CARRIER WAVE INTERACTION IN SOLIDS

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ABSTRACT

CARRIER WAVE INTERACTION IN SOLIDS

By

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The carrier wave characteristics of velocity-modulated electrons and holes in solids are studied. The analysis is based on the Maxwell's equations and the Boltzmann transport equations. By considering the carriers in solids as charged particles with effective mass m^*, a hydrodynamic model is adopted to describe the carrier behavior. Macroscopic equations of this model are derived. From the solution of the carrier wave equations, equivalent transmission lines for electron and hole motion in solids are developed.

A general expression of the propagation constant of the carrier waves in solids is obtained from the fundamental equations. The dispersion characteristics for the carrier waves in an extrinsic semiconductors are discussed in detail. The result shows that the thermal-to-drift velocity ratio plays an important role to the nature of the carrier wave while the collision between the carriers and the solid lattice determined the degree of wave attenuation.

The possibility of wave amplification is investigated by examining the kinetic power flow in solids surrounded by an electromagnetic slow-wave circuit. It is found that the essential condition for wave amplification is that the carrier wave which carries negative electrokinetic power is excited.

The normal modes of the collisionless carrier waves in an extrinsic semiconductor and an electromagnetic slow-wave circuit are defined. Using these results, a solid-state traveling-wave amplifier is studied by the coupled-mode analysis of wave interactions. Theoretically, for high gain operation, a high mean carrier drift velocity and low device operating temperature should be used.

CARRIER WAVE INTERACTION IN SOLIDS

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TO MY PARENTS MR. AND MRS. HAI-CHU CHENG

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INTRODUCTION

1.1 Space Charge Waves in Vacuum and the Carrier Waves in Solids

It is well known that there are fast and slow space-charge waves associated with an electron beam drifting in vacuum with an infinite homogeneous axial d-c magnetic field.[1] The existence of these waves depends on the space charge bunching produced by external longitudinal modulation. When the modulation of the beam is small and the signal frequency is much higher than the plasma frequency of the beam, the phase velocities of these waves are slightly faster and slower than the d-c drift velocity of the electron beam for the fast and slow space-charge waves, respectively. However, the group velocities of these waves are identically equal the beam's average drift velocity. When the beam has finite dimensions in the transverse direction and the applied axial d-c magnetic field is finite, more than one set of space-charge waves can exist in the system.[2]

Most electron beam devices operate according to the principle of circuit-beam wave interaction in which the electromagnetic wave propagating around the beam exchanges energy with one of its space-charge waves. If energy is removed from the beam and transferred to the circuit wave, the device will serve as an amplifier or oscillator. On the other hand, if signal energy is absorbed by the beam, the device will act as a passive element and is generally used as signal couplers.

As a result of rapid progress in solid state physics in the past decade, groups of new solid state electron devices such as transferred electron and avalanche transit time devices are receiving widespread attention. At present, these devices can only offer low power operation and have no competition, besides space and weight, for the vacuum beam devices such as the traveling-wave amplifier. However, their potential usefulness in the future is so great that intensive research in various aspects has been conducted almost everywhere.

It is natural to expect that there are similarities between electron streams in vacuum and the carrier streams in solids in charged carrier behavior. Furthermore, it is hoped that the principles of interaction used in the case of the electron beam in vacuum can be extended to the solid state devices. It has been shown by Wessel-Berg[3] that there are space-charge waves associated with drifting charged stream in a semiconductor. The nature of these waves in solids is identical to those in the electron beam except that they are heavily affected by the presence of collision and thermal effects. It has been shown by Ho[4] that when the d-c drift velocity of the carrier is small compared with its thermal velocity, a set of acoustic waves, similar to the electroacoustic waves in gaseous plasma, may exist in the semiconductor. These waves are electromechanical in nature, i.e. the wave propagation results from the interchange of kinetic energy of the stream charged carriers with stored energy in a-c electric field. For high operating frequency, these waves will propagate at the thermal velocity of the carriers. There is also a possibility of the existence of hybrid modes as a combination of the space-charge waves and surface waves propagating along the surface of the solid state plasma if the transverse dimension is finite.[5] All these waves are commonly known as "carrier waves" in a semiconductor.

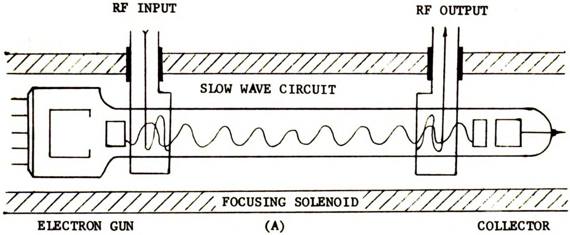
1.2 Previous Studies of the Carrier Waves in Solids

The studies of carrier waves in solid state plasmas was stimulated by Konstantinov and Perel[6] and also by Aigrain[7] in 1960. They showed that in the presence of magnetostatic fields, it is possible for electromagnetic waves to propagate in solids with a small attenuation. In 1961, Bowers, Legendy and Rose[8] performed a set of experiments to verify such possibilities. The investigation of wave interactions in solid state plasmas received greater attention and increased interest when the technological utilization in solid state microwave devices[9-14], such as the solid state traveling wave amplifier, was developed recently,

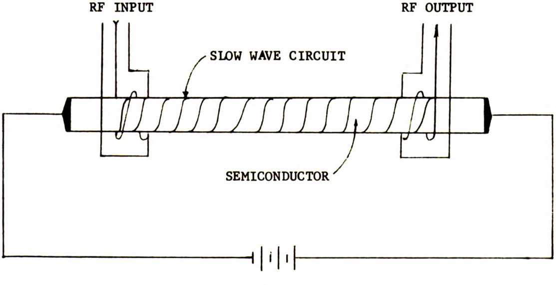
The power conservation theorem of electron beams has been investigated by several authors[15,16,17], for some time. An analogous study in solid state plasmas was given by Vural and Bloom[18] in 1967. From the Poynting theorem for a conducting medium, Vural and Bloom were able to obtain the effects of diffusion and collision to stored energy density and power flow of the carrier in solids. The electrokinetic power and energy density of the electron stream are examined in detail for several special cases and the conditions which lead to a condition where the stream's kinetic power become negative were discussed.

Recently, Kino[19] described the charged carrier motion in a semiconductor due to longitudinal modulation at low temperature through the expediency of a space-charge wave concept.

With the idea of replacing the electron beam in a traveling wave tube with drifting carriers in a semiconductor, as shown in Fig. 1-1 the interaction of drifting carriers in semiconductors with the traveling wave in an external slow-wave circuit was studied by Solymer and Ash[9] in a one-dimensional treatment. The conditions for amplification were derived by considering carrier momentum, thermal diffusion and collision effects. Sumi[10] made a three-dimensional investigation by ignoring the surface charge and current at the semiconductor surface. He derived a dispersion relation for the system and found that the characteristics of propagating waves are similar to those of ultrasonic wave amplification. For an n-type GA-As at room temperature, he predicted the attainable gain to be about 200db/mm. A further study including the surface effects was made by Vural and Steele[20] to consider the interaction with a generalized admittance wall. Although various slow-wave structures intended for use in solid-state traveling wave amplifiers were proposed by several authors[21,22], no successful experiment has yet been reported.



(A)



(B)

Fig. 1.1 (A) Vacuum Traveling-Wave Tube.
(B) Solid-State Analog of Traveling-Wave Tube

1.3 Objective and Outline of the Present Study

A review of the state-of-the art shows that most of the physical phenomena in solids, such as the Gunn oscillation and avalanche transit time oscillation, reported in the past few years have been studied rather extensively in two general aspects: the theoretical studies of the scattering mechanism in solids which produces instability under high field by quantum theory[23], and analytical studies, mostly using computer technique, of the charged particle dynamics based on the experimental velocity-field characteristics[24]. These approaches are either too theoretical or complicated in device application. None of these approaches has attempted to describe the various instabilities in solid state plasmas by the concept of wave interaction which was so successfully used in the electron beam devices[25]. The objective of this study is to develop an analytical technique, to study the basic properties of the carrier waves in solids. Furthermore, coupled mode theory will be used to examine all possible coupling between carrier waves and external electromagnetic waves.

In Chapter II, the general characteristics of the carrier waves in solids are described. Starting with Maxwell's equations and the Boltzmann transport equations, several fundamental equations are obtained to describe the behavior of the charged carriers in solids. After simplifying the fundamental equations by appropriate assumptions, the general wave equations for both electrons and holes with longitudinal modulation are derived. The attenuation and phase constants which describe the propagating characteristics of the carrier waves are obtained by solving those wave equations. Finally, considering a quasi-one-dimensional model and defining the kinetic voltages due to velocity modulation and thermal diffusion, two types of the transmission-line equivalent circuits in terms of a-c current density and the defined kinetic voltages for carrier waves in solids with the external slow wave circuit under consideration are deduced.

From the propagation constants obtained in Chapter II, the dispersion relations and wave characteristics for an extrinsic semiconductor are studied in detail in Chapter III. Several special cases are discussed. It is shown that the carrier waves are reduced to the space-charge waves of the electron beam in vacuum in the absence of collision and thermal diffusion. For a general case, the effects of longitudinal modulation, thermal diffusion, and collision upon the wave characteristics are examined. When the thermal-to-drift velocity ratio is changed from one extreme to another, it clearly shows that the fast and slow space-charge waves will gradually emerge into the electro-acoustic waves. It is also shown that collision between the carriers and solid lattice will cause attenuation in most cases. Several dispersion diagrams for simple cases are checked with those reported by Vural and Bloom[26] to confirm the validity of the theory developed.

In Chapter IV, the small signal kinetic power theorem for longitudinal carrier waves is presented. Using the fundamental equations and assumptions stated in Chapter II, a new form of Poynting theorem which includes the effects of diffusion and collisions is derived. The properties of the real power flow are investigated and the possibility of wave amplification and oscillation is examined in detail through the derived power and energy equation.

Chapter V presents the coupled mode analysis of carrier wave interactions. In order to apply the coupled mode technique, the normal modes of each carrier wave must first be obtained. In this chapter the normal modes of the collisionless carrier waves in solids are obtained in terms of the equivalent kinetic voltage and a-c current of the carriers. A coupled system which involves a slow electromagnetic wave circuit and a modulated carrier wave in the semiconductor is studied in detail. Using the derived normal modes and neglecting some of the weakly-coupled effects between the modes, an expression for gain is obtained.

Chapter VI contains a discussion of results and conclusions.

CHAPTER II

GENERAL CHARACTERISTICS OF THE CARRIER WAVES

2.1 Introduction

Since the individual carriers in solids have different velocities and energies distributed over a wide range, the characteristics of the carrier waves in solids are determined by the average behavior of the ensemble. Therefore, instead of trying to calculate the contribution of each electron individually, a statistical analysis is needed to derive the macroscopic equations describing the streaming carriers in solids. Generally, quantum-statistical analysis is used to describe the carrier motion inside solids, however, in the long-wavelength limit the quantum-mechanical description goes over to the classical description[20]. Here the general characteristics of carriers (electron and holes) in solids interacting with their self-created or externally imposed electric or electromagnetic field or both is investigated. The analysis is restricted to the long-wavelength exictations so that a classical statistical description can be applied.

In order to treat the carrier stream hydrodynamically, we further assume that the wavelength of any disturbance is much longer than the Debye length λ_0; the interactions of carriers with lattice vibrations are taken into account by introducing constant collision frequencies; the effect of band-to-band transitions is neglected due to the assumption that the energy and momentum changes per particle are small and the effect of the environment is taken into account by introducing effective masses for electrons and holes.

In this Chapter, a set of fundamental equations is introduced to describe the hydrodynamic model of the carrier stream in the solid. A general wave equation of the carriers is derived from those fundamental equations and the propagation constants for the longitudinal carrier waves in an extrinsic semiconductor are obtained from the wave equation. Using this result, the characteristics as well as the dispersion relations of the carrier waves in an extrinsic semiconductor can be studied and will be examined in detail in Chapter III. Equivalent transmission-lines for carrier waves in the solid including an external slow wave circuit surrounding the solid are also developed from the fundamental equations by defining proper kinetic voltages and equivalent current density. In our quasi-one-dimensional model, the longitudinal electrokinetic waves are coupled to the external electromagnetic waves through an ideal transformer which indicates a possibility of energy exchange between the carrier wave and the external slow wave circuit. The real power of the carrier wave dissipated by the collision effect between the carriers and the solid lattice is examined from the real power loss or the equivalent transmission-line, the result is checked with that obtained from the kinetic power theorem in Chapter IV. The suppose of this equivalent transmission-line is to introduce a circuit equivalence for the propagating carrier waves; when the transmission-line equivalence of the specific designed slow wave circuit is also developed, it would be possible to investigate the energy exchange and conditions of wave amplification by the circuit theory.

2.2 Fundamental Equations

The fundamental equations describing the average behavior of such carriers are Maxwell's equations and the macroscopic equations of the hydrodynamic model which are derived from the microscopic Boltzmann equation by taking moments of the velocity distribution. These equations can be written as follows:

$$\nabla \cdot \vec{E} = \frac{e}{\epsilon} (p-n) \tag{2.1}$$

$$\nabla \times \vec{E} = -\mu \frac{\partial \vec{H}}{\partial t} \tag{2.2}$$

$$\nabla \times \vec{H} = \vec{J}_h + \vec{J}_e + \epsilon \frac{\partial \vec{E}}{\partial t} \tag{2.3}$$

$$\nabla \cdot \vec{H} = 0 \tag{2.4}$$

$$\nabla \cdot \vec{J}_e - e \frac{\partial n}{\partial t} = 0 \tag{2.5}$$

$$\nabla \cdot \vec{J}_h + e \frac{\partial p}{\partial t} = 0 \tag{2.6}$$

$$\frac{d\vec{v}_e}{dt} = - \frac{e}{m_e^*} (\vec{E} + \vec{v}_e \times \mu \vec{H}) - \nu_e \vec{v}_e - \frac{v_{T-}^2}{n} \nabla n \tag{2.7}$$

$$\frac{d\vec{v}_h}{dt} = \frac{e}{m_h^*} (\vec{E} + \vec{v}_h \times \mu \vec{H}) - \nu_h \vec{v}_h - \frac{v_{T+}^2}{p} \nabla p \tag{2.8}$$

$$\vec{J}_e = -en\, \vec{v}_e \tag{2.9}$$

$$\vec{J}_h = ep\, \vec{v}_h \tag{2.10}$$

where $\vec{E}$ = the electric field intensity

$\vec{H}$ = the magnetic field intensity

μ = the permeability of the solid

ϵ = the permittivity of the solid

n = the electron density

p = the hole density

$\vec{J}_e$ = the electron current density

$\vec{J}_h$ = the hole current density

$\vec{v}_e$ = the electron velocity

$\vec{v}_h$ = the hole velocity

e = the electron charge = value

m_e^* = the effective mass of electrons in the solids

m_h^* = the effective mass of holes in the solids

ν_e = the collision frequency between the electrons and the solid lattice

ν_h = the collision frequency between the holes and the solid lattice

k = the Boltzmann constant = value

T = absolute temperature of the carriers

$v_{T-} = (\frac{3kT}{m_e^*})^{\frac{1}{2}}$ = mean thermal velocity of electrons in the solids

$v_{T+} = (\frac{3kT}{m_h^*})^{\frac{1}{2}}$ = mean thermal velocity of holes in the solids

Equations (2.1) thru (2.4) are the Maxwell's equations in the presence of charge and current inside the solid. Equations (2.5) and (2.6) are the zeroth moment of the Boltzmann transport equation which are commonly known as the equations of continuity for electrons and holes, respectively. Equations (2.7) and (2.8) are the first moments of the Boltzmann transport equation of the so-called equations of motion for electrons and holes. Equations (2.9) and (2.10) are the basic definitions of current density due to the drifting charged particles.

Solving the above fundamental equations with a given set of boundary conditions, the average behavior of the charged carriers in the solids can be described for a given excitation.

2.3 Basic Assumptions and the Simplified Fundamental Equations

Since the main purpose of this work is to try to obtain a general wave description of the carrier motion in solids, any second order effects will be ignored for simplicity, while the important phenomenological results will be retained in order to explore physical insights into the problem. With this intention in mind, the following assumptions and approximations are made:

(1) The carrier temperature is considered to be constant through the specimen,

(2) Each variable can be expressed as the sum of a time-independent (d-c) term and a time-dependent (a-c) term. The magnitude of the time-dependent term is small compared with that of the time-dependent term, so that a small signal analysis is used,

(3) All of the a-c components of the velocities of the carriers, densities of the carriers, electric field and magnetic

field have a periodic time-dependence with constant frequency ω,

(4) There is a strong homogeneous d-c magnetic focusing field in the longitudinal direction, namely the positive z direction; so that the carriers are confined to move in the longitudinal direction and thus a one-dimensional model is utilized. All the vectors are in the z-direction and all the variables are functions of z only.

By the above assumptions, the electric field intensity, the magnetic field intensity, the velocities of the carriers, the carrier densities and the current densities can be written in the following forms

$$\vec{E} = \vec{E}_0 + \vec{E}_1 = \hat{z}[E_0 + E_1(z,t)] \tag{2.11}$$

$$\vec{H} = \vec{H}_0 = \hat{z}\, H_0 \tag{2.12}$$

$$\vec{v}_e = \vec{u}_{e0} + \vec{v}_{e1} = \hat{z}[u_{e0} + v_{e1}(z,t)] \tag{2.13}$$

$$\vec{v}_h = \vec{u}_{h0} + \vec{v}_{h1} = \hat{z}[u_{h0} + v_{h1}(z,t)] \tag{2.14}$$

$$n = n_0 + n_1(z,t) \tag{2.15}$$

$$p = p_0 + p_1(z,t) \tag{2.16}$$

$$\vec{J}_e = \vec{J}_{e0} + \vec{J}_{e1} = \hat{z}[J_{e0} + J_{e1}(z,t)] \tag{2.17}$$

$$\vec{J}_h = \vec{J}_{h0} + \vec{J}_{h1} = \hat{z}[J_{h0} + J_{h1}(z,t)] \tag{2.18}$$

where

$$|E_0| \gg |E_1| \;,\quad |n_0| \gg |n_1| \;,\ \text{etc.} \tag{2.19}$$

$$E_1 = E_{10}\epsilon^{j\omega t},\ n_1 = n_{10}\epsilon^{j\omega t} \;,\ \text{etc.} \tag{2.20}$$

Here the subscript "0" and "1" denote the time-independent and time-dependent terms, respectively.

Substituting Equations (2.11) through (2.20) into Equations (2.1) through (2.10), neglecting all second order terms, separating the time-independent and time-dependent parts and considering the time-dependent parts only, the following system of scalar equations is obtained:

$$\frac{\partial E_1}{\partial z} = \frac{e}{\epsilon}(p_1 - n_1) \tag{2.21}$$

$$J_{e1} + J_{h1} + j\omega\epsilon E_1 = 0 \tag{2.22}$$

$$\frac{\partial J_{e1}}{\partial z} = j\omega e n_1 \tag{2.23}$$

$$\frac{\partial J_{h1}}{\partial z} = -j\omega e p_1 \tag{2.24}$$

$$(j\omega + \nu_e)v_{e1} + \frac{e}{m_e^*}E_1 + u_{e0}\frac{\partial v_{e1}}{\partial z} + \frac{v_{T-}^2}{n_0}\frac{\partial n_1}{\partial z} = 0 \tag{2.25}$$

$$(j\omega + \nu_h)v_{h1} - \frac{e}{m_h^*}E_1 + u_{h0}\frac{\partial v_{h1}}{\partial z} + \frac{v_{T+}^2}{p_0}\frac{\partial p_1}{\partial z} = 0 \tag{2.26}$$

$$J_{e1} = -e(n_0 v_{e1} + n_1 u_{e0}) \tag{2.27}$$

$$J_{h1} = e(p_0 v_{h1} + p_1 u_{h0}) \tag{2.28}$$

In summary, following the basic assumptions stated at the beginning of this section, the fundamental Equations (2.1) through (2.10) are reduced to a set of first order linear differential Equations (2.21) through (2.28) which can easily be handled in the subsequent developments.

2.4 Wave Equations and the Propagation Constants

In order to describe the carrier motion in solids under external modulation by the wave concept, it is desirable to obtain wave equations which express the voltage and current waves traveling along the sample. With this purpose in mind, differentiating Equation (2.25) with respect to z, we have:

$$(j\omega + \nu_e) \frac{\partial v_{e1}}{\partial z} + \frac{e}{m_e^*} \frac{\partial E_1}{\partial z} + u_{e0} \frac{\partial^2 v_{e1}}{\partial z^2} + \frac{v_{T-}^2}{n_0} \frac{\partial^2 n_1}{\partial z^2} = 0 \qquad (2.29)$$

Differentiating Equation (2.27) with respect to z and rearranging, yields:

$$\frac{\partial v_{e1}}{\partial z} = - \frac{1}{n_0} \left(\frac{1}{e} \frac{\partial J_{e1}}{\partial z} + u_{e0} \frac{\partial n_1}{\partial z}\right) \qquad (2.30)$$

Replacing $\frac{\partial J_{e1}}{\partial z}$ with the right hand side of (2.23), Equation (2.30) becomes:

$$\frac{\partial v_{e1}}{\partial z} = - \frac{1}{n_0} \left(j\omega n_1 + u_{e0} \frac{\partial n_1}{\partial z}\right) \qquad (2.31)$$

Substituting Equations (2.21) and (2.31) into Equation (2.29), one obtains:

$$(u_{e0}^2 - v_{T-}^2) \frac{\partial^2 n_1}{\partial z^2} + u_{e0}(\nu_e + j2\omega) \frac{\partial n_1}{\partial z} + \left(\frac{e^2}{m_e^*} \frac{n_0}{\epsilon} - \omega^2 + j\omega\nu_e\right)n_1$$
$$- \frac{e^2}{m_e^*} \frac{n_0}{\epsilon} p_1 = 0 \qquad (2.32)$$

This is the wave equation in terms of the a-c electron density. Similarly, the wave equation in terms of the a-c hole density can be obtained as:

$$(u_{h0}^2 - v_{T+}^2)\frac{\partial^2 p_1}{\partial z^2} + u_{h0}(\nu_h + j2\omega)\frac{\partial p_1}{\partial z} + \left(\frac{e}{m_h^*}\frac{p_0}{\epsilon} - \omega^2 + j\omega\nu_h\right)p_1$$

$$- \frac{e}{m_h^*}\frac{p_0}{\epsilon}\, n_1 = 0 \tag{2.33}$$

Introduce the following parameters:

$\omega_{p-} = e\left[\frac{n_0}{\epsilon m_e^*}\right]^{\frac{1}{2}}$ = radian electron plasma frequency

$\omega_{p+} = e\left[\frac{p_0}{\epsilon m_h^*}\right]^{\frac{1}{2}}$ = radian hole plasma frequency

$\beta_e = \omega/u_{e0}$ = phase constant for cold electron waves

$\beta_h = \omega/u_{h0}$ = phase constant for cold hole waves

$k_{T-} = v_{T-}/u_{e0}$ = thermal-to-drift velocity ratio of electrons

$k_{T+} = v_{T+}/u_{h0}$ = thermal-to-drift velocity ratio of holes

Equations (2.32) and (2.33) become

$$(1 - k_{T-}^2)\frac{\partial^2 n_1}{\partial z^2} + \left(\frac{\nu_e}{u_{e0}} + j2\beta_e\right)\frac{\partial n_1}{\partial z} + \left(\frac{\omega_{p-}^2}{u_{e0}^2} - \beta_e^2 + j\beta_e\frac{\nu_e}{u_{e0}}\right)n_1$$

$$- \frac{\omega_{p-}^2}{u_{e0}^2}\, p_1 = 0 \tag{2.34}$$

$$(1 - k_{T+}^2)\frac{\partial^2 p_1}{\partial z^2} + \left(\frac{\nu_h}{u_{h0}} + j2\beta_h\right)\frac{\partial p_1}{\partial z} + \left(\frac{\omega_{p+}^2}{u_{h0}^2} - \beta_h^2 + j\beta_h\frac{\nu_e}{u_{h0}}\right)p_1$$

$$- \frac{\omega_{p+}^2}{u_{h0}^2}\, n_1 = 0 \tag{2.35}$$

This is the wave equation for the carrier densities in solids with a longitudinal excitation. A general solution for the above simultaneous differential equations are of $e^{\Gamma z}$ form; there Γ is defined as the propagation constant for the carrier waves. From Equations (2.34) and (2.35), an equation for Γ is obtained as

$$[(1 - k_{T-}^2)\Gamma^2 + (\frac{\nu_e}{u_{e0}} + j2\beta_e)\Gamma + \frac{\omega_{p-}^2}{u_{e0}^2} - \beta_e^2 + j\beta_e \frac{\nu_e}{u_{e0}}] \cdot$$

$$[(1 - k_{T+}^2)\Gamma^2 + (\frac{\nu_h}{u_{h0}} + j2\beta_h)\Gamma + \frac{\omega_{p+}^2}{u_{h0}^2} - \beta_h^2 + j\beta_h \frac{\nu_h}{u_{h0}}] = [\frac{\omega_{p-}\omega_{p+}}{u_{e0}u_{h0}}]^2 \qquad (2.36)$$

Now let us consider the carrier waves propagating along extrinsic semiconductors which have a considerably large donor (n-type) or acceptor (p-type) concentration, in other words, the following conditions are assumed to be satisfied:

for n-type semiconductor $N_d^- \gg n_i$ or $n \gg p$

for p-type semiconductor $N_a^+ \gg p_i$ or $n \ll p$

where N_d, N_a, n_i and p_i are the concentration of donors, acceptors, intrinsic electron, and intrinsic holes, respectively. In such cases, the effects of minority carriers are neglected because of their relatively small concentration, and the wave equations reduce to

$$(1 - k_{T-}^2) \frac{\partial^2 n_1}{\partial z^2} + (\frac{\nu_e}{u_{e0}} + j2\beta_e) \frac{\partial n_1}{\partial z} + (\frac{\omega_{p-}^2}{u_{e0}^2} - \beta_e^2 + j\beta_e \frac{\nu_e}{u_{e0}})n_1 = 0 \qquad (2.37)$$

or

$$(1 - k_{T-}^2) \frac{\partial^2 p_1}{\partial z^2} + (\frac{\nu_h}{u_{h0}} + j2\beta_h) \frac{\partial p_1}{\partial z} + (\frac{\omega_{p+}^2}{u_{h0}^2} - \beta_h^2 + j\beta_h \frac{\nu_h}{u_{h0}})p_1 = 0 \qquad (2.38)$$

The propagation constants for longitudinal electrokinetic carrier waves in an extrinsic semiconductor in which the majority carriers greatly out-number the minority carriers can be written as:

$$\Gamma_{\pm} = -\frac{\omega}{u_0}\frac{1}{1-k_T^2}\left[\frac{\nu}{2\omega} + j \mp \Gamma_1\right] \tag{2.39}$$

where

$$\Gamma_1 = \left[\frac{\nu^2}{4\omega^2} - k_T^2 - \frac{\omega_p^2}{\omega^2}(1-k_T^2) + jk_T^2\frac{\nu}{\omega}\right]^{\frac{1}{2}} \tag{2.40}$$

Note the subscript "e" or "h" in Equations (2.39) and (2.40) was abbreviated for simplicity.

In general, the propagation constants $\Gamma_{\pm}$ are complex and can be expressed as

$$\Gamma_{\pm} = \alpha_{\pm} - j\beta_{\pm} \tag{2.41}$$

where $\alpha_{\pm}$ and $\beta_{\pm}$ are the real and imaginary parts of the propagation constant, and are known as the attenuation and phase constants of the longitudinal carrier waves, respectively.

2.5 The Equivalent Transmission-line for Carrier Waves in Solids Including an External Slow Wave Circuit

In general, if the electromagnetic wave from the external slow wave circuit is taken into account, we have to use the three dimensional analysis and the knowledge of the boundary conditions as well as the structure of the slow wave circuit are needed in detail. However when the magnitude of the electromagnetic wave is small compared with that of the electromechanical wave due to longitudinal modulation, a quasi-one-dimensional model is used. In this case, we express the electric and magnetic field in the rectangular coordinates as

$$\vec{E} = \vec{E}_0 + \vec{E}_1 = \hat{z}\,E_0 + \hat{x}\,E_{1x} + \hat{y}\,E_{1y} + \hat{z}\,E_{1z} \tag{2.42}$$

$$\vec{H} = \vec{H}_0 + \vec{H}_1 = \hat{z}\, H_0 + \hat{x}\, H_{1x} + \hat{y}\, H_{1y} + \hat{z}\, H_{1z} \tag{2.43}$$

Since the magnitude of the electromagnetic wave coupled to the system is small, the velocities and densities of the carriers are still strongly affected by the longitudinal modulation; therefore the transverse components of those variables are small compared with those of the longitudinal ones and can be neglected. Under this condition, the variables $\vec{v}_e$, $\vec{v}_h$, n, p, $\vec{J}_e$ and $\vec{J}_h$ are considered to be one dimensional as expressed in Equations (2.13) through (2.18). Substituting Equations (2.13)-(2.20), (2.42) and (2.43) into the fundamental equations (2.3), (2.5)-(2.10) and consider the a-c terms in the z-components of the vector equations, we have the following scalar equations as

$$\frac{\partial H_{1y}}{\partial x} - \frac{\partial H_{1x}}{\partial y} = J_{e1} + J_{h1} + j\omega\varepsilon\, E_{1z} \tag{2.44}$$

$$\frac{\partial J_{e1}}{\partial z} - j\omega e n_1 = 0 \tag{2.45}$$

$$\frac{\partial J_{h1}}{\partial z} + j\omega e p_1 = 0 \tag{2.46}$$

$$(j\omega + \nu_e)v_{e1} + \frac{e}{m_e^*} E_{1z} + u_{e0}\frac{\partial v_{e1}}{\partial z} + \frac{v_{T-}^2}{n_0}\frac{\partial n_1}{\partial z} = 0 \tag{2.47}$$

$$(j\omega + \nu_h)v_{h1} - \frac{e}{m_h^*} E_{1z} + u_{h0}\frac{\partial v_{h1}}{\partial z} + \frac{v_{T+}^2}{p_0}\frac{\partial p_1}{\partial z} = 0 \tag{2.48}$$

$$J_{e1} = -e(n_0 v_{e1} + n_1 u_{e0}) \tag{2.49}$$

$$J_{h1} = e(p_0 v_{h1} + p_1 u_{h0}) \tag{2.50}$$

In order to develop an equivalent transmission-line circuit for the system, the following kinetic voltages and current density are defined

$$V_{e1} = -\frac{m_e^*}{e} u_{e0} v_{e1} \tag{2.51}$$

$$V_{h1} = \frac{m_h^*}{e} u_{h0} v_{h1} \tag{2.52}$$

$$V_{T-} = -\frac{m_e^*}{e} \frac{n_1}{n_0} v_{T-}^2 \tag{2.53}$$

$$V_{T+} = \frac{m_h^*}{e} \frac{p_1}{p_0} v_{T+}^2 \tag{2.54}$$

$$\vec{J}_{m1} = \nabla \times \vec{H}_1 \tag{2.55}$$

V_{e1} and V_{h1} are the kinetic voltages corresponding to the velocity modulation of electrons and holes, respectively. V_{T-} and V_{T+} are the kinetic voltages corresponding to the thermal diffusion of electrons and holes, respectively. $\vec{J}_{m1}$ are the total equivalent a-c current density due to the electromagnetic field of the slow wave circuit which can also be expressed as

$$\vec{J}_{m1} = \vec{J}_{c1} + j\omega\varepsilon \vec{E}_1 \tag{2.56}$$

where $\vec{J}_{c1} = \vec{J}_{e1} + \vec{J}_{h1}$ is the conduction current density in the solid and $j\omega\varepsilon E_1$ is the displacement current density in the solid. Here, both the space-charge wave and the electromagnetic wave are taken into consideration, the conduction current density and the electric field intensity can be expressed as

$$\vec{J}_{c1} = (\vec{J}_{c1})_{SC} + (\vec{J}_{c1})_{EM} \tag{2.57}$$

$$\vec{E}_1 = (\vec{E}_1)_{SC} + (\vec{E}_1)_{EM} \tag{2.58}$$

where the subscript "SC" and "EM" refer to the effect due to the space-charge wave and the electromagnetic wave respectively. From Equation (2.22) we have

$$(\vec{J}_{e1})_{SC} + j\omega\epsilon(\vec{E}_1)_{SC} = 0 \tag{2.59}$$

Using Equations (2.57) through (2.59), Equation (2.56) becomes

$$\vec{J}_{m1} = (\vec{J}_{c1})_{EM} + j\omega(\vec{E}_1)_{EM} \tag{2.60}$$

The above expression gives a clearer physical picture for $\vec{J}_{m1}$ which is composed of both the conduction and displacement current densities due to the electromagnetic field of the slow wave circuit.

Using these four kinetic voltages and the equivalent current density defined above, Equations (2.44) through (2.50) can be rewritten in the following form

$$-j\omega\epsilon\, E_{1z} = J_{e1} + J_{h1} - J_{m1z} \tag{2.61}$$

$$\frac{\partial J_{e1}}{\partial z} = -j\omega\, \frac{\epsilon\omega_{p-}^2}{v_{T-}^2}\, V_{T-} \tag{2.62}$$

$$\frac{\partial J_{h1}}{\partial z} = -j\omega\, \frac{\epsilon\omega_{p+}^2}{v_{T+}^2}\, V_{T+} \tag{2.63}$$

$$(j\beta_e + \frac{\nu_e}{u_{e0}})V_{e1} - E_{1z} + \frac{\partial V_{e1}}{\partial z} + \frac{\partial V_{T-}}{\partial z} = 0 \tag{2.64}$$

$$(j\beta_h + \frac{\nu_h}{u_{h0}})V_{h1} - E_{1z} + \frac{\partial V_{h1}}{\partial z} + \frac{\partial V_{T+}}{\partial z} = 0 \tag{2.65}$$

$$J_{e1} = \frac{\epsilon\omega_{p-}^2}{u_{e0}}\, [V_{e1} + \frac{V_{T-}}{k_{T-}^2}] \tag{2.66}$$

$$J_{h1} = \frac{\epsilon\omega_{p+}^2}{u_{h0}}[V_{h1} + \frac{V_{T+}}{k_{T+}^2}] \tag{2.67}$$

where J_{m1z} are the z-component of $\vec{J}_{m1}$.

In order to develop an equivalent transmission-line equation in terms of the kinetic voltages of velocity modulation and the current density, V_{T-} can be put in terms of V_{e1} and J_{e1} from Equation (2.66)

$$V_{T-} = k_{T-}^2[-V_{e1} + \frac{u_{e0}}{\epsilon\omega_p^2} J_{e1}] \tag{2.68}$$

Substituting the V_{T-} obtained above into Equations (2.62) and (2.64) yields

$$\frac{\partial J_{e1}}{\partial z} = -j\beta_e J_{e1} + j\omega\epsilon \frac{\omega_{p-}^2}{u_{e0}^2} V_{e1} \tag{2.69}$$

$$\frac{\partial V_{e1}}{\partial z} = -j\beta_e V_{e1} - \frac{1}{1-k_{T-}^2}[\frac{\nu_e}{u_{e0}} + j2k_{T-}^2\beta_e]V_{e1} + j\omega \frac{k_{T-}^2}{1-k_T^2} \frac{1}{\epsilon\omega_{p-}^2} J_{e1} + \frac{E_{1z}}{1-k_{T-}^2} \tag{2.70}$$

For the sake of simplifying these equations, the following simple changes of variables are made:

$$J'_{e1} = J_{e1}e^{j\beta_e z} \tag{2.71}$$

$$V'_{e1} = (1-k_{T-}^2)V_{e1}e^{j\beta_e z} \tag{2.72}$$

$$E'_{1z} = E_{1z}e^{j\beta_e z} \tag{2.73}$$

Equations (2.69) and (2.70) are then reduced to:

$$\frac{\partial J'_{e1}}{\partial z} = - Y_{e1} V'_{e1} \tag{2.74}$$

$$\frac{\partial V'_{e1}}{\partial z} = -Z_{e1} J'_{e1} - A_{e1} V'_{e1} + E'_{1z} \tag{2.75}$$

where

$$Y_{e1} = -j\omega\epsilon \frac{\omega_{p-}^2}{u_{e0}^2} \frac{1}{1-k_{T-}^2} \tag{2.76}$$

$$Z_{e1} = -j\omega \frac{k_{T-}^2}{\epsilon\omega_{p-}^2} \tag{2.77}$$

$$A_{e1} = \frac{1}{1-k_{T-}^2} [\frac{\nu_e}{u_{e0}} + j\omega \frac{2k_{T-}^2}{u_{e0}}] \tag{2.78}$$

Starting from Equations (2.63), (2.65) and (2.67), and following a similar procedure, one obtains

$$\frac{\partial J'_{h1}}{\partial z} = -Y_h V'_{h1} \tag{2.79}$$

$$\frac{\partial V'_{h1}}{\partial z} = -Z_{h1} J'_{h1} - A_{h1} V'_{h1} + E'_1 \tag{2.80}$$

$$Y_{h1} = \frac{-j\omega\epsilon}{1-k_{T+}^2} \frac{\omega_{p+}^2}{u_{e0}^2} \tag{2.81}$$

$$Z_{h1} = -j\omega \frac{k_{T-}^2}{\epsilon\omega_{p-}^2} \tag{2.82}$$

$$A_{h1} = \frac{1}{1-k_{T+}^2} [\frac{\nu_h}{u_{h0}} + j\omega \frac{2k_+^2}{u_{h0}}] \tag{2.83}$$

The relationship between the electron and hole waves can be found from Equation (2.61) as

$$J_{e1} + J_{h1} - J_{m1z} = -j\omega\epsilon\, E_{1z} \tag{2.84}$$

From Equations (2.74) through (2.84), an equivalent transmission-line for carrier waves in solids is obtained and shown in Fig. 2.1. The elements of this equivalent line include capacitors, inductors, ideal transformers and two common base transistors with zero internal resistance and complex amplification factors. When $\nu \gg 2k_T^2\omega$ which is always true[10] for the case of semiconductors, the amplification factors per unit length reduce to

$$A_{e1} = \frac{1}{1-k_{T-}^2}\frac{\nu_e}{u_{e0}} \tag{2.85}$$

$$A_{h1} = \frac{1}{1-k_{T+}^2}\frac{\nu_h}{u_{h0}} \tag{2.86}$$

Since the two transistors are the only non-reactive elements in the equivalent transmission-line, the real power dissipated or created in the line per unit volume is given by

$$\begin{aligned} P_{e1} &= \frac{1}{2}\,\mathrm{Re}[A_e V'_{e1} J'^*_{e1} + A_h V'_{h1} J'^*_{h1}] \\ &= \frac{1}{2}\,\mathrm{Re}[\frac{\nu_e}{u_{e0}} V_{e1} J^*_{e1} + \frac{\nu_h}{u_{h0}} V_{h1} J^*_{h1}] \end{aligned} \tag{2.87}$$

The above expression may also be considered as the total real a-c kinetic power loss or gain in solids per unit volume. This result will be proved in Chapter IV from the equation of real power flow directly.

There is an alternative way of interpreting the equivalent transmission-line for the carrier waves in solids; that is, instead of presenting the equivalent-line in terms of the kinetic voltages due to velocity modulation and the a-c current densities, one can also develop an equivalent line in terms of the kinetic voltages due to thermal diffusion and the a-c current densities. For this purpose, expressing V_{e1} by V_{T-} and J_{e1} from Equation (2.66) and substituting the result in Equation (2.64) one obtains

$$\frac{\partial V'_{T-}}{\partial z} = -Z_{eT}J_{e1} - A_{eT}V'_{T-} + E_1 \qquad (2.88)$$

where

$$V'_{T-} = (1 - \frac{1}{k^2_{T-}})V_{T-} \qquad (2.89)$$

$$Z_{eT} = \frac{1}{\epsilon\omega^2_{p-}}(\nu_e + j\omega) = R_{eT} + jX_{eT} \qquad (2.90)$$

$$A_{eT} = \frac{1}{1-k^2_T}[\frac{\nu_e}{u_{e0}} + j2\beta_e] \qquad (2.91)$$

Rewriting Equation (2.62) we have

$$\frac{\partial J_{e1}}{\partial z} = -Y_{eT}V'_{T-} \qquad (2.92)$$

where

$$Y_{eT} = j\omega\frac{\epsilon\omega^2_{p-}}{v^2_{T-} - u^2_{e0}} \qquad (2.93)$$

Using Equations (2.63), (2.65) and (2.67) and following a similar procedure, the equivalent transmission-line equations for holes are obtained as

$$\frac{\partial V'_{T+}}{\partial z} = -Z_{hT}J_{h1} - A_{hT}V'_{T+} + E_1 \qquad (2.94)$$

$$\frac{\partial J_{h1}}{\partial z} = -Y_{hT}V'_{T+} \qquad (2.95)$$

where

$$V'_{T+} = (1 - \frac{1}{k_{T+}^2})V_{T+} \tag{2.96}$$

$$Z_{hT} = \frac{1}{\epsilon\omega_{p+}^2}(\nu_h + j\omega) = R_{hT} + jX_{hT} \tag{2.97}$$

$$Y_{hT} = j\omega \frac{\epsilon\omega_{p+}^2}{v_{T+}^2 - u_{h0}^2} \tag{2.98}$$

$$A_{hT} = \frac{1}{1-k_{T+}^2}[\frac{\nu_h}{u_{h0}} + j2\beta_h] \tag{2.99}$$

Now, from Equations (2.88) through (2.99) together with Equation (2.84), we have another form of the equivalent line analog in terms of V_{T-}, V_{T+}, J_{e1} and J_{h1} as shown in Fig. 2.2. In this case, R_{eT} and R_{hT} can be considered as the internal resistances of the two common base transistors, respectively. For the case of semiconductors in room temperature, the collision frequencies which usually have the order of 10^{12} cps are much higher than the operation frequency or $\nu \gg \omega$, therefore the amplification factors per unit length reduce to

$$A_{eT} = \frac{1}{1-k_{T-}^2}\frac{\nu_e}{u_{e0}} \tag{2.100}$$

$$A_{hT} = \frac{1}{1-k_{T+}^2}\frac{\nu_h}{u_{h0}} \tag{2.101}$$

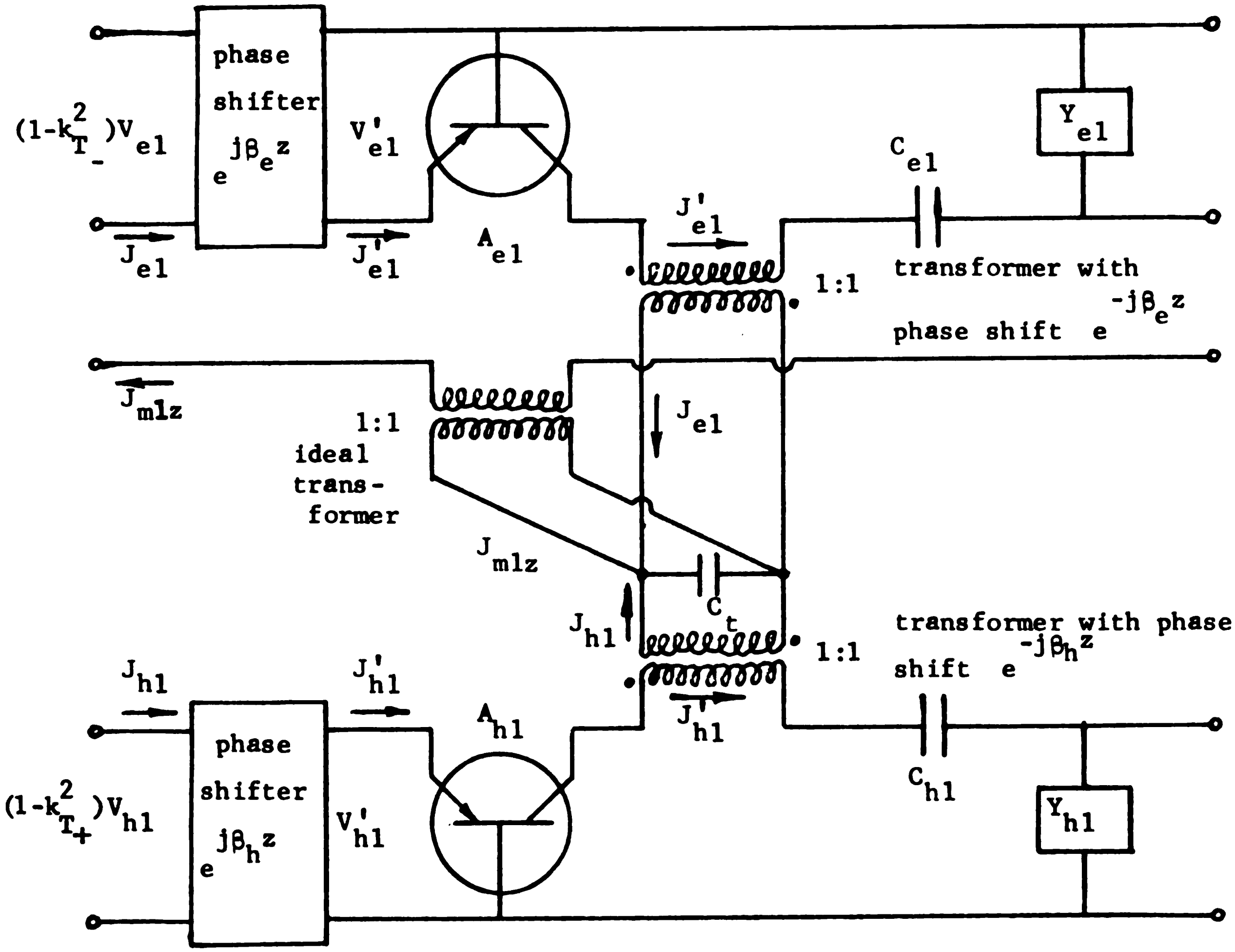
Similar to the previous case, the total real a-c power loss or gain in this equivalent line per unit volume can be evaluated by

$$P_{eT} = \frac{1}{2}\,\mathrm{Re}[A_{eT}V'_{T-}J_{e1}^* + R_{eT}J_{e1}J_{e1}^* + A_{hT}V'_{T+}J_{h1}^* + R_{hT}J_{h1}J_{h1}^*] \tag{2.102}$$

Using the Equations (2.89), (2.96), (2.100), (2.101), (2.66) and (2.67), the above equation can be simplified as

$$P_{eT} = \frac{1}{2} \mathrm{Re}\left[\frac{\nu_e}{u_{e0}} V_{e1} J_{e1}^* + \frac{\nu_h}{u_{h0}} V_{h1} J_{h1}^*\right] \qquad (2.103)$$

which is checked with both Equation (2.87) and the result obtained from the kinetic energy theorem developed in Chapter IV.

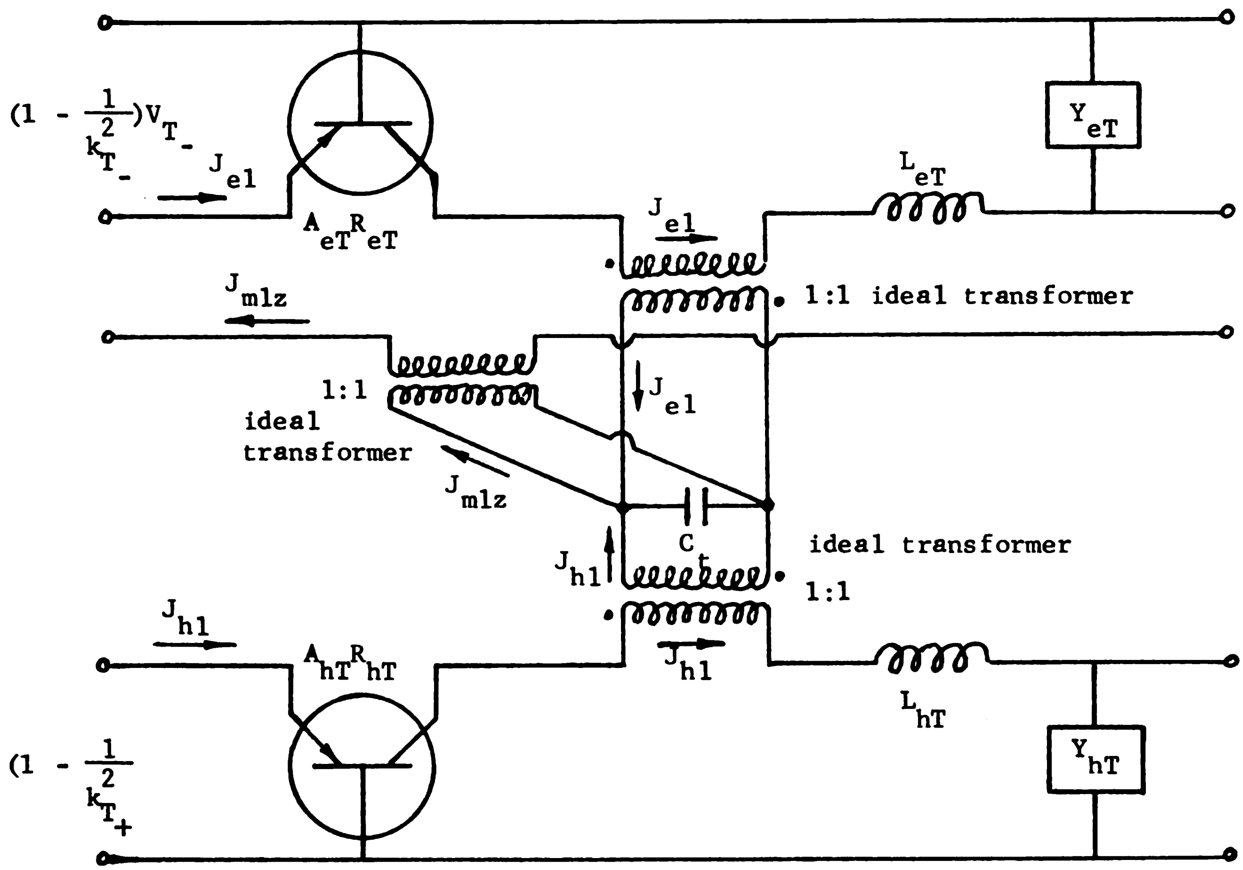


$$Y_{e1} = j\omega\epsilon \frac{\omega_{p-}^2}{u_{e0}^2} \frac{1}{1-k_{T_-}^2} \qquad C_{e1} = \epsilon\left(\frac{\omega_{p-}}{\omega k_{T_-}}\right)^2 \qquad A_{e1} = \frac{1}{1-k_{T_-}^2} \frac{1}{u_{e0}} [\nu_e + j2\omega k_{T_-}^2]$$

$$Y_{h1} = j\omega\epsilon \frac{\omega_{p+}^2}{u_{h0}^2} \frac{1}{1-k_{T_+}^2} \qquad C_{h1} = \epsilon\left(\frac{\omega_{p+}}{\omega k_{T_+}}\right)^2 \qquad A_{h1} = \frac{1}{1-k_{T_+}^2} \frac{1}{u_{h0}} [\nu_h + j2\omega k_{T_+}^2]$$

$$C_t = \epsilon$$

Fig. 2.1 Transmission Line Analog for the Carrier Waves in Solids Including the Effect of the External Slow Wave Circuit, Using the Current Densities J_{e1} and J_{h1} and the Kinetic Voltages V_{e1} and V_{h1}.



$$Y_{eT} = j\omega\varepsilon \frac{\omega_{p-}^2}{u_{e0}^2} \frac{1}{1-k_{T_-}^2} \qquad L_{eT} = \frac{1}{\varepsilon\omega_{p-}^2} \qquad A_{eT} = \frac{1}{1-k_{T_-}^2} \frac{1}{u_{e0}} [\nu_e + j2\omega]$$

$$Y_{hT} = j\omega\varepsilon \frac{\omega_{p+}^2}{u_{h0}^2} \frac{1}{1-k_{T_+}^2} \qquad L_{hT} = \frac{1}{\varepsilon\omega_{p+}^2} \qquad A_{hT} = \frac{1}{1-k_{T_+}^2} \frac{1}{u_{h0}} [\nu_h + j2\omega]$$

$$R_{eT} = \frac{\nu_e}{\varepsilon\omega_{p-}^2} \qquad R_{hT} = \frac{\nu_h}{\varepsilon\omega_{p+}^2} \qquad C_t = \varepsilon$$

Fig. 2.2 Transmission Line Analog for the Carrier Waves in Solids including the Effect of the External Slow Wave Circuit, Using the Current Densities J_{e1} and J_{h1} and the Kinetic Voltages V_{T_-} and V_{T_+}.

CHAPTER III

DISPERSION RELATIONS OF THE CARRIER WAVES IN AN EXTRINSIC SEMICONDUCTOR

3.1 Introduction

For any wave there is a functional relationship between the propagation constant and the operating frequency which is known as the dispersion relation. The instability as well as the nature and the propagation characteristics of waves can be examined from its dispersion relation. For carrier waves in solids, the dispersion relations strongly depend on the longitudinal modulation, thermal diffusion and the collision between the carriers and the solid lattice.

In general, the propagation constant of the longitudinal carrier waves in solids can be solved from Equation (2.36). However, instead of solving this fourth order equation for Γ, the specific propagation constant for an extrinsic semiconductor (in which $n \gg p$ or $p \gg n$) indicated in Chapter II will be investigated. From Equations (2.37) and (2.38), the real and imaginary parts of the propagation constant for longitudinal carrier waves in an extrinsic semiconductor can be expressed as

$$\alpha_{i\pm} = -\frac{\omega}{u_{0i}} \frac{1}{1-k_{Ti}^2} [\frac{\nu_i}{2\omega} \mp \alpha_i'] \tag{3.1}$$

$$\beta_{i\pm} = \frac{\omega}{u_{0i}} \frac{1}{1-k_{Ti}^2} [1 \mp \beta_i'] \tag{3.2}$$

where

$$\alpha_i'^2 - \beta_i'^2 = \frac{\nu_i}{4\omega^2} - \frac{\omega_{pi}^2}{\omega^2}(1 - k_{Ti}^2) - k_{Ti}^2 \tag{3.3}$$

$$\alpha_i' \beta_i' = k_{Ti}^2 \frac{\nu_i}{2\omega} \tag{3.4}$$

where the subscript "i" can be replaced by "e" or "h" to denote the electron of hole waves. However, this subscript will be dropped in later discussion for simplicity.

Taking the thermal-to-drift velocity ratio k_T and the collision-to-plasma frequency ratio ν/ω_p as parameters, various dispersion diagrams under different conditions are plotted in this Chapter. The effects of longitudinal modulation, thermal diffusion and collision to the wave characteristics are studied and discussed in view of these dispersion diagrams.

3.2 Equivalence of an Electron Beam in Vacuum

When the devices are operating at low power level with relatively high drift potential and extremely low temperature, both the thermal diffusion and collision effects can be neglected. This situation is similar to the electron stream in vacuum and the expression for attenuation and phase constants of Equations (3.1) and (3.2) reduce to

$$\alpha_{\pm} = 0 \tag{3.5}$$

$$\beta_{\pm} = \frac{1}{u_0}(\omega \mp \omega_p) \tag{3.6}$$

The group and phase velocities of these waves are

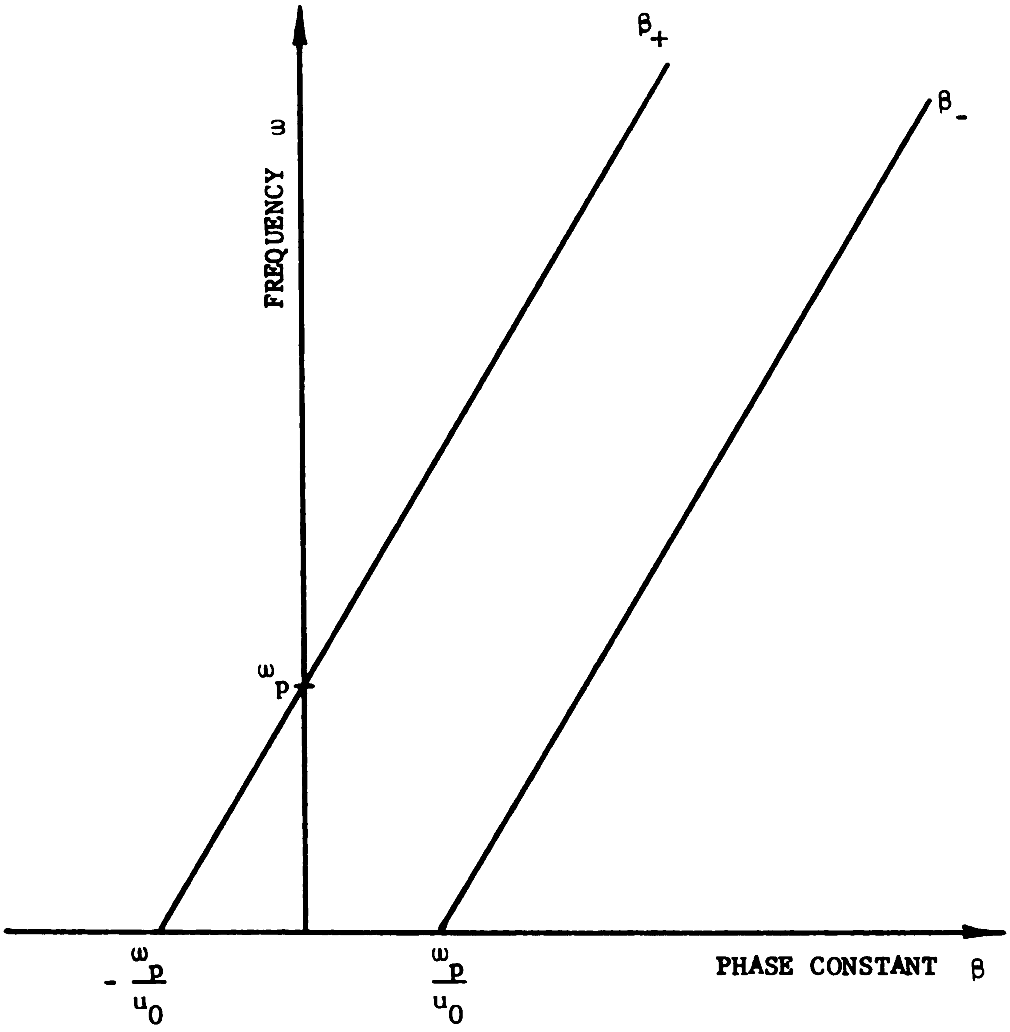


Fig. 3.1 Dispersion Diagram for the Longitudinal Carrier Waves in an Extrinsic Semiconductor with Thermal Diffusion and Collision Effects Neglected.

$$v_{g\pm} = \frac{d\omega}{d\beta_\pm} = u_0 \tag{3.7}$$

$$v_{p\pm} = \frac{\omega}{\beta_\pm} = \frac{u_0}{1 \mp \omega_p/\omega} \tag{3.8}$$

It can readily be seen that the propagating properties of these carrier waves are identical to the space-charge waves in vacuum. Hence, similar to a modulated electron beam in vacuum, the electron or hole bunching process in an extrinsic semiconductor can be analyzed by the concept of fast and slow space-charge waves whose phase velocities are faster and slower than the average drift velocity of the carriers respectively. Since the attenuation constant is zero, the fast and slow carrier waves will propagate along the extrinsic semiconductor with a constant amplitude provided there is no interaction with external circuit waves.

3.3 Cold Carrier Stream with Collision Effect

When the collision effect is taken into account while thermal diffusion is neglected, Equations (3.1) through (3.4) become

$$\alpha_\pm = -\frac{\omega}{u_0}\left[\frac{\nu}{2\omega} \mp \alpha'\right] \tag{3.9}$$

$$\beta_\pm = \frac{\omega}{u_0}(1 \mp \beta') \tag{3.10}$$

$$\alpha'^2 - \beta'^2 = -\frac{\omega_p^2}{\omega^2} + \frac{\nu^2}{4\omega^2} \tag{3.11}$$

$$\alpha'\beta' = 0 \tag{3.12}$$

Since both α' and β' are real quantities, two sets of solutions of α' and β' from Equations (3.11) and (3.12) can therefore be obtained depending upon the relative magnitude between the plasma frequency and the collision frequency. The propagation characteristics of the carrier waves will differ accordingly. Two general cases will be discussed as follows:

3.3.1 Slight Collision Case

If $\nu < 2\omega p$, the solution of α' and β' becomes

$$\alpha' = 0 \tag{3.13}$$

$$\beta' = \sqrt{\frac{\omega_p^2}{\omega^2} - \frac{\nu^2}{4\omega^2}} \tag{3.14}$$

Substituting these values into Equations (3.1) and (3.2), we have

$$\alpha_{\pm} = -\frac{\nu}{2u_0} \tag{3.15}$$

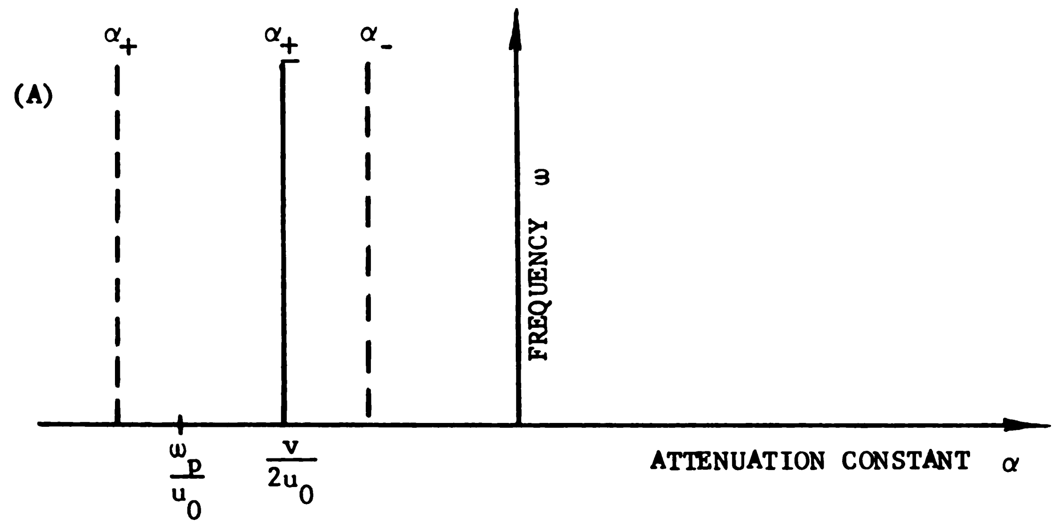
$$\beta_{\pm} = \frac{\omega}{u_0} \mp \frac{\omega_p}{u_0}\sqrt{1 - \left(\frac{\nu}{2\omega_p}\right)^2} \tag{3.16}$$

and the phase velocities become

$$v_{p\pm} = \frac{u_0}{1 \mp \frac{\omega_p}{\omega}\sqrt{1 - \left(\frac{\nu}{2\omega_p}\right)^2}} \tag{3.17}$$

The ω-α and ω-β diagrams for this case are shown in Fig. 3.2.A and Fig. 3.2.B respectively.

If one compares the propagation constants and phase velocities shown in Equations (3.15) through (3.17) with those of the cold, collisionless electron stream obtained in Section 3.2, one observes that the fast and slow space-charge wave



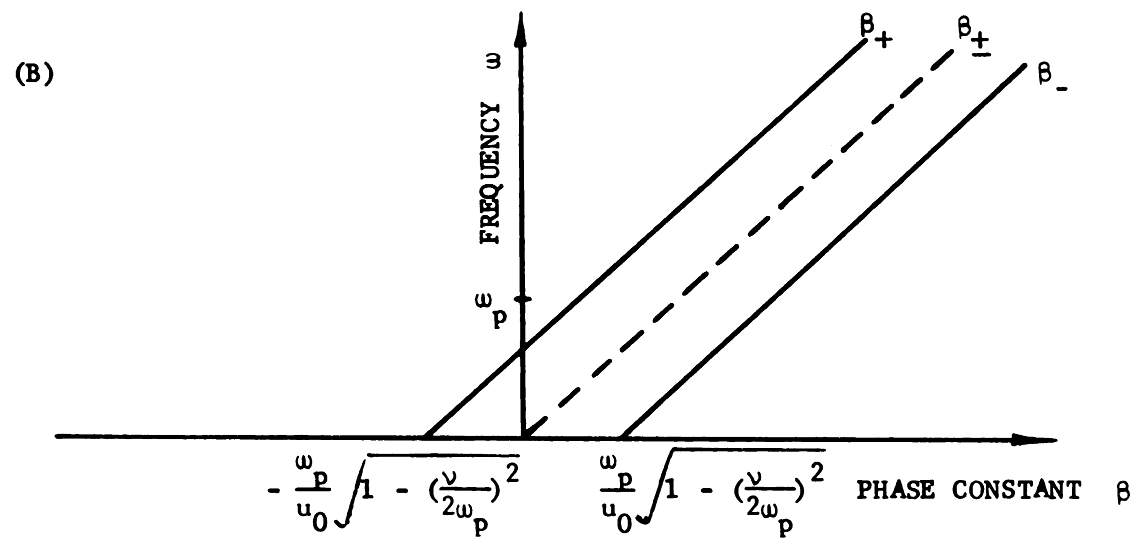


Fig. 3.2 Dispersion Diagrams for the Cold Longitudinal Carrier Waves in an Extrinsic Semiconductor with Collision

(A) ω vs α Plot

(B) ω vs β Plot

where the Solid Lines Refer to the Carrier Waves with $\nu < 2\omega_p$ and the Dashed Lines Refer to the Carrier Waves with $\nu > 2\omega_p$.

characteristics is still unchanged when the collision of the carriers is less frequent. However, the phase velocities of the fast and slow carrier waves approach each other due to collision. This phenomenon can be explained as follows: The collision between the drift carriers and solid lattice will disturb the drift velocities of the carriers. The result of the collision makes the carrier return to a random state such that the velocity modulation will be decreased. When the collision frequency is increased to $\nu = 2\omega_p$, both the phase velocities of the fast and slow carrier waves will approach the d-c drift velocity. Furthermore, the carrier waves will no longer be lossless in the presence of a slight collision. An attenuation constant is used to represent the collision effect. It is shown in Equation (3.15) that both the fast and slow waves will attenuate at the same rate which is directly proportional to the collision frequency.

3.3.2 Collision Dominated Cases

If $\nu > 2\omega_p$, the solution of α' and β' become

$$\alpha' = \frac{\nu^2}{4\omega^2} - \frac{\omega_p^2}{\omega^2} \tag{3.18}$$

$$\beta' = 0 \tag{3.19}$$

Using Equations (3.18) and (3.19) the attenuation and phase constants can be found as

$$\alpha_{\pm} = - \frac{\nu}{2u_0}\left[1 \mp \sqrt{1 - \left(\frac{2\omega_p}{\nu}\right)^2}\right] \tag{3.20}$$

$$\beta_{\pm} = \frac{\omega}{u_0} \tag{3.21}$$

The phase velocities of the carrier waves can be determined by Equation (3.21) and are found to be a constant which is equal to the d-c drift velocity. Physically, we can explain it in the following way: The fact that the velocities of the carriers after colliding with the heavy particles in solids are completely random, will deteriorate the bunching effect. Consequently, the phase velocities of the fast and slow waves will emerge to the d-c drift velocity as the collision frequency increases. When the collision frequency is high enough, the velocities of the carriers will become quite random in a short drifting range and thus no bunching effect will occur within the extrinsic semiconductor. In such a case, the carrier waves will propagate with the same velocity which is equal to the average drift velocity of the carriers. Since the electrons will give up some of their a-c energy by collision, the attenuation constant is directly proportional to the collision frequency in a slight collision case. It seems that there will be two distinct attenuation constants in the collision dominated case as indicated in Equation (3.20). For a fixed collision frequency, the attenuation for the fast wave increases with the ratio of collision-to-plasma frequency while that of the slow wave decreases with ν/ω_p according to Equation (3.20). It was pointed out by Vural and Bloom[26] that in the completely collision dominated limit, where $\frac{\omega}{\nu} \ll 1$, the "slow wave" seems to become lossless. We will look at this situation from another point of view. We have shown that the fast and slow waves emerge with the same phase and group velocities when $\nu > 2\omega_p$, a single attenuation constant for the carrier waves can be found by taking the arithmetic mean of the two waves. The result is identical to that given by Equation (3.15). Therefore, we may conclude that the attenuation constant is only a function of collision frequency for the cold carrier waves.

3.4 The Collisionless Carrier Waves

When the collision effect between the carriers and the solid lattice is neglected, the expression for attenuation and phase constants become

$$\alpha_{\pm} = 0 \tag{3.22}$$

$$\beta_{\pm} = \frac{\omega}{u_0} \frac{1}{1-k_{T-}^2} \left[1 \mp \sqrt{k_T^2 + \frac{\omega_p^2}{\omega^2} (1-K_T^2)} \right] \tag{3.23}$$

It can be figured out from the above equations when the average velocity of the carriers is smaller than its thermal velocity, i.e. $u_0 < v_T$, no lossless longitudinal carrier waves are excited if the operation frequency is sufficiently low. The cut off frequency ω_0 can be evaluated by letting the square root term of Equation (3.23) equal zero. The result is

$$\omega_0 = \omega_p \sqrt{1 - (\frac{u_0}{v_T})^2} \tag{3.24}$$

When the d-c drift velocity of the carriers is considerably lower than the thermal velocity, the cut-off frequency approaches the plasma frequency of the carriers by Equation (3.24).

According to Equation (3.23), there are lossless carrier waves excited even at a low operating frequency when the average drift velocity of the carrier is higher than its thermal velocity. The phase constant at the low frequency near zero can be obtained from Equation (3.23) as

$$\beta_{0\pm} = \mp \frac{\omega_p}{u_0} \frac{1}{1-k_T^2} \tag{3.25}$$

When the system is operated at high frequency such that $\omega \gg \omega_p$ Equation (3.23) reduces to

$$\beta_\pm = \frac{\omega}{u_0 \pm v_T} \tag{3.26}$$

In this case, two kind of the lossless carrier wave exist in the extrinsic semiconductor; one is the fast wave with both group and phase velocities larger than the d-c drift velocity of the carriers and the other is slow wave with both group and phase velocities smaller than the d-c drift velocity of the carriers. Note when $u_0 < v_T$, the slow wave will propagate in the backward direction.

The general sketch of the dispersion diagrams of the collisionless case is shown in Fig. 3.3 and a computer plot for the collisionless longitudinal carrier waves in an extrinsic semiconductor with various thermal-to-drift velocity ratio is shown in Fig. 3.4. In case of $u_0 > v_T$, it can be seen that the slow wave in an extrinsic semiconductor will become a backward wave when the operating frequency is lower than the plasma frequency of the carriers. It can also be seen when $u_0 \gg v_T$, the dispersion diagram of the collisionless carrier waves approaches that of the electron beam equivalence case which has been shown

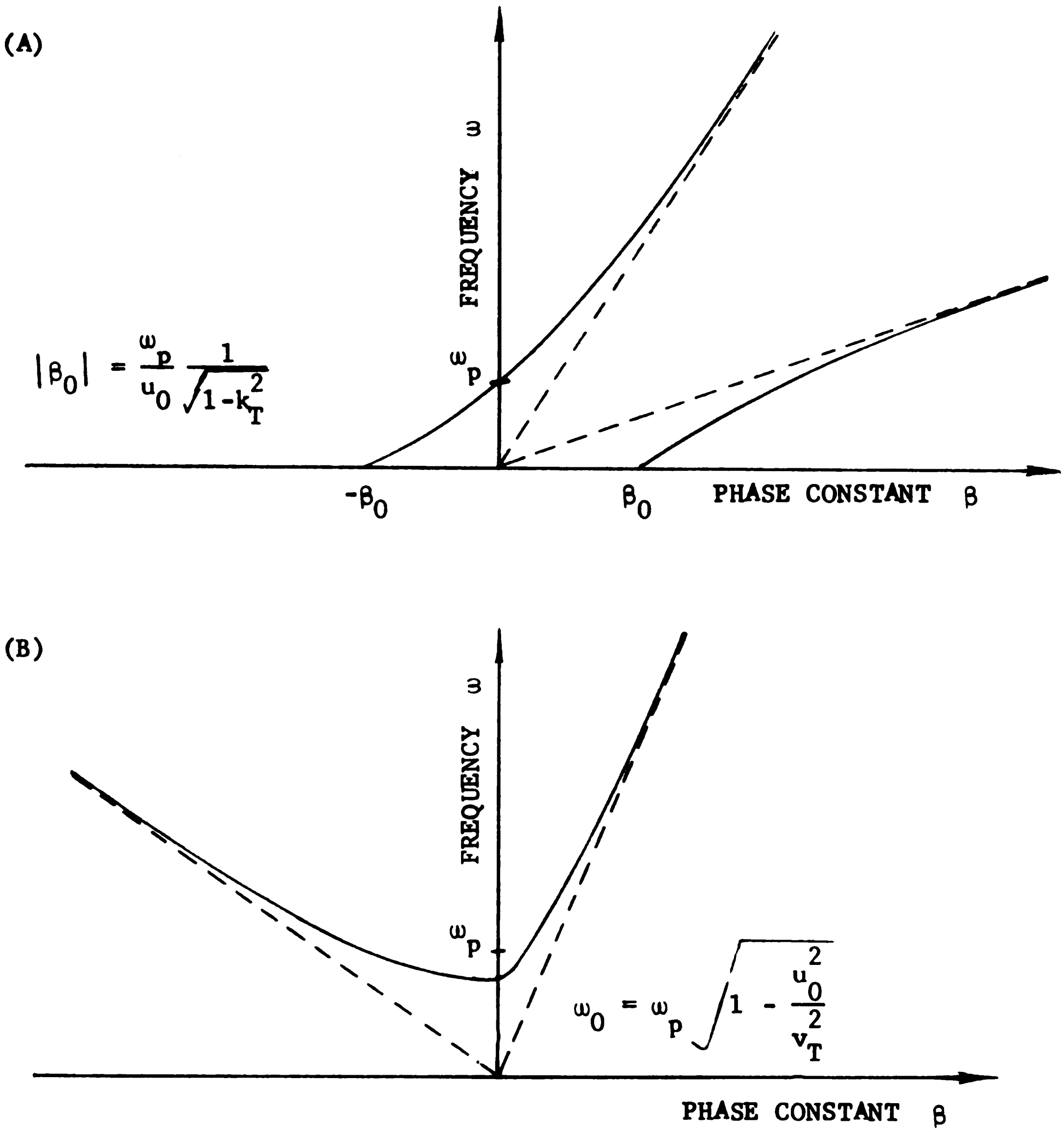


Fig. 3.3 Dispersion Diagrams for the Collisionless Longitudinal Carrier Waves in an Extrinsic Semiconductor with Thermal Diffusion under Consideration

(A) $u_0 > v_T$ (B) $u_0 < v_T$

where the Dashed Lines Refer to the Asymptotes of the Dispersion Diagrams at High Operating Frequency.

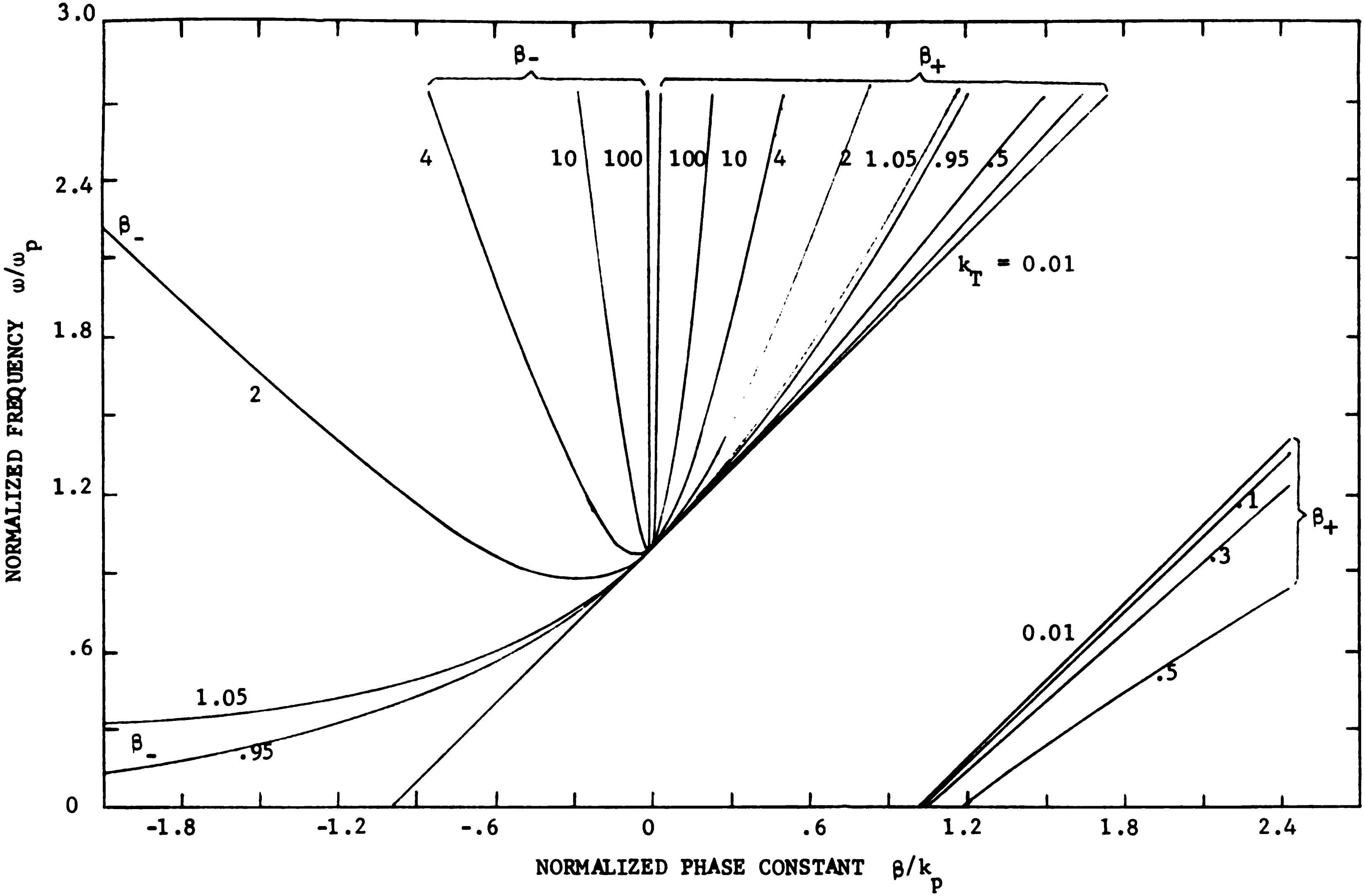


Fig. 3.4 $\omega - \beta$ Diagrams for the Collisionless Longitudinal Carrier Waves in an Extrinsic Semiconductor with Thermal-to-Drift Velocity Ratio as Parameters. Here $k_p = \omega/u_0$.

in Fig. 3.1; on the other hand, when $u_0 \ll v_T$, the dispersion diagram of the collisionless carrier waves approaches that of the electroacoustic waves which will be discussed in the next section.

3.5 The Electroacoustic Waves

When there is no axial d-c electric field applied across the solids, the average drift velocity of the carrier is zero. In such a case the carrier waves are affected by the a-c modulation only and are called electroacoustic waves. In this case, it is easier to obtain the propagation constant from the wave equation directly.

At $u_0 = 0$, the a-c carrier wave equation in an extrinsic semiconductor of (2.37) becomes

$$v_T^2 \frac{\partial^2 n_1}{\partial z^2} + (\omega^2 - \omega_p^2 - j\omega\nu) = 0 \tag{3.27}$$

Using the assumption that the solution of n_1 has a form of $e^{\Gamma z}$ (where $\Gamma = \alpha - j\beta$), the above equation can be degenerated as

$$\alpha^2 - \beta^2 = \frac{\omega_p^2 - \omega^2}{v_T^2} \tag{3.28}$$

$$\alpha\beta = \frac{-\omega\nu}{2v_T^2} \tag{3.29}$$

and the attenuation and phase constants for electroacoustic waves in an extrinsic semiconductor are obtained from Equations (3.28) and (3.29) as

$$\alpha_{\pm} = \begin{cases} \mp \dfrac{\sqrt{\omega_p^2 - \omega^2}}{\sqrt{2}\, v_T} \left[\sqrt{1 + \dfrac{\omega^2 \nu^2}{(\omega^2 - \omega_p^2)^2}} + 1 \right]^{\frac{1}{2}} & \omega < \omega_p \\ \mp \dfrac{\sqrt{\omega^2 - \omega_p^2}}{\sqrt{2}\, v_T} \left[\sqrt{1 + \dfrac{\omega^2 \nu^2}{(\omega^2 - \omega_p^2)^2}} - 1 \right]^{\frac{1}{2}} & \omega > \omega_p \end{cases} \tag{3.30}$$

$$\beta_{\pm} = \begin{cases} \pm \dfrac{\sqrt{\omega_p^2 - \omega^2}}{\sqrt{2}\, v_T} \left[\sqrt{1 + \dfrac{\omega^2 \nu^2}{(\omega^2 - \omega_p^2)^2}} - 1 \right]^{\frac{1}{2}} & \omega < \omega_p \\ \pm \dfrac{\sqrt{\omega^2 - \omega_p^2}}{\sqrt{2}\, v_T} \left[\sqrt{1 + \dfrac{\omega^2 \nu^2}{(\omega^2 - \omega_p^2)^2}} + 1 \right]^{\frac{1}{2}} & \omega > \omega_p \end{cases} \tag{3.31}$$

When the operating frequency is sufficiently high, i.e. $\omega \gg \omega_p$ and $\omega \gg \nu$, the above equations reduce to

$$\alpha_{\pm} = \mp \frac{\nu}{2v_T} \tag{3.32}$$

$$\beta_{\pm} = \pm \frac{\omega}{v_T} \tag{3.33}$$

On the other hand, when the operating frequency is sufficiently low, i.e. $\omega \ll \omega_p$ and $\omega \ll \nu$, Equations (3.30) and (3.31) become

$$\alpha_{\pm} = \mp \frac{\omega_p}{v_T} \tag{3.34}$$

$$\beta_{\pm} = \pm \frac{\omega\nu}{2v_T\omega_p} \tag{3.35}$$

According to Equation (3.31), two sets of the electroacoustic waves exist in a longitudinal modulated extrinsic semiconductor: one propagates forward and the other propagates backward as shown in Fig. 3.5. The dispersion diagrams of these two waves are symmetrical since no d-c drift potential is applied. Equation (3.33) indicates that the group and phase velocities of these

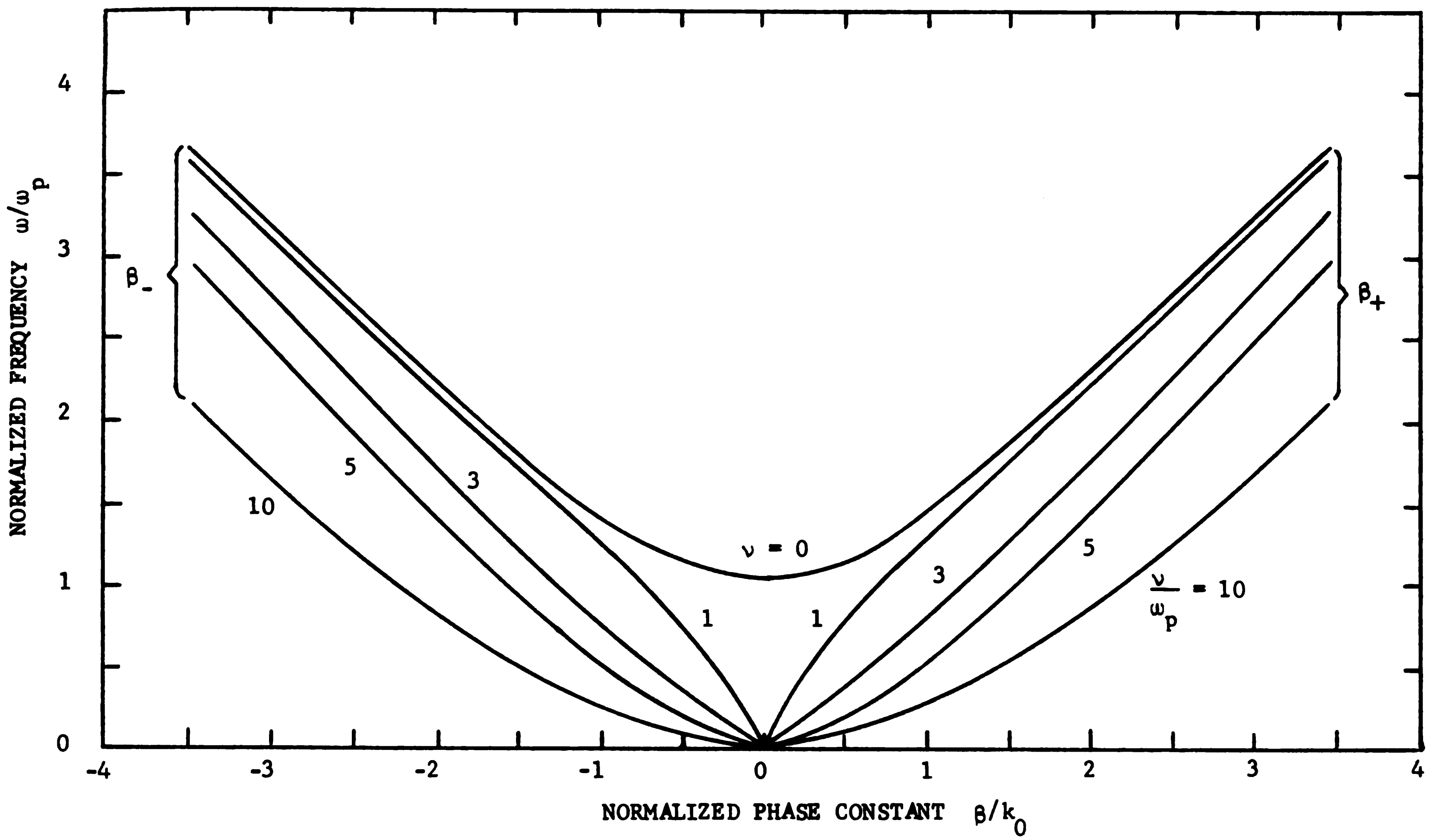


Fig. 3.5 $\omega - \beta$ Diagrams for the Longitudinal Carrier Waves in an Extrinsic Semiconductor Without D-C Drift Voltage Applied. Here $k_0 = \omega_p / v_T$.

waves approach a constant equal to the thermal velocity of the carriers in the +z or -z direction respectively. Note there exists a cut-off frequency which equals the plasma frequency of the carriers if the collision effect between the carriers and the solid lattice is neglected.

The attenuation constant given by Equation (3.30) is a function of the operating frequency, the plasma frequency, the collision frequency and the thermal velocity. However, it is noted by Equation (3.32) when the system is operated at high frequency, the wave attenuation is dominated by the collision effect between the carriers and the solid lattice. On the other hand, when the system is operated at low frequency, the plasma frequency play an important role to the wave attenuation according to Equation (3.34). The ω-α diagrams of the longitudinal carrier waves in an extrinsic semiconductor with only a-c driving source are shown in Fig. 3.6.

3.6 General Case for the Carrier Waves in an Extrinsic Semiconductor

The general dispersion characteristics for the longitudinal carrier waves in an extrinsic semiconductor is given by Equations (3.1) through (3.4). In this section, the dispersion diagrams are plotted from those equations with the aid of a digital computer and the dispersion relations in terms of various parameters are discussed.

Fig. 3.7 shows the ω-β diagram for the carrier waves in an extrinsic semiconductor with the thermal-to-drift velocity ratio as parameter. It has been indicated in the previous sections

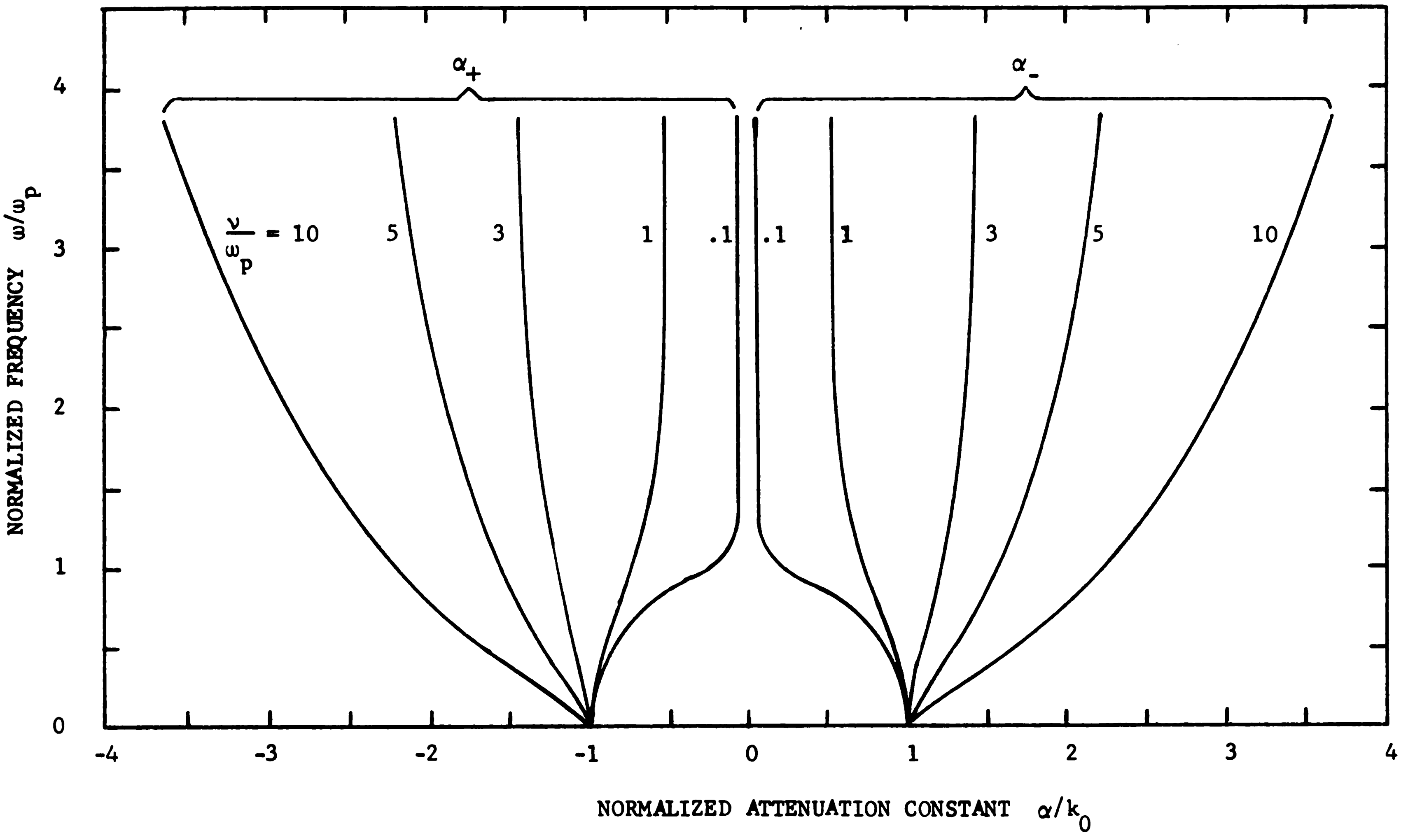


Fig. 3.6 $\omega - \alpha$ Diagrams for the Longitudinal Carrier Waves in an Extrinsic Semiconductor Without D-C Drift Voltage Applied. Here $k_0 = \omega_p/v_T$.

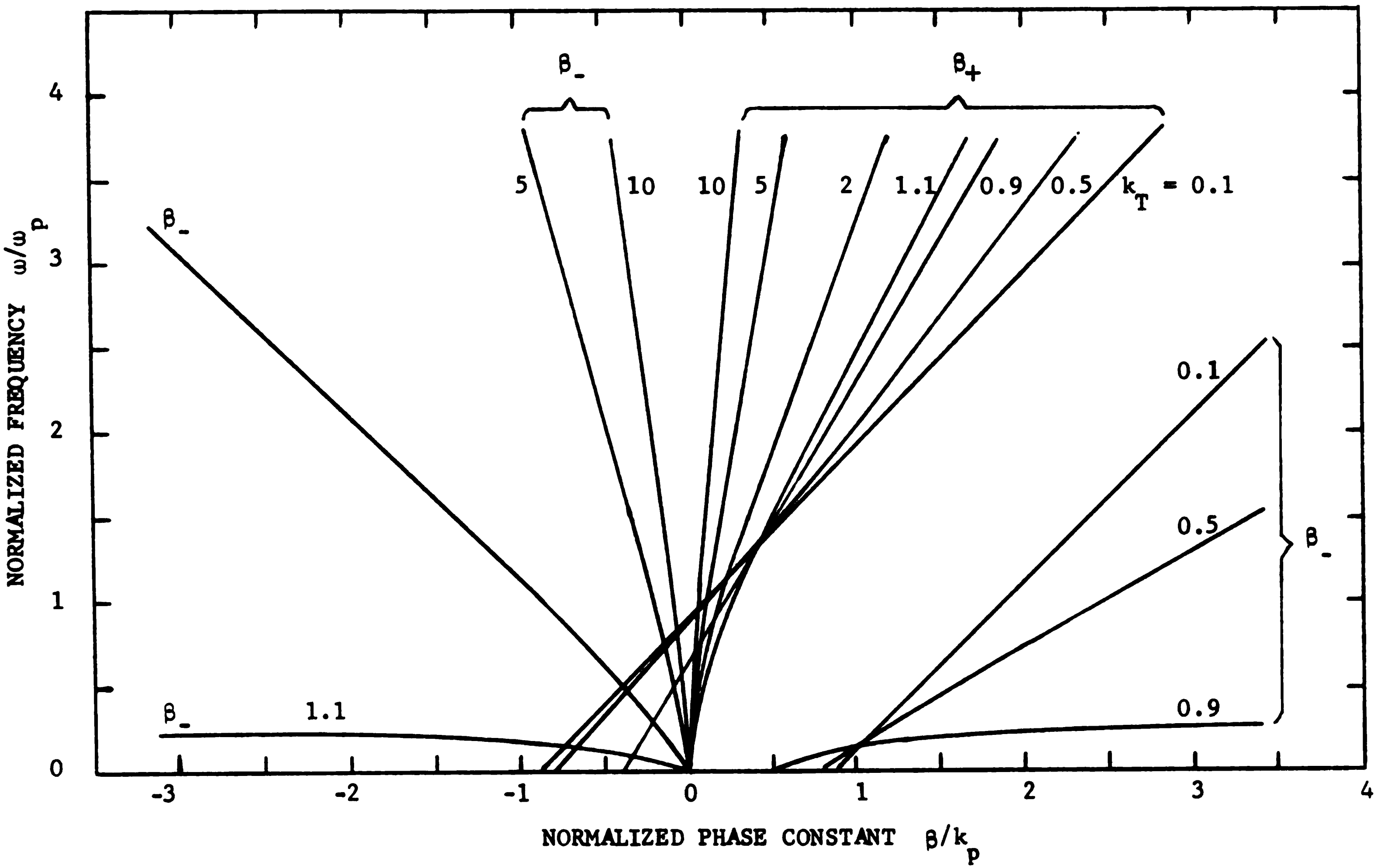


Fig. 3.7 ω - β Diagrams for the Carrier Waves in an Extrinsic Semiconductor With Thermal-to-Drift Velocity Ratio as Parameter for a Fixed Collision Frequency $\nu = \omega_p$. Here $k_p = \omega_p/u_0$.

that the longitudinal carrier waves in an extrinsic semiconductor will approach the space-charge waves when the thermal diffusion is neglected (i.e. $k_T = 0$); and will approach the electroacoustic waves when the average drift velocity of the carriers is zero (i.e. $k_T \to \infty$). Starting with a small thermal-to-drift velocity ratio ($k_T = 0.1$), the carrier waves approach the fast and slow space-charge waves propagated with nearly the same group velocity; when the thermal-to-drift velocity ratio increases, the phase and group velocities of the fast wave will speed up whereas that of the slow wave will slow down from its average drift velocity. The slow wave will propagate in the backward direction in case the average drift velocity of the carrier is smaller than its thermal velocity. When the thermal-to-drift velocity ratio increases further, both the phase and group velocities of the fast and slow waves will increase in the forward and the backward directions respectively; they will approach the thermal velocity of the carriers when k_T is sufficiently large. A general transition from space-charge waves to the electroacoustic waves in an extrinsic semiconductor by varying the thermal-to-drift velocity ratio is clearly shown by the shift of the dispersion diagrams in Fig. 3.7.

The same condition for a ω-α diagram is shown in Fig. 3.8. At first glance, we may find that all the attenuation constants are independent of the operating frequency when $\omega \gg \omega_p$; therefore, a high frequency operation is suggested for propagating carrier waves in solids in order to minimize the distortion. Secondly, it can be seen that there is less energy loss when the

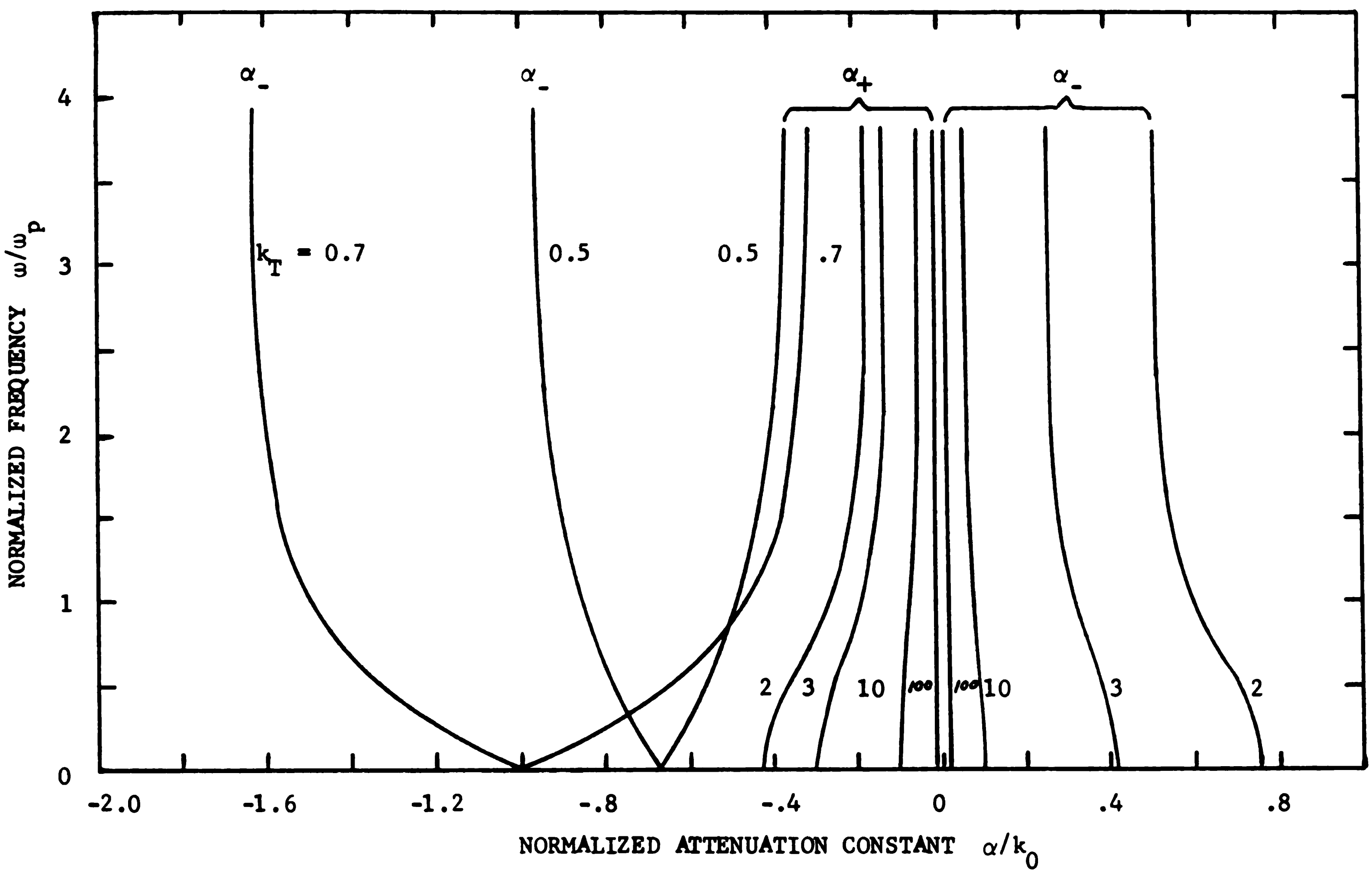


Fig. 3.8 $\omega - \alpha$ Diagrams for the Carrier Waves in an Extrinsic Semiconductor With Thermal-to-Drift Velocity Ratio as Parameter for a Fixed Collision Frequency $\nu = \omega_p$. Here $k_p = \omega_p/u_0$.

carrier waves approach pure electroacoustic waves or pure space-charge waves. In other words, the over all attenuation will decrease with an increasing thermal-to-drift velocity ratio when $u_0 < v_T$ and will increase with an increasing thermal-to-drift velocity ratio when $u_0 > v_T$. It is noted that the attenuation constants corresponding to the forward progressing carrier waves with positive group velocities are negative and those corresponding to the backward progressing carrier waves with negative group velocities are positive; therefore, no instability of the carrier waves will occur without coupling to an external circuit or applying high frequency pump source in a manner of parametric amplification.

Fig. 3.9 through 3.13 shows the dispersion diagrams of carrier waves in an extrinsic semiconductor with collision frequency as parameter while the thermal-to-drift velocity ratio is fixed. It can be seen that when the collision frequency increases, the attenuation constants increase rapidly whereas the phase constants stay almost constant. Physically, this means that the collision between carriers and the solid lattice will cause an energy loss; however, it will not affect the nature of the carrier wave to a significant degree.

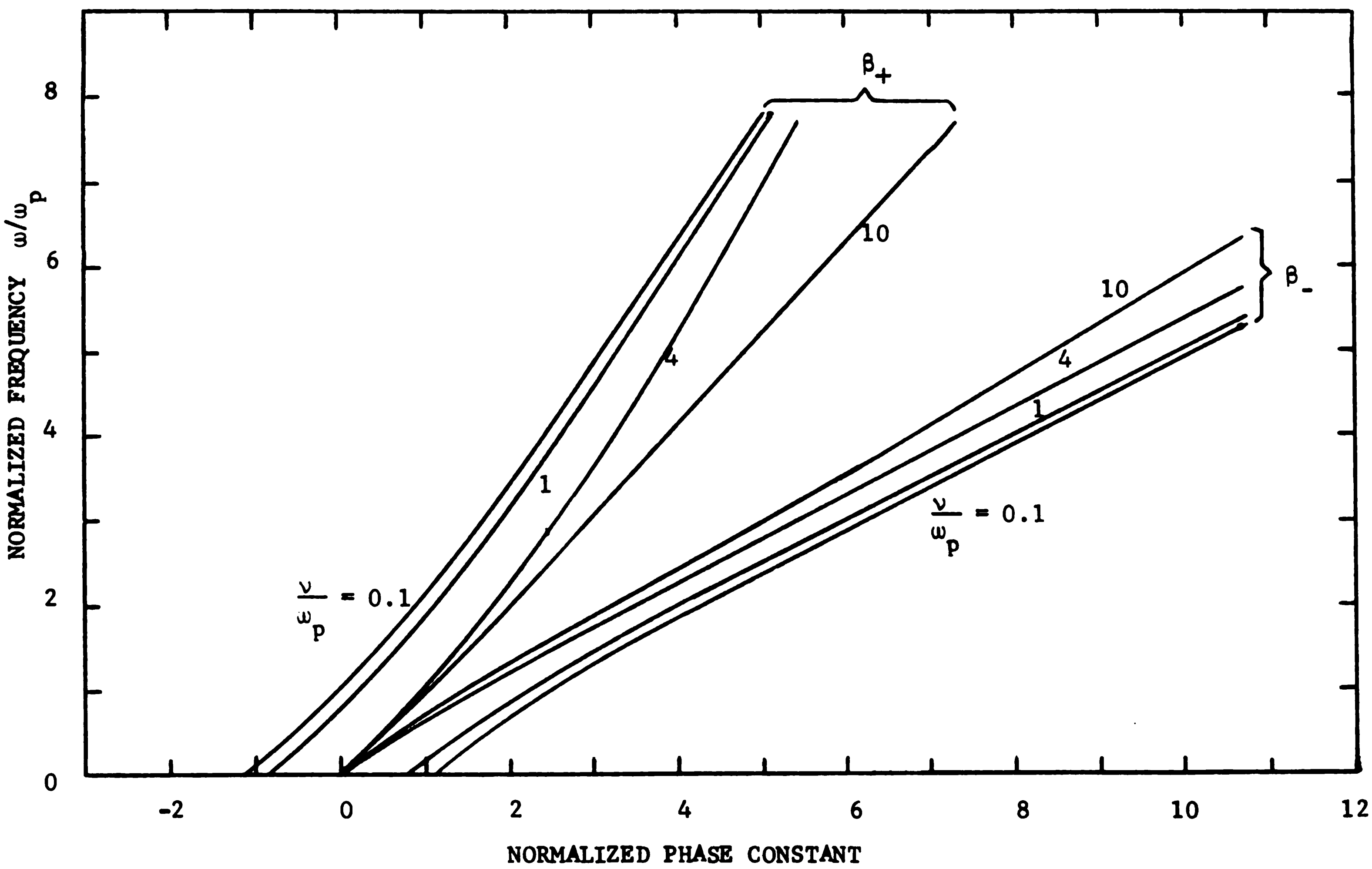


Fig. 3.9 $\omega - \beta$ Diagrams for the Carrier Waves in an Extrinsic Semicondcutor with Collision-to-Plasma Frequency Ratio as Parameter for a Fixed Thermal-to-Drift Velocity Ratio $k_T = 0.5$. Here $k_p = \omega_p/u_0$.

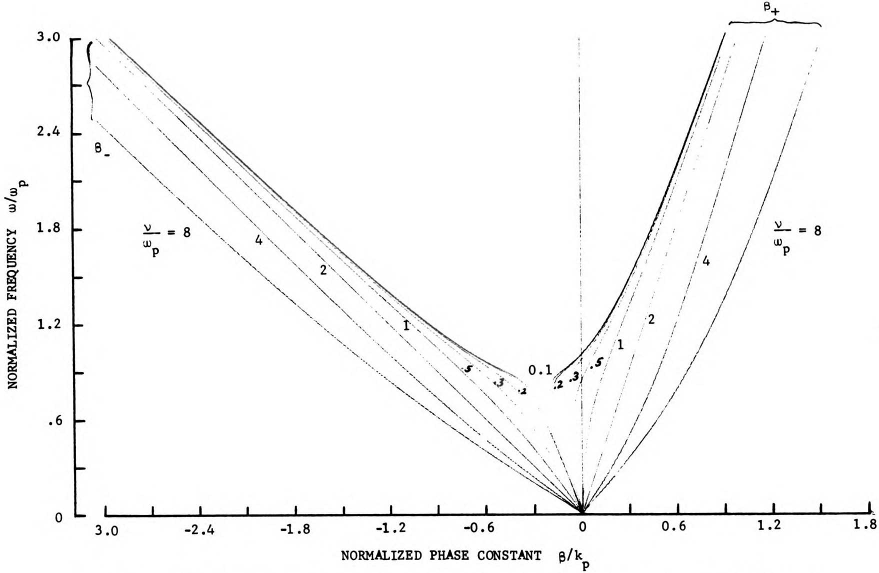


Fig. 3.10 ω - β Diagrams for the Carrier Waves in An Extrinsic Semiconductor with Collision-to-Plasma Frequency Ratio as Parameter for a Fixed Thermal-to-Drift Velocity Ratio $k_T = 2$. Here $k_p = \omega_p/u_0$.

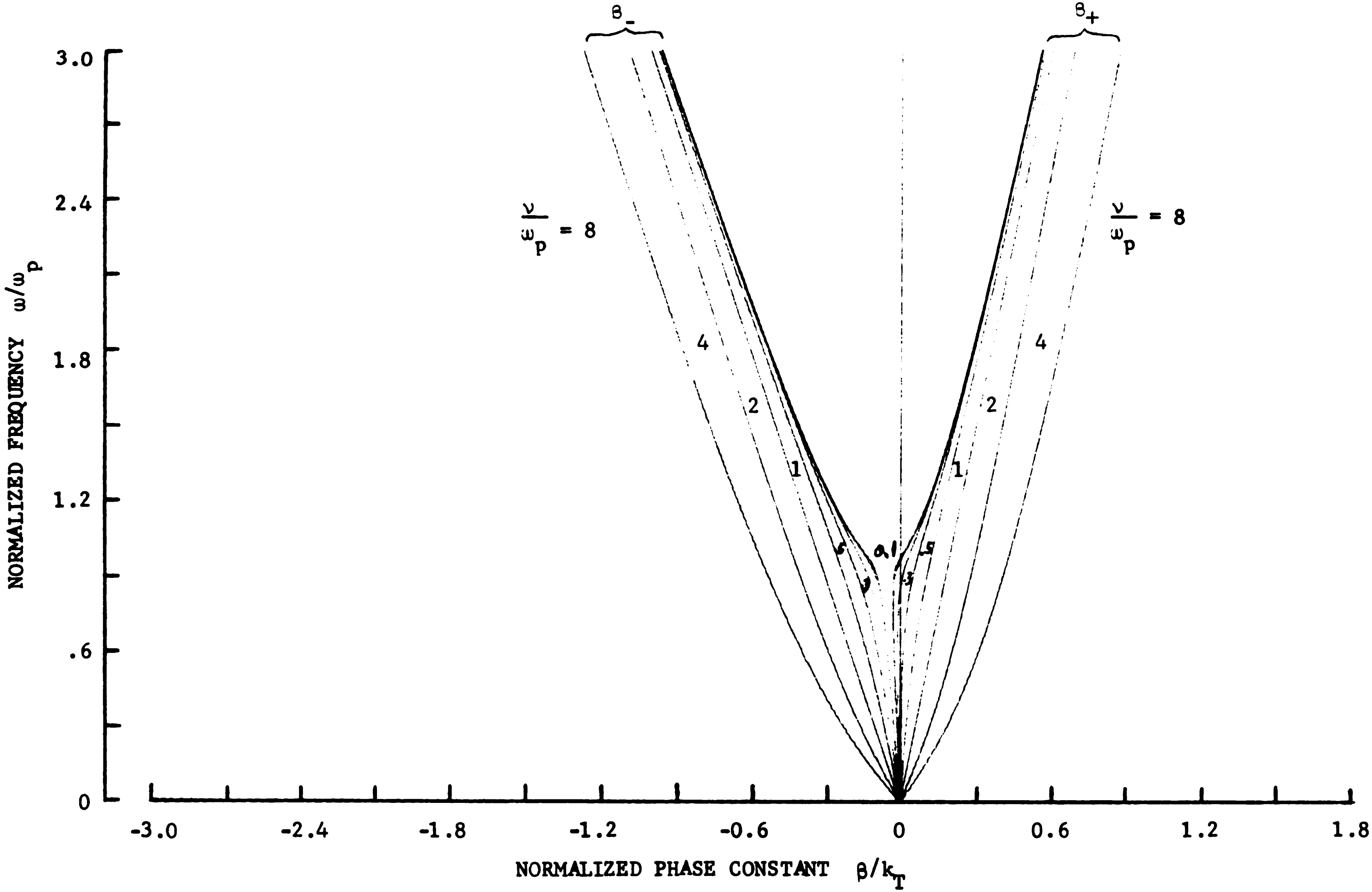


Fig. 3.11 ω - β Diagrams for the Carrier Waves in an Extrinsic Semiconductor with Collision-to-Plasma Frequency Ratio as Parameter for a Fixed Thermal-to-Drift Velocity Ratio $k_T = 4$. Here $k_p = \omega_p/u_0$.

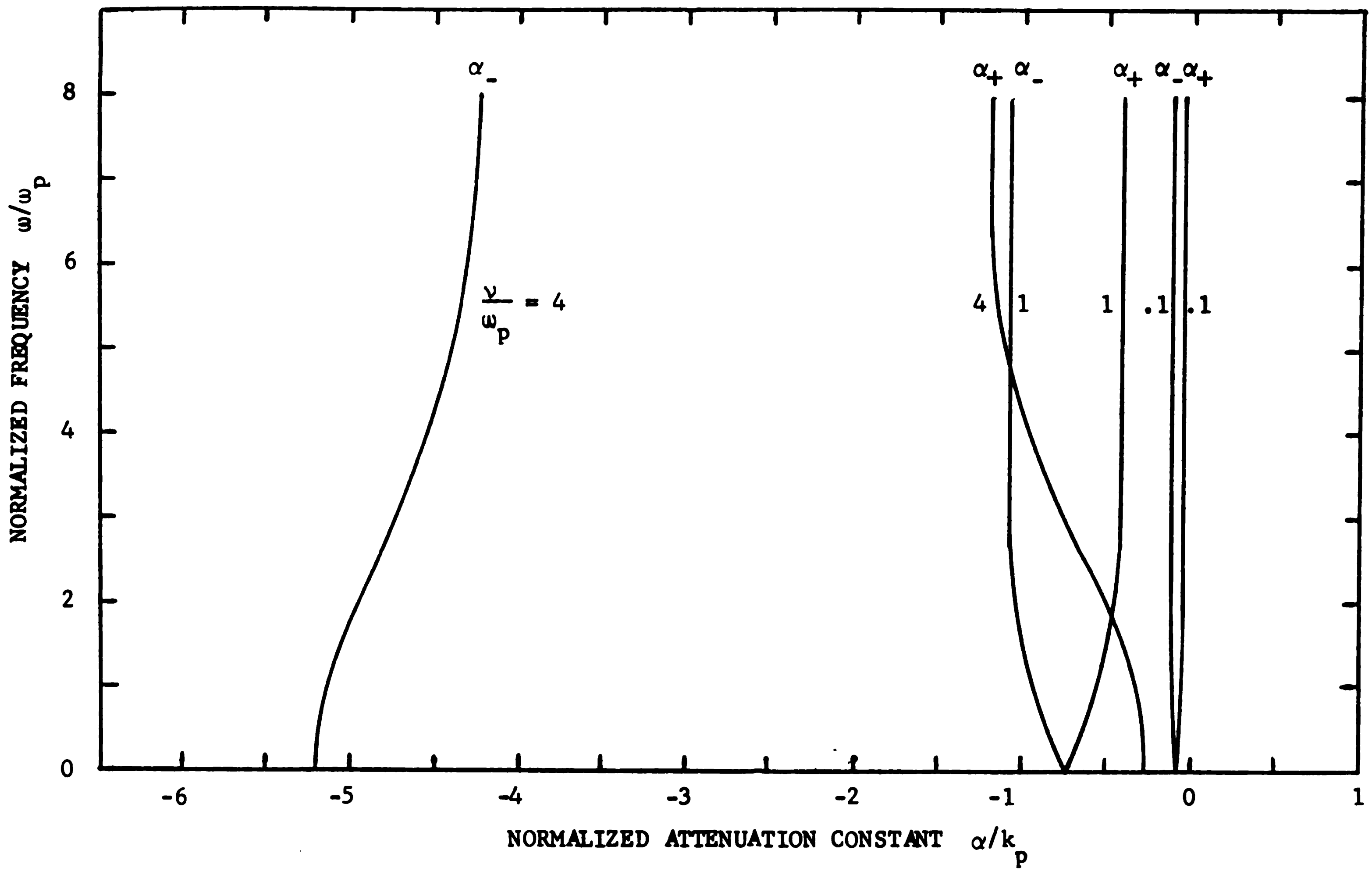


Fig. 3.12 $\omega - \alpha$ Diagrams for the Carrier Waves in an Extrinsic Semiconductor with Collision-to-Plasma Frequence Ratio as Parameter for a Fixed Thermal-to-Drift Velocity Ratio $k_T = 0.5$. Here $k_p = \omega_p/u_0$.

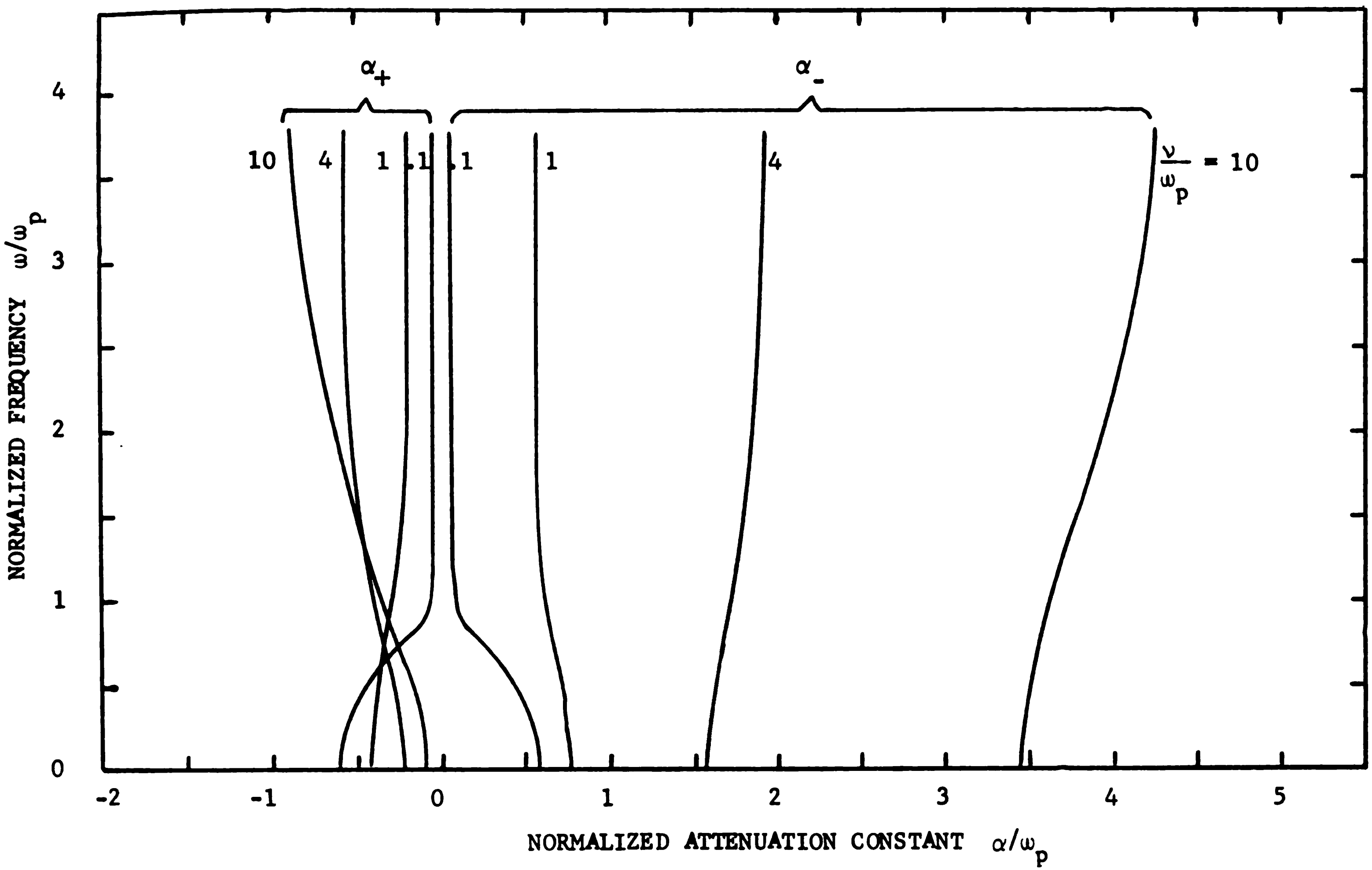


Fig. 3.13 ω - α Diagrams for the Carrier Waves in an Extrinsic Semiconductor with Collision-to-Plasma Frequency Ratio as Parameter for a Fixed Thermal-to-Drift Velocity Ratio $k_T = 2$. Here $k_p = \omega_p/u_0$.

CHAPTER IV

KINETIC POWER OF THE LONGITUDINAL CARRIER WAVES

4.1 Introduction

The function of most of the microwave electron device is to convert the d-c carrier stream energy into high frequency electromagnetic wave energy or vice versa. Consequently, the energy conversion principle for electromagnetic waves in the presence of carrier stream plays an important role in device physics. Once the power flow nature of the carrier waves is known, one can get a general idea about the characteristics of the interactions involving external circuit and stream carrier or carrier waves of the stream. The carrier stream can act as either a positive or negative resistive load to the external electromagnetic circuit wave depending upon which of the carrier waves is excited in the interaction. If the carrier stream acts as a positive resistive load, it will absorb energy from the electromagnetic fields around it. Signal couplers are examples of this kind. On the other hand, the carrier stream can supply energy to the electromagnetic fields when a negative energy-carrying wave is excited. Most of the amplifiers and oscillators work under this principle.

In this chapter the power and energy relations between the carrier stream and the external circuit are investigated. Starting with the macroscopic classical model described in

Chapter II, we derive a new form of the linearized Poynting theorem interpreting the power and energy flow of electromagnetic and longitudinal carrier waves in solids. Using the derived equation of real power flow, the conditions for wave amplification are discussed.

4.2 Derivation of Small Signal Kinetic Power Theorem for Longitudinal Carriers with External Surrounding Circuit

It was stated in Chapter II that the basic equations of solid state plasma considered as a conducting fluid are Maxwell's equations and the Boltzmann transport equations. From the Maxwell's equation we have

$$\nabla \times \vec{E} = -\mu \frac{\partial \vec{H}}{\partial t} \tag{4.1}$$

$$\nabla \times \vec{H} = e(p\vec{v}_n - n\vec{v}_e) + \epsilon \frac{\partial \vec{E}}{\partial t} \tag{4.2}$$

From the first moment of the Boltzmann transport equation we have

$$-\frac{v_{T-}^2}{n} \nabla n = \frac{\partial \vec{v}_e}{\partial t} + (\vec{v}_e \cdot \nabla)\vec{v}_e + \frac{e}{m_e^*} (\vec{E} + \vec{v}_e \times \vec{B}) + \nu_e \vec{v}_e \tag{4.3}$$

$$-\frac{v_{T+}}{p} \nabla p = \frac{\partial v_h}{\partial t} + (\vec{v}_h \cdot \nabla)\vec{v}_h - \frac{e}{m_h^*} (\vec{E} + \vec{v}_h \times \vec{B}) + \nu_h \vec{v}_h \tag{4.4}$$

In order to take the electromagnetic wave due to the external surrounding circuit into account, a quasi-one-dimensional model as stated in Section 2.6 is used. Substituting the expressions for the variables of this model of Equations (2.13) through (2.19), (2.91) and (2.92) into Equations (4.1) through (4.4) and considering the time-dependent terms only, we have

$$\nabla \times \vec{E}_1 = -\mu \frac{\partial \vec{H}_1}{\partial t} \tag{4.5}$$

$$\nabla \times \vec{H}_1 = \vec{J}_{e1} + \vec{J}_{h1} + \epsilon \frac{\partial \vec{E}_1}{\partial t} \tag{4.6}$$

$$-\frac{v_{T-}^2}{n_0} \nabla n_1 = \frac{\partial \vec{v}_{e1}}{\partial t} + u_{e0} \frac{\partial \vec{v}_{e1}}{\partial z} + \frac{e}{m_e^*} (\vec{E}_1 + \vec{v}_{e1} \times \vec{B}_0 + \vec{u}_{e0} \times \vec{B}_1) + \nu_e \vec{v}_{e1} \tag{4.7}$$

$$-\frac{v_{T+}^2}{p_0} \nabla p_1 = \frac{\partial \vec{v}_{h1}}{\partial t} + u_{h0} \frac{\partial \vec{v}_{h1}}{\partial z} - \frac{e}{m_h^*} (\vec{E}_1 + \vec{v}_{h1} \times \vec{B}_0 + \vec{u}_{h0} \times \vec{B}_1) + \nu_h \vec{v}_{h1} \tag{4.8}$$

where

$$\vec{J}_{e1} = -e(n_0 \vec{v}_{e1} + n_1 \vec{u}_{e0}) = \hat{z}\, J_{e1} \tag{4.9}$$

$$J_{h1} = e(p_0 \vec{v}_{h1} + p_1 \vec{u}_{h0}) = \hat{z}\, J_{h1} \tag{4.10}$$

Dot multiplying Equation (4.5) by $\vec{H}_1$, Equation (4.6) by $-\vec{E}_1$, Equation (4.7) by $\frac{m_e^*}{e} \vec{J}_{e1}$, Equation (4.8) by $-\frac{m_h^*}{e} \vec{J}_{h1}$ and adding together, one obtains

$$\begin{aligned} \nabla \cdot (\vec{E}_1 \times \vec{H}_1) = & -\mu \vec{H}_1 \cdot \frac{\partial \vec{H}_1}{\partial t} - \epsilon\, E_1 \cdot \frac{\partial \vec{E}_1}{\partial t} \\ & + \frac{m_e^*}{e} \vec{J}_{e1} \cdot \left[\frac{\partial \vec{v}_{e1}}{\partial t} + u_{e0} \frac{\partial \vec{v}_{e1}}{\partial z} + \frac{v_{T-}^2}{n_0} \nabla n_1 + \nu_e \vec{v}_{e1}\right] \\ & - \frac{m_h^*}{e} \vec{J}_{h1} \cdot \left[\frac{\partial \vec{v}_{h1}}{\partial t} + u_{h0} \frac{\partial \vec{v}_{h1}}{\partial z} + \frac{v_{T+}^2}{p_0} \nabla p_1 + \nu_h \vec{v}_{h1}\right] \end{aligned} \tag{4.11}$$

where the terms $\vec{J}_{e1} \cdot (\vec{v}_{e1} \times \vec{B}_0)$ etc. are vanished by vector identity. Rewriting Equation (4.11), we have

$$\nabla\cdot(\vec{E}_1 \times \vec{H}_1) = -\tfrac{1}{2}\frac{\partial}{\partial t}(\epsilon E_1^2 + \mu H_1^2)$$
$$+ \frac{m_e^*}{e} J_{e1}\left(\frac{\partial v_{e1}}{\partial t} + \nu_e v_{e1}\right) - \frac{m_h^*}{e} J_{h1}\left(\frac{\partial v_{h1}}{\partial t} - \nu_h v_{h1}\right)$$
$$+ \frac{m_e^*}{e}\left\{u_{e0}\left[\frac{\partial}{\partial z}(J_{e1}v_{e1}) - v_{e1}\frac{\partial J_{e1}}{\partial z}\right]\right.$$
$$\left. + \frac{v_{T-}^2}{n_0}\left[\frac{\partial}{\partial z}(J_{e1}n_1) - n_1\frac{\partial J_{e1}}{\partial z}\right]\right\}$$
$$- \frac{m_h^*}{e}\left\{u_{h0}\left[\frac{\partial}{\partial z}(J_{h1}v_{h1}) - v_{h1}\frac{\partial J_{h1}}{\partial z}\right]\right.$$
$$\left. + \frac{v_{T+}^2}{p_0}\left[\frac{\partial}{\partial z}(J_{h1}p_1) - p_1\frac{\partial J_{h1}}{\partial z}\right]\right\} \quad (4.12)$$

The a-c continuity equations for electrons and holes are

$$\frac{\partial J_{e1}}{\partial z} = e\frac{\partial n_1}{\partial t} \quad (4.13)$$

$$\frac{\partial J_{h1}}{\partial z} = -e\frac{\partial p_1}{\partial t} \quad (4.14)$$

Substituting Equations (4.13) and (4.14) into Equation (4.12) yields

$$\nabla\cdot(\vec{E}_1 \times \vec{H}_1) = -\tfrac{1}{2}\frac{\partial}{\partial t}(\epsilon E_1^2 + \mu H_1^2)$$
$$+ \frac{m_e^*}{e}\left[J_{e1}\frac{\partial v_{e1}}{\partial t} - eu_{e0}v_{e1}\frac{\partial n_1}{\partial t} - e\frac{v_{T-}^2}{n_0}n_1\frac{\partial n_1}{\partial t}\right]$$
$$- \frac{m_h^*}{e}\left[J_{h1}\frac{\partial v_{h1}}{\partial t} + eu_{h0}v_{h1}\frac{\partial p_1}{\partial t} + e\frac{v_{T+}^2}{p_0}p_1\frac{\partial p_1}{\partial t}\right]$$
$$+ \frac{m_e^*}{e}\left[u_{e0}\frac{\partial}{\partial z}(J_{e1}v_{e1}) + \frac{v_{T-}^2}{n_0}\frac{\partial}{\partial z}(J_{e1}n_1) + \nu_e J_{e1}v_{e1}\right]$$
$$- \frac{m_h^*}{e}\left[u_{h0}\frac{\partial}{\partial z}(J_{h1}v_{h1}) + \frac{v_{T+}^2}{n_0}\frac{\partial}{\partial z}(J_{h1}p_1) + \nu_h J_{e1}v_{e1}\right] \quad (4.15)$$

Using the kinetic voltages due to velocity modulation and thermal diffusion defined in Equations (2.51) through (2.54) together with the expression for the current densities of (4.9) and (4.10), the above equation becomes

$$\nabla\cdot(\vec{E}_1 \times \vec{H}_1) + \frac{\partial}{\partial z}(V_{e1}J_{e1} + V_{h1}J_{h1} + V_{T-}J_{e1} + V_{T+}J_{h1})$$

$$= -\tfrac{1}{2}\frac{\partial}{\partial t}(\epsilon E_1^2 + \mu H_1^2)$$

$$- m_e^*(n_0 v_{e1}\frac{\partial v_{e1}}{\partial t} + u_{e0}n_1\frac{\partial v_{e1}}{\partial t} + u_{e0}v_{e1}\frac{\partial n_1}{\partial t} + \frac{v_{T-}^2}{n_0}n_1\frac{\partial n_1}{\partial t})$$

$$- m_h^*(p_0 v_{h1}\frac{\partial v_{h1}}{\partial t} + u_{h0}p_1\frac{\partial v_{h1}}{\partial t} + u_{h0}v_{h1}\frac{\partial p_1}{\partial t} + \frac{v_{T+}^2}{p_0}p_1\frac{\partial p_1}{\partial t})$$

$$- \frac{\nu_e}{u_{e0}}V_{e1}J_{e1} - \frac{\nu_h}{u_{h0}}V_{h1}J_{h1} \qquad (4.16)$$

which can be further simplified as

$$\nabla\cdot(\vec{E}_1 \times \vec{H}_1) + \frac{\partial}{\partial z}(V_{e1}J_{e1} + V_{h1}J_{h1} + V_{T-}J_{e1} + V_{T+}J_{h1})$$

$$+ \tfrac{1}{2}\frac{\partial}{\partial t}[\epsilon E_1^2 + \mu H_1^2 + m_e^*(n_0 v_{e1}^2 + 2u_{e0}n_1v_{e1} + \frac{v_{T-}^2}{n_0}n_1^2)$$

$$+ m_h^*(p_0 v_{h1}^2 + 2u_{h0}p_1v_{h1} + \frac{v_{T+}^2}{p_0}p_1^2)]$$

$$= -\frac{\nu_e}{u_{e0}}V_{e1}J_{e1} - \frac{\nu_h}{u_{h0}}V_{h1}J_{h1} \qquad (4.17)$$

For the purpose of making a clearer physical picture, the following instantaneous kinetic powers and energy densities are defined

$$\vec{P}_{m1} = \vec{E}_1 \times \vec{H}_1 \qquad (4.18)$$

$$\vec{P}_{e1} = \hat{z}\, P_{e1} = \hat{z}\, V_{e1}J_{e1} \qquad (4.19)$$

$$\vec{P}_{h1} = \hat{z}\, P_{h1} = \hat{z}\, V_{h1} J_{h1} \tag{4.20}$$

$$\vec{P}_T = \hat{z}\, P_T = \hat{z}(V_{T-} J_{e1} + V_{T+} J_{h1}) \tag{4.21}$$

$$W_{m1} = \tfrac{1}{2}(\epsilon\, E_1^2 + \mu\, H_1^2) \tag{4.22}$$

$$W_1 = \tfrac{1}{2}(m_e^* n_0 v_{e1}^2 + m_h^* p_0 v_{h1}^2) + m_e^*(u_{e0} n_1 v_{e1} + m_h^* u_{h0} p_1 v_{h1}) \tag{4.23}$$

$$W_T = \tfrac{1}{2}[m_e^* \frac{v_{T-}^2}{n_0} n_1^2 + m_h^* \frac{v_{T-}^2}{p_0} p_1^2] \tag{4.24}$$

Using the above expressions, Equation (4.17) can be written as

$$\nabla\cdot(\vec{P}_{m1} + \vec{P}_{e1} + \vec{P}_{h1} + \vec{P}_T) + \frac{\partial}{\partial t}(W_{m1} + W_1 + W_T) = -\frac{\nu_e}{u_{e0}} P_{e1} - \frac{\nu_h}{u_{h0}} P_{h1} \tag{4.25}$$

Integration of Equation (4.25) over the whole space gives the instantaneous power and energy equation. It is

$$\int_s (\vec{P}_{m1} + \vec{P}_{e1} + \vec{P}_{h1} + \vec{P}_T)\cdot d\vec{s} + \frac{\partial}{\partial t}\int_v (W_{m1} + W_1 + W_T)dv = -\int_v \left(\frac{\nu_e}{u_{e0}} P_{e1} + \frac{\nu_h}{u_{h0}} P_{h1}\right)dv \tag{4.26}$$

The physical meaning of each term of Equation (4.26) are

$-\int_v\left(\frac{\nu_e}{u_{e0}} P_{e1} + \frac{\nu_h}{u_{h0}} P_{h1}\right)dv$ = the a-c power loss due to collision effect

$\vec{P}_{m1}$ = energy flow of the electromagnetic wave

$\vec{P}_{e1}$ = kinetic energy flow due to velocity modulation of electrons

$\vec{P}_{h1}$ = kinetic energy flow due to velocity modulation of holes

$\vec{P}_T$ = kinetic energy flow due to the thermal diffusion of the carriers

W_{m1} = sum of the electric and magnetic energy densities of the electromagnetic wave

W_1 = kinetic energy density of the carriers

W_T = thermal energy density of the carriers

The equation of real power flow of the system can be obtained by taking the time average of Equation (4.26). If all the time-dependent variables of the system have a periodic variation of constant frequency, we may map all the variables into a complex plane and the equation of real power flow of the system reduces to

$$\int_s \mathrm{Re}(\vec{\mathcal{P}}_{m1} + \vec{\mathcal{P}}_{e1} + \vec{\mathcal{P}}_{h1} + \vec{\mathcal{P}}_T)\cdot d\vec{s} = -\frac{\nu_e}{u_{e0}}\int_v \mathrm{Re}[\mathcal{P}_{e1}]dv - \frac{\nu_h}{u_{h0}}\int_v \mathrm{Re}[\mathcal{P}_{h1}]dv \tag{4.27}$$

where

$$\vec{\mathcal{P}}_{m1} = \tfrac{1}{2}(\vec{E}_1 \times \vec{H}_1) \tag{4.28}$$

$$\vec{\mathcal{P}}_{e1} = \hat{z}\ \tfrac{1}{2}\ V_{e1}J^*_{e1} \tag{4.29}$$

$$\vec{\mathcal{P}}_{h1} = \hat{z}\ \tfrac{1}{2}\ V_{h1}J^*_{h1} \tag{4.30}$$

$$\vec{\mathcal{P}}_T = \hat{z}\ \tfrac{1}{2}(V_{T-}J^*_{e1} + V_{T+}J^*_{h1}) \tag{4.31}$$

where $\vec{\mathcal{P}}_{m1}$ is the rf complex electromagnetic power density, $\vec{\mathcal{P}}_{e1}$ is the kinetic power density due to the velocity modulation of electrons, $\vec{\mathcal{P}}_{h1}$ is the kinetic power density due to the velocity modulation of holes and $\vec{\mathcal{P}}_T$ is the kinetic power density due to the thermal diffusion of the carriers.

Equation (4.27) can also be obtained from the complex rf fundamental equations directly; for this purpose, we map all the variables onto a complex plane and consider Equations (4.1) through (4.10) as complex equations. Taking complex conjugate of Equation (4.6) yields

$$\nabla \times \vec{H}_1^* = \vec{J}^*_{e1} + \vec{J}^*_{h1} - j\omega\epsilon\, \vec{E}_1^* \tag{4.32}$$

Dot multiplying Equation (4.5) by $\vec{H}_1^*$, Equation (4.32) by $-\vec{E}_1$, Equation (4.6) by $\frac{m_e^*}{e}\vec{J}^*_{e1}$, Equation (4.7) by $-\frac{m_h^*}{e}\vec{J}^*_{h1}$ and adding together, we have

$$\begin{aligned}
\nabla\cdot(\vec{E}_1 \times \vec{H}_1^*) &= -j\omega\mu\, H_1H_1^* + j\omega\epsilon\, E_1E_1^* \\
&+ \frac{m_e^*}{e}\vec{J}^*_{e1}\cdot(j\omega\, \vec{v}_{e1} + u_{e0}\frac{\partial \vec{v}_{e1}}{\partial z} + \frac{v_{T-}^2}{n_0}\nabla n_1 + \nu_e\vec{v}_{e1}) \\
&- \frac{m_h^*}{e}\vec{J}^*_{h1}\cdot(j\omega\, \vec{v}_{h1} + u_{e0}\frac{\partial \vec{v}_{h1}}{\partial z} + \frac{v_{T+}^2}{p_0}\nabla p_1 + \nu_h\vec{v}_{h1})
\end{aligned} \tag{4.33}$$

which can be rewritten as

$$\begin{aligned}
\nabla\cdot(\vec{E}_1 \times \vec{H}_1^*) &= -j\omega(\mu\, H_1H_1^* - \epsilon\, E_1E_1^* - \frac{m_e^*}{e} v_{e1}J^*_{e1} + \frac{m_h^*}{e} v_{h1}J^*_{h1}) \\
&+ \frac{m_e^*}{e}\{u_{e0}[\frac{\partial}{\partial t}(v_{e1}J^*_{e1}) - v_{e1}\frac{\partial J^*_{e1}}{\partial z}] + \nu_e v_{e1}J^*_{e1} \\
&\qquad + \frac{v_{T-}^2}{n_0}[\frac{\partial}{\partial z}(n_1J^*_{e1}) - n_1\frac{\partial J^*_{e1}}{\partial z}]\} \\
&- \frac{m_h^*}{e}\{u_{e0}[\frac{\partial}{\partial z}(v_{h1}J^*_{h1}) - v_{h1}\frac{\partial J^*_{h1}}{\partial z}] + \nu_h v_{h1}J^*_{h1} \\
&\qquad + \frac{v_{T+}^2}{p_0}[\frac{\partial}{\partial z}(p_1J^*_{h1}) - p_1\frac{\partial J^*_{h1}}{\partial z}]\}
\end{aligned} \tag{4.34}$$

From the rf complex continuity equations, we have

$$\frac{\partial J_{e1}}{\partial z} = j\omega e n_1 \tag{4.35}$$

$$\frac{\partial J_{h1}}{\partial z} = -j\omega e p_1 \tag{4.36}$$

Using Equations (4.9), (4.10), (4.35) and (4.36) together with the kinetic voltages of (2.45) through (2.48) defined in Chapter II, Equation (4.34) becomes

$$\begin{aligned}\nabla\cdot(\vec{E}_1 \times \vec{H}_1^*) &+ \frac{\partial}{\partial z}(V_{e1}J_{e1}^* + V_{h1}J_{h1}^* + V_{T-}J_{e1}^* + V_{T+}J_{h1}^*) \\ &+ j\omega[\mu\, H_1H_1^* - \epsilon\, E_1E_1^* + m_e^*(n_0 v_{e1}v_{e1}^* - \frac{v_{T-}^2}{n_0} n_1 n_1^*) \\ &\qquad + m_h^*(p_0 v_{h1}v_{h1}^* - \frac{v_{T+}^2}{p_0} p_1 p_1^*)] \\ &= -\frac{\nu_e}{u_{e0}} V_{e1}J_{e1}^* - \frac{\nu_h}{u_{h0}} V_{h1}J_{h1}^* \end{aligned} \tag{4.37}$$

With the defined complex power density (4.28) through (4.31), Equation (4.37) reduces to

$$\begin{aligned}\nabla\cdot(\vec{\mathcal{P}}_{m1} + \vec{\mathcal{P}}_{e1} + \vec{\mathcal{P}}_{h1} + \vec{\mathcal{P}}_T) &+ j\,\frac{\omega}{2}[\mu|H_1|^2 - \epsilon|E_1|^2 + m_e^* n_0|v_{e1}|^2 \\ &+ m_h^* p_0|v_{h1}|^2 - m_e^*\frac{v_{T-}^2}{n_0}|n_1|^2 - m_h^*\frac{v_{T+}^2}{p_0}|p_1|^2] \\ &= -\frac{\nu_e}{u_{e0}}\mathcal{P}_{e1} - \frac{\nu_h}{u_{h0}}\mathcal{P}_{h1}\end{aligned} \tag{4.38}$$

Integrating Equation (4.38) over the whole space gives the rf complex power equation of the system as

$$\int_s \mathrm{Im}(\vec{\mathcal{P}}_{m1} + \vec{\mathcal{P}}_{e1} + \vec{\mathcal{P}}_{h1} + \vec{\mathcal{P}}_T)\cdot d\vec{s}$$

$$+ \frac{\omega}{2}\int_v [\mu|H_1|^2 - \epsilon|E_1|^2 + m_e^*(n_0|v_{e1}|^2 - \frac{v_{T-}^2}{n_0}|n_1|^2)$$

$$+ m_h^*(p_0|v_{h1}|^2 - \frac{v_{T+}^2}{p_0}|p_1|^2)]dv$$

$$+ \frac{\nu_e}{u_{e0}}\int_v \mathrm{Im}[\mathcal{P}_{e1}]dv + \frac{\nu_h}{u_{h0}}\int_v \mathrm{Im}[\mathcal{P}_{h1}]dv = 0 \qquad (4.39)$$

The real part of the above equation which is identical to Equation (4.27) gives the expression of real power flow of the system and the imaginary part of Equation (4.39) gives the balance equation of the rective power. It is

$$\int_s \mathrm{Im}(\vec{\mathcal{P}}_{m1} + \vec{\mathcal{P}}_{e1} + \vec{\mathcal{P}}_{h1} + \vec{\mathcal{P}}_T)\cdot d\vec{s} + \frac{\omega}{2}\int_v [\mu|H_1|^2 - \epsilon|E_1|^2$$

$$+ m_e^*(n_0|v_{e1}|^2 - \frac{v_{T-}^2}{n_0}|n_1|^2) + m_h^*(p_0|v_{h1}|^2 - \frac{v_{T+}^2}{p_0}|p_1|^2)]dv$$

$$+ \frac{\nu_e}{u_{e0}}\,\mathrm{Im}\int_v \mathcal{P}_{e1}dv + \frac{\nu_h}{u_{h0}}\,\mathrm{Im}\int_v \mathcal{P}_{h1}dv = 0 \qquad (4.40)$$

It can be seen from Equation (4.27) that the sum of the electromagnetic power from external slow wave circuit and the kinetic powers due to velocity modulation and thermal diffusion of the carriers are conserved only when there is no collision in the process. Generally they are not conserved and will decrease or increase in an amount of $-\mathrm{Re}\int_v [\frac{\nu_e}{u_{e0}} P_{e1} + \frac{\nu_h}{u_{h0}} P_{h1}]dv$ depending upon the nature of the carrier wave whether it carries a positive or negative kinetic power respectively. Such amount of energy gain or loss of the system is checked with that obtained from the equivalent transmission-line analog in Chapter II.

In a particular case when the effects of both collision and thermal diffusion are neglected and the carriers are electrons only, Equation (4.27) is then reduced to

$$\int_s \mathrm{Re}[\vec{\mathcal{P}}_m + \vec{\mathcal{P}}_{e1}]\cdot d\vec{s} = 0 \qquad (4.41)$$

which is the same as the kinetic power theorem deduced by Chu[32] for the electron stream in vacuum coupling with a slow wave circuit.

4.3 Discussions for Possible Wave Amplification from Kinetic Power Theorem

The equation of real power flow of (4.27) in the last section shows the possibility of exchange energy between the longitudinal carrier waves in solids and the electromagnetic waves from a slow wave circuit. When the collision effects are neglected, the sum of the rf electromagnetic power from the surrounding slow wave circuit and the total kinetic power of the carriers is conserved. Similar to the case of an electron beam in vacuum, wave amplification arises when the carrier waves which carry a negative electrokinetic power are excited. When the collision effect is taken into consideration, it seems that an additional power of $-\int_v \mathrm{Re}[\frac{\nu_e}{u_{e0}} P_{e1} + \frac{\nu_h}{u_{h0}} P_{h1}]dv$ is generated along the stream when both P_{e1} and P_{h1} are negative and thus further amplification arises due to collision effect. Actually, the collision effect will reduce the amount of amplification; the reason is that the kinetic powers P_{e1} and P_{h1} themselves are functions of the collision frequency and will decrease when the collision frequency is raised.

Here we investigate the electrokinetic power densities of the one-dimensional model where the carrier waves propagating through an n-type semiconductor with negligible hole concentration. In such a case, the simplified fundamental equations (2.21), (2.23) and (2.25) reduce to

$$\Gamma E_1 = -\frac{e}{\epsilon} n_1 \tag{4.42}$$

$$\Gamma J_{e1} = j\omega e n_1 \tag{4.43}$$

$$(j\omega + \nu_e)v_{e1} + \frac{e}{m_e^*} E_1 + \Gamma u_{e0} v_{e1} + \frac{v_{T-}^2}{n_0} \Gamma n_0 = 0 \tag{4.44}$$

where $\Gamma = \alpha - j\beta$ is the propagation constant of the carrier waves defined in Chapter II.

Using the three equations stated above, the a-c velocity and density of the electrons can be expressed in terms of the electron current density as

$$v_{e1} = \frac{-\omega_{p-}^2 + v_{T-}^2(\alpha^2 - \beta^2 - j2\alpha\beta)}{j\omega e n_0[j\omega + \nu_e + u_{e0}(\alpha - j\beta)]} J_{e1} \tag{4.45}$$

$$n_1 = \frac{\alpha - j\beta}{j\omega e} J_{e1} \tag{4.46}$$

The real kinetic power densities due to velocity modulation and thermal diffusion in the system are obtained from Equations (4.29), (4.31), (2.45), (2.47), (4.45) and (4.46) as

$$\mathrm{Re}[P_{e1}] = \frac{(\beta_e-\beta)[\omega_{p-}^2 - v_{T-}^2(\alpha^2-\beta^2)] - 2\alpha\beta v_{T-}^2(\frac{\nu_e}{u_{e0}} + \alpha)}{(\beta_e - \beta)^2 + (\frac{\nu_e}{u_{e0}} + \alpha)^2} \frac{J_{e1}J_{e1}^*}{2\omega\epsilon\omega_{p-}^2} \quad (4.47)$$

$$\mathrm{Re}[P_T] = \frac{v_{T-}^2}{e\epsilon\omega_{p-}^2} \frac{1}{v_\phi} J_{e1}J_{e1}^* \quad (4.48)$$

where $v_\phi = \omega/\beta$ is the phase velocity of the carrier wave. When the thermal diffusion effect is neglected, i.e. $P_T = 0$; the total real kinetic power density is given by Equation (4.47) as

$$\mathrm{Re}[P_{e1}] = \frac{1 - u_{e0}/v_\phi}{(\beta_e-\beta)^2 + (\frac{\nu_e}{u_{e0}} + \alpha)^2} \cdot \frac{J_{e1}J_{e1}^*}{2\epsilon u_{e0}} \quad (4.49)$$

For slight collision case, $\nu_e < 2\omega_{p-}$, the phase velocities of the fast and slow waves are given by Equation (3.17) and the real electrokinetic powers due to velocity modulation are

$$\mathrm{Re}[P_{e1\pm}] = \pm \; 1 - (\frac{\nu_e}{2\omega_{p-}})^2 \frac{u_0}{\omega} \frac{J_{e1}J_{e1}^*}{2\epsilon\omega_p} \quad (4.50)$$

The above equation shows that the real electrokinetic power is positive for the fast wave and negative for the slow wave. It is also shown that the real electrokinetic power will decrease with an increasing collision frequency; when the collision frequency is sufficiently high, such that $\nu_e > 2\omega_{p-}$, there is no real electrokinetic power flow and thus no amplification arises since both the fast and slow waves are synchronous with the d-c drift motion.

When the thermal diffusion effect is taken into account, there exist a certain amount of real kinetic power due to thermal diffusion given by Equation (4.48). For the forward space-charge waves under consideration, the phase velocities of these waves are always positive and so are the $Re[P_T]$. Hence the real kinetic power flow due to thermal diffusion acts as a load in the energy conversion process. For a backward slow space-charge wave, $Re[P_T]$ becomes negative beeause of its negative phase velocity; therefore a possible application of solid state backward wave amplifier or oscillator is anticipated.

Consequently, the concept of wave amplification can be investigated by examining the energy exchange from the moving carriers to a surrounding circuit. From the equation of real power flow, the possibility of wave amplification arises when the slow carrier wave, which carries a negative kinetic energy, is excited. Although the sum of the rf electromagnetic power from the surrounding slow wave circuit and the total kinetic power of the carriers equal a positive amount of $-\int_V Re[\frac{\nu_e}{u_{e0}} P_{e1} + \frac{\nu_h}{u_{h0}} P_{h1}]dv$ when a negative kinetic energy carrying wave is applied; it seems that the collision effect will reduce the amoutn of amplification because the real kinetic power flow, $Re[P_{e1}]$ and $Re[P_{h1}]$, decrease rapidly in terms of an increasing collision frequency. A backward wave amplifier or oscillator is observed from the fact that the real kinetic power flow may become negative when the backward slow space-charge wave is excited.

CHAPTER V

COUPLED MODE ANALYSIS OF CARRIER WAVE INTERACTIONS

5.1 Introduction

Besides the three well-known forms of the equations of motion in classical mechanics, i.e. the Newtonian, the Hamiltonian and the Largrangian; there is another form called the normal mode form which is a set of first-order differential equations and is sometimes proven to be very useful in the theory of coupled systems.

When two or more systems are weakly coupled, that is, when the energy associated with the coupling is small compared with the energy contained in each system, we may analyze the equations of motion by finding the normal modes of the isolated systems and then express the coupled system by a slight perturbation on the motion of the isolated systems. In most of the physical systems, some of the modes of the isolated systems will play a minor role in the coupling mechanism. Thus the problem can be further simplified with good approximation by neglecting the coupling effect between those modes.

It is a necessary condition that the system be weakly coupled in order to take advantage of the couple mode method in which linearized equations are used. Should this not be the case, all the possible coupling effects between the modes have to be considered. Furthermore, the solutions of the coupled system will be significantly different from the uncoupled solutions such that a knowledge of the solution for the isolated system will not be useful.

In this Chapter we will work on a case that an extrinsic semiconductor with a relatively small minority carrier concentration is used as the propagation medium and the effect of collision between the majority carriers and the solid lattice in the semiconductor is neglected. Starting with the equivalent transmission-line equation of the collisionless longitudinal carrier waves in an extrinsic semiconductor, the normal modes of these waves are derived and then the coupled-mode approach is used to analyze the collisionless carrier waves in an extrinsic semiconductor with an external slow electromagnetic wave. A weak coupling condition is assumed for simplicity.

5.2 Derivation of an Equivalent Transmission-line Equation of the Collisionless Longitudinal Carrier Waves in Solids

In Chapter II, we have derived the propagation constant for the longitudinal carrier waves in extrinsic semiconductors. When the effect of the collision between the carriers and the solid lattice is neglected, the propagation constants from Equation (2.39) become:

$$\Gamma_{\pm} = -j \frac{\omega}{u_0} \frac{1}{1-k_T^2} \left[1 \mp \sqrt{k_T^2 + \frac{\omega_p^2}{\omega^2} (1-k_T^2)} \right] \tag{5.1}$$

With the kinetic voltages V_1 and V_T defined in Chapter II, Equations (2.23) and (2.27) can be rewritten as:

$$\frac{\partial J_1}{\partial z} = -j\omega\epsilon \frac{\omega_p^2}{v_T^2} V_T \qquad (5.2)$$

$$J_1 = \frac{e\omega_p^2}{u_0} V_1 + eu_0 \frac{\omega_p^2}{v_T^2} V_T \qquad (5.3)$$

Here, the subscripts denoting electrons or holes are dropped for simplicity.

As mentioned in Chapter II, the a-c current density and the two kinetic voltages have a solution of $\epsilon^{\Gamma z}$ type and can be expressed in the following way:

$$V_1 = V_{1\oplus}\epsilon^{\Gamma_+ z} + V_{1\ominus}\epsilon^{\Gamma_- z} \qquad (5.4)$$

$$V_T = V_{1\oplus}\epsilon^{\Gamma_+ z} + V_{T\ominus}\epsilon^{\Gamma_- z} \qquad (5.5)$$

$$J_1 = J_{1\oplus}\epsilon^{\Gamma_+ z} + J_{1\ominus}\epsilon^{\Gamma_- z} \qquad (5.6)$$

where the coefficients $V_{1\oplus}$, $V_{1\ominus}$, $V_{T\oplus}$, $V_{T\ominus}$, $J_{1\oplus}$ and $J_{1\ominus}$ are independent of z.

Substituting Equations (5.5) and (5.6) into Equation (5.2) we have

$$\Gamma_+ \; J_{1\oplus}\epsilon^{\Gamma_+ z} + \Gamma_- J_{1\ominus}\epsilon^{\Gamma_- z}$$

$$= -j\omega\epsilon \frac{\omega_p^2}{v_T^2} (V_{T\oplus}\epsilon^{\Gamma_+ z} + V_{T\ominus}\epsilon^{\Gamma_- z}) \qquad (5.7)$$

Equating the coefficients of $\epsilon^{\Gamma_+ z}$ and $\epsilon^{\Gamma_- z}$ separately on both sides of Equation (5.7) we obtain

$$V_{T\oplus} = \frac{-1}{j\omega\epsilon} \frac{v_T^2}{\omega_p^2} \Gamma_+ \; J_{1\oplus} \qquad (5.8)$$

$$V_{T\ominus} = \frac{-1}{j\omega\epsilon} \frac{v_T^2}{\omega_p^2} \Gamma_- \; J_{1\ominus} \tag{5.9}$$

Similarly, substituting Equations (5.4) through (5.6) into Equation (5.3), equating the coefficients of $\epsilon^{\Gamma_+ z}$ and $\epsilon^{\Gamma_- z}$ separately and using Equations (5.8) and (5.9) yields

$$V_{1\oplus} = \frac{u_0}{\epsilon\omega_p^2} \left(1 + \frac{u_0}{j\omega} \Gamma_+\right) J_{1\oplus} \tag{5.10}$$

$$V_{1\ominus} = \frac{u_0}{\epsilon\omega_p^2} \left(1 + \frac{u_0}{j\omega} \Gamma_-\right) J_{1\ominus} \tag{5.11}$$

The effects of longitudinal modulation and thermal diffusion can be expressed by a combined equivalent voltage as

$$\begin{aligned} V &= V_1 + V_T \\ &= \left[\frac{u_0}{\epsilon\omega_p^2} + \frac{\Gamma_+ u_0^2}{j\omega\epsilon\omega_p^2}(1 - k_T^2)\right] J_{1\oplus}\epsilon^{\Gamma_+ z} \\ &\quad + \left[\frac{u_0}{\epsilon\omega_p^2} + \frac{\Gamma_- u_0^2}{j\omega\epsilon\omega_p^2}(1 - k_T^2)\right] J_{1\ominus}\epsilon^{\Gamma_- z} \end{aligned} \tag{5.12}$$

where V can be considered as the total electrokinetic voltage of the collisionless longitudinal carrier wave in an extrinsic semiconductor since we have derived the total electrokinetic power for the collisionless longitudinal carrier waves in an extrinsic semiconductor from Equation (4.26) as $\int_s (V_1 + V_T)\vec{J}_1^* \cdot d\vec{s}$ or $\int_s V\vec{J}_1^* \cdot d\vec{s}$. Replacing $\Gamma_\pm$ with the right hand side of Equation (5.1) yields

$$V = \frac{u_0}{\epsilon\omega^2} \sqrt{k_T^2 + \frac{\omega_p^2}{\omega^2}(1 - k_T^2)} \cdot \left[J_{1\oplus}\epsilon^{\Gamma_+ z} - J_{1\ominus}\epsilon^{\Gamma_- z}\right] \tag{5.13}$$

Differentiating the above equation with respect to z gives

$$\frac{\partial V}{\partial z} = \frac{u_0}{\epsilon\omega_p^2}\sqrt{k_T^2 + \frac{\omega_p^2}{\omega^2}(1 - k_T^2)} \cdot [\Gamma_+ \cdot J_{1\oplus}\epsilon^{\Gamma_+ z} - \Gamma_- \cdot J_{1\ominus}\epsilon^{\Gamma_- z}] \tag{5.14}$$

Substituting Equation (5.1) into (5.14) and using Equations (5.6) and (5.13) one obtains

$$\frac{\partial V}{\partial z} = -j\beta_T V - Z_s J_1 \tag{5.15}$$

where

$$\beta_T = \frac{\omega}{u_0}\frac{1}{1-k_T^2} \tag{5.16}$$

$$Z_s = -j\frac{\omega}{1-k_T^2}\frac{1}{\epsilon\omega_p^2}[k_T^2 + \frac{\omega_p^2}{\omega^2}(1 - k_T^2)] \tag{5.17}$$

Similarly, differentiating Equation (5.6) with respect to z and using Equations (5.1) and (5.13) we have

$$\frac{\partial J_1}{\partial z} = -j\beta_T J_1 - Y_s V \tag{5.18}$$

where

$$Y_s = \frac{-j\omega}{1-k_T^2}\frac{\epsilon\omega_p^2}{u_0^2} \tag{5.19}$$

Equations (5.15) and (5.18) can be rearranged as

$$(\frac{\partial}{\partial z} + j\beta_T)V = -Z_s J_1 \tag{5.20}$$

$$(\frac{\partial}{\partial z} + j\beta_T)J_1 = -Y_s V \tag{5.21}$$

Making the following simple changes of variables

$$V = V' \epsilon^{-j\beta_T z} \tag{5.22}$$

$$J_1 = J_1' \epsilon^{-j\beta_T z} \tag{5.23}$$

Equations (5.20) and (5.21) are reduced to:

$$\frac{\partial V'}{\partial z} = -Z_s J_1' \tag{5.24}$$

$$\frac{\partial J_1'}{\partial z} = -Y_s V' \tag{5.25}$$

Equations (5.24) and (5.25) have the same form of the standard transmission-line equations with voltage V' along the line and current density J_1' through the line. However, the transmission-line analog is not perfect because of a phase shift introduced in Equations (5.22) and (5.23).

5.3 Normal Modes of the Collisionless Longitudinal Carrier Waves in Solids

5.3.1 Derivation of the Normal Mode Equation

The equivalent transmission-line Equations (5.20) and (5.21) derived in the last section can be considered as the Hamiltonian form of the equations of motion for the collisionless carrier waves in an extrinsic semiconductor due to longitudinal modulation and thermal diffusion. The equations show that the total voltage and current density are coupled, which allows for an interchange of electric and kinetic energy as the waves propagate down the line. However, the transmission-line analog can also be described by a particular form of first-order differential equations called the normal mode form of the equation of motion. The advantage of using such a form is that the decoupled differential equations can sometimes be easily handled. Furthermore, the kinetic power carried by the carriers can readily be obtained by using normal modes formulation. For the purpose of obtaining these forms, linear combinations of the Hamiltonian equations which will decouple the variables are to be sought. With this in mind, multiplying Equation (5.21) by an arbitrary constant Z and adding to Equation (5.20) we have:

$$\left(\frac{\partial}{\partial z} + j\beta_T\right)(V + Z\ J_1) = -Y_s Z\left(V + \frac{Z_s}{Y_s Z}\ J_1\right) \tag{5.26}$$

letting $Z_s/Y_s = Z^2$ and $Y_s Z = \Gamma_s$ Equation (5.26) becomes:

$$\left(\frac{\partial}{\partial z} + j\beta_T + \Gamma_s\right)(V + Z\ J_1) = 0 \tag{5.27}$$

where

$$Z = \pm \sqrt{\frac{Z_s}{Y_s}} = \pm \frac{u_0}{\epsilon\omega_p^2}\sqrt{k_T^2 + \frac{\omega_p^2}{\omega^2}(1-k_T^2)} = \pm Z_a \tag{5.28}$$

$$\Gamma_s = Y_s Z = \mp j\,\frac{\omega}{u_0}\,\frac{1}{1-k_T^2}\sqrt{k_T^2 + \frac{\omega_p^2}{\omega^2}(1-k_T^2)} = \mp j\beta_s \tag{5.29}$$

Since there are two solutions for Z and Γ_s, Equation (5.27) can be expressed as two separate equations. They are

$$\left[\frac{\partial}{\partial z} + j(\beta_T - \beta_s)\right](V + Z_a J_1) = 0 \tag{5.30}$$

$$\left[\frac{\partial}{\partial z} + j(\beta_T + \beta_s)\right](V - Z_a J_1) = 0 \tag{5.31}$$

Evaluating $\beta_T \mp \beta_s$ by replacing the right hand side of Equations (5.16) and (5.29) and comparing with Equation (5.1), one obtains that $j(\beta_T \mp \beta_s) = -\Gamma_{\pm}$. Consequently, Equations (5.30) and (5.31) can be expressed as

$$(\frac{\partial}{\partial z} - \Gamma_+)a_+ = 0 \tag{5.32}$$

$$(\frac{\partial}{\partial z} - \Gamma_-)a_- = 0 \tag{5.33}$$

where

$$a_{\pm} = K_{\pm}(V \pm Z_a J_1) \tag{5.34}$$

Equations (5.32) and (5.33) are called the normal mode form of equation of motion for the collisionless carrier stream in an extrinsic semiconductor. They describe the collisionless carrier waves in an extrinsic semiconductor in a different form from Equations (5.20) and (5.21). The quantities $a_{\pm}$ which are made up of a linear combination of the total kinetic voltage V and the a-c current density J_1 are called the normal modes of the carrier waves in an extrinsic semiconductor and $K_{\pm}$ are the proportionality constants which will be evaluated in the next section. Since the propagation constant of a_+ and a_- modes are Γ_+ and Γ_- respectively, we may call the a_+ mode the fast wave and the a_- mode the slow wave according to the statements discussed in Chapter III.

5.3.2 Evaluation of Normal Mode Amplitude Constants and Kinetic Energy Relation

It is well-known that the average power transmitted down a transmission-line is given by the time average of the product of the voltage across the line and the current flow through the line. Applying this to the transmission-line analog of the carrier waves in solids, we have the a-c power per unti cross section area as

$$P = \frac{1}{2} \text{Re}[\vec{V} \cdot \vec{J}_1^*] \tag{5.35}$$

For the purpose of evaluating the proportionality constant K_+ let us consider that only the a_+ mode is being excited along the sample; that is $a_- = 0$ or

$$Z_a = \frac{V}{J_1} \tag{5.36}$$

In such a case, the normal mode of the fast wave becomes

$$a_+ = 2K_+V \tag{5.37}$$

and the a-c power carried by the charged carriers when only the a_+ mode is excited can be written as

$$P_+ = \frac{1}{2} \frac{|V|^2}{Z_a} \tag{5.38}$$

Using Equation (5.37), the kinetic power carried by a_+ mode is

$$P_+ = \frac{1}{2} a_+a_+^* = 2K_+^2|V|^2 \tag{5.39}$$

Since $a_- = 0$, the kinetic power carried by the a_+ mode equals total a-c power of the collisionless carrier waves in solids; equating Equations (5.38) and (5.39), we have the expression for K_+ as

$$K_+ = \frac{1}{2\sqrt{Z_a}} \tag{5.40}$$

Similarly, the proportionality constant K_- for the a_- mode can be evaluated by setting $a_+ = 0$. The result is

$$K_- = \frac{1}{2\sqrt{Z_a}} \tag{5.41}$$

Therefore, the complete expression of the normal modes describing carrier waves in solids are obtained as

$$a_{\pm}(z,t) = \frac{1}{2\sqrt{Z_a}} [V(z) \pm Z_a J_1(z)] e^{j\omega t} \tag{5.42}$$

Using Equations (5.39) and (5.46), we may express the a-c power density in terms of normal modes when both fast and slow waves are present. It is

$$P = \tfrac{1}{2} \mathrm{Re}[V \cdot J_1^*] = \tfrac{1}{2}(|a_+|^2 - |a_-|^2) \tag{5.43}$$

The above equation implies that the a_+ mode carries positive kinetic power while the a_- mode carries negative kinetic power. The physical interpretation of positive and negative kinetic powers is as follows: On the average, the carrier stream carries a larger amount of kinetic energy than it carries in the d-c state when a fast mode is excited. In the other case, the carrier stream carries a smaller average kinetic energy than it carries in the d-c state when a slow mode is excited.

5.4 Normal Mode Application -- Traveling Wave Amplification of Carrier Wave in Solids

Solid state traveling-wave amplifier (STWA) has been studied by several workers recently[9-14]. Its most attractive feature is the extreme high gain, which makes it a potentially active device in a microwave integrated system. The practical difficulties are the device heating problem and the saturation of carrier drift velocity. However, with the rapid advances in solid state technology, it is feasible that the STWA might be able to operate at higher power and frequency range. The methods of analysis on the theoretical work published use either an extended classical Pierce's approach[27] or match the wave impedances on the slow wave semiconductor boundary[28]. Both approaches are rather lengthy. Quite often, some of the important aspects such as the coupling scheme, or the energy exchange between circuit and carrier wave may not be revealed explicitly. In this section, the coupled-mode approach is used to study the interaction between the carrier wave and the slow electromagnetic wave. This method appears to be simpler and clearer in describing the various possible interactions.

Here, a simple model of traveling wave amplifier is investigated. As shown in Fig. 5.1, the majority carriers in an extrinsic semiconductor drifting along a tightly coupled electromagnetic slow wave circuit are considered. The system is assumed to be lossless, that is, the real power loss due to collision between the carriers and the solid lattice, and the series and shunt resistances of the slow wave circuit are neglected. The equivalent transmission-line equations for a lossless carrier wave in an extrinsic semiconductor are given by Equations (5.24) and (5.25), and the normal modes of these carrier waves are derived in Equation (5.42) in the last section. For a lossless slow wave circuit, the transmission-line equations are

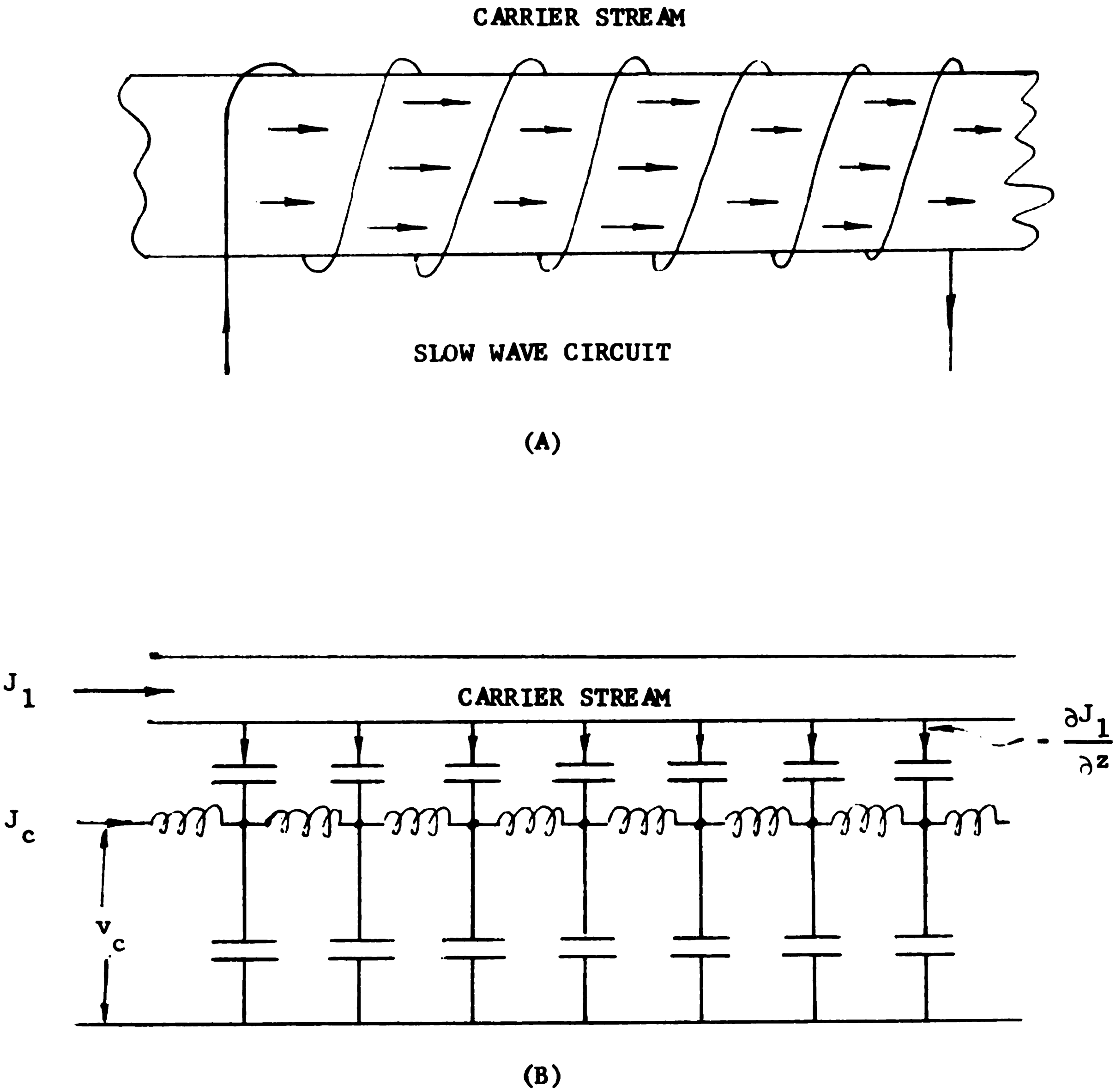


Fig. 5.1 (A) Carrier Stream Coupled to a Slow Wave Circuit

(B) Equivalent Circuit.

Carrier Stream is Capacitively Coupled to the Circuit and $-\frac{\partial J_1}{\partial z}$ is the Displacement Current Induced in the Circuit.

$$\frac{\partial V_c}{\partial z} = -j\omega L\, J_c \tag{5.44}$$

$$\frac{\partial J_c}{\partial z} = -j\omega C\, V_c \tag{5.45}$$

where the series inductance L and the shunt capacitance C are given per unit length.

The normal modes for a lossless transmission-line are[29]

$$a_{c\pm} = \frac{1}{2\sqrt{Z_c}} (V_c \pm Z_c J_c) \tag{5.46}$$

where

$$Z_c = \sqrt{\frac{L}{C}} \tag{5.47}$$

It is noted that a_{c+} is the wave propagating forwards, whereas a_{c-} is the wave propagating backwards and Z_c is the circuit characteristic impedance.

When the circuit and carrier stream are closely coupled as shown in Fig. 5.1, a displacement current will be induced in the circuit by the carrier stream; in the meantime, a force due to the circuit field will act upon the carrier stream. The modified transmission-line equations are called the coupled-mode equations which can be expressed as follows:[29]

For carrier wave:

$$\left(\frac{\partial}{\partial z} + j\beta_T\right)J_1 = -Y_s V \tag{5.48}$$

$$\left(\frac{\partial}{\partial z} + j\beta_T\right)V = -Z_s J_1 + \frac{\partial V_c}{\partial z} \tag{5.49}$$

For slow wave circuit:

$$\frac{\partial V_c}{\partial z} = -j\omega L\ J_c \tag{5.50}$$

$$\frac{\partial J_c}{\partial z} = -j\omega C\ V_c - \frac{\partial J_1}{\partial z} \tag{5.51}$$

Putting the voltage and current density in terms of normal modes, we have

$$V_c = \sqrt{Z_c}\ (a_{c+} + a_{c-}) \tag{5.52}$$

$$J_c = 1/\sqrt{Z_c}\ (a_{c+} - a_{c-}) \tag{5.53}$$

$$V = \sqrt{Z_a}\ (a_+ + a_-) \tag{5.54}$$

$$J_1 = 1/\sqrt{Z_a}(a_+ - a_-) \tag{5.55}$$

Using the above expressions together with Equations (5.1), (5.16), (5.17), (5.19) and (5.27), the coupled-mode Equations (5.48) through (5.51) become

$$(\frac{\partial}{\partial z} - \Gamma_+)a_+ - (\frac{\partial}{\partial z} - \Gamma_-)a_- = 0 \tag{5.56}$$

$$(\frac{\partial}{\partial z} - \Gamma_+)a_+ + (\frac{\partial}{\partial z} - \Gamma_-)a_- = \frac{Z_c}{Z_a}\frac{\partial}{\partial z}\ (a_{c+} + a_{c-}) \tag{5.57}$$

$$(\frac{\partial}{\partial z} + j\beta_c)a_{c+} + (\frac{\partial}{\partial z} - j\beta_c)a_{c-} = 0 \tag{5.58}$$

$$(\frac{\partial}{\partial z} + j\beta_c)a_{c+} - (\frac{\partial}{\partial z} - j\beta_c)a_{c-} = -\sqrt{\frac{Z_c}{Z_a}}\frac{\partial}{\partial z}\ (a_+ - a_-) \tag{5.59}$$

where

$$\beta_c = \omega\sqrt{LC} \tag{5.60}$$

β_c is the propagation constant for normal modes of the slow wave circuit.

Using the relation of Equations (5.56) and (5.58), the coupled-mode equations can be simplified as

$$\left(\frac{\partial}{\partial z} - \Gamma_{\pm}\right)a_{\pm} = j\,\frac{\beta_c}{2}\sqrt{\frac{Z_c}{Z_a}}\,(a_{c+} - a_{c-}) \tag{5.61}$$

$$\pm\left(\frac{\partial}{\partial z} \pm j\beta_c\right)a_{c\pm} = -\tfrac{1}{2}\sqrt{\frac{Z_c}{Z_a}}\,(\Gamma_{+}a_{+} - \Gamma_{-}a_{-}) \tag{5.62}$$

Note that the forward and backward coupled circuit modes a_{c+} and a_{c-} are not directly coupled. Similarly, the fast and the slow carrier modes a_+ and a_- are not directly coupled. Each of the fast and the slow carrier modes is directly coupled to the forward and backward circuit modes, and the circuit modes are directly coupled only to the carrier modes.

The basic concept of a solid state traveling-wave amplifier is to utilize drifting carriers in a solid surface adjacent to and interacting with a slow electromagnetic propagating circuit. There are many possible kinds of slow wave structures for STWA, typical ones are those such as helix, meander-line and interdigital circuit. A mosaic pattern was suggested by Solymar and Ash[9] and also by Hines[21]. One of the schematic representations of a STWA is shown in Fig. 5.2.

If the group and drift velocities of the circuit wave and the lossless longitudinal carrier waves are approximately synchronous, which is similar to the condition in the traveling wave amplification in beam devices, the slow circuit wave will interact strongly with moving carriers. Should this be the case, the electric field

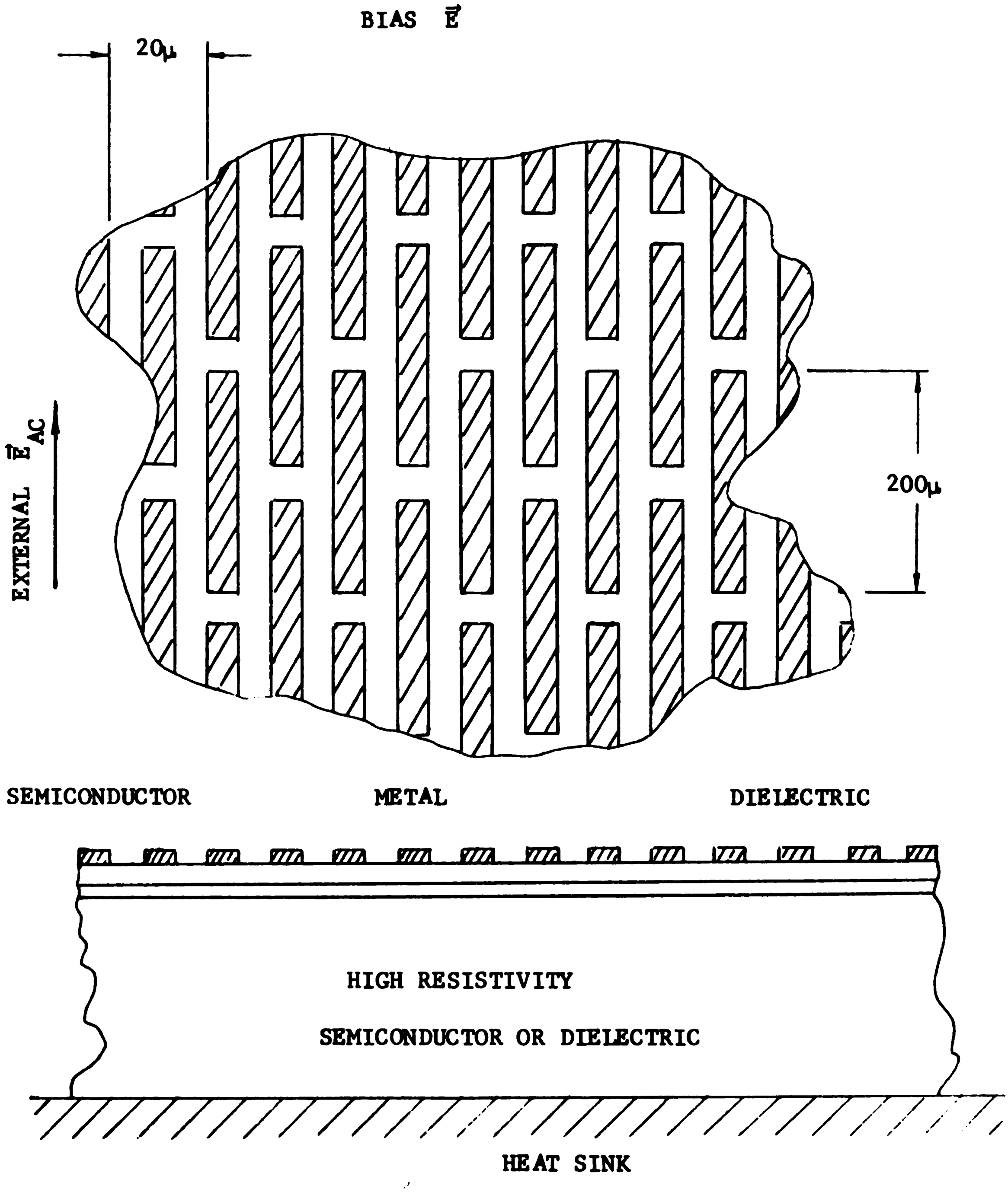


Fig. 5.2 One of the Many Possible Mosaic Patterns for Space-Harmonic Coupling Between Slow Semiconductor Space-Charge Waves and External Microwave Fields.

of the slow wave circuit slows the carrier down, and the loss of carrier kinetic energy is being transferred to the circuit wave. If the energy is continuously transferred from the drifting carriers to the slow wave, it will result in wave growth with distance along the circuit. It has been shown[29] that for a traveling-wave amplification, the forward circuit wave a_{c+} should be strongly coupled to the slow carrier wave a_-. The coupled-mode Equation (5.61) and (5.62) are then reduced to

$$\frac{\partial a_-}{\partial z} = C_{11}a_- + C_{12}a_{c+} \tag{5.63}$$

$$\frac{\partial a_{c+}}{\partial z} = C_{21}a_- + C_{22}a_{c+} \tag{5.64}$$

where

$$C_{11} = \Gamma_- \tag{5.65}$$

$$C_{12} = j\,\tfrac{1}{2}\sqrt{\frac{Z_c}{Z_a}}\,\beta_c \tag{5.66}$$

$$C_{21} = \tfrac{1}{2}\sqrt{\frac{Z_c}{Z_a}}\,\Gamma_- \tag{5.67}$$

$$C_{22} = -j\beta_c \tag{5.68}$$

Assuming that the z-dependent part of both a_- and a_{c+} have the form $e^{\Gamma_c z}$, Equations (5.63) and (5.64) become

$$(\Gamma_c - C_{11})a_- = C_{12}\,a_{c+} \tag{5.69}$$

$$(\Gamma_c - C_{22})a_{c+} = C_{21}\,a_- \tag{5.70}$$

Solving the above two equations for Γ_c and using the right hand side of Equation (5.65) through (5.68), the propagation factor is obtained as

$$\Gamma_c = \tfrac{1}{2}\left[(\Gamma_- - j\beta_c) \pm \sqrt{(\Gamma_- - j\beta_c)^2 + j4\beta_c\Gamma_-(1 + \frac{Z_c}{4Z_a})}\right] \qquad (5.71)$$

The propagation factor is generally a complex quantity. The two modes are said to be actively coupled if the Γ_c has a positive real part.

The transfer factor which is defined as the fraction of the total power transfer between modes[29] is found to be

$$F_{a_-,a_{c+}} = [1 + \frac{Z_a}{Z_c}\frac{1}{j\beta_c\Gamma_-}(\Gamma_- + j\beta_c)^2]^{-1} \qquad (5.72)$$

Close coupling occurs when two modes are synchronized and the transfer factor approaches unity; that is

$$\Gamma_- = -j\beta_c \qquad (5.73)$$

Substituting Equations (5.1) and (5.60) into Equation (5.73), the condition for synchronization is obtained as

$$u_0 = v_c(1 + \sqrt{\frac{v_T^2}{v_c^2} + \frac{\omega_p^2}{\omega^2}} \qquad (5.74)$$

where $v_c = \frac{1}{\sqrt{LC}}$ is the group velocity of the lossless slow circuit wave.

Equation (5.74) shows that in order to have the system synchronized, a higher drift velocity is needed at higher operation temperature and/or larger carrier concentration. Note that the effect of carrier concentration becomes less important if the system is operated at a higher frequency.

Maximum gain occurs when the synchronized condition given by Equation (5.73) is reached. Under such a condition, the positive real part of Γ_c is

$$[\alpha_c]_m = \tfrac{1}{2}\,\beta_c \sqrt{\frac{Z_c}{Z_a}} \tag{5.75}$$

Using Equations (5.28), (5.47), (5.60) and (5.75) the maximum gain per unit wavelength can be expressed as

$$G_m = \frac{[\alpha_c]_m}{\beta_e} = \frac{\omega_p}{2v_c}\left[\epsilon u_0\sqrt{\frac{L}{C}}\;\frac{1}{\sqrt{k_T^2 + \frac{\omega_p^2}{\omega^2}(1-k_T^2)}}\right]^{\frac{1}{2}} \tag{5.76}$$

When the system is operated at a high frequency, that is $\omega \gg \omega_p$, Equation (5.76) can be reduced to

$$G_m = \frac{u_0}{2v_c}\,\omega_p\left(\frac{\epsilon}{v_T}\right)^{\frac{1}{2}}\left(\frac{L}{C}\right)^{\frac{1}{4}} \tag{5.77}$$

A plot of the relative gain as a function of operation frequency with the thermal-to-drift velocity ratio as parameter is shown in Fig. 5.3. For a fixed k_T, the gain reaches its peak attainable value and then levels off as the operation frequency is considerably higher than the carrier plasma frequency. Fig. 5.3 also shows that a higher attainable gain can be achieved for a smaller k_T value. Therefore, it can be concluded that for a good solid state traveling wave amplifier, it is desirable to have high d-c carrier drift velocity, high frequency and low temperature operation. In reality, the highest possible drift velocity is limited by the hot-carrier effect and the lower temperature

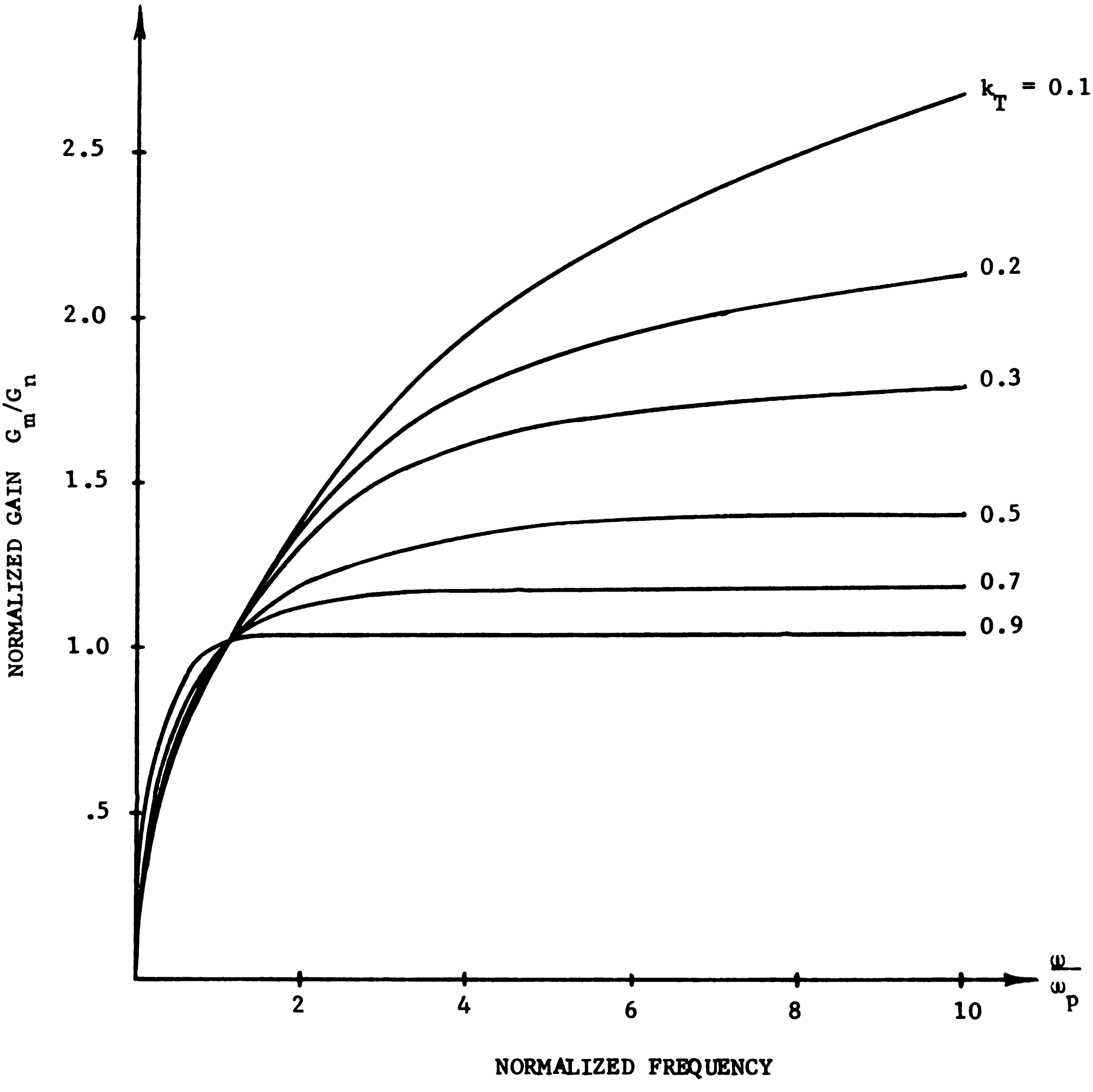


Fig. 5.3 Normalized Gain as a Function of Operation Frequency with Thermal-to-Drift Velocity Ratio as Parameter where the Gain is Normalized with Respect to

$G_n = \frac{\omega_p}{2v_c} (\varepsilon\ u_0)^{\frac{1}{2}} (\frac{L}{C})^{\frac{1}{4}}$.

operation is restricted by the complexity of experimental setup. Taking Indium Antimonide as an example, the highest drift velocity attainable before the Gunn-type instability occurs is about 1×10^7 cm/sec[30] and the lowest attainable value of k_T is about 0.7.[31]

CHAPTER VI

SUMMARY AND CONCLUSIONS

6.1 Summary and Conclusions

This investigation has attempted to provide a better understanding of the carrier waves propagating in solids due to longitudinal modulation. Restricting ourselves to the long-wavelength excitations, a classical statistical analysis is used to describe the electron and hole motion inside the solids. By several appropriate assumptions stated in Chapter II, the macroscopic equations of the carriers in solids are obtained from Maxwell's equations and the Boltzmann transport equations as the fundamental equations of the carrier stream considered as a conducting fluid. General wave equation is derived by the simplified fundamental equations to describe the wave characteristics of a stream of electrons and holes in solids. Special attention is paid to the extrinsic semiconductors since most of the commercial semiconductors belong to this type. The dispersion characteristics of the electron or hole waves propagating in these kind of semiconductors are obtained from the simplified wave equation.

A transmission-line analog of the carrier waves including the effect of an external slow wave circuit is developed by defining the kinetic voltages due to velocity and density modulation. Equivalent transmission-line circuits are constructed in terms of the kinetic voltages and the a-c electron and hole currents; besides the capacitors and inductors, the elements of these equivalent circuits include ground base transistors and ideal transformers which indicate the possible amplification and energy exchange between the carriers and the external circuit waves. The real power dissipated per unit length obtained from these equivalent transmission-line circuits checks closely with those obtained from the kinetic power theorem in Chapter IV. This analysis gives a possibility to investigate the coupling between the carriers and the surrounding slow wave circuit and to figure out the conditions of wave amplification by circuit theory provided that the equivalent transmission-line of a proper designed slow wave circuit is also developed.

The propagation characteristics of the electron or hole waves propagating in an extrinsic semiconductor have been examined in detail. It has been shown that in general two basic types of electromechanical waves exist in an extrinsic semiconductor with longitudinal modulation. When the average drift velocity of the carriers is higher than its thermal velocity, the space-charge waves are strongly excited. As the carrier thermal velocity exceeds its average drift velocity, the electroacoustic wave will become dominate. The fast and slow space-charge waves carry positive and negative kinetic power respectively. For a growing wave instability, the slow space-charge wave must be excited such that the kinetic energy of the carriers can be taken out and transferred to the circuit surrounding it. It has been shown that the collisions between the carriers and the solid lattice play the role of consuming carrier kinetic energy, since such collisions make the carriers return to a random state and cause them to lose velocity modulation. Under the condition when the space-charge waves are strongly excited, that is, $u_0 \gg v_T$, and the collision frequency is high enough, the fast space-charge wave and slow space-charge wave will emerge as one kind of carrier wave which is synchronous with the drift motion.

Starting with the fundamental equations which describe the carrier behavior in solids, an expression for real power flowing through the solids with longitudinal modulation is obtained. The result indicated that the electromagnetic power will grow along the longitudinal direction if a negative kinetic power carrying wave is excited in the interaction. This agrees with the argument that the amplification can occur when the slow space-charge wave is excited because the slow space-charge wave carries a negative kinetic power due to velocity modulation. In a special case when collisions and thermal diffusion are neglected, the result shows that the sum of the electromagnetic power and the kinetic power of velocity modulation propagated along the solid is conserved. In a degenerate case, the same result has been given by Chu[32] as the kinetic power theorem of an electron beam in vacuum.

Normal modes of the collisionless carrier waves in solids are evaluated from the equivalent transmission-line equations. Each mode is normalized by letting the a-c power carried by the carriers be equal to the product of the normal mode and its conjugated. Once the normal modes of the separated systems are defined, modes of a coupled system can easily be found by using coupled-mode theory.

The solid state traveling wave amplifier in which the slow space-charge carrier wave interacts with the surrounding lossless slow electromagnetic wave is used to demonstrate the application of the coupled-mode theory. The conditions for synchronization as well as a maximum gain expression were derived. In reality, the inevitable slow wave circuit loss, the collision effect between the charged carriers and the solid lattice and the surface effect of the solids (for example, the reduction of carrier mobility in a surface layer, which arises from random scattering of carriers at the surface) will definitely reduce the theoretical gain by a significant margin. However, since no such device has been built, its future potential is yet to be determined.

The electroacoustic wave in solids due to longitudinal modulation has never been observed or reported elsewhere. It is our belief that its general characteristics will be very much the same as those in gaseous plasma. They will certainly affect the propagation of electromagnetic waves in solids. It would be interesting to perform experiments to observe their dispersion characteristics and other phenomena such as the dipole resonance and Tonks-Dattner resonances.

The general study of longitudinal carrier waves cannot only lead to a clearer description of the existing interactions in solids, but also predict new interactions such as the possible solid state backward wave oscillator, the instability due to space-charge waves of carrier stream, the electroacoustic waves interact with background stationary plasma, and two carrier stream instability, etc.

Numerous microwave radiations from solid state materials, such as in the Gunn oscillator, the avalanche diode, and the TRAPPAT mode oscillator have been reported in the past few years. The analysis given for these devices were either too theoretical, using statistical quantum theory, or too experimental, using the experimental carrier velocity versus electric field intensity curve of the material to obtain the negative conductance characteristics. Quite often, some of the important aspects of the interaction such as the coupling scheme and energy exchange between circuit and carrier waves may not be revealed explicitly. By using the normal mode formulation developed here, most of those devices can be explained more clearly.

For the coupled mode analysis of carrier wave interactions, we have restricted our analysis to the weakly coupled systems in order to neglect some of the weakly couplings between the modes. However, for strongly coupled systems, we have to solve a more complex system. The effects of collisions and thermal diffusion upon the dispersion relation and carrier wave content can be studied in more detail with the aid of a digital computer using appropriate numerical analysis.

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