CONSIDERATION OF PLANT MATERIAL AS AN INTERACTING CONTINUUM

> Thesis for the Degree of Ph. D. MICHIGAN STATE UNIVERSITY GERALD HENRY BRUSEWITZ 1969



This is to certify that the

thesis entitled

CONSIDERATION OF PLANT MATERIAL AS AN INTERACTING CONTINUUM

presented by

Gerald Henry Brusewitz

has been accepted towards fulfillment of the requirements for

Ph.D. degree in Agricultural Engineering

Major professor

Date August 13, 1969

**O**-169





### ABSTRACT

### CONSIDERATION OF PLANT MATERIAL AS AN INTERACTING CONTINUUM

Вy

## Gerald Henry Brusewitz

Plant tissue was considered as an interacting mixture of solid and fluid in postulating factors contributing to its overall strength. Strength contributed by the solid portion is due to mechanical strength of the cell wall, structural strength of the cells as a framework, and connecting strength of the middle lamella. The fluid in plant material adds to the gross strength from the standpoint of being temporarily trapped or contained in particular regions. In addition, mechanical strength is determined by the interaction of these elements.

Consideration of the anatomy of plant materials as such led to a mathematical modeling of their mechanical properties based on the theory of interacting media. The three-dimensional continuum theory is presented for the case of a single solid and a single fluid. Methods of determining material properties as required in evaluating constants in the constitutive equations are also presented.

#### Gerald Henry Brusewitz

Experiments were conducted with sections from the potato tuber in an attempt to determine its compressibility and liquid permeability. Limits of the previously used bulk compression apparatus were discovered and prevented a comparison of jacketed and unjacketed compressibility. Experiments to measure the ease with which liquid water flows through potato tissue were inconclusive in determining the material's permeability. Uniaxial compression tests under different boundary conditions confirmed the interacting nature of the potato.

The use of the theory of interacting media in modeling the mechanical properties of biological materials is only suggested herein. Further research is needed to verify the theory and develop experimental techniques.

Approved

Major Professor

Belw, Jull Approved

Department Chairman

### CONSIDERATION OF PLANT MATERIAL

### AS AN INTERACTING CONTINUUM

Ву

Gerald Henry Brusewitz

### A THESIS

Submitted to Michigan State University in partial fulfillment of the requirements for the degree of

#### DOCTOR OF PHILOSOPHY

Department of Agricultural Engineering

#### ACKNOWLEDGMENTS

The author would like to express his gratitude for the encouragement and assistance of Dr. B. A. Stout (Agricultural Engineering) under whose supervision this investigation was conducted.

The assistance of Dr. R. W. Little (Metallurgy, Mechanics, and Materials Science) in suggesting the appropriate theoretical development is gratefully acknowledged.

Thanks are due Dr. F. W. Bakker-Arkema (Agricultural Engineering) and Dr. R. C. Hamelink (Mathematics) for serving on the guidance committee.

Thanks also go to Dr. C. W. Hall (Chairman of Agricultural Engineering) for obtaining the financial support in the form of a National Defense Education Act Fellowship which made this program feasible.

My wife, Susan, is recognized for her contribution of encouragement and patience during this study.

### TABLE OF CONTENTS

																					]	age	
ACKN	OWLEDGM	ENTS																				11	
LIST	OF TAE	BLES.																		•		v	
LIST	OF FIG	URES																				vi	
LIST	OF SYM	BOLS																				vii	
Chan	ton																						
chap	cer.																						
Ι.	INTRO	DUCTI	ON		•				•				•					•		•	•	1	
	1 1	The	Nee	đ																		1	
	1.2	Obje	eti	ve		:		:	:		:	:		:		:	:	:		:	:	2	
		00000			·																	-	
II.	DEFIN	ITION	OF	PI	ROI	BLE	EM	•	•	•	•			•	•		•	•	•	•	•	4	
	2 1	Plar	ot A	na	tor	w																4	
	2.2	Stre	engt	h	of	PI	lar	it	T1	.ss	ue		:	:	:	:	:	:	:	:	:	5	
	2.3	Mech	ani	ca	1 3	sti	rer	igt	h	of	. (	lel	i	Wa	11	Ls						8	
	2.4	Stru	ictu	ra	1 3	Sti	rer	igt	h													10	
	2.5	Flui	d S	tre	eng	str	ı															13	
	2.6	Gas	Str	en	gth	1																16	
	2.7	Flui	d F	101	wt	h	rou	ıgł	n F	or	01	ıs	Me	d	la							17	
	2.8	Inte	erac	ti	on	01	f M	lat	er	12	11	ΕJ	en	ner	nts	5	•	•	•	•	•	18	
III.	PROBL	EM SC	DLVI	NG	AI	PF	ROA	ACH	IES	5												21	
IV.	THEOF	RETICA	L D	EVI	ELO	PN	1EN	IT														23	
	4.1	Theo	ory	of	M	xt	ur	res	3													23	
	4.2	Bina	ary	M1:	κtι	ire	9	•	•	•	•	•	•	•		•			•	•	•	24	
	4.3	Stre	ss	and	d F	ζqι	11]	11	ri	un	n	•	•	•	•		•	•	•	•	•	24	
	4.4	Defi	nit	101	n o	of.	St	ra	air.	1-I	)is	spl	ac	en	ner	ıt	•	•	•	•	•	27	
	4.5	Cons	tit	ut:	1 Ve	e F	(e)	lat	:10	n	•	•	•	•	•	•	•	•	•	•	•	28	
	4.6	F.TOM	ке	Ta.		n	•	•	•	•	•	•	•	•	•	•	•	•	•	•	٠	30	
	4.1	Fiel	UE	qua	LUL	or	15	•	•	•	•	•	•	•	•	•	•	•	•	•	•	31	
	4.0	SIMP	III	TGS	203	or	15	•	•	•	•	•	•	•	•	•	•	•	•	•	•	35	

### Chapter

### Page

ν.	INTE	RPRET	TATIC	N	ANI		DE	ref	RMI	IN	AT ]	101	1 0	ΟF								27
	MAIL	RIAL	PROP	ER	1.11	20	•	•	•	•	•	•	•	•	•	•	•	•	•	•	•	31
	5.1	Inte	erpre	at	tio ior	on 1 d	of	f H Is	Equ	lat	ic	ons Le	5.	•	•	•	•	•	•	•	•	37
		Mate	erial	P	rop	bei	rt:	ies	з.													39
	5.3	Non-	-isot	ro	pic	e 1	Pro	ope	ert	ti	es											41
	5.4	Othe	er Fo	rm	ula	ati	ior	ns	•	•	•	•	•	•	•	•	•	•	•	•	•	42
VI.	EXPE	RIMEN	TATI	ON																		44
	6.1	Prep	arat	io	n	of	Po	ota	ato	o :	Spe	ect	ime	en								45
	6.2	Unia	axial	C	omp	ore	ess	510	on													45
	6.3	Bulk	c Con	ıpr	ess	sit	<b>1</b>	lit	уy						•							52
	6.4	Pern	neabi	11	ty	·	•	•	•	•	•	•	•	•	•	•	•	•	•	·	·	56
CONC	LUSIO	NS.		•																		66
SUGG	ESTIO	NS FO	OR FU	JRT	HEI	2 2	STU	UDY	ζ.					•		•						67
REFE	RENCE	s.,		•																		68
GLOS	SARY																					76

### LIST OF TABLES

Table

Page

A6.1 Uniaxial Compression during Successive Loadings of Potato 1.0 inch long . . . 51

### LIST OF FIGURES

Figure		Page
2.1	Response of Various Shape Plant Cells to Mechanical Forces	. 6
2.2	Offset Cell Wall Joints	. 11
2.3	Aligned Cell Wall Joints	. 11
6.1	Nonporous Flat Plate Loading	. 47
6.2	Porous Flat Plate Loading	. 47
6.3	Instron Uniaxial Compression Test with Porous Flat Plate Loading	. 47
6.4	Force-Deformation Curve for Uniaxial Compression Loading	• 49
6.5	Bulk Compression Apparatus	• 53
6.6	Holder for Flat Specimen	• 58
6.7	Holder for Reduced Area Specimen	. 58
6.8	Apparatus for Cutting of Reduced Area Permeability Specimen	. 60
6.9	Permeability Apparatus	. 60
6.10	Pressure-Flow Rate Relation from Fermeability Test	. 62
6.11	Flow Rate-Time Relation from Permeability Test at Constant Pressure	. 63

### LIST OF SYMBOLS

А	=constant in transverse isotropic constitutive
	equation
<sup>c</sup> ijkl	=constants in anisotropic constitutive equation
C	=constant in isotropic flow equation
C <sub>ij</sub>	=constants in anisotropic flow equation
е	=solid dilatation
<sup>e</sup> ij	=solid strain tensor component
f <sub>ij</sub>	=arbitrary function
F	=constant in transverse isotropic constitutive
	equation
Fi	=body force per unit mass
k	=permeability of solid
L	=constant in transverse isotropic constitutive
	equation
М	=constant in transverse isotropic constitutive
	equation

vii

N	=constant in transverse isotropic constitutive equation
р	=hydrostatic pressure
Р	=porosity
Q	=constant in transverse isotropic constitutive equation
R	=constant in transverse isotropic constitutive equation
t	=time
ui	=solid displacement component
vi	=fluid displacement component
V	=pore volume
v <sub>o</sub>	=bulk volume
x,y,z	=mutually perpendicular coordinate axes
α	=constant in isotropic constitutive equation
β	=constant in isotropic constitutive equation
δ <sub>ij</sub>	=Kronecker delta
ε	=fluid dilatation
€ij	=fluid strain tensor component

viii

λ	=constant in isotropic constitutive equation
μ	=constant in isotropic constitutive equation
μf	=fluid viscosity
ξ	=change in fluid content
ρf	=fluid mass per unit bulk volume
٩m	=mass density of bulk material
σ	=fluid stress tensor
σ <sub>ij</sub>	=solid stress tensor component
τ <sub>ij</sub>	=total stress tensor component
V	=differential operator

## 1. INTRODUCTION

# 1.1 The Need

The mechanical behavior of biological materials predicted by the most sophisticated methods available deviates significantly from that found experimentally. Classical theory of elasticity does not account for anomalies due to time, temperature, and moisture content. The time factor can be included by use of the theory of linear viscoelasticity which is a mere extension of elasticity. Although experimental evidence (Mohsenin, 1968) indicates nonlinear viscoelastic behavior for many biological products, the lack of well supported theory (Clark <u>et al.</u>, 1968) requires simplification and use of linear viscoelasticity. Thermal effects have been included to a limited extent in the elastic theory (Parkus, 1968). A theory (Schapery, 1964) has also been proposed to include both time and temperature effects.

Prediction of stress-strain behavior for metals by existing theory is more accurate than for biological materials. The recognized difference in microstructure between the two is a conceivable and logical source of the accuracy discrepancy. In addition to this general

difference, the structural make-up of biological materials is widely variable. It seems obvious that a thorough knowledge of the material microstructure is basic in developing theory to more precisely describe the behavior of biological materials subjected to mechanical loads.

Materials of a biological origin are unique in that mechanical behavior is dependent on moisture content. This factor has been considered by Hammerle (1968) and Hammerle and Mohsenin (1969) in a study of the cracking of a corn kernel subjected to both temperature and moisture gradients. They determined the time-temperature shift factor for corn horny endosperm and also proposed an analogous timemoisture shift factor. However, the theory most widely employed by engineers dealing with biological materials is that of linear viscoelasticity at constant temperature had moisture content.

# 1.2 Objectives

The overall goal of this study was to develop a method to more accurately predict the mechanical behavior of biological materials, particularly those of a plant origin. Various approaches were conceivable as a study could have been directed toward structure alone, or solely at various methods of mathematically modeling the response typically found experimentally, or some combination of these two. The objectives of this study were: (1) To develop a better understanding of the structure of plant



**"**,

material as needed for modeling gross mechanical behavior and (2) To apply appropriate mathematical theory in order to more accurately predict the mechanical behavior of plant tissue.

The first objective was realized by a study of the botanical literature and gleaning that portion yielding clues of how to model plant material. The resulting product was to indicate the nature or type of theory which could be used to best describe this material. Thus the second objective was highly dependent on the earlier results. If the required theory existed, the second portion of the study would be easy. If the required theory could not be found it would be necessary to develop or at least attempt to develop a new model.

## II. DEFINITION OF PROBLEM

### 2.1 Plant Anatomy

The following is a review of the anatomy of plant material pertinent to a better understanding of gross mechanical properties. Practically an unlimited source of references on plant material is available but one can most profitably begin with standard textbooks such as Esau (1960) or Ruhland (1955). Herein, the various constituents of plant material will be considered along with such characteristics as their chemistry, quantity, variability, distribution, and interaction with each other.

In general, biological material contains all three states of matter, i.e. solid, liquid, and gas. The solid state possesses sufficient rigidity so as to define a certain shape. In plant tissue this is represented mainly by the cell wall and somewhat less by certain elements of the protoplast\* such as starch grains, sugar crystals, protein chains, plastids, and waste product grains. This cell wall may be very thin and highly deformable as in young growing cells or it may be relatively thick and only slightly deformable as in the dead tracheid cells of wood.

<sup>\*</sup>See Glossary (p. 76) for definition of botanical terms such as this.

It is this cell wall that provides the majority, but not all, of the strength to the complete tissue. It is the conclusion of most cell wall researchers that the mechanical properties of the cell wall represent the mechanical properties of the entire tissue. Experiments reported by Frey-Wyssling (1952) showed individual ramie fiber cells and cotton hairs to be weaker than fiber strands. The question thus arises as to what factors contribute to tissue strength other than cell wall properties.

#### 2.2 Strength of Plant Tissue

Researchers, principally biologists, have measured cell wall strength by measuring tissue strength and yet, the problem of determining the strength of an individual cell is much more involved. Under certain conditions such as a very thick cell wall and a strong middle lamella the cell wall strength may clearly dictate the tissue strength. However, there are other cases where tissue strength is not equivalent to cell wall strength. Consider the parenchyma cells of common fruit which are thin-walled and nearly spherical. The stress-strain behavior of such material (Figure 2.1) would seem to be best described by the theory of thin shells. This approach has been taken by Akyurt (1969) in modeling the cell as a fluid-filled, eight-sided, thin-walled shell.

The response of long narrow cells of cellulose fiber to mechanical loading depends on the direction of





Í.

Figure 2.1.--Response of Various Shape Plant Cells to Mechanical Forces. The dotted lines indicate the resulting deformation.

load application. A force applied parallel to the length of the cell (Figure 2.1) would best be considered a stability problem while a transverse loading would be considered a bending load. The stability of the cell shape has been recognized by Frey-Wyssling (1952) as he noted that cell shape may be more important than the cell wall strength in determining gross mechanical strength.

Overall mechanical strength of plant tissue is also dependent on the connection strength or the bond between cells. A weak middle lamella could result in tissue failure by rupture between cells as noted by Rasmussen (1966) in a study of tobacco leaves. Weak leaves displayed failure by separation between cells whereas strong leaves failed by tearing through the cell wall. The difference was traced to the amount of pectin in the middle lamella which changed its strength. A strong middle lamella leads to tissue failure by breaking of the cell wall. This was noted by Huff (1967) in studying tensile properties of potatoes.

In general, the total mechanical strength of plant tissue contributed by the solid portion is due to mechanical strength of the cell wall itself, structural strength of the cells as a framework, and connection strength of the bonds between cells.

# 2.3 Mechanical Strength of Cell Walls

An examination of the anatomy of thick cell walls has revealed their non-isotropic nature. In a study of tracheid cell walls, Mark (1967) considered the cell wall as a composite body with an anisotropic framework surrounded by an isotropic matrix. The amorphous matrix, primarily lignin, is elastic at low moisture contents. As water is supplied, the matrix swells and loses its strength such that at high moisture contents the matrix becomes plastic. This is the portion of the cell wall whose mechanical properties vary significantly with moisture content. This contributes, in part at least, to the explanation of how and why the moisture content of a biological material affects the mechanical properties of the tissue. The cellulose framework on the other hand is not available to water. This framework consists of a large number of microfibrils with a definite orientation called the "fibril angle" and is best described by a helix. The fibril angle depends on the particular layer in the cell wall, the location in the cell, cell width, and wall thickness. The mechanical strength of tissue containing cells with secondary walls is largely dependent on the strength of the microfibrils. The importance of the fibril angle is thus indicated for an anisotropic framework.

Proposed models of the structure of wool generally have agreed with that given for wood. After making a

critique of existing models, Menefee (1968) proposed the extended matrix or honeycomb model. The suggested model is likened to reinforced concrete where the microfibrils play the part of the high compression strength cement and the matrix is similar to the high tensile strength reinforcing cables. These roles played by the microfibrils and the matrix are opposite to what was intuitively expected. According to this model the mechanical strength of the tissue may be dependent on the microfibril strength or the matrix strength depending on the type of load application.

The effect of cell wall thickness on tissue strength has been implied in studies of fruit firmness. Pressure tests with a 3/16 inch plunger on peach fruit conducted by Blake et al. (1931) indicated a direct correlation between flesh firmness and thickness of the cell walls. The cell wall thickness of apple flesh was found by Tetley (1931) to change only slightly from June to September. In a more thorough study of the apply fruit Tukey and Young (1942) described the development of the various tissues and the changes that occur such as increased cell wall thickness. The cell wall increased in thickness at different times and in different amounts depending on the tissue. Certain fruit display definite stages of growth during which the changes in cell wall thickness have been measured. Tukey and Young (1939) found the cell walls of the sour cherry to increase in thickness only during the first half of fruit development. Thus the geometric

characteristic of cell wall thickness is a recognized factor contributing to the strength of fruit.

# 2.4 Structural Strength

In order to ascertain the structural strength of biological material, further understanding is needed of those anatomical aspects such as cell shape, size, orientation, wall thickness and variations in each of these. A single unique cell shape does not exist. The most common shape found by Lewis (1946) is the fourteen-sided cubooctahedron. Other common shapes were noted to have ten or six faces. Sinnott and Bloch (1941) in investigating the relative position of cell walls, noted two distinct types of relative wall position. They observed that either three adjacent walls met at one point (offset arrangement) or four walls met at one point (aligned arrangement) as shown in Figures 2.2 and 2.3. It is evident that a physical arrangement like the latter would be more susceptible to failure when mechanical loads are applied. This is analogous to the bricks in a wall where the joints are offset for strength purposes. The strength of tissue having cell walls aligned is more dependent on the bond between cells. It was also noted that cells with many sides, approaching the shape of a sphere, had larger intercellular air spaces if the wall positioning was aligned rather than offset.

Plant breeders have investigated various cell properties in attempting to develop new varieties less prone





Figure 2.3.--Aligned Cell Wall Joints.

to failures such as bruising and rotting. They have found a positive correlation between cell size and the susceptibility to internal failure. This was found for apples by Jackson (1967), Letham (1961), and Martin et al. (1965). The development in size of an organ such as a fruit is often paralleled by similar changes in cell size. Houghtaling (1935) measured the size of tomato fruit and their cells and fitted the data to the formula  $y = bx^k$ . The data fit well as a plot of the logarithms of these two variables was nearly linear. A high correlation between apple fruit diameter and disorder breakdown was found by Martin (1954). Genetic characteristics of improved apple fruit were suggested by Martin and Lewis (1952) as "attempts to raise the mean fruit size without impairing keeping quality are most likely to succeed if cell number por fruit is increased and cell size is kept small." These studies found various correlations but not causeeffect relations and thus the actual contribution of cell wall thickness to tissue strength is not known. Nevertheless it appears quite evident that injury of apples as well as other fruit well along in their development is a structural type of failure.

First attempts at modeling the plant cell assume constant geometric properties for cell shape, cell size, and cell wall thickness. Finding these properties constant in any given tissue is rare. Variations in cell shape have already been discussed. Cells vary in size

by orders of magnitude depending on such variables as type of tissue, variety, specie and period of growth. The same argument follows for cell wall thickness. This variability is undoubtedly one of the major obstacles to realistic modeling of mechanical strength at the cellular level.

## 2.5 Fluid Strength

The fluid in biological material adds to the overall strength of the tissue from the standpoint that it is trapped or held within, either temporarily or permanently. This fluid is primarily water containing minute protoplasmic solid particles which alter the viscosity. Certain seeds high in oil content display a completely different type of fluid. Chemically bound fluid acts to alter the solid phase and such physical properties as volume, density, and mechanical strength. When this nearly incompressible fluid is totally surrounded by solid parts of the cell it behaves much like an elastic membrane containing a fluid. A model incorporating these ideas has been used by Falk <u>et al.</u> (1958) and Nilsson <u>et al.</u> (1958) to show the dependence of tissue strength on both turgor pressure and cell wall strength.

A thorough knowledge and understanding of the state of fluid in biological materials is a prerequisite to ascertain the fluid's contribution to mechanical strength. Water is the most abundant fluid in plant tissue and often

provides more of the weight than the solid material, The proportion of water contained within this solid-liquid-gas mixture known as tissue is commonly denoted as the moisture content. Conceptually the moisture content of a material is that part represented by water. In experimentally quantifying this concept, moisture content can best be defined in terms of the method of determination (Van Arsdel and Copley, 1963). The amount of moisture which can be removed from a product is dependent on the applied force. Stark (1932) calls free water that portion of the moisture which can be removed by mechanical pressure or freezing and bound water as that additional moisture removed by oven drying. After a study of water content of foods, Kuprianoff (1958) concluded that total moisture content should be determined by an internationally accepted method and that any adjec-1/e used before water content should indicate the method of determination. For instance, the term "unfreezable water" should be used rather than "bound water". This dependency of the moisture content on the method of determination or more specifically the amount of applied force, is not readily handled in modeling. This refinement must remain unattended until other more important factors have been included in the model.

After avoiding a precise definition of moisture content, consider now the location of water in biological tissue. This is well delineated and discussed in texts by Slayter (1967) and Ruhland (1955). Suffice it to say here

that water is distributed throughout the protoplasm, cell wall, and vacuoles, in order of increasing amounts. The holding forces are mainly osmotic and imbibitional forces. Osmotic forces are established due to concentration differences. Imbibition is the absorption of water over external surfaces and around the inside surface of capillaries resulting in swelling of the material. Pressures of a given magnitude will remove water from capillaries of a given size and larger. This will be considered in further detail later during discussion of pore volume and pore size distribution.

Osmotic forces are the main holding forces of water in the vacuoles and imbibitional forces hold the water in the cell wall and protoplasm. Because water is held by these forces, its free energy is reduced and led Ling (1968) to the conclusion that water in the cell is in a different physical state than water in an open container. This is the basis of the association-induction hypothesis describing the physical state of water in biological systems as protein and water exhibiting a cooperative phenomenon.

At equilibrium these forces hold water at the various locations in different amounts according to the free energy of the water at that particular location. An imbalance such as a change in water content surrounding the tissue causes the homeostasis phenomenon to occur and a tendency to return to the original balance. This phenomenon is

similar to a physical control system with feedback tending to correct for any disturbances.

# 2.6 Gas Strength

The gaseous portion of the solid-liquid-gas mixture of the model for plant material may contribute mechanical strength to the system in a direct sense but is considered here only indirectly in that it furthers knowledge of the solid portion. Any characteristics of the gaseous phase yield knowledge about the solid since this is the space within the mixture not occupied by solid or fluid. These voids may or may not be connected with the surrounding atmosphere. For example, a portion of the flesh of apple has connected voids whereas the entire apple with its nearly impervious surrounding skin has no voids connected with the atmosphere. A material having voids which are connected with the atmosphere is referred to as a porous material. However, a precise definition of a porous material required knowledge of the media which can be passed through, whether it be gas or liquid.

Marvin (1939) in compressing lead shot found that the resulting particle shape was like that found in plant cells. This would indicate that the size and shape of intercellular spaces is dependent on the pressure between cells. If this is true, a knowledge of the characteristics of the intercellular spaces would be helpful in a study of internal stresses.

# 2.7 Fluid Flow through Porous Media

The flow of liquids through porous media has been widely studied in the area of soil mechanics. For a comprehensive review of flow through porous media describing the various theoretical models and listing the important references see Coheidegger (1966). Basic in the analysis of flow of liquide through perous solids is Darcy's Law relating volume of fluid flow to creas-sectional area of the solid, r textial drop, and distance across the medium. This macroscopic hyprosch is prompted by a lack of knowledge of the complex pore make-up. It also makes mathematic formulations and descriptions easier.

Pores are referred to as those voids in the solid which are connected with the external surroundings. The definition is well appeal upon except the indefiniteness The system deconnectes?. Contrad to the case of liquid flow through a collo - As a certain applied pressure a certain number of pores in the colid allow flow of liquid while at a higher problem more point allow flow. This dependency of pore volume on the applied pressure leads to a nonlinearity in Darcy's Law. Perosity or pore volume is the air portion of the total volume of an air-solid mixture. Porosity and internal surface area are also important factors in the study of heat and mass transfer during drying of hygroscopic materials. The measurement of these properties is described by Gregg and Sing (1967). Knowledge gained in these allied areas will be useful in modeling
the mechanical response of the plant cell by considering it as a solid-liquid-gas mixture.

2.8 Interaction of Material Elements

Biological tissue has thus far been considered as an independent mixture of solid, liquid, and gas. The interdependency or interaction between these components is now of concern. In living tissue the liquid contents of a cell exert a hydrostatic pressure on the cell wall called the turgor pressure. Turgor pressure results from the differential osmotic concentration within the cell relative to outside. At equilibrium this turgor pressure is counteracted by the equal and opposite wall pressure. In growing cells the turgor pressure is slightly larger than the wall pressure to allow an increase in cell volume.

A correlation between cell growth rate and mechanical properties of the cell wall has prompted cell wall scientists such as Cleland (1967), Green (1968), Haughton <u>et al</u>. (1968), and Lockhart (1967) to investigate cell wall properties with the hope that this knowledge will contribute to their understanding of growth processes.

A study by Falk <u>et al</u>. (1958) of the parenchyma cells of potatoes indicated a direct linear relation between Young's modulus of the tissue and turgor of the cells. The relationship was first measured experimentally and later explained by a theoretical model. This increase in tissue strength due to increased turgor is an indication of the

important role played by the fluid in providing strength to the tissue. A mathematical relation for turgor pressure as a function of cell dimensions and wall strength was developed by Haines (1950). Turgor pressure was linearly related to elasticity of the cell wall and nonlinearly with the cell size.

Slices of potato tuber were mounted by Somers (1966) as cantilever beams for determination of the modulus of elasticity. The results were affected by some unmeasured property thought to be the turgor pressure which led him to say that "it appears from the above observations that the apparent value of E is the resultant of the interaction between the elastic cell walls and the turgor pressure of the cell contents." Turgor pressure of living tissue varies during the day and has been measured by changes in organ size. Barrs (1968) reviewed studies of diurnal changes in leaf thickness, fruit size, and tree trunk diameter. Measurements of size were used as indicators of the water status of the tissue.

The causes and effects of bruising of fruit have been widely studied for many years but an understanding of the phenomemon itself is limited. Greenham (1966) investigated the physiological aspects of bruising in apple fruit. He used electrical resistance measurements to show that bruising consists of an immediate cell breakdown and a delayed injury caused by the toxic effects of the liberated sap. This denotes a fluid-solid interaction of both a physical and chemical nature.

An indication of interaction between solid and liquid is illustrated in the results of a study of apple skin by Huelin and Gallop (1951). It was found that the natural presence of oils reduces the flow of liquids and gases through the tissue by reducing the pore volume. Differences in viscosity of these elements would be expected to cause variations in the pressure-pore volume relationship which illustrates an interaction between the various components of the solid-liquid-gaseous mixture.

This interaction between elements has been recognized as a possible source of error in experimental measurements but as yet has not been isolated or identified. The fact that different investigators present widely varying estimates of any given mechanical property throws doubt on the reported value and illustrates the need for an understanding of the property a particular test actually measures.

#### III. PROBLEM SOLVING APPROACHES

The current knowledge of the microstructure of biological tissue such as outlined here implies a stressstrain relationship based upon an interacting mixture of solid, liquid, and possibly gaseous phase. Previous approaches to developing constitutive relations have been from either a macro- or microscopic viewpoint. Microstructure theory attempts to explain gross behavior of a complex material based upon the known behavior of its individual elements. For a biological material it is logical to consider as elements the individual cells. The immediate limitation of this approach is the magnitude of different elements comprising even the simplest biological organ.

Reiner (1960) identifies structural analysis as the use of phenomenological data to postulate the microstructure. This is a reason for the popularity of mechanical models used in analyzing viscoelastic response of materials.

The macroscopic approach visualizes the phenomenological behavior as being more important than the microstructure. It does use a knowledge of microstructure in defining and idealizing the gross behavior. This so called macroanalytical approach (Lazan, 1962) could even be considered a combination of macro- and microscopic approaches.

Historically, the macroscopic formulation has been far more successful in predicting stress-strain behavior. For these reasons the macroscopic approach was used in devising mathematical theory to model the mechanical behavior of plant material.

#### IV. THEORETICAL DEVELOPMENT

# 4.1 Theory of Mixtures

The theory of mixtures deals with a medium consisting of any number of constituents each being a medium with unique properties. The simplest case is that of a single constituent. Conversely, the theory of mixtures can be thought of as a generalization of the classical theory of continuous media. The mixing or combining of the constituents produces a change in each of the components. The mixture is not obtained by merely superimposing its constituents. Each constituent not only contributes to the properties of the mixture but also affects that same property in the other constituents.

Utilizing a thermodynamic approach, Green and Naghdi (1967) have formulated a theory of mixtures. Two assumptions basic to the theory are:

- A. Each point in space may be occupied simultaneously by several different particles.
- B. Distinct quantities such as displacement and velocity can be assigned to each constituent at each point in space.

Physically, it is impossible for more than one substance to occupy the same point in space. Mathematically, these assumptions are equally as valid as that of material continuity. The constitutive equations for a binary mixture of Newtonian fluid plus elastic solid and fluid plus viscoelastic solid have recently been derived by Tabaddor (1968) using this theory.

### 4.2 Binary Mixture

A reduced case of the theory for a mixture of any number of components is Biot's (1941, 1955) theory of flow of fluids through a porous, deformable solid. Paria (1966) has reviewed the work on this subject by Biot, Jana, Paria, and others. The mixture in this case consists of a single solid and a single fluid. The solid is deformable and the liquid can be viscous or nonviscous, compressible or incompressible. The solid is the framework with the fluid filling the pores. The volume of the pores which are interconnected so as to eventually lead to the atmosphere is denoted by V. If the bulk volume of the solid-fluid mixture is given by  $V_0$  then the porosity P is defined as  $P = V/V_0$ . Void spaces not available to the fluid are considered part of the solid.

#### 4.3 Stress and Equilibrium

Consider a small cube of the solid-fluid mixture with orientation parallel to a set of three mutually

perpendicular coordinate axes (x, y, z). The stress tensor for the solid-fluid mixture is

$$\begin{bmatrix} \sigma_{xx} + \sigma & \sigma_{xy} & \sigma_{xz} \\ \sigma_{yx} & \sigma_{yy} + \sigma & \sigma_{yz} \\ \sigma_{zx} & \sigma_{zy} & \sigma_{zz} + \sigma \end{bmatrix}$$



Due to symmetry (i.e.  $\sigma_{ij} = \sigma_{ji}$ ) there are six independent stress components. The meaning of the subscript in the stress tensor  $\sigma_{ij}$  is as follows. The first subscript indicates the coordinate direction normal to the element of area on which that particular stress acts. The second subscript denotes the coordinate direction of that particular stress component. The components  $\sigma_{xx}$ ,  $\sigma_{yy}$  and  $\sigma_{zz}$  are the

normal stress components while the others are the shear components. The stress components  $\sigma_{ij}$  represent that part of the total stress applied to the solid portion only. A stress component,  $\sigma_{xx}$  for instance, does not act over the entire area  $\Delta y \Delta z$  but rather is reduced by the porosity to act over an area  $(1-P)\Delta y \Delta z$ . Note that although porosity is the volume portion due to voids, it also represents a ratio of areas. The normal tension force acting on the fluid portion of a cube's face is given by  $\sigma$  and is the same in all directions. This stress is related to the hydrostatic pressure p of the fluid in the pores by

$$\sigma = -Pp \tag{1}$$

At this point the assumption is made that the fluid's shear stress caused by viscosity is negligible relative to the fluid pressure.

Newton's second law is now utilized as in elasticity in a balance of linear momentum to yield the following equations of equilibrium

$$\frac{\partial}{\partial x}(\sigma_{xx}+\sigma) + \frac{\partial\sigma}{\partial y}xy + \frac{\partial\sigma}{\partial z}xz + \rho_m F_x = 0$$
(2a)

$$\frac{\partial \sigma}{\partial \mathbf{x}} \mathbf{y} \mathbf{x} + \frac{\partial}{\partial \mathbf{y}} (\sigma_{\mathbf{y}\mathbf{y}} + \sigma) + \frac{\partial \sigma}{\partial z} \mathbf{y} \mathbf{z} + \rho_{\mathbf{m}} \mathbf{F}_{\mathbf{y}} = 0$$
(2b)

$$\frac{\partial \sigma}{\partial x} zx + \frac{\partial \sigma}{\partial y} zy + \frac{\partial}{\partial z} (\sigma_{zz} + \sigma) + \rho_m F_z = 0$$
 (2c)

where  $\rho_m$  is the mass density of the bulk material and  $F_1$  is the body force per unit mass. This set of three equations

can be rewritten using tensor notation as the following single equation

$$(\sigma_{ij} + \sigma\delta_{ij}), j + \rho_m F_i = 0$$
(3)

where the comma denotes differentiation and  $\delta_{ij}$  is Kronecker's delta taking on the value of one when i=j and zero when  $i\neq j$ . Inertia terms which would appear on the right hand side of equation (2) or (3) have been neglected since this is a quasi-static formulation. Dynamic theory has been developed (Paria, 1966) beginning a step earlier with the equation of motion.

# 4.4 Definition of Strain-Displacement

The strain tensor for the material can be derived from the displacement components. The displacement compoments for the solid are  $u_x$ ,  $u_y$ ,  $u_z$  and for the fluid are  $v_x$ ,  $v_y$ ,  $v_z$ . The strain components are defined as

$$e_{j} = \frac{1}{2} (u_{j,j} + u_{j,j})$$
(4)

for the solid and

$$\varepsilon_{ij} = \frac{1}{2} \left( v_{i,j} + v_{j,i} \right)$$
(5)

for the liquid.

The cubical dilatation for the solid is defined in terms of displacement by

$$e = \nabla_{i} u_{i} \tag{6}$$

or in terms of strain components by

$$e = e_{XX} + e_{yy} + e_{ZZ}$$
(7)

Similarly for the fluid dilatation

$$\epsilon = \nabla_{i} \nabla_{i}$$
(8)

 $\mathbf{or}$ 

$$\varepsilon = \varepsilon_{XX} + \varepsilon_{yy} + \varepsilon_{zz} \tag{9}$$

The definition of strain in equations (4) and (5) is based on small strain components. If the displacements  $u_i$  and  $v_i$ are small and also their derivatives, the products of the partial derivatives in the nonlinear terms may be neglected leaving only the linear terms and thereby simplifying the strain-displacement relation.

## 4.5 Constitutive Relation

The stress and strain tensors have been defined sufficiently that a relation between them can now be formulated. This can be accomplished most succinctly by viewing the solid-fluid mixture as a new, single medium with seven stress components and seven strain components. This concept provides mathematic development allied with that of the classical continuum mechanics.

An elastic medium, held at constant temperature, is assumed to exhibit the one-to-one relation between the stress components and the strain components of

$$\sigma_{ij} = f_{ij} (e_{ij}) \tag{10}$$

An additional assumption requires that this function pass through the origin or that the unstrained condition correspond to the unstressed. Expansion of the functions  $f_{1j}$  in a power series in  $e_{1j}$  and omitting all terms higher than first order results in the linear relation

$$\sigma_{ij} = c_{ijkl} e_{kl} \tag{11}$$

The constants  $c_{ijkl}$  are assumed to be independent of position which means physically that the material is assumed to be elastically homogeneous. Equation (11) is the well known generalized Hooke's Law. The eighty-one constants  $c_{ijkl}$  can be reduced to twenty-one independent constants upon considerouter of symmetry.

Three-dimensional Hooke's Law for the solid-fluid mixture is given in matrix notation as

								-		-
σχχ		[C <sub>11</sub>	C <sub>12</sub>	C <sub>13</sub>	c <sub>14</sub>	c <sub>15</sub>	c <sub>16</sub>	C <sub>17</sub>	e <sub>xx</sub>	
σуу			C <sub>22</sub>	C <sub>23</sub>	<sup>C</sup> 24	C <sub>25</sub>	°26	с <sub>27</sub>	е уу	
σχχ				°33	с <sub>34</sub>	с <sub>35</sub>	с <sub>36</sub>	с <sub>37</sub>	e <sub>zz</sub>	
σyz	=				Сцц	с <sub>45</sub>	c <sub>46</sub>	c <sub>47</sub>	e <sub>yz</sub>	
σ <sub>zx</sub>						с <sub>55</sub>	°56	с <sub>57</sub>	ezx	
σ <sub>xy</sub>							c <sub>66</sub>	c <sub>67</sub>	e <sub>xy</sub>	
σ								C <sub>77</sub>	ε	(12)
									 	-

This stress-strain relation is assumed to be reversible in that there is identical response in either the positive or negative directions. This constitutive equation involving twenty-eight material constants serves as a base from which simplifications can be made for special cases. The two common cases of transverse and complete isotropy will be considered later.

# 4.6 Flow Relation

The equations presented so far would be sufficient to analyze a medium in equilibrium. In order to study transient behavior the equations of fluid flow are needed. The behavior of the flow of fluid is considered to follow that of generalized Darcy's Law which is

$$\sigma_{i} + \rho_{f} F_{i} = C_{ij} \frac{\partial}{\partial t} (v_{j} - u_{j})$$
(13)

where  $\rho_{\mathbf{f}}$  is the fluid mass per unit volume of bulk material and  $F_{\mathbf{i}}$  is the body force on the fluid. By symmetry the coefficient matrix  $C_{\mathbf{ij}}$  consists of six independent constants. The physical interpretation of these constants will be delayed to that time when special cases are outlined. Note that when all the  $u_{\mathbf{i}}$ 's vanish, equation (13) reverts to the standard form of Darcy's Law for fluid flow through an undeformed medium. If necessary, equation (13) can be rewritten in terms of fluid pressure p instead of stress  $\sigma$ by utilizing equation (1).

# 4.7 Field Equations

Solution of any problem is now feasible as we have a sufficient number of equations relative to the number of unknowns. The field equations are summarized here for the quasi-static loading of an anisotropic, interacting medium. The equilibrium equations are

$$(\sigma_{ij} + \sigma\delta_{ij}), j + \rho_m F_i = 0$$
(14)

Strain-displacement equations for the solid are

$$e_{ij} = \frac{1}{2} (u_{i,j} + u_{j,i})$$
 (15)

and similarly for the fluid are

$$\varepsilon_{ij} = \frac{1}{2} \left( v_{i,j} + v_{j,i} \right)$$
(16)

The constitutive equations are

_								1		1	
σχχ		C <sub>11</sub>	c <sub>12</sub>	c <sub>13</sub>	c <sub>14</sub>	c <sub>15</sub>	c <sub>16</sub>	C <sub>17</sub>	exx		
σуу			C <sub>22</sub>	c <sub>23</sub>	c <sub>24</sub>	C <sub>25</sub>	c <sub>26</sub>	C <sub>27</sub>	еуу		
σ <sub>zz</sub>				c <sub>33</sub>	c <sub>34</sub>	c <sub>35</sub>	c <sub>36</sub>	c <sub>37</sub>	e <sub>zz</sub>		
σ <sub>yz</sub>	=				Сцц	c <sub>45</sub>	c <sub>46</sub>	c <sub>47</sub>	eyz		
σ <sub>zx</sub>						°55	c <sub>56</sub>	c <sub>57</sub>	e <sub>zx</sub>		
σxy							<sup>C</sup> 66	<sup>C</sup> 67	e <sub>xy</sub>		
σ								C <sub>77</sub>	ε	(1	7

)

Finally, Darcy's flow equations are

$$\sigma_{j} + \rho_{f} F_{j} = C_{jj} \frac{\partial}{\partial t} (v_{j} - u_{j})$$
(18)

This set of twenty-five equations involve the following twenty-five unknowns; 1 fluid stress, 6 solid stresses, 6 fluid strains, 6 solid strains, 3 fluid displacements, and 3 solid displacements. Hence it is possible, at least theoretically, to completely solve the system once the boundary and/or initial conditions are specified. These equations may be combined in numerous ways depending on which variables are eliminated.

# 4.8 Simplifications

The solution of a set of twenty-five linear partial differential equations such as given here may be quite difficult. Simplifications provide for a reduction in the number of equations and thus increased ease of problem solution.

Transverse isotropy is material symmetry about one particular axis. If the z axis is arbitrarily selected, constitutive equation (17) with its twenty-eight constants is reduced to a new set with only eight material constants.

$$\sigma_{xx} = 2Ne_{xx} + A (e_{xx} + e_{yy}) + Fe_{zz} + M \varepsilon$$
(19a)

$$\sigma_{yy} = 2Ne_{yy} + A \left(e_{xx} + e_{yy}\right) + Fe_{zz} + M \varepsilon$$
(19b)

$$\sigma_{zz} = Be_{zz} + F (e_{xx} + e_{yy}) + Q \varepsilon$$
(19c)

$$\sigma_{yz} = L e_{yz}$$
(19d)

$$\sigma_{zx} = L e_{zx}$$
(19e)

$$\sigma_{xy} = N e_{xy} \tag{19f}$$

$$\sigma = M (e_{xx} + e_{yy}) + Q e_{zz} + R \epsilon$$
(19g)

The flow equation (18) with its six constants  $C_{ij}$  is likewise reduced to a system involving only the two constants  $C_{xx}$  or  $C_{yy}$  (Since  $C_{xx} = C_{yy}$ ) and  $C_{zz}$ . Thus

$$\frac{\partial \sigma}{\partial x} + \rho_{\mathbf{f}} F_{\mathbf{x}} = C_{\mathbf{x}\mathbf{x}} (\dot{\mathbf{v}}_{\mathbf{x}} - \dot{\mathbf{u}}_{\mathbf{x}})$$
(20a)

$$\frac{\partial \sigma}{\partial y} + \rho_{f} F_{y} = C_{xx} (\dot{v}_{y} - \dot{u}_{y})$$
(20b)

$$\frac{\partial \sigma}{\partial z} + \rho_{f} F_{z} = C_{zz} (\dot{v}_{z} - \dot{u}_{z})$$
(20c)

where the dot denotes differentiation with respect to time. The comma used earlier signified differentiation with respect to position. The number of equations for the transverse isotropic case is no less than that for the anisotropic but the number of material constants has been reduced from 34 to 10 and thus simplifies solving the problem.

A material is isotropic with regard to a particular property if at a point that property is the same in all directions. Under this assumption, the stress-strain equation (17) is reduced to the following

$$\sigma_{ij} = 2 \operatorname{Ne}_{ij} + \operatorname{Ae} + Q \varepsilon \qquad \text{for } i = j \qquad (21a)$$

$$\sigma_{ij} = Ne_{ij} \qquad \text{for } i \neq j \qquad (21b)$$

$$\sigma = Qe + R \varepsilon$$
 (21c)

Note that when Q and R are identically zero what remains is the standard three-dimensional Hooke's Law.

The flow equation (18) is reduced to contain the single constant C

$$\sigma_{,i} + \rho_{f} F_{i} = C \frac{\partial}{\partial t} (v_{i} - u_{i})$$
(22)

Equations (21) and (22) contain the five material constants N, A, Q, R, and C. These two sets of equations along with equations (14), (15), and (16) mathematically describe the quasi-static mechanical behavior of an isotropic fluid-solid mixture.

Simplification is further possible when it is valid to assume incompressibility. A condition of incompressibility can be applied to the entire mixture or any of the components. Only a binary mixture consisting of one solid and one fluid is considered here.

Incompressibility of the entire mixture means that the volume of the fluid squeezed out during a compressive loading is identically equal to the reduction in volume of the solid. In terms of the quantities already identified, this is

 $e (1 - P) + P \varepsilon = 0$ (23)

and must hold for all values of fluid stress  $\sigma$ . This condition when combined with the constitutive equations such as equation (21) reduces the number of material constants by two. This is the case commonly used during consolidation of a saturated soil.

The physical situation of a rigid solid and a fluid which only partially fills the void space would allow the incompressibility condition to be applied to the solids only. In the case where the fluid completely fills the voids and is many times stronger than the solid component the incompressibility condition may be imposed on the fluid component.

#### 4.9 Extensions

The theory presented here has considered some of the simplest cases of interacting media because of the introductory nature of this study. Should this theory show merit for further application to biological materials some of the following additional considerations and refinements would be appropriate.

As indicated at the onset the theory of mixtures is applicable to any number of components, each having widely varying properties. The solid, for instance, could be considered as behaving viscoelastically where the various

material constants are functions of time. Biot (1956) first used this approach while studying the settlement of a loaded column. Paria (1958) derived the resulting deformation of a porous viscoelastic circular cylinder when a load was applied to the solid portion only. Freudenthal and Spillers (1962) derived solutions for the infinite layer and the half space of poro-viscoelastic media.

Strains thus far have been assumed to be small and linearly related to stress. Biot and Willis (1957) applied the linear theory to systems with incremental stresses near a prestressed condition and found it directly usable when the stresses were defined in terms of the prestressed area rather than the final area.

A dynamic formulation as indicated earlier can be developed using a generalized equation (3) to include inertia terms. When this is done, it is found that there are additional terms due to the solid and fluid elements plus a third term due to the fluid-solid interaction. This differs from the case when generalizing from quasi-static to the dynamic for a single medium where the acceleration term can be directly inserted into the constitutive equations. Paria (1966) reviewed numerous articles where the dynamic theory was used in problems of wave propagation.

### INTERPRETATION AND DETERMINATION

#### OF MATERIAL PROPERTIES

## 5.1 Interpretation of Equations

The theory of mixtures can be used in solving actual problems only after the coefficients have been experimentally measured. Various forms of the equations have been expressed by Biot and Willis (1957) although only one is presented herein in an effort to suggest how the material properties could be measured.

The first case to be described is an isotropic binary mixture of solid and fluid. The constitutive equation (21) is the starting point for introducing different variables which aid in grasping the physical significance of the equations and particularly the coefficients. Let  $\tau_{ij}$  be defined as the total stress acting on a surface so that

$$\tau_{\mathbf{X}\mathbf{X}} = \sigma_{\mathbf{X}\mathbf{X}} + \sigma \tag{24a}$$

$$\tau_{yy} = \sigma_{yy} + \sigma \tag{24b}$$

$$\tau_{zz} = \sigma_{zz} + \sigma \tag{24c}$$

$$\tau_{yz} = \sigma_{yz} \tag{24d}$$

$$\tau_{zx} = \sigma_{zx} \tag{24e}$$

$$\tau_{xy} = \sigma_{xy} \tag{24f}$$

The change in fluid content  $\boldsymbol{\xi}$  is

$$\xi = P (e - \varepsilon)$$
(25)

New coefficients are defined as

-

$$\mu = N \tag{26}$$

$$\lambda = A - Q^2 / R \tag{27}$$

$$\beta = R/P^2 \tag{28}$$

$$\alpha = (Q/R + 1)P \tag{29}$$

Substituting equations (24-29) into equation (21) results in

$$\tau_{\mathbf{X}\mathbf{X}} + \alpha \mathbf{p} = 2_{\mu} \mathbf{e}_{\mathbf{X}\mathbf{X}} + \lambda \mathbf{e}$$
(30a)

$$\tau_{yy} + \alpha p = 2\mu e_{yy} + \lambda e$$
 (30b)

$$\tau_{zz} + \alpha p = 2\mu e_{zz} + \lambda e \qquad (30c)$$

$$\tau_{yz} = \mu e_{yz} \tag{30d}$$

$$\tau_{zx} = \mu e_{zx}$$
(30e)

$$\tau_{xy} = \mu e_{xy} \tag{30f}$$

$$\xi = (1/\beta) p + \alpha e \qquad (30g)$$

The first six of these equations show that a strain imposed on the solid produces either a stress in the solid, the fluid, or both. The last equation shows that there can be a net change in fluid content as a result of solid strain, fluid pressure, or both. No change in fluid content under a straining of the solid yields a proportional change in fluid pressure.

### 5.2 Determination of Isotropic Material Properties

The coefficients  $\alpha$ ,  $\beta$ ,  $\lambda$ , and  $\mu$  are characteristics of the particular medium under study. The coefficients  $\lambda$ and  $\mu$  are analogous to the Lamé constants of elasticity provided they are measured during negligible fluid pressure. It must be negligible in the sense that the fluid stress is relatively small compared to the solid stress. Thus the tests commonly used to measure elastic properties can be readily adopted except possibly the bulk compression experiment.

The coefficient  $\alpha$  could be measured during a jacketed compressibility test. In the jacketed compressibility test the specimen is surrounded by a very thin, flexible, impermeable jacket before the hydrostatic load is applied. The pressure in the specimen is isolated from the loading pressure by providing a fluid pathway between the inside of the jacket and the environment beyond the load. Then equation (30g) reduces to

$$\xi = \alpha \, e \tag{31}$$

and

$$\alpha = \xi/e \tag{32}$$

By measuring the amount of fluid flowing to or from the specimen and the volumetric strain in the solid the coefficient  $\alpha$  can be computed. If  $\xi$  indicates the change in pore volume then  $\alpha$  is the ratio of pore volume to volumetric strain. In equations (30a), (30b), or (30c),  $\alpha$  can be thought of as that part of the fluid pressure capable of producing a strain comparable to the total stress.

The coefficient  $\beta$  could be determined in various ways of which two will be described. The result of solving equation (30g) for  $\beta$  is

 $\beta = 1/(\xi/p - \alpha e/p)$ (33)

A conventional bulk compressibility test could be used here with measurements being made of fluid pressure, volumetric solid strain, and quantity of fluid flow. It would be necessary to use a compressing fluid identical to the fluid in the specimen. The assumption is made that the fluid pressure within the specimen and the pressure of the compressing fluid are identical. These three measurements along with the previously calculated  $\alpha$  allow  $\beta$  to be computed.

An alternate way of determining  $\beta$  would utilize a jacketed compressibility test with the specimen totally enclosed with no fluid outlets. In this case there would be no change in the fluid content and

$$\beta = -p/\alpha e \tag{34}$$

The fluid pressure within the specimen and the volumetric solid strain would be measured for various applied loads and along with the previously calculated  $\alpha$  be used to compute  $\beta$ .

The flow equation (22) for an isotropic material in the absence of body forces contains the single coefficient C which is

$$C = \mu_{f} / k \tag{35}$$

where  $\mu_{f}$  is the viscosity of the fluid and k is the permeability of the material. Measuring permeability is analagous to the common test for determining the Darcy coefficient except now the solid displacements must be taken into account.

#### 5.3 Non-isotropic Properties

The measurement of the properties of materials which are not isotropic follows the same format as for isotropic only there is an increase in the number of properties. The need for more tests requires added laboratory effort and increases the likelihood of error.

The constitutive equation (12) for an anisotropic solid-fluid mixture can be altered by substituting the last equation into the first six and obtaining a set of six equations which resemble those for an anisotropic elastic material. The coefficients will be identical when the fluid stress is zero. This is probably the best approach to arranging the equations to facilitate conceiving experimental techniques. Additional methods of measuring the coefficients in the constitutive equations for isotropic, transverse isotropic, and complete anisotropic case are described by Biot and Willis (1957).

#### 5.4 Other Formulations

The formulation of the equations presented here is but one of many possible approaches. Experience and success in the laboratory will bear out the usefulness of any particular formulation. Equation (30g) could be solved for the fluid pressure p in terms of the volumetric solid strain and change in fluid content. Substituting this result into equations (30a), (30b), and (30c) yields equations for the total stress in terms of the solid stress, fluid stress, and change in fluid content. In establishing laboratory tests it may be easier and a better assumption to require the change in fluid content to be zero rather than for the fluid pressure in the specimen to be zero.

The equations obtained here do not involve the porosity P. The usefulness of formulations which involve the porosity is dependent on the knowledge of this material property. The porosity or pore volume is a property which has been measured for many materials and thus would not limit one to a set of equations excluding the porosity factor.

#### VI. EXPERIMENTATION

The main purpose of the laboratory tests incorporated in this study was to indicate the validity of considering plant materials as interacting media. Because standard procedures and proven apparatus for the determination of the material properties of an interacting media do not yet exist the success obtained in the laboratory is at this time dependent largely on the successful operation of the equipment used. The approach taken was to attempt to use directly or with slight modification experimental equipment commonly used in measuring elastic properties. The decision with regard to the specific specimen was somewhat arbitrary as it was limited to the potato tuber. The potato was selected for its relative homogeneity and isotropy and because it has been the subject of numerous mechanical properties investigations which could serve as a basis of comparison. The void space of the specimen could be neglected since the potato has been reported by Davis (1962) to range in air volume from 1.0 - 1.7 per cent at the time of harvest and to be 4 - 30 per cent less than this after storage. Numerous other properties of the potato can be found in the text by Talburt and Smith (1967).

#### 6.1 Preparation of Potato Specimen

The potatoes used in this study were of the Ona variety. They were planted June 1, 1968 in sandy loam soil on a private farm at McBride, Michigan. Moisture during the growing season was slightly below normal. The fertilizer added per acre was 400 pounds of 8 - 32 - 16, 800 pounds of 22.5 - 0 - 30, and three pounds of borax. The potatoes were manually dug on October 25, transported by truck to the Michigan State University campus in East Lansing, and placed in storage all within five hours. During the first ten days the storage conditions were  $60 \pm 1^{\circ}F$  and 90 - 95 per cent relative humidity. After the initial ten-day period the temperature was lowered to 40°F for the remainder of the storage time. These were the conditions considered to be optimum for maintaining be potatoes until they were tested the following May. Huff (1967) found, during the first four months of storage, a change in potato strength which depended on the location within the potato. Potatoes were removed from storage as needed and placed in the open atmosphere of the laboratory at least twelve hours prior to testing. Room conditions during the tests fluctuated from 76 to 86°F for temperature and 30 to 50 per cent for relative humitity.

#### 6.2 Uniaxial Compression

One of the simplest elastic tests is the uniaxial tension or compression experiment. This test is applicable to interacting media when the fluid pressure

remains identically zero. The uniaxial compression test was used for this purpose and to show the effects of imposing different boundary conditions. Conditions could be altered by allowing or preventing fluid flow across the boundary. An elastic material would act the same under these two conditions while the interacting media would act differently. Figures 6.1 and 6.2 schematically illustrate the two conditions described and which were experimentally used.

The specimens were taken from potatoes which were nearly spherical in shape and about two inches in diameter. A large ordinary kitchen knife and a simple but specially made holder were used to make the two parallel cuts. The holder was made from 2.75 inch square tubing within which a flat plate stop was clamped one inch from the end of the tubing. To form a specimen the first cut was made with the potato lying on a table. The potato was then placed into the holder with the first cut against the stop, held in place with one hand, and the second cut made by moving the knife down along the end of the tubing.

Compression tests on the specimens were performed on an Instron model TM universal testing machine as shown in Figure 6.3. The procedure used for repeated loadings of a specimen was to alternate between the nonporous and porous flat places. The nonporous plates were those standard accessories provided for the testing machine. The porous loading was obtained by placing thin porous slabs





Figure 6.1.--Nonporous Flat Plate Loading.

Figure 6.2.--Porous Flat Plate Loading.



Figure 6.3.--Instron Unlaxial Compression Test with Porous Flat Plate Loading. (Photo 69-83)

between the specimen and the machine's plates. The slabs used were fritted glass 0.125 inch thick and 2.25 inches in diameter with a medium porosity. The deformation of the porous slabs during a fifty pound force application was found to be 0.0005 inch which was considered negligible compared to the sample deformations.

Preliminary testing revealed that successive loading curves displayed less deformation at the same force and that this effect increased with the magnitude of the applied force. For this reason a maximum force of fifty pounds was selected during which the resulting strain ranged from 2 to 5 per cent. A low loading rate of 0.1 inch per minute was used to provide time for any fluid pressure which might arise to be dissipated.

The testing machine recorded force-deformation curves of the nature shown in Figure 6.4. An initial length of 1.0 inch was always used so that the deformation reported could be easily converted to strain. The cross-sectional area of a specimen varied along the length and prevented calculation of actual stress. This required reporting of the values in terms of forces and deformations. An area suitable for stress computation might be the area at the loading plates. Using this area in computing a modulus of elasticity produced values in the 300 to 500 psi range. These values are lower than those given by Finney and Hall (1967), Huff (1967), and Timbers <u>et al.</u> (1966) probably because of the longer time in storage.



Figure 6.4.--Force-Deformation Curve for Uniaxial Compression Loading.

The clope of the force-deformation curve is the important characteristic as a linear slope is proportional to the modulus of elasticity. The more nonlinear portion of the force-deformation curve at the beginning is thought to be caused by nonalignment of the mating edges.

The resulting differential deformations between 10 and 50 pounds of force for ten samples are listed in Table A6.1. Permanent deformation prevents exact comparison of the data and requires viewing the trend during successive loadings. The results show the porous plate loading to have larger deformation under the same force. This reduced strength is expected for an interacting media because the fluid pressure is not allowed to build up and contribute to the overall strength.

The distribution of pore pressure for a similar *type* loading has been derived by Jana (1964/5). The pressure was numerically evaluated for different values of time for the case of a porous, undeformable die loading of a fluid saturated interacting half space. The graphic results show the expected high pore pressure in the neighborhood of the loaded surface.

This data for the potato subjected to uniaxial compression under different boundary conditions supports its consideration as an interacting medium. The loss of fluid from the sample during a single loading results in a new and different material with new and different properties. This may explain the cause of large permanent deformations in high moisture materials.

Table A6.1	Uniaxial Co	mpression dn	g Successive Load	ings of Potato 1.	.0 inch long.
		Incremental	Deformation x10 <sup>4</sup>	inch between 10 a	and 50 pounds
Specimen	Loading Plate	First Loading	Second Loading	Third Loading	Fourth Loading
C-1	snorodnon	11J	255	233	230
C - 2	porous nonporous	368	260	268	
C-3	porous ncnporous	323	260	255	233
C-4	porous nonporous	275	۲۰ ۲۰ ۵۵	225	
C-5	porous nonporous	258	265	215	
C-6	pcrous nonporous	188	230	175	220
C-7	porous nonporous	203	245	183	223
С – С	porous nonporous	208	303	175	258
C-9	porous nonporous	200	220	180	208
C-10	porous nonporous	220	240	198	220

### 6.3 Bulk Compressibility

Apparatus for measurement of bulk compression was modified with the hope that it could be used in jacketed and unjacketed compressibility experiments. The apparatus shown in Figure 6.5 was the same one used by Finney (1963) for potatoes and similar in principle to that used by Morrow (1965) for apples. For a test the specimen was placed in the opened chamber, the chamber was closed, liquid was added until the level reached near the top of the clear glass graduated column, fill valve was closed, air pressure was applied, and the drop in the liquid level was observed.

The chamber had a wall thickness of 0.25 inch and inside dimensions of six inches long by four inches in diameter. The glass column of twenty-two inch length was the diameter of 5.00  $\pm$  0.01 mm. A scale graduated in hundredths of an inch was taped to the back side of the tubing such that the water level could be read with the aid of a hand lens by viewing through the glass. Volume changes could be measured with a sensitivity of  $\pm$  0.00030 in<sup>3</sup>. The pressure was measured by a system consisting of a Kistler Model 701A quartz pressure transducer, Kistler Model 503M15 charge amplifier, and voltmeter. The charge amplifier was precalibrated such that its output could be read directly with full scale readings of either 1, 2, 5, 10, 20, 50, or 100 psi. The voltmeter was adjusted so the calibrated amplifier output produced a full



Figure 6.5.--Bulk Compression Apparatus.


scale, 100 division reading. Calibration of the system was checked with a dead weight tester and found over the 0 - 100 psi range to be linear and reading one per cent high.

The fluid used for this test was distilled water that had been boiled to remove the air. Care was taken during the filling of the chamber to prevent splashing of the water and thus introducing air.

It was found that a drop in the water level could be detected during a pressure increase of one psi even though the chamber contained only water. For this reason a series of calibration tests was required in order to obtain the volume change of the system alone. In measuring this volume change the variability of the calibration curve revealed the accuracy limitation of the system. Repeatability of the calibration curve was found by Finney (1963) to be within

(C per cent and by Morrow (1965) to be less than 5 per cent. A variation of the calibration curve of 10 per cent may be acceptable when the total volume change due to system and specimen is large. However, the jacketed and unjacketed compressibility tests require the detection of a difference between two large numbers. Preliminary tests for pressures up to 50 psi indicated that the difference in compressibility between jacketed and unjacketed tests was masked by the variability of the system itself and the specimen. Conducting a very large number of tests and using statistical procedures may have uncovered the facts. The logical approach to reduce the overall variability was to reduce that of the

system alone. The best solution would have been to eliminate the volume change of the apparatus and if this was not possible the next best solution would have been to reduce the variability in volume change to a negligible amount.

The volume change of the apparatus was due to expansion of the cylinder and compression of the fluid. Cylinder expansion could be reduced to nil if properly designed but a particular design should display highly repeatable pressure-expansion characteristics. An inconsistency in the compression of the fluid was found to be the problem source. In the 0 - 50 psi range the compressibility of water was found to vary as much as 50 per cent with the larger variations at lower pressures. In the 75 - 500 psi range the compressibility of water was found to be linear with a variation less than 2 per cent. Dorsey (1940) has compiled at a on the properties of water but none was found to verify the findings herein. Various filling techniques were tried in an attempt to reduce the variability to less than 2 per The only technique showing some success involved a cent. three day time lapse between the final filling of the chamber and pressure-volume measurements. This technique is not acceptable for use with a biological material whose properties may change during this time period. Hydraulic oil was tried as a possible fluid with the results being no more successful than for the water.

A factor of somewhat less importance is the volume of the fluid. A smaller volume of fluid compressed less

but its per cent variability was higher because the amount of variation remained nearly constant.

Another factor may become important when the volume of the sample is not negligible relative to the volume of the chamber. A precise calibration curve should be obtained using a fluid volume identical to that when the specimen is in place. The system calibration was determined by Finney (1963) and Morrow (1965) with the chamber completely filled with fluid. The error involved is dependent on the sample to fluid volume ratio and on the fluid compressibility in the relevant pressure range.

These results indicate the variability limitation of the bulk compressibility apparatus using fluids such as water and oil to determine the jacketed and unjacketed compressibility of materials with properties like that of the state. This apparatus could however be used for materials of higher strength where hydrostatic pressures less than 50 psi are only of minor importance.

# 6.4 Permeability

A permeability apparatus was constructed in order to measure the ease with which a fluid such as water would flow through potato tuber tissue. Fluid pressure was applied to a thin section of tuber which was mechanically supported by a highly porous plate and the resulting flow rate was measured. The bulk compressibility apparatus after slight modification was used as a source of controlled fluid

pressure. The specimen holder consisted of two parts between which the sample was held.

The first specimen holder design was built to accomodate a thin flat potato section as shown in Figure 6.6. The fluid pressure was applied over a circular area 0.50 inch in diameter. The porous support for the specimen was plaster of Paris formed in place. A very wet mix was used in forming the porous region to produce large voids and high permeability. This design could readily accomodate various specimen thicknesses. Preliminary testing with sections 0.25-0.50 inch thick cut from the center of potato tubers with a common kitchen knife displayed no throughflow for pressures up to 80 psi. Attempts with specimens ranging in thickness from 0.03-0.06 inch revealed a problem concerned with the preload applied to the specimen when it was first graced in the holder. If the two parts of the holder were not bolted together tightly leakage around the specimen occurred. In order to prevent leakage, the bolts had to be tightened to a degree which crushed the tissue. Flow through the tissue during any of these tests was immeasurable.

A second specimen shape as shown in Figure 6.7 was tried. The rationale for this specimen shape was that the reduced area section would be the true test region and the remainder of the specimen would help form a seal. This shape required a number of forming operations with the apparatus shown in Figure 6.8. The specimen was taken from the center of large tubers. Two parallel cuts about 0.75



i and

Figure 6.6.--Holder for Flat Specimen.



Figure 6.7.--Holder for Reduced Area Specimen.

inch apart were made with the knife. A 2 inch circular section was obtained using the apparatus shown at the left in Figure 6.8. The disk shaped specimen was then placed in the bottom of the permeability holder and a slice with the knife was taken to make the specimen flush with the holder. A blind hole was then drilled in the center of the specimen to within 0.125 inch of going completely through using the equipment shown at the right in Figure 6.8. A common 0.562 inch drill bit ground flat on the end was used to form a hole with a square bottom. A guide was placed atop the specimen and the holder bottom bolted to the holder top as shown in Figure 6.9. This design prevented leakage and provided at 75 psi a flow of one drop every few seconds. Throughflow was measured as the amount exiting the porous support. It was quantified by measuring the time in seconds between consecutive drops and the weight per drop.

The procedure for applying the pressure followed two schemes. According to the first scheme, pressure readings were taken at 0, 20, 40, 60, and 80 psi. After increasing the pressure to the desired level, the flow data were taken on the first drop which began to form. A release of the pressure and subsequent reloading revealed a dependence of the flow rate on time or rather the previous loading history. Accordingly, a second loading scheme was used in which the pressure was brought to a particular level, held constant, and the flow rate observed over a time period of 30 to 60 minutes. After the flow data on a particular sample had



Figure 6.8.--Apparatus for Cutting of Reduced Area Permeability Specimen. Press on left was used for initial external cut. Press on right was used to cut center blind hole. (Photo 69-77)



Figure 6.9.--Permeability Apparatus. Included are specimen holder, fluid supply, regulated pressure source, and pressure readout. (Photo 69-80)

been obtained the holder was unbolted and the sample thickness was measured. The specimen was visually checked using the low power stereo microscope for any signs of gross failure such as cracks or depressions.

An example of the flow data is shown in Figure 6.10 where  $T_1$ ,  $T_2$ , and  $T_3$  indicate loadings successively later in time and in Figure 6.11 as a time effect at constant pressure. The data for thirteen samples of the same thickness show a large variation in magnitude from that illustrated in Figure 6.10. Possible causes of this deviation are differences in product moisture content and mechanical preload between samples and the variation in sample length over the test cross section. The data of Figures 6.10 and 6.11 indicate the dependency of flow rate on both pressure and time. The dependency on time is as expected because of the deformability of the sample. In fact, the shape of the curve in Figure 6.11 resembles that of a stress relaxation curve for the same material. To compute a permeability coefficient from data like this requires consideration of the displacements of the solid in addition to the fluid displacement or flow.

The flow rates encountered herein were less than those measured by Görling (1956) during the combined application of fluid and mechanical pressures. The possibility of flow around rather than through the sample was recognized in that study and also herein.



Figure 6.10.--Pressure-Flow Rate Relation from Permeability Test. The three curves were recorded at successively later times from  $T_1$  to  $T_2$  to  $T_3$ .



Figure 6.11.--Flow Rate-Time Relation from Permeability Test at Constant Pressure.

Two methods of checking for leakage around the specimen were tried. A fluorescent dye was injected into the water at the beginning of a test and an attempt was made to trace its path. During the test the throughflow water was noted to be similar in color to that of the water at the beginning. After the test, the sample was dissected and microscopically viewed at magnifications up to 200X with fluorescent lighting. No traces of the dye could be found in the test region of the sample but some traces were found on the sample surfaces.

A second check was used to test for the presence of starch in the throughflow. Single drop samples of the throughflow were collected on microscope slides over a period of time starting from the first throughflow drop. These drops were then treated with a drop of starch-iodine colution and the resultant color change observed. Only the first one or two drops indicated any presence of starch in the throughflow. This source of starch precipitate may have been due to the initial mechanical compressive load imposed on the sample during assembly of the holder. These two tests although of only an indicative nature cast a shadow of doubt on the validity of the throughput data. The answer to the question of how much of the throughflow actually passed through the sample remains unknown and thus so does a quantitative measure of potato permeability.

The conclusion of these experiments is that this method for determination of permeability of potato tissue

is inadequate. An alternate method which might be considered is use of an osmometer as first suggested by Denny (1917) and recently modified by Dumbroff and Webb (1968) for measuring the permeability of plant membranes such as seed coats. According to the osmometer principle, flow through the sample is caused by a differential concentration of solutions.

### CONCLUSIONS

The following conclusions can be drawn from this study:

- 1. A study of plant anatomy indicates the potential for consideration of the stress-strain behavior of biological materials according to the theory of interacting media. As a first approximation, the potato tuber can be considered as a binary mixture of compressible solid and incompressible liquid.
- 2. Experimental tests with the potato showed:
  - a. significant differences in uniaxial compression
     with porous and nonporous flat plates as pre dicted by the interacting media theory.
  - b. lack of precision in calibrating previously used bulk compression equipment needed to determine jacketed and unjacketed compressibility.
  - c. inconclusive results of attempts to measure permeability by applying fluid water pressure.
- 3. There is need to develop and perfect experimental techniques to validate the interacting media theory on biological materials.



### SUGGESTIONS FOR FURTHER STUDY

The value of the theory will not be known until the stress-strain behavior can be predicted. At present, the lack of estimates of the material properties prevents a prediction study. Further studies should be made in the following areas:

- Conclusive data is needed with regard to the quantity of fluid flow through a sample subjected to fluid pressure. Knowledge of the actual fluid path would also be valuable information.
- 2. The bulk compression equipment needs improvement at pressures less than 50 psi in order to detect differences in compressibility during jacketed and unjacketed tests. An instrument or method needs to be devised whereby the pore pressure in the specimen can be measured.
- 3. Materials of both plant and animal origin need to be tried. Materials should be considered which have various amounts of fluid and degrees of saturation.
- 4. Attention should be given to more complex mathematical models such as a solid-liquid-gas mixture and a liquidviscoelastic solid mixture.



REFERENCES



#### REFERENCES

Akyurt, M.

- 1969 Constitutive relations for plant materials. Thesis for the degree of Ph. D. Purdue University, Lafayette. (Unpublished).
- Barrs, H. D.

1968 Determination of water deficits in plant tissues. pp. 235-368 in T.T. Kozlowski, Ed. Water Deficits and Plant Growth, Volume I. Academic Press, New York. 390 pp.

#### Biot, M. A.

- 1941 General theory of three-dimensional consolidation. Journal of Applied Physics 12:155-164.
- Biot, M.A.

1955 Theory of elasticity and consolidation for a porous anisotropic solid. Journal of Applied Physics 26:182-185.

- Biot, M. A.
  - 1956 Theory of deformation of a porous viscoelastic anisotropic solid. Journal of Applied Physics 27:459-467.
- Biot, M. A. and D. G. Willis

1957 The elastic coefficients of the theory of consolidation. Journal of Applied Mechanics 24:595-601.

- Blake, M. A., O. W. Davidson, R. M. Addoms, and G. T. Nightingale
  - 1931 Development and ripening of peaches as correlated with physical characteristics, chemical composition, and histological structure of the fruit flesh I. Physical measurements of growth and flesh texture in relation to the market and edible qualities of the fruit. New Jersey Agricultural Experiment Station Bulletin 525. 35 pp.

Clark, R. L., W. R. Fox, and G. B. Welch

1968 Representation of mechanical properties of nonlinear viscoelastic materials by constitutive equations. Paper No. 68-811 presented at the 1968 winter meeting, American Society of Agricultural Engineers, Chicago.



Cleland, R. 1967 Extensibility of isolated cell walls; measurement and changes during cell elongation. Planta (Berlin) 74:197-209.

- Davis, R. M.
  - 1962 Tissue air space in the potato; its estimation and relation to dry matter and specific gravity. American Potato Journal 39:298-305.
- Denny, F. E. 1917 Permeability of certain plant membranes to water. Botanical Gazette 63:373-397.
- Dorsey, N. E.
  - 1940 Properties of Ordinary Water-Substance. Reinhold Publishing Corp., New York. 673 pp.

Dumbroff, E. B. and D. P. Webb 1968 A modified Denny osmometer for permeability studies with plant membranes. Canadian Journal of Botany 46:1601-1603.

Esau, K. 1960 Anatomy of Seed Plants. John Wiley and Sons, Inc., New York. 376 pp.

Falk, S., C. H. Hertz, and H. I. Virgin 1958 On the relation between turgor pressure and tissue rigidity. I. Experiments on resonance frequency and tissue rigidity. Physiologia Plantarum 11:802-817.

Finney, E. E.

1963 The viscoelastic behavior of the potato, Solanum tuberosum, under quasi-static loading. Thesis for the degree of Ph. D. Michigan State University, East Lansing. (Unpublished).

Finney, E. E. and C. W. Hall 1967 Elastic properties of potatoes. Transactions of the ASAE 10:4-8.

Freudenthal, A. M. and W. R. Spillers

1962 Solutions for the infinite layer and the halfspace for quasi-static consolidating elastic and viscoelastic media. Journal of Applied Physics 33:2661-2668.

Frey-Wyssling, A.

<sup>1952</sup> Deformation and Flow in Biological Systems. Interscience Publishers, New York. 552 pp.

Görling, P. 1956 Untersuchungen zur Aufklärung des Trocknungsverhaltens pflanzlicher Stoffe. VDI-Forschungsheft 458. im Forschung auf dem Gebiete des Ingenieurwesens. Ausgabe  $\overline{B}$ , Band 22. 35 pp. Green, A. E. and P. M. Naghdi 1967 A theory of mixtures. Archive for Rational Mechanics and Analysis 24:243-263. Green, P. B. 1968 Growth physics in Nitella: a method for continuous in vivo analysis of extensibility based on a micro-manometer technique for turgor pressure. Plant Physiology 43:1169-1184. Greenham, C. G. 1966 Bruise and pressure injury in apple fruits. Journal of Experimental Botany 17:404-409. Gregg, S. J. and K. S. W. Sing 1967 Absorption, Surface Area and Porosity. Academic Press, New York. 371 pp. Haines, F. M. 1950 The relation between cell dimensions, osmotic pressure and turgor pressure. Annals of Botany, N. S. 14:385-394. Hammerle, J. R. 1968 Failure in a thin viscoelastic slab subjected to temperature and moisture gradients. Thesis for the degree of Ph. D. The Pennsylvania State University, University Park. (Unpublished). Hammerle, J. R. and N. N. Mohsenin 1969 The tensile relaxation modulus of corn horny endosperm as a function of time, temperature, and moisture content. Paper No. 69-343 presented at the 1969 summer meeting, American Society of Agricultural Engineers, Lafayette, Indiana. Haughton, P. M., D. B. Sellen, and P. D. Preston 1968 Dynamic mechanical properties of the cell wall of Nitella opaca. Journal of Experimental Botany 19:1-12. Houghtaling, H. B. A developmental analysis of size and shape in 1935

1935 A developmental analysis of size and shape in tomato fruits. Torrey Botanical Club Bulletin 62:243-252.



Huelin, F. E. and R. A. Gallop 1951 Studies in the natural coating of apples. Australian Journal of Scientific Research Series B 4:526-543.

- Huff, E. R.
  - 1967 Tensile properties of Kennebec potatoes. Transactions of the ASAE 10:414-419.
- Jackson, D. I.

1967 Relationship between cell size in the cortex and pith of apples and varietal susceptibility to internal breakdown and core flush. New Zealand Journal of Agricultural Research 10:319-322.

- Jana, R. N.
  - 1964/5 Indentation of a semi-infinite fluid-saturated poroelastic medium by an undeformable porous punch. Applied Scientific Research A 14:361-378.
- Kuprianoff, J.
  - 1958 'Bound water' in foods. pp. 14-23 in Fundamental Aspects of the Dehydration of Foodstuffs, Papers read at the Conference of the Society of Chemical Industry, London, England. 238 pp.
- Lazan, B. J.
  - 1962 Stress-strain-time relations for idealized materials. pp. 3-18 in Symposium on Stress-Strain-Time-Temperature Relationships in Materials. American Society for Testing and Materials Special Technical Publication 325. 128 pp.
- Letham, D. S.
  - 1961 Influence of fertilizer treatment on apple fruit composition and physiology I. Influence on cell size and cell number. Australian Journal of Agricultural Research 12:600-611.
- Lewis, F. T.
  - 1946 The shape of cells as a mathematical problem. American Scientist 34:359-369.

# Ling, G. N.

- 1968 The physical state of water in biological systems. Food Technology 22:1254-1258.
- Lockhart, J. A. 1967 Physical nature of irreversible deformation of plant cells. Plant Physiology 42:1545-1552.



Mark, R. E. 1967 Cell Wall Mechanics of Tracheids. Yale University Press, New Haven. 310 pp.

Martin, D.

- 1954 Variation between apple fruits and its relation to keeping quality II. Between-tree variations due to cropping factors. Australian Journal of Agricultural Research 5:9-30.
- Martin, D. and T. L. Lewis
  - 1952 The physiology of growth in apple fruits III. Cell characteristics and respiratory activity of light and heavy crop fruits. Australian Journal of Scientific Research Series B 5:315-327.
- Martin, D., N. S. Stenhouse, T. L. Lewis, and J. Cerny 1965 The interrelation of susceptibility to breakdown, cell size, and nitrogen and phosphorus levels in Jonathan apple fruits. Australian Journal Agricultural Research 16:617-625.
- Marvin, J. W.
  - 1939 The shape of compressed lead shot and its relation to cell shape. American Journal of Botany 26:280-288.
- Menefee, E.
  - 1968 A mechanical model for wool. Textile Research Journal 38:1149-1163.
- Mohsenin, N. N.
  - 1968 Physical Properties of Plant and Animal Materials. Part I of Volume I: Structure, Physical Characteristics and Rheological Properties. The Pennsylvania State University, University Park. 305 pp.
- Morrow, C. T.
  - 1965 Viscoelasticity in a selected agricultural product. Thesis for the degree of M. S. The Pennsylvania State University, University Park, (Unpublished.)

Nilsson, S. B., C. H. Hertz, and S. Falk

1958 On the relation between turgor pressure and tissue rigidity. II. Theoretical calculations on model systems. Physiologia Plantarum 11:818-837.

Paria, G.

1958 Deformation of a porous visco-elastic body containing a fluid under steady pressure. Bulletin of the Calcultta Mathematical Society 50:71-76.



Paria, G. 1966 Flow of fluids through porous deformable solids. pp. 901-907 in H. N. Abramson, Ed. Applied Mechanics Surveys. Spartan Books, Washington, D. C. 1198 pp. Parkus, H. 1968 Thermoelasticity. Blaisdell Publishing Co., Waltham, Massachusetts. 122 pp. Rasmussen, H. P. 1966 What holds leaf cells together ? p. 4 in Frontiers of Plant Science 18(2) Connecticut Agricultural Experiment Station, New Haven. 8 pp. Riener, M. 1960 Lectures on Theoretical Rheology. Interscience Publishers, Inc., New York. 158 pp. Ruhland, W. 1955 Encyclopedia of Plant Physiology. Volume I. Springer-Verlag, Berlin. 850 pp. Schapery, R. A. 1964 Application of thermodynamics to thermomechanical, fracture, and birefringent phenomena in viscoelastic media. Journal of Applied Physics 35:1451-1465. Ccheidegger, A. E. 1966 Flow through porous media. pp. 893-900 in H. N. Abramson, Ed. Applied Mechanics Surveys. Spartan Books, Washington, D. C. 1198 pp. Sinnott, E. W. and R. Bloch 1941 The relative position of cell walls in developing plant tissues. American Journal of Botany 28:607-617. Slatyer, R. O. 1967 Plant-Water Relationships. Academic Press, New York. 366 pp. Somers, G. F. 1966 The bending of potato-tuber slices mounted as cantilever beams. Journal of Experimental Botany 17:27-33. Stark, A. L. An apparatus and method for determining bound 1932 water in plant tissue. Proceedings American Society Horticultural Science 29:384-388.



Tabaddor, F. 1968 Constitutive equations and the solution of some problems of interacting continuous media. Thesis for the degree of Ph. D. Michigan State University, East Lansing. (Unpublished).

- Talburt, W. F. and O. Smith 1967 Potato Processing. Avi Publishing Co., Westport, Connecticut. 588 pp.
- Tetley, U.

1931 The morphology and cytology of the apple fruit with special reference to the Bramley's seedling variety. Journal of Pomology and Horticultural Science 9:278-297.

- Timbers, G. E., L. M. Staley, and E. L. Watson 1966 Some mechanical and rheological properties of the netted gem potato. Canadian Agricultural Engineering 8:15-18.
- Tukey, H. B. and J. O. Young 1939 Histological study of the developing fruit of the sour cherry. Botanical Gazette 100:723-749.
- Tukey, H. B. and J. O. Young 1942 Gross morphology and histology of developing fruit of the apple. Botanical Gazette 104:3-25.
- Van Arsdel, W. B. and M. J. Copley
  1963 Food Dehydration. Avi Publishing Co., Westport,
  Connecticut. 185 pp.

GLOSSARY

- cytoplasm -the least differentiated portion of the protoplasm which encloses all the other parts.
- homeostasis -a state of equilibrium or a tendency to return to such state.

imbibition -absorption of water over a surface.

- microfibril -relatively inert thread-like component of the cell wall.
- middle lamella -the layer of intercellular material cementing together walls of adjacent cells.
- osmosis -the diffusion proceeding between two solutions at different concentrations.
- parenchyma cell -a living, thin-walled cell varying widely in size, form, and wall structure.
- plastid -a cytoplasmic unit concerned with photosynthesis and food storage.

protoplasm -the living matter of a cell.

- protoplast -entire contents of the cell excluding the cell wall.
- tracheid -elongated, water conducting cell found in the xylum of vascular plants and functional as a dead element.
- turgor pressure -the pressure exerted by the cell contents on the cell wall.
- vacuole -a cavity in the cytoplasm filled with a watery fluid called the cell sap.








