SOME ACOUSTICAL PROPERTIES OF TRIANGLES AND CYMBALS AND THEIR RELATION TO PERFORMANCE PRACTICES

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ABSTRACT

SOME ACOUSTICAL PROPERTIES OF TRIANGLES AND CYMBALS AND THEIR RELATION TO PERFORMANCE PRACTICES

By

John Baldwin

Very little specific information is currently available concerning the vibrational aspects of triangles and cymbals. Further, much of the information that is available is based on subjective personal opinions which are often confusing and contradictory. The data from this study should provide a basis for predicting the sounds of various triangles and cymbals (when instrument size, implement size and material, striking point and angle, and dynamic level are known), thus eliminating excessive experimentation. The data was also used to substantiate (and sometimes invalidate) certain typical performance practices.

The study investigated, measured, and compared the overtone structures produced by six triangles (6" Abel, 6" Ludwig, 6" Sonor, 6" Zildjian, 6" Pigstail, 10" Pigstail), and five cymbals (16" Avedis Zildjian, 16" New K. Zildjian, 16" Old K. Zildjian, 17" Paiste, 20" Paiste).

The triangles were struck with three implements, each 9" in length: 7/32" drill rod; 5/32" drill rod; 7/32" cold-rolled steel. The cymbals were struck with three implements: Musser yellow yarn; Musser red yarn; Deagan brown cord with red stitching.

The triangles were struck at three points: near the top of the closed side; the middle of the bottom side; near the closed corner of the

bottom side. A 90° angle of incidence was used in all instances. However, at impact, the implement was perpendicular to the plane of the triangle at the top and bottom striking points (90° striking angle), and parallel to the plane of the triangle at the corner striking point (0° striking angle). The cymbals were struck at two points: near the edge; near the cup. A 90° angle of incidence and a 0° striking angle were used in all instances.

Three dynamic levels (ff, mf, and pp) were used on all triangles and cymbals.

The sounds produced by variations in instrument size, implement size and material, striking point and angle, and dynamic level were recorded with a Magnecord recorder, played back through an Ampex Recorder / Reproducer, and analyzed with a Bruël and Kjaer Frequency Analyzer. The resulting graphs were printed out with a Bruël and Kjaer Level Recorder.

A mathematical investigation of the triangles was made using the finite element method in conjunction with the Structural Analysis and Matrix Interpretive System (SAMIS), a computer program developed by the Philco Corporation. The program produced a set of predicted frequencies for each triangle plus the information necessary to diagram each frequency's mode of vibration.

These recommendations were made for high sounds on most triangles: use a relatively short and/or thick steel triangle; use an implement of either drill rod or cold-rolled steel; strike the triangle near one of the closed corners with a 90° striking angle. These recommendations were made for low sounds on most triangles: use a relatively large and/or thin steel triangle; use an implement of either drill rod or cold-rolled steel; strike the triangle on either open side with a 0° striking angle.

These recommendations were made for high sounds on most cymbals: use a relatively small and/or thick cymbal; use a hard yarn implement if playing at a mf or pp level; strike the cymbal near the cup. These recommendations were made for low sounds on most cymbals: use a relatively large and/or thin cymbal; use a soft yarn implement if playing at a mf or pp level; strike the cymbal near the edge.

Recommendations for further research included the determination of the influence of these variables: instrument material; lengths of the sides of a triangle; angles formed by the sides of a triangle; shape, size, and height of a cymbal's bow and cup; other implement materials and sizes; and aging of the instrument.

SOME ACOUSTICAL PROPERTIES OF TRIANGLES AND CYMBALS AND THEIR RELATION TO PERFORMANCE PRACTICES

By

John Baldwin

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The author also wishes to acknowledge his wife, Alison, for her constant encouragement and willing assistance in the preparation of this dissertation.

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I. INTRODUCTION

Statement of Purpose

> The tone quality [of percussion instruments] depends on such factors as the dimensions, character of the metal or other material, point of striking, material and shape or the striking point, and the manner of reinforcement.[1]

In addition, various types or brands of instruments produce different sounds (e.g., spindle and pigstail triangles; Avedis Zildjian and Paiste cymbals). But as yet, there is no body of organized knowledge available to provide the scientific basis for these different sounds and tone qualities.

As Cloyd Duff of the Cleveland Orchestra says,

> I do not know the technical points about cymbals and triangles . . . I only know them from a performance point. If they are good instruments and possess the qualities that I require in these instruments for my purpose, I use them, and if not they are poor instruments and not for my need.[2]

Thus, the performing percussionist is forced to grope rather blindly and gradually learn, through trial and error or oral tradition, how to utilize the various tonal capabilities of his instruments to produce the most appropriate sound at the proper musical moment.

This study investigated and measured the overtone structures produced by triangles and suspended cymbals. In addition, the mathematical predictions of a triangle's vibrational behavior (based on

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[1]Wilmer Bartholomew, Acoustics of Music, p. 130.

[2]Cloyd Duff, personal letter, October 17, 1967.

material, shape, and dimensions) were compared with the acoustical measurements of the actual sounds produced.

The following questions served as a guide to this investigation and measurement:

1. What are the comparative overtone structures produced by triangles when played with large and small implements of the same material?

2. What are the comparative overtone structures produced by cymbals when played with hard and soft implements of the same material?

3. What is the effect, if any, of the material of the implement on the overtone structures produced by triangles and cymbals?

4. What is the precise relationship between the striking angle and/or point and the predominantly high and low pitch areas within one triangle or cymbal?

5. What are the comparative overtone structures produced by triangles and cymbals when played at various dynamic levels?

6. What are the relative strengths or intensities of the overtones produced by triangles and cymbals?

7. What are the similarities, if any, among the overtone structures produced by different types of triangles (e.g., spindle and pigstail) and different brands of cymbals (e.g., Avedis Zildjian and Paiste)?

8. What are the modes of vibration of a triangle suspended at one corner?

The data which resulted from answering the above questions were then used to substantiate (and sometimes invalidate) certain typical

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performance practices used on triangles and suspended cymbals. The data also provided a basis for predicting the sounds that various triangles and cymbals produce (when material, shape, and dimensions are known), thus eliminating excessive experimentation. Another use of the data would be to point the way for further research into the actual design of triangles and cymbals.

Background of Problem

General discussions of the acoustics of membranes, rods, bars, tubes, plates, and bells are readily available. Unfortunately, these general discussions seem always to be in terms of the ideal vibrating body: uniform cross-section or thickness; homogeneous material; uniform tension--requirements which are rarely, if ever, met in actual percussion instruments. In other words, the specific acoustical properties of actual percussion instruments have been quite neglected, both in research and explanation. This is partially due to the fact that acousticians, other musicians, and "particularly percussionists as a whole have largely not attempted or had the means to explore more fully the acoustical bases and properties of these instruments."[1]

Since the fundamentals and overtone structures of many percussion instruments, notably triangles and cymbals, form what has been called "an indefinite noise mixture"[2] (rather than a regular harmonic series with integer ratios), some musical acousticians apparently justify their neglect of these instruments by saying that "it is easy to understand why

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[1]James Moore, "Percussion Acoustics: An Introductory Evaluation," Percussionist, V (October, 1967), 218.

[2]Harry Olson, Music, Physics and Engineering, p. 177.

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percussion instruments have only a limited use musically."[1] Indeed, Buck seems to sum up the apathy or disinterest of musical acousticians by saying that "the sounds that can be produced from rods, plates, bells, etc., are of interest and importance to physicists rather than to musicians."[2]

However, physicists are apparently not overly interested in percussion instruments either (Wood dismisses cymbals by saying that their sound is "called by no accident 'kitchen music'"[3]). In response to the author's inquiry concerning the availability of relevant bibliographical materials of either a musical or physical nature, an Instructor of Music Theory at the University of Illinois wrote:

> I have not only been trying to find information on tamtams but also have been checking out metal-plate percussion instruments in general. I have now exhausted most of the possibilities and can say the same thing about cymbals and triangles as I can about tamtams: I have found practically no information on their actual harmonic structures.[4]

Knowledgeable percussionists will take issue with Buck's inference that percussion sounds are not musically important. For them, it is very important to be able to accurately and consistently produce any of a number of musical sounds. However, with the present lack of information, it is necessary for the percussionists to experiment with various combinations of instruments, implements, and striking angles and/or points in order to be reasonably confident of producing a particular sound at any particular moment.

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[1]Bartholomew, Acoustics of Music, p. 130.

[2]Percy Buck, Acoustics for Musicians, p. 84.

[3]Alexander Wood, Acoustics, p. 448.

[4]Donald Andrus, personal letter, October 28, 1967.

The present acoustical investigation of the sounds of triangles and cymbals, coupled with the mathematical investigation of the triangles, should remove some of the uncertainty of percussion performance by making it possible to accurately predict the tonal results when the major performance variables are known (i.e., the dimensions of the instrument, the material of the instrument and implement, the striking angle and/or point, and the striking force).

Related Literature

The following information concerning triangles and cymbals has been compiled from pedagogical materials, magazine articles, books on percussion, musical and physical acoustics texts, and personal correspondence. Much of the information is subjective in nature or is based on tradition or personal experience. It should be noted that the questions presented in this study are not answered with any degree of scientific satisfaction.

Instrument Material

Triangles

Triangles made of aluminum are presently available, but they are almost always "avoided because they produce an inferior tonal response."[1] Although Hart vaguely reports that the "triangle is a piece of metal,"[2] most instruments in use today are made of steel. But there are only

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[1] James Ross, "The Triangle: Don't Underestimate It," The Instrumentalist, XIX (April, 1965), 84.

[2] William Sebastian Hart, "Percussion Clinic," The Instrumentalist, XIII (February, 1959), 69.

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general comments as to the specific kind of steel used: "very hard high tempered carbonized steel"[1] and "plated steel."[2]

The late Harold Thompson, former percussionist with the Boston Symphony, confirms that the presence of some kind of plating (usually nickel or chrome) is essential for the desired triangle sound: "I have not as yet found an unplated triangle that sounds as good as a highly plated one."[3]

The author was unable to find any definitive information as to why aluminum is inferior to steel, why some steel is better than other types, or why the plating is essential.

Cymbals

Regardless of the brand, all cymbals seem to consist of a "metal alloy, made from copper, tin, lead, and iron."[4] Quoting Zildjian, Flagler states that their cymbals are "'roughly eighty per cent copper and twenty per cent tin'" with a "'small amount of silver.'"[5] Peters reports the use of a specific combination of 78.55% copper, 20.28% tin, .54% lead, and .18% iron.[6] Two cymbal manufacturers state that the

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[1]Al Payson and Jack McKenzie, Music Educators' Guide to Percussion, p. 58.

[2]Harry Bartlett, Guide to Teaching Percussion, p. 110.

[3]Harold Thompson, personal letter, October, 1967.

[4]Charles Spohn, The Percussion, p. 47.

[5]J.M. Flagler, "Onward and Upward With the Arts," The New Yorker, December 6, 1958, p. 156.

[6]Gordon Peters, "Treatise on Percussion," p. 114.

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metal in their cymbals (no matter what the exact content) is very highly tempered.[1,2]

Implement Size and Material

Triangles

One percussion innovator "used a Stroboconn in an attempt to determine if different sizes of beaters caused different overtones to be more prominent. . . . [He] found no change in the overtones due to changing the size of the beater."[3]

Thompson feels that a "soft and dense stick will generate maximum overtones and long sound."[4] Ross states that the "best material to use is drill rod cut up in 9 inch lengths."[5] And Hart suggests that the "largest nails available"[6] should be used.

Spencer prefers stainless steel for the following reasons:

> I chose stainless steel because it combines a steel of medium hard temper with an attractive metal which doesn't require plating. . . . Cold rolled steel . . . dents more when played than does the stainless steel. Highly tempered steel has an excellent sound but is hard to cut and requires plating.[7]

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[1]Avedis Zildjian Co., Cymbal Notes.

[2]Phil Grant, Tested Tips for School Music Supervisors.

[3]H.R. Spencer, personal letter, December 2, 1968.

[4]Thompson, personal letter.

[5]Ross, "The Triangle," p. 84.

[6]Hart, "Percussion Clinic," p. 69.

[7]Spencer, personal letter.

Cymbals

Although the variety of usable implements is limited only by the percussionist's imagination, only two sources yielded any specific information as to the effect of the size and material of the implement on the sound produced. Firth states that "a larger, heavier stick will bring out the fundamental and its overtones much quicker and clearer."[1] And Bartlett reports that the tips of wooden snare drum implements "will cause the higher frequencies in the cymbal to predominate, producing a more tinkly sound."[2]

Striking Point

Triangles

Bartlett simply states that the "usual playing spot is on the outside,"[3] and Wildman rather vaguely says the triangle is "hit on the opposite side from the open corner."[4] Leidig is more specific when he recommends that the triangle be struck "on the upper third of [the] right side (corner with the opening to the left)."[5] When Tilles says to strike the "closed end of the triangle and have this side facing your right hand," his stated reason is "because the sound travels

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[1]Vic Firth, Percussion Symposium, p. 26.

[2]Harry Bartlett, Percussion Ensemble Method, p. 65.

[3]Bartlett, Teaching Percussion, p. 110.

[4]Louis Wildman, Practical Understanding of the Percussion Section, p. 71.

[5]Vernon Leidig, Contemporary Percussion Technique and Method, p. 12.

toward the ends of the instrument."[1] However, he includes no further explanation or supporting evidence for this recommendation.

Leach reports that "physical laws tell us it [the triangle] will sound better if struck in the middle"[2] (but then neglects to say what physical laws, how or why it will sound better, or what defines "the middle"). Spencer seems to agree with Leach, reporting that "generally . . . the middle of the closed sides produced the cleaner clearer tone."[3] Blades, however, feels that "the best tone is produced by striking the triangle on the outer [open] side near the top corner."[4]

Price emphatically disagrees with the above opinions and asserts that "the triangle is never struck on the outside, but always on the base."[5] Collins and Green corroborate this view, stating that the "beater should fall upon the center of the bottom angle of the triangle when single notes are played."[6] Although Gardner concurs with the use of the base, he includes a picture showing a striking point very near the open end.[7]

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[1]Bob Tilles, "The Bob Tilles Column," The Ludwig Drummer, VIII (Spring, 1968), 31.

[2]Joel Leach, Percussion Manual for Music Educators, p. 78.

[3]Spencer, personal letter.

[4]James Blades, Orchestral Percussion Techniques, p. 26.

[5]Paul Price, Techniques and Exercises for Playing Triangle, Tambourine, and Castanets, p. 7.

[6]Myron Collins and John Green, Playing and Teaching Percussion Instruments, p. 123.

[7]Carl Gardner, The Gardner Modern Method, p. 80.

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Hart and others[1] recommend a change of striking point to correspond with dynamic changes: "near the top for pianissimo work, toward the middle of the right hand [side] for louder work, and directly in the middle for loudest effect."[2]

It is readily apparent that there is no universal agreement among percussionists as to a single best striking point for all occasions. Various authorities have recommended "the outside," "the upper third of the right side," "the closed end," "the middle," "the outer side near the top corner," and "the center of the bottom angle." However, none of these striking points were defined with any type of accurate measurement, nor related to any specific sound. This means that, as of now, the percussionist still "must determine by experimentation which playing spot produces the best sound for that particular triangle and musical passage."[3]

Cymbals

While most authorities agree that "a number of sounds can be obtained, depending upon which part of the surface is struck,"[4] they also seem to agree that normally the best striking point is fairly close to the edge: "the cymbal, generally, should be struck close to the

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[1]Thomas Brown and Willard Musser, Percussion Studies I, p. 5.

[2]Hart, "Percussion Clinic," p. 69.

[3]Mitchell Peters, "Triangle Technique," The Instrumentalist, XXII (February, 1968), 79.

[4]Bartlett, Teaching Percussion, p. 71.

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edge;"[1] "about 2 1/2 inches from [the] edge;"[2] "stick striking near the edge of the cymbal;"[3] "for almost all strokes and rolls, the cymbal is played upon near the edge;"[4] and "the cymbal should be struck on top near the edge."[5] However, Thompson feels that if a cymbal "is activated on the extreme edges, the full spectrum of sound does not emerge."[6]

Denov agrees with the close-to-the-edge striking point, "unless a rapid rhythmic figure is to be played."[7] Leach goes on to say that "so each stroke can be easily distinguished, it is often necessary to play near to, or on the bell of the cymbal."[8]

Striking Angle

Almost without exception, the typical angle of incidence for playing both triangles and cymbals is 90°; that is, the plane of the implement's stroke is perpendicular to the playing surface. However, the term "striking angle" used in this study refers not to the angle of incidence, but rather to the angle of the implement to the plane of the

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[1]Sam Denov, The Art of Playing Cymbals, p. 7.

[2]Leidig, Contemporary Percussion Technique, p. 12.

[3]Collins and Green, Playing and Teaching, p. 121.

[4]Morris Goldenberg, Modern School for Snare Drum, p. 92.

[5]Bartlett, Percussion Ensemble Method, p. 65.

[6]Thompson, personal letter.

[7]Sam Denov, "Techniques of Cymbal Playing," The Instrumentalist, XIX (September, 1964), 58.

[8]Leach, Percussion Manual, p. 38.

instrument at the moment of impact. In other words, a striking angle of 90° indicates that the implement is perpendicular to the plane of the instrument at impact; and a striking angle of 0° indicates that the implement is parallel to the plane of the instrument at impact.

Triangles

Although most of the literature concerning the triangle shows the implement perpendicular to the plane of the triangle at impact (90° striking angle), Ross states that the triangle should be struck on the horizontal leg with the implement "at about a 45 degree angle."[1] Peters states that "striking the triangle with the beater perpendicular [i.e., parallel to the plane of the triangle--0° striking angle] will produce a more diffuse sound with more overtones."[2] Thompson states that he generally strikes the triangle with "the stick vertical--not against gravity."[3]

Cymbals

Very little information is available concerning the striking angle used on cymbals, but Denov and Bartlett include illustrations which indicate that the implement is parallel to the plane of the cymbal at impact--0° striking angle.[4,5]

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[1]Ross, "The Triangle," p. 84.

[2]Mitchell Peters, "Triangle Technique," p. 82.

[3]Thompson, personal letter.

[4]Denov, Playing Cymbals, pp. 18-19.

[5]Bartlett, Teaching Percussion, pp. 71-72.

Pitch Levels

Triangles

Although different triangles will produce varied pitch levels, the word pitch should not be used "in a sense of definite pitch, for triangles are not tuned."[1] Peters very explicitly states that "it is out of the question to talk of pitch."[2] Wood agrees, saying that the triangle "gives numerous strong partials and no definite pitch."[3] Although Briggs has found that some triangles exhibit a "trace of fundamental at about six per milisecond, say round about [sic] 6,000 c/s,"[4] Stauder reports a fundamental of about 700 cps, with the most intense partials occurring between 7000 and 9500 cps.[5]

Contrary to the apparent majority opinion, Spohn emphasizes the point that "because of the variety of triangles which are available and the variation in pitch of different triangles of the same size, the teacher and student should select triangles by pitch for specific compositions."[6] However, the author is unable to find any information telling what pitches are to be found in what sizes of triangles, nor any evidence of music written for triangles tuned to definite pitches.

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[1]Mitchell Peters, "Triangle Technique," p. 79.

[2]Gordon Peters, "Treatise on Percussion," p. 280.

[3]Alexander Wood, The Physics of Music, p. 149.

[4]G.A. Briggs, Musical Instruments and Audio, p. 96.

[5]Wilhelm Stauder, "Schlaginstrumente--Akustik," Die Musik in Geschichte und Gegenwart, XI (1963), 1747.

[6]Spohn, Percussion, p. 51.

Cymbals

It seems to be commonly accepted that good cymbals "'are the one kind of instrument that doesn't have positive pitch. Instead, they have a rough dominant pitch,'"[1] often called the bell tone. Burns agrees, saying that a "good cymbal will sound all pitches or their harmonics simultaneously, even though it cannot be tuned to a specific note."[2] In fact, one company states that "'if any single note does dominate a cymbal's tone, it's obviously an inferior instrument.'"[3] Sewrey found that he could raise the overall pitch level of the cymbal "by playing closer to the cup, in order to bring out the higher overtones."[4]

One of the few published acoustical studies on musical instruments includes this statement relative to the pitch level of a cymbal: "The spectrum is particularly rich in high frequencies, the higher peaks lying above 8,000 cps."[5] And although Briggs reports finding several cymbal sounds up to 25000 cps, he states that "most of the energy is located in the range above about 5 kc/s [5000 cps]."[6]

When questioned, one professional percussionist commented on the influence of the shape of the bow on the pitch level of a cymbal:

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[1]Flagler, "Onward and Upward," p. 136.

[2]Roy Burns, The Selection, Use and Care of Cymbals in the Stage and Dance Band, p. 3.

[3]Flagler, "Onward and Upward," p. 138.

[4]James Sewrey, "Percussion Clinic," The School Musician, XXXIII (January, 1962), 14.

[5]L. Sivian, H. Dunn, and S. White, "Absolute Amplitudes and Spectra of Certain Musical Instruments," Journal of the Acoustical Society of America, II (January, 1931), 353.

[6]Briggs, Musical Instruments and Audio, p. 73.

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"The shape of the arc or bow is the most determining factor of high or low . . . cymbal sound. [A] flat and relatively straight bow produces a lower more bodied sound as a rule. The noticeably rounded arc suppresses to an extent the lower vibrations, and therefore allows a higher sound."[1]

General Sound or Timbre

Triangles

The following subjective terms have all been used in attempts to describe the desired triangle sound: "tinkling, shrill sound;"[2] "delicate, extremely high pitched 'tinkle;'"[3] "shimmering metallic quality which has considerable brilliance."[4] White compares a good triangle sound to the "shimmering sparkle of exquisite jewels."[5] Hart goes farther, saying that "the sound of the triangle is a tinkle. This can be frivolous, thunderous, exciting, melancholy, according to the taste and artistry of the person playing it."[6] One physicist simply states that the triangle "produces a jangle of partial tones."[7]

Firth recognizes that the size of the triangle has some influence on the general sound when he reports that the sound of a smaller one is

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[1]Thompson, personal letter.

[2]Brown and Musser, Percussion Studies, p. 5.

[3]Price, Triangle, Tambourine, and Castanets, p. 7.

[4]Bartlett, Teaching Percussion, p. 110.

[5]Charles White, Drums Through the Ages, p. 57.

[6]Hart, "Percussion Clinic," p. 69.

[7]Wood, Acoustics, p. 425.

"thinner in texture."[1] Leach agrees, saying that the "larger the triangle, the more it tends to sound a 'bong' rather than a 'ting.'"[2] Ross feels that there is a definite relation between the striking point and the general sound: "the tonal response on the bottom of the triangle amplifies the lower overtones, the upper side, the higher overtones."[3] Spencer reports finding the same relationships in his experimentation with various triangles.[4] But neither Ross nor Spencer provide any explanations for these relationships.

Although discriminating percussionists eventually come to associate the above subjective terms with various triangle sounds, to date there has been no systematic attempt to accurately describe these sounds in terms of their partial or overtone structures.

Cymbals

In general the use of subjective terms also characterizes most attempts to describe a good cymbal sound: "unique shimmering sound;"[5] "brilliant, crashy tone;"[6] "a thick quality."[7] Spohn feels that the "sound of a good cymbal will tend to rise after it is struck."[8]

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[1]Firth, Percussion Symposium, p. 30.

[2]Leach, Percussion Manual, p. 78.

[3]Ross, "The Triangle," p. 84.

[4]Spencer, personal letter.

[5]Leach, Percussion Manual, p. 38.

[6]Bartlett, Teaching Percussion, p. 12.

[7]Denov, Playing Cymbals, p. 7.

[8]Spohn, Percussion, p. 47.

Perhaps this is what Lang refers to when he says "it is especially important that its highs 'come out.'"[1]

Payson and McKenzie report that "there are several brands of cymbals and each brand has its own particular sound."[2] Sewrey apparently agrees, saying that there is a "wide difference between a [new] 'K,' an old 'K,' and an 'A' Zildjian cymbal."[3] (Avedis Zildjian--made in America since 1929; New K. Zildjian--made in Turkey since 1929; and Old K. Zildjian--made in Turkey before 1929.) Lang finds that an Old "'K' is somewhat thicker and with a wider range of overtones."[4] Thompson finds that when New K.'s are compared with Avedis Zildjians, "the K's sound more 'lows' with the tendency to become kind of 'brashy' the louder played."[5]

One rather prevalent opinion is that the general sound depends upon the striking point. For example, "if struck on the edge, it has a 'splashy' sound. If struck on the cup, it has a hollow and 'clanging' sound."[6] Thompson writes that he finds "the tremelo when played on [the] edges of cymbal . . . a different color than if the roll is played more

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[1]Morris Lang, "Percussion Clinic," The School Musician, XXXIII (December, 1961), 16.

[2]Payson and McKenzie, Music Educators' Guide, p. 52.

[3]James Sewrey, "Percussion Clinic," The School Musician, XXXIII (February, 1962), 54.

[4]Lang, "Percussion Clinic," p. 16.

[5]Thompson, personal letter.

[6]Firth, Percussion Symposium, p. 26.

'amidships.'"[1] He then attempts to explain this observation by saying that "probably the outside is the more easy to vibrate as this surface suggests that the action flows in toward the cup--when struck at a halfway point the force must flow both directions."[2]

Denov recognizes the influence of the cymbal's size on the overall sound when he relates that the cymbal tone "is enhanced in direct proportion to the amount of metal contained in the cymbal. Plurality of overtones and sustaining quality increases as the quantity of metal increases."[3]

Summary

It has been pointed out that very little concrete information is known concerning the vibrational aspects of triangles and cymbals. Further, many of the opinions and much of the material pertaining to these instruments is confusing, misleading, and contradictory. This means that, at the present time, percussionists have no way of knowing exactly what sounds any given instrument will produce without actually experimenting with different implements, striking points and angles, and striking forces.

This acoustical study of triangles and cymbals, coupled with the mathematical investigation of the triangles, should remove some of the uncertainty of percussion performance by enabling the public school music

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[1] Thompson, personal letter.

[2] Thompson, personal letter.

[3] Sam Denov, "Equipping the Cymbalist," The Instrumentalist, XVIII (June, 1964), 60.

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director and the performing percussionist to 1) predict the tonal results when the major performance variables are known (i.e., the dimensions and material of the instrument, the size and material of the implement, the striking point and angle, and the striking force), and 2) select an instrument and implement to produce the appropriate musical sound at the proper musical moment.

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II. EQUIPMENT AND PROCEDURE

Equipment

Instruments

Six triangles were used: three commercial 6" models marketed by Ludwig Industries, Sonor Company, and Avedis Zildjian Company; one professional 6" model manufactured by Alan Abel; and two Pigstail triangles (one 6" model and one 10" model) manufactured by the Mound Tool Company.

Five cymbals were used: two 16" models marketed by the Avedis Zildjian Company (A.) and Fred Gretsch Manufacturing Company (New K.); one 16" model manufactured by Zildjian (Old K.); and two Paiste cymbals (one 17" model and one 20" model) marketed by Ludwig Industries.

As the majority of these instruments were selected from large stocks by experienced professional percussionists (including Alan Abel, Maurie Lishon, Dick Schory, James Sewrey, and the late Harold Thompson[1]),

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[1] Alan Abel: presently percussionist with the Philadelphia Orchestra, and percussion innovator and inventor. Maurie Lishon: presently owner of Franks Drum Shop, Inc., formerly professional percussionist with name bands and staff percussionist with CBS-WBBM Radio in Chicago. Dick Schory: presently Senior Vice-President of Ludwig Industries, director of the Percussion Pops Orchestra, percussion clinician, formerly percussionist with the Chicago Symphony Orchestra and free-lance arranger and studio percussionist. James Sewrey: presently Product Manager and Educational Director of Ludwig Industries and percussion clinician, formerly public school music director and university percussion instructor, percussionist with the Wichita (Kansas) Symphony Orchestra. Harold Thompson: formerly percussionist with the Boston Symphony Orchestra and consultant for the Avedis Zildjian Company.

it may be assumed that these instruments are of above-average quality according to present standards. Although the two Pigstail triangles are not commercially available today, the above instruments are typical in shape and size of the instruments used by both amateur and professional percussionists.

The generalized triangle in Figure 1 illustrates the various measurements made for this study. The lengths of the segments were measured from the open ends to the dotted lines. These numbers are placed inside the outline of the triangle. The diameters of the ends of each segment are located outside the outline. The angles formed by the legs of the triangle are indicated within the outline, as is the total length. All length measurements were made with a ruler graduated in tenths of inches and all diameters were measured with a micrometer graduated in thousandths of inches. The dimensions and distinguishing characteristics of each of the triangles used for this investigation are shown in Figures 2 through 7.

The generalized cymbal in Figure 8 illustrates the various measurements made for this study. The diameters of the hole, the cup, and the entire cymbal are given in the upper portion. In the lower portion, the height of the cup, the height of the bow, and the thickness are indicated. Except for the thickness (measured with a micrometer graduated in thousandths of inches), all measurements were made with a ruler graduated in tenths of inches. Dimensions and distinguishing characteristics of the cymbals used in this investigation are shown in Figures 9 through 13.

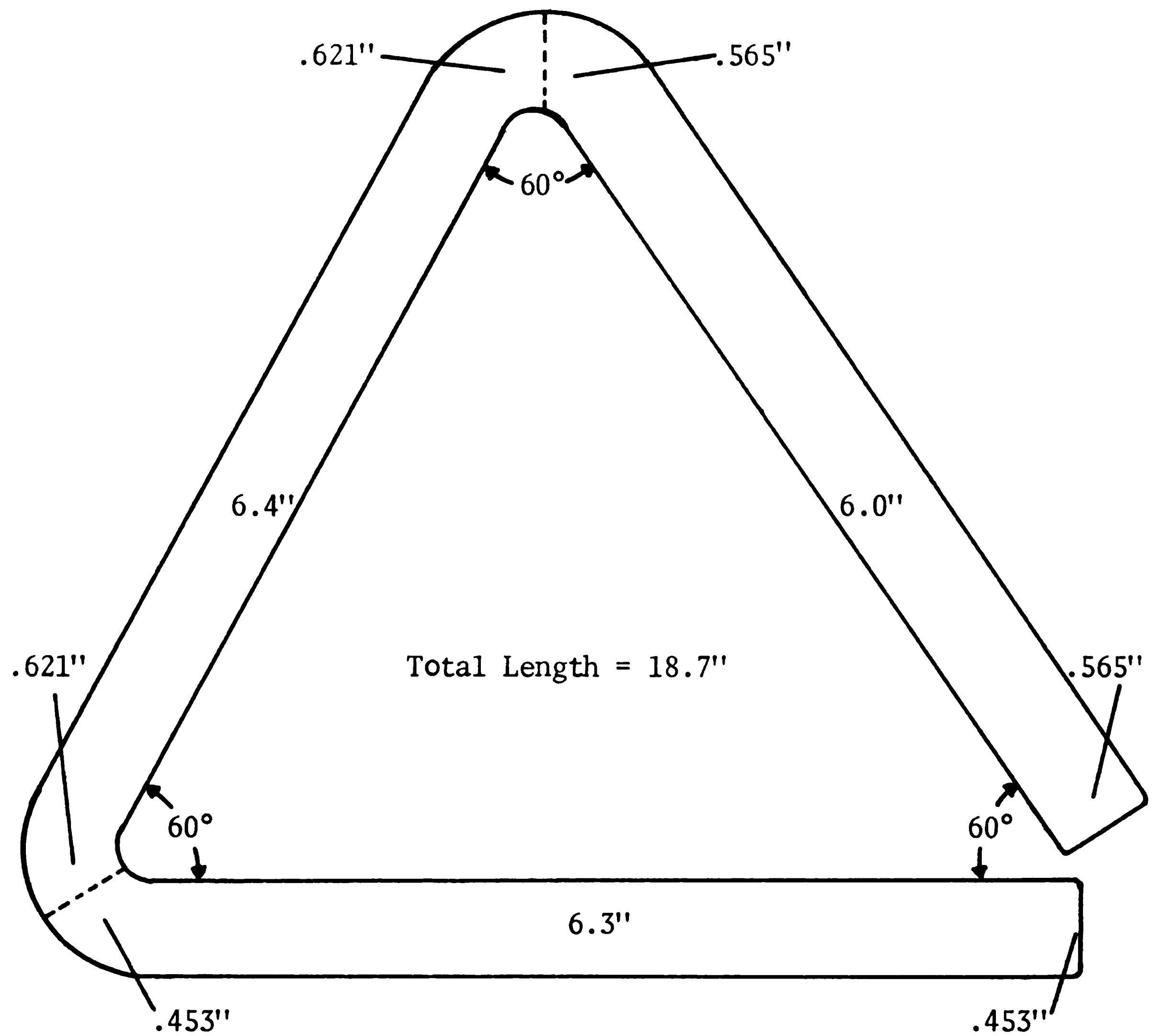


Figure 1.--Generalized Triangle

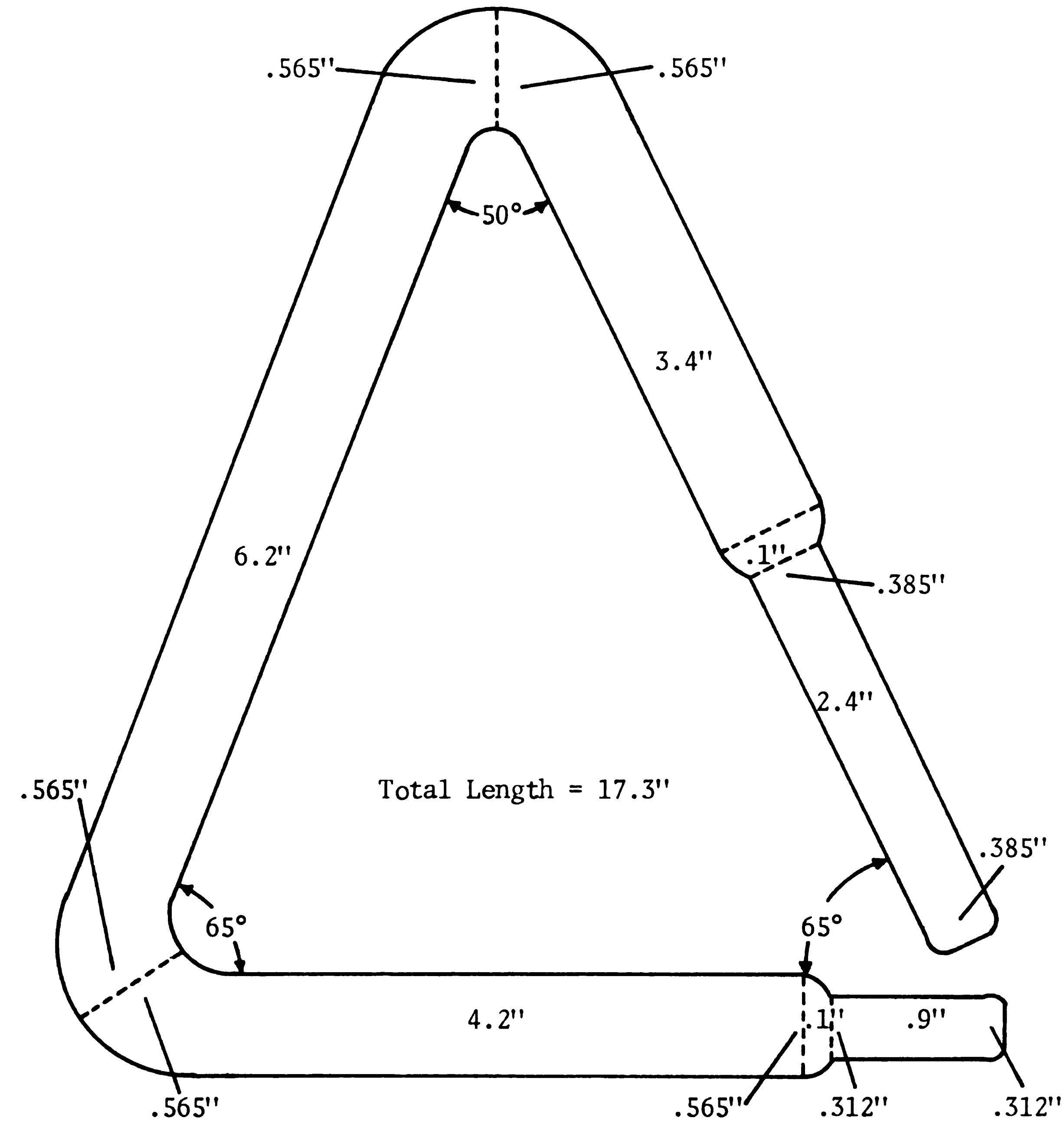


Figure 2.--Abel Triangle

Distinguishing characteristics: isocoles shape; smaller segments of unequal length and diameter; relatively large overall diameter.

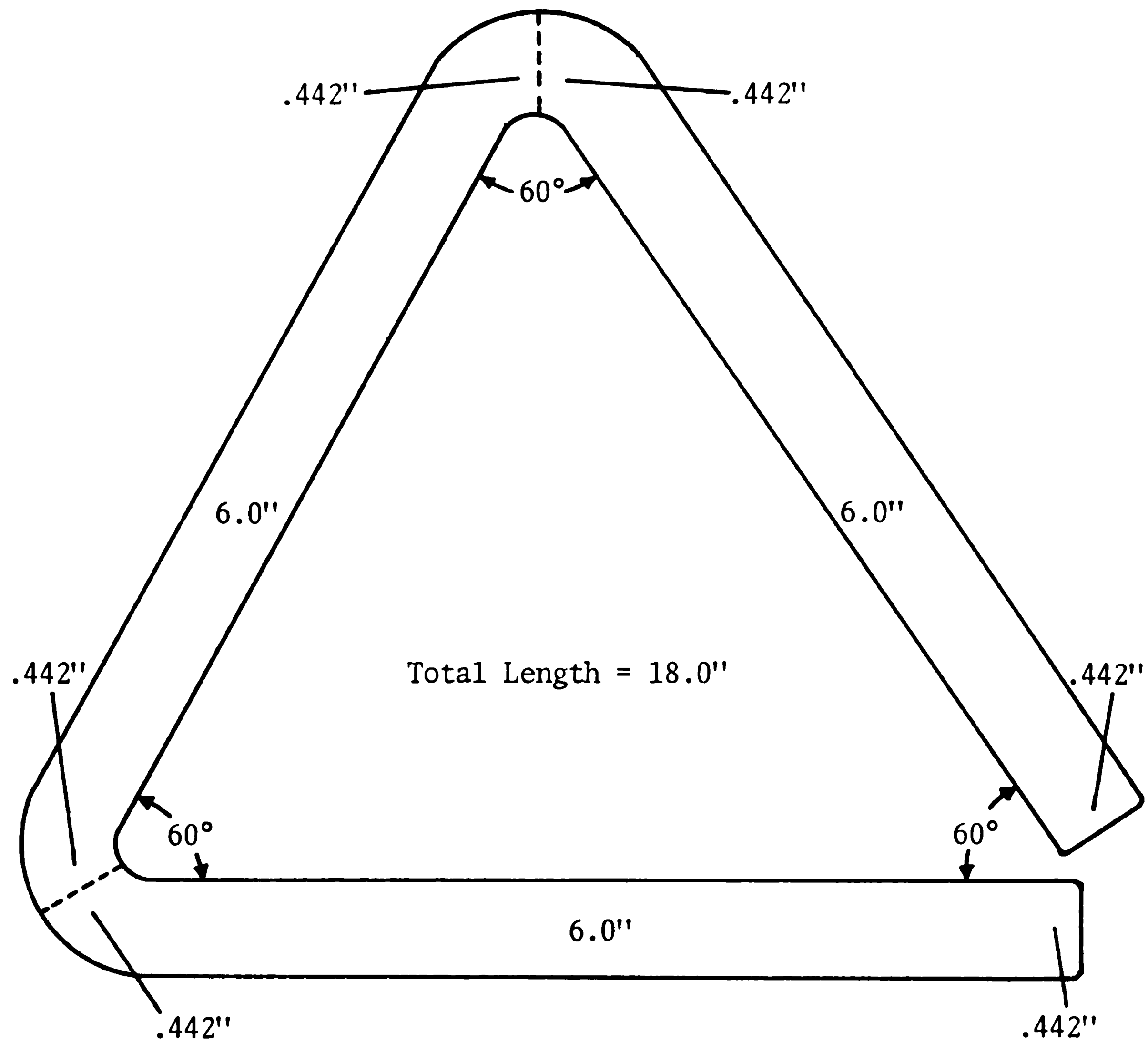


Figure 3.--Ludwig Triangle

Distinguishing characteristics: equality of all measurements.

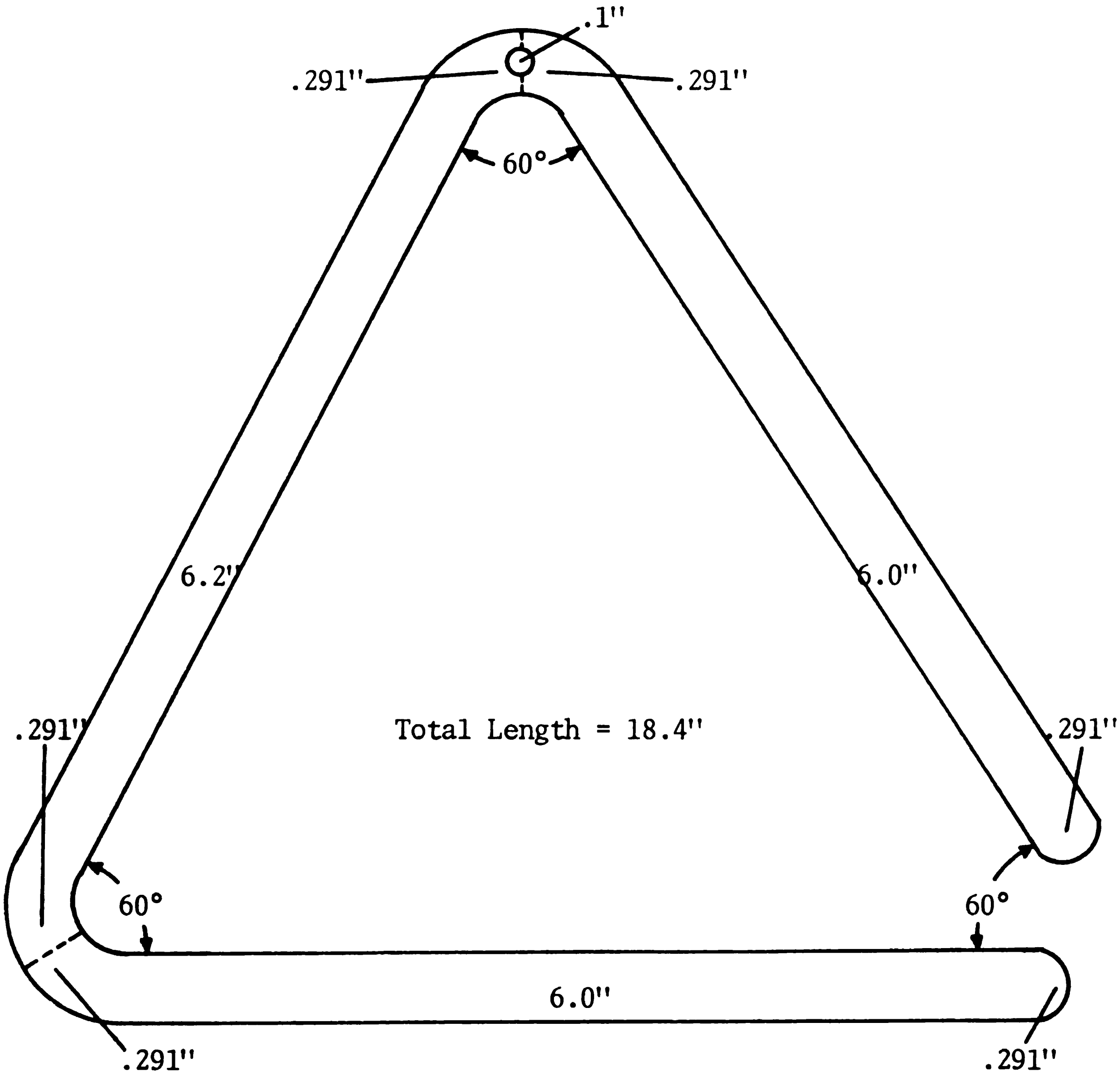


Figure 4.--Sonor Triangle

Distinguishing characteristics: equal angles; equal, and relatively small, diameter at all points; small hole in apex.

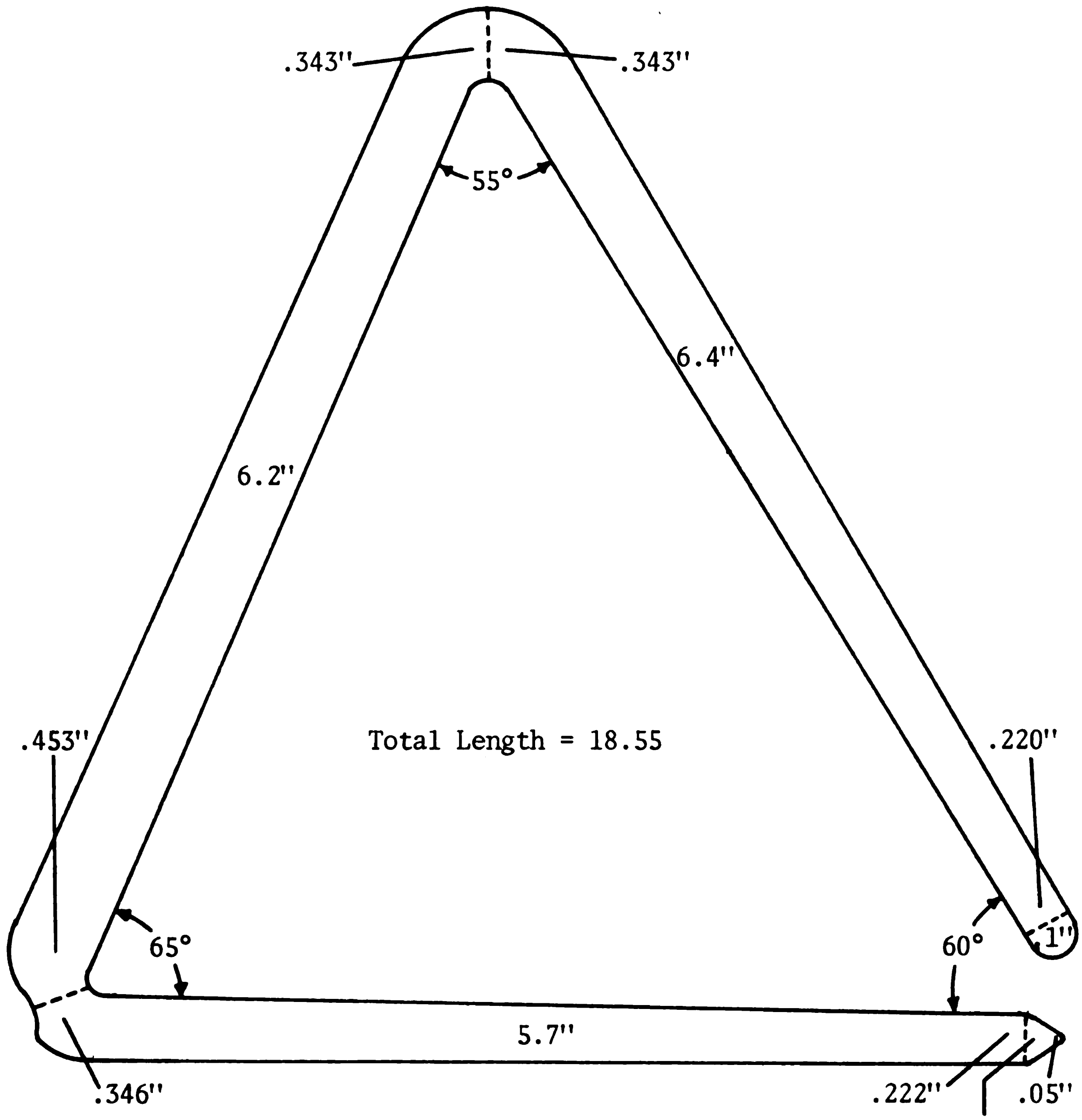


Figure 5.--Zildjian Triangle

Distinguishing characteristics: unequal angles; tapered segments of unequal lengths.

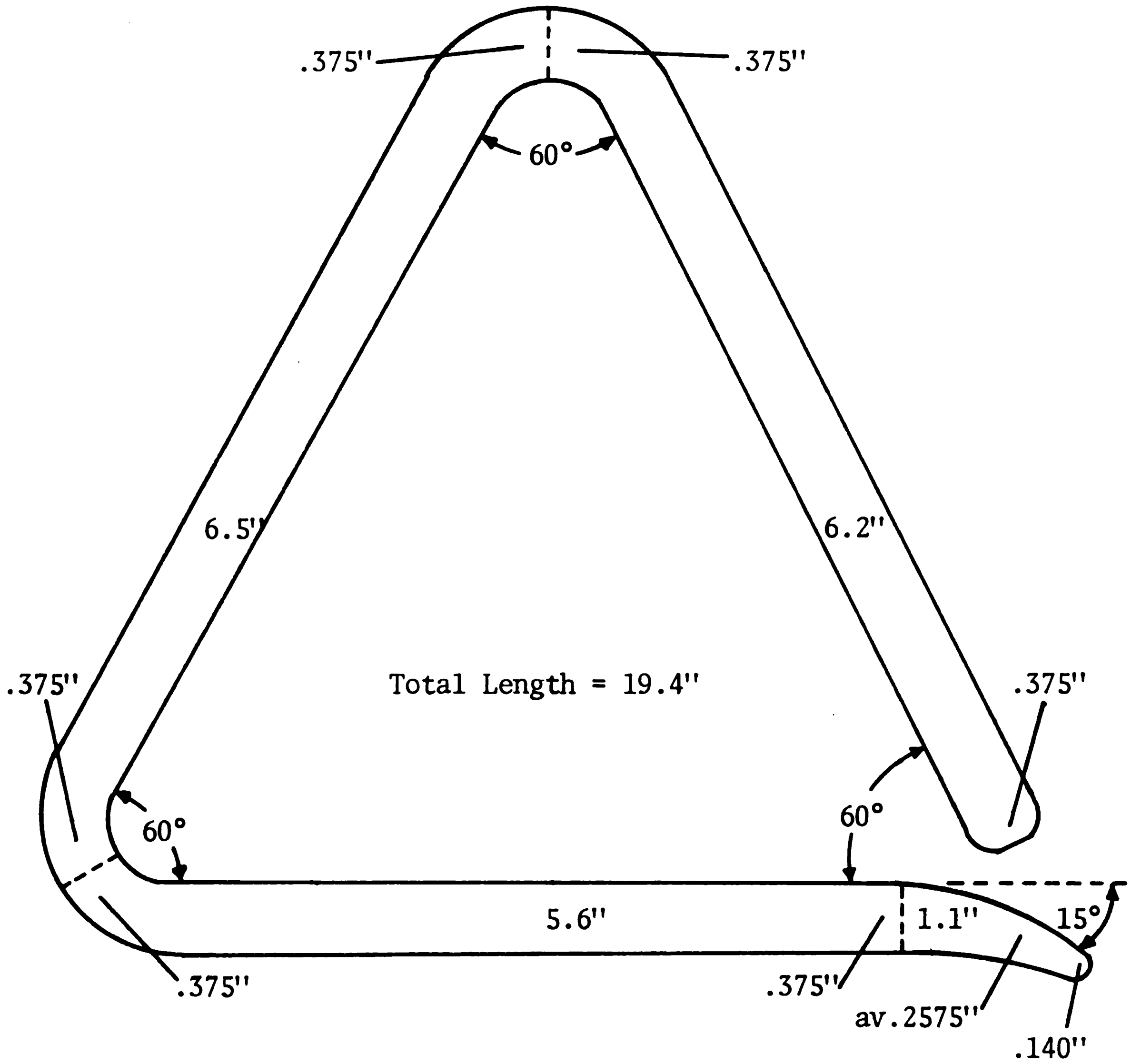


Figure 6.--6" Pigstail Triangle

Distinguishing characteristics: tapered segment on lower leg; equal angles; equality of diameter (with exception of tapered segment); downward angle of tapered segment.

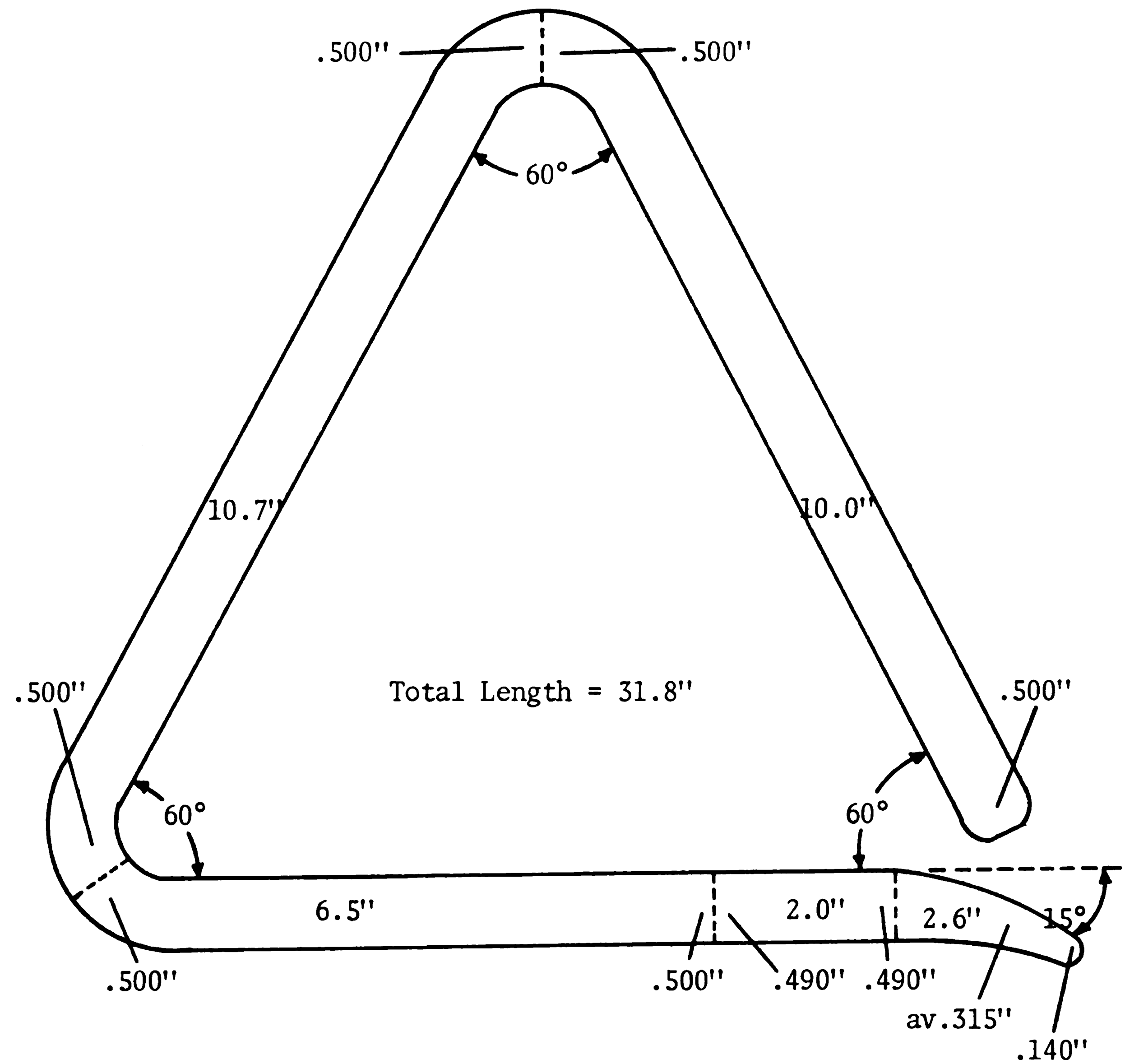
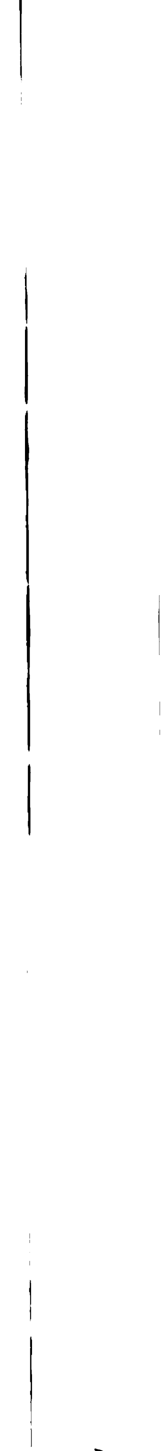


Figure 7.--10" Pigstail Triangle

Distinguishing characteristics: tapered segment on lower leg; equal angles; equality of diameter (with exception of tapered segment); downward angle of tapered segment.



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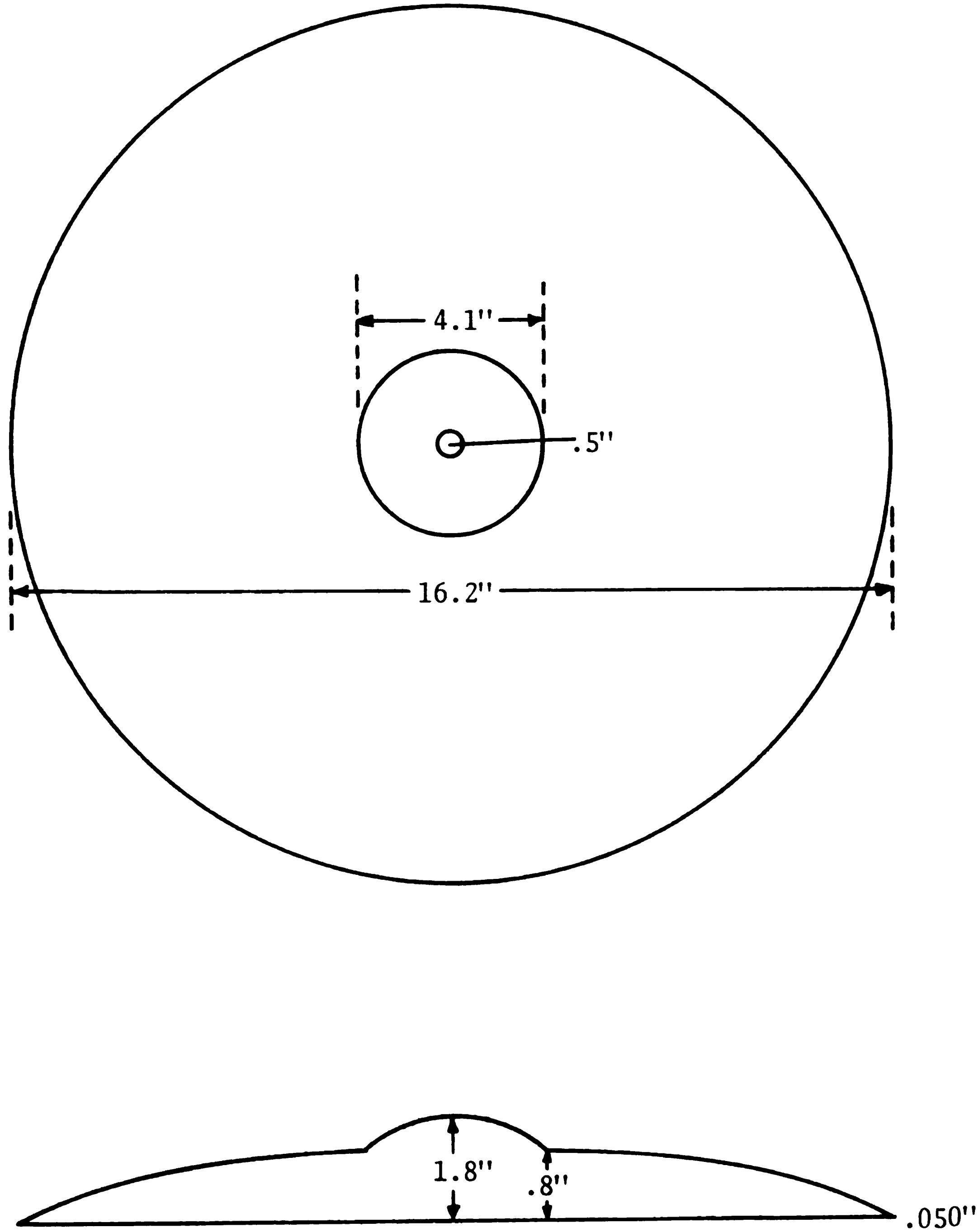

Figure 8.--Generalized Cymbal

Figure 9.--

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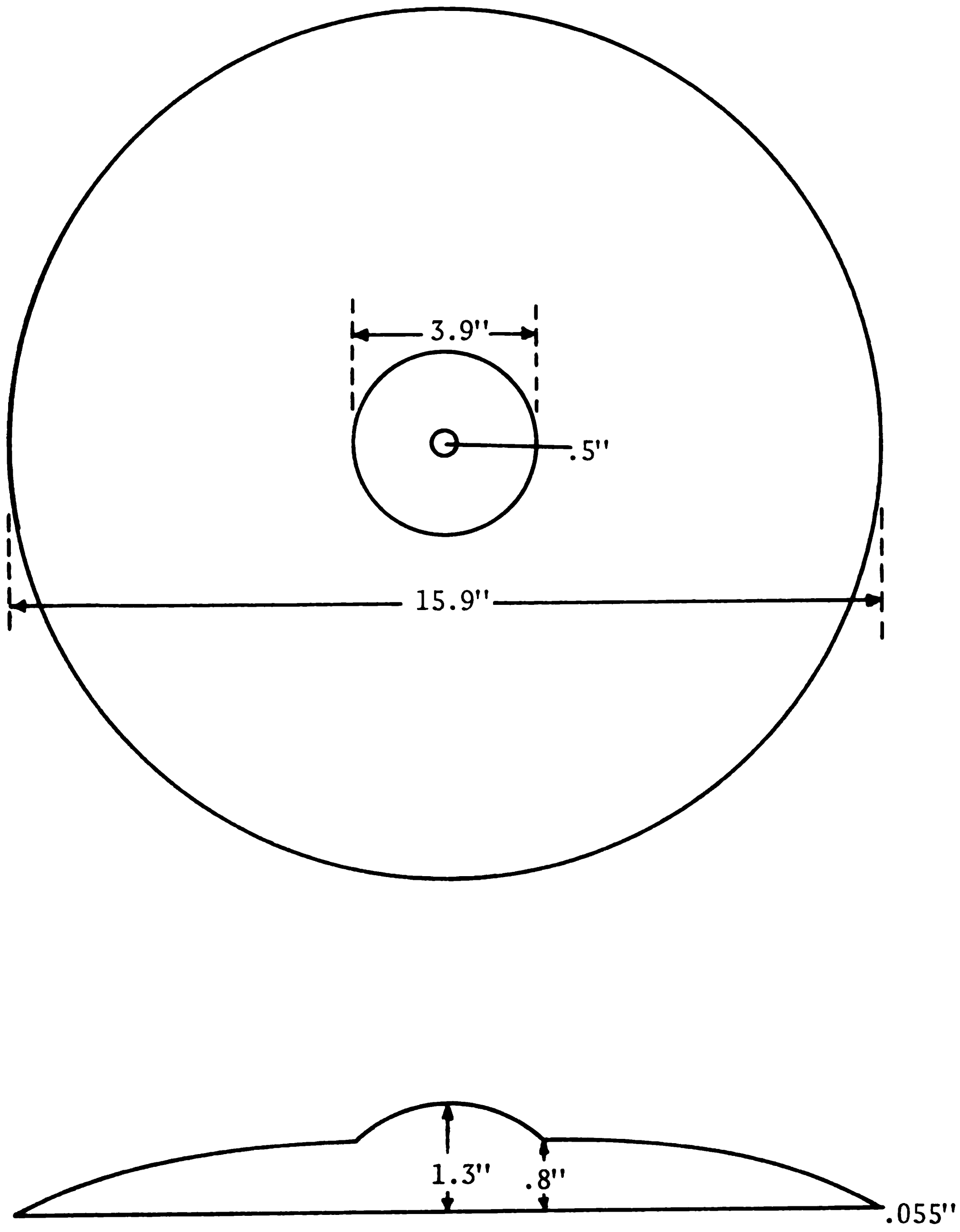

Figure 9.--Avedis Zildjian Cymbal

Distinguishing characteristics: no unique features compared to the other cymbals.

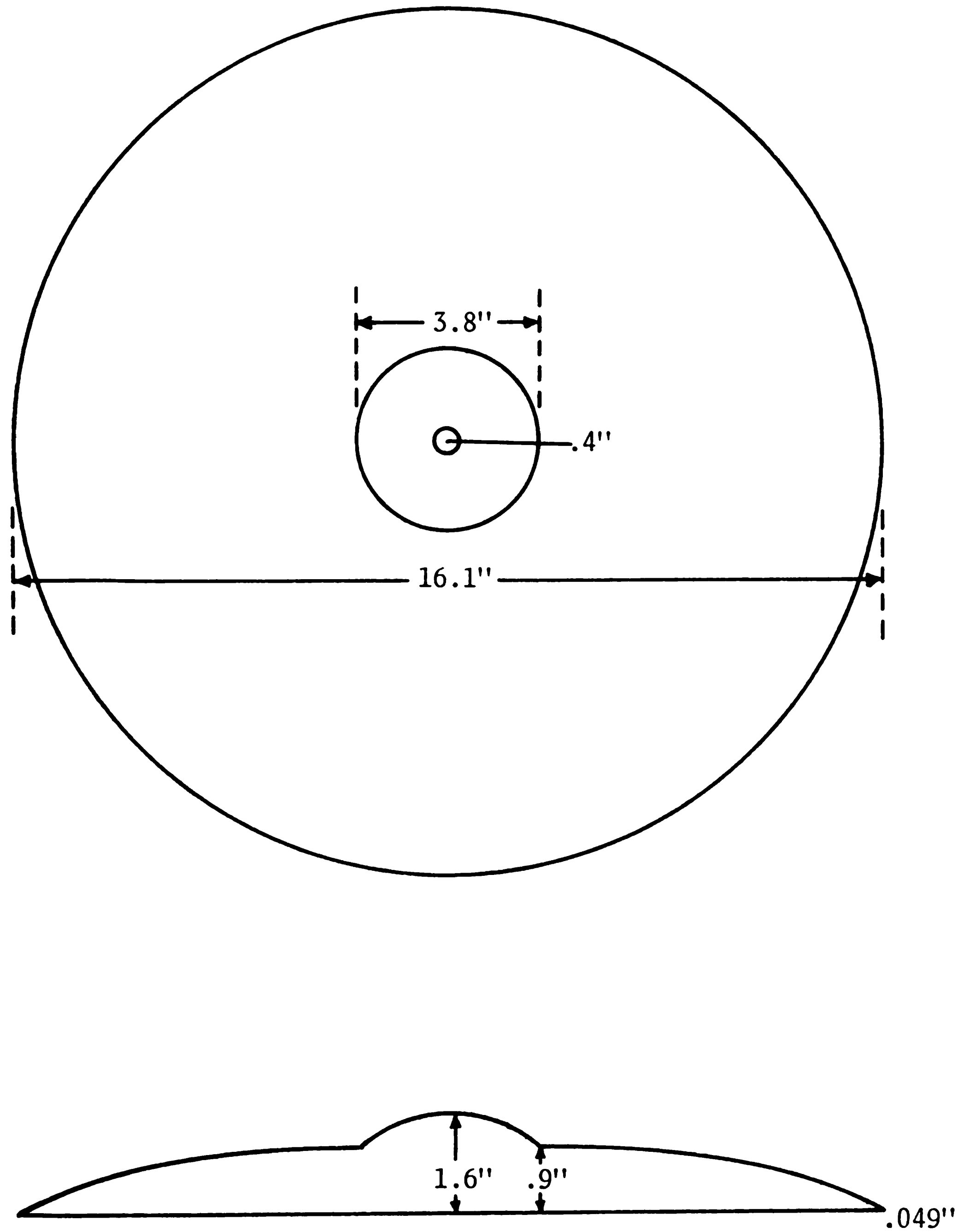

Figure 10.--New K. Zildjian Cymbal

Distinguishing characteristics: relatively small cup diameter; relatively high cup.

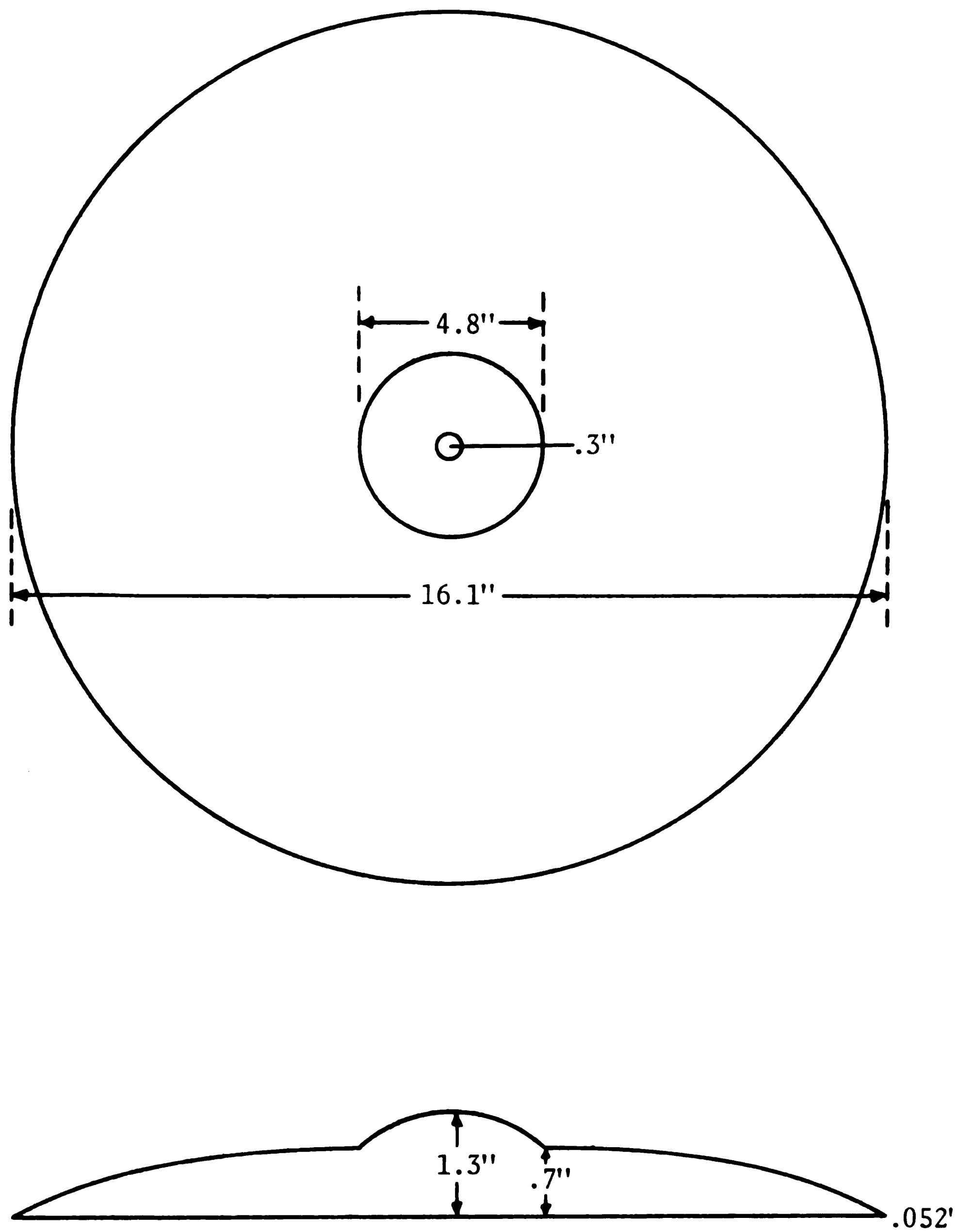

Figure 11.--Old K. Zildjian Cymbal

Distinguishing characteristics: relatively large cup diameter; flat bow; small hole.

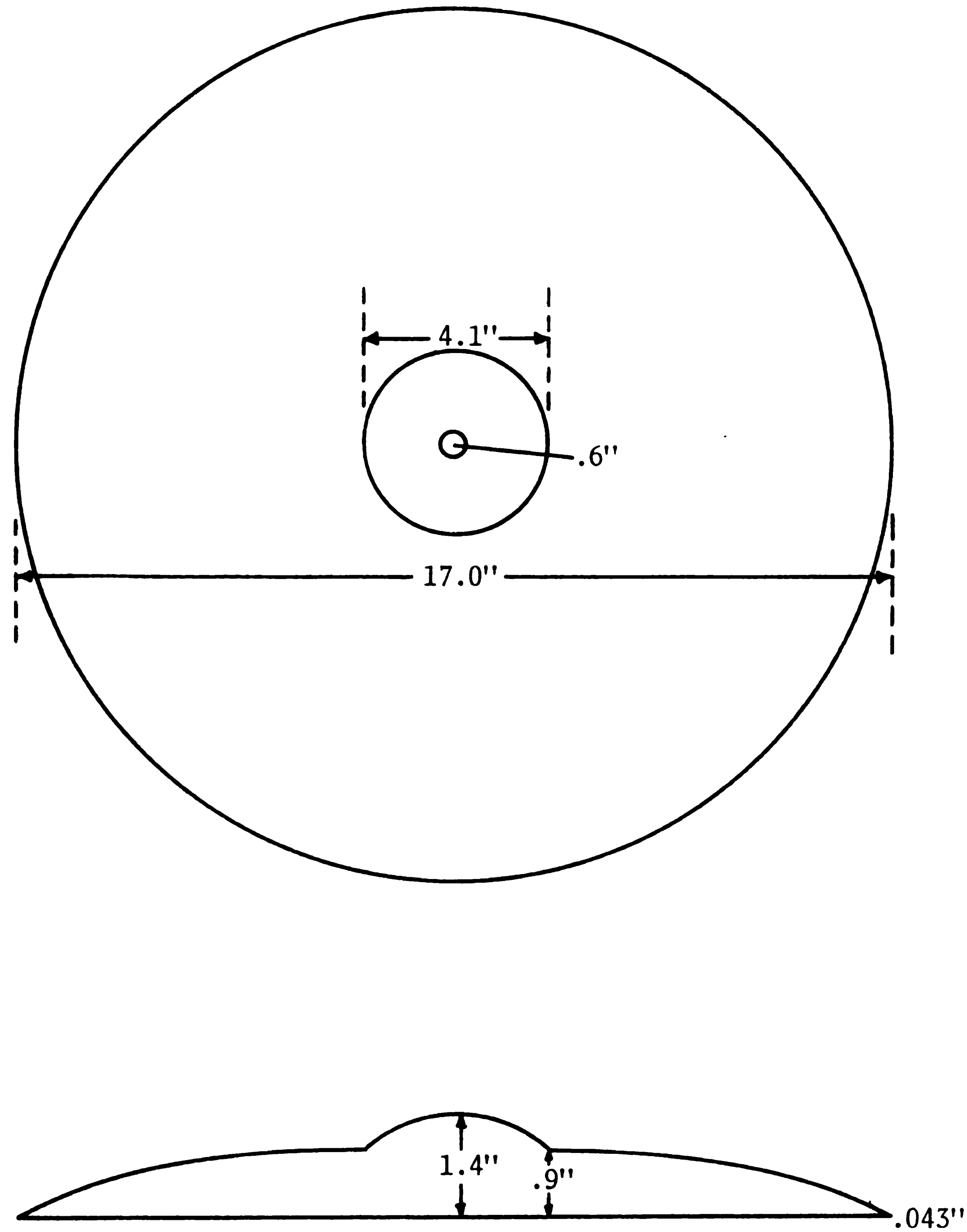

Figure 12.--17" Paiste Cymbal

Distinguishing characteristics: largest hole of all cymbals studied; thinnest of all cymbals studied.

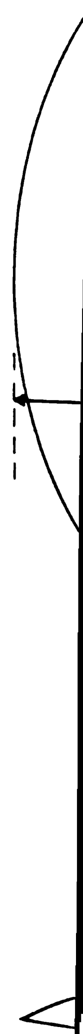

Figure 13.--

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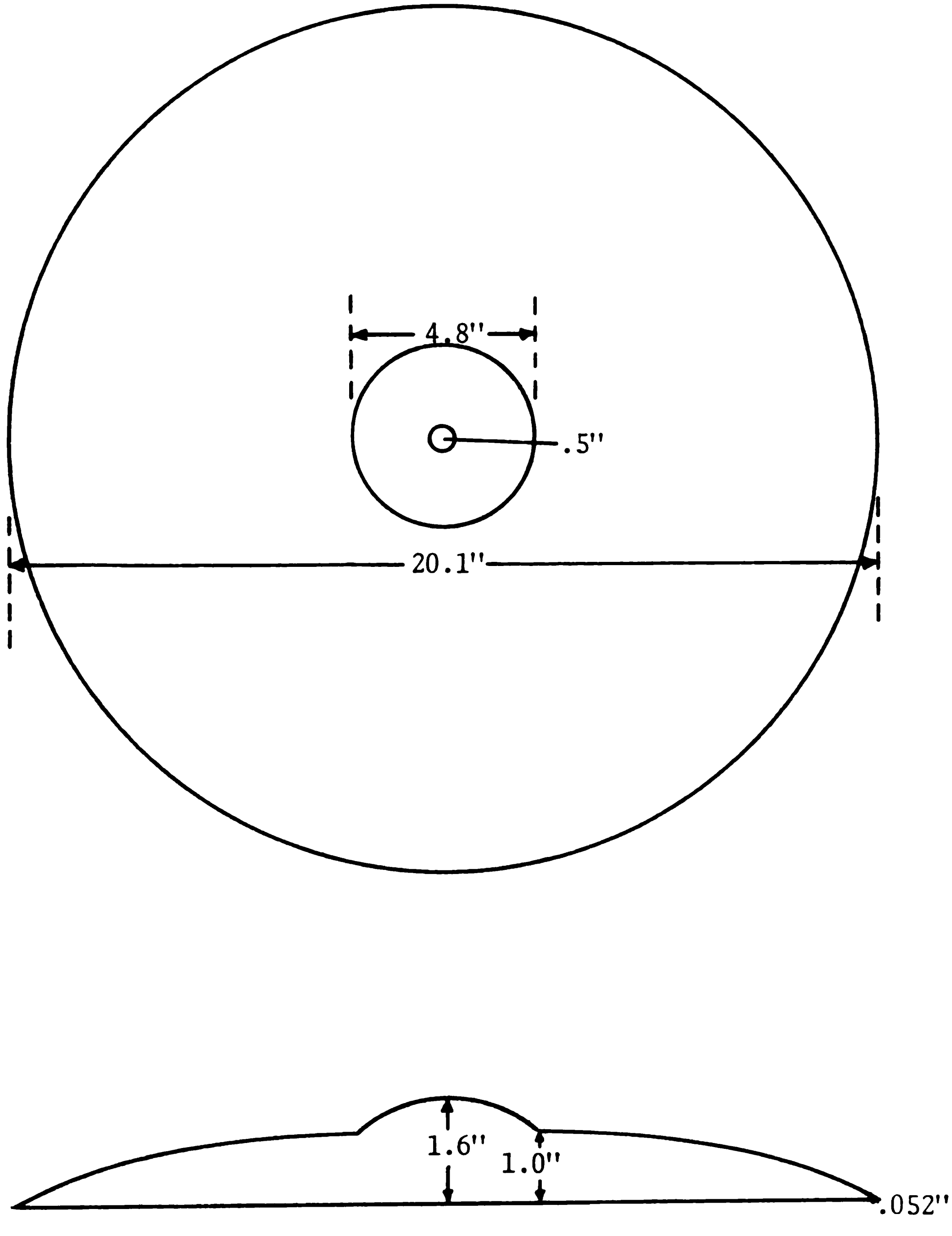

Figure 13.--20" Paiste Cymbal

Distinguishing characteristics: cup height same as New K. Zildjian; largest bow height; largest overall diameter of all cymbals studied.

Suspension Setups

The triangles were suspended by a heavy-wire "Podemski" holder, insulated with rubber tubing, with a supporting cord of dental floss. All triangles were held so that the top of each was five feet from the floor.

The cymbals were supported by a regular floor stand, consisting of a Premier flush-base with rubber feet, a metal shaft, and a Slingerland cymbal tilter. The tilter post was insulated with rubber tubing and the cymbal rested on a felt washer. The stand was adjusted so the edges of all the cymbals were three feet from the floor.

These suspension setups are typical of those used by both amateur and professional percussionists.

Implements

Three triangle implements were used: 7/32" (.218") x 9" in drill rod and cold-rolled steel, and 5/32" (.156") x 9" in drill rod. Cold-rolled steel is relatively soft with a carbon content of .15 to .25 per cent, while drill rod is much harder with a carbon content of at least .85 per cent.[1]

Three implements were used on the cymbals: Musser yellow yarn (M8); Musser red yarn (M6); and Deagan brown cord with red stitching (#2014-C). The Musser series of yarn implements is color and number coded, with the yellow (M8) being larger and softer than the red (M6). The Deagan series of brown cord implements is also color and number

[1]Ernest Edgar Thum and Richard Edward Grace, "Iron and Steel--Classification and Uses of Plain Carbon Steels," Encyclopaedia Britannica, 1963, XII, 666.

coded, with the #2014-C being the hardest of a set of four. Each implement head was mounted on a 3/8" x 13" birch shaft.

These implements are typical of those used by both amateur and professional percussionists.

Striking Mechanism

The basis of the striking mechanism (Figure 14) was an "Eaton's Vibration Demonstrator" (#3325), a flat-spring apparatus built by the Welch Scientific Company. This apparatus was mounted on a heavy metal stand, adjustable for the proper striking angles, and insulated with rubber tubing. Wooden blocks fastened to the springs were drilled out to accomodate the various implement shafts. An arbitrary scale was also fastened to the apparatus in order to help maintain continuity of the striking forces.

Striking Points and Angles

The three striking points used on the triangles are shown in Figure 15. The first point (<u>top</u>) was located 1" down on the closed side of the small triangles, and 1 1/2" down on the closed side of the 10" Pigstail triangle. The second point (<u>corner</u>) was located on the bottom side 1" from the closed corner on the small triangles, and 1 1/2" from the closed corner on the 10" Pigstail triangle. The third point (<u>bottom</u>) was located at the midpoint of the bottom side on all of the triangles.

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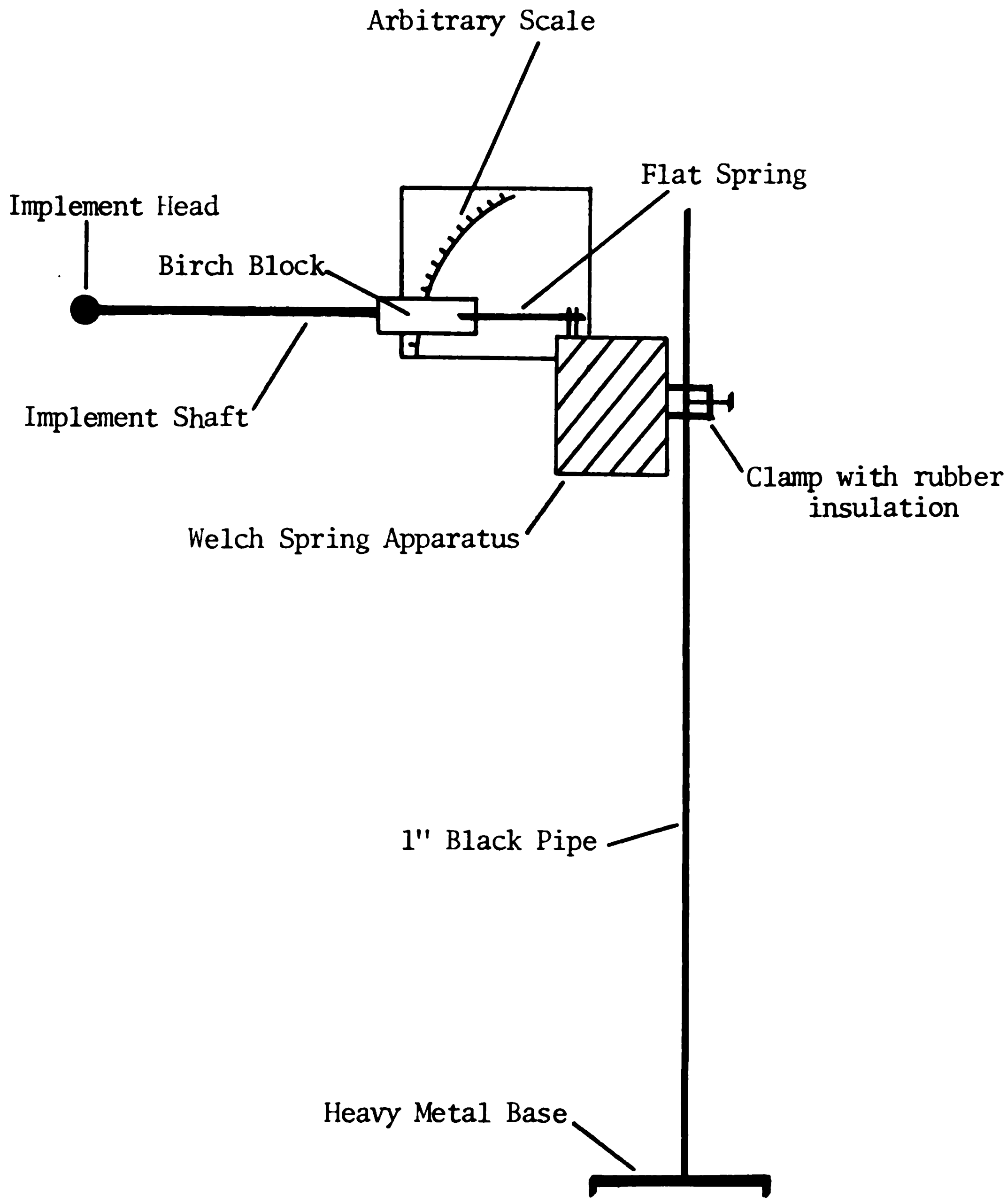

Figure 14.--Striking Mechanism

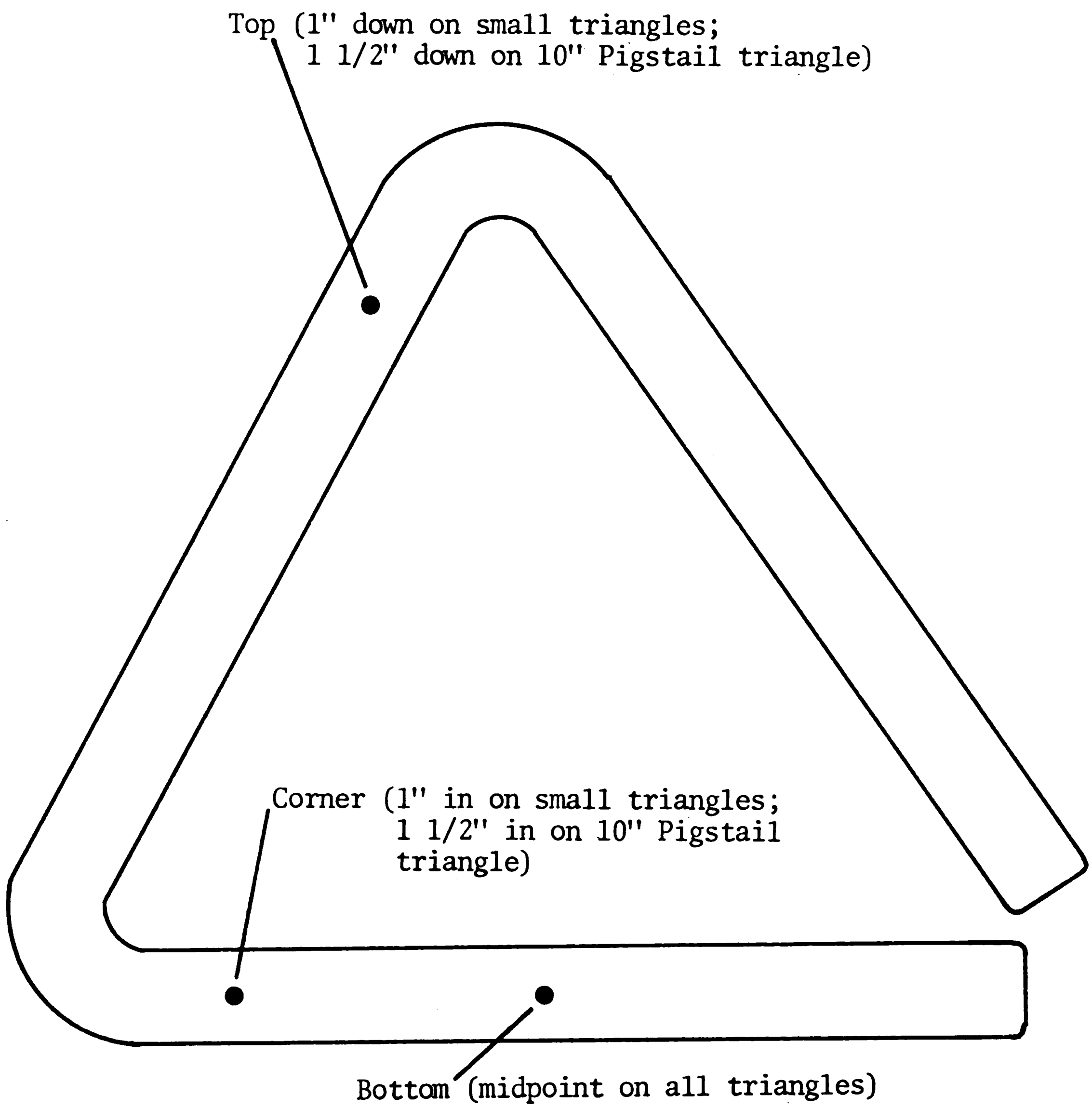

Figure 15.--Striking Points Used on Triangles

At the top and bottom striking points, the implement was perpendicular to the plane of the triangle at impact (90° striking angle). At the corner striking point, the implement was parallel to the plane of the triangle at impact (0° striking angle).

The two striking points used on the cymbals are shown in Figure 16. The first point (edge) was located 1" from the edge on the small cymbals, and 1 1/2" from the edge on the 20" Paiste cymbal. The second point (cup) was located near the cup, but was still measured from the edge: 5" from the edge on the Avedis Zildjian, the New K. Zildjian, and the Old K. Zildjian cymbals; 5 1/2" from the edge on the 17" Paiste cymbal; and 6 1/2" from the edge on the 20" Paiste cymbal.

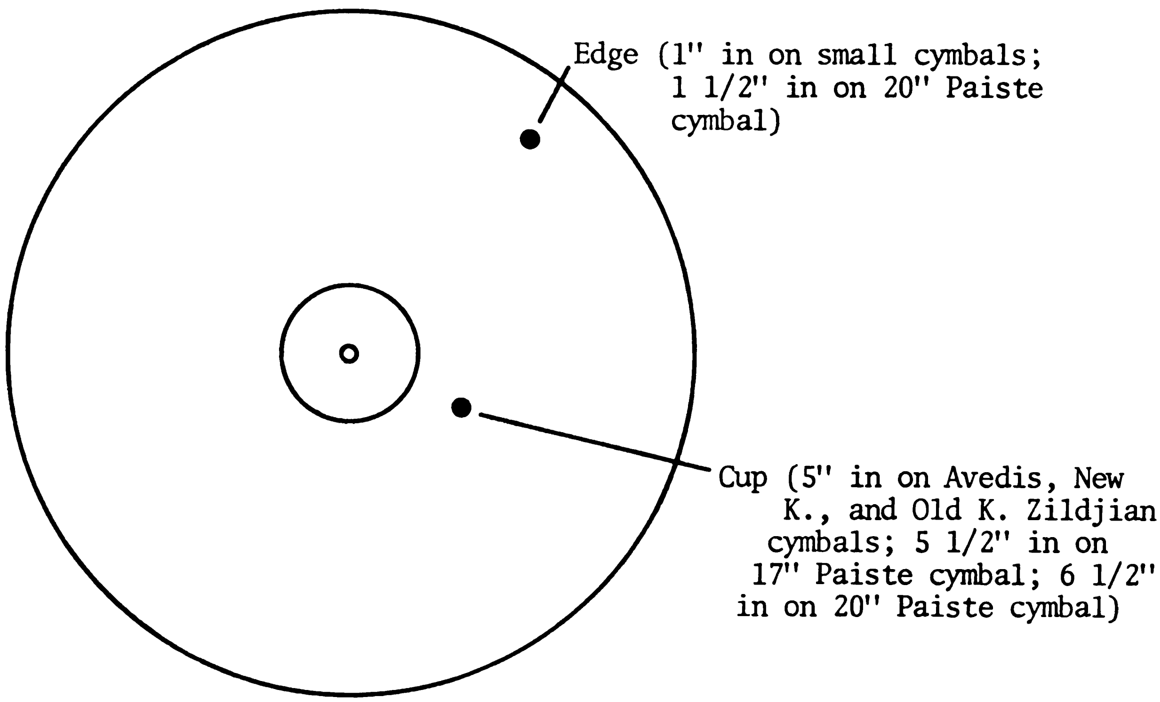

Figure 16.--Striking Points Used on Cymbals

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At both striking points, the implement was parallel to the plane of the cymbal at impact (0° striking angle).

These striking points and angles are typical of those used by both amateur and professional percussionists.

Striking Force or Loudness

The force or loudness of the various dynamic levels used (ff, mf, and pp) was measured by a sound level meter built by General Radio Company (#1551-C). The meter was held at the same position as the microphone--at a distance of 5 1/4' from the instruments and at a height of 6 1/2'. The meter was set on "fast" and the weighting was set on "C" (thus insuring equal influence from 20 to 20000 cps, limited only by the capabilities of the microphone).

The dynamic levels and their decibel equivalents at impact for both triangles and suspended cymbals are shown in Table 1.

Table 1.--Decibel Levels at Impact

Dynamic Level	Decibel Level	
	Triangles	Cymbals
ff	89	95
mf	80	85
pp	74	78

Consistency of these levels was maintained throughout the recording sessions by a combined use of the arbitrary scale on the striking mechanism and the VU meter on the recorder.

Recording Equipment and Studio

The microphone used was a Neuman condensor microphone (#C-47/64). For both the triangles and cymbals, the microphone was positioned 5 1/4' from the instruments at a height of 6 1/2'.

The recorder used was a Magnecord (#1028). The recording was done on "Channel 1," with the record level set at "4" and with a tape speed of 15" per second. The tape used was Scotch #210, cut into 31" loops spliced with Scotch splicing tape.

The overall dimensions of the recording studio were: length = 21', width = 17 1/4', and height = 8 1/4'. The locations of the microphone stand, the instrument stands, other equipment, and miscellaneous furniture, as well as the presence of various wall materials, are indicated in Figure 17. The numbers inside the outlines of the filing cabinets and bookshelves refer to the heights of these items.

Playback and Analyzing Equipment

The playback machine was an Ampex console-mounted Recorder/Reproducer (#AG-350). The playback levels used were "8" for the triangles and "7" for the cymbals. The sounds of the instruments were analyzed by a Bruël and Kjaer Frequency Analyzer (#2107) and the resulting graphs were printed out by a Bruël and Kjaer Level Recorder (#2305). Figures 18, 19, and 20 show the playback and analyzing setup, as well as the settings used on the analyzing equipment.

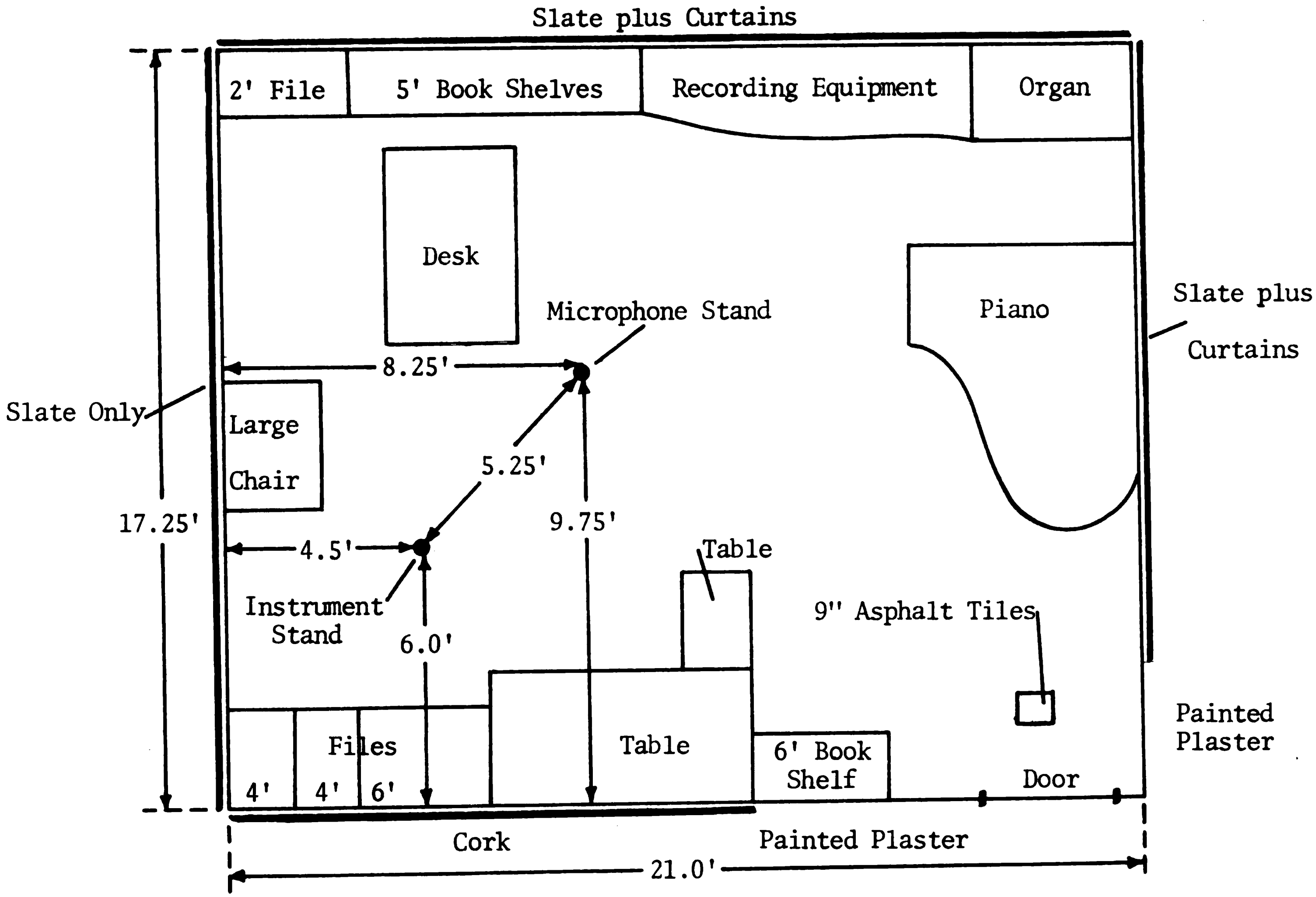

Figure 17.--Recording Studio

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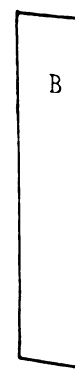

Figure 18.-

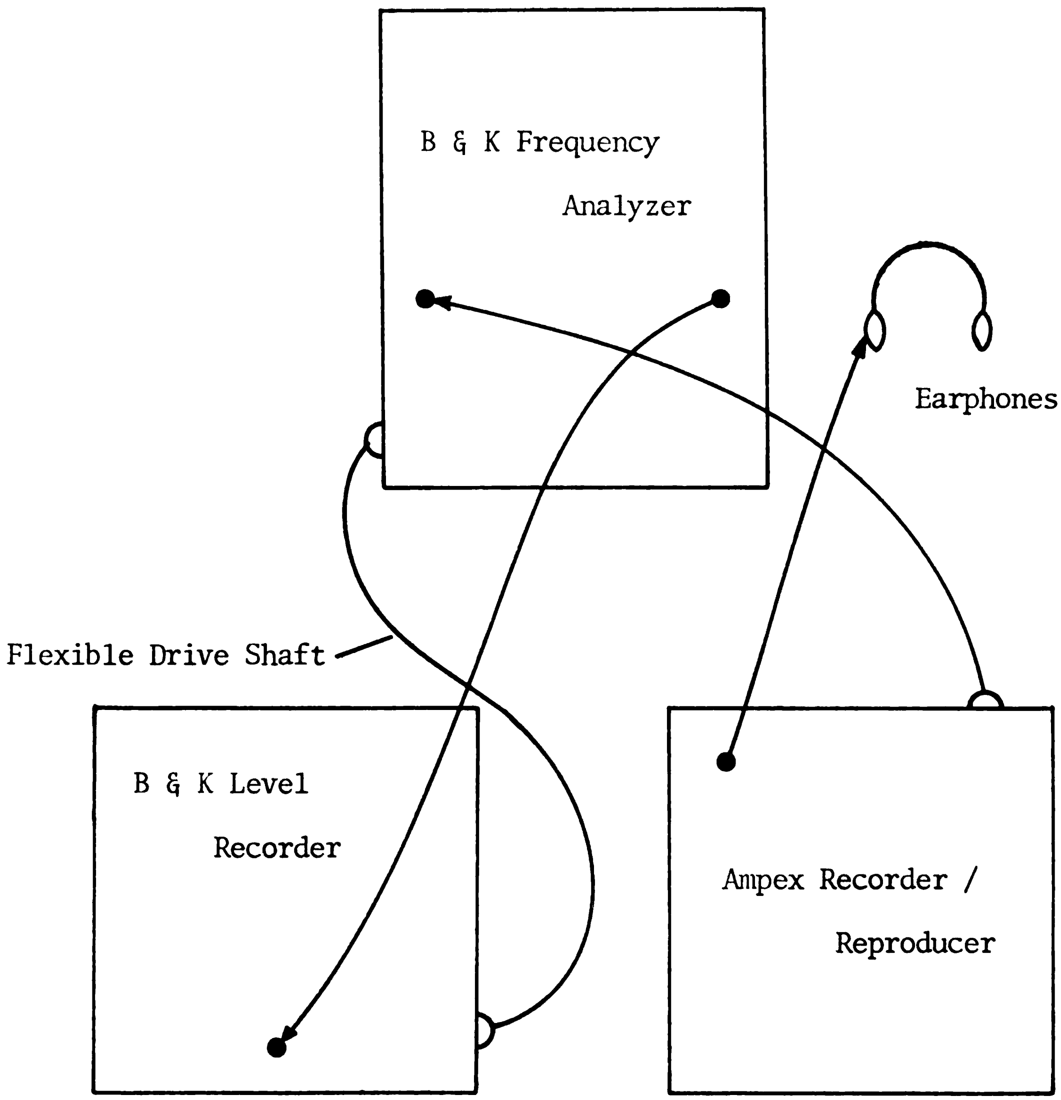

Figure 18.--Playback and Analyzing Equipment

Input Potentiometer

"5"

Meter Switch

"Fast - RMS"

"Direct"

Range Multiplier

"-20 dB x 0.1"

Meter Range

"80 dB SL
-40 dB
100 mV"

Frequency Rejection

"Balance"

Weighting Network

"Linear 20 - 40,000"

Frequency Analysis
Octave Selectivity

"40 dB"

Frequency Range - c/s

"200 - 630

630 - 2000

2000 - 6300

6300 - 20,000"

Function Selector

"Auto"

Figure 19.--Settings on Frequency Analyzer #2107

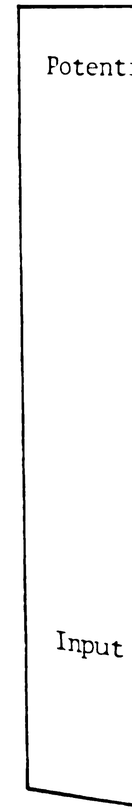

Figure 20

Potentiometer Range - dB

"50"

Lower Limiting
Frequency - c/s

"20"

Rectifier Response

"RMS"

Writing Speed -
mm/sec.

"100"

Paper Speed -
mm/sec.

"10_1"

Drive Shaft Speed -
rpm

"0.36"

Input Attenuator

"30"

Input Potentiometer

"4"

Figure 20.--Settings on Level Recorder #2305

Procedure

Experimental

Following the proper placement and adjustment of all necessary equipment, the various sounds that were produced on the triangles and cymbals were recorded. To avoid the influence of initial or impact transient sounds, the recorder was activated slightly less than one second after impact.[1] Within the limits of the operator's reflexes, there was little or no overlap of recorded sound on the tape loops. This meant that a full two seconds of analyzable sound was obtained.

The tape loops were then played back through a reproducer and the sounds analyzed by a frequency analyzer. The resulting graphs of the sounding partials (from 20 to 20000 cps) and their relative strengths or intensities in decibels were printed out by a level recorder (see Appendix B for examples of the graphs).

Mathematical

The mathematical investigation of the triangles' vibrational behavior was accomplished through the use of the finite element method. The principle of the finite element method is: "a structure may be satisfactorily represented by an assembly of discrete elements having simplified elastic properties,"[2] and which are connected with each other at a "finite number of nodal points."[3] In this investigation, each

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[1]Although the author recognizes that initial transients are an important aspect of instrumental timbre, their influence was intentionally avoided in this study.

[2]Robert W. Little, "Finite Element Method," p. 1.

[3]O.C. Zienkiewicz, The Finite Element Method in Structural and Continuum Mechanics, p. 1.

triangle was represented by fifteen elements having sixteen nodal points, with point sixteen fixed to eliminate rigid body motions (Figure 21).

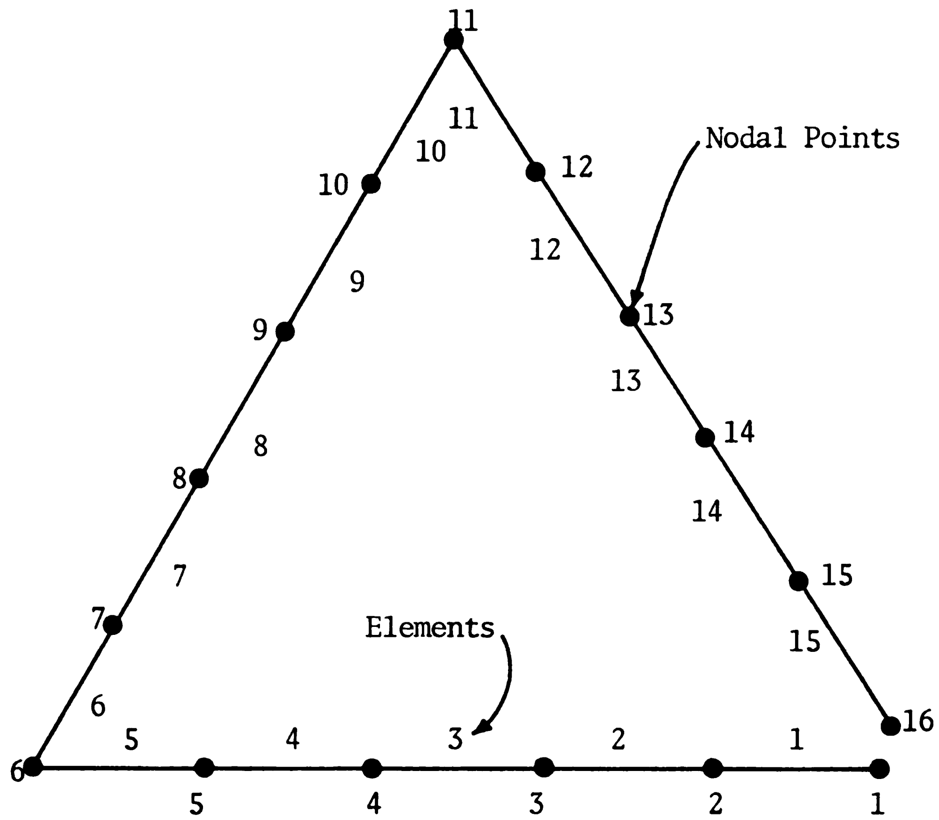

Figure 21.--Numbering of Elements and Nodal Points

On the basis of each element's physical characteristics and geometry, the nodal forces and displacements (axial, bending, and torsional) may be calculated for each individual element. The displacements, forces, stiffness characteristics, and mass relevant to each element are expressed in matrix form. When these individual matrices are properly assembled into large matrices for the entire triangle, the resulting matrix equation can then be solved for the resonant frequencies.

These resonant frequencies (expressed in cycles per second--cps) are functions of the expression $\sqrt{\frac{E\ I}{\rho\ A\ l^2}}$ (the constants which can be factored out of the matrices mentioned above). Two of these constants are material properties: Young's modulus of elasticity--E; and mass density--ρ (steel = $\frac{.283}{386}$). The other constants are geometric properties: the moments of inertia resisting twist about the y and z axes--I ($\frac{\pi d^4}{64}$ where d = diameter); the cross-sectional area--A; and the length--l. Therefore, with a minimum of calculation, it is possible to ascertain the relative influence on the frequency of a change in material, diameter, or length.

Another material property which should be noted here--damping--has been described by Wood: "Any source of sound if set in vibration and left to itself vibrates in its own natural frequency, producing a note which gradually dies away . . . but remains constant in pitch."[1] Thus, triangles made of metals with differing damping properties will produce sounds of varying duration. This particular property, however, is independent of any of the above material or geometric properties and does not affect the frequency in any way.

The actual generation and manipulation of the matrices was done by the Structural Analysis and Matrix Interpretive System (SAMIS) computer program developed by the Philco Corporation, Western Development Laboratories, under contract to and in association with the Jet Propulsion Laboratory. The objective of this program is "to automate analysis of structures composed of . . . line elements with uniform cross-sections.

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[1]Wood, Physics of Music, p. 23.

This includes predictions of deflections and stresses . . . and in addition, resonant frequencies can be obtained."[1]

The SAMIS program is a segmented system within the guidelines of the FORTRAN II computer language. The selection or sequencing of the various segments is controlled by the user and is accomplished by writing a set of pseudo instructions ("a pseudo instruction calls for a set of subprograms to perform a matrix operation rather than defining each step of the operation"[2]).

In addition to the pseudo instructions, the principal input data for this investigation included material tables and element data. The material tables define the mechanical properties of the material(s) used in the various elements, and the element data defines the "local geometry (member thickness, cross-sectional areas, moments of inertia), grid-points (numbers and locations), coordinate systems, temperature, weight, and pressure on each structural element."[3] (The works of Lang, and Melosh and Christiansen are recommended for further reference to the SAMIS program.[4,5])

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[1]M.E. Lakser, User's Guide--Structural Analysis and Matrix Interpretive System (SAMIS), p. 1-1.

[2]Theodore E. Lang, Summary of the Functions and Capabilities of the Structural Analysis and Matrix Interpretive System Computer Program #32-1075, p. 2.

[3]Robert J. Melosh, Philip A.Diether, and Mary Brennan, Structural Analysis and Matrix Interpretive System (SAMIS) Program Report #33-307, Revision #1, p. 107.

[4]Theodore E. Lang, Structural Analysis and Matrix Interpretive System (SAMIS) User Report #33-305.

[5]Robert J. Melosh and Henry N. Christiansen, Structural Analysis and Matrix Interpretive System (SAMIS) Program: Technical Report #33-311.

The principal output data was of two types. The first was a listing of the frequencies (in cps) of the predicted partials. The second was a matrix indicating the relative movements (in three-dimensional space) of each nodal point for each partial. From this data, diagrams were plotted showing the actual vibration patterns which were responsible for producing the predicted frequencies (see Appendix C).

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III. REPORT AND DISCUSSION OF RESULTS

Three graphs were printed to determine what, if any, sounds might be inherent in the recording studio, and the recording and playback equipment. A graph printed with no tape on the playback machine indicated the presence of detectable sounds at 60 and 500 cps. A graph printed from a blank tape indicated the same two frequencies, plus a band of "noise" from 0 to 200 cps. A third graph printed from a tape of the studio background "noise" indicated a broad band of "noise" from 0 to 2000 cps. However, these sounds and "noise" did not seem to influence the results of the investigation to any noticeable extent.

The fundamental resonant frequencies of the studio were 53.33 cps, 64.99 cps, and 125.76 cps. Higher frequencies having nodal or antinodal points at or near the microphone location are shown in Table 2. Several of the triangles and cymbals produced frequencies near those in Table 2. However, as all recording was done in the same studio, the extent (if any) of the inhibiting or reinforcing effects of the standing waves was not isolated.

Triangles

Experimental Results

The experimental results of the six triangles examined are listed in Tables 3 through 8 under these headings: implement (Imp)--7/32" drill rod (7/32 Drill), 7/32" cold-rolled steel (7/32 Cold), 5/32" drill rod

(5/32 Drill); striking point[1] (St Pt)--top (Top), corner (Cor), bottom (Bot); dynamic level (Lev)--ff, mr, pp; fundamental (Fund); upper limit (UL); and Energy Peaks in Decreasing Order of Intensity.

Table 2.--Frequencies Having Nodal or Antinodal Points At or Near the Microphone Location

Frequencies in cps	
Nodes	Antinodes
266.65	259.96
454.93	319.98
533.30	789.88
628.80	879.32
909.86	1013.27
1066.60	2369.64
1131.84	2637.96
1819.72	3039.81
2133.20	7108.92
2389.44	7913.88
3639.44	9117.43
4266.40	
4778.88	
7278.88	
8532.80	
9557.76	
14557.76	
17065.60	
19115.52	

The fundamentals, upper limits, and energy peaks are given in cycles per second (cps). The energy peaks are further identified by decibel ratings (dB)--e.g., 7150 / 33 indicates a frequency of 7150 cps at an intensity level of 33 dB. It should be noted that the energy peaks, or partials are listed in decreasing order of intensity and <u>not</u> in order of frequency.

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[1]It should be noted that the top and bottom striking points were used with a 90° striking angle, and the corner striking point was used with a 0° striking angle.

Each table is accompanied by a discussion of the effects (if any) on each triangle's overtone structures produced by changes in implement size and material, dynamic level, and striking point and angle. General trends in pitch levels and overtone strengths or intensities are also noted for each triangle.

Abel Triangle (Table 3)

Implement Size and Material

There were no consistent differences in the overtone structures produced by the two sizes of implements on the Abel triangle. There were no consistent changes in the overtone structures produced by the two kinds of implements used on the Abel triangle.

Dynamic Level

The ff level produced two effects on the Abel triangle's overtone structures: higher upper limits were produced; and the amplitudes of the partials were generally increased. In two instances (7/32" drill rod / top, and 7/32" drill rod / corner), more partials were produced at the pp level than at the ff level. And more partials were produced at the ff level than at the pp level in only two instances (5/32" drill rod / top, and 5/32" drill rod / bottom). The remaining five instances exhibited the same number of partials at both dynamic levels.

Striking Point and Angle

The two larger implements produced more partials at the bottom than at the top striking point. However, both of these implements produced a higher strongest partial at the top than at the bottom striking

Table 3.--Fundamentals, Upper Limits, and Energy Peaks of the Abel Triangle

Imp	St Pt	Lev	Fund	UL	Energy Peaks in Decreasing Order of Intensity																			
			cps	cps	cps	dB	cps	dB	cps	dB	cps	dB	cps	dB	cps	dB	cps	dB	cps	dB	cps	dB	cps	dB
	Top	ff	1900	20000+	5500	50	4000	48	7200	35	2650	32	8800	31	16100	20	1900	17						
		mf	1900	17000	5500	42	4000	40	8800	32	2650	27	1900	27	7000	24	13000	14	15750	12				
		pp	1850	16000	5400	40	4000	31	8700	30	1850	22	7000	21	2650	20	13000	13	15000	11				
7/32 Drill	Cor	ff	1400	20000+	8100	41	3900	36	14000	24	1900	19	1400	6										
		mf	1525	19000	4700	39	6300	38	8100	34	3800	30	1525	28	14000	27	2450	20						
		pp	1525	18000	8100	29	6300	27	3800	25	1525	21	2450	20	13000	17	15000	15						
	Bot	ff	1950	20000+	4000	47	2650	40	4600	39	6300	29	7200	28	1950	26	8100	24	13000	20	18000	11		
		mf	1950	17000	4000	33	2700	29	6300	27	2650	22	1950	21	9000	19	11000	28	13500	12	15500	11		
		pp	1975	15000	4000	42	2675	35	5500	27	6300	26	7200	22	1975	19	8900	16	11000	12	13000	12		
	Top	ff	1900	20000+	4000	50	5500	48	2650	40	7100	34	1900	29	8800	28	13000	19						
		mf	1925	17000	5500	37	4000	37	8800	30	1925	28	2650	27	7100	21	13000	15						
		pp	1950	16000	5500	38	8800	27	4000	27	7100	23	2650	20	1950	15	13000	15						
7/32 Cold	Cor	ff	1580	20000+	4800	44	3900	42	6300	40	1580	35	8300	34	2500	32	15500	20						
		mf	1575	19000	3900	42	6300	36	4750	35	8200	35	2475	20	14000	23	1575	18						
		pp	1550	18000	3900	34	8200	22	4750	22	14300	19	2475	18	6300	16	1550	15						
	Bot	ff	1800	18000	3800	40	5000	36	6300	35	1800	32	8500	31	6900	29	2550	27	12500	26	10300	23		
		mf	1900	17000	4000	36	2650	35	4700	35	5500	30	6300	28	7900	27	8100	27	11000	22	1900	22	13000	21
		pp	2000	20000	4000	42	2650	34	5500	33	7200	29	13000	26	8100	25	11000	19	2000	17	18000	15		
	Top	ff	1900	20000+	4000	43	5400	40	3200	40	8800	37	2650	34	7900	30	15500	26	9000	20	1900	17		
		mf	1950	20000	1950	42	3900	41	3200	39	4900	38	6300	31	7900	29	9000	26	10750	21	17000	15	14000	13
		pp	1900	16000	4000	37	5500	32	8800	26	7100	20	2650	20	1900	19	15500	9	13000	9				
5/32 Drill	Cor	ff	1550	20000+	4700	43	2450	35	3800	35	8100	34	6300	33	1550	32	14000	27						
		mf	1550	17000	3800	31	6300	29	4700	28	2450	23	1550	21	8100	17	15500	11						
		pp	1550	18000	3800	32	4750	29	6300	28	2450	19	15500	15	8100	13	1550	12						
	Bot	ff	1950	16000	5500	38	2650	30	7100	28	8100	27	4000	26	1950	25	10750	19	13000	16				
		mf	1900	16000	4000	44	5400	39	2650	38	7100	25	1900	21	10750	17	8600	15	13000	13				
		pp	1900	10000	4000	33	2650	28	5400	27	1900	16	7100	12										

point. The smaller implement produced more partials at the top striking point, and the most intense first partials were the same for both the top and bottom striking points.

The 0° striking angle consistently produced a lower fundamental (325 to 375 cps lower) than the 90° striking angle. With only one exception (7/32" cold-rolled steel / top), fewer partials were produced with the 0° striking angle than with the 90° striking angle.

Pitch Level and Overtone Intensities

The Abel triangle's partials seemed to fall into two sections: the majority of the partials occurred from 1500 to about 9000 cps; and a smaller number occurred from about 10500 to 16000 cps. One 17000 cps partial and two 18000 cps partials were found.

The Abel triangle produced two rather definite fundamentals: 1900 to 1950 cps at the top and bottom striking points; and 1525 to 1575 cps at the corner striking point with the 0° striking angle. These partials were rated as high as the fourth most intense partial in only seven of the eighteen examples, and were the weakest partial in another six instances. The most intense partial was the third partial in seventeen of the twenty-seven examples and the fourth partial in another eight instances. The two strongest energy peaks most frequently occurred near the frequencies of 4000 and 5500 cps.

Ludwig Triangle (Table 4)

Implement Size and Material

There was no consistent variation in the Ludwig triangle's overtone structures due to a change in implement size. A change in the

Table 4.--Fundamentals, Upper Limits, and Energy Peaks of the Ludwig Triangle

Imp	St Pt	Lev	Fund	UL	Energy Peaks in Decreasing Order of Intensity																			
			cps	cps	cps	dB	cps	dB	cps	dB	cps	dB	cps	dB	cps	dB	cps	dB	cps	dB	cps	dB	cps	dB
7/32 Drill	Top	ff	1400	18000	4500	43	6600	41	1400	39	2400	39	2100	36	8900	29	12000	16	15000	14				
		mf	1400	18000	4600	47	6600	41	1400	29	2400	25	9000	19	11000	15	12000	15	15000	13				
		pp	1400	16000	6600	30	2400	29	1400	28	4500	23	9000	18	2100	14	12500	12						
	Cor	ff	1200	14000	5500	42	4000	41	4900	40	11000	20	2450	16	2100	16	1900	15	1200	11				
		mf	1200	14000	5750	37	3900	36	4800	35	6900	25	8800	18	11000	14	13000	11	1450	11	2450	10	1200	8
		pp	1200	13000	5750	34	4950	31	6900	21	4000	18	11000	16	8900	12	2450	10	1200	9				
	Bot	ff	1400	14000	5500	39	4700	38	2400	32	1400	38	8700	13	12300	11								
		mf	1450	13000	4900	33	2450	31	6700	19	11250	14	1450	11	9200	10								
		pp	1400	12000	2400	30	5500	29	4600	28	1400	18	11000	12	9000	11								
7/32 Cold	Top	ff	1400	16000	2400	43	4600	41	1400	29	6600	38	5400	37	8900	22	11000	18	15000	10				
		mf	1400	16000	4500	40	6600	34	1400	25	2400	23	8900	15	15500	10	11000	8						
		pp	1425	16000	6600	28	1425	26	4600	25	2400	24	5500	19	9200	15	12250	10	15000	9				
	Cor	ff	1200	14000	3900	46	5500	43	6800	37	2400	24	2100	24	8700	23	10750	12	1200	7				
		mf	1250	14000	4900	39	5900	34	3900	31	6900	29	2100	12	2450	11	11000	11	1250	8				
		pp	1250	12000	3900	33	4900	32	5750	28	6900	23	8800	20	11000	12	1250	7						
	Bot	ff	1450	14000	2400	44	5500	41	4700	38	1450	33	3900	31	6500	28	9000	15	12500	15	18000	10		
		mf	1450	13000	5500	40	2400	33	6500	28	1450	25	3900	24	9000	19	11000	16						
		pp	1450	12000	1450	24	5400	22	2400	22	2150	21	4700	19	10750	13	8800	12	3900	10	14300	8		
5/32 Drill	Top	ff	1425	18000	4600	48	2400	42	1425	39	6600	31	8700	19	17000	14	11000	13	12750	11				
		mf	1425	16000	4600	46	2400	41	1425	35	6600	33	8700	17	11000	12	15500	11						
		pp	1400	14000	4600	38	6600	38	8900	21	1400	17	2400	16	11000	13	12000	12	15000	9				
	Cor	ff	1250	14000	5600	43	3900	41	4800	39	8700	25	2100	22	2400	21	11000	15	1250	10				
		mf	1200	14000	4800	36	5700	36	3900	29	8750	27	11000	21	2400	19	1400	10	1200	8				
		pp	1200	12000	5700	30	4800	29	3900	25	8700	12	11000	10	1200	8								
	Bot	ff	1425	15000	2400	40	5400	32	4600	29	1425	26	6600	24	3900	20	9000	19	11000	18				
		mf	1450	13000	5500	34	2400	24	2150	21	6500	21	11000	18	1450	17	9000	16						
		pp	1400	11000	2400	34	4700	20	2150	19	1400	11	9000	11										

implement material had no consistent effect on the overtone structures of the Ludwig triangle.

Dynamic Level

The louder dynamic level (ff) seemed to have two effects on the Ludwig triangle's overtone structures: the amplitudes of the partials were generally increased; and higher upper limits were produced. In one instance (7/32" cold-rolled steel / bottom), the pp level produced more partials than the ff level. The ff level produced more overtones than the pp level in four instances (7/32" drill rod / top, 7/32" cold-rolled steel / corner, 5/32" drill rod / corner, and 5/32" drill rod / bottom). The number of partials was the same at the two dynamic levels in the remaining four examples.

Striking Point and Angle

One noticeable effect of a change in striking point was the higher upper limits produced at the top striking point. The partials produced at the top striking point were also generally of greater intensity than those produced at the bottom striking point.

The fundamental frequencies produced by the 0° striking angle were consistently 150 to 200 cps lower than those produced by the 90° striking angle. In addition, the most intense partials produced by the 0° striking angle were generally higher than those produced by the 90° striking angle. With only two exceptions (5/32" drill rod / corner / pp, and 7/32" drill rod / corner / mf), the 0° striking angle produced a lower top partial (around 11000 cps) than the 90° striking angle (around 15000 cps).

Pitch Level and Overtone Intensities

Two main groups of partials were found for the Ludwig triangle. The majority of the partials occurred between 1400 and 7000 cps, with a small number occurring between about 8800 and 15000 cps. One 18000 cps partial, one 17000 cps partial, and two 15500 cps partials were found.

Two rather definite fundamental partials were produced on the Ludwig triangle: 1400 to 1450 cps at the top and bottom striking points, and 1200 to 1250 cps at the corner striking point with a 0° striking angle. However, these frequencies occurred as the most intense partial only once (7/32" cold-rolled steel / bottom / pp), and were usually the third most intense partial at the top striking point, the weakest at the corner striking point, and the fourth or fifth most intense partial at the bottom striking point.

There was no consistency as to which partial was the most intense on the Ludwig triangle. However, the three strongest energy peaks at the top striking point occurred most frequently around 4500, 6600, and 1400 cps. The three strongest energy peaks at the bottom striking point occurred most frequently around 2400, 5500, and 4700 cps. And the three strongest energy peaks at the corner striking point occurred most frequently around 5750, 3900, and 4800 cps.

Sonor Triangle (Table 5)

Implement Size and Material

A change of implement size produced no consistent changes in the overtone structures of the Sonor triangle. The use of different implement materials did not seem to have any consistent effect on the Sonor triangle's overtone structures.

Table 5.--*Fundamentals, Upper Limits, and Energy Peaks of the Sonor Triangle*

Table 5.--Fundamentals, Upper Limits, and Energy Peaks of the Sonor Triangle

Imp	St Pt	Lev	Fund	UL	Energy Peaks in Decreasing Order of Intensity																							
			cps	cps	cps	dB	cps	dB	cps	dB	cps	dB	cps	dB	cps	dB	cps	dB	cps	dB	cps	dB	cps	dB	cps	dB	cps	dB
	Top	ff	910	17000	4400	46	6000	43	3600	42	3000	40	1575	33	7300	33	10400	22	910	21	16000	11						
		mf	900	17000	6000	43	4400	40	2975	37	3600	33	8400	29	10200	23	1550	21	15750	10	900	6						
		pp	900	13000	4400	39	6000	32	8500	29	3600	27	3000	25	10500	18	1600	9	900	6								
7/32 Drill	Cor	ff	1250	12000	4800	32	6400	22	1600	15	2750	14	3650	12	8500	11	1250	9	11000	8								
		mf	1100	14000	6400	40	7400	34	3500	32	8400	30	4400	29	10400	18	13000	12	1100	6								
		pp	1250	16000	3500	24	6400	22	7400	21	4400	17	10000	13	13000	11	1250	8										
	Bot	ff	920	20000+	3000	38	4400	42	7500	31	3600	40	6000	38	8500	34	1600	33	10600	23	16000	13	19000	13	920	11
		mf	920	17000	4500	42	1600	36	8500	34	3000	34	7600	33	12000	14	16000	10	920	8						
		pp	920	14000	6000	39	7500	29	3000	28	4500	28	3600	26	1600	19	16000	14	12000	10	920	9				
	Top	ff	920	18000	6000	42	4400	39	3600	38	3000	36	8500	34	1600	33	7300	31	920	15	13250	11				
		mf	950	14000	4400	39	6000	38	3600	38	2950	33	8400	32	7300	23	10200	22	1575	20	950	5				
		pp	980	19000	4500	34	6000	28	3600	25	3000	23	7500	20	17000	14	11600	9	1600	6	980	5				
7/32 Cold	Cor	ff	1250	15000	6400	43	7500	33	3500	32	4900	31	3000	21	9800	20	13000	13	1600	9	1250	8				
		mf	1300	19000	3500	29	6400	27	8500	21	10500	20	4950	20	12750	17	17000	13	1300	6						
		pp	1250	11000	3500	31	7500	30	6400	28	4400	27	2750	16	1250	8	2230	7								
	Bot	ff	920	20000+	3600	42	6000	40	3000	39	4500	38	8500	33	7400	31	1600	30	10400	26	16500	15	920	12		
		mf	920	20000+	3000	26	3600	36	6000	35	7500	35	1600	32	4400	29	8500	27	10300	23	16000	17	18500	15	920	12
		pp	895	17000	6000	34	3000	31	3600	30	7500	26	8400	19	10400	16	1600	15	16000	11	1200	7	895	7		
	Top	ff	920	20000+	3000	48	4400	46	8500	39	7500	25	6000	34	16000	34	10400	27	920	19	17000	16				
		mf	920	14000	3000	43	[illegible]	[illegible]	3600	39	6000	38	1600	29	8500	28	10200	21	920	20	13200	10				
		pp	920	20000	6000	39	8500	33	3000	32	4500	30	3600	26	7250	25	10200	21	19000	14	13750	13	920	11	1600	9
5/32 Drill	Cor	ff	920	18000	4800	33	1600	31	6400	30	7400	26	3000	23	9750	22	3600	20	11600	14	16000	12	920	10		
		mf	1200	14000	3500	21	6400	20	7300	16	5400	15	4300	15	1600	13	8500	11	10300	9	13000	7	1200	6		
		pp	1100	12000	6400	36	3500	31	7300	26	8400	25	4400	25	10200	16	2750	14	1100	9						
	Bot	ff	920	18000	4400	50	6000	41	7400	39	8500	38	3000	35	1600	28	920	19	16000	14						
		mf	900	18000	7400	39	6000	34	4400	32	3000	27	3600	25	10200	19	1600	18	16000	14	1280	12	900	6		
		pp	900	12000	3600	40	4400	35	7500	34	6000	30	8500	29	1600	9	900	6								

Dynamic Level

The ff level consistently increased the amplitudes of the partials, but did not alter the basic overtone structures of the Sonor triangle. In all but two instances (7/32" drill rod / corner, and 7/32" cold-rolled steel / top), the ff level produced higher upper limits than did the pp level. The pp level produced more overtones than the ff level in only one instance (5/32" drill rod / top). The two levels produced equal numbers of overtones in two instances (7/32" cold-rolled steel / top, and 7/32" cold-rolled steel / bottom). In the remaining six instances, the ff level produced more overtones than the pp level.

Striking Point and Angle

With the two larger implements, the upper limits produced at the bottom striking point were higher than those produced at the top striking point. The 0° striking angle consistently produced a higher fundamental (about 300 cps higher) than the 90° striking angle. The upper limits produced by the 0° striking angle were generally lower than those with the 90° striking angle. With the exception of the 5/32" drill rod implement, the 0° striking angle produced fewer partials than did the 90° striking angle. The partials produced with the 0° striking angle were generally of lesser intensity than those produced with the 90° striking angle.

Pitch Level and Overtone Intensities

Although the Sonor triangle did produce two rather definite fundamentals (900 to 920 cps at the top and bottom striking points, and 1200 to 1250 cps at the corner striking point with a 0° striking angle), these frequencies were consistently one of the two weakest partials (nineteen were the weakest and eight were the next-to-weakest). The two most intense partials at the top and bottom striking points generally occurred around 4400 and 6000 cps, and those at the corner striking point generally occurred around 3500 and 6400 cps.

The majority of the Sonor triangle's partials occurred below 10500 cps. The few partials above that level were all weak and only slightly more intense than the fundamentals noted above. There was no consistency as to which partial was the most intense: the second partial twice; the third partial seven times; the fourth partial four times; the fifth partial seven times; the sixth partial six times; and the eighth partial once.

Zildjian Triangle (Table 6)

Implement Size and Material

No consistent changes in the Zildjian triangle's overtone structures could be attributed to a change of implement size. With only three exceptions (7/32" cold-rolled steel / top / mf, 7/32" cold-rolled steel / top / pp, and 7/32" cold-rolled steel / corner / pp), the cold-rolled steel implement produced more partials in each instance than did the drill rod implement.

Dynamic Level

The ff level produced higher upper limits in all but one instance (5/32" drill rod / bottom). The ff level produced more overtones than did the pp level in seven instances, and an equal number of overtones in the remaining two instances (7/32" drill rod / top, and 5/32" drill

Table 6.--Fundamentals, Upper Limits, and Energy Peaks of the Zildjian Triangle

Imp	St Pt	Lev	Fund	UL	Energy Peaks in Decreasing Order of Intensity																	
			cps	cps	cps	dB	cps	dB	cps	dB	cps	dB	cps	dB	cps	dB	cps	dB	cps	dB	cps	dB
7/32 Drill	Top	ff	1100	16000	4200	42	3250	41	6400	37	7900	31	1900	22	1100	21	10600	20	1500	15	12750	14		
		mf	1100	14000	3200	42	6400	37	4000	36	5000	30	7900	29	1900	26	9300	22	1100	14	12500	11		
		pp	1100	14000	4100	39	3200	34	6400	33	7900	19	1900	15	9600	11	13000	9	1250	8	1100	7		
	Cor	ff	980	14000	4700	38	3850	37	7700	30	6000	27	3200	24	9600	22	980	13	17000	10	1800	9		
		mf	900	12000	3200	33	3900	31	7750	25	6400	15	9400	14	900	8	1250	7						
		pp	920	12000	3800	30	6000	25	3200	24	7700	23	9700	13	1200	8	920	7						
	Bot	ff	1120	14000	8000	33	6400	31	3250	30	1950	30	5000	29	4100	28	1120	22	1500	22				
		mf	1120	11000	3250	31	6400	26	1120	22	1900	21	1800	17										
		pp	1120	11000	6400	28	4100	26	8000	22	1120	18	1950	16	5000	16	3300	11						
7/32 Cold	Top	ff	1100	18000	3250	46	4100	39	6400	29	8000	29	1100	25	1900	24	10600	18	1500	14	12500	13		
		mf	1100	14000	3200	42	4100	34	6400	33	7800	22	12500	12	1950	11	1100	11						
		pp	1100	12000	4100	31	6400	28	[illegible]	24	7900	20	2000	15	10700	10	1100	8						
	Cor	ff	970	17000	3800	37	4550	33	3200	30	6000	29	7700	26	2000	18	970	15	9500	15	16000	12	1475	11
		mf	920	12000	6000	35	3900	32	3200	26	4600	25	7700	23	9700	15	2000	9	1275	8	920	7		
		pp	950	12000	3900	22	3250	13	9700	9	1250	8	950	7	8000	6								
	Bot	ff	1120	16000	2000	36	8000	35	6400	33	3300	30	5000	27	4100	21	10600	21	1120	17	1500	17		
		mf	1120	13000	4100	28	6400	28	2000	22	5000	19	3250	18	7700	16	1500	16	1120	15	10800	13		
		pp	1110	9000	4200	27	3250	18	6400	16	2000	15	7000	15	[illegible]	13	1110	10	10700	8				
5/32 Drill	Top	ff	1120	14000	4100	48	3250	45	2000	32	6400	30	1120	29	[illegible]	21	7800	20	10750	19	9300	15		
		mf	1120	13000	4100	40	3200	39	6400	37	1950	25	7900	23	9200	16	1120	11	12500	9				
		pp	1100	13000	4000	36	3200	34	6400	29	7950	26	1900	17	11000	11	1350	10	1100	8				
	Cor	ff	980	14000	3900	38	3200	29	7700	28	2000	26	6000	21	9500	20	980	17	1500	13				
		mf	990	13000	3900	34	3200	31	7750	29	9750	24	6000	23	2000	16	990	8	1200	6				
		pp	950	11000	3900	20	3150	16	4600	13	9600	13	7700	12	6000	11	1200	8	950	6				
	Bot	ff	1120	12000	6400	38	7800	22	5000	21	1900	18	3250	16	1120	13	1500	13	9600	12				
		mf	1120	14000	6400	33	3200	28	7900	23	9700	23	4100	19	1950	13	1120	12						
		pp	1120	12000	6400	29	4100	27	3250	19	7900	18	1950	16	9250	12	1120	10						

rod / corner). The ff level also generally increased the amplitudes of the various partials without altering the basic overtone structures.

Striking Point and Angle

With only one exception (5/32" drill rod / top / mf), the overtone structures produced at the top striking point had higher upper limits than those produced at the bottom striking point. While partials with frequencies of 11000 to 13000 cps were consistently present in the overtone structures of the top striking point, the partials of the bottom striking point were consistently lower: none higher than 8000 cps with the large drill rod implement; none higher than 10800 cps with the cold-rolled steel implement; and none higher than 9700 cps with the small drill rod implement. With only two exceptions (7/32" cold-rolled steel / top / mf, and 7/32" cold-rolled steel / top / pp), more partials were produced at the top striking point than at the bottom striking point. The partials produced at the top striking point were generally of greater intensity than those produced at the bottom striking point.

A striking angle of 0° produced a consistently lower fundamental partial (about 300 cps lower) than the 90° striking angle. The upper limits produced by the 0° striking angle were always lower than, or the same as, the upper limits produced by the 90° striking angle. The partials produced with the 0° striking angle were generally of lesser intensity than those produced with the 90° striking angle.

Pitch Level and Overtone Intensities

The Zildjian triangle's partials seemed to occur in three sections: from 900 to 5000 cps; from 6000 to 10800 cps; and from 11000 to 17000 cps.

The first group (lower frequencies) contained the greatest number of partials, and the third group (higher frequencies) contained the least (only nine partials in all).

The Zildjian triangle produced two rather definite fundamental partials: 1100 to 1120 cps at the top and bottom striking points; and 900 to 920 cps at the corner striking point with a 0° striking sngle. However, these frequencies were rated higher than the sixth most intense partial only three times: the third most intense--7/32" drill rod / bottom / mf; the fourth most intense--7/32" drill rod / bottom / pp; and the fifth most intense--5/32" drill rod / top / ff. And these frequencies were the weakest partials in nine instances.

The three strongest energy peaks at the top and bottom striking points generally occurred near the frequencies of 4100, 3250, and 6400 cps. The three strongest energy peaks at the corner striking point were generally near the frequencies of 3200, 3900, and 7700 cps. There was no consistency as to which partial was the most intense. However, the fourth partial was the most intense nine times and the fifth partial was the most intense eleven times.

6" Pigstail Triangle (Table 7)

Implement Size and Material

A difference in implement size did not have any consistent effect on the overtone structures of the small Pigstail triangle. A difference in implement material did not seem to have any consistent effect on the small Pigstail triangle's overtone structures.

Table 7.--Fundamentals, Upper Limits, and Energy Peaks of the 6" Pigstail Triangle

Imp	St Pt	Lev	Fund	UL	Energy Peaks in Decreasing Order of Intensity																					
			cps	cps	cps	dB	dps	dB	cps	dB	cps	dB	cps	dB	cps	dB	cps	dB	cps	dB	cps	dB	cps	dB	cps	dB
7/32 Drill	Top	ff	970	18000	3150	47	4800	45	3850	41	6300	38	7900	29	9000	27	1650	23	970	20	1500	16	14000	15		
		mf	970	12000	4800	37	3800	34	6300	32	3100	30	7750	26	9000	20	1700	19	1500	16	970	10				
		pp	950	12000	3800	33	6300	32	3100	28	4800	22	9000	18	1750	16	10750	12	1200	8	950	7				
	Cor	ff	1800	13000	5000	37	6400	34	2800	32	7600	22	1800	19												
		mf	1800	10000	3600	37	2800	32	3200	25	6400	24	7750	22	12700	9	1800	7								
		pp	1800	14000	5300	29	6400	24	2800	24	3600	23	7800	20	13000	13	11000	10	1800	7						
	Bot	ff	980	19000	8000	35	3200	34	3900	33	1800	32	6500	32	5000	28	1530	26	980	17	10700	17	18000	10		
		mf	1000	15000	6400	40	5000	38	1800	30	3200	29	7800	28	1530	23	13750	13	1000	12						
		pp	950	12000	3900	30	1750	29	4800	25	6400	24	3200	21	1500	20	7900	18	11000	14	9000	13	950	8		
7/32 Cold	Top	ff	960	14000	3800	46	4600	42	6300	40	3100	37	1725	27	7700	27	9000	21	10750	19	1450	19	960	14		
		mf	970	13000	3800	33	3175	31	4900	28	6300	25	7900	22	9100	19	1750	17	10500	14	970	10				
		pp	970	12000	6300	37	3800	34	4800	32	3150	23	1725	18	9000	17	970	11	10800	11	14000	9	1250	7		
	Cor	ff	1800	20000	3600	37	5500	36	6400	34	7800	32	2800	31	13000	20	11000	19	1800	17	17800	14				
		mf	1800	20000	6400	37	3700	36	2850	36	5500	35	7800	28	13000	20	1800	17	11000	15	17800	14				
		pp	1800	14000	3600	33	2800	27	5400	25	6400	22	7800	18	9000	13	13000	10	11000	9	1800	6				
	Bot	ff	970	19000	3200	44	6300	42	4800	36	1750	34	7850	31	9000	23	14000	16	970	14	18000	13				
		mf	950	15000	3200	30	1750	29	4850	28	7800	27	6400	23	1500	20	950	12	14200	9						
		pp	900	19000	3150	35	4900	33	3900	28	6400	26	1750	25	7900	24	11000	12	17750	11	900	9				
5/32 Drill	Top	ff	970	16000	6300	43	4900	42	3800	41	1725	29	3150	36	7700	30	970	25	10500	15	15600	10				
		mf	970	11000	3800	37	4900	37	6300	31	3200	27	1750	23	970	20	7800	16	9100	14	1275	8				
		pp	970	19000	4900	36	6300	33	3800	28	3150	27	1750	22	9200	20	11000	18	15500	16	970	14	12600	9		
	Cor	ff	1800	14000	5300	36	6300	28	3850	20	1800	19	9900	18	7800	17	9000	17	12800	15	2800	14				
		mf	1800	12000	2800	33	3600	32	5400	30	6400	28	7800	21	10000	18	1800	8								
		pp	1800	15000	3600	32	2800	30	6400	27	5500	26	7800	20	10000	18	13000	17	1800	9						
	Bot	ff	970	18000	1750	37	3800	26	6400	33	3150	32	4800	29	970	25	9100	24	1950	23	10500	17	17500	11		
		mf	970	11000	4900	40	3850	29	6400	29	3150	24	1750	22	1500	20	7600	16	970	15						
		pp	980	14000	6400	24	1750	24	3200	21	9000	20	4800	19	3800	18	7900	17	1500	14	980	11	14000	9	1250	7

Dynamic Level

The ff level produced more partials than did the pp level in only two instances (7/32" drill rod / top, and 5/32" drill rod / corner), with the reverse occurring in three instances. The partials were of equal number in the other four examples. The ff level increased the amplitudes of the partials without actually altering the overall overtone structures of the small Pigstail triangle.

Striking Point and Angle

The partials under 2000 cps were of greater intensity when produced at the bottom striking point, but the partials above 2000 cps were generally of greater intensity when produced by striking at the top striking point. The top striking point produced six partials with frequencies of 11000+ cps, while the bottom striking point produced ten With only one exception (5/32" drill rod / bottom / pp), the bottom striking point produced upper limits which were higher than, or the same as, the upper limits produced by the top striking point.

The 0° striking angle produced a fundamental frequency almost double that of the 90° striking angle: 1800 cps as compared to 950 to 980 cps. The 0° striking angle also consistently produced fewer partials than the 90° striking angle. The partials produced by the 0° striking angle were generally of lesser intensity than those produced by the 90° striking angle.

Pitch Level and Overtone Intensities

The partials produced at the top and bottom striking points seemed to appear in three groups: 900 to 1800 cps; 3100 to 5000 cps;

and 6300 cps and above. The third group (higher frequencies) usually contained the most partials, with a few occurring as high as 18000 cps. The partials produced at the corner striking point (with a 0° striking angle) also seemed to occur in three groups: 1800 to 3850 cps; 5000 to 7800 cps; and 9000 cps and above. The third group (higher frequencies) contained the fewest partials with only a few partials occurring as high as 17800 cps.

The small Pigstail triangle produced two quite definite fundamentals: 950 to 980 cps at the top and bottom striking points; and 1800 cps at the corner striking point with a 0° striking angle. However, these frequencies were generally quite weak, being rated higher than the seventh most intense partial only four times. And these frequencies were the weakest partials fourteen times.

The three strongest energy peaks at the top and bottom striking points were near the frequencies of 3200, 3800, and 6300 cps, and the three strongest energy peaks at the corner striking point were near the frequencies of 5300, 3600, and 2800 cps. There was no consistency as to which partials were the most intense.

<u>10" Pigstail Triangle</u> (Table 8)

Implement Size and Material

With the exception of the ff level, the smaller implement (5/32" drill rod) produced slightly more intense partials than did the larger implement (7/32" drill rod). With only two exceptions (7/32" cold-rolled steel / top / ff, and 7/32" cold-rolled steel / top / mf), the cold-rolled steel implement produced slightly more intense partials than did the drill rod implement.

Table 8.--Fundamentals, Upper Limits, and Energy Peaks of the 10" Pigstail Triangle

Imp	St Pt	Lev	Fund	UL	Energy Peaks in Decreasing Order of Intensity																							
			cps	cps	cps	dB	cps	dB	cps	dB	cps	dB	cps	dB	cps	dB	cps	dB	cps	dB	cps	dB	cps	dB	cps	dB	cps	dB
	Top	ff	500	18000	4100	49	3400	46	5700	42	1700	42	2050	38	9400	31	7600	28	12750	23	16000	14	800	13	500	12	675	11
		mf	490	16000	5600	40	3350	37	4200	35	1650	32	2050	29	9500	28	12500	14	490	8								
		pp	510	14000	4250	31	5750	29	3400	27	1700	22	6400	21	9600	20	7600	18	10700	17	2050	16	510	16	13000	14	675	10
7/32 Drill	Cor	ff	440	17000	3500	43	2850	41	4750	37	6950	35	440	34	1900	32	5600	32	1675	29	13000	28	1450	27	10400	26	900	15
		mf	440	16000	3600	35	3100	32	4200	29	1490	24	1900	24	10750	22	440	20	12750	19	7000	18	14750	17	900	10		
		pp	440	16000	3500	29	10300	22	5600	20	12750	19	440	17	7000	14	2000	13	1450	13	1650	12	700	10				
	Bot	ff	510	18000	4800	45	4200	43	3150	38	9600	29	800	23	2000	22	900	20	510	15	15250	13	1500	10				
		mf	500	18000	4800	42	3150	38	9500	29	7500	27	2500	24	800	14	1900	13	500	11	16000	11						
		pp	510	13000	3400	37	4200	36	10000	21	7600	20	510	19	1700	17	700	11										
	Top	ff	520	16000	4750	46	2400	41	6600	40	1425	34	9000	22	11000	14	520	12	760	8								
		mf	520	14000	4100	29	3350	38	5600	35	1675	30	2050	16	8200	17	9200	15	10500	13	520	10	13500	9				
		pp	500	13000	4100	38	3350	36	5600	31	1675	27	2050	26	9600	15	500	14	700	9								
7/32 Cold	Cor	ff	440	18000	3600	46	3200	44	4200	40	7000	36	5800	35	1950	31	440	30	10200	29	1475	29	16000	19	900	18	770	12
		mf	440	18000	3500	39	4250	33	2850	32	5750	29	7000	27	1460	23	1900	22	10250	17	16000	14	12750	13	440	12	900	11
		pp	450	17000	3100	33	3500	27	5600	26	1900	23	7000	22	8500	20	15500	16	10250	15	12800	13	450	11	900	8		
	Bot	ff	500	18000	3400	44	4200	43	4900	42	9500	30	2050	30	7500	29	800	23	500	14	16000	13	650	12				
		mf	500	17000	3400	42	4200	40	4700	37	9500	28	8500	27	7600	26	2000	22	800	19	1650	19	900	17	15750	12	500	10
		pp	510	12000	4800	39	4250	38	3400	35	1700	17	2050	16	10500	15	7500	15	510	11	900	9						
	Top	ff	500	17000	3400	48	2050	42	4100	41	1650	37	5500	36	6400	31	10750	29	9500	28	790	13	15500	12	500	9		
		mf	500	17000	5600	42	3350	41	4100	40	1675	36	2050	35	9400	26	12500	13	500	12	16000	10	690	10				
		pp	500	17000	3350	36	5600	33	2050	32	4100	31	9500	22	800	15	500	14	12500	13	16000	12						
5/32 Drill	Cor	ff	440	17000	3500	48	4200	41	2850	35	1900	32	5600	30	6900	27	2450	25	10300	24	440	24	12500	18	16000	17	900	14
		mf	440	17000	3500	38	4200	31	2850	29	2000	25	5600	24	1450	22	1700	21	6900	21	440	17	15500	16	12750	13	10000	12
		pp	425	16000	3500	36	4200	29	2850	23	1900	22	5600	17	17000	16	10250	14	12750	13	425	12	15500	10	900	9		
	Bot	ff	440	17000	4200	42	4700	41	3400	37	5700	36	2500	32	7600	28	900	27	1650	26	8600	25	800	23	440	14	16000	11
		mf	510	16000	3400	40	4200	38	4700	37	5700	35	2500	34	10000	28	7600	29	900	24	800	21	510	13	16000	11		
		pp	500	13000	4200	38	4700	36	3350	26	5700	24	7500	20	9600	17	2500	16	1650	15	900	15	800	14	500	13		

Dynamic Level

In eight of the nine examples, the ff level produced a higher upper limit than did the pp level. With only two exceptions (7/32" drill rod / top, and 7/32" cold-rolled steel / top), the ff level produced more partials than did the pp level. In addition, the partials produced at the ff level were generally of greater intensity than the partials produced at the pp level.

Striking Point and Angle

The partials under 1000 cps were consistently more intense when produced at the bottom striking point; however, those partials between 1000 and 3000 cps were more intense when produced at the top striking point. The top striking point produced more partials between 9000 and 15000 cps than the bottom striking point did by a nineteen to eight margin. But the bottom striking point produced more partials above 15000 cps by a six to four margin.

The 0° striking angle produced a consistently lower fundamental partial (60 to 70 cps lower) than that produced by the 90° striking angle. The 0° striking angle also produced more partials up to 2000 cps by a thirty-six to thirty margin.

Pitch Level and Overtone Intensities

The partials produced at the top and bottom striking points seemed to fall into four groups, with decreasing numbers of partials: 490 to 2500 cps; 3100 to 5750 cps; 6400 to 8600 cps; and 9000 to 16000 cps. The partials produced at the corner striking point seemed to fall into three groups: 425 to 2000 cps; 2850 to 7000 cps; and 8500 to

16000 cps. The middle group had the most partials, and the last group (higher partials) had the fewest (including only seven partials above 13000 cps).

Although the large Pigstail triangle produced two rather definite fundamental partials (490 to 520 cps at the top and bottom striking points, and 440 cps at the corner striking point with a 0° striking angle), these frequencies were always relatively weak. They were rated higher than the seventh most intense partial only twice (7/32" drill rod / corner / ff, and 7/32" drill rod / corner / pp).

The most frequent strong partials at the top and bottom striking points were near the frequencies of 4200, 3400, 4700, and 5600 cps. The most frequent strong partials at the corner striking point were near the frequencies of 3500, 4200, and 2850 cps. The most intense partials were usually either the sixth or seventh partials, but there was no consistent pattern with regard to any of the variations in implements or dynamics.

Summary of Experimental Results and Related Research

Instrument Size and Material

Recalling that frequency is a function of the expression $\sqrt{\frac{E\ I}{\rho\ A\ l^2}}$ (see above on p. 48), it is evident that changes in the geometric properties (I, A, and l) affect the resonant frequency of the triangle. With all other properties constant, these relationships are valid: a longer length lowers the resonant frequency; and a larger diameter raises the resonant frequency (the A value involves a radius squared, while the I value involves a diameter to the fourth power).

Although the 10" Pigstail triangle was .07" thinner than the Abel triangle, it was felt that the large difference in the average lengths of the sides of the two triangles (the average length of the sides of the 10" Pigstail triangle was almost twice that of the Abel triangle) would effectively minimize the influence of this very slight difference in diameter. Thus, the result of the length principle may be seen in the following comparison: 10" Pigstail triangle (.5" thick and sides averaging 10.6" long)--490 to 520 cps at the top and bottom striking points, and 440 cps at the corner striking point; Abel triangle (.57" thick and sides averaging almost 5.8" long)--1900 to 1950 cps at the top and bottom striking points, and 1525 to 1575 cps at the corner striking point. The 10" Pigstail triangle also produced almost twice as many partials under 5000 cps than did the Abel triangle.

Although the average length of the sides of the Abel triangle was .4" shorter than that of the Sonor triangle, it was felt that the large difference in diameters (the Abel triangle was very nearly twice as thick as the Sonor triangle) would effectively minimize the influence of the slight difference in the average lengths of the sides. Thus, the results of the diameter principle may be seen in the following comparison: Abel triangle (.57" thick and sides averaging almost 5.8" long)--1900 to 1950 cps at the top and bottom striking points, and 1525 to 1575 cps at the corner striking point; Sonor triangle (.29" thick and sides averaging almost 6.2" long)--900 to 920 cps at the top and bottom striking points, and 1200 to 1250 cps at the corner striking point. The Abel triangle also produced fewer partials under 5000 cps than did the Sonor triangle.

Referring again to the expression $\sqrt{\frac{E\ I}{\rho\ A\ l^2}}$, it is evident that a change in the material, and thus the material properties E and ρ, will affect the resonant frequency of the triangle. As it is almost impossible to consider the E and ρ values separately, it is more valid to consider the ratio of the two values (E : ρ). Therefore, with all geometric properties equal, triangles made from metals with larger ratios than steel ($30 \times 10^6 : \frac{.283}{386}$), such as beryllium ($42 \times 10^6 : \frac{.066}{386}$), molybdenum ($49.3 \times 10^6 : \frac{.35}{386}$), or chromium ($34.1 \times 10^6 : \frac{.24}{386}$), would have higher resonant frequencies than steel. Conversely, triangles made from metals with smaller ratios than steel, such as brass ($16 \times 10^6 : \frac{.30}{386}$), silver ($11.42 \times 10^6 : \frac{.38}{386}$), or aluminum ($10.2 \times 10^6 : \frac{.10}{386}$), would have lower resonant frequencies than steel.

Although many authorities state that a plated triangle is superior in sound to an unplated one, by applying the expression $\sqrt{\frac{E\ I}{\rho\ A\ l^2}}$, it can be seen that the overall effect of a plating even several thousandths of an inch thick would be very minimal at best.

The damping property of the metal, although not affecting the frequency, should also be considered for its effect on the resonance or duration of the sound. However, damping properties vary considerably with chemical content, frequency, and heat, thus making it very difficult to readily obtain exact comparative figures for various metals.

Implement Size and Material

A change of implement size produced no consistent changes in the overtone structures of any of the five small triangles (Abel, Ludwig, Sonor, Zildjian, and 6" Pigstail). However, the smaller implement (5/32" drill rod) produced partials of slightly greater intensity than the larger implement (7/32" drill rod) on the 10" Pigstail triangle at the mf and pp dynamic levels.

A change of implement material produced no consistent changes in the overtone structures of the Abel, Ludwig, Sonor, and 6" Pigstail triangles. However, the cold-rolled steel implement produced more partials than the drill rod implement in six of the nine instances on the Zildjian triangle. And in seven of the nine instances on the 10" Pigstail triangle, the cold-rolled steel implement produced partials of slightly greater intensity than did the drill rod implement.

Striking Point and Angle

There was no consistent pattern among the six triangles in their reaction to a change from the top striking point to the bottom striking point.

The 0° striking angle produced lower fundamentals in four triangles (Abel, Ludwig, Zildjian, and 10" Pigstail), and higher fundamentals in the other two (Sonor and 6" Pigstail). Other general effects of the 0° striking angle were lower upper limits, fewer partials, and weaker partials.

Dynamic Level

Compared to the pp level, the ff level seemed to have three general effects on the overtone structures of the triangles: it produced higher upper limits in most cases; it consistently produced partials of greater intensity; and it produced more partials in twenty-eight of the fifty-four examples.

Pitch Level and Overtone Intensities

Each triangle produced two rather definite fundamental partials--one at the top and bottom striking points with a 90° striking angle, and one at the corner striking point with a 0° striking angle. The fundamentals produced at the top and bottom striking points on the five small triangles ranged from the 900 to 920 cps partials of the Sonor triangle to the 1900 to 1950 cps partials of the Abel triangle. The fundamentals produced at the corner striking point on the five small triangles ranged from the 900 to 990 cps partials of the Zildjian triangle to the 1800 cps partials of the 6" Pigstail triangle. The fundamentals of the 10" Pigstail triangle were considerably lower: 490 to 520 cps at the top and bottom striking points, and 440 cps at the corner striking point. No fundamentals were found that approached the 6000 cps level reported by Briggs (see above on p. 13).

With one exception (the corner striking point on the Zildjian triangle), the most frequent strong partials on all the triangles were below the 7000 cps level. A fundamental partial was the most intense partial only once (Ludwig / 7/32" cold-rolled steel / bottom / pp). And the fundamental partials were actually the weakest partials in sixty-one of the one hundred sixty-two examples. There was no consistency among the triangles as to which partials were the most intense.

Similarity of Overtone Structures

The partials and their frequencies presented above in Tables 3 through 8 form the basis of Figure 22. By plotting each partial's frequency on a horizontal scale marked in thousands of cycles per second (kcs), the overtone structures of the triangles may be compared.

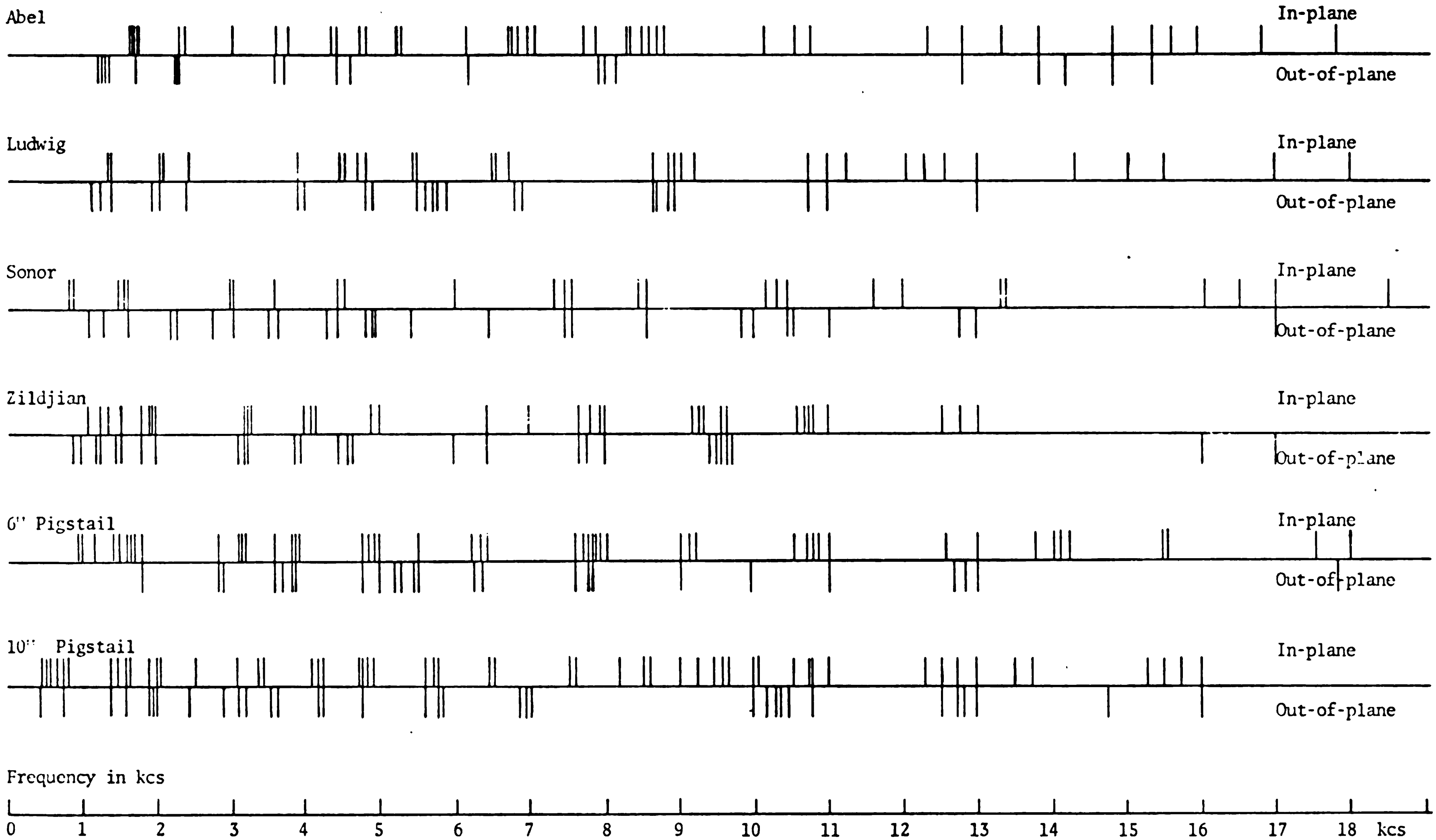

Figure 22.--Analyzed Overtone Structures for Triangles

Frequencies produced by in-plane vibration (transverse vibration within the plane of the triangle) are indicated by the vertical lines above the frequency scales, and those produced by out-of-plane vibration (transverse vibration perpendicular to the plane of the triangle) are indicated by the vertical lines below the frequency scales. Overtone intensities are not a factor in Figure 22.

The Sonor, Zildjian, and 6" Pigstail triangles were similar in that they exhibited a cluster of partials from about 900 to 2000 cps. The Ludwig triangle showed the same type of cluster of partials, but from about 1200 to 2400 cps. This cluster of low partials was followed by a wider group of partials on the Zildjian triangle (from about 3200 to 5000 cps), the 6" Pigstail triangle (from about 2800 to 3850 cps), and the Ludwig triangle (from about 3900 to 6000 cps).

The Zildjian and 6" Pigstail triangles shared another similarity--a narrow band of frequencies from about 7600 to 8000 cps. The Abel and 10" Pigstail triangles were similar in that their partials were more evenly distributed with fewer narrow clusters of partials. With the exception of the Zildjian triangle (17000 cps) and the 10" Pigstail triangle (16000 cps), the triangles all produced at least one partial of 18000+ cps.

Mathematical Results

Predicted Frequencies

Table 9 shows the sixteen partials predicted for each triangle by the SAMIS computer program. The frequencies (in cps) for each triangle are listed by partial number from the fundamental up (1 through 16). The column headings In and Out indicate whether the frequency listed was produced by in-plane or out-of-plane vibration.

Table 9.--Predicted Triangle Frequencies

Triangle	Partial Numbers and Frequencies in cps															
	1		2		3		4		5		6		7		8	
	In	Out	In	Out	In	Out	In	Out	In	Out	In	Out	In	Out	In	Out
Abel		1732	1777			2645	2778			3442	3472		5202			5303
Ludwig		1318	1324			1895	1930			2000	2132		4331			4380
Sonor		828	832.2			1214	1243			1390	1480			2750	2800	
Zildjian		908	922.3			1230	1276			1411	1554		3028			3045
6" Pigstail		1006	1009			1439	1481			1551	1657			3075	3223	
10" Pigstail	469.7			471.2		645.7	672.4			814.2	861.9			1226	1299	

Triangle	9		10		11		12		13		14		15		16	
	In	Out	In	Out	In	Out	In	Out	In	Out	In	Out	In	Out	In	Out
Abel	6821			7339		7607	7649		7872			9863	10440		11460	
Ludwig		4830	4893			5233	5589		7139			8477	8716			8558
Sonor		3139	3278			3763	3987			5432	5450			5851	6192	
Zildjian		3184	3391			3713	3872		5450			5751	6000			6016
6" Pigstail		3519	3676			4190	4396			5447	5631		6197			6411
10" Pigstail	1492			1502		1942	2034			2692	2779			3277	3501	

Although the triangles were not entirely consistent as to the correlation of partial number and in-plane or out-of-plane vibration, it is interesting to note that they were identical in six of the sixteen predicted partials--numbers 3, 4, 5, 6, 11, and 12. The Sonor triangle was the only triangle to exhibit strict alternation of in-plane and out-of-plane vibration. The others generally alternated but had some instances of two adjacent partials produced by the same type of vibration.

It should be noted that in almost all instances, and particularly with the fundamentals, the predicted frequencies of the "paired" partials (i.e., paired as to in-plane and out-of-plane vibration--1 and 2, 3 and 4, . . . 15 and 16) are relatively close. In the case of the 10" Pigstail triangle, the frequencies of partials 1 and 2 are only 1.5 cps apart.

It should also be noted that at no time does an individual triangle's series of either in-plane or out-of-plane partials even begin to approach the frequency series as given by Olson for a bar free at both ends: f_1 = fundamental; $f_2 = 2.756f_1$; $f_3 = 5.404f_1$; and $f_4 = 8.933f_1$.[1]

Predicted Overtone Structures

The predicted overtone structures for the six triangles are shown in Figure 23. The horizontal scale represents frequency in thousands of cycles per second (kcs). The frequencies produced by in-plane vibration are indicated by the vertical lines above the frequency scales, and the frequencies produced by out-of-plane vibration are indicated by the vertical lines below the frequency scales.

[1]Olson, Music, Physics and Engineering, p. 77.

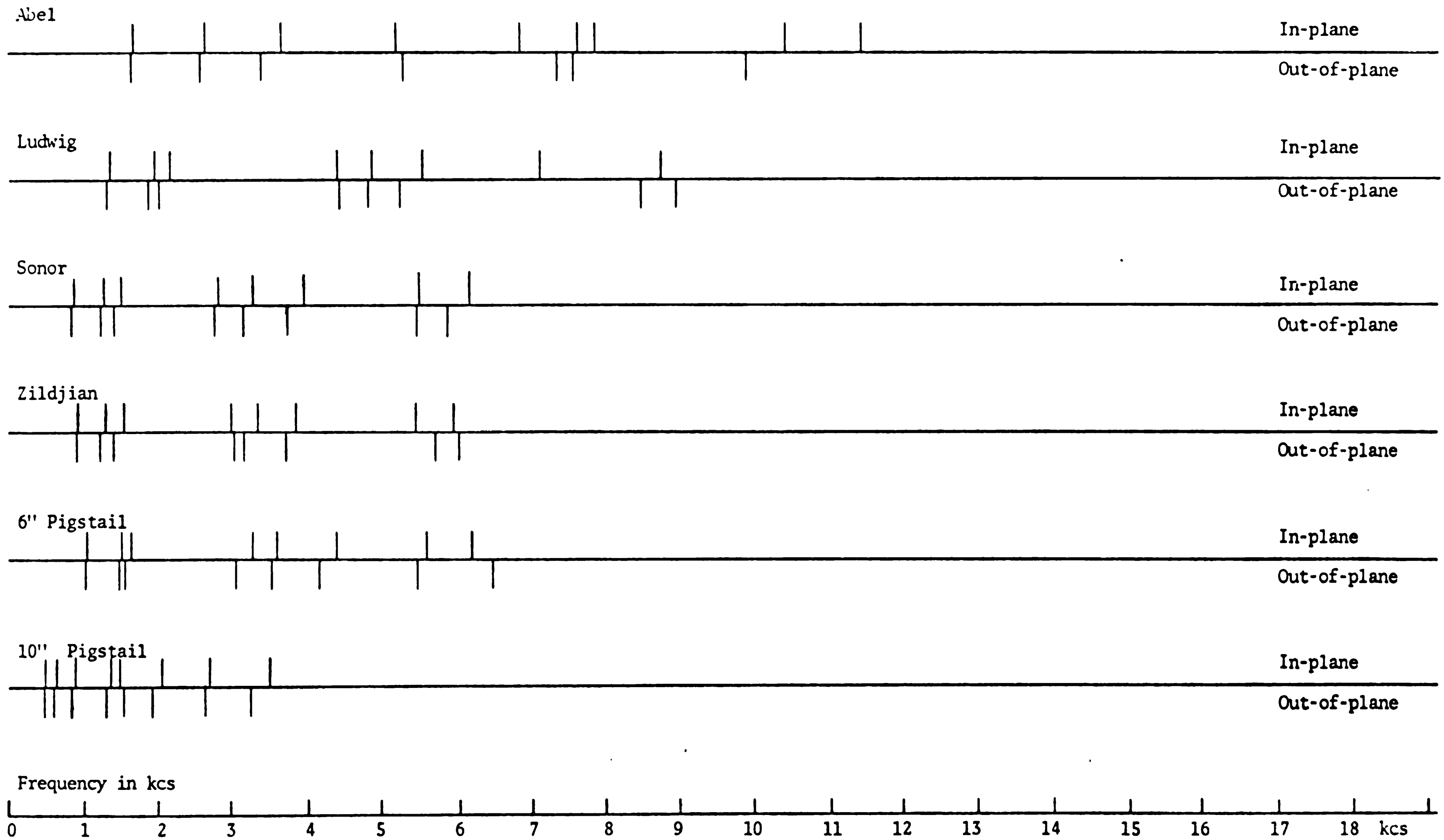

Figure 23.--Predicted Overtone Structures for Triangles

Three of the triangles (Sonor, Zildjian, and 6" Pigstail) exhibited strikingly similar overtone structures. The sixteen predicted partials were divided into three definite groups: six low partials from about 800 to 1600 cps; six intermediate partials from about 2700 to 4400 cps; and four high partials from about 5400 to 6400 cps. The Ludwig triangle's overtone structure was similar, but placed higher on the frequency scale with more space between the groups: six low partials from about 1700 to 2100 cps; six intermediate partials from about 4300 to 5600 cps; and four high partials from about 7100 to 8600 cps. The Abel and 10" Pigstail triangles exhibited overtone structures with less definitely grouped and more evenly distributed partials.

Predicted Modes of Vibration

All six triangles exhibited similar modes of vibration (see Appendix C for an explanation of the derivation of the vibrational modes). Depending upon the frequency, each side showed from 0 to 3 nodes during in-plane vibration, and from 0 to 4 nodes during out-of-plane vibration.

Although the triangles were not entirely consistent as to which partials were in- or out-of-plane, the vibrational modes for two of the six partials that were consistent with all of the triangles are shown in Figures 24 and 25. In both Figures 24 and 25, the shapes of the triangles are shown by the solid lines, and the deformities produced by vibration are shown by the dotted lines. As mentioned before (see above on p. 47), Point 16--the free end of the right leg of each triangle in the Figures--was fixed to eliminate rigid body motions, thus accounting for the apparent lack of vibration at the free ends of the right legs of the triangles.

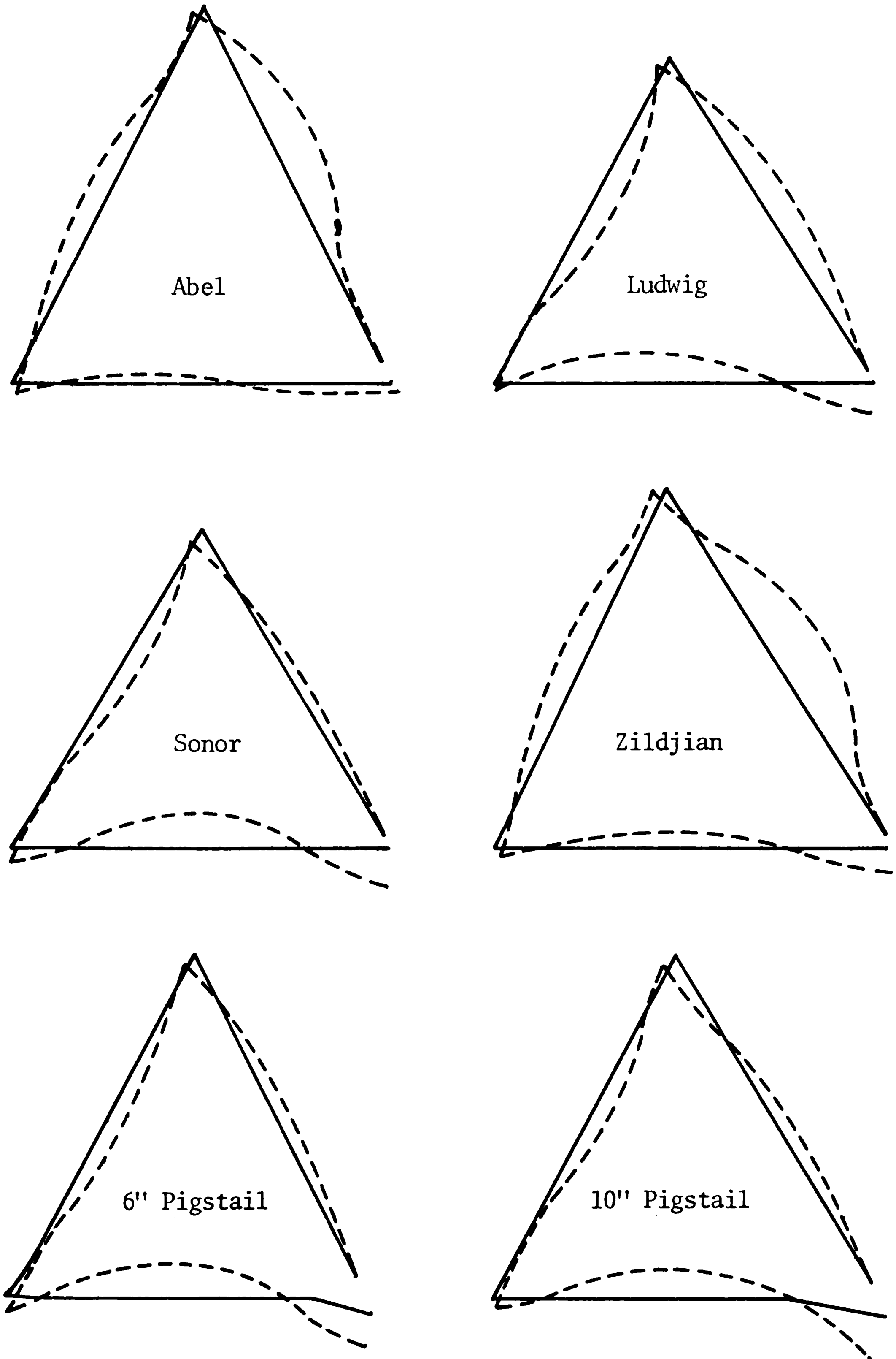

Figure 24.--In-plane Vibration--Partial 4

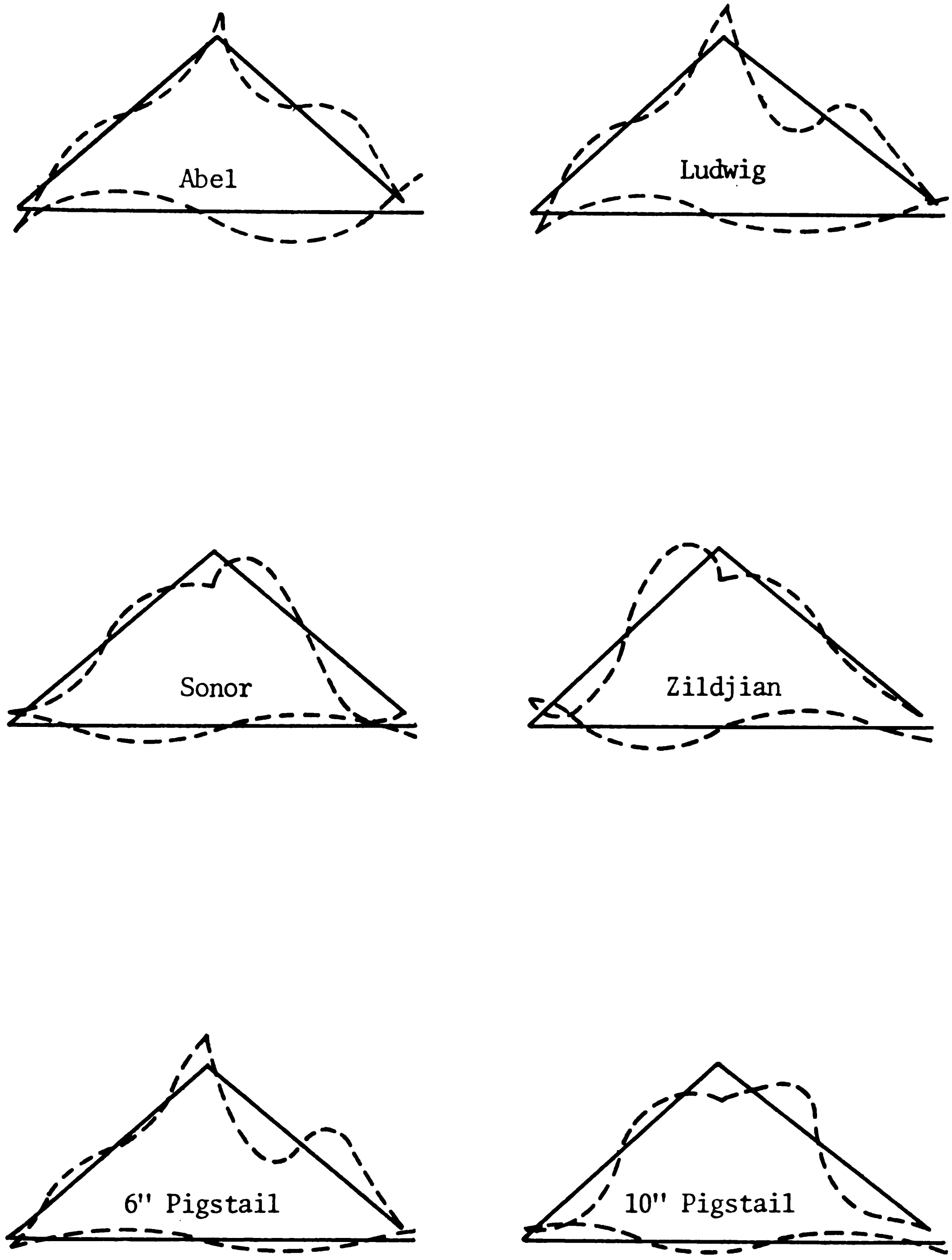

Figure 25.--Out-of-plane Vibration--Partial 11

In Figure 24, each triangle is shown as if observed from a point perpendicular to the plane of the triangle. In Figure 25, however, the point of observation is just above the plane of the triangle looking from the base to the apex.

Agreement of Mathematical and Experimental Results

There was no consistency among the triangles either as to which predicted partials did not appear, or as to which analyzed partials were not predicted. However, the agreement between the mathematical (predicted) and experimental (analyzed) results was quite high (at least 85% in all but two examples).

Table 10 shows the predicted partials and the corresponding analyzed partials that were produced by both in-plane and out-of-plane vibration. The intensities of the analyzed partials are also included, but there seemed to be no consistent relationship between intensity and the presence or absence of partials. Dashes in the "predicted" column indicate that the corresponding analyzed partials were not predicted; and dashes in the "analyzed" column indicate that the corresponding predicted partials were apparently not produced.

The predicted and analyzed overtone structures for all but one (Sonor) of the six triangles were similar. The manner in which the partials of the Ludwig, Zildjian, and 6" Pigstail triangles were grouped was quite similar, although the frequency ranges of the analyzed groupings were consistently greater and somewhat higher

Table 10.--Agreement Between Experimental and Mathematical Results

Triangle	In-plane Vibration			Out-of-plane Vibration		
	Predicted partials in cps	Analyzed partials in cps	Analyzed intensities in dB	Predicted partials in cps	Analyzed partials in cps	Analyzed intensities in dB
	1777	1800-2000	15-42	1732	1400-1900	6-35
	2778	2550-2700	20-40	2645	2450-2500	18-32
	----	3200	40	3442	3800-3900	25-42
	3472	3800-4000	40-50	5303	4700-4800	22-48
Abel	5202	4600-5500	35-50	----	6300	16-40
	6821	6300-7200	22-35	7339	----	----
	7649	----	----	7607	8100-8300	13-41
	7872	7900-9000	15-31	9863	----	----
	10440	10300	23			
	11460	10750-11000	12-28			
	1324	1400-1450	11-39	1318	1200-1450	7-11
	1930	----	----	1895	1900	15
	2132	2100-2450	14-39	2000	2100-2450	10-24
	4331	3900-4500	10-43	4380	3900-4000	25-46
Ludwig	4893	4600-4900	19-47	4830	4800-4950	31-40
	5589	5400-5500	19-41	5233	5500-5900	30-43
	7139	6500-6700	19-41	----	6800-6900	21-37
	8716	8700-9200	10-29	8477	----	----
				8558	8700-8900	12-27
	832.2	895-980	5-21	828	920	10
	1243	1200-1280	7-12	1214	1100-1250	6-9
	1480	1550-1600	6-36	1390	1300-1600	6-31
	2800	2950-3000	23-48	2750	2230-2750	7-16
Sonor	3278	----	----	3139	3000	21-23
	3987	3600-4500	25-50	3763	3500-3650	11-32
	5450	----	----	----	4300-4400	15-29
	6192	6000	28-43	5432	4800-5400	15-33
				5851	6400	20-43
	922.3	----	----	908	900-920	6-17
	1276	1100-1350	8-29	1230	1200-1275	6-8
	1554	1500	13-32	1411	1475-1500	11-13
	----	1800-2000	11-30	----	1800-2000	9-26
Zildjian	3028	----	----	3045	----	----
	3391	3200-3300	11-46	3184	3150-3250	13-33
	3872	4000-4200	19-48	3713	3800-3900	20-37
	5450	5000	16-30	----	4550-4700	13-38
	6000	6400	16-38	5751	----	----
				6016	6000-6400	11-35
	1009	900-1000	7-25	1006	----	----
	----	1200-1275	7-8	1439	----	----
	1481	1450-1530	14-23	1551	1800	6-19
6"	1657	1650-1950	16-37	3075	2800-2850	14-36
Pigstail	3223	3100-3260	21-47	3519	3600-3850	20-37
	3676	3800-3900	18-46	4190	----	----
	4396	4800-4900	19-45	5447	5000-5500	25-37
	5631	5000	26-38	6411	6300-6400	22-37
	6197	6300-6500	23-43			
	469.7	440-[illegible]	8-19	471.2	425-450	12-34
	672.4	650-760	8-12	645.7	700	10
	861.9	790-900	13-27	----	770	12
10"	1299			814.2	900	8-18
Pigstail	1492	1425-1700	10-42	1226	----	----
	2034	1900-2050	13-38	1502	1450-1700	12-29
	2779	2400-2500	16-41	1942	1900-2000	13-31
	3501	3150-3400	26-46	2692	2450-2850	23-41
				3277	3100-3600	27-48

than the comparable predicted groupings. Only the first group of the Sonor triangle's analyzed partials was similar to the predicted grouping, and it was also placed somewhat higher on the frequency scale. Both the predicted and analyzed partials of the Abel and 10" Pigstail triangles were less definitely grouped and more evenly distributed than those of the other four triangles.

Cymbals

Experimental Results

The experimental results of the five cymbals examined are listed in Tables 11 through 15 under these headings: implement (Imp--yellow yarn (Yel), red yarn (Red), brown cord (Cord); striking point[1] (St Pt)--edge (Edge), cup (Cup); dynamic level (Lev)--ff, mf, pp; fundamental (Fund); upper limit (UL); and Energy Peaks in Decreasing Order of Intensity.

The fundamentals, upper limits, and energy peaks are given in cycles per second (cps). The energy peaks are further inentified by decibel ratings (dB)--e.g., 1220 / 46 indicates a frequency of 1220 cps at an intensity level of 46 dB. It should be noted that the energy peaks, or partials, are listed in decreasing order of intensity, and not in order of frequency.

Each table is accompanied by a discussion of the effects (if any) on each cymbal's overtone structures produced by changes in implement

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[1]It should be noted that both striking points were used with a 0° striking angle.

hardness and material, dynamic level, and striking point. General trends in pitch levels and overtone strengths or intensities are also noted.

Avedis Zildjian Cymbal (Table 11)

Implement Hardness and Material

A change of implement hardness did not produce any consistent differences in the Avedis Zildjian cymbal's overtone structures. The cord implement produced more and more intense partials of 6000+ cps than the red yarn implement did by a nine to four margin.

Dynamic Level

With only one exception (cord / cup / ff), the ff level produced up to six more partials than did the pp level. The majority of these additional partials occurred above 4000 cps. The ff level consistently produced much higher upper limits (20000+ cps) than did the pp level (2200 to 7000 cps). Without exception, the partials at the ff level were of considerably greater intensity than those at the pp level.

Striking Point

With only one exception (yellow / edge / pp), the edge striking point produced upper limits higher than, or as high as, those produced at the cup striking point. The cup striking point consistently produced a fundamental 60 cps higher than the edge striking point. The edge striking point produced more partials below 1000 cps by a thirty-six to twenty-six margin, but only one more partial above 5000 cps.

Table 11.--Fundamentals, Upper Limits, and Energy Peaks of the Avedis Zildjian Cymbal

Imp	St Pt	Lev	Fund	UL	Energy Peaks in Decreasing Order of Intensity																									
			cps	cps	cps	dB	cps	dB	cps	dB	cps	dB	cps	dB	cps	dB	cps	dB	cps	dB	cps	dB	cps	dB	cps	dB	cps	dB	cps	dB
Yel	Edge	ff	340	20000+	780	49	1200	46	3700	45	540	43	2150	42	4400	42	2650	42	1700	41	6000	41	550	41	440	40	730	40	9000	32
		mf	340	7500	340	39	780	38	1700	37	1100	36	2150	32	440	30	3600	29	540	27	2700	26	4400	23	6000	13				
		pp	340	2200	770	35	340	32	430	30	540	25	1100	21	1700	15	2150	3												
	Cup	ff	400	20000+	790	49	3800	45	1100	43	1650	42	2150	21	530	41	2750	40	400	39	7200	36	5600	30	10200	23				
		mf	400	7500	750	38	400	36	2150	34	1100	32	4400	29	2900	28	3600	28	1700	27	5600	26								
		pp	400	5000	740	33	560	33	400	30	1000	26	1110	26	1700	21	2300	14	3100	9										
Red	Edge	ff	340	20000+	790	49	4000	45	1700	43	2700	42	1200	41	340	40	2150	39	6000	39	430	38	550	36	7500	32				
		mf	340	9000	770	39	340	35	3600	34	440	33	1200	33	1700	32	2700	30	2150	29	4400	27	550	26						
		pp	340	5000	760	37	340	31	1200	31	1700	30	425	28	540	25	2150	22	2700	16	3600	14	440	10						
	Cup	ff	400	20000+	800	47	2400	43	4400	42	4650	41	3600	41	2150	41	1700	40	400	39	1100	38	540	34	8800	25				
		mf	400	9000	300	38	1200	37	400	36	2150	36	1100	36	2800	35	3600	34	1700	33	4400	31	7400	10						
		pp	400	4500	1200	32	730	29	400	27	800	26	1675	21	540	19	2350	17	3600	6										
Cord	Edge	ff	340	20000+	760	50	4000	47	1700	43	1200	43	2850	42	2150	41	6000	41	2700	40	340	39	430	38	540	36	8900	35	12000	21
		mf	340	10000	760	42	3600	40	340	37	1200	37	1700	36	440	35	2350	35	2750	34	2150	33	4500	33	7400	16	8500	12		
		pp	340	7000	770	38	1650	34	1200	34	340	33	420	30	2100	29	530	28	2350	27	2700	25	3600	25	4300	19	5300	9		
	Cup	ff	400	20000+	790	48	1200	48	4000	44	1700	43	2600	42	2100	42	400	38	540	34	6500	34	8800	26						
		mf	400	8500	1200	37	8000	36	400	32	2650	31	3600	30	2150	29	1700	29	4400	29	550	23	7500	6						
		pp	400	6300	1200	38	730	34	920	31	400	28	1660	28	2350	27	2150	26	2650	25	4400	24	540	21	3600	18				

Pitch Level and Overtone Intensities

The Avedis Zildjian cymbal produced two definite fundamental partials: 340 cps at the edge striking point, and 400 cps at the cup striking point. There was no consistency as to the intensity of these frequencies--they appeared in positions one to four, six, and eight to nine.

The overtone structures produced at the edge striking point consistently exhibited three strong partials between 340 and 540 cps. However, with only one exception (yellow / edge / mf), the most intense partials produced at the edge striking point were immediately above this band (from 760 to 790 cps). The overtone structures produced at the cup striking point generally exhibited two strong partials: 400 and 540 cps. However, with only three exceptions (red / cup / pp, cord / cup / mf, and cord / cup / pp), the most intense partials produced at the cup striking point occurred between 740 and 800 cps.

New K. Zildjian Cymbal (Table 12)

Implement Hardness and Material

The harder red yarn implement consistently produced upper limits as high as, or higher than, the yellow yarn implement. The cymbal's overtone structures were not affected by a change in implement material.

Dynamic Level

With only one exception (cord / edge / ff), the ff level produced up to five more partials than the pp level. Most of these extra partials occurred above 5000 cps. The partials of the ff level were consistently of greater intensity than those of the pp level. The ff level produced

Table 12.--Fundamentals, Upper Limits, and Energy Peaks of the New K. Zildjian Cymbal

Imp	St Pt	Lev	Fund	UL	Energy Peaks in Decreasing Order of Intensity																							
			cps	cps	cps	dB	cps	dB	cps	dB	cps	dB	cps	dB	cps	dB	cps	dB	cps	dB	cps	dB	cps	dB	cps	dB	cps	dB
Yel	Edge	ff	235	20000+	4100	45	3200	44	2400	43	550	43	740	42	1700	41	6400	40	1100	39	980	39	7200	38	310	33	235	25
		mf	235	9500	550	42	3200	39	740	38	1650	37	1120	36	2350	35	4200	35	310	24	235	19						
		pp	235	3600	560	38	730	34	1100	27	1650	22	410	19	310	15	2450	15	235	12	3100	8						
	Cup	ff	440	20000+	4000	46	720	44	3150	43	600	43	2350	41	1800	40	7000	39	1050	37	1160	37	440	33				
		mf	370	10000	560	42	720	39	1600	37	2350	34	3150	33	4200	32	1240	32	370	24	7200	13						
		pp	340	2300	410	38	750	27	340	26	1220	20	1900	12														
Red	Edge	ff	235	20000+	3200	43	4100	42	560	42	730	41	1700	40	2350	40	6400	36	1100	36	410	29	310	21	235	12		
		mf	235	10000	560	41	740	38	420	37	2350	37	3200	26	1700	35	1100	34	980	33	310	25	235	14				
		pp	235	6000	560	40	740	36	1700	30	1020	27	2400	26	3200	24	410	23	4100	20	310	17	235	12				
	Cup	ff	360	20000+	4100	45	3200	44	560	43	2350	42	740	42	1700	41	1010	39	7200	36	410	35	360	22				
		mf	420	10000	570	41	3200	40	800	39	1750	38	4200	37	2400	36	420	35	1200	33	1000	32						
		pp	410	5500	540	34	740	32	410	28	970	27	1700	27	1090	26	2300	22	3200	19	4100	14						
Cord	Edge	ff	235	20000+	4100	44	3200	43	560	42	750	41	1650	41	2350	40	1100	39	310	29	235	22						
		mf	235	12000	560	41	740	38	3200	37	2350	36	4100	36	1700	35	1100	33	410	28	310	26	235	13				
		pp	235	6000	600	39	750	35	1550	32	1000	26	2500	26	3200	23	425	20	4250	16	320	12	235	10				
	Cup	ff	410	20000+	3200	42	4100	41	740	41	1700	39	2450	38	6400	36	1020	36	1110	36	410	30						
		mf	410	10000	3200	40	740	38	2250	37	1650	36	560	36	4100	35	1100	34	410	33	7000	20						
		pp	410	6300	560	35	970	32	750	32	1700	30	2350	27	3200	27	410	27	4100	21								

consistently higher upper limits (all were 20000+ cps) than the pp level (from 2300 to 6300 cps).

Striking Point

The cup striking point consistently produced a higher fundamental partial (from 85 to 185 cps higher) than the edge striking point. The cup striking point never produced more partials than the edge striking point in parallel instances. The edge striking point produced more partials under 1000 cps by a forty-five to thirty-one margin, but the cup striking point produced two more partials above 5000 cps.

Pitch Level and Overtone Intensities

The edge striking point consistently produced a fundamental partial of 235 cps, but the cup striking point varied from 340 to 440 cps. These frequencies occurred only twice in a position higher than the seventh most intense partial (yellow / cup / pp, and red / cup / pp).

The New K. Zildjian cymbal's overtone structures included a strong and relatively narrow band of three to six partials between 235 and 980 cps. Except for the six ff instances and one mf instance (cord / cup / mf), the most intense partial of each example occurred in this frequency band (usually between 550 and 570 cps).

Old K. Zildjian Cymbal (Table 13)

Implement Hardness and Material

Except for the ff level, the harder red yarn implement produced higher upper limits than the yellow yarn implement. With one exception (cord / edge / ff), the cord implement produced more partials than the red yarn implement did in parallel instances.

Table 13.--Fundamentals, Upper Limits, and Energy Peaks of the Old K. Zildjian Cymbal

Imp	St Pt	Lev	Fund	UL	Energy Peaks in Decreasing Order of Intensity																									
			cps	cps	cps	dB	cps	dB	cps	dB	cps	dB	cps	dB	cps	dB	cps	dB	cps	dB	cps	dB	cps	dB	cps	dB	cps	dB	cps	dB
Yel	Edge	ff	225	20000+	2400	49	3700	47	1350	46	3000	46	4700	44	780	43	400	42	520	41	6500	39	630	38	310	37	225	20		
		mf	225	6000	520	43	400	40	1900	39	760	38	1350	38	940	36	310	36	2850	33	2500	32	3700	27	225	16				
		pp	220	2100	440	36	520	33	780	32	630	27	310	26	1340	19	220	12	1900	7										
	Cup	ff	330	20000+	1250	50	2900	48	2850	45	750	44	410	42	1250	41	1450	39	640	38	330	37	550	35	8500	24				
		mf	330	5500	750	36	410	35	2000	34	1250	33	525	33	840	32	2900	29	330	29	1500	37	3800	18						
		pp	330	1400	410	31	330	26	700	24	1250	13	1090	11																
Red	Edge	ff	225	20000+	1950	47	2500	45	2900	44	3600	43	400	42	525	42	880	41	1200	41	1030	40	760	39	310	37	630	36	225	16
		mf	220	9000	410	42	520	41	2500	40	1900	40	775	38	1350	37	630	35	4000	34	1050	33	310	33	225	13				
		pp	220	3000	410	34	520	31	310	27	760	26	630	20	1090	20	1325	18	1900	13	220	12	2500	5						
	Cup	ff	330	20000+	1900	50	2850	48	3600	46	1300	45	400	43	850	40	760	40	330	37	630	36	520	34	8900	35				
		mf	330	6000	400	42	1250	38	2175	37	740	37	330	34	640	33	1100	33	950	31	2850	30	3650	24						
		pp	330	3000	410	32	700	30	330	26	1250	25	520	24	1100	23	1900	17	2850	6										
Cord	Edge	ff	220	20000+	2000	50	3000	47	3850	45	245	44	400	43	1310	42	1500	42	775	41	6500	29	310	38	500	37	630	36	220	16
		mf	225	8000	410	43	520	42	760	41	2500	40	1900	39	310	35	630	35	940	34	1075	34	1350	34	3600	29	4400	28	225	15
		pp	225	4500	410	36	520	36	750	31	1350	28	875	27	310	26	2050	24	1075	23	2500	22	2800	19	225	12	3600	11		
	Cup	ff	330	20000+	2000	47	2950	44	3700	43	1300	42	2500	41	400	41	1000	39	870	37	6500	36	330	34	520	34	770	34		
		mf	330	7500	400	42	840	36	2300	35	1900	35	1450	33	1325	32	1100	32	640	32	2800	31	530	31	330	28				
		pp	330	4500	700	34	410	33	1325	29	825	28	2000	27	930	26	1060	25	330	24	510	24	2850	21	2500	19	3700	12		

Dynamic Level

The ff level consistently produced much higher upper limits (20000+ cps) than the pp level (from 1400 to 4500 cps). With only one exception (cord / cup / ff), the ff level produced up to six more partials than the pp level. The majority of the additional partials occurred above 3000 cps. The partials produced by the ff level were consistently of greater intensity than those produced by the pp level.

Striking Point

The cup striking point consistently produced a fundamental 105 to 110 cps higher than the fundamental produced at the edge striking point. The edge striking point produced more partials in all but one instance (cord / edge / pp). The edge striking point also produced partials which were generally of greater intensity than those produced at the cup striking point. The upper limits at the edge striking point were consistently either equal to, or higher than, the upper limits at the cup striking point.

Pitch Level and Overtone Intensities

The two striking points produced two rather definite fundamental partials: 220 to 225 cps at the edge striking point; and 330 cps at the cup striking point. However, these frequencies were rated higher than the seventh most intense partial only three times: yellow / cup / pp, red / cup / mf, and red / cup / pp.

Each example of the Old K. Zildjian cymbal's overtone structures included a concentrated band of three to seven relatively strong partials in the range from 220 to 1000 cps. Except for the six ff examples and one mf example (red / cup / mf), the two most intense partials of each instance fell in this frequency band. The most intense partials of the ff examples ranged from 1900 to 2400 cps. There were only eight partials of 4000+ cps, and only two of these were above 7000 cps.

<u>17" Paiste Cymbal</u> (Table 14)

Implement Hardness and Material

Although both the red and yellow yarn implements produced upper limits of 20000+ cps at the ff level, the harder red yarn implement produced higher upper limits at the mf and pp levels. The cord implement consistently produced upper limits as high as, or higher than, the red yarn implement.

Dynamic Level

The ff level consistently produced higher upper limits (20000+ cps) than the pp level (2450 to 6000 cps). Without exception, the ff level produced up to six more partials than the pp level in parallel instances. Most of the additional partials occurred above 4000 cps. The partials produced at the ff level were consistently of greater intensity than those produced at the pp level.

Striking Point

The cup striking point consistently produced a higher fundamental (100 to 115 cps higher) than the edge striking point. With only one exception (cord / edge / pp), the edge striking point produced up to four more partials than the cup striking point in parallel instances. The edge striking point produced upper limits which were higher than, or at least equal to, the upper limits produced at the cup striking point.

Table 14.--Fundamentals, Upper Limits, and Energy Peaks of the 17" Paiste Cymbal

Imp	St Pt	Lev	Fund	UL	Energy Peaks in Decreasing Order of Intensity																									
			cps	cps	cps	dB	cps	dB	cps	dB	cps	dB	cps	dB	cps	dB	cps	dB	cps	dB	cps	dB	cps	dB	cps	dB	cps	dB	cps	dB
Yel	Edge	ff	285	20000+	740	47	3600	46	2850	45	1500	44	5000	43	540	43	920	42	7000	42	2100	41	1050	41	425	41	350	40	285	37
		mf	285	9500	700	42	1000	37	1500	37	525	35	425	34	2850	33	3700	32	355	30	4700	29	285	25						
		pp	285	3100	700	34	920	31	520	30	1500	27	360	22	285	18	2900	12	2200	10										
	Cup	ff	390	20000+	740	50	290	49	3600	48	920	45	2200	42	290	42	525	41	1475	21	1900	41	1125	40	8500	35	1100	28		
		mf	390	8000	630	48	750	43	390	38	920	38	1200	37	1600	36	2950	35	3900	35	2150	31								
		pp	400	2150	630	36	720	34	400	29	920	22	1200	19	1600	15														
Red	Edge	ff	285	20000+	700	48	2900	46	4100	45	4800	44	6800	43	1500	42	920	41	2100	40	500	49	330	36	11100	31	285	26		
		mf	285	11000	720	46	3600	42	1500	39	1000	39	920	39	530	38	2850	38	4700	37	425	35	2200	34	355	32	285	24		
		pp	290	5000	920	34	725	33	540	30	360	29	1100	28	1500	27	1300	24	2100	21	2850	19	290	18	3600	15				
	Cup	ff	390	20000+	630	50	3800	48	2900	47	2200	44	4700	43	1500	42	390	41	1040	40	1200	39	920	38	7800	38				
		mf	390	9000	630	44	730	38	1640	37	3900	35	1220	33	390	32	950	32	2800	28										
		pp	390	4500	630	40	710	37	900	33	1600	30	390	29	1100	28	2850	21	3850	15										
Cord	Edge	ff	290	20000+	700	49	2900	47	4000	46	1450	43	2050	41	540	41	1200	40	1100	39	910	38	330	36	420	35	11000	33	290	26
		mf	290	11000	740	41	2900	40	3700	39	1500	38	500	37	4750	36	1000	36	2160	35	630	35	430	32	360	30	290	18		
		pp	290	6000	720	34	920	34	1500	29	1100	38	540	28	2900	27	455	26	370	23	290	13								
	Cup	ff	390	20000+	630	50	3900	48	2900	46	740	44	5500	43	900	42	2150	41	1475	40	540	40	390	39	8000	36				
		mf	390	9500	630	45	730	42	1650	39	3900	36	1030	36	1200	34	3200	33	4600	32	390	31	8000	10	9000	7				
		pp	400	6000	630	39	700	37	920	35	1500	33	400	30	1100	29	2900	24	2200	23	3900	20								

The partials produced at the cup striking point were generally of greater intensity than those produced at the edge striking point. The edge striking point produced more partials under 1000 cps than the cup striking point by a fifty-three to thirty-six margin.

Pitch Level and Overtone Intensities

Two rather definite fundamental partials were produced on the smaller Paiste cymbal: 285 to 290 cps at the edge, and 390 to 400 cps at the cup. However, there was no consistency as to the strength of these frequencies. They appeared as the third most intense partial twice, the fifth most intense twice, the sixth most intense three times, the seventh most intense once, the ninth most intense twice, the tenth most intense three times, the twelfth most intense three times, and the thirteenth most intense twice.

The overtone structures included a band of strong partials from 285 to 1100 cps. Almost half of the total partials produced occurred within this frequency band. Without exception, the most intense partials in each example occurred within this range. Only eleven partials of 5000+ cps were found.

20" Paiste Cymbal (Table 15)

Implement Hardness and Material

The harder red yarn implement produced higher upper limits at the pp and mf levels (the upper limits at the ff level were the same). The red yarn implement also tended to produce partials of slightly greater intensity at the mf and pp levels. The cord implement produced upper limits equal to, or higher than, the upper limits produced by the red

Table 15.--Fundamentals, Upper Limits, and Energy Peaks of the 20" Paiste Cymbal

Imp	St Pt	Lev	Fund	UL	Energy Peaks in Decreasing Order of Intensity																									
			cps	cps	cps	dB	cps	dB	cps	dB	cps	dB	cps	dB	cps	dB	cps	dB	cps	dB	cps	dB	cps	dB	cps	dB	cps	dB	cps	dB
Yel	Edge	ff	330	20000+	3900	50	2300	49	3000	48	1700	47	440	46	1200	43	800	40	630	39	900	37	330	47	8500	34	11000	21		
		mf	330	7500	430	43	530	39	2300	37	1200	36	3200	35	1700	35	800	35	330	35	3900	31	1000	31	650	30	5200	23		
		pp	320	3500	430	40	540	34	320	33	800	29	660	28	1280	25	1700	22	2200	14	2900	8								
	Cup	ff	380	20000+	380	50	440	49	2900	48	2300	45	1200	42	1700	42	800	41	550	40	5200	40	890	39	11250	16				
		mf	380	6000	380	45	1160	37	560	34	760	34	900	32	1700	30	2250	29	3100	27	4100	19								
		pp	380	2000	380	40	550	26	1300	25	900	22	1000	19	740	19	1900	6												
Red	Edge	ff	325	20000+	3300	50	425	47	2250	43	800	42	1800	41	5100	41	1160	40	1500	40	325	38	630	37						
		mf	330	9000	425	43	1160	40	540	37	800	36	3200	36	2300	36	1700	35	2600	35	3000	35	900	34	3900	33	330	32	6600	31
		pp	320	5000	390	37	430	36	540	33	320	32	1275	31	800	30	2200	28	1700	27	650	27	2900	22	3300	20				
	Cup	ff	380	20000+	380	47	3000	46	2200	45	3800	44	1800	42	800	40	1200	39	570	38	890	38	650	35						
		mf	380	7000	380	41	1160	38	550	34	2300	33	900	32	1700	31	3000	31	3400	31	730	30	660	28	3800	27				
		pp	380	4000	380	40	1250	30	540	28	900	26	1700	25	2200	23	730	19	2850	17										
Cord	Edge	ff	320	20000+	430	49	2300	47	2900	45	3800	44	1200	43	8000	43	5250	42	1700	42	525	41	660	39	320	38	7700	32		
		mf	320	10000	430	42	2900	41	2800	40	8000	38	530	37	1250	37	3350	37	1700	35	1000	33	320	32						
		pp	320	6000	430	41	2200	34	800	33	530	32	1250	32	1800	31	320	31	2900	30	4000	23								
	Cup	ff	380	20000+	380	48	2900	46	2300	44	1200	42	4300	41	5200	40	1800	40	800	38	570	38	650	35	7500	25				
		mf	380	7000	380	42	900	35	520	35	2350	34	2900	33	1100	33	1800	32	3400	32	750	29	3800	27						
		pp	380	5000	380	29	1260	34	1700	29	550	28	2250	28	885	28	2600	26	2850	24	3300	20								

yarn implement. The cord implement also tended to produce partials of slightly greater intensity at each dynamic level.

Dynamic Level

With one exception (red / edge / ff), the ff level produced up to four more partials than did the pp level. The majority of the additional partials occurred above 3500 cps. The partials produced at the ff level were consistently of greater intensity than those produced at the pp level. The ff level also consistently produced higher upper limits (20000+ cps) than the pp level (2000 to 6000 cps).

Striking Point

The fundamental partials produced at the cup striking point on the 20" Paiste cymbal were consistently 50 to 60 cps higher than the fundamental partials produced at the edge striking point. The upper limits produced at the cup striking point were lower at the mf and pp levels than the upper limits produced at the edge striking point. The partials produced at the edge striking point were generally of greater intensity than the partials produced at the cup striking point. The edge striking point produced more partials of 4000+ cps than the cup striking point by a ten to six margin.

Pitch Level and Overtone Intensities

The larger Paiste cymbal produced two quite definite fundamental frequencies: 300 to 330 cps at the edge striking point; and 380 cps at the cup striking point. The 380 cps partial was the most intense partial in each of the overtone structures produced at the cup striking point.

However, the edge fundamental rated only as high as the third most intense partial once and the fourth most intense once. The remaining edge fundamentals appeared as the seventh through the twelfth most intense partials. The majority of the partials produced occurred in the frequency range of 320 to 1300 cps. With only two exceptions (yellow / edge / ff, and red / edge / ff), the most intense partial in each example occurred within this range. Only sixteen partials were found with frequencies of 4000+ cps.

Summary of Experimental Results and Related Research

Instrument Size and Material

The basic acoustical principles of circular plates provide a basis for comparing cymbals and their overtone structures. With all other properties constant, these relationships are valid for cymbals: an increased thickness raises the resonant frequency; and a larger diameter lowers the resonant frequency.

Although the "other properties" were rarely constant on the cymbals studied, the above principles were confirmed to a certain extent. The Avedis Zildjian cymbal (.055" thick, with a diameter of 15.9") did produce higher fundamental frequencies at both the edge and cup striking points--340 and 400 cps--than the Old K. Zildjian cymbal (.052" thick, with a diameter of 16.1")--225 and 330 cps. However, the New K. Zildjian and the 17" Paiste cymbals (with dimensions of .049" and 16.1", and .043" and 17.0" respectively) also produced higher fundamental frequencies than the thicker Old K. Zildjian cymbal: New K. Zildjian's edge = 235 cps and cup = 410 cps; and 17" Paiste's edge = 285 to 290 cps and cup = 390 cps.

The increased size of the 20" Paiste cymbal seemed to have no consistent effect on the frequency of the edge or cup fundamental partials.

In addition to the diameter and thickness, it would seem that the various other dimensions (cup width, cup height, and bow height) also affect the resonant frequencies of cymbals. However, except for the Old K. Zildjian cymbal (which had the flattest bow, the lowest fundamental, and the most partials under 1000 cps), the effect of the shape of the bow was not consistent with Thompson's statement that a "flat . . . bow produced a lower more bodied sound." The Avedis Zildjian cymbal had a flatter bow (.8") than the New K. Zildjian and 17" Paiste cymbals (each .9"), yet it produced a higher fundamental at both the edge (340 cps compared to 235 and 285 to 290 cps) and the cup (400 cps compared to about 400 and 390 cps). None of the various dimensions (diameter, thickness, cup width, cup height, and bow height) seemed to have any consistent effect on either the total number of partials or the number of partials below 1000 cps or above 5000 cps for any of the five cymbals studied.

As has been stated before, the leading cymbal manufacturers apparently make their cymbals from the same basic ingredients (copper and tin, with small amounts of iron, lead, or silver). The success or failure of each instrument is therefore attributed to the processing and/or the use of catalytic agents during the processing. These items are closely guarded secrets of each company.

Implement Hardness and Material

The harder red yarn implement produced higher upper limits at the mf and pp levels than the yellow yarn implement on the Old K. Zildjian, the 17" Paiste, and the 20" Paiste cymbals. There was no effect on the Avedis Zildjian cymbal's overtone structures due to a change in implement hardness. And on the New K. Zildjian cymbal, the red yarn implement consistently produced upper limits as high as, or higher than, the yellow yarn implement. In addition to the higher upper limits, the red yarn implement also produced partials of slightly greater intensity at the mf and pp levels on the 20" Paiste cymbal.

The cord implement produced upper limits equal to, or higher than, the upper limits produced by the red yarn implement on the two Paiste cymbals. On the Avedis Zildjian cymbal, the cord implement produced more and more intense partials of 6000+ cps. The cord implement generally produced more total partials on the Old K. Zildjian cymbal and generally partials of greater intensity on the 20" Paiste cymbal. A change of implement material had no consistent effect on the overtone structures of the New K. Zildjian cymbal.

Striking Point

The edge striking point produced upper limits equal to, or higher than, the upper limits produced by the cup striking point. All five cymbals consistently produced higher fundamentals when struck at the cup striking point than when struck at the edge striking point. The greatest difference occurred on the New K. Zildjian cymbal (up to 185 cps) and the smallest difference occurred on the Avedis Zildjian and the 20" Paiste cymbals (up to 60 cps).

Other noticeable effects of the edge striking point were: more partials below the 1000 cps level on the Avedis Zildjian, the New K. Zildjian, and the 17" Paiste cymbals; usually more and more intense partials on all cymbals; and more partials of 4000+ cps on the 20" Paiste cymbal.

Dynamic Level

The ff level consistently produced upper limits considerably higher (20000+ cps) than did the pp level (never higher than 7000 cps). The partials produced at the ff level were consistently of greater intensity than those produced at the pp level. With very few exceptions, the ff level produced from four to six more partials than did the pp level in parallel instances. Most of these additional partials occurred above the 3000 cps level on the Old K. Zildjian cymbal, the 3500 cps level on the 20" Paiste cymbal, the 4000 cps level on the Avedis Zildjian and 17" Paiste cymbals, and the 5000 cps level on the New K. Zildjian cymbal.

Pitch Level and Overtone Intensities

Each cymbal produced rather definite fundamental partials at each striking point. The fundamentals produced at the edge striking point on the smaller cymbals ranged from the 220 to 225 cps partials of the Old K. Zildjian cymbal to the 340 cps partials of the Avedis Zildjian cymbal. The fundamentals produced at the cup striking point on the smaller cymbals ranged from the 330 cps level of the Old K. Zildjian cymbal to the 340 to 440 cps partials of the New K. Zildjian cymbal.

The 20" Paiste cymbal's edge fundamental was 300 to 330 cps, and the cup fundamental was 380 cps.

The above fundamental partials were generally relatively weak. A fundamental partial occurred as the most intense partial only once on the four smaller cymbals (Avedis Zildjian / edge / yellow / mf). However, the fundamental partials at the cup striking point were consistently the most intense partials with all implements and all dynamic levels on the 20" Paiste cymbal. The fundamental partials were one of the two weakest partials in thirty-five of the fifty-four examples on the New K. Zildjian, the Old K. Zildjian, and the 17" Paiste cymbals. There was no consistency among the cymbals as to which partials were the most intense.

Except for the ff examples, the most intense partials on all the cymbals were rarely as high as 1000 cps; and in all examples, the strongest three partials rarely went as high as 4000 cps. Only twenty-one energy peaks were found above the 8000 cps level mentioned by Sivian, Dunn, and White (see above on p. 14). Even the 5000 cps level reported by Briggs (see above on p. 14) seemed too high, for only fifty-eight energy peaks were found above that level.

Similarity of Overtone Structures

The partials and their frequencies presented above in Tables 11 through 15 form the basis for Figure 26. By plotting each partial's frequency on a horizontal scale marked in thousands of cycles per second (kcs), the overtone structures of the cymbals may be observed and compared. Frequencies produced by striking at the edge of the cymbals are indicated by the vertical lines above the frequency scales, and those produced by striking at the cup of the cymbals are indicated by

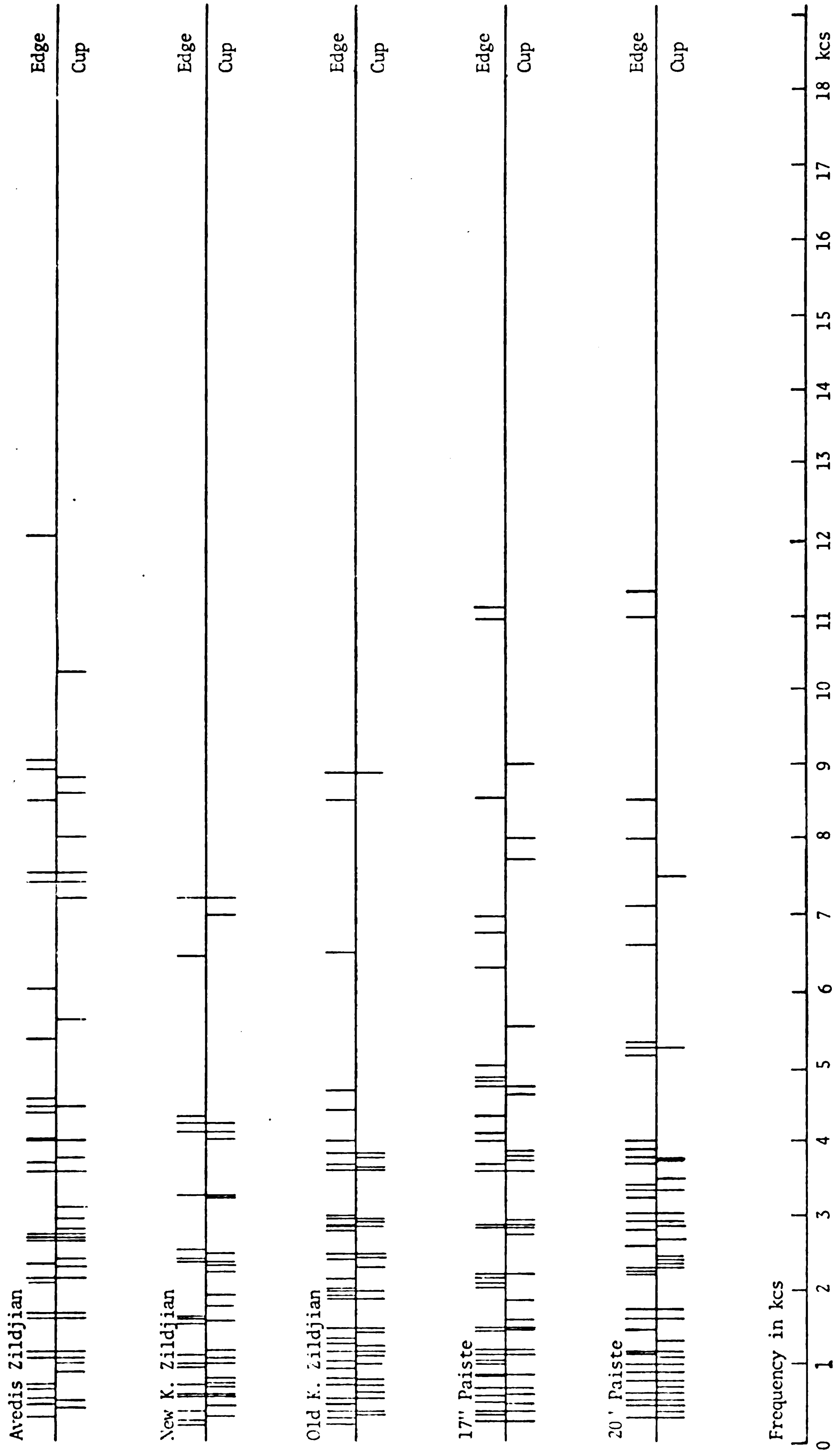

Figure 26.--Analyzed Overtone Structures for Cymbals

the vertical lines below the frequency scales. Overtone intensities are not a factor in Figure 26.

The five cymbals studied exhibited somewhat similar overtone structures, producing five rather definite groups of partials under 5000 cps. The 20" Paiste cymbal's groups were the lowest in frequency (320 to 1275 cps; 1500 to 1900 cps; 2200 to 2600 cps; 2850 to 3400 cps; and 3800 to 4300 cps), and the 17" Paiste cymbal's groups were the highest in frequency (285 to 1640 cps; 1900 to 2200 cps; 2850 to 2950 cps; 3600 to 4100 cps; and 4600 to 5000 cps).

A further similarity among the cymbals was the presence of relatively few partials above 5000 cps. The Avedis Zildjian cymbal had the most evenly distributed partials, and the New K. Zildjian cymbal had the most clearly grouped partials with wide spaces between the groups.

IV. CONCLUSIONS AND RECOMMENDATIONS

Conclusions

The questions presented in Chapter I are restated below accompanied by their answers as concluded from the results presented in Chapter III. Conclusions are also drawn from related research concerning instrument size and material, and pitch level.

What are the comparative overtone structures produced by triangles when played with large and small implements of the same material?

Based on the results of the study of the five small triangles (Abel, Ludwig, Sonor, Zildjian, and the 6" Pigstail), it must be concluded that a change in implement size from a diameter of 5/32" to one of 7/32" produces no consistent effect on the overtone structures. This confirms the findings of Spencer (no overtone changes due to implement size). However, the smaller implement did produce partials of slightly greater intensity on the 10" Pigstail triangle at the mf and pp dynamic levels.

What are the comparative overtone structures produced by cymbals when played with hard and soft implements of the same material?

On all but the Avedis Zildjian cymbal, a harder implement (red yarn) produces upper limits at the mf and pp levels higher than, or at least equal to, those produced by a softer implement (yellow yarn). At the ff level, both implements produce sound above 20000 cps. According to this study, a harder implement also produces partials of slightly greater intensity on the 20" Paiste cymbal than a softer implement.

What is the effect, if any, of the material of the implement on the overtone structures produced by triangles and cymbals?

Triangles

On the four small, untapered triangles (Abel, Ludwig, Sonor, and 6" Pigstail), a change of implement material from drill rod to cold-rolled steel does not affect the overtone structures in any consistent manner. However, a cold-rolled steel implement will generally produce more partials on the tapered Zildjian triangle, and partials of slightly greater intensity on the 10" Pigstail triangle than the drill rod implement.

Cymbals

Based on the results of this study, it must be concluded that a change of implement material from cord to yarn has no consistent effect on the overtone structures of cymbals. This seems to refute Bartlett's implication that harder implements cause the higher partials to predominate.

What is the precise relationship between the striking angle and/or point and the predominantly high and low pitch areas within one triangle or cymbal?

Triangles

As the top and bottom striking points were used with the implement perpendicular to the plane of the triangle (90° striking angle), both of these points produced in-plane, or transverse, vibration. Four of the six triangles studied (Abel, Ludwig, Zildjian, and 10" Pigstail) exhibited higher fundamentals in this type of vibration. The out-of-plane vibration produced by the combination of corner striking point and 0° striking angle induced torsional vibration in at least one side of the triangle (the closed side). Zahm's statement that the frequency produced by torsional vibration is lower than that produced by transverse vibration[1] is thus confirmed by the results of the Abel, Ludwig, Zildjian, and 10" Pigstail triangles. However, Peters' statement that a 0° striking angle "will produce a more diffuse sound with more overtones" seems to be refuted due to the general effects of this striking angle (lower upper limits; fewer partials; and weaker partials).

Although not confirmed by this study, acoustical principles state that "if the point of impact coincides with a node for a particular partial, that partial will tend to be absent or at minimum intensity, while if it coincides with an antinode, the partial will be present with greater strength."[2] Therefore, the percussionist could expect to elicit the highest sounds by striking the triangle at a point near one of the closed corners with a 90° striking angle. The middle of any side should produce slightly lower frequencies when struck at the same angle. And striking either open leg with a 0° striking angle should produce the lowest sounds. This substantiates the findings of Ross and Spencer that the sounds produced on the open sides seem to be lower than the sounds produced on the closed side.

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[1]J.A. Zahm, Sound and Music, 2nd ed., pp. 189-90.

[2]Wood, Acoustics, p. 94.

Cymbals

As the cup striking point consistently produced higher fundamental partials, and upper limits equal to or less than those produced by the edge striking point, it must be concluded that a relatively narrow band of sound is produced by striking near the cup and a relatively full sound is produced by striking near the edge of the cymbal. These findings confirm the statements of the authorities quoted in Chapter I (Denov, Leidig, Collins and Green, Goldenberg, Thompson, and Leach).

What are the comparative overtone structures produced by triangles and cymbals when played at various dynamic levels?

On both triangles and cymbals, a dynamic level of ff will have three effects on the overtone structures: higher upper limits; all partials will be of greater intensity; and more partials will be produced in the majority of instances.

What are the relative strengths or intensities of the overtones produced by triangles and cymbals?

Triangles

Three definite conclusions may be drawn concerning relative overtone strengths or intensities on triangles: the fundamental partials are usually rather weak; with few exceptions, the most frequent strong partials are below 7000 cps; and there is no consistency as to which partials are most intense.

Cymbals

The following conclusions may be drawn concerning relative overtone strengths or intensities on cymbals: the fundamental partials are usually relatively weak; the strongest three partials will usually be below 4000 cps; and there is no consistency as to which partials are most intense.

What are the similarities, if any, among the overtone structures produced by different types of triangles (e.g., spindle and pigstail) and different brands of cymbals (e.g., Avedis Zildjian and Paiste)?

Triangles

Based on the predicted and analyzed overtone structures, the four relatively thin triangles (Ludwig, Sonor, Zildjian, and 6" Pigstail) exhibited some degree of similarity in the way their partials were grouped. The relatively thick triangles (Abel and 10" Pigstail) were similar in that they produced partials which were more evenly distributed throughout the frequency range.

Cymbals

Although the frequency ranges of each group of partials were different for each cymbal, the five cymbals studied produced overtone structures which were similar in two ways: five groups of partials under 5000 cps, and relatively few partials above 5000 cps.

What are the modes of vibration of a triangle suspended at one corner?

A 90° striking angle induces primarily transverse, or in-plane, vibration in all three sides of the triangle; and a 0° striking angle induces primarily out-of-plane (but still transverse) vibration in all sides and torsional vibration in at least one side (the closed side).

In the mathematical study, no side exhibited more than three nodes for in-plane vibration, or more than four nodes for out-of-plane vibration. There is no consistency, either for any one triangle or among all the triangles, as to the number of nodes occurring in each side at any given frequency or partial number. Both closed corners and both free ends of each triangle are in motion, either from in-plane or out-of-plane vibration, at each frequency.

Instrument Size and Material

Triangles

The mathematical and experimental results confirmed that a longer length does indeed tend to produce a lower resonant frequency on a triangle: 10" Pigstail = 469.7 cps predicted and about 500 cps analyzed; and Abel = 1732 cps predicted and about 1550 cps analyzed. The results also confirmed that a larger diameter does tend to produce a higher resonant frequency: Abel = 1732 cps predicted and about 1550 cps analyzed; and Sonor = 828 cps predicted and about 900 cps analyzed.

As all the triangles studied were plated, no conclusions as to the effect of the plating can be drawn from the experimental results. However, the application of the expression $\sqrt{\frac{E\,I}{\rho\,A\,l^2}}$ indicates that the influence of a plating even several thousandths of an inch thick would be very minimal at best.

Triangles made from metals with larger E : ρ ratios than steel, such as beryllium, molybdenum, or chromium, will produce higher resonant frequencies than steel triangles. And triangles made from metals with ratios smaller than steel, such as brass, silver, or aluminum, will produce lower resonant frequencies than steel triangles. However, steel apparently has the combination of E and ρ values, plus the appropriate damping characteristics, most conducive to the production of a high resonant frequency with a relatively long duration of sound.

Cymbals

It must be concluded that, although the relationships of diameter to frequency and thickness to frequency are generally applicable, the principles did not provide an infallible basis for comparing the overtone structures of the five cymbals studied. No conclusions could be made concerning the influence of the heights of the cup and bow, and the width of the cup.

As the three leading cymbal manufacturers apparently make their instruments from the same basic ingredients (copper and tin, with small amounts of iron, lead, or silver), it must be concluded that the chemical makeup of the cymbal material does not affect the overtone structures of the cymbals.

Pitch Level

Given the right combination of triangle, implement, striking point and angle, and dynamic level, it is conceivable (though improbable) that some partial(s) could be emphasized to the extent of being noticeably stronger than other partials, thus approaching a comparatively "definite" pitch. And Bartholomew reports that it is possible, through careful selection of the striking point, stroke, and implement, to isolate certain of a cymbal's many simultaneously-sounding partials,[1] thus creating a somewhat well-defined pitch. These practices, however, would disregard

[1]Bartholomew, Acoustics of Music, p. 134.

the common opinion that good triangles and cymbals do not produce definite pitches. And no instrument examined did produce anything like a definite pitch, or even a noticeably strong band of frequencies which could be heard as definite pitches.

Thompson's conclusion that a New K. Zildjian cymbal seems to have more lows than an Avedis Zildjian cymbal is substantiated. However, the fact that the upper partials decayed more rapidly than the lower partials on all the cymbals seems to refute the statements of Spohn (the sound "will tend to rise") and Lang (it is important "that its highs 'come out'") concerning the projection and duration of the upper partials.

Recommendations for Performance

It should again be noted that this study is not concerned with the strike tones (or initial transients), but only with the overtone structures present after impact. The author recognizes that implement size, hardness, and material, striking point, and dynamic level all undoubtedly contribute significantly to the overall sound (impact plus sustaining sound) produced by a triangle or cymbal. With this in mind, the following recommendations for performance (including choice of instrument, choice of implement, and striking point and angle) are presented based on the preceding conclusions drawn from the results in Chapter III.

Instrument Size and Material

Triangles

For a high resonant frequency, a triangle with a combination of a relatively short length and a relatively large diameter is recommended.

For a lower resonant frequency, either a larger or thinner triangle is recommended.

Steel triangles are recommended, for, at the present time, steel apparently has the combination of material properties, plus the appropriate damping characteristics, most conducive to the production of high resonant frequencies with relatively long duration of sound. However, further research may contribute metals with Higher $E : \rho$ ratios plus superior damping qualities, thus resulting in triangles with even higher resonant frequencies and greater duration of sound.

Cymbals

Only two general recommendations can be made concerning cymbal size: given two cymbals of equal diameter, the thicker one should be used if higher sounds are desired; and given two cymbals of equal thickness, the larger one should be used if lower sounds are desired.

Based on this study, no recommendations can be made for choosing among Avedis Zildjian, New and Old K. Zildjian, or Paiste cymbals on the basis of material.

Implement Size or Hardness and Material

Triangles

As a change of implement size from 5/32" to 7/32" produced no consistent effects on the overtone structures of the triangles in this study, no recommendations can be made concerning implement size.

When playing one of the small, untapered triangles (Abel, Ludwig, Sonor, and 6" Pigstail), implements of either drill rod or cold-rolled steel are recommended (with some percussionists preferring the drill rod as it resists denting). However, a cold-rolled steel implement is recommended to produce a fuller sound (more partials) on a Zildjian triangle; and a drill rod implement is recommended for the 10" Pigstail triangle is partials of greater intensity are desired.

Cymbals

A harder yarn implement is recommended if the desired cymbal sound at a mf or pp dynamic level is to be as high as possible. As a change of implement material from cord to yarn produced no consistent effects on the overtone structures of the cymbals in this study, no recommendations can be made concerning implement material.

Striking Point and Angle

Triangles

The following recommendations are made for playing Abel, Ludwig, Zildjian, and 10" Pigstail triangles: strike near one of the closed corners with a 90° striking angle for the highest sounds; strike in the middle of any side with a 90° striking angle for slightly lower sounds; and strike on either open side with a 0° striking angle for the lowest sounds.

Cymbals

Striking near the cup of a cymbal is recommended if a high, relatively narrow sound is desired. Conversely, striking near the edge of a cymbal is recommended if low, full sounds are desired. Striking at both points simultaneously should help to bring out the total sound inherent in the cymbal.

Recommendations for Further Research

The following topics are presented to illustrate areas of research which would contribute the information to confirm or refute many of the theories and opinions not covered in this study relative to the selection of, and performance on, triangles and cymbals. It should be noted that similar studies of other percussion instruments (e.g., wood block, tambourine, tamtam, cowbell, temple blocks, claves) would also benefit public school music directors and performing percussionists.

Triangles

1. Construct triangles of equal geometric properties, but of different materials, in order to determine the exact influence of these materials on frequency and duration.

2. Determine the influence of the equality or inequality of the lengths of the sides of a triangle.

3. Determine the influence of the equality or inequality of the angles formed by the sides of a triangle.

4. Determine the influence of implements of more extreme sizes and other materials.

5. Determine the exact influence of the striking point and angle by using other points and angles in various combinations.

6. Determine the influence of implement size and material, striking point and angle, and dynamic level on the strike tones (or starting transients) of a triangle.

7. Determine the influence of aging on the acoustical properties of triangles.

8. Make comparative analyses of the acoustical properties of the favorite triangles of leading percussionists.

Cymbals

1. Determine the exact influence of diameter and thickness by constructing cymbals differing only in these dimensions.

2. Determine the influence of the shape, size, and height of the cup and bow.

3. Determine the influence of implements of more extreme sizes and hardnesses, as well as other materials.

4. Determine the influence of implement hardness, size, and material, striking point, and dynamic level on the strike tones (or starting transients) of a cymbal.

5. Determine the influence of aging on the acoustical properties of cymbals.

6. Make comparative analyses of the acoustical properties of the favorite cymbals of leading percussionists.

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APPENDICES

APPENDIX A

Letter of Inquiry

In an attempt to discover what, if any, scientific investigations had been (or were being) conducted on the sounds of triangles and cymbals, and also to gather the opinions of well-known and highly respected percussionists, the letter presented below was sent to thirty-nine sources from which the author received twenty-four replies.

Dear Sir,

While Instructor of Percussion at Michigan State University, I have been working on my Ph.D. in Theory. At present, I am beginning research for my dissertation, on the topic of "Some Acoustical Properties of Triangles and Cymbals and Their Relation to Performance Practices."

Some of the points I hope to investigate and draw some conclusions about include the following:

The general mode(s) of vibration in triangles and cymbals.

The average relative strengths of overtones created by triangles and cymbals.

The acoustical explanation of highs and lows within one cymbal.

The difference (if any) between the average overtone structures of different types or brands of triangles and cymbals.

The acoustical explanation of the different sounds produced by a triangle when struck parallel or perpendicular to its plane, and/or in different places.

The comparative overtone structures created by hard and and soft (for cymbals), and large and small (for triangles) implements.

The comparative overtone structures created by triangles and cymbals when played at various dynamic levels.

Any specific information you might be willing to relay to me concerning the above points, or any bibliographical sources (other than standard acoustics texts) to which you might direct me, will be greatly appreciated.

For your convenience, I have enclosed a reply sheet, plus a self-addressed, stamped return envelope.

Thank you in advance for your very kind consideration and prompt cooperation with this request for information.

Sincerely,

John Baldwin
Instructor of Percussion

APPENDIX B

Bruël and Kjaer Level Recorder Graphs

Figure A1 shows three graphs (identified as to instrument, implement, striking point, and dynamic level) printed by the Bruël and Kjaer Level Recorder. The complete graph consisted of six sections with frequency spans of: #1 = 0 to 63 cps; #2 = 63 to 200 cps; #3 = 200 to 630 cps; #4 = 630 to 2000 cps; #5 = 2000 to 6300 cps; and #6 = 6300 to 20000 cps. These frequencies are indicated by the numbers on the horizontal scale of the graphs (sections 3 and 4 must be multiplied by 10, and sections 5 and 6 must be multiplied by 100). It was determined that there was no instrument sound (either triangle or cymbal) in the first two sections of the graphs (0 to 200 cps). Therefore, only sections 3 through 6 are shown in Figure A1.

The vertical scale of the graphs represents decibel levels. Although two maximum decibel levels are indicated (25 and 50 dB), the 50 dB maximum was used in this study. The rather flat appearance of the peaks in section 5 of the top graph of Figure A1 is due to the fact that all the peaks and all the low points could not be obtained with only one setting of the Level Recorder controls. But this flattening effect occurred only in the graphs of the ff sounds, and then only occasionally. The length of the vertical portions of the lines indicates the comparative rapidity of decay of the sounds at various points in the frequency range. That is, the greater the vertical travel of the line, the faster the

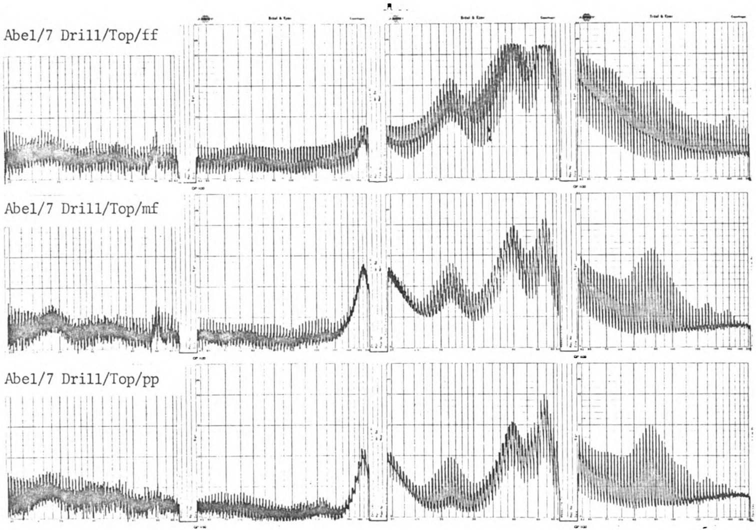


Figure A1.--Bruël and Kjaer Level Recorder Graphs decay. Thus, the frequencies represented in section 6 of Figure A1 decayed much more rapidly within the two seconds of recording time than the frequencies represented in section 5.

APPENDIX C

Modes of Vibration--Matrix and Diagrams

Table A1 shows the matrix developed by the SAMIS computer program for the vibrational modes of the Zildjian triangle. The partial numbers are listed in the first column. The next three columns are labeled points and locations. The numbers at the top of each column serve to identify the nodal points (1 through 16) and locations in three-dimensional space (1 = x, 2 = y, and 3 = z). That is, the numbers 11, 12, and 13 serve to locate nodal point 1 in the x, y, and z directions.

The partials with numbers in the first two columns only (i.e., in the x and y columns) were produced by in-plane vibration. The partials with a number in the third column only (i.e., in the z column) were produced by out-of-plane vibration. For in-plane vibration, "plus" numbers in the first two columns indicate positive directions for x (to the right) and y (up), while "minus" numbers indicate negative directions for x (to the left) and y (down). For out-of-plane vibration, "plus" numbers in the third column indicate a positive direction for z (above the plane of the triangle), while "minus" numbers indicate a negative direction for z (below the plane of the triangle).

The last three digits of the numbers indicate the placement of the decimal point: +001 indicates that the decimal point is to be moved to the right one place; and -002 indicates that the decimal point is to be moved to the left two places.

Table A1.--Vibrational Mode Matrix for Zildjian Triangle

Partial Numbers	Points and Locations		
	11	12	13
1	0.	0.	-0.3237+000
2	-0.8980-001	0.2665+000	0.
3	0.	0.	-0.3756+000
4	0.3925-001	-0.2950+000	0.
5	0.	0.	-0.3650+000
6	0.4471-001	0.4649+000	0.
7	-0.3715-001	-0.1173+000	0.
8	0.	0.	-0.1983+000
9	0.	0.	0.4142+000
10	-0.2451-001	-0.3739+000	0.
11	0.	0.	-0.1410+000
12	0.4351-001	-0.2387+000	0.
13	0.4054-001	-0.1069+000	0.
14	0.	0.	-0.1332+000
15	-0.5416-002	0.2745-001	0.
16	0.	0.	-0.2563+000

Partial Numbers	Points and Locations		
	21	22	23
1	0.	0.	0.1373+000
2	-0.1337+000	0.1158+000	0.
3	0.	0.	-0.6517-001
4	0.5841-001	-0.4519-001	0.
5	0.	0.	-0.3086-001
6	0.6650-001	-0.2702-001	0.
7	-0.5449-001	0.9894-001	0.
8	0.	0.	0.1618+000
9	0.	0.	-0.3755+000
10	-0.3756-001	0.3731+000	0.
11	0.	0.	0.1617+000
12	0.6416-001	0.2867+000	0.
13	0.5919-001	0.1914+000	0.
14	0.	0.	0.2516+000
15	-0.7870-002	-0.5402-001	0.
16	0.	0.	0.5056+000

Table A1.--(cont'd.)

Partial Numbers	Points and Locations		
	31	32	33
1	0.	0.	0.1287+000
2	-0.1461+000	-0.9703-001	0.
3	0.	0.	0.2693+000
4	0.6373-001	0.2133+000	0.
5	0.	0.	0.2760+000
6	0.7245-001	-0.4388+000	0.
7	-0.5921-001	0.1030+000	0.
8	0.	0.	0.1706+000
9	0.	0.	-0.3253+000
10	-0.4027-001	0.2530+000	0.
11	0.	0.	0.7312-001
12	0.6836-001	0.8966-001	0.
13	0.6138-001	-0.1020+000	0.
14	0.	0.	-0.1678+000
15	-0.8070-002	0.4155-001	0.
16	0.	0.	-0.3933+000

Partial Numbers	Points and Locations		
	41	42	43
1	0.	0.	0.2683+000
2	-0.1589+000	-0.1990+000	0.
3	0.	0.	0.3539+000
4	0.6916-001	0.2599+000	0.
5	0.	0.	0.2671+000
6	0.7846-001	-0.4223+000	0.
7	-0.6294-001	-0.6048-001	0.
8	0.	0.	-0.1124+000
9	0.	0.	0.3119+000
10	-0.4254-001	-0.3581+000	0.
11	0.	0.	-0.1516+000
12	0.7148-001	-0.3191+000	0.
13	0.6142-001	-0.1328+000	0.
14	0.	0.	-0.1356+000
15	-0.7931-002	0.2062-001	0.
16	0.	0.	-0.1935+000

Table A1.--(cont'd.)

Partial Numbers	Points and Locations		
	51	52	53
1	0.	0.	0.2463+000
2	-0.1719+000	-0.1563+000	0.
3	0.	0.	0.2023+000
4	0.7465-001	0.1079+000	0.
5	0.	0.	-0.2259-001
6	0.8445-001	-0.1411+000	0.
7	-0.6611-001	-0.9155-001	0.
8	0.	0.	-0.1789+000
9	0.	0.	0.4418+000
10	-0.4428-001	-0.3967+000	0.
11	0.	0.	-0.4389-001
12	0.7342-001	-0.2036+000	0.
13	0.5928-001	0.1522+000	0.
14	0.	0.	0.2269+000
15	-0.7443-002	-0.5600-001	0.
16	0.	0.	0.4922+000

Partial Numbers	Points and Locations		
	61	62	63
1	0.	0.	0.1063+000
2	-0.2145+000	0.1685-001	0.
3	0.	0.	-0.4879-001
4	0.9281-001	-0.1491+000	0.
5	0.	0.	-0.5130+000
6	0.1047+000	0.3118-001	0.
7	-0.7946-001	0.1473+000	0.
8	0.	0.	0.1570+000
9	0.	0.	-0.8990-001
10	-0.5263-001	0.5574-001	0.
11	0.	0.	0.3995+000
12	0.8574-001	-0.3893-001	0.
13	0.6355-001	0.5486-001	0.
14	0.	0.	-0.2439-001
15	-0.7625-002	-0.7199-002	0.
16	0.	0.	-0.1563+000

Table A1.--(cont'd.)

Partial Numbers	Points and Locations		
	71	72	73
1	0.	0.	-0.1148+000
2	0.4706-001	-0.1146+000	0.
3	0.	0.	-0.1712+000
4	-0.1491+000	-0.4964-001	0.
5	0.	0.	-0.3553-001
6	-0.3793-001	0.1084+000	0.
7	0.2847+000	-0.9087-002	0.
8	0.	0.	-0.1621-001
9	0.	0.	-0.2584+000
10	0.1817+000	-0.4416-001	0.
11	0.	0.	-0.3651+000
12	-0.2911+000	0.1401+000	0.
13	-0.2527+000	0.2045+000	0.
14	0.	0.	-0.9868-001
15	0.3329-001	-0.2536-001	0.
16	0.	0.	-0.8314-001

Partial Numbers	Points and Locations		
	81	82	83
1	0.	0.	-0.2652+000
2	0.2646+000	-0.2107+000	0.
3	0.	0.	-0.2075+000
4	-0.2783+000	0.1631-001	0.
5	0.	0.	0.3370+000
6	-0.2414+000	0.1996+000	0.
7	0.2940+000	-0.2128-001	0.
8	0.	0.	-0.1071+000
9	0.	0.	-0.1232+000
10	0.3680-001	0.2649-001	0.
11	0.	0.	-0.3557+000
12	-0.2910+000	0.1430+000	0.
13	0.3950-001	0.5142-001	0.
14	0.	0.	0.4428-001
15	-0.2371-001	0.4581-002	0.
16	0.	0.	0.7628-001

Table A1.--(cont'd.)

Partial Numbers	Points and Locations		
	91	92	93
1	0.	0.	-0.2823+000
2	0.3240+000	-0.2326+000	0.
3	0.	0.	-0.1092+000
4	-0.2505+000	0.9651-002	0.
5	0.	0.	0.3726+000
6	-0.2982+000	0.2218+000	0.
7	-0.6084-001	0.1343+000	0.
8	0.	0.	-0.1212-001
9	0.	0.	0.1783+000
10	-0.1994+000	0.1388+000	0.
11	0.	0.	0.3158+000
12	0.2035+000	-0.8477-001	0
13	0.3036+000	-0.9204-001	0.
14	0.	0.	0.8066-001
15	-0.4080-001	0.1631-001	0.
16	0.	0.	0.2824-001

Partial Numbers	Points and Locations		
	101	102	103
1	0.	0.	-0.1718+000
2	0.1997+000	-0.1691+000	0.
3	0.	0.	0.1138+000
4	-0.1307+000	-0.3985-001	0.
5	0.	0.	0.6328-001
6	-0.1495+000	0.1481+000	0.
7	-0.2786+000	0.2248+000	0.
8	0.	0.	0.1547+000
9	0.	0.	0.2287+000
10	-0.1503+000	0.1174+000	0.
11	0.	0.	-0.4527+000
12	0.4132+000	-0.1799+000	0.
13	-0.1371+000	0.9097-001	0.
14	0.	0.	-0.1015+000
15	-0.9190-002	0.5470-002	0.
16	0.	0.	-0.7844-001

Table A1.--(cont'd.)

Partial Numbers	Points and Locations		
	111	112	113
1	0.	0.	-0.5828-002
2	-0.5527-001	-0.4389-001	0.
3	0.	0.	0.4062+000
4	-0.9662-001	-0.4853-001	0.
5	0.	0.	-0.4031+000
6	0.5553-001	0.4714-001	0.
7	0.1687-001	0.7379-001	0.
8	0.	0.	0.2532+000
9	0.	0.	-0.3470-001
10	0.2110-001	0.3764-001	0.
11	0.	0.	-0.3819+000
12	0.1365+000	-0.4873-001	0.
13	-0.4682+000	0.2216+000	0.
14	0.	0.	-0.1586+000
15	-0.2804+000	0.1357+000	0.
16	0.	0.	0.1431+000

Partial Numbers	Points and Locations		
	121	122	123
1	0.	0.	0.3019+000
2	0.1900+000	0.1034+000	0.
3	0.	0.	-0.1496-001
4	0.1266+000	0.8157-001	0.
5	0.	0.	-0.1380+000
6	0.3381-001	0.3147-001	0.
7	-0.3931+000	-0.1805+000	0.
8	0.	0.	-0.4469+000
9	0.	0.	-0.1932+000
10	0.2038+000	0.1412+000	0.
11	0.	0.	0.1486-001
12	-0.5719-001	-0.1409+000	0.
13	-0.3590+000	0.2150+000	0.
14	0.	0.	0.4860+000
15	0.3247+000	0.4548+000	0.
16	0.	0.	-0.2382+000

Table A1.--(cont'd.)

Partial Numbers	Points and Locations		
	131	132	133
1	0.	0.	0.4780+000
2	0.3322+000	0.1897+000	0.
3	0.	0.	-0.3584+000
4	0.3899+000	0.2357+000	0.
5	0.	0.	0.3774-001
6	0.1106+000	0.7407-001	0.
7	-0.2471+000	-0.1063+000	0.
8	0.	0.	-0.3035+000
9	0.	0.	-0.1398+000
10	0.1921+000	0.1295+000	0.
11	0.	0.	0.1351+000
12	-0.1947+000	-0.1980+000	0.
13	-0.4895-001	0.3106+000	0.
14	0.	0.	-0.2789+000
15	-0.2558+000	0.5999-001	0.
16	0.	0.	0.9023-001

Partial Numbers	Points and Locations		
	141	142	143
1	0.	0.	0.4174+000
2	0.2978+000	0.1716+000	0.
3	0.	0.	-0.4271+000
4	0.4333+000	0.2597+000	0.
5	0.	0.	0.1039+000
6	0.1443+000	0.9112-001	0.
7	0.3146+000	0.2128+000	0.
8	0.	0.	0.4403+000
9	0.	0.	0.1478+000
10	-0.1623+000	-0.8531-001	0.
11	0.	0.	-0.8615-001
12	0.1247+000	0.1778-001	0.
13	-0.4508-001	0.2073+000	0.
14	0.	0.	-0.2800+000
15	-0.3567+000	-0.6421-001	0.
16	0.	0.	0.1665+000

Table A1.--(cont'd.)

Partial Numbers	Points and Locations		
	151	152	153
1	0.	0.	0.1707+000
2	0.1234+000	0.7075-001	0.
3	0.	0.	-0.2007+000
4	0.2013+000	0.1208+000	0.
5	0.	0.	0.5728-001
6	0.7309-001	0.4614-001	0.
7	0.3614+000	0.2272+000	0.
8	0.	0.	0.4984+000
9	0.	0.	0.1873+000
10	-0.2380+000	-0.1354+000	0.
11	0.	0.	-0.1669+000
12	0.2567+000	0.1235+000	0.
13	-0.2289+000	-0.1552-001	0.
14	0.	0.	0.6118+000
15	0.4921+000	0.3666+000	0.
16	0.	0.	-0.2953+000

Partial Numbers	Points and Locations		
	161	162	163
1	0.	0.	0.
2	0.	0.	0.
3	0.	0.	0.
4	0.	0.	0.
5	0.	0.	0.
6	0.	0.	0.
7	0.	0.	0.
8	0.	0.	0.
9	0.	0.	0.
10	0.	0.	0.
11	0.	0.	0.
12	0.	0.	0.
13	0.	0.	0.
14	0.	0.	0.
15	0.	0.	0.
16	0.	0.	0.

As the unit of measurement was not specified by the computer, the arbitrary scale chosen for the diagrams of the vibrational modes in Figure A2 was: a 5/16" square equaled a number of 0.2000+000. Referring to Table A1, the location of point 1 in the second mode of vibration (partial 2) was found by moving .0898 to the left (almost half a square) and .2665 up (slightly more than two and one-quarter squares). The same procedure was followed for each point for each partial, resulting in the accurate plotting of the vibrational modes for each partial.

In Figure A2, the shape of the triangle is shown by the solid lines, and the deformities produced by vibration are shown by the dotted lines. As mentioned before, point 16 was fixed to eliminate rigid body motions, thus accounting for the apparent lack of vibration at the free end of the upper right-hand leg.

The diagrams for in-plane vibration (partials 2, 4, 6, 7, 10, 12, 13, and 15) are shown as if the triangle were being observed from a point perpendicular to the plane of the triangle. The diagrams for out-of-plane vibration (partials 1, 3, 5, 8, 9, 11, 14, and 16) are shown as if the triangle were being observed from a point just above the plane of the triangle looking from the base toward the apex.

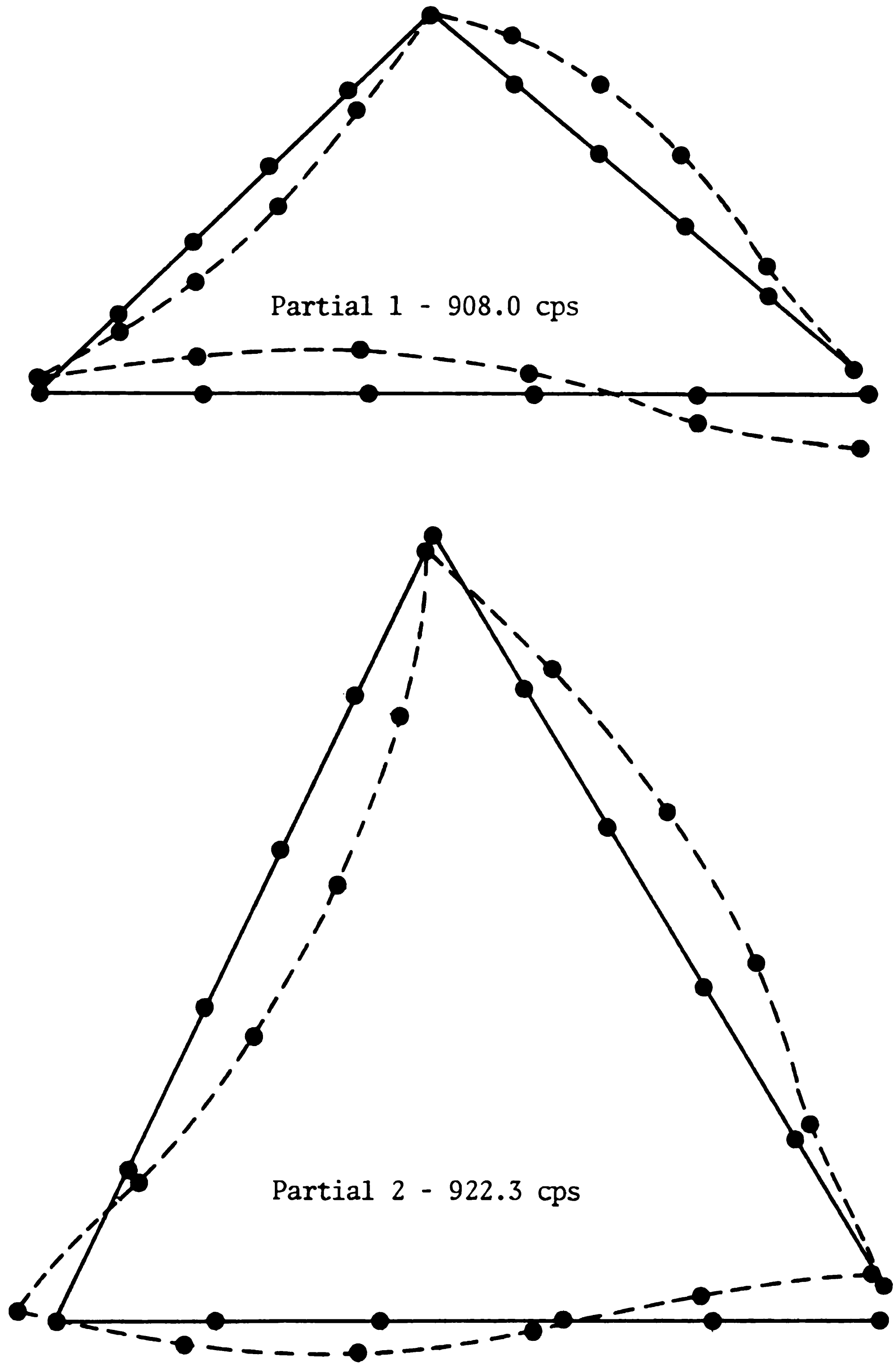
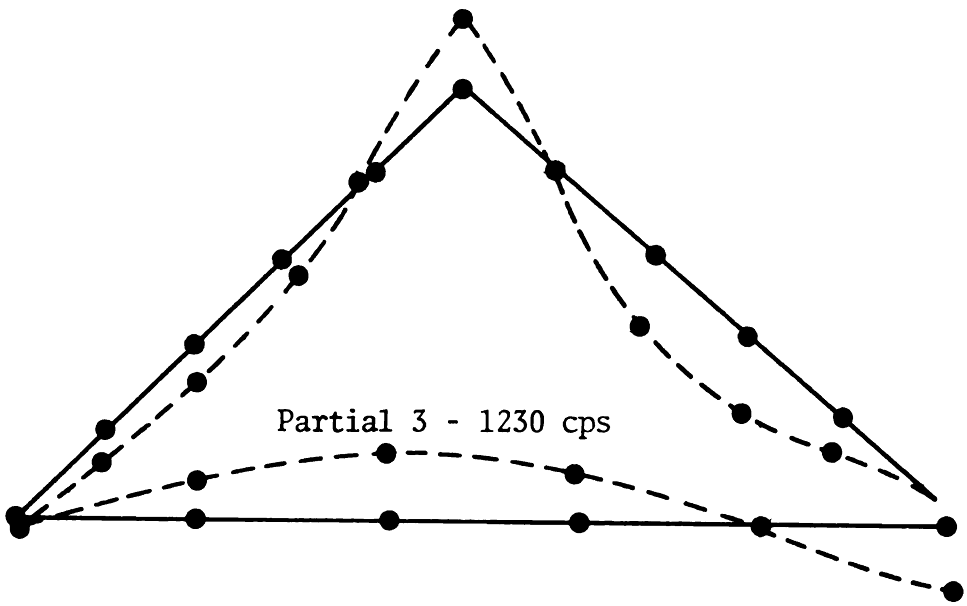


Figure A2.--Modes of Vibration for Zildjian Triangle



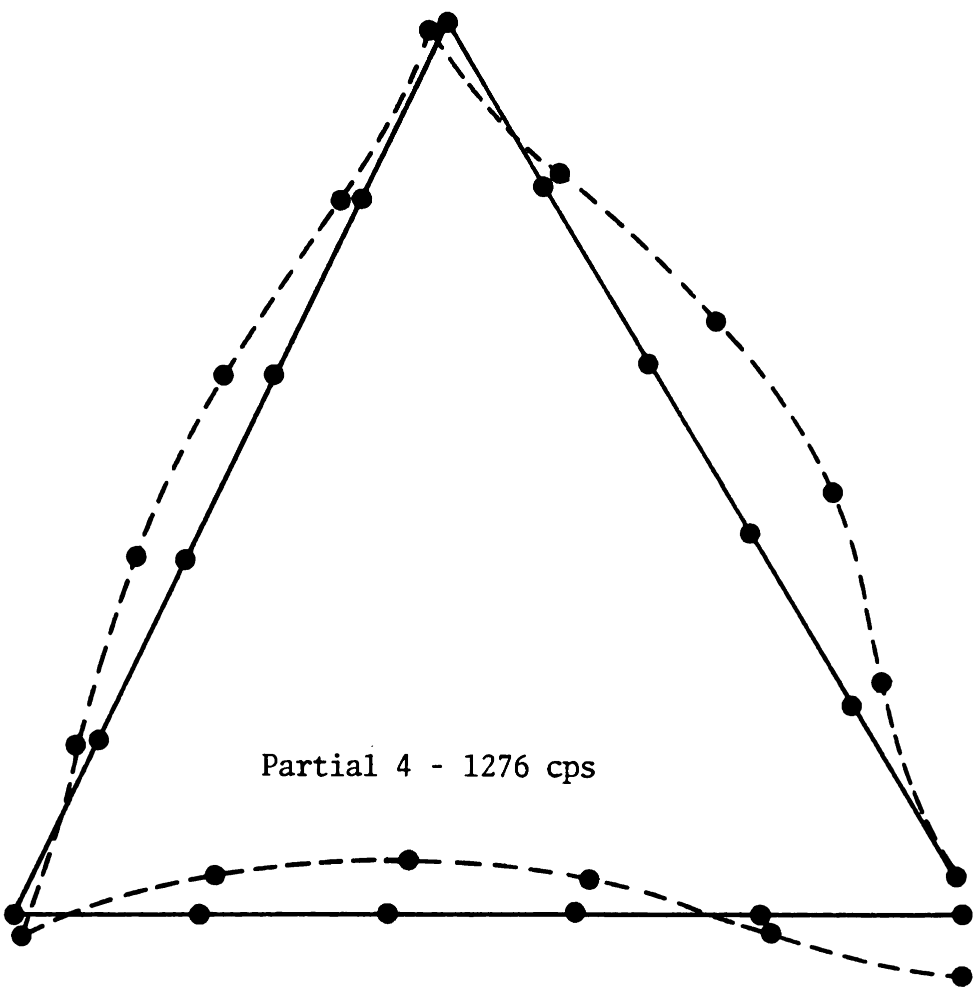
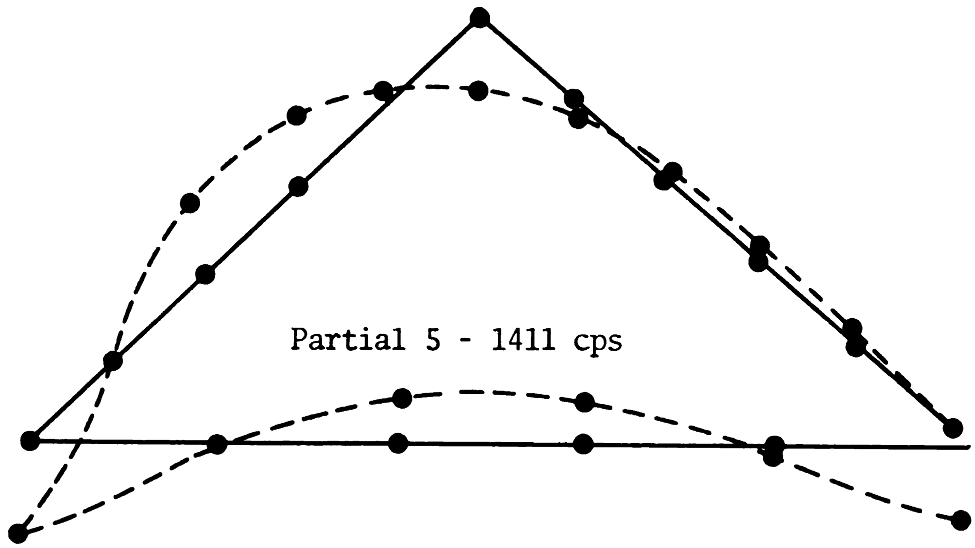


Figure A2.--(cont'd.)



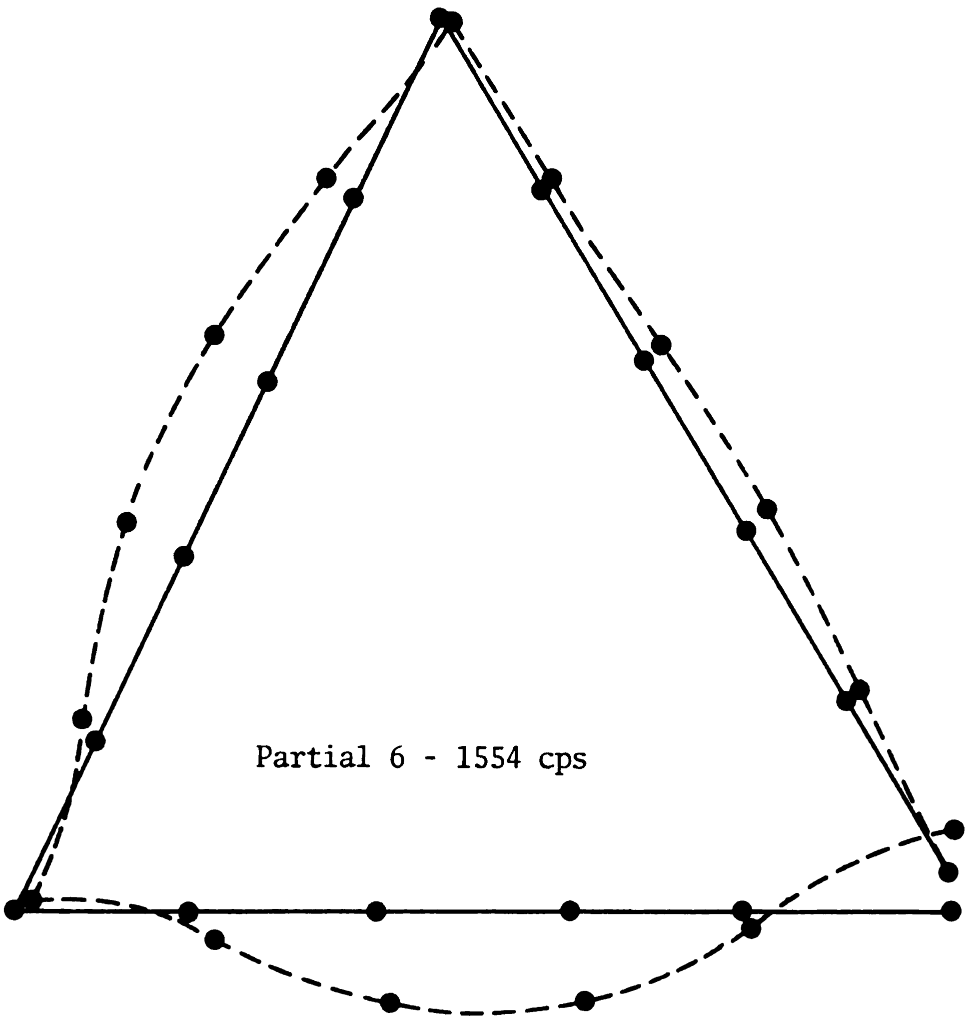


Figure A2.--(cont'd.)

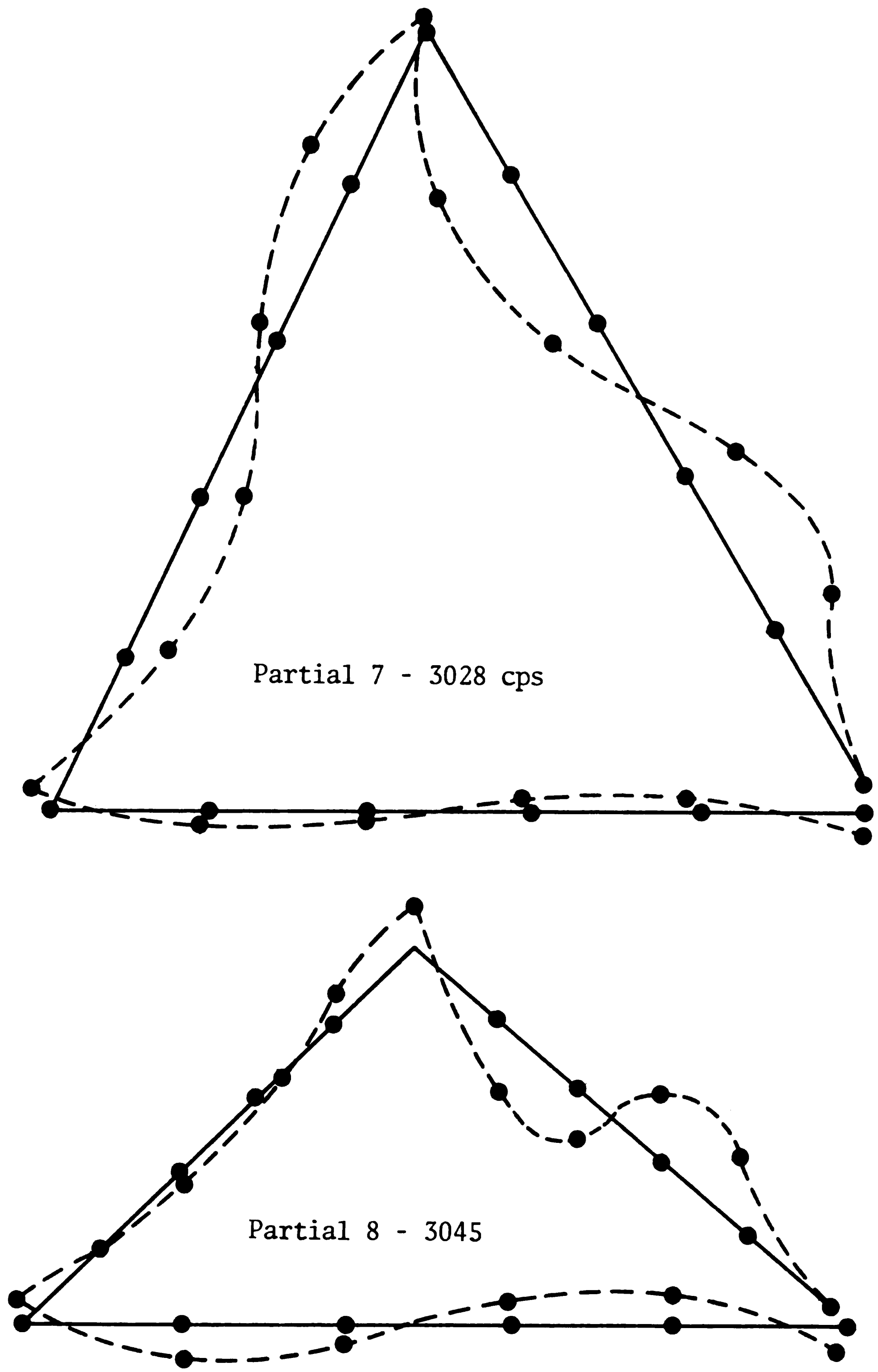
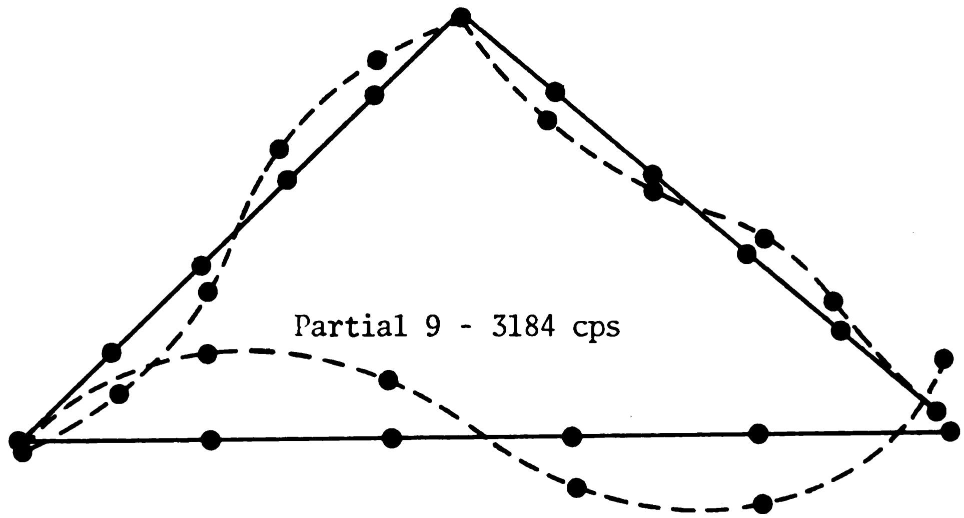


Figure A2.--(cont'd.)



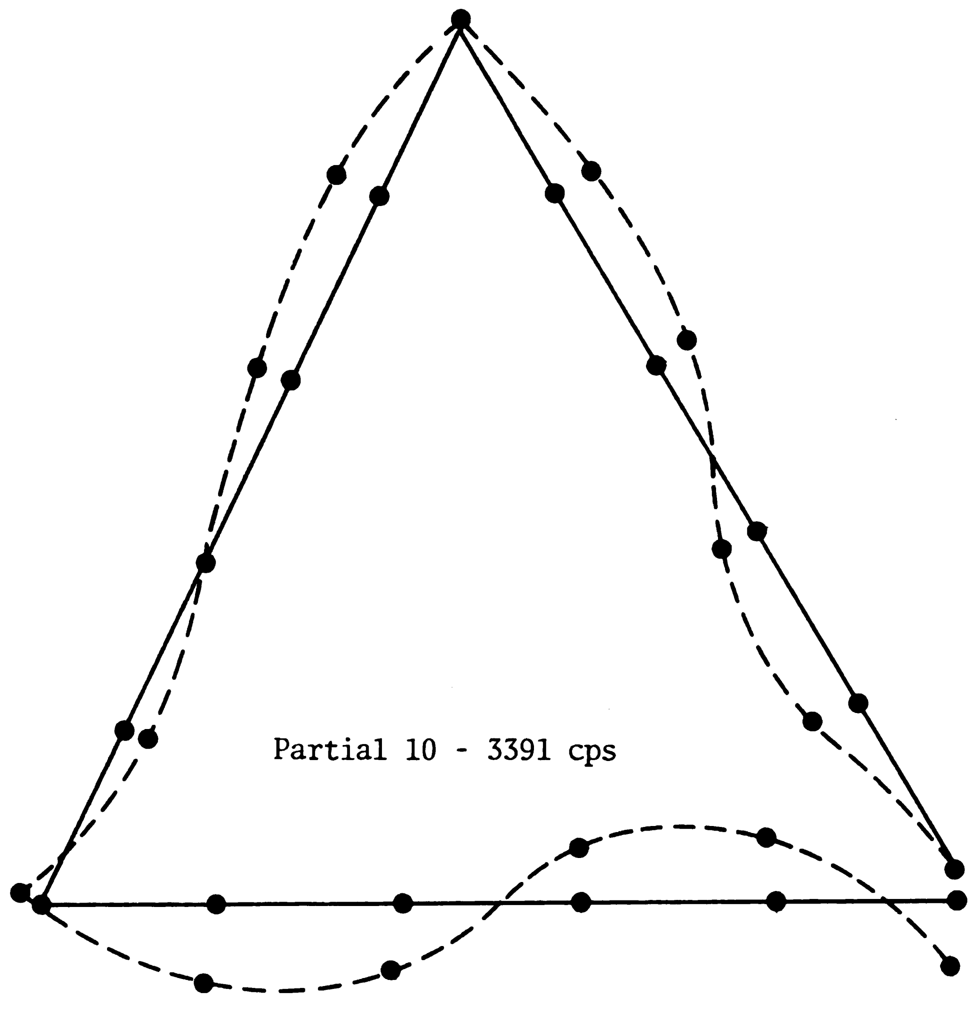
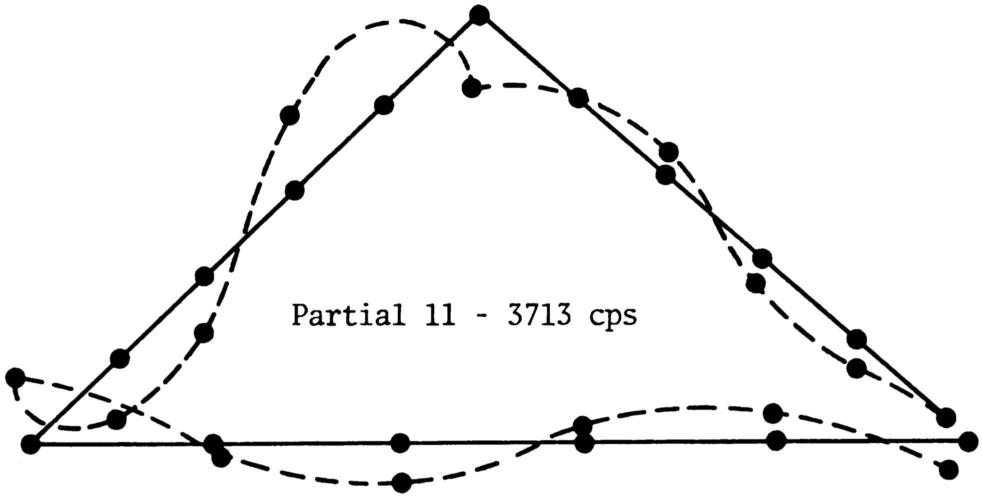


Figure A2.--(cont'd.)



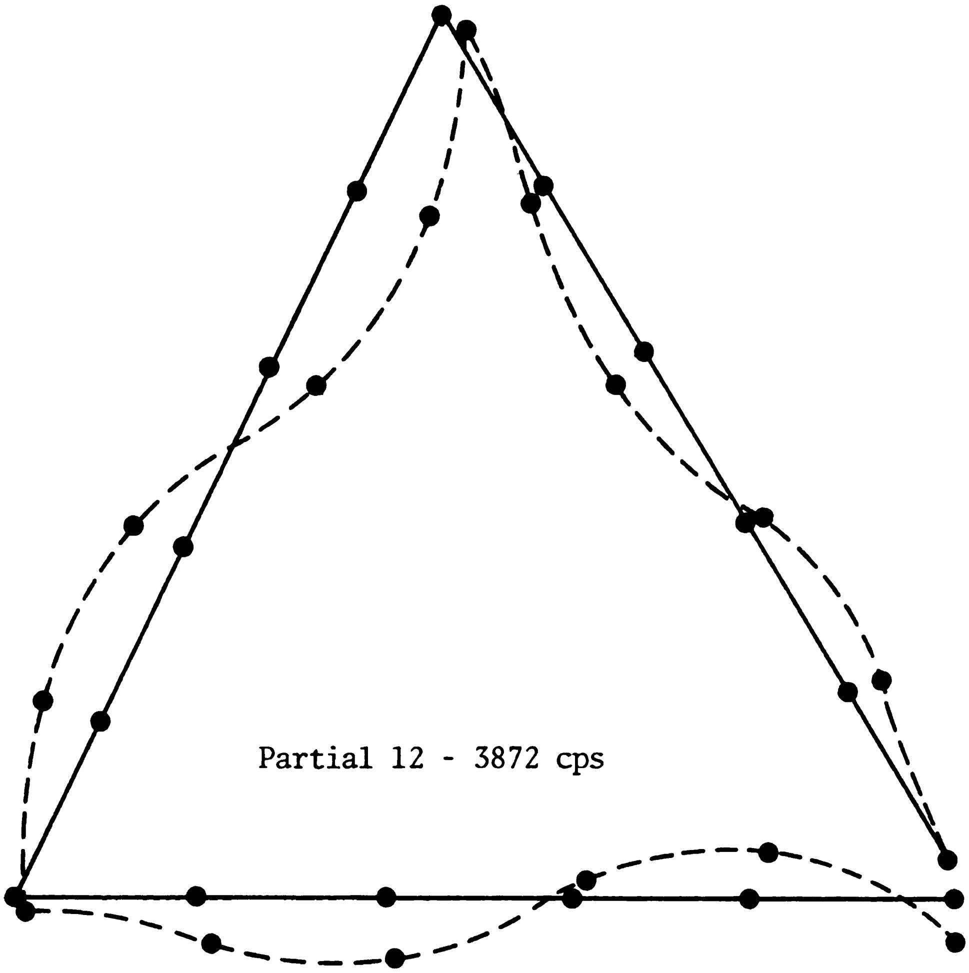
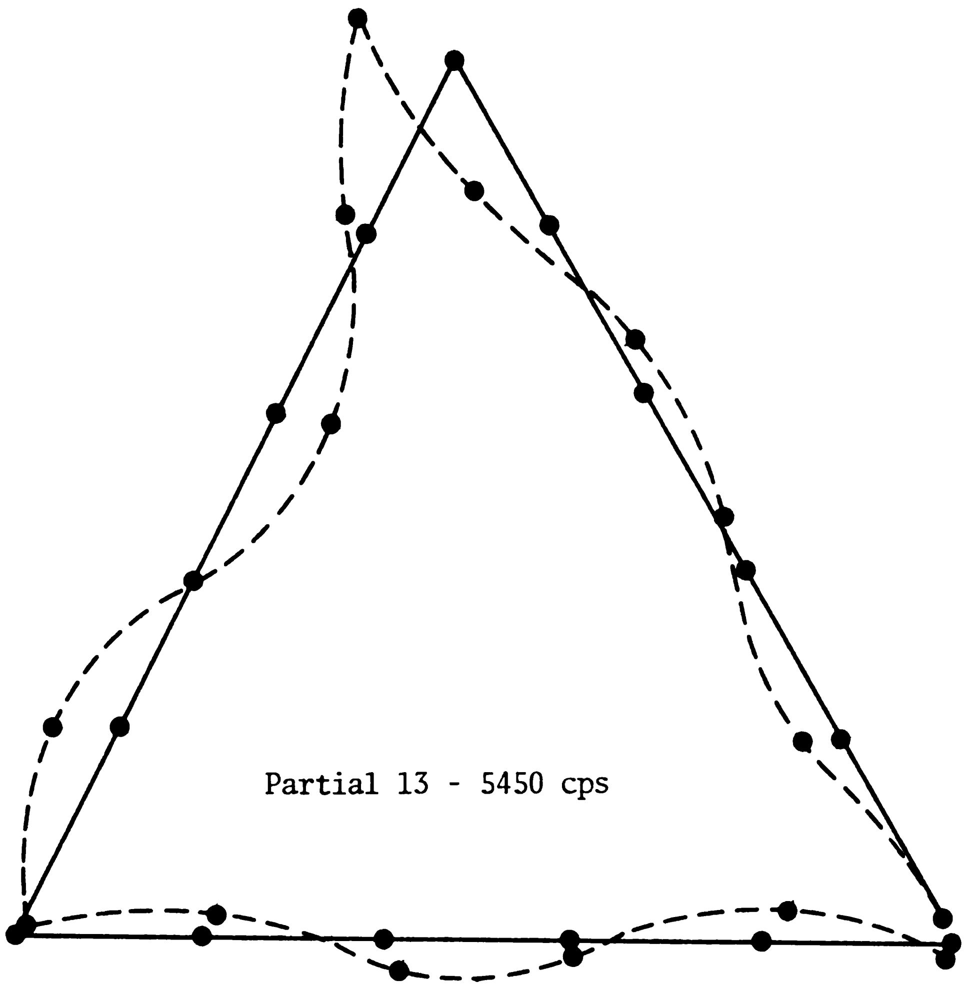


Figure A2.--(cont'd.)



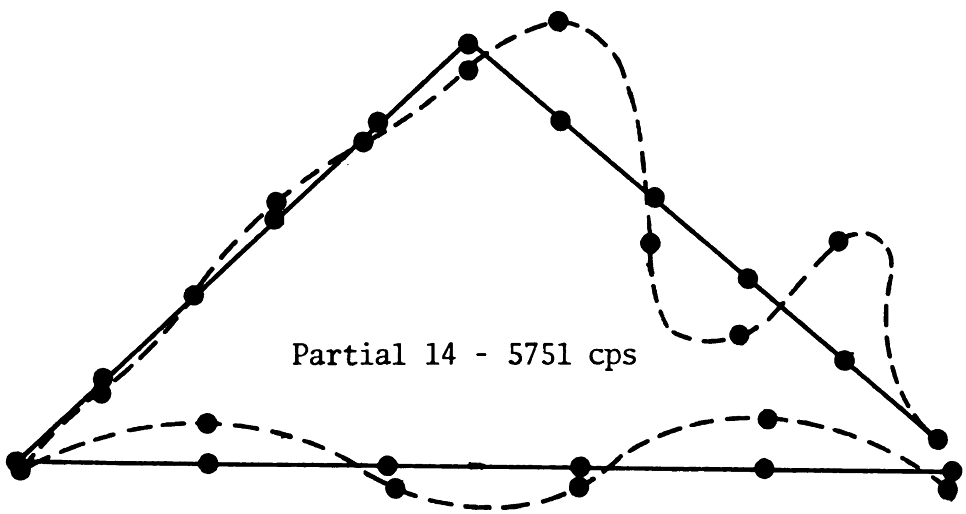
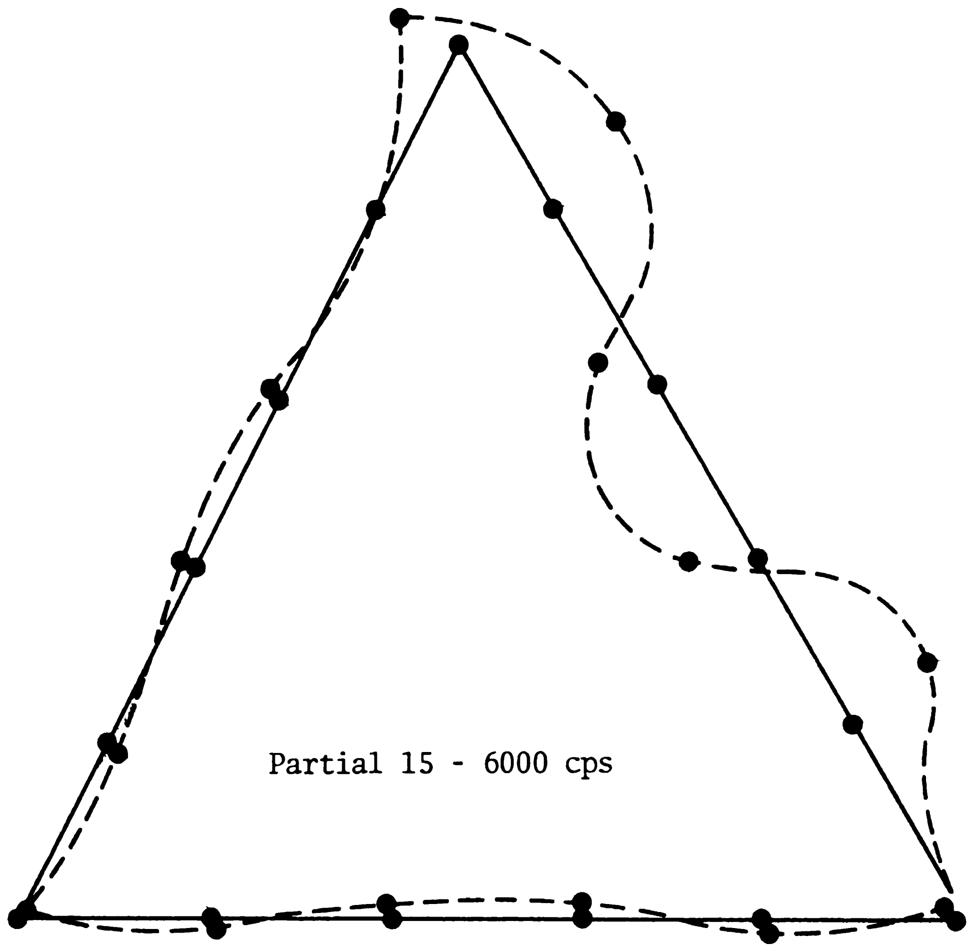


Figure A2.--(cont'd.)



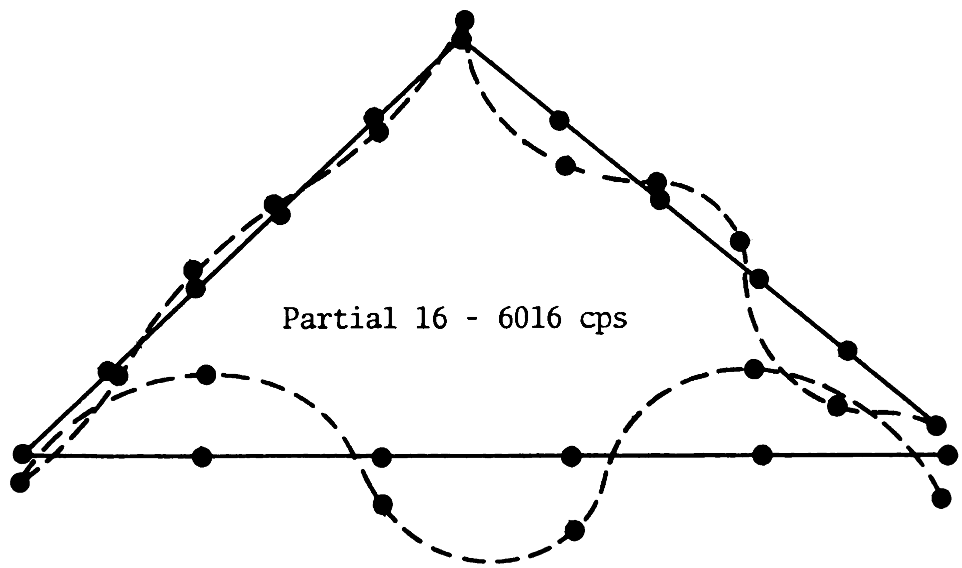


Figure A2.--(cont'd.)