such conversation cues provided contextual information about the dynamics of discussion and different roles played by the teachers in moving the conversations forward in these problem-sharing and solving situations.

For instance, in conversation 1, one category of actions that could not be captured in the transcript was how the teachers would point at a problem in a student's work and sometimes sketch what they wanted to show students in teaching the problem. These were actions to be used to describe the dynamics of the homework-marking and sharing. Another instance of frequently performed marking actions occurred when Tr. Wang turned to the cover of an unsatisfactory workbook to read the name of the student out loud. These actions not only suggest the public nature of student learning problems and their identity in Chinese schools but also how the teachers might be able to understand the learner more by knowing what mistakes he or she had made or how they might know more about the nature of the mistake by knowing who made it.

In conversation 2, however, the actions were not about how they shared, but how each took turns to initiate or move the discussion forward and what artifact was being used in the process. For example, Tr. Zhao often moved the conversation by citing counterexamples, and the artifact they referred to twice was the Teaching Reference Material, a symbol of authority. Furthermore, in coding the non-verbal cues, I chose to code more in the first conversation to highlight the stress or frustration involved in the sharing.

In analyzing the third conversation, I adapted and used Engestrom's categories that he employed in coding and analyzing the innovative learning taking place in work team meetings in industrial institutions (380-385)[6]. The problem-solving and learning involved in this conversation of three colleagues resembled in many ways how work teams studied by Engestrom solved problems and created knowledge in team meetings. More details will be provided in the discussion of case 3.

Case 1: Homework-marking conversation as a case of sharing stress and making sense of student learning together

> *On Monday morning after she finished teaching her first two class periods, Tr. Wang sat down at her desk and started marking homework. Tr. Zhao, just returning from his teaching, took his seat facing Tr. Wang and drank from his tea mug, as he started sharing what he had done in his class. For the rest of the morning, spanning two hours, they marked homework and talked intermittently.*

What is this conversation about? – Topics and dynamics

[6] Engestrom, Y. (1999). Innovative learning in work teams: Analyzing cycles of knowledge creation in practice. In Y. Engestrom and R. Miettinen (Eds), *Perspectives on activity theory.* Cambridge University Press

This conversation, as much as is about student homework, allows us to view what this work has to do with the opportunities that the two colleagues had to make sense of their student learning as well as what problems were on their minds. The conversation on November 18 involved 120 minutes of conversation that I have termed as 4 incidents. The following table breaks the conversation into turns and turn-taking between the interlocutors and gives a summary of the topics and their knowledge coverage.

Table 7.2. Coding of Conversation 1, a Collegial Homework-Marking Conversation

Turns[7]	Turn-taking	Topic[8](s)	Whether topic(s) are related to homework	Topic coverage		
				Cnt[9]	Tch	Ss lrng
01-05	ZW-ZWW	Z said he taught again how to add and write about adding an auxiliary. W was reminded of a mid-term exam question that asked students to draw an auxiliary. She said that students drew the auxiliary but failed to use it as a condition in the proof writing. She reduced their points to make them remember.	They often talked about adding and writing about adding an auxiliary as two difficult and important points. **Directly related to homework.*	Yes	Yes	Yes
06-12	Z-ZW-WZ-WZ-W	Z drew and showed W how a student in the homework wrote a proof in a tedious way—habit of using old theorem. Laughed together. They discussed this learning issue and shared how they modeled their "teacher" way of writing the proof to the student.	Tr. Z was always concerned with this issue. **Directly from homework.*	Yes	Yes	Yes
Incident1		A novice teacher came in to ask for the mentor form from Tr. W, who said she was too busy with marking to finish it for her so quickly.	Dynamic office life. **Not very related to homework.*	No	(Y)	(Y)
13-16 (Break)	ZW-WZ	During break, W and Z both summoned students for quick tutoring mostly on the issues they had just identified from the	Dynamic office life during break.	Yes	Yes	Yes

[7] Turns refer to numbered turns of talk in the discussion between Tr. Wang (W) and Tr. Zhao (Z).
[8] A topic refers to both the length of a general topic discussed as well as the content discussed. A topic usually consists of a number of turns and within one topic all turns are largely focused on the same topic or the same related sub-topics.
[9] The three sub-columns under "Topic coverage" are: Content of teaching, teaching, and student learning.

		homework. Then they showed each other "strange" errors in student construction and writing.	**Directly from homework.*			
17-19	WZ-W-	W shared specific errors twice and commented on and interpreted student learning. Z came up with the "eradicating bandits" metaphor.	**Directly from homework.*	Yes	Yes	Yes
20-24	ZW-ZW-Z	A brief but productive discussion on student confusion about the definition of an isosceles triangle.	**Directly from homework.*	Yes	Yes	Yes
25-30	WZ-ZW-ZW	Expressed frustration with students' poor homework habits and laughed together at how surprised the students looked when they arrived at the office for tutoring.	**Directly from homework.*	No	No	Yes
Incident2		Telephone rang to announce meeting for class director teachers at 12:30.	Dynamics of office life * *Not directly related to homework.*			
31-47	ZWW-ZW-ZW-ZZW-ZW-ZW-ZW-Z	Rambling talk about causes of student learning problems: lack of parental attention, content being too hard, high expectations for every learner, societal pressure on schools.	**Indirectly related to homework.*	Yes		Yes
Incident3		Physics teacher's visit to discuss how she had high expectations for completion and submission of homework and her view of the importance of homework.	**Directly related to homework.*	No	(Y)	(Y)
48-50	Z-W-Z	Z and W followed physics teacher's words and talked about the need to be strict with requiring students to submit work on time.	**Related to homework.*	No	(Y)	Yes
50-51	W and *Banzhuren* sitting closer by	W complained about a student's messy, careless work, and the *Banzhuren* teacher shared extra information about this student from her class.	**Directly from homework.*	No	No	Yes
Incident4 (Break)		W tutored a girl on her homework errors. She saw her in the office and called her for tutoring.	**Directly related to homework.*	(Y)	(Y)	(Y)
52-55	ZZ-WZ	Z and W shared what they planned to do for the week.	Common topic for W and Z * *Not directly related to homework.*	Yes	Yes	Yes

Homework, as a topic, was discussed during this conversation with high frequency. Out of 10 topics shared in the conversation, 8 (except for the first and last one) were related directly (7 topics) or indirectly (1 topic, turns 31-47) to homework, accounting for 80 percent of the total topics of conversation. Of the 7 topics directly related to homework, 86 percent of them were triggered by problems in student work being checked by the teachers. The 2 topics (the first and last) that were not related to homework were both instances of the teachers' discussion of their teaching and of their teaching schedule.

In terms of scope, since student homework is essentially a product of student learning, the homework-marking conversation touched upon issues related to student learning from beginning to end, including three of the four interlaced incidents. The topics shared in the earlier half of the conversation (up until turns 20-24, including the last topic) all bore relation to the content of their teaching, their own teaching experiences related to the topics, and student learning of the topics, as well as students' personal and learning traits (such as turns 23-30 and 50-51). Although they did not dwell on any single topic for long, given the ongoing flow of the work, when a certain student work or teaching problem was shared, they did not just mention it, but also interpreted the causes and often shared how they taught or wanted to teach the topic to their students. For instance, turns 01-05, 06-12, 17-19, 20-24, and 52-55 are all typical cases, of which 20-24 involved more thorough discussion of causes and analysis. This will be discussed in detail a little later.

Homework mediation not only characterized the content of the conversation topics but also the dynamics of the conversation. In terms of the turn-taking, the number of conversation turns for each teacher was similar, Tr. Wang taking 26 and Tr. Zhao taking 28. This suggests a high level of interaction and participation. As marking homework involved constant ticking, crossing or writing comments, their sharing with each other what they noticed often involved physical actions, such as pointing to the problem on the page of a student's work and drawing what one found on a piece of paper. When they shared, their words about the student work carried their tone of frustration, expressed often with a frown or wrinkling of their eyebrows. Once in a while, they laughed together at what they shared, half amused and half frustrated. On the one hand, marking homework involved stress and frustration at the obstacles faced in facilitating student learning and at the challenges to their teaching; sharing such frustrations, on the other hand, was an outlet for the stress as well as an opportunity for them to make sense of student learning problems together.

Opportunities to learn for teachers.

Both sides of complaints – Shared stress and sense-making. When Tr. Wang and Tr. Zhao were marking homework together, their talking and showing each other student homework problems seemed to provide an opportunity for them to do two things: share their frustrations and think and deliberate about those problems together. Let us take a look at these two roles of their homework-marking conversation by examining turns 17-30 from the previous table, presented in the transcript below.

(It was 10:40 AM. The new period had just started; the office was suddenly quiet.)

W: *(Turned over the workbook she was marking and read the name, a girl's name):* This girl, she does not seem to have the kind of thinking for doing the problem *(she said, showing signs of frustration).* Many students don't seem to be very clear about what they are doing, what's the purpose of the problem.

Z: No. *(After a little while)* We mark homework just like putting on a battle to track down and exterminate the bandits---wherever they are hiding, we track them down and wipe them out.

W: *(Started another homework book.)* Extend point A to CD intersecting at E. Extending a point? *(She laughed dryly, as did Zhao. Then she said to herself, whose is this? She flipped the book over to see the name on the cover.)*

Z: It seems that he still does not understand what it means for two sides of a triangle to be equal.

W: No. They confused them *(two sides are equal and two base angles are equal)* to be one thing; in fact they are two separate things.

Z: *(After a while)* Yes, they are.

W: In quite a few student workbooks, this problem has occurred.

Z: *(Still marking.)* It seems now the geometry proof has fallen into this dilemma: if you ask them *(students)* to talk about the proof and reasons orally, they can explain it clearly and fluently. Once they write the proof down on paper, they show various problems.

W: *(Marking another student workbook.)* This Lin XX, without writing the (words for) "Already Known" and "To prove." *(Required format for writing proof.) (She added it for him using her red pen.)* He's always doing things in such a rush but feels good about himself. When you ask him why he's wrong, he would ask in return in a matter-of-fact tone, "Why am I wrong?" *(Imitating this boy and laughing together with Zhao.)* When you explain to him, he would say, "Oh, I see!" in all such a sudden realization.

W+Z: *(Laughed again together.)*

The stress involved in this activity was often exhibited through the verbal comments Tr. Wang and her colleague made in the form of complaints about student work and its errors. These expressions of dissatisfaction were an outlet for her frustration and disappointment at students' learning problems. In the passage above, the brief interaction (in fact, the entire conversation) gives the impression that they never seemed to stop complaining. Tr. Wang grumbled over a girl's total missing of the point in doing an exercise that suggested her lack of the necessary thinking in doing it (lines 2-3); they laughed dryly at another student's silly way of describing how he or she drew the auxiliary by "extending a point" to a line (lines 9-10); and she accused a boy of failing to write according to the required format (lines 24-31) and laughed together with Tr. Zhao (line 32) about the student's lovable reactions to the teacher's feedback. On the one hand, they sound nagging and impatient about the seemingly trivial procedural problems. Her disappointment in students' failing to fulfill her basic requirements was also fully displayed. On the other side, however, her complaining should not be taken just as such; there is more beneath her grumbling.

Besides sharing stress, the two colleagues were also making sense of the student learning reflected in the homework they were marking. When Zhao, a colleague instructor turned middle school math teacher in his third year of teaching, reacted to Wang's frustration, his response was filled with fresh revelations and strong feelings towards the entire business of marking homework: it was as challenging and difficult as "putting on a battle to track down and exterminate the bandits---wherever they are hiding, we track them down and wipe them out" (lines 6-8). This metaphor fits the intense nature of the work and the arduous tasks it involves. Bandits are threats to social security and personal welfare, just as errors and misconceptions are threats to further understanding and the welfare of successful learning. The actions and measures these teachers took to deal with errors reflected how they viewed errors: frustrating but hard to let go. These errors were dealt with differently than bandits, needing to be explored, used, and then removed. Such opportunities to share their minds while marking homework have allowed these teachers to alleviate the stress of the work and to learn and benefit from each others' strengths in managing the dilemmas of teaching and learning, which is particularly valuable for the novice who had to get used to the norms and routines.

At the same time, their conversation also engaged them in making sense together of problems that students encountered in their homework. One of the most prominent homework issues that Tr. Wang was dealing with that day concerned students' failing to use the definition of an isosceles triangle correctly – they did not use the two equal sides, choosing instead to use the equal base angles. (The two equal sides defines an isosceles triangle, and that the two base angles are equal is derived from the definition.) In their conversation above, when Tr. Zhao confronted this same issue (that Tr. Wang had explained to her class) in his students' homework, they continued to deliberate on why students could fail to cite the definition as a necessary condition in writing the proof (lines 13-19). They concluded that students might have thought that the two sides of an isosceles triangle being equal is the same thing as the two base angles being equal[10]. But to the teachers, these are different. Tr. Zhao reacted to this problem as a dilemma in the teaching and learning of writing proofs in which students being able to express orally their thinking about writing a proof clearly does not mean that they could write coherent and logical proofs (lines 20-23).

Evaluating the errors – who is the owner? In the above collegial exchange, Tr. Wang's frustration led her to check which student owned the workbook and error(s). Her tendency to turn to the cover of the workbook to see the name of the student was apparently driven by her unhappiness with a student's work. On knowing whom the owner of the error(s) was, her knowledge about the learning capacity and characteristics of this student was brought into play to help inform her about why this particular student

[10] In the textbooks and teaching of geometry and geometry proof in China, the idea of "congruent angles" (either independent angles or angles belonging to a shape) is generally referred to as 'equal angles"; but the term "congruence" (meaning completely equal and overlap) is always used in referring to equal shapes, such as congruent triangles, and quadrilaterals.

failed here and whether this error was a general issue for all students. She sorted out those that needed immediate attention into a separate pile for tutoring during breaks. This process helped add specificity to how she made sense of the errors -- in terms of which student(s) made certain types of errors, why, and what types of errors they were. In explaining homework errors, she could often explain multiple dimensions of the same error or multiple errors within the same problem (as discussed in chapter five).

In the meantime, she seemed to be evaluating the errors against who the learner was. Such wonderings have two features: first, they associate the error(s) with the student committing them, which often made her think about how they behaved in the classroom, whether they showed enough effort, or other personal traits of that student, such as in the case of the boy called Lin (lines 24-31). Such association created by the marking of homework enabled her to know well where each student was in learning a topic and form a general picture of how different levels of students compared in their learning so that she could make informed decisions to better meet their needs. (See related discussion in Chapter Four on marking homework.) Later that day, she summoned Lin for additional tutoring. More often than not, such association often triggers other subject area teachers sitting close by to join the conversation and add in their knowledge about the same student. For example, Lin's *Banzhuren* (the head teacher of the class) told Tr. Wang about the strengths of the child in using his hands and as an athlete (turns 50-51 in the table).

Second, her wonderings about the causes of the errors seem to show Tr. Wang assessing the generalizability or severity of the situation. For instance, she generalized from the girl's error or incompetence in writing the proof to her frustrating finding that "many students do not seem to be clear about the purpose of the very problem/exercise" (lines 4-5). Her concern about student construction of drawings (in her second complaint above) and the format of geometry proof writing were also two of the issues she often dealt with in homework explanation and tutoring. Therefore, conversations between Tr. Wang and Tr. Zhao while marking student homework served as a tool both for stress management and for collaboratively making sense of student learning and students as learners.

Case 2: Mathematics entailed in homework-generated conversations – Collaborative problem-solving in teaching geometry proofs

> *Tr. Wang taught the concept of Converse Propositions and Theorems during the first period on the morning of November 15, 2002. Twenty minutes into the lesson, she finished teaching the concept, assigned seat work and also finished checking seat work and explaining seatwork exercises from the textbook that ask students to tell the premises and conclusions of the given propositions and converting them into their converse. In the remaining 10 minutes she asked them to take out Volume B and do the first two small exercises of the homework of the day which asks them to write the converse of the given propositions: (1) An isosceles triangle is an axial symmetric figure; and (2) Every angle of a rectangle is a right angle. When she checked on students' answers again, she found them disagreeing with each other on two different ways of writing the converses. In the first way of writing, students followed the rule faithfully and turned the proposition's premises into conclusions and the conclusions into premises, which produced false statements. In the second way, many of them made some qualifications (by making the figure specifically a "triangle") and turned the converses into true statements. While explaining, she found that both answers could be right in different lights. Returning to the office as soon as she finished the lesson, she asked Tr. Zhao's opinions, and they started a 6.5-minute heated discussion of the two exercises.*

What is this conversation about? – Coding of the conversation

The following table summarizes both the content of the conversation and the process of how the conversation was conducted in each phase, as well as the major actions of exchange.

Table 7.3. Coding of Conversation 2 by Turns, Turn-Taking Patterns, Phases, and Major Actions

Turns	Turn-Taking	Phase[11]	Major Actions
01-14	WZ WZ WZ WZ WZ WZ WZ	Questioning and discussing the first of the exercises that require students to turn the given propositions into their converses: *(1) An isosceles triangle is an axial symmetric figure.*	Disagreed; **consulted the key in the TRM;** sympathized with each other; one made a suggestion and the other followed it; laughed together…
15-38	WZ WZ WZ WZ WZ-Z WZ WZ WZ WZ WZ WZ	Questioning and discussing the second exercise: *(2) Every angle of a rectangle is a right angle.*	Compared with 2nd exercise; probed by questioning and citing an counterexample; probed and reiterated; **compared with key in TRM.**
	W - Me - W	Tr. Zhao turned to tutor a student and Tr. Wang continued to push her thinking, "Within which range is this problem discussed?" I offered a suggestion which she did not seem to hear.	Unsatisfied and kept wondering why; reasoned in her mind; complained about the ambiguity; opened up a new perspective.
39-57	WZ WZ WZ WZ WZ WZ WZ WZ WZ	Tr. Zhao joined the conversation again and they deliberated together and "found" causes for the inconsistencies of the textbook and the confusion of standard keys: "depending on what is the implied or hidden premise."	Agreed; built on each other's answers; added, proposed, kept building on each other; reaffirmed the ambiguity; **questioned the inconsistency of the text and TRM.**

The conversation flow consists of four clear phases. Phase 1 started with Tr. Wang eagerly posing the question to Tr. Zhao, who readily joined in and commented that writing the converse would depend how one understands it. When she shared the two ways that her students wrote the converse, he accepted the first one and she the second one. Tr. Wang was looking for the Teaching Reference Material (TRM) to check the standard key when Tr. Zhao suggested to write the given proposition into an "if…,

[11] A phase is where a task or question gets addressed or answered and the conversation is obviously transitioning into a different or new task or question.

then…" form before converting it to its converse. She followed his suggestion and realized that writing in this way by specifying the figure as a triangle produced a true converse (which was how some of her students did it in class).

Phase 2 started when Tr. Wang moved to compare the first exercise with the second exercise and felt that similar confusion did not seem to apply to the second exercise. (Ex. 2: Every angle of a rectangle is a right angle.) She found that some students did not indicate the figure as a "quadrilateral," and the converse is still true. Zhao offered a counter example to prove that the converse is not a true proposition. (A figure with all its angles being right angles is not necessarily a rectangle.) Then they read the key to both exercises and noticed that the key to Ex. 1 used the first students' answer (the "figure" in the premise refers to "any figure") and the key to Ex. 2 provided the second answer by specifically referring the figure in the premise as a "quadrilateral." They wondered aloud why the teaching materials treat this concept with such inconsistency.

In phase 3, Tr. Zhao turned to tutor a student (who was making up his homework at Zhao's desk) and Tr. Wang continued to wonder about the inconsistency to herself (half to me, as the listener/observer). She said twice with some frustration that "This problem does not seem to have a firm conclusion." She then moved what they discussed to a different level by thinking that this problem involved an issue of "what is the range of the discussion." In Ex. 1, is the range of discussion "a triangle" or "any axial symmetric figure"? She started to wonder how to explain this clearly to students. (At this point, I thought that she was half talking to me and that to remain silent would be rude, so I suggested letting students know both ways of writing a solution. She did not seem to hear this suggestion.) She then complained about the teaching materials being "sometimes acceptable whichever way you say it," meaning that the same problem could be answered or approached in different ways, all of which could be acceptable or reasonable. Hearing this comment, Zhao jumped in right away, repeating, "Yes, reasonable whichever way you say it." This is where phase 4 began.

In phase 4, they continued with the question Tr. Wang had just opened up, "what is the range of discussion," and when she asked "what's the hidden/implied premise?" their discussion seemed to have come to some "solution." They kept building on each other's thoughts in identifying the possible hidden premises for both exercises, whether it be the major premise or the minor one: if the hidden premise is a major one (such as "any figure"), its converse would be false, and when it is a minor one (such as "triangle"), its converse would be true. So they agreed that the key from the TRM does not always apply.

Each of the four phases of the conversation helped create an opportunity for Tr. Wang to connect her question and their discussion with what she had just taught in the previous period. In phase 1, Zhao's suggestion of converting the problem to an "If...then" form helped her to see how some of her students got answer 2. In phase 2, Tr. Zhao's counter example reminded her that what she told students was not necessarily right and could be confusing when she favored one of students' two answers over the other. In phase 3, when she probed persistently on her own, she accepted that "the answer" does not exist, and started to face the reality that here there were two possible answers that both make sense. At this point, she began to use her mathematics knowledge to probe and reason.

In the two early phases, Zhao was taking the major role in the discussion as he questioned, challenged and contradicted Wang to help move the inquiry forward. In the later two phases, Wang took the initiative and opened up an important perspective, "the range of discussion" and "what's the hidden premise (the major one)?" that carried the conversation to an "end." Therefore, in the brief conversation, with the help of her colleague and her own persistence, she was able to use more mathematics to clarify and challenge what she taught students in class, see better what explained students' solutions, and get a deeper understanding of how to deal with the ambiguity in the teaching materials.

In this case, her knowing and learning was mediated by her colleague's support and participation (note that the pattern of their turn-taking was almost always that of one following or building on the other's turns right away), a teaching material that was able to serve as a basis for the deliberation, and her own habit of puzzling over teaching content and student learning. Ball and Bass (2001) note that "We assume that the integration (of content and pedagogy) required to teach is simple and happens in the course of experience. In fact, however, this does not happen easily, and often does not happen at all" (86). This case shows that this integrating of content knowledge with practice happened in the context of Tr. Wang's work and very often was mediated by opportunities to probe into student homework.

Mathematics entailed in the conversation

In coding the categories of knowledge that the two colleagues used in this conversation, one should note that the purpose of the coding is not to see what and how many categories of knowledge there are in this conversation, but what kind(s) of knowledge gets used in their trying to "solve" the problem and how it is used, thus showing what kind of knowledge the teachers have opportunities to learn. Different from the knowledge reported by teachers in interviews, the knowledge here is knowledge-in-use or knowledge-in-action (Shön, 1983)[12]. In addition, the situation from which the problem arises, whether from a teaching situation or from marking homework, could make a difference in how the problem would be raised and managed.

As shown in the coding, most content that dominated the conversation was mathematical in nature (those coded as CK, about 60%); in other words, the two teachers chiefly drew on their mathematics knowledge and understanding to "solve" the problem. Although the mathematics problem came from student learning (the homework assignment used as seatwork in class) and classroom teaching, there are only two places coded as "knowledge of student learning" (SLK, about 2% of the coded utterances). In light of student learning and teaching, the conversation is also about how best to represent the knowledge and whether the ways of writing the converse of a proposition might be hard for teachers to handle or confusing. In these places, mathematics knowledge was used for potential curricular and pedagogical functions, and therefore is coded as PCK, encompassing about 40% of coded utterances. They also used mathematics and curricular knowledge to judge and assess the standard key. It should be noted that the numerous places when the standard keys were compared and critiqued are coded as PCK. The two places where Tr. Wang read directly the standard keys from the TRM are coded as "Making Reference to TRM," accounting for 2%. See the table below for more details.

---

[12] Shön, D. A. (1987). *Educating the reflective practitioner.* San Francisco: Jossey-Bass Publishers.

Table 7.4. Coding of Knowledge Categories in Conversation 2

Total sentences coded	**54 (100%)**	**Knowledge features**
Math Content Knowledge (CK)	31+ (59%)	Conventional representation, procedure, fact, counterexamples, arguments, logic, induction.
Pedagogical Content Knowledge (PCK)	19- (37%)	The "what" and "how" of the math entailed in helping students learn, as pedagogically useful; subtle CK used in appreciating and evaluating the usefulness and clarity of a particular representation of a math idea or concept. Verbs to describe the use of PCK: unpacking, decompressing, inducing, comparing, critiquing, reasoning, contradicting, representing.
Knowledge of Student Learning (SLK)	2 (2%)	Citing of student answers from class.
Making Reference to TRM	2 (2%)	***Many other instances that compared and critiqued the teaching materials and the key offered in the TRM are coded as PCK.

It is also noteworthy that there are places where the codings assigned to content knowledge (CK) and pedagogical knowledge (PCK) are very ambiguous, subjective, and arbitrary. However, the purpose of distinguishing between the CK and PCK is to facilitate analysis and not to draw a clear line between them. Whether the knowledge is more "purely" mathematical or more focused on curriculum or student learning, the coding shows that the knowledge used and entailed in the conversation is mathematical in nature. Just as Ball and Bass (2001) put it, "This kind of knowledge is quite clearly mathematical, yet formulated around the need to make ideas accessible to others" (99).

The mathematical problem that the two colleagues were eager "to solve" originated in student responses to teaching, in student learning of the mathematics involved and the way her explanations left her more uncertain. From the coding, however, the conversation was almost entirely coded as CK and PCK about the mathematics content knowledge, accounting for 96 percent. In comparison, knowledge about student learning (the two places coded as SLK) and teaching (PCK, but much of this is inferred from context) were minimal. It seems that, in order to assist students' learning, teachers have to know and use a lot of mathematics to work out the mathematics problems arising from their teaching. In learning the converse of a proposition, the curriculum requires a student to identify the premise and conclusion and reverse the two components. From the textbook, the mathematics entailed in writing the converse of the proposition provided in the teaching materials was more procedural: the premise of the original proposition becomes the conclusion of the converse proposition; and the conclusion of the original proposition becomes the premise of the converse proposition, so by reversing the premise/condition and the conclusion of a proposition, one finds the converse of the proposition.

From this case, it seems that teaching for students' understanding depends on whether the teacher(s) are able to make clear sense of the mathematics entailed in the problems. Procedural knowledge alone is unable to explain why students could have two ways of writing the converse of the same proposition, both of which are acceptable. In other words, the mathematics knowledge required to do the job is of a much higher level: it entails using mathematics to unpack the polished and compressed versions of the geometric propositions, a process of "decompression" (Ball and Bass, 2001: 98), which according to Tr. Zhao, is confusing because they are "over concise." To do so, the teachers used their knowledge about logic and proofs to figure out what was implied or hidden under premises (major or minor) when it undergoes the process of compression, what results would be caused if the hidden premises were major or minor, and what difference the hidden premise being major or minor would make in judging whether the converse of the proposition is true or false. Reasoning and giving counterexamples to disprove a seemingly true proposition is also what they needed to do in this problem-solving. Frequent use of such knowledge in their practice to solve real teaching problems and make sense of student learning produced rich opportunities for them to develop such knowledge.

Case 3: Role of curriculum material in homework-generated conversations – Collaborative problem-solving in teaching functions

What is the conversation about? - Coding of the conversation

Context of the conversation. A detailed description of this 28-minute long conversation between Tr. Wang and her two colleagues, Tr. Zhao and Tr. Li, was offered in the vignette opening this chapter. The conversation started at 9:10 AM on November 19, 2002, when Tr. Li, a young vice principal and 8th grade math teacher, visited the two colleagues who were marking homework in their office. This fairly long conversation can be broken down into the following five topics or phases (See the table below.): they shared what they were teaching to find that Tr. Li and Tr. Hu had just started teaching the functions unit while Tr. Wang and Tr. Zhao were finishing up with geometric proofs; they discussed a homework assignment on direct proportion functions that had different keys from different teaching references; they talked about their agendas at the Lesson Study Group Meeting in the afternoon and the school's higher "average class score" policy for the final exam; they shared lively student homework mistakes; and at the end, Tr. Li asked whether a topic had been taught before. The conversation ended when the music bell rang to announce the ending of this second period.

The second topic (turns 021-142 in the table below, taking up half of the entire conversation) and the fourth topic (turns 206-230) are related to homework. For this case, I mainly examine the problem-solving involved in the second topic of conversation and also mention of the fourth topic in the analysis.

Table 7.5. Composition of Conversation 3 by Turns and Topics

Turns	Topics
001 – 020	Sharing of what each was teaching. Tr. Li's question was followed by a brief discussion about the teaching of direct proportion functions.
021 – 142	Discussion of "Why does there exist different answers about whether y:x=1:4 is a direction proportion function?"
143 – 205	Discussed agendas for the meeting in the afternoon. Talked about how to understand the "high average class score" policy and how to handle students of different levels, particularly the weaker students, in the upcoming review for the final exam.
206 – 230	Shared student homework problems and their teaching experiences.
231 – 246	Tr. Li asked whether the property of a perpendicular bisector had been taught before, and a brief discussion ensued.

With this case, I aim at illustrating the problem-solving actions of a small work team and showing how the curriculum materials, as tools, mediated the problem-solving process. I used the categories devised by Engestrom (1999)[13] in studying the innovative learning actions in work teams. Engestrom coded work team meetings in two manufacturing companies working out plans for innovations in order to raise productivity. Although informal teacher conversations focused on solving the controversies of a routine homework problem are very different in nature from the manufacturing team meetings in terms of the goals and content, Engestrom's approach

[13] Engestrom, Y. (1999). Innovative learning in work teams: Analyzing cycles of knowledge creation in practice. In Y. Engestrom and R. Miettinen (Eds), *Perspectives on activity theory.* Cambridge University Press

allows us to observe the stages in the process of the teachers' activity and the role of tools in advancing teacher understanding.

Engestrom coded the meetings in terms of the following categories: *formulating/debating a problem, analyzing/debating a problem systematically, sharing sympathized knowledge, constructing operational knowledge, and creating concepts.* This type of coding of phases, turns and major actions can be found in the table below.

Table 7.6. Coding of Topic 2 of Conversation 3 by Turns, Phases and Major Actions

Turns (021-142)	Phases	Major Actions
021 – 040	Formulating/debating the problem	W finds and refers to the workbook, *Volume B*, for the controversial homework assignment; colleagues debate by citing different references as authorities to back up their own points.
041 – 052	Analyzing to arrive at the first cause of the problem	Building on Z's suggestion and on each other's ideas, W finds one angle to explain the course of the controversy: *domain* – that the domain of y:x=1:4 is different from that of a direct proportion function (DPF).
052 – 062	Analyzing to arrive at the second cause of the problem	Building on each others' ideas, W finds another angle – *graph* – that y:x=1:4 does not have a graph as a DPF does: a line passing the origin.
063 – 070	More analyzing of the first cause of the problem	Building on L's idea, group pursue the *domain* angle further by turning y:x=1:4 into y=1/4x and seeing how their domains and ranges are different.
071 – 092	Constructing operational knowledge	Group contributes ideas about how this is confusing and how to deal with it, agreeing to avoid pushing the controversy further in order to avoid confusing the students, beginners in learning functions.
093 – 103	Continued analyzing to bring out a new perspective	W adds another angle: *"a real world problem"* – the relationship between the perimeter of a square and its sides is that of a DPF but its domain is different from the general form of a DPF.
104 – 122	Systematically analyzing another dimension of the problem brought up by a colleague	Z adds that the textbook does not indicate the domain of x in a DPF; W purses and explains his opinion in terms of general and specific cases: y=kx as a general case of DPF, where x can be any real numbers, while the "real world

		problems" are specific cases and their domains have to be specifically indicated according to the context.
123 – 130	Constructing operational knowledge	Group agrees not to stress this distinction too much and that marking both answers (yes or no) as correct is okay because of these different but reasonable perspectives.
131 – 142	More analyzing of the causes of the problem	Group traces the confusion to the roughly defined definition and the curriculum's forced move from direct (and inverse) proportions to direct (and inverse) proportion functions.

Similar to Engestrom's coding of the team meetings, this part of the teacher conversation (turns 021-040) also started by formulating and debating the problem as soon as Tr. Wang brought up the homework assignment as an issue to discuss. The formulating and debating actions in this sequence mainly consisted of Tr. Wang and Tr. Li citing different teaching references as the authority to illustrate whether the given proportionality y:x=1:4 is a direct proportionality (y=kx). Through debating, they agreed that the relationship is of direct proportionality but is not a direct proportion function (DPF). Then the question was raised of why the Teaching Reference Material (TRP) stated that the function is a DPF while other references such as Same Step (another resource book from which the teachers assigned additional homework) stated that it is not. The colleagues then tried to figure out this controversy.

The sequences coded as "analyzing a problem" repeated several times in this teacher conversation, similar to what was found in the coding of Engestrom's team meetings. In fact, this problem-solving process was primarily analytical, given that six out of nine phases are coded as "analyzing a problem" (except for the first phase and two other phases in the middle coded as "constructing operational knowledge," turns 071-092 and turns 123-130). Although these are coded as actions of analyzing, in each phase of the actions, the three colleagues opened up a "new" angle to use in identifying and interpreting a different cause of the controversy.

For instance, the three colleagues viewed the controversy subsequently from a domain angle, a graph angle, a "real world problem" angle and a curricular angle. In turns 041-052, they found that the two expressions have different domains and, in turns 063-070, they returned to this angle and turned the given expression (y:x=1:4 or y/x=1/4) into y=1/4x, which allowed them to see concretely the difference in their domains: in the original form, x cannot be zero and in the DPF form, x can be any real number. In turns 053-062, they viewed the problem from the difference in graphs and found that in the "new" form, the DPF has a graph, a line passing through the origin, while the original form does not have a graph at all. In turns 093-103, Tr. Wang cited a "real world problem" (the relationship between the perimeter of a square and its sides) to contradict the idea that all DPFs have the same domain. In the next phase, this perspective was further developed by Tr. Wang when Tr. Zhao said that the textbook does not indicate the domain of a DPF. This problem-solving work ended with their complaining about the textbook doing a poor job in helping students understand direct and inverse proportions before they start learning direct and inverse proportion functions. Their analysis enriched their explanations to students about why this instance is not a direct proportion function and also made it possible for the teachers to have a deeper understanding of the difference between the two concepts, particularly helping to strengthen their understanding of direct proportion functions.

The coding of this collegial teacher conversation differs from Engestrom's coding of work team meetings in two major ways. First, the analyzing actions coded in the formal team meetings are accompanied with debate and thus labeled as "analyzing/debating a problem." Given the nature of seeking new plans and models of production and the team members' different work areas (machinists, quality control experts, the coordinator, etc.), debates between the old and new and different knowledge and skill areas abounded. If it was the debate in these industrial conversations (with good coordination) that moved their discussions forward, it was collaboratively building on one another's ideas that made the teachers' discussion productive.

Although Tr. Wang, the experienced teacher (or expert), almost always seemed to lead the analyses, her leading often built on the other two colleagues' suggestions. For instance, she was inspired by Tr. Zhao and later Tr. Li's suggestions to pursue the problem from a domain angle in turns 041-052 and 063-070. It was also Tr. Zhao's mention that the textbook does not mention domain in teaching about direct proportion functions (turns 104-122) that propelled her to think deeper about how the "real world problems" have different domains and therefore are different from a general function. As the experienced one in the team, however, her persistent thinking about a question pushed the conversation ahead, especially in turns 093-103 when she cited a "real world situation" to contradict the understanding they had just reached. This pushed their perspectives to a new level. Besides building upon one another's ideas, the collaborative dynamism of the conversation is also demonstrated by their active turn-taking patterns, completing one another's sentences and affirming one another's contributions.

The other difference between coding the work team meetings and coding the teachers' informal conversations is that the action of "creating concepts" did not occur during my observation. This is certainly due to the nature of teachers' deliberation over packaged curriculum materials. They interrogated and interpreted these materials rather than creating new routes of thinking about the problem and its possibility for generating student learning. The concepts they used, such as domain and graph, were part of their mathematics knowledge developed with their experience of teaching. Yet, during the later part of the entire conversation when the colleagues shared their own students' homework errors, they became very lively and creative. For instance, they shared their teaching metaphors and discussed how the errors could relate to problems of learning new content (turns 206-230). Such sharing is very similar to the category of "sharing sympathized knowledge" in Engestrom's coding even though this category is missing in the problem-solving sequence of the conversation.

Expansive learning and continuous improvement. Engestrom did his coding based on his theory of expansive learning, "the dialectics of ascending from the abstract to the concrete," which he developed to study the longer learning cycles of large institutions (382). *Abstract* "refers to partial, separated from the concrete whole" (383). In this theory, he viewed cycles of expansive learning as a five-step model -- "*questioning, analyzing, modeling, examining the model and implementing the model*" (383) in search of new objects and practices. He then applied this framework to the study of "small cycles of team based continuous development" (378) and found it equally useful in helping understand innovative learning in team meetings.

As my coding and the above discussion indicate, the collegial problem-solving processes allowed the teachers to move the small problem from its original, in Engestrom's terms, abstract form to different instantiations obtained from multiple perspectives. Although this process represents a tiny segment of the teachers' knowledge development through inquiry into their work practices, it is, according to Engstrom, a form of expansive learning as the object of the activity has expanded. The first two actions of the learning cycle, questioning and analyzing, were rich in the teachers' conversation. Even though there was no new model created, the "search actions" that the teachers took in seeking the different angles of interpretation of the problem were what encompassed "the creative part of the activity" (283)

In addition, I notice that the two phases coded as "constructing operational knowledge" (turns 071-092 and 123-130) can be regarded as developing a model agreed upon by the three teachers: avoid pushing distinctions too much so as to avoid confusing the students. This certainly was a decision, a strategy or work model far from "innovative or new." Tr. Wang explained the same assignment to students on November 21, 2002, when she started teaching functions. She shared with them the controversy involved in this assignment and the explanation of why the problem is not a direct proportion function. She converted the given proportion into the form of a direct proportion function (y=kx) and compared it with the original to demonstrate that their domains are different. She also shared with students that such ambiguity was usually avoided in exams (which was also mentioned in the teachers' conversation), since students felt concerned about this. In the context of learning from routine practice, this model aimed to bring understanding of the concept to students rather than creating new ways of understanding.

Similar to case 2, the mathematics used in this problem-solving segment was a main portion of the conversation. Although Tr. Wang only shared with students one angle of analyzing the problem, she and her colleagues had to know much more than this one angle. The inquiry stance of the teachers would also create opportunities for them to model this stance and disposition to their students.

Role of artifacts—the curriculum materials

Engestrom's coding of the actions in the team meetings is intended to capture the dynamics of the processes of how the teams reached an innovative solution. He (1990, 1996) used four categories of artifacts to describe the role of artifacts in mediating object formation and transformation. He used the *what* artifacts to "identify and describe objects"; the *how* artifacts "to guide and direct processes and procedures on, within, or between objects"; the *why* artifacts "to diagnose and explain properties and behavior of objects"; and the *where to* artifacts "to envision the future state or potential development of objects, including institutions and social systems" (381-382).

These four categories are also very helpful in explaining how artifacts were used in mediating the teachers' process of solving the controversial homework problem. Here I mainly look at the what and why artifacts in order to highlight the multiple roles the curriculum material played in the solution process. Chapter Three showed that the curriculum design was purposefully designed in ways that induce students to err and teachers to identify the errors and problems in order to help students learn. Here, the curricular materials were used by the teachers both to formulate and describe the problem/object and to diagnose and explain their problem in implementing the curriculum.

A number of artifacts were used in this conversation. The primary what artifacts included talk and debate to formulate or construct the object/problem and the solution. It is interesting that in the debate, the teachers cited different references as external authority to back up their own points. These references were used as why artifacts as they sought to explain the controversial problem. Tr. Wang cited the Teaching Reference Material (which stated that the given proportion was a direct proportion function), in which she always referred to the keys at the end of the book. Tr. Li cited the Same Step (which stated that the given proportion was not a direct proportion function). Tr. Li also cited a principal of a well-known middle school who was also a noted mathematics expert a number of times to back her point.

These authorities were useful in helping them formulate the problem, but merely citing them would not lead them far. They also used the textbook, referred to simply as "the book" in their search for the reasons for the controversy. Tr. Wang first looked the homework problem up in the Volume B workbook and then she looked around her desk and in her drawers for the TRM to check and confirm the key. In the problem-solving process, she was holding the textbook and constantly reading from it to find evidence to support an idea, to contradict the other two colleagues when they said that "the book" did not say this or that, or to confirm the textbook's inadequacy.

For instance, in turns 071-092 (constructing operational knowledge), they agreed that the controversy was confusing and that they should not to push students to make this distinction. At that moment, Tr. Wang was thinking and reading the textbook and suddenly came up with a counter example – the direct proportionality in the relationship between the perimeter of a square and its sides – to argue that the difference of domains does not only exist between the two expressions that they had discussed so far but also is indicated differently in those so-called context-dependent "real world problems" to make them meaningful (turns 093-103).

It was when Tr. Zhao's claim that "the book" did not indicate the domain of direct proportion functions that she read more from the textbook and confirmed that it was true that the textbook does not mention this until the next section on functions in general. But this confirming action also pushed her earlier idea further so that she began to explain the differences between domain in the "real world problems" and the canonical form of DPF in terms of special and general cases. The domains should be indicated clearly in those "real world problems" because they belong to those that the textbook refers to as "special cases" in which the domain and range determine whether the relationship between the variables exists or is meaningful, while the form y=kx represents those "general cases" in which x can be any real number (turns 104-122).

The what artifacts also include the concepts or knowledge about the concepts that the teachers used to view the problem from different angles: *domain, range, graph, general and special cases of functions.* These concepts are not created by them or learned in the professional development workshops but come from their own knowledge and are enriched by their teaching experiences. This situation shows that teachers' knowledge about the mathematics subject matter plays a major role in solving real teaching and learning problems. The opportunities for them to use their knowledge to solve practical problems made it possible to expand and enrich the problem (the object of their collegial conversations) through use. In this case, the textbook and curriculum materials provided them substantial information or triggered more thinking about the information as they evaluated, inquired and probed into the problems, moving in and out of the problem situations. From the perspective of the where-to artifact, the last of the four categories of Engestrom's artifacts, the knowledge and "solutions" (the agreement that they should not push the problem too much so as to avoid confusing the students) was derived from the problem-solving work in their daily practice and was returned to practice by use (for instance, Tr. Wang explained the problem that they had discussed to students a day later).

It also should be mentioned that the curricular materials' authoritative role is both empowering and limiting. The teachers interrogated the ambiguity and controversy in the curriculum materials, and their interrogation pushed them to inquire and understand more, unpacking the hidden mathematics in a problem that they needed to teach students. Nevertheless, the teachers never moved beyond such interrogation to reframe the problems or think in terms of what their own pedagogical purposes were in teaching or assigning students a certain problem. Instead, they always pursued it from the curriculum's pedagogical objectives and perspectives. In following this tradition, these teachers are faithful and productive curriculum implementers but they seldom make a move to rethink and create new ideas and new ways of helping students learn.

Summary

In this chapter, I crafted three cases of homework conversation to help the reader understand the roles of conversation in the work lives of Tr. Wang and her colleagues. In case 1, the dynamic homework-marking conversation between Tr. Wang and Tr. Zhao served as a tool to help them share student learning problems and thus alleviate their stressful concerns about student learning. In addition, their homework-marking conversation also provided them with opportunities to make sense of student learning and share their teaching experience and plans together.

In case 2, I described how different but reasonable responses given by students to a homework exercise assigned as seatwork during a lesson later triggered a lively discussion between two colleagues. Their discussions led them to question the ambiguity in the curricular material and pushed them to seek more understanding about why such ambiguity existed. In pursuing such understanding, they used their own mathematics knowledge to trace the problem to its root – to unveil those elements that have become hidden as the knowledge in the problem has come to be compressed. As Ball and Cohen also pointed out, "(b)ecause teachers must be able to work with content for students in its growing, not finished state, they must be able to do something perverse: work backward from mature and compressed understanding of the content to unpack its constituent elements (Cohen in preparation)" (98). The large share of mathematics coded in the entire conversation attests to the fact that teachers need more deep and thorough understanding of their subject matter in order to teach students to understand basic math. The use of such knowledge in practice to solve teaching problems and to make sense of student learning provides rich opportunities for developing such knowledge.

In case 3, I provided another scenario, teachers' collaborative problem-solving of a homework controversy through informal conversation among Tr. Wang and her two other colleagues. It corroborates the findings of case 2 in two ways: first, the dynamic collaboration helped teachers to come up with different ways to analyze the problem as they kept building upon each other's ideas; and second, their subject matter knowledge was widely applied and used in the process. It also highlights the important mediating role of the curriculum material in helping the teachers formulate and develop different perspectives in examining the problem. The curriculum served to both empower and limit teacher learning from practice. While making it possible to substantiate teacher knowing and develop their knowledge in their work practice, it subsumes such knowing under its premises and confines teacher thinking within its scope of authority.

Taken together, the three cases depict the collegial and collaborative dimension of Tr. Wang's work surrounding student homework. Interacting with her colleagues made homework practice less stressful. They worked together to construct and transform the problems arising from teaching and student learning. These collegial opportunities were sites where their knowledge about content, pedagogy and student learning grew and developed through use.

Chapter Eight:
Conclusions and Implications for Future Research

Chapter Overview

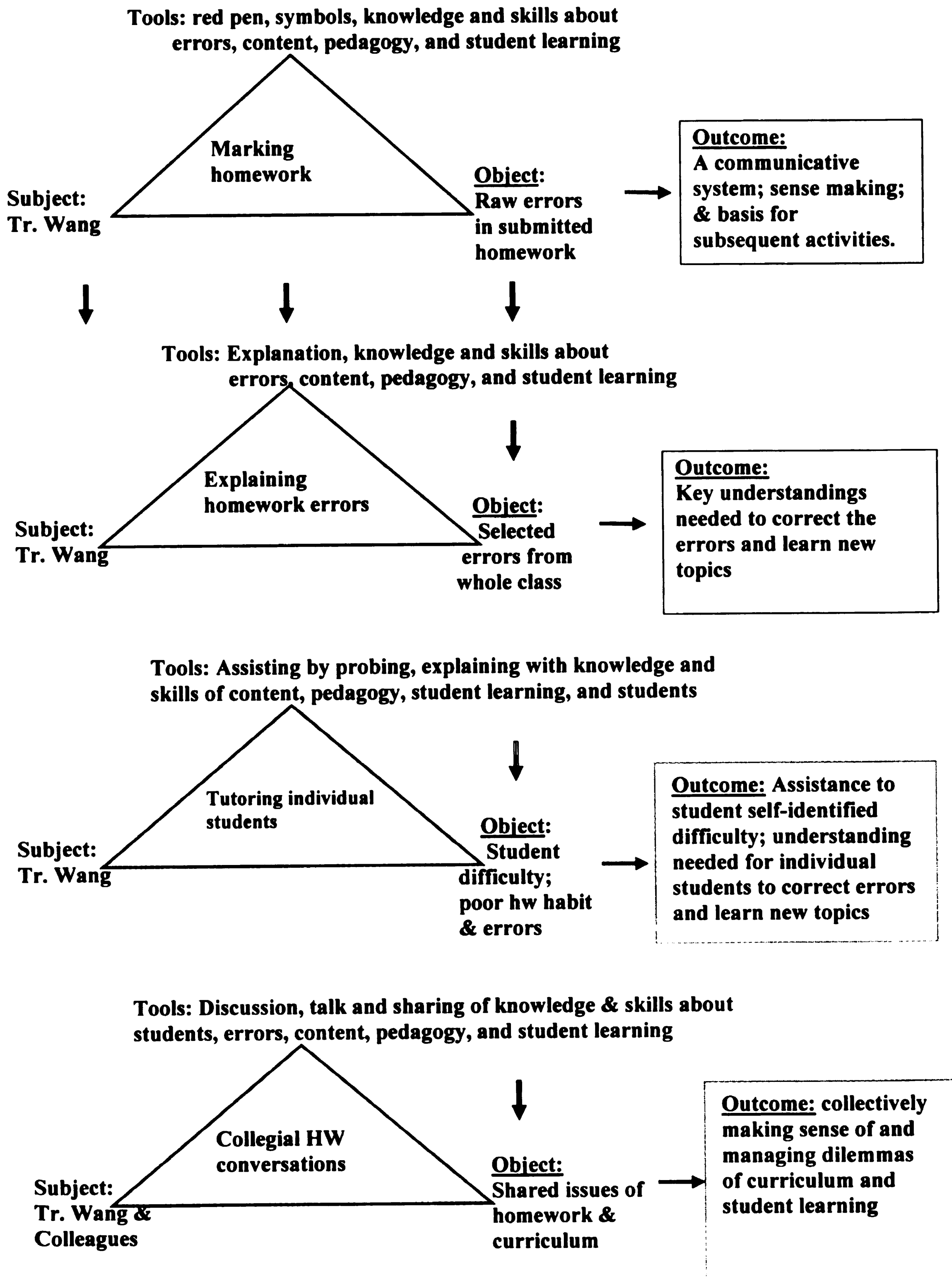
This chapter first summarizes the dissertation and then explores the implications of the study. In the summary, I work to put together the system of homework that has been dissected in the previous seven chapters. In implications, I consider the practice that the dissertation has presented as a social practice with a transparent technology and a cultural practice that is rooted in Confucian heritage.

Putting the System Back Together

As Chapter One and Two illuminate, student homework took up a lion's share of Tr. Wang's daily work hours and was always a highly visible object of her practice. From the trajectory of her routine interactions with homework and related student learning issues, she used homework as a tool to communicate with different audiences in different ways. Each use opened up a dynamic level of interaction between the teacher and the object – the problem issues mainly in the form of errors in student work, and between the teacher and her students and colleagues in utilizing the errors as resources for assisting student learning. To provide a rationale for such a practice, I use Chapter Three to locate homework in the curriculum and help the reader understand that the homework is designed with schemes to induce students to err so that teachers can make use of the errors to assist learning.

For analytical purposes and to allow readers to inspect each level of the tool-mediated interaction in detail, in the previous four chapters (Chapters 4-7), I artificially divided the system into four different activities based on the differentiable object formed at each level and dedicated a separate chapter to each activity: marking homework, explaining homework errors to the whole class, tutoring individual students on homework errors and collegial homework conversations. Along these four different levels I showcase the dynamics of the interactions in which the object was constructed and then transformed into the outcome that the teacher aimed to obtain. The following pulls together the schematic representation of such processes.

Figure 8.1 System of T. Wang's Homework Activities



A Social Practice with Transparent Technology

Together, these four levels of activities mark a significant part of Teacher Wang's practice with each activity, Tr. Wang constructed a different dimension of the object and produced different outcomes in her interaction with students and colleagues. In marking homework, she mined the raw errors by assigning to them signs and symbols and built a communicative system to inform students of their errors and remind them to make timely corrections. Mining raw errors and communicating them to students via their marked workbooks was obviously not her primary goal. Accompanying ticking and crossing were her actions of making sense of the errors, selecting and sorting and making decisions to follow up and address errors.

Such actions were directed at mining deeper into the errors as if she found the ores containing the precious stones. This process formed two more refined objects: errors representative of the learning obstacles for all students and those for individual students. Explaining the errors to the whole class offered her the opportunity to make the important mathematics and ways of learning them visible to students and cleared away their obstacles for new learning. Tutoring individual students on the difficulty they encountered and their errors in homework not only gave her the opportunity to meet the needs of individual learners of different levels but also enabled her to diagnose and treat individually manifested symptoms of learning. Talking with colleagues not only allowed her knowledge and experience about students' learning, content and her own teaching to be shared and examined with others but also, more importantly, offered opportunities for colleagues to collectively make sense of the uncertainty and ambiguity in teaching.

In short, this dissection allows us to see how the raw errors were being mined and polished at different levels of activities (indicated by the vertical arrows on the diagram) and eventually turned into shining gems of important pedagogical and learning opportunities for students as well for the teachers (indicated by the horizontal arrows). Located in such communities of practice, Tr. Wang was surrounded by persons, tools in which "participation in an activity system about which participants share understandings concerning what they are doing and what that means in their lives and for their communities" (Lave & Wenger, 1991, p.98). This activity setting constitutes the middle level between the systems and structures and the daily practice, which Engestrom (1998) regarded as missing. The activity system described for Tr. Wang stands as a visible middle level full of tool use, information flow and exchange, construction and transformation of errors for student learning of accurate mathematics.

This system of homework use constitutes the technology of practice in Tr. Wang's community of teaching (Lave & Wenger, 1991). From the above triangular representation, the mechanisms of the system at each level with its "inner workings" depicted in the proceeding chapters are open for the observer to view. In this way, it is almost a "glass box" (p. 102) – transparent.

Following the homework trajectory from Tr. Wang's office to classrooms, as an observer, I was able to record and describe her homework activities and make meaning out of the internal relationships and teacher actions involved. At the same time, I was also able to capture the detailed moments of Tr. Wang using homework as a tool to enter into students' minds, save, retrieve and adjust and use information she obtained for reasoning and taking actions. These become "further fields of transparency" that are "intricately tied to the cultural practice and social organization" (p. 102). Through these relationships, actions, and reasoning with tools and artifacts, the technology "fulfils a mediating function" that "involves specific forms of participation" (p. 102) by members of the communities.

This supporting technology of Tr. Wang's technology has a dual nature, "invisibility and visibility" that is best captured by Lave and Wenger's analogy to a "window":

> *A window's invisibility is what makes it a window, that is, an object through which the world outside becomes visible. The very fact, however, that so many things can be seen through it makes the window itself highly visible, that is, very salient in a room, when compared to, say, a solid wall. Invisibility of mediating technologies is necessary for allowing focus on, and thus supporting visibility of, the subject matter. Conversely, visibility of the significance of the technology is necessary for allowing its unproblematic – invisible – use. (p. 103)*

The technology of homework studied in this dissertation research has remained invisible to both outsiders and insiders for many reasons. Observers of mathematics teaching in the Chinese education system almost always focus on what is going on in the front stage – classroom instructional processes and rarely are interested in what happens in the backstage – the teachers' territory, their office, where dynamic teacher-student interactions still take place besides those between the colleagues, as the present study illustrates.

Even when I made a special trip to videotape the dynamic teacher interactions in the office in 2001, the focus did not start with homework but eventually it caught me by surprise when I noticed from data how much the teacher interactions and even those with students that happened there were focused on homework. Native researchers who have learned about my research on teachers' office and homework reacted initially with lukewarm interest – is there anything to see or are they worthy of study? They wondered.

To researchers, this invisibility of homework is largely due to the assumption that things that happen there do not have much to do with teaching[1]. Surprised teachers and students showed the same reaction – "What? Study our homework? What is there valuable in it for you to study?!" In many ways, homework is what students and teachers do everyday, so like the air they breathe, they don't feel it, which makes homework invisible or even unworthy of attention. The other reason is that homework is marked but not graded for any credit in exams. Yet, this invisibility to insiders was what allowed them to highly focus on and "support the visibility of, the subject matter" (Lave & Wenger, 1991, p. 103) of the practice – the errors and learning issues reflected from homework, which is what most of their interactions were about, from marking to selecting errors to explain and tutor to students and to collectively negotiate meaning of those homework ambiguities with colleagues. In this way, the technology puts student learning at the center of the practice. As Lave and Wenger termed it, this form of the invisibility allowed students' and teachers' "unproblematic interpretation and integration into activity" (p. 103).

At the same time, homework is a highly visible artifact traveling from student homes to the teacher's desktop as well as the topic of discipline or discussion when a student failed to submit it or turned in poor quality work and when disagreements occurred among colleagues. It opened a visible window onto the teachers' practice; it was through this window, light and air came in as information that fed their teaching and

[1] But Professor Gu's view point is very different – his 20 years of research and experiments in rural schools included carefully observing the change of effect of teacher feedback to homework.

lighted up the room of their practice to allow them to see and then deliberate on the different manifestations of problems bearing upon student learning. When the researcher was able to join the flow of life in there, I was able to open up the inner workings of this hidden visibility for more to view and argue about the significance of this technology in Tr. Wang's practice.

One salient aspect of this significance is the accountability system it created and supported. It was a rigorous system that closely tied the teachers up to helping as many students as possible to "get it right" in homework and eventually in the exams and in entering the next grade and senior high school for further learning. Many would argue that it was largely owing to this technology that the Chinese education system has achieved, especially in math, the goal of "two basics" set up in the past two decades – basic knowledge and basic skills, which has helped lay foundation for children's further learning. This system resembles an airplane cockpit (Hutchins, 1999), with its distributed knowledge and responsibilities in the technology of piloting its crew (Tr. Wang and her colleagues as well as the students) worked together to prevent foreseeable and unforeseeable mechanical failures and weather conditions (errors and learning difficulties) to safely deliver the passengers to its destination.

A Cultural Practice Rooted in Confucius Heritage

The directing role of examination. What is this destination the piloting crew is heading towards? In a cultural practice of teaching deeply rooted in Confucius heritage, the most immediate destination is passing the exams successfully. In Tr. Wang's daily practice, the examination exerts a powerful role "behind the scenes". In her talking with colleagues, the controversy of an ambiguous assignment involving multiple answers or solutions that were different from the official key often led to rich discussions on the causes of ambiguity, such as in Case 2 and Case 3 of Chapter Seven. While such collegial deliberations allowed teachers to gain better understanding about the controversial content and teaching related problems to better assist student learning, how far such deliberation could go was often determined by whether the exam would include such kind of problems. For instance, in Case 3 of Chapter Seven, they decided not to lay too much emphasis on the discussed problem for two reasons: first, they did not want to confuse the students, beginners in learning functions, by taking too much pain meddling with the conceptual implications; and second, because in general such ambiguity would be avoided in an examination.

One of the most often discussed topics among the teachers in the office or at meetings was the school's (district's) policy of "higher average class scores" in the final exams. This policy made teachers pay more attention to helping raise the performance of the weaker students so that they would not lower the class average score. This explained why most of the students receiving tutoring were average and below average learners in math. On the one hand, this caused anxiety for teachers, especially the novice ones like Tr. Zhao who worried about the feasibility of bringing up the level of weaker students; on the other hand, it pushed them to spend more time working with the weak learners and come up with more strategies to take care of all levels of learners, such as the strategy of "treating each level with due attention" that the teachers developed in their Lesson Planning Meeting in the afternoon of November 19 for the upcoming month-long review for the final exam.

This "average score" policy held by education in Shanghai reflected the "Confucian presumption that everyone is educable" (Lee, 1996, p. 29). Given that Confucius, as an educator, taught students from different backgrounds, he recognized differences in intelligence among learners, but he believed that these differences "do not inhibit one's educability, but the incentive and attitude to learn does" (Lee, 1996, p. 29). This tradition becomes controversial in the teaching and learning of mathematics facing large classrooms of over 50 students, at least in the practice of Tr. Wang and her colleagues. The situation that many of their students found it too hard to learn geometry proof made them think that the common curriculum set an unrealistic expectation for all students to learn well and have the ability to think mathematically.

While bearing grudges against such impossibility, in their teaching, they made effort to meet the different needs of all learners through additional teaching effort, such as explaining and tutoring. In both these two activities, Tr. Wang gave multiple and detailed explanations on student errors and engaged the students with questions in a process of going through the errors. This explanation giving practice was clearly teacher-led with the teacher as the knowledge giver and students, the receiver; yet, these explanations were not direct showing and telling because they entailed analysis, probing and different perspectives.

Two conflicting images of the practice. In analyzing and writing about these teaching segments, I always found two conflicting images of teaching mixed together: I oftentimes was left perplexed – what is teaching a practice of? Some could argue explaining and tutoring were constructive (such as Stigler & Stevenson, 1991) because they were based on or constructed from student learning problems from homework and developed through questions. Yet the teacher was the leader instead of the facilitator. They are rich and detailed in terms of the mathematics and elements of learning embedded. Yet they were guided by only one-right way, the seeking of and learning by the correct answer; activity focused on changing the wrong to the correct. They are inquiry-based learning because they involved chains of probing and questioning.Yet the teacher did look like an authoritarian figure with knowledge and power in her hand, and students did appear to be passively responding and receiving. They are transformational because errors were picked up, taken apart, examined, explained and turned into concrete embodiment of subject matter and ways of learning. Yet they never moved beyond the confine of the curricular expectations to reframe the problem and explore the "big ideas" and the "aha moments".

The question slowly dawns on me – this might have to do with the cultural frames that I choose to view this practice. Reading of research literature on teaching and education published in the West during my advanced training has somehow tempted me to view certain types of teaching as reform-minded and others as traditional. I am caught to be dichotomizing and I am unaware that I am doing that. Such dichotomizing also exists among Western scholars when they observed classroom teaching in East Asia. Ginsberg (1992), after a visit to China and Japan, reported:

> *In China, knowledge is not open to challenge and extension (by students arguing with their instructors) ... The teachers decides which knowledge is to be taught, and the students accept and learn that knowledge. The lecture is the authority, the repository of knowledge, leading the students forward to the knowledge, a respected elder transmitting to a subordinate junior.* (Ginsberg, 1992; p. 6)

Ginsberg's observation did echo one side of the mixed practice with which I have been bewildered. Stigler and Stevenson (1991), however, had opposite views after they observed classrooms in China, Taiwan, and Japan:

> *A common Western stereotype is that the Asian teacher is an authoritarian purveyor of information, one who expects students to listen and memorize correct answers and procedures rather than to construct knowledge themselves. This does not describe the dozens of elementary school teachers that we have observed.* (Stigler and Stevenson, 1991:43)

These two opposite observations represent very well the two sides of my own observation. Stigler & Stevenson saw the teachers they visited as "posing provocative questions, allowing reflection time, and varying techniques to suit individual students: Confucius' elicitation mode in full swing. They used the term 'constructive' purposefully to describe the commonest teaching approach they saw, an idea espoused by progressive Western educators and in practice realized only by the expert few " (Driver and Oldham, 1986; Tobin and Fraser, 1988 cited by Biggs, 1999:55-56) . The essence of this elicitation approach "lies in motivating and engaging the initiatives of the students" *(diaodong xuesheng de jijixing)* (Wang el al, 1992: 30).

Ausubel (1968) made a distinction between rote and meaningful learning in terms of the product: the formal generates verbatim content while the latter produces meaning capable of transformation. From this perspective, Tr. Wang's practice as mining errors was indeed transformational.

I argue that the pedagogy of student homework errors conducted by Tr. Wang and colleagues centered around students' initiatives in several ways: first, errors as problems of learning are owned by students. By taking these problems as problems of teaching, teachers not only turned them into opportunities for them to generate meaning from errors but also provided prompt and detailed feedback to students so that errors became resources for both teaching and learning. The process of explaining and tutoring got students into the teaching mode as the teacher probed with a series of questions. As mentioned earlier, it created a metacognitive learning experience that cultivated students' thinking about their own routes to learning (Shoenfeld, 1991).

Long time observers of learners in the Confucian learning contexts, such as Biggs and his colleagues (1999, p. 45) called such learning as "deep approach" versus "surface approach" and they commented that "(h)igh attainment and deep approaches are however complementary bedfellows; one of the reliable outcomes of a deep approach is a correct answer" (Biggs, 1996/1999, p. 45). Tr. Wang was leading students to correct answers; but correct answers were only the result, not the process: The meaningfulness of correct answers to teaching and learning lies not in the result but in those "search actions" that seek and define problems (Engestrom, 1999) from student routes to learning and understanding. Such a process might well be called "sticking probing" in which "the focus of the probing is typically a math error made by a particular student, which the teacher believes would be instructive to publicly unpack and reconstruct…" (Hess and Azuma, 1991, cited in Biggs, 1996/1999, p. 45).

This is not to say that these teachers are non-authoritarian. The correct answer seems to suggest there is only one "right way" and students must tread that path (Biggs, 1996/1999, p.54). Gardner's (1989) first impression of his study of the art and music education in China was that this was "mimetic" teaching (highly directive and imitative). His more prolonged observation led him to think that this is "transformational" teaching – "student centered and creative" (p. 15). His interpretation was that "the Chinese believe in skill development first" (involving repetitive practice) while the Westerners "believe in exploring first, then in the development of skill." The belief that art is both beautiful and morally good leads to the pursuit of the "right way" and the teaching is by "holding the hand" (Biggs, 1999, p.55) and teaching itself as endless rehearsal leading to virtuoso performance (Paine,1990). On the one hand, given this mixed observation, good international comparative studies in education could provide valuable insights by comparing and contrasting different systems of practices. But using it to argue for one version of reform-minded teaching could lead to further dichotomizing and stereotypes.

The limiting role of the technology. Traditionally, the technology of the practice as examination-oriented education has succeeded in laying a solid foundation for the next level of learning and created a system that promotes teacher quality and accountability. Especially in mathematics, students from the Confucian cultural contexts have achieved high academic performance in mathematics. However, tools both afford cultural practice in that they provide a means of action and constrain new action through the specific purposes suggested by prior use. Tools can thus be seen as liberating in their enabling function or limiting in that their historical uses may preclude new ways of thinking (Wertsch, 1991). I also found teachers in my study do not use student initiatives in correcting errors; lack initiative to use their own perspectives to reframe the curricular requirement to fit their own pedagogical needs; stay at the level of interpreting and problem solving within the curricular framework and rarely move beyond the confine of the curricular authority for new and different ideas of organizing and delivering the content.

This examination technology has been seriously challenged in the recent decades when it has posed more challenges and contradictions to meeting the needs of today's technology and global economy. The current curriculum reform that seeks to develop creativity and problem solving skills has incorporated many recent new ideas of mathematics content, cognitive theories and pedagogical approaches, such as student-centered approaches and teacher as facilitator (Wang, 2004). To meet the demand for reform, this technology of homework-based practice still has to play its routine roles while adapting itself to new ambitions for teaching and learning. Tr. Wang and her colleagues will experience learning from the conflict between the conventions and new thinking. It will be interesting to find out how this technology of practice is going to mediate this different set of experiences.

Implications for Future Research

This dissertation study on the homework-related activities of an experienced 8^{th} grade mathematics teacher in Shanghai helps raise questions about what teaching entails, how to organize teachers' work to place student learning in the central place of teaching and the many facets of homework in supporting that work. It also helps us rethink the role of homework in teaching and learning, in professional development as well as in teacher education.

Organizing teachers' work for both teacher and student learning. Tr. Wang's work days are filled with long hours of interactions around student homework. The real meaning of the long processes lies in the fact that it involves her continuously making sense of issues of student learning, collecting important and immediate information from student work to inform teaching and create more timely teaching opportunities and assistance to resolve student learning problems. This kind of organization of her work makes knowing and knowledge to be distributed in the tools, artifacts, persons that she worked with from moment to moment. Her role of a teacher is not just planning, teaching and grading; it encompassed continuously gathering information, analyzing and using the information to inform her on-line decision making for teaching and helping students learn. In this sense, the organization of her work makes her a researcher of her own teaching with the central purpose of ensuring that information from teaching and student learning is processed and fed back in the system of teaching for reuse. This system is resource- and information-rich and works efficiently for the central purpose of safeguarding student learning.

For a teacher who has worked in this system over time, her knowledge about content, pedagogy and student learning has become deep, thorough and well connected, as Ma (1999) documented in her study of a group of mathematics teachers in China. The implication for the organization of teacher's work in the U.S. is to encourage inquiry about both the organization of teacher's work in terms of how they spend their day and the tools and persons they have time to interact with informally. This study raises questions about the structuring of both the time and space of teacher's work and their roles as teachers.

Pedagogical role of homework. As we have discussed, homework in this system of teaching is a boundary object that is able to help cross the boundary of teaching and learning when used systematically for sense making and information gathering and using. It allows the teacher to both create and coordinate different interactions with students and colleagues around the central purpose of enabling informed teaching and result-oriented learning. When used in this way, homework becomes an important pedagogical tool besides student practice to retain what is learned. This helps us rethink the role of homework in school learning and how to design curriculum and homework in ways that allow the pedagogical value of homework to be mobilized.

Implications for professional development. Homework as a boundary tool for teaching has important implications for designing professional development initiatives that are workplace-based. While teachers need to equip themselves with new and continuously upgraded knowledge and skills to teach better, it is more important to base their learning and professional development firmly on what they do and what tools are available to use. This study poses the possibility of designing a boundary object that not only connects teaching with students learning but also links what a teacher does with their own knowing about student learning for the purpose of informing their practice and learning from what they do.

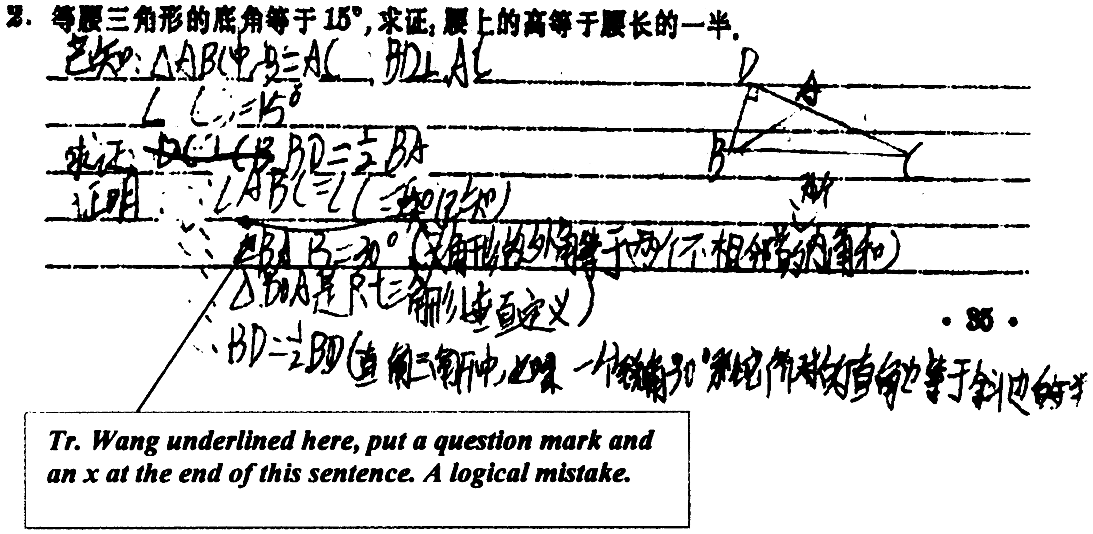
Implications for teacher education. Tr. Wang's case suggests that teacher education programs might consider educating future teachers to reconceptualize homework as an important pedagogical tool. It should create specific activities that help them move from the grading- and credit-earning type of homework use to designing and using homework purposefully as salient tools to connect students learning with teaching by informing their decision making and diagnosis of student learning.

APPENDIX A

SAMPLES OF MARKED STUDENT HOMEWORK
(GOING WITH CHAPTER FOUR)

Assisting students in understanding the deductive system.

Student Work Sample 1



2. An isosceles triangle with its base angle equal to 15°.
To prove: the altitude on its side is half of the side.

Known: in ΔABC, AB=AC, BD $\perp$ AC, $\angle$C=15°.

To prove: DC $\perp$ CB, BD=1/2AB (*The shaded perpendicularity was crossed off by the student.)

Proof: $\because$ <u>$\angle$C=$\angle$ABC =15° (Known)</u>

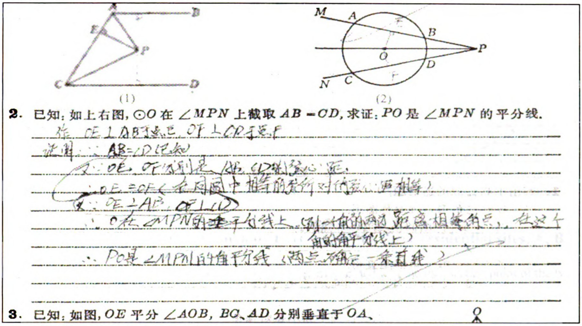
$\therefore$ $\angle$BAD =30° (An external angle of a triangle is the sum of the two distant angles of the triangle.)

$\because$ ΔBDA is Rt triangle (Definition of perpendicular lines/altitudes)

$\therefore$ BD=1/2AB

(This assignment was selected to explain to the whole class on November 15, 2002.)

Student Work Sample 2



2. 已知：如上右图，⊙O 在 ∠MPN 上截取 AB = CD，求证：PO 是 ∠MPN 的平分线.

作 OE⊥AB于点E，OF⊥CD于点F

证明：∵ AB=CD（已知）

又∵ OE、OF分别是AB、CD的弦心距，

∴ OE=OF（在同圆中相等的弦所对的弦心距相等）

又∵ OE⊥AB，OF⊥CD

∴ O在∠MPN的平分线上（到一个角的两边距离相等的点，在这个角的角平分线上）

∴ PO是∠MPN的角平分线（两点确定一条直线）

3. 已知：如图，OE 平分 ∠AOB，BC、AD 分别垂直于 OA、

***2. Known: As in the above drawing on the right, circle O intersects ∠MPN and AB=CD.**
To prove: PO is angle bisector of ∠MPN.

Draw OE ⊥ AB meeting AB at point E, OF ⊥ CD meeting CD at point F.

Proof: ∵ AB=CD (Known)

Also ∵ OE and OF are distances from the center of circle O respectively,

∴ OE=OF (In the same circle, the distances from equal chords to the center of the circle are equal.)

Also ∵ OE ⊥ AB, OF ⊥ CD (Tr. Wang circled this sentence and used an arrow to insert it in between AB=CD and OE and OF are DCCCs of AB and CD respectively.)

∴ Point O passes through the bisector of ∠MPN (All points that are equidistant to both sides of the angle are on the angle bisector.)

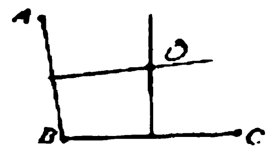
∴ PO is bisector of ∠MPN (Two points determine a straight line.)

(This exercise was chosen to explain to the whole class on November 20, 2002)

Student Work Sample 3

习 题 28.5(2)

· 说出如何找出一个点到如图的三个居民点 A、B、C 的距离都相等.



Incomplete writing.

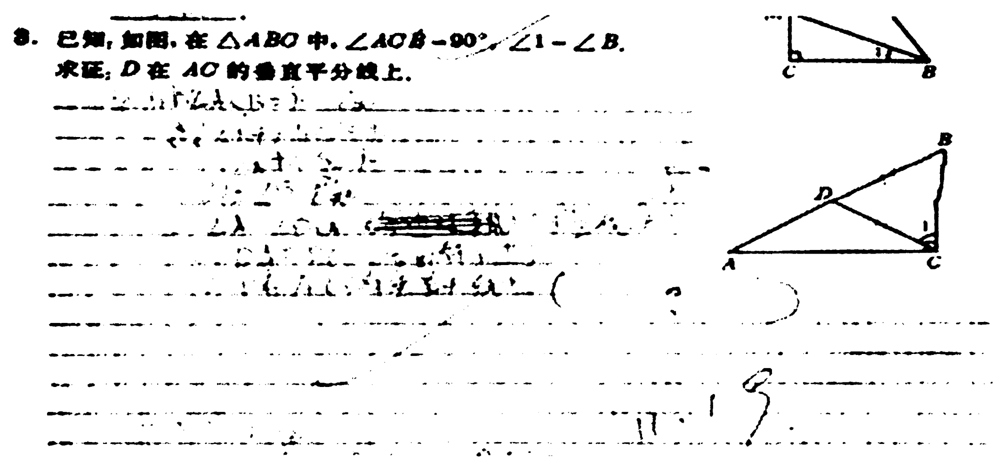
2. 填空:

1. **Say [Write] how to find a point reaching the three residential sites, A, B and C in equal distance.**

   Construct the perpendicular bisectors of segment AB and BC; the intersecting point of the two perpendicular bisectors is what is required to find.

(This assignment was explained to the whole class on November 19, 2002)

Student Work Sample 4


3. 已知，如图，在△ABC中，∠ACB＝90°，∠1＝∠B.
求证：D在AC的垂直平分线上.

3. **Known: See drawing: in Triangle ABC, Angle ACB=90 degrees, Angle 1=Angle B, To prove: D is on the perpendicular bisector of AC.**

Student Work Sample 5

Problem No. 2: drawing same as student work sample 2.

Proof: Connect MC

∵ M is midpoint of AB, Δ ABC is right triangle, **<u>∠ACB = 90° (added by Tr. Wang with her red pen)</u>**

∴ MB = MC (Median on the hypotenuse of a right triangle is half of the hypotenuse)

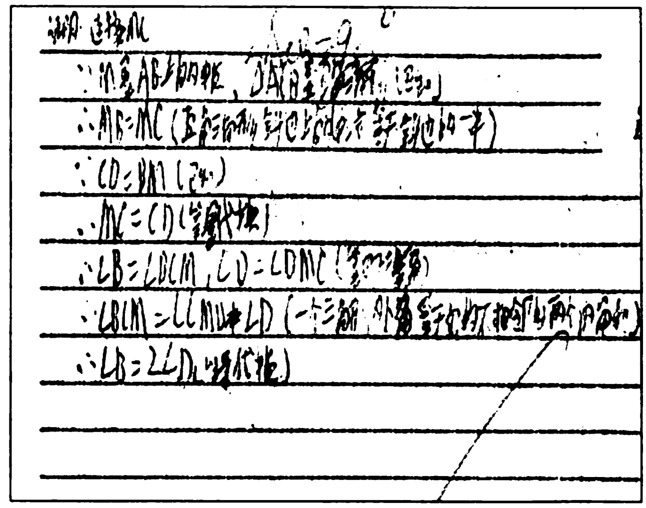
∵ CD=BM (Known)

∴ MC = CD (Substitution)

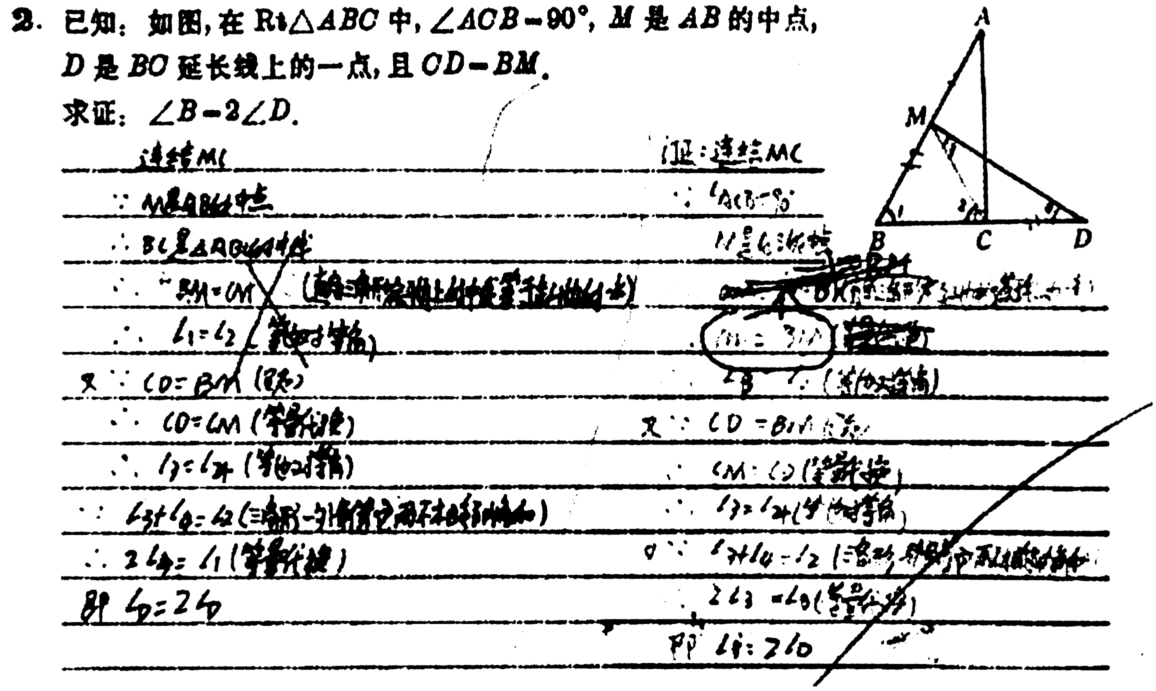
∴ ∠B = ∠BCM, ∠D = ∠DMC (Equal sides face equal angles)

∵ ∠BCM = ∠CMD + ∠D (An external angle of a triangle is the sum of the two distant angles.)

∴ ∠B = 2∠D (Substitution)



Student Work Sample 6


2. 已知：如图，在 Rt△ABC 中，∠ACB=90°，M 是 AB 的中点，
D 是 BC 延长线上的一点，且 CD=BM.
求证：∠B=2∠D.

2. **Known: As in the drawing, in Rt Δ ABC, ∠ACB = 90°, M is midpoint of M is midpoint of AB, D is a point on the extension of BC, and CD = BM.**
To prove: ∠B = 2∠D
Proof: Connect MC
∵ M is midpoint of AB
∴ MC is median of Δ ABC
*∴ BM = CM (Median on the hypotenuse of a Rt triangle is half of the hypotenuse)
∴ ∠1 = ∠2 (Equal sides face equal angles)
Again, ∵ CD=BM (Known)
∴ CD = CM (Substitution)
∴ ∠3 = ∠4 (Equal sides face equal angles)
∵ ∠3 +∠4 = ∠2 (An external angle of a triangle is the sum of the two distant angles)
∴ 2∠4 = ∠2, that is, ∠B = 2∠D

(Student redo with corrections made)
Correction: Connect MC
*∵ ∠ACB = 90°
M is midpoint of AB
*∴ CM =1/2 AB (Median on the hypotenuse of a Rt triangle is half of the hypotenuse.)
∴ CM = BM (Substitution).
∴ ∠B = ∠2 (Equal sides face equal angles)
Again, ∵ CD=BM (Known)
∴ CD = CM (Substitution)
∴ ∠3 = ∠4 (Equal sides face equal angles)
∵ ∠3 +∠4 = ∠2 (An external angle of a triangle is the sum of the two distant angles)
∴ 2∠4 = ∠2, that is, ∠B = 2∠D

Note: * The underlined step on the right side indicates the step marked as wrong in the original student proof. The underlined steps on the left side indicate the added corrections made by the students.

APPENDIX B

SAMPLE OF CODING ANALYSIS ON TRANSCRIBED EXPLAINING SEGMENTS

Time: 9:50 AM, Tuesday, November, 19, 2002
Place: Class 4
Topic: Explaining Assignment 1
Length of transcript: Review plus explanation total 7.5 minutes
T: Teacher (Tr. Wang) S: Students in Class 4
I: Teacher Initiation; R: Student Response; E: Teacher Evaluation

I	T*:	Where to find this point? *(Writing on board)* First, a point on a perpendicular bisector. **If a point is on a line segment's perpendicular bisector, then what characteristics does it have?**
R	S*:	*(Indistinctly)* It is of equal distance to the ends of the line segment.
E + I	T:	Ah (Yes), its distance to the ends of the line segment are equal. Is it so? Some of you are able to say it but unable to use it. **Conversely, the points that are of equal distance to the ends of a line segment** *(in a hushed and mysterious tone)***?**
R	S:	*(Continues the T's words)* are on this line bisector.
I	T:	**(They) certainly are on what?**
R	T+S	(They) certainly are on the perpendicular of this line segment.
I	T:	Then according to this converse theorem, we say, **if we want to find a point that is of equal distance to two known points, then where shall we go to find this point?**
R	S:	*(Some students)* On its perpendicular bisector.
I	T:	*(Pointing to the blackboards)* These are two known points A and B, I'd like to find a point whose distance to A and to B is equal. **Then could you say that I go and look for this point blindly (everywhere)?**
R	S:	No.
I	T:	*(In more mysteriously tone)* **Where to look for it?**
R	S:	*(Many students)* On the perpendicular bisector of AB.
S * … I	T: … Transition to homework	*(Pushing in the same tone again so that all students could join her.)* To where? The perpendicular bisector *(again emphatically)* of AB. Is it right? Then, all such points, aren't they all concentrated on its perpendicular bisector? Are there any such points elsewhere? *(Pause)* Right or not? This is how its converse says: all points that are of equidistance to two given points are on the perpendicular bisector of the line segment that the two points determine *(run through) (slowly, distinctively pronounce the words in the theorem).* Is this so? Is this what this converse theorem says about? Where is the point that is of equal distance to two fixed (given) points? It's on the perpendicular bisector of this segment, this segment. On the segment that is determined *(emphatically)* by the two points. Right? Then, the homework we did yesterday. There are three points given

035 036			here *(pointing to the board at the already drawn points)*, now we will find the point, so that its distances to these three points are all equal. Now here, there are three points *(emphatically)*, how should I solve this problem?
037	R	S	Connect two points…
038	I	T	**How many points I should consider first?**
039	R	S	*(A few students)* Two.
040 041 042	E + I	T	Two points. And then, I will consider another two. Is it like this? Then first let's look at here—first find the point whose distance to AB is equal. Then, **where should it be?**
043 044	R	S+T	On the middle perpendicular line (the perpendicular in the middle) *(zhong chuixian)* of the line segment, AB.
045	I	T	**The middle perpendicular line is?**
046	R	T+S	The perpendicular bisector.
047 **048** 049 050	*D* * I	T	Ah. Then, we, it says, have to say how to find it. How to find it? *(Some students were starting to answer).* **First,** draw line segment AB, connect AB, right? *(Pointing at the board).* Find the point of equal distance to A and B, **where are all of them concentrated?**
051	R	S	*(Some students)* On its perpendicular bisector.
052 053 054 055	*D* I	T	**Second,** draw its perpendicular bisector *(saying the term emphatically).* Perpendicular bisector. First find its midpoint, then draw the perpendicular line through it, draw its perpendicular line, right? *(Drawing while speaking).* Good. **Then, do what? Consider which two points?**
056	R	S	(B and C…)
057 058	E I	T	The points that are of equal distance to the two points B, C. **Where are they for sure?**
059	R	S	Its perpendicular bisector.
060 061	E I	T	The perpendicular of the segment, BC. Is it right? Then, our **third step** is, **connect?**
062	R	T+S	BC, draw the middle perpendicular line of BC.
063 064	E I	T	Ah, draw middle perpendicular line of BC. Good. Now, **have we found this point?**
065	R	S	Yes, we have.
066	I	T	**Found it or not?**
067	R	S	Yes. *(One student closer by me said, "Not yet.")*
068 069 070	E I	T	The intersecting point of this two perpendicular bisectors is this point that I want to find, is it right? *(Lower her voice mysteriously)* **Do we need to draw a third perpendicular for A and C?**
071	R	S	No need to.
072	I	T	*(Raise her voice).* **Do we need to consider this?**
073	R	S	No need.
074	I	T	Do we? *(Push again.)*
075	R	S	No.
076	*D*	T	Many students have drawn three perpendicular bisectors. Is this

I *D*		necessary? If you failed to draw any of the three accurately, they won't intersect at one point.
	S	*(A student)* They won't intersect.
I	T	Right? *(Particularly to this student)* Then, is this point of equal distance to all three points? **The perpendicular bisectors, OB and OC, are they equal?**
R	S	Yes.
I	T	**Are OB and OA equal?**
R	S	Yes, they are.
I	T	Then, **aren't OA, OB and OC, the point we have found are of equal distance to all the three points?**
D I	T	As the **<u>last step</u>, what do we need to say?** For construction exercises, what do we need to say?
R	S	Point O…
I	T	Conclusion, what? Point O is what is required to find. *(diant A wei suo qiu.)* **Have you written this?**
R	S	*(Nearly all answered in unison)* No.
S *D*	T Summarize <u>Generalize</u>	All constructions require you to write its conclusion! *(Emphatically).* This exercise also requires you to say how to get this point, that is, write the construction methods, right? How do we say our construction methods? **How many sentences?** In order to explain clearly about your construction methods, you need to go step by step. ***The first sentence***, draw line segment AB or connect AB. Or the first sentence does two tasks: connect AB and BC, right? ***The second sentence***, draw the perpendicular bisector of AB, OE and the perpendicular bisector of BC, OF. The two perpendicular bisectors intersect at point O. Right? ***So,*** point O is the point we need to find. **So for the construction exercises, we do as required to write the conclusion as well as the methods we draw it.** Of course, we need a clear understanding of the converse theorem of perpendicular bisector (in order to do it). As for how to make a perpendicular bisector, you can also observe from the drawing you construct. And when drawing, you have to have a ruler and a protractor so as to be accurate. *(The following sentences she said are too low to hear clearly).* Those who did not write complete, after I return your workbooks to you, please do it again and make it complete after lunch.

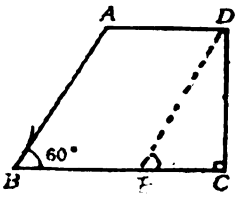
APPENDIX C

TUTORING A GIRL ON HER HOMEWORK ERRORS

Time: In between the 4th and 5th period in the morning of November 15, 2002.
Topic: Tutoring a girl that Tr. Wang noticed in the office and called to her desk for tutoring.
Note: This segment occurred right after Tr. Wang tutored the girl on her first two homework assignments.

4. Known: As in the drawing, AD // BC, ∠C=90°, ∠B=60°.
To prove: BC = AD + ½ AB

4. 已知，如图，AD∥BC，∠C=90°，∠B=60°.
求证：BC=AD+$\frac{1}{2}$AB.



T:	And also. You look, two lines parallel (to each other), the alternate angles are equal. This condition, you should point it out before you have this angle as 60 degrees. Ok?
G:	Yes.
T:	Because here there does not exist a 60 degrees. Right? The alternate angles are just equal. This equals 60 degrees (stress 60 degrees) because angle B (stress on B) is 60 degrees? Right? **Then how about this (stress on "this")?**
G:	This is…
T:	Angle DEC equals angle ADE.
G:	This is alternate angles are equal and then…
T:	These alternate angles, they are alternate angles because **who (which segment) is parallel to who (which segment)?**
G:	AD//BC.
T:	**Then,** this is wrong. This *(Diagnosing)*
G:	AB//DE
T:	**But** you said this is parallel to this.
G:	I want to omit…; I wanted to save some work.
T:	What? If you want the two corresponding angles (angle ABE and angle DEC) be equal, **you should have which two segments be parallel?**
G:	This (AB) should parallel to this (DE).
T:	Right. I should have this and this parallel and then this
T:	And also, why do you want these angles to be equal? What is a *(Diagnosing)* parallelogram? *(Music for the new lesson started.)*
G:	A parallelogram is a figure that the opposite sides are parallel to each other.

T:	Ah. A quadrilateral with two pairs of opposite sides respectively parallel to each other (the girl saying together with the teacher.) **Do you still need the angles?**
G:	No. No need.
T:	You don't need the angles. So you don't seem to clearly understand the definition of a parallelogram.
G:	I'll take it to correct (the mistakes).
	(The girl rushed to her classroom with her workbook in her hand.)
	(The counter stops at 036.) This occurred in between the fourth and fifth classes.

APPENDIX D

SAMPLE OF COLLEGIAL HOMEWORK-RELATED CONVERSATION BETWEEN TR. WANG AND TR. ZHAO (GOING WITH CASE 2, CHAPTER SEVEN)

Conversation		Dynamics and some context	Moves	Color coding of knowledge categories	Memo
J:	Jiang: Chen XX *(she addresses Tr. Chen by name because he's younger),* I have to ask you. Eh, this converse proposition, I have to ask you. An isosceles triangle is an axial symmetrical figure. How should you write its converse theorem?	Jiang's eager to know so that the conversation happened during break with a parent talking with a teacher, with students coming and going, noisy and busy	Eager to know and asks immediately arriving in the office.	Content Knowledge (CK): *Convention of representation* PCK (about **both** "what" and "how" of the math entailed in helping student learn. SLK(Knowledge of student learning)—this question arises from student answers in class.	Jiang "**hear" students** and her own inner conflicting moment in her teaching. She seeks to manage with it by consulting a colleague. Sensitivity to student ideas and mathematics entailed.
C:	Chen: This… it depends on how you understand it…	Chen answers immediately and shows his interests.	Immediate reaction with interest.	PCK: both about the "it" and the way to understand it in CK and teaching	He suggests that there is not only one answer/way.
J:	Jiang: Then this is false proposition.	States question and seeks confirmation.	Further states her question.	CK	
C:	Chen: Yes. But I think the first way of writing is right	He acknowledges the answer.	Disagrees and seeks external	CK	

J:	Jiang: I think the second one is. Can I use your Teaching Reference Material (TRM) (to see the key)? I don't know where I left mine. When you teach this…	They disagree on which way of writing is right; so seek the standard key in the Teaching Reference Material.	authority---the standard key.	Use of CK SSK (standard solution key) PCK	Jiang tends to look for the SSK in the TRM when she feels uncertain about the solutions of a math problem.
C:	Chen: It's vague, isn't it? What about you write it now into the form of "if…, then…'?	Chen finishes what Jiang wants to say and makes a suggestion.	Shares Jiang's feeling and gives suggestion	PCK A way of representing CK related to teaching and learning	The current textbook starts the Geometry proof Chapter with definition of Proposition, Axioms, & theorems. It suggests to change the proposition into "If…, then…" to identify the hypothesis and conclusion, the two parts of a proposition.
J:	Jiang: If according to me, if a figure is an axial symmetrical figure, it is an isosceles triangle. *(She thankfully took the TRM I handed her.)*	She answers Chen.	Answers	CK Use of CK to solve routine problem---to write the converse of a proposition	
C:	Chen: This is not a true proposition in itself.	Chen comments/judges the key.	Comments on or evaluates	CK	
J:	Jiang: This is a false proposition. The converse proposition is a false one.	Jiang restate with emphasis.	Repeats and stresses.	CK	

C: *	Chen: If I want to change the original proposition into "If…, then…." How do you say it?	Chen clarifies his suggestion.	Clarifies suggestion	CK PCK: pedagogically useful **Representation**	
J:	Jiang: If a triangle is an isosceles triangle, then it is an axial symmetric figure.	Jiang acts on his suggestions.	Acts on suggestion	CK: procedure, fact	
C: *	Chen: Yes. If you change this into its converse proposition, how do you change this sentence?	Chen continues his suggestion.	Continue his suggestion	CK: procedure PCK: pertain to math and way of representing (in teaching) **Representation**	
J:	Then change this sentence… (Jiang laughed. And they laughed together.)	Jiang sees the ambiguity—writing this way likely produces a true proposition. Chen confirms it by laughing together.	Understand what it is and understand each other		Disposition to appreciate ambiguity and uncertainty
C:	Is this so? Sometimes when the sentence is written in a simple concise form (such as the original given proposition), it is annoying/confusing.	Chen explains what he sees as a cause for the confusion—the teaching materials are too concise (math is highly compressed)	Explains a cause of confusion.	PCK: subtle CK in appreciating and evaluating the usefulness or tendency to confuse in representing math idea or concept Curricular K Unpacking **Decompress**	Math Geometry propositions and theorems are usually in highly polished and compressed form---concise and elegant ---**unpacking** it opens up different interpretations and ways of representation.

J:	Jiang: Yes. But for the second one (the second small exercise under Ex. 1 above), this kind of problem (confusion) does not exist. Some students wrote it this way: they did not mention that it is a quadrilateral. If a figure's all angles are right angles, then this figure is a rectangle.	Jiang agrees but feels that the second exercise does not have such confusion: **Write the converse of: Every angle of a rectangle is a right angle.** Without indicating the figure as a "quadrilateral", the converse still seems a true proposition.	Agrees and compares with the second exercise.	CK SSK: One version of student solutions	During class, Jiang accepted this answer that some students wrote (and said a figure with all right angles is a quadrilateral). But then she told the class that she thought the other way of writing in which some other students indicated the figure as "quadrilateral" in the "if" condition was better.This gave rise to a noisy reaction from students.
C:	Chen: Is this figure closed, I should ask you?	Chen gives a counterexample showing Jiang that the converse of the is not always true.	Probes by questioning and contradicting.	CK: using math knowledge to reason and argue.	But in the textbook: the range of figures discussed in middle school are all closed plane geometric figures.
J:	Jiang: Ah? (*she started laughing again*). Ha, ha, ha!	Jiang realizes right away what Chen suggested is reasonable.	Understands and agrees		
C:	Chen: Hey hey (*he's amused*). If you say so, then I have a question, is this figure closed?	Chen pushes further by reiterating what he says.	Probes and reiterates		
J:	Jiang: Eh? A polygon (*in Chinese a multilateral*), a polygon.	Jiang corrects herself by adding what she misses.	Understands, corrects, and adds.	CK: use of math knowledge to reason and argue	

C:	Chen: then, it still needs one more sentence, a premise, right?	Chen clarifies and adds what might be missing.	Clarifies, sums up	CK: use of math knowledge to make a point, induce.	
J:	Jiang: A polygon, polygon…	Jiang reassures her idea by repeating what she added.	Reassures, repeats	CK	
C:	Chen: (*Breaks in*) If you say all the angles are equal, then I can draw a figure not closed?	Chen reemphasizes his idea by repeating what he just said.	Reemphsizes and repeats point	CK: Using a counterexample to prove that the converse is not true.	Mathematical reasoning.
J:	Jiang: *(Still laughing.)* A polygon.	Finishes what she said.	Repeat		
C:	Chen: Isn't it ? *(he laughed.)* So for some things, if they are not written in complete version, it's got to be annoying; then it's a question about how we understand it. Right? According to my understanding, this is a quadrilateral. Why don't you say it's a quadrilateral instead of saying it a polygon? Because a square's all angles are also right angles.	Chen recapitulates all his points and questions Jiang's idea of "polygon".	Summarizes, questions again, and gives reason.	PCK: understanding the compressed mathematics in textbooks tends to be confusing and different ways (perspectives) to understand it give rise to different result CK: mathematics reasoning, making an argument and provide evidence.	

	Jiang did not answer. She was thinking and reading the key on the TRM.				
C:	Chen: *(after a pause)* The second one, I think the polygon…	Chen continues by wanting to move into the second exercise.	Continues and moves on to the next exercise.		
J:	Jiang: Then, it (the standard key) writes in the first exercise), the converse theorem is, **"An axial symmetric figure is an isosceles triangle."**	Jiang reads the standard key to the first exercise.	Seeks the external authority	CK Standard solution keys	
C:	Chen: Then it says purely in terms of words/language.	Comments on and evaluates the standard key.	Comments, and evaluates	PCK: critical insight of the SSK, the curricular content.	
J:	Jiang: An axial symmetric figure is an isosceles triangle, just as I said, if a figure is an axial symmetric figure, then it is an isosceles triangle.	Jiang lines up with what she said which was similar to the standard key.	Compares with standard key	CK: procedure (of writing of the converse of a proposition.)	
C:	Chen: Eh, this is a false proposition.	Chen agrees that this converse is a false	Agrees	CK: fact	
J:	Jiang: Eh (yes).	Jiang agrees.	Agrees		

C:	Chen: Then what about this? How does it say about the "rectangle" (in the second exercise)?	Chen asks Jiang to read and compare the key to the second exercise with the first one.	Probes by comparing with the standard key	PCK: compare the SSK	
J:	Jiang: In the case of the one with the "rectangle" (the second exercise), it still adds "a quadrilateral".	Jiang reads the standard key "**A quadrilateral with every of its four angles being a right angle is a rectangle**" and notices it is very specific about the hidden premise being "quadrilateral".	Compares the keys to both exercises together	PCK: compare the SSK	
C:	Chen: Then why it adds ("a quadrilateral") to this exercise here but not "a triangle" to one there (in the first exercise)?	Question the teaching material's inconsistency	Questions the standard key	PCK: discerning the differences of SSK given to similar kind of exercises.	
J:	Jiang: (*Laughed.*) This, is very hard to say (explain).	Jiang finds it impossible to answer (or may be she thinks there many reasons).	Agrees and wonders why		

C:	Chen: This, I find, sometimes it is not good if it is too simple (concise). Why should I say "rectangle"? If I say a figure with all its angles being right angles is a rectangle, this sentence is not right *(the converse is false)*.	Chen goes back to his former points still citing the exception which contradicts the key.	Keeps wondering why	PCK: Judging the quality/style of the content knowledge and how it might influence teaching and student learning. CK: reasoning and offering a counterexample. Repeats to emphasize.	
J:	Jiang: An axial symmetric figure is an isosceles triangle, then as I just said, if a figure is an axial symmetric figure, then this figure is an isosceles triangle. Right?	Jiang repeats the key to the first exercise. It seems that she's also wondering and still puzzled about why the keys of the two exercises are treated differently.	Repeats, keeps wondering why.	CK: procedure Repeats to emphasize.	Jiang has been seeking the "answer" and the math content in her part has been more procedural.
C:	Chen: Enh. (Indicating yes.)				
	Chen turned to the boy student who's making up for the homework he finished but forgot to bring to school and gave him some brief tutoring. Jiang was half talking to herself and half to me.				

J: *	Jiang: *(Jiang's still thinking. After a minute, she said to me:)* Now this thing doesn't have a firm conclusion, *(She laughs)* does it? This thing doesn't have a firm conclusion. In other words, within which range this problem is discussed, right? For this proposition, is it within the range of "triangles" or that of the "entire axial symmetric figures"?	Jiang's not satisfied with not having a definite answer. She wonders aloud why this is so and trying to come up with some explanations.	Not Satisfied, keep wondering why and identifying an "answer". Reasoning to herself	PCK: She's seeking that certainty in a "final solution" or "conclusion" driven by the elusiveness in how she reacted to the different ways that her student wrote the converses. PCK: Her persistent probe into why "there seems to be no solution" make her ask further question and reason by using her math knowledge and seeking a working explanation to this "either…or" "uncertainty".	It resembles the situation of "math problem solving in the practice of teaching" (Ball & Bass, 2001
Me	Then, maybe also give students the two situations: what happens when the range is triangles and what happens when the range is all axial symmetric figures.	I tried to say something in response when she's speaking to me.			
J:	*(After a while)*Sometimes the teaching materials do have this problem—no final conclusion; it is reasonable no matter how you say it.	Not sure what's the "answer", she finds what's wrong with the teaching materials.	Complains about the ambiguity of the teaching materials	PCK: critique the teaching materials leaving things open to interpretation, which brings difficulty to teach—(need both to unpack and trim?)	

C:	*(Hearing this, he turned to Jiang from tutoring his student)*. Right *(I agree)*. It's reasonable no matter how you say it.	Chen joins in readily when he heard what Jiang said about the ambiguity.	Still interested, joins in, agrees by repeating	PCK	
J:	Jiang: If as you said, first I change the proposition into "if..., then....", then very obviously its converse proposition tends to be a true proposition.	Jiang compares the results of Chen's early suggestion of changing into "If..., then..." format first.	Compares, and brings up again an early suggestion.	CK: procedure	
C:	Ayh, right. Yes.		Agrees		
J:	Writing like this (the first way), its converse proposition is a false proposition.	Jiang continues to finish her sentence.	Continues to provide view	CK: procedure	

C:	Chen: Right. A false proposition. Then next, how should we say this, say the other one with the rectangle? If you say it in terms of "figure" and "degrees of angles" (*referring to the second exercise that they also discussed above.*), it's not right either. What if the figure in not closed? Why should a figure be necessarily closed? So we need to say quadrilateral. But it adds (the specific shape of the figure in the problem) and sometimes it doesn't do this to other problems.??? I think problems like such sometimes are hard to handle. Do you agree? An isosceles triangle is an axial symmetric figure, it says this is right (a true proposition).	Chen confirms Jiang's comparison/explanation and wonders out loud why in the 2nd exercise it adds a "quadrilateral" and why it does not add a "triangle" in the 1st exercise. He concludes that such inconsistency is hard to deal with.	Confirms, agrees, wonder out loud about the inconsistency, and conclude that this is hard to deal with.	CK: use content knowledge to compare, reason, and argue mathematically. PCK: use content knowledge to compare, reason about the teaching materials (the SSK—standard solution key) and evaluate the materials pedagogically.	
J:	Jiang: *(Jiang did not answer right away, she's still thinking.)* What's its implied (hidden) premise, right?	Jiang opens up another perspective to wrestle with the problem—identifying the hidden premise.	Open up another perspective.	CK: use math knowledge to probe and reason.	

C:	Chen: Eh, what's its implied premise? Its implied premise is "figure" or "triangle".	Chen thinks about the question posed by Jiang and offers his answer.	Build on each other's answers.	CK:	
J:	Jiang: Or it is a closed polygon. Right?	Jiang adds to what Chen said.	Adds to the other's answer.	CK: use of math knowledge to probe and reason	
C:	Chen: Ayh (he agrees).				
J:	Jiang: Then, in the second one, it is about whether we are discussing this within the range of "polygons" or "all the figures". Right?	Jiang proposes the range of discussion in the second exercise.	Propose and keeps building on.	CK: use of math knowledge to probe and reason	
C: *	Chen: Ayh *(Yes)*. Such as in the first one, it says "figure" and does not say what?	Chen considers the first exercise as an example.	Build on by wanting to know more.	PCK: use math to inquiry about the teaching content (SSK)	
J:	Jiang: It says an axial symmetric figure is an isosceles triangle.	Jiang tells Chen the key.	Read and answer.	CK SSK	
C:	Chen: Then, there are many kinds of axial symmetric figures. So what's its premise?	Question the key and inquire into their common question---what's the hidden premise?	Questions, keeps building by probing further into the question.	CK: use math knowledge to reason and probe	

J:	Jiang: Its major premise.	Jiang adds/completes/clarifies Chen's question.	Add to make complete and clear	CK: use math knowledge to reason	
C:	Chen: Eh, what's its major premise? So sometimes when you say this is the standard key to the exercises (referring to the key provided in Teaching Reference Material) it's not going to be all applicable. Why does it say that the premise for the first one is "figure" and the second one, is "quadrilateral"?	Chen accepts Jiang's addition. He argues that the standard keys are not always applicable. He questions again about the keys' inconsistency.	Accepts addition, argues and questions the inconsistency.	PCK: Critique (assess) the curricular material with mathematical insight and probing with question.	
J:	Jiang: Right. These two exercises are obviously unclear.	Jiang agrees and reaffirms that ambiguity or confusion of the two exercises.	Agrees, reaffirms the ambiguity		
C:	Chen: I think that it would be better if there is a standard way like this: first turn it into "if…, then…" , next write its converse theorems, which seems better. Can't all the propositions be written into "if…, then" form?!	Chen still holds the way he suggested earlier that turn the original proposition into "if…, then…" first.	Hold early suggestion about how to write the converse of a proposition.	PCK: use the above mathematical reasoning to think about what is a better way in teaching the writing of the converse of a given proposition.	
	Jiang did not answer. She turned to mark her homework.				

REFERENCES

Anderson-Levitt, K. (2002). Teaching Cultural as National and Transnational: A Response to Teacher's Work. *Educational Researcher* 31(3), 19-21.

Ausubel, D. (1963). *The psychology of meaningful verbal learning.* New York: Grune & Stratton .

Ausubel, D. (1968). *Educational psychology, a cognitive view.* New York: Holt, Rinehart and Winston, Inc.

Ball, D. L., & Bass, H. (2000) Interweaving content and pedagogy in teaching and learning to teach: Knowing and using mathematics. In J. Boaler (Ed.), *Multiple perspectives on the teaching and learning of mathematics* (pp. 83-104). Westport, CT: Ablex.

Ball, D. L., Lubienski, S.T., & Mewborn, D.S. (2001). Research on teaching mathematics: The unsolved problem of teachers' mathematical knowledge. In V. Richardson (Ed.), *Handbook of research on teaching, 4th Edition.* (p. 433-456). Washington, D.C.: American Education Research Association.

Ball, D. L., & Cohen, D. K. (1996). Reform by the book: What is - or might be -the role of curriculum materials in teacher learning and instructional reform. *Educational Researcher*, 25(9), 6-8.

Bannon, L. and Kuutti, K. (1996). Shifting perspectives on organizational memory: From storage to active remembering. In *IEEE HICSS 96, Vol.II. (*p. 156-167). Hawaii: Information Systems- Collaboration Systems and Technology IEEE Computer Society Press.

Berliner, D.C. (1986). In pursuit of the expert pedagogue. *Educational Researcher*, 15 (7), 5-13.

Beveridge, M. & Rimmershaw, R. (1991) Teaching and tutoring systems: Explanatory dialogues in context. In P. Goodyear (Ed.) *Teaching knowledge and intelligent tutoring.* Norwood, NJ: Ablex Publishing Corporation, p. 279-293.

Biggs, J. (1996/1999). Western misconceptions of Confucian-heritage learning culture. In D.A. Watkins & J. Biggs (Eds.), *The Chinese learner: cultural, psychological and contextual influences.* Hongkong: Comparative Education Research Center.

Borasi, R. (1996). *Reconceiving mathematics instruction: A focus on errors.* Norwood, New Jersey: Ablex Publishing Corporation.

Bruner, J. (1960/1977). *The process of education.* Cambridge: Harvard University Press.

Bruner, J. (1966). *Toward a theory of instruction.* Cambridge: The Belknap Press of Harvard University Press.

Bruner, J. (1979). *On knowing – Essays for the left hand.* Cambridge, MA: Harvard University Press.

Cai, J. (1995). *A cognitive analysis of U.S. and Chinese students' mathematical performance on tasks involving computation, simple problem solving, and complex problem solving. (Journal for Research in Mathematics Education* monograph series 7), Reston, VA: National Council of Teachers of Mathematics.

Cazden, C. B. (1988). *Classroom discourse- The language of teaching and learning.* Portsmouth, NH: Heinemann.

Cobb, P., McClain, K, Lamberg, T., & Dean, C. (2003). Situating teachers' instructional practices in the institutional setting of the school and district. *Educational Researcher,* 32(6), 13-24.

Cole, M. & Engestrom, Y. (1993). A cultural historical approach to distributed cognition. In G. Salomon (Ed.) *Distributed cognitions: Psychological and educational considerations.* Cambridge: Cambridge University Press.

Collopy, R. (2003). Curriculum materials as a professional development tool: How a mathematics textbook affected two teachers' learning. *The Elementary School Journal*, 103(3), 227-311.

Cooper, H. (1989). *Homework.* White Plains, N.Y.: Longman.

Cooper, H. (1994). *The battle over homework.* Thousand Oaks, California: Corwin Press, Inc.

Darling-Hammond, L & Wise, A.E. (1985). Beyond standardization: State Standards and School Improvement. *Elementary School Journal*, 85, 313-335.

Davis, P.J. & Hersh, R. (1981). *The mathematical experience.* Boston: Houghton and Mifflin.

Engestrom, Y. & Miettinen, R. (1999). Introduction. In Y. Engestrom, R. Miettinen & R. L. Punamaki (Eds.), *Perspectives on activity theory.* Cambridge University Press.

Engestrom, Y. (1998). Reorganizing the motivational sphere of classroom culture: An activity theoretical analysis of planning in a teacher team. In F. Seeger, J. Voight, & U. Waschescio (Eds.), *The culture of the mathematics classroom (p.76-103).* New York: Cambridge University Press.

Engestrom, Y. & Middleton, D. (1996). Introduction: Studying work as mindful practice. In Y. Engestrom and D. Middleton (Eds.), *Cognition and communication at work*. Cambridge University Press.

Engestrom, Y. (1990). *Learning, working and imagining: Twelve studies in activity theory.* Helsinki: Orienta-Konsultit.

Fawcett, H. P. (1938). *The nature of proof: a description and evaluation of certain procedures used in a senior high school to develop an understanding of the nature of proof. (NCTM yearbook 1938)* New York: Teachers' College, Columbia University.

Fuys, D., Geddes, D., Lovett, C. J. & Tischler, R. (1988). The Van Hiele model of thinking in geometry among adolescents. *Journal for Research in Mathematics Education* [Monograph number 3]. Reston, VA, NCTM.

Gardner, H. (1989). *To open minds.* New York: Basic Books.

Ginsberg, E. (1992). Not just a matter of English. *HERDSA News*, 14(1), 6-8.

Glaser, B.G. & Strass, A.L. (1967). *The discovery of grounded theory.* Chicago: Aldine.

Goodyear, P. (Ed.). *Teaching knowledge and intelligent tutoring.* Norwood, New Jersey: Ablex Publishing Corporation, 1991.

Gu, L et al. (1999). Finding the middle zone – detailed dissection of a middle school math lesson. Paper presented at the Sino-U.S Advanced Seminar on Mathematic Education co-organized with delegation of Carnegie Foundations for the Advancement of Teaching chaired by Lee Shulman.

Gu, L. (1994). *On teaching experimentation – Study of methodology and teaching principles of Qingpu Experiment.* Beijing: Education Science Press.

Hashimoto, Y. (1987). Classroom practice of problem solving in Japanese elementary schools. In J. P. Becker & Miwa (Eds.) *Proceedings of the U.S.-Japan seminar on mathematics problem solving (pp. 94-119).* Carbondale, IL: Southern Illinois University.

Hess, R.D. & Azuma, M. (1991). Cultural support for schooling: contrasts between Japan and the United States. *Educational Researcher,* 20 (9), 2-8.

Hutchins, E. & Klausen, T. (1996). Distributed cognition in an airplane cockpit. In Y. Engestrom & D. Middleton (Eds.), *Cognition and communication at work*. Cambridge University Press.

Kralovec, E. and Buell, J. (2000). *The end of homework: How homework disrupts families, overburdens children, and limits learning.* Boston: Beacon Press

Mullis, I.V.S., et al (2000). *TIMSS 1999 International Mathematics Report: Readings from IEA's Repeat of the Third International Mathematics and Science Study at the Eighth Grade.* MA: ISC, the International Study Center, Boston, Lynch School of Education and IEA, the International Association for the Evaluation of Education Achievement.

Jackson, P.W. (1990). *Life in the classroom.* New York: Teachers College Press.

Kralovec, E. & Buell, J. (2000). *The end of homework: how homework disrupts families, overburdens children & limits learning.* Boston, Mass.: Beacon Press.

Kuutti, K. (1996). Activity theory as a potential framework for human-computer interaction research. In Nardi (ed.) *Context and consciousness: Activity theory and human-computer interaction.* Cambridge, MA: The MIT Press (17-44).

LeCompte, M.D. & Preissle, J. (1993). *Ethnography and qualitative design in educational research.* San Diego: Academic Press.

Leinhardt, G. & Greeno, J. (1991). The cognitive skill of teaching. In Peter Goodyear (ed.) *Teaching knowledge and intelligent tutoring.* Norwood, New Jersey: Ablex Publishing Corporation (233-268)

Leinhardt, G. (1990). Capturing Craft Knowledge in Teaching. *Educational Researcher,* No. 19, March 1990, p. 18-25.

Lektorsky, V.A. (1984). *Subject, object, cognition.* Moscow: Progress.

Leont'ev, A. N. (1977). *Activity, consciousness, personality.* Englewood Cliffs, NJ, Prentice Hall.
Also: http://www.marxists.org/archive/leontev/works/1977/leontev 1977.htm

LeTendre, G., Baker, D., Akiba, M., Goesling, B., and Wiseman, A. (2002). Response to K. Anderson-Levitt's Rejoinder, *Educational Researcher, Vol. 31, No. 3,* pp. 22–23

LeTendre, G., Baker, D., Akiba, M., Goesling, B., & Wiseman, A. (2001). Teachers' work: Institutional isomorphism and cultural variation in the U.S., Germany and Japan. *Educational Researcher,* 30(6), 3–15.

Lieberman, A. & Miller, L. (2000). Teaching and teacher development: A new synthesis for a new century. In R.S. Brandt (Ed.) *Education in a new era: ASCD Yearbook 2000, p. 47-66.*

Littman, D (1991). Tutorial planning schemas. In Peter Goodyear (ed.) *Teaching knowledge and intelligent tutoring.* New Jersey: Ablex Publishing Corporation, 1991.

Little, J. (2003). Inside Teacher Community: Representations of Classroom Practice. *Teachers' College Record,* 105(6), 913-945.

Ma, L. (1999). *Knowing and teaching elementary mathematics: Teachers' understanding of fundamental mathematics in China and the United States.* New Jersey: Lawrence Erlbaum Associates.

Merriam, S.B. (1988). *Case study research in education – A qualitative approach.* San Francisco: Jossey-Bass Publishers.

Paine, L. W., Fang, Y. & Wilson, S. (2003) Entering a culture of teaching. In Britton, T., Paine, L.W, Pimm, D. & Senta, R. (2003). *Comprehensive Teacher Induction: Systems for Early Career Learning.* The Netherlands: Kluwer Academic Publishers, pp. 20-82.

Paine, L. W. (1990). The teacher as virtuoso: A Chinese model for Teaching. *Teachers College Record,* Vol. 92, No. 1. Fall 1990.

Piaget, J. (1970). *Genetic epistemology.* New York: Columbia University Press

Record on the Subject of Education, *Hsio Ki (Xue Ji),* from *The Book of the Rites, The Li Ki (Li Ji), Part II.* Translated by James Legge (1885). In *Sacred Books of the East vol. 28 – The Sacred Books of China, vol. 5*

Ren Y. and Zhang, F. (2001). (*Ren Yong de zhongxue shuxue jiaoxue yishu yu yanjiu) Ren Yong's secondary mathematics teaching art and research.* Serials of Nationally known special rank teachers' teaching art and research. Important national publications projects in the 9th five-year plan. Jinan: Shandong Education Press.

Schoenfeld, H. A. (1991). On mathematics as sense making: An informal attack on the unfortunate divide of formal and informal mathematics. In J.F. Voss, D.N. Perkins and J.W.Segal (Eds.), *Informal reasoning and education.* P. 311-343. Hillsdale, NJ: Lawrence Erlbaum Associates, Publishers.

Schoenfeld, H.A. (1985). *Mathematical problem solving.* New York: Academic Press. Chapter 5 and 10.

Shulman, L. S. (1987). Knowledge and teaching: Foundations of the new reform. *Harvard Educational Review, 57,* 1-22.

Shulman, L. S. (1986). Those who understand: Knowledge growth in teaching. *Educational Researcher,* 15 (2), 4-14.

Star, S. L. (1989). The structure of ill-structured solutions. In Gasser, L. and Huhns, M. (Eds.), *Distributed artificial intelligence – Volume II.* Morgan Kaufmnn, 1989, p. 37-54.

Stigler, J. and Hiebert, J. (1999). *The teaching gap.* New York: The Free Press.

Stigler, J & Stevenson, H.W. (1991). How Asian teachers polish each other to perfection. *American Educator,* 15(1), 12-21 & 43-47.

Strauss, A. and Corbin, J. (1990). *Basics of qualitative research--- Grounded theory and procedures and techniques.* Sage Publications.

Tharp, R.G. & Gallimore, R. (1988). *Rousing minds to life: Teaching , learning, and schooling in social context.* Cambridge University Press.

Vygotsky, L.S. (1978). *Mind in society: The development of higher psychological processes.* Cambridge, MA: Harvard University Press.

Wang, J. and Paine, L. (2003). Learning to teaching with mandated curriculum and public examination of teaching as contexts. *Teaching and Teacher Education, 19 (1), 75-* 94.

Wang, Jiafeng et al. (1995). *Initial exploration in secondary school mathematics teaching psychology.* Shanghai Education Press.

Wang, B. (2004). The ICME 2004 Shanghai Conference - Proceedings. East China Normal University Press

Wang, B., Guo, J., Liu, D., He, X, & Gao, Q. (1994). *Brief history of Chinese education.* Beijing Normal University Press.

Wenger, E. (1998). *Communities of practice: learning, meaning, and identity* Cambridge, U.K.: Cambridge University Press.

Wertsch, J.V. (1991). Voices of the mind: a sociocultural approach to mediated action. Cambridge, Mass.: Harvard University Press.

Wertsch, J. V. (1985). *Vygotsky and the social formation of mind.* Cambridge, MA: Harvard University Press. http://wwwsv.cict.fr/cotcos/TheorecticalApproaches/Activity/ActivitypaperBannon.htm

Wertsch, J. (Ed). (1981). The concept of activity in Soviet psychology. Armonk, NY: Sharpe.

W.O. Lee (1996). The cultural context for Chinese learners: Conceptions of learning in the Confucian context. In D. A. Watkins & J. Biggs (Eds.), *The Chinese learner – Cultural, psychological and contextual influences.* Hong Kong University: Comparative Education Research Center.

Wolf, D., Bixby, J., Glenn, J., & Gardner. H. (1991) To use their minds well: Investigating new forms of student assessment. *Review of Research in Education*, 17, pp. 31-73.

Zhang, D., Tang, R. & Liu, H. (1991). *Mathematics pedagogy.* Jiangxi Education Press

Zhu, S. & Wang, Y. (1998). *On secondary school mathematics teaching.* Beijing: Education Science Press.

Zhong, S. (1988). On mathematical problem solving: A conversation with students. *Education in school subjects,* 2, 1-3.

Curriculum Materials:

Curriculum Standards of Nine-Year Compulsory Education. (1998). Shanghai Education Press.

Teaching Reference Material of Eighth Grade, First Semester. (1996). Shanghai Education Press

Volume A and Volume B – Textbook Materials for Homework Assignment. (1996). Shanghai Education Press