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THE EFFECTIVENESS OF A UNIT
IN TEACHING AND LEARNING OF GROWTH RELATIONS
IN THE SIXTH AND SEVENTH GRADES

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THE EFFECTIVENESS OF A UNIT IN TEACHING AND LEARNING OF GROWTH RELATIONS IN THE SIXTH AND SEVENTH GRADES

By

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ABSTRACT

THE EFFECTIVENESS OF A UNIT IN TEACHING AND
LEARNING OF GROWTH RELATIONS IN THE SIXTH AND SEVENTH GRADES

By

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The major motivation for the current study was a previous study by Fitzgerald and Shroyer involving the teaching of area and volume concepts using a unit entitled The Mouse and The Elephant (64). That study revealed encouraging pupil success rates in understanding area and volume. The teachers and the pupils reacted favorably to the unit. A replication study was recommended using an improved unit guide and well-trained teachers.

This study investigated five aspects of the unit instruction: 1) what actually happened in the classrooms; 2) pupil performance; 3) the performance of old (experienced) and new teachers; 4) the comparison of pupils in this and the previous study on the final and post evaluations; and 5) the teachers' and pupils' attitude toward the unit.

This study used a modified unit guide, two old and two new teachers, and 161 sixth and seventh grade pupils in an East Lansing, Michigan middle school. Data was collected in ten weeks from observations, achievement tests, and questionnaires.

Pre-, post- and retention tests were administered respectively on a day before, a day after and three weeks after the instruction of the unit.

The pre- and retention tests were investigator constructed. The post-test and post evaluation used were those in the previous study. Each teacher was observed in almost all the classes. All the tests, except the post evaluation, were administered by the class teacher and the investigator. One class was not given the retention test. The post evaluation was given by the investigator only to pupils who scored 76% or better on the final evaluation.

The unit was taught in 2 to 3 weeks for 3 classes and 5 weeks in 1 class. The pupil success rates on the tests, their mean scores in the old and new teachers classes were computed. The t-statistics was used to test for significant differences (for the two groups) on the three test means. The scores of only pupils who took all the tests were included for the t-test. Percents of pupils responses were categorized from the questionnaire. Teachers' responses were reported directly with comments.

The results showed that the teachers actually used only 2/3 to 3/4 of the class instruction time; over 50% of each instruction time was spent on exploration, and pupils were able to generate all the formulas needed to compute the concepts in the unit, but they stated them differently from their traditional format. On achievement tests, the success rates were: area - over 90%, volume 68%, perimeter 67%, and surface area 65% - which was the order of difficulty from easiest to hardest. There was no significant difference at 5% level between the two groups on the pre-test, but on the post- and retention tests there were. The differences were small for practical purposes. Pupils performed slightly higher in this study on the final evaluation but substantially higher on the post evaluation than in previous study.

Pupils preferred the unit to regular math and the teachers reportedly liked the unit.

It was concluded that 1) teachers did not seem to use all the instructional time on actual instruction, but seemed to agree that exploration by pupils should be given most of class time, 2) the unit seemed appropriate for the sixth and seventh graders and their teachers, 3) from the low performance on the retention test on surface area items, pupils seemed to forget substantially on some aspects of the unit after three weeks, 4) newness to the unit did not seem to pose problems for the teachers and 5) the modification of the unit seemed to have helped the pupils perform slightly higher in this study than the previous study.

This Thesis is Dedicated

to

Madam Messie Kumba, my mother;

to

Late Momoh Yaa Bockarie, my elder brother who
struggled very hard to finance my secondary education;

to

Late Madam Moiyatu Tewa, my father's second wife
for caring for me;

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ACRONYMS AND DEFINITIONS OF TERMS

CEEB	College Entrance Examination Board
CSMP	Comprehensive School Mathematics Project
CUMP	Committee on the Undergraduate Program in Mathematics
JRME	Journal for Research in Mathematics Education
NAEP	National Assessment of Educational Progress
NCES	National Center for Education Statistics
NCTM	National Council of Teachers of Mathematics
NIE	National Institute of Education
NLSMA	National Longitudinal Study of Mathematical Abilities
NSF	National Science Foundation
SAT	Scholastic Aptitude Test
SMSG	School Mathematics Study Group
SSM	School Science and Mathematics (Journal)
UICSM	University of Illinois Committee on School Mathematics
UMMP	University of Maryland Mathematics Project

Old Teacher - Teacher who is participating in the teaching of the Mouse and Elephant Unit for a second or third time with previous training on teaching the unit.

New Teacher - Teacher who is participating in the teaching of the unit for the first time with no previous training on teaching the unit.

CHAPTER I

THE PROBLEM

Introduction

In order to improve the teaching and learning of some topics in mathematics the investigator believes that it is important to assess pupils achievement, identify errors and determine possible causes of the errors, and investigate students' and teachers' attitudes toward their mathematics instruction.

The Mouse and Elephant Unit (Fitzgerald (64)) is a unit for teaching measurement of area and volume and how those measures change during growth.

The unit is thus a teaching tool designed to present some mathematical ideas to sixth and seventh graders in a useful, challenging and interesting way. In the introduction of the unit it is stated:

> This unit is designed to present interesting and useful mathematical ideas to people who do not know them.... The unit is designed to provide activity-oriented lessons in which an entire class can participate, accommodating the wide range of abilities and background. The central focus of the unit is to provide for children manipulative experiences with materials which embody the mathematical concepts to be learned...this kind of experience for children is necessary for the healthy development of the cognitive attainments we expect for children.

The main mathematical ideas and skills covered by the unit include three categories: definition of certain terms: rectangle, square, solid block, cube, edge (dimensions), faces, corners; measuring concepts: edges, perimeter, area, surface area, volume; and measuring skills:

counting, rule; and relationships among the different concepts.

There are three phases of instruction: a) launching period, b) exploratory period, c) summary time.

The unit clearly describes what is expected from the teacher and pupils during each phase of instruction in this way:

> During the launching the teacher follows the script very closely posing the questions and challenges in the sequence they are intended and presented. This sequence allows each student to be engaged in the task at his/her appropriate level with some degree of success.
>
> After the major challenge has been posed, the class can begin working individually or in small groups. The teacher can float around the class to keep abreast of developments. Some children will need additional help beginning the task as one presentation of the challenge is often not sufficient. Other children will need help maintaining progress toward the challenge. The teacher may spot errors the students have made and help the children correct their error. Still other children will finish the task and will need to be presented with an extra challenge to keep them working productively.
>
> Such a work period will result in the children being more different from each other than before. While all children have made progress, some have made much more than others. This is as it should be.
>
> However, it is desirable to bring the class together again to summarize the results of the activity. The orderly tabulation of results will allow children to recognize patterns and generate rules. Again, one should expect great differences among the children, but all can profit from a discussion of the generalizations which might surface from the group.

Previous investigations using the Mouse and Elephant Unit revealed that several pupils made such errors as calculating volume for surface area and vice versa. In computing perimeter of a rectangle such mistakes as just adding two sides or multiplying the sum of two sides by an integer other than two were common. Computational errors with regards to addition and multiplication occurred fairly often. A small percentage of the pupils called a 4 by 5 rectangle a triangle, triangular square or a square.

In an attempt to help reduce these types of errors the original Mouse and Elephant Unit was modified to make very explicit instructions to the teacher on what to do and when to do it. The format of the unit was changed, a summary section was added at the end of each activity and some new challenges were added while some old ones were deleted. The investigator intended to determine the effects of this modified unit when used by previously trained teachers and teachers with no training on the unit, and to compare such effects with previous findings on pupils achievement, errors and teachers' and students' attitudes.

A summary section has been added at the end of each activity to enable teacher and pupils to sum up what has been (or supposed to have been) taught and learned. In the previous investigation, the absence of such summary sections for each activity might have caused pupils to make many algorithmic errors as no opportunity to correct a preconceived idea was available after each activity. Summeries, where they were included, proved very useful in previous study.

The challenges with circle have been replaced by more rectangular figure challenges. A non-rectangular parallelogram has been added. The formula for calculating the area of a circle and volume of a sphere are believed to be in a different category of difficulty. Their deletion may help the focus on areas and perimeters of rectangles and thus minimize confusion. Approximate time for each stage of instruction recommended in the unit will help ensure that the teacher does not ignore any of the stages for each activity and that pupils will not be too carried away on a certain aspect of an activity. Pupils tended to be carried away with the cutting of jackets and the building of large blocks in previous investigation. Some pupils focused more on different shapes of jacket cut than the essence of the activity - calculating surface area.

Both the pre- and post-tests in a pilot showed that some pupils had not understood such differences as that between the top area and surface area of a rectangular block. Sentence construction may help eliminate this error and build pupils confidence.

Using the modified Mouse and Elephant Unit the director of this study intends, broadly speaking, to:

1. find out how the actual instruction goes on in the classroom. In particular, to determine how the instructional time is used, how the teachers and pupils maneuver in the class, and the questions pupils ask or answers they give during the summary stage, i.e. how involved are the teachers and the pupils in the unit?
2. assess pupils cognitive achievement on the content of the unit as measured by the pre- and post-tests, and also assess how much the pupils remembered three weeks after the instruction of unit as determined by the retention test; and compare the cognitive achievements of the pupils in the experienced teachers' classes and those in new teachers' classes.
3. compare the overall achievements of this group with those in the Fitzgerald-Shroyer study.
4. identify the kinds and patterns of errors pupils will make prior to the unit and after the unit, and compare the errors with those in the Fitzgerald-Shroyer study.
5. assess pupils' and teachers' attitudes about the project and compare them with similar data on previous participants attitudes.

Thus, the categories to be investigated and compared with previous data include: achievement of pupils, errors made by pupils and teachers' and pupils' attitudes. The investigator believes that instruction in school should mainly be geared to cognitive achievement and activities that may enhance such achievement should be investigated. Ebel (15) fully supports such a position recognizing cognitive achievement as the main function of schools and learning as primarily the responsibility of the learner. In part he puts it this way:

> "Schools are for learning. They should bend most of their efforts to the facilitation of learning. The kind of learning on which schools should concentrate most of their efforts is cognitive competence, the command of useful knowledge.
>
> Knowledge is a structure of relationships among concepts. It must be built by the learner himself as he seeks understanding of the information he has received....
>
> Schools should be held accountable for providing a good learning environment, which consists of a) capable, enthusiastic teachers, b) abundant and appropriate instructional materials, 3) formal recognition and reward of achievement, and d) a class of willing learners."

While new methods of instruction with potential to facilitate learning are tried, it is important to measure how much the learners achieve cognitively by tests and interviews.

Lesh (38), commenting on an article by Cox on the importance of an investigation on errors, states:

> Cox's study is a commendable first step in the isolation and analysis of systematic errors in computational algorithms. Further studies of this type are clearly needed. Further studies might attempt to determine how errors on one algorithm are related to errors on another algorithm, or how errors at one skill level are related to errors at another skill level for the same algorithm.... Finally, it is of great importance to find instructional experiences that can effectively deal with problems of the type described in Cox's study.

Shulman (51) points out how errors and achievement are related as observed by Piaget in the following statement:

> He (Piaget) was struck by the observation that the character of the error made by children held as much interest as the nature of their correct answers. In fact, there were systematic internal consistencies in the kinds of errors made by children of different ages. It was as if they were operating with their

own forms of logic which, though unlike adult logical forms, were regular and amenable to formal analysis.

Purpose of the Study

The purpose of this investigation is:

a) to study classroom activities:

1) to determine how much of each class period was spent on actual instruction; how many minutes were spent on an average at each of the three stages stated in the unit guide; the launching, the exploration and the summary each activity.

2) to determine what teachers added to their instruction not included in the unit.

3) to find the kinds of patterns, or rules pupils would state and the reasons they would give for the rules.

b) to study achievement in order:

1) to determine how much the pupils achieve cognitively in learning concepts and computations of areas and perimeters, volumes and surface areas; and the relations between area and perimeter, and volume and surface area.

2) to compare the average performances of the pupils of the teachers with previous experience with the unit and those of the teachers new to the unit.

3) to compare the performances of pupils in this project and those from previous project by comparing the post-test scores.

4) to determine how much the pupils remembered three weeks after the project from the retention test.

c) to study errors in order:

  1) to identify the kinds of errors pupils make during and at the end of the project, particularly algorithmic errors.

  2) to identify possible error patterns on the various sections of the unit.

  3) to compute percent of pupils making certain kinds of errors on the various concepts.

d) to study attitudes to assess teachers' and pupils' attitude toward the project and compare with similar attitudinal data in previous investigation.

Hypotheses

A priori hypotheses can be proposed at the start of this investigation. The hypotheses, particularly on classroom observations, will be derived mainly during the investigation which is primarily ethnographic in nature. Accordingly, these main hypotheses will be based on observations, interviews and reviews of pupils' and teachers' work. However, some tentative hypotheses, based on previous similar studies, Cox (1975), Robert (1968), Fitzgerald and Shroyer (1972), can be suggested. They fall into three categories:

Observations. It is hypothesized that:

1. Each teacher will spend at least 15 minutes on the summary stages and will teach the whole unit in three weeks.

2. All the teachers will have no problems using the unit and will use their own techniques during the instructions.

3. The pupils will spend the most time on the exploration stage.

Achievement. It is hypothesized that:

1. From pre-test to post-test, pupils will improve considerably in learning the topics covered by the unit. That is to say that the proportions of pupils who will be successful on the post-test on area, perimeter, surface area, volume and other items covered by the unit will be higher than those on similar items on the pre-test.

2. The mean score of pupils in the old teachers' classes will be significantly higher than that of the pupils of the new teachers' classes on the post-test and on the retention test.

3. The pupils in this study will perform about the same level as those in Fitzgerald-Shroyer study on the post-test and post evaluation test.

4. Three weeks after the unit is taught, the proportion of pupils successful on various items on the post-test will be about the same on similar items on the retention test.

5. In descending order (frequency error occurred among pupils) pupils errors will be as follows: computation, wrong algorithm, random errors, and space and language difficulties.

Attitudes: It is hypothesized that teachers and pupils will react as favorably as teachers and pupils did in the previous investigation.

Procedure

The investigation was limited to sixth and seventh graders and their teachers in one selected school, in East Lansing district, Michigan, over a 10 week period. The unit takes about two to three weeks for each class. Due to insufficient materials, conflicts on time-tables of teachers' and the investigators', and some teacher's preference it was necessary to stagger the data collection.

An investigator-constructed retention test was conducted three weeks after the final post-test. Teachers who had previously participated in the Mouse and Elephant Unit were asked to teach the unit in an attempt to achieve the desired outcome of the project. Also, two teachers who never saw the unit before taught the modified unit in order to compare the achievements of their pupils with those of the trained teachers. This was essential because the unit may later be used by teachers who have not been trained to teach the unit. The sixth and seventh graders and their teachers in this project were observed, interviewed and tested (only the pupils) before, during and the end of the project.

Thus, the main methods of investigation included:

a) observation during the instruction

b) interviews with teachers and students

c) paper and pencil testing

d) analysis of pupils errors, from written work and interviews

e) collection of attitude inventory from teachers and pupils.

The following steps were taken in the order listed to attempt to answer the questions stated in the statement of the problem. The unit takes about two to three weeks, but different classes started at different times due to insufficient materials and teacher preference. Data was collected on seven classes altogether.

1. Modifying the original unit.

   This was done by Fitzgerald and Shroyer prior to the start of this study.

2. Administration and scoring of pre-test.

3. Constant observation of teachers and pupils during teaching to collect data on teachers instructions, kinds of questions teachers

ask, pupils ask, and the corresponding responses each side receives (to determine possible explanation of some errors especially where pupils give reasons that "teachers told us to do it this way," and to also determine where and when the teachers modifies or deviates from the unit.)

4. Administration of post-test (and three weeks later retention test).
5. Identification of pupils errors.
   a) that research has shown to be important and persistent in mathematics at the lower and sixth grade levels,
   b) that were recognized from analysis of written work of pupils in previous efforts of the Mouse and Elephant Project.
6. Collection of data on attitudes (from teachers' and pupils' attitude inventory).
7. Analysis of tests and attitude inventory.

For the purposes of this investigation, errors will be broadly categorized into three main areas which seem to encompass the major sources of difficulties for pupils of this age group.

A. Language and Spatial Difficulties: Mathematical concepts, symbols and vocabulary is a 'foreign language' problem. This is particularly true in solving word problems. In the Mouse and Elephant Unit such words as rectangle, square, block, and cubes may pose difficulties for pupils. Also, their representations demand a lot from children's spatial abilities and capacity for visual discrimination. Such demands are less content-specific and errors judged as due to substantial individual differences in children's spatial imagery and spatial thinking, and language difficulties will be considered language and spatial difficulties.

B. <u>Prerequisite, Irrelevant Rules and Defective Algorithm Difficulties</u>: This includes all deficits in the content - and problem - specific knowledge necessary for the successful performance of the mathematical task, such as ignorance of algorithms, inadequate mastery of basic number facts like multiplication and incorrect procedures in applying mathematical techniques. It also includes errors to negative transfer of a mathematical fact like using the formulas for surface area to find the volume of a block. Pupils do develop cognitive operations and continue to use them though the fundamental conditions of the mathematical task have changed - some aspects of content and solution process persist in the mind, inhibiting the processing of new information. Thus, pupils may have in the mind a defective algorithm to find the maximum dimensions of a given rectangle with a fixed area.

C. <u>Others: careless, random, omissions</u>: Errors that cannot be explained or judged as belonging to any of the first two categories will be included here. If a pupil, for example, correctly solves a problem but interchanges answers, in recording in the appropriate spaces, this will be considered a careless error.

The difference between the percents of pupils getting the items correct on pre-test and that on post-test will be assumed to be the effect of the 'unit' primarily. Similar percent difference between the post-test score and the retention score will be a measure of how much they remember after three weeks. To assess the old and new teachers average class performance, the means of the pre-test, post-test, and retention test scores for pupils who take the tests will be compared. The difference between the class averages on post-test of the previous and current investigation will be attributed to unit modification.

The difference, if positive, between the class means of the trained teachers and those not trained in the same school will be attributed to advantage of pre-training, otherwise pre-training will be of no major advantage.

Need for Study

The need for this study can be stated in three categories or areas that overlap with each other:

1) Questions were raised by previous investigations such as the Mouse and Elephant Project and related investigations.
2) Importance of investigating alternative styles of teaching and learning that have potential to lead to increase in learning.
3) Recommendations by the Conference Board of Mathematical Sciences National Advisory Committee on Mathematical Education (hereafter referred to as the Board).

1. Previous Investigations - Mouse and Elephant Project

The investigators, Fitzgerald and Shroyer (19), in the Mouse and Elephant Project had measured pupil's achievement as determined by the tests they constructed without a priori giving the pupils a pre-test.

They also made certain observations before their ninth conclusion, "Much more effort is required to lead class discussions, provide challenges, and provide experiences with manipulative materials for a whole class in a problem solving mode. If we are going to expect teachers to teach good mathematics, such as growth relationships, and teach it well as described in this unit, we need to find more effective ways of helping them to do so." Their conclusions nine and ten state:

Conclusion 9: Given a carefully developed script with clearly described phases of instruction, experienced mathematics teachers need explicit instruction in the roles and techniques for successfully conducting the phases of instruction.

Conclusion 10: Each activity must be revised to conform to the three phases of instruction in a much more explicit and complete fashion. This would include the specific characterization of the roles and techniques which are appropriate for the successful conduct of each phase.

They assumed that different teaching would result in different learning in some important way. They believed that,

"With the development of a more refined script and a careful preparation period,...we can remove some questionable variability and unproductive behavior in the teaching."

They concluded in this respect that,

"A more careful study of the effects of teaching good mathematics well on the learning outcomes by students in grades six and seven can only be accomplished when the teaching has become standardized in several important ways."

They had some recommendations as to the ways teaching of the unit should be standardized in order to learn more about learning outcomes. They state them as:

1) The teacher must have a thorough understanding of the content of the unit.
2) The teacher must have a rapport with the class so as to be in control without the need for a strict authoritarian atmosphere.
3) The teacher must be able to develop and embellish a story in which the concepts and relationships assume a concrete meaning in a problem solving atmosphere.
4) The teacher must understand the model of instruction, its phases, roles, techniques, their purposes and interrelationships.
5) The teacher must be able to execute those roles and techniques in ways which provide for the orderly conduct of the activity.

If a teacher possessed the five characteristics described above and chose to teach the unit using the revised script, what would be the effects on the teaching of the unit? What would be the nature of the conduct of the activities? What would be the cognitive and affective learning outcomes for the students?

The pursuit of the answers to these questions seems to be the most reasonable next step.

Two of the teachers involved in the current investigation had worked previously with the Mouse and Elephant Unit. They are both regarded by previous investigator (Fitzgerald) as good and competent teachers.

Previous efforts of the project necessitate a new investigation.

2. It is important to investigate alternative styles with potential to lead to increased learning in mathematics. As new discoveries are unravelled in such areas as physical and social sciences, new materials are generally added to the existing ones to be learned. Mathematics is no exception. It is, therefore, necessary to constantly keep searching for potentially improved methods of teaching and learning (at least certain topics) of mathematics. Teachers need better ways to teach, Reys (45) tells concerned people of minimum competency,

> "It must be made perfectly clear to everyone that good teaching, not the establishment of a test of minimal competency, is the key to the achievement of better performances in mathematics."

3. The Board (see Literature Review (59)) outlined a report on current status of mathematics in school. As stated in the review of literature, the Board's report is not very encouraging. In part, the report reads:

> School mathematics is in an unusual state today. Long enshrined as a unique and well-supported discipline with a clear-cut and almost monolithic identity, it is suddenly beset with many troubles -- an identity crisis brought on by the usual causes: internal confusion and loss of clear-cut direction and external changes in familiar support and status structures....
>
> Young people who formerly flocked to mathematics because it was so important and had such national status now look elsewhere. The popularity of mathematics and its funding by public and private agencies has greatly waned. At the same time, a plateau has been reached in the two-decade series of developments referred to as the "new math." Whatever its achievements (and they are many), it also has enough problems and unfulfilled goals to generate a host of critics among educators, parents and even politicians.

The investigator using the Mouse and Elephant Unit, "to teach good mathematics well," observing teachers do it in their classrooms, assessing both pupils cognitive achievement and errors may help remove some of the troubles referred to above if good results are obtained. The goal of the Mouse and Elephant Unit is consistent with one of the areas the Conference Board recommends as a needed research area "research on objective means for identifying good teaching and the characteristic of the effective teacher."

The Mouse and Elephant Unit is based on teaching good mathematics well. Specifically, this investigation focuses on the effect of the unit guide on teachers in teaching the Mouse and Elephant Unit. As the review recognizes, good teaching does not come by accident. If we agree on a definition of 'good teaching', we need to demonstrate and verify some aspects of such teaching. The unit guide, as one such aspect, can be tested to find out how well pupils learn mathematics.

The unit guide provides teachers with content and mode of instruction. They (teachers) will be observed to determine when and why they vary from the unit guide. On teacher education, the Board states:

> Colleges of education, professional mathematics education organizations, accrediting agencies of teacher certification, and the mathematics community must cooperate to produce mathematics teachers knowledgeable in mathematics, aware of, oriented to, and practiced in a multitude of teaching styles and materials and philosophically prepared to make decisions about the best means to facilitate the contemporary, comprehensive mathematics education of their students.

The investigator, like in previous efforts, will work with entire classes in their 'normal' setting with no control groups. In current mathematics education research little is known or reported about what happens in mathematics classrooms. The overview admits this by stating that:

> "Appallingly little is known about teaching in any large fraction of U.S. classrooms.... The vacuum of data on classroom practices should give pause to those who present simplified cause and effect explanations.... We have reason to suspect that in many classes teachers very poorly related structural understanding to algorithms embodying the structures. In other classes, teachers made structure a royal road to skill and failed to provide any emphasis on computational practice. ...it appears to us that the case for decreased classroom emphasis on manipulative skills is stronger now than ever before."

From NAEP's findings, as stated in the Literature Review, measures of volume, area and perimeter are problem areas for sixth grade pupils also. The Mouse and Elephant Unit, based on this topic, is therefore, very relevant. More important, is an investigation on how well pupils achieve using the unit guide approach.

NAEP in the same report recommends that:

> Since many pupils appear to be unfamiliar with concepts of perimeter, area and volume, teachers may wish to provide them with experience in partitioning regions into unit squares and solids into unit cubes, and then counting the units. Pupils also need experience in measuring the distance around geometric shapes. Formulas should not be derived until pupils have gained some confidence in working with physical models and some insight into why the formulas work. Furthermore, when they are in doubt about a formula, pupils should be encouraged to use diagrams or other physical models for verification. To prevent pupils from applying formulas

> irrationally, they can be given two-step problems in which the first step is to decide which dimensions are needed to calculate an area or a volume. Such problems can curb the tendency to substitute numbers into formulas blindly.
>
> Specific attention should be devoted to the concept of a unit of measure. Much of the difficulty in area and volume exercises in the assessment appeared to result from pupils ignoring or incorrectly identifying the unit of measure.

So the use of tiles, and the problem solving mode in the Mouse and Elephant Unit are clearly in line with NAEP's recommendation. In closing, NAEP warns that:

> Measurement concepts do not develop naturally without experience, however, and even then their development requires substantial preparations. Therefore, above all, teachers are encouraged to provide an abundance of measurement experiences in every grade.

But the report could have also equally alerted researchers to investigate the effects of such efforts recommended for teachers.

NAEP also reports that on a problem like 38 x 9 = ____, 37% of 9-year-olds, 15% of 13-year-olds, 11% of 17-year-olds and 17% of adults gave unacceptable responses. Error analysis will help us determine what errors are not age related, which ones are carried over to adulthood. Error analysis will thus enlighten teachers on the kinds of errors all pupils make and certain ability groups make.

Assumptions

The investigator assumed that teachers will deviate, consciously or otherwise, from the unit guide as detected in previous trials of the unit, and that various consequences, good or bad, will result. In the process of learning and applying mathematical concepts, ideas and skills in the Mouse and Elephant Unit, pupils make errors (as a result of misunderstanding between the pupils and teacher, or between the pupil and his peer group, or the text). That is, errors in this project are not necessarily accidental, they may be a result of some instructional strategy or some 'logical' reasoning.

All tests were administered by the regular class teachers. Pupils were expected to attempt all tests independently. The tests were analyzed for reliability and for the appropriateness of the difficulty level.

The investigator believes that to teach good mathematics well and achieve good results depends on the interaction of several factors. Among these factors are: teacher, students, topic, materials available and procedure in instructional evaluation. Barr (4) has pointed out that:

> Teaching does not take place in a vacuum.... Effectiveness does not reside in the teacher per se but in the interrelationships among a number of vital aspects of a learning-teaching situation and a teacher.

And Begle (1979) clarifies this point in the following way:

> This point - that the outcome of teaching does not depend just on the teacher but rather is the result of a complex interaction between the teacher, the students, the subject matter, the instructional materials available, the instructional procedure used, the school and community, and who knows what other variables - is one that will receive further support....

CHAPTER II

LITERATURE REVIEW

Introduction

One of the goals of teachers of mathematics and mathematics educators is to facilitate the teaching and learning of mathematics. To achieve this common goal, teachers, mathematics educators and agencies such as the National Science Foundation (NSF) have tried several different approaches. Many curriculum materials have been produced and used by such groups as the School Mathematics Study Group (SMSG) and the University of Maryland Mathematics Project (UMMP). The University of Illinois Committee on School Mathematics (UICSM) produced, tested and revised units for grades 9-12. It also conducted institutes to train teachers in the use of the materials it produced. UICSM considered the teacher training aspect so important that "in fact, it would not let the units be used without adequate previous teacher preparation" (42). Demonstrations, films, speakers and conventions have existed since the 1950's to improve the teaching and learning of mathematics.

It seems that these efforts by far outweigh the hoped-for outcomes. Fitzgerald (18) sums it up this way:

> "With all efforts and good intentions we seem to fall short of finding the ways to what is needed to achieve desirable and possible ends."

Begle (5), in his overall reaction to the mass of information available about teachers, stated that:

"These numerous studies have provided us no promising leads. We are no nearer any answers to questions about teacher effectiveness than our predecessors were some generations ago. What is worse, no promising lines of further research have been opened up. Evidently our attempts to improve mathematics education would not profit from further studies of teachers and their characteristics. Our efforts should be pointed in other directions."

Begle dispelled the commonly held belief that elementary school teachers disliked mathematics from his analysis of NLSMA studies. He did not, however, state whether empirical evidence suggests that disliking mathematics (by teachers) is related to good teaching as determined by pupils cognitive achievement. The concern here now, therefore, is method of teaching preferred by mathematics teachers in order to do a good job. When Begle pointed out his overall reaction to the mass of information available about teachers as one of discouragement, criticizing that most of the studies were on teachers (in the Literature Review) he recommended other directions. To fulfill such an appeal, one possible direction is the use of a unit guide. As Romberg and Uprichard (47) have suggested, "teachers and mathematics educators need to approach research diagnosing mathematics difficulties from many different perspectives. If one's knowledge about diagnosis and instruction in mathematics is to improve, then we collectively must be more imaginative in our approach to inquiry." The approach in this study offers an alternative way of finding out about pupils difficulties.

Attitude, I.Q. and Achievement

Several studies have been done on attitudes and I.Q.'s of sixth and seventh graders to determine how they relate to achievement in mathematics (Gilbert (24), Walker (55), and Hungerman (32). Hogman (32) surveyed the pupils to determine the areas of mathematics they liked most. In the case of correlational studies on attitudes or I.Q. to achievement, the results have not always pointed to the same direction. For example, Walker and Hungerman did not agree on whether ability was related to achievement in mathematics.

It is generally believed that pupils' attitudes towards mathematics are somehow related to the instructional methods they have experienced. What is missing in most of the attitudinal studies reported below is any mention of the pedagogy by which pupils were taught. The investigator believes that attitudes should be investigated in the context of the method by which the topic has been taught, because a change in pedagogy may change the results of some of these studies.

Walker (55) reported from his studies that "competency in mathematics was not related to general ability and that high level of proficiency in one area is no guarantee of a correspondingly high level in any other area."

Hungerman (32), in contrast to Walker, found that group correlations for I.Q. in mathematics were significant.

Rose and Rose (48) and Gunderson and Felt (30) also found a high correlation between I.Q., school grades and achievement in mathematics.

There is clearly a contradiction among Walker, Hungerman, and Rose in their conclusions. The investigator believes that perhaps there is more to achievement in mathematics in schools than isolated variables such as I.Q. and general ability. Teachers instructional methods and the pupils' attitudes in mathematics classes all influence achievement.

Gilbert's study (24) was designed to combine the variables of ability, attitude, and teacher perceptions to determine their inter-relationships with reported achievement scores in mathematics of sixth grade pupils. He stated his purpose as to:

1) investigate what degree of relationship existed at the beginning and the end of the school year among students' rank order I.Q. scores, and the rank order of students in respect to their teachers' perceptions of their competency in mathematics within the group.

2) Determine the degree of interrelationship that existed at the end of the year among the student oriented variables - I.Q., achievement, and attitudes toward taking mathematics with the value of the teacher knowing the scores and rank order of students in their classroom in respect to each of the variables.

He found no significant correlations among sixth grade students' attitudes toward taking mathematics and the variables: general ability, achievement, or teachers' perceptions. He also found that "students' attitudes did not significantly vary from the beginning to the end of the year." In his conclusion, Gilbert conjectured that:

> "...contrary to popular belief, attitudes of students are irrelevant to achievement or ability, students don't reveal their attitudes, or teachers consider attitudes immaterial and don't respond to clues."

Gilbert's study leaves one important question unanswered and that is, what if one measure was given for the three variables...? Would there be a significant correlation with the attitude measure? Classroom teachers generally do not deal with general ability or I.Q., or pupils' attitudes as separate entities during instruction.

Hogman (33) investigated the degree of self-expressed student interest in a variety of particular mathematics topics and the changes in these interests across grade levels (grades 2 through 8) involving 13,000 students in 12 school systems from 10 different states. He found that:

> In the lower grades, the average percentage liking figures for all items combined is 58%, indicating a generally favorable attitude toward mathematics topic.

However, the percentage liking figures ranged from 40% to 80% across individual items. The topic favored most by lower grades includes:

1) measurement items such as money, how much coins are worth; telling

time; work with thermometers; measure things, find the longest thing; 2) basic numeration topic items such as count, count by 10's, and 3) story problem items such as do (story) problems. At the upper grades, the percentage liking figures for all items combined was under 40% when students responded on a three-point scale: like, indifferent, or dislike. Most favored items in the upper grades include computation items.

On students' attitudes across grade levels, Hogman concludes from his data that:

> In the lower grades, there is an increase in liking for doing addition problems and virtually no decrease in liking for doing subtraction problems. Many of the most favored items in numeration tend to remain fairly stable...leading the decline in interest are two types of items that were among the least favored on the whole: geometry and sets. Some, but not all, of the measurement items showed appreciable declines.
>
> In the upper grades, working with graphs and some computation items showed little loss or even slight gains in degree of student interest. Leading the decline in interest were items in geometry, word problems, and a variety of numeration items.

Hogman's results imply that even though student's overall attitude toward mathematics may decline, in certain topics they remain fairly stable or even increase. Hogman, from a practical view point, stated two implications of his studies:

> First, teachers should be made aware of the differential interest value of various mathematical topics.... And the specially liked and disliked topics are noteworthy; for example, computations are not the ogres they are often made out to be. Secondly, the differential patterns of change in liking for certain topics across grade levels suggest that program evaluations that incorporate assessment of attitudinal variables may be improved by analyzing changes in attitude toward specific mathematics topics.

What is probably missing is that how much a topic is liked could have been a result of previous instructional methods. Hogman did not mention pedagogy.

Campbell and Schoen (9) in their investigation tried to determine whether or not statistically significant relationships exist between students' perceptions of selected behaviors of pre-algebra teachers and selected student performance and attitude variables. The investigation consisted of 1602 pre-algebra students - 816 females and 786 males - attending 28 public schools and 73 pre-algebra teachers. They found that:

> "...students' grades in mathematics were significantly positively related to their attitudes toward mathematics. The strongest relation was found between the students' attitudes toward mathematics and their mathematics teachers. Students who made higher grades in mathematics than in their other courses tended to have positive attitudes toward mathematics and their mathematics teacher.... The correlation between students' attitudes toward their mathematics teacher and the teacher behavior "shows continuity of the mathematics curriculum" was the only correlation identified as being significant.

They observed that:

> The significant correlation between the Positive Teaching Orientation and the students' attitudes toward their mathematics teacher indicated that students who had positive attitudes toward their mathematics teachers perceived their teachers as those who 1) showed continuity in teaching the mathematics curriculum, 2) were willing to re-explain material to students who had misbehaved during the first explanation, 3) taught reading in mathematics class, and 4) retaught material when evaluation of student learning indicated the need.... Students who perceived their teachers as using individualized teaching techniques and mathematics related games in the classroom, reported liking mathematics and making better grades in mathematics than in other classes.

Rosenshine (49) in his studies on teacher effectiveness concludes that teacher effectiveness does vary over time.

Begle and Geesline (6) supported Rosenshine's conclusions in their analysis of NLSMA data. They discovered that:

> A substantial number of teachers who had been teaching fourth grade NLSMA students during the first year of the study were teaching fifth grade NLSMA students during the second year. We computed (using all the fifth grade and eighth grade second year students) the same kind of effectiveness scores for those teachers.

The correlations found varied from .35 for grade 4 and grade 5 to .15 for grade 7 and grade 8, and .01 for grade 4 and grade 5 to .28 for grade 7 and grade 8 for effectiveness on computations, and comprehensions respectively for high ability students.

Problem-Solving and a Unit

There are virtually no studies in the literature that have looked at a unit of instruction in a problem-solving mode structured like the Fitzgerald-Shroyer, Mouse and Elephant Unit. Several studies on problem-solving have tried to look at the processes involved in problem-solving with regards to say computation in arithmetic (Anthony and Hudgins (1)). Some studies have reported on how to use problem-solving to narrow gaps in achievement between two groups of pupils (Babard and Bashi (3)).

In the literature, problem-solving is used to mean everything from the solving of routine class exercises to the solution of applied problems. In the Fitzgerald, et al (60) proposal, problem-solving is used to mean applying mathematical ideas to find the solution to major challenges. Students go from what they know in a step by step way through mini challenges until they solve the major challenges.

The 1979 Michigan Educational Assessment Program (41) reports a 30% decline in the number of students from the fourth to the seventh grades who demonstrated minimum competency in the mathematics tested. Fitzgerald, et al, observed that current teaching practices in most mathematics classes are boring to the pupils, and involve too much paper-pencil computation. Doubt is expressed about the adequacy and appropriateness of present mathematics instruction in middle schools. The declining percentage of pupils performing at some expected standard, as reported in the Michigan Assessment Program, partly support their doubts.

Also, they state that:

> "To handle individual differences, many teachers move the better students on to the 'next' book, to be done on their own, and then to the 'next', etc. Missing is the excitement of tackling a significant and challenging problem with a group of peers."

They clearly support the current investigation which uses a unit written in the way the proposal suggests. In fact, the unit was written by two of the authors of the proposal. It is important to investigate how teachers, who were not trained in how to use the unit, will compare with those who have been trained on the mean achievement of their pupils, on their methods of instruction and on their attitude towards the unit.

Fitzgerald, et al (60), on individualized instruction, reminded us of Elwanger's "Benny," who was doing well on the program using incorrect rules and conclusions (17). Elwangér was able to determine Benny's reasons by interviewing him. However, on whole class instruction and textbooks for the middle grades, Fitzgerald, et al, states:

> "The practice of whole class teaching does not necessarily solve these social and mathematical problems (i.e. referring to the social problems of early adolescents). In many non-individualized classes, the students are moved along in a block through the same mathematics that they have seen year after year....
>
> The textbooks on which teachers must rely produce sterile, algorithmic drill oriented mathematics. At the middle level skills should be used to sudy applications, to generate strategies, to cement understandings of underlying concepts and to acquire new knowledge.... They (middle school teachers) need help, both in understanding the mathematics itself and in understanding how to teach the mathematics."

The current investigation will try to reveal how helpful the unit is to the teachers by looking at the pupils attitudes and achievements and teachers attitudes toward the unit.

Anthony and Hudgins (1) in their study on problem-solving processes of 5th grade arithmetic pupils, stated that no significant effect occurred for the poor problem solvers as a result of extensive training. From pilot study of this investigation, almost all the pupils were able to solve some of the items on the post achievement test. Previous study using the Mouse and Elephant Unit, showed that most of the sixth graders were able to do well on the achievement test. In the pilot study, computational errors continued to be a problem as in the Fitzgerald-Shroyer study. It is worth repeating the study with a new format to see if computational (arithmetic) errors can be reduced considerably.

In Israel, Barhard and Bashi (3) in their investigation on narrowing the performance gap in mathematical thinking between advantaged and disadvantaged children in the seventh grade concluded that:

> "Disadvantaged Israeli students scored as well as advantaged students on cryptarithmetic test following instruction on relevant problem solving strategies."

In effect, the Barhard and Bashi study seems to imply that given appropriate problem-solving environment with appropriate instructions, the so called disadvantage might be removed for some area of mathematics.

Carey (10) investigated sex differences in problem-solving performance as a function of attitude difference and reported that differences in problem-solving ability was attributable to attitudes rather than intelligence.

Keane (34) reported on the other hand, that economic environment affects student attitudes but not achievement.

Both Carey and Keane seem to agree that proper problem-solving environment has an effect on pupil's attitude. Since the Mouse and Elephant Unit is assumed to provide proper problem-solving environment, pupils and teachers are expected to have favorable attitudes toward the unit. This has to be investigated. Hence, the need for the current study.

Errors

In mathematics educational research much concern has been expressed about which questions have been responded to correctly as opposed to those questions answered incorrectly. Not much has been reported involving pupils' errors (43). Several efforts have been made to compare particular modes of instruction with others while holding certain variables constant. The results of such investigations as Begle (5) reports, have sometimes yielded contradictory results and are on the whole inconclusive.

Behne (7) found a significant growth in basic skills, problem-solving, deductive reasoning and algebra between successive grades, (grades 3, 5, 7, 9) but a decline in percentage meeting a criterion level in basic skills from grade to grade.

Korth (37), on the effects of formative evaluation methods on achievement in individualized seventh-grade mathematics found that:

> "On 16 of 20 comparisons, no significant difference was found between self-testing or regular testing."

Error analysis in teaching and learning of mathematics has focused mostly on computation. There have not been many studies that have specifically evaluated the effectiveness of a 'script' and analyzed errors for a specific 'chunk' of teachable mathematics using broader categories of difficulties:

a) language and space difficulties

b) prerequisite and irrelevant rule difficulties, and

c) others such as omission, random/careless errors,

and within each broad category to see the proportions of students who made systematic errors as defined by Cox in 1975 (14).

Grossnickle in 1935 (26) defined constant errors as:

> "Recurring incorrect responses to a specific number combination such as 7-5."

His report showed that in using the division algorithm the percentages of pupils who made constant errors were two and five in using subtraction and multiplication facts respectively to do the division.

Roberts in 1968 (46) reported four types of failure strategies: wrong operations, obvious computational errors, defective algorithm, and random response, for third grade pupils. His results indicated a steady decline of percentages of pupils who made errors (wrong computation) 25%, 19%, 15% and 11% for the quantities I, II, III and IV respectively, but for defective algorithm 29%, 37%, 43% and 39% were the percentages for the quantities I, II, III and IV, a fairly stable error pattern for the upper third of the class. And that (36%) defective algorithm was the category with the highest percentage of errors.

Cox reported, in his two year study of 744 children of grades 2 through 6 for 'normal' children and primary, intermediate and junior high pupils for the handicapped, each of the four arithmetic algorithms: addition, multiplication, subtraction and division, systematic error percentage for each grade level. And for the sixth grade the percentages were 0, 6, 1 and 3 for addition, multiplication, subtraction and division respectively. There was also a steady decline for each of the four operations in systematic erros for grades 3 through 6. And overall, systematic errors were made most in subtraction (13%).

Cox was only concerned with computation in grades 3 through 6. He stated an overall systematic error made in subtraction (13%) but he did not interview any of the pupils. An interview with the pupils would have probably identified what they (pupils) were doing particularly with regards to systematic errors. There is a need for a study that reports both what the investigator 'thinks' pupils are doing and what the pupils 'think' they are doing as the current investigation will attempt to do.

McCreddin in 1976 (40) points out the need for error analysis in language (mathematical terms) in the following statement:

> "One sadly neglected aim of mathematics education is to develop the ability to read and comprehend written mathematics. This neglect occurs at all levels: educational research, teacher education, curriculum planning and tragically, in the classrooms. The assumption that skills learnt under guise of 'English Studies' are sufficient is fallacious, and overall mathematics education is suffering badly because of it."

McCredin's (40) concern quoted above on a sadly neglected aim of mathematics education - reading and comprehending mathematics at all levels of mathematics education, suggests in part the need for assessment of mathematical language difficulties. Such an assessment as it is intended in this investigation is a good starting point to solve the dilemma McCredin refers to.

Ashlock 1972 (1) has written extensively on error analysis and has included items and the kinds of errors made in solving those items by pupils. He states:

> "Usual scoring techniques do not distinguish among procedures used to get correct answers, frequently they do not even distinguish between situations in which the child uses an incorrect procedure and situations in which the child does not know how to proceed.... It is better to spend your time as a professional in analyzing the written work of children and planning remedial instruction than it is to use what time you have for scoring."

In his analysis of written whole-number computation of sixth graders, Ellis in 1972 (16) concluded that "it was profitable to analyze errors as means of gathering data and planning individualized instruction."

On awareness of error analysis, Johnson (1979) points out that "we have become increasingly aware of the need for a careful delineation of the various factors which contribute to learning problems and the concommitant necessity for carefully matching both remedial theory and practice to the precise needs of the learning disabled pupil."

Lepore (61) recognizes the importance of error analysis in learning and believes that a teacher must be able to diagnose the child's problem to the best of his or her ability.

Pace (1959) found that the understanding of basic arithmetical operations is necessary before problem solving ability can be improved. In the current investigation, the unit takes cognizance of this by building in exercises on number facts. Errors cannot be properly analyzed merely by grading the child's papers. The techniques that the child employs may represent an extreme departure from proper mathematical reasoning. Interviewing and having the child solve problems 'thinking aloud' helps in error analysis.

Begle (1978) on reviewing NLSMA expressed concern over correlational studies in mathematics education. In part he states "a number of teacher opinions about mathematics, students themselves, etc., have been correlated with student mathematics learning. In most of these studies the correlation turned out to be rather low.... Descriptive statistics (from NLSMA) incidentally refute a commonly held belief that elementary school teachers dislike mathematics and are afraid of it...but at least for this large set of teachers the average attitude toward mathematics was at worst neutral."

Achievement and Instruction

In the overview and analysis of school mathematics grades K-12, the Conference Board of the Mathematical Sciences National Advisory Committee on Mathematics Education (1977) discussed the current school mathematics project. The report is not encouraging.

In it's report on mathematics assessment of 9-year-olds and 13-year-olds, done in 1972-73, MAEP (1975) found that on perimeter, area and volume:

> "Few 9-year-olds or 13-year-olds had a clear understanding of basic concepts of perimeter, area, and volume. Only 7 percent of the 9-year-olds could calculate correctly the length of fencing needed to go "all the way around (a) rectangular garden...9feet long and 5 feet wide." Forty-three percent of the 9-year-olds simply added the 9 and the 5; 8 percent multiplied them. Only 7 percent of the 13-year-olds could find the area of a square, given that the perimeter was 12 inches. One in three 13-year-olds apparently had a dim notion of a formula for perimeter or area but used an incorrect relationship, either multiplying the perimeter by four (20 percent) or squaring it (12 percent).
>
> Asked to compare rectangular regions cut into unit squares, only 44 percent of the 9-year-olds could select the region having the same area as a 4-by-4 square. The most common erroneous choice (38 percent) was a 3-by-5 region, which was similar in shape as well as equal in perimeter.
>
> Only 6 percent of the 9-year-olds and 21 percent of the 13-year-olds could compute correctly the volume of a pictured rectangular solid cut into unit cubes. A common erroneous choice (48 percent of the 9-year-olds, 27 percent of the 13-year-olds) was the number of cubes on the pictured faces, suggesting either a misunderstanding of the task or a tendency to rely on visual judgments instead of measurement processes. Some 13 percent of the 13-year-olds chose the number of unit squares on the six faces, suggesting a confusion of volume and surface area.

In his analysis of spatial-ability studies, Smith in 1964 (53) concluded that:

> Spatial ability is positively related to high-level mathematical conceptualization, that is people who can solve high-level mathematical problems generally have greater spatial ability than people who cannot solve these problems.

Thus, this assertion given by Smith offers some explanation of why sex differences favoring males are often found concurrently on tests of mathematical achievement and spatial ability.

Guay and McDaniel (29) investigated the relationship between school mathematics achievement and high and low-level spatial abilities among males and females in elementary school (grades 2 to 7). They defined low-level and high-level spatial abilities as "requiring the visualization of: two-dimensional configurations, (but no mental transformation of these images) and three-dimensional configurations (and the mental manipulation of these visual images)" respectively. Four experiment-developed spatial ability tests were administered to 90 children, 14 to 16 children selected from each of the six grades two through seven. An analysis of variance (mathematical achievement by sex by grade) was used. They found that:

> High mathematics achievers scored significantly higher ($p < .05$) that low mathematics achievers on all four spatial tests. Additionally, males scored significantly higher ($p < .05$) than females on the two tests measuring complex spatial ability.

In 1930 Whitcraft (1980) criticised high school mathematics teachers and the textbooks. Some of the criticisms are still valid today. Hence, the article was reprinted in the Mathematics Teacher January 1980 issue. He expressed concern that:

> There are a great many poor teachers in the secondary schools, teachers who do not know how to develop a topic or a lesson as it should be developed or who are too lazy to exert themselves to do a fair job of it.

His other concern, on textbooks, was on the weaknesses of the books. He cited the following as the main weaknesses of the books:

> The texts are especially weak in the following points: (a) the language is not that of the child but of the adult and the pupil does not understand what is meant; (b) the material is not well developed or rationalized; (c) the model examples do not always illustrate the various difficulties of the exercies which follow; and (d) there is no statement given or means suggested in the texts by which the pupil can know just how well he is doing or what he should do to bring about the needed improvement which he feels he should have. Teachers, therefore, must give a lot of

> work to make up for that which the textbook teaches poorly or not at all, and, in addition, make it possible for the pupils to know just how well they are doing at any time in the course. Teachers will look forward to the time when high school texts in the field of mathematics have remedied the defects which have been shown to exist in their makeup.

Since there are still poor teachers in both elementary and secondary schools and some poor textbooks that demand a lot of work from the teachers, these factors may contribute to a lack of satisfactory pupil achievement in the learning of mathematics. The new format of the Mouse and Elephant Unit is written in almost conversational format. It, thus, demands less from the teacher in terms of preparation or development of the lesson and explicitly states what pupils should do and should learn.

The calculation of area and perimeter is an important chunk of school mathematics. Area and perimeter are important because they are one of the most commonly used domains of measure in everyday life. Understanding the concept of area is vital for the child's success in mathematics because understandings about area are at the base of many models used by teachers and textbooks to explain numbers and number operations. For example, in multiplication of whole numbers, rectangles are drawn with the two adjacent sides representing the factors and the number of unit squares inside representing their product. As Carpenter, Coburn, Reys and Wilson (1975) found out from National Assessment, pupils have difficulty with the computation of area and perimeter and more fundamentally with understanding the concepts. They reported that:

> The results...show that fewer than 10% of 13-year-olds and about a fourth of the 17-year-olds and young adults could calculate the area of a square given its perimeter. Between 10 and 20 percent attempted to find the area by multiplying the perimeter by four and 12 to 25 percent simply squared the perimeter. The first error seems to be a rote application of the procedure for calculating the perimeter of a square, given the length of one side. The second seems to involve a rote application of the procedure

> for calculating the area of a square, given the length of one side...the incidence of the first type of error decreased with age while the second actually increased.... For each age group over 29 percent of the respondents mechanically applied a procedure designed to calculate area or perimeter. Many respondents...had no comprehension of when to apply them (perimeter and area formulas) or what dimension to apply them to.

They also observed that:

> In an exercise from Michigan Assessment (Zoet 1976) only 25% of the seventh graders tested could find the length of a side of a square given its perimeter.... The results of exercises from the State Assessment and from NAEP indicate that between one-third and one-half of all seventh graders cannot find the area of a rectangle given two adjacent sides.

In the Mouse and Elephant Unit, pupils suggest different rectangles and their areas when a fixed perimeter is given. They also suggest different rectangles and their perimeters when a fixed area is given. Most of these activities are taught in a concrete and pictorial mode. (See the appendix.) This is in line with what Carpenter, et al, suggests as interesting problem situations involving area and perimeter.

> For example, students might be asked to find the area of a number of different rectangles with a fixed perimeter and to generate some hypotheses about the relationship of the shape of the rectangle to the area.

In their investigation of 106 children in grades 3 through 6 in two schools in Columbus, Ohio and Austin, Texas, Heisten, Lamb and Osborne (63) studied 'how children incorporate number into their judgment of area'. They found that generally all the children had some concept about area and that children who appeared to understand area could shift between unit-counting/unit-covering approach and multiplication involving formulas for area. Such children exhibited partitioning and recombining approach behavior. The investigators claimed that their results were strong enough to warrant suggestion that:

> Young children need experiences designed to help them acquire that partitioning a figure and putting it back does not change its area.... Older students who are frequently discussed in terms "of learning the formulas too soon," who confuse such topics as perimeter and area, and who seldom gain control of measurement, have been introduced to area without attention being given to the more primitive idea of area being invariant on partitioning and recombining.

In the Mouse and Elephant Unit, pupils construct rectangles, find their area by filling them with tiles, and for a fixed area pupils learn that different shapes can have the same area. Putting tiles together to form bigger rectangles helps them learn that 'area is invariant under partitioning and recombining'.

Heisten, Lamb and Osborne also found five "common misconceptions" in their investigations. Pupils used: the length of one dimension to make area judgments, primitive compensation methods, point-counting for area, counting around the corner and point-counting linear units. It is hoped that the use of tiles in the unit will eliminate the point-counting for area and the use of one dimension to make judgments about area. The idea of banquet tables and number of people to sit around the table to teach perimeter will help to overcome the point-counting linear units misconceptions and the counting around the corner. Previous investigation showed that using the Mouse and Elephant Unit reduced the occurrence of these errors. A new format, for this activity used in the current investigation may help reduce these misconceptions even further.

Curriculum and Achievement

Measurement is an important aspect of school mathematics. Hence, in the NAEP survey, some items were set to test concepts of area, perimeter, volume and surface area. But as NAEP reports:

"They (9-year-olds and 13-year-olds) have a great deal of difficulty with many basic measurement concepts, especially those involving perimeter, area and volume.... 28% of the 9-year-olds could find the area of a rectangle that was divided into square units, 71 percent of the 13-year-olds could. Only 51 percent of the 13-year-olds, however, could calculate the area of the rectangle from dimensions of the sides.

...student's general knowledge of volume concepts was even poorer than their knowledge of area concepts...7 percent of the 9-year-olds and 24 percent of the 13-year-olds could find the volume of a rectangular solid cut into unit cubes. Forty-six percent of the 9-year-olds and 36 percent of the 13-year-olds simply counted the faces of the cubes shown in the picture or found the surface area of the solid."

From this survey it seems that even though 13-year-olds have been 'taught' these concepts, they might not have 'learnt' them well enough. An instructional unit that teachers can use, and that will pose challenges to the pupils may be useful, especially if its effectiveness is evaluated.

CHAPTER III

PROCEDURE

Pilot Study

A pilot study on the modified Mouse and Elephant Unit was conducted in November and December 1979 in a middle school in Okemos School District in Michigan comparable to the school in which the study was conducted. The modified unit was taught in three sixth grade classes. One of the classes was taught by an author of the unit, the other two classes were taught by a former participating teacher in the Mouse and Elephant Unit.

An investigator-constructed pre-test, approved by three experts, two professors from Mathematics Education and one from Measurement and Evaluation, was administered to seventy-four pupils in the pilot study. The test was checked for content validity only.

The results on the pre-test showed that: a) pupils could recognize and name a rectangle (over 98% of the pupils named the given figure correctly as rectangle), b) only 35.1% of the pupils could compute an area of a rectangle drawn with dimensions given (15.3% of the pupils simply gave one of the dimensions as the area), c) 25.5% of the pupils were able to compute perimeter correctly, and more than half of the pupils who got the item wrong simply "added the two dimensions" or "multiplied the two dimensions," d) 32.2% were able to compute volume of rectangular block sketched (5 x 4 x 3), 15.3% added the three dimensions, e) on surface area 4.2% of the pupils computed the surface area of the 5 x 4 x 3 block, but 48.6% of the pupils simply found "the area of the top of the block (5 x 4)," and 12.5% of them counted the squares on the faces shown of the block, f) 4.2% of the pupils were able to find "the number of sheets needed to cover a 20 x 20 x 20 cube if one sheet covered a 10 x 10 x 10 cube," and 75% of them gave 2 as the answer.

From the results of the pilot study, it appeared sixth graders were not generally familiar with the concept and computation of area and perimeter of rectangles and surface area and volume of rectangular blocks. It also appeared that pupils had a misconception of the meaning of the word surface area. They generally referred to the top of the block as the surface.

In the pilot study classroom observations were done for each class and a post-test was given at the end of two weeks (10 classes) of instruction. The results of the post-test indicated that there was a substantial gain on pupils' achievement on all the areas covered: eighty-nine percent and 60% of the pupils correctly computed area in concrete and abstract modes respectively on the post-test; 81% and 53% correctly computed perimeter in concrete and abstract modes respectively; 63% and 46% computed volume in semi-abstract (diagram) and abstract modes respectively; 42% and 43% computed surface area in semi-abstract (diagram) and abstract modes respectively. Also, the instructors indicated that the modified unit was easier to teach.

These results were encouraging enough to warrant a new investigation. Hence, the investigation in a comparable school was undertaken.

The Population

The investigation was conducted in a middle school, in the East Lansing, Michigan, school district.

The Fourth Report of the 1972-73 Michigan Educational Assessment Program by Michigan Department of Education (1973), compared the East Lansing school district with other school districts in the state. The school district was at the 97th percentile in human resources, with 60.1 professional instructional staff per 1,000 pupils; 92nd percentile with 49.1 teachers per 1,000 pupils; 70th percentile with 9.8 average years of teaching for the teachers; 98th percentile with 54.5 percent of teachers with Masters degree; 62nd percentile with $11,142 as average contracted salary of teachers. Under financial resources for the year 1971-72, East Lansing school district was at the 88th percentile with $26,688 state equalized valuation (SEV) per resident member (i.e. SEV is approximately 50 percent of the fair cash value of the real and personal property in the district); 98th percentile with $1,034 local revenue per pupil; 12th percentile with $242 as state school aid per pupil; 99th percentile with $894 as K-12 instructional expense per pupil; 94th percentile with $681 as Elementary instruction expense per pupil; 99th percentile with $1,232 as total current operating expense per pupil; 98th percentile with 34.50 total operating millage; 87th percentile with 9.0 percent of racial-thnic minority students; 22nd percentile with 3.0 school dropout rate. On basic skills achievement for seventh graders in January 1973, the school district had 55.1% of the pupils in the 92-95 percentile band on mathematics; 54.7% of the pupils in 96-98 percentile band on reading; 54.5% of them in the 95-96 percentile band on basic skills composite achievement in the state. And when pupils in the districts were placed in decile, East Lansing school district had 22 and 21 percents of their pupils in tenth decile for 4th and 7th graders respectively, 16 and 21 percents in the 9th decile, for the 4th and 7th grade, and 4% in the 1st decile for each grade level 4th and 7th.

From the descriptions above it seems that East Lansing school district is in an upper middle class community. The schools are rated high in achievement test scores conducted by the state, and the financial resources available to the schools are also rated high in the state. The middle school where this investigation was conducted could be described as average for the school district.

Four classes, two from seventh grade, one seventh and sixth grades combined, and one sixth grade were observed from January 1980 to March 1980. There were 161 pupils involved in the study, 84 boys and 78 girls. Only 148 pupils took any of the three tests: pre-test, post-test and retention tests and regularly attended school during the investigation. These pupils missed at most, two classes of instruction. The pupils ranged in age from twelve plus to fourteen plus with an average age of 13 years 5 months. They were predominantly white students with 13 blacks, 5 chicanos and 3 others.

The 1979-80 Test Report of the Michigan Assessment Program gives the distribution of the seventh graders in the school where the study was conducted, on their performances on mathematics and English as shown in the table below in quartiles compared with all seventh graders in the state.

Table 1

Distribution of seventh graders in this study on math and English on Michigan Assessment Test of 1979-80.

	Quartiles			
	First	Second	Third	Fourth
Mathematics	5.6	6.6	18.8	69.0
English	4.1	5.1	6.6	84.3

Eighty-three percent of all the seventh graders in the school could not "identify the shaded area of a figure with a fraction." Some of these seventh graders participated in the study. On the Assessment test, however, there were no items that required the pupils to compute perimeter of a rectangle or surface area of a rectangular block. The items on area were polygons drawn on a grid whose areas were to be estimated. The largest grid was a six by six grid. For volume, diagrams of rectangular blocks (with cubes drawn), the largest being three by four by five, were given; the students were asked to find the number of cubes needed to build each.

The Teachers

Four teachers agreed to teach the unit. They ranged in teaching experience after graduating with a bachelors degree from seven to ten years with mean and median of eight years. Only one of the teachers was a female. Two of the teachers had masters degrees.

Design of the Investigation

The investigation was mainly ethnographic in nature: observations were made, field notes and audio recordings were taken and made on the processes of instruction that went on in the classrooms.

Achievement tests were given: a pre-test to ascertain whether pupils were familiar with the concepts and skills included in the unit, post-test to determine whether they learned the concepts and skills taught, post evaluation to determine whether the pupils who scored high grades (above 76%) could handle harder problems, and retention test to determine how much pupils remembered about three weeks after the unit. The averages of the old teachers' pupils were compared with those of a new teacher's class on the pre-test, post-test and retention tests scores using t-statistics as shown in the diagram below.

	Old Teacher	New Teacher
Pre-test	$\bar{x}_1$	$\bar{x}_2$
Post-test	$\bar{x}_3$	$\bar{x}_4$
Retention Test	$\bar{x}_5$	$\bar{x}_6$

Hypotheses for the mean grades:

$$H_1: \ \mu_1 = \mu_2$$

$$H_2: \ \mu_3 = \mu_4$$

$$H_3: \ \mu_5 = \mu_6$$

Different questionnaires related to the investigation were given to teachers and pupils. The proportion of pupils giving certain answers were computed. Teachers' responses were reported individually, since there were four teachers, percent did not make sense.

Data Collection

Schedule and Pre-Test

The researcher visited each of the four teacher's mathematics class before the pre-test was given to the pupils. Due to insufficient materials (tiles and cubes), conflicts on time-tables of the teachers' and the investigators, and some teacher's preference, the teaching of the unit was staggered. The old teachers taught the unit first, before the new teachers did. The schedule followed was as follows:

Teachers	Pre-Test Administered	Instruction	Administered Post-Test/Post Evaluation	Retention Test	Investigator Received Teacher's Attitude Inventory
T1	Jan. 11	Jan. 14-Feb. 5 about 3 wks.	Feb. 6/Feb. 8	Feb. 28	April 4
T2	Jan. 7	Jan. 8-Jan. 27 about 3 wks.	Jan. 28/Feb. 1	Feb. 19	March 24
T3	Feb. 1	Feb. 4-Feb. 21 2 weeks	Feb. 22/Feb. 25	March 12	March 28
T4	Jan. 14	Jan. 15-Mar. 4 7 weeks	Mar. 5/none	None	March 31

During the pre-instruction visits, notes were not taken. Observation was made of the sitting arrangements and the math topic pupils were working on.

Each teacher administered an investigator-constructed pre-test to his/her class a day before the start of instruction of the unit. The investigator observed the test administration passively. Pupils who were absent were not allowed to take the test the next day because the teaching of the unit started then. The tests were not marked until later during the investigation in order to avoid prejudice on the part of the investigator on which pupil to ask what question on the test. Some pupils were selected at random (about five from each class) and were asked, individually or in small groups, to explain how they solved certain items on the test. Their answers and reasons were recorded. When they asked the investigator whether they were correct, only a yes or no response was given. The investigator did not elaborate because he did not want to influence the summary stages in the unit where pupils would be required to find patterns or formulas or rules. He often told the pupils:

> You will be able to solve all the problems on this test after the unit. I don't want to spoil any part of the unit for you.

Initially there was a problem of communication between the pupils and the investigator due to accent differences.

Instruction of the Unit

T1 and T2 both started teaching the unit one week before T3 and T4 started. Because of lack of availability of enough tiles and cubes, conflicts of the teachers' schedules with the investigator's and teachers preference, the staggering arrangement was necessary and convenient.

The investigator visited twenty-four classes for the entire period. He walked in on five classes and left before the classes were over. The total number of instruction periods was 38 for all the four teachers.

Teachers

The actual instruction time, number of math lessons, spent on the unit were 9, 9 and 10 for teachers T1, T2 and T3 respectively. The fourth teacher (T4) taught the unit over a five-week period. This was his first time of teaching the unit. However, the actual number of periods of instructions was 10. The first three activities were done in three consecutive class periods in the first week. Then in the second week activity 4 and part of activity 5 were taught in class periods on Tuesday and Friday. For the 3rd and 4th weeks one class period for each week was devoted to the unit.

In the fifth week, the investigator had informed him that all the post-test data for the other three classes were in. Then two days were set to finish the unit. The investogator could not visit the last two lessons because of lack of continuity in the presentation of the unit, and the teacher's deviations from the unit guide. The fourth teacher's class (T4) was analyzed separately. The teacher presented extra challenges to the entire class which generally led to frustration and a waste of class time. For instance, the pupils were asked to gather all their tiles to build a rectangle with perimeter of 100. T4 launched each activity similarily to the other teachers, but felt that there was "not enough time to do the summary." Pupils were carried away by such activities as cutting jackets and continued doing that until the class time was over (fifty to fifty-five minutes).

T1 and T2 had tuaght the unit before. T1's Friday math class was the last lesson of the day (2:00-2:55). Instruction on the Mouse and Elephant Unit was not done on Fridays (two Fridays) because T1 admitted "it is not a good time to teach math." Both T1 and T2 taught the unit very close to what was suggested in the outline.

However, in looking for patterns during summary, T1 always asked the pupils "to let the math do the work for you - if you are going to Detroit, you can walk, drive or fly." (Detroit is about 80 miles from the school). When the pupils were looking for patterns, he would refer to those who rely exclusively on counting as those "wanting to walk to Detroit, and not letting math do the job for them." There were no displays of the cut-outs in T1's class because he intended to teach the unit to his next class, on their request, later during the term. He felt a display would spoil the intention of the unit. The investigator agreed. T2 had displays in his class as suggested in the unit. With the exceptions noted, both T1 and T2, taught the unit in the same way.

T3 was the only female teacher. She was teaching the unit for the first time. She followed the unit guide closely. However, unlike T1 and T2, T3 during summary, always waited for absolute attention. It was amazing to see pupils park all their tiles to raise up their hands to signify "giving the teacher the floor." This usually lasted for three to four minutes. When most of the kids had their hands up, T2 proceeded to summary. The same hand-raising was practiced before each unit was launched. The displays of jacket cut-outs were placed on cards with a sketch of the jacket on a larger scale. These were easily visible from the rear of the class.

Pupils

Generally when the problem was presented: to determine the number of mice coats needed to cover an elephant and the number of mice needed to balance an elephant, the conversation went like this:

T - How many mice coats will cover an elephant?

P - You have to know how wide it is, how long it is.

At this point teachers generally refrained from saying that the width and length of the elephant were the same as the height by assumption.

Pupils generally laughed about the problem. However, teachers (all of them) did say that the same 'chunks' cut from a mouse and elephant would weight the same. The question was repeated.

P - Forty.

T - (Teacher according to the unit was looking for this response.) How did you get 40?

P - Well, divide 240 by 6, you get 40 (with confidence).

T - Will 40 mice or mice coats cover or balance an elephant?

There was generally an expression of doubt about the correctness of 40 mice on their faces (pupils). They all seemed to agree that the same number of mice needed to balance an elephant would provide enough coats to cover the elephant.

And at this point, the teachers asked the pupils to guess and write their guesses on the sheets provided. These guess sheets were put away until the final challenge was solved.

Pupils worked in groups from two to four in a group. All their group members were selected by themselves. T1 did change two group compositions because of too much talking. T2 did change the seats of only one class member during one of my observations.

In each class pupils moved to ask me, the teacher or other groups questions relating to an activity launched. Some pupils moved from group to group to exchange their tiles or cubes with those whose colors they preferred. This sometimes became a minor problem, which went (perhaps) unnoticed by the teacher. This problem could be solved in the future by giving each class the same color of tiles or cubes.

Observation

Each class period was about 55 minutes long. The investigator visited each class for six or seven full class periods during the teaching of the unit. Two other visits were made in each class at the beginning or during the lesson for about 15 to 20 minutes.

In class, notes were taken by the investigator on four different colors of 2 by 3 cards: pink cards for Teacher 1, yellow for Teacher 2, green for Teacher 3, and white for Teacher 4. The notes were on:
1) time the class and teacher spent on each stage of instruction using a hand watch. The time the launching started was noted, normally after pupils have taken their places. Launching was assumed ended if the teacher stopped talking and gave a go-ahead instruction for pupils to work on their own or in groups. For example, Teacher 1 read for the second time the challenge the pupils were to work on: "Use all twenty four tiles to find all banquet tables." Teacher 3 would invite questions for clarification on what they were supposed to do and would say "Go to work. You have only fifteen minutes to finish."

Exploration stage started immediately after launching was assumed over, and it ended when the teacher went to the front of the class and called for results or date. This was the beginning of the summary stage. The teacher recorded the data on the board or on an overhead projector. Summary ended when the teacher requested that a pupil collect the materials used in class. However, if the teacher stopped in the middle of the exploration stage to give instruction to the whole class on a point he felt pupils did not quite understand, that time was still considered part of exploration time instead of launching time. That did not happen very often.

2) Questions pupils and teachers asked, especially those that were different from those included in the unit and those that were related to the unit were noted. Also, any discussion and commands not directly related to the unit like 'raise you hand before you speak', 'go back to your place' and 'where are you from?' were noted.

3) Conclusions, formulas, rules or patterns that the pupils found, mostly during the summary stage, were noted as they verbalized them.

4) Teacher movements were noted especially where they stood or sat to give most of the instructions.

5) Comments on how comfortable pupils were on particular tasks were noted. Also, whether the teacher requested reassurances and the frequency of such assurances from the investigator while teaching were recorded. Notes also included recording errors made by the teacher, and any observable misconceptions pupils had.

Audiotape was used occasionally, mostly during the summary stages of the activities in the unit. The investigator listened to the recordings the following evening so he transcribed the in-class notes, to compare some of the discussions in class with the notes.

During the exploration stage, the investigator walked around the class to see what pupils were doing and to help them if they asked for help. Other activities like 'building bridges' with the tiles, houses and other shapes with the cubes were noted. Pupils were observed as they went through the class looking for tiles or cubes of their favorite colors.

An attitude inventory constructed during Fitzgerald-Shroyer study was modified (see appendix E). This ten minute inventory was given to the pupils just before the final post-test at the end of the teaching of the unit. Then a post-test constructed by Fitzgerald and Shroyer was administered by the class teacher the day following the completion of the unit. For teachers 2 and 1, the post-tests were conducted two days after the unit was completed due to electricity failure (black-out) in the school. The post-test was graded by the investigator and pupils who scored 78% or better were listed and given a final evaluation (see appendix F) test that was constructed during the previous investigation. Teacher 1 administered the Final Evaluation test in a reserved room in the school in the presence of the investigator. Two pupils at a time went in to sit in two separate corners in the room. The questions were first read to them and they were given enough time to write their responses in column I. Then models were presented and the pupils wrote their responses to the same question in column II. They were allowed on all occasions to write on the back of their test papers. There were models for questions 1 through 3 but no model was available for the dog and mouse problem. The models included:

1. a) a 2 x 2 square made of tiles
   b) a 6 x 6 square made of tiles
2. a) a 2 x 2 x 2 cube of cubes
   b) a 6 x 6 x 6 cube of cubes
3. a) a regular popcorn box
   b) a reduced (half dimension of each of the sides of the box in (a) above) popcorn box.

The post or Final Evaluation for teachers 2 and 3 were administered by the investigator in a special room. The whole post evaluation lasted for three days. There were no signs that pupils who were tested earlier conferred with those who were to be tested. No pupil in Teacher 4's class scored up to 78% on the post-test.

Three weeks after the post-test, a retention test (see appendix G) constructed by the investigator was given to all the pupils in Teachers 1, 2 and 3's classes. Only pupils who were present took the test. Teacher 4 was not able to give the retention test to his pupils due to lack of time because the teacher finished less than two weeks before school recess began. The third week when the pupils were to be given the test came during their holidays.

An attitude inventory (appendix H) of the previous study was modified and given to the four teachers three weeks to eight weeks after the completion of the unit.

Informal interviews were held with each during all stages of the data collection from time of pre-test to the time their attitude inventories were collected. The discussion centered on their feelings about the unit and how well pupils were doing.

Date Collected

The following data were collected in the order indicated below.

1. Pre-test scores from all the pupils who participated in the investigation.
2. Interview notes from a few pupils selected at random from each class. The questions asked were on how they attempted certain items on pre-test and why they did what they did.
3. Discussion (notes) with teachers before, during and at the end of the teaching of the unit.
4. Observation of the learning process during teaching, questions asked, pupils-teacher interaction, and pupil-pupil interactions.

5. Post-test scores and attitude inventory from all the pupils who completed the unit and were present on the day of the test. (Pupils who were absent for the three classes or more were considered as not completing the unit).

6. Interview notes from pupils; the questions were based on post-test items.

7. Post evaluation scores from only pupils who scored 16 or more out of 21 points (or better than 76%) on the post-test.

8. Retention test scores from all the pupils in three of the classes three weeks after the post-test. (No retention test scores from teacher number four who gave his post-test only a week before vacation started.)

9. Attitude inventory from all the four teachers. A questionnaire to each teacher and one-to-one discussion on the unit between each teacher and the investigator.

The Analysis

There were three kinds of data collected: field notes from the classes achievement test scores (and errors made from them) and attitudinal inventories from both teachers and pupils. There were, therefore, three categories of analysis done each in line with the data collected.

From the field notes the amount of time each class spent on each stage of the unit: launching, exploration and summary, was estimated for each class day observed. The time the launching started was after the routine activities of pupils taking their seats, getting their folders and teacher taking attendance. For the exploration time, the time spent in distributing materials to teach the unit was generally ignored, in the estimates. For the summary stage, the time spent in collecting folders and materials and cleaning up the desks was not included. The average number of minutes for stage, and for the whole unit were computed. Since some activities overlapped during the instruction, i.e. those activities whose summaries were not completed in one class day were continued the next class day, before the start of the following activities, the investigator found it easier to estimate the time by days rather than by activities.

Also, from the field notes, discernible patterns of teachers movement, like places from where he/she generally gave instructions, the kinds of questions pupils asked, and 'turning-on' 'turning-off' points of the units were described. 'Turning-on' points were places in the instructions pupils expressed delightful surprise. For example, pupils expressed surprise at the rate areas and volumes of squares or cubes were growing. Generally, they tried to cut the biggest square, or build the largest cubes. 'Turning-off' points were places the investigator felt that pupils thought it was either too easy, or less interesting. For example, after building some banquet tables, pupils did not generally like to build larger ones.

From the field notes, the ways pupils verbalized the rules, formulas or patterns during summary were arranged under the headings area, perimeter, volume and surface area and growing squares or cubes.

The Instruments

Each test used in the investigation was either constructed by the investigator, or constructed by Fitzgerald and Shroyer. The pre-test and retention test were constructed by the investigator. The post-test and post evaluation were constructed by Fitzgerald and Shroyer.

All of these tests were only tested for content validity. Because of the concern for content validity items that proved to be too easy or too difficult were not eliminated. Thus, the KR20 computed for each test was very low.

The attitudinal questionnaires for teachers and pupils were modifications by the investigator from previous studies on measurement by Fitzgerald and Shroyer.

From the achievement test scores, the percents of pupils correctly answering items on area, perimeter, volume, surface area, were computed. Pupils were distributed in various cells according to their responses on pre-test and post-test on say area items, i.e. pupils who correctly answered items on area were followed to see if they correctly answered area items in abstract or concrete modes on the post-test. Proportions of pupils who correctly answered certain items on pre-test, post-test and retention tests were compared to see whether there were any significant (meaningful significant) between the performances of the pupils on the tests.

Pupils were distributed according to their responses to area/perimeter items, volume/surface area to find out whether pupils who were successful with area items were successful with perimeter items or vice versa. The same was done for volume/surface area. In all the cells it was of concern to determine how pupils unsuccessful on any item fitted into the various cells.

From the test scores, group averages for old teachers and new teachers were computed for all the three tests. Teacher 4's averages (new teacher) were computed separately. Since all the other teachers taught the unit in about the same time (number of weeks spent from start to finish), and the number of pupils in the new teacher's class (not including teacher 4) and that in old teachers' classes were about equal, t-statistics was used to see if there were any significant differences between the averages.

Errors that were discernible from the written test according to the categories of errors were listed. The percent of certain pupils making certain kinds of errors was computed. Algorithm or procedural errors were of main concern to the investigator and, therefore, if a pupil made both computational error and algorithm error, he/she was recorded for algorithm error and not for computational errors. (It was reported that computational errors were made across all errors.)

For the attitudinal questionnaire, percent of pupils giving certain responses were computed. Pupils were also categorized in groups according to their responses to open ended questions on areas of the unit they liked best or disliked most.

The percents of pupils successful on various areas of the achievement tests and the post evaluation were compared with the same results from the Fitzgerald-Shroyer studies.

CHAPTER IV

RESULTS

Introduction

The results of the investigation, addressing the five questions, were in three parts: the ethnographic results from field notes, audio recording and interviews to try to assess "what went on in the classrooms"; the performance results from the achievement tests intended to find how much the pupils in different classes achieved after the unit was taught, and how the pupils were distributed among different achievement levels; the attitude results from questionnaires and observations designed to find out how the subjects felt about the investigation.

The old and new teachers were compared on the amount of time they spent on instruction proper and how they generally taught the unit. The teachers generally followed the three stages on instruction in the unit except for T4. The teachers spent over half of each class period on the exploration stage. Pupils' verbalized formulas, patterns and rules were reported as they said them.

On achievement tests, all the subjects as a group performed better on the post-test than they did on the pre-test as expected. There were significant differences between the means of the old and new teachers classes on the post- and retention tests but not on the pre-test. T4's class had the lowest mean on the post-test. Area and volume items posed less difficulty for the pupils than perimeter and surface area items. Relative surface area and relative volume items seemed very difficult on the post-test and seemed even more difficult on the retention test. Pupils who were generally successful on items on perimeter were successful with items on area; those successful on surface area items were successful on volume items. The converses of these statements were not necessarily true. The class means on the retention test were generally low. Pupils made more computational and algorithm errors on the retention test than they did on the post-test. Success rates on final evaluation were higher in this investigation than the Fitzgerald-Shroyer study.

Teachers and pupils reacted favorably toward the unit in all the classes. Pupils preferred the unit to regular mathematics in a ratio of better than two-to-one. For most pupils (over 50%) the best part of the unit was building tables or blocks and cutting jackets to cover the blocks. Three percent of the pupils did not like anything about the unit. The same three percent, however, did not like regular mathematics either.

Test Attendance:

All the pupils in the investigation did not take all of the achievement tests. Some pupils took two of the three tests. The table below shows the test attendance in percent and number of pupils for teachers T1, T2 and T3.

Table 2

Percent (number) of pupils who were present for the tests for T1, T2 and T3

Tests Taken	T1	T2	T3	Total
All 3 Tests	89.3 (25)	75.0 (18)	70.9 (39)	76.6 (82)
Pre- and Post-	96.4 (27)	75.0 (18)	83.6 (46)	85.0 (91)
Post- and Retention	89.3 (25)	91.7 (22)	80.0 (44)	85.0 (91)
Pre- and Retention	92.9 (26)	75.0 (18)	76.4 (42)	80.1 (4)
Pre-	100.0 (28)	79.2 (19)	90.2 (50)	90.7 (97)
Post-	96.4 (27)	91.7 (22)	92.5 (51)	93.5 (100)
Retention	92.9 (26)	95.8 (23)	85.8 (47)	89.7 (96)
Post Evaluation	28.6 (8)	25.0 (5)	60.9 (33)	43.9 (47)

From the table above, 89.3% of pupils or 25 pupils in T1's class took all the three tests: pre-, post- and retention, 75.0% of the pupils of 18 pupils of T2's class took the tests and 70.9% or 39 pupils of T3's class took the three tests. 76.6% or 82 pupils of the three classes took all the tests. For the post evaluation all the pupils who were to take the test took it. Therefore, 28.6% or 8 pupils, 25.0% or 5 pupils and 60.0% or 33 pupils qualified for the test in T1's, T2's and T3's classes respectively. They scored 76% or better on the post-test to take the post evaluation test.

The three test, pre-, post- and retention, were valid contentwise. The need for content validity meant that certain items were retained because of what they were measuring. They were not deleted even though they were easy like item 1 or difficult like item 6 on the post-test. On the retention test, item 3 was presumed difficult but was intended to measure the pupils ability to solve problems of relative surface areas on relative volume. KR20 was computed for each test. As was expected, each KR20 was low since K20 is a function of item difficulty and test length. For pre-test, post-test and retention test, the KR20's were .29, .36, .31 respectively.

Ethnographic Results

The results from the ethnographic data were in three parts: 1. instructional time, 2. teachers class maneuvers and 3. the pupils conclusions. These results were from the field notes and audio recordings. The data were collected to try to understand what instructional processes went on in the classrooms and how much time was spent on the instruction proper. The data was also intended to try to see how much agreements were among the teachers using the same modified unit and tools.

1. Instruction Time: Generally teachers spent about half of the instruction time on the exploration stage. Between 5 to 15 minutes of the 55 minute instruction time were spent on disciplinary or procedural matters during each class period. Teachers who could teach only when the entire class was 'ready' spent more time on disciplinary matters than those who could teach in noisy classrooms.

The table below shows the distribution of time in minutes, for each teacher, for each visit and at each stage of instruction.

Table 3

Estimates of time (in minutes) each class spent at each stage of instruction of the unit for the visits made by the investigator.

	Launching				Exploration				Summary				Total			
Visits	T1	T2	T3	T4	T1	T2	T3	T4	T1	T2	T3	T4	T1	T2	T3	T4
1	6	5	8	5	22	21	21	33	11	9	10	3	39	35	39	41
2	10	8	10	3	23	25	18	28	6	5	15	0	39	38	43	31
3	8	10	8	8	21	23	24	31	10	10	11	0	39	43	43	39
4	5	9	12	11	30	18	19	23	6	5	8	10	41	32	39	44
5	10	11	12	7	25	21	22	24	10	13	12	8	45	45	46	39
6	4	8	8	5	27	19	18	39	15	8	12	0	46	35	38	44
7			8	3			25	28			11	14			44	45
Total:	43	51	66	42	148	127	147	206	58	50	79	35	249	228	292	283
Average Per Visit	7.2	8.5	9.4	6.0	24.7	21.2	21.0	29.4	9.7	8.3	11.3	5.0	41.5	38.0	41.7	40.4
Class Period													330	330	385	385

Table 4

The average time spent on launching and summary compared with exploration time for the teachers from table (3) above.

	A	B	
Average	Launching + Summary	Exploration	Ratio Time (B:A)
T1	16.8	24.7	1.5:1
T2	16.8	21.2	1.3:1
T3	20.7	21.0	1.0:1
T4	11.0	29.4	2.7:1

T1, T2 and T3 consistently spent the smallest average time on launching, then on summary and most time on exploration. T4 spent the least time on summary, then launching and then exploration.

2. Teachers Maneuvers: A routine was established after about two classes of instruction of the unit. All the teachers said that they had at one time or another taught their classes in small groups. In all of the classes, when pupils walked into the classes they joined their group members and arranged their seats so that they could sit around them. Then the teacher taught the unit. A typical time scale on what the pupils and teachers did in each class went as follows. (The times were estimates and there were several overlappings.)

Time	Activities
First 5-15 minutes	Teachers in front of the class by the board or overhead projector. Teacher demanded order, quietness as pupils took their seats. (T3 took attendance, and pupils indicated readiness for class by putting up a hand.) (T1 stood quiet and let the pupils talk until they realized he wanted attention.) T2 and T4 asked pupils to keep quiet, and then started to launch the unit. T1 and T3 had to have quiet class before they started the launching. T2 and T4 started teaching the unit and then pupils kept quiet to listen to what was going on.

Next 15 minutes	Teacher started formal instruction - the launching of the unit. Teacher solicited or gave answers to questions about the concept in the activity. T3 and T4 generally read the questions as they were written in the unit and were a bit quick to supply the answers to some of the difficult questions. T1 and T2 had longer waiting times before they gave the answers. Teachers moved from group to group to see whether pupils were following the first instructions - cutting out the appropriate jackets or building the correct blocks. Teacher collected samples and showed the class, and gave the pupils instructions to make or build similar blocks or jackets and to record the dimensions.
Next 25 minutes	Pupils 'took over' the class. They built objects, discussed among themselves what they were building. (Conversations among pupils also included other things not related to mathematics - like 'the getting of bridge in a pupils mouth' or finding out where the investigator was from.) Teacher walked around the tables to help groups. T1 and T2 and T3 took toys away and T4 requested pupils to put their toys away.
Next 5 minutes	Teacher took over class, stood in front of class and requested order and quietness. Teacher asked for results, or asked pupils to bring their cut-out jackets, or to build some of the objects on the projector. Generally only correct samples of objects made were shown during the summary. Wrong data that pupils gave were said to be wrong. Conclusions pupils made were written. Those conclusions that were not directly related to the unit were given O.K. - remarks but not written on the board or the projector. Pupils were asked to find or show whether their patterns or rules worked by giving examples or demonstrations. Teacher generally furnished pupils with traditional method of writing some of the formulas. A pupil collected the materials together. Then they 'cleaned up' and the class ended.

3. Pupils Conclusions: The pedagogy in the unit required pupils to draw conclusions from their exploration and from the tables they constructed with the help of their teachers during the summary stages. Discussions and generalizations of the rules, formulas or patterns generated by pupils went on in all the classes. The formulas, rules and patterns that pupils found are listed below under the headings: area, perimeter, volume and surface area. Generally all these patterns or rules were found in all the classes. The differences among the classes were whether the formulas came early during the summary or later, whether the teacher pushed too much for the formulas or not and whether the teacher wrote all or some of the formulas on the board or on an overhead projector.

Area:

Pupils would say:

1. "Count in two's or three's to find the number of tiles (area)" covering a rectangle with one of the sides having two or three tiles.

2. With rectangles with even sides; pupils would count in two's, three's or multiples of one of the sides until they went half way and then doubled their results to find the number of tiles (area) covering the rectangle.

3. Multiply the side edge by the bottom edge = area.

For fixed perimeter, pupils found a pattern between the bottom edge and the side edge of the remaining rectangles once one of them was found. They would say:

4. Bottom edge goes up one, side edge goes down one, but you do not go to zero.

For example, when the exploration of making all rectangles with perimeter 24 was completed and if a pupil started with a rectangle with bottom edge 8 and side edge 4, he/she then proceeded like below.

Bottom Edge	Side Edge	Area
8	4	32
7	5	35
6	6	36
5	7	35
4	8	32
3	9	27
2	10	20
1	11	11
0	12	0 (12)

(a) When they wrote zero, teachers generally asked them to build the rectangles they listed. They then said that zero was not an area or area was not supposed to be zero, so they crossed it out. Teachers were satisfied with that reasoning and made no further comments.

5. Also from the area column they said that when the bottom edge and side edge were the same area was biggest, when bottom edge was one area was smallest for the perimeter 24.

Perimeter:

1. They counted all the way around the rectangle after marking their starting point of counting.

2. (bottom edge + bottom edge) + (side edge + side edge) = perimeter.

3. (bottom edge x 2) + (side edge x 2) = perimeter.

4. (side edge x 4) = perimeter for squares.

5. (side edge + bottom edge) x 2 = perimeter.

Generally new teachers generated discussion using result 3. 'show' pupils that 5. was also true. Old teachers were content with the first three and did not push for 5. At the end of summary they stated 5. and referred to their tables to illustrate that it worked.

Volume:

1. Number of cubes in the bottom flat or layer multiply by the number of layers or flats up = volume.

2. Area of bottom multiplied by the height.

3. Bottom front edge x bottom side edge x height = volume.

Three (3.) was generally pushed by the teachers as they asked how to find the bottom area of a rectangular block. Initially most pupils would say count the cubes on the bottom, before they came up with bottom side edge times bottom front edge with teachers help.

For fixed volume, pupils verbalized that when the bottom front edge, bottom side edge and the height were equal, you got the most economical package; i.e. more cubes and less surface area when all the three sides were equal.

Surface Area:

1. (bottom area + top area) + (front area + back area) + (left side area + right side area) = surface area.

2. (bottom area x 2) + (front area x 2) + (side area x 2) = surface area.

3. (bottom area + front area + side area) x 2.

As above, teachers pushed for this by illustrations using some blocks and tables.

On growing squares and cubes, there were patterns pupils found that received only an O.K. by the teacher with no further comments because the patterns did not fit into the unit very well. The conclusions on growing squares included:

1. For 1-square, 2-squares, 3-squares, the perimeter went up by four each time.

2. Side x 4 = perimeter.

3. On area, pupils said add 3, 5, 7, 11, 13 down to find the next area. This conclusion was acknowledged by the teacher as very good, but nothing else was done with it.

4. On area: multiply side by side.

Pupils were capable of generating several working formulas generally different in form from the traditional method of writing them except for area. Teachers generally always included the traditional forms of writing the formulas when pupils did not come up with them. It seems that the teachers always tried to take the pupils where they (teachers) wanted, to give them the formulas they were likely to find later in standard textbooks, so to speak.

It was observed that in post-test on area and perimeter, pupils generally went with their own formulas. It was difficult to tell the patterns they used for volume, but for surface area, over half of the pupils who answered the question right used formula 2 under surface area or a scheme derived by teacher 3 in her class. The scheme was:

BF BS HT

$$2[(BF \times BS) + (BF \times HT) + (BS \times HT)] = \text{Surface Area}$$

Actually this seemed to be a conventional format used in a diagramatic way.

The Achievement Tests Results:

Two of the questions of the investigation were: to determine whether the pupils would learn substantially from the modified unit, and how the old and new teacher's pupils compared on the achievement tests. Pre-test scores were analyzed item by item to determine the proportion of pupils successful on the various concepts that were to be taught. The proportions of pupils who got certain items correct on the pre-test but got others wrong were computed. Specifically the proportion of pupils successful on: area and perimeter, area but not perimeter, perimeter but not area, and on neither concept were computed. Similar tables for volume and surface area, relative surface area and relative volume were reported. On the post-test percents of pupils successful on the rectangle, area perimeter, volume and surface area items were reported. Percents of pupils who scored between 0-25%, 26-50%, 51-75% and 76-100% on the post-test were reported by classes to determine class proportions of students scoring low or high on the tests. The arithmetic means for the old and new teachers' classes were computed and compared statistically (t-statistics) for each of the three tests: pre-test, post-test, and retention test. Percent of pupils who got the volume-area, perimeter-surface area items right or wrong were reported. Post- and retention test scores were compared on the concepts: area, perimeter, volume and surface area. Percents of pupils who were successful on certain items on the Final Evaluation in this and Fitzgerald-Shroyer investigations were reported. Percents of pupils making certain kinds of errors were reported also.

Pre-Test Results and Conclusions:

The pre-test consisting of nine items was given to each class a day prior to the beginning of instruction of the unit. It covered recognition and naming of a rectangle, computation of area, perimeter, surface area, volume and relations between surface areas and volumes of rectangular blocks (see appendix 1).

Based on their performances on the pre-test pupils were categorized under three headings: a) Area and Perimeter, b) Surface Area and Volume and c) Relative Surface Area and Relative Volume. Most of the pupils (91.8%) who took the test recognized and named correctly a given figure as rectangle. For each heading (a, b, and c) and for each teacher, pupils were placed into four different cells depending on whether they could answer the items on both concepts (area and perimeter, surface area and volume, or relative surface area and relative volume) or only one concept but not the other, or missed both concepts in the category.

Each table below has a vertical and horizontal label for concept tested for each of the three main headings. The percent in Cell I is the proportion of those students who were identified as "knowing the two concepts" for a particular teacher (T1, T2, T3, T4). The percent in Cell II is for the proportion of students who answered correctly the item from the horizontal label, but missed the one from the vertical heading, Cell III is the proportion of pupils who got correctly the item from the vertical concept but missed the item from the horizontal dimension for each teacher. Cell IV members are those who got neither items correct. The table below gives an illustration (Table 5).

Table 5

Illustration of what the cells mean in the subsequent tables.

		Concept A	
		Right	Wrong
Concept B	Right	I	II
	Wrong	III	IV

For example, for the area and perimeter table - table 6 , 0% (in Cell I) of the pupils in teacher number one's class (T1) were able to answer the items on area and perimeter correctly, while 3.6% (in Cell II) could do perimeter item but not area and 3.6% (in Cell III) could do the area problem correctly but not the perimeter item. In Cell IV 92.8% of the pupils could do neither items correctly. (There was less than 3% of the pupils making computational or arithmetical errors on any one item on the pre-test, because the numbers did not pose addition or multiplication, division or subtraction difficulties. The numbers used were: (see appendix A) 2, 3, 4, 6, 7, 8 for computing area or perimeter, and 3, 4, 5 for computing volume and surface area, and 10, 20 for relative surface area and relative volume items.)

In examining the Cells for each pupil, it was found that several pupils had simply "multiplied or added the two dimensions" to find both area and perimeter. Consequently, when they multiplied and entered results for both area and perimeter, they were placed in Cell III for they got area correct, but if they chose to add they went to Cell IV as they got neither right. 27.4% of the pupils entered the same answers for both area and perimeter.

From table 6 it appears that at least 79.2% did not know how to compute area, 80.6% could not compute perimeter and only 8.8% of the pupils knew both concepts, area and perimeter well. For teacher number one (T1) 96.4% of all his seventh graders could not solve area problems and the same percent could not do perimeter problems. For T2, 75% could not do area or perimeter problems - they were both sixth and seventh graders. T1 and T2 seventh grade pupils were described as 'low' in math ability by their teachers. For T3 (mostly 'bright' sixth graders and some seventh graders), only 65.2% could not do area problems and 73.5% could not do perimeter problems. For T4 (sixth graders) 88% and 82% could not do the area and perimeter items correctly. It, therefore, appears that most of the pupils had not met or did not know the concepts of and how to compute area and perimeter.

From table 7 on surface area and volume, there were no pupils who could do surface area but not volume. Only T3 had pupils (8.2% of her pupils) who could do both items in surface area and volume. The most common algorithm error occurred when pupils multiplied two of the dimensions given for surface area (63.4%) and half of these multiplied the two top dimensions to get surface area. In a one-on-one interview, 3 out of the 8 I talked to confidently referred to the top as the surface. (All 8 got both the area and perimeter items correct.) The other 5 made 'gestures of not knowing' and could not elaborate.

Here again if the pupils "simply multiplied the three numbers" they got volume correct and were thus placed in Cell III. From table 7 most of the pupils could not do items on volume and surface area respectively. It, therefore, appears that most of the subjects in this investigation did not know the concept of volume or how to compute volume.

From table 8 most of the pupils (97.2%) were in Cell IV. Thus, we can conclude that they could not correctly answer relative surface area or relative volume problems.

From the pre-test, it appears that the pupils in the investigation either have not studied or have not learned the concepts included in the Mouse and Elephant Unit. Further justification for this claim is the fact that there were no area items that required computation, or perimeter, volume, surface area, relative surface area, or volume items in the 1978-79 Michigan State Assessment Test at the seventh grade level.

Table 6

Percentage of pupils according to their responses on area/perimeter items on pre-test (Items 3 and 5).

		AREA			
		I	II	III	IV
PERIMETER	T1	0	3.6	3.6	92.8
	T2	5.0	20.0	20.0	55.0
	T3	20.4	6.1	14.3	59.2
	T4	4.0	14.0	8.0	74.0
T1 + T2 + T3		11.3	8.2	12.4	68.1
Total (All)		8.8	10.2	10.9	69.7

Table 7

Percentage of pupils according to their responses on volume/surface area items on pre-test (Items 6 and 7).

		VOLUME			
		I	II	III	IV
SURFACE AREA	T1	0	0	10.7	89.3
	T2	0	0	47.4	52.6
	T3	8.2	0	32.7	59.2
	T4	0	0	16.0	84.6
T1 + T2 + T3		4.1	0	28.9	67.0
Total (All)		2.7	0	24.5	72.8

Table 8

Percentage of pupils according to their responses on relative surface area/relative volume items on pre-test (Items 8 and 9).

		RELATIVE SURFACE AREA (Growing Area)			
		I	II	III	IV
RELATIVE VOLUME (Growing Volume)	T1	0	0	3.6	96.4
	T2	0	0	0	100.0
	T3	0	2.0	6.1	93.9
	T4	0	0	0	100.0
T1 + T2 + T3		0	1.0	4.1	94.9
Total (All)		0	0.1	2.7	97.2

Table 9 gives a summary of the pupils performances by item on the pre-test. Item one clearly posed little or no difficulties. However, it also appears, from this summary, that pupils were not familiar with the concepts included in the unit except recognition and naming of rectangle.

Table 9

Percentage of pupils correctly answering each item on pre-test.

Percentage of Pupils Getting Each Item Correct on Pre-Test for Each Teacher

Items	T1	T2	T3	T4	Total
1	75	100.0	98.0	92.0	91.8
2	25	50.0	65.3	22.0	40.8
3	3.6	25.0	34.7	12.0	19.7
4	64.3	85.0	57.1	60.0	63.3
5	3.6	20.0	26.5	16.0	17.7
6	10.7	45.0	40.8	16.0	27.2
7	0.0	0.0	8.2	0.0	2.7
8	3.6	0.0	6.1	0.0	2.7
9	0.0	35.0	0.0	0.0	4.8

Post-Test Results

Overall Performance on Post-Test:

As table 10 below shows, over 86% of the pupils who took the post-test in each class were successful on the items on area and perimeter in the concrete mode and between 16.7% and 82.4% were successful in the abstract mode. The percent of pupils who successfully answered volume and surface area items with diagrams were 14.6 - 82.4 percent and

2.1 to 76.5 percent respectively. On the rectangle recognition, the percent of pupils who said that the 4 by 5 figure drawn was a rectangle ranged from 58.3 to 74.5 percent.

Table 10

Percent of pupils by class on the post-test items in concrete/abstract modes.

	Rectangle	Area	Perimeter	Volume	Surface Area
		Concrete/Abstract	Concrete/Abstract		
T1	59.3	88.9/59.3	81.5/51.9	51.9	37.0
T2	59.1	100.0/50.0	86.4/27.3	54.4	63.6
T3	74.5	94.1/82.4	96.1/80.4	82.4	76.5
T4	58.3	100.0/16.7	91.7/20.8	14.6	2.1
Total T1-T3	67.0	94.0/69.0	90.0/67.0	68.0	65.0
All Total	64.2	95.9/52.0	90.5/48.0	47.3	43.2

Table 11

Percent of pupils scoring 0-25%, 26-50%, 51-75% and 76-100% on the post-test.

Class	0-25%	26-50%	51-75%	76-100%
T1	11.1	22.2	48.1	18.5
T2	0.0	10.5	47.4	42.1
T3	0.0	10.9	21.7	67.4
T4	16.7	64.6	18.7	0.0
Total (T1, T2, T3)	3.3	14.1	34.8	47.8
Total (All Classes)	7.9	31.4	29.3	31.4

Over 82% of the pupils scored above 50% on the post-test for the three teachers who taught the unit within two weeks. About 19% of the pupils in teacher 4's class scored between 50% and 75% and no pupil scored more than 75%. Over 60% of all the pupils who took the post-test scored more than 50% and about 8% got less than 25% on the test. No pupil got 0% on the post-test (See table 11).

For T1, T2 and T3, over 66 percent of each class scored more than 50% on the post-test, and for T4 about 19% scored between 50 and 75%.

T4 taught the unit in similar manner to T1, T2 and T3 except that the unit was done over a five-week period instead of within three weeks. T1 did not teach the unit on Fridays.

It seems that pupils who had the unit in a constant flow did better than those who did not.

For both T2 and T3, there were no breaks once the unit was started. In those classes no pupil scored below 25% on the post-test, and over 88% of the pupils scored better than 50%. For T1, 11.1% scored less than 25%, and about 66% scored better than 50%. For T4, about 17% of the pupils scored less than 25%.

Since the main difference was the time the unit was taught, it seems that the fewer the number of breaks the better the pupils will achieve.

On achievement of pupils for the new and old teachers, differences were generally very small between average performances of pupils on the pre-, post- and retention tests. The table below shows the averages for each group of pupils in new and old teachers classes. Teacher 4 was a new teacher, but because of his instructional time length, his class averages were worked out separately.

The group averages were 32.2 and 36.0 percent on pre-test for old and new teachers respectively; 74.2 and 81.0 percent for the old and new teachers respectively on the post-test; 31.7 and 41.7 percent for the new and old teachers respectively on the retention test. For teacher 4, on pre-test and post-test, the class averages were 31.9 and 39.9 percent. The pre-test averages for all the three categories (old, new teachers and teacher 4) above were about the same. The post-test average for teacher 4 were about the same. The post-test average for teacher 4 was significantly lower than any of the averages for the other two groups: 39.9% compared with 81.0 percent or 74.2% (See table 12).

Table 12

The means of the old and new teachers on the three tests and the mean for T4. (The means are also given in percent.)

Tests	Groups	Number of Pupils Tested	Mean Score	Mean Score Percent	Max Score Possible
Pre-Test	Old Teachers (T1, T2)	47	2.90	32.2	9
	New Teacher T3	48	3.24	36.0	9
Post-Test	Old Teachers	49	15.6	74.2	21
	New Teacher	51	17.0	81.0	21
Retention Test	Old Teachers	49	1.9	31.7	6
	New Teacher	47	2.5	41.7	6
Pre-Test	T4 (New Teacher)	49	2.8	31.1	9
Post-Test		48	8.4	39.9	21

T4 did not give retention test to his class. On the pre-test all of the pupils performed about the same level. They scored between 31.1% to 36.0%. On the post-test T4's class had a mean percent of 39.9, the old teachers class scored 74.2 percent, mean, and the new teacher has 81.0% mean. The mean scores for the old and new were about equal. The difference in the means of retention test was substantial (31.7% and 41.7%) for old and new teachers classes respectively.

The design of the experiment fitted quasi experiment design 10 recommended by Campbell and Stanley in Experimental and Quasi-Experimental Designs for Research.

The old teachers were considered the 'control group' and the new teachers the 'experimental group'. The experimental treatment included all the activities each teacher employed to effectively teach the unit within three weeks. The only difference between the two groups was the newness of the unit to one group but familiar to the other. Thus, teaching styles were not controlled because the investigator felt it unrealistic to control these. Pupils were not assigned to classes at random. They remained in their regular classes and the unit was considered as part of the regular curriculum to be taught.

The tables below give the comparison of the averages of the two groups on the tests. T4's results were not included because of lack of continuity in his instruction. That the deviation was unacceptable to the investigator (See tables 13, 14, 15).

Table 13

Two-tailed t-statistic tests on pre-test.

Hypothesis: H_0: $\mu_1 - \mu_2 = 0$, H_1: $\mu_1 \neq \mu_2$

Groups	n	$\overline{X}$	S^2
Old Teachers	47	2.90	2.325939
New Teachers	48	3.24	3.1224

$t_{cal} = -1.011602$

$t_{93}(1 - \frac{\alpha}{2}) = 1.980$, $\alpha = .05$

The null hypothesis H_0 was not rejected.

Table 14

Two-tailed t-statistic test on post-test.

Hypothesis: H_0: $\mu_3 - \mu_4 = 0$, H_1: $\mu_3 \neq \mu_4$

Groups	n	$\overline{X}$	S^2
Old Teachers	49	15.6	23.49104537
New Teachers	51	17.0	15.7254902

$t_{cal} = -2.193727$

$t_{98}(1 - \frac{\alpha}{2}) = 1.980$, $\alpha = .05$

The null hypothesis H_0, was rejected.

Table 15

Two-tailed t-statistic test on retention test.

Hypothesis H_0: $\mu_5 - \mu_6 = 0$, H_1: $\mu_5 + \mu_6$

Groups	n	$\overline{X}$	S^2
Old Teachers	49	1.9	1.346938775
New Teachers	47	2.5	2.248109641

$t_{cal} = -2.50367066$

$t_{94}(1 - \frac{\alpha}{2}) = 1.980$, $\alpha = .05$

The null hypothesis H_0 was rejected.

The hypothesis of the investigation for each pair the means on the two tests was:

> There would be significant difference between the mean scores of the pupils in the old teachers' classes and those in the new teachers class on the achievement tests: post-test and retention test. (The difference would positively favor the old teachers.)

Thus, the null hypothesis for each of the tests was:

H_0: There would be no significant difference between the mean scores of the pupils in the old teachers class and those in the new teachers class on the achievement tests: post- and retention tests.

H_1: There would be significant differences between the two groups on the means of each of the two achievement tests.

The null hypothesis was rejected for each of the two tests, post and retention, at 5% significant level. The two groups were not significantly different on their means on the pre-test at the 5% level. Also, for practical purposes, the mean differences on the pre-test and post-test were not substantial, but the difference was substantial on the retention test.

The statistical significant differences could probably be explained by: the new teacher's style of teaching. The new teacher gave schemes to the pupils 'to help them remember' the formulas. The teacher expressedly criticized the unit for not including ways to have pupils remember the formulas.

Pre-Test and Post-Test Results

As stated above according to pupils performances on post-test, area appeared to be easier than perimeter, and volume seemed easier than surface area. In order to investigate how pupils who answered items on area correctly did on volume items, each pupil was placed in a cell according to his responses on the items. A similar table was made for volume and perimeter. The item used was item 4, which was an abstract item for area and perimeter and item 5 which required pupils to find volume and surface area.

It is apparent in tables 16 through 19a that more than 75% of pupils tended to either get both area and volume correct or missed both of them on both pre- and post-tests. Also, more than 75% had similar patterns of responses with regards to perimeter and surface area on the two tests. This analysis (for post-test) was done based on only teachers 1, 2 and 3 since T4 did not teach the unit within the two and a half week limit.

Table 16

Percentage of pupils getting area and volume items correct on the post-test for three teachers (T1, T2, T3).

		VOLUME	
		Right	Wrong
AREA	RIGHT	58	12
	WRONG	9	21

Table 17

Percentage of pupils getting perimeter and surface area items correct on the post-test for three teachers (T1, T2, T3).

		SURFACE AREA	
		Right	Wrong
PERIMETER	RIGHT	47	12
	WRONG	13	28

Table 18

Percentage of pupils getting area and volume items correct on the pre-test for three teachers (T1, T2, T3).

		VOLUME Right	VOLUME Wrong
AREA	RIGHT	17.2 13.5*	6.1 6.1*
	WRONG	15.2 13.5*	61.6 66.9*

Table 19a

Percentage of pupils getting perimeter and surface area items on pre-test for three teachers (T1, T2, T3).

		SURFACE AREA Right	SURFACE AREA Wrong
PERIMETER	RIGHT	1.0 0.7*	16.2 16.6*
	WRONG	4.0 2.8*	78.8 80.0*

*Percentage for all the four teachers.

The results of these tables forces one to question the standard approach of teaching area-perimeter then volume-surface area. Since area and volume seem to be a lot easier to understand and compute (they only involve multiplication) than perimeter-surface area

Table 19b

Percent of pupils who took both pre- and post-tests according to how they moved from different cells in (area/perimeter) pre-test to different cells on post-test (area/perimeter).

POST-TEST

Area and Perimeter

PRE-TEST	(Abstract-Item 4) I	Perimeter II	Area III	IV	(Concrete-Item 1) I	Perimeter II	Area III	IV
I	10.1	1.0			11.1			
Perimeter II	8.1				8.1			
Area III	7.1	1.0		4.0	10.1			2.0
IV	39.4		12.1	17.2	59.6	3.0	3.0	3.0

Table 19c

Percent of pupils who took both pre- and post-tests according to how they moved from different cells in (area/perimeter) pre-tests to different cells on post-test (volume and surface area).

POST-TEST

Volume and Surface Area

PRE-TEST	(Semi-Abstract-Diagram-Item 5) I	Surface Area II	Volume III	IV	(Abstract-Item 6) I	Surface Area II	Volume III	IV
I	11.1				11.1			
Perimeter II	8.1				5.0			3.0
Area III	10.1	1.0		1.0	1.0	8.1		4.0
IV	35.4	8.1	8.1	18.2	24.2		5.0	39.4

Table 19d

Percent of pupils who took both pre- and post-tests according to how they moved from different cells in (surface area/volume) pre-test to different cells on post-test (area and perimeter).

	POST-TEST							
	Area and Perimeter							
	(Abstract-Item 4)				(Concrete-Item 1)			
	I	Perimeter II	Area III	IV	I	Perimeter II	Area III	IV
PRE-TEST I	4.2				4.2			
Surface Area II								
Volume III	20.8	1.0	2.1	5.2	26.0		2.1	1.0
IV	38.5	2.1	12.5	13.5	59.4	3.1	3.1	1.0

Table 19e

Percent of pupils who took both pre- and post-tests according to how they moved from different cells in (surface area/volume) pre-test to different cells on post-test (volume and surface area).

	POST-TEST							
	Volume and Surface Area							
	(Semi-Abstract-Diagram-Item 5)				(Abstract-Item 6)			
	I	Surface Area II	Volume III	IV	I	Surface Area II	Volume III	IV
PRE-TEST I	4.2				4.2			
Surface Area II								
Volume III	22.9	3.1	3.1		18.8		3.1	7.3
IV	32.3	7.3	6.3	20.8	16.7		1.0	38.5

(they both involve multiplication and addition), it is probably worth considering teaching in the order area-volume and then perimeter-surface area.

To determine how pupil performance on the post-test compared with performances on the pre-test, each pupil in one of the four categories for area/perimeter on one hand and for surface area/volume on the other was followed to see where he/she was on the post-test. Tables 19b - 19d show these results in percentages.

From tables 19b, 19c, 11.1% of the pupils correctly answered the items on area and perimeter on the pre-test. On the post-test, all of these pupils had no problems with the items on area/perimeter and surface area/volume presented in abstract mode. The only error one of them made was in computation in calculating area (24 x 13) in the abstract mode (among the 11.1% of the pupils).

8.1% of the pupils correctly answered the item on perimeter on the post-test but missed the perimeter item on the pre-test. On the post-test all of these pupils (the 8.1%) correctly answered the area/perimeter items in both the concrete and abstract modes, and the item on surface area/volume with a diagram, but on the abstract surface area/volume items 5.0% got both items correct and 3.0% got neither correct.

12.1% of the pupils answered the area item correctly but missed the item on perimeter on the pre-test. On the post-test the 12.1% was distributed as follows: 7.1% and 10.1% answered area/perimeter items correctly in the abstract and concrete modes respectively, 4.0% missed both items on area/perimeter in both modes. Their errors were both computational and algorithmic. And on surface area/volume, 8.1% and

10.1% did the items correctly in abstract and diagramatic modes respectively, and again 4.0% could not do these items correctly in abstract mode. The errors were in computation, algorithm and definition. They (pupils) interpreted "8-cube to mean 8 cubes instead of 8 x 8 x 8 cube."

The fourth category of pupils on the pre-test included those who missed both the area/perimeter items - over 68% of the pupils were in this category. 39.4% and 59.6% could do the items on area/perimeter on the post-test successfully in abstract and concrete modes respectively, but 17.2% and 3.0% could not work correctly the same items in the abstract and concrete modes respectively. On surface area/volume, 24.2% and 35.4% were successful in the abstract and diagramatic modes respectively but 39.4% and 13.2% could not do any of the items in abstract and diagramatic modes respectively.

From tables 19b, 19c, it appears most of the pupils learned something from the unit. It also appears that pupils who knew how to compute perimeter had no problems at all in learning the unit. Knowing how to compute area on the pre-test did not seem to lead to comprehending the unit completely. Some pupils appeared to learn little or nothing. Some of them might have computed area correctly on the pre-test by guessing.

<u>Post and Retention Test Comparisons</u>:

The post- and retention test attendance is shown in table 2. There were eight pupils who took the post-test but did not take the retention test from the T1's, T2's, and T3's classes. The teachers explained the absenteeism as due to an outbreak of a flu. One pupil who was absent for the post-test took the retention test. Of the eight pupils who did not take both tests (post- and retention) two and six of them were from T1's and T3's classes respectively. (T4 did not give a retention test.) Both pupils from T1's class correctly answered the items on area and perimeter in the concrete and abstract modes on the post-test, but only one got the surface area and volume items correct in the abstract mode. The six pupils from T3's class were distributed on the post-test as follows: on area/perimeter one of them got all the items correct in both the concrete and abstract modes, two got area items in abstract mode, but missed the perimeter items in the same modes, and all the other three got all the area/perimeter items wrong in abstract mode. On surface area/volume, only one got all the items right, three got volume right but missed the surface area items, and the other two missed the items for surface area/volume.

Table 20

Percent of pupils according to their performances on area and perimeter items on post and retention tests.

PERIMETER		AREA: Right		AREA: Wrong		
		Post	Retention	Post	Retention	Post-Test/Retention
RIGHT	T1	46.4	23.8	3.6	28.6	50.0/52.4
	T2	26.3	19.0	5.3	4.8	31.6/23.8
	T3	74.0	56.1	6.0	0.0	80.0/56.1
	Total	56.7	38.6	5.2	8.4	61.0/47.0
WRONG	T1	14.3	19.6	35.7	28.6	
	T2	31.6	47.6	36.8	28.6	
	T3	6.0	29.3	14.7	14.6	
	Total (All Classes)	13.4	31.3	24.7	21.6	
Total - Each Class	T1	60.7	42.3			
	T2	57.9	66.6			
	T3	80.0	85.4			
Total		69.0	69.9			

Table 21

Percent of pupils according to their performances on volume/surface area items on post-test and retention tests.

			VOLUME				
			Right		Wrong		
			Post	Retention	Post	Retention	Post-Test/Retention
SURFACE AREA	RIGHT	T1	22.2	9.5	18.5	23.8	40.7/33.3
		T2	57.9	23.8	21.1	4.8	79.0/28.6
		T3	72.5	34.1	5.9	4.9	78.4/39.0
		Total	55.7	25.3	12.4	9.6	65.0/34.9
	WRONG	T1	18.5	38.1	40.7	28.6	
		T2	0.0	19.0	21.1	53.4	
		T3	7.8	19.5	13.7	41.5	
		Total	9.3	24.1	22.7	41.5	
		T1	40.7	47.6			
		T2	57.9	42.8			
		T3	80.3	53.6			
		Total	68.0	49.4			

From table 20 above, there was an overall decline in percent of pupils who were successful on items on perimeter from post-test to retention test (61.0 to 47.0) but no change on items on area (69.0 to 69.9). There were higher percents of success on area than on items on perimeter in both tests.

By classes the percent of seventh graders (T1's class) remained about the same on perimeter - 50% to 52.2%, but declined from 60.7% to 42.8% on area on the post- to retention tests. However, nearly three-fourths of the pupils causing the decline made computational errors. For the mixed class (seventh and sixth graders - T2's class) there was an increase on area items from 57.9% to 66.6% but a decline from 31.6% to 23.8% on perimeter items from post-test to retention test. Here pupils generally made both computational and algorithm errors. For the sixth graders the decline was substantial, from 80% to 56.1% on perimeter but small increase on area from 80% to 85.4% from the post-test to retention test.

On volume and surface area, there was a substantial decline in each case from post-test to retention test: 65.0% to 34.9% for surface area and 68.0% to 49.4% for volume (table 21).

By classes, the percent of seventh graders who succeeded on post-test to retention test were 60.7% to 33.3% on surface area and 40.7% to 47.6% on volume. For the mixed class and the sixth graders alone on surface area the percents were from 79.0% to 28.6% and 78.4% to 39.0% respectively. On volume, they were 57.9% to 42.8% and 80.3% to 53.6% respectively.

Errors pupils made on surface area were mostly in algorithms. About half of the pupils who got the items wrong computed the area of the three sides shown on the paper for the rectangular block on both tests.

The sixth and seventh grade pupils tended to forget how to compute surface area faster than how to compute volume, but seventh graders seem better than sixth graders at recalling methods of computing area and volume.

On the retention test on area, over three-fourths of the pupils over a diagram of a rectangle to find its area and perimeter. It appears that several of them relied more on diagrams in the absence of concrete models to compute area and perimeter.

Table 22

Percent of students successful on post evaluation for previous and present investigations.

	1 Squares		2 Cubes		3 Popcorn		4 Mouse/Dog Balance		4 Mouse/Dog Coats	
	Previous	Current	Previous	Current	Previous	Current	Previous	Current	Previous	Current
First Response (Abstract)	42.5	48.9	16.4	36.2	15.7	40.4	11.9	17.0	7.5	4.3
Second Response (Concrete)	49.3	46.8	30.6	42.6	18.7	36.2				
Total	91.8	95.7	47.0	78.8	34.3	74.5	11.9	17.0	7.5	4.3
Estimated % of Entire Population	35	45	18	37	13	35	5	8	3	2

Success Rates on Final Evaluation

The students performed well on three of the four items on the final evaluation in both the concrete and abstract modes (table 22). The proportion of students successful in both modes (abstract and concrete) was almost even for all the three items they did well on. The fourth item seemed very difficult for all the pupils.

Over ninety-five percent (95.7%) of the pupils who took the test were able to figure out the number of 2-squares in a 6-square. About of them did not require the models of a 2-square and a 6-square to do the item. All of the pupils tried to draw a diagram to answer the question when no models were presented.

Over seventy-eight percent (78.8%) were able to find the number of 2-cubes in a 6-cube, but a little over half of them needed models of a 2-cube and a 6-cube to answer the question. Here again each of the pupils drew a diagram of a 6-cube and counted the number of 2-cubes in the 6-cube.

About seventy-four percent of them were successful in solving the 'shrinking popcorn problem' (see appendix) and nearly half of the successful students did not require the popcorn models. However, most of them looked at the 6-cube model to figure out the number of smaller packets possible when the investigator read the problem to them.

On the Mouse and Dog problem, 17.0% and 4.3% were able to figure out the number of mice needed to balance a dog and the number of mouse coats needed to cover a dog.

The success rates on the same examination in previous investigation (1977) were lower on all the items (than in this investigation) except the second part of item four where pupils were to find the number of mouse coats needed to balance a dog. On the item on squares there was a slight increase from 91.8% to 95.7% over the previous study and those success rates translated into estimates of 35% to 45% of the entire population. On the items on cubes the increase was from 47.0% to 78.8% that translated into estimates of 18% to 37% of the population, an almost two-to-one increase. The increase was from 34.3% to 74.5% on the popcorn problem, and this translated into estimates of 13% to 35% of the entire population.

It seems that there was a substantial increase in the pupils achievement on the post evaluation in the current investigation over the previous one on the items where models were provided. It appears also that pupils in this investigation achieved about the same level as those in previous investigation on the Mouse and Dog item.

It appears that pupils were not able to transfer the ideas used to solve the Mouse and Elephant problem solved in class to the Mouse and Dog problem. The differences between the pupils achievement on square-item and Mouse/Dog (coats) item on the one hand and that between the cube-item and the Mouse/Dog (balance) item suggests that pupils probably saw no connestions between each pair of problems. The pupils might have been able to answer the balance part of item 4 if the question had asked about the number of 6-cubes in a 72-cube.

Comparison with the Previous Study (Fitzgerald-Shroyer)

The comparisons made were on overall achievement on the post-test (final evaluation) and post evaluation administered to only pupils who scored 76% or better on the post-test. The responses pupils gave to the open question 'What do you think about the unit?' were also compared. In previous study there were 324 pupils from 13 schools, and in the current study there were 147 pupils from one school. Of the 147 pupils in the study, 100 pupils' performances were compared with those in the previous study because T4 (in current study) taught the unit over 5 weeks with no continuity in the instruction. The comparative results thus should be read with caution.

The responses of students in the post-test (final evaluation) when building rectangles with perimeter 20, or building solid blocks with volume on the Fitzgerald-Shroyer study on this study were about the same. The tables below show the responses with their conditional probabilities (tables 23, 24).

Table 23

Responses of students on final evaluation on the two studies (when pupils built rectangles with perimeter 20).

Initial Responses	% of Students		Conditional Probabilities of Finding			
			Larger Area		Smaller Area	
	Previous	Now	Previous	Now	Previous	Now
1 x 9	13.6	14.5	86.1	91.2	86.1	91.2
2 x 8	21.1	50.0	68.7	81.1	65.1	80.0
3 x 7	12.1	9.2	74.8	78.0	68.5	73.1
4 x 6	30.9	13.2	40.2	42.1	59.7	80.7
5 x 5	22.3	13.2	52.5	35.0	57.6	40.2

Table 24

Responses of students on final evaluation on the two studies (when pupils were building a solid block with volume of 12).

Initial Responses	% of Students		Likelihood of Finding			
			Maximum		Minimum	
	Previous	Now	Previous	Now	Previous	Now
1 x 1 x 12	6.8	1.2	55.0	45.0	50.0	48.1
1 x 2 x 6	14.7	22.1	72.1	58.3	65.2	53.0
1 x 3 x 4	21.5	34.9	65.0	63.0	61.9	71.0
2 x 2 x 3	57.0	41.9	69.5	72.1	80.3	74.0

In demonstrating measurement skills on the final evaluation the pupils in the current study had a very slight edge over the pupils in the previous study at the abstract level. On area for example, 56.3% and 69.0% were successful on previous and present studies respectively; 44.0% and 61.0% were successful on perimeter (abstract) for previous and present studies respectively. As table 25 shows, the performances of the pupils were about the same at the concrete level for the two studies, but at the picture and abstract level, performances of the pupils in the current study were consistently and slightly higher than those in previous study. The FE's referred to the item number on Final Evaluation. For example, FE2 referred to item 2 on the post-test.

Table 25

Percent of students demonstrating measurement skills on final evaluation for the two studies.

Problem Representation	Area		Perimeter		Surface Area		Volume	
	Previous	Now	Previous	Now	Previous	Now	Previous	Now
Concrete	86.3	94.0 (FE2)			71.4	70.0 (FE3)		
Picture					48.9	65.0 (FE5)	43.1	68.0 (FE5)
Abstract Written Description	56.3	69.0 (FE4)	44.0	61.0 (FE4)	17.1	37.0 (FE6)	24.6	51.0 (FE6)

The rules reportedly used by students in previous study were the same as those they used in the present study. Several patterns of rules not related to computing the measure of the concepts in the unit were also verbalized by students in the current study. These were listed under the results from observation.

For the general attitude pattern comparison for the two studies, the responses of the pupils to the open question were reported. Table 26 below shows the percent of pupils giving certain responses.

Table 26

Student reactions to the general open question: 'What do you feel about the unit?'

Category	Number of Students		Percent	
	Previous	Now	Previous	Now
Positive - general/global	146	48	44.9	48.0
Positive - with specifics	82	28	25.2	28.0
Better than regular math	6	4	1.8	4.0
Mixed Response (Some positive) (Some negative)	31	10	9.5	10.0
Negative Comment	12	5	3.7	5.0
Repugnant - more violent negative	5	2	1.5	2.0
No Response	43	3	13.2	3.0
Total	325	100		

<u>The Errors</u>:

Over 25% of the pupils did not name correctly the 4 by 5 figure as rectangle from each class. As a whole group over 30% of the pupils failed on this item. The pupils who got the item wrong were nearly divided between those who called it a box and those who called it a tile in each class. No pupil called it anything else.

Less than 6%, 5% and 4% of them were not able to find the dimensions, area, and perimeter respectively of the 4 by 5 figure using tiles. All of these pupils called the figure a tile and give the dimensions as 1 by 1, areas as 1, and perimeter as 4.

Of the less than 31% of the pupils who were unable to compute area without using tiles, half of them made computational errors, and about one-tenth of them made mistakes in algorithm. Of the 39% who made errors in computing perimeter, nearly half of them made algorithm errors. On perimeter, pupils used one of the following procedures:

1. added the two dimensions to give perimeter.

2. added the two dimensions and multiplied the sum by one of sides given, or a number other than two.

3. multiplied the two dimensions first and then multiplied the product by two or four.

On area, the one-tenth who made algorithm errors used one of the two first procedures above.

Computational errors, especially on multiplication, were made generally across all algorithm errors.

There was no significant number of careless errors; that is, errors where pupils interchanged answers: area for perimeter and perimeter for area.

Thirty-five percent of the pupils in T1's, T2's and T3's classes were not able to compute the surface area and volume respectively on the post-test. Nine percent of the pupils interchanged the surface area answer with that for volume. Over half the pupils who got the surface area and volume items wrong made algorithm errors. The computational errors made in computing surface area included: the calculation of the surface area shown in the diagram; the finding of the area of one face and multiplied the result by 6 even though the block was not a cube, the multiplying of all the three dimensions and multiplying the result by six, and some pupils simply multiplied any two sides and multiplied the product by a 2, 4 or 8. Some computational errors were not discernible. For volume there were several computational errors ranging from addition to place value. The most common algorithm error was pupils multiplying the three sides and then doubling the result or multiplying it by six. Apparently, the surface area and volume formulas were confused somewhere along the line.

Attitudes

The attitudinal results were in two parts: the teachers and the pupils.

The teachers' attitudes toward the unit as reported on the questionnaires are reported below as they stated them. The percent of pupils responding the same way to their questionnaires are reported below.

The Teachers' Reactions to the Unit

Two of the teachers had Master's degrees and two had Bachelor's degrees. They ranged in teaching experience from seven to thirteen years and each had taught at the sixth grade level for at least six years and at most thirteen years. For two of the teachers, this was their first time to teach the unit. The other two teachers taught the unit once before.

The teachers described their classes as "bright sixth and seventh graders," "seventh graders" or "below seventh grade" in response to the question on where they could put their pupils according to math ability.

They varied in their responses to the 'What do you think about the Mouse and Elephant Unit?'

> Teacher 1: It was an enjoyable unit to teach. The language models were especially appropriate for middle Junior High School students, space armor for surface area, etc. This is a conceptual key for students to get into the content involved. Also, the sequence of beginning with concrete materials and ending with generalization is always delightful.

> Teacher 2: Very good unit, allows different students to get something out of unit.

> Teacher 3: I thought the concepts and the manipulative aids were great! The kids loved working with the tiles and cubes. However, I think more stress should have been put on teaching the kids to remember the formulas for perimeter, area, volume and surface area so they could use these to figure out problems with shapes just drawn on paper. Many of my kids could tell you perimeter or area when you used the tiles, but not when a shpae was drawn on paper and labeled.

> Teacher 4: The Mouse and Elephant Unit is a good unit for sixth and seventh graders to have as a background to geometry. The year I want to compare the difference in geometry scores with last year's students. I think this unit will be a great help during geometry.

Their responses to whether the unit influenced their teaching during the investigation and three weeks later were brief. Generally the unit did not require them to teach differently. They responded as follows: Did Mouse and Elephant Unit require you to teach differently?

> Teacher 1: Not really. I have used small groups a lot in my teaching. Also the use of manipulatives was already part of my instruction.

> Teacher 2: No.

Teacher 3: Yes, I couldn't be as relaxed because I didn't always know what I should be doing. The teacher's instruction book was difficult to understand at first. I had a hard time knowing what I was supposed to do.

There were not enough activities recommended for students to do when they finished their work. I have some very bright math students and they found the unit quite boring at times.

Teacher 4: No, not really. The students work in small groups during regular math.

The next question was: Did your experience with the unit effect how you now teach math?

Teacher 1: Not really. What it will effect is the curriculum sequence that I teach. The concepts of the Mouse and Elephant Unit will become an integral part of my geometry sequence.

Teacher 2: No.

Teacher 3: No.

Teacher 4: No.

Teachers were asked whether they would recommend the unit to other teachers and to give their reasons why or why not. They responded as follows:

Teacher 1: Yes. The unit is an interesting "hands on" unit which follows good pedagological sequencing. Each unit starts with a challenge the students can understand. This challenge is then explored using concrete materials to arrive at generalizations.

Teacher 2: Yes, good unit for sixth and seventh grades.

Teacher 3: I thought the activities were well planned.

Generally all the teachers liked the unit and thought that it was appropriate for sixth and seventh grades. One of the teachers felt that more challenges should be included for the very brilliant pupils. Teachers did not make specific comments on activities that proved too confusing or hard to understand.

Pupils' Attitudes Toward the Unit

While sixth and seventh graders in this study did not hate mathematics, they preferred the Mouse and Elephant Unit to 'Regular Math' by an over two-to-one margin. What they liked best about the unit were the activities involved, such as building blocks and banquet tables, cutting space armors and other shapes.

The attitude of the pupils in the study to the Mouse and Elephant Unit was generally very favorable. Those who did not like the Mouse and Elephant Unit thought the unit "can be taught in two or three classes," "was boring because it dragged on," or "you do the same things many times."

Pupils' responses to the question 'What part of the Mouse and Elephant Unit did you like?' varied as was expected. Their answers were placed broadly in seven categories as shown in table 27 below.

Table 27

Percent of pupils according to category of unit they liked best.

Category	Percent of Pupils
1. Building and playing with tiles/cubes (including making banquet tables)	36.4
2. Cutting shapes (mainly space armor)	20.2
3. All of the unit	14.1
4. Solving the main problem: number of mice coats and mice needed to cover and balance an elephant respectively	10.1
5. Computing surface area using cut-out jackets and wrapping	9.1
6. Tests and the extra challenges in the unit	7.1
7. Nothing	3.0

On a three point scale, pupils rated Regular Math and the Mouse and Elephant Unit as shown in the table below.

Table 28

Percent of pupils describing regular math and the Mouse and Elephant Unit as exciting, or boring.

	Regular Math	Mouse and Elephant Unit
Always Exciting	12.9	32.3
Sometimes Exciting and Sometimes Boring	78.5	59.1
Always Boring	8.6	8.6

In responding to the question 'Which do you like better, Regular Math or Mouse and Elephant Unit?' the pupils were distributed as follows: Regular Math - 33.0% and Mouse and Elephant Unit - 67.0% preferring each (table 28).

It appears that pupils prefer the Mouse and Elephant Unit to Regular Math more because of the cutting and building activities built into the unit, than because they learned more from the unit. It also appears that the main problems of the unit, which generated a lot of discussion during the first class, were no longer of key interest to the pupils. 10.1% of them liked the solving of those problems while 36.4% liked the building activities. One questions whether sixth and seventh grade pupils' interest in the solution to a problem will continue to be sustained over a period of two weeks. One also wonders if it would make a difference in pupils attitude and achievement to launch the main problems in the middle or toward the end of the unit, or to be coming back and forth to the main challenges as the unit goes on.

CHAPTER V

DISCUSSION AND CONCLUSIONS

Introduction

The discussion of the questions raised in the investigation is presented in three stages. The stages, based on the design of the data collection, were: the classroom observations, the achievement tests and the attitudinal questionnaires.

From observations, conclusions were drawn on "What actually went on in the classrooms during the instruction of the unit" with regards to the use of instructional time, the teachers' manuevers in the millieu of instruction and what rules or patterns pupils were capable of verbalizing. From the achievement tests, conclusions were drawn on what concepts in the unit pupils were able to learn and to remember three weeks after the unit was taught. In order to answer the questions on whether the pupils in the study learned anything from the unit, their post-test scores were compared with their pre-test scores. Test scores of various concepts were compared to determine the hierarchy of difficulty of understanding of the concepts treated in the unit. Conclusions were also drawn on the errors pupils made on various items. Teachers' and pupils' attitudes were assessed from observations, questionnaires and interviews by the investigator.

Observations

Each class period was distributed among the three instructional stages: launching, exploration and summary. All the classes spent over half of the daily instructional class period on the exploration stage. The distribution of timing between launching and summary was almost even. Between one-fifth and three-tenths of each period was spent on routine procedural matters such as registration, or minor disciplinary problems like asking pupils to keep quiet. In the classroom, teachers switched from one instruction stage to another reportedly for one of the following reasons: 1) "If they felt that pupils understood what they were to do," 2) "If time was running out for summary." Regardless of what reasons teachers switched from one stage to another, the amount of time they spent on instruction of the unit was less than the normal class period. Thus, for a total of six visits for T1 and T2, the teaching time was supposed to be 330 minutes but the actual instruction time was 249 minutes and 228 minutes respectively. For T3 and T4, the time available for teaching in the seven visits was 385 minutes but the actual teaching time was 292 and 283 respectively. Thus, it was concluded that:

> Conclusion 1: Actual class instructional time was less than the regular class period allocated to mathematics each day. Between 69% and 76% of the allocated class time over all the visits were spent on actual instruction of the unit and most of this time was spent on the exploration stage by all teachers.

T1, T2 and T3 taught the unit in less than three weeks. There was continuity in their instruction. Each of them taught the unit five days a week (T1 and T3) or four days a week (T2). T4 taught the unit over a five week period, with an average of two classes per week. All of the teachers wanted 'to finish teaching the unit'. Because of the difference in the time of instruction between T4 and the rest of the other teachers, T4's class results on the post-test were analyzed separately. T4's class mean was compared with the means of the other teachers. On the pre-test scores all the classes had about the same mean. On the post-test, T4's class mean was the lowest, even though T4 used the most time (about 76% of the class period) on actual instruction. T4's class average was 39.9% compared with 74.2%, the lowest average for the other classes. No pupil in T4's class qualified for the post evaluation which required a score of at least 78% on the test. Thus, it was concluded that:

> Conclusion 2: There seems to be a relationship between the level of achievement on the post-test and the continuity of instruction of the unit. It appears that pupils achieved higher scores when they were taught the unit continuously than when they were taught in a piece-meal fashion every week.

Teachers generally were consistent in giving the launching and summary in 'front' of the class. Both old and new teachers consistently went to the front of the class to give instructions, even though the pupils were sitting in groups and were not necessarily facing the board. In summary, pupils gave many patterns, formulas or rules for each concept covered. The ways these formulas were verbalized were not the same as the ways they are traditionally written.

> Conclusion 3: Generally, pupils appeared to generate more rules, patterns or formulas from the tables they constructed than those necessary to compute the concepts taught. For example, pupils found that: 'the next square is increased by the next odd number, starting with one and then add three, then add five and so on'.

Achievement Tests

From the pre-test scores, it appears that pupils were not familiar with concepts included in the unit. Discussions with their class teachers confirmed that the pupils had not been taught the areas covered in the unit in the middle school but did not rule out the possibility that they (pupils) might have had some of the ideas in elementary schools. The pupils generally scored substantially higher on the final evaluation (or post-test), especially on items in the concrete mode. The percent of pupils successful on the abstract items was better than 60% on area and perimeter on the post-test. It was better than 50% and 37% on relative volume and relative surface area items (in abstract mode) respectively. It was better than 65% on volume and surface area items with pictures.

The order of difficulty, according to the pupils' performances on the post-test (final evaluation), from easiest to the most difficult, was area and volume, perimeter and surface area. For example, 69.0%, 68.0%, 67.0% and 65.0% of the pupils were successful on area and volume, perimeter and surface area items respectively (when the area and perimeter items were in the abstract mode and volume and surface area items with pictures). Pupils also performed better on items in the concrete mode than those in the abstract mode. 94% and 90% of the pupils correctly answered area, perimeter items respectively in the concrete mode, but only 69.0% and 67.0% in the abstract mode. For volume and surface area, 68.0% and 65.0% were respectively successful when the items had pictures, but only 51.0% and 37.0% in the abstract mode of the two concepts. Thus, it was concluded that:

> Conclusion 4: It appears that the pupils in the study were capable of learning the concepts and the computations included in the unit, and they were more successful with the concrete and pictorial items than abstract ones. The order of difficulty of the concepts from easiest to most difficult seemed to be area, volume, perimeter and surface area.

The mean score on pre-test of old and new teachers' pupils were 32.2% and 36.0% respectively. The mean for the post-test scores were

74.2% and 81.0% for the pupils in the old and new teachers classes respectively. For the retention test, 31.7% and 41.7% were the means for the old and new teachers respectively. There was a statistically significant difference between the two means at the .5% level for the old and new teachers. But for instructional purposes the differences were small: 6.8% and 10.0% for the post-test and retention test respectively. The two groups did not differ statistically at the start of the study, but the little difference (3.8%) seemed to increase from pre-test to post-test to retention test. That probably and partly explained the reason for the differences between the groups on the post-test and retention test. The new teacher 'reinforced' the formulas by schemes 'to help pupils remember' as that teacher stated in the questionnaire. That could also probably explain the slight difference in favor of the new teacher's class on pupil performance on the test. Thus, it was concluded that:

> Conclusion 5: For instructional purposes, it seems that newness of the unit to the teacher did not result in less achievement of the pupils, and the teaching style used 'to enhance pupils remembering' might have been an advantage.

T4 did not give the retention test. The pre-test mean for T4's class was about the same as those for all the other classes. But the post-test mean was 39.9%, less than the means for the other new teacher's class of 81.0%, and the old teachers' classes of 74.2%.

The percent of pupils successful on area items on post-test -- retention test for T1, T2 and T3 were 60.7% -- 42.8%, 57.9% -- 66.6%, 80.0% -- 85.4% respectively. For perimeter for the T1, T2 and T3, the post-test -- retention test percent of success were 50.0 -- 52.4, 31.6 -- 23.8, and 80.0 -- 56.1 respectively. On area, it seems that pupils improved over their post-test performance in each class except

T1's class. In T1's class, pupils generally made computational errors. However, on the whole, improvement on area computation in the abstract mode was negligible 69.0% to 69.9%. On perimeter, except for T1'2 class, the pupils scores declined considerably from post-test to retention test. The overall decline was from 61.0 to 47.0%, i.e. on the post-test 61% of the pupils could do the perimeter item (in abstract mode) and 47.0% could do a comparable perimeter item in the same mode on the retention test. For volume and surface area, the decline was substantial overall: 65 to 49.4% and 65.0 to 34.9% for volume and surface area respectively (from post-test to retention test). For T1, T2 and T3 the percents of pupils successful on post-test to retention test items on volume were 40.7 to 47.6, 57.9 to 42.8, 80.3 to 53.6 respectively; for surface area 40.7 to 33.3, 79.0 to 28.6, 78.4 to 39.0 for T1, T2 and T3 respectively. Between 9.5% and 34.1% of the pupils of each of the three classes could answer correctly both volume and surface area items on retention test, but between 22.2% to 72.5% on the post-test. Between 26.3% and 74.0% of the pupils in each class could do the area and perimeter items on the post-test but 19.0% to 56.1% of them could do similar items on the retention test. Also, the mean scores for the pupils in the three classes, on the post-test and retention test, were 77.67% and 36.80% respectively. Thus, it was concluded:

> Conclusion 6: It appears that the pupils forgot several aspects of the unit three weeks after the instruction. However, on area there was a very slight improvement, whereas on surface area, almost half of the pupils could not remember the correct way to compute it in the abstract mode.

In comparing the performances of the pupils in this study and those in the Fitzgerald-Shroyer study, it was noted that: there were 324 pupils from 13 different schools who participated in that study, compared with

147 pupils from only one school in the present study. Therefore, no exact test of the statistical significance of the difference could be applied. By By simple percent comparison on the post-test, pupils in both studies performed about the same level on concrete items. On area (concrete mode) 86.3% and 94.0% of the pupils were successful on previous and current studies respectively. On surface area, in the concrete mode, the percents of pupils who answered the items correctly were 71.4 and 70.0, for previous and present studies, respectively. On pictorial and abstract items, the percents of pupils who were successful were slightly higher in this study than in previous study. The percents for the abstract items were: area 56.3 to 69.0, on perimeter 44.0 to 61.0, volume 24.6 to 51.0 and on surface area 17.1 to 37.0. For pictorial items, 48.9% to 65.0%, and 43.1 to 68.0% of the pupils were successful on the previous-to-present study on surface area and volume respectively. On the post evaluation for the pupils who scored better than 76%, the percent of pupils in this study who got the items correct compared with previous study were: 95.7 to 91.8 on the 'squares' item, 78.8 to 47.0 on the 'cubes' item, 74.5 to 34.3 on the 'popcorn' item, 17.0 to 11.9 on the mouse/dog balance item, and 7.5 to 4.3 on the mouse/dog coat item. The conclusion then was:

> Conclusion 7: Pupils in the present study performed consistently slightly higher than pupils in the previous study for the items in pictorial and abstract modes, but on the concrete mode they all performed about the same level.

For those pupils who scored 76% or better on the post-test, a higher proportion of them were successful on the squares, cubes and popcorn items than similar proportion in previous study. The dog/mouse item proved to be equally difficult for all the pupils in both studies.

Over 25% of the pupils were not able to name the 4 x 5 figure a rectangle. They referred to it as a tile or a box. This was a language difficulty. On area and perimeter, over half of the pupils who got the items wrong made computational errors. The algorithm difficulties pupils ran into were what to add and when. Several of the pupils added the two given dimensions and multiplied the sum by a factor, other than two, to find perimeter. On surface area and volume, most of the errors were in algorithm. Nine percent of the pupils computed surface area and volume correctly but interchanged the answers. The other errors included multiplying the three dimensions given and multiplying the product again by 6 to find volume. For surface area, several pupils simply multiplied any two sides and multiplied by 6 or added the three sides and multiplied by six. (The rectangular block was a 3 x 4 x 4 block.) Some pupils found the area shown in the diagram by simply counting the squares shown. For errors, computation was the most frequent. It was concluded that:

> Conclusion 8: The order of frequency errors occurred from most to least was: computation, algorithm, language and space, and careless or random errors.

Attitudinal Questionnaire

Generally, the pupils did not hate mathematics. They preferred the Mouse and Elephant unit to regular mathematics by an over two-to-one margin. What they liked best (56.4% of them) about the unit were the building and cutting activities. The teachers reacted favorably to the unit. A new teacher reported that some of the harder challenges were not hard enough for the bright class the teacher had. The conclusion on attitude, it seems, would be:

Conclusion 9: The pupils preferred the unit to regular math by better than a two-to-one margin and the teachers liked to teach using the unit.

Summary and Conclusion

Between two-thirds and three-fourths of the total class time was devoted to the actual unit instruction by each of the four teachers. Three of the teachers taught the unit continuously for about three weeks, and the fourth teacher taught the unit over a five week period. Pupils were generally capable of generating all the formulas required to compute area and perimeter of rectangles, volume and surface area of rectangular blocks. All the rules pupils verbalized, except that for area of rectangles, were not stated in the traditional way they are normally written. For example, on perimeter, pupils doubled the opposite sides of rectangle first before adding the results together. Traditionally, the length and width of a rectangle are added and then doubled to find perimeter. Symbolically the rules could be stated as:

Pupils: $P = (2 \times b.e) + (2 \times s.e)$

Traditionally $P = 2(b.e + s.e)$

The pupils were generally very much involved at the exploration stage. They seemed to like the activities on cutting space armors.

The three achievement tests for all the pupils in each class were analyzed for content validity only. The KR20's were computed and each was below .40. On the pre-test, the mean score of all the pupils was about 30%; on the post-test it was about 75% and on the retention test the mean was about 35%. 90% of the pupils who took the pre-test could identify a figure as a rectangle. 23.7% and 18.6% of them were able to compute area and perimeter of a rectangle drawn on the pre-test. 62.5% and 4.1% of them were able to compute the volume and surface area respectively of a 3 x 4 x 5 rectangular block drawn. On the post-test, better than 85% and 70% of the pupils were able to compute area and perimeter of a rectangle comparable to the one on the pre-test. 55% and 45.8% of the pupils correctly computed the volume and surface area of a 3 x 4 x 5 rectangular block on the retention test.

The mean scores on the pre-test, post-test and retention test of old teachers and those of the new teachers were compared using the t-statistics. On the pre-test, there was no significant difference between the two groups at 5% level. On the post-test and retention test, each of the two groups were significantly different at the 5% level. However, for instructional purposes, these differences (on the post and retention tests) were not very substantial.

The sixth and seventh graders in this study demonstrated the capability and willingness to learn the topics covered by the unit. The modified unit did not seem to pose any difficulties for the teachers who used the unit for the first time. The pupils satisfactorily demonstrated the ability to detect patterns and to make appropriate rules for the computation of concepts covered. The overall gain in performance of the pupils in this study over those in previous study was very small. That is, slightly higher percents of pupils were successful on the post-test (final evaluation) items in this study than in previous study. For example, on computation of area of rectangle (in abstract mode) 69.0% got it correct in this study compared with 56.3% of the pupils in previous study. However, the difference between the pupils who scored 76% or better on the post-test, in the two studies, was quite substantial on the post evaluation. On the square, cube and popcorn items, the present study-previous study percents of pupils successful were 95.7 - 91.8, 78.8 - 47.0, 74.5 - 34.3.

On the last item on the post evaluation, both studies had essentially the same results. It thus appears that the transition from Mouse-Elephant challenge to Mouse-Dog challenge did not take place. Thus, the Mouse-Dog problem seemed to be too difficult for even the pupils with a perfect score on the final evaluation.

Pupils in T4's class got about the same average on the pre-test as those in the other three classes. On the post-test, T4's pupils had a mean of less than 40%. The means of the other classes were above 72%. No pupil in T4's class scored 76% or higher on the post-test, and therefore, no pupil in that class qualified to take the post evaluation. The major instructional differences between T4 and each of the other teachers were: 1) There was no continuity in the instruction of T4. T4 taught the unit over a five week period. Some weeks he taught the unit twice or once. The other teachers taught the unit for at least four consecutive days each week. 2) T4 did not do summary in two of the days he was observed. Each other teacher always found some few minutes for summary. Generally, T4's pupils were as involved as the pupils in other classes. They went on with the exploration activities for longer periods. The lack of continuity in instruction and the failure to summarize was probably one of the important factors for the pupils low achievement on the post-test.

On attitude, pupils preferred the Mouse and Elephant unit to regular math by a margin of over two-to-one. The teachers liked the unit according to their reports on the questionnaire. There were some reasons to believe that the teachers liked the unit. After the teachers finished the instruction for the investigation, three other teachers in the school wanted to try the unit in their classes. However, some of the materials were needed for other classes at Michigan State University. One set of materials was left for them to use one after another. One of the teachers in this study stated that "Mouse and Elephant Unit is now part of my curriculum."

Further Questions

1. During the instruction process, teachers had to decide on when to have the class move from one instructional stage to another (Launching to Exploration to Summary). They decided how long each activity lasted, thus controlling the total time on instruction of the whole unit. This study reveals that teachers spent different amount of time at the different stages of instruction on different days on different activities. In response to an interview question 'When do you have pupils move from one stage (of instruction) to another', one of the teachers had to stop to think it over. The responses they gave ranged from "their feelings of most pupils having understood or finished" to "the running out of class period." But since the investigator asked this question at the end of the project, no clues were detected on why and when the teachers switched from one stage to another. Probably some teachers moved from one stage to another because the unit says so. One question of interest to include in future investigation is "What influences a teacher to move from stage to stage in the instruction?" Certainly that decision making point will control the amount of time to be spent on the unit. Maybe some activities may be expanded or condensed or left just the way they are now. From this study, it could be that T4 had completely different reasons for teaching the unit the way he did than T3, even though they were both new teachers. To ask such a question after the unit will only force the teacher to reflect. The investigator believes that such reflective information may not be as accurate as on the spot information during and after each class.

2. The three tests: pre-, post-, retention, the investigator used were not parallel or equivalent. They were assessed for content validity. But to be able to make a more substantive statement about the amount of knowledge gained as a result of the instruction using the modified unit, he believes that parallel tests should be used. That is, a pre- and retention test very similar to the current final evaluation could be constructed.

3. The setting of the study, due to insufficient materials, meant that only two teachers taught the same sections of the unit simultaneously. The materials were not enough for all the teachers to use at the same time. T3 taught the unit last. The investigator did not detect that pupils who took the post-test last had conferred with those who did theirs early. Also, to throw more light, a larger population (more schools and more classes, more new teachers) will be necessary for future studies. It would probably be better if all the teachers taught the unit at about the same time in each school. For classroom observation, the investigator believes that two or more participant observers for each class (they may rotate on daily basis) who will compare notes, may give more accurate information about the classroom activities.

4. One of the main problem areas in this study was the transfer of knowledge on the main challenge to a similar situation by the pupils. The pupils were excited in class to finally solve the Mouse-Elephant problem. When the pupils who scored 76% or higher were confronted with the Mouse-Dog problem, they seemed to be totally lost. To bridge this gap, the investigator will suggest the inclusion of such challenges as boy-father, mouse-cat and similar challenges into the unit. Probably if teacher requests, pupils may be able to formulate similar problems that may invariably bring out and facilitate the relative surface area and

volume concepts transfer. Such challenges can probably come after solving the main challenge - at the end of the unit on can they be given as assignments to the pupils.

5. The unit was taught for, at most, 10 class periods. On pupils questionnaire, some of them stated that the unit could be done in three days. One of the authors in the pilot study, taught the unit in seven class periods. (Incidentally, in his class, some of the time each day was spent deliberately on some games not included in the unit.) Can this unit be taught effectively in a week? What kinds of results will be obtained? Will the pupils still enjoy the unit? What is the optimal length of time to spend on the unit? These questions could be investigated.

6. All the pupils did not achieve equally on the final evaluation. Each of the teachers in the study expressed a surprise over how well some of their poor pupils in mathematics performed. One of the old teachers remarked that "each pupil gets at least something from this unit." One important question to ask is "What kinds of pupils benefit most from this kind of instruction and which ones might be deterred by it?"

The investigator believes that this study should be repeated with the above six points in mind.

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APPENDICES

APPENDIX A

PRE-TEST

APPENDIX A

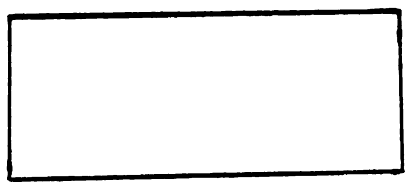
PRE-TEST

NAME________________________

SCHOOL______________________

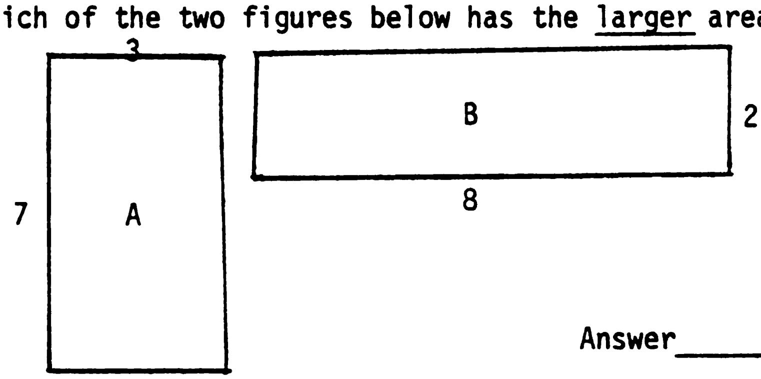
TEST - MEASUREMENT

1. What do we call the figure below?



Answer________________________

2. Which of the two figures below has the larger area?

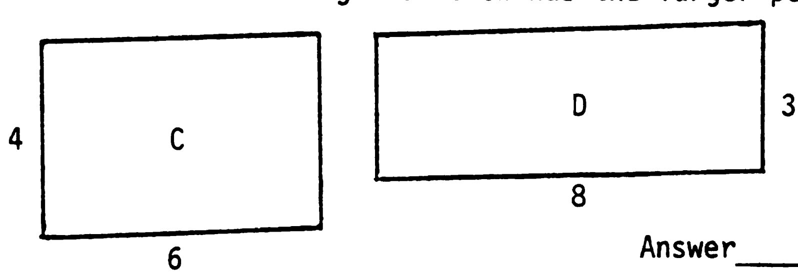


Answer________________________

3. Find the area of the figure you selected in Question 2.

Answer________________________

4. Which of the two figures below has the larger perimeter?

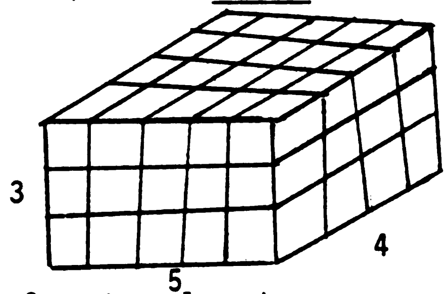


Answer________________________

5. Find the perimeter of the figure you selected in Question 4.

Answer______________________

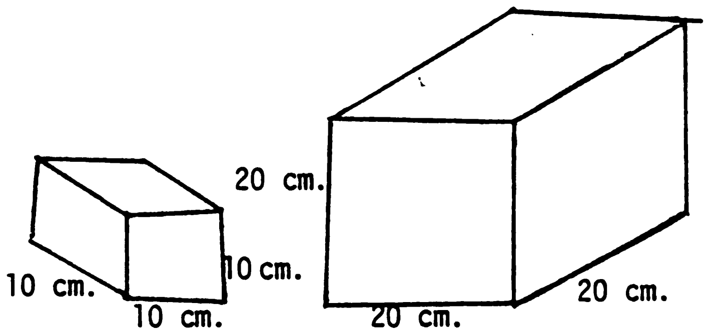
6. Compute the volume and surface area of the figure below.



Compute volume here

Compute surface area here

volume = ____________________ surface area = ____________________



7. Two parcels containing the same materials as shown above were wrapped.

   a) If one sheet of wrapping paper was used for the small parcel, how many sheets of wrapping paper would be used for the big parcel?

Answer______________________

   b) If the big parcel weighed 40 lb., how many pounds did the small parcel weigh?

Answer______________________

APPENDIX B

PUPILS' QUESTIONNAIRE

APPENDIX B

PUPILS' QUESTIONNAIRE

Name ____________________ Date: ____________________

Teacher: ____________________

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1. What do you think about Mouse and Elephant Unit?

2. On a scale of 1 to 3 where will you put Regular Math?

1	2	3
Always Exciting	Sometimes Exciting and Boring	Always Boring

3. Where do you put Mouse and Elephant Unit?

1	2	3
Always Exciting	Sometimes Exciting and Boring	Always Boring

4. Which do you like better Regular Math or Mouse and Elephant Unit?

5. What part of the Mouse and Elephant Unit did you like?

6. What part of the unit did you <u>not</u> like?

APPENDIX C

POST-TEST (FINAL EVALUATION)

APPENDIX C

POST-TEST (FINAL EVALUATION)

MOUSE & ELEPHANT EVALUATION

____
Name

teacher # ____

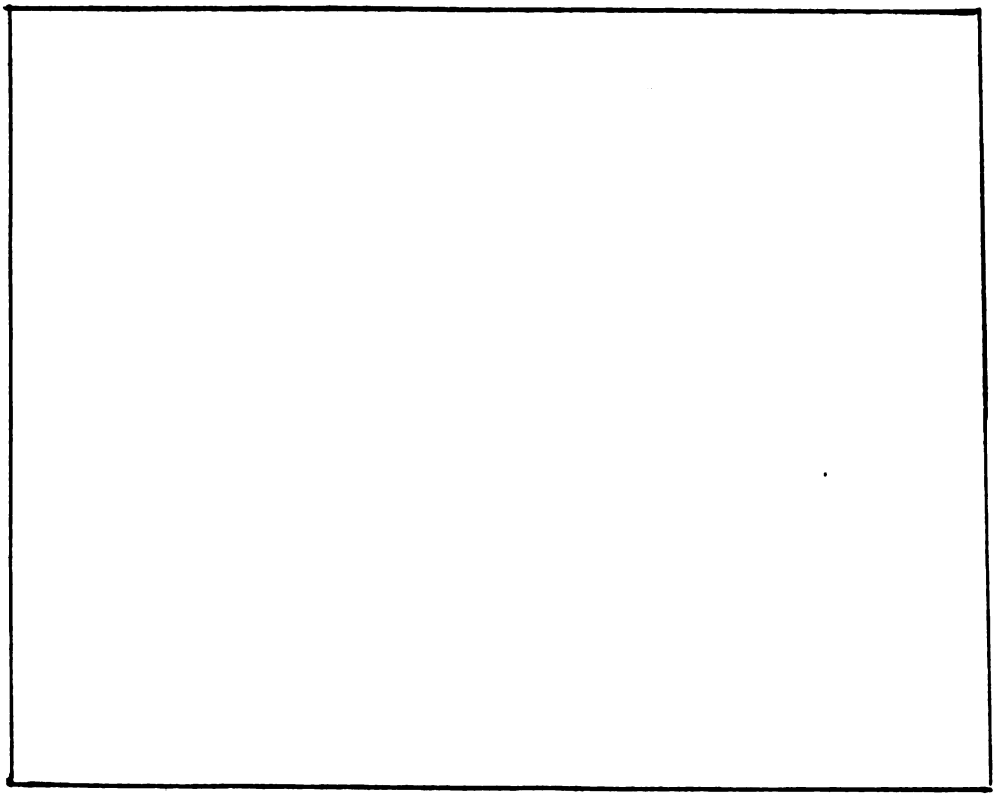
1. What do we call the figure drawn below? ______________________

   Use your tiles to help find:

   edges: front______ side______

   area __________

   perimeter __________



If you need to ask what area or perimeter mean -- raise your hand.

2. A. Use your tiles to make a rectangle with a perimeter of 20.

   What are its edges? side______ front______

   What is its area? ______

   B. Can you make a rectangle with a perimeter of 20 that has a <u>larger</u> area than the one you have just built? ______

   If you can, what are the edges? side______ front______

   What is the area? ______

   C. Can you make a rectangle with a perimeter of 20 that has a <u>smaller</u> area than the one you first built (part A)? ______

   If you can, what are the edges? side______ front______

   What is the area? ______

3. A. Assume you have 12 food pellets to take to Mars. You must wrap the food pellets in special Mars cloth which costs a dollar a square.

   Use 12 cubes to represent the food pellets. The square that costs a dollar is the same size as the square face of the cube.

   Describe one way you can stack all 12 food pellets (cubes) in one solid block by giving the edges:

   bottom front edge ______

   bottom side edge ______

   height ______

   What does it cost to wrap this package? ______

   B. What is <u>most</u> expensive way to wrap the 12 cubes (food pellets)?

   edges of the solid block: bottom front ______

   bottom side ______

   height ______

   cost of wrapping ______

   C. What is the <u>least</u> expensive way to wrap the 12 cubes (food pellets)?

   edges of the solid block: bottom front ______

   bottom side ______

   height ______

   cost of wrapping ______

4. Do NOT use tiles or cubes. Show all your arithmetic calculations!

   What is the area of a rectangle whose dimensions are 13 on the bottom edge and 24 on the side edge? ______(area)

   What is the perimeter of this rectangle? ______

5. Do not use cubes.

   Show arithmetic calculations you use in answering the questions.

   The picture below is of a solid block made of cubes.

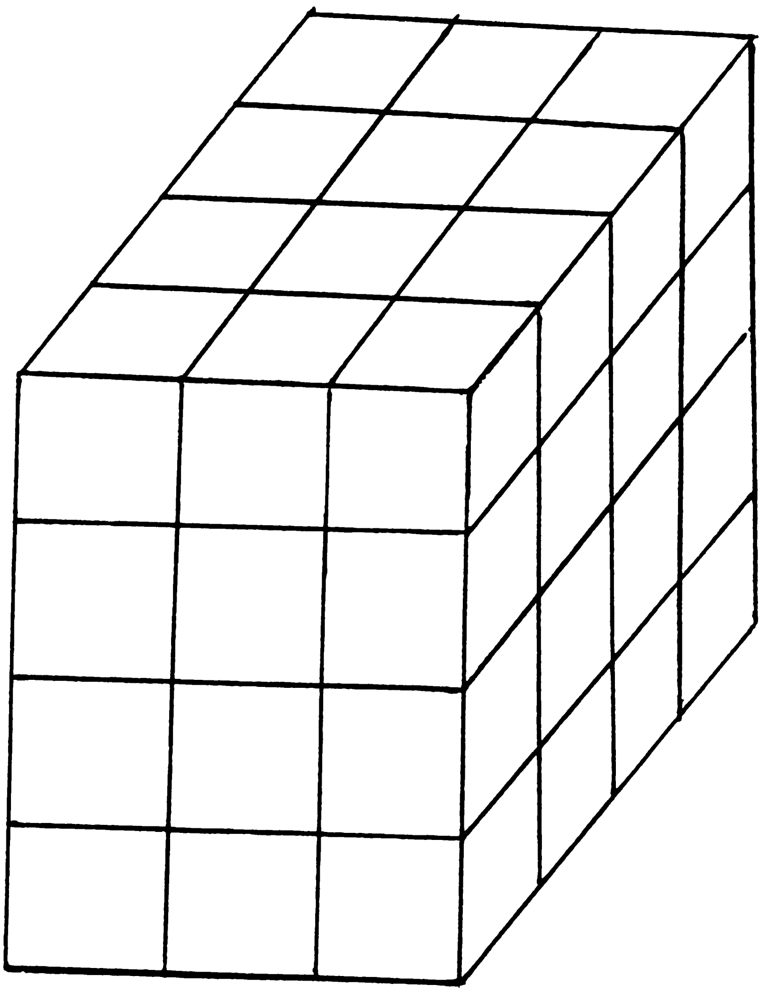
   What are the edges? bottom front ______

   bottom side ______

   height ______

   What is the volume? ______

   What is the surface area? ______



If you need to ask what volume or surface area mean, raise your hand.

6. How many 1-cubes does it take to make an 8-cube? ______

   How did you figure it out?

   How many squares (same size as found on one face of a 1-cube) does it take to cover an 8-cube? ______

   How did you figure it out?

   If you need to ask what a 1-cube or an 8-cube is, raise your hand.

APPENDIX D

POST EVALUATION

APPENDIX D

POST EVALUATION

All these items were read to the pupils.

Squares: How many 2-squares are in a 6-square?
1st response without model, 2nd response with model.

Cubes: How many 2-cubes are in 6-cube?

Popcorn: Kindergarten pupils wanted to have a party. They decided to buy popcorn box. They thought the regular popcorn box was too big for each one, so they decided to get smaller box by cutting each of the edges into two. If the big box cost 40¢, how much will each of the smaller box cost?

Name: ______________________ Teacher: ______________________

Date: ______________________

	1st Response	2nd Response
Square		
Cube		
Popcorn		
Dog	# Mouse Coats Work:	
	# Mice Work:	

The Mouse - Dog Problem

Name: ____________________ Teacher: ____________________

Date: ____________________

Before a mouse grows to be like an elephant, he grows to be like a St. Bernard Dog.

If the mouse is 6 cm high, and the St. Bernard is 72 cm high,

A) How many mice coats are needed to make a coat for a St. Bernard Dog?

B) How many mice are needed to balance a St. Bernard on the balance scale?

Write on this paper.

APPENDIX E

RETENTION TEST

APPENDIX E

RETENTION TEST

Name: ____________________________ Teacher's Name: __________________

Date: ____________________________

SHOW ALL YOUR ARITHMETIC CALCULATIONS

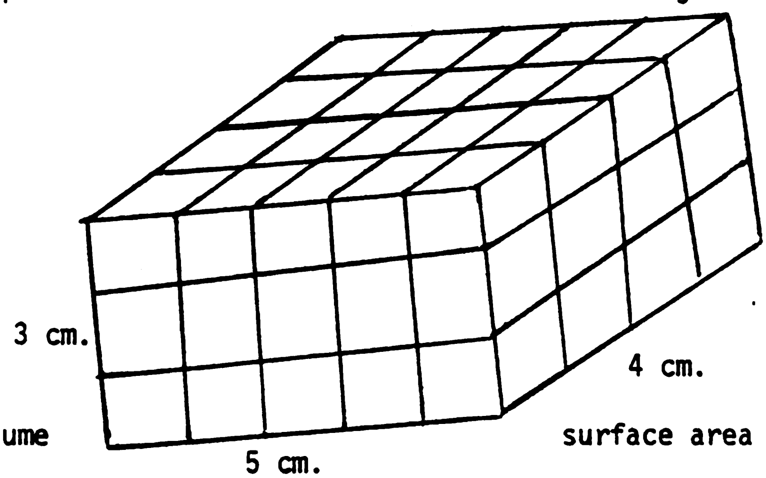
1. (a) What is the area of a rectangle whose dimensions are 18 on the bottom edge and 12 on the side edge.

area = ______________

(b) What is the perimeter of this rectangle.

perimeter = ______________

2. Compute the volume and surface area of the figure below.



volume ____________

surface area ____________

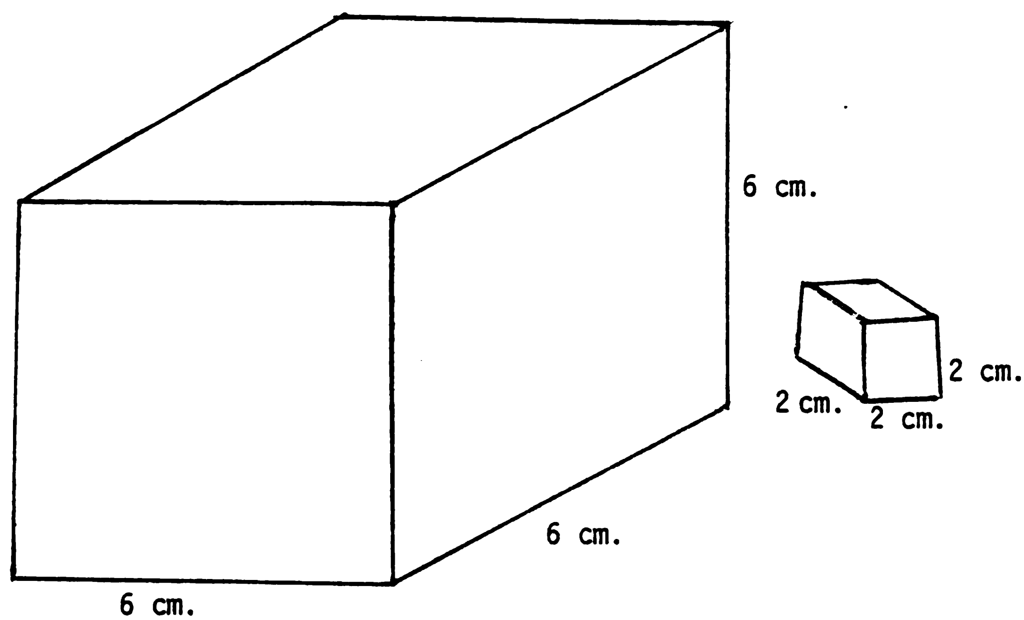
3. Two parcels containing the same materials as shown below were wrapped.

   (a) If one sheet of wrapping paper was used for the small parcel, how many sheets of wrapping paper would be used for the big parcel?

   Answer__________________

   (b) If the big parcel weighted 54 lb., how many pounds did the small parcel weigh?

   Answer__________________



APPENDIX F

TEACHER'S QUESTIONNAIRE

APPENDIX F

TEACHER'S QUESTIONNAIRE

Teacher Information

1. Name (optional) ______________________________
2. Degrees and Diplomas (and year) ______________________________
3. Number of years of teaching __________
4. Number of years of teaching at this grade level __________
5. This is my first, second or third time of teaching the unit (circle one).
6. Where do you generally put your students according to math ability? e.g. above ro below 7th grade, bright 6th grade, etc.
7. What do you think about the Mouse and Elephant Unit?
8. Did the Mouse and Elephant Unit require you to teach differently?
9. Did your experience with the unit effect how you now teach Math?
10. Can you recommend the unit to other teachers and under what circumstances or why not?

APPENDIX G

SAMPLE ERRORS ON THE POST-TEST (APPENDIX C)

APPENDIX G

SAMPLE ERRORS ON THE POST-TEST (APPENDIX C)

<u>Item 1</u>:

The only error pupils made was calling the figure a tile instead of a rectangle. This response was probably a result of the use of tiles in the project.

All the pupils were able to find the edges and area of the figure.

Less that 5% of the pupils got the perimeter wrong. They gave 20 (a product of 4 and 5) or 19, miscounting one tile twice.

<u>Item 2</u>:

In making a rectangle with perimeter 20, pupils generally gave the following incorrect dimensions: (i) 4 by 5, (ii) 2 by 10.

It seems that pupils were finding the dimensions of a rectangle with a fixed area 20.

Some pupils interchanged the rectangles with smaller area and that with larger area. For example: smaller area - 2 by 8, larger area - 1 by 9. Area of the dimensions of the rectangles given by pupils were generally correctly computed.

<u>Item 3</u>:

To find the dimension of a rectangular block of volume 12, the pupils gave incorrect answers such as (i) 4 by 4 by 3, (ii) 3 by 3 by 4, (iii) 12 by 1 by 2. To compute its surface area, some pupils wrote 12.

The other common mistake was pupils interchanged the blocks with smaller or larger surface area. For example: smaller surface area block 1 by 12 by 1, larger surface area block 3 by 4 by 1. Generally the surface areas were correctly computed.

Item 4:

The incorrect rules the pupils used in finding area or perimeter of a 13 by 24 rectangle include the following: 1) adding the two sides, 2) interchanging area and perimeter formulas, 3) computing area and then multiplying it by 2, 4 or 6, 4) computational errors were made in each of the above methods.

Examples on Area:

(a) computation errors included:

(i)	(ii)	(iii)	(iv)	(v)
24	13	13	13	13
13	24	24	24	24
72	52	56	52	52
24	130	26	160	36
96	182	72	212	412

(b) algorithm errors:

(i)	(ii)		(iii)
24	13 + 13 = 26	26	312
13		24	x6
312	24 + 24 = 48	1248	1872
x2			
624			

Examples on Perimeter:

(a) computation errors included:

(i)	(ii)
1	
26	26
48	48
94	114

(b) algorithm errors:

(i)	(ii)	(iii)	(iv)
24	13	13	13
24	24	24	24
13	37	37	312
13		x4	
24		148	
98			

Item 5:

Generally pupils had no problems with finding the dimensions of a 4 x 4 x 3 block drawn.

In computing the volume and surface area, pupils made algorithm, computation errors, and interchanging of the two answers. The wrong rules pupils used included: 1) adding two or all the three sides, 2) multiplying two of the dimensions, and then the product multiplied by 2, 3, 4, 6 or 8 in computing surface areas, 3) multiplying the three numbers and then multiplying the product by two in computing volume.

Examples on Surface Area:

Algorithm:

(i) 4 + 4 = 8 (ii) 4 + 4 + 3 = 11 (iii) 4 x 4 x 2 = 32

(iv) 4 x 4 x 4 = 64

Examples on Volume:

Algorithm:

(i) $4 \times 4 \times 3 = \begin{array}{r} 48 \\ \underline{\times 2} \\ \underline{96} \end{array}$ (ii) $4 \times 4 = \begin{array}{r} 16 \\ \underline{\times 6} \\ \underline{96} \end{array}$ (iii) $4 \times 4 \times 3 = \begin{array}{r} 48 \\ \underline{\times 3} \\ \underline{144} \end{array}$

(iv) 4 x 4 x 4 = 64 (v) 4 x 3 x 6 = 60

Item 6:

The wrong answers (for the number of cubes) given were 8, 64, and multiples of 8, from 16 to 56.

(i) 8, when pupils interpreted 8-cube to mean 8 cubes. The reasons they gave for this response were "count them,"

(ii) 64, when pupils multiplied 8 by 8. The reasons they gave were "same way to get area," "put 8 on top and 8 on bottom," "I stack them."

(iii) For the 2 x 8, 3 x 8, 4 x 8 to 7 x 8 responses, pupils stated their reasons: "It has 4 sides, counted all the block," "draw picture and count."

Examples of Number of Cubes:

Algorithm:

(i) 8 by counting (ii) $8 \times 8 = 64$ (iii) $8 \times 4 = 32$

(iv) $8 \times 8 \times 6 = 384$, interchanged answer with surface area

Examples of Surface Area:

Algorithm:

(i) computing volume $8 \times 8 \times 8 = 512$

(ii) $8 \times 2 = 16$, $8 \times 3 = 24$ to $8 \times 8 = 64$

(iii) $8 \times 8 \times 2 =$ to $8 \times 8 \times 5 =$