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ABSTRACT

A rigid hub-single flexible beam system is the focal point of this study. A time-dependent torque is applied at the hub to maneuver the tip position in space and time. Two approximate models are derived to describe the flexibility in the system. Approximate analytical and numerical solutions are obtained for a rectangular-pulse angular velocity using an approximate flexure model. It is shown that it is easy to eliminate the effect of any mode by a proper choice of the constant angular velocity.

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TABLE OF CONTENTS

TABLE OF FIGURES AND PLOTS

CHAPTER ONE

INTRODUCTION

In literature, a number of people are interested in the modeling and control of flexible structures. Jasinski [6], Sung [11] and Viscomi [13] developed the equations of motion of a slider crank mechanism. They started from kinematics to generate a set of two coupled equations in flexure and extension. Nachman [8] and Cannon [3] took simple rotating beams and developed the equations of motion by neglecting the effect of extension.

In the first part of this thesis, our interest was to obtain and examine the fundamental equations of motion for simple rotating beams. These equations describe the coupling between flexure and extension, and the torque applied at the hub. Consequently, these equations are of use in the control and positioning of the beam in space and time.

In the second part we were interested in one maneuvering problem using an approximate simple flexure model. The maneuvering test was confined to a certain range of angular velocities for which the model was valid. The model used was valid within 10% maximum deflection with respect to the length of the beam. Our physical system was a rigid-hub flexible beam mechanism (fig.1). A time-varying torque was applied at the hub to maneuver the beam tip position. This investigation was confined to planar motion. The experimental beam was made of aluminum and had a rectangular-cross section. The maneuvering problem consisted of rotating the hub through a finite angle with desired physical states of its tip position at the end time. For instance, a rest position of the beam is desired for accurate operations.

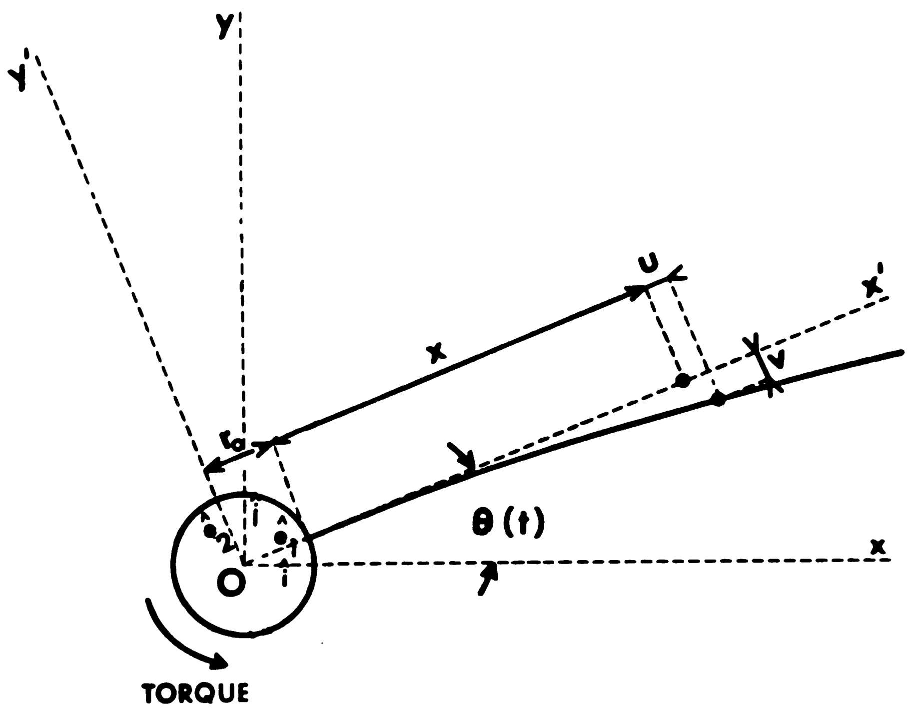


fig.1: Hub-beam system

CHAPTER TWO

EQUATIONS OF MOTION

The basic differential equations governing the axial and transverse displacements of the robot arm relative to its rotating undeformed position are developed by energy methods.

The plane mechanism (fig.1) consists of a rigid hub of radius r_0 and mass moment of inertia I_h, and a flexible beam of length L, cross-section area A (b×h), and constant material properties. The hub angular rotation is $\theta(t)$ measured counterclockwise from the x-axis. The transverse and longitudinal displacements v and u respectively, are measured with respect to the undeformed position of the beam, i.e. an observer is located at the origin O and rotates with the hub. It is assumed that plane sections remain plane during deformation. The effects of shear deformation and rotary iertia are assumed negligible. We shall consider two approximations to the physical problem.

(i) Rayleigh Beam

The extensional deformation can be simulated by measuring the distance PP'' (see fig.2) where P'' corresponds to the projection of P' on the x'-axis. According to Cannon [3] these approximations are valid for a maximum deflection less than 10% of the beam length.

If a small chunck of material is to be taken, then from (fig.2) in its undeformed configuration it lies on the x'-axis. A particle P located on the neutral axis (N.A.) moves to a new position P'. Any particle point Q at a distance s perpendicular to the neutral axis from P at its undeformed configuration moves to Q' at a distance s' normal to the neutral axis from P'.

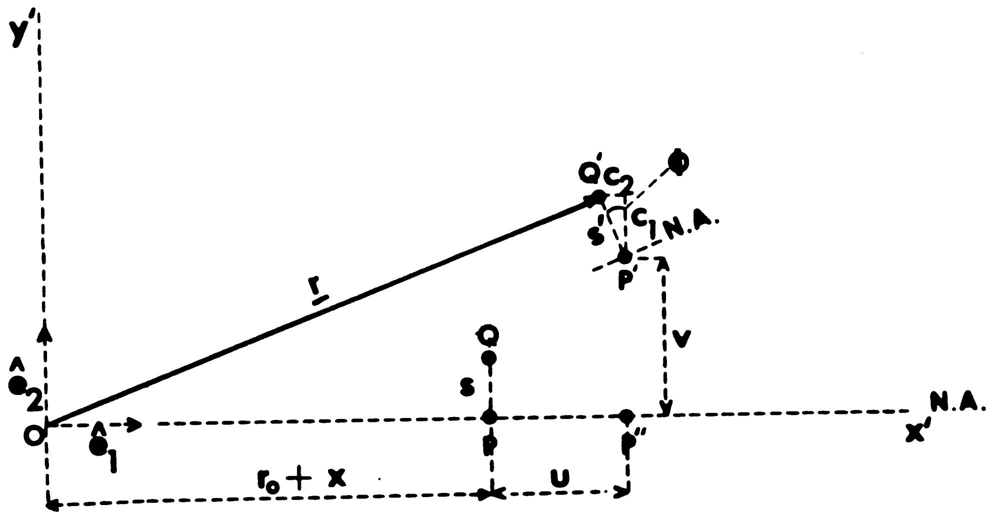


fig.2: Deflected configuration of a particle point

If the expansion and compression of any section are small during deformation, then it is reasonable to assume that s' and s are equal. Then the position vector

$$\underline{r} = \underline{R}_{Q'O} \tag{2.1}$$

can be written as

$$\underline{r}\ (x,s,t) = (r_0 + x + u - c_1)\hat{e}_1 + (v + c_2)\hat{e}_2 \tag{2.2}$$

The angle ϕ is assumed small and expressed as:

$$\phi \simeq \tan\phi \simeq \sin\phi \simeq \partial v/\partial x \tag{2.3.1}$$

and $\cos\phi \simeq 1.0$ (2.3.2)

Equation (2.3.1) is valid if the vector $\mathbf{R}_{P'O}$ is approximately parallel to the tangent at the neutral axis through P'. Then the position vector $\mathbf{r}$ is expressed as:

$$\mathbf{r}\ (x,s,t) = (r_0 + x + u - s\partial\phi/\partial x)\hat{e}_1 + (v + s)\hat{e}_2 \tag{2.4}$$

The velocity vector is obtained by differentiating (1.4) once with respect to time

$$\dot{\mathbf{r}}\ (x,s,t) = \left\{ \partial u/\partial t - s\partial^2 v/\partial x\partial t - (v + s)\dot{\theta} \right\}\hat{e}_1 + \left\{ (r_0 + x + u)\dot{\theta} - s\partial v/\partial x\dot{\theta} + \partial v/\partial t \right\}\hat{e}_2 \tag{2.5}$$

Where the differentiations of $\hat{e}_1$ and $\hat{e}_2$ with respect to time are $\dot{\theta}\ \hat{e}_2$ and $-\dot{\theta}\ \hat{e}_1$ respectively. The energy methods are then employed. The total kinetic energy of the system is:

$$T = T_{beam} + T_{hub} \tag{2.6}$$

where

$$T_{beam} = 0.5\rho\int_{-b/2}^{b/2}\int_{0}^{L'} \dot{\mathbf{r}} \cdot \dot{\mathbf{r}}\ dx\ (h\ ds) \tag{2.6.1}$$

$$T_{hub} = 0.5I_h\dot{\theta}^2 \tag{2.6.2}$$

h is the height of the beam cross section and L' corresponds to the deformed arc length of the beam. L' is assumed to be equal to the undeformed beam length L. The potential energy depends on the deformations u and v. The total potential energy is:

$$V = 0.5EA\int_0^L (\partial u/\partial x)^2 \, dx + 0.5EI\int_0^L (\partial^2 v/\partial x^2)^2 \, dx \tag{2.7}$$

where the first term corresponds to the total strain energy due to compression or tension, and the second term represents the total bending energy. The work done by the torque, τ, at the hub is expressed as:

$$W = \tau.\theta \tag{2.8}$$

Next, Hamilton's principle is employed where the variation is taken into the extensional and flexural deflections u and v and the rotating angle θ.

$$\int_{t_1}^{t_2} (\delta T - \delta V + \delta W) \, dt = 0 \tag{2.9}$$

The cross section area and the moment of inertia come from the following expressions

$$A = \int_{-b/2}^{b/2} h \, ds = bh \tag{2.10}$$

$$I = \int_{-b/2}^{b/2} hs^2 \, ds = hb^3/12 \tag{2.11}$$

The coupled equations of motion are then:

$$EA\, \partial^2 u/\partial x^2 + \rho A \left[(r_0+x+u)\dot{\theta}^2 + v\ddot{\theta} + 2\dot{\theta}\, \partial v/\partial t - \partial^2 u/\partial t^2\right] = 0 \qquad (2.12.1)$$

$$EI\, \partial^4 v/\partial x^4 - \rho I\, \partial^4 v/\partial x^2 \partial t^2 + \rho I \dot{\theta}^2\, \partial^2 v/\partial x^2$$

$$+ \rho A \left[-v\dot{\theta}^2 + (r_0+x+u)\ddot{\theta} + 2\dot{\theta}\, \partial u/\partial t + \partial^2 v/\partial t^2\right] = 0 \quad (2.12.2)$$

$$\tau(t) = \left\{ \rho IL + I_h + \int_0^L \left[\rho A v^2 + \rho A(r_0+x+u)^2 + \rho I(\partial v/\partial x)^2\right]dx \right\}\ddot{\theta}$$

$$\left\{ \int_0^L \left[2\rho A v \partial v/\partial t + 2\rho A(r_0+x+u)\partial u/\partial t + \rho I \partial v/\partial x\ \partial^2 v/\partial t \partial x\right]dx \right\}\dot{\theta}$$

$$\left\{ \int_0^L \left[-\rho A \partial^2 u/\partial t^2\ v + \rho I \partial^3 v/\partial x \partial t^2 + \rho A(r_0+x+u)\partial^2 v/\partial t^2\right]dx \right\}$$

(2.12.3)

The corresponding boundary conditions are:

$$u(0,t) = 0 \qquad (2.12.4)$$

$$\partial u/\partial x(L,t) = 0 \qquad (2.12.5)$$

$$v(0,t) = 0 \qquad (2.12.6)$$

$$\partial v/\partial x(0,t) = 0 \qquad (2.12.7)$$

$$\partial^2 v/\partial x^2(L,t) = 0 \qquad (2.12.8)$$

$$\partial^3 v/\partial x^3 (L,t) = 0 \qquad (2.12.9)$$

$$\dot{\theta}^2 \partial v/\partial x(L,t) - \ddot{\theta} - \partial^3 v/\partial x \partial t^2 (L,t) = 0 \qquad (2.12.10)$$

where (2.12.10) is a natural boundary condition. The Rayleigh beam model is valid for the case where the width of the beam is not small.

(ii) Bernoulli-Euler Approximation

In this case, the width is considered to be small, and in each section all particles have the same velocity. Hence the new position vector $\underline{r}$ for a particle point on the neutral axis becomes

$$\underline{r}\ (x,t) = (r_0 + x + u)\hat{e}_1 + v\hat{e}_2 \qquad (2.13)$$

The derivation of the governing equations of motion is accomplished in the same manner as in case (i).The final equations are:

$$EA\ \partial^2 u/\partial x^2 + \rho A\left[\ (r_0+x+u)\dot{\theta}^2 + v\ddot{\theta} + 2\dot{\theta}\ \partial v/\partial t - \partial^2 u/\partial t^2\ \right] = 0 \quad (2.14.1)$$

$$EI\ \partial^4 v/\partial x^4 + \rho A\left[\ -v\dot{\theta}^2 + (r_0+x+u)\ddot{\theta} + 2\dot{\theta}\ \partial u/\partial t + \partial^2 v/\partial t^2\ \right] = 0 \quad (2.14.2)$$

$$\tau(t) = \left\{ I_h + \int_0^L \left[\rho A v^2 + \rho A(r_0+x+u)^2\right]dx \right\}\ddot{\theta}$$

$$+\left\{ \int_0^L\left[\ 2\rho A v \partial v/\partial t + 2\rho A(r_0+x+u)\partial u/\partial t\ \right]dx \right\}\dot{\theta}$$

$$+\left\{ \int_0^L\left[\ -\rho A \partial^2 u/\partial t^2 v + \rho A(r_0+x+u)\partial^2 v/\partial t^2\right]dx \right\} \qquad (2.14.3)$$

The corresponding boundary conditions are:

$$u(0,t) = 0 \tag{2.14.4}$$

$$\partial u/\partial x(L,t) = 0 \tag{2.14.5}$$

$$v(0,t) = 0 \tag{2.14.6}$$

$$\partial v/\partial x(0,t) = 0 \tag{1.14.7}$$

$$\partial^2 v/\partial x^2 (L,t) = 0 \tag{2.14.8}$$

$$\partial^3 v/\partial x^3 (L,t) = 0 \tag{2.14.9}$$

Both approximations include two coupled equations in flexure and extension, and a torque dependent on u, v, θ, and the system's parameters.

The rigid body motion can be deduced from equations (2.14.1) through (2.14.3) by dividing equation (2.14.1) and (2.14.2) by Young's modulus E, then letting u and v go to zero and E go to infinity. The first two equations (2.14.1) and (2.14.2) become identities and (2.14.3) becomes of the form

$$\tau(t) = I_t \ddot{\theta} \tag{2.15}$$

$$\text{where } I_t = I_h + \rho A \int_0^L (r_0+x)^2 dx \tag{2.16}$$

The decoupled equations can be derived by going back to the position vector (2.13), set u to zero and get the simple flexure model in the same manner as case (i). The simple extension is obtained by setting v to zero. In equation (2.14.1) terms involving v and its derivative are treated as internal forces exciting the extensional vibration of the beam and vice versa in equation (2.14.2). The terms $\rho A(r_0 + x)\dot{\theta}^2$ and $\rho A(r_0 + x)\ddot{\theta}$ represent body forces in the first and second field equations respectively.

CHAPTER THREE

SIMPLE FLEXURE

It is difficult to get an analytical solution to the coupled equations of motion (2.14). In this Chapter, we shall deduce a simple approximate model of flexure. The equations of motion are obtained by setting u to zero in equation (2.13), writing the kinetic energy and potential energy expressions and using Hamilton's principle. This leads to the equations of motion

$$EI\ \partial^4 v/\partial x^4 + \rho A\left[-v\dot{\theta}^2 + (r_0+x)\ddot{\theta} + \partial^2 v/\partial t^2 \right] = 0 \tag{3.1.1}$$

$$\tau(t) = \left\{ I_h + \rho A\int_0^L\left[v^2 + (r_0+x)^2\right]dx \right\}\ddot{\theta} + \left\{ 2\rho A\int_0^L\left[v\partial v/\partial t\right]dx \right\}\dot{\theta}$$

$$+\left\{ \rho A\int_0^L\left[(r_0+x)\partial^2 v/\partial t^2\right]dx \right\} \tag{3.1.2}$$

The boundary conditions are

$$v(0,t) = \partial v/\partial x(0,t) = \partial^2 v/\partial x^2(L,t) = \partial^3 v/\partial x^3(L,t) = 0 \tag{3.1.3}$$

To generalize this problem, nondimensional equations of motion will be used. The nondimensional variables and parameters are described as the following

$$x^* = x/L \tag{3.2.1}$$

$$t^* = t\sqrt{E/\rho L^2} \tag{3.2.2}$$

$$v^* = v/L \tag{3.2.3}$$

$$r_0^* = r_0/L \tag{3.2.4}$$

$$\dot{\theta}^* = \dot{\theta}/\sqrt{E/\rho L^2} \tag{3.2.5}$$

$$\ddot{\theta}^* = \ddot{\theta}/(E/\rho L^2) \tag{3.2.6}$$

$$I^* = I/AL^2 \tag{3.2.7}$$

$$\tau^* = \tau/AEL \tag{3.2.8}$$

$$I_h^* = I_h/\rho AL^3 \tag{3.2.9}$$

Equations (3.1.1) and (3.1.2) then reduce to the following nondimensional boundary value problems

$$I^*\partial^4 v^*/\partial x^{*4} - v^*\dot{\theta}^{*2} + (r_0^*+x^*)\ddot{\theta}^* + \partial^2 v^*/\partial t^{*2} = 0 \tag{3.3.1}$$

$$\tau^*(t^*) = \left\{ I_h^* + \int_0^1 \left[v^{*2} + (r_0^*+x^*)^2 \right] dx^* \right\} \ddot{\theta}^*$$

$$+ \left\{ 2\int_0^1 \left[v^*\partial v^*/\partial t^* \right] dx^* \right\} \dot{\theta}^*$$

$$+ \left\{ \int_0^1 \left[(r_0^*+x^*)\partial^2 v^*/\partial x^{*2} \right] dx^* \right\} \tag{3.3.2}$$

The associated boundary conditions are:

$$v^*(0,t^*) = 0 \tag{3.3.3}$$

$$\partial v^*/\partial x^*(0,t^*) = 0 \tag{3.3.4}$$

$$\partial^2 v^*/\partial x^{*2}(1,t^*) = 0 \tag{3.3.5}$$

$$\partial^3 v^*/\partial x^{*3}(1,t^*) = 0 \tag{3.3.6}$$

Without the body force term, equation (3.3.1) is seperable. Let

$$v^*(x^*,t^*) = \Phi(x^*)q(t^*) \tag{3.4}$$

Then the eigenvalue problem is:

$$\Phi'''' - \beta^4\Phi = 0 \tag{3.5.1}$$

$$\Phi(0) = \Phi'(0) = \Phi''(1) = \Phi'''(1) = 0 \tag{3.5.2}$$

This eigenvalue problem is the same for a fixed cantilever beam. The solution of equations (3.5.1)-(3.5.2) is

$$\Phi_n(x^*) = \cosh\beta_n x^* - \cos\beta_n x^* - e_n(\sinh\beta_n x^* - \sin\beta_n x^*) \tag{3.6.1}$$

where

$$e_n = \frac{\cosh\beta_n + \cos\beta_n}{\sinh\beta_n + \sin\beta_n} \tag{3.6.2}$$

$$\cosh\beta_n \cos\beta_n = -1 \qquad n = 1,2,3,\ldots \tag{3.6.3}$$

Equations (3.6.1) and (3.6.3) are expressions of the problem eigenfunctions and eigencondition respectively.

By expanding the body force $(r_0^* + x^*)\ddot{\theta}^*$ about the eigenfuntions Φ_n's, an infinite set of ordinary differential equations results:

$$\ddot{q}_n + (w_n^{*2} - \dot{\theta}^{*2})q = -a_n\ddot{\theta}^* \qquad n = 1,2,3,\ldots \tag{3.7}$$

where

$$w_n^* = \beta_n^2/\sqrt{\bar{I}^*} \tag{3.8}$$

$$a_n = \frac{2(1 + r_0^*\,\beta_n\,e_n)}{\beta_n^2} \qquad n = 1,2,3,\ldots \tag{3.9}$$

The expansion of the body force does not lead to a uniform convergence of the series to $(r_0^* + x^*)\ddot{\theta}^*$. This is because the boundary condition at $x^* = 0$ is not consistent with the eigenfunctions $\Phi_n(x^*)$ which vanish at $x^* = 0$. However, the series represents a good approximation to the body force function away from $x^* = 0$.

CHAPTER FOUR

RECTANGULAR PULSE ANGULAR VELOCITY

In this case the angular velocity is a nonzero constant during the maneuvering time and zero after the final time. Mathematically, the angular velocity and acceleration are written as:

$$\dot{\theta}^*(t^*) = \dot{\theta}^*_m \left[H(t^*) - H(t^*-t^*_e) \right] \tag{4.1}$$

$$\ddot{\theta}^*(t^*) = \dot{\theta}^*_m \left[\delta(t^*) - \delta(t^*-t^*_e) \right] \tag{4.2}$$

where H and δ are the heaviside step and Dirac-delta functions respectively. Using zero initial conditions, i.e. the beam is initially at rest, the set of ordinary differential equations (3.7) are written as:

$$\ddot{q}_n + (w_n^{*2} - \dot{\theta}^{*2}(t^*))q_n = -a_n\ddot{\theta}^*(t) \tag{4.3.1}$$

with

$$q_n(0) = dq/dt^*(0) = 0 \tag{4.3.2}$$

By using an asymptotic approach, the nonhomogeneous term can be translated into the initial conditions. First (4.3.1) is integrated from time zero to ϵ

$$\int_0^{\epsilon} \ddot{q}_n(t^*)\, dt^* + w_n^{*2}\int_0^{\epsilon} q_n(t^*)\, dt^*$$

$$-\dot{\theta}_m^{*2}\int_0^{\epsilon}\Big[H(t^*) - H(t^* - t_e^*)\Big]^2 q_n(t^*)\, dt^*$$

$$= -a_n\dot{\theta}_m^*\int_0^{\epsilon}\Big[\delta(t^*) - \delta(t^* - t_e^*)\Big]\, dt^* \qquad (4.4.1)$$

which can be reduced to (as ϵ goes to zero)

$$\dot{q}_n(0^+) = -a_n\dot{\theta}_m^* \qquad (4.4.2)$$

Therefore the new problem becomes as:

$$\ddot{q}_n(t^*) + (w_n^{*2} - \dot{\theta}_m^{*2})q_n(t^*) = 0 \qquad 0 < t^* < t_e^* \qquad (4.5.1)$$

$$q_n(0) = 0 \qquad (4.5.2)$$

$$dq_n/dt^*(0) = -a_n\dot{\theta}_m^* \qquad (4.5.3)$$

The solution is

$$q_n(t^*) = -\frac{a_n\dot{\theta}_m^* \sin\kappa_n t^*}{\kappa_n} \qquad 0 < t^* < t_e^* \qquad (4.6.1)$$

where $\kappa_n = (w_n^{*2} - \dot{\theta}_m^{*2})^{1/2}$ $\qquad n = 1,2,3,\ldots \qquad (4.6.2)$

Equation (4.6.1) is valid only when $\dot{\theta}_m^* < w_n^*$ for any $n = 1,2,3,\ldots$

The same asymptotic approach is used at time t_e^*, where equation (4.3.1) is integrated from time $t_e^* - \epsilon$ to $t_e^* + \epsilon$. By letting ϵ go to zero and replacing q_n by r_n, the final equation becomes:

$$\ddot{r}_n + w_n^{*2} r_n = 0 \qquad t^* > t_e^* \qquad (4.7.1)$$

$$r_n(t_e^{*+} - t_e^*) = q_n(t_e^{*-}) = -\frac{a_n \dot{\theta}_m^* \sin\kappa_n t_e^*}{\kappa_n} \qquad (4.7.2)$$

$$dr_n/dt^*(t_e^{*+} - t_e^*) = dq_n/dt^*(t_e^{*-}) + a_n \dot{\theta}_m^*$$

$$= a_n \dot{\theta}_m^* \left[1 - \cos\kappa_n t_e^*\right] \qquad (4.7.3)$$

Therefore:

$$r_n(t^*) = \left[-\frac{a_n \dot{\theta}_m^* \sin\kappa_n t_e^*}{\kappa_n} \right] \cos w_n^*(t^* - t_e^*)$$

$$+\left[-\frac{a_n \dot{\theta}_m^*}{w_n^*}(1 - \cos\kappa_n t_e^*) \right] \sin w_n^*(t^* - t_e^*) \qquad t^* > t_e^* \qquad (4.8)$$

Therefore the formal solution of the initial boundary value problem (3.3) is :

$$v^*(x^*, t^*) = \sum_{n=1}^{\infty} \left\{ q_n(t^*) \left[H(t^*) - H(t^* - t_e^*) \right] \right.$$

$$\left. + r_n(t^*) \left[H(t^* - t_e^*) \right] \right\} \Phi_n(x^*) \qquad (4.9)$$

where $q_n(t^*)$, $r_n(t^*)$ and $\Phi_n(x^*)$ are defined in equations (4.6), (4.8) and (3.6.1) respectively.

Equation (4.9) is a generalized or approximate solution because of the nonuniform convergence of the body force series. This equation is valid as long as the Bernoulli-Euler approximation is not violated. The dynamics of the beam during maneuver is not very important. However, the dynamics after the final time are the focal point of the maneuvering problem. The latter consists of examining the behavior of $r_n(t^*)$ at time t_e^* and thereafter. A very interesting point to make is that the term by term in the series solution (4.9) can be made zero by making κ_n an integer multiple of $2\pi/t_e^*$, i.e.

$$\kappa_n = 2k\pi/t_e^* \qquad k=1,2,3,... \tag{4.10}$$

solving for $\dot{\theta}_m^* = (w_n^{*2} - 4k^2\pi^2/t_e^{*2})^{1/2}$ (4.11)

The relation between $\dot{\theta}_m^*$ and the final time t_e^* is

$$\dot{\theta}_m^* = \alpha/t_e^* \tag{4.12}$$

where α is the maneuvering angle. For example, if $\dot{\theta}_m^*$ is to be made such that there is no contribution from the first mode, then

$$\dot{\theta}_m^* = (w_1^{*2} - 4k^2\pi^2/t_e^{*2})^{1/2} = \alpha/t_e^* \qquad k = 1,2,3,... \tag{4.13}$$

The nondimensional natural frequency w_1^* can be written as:

$$w_1^* = 2\pi/T_1^* \tag{4.14}$$

where T_1^* is the first nondimensional natural period.

Introduce (4.14) into (4.13) and solve for t_e^*, then

$$t_e^* = T_1^* \left(4\pi^2 k^2 + \alpha^2\right)^{1/2} / 2\pi \qquad k = 1,2,3,\ldots \qquad (4.15)$$

Therfore t_e^* corresponds to an infinite set of critical final times such that the first term does not have any contribution to the free vibration response.

CHAPTER FIVE

NUMERICAL ANALYSIS

In this section, a general numerical scheme shall be constructed to solve the simple flexure problem. This scheme will be useful for any time-dependent angular velocity and acceleration. The nondimensional equation of motion (3.3.1) shall be used. The first term in this equation is approximated by a central difference on space and averaged on time between the previous and future times. The second and third terms are evaluated at the present time, and the fourth is approximated by a central difference. The number of steps in space and time are N and M respectively, where M is the number of time steps up to the final time t_e^*, i.e.

$$\Delta x^* = \frac{1}{N} \tag{5.1.1}$$

$$\Delta t^* = \frac{t_e^*}{M} \tag{5.1.2}$$

The finite difference approximation of the nondimensional flexure equation is given by

$$\frac{I^*}{\Delta x^{*4}} \left\{ \frac{1}{2} \left[v^{*j+1}_{i+2} + v^{*j-1}_{i+2} \right] - \frac{4}{2} \left[v^{*j+1}_{i+1} + v^{*j-1}_{i+1} \right] + \frac{6}{2} \left[v^{*j+1}_{i} + v^{*j-1}_{i} \right] \right.$$

$$-\frac{4}{2}\left[v^{*j+1}_{i-1} + v^{*j-1}_{i-1} \right] + \frac{1}{2}\left[v^{*j+1}_{i-2} + v^{*j-1}_{i-2} \right] \Big\}$$

$$+ (r_0^* + i\Delta x^*)\ddot{\theta}^*(j\Delta t^*) - v^{*j}_{i}\,\dot{\theta}^{*2}(j\Delta t^*)$$

$$+ \frac{1}{\Delta t^{*2}}\left\{ v^{*j+1}_{i} + v^{*j-1}_{i} - 2v^{*j}_{i} \right\} = 0 \tag{5.2}$$

(5.2) can be arranged as the following:

$$v^{*j+1}_{i+2} - 4v^{*j+1}_{i+1} + \left(6 + \frac{2\Delta x^{*4}}{I^*\Delta t^{*2}} \right)v^{*j+1}_{i} - 4v^{*j+1}_{i-1} + v^{*j+1}_{i-2}$$

$$= 2\,\frac{\Delta x^{*4}}{I^*\Delta t^{*2}}\left(2v^{*j}_{i} - v^{*j-1}_{i} \right) + 2\,\frac{\Delta x^{*4}}{I^*}\Big[-(r_0 + i\Delta x^*)\ddot{\theta}^*(j\Delta t^*)$$

$$+ v^{*j}_{i}\,\dot{\theta}^{*2}(j\Delta t^*) \Big] - \Big\{ v^{*j-1}_{i+2} - 4v^{*j-1}_{i+1} + 6v^{*j-1}_{i} - 4v^{*j-1}_{i-1}$$

$$+ v^{*j-1}_{i-2} \Big\} \tag{5.3.1}$$

where the terms of the left side of the equal sign are the future time unknowns, and i and j are defined as integers corresponding to space and time respectively. The approximated boundary and initial conditions are:

$$v^{*j}_{0} = 0 \tag{5.3.2}$$

$$v^{*j}_{1} = 0 \qquad (5.3.3)$$

$$v^{*j}_{N+1} + v^{*j}_{N-1} - 2v^{*j}_{N} = 0 \qquad (5.3.4)$$

$$v^{*j}_{N+2} - 3v^{*j}_{N+1} + 3v^{*j}_{N} - v^{*j}_{N-1} = 0 \qquad (5.3.5)$$

$$v^{*0}_{i} = 0 \qquad (5.3.6)$$

$$v^{*0}_{i} - v^{*-1}_{i} = 0 \qquad (5.3.7)$$

where v^{*-1}_{i} is a fictitious point at $t^{*} = -\Delta t^{*}$. The combination of equations (5.3.1)-(5.3.5) constitute a set of linear algebraic equations which are to be solved simultaneously at each time step. For progarmming convenience the linear equations are written in a matrix form:

$$\underline{M}\, \underline{v}^{*j}_{k} = \underline{B} \qquad (5.4.1)$$

where

$$\underline{v}^{*j}_{k} = \left[\underline{v}^{*M}_{2} \quad \underline{v}^{*M}_{3} \quad \underline{v}^{*M}_{4} \quad \cdots \quad \underline{v}^{*M}_{N} \right]^{T} \qquad (5.4.2)$$

$M = j+1,\ j,\ j-1$

$$\underline{B} = 2\frac{\Delta x^{*4}}{I^*}\left[-\frac{(r_0^* + I^*\Delta x^*)}{} \ddot{\theta}^*(j\Delta t^*) + \underline{v}_i^{*j}\,\dot{\theta}^{*2}(j\Delta t^*)\right]$$

$$+ 2\frac{\Delta x^{*4}}{I^*\Delta t^{*2}}\left[2\underline{v}_i^{*j} - \underline{v}_i^{*j-1}\right] - \underline{G} \qquad (5.4.3)$$

where $i = 2,3,\ldots,N$

$j = 0,1,2,3,\ldots,M,M+1,\ldots$

and

$$\underline{G} = \underline{v}_{i+2}^{*j-1} - 4\underline{v}_{i+1}^{*j-1} + 6\underline{v}_i^{*j-1} - 4\underline{v}_{i-1}^{*j-1} + \underline{v}_{i-2}^{*j-1} \qquad (5.4.4)$$

The boundary conditions (5.3.4) and (5.3.5) are accounted for in (5.4.4) when i is equal N-1 or N.

The matrix $\underline{M}$ is of the following format

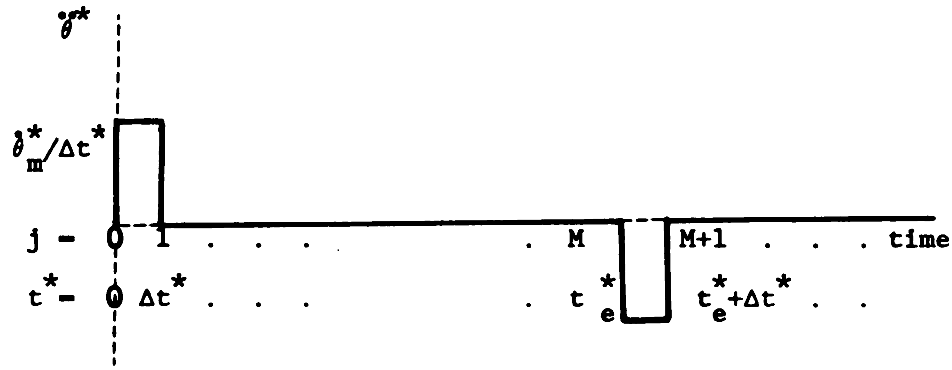
$$\underline{M} = \begin{bmatrix} \alpha & -4 & 1 & 0 & 0 & 0 & 0 & \cdots & 0 & 0 & 0 & 0 & 0 \\ -4 & \alpha & -4 & 1 & 0 & 0 & 0 & \cdots & 0 & 0 & 0 & 0 & 0 \\ 1 & -4 & \alpha & -4 & 1 & 0 & 0 & \cdots & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & -4 & \alpha & -4 & 1 & 0 & \cdots & 0 & 0 & 0 & 0 & 0 \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & & \cdot & \cdot & \cdot & \cdot & \cdot \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & \cdots & 1 & -4 & \alpha & -4 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & \cdots & 0 & 1 & -4 & \alpha-1 & -2 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & \cdots & 0 & 0 & 1 & -2 & \alpha-5 \end{bmatrix} \qquad (5.4.5)$$

where $\alpha = 6 + 2 \dfrac{\Delta x^{*4}}{I^{*}\Delta t^{*2}}$ (5.4.6)

and $\underset{\sim}{M}$ is an (N-1)×(N-1) matrix. The matrix $\underset{\sim}{M}$ accounts for the boundary conditions at the fixed and free ends.

Rectangular-Pulse-Angular Velocity

The above scheme is applied to a rectangular-pulse-angular velocity. The expression (4.2) can be approximated by



fig(3): Impulse approximation

If Δt^* is chosen as small as possible, then the impulses at $t^*=0$ and $t^*=t_e^*$ can be approximated by a step function of duration Δt^* and magnitude $\frac{\dot{\theta}_m^*}{\Delta t^*}$. Therefore, the angular acceleration is expressed as:

$$\ddot{\theta}^*(j\Delta t^*) = \begin{cases} \dot{\theta}_m^* & \text{if } j = 0 \\ 0 & \text{if } 0 < j < M \\ -\dot{\theta}_m^* / \Delta t^* & \text{if } j = M \\ 0 & \text{if } j > M \end{cases} \tag{5.5}$$

The angular velocity is defined as:

$$\dot{\theta}^*(j\Delta t^*) = \begin{cases} 0 & \text{if } j = 0 \\ \dot{\theta}_m^* & \text{if } 0 < j < M \\ 0 & \text{if } j = M \\ 0 & \text{if } j > M \end{cases} \tag{5.6}$$

The numerical scheme above is constructed using an implicit method. We shall study the stability of this scheme:

<u>Stability</u>

$$\text{Let } \underline{v}_i^{*j} = V_0 w^j e^{iK\hat{j}} \tag{5.7}$$

where $\hat{j} = \sqrt{-1}$

Substitute (5.7) into equation (5.2)

$$V_0 w \left\{ w^j e^{(i+2)K\hat{j}} - 4w^j e^{(i+1)K\hat{j}} + (6 + 2\alpha) w^j e^{iK\hat{j}} \right.$$

$$\left. - 4w^j e^{(i-1)K\hat{j}} + w^j e^{(i-2)K\hat{j}} \right\}$$

$$= 2\alpha V_0 \left(2w^j e^{iK\hat{j}} - w^{j-1} e^{iK\hat{j}} \right) + 2\alpha \dot{\theta}_m^{*2} \Delta t^{*2} V_0 w^j e^{iK\hat{j}}$$

$$-V_0 w^{-1} \left\{ w^j e^{(i+2)K\hat{j}} - 4w^j e^{(i+1)K\hat{j}} + 6w^j e^{iK\hat{j}} \right.$$

$$\left. -4w^j e^{(i-1)K\hat{j}} + w^j e^{(i-2)K\hat{j}} \right\} \tag{5.8}$$

Note that (5.8) corresponds to the homogeneous field equation of (5.2). Dividing through by $w^j e^{iK\hat{j}}$ and using the following identities:

$$\cos K = \frac{e^{K\hat{j}} + e^{K\hat{j}}}{2} \tag{5.9}$$

$$\sin K = \frac{e^{K\hat{j}} - e^{-K\hat{j}}}{2} \tag{5.10}$$

Then equation (5.8) becomes:

$$\left\{ 2(1 - \cos K)^2 + \alpha \right\} w^2 - 2\left\{ \alpha + \alpha \dot{\theta}_m^{*2} \Delta t^{*2}/2 \right\} w$$

$$+ \left\{ 2(1 - \cos K)^2 + \alpha \right\} = 0 \tag{5.11}$$

Let $\beta = 2(1 - \cos K)^2$ (5.12)

$$\alpha = \frac{\Delta x^{*4}}{I^{*}\Delta t^{*2}} \tag{5.13}$$

$$\gamma = \alpha(1 + \frac{\dot{\theta}_m^{*2}}{2}\ \Delta t^{*2}) \tag{5.14}$$

Then the roots of equation (5.11) are

$$w_{1,2} = \frac{\gamma}{\alpha + \beta} \pm \left[\left\{ \frac{\gamma}{\alpha + \beta} \right\}^2 - 1 \right]^{1/2} \tag{5.15}$$

For small angular velocity γ is approximately equal to α. Then it is easy to see that the term under the radical sign in equation (5.15) is negative. Therefore, the roots are complex and conjugate.

$$w_{1,2} = \frac{\gamma}{\alpha + \beta} \pm \hat{j} \left[1 - (\frac{\gamma}{\alpha + \beta})^2 \right]^{1/2} \tag{5.16}$$

It can be shown that the magnitude of w_1 and w_2 are

$$| w |^2 = 1 \tag{5.17}$$

Equation (5.17) proves that the scheme used is unconditionally stable, i.e. the stability of the scheme is independent of the choice of space and time step sizes.

CHAPTER SIX

NUMERICAL RESULTS

The nondimensional parameters chosen in the computer programs are the following

$I^* = 1.0288\ 10^{-7}$ (nondimensional moment of inertia of the beam)

$I_h^* = 0.091146$ (nondimensional mass moment of inertia of the hub)

$r_o^* = 0.125$ (nondimensional hub radius)

$L^* = 1$ (nondimensional beam length)

$\alpha = \pi/2$ (angle of rotation)

$t_e^* = 33{,}457.35$ (nondimensional final time)

Physical properties of, for example, an aluminum beam

$\rho = 2710\ kg/m^3$ (aluminum density)

$E = 71\ 10^9\ N/m^2$ (aluminum Young's modulus)

$b = 8.467\ 10^{-4}$ m (width)

$h = 1.905\ 10^{-2}$ m (heigth)

$L = 0.762$ m (length)

$A = 1.613\ 10^{-5}\ m^2$ (area)

$I = 9.6361\ 10^{-13}\ m^4$ (beam moment of inertia)

$I_h = 1.7628\ 10^{-3}\ kg/m^2$ (hub mass moment of inertia)

$r_0 = 0.09525$ m (hub radius)

The maximum static deflection (when deflected about its z-axix under its own inertia) = L/3

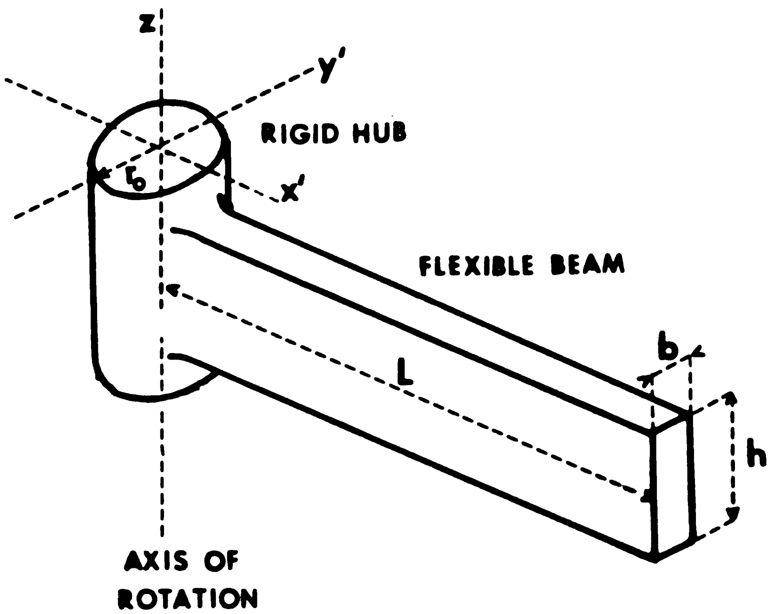
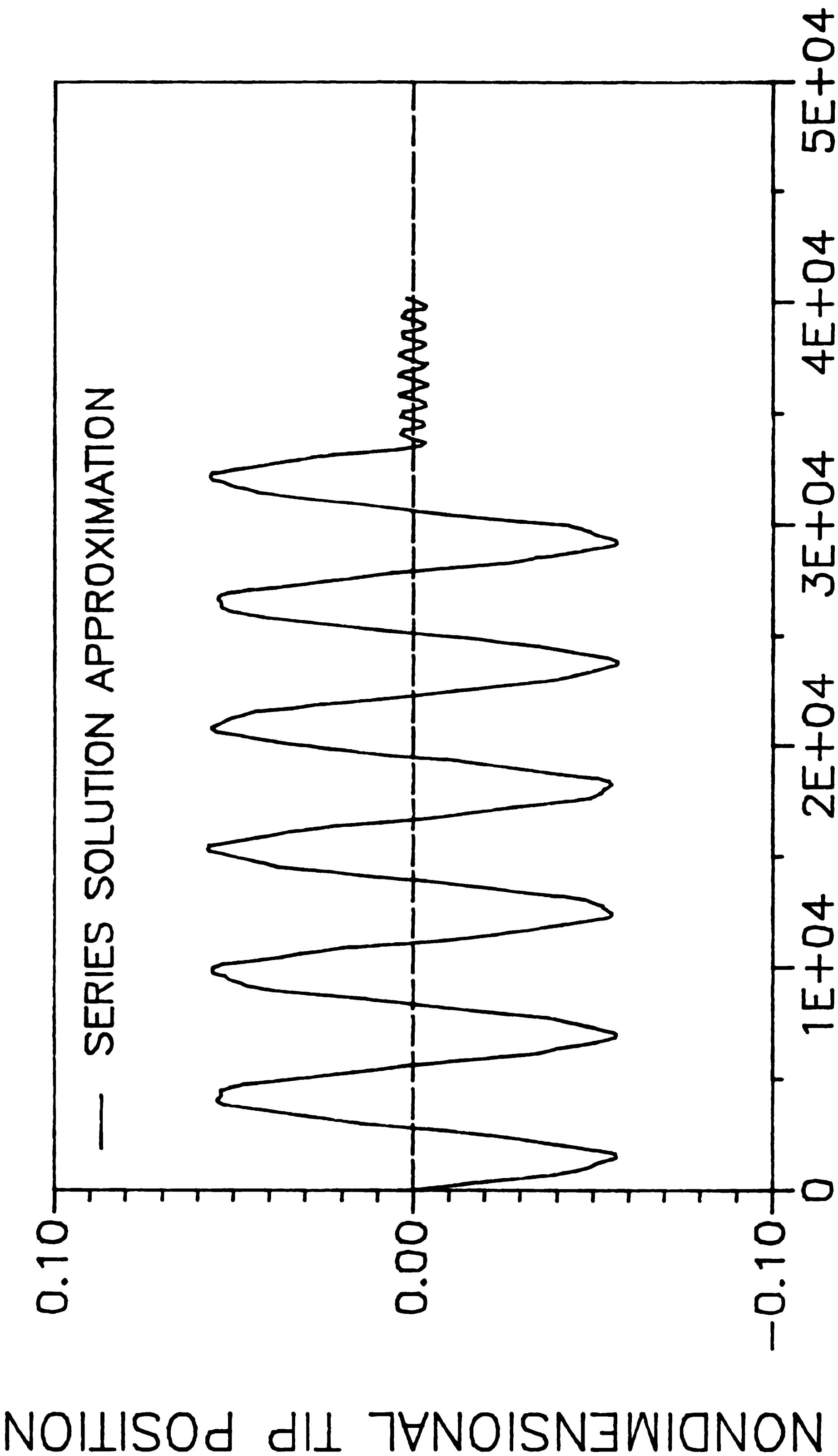
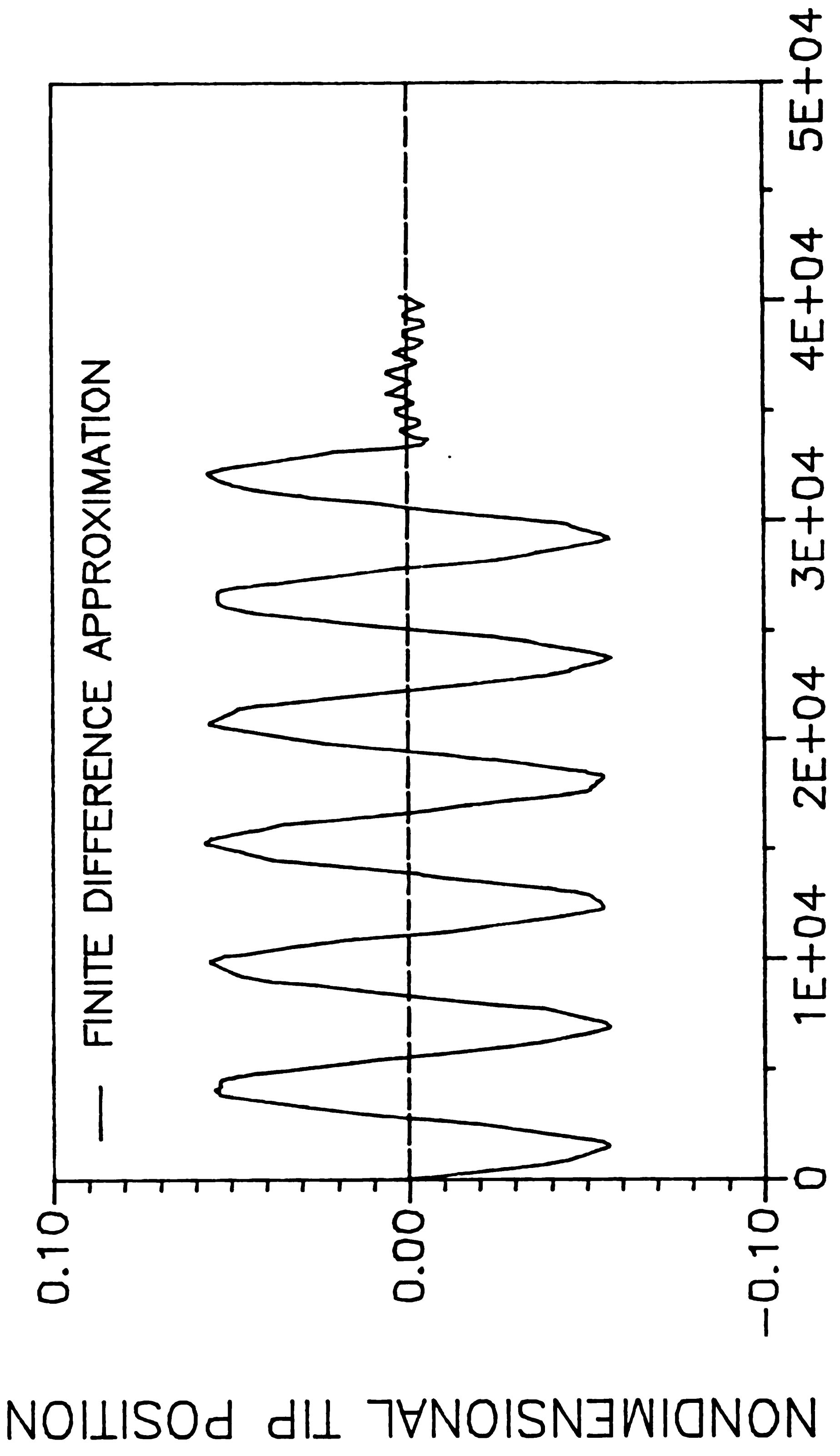


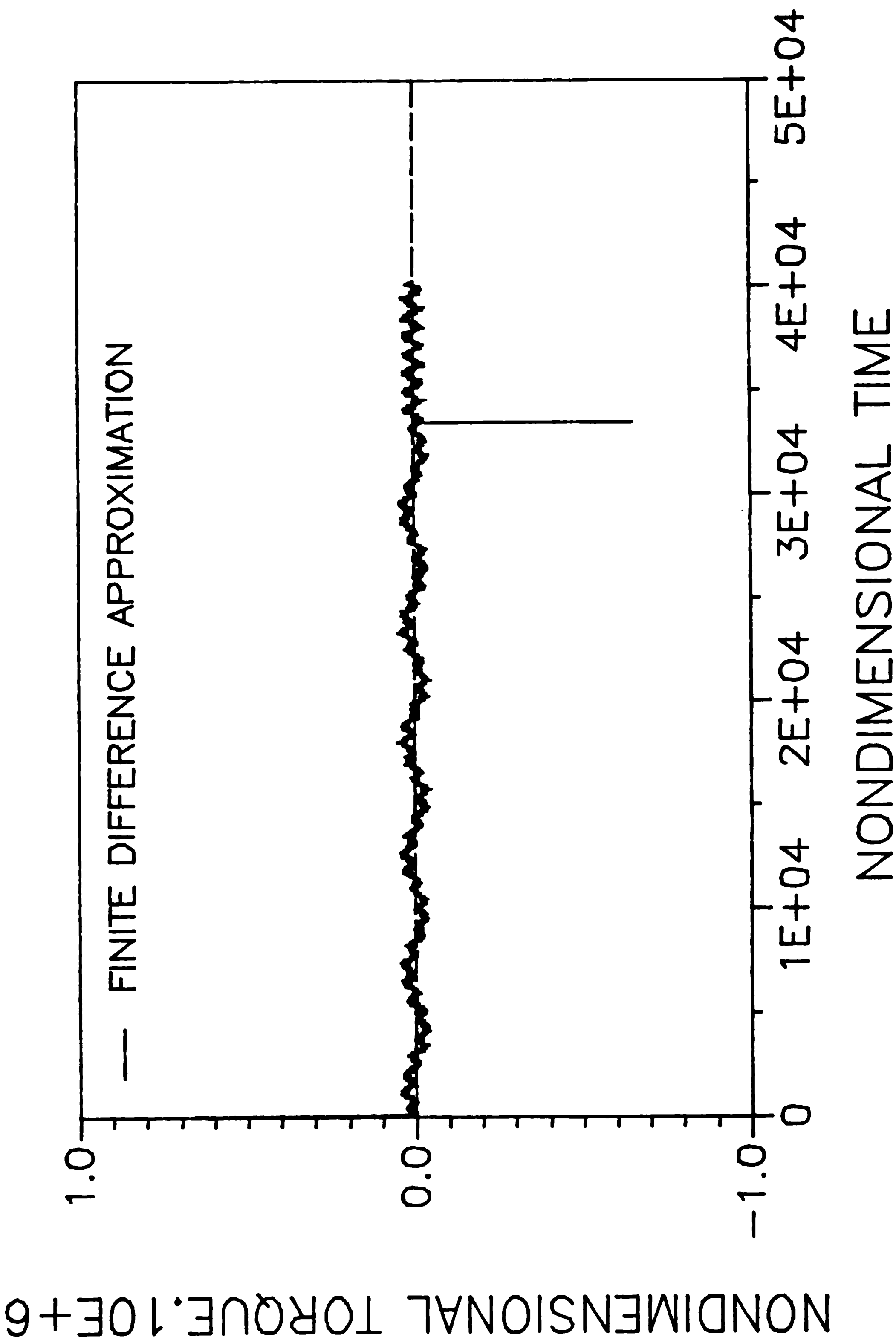
fig.4 : Hub-beam system in three dimensions



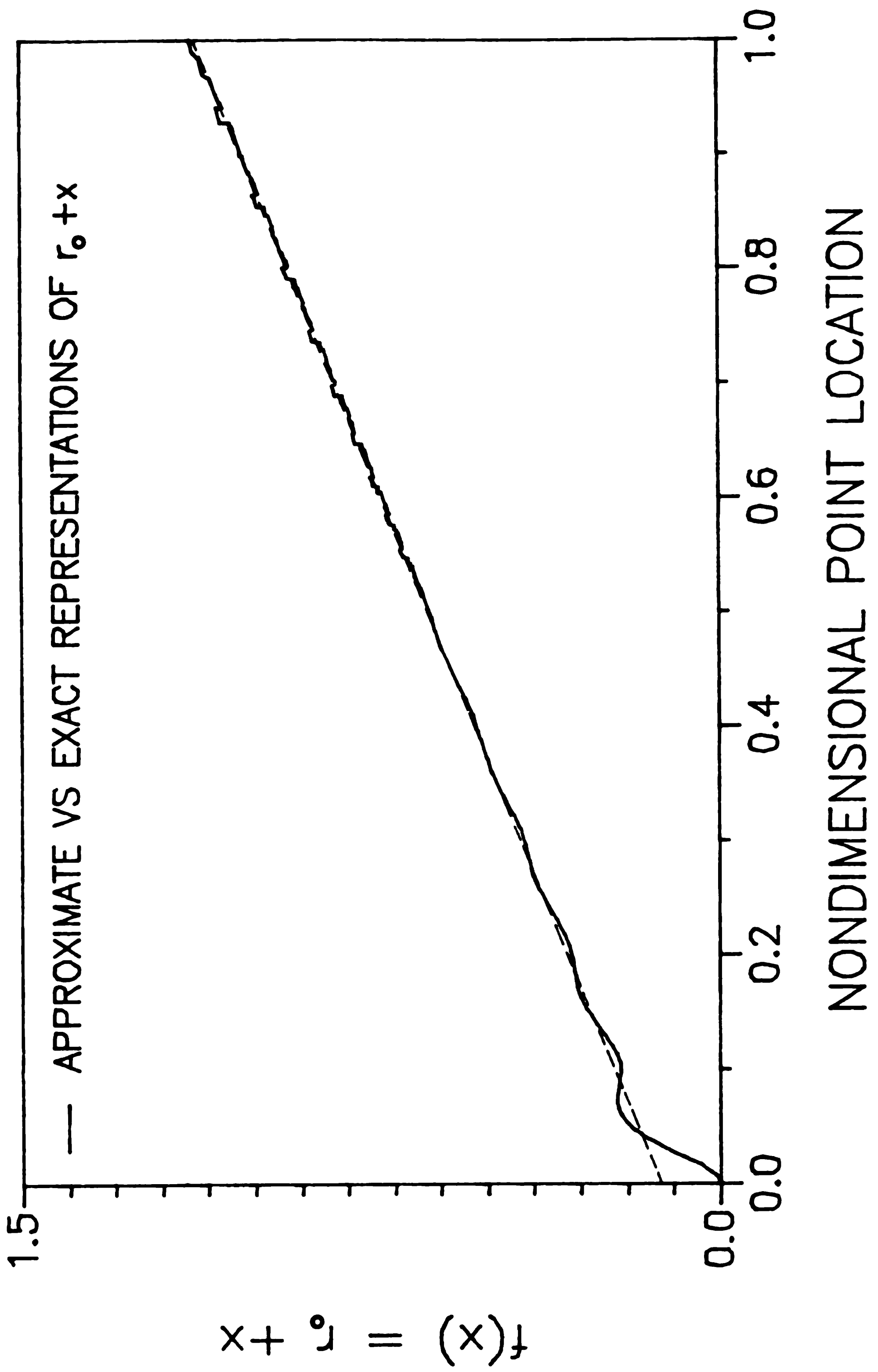
Plot 1: Tip displacement using the series solution



Plot 2: Tip displacement using the finite difference scheme



Plot 3: Torque applied at the hub



Plot 4: Comparison between exact and approximate representations of the body force

CHAPTER SEVEN

DISCUSSION OF RESULTS

The rectangular velocity has discontinuities at $t^* = 0$ and $t^* = t_e^*$. These discontinuities produce impulsive loadings at the begining and ending of the maneuver. These impulses occur in the field equation of motion as body forces. The sudden change of the angular velocity from zero to $\dot{\theta}_m^*$ at $t^* = 0$ and $\dot{\theta}_m^*$ to zero at $t^* = t_e^*$ produce deformation in the beam. In plots (1) and (2) there are two phases: forced-motion phase and free-motion phase. In the forced-motion phase, the number of oscillations from zero to t_e^* is equal to k (see equation (4.15)).
For $\alpha = \pi/2$, one can show that

$$t_e^* = T_1^* \left(k^2 + \frac{1}{16}\right)^{1/2} \simeq kT_1^* \tag{7.1}$$

For instance if $k = 6$, there will be six oscillations in the forced-motion phase. The free motion depends on the physical state of the beam at the final time. It also depends on the velocity jump which has an absolute value of $(r_0^* + x^*)\dot{\theta}_m^*$. This jump depends on the point location in space, and also on the magnitude of the maximum angular velocity.

Plots (1) and (2) correspond to the case where the first mode has no contribution to the overall response in the free vibration phase. Plot (1) was obtained by taking eight terms of the series solution (4.9). It was verified that 8 terms are adequate to represent the series solution. Plot (2) was obtained by using the finite difference scheme described in Chapter 5. Plots (1) and (2) are in good agreement. The minor difference occuring at $t^* = t_e^*$ and thereafter is due to an inaccurate estimation of the maximum angular velocity $\dot{\theta}_m^*$ by the series solution (4.9). The main error was generated from the representation of the body force by a nonuniform convergent series. Plot (4) describes very well the difference between the ideal function $(r_0^* + x^*)$ and the partial sum of the series. It was verified by taking more terms that the series is convergent to the function in plot (4). The numerical solution is a representation of the exact solution. It was verified that the curve in plot(2) is recovered for different step sizes.

Plot (3) was obtained using the numerical scheme. It represents the applied torque necessary to produce a constant angular velocity. Note the two impulses which produce the sudden changes in angular velocity at $t^* = 0$ and $t^* = t_e^*$. The vibrations, in the first and second phases, indicate that a torque has to be applied at the hub in order to kill the rotational vibrations of the rigid hub due to the nonzero moment at $x^* = 0$.

SUMMARY AND CONCLUSIONS

- Two approximate mathematical models were derived in order to describe the overall flexibility of a rotating beam.

- An approximate analytical solution was found and then used to estimate the necessary maximum angular velocity required to delete the effect of any desired mode in the free vibration phase.

- A general numerical scheme was developed, and shown to be unconditionally stable.

- The analytical and numerical solutions are in good agreement in killing the first mode in the free vibration phase.

APPENDIX A

APPENDIX A

```
C**************************************************************C
C                                                              C
C     THIS PROGRAM IS USED TO SOLVE THE NONDIMENSIONAL         C
C     SIMPLE FLEXURE FOR ANY TIME-DEPENDENT ANGULAR            C
C     VELOCITY ( THE ANGULAR PULSE ANGULAR VELOCITY            C
C     IS AN ILLUSTRATION IN THIS PROGRAM)                      C
C                                                              C
C**************************************************************C

      PROGRAM BACK
      DOUBLE PRECISION W(20,20),A(19,19),T,SI,THT2
      DOUBLE PRECISION V1(19),V2(19),DX,DT,C1,C2,D,SIH,TORQUE
     #,V3(19),TE,RO,B(19),PI,ALPHA,S1(19),S2(19),S3(19),THTT
     #,Y,THTM
      OPEN(100,FILE='AMIN1S')

C    DEFINITION OF PARAMETERS
C    ---------- -- ----------
C    SI: NONDIMENSIONAL MOMENT OF INERTIA OF THE BEAM
C    RO: NONDIMENSIONAL HUB RADIUS
C    Y: CONSTANT OF INTEGRATION
C    SIH: NONDIMENSIONAL MASS MOMENT OF INERTIA OF THE HUB
C    TE: NONDIMENSIONAL FINAL TIME AT WHICH ROTATION IS STOPPED
C    KK: THE NUMBER OF TIME STEPS UP TO TE
```

```
C     DT: NONDIMENSIONAL TIME INCREMENT
C     DX: NONDIMENSIONAL SPACE INCREMENT
C     ALPHA: FINAL ANGLE
C     THTM: NONDIMENSIONAL MAXIMUM ANGULAR VELOCITY

      SI=1.0288E-7
      RO=0.125
      Y=((RO+1)**3.0-RO**3.0)/3.0
      SIH=0.091145833
      D=(1.8751041)**2.0*SQRT(SI)*0.041630544
      TE=ACOS(-1.0)/(2.0*D)
      KK=5000
      P=KK
      DT=TE/P
      DX=1.0/20.0
      PI=ACOS(-1.0)
      ALPHA=PI/2.0
      THTM=ALPHA/TE

C**** ZERO ALL THE MATRIX ENTRIES, DEFINE INITIAL CONDITIONS

       DO 111 K1=1,19
        DO 112  K2=1,19
        A(K1,K2)=0.0
112     CONTINUE
111    CONTINUE
      DO 101 I=1,19
      V1(I)=0.0
      V2(I)=0.0
```

```
101   CONTINUE
      T=0.0
      THT2=0
      TORQUE=0.0
      WRITE(100,135)T,THT2,V2(19),TORQUE
135   FORMAT(F16.8,1X,F12.8,1X,F12.8,1X,F16.14)
      T=DT
      DO 102 J=1,6000

C**** DEFINE THE ANGULAR VELOCITY AND ACCELERATION

      IF(J.GE.1.AND.J.LT.(KK))THT2=THTM
      IF(J.EQ.(KK))THT2=0.0
      IF(J.GT.(KK))THT2=0.0
      IF(J.EQ.1)THTT=THTM/(DT)
      IF(J.NE.1)THTT=0.0
      IF(J.EQ.(KK))THTT=-THTM/(DT)
C**** DEFINE THE NONHOMOGENEOUS VECTOR B

       C2=DX**4.0/(SI)
       DO 103 I=1,19
       IF(I.EQ.1)G=V1(3)-4*V1(2)+6*V1(1)
       IF(I.EQ.2)G=V1(4)-4*V1(3)+6*V1(2)-4*V1(1)
       IF(I.GT.2.AND.I.LT.18)G=V1(I+2)-4*V1(I+1)+6*V1(I)
     # -4*V1(I-1)+V1(I-2)
       IF(I.EQ.18)G=-2*V1(19)+5*V1(18)-4*V1(17)+V1(16)
       IF(I.EQ.19)G=V1(19)-2*V1(18)+V1(17)
       B(I)=-2*C2*(R0+(I+1)*DX)*THTT
```

```
     # +2*C2*THT2*THT2*V2(I)-2*C2*(V1(I)-2*V2(I))/DT**2.0-G
103    CONTINUE

C**** RESET THE NONZERO ENTRIES IN THE MATRIX A

       C1=6+2*DX**4.0/(SI*DT**2.0)
       N=1
       DO 113 K1=3,17
        A(K1,N)=1.0
        A(K1,N+1)=-4.0
        A(K1,N+2)=C1
        A(K1,N+3)=-4.0
        A(K1,N+4)=1.0
        N=N+1
113    CONTINUE
       A(1,1)=C1
       A(1,2)=-4.0
       A(1,3)=1.0
       A(2,1)=-4.0
       A(2,2)=C1
       A(2,3)=-4.0
       A(2,4)=1.0
       A(18,16)=1.0
       A(18,17)=-4.0
       A(18,18)=C1-1.0
       A(18,19)=-2.0
       A(19,17)=1.0
       A(19,18)=-2.0
       A(19,19)=C1-5.0
```

```
C**** CALCULATE THE FUTURE TIME DISPLACEMENT VECTOR

      CALL DLINEQ(V3,B,A,W,19,20,IERR)

C**** CALCULATE THE TORQUE NECESSARY TO PRODUCE THE PRESCRIBED
C**** ANGULAR VELOCITY USING SIMPSON'S METHOD

      DO 161 I=1,19
      S1(I)=ABS(V3(I))**2.0
      S2(I)=2.0*(V3(I)-V2(I))*V3(I)/DT
      S3(I)=(RO+(I+1)*DX)*(V3(I)+V1(I)-2*V2(I))
     #/DT**2.0
161   CONTINUE
      SUM1=DX/3.0*(2.0*S1(1)+4*S1(2)+2*S1(3)+4*S1(4)+2*S1(5)+4*S1(6)
     #+2*S1(7)+4*S1(8)+2*S1(9)+4*S1(10)+2*S1(11)+4*S1(12)+2*S1(13)
     #+4*S1(14)+2*S1(15)+4*S1(16)+2*S1(17)+4*S1(18)+S1(19))
      SUM2=DX/3.0*(2.0*S2(1)+4*S2(2)+2*S2(3)+4*S2(4)+2*S2(5)+4*S2(6)
     #+2*S2(7)+4*S2(8)+2*S2(9)+4*S2(10)+2*S2(11)+4*S2(12)+2*S2(13)
     #+4*S2(14)+2*S2(15)+4*S2(16)+2*S2(17)+4*S2(18)+S2(19))
      SUM3=DX/3.0*(2.0*S3(1)+4*S3(2)+2*S3(3)+4*S3(4)+2*S3(5)+4*S3(6)
     #+2*S3(7)+4*S3(8)+2*S3(9)+4*S3(10)+2*S3(11)+4*S3(12)+2*S3(13)
     #+4*S3(14)+2*S3(15)+4*S3(16)+2*S3(17)+4*S3(18)+S3(19))
      TORQUE=((SIH+Y+SUM1)*THTT
     #+SUM2*THT2+SUM3)*1E+6

C**** PRINT THE RESPONSE

      WRITE(100,179)T,THT2,V3(19),TORQUE
179   FORMAT(F16.8,1X,F12.8,1X,F12.8,1X,F16.14)
```

```
C**** REINITIALIZE THE DISPLACEMENT VECTORS

      DO 106 I=1,19
      V1(I)=V2(I)
      V2(I)=V3(I)
106   CONTINUE
      T=T+DT
102   CONTINUE
      STOP
      END

C************************* END OF PROGRAM *************************C


C*****************************************************************C
C                                                                 C
C DEFINITION OF VARIBLES                                          C
C ---------- -- --------                                          C
C  T: NONDIMENSIONAL TIME                                         C
C  THT2: NONDIMENSIONAL ANGULAR VELOCITY                          C
C  THTT: NONDIMENSIONAL ANGULAR ACCELERATION                      C
C  TORQUE: NONDIMENSIONAL APPLIED TORQUE                          C
C  V1,V2,V3: NONDIMENSIONAL DISPLACEMENTS AT DIFFERENT TIME       C
C  A: MATRIX                                                      C
C  B: NONHOMOGENEOUS VECTOR                                       C
C                                                                 C
C*****************************************************************C
```

APPENDIX B

APPENDIX B

```
C************************************************************C
C                                                            C
C     THIS PROGRAM CALCULATE THE TIP RESPONSE USING 8 TERMS  C
C     FROM THE SERIES                                        C
C                                                            C
C************************************************************C

      PROGRAM SERIES
      DIMENSION V(8),VV(8),BE(8)
     #,PHI(8),W(8),WN(8),A(8),EE(8)
      DOUBLE PRECISION A,V,W,WN,T,BE,PHI,VV,EE,THTO
     #,Z,VT,VVT,DT,SI,ALPHA,TO,PI,RO
      OPEN(100,FILE='DELTA1')
      PI=ACOS(-1.0)

C**** DEFINE THE NONDIMENSIONAL WAVE LENGTH

      BE(1)=1.8751041
      BE(2)=4.6940911
      BE(3)=7.8547574
      BE(4)=10.9955407
      BE(5)=14.1371684
      BE(6)=17.2787595327
      BE(7)=20.420352251
```

```
      BE(8)=23.561944900414

C**** DEFINITION OF PARAMETERS  - ALPHA: ANGLE OF ROTATION
C****                           - SI: NONDIMENSIONAL MOMENT OF
C****                                 INERTIA
C****                           - RO: NONDIMENSIONAL HUB RADIUS
C****                           - TO: NONDIMENSIONAL FINAL TIME

      ALPHA=PI/2.0
      T=0.0
      SI=1.0288E-04
      RO=0.125
      DO 102 J=1,8
      EE(J)=(DCOSH(BE(J))+DCOS(BE(J)))/(DSINH(BE(J))+DSIN(BE(J)))
      PHI(J)=DCOSH(BE(J))-DCOS(BE(J))-EE(J)*(DSINH(BE(J))
     #-DSIN(BE(J)))
      A(J)=2.0*(EE(J)*RO+1.0/BE(J))/BE(J)
      W(J)=BE(J)**2.0*SQRT(SI)
102   CONTINUE
      CV=1.0/(SQRT(1.0+16.0*1.0))
      THTO=CV*W(1)
      TO=ALPHA/THTO
      DT=TO/5000.0
      PRINT *,DT
      DO 101 I=1,6000
        DO 105 J=1,8
      WN(J)=SQRT(W(J)**2.0-THTO**2.0)
```

```
C**** DEFINE THE DISPLACEMENT AND VELOCITY DURING THE FORCED
C**** VIBRATION MOTION

      IF(T.LE.TO)THEN
      V(J)=-A(J)*THTO*DSIN(WN(J)*T)/WN(J)*PHI(J)
      VV(J)=-A(J)*THTO*DCOS(WN(J)*T)*PHI(J)
      ENDIF

C**** DEFINE THE DISPLACEMENT AND VELOCITY DURING THE FREE
C**** VIBRATION MOTION

      IF(T.GT.TO)THEN
      V(J)=(-A(J)*THTO*DSIN(WN(J)*TO)*DCOS(W(J)*(T-TO))/WN(J)
     #+A(J)*THTO*(1.0-DCOS(WN(J)*TO))/W(J)*DSIN(W(J)*(T-TO)))*
     #PHI(J)
      VV(J)=(A(J)*THTO*DSIN(WN(J)*TO)*W(J)*DSIN(W(J)*(T-TO))/WN(J)
     #+A(J)*THTO*(1.0-DCOS(WN(J)*TO))*DCOS(W(J)*(T-TO)))*PHI(J)
      ENDIF
105    CONTINUE

C**** SUM OF THE FIRST EIGHT TERMS IN THE SERIES

      VT=V(1)+V(2)+V(3)+V(4)+V(5)+V(6)+V(7)+V(8)
      VVT=VV(1)+VV(2)+VV(3)+VV(4)+VV(5)+VV(6)+VV(7)+VV(8)
      WRITE(100,104)T,VT,VVT
104   FORMAT(F18.8,3X,F12.8,3X,F12.8)
      T=T+DT
101   CONTINUE
      PRINT *,A(1),A(2),A(3),A(4),A(5),A(6),A(7),A(8)
```

```
      STOP
      END

C************************ END OF PROGRAM ***********************C


C**************************************************************C
C   DEFINITION OF VARIABLES                                    C
C                                                              C
C   T: NONDIMENSIONAL TIME                                     C
C   V(I): NONDIMENSIONAL TIP DISPLACEMENT USING THE ITH TERM   C
C   VV(I): NONDIMENSIONAL TIP VELOCITY USING THE ITH TERM      C
C   W(J): NONDIMENSIONAL NATURAL FREQUENCY OF THE JTH MODE     C
C   PHI(J): NONDIMENSIONAL EIGEN-FUNCTION OF THE JTH MODE      C
C   A(J): NONDIMENSIONAL CONSTANT FROM THE EXPANSION OF THE    C
C         THE BODY FORCE ABOUT THE EIGENFUNCTIONS PHI(J)'S     C
C**************************************************************C
```

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REFERENCES

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