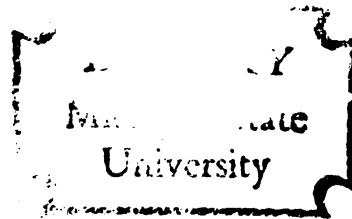


PRODUCTION FUNCTIONS FOR EDUCATION:  
AN EMPIRICAL STUDY

Thesis for the Degree of Ph. D.  
MICHIGAN STATE UNIVERSITY  
WILLIAM BARNETT  
1974



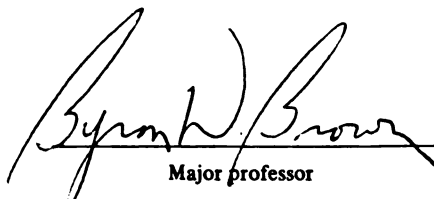
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## ABSTRACT

### PRODUCTION FUNCTIONS FOR EDUCATION: AN EMPIRICAL STUDY

By

William Barnett

This dissertation was designed to investigate the production function for education taking achievement in cognitive skills as the output, with particular emphasis on socioeconomic class and race as inputs.

Two models of the production function were developed: a short run model and a long run model. In these models education is viewed as a cumulative process.

In the short run model the inputs are prior achievement, home, school, and community influences in the relevant period. In the long run model the inputs are initial endowments at birth (assumed normally distributed) and home, school, and community influences cumulative from birth. The empirical models used to test hypotheses are linear in form. The data used are averages for schools. Socioeconomic status is considered a home input variable and the racial composition of a school is taken to be a school (or policy) input variable.

The results indicate the existence of multiple production functions for schools based upon the type of community within which a school is located.





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In general, prior achievement, home, and school inputs were found to be productive of achievement. The specific variables found to be significant for achievement were the lagged dependent variable, socioeconomic class, and the racial composition of the school.

The relationship between the short and long run models was derived and the statistical tests tended to confirm that this relationship does in fact exist.

Finally, we simulated cross-district busing for racial balance in the Detroit metropolitan area. The results of this simulation indicated that there would be gains in average achievement for schools in which the racial composition was altered to increase the percentage of white students, and losses in average achievement in those schools in which the percentage of white students declined. In general, the decreases in average achievement appear to offset the increases. The net effects, in general, are increases in average achievement of less than one-tenth of one standard deviation.

PRODUCTION FUNCTIONS FOR EDUCATION:  
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By

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## INTRODUCTION

With the publication of the Coleman Report a new era was born in the study of education.<sup>1</sup> The Report challenged certain nearly universally held assumptions. These assumptions were (1) most blacks go to schools that are significantly different, in terms of physical facilities, curricula, and teachers' characteristics, from the schools most whites attend; (2) physical facilities, curricula, and teachers' characteristics are important for education; and (3) therefore these differences are the cause of differences in black-white achievement in school.<sup>2</sup>

The Coleman Report, in essence, said schools make little if any difference.<sup>3</sup> School to school variations in achievement, from whatever source (community differences, variations in the average home background of the student body, or variations in

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<sup>1</sup> The Coleman Report is the popular name for Equality of Educational Opportunity by James S. Coleman, Ernest Q. Campbell, Carol J. Hobson, James McParland, Alexander M. Mood, Frederic D. Weinfeld, and Robert L. York. 2 volumes. (Washington, D.C.: Office of Education, U.S. Department of Health, Education, and Welfare, U.S. Government Printing Office, 1966). OE-38001. Superintendent of Documents Catalog No. FS 5.238:38001. Any reference in this thesis to the Coleman Report or The Report will be to the above work.

<sup>2</sup> See e.g., Godfrey Hodgson, "Do Schools Make a Difference?" The Atlantic Monthly, (March 1973), p. 36; or the Coleman interview in Southern Education Report, (November-December 1965).

<sup>3</sup> James S. Coleman, "Equal Schools or Equal Students?" The Public Interest, No. 4, (Summer 1966), pp. 73-74. See also: Coleman, Equality of Educational Opportunity, op. cit., p. 312, Table 3.24.1 and p. 302.



school factors), are much smaller than individual variations within the school, at all grade levels, for all racial and ethnic groups. This means that most of the variation in achievement could not possibly be accounted for by school differences, since most of it lies within the school.<sup>4</sup> Further, the data suggest that variations in school quality are not highly related to variations in achievement of pupils.<sup>5</sup> Taking all these results together, one implication stands out above all: that schools bring little influence to bear on a child's achievement independent of his background and general social context.<sup>6</sup> In fact, the Report found that the measurable differences between blacks' schools and whites' schools was very small.<sup>7</sup> The Report noted that:

Some careful study will reveal that there is not a wholly consistent pattern -- that is, minorities are not at a disadvantage in every item listed -- but there are nevertheless some definite and systematic directions of differences.<sup>8</sup>

and

At the same time, these differences in facilities and programs must not be overemphasized. In many cases, they are not large. Regional differences between schools are usually larger than minority-majority differences.<sup>9</sup>

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<sup>4</sup> Ibid., p. 296.

<sup>5</sup> Ibid., p. 297.

<sup>6</sup> Ibid., p. 325.

<sup>7</sup> Ibid., pp. 9 and 12.

<sup>8</sup> Ibid., pp. 9 and 12.

<sup>9</sup> Ibid., p. 122.

This report stimulated research in the area. The most obvious question to be answered was, "If schools don't make a difference for achievement, what does?" In fact, the Report itself offered a tentative answer: family background. Thus, the Report stated:

It is known that socioeconomic factors bear a strong relation to academic achievement. When these factors are statistically controlled, however, it appears that differences between schools account for only a small fraction of differences in pupil achievement.<sup>10</sup>

and,

...it is clear that no strong outside stimulus is making its impact felt in such a way as to interfere with the general relation of background to achievement; that is, it is clear that schools are not acting as a strong stimulus independent of the child's background, or the level of the student body. For if they were, there would be a decline in this correlation, proportional to the strength of such stimulus. This is not to say, of course, that schools have no effect, but rather that what effects they do have are highly correlated with the individual student's background, and with the educational background of the student body in the school; that is, the effects appear to arise not principally from factors that the school system controls, but from factors outside the school proper. The stimulus arising from variables independent of the student background factors appears to be a relative weak one.<sup>11</sup>

Further, it said:

It is known that socioeconomic factors bear a strong relation to academic achievement. When these factors are statistically controlled, however, it appears that differences between schools account for only a small fraction of differences in pupil achievement.<sup>12</sup>

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<sup>10</sup> Ibid., pp. 21-22.

<sup>11</sup> Ibid., pp. 311-312.

<sup>12</sup> Ibid., pp. 21-22.

Other answers were also forthcoming. Arthur Jensen pointed to the difference in average I.Q. between blacks and whites as the primary factor causing differences in achievement.<sup>13</sup> His point that "intelligence variation has a large genetic component" was documented by Herrnstein who, however, carefully avoided the racial issue.<sup>14</sup> Reexaminations of the Report's findings (using the same data) by Jencks, Armor, and Smith seemed to confirm the original findings.

Emerging from this were three positions: (1) that schools make little or no difference for achievement; (2) the agnostic position, that we don't know if schools make a difference for achievement; and (3) that schools may or may not make a difference for achievement, but that in any case this is not their primary function. Each of these basic positions has different branches. Within the first position we find those such as Drachler.<sup>15</sup> His point is that we should continue our massive resource allocation to education because even though schools make only a slight difference, that difference is worth the cost. We also find Jencks arguing that schools don't make much difference, but we should maintain large expenditures for education because people spend

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<sup>13</sup> A.R. Jensen, "How Much Can We Boost I.Q. and Scholastic Achievement?" Harvard Educational Review 39 (1969), pp. 1-123.

<sup>14</sup> Richard J. Herrnstein, I.Q. in the Meritocracy (Boston: Little, Brown and Company, 1971).

<sup>15</sup> Norman Drachler, Superintendent of the Detroit public school system from 1966 to 1971, as quoted by Hodgson in The Atlantic Monthly, op. cit.





large portions of their lives in school and that, therefore, the schools should be as pleasant as possible.<sup>16</sup> Moynihan agrees that schools don't make much difference, and believes that additional expenditure on education will benefit primarily the non-poor.<sup>17</sup> (He notes that 68 percent of school operating expenditures go to teachers, who are not deprived.) He would attack the problem of inequality of income and wealth directly via redistribution rather than indirectly via the schools. (Jencks would also subscribe to this solution to the problem of inequality in the distribution of income and wealth.<sup>18</sup>)

In the agnostic's corner (the second position) we can also discern several positions. First, there is that of Pettigrew (and, again, Jencks who seems to take both positions<sup>20</sup>) who say that integration may make a difference for the achievement of blacks, but we will not know unless and until we have truly integrated schools, not just desegregated schools.

The second main position would be that we need more research into the way education is produced before we can determine whether or not schools make a difference. This position is that of

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<sup>16</sup> Christopher Jencks, et al., Inequality: A Reassessment of Family and Schooling in America (New York: Basic Books, Inc., 1972).

<sup>17</sup> Daniel P. Moynihan, The Public Interest, Fall 1972.

<sup>18</sup> Jencks, op. cit.

<sup>19</sup> David K. Cohen, Thomas F. Pettigrew and Robert T. Riley, "Race and the Outcomes of Schooling," in On Equality of Educational Opportunity (eds.) Frederick Mosteller and Daniel P. Moynihan (New York: Vintage Books, 1972), pp. 343-368.

<sup>20</sup> Jencks, as quoted by Hodgson in The Atlantic Monthly, op. cit.

Hanushek<sup>21</sup>, Kain<sup>21</sup>, and Averch, et al.<sup>22</sup> This group sees a number of potential problems arising from lack of knowledge of the production function. The production function is a concept that shows, "the maximum amount of output that can be produced from any specified set of inputs, given the existing technology, or 'state of the art'."<sup>23</sup>

It may be that school resources are used to produce outputs other than achievement. Or, resources may be used inefficiently; non-productive resources may be purchased through ignorance. Any of these possibilities could lead to the conclusion that the schools make little or no difference for achievement. In effect the second group argues that schools make little or no difference for achievement, but that this may be due to lack of knowledge of the production function and, therefore, in the future it may be possible for schools to make a difference. If we can identify the production function for education maybe this knowledge will enable schools to make a difference.

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<sup>21</sup> Eric A. Hanushek and John F. Kain, "On the Value of Equality of Educational Opportunity as a Guide to Public Policy," in On Equality of Educational Opportunity, (eds.) Frederick Mosteller and Daniel P. Moynihan (New York: Vintage Books, 1972), pp. 116-145; also, Eric A. Hanushek, "Teacher Characteristics and Gains in Student Achievement: Estimation Using Micro Data," Papers and Proceedings of the 83rd Annual Meeting of the American Economic Association, Published in May 1971, pp. 280-288.

<sup>22</sup> Harvey A. Averch, et al., How Effective is Schooling?: A Critical Review and Synthesis of Research Findings, Prepared for President's Commission on School Finance, (Santa Monica: Rand, 1972).

<sup>23</sup> C.E. Ferguson, Microeconomic Theory, (Homewood, Ill.: Richard D. Irwin, 1966), p. 110.

The last position is that of Bowles, Brown and Gintis.<sup>24</sup>

This position is that the primary purpose of schools is not the production of achievement. Achievement may, however, be a by-product of the process. They take the view that schools, as institutions of society, would disappear if they did not adequately perform their function. They also note the apparent lack of relationship between schools and achievement as established by numerous studies. Thus, they rule out the possibility that the primary output of schools is academic achievement and look elsewhere for the primary output of education. In fact, they hold a theory which says that the primary function of schools is to condition students so that they grow up to be docile cogs in the "military-industrial complex," i.e., schools should and do turn out good workers and soldiers, who will not challenge the status quo, but rather will allow the ruling class to continue to rule.

I take the position of Hanushek, Kain, and Averch, et al.

This thesis is concerned with the identification of the production function of education.

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<sup>24</sup> See the following three sources:

S. Bowles and H. Levin, "The Determinants of Scholastic Achievement - An Appraisal of Some Recent Evidence," Journal of Human Resources. Vol. III, No. 1 (Winter 1968).

Byron W. Brown, "Achievement, Costs, and the Demand for Public Education," Western Economic Journal, Vol. X, No. 2 (June 1972).

Herbert Gintis, "Education, Technology, and the Characteristics of Worker Productivity," The American Economic Review (May 1971).

## CHAPTER I

### THE PROBLEM, METHODOLOGY, AND MODEL

#### 1. The Importance of the Problem

The primary concern of this work is the identification of the production function of education with particular emphasis on the existence and form of relationships among academic achievement, socioeconomic status (class), and race. The existence and form of the relationships among achievement, socioeconomic status, and race within the framework of a production function of education are important for several reasons.

First, we spend a lot on education in this country and inefficiencies in the production of education may involve the waste of vast quantities of resources.<sup>1</sup> There are two types of inefficiencies which may exist. One is the allocation of too many or too few resources to education, vis á vis other uses. If too many resources are used for education we produce (assuming the extra resources are not redundant) too much education and, due to the scarcity of resources, too little of some other things, given the values society places on these competing uses. Should, instead,

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<sup>1</sup> For estimation of resource costs of education in the U.S. see T. Schultz, Investment in Human Capital (New York: The Free Press, 1971), pp. 84, 85, 91, 92, 94, 95 and especially p. 100. See also F. Machlup, The Production and Distribution of Knowledge in the United States (Princeton, New Jersey, 1962), pp. 354 and 362.

too few resources be used for education, then, again, given the values of society, we produce too little education, and too much of other outputs (assuming the resources are not redundant). The other type of inefficiency is the use of too much or too little of a particular resource within the educational process. The results of this type of misallocation of resources is that, given the unit costs of the various resources, the same educational output could be produced at a lower total resource cost. Thus we use more resources than necessary to produce the given output. Knowledge of the educational production function is important, then, in order to allocate resources efficiently both between education and other uses, and within education. Such knowledge is, therefore, important for policy decisions.<sup>2</sup>

Second, there is widespread poverty in the U.S. and its elimination is a stated national goal.<sup>3</sup> Because of the existence of linkages between education and income and the racial pattern of poverty, the relationships among achievement, socioeconomic status, and race are important for policy decisions involved in attempting

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<sup>2</sup> S. Bowles, "Towards An Educational Production Function," in Education, Income, and Human Capital, ed. W. Hansen, NBER, New York, 1970, p. 12. See also H. Kiesling, Multivariate Analysis of Schools and Educational Policy, (Santa Monica, California: The Rand Corporation, March 1971), pp. 1-2.

<sup>3</sup> On the pervasiveness of poverty see M. Orshansky, "The Shape of Poverty in 1966," in R. Marshall and R. Perlman, An Anthology of Labor Economics: Readings and Commentary, (New York: Wiley and Sons, 1972), pp. 810-816. See also, "The March to Equality Marks Time," Time, September 3, 1973, pp. 74-75 according to which over one-third of all blacks are classified as poor as well as nine percent of whites.

to achieve the above mentioned national goal.<sup>4</sup> The socioeconomic linkages between education and income have been well established by a number of studies. Recent studies of this type have estimated rates of return on investment in education, based upon cost-benefit analysis of "human capital" models. In these studies education is one method of formation of human capital, and human capital is productive. Costs of education, returns to education (in the form of increases in future earnings), and rates of return are estimated. The results establish the linkages.<sup>5</sup>

Third, there is considerable discontent with public education in this country.<sup>6</sup> Undoubtedly, the causes are many but if more efficient allocations of resources are possible in the future this would serve to lessen the discontent. It would lessen the discontent by either yielding more educational achievement than we now get for the same cost, or reducing the cost of producing the level of achievement we now get. That is, taxpayers would either get more for their money or pay less for what they are now getting. Because of racial issues and busing, discontent is likely to grow. If there is a lessening of the commitment to public education with concomitant effects on resource availability, it will be even more

---

<sup>4</sup> Although there are more poor whites than non-whites, the percentage of non-whites who are poor far exceeds the percentage of whites who are poor. There is then a racial pattern to poverty.

<sup>5</sup> See Schultz, op. cit., pp. 132-156 and p. 173. See also W. Hansen, "Rates of Return to Investment in Schooling in the United States," in Economics of Education 1, ed. M. Blaug (Baltimore: Penguin Books, Inc., 1968), pp. 137-154.

<sup>6</sup> See e.g., Hodgson, The Atlantic Monthly, op. cit.

imperative that resources be used efficiently. In order to use resources efficiently we must know both the production function and the input supply functions so that we can maximize the output from a given budget.

## 2. Approaches to Research

There have been a number of empirical studies of achievement, socioeconomic status, and race in the recent past.<sup>7</sup> These studies have used a variety of measures of socioeconomic status (SES). Socioeconomic status is defined by a variety of characteristics such as income, values, attitudes, and cultural heritage.<sup>8</sup> Scores on various standard achievement tests were used as measures of academic achievement. The results of these studies may be summarized as follows. Invariably, they have shown correlations between SES and race, and between SES and achievement.<sup>9</sup> The studies of relationships

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<sup>7</sup> See e.g., Coleman, et al., Equality of Educational Opportunity, op. cit. See also Cohen, Pettigrew, and Riley, "Race and the Outcomes of Schooling,"; David J. Armor, "School and Family Effects on Black and White Achievement: A Reexamination of the USOE Data,"; M. Smith, "Equality of Educational Opportunity: The Basic Findings Reconsidered,"; and C. Jencks, "The Coleman Report and the Conventional Wisdom," in On Equality of Educational Opportunity, op. cit. See also Kiesling, op. cit. in which he summarizes the results of 15 studies of American schools. See also Bowles, op. cit. Although Bowles explicitly rejects the usefulness of SES as representative of non-school effects; the variables he actually uses for non-school effects are determinants of SES. See also F. Mosteller and D. Moynihan, "A Pathbreaking Report: Further Studies of the Coleman Report," in On Equality of Educational Opportunity, op. cit., p. 22ff.

<sup>8</sup> For a definition of SES, see Armor, Ibid., pp. 172-173. See also Mosteller and Moynihan, Ibid., p. 22.

<sup>9</sup> On the SES-achievement relationship see Kiesling, op. cit., p. 20ff. As to the race-SES relationship, although none of the authors speaks of the studies as establishing this fact, it is implicit in those that recognize the need to control for SES in attempts to identify race-achievement relationships. See Jencks in On Equality of Educational Opportunity, op. cit., pp. 70-71.

between race and achievement may be divided into two categories:

(1) those which attempted to identify direct relationships (partial correlations) between race and achievement, i.e., independent of indirect relationships via the race-SES and SES-achievement relationships;<sup>10</sup> and (2) those which do not separate the direct and indirect relationships.<sup>11</sup> The results for the studies falling in the second category have shown invariably a correlation between race and achievement.<sup>12</sup> The results for those studies falling in the first category, although somewhat mixed, run generally parallel to those for the second category of studies.<sup>13</sup>

These results may be interpreted in the following way. Certain of the characteristics which define an individual's SES may also have a direct causal effect on educational achievement, e.g., attitudes towards education.<sup>14</sup> In addition, some of these factors may have an indirect causal effect on educational achievement.<sup>15</sup> For example, if educational achievement is affected by

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<sup>10</sup> See e.g., Cohen, Pettigrew, and Riley, Ibid.

<sup>11</sup> See Coleman, et al., op. cit. On the failure to separate the effects of race and SES see Cohen, Pettigrew, and Riley, op. cit., pp. 344-345.

<sup>12</sup> See e.g., Jencks, in On Equality of Educational Opportunity, op. cit., pp. 69-71; Cohen, Pettigrew, and Riley, Ibid., pp. 344-345; and Mosteller and Moynihan, Ibid., p. 41.

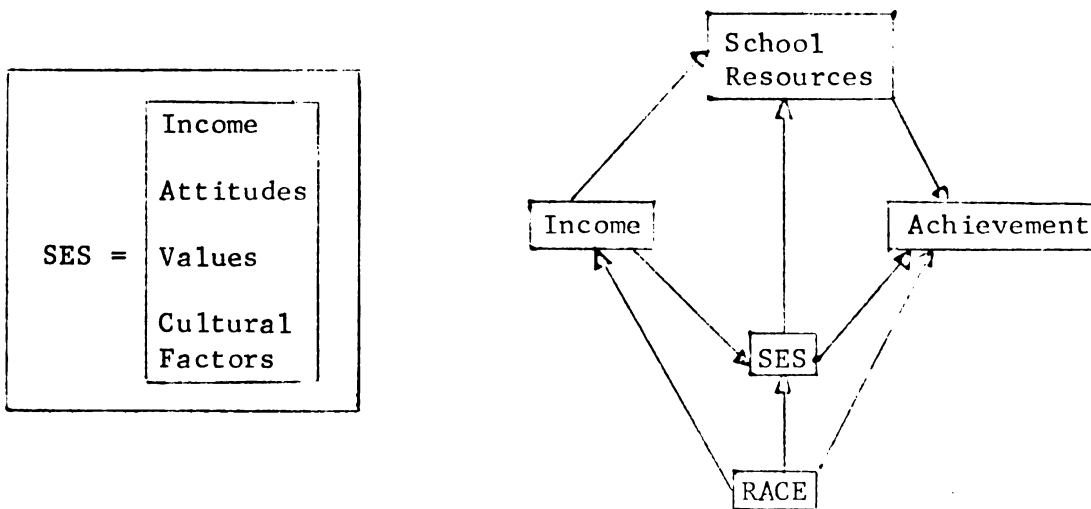
<sup>13</sup> See Armor, Ibid., pp. 222-225; Cohen, Pettigrew, and Riley, Ibid., pp. 363-366; and for a contrasting view see Jencks, Ibid., pp. 70-71.

<sup>14</sup> See Armor, Ibid., pp. 172-173.

<sup>15</sup> See Jencks, Ibid., pp. 70-71 where it is concluded that although this link exists, it is insignificant in terms of differences in resource allocations. This agrees in general with the findings of the Coleman Report, pp. 9 and 12.



the quality of teachers and if the availability of quality teachers depends upon community income, then SES has an indirect influence on educational production; in addition, if race affects achievement, socioeconomic status, and income then race affects achievement both directly and indirectly. This may be illustrated by a path diagram.



Thus the relationship between SES and academic achievement may exist because they are jointly determined, in part or in toto, i.e., there is a simultaneous equations problem. By jointly determined we mean that certain factors (e.g., income) affecting one thing (e.g., SES) may also affect another thing (e.g., achievement). The race-achievement relationship may be explained by either a direct causal effect of race on achievement or by an indirect causal effect of race on some of the factors which affect, directly or indirectly, achievement. That is, race, because of racism, may affect both income and attitudes.<sup>16</sup> The racism factor makes the

<sup>16</sup> See footnotes 9, 10, 11, and 12 for references on these points. In particular, see Cohen, Pettigrew, and Riley, Ibid.

link between race and both income and attitudes indirect. And, as noted above, the income-achievement and attitude-achievement relationships are indirect and direct, respectively. Thus, the race-achievement relationship may exist because of indirect causality. There are some who believe the relationship is direct through inherited natural abilities which differ between the races.<sup>17</sup> The race-achievement relationship may, on the other hand, be a direct relationship. It is well established that on the average blacks score approximately 15 points lower than whites on I.Q. tests. If, in fact, these I.Q. tests accurately measure mental abilities necessary for achievement, then the relationship between race and achievement is direct.

These results of these studies have been questioned on several grounds. The main bases of criticism have been: the statistical techniques used; the data used; and the models.

Statistical techniques that are criticized include simple correlation, stepwise regression, and analysis of variance.<sup>18</sup> Simple correlation has been criticized for its failure to take into account the complexities of the various relationships. This type of analysis is limited to determining the degree to which two variables covary linearly. It is misleading if the actual relationship is significantly different from linear, or if the actual relationship between the two variables is complicated by the presence

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<sup>17</sup> See Jensen, op. cit.

<sup>18</sup> For criticisms of statistical techniques, see S. Bowles and H. Levin, op. cit. See also Mosteller and Moynihan, op. cit., pp. 34-35.

of one or more variables which affect both of them. Stepwise regression is criticized because the results depend upon the order in which the variables are added to the regression. This is because it attributes the variance in the dependent variable, due to a pair of variables but which cannot be accounted for by either variable alone, to whichever of the variables is entered first. That is, it understates the importance of the latter variable. Thus the results of this type of analysis are ambiguous. Analysis of variance is criticized primarily for failure to lead to policy conclusions. This is because, although analysis of variance can attribute percentages of the variance in a variable to other variables, it gives no indication of what we economists call the marginal productivity. That is, we get no information as to how much of a change in one variable is required to produce a given change in another variable.

The data is suspect on a wide variety of grounds including, in many instances, inability to discriminate between inputs on the basis of quality; the inability of the data to adequately represent the variables actually called for by the model being estimated;<sup>19</sup> and the crudeness of the data gathering techniques, i.e., inaccuracy of the data.<sup>20</sup>

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<sup>19</sup> This is not an "errors in measurement" problem in that it is not a case of desiring to measure the  $X_i$ 's and actually measuring the  $X_i^*$ 's,  $X_i = X_i^* - V_i$ ,  $V_i \sim N(0, \sigma_v^2)$ . (In this case, the use of  $X_i^*$  as a dependent variable would lead to inconsistency. In such a case the method of instrumental variables would be appropriate.) Rather, we speak here of the problem that arises in that there may be no correlation between the variable used and the variable desired.

<sup>20</sup> For criticisms of the data see Kiesling, op. cit., pp. 5-12.

The criticisms of the models fall into two main groups. The first group criticizes the specification of the models.<sup>21</sup> There are two main criticisms of the specification of the models. One is with respect to the functional form. The linear form is attacked because the marginal products are constant, i.e., an input's productivity depends neither on its own level nor on the level of any other input. The Cobb-Douglas form is criticized because the cross partial derivatives of output are positive for any pair of inputs each of whose marginal product is positive.<sup>22</sup> That is, if:

$$Y = \sum_{i=1}^n X_i^a$$

then, for all:

$$\frac{\partial Y}{\partial X_i} > 0, \quad \frac{\partial Y}{\partial X_j} > 0 \quad j \neq i$$

the following must be true:

$$\frac{\partial^2 Y}{\partial X_i \partial X_j} > 0$$

where:  $Y$  is output, and the  
 $X_i$  ( $i = 1, \dots, n$ ) are inputs.

Thus, an increase in the quality of teachers would have a greater effect on high I.Q. students than on low I.Q. students, assuming both I.Q. and teachers to be inputs whose marginal products are

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<sup>21</sup> See e.g., Bowles, op. cit., pp. 33-34.

<sup>22</sup> See Bowles, op. cit., p. 19.

positive. The other and more serious criticism is with respect to the variables, particularly the inputs. According to this criticism we really don't know what the inputs into the educational process are. That is, we have no adequate theory of how one becomes educated. And, of course, if we don't know what the inputs are, we cannot properly specify the production function. However, as long as our knowledge of the educational process is as rudimentary as it is, all such models will be open to this criticism. The second group of criticisms is not actually directed at models, but rather at the lack of theoretical models. In fact, some of the studies present no such model, although in some cases it may be argued that the model is implicit.<sup>23</sup>

It is the task of this work to improve upon these studies. Specifically, an attempt is made: (1) to use statistical techniques, multiple regression analysis, superior to those that have been criticized; and, (2) to estimate an econometric model based upon, and adequately representative of, the theoretical model constructed herein.

Multiple regression is superior to simple correlation in that it can deal with complex relationships among more than two variables. It is superior to stepwise regression in that the results are not dependent upon the order in which the variables are entered. Finally, multiple regression is superior to analysis of variance in that it allows us to determine marginal products, i.e., we can estimate the effects of changes in one or more explanatory variables on the dependent variable.

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<sup>23</sup> See e.g., Hanushek and Kain, op. cit., pp. 123-124.

As mentioned earlier, the basic problem we are interested in is the existence and form of the production function of education. We are particularly interested in the effects on achievement of socioeconomic status and race.

The hypotheses to be tested are:

$$H_0: \frac{\partial A}{\partial I_i} = 0 \qquad H_A: \frac{\partial A}{\partial I_i} \neq 0 \quad (i = 1, \dots, n)$$

where  $A$  is academic achievement and the  $I_i$ 's are inputs into the educational process including socioeconomic class and race. These hypotheses will be tested by the use of multiple regression techniques applied to the models of the production function of education.

### 3. The Model

According to Averch et al. there are five basic approaches to educational research which they identify as: (1) input-output; (2) process; (3) organizational; (4) evaluation; and, (5) experimental.<sup>24</sup> They briefly describe these approaches as follows:

The input-output approach assumes that students' educational outcomes are determined by the quantities and qualities of the educational resources they receive. The Equality of Educational Opportunity survey -- known as the Coleman Report after its principal author, James Coleman -- is the best-known example of this, the educational economist's, approach to educational research.

The process approach includes most of the work done by educational psychologists, as well as certain studies by sociologists and clinical and experimental psychologists. These studies attempt to examine the processes and methods by which resources are applied to students.

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<sup>24</sup> Averch, et al., p. v.

The organizational approach consists of case studies of school systems that assume what is done in the school is not the result of a rational search for effective inputs or processes, but is a reflection of history, social demands, and organizational change and rigidity. These studies are typically done by political scientists or sociologists and focus on the ways in which the factors that influence or impinge on the various decisionmakers in the school system affect the behavior of the system.

Studies of relatively large-scale interventions in school systems are included in the evaluation approach. Examples include the evaluations of compensatory education programs for the disadvantaged, funded by Title I of the Elementary and Secondary Education Act (1965), and the evaluations of Head Start Programs. The central issue in these studies is whether broad-based interventions affect students' outcomes.

Finally, we include in the experiential approach the so-called "reform" literature. These are books and articles, typically written by teachers or advocates of educational reform, that describe how the school system works and what it does to those on the inside, particularly students. They share the view that what happens to the student in school is an end in itself, rather than a means toward some further end, such as the acquisition of specific skills.<sup>25</sup>

We shall use the "educational economist's approach" -- input-output, otherwise known as the production function.

In its most general form, i.e., multi-product, the production function is as follows:

$$E = f(I)$$

where:  $E$  = the output, or vector of outputs in cases of multiple products, and

$I$  = the vector of inputs .

The general problem in identifying a particular production function at the theoretical level is threefold. First, the output

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<sup>25</sup> Ibid., p. vi.

or vector of outputs must be specified. Second, the vector of inputs must be specified. Third, the exact relationships between inputs and output(s) must be specified. In general, not all three of these aspects will present problems.

For example, consider the specification of the output vector. Many firms and industries can readily specify the output vector. Thus, for a television manufacturer, output consists of so many of one type of television and so many of another. We can easily think of many other examples, but the foregoing should be sufficient to illustrate the point that for many firms and industries the specification of the output vector is not a serious problem. We can also think of many firms and industries for which the specification of the input vector is not too difficult. As an example I suggest a clothing manufacturer. For such a firm the inputs can be fairly easily identified. They include buttons, zippers, etc., raw materials (fabrics of different types), services of capital equipment (cutting and sewing machines), energy, and various types of labor. Again, this example should suffice as an indicator of the ease with which inputs into certain production processes may be specified. The specification of the exact relationships between inputs and outputs is, usually, the most difficult problem. In certain, usually highly technical, processes this problem is not too difficult. For example, in the production of electrical energy by use of natural gas powered steam generators engineers can tell us exactly the technical relationship between the amount of natural gas and water used in a given generating system as inputs and the number of kilowatt hours of electrical



energy of output produced. In addition, they can tell us how much and what types of labor are necessary to run and maintain the system. Such types of production processes are in the minority, however. Usually this is particularly true with respect to the production of services.

Returning to our consideration of the production function of education, we must recognize that all three aspects present problems. None is easily handled. We now consider each of them in turn.

#### Problems in the Specification of Output of the Educational Process

First, the output is multi-dimensioned.<sup>26</sup> That is, there is no one thing we can call the output of education. Rather, the output is a composite of several things. We shall refer to the different dimensions which comprise the output vector as attributes of the output. The following is but a partial listing of those dimensions (attributes) of the output of the educational process suggested by various authors:<sup>27</sup> academic achievement; retention rates; proportion going to college; income and occupation of graduates; happiness; educational quality; a sense of social dignity and place; a commitment to their community; knowledge of how to work and live with others; better student adjustment; motivation;

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<sup>26</sup> We must recognize two facets of output: normative, concerned with what the output should be, and positive, concerned with what the output is. In this paper we deal exclusively with the positive facet.

<sup>27</sup> See Mosteller and Moynihan, op. cit., pp. 6, 27, and 39; Armor, Ibid., pp. 170-171; Bowles, op. cit., p. 20; and H. Dyer, "The Measurement of Educational Opportunity," in On Equality of Educational Opportunity, op. cit., pp. 518 and 522.

good career choices; basic skills in reading, writing, and arithmetic; personal self-esteem; respect for law and the rights of others; understanding of and commitment to the national culture; self-understanding and acceptance; mastery of basic skills; social and vocational competence; physical well-being. It is possible that some of these are redundant because of semantics or the possibility that some are not independent of others.

Second, it is customary in economic theory to assume that all units of a given good are homogeneous. Indeed, this is how a good is usually defined. The level of output is the quantity of a good. In the case of multiple products the level of output is the vector of the quantities of the various products. This is so because any attempt to aggregate heterogeneous goods runs into an index number problem. In the case of education the output is in fact just such a vector of changes of the various attributes of the output. Such changes may, in fact, not be easily quantifiable. However, we assume that such changes are quantifiable, even if imperfectly. This study, then, will deal, on the theoretical plane, with the level of output as a vector of the various quantities of the different attributes of output.<sup>28</sup>

Third, components of the output vector as distinguished from the level of output may vary between producers.<sup>29</sup> In addition,

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<sup>28</sup> See Bowles, op. cit., pp. 20-25; and Kiesling, op. cit., pp. 6, 16, and 17. In our empirical work we shall use as our measure of output a single variable, composite achievement, which is the simple average of the level of achievement in three cognitive skills (reading, the mechanics of written English, and mathematics) which are attributes of output.

<sup>29</sup> See Bowles, op. cit., pp. 17 and 18.

producers differ in their abilities to translate given inputs into positive output. Therefore, outputs may vary between producers even with no difference observed in the level of inputs. This problem can be handled in constructing a theoretical model of the production function as follows. Construct a vector of output,  $E$ , that includes all attributes of output, such that:

$$E = (e_1, e_2, \dots, e_n)$$

where:  $e_i = i^{\text{th}}$  aspect of output .

That is,  $E$  is exhaustive, but for some individual schools or districts some  $e_i$  may be equal to zero.

For the purpose of this paper, the attributes of output which are of interest are cognitive skills. Our output vector is, then:

$$E = (e_1, e_2, \dots, e_n)$$

where:  $e_i, (i = 1, 2, \dots, j); (j < n)$ , are cognitive skills.

There are two major reasons why we limit our output vector in this way. First, there is no general agreement as to which non-cognitive attributes are produced. Different producers probably produce different non-cognitive attributes based upon their own value judgments.<sup>30</sup>

Second, non-cognitive attributes are much more difficult to quantify than cognitive attributes. This has resulted in a lack

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<sup>30</sup> The question of who is the producer of education is dealt with in the section on specification of the inputs. Suffice it to say that at this point one may consider students, parents, teachers, principals, or district superintendents as the producer.

of data concerning such attributes, while at the same time considerable quantities of data has been collected on the cognitive skills.

Of course, we must bear this in mind when interpreting the results of our study. This is so because measured differences in production functions between producers may be due to differences in choices among production of cognitive and non-cognitive attributes of output, rather than to differences in the actual production functions. For example, suppose we consider two separate groups of schools, identical but for the sets of principals. In the one group of schools the principals are particularly concerned with the development of reading skills, while in the other the principals are particularly concerned with instilling patriotic attitudes in the students. It is conceivable that in the first group of schools students are encouraged to spend study periods, recesses, spare lunch time and extracurricula time in the school library, reading. In addition, discussion groups focused on certain reading materials might be developed. History teachers might be swayed to place emphasis on reading ability as well as command of the subject matter. Proficiency in reading might be rewarded more than proficiency in other areas. In fact, in many ways and forms reading might be encouraged and rewarded. In the other group of schools emphasis would be on patriotism. Students might be encouraged to engage in performing patriotic plays or join the band and play patriotic songs. Portions of the recess or lunch period might be devoted to the recitation of the pledge of allegiance. History teachers might concentrate on certain particular events such as

the signing of the Declaration of Independence. In fact, ways might be found to reward more those students who manifest a higher level of patriotism. Now we can see that the diversion of resources in the one case to the development of reading might yield higher reading achievement scores for the children in that group of schools. The students in the other group of schools would probably have lower reading achievement, but a higher level of patriotism. Estimating production functions for the two sets of schools, using reading achievement as the dependent variable would yield different estimates of the production function. In truth, however, the production functions could be identical. That is, the results might be spurious. The difference in estimates might be due entirely to the difference in outputs produced. It could well be that if the second set of principals changes their emphasis from patriotism to the development of reading, we would then observe no difference in the estimated functions for the two sets of schools. We see, then, that differences in the estimated functions for the production of reading achievement might be in fact illusory due entirely to differing allocations of resources among competing attributes of output. In fact, the actual technical relationships between inputs and reading achievement might be identical.

#### Problems in the Specification of the Inputs of the Educational Process

There is no generally accepted theory of the production of education.<sup>31</sup> Ideas as to the inputs in the process differ

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<sup>31</sup> Ibid., pp. 11 and 13.

considerably, even over such fundamental questions as, "Do teachers make a difference?"<sup>32</sup> Although these are constraints within which the researcher must work, at present, the magnitude of the controversy concerning inputs permits significant freedom in the choice of inputs. The nature of the constraints is relatively simple. Without exception, all investigators of the subject agree that individual student characteristics, e.g., I.Q., are inputs into the educational process. The controversy is concerned with the identification of the relevant student characteristics. In addition, the controversy also extends to the identification of relevant home influences, as it is generally agreed that these also are inputs. Further, the controversy extends to the question of whether or not, in general, school, peer, and community influences are inputs. For those who accept some of these influences as inputs into the educational process, the controversy also covers the identification of the specific influences that are relevant.

The fact that the conclusions of any model are implicit in the assumptions, combined with the significant variation allowable in the assumptions regarding inputs, makes the choice of input variables critical.

The following is a partial listing of various inputs proposed by others:<sup>33</sup> quantity of verbal interaction with adults; motivation for achievement in school; richness of the physical environment (e.g., Does the school have a library, a physics lab?

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<sup>32</sup> See Smith, op. cit., pp. 303 and 304.

<sup>33</sup> See Hanushek and Kain, op. cit., pp. 116 and 123; and Bowles, op. cit., pp. 31 and 34.

Does the home have a set of encyclopedias, a typewriter?); home influence on learning (Do the parents care about the student's performance in school? Do they aid the student when possible?); general intelligence; initial endowments (including general intelligence, as well as the student's health); individual and family characteristics (including the family's socioeconomic status, as well as such personality characteristics as whether the student is introverted or extroverted. Is the father or mother domineering, etc?); student body characteristics such as socioeconomic and background factors of other students; school inputs such as physical facilities; curriculum; and personnel.

The definition of inputs depends upon the questions to be answered. This is true in most production function work. In estimating an aggregate production function for a nation in order, say, to make some statement about functional shares of income, it is customary to include only two general types of inputs, labor and capital; while in estimating production functions for specific industries for the purpose, say, of establishing marginal productivities of the various inputs, the inputs are usually divided into more specific categories. Likewise, with education. If the object of a study is the effectiveness of teachers as inputs into the development of mathematical skills, one could exclude the existence or adequacy, of athletic facilities. On the other hand, if one were estimating production functions in order to develop cost curves for a district, one could not exclude costly athletic facilities. Further, it is obvious from the aforementioned list of inputs that many of them are not homogeneous. Since educational

output depends on the quality of the inputs as well as their quantity, there is a problem similar to that of output.<sup>34</sup> If inputs are differentiated on the basis of quality, their number is greatly increased. Since the essence of model-building is to incorporate key variables in such a way as to be able to determine the effects of changes in them on the variables of interest, it is best to use as few variables as possible provided that the conclusions to be drawn are not affected in any essential way and that the loss of insights is minimized.<sup>35</sup> If differences in quality can be measured, we may treat inputs which differ only as to quality as if they differed in quantity, not quality. That is, a certain quality of a given input is taken as the standard; and different quality levels of the same input are considered to be different quantities of the standard input.

The inputs in education, except for the raw material (the student), are not physically embodied in the output. What is important is the services of the inputs. Thus, the inputs are really the services provided and the measure of services rendered. Now this is no different for education than it is for any other good or service. However, with education it is considerably more difficult to separate the input from its services than it is for many other production processes. For example, it is not too difficult to distinguish between the time a welder is physically present and the time he actually spends welding.

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<sup>35</sup> See E. Kane, Economic Statistics and Econometrics, (New York: Harper and Row, 1968), pp. 12 and 18; and H. Liebhafsky, The Nature of Price Theory, (Homewood, Ill.: The Dorsey Press, 1968), pp. 4, 5, and 21.



In the educational process, though, and particularly with respect to teachers, it is hard to separate the input from the services of the input. (Is a teacher providing teaching services only when lecturing?) When a teacher gives in-class reading assignments and then just watches the students study quietly at their desks, are teaching services being provided? There are no simple answers. Therefore, we must distinguish between the presence of a given level of inputs and the actual rendering of services by the inputs. One major aspect of this is that given a certain level of inputs, it is not necessary that the services rendered by the inputs be either zero, or the maximum capable of being rendered; but rather any level of services between zero and the maximum is possible. Thus knowledge of the level of inputs available is not the same as knowledge of the services of inputs used.<sup>36</sup> This presents a problem of the first magnitude of importance, and yet, given the state of the arts, no definitive solution is possible. For that reason, it is here assumed that all inputs render some average level of services and, therefore, that there is a unique correspondence between the level of inputs and the level of services provided by inputs.

In the case of education the inputs may be separated into logical groups which are of interest in our study. These groups are (1) student inputs; (2) home inputs; (3) school inputs; and, (4) community inputs. The separation of inputs into these

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<sup>36</sup> See W. Garner, "Discussion of Educational Production Relationships," Papers and Proceedings of the 83rd Annual Meeting of the AEA, Published May 1971, p. 300.

particular groups is particularly useful in examining questions related to the importance of home and school inputs for academic achievement. This framework allows us to isolate particular groups of variables. Thus, for example, we can explain the effects of school inputs, not individually but collectively. The same is true, of course, for the variables in the other groups.<sup>37</sup> We partition our input vector (I) as follows:

$$E = (i_1, i_2, \dots, i_j)$$

$$H = (i_{j+1}, i_{j+2}, \dots, i_k)$$

$$S = (i_{k+1}, i_{k+2}, \dots, i_m)$$

$$C = (i_{m+1}, i_{m+2}, \dots, i_n)$$

$$I = (E, H, S, C)$$

where:

E = vector of student inputs

H = vector of home inputs

S = vector of school inputs

C = vector of community inputs.

The next problem is the definition of the production unit. The problem here, again, is that there are no set answers. In industrial production, the production unit may be taken to be the individual plant, the firm, or the industry. Similarly, in education, the individual student, the class, the school, or the school

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<sup>37</sup> See e.g., the models in Smith, op. cit.; Hanushek and Kain, op. cit.; and Bowles, op. cit.

district may be taken as the production unit.<sup>38</sup> The decision, in both cases, as to what constitutes the production unit depends upon the questions to be answered. It is, perhaps, most reasonable in the case of education to treat the individual student as the production unit. This is so because unless the student enters school devoid of any of the output of education, we must recognize the existence of producing units outside of the school system. If, then, we also wish to consider some level of the school system (e.g., the class, the school, or the district) as a producing unit, we must deal with multiple types of producing units. That is, there would have to exist some other type of producing unit such as the family or community; and, prior to a student's entering school the educational output which the student had acquired would have to have been produced by such a unit. Since there would be no reason to expect that unit to cease production at that point, additional educational output would have to be considered the joint product of two (or more) production units. Such a formulation is unwieldy and would add more problems than it solved. As examples of the types of problems such a formulation would raise, consider: (1) the necessity of attributing part of the output to the one producing unit, and part to the other; and, (2) the necessity of attributing certain inputs such as peer influence, to a particular producing unit. That is, if a student increased his or her reading achievement by ten percent, how much should be attributed to each

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<sup>38</sup> This question is not dealt with adequately in the sources I have read. Most seem to consider the class as the appropriate unit, although some seem to believe it is the school, and some the district. See e.g., Bowles, op. cit.; Hanushek and Kain, op. cit.; Smith, op. cit.; Armor, op. cit.; and Brown, op. cit.

producing unit; and, if peer influence is an input, is it the same in both units? Is peer influence more or less important in school than out? In the empirical work to follow, problems such as these far outweigh any gains to be had from such a formulation.

For our purposes the most useful way to conceptualize the production unit is to consider the individual student as a single plant firm until entering school, at which time the firm is merged into a multi-plant firm (the school). For the purposes of this paper, then, the school is the unit of production.

Our input vector,  $I$ , is defined to be:

$$I = (E, H, S, C)$$

where:

- $E$  is a vector of student characteristics at the beginning of the process
- $H$  is a vector of home inputs
- $C$  is a vector of community inputs
- $S$  is a vector of school inputs.

### The Model

Let  $E_{ijkT}$  be the vector of educational output for the  $i^{\text{th}}$  student in the  $j^{\text{th}}$  school in the  $k^{\text{th}}$  district cumulative to time  $T$ . Let  $H_{ijkt}$ ,  $S_{ijkt}$ , and  $C_{ijkt}$  be the vectors of home, school, and community inputs of the  $i^{\text{th}}$  student in the  $j^{\text{th}}$  school in the  $k^{\text{th}}$  district in period  $t$ , the period ending at time  $T$ .

Then we write our model of the production function as:

$$1.1 \quad E_{ijkT} = E(E_{ijkT-1}, H_{ijkt}, S_{ijkt}, C_{ijkt}) .$$

That is, the cumulative education of a student to time  $T$  is a function of his or her education cumulative to the beginning  $(T-1)$  of the period  $(t)$  ending at  $T$ , and the home, school, and community inputs to the student's education in period  $t$ .

I assume that school inputs are evenly divided among the students in the school. That is, each student in a particular school receives school inputs equal to the average school inputs for all students in that school. Let  $N_{jkt}$  be the number of students in the  $j^{\text{th}}$  school in the  $k^{\text{th}}$  district in period  $t$ . Then:

$$1.2 \quad S_{ijkt} = \sum_{i=1}^n \frac{S_{ijkt}}{N_{jkt}} \quad \text{for all } i, j, k, t .$$

Whenever we replace an index with a dot it means the sum over the index that was replaced. Thus:

$$1.3 \quad S_{.jkt} = \sum_i S_{ijkt} .$$

Then:

$$1.4 \quad S_{ijkt} = \frac{S_{.jkt}}{N_{jkt}} \quad \text{for all } i, j, k, t .$$

The assumption that school inputs are evenly divided among the students in the school does not seem to be too unreasonable for the following reasons.

First, with respect to non-human inputs, e.g., books, libraries, and laboratories, as well as desks, blackboards, and water fountains, the exposure of students to these resources in any school is approximately equal. Second, teachers' services are the primary school input, at least in terms of cost. Each

child is exposed to roughly the same amount of teachers' classroom time. In addition, teachers' time spent preparing outside of the classroom, e.g., lectures and grading tests, at least potentially benefits all students approximately equally. Of course, the quality of teachers' services vary, and, therefore, even if each of a teacher's students receives the same amount of the teacher's services, students of different teachers may receive different amounts of teachers services. However, we have no specific knowledge on this matter, and, in lieu of a better assumption, we will use the foregoing one.

I assume that community inputs are goods which satisfy social wants and all students partake equally thereof. A necessary and sufficient condition for a social want to exist is that the good (or service) satisfying the want must not be subject to the exclusion principle. The essence of the exclusion principle is that someone who will not voluntarily pay the price of the good may be excluded from the enjoyment of the good. Any good satisfying the criteria that it is not subject to the exclusion principle must necessarily involve joint consumption. Therefore, a necessary but not sufficient condition for a social want is that the good which satisfies it must involve joint consumption. Joint consumption exists when one person's consumption of a good does not lessen the amount of the good available for others' consumption. (Joint consumption does not, however, imply that all consume equally, although they may.) For example, consider three goods; a pair of shoes, a circus, and national defense. A pair of shoes does not involve joint consumption in that when one wears them, no one else

can wear them at the same time, and they are subject to the exclusion principle in that if one refuses to pay for them, one cannot have them. Therefore, shoes do not satisfy social wants in excess of private wants of those willing to pay. A circus, however, does involve joint consumption as the enjoyment of watching the circus by one person does not preclude others from watching and enjoying it at the same time. On the other hand, if one does not pay the price of admission, one can be excluded from the circus. The exclusion principle does apply to circuses. Therefore, a circus does not satisfy a social want in excess of private wants to those willing to pay. Lastly, national defense involves joint consumption in that, if protection is provided for a person against a nuclear attack, the protection extends to others, and there is no less available for one because the other has it. In addition, the exclusion principle does not hold in that there is no way to protect one without protecting others. Therefore, national defense does satisfy a social want in excess of the private wants of those willing to pay.

Since community inputs are things like the intellectual and moral environment, attitudes and values, as well as the availability of libraries, it does not seem unreasonable to consider them as goods that satisfy social wants, even though there is the possibility that certain individual inputs, such as libraries, do not conform exactly to the definition. That is, the exclusion principle does apply to books checked out of the library. The part of the assumption that states that students partake equally of the community inputs is more questionable. However, our assumption is

probably not too far from the truth, and since we lack both actual knowledge about the allocation of community inputs and a better assumption about the allocation, we shall use the aforementioned assumption, i.e., students partake equally of community inputs. Thus,

$$1.5 \quad C_{ijkt} = \frac{C_{\cdot \cdot kt}}{N_{\cdot kt}} .$$

Substituting from equations 1.4 and 1.5 into equation 1.1 we get:

$$1.6 \quad E_{ijkT} = E(E_{ijkT-1}, H_{ijkt}, \frac{S_{\cdot jkt}}{N_{jkt}}, \frac{C_{\cdot \cdot kt}}{N_{\cdot kt}}) .$$

#### 4. The Problem of Simultaneous Equations

Our next concern is with respect to the problem of simultaneous equations models. If our production function equation is but one of a set of simultaneous equations, this will affect the method of estimation. As we shall see, our system of equations is recursive in nature. That is, the value of every input in period  $t$  is either determined exogenously to our system of equations, or is predetermined, i.e., determined prior to the beginning of the  $t^{\text{th}}$  period. We shall now explain why this is so.

First consider the inputs  $E_{ijkT-1}$ . This is the vector of outputs cumulative to the beginning of the  $t^{\text{th}}$  period, i.e., it is the beginning education of the student. Obviously, the values of these inputs are determined prior to the beginning of the  $t^{\text{th}}$  period. Next, consider the vectors of home inputs ( $H_{ijkt}$ ) and community inputs ( $\frac{C_{\cdot \cdot kt}}{N_{\cdot kt}}$ ). It is reasonable, I believe, to assume that home influences ( $H_{ijkt}$ ) are basically the influences



of parents and community influences  $\left(\frac{C_{..kt}}{N_{.kt}}\right)$  are essentially the influences of the adults in the community, (though, to be sure, they may be exerted on children primarily through other children). That is, community influences stemming from adult A may be exerted on child b primarily through child a (the child of adult A). We are not likely to see the effects of a child's education reflected in home and community influences until that child is an adult. We assume, then, that  $H_{ijkt}$  and  $\frac{C_{..kt}}{N_{.kt}}$  are functions not of the education of children, but rather of the education of adults.

We consider now the vector of school inputs  $\frac{S_{.jkt}}{N_{jkt}}$ . At this point it is useful to develop the rest of the equations of the system.

We introduce the following notation. The letter X preceeding a variable will refer to the expectation with respect to the value the variable will take on in the period (or at the time) designated by the appropriate subscript, such expectation being held in the immediately preceeding period (or point in time). For example, if at time T-1, there is an expectation as to the vector of the  $i^{th}$  student's educational output at time T, we write it as  $XE_{ijkt}$ . Throughout we shall assume that all expectations are realized.

I assume that each school district attempts to maximize its expected average output  $(XE_{..kT})$ . However, they are constrained in their attempts to do so primarily by their budget. I assume that the constraints are the same for all districts and that the form of the constraint is a zero net budget. That is, they are constrained to plan to have neither a surplus nor a

deficit in their budget.

$$1.7 \quad XB_{kt} = XQ_{kt}$$

where:  $B_{kt}$  is the total revenue of the  $k^{\text{th}}$  district in period  $t$   
and  $Q_{kt}$  is the total costs (expenses) of the  $k^{\text{th}}$  district in period  $t$ .

With respect to the budget, we assume the school administrators to have no control over their revenues; that is, their expected revenues are a given from their point of view. In fact, public schools receive their revenues from three sources: (1) the federal government; (2) the state government; and, (3) the local government -- quite often the school district itself, i.e., the district board of education. Now, although the administrators can lobby federal and state legislators, and can attempt to persuade local voters to provide more funds, they have no actual control over the amount of funds provided to their district.

On the cost side, administrators do have some control over the price they pay for some inputs, while having no control over others.<sup>39</sup> For example, they have little or no control over the prices of such things as desks, books, laboratory equipment, and similar items. On the other hand, they do have some control over

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<sup>39</sup> The section on input prices, especially with respect to salaries, is based upon innumerable discussions over the past fifteen years with many teachers, principals, and administrators -- particularly with Maurice Geisel, principal of Kohn Junior High, New Orleans, Louisiana, a former President of Orleans Education Association and for many years a member of the Executive Committee of the Louisiana Teachers' Association.

the prices of their major cost item, labor, particularly teachers. Teachers are, and have been for some time, one of the most thoroughly organized groups of people in our society. Teacher organizations take on many forms, including that of a union. It is not unreasonable to state that the teachers' salaries are determined in negotiations between teachers' organizations and school administrators; and it is in the negotiating process that the administrators can effect some control over the price of labor. The strength of most teachers' organizations, the public pressure on administrators to avoid strikes, and the readiness, until quite recently, of local taxpayers to provide funds for their school system almost without question, have placed severe limitations on the abilities of administrators to control teachers' salaries. On the other hand, there are limitations on the teachers' abilities to control their own salaries. Public pressure, the economic environment, the current oversupply of teachers, custom, and tradition are among the forces exerting leverage on the teachers.

Based on the foregoing discussion of input prices we shall treat them as endogenous to our model. That is, we assume that they are not subject to control of the school administration. Since both the suppliers of inputs (primarily teachers) and the demanders of inputs (school administrators) have some control over input prices, we shall treat the market for teachers as a competitive one in which the interaction of supply and demand determines both quantity and price.

Alternatively, we could treat price as determined exogenously by institutional forces. In this case we would have to allow for

the strong possibility that the institutionally determined prices might not be the market clearing prices. This opens up the possibility of existence of either a shortage or a surplus of any particular input. As long as no shortages would exist, i.e., each market would either clear or would be in a state of surplus, the use of this alternative market structure would not affect our problem, in any important way. That is, under these circumstances administrators could always purchase the quantity of inputs that they desired, given the prices; and, therefore, quantities of the various inputs in the production function would be the desired quantities. For our problem this means we would not have to add an additional constraint to the maximization problem. Should shortages exist, less than desired quantities of inputs would be purchased and this would affect our problem, substantially. That is, administrators would desire more inputs than were supplied at the given price. In order to properly account for these effects we would have to add an additional constraint to our problem. This constraint would have to be of the form:

$$1.8 \quad S_{..kt} = \bar{S}_{..kt}$$

where:  $\bar{S}_{..kt}$  is the vector of maximum quantities of inputs supplied at the given prices.

We have chosen to use the former market structure in our model because I believe that in the long run input supply functions are highly responsive to market conditions, i.e., surpluses/shortages, and, therefore, in the long run market forces, supply and demand, are more important than institutional forces. That is,

the latter market structure more closely resembles the short run markets, while the former more closely resembles the long run market structure. In addition, most problems are more easily solved in a longer period of time than a shorter period of time if for no other reason than institutional resistance to change, whether this resistance be due to fear, inertia, lethargy, or apathy.

We now develop the input demand and supply equations. The vector of demand for inputs is written as a function of the vector of prices of the inputs and the number of students in the district.

$$1.9 \quad X_{S \cdot kt} = F(X_{PS_{kt}}, X_{N \cdot kt})$$

where:  $X_{PS_{kt}}$  = vector of input prices.

We assume that input decisions are made in the period prior to the one in which the inputs are to be used. In period  $t-1$ , the demand for inputs for period  $t$  is a function of the number of students expected to be in the system in period  $t$ , and the prices which the inputs are expected to command in period  $t$ . Although the administrators do not control input prices, they do control the quantities of inputs purchased. However, the supply functions of inputs are not under their control and are exogenous to our model. The vector of input supply functions is:

$$1.10 \quad X_{S \cdot kt} = G(X_{PS_{kt}}, X_{PO_{kt}}, \frac{X_{C \cdot kt}}{X_{N \cdot kt}})$$

where:  $X_{PO_{kt}}$  = vector of prices of other goods and services.

That is, input supplies are functions of the input prices, the prices of other goods and services, and community influences. The rationale behind these functions is as follows. The major input, at least in terms of total cost, is teachers. The supply of teachers depends on the salary of teachers, the prices of other goods and services, and the average community influence. We included community influences on the ground that teachers prefer to work in pleasant schools and districts than in those in which, for example, police protection in the halls is required for physical safety. We should also note that the expected prices of other goods and services are exogenous to our model.

Finally, we assume that districts have no control over the number of students therein, but that they can control the allocation of students among schools in the district.

Thus, our complete model is:

$$1.11 \quad XE_{..kt} = E(XE_{ijkT-1}, XH_{ijkt}, \frac{XS_{.jkt}}{\bar{XN}_{jkt}}, \frac{\bar{XC}_{..kt}}{\bar{XN}_{.kt}})$$

( $i = 1, \dots, \bar{XN}_{jkt}$ ,  $j = 1, \dots, \bar{XL}_{kt}$ ) (Production Function)

$$1.7 \quad \bar{XB}_{kt} = XQ_{kt} \quad (\text{Budget Constraint})$$

$$1.12 \quad XB_{kt} = \bar{XB}_{kt} \quad (\text{Total Revenue})$$

$$1.13 \quad XQ_{kt} = XPS_{kt} \cdot XS_{..kt} \quad (\text{Total Costs})$$

$$1.9 \quad XS_{..kt} = F(XPS_{kt}, \bar{XN}_{.kt}) \quad (\text{Input Demand Function})$$

$$1.10 \quad XS_{..kt} = G(XPS_{kt}, \bar{XPO}_{kt}, \frac{\bar{XC}_{..kt}}{\bar{XN}_{.kt}}) \quad (\text{Input Supply Function})$$

$$1.14 \quad XN_{.kt} = \bar{XN}_{.kt} \quad (\text{Student Allocation Constraint})$$

where: A bar over a variable means it is exogenous to our model.

$L_{kt}$  is the number of schools in the  $k^{th}$  district in period  $t$

A dot ( $\cdot$ ) between two vectors refers to the inner (or dot) product .

At this point we make a short diversion to reformulate the input supply function. This reformulation will be useful when we deal with the solution of the input markets.

$$1.15 \quad XPS_{..kt} = F(XS_{..kt}, XPO_{kt}, \frac{XC_{..kt}}{XN_{..kt}}) .$$

Substituting into the cost function (1.13) we get:

$$1.16 \quad XQ_{kt} = XS_{..kt} \cdot F(XS_{..kt}, XPO_{kt}, \frac{XC_{..kt}}{XN_{..kt}})$$

which can be written as:

$$1.17 \quad XQ_{kt} = Q(XS_{..kt}, XPO_{kt}, \frac{XC_{..kt}}{XN_{..kt}}) .$$

Equation 1.17 means that the cost function is a function of the input supply functions, the prices of other goods and services, and community influences. We return now to the maximization problem.

As stated, the administrators desire to maximize the expected output subject to two constraints: the budget and the number of students in the district. They maximize with respect to the variables they can control. These are: (1) the number of students in each school save one. Since  $\sum_{j=1}^L N_{jkt} = N_{..kt}$ , there are only  $L-1$  independent variables, i.e., once given the number of students in  $L-1$  schools, the number of students in the  $L^{th}$

school is determined; (2) the quantity of each purchased input each school receives, save for one input in one school. The logic is similar to that for the allocation of students. Suppose that there are  $q$  different inputs. Once we know the quantity of  $q-1$  inputs going to each of the  $L$  schools and the quantity of the  $q^{\text{th}}$  input going to  $L-1$  of the schools the quantity of the  $q^{\text{th}}$  input going to the  $L^{\text{th}}$  school is determinate. This is so because the purchase of those inputs from a fixed budget will leave a fixed sum available for the purchase of the remaining input for the last school. Since input prices are taken as given, that quantity is determinate.

The maximization problem may be written as:

Maximize:

$$1.18 \quad V = XE_{..kt} \left( \frac{XS_{jkt}}{XN_{jkt}} \right) (j = 1, \dots, XL_{kt}) \\ - \lambda_1 (Q(XS_{..kt}) - XB_{kt}) - \lambda_2 \left( \sum_{j=1}^{XL_{kt}} XN_{jkt} - XN_{..kt} \right) .$$

Note that with respect to the maximization problem the variable corresponding to entering achievement, home, and community inputs, input prices, and other prices are not explicit in our functions. This is because they are not decision variables and are, therefore, treated as parameters. Later, when dealing with the input markets, the input prices will be treated as decision variables.

The first order conditions for a maximum are:<sup>40</sup>

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<sup>40</sup> The Lagrange multipliers are interpreted as:

$$\lambda_1 = \frac{\partial XE_{..kt}}{\partial XQ_{kt}(XT_{..kt})} .$$

That is,  $\lambda_1$  is the total change in a district's output due to a



$$1.19 \quad \frac{\partial XE_{..kt}}{\partial XS_{.jkt}} = \lambda_1 \frac{\partial XQ_{kt}}{\partial XS_{.jkt}} \quad (j = 1, \dots, XL_{kt})$$

$$1.20 \quad \frac{\partial XE_{..kt}}{\partial XN_{jkt}} = \lambda_2 \quad (j = 1, \dots, XL_{kt})$$

$$1.21 \quad XQ_{kt} = \bar{XB}_{kt}$$

$$1.22 \quad \sum_{j=1}^{L_{kt}} XN_{jkt} = \bar{XN}_{.kt}.$$

Since  $S_{.jkt}$  is a vector of inputs we can write it as:  $S_{.jkt} = (S^1_{.jkt}, S^2_{.jkt}, \dots, S^q_{.jkt})$  using superscripts to refer to the different elements of the vector. Then the first set of first order conditions (1.19) can be rewritten as:

$$1.23 \quad \frac{\partial XE_{..kt}}{\partial XS^h_{.jkt}} = \lambda_1 \frac{\partial XQ_{kt}}{\partial XS^h_{.jkt}} \quad (h = 1 \dots q) \\ (j = 1 \dots XL_{kt})$$

There are thus  $qL$  equations in the first set, but only  $qL-1$  of them are linearly independent.

$$1.24 \quad \frac{\frac{\partial XE_{..kt}}{\partial XS^1_{.kt}}}{\frac{\partial XQ_{kt}}{\partial XS^1_{.1kt}}} = \frac{\frac{\partial XE_{..kt}}{\partial XS^h_{.kt}}}{\frac{\partial XQ_{kt}}{\partial XS^h_{.jkt}}}$$

for  $(j = 1, h = 2, \dots, q \text{ and } j = 2, \dots, L, h = 1, \dots, q).$

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change in its total costs (or revenue).

$$\lambda_2 = \frac{\frac{\partial XE_{..kt}}{\partial L}}{\frac{\partial \sum_{j=1}^{L_{kt}} XN_{jkt}}{\partial L}}.$$

That is,  $\lambda_2$  is the total change in a district's output due to a change in the number of students in the district.

Similarly in the second set of first order conditions there are  $L$  equations, but only  $L-1$  are linearly independent.

$$1.25 \quad \frac{\partial XE \dots kT}{\partial XN_{1kt}} = \frac{\partial XE \dots kT}{\partial XN_{jkt}} \quad (j = 1, \dots, L) .$$

We have, then, a set of  $(qL-1) + (L-1) + 2$  equations in the  $(qL-1)$  unknown inputs +  $(L-1)$  unknown number of students in school +  $(2)$  unknown  $\lambda$ 's (Lagrange multipliers). Recall that:

$$1.26 \quad XE \dots kT = \sum_{j=1}^L XE \dots jkT .$$

Then:

$$1.27 \quad \frac{\partial XE \dots kT}{\partial XS_h \dots jkt} = \sum_{j=1}^L \frac{\partial XE \dots jkT}{\partial XS_h \dots jkt} \quad (j = 1, \dots, L; h = 1, \dots, q) .$$

That is, the marginal productivity of the  $h^{th}$  input in the  $j^{th}$  school is equal to the sum of the changes in output in every school in the district due to a change in the  $h^{th}$  input in the  $j^{th}$  school. But, the  $h^{th}$  input in the  $j^{th}$  school affects the output of the  $j^{th}$  school only. That is:

$$1.28 \quad \frac{\partial XE \dots jkT}{\partial XS_h \dots akt} \equiv 0 \quad (j \neq a; a = 1, \dots, L; h = 1, \dots, q) .$$

Therefore:

$$1.29 \quad \frac{\partial XE \dots kT}{\partial XS_h \dots jkt} = \frac{\partial XE \dots jkT}{\partial XS_h \dots jkt} \quad (h = 1, \dots, q; j = 1, \dots, L) .$$

That is, the change in output in the district due to a change in the  $h^{th}$  input in the  $j^{th}$  school is equal to the change in output in the  $j^{th}$  school due to a change in the  $h^{th}$  input in that school.

Thus, the first set of first order conditions,

$$1.30 \quad \frac{\frac{\partial XE}{\partial X S^1} \cdot 1kT}{\frac{\partial X Q}{\partial X S^1} \cdot 1kt} = \frac{\frac{\partial XE}{\partial X S^h} \cdot jkT}{\frac{\partial X Q}{\partial X S^h} \cdot jkt} \quad (j = 1, h = 2, \dots, q \text{ and } j = 2, \dots, L, h = 1, \dots, q)$$

may be interpreted as marginal cost conditions.

The marginal cost of producing output by changing an output in a school should be equal to the marginal cost of producing output by changing any other output in the same school, and by changing any output in any other school. In addition, since the price of the  $p^{th}$  output is the same for every school, these conditions may be interpreted as marginal productivity conditions.

$$1.31 \quad \frac{\frac{\partial XE}{\partial X S^h} \cdot 1kT}{\frac{\partial Q}{\partial X S^1} \cdot 1kt} = \frac{\frac{\partial XE}{\partial X S^h} \cdot jkT}{\frac{\partial Q}{\partial X S^1} \cdot jkt} \quad (j = 1, \dots, L; h = 1, \dots, q).$$

That is, the marginal productivity of an input in one school must be equal to the marginal productivity of the same input in any other school, and this for any input. Similarly:

$$1.32 \quad \frac{\frac{\partial XE}{\partial X N} \cdot jkT}{\frac{\partial XE}{\partial X N} \cdot jkt} = \frac{\frac{\partial XE}{\partial X N} \cdot jkT}{\frac{\partial XE}{\partial X N} \cdot jkt} \quad (j = 1, \dots, L).$$

That is, the change in the district's output due to a change in the number of students in the  $j^{th}$  school is equal to the change in output in the  $j^{th}$  school due to a change in the number of its students.

The second set of first order conditions is:

$$1.33 \quad \frac{\partial XE_{.1kt}}{\partial XN_{1kt}} = \frac{\partial XE_{.jkt}}{\partial XN_{jkt}} \quad (j = 2, \dots, L).$$

They are marginal productivity conditions. The change in output due to a change in the number of students in one school must be equal to the change in output due to a change in the number of students in any other school.

The final two first order conditions are:

$$1.21 \quad XQ_{kt} = XB_{kt}, \quad \text{and}$$

$$1.22 \quad \sum_{k=1}^L XN_{jkt} = XN_{.kt}.$$

The first is the zero net budget constraint. Total costs must be equal to total revenues. The second constraint, i.e., the sum of the number of students in each school must be equal to the number of students in the district.

The maximizing conditions are  $qL + L$  equations in  $qL + L$  unknowns ( $qL - 1$   $S_{.jkt}^h$ 's,  $L - 1$   $N_{jkt}$ 's, and  $2$   $\lambda$ 's). This system of simultaneous equations may be solved for the unknowns. These solutions are:

$$1.34 \quad \lambda_1 = \lambda_1(XPS_{kt}, XPO_{kt}, XE_{ijkT-1}, XH_{ijkt}, \frac{XC_{.kt}}{XN_{.kt}})$$

$$1.35 \quad \lambda_2 = \lambda_2(XPS_{kt}, XPO_{kt}, XE_{ijkT-1}, XH_{ijkt}, \frac{XC_{.kt}}{XN_{.kt}})$$

$$1.36 \quad XS_{.jkt}^h = S_j^h(XPS_{kt}, XPO_{kt}, XE_{ijkT-1}, XH_{ijkt}, \frac{XC_{.kt}}{XN_{.kt}})$$

$$(j = 1, \dots, L-1, h = 1, \dots, q; \text{ and } j = L, h = 1, \dots, q-1)$$

$$1.37 \quad XN_{jkt} = N_j(XPS_{kt}, XPO_{kt}, XE_{ijkT-1}, XH_{ijkt}, \frac{XC_{.kt}}{XN_{.kt}})$$

$$(j = 1, \dots, L-1).$$

We now wish to consider the solution to the input markets. The  $qL-1$  functions (1.36) are demand functions for inputs. These functions may be used in conjunction with the budget constraint (1.21) to determine the  $qL^{th}$  input demand functions, one for each input in each school. By properly summing the demand functions for any input, over the schools, we can derive the district demand function for any input.<sup>41</sup> There are  $q$  of these district demand functions:

$$1.38 \quad XS_{..kt}^h = X^h(XS_{.jkt}^h, j = 1, \dots, L) \quad (h = 1, \dots, q) .$$

The vector of input supply functions is:

$$1.9 \quad XS_{..kt} = G(XPS_{kt}, XPO_{kt}, \frac{XC_{..kt}}{XN_{..kt}}) .$$

This vector may be broken up into  $q$  individual supply functions, one for each input:

$$1.39 \quad XS_{..kt}^h = G^h(XPS_{kt}, XPO_{kt}, \frac{XC_{..kt}}{XN_{..kt}}) \quad (h = 1, \dots, q)$$

The equation (1.38) and (1.39) for a set of  $2q$  simultaneous equations in  $2q$  unknowns (the  $q$  input prices and the  $q$  input quantities). The solution to this set of equations is:

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<sup>41</sup> The proper method of summing demand functions is to add the various quantities demanded at some price to get the total quantity demanded at that price. This process is repeated for every price in the relevant price range. The resulting total quantities can be paired with their respective prices to form the new price-quantity relationship. This process is called horizontal summation by economists. The reason is that economists normally draw demand functions on graphs whose vertical axis is the price axis and horizontal axis is the quantity axis, and this process amounts to horizontal summation of the curves on such a graph.

$$1.40 \quad X_{..kt}^h = S(XPO_{kt}, XE_{ijkT-1} \quad (i = 1, \dots, N_{jkt} \quad \text{and} \quad j = 1, \dots, L_{kt}), \\ XH_{ijkt} \quad (i = 1, \dots, N_{jkt} \quad \text{and} \quad j = 1, \dots, L_{kt}), \frac{XC_{..kt}}{XN_{..kt}})$$

the equilibrium quantity of inputs; and,

$$1.41 \quad XPS_{kt} = P(XPO_{kt}, XE_{ijkT-1} \quad (i = 1, \dots, N_{jkt} \quad \text{and} \quad j = 1, \dots, L_{kt}), \\ XH_{ijkt} \quad (i = 1, \dots, N_{jkt} \quad \text{and} \quad j = 1, \dots, L_{kt}), \frac{XC_{..kt}}{XN_{..kt}})$$

which is the vector of equilibrium input prices. Substituting the equilibrium prices  $XPS_{kt}$ , into the individual demand function (1.36) enables us to determine the equilibrium vectors of inputs for each input, for each school, the  $X_{.jkt}^h$ 's. From these we can form the equilibrium vector of inputs for each school,  $XS_{.jkt}$ . These vectors are:

$$XS_{.jkt} = X_j(XPO_{kt}, XE_{ijkT-1} \quad (i = 1, \dots, N_{jkt} \quad \text{and} \quad j = 1, \dots, L_{kt}), \\ XH_{ijkt} \quad (i = 1, \dots, N_{jkt} \quad \text{and} \quad j = 1, \dots, L_{kt}), \frac{XC_{..kt}}{XN_{..kt}})$$

We note that the inputs  $(S_{.jkt}/N_{jkt})$  are a function only of exogenous and predetermined variables, and are themselves predetermined variables in our model. So far we have determined that  $(E_{ijkT-1})$  and  $S_{.jkt}/N_{jkt}$  are predetermined variables and  $(H_{ijkt})$  and  $(C_{..kt}/N_{..kt})$  are exogenous variables.

Returning now to our original model of the production function:

$$1.6 \quad E_{ijkT} = E(E_{ijkT-1}, H_{ijkt}, \frac{S_{.jkt}}{N_{jkt}}, \frac{C_{..kt}}{N_{..kt}})$$

Output ( $E_{ijkt}$ ) is a function only of predetermined ( $E_{ijkt-1}$ ,  $S_{.jkt}/N_{jkt}$ ) and exogenous ( $H_{ijkt}$ ,  $C_{..kt}/N_{.kt}$ ) variables; and, as stated previously, our system of equations is recursive. This is significant in that it means that the method of ordinary least squares is not precluded as a technique of estimation.

## CHAPTER II

### THE ECONOMETRIC MODEL, PROXY VARIABLES, AND DATA

#### 1. The Econometric Model and Estimation Techniques

We now consider the econometric specification of the model. In order to estimate it we must have an explicit form for the function. In addition, as we use school rather than individual data, we must aggregate over individual students in the school. We assume a linear form for the model. We do so for two reasons: (1) over small ranges of the dependent variable, any function may be approximated by a linear function; and, (2) the coefficients of the independent variables in the linear form of the production function are of particular interest to economists as they are the marginal productivities of the inputs. The primary drawback of this form is that it implies that the marginal productivity of each input is constant. That is, an increase (or decrease) of one unit of an input leads to a change in the level of output which is independent of the quantity of that or any other input, already in use, in effect denying the theory of diminishing marginal productivity. In spite of this drawback, it is not unreasonable to use this form as an approximation. Thus:

$$\begin{aligned} 2.1 \quad E_{ijkT} = & \alpha_{ij0} + \alpha_{ij1} E_{ijkT-1} + \alpha_{ij2} H_{ijkt} \\ & + \alpha_{ij3} \frac{S_{.jkt}}{N_{jkt}} + \alpha_{ij4} \frac{C_{.kt}}{N_{jkt}} + \epsilon_{ijkt} . \end{aligned}$$



We assume the production function is the same for all students.

That is, we assume:

$$2.2 \quad \alpha_{ijk} = \alpha_k \quad \text{for all } i, j, k$$

Summing over  $i$ , we get:

$$2.3 \quad E_{.jkt} = \alpha_0 N_{jkt} + \alpha_1 E_{.jkt-1} + \alpha_2 H_{.jkt} \\ + \alpha_3 S_{.jkt} + \alpha_4 N_{jkt} \frac{C_{.kt}}{N_{.kt}} + \epsilon_{.jkt} .$$

Because of the form our data is in, it is more convenient to use school averages rather than totals. Averaging, we obtain:

$$2.4 \quad \frac{E_{.jkt}}{N_{jkt}} = \alpha_0 + \alpha_1 \frac{E_{.jkt-1}}{N_{jkt}} + \alpha_2 \frac{H_{.jkt}}{N_{jkt}} \\ + \alpha_3 \frac{S_{.jkt}}{N_{jkt}} + \alpha_4 \frac{C_{.kt}}{N_{.kt}} + \frac{\epsilon_{.jkt}}{N_{jkt}} .$$

We assume that the random disturbances satisfy the classical assumptions for all individuals.

$$2.5 \quad \epsilon_{ijkt} \sim N(0, \sigma^2) \quad \text{and} \quad E(\epsilon_{ijkt} \epsilon_{wxyz}) = 0 \quad (i \neq w \text{ or } j \neq x \\ k \neq y \text{ or } t \neq z) .$$

Then:

$$2.6 \quad \frac{\epsilon_{.jkt}}{N_{jkt}} \sim N(0, \frac{\sigma^2}{N_{jkt}}) \quad \text{and} \quad E(\frac{\epsilon_{ijkt}}{N_{jkt}} \frac{\epsilon_{wxyz}}{N_{xyz}}) = 0 \\ (i \neq w \text{ or } j \neq x \text{ or } k \neq y \text{ or } t \neq z)$$

and

$$E(\frac{\epsilon_{.jkt}}{N_{jkt}} \frac{\epsilon_{.xyz}}{N_{xyz}}) = 0 \quad (j \neq x \text{ or } k \neq y \text{ or } t \neq z) .$$

We estimate the model as given in equation 2.4, weighting each observation (school) by  $(N_{jkt})^{\frac{1}{2}}$  in order to remove the heteroskedasticity arising from the use of grouped data.

In addition we estimate a different form of the model which is derived by successive substitution into equation 2.1 for the lagged dependent variable. This yields:

$$\begin{aligned}
 2.7 \quad E_{ijkT} = & \alpha_0 \sum_{h=0}^{t-1} \alpha_1^h + \alpha_1^t E_{ijk0} + \alpha_2 \sum_{h=1}^t \alpha_1^{t-h} H_{ijkh} \\
 & + \alpha_3 \sum_{h=V_{ijk}}^t \alpha_1^{t-h} \frac{S_{.jkh}}{N_{jkh}} + \alpha_4 \sum_{h=1}^t \alpha_1^{t-h} \frac{C_{..kh}}{N_{.kh}} \\
 & + \sum_{h=1}^t \epsilon_{ijkh} .
 \end{aligned}$$

Where  $V_{ijk}$  is the time at which a particular child enters school.

We assume:

$$2.8 \quad V_{ijk} = V \quad \text{for all } i, j, k .$$

Further, we assume that home inputs, average community inputs, average school inputs, and the number of students in a school change, if at all, very slowly over time. Then:

$$\begin{aligned}
 2.9 \quad H_{ijkt} & \approx H_{ijkt-1} \quad \text{and} \quad T_{.jkt} \approx T_{.jkt-1} \quad \text{and} \\
 C_{..kt} & \approx C_{..kt-1} \quad \text{and} \quad N_{jkt} \approx N_{jkt-1} \quad \text{for all } t .
 \end{aligned}$$

Introducing this assumption into the model yields:

$$\begin{aligned}
 2.10 \quad E_{ijkT} = & \alpha_0 \sum_{h=0}^{t-1} \alpha_1^h + \alpha_1^t E_{ijk0} + (\alpha_2 \sum_{h=1}^t \alpha_1^{t-h}) H_{ijkt} \\
 & + (\alpha_3 \sum_{h=v}^t \alpha_1^{t-h}) \frac{S_{.jkt}}{N_{jkt}} + (\alpha_4 \sum_{h=1}^t \alpha_1^{t-h}) \frac{C_{..kt}}{N_{.kt}} + t \epsilon_{ijkt} .
 \end{aligned}$$

Summing to the school level (over  $i$ ) and averaging, we get:

$$\begin{aligned}
 2.11 \quad \frac{E_{.jkt}}{N_{jkt}} &= \alpha_0 \sum_{h=0}^{t-1} \alpha_1^h + \alpha_1^t \frac{E_{.jk0}}{N_{jkt}} + (\alpha_2 \sum_{h=1}^t \alpha_1^{t-h}) \frac{H_{.jkt}}{N_{jkt}} \\
 &+ (\alpha_3 \sum_{h=v}^t \alpha_1^{t-h}) \frac{S_{.jkt}}{N_{jkt}} + (\alpha_4 \sum_{h=1}^t \alpha_1^{t-h}) \frac{C_{.kt}}{N_{.kt}} \\
 &+ \frac{t}{N_{jkt}} \epsilon_{.jkt} .
 \end{aligned}$$

Again, assuming that the random disturbances satisfy the classical conditions for individuals, we weight each observation by  $(N_{jkt})^{1/2}/t$  in order to remove the heteroskedasticity arising from the use of grouped data.

$E_{ijk0}$  is the vector of educational output for an individual at time 0, i.e., at birth. In fact then, this is the vector of initial endowments of the student. We assume:

$$2.12 \quad E_{ijk0} \sim N(m, \sigma^2) \quad \text{for all } j, \text{ for all } k.$$

That is, we assume that the vector of initial endowments for any child is a randomly distributed variable with mean  $m$  and variance  $\sigma^2$ . The average initial endowment for the children in any school, then, is  $m$ . That is:

$$2.13 \quad \sum_{i=1}^{N_{jkt}} \frac{E_{ijk0}}{N_{jkt}} = E_{.jk0} = m .$$

Substituting from (2.13) into (2.11) we get:

$$\begin{aligned}
 2.14 \quad \frac{E_{.jkt}}{N_{jkt}} &= \beta_1^* + \beta_2 \frac{H_{.jkt}}{N_{jkt}} + \beta_3 \frac{S_{.jkt}}{N_{jkt}} \\
 &+ \beta_4 \frac{C_{.kt}}{N_{.kt}} + \frac{t}{N_{jkt}} \epsilon_{.jkt}
 \end{aligned}$$

$$\text{where: } \beta_1^* = \alpha_0 \sum_{h=0}^{t-1} \alpha_1^h + m\alpha_1^t$$

$$\beta_2 = \alpha_2 \sum_{h=0}^{t-1} \alpha_1^h$$

$$\beta_3 = \alpha_3 \sum_{h=0}^{t-v} \alpha_1^h$$

$$\beta_4 = \alpha_4 \sum_{h=0}^{t-1} \alpha_1^h$$

$$\text{and } \frac{t}{N_{jkt}} \epsilon_{.jkt} \sim N(0, \frac{t}{N_{jkt}} \sigma^2) .$$

We note that assumption 2.12 transforms the term corresponding to the lagged dependent  $(\alpha_1^t \frac{E_{.jk0}}{N_{jkt}})$  variable into a constant  $(\alpha_1^t m)$ . Combining this term with the structural constant  $(\alpha_0 \sum_{h=0}^{t-1} \alpha_1^h)$  yields a new constant term  $(\alpha_0 \sum_{h=0}^{t-1} \alpha_1^h + \alpha_1^t m)$ , and eliminates the lagged dependent variable from the equation to be estimated. This procedure will, of course, give an upward bias to our estimate of the structural constant. It will not, however, have any effect upon the estimates of any of the other parameters.

## 2. The Variables

The proxies we shall use are the following:

1) For average output  $(E_{.jkT}/N_{jkt}$  and  $E_{.jkT-1}/N_{jkt})$  we shall use averages of a measure of composite achievement in reading, mechanics of written english, and mathematics (COMP T and COMP T-1) for 7th grade students in each school.

2) For average home influence  $(H_{.jkt}/N_{jkt})$  we shall use several variables: (1) average socioeconomic status of the students in the 7th grade of the school (SES-S); (2) average attitude of students in the 7th grade of the school towards the

importance of school achievement (M SCH-S); (3) average self-perception of students in the 7th grade of the school (SELF-S); and (4) average attitude of students in the 7th grade in the school towards school (SCH-S). The function of the variables we shall use is:

$$2.15 \quad H_{.jkt}/N_{jkt} = \gamma_1 \text{SES-S} + \gamma_2 \text{M SCH-S} + \gamma_3 \text{SELF-S} + \gamma_4 \text{SCH-S} .$$

3) For average school influence ( $S_{.jkt}/N_{jkt}$ ) we shall use several variables: (1) the number of students in the school (PUP-S); (2) the percentage of white students in the school (RACE-S); (3) the percentage of teachers in the school having five or more years experience (EXP); (4) the percentage of teachers in the school having a master's degree (MA); and (5) the teacher to pupil ratio for the school (T/P). The function of the variables we shall use is:

$$2.16 \quad S_{.jkt}/N_{jkt} = \gamma_5 \text{PUP-S} + \gamma_6 \text{RACE-S} + \gamma_7 \text{EXP} + \gamma_8 \text{MA} + \gamma_9 \text{T/P} .$$

4) For community influence ( $C_{..kt}/N_{.kt}$ ) we shall use several variables: (1) the number of students in the school district (PUP-D); (2) the percentage of white students in the district (RACE-D); (3) the average socioeconomic status of students in the 7th grade in the district (SES-D); (4) the average attitude of students in the 7th grade in the district towards the importance of school achievement (M SCH-D); (5) the average self-perception of students in the 7th grade in the district (SELF-D); (6) the average attitude of students in the 7th grade in the district towards school (SCH-D); and (7) a set of three binary variables

representing geographical regions (DETROIT, Southern Lower Peninsula (SOL), Northern Lower Peninsula (NOL)). We omit a variable for the fourth region, the upper peninsula.<sup>1</sup> The geographical regions correspond roughly to the Detroit metropolitan area (DETROIT), the remainder of the southern portion of the lower peninsula of Michigan (SOL), the northern portion of the lower peninsula (NOL), and the Upper Peninsula (the one for which the dummy variable was omitted). For an exact specification of the regions see footnote 3. The function of the variables we shall use is:

$$2.17 \quad C_{..kt} = \gamma_{10}^{PUP-D} + \gamma_{11}^{RACE-D} + \gamma_{12}^{SES-D} + \gamma_{13}^{M SCH-D} \\ + \gamma_{14}^{SELF-D} + \gamma_{15}^{SCH-D} + \gamma_{16}^{DETROIT} + \gamma_{17}^{SOL} + \gamma_{18}^{NOL}.$$

The equations to be estimated then are:

$$2.18 \quad COMP T = \alpha_0 + (\alpha_1^{COMP T-1}) + (\alpha_2^{SES-S} + \alpha_3^{M SCH-S} \\ + \alpha_4^{SELF-S} + \alpha_5^{SCH-S}) + (\alpha_6^{PUP-S} + \alpha_7^{RACE-S} \\ + \alpha_8^{EXP} + \alpha_9^{MA} + \alpha_{10}^{T/P}) + (\alpha_{11}^{PUP-D} + \alpha_{12}^{RACE-D} \\ + \alpha_{13}^{SES-D} + \alpha_{14}^{M SCH-D} + \alpha_{15}^{SELF-D} + \alpha_{16}^{SCH-D} \\ + \alpha_{17}^{DETROIT} + \alpha_{18}^{SOL} + \alpha_{19}^{NOL}) + \epsilon$$

and

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<sup>1</sup> Inclusion of a dummy variable for the fourth region would lead to singularity of the data matrix as the data vectors for the four dummy variables and the constant term would not be linearly independent. On this point see Jan Kmenta, Elements of Econometrics, (New York: The Macmillan Company, 1971), p. 413.

$$\begin{aligned}
2.19 \quad \text{COMP } T = & \beta_1^* + (\beta_2^{\text{SES-S}} + \beta_3^{\text{M SCH-S}} + \beta_4^{\text{SELF-S}} + \beta_5^{\text{SCH-S}}) \\
& + (\beta_6^{\text{PUP-S}} + \beta_7^{\text{RACE-S}} + \beta_8^{\text{EXP}} + \beta_9^{\text{MA}} + \beta_{10}^{\text{T/P}}) \\
& + (\beta_{11}^{\text{PUP-D}} + \beta_{12}^{\text{RACE-D}} + \beta_{13}^{\text{SES-D}} + \beta_{14}^{\text{M SCH-D}} \\
& + \beta_{15}^{\text{SELF-D}} + \beta_{16}^{\text{SCH-D}} + \beta_{17}^{\text{DETROIT}} + \beta_{18}^{\text{SOL}} \\
& + \beta_{19}^{\text{NOL}}) + \mu .
\end{aligned}$$

It is useful at this point to discuss again the relationship between the two models. First let us think of the educational process as a long run process. Let us assume that cumulative achievement to time  $T$  is a linear function of students' initial endowments, and home, school, and community inputs from birth (time 0) to time  $T$ . Using the notation developed previously we can write this as:

$$\begin{aligned}
2.20 \quad E_{ijkT} = & \beta_0 + \beta_1^E E_{ijk0} + \sum_{h=1}^t \beta_{2h}^H H_{ijkh} + \sum_{i=v}^t \beta_{3h}^S S_{ijkh} \\
& + \sum_{i=1}^t \beta_{4h}^C C_{ijkh} .
\end{aligned}$$

Alternatively, let us think of the educational process as a short run process. Let us assume the cumulative achievement to time  $T$  is a linear function of students' achievement at time  $T-1$ , and the home, school, and community inputs between time  $T-1$  and time  $T$ , i.e., in period  $t$ . We can write this as:

$$2.21 \quad E_{ijkT} = \alpha_0 + \alpha_1^E E_{ijkT-1} + \alpha_2^H H_{ijkt} + \alpha_3^S S_{ijkt} + \alpha_4^C C_{ijkt} .$$

Suppose that either view of the educational process is a valid one. Then, these two models are but two different representations of the same process. That this is so can be seen by making

successive substitutions into the short run model (2.21) for the lagged dependent variable, achievement. This results in the following model.

$$2.22 \quad E_{ijkT} = \alpha_0 \sum_{h=0}^{t-1} \alpha_1^h + \alpha_1^t E_{ijk0} + (\alpha_2 \sum_{h=1}^t \alpha_1^{t-h}) H_{ijkh} \\ + (\alpha_3 \sum_{h=v}^t \alpha_1^{t-h}) S_{ijkh} + (\alpha_4 \sum_{h=1}^t \alpha_1^{t-h}) C_{ijkh} .$$

Now equation (2.22) is the same as (2.20) except for the designation of the parameters. That is the following relationships hold among the parameters:

$$2.23 \quad \beta_0 = \alpha_0 \sum_{h=0}^{t-1} \alpha_1^h \quad \alpha_1 = \frac{\beta_0}{\sum_{h=0}^{t-1} \beta_1^{h/t}} \\ \beta_1 = \alpha_1^t \quad \alpha_1 = \beta_1^{1/t} \\ \beta_{2h} = \alpha_2 \alpha_1^{t-h} \quad (h = 1, \dots, t) \quad \alpha_2 = \frac{\beta_{2h}}{\beta_1^{1-(h/t)}} \quad (h = 1, \dots, t) \\ \beta_{3h} = \alpha_3 \alpha_1^{t-h} \quad (h = v, \dots, t) \quad \alpha_3 = \frac{\beta_{3h}}{\beta_1^{1-(h/t)}} \quad (h = v, \dots, t) \\ \beta_{4h} = \alpha_4 \alpha_1^{t-h} \quad (h = 1, \dots, t) \quad \alpha_4 = \frac{\beta_{4h}}{\beta_1^{1-(h/t)}} \quad (h = 1, \dots, t) .$$

We assumed on page 54 that home, school, and community inputs change, if at all, only slowly over time. That is:

$$2.24 \quad H_{ijkT} = H_{ijkh} \quad (h = 1, \dots, t-1) \\ S_{ijkT} = S_{ijkh} \quad (h = v, \dots, t-1) \\ C_{ijkT} = C_{ijkh} \quad (h = 1, \dots, t-1) .$$



Then, we can rewrite the long run model as:

$$2.25 \quad E_{ijkT} = \beta_0 + \beta_1 E_{ijk0} + \left( \sum_{h=1}^t \beta_{2h} \right) H_{ijkT} \\ + \left( \sum_{h=v}^t \beta_{3h} \right) S_{ijkT} + \left( \sum_{h=1}^t \beta_{4h} \right) C_{ijkT}.$$

Summing both models to the school (over i) and taking averages yields for the short and long runs, respectively:

$$2.26 \quad \frac{E_{.jkt}}{N_{jkt}} = \alpha_0 + \alpha_1 E_{.jkt-1} + \alpha_2 \frac{H_{.jkt}}{N_{jkt}} + \alpha_3 \frac{S_{.jkt}}{N_{jkt}} + \alpha_4 \frac{C_{.kt}}{N_{.kt}}.$$

(This corresponds to the first equation to be estimates (2.18)).

$$2.27 \quad \frac{E_{.jkt}}{N_{jkt}} = \beta_0 + \beta_1 \frac{E_{.jk0}}{N_{jkt}} + \beta_2 \frac{H_{.jkt}}{N_{jkt}} + \beta_3 \frac{S_{.jkt}}{N_{jkt}} + \beta_4 \frac{C_{.kt}}{N_{.kt}}$$

where:

$$\beta_2 = \sum_{h=1}^t \beta_{2h}$$

$$\beta_3 = \sum_{h=v}^t \beta_{3h}$$

$$\beta_4 = \sum_{h=1}^t \beta_{4h}.$$

Finally, we assumed, on page 55, that the initial endowments ( $E_{ijk0}$ ) are normally distributed with mean  $m$ . Substituting  $m$  into (2.27) for  $E_{.jk0}/N_{jkt}$  yields:

$$2.28 \quad \frac{E_{.jkt}}{N_{jkt}} = \beta_0 + \beta_1 m + \beta_2 \frac{H_{.jkt}}{N_{jkt}} + \beta_3 \frac{S_{.jkt}}{N_{jkt}} + \beta_4 \frac{C_{.kt}}{N_{.kt}}.$$

This can be rewritten as:

$$2.29 \quad \frac{E_{.jkt}}{N_{jkt}} = \beta_1^* + \beta_2 \frac{H_{.jkt}}{N_{jkt}} + \beta_3 \frac{S_{.jkt}}{N_{jkt}} + \beta_4 \frac{C_{.kt}}{N_{.kt}}$$

where:  $\beta_1^* = \beta_0 + \beta_1^m$ .

This model corresponds to the second equation (2.19) to be estimated. The second equation to be estimated is but a different form of the first equation to be estimated (2.18), given our assumptions that home, school, and community influences change very slowly over time, and that initial endowments are normally distributed. This is so because both equations to be estimated (2.18 and 2.19) may be derived from the same equation (2.21). The two equations to be estimated, then, represent a short and a long run view of the educational process.

We test the relationship between the two models. This is done in the following manner. First, we estimate the coefficients for the short run model. Next, we use these estimates and the relationships between the parameters of the two models (2.23) to predict the values of the coefficients of the long run model. Then we estimate the coefficients of the long run model. Finally, we use the t test to test the null hypotheses that the predicted values are equal to the estimated values.

One reminder before we go on to a consideration of the data. As stated previously, we correct for heteroskedasticity due to the use of grouped data by weighting each observation. Since our data are in fact for the 7th grade, we weight each observation (school) in the first regression (2.18) by the square root of the number of pupils in the 7th grade in that school for whom data were collected. The weights for the second regression (2.19) are those used for the first, divided by 12, i.e., the

weights for the first regression are  $(N_{jkt})^{\frac{1}{2}}$  and those for the second are  $(N_{jkt})^{\frac{1}{2}}/12$ . This can be seen from our considerations of the two error terms on pages 53, 54, and 55. The error term for the first model is:

$$\frac{\epsilon_{.jkt}}{N_{jkt}} \sim N(0, \frac{\sigma^2}{N_{jkt}})$$

and for our second model:

$$t \frac{\epsilon_{.jkt}}{N_{jkt}} \sim N(0, \frac{t^2 \sigma^2}{N_{jkt}})$$

The weighting factors necessary to remove heteroskedasticity are, respectively:

$$N_{jkt}^{\frac{1}{2}} \quad \text{and} \quad \frac{N_{jkt}^{\frac{1}{2}}}{t}$$

Since our observations are on 7th grade students we take  $t = 12$ . The two sets of weights differ, then, only by a constant; and, therefore, the relative weights among observations are the same in both sets. Since it is only the relative weights of the observations which affect the estimates it makes no difference which of the sets of weights we use.

### 3. The Data

The data we shall be using are from the Michigan Educational Assessment Program for the academic year 1970-71. This program is relatively new having been initiated in 1969 and information on certain series is available for the academic year 1970-71 only, eliminating the possibility of any time series work.

The survey collected data on the achievement of 4th and 7th grade students by means of a set of four tests: vocabulary, reading, mechanics of written English, and mathematics. The scores on these tests were standardized, i.e., fitted to a function  $X$  such that  $X \sim N(50,100)$ . The simple averages of the standardized scores for reading, mechanics of written English, and mathematics for the 7th grade students for the academic years 1970-71 and 1969-70 are the data used for COMP T and COMP T-1, respectively.

A pupil background and attitude questionnaire was also given to the students in the 4th and 7th grades. From these questionnaires scores were developed for the 4th and 7th grades of each school and district, for socioeconomic status, importance of school achievement, self-perception, and attitude towards school. These scores were then standardized, i.e., fitted to a function  $X$  such that  $X \sim N(50,100)$ . These school (district) scores are the data used for SES-S (SES-D), M SCH-S (M SCH-D), SELF-S (SELF-D), and SCH-S (SCH-D), respectively.

Data were also collected on the number of full-time equivalent students in each school, the percentage of teachers with five or more years experience and the percentage of teachers with master's degrees in each school, and the teacher-pupil ratio in each school. These are the data used for PUP-S, EXP, MA, and T/P, respectively.

Information on the percentage of non-racial-ethnic minorities students in each school and district was also collected. These are the data used for RACE-S and RACE-D, respectively. Data were

also collected on the state aid membership in each district.<sup>2</sup>

These are the data used for PUP-D.

Each district was also classified according to its regional location, i.e., whether it is in the Detroit area, the remainder of the southern portion of the lower peninsula, the northern portion of the lower peninsula, or in the upper peninsula.<sup>3</sup> We assigned to each school, the regional location of its district. These are the data used for DET, SOL, and NOL. In addition, each school was classified as to community type: metropolitan core city, urban fringe, city, town, or rural.<sup>4</sup> We assigned to each school the

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<sup>2</sup> State aid membership is defined as the total number of pupils legally enrolled in the district at the close of school on the fourth Friday following Labor Day of the school year.

<sup>3</sup> The Detroit region is composed of Wayne, Oakland, and Macomb counties. The SOL region includes Berrien, Cass, St. Joseph, Branch, Hillsdale, Lenawee, Monroe, Washtenaw, Jackson, Calhoun, Kalamazoo, Van Buren, Allegan, Barry, Eaton, Ingham, Livingston, St. Clair, Lapeer, Genesee, Shiawassee, Clinton, Ionia, Kent, Ottawa, Muskegon, Montcalm, Gratiot, Saginaw, Tuscola, Sanilac, Huron, Bay and Midland counties. The NOL region includes Oceana, Newaygo, Mecosta, Isabella, Arenac, Gladwin, Clare, Osceola, Lake, Mason, Manistee, Wexford, Missaukee, Roscommon, Ogemaw, Iosco, Alcona, Oscoda, Crawford, Kalkaska, Grand Traverse, Benzie, Leelanau, Antrim, Otsego, Montmorency, Alpena, Presque Isle, Cheboygan, Charlevoix and Emmet counties. The UP region includes Chippewa, Mackinac, Luce, Schoolcraft, Delta, Alger, Menominee, Dickinson, Marquette, Iron, Baraga, Keweenaw, Houghton, Ontonagon, and Gogebic counties.

<sup>4</sup> Metropolitan Core Cities are defined as one or more adjacent cities with a population of 50,000 or more which serve as the economic focal point of their environs. Cities are communities of 10,000 to 50,000 that serve as the economic focal point of their environs. Towns are communities of 2,500 to 10,000 that serve as the economic focal point of their environs. Rural Communities are communities of 2,500 or less. Suburbs are communities of any size that have as their economic focal point a metropolitan core city, or a city.

community type of its district. In addition, we further subdivided metropolitan core city into two groups: (1) those in the Detroit school district and (2) those in the rest of the metropolitan core city districts. We used this six-way community type classification as the basis for dividing our data into subsamples.

## CHAPTER III<sup>1</sup>

### MULTIPLE PRODUCTION FUNCTIONS AND REGIONAL DIFFERENCES

#### 1. Production Functions for Subsamples

The primary purpose of this study is to examine the production function for education empirically, particularly with respect to socioeconomic class and race taken as inputs. In order to pursue this goal it was necessary to determine if there is a production function for education; or, if in fact, there is more than one such function. To this end the data are grouped as follows, and the production function estimated for each group:

- 1) Group I - all 701 schools for which we had data (referred to hereinafter as TOTAL)
- 2) Group II - schools from the Detroit School District only - there were 72 such schools - (referred to hereinafter as DET)
- 3) Group III - schools from metropolitan school districts<sup>2</sup> excluding those in the Detroit School District (referred to hereinafter as METRO)
- 4) Group IV - schools from suburban school districts - there were 187 such schools - (referred to hereinafter as SUBURB)

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<sup>1</sup> Wheresoever possible, we shall compare our results with those of Averch, et al. We do this because they base their conclusions not on one, but upon 19 different studies of the production function of education. Their book is the best single source on production functions for education.

<sup>2</sup> There were a total of 134 metropolitan core city schools for which we had data. Of these 62 were not in the Detroit school district. For definitions of the various classifications see footnote 5 in Chapter II.

- 5) Group V - schools from city school districts - there were 43 such schools - (referred to hereinafter as CITY)
- 6) Group VI - schools from town school districts - there were 103 such schools - (referred to hereinafter as TOWN)
- 7) Group VII - schools from rural school districts - there were 234 such schools - (referred to hereinafter as RURAL)
- 8) Group VIII - all schools except those in DET (referred to hereinafter as NON DET)
- 9) Group IX - all schools except those in DET or METRO (referred to hereinafter as NON DET/METRO)
- 10) Group X - all schools in DET or METRO (referred to hereinafter as DET/METRO)
- 11) Group XI - all schools in METRO or SUBURB (referred to hereinafter as METRO/SUBURB)
- 12) Group XII - all schools in CITY or TOWN (referred to hereinafter as CITY/TOWN)
- 13) Group XIII - all schools in TOWN or RURAL (referred to hereinafter as TOWN/RURAL).

Both models (including and excluding the lagged dependent variable) were estimated twice, once including the community variables and once excluding the community variables except for the DET group which was estimated only once, excluding community variables as these variables are constant for all observations in that group. The estimated coefficients, the  $t$  values and the  $R^2$ 's for these regressions are presented in Tables 1 through 4. We then tested a series of hypotheses in order to determine if there is one or more production functions. The results of these tests are presented in Table 5. The first hypothesis we tested was:

$$\beta_D = \beta_M = \beta_S = \beta_C = \beta_T = \beta_R$$



Table 1. Estimated coefficients, (t values), and  $R^2$ 's for models including the lagged dependent variable, home, and school variables, by subsamples.\*

| Item     | TOTAL            | DETROIT        | METRO          | SUBURB          |
|----------|------------------|----------------|----------------|-----------------|
| Constant | 5.15             | -2.18          | 27.67          | .63             |
| COMP T-1 | .60<br>(21.8)**  | .47<br>(4.13)  | .69<br>(4.76)  | .62<br>(12.7)   |
| SES-S    | .19<br>(10.2)    | .22<br>(3.86)  | .30<br>(3.25)  | .19<br>(5.35)   |
| M SCH-S  | .03<br>(1.13)    | .07<br>(.82)   | .02<br>(.16)   | .08<br>(1.42)   |
| SELF-S   | -.01<br>(.44)    | .15<br>(1.35)  | -.18<br>(1.50) | -.06<br>(1.01)  |
| SCH-S    | .08<br>(3.34)    | .08<br>(.88)   | -.06<br>(.60)  | .09<br>(2.21)   |
| PUP-S    | -.0002<br>(1.46) | .0003<br>(.53) | .0002<br>(.53) | -.0001<br>(.58) |
| RACE-S   | 3.16<br>(10.6)   | 4.36<br>(4.68) | .82<br>(.79)   | 3.47<br>(4.20)  |
| EXP      | -.49<br>(1.46)   | -1.47<br>(.87) | .05<br>(.04)   | -.87<br>(1.43)  |
| MA       | .90<br>(2.30)    | .76<br>(.34)   | .67<br>(.53)   | .84<br>(1.27)   |
| T/P      | 6.93<br>(.93)    | -26.0<br>(.51) | 11.1<br>(.46)  | 25.7<br>(1.67)  |
| $R^2$    | <u>.8917</u>     | <u>.9350</u>   | <u>.9548</u>   | <u>.8911</u>    |

"Continued"

Table 1. Continued

| Item           | CITY            | TOWN            | RURAL            | NON-DETROIT      |
|----------------|-----------------|-----------------|------------------|------------------|
| Constant       | 2.33            | 13.45           | 10.54            | 4.96             |
| COMP T-1       | .62<br>(5.46)   | .64<br>(7.28)   | .54<br>(10.7)    | .61<br>(20.9)    |
| SES-S          | .15<br>(1.31)   | .22<br>(2.70)   | .15<br>(2.95)    | .21<br>(9.27)    |
| M SCH-S        | -.01<br>(.08)   | .02<br>(.16)    | -.004<br>(.07)   | .01<br>(.48)     |
| SELF-S         | -.006<br>(.06)  | -.18<br>(1.91)  | .03<br>(.53)     | -.04<br>(1.16)   |
| SCH-S          | .17<br>(1.68)   | .12<br>(1.66)   | .03<br>(.64)     | .07<br>(2.52)    |
| PUP-S          | -.0002<br>(.42) | -.0003<br>(.66) | -.0005<br>(1.66) | -.0003<br>(1.99) |
| RACE-S         | 3.59<br>(1.89)  | -3.56<br>(1.06) | 2.81<br>(1.59)   | 2.58<br>(6.09)   |
| EXP            | 1.38<br>(.80)   | .13<br>(.13)    | -.47<br>(.66)    | -.30<br>(.83)    |
| MA             | 1.05<br>(.66)   | .56<br>(.58)    | 1.51<br>(1.62)   | .88<br>(2.19)    |
| T/P            | -53.2<br>(1.15) | -8.42<br>(.32)  | -3.63<br>(.28)   | .49<br>(.06)     |
| R <sup>2</sup> | <u>.9125</u>    | <u>.6555</u>    | <u>.4772</u>     | <u>.8446</u>     |

"Continued"

Table 1. Continued.

| Item     | NON-<br>METRO    | DETROIT/<br>METRO | METRO/<br>SUBURB | CITY/<br>TOWN    | TOWN/<br>RURAL   |
|----------|------------------|-------------------|------------------|------------------|------------------|
| Constant | 3.76             | -2.13             | 3.65             | 4.19             | 8.47             |
| COMP T-1 | .60<br>(20.2)    | .55<br>(6.69)     | .63<br>(13.8)    | .63<br>(9.43)    | .58<br>(13.5)    |
| SES-S    | .19<br>(7.91)    | .21<br>(5.17)     | .22<br>(6.85)    | .18<br>(2.93)    | .18<br>(4.26)    |
| M SCH-S  | .03<br>(.86)     | .08<br>(1.30)     | .05<br>(1.00)    | -.006<br>(.08)   | .01<br>(.28)     |
| SELF-S   | -.02<br>(.66)    | .06<br>(.85)      | -.08<br>(1.58)   | -.07<br>(1.11)   | -.006<br>(.13)   |
| SCH-S    | .08<br>(2.81)    | .05<br>(.78)      | .06<br>(1.61)    | .15<br>(2.67)    | .05<br>(1.44)    |
| PUP-S    | -.0003<br>(2.13) | .0003<br>(1.05)   | -.00006<br>(.32) | -.0003<br>(1.15) | -.0003<br>(1.35) |
| RACE-S   | 3.24<br>(5.06)   | 3.34<br>(5.42)    | 2.51<br>(5.16)   | 3.12<br>(2.28)   | 1.44<br>(.93)    |
| EXP      | -.18<br>(.46)    | -.29<br>(.30)     | -.86<br>(1.78)   | .35<br>(.43)     | .09<br>(.16)     |
| MA       | .88<br>(2.06)    | 1.00<br>(.87)     | .88<br>(1.57)    | .66<br>(.83)     | 1.16<br>(1.74)   |
| T/P      | -.47<br>(.05)    | 23.4<br>(1.45)    | 20.6<br>(1.65)   | -23.1<br>(1.10)  | -6.58<br>(.57)   |
| $R^2$    | .7850            | .9420             | .9284            | .7989            | .5476            |

\* No statistics are available for constant terms other than the estimated value of the coefficient due to the nature of the statistical routines used on the MSU CDC 6500 computer.

\*\* The t values given are absolute values. The sign of each t value corresponds to that of the estimated coefficient.

Table 2. Estimated coefficients (t values), and  $R^2$ 's for models including home and school variables, by subsamples.\*

| Variable | TOTAL            | DETROIT          | METRO           | SUBURB           |
|----------|------------------|------------------|-----------------|------------------|
| Constant | 11.4             | -4.55            | 35.5            | 10.0             |
| SES-S    | .44<br>(22.8)**  | .40<br>(8.96)    | .66<br>(10.6)   | .49<br>(13.6)    |
| M SCH-S  | .04<br>(1.24)    | .19<br>(2.29)    | -.07<br>(.52)   | .05<br>(.70)     |
| SELF-S   | .04<br>(.95)     | .29<br>(2.34)    | -.21<br>(1.45)  | -.01<br>(.18)    |
| SCH-S    | .11<br>(3.55)    | .07<br>(.67)     | -.18<br>(1.43)  | .13<br>(2.18)    |
| PUP-S    | -.0004<br>(2.19) | -.00008<br>(.13) | -.0005<br>(.90) | -.0004<br>(1.39) |
| RACE-S   | 6.28<br>(18.5)   | 7.02<br>(9.30)   | 4.30<br>(4.99)  | 6.97<br>(6.49)   |
| EXP      | 1.32<br>(3.11)   | -.25<br>(.13)    | .70<br>(.52)    | -.38<br>(.46)    |
| MA       | 1.87<br>(3.68)   | 1.95<br>(.77)    | 2.14<br>(1.46)  | 2.42<br>(2.73)   |
| T/P      | 19.2<br>(1.98)   | 26.6<br>(.48)    | 6.17<br>(.21)   | 27.5<br>(1.29)   |
| $R^2$    | .8173            | .9168            | .9348           | .7918            |

"Continued"

Table 2. Continued.

| Variable | CITY           | TOWN             | RURAL            | NON-DETROIT      |
|----------|----------------|------------------|------------------|------------------|
| Constant | 16.0           | 29.8             | 28.6             | 17.5             |
| SES-S    | .54<br>(4.47)  | .50<br>(5.72)    | .25<br>(3.97)    | .49<br>(20.6)    |
| M SCH-S  | -.02<br>(.09)  | -.15<br>(1.32)   | .002<br>(.04)    | -.03<br>(.79)    |
| SELF-S   | -.10<br>(.67)  | -.23<br>(1.94)   | .003<br>(.04)    | -.04<br>(.96)    |
| SCH-S    | .17<br>(1.18)  | .22<br>(2.44)    | .07<br>(1.21)    | .10<br>(2.97)    |
| PUP-S    | .0002<br>(.21) | -.00002<br>(.04) | -.0007<br>(1.97) | -.0004<br>(2.23) |
| RACE-S   | 5.62<br>(2.20) | .77<br>(.19)     | 4.90<br>(2.27)   | 5.81<br>(11.3)   |
| EXP      | 4.57<br>(2.06) | 2.48<br>(2.09)   | 1.91<br>(2.32)   | 1.57<br>(3.51)   |
| MA       | .44<br>(.20)   | 1.25<br>(1.04)   | 2.06<br>(1.81)   | 1.85<br>(3.56)   |
| T/P      | -56.9<br>(.90) | 33.3<br>(1.02)   | 4.63<br>(.29)    | 15.2<br>(1.45)   |
| $R^2$    | .8173          | .9168            | .9348            | .7918            |

"Continued"

Table 2. Continued.

| Variable       | NON -<br>METRO   | DETROIT/<br>METRO | METRO/<br>SUBURB | CITY/<br>TOWN    | TOWN/<br>RURAL   |
|----------------|------------------|-------------------|------------------|------------------|------------------|
| Constant       | 15.6             | 1.75              | 15.2             | 18.8             | 25.4             |
| SES-S          | .47<br>(17.7)    | .42<br>(13.6)     | .52<br>(17.1)    | .52<br>(7.76)    | .34<br>(6.77)    |
| M SCH-S        | -.01<br>(.24)    | .18<br>(2.68)     | .01<br>(.20)     | -.10<br>(1.09)   | -.03<br>(.45)    |
| SELF-S         | -.03<br>(.57)    | .23<br>(2.77)     | -.03<br>(.48)    | -.15<br>(1.81)   | -.02<br>(.40)    |
| SCH-S          | .12<br>(3.36)    | -.006<br>(.08)    | .08<br>(1.47)    | .22<br>(2.98)    | .10<br>(2.17)    |
| PUP-S          | -.0004<br>(2.06) | -.0002<br>(.42)   | -.0004<br>(1.51) | -.00008<br>(.23) | -.0003<br>(1.20) |
| RACE-S         | 6.39<br>(7.84)   | 6.39<br>(13.2)    | 5.58<br>(9.65)   | 6.23<br>(3.66)   | 3.90<br>(2.03)   |
| EXP            | 1.75<br>(3.56)   | .31<br>(.28)      | .03<br>(.05)     | 2.90<br>(2.93)   | 2.59<br>(3.93)   |
| MA             | 1.75<br>(3.13)   | 2.63<br>(2.01)    | 2.41<br>(3.28)   | .97<br>(.96)     | 1.86<br>(2.25)   |
| T/P            | 14.2<br>(1.25)   | 22.4<br>(1.19)    | 25.3<br>(1.52)   | 9.54<br>(.36)    | 7.41<br>(.52)    |
| R <sup>2</sup> | .6278            | .9209             | .8711            | .6664            | .2961            |

\* No statistics are available for constant terms other than the estimated value of the coefficient due to the nature of the statistical routines used on the MSU CDC 6500 computer.

\*\* The t values given are absolute values. The sign of each t value corresponds to that of the estimated coefficient.

Table 3. Estimated coefficients, (t values), and  $R^2$ 's for models including the lagged dependent variable, home, school, and community variables, by subsamples.\*

| Variable | TOTAL              | METRO            | SUBURB           | CITY             |
|----------|--------------------|------------------|------------------|------------------|
| Constant | 3.69               | 27.2             | -.71             | -18.1            |
| COMP T-1 | .60<br>(21.5)**    | .62<br>(3.64)    | .63<br>(12.7)    | .59<br>(5.23)    |
| SES-S    | .17<br>(6.61)      | .33<br>(3.01)    | .11<br>(1.50)    | .12<br>(.86)     |
| M SCH-S  | .11<br>(2.40)      | -.06<br>(.39)    | .14<br>(1.29)    | -.11<br>(.55)    |
| SELF-S   | -.02<br>(.34)      | -.12<br>(.77)    | -.05<br>(.49)    | -.10<br>(.78)    |
| SCH-S    | .02<br>(.30)       | -.07<br>(.53)    | .02<br>(.36)     | .13<br>(.83)     |
| PUP-S    | -.0002<br>(1.45)   | .0002<br>(.39)   | -.0002<br>(.86)  | .00004<br>(.08)  |
| RACE-S   | 2.99<br>(7.69)     | .61<br>(.47)     | 2.64<br>(1.63)   | 2.22<br>(.98)    |
| EXP      | -.29<br>(.84)      | -.18<br>(.13)    | -.68<br>(1.09)   | 3.75<br>(2.01)   |
| MA       | .98<br>(2.45)      | 1.57<br>(1.06)   | .74<br>(1.11)    | -.12<br>(.07)    |
| T/P      | -.48<br>(.06)      | -2.69<br>(.09)   | 21.0<br>(1.31)   | -24.9<br>(.57)   |
| PUP-D    | -.000001<br>(1.17) | -.00002<br>(.99) | .000007<br>(.84) | -.0001<br>(3.05) |
| RACE-D   | -.11<br>(.17)      | .47<br>(.26)     | .84<br>(.51)     | 2.89<br>(.83)    |
| SES-D    | .04<br>(1.38)      | .21<br>(1.07)    | .08<br>(1.20)    | -.08<br>(.45)    |
| M SCH-D  | -.10<br>(1.85)     | .30<br>(1.05)    | -.07<br>(.60)    | .34<br>(1.17)    |
| SELF-D   | -.005<br>(.08)     | -.51<br>(1.25)   | -.03<br>(.19)    | .44<br>(1.55)    |
| SCH-D    | .08<br>(1.43)      | -.25<br>(.78)    | .12<br>(1.32)    | -.05<br>(.21)    |
| $R^2$    | .8936              | .9576            | .8937            | .9420            |

"Continued"

Table 3. Continued.

| Variable       | TOWN             | RURAL             | NON-<br>DETROIT  | NON-<br>METRO     |
|----------------|------------------|-------------------|------------------|-------------------|
| Constant       | 13.0             | 10.4              | 3.13             | 2.41              |
| COMP T-1       | .60<br>(6.77)    | .54<br>(10.5)     | .61<br>(20.9)    | .60<br>(20.2)     |
| SES-S          | -.0009<br>(.004) | .29<br>(.93)      | .22<br>(5.14)    | .13<br>(2.22)     |
| M SCH-S        | .37<br>(1.18)    | .45<br>(1.42)     | .08<br>(1.18)    | .11<br>(1.30)     |
| SELF-S         | -.71<br>(1.79)   | -.23<br>(.78)     | -.09<br>(1.38)   | -.06<br>(.91)     |
| SCH-S          | -.18<br>(.53)    | -.05<br>(.26)     | -.02<br>(.40)    | -.001<br>(.02)    |
| PUP-S          | -.00007<br>(.17) | -.0003<br>(.70)   | -.0003<br>(1.72) | -.0003<br>(1.92)  |
| RACE-S         | 13.3<br>(.96)    | 1.80<br>(.22)     | 2.18<br>(3.24)   | 2.89<br>(2.26)    |
| EXP            | -.21<br>(.21)    | -.50<br>(.71)     | -.24<br>(.66)    | -.18<br>(.49)     |
| MA             | 1.06<br>(1.10)   | 1.63<br>(1.70)    | .97<br>(2.37)    | 1.09<br>(2.21)    |
| T/P            | -12.1<br>(.46)   | -5.12<br>(.39)    | -.27<br>(.03)    | -1.84<br>(.21)    |
| PUP-D          | -.0002<br>(2.55) | -.00004<br>(1.05) | .000003<br>(.45) | -.000005<br>(.62) |
| RACE-D         | -18.1<br>(1.21)  | .97<br>(.12)      | .64<br>(.81)     | .30<br>(.23)      |
| SES-D          | .23<br>(.92)     | -.13<br>(.40)     | -.01<br>(.33)    | .07<br>(1.20)     |
| M SCH-D        | -.35<br>(1.07)   | -.47<br>(1.44)    | -.07<br>(.95)    | -.09<br>(1.00)    |
| SELF-D         | .62<br>(1.53)    | .27<br>(.88)      | .07<br>(.99)     | .06<br>(.75)      |
| SCH-D          | .29<br>(.83)     | .09<br>(.44)      | .11<br>(1.82)    | .09<br>(1.37)     |
| R <sup>2</sup> | .6985            | .4882             | .8464            | .7873             |

"Continued"



Table 3. Continued.

| Variable       | DETROIT/<br>METRO | METRO/<br>SUBURB | CITY/<br>TOWN    | TOWN/<br>RURAL    |
|----------------|-------------------|------------------|------------------|-------------------|
| Constant       | 32.1              | 1.89             | 1.01             | 7.86              |
| COMP T-1       | .52<br>(6.01)     | .64<br>(13.8)    | .59<br>(8.97)    | .56<br>(13.2)     |
| SES-S          | .21<br>(4.82)     | .23<br>(4.66)    | .17<br>(1.55)    | .12<br>(.79)      |
| M SCH-S        | .09<br>(1.37)     | .10<br>(1.17)    | -.06<br>(.41)    | .25<br>(1.36)     |
| SELF-S         | .08<br>(.95)      | -.10<br>(1.13)   | -.12<br>(1.12)   | -.36<br>(1.73)    |
| SCH-S          | .03<br>(.45)      | -.003<br>(.04)   | .07<br>(.51)     | -.15<br>(.97)     |
| PUP-S          | .0001<br>(.32)    | -.0001<br>(.60)  | -.00006<br>(.20) | -.00003<br>(.10)  |
| RACE-S         | 3.47<br>(5.01)    | 2.17<br>(3.03)   | 3.29<br>(1.56)   | 5.29<br>(.79)     |
| EXP            | -.59<br>(.58)     | -.79<br>(1.58)   | .65<br>(.78)     | .02<br>(.04)      |
| MA             | 1.09<br>(.86)     | .88<br>(1.55)    | .71<br>(.89)     | 1.26<br>(1.87)    |
| T/P            | 2.83<br>(.11)     | 19.2<br>(1.47)   | -5.89<br>(.28)   | -10.1<br>(.88)    |
| PUP-D          | .000003<br>(1.65) | .000004<br>(.59) | -.0001<br>(3.60) | -.00008<br>(2.27) |
| RACE-D         | -2.16<br>(1.40)   | .72<br>(.86)     | -1.06<br>(.43)   | -4.22<br>(.61)    |
| SES-D          | .37<br>(1.98)     | -.02<br>(.42)    | .03<br>(.23)     | .07<br>(.44)      |
| M SCH-D        | .02<br>(.10)      | -.07<br>(.70)    | .11<br>(.67)     | -.24<br>(1.29)    |
| SELF-D         | -.69<br>(1.80)    | .02<br>(.14)     | .11<br>(.77)     | .38<br>(1.78)     |
| SCH-D          | -.30<br>(1.17)    | .12<br>(1.52)    | .06<br>(.38)     | .20<br>(1.32)     |
| R <sup>2</sup> | .9448             | .9297            | .8194            | .5476             |

\* No statistics are available for constant terms other than the estimated value of the coefficient due to the nature of the statistical routines used on the MSU CDC 6500 computer.

\*\* The t values given are absolute values. The sign of each t value corresponds to that of the estimated coefficient.

Table 4. Estimated coefficients, (t values), and  $R^2$ 's for models including home, school, and community variables by subsamples.\*

| Variable | TOTAL              | METRO             | SUBURB           | CITY             |
|----------|--------------------|-------------------|------------------|------------------|
| Constant | 16.4               | 71.4              | 11.3             | 1.11             |
| SES-S    | .41<br>(14.0)**    | .66<br>(9.48)     | .45<br>(4.94)    | .54<br>(3.34)    |
| M SCH-S  | .21<br>(3.68)      | -.21<br>(1.15)    | .12<br>(.81)     | .02<br>(.09)     |
| SELF-S   | .08<br>(1.27)      | -.12<br>(.67)     | .06<br>(.42)     | -.09<br>(.48)    |
| SCH-S    | .03<br>(.59)       | -.07<br>(.52)     | .07<br>(.69)     | .002<br>(.008)   |
| PUP-S    | -.0003<br>(1.96)   | -.00009<br>(.13)  | -.0004<br>(1.32) | .00007<br>(.09)  |
| RACE-S   | 6.40<br>(14.0)     | 3.29<br>(2.72)    | 7.92<br>(3.64)   | 4.94<br>(1.60)   |
| EXP      | 1.41<br>(3.19)     | .75<br>(.51)      | -.44<br>(.51)    | 5.07<br>(1.95)   |
| MA       | 2.05<br>(4.01)     | 2.98<br>(1.85)    | 2.47<br>(2.69)   | -.19<br>(.08)    |
| T/P      | 16.9<br>(1.63)     | -13.1<br>(.40)    | 28.6<br>(1.29)   | -4.59<br>(.07)   |
| PUP-D    | -.0000001<br>(.11) | -.00003<br>(1.53) | .000004<br>(.31) | -.0002<br>(2.99) |
| RACE-D   | -.40<br>(.51)      | 1.23<br>(.61)     | -1.08<br>(.48)   | 1.71<br>(.35)    |
| SES-D    | .07<br>(1.81)      | .29<br>(1.32)     | .05<br>(.51)     | -.08<br>(.31)    |
| M SCH-D  | -.26<br>(3.62)     | .55<br>(1.79)     | -.10<br>(.57)    | .14<br>(.35)     |
| SELF-D   | -.10<br>(1.29)     | -.80<br>(1.77)    | -.12<br>(.64)    | .10<br>(.27)     |
| SCH-D    | .10<br>(1.41)      | -.79<br>(2.45)    | .09<br>(.76)     | .21<br>(.63)     |
| $R^2$    | .8221              | .9451             | .7933            | .8811            |

"Continued"

Table 4. Continued.

| Variable       | TOWN             | RURAL             | NON -<br>DETROIT  | NON -<br>METRO   |
|----------------|------------------|-------------------|-------------------|------------------|
| Constant       | 25.4             | 28.5              | 15.8              | 14.6             |
| SES-S          | .23<br>(.74)     | .66<br>(1.75)     | .54<br>(10.4)     | .46<br>(6.44)    |
| M SCH-S        | .04<br>(.11)     | .68<br>(1.72)     | .04<br>(.43)      | .08<br>(.73)     |
| SELF-S         | -.89<br>(1.82)   | -.57<br>(1.56)    | -.11<br>(1.33)    | -.08<br>(.88)    |
| SCH-S          | -.25<br>(.60)    | .13<br>(.54)      | .007<br>(.10)     | .03<br>(.43)     |
| PUP-S          | .0002<br>(.48)   | -.0004<br>(.88)   | -.0004<br>(1.85)  | -.0004<br>(1.80) |
| RACE-S         | 21.4<br>(1.27)   | 12.1<br>(1.22)    | 5.51<br>(6.45)    | 7.04<br>(4.24)   |
| EXP            | 1.68<br>(1.41)   | 1.75<br>(2.10)    | 1.57<br>(3.42)    | 1.69<br>(3.38)   |
| MA             | 1.92<br>(1.63)   | 2.04<br>(1.74)    | 1.95<br>(3.69)    | 1.85<br>(3.22)   |
| T/P            | 22.2<br>(.70)    | 2.23<br>(.14)     | 15.9<br>(1.49)    | 14.3<br>(1.24)   |
| PUP-D          | -.0002<br>(2.77) | -.00006<br>(1.24) | -.000004<br>(.59) | .000006<br>(.55) |
| RACE-D         | -22.9<br>(1.25)  | -7.71<br>(.75)    | .41<br>(.39)      | -.85<br>(.48)    |
| SES-D          | .29<br>(.91)     | -.40<br>(1.05)    | -.06<br>(1.04)    | .008<br>(.11)    |
| M SCH-D        | -.14<br>(.34)    | -.68<br>(1.72)    | -.08<br>(.79)     | -.11<br>(.93)    |
| SELF-D         | .79<br>(1.57)    | -.59<br>(1.58)    | .11<br>(1.08)     | .08<br>(.78)     |
| SCH-D          | .45<br>(1.04)    | -.06<br>(.25)     | .12<br>(1.48)     | .10<br>(1.13)    |
| R <sup>2</sup> | .5377            | .2288             | .7367             | .6301            |

"Continued"

Table 4. Continued.

| Variable       | DETROIT/<br>METRO | METRO/<br>SUBURB | CITY/<br>TOWN    | TOWN/<br>RURAL   |
|----------------|-------------------|------------------|------------------|------------------|
| Constant       | 68.7              | 15.8             | 14.0             | 23.8             |
| SES-S          | .41<br>(12.8)     | .55<br>(9.68)    | .57<br>(4.37)    | .26<br>(1.35)    |
| M SCH-S        | .20<br>(2.70)     | .04<br>(.34)     | -.12<br>(.68)    | .14<br>(.61)     |
| SELF-S         | .24<br>(2.54)     | -.02<br>(.18)    | -.24<br>(1.70)   | -.54<br>(2.12)   |
| SCH-S          | .01<br>(.18)      | .03<br>(.36)     | -.05<br>(.31)    | -.09<br>(.50)    |
| PUP-S          | -.0003<br>(.82)   | -.0004<br>(1.56) | .0003<br>(.66)   | .00003<br>(.08)  |
| RACE-S         | 6.50<br>(12.0)    | 5.53<br>(6.11)   | 6.81<br>(2.59)   | 15.9<br>(1.93)   |
| EXP            | -.004<br>(.003)   | -.11<br>(.16)    | 2.88<br>(2.85)   | 2.39<br>(3.62)   |
| MA             | 2.36<br>(1.67)    | 2.47<br>(3.29)   | 1.48<br>(1.28)   | 1.95<br>(2.34)   |
| T/P            | 18.7<br>(.63)     | 28.1<br>(1.60)   | 29.8<br>(1.13)   | 1.25<br>(.09)    |
| PUP-D          | .000002<br>(1.15) | .000005<br>(.66) | -.0001<br>(3.75) | -.0001<br>(2.54) |
| RACE-D         | -2.42<br>(1.38)   | .25<br>(.22)     | -2.37<br>(.76)   | -13.1<br>(1.52)  |
| SES-D          | .52<br>(2.44)     | -.04<br>(.55)    | -.06<br>(.39)    | .10<br>(.50)     |
| M SCH-D        | -.09<br>(.39)     | -.05<br>(.38)    | .07<br>(.33)     | -.16<br>(.68)    |
| SELF-D         | -1.14<br>(2.65)   | -.04<br>(.23)    | .17<br>(.95)     | .57<br>(2.14)    |
| SCH-D          | -.60<br>(2.11)    | .08<br>(.80)     | .27<br>(1.42)    | .19<br>(.98)     |
| R <sup>2</sup> | .9278             | .8722            | .7069            | .3256            |

\* No statistics are available for constant terms other than the estimated value of the coefficient due to the nature of the statistical routines used on the MSU CDC 6500 computer.

\*\* The t values given are absolute values. The sign of each t value corresponds to that of the estimated coefficient.

Table 5. Tests of Hypotheses that the Vectors of Coefficients  
For Different Subsamples are Equal.

| Null Hypothesis                                                  | Variables<br>Included* | F<br>Value | Degrees of<br>Freedom | Critical Value<br>of F at the<br>.05 level | Hypotheses<br>Accepted (A) or<br>Rejected (R) |
|------------------------------------------------------------------|------------------------|------------|-----------------------|--------------------------------------------|-----------------------------------------------|
| $\beta_D^{**} = \beta_M = \beta_C = \beta_T = \beta_S = \beta_R$ | L, H, S                | 1.59       | 55/635                | 1.36                                       | R                                             |
| $\beta_M = \beta_C = \beta_T = \beta_S = \beta_R$                | "                      | 1.55       | 44/574                | 1.40                                       | R                                             |
| $\beta_C = \beta_T = \beta_S = \beta_R$                          | "                      | 1.50       | 33/523                | 1.46                                       | R                                             |
| $\beta_D = \beta_M$                                              | "                      | 2.12       | 11/112                | 1.87                                       | R                                             |
| $\beta_M = \beta_S$                                              | "                      | 1.31       | 11/227                | 1.83                                       | A                                             |
| $\beta_C = \beta_T$                                              | "                      | .77        | 11/124                | 1.86                                       | A                                             |
| $\beta_T = \beta_R$                                              | "                      | 1.65       | 11/315                | 1.82                                       | A                                             |
| $\beta_D = \beta_M = \beta_C = \beta_T = \beta_S = \beta_R$      | H, S                   | 2.26       | 50/641                | 1.37                                       | R                                             |
| $\beta_M = \beta_C = \beta_T = \beta_S = \beta_R$                | "                      | 2.04       | 40/579                | 1.42                                       | R                                             |
| $\beta_C = \beta_T = \beta_S = \beta_R$                          | "                      | 2.01       | 30/527                | 1.49                                       | R                                             |
| $\beta_D = \beta_M$                                              | "                      | 2.50       | 10/114                | 1.91                                       | R                                             |
| $\beta_M = \beta_S$                                              | "                      | 1.42       | 10/229                | 1.87                                       | A                                             |
| $\beta_C = \beta_T$                                              | "                      | .70        | 10/126                | 1.90                                       | A                                             |
| $\beta_T = \beta_R$                                              | "                      | 2.21       | 10/317                | 1.86                                       | R                                             |

"Continued"

Table 5. Continued.

| Null Hypothesis                                   | Variables Included* | F Value | Degrees of Freedom | Critical Value of F at the .05 level | Hypotheses Accepted (A) or Rejected (R) |
|---------------------------------------------------|---------------------|---------|--------------------|--------------------------------------|-----------------------------------------|
| $\beta_M = \beta_C = \beta_T = \beta_S = \beta_R$ | L, H, S, C          | 1.50    | 68/544             | 1.34                                 | R                                       |
| $\beta_C = \beta_T = \beta_S = \beta_R$           | "                   | 1.60    | 51/499             | 1.38                                 | R                                       |
| $\beta_M = \beta_S$                               | "                   | 1.01    | 17/215             | 1.67                                 | A                                       |
| $\beta_C = \beta_T$                               | "                   | 1.17    | 17/112             | 1.72                                 | A                                       |
| $\beta_T = \beta_R$                               | "                   | 1.54    | 17/303             | 1.66                                 | A                                       |
| $\beta_M = \beta_C = \beta_T = \beta_S = \beta_R$ | H, S, C             | 1.92    | 64/549             | 1.35                                 | R                                       |
| $\beta_C = \beta_T = \beta_S = \beta_R$           | "                   | 1.97    | 48/503             | 1.39                                 | R                                       |
| $\beta_M = \beta_S$                               | "                   | 1.25    | 16/217             | 1.69                                 | A                                       |
| $\beta_C = \beta_T$                               | "                   | 1.11    | 16/114             | 1.74                                 | A                                       |
| $\beta_T = \beta_R$                               | "                   | 2.00    | 16/305             | 1.68                                 | R                                       |

\* Variables included: L, H, S, C

L = Lagged dependent variable (COMP T-1)

H = Home influence variables (SES-S, M SCH-S, SELF-S, SCH-S)

S = School influence variables (PUP-S, RACE-S, EXP, MA, T/P)

C = Community influence variables (PUP-D, RACE-D, SES-D, M SCH-D, SELF-D, SCH-D)

\*\*  $\beta_i$  (i = D, M, C, T, S, R,) = vector of coefficients for the DETROIT, METRO, CITY, TOWN, SUBURB, and RURAL subsamples.

where:  $\beta_i$ ,  $i = D, M, S, C, T, R$  refers to the vectors of coefficients of explanatory variables for the DET, METRO, SUBURB, CITY, TOWN, and RURAL groups, respectively.<sup>3</sup>

The hypothesis we tested was that coefficients for all groups came from the same population. Since the DET group was included we could only test for the models excluding community variables. We rejected the hypothesis, at the .05 level, for both the model including and the model excluding the dependent variable. We therefore concluded that there is more than one production function for education and we could not put all schools in one group. The next step was to determine just how many there are.

Our next step was to exclude one group and retest the same hypothesis as applied to the remaining groups. It seemed logical to exclude the DET group on several grounds: (1) it is by far the largest group; (2) it is the only group for which we cannot use community variables; and, (3) it is a perfect example of the type of large school system experiencing severe problems which are constantly in the news. This time, as the DET group was excluded we could test both models each way. That is, we could test the model including the lagged dependent variable both with and without the community variables, and we could do likewise for the model excluding the community variables. The hypothesis tested is:

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<sup>3</sup> This set of tests was run excluding community variables. This was necessary because of the presence of the DET subsample for which the community variables are constant for all observations. With one exception noted later, the other tests were run both including and excluding community variables.

$$\beta_M = \beta_S = \beta_C = \beta_T = \beta_R$$

where:  $\beta_i$ ,  $i = M, S, C, T, R$  refers to the vectors of coefficients of explanatory variables for the METRO, SUBURB, CITY, TOWN, and RURAL groups, respectively.

We rejected the hypothesis, at the .05 level, for all four tests. That is to say we rejected the hypothesis that all five groups came from the same population.

We then split off the METRO group in order to test two hypotheses: (1) there is no difference between the production functions for the DET group and that for the METRO group, and (2) there is no difference among the production functions for the remaining groups. The hypotheses we tested, then, are:

$$\beta_D = \beta_M^4 \quad \text{and} \quad \beta_S = \beta_C = \beta_T = \beta_R$$

where:  $\beta_i$ ,  $i = D, M$  and  $\beta_i$ ,  $i = S, C, T, R$  refers to the vectors of coefficients of explanatory variables for the DET, METRO and SUBURB, CITY, TOWN, and RURAL groups, respectively.

Again, because of the inclusion of the DET group the former hypothesis could not be tested for the models including the community variables. This restriction did not apply to the tests of the latter hypothesis. The former hypothesis was rejected at the .05 level, in both tests, while the latter hypothesis was rejected at the .05 level in all four tests. That is, we rejected both the hypothesis that the DET group and the METRO group came

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<sup>4</sup> This is the exception mentioned in footnote 3, q.v.



from the same population and the hypothesis that the SUBURB, CITY, TOWN, and RURAL groups all came from the same population.

At this point we continued only with our pairwise testing of groups as this appeared to be the most efficient way to proceed. We tested the hypothesis that the METRO group and the SUBURB group came from the same population. That is, we tested the hypothesis:

$$\beta_M = \beta_S$$

where:  $\beta_i$ ,  $i = M, S$  refers to the vectors of coefficients of explanatory variables for the METRO and SUBURB groups, respectively.

We could not reject this hypothesis at the .05 level for any of the four tests.

Next we tested the hypothesis that the CITY group and the TOWN group came from the same population. That is, we tested the hypothesis:

$$\beta_C = \beta_T$$

where:  $\beta_i$ ,  $i = C, T$  refers to the vectors of coefficients of explanatory variables for the CITY and TOWN groups, respectively.

Again, we could not reject the hypothesis at the .05 level for any of the four tests.

Finally, we tested the hypothesis that the TOWN and RURAL groups came from the same population. That is, we tested the hypothesis:

$$\beta_T = \beta_R$$

where:  $\beta_i$ ,  $i = T, R$  refers to the vectors of coefficients of explanatory variables for the TOWN and RURAL groups, respectively.

This was the only hypothesis for which the results were mixed. We could not reject the hypothesis at the .05 level for the two tests of the model including the lagged dependent variable, i.e., both including and excluding the community variables. On the other hand, the hypothesis was rejected at the .05 level for the two tests of the model excluding the lagged dependent variable. Now, as we have seen, the models including the lagged dependent variable are short run models and those excluding the lagged dependent variable are long run models. As education is essentially a long run process, we decided to rely on the results for the long run models. Therefore, although the results were mixed, we rejected the hypothesis.

Based upon the entire series of tests it is not unreasonable to conclude that there are four different production functions for education -- one for each of the following groups of schools: (1) Detroit (DET) schools; (2) metropolitan and suburban (METRO and SUBURB) schools; (3) city and town (CITY and TOWN) schools; and (4) rural (RURAL) schools. (The other possibility is that we have piecewise estimates of a single non-linear function.)

One can only speculate as to the reasons for the existence of multiple production functions for education. Ranking the groups according to either the mean number of pupils in the school or in

the district yields identical results. Detroit is the largest followed in order by Metropolitan, Suburban, City, Town, and Rural schools. Such a ranking is not inconsistent with our separation into four groups: DET, METRO/SUBURB, CITY/TOWN, and RURAL. Further, this result appears consistent with the results from other studies. Benson, using the school district as the level of analysis, divided the sample into groups according to district size.<sup>5</sup> The result was that the sets of significant variables differed for each subsample. His dependent variable was a reading achievement score and he used 25 explanatory variables. Kiesling, using the school district as the level of analysis divided his sample into two groups: urban and non-urban.<sup>6</sup> Using six explanatory variables, he estimated 54 regression equations for each subsample. The sets of significant variables differed between the urban and non-urban samples. It is likely that the urban districts in his study were larger than the non-urban districts.

It is not clear, however, that size is the determining factor. It may well be that social and cultural values differ as between types of communities and that such values are correlated with the size of the community; and, in fact, it is this divergence of values that yields different production functions. Obviously, more research on this topic is required before we begin to have

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<sup>5</sup> Charles Benson, et al., State and Local Fiscal Relationships in Public Education in California, Report of the Senate Fact Finding Committee on Revenue and Taxation, Senate of the State of California, (Sacramento: March 1965).

<sup>6</sup> Kiesling, op. cit.

any confidence in our tentative answers. Certainly, more work is necessary on the whole area of multiple production functions, particularly with respect to individual personality differences among students and differing values among communities.

## 2. Regional Differences

The models for METRO/SUBURB, CITY/TOWN, and RURAL were also estimated including sets of dummy variables for differing regions: the Detroit region, the rest of the southern portion of the lower peninsula, the northern portion of the lower peninsula, and the upper peninsula.<sup>7</sup> The results of the estimations are given in Table 6a and 6b. For the METRO/SUBURB schools we found the coefficient for DETROIT to be insignificantly different from zero. (The omitted dummy variable was for SOL.) This implies that for the METRO/SUBURB schools it makes no difference whether they are in the Detroit region or in the SOL region. For the CITY/TOWN and RURAL schools all of the dummy variables are significantly different from zero. Since for these samples the dummy variable for the Upper Peninsula was the one omitted; this indicates that, ceteris paribus, the other regions do not fare as well academically as the Upper Peninsula. We tested the hypotheses that the vector of coefficients for the regional dummy variables was equal to zero. We rejected the hypotheses for CITY/TOWN and RURAL schools, but did not reject it for METRO/SUBURB schools. We then tested the hypotheses that the coefficients of the regional dummy variables

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<sup>7</sup> For the METRO/SUBURB subsample only one dummy variable (for the Detroit region) was included as there are no metropolitan or suburban schools in NOL or the UP. The complete set of dummy variables was used for both the CITY/TOWN and RURAL subsamples.

Table 6a. Estimated coefficients, (t values), and  $R^2$ 's for models including the lagged dependent variable, home, school, community, and region variables, by subsamples.\*

| Variable | METRO/<br>SUBURB | CITY/<br>TOWN     | RURAL            |
|----------|------------------|-------------------|------------------|
| Constant | 1.81             | 3.00              | 13.5             |
| COMP T-1 | .64<br>(13.7)**  | .54<br>(8.05)     | .45<br>(8.41)    |
| SES-S    | .22<br>(4.63)    | .20<br>(1.87)     | .39<br>(1.31)    |
| M SCH-S  | .10<br>(1.17)    | -.08<br>(.56)     | .53<br>(1.69)    |
| SELF-S   | -.10<br>(1.13)   | -.10<br>(.95)     | -.49<br>(1.63)   |
| SCH-S    | -.003<br>(.05)   | .06<br>(.49)      | -.07<br>(.36)    |
| PUP-S    | -.0001<br>(.59)  | .00002<br>(.08)   | -.00009<br>(.25) |
| RACE-S   | 2.17<br>(3.03)   | 3.52<br>(1.71)    | 5.39<br>(.66)    |
| EXP      | -.82<br>(1.57)   | .08<br>(.10)      | -.75<br>(1.04)   |
| MA       | .91<br>(1.55)    | .55<br>(.70)      | 1.00<br>(1.04)   |
| T/P      | 18.9<br>(1.42)   | -11.7<br>(.56)    | -17.7<br>(1.32)  |
| PUP-D    | .000004<br>(.59) | -.00009<br>(2.75) | -.00003<br>(.49) |
| RACE-D   | .73<br>(.86)     | -1.67<br>(.69)    | -3.47<br>(.41)   |
| SES-D    | -.02<br>(.43)    | .06<br>(.49)      | -.15<br>(.50)    |
| M SCH-D  | -.07<br>(.71)    | .15<br>(.93)      | -.56<br>(1.77)   |
| SELF-D   | .02<br>(1.52)    | .05<br>(.35)      | .54<br>(1.77)    |
| SCH-D    | .16<br>(1.52)    | .06<br>(.41)      | .10<br>(.50)     |

"Continued"

Table 6a. Continued.

| Variable | METRO/<br>SUBURB | CITY/<br>TOWN   | RURAL           |
|----------|------------------|-----------------|-----------------|
| DET      | -.03<br>(.19)    | -1.15<br>(2.73) | 2.04<br>(2.96)  |
| SOL      |                  | -.93<br>(2.84)  | -1.60<br>(3.90) |
| NOL      |                  | -1.04<br>(2.72) | -1.16<br>(2.89) |
| $R^2$    | .9297            | .8329           | .5234           |

\* No statistics are available for constant terms other than the estimated value of the coefficient due to the nature of the statistical routines used on the MSU CDC 6500 computer.

\*\* The t values given are absolute values. The sign of each t value corresponds to that of the estimated coefficient.

Table 6b. Estimated coefficients, (t values) and  $R^2$ 's for models including home, school, community and region variables, by subsamples.\*

| Variable | METRO/<br>SUBURB | CITY/<br>TOWN    | RURAL            |
|----------|------------------|------------------|------------------|
| Constant | 16.2             | 14.0             | 29.1             |
| SES-S    | .55<br>(9.68)**  | .57<br>(4.67)    | .75<br>(2.17)    |
| M SCH-S  | .04<br>(.32)     | -.15<br>(.90)    | .74<br>(2.08)    |
| SELF-S   | -.02<br>(.16)    | -.20<br>(1.48)   | -.94<br>(2.77)   |
| SCH-S    | .03<br>(.37)     | -.05<br>(.33)    | .04<br>(.20)     |
| PUP-S    | -.0004<br>(1.55) | .0002<br>(.63)   | -.00004<br>(.09) |
| RACE-S   | 5.49<br>(6.06)   | 6.63<br>(2.68)   | 15.8<br>(1.71)   |
| EXP      | .07<br>(.11)     | 1.49<br>(1.48)   | .69<br>(.85)     |
| MA       | 2.28<br>(2.95)   | 1.01<br>(1.06)   | .75<br>(.68)     |
| T/P      | 30.6<br>(1.72)   | 13.3<br>(.53)    | -22.3<br>(1.46)  |
| PUP-D    | .000005<br>(.65) | -.0001<br>(2.47) | -.00004<br>(.60) |
| RACE-D   | .22<br>(.19)     | -3.20<br>(1.08)  | -13.3<br>(1.40)  |
| SES-D    | -.03<br>(.51)    | -.02<br>(.11)    | -.37<br>(1.06)   |
| M SCH-D  | -.05<br>(.35)    | .18<br>(.90)     | -.79<br>(2.16)   |
| SELF-D   | -.05<br>(.35)    | .11<br>(.62)     | 1.00<br>(2.87)   |
| SCH-D    | .08<br>(.78)     | .24<br>(1.33)    | .002<br>(.01)    |

"Continued"

Table 6b. Continued

| Variable | METRO/<br>SUBURB | CITY/<br>TOWN   | RURAL           |
|----------|------------------|-----------------|-----------------|
| DET      | .19<br>(1.03)    | -2.09<br>(4.19) | -3.56<br>(4.66) |
| SOL      |                  | -1.59<br>(4.10) | -2.94<br>(6.73) |
| NOL      |                  | -1.34<br>(2.88) | -2.17<br>(4.90) |
| $R^2$    | .8728            | .7470           | .3657           |

\* No statistics are available for constant terms other than the estimated value of the coefficient due to the nature of the statistical routines used on the MSU CDC 6500 computer.

\*\* The t values given are absolute values. The sign of each t value corresponds to that of the estimated coefficient.



are equal. The results of these tests are given in Tables 7a and 7b. We found that we could not reject the hypothesis that these coefficients are the same. Based on this information, we conclude that for each group of schools there is no regional difference within the lower peninsula; but, there is such a difference between Upper Peninsula schools and lower peninsula schools. These results are basically consistent with those of Brown who found that the region in which a district was located did affect the production of achievement.<sup>8</sup>

Again, we can only speculate as to the underlying causes of this phenomenon. One hypothesis is that we are dealing with people whose values differ. It may well be that some of the factors underlying multiple production functions by community type also cause regional differences to be significant. That is, just as the cause of differing production functions by community types may be that values differ by community types, so too these same values may differ by region. It would seem logical, on the basis of this hypothesis, to search for multiple production functions by region as well as by community types. We did not so search because, although in Michigan there is a relationship between community types and regions, this is not generally the case in other states. Therefore, even had we searched for and found multiple production functions by regions or a combination of regions and community types, we would not view the results as having any general validity. Rather, they would be strictly limited to the State of Michigan. On the other hand, multiple production functions by community type are in general valid.

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<sup>8</sup> Brown, op. cit.

Table 7a. Tests of Hypotheses Concerning the Sets of Dummy Variables for Regions.

| Sample       | Variables Included* | Null Hypotheses                                                        | F Value | Degrees of Freedom | Critical Values of F at .05 | Accept (A) Reject (R) |
|--------------|---------------------|------------------------------------------------------------------------|---------|--------------------|-----------------------------|-----------------------|
| METRO/SUBURB | L,H,S,C,R           | $\beta_{\text{DETROIT}}^{**} = 0$                                      | 0       | 1/231              | 3.89                        | A                     |
| CITY/TOWN    | "                   | $\beta_{\text{DETROIT}} = \beta_{\text{SOL}} = \beta_{\text{NOL}} = 0$ | 3.39    | 3/126              | 2.68                        | R                     |
| RURAL        | "                   | $\beta_{\text{DETROIT}} = \beta_{\text{SOL}} = \beta_{\text{NOL}}$     | 5.27    | 3/214              | 2.65                        | R                     |
| METRO/SUBURB | H,S,C,R             | $\beta_{\text{DETROIT}} = 0$                                           | 1.09    | 1/232              | 3.89                        | A                     |
| CITY/TOWN    | "                   | $\beta_{\text{DETROIT}} = \beta_{\text{SOL}} = \beta_{\text{NOL}} = 0$ | 6.71    | 3/127              | 2.68                        | R                     |
| RURAL        | "                   | $\beta_{\text{DETROIT}} = \beta_{\text{SOL}} = \beta_{\text{NOL}}$     | 15.47   | 3/215              | 2.65                        | R                     |

\* Variables included: L,H,S,C,R

L = Lagged dependent variable (COMP T-1)

H = Home influence variables (SES-S, M SCH-S, SELF-S, SCH-S)

S = School influence variables (PUP-S, RACE-S, EXP, MA, T/P)

C = Community influence variables (PUP-D, RACE-D, SES-D, M SCH-D, SELF-D, SCH-D)

R = Region Variables (DET, SOL, NOL)

\*\*  $\beta_i$  = vector of coefficients for the  $i^{\text{th}}$  subsample.

Table 7b. Tests of hypotheses concerning the dummy variables for regions.

| Sample    | Variables Included | Null Hypotheses                   | t Value | Degrees of Freedom |
|-----------|--------------------|-----------------------------------|---------|--------------------|
| CITY/TOWN | L,H,S,C,R**        | $\beta_{DETROIT}^* = \beta_{SOL}$ | .32***  | 126                |
|           | "                  | $\beta_{SOL} = \beta_{NOL}$       | .18     | 126                |
|           | "                  | $\beta_{DETROIT} = \beta_{NOL}$   | .16     | 126                |
| CITY/TOWN | H,S,C,R            | $\beta_{DETROIT} = \beta_{SOL}$   | .82     | 127                |
|           | "                  | $\beta_{SOL} = \beta_{NOL}$       | .78     | 127                |
|           | "                  | $\beta_{DETROIT} = \beta_{NOL}$   | .84     | 127                |
| RURAL     | L,H,S,C,R          | $\beta_{DETROIT} = \beta_{SOL}$   | .45     | 214                |
|           | "                  | $\beta_{SOL} = \beta_{NOL}$       | .97     | 214                |
|           | "                  | $\beta_{DETROIT} = \beta_{NOL}$   | .92     | 214                |
| RURAL     | H,S,C,R            | $\beta_{DETROIT} = \beta_{SOL}$   | .58     | 214                |
|           | "                  | $\beta_{SOL} = \beta_{NOL}$       | .93     | 215                |
|           | "                  | $\beta_{DETROIT} = \beta_{NOL}$   | 1.28    | 215                |

\*  $\beta_i$  (i = DETROIT, SOL, NOL) = the coefficient of the i<sup>th</sup> variable.

\*\* Variables included: L,H,S,C,R

L = Lagged dependent variable (COMP T-1)

H = Home influence variables (SES-S, M SCH-S, SELF-S, SCH-S)

S = School influence variables (PUP-S, RACE-S, EXP, MA, T/P)

C = Community influence variables (PUP-D, RACE-D, SES-D, M SCH-D, SELF-D, SCH-D)

R = Region Variables (DETROIT, SOL, NOL)

\*\*\* The t values given are absolute values. The sign of each t value corresponds to that of the estimated coefficient.

## CHAPTER IV

### THE IMPORTANCE OF GROUPS OF VARIABLES, INDIVIDUAL VARIABLES, AND THE RELATIONSHIP BETWEEN THE SHORT RUN AND LONG RUN MODELS

As noted earlier, we cannot estimate the model for DET schools using community variables as they are constant over all observations. Further, the degree of multicollinearity is high in the models for RURAL schools when community variables are included. Multicollinearity refers to the condition whereby one regressor is a linear combination of one or more other regressors. Perfect multicollinearity means that a regressor is an exact linear function of one or more other regressors. When this condition occurs estimation is impossible because the data matrix is singular and cannot be inverted. Although we may not have an exact linear function, it may be close. One way of checking for multicollinearity is to run a regression treating one of the original regressors as the dependent variable and the other regressors as the explanatory variables. The higher the  $R^2$ , the higher the degree of multicollinearity; and, with multicollinearity, it is not a matter of existence or non-existence but, rather, a matter of degree.<sup>1</sup> The problem with multicollinearity is that it affects the estimated variances of the parameters. In fact, the higher the multicollinearity, the larger the variances. This presents problems in testing hypotheses as we will reject hypotheses more frequently than they should be rejected.

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<sup>1</sup> Kmenta, op. cit., p. 380.

The reason a high degree of multicollinearity exists in the RURAL schools is that in cases where there is only one school in a district, the values of the community variables are identical to the values of the corresponding home and school variables. Though there are 234 RURAL schools, there are 214 RURAL districts, so there is a maximum of 20 (out of 234) schools for which the community variables are not identical to the corresponding home and school variables. Therefore, for the remainder of this study we shall consider only models excluding community variables for both the DET and the RURAL schools. We shall, however, continue to consider models with community variables for both METRO/SUBURB and CITY/TOWN schools.

#### 1. Home, School, and Community Variables as Groups

The next tests were with regard to the inclusion of the sets or proxy variables for the following: entering achievement, home, school, and (where appropriate) community influences. The object of these tests is to determine whether or not each of these groups of variables taken as a whole is significant in the production process. The maintained hypotheses are that the coefficients for any set of proxy variables are equal to zero. In other words, we are testing the hypotheses that all of the variables in any group (entering achievement, home, school, or community) could be excluded without significantly affecting the model, i.e., the contribution of these variables in explaining the dependent variable is insignificant. The results of these tests are given in Table 9 and the results of the estimations upon which they are based are given in Table 8.

Table 8. Estimated coefficients, (t values), and  $R^2$ 's for the following models:\*

- (1) For DETROIT and RURAL schools -- L,H,S; L,H; H,S; H; S.  
 (2) For METRO/SUBURB and CITY/TOWN schools -- L,H,S,C; L,H,S; L,H,C; L,S,C; H,S,C; H,S; H,C; S,C.

| Variable | DETROIT Schools |                |                 |                  |                |                 |
|----------|-----------------|----------------|-----------------|------------------|----------------|-----------------|
|          | L,H,S           | L,H            | L,S             | H,S              | H              | S               |
| Constant | -2.18           | -3.22          | 10.1            | -4.55            | -8.81          | 39.1            |
| COMP T-1 | .47<br>(4.13)** | .90<br>(13.2)  | -.80<br>(10.7)  |                  |                |                 |
| SES-S    | .22<br>(3.86)   | .07<br>(1.34)  |                 | .40<br>(8.96)    | .61<br>(9.46)  |                 |
| M SCH-S  | .07<br>(.82)    | -.09<br>(1.12) |                 | .19<br>(2.29)    | -.19<br>(1.17) |                 |
| SELF-S   | .15<br>(1.35)   | .06<br>(.48)   |                 | .29<br>(2.34)    | .60<br>(2.54)  |                 |
| SCH-S    | .08<br>(.88)    | .12<br>(1.10)  |                 | .07<br>(.67)     | .13<br>(.66)   |                 |
| PUP-S    | .0003<br>(.53)  |                | .0004<br>(.75)  | -.00008<br>(.13) |                | -.0004<br>(.42) |
| RACE-S   | 4.36<br>(4.68)  |                | 2.35<br>(3.00)  | 7.02<br>(9.30)   |                | 5.95<br>(5.07)  |
| EXP      | -1.47<br>(.87)  |                | -.14<br>(.08)   | -.25<br>(.13)    |                | 8.36<br>(3.31)  |
| MA       | .76<br>(.34)    |                | -.04<br>(.02)   | 1.95<br>(.77)    |                | 3.59<br>(.90)   |
| T/P      | -26.0<br>(.51)  |                | -84.2<br>(1.64) | 26.6<br>(.48)    |                | -50.4<br>(.60)  |
| $R^2$    | .9350           | .8992          | .9190           | .9168            | .6317          | .7750           |

"Continued"

Table 8. Continued.

| Variable       | <u>METRO/SUBURB Schools</u> |                 |                  |                  |
|----------------|-----------------------------|-----------------|------------------|------------------|
|                | L,H,S,C                     | L,H,S           | L,H,C            | L,S,C            |
| Constant       | 1.89                        | 3.65            | .30              | -1.41            |
| COMP T-1       | .64<br>(13.8)               | .63<br>(13.8)   | .69<br>(16.6)    | .77<br>(19.5)    |
| SES-S          | .23<br>(4.66)               | .22<br>(6.85)   | .25<br>(5.30)    |                  |
| M SCH-S        | .10<br>(1.17)               | .05<br>(1.00)   | -.002<br>(.03)   |                  |
| SELF-S         | -.10<br>(1.13)              | -.08<br>(1.58)  | -.14<br>(1.57)   |                  |
| SCH-S          | -.003<br>(.04)              | .06<br>(1.61)   | .02<br>(.31)     |                  |
| PUP-S          | -.0001<br>(.60)             | .00006<br>(.32) |                  | -.001<br>(.50)   |
| RACE-S         | 2.17<br>(3.03)              | 2.51<br>(5.16)  |                  | 2.24<br>(3.27)   |
| EXP            | -.79<br>(1.58)              | -.86<br>(1.78)  |                  | -.68<br>(1.28)   |
| MA             | .88<br>(1.55)               | .88<br>(1.57)   |                  | .50<br>(.84)     |
| T/P            | 19.2<br>(1.47)              | 20.6<br>(1.65)  |                  | 13.4<br>(.98)    |
| PUP-D          | .000004<br>(.59)            |                 | .000001<br>(.18) | .000004<br>(.67) |
| RACE-D         | .72<br>(.86)                |                 | 2.44<br>(4.36)   | .08<br>(.10)     |
| SES-D          | -.02<br>(.42)               |                 | -.07<br>(1.48)   | .14<br>(3.91)    |
| M SCH-D        | -.07<br>(.70)               |                 | .03<br>(.33)     | .04<br>(.75)     |
| SELF-D         | .02<br>(.14)                |                 | .09<br>(.82)     | -.08<br>(1.09)   |
| SCH-D          | .12<br>(1.52)               |                 | .09<br>(1.18)    | .11<br>(2.08)    |
| R <sup>2</sup> | .9297                       | .9284           | .9255            | .9201            |

"Continued"

Table 8. Continued.

| Variable       | <u>METRO/SUBURB (cont.)</u> |                  |                   |                  |
|----------------|-----------------------------|------------------|-------------------|------------------|
|                | H,S,C                       | H,S              | H,C               | S,C              |
| Constant       | 15.8                        | 15.2             | 14.3              | 14.7             |
| COMP T-1       |                             |                  |                   |                  |
| SES-S          | .55<br>(9.68)               | .52<br>(17.1)    | .75<br>(14.1)     |                  |
| M SCH-S        | .04<br>(.34)                | .01<br>(.20)     | -.32<br>(2.80)    |                  |
| SELF-S         | -.02<br>(.18)               | -.03<br>(.48)    | -.09<br>(.69)     |                  |
| SCH-S          | .03<br>(.36)                | .08<br>(1.47)    | .14<br>(1.57)     |                  |
| PUP-S          | -.0004<br>(1.56)            | -.0004<br>(1.51) |                   | -.0007<br>(2.02) |
| RACE-S         | 5.53<br>(6.11)              | 5.58<br>(9.65)   |                   | 9.68<br>(10.5)   |
| EXP            | -.11<br>(.16)               | .03<br>(.05)     |                   | .94<br>(1.11)    |
| MA             | 2.47<br>(3.29)              | 2.41<br>(3.28)   |                   | 2.39<br>(2.52)   |
| T/P            | 28.1<br>(1.60)              | 25.3<br>(1.52)   |                   | 7.50<br>(.34)    |
| PUP-D          | .000005<br>(.66)            |                  | .0000002<br>(.03) | .00001<br>(.96)  |
| RACE-D         | .25<br>(.22)                |                  | 4.70<br>(5.91)    | -3.55<br>(2.75)  |
| SES-D          | -.04<br>(.55)               |                  | -.21<br>(3.07)    | .50<br>(10.6)    |
| M SCH-D        | -.05<br>(.38)               |                  | .27<br>(1.95)     | .02<br>(.17)     |
| SELF-D         | -.04<br>.23                 |                  | .09<br>.54        | -.03<br>.23      |
| SCH-D          | .08<br>(.80)                |                  | .01<br>(.06)      | .10<br>(1.08)    |
| R <sup>2</sup> | .8722                       | .8711            | .8387             | .7915            |

"Continued"



Table 8. Continued.

| Variable       | <u>CITY/TOWN Schools</u> |                  |                  |                  |
|----------------|--------------------------|------------------|------------------|------------------|
|                | L,H,S,C                  | L,H,S            | L,H,C            | L,S,C            |
| Constant       | 1.01                     | 4.19             | -.16             | .65              |
| COMP T-1       | .59<br>(8.97)            | .63<br>(9.43)    | .65<br>(10.9)    | .64<br>(10.5)    |
| SES-S          | .17<br>(1.55)            | .18<br>(2.93)    | .20<br>(1.93)    |                  |
| M SCH-S        | -.06<br>(.41)            | -.006<br>(.08)   | -.08<br>(.58)    |                  |
| SELF-S         | -.12<br>(1.12)           | -.07<br>(1.11)   | -.13<br>(1.15)   |                  |
| SCH-S          | .07<br>(.51)             | .15<br>(2.67)    | .06<br>(.46)     |                  |
| PUP-S          | -.00006<br>(.20)         | -.0003<br>(1.15) |                  | -.0001<br>(.35)  |
| RACE-S         | 3.29<br>(1.56)           | 3.12<br>(2.28)   |                  | 3.82<br>(1.88)   |
| EXP            | .65<br>(.78)             | .35<br>(.43)     |                  | .41<br>(.49)     |
| MA             | .71<br>(.89)             | .66<br>(.83)     |                  | .76<br>(.97)     |
| T/P            | -5.89<br>(.28)           | -23.1<br>(1.10)  |                  | -11.2<br>(.54)   |
| PUP-D          | -.0001<br>(3.60)         |                  | -.0001<br>(3.79) | -.0001<br>(3.47) |
| RACE-D         | -1.06<br>(.43)           |                  | 1.49<br>(1.02)   | -1.66<br>(.69)   |
| SES-D          | .03<br>(.23)             |                  | -.001<br>(.01)   | .17<br>(2.56)    |
| M SCH-D        | .11<br>(.67)             |                  | .14<br>(.86)     | .05<br>(.58)     |
| SELF-D         | .11<br>(.77)             |                  | .10<br>(.70)     | -.009<br>(.11)   |
| SCH-D          | .06<br>(.38)             |                  | .06<br>(.43)     | .12<br>(1.81)    |
| R <sup>2</sup> | .8194                    | .7989            | .8119            | .8128            |

"Continued"

Table 8. Continued.

| Variable       | CITY/TOWN Schools (cont.) |                  |                  |                  |
|----------------|---------------------------|------------------|------------------|------------------|
|                | H,S,C                     | H,S              | H,C              | S,C              |
| Constant       | 14.0                      | 18.8             | 17.8             | 14.8             |
| COMP T-1       |                           |                  |                  |                  |
| SES-S          | .57<br>(4.37)             | .52<br>(7.76)    | .70<br>(5.45)    |                  |
| M SCH-S        | -.12<br>(.68)             | -.10<br>(1.09)   | -.20<br>(1.06)   |                  |
| SELF-S         | -.24<br>(1.70)            | -.15<br>(1.81)   | -.24<br>(1.63)   |                  |
| SCH-S          | -.05<br>(.31)             | .22<br>(2.98)    | -.09<br>(.49)    |                  |
| PUP-S          | .003<br>(.66)             | -.00009<br>(.23) |                  | .0003<br>(.63)   |
| RACE-S         | 6.81<br>(2.59)            | 6.23<br>(3.66)   |                  | 10.7<br>(4.12)   |
| EXP            | 2.88<br>(2.85)            | 2.90<br>(2.93)   |                  | 2.86<br>(2.66)   |
| MA             | 1.28<br>(1.28)            | .97<br>(.96)     |                  | 1.38<br>(1.31)   |
| T/P            | 29.8<br>(1.13)            | 9.54<br>(.36)    |                  | 21.1<br>(.76)    |
| PUP-D          | -.0001<br>(3.75)          |                  | -.0001<br>(3.02) | -.0001<br>(3.45) |
| RACE-D         | -2.37<br>(.76)            |                  | 1.89<br>(.94)    | -5.75<br>(1.80)  |
| SES-D          | -.06<br>(.39)             |                  | -.08<br>(.55)    | .50<br>(6.08)    |
| M SCH-D        | .07<br>(.33)              |                  | .11<br>(.49)     | -.07<br>(.59)    |
| SELF-D         | .17<br>(.95)              |                  | .07<br>(.37)     | -.05<br>(.45)    |
| SCH-D          | .27<br>(1.42)             |                  | .37<br>(1.89)    | .21<br>(2.41)    |
| R <sup>2</sup> | .7069                     | .6664            | .6451            | .6569            |

"Continued"

Table 8. Continued.

| Variable       | RURAL Schools    |                |                  |                  |               |                  |
|----------------|------------------|----------------|------------------|------------------|---------------|------------------|
|                | L,H,S            | L,H            | L,S              | H,S              | H             | S                |
| Constant       | 10.5             | 10.9           | 17.6             | 28.6             | 33.0          | 41.5             |
| COMP T-1       | .54<br>(10.7)    | .56<br>(12.2)  | .57<br>(11.4)    |                  |               |                  |
| SES-S          | .15<br>(2.95)    | .17<br>(3.49)  |                  | .25<br>(3.97)    | .29<br>(4.69) |                  |
| M SCH-S        | -.004<br>(.07)   | -.002<br>(.04) |                  | .002<br>(.04)    | -.03<br>(.46) |                  |
| SELF-S         | .03<br>(.53)     | .02<br>(.43)   |                  | .003<br>(.04)    | -.01<br>(.17) |                  |
| SCH-S          | .03<br>(.64)     | .03<br>(.75)   |                  | .07<br>(1.21)    | .10<br>(1.80) |                  |
| PUP-S          | -.0005<br>(1.66) |                | -.0004<br>(1.33) | -.0007<br>(1.97) |               | -.0006<br>(1.74) |
| RACE-S         | 2.81<br>(1.59)   |                | 4.70<br>(2.78)   | 4.90<br>(2.27)   |               | 8.06<br>(3.87)   |
| EXP            | -.47<br>(.66)    |                | -.66<br>(.95)    | 1.91<br>(2.32)   |               | 1.85<br>(2.24)   |
| MA             | 1.51<br>(1.62)   |                | 1.57<br>(1.67)   | 2.06<br>(1.81)   |               | 2.16<br>(1.85)   |
| T/P            | -3.63<br>(.28)   |                | -1.73<br>(.13)   | 4.63<br>(.29)    |               | 7.64<br>(.46)    |
| R <sup>2</sup> | .4772            | .4578          | .4510            | .2068            | .1067         | .1378            |

\*

Variables included: L,H,S,C

L = Lagged dependent variable (COMP T-1)

H = Home influence variables (SES-S, M SCH-S, SELF-S, SCH-S)

S = School influence variables (PUP-S, RACE-S, EXP, MA, T/P)

C = Community influence variables (PUP-D, RACE-D, SES-D,  
M SCH-D, SELF-D, SCH-D)

No statistics are available for constant terms other than the estimated value of the coefficient due to the nature of the statistical routines used on the MSU CDC 6500 computer.

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The t values given are absolute values. The sign of each t value corresponds to that of the estimated coefficient.

Table 9. Tests of Hypotheses Concerning the Inclusion of the Lagged Dependent Variable, and of Sets of Variables for Home, School, and Community Influences by subsample.

| Sample           | Models Compared*  | Null Hypothesis | F Value | Degrees of Freedom | Critical Value of F at the .05 level | Hypothesis Accepted (A) Rejected (R) |
|------------------|-------------------|-----------------|---------|--------------------|--------------------------------------|--------------------------------------|
| DETROIT          | L,H,S vs. L,H     | $\beta_S = 0$   | 7.06    | 5/61               | 2.37                                 | R                                    |
|                  | L,H,S vs. L,S     | $\beta_H = 0$   | 4.36    | 4/61               | 2.52                                 | R                                    |
|                  | L,H,S vs. H,S     | $\beta_L = 0$   | 17.08   | 1/61               | 4.00                                 | R                                    |
|                  | H,S vs. H         | $\beta_S = 0$   | 50.25   | 5/62               | 2.37                                 | R                                    |
|                  | H,S vs. S         | $\beta_H = 0$   | 27.33   | 4/62               | 2.52                                 | R                                    |
|                  | L,H,S,C vs. L,H,S | $\beta_C = 0$   | 0.72    | 6/232              | 2.14                                 | A                                    |
| METRO/<br>SUBURB | L,H,S,C vs. L,H,C | $\beta_S = 0$   | 2.77    | 5/232              | 2.26                                 | R                                    |
|                  | L,H,S,C vs. L,S,C | $\beta_H = 0$   | 7.92    | 4/232              | 2.41                                 | R                                    |
|                  | L,H,S,C vs. H,S,C | $\beta_L = 0$   | 189.76  | 1/232              | 3.89                                 | R                                    |
|                  | H,S,C vs. H,S     | $\beta_C = 0$   | 0.33    | 6/233              | 2.14                                 | A                                    |
|                  | H,S,C vs. H,C     | $\beta_S = 0$   | 12.20   | 5/233              | 2.26                                 | R                                    |
|                  | H,S,C vs. S,C     | $\beta_H = 0$   | 36.79   | 4/233              | 2.41                                 | R                                    |

"Continued"

Table 9. Continued.

| Sample        | Models Compared*       | Null Hypothesis | F Value | Degrees of Freedom | Critical Value of F at the .05 level | Hypothesis Accepted (A) Rejected (R) |
|---------------|------------------------|-----------------|---------|--------------------|--------------------------------------|--------------------------------------|
| CITY/<br>TOWN | L, H, S, C vs. L, H, S | $\beta_C = 0$   | 2.44    | 6/129              | 2.17                                 | R                                    |
|               | L, H, S, C vs. L, H, C | $\beta_S = 0$   | 1.07    | 5/129              | 2.29                                 | A                                    |
|               | L, H, S, C vs. L, S, C | $\beta_H = 0$   | 1.18    | 4/129              | 2.44                                 | A                                    |
|               | L, H, S, C vs. H, S, C | $\beta_L = 0$   | 80.36   | 1/129              | 3.92                                 | R                                    |
|               | H, S, C vs. H, S       | $\beta_C = 0$   | 2.99    | 6/130              | 2.17                                 | R                                    |
|               | H, S, C vs. H, C       | $\beta_S = 0$   | 4.57    | 5/130              | 2.29                                 | R                                    |
| RURAL         | H, S, C vs. S, C       | $\beta_H = 0$   | 5.50    | 4/130              | 2.44                                 | R                                    |
|               | L, H, S vs. L, H       | $\beta_S = 0$   | 1.66    | 5/223              | 2.26                                 | A                                    |
|               | L, H, S vs. L, S       | $\beta_H = 0$   | 2.79    | 4/223              | 2.41                                 | R                                    |
|               | L, H, S vs. H, S       | $\beta_L = 0$   | 30.03   | 1/223              | 3.89                                 | R                                    |
|               | H, S vs. H             | $\beta_S = 0$   | 5.74    | 5/224              | 2.26                                 | R                                    |
|               | H, S vs. S             | $\beta_H = 0$   | 19.49   | 4/224              | 2.41                                 | R                                    |

\* Designated by sets of variables included:

L = Lagged dependent variable (COMP T-1)

H = Home influence variables (SES-S, M SCH-S, SELF-S, SCH-S)

S = School influence variables (PUP-S, RACE-S, EXP, MA, T/P)

C = Community influence variables (PUP-D, RACE-D, SES-D, M SCH-D, SELF-D, SCH-D)

The results may be summarized as follows for the models which include the lagged dependent variable:

1) DET schools - we reject (at the .05 level) the hypotheses that the coefficients of the set of variables corresponding to beginning achievement or home influence or school influence is zero. That is, each of these groups of variables, taken as a whole, contributes something as an input. We cannot exclude any of these groups without significantly altering our results.

2) METRO/SUBURB schools - we reject (at the .05 level of significance) the hypotheses that each of the coefficients of the set of variables corresponding to beginning achievement or home influence or school influence is zero. We do not reject the hypothesis for the coefficients of the community variables. That is, taken as groups, the variables corresponding to beginning achievement, home, and school influences cannot be excluded from the model without significantly changing it. On the other hand, there is no significant difference between the model including the community variables as a group and the model excluding the community variables.

3) CITY/TOWN schools - we reject the hypotheses that the coefficients for beginning achievement or community influence are zero. We do not reject the hypotheses that the coefficients for home influence or school influence are zero.

4) RURAL schools - we reject the hypotheses that the coefficients for beginning achievement or home influence is zero. We do not reject the hypothesis that the coefficients for school influence are zero.

For the models excluding the lagged dependent variable the results may be summarized as follows:

1) DET schools - we reject the hypotheses that the coefficients for home influence or school influence are equal to zero.

2) METRO/SUBURB schools - we reject the hypotheses that the coefficients for home influence or school influence are zero. We do not reject the hypothesis that the coefficients of community influence are zero.

3) CITY/TOWN schools - we reject the hypotheses that the coefficients for home influence or school influence or community influence are zero.

4) RURAL schools - we reject the hypotheses that the coefficients for home influence or school influence are zero.<sup>2</sup>

#### Summary of Results for Groups of Variables

We find that the groups of variables corresponding to beginning achievement, home and school influences are, usually, important (in the sense that the estimated coefficients are significantly different from zero). The results with respect to the community influence variables as a group are mixed. Before beginning a discussion of the individual variables it should be pointed out that, in order to give the reader a feel for the magnitudes involved, I have relied primarily upon the application of the results as presented in Chapter V.

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<sup>2</sup> The conclusions with respect to home influences are to be expected. Further, the  $\beta$  weights (see Tables 10a and 10b) conform to the theory that schools make little or no difference for achievement. The only school variable with a  $\beta$  weight greater than 0.1 is the percentage of white students in the school. These results accord well with those found by Averch, et al., quoted here on pages 111 and 112.

## 2. Individual Variables

### School or Policy Variables

We now consider those variables which are policy variables for individual school districts. These are the school influence variables: (1) the number of pupils in each school, (2) the percentage white pupils in each school, (3) the percentage teachers with five or more years experience in each school, (4) the percentage teachers with master's degrees in each school, and (5) the teacher-pupil ratio in each school.

Considering first the number of pupils in each school, we find that this is significant only for RURAL schools and then only in the long run.<sup>3</sup> The percentage of teachers with five or more years experience is significant only for CITY/TOWN and RURAL schools and then only in the long run. The percentage of teachers with a master's degree is significant only for METRO/SUBURB schools and then only in the long run. The teacher-pupil ratio is never significant within the range of observations. The percentage of white students in the school is significant for DET and METRO/SUBURB schools in the short run and for all schools in the long run. That is, in the short run, none of the policy variables are consistently significant for achievement, while in the long run only one of the policy variables, the percentage of white students in school, is consistently significant for achievement.

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<sup>3</sup>

In order to give the reader, at this point, some idea of the relative importance of the variables, the  $\beta$  weights are given in Tables 10a and 10b.



Table 10a. Beta weights for models including the lagged dependent variable, by subsamples.

| Variable | DETROIT | METRO/<br>SUBURB | CITY/<br>TOWN | RURAL |
|----------|---------|------------------|---------------|-------|
| COMP T-1 | .46     | .62              | .58           | .59   |
| SES-S    | .28     | .30              | .19           | .16   |
| M SCH-S  | .04     | .04              | -.04          | -.004 |
| SELF-S   | .05     | -.05             | -.09          | .03   |
| SCH-S    | .03     | -.001            | .05           | .03   |
| PUP-S    | .02     | -.01             | -.008         | -.09  |
| RACE-S   | .39     | .13              | .14           | .08   |
| EXP      | -.06    | -.04             | .04           | -.04  |
| MA       | .02     | .04              | .04           | .09   |
| T/P      | -.02    | .03              | -.01          | -.01  |
| PUP-D    |         | .01              | -.16          |       |
| RACE-D   |         | .04              | -.04          |       |
| SES-D    |         | -.03             | .03           |       |
| M SCH-D  |         | -.03             | .06           |       |
| SELF-D   |         | .006             | .06           |       |
| SCH-D    |         | .04              | .04           |       |

Table 10b. Beta weights for models excluding the lagged dependent variable, by subsamples.

| Variable | DETROIT | METRO/<br>SUBURB | CITY/<br>TOWN | RURAL |
|----------|---------|------------------|---------------|-------|
| SES-S    | .50     | .73              | .64           | .27   |
| M SCH-S  | .11     | .02              | -.07          | .002  |
| SELF-S   | .10     | -.01             | -.17          | .002  |
| SCH-S    | .03     | .01              | -.04          | .07   |
| PUP-S    | -.007   | -.04             | .03           | -.13  |
| RACE-S   | .63     | .32              | .29           | .15   |
| EXP      | -.009   | -.005            | .17           | .16   |
| MA       | .04     | .10              | .08           | .12   |
| T/P      | .02     | .04              | .06           | .02   |
| PUP-D    |         | .02              | -.21          |       |
| RACE-D   |         | .01              | -.09          |       |
| SES-D    |         | -.04             | -.06          |       |
| M SCH-D  |         | -.02             | .03           |       |
| SELF-D   |         | -.01             | .09           |       |
| SCH-D    |         | .03              | .17           |       |

This conclusion is not inconsistent with that of Averch, et al.<sup>4</sup> Their findings, based on a survey of the literature, are (1) "School resources are seldom important determinants of student outcomes," and (2) "No school resource is consistently related to student outcomes."

Our finding with respect to the percentage of white students in the school is, again, consistent with Averch, et al.<sup>5</sup> They state:

1. There is no strong evidence that student-body effects exist. In particular, there is no evidence that the racial composition of a student body affects the performance of individual members of that student body.

and

2. There is no strong evidence to the contrary. Many researchers have argued that alternative and more likely hypotheses have led to the results' being interpreted as student effects. But no researcher has shown that student-body effects do not exist.

This is so because we cannot determine from our data if the increase in average achievement associated with increases in percentage white in the school means that all students are doing better, or rather that white students do better and the mix changes the average, even if no individual scores change.

#### Home Influence Variables

With respect to home influence (SES, M SCH-S, SELF-S, SCH-S) we find the following: (1) socioeconomic status is significant for all schools in both the short and long runs;

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<sup>4</sup> Averch, et al., op. cit.

<sup>5</sup> Averch, et al., op. cit.

(2) attitude towards the importance of doing well in school is significant for DET schools only and then only in the short run;  
 (3) self-perception: the results are identical with those for attitude towards the importance of doing well in school; and  
 (4) attitude towards school: this variable was never significant.  
 These findings are not inconsistent with those of Averch, et al., who state:<sup>6</sup>

Background factors are always important  
determinants of educational outcomes.

The socioeconomic status of a student's  
family and community is consistently  
related to his educational outcome.

#### Community Influence Variables.

The community influences were included in the models for METRO-SUBURB and CITY/TOWN only. Of the six variables representing this influence the only one that is ever significant is the number of students in the district. This number is significant for CITY/TOWN schools only in both the long and short runs.

#### Summary of Results for Individual Variables.

We find, then, that in DET schools in the short run only entering achievement, socioeconomic status, and the percentage of white in the school are important, while in the long run both attitudes towards the importance of doing well in school and self-perception are important along with socioeconomic status and percentage white in the school.

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<sup>6</sup> Averch, et al., op. cit.

For METRO/SUBURB schools in the short run the set of important variables is the same as for DET schools, while in the long run the percentage of teachers with a master's degree becomes important as well.

Entering achievement and the number of pupils in the district are the only important variables in the short run for CITY/TOWN schools, while in the long run socioeconomic status, percentage white in the school, and percentage teachers with five or more years experience become important as well.

The only important variables in the short run for RURAL schools are entering achievement and socioeconomic status, while in the long run the number of pupils in the school, the percentage of white students in the school, and the percentage of teachers with five or more years experience are important.<sup>7</sup>

We infer the following from our analysis:

1) There may be multiple production functions for education.

2) Regional differences affect the output of education.

3) Home, school, and community influences on achievement accumulate over time.

4) Home influences are significant in the short run for DET, METRO/SUBURB, and RURAL schools; in the long run they are significant for all schools.

5) School influences are significant for DET and METRO/SUBURB schools in the short run, and for all schools in the long run.

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<sup>7</sup> These results are not inconsistent with those of Averch, et al.

6) Community influences are significant in both the short run and long run for CITY/TOWN schools. They are not significant for METRO/SUBURB schools. We lack evidence of their effects on DET and RURAL schools.

### 3. Relationship Between Coefficients in Models Including Lagged Dependent Variables and Models Excluding Lagged Dependent Variables

We also compared the coefficients from the incremental model with those from the model in which the lagged dependent variable was not included. Based on our theory and assumptions we note that the coefficients for home influences and community influences in the models excluding the lagged dependent variable should be  $k_1 = \sum_{h=0}^{11} \alpha_1^h$  ( $\alpha_1$  = coefficient of the lagged dependent variable in the incremental model) times those in the incremental model. The coefficients for school influences should be  $k_2 = \sum_{h=0}^7 \alpha_1^h$  ( $\alpha_1$  as defined above) times those in the incremental models. These relationships were fully developed on pages 52 to 56 and 58 to 62, using the assumption that Home, School, and Community influences vary, if at all, only slightly over time. Then, by successive substitutions for the lagged dependent variable the above relationships were generated. We tested these relationships in order to see if the results tend to confirm or deny both our theory and our assumptions. A lack of significant differences between the estimated values of the coefficients and the predicted values of the coefficients tends to confirm, whereas the presence of significant differences tends to deny the existence of these relationships. This is so because the relationships were based on both the theory and the assumptions. Suppose, for example, we were far wrong in theorizing

that a linear function was a close approximation to reality. Consider instead that some far different function was proper. It is difficult to believe that the relationship postulated above would hold also for the other function when we had substituted for the lagged dependent variable and used the same assumption about the course of the values of the variables over time.

We tested to determine if the estimated coefficients from the models excluding the lagged dependent variable were the same as those we would have predicted based upon the estimated coefficients in the incremental models. The results are given in Tables 11 through 14.

1) For the DET schools none of the nine estimated coefficients (of which four were significantly different from zero) were significantly different from the predicted coefficients at the .05 level.

2) For the METRO/SUBURB schools - Of the three stimated coefficients which were significantly different from zero, none was significantly different from the predicted coefficients at the .05 level. Of the other 12 estimated coefficients, three were significantly different from the predicted coefficients at the .05 level, of which three only one differs significantly at the .01 level.

3) For the CITY/TOWN schools - None of the 15 estimated coefficients (of which four were significantly different from zero) is significantly different from the predicted coefficients at the .05 level.

Table 11. Comparison of estimated coefficients and predicted coefficients for the DETROIT Schools.

$$\alpha_1 = .47 \quad \sum_{h=0}^7 \alpha_1^h = 1.89 \quad \sum_{h=0}^{11} \alpha_1^h = 1.89$$

| Variable | Predicted Value<br>of Coefficients | Estimated Value<br>of Coefficients | t<br>Value* |
|----------|------------------------------------|------------------------------------|-------------|
| SES-S    | .42                                | .40                                | .5          |
| M SCH-S  | .13                                | .19                                | .75         |
| SELF-S   | .28                                | .29                                | .08         |
| SCH-S    | .15                                | .07                                | .8          |
| PUP-S    | .00014                             | -.00008                            | .37         |
| RACE-S   | 8.24                               | 7.02                               | 1.63        |
| EXP      | -2.78                              | -.25                               | 1.36        |
| MA       | 1.44                               | 1.95                               | .2          |
| T/P      | -49.16                             | 16.57                              | 1.38        |

\* The t values given are absolute values. The sign of each t value corresponds to that of the estimated coefficient.



Table 12. Comparison of estimated coefficients and predicted coefficients for the METRO/SUBURB Schools.

$$\alpha_1 = .64 \quad \sum_{h=0}^7 \alpha_1^h = 2.67 \quad \sum_{h=0}^{11} \alpha_1^h = 2.74$$

| Variable | Predicted Value<br>of Coefficients | Estimated Value<br>of Coefficients | t<br>Value* |
|----------|------------------------------------|------------------------------------|-------------|
| SES-S    | .63                                | .55                                | 1.33        |
| M SCH-S  | .27                                | .04                                | 1.92        |
| SELF-S   | -.27                               | -.02                               | 2.08        |
| SCH-S    | -.0071                             | .0218                              | .38         |
| PUP-S    | -.000035                           | -.00044                            | .30         |
| RACE-S   | 5.79                               | 5.53                               | .29         |
| EXP      | -2.11                              | -.11                               | 2.94        |
| MA       | 2.35                               | 2.47                               | .16         |
| T/P      | 51.37                              | 28.14                              | 1.32        |
| PUP-D    | .0000095                           | .0000053                           | .53         |
| RACE-D   | 1.97                               | .25                                | 1.52        |
| SES-D    | -.05                               | -.04                               | .14         |
| M SCH-D  | -.19                               | -.05                               | 1.00        |
| SELF-D   | .05                                | -.04                               | .60         |
| SCH-D    | .33                                | .08                                | 2.27        |

\* The t values given are absolute values. The sign of each t value corresponds to that of the estimated coefficient.

Table 13. Comparison of estimated coefficients and predicted coefficients for the CITY/TOWN Schools.

$$\alpha_1 = .59 \quad \sum_{h=0}^7 \alpha_1^h = 2.42 \quad \sum_{h=0}^{11} \alpha_1^h = 2.46$$

| Variable | Predicted Value<br>of Coefficients | Estimated Value<br>of Coefficients | t<br>Value* |
|----------|------------------------------------|------------------------------------|-------------|
| SES-S    | .42                                | .57                                | 1.15        |
| M SCH-S  | -.15                               | -.12                               | .17         |
| SELF-S   | -.30                               | -.24                               | .43         |
| SCH-S    | .17                                | -.05                               | 1.29        |
| PUP-S    | .00015                             | .00025                             | .25         |
| RACE-S   | 7.69                               | 6.81                               | .44         |
| EXP      | 1.57                               | 2.88                               | 1.30        |
| MA       | 1.72                               | 1.28                               | .44         |
| T/P      | -14.25                             | 29.81                              | 1.65        |
| PUP-D    | -.00020                            | -.00015                            | 1.25        |
| RACE-D   | -2.61                              | -2.37                              | .08         |
| SES-D    | .07                                | -.06                               | .87         |
| M SCH-D  | .27                                | .07                                | .95         |
| SELF-D   | .27                                | .17                                | .56         |
| SCH-D    | .15                                | .27                                | .63         |

\* The t values given are absolute values. The sign of each t value corresponds to that of the estimated coefficient.

Table 14. Comparison of estimated coefficients and predicted coefficients for the RURAL Schools.

$$\alpha_1 = .54 \quad \sum_{h=0}^7 \alpha_1^h = 2.17 \quad \sum_{h=0}^{11} \alpha_1^h = 2.18$$

| Variable | Predicted Value<br>of Coefficients | Estimated Value<br>of Coefficients | t<br>Value* |
|----------|------------------------------------|------------------------------------|-------------|
| SES-S    | .33                                | .25                                | 1.33        |
| M SCH-S  | -.0085                             | .0023                              | .15         |
| SELF-S   | .065                               | .0025                              | .90         |
| SCH-S    | .063                               | .067                               | .08         |
| PUP-S    | -.0011                             | -.00072                            | .95         |
| RACE-S   | 6.10                               | 4.90                               | .56         |
| EXP      | -1.02                              | 1.91                               | 3.57        |
| MA       | 3.28                               | 2.06                               | 1.07        |
| T/P      | -7.88                              | 4.63                               | .78         |

\* The t values given are absolute values. The sign of each t value corresponds to that of the estimated coefficient.

4) For the RURAL schools - Of the four estimates coefficients which were significantly different from zero, one was significantly different from its predicted value. None of the other five estimated coefficients was significantly different from its predicted value.

In general, these results seem to confirm the relationship between the two types of models. This is an interesting result in that it tends to confirm the idea that home, school, and community influences on achievement accumulate. That is, there is a carryover from one period to the next, with the whole effect of the influence not being spent in the period in which it is exerted.

The results also shed light on the nature of our two models. The lagged model is a short-run model and shows the initial effects of changes in the variables. The model without the lagged dependent variable is a longer-run model and shows the full effects of a changed level of input, when the new level is maintained in each period after the change.<sup>8</sup> We note that, in general, the long run effects are between 1.9 and 2.8 times the short run effects. Therefore, the full effects of a permanent change in the level of a variable will not be felt immediately. In fact, the effects of such a change will continue to increase over time, although at a decreasing rate. This indicates that there is a type of production relation somewhat analogous to the standard one which exhibits

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<sup>8</sup> To the best of my knowledge this is the first attempt to relate models dealing with achievement in such a fashion. Success here may be of some value in overcoming data deficiencies, particularly with respect to the lack of adequate time series.

diminishing marginal productivity. That is, if we, say, doubled an input in any time period we would expect, based on our estimates of a linear function, that the marginal productivity of each additional unit of the input would be the same as for that of each of the starting units of the input. In addition, the total additional productivity of the additional units of input would be equal to the total productivity of the starting units of the input. That is, doubling the units of an input doubles the productivity of an input. Thus, within a given period we do not expect to find diminishing marginal productivity. If on the other hand we, say, doubled a particular input, not by adding the extra units of the input in the same time period, but rather by adding them in the next time period, the results to be expected are considerably different. In this case the marginal productivity of the additional units is equal to that of the original units in the starting period. But, the productivity of the original units is diminished in the succeeding period so that the total productivity of all of the units of input is less than twice that of the original units of input. Thus there is a type of intertemporal diminishing marginal productivity. In the standard case diminishing marginal productivity is due to an increase in some input given a fixed amount of some one or more other inputs. In our case the result does not depend on the levels, or changes in the levels, of the inputs but, rather, on the fact that the impact of an input diminishes over time. That is, in our model prior output enters into future production as an input. However, since the coefficients of prior output are less than one, the full value of the prior output is not retained as such, i.e., the

full value is not converted to future output.<sup>9</sup> Since other prior inputs enter into future output via their incorporation into prior output, so too the full impact of prior inputs is not sustained. An example may clarify this matter.

Consider the case of a man digging a well. Suppose he can dig continuously at the rate of one cubic yard per hour. If we take as our unit of labor a one hour period, then one man hour of labor is one unit of labor; and, the marginal productivity of labor in this undertaking is one cubic foot of a well. Assume the normal work day is eight hours. Suppose, now, that every night there is a wind storm or an earth tremor which causes some earth to fall back into the well. Looking at the project at the end of the first work day, we find, a hole measuring eight cubic yards; the marginal productivity of labor has been constant at the rate of one cubic yard per hour. At the end of the second work day we find, however, that the hole is not 16 cubic yards in volume, but rather somewhat less due to the working of the natural forces. Nevertheless, the marginal productivity for any given time period, viewed at the end

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<sup>9</sup> We tested to see if the coefficients of the lagged dependent variable were significantly less than one. The hypotheses were:

$$H_0: 1 - \alpha_1 = 0$$

$$H_a: 1 - \alpha_1 > 0$$

where:  $\alpha_1$  is the coefficient of the lagged dependent variable. We used a one-tailed t test. The t values were as follows:

For the Detroit model, 4.27;  
 For the METRO/SUBURB model, 13.91;  
 For the CITY/TOWN model, 8.94; and  
 For the RURAL model, 10.30.

We rejected the hypothesis that the coefficients of the lagged dependent variable were equal to one.

of that time period, is the same as for any other time period. Yet, the sum of the marginal products is not equal to the total product. In fact, viewed from the perspective of the end of the second work day, the total product of the first work day does not appear to have been eight cubic yards and, of course, it does not appear that the marginal productivity on the first day was constant at the rate of one cubic yard. Instead, the total product appears to have been less than eight cubic yards, and assuming one viewed the results knowing the marginal product of labor to be constant, the marginal productivity appears to be less than one cubic yard. This effect was caused in our example by natural forces. In general we would observe this effect whenever there is depreciation of the output, before it (the output) is complete, i.e., during the production process. In other words, whenever the depreciation of an output commences before production of the output is complete, we would observe this effect which appears as what may be called intertemporal diminishing marginal productivity.

This effect is apparent, inter alia, in the educational process. In the case of education, however, it is not wind storms or earth tremors that cause the erosion of output, rather, it is a phenomenon common to everyone. Everyone has experienced the loss of some piece of knowledge. In fact, it is interesting to speculate somewhat on this point. Should we consider as wasted any efforts made to acquire knowledge which is later lost? Is the marginal product of those efforts zero? Since one rarely knows exactly which knowledge will be lost and, since future knowledge builds on past knowledge, it seems we should not consider such efforts wasted.

But, in specific cases, if one knows a certain bit of knowledge will be lost before it can be built upon, perhaps one should consider the efforts expended in acquiring it wasted. I think for example of the student who memorizes a date for a history test fully expecting (and having his expectation realized) that he will forget the moment the exam is over.



## CHAPTER V

### SIMULATION OF CROSS DISTRICT BUSING FOR RACIAL BALANCE

The purpose of this chapter is to illustrate our results through an application. I intend to show the effects, as predicted by our results, of certain changes in schools. More specifically, we shall look at the predicted effects of cross-district busing for the purpose of racial balance in a large metropolitan area, namely the Detroit metropolitan area.

I have chosen to illustrate the results in this manner for two reasons: (1) cross-district busing for racial balance is a major legal issue of the present time; and, in addition, it has important political, economic, social, and educational overtones; and (2) at the time this is being written a major decision concerning this issue and involving the Detroit metropolitan area is under appeal to the U.S. Court of Appeals for the Sixth Circuit in the case generally referred to as Bradley v. Milliken.<sup>1</sup>

#### 1. Bradley v. Milliken

At this point I will briefly summarize the state of the case as it exists at the time of this writing, based upon the

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<sup>1</sup> The official name of the case is Ronald Bradley, et al., Plaintiffs v. William G. Milliken, et al., Defendants and Detroit Federation of Teachers, Local 231, American Federation of Teachers, AFL-CIO, Defendant - Intervenor and Denise Magdowski, et al., Defendants - Intervenor et al. United States District Court, Eastern District of Michigan, Southern Division. Civil Action No: 35257.

rulings and orders of the Federal Courts as explained to me by Mr. George McCargar of the Office of the Attorney General of the State of Michigan.

The Honorable Stephen J. Roth, the judge who presided over the case in District court, ruled on September 27, 1971 that, "illegal segregation exists in the public schools of the city of Detroit as a result of a course of conduct on the part of the State of Michigan and the Detroit Board of Education."<sup>2</sup> He then:

directed the school board defendants, City, and State to develop and submit plans of desegregation, designed to achieve the greatest possible degree of actual desegregation taking into account the practicalities of the situation.<sup>3</sup>

Although a number of plans for desegregation involving the City only were submitted, they were all rejected on the grounds that "none of the plans would result in the desegregation of the public schools of the Detroit School District."<sup>4</sup>

Similarly, a number of plans for desegregation of the metropolitan area were submitted. These plans proposed, "to incorporate, geographically, most -- and in one instance, all -- of the three-county area of Wayne, Oakland, and Macomb."<sup>5</sup> However, none of these proposals was completely acceptable to the court. The court

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<sup>2</sup> Bradley v. Milliken; Ruling on Desegregation Area and Order for Development of Plan of Desegregation.

<sup>3</sup> Ibid.

<sup>4</sup> Ibid.

<sup>5</sup> Ibid.

therefore decided to "draw upon the resources of the parties to devise, pursuant to its direction, a constitutional plan of desegregation of the Detroit public schools."<sup>6</sup>

A plan of desegregation, involving 52 of the 86 school districts in the three county metropolitan area, is under appeal in the Federal Courts. The object of this appeal is to prevent cross-district busing, i.e., to reverse the order, at least with respect to the establishment of a desegregation plan involving districts other than the Detroit School District. This plan requires the racial composition of the students in each involved school to be almost exactly equal to the racial composition of the students in the involved areas as a whole. That is, the ratio of white to minority students in each affected school must be almost exactly equal to the ratio of white to minority students in affected areas as a whole. In the entire area covered by this plan white students comprise approximately 75% of the school population. Since the aforementioned appeal was made, an additional appeal has been made. Unlike the former appeal, this one seeks not to destroy the cross-district busing plan, but rather to expand the geographical area involved. This latter appeal seeks to extend the plan to cover all 86 of the school districts in the three county metropolitan area. In the area covered by this plan, white students comprise approximately 80% of the student population. We note that for the 1970-1971 school year, the year to which the data we have used pertain, the mean percentage of white students in each school in

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<sup>6</sup> Ibid.

the Detroit School District was approximately 56.7%; whereas, the mean percentage of white students in the Detroit district as a whole was approximately 35%.<sup>7</sup>

## 2. The Method of the Simulation

The primary purpose of this chapter is simply to illustrate our results by means of a simulation of cross-district busing. Either plan would serve as a satisfactory basis for the simulation. The differences between the plans, in terms of the area-wide percentages of white students, are not great. The choice is between using the second plan as a basis for our application and, perhaps, thereby misleading the reader by exaggerating the magnitudes of the effects of cross-district busing; or using the first plan and, possibly, understating the effects of cross-district busing. I prefer and, therefore, use the second plan as the basis. My reason is as follows. The use of the second plan will exaggerate the effects for any school, be they positive (an increase in average achievement) or negative (a decrease in average achievement). In addition, our results are accurate enough, i.e., the variances of the forecast errors of our production functions are small enough, to insure that the direction of the predicted effects are correct, at least.

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<sup>7</sup> This means, of course, that there are more schools whose percentage of white students is greater than 35 percent (the percentage of white students in the district as a whole), than there are schools whose percentage of white students is less than 35 percent. Two further conclusions necessarily follow: (1) on the average, white students go to smaller schools than do minority students; and, (2) on the average, those schools whose percentage of white students exceeds that for the district as a whole have fewer students than do those whose percentage of white students falls short of that for the district as a whole.

The standard deviation of the forecast error depends on (1) the standard error of the estimate; (2) the number of observations used in estimating the parameters; (3) the variances and covariances of the estimated coefficients; and, (4) the differences between the values of the explanatory variables used in the forecast and the mean values of the explanatory variables.<sup>8</sup> With the exception of the final factor, all of the others are very small in our forecasts. Therefore, by using the exaggerated effects generated by the second plan we can bracket the "actual" effects as lying in the interval between zero (i.e., no change in average achievement for a school) and the magnitudes of the exaggerated effects.

Having made our choice of the plan to be used, we proceed as follows. We selected five schools from the Detroit School District on the basis of the percentage of white students therein. We selected our five schools, each of whose relevant percentages most nearly equalled 5%, 20%, 35%, 50%, and 65%. This was done

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<sup>8</sup> The formula for an unbiased estimator of the variance of the forecast error ( $s_F^2$ ) is:

$$s_F^2 = s^2 + \frac{s^2}{n} + \sum_{k=1}^K (X_{0k} - \bar{X}_k)^2 \text{Var}(\hat{\beta}_k) \\ + 2 \sum_{j < k} (X_{0j} - \bar{X}_j)(X_{0k} - \bar{X}_k) \text{Cov}(\hat{\beta}_j, \hat{\beta}_k)$$

where:  $s^2$  is the estimated standard error of the estimation  
 $n$  is the number of observations  
 $X_{0k}$  ( $k = 1, \dots, K$ ) is the value, of the  $k^{\text{th}}$  explanatory variable, used in making the forecast  
 $\bar{X}_k$  ( $k = 1, \dots, K$ ) is the mean of the  $k^{\text{th}}$  explanatory variable  
 $\hat{\beta}_k$  is the estimated value of the coefficient of the  $k^{\text{th}}$  explanatory variable.

in order to provide a range of schools within the Detroit School District. On this basis we selected the following five schools (the percent white students in each school is also given):

(1) Foch Junior High (4.5%); (2) Ellis Elementary and Special School (22.5%); (3) Winship Junior High (36.2%); (4) Carstens School (51.0%); and (5) Earhart Junior High (63.4%).

We then selected four schools from districts, other than the Detroit City School District, located within the plan area. The reason we chose four schools was in order to have a range of socioeconomic classes represented. That is, we wanted one each of the following four types of schools: (1) a lower middle class school in a lower middle class district; (2) a middle class school in a middle class district; (3) an upper middle class school in an upper middle class district; and, (4) an upper class school in an upper class district. As criteria for distinguishing schools and districts by socioeconomic class, we used our data on the average socioeconomic status of the children in the school and district, respectively. Both of these variables (SES-S and SES-D) are distributed as  $N(50,100)$ . We took the mean value, 50, as representative of the middle class, the values one-half of a standard deviation above (55) and below (45) the mean as representative of the upper and lower middle classes, respectively, and, finally, the value one standard deviation above the mean (60) as representative of the upper class.

We had data on 153 schools in 74 districts located within the plan area, not counting the Detroit City School District. In 85% of these schools and 82% of these districts white students

made up 97% or more of the school or district students population. In addition, in only 7% of the schools and 4% of the districts were the white students less than 70% of the student body. On the basis of this information we decided to consider only those schools whose student bodies were 97% or more white, and which were located in districts whose student populations were at least 97% white.

Based on these criteria, i.e., the socioeconomic status of the school and district and the racial composition of the school and district, we selected the following four schools (the average socioeconomic status of the students in each school, and its district are also given): (1) Howard Beecher Junior High (45.5), Hazel Park City School District (45.6); (2) Burger Junior High (50.0), Garden City School District (50.0); (3) West Junior High (55.3); Rochester Community School District (54.8); and, (4) Berkshire Junior High (60.6), Birmingham City School District (60.1). Having selected the schools to be used, we proceeded in the following manner.

First, we assumed that the following variables would not change due to the initiation of the plan: (1) the number of students in each school (PUP-S); (2) the number of students in each district (PUP-D); (3) the percentage of teachers in each school with five or more years experience (EXP); (4) the percentage of teachers in each school with a master's degree (MA); and, (5) the teacher/ pupil ratio in each school (T/P). Second, we assumed, except with respect to race, that the children actually being bused would comprise a random sample of the students in the school which they originally

attended. Third, we assumed that, except with respect to race, the children actually bused would comprise a random sample of the students in the district in which they originally attended. Taken together these assumptions allow us to determine new values for the explanatory variables.

We illustrate the way this is done using the following example. (We use only one home and one community variable in order to simplify the example, but the procedure is exactly the same for the other variables.) Suppose we have the following data on a Detroit city school and one non-Detroit city school within the plan area.

| <u>Variable</u> | <u>Detroit city school</u> | <u>non-Detroit city school</u> |
|-----------------|----------------------------|--------------------------------|
| SES-S           | 50                         | 60                             |
| SES-D           | 40                         | 60                             |
| RACE-S          | .20                        | 1.00                           |
| PUP-S           | 100                        | 50                             |

In order to see the effects of busing on the values of the variables for the non-Detroit city school, we see that we would have to remove ten of the present pupils and replace them with ten non-white students from the Detroit city school. And, in order to see these same effects on the values of the variables for the Detroit city school, we would have to remove 60 non-white students from the Detroit city school and replace them with 60 white students from the non-Detroit city school. Obviously, we cannot transfer ten students from school A to school B and 60 students from school B to A and retain the original number of



students in each school. However, this simulation is solely for the purpose of illustration, and our schools were chosen as representatives of different types of schools. Therefore, we can assume that there exist a number of schools similar to those chosen. Thus, for the above example to work out in every detail, we would have to have not one non-Detroit city school with the given characteristics, but six. Then, by transferring ten students from each of the six schools (the correct number for each of them) to the Detroit city school, we would be transferring in the required 60 students. By transferring to each of the six non-Detroit city schools ten non-white students (the correct number for each of them) from the Detroit city school, we would be transferring out the required 60.

On the basis of our assumption we can "do" this for the non-Detroit city school by taking 80 percent of the school variable SES-S for the non-Detroit city school and adding to it 20 percent of the school variable SES-S for the Detroit city school, i.e.,  $(.8 \times 60) + (.2 \times 50)$ ; we follow the same procedure for the district variable SES-D, i.e.,  $(.8 \times 60) + (.2 \times 40)$ . Similarly, for the Detroit city school we follow the same procedure, except that in this case we take 40 percent of the school and district variables for the Detroit city school and 60 percent of the variables for the non-Detroit city school and district, i.e.,  $(.4 \times 50) + (.6 \times 60)$  and  $(.4 \times 40) + (.6 \times 60)$ , respectively. The new values of the variables are:

| <u>Variable</u> | <u>Detroit city school</u> | <u>non-Detroit city school</u> |
|-----------------|----------------------------|--------------------------------|
| SES-S           | 56                         | 58                             |
| SES-D           | 52                         | 56                             |
| RACE-S          | .80                        | .80                            |
| PUP-S           | 100                        | 50                             |

We use the foregoing procedure to determine new sets of values for the explanatory variables. This procedure is repeated twenty times, i.e., we determine twenty new sets of values, one for each possible combination of a Detroit city school with a non-Detroit city school. Table 15a contains the original values of the variables for each of the nine schools used in this simulation. Table 15b contains the twenty sets of new values.

Finally, for each school we take the difference between the new values of the variables and the original values, multiply these differences by the appropriate marginal productivities, and sum. The result is the forecast value of the change in the dependent variable, achievement.

We make long run and short run forecasts for each case. There is a slight difference between the way the long and short run forecasts are made. Recall that we have two models, a long and a short run model, and further, that we assumed the value of each explanatory variable to be constant over time. The long run model was estimated based on this assumption, and, indeed the relationship among the long and short run parameters were confirmed. This has ramifications for the forecasts.

First, we may get two long run forecasts, one from each model. We get a long run forecast from the long run model by

Table 15a. Original Values of the Variables for the Schools Used in the Simulation

| Variable | <u>Detroit City Schools</u> |       |         |          |         | <u>non-Detroit City Schools</u> |        |       |           |
|----------|-----------------------------|-------|---------|----------|---------|---------------------------------|--------|-------|-----------|
|          | Foch                        | Ellis | Winship | Carstens | Earhart | Howard Beecher                  | Burger | West  | Berkshire |
| COMP T-1 | 41.0                        | 40.8  | 49.9    | 44.4     | 43.3    | 49.1                            | 51.5   | 55.3  | 57.0      |
| SES-S    | 39.3                        | 41.3  | 43.7    | 38.8     | 36.9    | 45.5                            | 50.0   | 55.3  | 60.6      |
| M SCH-S  | 51.4                        | 53.3  | 47.2    | 52.1     | 47.4    | 49.2                            | 48.0   | 52.0  | 50.2      |
| SELF-S   | 47.4                        | 47.4  | 51.6    | 49.0     | 46.6    | 49.8                            | 51.2   | 51.5  | 51.5      |
| SCH-S    | 51.1                        | 51.7  | 49.2    | 50.0     | 47.5    | 48.7                            | 47.1   | 48.4  | 49.6      |
| PUP-S    | 1640                        | 756   | 472     | 2143     | 1111    | 813                             | 917    | 825   | 1036      |
| RACE-S   | 0.045                       | 0.225 | 0.362   | 0.510    | 0.634   | 0.99                            | 0.99   | 1.00  | 0.99      |
| EXP      | 0.36                        | 0.54  | 0.82    | 0.41     | 0.51    | 0.50                            | 0.79   | 0.65  | 0.48      |
| MA       | 0.18                        | 0.21  | 0.41    | 0.30     | 0.40    | 0.32                            | 0.55   | 0.43  | 0.33      |
| T/P      | 0.037                       | 0.032 | 0.046   | 0.031    | 0.041   | 0.042                           | 0.037  | 0.040 | 0.046     |
| PUP-D    | 29382                       | 29382 | 29382   | 29382    | 29382   | 8379                            | 14031  | 8385  | 17528     |
| RACE-D   | 0.35                        | 0.35  | 0.35    | 0.35     | 0.35    | 0.99                            | 0.99   | 1.00  | 0.99      |
| SES-D    | 44.7                        | 44.7  | 44.7    | 44.7     | 44.7    | 45.6                            | 50.0   | 54.8  | 60.1      |
| M SCH-D  | 51.3                        | 51.3  | 51.3    | 51.3     | 51.3    | 49.0                            | 48.2   | 51.3  | 51.0      |
| SELF-D   | 49.0                        | 49.0  | 49.0    | 49.0     | 49.0    | 48.7                            | 50.7   | 51.3  | 51.9      |
| SCH-D    | 49.5                        | 49.5  | 49.5    | 49.5     | 49.5    | 50.3                            | 49.3   | 49.2  | 50.8      |

Table 15b. Simulated Values of the Variables

| Variables                                 | <u>Detroit City Schools</u> *          |        |      |           |
|-------------------------------------------|----------------------------------------|--------|------|-----------|
|                                           | ----- For Foch when paired with: ----- |        |      |           |
|                                           | Howard<br>Beecher                      | Burger | West | Berkshire |
| COMP T-1                                  | 47.1                                   | 48.9   | 51.0 | 53.0      |
| SES-S                                     | 44.0                                   | 47.2   | 51.4 | 55.4      |
| M SCH-S                                   | 49.7                                   | 48.8   | 51.9 | 50.5      |
| SELF-S                                    | 49.2                                   | 50.3   | 50.5 | 50.5      |
| SCH-S                                     | 49.3                                   | 48.1   | 49.0 | 49.9      |
| ----- For Ellis when paired with: -----   |                                        |        |      |           |
| COMP T-1                                  | 45.5                                   | 46.9   | 48.5 | 50.0      |
| SES-S                                     | 43.7                                   | 46.3   | 49.3 | 52.3      |
| M SCH-S                                   | 50.9                                   | 50.3   | 52.5 | 51.5      |
| SELF-S                                    | 48.8                                   | 49.6   | 49.8 | 49.8      |
| SCH-S                                     | 50.1                                   | 49.1   | 49.9 | 50.6      |
| ----- For Winship when paired with: ----- |                                        |        |      |           |
| COMP T-1                                  | 49.5                                   | 50.6   | 51.8 | 53.0      |
| SES-S                                     | 44.5                                   | 46.5   | 48.8 | 51.2      |
| M SCH-S                                   | 48.0                                   | 47.5   | 49.3 | 48.5      |
| SELF-S                                    | 50.8                                   | 51.4   | 51.6 | 51.6      |
| SCH-S                                     | 49.0                                   | 48.3   | 48.9 | 49.4      |

"Continued"

Table 15b. Continued.

| <u>Detroit City Schools</u> *                    |                   |        |         |           |         |
|--------------------------------------------------|-------------------|--------|---------|-----------|---------|
| ----- For Carstens when paired with: -----       |                   |        |         |           |         |
| Variables                                        | Howard<br>Beecher | Burger | West    | Berkshire |         |
| COMP T-1                                         | 45.7              | 46.4   | 47.2    | 48.0      |         |
| SES-S                                            | 40.7              | 42.0   | 43.5    | 45.1      |         |
| M SCH-S                                          | 51.3              | 50.9   | 52.1    | 51.6      |         |
| SELF-S                                           | 49.2              | 49.6   | 49.7    | 49.7      |         |
| SCH-S                                            | 49.6              | 49.2   | 49.5    | 49.9      |         |
| ----- For Earhart when paired with: -----        |                   |        |         |           |         |
| COMP T-1                                         | 44.2              | 44.7   | 45.1    | 45.8      |         |
| SES-S                                            | 38.3              | 39.1   | 40.0    | 40.9      |         |
| M SCH-S                                          | 47.7              | 47.5   | 48.1    | 47.8      |         |
| SELF-S                                           | 47.2              | 47.4   | 47.5    | 47.5      |         |
| SCH-S                                            | 47.7              | 47.4   | 47.6    | 47.8      |         |
| <u>non-Detroit City Schools</u> **               |                   |        |         |           |         |
| ----- For Howard Beecher when paired with: ----- |                   |        |         |           |         |
|                                                  | Foch              | Ellis  | Winship | Carstens  | Earhart |
| COMP T-1                                         | 47.6              | 47.6   | 49.3    | 48.2      | 48.0    |
| SES-S                                            | 44.4              | 44.7   | 45.2    | 44.3      | 43.9    |
| M SCH-S                                          | 49.7              | 50.0   | 48.9    | 49.8      | 48.9    |
| SELF-S                                           | 49.3              | 49.3   | 50.1    | 49.6      | 49.2    |
| SCH-S                                            | 49.1              | 49.2   | 48.7    | 48.9      | 48.4    |
| SES-D                                            | 45.4              | 45.4   | 45.4    | 45.4      | 45.4    |
| M SCH-D                                          | 49.4              | 49.4   | 49.4    | 49.4      | 49.4    |
| SELF-D                                           | 48.7              | 48.7   | 48.7    | 48.7      | 48.7    |
| SCH-D                                            | 50.1              | 50.1   | 50.1    | 50.1      | 50.1    |

"Continued"

Table 15b. Continued.

| Variables                              | non-Detroit City Schools**               |       |         |          |         |
|----------------------------------------|------------------------------------------|-------|---------|----------|---------|
|                                        | ----- For Burger when paired with: ----- |       |         |          |         |
|                                        | Foch                                     | Ellis | Winship | Carstens | Earhart |
| COMP T-1                               | 49.5                                     | 49.5  | 51.2    | 50.1     | 49.9    |
| SES-S                                  | 48.0                                     | 48.3  | 48.8    | 47.9     | 47.5    |
| M SCH-S                                | 48.7                                     | 49.0  | 47.9    | 48.8     | 47.9    |
| SELF-S                                 | 50.5                                     | 50.5  | 51.3    | 50.8     | 50.4    |
| SCH-S                                  | 47.9                                     | 48.0  | 47.5    | 47.7     | 47.2    |
| SES-D                                  | 49.0                                     | 49.0  | 49.0    | 49.0     | 49.0    |
| M SCH-D                                | 48.7                                     | 48.7  | 48.7    | 48.7     | 48.7    |
| SELF-D                                 | 50.4                                     | 50.4  | 50.4    | 50.4     | 50.4    |
| SCH-D                                  | 49.3                                     | 49.3  | 49.3    | 49.3     | 49.3    |
| ----- For West when paired with: ----- |                                          |       |         |          |         |
| COMP T-1                               | 51.6                                     | 51.6  | 53.4    | 52.3     | 52.1    |
| SES-S                                  | 52.1                                     | 52.5  | 52.9    | 52.0     | 51.6    |
| M SCH-S                                | 51.9                                     | 52.3  | 51.0    | 52.0     | 51.1    |
| SELF-S                                 | 50.7                                     | 50.7  | 51.5    | 51.0     | 50.5    |
| SCH-S                                  | 48.9                                     | 49.0  | 48.5    | 48.7     | 48.2    |
| SES-D                                  | 52.7                                     | 52.7  | 52.7    | 52.7     | 52.7    |
| M SCH-D                                | 51.3                                     | 51.3  | 51.3    | 51.3     | 51.3    |
| SELF-D                                 | 50.8                                     | 50.8  | 50.8    | 50.8     | 50.8    |
| SCH-D                                  | 49.3                                     | 49.3  | 49.3    | 49.3     | 49.3    |

"Continued"

Table 15b. Continued.

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| Variables | <u>non-Detroit City Schools</u> **          |       |         |          |         |
|-----------|---------------------------------------------|-------|---------|----------|---------|
|           | ----- For Berkshire when paired with: ----- |       |         |          |         |
|           | Foch                                        | Ellis | Winship | Carstens | Earhart |
| <hr/>     |                                             |       |         |          |         |
| COMP T-1  | 54.0                                        | 54.0  | 55.7    | 54.6     | 54.4    |
| SES-S     | 56.6                                        | 56.9  | 57.4    | 56.5     | 56.1    |
| M SCH-S   | 50.5                                        | 50.8  | 49.7    | 50.6     | 49.7    |
| SELF-S    | 50.7                                        | 50.7  | 51.5    | 51.0     | 50.6    |
| SCH-S     | 49.9                                        | 50.0  | 49.5    | 49.7     | 49.2    |
| SES-D     | 57.2                                        | 57.2  | 57.2    | 57.2     | 57.2    |
| M SCH-D   | 51.0                                        | 51.0  | 51.0    | 51.0     | 51.0    |
| SELF-D    | 51.3                                        | 51.3  | 51.3    | 51.3     | 51.3    |
| SCH-D     | 50.5                                        | 50.5  | 50.5    | 50.5     | 50.5    |

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\* The simulated value of RACE-S is 0.80 in every case. Simulated values are not given for these variables assumed to be unaffected by busing, e.g., the teacher-pupil ratio (T/P). The simulated values of the district variables are not given as those variables do not enter into the production function.

\*\* The simulated values of RACE-S and RACE-D are 0.80 in every case. Simulated values are not given for those variables assumed to be unaffected by busing.

applying the procedure described above to the home variables, i.e. SES-S, M SCH-S, SELF-S, and SCH-S; to the only school variable that changes, RACE-S; and, to the community variables that change, i.e., RACE-D, SES-D, M SCH-D, SELF-D and SCH-D. Due to the fact that the production functions for the Detroit city schools do not include community variables, it is meaningless to calculate new values of the community variables for them. And, also, in the case of the Detroit city schools, it is meaningless to attempt to employ the rest of the procedure insofar as it applies to community variables. We get the other long run forecast from the short run model by using the same procedure, but this time we include the change in the value of the lagged dependent variable. The reason that this is possible is that by using the change in the value of the lagged dependent variable, COMP T-1, we are in effect simulating a long, not a short, run change. That is, when we generate the new set of values, we generate, inter alia, a new value of the lagged dependent variable, and therefore, a change in the value of the dependent variable. Now, given our two models, the only way that the change in the values of all the other explanatory variables, i.e., other than the lagged dependent variable, could be the same in both models is if the changes in the values of each of them is constant over time. Thus, implicit in the change in the value of the lagged dependent variable is the assumption that the changes in values of the other variables have been constant over time. Thus, this is but a reformulation of the long run model; and, therefore, we expect the forecast values of the dependent variable, COMP T, to be the same for both.



Now, although the procedure is the same for both forecasts and the changes in the values of the variables are the same for both forecasts, the parameters by which they (the changes in the values of the variables) are multiplied differ between the models. And, of course, since the relationship between the models is confirmed, we expect the forecasts to be the same. We did not test the hypotheses that there is no difference between the two forecasts; however, an inspection shows that they are in fact very similar. These results are given in Tables 16a and 16b for the Detroit city schools and Tables 16c and 16d for the non-Detroit city schools. Note that these results are in terms of changes in cumulative achievement over the entire period (from time 0 to time T) due to changed values of the inputs which are maintained over the entire period. We postpone further consideration of these results until we complete the development of the method for generating short run results at which time we will consider the results of both the long and the short run forecasts together.

Second, besides being used to produce a long run forecast, the short run model may be used to produce a short run forecast. The procedure is identical to that for the long run forecast, except that instead of including the change in the value of the lagged dependent variable, we assume it to be zero. That is, in order to make a short run forecast, we treat only changes in the home, school, and community variables. The logic is as follows. If there has been no change in the home, school, or community variables prior to time T-1, then cumulative achievement to time T-1 is unchanged. Therefore, the short run effects of a change are

Table 16a. Long Run Changes in Average Achievement in Detroit City Schools Based on the Long Run Model.

| Detroit City Schools * | non-Detroit City Schools ** |      |       |       |
|------------------------|-----------------------------|------|-------|-------|
|                        | 45.5                        | 50.0 | 55.3  | 60.6  |
| .045                   | 6.21                        | 8.60 | 10.99 | 12.73 |
| .225                   | 5.20                        | 5.89 | 7.62  | 8.68  |
| .362                   | 3.32                        | 4.15 | 5.55  | 6.35  |
| .510                   | 2.68                        | 3.20 | 4.08  | 4.65  |
| .634                   | 1.99                        | 2.31 | 2.83  | 3.15  |

Table 16b. Long Run Changes in Average Achievement in Detroit City Schools Based on the Short Run Model.

| Detroit City Schools * | non-Detroit City Schools ** |      |       |       |
|------------------------|-----------------------------|------|-------|-------|
|                        | 45.5                        | 50.0 | 55.3  | 60.6  |
| .045                   | 6.66                        | 8.76 | 10.99 | 12.78 |
| .225                   | 5.36                        | 6.37 | 8.02  | 9.37  |
| .362                   | 1.83                        | 2.79 | 4.10  | 5.14  |
| .510                   | 2.23                        | 2.94 | 3.68  | 4.40  |
| .634                   | 1.60                        | 2.00 | 2.47  | 2.99  |

\* The Detroit City Schools are identified by their original racial composition, i.e., the original value of RACE-S.

\*\* The non-Detroit City Schools are identified by their original socioeconomic class, i.e., the original value of SES-S.

Table 16c. Long Run Changes in Average Achievement in non-Detroit City Schools Based on the Long Run Model.

| Detroit<br>City Schools * | non-Detroit City Schools ** |      |      |      |
|---------------------------|-----------------------------|------|------|------|
|                           | 45.5                        | 50.0 | 55.3 | 60.6 |
| .045                      | 1.70***                     | 2.12 | 2.47 | 3.14 |
| .225                      | 1.51                        | 1.94 | 2.30 | 2.97 |
| .362                      | 1.32                        | 1.73 | 2.32 | 2.76 |
| .510                      | 1.76                        | 2.18 | 2.60 | 3.21 |
| .634                      | 2.02                        | 2.44 | 2.57 | 3.47 |

Table 16d. Long Run Changes in Average Achievement in non-Detroit City Schools Based on the Short Run Model.

| Detroit<br>City Schools * | non-Detroit City Schools ** |      |      |      |
|---------------------------|-----------------------------|------|------|------|
|                           | 45.5                        | 50.0 | 55.3 | 60.6 |
| .045                      | 1.71***                     | 2.18 | 2.49 | 3.27 |
| .225                      | 1.61                        | 2.08 | 2.46 | 3.17 |
| .362                      | 0.60                        | 1.07 | 1.41 | 2.16 |
| .510                      | 1.38                        | 1.84 | 2.18 | 2.93 |
| .634                      | 1.64                        | 2.01 | 2.28 | 3.20 |

\* The Detroit City Schools are identified by their original racial composition, i.e., the original value of RACE-S.

\*\* The non-Detroit City Schools are identified by their original socioeconomic class, i.e., the original value of SES-S.

\*\*\* All values in this table are negative values.

forecast by using the short run model, but only for changes in the values of the home, school, and community variables.

The results of the short run forecasts are given in Tables 17a and 17b for Detroit city schools and non-Detroit city schools, respectively. Just as for the long run forecasts, these results refer to changes in the value of cumulative achievement over the entire period. However, since cumulative achievement to time T-1 is assumed constant, the change in cumulative achievement over the entire period is due entirely to the change in the final period, i.e., the short run change. That is, these results refer to the change in cumulative achievement over the entire period due to a change in the inputs in the final period, their values having been constant prior to the final period change. And, since all of the change in achievement comes in the final period, the change in the cumulative achievement over the entire period is due solely to and is, therefore, equal to the change in achievement for the final period.

### 3. Results of the Simulation

Throughout the discussion of the results of the simulation we shall use the terms "greater than" and "less than" to refer to the absolute values, i.e., to refer to magnitudes, regardless of direction.

There are two sets of long run results, one set based on the long run model and one set based on the short run model. As the two sets of results are very similar, we shall discuss the long run results only in terms of one of them, those based on the long run model. There are two reasons for this choice.

Table 17a. Short Run Changes in Average Achievement in Detroit City Schools.

| Detroit City Schools * | non-Detroit City Schools ** |      |      |      |
|------------------------|-----------------------------|------|------|------|
|                        | 45.5                        | 50.0 | 55.3 | 60.6 |
| .045                   | 3.79                        | 5.05 | 6.29 | 7.14 |
| .225                   | 3.15                        | 3.50 | 4.40 | 5.05 |
| .362                   | 2.02                        | 2.46 | 3.21 | 3.68 |
| .510                   | 1.62                        | 1.91 | 2.36 | 2.71 |
| .634                   | 1.18                        | 1.34 | 1.62 | 1.81 |

\* The Detroit City Schools are identified by their original racial composition, i.e., the original value of RACE-S.

\*\* The non-Detroit City Schools are identified by their original socioeconomic class, i.e., the original value of SES-S.

Table 17b. Short Run Changes in Average Achievement in non-Detroit City Schools.

| Detroit City Schools * | non-Detroit City Schools ** |      |      |      |
|------------------------|-----------------------------|------|------|------|
|                        | 45.5                        | 50.0 | 55.3 | 60.6 |
| .045                   | 0.75***                     | 0.90 | 0.95 | 1.35 |
| .225                   | 0.65                        | 0.80 | 0.92 | 1.25 |
| .362                   | 0.73                        | 0.88 | 0.90 | 1.33 |
| .510                   | 0.80                        | 0.94 | 1.04 | 1.39 |
| .634                   | 0.94                        | 1.09 | 1.03 | 1.54 |

\* The Detroit City Schools are identified by their original racial composition, i.e., the original value of RACE-S.

\*\* The non-Detroit City Schools are identified by their original socioeconomic class, i.e., the original value of SES-S.

\*\*\* All values in this table are negative values.

First, although the results are similar, they do differ somewhat. It is reasonable to assume that the long run results based on the long run model are somewhat better than those based on the short run model which are in effect, extrapolated.

Second, one of the long run results based on the short run model is paradoxical. The paradox is that when Winship Junior High, from the Detroit city school district, is paired with Howard Beecher Junior High, from the Hazel Park city school district, the short run effects are greater than the long run effects for both of the schools. Such a result appears contradictory given the nature of our models, but it is not. The paradoxical result is due primarily to the fact that SES-S decreased while RACE-S increased; thus, the positive effect of the increase in RACE-S was partially offset. Since the ratio of long run to short run coefficients for home variables is greater than that for school variables, the offsetting effect is even greater in the long run, thus yielding the seemingly contradictory results. With the above exception all of the long run results are greater than the short run results for both sets of long run results.

Finally, before we go on to the analysis of the results, a word about terminology. I use the words "pairing" to refer to the simulation of busing between two schools, a Detroit city school and a non-Detroit city school.

#### Effects on Detroit City Schools

We are interested in establishing patterns among the forecasts. With respect to the Detroit city schools, we find the following patterns.

First, the average achievement in each of the four Detroit city schools increases no matter which of the five non-Detroit city schools it is paired with. This result holds in every case, in both the short and long runs. The major sources of these increases are the changes in two explanatory variables: (1) socioeconomic class, SES-S; and, (2) racial composition, RACE-S. That this is so can be seen from the contributions of the rest of the variables to the increases. These contributions are summarized in Table 18.

Table 18. Changes in Average Achievement in Detroit Schools Due to Variables Other Than SES-S and RACE-S.

|         | Short Run | Long Run |
|---------|-----------|----------|
| Maximum | .34       | .97      |
| Minimum | -.53      | -.97     |
| Mean    | .03       | .16      |

Second, the long run gains are greater than the short run gains in every case. This result is to be expected. In addition, for each Detroit city school the differences between the long and short run gains are greater the higher is the socioeconomic status of the non-Detroit city school with which it is paired. Further, for any pairing of a given non-Detroit city school with the Detroit city school this difference is greater the lower is the original percentage of white students in the Detroit city school. And these results hold without exception. These differences are given in Table 19.

Table 19. Differences Between Long and Short Run Changes in Average Achievement for the Detroit City Schools.

| Detroit<br>City Schools * | <u>non-Detroit City Schools</u> ** |      |      |      |
|---------------------------|------------------------------------|------|------|------|
|                           | 45.5                               | 50.0 | 55.3 | 60.6 |
| .045                      | 2.42                               | 3.55 | 4.70 | 5.59 |
| .225                      | 2.05                               | 2.39 | 3.22 | 3.63 |
| .362                      | 1.70                               | 1.69 | 2.34 | 2.67 |
| .510                      | 1.50                               | 1.29 | 1.72 | 1.94 |
| .634                      | 0.81                               | 0.97 | 1.21 | 1.34 |

\* The Detroit City Schools are identified by their original racial composition, i.e., the original value of RACE-S.

\*\* The non-Detroit City Schools are identified by their original socioeconomic class, i.e., the original value of SES-S.



Third, for each Detroit city school, the increase in achievement, i.e., the gain, is greater the higher is the socioeconomic class of the school with which it is paired. The major source of the additional gain is the change in the SES-S variable. That is, no matter which non-Detroit city school it is paired with, the change in the racial composition is the same for a given Detroit city school. Thus, the change in achievement due to a change in the racial composition of a Detroit city school is the same no matter which non-Detroit city school it is paired with. Since the change due to the other variables is very small, the major factor must be the change in socioeconomic class. This result holds without exception in both the long and short runs.

Fourth, comparing pairings of a given non-Detroit city school with each of the Detroit city schools, the gain is greater the smaller the original percentage of white students in the Detroit school. This is true no matter which non-Detroit city school we pair against the Detroit city school. This holds for both the long and short runs. This pattern is due primarily to the differences in the changes in the racial composition of the Detroit city schools, i.e., the lower the original percentage white students, the greater the change in the percentage of white students. Differences in the changes in socioeconomic status do not contribute to the pattern. This is so because the ordering of the Detroit schools by socioeconomic class differs from the ordering by racial composition.

Fifth, the ratio of long run to short run gains for all twenty cases is approximately constant. The mean of the ratios

is 1.70 and the variance is 0.002. All twenty of the values are between 1.64 and 1.78. These ratios are given in Table 20.

#### Effects on the non-Detroit City Schools

The long and short run results for the non-Detroit city schools are given in Tables 16b and 17b. These results are in terms of changes in average achievement for the school. Before we discuss the effects on the non-Detroit city schools, we must develop one more point.

Suppose, for example, that a Detroit city school has 100 students, 80 black students and 20 white students. Then consider it to be paired with a non-Detroit city school having 100 students, all white. In order to achieve a racial composition of 80 percent white and 20 percent black in each school, 60 of the black students in the Detroit city school would have to be replaced by 60 white students. However, the non-Detroit city school could only supply 20 of the white students to, and receive 20 of the black students from the Detroit city school. Any other transfer would mean that the 80 percent-20 percent figures would not hold for the non-Detroit city school. Thus, in order to make such a balancing of student populations work, there would have to be three times as many students in the affected non-Detroit city school as in the Detroit city school. It would make no difference whether there was one non-Detroit city school with 300 students, or three non-Detroit city schools each with 100 students, or any other combination, so long as the total number of students in the affected non-Detroit city school(s) was 300. The importance of this point is that the relationship between the average change in achievement

Table 20. Ratios of Changes in Long Run Average Achievement to Changes in Short Run Average Achievement for Detroit City Schools.

| Detroit<br>City Schools * | non-Detroit City Schools ** |      |      |      |
|---------------------------|-----------------------------|------|------|------|
|                           | 45.5                        | 50.0 | 55.3 | 60.6 |
| .045                      | 1.64                        | 1.70 | 1.75 | 1.78 |
| .225                      | 1.65                        | 1.68 | 1.73 | 1.72 |
| .362                      | 1.64                        | 1.69 | 1.73 | 1.73 |
| .510                      | 1.65                        | 1.68 | 1.73 | 1.72 |
| .634                      | 1.69                        | 1.72 | 1.75 | 1.74 |

\* The Detroit City Schools are identified by their original racial composition, i.e., the original value of RACE-S.

\*\* The non-Detroit City Schools are identified by their original socioeconomic class, i.e., the original value of SES-S.

for Detroit city schools and non-Detroit city schools does not adequately reflect the relationship between the total change in achievement for the Detroit city schools and for the non-Detroit city schools. In order to accurately reflect the relationship between the total changes in achievement, we must adjust the average changes in achievement for the non-Detroit city schools so that they properly reflect the number of students involved. That is, we must apply an adjustment factor to the average achievement of the non-Detroit city schools. The formula for the adjustment factor,  $k$ , is:

$$k = \frac{(B_D - .20)}{(.20 - B_N)}$$

where:  $B_D$  is the percentage of black students in the Detroit city school, written as a fraction

$B_N$  is the percentage of black students in the non-Detroit city school, written as a fraction

The adjustment factors are given in Table 21.

Table 21. Adjustment Factors<sup>\*</sup>

| $B_D$ | $B_N$ |      |
|-------|-------|------|
|       | 0.00  | 0.01 |
| .045  | 3.97  | 3.78 |
| .225  | 3.03  | 2.88 |
| .362  | 2.31  | 2.19 |
| .510  | 1.53  | 1.45 |
| .634  | 0.87  | 0.83 |

\* The factors for  $B_N = 0.00$  were used for the lower middle class, middle class, and upper class schools. The factors for  $B_N = 0.01$  were used for the upper middle class schools.

The analysis of the results for non-Detroit city schools will be in terms of the adjusted values, which we shall refer to as the adjusted average achievement. The adjusted values of the changes in average achievement for the non-Detroit city schools are given in Tables 22a and 22b, for the long and short run, respectively. Note that the adjusted values for the long run are based on the long run model. Again, we are interested in establishing patterns among the forecasts. In the cases of the non-Detroit city schools we find the following patterns.

First, there is a loss in adjusted average achievement for each non-Detroit city school, no matter which Detroit city school it is paired with. This result holds in every case, both in the short and long runs. Just as for the Detroit city school, the major sources of these losses are the changes in the two variables: (1) socioeconomic class, SES-S; and, (2) racial composition, RACE-S. That these are the major sources can be seen from the contributions of the rest of the variables to the changes in adjusted average achievement. These contributions are summarized in Table 23.

Second, the long run losses are greater than the short run losses in every case. In addition, for each non-Detroit city school the difference between the long and short run loss is greater the lower the percentage of white students in the Detroit city school with which it is paired. Further, for any pairing of a given Detroit city school with a non-Detroit city school, the difference is greater the higher the socioeconomic status of the non-Detroit city school. These results hold without exception. These differences are given in Table 24.

Table 22a. Changes in Short Run Adjusted Average Achievement for non-Detroit City Schools.

| Detroit<br>City Schools * | non-Detroit City Schools ** |      |      |      |
|---------------------------|-----------------------------|------|------|------|
|                           | 45.5                        | 50.0 | 55.3 | 60.6 |
| .045                      | 2.98***                     | 3.57 | 3.59 | 5.36 |
| .225                      | 1.96                        | 2.42 | 2.65 | 3.78 |
| .362                      | 1.68                        | 2.02 | 2.03 | 3.06 |
| .510                      | 1.23                        | 1.44 | 1.51 | 2.14 |
| .634                      | 0.82                        | 0.95 | 0.85 | 1.34 |

Table 22b. Changes in Long Run Adjusted Average Achievement for non-Detroit City Schools.

| Detroit<br>City Schools * | non-Detroit City Schools ** |      |      |       |
|---------------------------|-----------------------------|------|------|-------|
|                           | 45.5                        | 50.0 | 55.3 | 60.6  |
| .045                      | 6.75***                     | 8.41 | 9.34 | 12.46 |
| .225                      | 4.57                        | 5.87 | 6.62 | 8.98  |
| .362                      | 3.30                        | 3.98 | 5.08 | 6.34  |
| .510                      | 2.70                        | 3.35 | 3.77 | 4.93  |
| .634                      | 1.76                        | 2.13 | 2.14 | 3.02  |

\* The Detroit City Schools are identified by their original racial composition, i.e., the original value of RACE-S.

\*\* The non-Detroit City Schools are identified by their original socioeconomic class, i.e., the original value of SES-S.

\*\*\* All values in this table are negative values.

Table 23. Changes in Adjusted Average Achievement in non-Detroit City Schools Due to Variables Other than SES-S and RACE-S.

|         | Short Run | Long Run |
|---------|-----------|----------|
| Maximum | -0.76     | -0.91    |
| Minimum | 0.30      | 0.13     |
| Mean    | -0.12     | -0.23    |

Table 24. Differences Between Long and Short Run Changes in Adjusted Average Achievement for the non-Detroit City Schools.

| Detroit<br>City Schools * | non-Detroit City Schools ** |      |      |      |
|---------------------------|-----------------------------|------|------|------|
|                           | 45.5                        | 50.0 | 55.3 | 60.6 |
| .045                      | 3.97***                     | 4.84 | 5.75 | 7.10 |
| .225                      | 2.61                        | 3.45 | 3.97 | 5.20 |
| .362                      | 1.62                        | 1.96 | 3.02 | 3.28 |
| .510                      | 1.47                        | 1.91 | 2.26 | 2.79 |
| .634                      | 0.94                        | 1.18 | 1.29 | 1.68 |

\* The Detroit City Schools are identified by their original racial composition, i.e., the original value of RACE-S.

\*\* The non-Detroit City Schools are identified by their original socioeconomic class, i.e., the original value of SES-S.

\*\*\* All values in this table are negative values. This is so because the change in every case is negative, with the long run changes greater in magnitude (absolute value) than the short run changes.

Third, for each non-Detroit city school the loss in adjusted average achievement is greater the lower is the original percentage of white students in the Detroit city school with which it is paired. This result holds in every case, in both the long and short runs. This pattern is due primarily to the combination of the change in the racial composition of the non-Detroit city school and the adjustment factor to correct for the number of non-Detroit city school students involved. That is, no matter which Detroit city school we pair with a non-Detroit city school, the change in the racial composition of the non-Detroit city school is the same. However, the lower the original percentage of white students in the Detroit city school, the greater is the number of non-Detroit city school students to whom the average change applies. Differences in changes in socioeconomic class do not affect the pattern because the ordering of Detroit city schools by racial composition is different from their ordering by socioeconomic class.

Fourth, comparing pairings of a given Detroit city school with each of the non-Detroit city schools, the loss in adjusted average achievement is greater, the higher is the socioeconomic class of the non-Detroit city school with which it is paired. This result holds, without exception, in both the long and short run. This pattern is due primarily to the differences in the changes in socioeconomic class. This is so because the change in adjusted average achievement due to a change in racial composition is exactly the same for three of the non-Detroit city schools. For the fourth non-Detroit city school, the upper middle class school -- West Junior High, it differs slightly for two reasons: (1) the original



percentage of white students in this school is 99 percent, while in the other three it is 100 percent; and, (2) this school is classified as CITY/TOWN school, while the other three are classified as METRO/SUBURB, and since the production functions differ as between CITY/TOWN and METRO/SUBURB schools the marginal product of RACE-S and RACE-D are somewhat different. The difference in adjusted average achievement due to the differences in the change in racial compositions depends on the Detroit city school which is used in the pairing. However, as we stated the effects are small. That this is so can be seen in Table 25 where we give the differences between the actual change in adjusted average achievement in West Junior High and the adjusted average achievement it would have if the two aforementioned differences did not exist, i.e., if it both had 100% white students originally and was a METRO/SUBURB school.

Fifth, the ratios of long run to short run gains in all twenty cases are fairly constant. The mean of these ratios is 2.30 and the variance is 0.04. All twenty of the values are between 1.80 and 2.60. These ratios are given in Table 26.

#### Net Effects

This section is concerned with the net effects of the pairings of schools. By the term, "net effect," we mean the difference between the gain in average achievement for a Detroit city school minus the loss in adjusted average achievement for a non-Detroit city school when the two are paired. The following patterns (or lack thereof) were found.

Table 25. Differences Between Forecasted Changes in Adjusted Average Achievement for West Junior High and That Which Would Have Been Forecast if West Junior High had Been a METRO/SUBURB School With 99 Percent White Student Population.

| Detroit<br>City Schools * | West Junior High |          |
|---------------------------|------------------|----------|
|                           | Short Run        | Long Run |
| .045                      | -0.67            | -0.49    |
| .225                      | -0.51            | -0.37    |
| .362                      | -0.39            | -0.28    |
| .510                      | -0.31            | -0.18    |
| .634                      | -0.15            | -0.11    |

\* The Detroit City Schools are identified by their original racial composition, i.e., the original value of RACE-S.

Table 26. Ratios of Changes in Long Run Adjusted Average Achievement to Changes in Short Run Adjusted Average Achievement for non-Detroit City Schools.

| Detroit<br>City Schools * | non-Detroit City Schools ** |      |      |      |
|---------------------------|-----------------------------|------|------|------|
|                           | 45.5                        | 50.0 | 55.3 | 60.6 |
| .045                      | 2.27                        | 2.36 | 2.60 | 2.32 |
| .225                      | 2.33                        | 2.43 | 2.50 | 2.38 |
| .362                      | 1.80                        | 1.97 | 2.50 | 2.07 |
| .510                      | 2.20                        | 2.33 | 2.50 | 2.30 |
| .634                      | 2.15                        | 2.24 | 2.52 | 2.25 |

\* The Detroit City Schools are identified by their original racial composition, i.e., the original value of RACE-S.

\*\* The non-Detroit City Schools are identified by their original socioeconomic class, i.e., the original value of SES-S.

First, there was a positive net effect, i.e., gains exceed losses, for every pairing of schools, in the short run. The short run net effects are given in Table 27a.

Second, in the long run the net effects were mixed. In fact, in 25 percent of the cases the net effects were negative. This means, of course, that the ratio of long run losses to short run losses had to be greater than the ratio of long run gains to short run gains. And so they are. We noted previously that the average ratio of long run to short run gains is 1.7, while the average ratio for long run losses to short run losses is 2.3. And, not only is the average ratio for losses greater than that for gains, but for every case, without exception, the ratio for losses is greater than for gains. Given this condition concerning the ratios of long to short run gains and losses, all that is required to turn a short run net gain into a long run net loss is for the short run net gain to be relatively small. The long run net effects are given in Table 27b.

Third, a look at the magnitudes of the net effects indicates that, at least with respect to achievement, the net benefits of busing will not be large. Consider the magnitudes of the net effects. For the short run, the maximum value was 2.70, the minimum was 0.34, and the mean was 0.95; while for the long run, the maximum was 1.65, the minimum was -0.54, and the mean was 0.23. Even the largest value is only 27 percent of 1 standard deviation in achievement (standard achievement is distributed  $N(50,100)$ ).

Table 27a. Net Changes in Short Run Average Achievement.\*

| Detroit<br>City Schools ** | non-Detroit City Schools *** |      |      |      |
|----------------------------|------------------------------|------|------|------|
|                            | 45.5                         | 50.0 | 55.3 | 60.6 |
| .045                       | 0.81                         | 1.48 | 2.70 | 1.78 |
| .225                       | 1.19                         | 1.08 | 1.75 | 1.27 |
| .362                       | 0.34                         | 0.44 | 1.18 | 0.62 |
| .510                       | 0.39                         | 0.47 | 0.85 | 0.57 |
| .634                       | 0.36                         | 0.39 | 0.77 | 0.47 |

Table 27b. Net Changes in Long Run Average Achievement.\*

| Detroit<br>City Schools ** | non-Detroit City Schools *** |       |      |       |
|----------------------------|------------------------------|-------|------|-------|
|                            | 45.5                         | 50.0  | 55.3 | 60.6  |
| .045                       | -0.54                        | 0.19  | 1.65 | 0.27  |
| .225                       | 0.63                         | 0.02  | 1.00 | -0.30 |
| .362                       | 0.02                         | 0.17  | 0.47 | 0.01  |
| .510                       | -0.02                        | -0.15 | 0.31 | -0.28 |
| .634                       | 0.23                         | 0.18  | 0.69 | 0.13  |

\* The net changes were calculated by algebraically summing the changes in average achievement for the Detroit City Schools and the changes in adjusted average achievement for the non-Detroit City Schools.

\*\* The Detroit City Schools are identified by their original racial composition, i.e., the original value of RACE-S.

\*\*\* The non-Detroit City Schools are identified by their original socioeconomic class, i.e., the original value of SES-S.

Summary of Results of Simulation

We may summarize the results of the simulation as follows.

1) Detroit city schools will gain no matter which non-Detroit city school they are paired with; and the gains are greater the higher is the original socioeconomic status of the non-Detroit city school they are paired with and the lower is the original percentage of white students in the Detroit city school. And, the gains will always be greater in the long run than in the short run.

2) non-Detroit city schools will lose no matter which Detroit city school they are paired with; and the losses are greater the higher is the original socioeconomic status of the non-Detroit city school and the lower is the original percentage of white students in the Detroit city school with which they are paired. And, the long run losses are always greater than the short run losses.

3) There is a net gain for every pairing of schools in the short run. However, in the long run the results are mixed, with net losses occurring in 25 percent of the cases.

4) The magnitudes of the net effects -- the single highest net gain was only 27 percent of 1 standard deviation -- are not very large.

## CHAPTER VI

### SUMMARY AND CONCLUSIONS

This thesis has been concerned with the production function of education. In Chapter One we developed a model of the production function of education for the individual student in which the education of a student cumulative to some point in time ( $T$ ) is a function of the student's education cumulative to a prior point in time ( $T-1$ ), and the home, school, and community inputs received by the student in the period ( $t$ ) between ( $T-1$ ) and ( $T$ ).

We then developed a more general model of the education process, of which the production function is but one equation. This was a model of school district behavior in which the output of the educational process is maximized subject to two constraints -- a zero net budget and a given number of students. The purpose of the model of district behavior was to show why we consider school inputs as predetermined variables. Further, we explained why we consider home and community inputs as exogenous variables. We concluded that our model of the production function was one in which all of the explanatory variables were either exogenous or predetermined, and therefore that the method of ordinary least squares was not precluded as a technique of estimation.

In Chapter Two we developed the econometric models to be estimated which, of course, were based on the theoretical model of the first chapter. We used a linear form for the production function. In addition, we aggregated the production function to the school level as our data is for schools, not individuals. We then developed a second model of the production function of education for an individual student in which the student's education cumulative to time (T) is a function of his initial endowments, i.e., his characteristics at birth (time 0), and all of the home, school, and community inputs the student receives from time 0 to time (T). We developed and explained the relationships between the two models. This relationship is that of a long run to a short run model of the same production process. We then discussed the variables we used and the data. For educational output we used a measure of composite achievement in reading, writing, and mathematics. For home inputs we used a measure of socioeconomic status for the school and three attitudinal measures. For the student inputs we used the number of students in the school, the percentage of teachers with master's degrees, the percentage of teachers with five or more years experience, the teacher-pupil ratio, and the racial composition of the school. For community inputs we used a measure of socioeconomic status for the school district, three measures of attitudes for the school district, the number of students in the district, the racial composition of the district and a set of dummy variables for regions.

Chapters Three and Four were devoted to the results of our estimations and the test of various hypotheses. First, we tested

a set of hypotheses developed to determine if there is one or more production functions for education. We concluded that either there are multiple production functions or we have a set of piecewise estimates of a single nonlinear function. It should be noted that the only possible explanation for multiple production functions is that some relevant explanatory variable(s) have been omitted from the models.

We then tested a set of hypotheses designed to determine if there were regional differences in the production functions. Based on these tests we concluded that it makes no difference for METRO/SUBURB schools which region they are in. It should be noted that all of the schools in this category are in either the Detroit area or the remainder of the southern part of the lower peninsula. Thus, whether or not regional differences would appear if there was a school of this type in one of the other regions we cannot say. For both the CITY/TOWN and RURAL schools we concluded that there are no regional differences among the regions of the lower peninsula, but that there is a difference between the lower peninsula and the upper peninsula. The nature of this difference is such that, ceteris paribus, the upper peninsula schools have higher achievement scores.

We then tested hypotheses to determine the significance of variables individually and in groups. Based on these tests we conclude that there are only three individual variables which are consistently significant, i.e., the exclusion of which would affect the level of achievement. These are entering achievement (COMP T-1), socioeconomic status of the students in the school



(SES-S), and the percentage of white students in the school (RACE-S). Further, as groups: (1) the home variables are significant in the short run for DETROIT, METRO/SUBURB, and RURAL schools, and in the long run for all schools; (2) the school variables are significant in the short run for DETROIT and METRO/SUBURB schools and in the long run for all schools; (3) the community variables are significant in the short and long runs for CITY/TOWN schools, and are not significant for METRO/SUBURB schools. (We lack evidence of their effects on DETROIT and RURAL schools.)

These results are, in general, consistent with those of earlier studies, including the Coleman Report. Specifically, socioeconomic class is a significant variable in every study in which it is used as an explanatory variable. Further, in every study which includes some measure of family background, or in our terminology home inputs, the results have shown family background to be significant.

With respect to school inputs, however, our results appear to be at odds with those of earlier studies. Earlier studies conclude that school inputs are insignificant in the production of achievement, while this study indicates the opposite. This apparent conflict arises because in those earlier studies in which race is used as an input it is treated as a characteristic of the individual student. That is, the earlier studies were based on data for individual students, and the student's race was considered as an individual characteristic. In this study we use school average data, and the racial composition of the school is considered to be a school or policy variable. In the earlier studies

race is consistently found to be a significant variable just as it is in this study. The apparent conflict in results stems strictly from the fact that the race variable is treated differently in this study. That this is so is evidenced by the fact that our results with respect to the rest of the school variables coincide exactly with the conclusions of the earlier studies, i.e., we find no school variable (other than the race variable) to be consistently significant for the production of achievement.

We then tested a set of hypotheses designed to determine if the relationship between the short and long run models is as we theorized. Based on these tests we concluded that the relationships were as we theorized. Since this relationship depends on the functional form of the models, we further concluded that the linear form is a good approximation of the true form.

Chapter Five was given over to a simulation of cross-district busing for racial balance based on our models and recent court rulings. We simulated busing between schools in the Detroit City School District and schools in the metropolitan Detroit area, but not in the Detroit City School District. For this we selected five Detroit City Schools in order to get a cross-section of racial composition and four non-Detroit City Schools in order to get a cross-section of socioeconomic classes. Based on the results of our simulation we concluded that: (1) The Detroit City Schools would show an increase in achievement in both the long and short runs after busing, in all cases. The ratio of long run to short run gain being approximately 1.70. The major sources of these increases are the changes in the socioeconomic class (SES-S) and

racial composition (RACE-S) variables. These increases in achievement would be greater the higher the original socioeconomic class of the non-Detroit City School with which a given Detroit City School is paired, and the lower is the original percentage of white students in the Detroit City School.

(2) The non-Detroit City School would show a decrease in achievement in both the long and short runs, after busing in all cases, -- the ratio of long run to short run losses being approximately 2.30. As with the Detroit City Schools the major sources of these changes are the changes in socioeconomic status (SES-S) and racial composition (RACE-S). The decrease in achievement would be greater the higher the original socioeconomic status of the non-Detroit City School and the lower the original percentage of white students in the Detroit City School with which it is paired.

(3) The net effects are positive, in every case in the short run. In the long run the net effects are positive in 75 percent of the cases and negative in 25 percent of the cases. The magnitudes are such that the net changes, if any, will be small and possibly negative.

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## APPENDICES

# APPENDIX A

## MEANS AND (STANDARD DEVIATIONS) OF VARIABLES BY SUBSAMPLES

| <u>Variable</u> | <u>D</u>                | <u>MS</u>          | <u>CT</u>        | <u>R</u>       |
|-----------------|-------------------------|--------------------|------------------|----------------|
| COMP T          | 46.4<br>(4.6)           | 51.0<br>(3.5)      | 51.6<br>(2.6)    | 50.9<br>(2.2)  |
| COMP T-1        | 47.3<br>(4.5)           | 50.8<br>(3.4)      | 51.3<br>(2.5)    | 50.5<br>(2.4)  |
| SES-S           | 46.1<br>(5.4)           | 50.9<br>(4.6)      | 50.4<br>(2.8)    | 49.8<br>(2.4)  |
| M SCH-S         | 51.2<br>(2.3)           | 50.3<br>(1.6)      | 49.3<br>(1.8)    | 49.3<br>(2.1)  |
| SELF-S          | 49.2<br>(1.6)           | 50.4<br>(1.7)      | 49.7<br>(2.1)    | 48.9<br>(2.0)  |
| SCH-S           | 49.6<br>(2.5)           | 50.0<br>(1.7)      | 50.2<br>(1.9)    | 50.6<br>(2.4)  |
| PUP-S           | 952<br>(410)            | 861<br>(324)       | 721<br>(321)     | 463<br>(295)   |
| RACE-S          | .57<br>(.40)            | .90<br>(.20)       | .95<br>(.10)     | .97<br>(.06)   |
| EXP             | .61<br>(.17)            | .53<br>(.16)       | .59<br>(.15)     | .50<br>(.17)   |
| MA              | .29<br>(.10)            | .32<br>(.14)       | .28<br>(.15)     | .22<br>(.13)   |
| T/P             | .036<br>( $\approx 0$ ) | .044<br>(.009)     | .042<br>(.006)   | .045<br>(.007) |
| PUP-D           |                         | 14,097<br>(11,916) | 4,581<br>(3,561) |                |
| RACE-D          |                         | .90<br>(.17)       | .95<br>(.10)     |                |
| SES-D           |                         | 50.9<br>(4.2)      | 50.4<br>(2.5)    |                |
| M SCH-D         |                         | 50.3<br>(1.4)      | 49.3<br>(1.3)    |                |
| SELF-D          |                         | 50.4<br>(1.5)      | 49.9<br>(.80)    |                |
| SCH-D           |                         | 49.9<br>(1.3)      | 50.2<br>(1.7)    |                |

## APPENDIX B

### DEVELOPMENT OF MEASURES OF SOCIOECONOMIC STATUS AND PUPIL ATTITUDES\*

The percent of pupils selecting each response is shown at the left separately for grades four and seven. These percents are based upon representative samples of approximately 10,000 pupils at each grade level.

The weighting of items to determine the socioeconomic status and attitude scores reported to local schools and districts is indicated using the following short names and symbols:

SES - ADVANTAGE: Educational-economic advantage component  
of socioeconomic status

SES - Solidarity: Family solidarity component of socio-  
economic status

NOTE: Scores on the above two components were average together  
to derive the reported score on SES

ACHIEVEMENT: Importance of School Achievement

SELF: Self perception

SCHOOL: Attitude toward School

An underline indicates that the item received a heavy weight of .25 or more on the scale indicated; a moderate weight between .15 and .25 is indicated by listing the short name of the scale without an underline. If a weight less than .15 was used the short name of that scale was omitted. Items that received no weight of .15 or more on any scale contain no scale identification whatever. However their appearance in the list of items included in the scale would indicate that some weight less than .15 might have been given.

As asterisk (\*) indicates that the weight for the item was negative on the scale named.

NOTE: Items 39 and 51 were scored for a "NO" response. All others were scores for the order in which the responses were listed.

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\* Reprinted from the Technical Report of the 1970-71 Michigan Educational Assessment Battery, Michigan Department of Education, June 1972.

## 1970-71 PUPIL BACKGROUND QUESTIONS

Biographical Information

| Percent      |              |                             |
|--------------|--------------|-----------------------------|
| <u>Grade</u> | <u>Grade</u> |                             |
| <u>Four</u>  | <u>Seven</u> |                             |
|              |              | 1. Are you a girl or a boy? |
| 49           | 50           | (A) Girl                    |
| 51           | 50           | (B) Boy                     |
| 0            | 0            | Omit                        |
|              |              | 2. What grade are you in?   |
| 99           | 0            | (A) 4th grade               |
| 0            | 100          | (B) 7th grade               |
| 0            | 0            | (C) Some other grade        |
| 0            | 0            | Omit                        |

Socioeconomic Status: Educational Attainment of Parents

Items 3 - 6 were scored as a unit and weighted on (SES-ADVANTAGE)

|    |    |                                        |
|----|----|----------------------------------------|
|    |    | 3. Did your father go to college?      |
| 32 | 26 | (A) Yes                                |
| 22 | 49 | (B) No                                 |
| 45 | 24 | (C) I don't know                       |
| 0  | 0  | Omit                                   |
|    |    | 4. Did your father finish high school? |
| 56 | 61 | (A) Yes                                |
| 9  | 19 | (B) No                                 |
| 34 | 20 | (C) I don't know                       |
| 1  | 0  | Omit                                   |
|    |    | 5. Did your father go to high school?  |
| 71 | 80 | (A) Yes                                |
| 4  | 7  | (B) No                                 |
| 24 | 12 | (C) I don't know                       |
| 1  | 1  | Omit                                   |

Socioeconomic Status: Educational Attainment of Parents con't.

| Percent      |              |
|--------------|--------------|
| <u>Grade</u> | <u>Grade</u> |
| <u>Four</u>  | <u>Seven</u> |

6. Did your father finish the 8th grade?

|    |    |                  |
|----|----|------------------|
| 77 | 85 | (A) Yes          |
| 3  | 4  | (B) No           |
| 20 | 11 | (C) I don't know |
| 1  | 1  | Omit             |

Items 7 - 10 were scored as a unit and weighted on (SES-ADVANTAGE)

7. Did your mother go to college?

|    |    |                  |
|----|----|------------------|
| 38 | 23 | (A) Yes          |
| 27 | 58 | (B) No           |
| 35 | 19 | (C) I don't know |
| 1  | 1  | Omit             |

8. Did your mother graduate from high school?

|    |    |                  |
|----|----|------------------|
| 58 | 65 | (A) Yes          |
| 10 | 19 | (B) No           |
| 32 | 15 | (C) I don't know |
| 1  | 1  | Omit             |

9. Did your mother go to high school?

|    |    |                  |
|----|----|------------------|
| 75 | 85 | (A) Yes          |
| 4  | 5  | (B) No           |
| 20 | 9  | (C) I don't know |
| 1  | 1  | Omit             |

10. Did your mother finish the 8th grade?

|    |    |                  |
|----|----|------------------|
| 81 | 89 | (A) Yes          |
| 2  | 3  | (B) No           |
| 16 | 8  | (C) I don't know |
| 1  | 1  | Omit             |

Socioeconomic Status: Quality of Housing11. Do your parents rent the house or apartment you live in? (SES-SOLIDARITY)

|    |    |                  |
|----|----|------------------|
| 17 | 14 | (A) Yes          |
| 71 | 81 | (B) No           |
| 12 | 4  | (C) I don't know |
| 1  | 1  | Omit             |

Socioeconomic Status: Quality of Housing con't.

| Percent      |              |
|--------------|--------------|
| <u>Grade</u> | <u>Grade</u> |
| <u>Four</u>  | <u>Seven</u> |

12. Does your house or apartment have a dining room outside the kitchen? (SES-ADVANTAGE)

|    |    |                  |
|----|----|------------------|
| 54 | 62 | (A) Yes          |
| 41 | 35 | (B) No           |
| 5  | 2  | (C) I don't know |
| 0  | 0  | Omit             |

13. How many bedrooms does your house or apartment have? (SES-ADVANTAGE)

|    |    |                  |
|----|----|------------------|
| 1  | 1  | (A) None or one  |
| 11 | 9  | (B) Two          |
| 51 | 49 | (C) Three        |
| 26 | 30 | (D) Four         |
| 10 | 12 | (E) Five or more |
| 1  | 1  | Omit             |

Socioeconomic Status: Family Structure and Stability

14. How many grownups live in your house or apartment? (SES-SOLIDARITY)

|    |    |               |
|----|----|---------------|
| 7  | 8  | (A) 1         |
| 75 | 75 | (B) 2         |
| 10 | 11 | (C) 3         |
| 5  | 3  | (D) 4         |
| 3  | 2  | (E) 5 or more |
| 1  | 1  | Omit          |

15. How many children live in your house or apartment? (counting yourself)

|    |    |                 |
|----|----|-----------------|
| 5  | 7  | (A) 1 (only me) |
| 19 | 19 | (B) 2           |
| 27 | 24 | (C) 3           |
| 21 | 21 | (D) 4           |
| 28 | 29 | (E) 5 or more   |
| 1  | 0  | Omit            |

16. Did your family move from one house or apartment to another last year? (SES-SOLIDARITY)

|    |    |                  |
|----|----|------------------|
| 17 | 14 | (A) Yes          |
| 75 | 83 | (B) No           |
| 8  | 3  | (C) I'm not sure |
| 1  | 1  | Omit             |

Socioeconomic Status: Family Structure and Stability con't.

| Percent      |              |  |
|--------------|--------------|--|
| <u>Grade</u> | <u>Grade</u> |  |
| <u>Four</u>  | <u>Seven</u> |  |

## 17. Who acts as your father? (SES-SOLIDARITY)

|    |    |                      |
|----|----|----------------------|
| 83 | 83 | (A) My real father   |
| 6  | 7  | (B) My stepfather    |
| 1  | 1  | (C) A foster father  |
| 2  | 2  | (D) Some other adult |
| 7  | 6  | (E) No one           |
| 1  | 1  | Omit                 |

## 18. Who acts as your mother? (SE-SOLIDARITY)

|    |    |                      |
|----|----|----------------------|
| 92 | 94 | (A) My real mother   |
| 2  | 2  | (B) My stepmother    |
| 1  | 1  | (C) A foster mother  |
| 1  | 1  | (D) Some other adult |
| 3  | 1  | (E) No one           |
| 1  | 1  | Omit                 |

19. How many different schols have you gone to since you started the first grade? Count only the schools which you went to during the day. (SES-SOLIDARITY)

|    |    |                          |
|----|----|--------------------------|
| 53 | 17 | (A) One -- only this one |
| 28 | 35 | (B) Two                  |
| 11 | 25 | (C) Three                |
| 4  | 12 | (D) Four                 |
| 4  | 11 | (E) Five or more         |
| 1  | 1  | Omit                     |

Socioeconomic Status: Occupation, Income and Possessions

## 20. How many cars and trucks that run does your family have? (SES-SOLIDARITY)

|    |    |                 |
|----|----|-----------------|
| 8  | 5  | (A) None        |
| 33 | 29 | (B) One         |
| 59 | 65 | (C) Two or more |
| 1  | 1  | Omit            |

## 21. Does your family regularly take a newspaper? (SES-SOLIDARITY, SES-ADVANTAGE)

|    |    |                  |
|----|----|------------------|
| 81 | 87 | (A) Yes          |
| 13 | 11 | (B) No           |
| 5  | 2  | (C) I don't know |
| 0  | 1  | Omit             |

Socioeconomic Status: Occupation, Income and Possessions con't.

| Percent                                                                                 |              |                  |
|-----------------------------------------------------------------------------------------|--------------|------------------|
| <u>Grade</u>                                                                            | <u>Grade</u> |                  |
| <u>Four</u>                                                                             | <u>Seven</u> |                  |
| 22. Does your father have a job? (SES-SOLIDARITY)                                       |              |                  |
| 90                                                                                      | 90           | (A) Yes          |
| 6                                                                                       | 6            | (B) No           |
| 3                                                                                       | 3            | (C) I don't know |
| 1                                                                                       | 1            | Omit             |
| 23. Does your mother have a job?                                                        |              |                  |
| 41                                                                                      | 45           | (A) Yes          |
| 56                                                                                      | 53           | (B) No           |
| 2                                                                                       | 2            | (C) I don't know |
| 1                                                                                       | 1            | Omit             |
| 24. Did you attend nursery school? (SES-SOLIDARITY*, SES-ADVANTAGE)                     |              |                  |
| 19                                                                                      | 12           | (A) Yes          |
| 62                                                                                      | 74           | (B) No           |
| 18                                                                                      | 13           | (C) I don't know |
| 1                                                                                       | 1            | Omit             |
| 25. Did your family go away on a vacation last year? (SES-ADVANTAGE)                    |              |                  |
| 67                                                                                      | 68           | (A) Yes          |
| 26                                                                                      | 28           | (B) No           |
| 7                                                                                       | 3            | (C) I don't know |
| 1                                                                                       | 1            | Omit             |
| 26. Does your family have a dishwashing machine? (SES-ADVANTAGE)                        |              |                  |
| 33                                                                                      | 33           | (A) Yes          |
| 66                                                                                      | 66           | (B) No           |
| 1                                                                                       | 1            | (C) I don't know |
| 1                                                                                       | 1            | Omit             |
| 27. Has anyone in your family traveled in an airplane in the last year? (SES-ADVANTAGE) |              |                  |
| 34                                                                                      | 42           | (A) Yes          |
| 56                                                                                      | 52           | (B) No           |
| 10                                                                                      | 5            | (C) I don't know |
| 1                                                                                       | 1            | Omit             |



Socioeconomic Status: Occupation, Income and Possessions con't.

| Percent      |              |
|--------------|--------------|
| <u>Grade</u> | <u>Grade</u> |
| <u>Four</u>  | <u>Seven</u> |

28. Do you own your own wrist watch?

|    |    |         |
|----|----|---------|
| 64 | 79 | (A) Yes |
| 35 | 20 | (B) No  |
| 1  | 1  | Omit    |

29. Does your family have a typewriter? (SES-ADVANTAGE)

|    |    |                  |
|----|----|------------------|
| 62 | 69 | (A) Yes          |
| 35 | 29 | (B) No           |
| 3  | 1  | (C) I don't know |
| 1  | 1  | Omit             |

## 1970-71 PUPIL ATTITUDE QUESTIONS

Attitude A: Importance of School Achievement

| Percent                                                                         |              |                                          |
|---------------------------------------------------------------------------------|--------------|------------------------------------------|
| <u>Grade</u>                                                                    | <u>Grade</u> |                                          |
| <u>Four</u>                                                                     | <u>Seven</u> |                                          |
| 30. How good a student does your mother want you to be in school? (ACHIEVEMENT) |              |                                          |
| 56                                                                              | 41           | (A) One of the best students in my class |
| 10                                                                              | 27           | (B) Above the middle of my class         |
| 5                                                                               | 12           | (C) In the middle of my class            |
| 10                                                                              | 4            | (D) Just good enough to get by           |
| 18                                                                              | 15           | (E) I don't know                         |
| 1                                                                               | 1            | Omit                                     |
| 31. How good a student does your father want you to be in school? (ACHIEVEMENT) |              |                                          |
| 58                                                                              | 44           | (A) One of the best student in my class  |
| 10                                                                              | 25           | (B) Above the middle of my class         |
| 4                                                                               | 10           | (C) In the middle of my class            |
| 8                                                                               | 4            | (D) Just good enough to get by           |
| 19                                                                              | 18           | (E) I don't know                         |
| 1                                                                               | 1            | Omit                                     |
| 32. How good a student do you want to be in school? (ACHIEVEMENT)               |              |                                          |
| 69                                                                              | 45           | (A) One of the best students in my class |
| 11                                                                              | 31           | (B) Above the middle of my class         |
| 5                                                                               | 13           | (C) In the middle of my class            |
| 9                                                                               | 5            | (D) Just good enough to get by           |
| 5                                                                               | 4            | (E) I don't know                         |
| 1                                                                               | 0            | Omit                                     |
| 33. How good a student are you? (SELF)                                          |              |                                          |
| 17                                                                              | 12           | (A) One of the best students in my class |
| 22                                                                              | 28           | (B) Above the middle of my class         |
| 21                                                                              | 34           | (C) In the middle of my class            |
| 14                                                                              | 16           | (D) Just good enough to get by           |
| 25                                                                              | 11           | (E) I don't know                         |
| 1                                                                               | 1            | Omit                                     |
| 34. Is being a good student important? (SELF)                                   |              |                                          |
| 91                                                                              | 88           | (A) Yes                                  |
| 2                                                                               | 4            | (B) No                                   |
| 7                                                                               | 7            | (C) I'm not sure                         |
| 0                                                                               | 1            | Omit                                     |

Attitude A: Importance of School Achievement con't.

| Percent      |              |
|--------------|--------------|
| <u>Grade</u> | <u>Grade</u> |
| <u>Four</u>  | <u>Seven</u> |

35. How good a student do you want to be in reading?  
(ACHIEVEMENT)

|    |    |                                          |
|----|----|------------------------------------------|
| 65 | 44 | (A) One of the best students in my class |
| 16 | 32 | (B) Above the middle of my class         |
| 7  | 15 | (C) In the middle of my class            |
| 7  | 4  | (D) Just good enough to get by           |
| 4  | 3  | (E) I don't know                         |
| 1  | 1  | Omit                                     |

36. How good a student do you want to be in mathematics?  
(ACHIEVEMENT)

|    |    |                                          |
|----|----|------------------------------------------|
| 65 | 49 | (A) One of the best students in my class |
| 16 | 30 | (B) Above the middle of my class         |
| 7  | 14 | (C) In the middle of my class            |
| 7  | 5  | (D) Just good enough to get by           |
| 4  | 2  | (E) I don't know                         |
| 0  | 0  | Omit                                     |

37. How good a student do you want to be in English?  
(ACHIEVEMENT)

|    |    |                                          |
|----|----|------------------------------------------|
| 55 | 39 | (A) One of the best students in my class |
| 20 | 34 | (B) Above the middle of my class         |
| 10 | 18 | (C) In the middle of my class            |
| 10 | 6  | (D) Just good enough to get by           |
| 6  | 3  | (E) I don't know                         |
| 0  | 1  | Omit                                     |

Attitude B: Self Perception

38. Can you do many things well? (SELF)

|    |    |                  |
|----|----|------------------|
| 64 | 76 | (A) Yes          |
| 10 | 7  | (B) No           |
| 25 | 17 | (C) I'm not sure |
| 1  | 1  | Omit             |

39. Do you sometimes feel you just can't learn?  
(SELF, SCHOOL)

|    |    |                  |
|----|----|------------------|
| 52 | 57 | (A) Yes          |
| 33 | 34 | (B) No           |
| 14 | 9  | (C) I'm not sure |
| 1  | 1  | Omit             |

Attitude B: Self Perception con't.

| Percent      |              |
|--------------|--------------|
| <u>Grade</u> | <u>Grade</u> |
| <u>Four</u>  | <u>Seven</u> |

40. Do most of your classmates like you? (SELF, SCHOOL\*)

|    |    |                  |
|----|----|------------------|
| 53 | 63 | (A) Yes          |
| 13 | 6  | (B) No           |
| 33 | 30 | (C) I'm not sure |
| 1  | 1  | Omit             |

41. Do you have a good chance to be successful in life?  
(SELF)

|    |    |                      |
|----|----|----------------------|
| 72 | 79 | (A) I think so       |
| 6  | 4  | (B) I don't think so |
| 21 | 16 | (C) I'm not sure     |
| 1  | 1  | Omit                 |

42. Do you like your classmates?

|    |    |                  |
|----|----|------------------|
| 87 | 89 | (A) Yes          |
| 3  | 3  | (B) No           |
| 10 | 8  | (C) I'm not sure |
| 1  | 1  | Omit             |

43. Do you feel that you succeed at most things?  
(SELF)

|    |    |                  |
|----|----|------------------|
| 58 | 70 | (A) Yes          |
| 13 | 11 | (B) No           |
| 28 | 18 | (C) I'm not sure |
| 1  | 1  | Omit             |

44. Are you happy most of the time?

|    |    |                  |
|----|----|------------------|
| 78 | 82 | (A) Yes          |
| 14 | 11 | (B) No           |
| 7  | 6  | (C) I'm not sure |
| 1  | 1  | Omit             |

Attitude C: Attitude Toward School

45. Do you like school? (SCHOOL)

|    |    |                  |
|----|----|------------------|
| 66 | 60 | (A) Yes          |
| 21 | 23 | (B) No           |
| 12 | 16 | (C) I'm not sure |
| 1  | 1  | Omit             |

Attitude C: Attitude Toward School con't.

| Percent      |              |
|--------------|--------------|
| <u>Grade</u> | <u>Grade</u> |
| <u>Four</u>  | <u>Seven</u> |

46. How often do you tell your parents about things that happen in school? (SCHOOL)

|    |    |                                 |
|----|----|---------------------------------|
| 55 | 46 | (A) Just about every day        |
| 18 | 21 | (B) Once or twice a week        |
| 15 | 22 | (C) Occasionally, but not often |
| 12 | 10 | (D) Never or hardly ever        |
| 1  | 1  | Omit                            |

47. Do you like to talk to your parents about school work? (SCHOOL)

|    |    |                  |
|----|----|------------------|
| 64 | 52 | (A) Yes          |
| 24 | 35 | (B) No           |
| 12 | 13 | (C) I'm not sure |
| 1  | 1  | Omit             |

48. Do you like the time you spend in school on mathematics? (SCHOOL)

|    |    |                  |
|----|----|------------------|
| 64 | 54 | (A) Yes          |
| 24 | 33 | (B) No           |
| 12 | 12 | (C) I'm not sure |
| 1  | 1  | Omit             |

49. Do you like the time you spend in school on reading? (SCHOOL)

|    |    |                  |
|----|----|------------------|
| 73 | 59 | (A) Yes          |
| 17 | 27 | (B) No           |
| 10 | 13 | (C) I'm not sure |
| 1  | 1  | Omit             |

50. Do you like the time you spend in school on writing, spelling, and grammar? (SCHOOL)

|    |    |                  |
|----|----|------------------|
| 61 | 47 | (A) Yes          |
| 22 | 35 | (B) No           |
| 17 | 17 | (C) I'm not sure |
| 1  | 1  | Omit             |

51. If you had your choice, would you rather go to a school other than this one? (SELF\*)

|    |    |                  |
|----|----|------------------|
| 18 | 20 | (A) Yes          |
| 66 | 64 | (B) No           |
| 14 | 16 | (C) I'm not sure |
| 1  | 1  | Omit             |

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