ABSTRACT

AN EMPIRICAL EXPLORATION OF THREE POSSIBLE SOURCES OF FACTORIAL VARIATION

by Lawrence Massud Aleamoni

The major purpose of this study was to determine how factors (resulting from certain error introductions, certain sampling procedures, and certain permutations of unrotated factors) computed for a fixed set of variables and individuals would vary in terms of number of factors, size of factor loadings, factorial configuration, positions of salient variables, and maximum sum of fourth powers of the factor loadings. In addition, a comparison of two separate methods of factor comparison was included.

The data used in the study consisted of: (1) Thurstone's Primary Mental Abilities correlation matrix; (2) Holzinger and Swineford's correlation matrix; and (3) A Michigan State University (MSU) freshman sample.

Problem I, the influence of random fluctuations of the correlation coefficients on the principal axes and final rotated factor matrices, was investigated by selecting all, 5%, and then 1%, of the total number of correlations and varying these randomly by either increasing or decreasing them within their respective standard errors.

Problem II, the influence of sampling on the principal axes and final rotated factor matrices, was investigated by randomly selecting samples of 1600, 400, 100, 25, and 17 from a fixed population of 2,322 individuals.

Problem III, the influence on the final rotated factor solution of permutations on the order of the unrotated principal axes factors, was investigated by: (1) Administering seven different permutations to Thurstone's data and then rotating 15 factors by the quartimax and varimax methods; and (2) Administering four of the seven permutations to Thurstone's and Holzinger and Swineford's data and then rotating six factors by the quartimax and varimax methods.

The two separate methods of factor comparison used in the study were: (1) The Coefficient of Congruence; and (2) The Root Mean Square.

The results of the random error part of the study indicated that when random error was introduced first into all and then a small number of the correlation coefficients, the number of factors, size of the factor loadings, factorial configuration, and positions of salient variables were all changed in varying degrees. As the amount of error increased so did the degree of dissimilarity between matched pairs of factors. The quartimax rotational solution appeared to be more stable than the varimax.

The results of the sampling part of the study indicated that when samples of varying sizes were selected from 2,322 MSU freshmen, the number of factors, size of factor loadings, factorial configuration, and positions of salient variables were increasingly changed as the sample size decreased. An indication as to how large a sample would be needed to depict a finite population's factor structure was obtained. Here, however, the varimax rotational solution appeared to be more stable than the quartimax.

When the permutations were applied to the unrotated principal axes factors of Thurstone's data, the size of the factor loadings, factorial configurations, positions of salient variables, and the maximum sum of fourth powers of the factor loadings were all changed in varying degrees and did not conform to any assumptions of invariance.

When the same set of permutations was applied to Holzinger and Swineford's principal axes factors the results supported the evidence obtained from Thurstone's data when 15 factors were employed.

The comparison of the two separate methods of factor comparison indicated that both were doing an almost equivalent job in identifying highly similar and highly dissimilar factors within a fixed sample size.

AN EMPIRICAL EXPLORATION OF THREE POSSIBLE SOURCES OF FACTORIAL VARIATION

By

Lawrence Massud Aleamoni

A THESIS

Submitted to
Michigan State University
in partial fulfillment of the requirements
for the degree of

DOCTOR OF PHILOSOPHY

Department of Education

1966

This THESIS for the DOCTOR OF PHILOSOPHY degree

by

Lawrence Massud Aleamoni

has been approved

Charles F. Wrigley	Thesis Chairman, Guidance Committee
Robert L. Ebel	Chairman, Guidance Committee
John F. Vinsonhaler	Reader, Guidance Committee
Willard G. Warrington	Reader, Guidance Committee
Bernard R. Corman	Reader, Guidance Committee

ACKNOWLEDGEMENTS

The author is indebted to the members of his guidance committee, Professor Charles F. Wrigley (Thesis Chairman), Professor Robert L. Ebel (Chairman), Professor John F. Vinsonhaler, Professor Willard G. Warrington and Professor Bernard R. Corman for their advice and counsel during the course of this study. The author is especially indebted to his major professor, Charles F. Wrigley. His keen personal interest, stimulating discussions, perceptive criticisms, generous grants of time, and confidence are gratefully acknowledged. Professor John F. Vinsonhaler's helpful guidance in the initial stages of the study was deeply appreciated. To Mr. Alan M. Lesgold of the Computer Institute for Social Science Research the author's deepest thanks go for the many hours spent in consultation and program preparation.

Appreciation and thanks are extended to the members of the Office of Evaluation Services and in particular to Professors Willard G. Warrington and Arvo E. Juola for providing office space and data for use in the study.

The author's sincere appreciation and gratitude is extended to Miss Violet J. Braun for the typing of the many tables.

The author's sincerest appreciation is extended to his wife Margi, whose patience, faith, and encouragement throughout the study was a constant source of inspiration. To her parents, for their confidence and encouragement, many thanks are also offered.

To his mother, for her encouragement, determination, self-sacrifice, and confidence throughout his life, goes the author's humblest appreciation.

Finally, to many of the former faculty and staff members of Westminster College in Salt Lake City, Utah and in particular to Professor Joseph Salvatore, former chairman of the department of psychology, for his foresight, encouragement, and perceptive guidance, this thesis is dedicated.

TABLE OF CONTENTS

LIST OF TABLES

Table		Page

Table		Page

CHAPTER I

PROBLEM

The major aim of factor analysis is to identify the underlying dimensions in a domain of variables. The reason factorists attempt to identify these underlying dimensions is to discover a unique set of dimensions (of mental abilities, for example) which are reproducible using different samples from the same population and, hopefully, using samples from different populations. If this identification is achieved and a unique simple structure solution obtained, there is still no sufficient proof that the factors obtained actually represent the primary underlying dimensions. The design and selection of variables, even if carefully made, could produce a few factors which are artificial products of the domain of variables selected. According to Henrysson (1957), a single factor analysis yields relatively unverified hypotheses about factors which must be proved invariant through more factor studies employing other variables and other populations, before it can be said that the factors found represent the primary underlying dimensions of the domain of variables.

Although Henrysson's approach to the study of factorial invariance is worthwhile, it is incomplete; it should be preceded by more factor studies employing the same variables on the same population, before employing other variables and other populations.

According to Holzinger and Harman (1948), the form of any factor solution, ultimately selected, depends upon the following features: (a) the group of individuals measured; (b) the set of variables and all their correlations; (c) statistical standards, such as those found in a particular method of factor calculation; and (d) outside criteria from the particular field of investigation. They considered the attempts made in the field of psychology to formulate invariant solutions, as well as the theory underlying such invariant factors, involved the arbitrary specification of the four aspects mentioned above. Thus, by fixing the population and the set of variables, and agreeing upon the form of solution, a fundamental set of factors may be obtained. Then, they felt, all other variables in the given field may be expressed in terms of these factors.

Thurstone (1947, p. 363) first distinguished two types of factorial invariance: (1) The simplest type was called metric (or numerical). This would occur when the same method of factoring was used on different samples from the same population. (2) A different type, called configural, appears in relation to the selection of subjects to whom a test battery is given. The factor loadings might change markedly from one population to another; but if the same test battery were used on both populations, the configuration should be invariant. (Configurational invariance means that the zero factor loadings would be in the same position for similar factors from two different populations.) He felt that this was far more important than the numerical invariance of the factor loadings.

In keeping with this emphasis, Thurstone pointed out that the numerical values should be regarded as being of three kinds, namely, those that are significantly positive, those that vanish or nearly vanish, and those that are significantly negative. Another way of stating his same idea is to point out that it is the <u>factorial configuration</u> that is of importance rather than the factor matrix with its numerical entries.

Thurstone did not expect factor loadings to be invariant from one population to a different one. He considered any criterion of numerical invariance in factor analysis assumed that it was applied to analyses on the same or equivalent populations. In other words, if we limited ourselves to analyses that were made with the same battery of tests for several samples from the same population, then he expected the factor loadings to remain numerically invariant for the different samples within sampling errors.

Henrysson (1957), in his discussion of factorial invariance, began with Thurstone's four cases of invariance, and then, by dividing each case of a changed battery into cases of partial and entire change, distinguished six cases of factorial invariance, each of which gave rise to different problems:

1.--When the same test battery is used for the same population, there should be numerical invariance in all samples of the population, which means that the factor structure (matrix of correlations between tests and factors) found in the different samples should be identical within the limits of sampling errors.

2.--If the same test battery is used for different populations, then configurational invariance is hypothesized.

3.--If the test batteries, used on samples from the same population, include tests of which only some are the same, the hypothesis of numerical invariance requires that each test common to two or more batteries retains its factorial configuration.

4.--If partly different test batteries are used on different populations, then configurational invariance is hypothesized.

5.--If entirely different test batteries are used on the same population, then configurational invariance is hypothesized.

6.--If entirely different test batteries are used on different populations, then configurational invariance is hypothesized.

Although several approaches to investigating factorial invariance have been mentioned, they all seem to be dependent upon the general assumption that given a particular battery of variables, a particular population of individuals, and a particular method of factoring, the factors obtained should be at least configurationally invariant if they were calculated on samples of this particular population of individuals.

Statement of the Problem

The present study addresses itself to three major problems found in the case when the same test battery is used for the same population, the first two of which are actually related to sampling and random error effects on the factor loadings.

Problem I. The first problem will be concerned with the effect that the introduction of minor changes in the correlation coefficients (within the standard error of each correlation coefficient) have on the resulting unrotated and rotated factor solutions. Since the statistics we employ for evaluating observed data assume that the measurements are fallible and that a certain bound can be determined for this fallibility (within which the measurement is still considered to be accurate), this problem appeared worthy of investigation. This problem was divided into three sub-problems which would make use of the bounds of each correlation coefficient to introduce in a standard error. The first sub-problem was to determine what effects the introduction of these minor changes would have if they were administered to all of the correlations. The second was to determine what effects these changes would have if they were administered to only 5% of the total number of correlations. And the third was to determine what effects these changes would have if they were administered to only 1% of the total number of correlations.

The reason this problem appeared worthy of investigation was that factor analysis is assumed to be able to identify the underlying true configuration of relationships (of variables, in this case) and, therefore, small fluctuations in the correlation coefficients, within their standard errors, should have no real effect on this identification.

Problem II. The second problem, like the first, is concerned with how random error affects the unrotated and rotated factor solutions. The difference here is that the error is not introduced artificially, but is a result of taking several samples of varying sizes from a fixed population, thereby producing slightly or largely different correlation matrices. The results of the individual samples are then compared to those of the total group.

Problem III. The third problem will be concerned with the effect the certain permutations on the order of the unrotated factors have on the resulting rotated factor matrices. This problem arises from the fact that almost all factor analysis programs order the unrotated factors from highest to lowest, in terms of contribution to variance, and then commence with either a quartimax or varimax rotational solution. Each of these rotations is accomplished by successive pairings of the ordered factors (1 with 2, 1 with 3, . . ., 1 with k, k-1 with k) until a certain maximum or minimum distribution (of the sums of fourth powers of the rotated factor loadings, for example) is achieved for the final solution. The question arises that maybe there could be a better method of ordering the unrotated factors so that the final rotated solution would meet the criterion better.

The above three problems gave rise to two by-product problems:

1.--This problem is concerned with how the quartimax and varimax methods of rotation are affected by the methods of the above three problems. There are four sub-problems under this heading: (a) What effect will the error introduction have on the quartimax and varimax methods? (b) What effect will sampling have on the quartimax and varimax methods? (c) What effect will permutations have on the quartimax and varimax methods? and (d) Can a better solution be reached for the quartimax or varimax method, in terms of producing a higher sum of fourth powers of the final rotated factor loadings, by using a particular permutation?

2.--The statistics used to make the comparisons, in most of the problems and sub-problems stated above, gave rise to the final problem of concern in this study. That is, how similar are the two different methods of comparing factors?

<u>Purpose of the Study</u>

Broadly stated, the main point of emphasis is that before investigators can attempt to show that factorial invariance exists for Cases 2 through 6 of Henrysson's scheme, the methods of factorial analysis must show consistent results when using a fixed set of variables on samples from a fixed population.

This study will not attempt to be definitive. Rather, it will attempt to investigate several problem areas and indicate where more work and further research is needed. For the above reason, actual data was selected for use rather than randomly generated data and, therefore, any generalizations that could be made must take this into account.

In order to simplify the problem as much as possible, unities were used in the diagonals of the correlation matrices, only orthogonal rotations were used, and the error was introduced randomly rather than systematically into the correlation matrix.

Studies on Factorial Invariance

There appears to have been only a few articles discussing the invariance of factors. Most of these have tried to show that similar sets of factors can be obtained when a fixed set of variables is applied to samples from different populations. Very few have dealt with the effects of selection from the same population on a resulting set of factors, and those that did, imposed certain constraints on one or more of the variables used on the selected sample. None have dealt with the effects of errors in the particular correlation coefficients, which could also result from sample selection, on the resulting rotated and unrotated factor matrices. And, finally, none have dealt with the effect of permuting the order of the unrotated factors before rotating them to a final solution.

In a series of articles by Ledermann (1938), Thomson (1938), and Thomson and Ledermann (1939), an attempt was made to show that regardless of whether a factor analysis was conducted on a total population of subjects or on a subset thereof, the rank of the correlation matrix would not change. In other words, one would still obtain the same number of factors even if the correlation coefficients were not the same in the two groups of subjects. This was demonstrated by using Thurstone's (1947, p. 453) data on univariate selection and is discussed below by Ahmavaara (1954).

In contrast to the position he had taken with Ledermann, Thomson (1948, p. 168) indicated that sampling errors in the correlation coefficients would produce not only unique factors but, in general, they would produce new common factors, because the sampling errors of correlation coefficients are themselves correlated. He cited Pearson and Filon (1898) as giving the formulae for such a correlation:

> The correlation coefficient of the sampling errors of r_{12} and r_{13} (where one of the tests occurs in each correlation) is given by-- $r_{r_{12}r_{13}} = r_{23}$ - (a complicated function of r_{12}, r_{13}, r_{23}) and is roughly somewhat less than r_{23}, for positive correlations. The correlation coefficient of the sampling errors of r_{12} and r_{34}, on the other hand, is a much smaller quantity of the second order only.

Thomson, therefore, considered that sampling errors tend to produce, not irregular ups and downs of the correlations, but a rigid effect, with a general upward or a general downward tendency. Restated, he was indicating that the error factors are, or include, common factors and that not only some of the unique variance of any test could be due to sampling errors, but so would some of its communality.

The term "errors," as Thomson uses it, was defined to include not only sampling errors due to the particular set of persons tested, but also variable chance errors in the performance of the individuals, and even sheer blunders such as mistakes in recording results.

Ahmavaara (1954) demonstrated first, the invariance of the number of common factors under selection of samples from one population and second, the invariance of factor loadings under the same selection, which was based on the Thomson-Ledermann theorems. He used, as an illustration of the theorems, Thurstone's (1947, p. 453) example in which two factor matrices, consisting of three factors each and employing 10 variables, were derived from the correlation matrices of two populations. Both of the populations were supposed to be similar except that the standard deviation of the first variable was changed from unity, in the first population, to .60, in the second population. The results showed that the same number of factors was derived in each case and that the loadings on the one set were proportional to the loadings on the other. Incidentally, one unusual thing that Ahmavaara (1954, p. 33) did, in reporting Thurstone's results, was to mistakingly interchange the set of factors representing the sample with the altered standard deviation with the set representing the unaltered standard deviation sample.

Kenney and Coltheart (1960) were concerned with the problem treated by Thomson and Thurstone. The problem is that a set of correlations is disturbed when sampling restricts the variance of one or more of the correlated variables. They went on to state two assumptions: (1) That certain patterns of mental performance can commonly be represented by mental test score correlations, these being subject to unreliability of the measuring devices and of their applications. The particular power of factor analysis lies in the fact that it can represent the underlying pattern of relationships geometrically as the angular disposition in common factor space of the vectors representing mental tests. The authors maintain that this angular disposition is not distrubed by unreliability of the tests (Kenney, 1958). They argue that the random contributions to test correlations, such as unreliability, are represented factorially merely by a shortening of test vectors, standing for a decrease in test communality which is not correlated with changes in the communality of any other test. Their next assumption is (2) by letting "factor patterns" refer only to systematic relationships, in other words only to the angular disposition of test vectors in common factor space (and not to the size of the factor loadings), factor patterns are not considered to be disturbed by unreliability or by measurement error. Thurstone's factorial configuration is the same as Kenney and Coltheart's factor patterns.

Kenney and Coltheart then computed a correlation matrix on a sample of 255 cases, using eight variables, and derived three unrotated factors. A second correlation matrix was computed on 105 cases, using the same eight variables, selected from the original 255 cases so that the range of scores on the third variable was considerably reduced, and from this derived three unrotated factors. It was at this point that their study differed from Thomson and Thurstone's studies in that Thurstone's reduced standard deviation correlation matrix was produced by using certain selection formulas (1947, p. 447) rather than drawing a sample from the original population. They found that after restricting the variance of the one variable all the correlation coefficients changed and so did the resulting factor patterns. The change in the factor patterns was not in the linear fashion that Thomson and Thurstone had expected. It is of interest to note here that the differences in the correlation coefficients between the total population and sample matrices exceeded that which would have been obtained if the standard error of each respective correlation coefficient had been used as an upper or lower bound on their variability.

In his notes on factorial invariance, Meredith (1964) suggests that if a simple structure factor pattern can be satisfactorily determined in a particular population, the same simple structure can be found in any subpopulation of it, derivable by selection. He noted that before satisfactory factor matching between two sets of factors can be achieved, (1) each measure needs to be expressed on the same unit of measurement over the populations employed and (2) each orthogonal factor structure matrix should first be rotated to the best fitting "simple structure."

Heermann (1964) talks about factor score indeterminacy and rotational indeterminacy. His main thesis is that there is no unique solution in either of these cases when we use an indeterminate factor model (i.e., using communalities in the main diagonal). On the other hand, he does not advocate using a determinate factor model (i.e., using unities in the main diagonal) to achieve unique solution. His main conclusion is that unique factor measures can be obtained by using a determinate model, but the factors are always contained in the test space and hence cannot be expected to represent anything which goes beyond the original measures. If one considers factor analysis to be a tool for generating new measures which are more fundamental than the original measures, he then feels that the only choice is to retain the indeterminate factor model.

Summary of Factorial Invariance Studies

Most of the studies cited on factorial invariance seem to be preoccupied with the problem of defining a particular factor structure for a particular set of variables, regardless of the sample of persons. Kenney and Coltheart (1960) appear to be the only ones that attack this idea of factorial invariance empirically, but even they can be observed to agree that, given the same set of variables, random samples from the same population, and the same set of communalities, configurational invariance can be achieved.

Thomson (1948) stands out as the only author to speculate on the effects of random or chance error on a resulting factor structure, but he provides no evidence to support his view.

Another point worth emphasizing here is that most of the authors cited above tended to agree with Thurstone's hypothesis that factor analyses on selected sub-samples of one large population would have essentially the same factor structure as the parent population, yet no evidence has been provided to support this idea. Furthermore, when selection was conducted empirically on a fixed population, the standard deviation of one of the variables would purposely be altered and the resulting correlations would not be the same as those in the original analysis. In fact, the differences between the original and altered correlation matrices exceeded that which one would expect if he had used the standard error of each original correlation coefficient as an upper or lower limit for variability.

CHAPTER II

PROBLEM I: ERROR

Method

Data

Since the focus of the present study is on the methodological aspects of factor analysis, a correlation matrix was selected which would be familiar to most psychometric investigators. The correlation matrix chosen was that for Thurstone's (1938) Primary Mental Abilities test research. In his study, a battery of 57 tests was administered to 240 college students. The correlation matrix was computed on the 57 tests, using tetrachoric correlations, and then centroid factors were derived. Since Thurstone first presented his data there have been many reanalyses: Burt (1950); Eysenck (1939); Holzinger and Harman (1938); Wrigley, Saunders, and Neuhaus (1958); and Zimmerman (1953).

The names of the tests used by Thurstone can be found on page 22 of his monograph.

Data Matrices

The data matrices for this part of the study consisted of: (1) Thurstone's original correlation matrix; (2) The unrotated principal axes factor matrices derived from Thurstone's data; and (3) The random error matrices, described below.

Random Error Data Matrices. Three separate random error data matrices were produced. The first had random error introduced into all of the correlation coefficients in the correlation matrix, excluding the main diagonal entries. The second had the random error introduced into 5% of the correlation coefficients. The third had the random error introduced into 1% of the correlation coefficients.

Random Error Matrix. In the random error matrix, the original Thurstone correlation matrix was used. Each of the 1,596 correlations appearing on either side of the main diagonal, was systematically varied, by raising or lowering each by one standard error, the direction of change being determined by a table of random numbers. If we let n_i represent the original correlation coefficients $i = 1, 2, \ldots, 1{,}596$, p_i represent the random selection of 50% of the numbers within the bounds of i, and SE represent the standard error, then our formula for direction of change (DC) would be:

$$DC = n_i + SE\,(n_i), \quad \text{when } p_i = n_i,$$
$$DC = n_i - SE\,(n_i), \quad \text{when } p_i \neq n_i.$$

The formula for computing the standard error of the correlation coefficient was taken from Guilford (1956) and Lindquist (1951), where r is the correlation coefficient and

$$\sigma_r = \frac{(1 - r^2)}{(N - 1)^{\frac{1}{2}}}$$

is considered as a approximation to the corresponding correlation coefficient, ρ, in the population, from which the sample of N may be considered to be randomly drawn.

Five-Per Cent Random Error Matrix. In the 5% random error matrix the same procedure was followed as in the case of the random error, except that here 81 correlation coefficients in the lower left corner of the correlation matrix were changed. The 81 correlation coefficients used were the correlation of variables 49, 50, 51, 52, 53, 54, 55, 56, and 57 with variables 1, 2, 3, 4, 5, 6, 7, 8, and 9.

One-Per Cent Random Error Matrix. The same procedure was followed here as was done in the above two cases, except that here 16 correlation coefficients were changed in the lower left corner of the correlation matrix. The 16 correlation coefficients used were the correlation of variables 54, 55, 56, and 57 with variables 1, 2, 3, and 4.

Data Analyses

Factor Analysis Program. The FANIM 3 program, available at the Michigan State University Computer Library, provides as output, eigenvalues, principal axes factor loadings, quartimax or varimax rotated factor loadings, proportion of the total variance represented by each rotated factor, and the "observed communality" of each test (the proportion of variance of each test accounted for by the factors). The capacity for the program is 101 variables. This is written in FORTRAN-60 language.

The MINAF 3 program is identical to the FANIM 3 program except that a punchout subroutine was included, enabling the final rotated solution to be punched out on IBM cards.

Quartimax Method of Rotation. This method, which is included in the FANIM 3 program, provides a rotation of axes in order to reduce the complexity of the factorial description of the variables. In other words, a transformation is desired which will tend to increase the larger factor loadings and decrease the smaller ones for each variable of the original factor matrix. This means that the quartimax rotation is concentrating on the rows of the factor matrix. According to Harman (1960), the object of the quartimax method is to determine an orthogonal transformation, T, which will carry the original factor matrix, F, into a new factor matrix, B, for which the variance of the squared factor loadings is a maximum. The formula for this maximum is:

$$Q = \sum_{j=1}^{n} \sum_{p=1}^{m} b_{jp}^{4} ,$$

where b represents the rotated factor loading, p represents the number of factors 1, 2, . . ., m, and j represents the number of variables 1, 2, . . ., n.

Varimax Method of Rotation. In contradistinction to the quartimax method of simplifying the description of each row, or variable, of the factor matrix, this method concentrates on simplifying the columns, or factors, of the factor matrix in an attempt to approximate simple structure more closely. To achieve a "normal" varimax criterion the loadings in each row of the factor matrix are divided by the square root of the communality for each row, respectively.

The computing procedure for a varimax solution is quite similar to that employed for a quartimax, except that varimax requires that

$$V = n \sum_{p=1}^{m} \sum_{j=1}^{n} (b_{jp} / h_{j})^{4} - \sum_{p=1}^{m} (\sum_{j=1}^{n} b_{jp}^{2} / h_{j}^{2})^{2}$$

be maximized instead of Q. Here b, p, and j are the same as was noted above and h represents the communality.

Harman (1960, p. 307) thinks that the varimax solution is likely to yield a more factorially invariant set of factors, in Thurstone's sense of invariance, than the quartimax solution.

Factor Comparison Program. A factor comparison computer program was prepared which would allow one to make two different comparisons between the individual factors of two sets of factors. Each method of comparison is summarized by a single coefficient. The first method, called the "Coefficient of Congruence" by Tucker (1951) and the "Coefficient of Similarity" by Barlow and Burt (1954), approximates the correlation coefficient. The reason it is not a correlation, according to Harman (1960, p. 258), is that the factor loadings used in the formula are not deviates from their respective means and the summations are over the number of variables rather than the number of individuals. This method, which will be called the Coefficient of Congruence (CC) has been recommended by Burt (1949, p. 85), Wrigley and Neuhaus (1955), and Pinneau and Newhouse (1964) and is described as:

$$CC_{ij} = \frac{\sum_{k=1}^{m} a_{ki}\, b_{kj}}{\left(\sum_{k=1}^{m} a_{ki}^2 \sum_{k=1}^{m} b_{kj}^2\right)^{\frac{1}{2}}}$$

where a and b refer to the rotated factor loadings, i and j refer to the two factors to be compared, and k refers to the variables (1, 2, . . ., m) in each factor.

The second method simply yields a mean square of the differences between the loadings of two factors. This statistic is called the Root Mean Square (RMS) by Harman (1960, p. 257):

$$RMS_{(i-j)} = \left(\sum_{k=1}^{m} (a_{ki} - b_{kj})^2 / m \right)^{\frac{1}{2}}$$

where _a_ and _b_ refer to the rotated factor loadings, _i_ and _j_ refer to the two factors to be compared, and _k_ refers to the variables (1, 2, . . ., m) in each factor.

Design

Effects of Random Error on Factor Calculation. Thurstone's regular data matrix was factor analyzed by the FANIM 3 program. Then the total, 5%, and 1% random error matrices were factor analyzed by the FANIM 3 program. The 15 principal axes factors, with the largest eigenvalue, in each of the four factor analyses, were then punched in IBM cards and submitted with the factor comparison program.

Effects of Random Error on Factor Rotation. Thurstone's regular data matrix was factored by the MINIF 3 program, using four different rotational solutions. First, a quartimax rotation was performed using the Kiel-Wrigley (K-W) (1960) criterion for number of factors to be rotated, with a single high loading per factor designated. Second, a quartimax rotation was made with 15 factors. Third, a varimax rotation was performed using the (K-W) criterion with a single high loading per factor designated. Fourth, a varimax rotation was made with 15 factors.

The total, 5%, and 1% random error matrices were subjected to the same four analyses.

The factor comparison program was then used to compare the rotated factor matrices of the total, 5%, and 1% random error matrices to that of the regular data matrix, restricting the comparison to the particular type of rotation. In other words, the solutions resulting from, for example, the quartimax (K-W) rotations were compared with one another, but not with those from other rotational procedures.

Relation Between the Two Methods of Factor Comparison. A correlation was computed between the CCs and RMSs for all the pairs of factors that exhibited large fluctuations in their high loadings in order to see how similar they were in identifying these particular factors.

Results

Effects of Random Error on Factor Calculation

Comparisons were first made between the original errorless principal axes factors and those resulting from the introduction of total, 5%, and 1% random error. In Tables 1 and 2, the 15 highest eigenvalues in each category are presented. All other tables that follow will use letters to represent the factors. The factor comparison program was applied to these four solutions and Table 3 summarizes those factors that showed the highest relationship. One rectangle contains the Coefficients of Congruence (CC) and another the Root Mean Squares (RMS) for the most similar factors in the errorless and modified solution. The letters indicate the factors so paired. The average (M) CC and average (M) RMS are presented on the right.

Table 1. Eigenvalues

Errorless		Random Error	
(1)	19.6792	(1)	19.6663
(17)	5.1907	(17)	5.2728
(28)	3.3526	(28)	3.3720
(5)	2.3534	(3)	2.3882
(3)	2.1761	(6)	2.3249
(6)	1.9587	(10)	2.0510
(14)	1.7449	(5)	1.8873
(23)	1.5963	(9)	1.7056
(18)	1.5246	(12)	1.6583
(9)	1.4512	(16)	1.6155
(26)	1.2802	(25)	1.4562
(43)	1.2313	(14)	1.4415
(51)	1.1485	(23)	1.3720
(24)	1.0568	(45)	1.2821
(33)	.9847	(21)	1.1629
$\sum$=.46.7292		$\sum$=48.6566	

Table 2. Eigenvalues

Errorless		1% Random Error		5% Random Error	
(1)	19.6792	(1)	19.6784	(1)	19.6681
(17)	5.1907	(17)	5.1875	(17)	5.2154
(28)	3.3526	(28)	3.3555	(28)	3.3496
(5)	2.3534	(5)	2.3403	(5)	2.3672
(3)	2.1761	(3)	2.1739	(3)	2.1851
(6)	1.9587	(6)	1.9592	(6)	1.9728
(14)	1.7449	(14)	1.7391	(14)	1.7589
(23)	1.5963	(9)	1.6010	(23)	1.5940
(18)	1.5246	(27)	1.5318	(9)	1.5354
(9)	1.4512	(18)	1.4498	(27)	1.4935
(26)	1.2802	(23)	1.2731	(11)	1.2906
(43)	1.2313	(26)	1.2423	(26)	1.2550
(51)	1.1485	(22)	1.1575	(22)	1.1489
(24)	1.0568	(43)	1.0545	(25)	1.0811
(33)	.9847	(25)	.9828	(43)	.9775
	$\sum=46.7292$		$\sum=46.7167$		$\sum=46.8931$

Table 3. Factor Comparison Between
Errorless, Random, 1%, and 5% Error Principal Axes Factors

Errorless with Random Error

	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O
CC:	.9999	.9950	.9811	.9418	.9249	.9358	-.9282	.7270	-.8715	.8165	-.6141	.7651	.6714	-.5026	.4422

M=.8078

RMS:	.0091	.0305	.0472	.0696	.0772	.0673	.0680	.1259	.0843	.1005	.1359	.1058	.1154	.1432	.1494
	A	B	C	D	E	F	G	H	J	I	L	K	O	N	N

M=.0886

Errorless with 1% Error

	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O
CC:	1.0000	1.0000	1.0000	.9994	.9996	.9998	.9973	-.9971	.9959	.9986	.9988	-.9849	-.9820	.9955	-.9848

M=.9956

RMS:	.0010	.0019	.0021	.0071	.0054	.0035	.0128	.0127	.0148	.0086	.0072	.0256	.0270	.0129	.0229
	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O

M=.0110

Errorless with 5% Error

	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O
CC:	1.0000	.9997	.9996	.9922	.9905	.9963	.9898	.9673	.8662	-.8621	-.9710	-.9834	.9848	.9849	-.9727

M=.9707

RMS:	.0024	.0077	.0070	.0255	.0269	.0161	.0251	.0428	.0848	.0844	.0362	.0270	.0247	.0238	.0307
	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O

M=.0310

In the case of the errorless and random error comparison, the later factors agree less. Furthermore, even though the loadings are, on the average, smaller in the later factors, the RMSs become larger, indicating greater discrepancies. The factors exhibiting large fluctuations in their high loadings are presented in Table 4. Variables are included whenever one or the other loading is above the value (in the range from .40 to .25) stated at the head of each comparison. These are taken to be the salient variables on the factor. An asterisk indicates that one loading or the other has fallen below the arbitrary cut-off figure. The underlined loadings represent the highest loadings for each factor. The asterisks and underlining in all similar tables will have the same meaning as here.

In the case of the errorless and 5% error comparison, only four of the paired factors exhibited large enough fluctuations in their high loadings to be presented in Table 5. None of the four negative CCs represented a complete reversal in factor loading signs.

In the case of the errorless and 1% error comparison, inspection of the actual paired factors showed that the differences, detected by both the CC and the RMS, were not large enough to change the positions of the high loadings. Of the four negative CCs, reported in Table 3 for the errorless with 1% error comparison, only one, Factor L with Factor L, actually represented a complete reversal in signs of the factor loadings in each factor.

As can be observed from Tables 4 and 5, the interpretation of the factors are affected by the introduction of random and 5% error into the original correlation matrix.

Table 4. Comparisons of High Factor Loadings on Selected Pairs of Factors from Errorless Principal Axes with Random Error Principal Axes

Factor D with Factor D
All loadings above .40 reported

Name of Variable	Loadings	
	Errorless	Random Error
23. Identical Forms	.4128	.4323
37. Reasoning	-.4176	-.3644*
48. Picture Recall	.5418	.4367
56. Word Count	.4403	.2646*

Factor E with Factor E
All loadings above .40 reported

Name of Variable	Loadings	
	Errorless	Random Error
10. First and Last Letters	.4251	.5435
45. Number-Number	-.4763	-.3787*
48. Picture Recall	-.3747*	-.4647
51. Rhythm	.4171	.3703*

Factor F with Factor F
All loadings above .35 reported

Name of Variable	Loadings	
	Errorless	Random Error
1. Reading I	.3079*	.3719
12. Anagrams	-.4274	-.3828
43. Word-Number	-.3840	-.4261
44. Initials	-.4301	-.3876
45. Number-Number	-.3046*	-.3891
47. Figure Recognition	-.3526	-.2834*
50. Hands	-.3505	-.3146*

Factor G with Factor G
All loadings above .30 reported

Name of Variable	Loadings	
	Errorless	Random Error
5. Figure Classification	-.4219	.4750
14. Block Counting	.3181	-.2772*
33. Estimating	.4955	-.4736
51. Rhythm	-.3842	.4593

Table 4. (Continued)

Factor H with Factor H
All loadings above .30 reported

Name of Variable	Loadings	
	Errorless	Random Error
9. Disarranged Words	.2389*	.4029
11. Disarranged Sentences	-.3594	-.1702*
22. Mechanical Movements	.3122	.0937*
33. Estimating	-.5346	-.4914
35. Numerical Judgement	.1990*	.3886
44. Initials	-.0261*	-.3155
55. Vocabulary (Chicago)	.3273	.4008

Factor I with Factor J
All loadings above .30 reported

Name of Variable	Loadings	
	Errorless	Random Error
5. Figure Classification	.4133	-.3777
22. Mechanical Movements	.2747*	-.3383
43. Word-Number	-.1885*	.3868
44. Initials	-.1299*	.3031
51. Rhythm	-.3424	.3961
55. Vocabulary (Chicago)	-.3857	.2197*

Factor J with Factor I
All loadings above .25 reported

Name of Variable	Loadings	
	Errorless	Random Error
6. Controlled Association	-.4072	-.4706
9. Disarranged Words	.2804	.1392*
11. Disarranged Sentences	.0717*	.2772
22. Mechanical Movements	-.1066*	-.2674
28. Addition	-.2666	-.3026
35. Numerical Judgement	.3620	.1691*
48. Picture Recall	.2903	.3022
51. Rhythm	.2673	.2236*
52. Sound Grouping	.2384*	.3179
55. Vocabulary (Chicago)	-.0355*	-.2536

Table 4. (continued)

Factor K with Factor L
All loadings above .30 reported

Name of Variable	Loadings	
	Errorless	Random Error
4. Word Grouping	-.0401*	.3058
6. Controlled Association	.3205	-.1638*
12. Anagrams	.2067*	-.3161
16. Lozenges A	-.3299	.0799*
19. Lozenges B	-.1663*	.3213
24. Pursuit	.3523	-.2557*
46. Word Recognition	-.2208*	.3601
56. Word Count	-.2483*	.3549

Factor L with Factor K
All loadings above .25 reported

Name of Variable	Loadings	
	Errorless	Random Error
23. Identical Forms	.3954	.5116
26. Areas	-.2954	-.3707
34. Number Series	-.0423*	.2894
45. Number-Number	.0543*	.2588
48. Picture Recall	-.2506	-.2756
53. Spelling	.3735	.3849
56. Word Count	-.2734	-.3176

Factor M with Factor O
All loadings above .30 reported

Name of Variable	Loadings	
	Errorless	Random Error
43. Word-Number	.3281	.2757*
47. Figure Recognition	-.3420	-.3881
56. Word Count	.3282	.2507*

Table 4. (Continued)

Factor N with Factor N
All loadings above .25 reported

Name of Variable	Loadings	
	Errorless	Random Error
17. Flags	-.2597	.1922*
25. Copying	.2920	-.1094*
29. Subtraction	.2826	-.2488*
34. Number Series	.3310	-.4303
38. Verbal Analogies	.0970*	-.3197
39. False Premises	-.2225*	.3630
42. Syllogisms	-.1042*	.3237
47. Figure Recognition	.1587*	-.2773

Factor O with Factor N
All loadings above .25 reported

Name of Variable	Loadings	
	Errorless	Random Error
24. Pursuit	.2925	-.0235*
32. Tabular Completion	-.3099	.2315*
39. False Premises	.2903	.1035*
40. Code Words	-.2149*	-.2913
43. Word-Number	.0012*	.2757
44. Initials	.0579*	-.2776
46. Word Recognition	.2924	.0156*
47. Figure Recognition	-.1087*	-.3881
56. Word Count	.2248*	.2507

Table 5. Comparisons of High Factor Loadings on Selected Pairs of Factors from Errorless Principal Axes with 1% and 5% Error Principal Axes

Factor I with Factor I
All loadings above .30 reported

Name of Variable	Loadings	
	Errorless	5% Error
5. Figure Classification	.4133	.3349
9. Disarranged Words	.1423*	.3327
51. Rhythm	-.3424	-.2638*
55. Vocabulary (Chicago)	-.3857	-.3193

Factor J with Factor J
All loadings above .25 reported

Name of Variable	Loadings	
	Errorless	5% Error
6. Controlled Association	-.4072	.3189
9. Disarranged Words	.2804	-.2154*
10. First and Last Letters	-.2487*	.3168
28. Addition	-.2666	.2755
35. Numerical Judgment	.3620	-.3913
38. Verbal Analogies	-.2077*	.2558
48. Picture Recall	.2903	-.2403*
51. Rhythm	.2673	-.3559

Factor K with Factor K
All loadings above .30 reported

Name of Variable	Loadings	
	Errorless	5% Error
6. Controlled Association	.3205	-.2635*
16. Lozenges A	-.3299	.3455
24. Pursuit	.3523	-.3354
56. Word Count	-.2483*	.3041

Factor O with Factor O
All loadings above .25 reported

Name of Variable	Loadings	
	Errorless	5% Error
24. Pursuit	.2925	-.3545
32. Tabular Completion	-.3099	.3137
39. False Premises	.2903	-.3069
46. Word Recognition	.2924	-.2027*

Effects of Random Error on Factor Rotation

Comparisons were then made between the errorless rotated principal axes factors and those resulting from the random, 5%, and 1% error rotated principal axes factors. Four separate rotated solutions were employed. In Tables 6 and 7 the proportions of variance accounted for by each factor under the quartimax rotation using the (K-W) criterion, varimax rotation using the (K-W) criterion, quartimax rotation using 15 factors, and the varimax rotation using 15 factors are presented.

As can be observed from Tables 6 and 7, the introduction of random error into the correlation matrix had a definite effect on the number of factors extracted using the rotational solutions with the (K-W) criterion.

The factor comparison program was applied to the four separate rotational solutions comparing the errorless rotation to each of the random, 5%, and 1% error rotations in each of the four categories. Tables 8 and 9 summarize the factors that had the highest CCs and those that had the lowest RMSs. As was indicated about Table 3, the row of letters directly above each pair of rectangles represent the factors from the errorless rotation and those directly below each pair of rectangles represent the rotated factors using the error introduction.

If we compute the mean of the means reported in Tables 8 and 9, for the quartimax rotations with random, 5%, and 1% error we obtain mean CCs of:

.8016, .9557, and .9635.

Table 6. Proportion of Variance Accounted for by Rotated Factors

	Quartimax, Errorless	Rotated (K-W) Random Error	Varimax, Errorless	Rotated (K-W) Random Error
A	.2342	.2597	.1590	.1148
B	.1768	.1496	.1524	.1604
C	.0721	.0670	.0752	.0805
D	.0402	.0356	.0406	.0931
E	.0408	.0352	.0372	.0719
F	.0336	.0358	.0608	.0499
G	.0311	.0310	.0341	.0337
H	.0299	.0283	.0339	.0312
I	.0312	.0327	.0529	.0375
J	.0298	.0301	.0417	.0627
K		.0317	.0238	
L		.0367	.0283	
M		.0298	.0355	
N		.0299	.0270	
	$\sum^{10}$=.7197	$\sum^{14}$=.8331	$\sum^{14}$=.8024	$\sum^{10}$=.7357

	Quartimax, Errorless	Rotated (15) Random Error	Varimax, Errorless	Rotated (15) Random Error
A	.2511	.2530	.1424	.1345
B	.1557	.1535	.1481	.1487
C	.0687	.0662	.0779	.0760
D	.0339	.0344	.0381	.0456
E	.0290	.0350	.0365	.0705
F	.0333	.0322	.0583	.0351
G	.0302	.0310	.0351	.0349
H	.0282	.0295	.0351	.0335
I	.0307	.0325	.0546	.0420
J	.0308	.0303	.0372	.0362
K	.0224	.0312	.0225	.0380
L	.0271	.0361	.0319	.0401
M	.0312	.0297	.0354	.0320
N	.0264	.0307	.0432	.0467
O	.0210	.0282	.0235	.0399
	$\sum$=.8197	$\sum$=.8535	$\sum$=.8198	$\sum$=.8537

Table 7. Proportion of Variance Accounted For by Rotated Factors

Quartimax, Rotated (K-W)

	Errorless	1% Error	5% Error
A	.2342	.2351	.2405
B	.1768	.1747	.1695
C	.0721	.0741	.0744
D	.0402	.0406	.0411
E	.0408	.0415	.0406
F	.0336	.0346	.0353
G	.0311	.0322	.0326
H	.0299	.0301	.0298
I	.0312	.0313	.0317
J	.0298		
K			
L			
M			
N			
O			
	$\sum^{10}$ =.7197	$\sum^{9}$ =.6942	$\sum^{9}$ =.6955

Varimax, Rotated (K-W)

	Errorless	1% Error	5% Error
A	.1590	.1363	.1316
B	.1524	.1462	.1558
C	.0752	.0768	.0768
D	.0406	.0364	.0366
E	.0372	.0359	.0414
F	.0608	.0383	.0661
G	.0341	.0348	.0356
H	.0339	.0356	.0343
I	.0529	.0545	.0698
J	.0417	.0387	.0426
K	.0238	.0227	.0250
L	.0283	.0598	.0388
M	.0355	.0348	.0321
N	.0270	.0455	
O		.0234	
	$\sum^{14}$ =.8024	$\sum^{15}$ =.8197	$\sum^{13}$ =.7865

Quartimax, Rotated (15)

	Errorless	1% Error	5% Error
A	.2511	.2505	.2463
B	.1557	.1550	.1581
C	.0687	.0684	.0702
D	.0339	.0333	.0342
E	.0290	.0294	.0288
F	.0333	.0329	.0331
G	.0302	.0303	.0306
H	.0282	.0283	.0280
I	.0307	.0314	.0317
J	.0308	.0313	.0317
K	.0224	.0225	.0227
L	.0271	.0278	.0270
M	.0312	.0312	.0315
N	.0264	.0261	.0225
O	.0210	.0212	.0264
	$\sum$ =.8197	$\sum$ =.8196	$\sum$ =.8228

Varimax, Rotated (15)

	Errorless	1% Error	5% Error
A	.1424	.1363	.1244
B	.1481	.1462	.1450
C	.0779	.0768	.0792
D	.0381	.0364	.0369
E	.0365	.0359	.0389
F	.0583	.0383	.0609
G	.0351	.0348	.0362
H	.0351	.0356	.0360
I	.0546	.0545	.0535
J	.0372	.0387	.0389
K	.0225	.0227	.0224
L	.0319	.0598	.0387
M	.0354	.0348	.0350
N	.0432	.0455	.0489
O	.0235	.0234	.0278
	$\sum$ =.8198	$\sum$ =.8197	$\sum$ =.8227

Table 8. Factor Comparison Between
Errorless and Random Error Rotated Factors

Quartimax, Rotated (K-W) with Random Error

	A	B	C	D	E	F	G	H	I	J	D	E	I	J
CC:	.9952	.9906	.9672	-.8137	.8559	.8729	-.8924	.7521	-.7875	-.7469	.6082	.5325	-.5028	.5212

M = .7742

RMS:	.0550	.0648	.0682	.1191	.1058	.0949	.0830	.1229	.1141	.1294	.1688	.1830	.1742	.1668
	A	B	C	E	F	L	I	G	J	D	K	M	N	H

M = 1179

Varimax, Rotated (K-W) with Random Error

	A	B	C	D	E	F	G	H	I	J	K	L	M	N
CC:	.9627	.9965	.9729	-.8501	.7215	-.9391	-.9454	.8445	-.8592	-.5932	-.6132	-.5691	-.8478	.5344

M = .8035

RMS:	.1169	.0348	.0656	.1324	.1580	.0923	.0631	.1026	.1594	.2467	.1469	.2076	.1185	.2579
	A	B	C	J	F	E	I	G	D	D	H	J	F	D

M = .1359

Table 8. (Continued)

Quartimax, Rotated (15) with Random Error

	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O
CC:	.9977	.9978	.9844	-.8854	.8405	.8580	-.9508	.8367	-.7816	-.8599	-.6619	.7910	-.8896	.7400	.3591

M = .8290

RMS:	.0341	.0265	.0461	.0889	.0955	.0955	.0559	.0987	.1158	.0961	.1338	.1110	.0837	.1218	.1807
	A	B	C	E	O	L	I	G	N	D	H	K	F	J	H

M = .0925

Varimax, Rotated (15) with Random Error

	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O
CC:	.9792	.9949	.9867	-.9042	.8787	-.9435	-.9545	.8946	-.8820	-.8830	-.5119	.8494	-.8915	-.8450	.4361

M = .8557

RMS:	.0766	.0389	.0454	.0867	.0966	.0885	.0570	.0860	.1104	.0969	.1670	.1037	.0875	.1183	.3301
	A	B	C	L	O	E	J	G	D	I	H	K	F	N	A

M = .1060

Table 9. Factor Comparison Between Errorless, 1% Error, and 5% Rotated Factors

Quartimax, Rotated (K-W) with 1% Error

	A	B	C	D	E	F	G	H	I	J	
CC:	.9998	.9998	.9943	.9855	.9986	.9588	.9160	-.9935	.9982	.4474	M=.9292
RMS:	.0084	.0095	.0291	.0343	.0110	.0531	.0730	.0197	.0107	.1851	M=.0474
	A	B	C	D	E	F	G	H	I	G	

Quartimax, Rotated (K-W) with 5% Error

	A	B	C	D	E	F	G	H	I	J	
CC:	.9995	.9994	.9957	.9912	.9945	.9771	.8983	.9728	.8736	.4629	M=.9165
RMS:	.0174	.0168	.0253	.0268	.0211	.0400	.0805	.0403	.0891	.1831	M=.0540
	A	B	C	D	E	F	G	H	I	G	

Varimax, Rotated (K-W) with 1% Error

	A	B	C	D	E	F	G	H	I	J	K	L	M	N	K
CC:	.9941	.9974	.9991	.9600	.9958	.9975	.9866	-.9919	.9788	.9904	.8316	-.8642	-.9977	.8136	.5657
													M=.9310		
RMS:	.0511	.0290	.0121	.0565	.0179	.0175	.0305	.0241	.0479	.0288	.0885	.0955	.0129	.1243	.1432
	A	B	C	D	F	L	G	H	I	J	K	L	M	N	O
													M=.0520		

Varimax, Rotated (K-W) with 5% Error

	A	B	C	D	E	F	G	H	I	J	K	L	M	N
CC:	.9902	.9978	.9987	.9783	.9854	.9928	.9909	.9852	.9052	-.9864	-.8568	-.9170	.9906	-.6334
													M=.9435	
RMS:	.0642	.0263	.0146	.0422	.0355	.0321	.0255	.0318	.1127	.0339	.0837	.0796	.0269	.2045
	A	B	C	D	E	F	G	H	I	J	K	L	M	I
													M=.0542	

Table 9. (continued)

Quartimax, Rotated (15) with 1% Error

	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O
CC:	1.0000	.9999	.9998	.9958	.9961	.9991	.9989	-.9994	.9986	.9984	.9955	-.9937	-.9994	.9966	-.9954
														M=.9978	
RMS:	.0043	.0045	.0055	.0170	.0152	.0077	.0081	.0060	.0094	.0099	.0143	.0188	.0062	.0135	.0139
	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O
														M=.0103	

Quartimax, Rotated (15) with 5% Error

	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O
CC:	.9998	.9999	.9993	.9941	.9965	.9979	.9939	.9971	.9955	-.9956	-.9863	-.9868	.9965	.9937	.9898
														M=.9948	
RMS:	.0115	.0070	.0103	.0200	.0143	.0119	.0192	.0128	.0171	.0168	.0249	.0268	.0147	.0182	.0216
	A	B	C	D	E	F	G	H	I	J	K	L	M	O	N
														M=.0165	

Varimax, Rotated (15) with 1% Error

	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O
CC:	.9995	.9998	.9998	.9962	.9973	.9996	.9995	-.9989	.9993	.9983	.9966	-.9893	-.9991	.9978	-.9959
														M=.9978	
RMS:	.0140	.0075	.0064	.0174	.0148	.0076	.0061	.0088	.0089	.0119	.0124	.0291	.0081	.0149	.0138
	A	B	C	D	E	L	G	H	I	J	K	E	M	N	O
														M=.0121	

Varimax, Rotated (15) with 5% Error

	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O
CC:	.9962	.9997	.9994	.9913	.9959	.9981	.9949	.9961	.9979	-.9970	-.9834	-.9750	.9969	.9937	-.9831
														M=.9932	
RMS:	.0402	.0108	.0099	.0258	.0186	.0158	.0192	.0168	.0151	.0157	.0273	.0457	.0149	.0275	.0322
	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O
														M=.0224	

For the varimax rotations with random, 5%, and 1% error we obtain mean CCs of:

.8296, .9684, and .9644.

For the quartimax rotations with random, 5%, and 1% error we obtain mean RMSs of:

.1052, .0353, and .0289.

Finally, for the varimax rotations with random, 5%, and 1% error we obtain mean RMSs of:

.1210, .0383, and .0321.

Since the number of factors extracted by the (K-W) error rotations in Table 7 varied only by one from the errorless case, the mean of the mean CCs for both the 1% and 5% error without the last (and most dissimilar) factor comparison in each rectangle of Table 9 was computed for the (K-W) rotations in order to see if the same results would be observed. For the 5% error case, quartimax has a mean CC of .9809 and varimax has a mean CC of .9803. For the 1% error case, quartimax has a mean CC of .9903 and varimax has a mean CC of .9775. The mean RMSs also showed wider gaps between the varimax and quartimax solutions, but still in the same direction as before.

From these results it appears that the introduction of random error into the correlation matrix yields a lower CC for the quartimax solution than for the varimax, unless we compute the mean CC without the last (and most dissimilar) factor comparison, but it also yields a higher RMS for the varimax solution than for the quartimax.

We will now look at the factor matrices under each of the four rotations. The first set will be those resulting from the quartimax rotation using the Kiel-Wrigley criterion.

In the case of the quartimax (K-W) with random error comparison, the later factors agree less. This is reasonable, considering that we are comparing 10 factors to 14. The ten pairs of factors which exhibited large fluctuations in their high loadings are presented in Table 10. None of the five negative CCs represented a complete reversal in factor loading signs.

In the case of the quartimax, (K-W) with 5% error comparison, Factors G, I, and J with Factors G, I, and G, respectively, have the lowest CCs and the highest RMSs. Only these three, out of the 10 pairs of factors, exhibited large fluctuations in their high loadings. These are presented in Table 11. The large differences in the first two pairs of factors, above, can also be seen to be affected by the fact that Factor J has a CC of .4629 with Factor G and one of .4239 with Factor I.

In the case of the quartimax (K-W) with 1% error comparison, Factor G with Factor G and Factor J with Factor G are the only two, out of the 10 pairs of factors, which exhibit large fluctuations in their high loadings. These are presented in Table 12. The larger differences exhibited by Factor F with Factor F and Factor G with Factor G can probably be better understood if one realizes that Factor J has a CC of .3508 with Factor F and one of .4474 with Factor G. The one negative CC did not represent a complete reversal in factor loading signs.

Secondly, we will look at the factor matrices of the varimax rotation using the Kiel-Wrigley criterion.

For the varimax, (K-W) with random error comparison, the ten pairs of factors which exhibited large fluctuations in

Table 10. Comparisons of High Factor Loadings on Selected Pairs of Factors from Quartimax, Rotated (K-W) with Random Error

Factor D with Factor E
All loadings above .40 reported

Name of Variable	Loadings	
	Errorless	Random Error
11. Disarranged Sentences	.4344	-.3219*
23. Identical Forms	.5073	-.1321*
48. Picture Recall	.7543	-.8510
56. Word Count	.5453	-.2990*

Factor E with Factor F
All loadings above .50 reported

Name of Variable	Loadings	
	Errorless	Random Error
43. Word-Number	-.6151	-.7888
44. Initials	-.5578	-.4834*
45. Number-Number	-.6777	-.2634*
46. Word Recognition	-.4655*	-.5357
47. Figure Recognition	-.5222	-.3176*

Factor H with Factor G
All loadings above .27 reported

Name of Variable	Loadings	
	Errorless	Random Error
5. Figure Classification	.2523*	.4725
11. Disarranged Sentences	-.2774	.0045*
25. Number Code	-.2871	-.1818*
33. Estimating	-.7074	-.7526
50. Hands	-.3444	-.1584*
56. Word Count	.2644*	.5852

Table 10. (Continued)

Factor I with Factor J
All loadings above .40 reported

Name of Variable	Loadings	
	Errorless	Random error
5. Figure Classification	.5488	-.3350*
34. Number Series	.4074	-.0648*
39. False Premises	.4084	-.4152
55. Vocabulary (Chicago)	-.5430	.6772

Factor J with Factor D
All loadings above .30 reported

Name of Variable	Loadings	
	Errorless	Random error
6. Controlled Association	-.4568	.1754*
10. First and Last Letters	-.3662	.3327
26. Areas	.3272	-.3031
31. Division	.2234*	-.3746
32. Tabular Completion	.1850*	-.3782
35. Numerical Judgement	.5925	-.6774
36. Arithmetical Reasoning	.3605	-.5441

Factor D with Factor K
All loadings above .40 reported

Name of Variable	Loadings	
	Errorless	Random error
11. Disarranged Sentences	.4344	.2059*
23. Identical Forms	.5073	.8247
26. Areas	-.1111*	-.4023
48. Picture Recall	.7543	.0811*
56. Word Count	.5453	.1041*

Table 10. (Continued)

Factor E with Factor M
All loadings above .50 reported

Name of Variable	Loadings	
	Errorless	Random Error
43. Word-Number	-.6151	-.1678*
44. Initials	-.5578	-.1593*
45. Number-Number	-.6777	-.8187
47. Figure Recognition	-.5222	-.1107*

Factor I with Factor N
All loadings above .40 reported

Name of Variable	Loadings	
	Errorless	Random Error
5. Figure Classification	.5488	-.2794*
34. Number Series	.4074	-.6650
39. False Premises	.4084	.0611*
55. Vocabulary (Chicago)	-.5430	.1084*

Factor J with Factor H
All loadings above .30 reported

Name of Variable	Loadings	
	Errorless	Random Error
6. Controlled Association	-.4568	-.6272
10. First and Last Letters	-.3662	-.0741*
16. Lozenges A	.1029*	.4043
24. Pursuit	-.1348*	-.3570
26. Areas	.3272	-.0904*
35. Numerical Judgement	.5925	.1058*
36. Arithmetical Reasoning	.3605	.0106*
44. Initials	-.2952*	-.3550
47. Figure Recognition	.1505*	.3416

Table 11. Comparisons of High Factor Loadings on Selected Pairs of Factors from Quartimax, Rotated (K-W) with 5% Error

Factor G with Factor G
All loadings above .30 reported.

Name of Variable	Loadings	
	Errorless	5% Error
22. Mechanical Movements	.5132	.5229
36. Arithmetical Reasoning	.2860*	.3844
51. Rhythm	-.6722	-.5750
52. Sound Grouping	-.5255	-.4866

Factor I with Factor I
All loadings above .40 reported

Name of Variable	Loadings	
	Errorless	5% Error
5. Figure Classification	.5488	.4223
6. Controlled Association	-.1968*	-.4810
34. Number Series	.4074	.3171*
39. False Premises	.4084	.3564*
55. Vocabulary (Chicago)	.5430	-.5979

Factor J with Factor G
All loadings above .30 reported

Name of Variable	Loadings	
	Errorless	5% Error
6. Controlled Association	-.4568	-.0883*
10. First and Last Letters	-.3662	-.0651*
22. Mechanical Movements	.1072*	.5229
26. Areas	.3272	.1318*
35. Numerical Judgement	.5925	.2704*
36. Arithmetical Reasoning	.3605	.3844
51. Rhythm	.0492*	-.5750
52. Sound Grouping	.0052*	.4866

Table 12. Comparisons of High Factor Loadings On Selected Pairs of Factors from Quartimax, Rotated (K-W) with 1% Error

Factor G with Factor G
All loadings above .30 reported

Name of Variable	Loadings	
	Errorless	1% Error
22. Mechanical Movements	.5132	.5457
36. Arithmetical Reasoning	.2860*	.3850
51. Rhythm	-.6722	-.5650
52. Sound Grouping	-.5255	-.4570

Factor J with Factor G
All loadings above .30 reported

Name of Variable	Loadings	
	Errorless	1% Error
6. Controlled Association	-.4568	-.1358*
10. First and Last Letters	-.3662	-.0266*
22. Mechanical Movements	.1072*	.5457
26. Areas	.3272	.1217*
35. Numerical Judgement	.5925	.2400*
36. Arithmetical Reasoning	.3605	.3850
51. Rhythm	.0492*	-.5650
52. Sound Grouping	.0052*	-.4570

their high loadings are presented in Table 13. The greater disagreement in the later factors can be attributed to the fact that we are comparing 14 to 10 factors. None of the eight negative CCs represented a complete reversal in factor loading signs.

For the varimax, (K-W) with 5% error comparison, four pairs of factors, exhibited large fluctuations in their high loadings and are presented in Table 14. None of the four negative CCs represent a complete reversal in factor loading signs.

For the varimax, (K-W) with 1% error comparison, five pairs of factors, exhibited large fluctuations in their high loadings and are presented in Table 15. None of the three negative CCs represented a complete reversal in factor loading signs.

Thirdly, we will look at the factor matrices of the quartimax rotation using 15 factors.

In the case of the quartimax,(15) with random error comparison, nine pairs of factors exhibited large fluctuations in their high loadings and are presented in Table 16. None of the six negative CCs represented a complete reversal in factor loading signs.

When we look at the actual factor loadings, for the quartimax (15) with 5% error comparison, only Factor L with Factor L exhibits a large fluctuation in its high loadings and is presented in Table 17. None of the three negative CCs represented a complete reversal in factor loading signs.

In the case of the quartimax, (15) with 1% error comparison, inspection of the paired factors having the lowest CCs

Table 13. Comparisons of High Factor Loadings
On Selected Pairs of Factors
From Varimax, Rotated (K-W) with Random Error

Factor A with Factor A
All loadings above .70 reported

Name of Variable	Loadings	
	Errorless	Random Error
1. Reading I	.7996	.6685*
2. Reading II	.8529	.6472*
55. Vocabulary (Chicago)	.8729	.9862
57. Vocabulary (Thorndike)	.7382	.6386*

Factor D with Factor J
All loadings above .50 reported

Name of Variable	Loadings	
	Errorless	Random Error
3. Verbal Classification	.2828*	-.5033
11. Disarranged Sentences	.3675*	-.6062
23. Identical Forms	.3154*	-.5492
48. Picture Recall	.7881	-.7834
56. Word Count	.7300	-.4183*

Factor E with Factor F
All loadings above .40 reported

Name of Variable	Loadings	
	Errorless	Random Error
43. Word-Number	-.1381*	-.7954
44. Initials	-.4204	-.6072
45. Number-Number	-.4982	-.5230
46. Word Recognition	-.4237	-.5375
47. Figure Recognition	-.7869	-.4127

Table 13. (Continued)

Factor F with Factor E
All loadings above .40 reported

Name of Variable	Loadings	
	Errorless	Random Error
7. Inventive Opposites	-.3770*	.4149
9. Disarranged Words	-.5746	.6330
10. First and Last Letters	-.7026	.8223
12. Anagrams	-.7957	.6876
13. Inventive Synonyms	-.5048	.5723
54. Grammar	-.4701	.5338

Factor H with Factor G
All loadings above .30 reported

Name of Variable	Loadings	
	Errorless	Random Error
6. Controlled Association	.1773*	.4085
11. Disarranged Sentences	-.3728	-.1154*
33. Estimating	-.8432	-.7731
38. Verbal Analogies	-.3078	-.1068*
50. Hands	-.3052	-.2686*
56. Word Count	.1691*	.4257

Factor I with Factor D
All loadings above .30 reported

Name of Variable	Loadings	
	Errorless	Random Error
1. Reading I	.1927*	-.5493
2. Reading II	.1173*	-.5126
5. Figure Classification	.6695	-.3782*
25. Copying	.4847	-.3377*
26. Areas	.2826*	-.5041
32. Division	.4307*	-.5166
34. Number Series	.7091	-.6888
37. Reasoning	.3171*	-.6623
39. False Premises	.3021*	-.6617

Table 13. (Continued)

Factor J with Factor D
All loadings above .55 reported

Name of Variable	Loadings	
	Errorless	Random Error
34. Number Series	.1877*	-.6888
35. Numerical Judgement	.7537	-.3631*
36. Arithmetical Reasoning	.5529	-.4290*
37. Reasoning	.1218*	-.6623
39. False Premises	.1535*	-.6617

Factor K with Factor H
All loadings above .35 reported

Name of Variable	Loadings	
	Errorless	Random Error
6. Controlled Association	.3733	-.5290
9. Disarranged Words	-.4320	.3881
24. Pursuit	.3910	-.3323*
35. Numerical Judgement	.0061*	.4978
44. Initials	.2138*	-.4231
46. Word Recognition	-.3604	.1992*

Factor L with Factor J
All loadings above .50 reported

Name of Variable	Loadings	
	Errorless	Random Error
3. Verbal Classification	.3830*	-.5033
11. Disarranged Sentences	.4070*	-.6062
23. Identical Forms	.5263	-.5492
48. Picture Recall	.1149*	-.7834

Table 13. (Continued)

Factor M with Factor F
All loadings above .50 reported

Name of Variable	Loadings	
	Errorless	Random Error
43. Word-Number	.7923	-.7954
44. Initials	.4005*	-.6072
45. Number-Number	.4123*	-.5230
46. Word Recognition	.3146*	-.5375

Factor N with Factor D
All loadings above .50 reported

Name of Variable	Loadings	
	Errorless	Random Error
1. Reading I	-.1844*	-.5493
2. Reading II	-.1489*	-.5126
26. Areas	-.0626*	-.5041
32. Tabular Completion	-.1452*	-.5166
34. Number Series	-.0601*	-.6888
37. Reasoning	-.3470*	-.6623
39. False Premises	-.5497	-.6617

Table 14. Comparison of High Factor Loadings On Selected Pairs of Factors from Varimax, Rotated (K-W) with 5% Error

Factor I with Factor I
All loadings above .40 reported

Name of Variable	Loadings	
	Errorless	5% Error
1. Reading I	.1940*	.4284
5. Figure Classification	.6695	.5713
25. Copying	.4847	.2391*
32. Tabular Completion	.4307	.4817
34. Number Series	.7091	.6290
37. Reasoning	.3171*	.5879
39. False Premises	.3021*	.6436

Factor K with Factor K
All loadings above .30 reported

Name of Variable	Loadings	
	Errorless	5% Error
6. Controlled Association	.3733	-.5468
9. Disarranged Words	-.4320	.2980*
16. Lozenges A	-.3472	.3084
24. Pursuit	.3910	-.3489
45. Number-Number	.3075	-.1689*
46. Word Recognition	-.3604	.2369*
53. Spelling	-.3241	.3207

Table 14. (continued)

Factor L with Factor L
All loadings above .30 reported

Name of Variable	Loadings	
	Errorless	5% Error
3. Verbal Classification	.3850	-.5674
4. Word Grouping	.3574	-.4300
11. Disarranged Sentences	.4070	-.4617
15. Cubes	.2483*	-.3112
21. Punched Holes	-.3264	.2213*
23. Identical Forms	.5263	-.6607
26. Areas	-.3588	.2493*
53. Spelling	.3953	-.3295

Factor N with Factor I
All loadings above .30

Name of Variable	Loadings	
	Errorless	5% Error
1. Reading I	-.1844*	.4284
5. Figure Classification	-.1628*	.5713
6. Controlled Association	.3010	-.0032*
18. Form Board	.3240	.0002*
32. Tabular Completion	-.1452*	.4817
34. Number Series	-.0601*	.6290
37. Reasoning	-.3470	.5879
39. False Premises	-.5497	.6436

Table 15. Comparison of High Factor Loadings On Selected Pairs of Factors from Varimax, Rotated (K-W) with 1% Error

Factor D with Factor D
All loadings above .30 reported.

Name of Variable	Loadings	
	Errorless	1% Error
11. Disarranged Sentences	.3675	.3629
23. Identical Forms	.3154	.1692*
48. Picture Recall	.7881	.7782
56. Word Count	.7300	.7835

Factor K with Factor K
All loadings above .30 reported.

Name of Variable	Loadings	
	Errorless	1% Error
6. Controlled Association	.3733	.4034
9. Disarranged Words	-.4320	-.5197
13. Inventive Synonyms	-.1753*	-.3429
16. Lozenges A	-.3472	-.2549*
24. Pursuit	.3910	.4729
45. Number-Number	.3075	.1786*
46. Word Recognition	-.3604	-.1528*
53. Spelling	-.3241	-.0948*

Factor K with Factor O
All loadings above .30 reported.

Name of Variable	Loadings	
	Errorless	1% Error
6. Controlled Association	.3733	.0317*
9. Disarranged Words	-.4320	-.1221*
16. Lozenges A	-.3472	-.1031*
24. Pursuit	.3910	.0160*
45. Number-Number	.3075	.2527*
46. Word Recognition	-.3604	-.2811*
53. Spelling	-.3241	-.6242

Table 15. (continued)

Factor L with Factor E
All loadings above .30 reported.

Name of Variable	Loadings	
	Errorless	1% Error
3. Verbal Classification	.3850	-.4991
4. Word Grouping	.3574	-.4315
11. Disarranged Sentences	.4070	-.3835
15. Cubes	.2483*	-.3088
21. Punched Holes	-.3264	.2088*
23. Identical Forms	.5263	-.7247
26. Areas	-.3588	.2871*
53. Spelling	.3953	-.1542*

Factor N with Factor N
All loadings above .30 reported.

Name of Variable	Loadings	
	Errorless	1% Error
1. Reading I	-.1844*	-.3258
6. Controlled Association	.3010	.1126*
18. Form Board	.3240	.1402*
37. Reasoning	-.3470	-.6006
39. False Premises	-.5497	-.7875
54. Grammar	-.2597*	-.4325

Table 16. Comparisons of High Factor Loadings
On Selected Pairs of Factors
From Quartimax, Rotated (15) with Random Error

Factor D with Factor E
All loadings above .30 reported

Name of Variable	Loadings	
	Errorless	Random Error
11. Disarranged Sentences	.3374	-.3930
48. Picture Recall	.7902	-.8597
56. Word Count	.7306	-.4445

Factor E with Factor O
All loadings above .30 reported

Name of Variable	Loadings	
	Errorless	Random Error
21. Punched Holes	-.0768*	-.3201
44. Initials	-.3362	-.3389
45. Number-Number	-.4719	-.0952*
46. Word Recognition	-.3124	-.3675
47. Figure Recognition	-.7471	-.7836
56. Word Count	.3073	.0859*

Factor H with Factor G
All loadings above .30 reported

Name of Variable	Loadings	
	Errorless	Random Error
5. Figure Classification	.2419*	.4600
11. Disarranged Sentences	-.3440	-.0342*
33. Estimating	-.7879	-.7875
50. Hands	-.3193	-.1822*
55. Vocabulary (Chicago)	.3003	.2144*
56. Word Count	.2141*	.5140

Table 16. (Continued)

Factor I with Factor N
All loadings above .40 reported

Name of Variable	Loadings	
	Errorless	Random Error
5. Figure Classification	.5308	-.2896*
25. Copying	.4451	-.2613*
32. Tabular Completion	.4250	-.3837*
34. Number Series	.6445	-.7277

Factor K with Factor H
All loadings above .30 reported

Name of Variable	Loadings	
	Errorless	Random Error
6. Controlled Association	.4637	-.6442
9. Disarranged Words	-.5351	.2344*
13. Inventive Synonyms	-.3103	-.0495*
16. Lozenges A	-.2175*	.3694
24. Pursuit	.4591	-.2392*
44. Initials	.2144*	-.5984

Factor L with Factor K
All loadings above .30 reported

Name of Variable	Loadings	
	Errorless	Random Error
3. Verbal Classification	.3273	.4173
21. Punched Holes	-.3205	-.0724*
23. Identical Forms	.6472	.8665
26. Areas	-.4072	-.3175
37. Reasoning	-.2491*	-.3056
46. Word Recognition	-.3342	.0111*

Table 16. (Continued)

Factor M with Factor F
All loadings above .30 reported

Name of Variable	Loadings	
	Errorless	Random Error
19. Lozenges B	.2588*	-.3444
24. Pursuit	-.1113*	.3277
43. Word-Number	.7718	-.8440
44. Initials	.3732	-.2333*
45. Number-Number	.3959	-.2291*
46. Word Recognition	.2905*	-.4866

Factor N with Factor J
All loadings above .30 reported

Name of Variable	Loadings	
	Errorless	Random Error
5. Figure Classification	-.2432*	-.3448
18. Form Board	.3033	.2317*
26. Areas	-.0702*	-.3025
37. Reasoning	-.3456	-.0419*
39. False Premises	-.5990	-.3996
55. Vocabulary (Chicago)	.4015	.6712

Factor O with Factor H
All loadings above .30 reported

Name of Variable	Loadings	
	Errorless	Random Error
6. Controlled Association	-.0793*	-.6442
16. Lozenges A	.0719*	.3694
40. Code Words	-.3213	-.0556*
44. Initials	-.1402*	-.5984
53. Spelling	.5617	.2637*

Table 17. Comparison of High Factor Loadings
On a Selected Pair of Factors from
Quartimax, Rotated (15) with 5% Error

Factor L with Factor L
All loadings above .30 reported

Name of Variable	Loadings	
	Errorless	5% Error
3. Verbal Classification	.3273	-.4054
21. Punched Holes	-.3205	.2891*
23. Identical Forms	.6472	-.6807
26. Areas	-.4072	.3360
46. Word Recognition	-.3342	.2945*

and the highest RMSs did not show large fluctuations in their high loadings. None of the four negative CCs represented a complete reversal in factor loading signs.

Finally, we will look at the factor matrices of the varimax rotation using 15 factors.

For the varimax, (15) with random error comparison, ten pairs of factors exhibited large fluctuations in their high loadings and are presented in Table 18. None of the eight negative CCs represented a complete reversal in factor loading signs.

When we look at the actual factor loadings, for the varimax (15) with 5% error comparison, only Factor K with Factor K and Factor O with Factor O exhibited large fluctuations in their high loadings and are presented in Table 19. None of the four negative CCs represented a complete reversal in factor loading signs.

For the varimax, (15) with 1% error comparison, Factor L with Factor E was the only pair to have a low CC and a high RMS. The high loadings of this pair of factors are shown in Table 20. None of the four negative CCs represented a complete reversal in factor loading signs.

<u>Comparison of the Two Methods of Factor Comparison</u>

In order to determine the extent to which both the CC and the RMS were related in identifying those pairs of factors that exhibited large fluctuations in their high loadings, a correlation coefficient, -.8391, indicates that there is a very high negative relationship between the CC and the RMS with

Table 18. Comparisons of High Factor Loadings
On Selected Pairs of Factors
From Varimax, Rotated (15) with Random Error

Factor D with Factor L
All loadings above .30 reported

Name of Variable	Loadings	
	Errorless	Random Error
11. Disarranged Sentences	.3670	-.4478
41. Pattern Analogies	.2661*	-.3398
46. Word Recognition	.3042	-.0709*
48. Picture Recall	.7739	-.8598
56. Word Count	.7782	-.5367

Factor E with Factor O
All loadings above .40 reported

Name of Variable	Loadings	
	Errorless	Random Error
21. Punched Holes	-.1514*	-.4031
44. Initials	-.4060	-.3939*
45. Number-Number	-.4881	-.1295*
46. Word Recognition	-.3874*	-.4485
47. Figure Recognition	-.7916	-.8297

Factor F with Factor E
All loadings above .40 reported

Name of Variable	Loadings	
	Errorless	Random Error
9. Disarranged Words	-.5304	.7212
10. First and Last Letters	-.7013	.7062
12. Anagrams	-.8014	.7553
13. Inventive Synonyms	-.4806	.6354
54. Grammar	-.4418	.5265

Factor H with Factor G
All loadings above .35 reported

Name of Variable	Loadings	
	Errorless	Random Error
11. Disarranged Sentences	-.4138	-.1395*
33. Estimating	-.8376	-.8532
56. Word Count	.1667*	.3905

Table 18. (Continued)

Factor I with Factor D
All loadings above .50 reported

Name of Variable	Loadings	
	Errorless	Random Error
5. Figure Classification	.5968	-.3911*
25. Copying	.5526	-.3280*
32. Tabular Completion	.5195	-.4796*
34. Number Series	.7271	-.7945

Factor J with Factor I
All loadings above .35 reported

Name of Variable	Loadings	
	Errorless	Random Error
26. Areas	.3935	-.4281
31. Division	.4198	-.3710
35. Numerical Judgement	.7299	-.7233
36. Arithmetical Reasoning	.4836	-.6087
57. Vocabulary (Thorndike)	.3653	-.3400*

Factor K with Factor H
All loadings above .40 reported

Name of Variable	Loadings	
	Errorless	Random Error
6. Controlled Association	.4196	-.6785
9. Disarranged Words	-.5139	.1602*
24. Pursuit	.4937	-.2353*
44. Initials	.1872*	-.6569

Table 18. (Continued)

Factor M with Factor F
All loadings above .40 reported

Name of Variable	Loadings	
	Errorless	Random Error
43. Word-Number	.7902	-.8581
44. Initials	.4105	-.2513*
45. Number-Number	.4305	-.2421*
46. Word Recognition	.3139*	-.5098

Factor N with Factor N
All loadings above .40 reported

Name of Variable	Loadings	
	Errorless	Random Error
3. Verbal Classification	-.2029*	.4252
26. Areas	-.1462*	.4473
37. Reasoning	-.5762	.3014*
39. False Premises	-.7749	.6640
42. Syllogisms	-.2523*	.5337
54. Grammar	-.4169	.2645*

Factor O with Factor A
All loadings above .60 reported

Name of Variable	Loadings	
	Errorless	Random Error
1. Reading I	-.1321*	.6801
2. Reading II	-.0490*	.7696
7. Inventive Opposites	.1837*	.7011
8. Completion	.0509*	.6631
49. Theme	.2936*	.6677
53. Spelling	.6128	.6068
55. Vocabulary (Chicago)	.2046*	.9344
57. Vocabulary (Thorndike)	.0174*	.7255

Table 19. Comparison of High Factor Loadings
On a Selected Pair of Factors from
Varimax, Rotated (15) with 5% Error

Factor K with Factor K
All loadings above .30 reported

Name of Variable	Loadings	
	Errorless	5% Error
6. Controlled Association	.4196	-.3605
9. Disarranged Words	-.5139	.4685
13. Inventive Synonyms	-.3371	.3248
24. Pursuit	.4937	-.5434

Factor O' with Factor O
All loadings above .30 reported

Name of Variable	Loadings	
	Errorless	5% Error
46. Word Recognition	.3256	-.2898*
49. Theme	.2936*	-.3732
53. Spelling	.6128	-.6826

Table 20. Comparison of High Factor Loadings on a Selected Pair of Factors from Varimax, Rotated (15) with 1% Error

Factor L with Factor E
All loadings above .30 reported

Name of Variable	Loadings	
	Errorless	1% Error
3. Verbal Classification	.4422	-.4991
4. Word Grouping	.3636	-.4315
11. Disarranged Sentences	.3669	-.3835
15. Cubes	.2977*	-.3088
23. Identical Forms	.7077	-.7277
26. Areas	-.3423	.2871*

respect to the 73 pairs of factors showing large fluctuations in their high loadings. This result seems to be in harmony with the purposes of the two statistics, the CC is a measure of similarity and the RMS is a measure of dissimilarity.

CHAPTER III

PROBLEM II: SAMPLING

Method

Data

For the sampling or selection part of the study, the scores of 2,322 freshmen at Michigan State University (MSU) for Fall, 1964, using 15 variables, were obtained from the University Office of Evaluation Services. There were 1,198 males and 1,124 females in the sample.

The 15 variables consisted of five orientation test scores, four standardized test scores, four consolidated high school grades, the parent's educational level, and the MSU Fall Term grade point average. The tests were described as follows: (1) English Grammar, (2) Reading Comprehension, (3) Vocabulary, (4) General Information, (5) Numerical, (6) English (ACT, The American College Testing Program), (7) Mathematics, (8) Social Studies, (9) Natural Science, (10) English (HS Grade), (11) Mathematics (HS Grade), (12) Social Studies (HS Grade), (13) Natural Science (HS Grade), (14) Parent's Educational Level, and (15) MSU Grade Point Average.

Data Matrices

The data matrices for this part of the study consisted of: (1) The freshman score matrix for the 2,322; (2) The principal axes factor matrices derived from the data; and (3) The sampling data matrices, described below.

Sampling Data Matrices. Five samples were selected from the total freshman group, made up of 1600, 400, 100, 25, and 17 randomly selected individuals, respectively. Since the number of males and females in the total group was not equal, all samples were selected so as to preserve the same ratio as that in the total group. To ensure random selection, random numbers were assigned to each of the male and female groups. Then each group was ordered from high to low on the basis of the random numbers, and renumbered from 1 to 1,124 and 1,198, respectively.

Sample of 1600. To obtain this sample, a total of 722 males and females were randomly selected out of the total 2,322. In the male group 372 were selected out and in the female group 350. The method of selection was simply to go to a table of random numbers, such as that found in Li (1957), and select the first 372 numbers within the limits of 1 to 1,198, for example.

Sample of 400. This sample was obtained by selecting 206 males and 194 females from the total group, using the procedure described above.

The other three samples were obtained in the same manner.

Data Analyses

Factor Analysis Program. The FACTOR A program, available in the Michigan State University Computer Library, provides means, standard deviations, correlation, eigenvalues, principal axes factor loadings, quartimax and/or varimax rotated factor loadings, proportions of the total variance represented by each rotated factor, and the "observed communality" of each test (the proportion of variance of each test accounted for by the factors). The program is written in 3600 FORTRAN language and will accept either test scores or correlations.

The quartimax and varimax methods of rotation and the factor comparison program have been described in Chapter II.

Design

Effects of Sampling on Factor Calculation. The total freshman score matrix was factor analyzed by the FACTOR A program. Then the samples of 1600, 400, 100, 25, and 17 were factored by the FACTOR A program. The six largest principal axes factors in each of the six factor analyses with the largest eigenvalues, were then punched in IBM cards and submitted with the factor comparison program.

Effects of Sampling on Factor Rotation. The total freshman score matrix was factored by the FACTOR A program, using four different rotational solutions: (1) The quartimax rotation was performed using the (K-W) criterion for number of factors to be rotated, with a single high loading per factor designated; (2) The quartimax rotation was performed with six factors to be rotated; (3) The varimax rotation was performed using the (K-W) criterion with a single high loading per factor designated; and (4) The varimax rotation was performed with six factors to be rotated.

Each of the five samples of the total freshman score matrix was subjected to the same four analyses as was indicated above and then the results were compared to those of the total.

Relation Between the Two Methods of Factor Comparison. A correlation was computed between the CCs and RMSs for all the pairs of factors that exhibited large fluctuations in their high loadings in order to see how similar they were in identifying these particular factors.

Results

Effects of Sampling on Factor Calculation

Comparisons were made between the total principal axes factors and those obtained from the samples of 1600, 400, 100, 25, and 17. In Table 21 the six highest eigenvalues in each of the samples are presented. The numbers 1 through 6, used to designate the eigenvalues, correspond to the letters A through F, used to designate their respective factors. An inspection of Table 21 shows how the eigenvalues change from sample to sample. Interestingly enough, as we reduce the size of the sample there seems to be a steady increase in the size of the eigenvalues.

The factor comparison program was applied to the six principal axes solutions. Table 22 summarizes the factors that had the highest CCs and those that had the lowest RMSs. The format and notation will be in keeping with similar tables reported above. An inspection of this table reveals that as the samples grow smaller, the mean RMSs grow larger. The four negative CCs did not represent a complete reversal in factor loading signs.

In the case of the comparison of the total group with the 1600 sample, Factor F with Factor F has the lowest CC and the highest RMS. But this one, along with the other five pairs

Table 21. Eigenvalues

	Total Group N=2322	Sample of N=1600	Sample of N=400
(1)	6.4480	6.5338	5.9862
(2)	1.5411	1.5426	1.6135
(3)	1.3223	1.2866	1.5061
(4)	.9461	.9513	.9641
(5)	.8691	.8407	.9318
(6)	.5984	.5888	.6484
	$\sum$=11.7250	$\sum$=11.7438	$\sum$=11.6501

	Sample of N=100	Sample of N=25	Sample of N=17
(1)	6.2445	5.6105	8.8308
(2)	1.9287	2.5548	2.0170
(3)	1.4126	1.5964	1.1415
(4)	1.0193	1.0596	.8811
(5)	.9780	.8878	.6705
(6)	.6463	.7457	.4854
	$\sum$=12.2394	$\sum$=12.4548	$\sum$=14.0263

Table 22. Factor Comparison Between Total Group and Samples of N=1600, N=400, N=100, N=25, and N=17 Principal Axes Factors

Total Group with N=1600

	A	B	C	D	E	F	
CC:	.9999	.9973	.9968	.9958	.9985	.9394	M=.9880
RMS:	.0091	.0235	.0238	.0230	.0137	.0693	M=.0271
	A	B	C	D	E	F	

Total Group with N=400

	A	B	C	D	E	F	
CC:	.9985	.9904	.9865	.8660	.8753	.9428	M=.9433
RMS:	.0427	.0455	.0543	.1295	.1240	.0694	M=.0776
	A	B	C	E	D	F	

Total Group with N=100

	A	B	C	D	E	F	
CC:	.9898	.9622	.8627	.6036	.6185	-.6839	M=.7868
RMS:	.0933	.1007	.1585	.2280	.2197	.1880	M=.1647
	A	B	C	D	D	E	

Total Group with N=25

	A	B	C	D	E	F	
CC:	.9701	-.6437	.7805	.6490	.6734	.6038	M=.7201
RMS:	.1611	.3206	.1887	.2003	.2421	.2010	M=.2190
	A	B	D	F	C	E	

Total Group with N=17

	A	B	C	D	E	F	
CC:	.9920	.8255	.6643	.4387	-.8240	-.4221	M=.6944
RMS:	.1433	.2078	.2355	.2474	.1433	.2212	M=.1998
	A	B	C	E	D	E	

of factors, exhibited no large fluctuations in their high loadings.

Tables 23 through 26 contain the factors exhibiting large fluctuations in their high loadings for the 400, 100, 25, and 17 samples. These comparisons indicate that as the sample size is decreased the number of similar factors are also decreased.

Effects of Sampling on Factor Rotation

Comparisons were then made between the total rotated principal axes factors and those obtained from the samples of 1600, 400, 100, 25, and 17. Three separate rotated solutions were employed. In Table 27 the proportions of variance accounted for by each factor under the quartimax rotation using the (K-W) criterion, varimax rotation using the (K-W) criterion, and the quartimax rotation using six factors are presented. Here we see that as the sample size is reduced the total proportion of variance is increased.

As can be observed from Table 27, the sampling affected the number of factors extracted using the Quartimax rotation with the (K-W) criterion. In the case of the sample of N = 400, the number of factors extracted was reduced from 6 to 5. In the case of the sample of N = 17, the number of factors extracted was also reduced from 6 to 5.

The factor comparison program was applied to the three separate rotational solutions comparing the total group to those of each of the samples of 1600, 400, 100, 25, and 17 in each of the three categories. Tables 28, 29, and 30 summarize the factors that had the highest CCs and the lowest RMSs.

Table 23. Comparisons of High Factor Loadings on Selected Pairs of Factors from Total Group with N=400 Principal Axes Factors

Factor C with Factor C
All loadings above .45 reported

Name of Variable	Loadings	
	Total	N=400
10. English (HS Grade)	.5543	.5351
11. Mathematics (HS Grade)	.3717*	.4579
12. Social Studies (HS Grade)	.5074	.4696
13. Natural Science (HS Grade)	.4554	.5686

Factor D with Factor E
All loadings above .35 reported

Name of Variable	Loadings	
	Total	N=400
12. Social Studies (HS Grade)	.0904*	.3592
14. Parent's Education Level	.9064	.6732

Factor E with Factor D
All loadings above .35 reported

Name of Variable	Loadings	
	Total	N=400
1. English Grammar	.3866	.4171
6. English (ACT)	.3848	.3204
12. Social Studies (HS Grade)	.4718	.3817
14. Parent's Education level	-.1849*	-.5897

Table 24. Comparisons of High Factor Loadings on Selected Pairs of Factors from Total Group with N=100 Principal Axes Factors

Factor B with Factor B
All loadings above .40 reported

Name of Variable	Loadings	
	Total	N=100
3. Vocabulary	-.4300	-.4147
5. Numerical	.5549	.5656
6. English (ACT)	-.3411*	-.4421
7. Mathematics	.5377	.5363
11. Mathematics (HS Grade)	-.4736	-.6588
13. Natural Science (HS Grade)	-.2343*	-.4043

Factor C with Factor C
All loadings above .45 reported

Name of Variable	Loadings	
	Total	N=100
10. English (HS Grade)	.5543	.4272*
12. Social Studies (HS Grade)	.5074	.6210
13. Natural Science (HS Grade)	.4554	.5666

Factor D with Factor D
All loadings above .35 reported

Name of Variable	Loadings	
	Total	N=100
4. General Information	-.0810*	-.3601
9. Natural Science	-.0833*	-.3642
14. Parent's Education Level	.9064	.4075

Table 24. (continued)

Factor E with Factor D
All loadings above .35 reported

Name of Variable	Loadings	
	Total	N=100
1. English Grammar	.3866	.3007*
4. General Information	-.3192*	-.3601
6. English (ACT)	.3848	.2839*
9. Natural Science	-.1615*	-.3642
12. Social Studies (HS Grade)	.4718	.2434*
14. Parent's Education level	-.1849*	.4075

Factor F with Factor E
All loadings above .35 reported

Name of Variable	Loadings	
	Total	N=100
12. Social Studies (HS Grade)	-.3817	.5343
13. Natural Science (HS Grade)	.5468	-.3653
14. Parent's Education level	.0165*	-.4830

Table 25. Comparisons of High Factor Loadings on Selected Pairs of Factors from Total Group with N=25 Principal Axes Factors

Factor A with Factor A
All loadings above .75 reported

Name of Variable	Loadings	
	Total	N=25
2. Reading Comprehension	.7935	.7703
4. General Information	.7511	.8538
5. Numerical	.6832*	.7921
8. Social Studies	.7806	.8084
9. Natural Science	.7610	.6328*
15. Grade Point Average	.6763*	.8248

Factor B with Factor B
All loadings above .40 reported

Name of Variable	Loadings	
	Total	N=25
1. English Grammar	-.2740*	.6367
3. Vocabulary	-.4300	.4853
5. Numerical	.5549	-.3223*
6. English (ACT)	-.3411*	.6936
7. Mathematics	.5377	-.3580*
9. Natural Science	.1091*	-.4554
10. English (HS Grade)	.2978*	-.6529
11. Mathematics (HS Grade)	-.4736	-.0902*
13. Natural Science (HS Grade)	-.2343*	.4316

Factor C with Factor D
All loadings above .45 reported

Name of Variable	Loadings	
	Total	N=25
10. English (HS Grade)	.5543	.3127*
11. Mathematics (HS Grade)	.3717*	.5706
12. Social Studies (HS Grade)	.5074	.3936*
13. Natural Science (HS Grade)	.4554	.3554*

Table 25. (continued)

Factor D with Factor F
All loadings above .35 reported

Name of Variable	Loadings	
	Total	N=25
11. Mathematics (HS Grade)	-.1644*	-.4686
14. Parent's Education level	.9064	.5205

Factor E with Factor C
All loadings above .35 reported

Name of Variable	Loadings	
	Total	N=25
1. English Grammar	.3866	.3625
6. English (ACT)	.3848	.0333*
7. Mathematics	.1467*	.4528
11. Mathematics (HS Grade)	-.2114*	-.5360
12. Social Studies (HS Grade)	.4718	.6359
14. Parent's Education Level	-.1849*	-.6023

Factor F with Factor E
All loadings above .35 reported

Name of Variable	Loadings	
	Total	N=25
2. Reading Comprehension	-.0269*	.3534
12. Social Studies (HS Grade)	-.3817	-.3399*
13. Natural Science (HS Grade)	.5468	.5729
14. Parent's Education Level	.0168*	-.3851

Table 26. Comparisons of High Factor Loadings on Selected Pairs of Factors from Total Group with N=17 Principal Axes Factors

Factor A with Factor A
All loadings above .75 reported

Name of Variable	Loadings	
	Total	N=17
1. English Grammar	.7288*	.8445
2. Reading Comprehension	.7935	.8606
3. Vocabulary	.7433*	.8556
4. General Information	.7511	.9209
5. Numerical	.6832*	.7816
6. English (ACT)	.7078*	.8400
7. Mathematics	.6938*	.7595
8. Social Studies	.7806	.8463
9. Natural Science	.7610	.7734
13. Natural Science (HS Grade)	-.5491*	-.8861
15. Grade Point Average	.6763*	.8669

Factor B with Factor B
All loadings above .40 reported

Name of Variable	Loadings	
	Total	N=17
3. Vocabulary	-.4300	-.3057*
5. Numerical	.5549	.3267*
7. Mathematics	.5377	.4908
9. Natural Science	.1091*	.4359
11. Mathematics (HS Grade)	-.4736	-.4720
14. Parent's Education Level	-.3012*	.8104

Factor C with Factor C
All Loadings above .45 reported

Name of Variable	Loadings	
	Total	N=17
10. English (HS Grade)	.5543	.4759
11. Mathematics (HS Grade)	.3717*	.5750
12. Social Studies (HS Grade)	.5074	.4762
13. Natural Science (HS Grade)	.4554	-.1563*

Table 26. (continued)

Factor D with Factor E
All loadings above .35 reported

Name of Variable	Loadings	
	Total	N=17
11. Mathematics (HS Grade)	-.1644*	-.3530
12. Social Studies (HS Grade)	.0904*	.3707
14. Parent's Education Level	.9064	.3635

Factor E with Factor D
All loadings above .35 reported

Name of Variable	Loadings	
	Total	N=17
1. English Grammar	.3866	-.2911*
5. Numerical	.1182*	-.3790
6. English (ACT)	.3848	-.3962
12. Social Studies (HS Grade)	.4718	-.4403

Factor F with Factor E
All loadings above .35 reported

Name of Variable	Loadings	
	Total	N=17
11. Mathematics (HS Grade)	.0237*	-.3530
12. Social Studies (HS Grade)	-.3817	.3707
13. Natural Science (HS Grade)	.5468	-.0347*
14. Parent's Education Level	.0165*	.3635

Table 27. Proportion of Variance Accounted for by Rotated Factors

Quartimax, Rotated (K-W)

	Total Group N=2322	Sample of N=1600	Sample of N=400	Sample of N=100	Sample of N=25	Sample of N=17
A	.3605	.3659	.3140	.3422	.3268	.5696
B	.1311	.1367	.1462	.1560	.1880	.1112
C	.0743	.0700	.1396	.0786	.0978	.0873
D	.0662	.0662	.0657	.0767	.0740	.0593
E	.0824	.0632	.0680	.0858	.0685	.0754
F	.0673	.0809		.0757	.0752	
	$\sum$=.7818	$\sum$=.7829	$\sum$=.7335	$\sum$=.8160	$\sum$=.8303	$\sum$=.9028

Varimax, Rotated (K-W)

	Total Group N=2322	Sample of N=1600	Sample of N=400	Sample of N=100	Sample of N=25	Sample of N=17
A	.2124	.1925	.1691	.2569	.2961	.2898
B	.1575	.1580	.1617	.1729	.1892	.1018
C	.0919	.0930	.1011	.0862	.0758	.0962
D	.0672	.0671	.1998	.1104	.1113	.2238
E	.1710	.1819	.0690	.0915	.0799	.1019
F	.0817	.0905	.0760	.0982	.0781	.1215
	$\sum$=.7817	$\sum$=.7830	$\sum$=.7767	$\sum$=.8161	$\sum$=.8304	$\sum$=.9350

Quartimax, Rotated (6)

	Total Group N=2322	Sample of N=400	Sample of N=17
A	.3605	.3218	.5684
B	.1311	.1447	.1034
C	.0743	.0821	.0830
D	.0662	.0911	.0564
E	.0824	.0680	.0755
F	.0673	.0689	.0484
	$\sum$=.7818	$\sum$=.7766	$\sum$=.9351

Table 28. Factor Comparison Between
Total Group and Samples of N=1600, N=400, N=100, N=25, and N=17 Using Quartimax, Rotated (K-W) Factors

Total Group with N=1600

	A	B	C	D	E	F	
CC:	.9986	.9965	.9872	.9977	.9285	.9338	M=.9737
RMS:	.0322	.0317	.0436	.0175	.1076	.1019	M=.0558
	A	B	C	D	E	F	

Total Group with N=400

	A	B	C	D	E	F	
CC:	.9867	.9290	.7540	.9792	.7798	.7974	M=.8710
RMS:	.1027	.1416	.2456	.0529	.1826	.2287	M=.1590
	A	B	C	E	D	C	

Total Group with N=100

	A	B	C	D	E	F	
CC:	.9889	.9799	.9122	.7872	-.7795	.8668	M=.8858
RMS:	.0895	.0833	.1202	.1752	.1870	.1408	M=.1327
	A	B	E	D	F	C	

Total Group with N=25

	A	B	C	D	E	F	
CC:	.8674	.6945	.8250	.8992	.7857	.8566	M=.8214
RMS:	.3031	.4128	.1773	.1205	.2735	.1396	M=.2378
	A	A	C	F	B	E	

Total Group with N=17

	A	B	C	D	E	F	
CC:	.9708	-.7799	.8385	-.7260	-.5938	-.4627	M=.7286
RMS:	.2243	.2270	.1555	.2298	.2423	.6751	M=.2923
	A	C	E	B	D	A	

Table 29. Factor Comparison Between
Total Group and Samples of N=1600, N=400, N=100, N=25,
and N=17 Using Varimax, Rotated (K-W) Factors

Total Group with N=1600

	A	B	C	D	E	F	
CC:	.9974	.9984	.9875	.9980	.9960	.9761	M=.9922
RMS:	.0394	.0224	.0481	.0163	.0397	.0659	M=.0386
	A	B	C	D	E	F	

Total Group with N=400

	A	B	C	D	E	F	
CC:	-.9932	.9953	.9799	.9805	.9965	.9795	M=.9875
RMS:	.0548	.0392	.0640	.0516	.0345	.0577	M=.0503
	D	B	C	E	A	F	

Total Group with N=100

	A	B	C	D	E	F	
CC:	.9730	.9817	.8994	.7158	-.8033	.8995	M=.8788
RMS:	.1213	.0800	.1358	.2329	.2470	.1301	M=.1579
	A	B	E	D	F	C	

Total Group with N=25

	A	B	C	D	E	F	
CC:	.8766	.8052	.8674	.8934	.9256	.8360	M=.8674
RMS:	.2623	.3253	.1666	.1259	.1650	.1628	M=.2013
	A	A	D	F	B	E	

Total Group with N=17

	A	B	C	D	E	F	
CC:	.9362	.9277	.8853	-.7885	-.9384	-.6135	M=.8483
RMS:	.1941	.1494	.1499	.1964	.1663	.4275	M=.2139
	A	F	E	B	D	A	

Table 30. Factor Comparison Between Total Group and Samples of N=400 and N=17 Using Quartimax, Rotated (6) Factors

Total Group with N=400

	A	B	C	D	E	F	
CC:	.9954	.9915	.9753	.9782	.9866	.9793	M=.9844
RMS:	.0649	.0517	.0637	.0541	.0504	.0532	M=.0563
	A	B	C	E	D	F	

Total Group with N=17

	A	B	C	D	E	F	
CC:	.9719	-.7322	.8363	-.7778	-.7348	-.4594	M=.7521
RMS:	.2213	.2476	.1566	.2022	.1965	.6752	M=.2832
	A	C	E	B	D	A	

If we look at the means of the CCs and RMSs in Table 28, 29, and 30, we observe that for all samples except N = 100 the varimax solution has a higher CC and a lower RMS than the quartimax solution.

We will now look at the factor matrices of each sample under each of the three rotations. The first set will be those resulting from the samples using quartimax, rotated (K-W) factors.

Tables 31 through 35 contain those factors which exhibit large fluctuations in their high loadings for all five samples. For the 1600 and 400 samples, more factors exhibit large fluctuations in their high loadings in the rotated than unrotated solution. In the case of the 100, 25, and 17 samples, the opposite is true.

None of the five negative CCs, for this rotation, represented a complete reversal in factor loading signs.

Secondly we will look at the factor matrices of the samples under varimax, rotated (K-W) factors.

Here, samples 400, 100, 25, and 17 had factors which exhibited large fluctuations in their high loadings. These are presented in Tables 36 through 38. There were less factors that exhibited large fluctuations in their high loadings for this rotated solution when compared to the unrotated.

Three of the five negative CCs represented a complete reversal in factor loading signs.

Finally, we will look at the factor matrices of the samples under quartimax, rotated (6) factors.

Table 31. Comparisons of High Factor Loadings on a Selected Pair of Factors from the Total Group Quartimax, Rotated (K-W) with N=1600

Factor E with Factor E
All loadings above .50 reported

Name of Variable	Loadings	
	Total	N=1600
1. English Grammar	.5485	.4757*
6. English (ACT)	.5237	.4555*
10. English (HS Grade)	-.5843	-.3595*

Table 32. Comparisons of High Factor Loadings on Selected Pairs of Factors from the Total Group Quartimax, Rotated (K-W) with N=400

Factor C with Factor C
All loadings above .30 reported

Name of Variable	Loadings	
	Total	N=400
10. English (HS Grade)	.3273	.5836
11. Mathematics (HS Grade)	.1444*	.6548
12. Social Studies (HS Grade)	.8656	.6117
13. Natural Science (HS Grade)	.1275*	.7176
15. Grade Point Average	-.4205	-.5633

Factor E with Factor D
All loadings above .50 reported

Name of Variable	Loadings	
	Total	N=400
1. English Grammar	.5485	.4530*
6. English (ACT)	.5237	.4171*
10. English (HS Grade)	-.5843	-.3003*

Factor F with Factor C
All loadings above .30 reported

Name of Variable	Loadings	
	Total	N=400
10. English (HS Grade)	.3159	.5836
11. Mathematics (HS Grade)	.3816	.6548
12. Social Studies (HS Grade)	.0964*	.6117
13. Natural Science (HS Grade)	.8404	.7176
15. Grade Point Average	-.1020*	-.5633

Table 33. Comparisons of High Factor Loadings on a Selected Pair of Factors from the Total Group Quartimax, Rotated (K-W) with N=100

Factor E with Factor F
All loadings above .50 reported

Name of Variable	Loadings	
	Total	N=100
1. English Grammar	.5485	-.3463*
6. English (ACT)	.5237	-.1866*
10. English (HS Grade)	-.5843	.8232

Table 34. Comparisons of High Factor Loadings on Selected Pairs of Factors from the Total Group Quartimax, Rotated (K-W) with N=25

Factor A with Factor A
All loadings above .75 reported

Name of Variable	Loadings	
	Total	N=25
2. Reading Comprehension	.8350	.7203*
3. Vocabulary	.8486	.4332*
4. General Information	.7886	.8653
5. Numerical	.4757*	.8850
7. Mathematics	.4929*	.8295
8. Social Studies	.8360	.6682*
9. Natural Science	.7955	.7099*
15. Grade Point Average	.5229*	.8523

Factor B with Factor A
All loadings above .65 reported

Name of Variable	Loadings	
	Total	N=25
2. Reading Comprehension	.0259*	.7203
4. General Information	.1777*	.8653
5. Numerical	.7915	.8850
7. Mathematics	.7792	.8295
8. Social Studies	.0677*	.6682
9. Natural Science	.2340*	.7099
11. Mathematics (HS Grade)	-.6725	-.2000*
15. Grade Point Average	.2883*	.8523

Factor E with Factor B
All loadings above .50 reported

Name of Variable	Loadings	
	Total	N=25
1. English Grammar	.5485	.7424
3. Vocabulary	.2231*	.7189
6. English (ACT)	.5237	.8905
10. English (HS Grade)	-.5843	-.7412

Table 35. Comparisons of High Factor Loadings on Selected Pairs of Factors from the Total Group Quartimax, Rotated (K-W) with N=17

Factor A with Factor A
All loadings above .75 reported

Name of Variable	Loadings	
	Total	N=17
1. English Grammar	.6580*	.7886
2. Reading Comprehension	.8350	.8626
3. Vocabulary	.8486	.8681
4. General Information	.7886	.9389
5. Numerical	.4757*	.7619
6. English (ACT)	.6868*	.8084
8. Social Studies	.8360	.8995
9. Natural Science	.7955	.8205
13. Natural Science (HS Grade)	-.3379*	-.9048
15. Grade Point Average	.5229*	.8716

Factor B with Factor C
All loadings above .65 reported

Name of Variable	Loadings	
	Total	N=17
5. Numerical	.7915	-.3240*
7. Mathematics	.7792	-.3878*
11. Mathematics (HS Grade)	-.6725	.8760

Table 35. (continued)

Factor E with Factor D
All loadings above .50 reported

Name of Variable	Loadings	
	Total	N=17
1. English Grammar	.5485	-.4503*
6. English (ACT)	.5237	-.4554*
10. English (HS Grade)	-.5843	-.0004*

Factor F with Factor A
All loadings above, 75 reported

Name of Variable	Loadings	
	Total	N=17
1. English Grammar	.0516*	.7886
2. Reading Comprehension	-.0243*	.8626
3. Vocabulary	.0201*	.8681
4. General Information	-.1295*	.9389
5. Numerical	.0044*	.7619
6. English (ACT)	.0506*	.8084
8. Social Studies	-.0153*	.8995
9. Natural Science	-.1200*	.8205
13. Natural Science (HS Grade)	.8404	-.9048
15. Grade Point Average	-.1020*	.8716

Table 36. Comparisons of High Factor Loadings on Selected Pairs of Factors from the Total Group Varimax, Rotated (K-W) with N=100

Factor A with Factor A
All loadings above .60 reported

Name of Variable	Loadings	
	Total	N=100
2. Reading Comprehension	.6365	.7567
3. Vocabulary	.6253	.8249
4. General Information	.7996	.7647
8. Social Studies	.7391	.7455
9. Natural Science	.7725	.7977

Factor D with Factor D
All loadings above .50 reported

Name of Variable	Loadings	
	Total	N=100
1. English Grammar	.0495*	.5076
6. English (ACT)	.0754*	.6424
14. Parent's Education Level	.9922	.8295

Factor E with Factor F
All loadings above .50 reported

Name of Variable	Loadings	
	Total	N=100
1. English Grammar	.7863	-.4383*
2. Reading Comprehension	.5081	-.0147*
3. Vocabulary	.6209	-.3128*
6. English (ACT)	.7943	-.2778*
10. English (HS Grade)	.5921	.8750

Table 37. Comparisons of High Factor Loadings on Selected Pairs of Factors from the Total Group Varimax, Rotated (K-W) with N=25

Factor A with Factor A
All loadings above .60 reported

Name of Variable	Loadings	
	Total	N=25
2. Reading Comprehension	.6365	.6893
3. Vocabulary	.6253	.3727*
4. General Information	.7996	.8253
5. Numerical	.3471*	.8890
7. Mathematics	.3521*	.8662
8. Social Studies	.7391	.6267
9. Natural Science	.7725	.6550
15. Grade Point Average	.3186*	.7905

Factor B with Factor A
All loadings above .65 reported

Name of Variable	Loadings	
	Total	N=25
2. Reading Comprehension	.1355*	.6893
4. General Information	.2792*	.8253
5. Numerical	.8493	.8890
7. Mathematics	.8397	.8662
9. Natural Science	.3380*	.6550
11. Mathematics (HS Grade)	-.6604	-.1760*
15. Grade Point Average	.3405*	.7905

Table 38. Comparisons of High Factor Loadings on Selected Pairs of Factors From the Total Group Varimax, Rotated (K-W) with N=17

Factor A with Factor A
All loadings above .60 reported

Name of Variable	Loadings	
	Total	N=17
2. Reading Comprehension	.6365	.6492
3. Vocabulary	.6253	.5081*
4. General Information	.7996	.8176
8. Social Studies	.7391	.8277
9. Natural Science	.7725	.8536
13. Natural Science (HS Grade)	-.2405	-.8067

Factor E with Factor D
All loadings above .50 reported

Name of Variable	Loadings	
	Total	N=17
1. English Grammar	.7863	-.8464
2. Reading Comprehension	.5081	-.6271
3. Vocabulary	.6209	-.6988
6. English (ACT)	.7943	-.7648
10. English (HS Grade)	-.5921	.3826*
15. Grade Point Average	.2850*	-.6239

Factor F with Factor A
All loadings above .80 reported

Name of Variable	Loadings	
	Total	N=17
4. General Information	-.1387*	.8176
3. Social Studies	-.0471*	.8277
9. Natural Science	-.1407*	.8536
13. Natural Science (HS Grade)	.8698	-.8067

Table 39 contains those factors which exhibit large fluctuations in their high loadings for the sample of 17.

None of the four negative CCs represented a complete reversal in factor loading signs.

Comparison of the two Methods of Factor Comparison

In order to determine the extent to which both the CC and the RMS were related in identifying those pairs of factors that exhibited large fluctuations in their high loadings, a correlation coefficient was computed between them. The correlation coefficient, -.6108, indicates that there is a fairly high negative relationship between the CC and the RMS with respect to the 44 pairs of factors showing large fluctuations in their high loadings. This result seems to be in harmony with the purpose of the two statistics, the CC is a measure of similarity and the RMS is a measure of dissimilarity.

Table 39. Comparisons of High Factor Loadings on Selected Pairs of Factors from the Total Group Quartimax, Rotated (6) with N=17

Factor A with Factor A
All loadings above .75 reported

Name of Variable	Loadings	
	Total	N=17
1. English Grammar	.6580*	.8007
2. Reading Comprehension	.8350	.8772
3. Vocabulary	.8486	.8726
4. General Information	.7886	.9357
6. English (ACT)	.6868*	.8117
8. Social Studies	.8360	.8986
9. Natural Science	.7955	.8123
13. Natural Science (HS Grade)	-.3379*	-.8925
15. Grade Point Average	.5229*	.8825

Factor B with Factor C
All loadings above. 65 reported

Name of Variable	Loadings	
	Total	N=17
5. Numerical	.7915	-.2690*
7. Mathematics	.7792	-.3110*
11. Mathematics (HS Grade)	-.6725	.8787

Table 39. (continued)

Factor E with Factor D
All loadings above .50 reported

Name of Variable	Loadings	
	Total	N=17
1. English Grammar	.5485	-.5072
6. English (CQT)	.5237	-.3918*
10. English (HS Grade)	-.5843	.1053*

Factor F with Factor A
All loadings above .84 reported

Name of Variable	Loadings	
	Total	N=17
2. Reading Comprehension	-.0243*	.8772
3. Vocabulary	.0201*	.8726
4. General Information	-.1295*	.9357
8. Social Studies	-.0153*	.8986
13. Natural Science (HS Grade)	.8404	-.8925
15. Grade Point Average	-.1020	.8825

CHAPTER IV

PROBLEM III: PERMUTATION

Method

Data

The permutation problem will mainly be concerned with the effect that certain permutations on the order of the unrotated factors have on the resulting rotated factor matrices.

For the permutation part of the study, two correlation matrices were selected which would be familiar to most psychometric investigators. The first was Thurstone's (1938) Primary Mental Abilities test battery already described in Chapter II.

The second correlation matrix selected was taken from Holzinger and Swineford's (1939) "A Study in Factor Analysis." In their study a battery of 24 psychological tests was administered to 145 children of the Grant-White School of Forest Park, Illinois.

The names of the 24 tests used by Holzinger and Swineford can be found on page 136 of Harman's (1960) text.

Data Matrices

The data matrices for this part of the study consisted of: (1) Thurstone's original correlation matrix; (2) Holzinger and Swineford's original correlation matrix; and (3) The unrotated principal axes factor matrices derived from both Thurstone's and Holzinger and Swineford's data.

Data Analyses

Factor Analysis Program. The computer factor analysis program, called MINAF 3 (described in Chapter II), was modified so that it would take the unrotated principal axes factors and their corresponding eigenvalues and order them from high to low, respectively. After this ordering had taken place, a subroutine would perform certain determined permutations of factors, when designated to do so, before transformation of the principal axes factors to a rotated solution. This program was called MINAC 3.

The permutations that were performed on the Thurstone loadings can be classed into seven categories. We will assume that the highest eigenvalue and corresponding factor each are assigned the letter A, the second highest eigenvalue and corresponding factor each are assigned the letter B, etc. In describing the different permutations only the factor permutation will be mentioned, but it should be understood that the particular permutations also apply to the eigenvalues corresponding to the factors, since both need to correspond so that the factor program will function properly.

The first permutation permuted Factor A with Factor N.

The second permuted Factors A and B with Factors N and M, respectively.

The third permuted Factors A, B, and C with Factors N, M, and L, respectively.

The fourth permuted Factors A and H with Factors G and N, respectively.

The fifth permuted Factors A, B, H, and I with Factors G, F, N, and M, respectively.

The sixth permuted Factors A, B, C, H, I, and J with Factors G, F, E, N, M, and L, respectively.

The seventh permuted Factors A, B, C, D, E, F, and G with Factors N, M, L, K, J, I, and H, respectively.

The quartimax and varimax methods of rotation and the factor comparison program are described in Chapter II.

Fourth Power Calculation Program. A program was prepared which would enable the calculation of the fourth power of the rotated factor matrix. This program calculated the fourth power of each factor loading in a given factor matrix and then sums these fourth powers yielding a single statistic which enables one to determine whether or not a particular factor solution has achieved a maximum for a given set of data (Harman, 1960). This statistic was denoted in Chapter II as Q.

Design

Effects of Permutation on Factor Rotation. The MINAC 3 program was used to perform seven permutations on Thurstone's regular data principal axes solution. The full set of seven permutations was used in comparing: (a) quartimax rotations on 15 factors and (b) varimax rotations on 15 factors. A reduced set of permutations was used with: (a) quartimax rotation on the six largest factors and (b) varimax rotation on the six largest factors. The reduced set included the first, second, third, and fourth permutations as described above. The reason only these four were used was due to the fact that the permutations were restricted to the six highest eigenvalue and factor combinations.

For the Holzinger and Swineford data, only the set of four permutations was used since six factors seemed best to describe the data.

The factor comparison program was then used to compare the unpermuted rotated factors with the permuted rotated factors. The comparisons were restricted to the particular type of rotational solution employed, as well as to the same number of factors rotated.

The fourth power calculation program was also employed here to determine a maximum statistic for each final rotated solution in order to see if it would be possible to obtain a permutation solution that would satisfy the rotational criterion better than the unpermuted solution.

<u>Relation Between the Two Methods of Factor Comparison</u>. A correlation was computed between the CCs and RMSs for all the pairs of factors that exhibited large fluctuations in their high loadings in order to see how similar they were in identifying these particular factors.

Results

Effects of Permutation on Factor Rotation

The four sets of permutations, as described in Chapter II, were applied to the unrotated factors of Thurstone's correlation matrix and then subjected to the four rotations described above. In Tables 40 and 41 the proportions of variance accounted for by each factor under the unpermuted and permuted conditions are presented. As can be observed in both tables, the permutations affect the size of the ptroportions of variance accounted for, but the total amount of variance accounted for by each set of factors is relatively stable.

The factor comparison program was applied to each of the four separate rotational solutions comparing the unpermuted rotation to each of the permuted rotations. The results for the four sets of comparisons are summarized in Tables 42, 43, 44, and 45. The format and notation will be in keeping with similar tables reported above. The only change here is that the letters directly below the pairs of rectangles represent the rotated factors after each particular permutation.

If we compute the mean of the means reported in these four tables we have, for Table 42, a mean CC of .9876 and a mean RMS of .0187. For Table 43 we have a mean CC of .9983 and a mean RMS of .0107. For Table 44 we have a mean CC of .9997 and a mean RMS of .0054. And for Table 45 we have a mean CC of .9996 and a mean RMS of .0073.

From these results it appears that the permutations are having more of an effect on the quartimax solution than the varimax, when 15 factors are rotated. But when only six factors are rotated, it appears that the permutations have more (even though very slightly so) effect on the varimax solution.

We will now look at the factor matrices of each permutation under each of the four rotations. The first set will be those resulting from the permutations using quartimax, rotated (15) factors.

Table 40. Proportions of Variance Accounted For by Rotated Factors after Permutation

Quartimax, Rotated (15)

	Un-permuted	Per-mute 1	Per-mute 2	Per-mute 3	Per-mute 4	Per-mute 5	Per-mute 6	Per-mute 7
A	.2511	.0261	.0306	.0312	.0301	.0302	.0301	.0302
B	.1557	.1502	.0315	.0261	.1510	.0334	.0330	.0280
C	.0687	.0680	.0709	.0271	.0691	.0690	.0287	.0312
D	.0339	.0340	.0357	.0342	.0347	.0340	.0345	.0263
E	.0290	.0312	.0347	.0303	.0285	.0308	.0685	.0363
F	.0333	.0331	.0282	.0315	.0336	.1511	.1508	.0230
G	.0302	.0302	.0305	.0290	.2543	.2542	.2555	.0278
H	.0282	.0281	.0278	.0282	.0261	.0262	.0225	.0308
I	.0307	.0318	.0225	.0329	.0313	.0318	.0316	.0225
J	.0308	.0225	.0230	.0309	.0312	.0269	.0267	.0340
K	.0224	.0286	.0225	.0226	.0262	.0286	.0308	.0320
L	.0271	.0216	.0308	.0682	.0215	.0215	.0214	.0698
M	.0312	.0317	.1443	.1481	.0316	.0313	.0314	.1436
N	.0264	.2558	.2607	.2584	.0279	.0281	.0281	.2606
O	.0210	.0270	.0260	.0213	.0226	.0226	.0261	.0238
	∑=.8197	∑=.8199	∑=.8197	∑=.8200	∑=.8197	∑=.8197	∑=.8197	∑=.8199

Varimax, Rotated (15)

	Un-permuted	Per-mute 1	Per-mute 2	Per-mute 3	Per-mute 4	Per-mute 5	Per-mute 6	Per-mute 7
A	.1424	.0446	.1276	.1325	.0547	.0346	.0345	.0352
B	.1481	.1477	.0354	.0490	.1477	.0584	.0633	.0373
C	.0779	.0763	.0765	.0351	.0773	.0773	.0368	.0349
D	.0381	.0381	.0375	.0375	.0369	.0325	.0380	.0522
E	.0365	.0374	.0230	.0374	.0371	.0367	.0772	.1232
F	.0583	.0608	.0373	.0349	.0581	.1469	.1476	.0656
G	.0351	.0346	.0353	.0351	.1429	.1409	.1344	.0366
H	.0351	.0356	.0363	.0355	.0435	.0543	.0451	.0522
I	.0546	.0560	.0651	.0608	.0351	.0229	.0356	.0363
J	.0372	.0371	.0373	.0226	.0373	.0357	.0335	.0385
K	.0225	.0222	.0222	.0350	.0232	.0448	.0371	.0371
L	.0319	.0327	.0355	.0769	.0329	.0387	.0224	.0772
M	.0354	.0350	.0537	.1470	.0356	.0375	.0555	.0222
N	.0432	.1386	.1489	.0575	.0349	.0350	.0360	.1482
O	.0235	.0232	.0480	.0229	.0227	.0236	.0230	.0229
	∑=.8198	∑=.8199	∑=.8196	∑=.8197	∑=.8199	∑=.8198	∑=.8200	∑=.8196

Table 41. Proportions of Variance Accounted For by Rotated Factors after Permutation

Varimax, Rotated (6)

	Unpermuted	Permute 1	Permute 2	Permute 3	Permute 4
A	.1778	.0645	.0509	.0500	.0937
B	.1805	.1772	.0625	.0626	.1784
C	.0916	.0922	.0923	.0464	.1768
D	.0466	.0463	.0471	.0944	.0642
E	.0644	.0476	.1776	.1769	.0495
F	.0482	.1812	.1784	.1788	.0464
	$\sum$=.6091	$\sum$=.6090	$\sum$=.6088	$\sum$=.6091	$\sum$=.6090

Quartimax, Rotated (6)

	Unpermuted	Permute 1	Permute 2	Permute 3	Permute 4
A	.2096	.0433	.0364	.0364	.0737
B	.2069	.2031	.0432	.0430	.2015
C	.0721	.0731	.0722	.0408	.2132
D	.0408	.0409	.0408	.0728	.0363
E	.0433	.0362	.2083	.2083	.0433
F	.0362	.2123	.2080	.2078	.0409
	$\sum$=.6089	$\sum$=.6089	$\sum$=.6089	$\sum$=.6091	$\sum$=.6089

Table 42. Factor Comparison Between Unpermuted and Permuted 1,2,3,4,5,6, and 7 Using Quartimax, Rotated (15) Factors

Unpermuted with Permuted 1

	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O
CC:	.9998	.9994	.9997	.9991	.9947	.9981	.9974	.9993	.9983	-.9970	-.9936	-.9978	.9991	.9966	.9948
													M=.9976		
RMS:	.0106	.0149	.0064	.0080	.0175	.0112	.0126	.0065	.0106	.0136	.0169	.0109	.0076	.0135	.0151
	N	B	C	D	K	F	G	H	I	E	J	O	M	A	L
													M=.0117		

Unpermuted with Permuted 2

	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O
CC:	.9992	.9977	.9979	.9897	.9912	-.9700	.9964	.9795	.9917	-.7699	.9877	.6758	.9959	-.9924	.8943
													M=.9486		
RMS:	.0224	.0302	.0177	.0271	.0228	.0453	.0148	.0339	.0295	.0191	.0235	.0274	.0160	.0199	.0685
	N	M	C	D	F	E	G	H	A	L	K	I	B	O	J
													M=.0407		

Unpermuted with Permuted 3

	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O
CC:	.9997	.9991	.9998	.9994	.9969	-.9985	-.9954	.9989	.9985	.9990	.9972	.9973	-.9995	-.9973	.9985
													M=.9983		
RMS:	.0144	.0194	.0050	.0066	.0135	.0099	.0167	.0080	.0098	.0079	.0112	.0121	.0059	.0119	.0079
	N	M	L	D	G	I	E	H	A	J	K	C	F	B	O
													M=.0107		

Unpermuted with Permuted 4

	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O
CC:	.9998	.9995	.9997	.9981	.9974	.9975	.9983	.9964	.9984	.9960	.9962	.9916	.9992	.9977	.9917
													M=.9972		
RMS:	.0094	.0133	.0070	.0116	.0124	.0130	.0101	.0143	.0101	.0158	.0131	.0213	.0071	.0110	.0189
	G	B	C	D	E	F	A	N	I	J	O	K	M	H	L
													M=.0126		

Table 42. (continued)

Unpermuted with Permuted 5

	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O
CC:	.9999	.9997	.9998	.9992	.9948	.9979	.9989	.9993	.9990	-.9976	.9921	-.9991	.9990	.9968	.9945
MS:	.0074	.0115	.0057	.0075	.0173	.0117	.0083	.0063	.0080	.0122	.0189	.0070	.0080	.0130	.0154
	G	F	C	D	K	B	A	N	M	E	O	J	I	H	L

M=.9978

M=.0105

Unpermuted with Permuted 6

	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O
CC:	.9999	.9996	.9997	.9976	.9957	.9986	.9979	.9988	.9982	.9982	.9957	.9979	.9989	-.9976	.9961
MS:	.0097	.0130	.0059	.0129	.0158	.0097	.0113	.0081	.0107	.0105	.0139	.0108	.0084	.0113	.0129
	G	F	E	D	C	B	A	N	M	K	H	J	I	O	L

M=.9980

M=.0110

Unpermuted with Permuted 7

	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O
CC:	.9992	.9976	.9993	-.9847	.9868	-.9844	-.9963	.9767	-.9951	-.9256	.9905	-.8295	.9955	-.9872	.9269
MS:	.0220	.0310	.0103	.0334	.0275	.0325	.0149	.0361	.0173	.0634	.0207	.0936	.0168	.0260	.0570
	N	M	L	E	B	J	A	G	H	K	I	O	C	D	F

M=.9717

M=.0338

Table 43. Factor Comparison Between Unpermuted and Permuted 1,2,3,4,5,6, and 7 Using Varimax, Rotated (15) Factors

Unpermuted with Permuted 1

	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O
CC:	.9998	.9999	.9997	.9995	.9995	.9991	.9996	.9995	.9996	.9997	.9963	.9990	.9996	.9993	.9995
													M=.9993		
MS:	.0087	.0052	.0069	.0059	.0063	.0114	.0057	.0061	.0073	.0045	.0129	.0084	.0054	.0086	.0048
	N	B	C	D	E	F	G	H	I	J	K	L	M	A	O
													M=.0072		

Unpermuted with Permuted 2

	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O
CC:	-.9983	.9999	.9997	.9991	.9983	-.9940	.9989	.9969	-.9934	.9994	.9797	.9926	.9990	-.9968	.9989
													M=.9967		
MS:	.0294	.0061	.0068	.0085	.0114	.0304	.0087	.0152	.0132	.0067	.0302	.0244	.0084	.0204	.0072
	A	N	C	D	F	I	G	H	M	J	K	L	B	O	E
													M=.0151		

Unpermuted with Permuted 3

	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O
CC:	-.9990	.9997	.9998	.9994	.9989	-.9992	.9985	.9984	.9970	.9974	-.9983	.9933	-.9993	-.9966	.9961
													M=.9981		
MS:	.0214	.0093	.0060	.0069	.0091	.0108	.0102	.0108	.0193	.0148	.0087	.0230	.0072	.0222	.0136
	A	M	L	D	E	I	G	H	N	K	J	C	F	B	O
													M=.0129		

Unpermuted with Permuted 4

	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O
CC:	.9999	.9998	.9998	.9994	.9997	.9998	.9996	.9996	-.9996	.9998	.9982	.9990	.9999	.9997	-.9994
													M=.9995		
MS:	.0039	.0067	.0050	.0074	.0052	.0051	.0054	.0051	.0063	.0042	.0090	.0086	.0032	.0052	.0052
	G	B	C	D	E	F	I	N	A	J	O	L	M	H	K
													M=.0057		

Table 43. (continued)

Unpermuted with Permuted 5

	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O
CC:	.9999	.9999	.9998	-.9993	-.9984	.9995	.9997	.9997	.9998	.9995	-.9944	.9987	-.9990	.9992	.9991
													M=.9991		
MS:	.0056	.0058	.0060	.0074	.0108	.0074	.0048	.0048	.0049	.0065	.0160	.0093	.0093	.0092	.0067
	G	F	C	L	J	B	A	N	H	M	I	D	E	K	O
													M=.0076		

Unpermuted with Permuted 6

	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O
CC:	.9994	.9997	.9999	.9998	.9997	.9977	.9989	.9985	.9994	.9999	-.9945	.9971	.9996	.9990	.9991
													M=.9988		
MS:	.0171	.0088	.0048	.0043	.0047	.0194	.0088	.0106	.0081	.0031	.0157	.0145	.0056	.0105	.0068
	G	F	E	D	C	B	A	N	M	K	L	J	I	H	O
													M=.0095		

Unpermuted with Permuted 7

	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O
CC:	-.9968	.9999	.9993	.9985	.9993	-.9944	-.9995	.9962	-.9989	-.9983	.9895	.9888	.9989	-.9926	.9984
													M=.9966		
MS:	.0392	.0063	.0108	.0108	.0072	.0301	.0058	.0170	.0121	.0118	.0217	.0301	.0089	.0336	.0088
	E	N	L	K	B	F	A	G	H	J	M	I	C	D	O
													M=.0169		

Table 44. Factor Comparison Between Unpermuted and Permuted 1,2,3, and 4 Using Quartimax, Rotated (6) Factors

Unpermuted with Permuted 1

	A	B	C	D	E	F	
CC:	1.0000	.9999	.9997	1.0000	.9998	-.9999	M=.9999
RMS:	.0051	.0074	.0066	.0018	.0038	.0024	M=.0045
	F	B	C	D	A	E	

Unpermuted with Permuted 2

	A	B	C	D	E	F	
CC:	-1.0000	1.0000	1.0000	.9993	.9998	.9986	M=.9996
RMS:	.0031	.0032	.0022	.0077	.0038	.0100	M=.0050
	E	F	C	D	B	A	

Unpermuted with Permuted 3

	A	B	C	D	E	F	
CC:	-1.0000	1.0000	.9999	.9993	.9996	.9989	M=.9996
RMS:	.0031	.0029	.0036	.0074	.0055	.0091	M=.0053
	E	F	D	C	B	A	

Unpermuted with Permuted 4

	A	B	C	D	E	F	
CC:	.9999	.9998	.9995	.9998	.9998	.9995	M=.9997
RMS:	.0066	.0101	.0092	.0037	.0046	.0058	M=.0067
	C	B	A	F	E	D	

Table 45. Factor Comparison Between Unpermuted and Permuted 1,2,3, and 4 Using Varimax, Rotated (6) Factors

Unpermuted with Permuted 1

	A	B	C	D	E	F	
CC:	-1.0000	1.0000	.9999	.9997	1.0000	-.9997	M=.9999
RMS:	.0037	.0029	.0044	.0049	.0025	.0051	M=.0039
	B	F	C	D	A	E	

Unpermuted with Permuted 2

	A	B	C	D	E	F	
CC:	-.9999	1.0000	.9996	.9995	.9997	.9983	M=.9995
RMS:	.0071	.0049	.0092	.0067	.0076	.0145	M=.0083
	E	F	C	D	B	A	

Unpermuted with Permuted 3

	A	B	C	D	E	F	
CC:	-.9999	.9999	.9989	.9994	.9992	.9984	M=.9993
RMS:	.0049	.0048	.0152	.0076	.0105	.0130	M=.0093
	E	F	D	C	B	A	

Unpermuted with Permuted 4

	A	B	C	D	E	F	
CC:	.9999	1.0000	.9992	.9996	.9997	-.9990	M=.9996
RMS:	.0057	.0047	.0125	.0064	.0058	.0104	M=.0076
	C	B	A	F	D	E	

Tables 46 through 48 contain the factors which exhibit large fluctuations in their high loadings for the unpermuted with permutations 2, 4, and 7 comparison. Factor L appears to be the one exhibiting the largest fluctuation and RMS.

Only one of the 19 negative CCs represented a complete reversal in factor loading signs.

Secondly, we will look at the factor matrices of the permutations under varimax, rotated (15) factors.

Here, the unpermuted with permutations 2 and 7 comparison were the only ones to exhibit large fluctuations in their high loadings and are presented in Tables 49 and 50. The differences in the factor loadings are much less than in the quartimax case.

Only three of the 22 negative CCs represented a complete reversal in factor loading signs.

Thirdly, we will look at the factor matrices of the permutations under quartimax, rotated (6) factors.

There were no factors which exhibited large enough fluctuations in their high loadings to be reported.

All three of the negative CCs represented a complete reversal in factor loading signs.

Finally, we will look at the factor matrices of the permutations under varimax, rotated (6) factors.

Tables 51 and 52 contain the factors exhibiting large fluctuations in their high loadings for the unpermuted with permutations 2 and 3 comparison. However, these fluctuations are small enough not to affect the interpretability of the factors.

Table 46. Comparisons of High Factor Loadings On Selected Pairs of Factors from Unpermuted Quartimax, Rotated (15) with Permuted 2

Factor B with Factor M
All loadings above .70 reported

Name of Variable	Loadings	
	Unpermuted	Permuted 2
15. Cubes	.7636	.7775
17. Flags	.8291	.8425
18. Form Board	.7356	.6952*
19. Lozenges B	.7546	.7383
21. Punched Holes	.7177	.6545*

Factor E with Factor F
All loadings above .30 reported

Name of Variable	Loadings	
	Unpermuted	Permuted 2
44. Initials	-.3362	-.3035
45. Number-Number	-.4719	-.4499
46. Word Recognition	-.3124	-.2554*
47. Figure Recognition	-.7471	-.7412
56. Word Count	.3073	.3867

Factor F with Factor E
All loadings above .30 reported

Name of Variable	Loadings	
	Unpermuted	Permuted 2
6. Controlled Association	-.2236*	.3000
9. Disarranged Words	-.3592	.3363
10. First and Last Letters	-.5472	.6331
12. Anagrams	-.7404	.7241
44. Initials	-.2507*	.3263

Table 46. (continued)

Factor J with Factor L
All loadings above .30 reported

Name of Variable	Loadings	
	Unpermuted	Permuted 2
10. First and Last Letters	-.3008	.1280*
23. Identical Forms	-.0623*	.4570
26. Areas	.3494	-.5802
31. Division	.3806	-.1838*
35. Numerical Judgement	.7049	-.5203
36. Arithmetical Reasoning	.4384	-.3542

Factor L with Factor I
All loadings above .30 reported

Name of Variable	Loadings	
	Unpermuted	Permuted 2
3. Verbal Classification	.3273	.0906*
16. Lozenges A	-.1695*	-.3258
21. Punched Holes	-.3205	-.3258
23. Identical Forms	.6472	.3183
26. Areas	-.4072	-.0853*
46. Word Recognition	-.3342	-.5173

Factor O with Factor J
All loadings above .25 reported

Name of Variable	Loadings	
	Unpermuted	Permuted 2
35. Numerical Judgement	.0789*	.2516
38. Verbal Analogies	-.2354*	-.2739
40. Code Words	-.3213	-.3217
45. Number-Number	-.2538	-.1515*
46. Word Recognition	.2925	.1305*
53. Spelling	.5617	.6089
54. Grammar	.1985*	.2725

Table 47. Comparisons of High Factor Loadings on a Selected Pair of Factors from Unpermuted Quartimax, Rotated (15) with Permuted 4

Factor L with Factor K
All loadings above .30 reported

Name of Variable	Loadings	
	Unpermuted	Permuted 4
3. Verbal Classification	.3273	.2905*
21. Punched Holes	-.3205	-.3411
23. Identical Forms	.6472	-.6122
26. Areas	-.4072	-.3766
46. Word Recognition	-.3342	-.3888

Table 48. Comparisons of High Factor Loadings On Selected Pairs of Factors from Unpermuted Quartimax, Rotated (15) with Permuted 7

Factor B with Factor M
All loadings above .70 reported

Name of Variable	Loadings	
	Unpermuted	Permuted 7
15. Cubes	.7636	.7762
17. Flags	.8291	.8393
18. Form Board	.7356	.6968*
19. Lozenges B	.7546	.7376
21. Punched Holes	.7177	.6556*

Factor H with Factor G
All loadings above .30 reported

Name of Variable	Loadings	
	Unpermuted	Permuted 7
11. Disarranged Sentences	-.3440	-.2813*
33. Estimating	-.7879	-.8091
50. Hands	-.3193	-.2844*
55. Vocabulary (Chicago)	.3003	.2537*

Factor J with Factor K
All loadings above .30 reported

Name of Variable	Loadings	
	Unpermuted	Permuted 7
10. First and Last Letters	-.3008	.2227*
26. Areas	.3494	-.5142
31. Division	.3806	-.2946*
35. Numerical Judgement	.7049	-.6513
36. Arithmetical Reasoning	.4384	-.4262

Table 48. (continued)

Factor L with Factor O
All loadings above .30 reported

Name of Variable	Loadings	
	Unpermuted	Permuted 7
3. Verbal Classification	.3273	-.1473*
21. Punched Holes	-.3205	.3526
23. Identical Forms	.6472	-.4217
26. Areas	.4072	.2278*
46. Word Recognition	-.3342	.5420

Factor N with Factor D
All loadings above .30 reported

Name of Variable	Loadings	
	Unpermuted	Permuted 7
18. Form Board	.3033	-.3431
37. Reasoning	-.3456	.2986*
39. False Premises	-.5990	.5526
45. Number-Number	-.2776*	.3207
55. Vocabulary (Chicago)	.4015	-.4431

Factor O with Factor F
All loadings above .25 reported

Name of Variable	Loadings	
	Unpermuted	Permuted 7
40. Code Words	-.3213	-.3263
45. Number-Number	-.2538	-.1907*
46. Word Recognition	.2925	.1368*
53. Spelling	.5617	.6154
54. Grammar	.1985*	.2763

Table 49. Comparisons of High Factor Loadings on Selected Pairs of Factors from Unpermuted Varimax, Rotated (15) with Permuted 2

Factor F with Factor I
All loadings above .40 reported

Name of Variable	Loadings	
	Unpermuted	Permuted 2
7. Inventive Opposites	-.3737*	.4147
8. Completion	-.3547*	.4181
9. Disarranged Words	-.5304	.6069
10. First and Last Letters	-.7013	.6924
12. Anagrams	-.8014	.7893
13. Inventive Synonyms	-.4806	.5493
54. Grammar	-.4418	.4678

Factor K with Factor K
All loadings above .30 reported

Name of Variable	Loadings	
	Unpermuted	Permuted 2
6. Controlled Association	.4106	.4793
9. Disarranged Words	-.5139	.4481
13. Inventive Synonyms	-.3371	-.2671*
24. Pursuit	.4937	.4833

Table 50 Comparisons of High Factor Loadings On Selected Pairs of Factors from Unpermuted Varimax, Rotated (15) with Permuted 7

Factor A with Factor E
All loadings above .60 reported

Name of Variable	Loadings	
	Unpermuted	Permuted 7
1. Reading I	.7613	-.7228
2. Reading II	.8180	-.7771
6. Controlled Association	.5812*	-.6025
7. Inventive Opposites	.6537	-.6151
8. Completion	.6791	-.6164
49. Theme	.6409	-.6409
55. Vocabulary (Chicago)	.9155	-.9255
57. Vocabulary (Thorndike)	.6904	-.6162

Factor K with Factor M
All loadings above .30 reported

Name of Variable	Loadings	
	Unpermuted	Permuted 7
6. Controlled Association	.4196	.4165
9. Disarranged Words	-.5139	-.4680
13. Inventive Synonyms	-.3371	-.2989*
24. Pursuit	.4937	.5069

Table 51. Comparisons of High Factor Loadings on a Selected Pair of Factors from Unpermuted Varimax, Rotated (6) with Permuted 2

Factor F with Factor A
All loadings above .40 reported

Name of Variable	Loadings	
	Unpermuted	Permuted 2
10. First and Last Letters	-.6111	-.6202
12. Anagrams	-.6531	-.6603
52. Sound Grouping	-.3869*	-.4061
54. Grammar	-.4056	-.4272

Table 32. Comparisons of High Factor Loadings
on Selected Pairs of Factors from
Unpermuted Varimax, Rotated (6) with Permuted 3

Factor C with Factor D
All loadings above .55 reported

Name of Variable	Loadings	
	Unpermuted	Permuted 3
27. Number Code	.5936	.5967
28. Addition	.7527	.7435
29. Subtraction	.6964	.6830
30. Multiplication	.8179	.8060
31. Division	.7562	.7575
32. Tabular Completion	.5592	.5825
35. Numerical Judgement	.5982	.6190
36. Arithmetical Reasoning	.5427*	.5707

Factor F with Factor A
All loadings above .40 reported

Name of Variable	Loadings	
	Unpermuted	Permuted 3
10. First and Last Letters	-.6111	-.6151
12. Anagrams	-.6531	-.6579
52. Sound Grouping	-.3869*	-.4008
54. Grammar	-.4056	-.4216

Three of the five negative CCs represented a complete reversal in factor loading signs.

The final step in evaluating the effects of permutation on factor rotation was to compute the sum of fourth powers of the factor loadings of each of the rotated solutions obtained by permutation and also to determine this sum for the unpermuted rotated solutions. The values obtained are presented in Table 53. The asterisks designate values greater than those for the particular unpermuted rotated solution. As can be observed in Table 53, for the quartimax, rotated (15), a sum of fourth powers greater than that for the unpermuted case was obtained in the first and third permutations. For the quartimax, rotated (6), a sum of fourth powers greater than that for the unpermuted case was obtained in the first, third, and fourth permutations. For both the varimax solutions none of the sums of fourth powers under the permutations exceeded that of the unpermuted case.

In order to see if it was possible to obtain similar results using the same permutational scheme on a different matrix, Holzinger and Swineford's 24-variable correlation matrix was employed.

The six highest eigenvalues of Holzinger and Swineford's matrix are presented in Table 54. The permutations that were used on the Thurstone principal axes solution when only six factors were rotated will be employed here. Each of the four permutations were performed in conjunction with a quartimax and varimax rotation using the six factors. The proportions

Table 33. Sum of Fourth Powers
of Final Rotated Factor Loadings

	Unpermuted	Permuted 1	Permuted 2	Permuted 3	Permuted 4	Permuted 5	Permuted 6	Permuted 7
uartimax otated (15)	15.6980462	* 15.7019142	15.5950886	* 15.7027003	15.6915350	15.6977664	15.6950441	15.6295680
arimax otated (15)	14.4788453	14.4429415	14.3464797	14.3727603	14.4775574	14.4385618	14.3935787	14.2888275
uartimax otated (6)	12.3170327	* 12.3256749	12.3147659	* 12.3187169	* 12.3278936			
arimax otated (6)	11.6797762	11.6781400	11.6468473	11.6443567	11.6361416			

Table 54. Eigenvalues

(5)	8.1423
(1)	2.1006
(11)	1.6889
(2)	1.5022
(12)	1.0180
(15)	.9479

$$\sum = 15.3999$$

of variance accounted for by each factor under the unpermuted and permuted conditions are presented in Table 55. As was observed above, the permutations affect the size of the proportions of variance accounted for, but the total amount of variance accounted for by each set of factors is quite stable.

The factor comparison program was applied to both of the separate rotational solutions comparing the unpermuted rotation to each of the permuted rotations. The results for the two sets of comparisons are summarized in Tables 56 and 57.

If we compute the mean of the means reported in these two tables we have, for the quartimax results, a mean CC of .9994 and a mean RMS of .0099. For the varimax results we have a mean CC of .9999 and a mean RMS of .0051.

From these results it appears that the permutations are having more of an effect (even though small) on the quartimax than the varimax solution.

We will now look at the factor matrices of each permutation under each of the rotations, beginning with those resulting from the permutations using quartimax, rotated (6) factors.

When we looked at the actual factor loadings, for the unpermuted with permuted 1, 2, 3, and 4 comparisons, no pair of factors was found exhibiting large fluctuations in its high loadings.

Two of the four negative CCs represented a complete reversal in factor loading signs.

Table 55. Proportions of Variance Accounted for by Rotated Factors After Permutation

Quartimax, Rotated (6)

	Unpermuted	Permute 1	Permute 2	Permute 3	Permute 4
A	.1981	.0456	.1431	.0653	.1197
B	.1213	.1925	.0650	.1366	.1340
C	.1356	.1213	.0459	.0745	.2013
D	.0771	.0756	.0760	.0460	.0459
E	.0642	.0637	.1202	.1200	.0661
F	.0453	.1429	.1915	.1992	.0747
	$\sum$=.6416	$\sum$=.6416	$\sum$=.6417	$\sum$=.6416	$\sum$=.6417

Varimax, Rotated (6)

	Unpermuted	Permute 1	Permute 2	Permute 3	Permute 4
A	.1748	.1231	.1251	.0786	.0585
B	.1239	.1745	.0772	.1236	.1752
C	.1231	.0588	.0587	.0823	.1233
D	.0841	.0830	.0828	.0586	.1244
E	.0770	.0777	.1733	.1231	.0767
F	.0587	.1245	.1245	.1753	.0836
	$\sum$=.6416	$\sum$=.6416	$\sum$=.6416	$\sum$=.6415	$\sum$=.6417

Table 56. Factor Comparison Between Unpermuted and Permuted 1,2,3, and 4 Using Quartimax, Rotated (6) Factors

Unpermuted with Permuted 1

	A	B	C	D	E	F	
CC:	.9998	.9998	.9992	.9998	.9994	.9991	M = .9995
RMS:	.0112	.0065	.0179	.0067	.0085	.0093	M = .0100
	A	B	C	D	E	A	

Unpermuted with Permuted 2

	A	B	C	D	E	F	
CC:	.9997	.9997	.9991	.9997	.9990	-.9984	M = .9993
RMS:	.0131	.0087	.0185	.0066	.0113	.0120	M = .0117
	F	E	A	D	B	C	

Unpermuted with Permuted 3

	A	B	C	D	E	F	
CC:	1.0000	.9998	-.9998	.9994	.9990	-.9984	M = .9994
RMS:	.0035	.0069	.0072	.0109	.0113	.0123	M = .0087
	F	E	B	C	A	D	

Unpermuted with Permuted 4

	A	B	C	D	E	F	
CC:	.9999	.9998	-.9999	.9995	.9989	.9983	M = .9994
RMS:	.0063	.0073	.0065	.0100	.0124	.0124	M = .0092
	C	A	B	F	E	D	

Table 57. Factor Comparison Between Unpermuted and Permuted 1,2,3, and 4 Using Varimax, Rotated (6) Factors

Unpermuted with Permuted 1

	A	B	C	D	E	F	
CC:	1.0000	1.0000	.9999	.9998	.9999	-.9997	M = .9999
RMS:	.0026	.0036	.0051	.0059	.0041	.0056	M = .0045
	B	A	F	D	E	C	

Unpermuted with Permuted 2

	A	B	C	D	E	F	
CC:	.9999	.9999	.9999	.9998	.9999	-.9997	M = .9999
RMS:	.0051	.0041	.0062	.0062	.0047	.0061	M = .0054
	E	A	F	D	B	C	

Unpermuted with Permuted 3

	A	B	C	D	E	F	
CC:	.9999	.9998	-.9998	.9998	.9998	-.9996	M = .9998
RMS:	.0044	.0070	.0067	.0064	.0067	.0068	M = .0063
	F	E	B	C	A	D	

Unpermuted with Permuted 4

	A	B	C	D	E	F	
CC:	.9999	1.0000	1.0000	.9998	.9999	-.9999	M = .9999
RMS:	.0049	.0023	.0031	.0060	.0038	.0037	M = .0040
	B	D	C	F	E	A	

Next we will look at the factor matrices of the permutations under varimax, rotated (6) factors.

For the cases of the unpermuted with permuted 1, 2, 3, and 4 comparisons, we also found no pair of factors exhibiting large fluctuations in its high loadings.

Two of the five negative CCs represented a complete reversal in factor loading signs.

The final step in evaluating the effects of permutation on factor rotation was to compute the sum of fourth powers of the factor loadings of each of the rotated solutions obtained by permutation and also to determine this sum for the unpermuted rotated solutions. The values obtained are presented in Table 58, and the notation will be the same as that used in Table 53. As can be observed in Table 58, for the quartimax, rotated (6), a sum of fourth powers greater than that for the unpermuted case was obtained in the third and fourth permutations. For the varimax, rotated (6), a sum of fourth powers greater than that for the unpermuted case was obtained in all four of the permutations.

Comparison of the Two Methods of Factor Comparison

In order to determine the extent to which both the CC and the RMS were related in identifying those pairs of factors that exhibited large fluctuations in their high loadings, a correlation coefficient was computed between them. The correlation coefficient, -.9538, indicates that there is a very high negative relationship between the CC and the RMS with respect

Table 58. Sum of Fourth Powers of Final Rotated Factor Loadings

	Unpermuted	Permuted 1	Permuted 2	Permuted 3	Permuted 4
Quartimax Rotated (6)	6.05278094	6.04077410	6.03836025	* 6.05503227	* 6.05842989
Varimax Rotated (6)	5.97695236	* 5.97697049	* 5.97702902	* 5.97915706	* 5.98258536

to the 20 pairs of factors showing large fluctuations in their high loadings. This result seems to be in harmony with the purposes of the two statistics, the CC is a measure of similarity and the RMS is a measure of dissimilarity.

CHAPTER V

DISCUSSION

Problem I: Error

Effects of Random Error on Factor Calculation

From the results presented in Tables 3, 4, and 5 it appears that when random error is introduced into the entire correlation matrix, all but the first three pairs of factors have large fluctuations in their high loadings. Of these 13 pairs of factors almost all, with the possible exception of Factor L with Factor K, would have differed, psychologically. In the case of the 5% error introduction, four pairs of principal axes factors were observed to have large fluctuations in their high loadings and in all of these the interpretations would probably be psychologically different for the members of each pair. When only 1% of the total number of correlation coefficients are subject to certain error introductions, changes in the principal axes factor loadings hardly seem enough to affect their interpretability.

Another interesting and expected observation about Table 3 is that as the number of correlations, influenced by random error, was decreased, so was the mean of the root mean squares for their corresponding factor matrices.

This suggests that if the correlations in some matrix vary because of error in some random fashion, then only the first few principal axes factors would be psychologically the same as those derived from the errorless correlation matrix. But, if these errors can be detected to affect 5% of the correlations then a few factors, probably in the last half of the principal axes factor matrix, should start showing wide differences from the comparable factors in the errorless principal axes factor matrix. And finally, if these errors can be detected to affect 1% of the total number of correlation coefficients of a given matrix, the technique of factor analysis should yield a set of principal axes factors psychologically the same as those derived from the same correlation matrix without the error influence.

There are several reasons why (a) negative coefficients of congruence (CCs) are presented in the results, (b) references are made as to whether or not the negative CCs represented a complete reversal in factor loading signs, and (c) the high loadings having opposite signs on comparative factors are presented:

1.--When looking for factorial variation, the factorial configuration (pattern of positive, zero, and negative factor loadings) was of prime consideration. In order to reduce the large number of comparisons that could be made, investigations of configurational variability were made on all pairs of factors having negative CCs.

2.--Whether or not the signs of similar factors are complete reversals of one another does not have any effect on the CC, but it does have an inflationary effect on the root mean square (RMS). Therefore, the negative CCs were needed to determine those factors that should have one of their pair reflected in order to calculate an accurate RMS.

3.--The signs of the high loadings on selected pairs of factors were presented as they were calculated in order to emphasize that even when the high loadings on two factors would have opposite signs, the rest of their loadings would not necessarily follow the same pattern.

The fact that only one of the 12 negative coefficients of congruence, found in Table 3, represented a complete reversal in factor loading signs, indicates that the factorial configurations of the random, 5% and 1% error factor matrices are not remaining invariant when Thurstone (1947) would have maintained that they should.

Effects of Random Error on Factor Rotation

After looking at the effects of the 5% and 1% error on the rotated factor solutions, the first thing that was noticed in Table 7 was that the number of rotated factors extracted, using the Kiel-Wrigley criterion, varied more with the varimax than the quartimax solution. But when the mean of the mean CCs was calculated from Table 9 for both the 5% error and 1% error case, varimax appeared to be the more stable of the two. Therefore, the mean of the mean CCs for both the 5% error and the 1% error without the last (and most dissimilar) factor comparison in each rectangle of Table 9 was computed for the Kiel-Wrigley rotations in order to see if the same results would be observed. This latter computation pointed out that the varimax rotated solution was more affected by the error introduction than the quartimax and, therefore, supported the conclusions made from Table 7.

After looking at the actual factor loading comparisons it was found that this same pattern was holding up on all three levels of error introduction. At the random error level using the quartimax rotation, only 5 of the 18 pairs of factors reported in Tables 10 and 16, would be psychologically the same. But, of the 21 pairs of factors for the varimax rotation reported in Tables 13 and 18, only 3 of them would be psychologically the same. At the 5% error level using the quartimax rotation, only two of the four pairs of factors reported in Tables 11 and 17, would be psychologically the same. But, only one (Factor O with Factor O) of the six pairs of factors for the varimax rotation reported in Tables 14 and 19, would be the same. At the 1% error level using the quartimax rotation only one of the two factors reported in Table 12, would be the same. But, of the six pairs of factors for the varimax rotation reported in Tables 15 and 20, four would be the same.

The method used to determine whether or not two factors would be interpreted as psychologically the same was to select a certain cut off level that would define the salient variables in the errorless factor and then apply that same cut off level to the error factor and see how the size and positions of the loadings of the salient variables compared. Admittedly, this was a somewhat arbitrary technique of determining whether or not the factors were psychologically the same, but the literature on factor analysis (e.g. Thurstone, 1947) indicates no better method of determining this. As a means of adding some statistical sophistication to this subjective approach, Spearman's rank correlation coefficient (Siegel, 1956, p. 202) was used on the salient variables reported in Tables 4 through 20. Of the 17 pairs of factors purported to be psychologically the same by the subjective method above, only four were found to have significant (two at the .01 level and two at the .05 level) rank correlations indicating that they were the most similar factors. Four other pairs of factors, not considered to be psychologically the same, were found to have significant (one at the .01 level and three at the .05 level) rank correlations. Of the eight significantly related pairs of factors only one was found in the principal axes results and that was Factor L with Factor K presented in Table 4.

None of the 50 negative coefficients of congruence, found in Tables 8 and 9, represented a complete reversal in factor loading signs, indicating again that the factorial configurations are not remaining invariant.

The results of the error introduction on factor calculation and rotation has pointed out that the number of rotated factors, the interpretations of a varying number of factors, and the factorial configurations do change when certain minor changes are imposed on the correlation coefficients. The assumption that factor analysis is invariant when it is carried out on a fixed set of variables and individuals does not hold even if we alter only a small number of correlations within their respective standard errors. It is also of interest to note that in both the principal axes and rotated solutions the degree of dissimilarity between the factor matrices (as determined by the RMS) was increased as the amount of error was increased. This can best be exemplified by pointing out that the mean increase in differences for the principal axes factor loadings was from .0110 to .0886 and for the rotated factor loadings it was from .0305 to .1112. More specifically, the error introductions ranged from .01 to .06, but the factor loading differences for the salient variables ranged from .00 to .54 for the principal axes factors and from .00 to .82 for the rotated factors.

A definite pattern has, also, been established for the effect of random error on the quartimax and varimax rotational solutions. The random, 5%, and 1% error introduction affect the results of both of the rotational techniques, but the effect on the quartimax is much smaller than the effect on the varimax rotation.

The attempt here to show the effect of error introduction was not meant to be exhaustive. It was simply one way of trying to see how the factor calculation would be affected by altering first, all and then only a few of the correlation coefficients. All the generalizations that might be made must take into account the specific correlation matrix used here as well as the particular method of error introduction.

Since so little has been done in investigating the effect of error on a fixed set of variables and individuals, the present attempt might simply serve as a stimulus or directive to further explore how the "random errors" of correlation coefficients can change factorial interpretation. A few of the problems that might bear investigating are:

1.--Exactly what percent of the correlations must be changed before the interpretations of the resulting rotated factors would be altered?

2.--How would the introduction of a small percent of each correlation coefficient's standard error affect the resulting rotated factorial interpretation?

3.--How would the number of variables in a correlation matrix influence the above two problems and the results of the present discussion?

4.--Would the results of the above three problems support the conclusion that varimax rotation is affected more by random error than quartimax rotation?

Looking at the total random error introduction from the viewpoint of sampling or selection, could mean that if one was to draw several samples of the same size from the same population, the errors that could be attributed to sampling, test unreliability, etc. would affect the outcome of most of the resulting factors beyond the first three or four. With this in mind the next logical step appeared to be an investigation of how many factors would be affected by a sampling approach. This could then allow one to see if the influence of error, artifically introduced, would be the same as that resulting from empirical sampling procedures.

Problem II: Sampling

Effects of Sampling on Factor Calculation

From the results presented in Tables 21 through 26, it appears that when a sample of 1600 is selected from a total population of 2322 the differences in the principal axes factor loadings are not large enough to affect the psychological interpretability of the factors. In the case of the sample of 400, one (Factor D with Factor E) of the three pairs reported in Table 23 would be psychologically the same. This one was also found to be significantly similar by Spearman's rank correlation method. In the case of the sample of 100, only Factor A with Factor A would have been similarly interpreted, but Factor D with Factor D had the significant rank correlation. Finally, in the case of both the samples of 25 and 17, all pairs of factors would have had different psychological interpretations for each of the members, but Factor D with Factor F in Table 25 was found to be significantly similar by Spearman's rank correlation method.

None of the four negative coefficients of congruence, found in Table 22, represented a complete reversal in factor loading signs.

Looking at the mean differences (via the RMS) for the principal axes factor loadings, one sees that they increase from .0271 for the sample of 1600 to .2190 for the sample of 25 with a slight decrease to .1998 for the sample of 17.

These results seem to indicate that if one had a defined population of 2322 individuals, taking 400 (or 17%) of this population as a sample would not yield the same factor pattern as would be observed on the total population. In fact the smaller the sample the more dissimilar the patterns.

Effects of Sampling on Factor Rotation

After looking at the effects of sampling on the rotated factor solutions the first thing that was noticed from Table 27 was that the number of rotated factors extracted, using the (K-W) criterion, varied only with the quartimax solution. Even when the mean of the mean CCs for both the 400 and 17 samples were calculated, without the last (and most dissimilar) factor comparison in each rectangle of Table 28 for the two (K-W) rotations, it was still found that for all samples, except N = 100, the varimax solution had higher coefficients of congruence and lower root mean squares. This indicated that the quartimax rotated solution was more affected by sampling than the varimax.

After looking at the actual factor loading comparisons, it was found that this same pattern was holding up on all five samples. In the sample of 1600, using the quartimax rotation, only Factor E with Factor E would have different psychological interpretations for the members of the pair. But, all of the members of the pairs of factors for the varimax rotation would have the same psychological interpretations. In the sample of 400, using the quartimax rotation, the three pairs of factors reported in Table 32 would have different psychological interpretations for the members of each pair. But, all of the members of the pairs of factors for the varimax rotation would have the same interpretations. In the sample of 100, using the quartimax rotation, the one pair of factors reported in Table 33 would have different psychological interpretations for the members of the pair, but it also was found to be significantly similar by Spearman's rank correlation method. However, the three pairs of factors for the varimax rotation reported in Table 36 would have different psychological interpretations for the members of each pair, but Factor D with Factor D was found to have a significant rank correlation. In the sample of 25, using the quartimax rotation, the three pairs of factors reported in Table 34 would have different psychological interpretations for the members of each pair. However, only two pairs of factors for the varimax rotation reported in Table 37 would have different psychological interpretations for the members of each pair. In the sample of 17, using the quartimax rotation, the eight pairs of factors reported in Tables 34 and 39 would have different psychological interpretations for the members of each pair. However, only six pairs of factors (using Table 38 also for varimax, rotated (6) factors) for the varimax rotation reported in Table 38 would have different psychological interpretations for the members of each pair.

If one had, on the other hand, only looked at the actual factor loading comparisons in Tables 31 through 39 in which at least six factors had been used from each sample, then the ratio of the psychologically different factors in the quartimax and varimax rotations would have been nine to eight instead of 16 to 11.

Of the 24 pairs of factors purported to be psychologically different by the subjective method, only two were found to have significant rank correlations between their salient variables. The two pairs of factors which had significant rank correlations, indicating that they were the most similar factors, would definitely not have been judged to be psychologically the same.

Looking at the mean differences (via the RMS) for the quartimax factor loadings one can see that they increase from .0558 for the sample of 1600 to .2923 for the sample of 17. In the case of the varimax factor loadings, the mean differences increase from .0386 for the sample of 1600 to .2139 for the sample of 17.

It is also interesting to note that none of the five negative coefficients of congruence reported in Table 28 represented a complete reversal in factor loading signs. However, two of the four negative coefficients of congruence reported in Table 29 did represent a complete reversal in factor loading signs.

The results of sampling on factor calculation and rotation has again pointed out that the number of rotated factors, the interpretations of a varying number of factors, and the factorial configurations do change when different sample sizes are selected from one large population. The assumption, therefore, that at least configurational invariance should exist when drawing samples from a particular population, does not find support here.

What this aspect of the study seems to be saying is that there probably exists an optimal sample level which would allow one to be able adequately to depict the loadings for the entire population. According to the results presented here, that level is somewhere above 400, which is about 17% of the total population, for the principal axes solution and varies for the two rotational solutions. On the other hand, if samples are selected which are equal to or less than 17% of the total population, we can expect at least 50% of the factors to change.

A definite pattern has, also, been established for the effect of sampling on the quartimax and varimax rotational solutions, with varimax being the more stable of the two.

A point worth emphasizing here is that in both the error introduction and sampling approaches there were more psychologically different factors in the principal axes case than in the rotated factor solutions. Also, the observation that the quartimax rotation was more stable than the varimax in the error introduction case, reversed itself in the sampling case.

This approach, to show the effect of sampling error on factorial invariance, was an attempt to check some of the conclusions from the error introduction part of this study as well as to see if the assumptions put forward by Thurstone, Henrysson, and others would hold in this particular case. It was by no means an exhaustive investigation and before it could become a definitive one it would have to:

1.--Select several samples of the same size in order to obtain the most descriptive sample for a particular sample size.

2.--Determine exactly what sample size of the fixed population allows one to reproduce the total population factor structure.

3.--Determine the effect of the number of variables.

Problem III: Permutation

Effects of Permutation on Factor Rotation

From the results presented in Tables 40 and 41 for Thurstone's matrix one can see that each set of permutations changes the exact proportions of variance accounted for by each factor. Inspection of each of the resulting rotated factor matrices after permutation had taken place showed that the size of the loadings in each factor matrix differed from all others including the unpermuted one. These differences, however, were not constant and, therefore, could not be accounted for by a transformation. Also, the factorial configurations of all of the factor matrices derived after permutation were different from those derived on the unpermuted cases.

No definite trend was observed for the mean differences, reported in Tables 42 and 43, of both the quartimax and varimax factor loadings.

After looking at the results of the factor comparison program one can get some idea about which rotational solution, quartimax or varimax, is more affected by the permutations performed. As was pointed out in Chapter IV, the mean of the CCs and RMSs for the quartimax rotation on 15 factors indicated that the permutations were having a greater effect on the quartimax than the varimax rotation. If we look at Table 42 more closely we find that for the quartimax rotated (15) factors, the two permutations exhibiting the widest differences are: (1) permutation 2 which has a mean CC of .9486 and a mean RMS of .0407, and (2) permutation 7 which has a mean CC of .9717 and a mean RMS of .0338. Looking at Table 43 we find that for the varimax rotated (15) factors, the two permutations exhibiting the widest differences are: (1) permutation 2 which has a mean CC of .9967 and a mean RMS of .0151, and (2) permutation 7 which has a mean CC of .9966 and a mean RMS of .0169. These results lend further support to the conclusion that the varimax rotation is the more stable one.

After looking at the actual factor loading comparisons it was possible to see more clearly how the permutations affected the factor interpretations.

In the quartimax rotated (15) factors comparison, permutations 2 and 7 produced five pairs of factors that would have different psychological interpretations for the members of each pair. Of the seven pairs of factors judged to be psychologically the same in Tables 46 through 48, only three were found to have significant (at the .01 level) rank correlations indicating similarity.

In the varimax rotated (15) factors comparison, permutations 2 and 7 produced two pairs of factors that would have different psychological interpretations for each of the members. Of the two pairs of factors judged to be psychologically the same in Tables 49 and 50, both had significant (at the .01 level) rank correlations indicating they were the same.

In Chapter III it was further pointed out that the mean of the CCs and RMSs for the varimax rotation on six factors indicated that the permutations were having a greater effect (even if only slightly) on the varimax than the quartimax rotation.

When we look at the actual factor loading comparisons it was possible to see, more clearly, how the permutations affected the factor interpretations.

In the quartimax rotated (6) factors comparison, all the permutations produced psychologically similar factors.

In the varimax rotated (6) factors comparison, all the permutations produced psychologically similar factors. Of the three pairs of factors judged to be psychologically the same in Tables 51 and 52, all had significant (at the .01 level) rank correlations.

From the results and discussion presented above, it would be reasonable to conclude that the varimax rotational solution is more stable than the quartimax, under the seven permutations designated above, when we are rotating 15 factors. But if we are rotating only six factors, then the quartimax rotational solution appears to be slightly more stable than the varimax.

The results of the set of four permutations on Holzinger and Swineford's matrix supported the evidence obtained from Thurstone's matrix when 15 factors were employed.

The results of this section bring to light an important point in the discussion of factorial variation and that is; we have been able to demonstrate here that simply by changing the order of a fixed set of unrotated factors we were able to change the resulting rotated factor loadings, factorial configurations, and even the interpretations of some of the factors. This point is a very important step along the path of a more thorough investigation of factorial invariance on a fixed set of variables and individuals.

The evidence which served to place more of an emphasis on this point was that resulting from the computation of the sum of fourth powers of the final rotated factor loadings under each permutation and each separate rotation. The results which were presented in Table 53 from Thurstone's matrix, showed that a higher sum of fourth powers was obtained by using permutations 1 and 3 for the quartimax rotation on 15 factors and by using permutations 1, 3, and 4 for the quartimax rotation on six factors. In comparison, the results which were presented in Table 58 from Holzinger and Swineford's matrix, showed that a higher sum of fourth powers was obtained by using permutations 3 and 4 for the quartimax rotation on six factors and by using permutations 1, 2, 3, and 4 for the varimax rotation on six factors. This means that we have been able to achieve a "better" final rotated solution for both the quartimax and varimax method by placing the eigenvalues and corresponding factors in an order other than from high to low. The results further pointed out that there should be a certain level, above 15 variables, at which the varimax method will not be affected by permutations and that permutations 1 and 3 seemed to be the most consistent ones for providing higher sums of fourth powers for the quartimax method on the Thurstone matrix with varying sizes of factors rotated, but permutations 3 and 4 seemed to be the most consistent ones for providing higher sums of fourth powers for the quartimax method on the two different samples with varying number of variables.

In light of all that has been said, therefore, the next logical step, which is beyond the scope of the present study, would be:

1.--To make sure that the computer programs are carrying out the calculations according to the underlying theory and not making any modifications that could yield different results.

2.--If the programs do reflect the underlying theory and calculations then perform a thorough investigation of the effects of all possible permutations for different numbers of rotated factors and variables.

3.--Then establish a permutation or ordering algorithm which would be able to ensure that the final rotated solution can provide the highest sum of fourth powers, taking into account the number of factors rotated as well as the varying number of variables.

Comparison of the Two Methods of Factor Comparison

The correlation coefficients of -.8391, -.9538, and -.6108 between the CCs and RMSs on the 73, 20, and 44 pairs of factors, respectively, from each of the three problems seem to indicate that both statistics can do an almost equal job in indicating the similarity and dissimilarity, respectively, of those pairs of factors within a fixed sample size. On the basis of the results of the present study it appears that the use of both of these statistics would provide the most accurate means of comparing factors. The reason for recommending the use of both is that the coefficient of congruence can yield a measure of similarity of, for example, .9978 for several different combinations of factors when the actual mean differences (RMSs) of their loadings vary from .0109 to .0263. If it actually came to a choice as to which one to use, the RMS appeared to be the most stable indicator in the present study as long as the signs of one factor were not a reflection of another.

CHAPTER VI

SUMMARY

The major purpose of the present study was to determine how factors (resulting from certain error introductions, certain sampling procedures, and certain permutations of unrotated factors) computed for a fixed set of variables and individuals would vary in terms of number of factors, size of factor loadings, factorial configuration, positions of salient variables, and maximum sum of fourth powers of the factor loadings. In addition, a comparison of two separate methods of factor comparison, was included.

Effects of Random Error on Factor Structure

For the random error part of the study, the question of how much error was needed to create large differences between the errorless and error solution was answered by observing that as the error was introduced into 5% of the correlation coefficients, at least four (out of a total of 15) factors were changed in the principal axes results. The rotated solutions were similarly affected. On the other hand, if error was introduced into the entire correlation matrix then at least 10 (out of a total of 15) factors were changed for the principal axes solution. The rotated solutions were somewhat less affected.

When the factors are said to be changed due to error introduction, this was meant to include changes in the psychological interpretation of the factors. In an attempt to check the reasonableness of using subjective judgements about the salient variables to determine psychological similarity of matched pairs of factors, Spearman's rank correlation coefficient was employed. The results indicated that, of the 73 pairs of factors reported to have large fluctuations in their salient variables, 17 were purported to be psychologically the same, but only eight were found to be significantly similar.

As to how much the factor loadings varied under each level of error introduction, in the principal axes case, the total random error case showed wider deviations in the factor loadings as you went on to the later factors. There was no definite pattern for the 5% and 1% error cases. For the rotated solutions, essentially the same thing was observed.

In both the principal axes and rotated solutions, the degree of dissimilarity between the factor loadings (as determined by the root mean square) was increased as the amount of error was increased. This can best be exemplified by pointing out that the mean increase in differences for the principal axes factor loadings was from .0110 to .0886 and for the rotated factor loadings it was from .0305 to .1112. More specifically, the standard errors of the correlation coefficients ranged from .01 to .06, but when the correlations were varied within these limits the differences in the factor loadings for the salient variables ranged from .00 to .54 for the principal axes and from .00 to .82 for the rotated factors.

In general, the quartimax rotational solution appeared to be more stable than the varimax under the three levels of error introduction.

Inspection of the factorial configurations of the factor structure indicated that they were definitely being affected by the error introductions.

Effects of Sampling on Factor Structure

For the sampling part of the study, the answer to the question of how large a sample size should be used to adequately depict a finite population's factor structure, has been attempted in a limited way. It was found that by selecting five separate random samples of varying sizes, a sample somewhere above 17%, but below 70%, of the total population should reproduce, fairly closely, the total population factor structure.

When six principal axes factors, from each of the five samples, were compared to those of the total population, 19 matched pairs would have had different psychological interpretations. Two of these occurred in the 400 sample, five in the 100, and all six in both the 25 and 17 samples. Of these 19 pairs, two were found to have significant rank correlations indicating similarity when, in fact, the factors were not similar.

When the same comparisons were made with the rotated factors, nine pairs would have had different psychological interpretations for the quartimax rotation and eight would have for the varimax rotation. One significant rank correlation was found in the quartimax and varimax results. However, these two pairs of factors still would be interpreted differently.

In both the principal axes and rotated solutions, the mean differences of the factor loadings (as determined by the root mean square) increased as the sample size was decreased. For the principal axes case they increased from .0271 for the 1600 sample to .2190 for the sample of 25 with a slight decrease to .1998 for the sample of 17. For the rotated solutions, the quartimax factor loadings increased from .0558 for the 1600 sample to .2923 for the sample of 17, but the varimax loadings increased from .0386 for the 1600 sample to .2139 for the sample of 17.

As in the error part of the study, the principal axes solution was found to have a larger number of dissimilar factors than the rotated solutions even though the mean differences of the principal axes factor loadings were lower than the rotated loadings. Also, as the size of the sample was reduced so was the similarity of the factor matrices.

In contrast to the error part of the study, the varimax rotational solution appeared to be the most stable when samples are drawn from one large population.

Here, again, inspection of the factorial configurations of the factor structures indicated that they were definitely being affected by the sampling procedure.

Effects of Permutation on Factor Structure

When certain sets of permutations were applied to the unrotated factors of Thurstone's data, definite changes were observed in the rotated factor loadings.

Of the 13 pairs of factors reported to have large fluctuations in their salient variables, for the quartimax rotations, seven were purported to be psychologically the same and three of those were found to have a significant rank correlation indicating similarity.

Of the seven pairs of factors reported to have large fluctuations in their salient variables, for the varimax rotations, five were purported to be psychologically the same and all five were found to have significant rank correlations.

The results of a set of four permutations on Holzinger and Swineford's data supported the evidence obtained from Thurstone's data.

The mean differences of the factor loadings exhibited no definite trend via the permutation employed and ranged from .0039 to .0407 for Thurstone's data and from .0040 to .0117 for Holzinger and Swineford's data.

The evidence resulting from the calculation of the sum of fourth powers of the rotated factor loadings indicated that ordering the eigenvalues and corresponding unrotated factors from high to low, did not yield the highest sum of fourth powers needed to satisfy the quartimax rotational criterion. What particular order is required to produce the highest sum of fourth powers was not determined, but several orderings used in the study did produce higher sums.

In agreement with the sampling part of the study, the varimax rotational solution was found to be the most stable and the factorial configurations were being affected as a result of the permutations.

Comparison of the Two Methods of Factor Comparison

The comparison of the two separate methods of factor comparison indicated that both were doing an almost equivalent job in identifying highly similar and highly dissimilar factors within a fixed sample size.

Questions Fostered by the Study

Some of the questions for further investigation fostered by this study are:

1.--Exactly what percent of the total number of correlations must be changed before the interpretations of the resulting rotated factors would be altered?

2.--How would the introduction of a small percent of each correlation coefficient's standard error affect the resulting rotated factorial interpretation?

3.--How would the number of variables in a correlation matrix influence the above two questions and the results of the present study?

4.--Would the results of the above three questions support the conclusion that the varimax rotation is affected more by random error than the quartimax rotation?

5.--Would the selection of several samples of the same size still produce the same results noted in this study?

6.--Exactly what sample size of a fixed population would allow reproduction of the total population factor structure?

7.--Do varying numbers of variables also have an effect on the size of samples needed to reproduce the total population factor structure?

8.--Are the computer programs carrying out the calculations for the rotational solutions according to the underlying theory?

9.--If the answer to Question 8 is yes, then what are the effects of all possible permutations for different numbers of rotated factors and variables?

10.--Can a permutation or ordering algorithm be established for the quartimax method which would be able to ensure that the final rotated solution would provide the highest sum of fourth powers, taking into account the number of factors rotated as well as the varying number of variables?

BIBLIOGRAPHY

Ahmavaara, Y. The mathematical theory of factorial invariance under selection. *Psychometrika*, 1954, 19, 27-58.

Barlow, J.A. & Burt, C. The identification of factors from different experiments. *Brit. J. Statist. Psychol.*, 1954, 7, 52-56.

Burt, C. Group factor analysis. *Brit. J. Statist. Psychol.*, 1950, 3, 40-75.

Eysenck, H.J. Review of L.L. Thurstone's "Primary mental abilities." *Brit. J. educ. Psychol.*, 1939, 9, 270-275.

Guilford, J.P. *Fundamental statistics in psychology and education.* New York: McGraw-Hill, 1956.

Harman, H.H. *Modern factor analysis.* Chicago: Univer. Chicago Press, 1960.

Heerman, E.F. The geometry of factorial indeterminancy. *Psychometrika*, 1964, 29, 371-381.

Henrysson, S. *Applicability of factor analysis in the behavioral sciences.* Stockholm: Almqvist & Wiksell, 1957.

Holzinger, K.J. & Harman, H.H. Comparison of two factorial analyses. *Psychometrika*, 1938, 3, 45-60.

Holzinger, K.J. & Harman, H.H. *Factor analysis.* Chicago: Univer. Chicago Press, 1948.

Holzinger, K.J. & Swineford, F. A study in factor analysis: The stability of a bi-factor solution. *Suppl. Educ. Monogr.*, No. 48, Chicago: Dept. of Educ., Univer. Chicago, 1939. Cited by H.H. Harman, *Modern factor analysis.* Chicago: Univer. Chicago Press, 1960. P. 135.

Kenney, P.B. A note on the formal requirements for factorial studies of differentiation of abilities with age. *Aust. J. Educ.*, 1958, 2, 121-122. Cited by P.B. Kenney & F.M.M. Coltheart, The effect of sampling restrictions on factor patterns. *Aust. J. Psychol.*, 1960, 12. P. 58.

Kenney, P.B. & Coltheart, F.M.M. The effect of sampling restrictions on factor patterns. *Aust. J. Psychol.*, 1960, 12, 58-69.

Kiel, D.F. & Wrigley, C.F. Effects upon the factorial solution of rotating varying numbers of factors. Amer. Psychologist, 1960, 15, 487.

Ledermann, W. Note on professor Godfrey H. Thomson's article "The influence of univariate selection on factorial analysis of ability." Brit. J. Psychol., 1938, 29, 69-73.

Li, J.C.R. Introduction to statistical inference. Ann Arbor: Edwards Bros., 1957.

Lindquist, E.F. Educational measurement. Washington, D.C.: Amer. Council on Educ., 1951.

Meredith, W. Notes on factorial invariance. Psychometrika, 1964, 29, 177-186.

Pearson, K. & Filon, L.N.G. On the probable errors of frequency constants and on the influence of random selection on variation and correlation. Phil. Trans. Roy. Soc. London, 1898, 191, A, 229-311. Cited by G.H. Thomson, The factorial analysis of human ability. New York: Houghton-Mifflin, 1949. P. 168.

Pinneau, S.R. & Newhouse, A. Measures of invariance and comparability in factor analysis for fixed variables. Psychometrika, 1964, 29, 271-282.

Siegel, S. Nonparametric statistics. New York: McGraw-Hill, 1956.

Thomson, G.H. The influence of univariate selection on the factorial analysis of ability. Brit. J. Psychol., 1938, 28, 451-459.

Thomson, G.H. The factorial analysis of human ability. New York: Houghton-Mifflin, 1948.

Thomson, G.H. & Ledermann, W. The influence of multivariate selection on the factorial analysis of ability. Brit. J. Psychol., 1939, 29, 288-305.

Thurstone, L.L. Primary mental abilities. Psychometr. Monogr. No. 1, 1938. Chicago: Univer. Chicago Press.

Thurstone, L.L. Multiple factor analysis. Chicago: Univer. Chicago Press, 1947.

Tucker, L.R. A method for synthesis of factor analysis studies. Personnel Res. Section Rep., No. 984. Washington, D.C.: Dept. of the Army, 1951. Cited by S.R. Pinneau & A. Newhouse, Measures of invariance and comparability in factor analysis for fixed variables. Psychometrika, 1964, 29. P. 275.

Wrigley, C.F. & Neuhaus, J.O. The matching of two sets of factors. Contract Memorandum Rep., A-32. Urbana, Illinois: Univer. Illinois, 1955. Cited by H.H. Harman, Modern factor analysis. Chicago: Univer. Chicago Press, 1960. P. 257.

Wrigley, C.F. & Neuhaus, J.O. The matching of two sets of factors. Amer. Psychologist, 1955, 10, 418-419.

Wrigley, C.F., Saunders, D.R., & Neuhaus, J.O. Application of the quartimax method of rotation to Thurstone's primary mental abilities study. Psychometrika, 1958, 23, 151-169.

Zimmerman, W.S. A revised orthogonal rotational solution for Thurstone's original primary mental abilities test battery. Psychometrika, 1953, 18, 77-93.

APPENDIX A

The MINAC 3 Factor Analysis Program

Table 59. MINAC 3 Program

```
      PROGRAM START
      CALL MINAC 3
      END
      SUBROUTINE MINAC 3
      DIMENSION R(60,60),F(53),IX(101),D(101),A(101),S(101),
     1G(101),B(60,60),FMT(10),SQ(101)
      COMMON R,F,IX,D,A,S,G,B,SQ
   12 LESGO =0
      DO 10000 I=1,60
      DO 10000 J=1,60
10000 B(I,J)=0.0
      DO 10001 I=1,60
      DO 10001 J=1,60
10001 R(I,J)=0.0
      DO 10002 I=1,101
      IX(I)=0
      D(I)=0.0
      SQ(I)=0.0
      A(I)=0.0
      S(I)=0.0
10002 G(I)=0.0
      DO 10003 I=1,53
10003 F(I)=0.0
      DO 10004 I=1,10
10004 FMT(I)=0.0
      READ INPUT TAPE 2,15,(F(I),I-1,7),IND,INK,NS,NV,KOD,KDIAG,
     1KSTP,KSEL,NP,NFR,KPRT
   15 FORMAT  (7A8,I1,I2,5X,I5,I3,5I1,I2,I1)
      WRITE OUTPUT TAPE 3,80
   80 FORMAT  (1H1)
      IF(NV) 99,99,17
   99 STOP
   17 WRITE OUTPUT TAPE 3,20, (F(I),  I=1,7)
   20 FORMAT  (2X,7A8///)
      READ INPUT TAPE 2,5,FMT
    5 FORMAT  (10A8)
      WRITE OUTPUT TAPE 3,6,FMT
    6 FORMAT  (//13H DATA FORMAT 10A8)
      IF(NO) 21,21,22
   21 KNO = 1
      GO TO 23
   22 KNO = 2
   23 DO 100  I=1,NV
  100 READ INPUT TAPE 2, FMT, (R(I,J),  J=1,NV)
      IF(KPRT-1) 81,81,104
   81 WRITE OUTPUT TAPE 3,85
   85 FORMAT  (25H INTERCORRELATION MATRIX.)
      CALL MATOUT  (NV,NV)
      WRITE OUTPUT TAPE 3, 80
```

Table 59. (continued)

```
104 EPS1=.0000001
    H=.5
    DO 105 K=1,NV
105 B(K,K)=1.0
    N1=NV-1
    SUM1=0.0
    DO 110 I=1,NV
    DO 110 J=I,NV
110 SUM1=SUM1+R(I,J)*R(I,J)
    EPS2=5.0E-7
    L=1
115 M=1
    N=2
120 IF(ABSF(R(M,N))-H)230,125,125
125 IF (R(M,M)-R(N,N))135,130,135
130 C=0.70710678
    SS=C
    GO TO 145
135 X=R(M,N)/(R(M,M)-R(N,N))
    Y = SQRTF(1.0/)L.0+4.0*X*X))
    C = SQRTF((1.0+Y)/2.0)
    SS = SQRTF((1.0-Y)/2.0
    IF(X) 140,230,145
140 SS=-SS
145 C2 = C*C
    S2 = SS*SS
    CSS = C*SS
    RHO = C2 + S2 - 10
    IF(ABSF(C2+S2-1.0) - EPS2) 160,150,150
150 WRITE OUTPUT TAPE 3,155,L,N,M,R,(M,N),R(M,M),R(N,N),C,SS
155 FORMAT  (3I5,5F15.6)
    STOP 156
160 DO 225 K=1,NV
    IF (K-M) 165,220,170
165 ARK=R(K,M)
    GO TO 175
170 ARK=R(M,K)
175 IF (K-N) 180,220,185
180 ASK=R(K,N)
    GO TO 190
185 ASK=R(N,K)
190 AIRK=C*ARK+SS*ASK
    AISK=C*ASK-SS*ARK
    IF(K-M) 195,220,200
195 R(K,M)=AIRK
    GO TO 205
200 R(M,K)=AIRK
205 IF (K-N) 210,220,215
210 R(K,N)=AISK
```

Table 59. (continued)

```
      GO TO 220
  215 R(N,K)=AISK
  220 BR=B(K,M)
      BS=B(K,N)
      B(K,M)=C*BR+SS*BS
  225 B(K,N)=C*BS-SS*BR
      AR=R(M,M)
      AS=R(N,N)
      ARS=R(M,N)
      TEMP = C2-S2
      R(M,M) = C2*AR+S2*AS+2.0*CSS*ARS
      R(N,N) = C2*AS+S2*AR-2.0*CSS*ARS
      R(M,N) = CSS*AS+ARS*TEMP-CSS*AR
  230 N=N+1
      IF (N-NV) 120,120,235
  235 M=M+1
      IF (M-N1) 240,240,245
  240 N=M+1
      GO TO 120
  245 L=L+1
      SUM=0.0
      DO 250 I=1,N1
      K=I+1
      DO 250 J=K,NV
  250 SUM=SUM+R(I,J)*R(I,J)
      IF (ABSF(SUM/SUM1)-EPS1) 260,255,255
  255 H=H/10.0
      GO TO 115
  260 DO 265  I=1,NV
      S(I)=R(I,I)
  265 A(I) = SQRTF(ABSF(S(I)))
      DO 280 I=1,NV
      DO 280 J=1,NV
  280 R(I,J)=B(I,J)*A(J)
      XN=NV
      NFM = NV/3 + 1
      IF (KPRT)  291,291,1291
 1291 IF (KPRT-2) 300,291,300
  291 WRITE OUTPUT TAPE 3,292
  292 FORMAT (25H PRINCIPAL AXIS ANALYSIS.////13H EIGENVALUES.)
      WRITE OUTPUT TAPE 3,71,(I,S(I),I=1,NV)
   71 FORMAT  (/3X,6)I6,F12.4))
  293 FORMAT (///25H PROPORTIONS OF VARIANCE.)
  294 FORMAT (/// 18H HIGHEST LOADINGS.)
      WRITE OUTPUT TAPE 3,295
  295 FORMAT (///23H FACTOR LOADING MATRIX.)
      CALL MATOUT (NV,NV)
      IF (KSTP-1) 300,12,300
  300 DO 1296  I=1,NV
```

Table 59. (continued)

```
1296 A(I) = S(I)
     DO 1301  I=1,NV
     BIGA = 0.0
     DO 1299  J=1,NV
     IF(BIGA - A(J)) 1298,1299,1299
1298 BIGA = A(J)
     K = J
1299 CONTINUE
     A(K) = 0.0
     SQ(I)=S(K)
     DO 1300  J=1,NV
1300 B(J,I) = R(J,K)
1301 CONTINUE
     GO TO (1304,1303),KNO
1304 NF = NFR
     GO TO 1305
1303 NF = 1
1302 NF = NF + 1
1305 DO 296  I=1,NV
     DO 296  J=1,NF
 296 R(I,J) = B(I,J)
     IF (LESGO) 4000,4001,4000
4001 WRITE OUTPUT TAPE 3,292
     WRITE OUTPUT TAPE 3,71,(I,SQ(I),I=1,NV)
     WRITE OUTPUT TAPE 3,295
     CALL MATOUT (NV,NV)
     CALL PERMUTE (NV,NFR,NO,IND,INK)
     WRITE OUTPUT TAPE 3,292
     WRITE OUTPUT TAPE 3,71,(I,SQ(I),I=1,NV)
     WRITE OUTPUT TAPE 3,295
     CALL MATOUT (NV,NV)
     LESGO=1
4000 DO 325  I=1,NV
     A(I)=0.0
     DO 320 J=1,NF
 320 A(I)=A(I)+R(I,J)*R(I,J)
 325 S(I) = SQRTF(A(I))
     DO 330 I=1,NV
     DO 330 J=1,NF
 330 R(I,J)=R(I,J)/S(I)
     CALL CALCV (NV,NF,V)
 335 VCHECK=V
     DO 385 M=1,NF
     J=M+1
     DO 385 N=J,NF
     E=0.0
     BB=0.0
     C=0.0
     DTEMP=0.0
```

Table 59. (continued)

```
      DO 340 I=1,NV
      D(I)=R(I,M)*R(I,M)-R(I,N)*R(I,N)
      G(I)=2.0*R(I,M)*R(I,N)
      E=E+D(I)
      BB=BB+G(I)
      C=C+D(I)*D(I)-G(I)*G(I)
  340 DTEMP=DTEMP+D(I)*G(I)
      DD=2.0*DTEMP
      IF(KSEL) 342,343,342
  342 XNUM = DD
      XDEN = C
      GO TO 344
  343 XNUM = DD-((2.0*E*BB)/XN)
      XDEN=C-((E*E-BB*BB)/XN)
  344 Y = ATANF(XNUM/XDEN)
      IF (XNUM)350,345,345
  345 IF (XDEN)365,355,355
  350 IF (XDEN)360,355,355
  355 FOURA=Y
      GO TO 370
  360 FOURA=-3.14+Y
      GO TO 370
  365 FOURA=1.57+ABSF(Y)
  370 ALP=FOURA/4.0
      IF (ABSF(ALP)-.01) 385,385,375
  375 Z1=SINF(ALP)
      Z2=COSF(ALP)
      Z3 = -Z1
      DO 380 I=1,NV
      SUM1=R(I,M)*Z2+R(I,N)*Z1
      SUM2=R(I,M)*Z3+R(I,N)*Z2
      R(I,M)=SUM1
  380 R(I,N)=SUM2
  385 CONTINUE
      CALL CALCV (NV,NF,V)
      IF (V-VCHECK-.0001) 390,390,335
  390 DO 400 I=1,NV
      DO 400 J=1,NF
  400 R(I,J)-R(I,J)*S(I)
  401 DO 410 J=1,NF
      S(J)=0.0
      D(J)=0.0
      DO 405 I=1,NV
      IF (ABSF(D(J))-ABSF(R(I,J))) 402,405,405
  402 D(J)=R(I,J)
  405 S(J)=S(J)+R(I,J)*R(I,J)
  410 S(J)=S(J)/XN
 2410 IF(KSEL) 2412,2411,2412
 2411 WRITE OUTPUT TAPE 3,411
```

Table 59. (continued)

```
  411 FORMAT (27H VARIMAX ROTATION ANALYSIS.)
      GO TO 2222
 2412 WRITE OUTPUT TAPE 3,2413
 2413 FORMAT (29H QUARTIMAX ROTATION ANALYSIS.)
 2222 WRITE OUTPUT TAPE 3,293
      WRITE OUTPUT TAPE 3,71,(I,S(I),I=1,NF)
      WRITE OUTPUT TAPE 3,294
      WRITE OUTPUT TAPE 3,71,(I,D(I),I=1,NF)
      WRITE OUTPUT TAPE 3,412
  412 FORMAT (///15H COMMUNALITIES.)
      WRITE OUTPUT TAPE 3,71,(I,A(I),I=1,NV)
      WRITE OUTPUT TAPE 3,413
  413 FORMAT (/// 25H ROTATED FACTOR LOADINGS.)
      CALL MATOUT (NV,NF)
80001 GO TO (80002,3080),KNO
80002 CALL PUNCHOU (NF)
      GO TO 12
 3080 DO 3074  I=1,NF
 3074 IX(I) = 0.0
      DO 3079  I=1,NV
      BIGR =0.0
      DO 3075  J=1,NF
      IF (BIGR - ABSF(R(I,J))) 3076,3075,3075
 3076 BIGR = ABSF(R(I,J))
 3075 CONTINUE
      DO 3073  J=1,NF
      IF(BIGR - ABSF(R(I,J))) 3073,3077,3073
 3077 IX(J) = IX(J) + 1
      GO TO 3079
 3073 CONTINUE
 3079 CONTINUE
      DO 3078  I=1,NF
      IF(IX(I) - NO) 3028,3078,3078
 3078 CONTINUE
      IF(NFM - NF) 8888,80002,1302
 3028 IF(NF - 2) 8888,3503,80002
 8888 CALL PUNCHOU (NF)
      STOP 6666
 3503 WRITE OUTPUT TAPE 3,3015
 3015 FORMAT  (51H THEOWRIGLEY-KIEL CRITERION HAS NOT BEEN
     1SATISFIED.)
 3501 GO TO 12
      END
      SUBROUTINE MATOUT (M,K)
      DIMENSION R(60,60),F(53),IX(101),D(101),A(101),S(101),
     1G(101),B(60,60),FMT(10),SQ(101)
      COMMON R,F,IX,D,A,S,G,B,SQ
      KM-(((K-1)/10+1)*10)
      DO 30 L=10,KM,10
      N=L-9
```

Table 59. (continued)

```
      IF (L-KM) 10,5,10
    5 L=K
   10 WRITE OUTPUT TAPE 3,15,(I,I=N,L)
   15 FORMAT (//7X,10I10)
      DO 20 I=1,M
   20 WRITE OUTPUT TAPE 3,25,I,(R(I,J),J=N,L)
   25 FORMAT (I6,F13.4,9F10.4)
   30 WRITE OUTPUT TAPE 3,35
   35 FORMAT (1H1)
      RETURN
      END
      SUBROUTINE CALCV (K,L,V)
      DIMENSION R(60,60),F(53),IX(101),D(101),A(101),S(101),
     1G(101),B(60,60),FMT(10),SQ(101)
      COMMON R,F,IX,D,A,S,G,B,SQ
      X=0.0
      DO 5 I=1,K
      DO 5 J=1,L
    5 X=X+R(I,J)**4
      XK=K
      X=XK*X
      YT=0.0
      DO 15 J=1,L
      F(J)=0.0
      DO 10 I=1,K
   10 F(J)-F(J)+R(I,J)*R(I,J)
      F(J)-F(J)*F(J)
   15 YT=YT+F(J)
      V=X-YT
      RETURN
      END
      SUBROUTINE PERMUTE (NV,NC1,NC2,I,NC)
      DIMENSION R(60,60),F(53),IX(101),D(101),A(101),Q(101),
     1G(101),B(60,60),FMT(10),S(101)
      COMMON R,F,IX,D,A,Q,G,B,S
      IF (I-7) 401,309,404
  404 STOP
  401 IF (I-3)303,303,304
  303 DO 101 J=I,NV
      TEMP=R(J,-)
      R(J,1)=R(J,NC)
  101 R(J,NC)=TEMP
      TEMP=S(1)
      S(1)=S(NC)
      S(NC)=TEMP
      IF(I-2)402,305,305
  305 NC=NC-1
      DO 102 J=1,NV
      TEMP=R(J,2)
```

Table 59. (continued)

```
    R(J,2)=R(J,NC)
102 R(J,NC)=TEMP
    TEMP=S(2)
    S(2)=S(NC)
    S(NC)=TEMP
    IF(I-3)402,306,306
306 NC-NC-1
    DO 103 J=1,NV
    TEMP=R(J,3)
    R(J,3)=R(J,NC)
103 R(J,NC)=TEMP
    TEMP=S(3)
    S(3)=S(NC)
    S(NC)=TEMP
402 RETURN
304 MID=NC/2
    MIDD=MID+1
    DO 104 J=1,NV
    TEMP=R(J,1)
    R(J,1)=R(J,MID)
    R(J,MID)=TEMP
    TEMP=R(J,MIDD)
    R(J,MIDD)=R(J,NC)
104 R(J,NC)=TEMP
    TEMP=S(1)
    S(1)=S(MID)
    S(MID)=TEMP
    TEMP=S(MIDD)
    S(MIDD)=S(NC)
    S(NC)=TEMP
    IF(I-5)403,307,307
307 NC=NC-1
    MID=MID-1
    MIDD=MIDD+1
    DO 105 J=1,NV
    TEMP=R(J,2)
    R(J,2)=R(J,MID)
    R(J,MID)=TEMP
    TEMP=R(J,MIDD)
    R(J,MIDD)=R(J,NC)
105 R(J,NC)=TEMP
    TEMP=S(2)
    S(2)=S(MID)
    S(MID)=TEMP
    TEMP=S(MIDD)
    S(MIDD)=S(NC)
    S(NC)=TEMP
    IF(I-6)403,308,308
308 NC=NC-1
```

Table 59. (continued)

```
      MID=MID-1
      MIDD=MIDD+1
      DO 106 J=1,NV
      TEMP=R(J,3)
      R(J,3)=R(J,MID)
      R(J,MID)=TEMP
      TEMP=R(J,MIDD)
      R(J,MIDD)=R(J,NC)
  106 R(J,NC)=TEMP
      TEMP=S(3)
      S(3)=S(MID)
      S(MID)=TEMP
      TEMP=S(MIDD)
      S(MIDD)=S(NC)
      S(NC)=TEMP
  403 RETURN
  309 IQX=NC/2
      DO 310 K=1,IQX
      L=NC-K+1
      DO 311 J=1,NV
      TEMP=R(J,K)
      R(J,K)=R(J,L)
  311 R(J,L)=TEMP
      TEMP=S(L)
      S(L)=S(K)
  310 S(K)=TEMP
      RETURN
      END
      SUBROUTINE PUNCHOU (NF)
      DIMENSION R(60,60),F(53),IX(101),D(101),A(101),S(101),
     1G(101),B(60,60),FMT(10),SQ(101)
      COMMON R,F,IX,D,A,S,G,B,SQ
      PRINT 200
  200 FORMAT (16H-PUNCHOU-REACHED)
      DO 101 I=1,57
  101 PUNCH 1000,(R(I,J),J=1,NF)
      PRINT 1000,(R(I,J),J=1,NF)
 1000 FORMAT (8X,12F6.4)
      RETURN
      END
      END
```