AN EVALUATION OF THE "CAMBRIDGE CRITICISM" OF NEOCLASSICAL THEORY

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AN EVALUATION OF THE "CAMBRIDGE CRITICISM" OF NEOCLASSICAL THEORY

By

Robert Francis Allen

AN ABSTRACT OF A THESIS

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ABSTRACT

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OF NEOCLASSICAL THEORY

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The chief characteristic of the neoclassical, real-capital model is the assumption that there exists a function $Q = F(K,L)$ that is homogeneous of degree one in the single homogeneous capital good (K) and a single homogeneous labor force (L). This implies (a) that the real wage varies inversely with the interest rate, and (b) that the real capital-labor ratio varies directly with the wage-interest ratio. Hence, an increase in the real wage leads to an increase in the capital intensity of techniques, and permanently sustainable net national product increases as the rate of interest declines.

Recently, Pasinetti, Morishima, Bruno-Burmeister-Sheshinski and others proved it is possible for a technique that is the most profitable of all feasible techniques at some relatively high wage rate to also be the most profitable at some relatively low wage rate. The technique "comes back" as the wage falls monotonically toward zero, with the consequence that an unambiguous ordering of techniques is impossible. This phenomenon (labeled the reswitching of techniques) is said to establish the simple technological fact that there may exist a "set of blueprints" for which there is no one-to-one correspondence

between the real capital-labor ratio and the wage-interest ratio. If so, it is not generally true that a fall in the interest rate entails the adoption of more capital-intensive techniques with the consequence that the permanently sustainable consumption stream is greater.

With minor exceptions, recent writers have employed two-sector, fixed-proportions, heterogeneous capital models in which the labor-output ratio of one of the sectors is used as an independent index of capital intensity (i.e., as the aggregate capital-labor ratio). This study analyzes several real-capital models and concludes that the use of the labor-output ratio as an independent index of capital intensity is definitely unwarranted.

The basic error implicit in the use of this index is the assumption that a technique of production represents a unique aggregate capital-labor ratio. This will not generally be the case. Changes in the wage-interest ratio imply changes in relative commodity price. This in turn implies changes in the composition of demand between sectors. When factor intensities differ between sectors, the assumption of full employment (common to all these discussions) implies that the economy's endowment of capital and/or labor is a variable within the context of a given technique.

To allow for changes in the composition of demand, this study introduces a simple demand relation into the reswitching model. More specifically, in a two good,

heterogeneous capital model, the demand relation is $\frac{M}{C} = f(p)$ where M is a capital good, C a consumption good, and p the relative price of the capital good in terms of the consumption good. The comparative static properties of the model are investigated and the implications of changing relative commodity price for the reswitching phenomenon set out.

Two fundamental points emerge from this analysis: first, the real capital embodied in a technique is a decreasing function of the rate of interest, and second, a switch of techniques exerts an independent influence on relative commodity price.

The first point implies that for any two techniques there will generally be some critical rate of interest for which the aggregate capital-labor ratio is the same for both techniques. Thus whether reswitching means that the aggregate capital-labor ratio has reversed itself or not depends first of all on whether the economy has passed the critical interest rate before reswitching occurs.

The second point means that, at any specified wage-interest ratio, relative commodity price may fall, remain unchanged, or rise as a result of entrepreneurs' switching techniques. When the switch itself causes relative commodity price to fall, it is virtually impossible for reswitching to destroy the neoclassical link between the aggregate capital-labor ratio and the wage-interest ratio.

This study concludes that when the interrelations between factor prices, factor endowments, and relative

commodity price are fully set out, the Cambridge criticism does not appear as serious as recent discussions would have one believe.

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TABLE OF CONTENTS

LIST OF TABLES

LIST OF FIGURES

CHAPTER I

NEOCLASSICAL THEORY AND THE "CAMBRIDGE CRITICISM"

I. Introduction

Capital theory has an ancient, if not entirely honorable, heritage. I think it is fair to say, however, that serious theorizing began with Bohm-Bawerk,[1] who formulated the period of production model and grappled with the concept of an "average period of production". He was followed in this endeavor by Wicksell who, it should be noted, finally abandoned the notion of an average period of production.[2] Nonetheless, the "average period" was crucial to their theory.

Bohm-Bawerk regarded capital as a "subsistence fund" or what in classical economic literature was called a wages fund. Its function was to support labor during the interval of time that must elapse between the first application of inputs and the later emergence of output. With a given amount of labor, the longer the production period the more

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[1]Eugen von Bohm-Bawerk, The Positive Theory of Capital, trans. William Smart (London: MacMillan and Company, 1891).

[2]Knut Wicksell, Lectures on Political Economy, vol. I, trans. E. Classen (New York: The Macmillan Company, 1934).

capital per man that would be required. Conversely, the more capital that is made available the longer could be the production period for any given amount of labor. If we think of the "subsistence fund" as allowing the "roundabout" use of labor, we may say that a longer period of production represents a more "roundabout" or capital-intensive method of production.

The basic idea to be conveyed by the "average period" is that for a given labor force any increase in the period of production means that the ratio of real capital to real labor has risen. In other words, the "average period" is to be taken as an independent index of the capital intensity of production, and this is what makes the concept of the "average period" so important.[3]

Thus the average period of production is really a measure of the quantity of real capital available in the

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[3]On the interpretation of the "average period" we rely on Kaldor, though the question is an open one. See for example Nicholas Kaldor, "On The Theory Of Capital: A Rejoinder To Professor Knight," Econometrica, VI (April, 1938), pp. 163-176. Kaldor has argued that whatever validity attaches to the "average period" is due to its ability to serve as such an index. p. 169. He further takes as a premise of Austrian theory the idea that " . . . with the aid of the concept of the 'investment period', the heterogeneous mass of capital goods can be reduced to homogeneity, and thus 'capital' can be treated as a quantity per se." p. 163. Hicks has taken essentially the same position as Kaldor but attempts to adjust the concept for Knight's objections about the consistency of the index. John R. Hicks, Value And Capital (2d ed.; Oxford: Clarendon Press, 1962), pp. 213-226. On the other hand see Joseph A. Schumpeter, History of Economic Analysis, ed. Elizabeth B. Schumpeter (New York: Oxford University Press, 1963), pp. 903-909, but especially p. 901, ft. 26, where Mr. Kaldor is chastised.

economy. It is the means by which we are able to reduce the heterogenous "subsistence fund" to a homogeneous quantity per se. The "average period" can then be combined with a homogeneous labor factor and a theory of production formulated. The most important relations that can be shown to hold in this framework are those between the rate of interest, the real wage rate, and the length of the period of production. In particular it can be shown that every fall in the rate of interest leads to more capital intensive production.[4] This is a result of great importance for it is upon it that the most basic propositions of modern neoclassical capital theory are made to stand.[5]

Yet, as everyone knows, the Austrian view of the production process is not the one that has gained favor for purposes of enunciating a theory of production and distribution. This is presumably due to the difficulties inherent in the measurement of the "average period".[6] Consequently our understanding of much of the real economic world has come to be based on what Professor Samuelson calls the

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[4]Robert Dorfman, "A Graphical Exposition of Bohm-Bawerk's Interest Theory," Review of Economic Studies, XXVI (February, 1959), pp. 153-58.

[5]The truth of this statement is very carefully and completely brought out in Chapter II of the present paper.

[6]As Kaldor has pointed out " . . . the question of whether the concept has meaning should be kept rigidly separate from the question whether it is relevant." Kaldor, op. cit., p. 164, the italics in original. He has shown its relevance.

"J. B. Clark Neoclassical fairy tale".[7] This fairy tale consists of a set of propositions that may be rigorously derived from the assumption that there exists a function

(1) $Q = F(K,L)$

that is homogeneous of degree one in a single homogeneous capital good (K) and a single homogeneous real labor force (L).

The assumption that such a function exists is the chief characteristic of the neoclassical, real-capital model. The relations between the rate of interest, the real wage rate, and the capital intensity of production that can be shown to hold in this framework are identical to those of the Austrian model. It may therefore seem to be a matter of indifference that the production function with its J. B. Clark capital has been generally preferred to the Austrian model with its Bohm-Bawerkian "average period" for purposes of passing on this body of theory. Yet the difference between them does appear substantial. The Austrian model views the process of production as a process that takes place over <u>time</u>. It is the <u>element of time</u> that is the single most important characteristic of the Austrian model.[8]

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[7]Paul A. Samuelson, "Parable and Realism in Capital Theory: The Surrogate Production Function," <u>Review of Economic Studies</u>, XXIX (June, 1962), pp. 193-207.

[8]"From the technical point of view, the essentially new contribution of Bohm-Bawerk's system to economic theory is the mastery of the element of time." Joseph A. Schumpeter, "Schumpeter on Bohm-Bawerk," <u>The Development of Economic Thought</u>, editor, Henry W. Spiegel (New York: John Wiley and Sons, 1964), p. 376.

The real capital model pretends that time does not matter. This difference in the models is actually quite critical and has in fact been responsible for the emergence of what is now called the "Cambridge Criticism" of neoclassical theory.[9]

Recently there has appeared a symposium[10] devoted to the Cambridge criticism. The upshot of the symposium is this: what the critics have been trying to say is both correct and important. It is important because it shows that the parables that collectively make up simple neoclassical theory are not generally valid.

This may be a very difficult pill to swallow. Yet as Professor Samuelson says "we must respect, and appraise, the facts of life."[11] As a first step in this direction let us set out completely the theory in question and the essential nature of the Cambridge criticism. Doing so will bring out the truth of the above statement concerning the element of time in production.

II. Simple Neoclassical Theory

One of the more famous analogies to be found in the history of economic analysis is contained in the writings

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[9]Section two below demonstrates that it is Mrs. Robinson's attempt to resurrect Bohm-Bawerk's view of production that leads to the criticism.

[10]"Paradoxes in Capital Theory: A Symposium," The Quarterly Journal of Economics, LXXX (November, 1966), pp. 503-83.

[11]Paul A. Samuelson, "A Summing Up," The Quarterly Journal of Economics, LXXX (November, 1966), p. 583.

of J. B. Clark. For Clark capital is like a waterfall and capital goods like its individual drops of water.[12] The individual drops of water may come and go but the waterfall will always be the waterfall. The interpretation that has been placed on this by the weight of authority is that there is a real substance called capital whose depletion is continuously replaced so that the substance itself remains homogeneous.[13]

In a somewhat similar fashion Clark drew a distinction between Pure Labor and individual laborers.[14] With the aid of this real homogeneous capital and labor, Clark developed his central thesis that in a static state there is a single, natural, and universal law that governs the distribution of the product between wages and interest. This is, of course, the law of marginal productivity.[15]

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[12]John B. Clark, The Distribution of Wealth (New York: The MacMillan Company, 1927), pp. 116-40. "The articles that embody the fund are, like particles of water in a river, vanishing things; while the fund itself, like the river, is the abiding thing." p. 157. The waterfall is Schumpeter's, History, op. cit., p. 902.

[13]For example, Schumpeter, ibid. The question of just what Clark did mean by capital is, like the "average period", an open question. Clark wavers between a value concept and the physical concept. For example, in reference to capital, he says "It is value embodied in goods the identity of which is perpetually changing, ..." p. 120, ft. 1, op. cit. But this is a question we cannot pursue here.

[14]Clark, op. cit., p. 157. "Men are as perishable as are capital-goods, but labor is as permanent as is capital."

[15]Ibid., p. 200. "One law governs wages and interest--the law of final productivity."

But our interest in Clark is not due to his law of distribution, though this was an important contribution to the development of economic analysis. Rather our interest stems from his notion of capital, which has proven to be a very fruitful simplification. When capital--defined as a single kind of physically homogeneous capital good--is combined with labor in a simple neoclassical production function, a truly imposing list of propositions or parables can be made to appear. In particular it can be shown that, given competitive markets, (a) the wage rate varies inversely with the interest rate, and (b) the real capital-labor ratio varies directly with the wage-interest ratio, so (c) distributive shares are well defined; and (d) net national product per worker, or the permanently sustainable consumption stream, varies inversely with the rate of interest.

That is quite a bit to prove; but having proved it, neoclassical theory establishes an orderly relation between the physical realm of production and the commodity and factor markets. Let us now set out the basic, rigorous model in its most demanding detail.[16] To this end, assume that there exists a production function

(2) $Q = F(K,L)$

that is homogeneous of degree one in the single homogeneous

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[16]For an expanded version see C. E. Ferguson, The Neoclassical Theory of Production and Distribution, forthcoming from Cambridge University Press.

capital good (K) and labor (L). By its homogeneity property

(3) $tQ = tF(K,L) = F(tK,tL)$

and letting $t = \frac{1}{L}$

(4) $\frac{Q}{L} = F(\frac{K}{L}) = F(\frac{K}{L},1)$.

Thus we may write the production function as:

(5) $Q = LF(\frac{K}{L})$.

The operation of diminishing returns ensures that

(6) $F''(\frac{K}{L}) < 0$,

whereas the economic operation of enterprise requires

(7) $F'(\frac{K}{L}) > 0$.

We further require competitive imputation, so

(8) $w = \frac{\partial Q}{\partial L} = F(\frac{K}{L}) - (\frac{K}{L})F'(\frac{K}{L}) > 0$,

and

(9) $r = \frac{\partial Q}{\partial K} = F'(\frac{K}{L}) > 0$.

Now differentiate (8) and (9) to obtain

(10) $\frac{dw}{d(\frac{K}{L})} = -(\frac{K}{L})F''(\frac{K}{L}) > 0$,

and

(11) $\frac{dr}{d(\frac{K}{L})} = F''(\frac{K}{L}) < 0$.

Then dividing (10) by (11) we obtain

(12) $\frac{\frac{dw}{d(\frac{K}{L})}}{\frac{dr}{d(\frac{K}{L})}} = \frac{dw}{dr} = \frac{-K}{L} < 0$.

Finally, the Marshallian elasticity of (12) is given by:

(13) $-\frac{r}{w}\frac{dw}{dr} = \frac{rK}{wL}$ = ratio of aggregate relative shares.

Now we have established the essential results of neo-classical theory. Let us look at it graphically. Panel a, Figure 1, is a graph of inequality (10). It shows that the

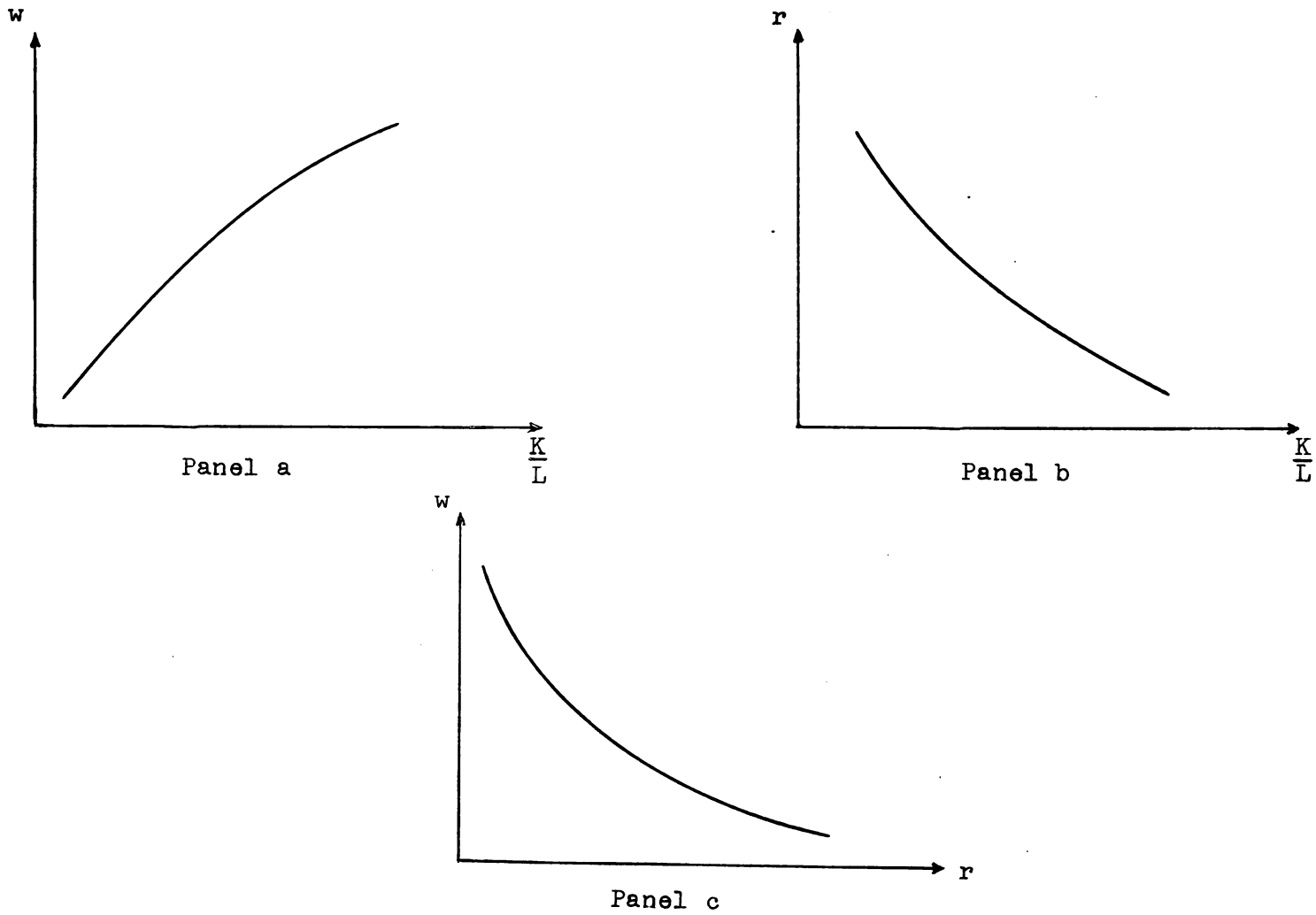

Figure 1. The Surrogate Factor-Price Frontier

wage rate varies directly with the capital-labor ratio. Similarly, Panel b illustrates inequality (11): the rate of interest varies inversely with the capital-labor ratio. The two relations are brought together in Panel c, which depicts inequality (12) and emphasizes that the wage rate must vary inversely with the rate of interest. Thus proposition (a) is established.

The curve in Panel c is what Samuelson calls the Surrogate Factor-Price Frontier.[17] By equation (13) the Marshallian elasticity of this curve is the ratio of aggregate relative shares. Thus we have proposition (c).

The Surrogate Frontier also verifies our propositions (b) and (d). We shall set these out in terms of the capital intensity of the economy's technique of production. The reason for so doing is that these concepts (capital intensity and the technique of production) have played a very prominent role in the recent criticism of neoclassical theory. It will be useful to begin with a clear picture of what these concepts mean.[18]

With a given technology and a large number of firms each producing a single product, we may view each firm as having a production constraint of the form:

(14) $y = f(l,k)$,

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[17]Paul A. Samuelson, "Parable and Realism in Capital Theory: The Surrogate Production Function," loc. cit.

[18]The following discussion is based on R. G. D. Allen, Mathematical Economics (London: MacMillan and Company, 1957), pp. 332-41.

where y stands for output and l,k represent the services of the firm's variable production inputs. This function summarizes the efficient combinations of input services open to the firm, and it is often assumed that the function is characterized by constant returns to scale. The feature of the production function of immediate interest is the implied continuous substitutability of inputs. Each combination of inputs represents one efficient method of producing the given output; hence we may refer to any particular combination as a method of production. If we call l labor and k capital, a method of production is simply an efficient capital-labor ratio, and we have a large number of production methods that are easily substitutable for one another.

Since each firm is a single-product firm, we may group them by product and speak unambiguously about m distinct industries (one for each product). If an arbitrary set of prices for labor and capital is specified, each industry would be observed to adopt a best method of production (i.e., capital-labor ratio), and the aggregate of these methods would constitute what is called the "technique of production". Hence a technique of production is the aggregate capital-labor ratio that contains one method of production for each of the m industries in the economy.

It is now clear from equation (12) that the slope at any point on the frontier of Panel c, Figure 1, corresponds to one technique of production open to society. Furthermore, the quantity of capital embodied in any

technique is simply the slope of the frontier at any point multiplied by the amount of labor utilized at that point. So we have the proposition that an increase in the equilibrium wage (or decrease in the rate of interest) will bring about an increase in the capital intensity of the technique "employed" by the economy. This is the critical point contained in proposition (b).

Finally equation (4) together with equations (6) and (7) yields proposition (d): a fall in the rate of interest will lead to an increase in the productivity of labor though at a decreasing rate.

On such simple relations as these do we base much of our understanding of factor pricing, product distribution, and the very size of the product that is to be distributed.

III. The Cambridge Criticism

What we now call the Cambridge criticism of simple neoclassical theory first appeared explicitly in Mrs. Robinson's article on the Production Function.[19] At the time, it appeared as though she did not realize how really fundamental and important her discovery was.[20] Her primary

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[19]Joan Robinson, "The Production Function and the Theory of Capital," Review of Economic Studies, XXI (1953), pp. 81-106.

[20]Perhaps it would be more accurate to say that she did not then, and does not now, feel that the criticism is as fundamental as others would have it appear. In the recent edition of her Accumulation she still maintains that

concern, at least ostensibly, was with J. B. Clark's capital[21] and the practice of thinking about an equilibrium position as though it were a position towards which the economy is moving.[22] Mrs. Robinson was irritated because the neoclassical economist insists on pretending that he can compare different steady-state, equilibrium ratios of capital to labor and thereby come to an understanding of the actual changes that take place between factor ratios in time.[23] She is interested in the problems of accumulation and her suspicion is that the neoclassical economist has been throwing mud in the water.

Like Bohm-Bawerk before her, Mrs. Robinson is struck by the following observation:[24]

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the "perverse" case is " . . . not of great importance." p. 109, ft. 1, Joan Robinson, The Accumulation of Capital (2d ed., New York: St. Martin's Press, 1966).

[21]Robinson, op. cit., p. 81. "The student of economic theory is taught to write O = f(L,C) where L is a quantity of labour, C a quantity of capital and O a rate of output. . . ." but he is never told "in what units C is measured."

[22]Ibid., p. 85, "The neoclassical economist thinks of equilibrium as a position towards which the economy is tending to move as time goes by. But it is impossible for a system to get into a position of equilibrium, for the very nature of equilibrium is that the system is already in it, and has been in it for a certain length of past time." Italics in original.

[23]This is a " . . . profound methodological error, which makes the major part of neo-classical doctrine spurious." Ibid., p. 84.

[24]Ibid., p. 82. Italics added. This no doubt is what Bohm-Bawerk had in mind when he said "That roundabout methods lead to greater results than direct methods. . . ." but "It must be emphatically stated that the only basis for

> To-day, in country Alpha, a length of roadway is being cleared by a few men with bulldozers; in Beta a road (of near enough the same quality) is being made by some hundreds of men with picks and ox-carts. In Gamma thousands of men are working with wooden shovels and little baskets to remove the soil. When all possible allowances have been made for differences in national character and climate, and for differences in the state of knowledge, it seems pretty clear that the main reason for this state of affairs is that capital in some sense is more plentiful in Alpha than in Gamma. Looked at from the point of view of an individual capitalist, it would not pay to use Alpha methods in Gamma (even if unlimited finance were available) at the rate of interest which is ruling, and looked at from the point of view of society, it would need a prodigious effort of accumulation to raise all the labor available in Gamma even to the Beta level of technique.

Thus there is a real problem with which neoclassical analysis pretends to cope. Indeed as Mrs. Robinson says there is even an "element of common sense" entangled in the production function that has been brought to bear on the problem.[25] In an effort to rescue it (the element of common sense) Mrs. Robinson is willing to play the neoclassical game though "with due regard to its limitations." Thus she proceeds to re-examine the steady-state equilibrium relations between the quantity of capital, the labor force, and the state of technical knowledge.

It is during this re-examination that Mrs. Robinson stumbles upon what appears now to be the Achilles heel of simple neoclassical theory. What she discovered is very

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this proposition is the experience of practical life." Bohm-Bawerk, op. cit., p. 20.

[25] Ibid., p. 83. "We cannot abandon the production function without an effort to rescue the element of common sense that has been entangled in it."

simply stated as follows: under the postulate of a given "state of knowledge," if we observe the changes that take place in the technique of production as the real wage rate rises (and interest rate falls), we may find that the technique being adopted is less capital intensive rather than more capital intensive.

Such a possibility is labeled "perverse" by Mrs. Robinson and she devotes very little attention to it.[26] But as we now know, it is a very important seed that has been planted here. For--and this is the Cambridge criticism--once this is admitted our propositions (b) and (d) of the preceding section have to go.

It will now be useful to look at Mrs. Robinson's argument in greater detail. Doing so will help to clarify the above statement. In addition it will bring out a point which is of some interest in itself: recall that the basic characteristic of the Austrian model is the element of time whereas the basic characteristic of the Clark model is a timeless production function. Mrs. Robinson's chief contribution, judging from all that has transpired since the appearance of her article, stems from

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[26]It is interesting to note how one of the most persistent of the critics is unable to admit to the basic flaw in the theory in question. Perhaps the explanation is that the flaw turns out to be the very "element of common sense" that we should like to preserve. Or perhaps it is merely that "A good deal of exploration . . . is needed before we can say whether the above is a mere theoretical rigamarole, or whether there is likely to be anything in reality corresponding to it." Robinson, ibid., p. 106.

the fact that she had put time back into the analysis of production. For although the result was a very "inconvenient" production function, it threw open for all to see the very doubts that had previously plagued Wicksell and were destined to lead to what we now call the Cambridge criticism.[27]

To re-examine the neoclassical argument about the movement of the capital-labor ratio, Mrs. Robinson found that it was first necessary to put definite meaning to the concept of a "quantity of capital". This is an index number problem[28] and it presupposes that there is something meaningful that is called capital. After some preliminary discussion lamenting the very notion of capital-in-general, she decides that "to treat capital as a quantity of labour time expended in the past is congenial to the production-function point of view, for it corresponds to the essential nature of capital regarded as a factor of production."[29] But, and here is the critical point, "the past labour time

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[27]In the Lectures he wrote that " . . . it appears inconceivable a priori that an increase of capital could, ceteris paribus, coincide with a decrease of both wages and rent--though the question should be further investigated." op. cit., p. 183. Italics added.

[28]This is the message of those who were first to react to Mrs. Robinson's article. See David G. Champernowne, "The Production Function and the Theory of Capital: A Comment," Review of Economic Studies, XXI (1953-1954), pp. 112-135. Also Robert M. Solow, "The Production Function and the Theory of Capital," Review of Economic Studies, XXIII (1955), pp. 101-08.

[29]Robinson, "The Production Function and the Theory of Capital," op. cit., p. 82.

which produced to-day's capital goods was itself operating upon preexisting capital goods or natural resources. . . . At any moment when work is being done to-day's labour is being added to the product of past labour, which in its own day was added to the product of still earlier labour."[30]

Thus if we are to take labor time embodied in capital goods to establish a concept of capital-in-general, we must recognize that it is compound labor time that is relevant. So Mrs. Robinson defines capital--the factor of production--to be past labor time, compounded at interest, embodied in the stock of capital goods.[31] As it turns out, this is a surprisingly critical step. For from this definition comes her shifting production function--called a productivity curve by her. Let us take a look at it.

First we must suppose that we can draw up a set of blueprints containing all the techniques of production that

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[30]Robinson, Accumulation, op. cit., p. 121. This point is less successfully made in her article on the Production Function, op. cit., p. 82.

[31]Robinson, Accumulation, op. cit., p. 123. Here it is clear that this is what she means by capital-in-general. It is confusing, but perfectly legitimate, to multiply our definitions according to our needs. Thus Mrs. Robinson calls " . . . the stock of goods in existence at any moment physical capital. The value of these goods in terms of a unit of output we call capital simpliciter. Capital valued in terms of wage units we call real capital; though it must be observed that there is a slightly misleading flavour about this term, since the cost of capital goods, in terms of wage units, includes interest over the time required to construct them and to use them in production." Robinson, "Production Function and the Theory of Capital," op. cit., p. 86. Italics in original. It is her real capital that interests us.

are known to producers. Each technique is defined as a specific set of capital goods that together with a given amount of labor will produce a certain output. These are strictly engineering relations. Next we must know how much capital is represented by each set of capital goods in order that we might avoid using a technique that involves more capital per man but the same or less output than some other known technique (in her terminology this is the problem of "costing" the capital goods).

Suppose we have done the above and are left with a set of techniques that may be represented by a curve such as shown in Figure 2.[32] The vertical axis measures output per man and the horizontal axis the ratio of real capital to labor. We have four distinct techniques labeled Alpha, Beta, Gamma, and Delta arranged in terms of increasing capital intensity. Thus Alpha represents more capital per man than Beta, Beta more than Gamma, and Gamma more than Delta.

Now suppose that when the wage rate is given as OW the techniques Gamma and Beta are equally profitable and the rate of profit on either technique (equals the rate of interest in long run competitive equilibrium) is given by $\frac{1}{ON}$.[33] The question is: what will happen to the capital

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[32]The following is based on the discussion in her Accumulation, op. cit., pp. 411-18.

[33]For the proof of these relations see ibid. Our interest is solely in the fact that the function must shift.

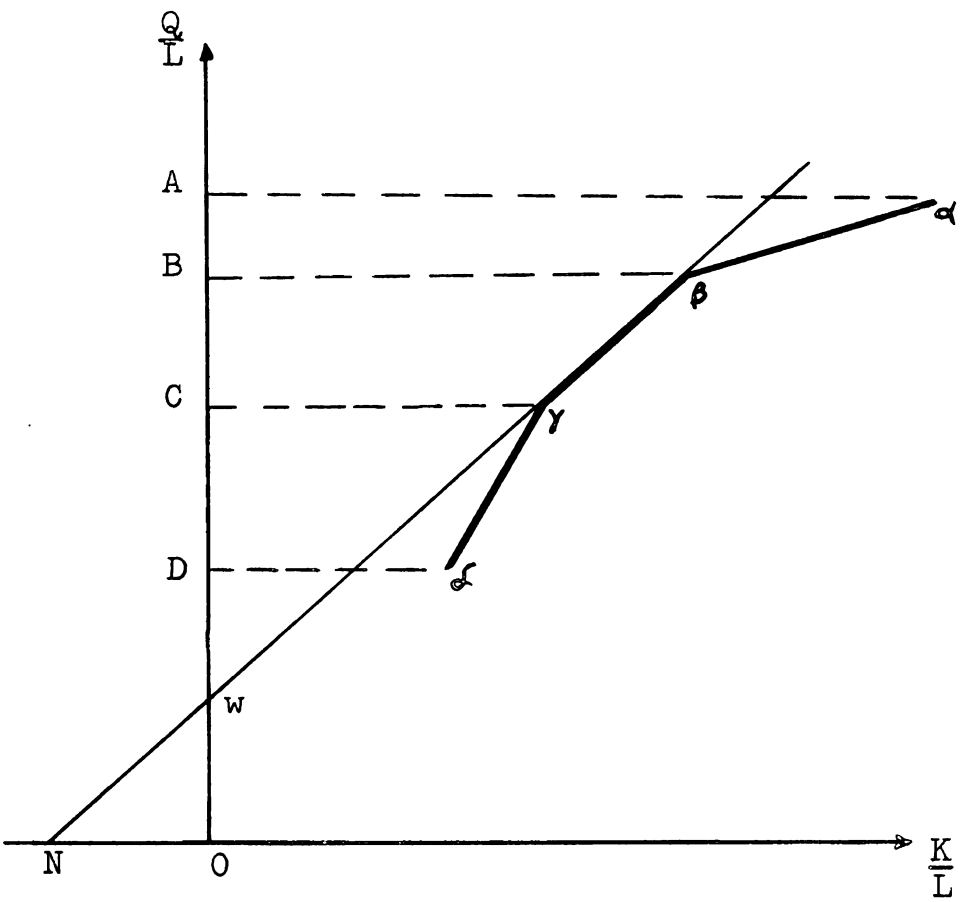

Figure 2. Joan Robinson's Productivity Curve

intensity of the technique in use as the real wage rises? The answer: it depends! Remember that capital is defined as past labor time, compounded at interest, embodied in the capital goods. Thus as the wage rate rises and the interest rate falls each and every set of capital goods appears as less capital.[34] The production function shifts to the left.

It may be "reasonable" to suppose that, as the wage rate rises, the production function shifts so that each new equilibrium wage-interest ratio corresponds to a more capital-intensive technique. This possibility is shown in Panel a, Figure 3. But it is also possible, as Miss Ruth Cohen has noticed, that the production function will shift such that the technique brought into use will be less capital intensive.[35] This possibility is shown in Panel b, Figure 3. When the real wage is W_1 both Gamma and Beta techniques are profitable. As the wage rate rises to W_2 the Gamma technique becomes gradually replaced by the more capital intensive Beta technique which alone is the most profitable at wage rates between W_1 and W_2. At the wage

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[34]That the interest rate must fall follows from the assumptions of a given fully employed labor force and the division of all product between laborers and capitalists. Robinson, "The Production Function and the Theory of Capital," op. cit., pp. 85-86. See below Chapter II where this relation is fully discussed for a somewhat different real capital model.

[35]Ibid., p. 94. "The relation between one curve and the next depends upon the reaction of the cost of various outfits of equipment to differences in the rate of interest, and this depends, as we have seen, in a complicated way, upon the gestation period and length of life of items of equipment."

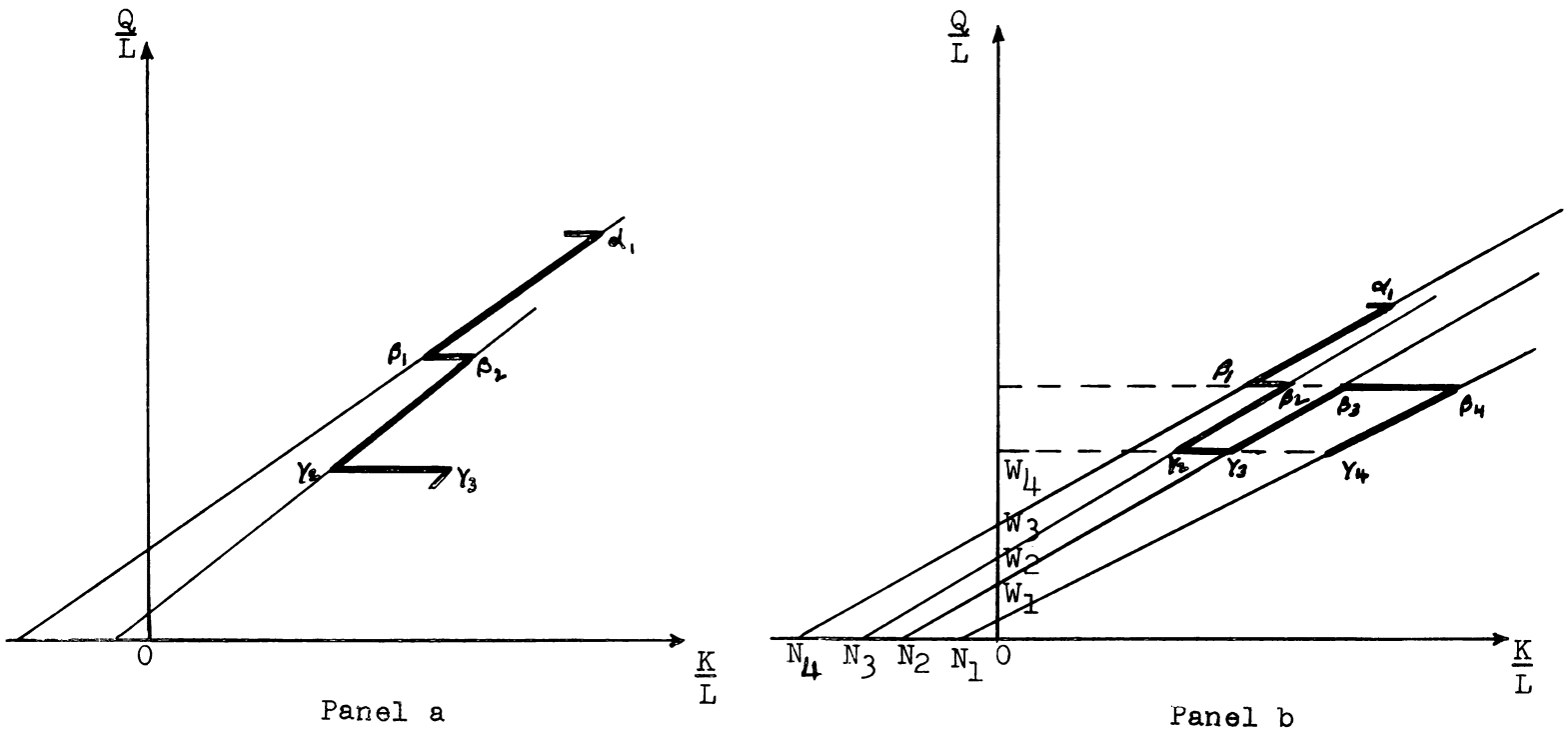

Figure 3. Effect of an Increase in the Real Wage on the Capital Intensity of Techniques

rate W_2 both techniques are again profitable and as the wage rate rises even higher, toward W_3, we find that the Gamma technique has become the most profitable technique. Thus we have moved from a more to a less capital-intensive technique.

If we search for an explanation of this "perverse" reaction to a change in the wage-interest ratio, we are led straight to the element of compound interest. In Mrs. Robinson's words: "This might occur if the plant required for less mechanized techniques had a much longer gestation period or working life, so as to be much more sensitive to the interest rate than that for mechanized techniques."[36] What this means is that the less capital-intensive technique has been more heavily weighted by the compound interest factor.[37]

To sum up, the foregoing demonstrates that the criticism of neoclassical theory (a-la-Robinson) really comes down to this:[38] time plays an important role as an

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[36] Ibid., p. 106.

[37] In other words, her measure of capital is subject to the same defect that was present in the Austrian measure (i.e., the average period). A lower rate of interest would shorten the average period even if production processes were unchanged if the average period did not neglect compound interest. This is Wicksell's criticism of Bohm-Bawerk's average period. Actually Bohm-Bawerk did not include even simple interest in his definition. On this see Samuelson, "A Summing Up," op. cit., p. 570.

[38] Perhaps I am over-reacting to Mrs. Robinson's capital concept. Nevertheless, I believe that this is the key to what really disturbs her. She is anxious, as was Bohm-Bawerk, to master the element of time. The criticisms

element in production. In particular it is crucial to the notion of capital and the productivity of labor. Yet neo-classical theory pretends that time does not matter. This is likely to lead to error.

Surprisingly the error, if there is one, does not concern time per se. Rather it concerns the link that the theory has erected between the wage-interest ratio and the capital intensity of production (i.e., the capital-labor ratio). In particular the rate of interest may not be a single-valued function of the capital-labor ratio.

The immediate reaction to this was to some extent beside the point, concentrating as it did on the index number problem.[39] But the "perverse" reaction of the capital-labor ratio did bother those who came upon it. For example, Mr. Champernowne, who was anxious to replace Mrs. Robinson's capital with something more "convenient", is quite careful to point out that the "perverse" case can only be made to disappear by assumption.[40]

But it is Piero Sraffa who finally announces with some degree of confidence that the "perverse" case must be taken seriously:[41]

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that we have mentioned above are repeated by Mrs. Robinson in "Accumulation and the Production Function," The Economic Journal (September, 1959), pp. 433-42. See especially p. 437 and p. 441.

[39]Champernowne, loc. cit., and Solow, loc. cit.

[40]Champernowne, ibid., p. 119. By convenience, he had in mind a function that would yield nice aggregate relative shares.

[41]Piero Sraffa, Production of Commodities by Means

> We have seen that as the rate of profits rises there may be several intersections between the prices at which the two methods produce, with as many switchings backwards and forwards from one method to the other. . . . In view of this possibility we cannot (contrary to what one might have expected) say in general that, of two alternative methods of production, the one that corresponds to a Standard system with a higher ratio of product to means of production . . . will be the most profitable when the rate of profits is comparatively high, and the least profitable when it is comparatively low.

The critics can no longer be ignored or passed over by a redefinition of terms. Henceforth we shall have to qualify our statements concerning the wage-interest ratio as it relates to the capital intensity of production.

The possibility that the same technique may be observed at two substantially different wage rates (interest rates) has been labeled the "Ruth Cohen Curiosum" (Joan Robinson) or more recently the "reswitching of techniques" (Sraffa-Pasinetti). It is, to repeat, reswitching and its implications that constitute the Cambridge criticism of simple neoclassical theory. This implication is of great importance for capital theory: no longer can one say that a fall in the rate of interest generally entails the adoption of more capital-intensive techniques

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of Commodities (Cambridge: Cambridge University Press, 1963), p. 84. Italics added. The entire thrust of this work is to determine the effects of distribution on the relative prices of commodities. The model employed is a fixed proportions circulating capital model, and the price of every commodity is determined by the production conditions. The cost of commodities is reduced to past labor compounded at interest. The critical point in the book comes with the demonstration that, as wages rise and interest falls, relative prices may reverse themselves. This is used to argue the above point by way of analogy.

of production with the consequence that the permanently sustainable consumption stream is greater. A lower rate of interest may be associated with a lower rather than a higher consumption plateau. And though reswitching reveals this possibility, even it (reswitching) is not necessary.[42]

The exponents of neoclassical theory were not long in replying to this criticism. The first response came in the form of a nonswitching theorem.[43] Here neoclassical theory was willing to concede reswitching in the production of a single good; but not for the economy as a whole. However, under closer examination, the proof of the theorem was itself shown to be erroneous and so the theorem had to go.[44] In its place we find a number of numerical examples of reswitching--each designed to prove that this phenomena is indeed possible for the whole basis of production. Beyond this everyone has been most anxious to spell out the above-mentioned implication of reswitching for capital

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[42]See, for example, Michael Bruno, Edwin Burmeister, and Dytan Sheshinski, "The Nature and Implications of the Reswitching of Techniques," The Quarterly Journal of Economics, LXXX (November, 1966), pp. 548-49. Perhaps it should be noted again that neither Mrs. Robinson nor her immediate critics seem to have realized the importance of the "perverse" case. For this the credit must go to Pasinetti. See Luigi L. Pasinetti, "Changes in the Rate of Profit and Switches of Techniques," ibid., pp. 503-17.

[43]David Levhair, "A Nonsubstitution Theorem and Switching of Techniques," The Quarterly Journal of Economics, LXXIX (February, 1965), pp. 98-105.

[44]David Levhari and Paul A. Samuelson, "The Nonswitching Theorem is False," The Quarterly Journal of Economics, LXXX (November, 1966), pp. 518-19.

theory. In the next chapter we shall explore in greater detail the neoclassical reaction to the Cambridge criticism.

CHAPTER II

SWITCHES OF TECHNIQUES AND STEADY-STATE CONSUMPTION LEVELS

I. Introduction

The simple neoclassical theory of the preceding chapter is based on the two fundamental assumptions of (a) instantaneous production (i.e., a timeless production function or the absence of "roundaboutness"), and (b) a single homogeneous real capital good. But a roundabout method of production may be viewed as an instantaneous production process by treating goods-in-process of different ages as different goods (i.e., as so many fictitious heterogeneous capital goods); hence it is the assumption of a homogeneous capital good that is the more basic assumption.[45] As we shall see below, once the heterogeneity of

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[45]At least insofar as reswitching is concerned, see Michio Morishima, "Refutation of the Nonswitching Theorem," The Quarterly Journal of Economics, LXX (November, 1966), p. 524. Actually, there are two aspects of a production process with a time lag that are important for reswitching. One is the element of compound interest; the other is the distribution of labor in time. It is the latter element that corresponds most closely to the heterogeneity aspect of capital goods in an instantaneous production process. On the importance of the distribution of labor in time see Maurice McManus, "Process Switching in the Theory of Capital," Economica, XXX (May, 1963), pp. 109-21. See especially p. 112 for some necessary conditions for switching that reflect this point.

capital goods is allowed for reswitching can be shown to be a definite possibility.[46]

It is this instantaneous production model with heterogeneous capital goods that dominates the neoclassical reply to the Cambridge criticism.[47] After some preliminary discussion, we will employ such a model to bring out the relation between switches in techniques and steady-state consumption levels.

Levhari's Nonswitching Theorem

It is necessary to begin by commenting briefly on the argument of David Levhari, whereby he is led to conclude that the "Ruth Cohen Curiosum" " . . . may indeed be observed in the production of a single good. But . . . it is impossible with the whole basis of production."[48] Our concern is

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[46]Recall that this is precisely what Mrs. Robinson was attempting to do when she came forward with her hybrid production function. But, as we emphasized, it was the element of time in her measure of capital that was critical in revealing the reswitching possibility. As we shall see below, the most recent discussions do not employ any concept of capital, i.e., they focus exclusively on capital goods.

[47]David Levhari, "A Nonsubstitution Theorem and Switching of Techniques," The Quarterly Journal of Economics, LXXIX (February, 1965), pp. 98-105. Also "Paradoxes in Capital Theory: A Symposium," The Quarterly Journal of Economics, LXXX (November, 1966), pp. 503-83. Levhari uses a one period circulating capital model. It is here classified as an instantaneous production model since it cannot be said to involve the element of time in any essential way.

[48]Levhari, op. cit., p. 99.

not with Levhari's proof per se but with the meaning of his "whole basis of production."[49] This he defines as an (n x n) matrix of commodity-input coefficients together with a (1 x n) row vector of labor-input coefficients. Each column vector is then interpreted as one method for producing a unit of output in one industry.[50] Thus the whole basis of production appears as the aggregate of the methods of production (one for each industry) when operated at unit level.

What Levhari tried to prove is that once any given matrix is left behind as the rate of interest falls (wage rate rises) it can never be readopted as the interest rate falls still further. Once this is proven it may be said that the matrices are unambiguously ordered with respect to the rate of interest. That, however, is all one could say.

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[49]His proof, being in error, served as the catalyst for the recent reswitching discussion. On the error in his proof see P. Garegnani, "Switching of Techniques," The Quarterly Journal of Economics, LXXX (November, 1966), pp. 554-561. Levhari has attempted " . . . to demonstrate that a semi-positive vector $x^* = (x_1^*, \ldots x_n^*)$ of levels of production of the various industries always exists, such that, at those levels of activity either system A requires more of all kinds of commodity-inputs than B, or the contrary is true, or, finally, A and B require the same quantities of all those inputs." p. 556. Italics in original. The error concerns the existence of this vector. I am not qualified to comment on the mathematics of Levhari's proof, but others have shown that this vector can be of no use to him. The emphasis, however, has been placed on numerical demonstration and it is that level of argument which dominates this paper.

[50]Levhari, op. cit., pp. 99-100. "Let there be k_1 activities which can be used to produce good 1, a^{11}, a^{12}, . . . , a^{1k_1}; k_2 activities to produce good 2, . . ., and k_n for good n. Each activity is composed of n + 1 elements. The first element . . . gives the labor requirements, and the remaining n components give the requirements of inputs of goods to produce one unit of gross output of the given commodity."

But, as Levhari clearly recognizes, the Cambridge criticism concerns the capital intensity of production.[51] Even if his matrices were unambiguously ordered with respect to the rate of interest, would this demonstrate that the rate of interest is a single-valued function of the capital-labor ratio? Perhaps. The point is, before anyone could say, it would first be necessary to spell out the exact sense in which these matrices could be said to be comparable to the aggregate capital-labor ratios of the neoclassical model.

Yet this was not done, either by Levhari or those who have subsequently shown that his proof is in error.[52] This seems most unwarranted in view of the fact that it is the capital intensity of production that is the critical variable in the Cambridge criticism.

Capital Intensity in a Heterogeneous Real Capital Model

Let us here set out the equivalence between the capital intensity of production in a heterogeneous

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[51] In referring to the Cambridge criticism he says, "This would have the unfortunate consequence that we could no longer say that the lowering of the interest rate brings about a process of 'deepening' and each process is more capital-intensive than its predecessors." Ibid., p. 99.

[52] The tendency in the literature is to associate directly the "non-comingback" of a matrix with the capital intensity of production. This, however, is not warranted. It is not obvious that when a matrix comes back it necessarily represents the same aggregate capital-labor ratio. This is the question of capital-intensity along the grand factor-price frontier that will be fully explored in the next chapter.

capital-goods model and the aggregate capital-labor ratios of the neoclassical model.[53] To begin, we assume a given technology and a large number of firms each producing a single product.

Now suppose each single-product firm is confronted with a production constraint of the form:

$$L_r = a_{Lr}y$$

$$(15) \qquad y = \min \left(\frac{L_r}{a_{Lr}}, \frac{K_r}{a_{Kr}}\right) \qquad (r = 1,2, \ldots n)$$

$$K_r = a_{Kr}y$$

where y, L, and K represent output and the services of labor and capital respectively. There are n different processes available to the firm, and a_{ir} (i = L,K) represents the additional assumption of fixed minimum unit production requirements for labor and capital. Less of either input will yield less output and more will be superfluous.

Strictly speaking we do not have a production function but a number of production functions, one for each of the n different processes. This complete set of production functions may be viewed as the technology of the firm. Let us refer to each production function as one method of production. Then, as above,[54] we may speak unambiguously of m distinct industries, each of which selects one method of

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[53]The following discussion is based on the treatment in Robert Dorfman, Paul A. Samuelson, and Robert M. Solow, Linear Programming and Economic Analysis (New York: McGraw-Hill, 1958), Chapter 11. See especially pp. 286-88 and pp. 300-02.

[54]See Chapter I, pp. 10-12.

production. The aggregate of these methods is referred to as a technique of production.[55]

Thus, except for the additional assumption of fixed coefficients, a technique of production in a heterogeneous, real-capital model is nothing but the aggregate capital-labor ratio of the neoclassical model. As long as at least one of the methods of production is different between any two arrays of industry methods (i.e., production functions), the techniques will be distinct, and we may think of a large number of such discrete capital-labor ratios as representing the technology of the economy.

As an extreme illustration consider a two-sector economy--a consumption sector (C) and a capital-good sector (M)--each of which has the following method of production:

$$C = \min\left(\frac{L_C}{a_{LC}}, \frac{K_C}{a_{KC}}\right) = \left(\frac{L_C}{2}, \frac{K_C}{4}\right)$$

(16)

$$M = \min\left(\frac{L_M}{a_{LM}}, \frac{K_M}{a_{KM}}\right) = \left(\frac{L_M}{2}, \frac{K_M}{4}\right)$$

The technique of production is then the aggregate capital-labor ratio given by $\frac{a_{KC}}{a_{LC}}$ or $\frac{a_{KM}}{a_{LM}}$, each of which equals 2.[56]

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[55]Thus Levhari's matrix is one technique of production (supra p. 29). The same meaning attaches to all of the so-called technology matrices to be found in the reswitching literature. See Michael Bruno, Edwin Burmeister, and Dytan Sheshinski, "The Nature and Implications of the Reswitching of Techniques," The Quarterly Journal of Economics, LXXX (November, 1966), pp. 528-31.

[56]Notice that $a_{LC} = a_{LM}$ and $a_{KC} = a_{KM}$. As explained

The simple but important point to be drawn from this discussion is that a technique of production must be thought of as the aggregate capital-labor ratio for the economy. This will be the case regardless of the capital model that one chooses for discussing the reswitching phenomenon or any of its implications.

Thus when Levhari argued that his matrices were unambiguously ordered with respect to the rate of interest, he really meant to say that the aggregate capital-labor ratio was so ordered.[57] In the reswitching literature that has appeared since his article, many writers have demonstrated that these matrices are not unambiguously ordered with respect to the rate of interest. The Cambridge criticism was subsequently reasserted by these writers: the aggregate capital-labor ratio is not a single-valued function of the rate of interest. Furthermore it was shown that this implied unorthodox behavior of steady-state consumption levels. But--and this is why we have taken so much time with this introduction--notice that these propositions concern the aggregate capital-labor ratio. In what follows it will be seen that the aggregate

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below, Chapter III, pp. 71-78, this assumption on the coefficients means that our so-called two-sector economy is in fact a single-sector economy. Thus the aggregate capital-labor ratio is the sector capital-labor ratio.

[57] Supra, n. 51.

capital-labor ratio has, in fact, been lost in the arguments surrounding the switches of techniques.

II. Switches of Techniques in a Two-Sector Heterogeneous Real Capital Model

The switch and reswitch in production techniques may be most easily discussed within the framework provided by Samuelson in his paper on the Surrogate Production Function.[58] The model we wish to employ consists of the following set of assumptions:

a) All firms are single-product firms (i.e., absence of joint production).

b) Two goods are produced, a consumption good and a capital good (denoted C and M respectively).

c) There is one primary unproduceable homogeneous input, Labor.

d) Each industry uses labor of a given amount together with a specific and fixed complement of the capital good to produce a unit of output (i.e., fixed coefficients of production).

e) Labor Employed $\leq$ Labor Supply, Capital Employed $\leq$ Capital Supply (i.e., either factor may be redundant).

f) Each industry has a finite number of production methods as defined above.

g) Competition is perfect in all markets.

This model is then represented by the following price equations:

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[58] Paul A. Samuelson, "Parable and Realism in Capital Theory: The Surrogate Production Function," Review of Economic Studies, XXIX (June, 1962), pp. 193-207.

$$1 = a_{LC}w + a_{KC}rP \quad ,$$

(17)

$$P = a_{LM}w + a_{KM}rP \quad ,$$

where P is the equilibrium price of machines in terms of consumption goods, w is the real wage rate, and rP = q is the quasi-rent of the capital good. There are two equations in the three unknowns: P, w, and r. We may eliminate the relative price of the capital good and in terms of the consumption good as numeraire obtain a single equation in the two unknowns w and r:

$$(18) \qquad w = \frac{1 - a_{KM}r}{a_{LC} + Ar} \quad ,$$

where we have taken

$A = (a_{KC}a_{LM} - a_{LC}a_{KM}) = \det. \begin{bmatrix} a_{LM} & a_{LC} \\ a_{KM} & a_{KC} \end{bmatrix}$, and we recognize this matrix to be one page from the economy's "book of blueprints" as discussed above.

The Factor-Price Frontier

It is this trade-off between the feasible long-run equilibrium wage and interest rates[59] that stands at the center of the recent reswitching discussion. Every technique yields a unique trade-off between these two variables. Let us suppose that A = 0, which amounts to assuming that

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[59]Under our assumptions the rate of interest is equal to the profit rate and we shall ask that the reader keep this in mind as we will make explicit reference to the rate of interest only.

both sectors use factors in the same proportions (i.e., $\frac{a_{KC}}{a_{LC}} = \frac{a_{KM}}{a_{LM}}$). We then have

$$(19) \qquad w = \frac{1 - a_{KMr}}{a_{LC}} .$$

This linear relation between the real wage and the rate of interest is illustrated in Figure 4.

The maximum real wage that may be obtained under this technique is seen to be determined by the unit production requirements in the consumption-good sector. The more labor required to produce a unit of the consumption good, the lower will be the maximum long-run wage rate. Thus the intercept is the average productivity of labor in this sector (this is an important relation that must be used in interpreting the graphs appearing in the remainder of this Chapter).

The maximum rate of interest with this technique is likewise seen to be completely determined by the technical production requirements of the capital-good sector. The less capital needed to produce a unit of itself, the higher the rate of return to the capital good.

Finally, note the inverse relation that must hold between the real wage and the interest rate under these assumptions.[60] When capital is redundant, the total product obtainable with this technique is $L_t \times \frac{1}{a_{LC}}$,

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[60]For a more rigorous treatment of this relation see, for example, John Hicks, Capital and Growth (New York: Oxford University Press, 1965), pp. 317-18.

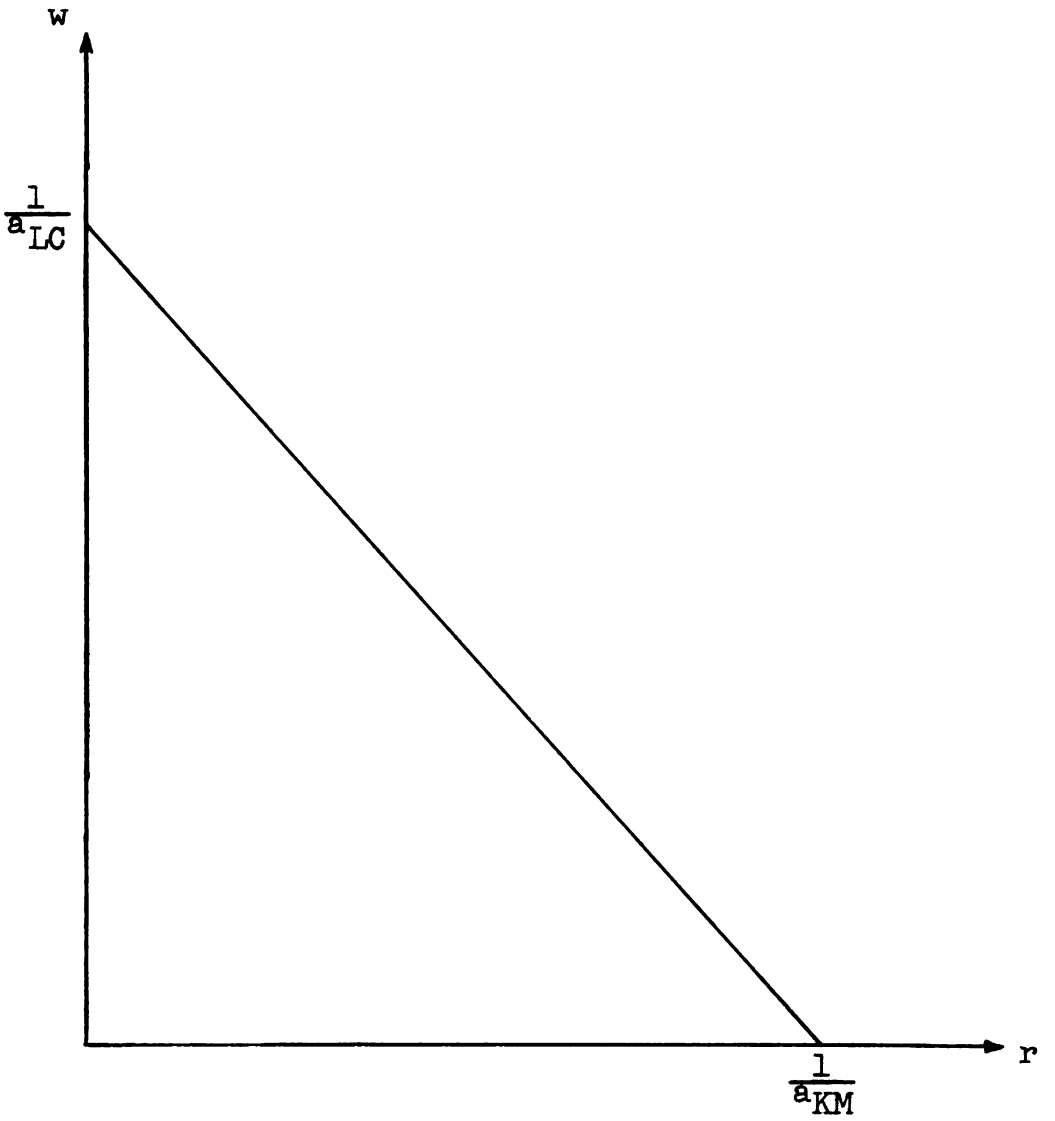

Figure 4. The Factor-Price Frontier

a constant; and the total product goes to the wage earners. If the wage rate falls, some of the product is freed from the wage bill and goes to pay for the now relatively scarce capital good. Since any movement in the wage or interest rate takes place <u>within</u> the context of an unchanged production technique, either factor gains only what the other loses. If the rate of interest is to rise, the real wage has to fall.

By arbitrarily varying the production coefficients, we may generate any number of such trade-offs between the real wage and the interest rate, called a "factor-price frontier" by Samuelson and a "wage curve" by Hicks.[61] Consider another technique (b_{LC}, b_{KC}, b_{LM}, b_{KM}) such that $b_{LC} > a_{LC}$ and $b_{KM} < a_{KM}$ and $B = 0$. The factor-price frontier is given by

$$(20) \quad w = \frac{1 - b_{KM}r}{b_{LC}} .$$

Equations (19) and (20) are brought together in Panel a, Figure 5.

It is assumed that given the wage rate, the technique that allows the greater return to capital is "chosen" by the economy. The critical wage rate is w*, with Alpha technique chosen for higher real wage rates and Beta chosen for rates below w*. Consider the wage falling monotonically toward zero: the economy is said to switch from Alpha technique

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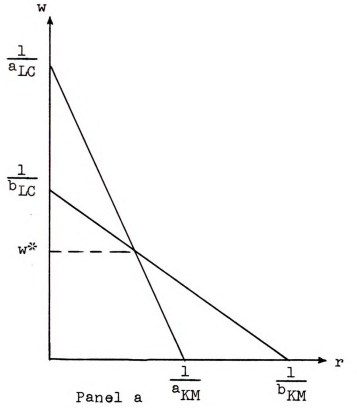
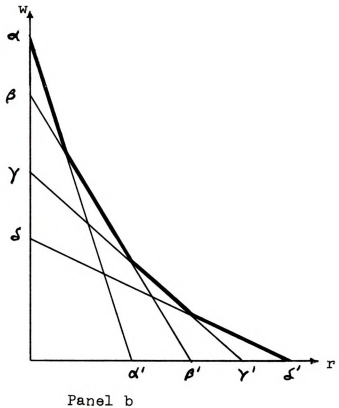
[61]Hicks, <u>ibid</u>., p. 140, and Samuelson, <u>op</u>. <u>cit</u>., p. 196.



Panel a



Panel b

Figure 5. The Grand Factor-Price Frontier

to Beta technique at the wage w*. A number of switches are shown in Panel b of Figure 5. The envelope produced by such switching of techniques (i.e., the heavily shaded outermost line called the grand factor-price frontier) will in this case possess properties identical to those of any individual factor-price frontier. In particular our conclusion concerning the relation of the real wage rate and the rate of interest is unaltered by such switching. We will always be operating on the envelope of the individual factor-price frontiers by virtue of our profit maximization hypothesis. In addition, with given money wages and perfect competition, market prices will ensure that a fall in the rate of interest is accompanied by an increase in real wages.[62]

Reference to Panel b of Figure 5 is sufficient to demonstrate that as the wage rate falls monotonically from its maximum to zero, the economy can never "readopt" a technique once it has been left behind. The techniques are unambiguously ordered with respect to the wage rate (interest rate). This is all we need concern ourselves with at present for it is precisely the question of the

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[62]Paul A. Samuelson, "A Summing Up," The Quarterly Journal of Economics, LXXX (November, 1966), p. 575. "Under perfect competition, either workers can hire capital goods or capitalists hire workers. At a lower interest rate, or cost of capital, workers can always pay themselves a higher real wage even without changing techniques; so capitalists will have to match up." With perfect competition, the result is the same even if capitalists do all the hiring.

"readoption" of a technique once it has been left behind that constitutes the issue of reswitching.

Reswitching on the Grand Factor-Price Frontier

It is possible that a technique which is the most desirable of all feasible techniques at some relatively high rate can once again become the most desirable at a relatively low wage rate. As shown by numerous writers, a "readoption" of techniques is a possibility both for the individual firm and/or the economy as a whole. Consider the two techniques of Figure 6.[63]

The critical wage rates are w_1^* and w_2^*. At any wage rate above w_2^*, the economy will adopt the Beta technique because it yields the higher rate of return. As the wage rate falls below w_2^*, the Alpha technique becomes the more profitable; but as the wage falls still farther, going below w_1^*, Beta technique again wins out in a competitive market. Hence it is demonstrated that the economy may switch back to the original technique before the wage rate has fallen to zero.

The literature on reswitching contains several numerical examples of the behavior depicted in Figure 6,

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[63]Note that the Beta technique gives rise to a nonlinear factor-price frontier. The following discussion shows that reswitching is impossible if all techniques give rise to linear factor-price frontiers. The frontier will be nonlinear whenever the intrinsic factor intensities of the capital and consumption good differ. See below, Chapter III, pp. 86-87.

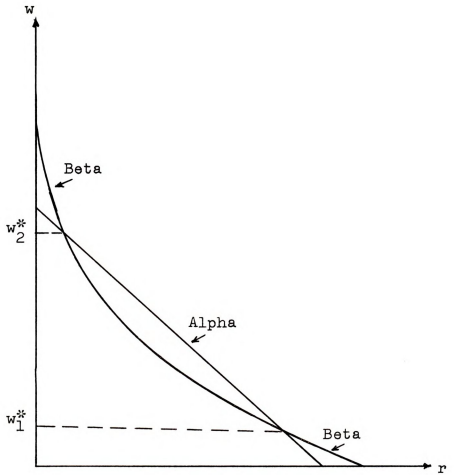

Figure 6. Reswitching of Techniques

and we shall on occasion want to examine such examples in great detail. The more immediate and interesting question, however, is: granting the possibility of reswitching, what are the conditions that must be satisfied if we are either to ensure the behavior of Figure 6 or prevent it?

Conditions sufficient to prevent the return of a technique have now been formulated by a number of writers.[64] Attention has centered primarily on the two-sector capital model of this section. We have, for example, the obvious conditions that for any two techniques, if there is one and only one intersection in the positive quadrant, reswitching is not possible (see e.g., Figure 5, Panel a). More precisely we require either that:[65]

$$b_{KM} > a_{KM} \text{ while } a_{LC} > b_{LC} ,$$

(21) or

$$b_{KM} < a_{KM} \text{ while } a_{LC} < b_{LC} .$$

That is, the technique with the lower (higher) labor-consumption coefficient must also be the one with the higher (lower) capital-capital coefficient.

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[64]Piero Sraffa, Production of Commodities by Means of Commodities (Cambridge: Cambridge University Press, 1963), p. 94; Murray Brown, "Substitution-Composition Effects, Capital Intensity Uniqueness and Growth," Discussion Paper No. 2, Department of Economics, SUNY at Buffalo, Buffalo, N.Y., 1967, pp. 14-18; Hicks, op. cit., p. 154; Morishima, op. cit., p. 523, and Bruno, et al., op. cit., pp. 534 and 545.

[65]This condition is stated by Sraffa, ibid., Hicks, ibid., and Morishima, ibid., but most concisely by Bruno, et al., ibid., p. 534.

In addition to such simple and direct formulations as these, Bruno, Burmeister, and Sheshinski have attempted to formulate similar conditions for a general n-capital good model.[66] Unfortunately, as they emphasized, little economic meaning can be attached to their results. Nevertheless, the method employed is worthy of comment for at least two reasons. First, it appears that the authors themselves have in the final analysis disregarded their own warnings.[67] Second, an examination of the method will, I think, provide us with some justification for our own preoccupation with numerical analysis.

Let the factor-price frontiers for the Alpha and Beta techniques of Figure 6 be:

$$(22) \qquad w = \frac{1 - {}^{a}_{KM}r}{{}^{a}_{LC}}; \qquad w = \frac{1 - {}^{b}_{KM}r}{{}^{b}_{LC} + Br}.$$

Set the two wage rates equal so as (hopefully) to determine the two switch points. One then obtains the following quadratic in r:

$$(23) \qquad r^2 - \frac{(B + {}^{a}_{LC}{}^{b}_{KM} - {}^{b}_{LC}{}^{a}_{KM})}{B\,{}^{a}_{KM}}\, r + \frac{({}^{a}_{LC} - {}^{b}_{LC})}{B\,{}^{a}_{KM}} = 0.$$

The real positive roots of this quadratic will be the switch points if there are any.[68] Presumably we can impose

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[66] Bruno, et al., op. cit., pp. 538-46 but especially pp. 544-45.

[67] They tend to put too much weight on their results (ibid., p. 527 and p. 545), though they are in agreement with the points which follow (ibid., p. 549).

[68] For a complete technical analysis concerning the possible number of switch points in a given model, see McManus, loc. cit.

restrictions on the coefficients so as either to ensure the behavior of Figure 6 or to prevent it. It is customary (but not necessary) to suppose that $B > 0$,[69] and we shall make the additional assumption that any solutions of equation (23) will be real.[70] We turn first to the sufficient conditions for reswitching.

According to Descartes' Rule of Signs, a condition for two positive real roots is given by

(24) $(a_{LC} - b_{LC}) > 0$ and $(a_{LC}b_{KM} - b_{LC}a_{KM}) > 0$,

which requires either

$b_{KM} = a_{KM}$ and $a_{LC} > b_{LC}$,

or $b_{KM} > a_{KM}$ and $a_{LC} > b_{LC}$,

or $a_{LC} > b_{LC}$ by more than $a_{KM} > b_{KM}$.

It seems that we have three distinct sets of restrictions that might be placed on the coefficients of equation (23) so as to yield reswitching in this model. However, as shown in Figure 7, only one set does, in fact, yield reswitching: the case where $a_{LC} > b_{LC}$ by more than $a_{KM} > b_{KM}$. Both Panel a and b demonstrate that as the wage rate falls from its maximum of $\frac{1}{b_{LC}}$ to zero, the switch of techniques

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[69] Thus requiring that production in the consumption-good sector be capital-intensive relative to production in the capital-good sector.

[70] This will be the case so long as the discriminant of (23) is positive. Thus we are assuming that $(B + a_{LC}b_{KM} - b_{LC}a_{KM})^2 > 4(a_{LC})(a_{LC} - b_{LC})$. This condition is necessary for reswitching in this model, and assuming its presence allows us to formulate some simple sufficient conditions that will also be necessary. Without this assumption, the conditions we formulate are necessary but not sufficient.

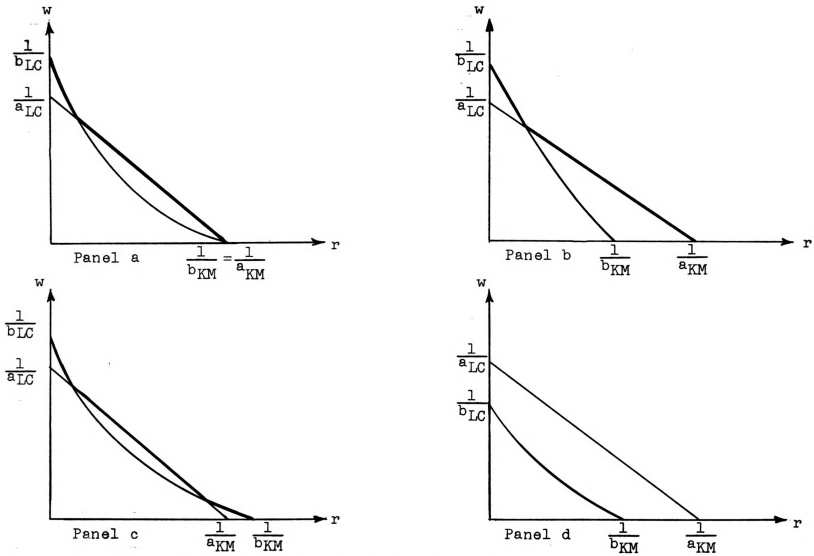

Figure 7. Reswitching and Descartes' Rule of Signs

is from Beta to Alpha. The economy will not return to the Beta technique before the wage rate reaches zero. Furthermore, as a comparison of Panel b with equation (21) makes clear, we have involved ourselves in a contradiction. The so-called obvious conditions for preventing the return of a technique now appear as sufficient conditions for the return of a technique. What has gone wrong?

The difficulty is in our straight-forward application of the method of Descartes to the problem of reswitching. Descartes' Rule of Signs states that the number of positive real roots of $f(r) = 0$ is either equal to the number of variations of sign in $f(r) = 0$ or less by multiples of two.[71] All our restrictions can guarantee is that equation (23) will have two variations of sign. Whether this yields reswitching cannot be determined without additional information.

Descartes' Rule of Signs may be applied to the problem of switching to determine the maximum possible number of switches. But there may be none. Thus the method that we have applied has an upward bias in the sense that the number of variations of sign often exceeds the actual switches of techniques. It is this important point that is demonstrated in Panels a and b of Figure 7.

Turning to the sufficient conditions to prevent reswitching, we must select our coefficients so as to allow

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[71] See, for example, Leonard E. Dickson, New First Course in the Theory of Equations (New York: Wiley, 1939), pp. 76-9.

at most one variation of sign in (23). We therefore let

(25) $(a_{LC} - b_{LC}) < 0$ and $(a_{LC}b_{KM} - b_{LC}a_{KM}) > 0$,

which requires that

$b_{KM} > a_{KM}$ by more than $b_{LC} > a_{LC}$.

As shown in Panel d, Figure 7, we have restricted ourselves to the irrelevant case in which one of the techniques is always dominant. This merely points up the fact that one variation of sign does not mean one switch point. One variation of sign means there can be at most one switch point. There may be none.

The point again to be emphasized is that the method used to establish our sufficient conditions is biased toward the reswitching result. We cannot, of course, say anything about the magnitude of the bias; nevertheless, we ought not lose sight of this bias when attempting to evaluate the stringency of our conditions. This point is especially important if we are to apply the method to a more general capital model. For then we are almost certainly going to overstate the case for reswitching.[72]

Now recall that we are investigating these sufficiency conditions in order that we might be in a position to say something about the possibility (likelihood?) of

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[72]There is, I think, a very real danger of putting too much weight on generality. See, for example, Bruno, et al., op. cit., p. 527. I for one would be reluctant to give much weight to the realization that switch points are related to the roots of n^{th} - degree polynomials. Unfortunately, there just isn't much one can say, in general, about these polynomials. McManus, loc. cit.

reswitching. Should we not then question the significance of an exercise which, when all is said and done, leaves us with the rather vacuous statement that " . . . reswitching is, at least theoretically, a perfectly acceptable case in the discrete capital model."[73] After all, everyone will agree that this possibility is demonstrated by the simple and more direct method of numerical example.

But, let us not be too hasty. We had better check to see if the conditions derivable by the method of Descartes (or any similar method such as Budan's Theorem), although shown above to be overly strong so far as reswitching is concerned, are not in fact of some importance when we come to a consideration of the implication that has been of much interest since the demonstrated possibility of the return of a technique for the economy as a whole.[74] I refer, of course, to the behavior of the permanently sustainable consumption stream available to society as the rate of interest falls.

III. Steady-State Consumption Levels

Our attention has been centered on the "choice" of production technique as the interest rate is allowed to fall (rise). The "choice" concerning which of several techniques is "best" carries with it society's decision

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[73]Bruno, et al., op. cit., p. 545.

[74]This is apparently the position of Bruno, et al., ibid., p. 549.

concerning which consumption stream is preferred over all that are technologically feasible. We may either suppose that society chooses directly between alternative consumption streams, or we may suppose that the choice is indirectly made through a decision to produce more or less "productive stuff", i.e., to add to or to subtract from the capital base.

The point of immediate interest is that a very definite relation has long been thought to hold between what we call the economy's technique of production, the interest rate, and steady-state consumption.[75] The latter refers to a plateau of consumption that may be attained and maintained indefinitely given an unchanging population and technology. The traditional triad will be briefly indicated by use of the neoclassical production model. We then spell out the logical equivalence between the economy's grand factor-price frontier and its steady-state consumption curve.[76] Finally the reswitching discussion is related to the behavior of steady-state consumption levels.

The Traditional Ordering of Consumption and the Interest Rate

Consider the smooth, twice differentiable, linearly homogeneous production function of the neoclassical

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[75]See, for example, Dan Usher, "Traditional Capital Theory," Review of Economic Studies, XXXII (1965), pp. 169-86. Our steady-states will be stationary states as the next sentence will make clear.

[76]This equivalence is noted by Usher, ibid., p. 173, n. 2.

model:[77]

$$(26) \quad C = F(K,L) \equiv LF\left(\frac{K}{L}\right) .$$

Thus

$$(27) \quad \frac{C}{L} = F\left(\frac{K}{L}\right) ,$$

where we have taken the national product (Y) to be net income (NNP) and thus equal to consumption (C) inasmuch as we are interested in the stationary state relations. The operation of diminishing returns ensures that

$$(28) \quad F''\left(\frac{K}{L}\right) < 0 ,$$

whereas the economic operation of enterprise requires

$$(29) \quad F'\left(\frac{K}{L}\right) > 0 .$$

We further require competitive imputation, so

$$(30) \quad w = \frac{\delta Y}{\delta L} = \frac{\delta C}{\delta L} = F\left(\frac{K}{L}\right) - \left(\frac{K}{L}\right)F'\left(\frac{K}{L}\right) > 0 ,$$

$$(31) \quad r = \frac{\delta Y}{\delta K} = \frac{\delta C}{\delta K} = F'\left(\frac{K}{L}\right) > 0 .$$

The relation of interest to us may be obtained from (26) and (31), parametric equations for the steady-state curve. Differentiate (31) with respect to $\left(\frac{K}{L}\right)$ to obtain

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[77]The following argument is offered as a simplification of the link between consumption flows and the changing augmentable capital factor as discussed by Usher, ibid., pp. 177-82. The basic difference is that his analysis allows for the element of time. It centers on what is called the "marginal input-output equation" whereas ours centers on the economy-wide production function. As he says, "The former is an equilibrium condition of a system describing the passage of the economy through time. Valid in a wide variety of circumstances, it will be shown to be the link between the production process and the determination of the rate of interest. The latter is one equation in a complete system. It requires for its existence a much more restrictive set of assumptions than does the marginal input-output equation, ...", p. 177.

$$(32) \quad \frac{dr}{d(\frac{K}{L})} = F''(\frac{K}{L}) < 0.$$

Next, differentiate (26) with respect to $(\frac{K}{L})$, obtaining

$$(33) \quad \frac{dC}{d(\frac{K}{L})} = LF'(\frac{K}{L}) > 0.$$

Dividing (33) by (32) yields the desired result:

$$(34) \quad \frac{dC}{d(\frac{K}{L})} \div \frac{dr}{d(\frac{K}{L})} = \frac{dC}{dr} = \frac{LF'(\frac{K}{L})}{F''(\frac{K}{L})} < 0.$$

The relations (32), (33), and (34) are shown in Panels a, b, and d of Figure 8. Panel d depicts what is called the steady-state consumption curve; it shows the relation between the rate of interest and the level of consumption for alternative stationary states. The steady-state curve has been drawn on the assumption that there is both a positive minimum consumption stream obtainable (C_M), and a finite level of consumption available at bliss (C_B). (The latter defined as the maximum consumption attained in any feasible stationary state).

Panel c depicts several alternative time paths of consumption. Starting at a common consumption level at time zero (C_0), each is assumed to end at a different stationary state. As society foregoes consumption today in favor of more consumption tomorrow, resources are diverted into the production of additional quantities of the homogeneous physical capital good. This capital is said to return to society not only the original consumption sacrificed, but a bonus that reflects the technical fact that

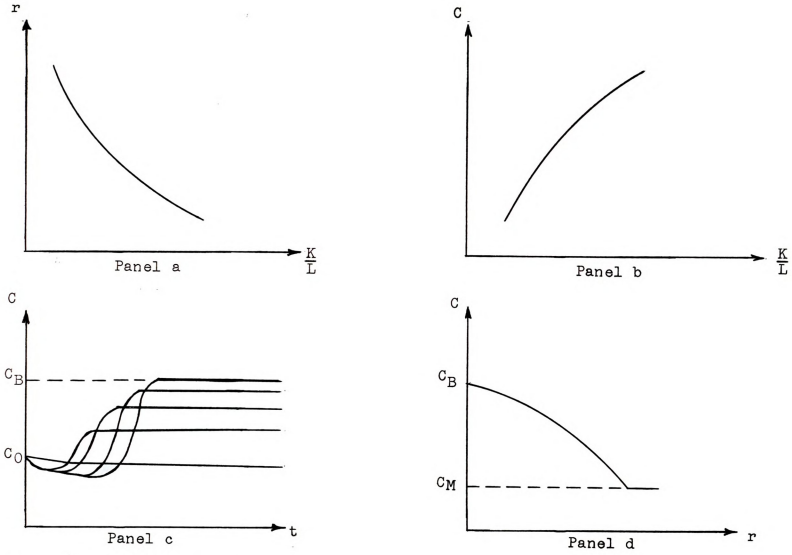

Figure 8. Steady-State Consumption and the Rate of Interest

capital has a net productivity. Each additional sacrifice of consumption will, therefore, move society to a new consumption plateau, the height of which will depend on the behavior of the net productivity of capital. With the assumption of diminishing returns to capital, the greater the level of the consumption stream, the larger the sacrifice of present consumption that is necessary to bring about a given increase in the consumption plateau.

It is now clear that the behavior of C(r) is a consequence of the net productivity of capital as society decides to trade present for future consumption. This net productivity is seen to depend, in turn, on the ratio of our physical capital to labor, i.e., the aggregate capital-labor ratio. But this is what we have referred to as the economy's technique of production. We may therefore directly relate the choice of technique and the consumption plateau implied by this choice.

Panel a, Figure 9 is the Surrogate Frontier derived above;[78] Panel b is the steady-state consumption curve. The slope of the Frontier is the capital-labor ratio; and every point on the Frontier represents a technique of production open to society. If we now think of the interest rate as falling from its maximum to zero, the economy may be said to "choose" successive techniques each of which displays a higher ratio of capital to labor. This will, so

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[78]Chapter I, pp. 7-10.

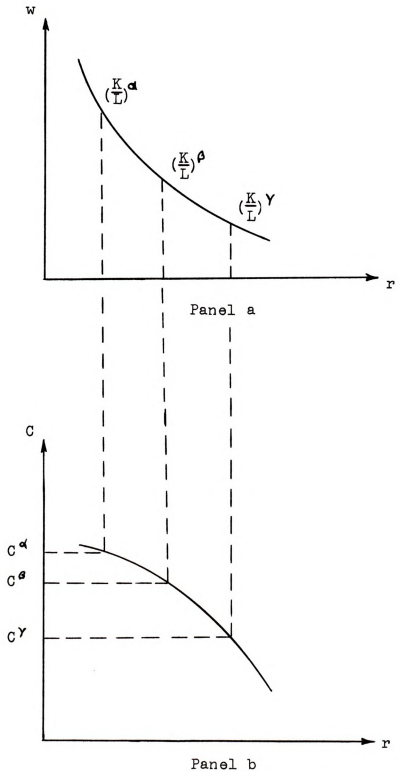

Figure 9. The Surrogate Frontier and Steady-State Consumption

long as capital can be said to have a net productivity, move the economy to higher steady-state consumption levels. But the operation of diminishing returns tells us that the rate of interest associated with higher consumption streams will be lower.

It thus appears that the unidirectional movement of our consumption plateau is based on two elements: a) the unambiguous ordering of techniques over the range of interest rates and b) the operation of diminishing returns to capital. Let us assure ourselves that this traditional behavior of C(r) does not depend upon our choice of a neo-classical production model.

The Consumption Plateau in a Heterogeneous Real Capital Model

Consider the heterogeneous capital good model of Section II. First we need a way of directly relating our factor-price frontier and the steady-state consumption curve in this model. We know that Y = C = NNP in the stationary state. Let the labor constraint be given by

(35) $L = a_{LC}Y$.

The maximum real wage is known to depend only on the production coefficients of the consumption sector;[79] and a typical price equation for the consumption good is

(36) $P_C = a_{LC}w + a_{KC}rP$.

If we now let r = 0 we have

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[79] Supra, p. 36.

(37) $P_C = a_{LC}w$.

Then with the consumption good as our numeraire, i.e., $P_C \equiv 1$, substituting (35) in (37) yields

(38) $\frac{C}{L} = w$,

i.e., the maximum wage rate is attained when the marginal and average products of labor are equal. Thus we may observe directly the steady-state behavior of per capita consumption over the range of relevant interest rates as the economy's technique of production varies.[80] Consider Figure 10.

Panel a, Figure 10, contains three discreet techniques Alpha, Beta, and Gamma all possessing properties identical to those of the techniques discussed and illustrated above.[81] Panel b is the steady-state consumption curve. It shows the per capita consumption that can be attained and maintained indefinitely under each of the three techniques.

At very high rates of interest, $r > r_2^*$, the Gamma technique is chosen as it is the most profitable. With this technique in use, the maximum per capita consumption that can be attained and maintained indefinitely is given by $(\frac{C}{L})^{\lambda}$ Thus for very high interest rates steady-state consumption is shown in Panel b as $(\frac{C}{L})^{\lambda}$

Now let the interest rate fall and wages rise, so

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[80] Essentially the same result, taking a slightly different route, is contained in Bruno, et al., op. cit., pp. 546-49.

[81] Supra, pp. 35-38.

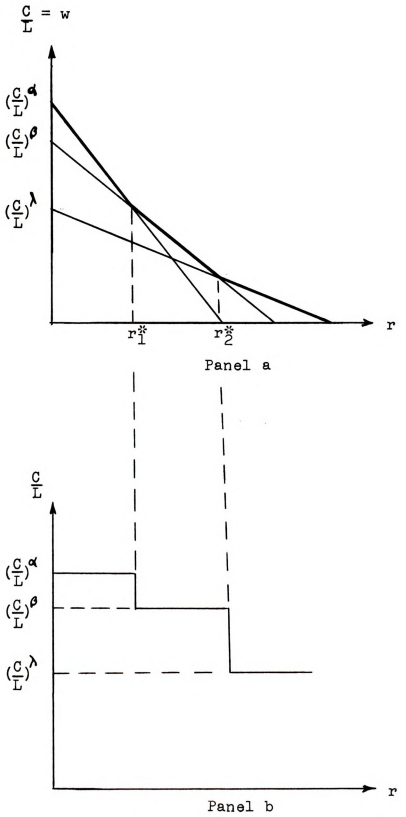

Figure 10. The Factor-Price Frontier and Steady-State Consumption

that $r_1^* < r < r_2^*$. Entrepreneurs "switch" to the Beta technique; and steady-state consumption rises to $(\frac{C}{L})^{\beta}$.

Finally, let the rate of interest fall to very low levels, $0 < r < r_1^*$. There is now a "switch" to the Alpha technique that raises the steady-state consumption to the even higher level of $(\frac{C}{L})^{\alpha}$.

What this demonstration tells us is that the question of the homogeneity of capital is of no consequence so far as an investigation of steady-state consumption patterns is concerned. In this model (as indeed for any model) it is the movement of the net productivity of capital that is paramount. Here we have merely submerged this relation. The variable that attracts our attention in this model is the average productivity of labor (the ordinate intercept). As the rate of interest falls we switch to techniques that raise the average productivity of labor (in the consumption sector), which means that steady-state consumption is raised as well. However, with constant technology, the average productivity of capital is usually taken to be simultaneously falling.[82] In other words, this model contains the assumption of diminishing returns to the capital factor.[83]

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[82]Actually, all we can say here is that the average productivity of capital in the capital-goods sector is falling.

[83]Thus diminishing returns is, as Samuelson has long maintained, independent of the nature of the capital stock present in the model. But, and there seems to be some confusion on this point, the reason is not due to any "technical fact." It is simply a matter of assumption. See Paul A. Samuelson, Economics (6th ed.; New York: McGraw-Hill,

The conclusion to be reached is that the unidirectional movement in the steady-state consumption pattern is a consequence of the unique ordering of techniques, together with the assumption of diminishing returns to capital.

Unorthodox Behavior of Steady-State Consumption

But now consider the reswitching case of Section II, as illustrated in Figure 11. When a technique chosen at a relatively high rate of interest is replaced by another technique as the interest rate falls and then returns at an even lower rate of interest, the traditional ordering of C(r) is lost. One can no longer say that a fall in the rate of interest generally leads to a higher permanently sustainable consumption stream. As shown in Panel b, Figure 11, when entrepreneurs "switch" from Beta technique to Alpha technique at the interest rate r_2^*, the new steady-state consumption level will fall precisely because the average product of labor in the consumption sector decreases (as indicated by the ordinate intercepts).

This unconventional behavior of C(r) is not dependent on the actual return of any given technique. This may be seen in Figure 12. In Panel a, Figure 12, we have three discreet techniques Alpha, Beta, and Gamma.[84] These

1964), pp. 595-96. (As Samuelson says, the situation is more complex than we are accustomed to supposing. Samuelson, "A Summing Up," op. cit., pp. 579-82.)

[84]Notice that none of these techniques gives rise to

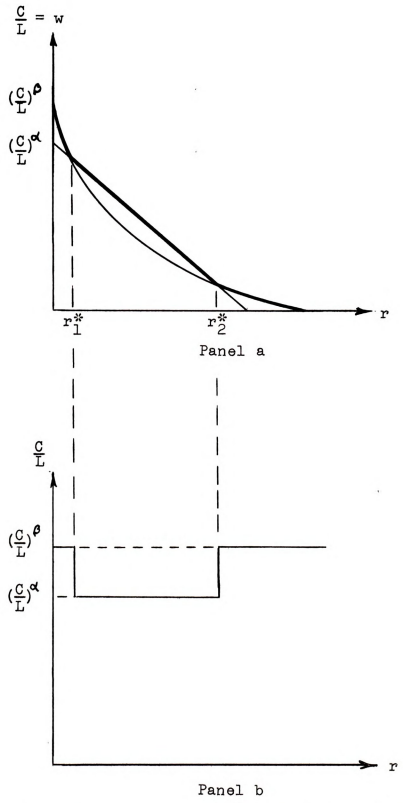

Figure 11. Reswitching and Steady-State Consumption

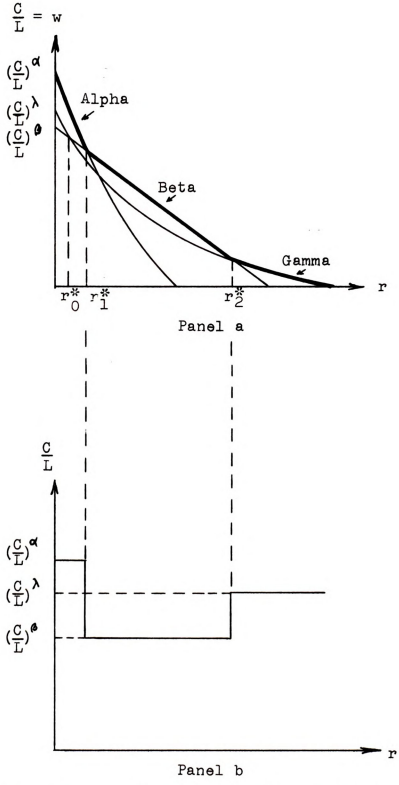

Figure 12. "Perverse" Ordering of Steady-State Consumption

techniques are unambiguously ordered with respect to the rate of interest. Thus at the relatively high interest rates $r > r_2^*$, Gamma technique is chosen by entrepreneurs. For intermediate rates of interest, $r_1^* < r < r_2^*$, Beta technique will be chosen; and when the interest rate is very low, $0 < r < r_1^*$, Alpha technique will be the most profitable technique. There is no question of reswitching here.

The maximum per capita consumption that can be attained and maintained indefinitely under each technique is measured by the ordinate intercepts in Panel a. This is then related directly to the various rates of interest by the steady-state consumption curve of Panel b.

Panel b illustrates the important possibility that, even in the absence of reswitching, a lower rate of interest may be associated with a lower steady-state consumption plateau. This occurs as entrepreneurs "switch" from Gamma technique to Beta technique at the interest rate r_2^*.

It is this generalization of the reswitching result that makes the reswitching literature more than a mere curiosum itself. If we take the rate of interest as reflecting the net productivity of capital, the net productivity of capital as reflecting the technique of

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a linear factor-price frontier. We have discussed above (ft. 19) the reason for convexity (concavity) of the frontier. There is the further question, now raised by the lack of linearity, of whether this has any bearing on the validity of our measure of the capital intensity of production. Yes it does, but we will delay discussion of this until the next chapter.

production, and the choice of technique as reflecting the decision by society to consume more or less today in favor of tomorrow, then we are faced with this startling proposition: the consuming public will step up consumption today in order to disaccumulate capital (select another technique) so that they may have tomorrow a permanently lower consumption stream! Surely something has gone wrong.[85] Otherwise stated, what characteristics of the model might help explain this unorthodox movement in the consumption plateau?

For Bruno et al., it is the fact that the individual factor-price frontiers cross below the outer envelope.[86] Figure 12 on page 62 parallels theirs, and it is clear that the movement of the consumption plateau in the vicinity of r_2^* results from the fact that the two factor-price frontiers do indeed intersect below the outer envelope, at r_o^*. But it is important to note that this intersection occurs to the left of the relevant switch point.

Both Panels a and b of Figure 13 illustrate the crucial point that it is the intersection of the factor-price

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[85]Mere demonstration is hardly sufficient, but see Samuelson, "A Summing Up," op. cit., p. 578. For a numerical example of such disaccumulation, see ibid., p. 581. Bruno, et al., op. cit., pp. 552-53, have suggested that such behavior is explainable in terms of the capitalist "splashing" himself with consumption today by an amount just equal to the then permanently lower consumption he will experience tomorrow. The crucial assumption here is that wage earners spend all their income on consumption.

[86]Their statement is not altogether clear. Presumably they mean no two individual functions may cross twice if one of their intersections is on the outer envelope. Bruno, et al., ibid., p. 549.

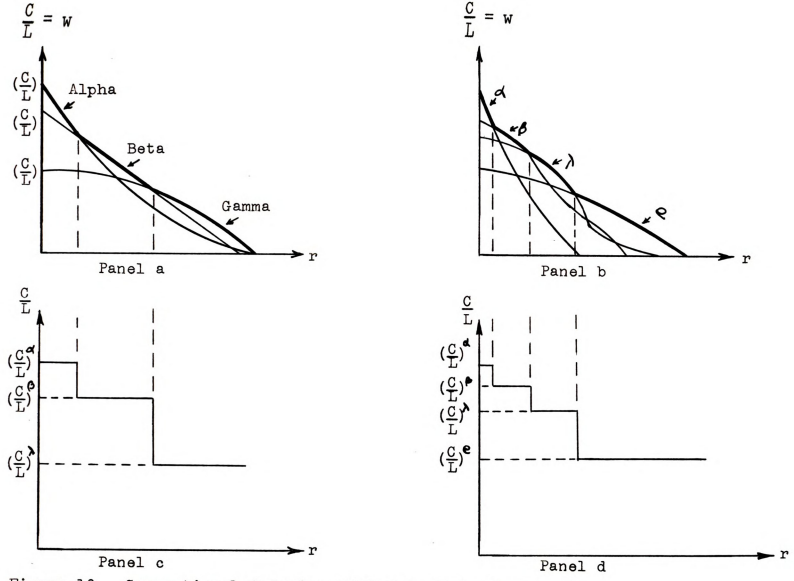

Figure 13. Conventional Ordering of Steady-State Consumption

frontiers below the outer envelope to the left of their intersection on the outer envelope that is, at least ostensibly, the culprit in all this. Notice that the conventional ordering of C(r) is maintained (Panels c and d) even though the relevant individual factor-price frontiers do intersect below the outer envelope. Thus the most we may say is the following:

a. If production techniques can be unambiguously ordered in terms of "capital-intensity", reswitching is not possible.

b. If we can interpret the ordinate intercepts as an accurate reflection of the "capital-intensity" of production techniques, the requirement that no two factor-price frontiers intersect below the outer envelope to the left of their intercept on the envelope is to require that (a) be met.

Thus the conditions previously said to be sufficient for preventing the return of a technique (so long as interpretable as "a") may be said to be necessary and sufficient for the preservation of the traditional ordering of C(r).[87]

IV. Conclusion

The basic conclusion emerging from a consideration of the reswitching discussion is that the "perverse"

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[87]Notice, however, that we are not saying the traditional ordering of C(r) will always be found to contain at most one intersection of any two frontiers (i.e., this is not a necessary condition for the traditional ordering of C(r)). Compare with Bruno, et al., ibid., p. 534.

behavior of C(r) is independent of the actual return to any given production technique. Yet there is a single common denominator present in these discussions that is, in fact, the critical variable in much that is of importance so far as reswitching and traditional capital theory are concerned. This critical variable is the capital intensity of the economy's production techniques.

The capital intensity of production must be taken to refer to the ratio of capital (in some sense) to labor (in some sense) embodied in the production techniques, i.e., to the aggregate capital-labor ratio. Furthermore, it is the ratio of capital to labor along the grand factor-price frontier (i.e., the envelope of the individual frontiers) that is of paramount importance. Clearly the questions raised and discussed above concern the capital intensity of production as we move along this frontier. Yet, as the reader will have noticed, the reswitching discussion does not make explicit reference to the aggregate capital-labor ratio along the grand frontier.

This failure has been noted by Hicks and he has put the issue as follows:[88]

> It already appears that to distinguish techniques by reference to the maximum rate of profit they would permit, at a wage of zero, is not a very interesting way of distinguishing them; but is labor intensity, which amounts to distinguishing them on the basis of profit being zero, so very interesting either? May not the whole trouble have arisen because we have

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[88]Hicks, op. cit., p. 166.

> asked the wrong question? Instead of looking **at** what might happen in extreme positions, at the 'ends' of the curves ought we not to fix our attention on what happens in the neighbourhood of the actual position where the change takes place?

In other words, what we really need to look at is the movement of the aggregate capital-labor ratio as wages rise and interest rates fall. In the next chapter we shall investigate several real capital models to see just what can be said about changes in the capital intensity of production as the economy moves along its grand factor-price frontier in response to the changes in the rate of interest.

CHAPTER III

CERTAINTIES AND AMBIGUITIES IN THE RESWITCHING DISCUSSIONS

I. Introduction

In the preceding chapter emphasis was placed on the switches of techniques in a two-sector, heterogeneous, real-capital model. We saw that the movement of the aggregate capital-labor ratio was the critical issue. In a world of fixed proportions processes and heterogeneous capital goods the simple neoclassical results concerning the relation between production and input and output markets may not hold. And it may not hold irrespective of reswitching, although reswitching provides a stronger case against simple neoclassical theory. Fundamentally, the simple neoclassical results hold if, and only if, the capital-labor ratio varies directly with the wage-interest ratio.

It is clear from the reswitching discussions that one cannot in general order "activities" (whether this is taken to mean a single activity or a whole matrix of activities) with respect to the wage-interest ratio. However, it is not clear that this means that the relation between

the aggregate capital-labor ratio and the wage-interest ratio is similarly affected.

We saw that the propositions surrounding the re-switching phenomena are based on the ordinate intercepts of the so-called canonical model introduced by Samuelson.[89] This means, first, that the labor-output ratio in the consumption sector is being used as an index of capital intensity (i.e., as the aggregate capital-labor ratio), and, second, that a technique of production is being identified with a unique aggregate capital-labor ratio. (Unique in the sense that it is a constant along the process frontier.)

There is no doubt that a technique of production (i.e., matrix of activities) is an aggregate capital-labor ratio.[90] What is uncertain is whether, in a multi-sector capital model, the ratio is unique; and, if it is, whether the labor-output ratio in one sector serves as an adequate index of it.

To help answer these questions three different heterogeneous capital models are analyzed below. It will be seen that the labor-output ratios prominent in the reswitching literature are less capable of measuring the capital intensity of techniques the closer one approaches a bona fide two-sector capital model.

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[89]Paul A. Samuelson, "Parable and Realism in Capital Theory: The Surrogate Production Function," Review of Economic Studies, XXIX (June, 1962), pp. 193-207.

[90]Supra, Chapter II, pp. 30-33.

II. Samuelson's "Parable"[91]

In 1962 Samuelson set out to show that the simple neoclassical theorems are valid in a world of fixed proportions and heterogeneous capital goods. To that end, he retained the neoclassical assumptions of constant returns to scale, competitive imputation, and one primary non-reproducible factor of production. However, he assumed that there are many, but not an infinite number of different kinds of capital goods, each of which, when combined with a very definite amount of labor (i.e., fixed production coefficients), can be used to produce either a unit of final output or a unit of itself. His model may be represented by the following price equations:

$$P_C = a_{LC}w + a_{KC}rP_M \,,$$

(39)

$$P_M = a_{LM}w + a_{KM}rP_M \,.$$

All terms in the expressions are as defined above.[92] Samuelson now makes the crucial assumption that $a_{LC} = a_{LM}$ and $a_{KC} = a_{KM}$ (notice that the assumption is much stronger than that of equal proportions production).[93]

For each of the finite number of different capital goods there is a set of price equations as above. Each capital good thus gives rise to a separate factor-price

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[91] Ibid.

[92] Supra, Chapter II, p. 35.

[93] Although this assumption contains the less stringent one of equal proportions, the reverse is not true (i.e., $\frac{a_{KC}}{a_{LC}} = \frac{a_{KM}}{a_{LM}}$ does not require, for example, $a_{KC} = a_{KM}$).

frontier represented by an equation of the form

$$(40) \qquad w = \frac{1}{a_L} - \frac{a_K}{a_L}(r) \ ,$$

where we have dropped the sector notation in view of the above assumption. The envelope of these individual frontiers is the grand factor-price frontier whose characteristics we wish to investigate.

First, take for the unit of output the net national product or national income of the technique and use this as numeraire, thereby adding an additional equation to the system but no new unknowns. We may then represent the frontier by

$$(41) \qquad w = \frac{1}{A_L} - \frac{A_K}{A_L}(r) \ .$$

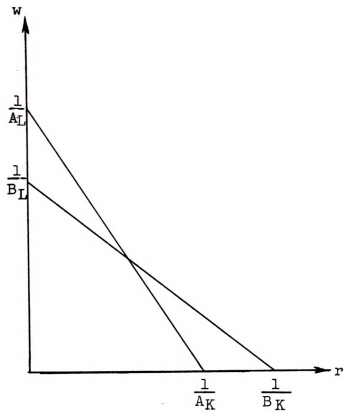
The A_K and A_L now represent the total requirements of capital and labor under this technique.

Let us call equation (41) the Alpha factor-price frontier. It is plotted in Panel a, Figure 14, as the straight line whose ordinate intercept is $\frac{1}{A_L}$ and whose abcissa intercept is $\frac{1}{A_K}$. It is important to notice that

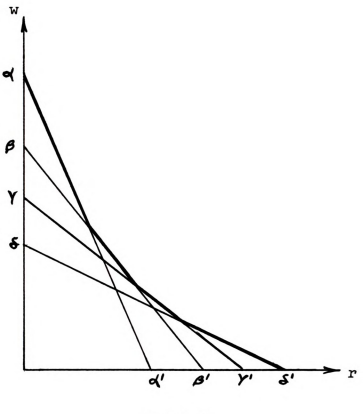
$$(42) \qquad -\frac{dw}{dr} = \frac{A_K}{A_L} = \frac{K}{L} \ .$$

That is, the slope of the Alpha factor-price frontier is the aggregate capital-labor ratio.

Now add many capital goods, Alpha, Beta, Gamma, Delta, et cetera. Their factor-price frontiers are shown in Panel b as $\alpha\alpha'$, $\beta\beta'$, $\gamma\gamma'$, and $\delta\delta'$. The heavily shaded outermost line comprises the grand factor-price frontier. Since the slope of each individual factor-price



Panel a



Panel b

Figure 14. Samuelson's Special Capital Model

frontier is necessarily the aggregate capital-labor ratio as well.

Thus, with the exception of the kinks, we have here a striking similarity to the Frontier yielded by the J. B. Clark neoclassical capital model.[94] In particular, we have a special validation of the proposition that a fall in the rate of interest will raise the capital-intensity of the economy's production techniques.

Now consider the relation between the capital intensity of techniques and the labor-output ratio in the consumption sector. Look at the ordinate in Panel a, Figure 14: $\frac{1}{A_L} > \frac{1}{B_L}$ so $B_L > A_L$. Since $A_L = \frac{L}{C}$, we can unequivocally say that the labor-output ratio in the consumption sector is greater when Beta is used than when Alpha is used. This result is the same as that yielded by equation (42); therefore, one may, without hesitation, use the labor-output ratio in the consumption sector as an index of the capital intensity of techniques in this model.

But notice why this is so. The assumption of identical unit production requirements in each industry means that $A_L = \frac{L}{C} = \frac{L}{M}$. That is, the labor-output ratio in the capital sector is the same as that of the consumption sector. Therefore the labor-output ratio in the consumption

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[94] Though potentially important, the kinks in the grand frontier are of no real consequence in this model. The discontinuity means that the aggregate capital-labor ratio falls discreetly as the interest rate rises. The meaning of this discontinuity for more general capital models is discussed below in Section III, Chapter IV.

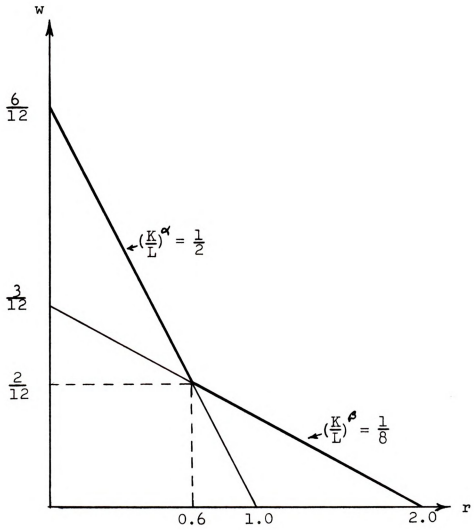

Figure 15. A Switch of Techniques in Samuelson's Model

sector <u>is</u> the labor-output ratio in the economy.[95] In this model, no distinction is made between the two sectors or between either of them and the economy as a whole. Otherwise stated, this is a single-sector capital model.

We may talk about a consumption-good industry and a capital-good industry but the distinction is one of labels only. In fact, the economy is characterized by a single production function.[96] The level of aggregation is the same as that of the variable-proportions neoclassical model. Of course, the level of aggregation does not itself validate the use of the labor-output ratio in the consumption sector as an index of the capital intensity of techniques. The labor-output ratio must be a single-valued function of the aggregate capital-labor ratio. Thus there is implicit in this model the assumption that increases in the aggregate capital-labor ratio raise the average productivity of labor in the economy.[97]

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[95]The labor-output ratio in the economy is a weighted arithmetic mean of the labor-output ratio in each sector considered separately. Thus we have $\frac{L}{Q} = \omega_C(\frac{L}{C}) + \omega_M(\frac{L}{M})$. By assumption $\frac{L}{C} = \frac{L}{M}$, so $\frac{L}{Q} = \frac{L}{C}(\omega_C + \omega_M)$, and since $(\omega_C + \omega_M \equiv 1)$ we have $\frac{L}{Q} = \frac{L}{C}$.

[96]Or to be more precise, the economy is characterized by a single production function given some equilibrium wage-interest ratio. For then only one of the capital goods is ever relevant.

[97]In the reswitching literature, this aspect of simple neoclassical theory is taken for granted by critics and exponents alike. However, for an example of switching that casts some doubt on this relation see below p. 83, and the comment in footnote 103.

A numerical example brings out clearly the special nature of Samuelson's model. Let the production relations for a particular capital good (say Alpha) be given by

$$(43) \qquad \begin{aligned} C &= \min(\tfrac{1}{2}L \; ; \; K) \; , \\ M &= \min(\tfrac{1}{2}L \; ; \; K) \; . \end{aligned}$$

Thus we see that the production functions are really a production function; the two goods are economically identical in that their equilibrium price must be identical.

Now add a different capital good (e.g., Beta) with its production relations

$$(44) \qquad \begin{aligned} C &= \min(\tfrac{1}{4}L \; ; \; 2K) \; , \\ M &= \min(\tfrac{1}{4}L \; ; \; 2K) \; . \end{aligned}$$

The corresponding factor-price frontiers are, respectively,

$$(45) \qquad w = \tfrac{1}{2} - \tfrac{1}{2}(r) \; , \qquad w = \tfrac{1}{4} - \tfrac{1}{8}(r) \; .$$

The frontiers are shown graphically in Figure 15. As the wage falls from its maximum toward zero, the economy switches from the Alpha technique to the Beta technique at the real wage of two-twelfths. The labor-output ratio in the consumption sector rises and the aggregate capital-labor ratio falls from one-half to one-eighth in accordance with the neoclassical parable.

Samuelson's model, like the J. B. Clark neoclassical model, leaves no doubt about the capital intensity of production as the wage-interest ratio varies.[98] And it is

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[98]Perhaps it should also be mentioned at this time that Samuelson's model, like the J. B. Clark neoclassical

equally clear that, in his model, the labor-output ratio (in the consumption sector) is an adequate index of the capital intensity of techniques. Fundamentally, these results flow from what is essentially a single-sector assumption.[99]

III. Hicks' Two Rules[100]

Let us now consider a model that has been emphasized by Hicks. It differs only slightly from Samuelson's model; but there is a significant difference in results. Hicks extended Samuelson's model by relaxing the assumption on the fixed coefficients so as to require equal-proportions production only. That is, the production relations are described by

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model, leaves no doubt as to how entrepreneurs will behave as the wage-interest ratio varies. Any "perverse" behavior is completely ruled out by the assumption on the production coefficients. That is, it is impossible for entrepreneurs to adopt more capital-intensive processes as the rate of interest rises.

[99]The above suggests that Mrs. Robinson's irritation with the neoclassical production function might equally well have been directed toward the level of aggregation implied by it. This appears to be the really "crucial" assumption so far as the results of simple neoclassical theory are concerned.

[100]John Hicks, Capital and Growth (New York: Oxford University Press, 1965), pp. 153-54.

$$M = \min\left(\frac{1}{a_{KM}}K_M , \frac{1}{a_{LM}}L_M\right) ,$$

$$(46) \qquad C = \min\left(\frac{1}{a_{KC}}K_C , \frac{1}{a_{LC}}L_C\right) ,$$

$$\frac{a_{KM}}{a_{LM}} = \frac{a_{KC}}{a_{LC}} , \; a_{KM} \neq a_{KC} .$$

Each set of relations (46) describes one technique. Again it is easy to show that each technique yields a factor-price frontier whose equation has the general form

$$(47) \qquad w = \frac{1}{a_{LC}} - \frac{a_{KM}}{a_{LC}}(r) .$$

Hence

$$(48) \qquad -\frac{dw}{dr} = \frac{a_{KM}}{a_{LC}} = \frac{K_M}{L_C} .$$

The slope of the frontier does not give the aggregate capital-labor ratio; and hence it is possible that the neo-classical relation between factor proportions and relative factor prices will not hold. Let us see what can be said about the capital-intensity of techniques in this model.

Consider Figure 16 in which two linear processes are shown. Visually, $\frac{1}{a_{LC}} > \frac{1}{b_{LC}}$. Hence $^{b}_{LC} > {}^{a}_{LC}$. Since $^{a}_{LC} = \frac{L}{C}$, we can unequivocally say that Beta is labor intensive relative to Alpha in the weak sense that the labor-output ratio in the consumption sector is greater when Beta is used than when Alpha is used. This is the first of Hicks' two rules.

Now look at the abscissa in Figure 16: $\frac{1}{b_{KM}} > \frac{1}{a_{KM}}$, which implies that $^{a}_{KM} > {}^{b}_{KM}$. The capital-output ratio in

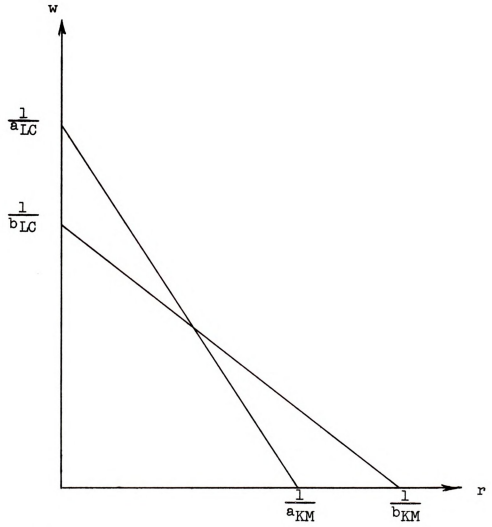

Figure 16. An Equal-Proportions Production Capital Model

the capital-good sector is greater with Alpha technique than with Beta. So again in a weak sense we may say that Alpha is capital intensive relative to Beta. This is the second rule.

In this model, the technique that is labor intensive in one sector is always less capital intensive in the other. Hence as the wage-interest ratio rises the techniques become more capital intensive in the weak sense that the capital-output ratio in the capital-good sector increases and/or the labor-output ratio in the consumption-good sector falls.

Thus the two rules are consistent with one another and it makes absolutely no difference which of them is used to classify techniques. In the reswitching literature the first rule is implicitly adopted.[101] We shall now show that even in this highly simplified model, a classification of techniques according to this rule may be inconsistent with the associated aggregate capital-labor ratios; and these are really the proper criteria for classification.

First a word about the aggregate capital-labor ratio itself. In a multi-sector capital model the aggregate

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[101] In the sense that statements about capital intensity and/or steady-state consumption rest on the ordinate intercepts which measure net final product per man. This, of course, is the reciprocal of the labor-output ratio in the consumption sector. See, for example, Luigi L. Pasinetti, "Changes in the Rate of Profit and Switches of Techniques," The Quarterly Journal of Economics, LXXX (November, 1966), pp. 513-14, or Michael Bruno, Edwin Burmeister, and Eytan Sheshinski, "The Nature and Implications of the Reswitching of Techniques," ibid., pp. 548-49.

capital-labor ratio is a weighted arithmetic mean of the proportion of capital to labor in each sector considered separately. Thus we are interested in

$$(49) \qquad \frac{K}{L} = \omega_C \left(\frac{a_{KC}}{a_{LC}}\right) + \omega_M \left(\frac{a_{KM}}{a_{LM}}\right).$$

The weights are simply the value shares of each sector in the value of total output

$$(50) \qquad \omega_C = \frac{C}{PM + C}, \omega_M = \frac{PM}{PM + C}.$$

These weights will, in general, depend on both the relative prices of the two goods and the level of income under the various techniques. As the price of machines falls relative to the consumption good "workers" may become "capitalists" and thereby substitute the relatively cheaper for the relatively more expensive good. Thus there is a market substitution effect that will accompany a change in the wage-interest ratio. In addition, there may be an income effect due to a lower (higher) income accompanying the adoption of less (more) capital-intensive techniques by individual entrepreneurs.

In view of the additional relations that one must consider in a multi-sector capital model, there is little reason to believe that the capital intensity of techniques is adequately indexed by a rule that fails to account for them. However, it is precisely these considerations that are eliminated by the equal-proportions production

assumption.[102] Hence one might reasonably expect that the rule will work for this model. But now look where it takes us.

Let the production relations for two different techniques be given by

$$
\begin{aligned}
C &= \min(L\ ;\ \tfrac{1}{3}K)\ , \\
M &= \min(\tfrac{1}{2}L\ ;\ \tfrac{1}{6}K)\ , \\
C &= \min(\tfrac{1}{2}L\ ;\ \tfrac{1}{4}K)\ , \\
M &= \min(L\ ;\ \tfrac{1}{2}K)\ .
\end{aligned}
\tag{51}
$$

Each individual factor-price frontier is shown in Panel a, Figure 17. The Alpha frontier has a slope of six, the Beta frontier a slope of one. The two techniques are equally profitable at a real wage of four-tenths. As the real wage falls from one to zero entrepreneurs switch from Alpha to Beta. The labor-output ratio in the consumption sector increases and the aggregate capital-labor ratio falls from three to two. The rule does appear to work.

Now let the production relations be

$$
\begin{aligned}
C &= \min(L\ ;\ \tfrac{1}{2}K)\ , \\
M &= \min(\tfrac{1}{2}L\ ;\ \tfrac{1}{4}K)\ , \\
C &= \min(\tfrac{1}{2}L\ ;\ \tfrac{1}{6}K)\ , \\
M &= \min(L\ ;\ \tfrac{1}{3}K)\ .
\end{aligned}
\tag{52}
$$

---

[102] By assumption $(\frac{K}{L})_C = k = (\frac{K}{L})_M$; hence one may write equation (49) of text as $\frac{K}{L} = \omega_C(\ k\ ) + (1-\omega_C)\ (\ k\ )$ inasmuch as $\omega_C + \omega_M \equiv 1$. Thus $\frac{K}{L} = k$.

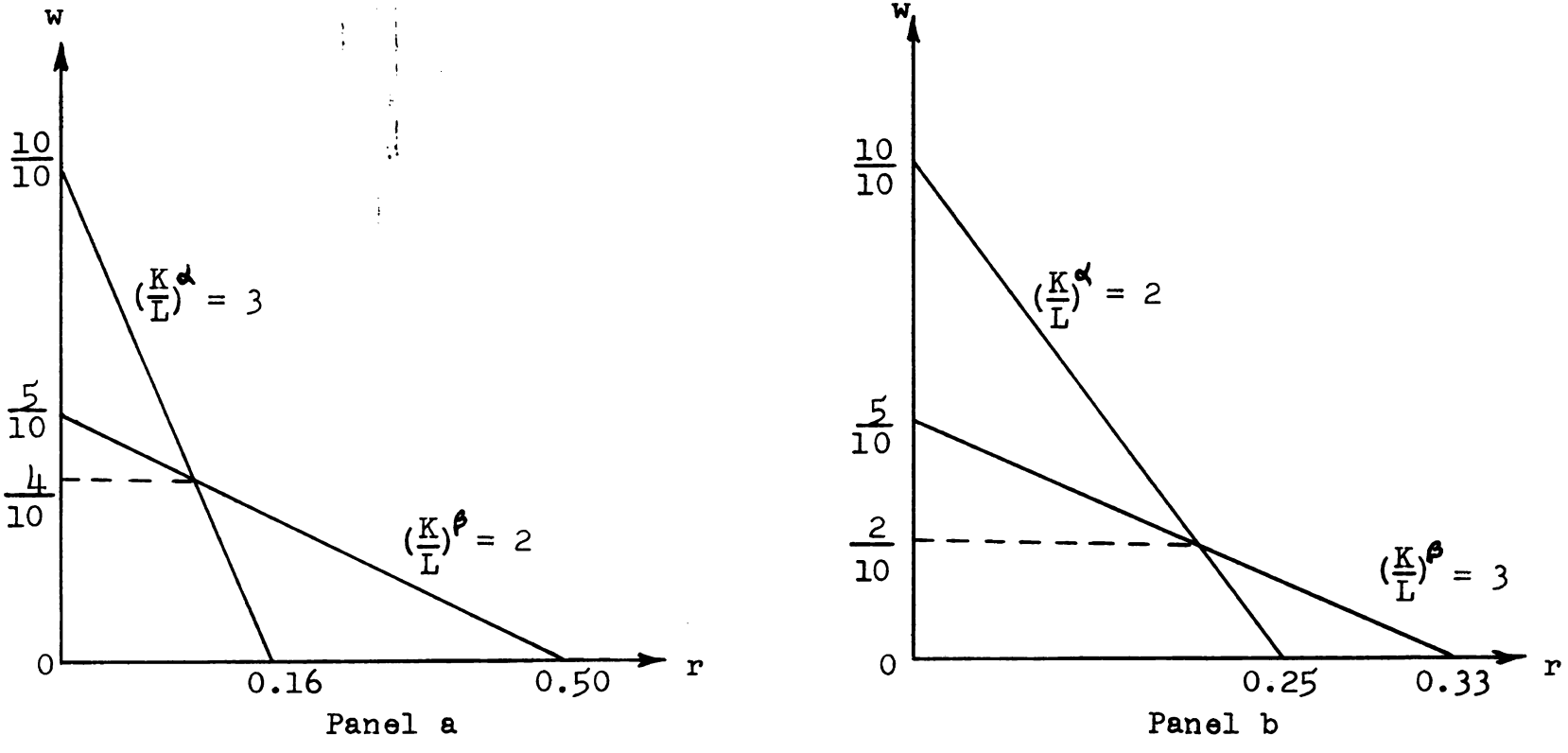

Figure 17. Switches of Techniques in an Equal-Proportions Production Model

The factor-price frontiers are shown graphically in Panel b Figure 17. The Alpha frontier has a slope of four and the Beta frontier of one and one-half. The techniques are equally profitable at the real wage of two-tenths. As the wage-interest ratio falls, entrepreneurs switch from Alpha to Beta and the labor-output ratio in the consumption sector increases. But in this example the aggregate capital-labor ratio <u>increases</u> also, going from two to three. Beta is definitely capital intensive relative to Alpha. Nevertheless, the rule classifies Beta as labor intensive relative to Alpha because the labor-output ratio in the consumption sector is greater for Beta than for Alpha.

This model is consistent with either an increase or a decrease in the capital intensity of techniques as entrepreneurs switch techniques in response to changes in the wage-interest ratio. The above examples clearly show that the result depends on the <u>assumed set of technical alternatives</u>. Hicks' two rules may be used to classify techniques so long as the technology is consistent with neoclassical switching. But as Mrs. Robinson and Mr. Sraffa have made clear, it is possible that the technology is as indicated in equations (52). This is sufficient, by itself, to prove that one cannot, in general, infer anything about the aggregate capital-labor ratio from the labor-output ratio in the consumption sector. Thus the implicit adoption of the first of Hicks' two rules as the basis for a) generalizing the reswitching result, and b) drawing inferences about the

behavior of steady-state consumption levels is definitely unwarranted.[103]

III. The Implications Of A Nonlinear Process Frontier

Let us now change Hicks' assumption slightly so as to allow the intrinsic factor intensities of the capital and consumption good to differ. Specifically, assume that production in the consumption-good sector is more capital intensive than production in the capital-good sector. Thus the production relations are described by

$$
\begin{aligned}
& C = \min\left(\frac{1}{a_{KC}}K_C \ , \ \frac{1}{a_{LC}}L_C\right) \ , \\
(53) \qquad & M = \min\left(\frac{1}{a_{KM}}K_M \ , \ \frac{1}{a_{LM}}L_M\right) \ , \\
& \frac{a_{KC}}{a_{LC}} > \frac{a_{KM}}{a_{LM}} \ .
\end{aligned}
$$

Each set of relations (53) describes one technique. Each technique yields a factor-price frontier whose equation has the general form

$$
(54) \qquad w = \frac{1 - a_{KM}r}{a_{LC} + Ar} \ ,
$$

---

[103] The equations (52) also point up what is perhaps the most fundamental question raised by the reswitching discussions. In the analysis of steady-state consumption levels, the ordinate $(\frac{1}{a_{LC}})$ measures net consumption per man. Thus, referring to Panel b Figure 17, an increase in the rate of interest raises the aggregate capital-labor ratio and steady-state consumption falls. This suggests a surprising possibility. It implies that even if the aggregate capital-labor ratio is a single-valued function of the wage-interest ratio, a higher steady-state consumption level may be associated with a higher rather than a lower rate of interest.

where $A = a_{LM}a_{KC} - a_{LC}a_{KM}$.

The frontier is negatively sloped inasmuch as

$$(55) \quad \frac{dw}{dr} = -\frac{a_{KM}a_{LC} + A}{(a_{LC} + Ar)^2} = -\frac{a_{LM}a_{KC}}{(a_{LC} + Ar)^2} < 0 .$$

It is convex to the origin by virtue of our assuming that $A > 0$. To see this first write the price equation for the capital good:

$$(56) \quad P_M = a_{LM}w + a_{KM}rP_M .$$

Now divide both sides by P_M to obtain

$$(57) \quad 1 - a_{KM}r > 0 .$$

Then from equation (54) it is seen that

$$(58) \quad a_{LC} + Ar \geq 0 .$$

Hence the frontier is convex or concave to the origin according as $A \gtrless 0$ inasmuch as

$$(59) \quad \frac{d^2w}{dr^2} = \frac{2A(a_{LC} + Ar)\, a_{LM}a_{KC}}{(a_{LC} + Ar)^4} .$$

The fact that the frontier is nonlinear is very important. Assuming continuous full employment of capital and labor, it implies that the economy's endowment of capital and/or labor is changing as we move along the process frontier.[104] The frontier is based on given, unchanging proportions of capital to labor in each of the two sectors considered separately. These sector ratios are what they are regardless of any variation in the wage-interest ratio.

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[104]In the absence of this assumption it implies that capital and/or labor is in various stages of idleness.

However, as the wage-interest ratio falls the price of the relatively labor-intensive good falls relative to the price of the relatively capital-intensive good.[105] This implies a change in the composition of output in favor of the relatively labor-intensive sector. Thus, referring to equation (49), it is clear that the aggregate capital-labor ratio must fall. The reason is that if the relatively labor-intensive sector is to absorb all the capital released by the relatively capital-intensive sector, the economy's endowment of labor must rise. Or, if the labor force is taken to be a constant, the change in output composition implies that capital has been disaccumulated. Either way, the point is that strict adherence to the full employment assumption implies that the factor endowment ratio is a variable along any given nonlinear, fixed-proportions process frontier.[106]

Now consider Figure 18 in which two nonlinear frontiers are shown. Visually, $\frac{1}{a_{LC}} > \frac{1}{b_{LC}}$ and $\frac{1}{b_{KM}} > \frac{1}{a_{KM}}$. Hence $a_{LC} < b_{LC}$ and $a_{KM} > b_{KM}$. Following the rules of the preceding section, technique Beta is classified as labor intensive relative to technique Alpha. But again it would be easy to construct an example in which, at any wage-interest ratio, the aggregate capital-labor ratio implied by technique Beta is greater than that implied by technique

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[105] A more rigorous proof of this statement is given in the next chapter.

[106] Ibid.

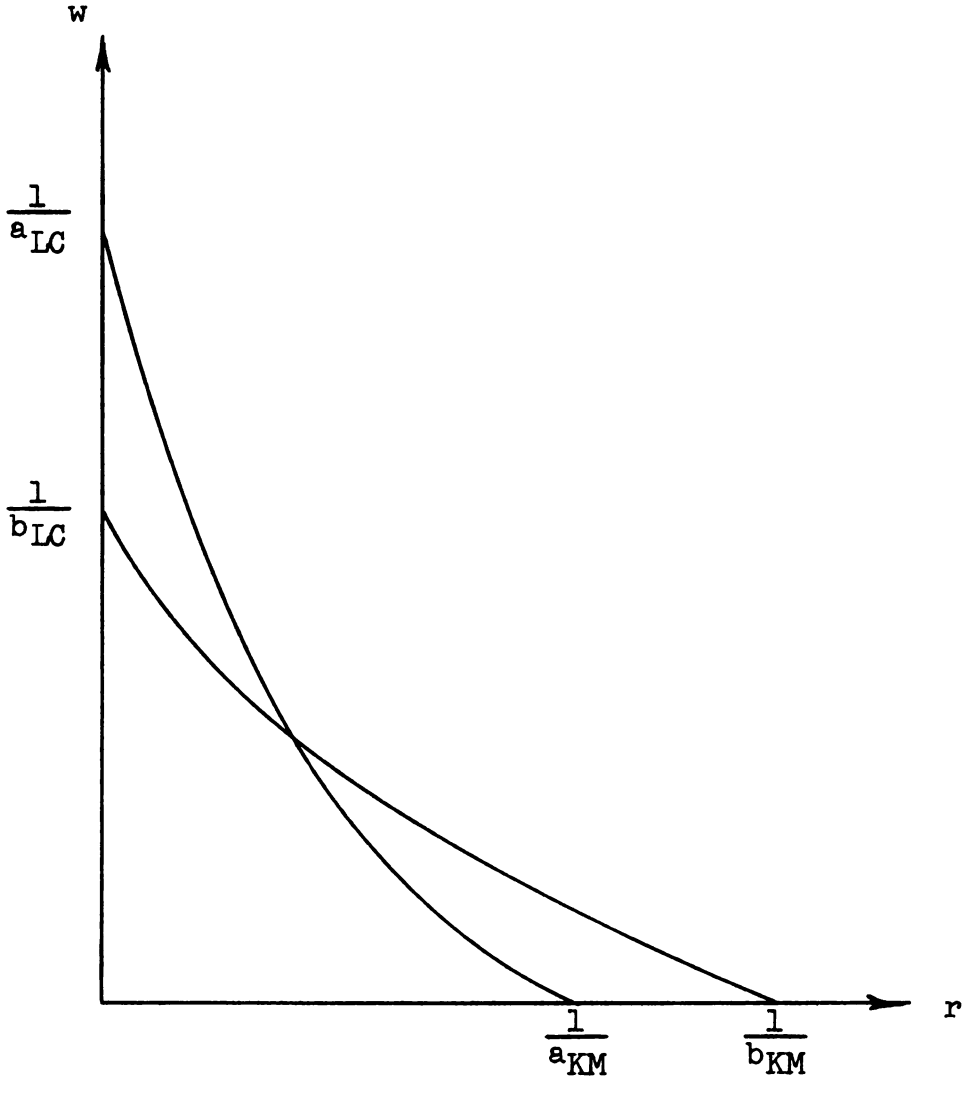

Figure 18. A Nonlinear Capital Model

Alpha (below we offer such an example). In such a case one would classify Beta as capital intensive relative to Alpha.

Suppose it is known that for each technique considered separately the aggregate capital-labor ratio is always greater for Beta than it is for Alpha. Does this mean that a switch from Alpha to Beta raises the capital intensity of techniques? In an equal-proportions production model it certainly does. However, the interesting aspect of this model is that the outcome is uncertain. It depends on what happens <u>as a result of the switch</u>.[107]

To illustrate, consider the techniques whose production relations are

$$(60)\qquad \begin{aligned} C &= \min(L, \tfrac{1}{5}K), \\ M &= \min(L, \tfrac{1}{2}K), \end{aligned}$$

and

$$\begin{aligned} C &= \min(\tfrac{1}{2}L, \tfrac{1}{12}K), \\ M &= \min(\tfrac{1}{3}L, K). \end{aligned}$$

---

[107] This is an important proposition as it highlights what is perhaps the basic fallacy contained in the rules enumerated by Hicks and implicitly adopted in the reswitching literature. Under the best of circumstances, the rules would classify techniques in a manner that is consistent with a classification according to the associated aggregate capital-labor ratios in the sense that, if Beta were known to imply a higher aggregate capital-labor ratio than Alpha for all rates of interest, the rules would classify Beta as the relatively capital-intensive technique. But any such classification is based on the implicit assumption that a switch of techniques does not itself affect the aggregate capital-labor ratio. The basic point developed in this section is that such an assumption is unwarranted in a fixed-proportions, heterogeneous capital model that allows the intrinsic factor intensities of the goods to differ.

The two techniques are plotted in Figure 19. Both process frontiers are convex to the origin because the consumption sector is assumed to be the relatively capital-intensive sector. The two techniques can be shown to be equally profitable at a real wage of three-tenths.

As the real wage falls from its maximum toward zero, entrepreneurs switch from Alpha to Beta and both sectors become more capital intensive. The question is whether the total proportion of capital to labor has increased.

Look at Table I. Notice that the aggregate capital-labor ratio is greater for Beta than it is for Alpha. Demand is assumed to shift from the capital-intensive sector to the labor-intensive sector as the wage-interest ratio falls. Hence the aggregate capital-labor ratio is falling for movements down each frontier considered separately. Now consider the point ($S_{\alpha\beta}$) in Figure 19. As entrepreneurs switch to the Beta technique, the weight going to the relatively labor-intensive sector falls, remains unchanged, or increases depending on the movement of relative commodity price <u>at the switch point</u>.

If, as in this example, the switch to Beta causes relative commodity price to fall,[108] the economy's technique need not become more capital intensive even though

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[108]As the reader may easily verify, relative commodity price ($\frac{P_M}{P_C}$) falls from .58 to .12 as a result of the switch from Alpha to Beta. A simple expression for relative commodity price is given in the following chapter, p. 114.

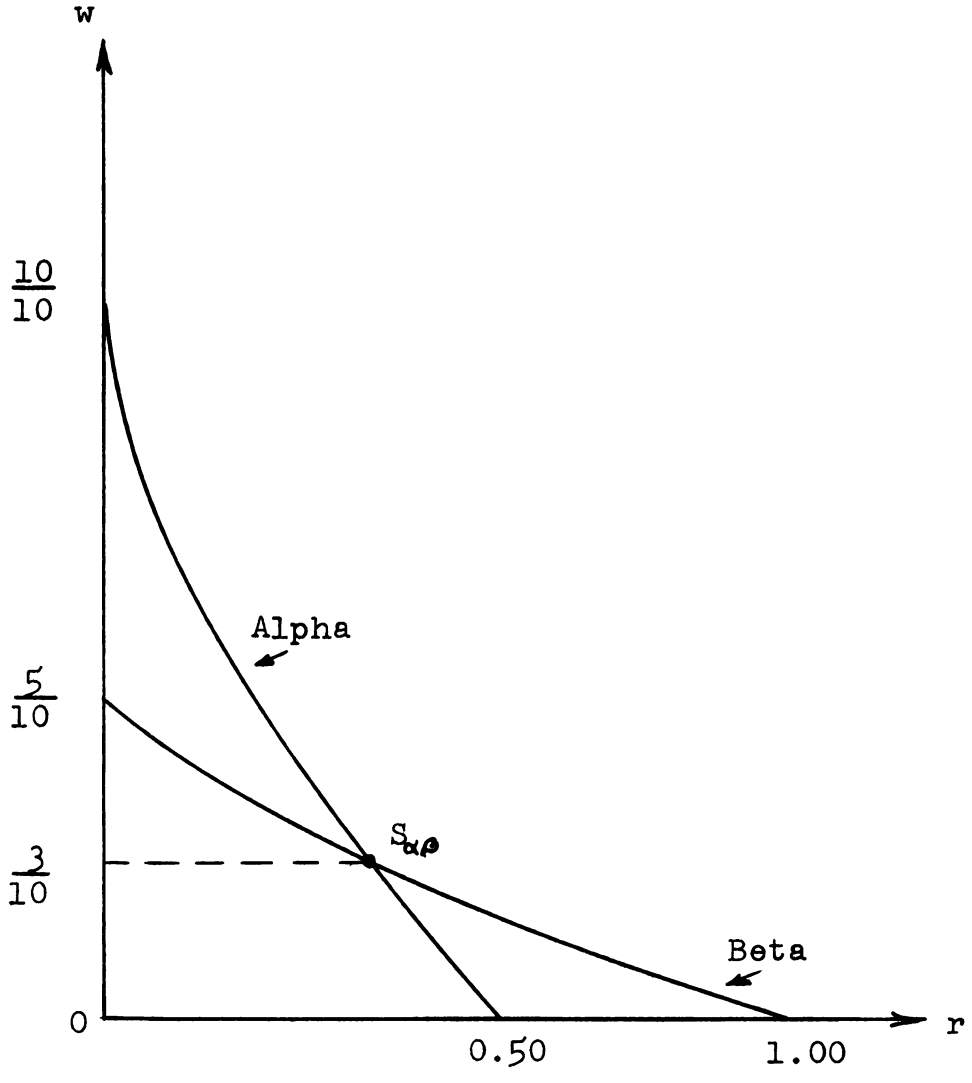

Figure 19. A Switch of Techniques in a Nonlinear Model

TABLE I

AGGREGATE CAPITAL-LABOR RATIOS FOR TWO TECHNIQUES

Output Composition: ω_C	ω_M	Aggregate Capital-Labor Ratio: Technique Alpha $(\frac{K}{L})$	Aggregate Capital-Labor Ratio: Technique Beta $(\frac{K}{L})$
1.0	0.0	5.0	6.0
0.9	0.1	4.6	5.7
0.8	0.2	4.4*	5.5
0.7	0.3	4.1	5.2
0.6	0.4	3.8	4.9
0.5	0.5	3.5	4.7
0.4	0.6	3.2	4.4*
0.3	0.7	2.9	4.1
0.2	0.8	2.6	3.9
0.1	0.9	2.3	3.6
0.0	1.0	2.0	3.3

*Referred to in text, p. 94.

all entrepreneurs adopt more capital-intensive techniques. For example, referring to Table I, suppose the fall in relative commodity price implies a shift in the composition of output from ($\omega_C = .8; \omega_M = .2$) to ($\omega_C = .4 ; \omega_M = .6$). Then the aggregate capital-labor ratio is unchanged at the switch point ($S_{\alpha\beta}$) though the proportion of capital to labor is higher in each sector considered separately. The explanation for this apparent paradox is contained in equation (49). At the same time that entrepreneurs are switching to the Beta technique, relative commodity price is changing so as to automatically increase the weight going to the relatively labor-intensive sector.[109] The aggregate capital-labor ratio need not rise so long as substitution in consumption is sufficient to overcome "perverse" substitution in production.

Thus it is demonstrated that the aggregate capital-labor ratio may fall continuously for movements down the grand factor-price frontier even though all entrepreneurs adopt more capital-intensive processes. The outcome depends on the movement of relative commodity price and the readiness of individuals to substitute goods in consumption.[110] The relation between factor prices, factor

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[109]For an interesting graphical demonstration that the aggregate capital-labor ratio may remain unchanged when both sectors become more capital intensive, see Wolfgang F. Stolper and Paul A. Samuelson, "Protection and Real Wages," Review of Economic Studies, XI (November, 1941), pp. 66-69.

[110]Our discussions implicitly assume that the income effect of a change in techniques is of the "right" kind. Thus, in the above example the income effect is assumed to

endowments, and relative commodity price in both the variable and fixed coefficient two-sector production model is examined in greater detail in the following chapter.

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favor the relatively labor-intensive sector. But even if the income effect favors the capital-intensive sector, the above statements are valid so long as the market substitution effect dominates the income effect. Thus it is the market substitution effect that is most fundamental.

CHAPTER IV

INTERRELATIONS BETWEEN FACTOR PRICES, FACTOR ENDOWMENTS, AND RELATIVE COMMODITY PRICE

I. Introduction

The economy's technique of production is the prevailing aggregate capital-labor ratio. This ratio, however, is a value-weighted arithmetic mean of the proportion of capital to labor utilized in each sector separately. Thus, in a disaggregated capital model, the capital intensity of techniques depends in general on (1) the technology of the economy, and (2) the distribution of demand between sectors. When one makes the two fundamental assumptions of full employment of both factors and perfect competition, switches of techniques imply: (a) factor prices are changing, (b) the proportion of capital to labor in one or more sectors is changing, (c) relative commodity price is changing, and (d) the composition of output is changing.

The reswitching discussions implicitly assume that the distribution of demand is fixed. In addition the "normal" switch is taken to mean that there has been a change in the proportion of capital to labor in one sector

only.[111] Thus when a switch occurs it implies that the economy's endowment of capital and/or labor is changing. Below it is shown analytically, for both the Clark and heterogeneous capital models, that in general movements along the factor-price frontier imply a change in the economy's endowment of capital and/or labor. When the labor force is taken as a constant (the usual procedure in these discussions), any movement down the frontier implies that capital is being disaccumulated. However, switches of technique in a heterogeneous capital model involve a movement from one frontier to another. Thus, as will be seen below, a switch of techniques need not mean that the factor endowment ratio has changed. When it has, it may have moved in either direction.

With technology specified, changes in the factor endowment ratio have a predictable impact on relative commodity price. In general, it can be shown that, for movements down the frontier, the price of the relatively labor-intensive good falls relative to the price of the relatively capital-intensive good. But in a world of fixed-proportions processes and heterogeneous capital goods, the question is what happens to relative commodity price

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[111] Luigi L. Pasinetti, "Changes in the Rate of Profit and Switches of Techniques," The Quarterly Journal of Economics, LXXX (November, 1966), p. 511. See also, Michael Bruno, Edwin Burmeister, and Eytan Sheshinski, "The Nature and Implications of the Reswitching of Techniques," ibid., p. 542, where this proposition is presented as a theorem.

at the point where entrepreneurs are switching techniques? A priori, little can be said about the behavior of relative commodity price at this point. It may fall, remain unchanged, or rise. An example of each is offered below.

Finally, the reswitching phenomenon is reconsidered taking into account the behavior of relative commodity price at the switch point. It is shown that the neoclassical relation between the aggregate capital-labor ratio and the wage-interest ratio is virtually impossible to destroy so long as reswitching occurs after the economy has passed some critical rate of interest and/or relative commodity price falls as a consequence of entrepreneurs returning to some previous technique.

II. The Model: Variable Coefficients[112]

The technology is described by the columns of the A matrix

$$(62) \qquad A = \begin{bmatrix} a_{LM} & a_{LC} \\ a_{KM} & a_{KC} \end{bmatrix}$$

where the a_{ij} denote the quantity of factor i required to produce a unit of commodity j. The input coefficients are assumed to vary neoclassically with the wage-rental ratio:

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[112]For convenience the complete model is reproduced here. A similar model using machine rental rather than the interest rate can be found in Ronald W. Jones, "The Structure of Simple General Equilibrium Models," The Journal of Political Economy, LXXIII (December, 1965), pp. 557-72. The present chapter, especially parts II and III, is heavily indebted to this very instructive article by Jones.

(63) $a_{ij} = a_{ij}(\frac{w}{q})$, $\quad i = L, K, \quad j = C, M.$

By definition

(64) $q = rP_M$.

That is, the quasi-rent of the capital good equals the product of the rate of interest and the price of the capital good.

The technology is assumed to exhibit constant returns to scale so factor demands are given by the product of the unit production requirements and the level of output in each industry. Assuming full employment of both factors gives

(65) $L = a_{LC}C + a_{LM}M$,

(66) $K = a_{KC}C + a_{KM}M$.

The sum of the sector demands for each factor must absorb the total supply of each factor.

Assuming perfect competition, the equilibrium prices of the two goods are given by

(67) $p = a_{LM}w + a_{KM}q$,

(68) $1 = a_{LC}w + a_{KC}q$,

where p is the consumption-good price of machines (i.e., $P_C \equiv 1$). Equations (67) and (68) state that the equilibrium market prices of the goods reflect their unit costs of production. Finally, assume that the ratio of commodities demanded depends solely on relative commodity price.[113]

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[113] In this paper we are primarily interested in the implications of changing relative commodity price for the

(69) $\frac{M}{C} = f(p)$.

The basic structure of the model is contained in equations (65) - (69), and the immediate problem is to convert these into equations of change. The latter can then be used to bring out the nature of the relation between factor prices, factor endowments, and relative commodity price along the grand factor-price frontier.

Let $\hat{x}$ denote the relative change in the variable x, i.e., $\hat{x} = \frac{dx}{x}$. Taking the proportionate rate of change of both sides of equations (65) - (68) we obtain:[114]

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reswitching argument and its derivatives. Hence the above demand specification. However, in general, there will be a change of income associated with a change of techniques, thus one could write equation (69) as $\frac{M}{C} = f(p, \phi)$ where ϕ is to be interpreted as a shift parameter representing the level of real income in the economy. Then in place of equation (84) below we would have, after taking the proportionate changes of both sides of equation (69), $(\hat{M} - \hat{C}) = -\sigma_D \hat{p} + \varepsilon_y \hat{y}$, where σ_D represents the change in demand composition as a result of a one percent change in relative commodity price:

$$\sigma_D = -\left(\frac{P}{\frac{M}{C}}\right) \frac{\partial(\frac{M}{C})}{\partial P} ,$$

and ε_y represents the change in the composition of demand as a result of a one percent change in real income:

$$\varepsilon_y = \left(\frac{Y}{\frac{M}{C}}\right) \frac{\partial(\frac{M}{C})}{\partial Y}$$

Thus (if in general $\varepsilon_y > 0$) demand is biased toward the M sector or the C sector according as a change of techniques raises or lowers the level of real income in the economy. This more general demand specification follows closely that of Akihiro Amano, "Determinants of Comparative Costs: A Theoretical Approach," Oxford Economic Papers, XVI (November, 1964), pp. 395-97.

[114]The procedure used here has been set out in terms of finite differences in Chapter Two of J. E. Meade, A Neo-classical Theory of Economic Growth (revised edition; London: George Allen and Unwin Ltd., 1962). One must take

$$(70)\quad \lambda_{LM}\hat{M} + \lambda_{LC}\hat{C} = \hat{L} - (\lambda_{LM}\hat{a}_{LM} + \lambda_{LC}\hat{a}_{LC}) ,$$

$$(71)\quad \lambda_{KM}\hat{M} + \lambda_{KC}C = \hat{K} - (\lambda_{KM}\hat{a}_{KM} + \lambda_{KC}\hat{a}_{KC}) ,$$

$$(72)\quad \theta_{LM}\hat{w} + \theta_{KM}\hat{q} = \hat{p} - (\theta_{LM}\hat{a}_{LM} + \theta_{KM}\hat{a}_{KM}) ,$$

$$(73)\quad \theta_{LC}\hat{w} + \theta_{KC}\hat{q} = - (\theta_{LC}\hat{a}_{LC} + \theta_{KC}\hat{a}_{KC}) .$$

The λ coefficients measure the proportion of factor i used in sector j.[115] Thus, for example,

$$(74)\quad \lambda_{LC} + \lambda_{LM} = 1 .$$

The fraction of labor used in the consumption sector plus the fraction of labor used in the capital sector must add to unity by the full employment assumption. Similarly for λ_{KC} and λ_{KM}.

These terms may be used to define a λ matrix

$$(75)\quad \lambda = \begin{bmatrix} \lambda_{LM} & \lambda_{LC} \\ \lambda_{KM} & \lambda_{KC} \end{bmatrix}$$

whose determinant[116]

$$(76)\quad |\lambda| = \lambda_{LM} - \lambda_{KM} = \lambda_{KC} - \lambda_{LC} ,$$

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the total differential and convert this into proportional terms.

[115]For example, we have: $L = a_{LC}C + a_{LM}M$. Taking the total differential gives $dL = a_{LC}dC + Cda_{LC} + a_{LM}dM + Mda_{LM}$, which in proportional terms is $\frac{dL}{L} = \frac{(a_{LC}C)}{L}\frac{dC}{C} + \frac{(a_{LC}C)}{L}\frac{da_{LC}}{a_{LC}} + \frac{(a_{LM}M)}{L}\frac{dM}{M} + \frac{(a_{LM}M)}{L}\frac{da_{LM}}{a_{LM}}$. Rearranging terms then gives equation (70) of text.

[116]Since each row sum is unity we have, for example, $\lambda_{KC} = (1 - \lambda_{KM})$ thus:

$$\begin{aligned} |\lambda| &= \lambda_{LM}\lambda_{KC} - \lambda_{KM}\lambda_{LC} \\ &= \lambda_{LM}(1 - \lambda_{KM}) - [\lambda_{KM}(1 - \lambda_{LM})] \\ &= \lambda_{LM} - \lambda_{KM} . \end{aligned}$$

is positive or negative according as production in the capital-good sector is labor intensive or capital intensive relative to production in the consumption-good sector.

The θ coefficients, by contrast, measure the relative share of factor i in sector j.[117] Hence we have, for example,

(77) $\theta_{LC} + \theta_{KC} = 1.$

The share of labor in the consumption sector plus the share of capital in this sector must add to unity by the zero profit condition (equations 67 and 68). Similarly for θ_{LM} and θ_{KM}. These coefficients serve to define a θ matrix

(78) $$\theta = \begin{bmatrix} \theta_{LM} & \theta_{LC} \\ \theta_{KM} & \theta_{KC} \end{bmatrix}$$

whose determinant is

(79) $$|\theta| = \theta_{LM} - \theta_{LC} = \theta_{KC} - \theta_{KM} .$$

Conditions sufficient to determine the $\hat{a}_{ij}$ may be formulated,[118] and substituting for the $\hat{a}_{ij}$ in equations (70) -

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[117] For example, $p = a_{LM}w + a_{KM}q$ so that in proportional terms

$$\frac{dp}{p} = \frac{a_{LM}w}{p}\left(\frac{dw}{w}\right) + \frac{wa_{LM}}{p}\left(\frac{da_{LM}}{a_{LM}}\right) + \frac{a_{KM}q}{p}\left(\frac{dq}{q}\right) + \frac{qa_{KM}}{p}\left(\frac{da_{KM}}{a_{KM}}\right).$$

Rearranging terms gives equation (72) of text.

[118] The $\hat{a}_{ij}$ may be determined in the following manner. $(\lambda_{LM}\hat{a}_{LM} + \lambda_{LC}\hat{a}_{LC})$ shows the percentage change in the total quantity of labor required by the economy as a result of changing factor proportions in both industries at a given level of output. The extent of this change depends on the elasticities of substitution between factors in each industry. In relative terms these may be defined as $\sigma_M = \frac{\hat{a}_{KM} - \hat{a}_{LM}}{\hat{w} - \hat{q}}$, and $\sigma_C = \frac{\hat{a}_{KC} - \hat{a}_{LC}}{\hat{w} - \hat{q}}$. Now consider the θ

(73) yields:

$$(80) \quad \lambda_{LM}\hat{M} + \lambda_{LC}\hat{C} = \hat{L} + \delta_L(\hat{w} - \hat{r} - \hat{p}) ,$$

$$(81) \quad \lambda_{KM}\hat{M} + \lambda_{KC}\hat{C} = \hat{K} - \delta_K(\hat{w} - \hat{r} - \hat{p}) ,$$

$$(82) \quad \theta_{LM}\hat{w} + \theta_{KM}\hat{r} = \hat{p}(1 - \theta_{KM}) ,$$

$$(83) \quad \theta_{LC}\hat{w} + \theta_{KC}\hat{r} = -\theta_{KC}\hat{p} ,$$

where $\delta_L = \lambda_{LM}\theta_{KM}\sigma_M + \lambda_{LC}\theta_{KC}\sigma_C$,

$\delta_K = \lambda_{KM}\theta_{LM}\sigma_M + \lambda_{KC}\theta_{LC}\sigma_C$.

The σ's denote the elasticities of substitution between factors in each industry. They are defined, in relative terms, as:

$$\sigma_j = \frac{\hat{a}_{Kj} - \hat{a}_{Lj}}{(\hat{w} - \hat{q})} \quad ; \qquad j = M, C .$$

One may interpret the δ's as reflecting the aggregate percentage saving in labor (δ_L) or capital (δ_K) inputs resulting from the substitution of one factor for another in response to changing factor prices at unchanged outputs.[119]

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weighted average of the $\hat{a}_{ij}$'s. For example, ($\theta_{LC}\hat{a}_{LC} + \theta_{KC}\hat{a}_{KC}$) shows the percentage change in the total costs of producing the consumption good as a result of changing factor proportions at a given level of output. Thus for both industries we must have ($\theta_{LM}\hat{a}_{LM} + \theta_{KM}\hat{a}_{KM}$) = 0, ($\theta_{LC}\hat{a}_{LC} + \theta_{KC}\hat{a}_{KC}$) = 0. That is, the θ weighted average of changes in the coefficients must equal zero in each industry. This result follows from equations (63) and (77) of the text. Solving these equations in pairs then yields the desired result:

$$\hat{a}_{Lj} = -\theta_{Kj}\sigma_j(\hat{w} - \hat{r} - \hat{p}); \qquad j = M, C.$$

$$\hat{a}_{Kj} = \theta_{Lj}\sigma_j(\hat{w} - \hat{r} - \hat{p}); \qquad j = M, C.$$

[119] Perhaps this is better seen by writing directly, for example,

$$\delta_L = -\frac{(\lambda_{LM}\hat{a}_{LM} + \lambda_{LC}\hat{a}_{LC})}{(\hat{w} - \hat{q})} .$$

It is important to notice that these equations no longer contain the quasi-rent of the capital good ($\hat{q}$). In relative terms equation (64) becomes $\hat{q} = \hat{r} + \hat{p}$. We have substituted for $\hat{q}$ since it is the rate of interest that will concern us in what follows.

Finally, taking the proportionate changes of both sides of equation (69) serves to define the elasticity of substitution between commodities on the demand side:

$$(84) \qquad \sigma_D = -\frac{(\hat{M} - \hat{C})}{\hat{p}} .$$

The comparative static properties of the model are now exhibited in equations (80) - (83). Given the wage-interest ratio, and hence relative commodity price, the λ matrix serves to link factor endowments to commodity outputs in the same way as the Θ matrix links relative commodity price to the wage-interest ratio given factor endowments.[120]

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[120] The link is a technological one inasmuch as the rates of growth of outputs depends on the sector capital-labor ratios whenever capital and labor grow at different rates. See Jones, op. cit., pp. 561-62. Also below, footnote 114. Similarly, the relation of factor prices to relative commodity price depends on the technology specified. Solving equations (82) and (83) directly for $\hat{w}$ and $\hat{r}$ gives:

$\hat{w} = \frac{\Theta_{KC}}{|\Theta|}\hat{p}$, $\hat{r} = \frac{-(\Theta_{LC} + |\Theta|)}{|\Theta|}\hat{p}$ which, in view of equation (79) of the text, may be written as

$(\hat{w} - \hat{p}) = \frac{\Theta_{KM}}{|\Theta|}\hat{p}$, $\hat{r} = \frac{-(\Theta_{LC} + |\Theta|)}{|\Theta|}\hat{p}$.

Thus if the capital sector is the relatively labor-intensive sector, a rise in the price of machines relative to the price of the consumption good raises the real wage by an even greater relative amount and the rate of interest falls. That is, $\hat{w} > \hat{p} > \hat{r}$ where $\hat{r} < 0$. One might say that the

The model may now be used to show: (1) with the consumption sector relatively capital-intensive, relative commodity price varies directly with the factor endowment ratio (capital/labor) and (2) the factor endowment ratio varies directly with the wage-interest ratio. Together (1) and (2) imply that relative commodity price falls continuously as one moves down the grand factor-price frontier.[121]

Solving equations (80) and (81) one may obtain the change in the ratio of outputs produced:

$$(85) \qquad (\hat{M} - \hat{C}) = \frac{(\hat{L} - \hat{K})}{|\lambda|} + \frac{\delta_L + \delta_K}{|\lambda|} (\hat{w} - \hat{r} - \hat{p}).$$

Subtracting (83) from (82) gives the change in the wage-interest ratio:

$$(86) \qquad (\hat{w} - \hat{r}) = \frac{1 + |\theta|}{|\theta|} \hat{p} ,$$

so that, by substitution,[122]

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change in relative commodity price has a magnified impact on factor prices. Jones, op. cit. In terms of our discussions this simply means that as we move along the factor-price frontier relative commodity price behaves as shown in equation (86) of the text.

All of this implies that factor endowments have no effect on factor prices (the factor-price equalization theorem). Paul A. Samuelson, "Prices of Factors and Goods in General Equilibrium," Review of Economic Studies, XXI (1953-54), pp. 5-6. However, as Samuelson notes on page 7, ibid., with commodity prices determined within the model, factor endowments will influence factor prices. Hence the qualification in the text. This last fact is also reflected in equations (89) and (97) of the text.

[121]The relation of the wage-interest ratio to relative commodity price is given directly by equation (25) of the text.

[122]Substituting $(\hat{w} - \hat{r})$ for $(\hat{w} - \hat{r} - \hat{p})$ gives

(87) $(\hat{M} - \hat{C}) = \frac{(\hat{L} - \hat{K})}{|\lambda|} = \sigma_s \hat{p}$,

where $\sigma_s = \frac{1}{|\theta|\,|\lambda|}(\delta_L + \delta_K)$,

may be interpreted as the elasticity of substitution between commodities on the supply side.[123] The change in relative commodity price is then given by the mutual interaction of supply (equation 87) and demand (equation 84):

(88) $\hat{p} = \frac{(\hat{K} - \hat{L})}{|\lambda|\,(\sigma_s + \sigma_D)}$.

Equation (88) shows that an increase in the economy's endowment of capital relative to labor implies an increase in the price of the relatively labor-intensive good relative to the price of the relatively capital-intensive good. With technology given, the extent of the change in relative commodity price depends on the substitution properties of the system.

There are two essentially equivalent explanations for this impact of changing factor endowments on relative commodity price. First, an increase in capital relative to labor raises the output of the relatively capital-intensive sector relative to the output of the relatively labor-intensive sector.[124] Thus relative commodity price moves

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$\frac{1 + |\theta|}{|\theta|}\,\hat{p} - \hat{p}$ so we have $\frac{1}{|\theta|}\,\hat{p}$.

[123] Along the transformation schedule. σ_s is a composite of the elasticities of substitution between factors in each sector. Supra, p. 103.

[124] Jones, op. cit. At constant commodity prices equations (19) and (20) may be solved directly for $\hat{M}$ and $\hat{C}$. The solutions may be written as $(\hat{M} - \hat{L}) = \frac{\lambda_{LC}}{|\lambda|}(\hat{L} - \hat{K})$, and

in the direction indicated by equation (88). Second, so long as there is any substitution in consumption, the increase in the factor endowment ratio implies that the price of the relatively more abundant factor (capital) has fallen relative to the less abundant factor (labor).[125] Thus the equilibrium price of the relatively labor-intensive good will be higher relative to that of the relatively capital-intensive good.

Substituting equation (88) into (86) gives directly the change in the factor endowment ratio that is implied by a changing wage-interest ratio:

$$(89) \qquad (\hat{K} - \hat{L}) = \frac{|\Theta|\,|\lambda|\,(\sigma_S + \sigma_D)}{1 + |\Theta|}\,(\hat{w} - \hat{r}) \ .$$

Equation (89) shows that, regardless of the technology,[126] the factor endowment ratio (capital to labor) declines for movements down the grand factor-price frontier. Suppose that the labor force is given and constant (the usual

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$(\hat{C} - \hat{K}) = \frac{\lambda_{KM}}{|\lambda|}(\hat{K} - \hat{L})$. Thus the sign of $|\lambda|$ is crucial. With $|\lambda| > 0$ an increase of capital relative to labor has what Jones calls a magnification effect on outputs. In this case $\hat{C} > \hat{K} > \hat{L} > \hat{M}$. That is, the output of the relatively capital-intensive sector is growing faster than the endowment of either factor alone. These magnification effects are dampened by price changes but in no case can the direction be reversed.

[125] With substitution in consumption, it will generally be impossible for all of the relatively greater quantity of capital to be absorbed solely by an expansion of the capital-intensive sector. Thus some alteration in factor proportions would be indicated.

[126] Since, as reference to equation (79) in the text shows, the denominator is necessarily non-negative.

assumption in much of the reswitching literature).[127] Then every fall in the equilibrium rate of interest (rise in the real wage rate) implies that capital has been accumulated by an amount that depends on: (a) the elasticity of substitution between commodities in production, (b) the elasticity of substitution between commodities in consumption, (c) the differential between the proportions of labor and capital used in each sector, and (d) the differential between the relative share of labor in each sector. These magnitudes cannot be specified a priori but it is clear from equation (89) that, so long as there is substitution between commodities either in production or consumption, every fall in the rate of interest implies that capital has been accumulated.[128]

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[127]See, for example, Bruno, Burmeister, and Sheshinski, op. cit., p. 546.

[128]When the two sectors are equally capital intensive, equation (89) of the text breaks down. We have shown in Chapter III that when this is the case information about the sector capital-labor ratios is all we need. Supra, footnote 102. Under the above assumptions, an increase in the wage-interest ratio raises the proportion of capital to labor utilized in each sector separately. Hence the aggregate capital-labor ratio must have risen.

The relation between the wage-interest ratio and the factor endowment ratio implied by equation (89) does, of course, presuppose a definite kind of mechanism whereby the economy moves from one factor endowment configuration to another. One possibility, consistent with the above and intuitively reasonable, is that a rise in the real quasi-rent from an equilibrium position induces further net investment while a decline results in net disinvestment via natural depreciation. This is one specification prominent in the two-sector growth story. See, for example, Hirofumi Uzawa, "On A Two-Sector Model of Economic Growth," Review of Economic Studies, XXIX (October, 1961), p. 43, equation 21. Or more recently, Akira Takayama, "On a Two-Sector

Together, equations (88) and (89) show, for the Clark type capital model, that every fall in the wage-interest ratio lowers the capital intensity of techniques. The ratio of real capital to real labor is falling as one moves down the frontier, and the weight going to the relatively labor-intensive sector is rising as the price of the relatively labor-intensive good falls relative to the price of the relatively capital-intensive good.[129]

III. The Model: Fixed Coefficients

With fixed coefficients of production, equations (80) - (83) are greatly simplified as every $\hat{a}_{ij}$ and, therefore, the $\mathcal{S}$'s reduce to zero. The structure of the production model is then exhibited in the following equations:[130]

Model of Economic Growth: A Comparative Statics Analysis," Review of Economic Studies, XXX (October, 1963), p. 100, equation 25.

[129]The change in the relative importance of the two sectors may be related directly to the change in the factor endowment ratio by substituting equation (88) into equation (84) thus obtaining:

$$(\hat{M} - \hat{C}) = \frac{\sigma_D}{|J|\,(\sigma_S + \sigma_D)}(\hat{L} - \hat{K}) .$$

Notice that while it is large values of σ_D that serve to increase the demand-composition effect of a change in the wage-interest ratio, it is a small value of σ_S (and the underlying elasticities of substitution in each sector) that accomplishes the same end.

[130]Notice that the price equations are identical for both models. This is due to the fact that the equations represent equilibrium points only. At an equilibrium point the relation of factor prices to commodity prices must be the same whether the a_{ij} are fixed or variable. In between such points, the variable coefficients would allow lower prices (costs) in response to changing factor prices than

$$(90)\quad \lambda_{LM}\hat{M} + \lambda_{LC}\hat{C} = \hat{L} ,$$

$$(91)\quad \lambda_{KM}\hat{M} + \lambda_{KC}\hat{C} = \hat{K} ,$$

$$(92)\quad \theta_{LM}\hat{w} + \theta_{KM}\hat{r} = \hat{p}(1 - \theta_{KM}) ,$$

$$(93)\quad \theta_{LC}\hat{w} + \theta_{KC}\hat{r} = -\theta_{KC}\hat{p} .$$

With the coefficients of production fixed, it is necessary to distinguish between the process factor-price frontier and the economy's grand factor-price frontier (the envelope of the process frontiers).[131] Each process frontier corresponds to one technique of production. A switch of techniques occurs as we move from one fixed-proportions process frontier to another.

In the preceding chapter it was argued that with the consumption sector relatively capital intensive both relative commodity price and the factor endowment ratio (capital to labor) fall for movements down any non-linear process frontier. We can now give a more rigorous demonstration of these propositions.

Solving equations (92) and (93) one may obtain the relation between relative commodity price and the wage-interest ratio for movements along the process frontier:

$$(94)\quad \hat{p} = \frac{|\theta|}{1 + |\theta|}(\hat{w} - \hat{r}) .$$

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would be possible with coefficients fixed. But these points are irrelevant anyway or as Samuelson puts it: "The Substitution effects are of a higher order of smallness, influencing curvatures rather than first-order slopes." Samuelson, "Prices of Factors and Goods in General Equilibrium," op. cit., p. 5. This is an example of what Samuelson calls the Wong-Viner envelope theorem.

[131] Supra, Chapter II for discussion of these points.

For movements down any nonlinear frontier, equation (94) shows that the price of the relatively labor-intensive good must fall relative to that of the relatively capital-intensive good.

Subtracting equation (91) from (90) yields the change in the ratio of outputs produced:

$$(95) \quad (\hat{M} - \hat{C}) = \frac{(\hat{L} - \hat{K})}{|\lambda|} .$$

Thus by using equation (84), one may relate the change in the factor endowment ratio to the change in relative commodity price:

$$(96) \quad (\hat{K} - \hat{L}) = |\lambda| \, \sigma_D \, \hat{p} .$$

Finally, substituting equation (94) into equation (96) one obtains the change in the factor endowment ratio that is implied by movements down the process frontier:

$$(97) \quad (\hat{K} - \hat{L}) = \frac{|\theta| \quad |\lambda|}{1 + |\theta|} \, \sigma \, (\hat{w} - \hat{r}) .$$

Equation (97) clearly shows that the economy's endowment of capital is falling relative to its endowment of labor as one moves down the process frontier.

The factor-price frontiers that are central to the reswitching discussions show the equilibrium real wage and interest rate combinations that are feasible under a specified technique. Equations (94) and (97) show how, under a specified technique, equilibrium commodity price and the economy's factor endowment ratio must vary in response to changes in the equilibrium wage-interest ratio. Thus, for example, in Figure 20, relative commodity price is lower

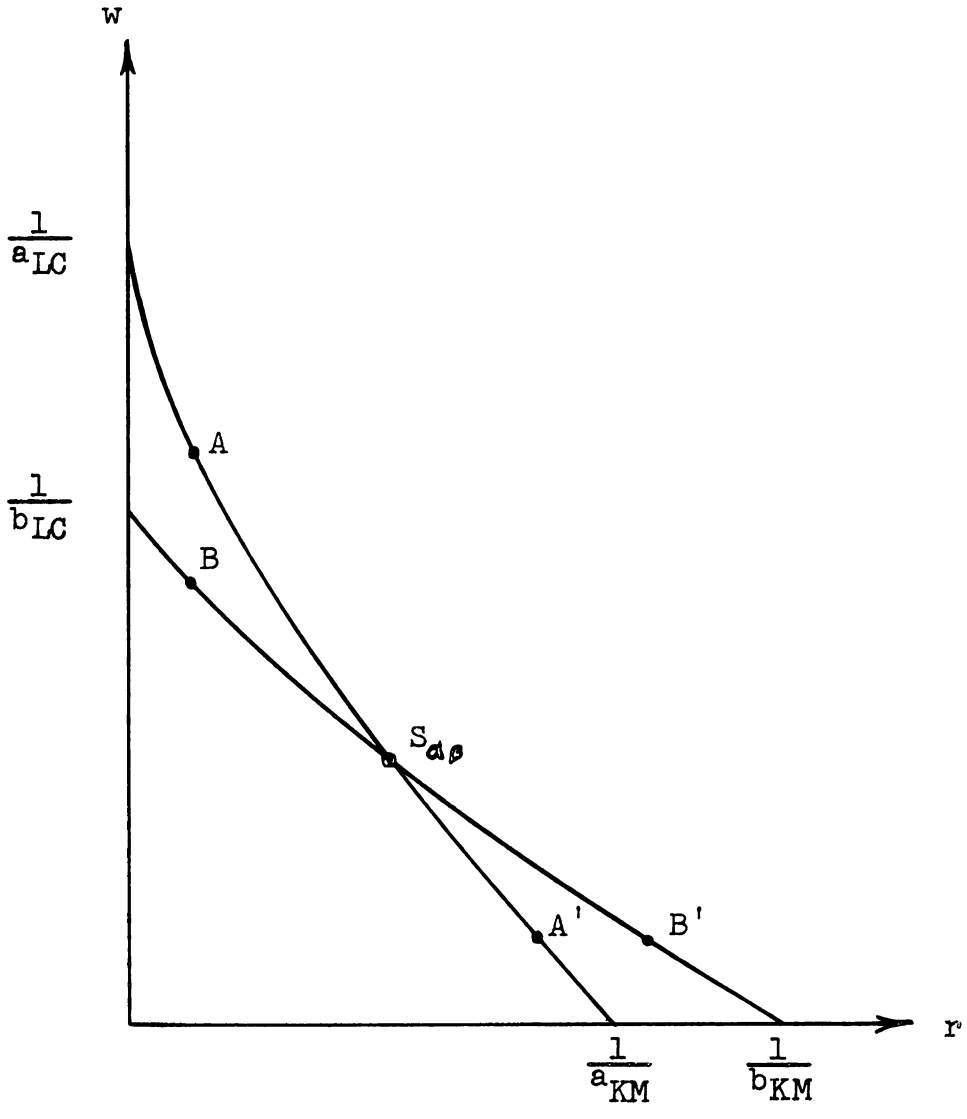

Figure 20. The Nonlinear Factor-Price Frontier

at A' than it is at A, and it is lower at B' as compared to B. The same holds for the factor endowment ratio. In fact, as equation (96) shows, with the consumption sector capital intensive, every fall in relative commodity price implies that the economy's endowment of labor has risen relative to its endowment of capital.

Now consider the switch point $S_{\alpha\beta}$. The equations of change do not (indeed, cannot) tell us how relative commodity price behaves at the precise point where entrepreneurs are adopting the Beta technique. In marked contrast to the production model with variable coefficients, the grand factor-price frontier of Figure 20 is kinked at the switch point. This kink is important as it means that when the switch occurs relative commodity price may "jump." If it does, there is no reason why it should not "jump" upward. That is, relative commodity price may be higher under the Beta technique than it is under the Alpha technique when the wage-interest ratio is that relevant to the point $S_{\alpha\beta}$.

For any point on the grand factor-price frontier, relative commodity price is given by[132]

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[132] Write the price equations as $P_M = a_{LM}w + a_{KM}rP_M$, and $P_C = a_{LC}w + a_{KC}rP_M$, so that we have $P_M = \frac{a_{LM}w}{1 - a_{KM}r}$, and $P_C = \frac{wa_{LC} + Awr}{1 - a_{KM}r}$, where $A = a_{LM}a_{KC} - a_{LC}a_{KM}$. Thus relative commodity price $(\frac{P_M}{P_C})$ is given by equation (98) of text.

(98) $$p = \frac{a_{LM}}{a_{LC} + Ar} .$$

Thus relative commodity price is lower, unchanged, or higher under the Beta technique at the point $S_{\alpha\beta}$ according as

(99) $$a_{LM}(b_{LC} + Br) \gtrless b_{LM}(a_{LC} + Ar) .$$

In the special case where the factor proportions are equal between sectors (A and B equal zero), equation (99) can be shown to reduce to[133]

(100) $$\lambda^{\beta}_{LC} \gtrless \lambda^{\alpha}_{LC} .$$

The λ_{LC} refer to the proportion of labor employed in the consumption sector under each technique. Thus if the proportion of labor employed in the consumption sector rises when the new technique is adopted, the price of machines will rise relative to the price of the consumption good at the switch point.

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[133] When A = B = 0 relative commodity price is equal under the two techniques if $\frac{a_{LM}}{a_{LC}} = \frac{b_{LM}}{b_{LC}}$ or $\frac{a_{LM}}{b_{LM}} = \frac{a_{LC}}{b_{LC}}$. By definition $\lambda^{\alpha}_{LM} = \frac{a_{LM}M}{L}$ and $\frac{\lambda^{\alpha}_{LM}}{\lambda^{\beta}_{LM}} = \frac{\frac{a_{LM}M}{L}}{\frac{b_{LM}M}{L}} = \frac{a_{LM}}{b_{LM}}$. Thus we may write $\frac{\lambda^{\alpha}_{LM}}{\lambda^{\beta}_{LC}} = \frac{\lambda^{\alpha}_{LC}}{\lambda^{\beta}_{LC}}$ or $\frac{\lambda^{\alpha}_{LM}}{\lambda^{\alpha}_{LC}} = \frac{\lambda^{\beta}_{LM}}{\lambda^{\beta}_{LC}}$. Adding one to both sides then gives $\frac{\lambda^{\alpha}_{LM} + \lambda^{\alpha}_{LC}}{\lambda^{\alpha}_{LC}} = \frac{\lambda^{\beta}_{LM} + \lambda^{\beta}_{LC}}{\lambda^{\beta}_{LC}}$. By assumption of full employment $\lambda_{LM} + \lambda_{LC} = 1$ so $\frac{1}{\lambda^{\alpha}_{LC}} = \frac{1}{\lambda^{\beta}_{LC}}$ or $\lambda^{\beta}_{LC} = \lambda^{\alpha}_{LC}$.

When we allow for different sector factor intensities, expression (99) is more difficult to interpret. All one can say is that so long as the new technique is no less capital intensive in the consumption sector than was the old technique, the behavior of relative commodity price at the switch point depends on changes that take place in the proportion of labor utilized in each sector as a result of the switch.

However, the point to be emphasized is that a priori one cannot say anything about the behavior of relative commodity price at the point $S_{\alpha\beta}$ in Figure 20. Relative commodity price may fall, be unchanged, or rise due to the switch of techniques. Thus all entrepreneurs may be adopting more capital-intensive techniques, but the aggregate capital-labor ratio may remain unchanged or even fall.

To complete this section, a numerical example of each type of price behavior is offered as definite proof that relative commodity price can move in any direction when a switch of techniques occurs in a fixed-proportions, heterogeneous, real capital model.

I. Relative commodity price falls. Two possible techniques of production are:

$$\text{Alpha}_1: \quad C = \min(L, \tfrac{1}{2}K), \quad M = \min(\tfrac{1}{4}L, \tfrac{1}{2}K),$$

and

$$\text{Beta}_1: \quad C = \min(\tfrac{1}{2}L, \tfrac{1}{3}K), \quad M = \min(\tfrac{1}{3}L, K).$$

Some values of p^{α} and p^{β} are computed in Table II. Entrepreneurs switch from the $Alpha_1$ technique to the $Beta_1$ technique at $r = 0.2500$. Relative commodity price falls from 1.600 to 0.800.

II. Relative commodity price is unchanged. Two possible techniques are:

$$\text{Alpha}_2 : \quad C = \min(L, \tfrac{1}{6}K), \quad M = \min(L, \tfrac{1}{2}K),$$

and

$$\text{Beta}_2 : \quad C = \min(\tfrac{1}{2}L, \tfrac{1}{5}K), \quad M = \min(\tfrac{1}{2}L, K).$$

Some values of p^{α} and p^{β} are computed in Table III. Entrepreneurs switch techniques at $r = 0.33$ and relative commodity price remains unchanged at 0.431.

III. Relative commodity price rises. Two possible techniques are:

$$\text{Alpha}_3 : \quad C = \min(L, \tfrac{1}{6}K), \quad M = \min(\tfrac{1}{3}L, \tfrac{1}{2}K),$$

and

$$\text{Beta}_3 : \quad C = \min(\tfrac{1}{2}L, \tfrac{1}{3}K), \quad M = \min(\tfrac{1}{5}L, K).$$

Some values of p^{α} and p^{β} are computed in Table IV. Entrepreneurs switch from the $Alpha_3$ technique to the $Beta_3$ at $r = 0.135$. Relative commodity price increases from 0.949 to 1.332.

The factor-price frontiers implied by these examples are shown graphically together with the trace of relative commodity price over the range of feasible interest rates

TABLE II

COMPUTED VALUES OF RELATIVE COMMODITY PRICE FOR TECHNIQUE ALPHA$_1$ (p^{α}) AND TECHNIQUE BETA$_1$ (p^{β})

r	p^{α}	p^{β}
0.00	4.000	1.550
0.05	3.076	1.276
0.10	2.500	1.111
0.15	2.105	0.983
0.20	1.808	0.882
0.25	1.600	0.800
0.30	1.428	0.732
0.40	1.176	0.625
0.60	0.869	0.484
0.90	0.625	0.361
1.00	0.571	0.333

Note: Entrepreneurs switch techniques at r = 0.25 and relative commodity price falls from 1.600 to 0.800.

TABLE III

COMPUTED VALUES OF RELATIVE COMMODITY PRICE FOR TECHNIQUE ALPHA$_2$ (p^{α}) AND TECHNIQUE BETA$_2$ (p^{β})

r	p^{α}	p^{β}
0.00	1.000	1.000
0.05	0.833	0.833
0.20	0.555	0.555
0.25	0.500	0.500
0.30	0.454	0.454
0.33	0.431	0.431
0.35	0.417	0.417
0.45	0.357	0.357
0.75	0.250	0.250
0.85	0.227	0.227
1.00	0.200	0.200

Note: Entrepreneurs switch techniques at r = 0.33 and relative commodity price is unchanged at 0.431.

TABLE IV

COMPUTED VALUES OF RELATIVE COMMODITY PRICE FOR TECHNIQUE ALPHA$_3$ (p^{α}) AND TECHNIQUE BETA$_3$ (p^{β})

r	p^{α}	p^{β}
0.000	3.000	2.50
0.055	1.596	1.824
0.105	1.119	1.488
0.135	0.949	1.332
0.205	0.701	1.073
0.405	0.401	0.689
0.505	0.330	0.584
0.705	0.244	0.448
0.805	0.216	0.401
0.905	0.194	0.363
1.000	0.177	0.333

Note: Entrepreneurs switch technique at r = 0.135 and relative commodity price rises from 0.949 to 1.332.

in Figure 21. Panel a, Figure 21, shows case I with relative commodity price falling when the switch occurs; Panel b, case II with relative commodity price rising at the switch point. Case III, in which relative commodity price falls continuously for movements down the grand factor-price frontier, is shown in Panel c, Figure 21. Notice that the factor-price frontiers have common ordinate and abscissa intercepts. The wage-interest ratio is, of course, different for each case at the switch point.

IV. Reswitching and the Aggregate Capital-Labor Ratio

An example of reswitching, discussed at length in Chapter II, is shown graphically in Figure 22. Given a choice of techniques, entrepreneurs are assumed to select the technique that yields the greatest rate of profit. Thus at very high wage rates, $w_2 < w < w_3$, the Beta technique is selected. At intermediate wage rates, $w_1 < w < w_2$, the Alpha technique is the most profitable technique, and for very low wage rates, $0 < w < w_1$, the Beta technique is again selected as it has once more become the most profitable of the two possible techniques.

The conclusion that has been drawn from examples such as this is that, if the economy goes over to "less capital" at the first switch point ($S_{\beta\alpha}$), it is necessarily coming back to "more capital" at the second switch

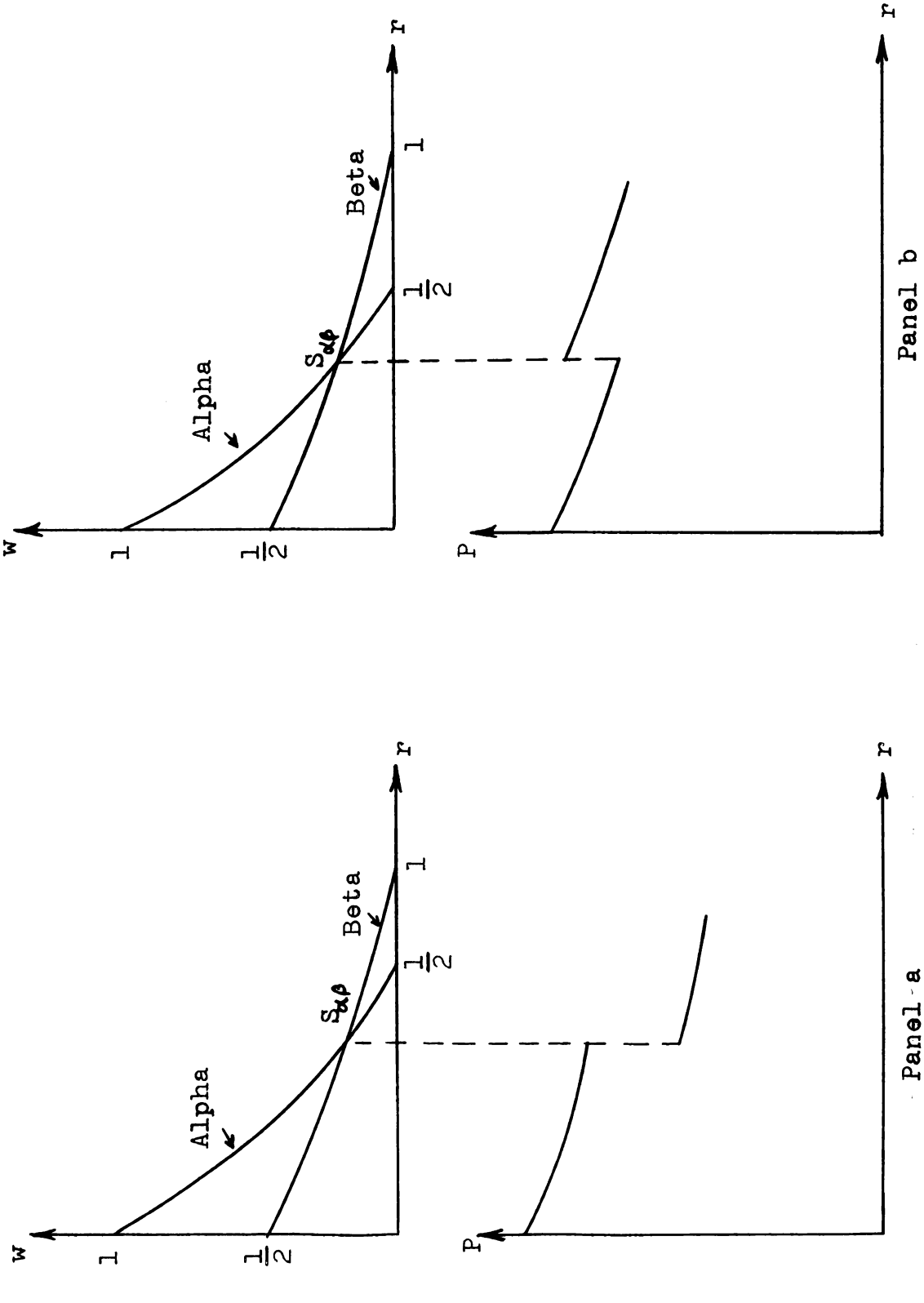

Figure 21. Effect of Switches of Techniques on Relative Commodity Price

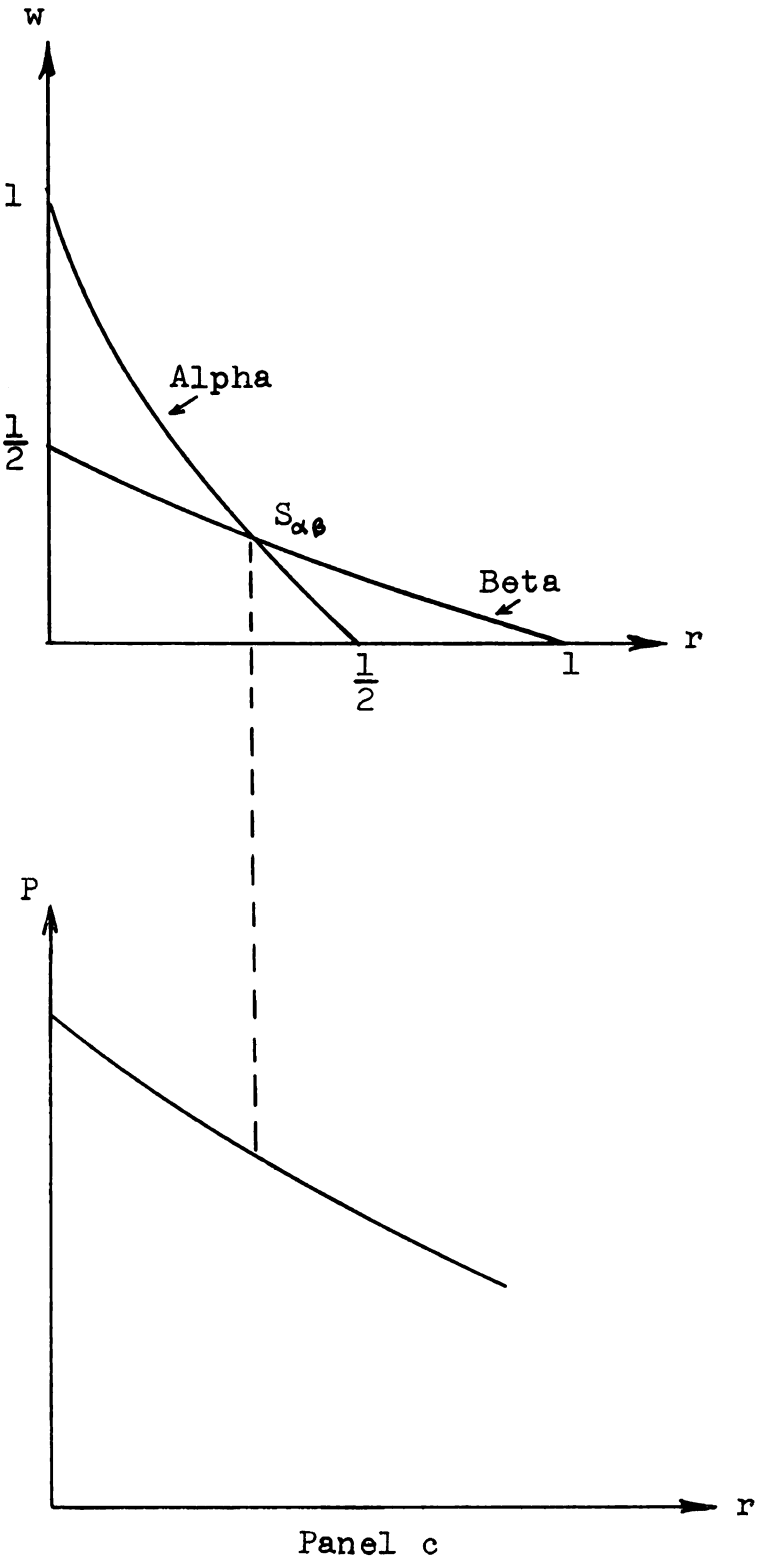

Figure 21. (continued)

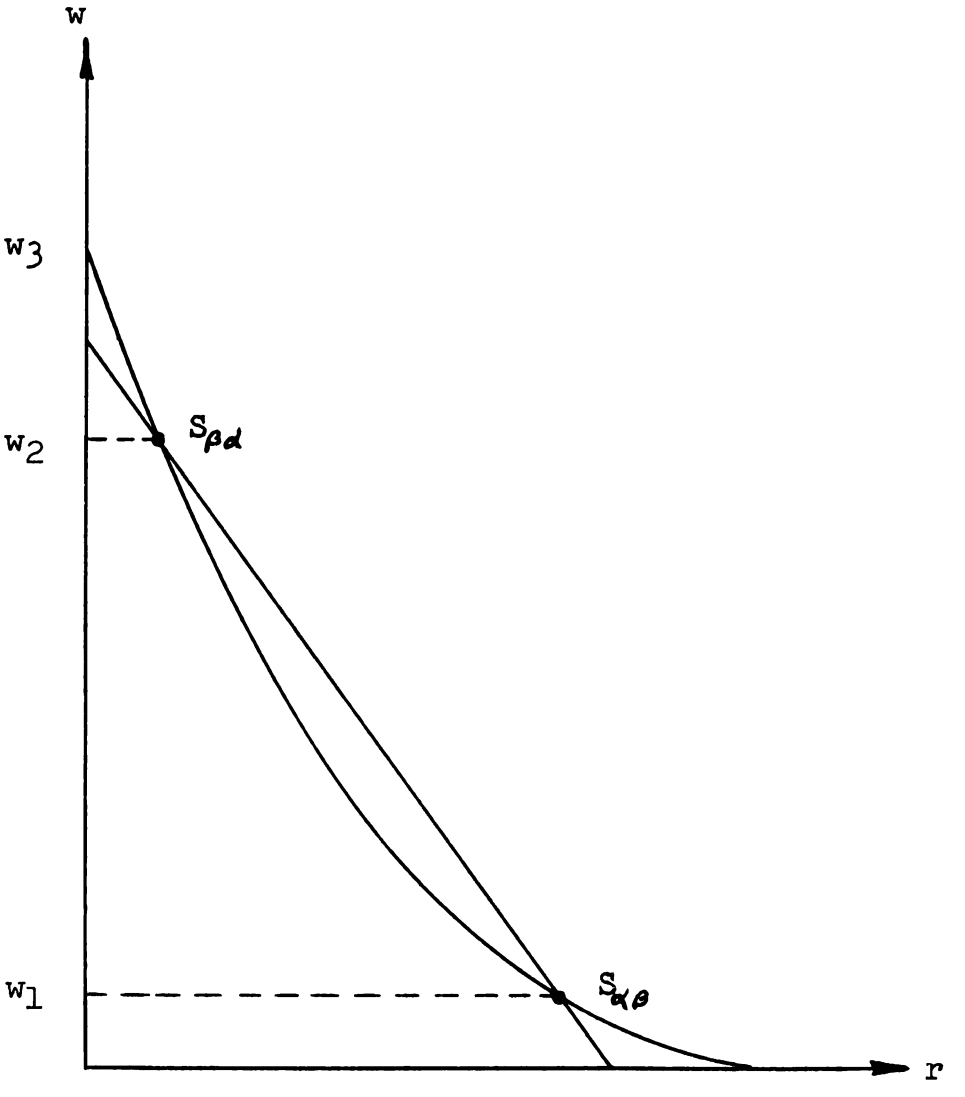

Figure 22. Reswitching of Techniques

point ($S_{\alpha\beta}$).[134] Thus it is said that, with a given, fully-employed labor force, the aggregate capital-labor ratio first falls (at the point $S_{\beta\alpha}$), then rises (at the point $S_{\alpha\beta}$), though the rate of interest is rising continuously.[135]

In the preceding section we saw that, for any technique giving rise to a nonlinear process frontier, both the factor endowment ratio and relative commodity price are necessarily lower at the point $S_{\alpha\beta}$ than they are at the point $S_{\beta\alpha}$. Hence it is clear that when the Beta technique "comes back" it does not represent the same ratio of capital to labor as it did when it went out of use.[136] It represents a smaller aggregate capital-labor ratio. Now suppose that, at the first switch point of Figure 22 ($S_{\beta\alpha}$), the economy goes over to a smaller aggregate capital-labor ratio as a result of all entrepreneurs adopting less capital-intensive processes. Suppose also that the new technique is one that requires equal-proportions production throughout the economy. The latter assumption ensures us that the factor endowment ratio is a constant for movements down the process frontier implied by the new technique. The question then is whether the capital-labor ratio under the Beta technique is greater at the second switch point

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[134] Pasinetti, op. cit., p. 514.

[135] Bruno, et al., pp. 548-49.

[136] cf. ante, Chapter II, footnote 84.

($S_{\alpha\beta}$) than that of the Alpha technique. It may be. But the point is that it need not be. All one can say for certain is that, if all entrepreneurs adopt less capital-intensive processes at $S_{\beta\alpha}$, all entrepreneurs adopt more capital-intensive processes at $S_{\alpha\beta}$. Beyond this, one must know how relative commodity price behaves at the second switch point before any inferences about aggregate capital-labor ratios are warranted.[137] Unfortunately, there is little one can say a priori about how relative commodity price behaves at the switch point. As we saw, given any wage-interest ratio, relative commodity price may fall, remain unchanged, or rise as a result of entrepreneurs' switching techniques.

Let us plot the relation between the aggregate capital-labor ratio and the rate of interest that is implied by movement down the grand factor-price frontier of Figure 22. We know from equation (97) that the aggregate capital-labor ratio falls continuously for movement down the Beta process frontier. Also from previous discussions we know that the aggregate capital-labor ratio is given by the sector capital-labor ratios for any equal-proportions

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[137]The behavior of relative commodity price at the first switch point is of no consequence in view of the fact that we are switching to an equal-proportions production technique at this point. Supra, Chapter III, footnote 102. In the more general case of different sector factor intensities under the Alpha technique and the Beta technique, the behavior of relative commodity price at both switch points would have to be considered.

production technique.[138] Thus, with all entrepreneurs adopting less capital-intensive techniques at the switch point ($S_{\beta\alpha}$), the aggregate capital-labor ratio is necessarily lower under the Alpha technique than it is under the Beta technique at the point ($S_{\beta\alpha}$). Finally, we know that the aggregate capital-labor ratio is a constant for movement along the Alpha process frontier.

Figure 23 shows how the aggregate capital-labor ratio must behave as the wage-interest ratio falls and one moves down the grand factor-price frontier of Figure 22. We assume a given fully employed labor force. At a zero rate of interest, the Beta technique represents a larger aggregate quantity of capital per man than does the Alpha technique. This fact may be inferred from the ordinate intercepts of Figure 22 alone.[139] (One might say that the Beta technique represents a larger (aggregate) plant than does the Alpha technique). As the rate of interest rises, the quantity of capital per man falls continuously under the Beta technique as shown by the curve k_b. The aggregate capital-labor ratio for the Alpha technique is given by the line k_a. At some rate of interest the aggregate capital-labor ratio implied by the Beta technique must be equal to that of the Alpha technique. This critical

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[138] See preceding reference.

[139] Provided we accept the neoclassical parable that says the real wage is higher the greater is the capital per man. cf. ante, Chapter III, footnote 97.

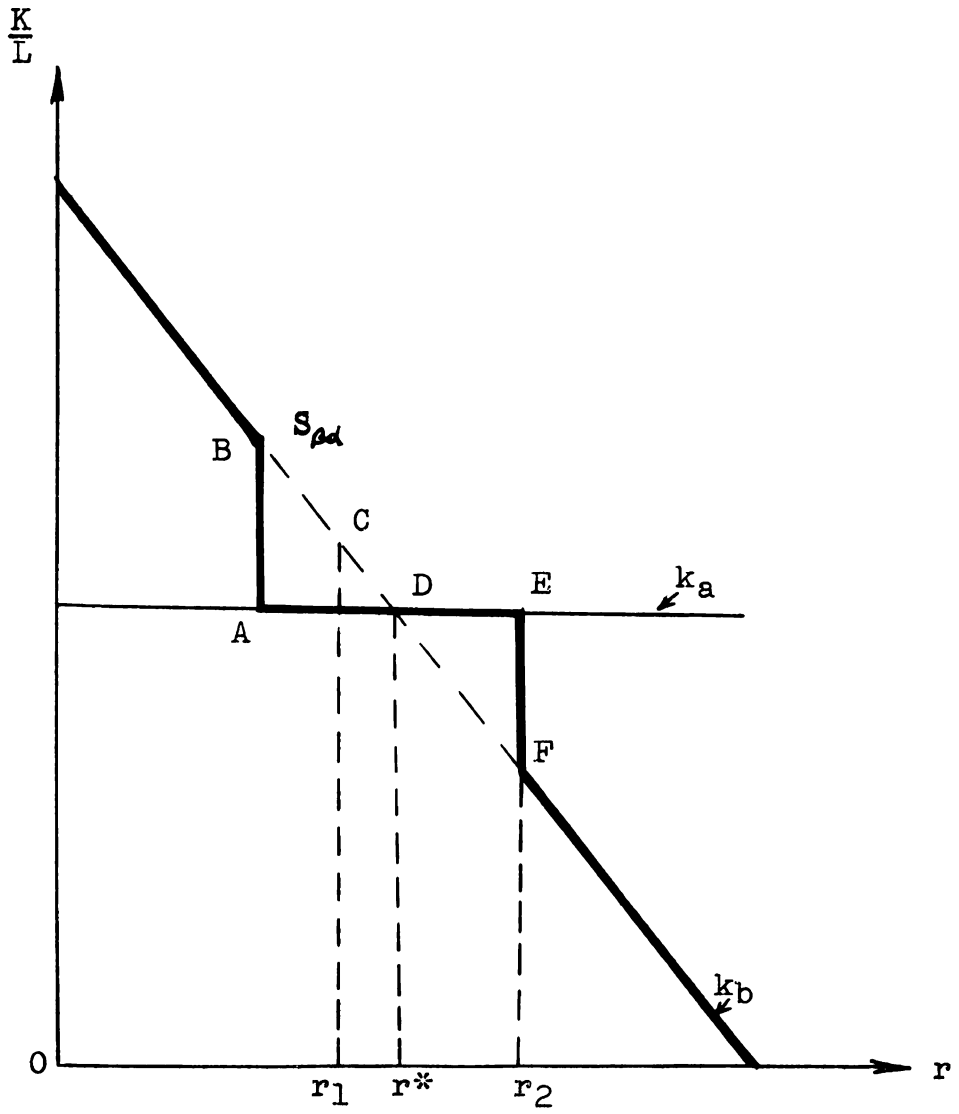

Figure 23. Effect of Reswitching on Capital Intensity: Relative Commodity Price Constant

rate of interest is labeled (r*) in Figure 23. Furthermore, at some interest rate below this critical rate, entrepreneurs will have switched from the Beta technique to the Alpha technique. Let the switch occur at the point ($S_{\beta\alpha}$) in Figure 23. Increases in the interest rate beyond the point ($S_{\beta\alpha}$) eventually result in a return to the Beta technique. There are now three types of price behavior that must be considered: relative commodity price is unaffected by the switch of techniques, it falls, or it rises. Case I: relative commodity price is unchanged at the point where reswitching occurs.

This case is already implied in Figure 23. Let the return to the Beta technique occur at one of the three interest rates labeled r_1, r*, or r_2. If the switch takes place at the critical interest rate r*, the capital-labor ratio is, of course, unchanged by the return to the Beta technique. In Figure 23, the aggregate capital-labor ratio moves from B to A, is unchanged as reswitching takes place at D, then falls toward F as the interest rate rises further.

If the switch occurs to the right of the critical rate r*, for example, at the rate of interest r_2, the aggregate capital-labor ratio must fall as a result of entrepreneurs returning to the Beta technique. Thus, referring to Figure 23, the aggregate capital-labor ratio falls from B to A as the Alpha technique replaces the Beta technique. It is then unchanged until entrepreneurs return to

the Beta technique at r_2. At r_2 the aggregate capital-labor ratio again falls going from E (=A) to F.

Finally, if entrepreneurs return to the Beta technique before the critical rate has been reached, for example at the interest rate r_1, the aggregate capital-labor ratio will rise when the switch occurs. This possibility is shown in Figure 23 by the dotted line rising from k_a to the point C at the interest rate r_1.

Case II: relative commodity price falls as entrepreneurs return to the Beta technique.

This case is shown in Figure 24. The fall in relative commodity price has the effect of shifting the k_b curve downward at the point of reswitching (from k_b to k_b'). The explanation for this is that more weight is automatically being given to the relatively labor-intensive sector when the switch occurs. Again suppose that entrepreneurs return to the Beta technique at one of the three interest rates r_1, r^*, or r_2.

The aggregate capital-labor ratio may now fall as entrepreneurs return to the Beta technique even when the switch occurs at rates below the critical interest rate (r^*).[140] In Figure 24, the aggregate capital-labor ratio is shown as unchanged when the switch occurs at $r_1 < r^*$.

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[140]There is some ambiguity here as one could view the fall in relative commodity price as meaning that the critical rate has been lowered thereby reducing the range of interest rates for which a return to the Beta technique could imply a higher aggregate capital-labor ratio. However, the results are unaffected by this choice.

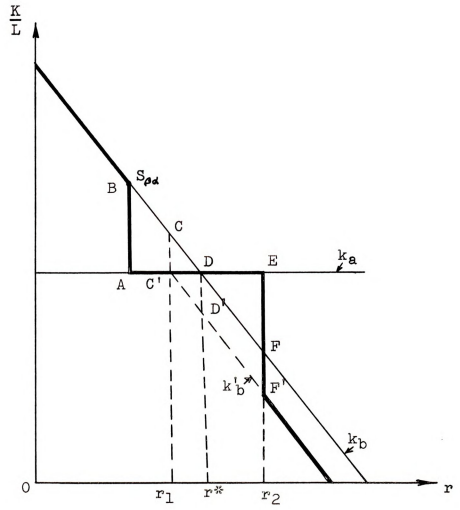

Figure 24. Effect of Reswitching on Capital Intensity: Relative Commodity Price Falling

At the switch point ($S_{\rho\alpha}$), the aggregate capital-labor ratio falls from B to A. As entrepreneurs return to the Beta technique at the interest rate r_1 the fall in relative commodity price causes the point C of Figure 23 to fall to C' in Figure 24. Thus the aggregate capital-labor ratio is unchanged by the return to the Beta technique. Further increases in the interest rate are associated with a falling capital-labor ratio as shown by the dotted line from C' to F'.

We cannot, of course, be certain that the aggregate capital-labor ratio will never rise for this case. However, the possibility is greatly reduced when the switch itself lowers relative commodity price. When reswitching occurs at r* or any higher rate of interest such as r_2, the aggregate capital-labor ratio will always fall as a consequence of returning to the Beta technique.

Case III: relative commodity price rises as the Beta technique returns.

This case is shown graphically in Figure 25. Of the three cases, it presents the greatest possibility that a return to the Beta technique will raise the aggregate capital-labor ratio.[141] The rise in relative commodity

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[141] This appears to be the case implied by the reswitching literature. See, for example, Samuelson's discussion of the influence of the interest rate on relative costs (prices) as he offers an explanation of why reswitching can occur. His explanation is that relative costs (prices) reverse themselves. Paul A. Samuelson, "A Summing Up," The Quarterly Journal of Economics, LXXX (November, 1966), pp. 571-73.

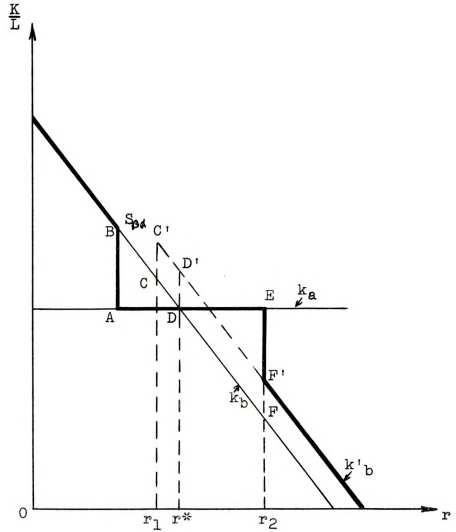

Figure 25. Effect of Reswitching on Capital Intensity: Relative Commodity Price Rising

price automatically increases the weight of the relatively capital-intensive sector as the entrepreneurs return to the Beta technique. Thus the cruve k_b shifts upward at the point of reswitching (from k_b to k_b' in Figure 25).

If the return to the Beta technique occurs at r* or any lower rate of interest such as r_1, the aggregate capital-labor ratio must rise. For example, as the interest rate increases from zero, we move along the curve k_b of Figure 25 and the aggregate capital-labor ratio falls. At the switch point $S_{\beta\alpha}$, entrepreneurs switch from the Beta to the Alpha technique. The aggregate capital-labor ratio falls from B to A. The aggregate capital-labor ratio is then unchanged for further increases in the rate of interest until the interest rate r_1 is reached. At this point, entrepreneurs switch back to the Beta technique, relative commodity price rises as a consequence of the switch of techniques, and the aggregate capital-labor ratio increases from A to C'. The aggregate capital-labor ratio then falls continuously (from C' toward F') until the interest rate reaches the maximum possible under the Beta technique. The argument is similar when the switch occurs at r*.

But notice that, even with relative commodity price rising as a result of entrepreneurs returning to the Beta technique, the aggregate capital-labor ratio need not necessarily rise. There may be a number of feasible interest rates (such as r_2 in Figure 25) where a return to the

Beta technique is accompanied by a fall in the aggregate capital-labor ratio. In Figure 25, return to the Beta technique at the interest rate r_2 causes the aggregate capital-labor ratio to fall from E to F'. It is therefore clear that even under the most unfavorable conditions the aggregate capital-labor ratio may fall though entrepreneurs are returning to techniques that were in use at some previous wage-interest ratio.

To sum up, the foregoing demonstrates that reswitching cannot be taken to mean that the aggregate capital-labor ratio is necessarily reversing itself. For any two techniques, there will generally be some critical rate of interest for which the aggregate capital-labor ratio is the same for both techniques.[142] Thus whether reswitching means that the aggregate capital-labor has reversed itself or not depends first of all on whether the economy has passed this critical interest rate before reswitching occurs. Once this critical rate is passed, it is virtually impossible for the aggregate capital-labor ratio to rise as a consequence of a fall in the wage-rate of interest ratio. When reswitching occurs before the economy has passed this critical rate, but relative commodity price falls as a result of reswitching, it is equally unlikely that the

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[142]When the sector factor intensities differ under both techniques, it will be necessary for the technique that represents the larger aggregate quantity of capital at a zero interest rate to react more strongly to changes in the rate of interest than the technique that represents the smaller quantity of capital.

aggregate capital-labor ratio has risen. It is only when reswitching occurs before the critical rate is passed and relative commodity price is unchanged or rises as reswitching takes place that the aggregate capital-labor ratio is necessarily higher.[143]

Whatever the behavior of relative commodity price at the point of reswitching, it is clear from the above that the relation of the aggregate capital-labor ratio to the wage-interest ratio depends critically on the interest rate itself. This is the one, single, unalterable fact made clear by the reswitching phenomenon. And it is important to record that though reswitching reveals this dependence, it is itself independent of any actual reswitching.[144]

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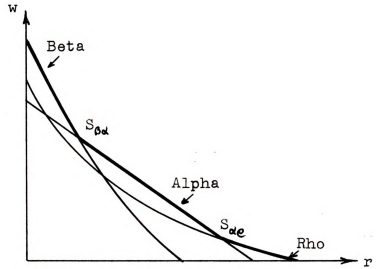
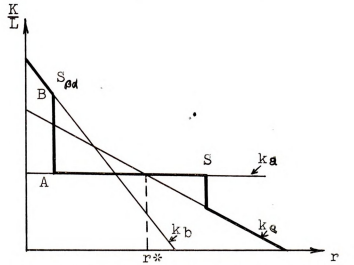
[143]The above demonstrates that reswitching may be accompanied by a reversal of relative commodity prices but it is important to notice that this need not be the case. Thus reversal of relative commodity prices implies reswitching but the reverse is not true.

[144]We have concentrated on reswitching because it presents the strongest case against simple neoclassical theory. If the neoclassical link between the aggregate capital-labor and the wage-rate of interest ratio can be shown to hold in the face of reswitching, it will hold a fortiori for more general cases. For example, consider again Figure 12, page 62 of the text:

In an Austrian capital model, a model that does not



At ($S_{\beta\alpha}$) entrepreneurs switch from the Beta technique to the Alpha technique and at ($S_{\alpha\rho}$) from the Alpha technique to the Rho technique. Plotting the relation between the aggregate capital-labor ratio and the interest rate as r rises from zero to its maximum under the Rho technique we have:



Following the notation and analysis of the text the aggregate capital-labor ratio falls from B to A at the switch point ($S_{\beta\alpha}$). At the second switch point ($S_{\alpha\rho}$) the aggregate capital-labor ratio may again fall, remain unchanged, or rise in accordance with the analysis of the text. Notice that the aggregate capital-labor ratio under the Rho technique is greater than that of the Alpha technique when r = 0. The interest rate that will be critical in this scheme is again labeled r* and corresponds to the

allow reswitching,[145] this dependence is revealed by the fact that the "average period" (the "independent" measure of capital) can be smaller or larger depending on whether the interest rate is relatively low or relatively high even though production processes are themselves unchanged.

In a heterogeneous capital model such as the one used in our discussions, this dependence is revealed by the fact that for any two techniques of production there is some viable interest rate at which the two techniques represent the same aggregate quantity of capital.

Precisely what this new (old) dependence between the interest rate and the quantity of capital implies for simple neoclassical theory is difficult to say. We have seen that it does not mean that the neoclassical relation between the aggregate capital-labor ratio and the wage-interest ratio is necessarily broken. There is a possibility that at some wage-interest ratio the neoclassical relation will not hold. However, when one considers the interrelations between factor prices, factor endowments, and relative commodity price the possibility does not appear to be a strong one. Mrs. Robinson's paradox is more likely to be to neoclassical production and/or capital

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interest rate at which the aggregate capital-labor ratio under the Rho technique is equal to that of the Alpha technique. At constant relative commodity price, the aggregate capital-labor ratio must fall when the switch to the Rho technique occurs after the economy has passed the interest rate r*.

145For the analysis of this model and its relation to reswitching, see Samuelson, "A Summing Up," op. cit., pp. 568-83.

theory what Giffen's paradox is to neoclassical demand theory. The relation is certainly more complicated than the simple parables would have us believe. But that, I think, is all one is entitled to conclude.

CHAPTER V

SUMMARY AND CONCLUSIONS

The chief characteristic of the neoclassical, real-capital model is the assumption that there exists a function $Q = F(K,L)$ that is homogeneous of degree one in a single homogeneous capital good (K) and a single homogeneous labor force (L). As we saw, this implies, (a) that the real wage varies inversely with the interest rate, and (b) that the real capital-labor ratio varies directly with the wage-interest ratio. Hence, an increase in the real wage leads to an increase in the capital intensity of techniques, and permanently sustainable net national product increases as the rate of interest declines.

The critics of simple neoclassical theory (especially Mrs. Robinson) have long maintained that the above assumption is "crucial" in the sense that a very elaborate set of propositions are being made to stand on what is in some respects a very dubious assumption. The trick, of course, was to show exactly how the results of simple neoclassical theory depended sensitively on this assumption.

Initially, much attention was given to the "physical jelly" that went into the production function under the

label of capital. Of course, no one (especially J. B. Clark) denied the existence and/or importance of heterogeneous capital goods; the question was how best to quantify them. For Mrs. Robinson, the answer was that it could not be done but if one insisted on doing it anyway, the element of time in the production, construction, and service life of these goods should be taken into consideration. Thus Mrs. Robinson replaced the "physical jelly" of the neoclassical production function with past labor time, compounded at interest, embodied in the stock of capital goods. The most significant result of this modification was that the ratio of real capital to labor was made to depend on the interest rate. Each technique (i.e., each particular set of capital goods) now appeared as a greater or lesser amount of real capital according as the interest rate was relatively high or relatively low.

A surprising possibility was revealed by this dependence. It implied that a decrease in the rate of interest may be accompanied by a change from more to less capital-intensive techniques. This was contrary to what had always been taken for granted and so there developed a tendency in the literature to label the possibility as "perverse," "anomalous," or "exceptional." Ultimately the fallacy of composition was invoked, and it was agreed that the "curiosum" (as Mrs. Robinson called it) may indeed be found at the level of the industry but never for the economy as a whole.

Subsequently the proof offered to support this contention was shown to be in error, and the "reswitching of techniques" became a "fact of life." Pasinetti, Morishima, Bruno-Burmeister-Sheshinski and others definitely proved it is possible for a technique that is the most profitable of all feasible techniques at some relatively high wage rate to also be the most profitable at some relatively low wage rate. The technique "comes back" as the wage falls monotonically toward zero, with the consequence that an unambiguous ordering of techniques is impossible. Thus, it seemed a simple technological fact that there could exist a "set of blueprints" for which there is no one-to-one correspondence between the capital-labor ratio and the wage-interest ratio. If so, it is not necessarily true that a fall in the rate of interest entails the adoption of more capital-intensive techniques, with the consequence that the permanently sustainable consumption stream is greater.

The problems posed by the critics of simple neoclassical theory concern the relation between the aggregate capital-labor ratio and the wage-interest ratio. Therefore, it is a serious criticism of recent discussions that a quantity of capital has come to be identified with a matrix of activities with no attempt to demonstrate how the columns of the matrix relate to the aggregate capital-labor ratio of the neoclassical model. That omission has had two unfortunate consequences: first, it has resulted in an

incomplete appreciation of the basis of the criticism, and second, it has allowed the role of relative prices to be completely ignored. By letting the aggregate capital-labor ratio represent itself in the Cambridge criticism, the present investigation demonstrates that the criticism is less important than recent discussions would have one believe.

As we saw, recent discussion has focused exclusively on capital goods. If a particular set of capital goods (i.e., matrix of activities) "comes back" as the wage-interest ratio falls monotonically toward zero, the aggregate capital-labor ratio is said to reverse itself regardless of how one might choose to measure capital. The only basis for this assertion is the labor-output ratio in the consumption sector relevant to the various techniques at a zero rate of interest (i.e., the labor-consumption coefficient). And this comes down to using the labor-output ratio of the consumption sector as an independent index of capital intensity.

There are two objections that one may raise against this index. First, and most obvious, is the fact that the labor-output ratio of the consumption sector is relevant only at a zero rate of interest. This is the major implication of a nonlinear factor-price frontier. Second, and perhaps more important, the labor-output ratio relevant at a zero rate of interest may not be a single-valued function of the aggregate capital-labor ratio. As we saw,

the relation between the ratios depends on the assumed set of technical alternatives. If entrepreneurs switch neoclassically, the two ratios are inversely related. However, as the reswitching literature makes clear, entrepreneurs may behave "perversely." In this case the ratios are directly related. It is possible for an increase in the labor-output ratio to be associated with either an increase or decrease in capital intensity. Hence, the index is inconsistent.

Beyond this, there is a rather subtle confusion present in the preoccupation with the labor-output ratio of the consumption sector. To suggest that the capital intensity of techniques is reflected in the labor-output ratio of the consumption sector at a zero rate of interest, is to implicitly assume that the aggregate capital-labor ratio is unique for a given technique (i.e., matrix of activities). That is, it implies that a technique embodies a given amount of real capital irrespective of the level of the wage-interest ratio. This, of course, presupposes an independence between the aggregate capital-labor ratio and the wage-interest ratio. Thus the basic relation on which the Cambridge criticism rests is lost in the literature devoted to the switches of techniques.

Reswitching and its implications are theoretical possibilities precisely because this interdependence exists. That is why the reswitching possibility was revealed only after the interdependence was incorporated

into the analysis of alternative, steady-state, capital-labor ratios. And it is also why reswitching is not necessary for the unconventional behavior of steady-state consumption. Reswitching and its so-called implications are all rooted in the fact that the aggregate capital-labor ratio is not independent of the wage-interest ratio.

As Mrs. Robinson says, this point, previously made by Wicksell in connection with Bohm-Bawerk's "average period," does not seem to have been properly digested into the teaching of the neoclassical doctrine.[146] I think it is fair to say that this point has not been properly developed in the most recent debates concerning the doctrine.[147] Perhaps this is due to the reluctance of recent writers to employ overtly the notion of a quantity of capital.[148] Whatever the reason(s), the fact remains that

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[146]"Wicksell points out that the length of the period of production does not by itself determine the ratio of capital to labour, because the value of capital required for a given method of production depends on the real-wage rate. This is a much more fundamental criticism of Bohm-Bawerk's theory than the objection that the length of the period of production is an over-simplified way of representing the real-capital ratio." Joan Robinson, The Accumulation of Capital (2d. ed., New York: St. Martin's Press, 1966), p. 396. Also ibid., p. 391.

[147]For the only explicit recognition of this point in the recent Symposium, see P. Garegnani, "Switching of Techniques," The Quarterly Journal of Economics, LXXX (November, 1966), p. 562, n. 3.

[148]It should be recorded that the failure to develop this point cannot be attributed to the type of model chosen to demonstrate the reswitching phenomena. As others have rightly pointed out (Bruno, et al., "The Nature and Implication of the Reswitching of Techniques," ibid., pp. 528-31), the instantaneous production model is the more general

the technology matrix is an aggregate capital-labor ratio. The point that needed to be developed is that the aggregate capital-labor ratio is not unique for a specified technique. This, of course, is the point contained in Mrs. Robinson's shifting productivity curve.

In Mrs. Robinson's analysis, the quantity of real capital embodied in a specified technique is dependent on the wage-interest ratio because of the element of interest in the cost (in terms of labor time) of capital goods. An increase in the rate of interest (fall in the real wage) raises her real-capital ratio. In the analysis of this paper, the quantity of real capital embodied in a specified technique is dependent on the wage-interest ratio because of a strict adherence to the full employment assumption. A fall in the wage-interest ratio lowers the price of the relatively labor-intensive good relative to that of the relatively capital-intensive good. And so long as demand is sensitive to relative commodity price, the full employment assumption requires either an increase in the economy's labor endowment or a decrease in its capital endowment. The result is a fall in the ratio of real capital to real labor embodied in the technique.

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model. And as we saw, a careful consideration of the implications of the nonlinear factor-price frontier of this model reveals the interdependence. On the other hand, as Samuelson has shown (ibid., "A Summing Up," pp. 568-83) it is quite evident in an Austrian-type production model that reswitching is in some sense an interest rate phenomenon. However, as Wicksell realized, one must go beyond the "average period" to get at the interdependence.

When the number of techniques exceeds one, the dependence of the factor endowment ratio on the wage-interest ratio implies that, for a given labor force, there is generally some interest rate (wage rate) at which any two techniques embody the same quantity of real capital. As we saw, this interest rate is critical insofar as the reswitching argument is concerned. Whether reswitching means that the aggregate capital-labor ratio has reversed itself or not depends in the first instance on whether, in the process of accumulation, the economy has passed the particular rate of interest that equates the real capital of the displaced technique with the real capital of the technique being adopted.

Finally as Mr. Sraffa has made clear,[149] one cannot examine the ratio of capital to labor independently of either the wage-interest ratio or relative prices. The latter are important to the problems posed by reswitching because the aggregate capital-labor ratio is a value-weighted mean of the ratio of capital to labor utilized in each sector of the economy considered separately. In a multi-sector capital model it is necessary to examine the

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[149] "The reversals in the direction of the movement of relative prices, in the face of unchanged methods of production, cannot be reconciled with any notion of capital as a measurable quantity independently of distribution and prices." Italics in original. Piero Sraffa, Production of Commodities by Means of Commodities (Cambridge: Cambridge University Press, 1963), p. 38.

interrelations of factor endowments, factor prices, and relative commodity price.

In a simple two-sector J. B. Clark neoclassical capital model, the relations between the wage-interest ratio, the factor endowment ratio, and relative commodity price are quite straightforward. The most important characteristic of the model is the continuous factor price frontier. As the wage-interest ratio falls one moves down the frontier, the ratio of real capital to real labor falls, and the price of the relatively labor-intensive good falls relative to the price of the relatively capital-intensive good. Hence, the aggregate capital-labor ratio is directly related to the wage-interest ratio and the neoclassical results follow.

In a fixed-proportions, heterogeneous capital model, one must distinguish between the process factor-price frontier and the economy's grand factor-price frontier (the envelope of the process frontiers). For an individual process frontier, the relations between the wage-interest ratio, the factor endowment ratio, and the relative commodity price are precisely those implied by movements along the continuous factor-price frontier of the J. B. Clark neoclassical capital model. Thus, for each process frontier considered separately, a decrease in the wage-interest ratio implies: (a) that the ratio of real capital to real labor has fallen and (b) that the price of the relatively labor-intensive good has fallen relative to the price of the

relatively capital-intensive good. Hence, the aggregate capital-labor ratio declines for movements down the process frontier.

When entrepreneurs switch techniques in a heterogeneous capital goods model, they move from one fixed-proportions process frontier to another. The result is a kink in the grand factor-price frontier at the point of switch. The kink is important because it implies that a switch of techniques has an independent influence on relative commodity price. A priori there is little one can say about the behavior of relative commodity price at the switch point. It may fall, remain unchanged, or increase as a result of entrepreneurs' switching techniques.

The introduction of relative prices into the analysis of reswitching substantially affects the Cambridge criticism insofar as that is directed toward the neoclassical link between the aggregate capital-labor ratio and the wage-interest ratio. It means that, contrary to the assertions in the literature, reswitching does not immediately imply a reversal in the capital intensity of techniques. If individuals are quite ready to substitute goods in consumption, and if reswitching causes relative commodity price to fall, the aggregate capital-labor ratio may be a single-valued function of the wage-interest ratio though individually entrepreneurs are adopting more capital-intensive processes.

In conclusion, one can only record that the neoclassical relation between factor proportions and factor prices depends in a complicated way on both the interest rate and relative commodity price. This, I think, is the simple but important kernel in the Cambridge criticism of simple neoclassical theory. The quantity of real capital embodied in any specified technique is not insensitive to the rate of interest, and relative prices are not insensitive to switches of techniques. It is possible that these interrelations may work so as to increase the capital intensity of techniques subsequent to a fall in the wage-interest ratio. I have shown how these interrelations may work so as to bring about a decrease in the aggregate capital-labor ratio in full accordance with the simple J. B. Clark neoclassical fairy tale. The invariant neoclassical relation is an empirical question that depends ultimately on the readiness of individuals to substitute goods in consumption. Unfortunately, it is an empirical question that has no empirical answer because economies do not proceed along smooth exponential growth paths.

BIBLIOGRAPHY

BIBLIOGRAPHY

A. BOOKS

Allen, R.G.D. Mathematical Economics. London: MacMillan and Company, 1957.

Bohm-Bawerk, Eugen von. The Positive Theory of Capital. Trans. William Smart. London: MacMillan and Company, 1891.

Clark, John B. The Distribution of Wealth. New York: The MacMillan Company, 1927.

Dickson, Leonard E. New First Course in The Theory of Equations. New York: Wiley, 1939.

Dorfman, Robert, Paul A. Samuelson, and Robert M. Solow. Linear Programming and Economic Analysis. New York: McGraw-Hill, 1958.

Ferguson, C. E. The Neoclassical Theory of Production and Distribution. Forthcoming from Cambridge University Press.

Hicks, John R. Capital and Growth. New York: Oxford University Press, 1965.

Hicks, John R. Value and Capital. Second edition. Oxford: Clarendon Press, 1962.

Meade, J. E. A Neoclassical Theory of Economic Growth. Revised edition. London: George Allen and Unwin Ltd., 1962.

Samuelson, Paul A. Economics. Sixth edition. New York: McGraw-Hill, 1964.

Robinson, Joan. The Accumulation of Capital. Second edition. New York: St. Martin's Press, 1966.

Schumpeter, Joseph A. History of Economic Analysis. Edited by Elizabeth B. Schumpeter. New York: Oxford University Press, 1963.

Spiegel, Henry W. The Development of Economic Thought. New York: John Wiley and Sons, 1964.

Sraffa, Piero. Production of Commodities by Means of Commodities. Cambridge: Cambridge University Press, 1963.

Wicksell, Knut. Lectures on Political Economy. Volume one. Trans. E. Classen. New York: The Macmillan Company, 1934.

B. PERIODICALS

Amano, Akihiro. "Determinants of Comparative Costs: A Theoretical Approach," Oxford Economic Papers, XVI (November, 1964), pp. 389-400.

Brown, Murray. "Substitution-Composition Effects, Capital Intensity Uniqueness and Growth," Discussion Paper Number 2, Economic Research Group, State University of New York at Buffalo, July, 1967.

Bruno, Michael, Edwin Burmeister, and Eytan Sheshinski. "The Nature and Implications of the Reswitching of Techniques," The Quarterly Journal of Economics, LXXX (November, 1966), pp. 526-553.

Champernowne, David G. "The Production Function and the Theory of Capital: A Comment," Review of Economic Studies, XXI (1953-1954), pp. 112-135.

Dorfman, Robert. "A Graphical Exposition of Bohm-Bawerk's Interest Theory," Review of Economic Studies, XXVI (February, 1959), pp. 153-158.

Garegnani, P. "Switching of Techniques," The Quarterly Journal of Economics, LXXX (November, 1966), pp. 554-561.

Jones, Ronald W. "The Structure of Simple General Equilibrium Models," The Journal of Political Economy, LXXIII (December, 1965), pp. 557-572.

Kaldor, Nicholas. "On The Theory Of Capital: A Rejoinder to Professor Knight," Econometrica, VI (April, 1938), pp. 163-176.

Levhari, David. "A Nonsubstitution Theorem and Switching of Techniques," The Quarterly Journal of Economics, LXXIX (February, 1965), pp. 98-105.

Levhari, David and Paul A. Samuelson. "The Nonswitching Theorem Is False," The Quarterly Journal of Economics, LXXX (November, 1966), pp. 518-519.

McManus, Maurice. "Process Switching in the Theory of Capital," Economica, XXX (May, 1963), pp. 109-121.

Morishima, Michio. "Refutation of the Nonswitching Theorem," The Quarterly Journal of Economics, LXXX (November, 1966), pp. 520-525.

Pasinetti, Luigi L. "Changes in the Rate of Profit and Switches of Techniques," The Quarterly Journal of Economics, LXXX (November, 1966), pp. 503-517.

Robinson, Joan. "Accumulation and the Production Function," The Economic Journal, LXIX (September, 1959), pp. 433-442.

Robinson, Joan. "The Production Function and the Theory of Capital," Review of Economic Studies, XXI (1953), pp. 81-106.

Samuelson, Paul A. "Parable and Realism in Capital Theory: The Surrogate Production Function," Review of Economic Studies, XXIX (June, 1962), pp. 193-207.

Samuelson, Paul A. "Prices of Factors and Goods in General Equilibrium," Review of Economic Studies, XXI (1953-1954), pp. 1-20.

Samuelson, Paul A. "A Summing Up," The Quarterly Journal of Economics, LXXX (November, 1966), pp. 568-583.

Solow, Robert M. "The Production Function and the Theory of Capital," Review of Economic Studies, XXIII (1955), pp. 101-108.

Stolper, Wolfgang F. and Paul A. Samuelson, "Protection and Real Wages," Review of Economic Studies, XI (November, 1941), pp. 58-73.

Takayama, Akira. "On a Two-Sector Model of Economic Growth: a Comparative Statics Analysis," Review of Economic Studies, XXX (October, 1963), pp. 95-106.

Usher, Dan. "Traditional Capital Theory," Review of Economic Studies, XXXII (1965), pp. 169-186.

Uzawa, Hirofumi. "On A Two-Sector Model of Economic Growth," Review of Economic Studies, XXIX (October, 1961), pp. 40-47.