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A Transient Coherency Measure and Its Application to Transient Security Enhancement of Electric Energy Systems- A Systems and Operations Research Approach

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A TRANSIENT COHERENCY MEASURE AND ITS APPLICATION TO TRANSIENT SECURITY-ENHANCEMENT OF ELECTRIC ENERGY SYSTEMS - A SYSTEMS AND OPERATIONS RESEARCH APPROACH

By

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ABSTRACT

A TRANSIENT COHERENCY MEASURE AND ITS APPLICATION TO TRANSIENT SECURITY-ENHANCEMENT OF ELECTRIC ENERGY SYSTEMS - A SYSTEMS AND OPERATIONS RESEARCH APPROACH

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A transient coherency measure is developed and shown to be an excellent tool for analyzing the transient response of an electric energy system in order to obtain insight into the dynamic structure of the interconnected system following a contingency. The transient response is disected into a sequence of events by showing the times at which even numbered terms of a Taylor-series approximation are added to the transient coherency measure to indicate the instants that successive stages of generators further from the disturbance begin to accelerate. The measure is evaluated for a deterministic and probabilistic disturbance and indicates the lines and generators that are affected by the disturbance and the relative stiffness of the interconnection from the disturbed generator to the portion of the system affected during each time interval of propagation of the disturbance. The transient coherency measure is thus shown to assist in the identification of weaknesses in dynamic system structure and the changes in

the dynamic structure of an electric energy system as the length of the observation interval increases.

The transient coherency measure is also shown to be an excellent measure of system security and reliability with the use of the equal-area criterion and the concept of an equivalent-line connected to an infinite bus. It is further shown to be a stability measure by showing that the transient coherency measure summed over all pairs of internal generator buses is proportional to the sum of the square of the eigenvalues of the system with zero damping.

The transient coherency measure in its use as a security measure is shown to have potential for on-line application and thus provide the industry with a valuable tool for on-line transient security enhancement. In this direction, the off-line dispatch problem is reformulated as an on-line linearized tracking secure dispatch problem. By viewing the security problem in terms of five operating states, the associated sub-control problems are formulated and shown to have the familiar linear-programming or quadratic programming format from operations research, depending upon the addition of the transient security measure to the performance index. An algorithmic control structure is proposed for solving the sub-control problems and it is shown that with the help of appropriate weightings, the reformulations can significantly enhance the security, reliability and stability of the system.

Thus the measure is shown to move the system in a direction to (1) stiffen weak lines, (2) reduce the vulnerability of any particular bus to a loss of synchronization or stability, (3) improve the overall stiffness of the interconnected network and (4) prepare the system for controlled islanding when it is moving toward total collapse.

IN THE NAME OF ALLAH

THE BENEFICIENT, THE COMPASSIONATE, THE MERCIFUL

بسم الله الرحمن الرحيم

To my Wife

Yosria

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CHAPTER 1

INTRODUCTION

Scenario:

Increased and steadily increasing system interconnections have created rather complex networks in the United States having load areas located far from the generating plants. The NY blackout of 1965 initiated a serious interest in reliability and security studies of Electric Energy Systems. However, the recent 1977 NY outage provides sufficient evidence of the inadequacy of these efforts and their achievement, especially so far as on-line methods are concerned.

To enable the systems planning division of the modern utility to meet existing requirements on system security, evaluate existing systems through transient stability studies and plan future development of the network, it is imperative that improved models be developed which adequately represent the real power system without including or deleting more than necessary from the model.

For a better assessment of this situation, a review of some of the characteristics of an electric energy system is in order [1]:

(i) Power plants will have to transmit power to distant load centers and economic growth will mean the increase in the size of these plants. The size of the plants is therefore dictated essentially by three factors which are

- geographical
- environmental
- economic realities

(ii) Due to increased demand coupled with delays in commissioning of new units such as nuclear power plants, the <u>stress</u> on existing units is increasing and transmission corridors are becoming more crowded with the increase in large amounts of power being transmitted over large distances. This will result in an increased probability of system instability as transmission lines carry nearly full rated power.

(iii) As EHV interconnections increase, the effects of the disturbances will be felt in more and more remote parts of the system, thus increasing the severity of any disturbance.

It is with these characteristics in perspective, then, that the modern utility spends a considerable amount of effort in terms of time and money to perform transient stability studies. The objective of their study is to aid the systems planners

- to evaluate a power system's ability to withstand large disturbances

. to perform routine long range and short range planning of electric energy systems which could involve solution schemes for various combinations of proposed generator and transmission configurations, solutions for coordination of protection schemes, and design of controls for existing systems.

Given this background, in any physical situation, most of the major disturbances can propagate through tie-lines to neighboring systems with the result that it becomes important to represent in the power system model, not only the power system in question, hereafter referred to as the internal or study system, but also the neighboring system, hereafter referred to as the external system. However, the extensive nature of the interconnection makes the representation of the external system a difficult chore. For example, to study the Michigan system, it becomes necessary to represent systems as far north as Ontario, Canada and as far east as New England. It is no wonder, then, that a single transient stability run requires large computers, a significant amount of computer time, and consequently high costs. In order to circumvent some of these difficulties, the concept of <u>coherency</u> in power system models comes into the picture.

<u>What is Coherency</u>:

Coherency is a relatively new area of interest in power systems studies. As befits the introduction of such a topic, the level of presentation is generally monotone, increasing from section to section and chapter to chapter.

Qualitatively, coherency is a quality or state of systematic connectedness or interrelatedness, especially when governed by logical principles. It is, then, this state of connectedness and interrelatedness by logical principles which form the basis of this dissertation, by the integration of machines into congruent sets based on a consistent pattern.

Quantitatively, coherency is ideally defined to hold for any two buses for which the ratio of their complex bus voltages is constant over time. This definition of coherency for a particular fault or disturbance implies that for two buses to be coherent, the following two conditions are satisfied [4]:

- the difference in voltage angles at the two buses, i and j, is constant
- the ratio of voltage magnitudes is constant for buses i and j.

Mathematically, therefore, this may be expressed as:

$$\underset{t \in [0,\hat{T}]}{C_{ij}} = \frac{V_i^*}{V_j^*} = \frac{|V_i| e^{j|\underline{\delta_i}}}{|V_j| e^{j|\underline{\delta_j}}} = \text{constant}$$

Ideal coherency is sufficient for a mathematically rigorous procedure for replacing a group of coherent buses by a single equivalent bus. However, the model which results cannot be represented in terms of normal power system components and thus could not be used in conventional transient stability programs. It has been shown that accurate dynamic equivalents can be developed using the above procedure for the case where buses that are to be equivalenced are determined based on a relaxed definition of coherency which depends only on differences in voltage angles and is independent of voltage magnitude.

Relaxed Coherency, then, refers to two buses as being coherent if the difference of their voltage angles remains constant to a certain prespecified tolerance, ε, over time.

$$\underset{t\in[0,\hat{T}]}{C_{ij}} = \delta_i(t) - \delta_j(t) < \varepsilon$$

Thus groups of generators that "swing-together" when disturbed are called coherent and knowledge of coherent behavior allows a simpler and cheaper representation of machines in large stability studies.

Coherency Based Modelling:

Several methods for developing Coherency Based Dynamic Equivalents have been proposed [3,7]. They use various definitions of electrical distance and various clustering altorithms for determining coherent groups. These electrical distance methods were heuristically based, requiring tuning of parameters for each system studied, and required validation of the equivalent against the results of the step by step integration of the system nonlinear differential equations during fault and post fault conditions, in a base case transient stability.

In an effort to overcome the above difficulties, Systems Control Incorporated (SCI) in Palo Alto, California has developed a method of producing coherency based dynamic equivalents which accurately reproduces the effects of the external system when the internal or study system is subjected to a specific disturbance. This method comprises of the following steps:

1) Classify groups of generators to be equivalenced into coherent groups by use of the relaxed coherency approach for processing swing curves from base case transient stability studies
2) Aggregate the generators in each coherent group to form one or more equivalent generators.

The result is a reduced equivalent network in the form of normal power system components which can be used without modifying present transient stability programs. This procedure, however, is beset by severe limitations which are [5]:

- The base case transient stability studies are expensive in terms of computer requirements. Consequently a large initial effort is required to form the equivalent.
- The advantageous applications of the method are limited to studies in which multiple transient

stability cases are to be simulated with faults concentrated in a local area.

. The coherency-based equivalencing method is not directly usable by utilities with small computers and with transient stability programs which cannot handle the full system representation.

The two major limitations of the coherency based dynamic equivalents method are:

(1) the equivalent is determined based on a single contingency in a particular location. The equivalent can thus only be used to study similar contingencies occurring in approximately the same local area.

(2) there is no theoretical justification for the coherency based method of producing dynamic equivalents when coherency is defined based strictly on the difference in voltage angles.

Two coherency measures considered by SCI were the max-min measure

$$C^1_{k\ell}(T) = \max_{t\in[0,T]} \{\delta_k(t) - \delta_\ell(t)\} - \min_{t\in[0,T]} \{\delta_k(t) - \delta_\ell(t)\}$$

and the rms measure

$$C^2_{k\ell}(T) = \mathrm{SQRT}\ \frac{1}{T} \int_0^T \{[\delta_k(t) - \delta_k(0)] - [\delta_\ell(t) - \delta_\ell(0)]\}^2 dt.$$

The max-min measure was chosen by SCI over the rms measure.

The following discussion attempts to establish the restrictive conditions under which the max-min measure is a good choice and to show the conditions under which an rms or mean square coherency measure is a better choice for

(1) analyzing the propagation of disturbances

(2) analyzing changes in power system dynamic structure

(3) use as a measure of power system security.

The max-min measure in its present form is a transient coherency measure as it selects the maximum change in the angular difference which will occur in the transient interval $[0,T]$ and depends upon T, the disturbance, and that portion of the system dynamics which is affected by the disturbance when the angular difference achieves its maximum or minimum values.

For this coherency measure to give an accurate indication of the coherent groups and dynamic structure, it must be assumed that the coherent groups do not change during the short interval in which the angular differences on every pair of buses achieve their maximum and minimum values. If the observation interval is short enough, this assumption that the coherent groups do not change is certainly valid, but the time-interval over which the coherency measure and any dynamic equivalent derived based on it are confined to the short interval.

On the other hand, an rms or mean square coherency measure averages the angular difference in the interval and thus the rms coherent equivalent may not be as accurate as an appropriate min-max measure over a short interval but would likely be a better equivalent over a longer interval where the coherent groups and dynamic structure have changed. The mean square measure is capable of providing a probabilistic description of the disturbance in terms of system parameters and is not dependent upon the location of the disturbance. It can also provide a deterministic description for a specific disturbance in a specific location.

The mean square measure has potential for application in the comparison of modal and coherent equivalents or comparison based on coherent equivalents derived on different observation intervals. In addition, it has potential for use as a security measure similar to the measure used by Byerly et al. [28, 29]

$$\max\ (\theta_i - \theta_j)$$

which is similar to the max-min coherency measure and is dependent upon the fault and its location. The transient coherency measure is proposed as a security measure in this research and is shown to have potential for on-line application because it

- provides a better measure of security
- provides a better measure of reliability
- provides a better measure of stability
- is a closed form analytical expression

- stiffens the network adaptively
- enables the analysis of the propagation of disturbances in the system
- permits the analysis of changes in power system dynamic structure
- does not depend upon type and location of the disturbance and can permit the representation of different disturbances in different locations both deterministically and probabilistically.

The specific topics in the development of a transient coherency measure and its application to transient security enhancement are now introduced.

A linearized power system state model and disturbance model are developed to permit an analysis of the spread of a contingency, deterministic or probabilistic, in the power system. The linearized models developed are used to represent dynamic fluctuations in both the internal system and in the external system which is far from the internal system where the disturbances occur and the models developed are accurate. Contingencies such as load shedding, generator dropping, electrical faults, and line switching can be handled easily by these models.

In order to determine coherent groups using these models, a generalized mean square coherency measure is developed. This measure of coherency between internal generator buses is capable of handling the set of deterministic and probabilistic disturbances already described.

The mean square coherency measure is expressed as a Taylor series expansion and its coefficients are analyzed. Computer programs are written to obtain the coherency matrix for a system based on these coefficients which are found to be recursive.

This Taylor series expansion is then used to define a transient coherency measure and thereby analyze the transient response of a power system for obtaining better insight into dynamic structure and associated changes in the interval following the disturbance. The analysis of the power system transient response is made possible by defining the aforementioned transient coherency measure in which the number of terms in the Taylor series increases with the observation interval in order to keep the approximation error bounded.

The transient response of a power system is subdivided into a finite sequence of events by showing the time instants at which even numbered terms are added to the measure, and signifying the time instants at which successive sets of generators or stages further from the disturbance location begin to accelerate.

The propagation of the disturbance in the power system with the help of this transient coherency measure is thus used to indicate the lines and generators that are affected by the disturbance and the relative stiffness of the interconnection from the disturbed generator to the portion of the system affected by the disturbance in any interval, which begins and ends when successive even numbered terms are added in the series as the disturbance propagates to successive stages of generators. The chief advantage of this analysis of the transient coherency measure for a specific disturbance is to aid in determining the cause of major fluctuations at a particular location and at a particular time which could lead to system instability. Consequently, the usefulness of the analysis is found in (a) determining the effects on transient stability of adding generation or transmission capacity at a particular location and (2) performing contingency analysis for systems operations and planning.

It is hypothesized and later supported by computational results on example systems that the transient coherency measure, evaluated for either a deterministic or probabilistic disturbance, can be used to determine coherent groups, dynamic system structure, and weaknesses in the associated structure. Changes in these are shown to occur due to changes in the stiffness of the interconnection of that portion of the system which is affected by the disturbance caused by the spread of the initial contingency.

When the transient coherency measure is evaluated for a deterministic contingency, the changes in associated coherent groups and structural weaknesses can be analyzed.

In order to analyze changes in coherent groups and weaknesses in dynamic structure that are independent of the location of the disturbance or its magnitude, a zero mean independent identically distributed (ZMIID) step change in shaft acceleration on all generators is used.

When the transient coherency measure is evaluated for the ZMIID probabilistic disturbance, it is shown that it is an excellent transient security measure. Following a comparison between the structural information contained in the transient and dynamic measures, these measures are shown to provide the information needed by system planning divisions and operators to obtain a realistic assessment of system security caused by weaknesses in dynamic structure which could be corrected by a change in generation scheduling and network configuration.

The transient coherency measure is then used to reformulate the off-line dispatch problem as an on-line linearized tracking secure dispatch problem with a transient security measure which augments the formulation in the form of an approximated quadratic performance index.

The transient security measure is justified as a measure of security, reliability and stability to enable its usage in the performance index of the reformulated on-line secure dispatch problem. This is done by using the concept of an equivalent line connecting a specific generator to an infinite bus representing the rest of the system. The equal-area criterion is used as a tool in the analysis for providing the justification of security and reliability. This measure is shown to be a stability measure by expressing the transient coherency measure as a function of the eigenvalues of the system matrix. By using the analogy of a harmonic oscillator the stability property of the index is then confirmed.

The power system operation is subdivided into five operating states and the sub-control problems are formulated. An algorithmic structure to solve these sub-problems is proposed and the sub-problems are then reformulated with the addition of the transient security measure. Time frames for controls are discussed. By assigning appropriate weighting coefficients it is shown that the transient security measure along with the reformulations can assist the operator or automatic control system to move the system in a direction which improves system security by stiffening weak parts of the system in the alert insecure mode when postulated next contingency tests are being done. In addition, it enables the system to prevent cascading by continually stiffening the network in the face of an actual transient emergency, provides the system operator with a tool for lowering generation and load to assist in controlled islanding when the system is under extremsis and system tearing is a strong likelihood and assists the operator in resynchronization of areas in the optimal load restoration process. In addition, it protects those parts of the system which have already been restored from the effects of any further contingencies. Thus the transient coherency measure in its use as a security measure is seen to provide a very useful measure of security in each power system operating state -- a feature which was not available to the industry before.

CHAPTER 2

A GENERALIZED STATE MODEL AND DISTURBANCE MODEL

The principal objective of this chapter is to develop a generalized power system state model and a generalized disturbance model which can be used to obtain an analytical expression for the mean square coherency measure [6]. This objective will be met in the following development which

(i) defines the power system

(ii) briefly reviews the need for the model

(iii) describes and develops the detailed state space model of interest

(iv) describes and develops the disturbance model appropriate for a mean square coherency measure used to describe the spread of a disturbance.

The power system being modeled consists of

(1) an electrical network consisting of transmission lines and transformers that connect generation and load. The equations that describe power flows in this network are the network equations of Kirchoff

(2) dynamic models of the generators that produce the power injections into the network. The model would generally include models of the generator, voltage regulator, and governor turbine energy system.

The system model and disturbance model developed in this chapter are not intended to accurately describe the effects of a particular contingency but rather to permit an analysis of the spread of this contingency through a power system. The system and disturbance models developed here are similar to ones developed for an rms coherency measure used to obtain equivalents for an external system which is distant from the location of the disturbance. These system and disturbance models will be slightly modified and used to represent the power system dynamic fluctuations in both the internal system and in the external system far from the disturbance where these models [6] are known to be accurate. The linearization of the classical transient stability model, the elimination of Q-V dynamics, the elimination of load dynamics, and the approximations used for disturbance are all justified based on the need to analyze the dependence on system structure and time sequence of the spread of the contingency in both the internal and external system. The accuracy of the model for the portion of the system close to the disturbance is therefore sacrificed.

Having established the raison d'etre of obtaining a simplified power system model, the next step is to specify the assumptions needed to meet this objective and finally to quantify the model in terms of state space notation.

State Model Development

The recent SCI work on coherency based dynamic equivalents has shown that the coherency can be evaluated using a model based on the following assumptions:

> The coherent groups of generators are independent of the size of the disturbance. Therefore, coherency can be determined by considering a linearized system model.
>
> The coherent groups are independent of the amount of detail in the generating unit models. Therefore, a classical synchronous machine model is considered and the excitation and turbine-governor systems are ignored.
>
> The effect of a fault may be reproduced by considering the unfaulted network and pulsing the mechanical powers to achieve the same accelerating powers which would have existed in the faulted network.

The first assumption may be confirmed by considering a fault on a certain bus, and observing that the coherency behavior of the generators is not significantly changed as the fault clearing time is increased. The second assumption is based upon the observation that although the amount of detail in the generating unit models has a significant effect upon the swing curves, particularly the damping, it does not radically affect the more basic characteristics such as the natural frequencies and mode shapes. The third assumption recognizes that the generator accelerating powers are approximately constant during faults with typical clearing times. These above assumptions and their justification are quoted from [5]. These assumptions were made and justified in a study that used the coherency measure as a basis for developing dynamic equivalents for an external system which is far from the location of the contingency. However, since the assumptions are reasonable in most cases even in the internal system close to the location of the contingency and since a linear simple model is required to perform the desired analysis of the mean square coherency measure, the above assumptions are also made in this study.

It is assumed that the power system includes N generator buses, K load buses and an infinite bus used as the synchronous reference.

The mechanical equations of motion for each synchronous generator are

$$
\begin{aligned}
M_i \frac{d\omega_i}{dt} &= PM_i - PG_i - D_i(\omega_i - \omega_s) \\
\frac{d\delta_i}{dt} &= \omega_i - \omega_s
\end{aligned}
\qquad i = 1,2,\ldots,N \qquad (2.1)
$$

where

i	subscript for generator i
	indicates that this variable represents a deviation from a specified steady-state operating point
M_i	inertia constant - p.u.
ω_i	speed of generator rad/sec
δ_i	generator rotor angle - radians
D_i	damping constant - p.u.

ω_s synchronous frequency - radians/sec

PM_i mechanical input power - p.u.

PG_i electrical output power - p.u.

Writing a deviational model for these generators around an operating point δ_i°, ω_s, PM_i°, and PG_i° where

$$\Delta\delta_i = \delta_i - \delta_i^\circ$$
$$\Delta\omega_i = \omega_i - \omega_s$$
$$\Delta PM_i = PM_i - PM_i^\circ$$
$$\Delta PG_i = PG_i - PG_i^\circ$$

the equations (1) become

$$M_i \frac{d\Delta\omega_i}{dt} = \Delta PM_i - \Delta PG_i - D_i\Delta\omega_i$$
$$\frac{d\Delta\delta_i}{dt} = \Delta\omega_i \qquad i = 1,2,\ldots,N \tag{2.2}$$

The network equations in polar form are linearized with the real power equations decoupled from the reactive power equations to obtain:

$$\begin{bmatrix} \Delta\underline{PG} \\ \Delta\underline{PL} \end{bmatrix} = \begin{bmatrix} \partial\underline{PG}/\partial\underline{\delta} & \partial PG/\partial\underline{\Theta} \\ \partial\underline{PL}/\partial\underline{\delta} & \partial\underline{PL}/\partial\Theta \end{bmatrix} \begin{bmatrix} \Delta\underline{\delta} \\ \Delta\Theta \end{bmatrix} \tag{2.3}$$

where

$$\underline{PG}^T = [PG_1, PG_2, \ldots, PG_N]$$
$$\underline{PL}^T = [PL_1, PL_2, \ldots, PL_K]$$
$$\underline{\delta}^T = [\delta_1, \delta_2, \ldots, \delta_N]$$
$$\underline{\Theta}^T = [\Theta_1, \Theta_2, \ldots, \Theta_K]$$

ΔPL_i - deviation in real power at load bus i - p.u.

$\Delta\Theta_L$ - deviation in voltage angle at load bus i - radians

This model can be expressed in state space form by using the network equations in (2.3) to obtain

$$\underline{\Delta PG} = \frac{\partial \underline{PG}}{\partial \underline{\delta}} \underline{\Delta\delta} + \frac{\partial \underline{PG}}{\partial \underline{\Theta}} \left[\frac{\partial \underline{PL}}{\partial \underline{\Theta}} \right]^{-1} \left[\underline{\Delta PL} - \frac{\partial \underline{PL}}{\partial \underline{\delta}} \underline{\Delta\delta} \right]$$

The state model becomes

$$\dot{\underline{x}}(t) \quad \underline{A}\ \underline{x}(t) + \underline{B}\ \underline{u}(t) \tag{2.4}$$

where

$$\underline{x} = \begin{bmatrix} \underline{\Delta\delta} \\ \underline{\Delta\omega} \end{bmatrix} \qquad \underline{u} = \begin{bmatrix} \underline{\Delta PM} \\ \underline{\Delta PL} \end{bmatrix} \tag{2.5}$$

$$\underline{A} = \begin{bmatrix} \underline{0} & \underline{I} \\ -\underline{M}^{-1}\underline{T} & -\underline{M}^{-1}\underline{D} \end{bmatrix}; \quad \underline{B} = \left[\begin{array}{c|c} \underline{0} & \underline{0} \\ \hline \underline{M}^{-1} & \underline{M}^{-1}\underline{L} \end{array}\right]$$

$$\underline{M} = \text{Diag}\{M_1\ M_2\ \ldots\ M_n\}$$

$$\underline{D} = \text{Diag}\{D_1\ D_2\ \ldots\ D_N\}$$

$$\underline{T} = \frac{\partial \underline{PG}}{\partial \underline{\delta}} - \frac{\partial \underline{PG}}{\partial \underline{\Theta}} \left[\frac{\partial \underline{PL}}{\partial \underline{\Theta}} \right]^{-1} \frac{\partial \underline{PL}}{\partial \underline{\delta}} \tag{2.6}$$

$$\underline{L} = -\frac{\partial \underline{PG}}{\partial \underline{\delta}} \left[\frac{\partial \underline{PL}}{\partial \underline{\Theta}} \right]^{-1}$$

Disturbance Model

The following disturbance model is chosen so that a deterministic as well as probabilistic contingency set could be handled.

The initial conditions are assumed random with

$$E\{\underline{x}(0)\} = \underline{0}$$
$$E\{\underline{x}(0)\ \underline{x}^T(0)\} = \underline{V}_x(0) \qquad (2.7)$$

since the expected deviations from any operating state is zero but the variance of such deviations is nonzero. The coherency measure to be developed will be shown to depend on this $\underline{V}_x(0)$. The initial conditions are included not to reflect any specific type of disturbance but rather the effects on the state from some hypothetical disturbance whose statistics (2.7) may be inferred from internal and external operating conditions.

The input, composed of the deviations in the mechanical input power $\underline{\Delta PM}$ and the deviation in load power $\underline{\Delta PL}$, can be used to model

i) loss of generation due to generator dropping

ii) loss of load due to load shedding

iii) line switching

iv) electrical faults

These contingencies can be modeled by an input $\underline{u}(t)$ that has the following form

$$\underline{u}(t) = \underline{u}_1(t) + \underline{u}_2(t) \qquad (2.8)$$

The vector function

$$\underline{u}_1(t) = \begin{cases} \underline{u}_1 & t \geq 0 \\ \underline{0} & t < 0 \end{cases} \tag{2.9}$$

represents

(i) the loss of generation due to generator dropping

(ii) the loss of load due to load shedding

(iii) changes in load injections due to line switching

The modeling of these three disturbances requires determination of $\underline{u}_1$ and possible modification of the network before determination of matrices $\underline{A}$ and $\underline{B}$. The procedure used is a modification of one used in [6] when the coherency measure was used for developing an equivalent for the external system.

<u>generator dropping</u> - the transient reactance of the generator dropped is omitted from the network and the deviation in the generator output PM_i of the generator dropped is set equal to the loss of generation.

<u>load shedding</u> - the load deviation PL_k for all buses k where load is shed should be set equal to the change in load caused by the load shedding operation.

<u>line switching</u> - the network is modified to represent the system after the line switching operation is performed. The load deviations, PL_k and PL_m, at buses to which this line is connected, are set equal to the changes at that bus which occur due to the particular line switching operation.

Note that in each case above all variables in $\underline{u}_1$ are zero unless otherwise specified. The operating point used to obtain matrices $\underline{A}$ and $\underline{B}$ is that obtained from the load flow after network changes are made rather than the base case load flow as in the case of using the coherency measure for producing external system dynamic equivalents because in this case the coherency measure will be used to investigate effects of a contingency in the internal system where the load flow conditions are important.

The vector function

$$\underline{u}_2(t) = \begin{cases} \underline{0} & t > T_1 \\ \underline{u}_2 & 0 \leq t \leq T_1 \\ \underline{0} & t < 0 \end{cases} \tag{2.10}$$

represents the affects of electrical faults where T_1 represents the fault clearing time and

$$\underline{u}_2 = \begin{bmatrix} \underline{\Delta PM} \\ \\ \underline{0} \end{bmatrix}$$

represents the step change in generation output equivalent to the accelerating powers due to a particular fault. This change of mechanical powers $\underline{\Delta PM}$, which equal the accelerating powers of generators due to a particular fault calculated by an ACCEL program [5], has been shown to adequately model the effects of that fault when a linearized model (2.2, 2.3) in the case of using the coherency measure to produce dynamic equivalents of the external system. The same electrical fault model is used because it is reasonable in many cases for studying the effects of faults on generators buses in the internal system and because it makes the model simpler for developing the coherency measure.

The above model can be generalized to model the uncertainty of any particular disturbance and yet handle specific deterministic disturbance as a special case. If the size and location of an electrical fault is not known and if the clearing time T_1 for this fault is not known, then a probabilistic description of this electrical fault is

$$E\{\underline{u}_2\} = \begin{bmatrix} \underline{m}_2^1 \\ 0 \end{bmatrix} = \underline{m}_2$$
$$E\{[\underline{u}_2 - \underline{m}_2][\underline{u}_2 - \underline{m}_2]^T\} = \begin{bmatrix} \underline{R}_2^1 & 0 \\ 0 & 0 \end{bmatrix} = \underline{R}_2 \qquad (2.11)$$

where $\underline{m}_2^1$ and $\underline{R}_2^1$ describe the uncertainty in accelerating power on all generators due to this electrical fault. This mean and variance should be determined based on observed historical records or hypothesized based on the present network and present internal and external conditions. If $\underline{R}_2 = \underline{0}$, and $\underline{m}_2^1 = \underline{\Delta PM}$ for a specific fault, this generalized model then reverts to the deterministic model of a specific electrical fault.

The uncertainty due to a generator dropping, line switching, and load shedding disturbance could be modeled by

$$E\{\underline{u}_1\} = \begin{bmatrix} \underline{m}_{11} \\ \\ \underline{m}_{12} \end{bmatrix} = \underline{m}_1$$

$$E\{[\underline{u}_1 - \underline{m}_1][\underline{u}_1 - \underline{m}_1]^T\} = \begin{bmatrix} \underline{R}_{11} & \underline{0} \\ \\ \underline{0} & \underline{R}_{22} \end{bmatrix} = \underline{R}_1 \tag{2.12}$$

where

(1) $\underline{m}_{11}$ and $\underline{R}_{11}$ can describe the uncertainty in generation changes due to generator dropping when the particular station, the generator in the station, and the power produced on the generator are unknown

(2) $\underline{m}_{12}$ and $\underline{R}_{22}$ describe the uncertainty in the location and magnitude of the load being dropped by any manual or automatic load shedding operation

(3) $\underline{m}_{12}$ and $\underline{R}_{22}$ describe the uncertainty in the location and the change in injections on buses due to any line switching operation

It should be noted that $\underline{\Delta PM}$ and $\underline{\Delta PL}$ are assumed uncorrelated because this model is to represent only one specific type of contingency at a time. For the same reason $\underline{u}_1$ and $\underline{u}_2$ are assumed uncorrelated with initial conditions and

$$E\{\underline{x}(0)\ \underline{u}_1^T\} = \underline{0}$$

$$E\{\underline{x}(0)\ \underline{u}_2^T\} = \underline{0} \tag{2.13}$$

The uncertain model of $\underline{u}_1$ can handle the case of a specific deterministic disturbance by setting $\underline{R}_1 = \underline{0}$ and $\underline{m}_1 = \underline{u}_1$ for the particular disturbance.

Care must be taken with this probabilistic model for generator dropping and line switching contingencies in order to make sure the network changes associated with the set of such contingencies being modeld probabilistically are properly performed.

CHAPTER 3

A GENERALIZED MEAN SQUARE COHERENCY MEASURE

The objective of this chapter is to develop a generalized measure of coherency which best reflects the overall system dynamics and which is broad enough to encompass the deterministic and probabilistic contingency set already discussed in Chapter 2.

This development is the mean square coherency measure based on a similar development [6] on the rms coherency measure.

The mean square measure of coherency between generator internal buses k and ℓ based on the uncertain description of disturbances is

$$C_{k\ell} = \frac{1}{T^n} E\{\int_0^T [\Delta\delta_k(t) - \Delta\delta_\ell(t)]^2 dt$$

$$= \frac{1}{T^n} E\{\int_0^T [(\Delta\delta_k(t) - \Delta\delta_s(t)) - (\Delta\delta_\ell(t) - \Delta\delta_s(t))]^2 dt\}$$

$$= \frac{1}{T^n} E\{\int_0^T \underline{x}^T(t)\underline{Q}_{k\ell}\underline{x}(t)dt\} \qquad (3.1)$$

where $\underline{Q}_{k\ell}$ is a 2N-dimensional square matrix

$$\underline{Q}_{k\ell} = \begin{bmatrix} \underline{Q}_{k\ell} & \underline{0} \\ \underline{0} & \underline{0} \end{bmatrix} \qquad (3.2)$$

and $\underline{Q}_{k\ell}$ is a N-dimensional square matrix where the ijth elements are

$$\{\underline{Q}_{k\ell}\}_{ij} = \begin{cases} 1 & i = j = k \text{ and } i = j = \ell \\ -1 & i = k,\ j = \ell \text{ and } i = \ell,\ j = k \\ 0 & \text{elsewhere} \end{cases} \tag{3.3}$$

Taking expectation inside the integral and recognizing that the coherency measure between internal generator buses k and ℓ is an element of a matrix $\underline{C}$, the coherency measure becomes

$$\{\underline{C}\}_{k\ell} = C_{k\ell} = \frac{1}{T^n} \int_0^T \mathrm{Tr}\{\underline{Q}_{k\ell}\ \underline{P}_x(t)\}dt$$

$$= \mathrm{Tr}\{\underline{Q}_{k\ell}[\frac{1}{T^n} \int_0^T \underline{P}_x(t)\ dt]\} \tag{3.4}$$

where $\mathrm{Tr}\{\cdot\}$ is the trace operator (sum of the diagonal elements of the argument matrix) and $\underline{P}_x(t)$ is

$$E\{\underline{x}(t)\ \underline{x}^T(t)\} = \underline{P}_{\underline{x}}(t) = \underline{V}_x(t) + \underline{m}_x(t)\ \underline{m}_x^T(t)$$

Since the objective is to compute the $N \times N$ matrix $\{\underline{C}\}_{k\ell} = C_{k\ell}$ and since

$$C_{k\ell} = \mathrm{Tr}\{\underline{Q}_{k\ell}\ \underline{S}_x(T)\}$$

$$= \{\underline{S}_x(T)\}_{kk} + \{S_{\underline{x}}(T)\}_{\ell\ell} - \{\underline{S}_x(T)\}_{\ell k} - \{S_{\underline{x}}(T)\}_{k\ell} \tag{3.5}$$

where

$$\underline{S}_x(T) = \frac{1}{T^n} \int_0^T \underline{P}_x(t)dt$$

It thus can be seen that the coherency matrix $\underline{C}$ is easily determined knowing $\underline{S}_x(T)$. The matrix $\underline{S}_x(T)$ is easily computed by substituting

$$\underline{x}(t) = \begin{cases} \varepsilon^{\underline{A}t}\underline{x}(0) + \int_0^t \varepsilon^{\underline{A}v}dv\ \underline{B}(\underline{u}_1 + \underline{u}_2) & t < T_1 \\ \varepsilon^{\underline{A}t}\underline{x}(0) + \int_0^t \ {}^{\underline{A}v}dv\ \underline{B}\ \underline{u}_1 & \\ \qquad + \varepsilon^{\underline{A}(t-T_1)} \int_0^{T_1} \varepsilon^{\underline{A}v}dv\ \underline{B}\ \underline{u}_2 & t > T_1 \end{cases} \tag{3.6}$$

$$\underline{S}_x(T) = \frac{1}{T^n} \int_0^T E\{\underline{x}(t)\ \underline{x}^T(t)\}dt \tag{3.7}$$

and taking expectation term by term using (2.7, 2.11, 2.12, 2.13) to obtain

$$\begin{aligned} \underline{S}_x(T) &= \frac{1}{T^n} \int_0^T \varepsilon^{\underline{A}}\ \underline{V}_x(0)\varepsilon^{\underline{A}^T\tau}d\tau \\ &+ \frac{1}{T^n} \int_0^{T_1} [\int_0^\tau \varepsilon^{\underline{A}v}dv\ \underline{B}][\underline{R}_1 + \underline{R}_2 + \underline{m}_1\underline{m}_1^T + \underline{m}_2\underline{m}_2^T + \underline{m}_1\underline{m}_2^T + \underline{m}_2\underline{m}_1^T] \\ &\qquad \times [\int_0^\tau \varepsilon^{\underline{A}v}dv\ \underline{B}]^T d\tau \\ &+ \frac{1}{T^n} \int_{T_1}^T ([\int_0^\tau \varepsilon^{\underline{A}v}dv\ \underline{B}][\underline{m}_1\underline{m}_1^T + \underline{R}_1][\int_0^\tau \varepsilon^{\underline{A}v}dv\ \underline{B}]^T)d\tau \\ &+ \frac{1}{T^n} \int_{T_1}^T ([\varepsilon^{\underline{A}(\tau-T_1)} \int_0^{T_1} \varepsilon^{\underline{A}v}dv\ \underline{B}][\underline{m}_2\underline{m}_2^T + \underline{R}_2] \\ &\qquad \times [\varepsilon^{\underline{A}(\tau-T_1)} \int_0^{T_1} \varepsilon^{\underline{A}v}dv\ \underline{B}]^T)d\tau \\ &+ \frac{1}{T^n} \int_{T_1}^T ([\varepsilon^{\underline{A}(\tau-T_1)} \int_0^{T_1} \varepsilon^{\underline{A}v}dv\ \underline{B}][\underline{m}_2\underline{m}_1^T][\int_0^\tau \varepsilon^{\underline{A}v}dv\ \underline{B}]^T)d\tau \\ &+ \frac{1}{T^n} \int_{T_1}^T ([\int_0^\tau \varepsilon^{\underline{A}v}dv\ \underline{B}][\underline{m}_1\underline{m}_2^T][\varepsilon^{\underline{A}(\tau-T_1)} \int_0^{T_1} \varepsilon^{\underline{A}v}dv\ \underline{B}]^T)d\tau \end{aligned} \tag{3.8}$$

The integer n is chosen to be one if a load shedding, line switching, or generator dropping contingency occurs and zero if an electrical fault occurs. This integer is chosen as one or zero so the integral will be finite and non-zero for an infinite observation interval.

The matrix $\underline{S}_x(T)$ becomes

$$\underline{S}_x(T) = \frac{1}{T} \int_0^T [\int_0^\tau \varepsilon^{\underline{A}v} dv \ \underline{B}] \underline{u}_1 \underline{u}_1^T [\int_0^\tau \varepsilon^{\underline{A}v} dv \ \underline{B}]^T) d\tau \qquad (3.9)$$

if a specific load shedding, loss of generation or line switching disturbance occurs since in this case

$\underline{R}_1 = \underline{R}_2 = \underline{0}$, $\underline{m}_2 = \underline{0}$, $\underline{m}_1 = \underline{u}_1$, and $\underline{V}_x(0) = \underline{0}$ and $n = 1$.

The matrix $\underline{S}_x(T)$ has the form

$$\underline{S}_x(T) = \int_0^{T_1} [\int_0^\tau \varepsilon^{\underline{A}v} dv \ \underline{B}] \underline{u}_2 \underline{u}_2^T [\int_0^\tau \varepsilon^{\underline{A}v} dv \ \underline{B}]^T d\tau$$

$$+ \int_{T_1}^T ([\varepsilon^{\underline{A}(\tau - T_1)} \int_0^{T_1} \varepsilon^{\underline{A}v} dv \ \underline{B}] \underline{u}_2 \underline{u}_2^T [\varepsilon^{\underline{A}(\tau - T_1)} \int_0^{T_1} \varepsilon^{\underline{A}v} dv \ \underline{B}]^T) d\tau$$

if the specific deterministic disturbance is an electrical fault since $\underline{R}_1 = \underline{R}_2 = \underline{0}$, $\underline{V}_x(0) = \underline{0}$, $\underline{m}_1 = \underline{0}$ and $\underline{m}_2 = \underline{u}_2$ and $n = 0$.

This generalized mean square coherency measure can handle both deterministic as well as probabilistic descriptions of power system disturbances.

CHAPTER 4

A TAYLOR SERIES EXPANSION OF THE MEAN-SQUARE COHERENCY MEASURE

The purpose of this chapter is to derive a Taylor series expression for the mean square coherency measure as a function of the observation interval T over which the measure is evaluated. The Taylor series expression for matrix $\underline{C}^s(T)$ will be derived by first deriving a Taylor series expression for $\underline{S}_x(T)$, and analyzing the coefficient matrices in this series. The transient coherency measure will be defined and analyzed in the next chapter based on the Taylor series expansion of the coherency measure developed in this chapter.

The mean square coherency measure can be computed using the matrix $\underline{S}_x(T)$, which is defined in (3.7). If the observation interval T is less than the fault clearing time T_1, vector $\underline{x}(t)$ is

$$\underline{x}(t) = \varepsilon^{\underline{A}t}\,\underline{x}(0) + \int_0^t \varepsilon^{\underline{A}v}dv\,\underline{B}(\underline{u}_1 + u_2) \tag{4.1}$$

and upon substitution the matrix $\underline{S}_x(T)$ becomes

$$\begin{aligned}\underline{S}_x(T) = \frac{1}{T^n}\int_0^t \varepsilon^{\underline{A}\tau}\,\underline{V}_x(0)\,\varepsilon^{\underline{A}^T\tau}d\tau \\ + \frac{1}{T^n}\int_0^t [\int_0^\tau \varepsilon^{\underline{A}v}dv]\underline{B}\,\underline{P}_u(0)\underline{B}^T[\int_0^\tau \varepsilon^{\underline{A}v}dv]^T d\tau\end{aligned} \tag{4.2}$$

where

$$\underline{V}_x(0) = E\{\underline{x}(0)\underline{x}^T(0)\} \tag{4.3}$$

$$\underline{P}_u(0) = E\{\underline{u}(0)\underline{u}^T(0)\} = \underline{R}_1 + \underline{R}_2 + (\underline{m}_1 + \underline{m}_2)(\underline{m}_1 + \underline{m}_2)^T \tag{4.4}$$

since $\underline{u}(0) = \underline{u}_1 + \underline{u}_2$ for $t < T_1$. Substituting

$$\begin{aligned} \varepsilon^{\underline{A}\tau} &= \sum_{k=0}^{\infty} \frac{\underline{A}^k \tau^k}{k!} \\ \int_0^\tau \varepsilon^{\underline{A}v} dv &= \sum_{k=0}^{\infty} \frac{\underline{A}^k \tau^{k+1}}{(k+1)!} \end{aligned} \tag{4.5}$$

into (4.2) and integrating, the matrix $\underline{S}_x(T)$ becomes

$$\begin{aligned} \underline{S}_x(T) &= \frac{1}{T^n} \sum_{j=0}^{\infty} \sum_{m=0}^{\infty} [\underline{A}^j \underline{V}_x(0) \underline{A}^{T^m}] \frac{T^{j+m+1}}{(j+m+1)j!m!} \\ &+ \frac{1}{T^n} \sum_{j=0}^{\infty} \sum_{m=0}^{\infty} [\underline{A}^j \underline{B}\ \underline{P}_u(0) \underline{B}^T \underline{A}^{T^m}] \frac{T^{j+m+3}}{(j+m+3)(j+1)!(m+1)!} \end{aligned} \tag{4.6}$$

Letting $K = j+m$, the $\underline{S}_x(T)$ matrix becomes

$$\begin{aligned} \underline{S}_x(T) &= \frac{1}{T^n} \sum_{K=0}^{\infty} \sum_{j=0}^{k} [\underline{A}^j\ \underline{V}_x(0) \underline{A}^{T\ K-j}] \frac{T^{K+1}}{(K+1)j!(K-j)!} \\ &+ \frac{1}{T^n} \sum_{K=0}^{\infty} \sum_{j=0}^{k} [\underline{A}^j \underline{B}\ \underline{P}_u(0) \underline{B}^T \underline{A}^{T\ K-j}] \frac{T^{K+3}}{(K+3)(j+1)!(K-j+1)!} \end{aligned} \tag{4.7}$$

Multiplying numerator and denominator of each term in the expression by appropriate factors, matrix $\underline{S}_x(T)$ can be expressed as

$$\underline{S}_x(T) = \frac{1}{T^n} \sum_{K=0}^{\infty} \underline{G}_K \frac{T^{K+1}}{(K+1)!} + \underline{L}_K \frac{T^{K+3}}{(K+3)!} \tag{4.8}$$

where coefficients

$$\underline{G}_K = \sum_{j=0}^{K} \binom{K}{j} \underline{A}^j \underline{V}_x(0) \underline{A}^{T\ K-j} \tag{4.9}$$

$$\underline{L}_K = \sum_{j=0}^{K} \binom{K+2}{j+1} \underline{A}^j \underline{B}\ \underline{P}_u(0) \underline{B}^T \underline{A}^{T\ K-j} \tag{4.10}$$

The matrices $\underline{L}_K$ and $\underline{G}_K$ can easily be shown to satisfy the following recursive formulas:

$$\underline{G}_{K+1} = \underline{A}\ \underline{G}_K + (\underline{A}\ \underline{G}_K)^T \quad K = 1,2,3,\ldots$$

$$\underline{G}_0 = \underline{V}_x(0) \tag{4.11}$$

$$\underline{L}_{K+1} = \underline{A}(\underline{L}_K + \underline{A}^K \frac{\underline{L}_0}{2}) + [\underline{A}(\underline{L}_K + \underline{A}^K \frac{\underline{L}_0}{2})]^T$$

$$\underline{L}_0 = 2\underline{B}\ \underline{P}_u(0) \underline{B}^T \tag{4.12}$$

The derivation of the recursive formula (4.11) follows the derivation found in [9]. Since

$$\underline{G}_K = \sum_{j=0}^{K} \binom{K}{j} \underline{A}^j\ \underline{V}_x(0) \underline{A}^{T\ K-j} \tag{4.13}$$

$$\underline{G}_K \underline{A}^T = \sum_{j=0}^{K} \binom{K}{j} \underline{A}^j\ \underline{V}_x(0) \underline{A}^{T\ K+1-j} \tag{4.14}$$

Replacing j by j-1 in (4.13),

$$\underline{G}_K = \sum_{j=1}^{K+1} \binom{k}{j-1} \underline{A}^{j-1} \underline{V}_x(0) \underline{A}^{T\ K+1-j} \tag{4.15}$$

and multiplying by $\underline{A}$ gives

$$\underline{A}\ \underline{G}_K = \sum_{j=1}^{K+1} \binom{k}{j-1} \underline{A}^j \underline{V}_x(0) \underline{A}^{T\ K+1-j} \tag{4.16}$$

The sum of (4.14) and (4.16) is

$$\begin{aligned} \underline{G}_K\underline{A}^T + \underline{A}\ \underline{G}_K = & \sum_{j=0}^{K} \binom{K}{j} \underline{A}^j \underline{V}_x(0) \underline{A}^{T\ K+1-j} \\ & + \sum_{j=1}^{K+1} \binom{k}{j-1} \underline{A}^j \underline{V}_x(0) \underline{A}^{T\ K+1-j} \end{aligned} \tag{4.17}$$

Since $\binom{K}{j} + \binom{K}{j-1} = \binom{K+1}{j}$ and $\underline{V}_x(0) = \underline{G}_0$,

$$\underline{G}_K\underline{A}^T + \underline{A}\ \underline{G}_K = \underline{G}_0\underline{A}^{T\ K+1} + \sum_{j=1}^{K} \binom{K+1}{j} \underline{A}^j \underline{G}_0 A^{T\ K+1-j} + \underline{A}^{K+1}\underline{G}_0 \tag{4.18}$$

The terms on the right hand side of (4.18) can be expressed as a single summation

$$\sum_{j=0}^{K+1} \binom{K+1}{j} \underline{A}^j \underline{G}_0 \underline{A}^{T\ K+1-j} = \underline{G}_{K+1} \tag{4.19}$$

and therefore

$$\underline{G}_{K+1} = \underline{G}_K\underline{A}^T + \underline{A}\ \underline{G}_K \tag{4.20}$$

Using the symmetric property of $\underline{G}_K$, the expression (4.20) becomes

$$\underline{G}_{K+1} = \underline{A}\ \underline{G}_K + (\underline{A}\ \underline{G}_K)^T \tag{4.21}$$

Similarly the second recursive formula (4.12) is derived as follows:

$$\underline{L}_K = \sum_{j=0}^{K} \binom{K+2}{j+1} \underline{A}^j \underline{B}\ \underline{P}_u(0) \underline{B}^T \underline{A}^{T\ K-j} \tag{4.22}$$

$$\underline{L}_K\underline{A}^T = \sum_{j=0}^{K} \binom{K+2}{j+1} \underline{A}^j \frac{\underline{L}_0}{2} \underline{A}^{T\ K+1-j} \tag{4.23}$$

Replacing j by j-1 in (4.22)

$$\underline{L}_K = \frac{1}{2} \sum_{j=1}^{K+1} \binom{K+2}{j} \underline{A}^{j-1}\underline{L}_0\underline{A}^{T\ K+1-j} \tag{4.24}$$

$$\underline{A}\ \underline{L}_K = \frac{1}{2} \sum_{j=1}^{K+1} \binom{K+2}{j} \underline{A}^j\underline{L}_0\underline{A}^{T\ K+1-j} \tag{4.25}$$

The sum of (4.23) and (4.25) is

$$\underline{A}\ \underline{L}_K + \underline{L}_K\underline{A}^T = \frac{1}{2} \sum_{j=0}^{K} \binom{K+2}{j+1} \underline{A}^j\underline{L}_0\underline{A}^{T\ K+1-j}$$

$$+ \frac{1}{2} \sum_{j=1}^{K+1} \binom{K+2}{j} \underline{A}^j\underline{L}_0\underline{A}^{T\ K+1-j} \tag{4.26}$$

Using $\binom{K+2}{j+1} + \binom{K+2}{j} = \binom{K+3}{j+1}$, (4.26) becomes

$$\underline{A}\ \underline{L}_K + \underline{L}_K\underline{A}^T = \frac{1}{2}\binom{K+2}{1} \underline{A}^j\underline{L}_0\underline{A}^{T\ K+1-j} + \frac{1}{2} \sum_{j=1}^{K} \binom{K+3}{j+1} \underline{A}^j\underline{L}_0\underline{A}^{T\ K+1-j} \tag{4.27}$$

$$+ \frac{1}{2} \binom{K+2}{K+1} \underline{A}^j\underline{L}_0\underline{A}^{T\ K+1-j}$$

$$= \frac{1}{2}(K+2)\underline{L}_0\underline{A}^{T\ K+1} + \frac{1}{2} \sum_{j=1}^{K} \binom{K+3}{j+1} \underline{A}^j\underline{L}_0\underline{A}^{T\ K+1-j} \tag{4.28}$$

$$+ \frac{1}{2}\ (K+2)\underline{A}^{K+1}\underline{L}_0$$

Now $\underline{L}_{K+1}$ can be expressed as the summation

$$\underline{L}_{K+1} = \frac{1}{2} \sum_{j=0}^{K+1} \binom{K+3}{j+1} \underline{A}^j\underline{L}_0\underline{A}^{T\ K+1-j} \tag{4.29}$$

$$= \frac{1}{2}\binom{K+3}{1}\underline{A}^{j}\underline{L}_0\underline{A}^{T\ K+1-j} + \frac{1}{2}\sum_{j=1}^{K}\binom{K+3}{j+1}\underline{A}^{j}\underline{L}_0\underline{A}^{T\ K+1-j}$$

$$+ \frac{1}{2}\binom{K+3}{K+2}\underline{A}^{j}\underline{L}_0\underline{A}^{T\ K+1-j} \tag{4.30}$$

$$= \frac{1}{2}(K+3)\underline{L}_0\underline{A}^{T\ K+1} + \frac{1}{2}\sum_{j=1}^{K}\binom{K+3}{j+1}\underline{A}^{j}\underline{L}_0\underline{A}^{T\ K+1-j}$$

$$+ \frac{1}{2}(K+3)\underline{A}^{K+1}\underline{L}_0 \tag{4.31}$$

Using (4.28), equation (4.31) becomes

$$\underline{L}_{K+1} = \underline{A}\ \underline{L}_K + \underline{L}_K\underline{A}^T + \frac{1}{2}\ \underline{L}_0\underline{A}^{T\ K+1} + \frac{1}{2}\ \underline{A}^{K+1}\underline{L}_0 \tag{4.32}$$

$$\underline{L}_{K+1} = \underline{A}(\underline{L}_K + \frac{1}{2}\ \underline{A}^K\underline{L}_0) + [\underline{A}(\underline{L}_K + \frac{1}{2}\ \underline{A}^K\underline{L}_0)]^T \tag{4.33}$$

The first assumption made in deriving specific forms for the $\{\underline{G}_K\}_{K=0}^{\infty}$, $\{\underline{L}_K\}_{K=0}^{\infty}$ sequences is that the initial condition $\underline{V}_x(0)$ is assumed to have zero initial frequency deviations because the initial conditions are intended to represent some disturbed condition in the network and is not intended to represent the state of the system after any specific contingency.

The second assumption made in deriving specific forms for the $\{\underline{G}_K\}_{K=0}^{\infty}$ and $\{\underline{L}_K\}_{K=0}^{\infty}$ sequences is that the damping contribution from the generator, load, and governor turbine energy system can be neglected. This assumption is justified because the length of the observation interval T is so small that effects of damping will be small and because the spread of the disturbance should depend principally on the generator inertias and transmission line synchronizing torque coefficients of the power system in question. The system matrix should thus have the form

$$\underline{A} = \begin{bmatrix} \underline{0} & \underline{I} \\ \underline{X} & \underline{0} \end{bmatrix} \tag{4.34}$$

where

$$\{\underline{X}\}_{ij} = \{-\underline{M}^{-1}\underline{T}\}_{ij} = \begin{cases} -\sum\limits_{\substack{k=1 \\ k\neq i}}^{N_g} \dfrac{T_{ik}}{M_i} & i=j \\ +\dfrac{T_{ij}}{M_i} & i\neq j \end{cases} \tag{4.35}$$

where T_{ij} is the synchronizing torque coefficient of the equivalent line connecting internal generator buses i and j for $i,j = 1,2,\ldots,N_g$.

The initial matrices $\underline{G}_0$ and L_0 are

$$\underline{G}_0 = \underline{V}_x(0) = \begin{bmatrix} \underline{V}_0 & \underline{0} \\ \underline{0} & \underline{0} \end{bmatrix} \tag{4.36}$$

$$\underline{L}_0 = 2\ B\ P_u(0)\underline{B}^T = 2\begin{bmatrix} \underline{0} & \underline{0} \\ \underline{0} & \underline{P} \end{bmatrix} \tag{4.37}$$

where

$$\underline{P} = \underline{M}^{-1}\underline{P}_u(0)_{11}\ \underline{M}^{-1^T} + \underline{M}^{-1}\ \underline{P}_u(0)_{12}\underline{L}^T\underline{M}^{-1^T} + \underline{M}^{-1}L\ \underline{P}_u(0)_{21}\underline{M}^{-1^T} + \underline{M}^{-1}\underline{L}\ \underline{P}_u(0)_{22}\underline{L}^T\underline{M}^{-1^T} \tag{4.38}$$

since

$$\underline{B} = \left[\begin{array}{c|c} \underline{0} & \underline{0} \\ \hline \underline{M}^{-1} & \underline{M}^{-1}\underline{L} \end{array}\right] \tag{4.39}$$

and

$$\underline{P}_u(0) = \begin{bmatrix} \underline{P}_u(0)_{11} & \underline{P}_u(0)_{12} \\ \underline{P}_u(0)_{21} & \underline{P}_u(0)_{22} \end{bmatrix} \tag{4.40}$$

$$= \underline{R}_1 + \underline{m}_1\underline{m}_1^T + \underline{R}_2 + \underline{m}_2\underline{m}_2^T + \underline{m}_1\underline{m}_2^T + \underline{m}_2\underline{m}_1^T$$

$$= \begin{bmatrix} \underline{R}_{11}+\underline{m}_{11}\underline{m}_{11}^T+\underline{R}_2^1+\underline{m}_2^1\underline{m}_2^{1^T}+\underline{m}_{11}\underline{m}_2^{1^T}+\underline{m}_2^1\underline{m}_{11}^T & \underline{m}_{11}\underline{m}_{12}^T+\underline{m}_2^1\underline{m}_{12}^T \\ \underline{m}_{12}\underline{m}_{11}^T + \underline{m}_{12}\underline{m}_2^{1^T} & \underline{R}_{22} + \underline{m}_{12}\underline{m}_{12}^T \end{bmatrix} \tag{4.41}$$

from Chapter 2. The matrix $\underline{P}_u(0)$ can be used to represent a generator dropping, load shedding, line switching or electrical fault either deterministically or probabilistically as described in Chapter 2. The following page summarizes the $\underline{P}_u(0)$ matrices and associated contingency profile.

	CONTINGENCY Description	Statistics	$\underline{P}_u(0)$
STOCHASTIC	Load Shedding Gen. Dropping Line Switching (STEP)	$\underline{R}_2 = 0;\ \underline{m}_2 = 0$	$\begin{bmatrix} \underline{R}_{11} + \underline{m}_{11}\underline{m}_{11}^T & \underline{m}_{11}\underline{m}_{12}^T \\ \underline{m}_{12}\underline{m}_{11}^T & \underline{R}_{22} + \underline{m}_{12}\underline{m}_{12}^T \end{bmatrix}$
	Electrical Fault (PULSE)	$\underline{R}_1 = 0;\ \underline{m}_1 = 0$	$\begin{bmatrix} \underline{R}_2^1 + \underline{m}_2^1\underline{m}_2^{1T} & \underline{0} \\ \underline{0} & \underline{0} \end{bmatrix}$
DETERMINISTIC	Load Shedding Gen. Dropping Line Switching	$\underline{R}_1 = 0;\ \underline{\mu}_1 = m_1$ $\underline{R}_2 = 0;\ \underline{m}_2 = 0$	$\begin{bmatrix} \underline{m}_{11}\underline{m}_{11}^T & \underline{m}_{11}\underline{m}_{12}^T \\ \underline{m}_{12}\underline{m}_{11}^T & \underline{m}_{12}\underline{m}_{12}^T \end{bmatrix}$
	Electrical Fault	$\underline{R}_1 = 0;\ \underline{m}_1 = 0$ $\underline{R}_2 = 0;\ \underline{\mu}_2 = \underline{m}_2$	$\begin{bmatrix} \underline{m}_2^1\underline{m}_2^{1T} & \underline{0} \\ \underline{0} & \underline{0} \end{bmatrix}$

The matrices $\{\underline{L}_K\}_{K=1}^{8}$ can be obtained from the recursive equations and have the form

$$\underline{L}_1 = \begin{bmatrix} \underline{0} & 3\underline{P} \\ 3\underline{P} & \underline{0} \end{bmatrix} \tag{4.42}$$

$$\underline{L}_2 = \begin{bmatrix} 6\underline{P} & \underline{0} \\ \underline{0} & 4\underline{X}\,\underline{P}+10\underline{P}\,\underline{X}^T \end{bmatrix} \tag{4.43}$$

$$\underline{L}_3 = \begin{bmatrix} \underline{0} & 5\underline{X}\,\underline{P}+10\underline{P}\,\underline{X}^T \\ 10\underline{X}\,\underline{P}+5\underline{P}\,\underline{X}^T & \underline{0} \end{bmatrix} \tag{4.44}$$

$$\underline{L}_4 = \left[\begin{array}{c|c} 15\underline{X}\,\underline{P}+15\underline{P}\,\underline{X}^T & \underline{0} \\ \hline \underline{0} & 6\underline{X}^2\underline{P}+20\underline{X}\,\underline{P}\,\underline{X}^T+6\underline{P}\,\underline{X}^{T2} \end{array}\right] \tag{4.45}$$

$$\underline{L}_5 = \left[\begin{array}{c|c} \underline{0} & 7\underline{X}^2\underline{P}+21\underline{P}\,\underline{X}^{T2}+35\underline{X}\,\underline{P}\,\underline{X}^T \\ \hline 21\underline{X}^2\underline{P}+7\underline{P}\,\underline{X}^{T2}+35\underline{X}\,\underline{P}\,\underline{X}^T & \underline{0} \end{array}\right] \tag{4.46}$$

$$\underline{L}_6 = \left[\begin{array}{c|c} 28\underline{X}^2\underline{P}+28\underline{P}\,\underline{X}^{T2}+70\underline{X}\,\underline{P}\,\underline{X}^T & \underline{0} \\ \hline \underline{0} & 8\underline{X}^3\underline{P}+8\underline{P}\,\underline{X}^{T3}+56\underline{X}^2\underline{P}\,\underline{X}^T+56\underline{X}\,\underline{P}\,\underline{X}^{T2} \end{array}\right] \tag{4.47}$$

$$\underline{L}_7 = \left[\begin{array}{c|c} \underline{0} & 9\underline{X}^3\underline{P}+36\underline{P}\,\underline{X}^{T3}+84\underline{X}^2\underline{P}\,\underline{X}^T+126\underline{X}\,\underline{P}\,\underline{X}^{T2} \\ \hline 36\underline{X}^3\underline{P}+9\underline{P}\,\underline{X}^{T3}+126\underline{X}^2\underline{P}\,\underline{X}^T+84\underline{X}\,\underline{P}\,\underline{X}^{T2} & \underline{0} \end{array}\right] \tag{4.48}$$

$$\underline{L}_8 = \left[\begin{array}{c|c} 45\underline{X}^3+45\underline{P}\,\underline{X}^{T3}+210\underline{X}^2\underline{P}\,\underline{X}^T+210\underline{X}\,\underline{P}\,\underline{X}^{T2} & \underline{0} \\ \hline \underline{0} & 10\underline{X}^4\underline{P}+120\underline{X}\,\underline{P}\,\underline{X}^{T3}+120\underline{X}^3\underline{P}\,\underline{X}^T+252\underline{X}^2\underline{P}\,\underline{X}^{T2}+10\underline{P}\,\underline{X}^{T4} \end{array}\right] \tag{4.49}$$

The partial matrix sequence $\{\underline{L}_k\}_{K=1}^{8}$, indicates that matrices $\{\underline{L}_{2K}\}_{K=0}^{\infty}$ have zero off diagonal submatrices and that matrices $\{\underline{L}_{2K+1}\}_{K=0}^{\infty}$ have zero $N \times N$ diagonal submatrices. Moreover it is shown that the upper $N \times N$

diagonal submatrix in the $\underline{L}_{2K}$ matrices have the form

$$\hat{\underline{L}}_{2K} = \sum_{\ell=0}^{K-1} a_{K\ell} \underline{X}^{K-\ell-1} \underline{P} \underline{X}^{\ell^T} \qquad (4.50)$$

These properties of the sequence $\{\underline{L}_K\}_{K=0}^{\infty}$ and $\{\underline{L}_{2K}\}_{K=0}^{\infty}$ can be formally proved using the definition (4.10) and

$$\underline{A}^j = \begin{cases} \begin{bmatrix} \underline{X}^K & \underline{O} \\ \underline{O} & \underline{X}^K \end{bmatrix} & j = 2K \\ \begin{bmatrix} \underline{O} & \underline{X}^K \\ \underline{X}^{K+1} & \underline{O} \end{bmatrix} & j = 2K+1 \end{cases} \qquad (4.51)$$

A Taylor series expansion of the coherency measure can now be derived based on the Taylor series expansion of $\underline{S}_x(T)$ and the analysis of the coefficient matrix sequences $\{\underline{L}_K\}_{K=0}^{\infty}$. Since the coherency measure

$$C^s_{k\ell}(T) = \{\underline{S}_x(T)\}_{kk} + \{\underline{S}_x(T)\}_{\ell\ell} - \{\underline{S}_x(T)\}_{k\ell} - \{\underline{S}_x(T)\}_{\ell k} \qquad (4.52)$$

depends on the upper N × N diagonal submatrix of $\underline{S}_x(T)$ because the coherency measure is only defined for $k,\ell = 1,2,\ldots,N_g$, the square coherency measure can be written as

$$C^s_{k\ell}(T) = \hat{\underline{e}}_k^T \, [\hat{\underline{S}}_x(T)] \hat{\underline{e}}_{k\ell} \qquad (4.53)$$

where $\hat{\underline{S}}_x(T)$ is the upper N × N diagonal submatrix of $\underline{S}_x(T)$, $\underline{e}_{k\ell}$ is the upper N_g vector of $\underline{e}_k$ and thus is a N_g vector with all zero elements except for 1 in the k^{th} element and -1 in the ℓ^{th} element. Matrix $\underline{S}_x(T)$ can be expressed as

$$\hat{\underline{S}}_x(T) = \frac{1}{T^n} \sum_{K=0}^{\infty} \hat{\underline{L}}_{2K} \frac{T^{2K+3}}{2K+3!} \tag{4.54}$$

where $\{\hat{\underline{L}}_{2K}\}_{K=0}^{\infty}$ are the upper $N_g \times N_g$ diagonal submatrices of $\{\underline{L}_{2K}\}_{K=0}^{\infty}$. The upper $N_g \times N_g$ diagonal matrix of $\{\underline{L}_{2K+1}\}_{K=0}^{\infty}$ have been shown to be null, and thus are omitted. The Taylor series expansion for the mean square coherency measure is thus defined by (4.53) and (4.54).

CHAPTER 5

THE TRANSIENT COHERENCY MEASURE AND POWER SYSTEM TRANSIENT RESPONSE

The purpose of this chapter is

(1) to define the transient coherency measure as a means of analyzing the power system's transient response to a disturbance

(2) to analyze the spread of a deterministic disturbance from a single bus by analyzing the additional terms that enter the transient coherency measure as the observation interval increases. Rules for predicting (1) the time instants that the disturbance begins to accelerate each successive set of generators or stage further from the disturbance location, (2) the generators and equivalent lines in each stage; (3) the relative stiffness of the interconnection from the disturbed generator to that portion of the system affected by the disturbance at any particular time; and (4) the effects of particular lines and generators on the coherency measure at a particular time and location are given

(3) to show how the coherency measure, coherent groups, and the weaknesses in dynamic structure can change as the stiffness of the interconnection affected by the disturbance changes due to the very spread of that disturbance. This analysis is carried out using the transient coherency measure evaluated for a deterministic and probabilistic disturbance. The transient coherency measure evaluated for a specific probabilistic disturbance is shown to be an excellent transient security measure. A comparison of the structural properties measured by the transient and dynamic coherency measures is also made. The use of this information by system operators and planners to assess dynamic and transient security of their system is also discussed.

5.1 The definition and analysis of the transient coherency measure is based on disturbance models of line switching, load shedding, generator dropping, and electrical faults but does not include the disturbances which might be expressed knowing only the statistics of the initial state at some instant. This definition and analysis could be extended to include such disturbances if desired.

The transient coherency measure will be defined after the following theorem is proved. The analysis of the spread of a disturbance for both deterministic and probabilistic disturbances through a power system, which is known as the ripple effect, will follow the definition of the transient coherency measure.

Theorem 1 The mean square coherency measure between internal generator buses k and ℓ is approximated by an N+2 term Taylor series

$$\hat{C}^{N}_{k\ell}(T) = \frac{1}{T^{n}} \sum_{K=0}^{N+1} (\hat{\underline{e}}^{T}_{k\ell}\, \hat{\underline{L}}_{2K\ell}\hat{\underline{e}}_{k\ell}) \frac{T^{2K+3}}{2K+3!} \tag{5.1}$$

with maximum error ε over interval $[0, T^{N}_{k\ell}(\varepsilon)]$

$$\sup_{T\varepsilon[0,\ T^{N}_{k\ell}(\varepsilon)]} \{|C^{s}_{k\ell}(T) - C^{N}_{k\ell}(T)|\} = \varepsilon \tag{5.2}$$

where $T^{N}_{k\ell}(\varepsilon)$ is constrained to satisfy

$$T^{N}_{k\ell}(\varepsilon) \leq T^{*} \tag{5.3}$$

and thus ε and $T^{N}_{k\ell}(\varepsilon)$ are dependent on T^{*}; $\varepsilon(T^{*})$. If T^{*} is sufficiently small, the length of the observation intervals, $\{T^{N}_{k\ell}(\varepsilon)\}^{\infty}_{N=1}$ for this sequence of approximations is a monotone increasing sequence.

Proof The error in the N+2 term approximation to the coherency measure $C^{s}_{k\ell}(T)$ is

$$C^{s}_{k\ell}(T) - C^{N}_{k\ell}(T) = \frac{1}{T^{n}} \sum_{K=N+2}^{\infty} (\hat{\underline{e}}^{T}_{k\ell}\, \hat{\underline{L}}_{2K\ell}\hat{\underline{e}}_{k\ell}) \frac{T^{2K+3}}{2K+3!} \tag{5.4}$$

If T^* and thus T is sufficiently small, this error for the N+2 and N+3 term approximations satisfy

$$\left| C^{s}_{k\ell}(T) - C^{N}_{k\ell}(T) \right| \simeq \frac{1}{T^n} \left| \hat{\underline{e}}^{T}_{k\ell} \; \hat{\underline{L}}_{2N+4} \; \hat{\underline{e}}_{k\ell} \right| \frac{T^{2N+7}}{2N+7!} \tag{5.5}$$

$$\left| C^{s}_{k\ell}(T) - C^{N+1}_{k\ell}(T) \right| \simeq \frac{1}{T^n} \left| \hat{\underline{e}}^{T}_{k\ell} \; \hat{\underline{L}}_{2N+6} \; \hat{\underline{e}}_{k\ell} \right| \frac{T^{2N+9}}{2N+9!} \tag{5.6}$$

for all $T < T^*$ because $C^{s}_{k\ell}(T)$ is analytic. Since the error for both approximations is a monotone increasing function of T, the maximum error occurs at $T^{N}_{k\ell}(\varepsilon)$ and $T^{N+1}_{k\ell}(\varepsilon)$ for the N+2 and N+3 term approximation respectively

$$\varepsilon = \left| C^{s}_{k\ell}(T) - \underline{C}^{N}_{k\ell}(T) \right|_{T=T^{N}_{k\ell}(\varepsilon)} \simeq \frac{1}{T^n} \left| \hat{\underline{e}}^{T}_{k\ell} \; \hat{\underline{L}}_{2N+4} \; \hat{\underline{e}}_{k\ell} \right| \left. \frac{T^{2N+7}}{2N+7!} \right|_{T=T^{N}_{k\ell}(\varepsilon)} \tag{5.7}$$

$$\varepsilon = \left| C^{s}_{k\ell}(T) - \underline{C}^{N+1}_{k\ell}(T) \right|_{T=T^{N+1}_{k\ell}(\varepsilon)} \simeq \frac{1}{T^n} \left| \hat{\underline{e}}^{T}_{k\ell} \; \hat{\underline{L}}_{2N+6} \; \hat{\underline{e}}_{k\ell} \right| \left. \frac{T^{2N+9}}{2N+9!} \right|_{T=T^{N+1}_{k\ell}(\varepsilon)} \tag{5.8}$$

Since from (5.6) above,

$$\left| C^{s}_{k\ell}(T) - C^{N}_{k\ell}(T) \right|_{T=T^{n}_{k\ell}(\varepsilon)} > \left| C^{s}_{k\ell}(T) - C^{N+1}_{k\ell}(T) \right|_{T=T^{N}_{k\ell}(\varepsilon)} \tag{5.9}$$

and since $C^{s}_{k\ell}(T) - C^{N+1}_{k\ell}(T)$ is a monotone increasing function of T, it follows that

$$T^{N+1}_{k\ell}(\varepsilon) \geq T^{N}_{k\ell}(\varepsilon) \qquad (5.10)$$

and the theorem is proved.

The transient coherency measure is now defined

$$C^{*}_{k\ell}(T) = \begin{cases} C^{1}_{k\ell}(T) & 0 \leq T \leq T^{1}_{k\ell}(\varepsilon) \\ C^{N}_{k\ell}(T) & T^{N-1}_{k\ell}(\varepsilon) \leq T \leq T^{N}_{k\ell}(\varepsilon) \\ & N=2,3,\ldots \end{cases} \qquad (5.11)$$

This transient coherency measure is an approximation of the mean square coherency measure over a sufficiently small interval $0 \leq T \leq T^*$ with a maximum error ε. The number of terms (N) used in the approximation increase as T increases. This property of the transient coherency measure is necessary to analyze the changes in the power system dynamics after a disturbance occurs. This analysis of the changes in power system dynamic structure will be carried out for both a deterministic generator dropping disturbance at a single bus

$$\underline{m}_1 = \underline{e}_1 M_1, \; \underline{m}_2 = \underline{0}, \; \underline{R}_1 = \underline{0}, \; \underline{R}_2 = \underline{0} \qquad (5.12)$$

where

$$\underline{e}_1^T = (1,0, \ldots, 0)$$

such that the matrix $\underline{P}$ (4.37) is

$$\underline{P} = \text{diag}(1,0, \ldots, 0) \qquad (5.13)$$

and a probabilistic disturbance such that

$$\underline{P} = \underline{I} \tag{5.14}$$

This probabilistic disturbance represents a zero mean IID step change in initial shaft accelerations on all generators and could be produced by the generator dropping, line switching, load shedding, and electrical fault disturbance models.

The explicit values of matrices $\underline{L}_0$, $\underline{L}_2$, $\underline{L}_4$, $\underline{L}_6$, evaluated from (4.42), which are required for this analysis, of the transient coherency measure for the deterministic disturbance (5.13) are

$$\underline{L}_0 = \underline{0}$$

$$\{\hat{\underline{L}}_2\}_{mn} = \{6\underline{P}\}_{mn} = \begin{cases} 6 & m=n=i \\ 0 & \text{otherwise} \end{cases}$$

$$\{\underline{L}_4\}_{mn} = \{15(\underline{X}\,\underline{P} + \underline{P}\,\underline{X}^T)\}_{mn} = \begin{cases} 0 & m\neq i,\ n\neq i \\ -30 \sum_{j=1}^{N} \frac{T_{ij}}{M_i} & m=n=i \\ +15 \frac{T_{ni}}{M_n} & m=i,\ n\neq i \\ +15 \frac{T_{mi}}{M_m} & m\neq i,\ n=i \end{cases} \tag{5.15}$$

$$\{\underline{L}_6\}_{mn} = \{28\underline{X}^2\ \underline{P} + 70\underline{X}\ \underline{P}\ \underline{X}^T + 28\ \underline{P}\ \underline{X}^{2^T}\}_{mn}$$

$$= \begin{cases} +70\ \dfrac{T_{mi}}{M_m}\ \dfrac{T_{ni}}{M_n} & m\neq i,\ n\neq i \\[2ex] 70\ \sum\limits_{j=1}^{N_g} \dfrac{T_{ij}}{M_i}\ \sum\limits_{j=1}^{N_g} \dfrac{T_{ij}}{M_i} + 56\left[\sum\limits_{\substack{j=1\\ j\neq i}}^{N_g} \dfrac{T_{ij}}{M_i}\ \dfrac{T_{ji}}{M_j} + \left(\sum\limits_{j=1}^{N_g} \dfrac{T_{ij}}{M_i{}^2}\right)^2\right] & n=m=i \\[2ex] -70\ \sum\limits_{j=1}^{N_g} \dfrac{T_{ij}}{M_i}\ \dfrac{T_{ni}}{M_n} - 28\left[\sum\limits_{\substack{j=1\\ j\neq i\\ j\neq n}}^{N_g} \dfrac{T_{nj}}{M_n}\ \dfrac{T_{ji}}{M_j} - \left(\sum\limits_{j=1}^{N_g} \dfrac{T_{nj}}{M_n}\right)\dfrac{T_{ni}}{M_n} - \dfrac{T_{ni}}{M_n}\left(\sum\limits_{j=1}^{N_g} \dfrac{T_{ij}}{M_i}\right)\right] & m=i,\ n\neq i \\[2ex] -70\ \dfrac{T_{mi}}{M_m}\left(\sum\limits_{j=1}^{N_g} \dfrac{T_{ij}}{M_i{}^2}\right) + 28\left[\sum\limits_{\substack{j=1\\ j\neq i\\ j\neq m}}^{N_g} \dfrac{T_{mj}}{M_m}\ \dfrac{T_{ji}}{M_j} - \left(\sum\limits_{j=1}^{N_g} \dfrac{T_{mj}}{M_m}\right)\dfrac{T_{mi}}{M_m} - \dfrac{T_{mi}}{M_m}\left(\sum\limits_{j=1}^{N_g} \dfrac{T_{ij}}{M_o}\right)\right] & \end{cases}$$

The transient coherency measure for $T\varepsilon[0,T^2_{k\ell}(\varepsilon)]$ and the deterministic disturbance (5.13) is now analyzed. This transient coherency measure evaluated over the first interval $0\leq T\leq T^1_{k\ell}(\varepsilon)$ when n=1 is

$$(T) = C'_{k\ell}(T) = \hat{\underline{e}}^T_{k\ell}(\hat{L}_c\frac{T^2}{3!} + \hat{L}_2\frac{T^4}{5!} + \hat{L}_4\frac{T^6}{7!})\hat{\underline{e}}_{k\ell} \qquad (5.16)$$

$$= \begin{cases} 0 & k\neq i,\ \ell\neq i \\ 0 & k=i,\ \ell=i \\ 6\frac{T^4}{5!} - 30\left(\frac{\sum_{j=1}^{N} T_{ij}}{M_i} + \frac{T_{\ell i}}{M_\ell}\right)\frac{T^6}{7!} & k=i,\ \ell\neq i \\ 6\frac{T^4}{5!} - 30\left(\frac{\sum_{j=1}^{N} T_{ij}}{M_i} + \frac{T_{ki}}{M_k}\right)\frac{T^6}{7!} & k\neq i,\ \ell=i \end{cases}$$

The coherency measure between generators k and ℓ, where neither k nor ℓ is the disturbed generator (i), indicates that all these generators $k\neq i$ swing coherently as a single generator over the first interval. The coherency measure $C^*_{ik}(T) = C^*_{ki}(T)$ is non zero over this interval and indicates all generators $k\neq i$ swing together as an infinite bus against generator i that is accelerated by the disturbance. Generator i is a stage 1 bus since it is the only generator accelerated in the first interval. The part of the system dynamics not directly connected to bus i can be neglected over this interval because it does not affect the coherency measure (5.16)for any pair of buses in that system.

The transient coherency measure over the second interval $T^1_{k\ell}(\varepsilon) \leq T \leq T^2_{k\ell}(\varepsilon)$ for n=1 is

$$C^*_{k\ell}(T) = \hat{\underline{e}}^T_{k\ell}(\hat{\underline{L}}_0 \frac{T^2}{3!} + \hat{\underline{L}}_2 \frac{T^4}{5!} + \hat{\underline{L}}_4 \frac{T^6}{7!} + \hat{\underline{L}}_6 \frac{T^8}{9!})\, \hat{\underline{e}}_{k\ell} \tag{5.17}$$

$$\underline{C}^*_{k\ell}(T) = \begin{cases} 70\left(\frac{T_{k\ell}^{\;2}}{M_k^2} + \frac{T_{\ell i}^{\;2}}{M_\ell^2} + \frac{2T_{k\ell}}{M_k}\frac{T_{\ell i}}{M_\ell}\right)\frac{T^8}{9!} & k\neq i,\ \ell\neq i \\ 0 & k=\ell=i \\ C^1_{i\ell}(T) + [\{\hat{\underline{L}}_6\}_{ii} + \{\hat{\underline{L}}_6\}_{\ell\ell} - \{\hat{\underline{L}}_6\}_{i\ell} - \{\hat{\underline{L}}_6\}_{\ell i}]\frac{T^8}{9!} & k=i,\ \ell\neq i \\ C^1_{ki}(T) + [\{\hat{\underline{L}}_6\}_{kk} + \{\hat{\underline{L}}_6\}_{ii} - \{\hat{\underline{L}}_6\}_{ki} - \{\hat{\underline{L}}_6\}_{ik}]\frac{T^8}{9!} & k\neq i,\ \ell=i \end{cases} \tag{5.18}$$

where $\underline{L}_{6\,mn}$ are taken from (5.15).

This transient coherency measure between generators k and ℓ, where neither k nor ℓ is the disturbed generator, is dependent on the relative stiffness of equivalent lines

$$\frac{T_{ki}}{M_i}\ ,\ \frac{T_{\ell i}}{M_i}$$

connecting generators k and i and ℓ and i. If there are no equivalent lines connecting generator i to generator k and to generator ℓ,

$$T_{ki} = T_{\ell i} = 0$$

then the transient coherency measure $C^*_{k\ell}(T)$ continues to be zero over the second interval just as it was over the first interval. However if there is an equivalent line connecting generator k to generator i,

$$T_{ki} \neq 0$$

then $C^{*}_{k\ell}(T)$ is non zero over the second interval for all generators ℓ. Thus, the set of all generators k connected to i by equivalent lines

$$T_{ki} \neq 0$$

will leave the group acting as an infinite bus swinging against generator i during the first interval and be accelerated by the disturbance over this second interval. This set of generators are called stage 2 generators. It should be noted that all generators (k) not connected to generator i by an equivalent line

$$T_{ki} = 0$$

will not have been affected by the disturbance in the first and second interval $0 \leq T \leq T^{2}_{k\ell}(\varepsilon)$ and act as an infinite bus. That portion of the system not in stage 1 or 2 or connected to stage 2 but not in stage 1 (stage 3) does not affect the coherency measure during the second interval.

If the above analysis were carried out for the N^{th} interval for each N it could be shown that

(1) stage 1 generators are those that feel the affect of the disturbance immediately at $t = 0$;

(2) stage N+1 generators are connected to stage 1 generators in the shortest path by N branches or through (N-1) generators as shown

(3) the $N+1^{st}$ stage is accelerated beginning at $T^{2N-1}_{k\ell}(\varepsilon)$ since $C^{2N}_{K\ell}(T)^{N}$ are nonzero for any generator K in stage N+1 but lower

order approximation $C_{kj}^{K}(T)$ for $K = 1,2,\ldots,2N$ are zero when j belongs to stage $N+1, N+2, \ldots$, $\{C_{kj}^{2N}(T)\}_{j=1}^{N}$ are all non zero for any k in stage $N+1$ since every term $\{C_{kj}^{2N}(T)\}_{j=1}^{N}$ depend on $\{\underline{L}_{4N+2}\}_{kk}$ which is not zero because

$$\{\underline{L}_{4N+2}\}_{kk} \approx (\underline{X}_{kL}^{N})^2$$

$$\{X^N\}_{ki} = \left(\sum_{j2}^{N} \sum_{j3} \cdots \sum_{jN} x_{ij_2} x_{j_2j_3} x_{j_3j_4} \cdots x_{j_Nk}\right) \qquad (5.19)$$

and because $\{\underline{X}^N\}_{ki}$ is nonzero for any k in $N+1$. $\{X^N\}_{k\ell}$ indicates both the generators and lines affected by the disturbance and the stiffness of the interconnection between the disturbed generator in stage 1 and any generator k in stages 2 to $N+1$ in the $2N^{th}$ interval. Note that $\{\underline{X}^N\}_{ki}$ is the sum of all N branch connections between k and i and thus relative stiffness of the interconnection between k and i depends on an increasing number of paths and a larger portion of the system as T and thus N increases.

This information is important because it

(1) disects the transient response into discrete intervals $T_{k\ell}^{2N-1}(\varepsilon) \leq T \leq T_{k\ell}^{2N+1}(\varepsilon)$ when the $N+1^{st}$ stage is accelerated

(2) indicates the portion of the system affected by the disturbance and provides a measure of the relative stiffness of that interconnection

$\{\underline{X}^N\}_{ki}$ between the disturbed generators and the portion of the system affected by the disturbance during each of these intervals which begin when even number terms $\underline{L}_{2N+2}$ are added to the coherency measure approximation.

(3) permits the cause of significant oscillation at any location in any time interval, which could lead to loss of stability, to be easily determined. This could be accomplished without the guesswork since contributions to the transient response at any time and location can be easily determined analytically without repeated simulation.

This type of analysis would be helpful in transmission or generation expansion planning to isolate the effects on transient stability of adding lines or generation. It would also be helpful in contingency analysis for operation planning.

This completes the analysis of the power system transient response for a deterministic disturbance at a single bus. The above results, which are confirmed using computational results in the next section, also indicate that changes in dynamic system structure, weakness in that structure, and coherent groups could be expected due to the changes in the stiffness of the interconnection which occur because the disturbance has spread. This can be clearly seen for the deterministic disturbance at a single generator but the results obtained then depend on the location and magnitude of that disturbance. A more complete understanding of the changes in the coherency measure, coherent groups, and weakness in dynamic structure as a function of time is possible if the transient coherency measure is analyzed for a probabilistic disturbance, which is chosen so that the transient coherency measure only depends on $\underline{X} = -\underline{M}^{-1}\underline{T}$ and does not depend on the disturbance. The disturbance model that makes this possible is from (4.37)

$$\underline{P} = \underline{I}$$
$$\underline{B}\underline{P}_u(0)\underline{B}^T = \underline{B}\left[\underline{R}_1+\underline{R}_2 + (\underline{m}_1+\underline{m}_2)(\underline{m}_1+\underline{m}_2)^T\right]\underline{B}^T = \begin{bmatrix} 0 & 0 \\ \underline{0} & \underline{I} \end{bmatrix} \tag{5.20}$$

This disturbance produces a zero mean independent, identically distributed (IID) step change in shaft acceleration at all generators in the system and is thus an excellent disturbance to investigate the dynamic structure and structural weaknesses during the transient interval. This disturbance (5.20) could be produced by a generator dropping, load shedding, line dropping or electrical fault disturbance model. The generator dropping disturbance required to obtain $\underline{P} = \underline{I}$ is

$$\underline{m}_1 = \begin{bmatrix} \underline{m}_{11} \\ \underline{0} \end{bmatrix} \quad \underline{m}_2 = \underline{0} \qquad (5.21) \qquad \underline{R}_{11} = \text{diag}\{M_1^2, M_2^2, \ldots, M_N^2\} \quad (5.23)$$

$$\underline{R}_1 = \begin{bmatrix} \underline{R}_{11} & \underline{0} \\ \underline{0} & \underline{0} \end{bmatrix} \quad \underline{R}_2 = \underline{0} \quad (5.22) \qquad \underline{m}_{11} = \underline{0} \qquad (5.24)$$

The generator dropping disturbance that produces zero mean IID step change in shaft acceleration on all generators requires the mean of the step deviation in mechanical power on each generator to be zero, the correlation between the step deviation on any two generators to be zero, and the variance of the step change in mechanical power deviation on any generator to be proportional to its inertia squared.

The transient coherency measure over the first interval for this probabilistic disturbance at every generator is now given as:

$$C^*_{k\ell}(T) = \hat{\underline{e}}^T_{k\ell}[6\underline{I}\,\frac{T^4}{5!} + 15(\underline{X}+\underline{X}^T)\frac{T^6}{7!}]\hat{\underline{e}}_{k\ell} \quad 0 \le t \le T^1_{k\ell}(\varepsilon)$$

$$= \frac{12T^4}{5!} - 30\left(\sum_{\substack{j=1\\j\neq k}}^{N_g} \frac{T_{kj}}{M_k} + \sum_{\substack{j=1\\j\neq \ell}}^{N_g} \frac{T_{\ell j}}{M_\ell} + T_{k\ell}\left(\frac{1}{M_k} + \frac{1}{M_\ell}\right)\right)\frac{T^6}{7!} \qquad (5.25)$$

This transient coherency measure reflects the stress on a power system during an initial interval where the disturbance at each generator has not yet propagated to neighboring generators. This coherency measure is quite independent of the kind of disturbance or the magnitude or correlation between disturbances so that the true system structure and weakness in that structure are indicated for this first interval.

The structural coherency of generators k and over the initial interval, that is independent of the disturbance and only dependent on system dynamic structure, not only requires a relatively stiff interconnection of k and ℓ measured by

$$T_k \left(\frac{1}{M_k} + \frac{1}{M_\ell}\right) \tag{5.26}$$

but also that these generators be stiffly connected to the system

$$\sum_{j=1}^{N_g} \frac{T_{kj}}{M_k} \qquad \sum_{j=1}^{N_g} \frac{T_{\ell j}}{M_\ell} \tag{5.27}$$

A group of generators will only be mutually coherent if all the generators k are stiffly tied to most of the generators in the group because then both (5.26) and (5.27) will be large for all members of that group. However, a generator could be coherent with one member of a group and not other members of the group if it is very stiffly tied to that one member.

The form of the transient coherency measure $C^*_{k\ell}(T)$ over the first interval also shows how the first swing stability of generators k and ℓ also depends on the relative stiffness of the equivalent line connecting k and ℓ (5.26) and the relative stiffness of all lines connected to k and to ℓ. This is extremely important information since it is not disturbance dependent and indicates regions where there is relative structural weakness and susceptibility to transient stability problems over that initial interval where first swing stability is determined.

These expressions (5.26) and (5.27) can be considered measures of the relative stiffness of any equivalent line in the system and the stiffness of the interconnection to any bus k or ℓ respectively. These could be used to determine weakness of any equivalent line or the interconnection to any generator bus.

The transient coherency measure over the $2N^{th}$ interval $T_k^{2N-1}(\varepsilon) \leq T \leq T_k^{2N}(\varepsilon)$ is

$$C^*_{k\ell}(T) = \frac{1}{T^n} \hat{\underline{e}}_{k\ell} \Big[6\underline{I} \frac{T^4}{5!} + 15[\underline{X}+\underline{X}^T] \frac{T^6}{7!} + [28\underline{X}^2+28\underline{X}^{T^2}+70\underline{X}\,\underline{X}^T] \frac{T^8}{9!}$$

$$\cdots [\sum_{j=0}^{N} a_{Nj}\underline{X}^{N-j}\underline{X}^{T^j}] \frac{T^{2N+4}}{2N+5!} \Big] \hat{\underline{e}}_{k\ell} \quad (5.28)$$

This transient coherency measure for the $2N^{th}$ interval has terms that have form $\underline{X}^N\ \underline{X}^{NT}$ which indicates the disturbance on each generator has propagated to N+1 stage generators for each disturbed generator and thus the transient coherency measure depends on generator inertias in stage 1 to N+1 for each disturbed generator and the synchronizing torque coefficients of lines that connect them. The last term added to the coherency measure

$$\hat{\underline{L}}_{2N+2} = \sum_{j=1}^{N} a_{Nj} \, \underline{X}^{N-j} X^{T^j} \tag{5.29}$$

should indicate the relative stiffness of the interconnection during the $2N^{th}$ interval as did $\{\underline{X}^N\}_{ki}$ for the deterministic disturbance. Thus, the transient coherency measure for the probabilistic disturbance indicates structural weakness, the relative stiffness of the interconnection between generators and the coherent groups, which are independent of disturbance location at the N^{th} interval for each N.

The dynamic coherency measure defined as

$$C_{k\ell}^{*x} = C_{k\ell}^{s}(T)\Big|_{T=\infty} \tag{5.30}$$

measures the power system structure in steady state where the disturbance propagation and reverberations have subsided. This dynamic coherency measure can not be evaluated for the synchronous frame power system model (4,6) since the $\underline{A}$ matrix is singular. The dynamic coherency measure will now be developed and compared with the transient coherency measure. The dynamic coherency measure is evaluated for a uniform machine angle frame model

$$\underline{X}\,\dot{\underline{y}} = \underline{F}\,\underline{y} + \underline{G}\,\underline{u}$$

$$y = \begin{bmatrix} \hat{\delta} \\ \underline{\hat{\omega}} \end{bmatrix}$$

$$\underline{F} = \begin{bmatrix} \underline{0} & \underline{I} \\ -\underline{M}^{-1}\underline{E}_1\underline{T}\,\underline{E}_2 & -\alpha\underline{I} \end{bmatrix} \qquad \underline{G} = \begin{bmatrix} 0 & 0 \\ -\underline{M}^{-1}\underline{E}_1 & -\underline{M}^{-1}\underline{E}_1\underline{L} \end{bmatrix}$$

$$\hat{\delta}^T = (\hat{\delta}_1, \hat{\delta}_2, \ldots, \hat{\delta}_{N-1})$$

$$\underline{\hat{\omega}} = (\hat{\omega}_1, \hat{\omega}_2, \ldots, \hat{\omega}_{N-1})$$

$$\hat{\delta}_i = \delta_i - \delta_N$$

$$\hat{\omega}_i = \omega_i - \omega_N$$

$$\underline{E}_1 = \left[\begin{array}{c|c} & -1 \\ & -1 \\ \underline{I}_{N-1} & -1 \\ & \vdots \\ & -1 \end{array}\right]$$

$$\underline{E}_2 = \begin{bmatrix} \underline{I}_{N-1} \\ \bar{0}\;\bar{0}\;\bar{0}\;\bar{0}\;\bar{.}\;\bar{.}\;\bar{.}\;\bar{.}\;\bar{0} \end{bmatrix}$$

where $\underline{M}$, $\underline{T}$ and $\underline{L}$ are defined in (2).

The dynamic coherency measure [39] for this model and the step disturbance model

$$\underline{V}_y(0) = \underline{0}$$

$$\underline{G}\ \underline{P}_u(0)\underline{G}^T = \begin{bmatrix} 0 & 0 \\ 0 & \underline{P} \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & \underline{I} \end{bmatrix}$$

$$P_u(0) = \underline{R}_1 + \underline{m}_1\underline{m}_1^T$$

has the form

$$\underline{C}_{k\ell}^{xx} = \tilde{\underline{e}}_{k\ell}\ \underline{S}_y(\infty)\ \tilde{\underline{e}}_{k\ell}$$

where

$$\{\tilde{\underline{e}}_{k\ell}\}_j = \begin{cases} \begin{cases} 1 & j=k \\ -1 & j=\ell \\ 0 & j\neq k,\ell \end{cases} & k\neq N \quad \ell\neq N \\ \begin{cases} 1 & j=k \\ 0 & j\neq k \end{cases} & k\neq N \quad \ell=N \\ \begin{cases} 1 & j=\ell \\ 0 & j\neq \ell \end{cases} & k=N \quad \ell\neq N \end{cases}$$

$$\begin{aligned} \underline{S}_y(\infty) &= \lim_{T\to\infty} \frac{1}{T}\int_0^T E\{\underline{y}(t)\underline{y}^T(t)\}dt \\ &= \frac{1}{T}\int_0^T [\int_0^\tau \varepsilon^{Av}dv]\underline{G}\ \underline{P}_u(0)\underline{G}^T \int_0^\tau \varepsilon^{\underline{A}^T v}dv\ d\tau \\ &= \underline{F}^{-1}\underline{G}\ \underline{P}_u(0)\underline{G}^T\underline{F}^{-1^T} \\ &= \left.\begin{bmatrix} [\underline{M}^{-1}\underline{E}_1\underline{T}\ \underline{E}_2]^{-1}\underline{P}[\underline{M}^{-1}\underline{E}_1\underline{T}\ \underline{E}_2]^{-1^T} & 0 \\ 0 & 0 \end{bmatrix}\right|_{\underline{P}=\underline{I}} \end{aligned}$$

The dynamic coherency measure is proportional to $[\underline{M}^{-1}\underline{E}_1\underline{T}\ \underline{E}_2]^{-1}$ rather than $[\underline{M}^{-1}\underline{T}]^{i}$ for the transient coherency measure. The generators will be coherent in the transient coherency measure if the equivalent line which connects them is stiff since this measure depends on $\underline{M}^{-1}\underline{T}$. However a pair of generators will only be coherent for the dynamic coherency measure if (1) the two generators are extremely stiffly connected that the first generator will be coherent with all generators in the group to which the generator is stiffly connected to even though this first generator is not stiffly connected to the other generators in that group or (2) both generators are stiffly connected to most if not all of the generators in the coherent group to which they both belong. In both cases all generators in a coherent group for the dynamic coherency measure should be mutually coherent since the dynamic coherency measure depends on $[\underline{M}^{-1}\underline{E}_1\underline{T}\ \underline{E}_2]^{-1}$ and not on $\underline{M}^{-1}\underline{T}$.

This transient and dynamic coherency measure can be used to indicate power system structure and weaknesses in the dynamic structure for some T. It should be noted that the transient and dynamic coherency measures and thus the system structure depends on the unit committment network configuration, load flow conditions, and interval T given. With the disturbance, unit committment, network configuration, load flow, and interval specified, the dynamic or transient coherency measure can be used to

(1) determine the groups of generators that are coherent and thus are relatively stiffly tied together. These coherent groups and of course incoherency of the generators in different groups defines the structure of the system;

(2) determine the relative weakness in the structure by examining the coherency measure $C_{k\ell}(T)$ on equivalent lines that connect internal generator buses k and ℓ that lie in two different coherent groups. If their coherency measure $C_{k\ell}(T)$ is very large, the apparent weakness in the equivalent lines, measured by

$$T_{k\ell}\left(\frac{1}{M_k} + \frac{1}{M_\ell}\right) \tag{5.31}$$

for the transient and dynamic time frames respectively, connecting the internal generator buses could be diagnosed as due to the

(i) overload on the equivalent lines

(ii) relatively small capacity of the lines for the generator groups they are expected to connect;

The stiffness of the interconnection to generator k for the transient and dynamic coherency measures are

$$[M^{-1}T]$$

and

$$[\underline{M}^{-1}\underline{E}_1\ \underline{T}\ \underline{E}_2]^{-1}$$

The former measures the size of the contingency that a bus could withstand without loss of synchronism for the transient interval. Weakness in the interconnection to a generator in the transient or dynamic time frame could be due to loss of lines due to a contingency, maintainance, or due to an inadaquate design.

This transient coherency measure (5.25) over the first interval can be shown to be an excellent transient security measure because

(1) it is defined for the initial interval where first swing stability is determined

(2) it measures weaknesses in system structure for this interval which are independent of magnitude location, correlation, or kind of disturbance. It could also be evaluated for a probabilistic or deterministic decsription of a specific contingency if that were desired.

(3) each term T_{kj} can be shown to be proportional

to the energy capacity of that line to withstand a fault. This will be done later in Chapter 7 using an equal area criterion argument because all generators connected to the disturbed generator act as an infinite bus for the disturbance during the first interval.

(4) it depends on the present unit committment, network, and load flow condition in the system.

This transient security measure (5.25) can be used as a performance index for optimal power dispatch or optimal load shedding problems to force a more secure transient stability related operating condition out of the optimal power or load dispatch in the alert and emergency operating states in a manner similar to that used in (28, 29). The use of the transient coherency measure as a transient security performance measure and for developing transient security constraints for the optimal dispatch and load shedding problems will be discussed in a subsequent publication.

This transient and dynamic rms coherency measure could also be evaluated off-line for system planning purposes

(1) to assess the security and structural weaknesses for the present power system with a particular network configuration, unit committment and load flow, which are greatly

affected by forced outages and maintainance schedules.

(2) assess dynamic and transient system security and structural weakness for particular transmission and generation expansion alternatives.

(3) assess particular relaying strategies and their effect on system security and structural weaknesses.

COMPUTATIONAL RESULTS

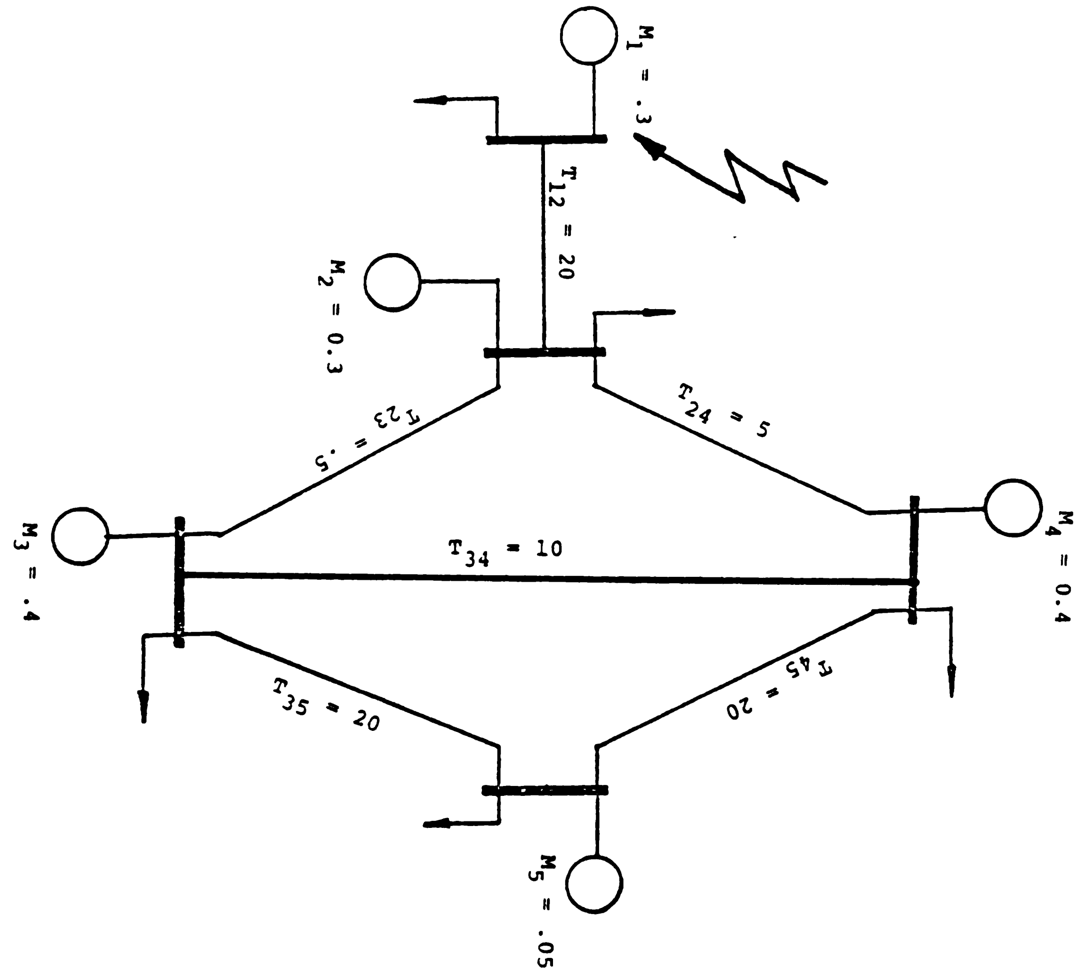
5.2 The purpose of this section is to use a computer program, which can calculate the Taylor series approximation of any order for any observation interval, to (1) analyze and justify the basis for using the transient coherency measure to analyze the transient response of power systems and (2) confirm the difference in the structural properties of a power system measured by the transient and dynamic coherency measures.

The justification of the basis for using a transient coherency measure to analyze propagation of disturbances will be accomplished using the simple system shown in Figure 0 with system matrices

$$\underline{X}=-\underline{M}^{-1}\underline{T}=\begin{bmatrix} -66.66 & 66.66 & -0 & -0 & -0 \\ 66.66 & -85.00 & 1.66 & 16.66 & -0 \\ -0 & 1.25 & -76.25 & 25.00 & 50.00 \\ -0 & 12.50 & 25.00 & -87.50 & 50.00 \\ -0 & -0 & 400.00 & 400.00 & -800.00 \end{bmatrix}$$

$$\underline{M}^{-1} = \text{diag}\{3.334, 3.334, 2.5000, 2.5000, 20.0016\}$$

and the deterministic step change in shaft acceleration at generator 1.



$$\underline{X} = \begin{bmatrix} -\frac{T_{12}}{M_1} & \frac{T_{12}}{M_1} & 0 & 0 & 0 \\ \frac{T_{12}}{M_2} & -\frac{(T_{12}+T_{23}+T_{24})}{M_2} & \frac{T_{23}}{M_2} & \frac{T_{24}}{M_2} & 0 \\ 0 & \frac{T_{23}}{M_3} & -\frac{(T_{23}+T_{34}+T_{35})}{M_3} & \frac{T_{34}}{M_3} & \frac{T_{35}}{M_3} \\ 0 & \frac{T_{24}}{M_4} & \frac{T_{34}}{M_4} & -\frac{(T_{24}+T_{34}+T_{45})}{M_4} & \frac{T_{45}}{M_4} \\ 0 & 0 & \frac{T_{35}}{M_5} & \frac{T_{45}}{M_5} & -\frac{(T_{45}+T_{35})}{M_5} \end{bmatrix}$$

$$\underline{P} = \begin{bmatrix} \frac{1}{M_1^{\;2}} & \underline{O} \\ \underline{O} & \underline{O} \end{bmatrix}$$

Figure 0

For this disturbance and power system model, the analysis based on the transient coherency measure indicates

(1) generator 1 is a stage 1 generator accelerated over the first interval and this acceleration is observed in $C^{1}_{k\ell}(T)$,

(2) generator 2 is a stage 2 generator accelerated beginning in the second interval and this acceleration is first observed in $C^{2}_{k\ell}(T)$;

(3) generator 3 and 4 are stage 3 generators and accelerated beginning in the fourth interval and this acceleration is first observed in $C^{4}_{k\ell}(T)$;

(4) generator 5 is a stage 4 generator and accelerated beginning in the 6th interval and this acceleration is first observed in $C^{6}_{k\ell}(T)$.

because stage N+1 was shown to be first accelerated over the $2N^{th}$ interval and this acceleration first appears in $C^{2N}_{k\ell}(T)$.

The Taylor series approximation for the mean square coherency measure of orders N=1,2,...,9 for T=0.025, 0.050, 0.075, and 0.100 is shown in Table 1. The results shown in this table show that

(1) stage N+1 is accelerated in $C^{2N}_{k\ell}(T)$ but is not accelerated in $\{C^{K}_{k\ell}(T)\}_{K=1}^{2N-1}$ for every T.

The order of the approximation at which acceleration of a stage begins to appear is found by noting the order of the approximation when the coherency measure between buses in this stage and every other stage are non zero for stages 1, 2, and 3.

(2) that all stages are accelerated at any value of T no matter how small if the order of the approximation is high enough;

(3) that the effective movement between buses in stage N+1 is significantly less than in stage N for all N as measured by $C^s_{k\ell}(T)$ when T is very small and that the motion in each stage N begins to become significant $(> \hat{\varepsilon})$ as T increases. This is the basis upon which the definition of the transient coherency measure is based. This definition adds terms to the coherency measure approximation as their effect becomes significant with increasing T. Neglecting the very small motion and acceleration in stages far from the disturbance location until their effect becomes significant permits (1) disecting the transient response into the discrete sequence of events which are the acceleration of successive stages, (2) the determination of lines and generator in each stage, (3)

the determination of stiffness of the interconnection $\{\underline{X}^N\}_{ki}$ between the disturbed generator and any generator k in stages 2 to N+1 that is affected by the disturbance in the $2N^{th}$ interval and (4) the effect of any particular lines synchronizing torque coefficient or generator inertias on the transient response at any time and location.

(4) the determination of the exact values of $T^N_{k\ell}(\varepsilon)$ is difficult for any k, but can easily be approximated even from Table 1. It is clear stage 2 does not become significant (ε=30x10^{-13}) until $T^1_{k\ell}(\varepsilon) \simeq 0.25$ seconds and that stage 3 does not become significant for the same ε until $T^3_{k\ell}(\varepsilon)=.10$ seconds. $T^1_{k\ell}(\varepsilon)$, which is the time that stage 2 (generator 2) begins to accelerate, is found by noting when $C^2_{2\ell}(T)$ is greater than ε for $\ell=3,4,5$. $T^3_{k\ell}(\varepsilon)$, which is the time stage 3 (generators 3 and 4) begin to accelerate is found by noting when $C^4_{3\ell}(T)$ is greater than ε for $\ell=5$.

(5) the approximation for the 2N model with zero damping, which has two zero eigenvalues, breaks down for T=0.15 as would be expected as can be seen from the convergence difficulties for $C^4_{34}(T)$. The value T* for which a Taylor series

approximation would be satisfactory should only be $T^* \leq .10$ seconds if N is small.

TABLE 1a

T=.025								
k-i	C^1_{ki}	C^2_{ki}	C^3_{ki}	C^4_{ki}	C^5_{ki}	C^6_{ki}	C^7_{ki}	C^8_{ki}
1-2	53900-08	53901-08	53901-08	53901-08	53901-08	53901-08	53901-08	53901-08
1-3	54115-08	54115-08	54115-08	54115-08	54115-08	54115-08	54115-08	54115-08
1-4	54115-08	54115-08	54115-08	54115-08	54115-08	54115-08	54115-08	54115-08
1-5	54115-08	54115-08	54115-08	54115-08	54115-08	54115-08	54115-08	54115-08
2-3	0	31142-13	31029-13	31030-13	31030-13	31030-13	31030-13	31030-13
2-4	0	31142-13	31018-13	31019-13	31019-13	31019-13	31019-13	31019-13
2-5	0	31142-13	31031-13	31031-13	31031-13	31031-13	31031-13	31031-13
3-4	0	0	0	11305-20	11260-20	11260-20	11260-20	11260-20
3-5	0	0	0	13957-22	12827-22	12858-22	12856-22	12858-22
4-5	0	0	0	13957-20	13793-20	13795-20	13795-20	13795-20

TABLE 1b

T=.05								
k-i	C^1_{ki}	C^2_{ki}	C^3_{ki}	C^4_{ki}	C^5_{ki}	C^6_{ki}	C^7_{ki}	C^8_{ki}
1-2	16859-06	16867-06	16867-06	16867-06	16867-06	16867-06	16867-06	16867-06
1-3	17134-06	17137-06	17137-06	17137-06	17137-06	17137-06	17137-06	17137-06
1-4	17134-06	17137-06	17136-06	17136-06	17136-06	17136-06	17136-06	17136-06
1-5	17134-06	17137-06	17137-06	17137-06	17137-06	17137-06	17137-06	17137-06
2-3	0	15945-10	15674-10	15677-10	15677-10	15677-10	15677-10	15677-10
2-4	0	15945-10	15651-10	15654-10	15654-10	15654-10	15654-10	15654-10
2-5	0	15945-10	15677-10	15679-10	15679-10	15679-10	15679-10	15679-10
3-4	0	0	0	92609-17	90948-17	90963-17	90962-17	90962-17
3-5	0	0	0	11433-18	77097-19	81219-19	81044-19	81043-19
4-5	0	0	0	11433-16	10876-16	10893-16	10893-16	10893-16

TABLE 1c

T=.075								
k-ℓ	$c^1_{k\ell}$	$c^2_{k\ell}$	$c^3_{k\ell}$	$c^4_{k\ell}$	$c^5_{k\ell}$	$c^6_{k\ell}$	$c^7_{k\ell}$	$c^8_{k\ell}$
1-2	12297-05	12332-05	12331-05	12331-05	12331-05	12331-05	12331-05	12331-05
1-3	12768-05	12779-05	12779-05	12779-05	12779-05	12779-05	12779-05	12779-05
1-4	12768-05	12778-05	12778-05	12778-05	12778-05	12778-05	12778-05	12778-05
1-5	12768-05	12779-05	12779-05	12779-05	12779-05	12779-05	12779-05	12779-05
2-3	0	61297-09	58843-09	58896-09	58896-09	58896-09	58896-09	58896-09
2-4	0	61297-09	58645-09	58707-09	58706-09	58706-09	58706-09	58706-09
2-5	0	61297-09	58865-09	58910-09	58910-09	58910-09	58910-09	58910-09
3-4	0	0	0	18024-14	17270-14	17286-14	17286-14	17286-14
3-5	0	0	0	22251-16	59140-17	99969-17	96037-17	96272-17
4-5	0	0	0	22251-14	19780-14	19942-14	19942-14	19942-14

TABLE 1d

T=0.10								
k-ℓ	$c^1_{k\ell}$	$c^2_{k\ell}$	$c^3_{k\ell}$	$c^4_{k\ell}$	$c^5_{k\ell}$	$c^6_{k\ell}$	$c^7_{k\ell}$	$c^8_{k\ell}$
1-2	48811-05	49296-05	49272-05	49273-05	49273-05	49273-05	49273-05	49273-05
1-3	52338-05	52491-05	52485-05	52486-05	52486-05	52486-05	52486-05	52486-05
1-4	52338-05	52483-05	52478-05	52478-05	52478-05	52478-05	52478-05	52478-05
1-5	52338-05	52492-05	52486-05	52486-05	52486-05	52486-05	52486-05	52486-05
2-3	0	81637-08	75690-08	75926-08	75920-08	75920-08	75920-08	75920-08
2-4	0	81637-08	75222-08	75495-08	75487-08	75487-08	75487-08	75487-08
2-5	0	81637-08	75742-08	75945-08	75943-08	75942-08	75942-08	75942-08
3-4	0	0	0	75866-13	70130-13	70345-13	70340-13	70340-13
3-5	0	0	0	93661-15	28713-15	25746-15	16391-15	17386-15
4-5	0	0	0	93661-13	75051-13	77337-13	77124-13	77140-13

Table 1. Taylor series approximations of the mean square coherency measure evaluated for observation intervals T=0.025, 0.050, 0.075 and 0.100 seconds.

i-j	Dynamic Coherency Measure	Transient Coherency Measure T=.25 sec.
1-2	201549-01	63503-04
1-3	169508-01	53364-04
1-4	233710-01	68186-04
1-5	199762-01	58412-04
1-6	198108-01	57644-04
1-7	169895-01	48244-04
2-3	103027-01	47835-04
2-4	164842-01	59355-04
2-5	101774-01	49620-04
2-6	102282-01	48980-04
2-7	861094-02	37962-04
3-4	784689-02	45758-04
3-5	305593-02	37054-04
3-6	288078-02	35828-04
3-7	169590-02	29936-04
4-5	637864-02	48205-04
4-6	636804-02	47248-04
4-7	910123-02	42924-04
5-6	188861-03	30501-04
5-7	330258-02	28038-04
6-7	318956-02	28047-04

Table 2. Transient and Dynamic Coherency Measures for the MECS example system.

A second example is used to show the difference in the structural properties measured by the transient and dynamic coherency measures. These coherency measures are evaluated for the seven station model of the Michigan Electric Coordinated Systems [10] that was used to derive modal [4] and coherency [39] based equivalents based on the rms coherency measure evaluated for an infinite observation interval. The system matrices for this model are

$$\underline{X} = -\underline{M}^{-1}\underline{T} = \begin{bmatrix} -38.51 & 5.33 & 8.41 & 3.12 & 4.55 & 4.97 & 12.11 \\ 22.45 & -89.70 & 9.88 & 4.92 & 8.80 & 9.36 & 34.25 \\ 31.12 & 8.68 & -145.90 & 23.92 & 24.40 & 27.14 & 30.98 \\ 8.79 & 3.29 & 18.02 & -80.87 & 16.14 & 17.72 & 16.88 \\ 12.86 & 5.89 & 18.62 & 16.15 & -137.50 & 45.57 & 37.97 \\ 14.03 & 6.27 & 28.72 & 17.73 & 45.37 & -140.80 & 36.47 \\ 34.23 & 22.99 & 23.68 & 16.91 & 38.02 & 36.21 & -172.00 \end{bmatrix}$$

$$\underline{M}^{-1} = \text{diag}(1.2375, 5.4228, 4.7612, 3.6319, 3.6340, 3{,}6340, 3.6340)$$

and the disturbance used is the zero mean IID step change in shaft accelerations (5.20).

From analysis of the Matrix $\underline{X}$, one would conclude (3,5,6,7) would be mutually coherent and generator 2 would be coherent with generator 7 but not with other generators in that group (3,5,6) the transient interval. This is confirmed by analyzing the Taylor series approximations of the mean square coherency measure for various values of T.

The analysis of the previous section would indicate that the equivalent line connecting generator 2 and 7 would have to be extremely stiff for generators 2 and 7 to be coherent in the dynamic coherency measure because generator 2 is not very stiffly connected to generators 3,5, and 6. This analysis would also suggest that if 2 and 7 were coherent in the dynamic coherency measure, generator 2 would be coherent with 3,5 and 6. The results from Table 2 shows that generator 2 is not coherent with 3,5 and 6 in the dynamic coherency measure. This would suggest that generator 2 and 7 are not extremely stiffly connected which is confirmed by observing the (2,7) element of the system matrix $\underline{X}$.

CONCLUSIONS

This chapter discusses

(1) a definition of the transient coherency measure which is a Taylor series approximation of the mean square where the order of the approximation increases with the observation interval in order to keep the approximation error within some bound

(2) the use of the transient coherency measure for a deterministic disturbance at a single bus to
  (a) disect the transient response into discrete events which are the acceleration of successive stages further from the disturbance location;
  (b) determine the lines and generator in each

stage; (c) determine the stiffness of the interconnection $\underline{x}^{N}{}_{ki}$ between the disturbed generator and any generator k in stage 2 to N+1 in the 2Nth interval; and (d) the effect of any line synchronizing torque coefficient or generator inertia on the transient response in any location and any interval.

(3) the use of the transient coherency measure for analysis of changes in dynamic structure weakness in that structure and coherent groups as the observation interval increases for the transient coherency measure. Comparison of these structural properties for the transient and coherency measures is also made

(4) the use of the transient and dynamic coherency measures as transient and dynamic security measures for security assessment in both system planning and system operation application. The use of the transient security measure as a performance index and as transient security constraints for the optimal power dispatch-optimal load shedding problem.

A very important application of the transient coherency measure, which has not been mentioned but is extremely important, is the development of coherency based equivalents for transient stability application.

The differences between the transient and dynamic coherency measures determined in this chapter could be used to better understand the differences and the appropriate applications for modal [4] and coherent [39] equivalents derived based on the dynamic coherency measure and the transient coherent equivalents derived based on the coherency based aggregation procedure and either the max-min [5] or transient coherency measures.

CHAPTER 6

AN OPTIMAL COHERENCY BASED SECURE DISPATCH FORMULATION

OBJECTIVES:

The principal objective of this part of the research is to show how an improvement in the formulation of the optimal secure dispatch problem can be achieved by including the transient coherency measure into the formulation of the optimal dispatch problem.

METHODOLOGY:

The methodology of this chapter shall consist of meeting the above objective by

(i) identifying the deficiencies in the current formulation of the optimal secure dispatch problem through documentation of the literature.

(ii) selecting a formulation approach (nonlinear vs. linear programming) based on both the trends in the current literature and the requirements for real time implementation. The limitations of the chosen approach are also mentioned.

(iii) proposing a formulation of the optimal secure dispatch problem which includes a coherency based performance index that should help improve the security, reliability and stability of the power system.

This development is now pursued in chronological order in the following subsections of this chapter.

6.1 THE NEED FOR AN IMPROVED DISPATCH:

The literature [12, 13] substantiates the fact that there exists the need for an optimization criterion for optimal dispatch which incorporates comprehensively the properties of security, reliability and stability. This need for a better index of performance is supported by Hajdu and Podmore [13] who indicate that for enhancing the security of a power system, it is desirable to express the results of on-line transient-stability studies in a more concise form such as a transient - security index, for reasons which are directly quoted from [13]:

- The evaluation of transient stability from visual inspection of swing curves is a time-consuming task and requires a certain degree of judgement, particularly for systems with many generators.
- In case of transient insecurity, the swing curves do not give the operator an appreciation of the nature or the severity of a potential stability problem. This is unlike the case of steady-state insecurity where the magnitudes of the potential overloads caused by a certain contingency give the operator an immediate appreciation of the severity of the problem and its location.

Byerly and Sherman [28] have proposed a stability index

$$\hat{\theta}_k = \max_t \max_{i,j} \theta_{ijk}(t), \qquad (6.1)$$

with $\theta_{ijk}(t) \triangleq$ angular displacement between generator i and j at time t due to fault k

$\hat{\theta}_k$ = max value depending upon system

which is a maximum angular swing between a pair of generators and is similar to the max-min coherency measure [5]. However, this index is beset by the difficulty that the critical value which separates the stable and unstable cases is dependent upon the fault and therefore upon the operating conditions as a result of that fault. This dependence on the fault is undesirable because a stability index should be able to handle not only a specific deterministic disturbance but also a probabilistic description of any disturbance. In addition, this max criterion does not yield any significant meaning in the case of instability.

El-Abiad, et al [32] and Saito, et al [33] have also proposed transient security indices using the method of pattern recognition. The pattern recognition methods, however, are heuristically based and further advances in the accuracy and efficiency of Liapunov methods are necessary for making these tools useful for on-line studies.

The deficiencies in the current formulation are further accentuated in [13] by the lack of a definition of proper constraints for transient system security. This "much more difficult, and to date unresolved problem" is according to current practice handled by formulating transient system security constraints on line phase-angle differences or power flow on lines.

In addition, Dyliacco, et al [14] point out that using a phase angle constraint is insufficient to prevent thermal overloading and that determining the bound on the angular difference $|\theta_i - \theta_j|$ which ensures a stable transient in case of a subsequent fault or combination thereof remains "a difficult and largely unsolved problem."

Consequently the overall approach will be one in which an effort is made to address some of the aforementioned difficulties by justifying the transient coherency measure (5.25) as a transient security measure and modifying the current formulation of the optimal secure dispatch problem to include a transient coherency measure which is shown to enhance the transient security of the system.

Accordingly, the transient coherency measure (5.25) over $[0, T'_{k\ell}(\varepsilon)]$ will be shown to be a useful security index since it improves system security by

(1) dispatching to decrease angular differences δ_{ij} and thereby move the system further below the static stability limit

(2) increase the system stiffness, and

(3) move the system away from thermal overload by limiting the power flow, P_{ij}.

Security enhancement of the system is thus shown to be achieved through preventive and corrective controls respectively which are described in Chapter 8.

6.2 <u>SELECTING A GENERIC FORMULATION</u>:

In formulating the optimal dispatch problem for power system security control applications, the choice of method lies essentially between two generic approaches [15]:

(a) Nonlinear Programming and

(b) Linear Programming (and its extensions to Quadratic and Convex programming).

The techniques used in category (a) comprise essentially of nonlinear system models handled by nonlinear solution techniques such as the elegant nonlinear programming formulations which shall be briefly surveyed later. This approach is, however, beset by three major impediments to on-line implementation which concern, as quoted from [13]:

(1) the computational and memory requirements of the ac-power flow program
(2) the relatively slow convergence properties of the various nonlinear optimization techniques, and
(3) the large real-time data base requirements at a central location.

The techniques used in category (b) comprise essentially of linearized system models handled by linear solution techniques such as the various linear programming formulations which shall also be surveyed later. The common feature of these LP formulations is that they enjoy [16]

(i) complete computational reliability
(ii) very high speed
(iii) ability to track deviations accurately

making it suitable for both real-time or study mode purposes.

Since the "fundamental requirement", as quoted from [17], for on-line application of an optimal dispatch strategy, is for the technique to be "computationally very fast", the choice of an LP formulation for on-line implementation becomes a very practical proposition.

In selecting linear programming as the generic approach for formulating the optimal dispatch problem, it is important to point out that this choice, reflecting the more favorable convergence characteristics and guaranteed solutions to feasible configurations, is subject to important modeling simplifications.

These simplifications are essentially the linearization of both the performance index and constraints. This linearization places the following limitations [22]

(i) since a linearized model is used, the accuracy of the model is sacrificed

(ii) no attempt should be made to model large disturbances such as losing a large block of generation

(iii) only small dimensional load-shedding problems should be attempted.

In addition to these model limitations, one recommendation regarding reactive power optimization is that only real power optimization should be attempted as the extra computational effort involved in including reactive optimization buys little additional information [23].

6.3. THE FORMULATION

Having selected the generic approach for reformulation, an improved LP formulation of the optimal secure dispatch problem is proposed.

This formulation is referred to as a quadratic programming (QP) formulation when an addition to the LP objective function of a quadratic transient security index is demanded by the security assessment functions.

The proposed formulation reflects the work of many eminent researchers beginning with the early work in this area by SRI in the application of Operations Research techniques to planning and reliability studies of the BPA grid. The many contributions and refinements including those of Stott which should permit effective real-time application of the LP approach, are given in a most generalized version in a recent publication by him [16].

In this formulation, which is based on the inclusion of the transient coherency measure in the performance index, only the more important contributions in Stott, et al [16], [17] which can affect an improvement of system security, stability and reliability, are included.

In addition, the formulation being developed applies only to thermal and nuclear generation and excludes hydrostations for obvious reasons.

The development of the formulation is divided into three parts which consist of:

(1) The general framework

(2) The linearized performance index

(3) The linearized constraints

6.3.1 The General Framework

In order to proceed with the development of the aforementioned parts, it is necessary to provide a conceptual framework around which the formulation of the optimum secure dispatch is completed. To this end, a brief statement identifying the various technical and mathematical components of the optimum dispatch problem is provided.

6.3.1.1 The Optimum Dispatch Problem

Minimize the cost of real power generation

$$\min_{PG_i, QG_i, V_i, \delta_i} \left\{ \sum_{i=1}^{NG} F_i(PG_i) = \sum_{i=1}^{NG} a_i + b_i PG_i + c_i PG_i^2 \right\} \tag{6.2}$$

subject to equality constraints due to power system energy balance requirements that the real and reactive power demanded be supplied by the generations

$$PG_i - PL_i = P_i^N = V_i \sum_{j=1}^{N} Y_{ij} V_j \cos(\delta_i - \delta_j - \alpha_{ij}) \tag{6.3}$$

$$QG_i - QL_i = Q_i^N = V_i \sum_{j=1}^{N} Y_{ij} V_j \sin(\delta_i - \delta_j - \alpha_{ij}) \tag{6.4}$$

and the inequality constraints which arise due to machine rating and service quality requirements

$$PG_i^m \le PG_i \le PG_i^M \quad \text{real generations} \tag{6.5}$$

$$QG_i^m \le QG_i \le QG_i^M \quad \text{reactive generations} \tag{6.6}$$

$$PL_i^m \le PL_i \le PL_i^M \quad \text{real loads} \tag{6.7}$$

$$QL_i^m \le QL_i \le QL_i^M \quad \text{reactive loads} \tag{6.8}$$

$$V_i^m \le V_i \le V_i^M \quad \text{voltage magnitudes} \tag{6.9}$$

$$t_{ij}^m \le t_{ij} \le t_{ij}^M \quad \text{tap changers} \tag{6.10}$$

$$\phi_{ij}^m \le \phi_{ij} \le \phi_{ij}^M \quad \text{phase shifters} \tag{6.11}$$

$$S_{ij}^m \le S_{ij} \le S_{ij}^M \quad \text{thermal (short lines)} \tag{6.12}$$

$$|\delta_i - \delta_j| \le \Psi_{ij}^M \quad \text{stability (long lines)} \tag{6.13}$$

where

m,M	are the lower and upper limits on variables
PG, QG	are the real and reactive generations
PL, QL	are the real and reactive loads
V, δ	are the voltage magnitude and its angle
Y, α	are the admittance magnitude and its angle
t, ϕ	are the values for tap changing and phase shifting transformers
S, Ψ	are the line flows and phase angle differences
NG, N	are the number of generating stations and number of nodes

Superscript N denotes net power injection.

6.3.1.2 Modus Operandi of Optimum Dispatch

Having stated the optimum dispatch problem (6.2 - 6.13) in its most general form, it is desired to complete the diagnosis of this problem before the details of the proposed on-line tracking secure formulation are developed in parts 2 and 3.

This objective is realized along the lines of the following diagnostic steps:

(a) Statement of the optimum dispatch problem in compact mathematical programming (MP) notation

(b) introduction to the tools (constraints) for classifying the security level of operation of the power system

(c) discussion of current dispatching practices in the utility industry in the context of system security

(d) discussion of operator limitations when system security is completely under the control of the operator

(e) identification of the features of an on-line tracking implementation of this optimal secure dispatch problem

(f) off-line to on-line conversion characteristics and an overview of parts 2-3.

6.3.1.2(a) MP Statement of the Optimum Dispatch Problem

This formulation framework is now stated in mathematical programming format using compact state space notation.

$$\min_{u} F(x,u) \tag{6.14}$$

s.t.

$$g(x,u,p) = 0 \tag{6.15}$$

$$h(x,u,p) \leq 0 \tag{6.16}$$

where

x is a vector of dependent controlled variables

u is a vector of independent control variables

p is a vector of perturbations or disturbances.

The optimum dispatching problem stated in this form represents a constrained optimization problem having a nonlinear objective function subject to nonlinear equality and inequality constraints. This compact mathematical programming format will be adhered to hereafter as it greatly facilitates in the formulation of an on-line secure dispatch algorithm. It will also be used extensively in Chapter 8 for formulating the sub-control problems in terms of power system operating states.

6.3.1.2(b) Tools for Security Level Classification

It is appropriate at this juncture, therefore, to precisely define the constraints (6.15) and (6.16) which have hitherto been dealt with only in general terms. In addition, it is necessary to introduce the concept of security constraints, which form the backbone of the off-line security assessment functions, as well as the on-line tracking problem under formulation. With this motivation, the constraints (6.15, 6.16) are now defined and described in terms of the role they play in maintaining the integrity of the power system.

The load constraints

The load constraints are represented mathematically by the equality constraint (6.15) and are the power system network flow equations for a particular network configuration. In terms of an energy balance, these represent the conservation of energy in the interconnected power system and impose the requirement that load demands be met by supplied generations. The load constraints are, perhaps, the most important constaints as it is a discrepancy between supply and demand which generally leads to power imbalances and possible cascading outages.

The operating constraints

The operating constraints are represented mathematically by the inequality constraint (6.16) and are the machine rating and service quality requirements demanded by the power system for reliability. In physical terms, these constraints impose upper or lower limits on the range of operation on variables associated with the component parts of the system. Accordingly, these constraints are mathematically expressed by inequalities on such quantities as equipment loading, bus voltage, generator real and reactive powers and phase angle differences. The operating constraints are further classified into two categories which are

. soft constraints

. hard constraints

The soft constraints are associated with those variables whose inexcessive violations may be generally allowed on a short time scale. Therefore, these include bus voltage limits (6.9) and thermal line loading limits (6.12).

The hard constraints are associated with those variables whose violations are undesirable on any time scale as they could cause equipment damage or lead to eventual system break-up. These, therefore, include tap changer limits (6.10) which cannot be exceeded due to mechanical limitations; power transmission limits (6.13), which could lead to loss of steady state stability or transient stability; and power generation limits (6.5).

The security constraints

The security constraints are the additional constraints on the present operating condition which are needed to include security into the formulation of the on-line secure dispatch problem. These are, therefore, none other than the additional load and operating constraints considered vulnerable based on engineering experience with the particular power system or off line security assessment programs. These constraints are imposed in order to ensure that there will be no violation of the current operating conditions if a contingency were to occur.

In the security context, the soft security constraints (6.12) represent steady state operating limits on power transfer across short transmission lines to account for thermal overloading and voltage constraints that prevent over or under voltages on system components. Since a system is steady state stable only if it is transient stable, the hard security constraints represent transient stability limits on line phase angle differences to ensure there is no loss of synchronism or oscillations increasing in amplitude, leading to cascading and eventual system splitting. The current industry practice in this regard is to incorporate these security constraints by imposing empirical steady state limits on line phase angle differences across selected transmission lines. This approach, although partially adequate for off-line security studies of small systems does not include a broad class of possible contingencies and is insufficient to meet the growing level of security desired of modern interconnected systems. The lack of appropriate transient security constraints that ensure system stability, security and reliability has been documented previously.

Security constraints are generaged by outage simulation tests or postualted next contingencey tests, forming an integral part of the security assessment functions,desired for providing information to the operator as to the security level of the power system. In this context, therefore, the security constraints are the set of additional constraints declared vulnerable by the security assessment functions.

6.3.1.2(c) Present Security Approach:

The current utility approach to power system security is characterized by two essential features which are

- off-line economic dispatch
- operator controlled security

since the off-line secure dispatch is used in only a few systems. The objective of the off-line economic dispatch is to minimize fuel cost as follows

$$\min_{u} F(x,u) \qquad (6.17)$$

$$\text{s.t.} \qquad g(x,u,p) = 0.$$

The solution to this problem is easily obtained by using the classical Lagrange multiplier technique for converting the constrained optimization problem (6.17) to an unconstrained one, and solving the necessary conditions thereof to obtain what is popularly referred to as the optimum incremental "lambda" dispatch.

6.3.1.2(d) Limitations of Off-Line Secure Dispatch:

It is important to observe that the off-line economic dispatch formulation (6.17) does not include the inequalities (6.16). Security of operation is, therefore, left up to the system operator, who modifies the economic dispatch depending upon the violations of the inequality constraints indicated by the security monitor. In this off-line mode, security is clearly not the chief objective and security of the system is totally the responsibility of the operator whose control actions are attributed to operator limitations which are characterized by

- insufficient information as to the operating state of his system

- lack of comprehension of the changing control requirements and controls which are suitable to the changes in the operating state of the system as it moves continuously in response to the changing security level of the power system.

6.3.1.2(e) Identification of Desired On-Line Tracking Features for Security:

These limitations of the operator point to the need for an on-line secure dispatch approach having the following features in that it should possess

- complete system information from a static state estimator
- ability to change the control objectives from economic to security based objectives in response to the changing level of security
- ability to change the controls from generation dispatch to load shedding, etc. as the security level of the system diminishes

The proposed on-line secure dispatch is aimed at addressing essentially the need for a better performance index while the need for improved constraints are dealt with indirectly as a result of the revised performance index.

6.3.1.2(f) Off-Line to On-Line Conversion Characteristics

The off-line secure dispatch as previously formulated is an improvement over economic dispatch operator controlled security because security constraints can be handled automatically with minimum change in fuel cost. Operating constraints can not be handled in the off-line secure dispatch problem since the time required to solve the dispatch problem is large compared with the time frame required to perform corrective action. An on-line secure dispatch problem is not only required to make possible corrective action for operating constraint violations but should also be characterized by:

- converting off-line problem to an on-line tracking problem
- decomposition of the real and reactive problems
- addition of a coherency based performance index
- monitoring deviations from base case economic dispatch
- eliminating need to re-run complete load flows at each iteration of on-line security dispatch

An overview of the need for some of these on-line tracking characteristics is now provided.

In view of the recent power outage in New York, it is clear that current operating practices are insufficient to prevent future outages and that the security of power system operations will have an additional and more profound impact on the total economic dispatch problem as a whole [15]. However, the inclusion of rigorous modelling techniques cannot be justified based solely on economic incentives as the results of recent studies [15] point out that no dollar savings are involved when cost minimization is the chief objective. Consequently the use of more advanced techniques is justified based on the need for improved system security for which more rigorous models are required in order to execute the different functions associated with operating security. As a result, the approach taken here is one of designing or formulating a strategy in which the security of operations receives a high priority when the system is declared vulnerable by existing violations or by the security assessment functions and one which yields the economic dispatch solution as a by product of the optimum secure dispatch when no load security or operating constraints would be violated.

The rest of this chapter will deal primarily with a reformulation of the above off-line problem into an on-line tracking problem. In order to achieve this goal it becomes necessary to modify the performance index and to incorporate the operating and security constraints into the optimum secure dispatch formulation. A brief overview of the performance index for the on-line secure dispatch problem is now provided before getting into its details.

The performance index of the economic dispatch problem is a quadratic cost function and is assumed dependent only upon the real power generation. This assumption does not result in any loss of generality because the reactive generations do not have any measurable or significant effect on fuel cost as a result of the decoupling between real and reaetive powers. Fixed costs are excluded from this economic objective. The sole objective of this criterion is to minimize the total fuel cost while simultaneously satisfying the demands.

The augmentation of the off line economic objective function for the on line secure dispatch problem is executed when the system is declared vulnerable by the security assessment functions. Depending upon the level of security the power system is operating at, preventive or corrective rescheduling is introduced in the form of penalty functions which minimize the deviation from the economic generation schedules. This rescheduling tells the system operator where to introduce generation shift and load curtailment controls. It is to be noted that load curtailment or load shedding is designed in this formulation framework as a last resort action by introducing large coefficients into the corresponding penalty functions.

While these preventive and corrective controls are useful in optimal rescheduling for improved security, they still lack a comprehensive capability of enhancing the security of the system as has been documented earlier. Therefore, a coherency based performance index is introduced into the objective function. This addition, as will be theoretically shown in Chapter 7 , has a profound impact on enhancing the transient security of the power system by adaptively improving the security, reliability and stability of the system, a comprehensive feature heretofore not found in previous work. The net effect of the introduction of the coherency based performance index following a postulated transient next contingency or an actual emergency condition, is to increase the security margin by selecting the generation and load corrections that result in stiffening the ties within the system or between connecting arease thus raising the level of security the power system is operating at.

In the reformulation it is assumed that a base case is available for the off line problem. This offline problem in then converted to an on-line tracking dispatch with the aid of an on-line estimator which eliminates the need for solving load flow equations (6.3, 6.4) to solve the secure dispatch problem as long as (1) an equation requiring total generation satisfies total load is included and (2) the inequality constraints are expressed in terms of deviations in bus injections using the Jacobian matrix generated by the static state estimator. The power flow is decoupled and the reactive power flow and associated voltage and reactive power constraints are eliminated because the reactive power and voltage have little effect on results of the secure dispatch problem and the voltage and reactive power dispatch is handled automatically in present system implementation.

Having converted the off-line problem to an on-line problem, it is possible to enhance the accuracy of the model by successive linearizations about the operating point wherein the triangular factorization of the Jacobian available from the base case is obtained using sparsity techniques [33]. Sensitivity could also be incorporated into the model, making it possible to update the state of the network continuously without the use of repeated load flows.

Control Variables and Priorities

Since real power control is used in this formulation, the set of controllable activities considered within the domain of real power control consists of those control actions which can be represented analytically as bus real power injection changes or equivalent real power generation changes. All such controls which are considered here are listed in order of priority as:

(1) thermal units on control
(2) thermal units off control
(3) emergency start-up
(4) load shedding

6.3.2 Development of the Linearized Performance Index:

In this step of the formulation, the given quadratic objective function (6.2) of the off-line economic dispatch problem is augmented by additional performance indices which are needed in order to convert it to an on-line tracking problem and to improve the level of security of the power system. These additions comprise of

- a penalty function for generation shifts
- a penalty function for load curtailment
- a transient security index.

Designing a proper objective function is philosophically the most important aspect of any optimization strategy. The objective being designed here will involve a careful tradeoff between two factors

(a) Economy (b) Security

A generalized objective incorporating this tradeoff is now formulated as follows:

$$\text{MIN } J = \sum_{i=1}^{NG} F_i(PG_i) + \sum_{i=1}^{N} \rho_i |\Delta PG_i| + \sum_{i=1}^{k} \sigma_i |\Delta PL_i| + \sum_{k=1}^{N} \sum_{\ell=1}^{N} \alpha_{k\ell} C_{k\ell} \qquad (6.18)$$

where

$$\underline{C}_{k\ell} = 12\,\frac{T^4}{5!} - 30\Big[\sum_{\substack{j=1\\ j\neq k}}^{N} \frac{T_{kj}}{M_k} + \sum_{\substack{j=1\\ j\neq \ell}}^{N} \frac{T_{\ell j}}{M_\ell} + T_{k\ell}\Big(\frac{1}{M_k} + \frac{1}{M_\ell}\Big)\Big]\frac{T^6}{7!}$$

$T_{ij} = \frac{V_i V_j}{X_{ij}} \cos(\delta_i - \delta_j)$ - line stiffness (6.18a) connecting internal generator buses i and j

ΔPG_i = a vector of changes in controllable power generations or equivalent bus generation shifts

ΔPL_i = a vector of changes in interruptible loads

$\delta_i - \delta_j$ = a vector of differences in bus voltage angles

$\rho_i, \sigma_i, \alpha_{ij}$ = positive weighting cost coefficients

Pending a discussion of the control objectives of each term in (6.18) in terms of the role it plays in enhancing the security of operation of the power system, the linearization of the performance index is conducted.

Linearization of the performance index

Clearly the performance index (6.18) is nonlinear owing to the quadratic nature of the economic objective, the discontinuous nature of the two penalty functions and the inherently nonlinear characteristic of the coherency term.

Therefore, in order to use (6.18) in an LP algorithm, it is necessary to linearize it. Since the incremental changes in the power system are small owing to the fact that changes in load occur in small steps, linearization is a viable assumption and the expression (6.18) is now linearized about the operating point.

From (6.2) the economic dispatch objective in its original quadratic form is given by the cost of generation per hour as

$$J_1 = \sum_{i=1}^{N} F_i(PG_i) = \sum_{i=1}^{N} a_i + b_i PG_i + c_i PG_i^2 \, . \qquad (6.19)$$

Linearizing (6.19) and using the relationship

$$\Delta J_1 = \frac{\partial J_1}{\partial PG_i} \Delta PG_i \qquad (6.20)$$

the equation (6.20) can be written as

$$\Delta J_1 = \sum_{i=1}^{N} (b_i + 2c_i PG_i) \Delta PG_i \qquad (6.21)$$

Evaluating the coefficients at the operating point and representing the resulting coefficient by K_i, the equation (6.21) becomes

$$\Delta J_1 = \sum_{i=1}^{N} K_i \Delta PG_i \qquad (6.22)$$

which is the first part of (6.18).

From (6.18), the second part of the objective is given by

$$J_2 = \sum_{i=1}^{N} \rho_i |\Delta PG_i| \, . \qquad (6.23)$$

Penalty function (6.23) represents a discontinuous function and must be linearized before it can be used in the

LP algorithm. Since ΔPG_i represents a free variable, it must first be converted into a nonnegative variable to comply with the nonnegativity requirements of a linear program. Accordingly, ΔPG_i should be converted into a difference of two nonnegative variables as follows

$$\Delta PG_i = \Delta PG_i^+ - \Delta PG_i^- \qquad (6.24)$$

where

$$\Delta PG_i^+ \geq 0$$

$$\Delta PG_i^- \geq 0.$$

Equation (6.24) is then rewritten as

$$J_2 = \sum_{i=1}^{N} \rho_i |\Delta PG_i^+ - \Delta PG_i^-| \qquad (6.25)$$

Applying the triangle inequality

$$|A - B| \leq |A| + |B| \qquad (6.26)$$

to equation (6.25), the result is

$$J_2 = \sum_{i=1}^{N} \rho_i (\Delta PG_i^+ + \Delta PG_i^-) \qquad (6.27)$$

It is to be noted that the mathematical formalism of (6.24-6.27) is equivalent to recognizing that (6.23) can be intuitively represented by the sum of two simple linear functions in two variables ΔPG_i^+ and ΔPG_i^-,

$$J_2 = \sum_{i=1}^{N} (\rho_i \Delta PG_i^+ - \rho_i \Delta PG_i^-) \qquad (6.27a)$$

where

$$\Delta PG_i^+ \geq 0, \qquad \Delta PG_i^- \leq 0.$$

Equation (6.27a) can directly be converted to meet the nonnegative requirement resulting in (6.27).

Similarly, linearization of the third term in (6.18) results in

$$J_3 = \sum_{i=1}^{k} \sigma_i (\Delta PL_i^+ + \Delta PL_i^-) \qquad (6.28)$$

where

$$\Delta PL_i^+ \geq 0, \ \Delta PL_i^- \geq 0$$

From (6.18), the last part of the objective which is the transient security index is given by

$$J_4 = \sum_{k=1}^{N} \sum_{\ell=1}^{N} \alpha_{k\ell} \ C_{k\ell} \qquad (6.29)$$

From (5.25), this weighted coherency term can be written in terms of the synchronizing torque coefficients, T_{ij}, as

$$J_4 = \sum_{k=1}^{N} \sum_{\ell=1}^{N} \alpha_{k\ell} \{12\frac{T^4}{5!} - 30[\sum_{\substack{j=1 \\ j\neq k}}^{N} \frac{T_{kj}}{M_k} + \sum_{\substack{j=1 \\ j\neq}}^{N} \frac{T_{\ell j}}{M_\ell} + T_{k\ell}(\frac{1}{M_k} + \frac{1}{M_\ell})]\frac{T^6}{7!}\} \qquad (6.30)$$

Expanding (6.30) the result is

$$J_4 = \sum_{k=1}^{N} \sum_{\ell=1}^{N} \{\alpha_{k\ell} 12 \frac{T^4}{5!} - \alpha_{k\ell} 30 [\sum_{\substack{j=1 \\ j \neq k}}^{N} \frac{T_{kj}}{M_k} + \sum_{\substack{j=1 \\ j \neq \ell}}^{N} \frac{T_{\ell j}}{M_\ell} + T_{k\ell} (\frac{1}{M_k} + \frac{1}{M_\ell})] \frac{T^6}{7!} \} \tag{6.31}$$

Recognizing that constants do not affect the optimization process, the effective term needed from (6.31) is given by

$$J_4 = - \sum_{k=1}^{N} \sum_{\ell=1}^{N} \alpha_{k\ell} 30 [\sum_{\substack{j=1 \\ j \neq k}}^{N} \frac{T_{kj}}{M_k} + \sum_{\substack{j=1 \\ j \neq \ell}}^{N} \frac{T_{\ell j}}{M_\ell} + T_{k\ell} (\frac{1}{M_k} + \frac{1}{M_\ell})] \frac{T^6}{7!} \tag{6.32}$$

where the synchronizing torque coefficients $T_{k\ell}$, $T_{\ell j}$, $T_{k\ell}$ represent line stiffness as defined in (6.18a) which is

$$T_{ij} = \frac{V_i V_j}{X_{ij}} \cos(\delta_i - \delta_j) \tag{6.33}$$

Approximating the cosine in (6.33) with the first two terms of the power series, each synchronizing torque coefficient above may be approximated

$$T_{ij} = \frac{V_i V_j}{X_{ij}} \{1 - \frac{(\delta_i - \delta_j)^2}{2!}\} \tag{6.34}$$

which makes (6.32) a quadratic.

The function J_4 is the quadratic transient security index which when combined with the linearized J_1, J_2 and J_3, gives the following complete form of the security oriented objective function

The function J_4 is the quadratic transient security index which when combined with the linearized J_1, J_2 and J_3. gives the following complete form of the security oriented objective function

$$\min_{\substack{\Delta PG^+, \Delta PG^- \\ \Delta PL^+, \Delta PL^- \\ \delta_i - \delta_j}} J = \sum_{i=1}^{N} k_i(\Delta PG_i^+ - \Delta PG_i^-) + \sum_{i=1}^{N} \rho_i (\Delta PG_i^+ + \Delta PG_i^-)$$

$$+ \sum_{i=1}^{k} \sigma_i (\Delta PL_i^+ + \Delta PL_i^-)$$

$$- \sum_{k=1}^{N} \sum_{\ell=1}^{N} \alpha_{k\ell} 30 \left[\sum_{\substack{j=1 \\ j \neq k}}^{N} \frac{T_{kj}}{M_k} + \sum_{\substack{j=1 \\ j \neq \ell}}^{N} \frac{T_{\ell j}}{M_\ell} + T_{k\ell}\left(\frac{1}{M_k} + \frac{1}{M_\ell}\right)\right] \frac{T^6}{7!} \qquad (6.35)$$

Control Objectives:

The first term in the objective function (6.18) reflects the desire to minimize fuel costs subject to equipment limitations and predetermined schedules. In this context this portion of the objective function represents the usual economic dispatch objective. Since each terms depends only upon one independent variable, it is often referred to as a separable economic objective function, the cost coefficients a_i, b_i, c_i being determined empirically [24].

The second term in the objective function (6.18) is a penalty function for the generation shifts which reflects the desire to bring a vulnerable system into a secure operating condition in the least costly manner. The cost coefficient ρ_i is chosen depending upon the power system operating state and the criteria for this choice shall be discussed in Chapter 8.

Similarly the third term in (6.18) is a weighted penalty function for the load shifts and is used to provide the least costly load correction based on a priority σ_i, the choice of which will also be discussed in Chapter 8.

Earlier in this chapter, a diagnosis of the optimum dispatch problem (6.1-6.13) revealed that the performance index was lacking in its ability to respond effectively to transient insecurities and that there existed a need for a comprehensive security, reliability and stability measure.

This improvement in the objective function is realized by the addition to the performance index (6.18) of a measure which is based on stiffening the transmission network and thereby adaptively raising the security level of the power system. The specific controls for altering the stiffness of the power system are discussed in Chapter 8 within the context of the overall power system security problem.

The augmentation of the dispatch strategy is executed only when the security assessment functions indicate or insecure condition with respect to a possible next contingency or an actual violation of the transient security operating constraints.

The objective (6.35) subject to linear constraints poses itself as a quadratic programming (QP) problem which can be solved by various QP algorithms, typically the Wolfe's method which converts the QP to a linear program using Kuhn Tucker conditions. The use of a QP approach for the economic dispatch problem has also been proposed by Nicholson and Sterling [31] but on-line implementation was impractical due to the memory requirements of the computer program. However, recent advances in computational techniques by Dayal, et al [31] have made the use of Quadratic Programming a viable proposition for on-line implementation although not as fast as the conventional LP packages.

6.3.3 The Linearized Constraints

Having formulated the linearized performance index, the next step in the formulationis to write the linearized constraints which comprise of the

- linearized load flow constraints
- linearized operating constraints
- linearized security constraints

In order to proceed with this formulation, it is necessary to recognize first that the constraints (6.3 - 6.13) which were formulated for the general optimum dispatch problem, represent in effect a full set of constraints comprising of two generic categories which are:

. Real Power - Phase Angle Constraints

. Reactive Power - Voltage Constraints

Of this full set, the load flow contraints which are represented by (6.3, 6.4), amount to 2N in number for an N bus system. For any optimization algorithm, the task of selecting an optimum generation schedule and simultaneously satisfying the 2N contraints translates into a rather formidable computational requirement, especially for larger networks. However, this difficulty is alleviated by making the observation that the full set of constraints are necessary only in an environment in which the dispatch objectives are to minimize fuel costs and to minimize losses while assuring that all equipment is or will, for a set of possible contingencies, operate within present voltage and thermal limits, generation, load and line capacity and stiffness constraints.

As suggested by Carpentier [18], for small perturbations, the interaction between real power and bus voltage, and between reactive power and phase angle is weak, thereby allowing a decoupling of the real and reactive problems into two separate problems which deal independently with

. Real Power Optimization

. Reactive Power Optimization

Recognizing that the reactive component does not affect in any major way the fuel costs or security, the decoupled reactive power flow can be omitted from the on-line tracking problem formulation. Accordingly, the modeling process is freed of the need for monitoring constraints associated with the reactive power balance (6.4) and associated voltage profile constraints (6.8, 6.9, 6.10). The constraint set (6.3 - 6.13) is thereby reduced to (6.3, 6.5, 6.7, 6.12, 6.13). Note that the phase shifter constraint (6.11) has been excluded here since only generation shifting and load shedding is being considered for controlling the security level of the power system. The voltage levels used for the on-line secure dispatch problem are thus obtained from the static state estimator and actions required to adjust voltage profile and real power flows are done automatically by existing voltage control equipment and methods.

Having stated and discussed the conditions under which the full optimization problem (6.2-6.13), consisting of the real and reactive dispatch, can be reduced to an on-line secure dispatch, it is important to recognize that the solution to the decoupled real power flow optimization represents a trade-off between computational requirements, accuracy, and the information content of the solution when compared to the rigorous full ac-network solution.

With the aforementioned decoupling process in perspective, the on-line real power dispatch may be stated as

$$\min_{\substack{\Delta PG^{+},\Delta PG^{-}\\ \Delta PL^{+},\Delta PL^{-}\\ \delta_i-\delta_j}} J = \sum_{i=1}^{N} k_i(\Delta PG_i^{+} - \Delta PG_i^{-}) + \sum_{i=1}^{N} \rho_i(\Delta PG_i^{+} + \Delta PG_i^{-}) + \sum_{i=1}^{k} \sigma_i(\Delta PL_i^{+} + \Delta PL_i^{-})$$

$$- \sum_{k=1}^{N} \sum_{\ell=1}^{N} \alpha_{k\ell} 30 [\sum_{\substack{j=1\\ j\neq k}}^{N} \frac{T_{kj}}{M_k} + \sum_{\substack{j=1\\ j\neq \ell}}^{N} \frac{T_{\ell j}}{M_\ell} + T_{k\ell}(\frac{1}{M_k} + \frac{1}{M_\ell})] \frac{T^6}{7!} \quad (6.36)$$

where

$$T_{ij} = \frac{V_i V_j}{X_{ij}} \{1 - \frac{(\delta_i - \delta_j)^2}{2!}\}$$

s.t.

$$PG_i - PL_i = P_i^N = V_i \sum_{j=1}^{N} Y_{ij} V_j \cos(\delta_i - \delta_j - \alpha_{ij}) \quad (6.37)$$

$$PG_i^m \leq PG_i \leq PG_i^M \quad (6.38)$$

$$PL_i^m \leq PL_i \leq PL_i^M \quad (6.39)$$

$$S_{ij}^m \leq S_{ij} \leq S_{ij}^M \quad (6.40)$$

$$|\delta_i - \delta_j| \leq \Psi_{ij}^M \quad (6.41)$$

The next requirement points to the need for eliminating the need for a recomputation of the load-flow at each iteration of the on-line tracking dispatch whenever a preventive or corrective control action is desired in response to the need for improving the security level of the power system. The objective is then achieved by

- converting the real power load flow constraint (6.3) into a linearized real power balance constraint for an area
- relating the real power injection changes in the network to the changes in the line phase angle differences representing real power flow along transmission lines by obtaining a linearized network model
- formulating the linearized operating and security constraints for the on-line problem using this linearized network model.

6.3.3.1 <u>The Linearized Load Flow Constraints</u>

The decoupled on-line secure dispatch (6.36-6.41) resulted in a reduction of the power flow equations from 2N to the N constraints on real power flow represented by (6.37). Since the on-line algorithm obtains its load flow information from a static state estimator, it will now be shown that it is not necessary to resolve (6.37) each time a preventive or corrective control action to reschedule the power system is desired for improved security.

However, any corrective action in terms of generation shifts, ΔPG_i, or load corrections, ΔPL_i, must be such that it maintains a balance between supply and demand in the interconnected system. In this regard it is imperative to restate that it is precisely the discrepancy between supply and demand which leads to a drop in system frequency and consequential deterioration of the security level of the power system. Obviously, even though the state estimator is providing the necessary state information, neglecting the power balance constraint would dispatch in a manner to compound the security problems of the network. Accordingly, a preservation of that balance for the on-line tracking problem must necessarily be reflected in an incremental relationship between the corrective actions, ΔPG_i and ΔPL_i.

With the supply and demand power balance relationship in focus, the constraints (6.37) can be shown to satisfy this balance relationship by expressing (6.37) as

$$PG_i = PL_i + V_i \sum_{j=1}^{N} Y_{ij} V_j \cos(\delta_i - \delta_j - \alpha_{ij}) \qquad (6.42)$$

Summing the generation and demand for the entire network, (6.42) can be expressed as

$$\sum_{i=1}^{N} PG_i = \sum_{i=1}^{k} PL_i + P_{Losses} \qquad (6.43)$$

Clearly this relationship states that the sum of the real generation equals the sum of the real demand plus the real losses.

Neglecting the losses as they amount only to a few percent of the total demand, (6.43) becomes

$$\sum_{i=1}^{N} PG_i - \sum_{i=1}^{k} PL_i = 0 \qquad (6.44)$$

Constraint (6.44) represents only one constraint compared to the N constraints (6.37). In addition, in this formulation in which the control actions are composed of generation shifts, ΔPG, and load corrections, ΔPL, this energy balance requirement translates to a conservation of power in the interconnected network by representing (6.44) in incremental form which is the desired power balance equation for the on-line dispatch problem

$$\sum_{i=1}^{N} \Delta PG_i - \sum_{i=1}^{k} \Delta PL_i = 0 \qquad (6.45)$$

In order to use the constraint (6.45) in an LP algorithm, the free variables are converted into nonnegative variables and the resulting constraint is

$$\left(\sum_{i=1}^{N} \Delta PG_i^+ - \sum_{i=1}^{N} \Delta PG_i^-\right) - \left(\sum_{i=1}^{k} \Delta PL_i^+ - \sum_{i=1}^{k} \Delta PL_i^-\right) = 0 \qquad (6.46)$$

Rearranging (6.46) the resulting linearized real power balance constraint corresponding to (6.37) of the decoupled model is

$$\sum_{i=1}^{N} \Delta PG_i^+ - \sum_{i=1}^{k} \Delta PL_i^+ - \sum_{i=1}^{N} \Delta PG_i^- + \sum_{i=1}^{k} \Delta PL_i^- = 0 \qquad (6.47)$$

where

$$PG_i^+, \quad PG_i^-, \quad PL_i^+, \quad PL_i^- \geq 0$$

6.3.3.2 The Linearized Operating Constraints

The objective of this section is to write incremental constraints corresponding to

- real generation (6.38)
- real load (6.39)
- line loading (6.40, 6.41)

The incremental constraints on generation and load are simple constraints and do not require any information besides their maximum and minimum limits on system operation. These constraints are of the general form:

$$\Delta PG^{MIN} \leq \Delta PG \leq \Delta PG^{MAX} \qquad (6.48a)$$

$$\Delta PL^{MIN} \leq \Delta PL \leq \Delta PL^{MAX} \qquad (6.48b)$$

The incremental constraints on line loading, however, are not as simple and require establishing a relationship between the line phase angle differences and injection changes. The general form of these constraints is

$$\underline{\Delta\Psi}^{MIN} \leq \underline{\Delta\Psi} \leq \underline{\Delta\Psi}^{MAX} \qquad (6.49)$$

with

$$\underline{\Delta\Psi} = \underline{A}\,\underline{\Delta I} \qquad (6.50a)$$

where

$\Delta\Psi$ = a vector of r line phase angle differences

A = r × n network matrix

ΔI = a vector of n bus injections

In order to find the matrix $\underline{A}$, a linearized network model is developed. This is done by

(a) obtaining the voltage angle differences in terms of the bus injection changes using a decoupled dc-model which gives

$$\underline{\Delta\theta} = \underline{H}^{-1}\underline{\Delta I} \qquad (6.50b)$$

where $\underline{H}$ = Jacobian matrix

(b) relating the voltage angles to the line phase angle differences by a bus incidence matrix giving

$$\underline{\Delta\Psi} = \underline{B}^{T}\,\underline{\Delta\theta} \qquad (6.51)$$

where $\underline{B}$ = n × r bus incidence matrix

The remainder of this section is concerned with

(1) formulating d.c. load flow (6.50b)

(2) form linearized network model (6.50a)

(3) derive the limits ΔPG^m, ΔPG^M, ΔPL^m, ΔPL^M, $\Delta\Psi^m$, $\Delta\Psi^M$ for the incremental constraints (6.48a, 6.48b, 6.49).

6.3.3.2(a) This component of the modelling process is equivalent to obtaining a decoupled model linearized about the operating point $(V°, \theta°)$ which would conform with two requirements

- the LP requirement of obtaining a linear network model
- the on-line tracking dispatch requirement of obtaining a deviational, incremental type of a model.

Writing the ac power flow equations (6.3, 6.4) in functional form

$$\begin{aligned} P_i^N &= f_1(V,\theta) \\ Q_i^N &= f_2(V,\theta) \end{aligned} \qquad (6.52)$$

where

f_1, f_2 = nonlinear functions of the network parameters

Relating small changes in nodal power injections to small changes in complex node voltages [17] and obtaining the operating point from either

- a static state estimator for real time studies,

or

- an ac-load flow for study mode purposes, the linearized system of equations in vector form are:

$$\begin{bmatrix} \underline{\Delta P}^N \\ \\ \underline{\Delta Q}^N \end{bmatrix} = \begin{bmatrix} \frac{\partial P}{\partial \theta} & \frac{\partial P}{\partial V} \\ \\ \frac{\partial Q}{\partial \theta} & \frac{\partial Q}{\partial V} \end{bmatrix}^{\circ} \begin{bmatrix} \underline{\Delta \theta} \\ \\ \underline{\Delta V} \end{bmatrix} \tag{6.53}$$

Recognizing the Jacobian matrix, (6.53) may be written as

$$\begin{bmatrix} \underline{\Delta P}^N \\ \\ \underline{\Delta Q}^N \end{bmatrix} = \begin{bmatrix} \quad & \underline{J}^{\circ} & \quad \end{bmatrix} \begin{bmatrix} \underline{\Delta \theta} \\ \\ \underline{\Delta V} \end{bmatrix} \tag{6.54}$$

where

$\underline{J}^{\circ}$ = Jacobian matrix evaluated at operating point $(V^{\circ}, \theta^{\circ})$

Recalling from section (6.3.3) that a decoupled model can be obtained for small perturbations, the decoupled model is:

$$\begin{aligned} \underline{\Delta P}^N &= \underline{H}\ \underline{\Delta \theta} \\ \underline{\Delta Q}^N &= \underline{L}\ \underline{\Delta V} \end{aligned} \tag{6.55}$$

where

$$\underline{J}^{\circ} = \begin{bmatrix} \underline{H} & \underline{N} \\ \underline{J} & \underline{L} \end{bmatrix} \approx \begin{bmatrix} \underline{H} & \underline{O} \\ \underline{O} & \underline{L} \end{bmatrix}$$

Since the decision to eliminate the reactive model has already been made earlier in this chapter, the real part of (6.55) may be written in terms of the notation of (6.50) as

$$\underline{\Delta I} = \underline{H}\ \underline{\Delta\theta} \qquad (6.56)$$

The system (6.56) is now rewritten without the slack bus as

$$\underline{\Delta\hat{I}} = \underline{\hat{H}}\ \underline{\Delta\hat{\theta}} \qquad (6.57)$$

where

$\underline{\Delta\hat{I}}$ = a sparse (n-1) vector of all bus injection changes except slack whose nonzero elements are the controlled bus generation shifts or load shed (or phase shifts if desired)

$\underline{\hat{H}}$ = a (n-1) × (n-1) Jacobian submatrix of H

Solving (6.57) for $\underline{\Delta\hat{\theta}}$, the resulting equations are

$$\underline{\Delta\hat{\theta}} = \underline{\hat{H}}^{-1}\ \underline{\Delta\hat{I}} \qquad (6.58)$$

This result gives the changes in voltage angles, $\underline{\Delta\hat{\theta}}$, in response to a shift in bus injections, ΔI.

6.3.3.2(b) To relate the bus injections to the line phase angle differences as in (6.51), it is important to recognize that the assumption of a lossless network results in the branch power flows being directly proportional to the branch phase angle differences. This requirement is expressed mathematically by

$$\underline{\Delta\hat{\Psi}} = \hat{B}^{T}\ \underline{\Delta\hat{\theta}} \qquad (6.59)$$

where

$\underline{\Delta\hat{\Psi}}$ = a vector of r line phase angle differences

$\underline{\Delta\hat{\theta}}$ = a vector of (n-1) bus voltage angle changes

$\underline{\hat{B}}$ = (n-1) × r bus incidence matrix

Here the elements corresponding to the slack bus have been omitted by eliminating the nth row resulting in (n-1) linear independent equations.

Substituting (6.58) into (6.59), the model becomes

$$\underline{\Delta\hat{\Psi}} = \underline{\hat{B}}^T \hat{H}^{-1} \Delta\hat{I} \tag{6.60}$$

$$[r \times 1] = [r \times (n-1)][(n-1) \times (n-1)][(n-1) \times 1]$$

Expressing (6.60) in compact form

$$\underline{\Delta\hat{\Psi}} = \underline{\hat{A}}\ \underline{\Delta\hat{I}} \tag{6.61}$$

where

$$\underline{\hat{A}} = \underline{\hat{B}}^T\underline{\hat{H}}^{-1}, \text{ an } r \times (n-1) \text{ matrix}$$

Comparing (6.61) with (6.51), the desired network matrix $\underline{A}$ is given by

$$\underline{A} = \left[\begin{array}{c|c} & 0 \\ \underline{\hat{A}} & \vdots \\ & 0 \end{array}\right] = \left[\begin{array}{c|c} & 0 \\ \underline{\hat{B}}^T\underline{\hat{H}}^{-1} & \vdots \\ & 0 \end{array}\right] \tag{6.62}$$

Representing this matrix in the general form of the desired model (6.51), the resulting model is

$$\underline{\Delta\Psi} = \left[\begin{array}{c|c} & 0 \\ \underline{\hat{B}}^T\underline{\hat{H}}^{-1} & \vdots \\ & 0 \end{array}\right] \underline{\Delta I} \tag{6.63}$$

It is important to point out that the triangular factorization of $\hat{\underline{H}}$, in the definition of $\underline{A}$ is obtained by sparsity techniques developed by Tinney et al. [33]. However, since the optimal power flow is being operated on-line wherein state estimation is being used to provide the data base, the Jacobian is automatically available and a single factorization of $\hat{\underline{H}}$ is sufficient for the contingency tests to be discussed later. In addition, the network matrix $\underline{A}$ is dependent only upon existing network conditions which makes it possible to utilize (6.63) for writing the linearized operating and security constraints.

The Operating Constraints:

The operating constraints, which are the constraints on physical limitations on equipment, before the occurrence of an actual contingency consist of two parts:

- constraints on correction limits
- constraints on line loading

Correction limit constraints

All such constraints are expressible in general form by the vector inequalities

$$\Delta PG^{MIN} \leq \Delta PG \leq \Delta PG^{MAX}$$
$$\Delta PL^{MIN} \leq \Delta PL \leq \Delta PL^{MAX} \tag{6.64}$$

Let the system operating point for the real power generation be given by $PG°$ and let ΔPG be the corresponding generation shift or correction needed to improve the security level of the power system. Representing this correction in terms of upper and lower physical limitations on generation, the appropriate constraint is

$$PG^{MIN} \leq PG^{\circ} + \Delta PG \leq PG^{MAX} \tag{6.65}$$

Subtracting PG° from (6.65)

$$PG^{MIN} - PG^{\circ} \leq \Delta PG \leq PG^{MAX} - PG^{\circ} \tag{6.66}$$

Expressing (6.66) in terms of deviations

$$\Delta PG^{MIN} \leq \Delta PG \leq \Delta PG^{MAX} \tag{6.67}$$

Substituting into (6.67) the relationship

$$\Delta PG = \Delta PG^{+} - \Delta PG^{-}$$

the resulting constraint is

$$\Delta PG^{MIN} \leq \Delta PG^{+} - \Delta PG^{-} \leq \Delta PG^{MAX} \tag{6.68}$$

Expressing (6.68) as two separate constraints which agree with physical intuition

$$\begin{aligned} 0 \leq \Delta PG^{+} \leq \Delta PG^{MAX} \\ \Delta PG^{MIN} \leq \Delta PG^{-} \leq 0 \end{aligned} \tag{6.69}$$

Substituting into (6.69) the relationships

$$\Delta PG^{MAX} = PG^{MAX} - PG^{\circ}$$

$$\Delta PG^{MIN} = PG^{MIN} - PG^{\circ}$$

the resulting constraint set is

$$\begin{aligned} 0 \leq \Delta PG^{+} \leq PG^{MAX} - PG^{\circ} \\ PG^{MIN} - PG^{\circ} \leq \Delta PG^{-} \leq 0 \end{aligned} \tag{6.70}$$

Constraints (6.70) may be rewritten in a more convenient form as

$$\begin{aligned} \Delta PG^{+} \leq PG^{MAX} - PG^{\circ} \\ -\Delta PG^{-} \leq PG^{\circ} - PG^{MIN} \end{aligned} \tag{6.71}$$

Similarly, the range of corrections on the load is given by the constraints

$$\begin{aligned} \Delta PL^{+} \leq PL^{MAX} - PL^{\circ} \\ -\Delta PL^{-} \leq PL^{\circ} - PL^{MIN} \end{aligned} \tag{6.72}$$

Constraints (6.71) and (6.72) constitute the correction limit constraints and correspond to (6.38, 6.39) of the decoupled model (6.36). There would be 2N constraints of the type (6.71) since the network could have at most N generation shifts i.e. N constraints for PG^{+} and N for PG^{-}. Similarly there would be 2K constraints of the type (6.72).

Line Loading Constraints

These incremental constraints, which must be satisfied before the occurrence of an actual postulated next contingency, are of the general form

$$\underline{\Delta\Psi}^{MIN} \leq \underline{A}\ \underline{\Delta I} \leq \underline{\Delta\Psi}^{MAX} \tag{6.73}$$

and correspond to constraints (6.40, 6.41) of the decoupled on-line dispatch (6.36).

The limitations on the range of corrections is given by

$$\Psi_i^{MIN} \leq \Psi_i^{\circ} + \Delta\Psi_i \leq \Psi_i^{MAX} \quad i = 1,\ldots,r \tag{6.74}$$

Subtracting Ψ_i° from (6.74)

$$\Psi_i^{MIN} - \Psi_i^{\circ} \leq \Delta\Psi_i \leq \Psi_i^{MAX} - \Psi_i^{\circ} \quad i = 1,\ldots,r \tag{6.75}$$

Expressing (6.75) in terms of deviations

$$\Delta\Psi_i^{MIN} \leq \Delta\Psi_i \leq \Delta\Psi_i^{MAX} \qquad i = 1,\ldots,r \tag{6.76}$$

Invoking the network model (6.63)

$$\underline{\Delta\Psi} = \underline{A}\ \underline{\Delta I}$$

(6.76) may be expressed in vector form as

$$\underline{\Delta\Psi}^{MIN} \leq \underline{A}\ \underline{\Delta I} \leq \underline{\Delta\Psi}^{MAX} \tag{6.77}$$

It is desired to represent the injection changes in terms of the generation and load shifts in the corrected normal state. This is done by defining the normal scheduled injection vector and the corrected injection vector

$\underline{I}°$ = normal scheduled injection vector

$\underline{I}$ = corrected injection vector

Expressing I as a linear function of I° and the correction vectors and observing that the net injection must equal the net correction to maintain a balance, the resulting expression is

$$\underline{I} - \underline{I}° = \underline{\Delta PG} - \underline{\Delta PL} \tag{6.78}$$

Representing (6.78) in terms of deviations

$$\underline{\Delta I} = \underline{\Delta PG} - \underline{\Delta PL} \tag{6.79}$$

Substituting (6.79) into (6.77) the constraint becomes

$$\underline{\Delta\Psi}^{MIN} \underline{<} \underline{A}(\underline{\Delta PG} - \underline{\Delta PL}) \underline{<} \underline{\Delta\Psi}^{MAX} \tag{6.80}$$

Observing from (6.75) and (6.76) that

$$\begin{aligned} \Delta\Psi^{MIN} &= \Psi^{MIN} - \Psi° \\ \Delta\Psi^{MAX} &= \Psi^{MAX} - \Psi° \end{aligned} \tag{6.81}$$

constraint (6.80) can be expressed as

$$\underline{\Psi}^{MIN} - \underline{\Psi}° \underline{<} \underline{A}(\underline{\Delta PG} - \underline{\Delta PL}) \underline{<} \Psi^{MAX} - \Psi° \tag{6.81a}$$

Substituting $\Delta PG = \Delta PG^{+} - \Delta PG^{-}$ and $\underline{\Psi}° = \underline{A}\ \underline{I}°$

$$\Psi^{MIN} - AI° \underline{<} A\Delta PG^{+} - A\Delta PG^{-} - A\Delta PL^{+} + A\Delta PL^{-} \underline{<} \Psi^{MAX} - AI° \tag{6.82}$$

Expressing (6.82) as simple inequalities

$$A\Delta PG^{+} - A\Delta PG^{-} - A\Delta PL^{+} + A\Delta PL^{-} \leq \Psi^{MAX} - AI°$$
$$-A\Delta PG^{+} + A\Delta PG^{-} + A\Delta PL^{+} - A\Delta PL^{-} \leq -\Psi^{MIN} + AI° \qquad (6.83)$$

Since there are r branches in the network there will be $2r$ constraints of the type (6.83).

6.3.3.3 <u>The Security Constraints</u>

The incremental security constraints are composed of

. line outage constraints

. generator outage constraints

The security constraints are, therefore, the incremental line loading operating constraints which have been declared vulnerable by the security assessment function based typically on a violation of 90% of the bound. Specifically these would include the constraints given by (6.83) and would be activated based on information provided by the assessment functions.

<u>Line outage constraints</u>

Let the system be subjected to "L" line outages. The matrix $\underline{A}$ is obviously affected by such outages by a change in the network configuration and the constraint for each such outage is given generally by

$$\Delta\Psi^{MIN} \leq A_j \Delta I \leq \Delta\Psi^{MAX} \qquad j = 1,\ldots,L \qquad (6.84)$$

Using a development similar to (6.83) the "L" line outage constraints are

$$\Psi^{MIN} - A_j I^\circ \leq A_j \Delta PG^+ - A_j \Delta PG^- - A_j \Delta PL^+ + A_j \Delta PL^- \leq \Psi^{MAX} - A_j I^\circ \quad (6.85)$$

Expressing (6.85) as simple inequalities

$$\begin{aligned} A_j \Delta PG^+ - A_j \Delta PG^- - A_j \Delta PL^+ + A_j \Delta PL^- &\leq \Psi^{MAX} - A_j I^\circ \\ -A_j \Delta PG^+ + A_j \Delta PG^- + A_j \Delta PL^+ - A_j \Delta PL^- &\leq -\Psi^{MIN} + A_j I^\circ \end{aligned} \quad (6.85a)$$

There would be up to 2L constraints (6.85a) for "L" line outages.

<u>Generator outage constraints</u>

Let the system be subjected to NG generator outages. Obviously the $\underline{A}$ matrix is unaffected but the injections undergo changes and the appropriate constraint is of the form

$$\Delta\Psi^{MIN} \leq A \Delta I_j \leq \Delta\Psi^{MAX} \qquad j = 1, \ldots, NG \quad (6.86)$$

In this case it is important to assume that the frequency and voltage dependence of the loads is neglected otherwise the injection relationships under discussion would be nonlinear.

In a manner similar to (6.79) the corrected injection vector is defined in terms of the scheduled normal injection, I°, the correction, ΔPG, ΔPL and the vector

$\underline{G}^j$ of loss in generation due to the jth generator outage

$$I = I° - G^j + \Delta PG - \Delta PL \qquad (6.87)$$

The constraints are obtained using a procedure similar to (6.85) and they are

$$\Psi^{MIN} - A(I° - G^j) \leq A\Delta PG^+ - A\Delta PG^- - A\Delta PL^+ + A\Delta PL^-$$
$$\leq \Psi^{MAX} - A(I° - G^j) \qquad (6.88)$$

Expressing (6.88) as simple inequalities

$$A\Delta PG^+ - A\Delta PG^- - A\Delta PL^+ + A\Delta PL^- \leq \Psi^{MAX} - A(I° - G^j)$$
$$-A\Delta PG^+ + A\Delta PG^- + A\Delta PL^+ - A\Delta PL^- \leq -\Psi^{MIN} + A(I° - G^j) \qquad (6.88a)$$

Similarly (6.88a) would constitute 2NG constraints.

Summarizing, the on-line secure dispatch formulation with a quadratic transient security index is given in mathematical programming format by:

$$\min_{\substack{\Delta PG^+,\Delta PG^- \\ \Delta PL^+,\Delta PL^- \\ \delta_i - \delta_j}} J = \sum_{i=1}^{N} K_i (\Delta PG_i^+ - \Delta PG_i^-) + \sum_{i=1}^{N} P_i (\Delta PG_i^+ + \Delta PG_i^-) + \sum_{i=1}^{k} \sigma_i (\Delta PL_i^+ + \Delta PL_i^-) \tag{6.89}$$

$$- \sum_{k=1}^{N} \sum_{\ell=1}^{N} \alpha_{k\ell} 30 \left[\sum_{\substack{j=1 \\ j\neq k}}^{N} \frac{T_{kj}}{M_k} + \sum_{\substack{j=1 \\ j\neq \ell}}^{N} \frac{T_{\ell j}}{M_\ell} + T_{k\ell}\left(\frac{1}{M_k} + \frac{1}{M_\ell}\right)\right] \frac{T^6}{7!}$$

where

$$T_{ij} = \frac{V_i V_j}{X_{ij}} \left\{1 - \frac{(\delta_i - \delta_j)^2}{2!}\right\}$$

S.T.

$$\sum_{i=1}^{N} \Delta PG_i^+ - \sum_{i=1}^{k} \Delta PL_i^+ - \sum_{i=1}^{N} \Delta PG_i^- + \sum_{i=1}^{k} \Delta PL_i^- = 0 \tag{6.90}$$

$$\left.\begin{aligned} \Delta PG^+ &\leq PG^{MAX} - PG^\circ \\ -\Delta PG^- &\leq PG^\circ - PG^{MIN} \\ \Delta PL^+ &\leq PL^{MAX} - PL^\circ \\ -\Delta PL^- &\leq PL^\circ - PL^{MIN} \end{aligned}\right\} \tag{6.91}$$

$$\left.\begin{aligned} A\Delta PG^+ - A\Delta PG^- - A\Delta PL^+ + A\Delta PL^- &\leq \psi^{MAX} - AI^\circ \\ -A\Delta PG^+ + A\Delta PG^- + A\Delta PL^+ - A\Delta PL^- &\leq -\psi^{MIN} + AI^\circ \end{aligned}\right\} \tag{6.92}$$

$$\left.\begin{aligned} A_j\Delta PG^+ - A_j\Delta PG^- - A_j\Delta PL^+ + A_j\Delta PL^- &\leq \psi^{MAX} - A_j I^\circ \\ -A_j\Delta PG^+ + A_j\Delta PG^- + A_j\Delta PL^+ - A_j\Delta PL^- &\leq -\psi^{MIN} + A_j I^\circ \end{aligned}\right\} \tag{6.93}$$

$$\left.\begin{aligned} A\Delta PG^+ - A\Delta PG^- - A\Delta PL^+ + A\Delta PL^- &\leq \psi^{MAX} - A(I^\circ - G^j) \\ -A\Delta PG^+ + A\Delta PG^- + A\Delta PL^+ - A\Delta PL^- &\leq -\psi^{MIN} + A(I^\circ - G^j) \end{aligned}\right\} \tag{6.94}$$

$$\Delta PG^+,\ \Delta PG^-,\ \Delta PL^+,\ \Delta PL^- \leq 0$$

Constraint (6.90) is the incremental load flow or power balance constraint and is a single constraint.

Constraints (6.91) are the operating constraints associated with the limitations on the range of corrections and are $(2N + 2k)$ in number.

Constraints (6.92) are the operating constraints associated with line loading and are $2r$ in number.

Constraints (6.93) are the security constraints associated with line outages and amount to $2L$ in number.

Constraints (6.94) are the security constraints associated with generator outages and amount to $2NG$ in number.

The maximum number of possible constraints is therefore $(1 + 2N + 2k + 2r + 2L + 2NG)$ and there exist several efficient computer packages for handling this large number efficiently.

CHAPTER 7

PERFORMANCE INDEX JUSTIFICATION FOR SECURITY DISPATCH

OBJECTIVES:

The principal objective of this part of the research is to show that the transient coherency measure is an improved measure for power system security and thereby raises the security margin of the electric energy system.

METHODOLOGY:

The methodology of this chapter shall consist of meeting the above objectives by

(i) developing of the equivalent-line concept and appropriate relationships for use in the analytical development

(ii) analytically justifying the addition of the new performance index and showing it to be a measure of

(a) security and reliability

(b) stability

7.1 The Equivalent Line

The concept of an "equivalent-line" connecting a specific generator to an infinite bus, which is appropriate over the interval $0 \leq T \leq T^1_{k\ell}(\varepsilon)$, is used in this chapter for the analytical development, leading to the proposed justification of the performance index as a measure of security, reliability and stability.

The use of this concept is, therefore, explained and the appropriate expressions are developed within the context of this development.

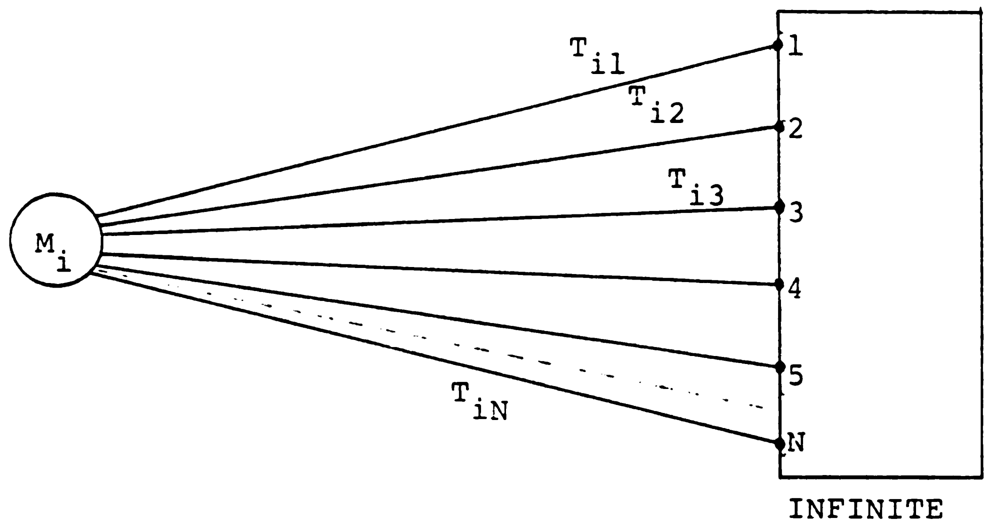


Figure 1

From elementary electric energy systems theory, the synchronizing torque coefficient between buses i and j, representing the stiffness of the transmission line, is given as in (6.18a) by the expression

$$T_{ij} = \frac{V_i V_j}{X_{ij}} \cos(\delta_i - \delta_j) \ . \tag{7.1}$$

However, if more than one transmission line connects the rest of the system to the disturbed generator i, the equivalent synchronizing torque coefficient, connecting bus i to the rest of the system, is given by the sum of the synchronizing torques for each connection k, weighted by the inertia of the disturbed bus i as

$$\frac{1}{M_i} \sum_{k=1}^{N} T_{ik} = \frac{1}{M_i} \sum_{k=1}^{N} \frac{V_i V_k}{X_{ik}} \cos(\delta_i - \delta_k) \qquad (7.2)$$

Representing the rest of the system by an infinite bus (or equivalent bus), as in Figure 1, it is desired to represent buses k by an infinite bus as

$$\frac{1}{M_i} \sum_{k=1}^{N} T_{ik} = \frac{1}{M_i} \frac{V_i V_j}{X_{ij}} \cos(\delta_i - \delta_j) \qquad (7.3)$$

where

j is the infinite bus.

Note that if the voltage magnitudes and angles in equation (7.2) were all equal, the equivalent synchronizing torque would reduce to the simple problem of obtaining a parallel equivalent as in classical network theory. However, this is generally not the case.

In developing the equivalent-line concept, the problem, therefore, reduces to one in which it is desired to find the appropriate expressions for V_j, X_{ij}, and δ_j in (7.3).

The solution to this is obtained by equating (7.2) and (7.3) and noting

$$\sum_{k=1}^{N} \frac{V_k}{X_{ik}} \cos(\delta_i - \delta_k) = \frac{V_j}{X_{ij}} \cos(\delta_i - \delta_j) \ . \qquad (7.4)$$

Invoking the trigonometric identities

$$\cos(\delta_i - \delta_k) = \cos\delta_k \cos\delta_i - \sin\delta_k \sin\delta_i$$
$$\cos(\delta_i - \delta_j) = \cos\delta_j \cos\delta_i - \sin\delta_j \sin\delta_i \tag{7.5}$$

and substituting into (7.4) the result is

$$\sum_{k=1}^{N} [\frac{V_k}{X_{ik}} \cos\delta_k]\cos\delta_i - \sum_{k=1}^{N} [\frac{V_k}{X_{ik}} \sin\delta_k]\sin\delta_i$$
$$= [\frac{V_j}{X_{ij}} \cos\delta_j]\cos\delta_i - [\frac{V_j}{X_{ij}} \sin\delta_j]\sin\delta_i \tag{7.6}$$

Comparing coefficients in (7.6) the result is

$$\sum_{k=1}^{N} \frac{V_k}{X_{ik}} \cos\delta_k = \frac{V_j}{X_{ij}} \cos\delta_j \tag{7.7}$$

and

$$\sum_{k=1}^{N} \frac{V_k}{X_{ik}} \sin\delta_k = \frac{V_j}{X_{ij}} \sin\delta_j \tag{7.8}$$

Taking ratios of (7.7) and (7.8) the result is

$$\tan\delta_j = \frac{\sin\delta_j}{\cos\delta_j} = \frac{\sum_{k=1}^{N} \frac{V_k}{X_{ik}} \cos\delta_{ik}}{\sum_{k=1}^{N} \frac{V_k}{X_{ik}} \sin\delta_{ik}} \tag{7.9}$$

and solving (7.7) or (7.8) for $\frac{V_j}{X_{ij}}$ the result is

$$\frac{V_j}{X_{ij}} = \frac{\sum_{k=1}^{N} \frac{V_k}{X_{ik}} \cos\delta_k}{\cos\delta_j} = \frac{\sum_{k=1}^{N} \frac{V_k}{X_{ik}} \sin\delta_k}{\sin\delta_j} \tag{7.10}$$

Equations (7.9) and (7.10) give the desired relationships defining the equivalent line which are written in final form as

$$\delta_j = \arctan\left\{\frac{\sum_{k=1}^{N} \frac{V_k}{X_{ik}} \cos \delta_{ik}}{\sum_{k=1}^{N} \frac{V_k}{X_{ik}} \sin \delta_{ik}}\right\}$$

(7.11)

$$\frac{V_j}{X_{ij}} = \frac{\sum_{k=1}^{N} \frac{V_k}{X_{ik}} \cos \delta_k}{\cos \delta_j} .$$

7.2a From the analysis of the transient coherency measure as a function of T, it is clear that generators which are not directly connected to the disturbed generator act as infinite busses in the interval right after the disturbance.

Since the generators k act as infinite bus j for any generator i over this initial interval, the "equal area criterion" can be applied. This criterion which deals with the principle by which stability under transient conditions is determined does not require the need to use the digital computer for solving the swing equation. Although not applicable to the study of multi-machine systems, it does provide tremendous insight and understanding of the stability and dynamics involved in a system where one machine is swinging with respect to an infinite bus.

Having justified the use of the equal-area criterion during the initial interval, the analysis then proceeds as follows using Figure 2.

Assume θ'_{ij} and P'_{ij} are the present angle at bus i and the power over the equivalent line connecting generator i and j, the power being given by the expression

$$P_{ij} = \frac{V_i V_j}{X_{ij}} \sin(\theta_i - \theta_j) \qquad (7.12)$$

where

V_i = voltage at generator i

V_j = voltage at generator j

X_{ij} = reactance of equivalent line ij.

Furthermore, assume that the electrical fault very near bus i is cleared when the power on the line connecting ij is P''_{ij} and angle on generator i is θ''_i.

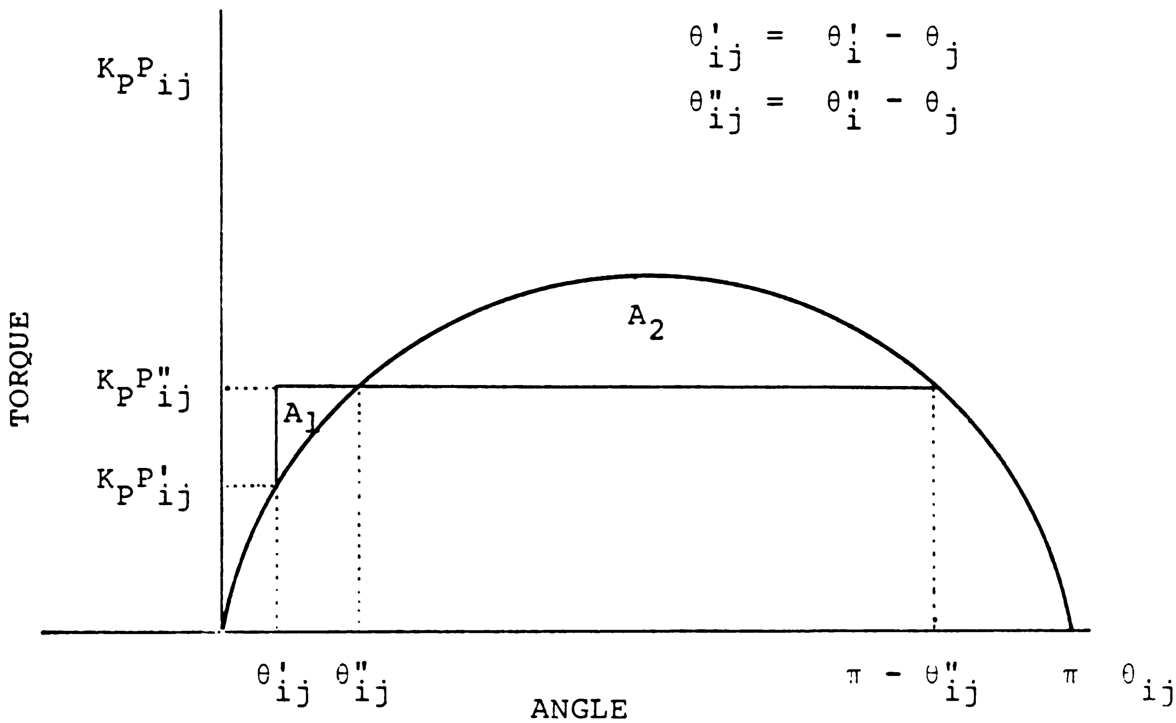


Figure 2

The equal area criterion would state that A_2 must be greater than or equal to A_1 in Figure 2 in order not to lose synchronism.

In order to meet the objective (iii) it is necessary to show that mean square coherency is a measure of the capacity of the system to withstand a disturbance. From elementary mechanics and Figure 2

$$\text{ENERGY} = \int \tau d\theta; \qquad \tau = P/2\pi N \tag{7.13}$$

where

τ is the net torque on the machine

θ is the angular displacement

N is the speed constant

P is the net power on the machine.

Letting $K_P = \frac{1}{2\pi N}$ the above energy equation is rewritten as

$$\text{ENERGY} = K_P \int P d\theta \ . \tag{7.14}$$

This is the fundamental equation which will be used in the development of the relationships for A_2 and A_1 in Figure 2.

Since the analysis is being conducted in terms of energy it is instructive to introduce the concepts developed by Podmore et al [25] and Magnusson [26] to assist in the justification of the transient coherency measure as a measure of security and reliability.

The concept of transient energy has unparalleled intuitive appeal in that it provides a very useful physical interpretation of the mathematical equations used in the study of power system stability.

This transient energy consists of two components - a kinetic energy term dependent upon rotor speeds and a potential energy term dependent on rotor angles. Consistent with equation (7.13), the integral of a torque with respect to the angle through which it acts is the work done, or energy transferred, in the process. Thus the area A_1 represents the potential energy inherent in the initial displacement of θ from θ'_{ij} to θ''_{ij} at the beginning of the transient. This potential energy is transformed into kinetic energy when θ equals θ''_{ij}, and is returned to the potential form as θ increases toward $\pi - \theta''_{ij}$. Note, however, that $\pi - \theta''_{ij}$ is simply a theoretical upper limit whereas the real upper limit would be at the point where the machine once again reaches synchronous speed. Angle θ then will decrease back to θ'_{ij}, during which time the energy goes into the kinetic form again, and then back to potential.

With these preliminary concepts in perspective it becomes quite simple to understand the physical meaning of the analysis in which it is desired to show that the ability of a system to withstand a disturbance is governed by the upper limit of its available energy capacity.

Expressions are now written for areas A_1 and A_2 in the post-fault state. Following this a relationship between A_1 and A_2 will be developed into an equation for A_2. This resulting equation for A_2 in the post fault state will then be compared to the A_2 equation for the pre-fault state and the results analyzed to lead to the objective (ii).

The area A_1 is given using Figure 2

$$A_1 = K_P P''_{ij}[(\theta''_i - \theta_j) - (\theta'_i - \theta_j)] - K_P \int_{\theta'_i-\theta_j}^{\theta''_i-\theta_j} P_{ij} d\theta_{ij} \quad (7.15)$$

$$= K_P P''_{ij}(\theta''_{ij} - \theta'_{ij}) - K_P \int_{\theta'_i-\theta_j}^{\theta''_i-\theta_j} \frac{E_i E_j}{X_{ij}} \sin \theta_{ij} d\theta_{ij} \quad (7.16)$$

$$= K_P P''_{ij}(\theta''_{ij} - \theta'_{ij}) + K_P \frac{E_i E_j}{X_{ij}} \cos \theta_{ij} \Big|_{\theta'_{ij}}^{\theta''_{ij}} \quad (7.17)$$

$$= K_P P''_{ij}(\theta''_{ij} - \theta'_{ij}) + K_P \frac{E_i E_j}{X_{ij}} \{\cos\theta''_{ij} - \cos \theta'_{ij}\} \quad (7.18)$$

and area A_2 is given by

$$A_2 = K_P \int_{\theta''_i-\theta_j}^{\pi-(\theta''_i-\theta_j)} P_{ij} d\theta_{ij} - K_P P''_{ij}\{\pi - (\theta''_i-\theta_j) - (\theta''_i-\theta_j)\} \quad (7.19)$$

$$= \frac{2K_P E_i E_j}{X_{ij}} \cos \theta''_{ij} - K_P P''_{ij}(\pi - 2\theta''_{ij}) \quad (7.20)$$

In order to develop the relationship between A_1 and A_2, it is desired to express A_2 in terms of A_1 as follows

$$A_2 = 2K_P \frac{E_iE_j}{X_{ij}} \cos\theta''_{ij} - K_P P''_{ij}(\pi - 2\theta''_{ij}) \qquad (7.21)$$

$$- 2\{K_P P''_{ij}(\theta''_{ij} - \theta'_{ij}) + K_P \frac{E_iE_j}{X_{ij}} (\cos\theta''_{ij} - \cos\theta'_{ij})\} + 2A_1 .$$

Eliminating terms, A_2 is rewritten as

$$A_2 = 2K_P \frac{E_iE_j}{X_{ij}} \cos\theta'_{ij} - K_P P''_{ij}(\pi - 2\theta'_{ij}) + 2A_1 . \qquad (7.22)$$

Since the pre-fault synchronizing torque coefficient is given by

$$T'_{ij} = \frac{E_iE_j}{X_{ij}} \cos\theta'_{ij}$$

the expression (7.22) may be rewritten in terms of T'_{ij} as

$$A_2 = 2K_P T'_{ij} - K_P P''_{ij}(\pi - 2\theta'_{ij}) + 2A_1 . \qquad (7.23)$$

Consistent with the energy concepts thus far introduced, A_2 represents the energy capacity of the line to withstand a fault, T'_{ij} is proportional to the energy capacity of the line before a loss of synchronism can occur, $K_P P''_{ij}(\pi - 2\theta'_{ij})$ represents the energy of the fault which does not get transferred, and A_1 represents the fault energy transferred onto the line as a result of the contingency.

The above expression for A_2 after a fault with energy A_1 has occurred can be written as

$$A_2 = \{2K_P T'_{ij} - K_P P'_{ij}[\pi - 2\theta'_{ij}]$$

$$- \{K_P(P''_{ij} - P'_{ij})(\pi - 2\theta'_{ij}) - 2A_1\} \qquad (7.24)$$

where

$$2K_P T'_{ij} - K_P P'_{ij}[\pi - 2\theta'_{ij}] \qquad (7.25)$$

represents the prefault energy the line can absorb due to a fault without loss of stability and

$$K_P(P''_{ij} - P'_{ij})(\pi - 2\theta'_{ij}) - 2A_1 \qquad (7.26)$$

represents the reduction in the fault energy absorption capacity of the line due to the fault.

From the pre-fault equation (7.25) it is clear that $P'_{ij}(\pi - 2\theta'_{ij})$ is relatively small in practice compared to $2T'_{ij}$ due to the fact that the pre-fault operating angle θ'_{ij} is chosen as small as possible in practice. This approximation translates to the following important conclusion that "the pre-fault energy capacity is essentially proportional to the pre-fault synchronizing torque coefficient, T'_{ij}, representing the stiffness of the equivalent line ij." Furthermore, since the post fault equation (7.24) reduces to the pre-fault (7.25) in the absence of a fault and since the reduction in the energy capacity due to any fault will be uncertain before any fault occurs, the energy capacity post-fault is proportional to the energy capacity pre-fault which has already been argued to be proportional to pre-fault line stiffness.

In conclusion, therefore, it is stated that the pre-fault line stiffness T'_{ij} represents the energy capacity of the line to withstand a fault. The significance of this conclusion is demonstrated by the fact that any design criteria for line stiffness would be specified before the fact and not after i.e. prior to the occurrence of a fault.

From the above analysis, it is clear that the transient coherency measure is an excellent measure of security because

(1) the transient coherency measure decreases proportional to the increase in stiffness of the equivalent line connecting internal generator buses i and j, and this pre-fault stiffness coefficient was shown to be proportional to the energy capacity of that line to withstand a contingency without loss of synchronism in the initial interval $0 \leq T \leq T^1_{k\ell}(\varepsilon)$ after a contingency. This shows that the transient coherency measure is proportional to the energy capacity to withstand a fault and is thus a measure of security and reliability.

(2) the transient coherency measure decreases proportional to the increase in stiffness of the equivalent line connecting generators k and ℓ and this stiffness increase is proportional to the decrease in the dif-

ference in phase angles at the two internal generator buses. Thus minimizing the coherency measure will increase stiffness of the equivalent line and reduce the pre-fault angle difference further below the static stability limit.

7.2b In the second part of the analysis it will be shown how the transient coherency measure is a stability measure as well. By showing that

$$\sum_i \sum_j C_{ij} = K_1 + K_2 \sum_{i=1}^{2N} \lambda_i^2 \qquad (7.27)$$

and thus decreasing C_{ij} amounts to increasing the complex part of the eigenvalues of the system indicating that reduction of this coherency measure increases the stiffness of the electrical network and thus the stability of the system.

The forementioned result is now derived.

First a relationship between the eigenvalues of $\underline{A}$ and $-\underline{M}^{-1}\underline{T}$ is established.

$$A = \left[\begin{array}{c|c} \underline{0} & \underline{I} \\ \hline \underline{X} & \underline{0} \end{array}\right] \; ; \; \underline{X} = -\underline{M}^{-1}\underline{T} \qquad (7.28)$$

The eigenvalues of $\underline{A}$ are obtained from the characteristic equation derived from equating the determinant of the matrix $[\lambda\underline{I} - \underline{A}]$ to zero, and solving for the values λ_i that solve the equation, i.e.

$$\det \lambda \underline{I} - \underline{A} = 0 \tag{7.29}$$

$$\det \begin{bmatrix} \lambda \underline{I} & -\underline{I} \\ -\underline{X} & \lambda \underline{I} \end{bmatrix} = \det \underline{M} = 0 \tag{7.30}$$

where $\{\lambda_i\}_{i=1}^{2N}$ are the eigenvalues of $\underline{A}$. Since the matrix $\underline{M}$ is written in the above $N \times N$ block matrix form, the $2N$ eigenvalues of $\underline{A}$ can be computed using

$$|M| = |\lambda^2 \underline{I} - \underline{X}| = 0 \tag{7.31}$$

Similarly the N eigenvalues $\hat{\lambda}_i$ of $\underline{X}$ are computed by solving the following equation

$$|N| = |\hat{\lambda} \underline{I} - \underline{X}| = 0 \; . \tag{7.32}$$

Observing that

(i) $\underline{X} = -\underline{M}^{-1}\underline{T}$

(ii) $\lambda_1^2, \lambda_2^2, \ldots, \lambda_{2N}^2$ are the solutions of $|M| = 0$

(iii) $\hat{\lambda}_1, \hat{\lambda}_2, \ldots, \hat{\lambda}_N$ are the solutions of $|N| = 0$

equation (7.31) can be rewritten as

$$|\lambda^2 \underline{I} - \underline{X}| = 0 = |\hat{\lambda} \underline{I} - \underline{X}|$$

so that

$$\lambda^2 = \hat{\lambda} \tag{7.33}$$

Thus the square of the eigenvalues of $\underline{A}$ equals the eigenvalues of $\underline{X}$ or $-\underline{M}^{-1}\underline{T}$.

Secondly a relationship between the synchronizing torque coefficients and the system eigenvalues is established.

Invoking the well known result from matrix analysis relating the trace of a matrix $\underline{W}$ to its eigenvalues

$$\mathrm{Tr}\{\underline{W}\} = \sum_{i=1}^{N} \lambda_i \tag{7.34}$$

and applying it to (7.31)

$$\mathrm{Tr}\{\underline{X}\} = \sum_{k=1}^{N} \hat{\lambda}_k \tag{7.35}$$

and using the relationship (7.33), equation (7.35) can be rewritten as

$$\mathrm{Tr}\{\underline{X}\} = \sum_{k=1}^{2N} \lambda_k^2 \tag{7.36}$$

Since $\underline{X} = -\underline{M}^{-1}\underline{T}$, $\mathrm{Tr}\{\underline{X}\}$ becomes

$$\mathrm{Tr}\{\underline{X}\} = -\mathrm{Tr}\{\underline{M}^{-1}T\}$$

$$= - \sum_{k=1}^{N} \sum_{\substack{j=1 \\ j\neq k}}^{N} \frac{T_{kj}}{M_k} \tag{7.37}$$

Substituting (7.37) into (7.36) the following expression is obtained

$$\sum_{k=1}^{N} \sum_{\substack{j=1 \\ j\neq k}}^{N} \frac{T_{kj}}{M_k} = - \sum_{k=1}^{2N} \lambda_k^2 \tag{7.38}$$

From (5.25), the transient coherency measure for a disturbance which produces a zero mean, independent identically distributed (IID) step change in shaft accelerations at all generators is with $\underline{P} = \underline{I}$ given by

$$\underline{C}_{k\ell} = 12\,\frac{T^4}{5!} - 30\left[\sum_{\substack{j=1\\j\neq k}}^{N} \frac{T_{kj}}{M_k} + \sum_{\substack{j=1\\j\neq \ell}}^{N} \frac{T_{\ell j}}{M_\ell} + T_{k\ell}\left(\frac{1}{M_k} + \frac{1}{M_\ell}\right)\right]\frac{T^6}{7!} \quad (7.39)$$

This transient coherency measure may now be expressed in terms of the eigenvalues of the system matrix $\underline{A}$ by summing over all ℓ and k and by making the observation that the sum of the diagonal elements of the $\underline{M}^{-1}\underline{T}$ matrix equal the negative of the sum of the off diagonal elements, so that (7.39) becomes

$$\sum_{\substack{k=1\\k\neq \ell}}^{N} \sum_{\ell=1}^{N} C_{k\ell} = 12(N^2-N)\frac{T^4}{5!} - 30(2N+2)\left\{\sum_{k=1}^{N} \sum_{\substack{\ell=1\\ \ell\neq k}}^{N} \frac{T_{k\ell}}{M_k}\,\frac{T^6}{7!}\right\} \quad (7.40)$$

Substituting (7.38) into (7.40) this becomes

$$\sum_{\substack{k=1\\k\neq \ell}}^{N} \sum_{\ell=1}^{N} C_{k\ell} = 12(N^2-N)\frac{T^4}{5!} + 30(2N + 2)\frac{T^6}{7!} \sum_{k=1}^{2N} \lambda_k^2 \quad (7.41)$$

The result shows the transient coherency measure summed over all pairs of internal generator buses is therefore proportional to the sum of the square of the eigenvalues of the power system with zero damping. Since these eigenvalues are imaginary and the imaginary part of these eigenvalues measure system stiffness much like the eigenvalues $\lambda_1, \lambda_2 = \pm j\sqrt{\frac{K}{M}}$ for a harmonic oscillator

$$M\ddot{X} + KX = 0 \quad (7.42)$$

the sum of square of the eigenvalues and thus the coherency measure is a measure of the overall stiffness of the system.

Since the loss of synchronism and loss of stability at any point in the system due to any possible contingency is dependent on the stiffness of the overall system, the probability of a loss of stability in a system for any location and any shaft accelerating contingency is thus measured by summing the coherency measure over all possible pair of generator buses in the system.

CHAPTER 8

THE SECURITY CONTROL PROBLEM

OBJECTIVES:

The principal objective of this chpater is to provide an integration of the on-line tracking secure dispatch formulation within the scope of the overall power system security problem.

METHODOLOGY:

The methodology of this chapter shall consist of meeting the above objectives by

(1) identification of the power system security control problem in terms of the generic operating states of a power system

(2) representation of the overall control problem in terms of sub-control problem formulations associated with each of the operating states

(3) formulating an on-line tracking algorithmic structure for solving the optimal secure dispatch problem

(4) identification of the time-frame for controls involved in the integrated power system security control problem

(5) reformulating the sub-control problems in (2) using the augmented transient coherency measure and discussing the choice of weighting coefficients.

8.1 IDENTIFICATION OF THE SECURITY PROBLEM:

Power System Operating States:

The security identification problem is presented by representing the power system operating conditions in terms of power system operating states. The literature [12, 13] makes reference explicitly to three operating states - normal, emergency and restorative, and includes a normal insecure or alert state implicitly as an insecure subset of the normal state. However, a better understanding of the security problem has led to the representation of the power system operating conditions in terms of five operating states [34] which are:

- normal operating state
- alert operating state
- emergency operating state
- extremesis operating state
- restorative operating state

Operating State Description Tools:

The power system operation is governed by three sets of generic equations [12] which are:

(i) differential equations

(ii) algebraic equalities

(iii) algebraic inequalities

The set of differential equations (i) are those which describe the dynamics of the system components in terms of well known physical laws.

The algebraic equalities (ii) describe the energy balance and thus refer to the system's total load and generation. The algebraic inequalities (iii) state the upper and lower limits on the system's components and thus refer to limitations on system variables such as voltages and currents.

However, only the algebraic equations are useful in describing the five operating states and therefore these are translated respectively into

- load constraints $g(x,u,p) = 0$
- operating constraints $h(x,u,p) \leq 0$
- security constraints $s(x,u,p) \leq 0$

These constraints have been discussed in Chapter 6 and are stated here as they form a very important part of the operating state identification process.

Having shifted the scenario to the algebraic equations dealing with load, operating and security constraints, it is now possible to describe the five states in terms of those three constraints with the help of mathematical programming nomenclature.

Normal State: The power system is said to be in this state when the load and operating constraints are satisfied, i.e.

$$g(x,u,p) = 0 \qquad (8.1a)$$

$$h(x,u,p) \leq 0 \qquad (8.1b)$$

Consequently the intersection of (8.1a) and (8.1b) defines the set of all feasible normal operating states in which the power system may be operated. When the system moves from one normal state to another in response to the continuously changing load profile, it is referred to as being in a quasi-steady state condition.

Alert State: The power system is said to be in this state when the load and operating constraints are completely satisfied but the security constraints are violated, i.e.

$$g(x,u,p) = 0 \quad (8.2a)$$

$$h(x,u,p) \leq 0 \quad (8.2b)$$

$$s(x,u,p) \nleq 0 \quad (8.2c)$$

Emergency State: The power system is said to be in this state when some of the operating constraints are in actual violation, i.e.

$$g(x,u,p) = 0 \quad (8.3a)$$

$$h(x,u,p) \nleq 0 \quad (8.3b)$$

$$s(x,u,p) \nleq 0 \quad (8.3c)$$

This type of violation takes place generally as a result of two types of contingencies:

(a) steady state (b) transient

Violation of soft operating constraints such as thermal overloading, which have already been discussed in detail in Chapter 6, lead to steady-state emergencies. Similarly, violation of hard operating constraints, generally lead to transient emergencies.

Extremesis State: The power system is said to be in this state when both load and operating constraints have been violated, i.e.

$$g(x,u,p) \neq 0 \qquad (8.4a)$$

$$h(x,u,p) \nleq 0 \qquad (8.4b)$$

$$s(x,u,p) \nleq 0 \qquad (8.4c)$$

This generally happens if emergency control measures in the emergency state, subject to time constraints, are unable to respond fast enough and the system is in the midst of complete collapse.

Restorative State: The power system is said to be in this state once action has been taken to halt the disintegration in the extremesis state. In this state, once again, the load and operating constraints are described by i.e.

$$g(x,u,p) \neq 0 \qquad (8.5a)$$

$$h(x,u,p) \leq 0 \qquad (8.5b)$$

$$s(x,u,p) \leq 0 \qquad (8.5c)$$

Having represented the power system conditions in terms of operating states, it is now possible to state the power system security control problem.

Statement of the Security Control Problem:

Given an insecure system, to find the best secure operating point satisfying the load, operating and security (logical) constraints for the constrained optimization problem formulated in general form as

$$
\begin{aligned}
& \min f(x,u) && \text{objective function} \\
\text{s.t.} \quad & g(x,u,p) = 0 && \text{load constraints} \\
& h(x,u,p) \leq 0 && \text{operating constraints} \qquad (8.6) \\
& s(x,u,p) \leq 0 && \text{security constraints}
\end{aligned}
$$

where

x is a vector of dependent state variables

u is a vector of independent control variables

p is a vector of disturbances or perturbations

Some examples of these variables

$x = \Delta\theta,\ \Delta V$ (angles, voltages)

$u = \Delta PG,\ \Delta QG,\ \Delta t,\ \Delta\phi$ (gen. shift, tap changers, phase shifters)

$p = \Delta PL,\ \Delta QL$ (load shift)

The overall objective of stating and identifying the security problem in this way is to enhance the security of the power system by utilizing the available generation and transmission capacity, spinning reserves and tie-line interchange capacility in an optimal fashion.

Figure 3 provides a pictorial view of various concepts and the operating states thus far described [34].

8.2 THE SUB-CONTROL PROBLEMS:

With the overall objective and the five operating states in perspective, it is now possible to decompose the

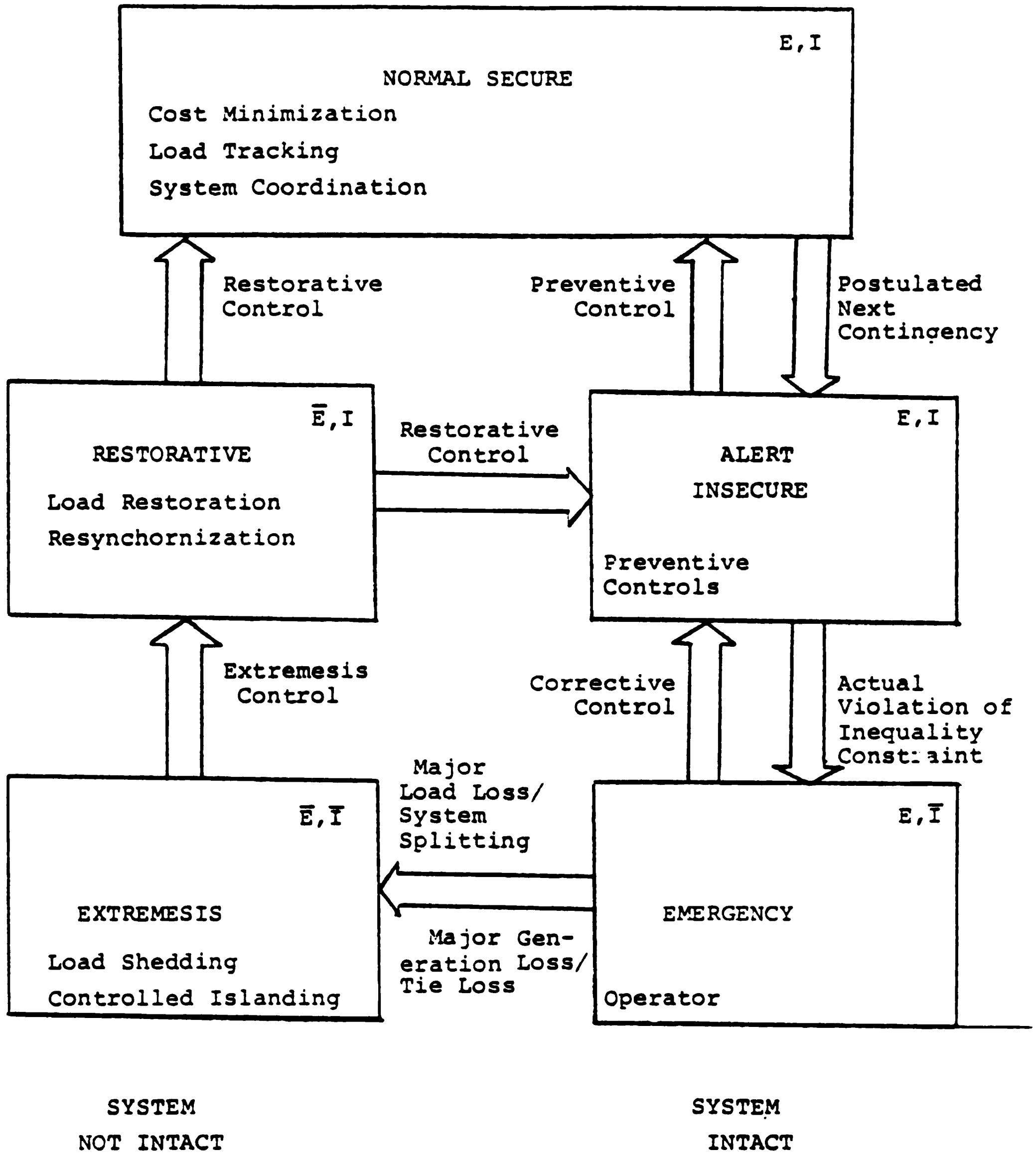


Figure 3

STATE TRANSITION DIAGRAM

security problem into sub-control problems which deal independently with distinct control objectives depending upon the level of security and power system is operating at. These sub-problems are illustrated in Figure 4 and the discussion of each problem formulation follows.

Decomposition of the Security Control Problem:

Subproblem I: Economic Dispatch

This subproblem deals with the security level in the normal state. Since the system is operating free of contingencies, the chief objective is to dispatch the generation based solely on economic considerations. Mathematically, this can be formulated as

$$\min_{\Delta u} f(x,u) = \sum_{i=1}^{N} K_i \Delta u_i \qquad \text{s.t.} \quad g(x,u,p) = 0 \tag{8.7}$$

where

Δu_i = a vector of changes in controllable generations

Subproblem II: Economic Dispatch with Preventive Control

This subproblem deals with the security level in the alert state. In this stage the security assessment functions have indicated a greater probability of an actual violation of the operating constraints via a series of postulated next contingency tests. The chief objective of the dispatch is to execute the appropriate preventive control in anticipation of a potentially dangerous situation. But since the contingency has not actually taken place,

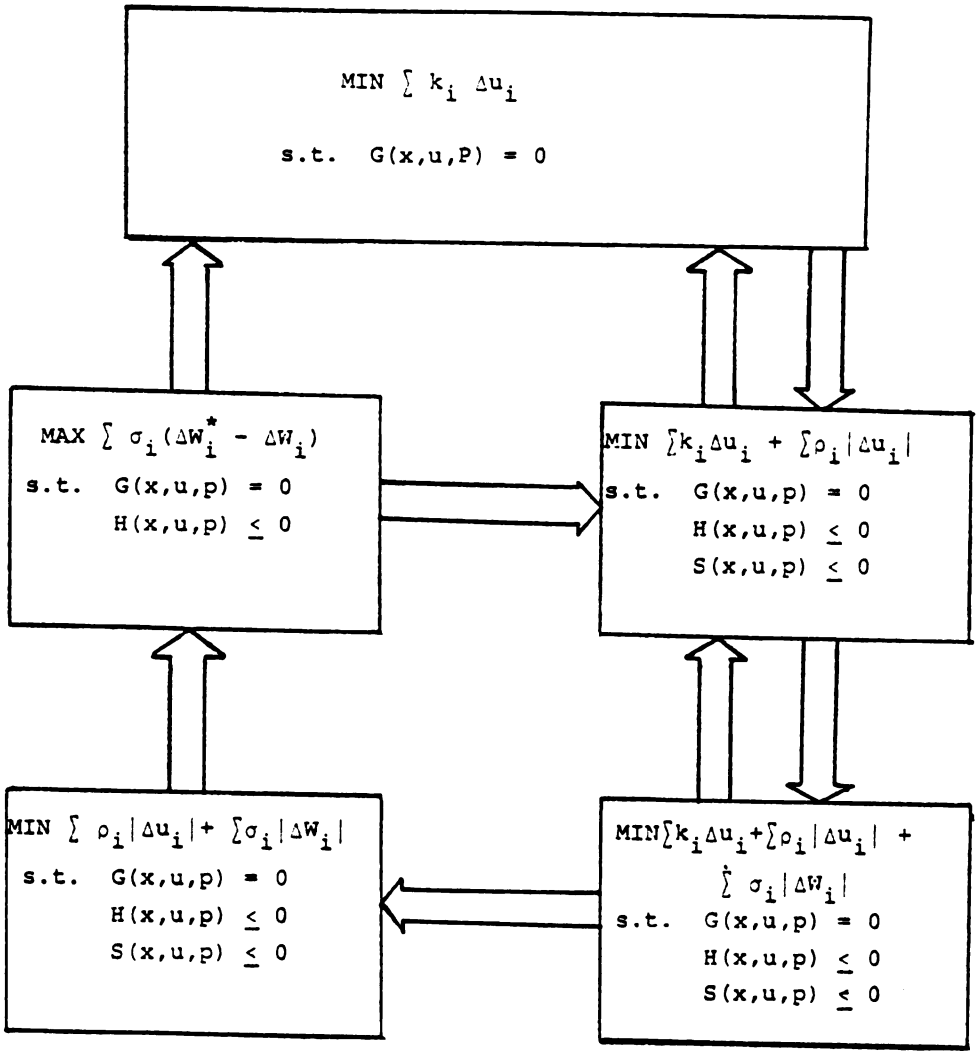


Figure 4

SUB-CONTROL PROBLEMS

there is time for preventive action by implementing preventive strategies which minimize the deviation from the most economic generation point. Mathematically, this entails superimposing on the economic dispatch formulation, the set of violated security constraints and augmenting the performance index with a penalty function for minimum deviation from economic generation as follows:

$$\min_{u} f(x,u) = \sum_{i=1}^{N} K_i \Delta u_i + \sum_{i=1}^{N} \rho_i |\Delta u_i|$$

$$\text{s.t.} \quad g(x,u,p) = 0$$

$$h(x,u,p) \leq 0 \qquad (8.8)$$

$$s(x,u,p) \leq 0$$

where

ρ_i positive cost coefficients of the penalty function.

Subproblem III: Economic Dispatch with Preventive-Corrective Control

This subproblem deals with the security level in the emergency state. At this point an actual violation of the operating constraints has taken place and the appropriate control strategy is to take corrective action as fast as possible. Since time is the essential ingredient in the emergency state, the dispatch is based essentially on removal of the insecure condition and therefore economics takes a low priority. Load shedding is thus a possible control action in addition to generation dispatch due to the need for rapid control action and a lower priority on economics. Mathematically this may be stated as

$$\min_{\Delta u, \Delta w} f(x,u) = \sum_{i=1}^{N} K_i \Delta u_i + \sum_{i=1}^{N} \rho_i |\Delta u_i| + \sum_{i=1}^{K} \sigma_i |\Delta w_i|$$

$$\text{s.t.} \quad g(x,u,p) = 0$$

$$h(x,u,p) \leq 0 \qquad (8.9)$$

$$s(x,u,p) \leq 0$$

where

$|\Delta w_i|$ = a vector of changes in interruptible loads

σ_i = a positive cost coefficient

This formulation provides the system with the capability of executing least costly preventive and corrective control actions in a manner similar to that discussed for the alert state formulation.

Subproblem IV:

This problem deals with the security level in the extremis state. In this state the emergency control is directed at saving as many pieces of the system as possible from total collapse. This can be stated mathematically as

$$\min_{\Delta u, \Delta w} f(x,u) = \sum_{i=1}^{N} \rho_i |\Delta u_i| + \sum_{i=1}^{K} \sigma_i |\Delta w_i|$$

$$\text{s.t.} \quad g(x,u,p) = 0$$

$$h(x,u,p) \leq 0 \qquad (8.10)$$

$$s(x,u,p) \leq 0$$

with load shedding taking a higher priority. This part of the control is mostly operator oriented.

Subproblem V:

This subproblem deals with the security level in the restorative state. In this state extremis has been avoided through saving some parts of the system and the objective is to resume normal operation by slowly rescheduling the whole system. Since the objective is to maximize the supplied demand, this can be stated mathematically as

$$\min_{\Delta w} \sum_{i=1}^{K} \sigma_i (\Delta w_i^* - \Delta w_i) \quad \text{or} \quad \max \sum_{i=1}^{K} \sigma_i \Delta w_i$$

$$\text{s.t.} \quad g(x,u,p) = 0 \qquad (8.11)$$

$$h(x,u,p) \leq 0$$

where

Δw_i^* is the connected demand.

An on-line secure dispatch tracking algorithmic structure to solve these sub-control problems is presented in the next section.

8.3 ON-LINE TRACKING ALGORITHM:

In order to solve the optimal on-line secure dispatch, an algorithmic structure is proposed as in Figure 5. The algorithmic steps are:

STEP 1: Assume base case load flow (x°,u°) from static state esimator is available

STEP 2: Obtain Jacobian matrix J from static state estimator

STEP 3: Obtain J^{-1} matrix by factorization techniques exploiting sparsity of Y_{BUS} matrix

STEP 4: Solve fast decoupled load flow $\Delta x = J^{-1}\Delta u$

STEP 5: Apply Δx to optimization process which provides the control correction Δu.

STEP 6: Using the fast decoupled d.c. load flow from step 5 to determine constraints, solve the on-line secure dispatch problem. If cost $> \varepsilon$ reiterate thru step 4

STEP 7: Reschedule power system

STEP 8: If any more insecurities detected by security assessment, go to step 2. Otherwise STOP.

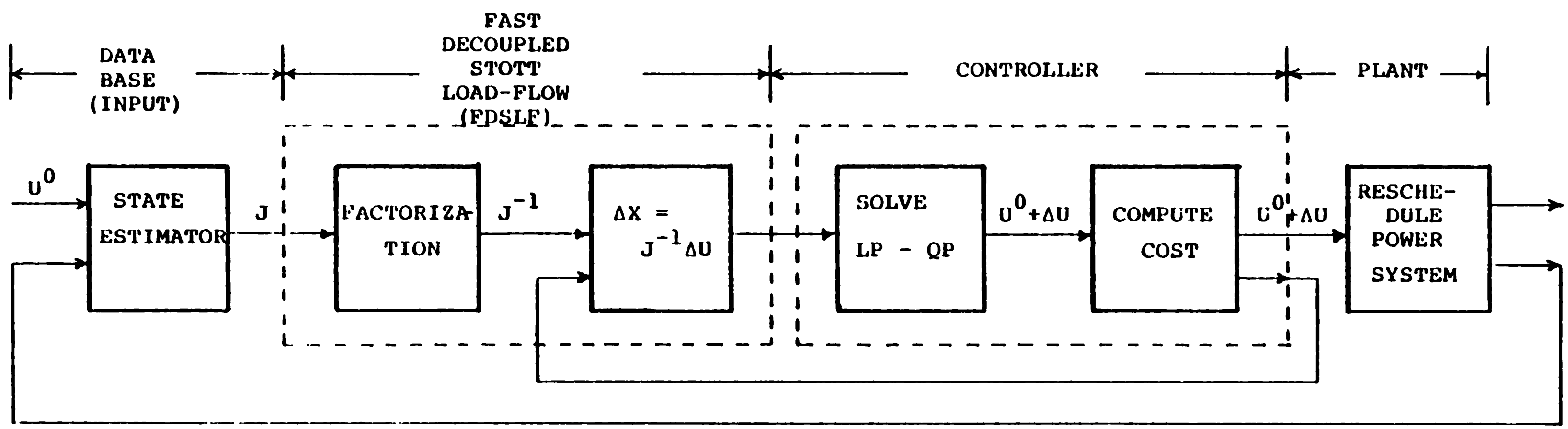


Figure 5

ON-LINE TRACKING ALGORITHMIC STRUCTURE

8.4 TIME-FRAME FOR CONTROLS

Significance of Transient Security Monitoring:

Present operating practices are aimed at static security monitoring of power systems wherein the bulk of the methods in prevalent use today are the off-line security assessment and associated, heretofore discussed, off-line operator techniques, which are used to compensate for system insecurities resulting from

- energy supply deficiencies
- energy demand overloads
- steady state insecurities such as thermal overloading

Static security monitoring techniques are well developed but dynamic security monitoring techniques are into their early stages of consideration. This research is aimed at addressing the transient security component of the latter. Quoting from a 1978 IFAC conference proceedings reference [36] on the prospects of power system security enhancement:

> "Dynamic security monitoring will form part of the overall centralized security scheme"

Stability Factors Affecting Transient Security:

Dynamic security is concerned with two types of stability considerations

- steady state stability
- transient stability

Since a system is steady state stable only if it is transient stable [32], the transient security question, within the context of dynamic security monitoring, is the more important aspect of the two problems.

Transient Security and Time-Frame for Controls:

Transient security is concerned with transient stability which describes the stability of the system following any change in system operating conditions, small or large, but in particular following short-circuit faults which can give rise to

- large electrodynamic oscillations between generator rotors
- loss of synchronism of generators leading to pulling apart and eventual damage to the machine

In addition to these short-circuit faults, the other major transient disturbances are

- line switching
- load shedding
- generator dropping
- loss of tie-lines

The transients caused by each of these contingencies must be survived without loss of synchronism on the loss of any major equipment for the system to be declared transient stable.

The time-frame involved in transient security monitoring is of paramount importance because the oscillations in rotor angles and the associated changes in frequency are of relatively short duration, typically a few seconds. No control of the transient disturbances is proposed using the on-line secure dispatch problem since the events are too fast to be directly controlled by any global system control such as the on-line secure dispatch and must, therefore, be compensated or controlled for by using local controls, if at all. Much research is aimed at developing discrete controls for such emergency transient conditions such as [37]

- Dynamic Braking
- Fast Valving
- Capacitor Switching
- Line Switching

Although an on-line secure dispatch could not directly control for such transients in the initial few seconds, it could through the use of such transient security controls as [13]

- rescheduling generation
- voltage and area-interchange controls
- changing phase-shifter tap settings
- modifying the network configuration
- readjusting relay settings
- changing the logic for emergency-state control devices
- rescheduling load

help prevent the cascading effects of the initial disturbance by effectively stiffening the network in the neighborhood by

(a) unloading weak lines that connect stiffly connected (coherent) groups of generators

(b) helping to maintain transmission lines in service that may otherwise be tripped out through present relaying practice

(c) generator rescheduling and load shedding to reduce the accelerating power and the size of the transients for a specific contingency

which also have a very strong effect on the steady-state security of the system.

Stiffness Controls:

Of the available set of transient security controls [13], the controls used within the context of this dissertation are

- generation-rescheduling and
- load-shedding

These controls are very effective tools for providing the appropriate stiffness which is needed by the coherency measure in its use as a security measure for transient security enhancement. Based on information as to which groups of generators are coherent as a result of a transient, these controls coupled with the appropriate wieghting factors to be discussed later, selectively tune the network to provide the desired degree of stiffness in each segment of the system needed to enable the system to withstand the cascading effects already mentioned.

In this context, the use of gneeration rescheduling as a control stiffens weak portions of an area by decreasing the power flow on a line which become overloaded in power due to contingencies such as the loss of an important tie-line. Load shedding also stiffens the network by relieving the stress on lines which become overloaded due to similar contingencies.

The result of the use of these controls in this way is to effectively isolate the disturbance by enabling the operator or automatic control system to orient remaining portions of the network so that the cascading effects of that disturbance or disturbances are better withstood.

Control-usage in Steady-State Security Concept:

Whenever a power system is subject to a transient disturbance which results in power imbalances that could trigger system-cascading, the present practice is to rely heavily on power interchange capabilities through area tie-lines to remove these imbalances and then if necessary, to readjust generation and load. This dispatch of generation and load is accomplished through the present STOTT formulation of the on-line steady-state secure dispatch problem. Obviously, this represents a corrective measure of sorts but, nevertheless, lacks the therapeutic capability needed to prevent the severely deteriorating effects which could result if the system and network were to become subject to multiple contingencies, a case in point being the NY outage of 1977.

Accordingly, these controls used in Stott [16] enhance only the steady-state security of the power system by executing different control strategies which are limited to the long term effects of the disturbances in the interconnected network and do little to improve the transient security of the system.

As has been stated earlier, a system is steady-state secure only if it is transient secure, and therefore the use of controls to remove imbalances in the steady-state is insufficient to improve the power system security. Accordingly transient security controls are proposed which enhance the transient as well as the steady-state security of the system under study.

8.5 REFORMULATION OF SUB-CONTROL PROBLEMS

In this section the sub-control problems formulated earlier in section (8.3) are reformulated by

(a) identifying the deficiencies in the formulations (8.7-8.11).

(b) reformulating the sub-control problems by adding a transient security measure and discussing the weightings of the coefficients of the performance indices.

(a) Identification of Formulation Deficiencies:

The formulations (8.7-8.11) provide the system with a capability of executing a least costly preventive or corrective control action by minimizing the deviation from the most economic generation schedule at which the system was operating prior to the postulated next contingency or actual violation. In this context, this formulation provides the system operator with the appropriate information in terms of a quantitative megawatt shift which would be required to relieve the system of the particular insecurity which was detected by the security assessment functions. Through an appropriate choice of weighting factors K_i, σ_i and ρ_i, the different indices take priority depending upon the level of security, reflected by the constraint violations, at which the power system is operating.

When there are soft constraint violations such as thermal overloading, they can be withstood for some time if it became an actual emergency because the system can operate for a short time with this type of emergency. However, when there are hard constraint violations such as the stability constraints for transient security, the severity of the transient disturbance in terms of ultimate system collapse and the speed at which such a transient develops, justifies more than just using constraints to correct the problem; an additional performance index should be used that stiffens the system which is shown to be weak by the constraint violation.

Since the transient disturbance is the more important one as it can have devastating effects, the operating states are further subdivided into

. transient alert . steady state alert

The emergency state must also be viewed as two sub-states

. steady state emergency . transient emergency

since the controls provided by the two penalty functions do not assure that the system will not actually make a transition to the extremesis state in case of a hard constraint violation which constitutes a transient emergency. Accordingly, an additional performance index, which enhances the capability of the system to withstand additional disturbances which could lead to cascading and system disintegration, is required. This would provide the operator with some criteria by which he could save the system from a major loss of generation, loss of ties connecting his system to other parts of a coordinated interconnection scheme and would therefore remove the need for performing major load shedding.

Besides the deficiencies in the performance index in the transient alert and transient emergency, the deficiencies in the extremesis point to the inability of the operator of having at his disposal any criteria by which he can optimally lower the generation and load level of operation and then proceed to perform controlled islanding in an effort to save as many parts of the system as possible. In addition, in the restorative state there exists a lack of sufficient criteria to resynchronize parts of the system which have been saved in order to reach normal operation and also to protect restored parts from the effects of any contingencies in the load restoration process.

(b) Reformulation of Sub-control Problems:

The operating states which could be improved by the addition of the transient coherency measure are

. transient alert

. transient emergency

. extremesis

. restorative

The associated scenario is portrayed using the state transition diagram in Figure 6 which shows the improved sub-problems.

Each state is now discussed in terms of the contribution of the coherency measure and the weightings of the performance index needed for improved system security.

Weightings

Transient Alert: Emphasis in this state is still on economics. However, the transient nature of this state indicates a high probability of an actual transient emergency condition. Accordingly, $\underline{K_i}$ should have a high weighting to emphasize economic dispatch, $\underline{\rho_i}$ should also have a high weighting to obtain a least costly generation shift correction and $\underline{\alpha_{k\ell}}$ should have a moderate weighting to stiffen the system so that if there is an actual transient, the system would be able to withstand it better.

Of specific importance is the choice of the weighting coefficients $\alpha_{k\ell}$ in the transient alert state. $\alpha_{k\ell}$

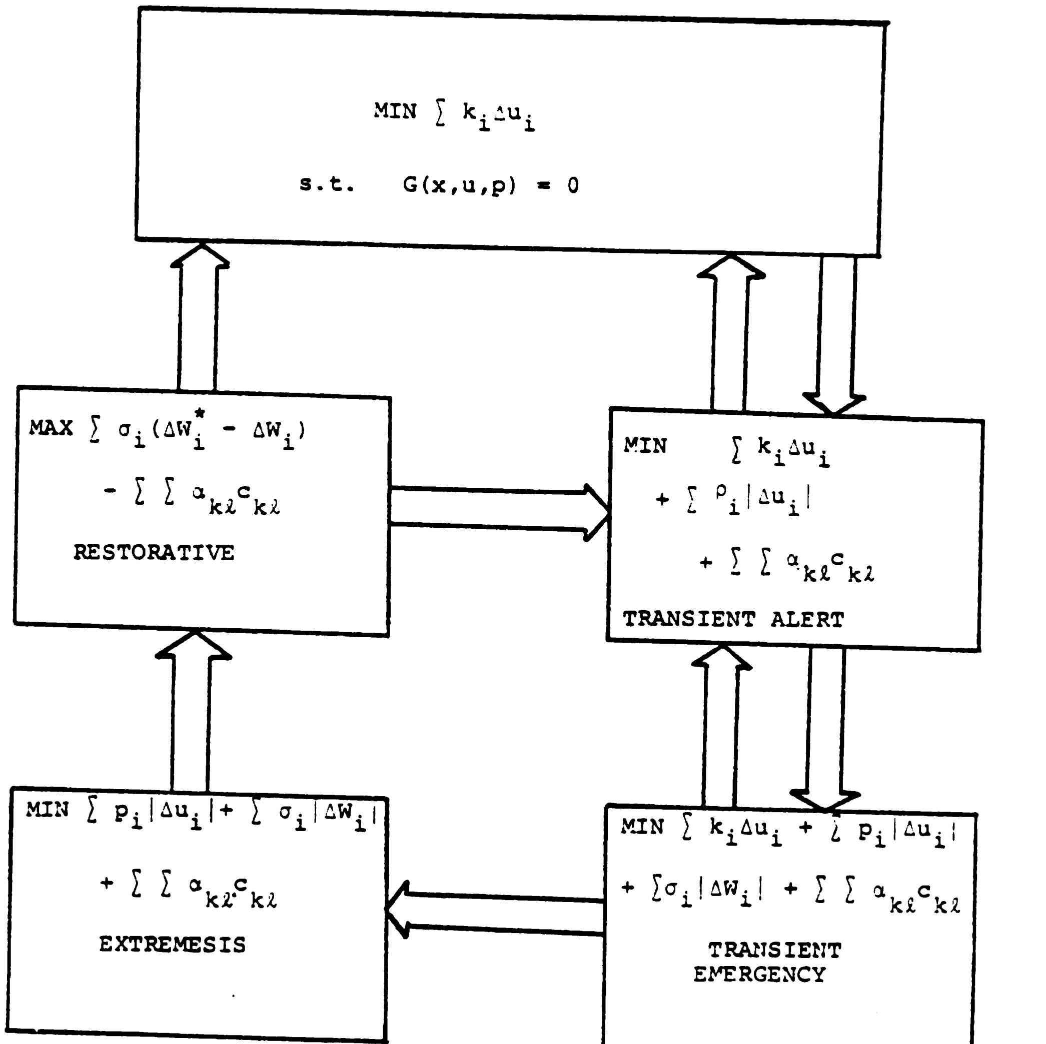

MIN ∑ k_i Δu_i
s.t. G(x,u,p) = 0
MAX ∑ σ_i (ΔW*_i - ΔW_i)
- ∑ ∑ α_kℓ c_kℓ
RESTORATIVE
MIN ∑ k_i Δu_i
+ ∑ ρ_i |Δu_i|
+ ∑ ∑ α_kℓ c_kℓ
TRANSIENT ALERT
MIN ∑ p_i |Δu_i| + ∑ σ_i |ΔW_i|
+ ∑ ∑ α_kℓ c_kℓ
EXTREMESIS
MIN ∑ k_i Δu_i + ∑ p_i |Δu_i|
+ ∑σ_i |ΔW_i| + ∑ ∑ α_kℓ c_kℓ
TRANSIENT
EMERGENCY

Figure 6

is chosen to stiffen the particular equivalent lines only in the neighborhood where the constraint violation occurred. The weightings on the lines in regions where no constraint violations occurred are set to zero.

The use of the transient security index in the alert state is not used to correct a particular violation. The corrective measures required to take care of specific violations consist of a combination of properly chosen stability constraints and generation rescheduling to relieve the system of that postulated next contingency.

The transient coherency measure is, therefore, seen to be useful in its ability to help stiffen the interconnected network in a sub-section or vicinity of the network in which the apparent weakness was detected. In addition, the usefulness of this measure in the alert state is to indicate the direction in which changes in generator rescheduling would stiffen the weakened subsection.

Transient Emergency: In this state the security level of operation of the system has become reduced to a point at which the likelihood of a transition to the most undesirable extremesis state could become a realistic proposition unless appropriate measures are taken. This diminishing security level is apparently due to

(a) appearance of more operating constraint violations

(b) appearance of more security constraints

(c) the size of contingencies or violations

Since an actual transient has occurred, the approach is to lower the weightings on economics since security of operation is now more important than simply obtaining economic generation schedules. Due to the fact that generation shifting is slow, typically 1% per minute, the changes in generation would not be fast enough to correct the insecure state of operation. Accordingly the weightings on k_i and ρ_i are lowered as the security level diminishes and the weightings on the load shedding coefficients, σ_i, are increased since load can be shed faster.

The transient security index coefficeints, $\alpha_{k\ell}$, continue to be chosen so that weak transmission links are continually stiffened as indicated by both operating and security constraints and, in addition, the weightings are increased with an increasing priority as the security level of operation decreases.

If load shedding is used to stiffen the network in a sub-section where constraint violations occur, generation will be simultaneously cut due to the power balance constraint. Load shedding can result in acceleration of the machines due to the speed of the response and the generation which is cut is generally the one related to the tie-line interchange power flow. It should be noted, however, that the use of load shedding is the least desirable due to the resulting interruption of continued service to the customer and must therefore be used as a last resort. Instead, generation rescheduling should be relied on as much as possible as it will stiffen the network by adjusting the direction of power flow to any load bus to unload weak transmission links. Accordingly, generation rescheduling is preferable to load shedding until the security level diminishes to a point at which generation rescheduling is not fast enough to correct the situation.

Extremesis: In this state the weighting on economic operation, k_i, is certainly zero as the priority is to save the system from total collapse. The weightings on load shedding, σ_i, are raised to a high level as are the weightings on $\alpha_{k\ell}$ for stiffening the network. However, the weightings on generation rescheduled are reduced to a low level.

The criteria used to set the weightings on $\alpha_{k\ell}$ will now depend not only upon the transient operating and transient emergency constraint violations but also upon the need to stiffen the lines between coherent groups. This approach has the following effects:

(1) it stiffens the weakest parts of the system (connections between groups)

(2) it will encourage both load shedding and generator dropping to reduce the accelerating powers and thus the size of the contingencies

(3) it will prepare the system for controlled islanding around coherent groups by reducing the power on the lines connecting coherent groups. Thus the mismatch in any coherent group after the contingency would be lower.

It is important to note that one of the most important contributions of the transient security index lies in this extremesis state, through appropriate choice of $\alpha_{k\ell}$. In this regard, it should be noted that the coherency measure will now permit reduction of the overall generation and load to a level at which controlled islanding would be more feasible. This important feature was not available before.

Restorative:

In this state σ_i is high to maximize supplied demand. A high $\alpha_{k\ell}$ should be used only in those parts of the system which have been restored to normal operation so that they are not subject to further collapse. If two parts of the system have been restored to normal operation, by weighting $\alpha_{k\ell}$ high between those areas, synchronization can be assisted so that they start operating once again as a composite system. In this way, the operator can be assisted in optimal system restoration.

CHAPTER 9

CONCLUSIONS AND FUTURE INVESTIGATION

9.1 This research has resulted in three important contributions in the area of Electric Energy Systems Theory:

(1) A transient coherency measure has been developed in an analytical form which was not available prior to this research. In addition, the transient coherency measure has been shown to provide a better understanding of dynamic system structure as the disturbance propagates. This measure has been investigated for a deterministic as well as probabilistic disturbance and was used to

   (a) disect the transient response into discrete events which consist of the acceleration of successive stages of generators from the disturbance location

   (b) determine the lines and generators in each stage which are affected

   (c) determine the stiffness of the interconnection between the disturbed generator and any generator

   (d) determine the effect of the synchronizing torque coefficients and inertias on the transient response in any location and any interval.

(2) A transient security measure has been developed which aids in improving the (a) security, (b) reliability and (c) stability of the power system. Thus a measure for transient security assessment is now available for both system planning and system operation.

In addition, it was shown that the transient security measure, summed over all buses connected to a faulted generator bus, is proportional to the energy capacity of an equivalent line connected to that bus to withstand a fault and thus acts as a measure of security and reliability. It was also shown that the transient coherency measure is a stability measure by showing that this measure summed over all pairs of internal generator buses is proportional to the sum of the square of the eigenvalues of the system with zero damping. In this regard the stiffness property of the transient coherency measure was further verified by using an analogy to show that these eignevalues measure system stiffness much like the eigenvalues for a harmonic oscillator. The important difference in the two developments in Chapter 7 using the equal area criterion and the eigenvalue analysis is that the former takes into account the local energy imbalances and tries to improve stiffness whereas the latter tries to correct the global stiffness of the network.

(3) The research on off-line dispatch techniques has been integrated with the help of the transient coherency measure into an on-line tracking secure dispatch which can dramatically improve system stability, security and reliability by correcting for weak transmission links, buses which are weakly connected to the rest of the system, and weaknesses in the overall network. This was made possible by augmenting the performance index with the transient coherency measure and using appropriate weightings on this measure in the performance index depending upon the security level of operation of the power system. Thus the use of this measure in transient security enhancement was heuristically shown to

(a) stiffen weak links in the network in the transient alert state when an insecure operating condition was detected by the security assessment functions

(b) prevent cascading in the transient emergency state by continuing to stiffen weak links, lowering economic priorities, increasing load shedding, and lower generation rescheduling depending upon the severity of the contingency.

(c) assist in stiffening the overall network by reducing further the generation and load. This general network stiffening has the effect of preparing for islanding by unloading lines between the coherent groups of generators to be islanded thus reducing the mismatch in generation and load when islanding is actually accomplished.

(d) provide an optimal load restoration feature through resynchronization of areas and prevention of further deterioration of areas which have been restored. Thus the measure provides a means of assessing weaknesses in the dynamic system structure and the coherent groups as observation interval increases.

Having summarized the contributions, the conclusions are dealt with in some detail.

The first major problem facing the utility industry was the need for the development of an analytical expression for the determination of coherent groups which would be based on a broad class of disturbances and which eliminated the need for the computation of swing curves. Since the technique of identifying coherency is crucial to forming dynamic equivalents in the representation of large power systems by their equivalents, the analytical expression obtained satisfies that requirement in a meaningful way. In addition, the understanding of power system dynamics has, heretofore, been done greatly through the use of transient stability simulations which besides their high cost do not provide in any meaningful way insight into the propagation of a disturbance in a power system so far as the improvement of interconnections in as much as stiffness is concerned . The transient coherency measure provides the system planning division of the modern utility with a tool for understanding system dynamic structure through

(a) propagation of disturbance

(b) changes in coherent groups and structure due to changes as a result of the propagated disturbance

(c) differences in dynamic and transient coherency or dynamic structure.

In this research, there were two routes to follow as far as the application of the transient coherency measure is concerned.

(1) dynamic equivalents formation

(2) transient security.

The use of the transient coherency measure for obtaining dynamic equivalents is not investigated in this thesis due to lack of time and is a subject for future research. The use of the transient coherency measure as a transient security measure was investigated and justified in this thesis.

At the beginning of Chapter 6, two significant problems facing the utility industry in the area of transient security enhancement were identified. The first of these was concerned with a deficiency in the performance index component of the optimal dispatch formulation to which an improved transient security index using the transient coherency measure was proposed. This measure, although, it does not solve the performance index problem in its totality, does provide a very significant improvement in the formulation, heretofore, not found in the literature.

The transient security measure was justified to be a measure of the system security, reliability and stability by the use of the equal area criterion and the concept of an equivalent line connected to an infinite bus. It was found that minimizing the coherency measure will increase system stiffness, reduce the pre-fault angle difference further below the static stability limit. In addition, it was found that the transient coherency measure summed over all pairs of internal generator buses is proportional to the sum of square of the eigenvalues of the power system with zero damping and that these eigenvalues measure stiffness much like the eigenvalues for a harmonic oscillator. It was concluded therefore, that, the sum of the square of the eigenvalues and thus the coherency measure is a measure of overall stiffness of the system.

The transient security measure developed for use as a performance index provides the following features

(1) stiffens weak links

(2) allows reduction in generation and load so that the network is stiff for load and generation which is connected

(3) prepares system for controlled islanding

(4) provides an adaptive direction for stiffening

The limitations of this security measure are:

(1) no quantitative idea of security margin is provided

(2) for any particular contingency the question of survival is not answered and therefore an additional energy based measure may be needed.

9.2 Future Investigation

Future research shows a lot of promise for implementing the results of this work. However, to obtain a realistic assessment and evaluation, specifically of the optimal secure dispatch formulation, the cooperation of utilities must be solicited in testing these algorithms on-line.

The second problem which was identified in Chapter 6 pertaining to the constraint problem also has significant prospects for future research. To recall, this problem was associated with the lack of properly defined transient security constraints wherein the current practice was to approach this difficulty by imposing line phase angle differences or power flows on lines based on empirical limits. It appears that if the constraints are addressed from an energy capacity point of view coupled with augmenting the transient coherency measure with an energy capacity index at each generator bus, the formulations could be further improved.

The study of coherency based equivalents based on this transient coherency measure is a subject of future research. The comparison of these equivalents with those derived using the max-min coherency or the dynamic coherency measure is currently being investigated in another thesis which is based on results of this thesis.

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APPENDIX

APPENDIX I

ONE VERSION OF THE STMULATION ALGORITHMS USED

```
      PROGRAM COSIM(INPUT,OUTPUT)
C             ***********************************
C             *  COHERENCY SIMULATION PROGRAM   *
C             ***********************************
      LOGICAL SWITCH
      DIMENSION A(10,10),GK(10,10),B2(5,5),RB2(5,5),R(5,5),TALK(10,10)
      REAL    LK(10,10),ALK(10,10),SUM1(10,10),SUM2(10,10),MM(5,5)
      REAL  LZB2(10,10),ALZB2(10,10),AK(10,10),AL(10,10)
      DIMENSION SX2(5,5)
      DIMENSION C1(5,5),C2(5,5),AGK(10,10),TAGK(10,10),P(5,5),H(5,5)
      DIMENSION GKO(10,10),OLDC2(5,5)
      REAL LKO(10,10)
      INTEGER OLDNT2
C     DIMENSION A(14,14),GK(14,14),B2(7,7),RB2(7,7),R(7,7),TALK(14,14)
C     REAL        LK(14,14),ALK(14,14),SUM1(14,14),SUM2(14,14),MM(7,7)
C     REAL        LZB2(14,14),ALZB2(14,14),AK(14,14),AL(14,14)
C     DIMENSION SX2(7,7)
C     DIMENSION C1(7,7),C2(7,7),AGK(14,14),TAGK(14,14),P(7,7),H(7,7)
C     DIMENSION GKO(14,14),OLDC2(7,7)
C     REAL LKO(14,14)
      SWITCH=.FALSE.
C             ****************************
C             * INITIALIZATION SECTION *
C             ****************************
C     PROGRAM DESIGNED TO HANDLE THE MS AND RMS MEASURES ONE AT A TIME
C     SPECIFY NUMBER OF GENERATORS(STATIONS),N
      READ 1,N,MS,RMS
      M=2*N
      KLOCK=0
C     SPECIFY A MATRIX(2NX2N)
C     T MATRIX USED WITHIN A AVAILABLE AS B FROM SIMULATION
      READ 3,((A(I,J),J=1,M),I=1,M)
C     MAKE DIAGONALS OF T NEGATIVE  TO CONFORM WITH STATE MODEL
      DO 40 K=1,N
      KD=K+N
 40   A(KD,K)=-A(KD,K)
C     SPECIFY INITIAL VALUE OF GK MATRIX(2N X 2N)
C         GO=G=E(XX*)
      READ 3,((GKO(I,J),J=1,M),I=1,M)
C             SPECIFY INITIAL R MATRIX(2NX2N)
      READ 4,((R(I,J),J=1,N),I=1,N)
C             SPECIFY  B2  MATRIX
      READ 5,((H(I,J),J=1,N),I=1,N)
      PI=3.141592
      DO 51 I=1,N
         DO 51 J=1,N
         MM(I,J)=H(I,J)/(PI*60.0)
         B2(I,J)=0.0
         IF(MM(I,J).NE.0.0) B2(I,J)=1.0/MM(I,J)
 51   CONTINUE
C             COMPUTE P=B2RB2*,  B2=B2*
      CALL MULT(R,B2,RB2,N,N,N)
      CALL MULT(B2,RB2,P,N,N,N)
C     FORCE P MATRIX TO BE IDENTITY FOR MSQUARE CONTINGENCY
C     THIS WILL ENSURE THE CONTINGENCY IS BEING PASSED ON CORRECTLY
C     DO 83I=1,N
C83   P(I,I)=1.0
      CALL SCALMA(2.0,P,P,N,N)
C         SET LKO WITH 2P ON SE QUADRANT AND REST ZERO
      DO 6I=1,M
      DO 6J=1,M
 6    LKO(I,J)=0.0
C     DO 7I=8,14
C     DO 7J=8,14
C7    LKO(I,J)=P(I-7,J-7)
      DO 7I=6,10
      DO 7J=6,10
 7    LKO(I,J)=P(I-5,J-5)
              DO 89I=1,M
              DO 89J=1,M
 89   LZB2(I,J)=LKO(I,J)/2

C     SPECIFY COHERENCY EVALUATION INTERVAL,T, IN SECONDS
      READ 2,T,NTERM1,NTERM2,LIMNT2
      READ 396,ALPHA,SIMTIM,DELT
      SIMTIM=SIMTIM+DELT
      DO 1111I=1,M
      DO 1111J=1,M
 1111 LK(I,J)=LKO(I,J)
      DO 2222I=1,M
      DO 2222J=1,M
 2222 GK(I,J)=GKO(I,J)
      PRINT 899,N
      PRINT 395,(I,M(I,I),I=1,N)
```

```
      PRINT 909,(I,MM(I,I),I=1,N)
      PRINT 931
      PRINT 902,((A(I,J),J=1,M),I=1,M)
      PRINT 903
      PRINT 904,((B2(I,J),J=1,N),I=1,N)
      PRINT 906
      PRINT 907,((GK0(I,J),J=1,M),I=1,M)
      PRINT 908
      PRINT 904,((R(I,J),J=1,N),I=1,N)
      PRINT 754
      PRINT 755,((LK0(I,J),J=1,M),I=1,M)
      DO 331 I=1,N
      DO 331 J=1,N
      C1(I,J)=0.0
 331  CONTINUE
C              ************************
C              *  PROCESSING  SECTION  *
C              ************************

C        COMPUTE C1, THE  FIRST PART OF THE COHERENCY EXPRESSION
C        TO ANALYZE THE EFFECT OF INITIAL CONDITIONS

C        THE C1 MATRIX IS THE COHERENCY MATRIX CORRESPONDING TO
C        SOME INITIAL CONDITION ON ANGLES SET BY SYSTEMS PLANNING
 88   CONTINUE
      IF(SWITCH)90,87
 90   CONTINUE

C              SPECIFY LENGTH OF SERIES DESIRED

      K=NTERM1
      DO 9 IN=1,K
              CALL FACT(IN,FAC)
              TK1=(T**IN)/FAC
              DO 10 J=1,M
              DO 10 L=1,M
 10           SUM1(J,L)=SUM1(J,L)+GK(J,L)*TK1
      IF(IN.EQ.K) GO TO 9
              CALL MULT(A,GK,AGK,M,M,M)
              CALL TRANS(AGK,TAGK,M,M)
              CALL SUM(AGK,TAGK,GK,M,M)
 9        CONTINUE
      CALL PROCES(SUM1,C1,M,M,N)
          DO 12 JJ=1,N
          DO 12 II=1,N
      IF(II.NE.JJ) GO TO 113
 112  C1(II,JJ)=0.0
      GO TO 12

C     USE MS OR  RMS MEASURES AS DESIRED

 113  IF(MS.NE.1)  GO TO 1113
      C1(II,JJ)=C1(II,JJ)/T
      GO TO 12
 1113 C1(II,JJ)=SQRT(C1(II,JJ)/T)
 12   CONTINUE
      PRINT 900,T,ALPHA,RMS
      PRINT 18
      PRINT 444,NTERM1
      PRINT 19
      PRINT 20,((C1(I,J),J=1,N),I=1,N)

C        COMPUTE SECOND PART OF COHERENCY EXPRESSION
C        TO ANALYZE THE EFFECT OF THE CONTINGENCY


C     SPECIFY NO OF TERMS DESIRED IN SERIES EXPANSION

   87 CONTINUE
      K=NTERM2
              DO 15 IN=1,K
              NF=IN+2
          CALL FACT(NF,FAC)
          TK2=(T**NF)/FAC
              DO 16J=1,M
              DO 16L=1,M
 16           SUM2(J,L)=SUM2(J,L)+LK(J,L)*TK2
              IF(IN.EQ.K) GO TO 15
      IF(IN.EQ.3) GO TO 96
      IF(IN.EQ.4) GO TO 97
      CALL SUM(LK,LZB2,LK,M,M)
      GO TO 1116
 96   CALL MULT(A,LZB2,ALZB2,M,M,M)
              DO 33I=1,M
              DO 33J=1,M
 33           AK(I,J)=A(I,J)
      CALL SUM(LK,ALZB2,LK,M,M)
```

```
      GO TO 1116
 97   CALL MULT(A,AK,AL,M,M,M)
                DO 43I=1,M
                DO 43J=1,M
 43             AK(I,J)=AL(I,J)
      CALL MULT(AK,LZB2,ALZB2,M,M,M)
      CALL SUM(LK,ALZB2,LK,M,M)
 1116           CALL MULT(A,LK,ALK,M,M,M)
                CALL TRANS(ALK,TALK,M,M)
                CALL SUM(ALK,TALK,LK,M,M)
 15         CONTINUE
      DO 124 J=1,M
      DO 124 L=1,M
 124  SX2(J,L)=SX2(J,L)+(SUM2(J,L)/T)
      CALL PROCES(SUM2,C2,M,M,N)
      DO 17JJ=1,N
      DO 17II=1,N
      IF(II.NE.JJ) GO TO 117
 116  C2(II,JJ)=0.0
      GO TO 17

C     USE MS OR RMS MEASURE AS DESIRED

 117  IF(MS.EQ.1) GO TO 17
 119  C2(II,JJ)=SQRT(C2(II,JJ))
 17   CONTINUE
C     PRINT 125
C     PRINT 126,((SX2(I,J),J=1,M),I=1,M)
C     REMOVE THIS CARD WHEN RUNNING C1 AS WELL
      PRINT 990,T,ALPHA,RMS
      PRINT 21
      PRINT 999,NTERM2
      PRINT 22
      PRINT 23,((C2(I,J),J=1,N),I=1,N)
C                        ************************
C                        *  BEGIN  SIMULATION   *
C                        ************************
      KLOCK=KLOCK+1
      IF(KLOCK-2)61,71,72
 71   OLDNT2=NTERM2
 72   DO 2134 I=1,N
      DO 2134 J=1,N
      IF(ABS(OLDC2(I,J)-C2(I,J))-ALPHA) 2134,37,37
 2134 CONTINUE
C     SAVE C2 AS OLD
 61   DO 1234 I=1,N
      DO 1234 J=1,N
 1234 OLDC2(I,J)=C2(I,J)
      NTERM1=NTERM1+2
      NTERM2=NTERM2+2
      IF(NTERM2.EQ.LIMNT2) GO TO 237
      GO TO 38
C     RESET NTERM2
 37   NTERM2=NTERM2-2
      PRINT 397,NTERM2,T,ALPHA
      GO TO 238
 237  PRINT 397,OLDNT2,T,ALPHA
 238  NTERM1=1
      NTERM2=3
      IF(T.GE.SIMTIM) GO TO 123
      T=T+DELT
      KLOCK=0
 38             DO 8888I=1,N
                DO 8888J=1,N
                C1(I,J)=0.0
 8888           C2(I,J)=0.0
      CONTINUE

C     REINITIALIZE ARRAYS USED IN COHERENCY COMPUTATIONS
                DO 7777I=1,M
                DO 7777J=1,M
                AGK(I,J)=0.0
 7777           TAGK(I,J)=0.0
      DO 6666 I=1,M
      DO 6666J=1,M
                TALK(I,J)=0.0
 6666 ALZB2(I,J)=0.0
                DO 9999I=1,M
                DO 9999J=1,M
                GK(I,J)=0.0
                LK(I,J)=0.0
 9999 ALK(I,J)=0.0
                DO 3333I=1,M
                DO 3333J=1,M
                SX2(I,J)=0.0
                AK(I,J)=0.0
 3333           AL(I,J)=0.0
      DO 2J=1,M
```

```
 8      SUM1(J,L)=0.0
        DO 14J=1,M
        DO 14L=1,M
 14     SUM2(J,L)=0.0
C       RESET GK AND LK TO INITIAL VALUES
        DO 7676 I=1,M
        DO 7676J=1,M
        LK(I,J)=LK0(I,J)
        GK(I,J)=GK0(I,J)
 7676   CONTINUE
        GO TO 88
 123    STOP
C               * FORMAT  SECTION  *
 1       FORMAT(3I5)
 2      FORMAT(F10.4,3I5)
C3      FORMAT(14F5.2)
 3      FORMAT(10F5.2)
C4      FORMAT(7F10.6)
 4      FORMAT(5F10.6)
C5      FORMAT(7F10.4)
 5      FORMAT(5F10.4)
 796    FORMAT(3F10.6)
 797    FORMAT(///,T10,≠C2 MATRIX APPROX GOOD FOR NTERM2=≠,I2,≠ COHERENCY
       CEVAL INTVL=≠,F5.2,≠ ALPHA=≠,F10.6)
 18     FORMAT(//,30X,≠  C1 MATRIX  ≠)
 19     FORMAT(≠0≠,25X,≠EFFECT OF INITIAL CONDITIONS ≠)
C20     FORMAT(≠0≠,7F10.4)
 20     FORMAT(≠0≠,5F10.4)
 21     FORMAT(////,32X,≠ C2 MATRIX ≠)
 22     FORMAT(≠0≠,25X,≠ EFFECT OF CONTINGENCY ≠)
C23     FORMAT(≠0≠,7E12.5)
 23     FORMAT(≠0≠,5E12.5)
C125    FORMAT(1H1,T50,≠ SX2  MATRIX  ≠)
C126    FORMAT(≠0≠,5X,7E12.5/6X,7E12.5)
 444    FORMAT(≠0≠,32X,≠NTERM1=≠,I2)
 999    FORMAT(≠0≠,32X,≠NTERM2=≠,I2)
 754    FORMAT(///,T50,≠ L0 MATRIX ≠)
C755    FORMAT(≠0≠,14F7.4)
 755    FORMAT(≠0≠,10F7.4)
 899    FORMAT(1H1,T25,≠ DATA BASE= MECS POWER SYSTEM ≠,T60,≠ NO OF GENERA
       CTORS=≠,I2)
 900    FORMAT(1H1,T25,≠ COHERENCY EVALUATION INTERVAL =≠,F5.2,≠ SECONDS≠,
       CT75,≠ALPHA=≠,F20.10,T110,≠RMS=≠,I2)
 901    FORMAT(///,T50,≠ A MATRIX ≠)
C902    FORMAT(≠0≠,5X,14F7.2)
 902    FORMAT(≠0≠,5X,10F7.2)
 903    FORMAT(///,T45,≠ B2  MATRIX  ≠)
C904    FORMAT(≠0≠,15X,7F10.4)
 904    FORMAT(≠0≠,15X,5F10.4)
C905    FORMAT(///,5X,7(≠H≠,I1,≠=≠,F10.4,5X))
 905    FORMAT(///,5X,5(≠H≠,I1,≠=≠,F10.4,5X))
 906    FORMAT(1H1,50X,≠ INITIAL CONDITION MATRIX G0 ≠)
C907    FORMAT(≠0≠,35X,14F5.2)
 907    FORMAT(≠0≠,35X,10F5.2)
 908    FORMAT(///,40X,≠  CONTINGENCY MATRIX  R ≠)
C909    FORMAT(///,5X,7(≠M≠,I1,≠=≠,F10.4,5X))
 909    FORMAT(///,5X,5(≠M≠,I1,≠=≠,F10.4,5X))
        END
        SUBROUTINE FACT(N,FAC)
        FAC=1
        IF(N)6,5,6
 6      DO 3 I=1,N
        FI=I
 3      FAC=FAC*FI
        GO TO 7
 5      FAC=1
 7      RETURN
        END
        SUBROUTINE TRANS(A,C,N,L)
        DIMENSION A(N,L),C(L,N)
C               C IS TRANSPOSE OF  A
                DO 10I=1,N
                DO 10J=1,L
 10     C(J,I)=A(I,J)
        RETURN
        END
        SUBROUTINE MULT(A,B,C,N,M,L)
        DIMENSION A(N,M),B(M,L),C(N,L)
C               C=A*B     A=NXM    B=MXL     C=NXL
                DO 10I=1,N
                DO 10J=1,L
                C(I,J)=0.0
                DO 10K=1,M
                C(I,J)=C(I,J)+A(I,K)*B(K,J)
 10             CONTINUE
        RETURN
        END
```

```
      SUBROUTINE SCALMA(SCAL,P,SK1,N,M)
      DIMENSION P(N,M),SK1(N,M)
C             MULTIPLICATION  OF A MATRIX BY A SCALAR
              DO 10J=1,N
              DO 10K=1,M
 10   SK1(J,K)=SCAL*P(J,K)
      RETURN
      END
      SUBROUTINE SUM(A,B,C,N,M)
      DIMENSION A(N,M),B(N,M),C(N,M)
C             C=A+B
              DO 10J=1,N
              DO 10K=1,M
 10           C(J,K)=A(J,K)+B(J,K)
      RETURN
      END
      SUBROUTINE PROCES(GK1,TRFIJ,N,M,NTG)
      DIMENSION GK1(N,M),TRFIJ(NTG,NTG)
      DO 500I=1,NTG
      DO 500J=1,NTG
C             TRACE OF FIJ TRANSPOSE FIJ OPERATION ON SERIES
 500  TRFIJ(I,J)=GK1(I,I)+GK1(J,J)-GK1(I,J)-GK1(J,I)
      RETURN
      END
      CALL GROUP(C2,ALPHA2,*C2*)
      SUBROUTINE GROUP(C1,ALPHA,IC)
      INTEGER SAVE,PURE
      DIMENSION C1(7,7),SAVE(7,7),PURE(7,7)
      DATA SAVE/49*0/,PURE/49*0/
      PRINT 3,IC
 3    FORMAT(*0*,T10,* COHERENT GROUP ANALYSIS FOR MATRIX *,A2)
      PRINT2,ALPHA
 2    FORMAT(*1*,30X,*ALPHA=*,F20.10)
C     ****************************************************************
C     COHERENCY ALGORITHM BEGINS
C     FIRST OBTAIN COHERENT GROUPS FOR EACH BUS
C     ****************************************************************
      DO 5J=1,7
      DO 6I=1,7
C*****IF EACH ELEMENT BEING SEARCHED VERTICALLY IN COLUMN J IS LESS THAN
C*****SOME CRITERION ALPHA THEN IT IS PART OF THE COHERENT GROUP FOR J
      IF(C1(I,J).LE.ALPHA) SAVE(I,J)=I
 6    CONTINUE
      PRINT 7,J
 7    FORMAT(*0*,10X,* COHERENT GROUP FOR BUS *,I2,* IS *)
      PRINT 8,(SAVE(I,J),I=1,7)
 8    FORMAT(*0*,20X,7(I2,2X))
 5    CONTINUE
      PRINT 98
 98   FORMAT(*0*,30X,*SAVE MATRIX*)
      PRINT 99,((SAVE(I,J),J=1,7),I=1,7)
 99   FORMAT(*C*,7I10)
C     ****************************************************************
C     START IDENTIFICATION OF PURE COHERENT GROUPS
C     ****************************************************************
      DO 40J=1,7

C*****DO NOT SEARCH COLUMNS ALREADY ASSIGNED TO MATRIX CALLED PURE
      DO 35 IR=1,7
      DO 35 IQ=1,7
      IF(J.EQ.PURE(IQ,IR)) GO TO 40
 35   CONTINUE
      DO 30I=1,7
C*****SET COMPARISON INDEX
      L=SAVE(I,J)
      IF(L.EQ.J) GO TO 30
      IF(L.EQ.0) GO TO 30
C*****COMPARE COLUMN J TO COLUMN L. IF UNEQUAL,GO TO NEXT COLUMN
      DO 20 IS=1,7
      IF(SAVE(IS,J).NE.SAVE(IS,L)) GO TO 40
 20   CONTINUE
C*****IF COLUMN J AND COLUMN WITH COMPARISON INDEX L ARE EQUAL THEN WE
C*****CONCLUDE THAT L IS PART OF THE PURE GROUP BEING COMPUTED
      PURE(L,J)=L
 30   CONTINUE
      PRINT 38
 38   FORMAT(*0*,10X,* PURE COHERENT GROUP IS *)
      PRINT 39,J,(PURE(I,J),I=2,7)
 39   FORMAT(*0*,20X,7(I2,3X))

 40   CONTINUE
      RETURN
      END
```