THE PLANNING OF MOTION

by

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ABSTRACT

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Existing theories of animal motion, such as S-R theory and Lewin's field theory, are explored in the two dimensional case and found to make unlikely predictions about motion in certain situations. In detour problems, which real animals can solve, the theoretical predictions lead to stalling at an intermediate point rather than going around the barrier. Animals are also predicted to stall at intermediate points of relative safety rather than escape between predators. Animals are predicted to be unable to go out of their way to make use of shelters such as tall grasslands or other cover. As an alternative, a theory is developed based upon the idea that animals choose optimal paths, minimizing their expected exposure to fear before engaging in motion. In the situations that are problematic for prior theories, this new theory makes predictions which match empirical observations. A neural circuit is constructed which can solve for the required optimal paths approximately.

DEDICATION

To Pretty Kitty and Mrs. Magoo, who through their patient cooperation with numerous observations and manipulations, made the idea of this research come alive.

ACKNOWLEDGMENTS

The author would like to thank his graduate committee, particularly John Hunter and Ray Denny, for many interesting and helpful discussions. The spirit of Stanley Ratner was also an important impetus to the present work. Last, but not least, I would like to acknowledge the patient and careful assistance of my typist, Geri Wilson.

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F_A - aversive force

F_G - attractive goal force

F_R - resultant force

v - velocity

E - equilibrium point

curl F - $\nabla \times F$ the vector curl of a vector field F

F_H - force of avoidance generated by a hole

T - in figures--denotes a trajectory

Es - in figures--denotes an escape trajectory

x - ordinary Cartesian coordinate

y - ordinary Cartesian coordinate

J - total exposure to fear

f - local (anticipated) fear function

T - in equations--the total time to execute a path

t - time

f_1 - the spatial component of fear

f_r - fear for the reward

w_p - the time premium weight

J_{ij} - the total cost of a leg of a path between positions i and j

*in order of occurrence

CUM	- the cumulative exposure to fear up to a point in space, while the animal pursues an optimal path to get there
BACK	- an array (in the Moore Algorithm) containing the best way to get to a node as an index of another node
FRONTIER	- a recursively defined working set in the Moore Algorithm
Δ	- a small increment of cost
STAGE	- a dynamic programming stage
s	- a parameter used to parametrize paths
x_1	- the x-coordinate of a starting position
x_2	- the x-coordinate of the goal
y_1	- the y-coordinate of a starting position
y_2	- the y-coordinate of the goal
$'$	- derivative by parameter s
T	- the tangent unit vector
U	- the unit normal to the path
Grad	- the vector gradient of a scalar
K	- the curvature vector
ℓn	- the natural logarithm
$\mathrm{Proj}_{\perp}$	- the projection operator that projects on the line perpendicular to the path at a given point
∇	- the gradient operator: same as Grad
f_0	- net goal fear
ℓ	- path length
r	- the radius coordinate in polar coordinates
θ	- the angle coordinate in polar coordinates

r_1 - the radius coordinate of the starting position

r_2 - the radius coordinate of the goal

c - a constant of integration

θ_0 - a constant of integration

α - a constant of integration

r_0 - a constant of integration

cos - cosine

$F(r,\theta,\dot{\theta})$ - the Lagrangian function of a variational problem

C_0, C_1, C_2 - constants involving energy expenditure

$\mathcal{E}$ - total energy consumption in executing a path

μ - a constant giving energy expenditure in the same scale as fear expenditure

γ - a parameter of velocity-dependent fear--see Eq. (3.2.4)

b - a parameter of velocity-dependent fear--see Eq. (3.2.4)

$\mathcal{F}$ - the total Lagrangian function of the variable velocity case

f^* - the equivalant constant velocity fear function for the variable velocity case

V_{MAX} - the maximum running velocity of the animal

V_{MIN} - the minimum velocity of an animal in a variable-velocity model

Q - a term in a special case of the variable-velocity model

ξ - the angle between the trajectory and an iso-aversion line

STEP - a minimum path increment in the Bell-Dolson network

NODE - a particular point on a path

N - the total number of nodes in a discrete network

$\| \ \|$ - absolute scalar length of a vector

INTRODUCTION

This work begins with a review of the literature in psychology on animal motion. The review discusses the theories of Loeb and Lewin. Lewin's theories provide the most appropriate present psychological theory which might be applied to explain the motion behavior addressed in this paper. In a subsequent chapter, we present a critique of Lewin's vector psychology, showing cases where it predicts very unlikely behavior. We then assume an optimal choice of path for the minimization of anticipated fear, an optimal planning model. Some detailed predictions are derived from the optimal planning model, including solutions to the problems of Lewin's theory.

This paper will present a theory of optimal path choice. This theory will assume that an animal chooses its path in such a way as to minimize its exposure to fear. The key to the theory is the idea of planning ahead. An animal must make some rather complex preliminary calculations and comparisons before it starts its motion. Otherwise, it might start in a direction which would initially seem attractive, but would subsequently be much less attractive than some alternate path. The present paper ignores the problem of choice of goal and considers only the optimal path to that goal once the goal has been chosen. In this theory the animal considers anticipated fear and chooses a path that will minimize its future exposure to fear. For reasons of mathematical simplicity, our model assumes optimal planning. Although suboptimal planning is a more realistic concept, it is beyond the scope of the present paper.

We will consider an animal in a two dimensional field of motion which has decided to go to a specific goal. The animal must choose its path in such a way as to avoid regions of elevated fear. The specific model is that the animal chooses its path in such a way that the total time weighted exposure to fear is minimized. We will present a neural network model which will show that an animal could compute the trajectories generated by the behavioral model.

Our investigations of the consequences of the optimal planning theory provide some results that agree well with intuition: for example, an animal that follows our theory will go in a much straighter path to an object if it is afraid that something might happen to the object; e.g., a cat that fears for its kittens. A key example is the predicted path a mother cat would choose to get to its kittens in the presence of a sleeping dog. We call this the sleeping dog problem. Our presentation deals only with static fields of fear such as may be experienced by an animal going by a sleeping predator.

This work was inspired by the work of Menzel [1973]. Menzel showed that chimpanzees will take a nearly optimal path around a field to pick up pieces of food that had been left there. That is, the chimps choose the sequence of goals approximating optimality in terms of minimum distance travelled. This experiment suggests that animals may plan optimally and suggested the present approach to the avoidance of anticipated fear.

CHAPTER 1

EVALUATION OF PSYCHOLOGICAL THEORIES OF MOTION

Section 1.1 Review of the Theoretical Literature on Animal Motion in Psychology

There is a large body of literature on elicited animal behavior involving locomotion. In an early book on the subject of animal motion, Loeb [1918] discussed motion caused by asymmetries in the surround of the animal. He gave particular attention to asymmetries of the physical characteristics of the environment, such as salinity, temperature, and hydrogen ion concentration. This line of work has since been extended by numerous authors to give rise to the study of tropisms, taxes, and compass reactions on a broad scale. For general review chapters, see the textbooks by Denny and Ratner [1970] and by Hinde [1970], as well as the encyclopaedic reference by Grzimek [1977]. There are also works, such as that of Schmidt-Koenig and Keeton [1978] on animal navigation, which deal with specialized subtopics of the field of animal locomotion.

Loeb's work does not take consideration of goals that the animal may have, or of distant sources of attraction or aversion. In fact, it does not take consideration of motivation, in the psychological sense, other than immediate motivation due to bodily sensations. By contrast, we are interested in the planning of motion, relative to distant sources of fear and attraction. We are also interested in behavior that arises from the animal itself, rather than that behavior elicited by immediate environmental stimuli. For this reason we will turn elsewhere for a beginning point from which to present our theory.

The present work concerns emitted or operant behavior, rather than elicited behavior. Therefore, we have chosen as a point of reference the theoretical work of Kurt Lewin on the organization of psychological space. In particular, his Principles of Topological Psychology (Lewin [1936]) and Field Theory in Social Science (Lewin [1951]) provide the main theoretical framework which suggested the present approach. In the present paper, we will be investigating the internal image of space and time using a different mathematical formulation than that of Lewin; however, these models find their conceptual roots in Lewin's works mentioned above. Lewin considers that psychological space--that is, space as the animal perceives it, or life-space--depends on the motivation of the animal, as well as upon the objective structure of physical space. (Actually, Lewin takes life-space to be an abstract generalization of the perceptual image of the physical environment, of which the physical aspect--the quasi-physical space--is only one facet.

Since motion is difficult to quantify along the non-physical dimensions of the life-space, we will specialize on the spacelike- and timelike- coordinates of life-space.)

In topological psychology, Lewin discusses how motivations warp the structure of space. The life-space is bounded by those objects which are most distant among those that possess a valance emotionally, for the animal at that particular time. A valance is a positive or negative goal-value attached to an object. Every important object in the life space has some valance. Goals to be sought-after and objects to be avoided have positive and negative valances respectively. In Lewin's work on topological psychology, the plasticity of space plays a key role, so that objects which are of intense interest appear nearer than they would otherwise be, and objects which are obstructions may appear directly between the actor and the goal. There are many instructive diagrams concerning these matters in Lewin [1936].

Goals and other animals are substantial determiners of spatial organization in Lewin's models. See Lewin [1936], Chapter 4. Lewin's conceptual basis is the primary antecedent for the present paper.

In his topological psychology, Lewin gave us no clear algorithm or formula for determining what an animal's motion would be in a physical space. Lewin later remedied this in his vector psychology. The reader should in particular see Lewin [1951], pages 238-303. In this work he dealt with what he called his vector psychology. In the vector psychology space is Euclidean and there are assumed to be forces acting in the animal's mind which determine motion by their vector summation. The net force which determines the animal's instantaneous motion is the vectorial summation of forces directed away from each object of negative valence and forces directed toward each object of positive valence. See Figures 1.1.1 and 1.1.2. Lewin is specific that these forces are central forces: that is, that they act in the direction of a line connecting the acting animal with the object which possesses the valence. See Figure 1.1.3. Moreover, motion is assumed to be in the direction of the resulting force and of a velocity monotonic in its strength. What this means is that Lewin's vector psychology predicts motion on the basis of a vector field operating on the animal. Lewin's primary discussion of the vector psychology is contained in Section 4, pages 256-269 of Lewin [1951]. The specific reference to central fields of force is contained in pages 256-257 of the same work.

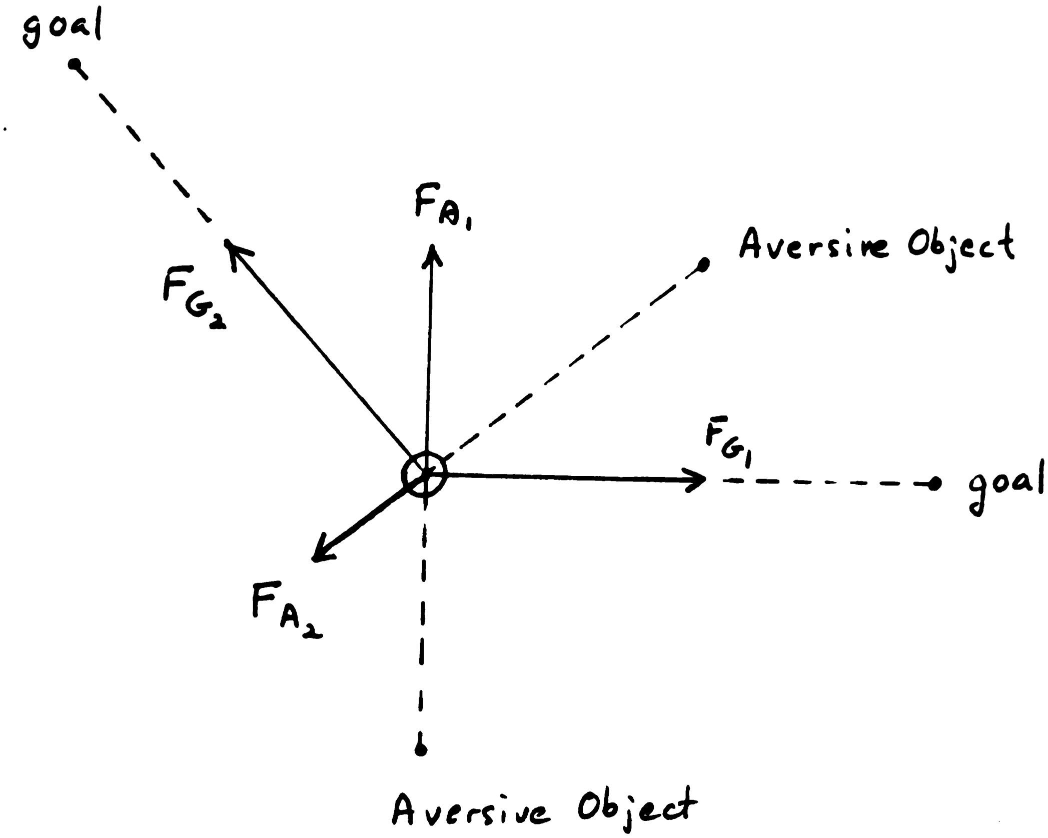


FIGURE 1.1.1

Generation of Forces in Lewin's Theory

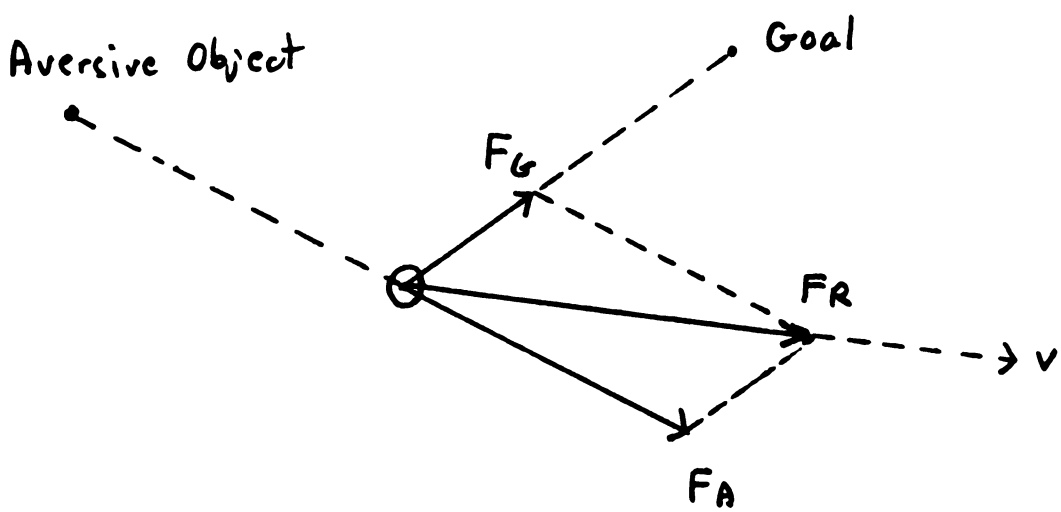


FIGURE 1.1.2

Summation of Forces in Lewin's Theory

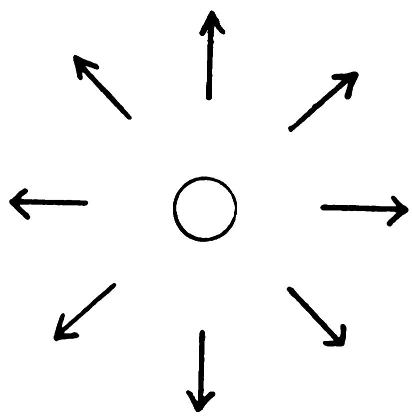


FIGURE 1.1.3

Symmetric Force Field of Aversive Object

In Lewin's vector psychology an animal can sometimes circumnavigate an object that is undesirable to it in order to get to a goal. An example of the vectorial structure of this problem is shown in Figure 1.1.4. This property of the theory is based purely upon the vector composition of forces generated by the valences of various objects.

The vector psychology is used by personality theorists to explain certain phenomena related to conflict (Hall and Lindsey, 1970 , pages 232-234). In personality theory the spaces are frequently abstract or "metaphorical," in that they are not ordinary three-dimensional space, but some analogous space in which emotions are assumed to operate.

Lewin [1931], Miller [1944], and Dollard and Miller [1950] have used Lewin's vector psychology to describe four basic types of emotional conflict in a situation of one-dimensional motion. The four basic types of conflict are approach-approach, approach-avoidance, avoidance-avoidance, and double approach-avoidance. The approach-approach conflict is characterized by having two attractive goals. Approach-avoidance conflict is a problem posed by having an attractive goal with an object to be avoided interposed between the actor and the goal. Avoidance-avoidance conflict occurs in a situation where the actor is forced to make a choice between two unattractive goals. The

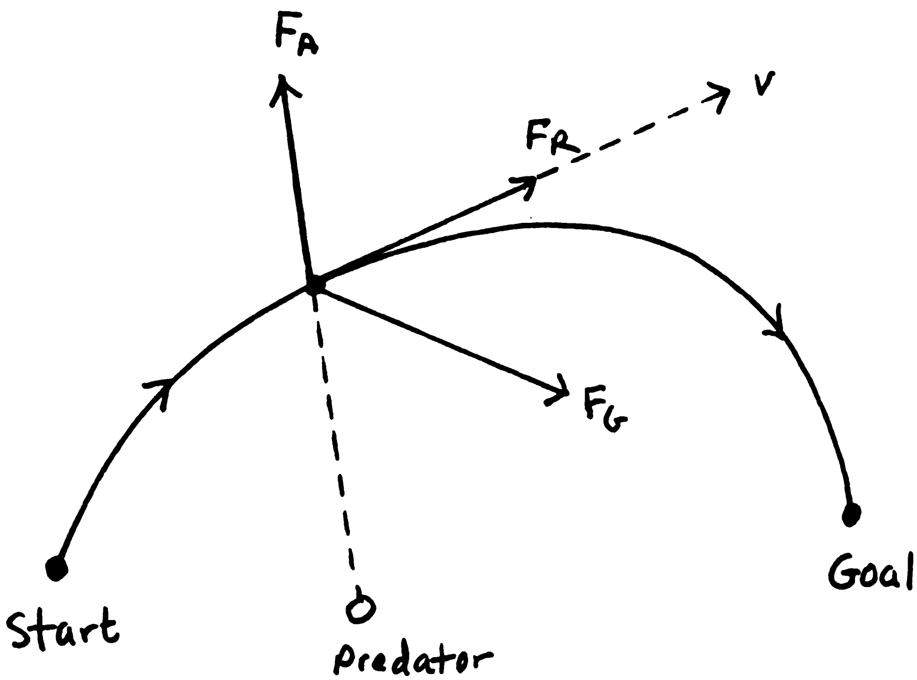


FIGURE 1.1.4

Lewinian Trajectory and Forces

double approach-avoidance conflict has in each direction of motion an object to be avoided and an attractive object. These types of conflict are summarized concisely in Mahl [1971] and Lewin [1951].

The approach-avoidance conflict is featured in the situation shown in Figure 1.1.5. Here the animal is in a one-dimensional tunnel and there is a goal at the end of the tunnel and a feared object intermediate between the animal and the goal. According to Lewin (e.g., 1951, page 263), the force of repulsion from the object which has negative valence dies-out more quickly with distance than does the force of approach toward the goal object. This results in an equilibrium at an intermediate point as shown at E in Figure 1.1.5. According to the Lewin-Miller theory, the animal will stall at E where the motion has a stable zero of velocity. Small deviations in the motion around the point E will lead the animal to return to the point E. It is not yet experimentally clear whether such equilibria exist for real animals. Lewin [1951, p. 264] says that children in such a situation will waver around the point of equilibrium until one force becomes dominant "as a result of changes in the circumstances or of a decision." Such statements seem to obfuscate the theory rather than amplify it as the constructs employed are not intrinsic to the vector psychology. In any case, an animal in the real world of survival would likely be in more danger, rather than less, if it tended to stall

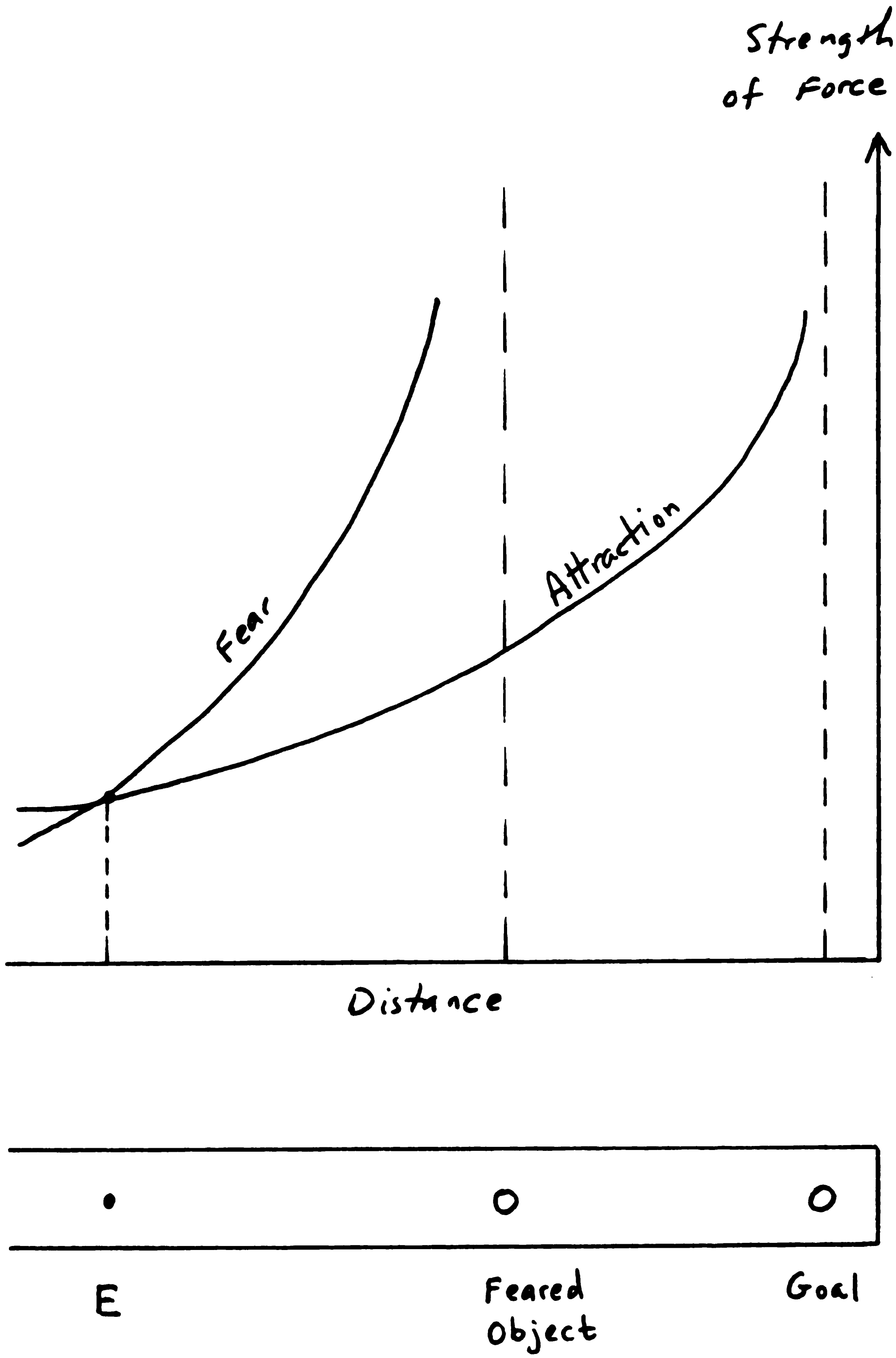


FIGURE 1.1.5

Approach-Avoidance Conflict in Straight Alley According to Lewinian Theory for long periods of time at fixed locations due merely to a balance of psychological forces. We will discuss this further in the next section.

In a one dimensional setting an avoidance-avoidance conflict, as pictured in Figure 1.1.6, is also stable. However, the psychological reality of this model, where the animal is caught in a tunnel between two aversive objects, is entirely based on the one dimensional constraint on motion imposed by the tunnel. Indeed, in a two dimensional space, as shown in Figure 1.1.7, it seems entirely likely that the animal would escape out the side. In fact this equilibrium is unstable in two dimensions, as shown by the trajectories in Figure 1.1.7. The first order differential equation of the Lewinian motion in Figure 1.1.7 has a saddlepoint at E and so describes an unstable equilibrium.

The approach-approach conflict, pictured in Figure 1.1.8, is always unstable, even in one dimension a small perturbation in either direction leading to approach to the goal in that direction. The animal, therefore, will be expected eventually to approach one of the goals and move all of the way to that goal.

The double approach-avoidance conflict, although seemingly a mere superposition of two approach-avoidance conflicts, is of sufficient complexity that it cannot be analyzed even in the one dimensional case without specific choice of the mathematical force gradients

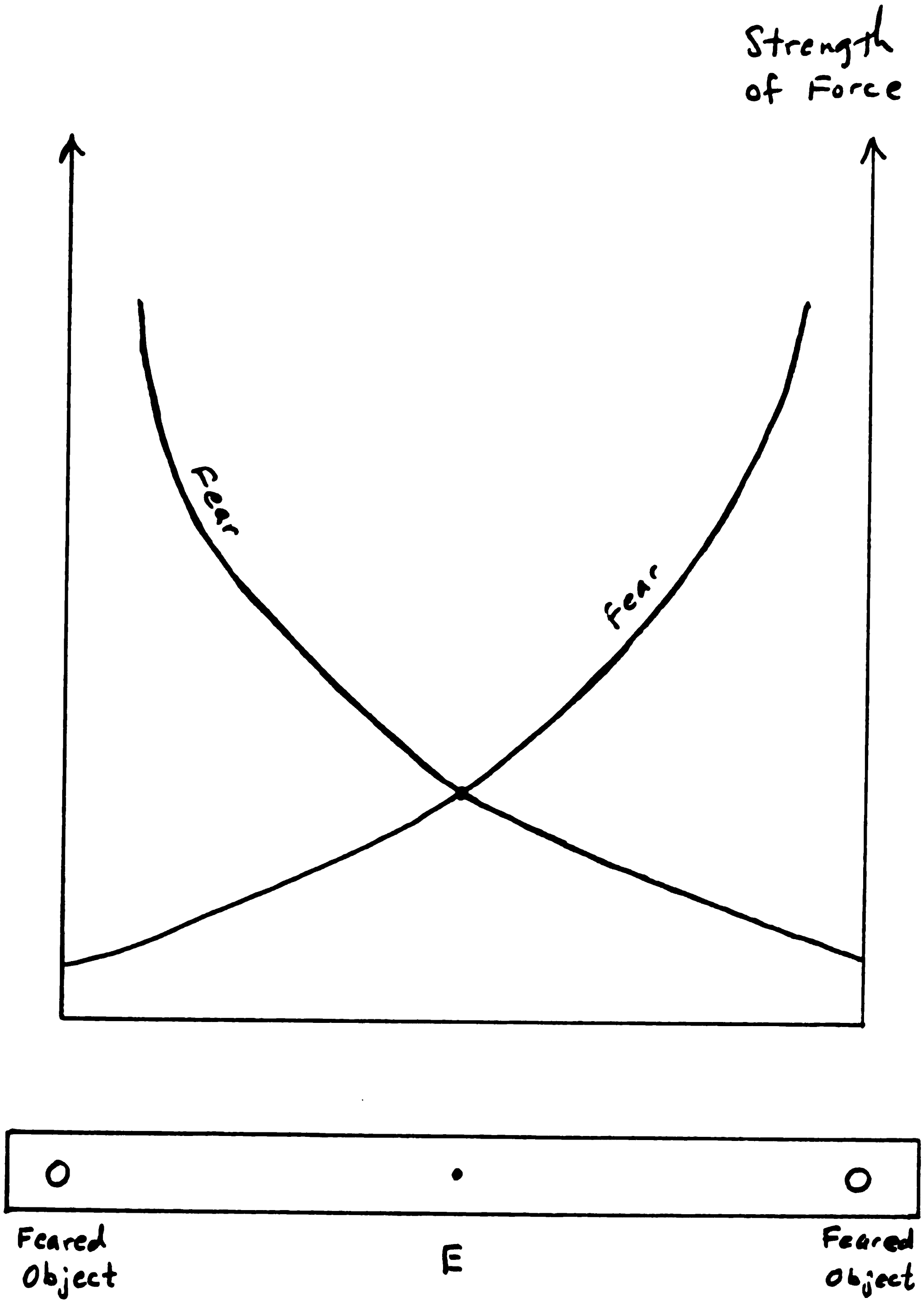


FIGURE 1.1.6

Avoidance-Avoidance Conflict in Straight Alley According to Lewin's Theory

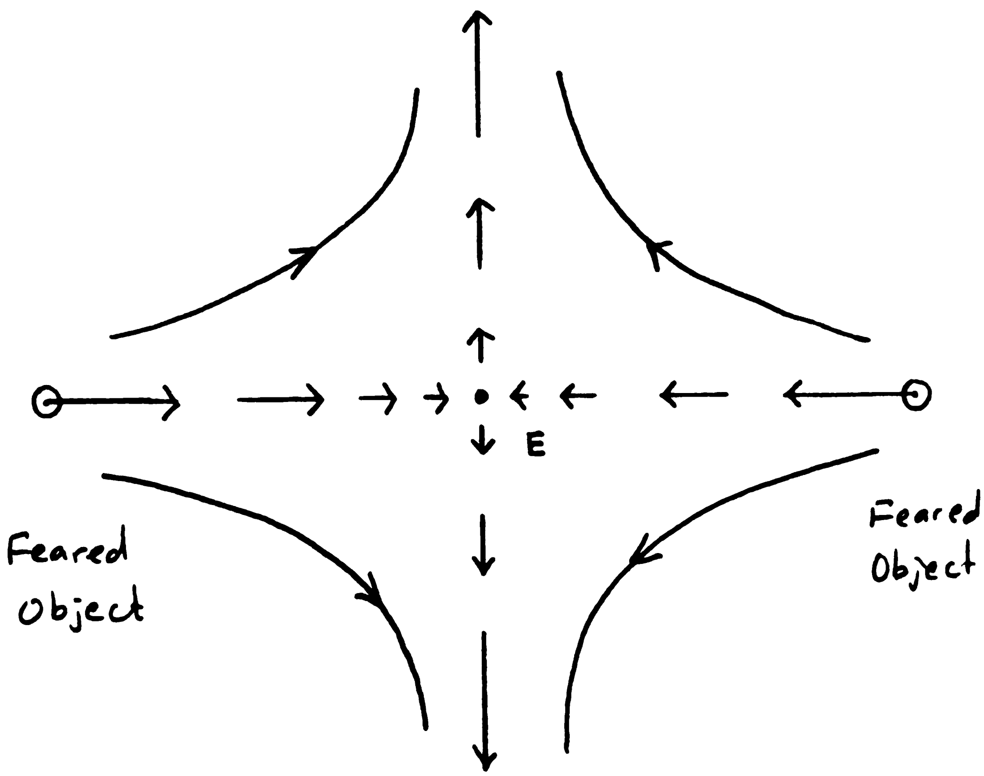


FIGURE 1.1.7

Two Dimensional Avoidance-Avoidance Conflict in Lewin's Theory

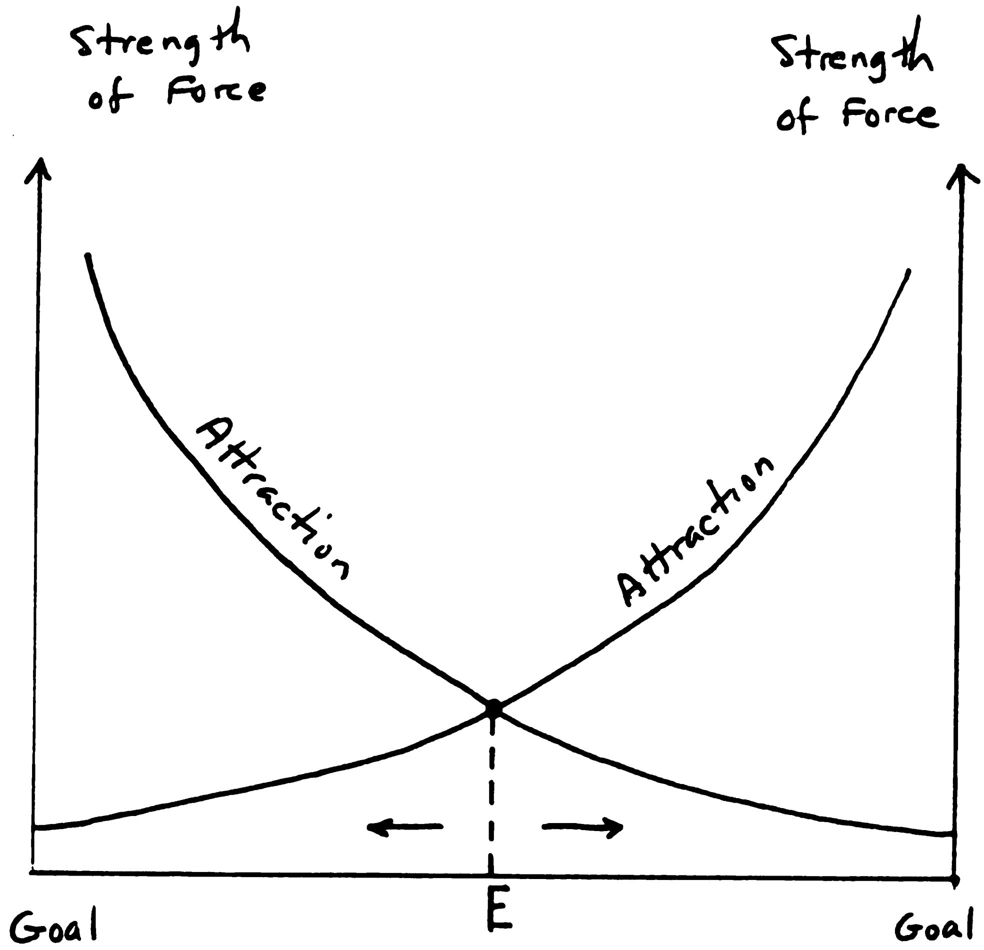


FIGURE 1.1.8

Approach-Approach Conflict in One Dimensional Lewinian Model involved. Depending on the shape of the approach-tendency and avoidance-tendency curves, the equilibrium may be either stable or unstable in the one dimensional double approach-avoidance conflict.

Section 1.2 Failures of the Lewin-Miller Theory in Two Dimensions

In the approach-avoidance setting, the Lewin-Miller theory predicts stalling in a one dimensional tunnel. See Figure 1.2.1. This stalling is due to the fact that the avoidance force dies out more quickly than the attractive force. We will now generalize this situation to the one shown in Figure 1.2.2 where we have a cat trying to get past the dog to its kittens in a two dimensional space. In this case, the cat can circumnavigate the dog to reach the kittens. In fact, according to the Lewin-Miller theory, it should do so providing it can once leave the equilibrium point shown as E in Figure 1.2.3. The equilibrium at E is unstable and leads, in the case of any small perturbation, to trajectories that go toward the kittens. Thus, the animal is not really trapped at E by its own motivations, since it will leave by one of the side trajectories if there is any small deviation from its initial position. On the other hand, this basic configuration can be generalized to cases where, even in two dimensions, the Lewin-Miller theory will lead to stalling. For example, we may consider that, interposed between the cat and its kittens,

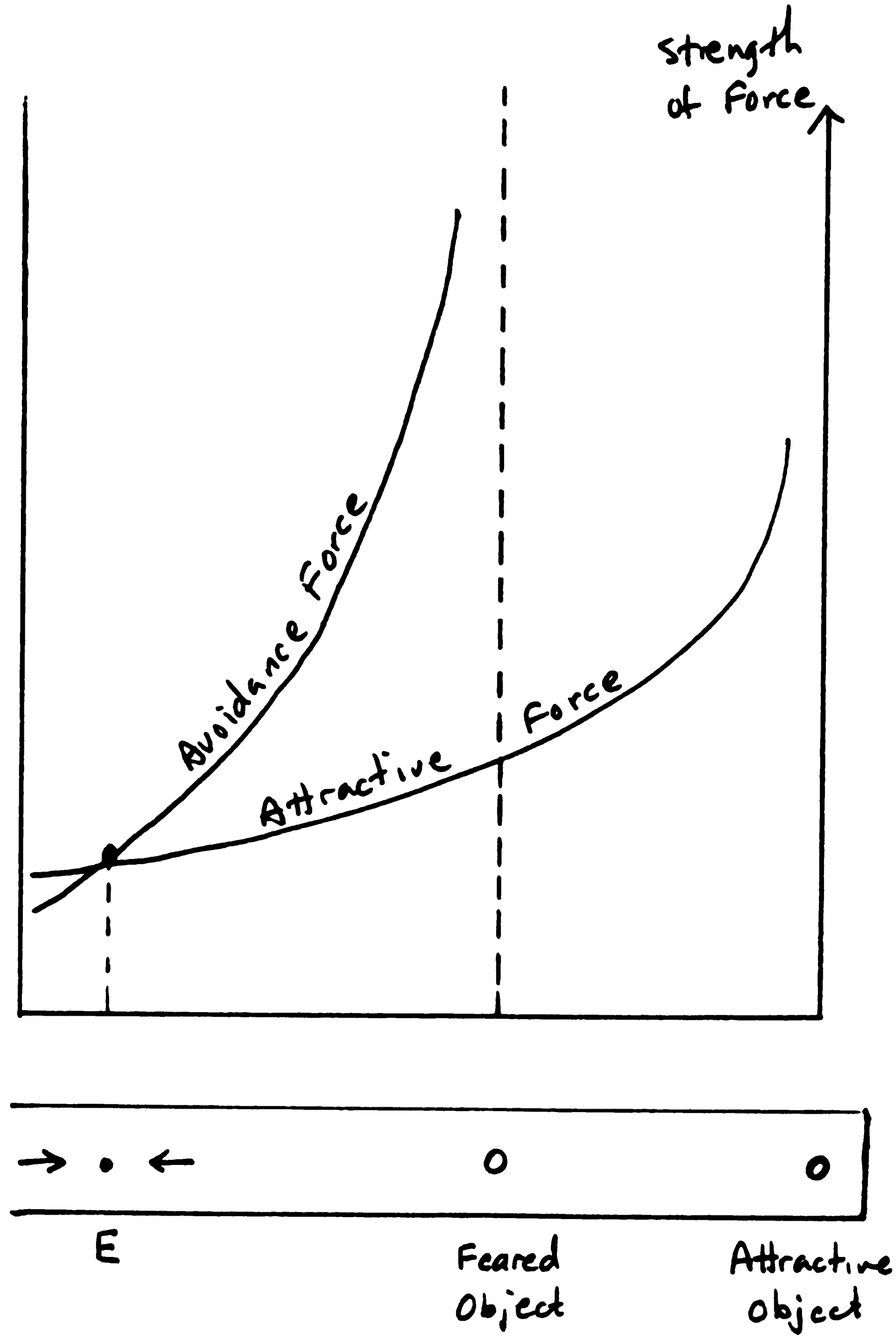


FIGURE 1.2.1

Stable Equilibrium in One Dimensional
Lewinian Approach-Avoidance Contingency

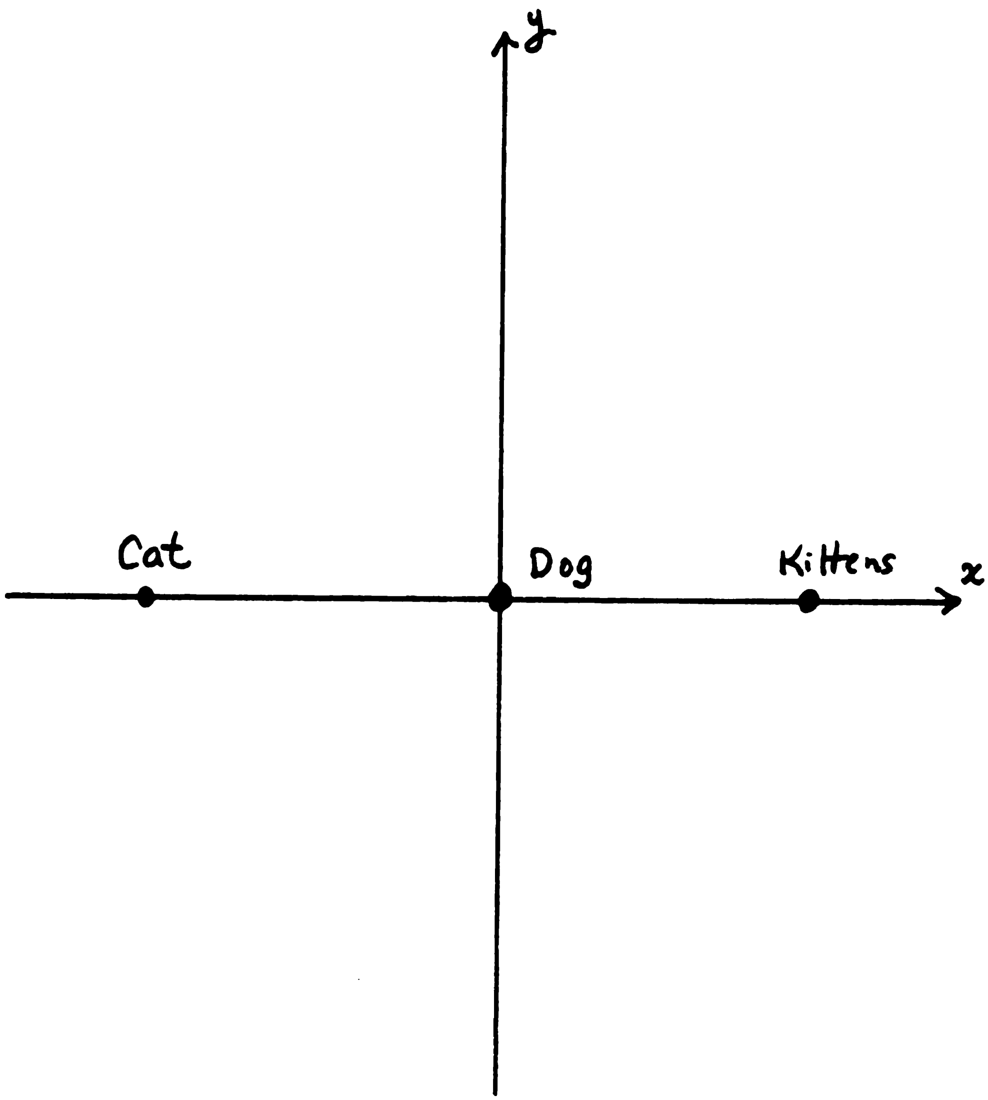


FIGURE 1.2.2

Configuration of Animals for Two Dimensional Approach-Avoidance Conflict

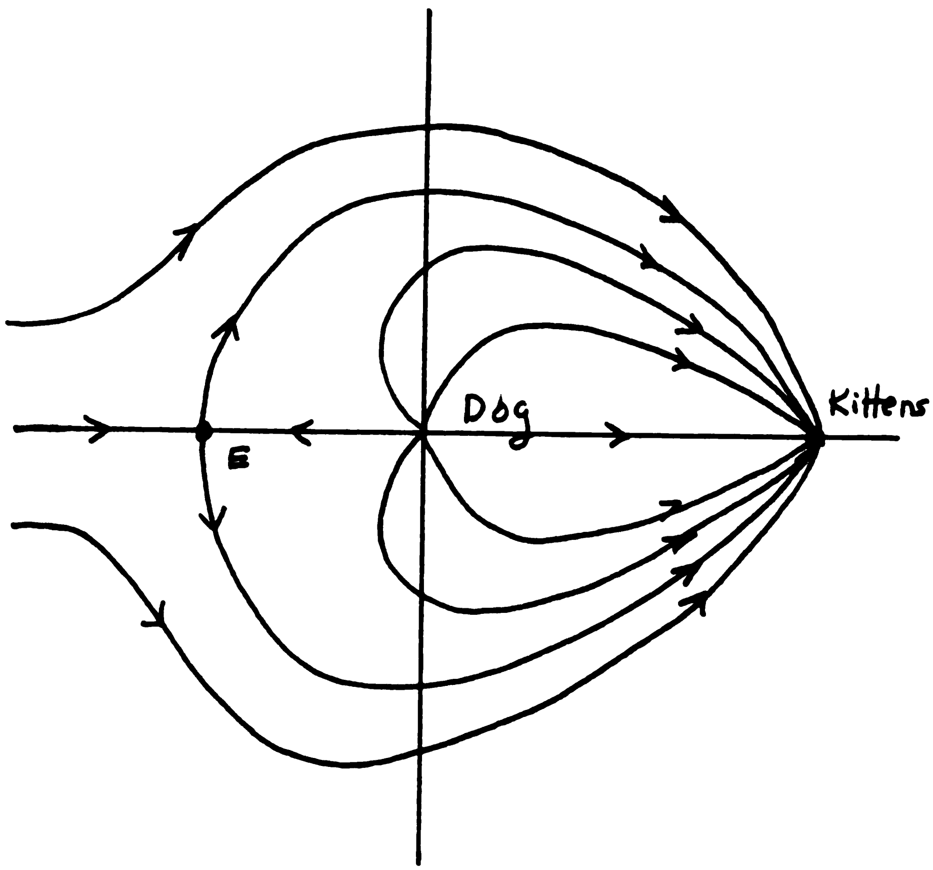


FIGURE 1.2.3

Lewinian Trajectories in Two Dimensional Approach-Avoidance Conflict there is a slit-shaped hole transverse to the direct path to the kittens, as shown in Figure 1.2.4. In this case, if the cat starts within the region subtended by the slit-hole with the kittens as the center (see Figure 1.2.4), the cat will reach a stable equilibrium at E where it is entrapped by its own motivations on the opposite side of the hole from the kittens. There is ample room for the cat to move safely to its kittens, but the vectorial theory predicts that the cat will not do so. For a real animal, the path shown in Figure 1.2.5 seems more probable: that is, the animal should circumnavigate the barrier regardless of the exact geometrical configuration.

In the experimental literature there is considerable evidence that even lower animals such as goldfish can circumnavigate a transverse obstacle. Lorenz [1982, pp. 237-241] discusses this behavior in goldfish. If there is an obstacle between the goldfish and its goal, the goldfish will execute the detour path. There are species-specific and age-specific differences in these behaviors. For example, according to Sholes [1965], an adult domestic chicken will not learn the detour behavior even when guided through the "correct" path by the experimenter. On the other hand, nine day old chicks will learn detour routing to a goal if they are coached along the way. According to Spigel [1964], turtles will accomplish the detour path behavior by a process that seems to involve much trial and error

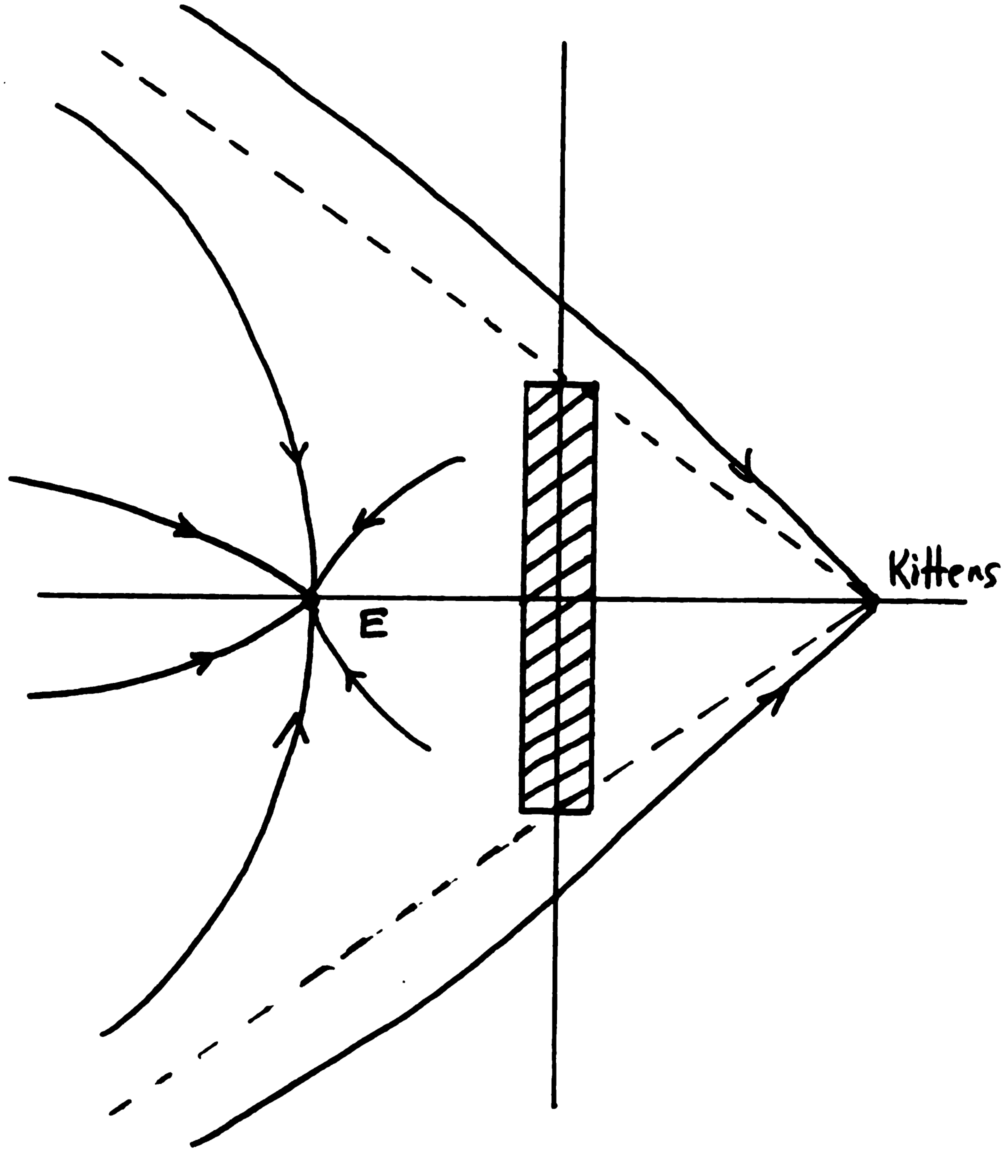


FIGURE 1.2.4

Entrapment in Two Dimensional Lewinian
Approach-Avoidance Situation

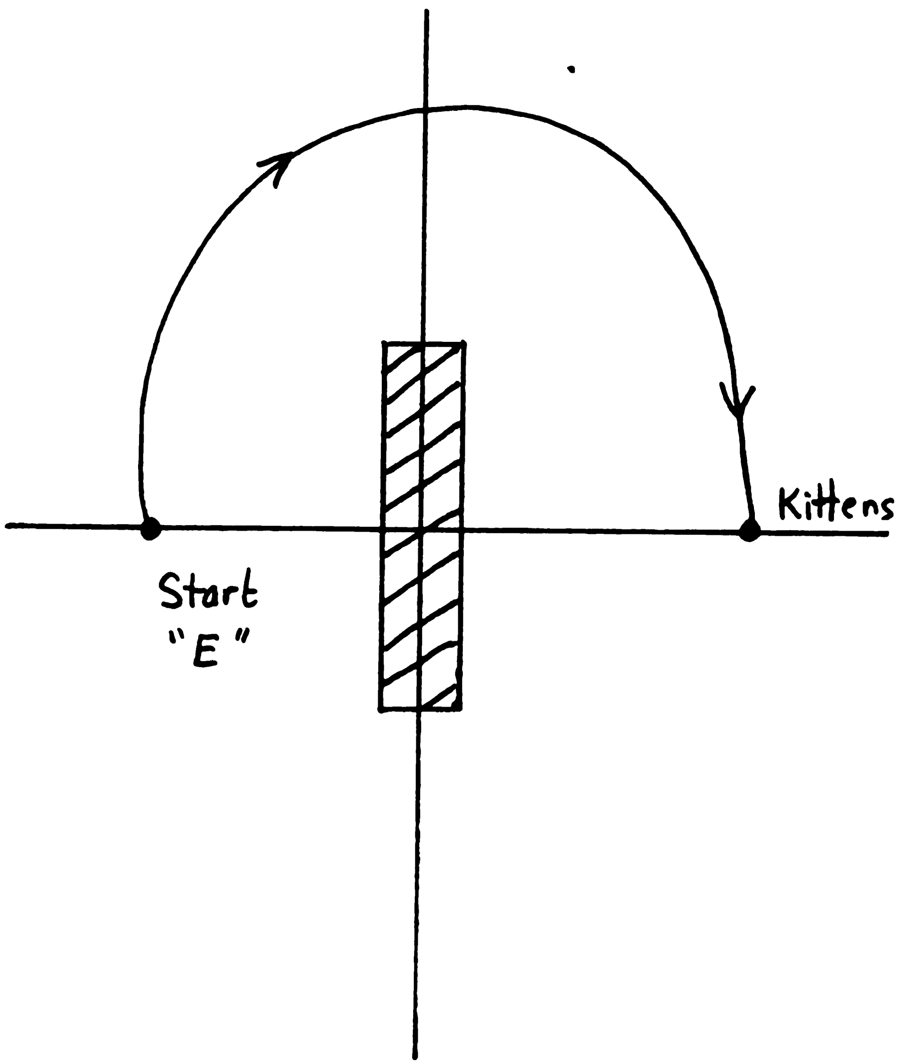


FIGURE 1.2.5

"Probable" Trajectory of Real Cat in Two Dimensional Approach-Avoidance Setting locomotion. In any case, they do not stall at the equilibrium point.

Dogs of various breeds and ages vary in their ability to solve detour problems. As reported by Scott and Fuller [1965, pp. 226-229], only 8 of 203 puppies solved the detour problem on the first test at six weeks of age. The breeds tested were Basenji, Beagle, Cocker, Sheltie, and Fox Terrier. Basenji's far outperformed the other breeds, with 18 out of 44 solving the second test on the first trial. The authors state that adult dogs have no problem whatsoever solving comparable detour problems. Thus there are learning and/or developmental factors operative in barrier test performance.

According to Scott [1958], there seem to be two varieties of behavior with respect to the detour problem. One involves frustration, much vocalization, exploratory behavior, and an occasional solution to the problem. In the other mode, the dog is quiet and the solution is comprehended in one step and immediately executed thereafter. Thus Scott [1958, page 152] speaks of a primary adaptive response consisting of frustrated exploration, competing with the immediate gestalt solution mode.

The survival importance of immediate solution of barrier problems is underlined by the work of Frank and Frank [1982] comparing the behavior of wild timber wolves (Canis lupis lycaon) to the results obtained by Scott and

Fuller. They tested four six week old wolf pups in a manner closely replicating the tests of dogs by Scott and Fuller [1965]. The initial solution by wolf pups was much superior to the performance of the domestic dog puppies. The error scores were 6.5 for the wolf pups as compared with 50.0 for the dogs. This result is for a U-shaped barrier where the animal is originally trapped inside of a large U-shaped wall and must reach food on the other side of the wall to solve the problem. In the test, the animal can see the food through a window in the barrier from its starting position, but can't reach the food. Other tests were also heavily in favor of wolf pups as better solvers of barrier circumnavigation problems than domestic puppies. This is also consistent with the result on the puppies alone. The Basenji's solved the problem better than other domestic dog pups, and the Basenji's are much more recently domesticated.

Frank and Frank [1982] again suggest that there are two problem solving modes at work: the primitive trial and error exploration mode and the immediate gestalt "contemplative" mode. In any case, none of the animals except domestic chickens were indefinitely trapped on the wrong side of the barrier by their own motivational system.

Next we will consider the avoidance-avoidance conflict of the Lewin-Miller theory as generalized to two dimensions. Here it helps to introduce some mathematical sophistication. In particular, we shall ask the question of whether or not a Lewinian force-field is irrotational. If a central field of force is the same strength in all directions, at an equal distance from the aversive object, the field is irrotational:

(1.2.1) curl (Force Vector) = 0

Any summation of irrotational force fields is again irrotational, so that if this property holds for all objects generating an emotional force, then it also holds for all of them together (Arfken, 1966, pp. 49-55). An irrotational field of force presents a particularly simple structure mathematically. The motion will always be represented by a scalar potential field with motion along the path of steepest descent. Thus, if we assume our Lewinian forces to be irrotational, we have the simple problem of an animal following a gradient of a scalar potential to a relative minimum. This potential function will be called the "avoidance potential" or "attractive potential" in the cases of aversive and attractive objects respectively. In the case of the superposition of a large number of such irrotational fields of force, we will call the sum of the potentials the "motivational potential." In algebraic sign, attractive potentials are counted as negative and repulsive potentials are positive.

Consider a cat surrounded by four dogs at the corners of a square centered on the origin (see Figure 1.2.6). Let us assume that the avoidance potential of each dog takes the simple form of one divided by the distance from that dog. In this case, the field of force of each dog is an inverse square field. The potential will have a relative minimum at the origin and the cat will be entrapped there by its own motivations. The region of entrapment is shown in Figure 1.2.7. The equilibrium point of the entrapment is at the origin and any starting position within the cusp-shaped region shown in Figure 1.2.7 will result in the animal eventually ending up at the origin.

In this example, and many that will follow, the mathematical argument leading to the entrapment of the Lewinian animal does not depend on the spatial scale of the configuration of objects to which the animal is responding. If the square in Figure 1.2.6 were four meters on a side, a real animal might indeed become trapped. However, if the square were one hundred meters on a side, entrapment is relatively unlikely. However, the Lewinian treatment of both is the same and leads to the same results. The same argument can be applied to most of the examples which we will discuss. Therefore, the reader is encouraged to envision the scale of diagrams flexibly. In the present example, for large spatial

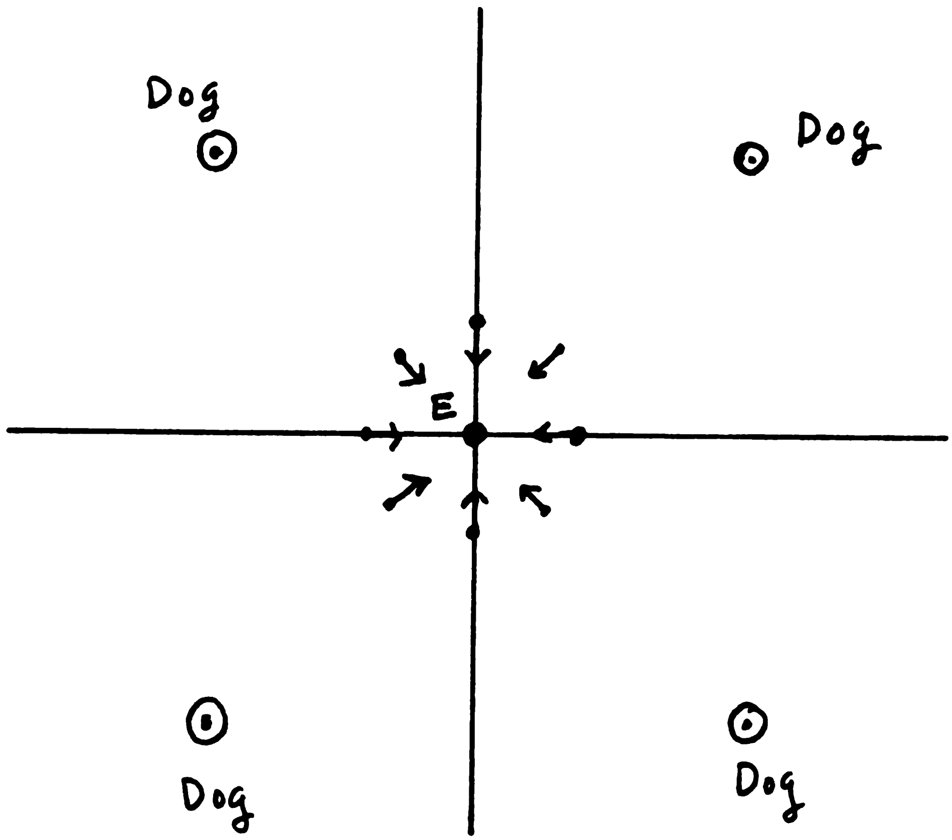


FIGURE 1.2.6

Stable Equilibrium in Two-Dimensional
Avoidance-Avoidance Setting

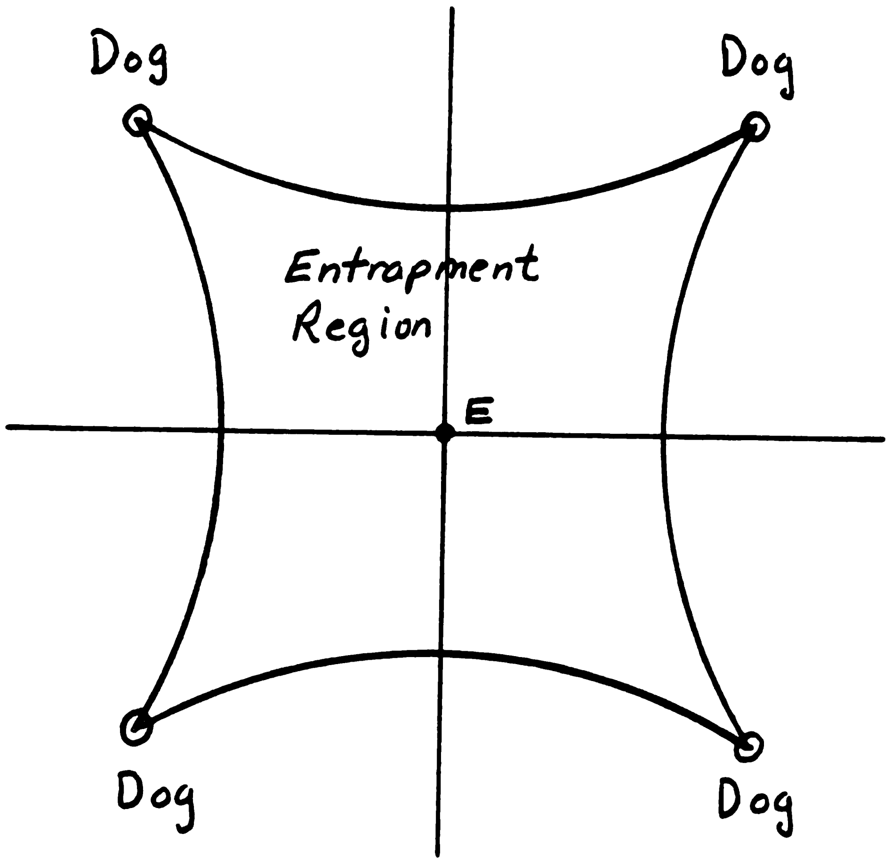


FIGURE 1.2.7

Entrapment Region in Two Dimensional
Avoidance-Avoidance Model

scale, we would expect the cat to escape via the kind of route shown in Figure 1.2.8.

Again consider the problem of entrapment. Figure 1.2.9 shows a dog which has trapped a cat in the cul-de-sac formed by a curved trench. In this case, given that the potential of the hole is a simple function of distance from the hole, such as its reciprocal, and similarly for the dog, entrapment again is predicted. In this case, the cat is trapped at the equilibrium point shown as E in Figure 1.2.9, according to the Lewin-Miller theory. The mathematical reason for this is shown in Figure 1.2.10, where the composition of forces is analyzed. Indeed, the entrapment region is obtained by dropping perpendiculars from the dog to the curved trench, as shown by the dotted lines in Figure 1.2.9. And again, we would expect that any real animal, at least as long as the dimensions of the trench were large, would escape as shown in Figure 1.2.11. However, this is not the prediction of the Lewin-Miller theory. Therefore, it is seen that the Lewin-Miller theory does not do well in the two dimensional avoidance-avoidance problem either.

The use of potentials to represent Lewinian theory is valid only if the fields themselves are irrotational in the sense of Equation (1.2.1). A Lewinian force field can be rotational. In Figure 1.2.12, schematic dogs are shown with (a) rotational and (b) irrotational central fields of force around them. In Figure 1.2.12(a), the

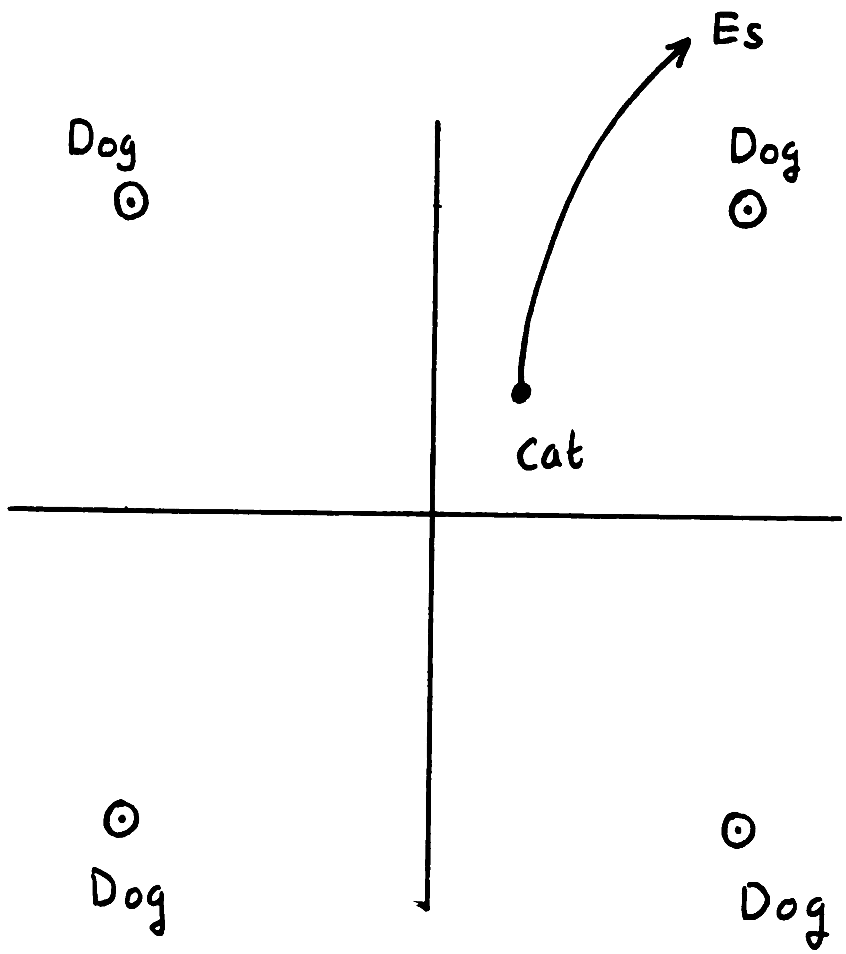


FIGURE 1.2.8

"Probable" Escape Route for Real Cat
in Avoidance-Avoidance Model

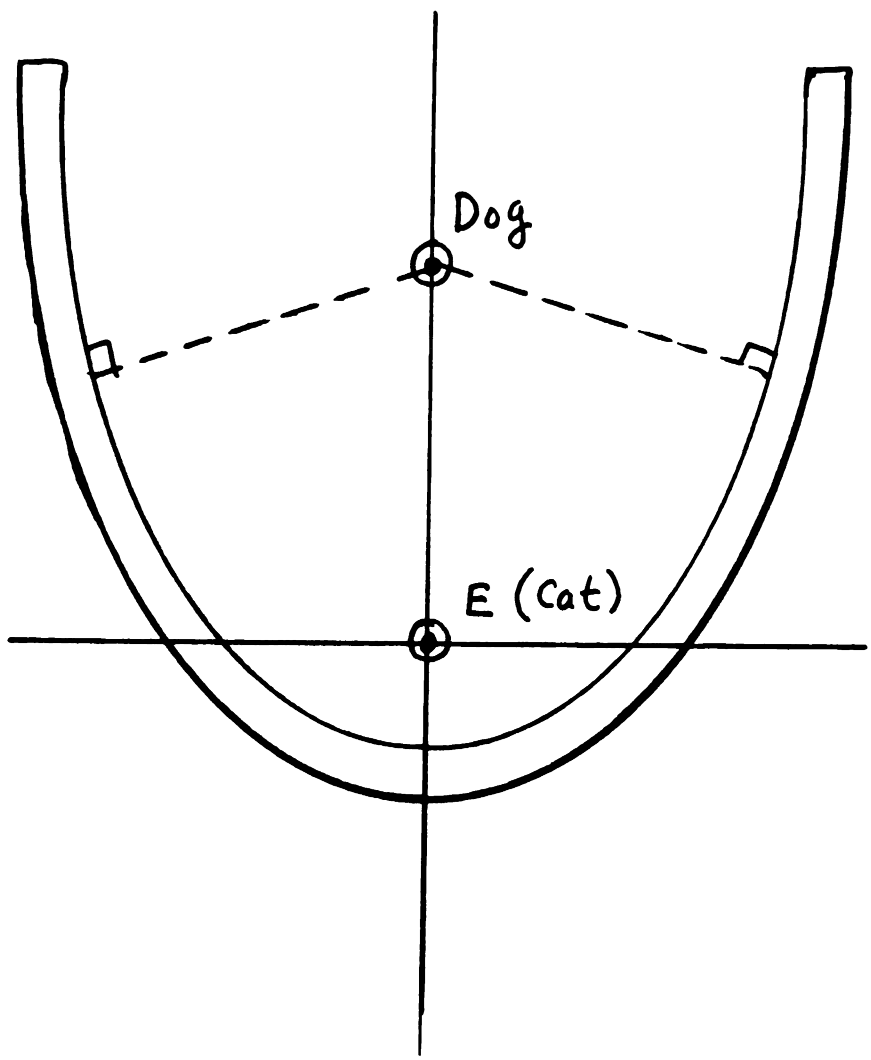


FIGURE 1.2.9

Cat Entrapped in a Cul-de-sac By
a Dog--Lewinian Model

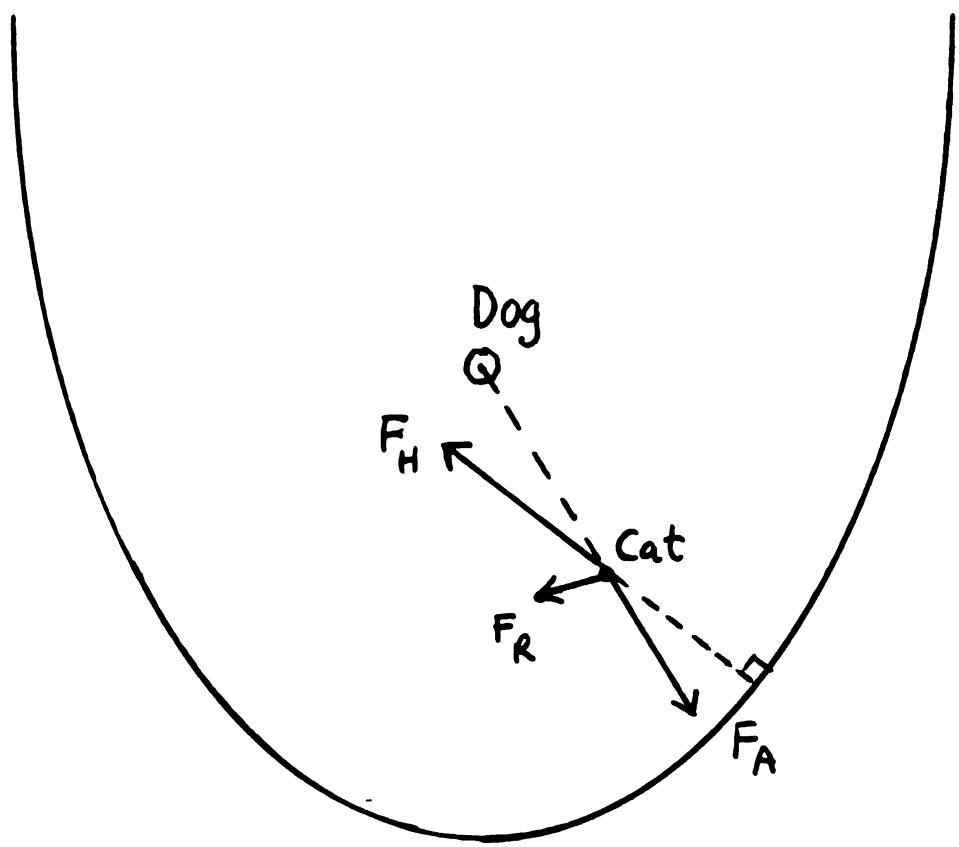


FIGURE 1.2.10

Summation of Psychological Forces for the Cat in the Cul-de-sac

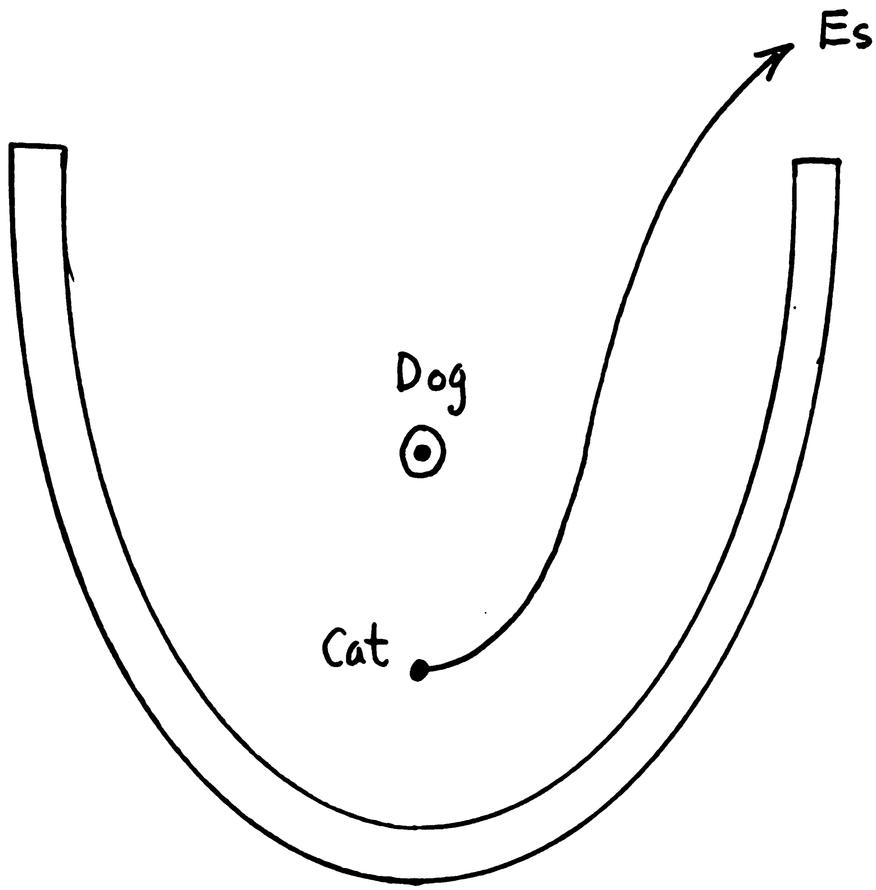
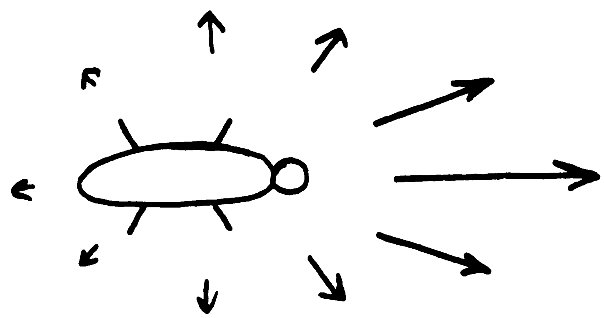


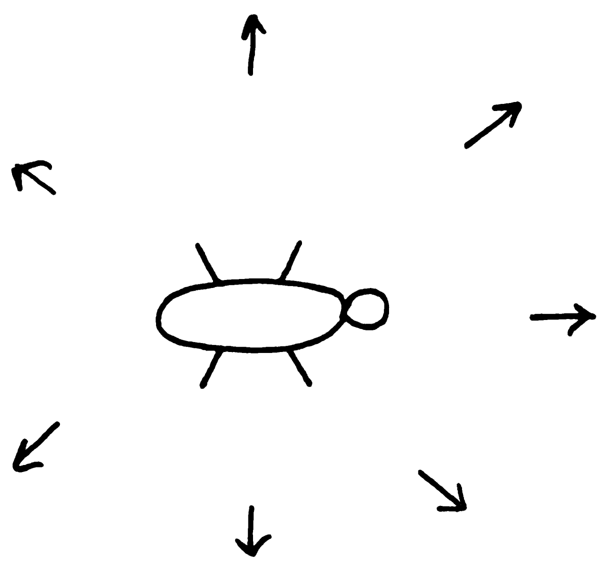
FIGURE 1.2.11

"Probable" Escape Route for Real Cat
From the Cul-de-sac



curl $F \neq 0$

(a)



curl $F = 0$

(b)

FIGURE 1.2.12

Rotational (a) and Irrotational (b) Fields of Force About a Predator force is stronger in front of the dog than behind the dog, and thus is not equal in all directions.

As might be expected from analogous phenomena in physics, such a rotational field may give rise to circulatory motion, that is, motion where the animal goes around and around a path indefinitely. Consider again four dogs surrounding a cat as shown in Figure 1.2.13. Let each dog's field of force be primarily in the forward direction and have each dog face counterclockwise around the square formed by the dogs. In this situation of asymmetric forces, the cat can be entrapped in a looped path, such as the trajectory shown as T, where it will circulate indefinitely. Thus, we certainly do not save the Lewin-Miller theory by allowing the force fields to be radially asymmetric and hence rotational. In fact, this model shows an utterly nonadaptive behavior as a very simple consequence of an asymmetric field of force. Certainly, if the scale of the square were large, we would in reality expect the animal to escape along some path such as Es in Figure 1.2.13. However, according to the Lewin-Miller theory, it cannot escape from the circulatory path shown as T.

Another problem with Lewin's theory relates to the existence of runways. A great many animals have runways in areas of cover, such as grasslands or forest. These are cleared pathways that they follow to get from one place to another. A simple set of runways is

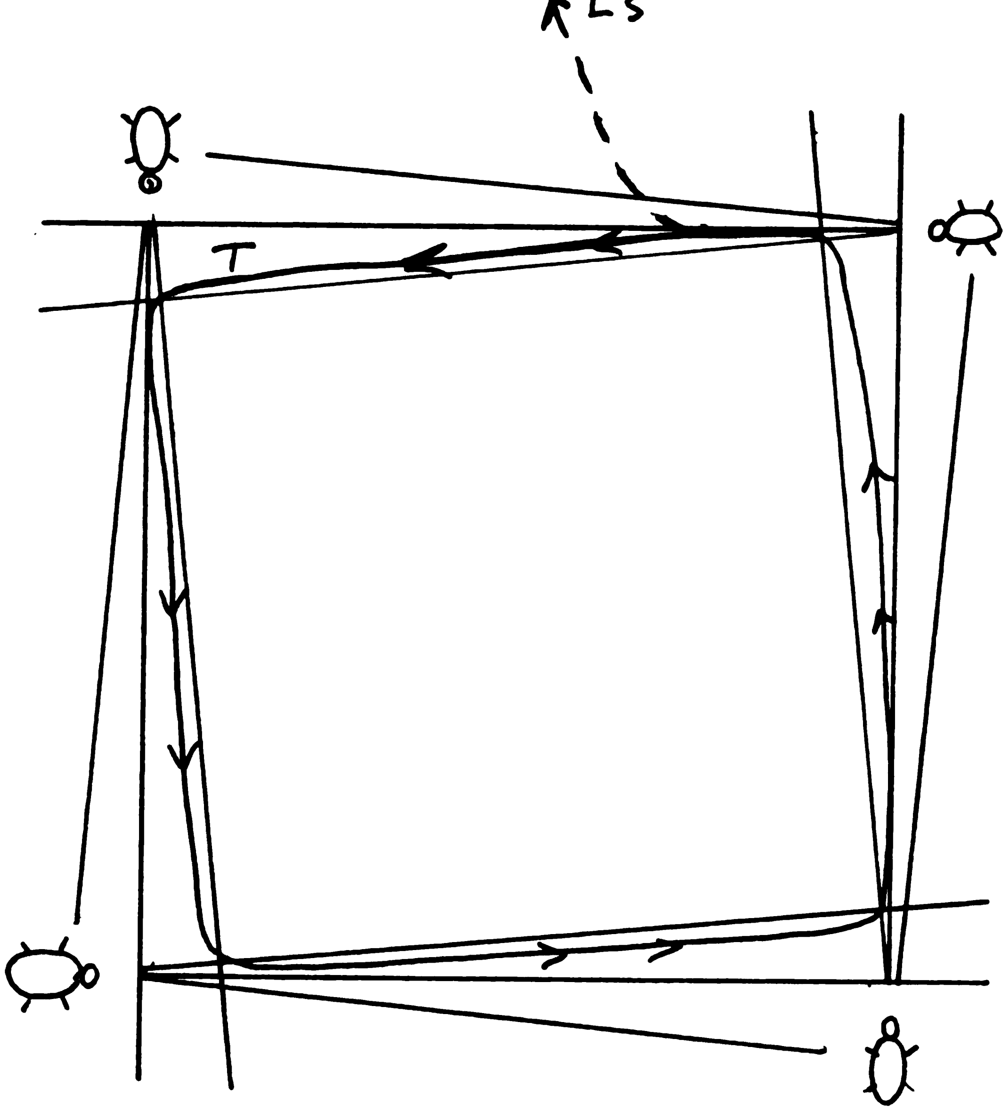


FIGURE 1.2.13

Circulatory Paths Can Occur in a
Rotational Lewinian Force Field

depicted in Figure 1.2.14. Envision a forest with low cover for the animals and a predator, such as an owl sitting in a tree, at the place marked in the diagram. The obvious interpretation of Lewin concerning such a case is that the psychological force vector is projected on the direction of each runway and the runway chosen at any choice point is that with the largest component of force in the direction away from the animal down the runway. As shown in Figure 1.2.14, this process can result in the animal taking a longer, and perhaps more dangerous, path than it would have taken if it followed some other rule.

In the theory which we will present, each of the problems illuminated in this section will be resolved by making a mathematical model of the animal's decision process that differentiates more clearly between goals and the surrounding field of aversions.

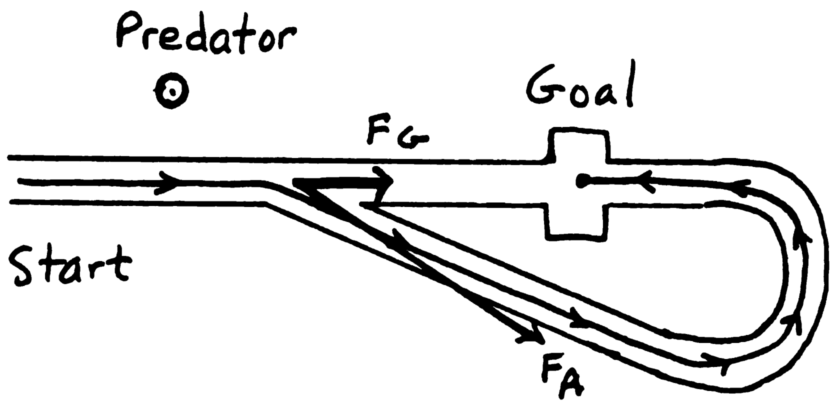


FIGURE 1.2.14

Deflection of an Animal's Path by a Lewinian Force Field Can Cause Unnecessary Detours in a Two Dimensional Runway Setting p

w

r

t

T

e

g

m

d

i

pe

dj

go

ch

CHAPTER 2

OPTIMAL MOTION PLANNING

Section 2.1 The Planning Process

Motion during instrumental behavior results from two cognitive processes: perception and action. According to classic behavioristic theories both processes are very crude. Perception consists of scanning the environment for positive objects which satisfy needs and negative objects which produce damage or pain. Action consists of either running from the object of greatest fear or moving toward the object of greatest need.

Lewin greatly elaborated the model of perception. The animal is assumed to create an internal model of the environment which preserves critical features of the geometry such as angles and distance. However, Lewin's model of action is still crude, the animal moves in a direction determined by immediate needs and fears. There is no anticipation of changes in needs of fears.

The theory to be developed uses Lewin's model of perception, but elaborates the model of action by introducing planning. Planning is introduced at two levels: goal choice and optimal movement. The model for goal choice is contemporary decision theory:

1. The animal generates a list of potential goals
2. The animal evaluates each goal in terms of the costs and benefits which derive from the attainment of that goal
3. The animal chooses the optimal goal, be that the best available good or the least of evils.

The focus of the present paper is on the costs incurred in achieving a goal, namely those costs which arise from the motion required to reach the goal.

There are two kinds of costs associated with motion. First, to the extent that the goal is urgent, long paths to the goal are more costly than short ones. Second, travel often requires exposure to danger, especially attack from nearby predators. The optimal path is that which minimizes the exposure to fear while keeping the length within reasonable limits.

The focus of the present theory is on the planning of motion. Thus the goal will always be taken as given. However, planning of motion enters into the goal selection process; the animal cannot know the cost of a goal until the optimal path to that goal has been planned. If the optimal path to a goal is too expensive, then that goal may be rejected. In this case, the optimal path will never be followed.

The theory will be presented in three parts: discussion of how to minimize exposure to danger, modification of the optimization criterion to take account of the urgency of the goal and consideration of how to compute the optimal path. Given the Lewin model of perception, the animal can minimize exposure to danger by minimizing anticipated exposure to fear. This leads to defining the optimal path as one which minimizes the integral of anticipated fear along the path. The urgency of the goal can be taken into account by modifying the fear function which is integrated and then minimizing the modified integral. Three methods of computing the optimal path will be discussed: the Moore algorithm, dynamic programming, and calculus of variations. The Moore algorithm is important because there is a neural network which could carry out the Moore algorithm very efficiently. Calculus of variations is important because it is the mathematical method of choice for obtaining the predictions of the theory. Dynamic programming provides the bridge between these two formalisms.

Suppose that a wild animal lives in a field of fear created by the presence of predators. For example, Figure 2.1.1 shows the field of fear due to two predators in terms of contours of equal fear (iso-aversion contours). We symbolize the value of the fear at a point (x,y) in the plane as

$$(2.1.1) \quad f(x,y) = \text{value of fear at point } (x,y)$$

This fear is an anticipated fear. The animal associates the value of $f(x,y)$ with the fear that would apply if the animal were at the point (x,y). It needn't be at (x,y)

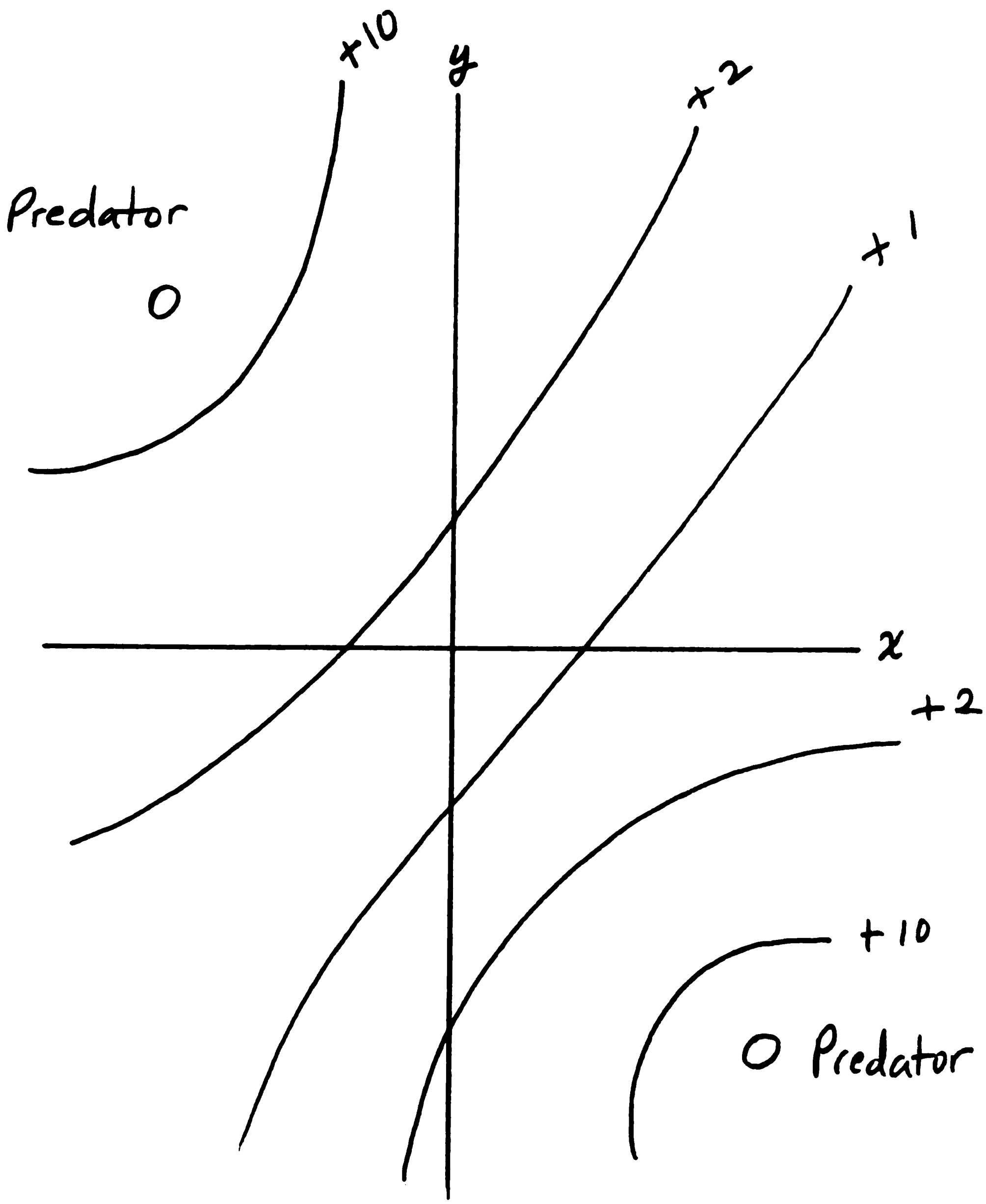


FIGURE 2.1.1

Iso-Aversion Contours Generated
by Two Predators

to do so, and in fact usually will not be. We will refer to f(x,y) as the fear function.

In considering various paths through a field of fear, we will define optimal paths. Optimality will be defined in terms of exposure to fear. Exposure to fear will be quantified as the integral of the fear function over the path of the animal with respect to time:

$$J = \text{exposure to fear} = \int_0^T f(x,y)\,dt \qquad (2.1.2)$$

Here the integral extends over the entire path and is the continuous analog of the following sum:

$$J = \text{exposure to fear} \cong \sum f(x,y)\,\Delta t \qquad (2.1.3)$$

In the case of discrete pathways, such as in a problem involving runways in a forest, the sum will suffice. In fact, we will have the following sum as the exposure to fear:

$$J = \sum_{\text{legs of path}} f_i\,\Delta T_i \qquad (2.1.4)$$

Fear is constant on each leg of the path, or may be considered to be so by taking averages, so that f_i represents the (average) fear level on the i^{th} leg of the path. In Equation 2.1.4, the fear on the i^{th} leg of the path is weighted by the time ΔT_i taken to traverse this leg of the path. The expressions 2.1.2, 2.1.3, and 2.1.4 all express exposure to fear which we symbolize by J. In each case, we have a time weighted sum of fear along the path.

For a given beginning and ending point an optimal path will be a path which minimizes the exposure to fear. Thus in the continuous case we pose the minimization problem

$$\text{Min } J = \text{Min} \int_0^T f(x(t), y(t))\, dt \tag{2.1.5}$$

The minimization is to be carried out with respect to all paths with the prescribed beginning and ending points, i.e., P_1 = starting point = (x(0), y(0)) and P_2 = ending point = (x(T), y(T)).

Section 2.2 Time Penalty

In this section we will consider the time dimension of the boundary of the life space and the corresponding time horizon. This will result in some adjustments to the value of the goal and to the fear function f(x,y). For example, if the planning is assumed to extend only to the point where the goal is reached and the animal has a fear f_r for the reward's safety, then we may write:

(2.2.1) $$f(x,y) = f_1(x,y) + f_r$$

This formula expresses total fear at (x,y) in terms of the anticipated fear of the moving animal for itself f_1 and the fear for the reward f_r. In other cases the animal may have some independently determined priority for getting to the goal in a timely manner. For example, it may also put a premium on time spent getting to the present goal because it wishes to explore other goals. Then we will have:

(2.2.2) $$J = \int_0^T [f_1(x,y) + f_r]\, dt + w_p T$$

Since the time weight w_p expressing this premium on time is in the same units as fear, we can rearrange our expression to get

(2.2.3) $$f(x,y) = f_1(x,y) + f_r + w_p$$

At this point, we note that the general form of the integrand can be taken to be

(2.2.4) $$f(x,y) = f_1(x,y) + f_0$$

where f_0 is a lumped constant expressing the net time premium in units of fear. We will call the term f_0 the net goal fear since it usually consists of f_r and some other terms. For example, in the above discussion

$$f_o = f_r + w_p \tag{2.2.5}$$

Section 2.3 The Moore Algorithm and Dynamic Programming

We will propose a method of solution of the minimization problems for finding the minimum path discussed in Section 2.1 which is based on purely combinatorial process. The reason for our choice is that the combinatorial method proposed can be implemented approximately in neural networks. Our solution method will be a variety of dynamic programming (see Bellman, 1957 and 1961; Larson and Casti, 1978; Wagner, 1975). To make our method clear, we will first present a variant known as the Moore Algorithm (Moore, 1957).

The Moore algorithm allows us to find the minimum path through a network (formally a digraph) given the costs of getting from one node to another. At this point we should introduce a few terms from discrete optimization theory: a <u>network</u> is a group of points, called nodes, connected by means of transition from one node to another, called <u>links</u>. In optimization theory, each link is associated with the <u>cost</u> of getting from its starting node to its terminal node. A <u>path</u> going from node A to node B is a series of links, starting at A and concatenating in such a fashion that the end of each link is the start of the next, with the end of the last link in the chain being B. A network is said to be connected if there is some path in it leading from any node to any other. That is, for any two nodes A and B, there is a path which begins at A and ends at B. In the language of optimization, our concept of fear exposure is the analog of cost. Thus the problem is that of minimizing the total cost,

$$J = \sum_{path} J_{ij} \tag{2.3.1}$$

where the i^{th} node is the starting point of a particular link in the network and the j^{th} node is its terminal point. The first value of i is the start node and the last value of j is the goal. J_{ij} is the cost of getting from node i to node j. We will, in addition, assume that the i's and j's chosen represent a path (see Figure 2.3.1).

The Moore Algorithm is best illustrated by providing a schematic computer program for its implementation. In the following we will have the elemental costs of links in the array J(N,M) with infinity assigned if the points N and M are not directly connected. The array CUM will contain the provisional minimum cost of getting from the START to the node I. That is, the provisional cost of getting to node I will be CUM (I). The back node, being the best node to come from to reach node N, will be given

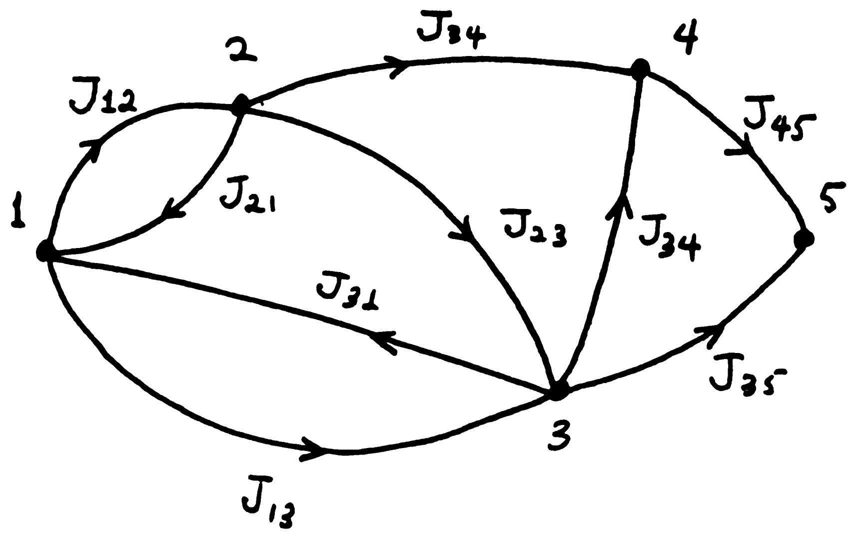


FIGURE 2.3.1

Example of a Network. The Series of Nodes 1, 3, 4, 5 Constitutes a Path J_{13}, J_{34}, J_{45}

by BACK (N). The FRONTIER will be a set of nodes that are being evaluated as jumping off places for the continuation of the trajectories. The computer program, in purely symbolic form, follows:

INITIALIZATION STEP: Make the FRONTIER set contain only the START. Set BACK (START) = 0. Set CUM (START) = 0. Set CUM (I) = infinity or a very big number for all other nodes.

FORWARD RECURSION: DO UNTIL FRONTIER SET EMPTY:
Take any node in the FRONTIER. Call it I. For each other node K, for which J(I,K) is less than infinity, compute CUM (I) + J(I,K). Compare this to CUM (K). If it is greater than CUM (K), then go to the next test K; otherwise assign CUM (K) = CUM (I) + J(I,K), add node K to the FRONTIER and assign BACK (K) = I. When you have evaluated all prospective nodes K for the node I, then remove I from the FRONTIER. Loop back.

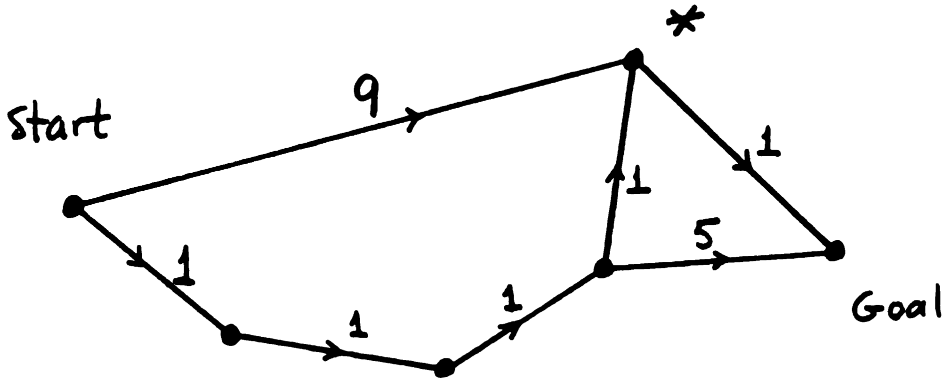
BACKWARD RECURSION: To get minimum paths the following will suffice: choose the desired GOAL node, G. Then compute the sequence: G, BACK (G), BACK(BACK (G)), BACK(BACK(BACK (G))), etc. until the START node is reached. Then the reverse sequence of nodes is optimal as a path to get from the START to the GOAL.

END.

A computer program implementing the Moore Algorithm in BASIC is given in Appendix A.

In the case that the process of path selection can be made into a one way process consisting of STAGES, however arbitrarily defined, then dynamic programming can be used. In dynamic programming we don't need to consider the FRONTIER as a dynamic entity and simply use the stages as frontiers, one at a time, without the possibility of a node re-entering the FRONTIER. This approach is of limited value in our calculations since situations such as are represented in Figure 2.3.2 can exist which make it impossible.

However, stages were used in the neural network of Dolson and Bell [1981] which can be used as the basis for a neural network solution of the minimum path problem. In the Dolson-Bell solution for the discrete case of a finite network, all link costs J_{ik} are made multiples of some common increment Δ. That is, all link costs were approximated as integers. In this case, we may define a STAGE as being all points n units out from the origin, i.e., an equidistant set as measured in cost from the START. For a stage defined this way, there is no possibility of nodes being reentered. This greatly simplifies neural networks, but would make a great burden on a sequential machine. The only reason why it is practical at all is that neural networks are parallel processors



Notes:
1.) Cost shown by links.
2.) Node (*) enters the FRONTIER at computation stage 1 and re-enters at stage 4.

FIGURE 2.3.2

Re-entry of Node into the FRONTIER is Caused by Certain Combinations of Link Costs so that all paths may be computed in this manner concurrently (see Dolson and Bell, 1981).

Section 2.4 Computation and Analysis in the Continuous Case

The Moore Algorithm and dynamic programming can both be made continuous in a variety of ways. Bellman [1961] shows how to make the dynamic programming method work in the continuous case in a number of ways. Some of these same methods are explained in more detail in other works (Bellman, 1957; Larson and Casti, 1978). These methods inherently make use of continuity of some of the functions involved and employ interpolative techniques. Moreover, they are highly dependent on the digital computer for both numerical and combinatorial features of the algorithms. We will use one such program employing continuous dynamic programming to get solutions to our continuous case problem of minimizing paths in the sense of the integral

$$\text{Min } J = \text{Min} \int_0^T f(x,y)\,dt \tag{2.4.1}$$

for the purpose of illustrating the consequences of our theory. The relevant BASIC computer program is given in Appendix A.

However, most of the time, for the purposes of illustrating the consequences of our theory, the calculus of variations will be used to find properties of the paths predicted by Equation 2.4.1. The connection between the dynamic programming type of solution and the calculus of variations is given by Bellman [1961, Chapter 4].

For the purpose of suggesting a neural solution process, we will again revert to the Moore Algorithm. In this case we will proceed by making the network so fine that it approximates the continuous case. The resulting approximation is never exceedingly close, but may be good enough as an approximation to be used by animals in their everyday activity planning.

CHAPTER 3

MATHEMATICAL EXAMINATION OF THE PLANNING THEORY: TRAJECTORIES

Section 3.1 The Variational Form of the Trajectories for Static Fear and Maximum Velocity

We will now discuss the case of motion in a static field of fear at maximum velocity. In this case the paths can easily be derived by the calculus of variations (Bellman, 1961, Chapter IV; Arthurs, 1975), providing that the fear fields and trajectories obey certain conditions of analytic regularity.

First of all, the assumption that animals would move at maximum velocity in a field of fear should be justified. Drawing data from many empirical sources (Brody, 1964; Oron et. al., 1981; Sonne and Galbo, 1980; Chassin et. al., 1976; Taylor et. al, 1970; and Taylor et. al., 1974), we can state with some confidence that the energy needed to move a specified distance decreases with running speed for most four-legged animals. Therefore, it is not unreasonable to assume that an animal in a state of fear would run at its maximum velocity, providing its fear level is independent of its speed (the contrary assumption will be dealt with in the next section). The reason for maximum velocity is immediately apparent in terms of the assumption (Section 2.1) that the animal is choosing trajectories so as to minimize its exposure to fear, given by the integral:

$$(3.1.1) \qquad J = \int_0^T f(x,y)\,dt$$

where x(t) and y(t) describe the trajectory in space in terms of the time t. In fact, if the trajectory is assumed to be fixed in spatial form, then the fear exposure will be inversely proportional to velocity. Thus both energy expenditure considerations and fear exposure mitigate in the direction of increased velocity.

As expressed by Equation 3.1.1, the fear minimization problem is not yet cast in a way that the calculus of variations can be used directly to determine the spatial path. This is because the endpoint in time, T, is not fixed in advance, although the spatial endpoints $(x_1,y_1) = P_1$ = the starting position, and $(x_2,y_2) = P_2$ = the position of the goal, are fixed. For this reason, we will re-parametrize the trajectory problem by introducing a "dummy" parameter, s, which is zero at P_1 and equal to 1 at P_2. Then the problem becomes that of minimizing the integral:

$$(3.1.2) \qquad J = \int_0^1 f(x(s), y(s)) \cdot \frac{dt}{ds} \cdot ds$$

The last equation is gotten from Equation 3.1.1 by a direct change of variable. Now, due to the fact that the animal is assumed to run at maximum velocity, which is a constant, we can express dt in terms of the length of arc traversed in infinitesimal time as follows:

$$dt = \frac{d\,(\text{path length})}{V_{MAX}} \tag{3.1.3}$$

We can then exploit the relationship between arc length and the differentials of x(s) and y(s) to eliminate t entirely from our model formulation 3.1.2. We proceed as follows:

$$\begin{aligned} d\,(\text{path length}) &= \sqrt{dx^2 + dy^2} \\ &= \sqrt{x'(s)^2 + y'(s)^2}\; ds \end{aligned} \tag{3.1.4}$$

Then combining (3.1.3) and (3.1.4), we obtain:

$$\frac{dt}{ds} = \frac{1}{V_{MAX}} \cdot \sqrt{x'(s)^2 + y'(s)^2} \tag{3.1.5}$$

This expression can be substituted into (3.1.2) to yield the following:

$$J = \frac{1}{V_{MAX}} \int_0^1 f\,(x(s), y(s)) \sqrt{x'(s)^2 + y'(s)^2}\; ds \tag{3.1.6}$$

as the exposure to fear on the path between P_1 and P_2. Then, omitting the constant v_{MAX}, we have a classical variational problem of minimization:

$$\text{(3.1.7)} \quad \text{Min} \int_0^1 f(x(s), y(s)) \cdot \sqrt{x'(s)^2 + y'(s)^2} \cdot ds$$

with the two independent variables x(s) and y(s) being varied between the fixed endpoints,

$$\text{(3.1.8)} \quad P_1^* = (0, x_1, y_1) \text{ and } P_2^* = (1, x_2, y_2)$$

Here it is understood that the endpoints are fixed, but that the shape of the path can be varied between them to minimize the expression in (3.1.7)

In the variational problem (3.1.7), we have two dependent variables, x(s) and y(s). Therefore, in our calculus of variations solution, there will be two Euler equations. These are as follows:

$$\text{(3.1.9)} \quad \sqrt{(x')^2+(y')^2}\ \frac{\partial f}{\partial x} = \frac{d}{ds}\left\{ f(x,y)\, \frac{\partial}{\partial x'} \sqrt{(x')^2+(y')^2} \right\}$$

and

$$\text{(3.1.10)} \quad \sqrt{(x')^2+(y')^2}\ \frac{\partial f}{\partial y} = \frac{d}{ds}\left\{ f(x,y)\, \frac{\partial}{\partial y'} \sqrt{(x')^2+(y')^2} \right\}$$

The symmetry of these equations, with respect to interchange of x and y, guarantees that these equations and equations that follow are identical with x and y interchanged throughout. Therefore, we will proceed to solve 3.1.9 and assume that the solution of 3.1.10 is the same, with x and y interchanged. Writing out the derivative indicated within the braces on the righthand side of 3.1.9, we have:

$$\frac{\partial}{\partial x'} \sqrt{(x')^2+(y')^2} = \frac{x'}{\sqrt{(x')^2+(y')^2}} \tag{3.1.11}$$

Multiplying 3.1.11 by f and differentiating by s to get the entire expression on the righthand side of 3.1.9, we obtain:

$$\frac{d}{ds} \frac{x' f}{\sqrt{(x')^2+(y')^2}} = \frac{(x')^2}{\sqrt{(x')^2+(y')^2}} \cdot \frac{\partial f}{\partial x} + \frac{x' y'}{\sqrt{(x')^2+(y')^2}} \frac{\partial f}{\partial y} + \frac{(y')^2 x'' - x' y' y''}{\left[(x')^2+(y')^2\right]^{3/2}} \cdot f \tag{3.1.12}$$

The lefthand side of equation 3.1.9 can be expressed as follows:

$$\frac{(x')^2+(y')^2}{\sqrt{(x')^2+(y')^2}} \frac{\partial f}{\partial x} \tag{3.1.13}$$

Substituting 3.1.13 and 3.1.12 into Equation 3.1.9, we get, with very minimal rearrangement, the following:

$$\frac{(y')^2}{\sqrt{(x')^2+(y')^2}}\cdot\frac{\partial f}{\partial x} - \frac{x'y'}{\sqrt{(x')^2+(y')^2}}\cdot\frac{\partial f}{\partial y} = f\,\frac{(y')^2 x'' - x'y'y''}{[(x')^2+(y')^2]^{3/2}} \tag{3.1.14}$$

Dividing 3.1.14 by y'f, and changing signs throughout, we obtain:

$$\frac{x'}{\sqrt{(x')^2+(y')^2}}\cdot\frac{1}{f}\cdot\frac{\partial f}{\partial y} - \frac{y'}{\sqrt{(x')^2+(y')^2}}\cdot\frac{1}{f}\cdot\frac{\partial f}{\partial x} = \frac{x'y'' - y'x''}{[(x')^2+(y')^2]^{3/2}} \tag{3.1.15}$$

Now we introduce several simplifications: these are based on the following well known identities. Namely we have the following substitutions,

$$\frac{1}{f}\frac{\partial f}{\partial x} = \frac{\partial}{\partial x}\ln f, \quad \frac{1}{f}\frac{\partial f}{\partial y} = \frac{\partial}{\partial y}\ln f \tag{3.1.16}$$

$$T_x = \frac{x'}{\sqrt{(x')^2+(y')^2}}, \quad T_y = \frac{y'}{\sqrt{(x')^2+(y')^2}} \tag{3.1.17}$$

(3.1.18) $$U_x = -\frac{y'}{\sqrt{(x')^2 + (y')^2}}\,,\quad U_y = \frac{x'}{\sqrt{(x')^2+(y')^2}}$$

where the T vector is the unit tangent to the trajectory, in the direction of motion, and the U vector is the unit normal to the path. Moreover, the expression on the righthand side of 3.1.15 is in fact the curvature of the path of motion, in parametric form:

(3.1.19) $$K = \frac{x'y'' - y'x''}{[(x')^2 + (y')^2]^{3/2}}$$

Therefore, we can write the following vectorial equation:

(3.1.20) $$\underset{\sim}{K} = \underset{\sim}{T} \times \mathrm{Grad}(\ln f) = \underset{\sim}{U} \cdot \mathrm{Grad}(\ln f)$$

using these substitutions in 3.1.15. The x between T and the gradient of log f is the vector cross-product, and the dot between U and gradient of log f is the vector dot product (scalar product). This result can be summarized much more simply as follows: the curvature of the path is the perpendicular projection of the gradient of the logarithm of f:

(3.1.21) $$\underset{\sim}{K} = \mathrm{Proj}_{\perp} \nabla \ln f$$

That is, if the gradient of the logarithm of the fear function is obtained, then its component perpendicular to the path of motion is the curvature vector of the path, defined canonically as the reciprocal of the radius of curvature times a unit vector pointing in the direction of the center of curvature of the path. Our other variational equation, 3.1.10, yields exactly the same result under the corresponding operations with x and y interchanged. Therefore, it yields no additional information as to the nature of the path. Equation 3.1.21 is, in highly compact symbolic form, the equation of the path of the animal predicted by our model. In this form it is not too useful for obtaining the path (for example, by integration), but it is very useful for deducing the consequences of our theory of animal motion at constant velocity in a field of fear.

First of all, we note that the curvature vector K points in the direction of the center of curvature and has a magnitude equal to the rate of curvature of the path. Thus, the path curves toward the upward gradient of fear. That is, since fear generally increases with closeness to the feared object, the path generally curves toward the feared object. See Figure 3.1.1 for an illustration of these concepts and relationships. The interpretation of Equation 3.1.21 is very straightforward in many respects. Since only the derivative of the logarithm of the fear appears in the equation, we can at once observe that this

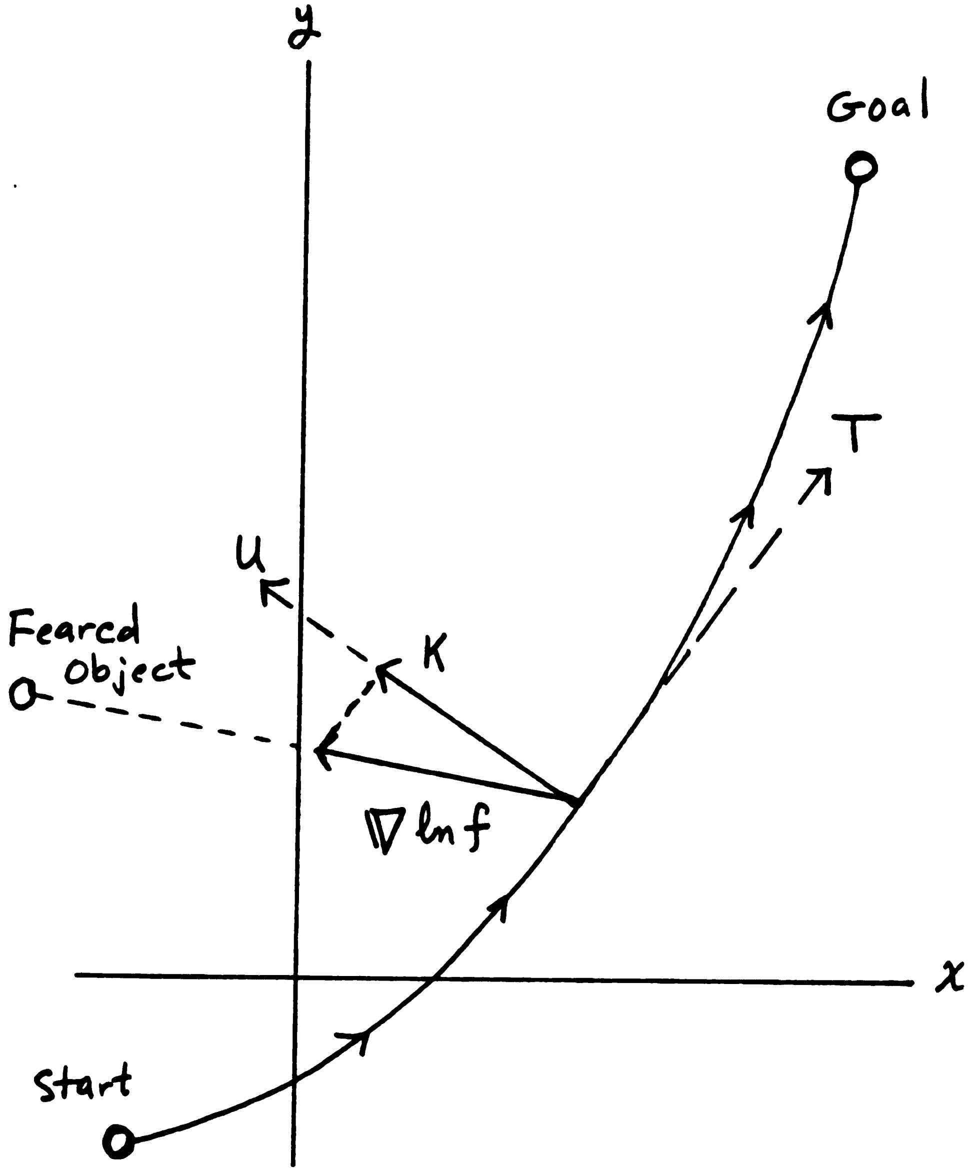


FIGURE 3.1.1

Variational Path, Showing Vectors
U, T, K, and $\nabla \ell n f$

equation expresses the relationships of fear in a unitless manner; that is, multiplying fear by a constant throughout does not make any difference. In other words, the level of fear is determined only up to a multiplicative constant by our model. In contrast to Lewin's theory discussed previously, Equation 3.1.21 implies that the animal that is running in a field of fear turns toward the feared object, rather than being pushed away from it. At first this might seem counterintuitive. However, what it means in terms of trajectories is rather direct: the animal must start out its path in a direction deflected from the source of fear and then curve around the source.

Another thing we can investigate directly by means of Equation 3.1.21 is the effect of adding a constant fear for the goal (ex., for the kittens) to the fear the cat experiences for itself. Let f_0 be a constant fear added to the spatially dependent fear $f_1(x,y)$. In other words, let the total fear be

$$f(x,y) = f_1(x,y) + f_0 \tag{3.1.22}$$

Then the gradient of the logarithm of the fear will be reduced in magnitude:

$$\text{Grad} \ln [f_1(x,y) + f_0] = \frac{\text{Grad } f_1(x,y)}{f_1(x,y) + f_0} \tag{3.1.23}$$

The more the constant fear f_0, the smaller the gradient, and hence the smaller the curvature of the path, according to Equation 3.1.21. In the limit, as the constant fear becomes very large, the curve becomes a straight line (its curvature is zero). See Figure 3.1.2. Later on in this section we will provide an explicit computational example of this flattening of the trajectory caused by the addition of a constant fear term.

Although Equation 3.1.21 has been derived under general assumptions as the solution of the minimization problem in Equation 3.1.7, it is not convenient for computational purposes. It is similarly unsuited for detailed analytic investigation of solutions. For these two purposes we will rewrite Equation 3.1.7 in polar coordinates, assuming that the feared object is at the origin. In this manner we will not have to worry about singular derivatives of the coordinate transformation at the origin, since the running animal will never actually go through the origin. Noting that the increment of path length can be written as follows,

$$dl = \sqrt{x'(s)^2 + y'(s)^2}\, ds \tag{3.1.24}$$

it can also be written in polar coordinates according to

$$dl = \sqrt{1 + r^2 \dot{\theta}^2}\, dr \tag{3.1.25}$$

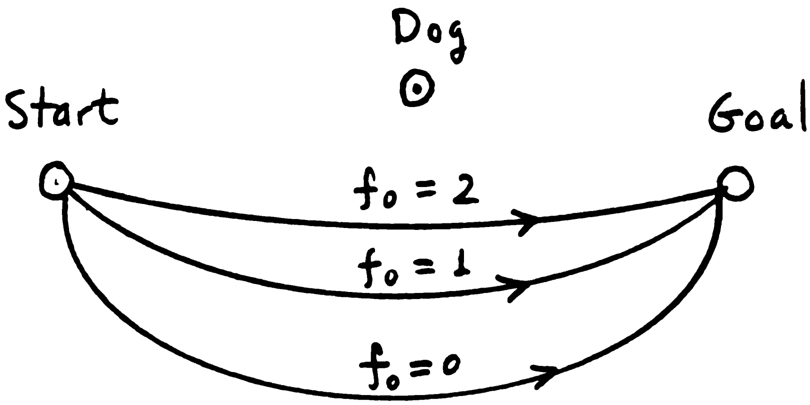


FIGURE 3.1.2

Increasing Fear for the Goal
Flattens an Animal's Path

where we have taken r as the independent variable and theta as the dependent variable. In the case of a radially symmetric fear function,

$$f = f(r) = f\left(\sqrt{x^2+y^2}\right) \tag{3.1.26}$$

we can rewrite the basic minimization problem, 3.1.7, as the following minimization problem:

$$\text{Min } J = \int_{r_1}^{r_2} f(r)\sqrt{1+r^2\dot{\theta}^2}\, dr \tag{3.1.27}$$

$$\text{where} \quad \dot{\theta} = \frac{d\theta}{dr}$$

We may have problems with the endpoints r_1 and r_2 in that the trajectory may pass through the same radial distance from the origin multiple times. In this case, we will simply splice pieces of curve together, as analytic splines, so that the trajectory remains an extremal of the variational problem (first derivative continuous). The Euler equation for 3.1.27, with theta as the dependent variable, takes the following very simple form:

$$\frac{\partial}{\partial \dot{\theta}} f(r)\sqrt{1+r^2\dot{\theta}^2} = \text{Const.} \tag{3.1.28}$$

(since theta does not appear explicitly in the integrand). Doing the indicated differentiation, we get the following:

$$\frac{f(r)\, r^2 \dot{\theta}}{\sqrt{1 + r^2 \dot{\theta}^2}} = \text{Const} \tag{3.1.29}$$

Solving for $\dot{\theta}$ and integrating formally, we obtain,

$$\theta(r) = \int^r \frac{c\, dr}{r\sqrt{f^2 r^2 - c^2}} \tag{3.1.30}$$

expressing theta as an integral including in its expression the fear function f(r).

For the simple case,

$$f(r) = \frac{1}{r} + f_0 \tag{3.1.31}$$

where the fear is dependent on r and contains an arbitrary static fear f_0, such as a fear for the goal-object, we obtain a family of curves depending on f_0, the static fear, which are illustrated in Figure 3.1.3 and in Figure 3.1.4. These are exact curves based on formal calculations much facilitated by the simple forms of Equation 3.1.30 and Equation 3.1.31. These curves clearly illustrate the effect of a static fear, such as a cat's fear for its kittens, in flattening the trajectories. The curves, in this case, are given by the functions that

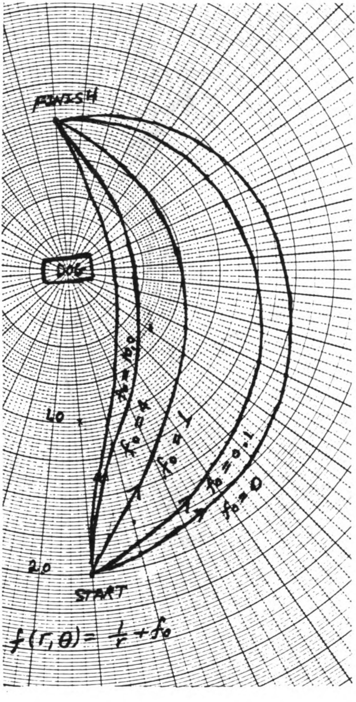


FIGURE 3.1.3

Computed Optimal Paths Show Flattening With Increasing Fear for the Goal

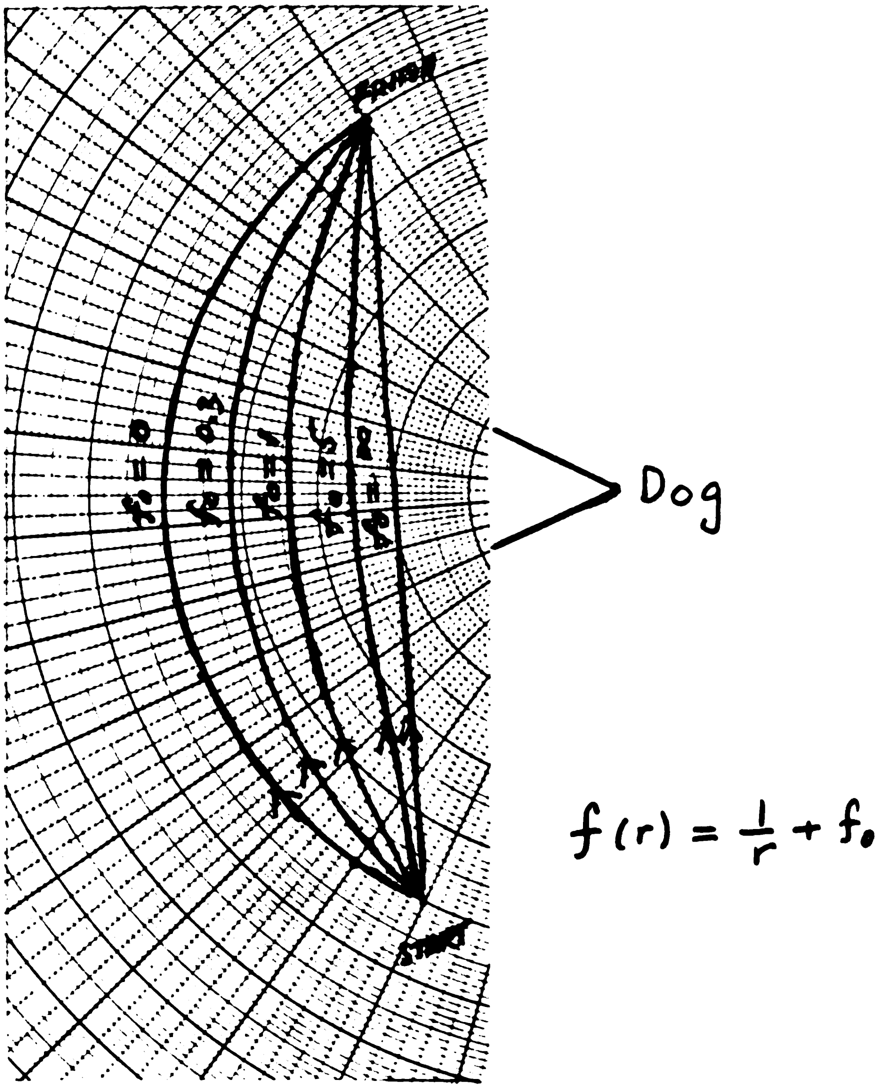


FIGURE 3.1.4

Computed Optimal Paths

follow, which are integrals 3.1.30 for the substitution in 3.1.31:

$$r = r_0 e^{\alpha(\theta - \theta_0)} \qquad (f_0 = 0) \tag{3.1.32}$$

$$r(\theta) = \frac{c^2 - 1}{f_0 \left[1 + c \cos \frac{\sqrt{c^2 - 1}}{c} (\theta - \theta_0) \right]} \qquad (f_0 > 0) \tag{3.1.33}$$

The constants θ_0 and alpha in Equation 3.1.32 are constants of integration, as is r_0. These constants must be determined by the endpoint conditions. In 3.1.33, c and θ_0 are the constants to be determined by the endpoint conditions. The boundary conditions consisting of the requirement that the trajectory must go through the specified endpoints, the first being the point where the motion begins and where the planning is carried out mentally by the animal and the second being the position of the goal, must be satisfied by the solution specified by choice of these constants of integration. This choice represents a complex computational process, involves solving transcendental equations, and is best left to the computer. The results in Figures 3.1.3 and 3.1.4 were determined by exact computation by means of one of the computer programs in Appendix A. In very few cases is the integral 3.1.30

solvable in quadratures. In most cases, it must be integrated numerically. However, it does have the advantage of reducing the problem to a one-variable integration.

Let us now turn to the question of whether or not we have a true absolute minimum when we solve the Euler differential equation (Solution of 3.1.21 or of the integral 3.1.30, for example). When we solve a minimization problem such as 3.1.27:

$$\text{(3.1.34)} \qquad \text{Min } J = \text{Min} \int f(r) \sqrt{1 + r^2 \dot{\theta}^2}\, dr$$

by means of the Euler equation we are not guaranteed a minimum, but may instead be a maximum or some complex "saddle" condition. For the purpose of determining that we have a local or relative minimum, we must use the Legendre conditions (Courant and Hilbert, 1937, pages 214-216). Given the function of r, θ, and $\dot{\theta}$:

$$\text{(3.1.35)} \qquad F(r, \theta, \dot{\theta}) = f(r) \cdot \sqrt{1 + r^2 \dot{\theta}^2}$$

we must prove that,

$$\text{(3.1.36)} \qquad \frac{\partial^2 F}{\partial \dot{\theta}^2} > 0$$

while

(3.1.37) $$\frac{\partial^2 F}{\partial \dot{\theta}^2} \cdot \frac{\partial^2 F}{\partial \theta^2} - \left(\frac{\partial^2 F}{\partial \theta \, \partial \dot{\theta}} \right)^2 \geq 0$$

However, differentiating 3.1.35 twice gives the following result:

(3.1.38) $$\frac{\partial F}{\partial \dot{\theta}} = f(r) \frac{r^2 \dot{\theta}}{\sqrt{1 + r^2 \dot{\theta}^2}}$$

(3.1.39) $$\frac{\partial^2 F}{\partial \dot{\theta}^2} = f(r) \frac{r^2}{[1 + r^2 \dot{\theta}^2]^{3/2}} > 0$$

$$\text{for} \quad f(r) > 0, \quad r > 0$$

Moreover, differentiating 3.1.35 by theta gives zero, so the other partial derivatives are zero as follows:

(3.1.40) $$\frac{\partial F}{\partial \theta} = 0 \Rightarrow \frac{\partial^2 F}{\partial \theta^2} = 0 \text{ and } \frac{\partial^2 F}{\partial \theta \, \partial \dot{\theta}} = 0$$

We can conclude that in the region of the plane excluding the origin for radially symmetric fear functions, f(r), the condition for a strong relative minimum is satisfied as long as fear is greater than zero throughout the region of the solution:

(3.1.41) $f(r) > 0$, in a region containing the solution.

If net fear function is zero in the region of the trajectory, it is easy to verify heuristically that the trajectory is also indeterminant in this region (see Figure 3.1.5). This occurs because of the integrand in the variational problem 3.1.7 or 3.1.34 will be zero identically, regardless of the choice of path in such a zero-fear region. Of course, a positive net goal fear will obviate this condition and will guarantee that the solution is always a relative minimum. These results carry over to the general case 3.1.7 as well, but the algebraic manipulations become too unwieldy to present here. A word of caution is appropriate here about calculus of variations terminology; a "strong minimum" (guaranteed by the Legendre conditions) is a relative minimum against all paths sufficiently close to the chosen path, but does not guarantee an absolute minimum against all other choices, as is required by our theory. Usually, if we use the calculus of variations, we must make subsidiary arguments to establish an absolute minimum. The crucial matter in such a case, for example, may be to sort-out two alternative minima, as illustrated in Figure 3.1.6. If both paths are absolute minima, then an arbitrary choice can be taken.

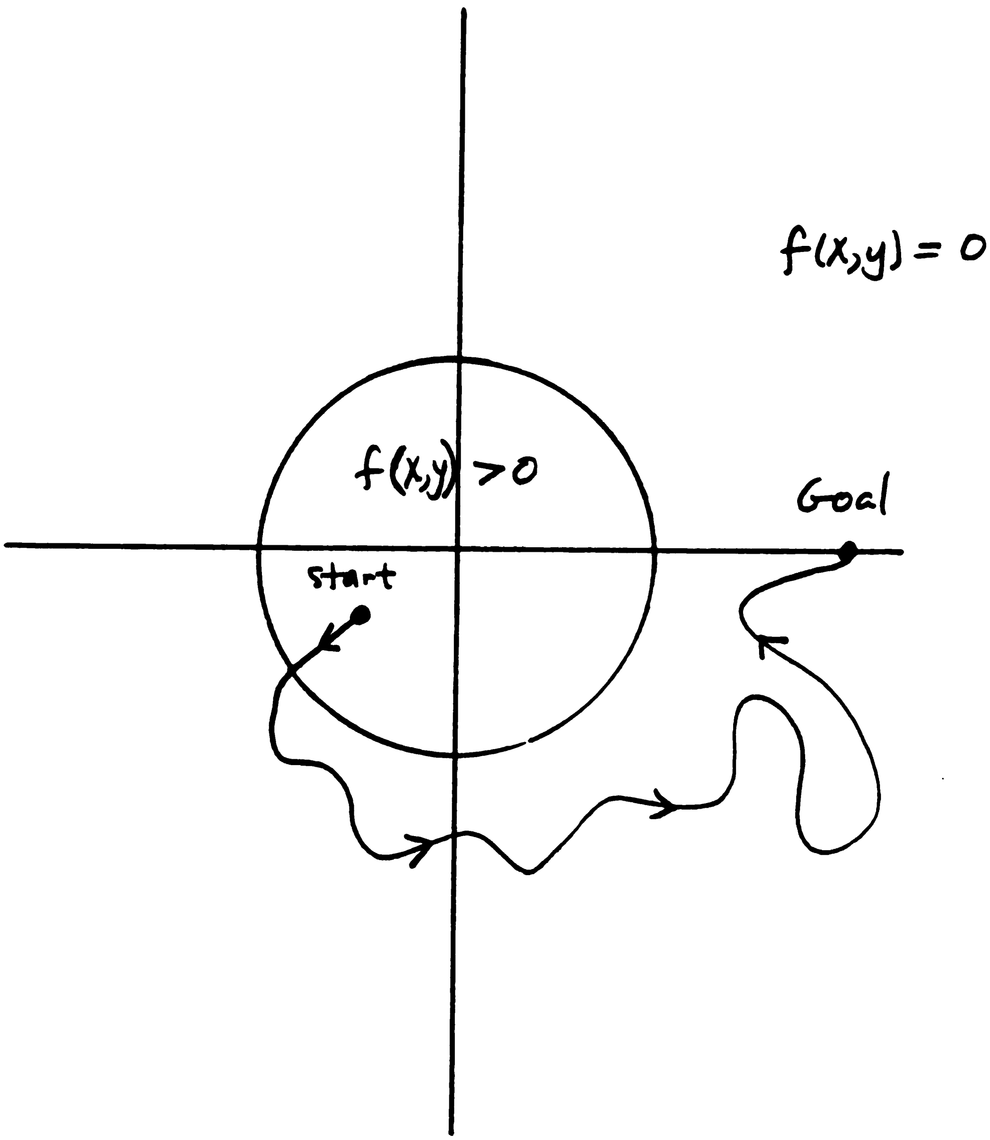


FIGURE 3.1.5

An Arbitrary Path Occurs in Regions
of Zero Total Fear

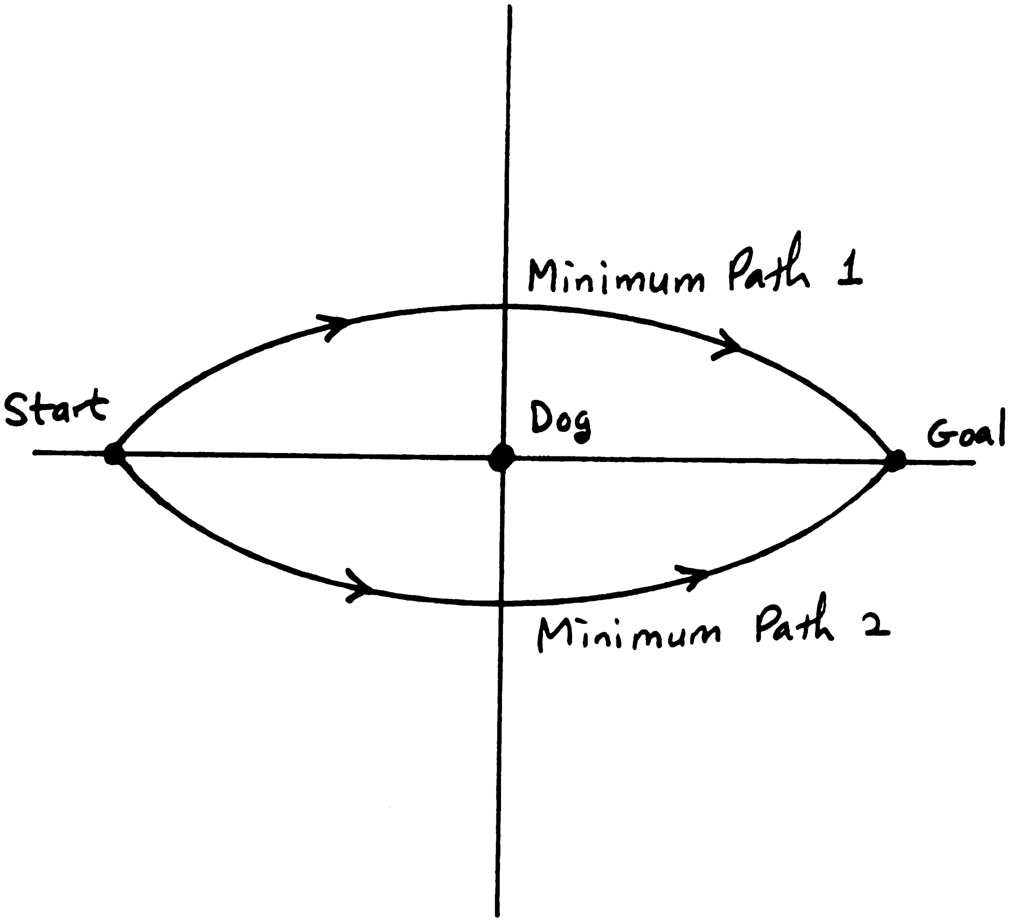


FIGURE 3.1.6

There Can be Two or More Minimal Paths

In another case we may have two minima, one of which is absolute and the other of which is relative, as in Figure 3.1.7. In this case, the top curve is a relative minimum and the bottom curve is an absolute minimum. Such a case can be sorted-out only by explicitly computing the appropriate integral, such as 3.1.7, and making an actual quantitative comparison of the integral for the two paths. In other cases where we put two dogs in the plane or otherwise make the topography of fear complex, there will be many multiple trajectories all of which are relative minima and only one of which is the absolute minimum. In such a case neither the Euler equation nor the Legendre conditions tell us which is the absolute minimum and we must use other techniques such as comparison of integrands or actual computation of the integrals along the candidate paths to determine which is the true minimum. Our theory of the behavior of animals in a field of fear, however, assumes that an animal seeks the true minimum in planning its trajectory. The Dynamic Programming solution, which we shall propose as the neural solution, has the property of yielding an absolute minimum directly which the calculus of variations does not. There is another problem hidden in the calculus of variations as a method of solution. Implicit in its formulation, the calculus of variations allows only such alternate paths as have a continuous tangent. This means that paths with "kinks" are not considered. With this

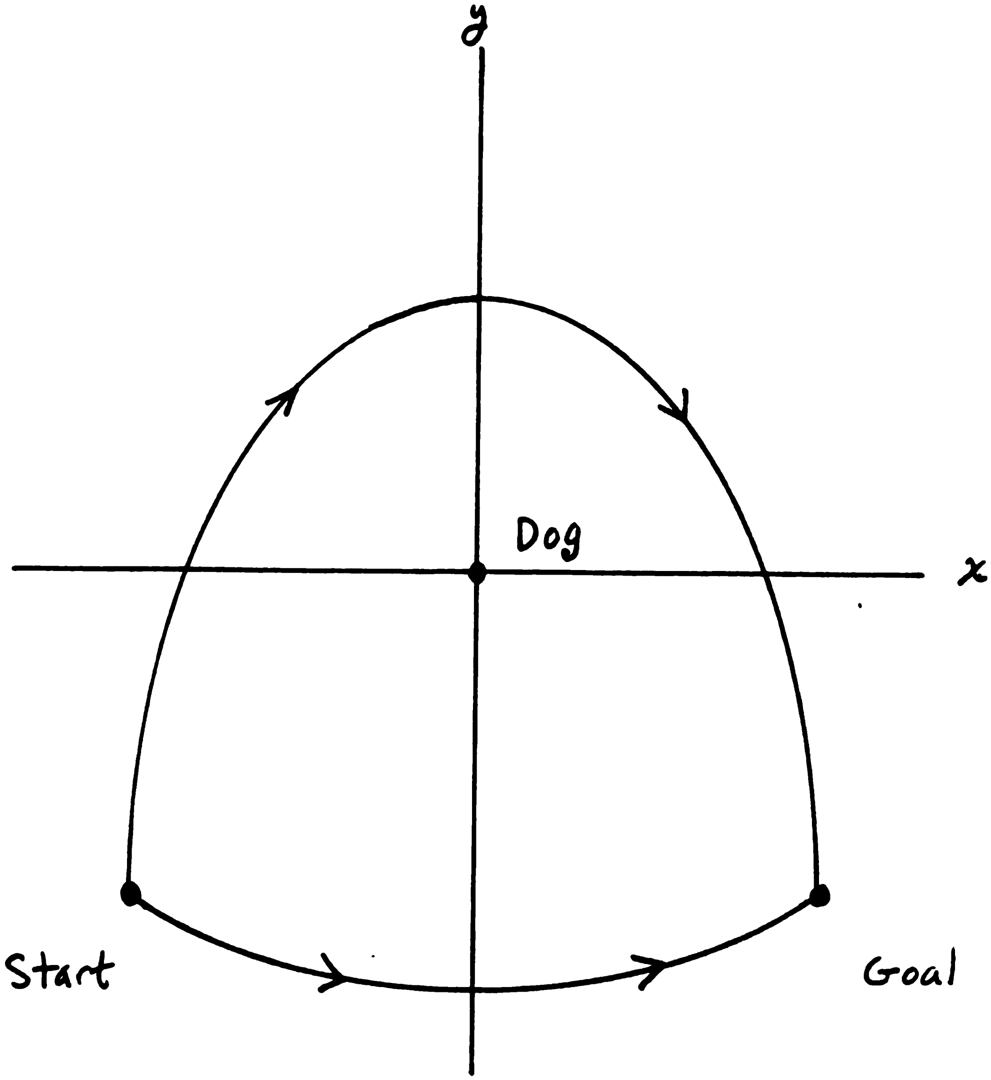


FIGURE 3.1.7

There Can be an Absolute Minimum
Path and Relative Minima

remark we conclude our discussion of the mathematics of the model which supposes minimum exposure to fear in the case of constant-velocity motion in the plane.

Section 3.2 Summary of a Variable Velocity Case

There may be cases where an animal would vary its velocity. Consider the energy needed for locomotion. Chassin et al. [1976], Taylor et. al. [1970], and Taylor et al. [1974] assert that the rate of energy consumption for locomotion is a linear function of velocity with a zero-velocity intercept higher than the resting metabolism:

$$\frac{d\mathcal{E}}{dt} = \begin{cases} c_1 + c_2 v & \text{(motion)} \\ c_0 & \text{(resting)} \end{cases} \tag{3.2.1}$$

with $c_0 < c_1$. Thus the energy needed to traverse a path is given by,

$$\mathcal{E} = \int_0^T (c_1 + c_2 v)\, dt \tag{3.2.2}$$

In the trajectory problem which follows, we will introduce a constant μ, which expresses the proportionality of the cost associated with energy expenditure to the cost of fear exposure. Thus we will be seeking a minimum of

$$J + \mu \mathcal{E} \tag{3.2.3}$$

Since we are considering variable velocity, we will also consider the dependence of fear on velocity at each point along the path. In particular, we will consider the case where the fear function is given by:

$$f(x,y) = (b + v^{\gamma})\, f_1(x,y) + f_0 \tag{3.2.4}$$

In this expression the first term expresses the fear of the moving animal for its own safety and the second term is the net goal fear. The expression $b + v^{\gamma}$ has been chosen for the model as a function that can be made to rise very sharply from a minimum at zero velocity. This choice is consistent with the fact that most predators have much greater visual sensitivity for fast moving objects than for objects that move slowly (Grzimek, 1977, p. 97; Suthers and Gallant, 1973, p. 340; Braitenberg, 1977, Chapter 7; Thompson, 1975, p. 205-213; Wallace, 1973, p. 130; and Clements and Dunstone, 1984). Thus we will expect the value of gamma to be greater than zero. For gamma having the value of 2 the graph of fear versus velocity is shown in Figure 3.2.1, assuming a constant value of x and y.

Reparametrizing in terms of s which ranges from zero to one, as in the last section, we obtain:

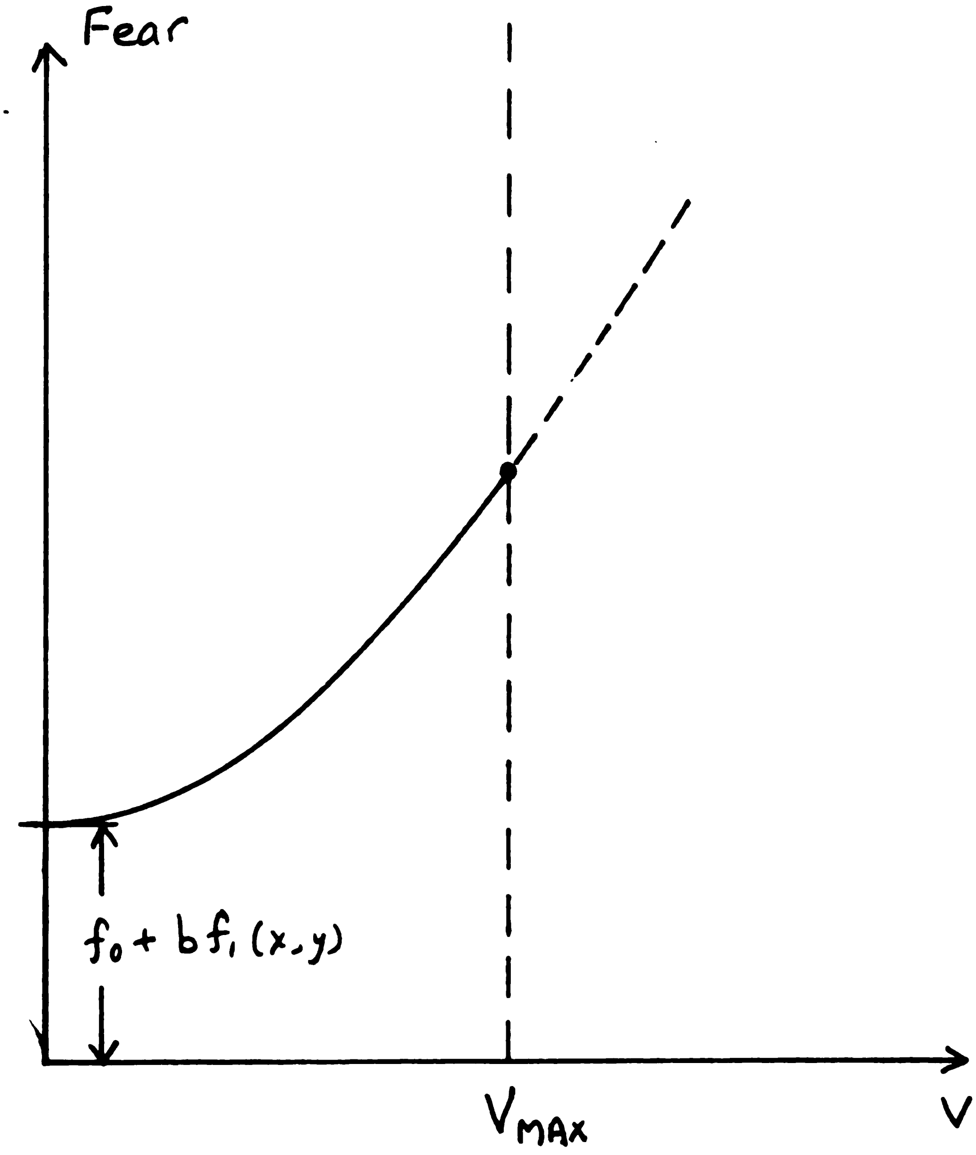


FIGURE 3.2.1

Fear as a Function of Velocity
at a Fixed Position

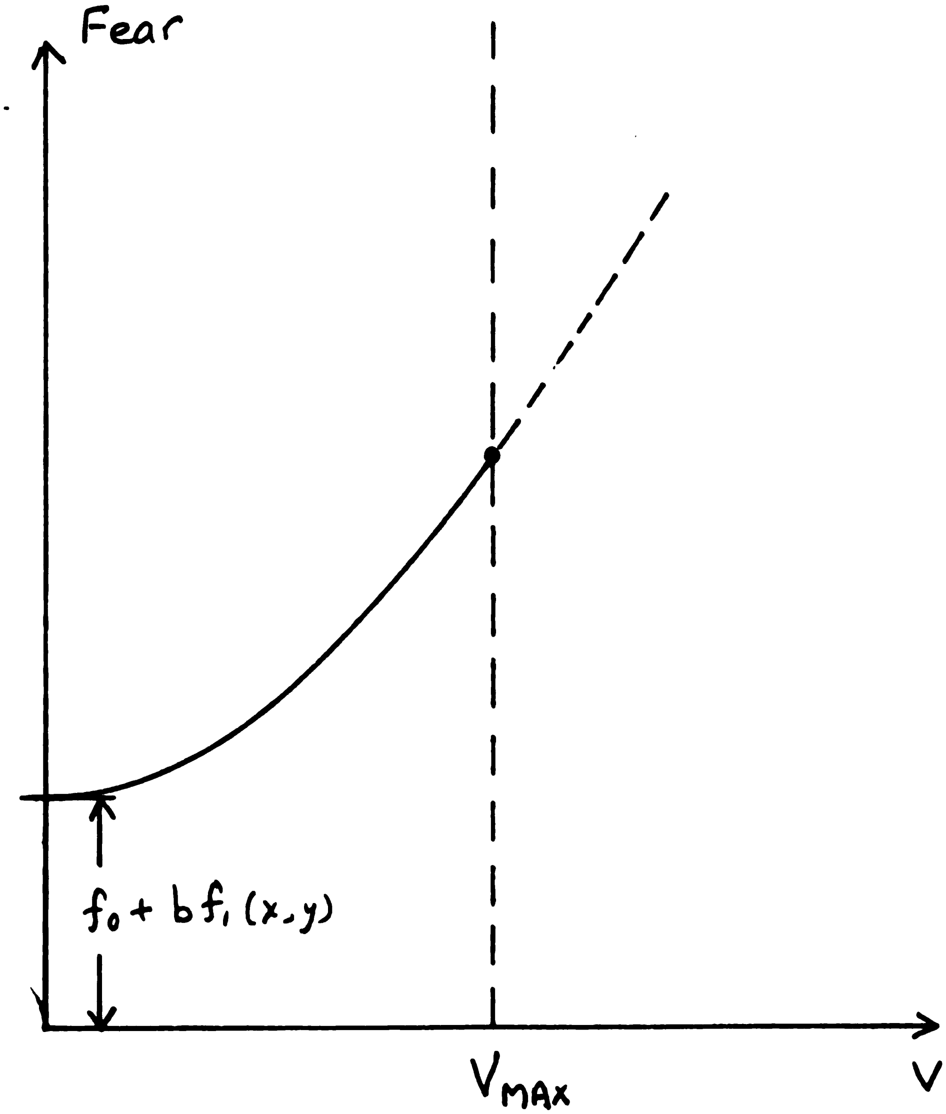


FIGURE 3.2.1

Fear as a Function of Velocity
at a Fixed Position

(3.2.5) $$\mathcal{E} = \int_0^1 [c_1 + c_2 v] \cdot \frac{\sqrt{(x')^2 + (y')^2}}{v} \, ds$$

and

(3.2.6) $$J = \int_0^1 [(b + v^\gamma) f_1(x, y) + f_0] \frac{\sqrt{(x')^2 + (y')^2}}{v} \, ds$$

as the integrals of our variational problem,

(3.2.7) $$\text{Min } \mathcal{F} = \text{Min } (J + \mu \mathcal{E})$$

Here, the independent variable of the variational problem is s and the dependent variables are x(s), y(s), and v(s). The spatial path is described by x and y and the velocity profile by v.

Obtaining the Euler equations for the variational problem 3.2.7 and applying the variable-endpoint conditions for v (These conditions, which are in fact void in this case, are required because v does not have a specified value at the endpoints; see Arthurs, 1975, page 24, Equation 7.8), we obtain after much algebraic manipulation,

(3.2.8) $$v = \left[\frac{f_0 + \mu c_1 + b f_1(x, y)}{(\gamma - 1) f_1(x, y)} \right]^{1/\gamma}$$

and curvature is given by:

(3.2.9) $$K = \text{Proj}_\perp \nabla \ln f^*(x, y)$$

where

$$(3.2.10) \quad f^*(x,y) = \gamma \left[\frac{f_0 + \mu c_1 + b f_1(x,y)}{(\gamma - 1) f_1(x,y)} \right]^{\frac{\gamma-1}{\gamma}} f_1(x,y) + \mu c_2$$

We note at once that the Euler equations couple the positional and velocity parts of the solution through Equation 3.2.8. The derivation leading to Equations 3.2.8-10 is valid as long as $\gamma \neq 1$.

The equations above predict that velocity decreases as the animal nears the predator, with a minimum velocity given by

$$(3.2.11) \quad V = V_{MIN} = \left(\frac{b}{\gamma - 1} \right)^{\frac{1}{\gamma}}$$

Moreover, trajectories tend to flatten (spatially) as f_0 gets larger, but now always have a positive curvature, as long as the component of the gradient of $f_1(x,y)$ perpendicular to the path is positive. There is, in addition, a caveat concerning maximum velocity: Equation 3.2.8 may give a magnitude of velocity exceeding the actual maximum running speed of the animal; in this case the velocity is truncated at the value v_{MAX} and trajectories are different than those given by the above spatial equations (2.3.10-11). In fact, over such a segment of trajectory the behavior would be predicted by our original model (See Section 3.1) with

$$f^*(x,y) = (b + V_{MAX}^{\gamma})\, f_1(x,y) + f_0 \tag{3.2.12}$$

and the energy equation ignored. Curves from the two models would have to be spliced in such a way as to obtain a trajectory with a continuous tangent and with a continuous velocity profile.

We note that Equations 3.2.9-10 allow the resulting spatial trajectory to be gotten directly by the simple spatial minimization problem:

$$\text{Min} \int_0^1 f^*(x,y) \sqrt{(x')^2 + (y')^2}\; ds \tag{3.2.13}$$

if we have some way of transforming the problem to get f*, since Equation 3.2.9 is exactly the same as the Euler equation for a simple spatial fear function and v = constant, once the substitution f = f* has been made. See Section 3.1, Equation 3.1.21.

As a sample problem, we consider the case where the predator is at the origin and the fear component f_1 is inversely proportional to the distance from the predator. In this case,

$$f(x,y) = (b + v^2)\,\frac{1}{r} + f_0 \tag{3.2.14}$$

where the constant gamma = 2. The resultant velocity as a function of the distance r from the predator is

(3.2.15) $$V = \sqrt{(f_0 + \mu c_1) + b}$$

and the curvature of the path is given by

(3.2.16) $$K = \frac{(f_0 + \mu c_1) r + b}{2rQ + \mu c_2 r^2 \sqrt{Q}} \cdot \cos \xi$$

where

(3.2.17) $$Q = (f_0 + \mu c_1) r + b$$

The angle ξ is the angle between the trajectory and a circle about the origin passing through the same point (see Figure 3.2.2). The sign of the curvature is such that the curvature is toward the center of fear at the origin.

Section 3.3 Direct Consequences of the Principle of Optimality

According to Bellman, the Principle of Optimality can be stated as follows: "an optimal policy has the property that whatever the initial state and the initial decision are, the remaining decisions must constitute an optimal policy with regard to the state resulting from the first decision." (see Bellman, 1961, page 57). The mathematical structure of our behavioral planning models is such that the Principle of Optimality must hold. This implies at once that extensions of our optimal trajectories from any intermediate point must also be

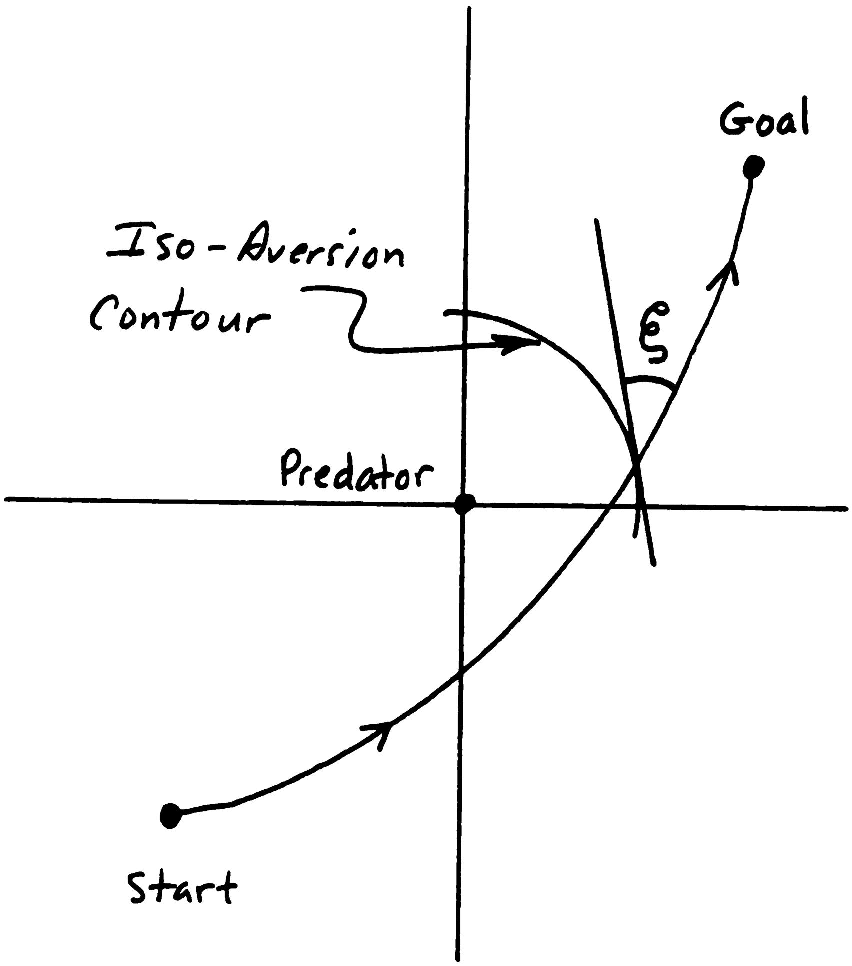


FIGURE 3.2.2

The Angle ξ is the Angle Between the Tangent to the Path and an Iso-Aversion Line optimal, given the intermediate status of the animal (position, time, and in the energy budget models energy expended). In turn, this immediately implies that looped trajectories, such as that shown in Figure 3.3.1, are impossible in our theory. The reason is obvious: the trajectory BC and the trajectory BD cannot both be optimal given the starting point B. In fact, the animal can always save both energy resources and fear exposure by eliminating the loop BDEB in the figure. The same reasoning applies to the looped paths predicted in Section 1.2 as a consequence of Lewinian theory. In the endless loop shown in Figure 3.3.2, consisting of the path ABCABCABCABC. . ., we can reason on the basis of the Principle of Optimality that AB and ABCAB cannot both be optimal paths from A to B. Hence the impossibility of this situation in an optimality theory.

Section 3.4 Behavior in Relation to Shelters of Passage

We will define a shelter of passage to be a region in the life space of the animal through which it can pass in relative safety. For a cat such a region might be a field of tall grass. In this case the Lewinian model has difficulty predicting that the animal would use the shelter of passage since it has no attractive value of its own as a goal. In Figure 3.4.1, a trajectory is shown which goes through a region of tall grass toward the goal. We may picture a cat trying to get to its

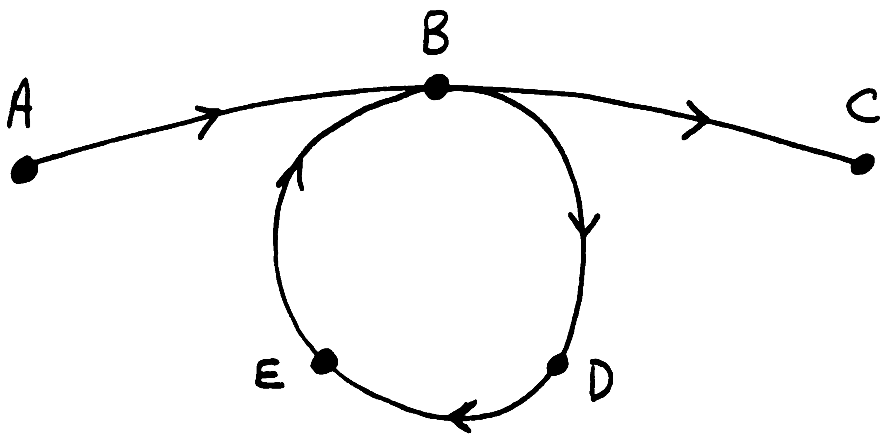


FIGURE 3.3.1

Circulatory Paths are not Possible in an
Optimal Planning Theory According to
the Principle of Optimality

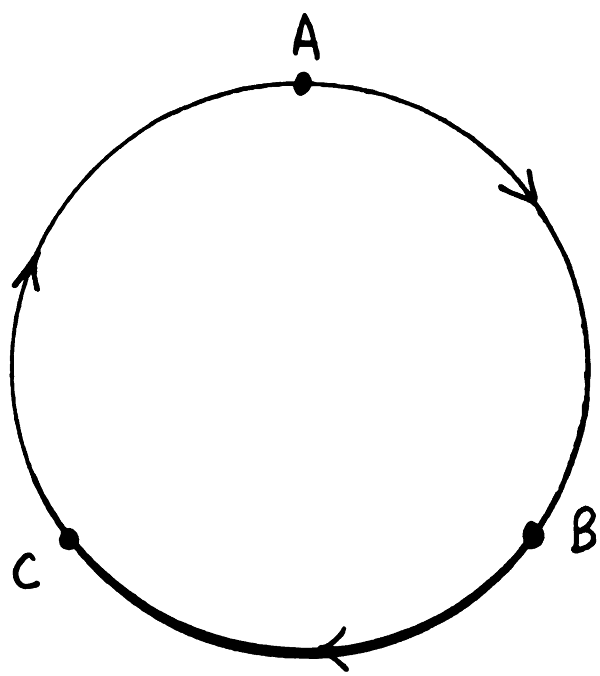


FIGURE 3.3.2

Circulatory Path

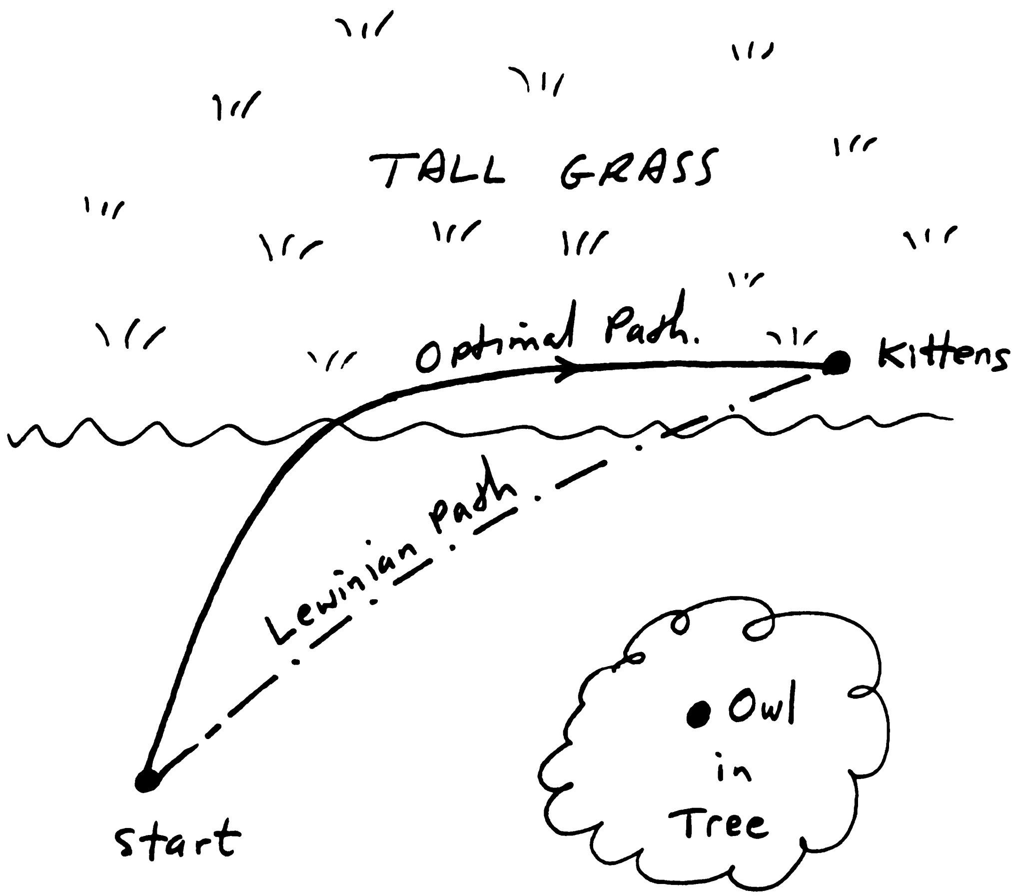


FIGURE 3.4.1

Animals may be Expected to use Shelters
of Passage According to the
Optimal Planning Theory

kittens which are hidden at the far end of a field of tall grass and the starting position of the cat being in a cleared area overlooked by an owl in a tree. In this case our model predicts that the cat will immediately head for the tall grass, where the fear function is very small, and then go through the grass to its kittens in relative safety. Here the function of planning is evident: the animal must plan ahead for the time it will be in the area of low fear; it cannot simply react on the basis of its present experience of fear. The Lewinian path would be a much more direct route to the kittens, not taking advantage of the shelter of the tall grass.

Making use of a simplified radially symmetric shelter of passage, we were able to explicitly compute trajectories for the following situation: an animal is in a large field in the hunting area of a hawk with a tree at the origin providing shelter under which it cannot easily be seen. To derive numerical trajectories the dynamic programming algorithm was used by means of executing the BASIC program given in Appendix A. The fear function used was:

$$f(r) = 0.5 + \frac{r^2}{0.25 + r^2} \qquad (3.4.1)$$

in polar coordinates. Figure 3.4.2 shows the resulting trajectories for two starting positions.

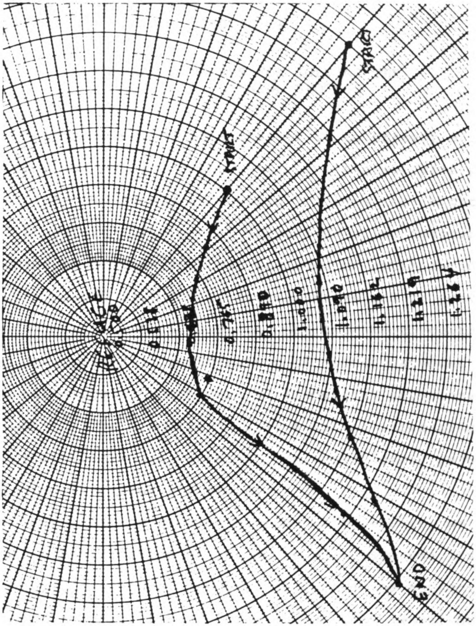


FIGURE 3.4.2

Computed Behavior in Relation to Shelter:
Dynamic Programming Solution With

$$f(r) = 0.5 + \frac{r^2}{0.25 + r^2}$$

CHAPTER 4

NEURAL NETWORKS FOR SOLVING THE OPTIMAL PLANNING PROCESS

The purpose of this chapter is to present a neural network which can solve the continuous minimum path problem and certain runway and hybrid problems that might occur in nature. The neural network solution described is presented to demonstrate that the idea of optimization is not an impossibility for living organisms and to provide neuroscientists with a guide to what type of circuitry might be involved. Consideration of neural networks might also suggest the nature of suboptimal solutions. It might then be possible to find a set of increasingly valid solutions which represent the evolutionary path of development of these structures.

In order to provide a frame of reference, we first discuss a modification of the Moore Algorithm and its embodiment in a neural network, following the presentation in Dolson and Bell [1981]. The Dolson-Bell neural network solves the minimum cost problem for arbitrary discrete networks of runways. However, we are primarily concerned with continuous space optimization in two dimensions. Therefore, we will adapt the Dolson-Bell solution to the continuous case by means of a neural network which is based on hexagonal tiling of the plane. This new neural network will not solve arbitrary network problems; it is specialized to solve minimum path problems that can be embedded in the plane. In the context of planar problems, it will solve both continuous and discrete problems and hybrid problems that contain regions where both kinds of solution are required and must be properly joined together. See Figure 4.0.1.

Section 4.1 An Arbitrary Network Solver in Abstract Form

Dolson and Bell [1981] provide a neural network solution to the minimum path problem through a discrete network of directed paths with assigned link costs. This network solution is based on a revision of the Moore Algorithm. The revision of the Moore Algorithm consists of integerizing the cost of links so that each link may be thought of as having a finite number of STEPs, each of length Δ. Since the costs in a network are always scalable by a constant, we can then think of the network as comprised of links of integer cost. Assume that we have created artificial new nodes between each STEP of an old link. Then we may go through the steps of the Moore Algorithm treating the elementary STEPs rather than the original links as the basic units of traversal. The consequence of this for practical computation is that we may proceed outward from the START in increments of one unit and move the FRONTIER out by one unit at a time

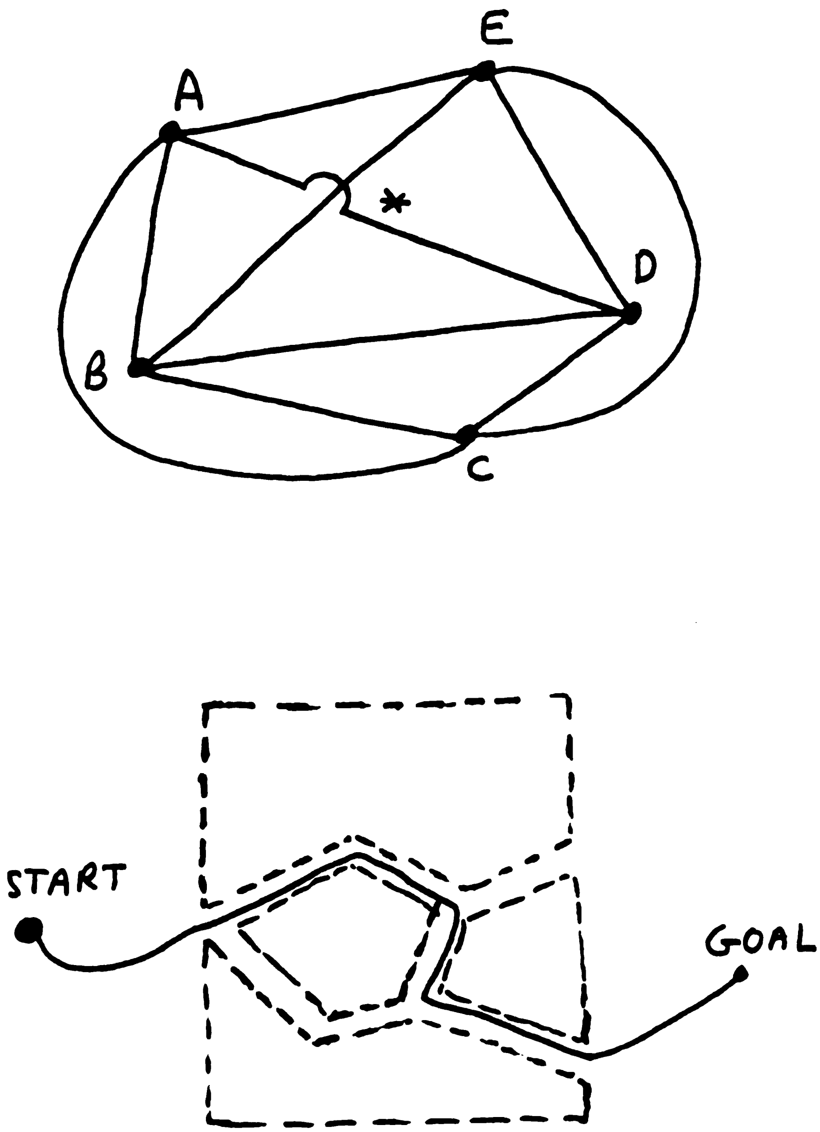


FIGURE 4.0.1

A Non-planar Maze Cannot be Embedded in the Plane; However, a Planar Hybrid Path Problem Can be Embedded without having to reevaluate the cumulative costs. The algorithm can be stated briefly as follows:

Step 1. Initialization. Divide all links into an integer number of artificial STEPs of length Δ, rounding as necessary. Set BACK [I] = 0 for all nodes; set CUM [I] = infinity for all nodes except the START and set CUM [START] = 0.

Step 2. Forward Loop. From N = 0 step by 1 until network exhausted: let the FRONTIER be the set of all nodes with CUM [node] = N. For each node in the FRONTIER, identify all nodes not yet reached (BACK [node] = 0 or CUM [node] = infinity) that are one STEP beyond the FRONTIER. Let the FRONTIER node be I and a reachable node be J. For such a node J, set CUM [J] = N + 1 and set BACK [J] = I. In the case that there are two or more nodes from which the new node can be reached in one STEP, choose the BACK node arbitrarily from among the various candidates.

Step 3. Backward Loop. If we now desire to find the minimum path to any destination, we use the reverse recursion: $node_{n+1}$ = BACK [$node_n$] to go computationally from the GOAL to the START.

The most significant feature of this new algorithm is that Step 2, which is computationally the most intensive step, can be performed in parallel within each value of N. For a very dense network, this saves an enormous cost in computation time if the necessary circuitry is available to do the parallel computation. The main modification to the Moore process consists of the definition of the artificial links of length one (or Δ, if unscaled), which basically makes the network one of links (called STEPs) of uniform size, so that the FRONTIER steps out by uniform increments of cost 0, 1, 2, 3, . . . from the START in the scaled cost metric. That is, the entire FRONTIER has cumulative cost 0Δ, 1Δ, 2Δ, 3Δ, etcetera, as we proceed through the calculation. The original links define the topology of the network, while the subdivision into STEPs has the property of providing that the entire FRONTIER is at a uniform distance from the START (in the cost metric) at any one stage in the processing. This means that we have a direct dynamic programming problem in which each stage is just the set of points a distance $N\Delta$ out from the START node. Hence the FRONTIER in the sense of the Moore Algorithm, which is used as a bookkeeping device, becomes unnecessary, allowing the computation to be done in a much simpler fashion as outlined above. The artificial nodes that must be defined between the STEPs would mean considerable unnecessary overhead for a sequential digital computer. However, these artificial nodes greatly facilitate the computational process by parallel methods.

In the Dolson-Bell neural network, the artificial nodes for each link are embedded in a special circuit called a "pulse counter." If the inner structure of the pulse counter is ignored, then the artificial nodes in the Dolson-Bell network can also be ignored. Only the original nodes are represented by node-like neural analysis (called "lockout circuits"). The principal use of the concept of artificial node is to permit a proof that the Dolson-Bell network carries out the Moore Algorithm.

Section 4.2 The Dolson-Bell Network for Discrete Networks

For a discrete maze, the Dolson-Bell approach to solution is to make a nerve impulse analog to the minimum path problem. The minimum path is gotten by allowing a series of nerve impulses from a neural clock to follow the path which minimizes the time of propagation to the goal, and then tracing the impulses back to their origin. This is accomplished by a set of pulse counters and special circuits which we will call "lockout circuits." The pulse counters are placed in the neural analogs of the links of the network, while the lockout circuits are placed at the nodes. The pulse counters can be preprogrammed to accept exactly a given number of pulses before outputting any pulses; thereafter they output one pulse per input pulse. The count programmed into a counter corresponds to the cost of the link it represents in the original network. Thus the programmed count corresponds to the number of STEPs between nodes in the algorithm of Section 4.1. The lockout circuit serves the purpose of the BACK array in the algorithm of Section 4.1. Each link coming into a node goes through one channel of the lockout circuit at that node in the neural circuitry. The lockout circuit is so designed that it prevents impulses from passing through a node from one direction if they have previously passed through from another. In operation, the number of pulses which have been emitted by the clock into the start node of the neural network corresponds to the dynamic programming STAGE. The FRONTIER is physically the farthest reach of the pulse train emitted from the clock as it travels through the circuitry. However, since the STEPs are all of the same size, the FRONTIER is always a uniform distance from the START in the cost metric, so it can never cut through itself. As a consequence, the BACK node of a given node never need be reevaluated as in the Moore Algorithm, with the consequences that the lockout circuits can lock the incoming path channel once and for all on a given network evaluation. The FRONTIER set is not explicitly represented in the circuitry because it is automatically accounted for by the way the circuitry works.

In the Dolson-Bell solution, the network is preformed as a general network for solving problems up to a certain number of nodes. Since the brain cannot be expected to have a prebuilt network of counters and lockout circuits corresponding to each possible network, the Dolson-Bell solution assumes that the entire network is preformed as a maximally interconnected net: the network of N nodes has all possible N(N - 1) links with their associated counters and lockout circuits. An outside neural circuit programs this general all purpose circuit to match the particular network to be solved. All links are assumed to be inherently one-way. An unused link is deleted by the activation of an inhibitory blocking cell. Two-way links are represented by having the links in both directions. The cost of the link is represented by setting the value of the counter in the counter circuit for that link.

The programmable counter circuits needed to implement the Dolson-Bell solution are extensively discussed in Dolson & Bell [1981]. The only other circuit element needed to compute the forward pass in the modified Moore Algorithm is the lockout circuit which prevents impulses from passing through a node from one direction if they have previously passed through from another direction. There is a lockout circuit at each of the original nodes. All incoming pulses go through some channel of the lockout circuit, but only the first channel to be activated actually transmits succeeding pulses. Later channels to be activated are "locked out." In the case of a tie, such that the initial input pulse to two channels of a lockout circuit are simultaneous, then the tie is resolved by activating both channels. A two-channel lockout circuit is shown in Figure 4.2.1. Lockout circuits with more than two channels are straightforward

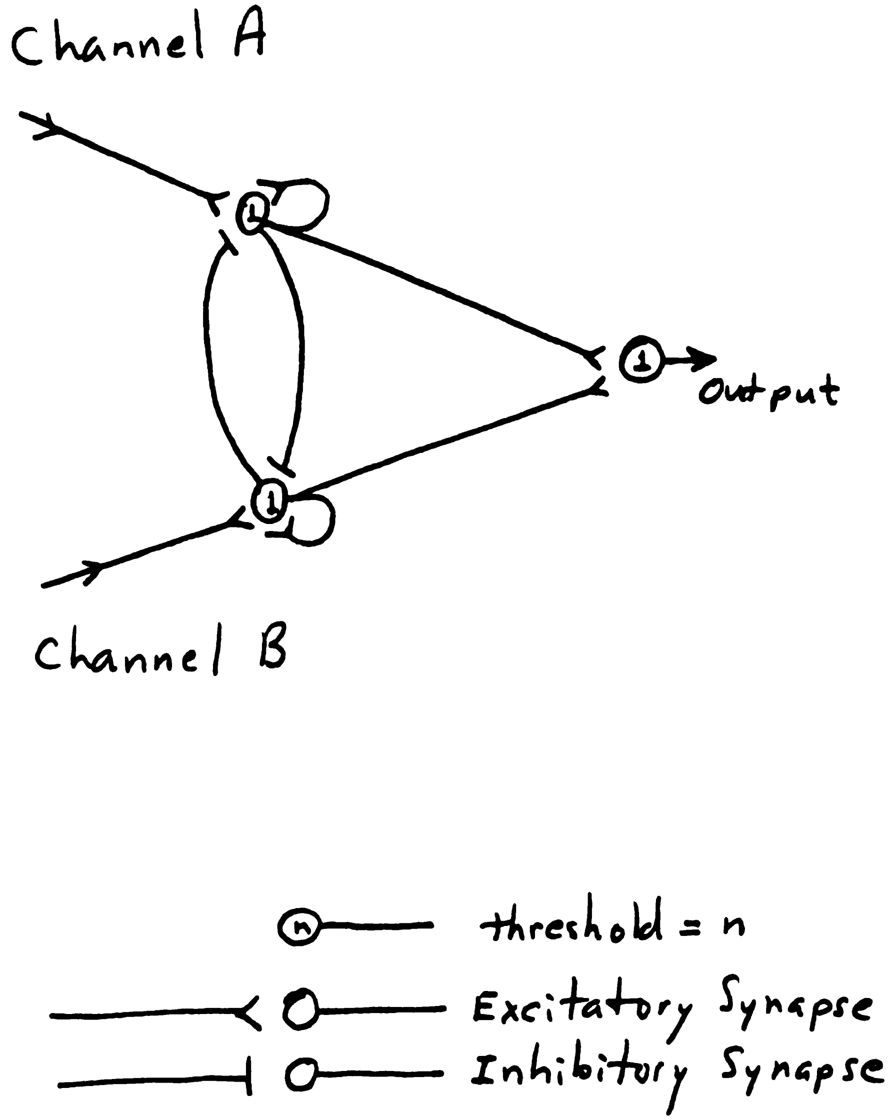


FIGURE 4.2.1

Two-Channel Lockout Circuit
Neuron Threshold = 1

generalizations of this two channel circuit. A six channel lockout circuit is given in Section 4.3.

In operation, the lockout circuits record the direction from which the minimum path entered each node. Therefore, once the forward pass of the circuitry is completed, they can be scanned to record the minimum routing. This can be done by a traceback circuit in a plane parallel to that of the forward pass circuitry. Such a traceback circuit is shown in detail in Figure 4.2.2 for a two-way lockout circuit. In operation, during the backward pass of our algorithm (see Section 4.1), clock pulses enter the traceback network at the neuron corresponding to the goal, and feed backward to the start neuron, traversing the choice taken at each lockout circuit but in the reverse direction. The result is that the optimal path is described by the path taken by the reverse process, and the question of which nodes are on that path can be answered by noting if clock pulses are present at any node neuron in the traceback circuitry.

In the lockout circuits shown in Figures 4.2.1 and 4.2.2 self stimulatory interconnections are shown. These are present merely to resolve ties in the time of arrival of the first pulses from the clock via two different channels. For example the circuit of Figure 4.2.1, there are two input channels and the circuit activates the first of them, while activating both only in the case of a tie.

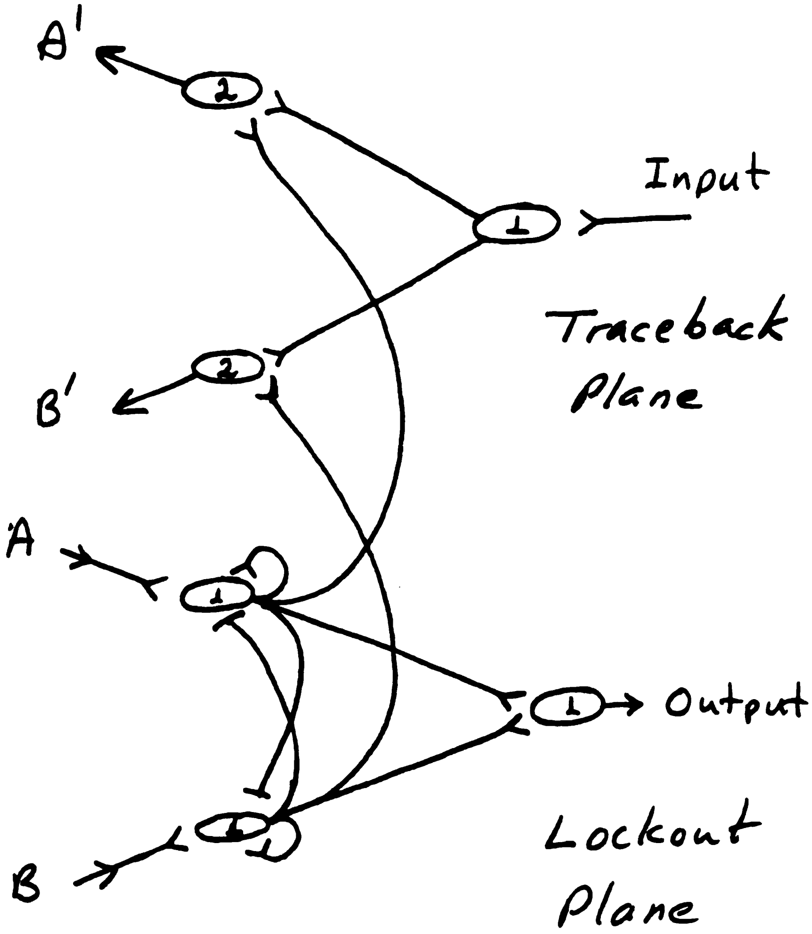


FIGURE 4.2.2

Traceback and Lockout Circuits
(Two-Channel)

Section 4.3 A Neural Network to Compute Optimal Paths

The basic idea for this network is to have a chain of neural impulses follow the optimal path. An auxillary neural circuit then reads off the optimal path and its associated fear exposure value. As a first step, the life space with its fear gradients is mapped onto a neural network by an auxillary circuit. Neural impulses are then started at the START node and fan out from the START node following optimal paths to all other nodes in the life space. The read out auxillary circuit then traces the path back from the GOAL node to obtain the optimal path and its fear value.

A neural network for the life space can be formed by "tiling" two dimensional space with regular hexagons. Figure 4.3.1 shows a section of two dimensional space tiled by hexagons. Each hexagon is treated as a node in the network. The nodes are linked if they have an adjacent side--as shown in Figure 4.3.2. The fear function $f(x,y)$ or $f^*(x,y)$ is represented by assigning a fear value to each node; i.e., the average of fear over the corresponding hexagon. The precision of the neural network depends on the number of neurons available for the network; the greater the number, the smaller the spatial area represented by each node (there is also an inherent error due to discretization of angles). The total fear

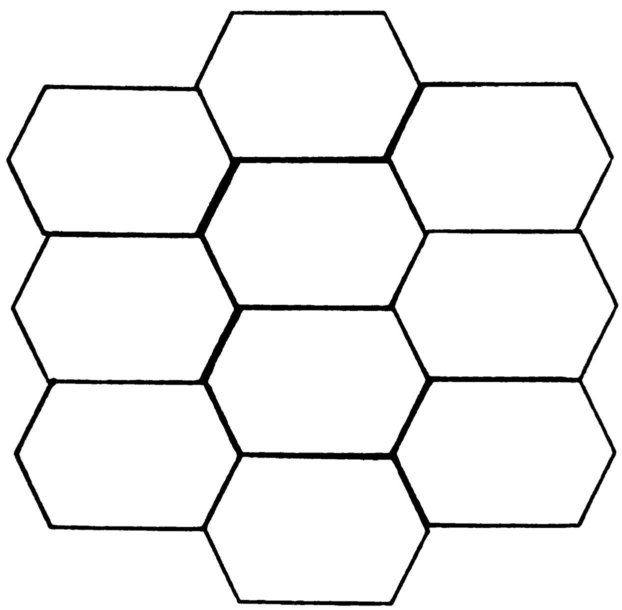


FIGURE 4.3.1

A Section of Two Dimensional Space
"Tiled" by Hexagons

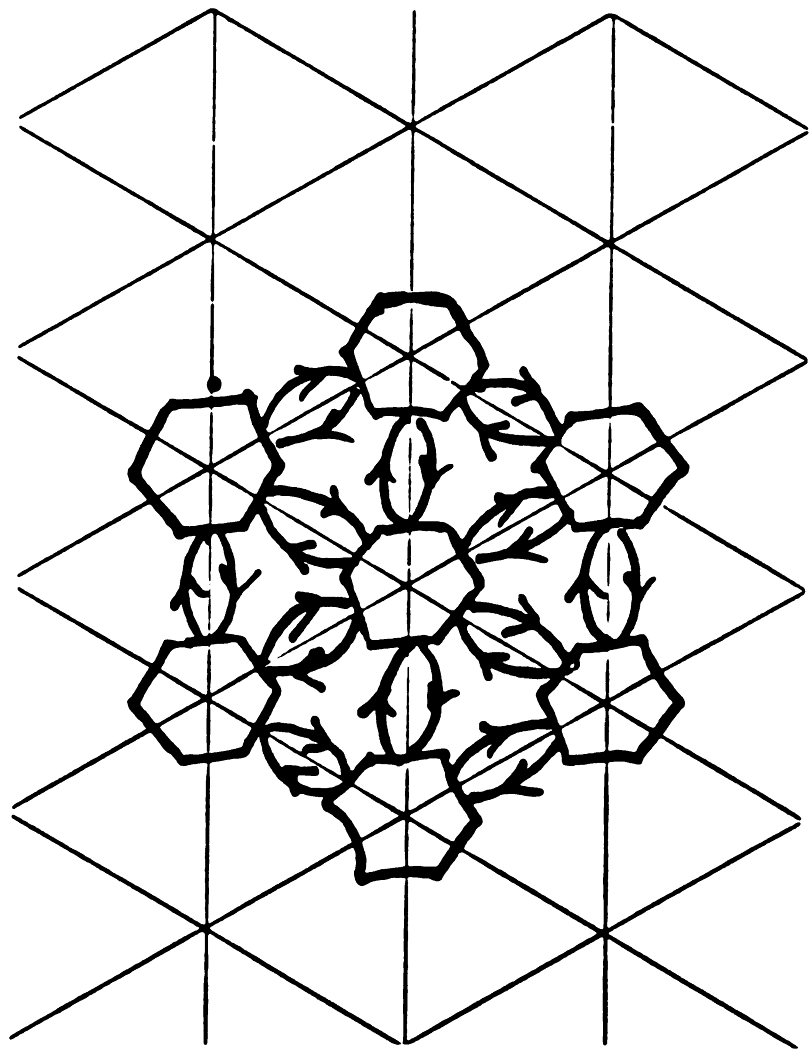


FIGURE 4.3.2

Triangulation Showing Interconnection of Computing Elements exposure associated with any path is simply the sum of the fear values for the nodes which make up the path.

The units of the neural network were taken from Dolson and Bell [1981]. Each node in the circuit is composed of a pulse counter and a six way lockout circuit. The pulse counter contains the fear value for that node. The six way lockout circuit permits the first impulse from one of the adjacent nodes to be transmitted along while ignoring all later impulses. Thus only the pulses which come from optimal paths are continued; other paths are discontinued. Each node will also have two auxillary circuits: one which programs the fear value into the pulse counter and one which provides for the back trace to read out the optimal path.

The basic scheme of interconnection of computing elements is shown in Figure 4.3.2. An individual computing element consists of a columnar arrangement of a counter, a lockout circuit, and a traceback circuit. See Figure 4.3.3. In our model, the cost is associated with a location rather than with a link between locations. Therefore, instead of having a counter on each link, we have a counter at each node. Each computing element forms a column perpendicular to the plane of the hexagonal tiling shown in Figure 4.3.2.

The interconnections of the computing elements in the plane of the triangulation are shown in Figure 4.3.2. However, these interconnections are actually three

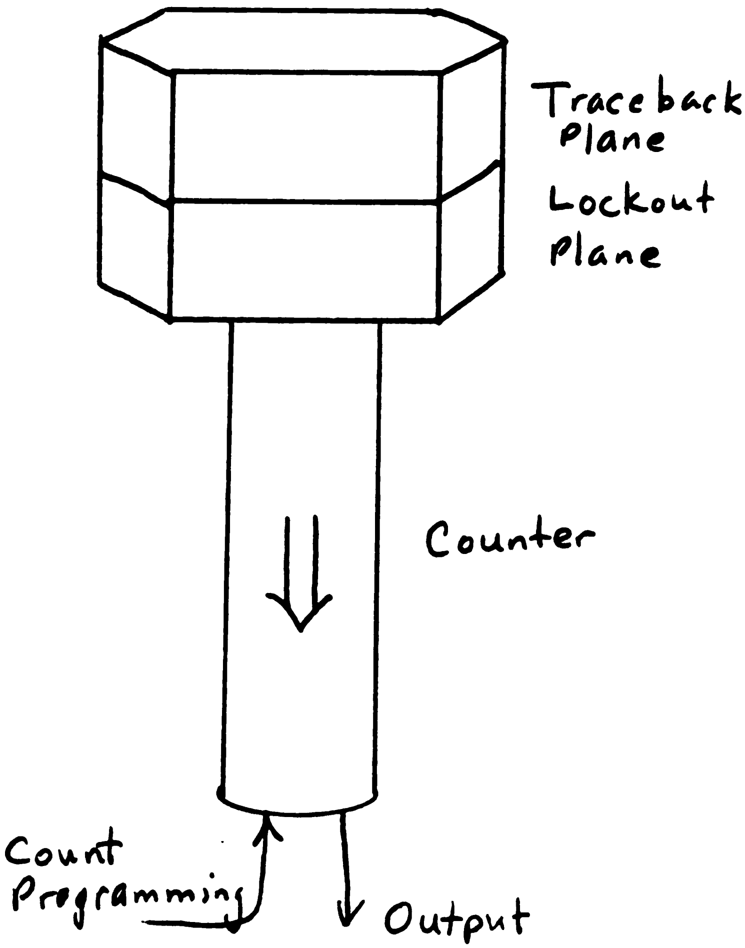


FIGURE 4.3.3

A Single Hexagonal Computing Element

dimensional, leading from the output at the bottom of each counter to the lockout circuit of each of its six neighboring computing elements. See Figure 4.3.4. The idea of this circuitry is that the cost associated with each node is fed into the counter for that node as its programmed count. The linkages between columns guarantees that if a counter puts out output pulses, these will be fed into all adjoining columns through their lockout circuits.

For reference, a six way lockout circuit is shown in Figure 4.3.5. The idea of this lockout circuit is that a hexagon of cells are formed around the first cell of the counter; each of these cells is inhibitory to each other while each is self-exciting to an extent sufficient to overcome that inhibition when tie breaking is necessary. Due to the fact that one clock pulse is spent in the lockout circuit, the counters must be programmed for one less count than the true count they represent.

In operation, perceptual circuitry would feed the function f(x,y) of Section 3.1 or f*(x,y) of Section 3.2 into the appropriate counters, after suitable integerization. Once the counters are set, the circuit can solve for a minimum path. Counter pulses would be fed into the top of the counter representing the START and allowed to traverse the entire net. Then pulses would be fed into the traceback circuit of the node corresponding to the GOAL. The traceback circuitry would then give

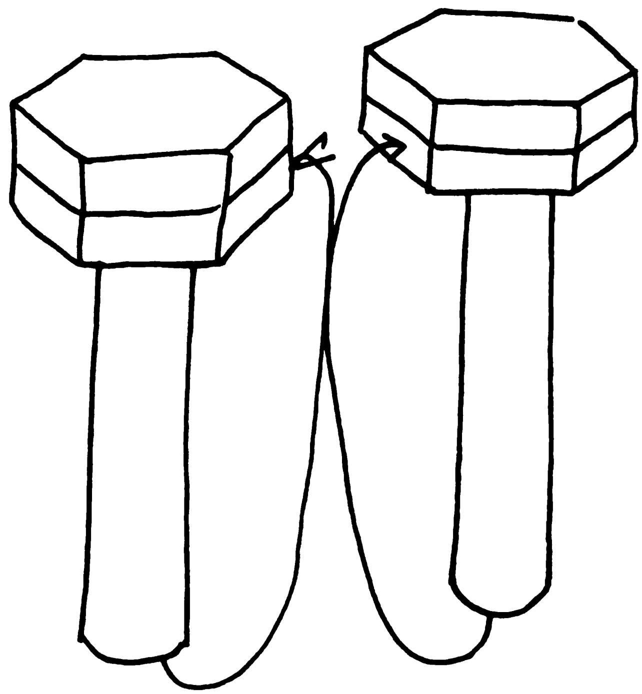
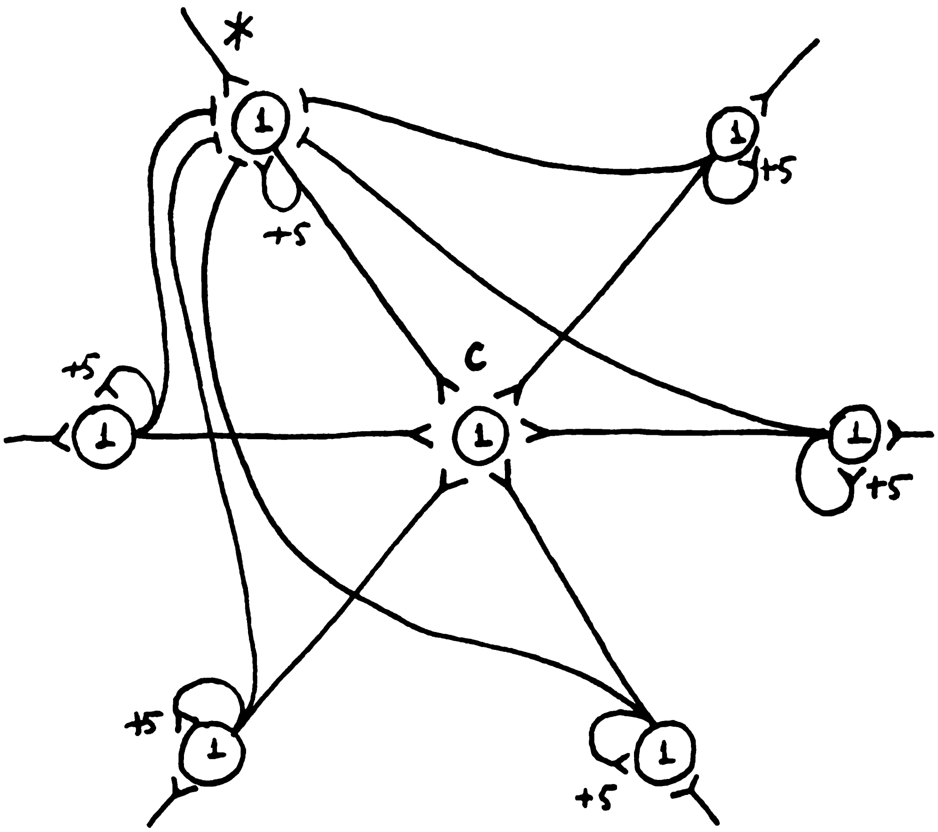


FIGURE 4.3.4

The Interconnection of Hexagonal Computing Elements
is From the Output of Each Neighbor
to the Lockout Circuit of the Other



"C" is the first counter cell of a descending column in the middle at right angles to the plane of the paper. "*" is shown with mutual inhibitory links; in fact all six lockout channels have them.

FIGURE 4.3.5

Six-way Lockout Circuit

the optimal path connecting the goal and the starting position.

To represent absolute barriers, an inhibitory synapse at the top of each corresponding counter can be added. This synapse would be of sufficient strength (a McCulloch-Pitts strength of -6 would suffice) to block an input. Such synapses would be programmed by the perceptual part of the brain. By this means the circuitry could find approximate solutions to problems involving both the traversal of open space and runways.

In analyzing the errors inherent in our network solution to the continuous minimum path problem, it is useful to think in terms of the triangulation of space produced by interconnecting the centers of adjoining hexagons in our tiling (see Figure 4.3.2). It is well known that approximate solutions to many continuous problems can be arrived at by means of sufficiently fine triangulations (see, in the field of topology, Cairns, 1961; in the field of partial differential equations, Pearson, 1974, pp. 1086-1096). This general approach will be taken here in providing an error analysis of our neural network solution.

The error analysis of such neural circuitry follows. First of all there is the discretization error associated with the integerization of the local cost function. If the counter span is N, so that the minimum count representable is 1 and the maximum is N, then there will be a relative error per step of about

(4.3.1) $$\frac{1}{2N}$$

However, since this error is random in the sense that rounding error is random, the total error due to this cause should be of the order of

(4.3.2) $$\frac{1}{2N\sqrt{L}}, \quad L = \text{links in path.}$$

as a fraction of the total cost of the path. If N were 10, and there are 400 links in the optimal path, then we would expect a relative error of 1/(2x10x20) or 0.0025 in the cost of the path. This is not likely to be a serious source of error. However, there is a much more stringent limitation on these circuits. Due to the fact that path angles of 0°, 60°, 120°, 180°, 240°, and 300° are the only ones explicitly represented, there is an angle-dependent error in the assessment of the path cost. This amounts to the difference between

(4.3.3) $\cos 0^\circ = 1$ and $\cos 30^\circ = 0.866$,

depending on whether the path is along one of the cardinal directions of the hexagonal grid or, at worst, midway between them. Since we can always scale cost to a value midway between the two geometric scales represented in

Equation 4.3.3, the actual relative error can be reduced to

$$(4.3.4) \qquad \tfrac{1}{2}(1 - \cos 30^\circ) = 6.7\%$$

taken in the sense of an error in either direction about the true value. This seems to be an absolute limitation of the circuitry presented which could be remedied only by finding a better way to tile the plane. However, we are already using hexagonal close packing which is known to be the most dense in both distance and angle. Thus it seems unlikely that the circuit can be improved by superior tiling. But some completely different and more accurate representation may exist.

CHAPTER 5

DIRECTIONS FOR FURTHER RESEARCH

Section 5.1 Experimental Tests of the Optimal Planning Theory

We will summarize here the basic predictions of the theory of optimal planning of trajectories in the presence of fear, provide experimental tests, and suggest directions for further research.

The main experimentally testable predictions of the theory of optimal planning follow:

(1) The spatial trajectory is flattened if there is fear for the reward. This means that the more fear there is for the goal object, the straighter will be the trajectory taken to it.

(2) If the fear function is always positive, the animal will never cross its own path on the way to a goal.

(3) The animal will not choose a compromise path if there are multiple consummatory goals. According to Lewinian vector theory, an animal confronted by the situation of multiple consummatory goals will choose a compromise path initially, which goes in the general direction of all of them. According to the optimal planning model presented here, the animal will be more

decisive, and can be expected to choose exactly one of the goals to go to.

(4) The animal will take advantage of shelters of passage that have no consummatory value. In general, the animal will seek and use shelters of passage that may allow it to get to its goal with less exposure to fear than it would otherwise experience. The exact results expected depend on the precise physical layout of the shelters in relation to the positions of the goal, predators, etc.

(5) The animal is never trapped by its own motivations in an avoidance-avoidance situation that will lead to capture.

(6) According to the variable velocity model in Section 3.2, the animal will slow down in proximity to a resting or sleeping predator. This phenomenon seems to be robust under minor changes in the model as long as the energy consumption is a linear funciton of velocity.

Section 5.2 Extensions of the Present Work

Some possible extensions of the present work are:

(1) The quantification of fear from observed paths of animals. Trajectories of an animal on the ground could easily be obtained by photographic means. In this case, Equations 3.1.21 or 3.2.8-10 could be used to quantify fear expectations. In particular, one would have to know the source of fear, such as a sleeping animal, so

that the angle between the trajectory and the iso-aversion line passing through the location of the running animal could be measured. Further, one would have to measure the curvature of the running animal's path (this task could no doubt be much facilitated by computer smoothing of the photographically recorded path). Then, given that the above mentioned angle is denoted by ξ, the gradient of the logarithm of f or f* would be given by:

$$\|\text{Grad}(\ln f)\| = \|K\| \text{ secant } \xi \tag{5.2.1}$$

(In the case where the variable velocity case is being studied, replace f by f* in the above equation). If the fear expectation f(x,y) is further assumed to be circularly symmetric, then the knowledge of the gradient of ln (f) or ln (f*) will allow the computation of the value of ln f by means of numerical integration, since in polar coordinates a circularly symmetric function allows the formula

$$\|\text{Grad } g\| = \left|\frac{dg}{dr}\right| \tag{5.2.2}$$

In the variable velocity case, further measurements or assumptions concerning the other constants (see Section 3.2: the constants c_1, c_2 of the rate of energy expenditure and the constant gamma of the velocity dependence of fear) would have to be made. In the variable velocity case, the constant μ would simply appear as a multiplicative constant of the fear function and allow it to be determined up to a multiplicative constant only, as usual.

(2) A similar framework to that presented would allow modelling the movements of prey and predator in chase and escape behavior. If both are optimal, then their joint behavior could be computed using differential game theory.

(3) Since all neural solution methods are inherently approximate, it is of interest to investigate the behavioral consequences of the level of approximation to true optimality. More accurate and efficient neural solvers for the minimum path problem may exist.

(4) Computer programs and hard-wired models of the hexagonal network solution would provide an interesting confirmation of the present work.

CHAPTER 6

SUMMARY

There are fundamental problems in existing psychological theories of motion. Lewin's life space theory appears to accurately describe perception of the situation. However, there are problems with his assumption that motion follows the vector sum of psychological forces. Instead we propose that the animal chooses its path to a goal so as to minimize its exposure to fear. The mathematics of optimization is used to derive properties of the animal's path choices which are predicted by this theory.

We present a neural circuit which can compute optimal paths to a reasonable degree of accuracy. This network shows that near optimality of motion is not an unattainable goal. The neural circuitry is not exorbitant in terms of neural structure, either from the point of view of complexity or from the point of view of the number of neurons needed.

APPENDIX A

COMPUTER PROGRAMS

Program 1. This program, provided in BASIC, solves the endpoint problem for the calculus of variations solution in Equations 3.1.32 and 3.1.33. This is done by a technique of varying parameters inside of a Newton - Rapheson loop.

```
10000 GO TO 62820
20010 REM PROGRAM TO FIND SOLUTIONS
20020 REM OF THE EULER EQUATION FOR
20030 REM FEAR = 1/R + F0
20040 REM BY MEANS OF VARIATIONALLY
20050 REM EMBEDDED NEWTON-RAPHESON.
20060 REM THE PROBLEM IS
20070 REM SOLVED BY N-R WHILE THE
20080 REM PARAMETERS ARE VARIED.
20090 REM THE VARIATIONAL EMBEDDING
20100 REM IS ACCOMPLISHED BY THE
20110 REM VARING THE WEIGHT W,
20120 REM IN THE IW LOOP.
20130 REM THE MAIN N-R LOOP IS AT
20140 REM 60000-61400.
20150 REM THE N-R ITERATIONS ARE
20160 REM CONTROLLED AT 62400-62500
20170 REM INSIDE THE VARIATIONAL
20180 REM LOOP AT 62000-62700 WHICH
20190 REM CHANGES THE PARAMETER W
20200 REM IN SUCH A WAY THAT THE N-R
20210 REM SOLUTION APPROACHES THE
20220 REM THE SOLUTION OF THE ACTUAL
20230 REM VARIATIONAL CALCULUS PROBLEM
20235 REM WHICH IS A
20240 REM BOUNDARY VALUE PROBLEM
20250 REM AS SPECIFIED BY THE EULER
20260 REM EQUATION.  THE CODE AT
20270 REM 62960 REFINES THE SOLUTION
20280 REM BY 10 N-R LOOPS AFTER W = 1,
20290 REM TO PROVIDE AN EXACT ANSWER.
20300 REM THE CODE AT 62962-63045
20310 REM PROVIDES A PRINTOUT OF THE
20320 REM RESULTING TRAJECTORY IN
20330 REM POLAR COORDINATE FORM.
```

```
20340 REM
50000 T0 = SQR (C↑2 - 1) / C
50100 T1 = SIN (T0 * (U1 - U0))
50200 T2 = SIN (T0 * (U2 - U0))
50300 T3 = COS (T0 * (U1 - U0))
50400 T4 = COS (T0 * (U2 - U0))
50500 T5 = 1 / SQR (C↑2 - 1)
50600 P1 = R1 * F0 * (C*T3+1) - C↑2 + 1
50700 P2 = R2 * F0 * (C*T4+1) - C↑2 + 1
50800 RETURN
60000 GOSUB 50000
60100 M(1,1) = R1 * F0 * T1 / T5
60200 M(2,1) = R2 * F0 * T2 / T5
60300 X = R1 * F0 * T3 - 2 * C
60400 M(1,2) = X-R1*F0*C*(U1-U0)*T5*T1
60500 X = R2 * F0 * T4 - 2 * C
60600 M(2,2) = X-R2*F0*C*(U2-U0)*T5*T2
60700 D = M(1,1)*M(2,2) - M(1,2)*M(2,1)
60800 Q(1,1) = M(2,2) / D
60900 Q(2,2) = M(1,1) / D
61000 Q(1,2) = - M(1,2) / D
61100 Q(2,1) = - M(2,1) / D
61200 U0 = U0 - Q(1,1)*P1 - Q(1,2)*P2
61300 C = C - Q(2,1)*P1 - Q(2,2)*P2
61320 PRINT "ERROR"; ER
61400 RETURN
61500 T0 = SQR (C↑2 - 1) / C
61600 T3 = COS (T0 * (U1 - U0))
61700 T4 = COS (T0 * (U2 - U0))
61800 G1 = (C↑2-1)/(F0*(C*T3+1))
61900 G2 = (C↑2-1)/(F0*(C*T4+1))
62000 FOR IW = 0 TO 100: W = IW / 100
62100 R1 = W * RA + (1 - W) * G1
62200 R2 = W * RB + (1 - W) * G2
62300 PRINT "START";W;U0*180/π; C
62400 GOSUB 60000
62500 IF ER > 10↑(-3) THEN GOTO 62400
62600 PRINT "FINISH";U0;C
62700 NEXT IW
62800 RETURN
```

```
62820 PRINT "INPUT R1,2;THETA1,2;F0"
62900 INPUT RA,RB,W1,W2,F0
62910 C = 1.1
62920 U0 = 0
62930 U1 = W1 * π / 180
62940 U2 = W2 * π / 180
62950 GOSUB 61500
62960 FOR I = 1 TO 10: GOSUB 60000:NEXT
62962 OPEN3,4: CMD3: REM FOR PRINTER
62964 PRINT "RANGE OF THETA";W1;W2
62966 PRINT "START AND END R";RA;RB
62968 PRINT "FEAR FOR GOAL";F0
62970 PRINT "TRAJECTORY FOLLOWS"
62980 FOR TH = W1 TO W2 STEP 10
62990 T = π * TH / 180
63000 T0 = SQR (C↑2 - 1) / C
63010 T3 = COS (T0 * (T - U0))
63020 R = (C↑2 - 1) / (F0*(C*T3+1))
63030 PRINT TH, R
63040 NEXT TH
63045 PRINT#3: CLOSE3: REM FOR PRINTER
63047 GO TO 62820
63050 STOP
```

Program 2. The following computer program solves the discrete minimum path problem by the Moore Algorithm. It is written in BASIC.

```
10 REM OPERATES ON DIGRAPH
20 REM TO PRODUCE MINIMUM PATHS
30 DIM FR(100):REM FRONTIER
40 DIM CM(100):REM CUMULATIVE COST
50 DIM CT(1000):REM LINK COST
60 DIM EN(1000):REM LINK ENDPOINT
70 DIM PT(100):REM PATH OF NODES
80 DIM IN(100):REM FIRST LINK FOR NODE
90 DIM BK(100):REM BACKTRACE ARRAY
100 REM ND=NUMBER OF NODES
110 REM NF=NUMBER IN FRONTIER
120 REM NS=STARTING NODE
130 REM NT=TERMINAL NODE
140 REM NC=CHOSEN REFERENCE NODE
150 REM NL=NUMBER OF LINKS
160 REM CS=TRIAL COST
170 REM ET=ENDPOINT OF TRIAL LINK
180 GO TO 540
190 REM FORWARD LOOP OF MOORE ALGORITHM
200 FOR I = 1 TO ND
210 CM (I) = 10↑10
220 FR (I) = 0
230 NEXT I
240 NF = 1
250 FR (NS) = 1
260 CM (NS) = 0
270 REM BEGIN FORWARD RECURSION
280 FOR I = 1 TO ND
290 IF FR (I) = 0 THEN GO TO 435
300 NC = I
310 J = IN (NC)
320 IF NC < ND THEN K = IN (NC+1) - 1
330 IF NC = ND THEN K = NL
340 FOR L = J TO K
350 CS = CM (NC) + CT (L)
360 ET = EN (L)
370 IF CS >= CM (ET) THEN GO TO 410
380 CM (ET) = CS
390 BK (ET) = NC
395 IF FR (ET) = 1 THEN GO TO 410
400 FR (ET) = 1
```

```
405 NF = NF + 1
410 NEXT L
420 FR (NC) = 0
430 NF = NF - 1
435 NEXT I
440 IF NF > 0 THEN GO TO 280
450 RETURN
460 REM BACKWARD RECURSION
470 PT (1) = NT
480 I = 1
490 IF PT (I) = NS THEN NZ = I:RETURN
500 PT (I + 1) = BK (PT (I))
510 I = I + 1
520 GO TO 490
530 REM INPUT & CONTROL LOOPS FOLLOW
540 PRINT "INPUT NUMBER OF NODES"
550 INPUT ND
560 I = 1
570 NL = 0
580 FOR I = 1 TO ND
590 IN (I) = NL + 1
600 PRINT "START = ";I;"INPUT EN,CT"
610 NL = NL + 1
620 INPUT EN (NL), CT (NL)
630 IF EN(NL) <> 0 THEN GO TO 610
640 NL = NL - 1
650 NEXT I
660 PRINT "INPUT STARTING NODE"
670 INPUT NS
680 GOSUB 200
690 PRINT "INPUT TERMINAL NODE NUMBER"
700 INPUT NT
710 GOSUB 470
720 LN = 0
730 PRINT "PATH FOLLOWS: NODE, COST
740 FOR I = NZ TO 1 STEP -1
750 PRINT PT (I), CM (PT (I))
760 LN = LN + 1
770 L1 = INT (LN / 10)
780 IF L1 * 10 <> LN THEN GO TO 810
790 PRINT "RETURN KEY TO CONTINUE"
800 INPUT
810 NEXT I
820 GO TO 690
830 END
```

Program 3. The following computer program (in BASIC) solves for optimal trajectories in the continuous case. It does so by use of continuous Dynamic Programming, as discussed by Bellman (1961, Chapters 5 and 7). It has very little internal documentation, because it is assumed that the reader can make use of Bellman's detailed explanations if he wishes to understand the details of the continuous dynamic programming model. The dynamic programming in the program is done with theta as the independent variable in polar coordinates. The program is interactive, but is written to output the trajectory which is optimal to the printer.

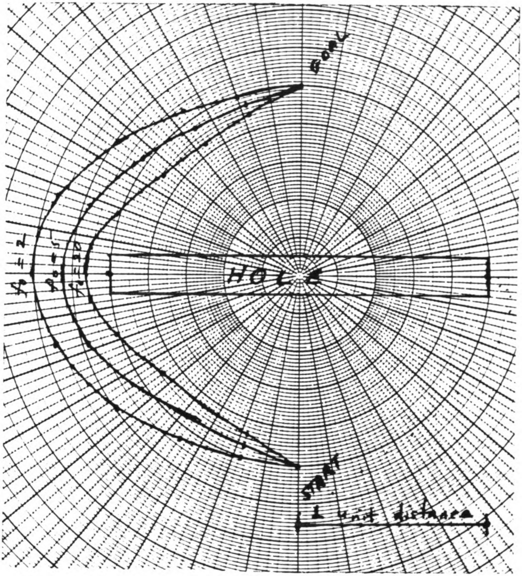
```
10 REM DYNAMIC PROGRAMMING ROUTINE
20 DIM G (50,50)
30 DIM U (50,50)
50 DIM R (50)
55 GO TO 610
60 REM DEFINES AVOIDANCE FUNCTION
61 JX = N - JR
62 XR = IR * DM * COS (T1*π/180+JX*DN)
64 YR = IR * DM * SIN (T1*π/180+JX*DN)
66 REM DEFINE FEAR IN TERMS OF X, Y
68 XS = ABS (XR): YS = ABS (YR)
69 W = 0.1
70 IF XS<=W AND YS<=1 THEN Q=10↑8
72 IF XS > W AND YS <=1 THEN Q=1/(XS-W)
74 IF XS <=W AND YS > 1 THEN Q=1/(YS-1)
76 IF XS <=W OR YS <=1 THEN GO TO 80
78 Q = 1/SQR ((XS-W)↑2+(YS-1)↑2)
80 REM BASIC FEAR FUNCTION GENERATED
90 Q = Q + Q0: REM ADD FEAR FOR GOAL
95 RETURN
100 FOR I = 0 TO M
110 G (0, I) = 0
130 RX = I * DM: IR = I: JR = 1
132 VX = (R2 - RX) / DN
134 GOSUB 60
136 U(1,I)=VX
138 G (1,I)=Q*SQR (RX↑2 + VX↑2)
140 NEXT I
```

```
200 FOR J = 2 TO N
210 FOR K = 0 TO M
215 REM PRINT "LOOP J = ",J," K = ",K
220 GT = 10↑10
225 IC = 2: LC = -10: HC = 10
230 UT = 0
232 JR = J: IR = K
234 GOSUB 60
240 FT = Q
250 RA = K * DM
260 FOR U1 = LC TO HC STEP IC
270 K1 = U1 * DN / DM
275 IF K + K1 < 0 THEN GO TO 360
280 K2 = INT (K + K1)
285 IF K2 + 1 > M THEN GO TO 360
290 X = K + K1 - K2
300 GX = G (J-1,K2) * (1 - X)
310 GX = GX + G (J-1, K2 + 1) * X
320 GX = GX + FT * SQR (RA↑2+U1↑2)*DN
325 REM DYNAMIC PROG CORE
330 IF GX >= GT THEN GO TO 360
340 GT = GX
350 UT = U1
355 REM PRINT "MIN CRITERION", GT,UT
360 NEXT U1
365 LC = UT-IC: HC = UT+IC: IC = IC/5
367 IF IC > 0.02 THEN GO TO 260
370 G (J,K) = GT
380 U (J,K) = UT
390 NEXT K
400 PRINT "END OF J LOOP", J
410 NEXT J
420 RETURN
430 REM COMPUTE OPTIMAL PATH
440 R (0) = R1
450 C = R1
460 FOR I = N TO 1 STEP -1
470 X = C / DM
480 J = INT (X)
490 X = X - J
500 V = U(I,J)*(1-X) + U(I,J+1)*X
510 C = C + V * DN
520 R (N - I + 1) = C
530 NEXT I
540 RETURN
```

```
600 REM DRIVER
610 PRINT "ENTER N, M"
620 INPUT N, M
650 PRINT "ENTER RANGE OF THETA, DEG"
660 INPUT T1, T2
670 DN = ((T2-T1)*π/180)/N
680 PRINT "ENTER MAXIMUM RADIUS"
690 INPUT RM
700 DM = RM / M
702 PRINT "ENTER FEAR FOR GOAL"
704 INPUT Q0
710 PRINT "ENTER ENDING R"
720 INPUT R2
730 GOSUB 100
740 PRINT "ENTER STARTING R"
750 INPUT R1
760 GOSUB 440
765 OPEN3,4: CMD3
767 PRINT "FEAR FOR GOAL = ";Q0
770 FOR I = 0 TO N
780 T = T1 + I * DN * 180 / π
782 L = INT (I): L1 = 10 * INT (L/10)
784 REM IF L1 <> L THEN GO TO 790
785 GO TO 790
786 PRINT "RETURN KEY TO CONTINUE"
788 INPUT
790 PRINT T, R(I)
800 NEXT I
805 PRINT#3: CLOSE3
810 GO TO 740
820 END
```

APPENDIX B

EXAMPLES OF TRAJECTORIES



$$\text{Fear} = f(x,y) = \frac{1}{\text{distance from hole}} + f_0$$

Trajectories are shown for the slit hole problem, where Lewinian theory predicts stalling.

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