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THERMALLY INDUCED PRESSURES AND STRESSES
IN CYLINDRICAL GRAIN SILOS

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THERMALLY INDUCED PRESSURES AND STRESSES IN CYLINDRICAL GRAIN SILOS

By

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ABSTRACT

THERMALLY INDUCED PRESSURES AND STRESSES IN CYLINDRICAL GRAIN SILOS

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A theoretical study of gravity induced stresses in cylindrical grain silos was conducted in order to determine the base conditions to which thermal effects are an increment. A closed form solution was determined for a silo with various bottom edge connections. The study showed that shear and moment at the bottom can nearly be eliminated by providing a sliding bearing at the edge.

An incremental model was developed to predict pressures and stresses caused by an ambient temperature drop. A modulus of elasticity in horizontal direction for wheat was used. This modulus of elasticity depends on both horizontal and vertical pressure levels in the grain. The model represents the non-linear increase in grain stiffness that may occur as the bin wall contracts. Sensitivity of thermally induced pressure to various parameters was studied. Radius and pressure ratio are found to have a

significant effect. The model predicts increased hoop tension and increased bending moments in bin wall when the grain bin is subject to a uniform drop in temperature and a temperature gradient through the wall. A validation of the model was carried out. The model predicts the same increase in lateral pressure as Andersen's equation at points away from bin edges.

TO MY FATHER

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NOMENCLATURE

d	Width of the bin
D	Flexural rigidity of bin wall
E	Modulus of elasticity of bin wall
E_g	Modulus of elasticity of grain in compression
E_h	Modulus of elasticity of grain in a horizontal direction
h	Bin wall thickness
H	Height of bin
h_o	Height of surcharge cone
I	Moment of inertia
k	Ratio of lateral to vertical pressure of grain
k_x	Grain stiffness
M_x	Bending moment about x axis
M_y	Bending moment about y axis
n	Ratio of modulus of elasticity of grain to modulus of elasticity of bin wall material
N_x	Membrane force in x direction
N_y	Membrane force in circumferential direction
p_x	Vertical frictional pressure of grain in wall
p_z	Lateral pressure of grain
Q_x	Shearing force
r	Radial distance measured from bin axis
R	Bin radius
s	Shear stress

NOMENCLATURE (cont'd)

t	Time
T	Temperature change or temperature
T_a	Annual amplitude of temperature variation
T_d	Diurnal amplitude of temperature variation
T_e	Temperature of external face of bin wall
T_i	Temperature of internal face of bin wall
u	Deflection in the x direction
V_1	Vertical pressure within the grain
w	Deflection in the z direction
x	Distance along the vertical axis
y	Distance along circumferential direction
z	Distance along the radial direction
α	Coefficient of thermal expansion of wall material
α_g	Coefficient of thermal expansion of grain
β_1	Coefficient of friction of grain on bin wall
β_2	Coefficient of friction of grain on grain
Γ_s	Unit weight of wall material
Γ_g	Unit weight of grain
ΔT	Temperature gradient through bin wall
Φ	Thermal diffusivity of grain
ϵ_h	Strain in horizontal direction in the grain mass
ϵ_v	Strain in vertical direction in the grain mass
ϵ_x	Strain in vertical direction in bin wall
ϵ_y	Hoop strain in bin wall
ϵ_a	Annual phase angle

NOMENCLATURE (cont'd)

ϵ_d Diurnal phase angle

θ_1 Angle of friction of grain on bin wall

θ_2 Angle of internal friction of grain

μ Poisson's ratio of wall material

μ_g Poisson's ratio of grain

σ_h Stress in horizontal direction in the grain

σ_x Axial stress in bin wall

σ_v Stress in vertical direction in the grain

σ_y Stress in circumferential direction of the wall

Ω_a Annual frequency

Ω_d Diurnal frequency

χ_x Curvature along the axial direction

χ_y Curvature along the circumferential direction

CHAPTER 1

INTRODUCTION

1.1 Background

The state-owned and cooperative sectors of Morocco plan to erect numerous silos in order to stabilize grain supply and reduce losses. This study, conducted mostly in Morocco, is meant as a contribution to the body of knowledge concerning design of these vital but costly structures.

Accurate prediction of the pressure applied by grain and other bulk materials on storage structures is necessary if such structures are to be designed to provide acceptable levels of safety and economy. In a recent paper, Bishara et al. (1983) concluded that: "there is a dire need for a theoretically more rigorous method for evaluating lateral and vertical pressures in silos storing granular materials."

One of the deficiencies of existing formulas for pressure in bins is that no provision is made for pressures caused by changes in temperature or moisture content of the grain or by changes in ambient temperature.

Rowe (1959) carried out experiments to determine the causes of cracking in a silo containing cement. This reinforced concrete silo was 90 ft high, 30 ft in diameter and had a 6 in thick wall through which he applied a temperature difference of about 33 C. He concluded: "It is clear that the cracking was due to a combination of the effects of both pressure and temperature."

Weiland (1964), analyzing the causes of failure of eight steel grain storage tanks, stated: "It is reasonably safe to conclude that a temperature drop of 8 F or more per day, coupled with a low of less than 15 F may be an omen of catastrophy." De Serio (1970) pointed out that: "One of the most common type of structures that show (sic) signs of distress due to temperature are concrete silos."

He reported the case of twelve circular silos located near one of the Great Lakes where the seasonal temperatures vary from +90 F to -20 F. These silos, which were used to store hot Portland cement (160 to 200 F), showed several vertical and horizontal cracks that reopened after they had been repaired. The Reimbert brothers (1974) indicated that due to temperature variation "Numerous disasters have happened in the world to reinforced concrete silos..." with preexisting cracks, particularly those built with the sliding coffering technique. Zakerzewski (1959), Andersen (1966), Britton (1973), Manbeck (1982) and Thompson and Ross (1982) also emphasized the effect of temperature variation in silo failure.

In addition to contributing to structural failure, temperature variations may cause the formation of new cracks or increase the size of existing ones. Cracks reduce the strength of a structure by accelerating corrosion of reinforcement, cause excessive maintenance and repair expense, and allow water to damage the stored product. Kellner (1938) indicated that weevils, which cause deterioration of grain, can live in cracks.

Temperature changes in grain silos have two different origins, a combination of which may occur during the life of the structure. First there is an internal source. Zakerzewski (1959) and Issacson (1963) mentioned that the biological activity of grain, which continually absorbs or generates heat, moisture and gases, causes a temperature gradient between the two faces of the bin wall, giving rise to thermal stresses. Issacson reported that: "This phenomenon is particularly evident in reinforced concrete bins without insulation, in cold geographical regions." The development of internal heat may be a symptom of deterioration of the grain and can be reduced by drying and ventilation of the stored product. The second origin of temperature variation comes from temperature fluctuations in the air surrounding the silo. Following a temperature drop, the silo wall cools and tends to contract but is prevented from doing so by the stored grain. Hoop stresses therefore appear in the restrained wall that add to those due to the static pressure of the grain. This combined effect can provoke rupture in some highly stressed sections of the wall. Weiland (1964) noted that insulation to avoid temperature induced failures of steel silos may not be feasible because of its vulnerability to mechanical damage and construction error.

Because external temperature variation cannot be prevented by improved operation or construction and because thermally induced stresses are known to contribute to failure and performance deficiencies in silos, it is important that silos be designed to resist such stresses. This investigation attempts to provide the means to achieve that goal.

1.2 Objectives

The specific objectives of this investigation are to:

1. Develop a method to determine stresses in the walls of cylindrical grain silos caused by static grain pressure and the effects of design variables on these stresses.

2. Estimate pressures and stresses on bin walls caused by changes in ambient air temperature and determine the relative magnitude of the effects of design variables on thermally induced lateral grain pressure and wall forces.

1.3 Approach

Thermally induced stresses in silo walls occur in combination with stresses caused by gravity (that is, induced by static pressure) but were anticipated to be of smaller magnitude. Therefore, the first step was to develop a method to predict gravity induced stresses. The usual properties of construction materials (homogeneity, isotropy and linear elasticity) and a generally accepted grain pressure distribution formula (Janssen's) were adopted, and an analytic solution was found. This formulation, based on thin shell theory, includes bending of the walls and is in itself useful for investigating the consequences of restraint conditions at the ends of the silo.

Because thermally induced deformations are resisted in part by grain, which is nonlinearly elastic for relatively small strains, it appeared that the analytic solution for gravity induced stresses could not be extended to include thermally induced stresses. A numeric solution using power series was attempted but found to be awkward in this application. An energy formulation was considered but abandoned because it required sacrificing the convenience of superposition. The technique finally chosen was one in which the pressure distribution and temperature change are discretized so that the stiffness of the grain could be considered constant over small increments of both depth and temperature. Assuming linear elasticity of the silo wall, total stresses are the sums of gravity induced stresses and the incremental stresses corresponding to temperature steps. The formulation was validated by extending it to include gravity loads and then comparing results to those of the analytic solution for an empty silo, and to those of an approximate analysis based on the membrane theory.

The incremental formulation was used to investigate the significance of design variables and of errors due to discretization.

CHAPTER 2
LITERATURE REVIEW

Engineering studies of grain storage began nearly a century ago. Theoretical and experimental investigations have covered various aspects pertinent to this study, including estimation of grain pressure on bin walls, mechanical and thermal properties of grain and heat transfer in grain silos.

2.1 Static and Dynamic Grain Pressures

Evaluation of grain pressure in bins started with two theories that proved inadequate. The most elementary approach is the fluid pressure theory, which results in lateral pressures much higher than the actual ones and which does not account for friction induced vertical load on bin walls. The earth pressure formulas of Coulomb and Rankine offer more realistic analyses but apply to shallow bins only. Rankine's theory, for example, neglects friction on walls.

Early experiments on grain pressure started in England with the work of Roberts (1882-1884). He studied model bins with circular and rectangular cross-sections and concluded that the lateral pressure grain exerts on bin wall ceases to increase beyond a depth of grain 2.5 to 3 times the width of the bin. During the three decades before 1920, extensive research was conducted in the following countries: Germany (Janssen, 1895; Prante, 1896; Pleissner, 1906); Canada (Toltz, 1897-1903; Bovey, 1903; Jamieson, 1905); England (Airy, 1897); USA (Ketchum, 1902-1903, 1919) and Argentina (Lufft, 1904). A summary of this work was given by Ketchum (1919), who concluded:

1. Grain pressure on bin walls is not predicted by the law of fluid pressure because the walls support a substantial part of the weight of the grain

2. Lateral pressure approaches maximum at a depth of grain 2.5 to 3 times the width or diameter of the bin.

3. The ratio of lateral to vertical pressure of grain ranges from 0.3 to 0.6 and varies with different grains and bins. It can be determined only from experiments.

4. The pressure of moving grain on bin walls is about 10% greater than that of grain at rest.

5. When discharge gates are located on the side of a bin, pressure of moving grain on the sides opposite the gates can be 2 to 4 times that of grain at rest. Therefore discharge gates should be located on the bottom of bins

as close as possible to the center.

6. The pressure is higher immediately after filling than when the grain has compacted under its own weight Pressure of grain in a bin filled rapidly is larger than in a bin filled slowly. In deep bins, the maximum lateral pressure occurs during filling, due to the impact of moving grain.

7. Pressures measured on small surfaces are in close agreement with pressures on large surfaces.

8. Janssen's and Airy's formulas yield pressures which agree very closely with actual pressure for static grain.

Janssen's equation, published in 1897, is based on vertical equilibrium of a small horizontal slice of silo and gives lateral pressure of grain at rest for a bin of circular cross-section as

$$p_z = \Gamma g R(1 - e^{-2kx\beta_1/R})/2\beta_1 \qquad \text{II.1}$$

k and β_1 are assumed constant in Janssen's theory. The vertical pressure V_1 within the grain is

$$V_1 = p_z/k \qquad \text{II.2}$$

The vertical traction p_x exerted by the grain on the wall is obtained from

$$p_x = p_z \beta_1 \qquad \text{II.3}$$

p_z, V_1 and p_x are assumed to be constant for all points on a horizontal plane. Janssen obtained the value of .67 for k for wheat stored in a wooden bin. Koenen (1896) suggested that k be taken as the Rankine coefficient of active earth pressure:

$$(1 - \sin\theta_2)/(1 + \sin\theta_2) \qquad \text{II.4}$$

Ketchum's summary reported values of k between 0.3 and 0.6. Expression II.1 shows that the grain pressure is affected by factors such as bin geometry (R), grain characteristics (β_1, θ_2, k, Γ_g), bin material (β_1) and depth of grain X. Computed pressures approach a maximum value as depth increases, as confirmed by earlier experimental findings. This value is independent of the ratio k.

In 1897, as reported by Ketchum, Airy derived separate equations for shallow and deep bins. His theory attributes the pressure on the wall to a sliding wedge of grain between the wall and the plane of rupture. For shallow bins the trace of the plane of rupture intersects the grain surface whereas for deep bins it intersects the wall. The lateral pressures in shallow and deep bins, respectively, are given by:

$$p_z = \Gamma_g x/(((\beta_2 + \beta_1)\beta_2)^{.5} + (1 + (\beta_2)^2)^{.5})^2 \qquad \text{II.5}$$

and

$$p_z = \Gamma_g d/(\beta_1+\beta_2)(1-(1+(\beta_2)^2)^{.5}/(2x(\beta_1+\beta_2)/d+1 -\beta_1\beta_2)^{.5}) \qquad \text{II.6}$$

The vertical pressures within the grain and on the wall can be obtained by respectively using expressions II.2 and II.3.

During the thirty years from 1920 to 1949 work consisted mainly of examining and assessing previous findings. Investigations were made by Frohlich (1934) and Dorr (1936) in Germany; Fordham (1937) in England; Kellner (1938) and M. Reimbert (1941-1943) in France; Takhtamishev (1938,1939) and Kim (1948) in the USSR; Jaky (1948) in Hungary and Hay (1920,1928), Long (1931), Mc Calmont and Ashby (1934,1938) and Amundson (1945) in the USA. The following results were reported:

1. Janssen's equation may safely be used to determine static pressures in deep bins.

2. For shallow bins Airy's equation and Coulomb's theory are more accurate than Janssen's equation which gives pressures higher than actual.

3. Grain pressure against bin wall can vary along the circumference.

4. Pressure in stored material can increase due to temperature, vibration or consolidation.

5. Charging and discharging operations create pressures higher than those of grain at rest. The difference is greatest at mid-height of bins.

6. The use of large surfaces to measure the pressure of coarse materials is more accurate than the use of small surfaces.

7. The ratio k varies with depth and with the shape of the bin cross-section.

M. Reimbert (1943) presented a new method to predict static granular pressures in deep bins. Reimbert measured the total load on bin bottoms from which he deduced the loads carried by the bin walls. He found that the variation of the latter with depth can be best represented by a hyperbolic function of depth. The ratio between the lateral pressure at a certain depth and the average vertical pressure at the same depth was found to increase with depth to an asymptotic limit given by expression II.4.
For bins of circular cross-section the equations are:

$$p_z = \Gamma_g R/2\beta_1 (1 - 1/(x/c + 1)^2) \qquad \text{II.7}$$

and

$$V_1 = \Gamma_g (x/(x/c + 1) + h_0/3) \qquad \text{II.8}$$

where $c = R/2\beta_1 \tan^2(45-\theta_2/2) - h_0/3$ II.9
represents the characteristic abscissa for a cylindrical silo.
The vertical wall pressure p_x is obtained using II.3. The asymptotic limit of lateral pressure given by II.7 is identical to the one given by II.1. In comparison to Janssen's formula II.1, Reimbert's predicts higher pressures at shallow depths and lower pressures near the bottom of the grain mass.

Jaky (1948) questioned the validity of a constant coefficient of grain-wall friction as assumed by Janssen and proposed an analysis in which the coefficient increases with depth up to a maximum value beyond which it remains constant. He observed that this assumption is in agreement with the shearing resistance curve of loose granular materials. Jaky suggested using

$$k = 1 - \sin\theta_2 \qquad \text{II.10}$$

for the ratio of the horizontal to vertical pressures along the bin axis.

After 1950, investigators concentrated on overpressures caused by moving grain and on relationships between the various factors that affect grain pressure. The findings of Takhtamitshev (1938-1939), and M. Reimbert (1941,1943) on dynamic pressure of grain were enhanced by additional work of the Reimbert brothers (1954,1956,1960) in France; Kim (1943-1953), Kovtun and Platonov (1959) in the USSR, and Weiland (1961,1964) and Jenike (1954) in the USA. It was found that:

1. In the process of unloading deep bins through centric discharge gates, two types of flow can take place, namely mass flow where a coherent mass of material moves toward the discharge opening causing appreciable overpressure on bin walls and hoppers, and funnel flow where flow occurs within a channel surrounded by material at rest and where no wall pressure increment was observed.

2. The flow pattern during emptying is affected by various factors such as loading procedure, properties of stored material, nature of wall surface, position of the discharge aperture, rate of discharge and shape and dimensions of bin walls and hoppers.

3. The magnitude and distribution of the dynamic pressure are variable with depth and along the bin circumference circumference. Dynamic pressure may be higher than twice the static pressure given by Janssen's equation.

4. Dynamic pressure should be considered in bin design either by applying a magnification factor to static pressures or by incorporating in the bin a device to prevent mass flow, such as a perforated pipe placed along the axis of the bin or horizontal rings attached to the walls throughout the bin depth.

5. Pressures observed during loading were generally smaller than those at unloading.

Analytical approaches to the prediction of moving grain pressure were developed in France by Caquot (1957), in Germany by Nanninga (1956), in the USSR by Geniev (1958) and in the USA by Jenike (1961).

Prompted by Reimbert's observations, Caquot put forward equations to predict grain pressure for bin filling and emptying. The stored material was assumed homogeneous and noncohesive, and Rankine's conjugate stresses were used. Caquot expressed the condition of equilibrium of the granular material by hypothesizing that at any point in the mass, the ratio of the vertical stress to its conjugate on a vertical plane is bounded. The lower bound value corresponds to the active limit state that occurs during loading and the upper bound value corresponds to the passive limit state which occurs during unloading. The equation for the lateral grain pressure in a circular silo for loading and stabilization after loading is

$$p_z = \Gamma_g R(1 - e^{-(k_0 x \sin 2\theta_1 / R)})/2\beta_1 \qquad \text{II.11}$$

where $$k_0 = (1 + \mu_0 \sin\theta_2)/(1 - \mu_0 \sin\theta_2) \qquad \text{II.12}$$

and $$\mu_0 = (1 - \tan^2\theta_1/\tan^2\theta_2)^{.5} \qquad \text{II.13}$$

The formula for pressure during discharge through side and centric gates is also exponential and is not reported here. For the case of unloading through centric gates, an expression based on the equilibrium of forces around tunnels was first derived by Caquot and Kerisel(1956). It predicts the greatest pressures in the upper parts of the silos.

Comparing the theories of Janssen II.1 and Caquot II.11, Despeyroux (1958) observed that the latter does not require the assumptions that:

(i) the lateral pressure and the vertical pressure within the stored material at a given depth are principal stresses.

(ii) the stored material is in a state of limit equilibrium at every point.

Lecenzer (1963) observed that Caquot's formula for emptying through a central outlet gives results that are far in excess of observed values. He noted that the base pressure distribution at rest is nearly uniform for all depths of fill with sand and that it exhibits a maximum near the center of the bin for cement.

Turitzin (1963) reported on the work carried out by Geniev (1958) to predict the pressure increase caused by motion of the granular mass. Assuming that the density of the granular mass increases with depth and that the angle of internal friction is independent of density, Geniev derived an expression for pressure of the moving granular mass similar to Bernoulli's equation for liquids. Geniev predicted that the largest pressure will occur in the lower parts of the bins.

Jenike (1961) published an analysis of the phenomenon of discharging of bins. In a subsequent publication, Jenike and Johanson (1968) studied the filling and discharging of bins and provided a detailed bibliography on bin loads. The active state of pressure within the stored granular material was characterized by the major principal pressure acting in a vertical or nearly vertical direction, and the passive state by the major principal pressure acting in a horizontal or nearly horizontal direction.

Jenike and Johanson supported the observation made by other researchers that during filling, an active pressure field develops whereas in unloading a passive pressure field occurs. Pressures on the bin for loading and unloading conditions were evaluated using Janssen's equation. To derive the expression of the pressure within hoppers, the granular material was assumed to be nonlinearly elastic under initial loading and to flow plastically during discharging. For mass flow to develop, two conditions were required

(i) a sufficiently steep and smooth hopper, and
(ii) vertical pressures greater than or equal to radial pressures within the hopper.

The grain pressure increment observed during discharging was attributed to the change from an active to a passive state of pressure. This overpressure at the walls is necessary to satisfy equilibrium for the granular mass. The effect of changing from active to passive pressure might be unnoticeable with the funnel flow condition because stationary grain prevents the passive pressure conditions from reaching the bin wall. The transition phenomenon, which starts when the discharge gate is opened, moves upward as more grain flows out of the bin and results in a transient local pressure, which was approximated as concentrated force. Jenike and Johanson noted "These pressures are difficult to measure, because they act over a very narrow band of the wall, narrower than most pressure gages which are located in the wall." This was their explanation of the wide divergence of results obtained by experimenters who measured wall pressures.

Jakobson and Despeyroux (1958) noted that the expression II.4 is valid only when the lateral and vertical pressures are principal stresses and the granular material is in a state of limit equilibrium. Therefore it cannot be used near the bin wall where the grain-wall friction gives non-zero shear stress on a vertical plane. The ratio k adjacent to the wall was found to vary with the angle of grain-wall friction. The use of expression II.4 in Janssen's theory results in an underestimation of grain pressures at shallow depths. The maximum value of k given by

$$(1 - \sin^2\theta_2)/(1 + \sin^2\theta_2) \qquad \text{II.14}$$

is obtained when the angle of grain-wall friction equals the angle of internal friction of the granular material.

Jenike and Johanson (1968) observed that for a cohesive granular material, the yield criterion for the active case is obtained by using the effective angle of friction instead of the internal angle of friction in expression II.4.

Hamilton (1964) studied passive and active pressures in a model grain bin. The passive pressures were obtained by using a device that contracts the bin circumference. The strains due to passive pressures measured at the bottom of the model bin were 2.7 times those due to active pressures. Hamilton stated that passive pressures result from relative movement of the side wall and grain mass. Such a movement can be caused by a change in moisture content or temperature of the grain.

Issacson (1963) showed that both Janssen's II.1 and Reimbert's II.7 formulas can be deduced from the same first order ordinary differential equation. Constant coefficients in the equation lead to Janssen's formula whereas variable coefficients lead to Reimbert's formula.

Lvin (1970) published a study on the analytical evaluation of static and dynamic pressures on silo walls. Unlike Janssen's theory, Lvin's did not assume that the vertical pressure was constant in a horizontal plane. Concentric rings were taken as the basic elements and the equilibrium condition was written, assuming the inner rings slide downward relatively to the outer ones. For the active state, a dimensionless partial differential equation was derived with independent variables related to the vertical and radial directions. Lvin obtained a family of solutions and concluded that his formula represents the upper bound solution and Janssen's the lower. Two pressure regions were obtained for a circular silo for $x < r/k$ $\qquad p_z = (xk\Gamma_g/\beta_1)(1 - kx/2r)$ $\qquad$ II.15

and

for $x > r/k$ $\qquad p_z = \Gamma_g r/2\beta_1$ $\qquad$ II.16

The vertical pressure within the granular material at a given point is obtained from equation II.2. The pressures at the wall are obtained with r=R. Expression II.16, for r=R, is identical to the asymptotic limit of Janssen's formula II.1. Lvin stated that the pressures are affected when the ratio k varies with depth. He reported that this phenomenon may be induced by unloading the bin, temperature changes, cessation of unloading and other causes. Assuming that the ratio k varies linearly between the parts of the silo where the state of equilibrium passes from active limit to passive limit, Lvin developed pressure expressions involving the ratio m_o of the passive k to the active k. For $m_o=4.5$ the passive pressure is twice the active one. Moysey (1979) reported this would occur with smooth-walled bins and that, if the walls are fully rough, $m_o=1$ and no increase in lateral pressure takes place. Using the properties of Mohr's circle, Moysey suggested that when the stored granular material is in a state of passive limit equilibrium, the upper and lower bound values of k at the wall are given respectively by the folowin expression

$(1 + \sin\theta_2)/(1 - \sin\theta_2)$ $\qquad$ II.17

and expression II.14.

In recent years pressures in silos have been investigated using finite element method. Bishara and Mahmoud (1976) developed a finite element model to analyze the deformation and stress histories in farm silage silos. The silage was characterized as a piecewise linear, viscoelastic, isotropic material. They found that the lateral pressure reaches a maximum near the bin bottom and that the vertical pressure on the silo floor is higher at the bin axis than near the wall.

Jofriet et al. (1977) presented a finite element method that incorporates the friction of granular material against bin wall.
The method allows for variation of friction with depth, which is not taken into account in Janssen's theory. They reported that, in general, excellent agreement was found with Janssen's theory. Differences were observed for pressures near the bottom of deep silos and for shallow bins where Janssen's assumption of uniform vertical pressure was said to be far from true.

Lumbroso A. (1977) established that the theoretical values of the ratio k of horizontal to vertical pressure within a stored grain mass fall in the interval defined by the following bounds:

$$m_1 = ((1-q\sin\theta_2)/(1+q\sin\theta_2))\cos^2\theta_1 \qquad \text{II.18}$$

for the lower limit equilibrium and

$$m_2 = ((1+q\sin\theta_2)/(1-q\sin\theta_2))\cos^2\theta_1 \qquad \text{II.18 1/2}$$

for the upper limit equilibrium.
where

$$q = (1-\tan^2\theta_1/\tan^2\theta_2)^{.5} \qquad \text{II.19}$$

Mahmoud and Sayed (1979) published a study on the effect of the flexibility of cold-formed steel walls on pressures in shallow bins. A finite element program was written assuming an axisymmetric system and nonlinear grain properties. Reasonable agreement was obtained with an experimental study of a model silo filled with sand.

Marchant (1980) presented a new technique for measuring the pressure of granular material on retaining structures. He showed that such measurements, when taken by means of force transducers, are accompanied by deflection of the measuring device, which results in underestimation of the pressures. He stated that the reduction of measured pressures is a linear function of the deflection over a limited range and proposed a method to correct the error in measured pressures.

Bishara et al. (1983) used a finite element method to determine static pressures in circular concrete silos storing granular materials. The granular material was characterized as nonlinearly elastic. They obtained a nearly parabolic distribution for the vertical pressure over the bin cross section with the maximum at the center and the minimum near the walls. The lateral pressures and the average vertical pressures obtained by the finite element solution lie between those given by Janssen's and Reimbert's methods for depths less than the silo diameter. Beyond this depth the finite element solution produced values exceeding Janssen's by 10 to 15% and Reimbert's by 20 to 25%. This excess varies with the stored material. Bishara et al. suggested expressions for granular pressures which were obtained by nonlinear least square analysis of the results of finite element studies of a number of circular silos.

Briassoulis and Curtis (1985) analyzed stresses at the ends of cylindrical silos. Their formulation includes a generalized treatment of rotational and translational end restraint stiffnesses but neglects the vertical frictional force of grain on walls. Grain pressure was represented by a modified form of Janssen's equation.

In North American practice, ACI standard 313-77 recommends either Janssen's II.1 or Reimbert's II.7 equation, the Canadian farm building code (NRCC 1977) recommends Janssen's equation for deep bins, as do the ASAE (American Society of Agricultural Engineers) year book (1983) and the Midwest Plan Service Structures and Environment Handbook (MWPS-1, 1983). In the latter it is noted that, for thermal stresses in silos, no theory is available on which design equations can be based.

2.2 Forces Caused by Temperature Changes

Design criteria based on theories of Janssen, Airy and Reimbert were found insufficient to ensure the resistance of bin walls to thermal stresses. Zarkewski (1959) reported: "Walls must also be reinforced to resist bending moments produced by continuity of the structure and variation of temperature. In certain parts of the silo, the latter may be the critical factor."

Kent (1931) studied the effect of temperature gradient on thin-walled cylinders of finite length and found that large circumferential forces are caused at a free edge by a temperature gradient in the radial direction.

Weiland (1964), assuming the grain incompressible, used the expression

$$\sigma_y = E\alpha T \qquad \text{II.20}$$

to estimate the hoop stress in the wall of a steel grain silo caused by a drop in external temperature. He emphasized that the true value would be a function of the ratio of the modulus of elasticity of the grain mass in compression to the modulus of elasticity of the wall material.

Andersen (1966) derived the following formula

$$p_z = \alpha T E_g / (Rn/h + 1 - \mu_g) \qquad \text{II.21}$$

to estimate the lateral pressure increment on the wall of a steel silo caused by a drop in external temperature T, assuming a two dimensional state of stress. He equated the reduction in diameter of a circular disk representing a horizontal slice of the compacted grain under an inward radial pressure to the net reduction in the diameter of a steel band. He pointed out the dependency of the modulus of elasticity and Poisson's ratio of the stored grain on the degree of confinement.

In a report of ACI committee 313 on silos chaired by Hahn (1968), paragraph 4.7 states "Load forces due to thermal effects of the stored material and of exterior thermal changes shall be evaluated."

Safarian and Harris (1968), in a paper related to the design of the temperature steel in conventionally reinforced circular concrete silos, assumed plane strain conditions and a large radius to wall thickness ratio and proposed the following expression for bending moment in the silo wall in the horizontal plane due to a linear thermal gradient ΔT through the wall

$$M_x = Eh^2 \alpha \Delta T / (1 - \mu) \qquad \text{II.22}$$

Saxena (1970) measured the strains caused by changes in ambient temperature in a model silo filled with soybeans. He reported that if the temperature of the walls drops from 110F to 80 F while that of the grain remains at 110 F, an increase in lateral pressure occurs.

The Reimberts (1970), accounting for partial restraint offered by the grain when the bin walls contract following a drop T in temperature, suggested the following formula for the thermal tensile hoop stress in the wall:

$$\sigma_y = E\alpha T/3 \qquad \text{II.23}$$

They evaluated the additional hoop tension per meter of wall height caused by a temperature change of 15 C on a 150 mm thick reinforced concrete silo wall and on a 3 mm thick steel wall and obtained respectively 150000 kN and 36000 kN.

Britton (1973), in order to estimate the strains in deep bins induced by an ambient temperature decrease, used a model steel bin 1.5 m in diameter and 3.1 m high containing grain sorghum. He reported increases in circumferential strains of 20 t0 25% near the base caused by an 11 C temperature drop. His analytical model for estimating temperature induced wall deflection was based on the theory of beams on elastic foundation. He recommended for further studies that: "The magnitude of stress... due to temperature decrease needs to be determined..."

Myers (1974) measured increases in circumferential forces in the wall of a bin filled with wheat following a decrease in ambient temperature.

The ACI standard 313-77 and commentary (1977) give in paragraph 4.5.4 an expression based on equation II.22 to determine the thermal bending moment per unit of wall height due to thermal effects of stored hot or cold granular materials. In the commentary it is added that the temperature distribution within the hot stored granular materials depends on several varaiables such as daily and seasonal temperature fluctuations and that, in the absense of rigorous analysis of these variables, approximations are usually used. Thermal forces in silo walls are directly affected by this temperature distribution.

Manbeck (1982), using equation II.21 and a stress-strain law for wheat in mass determined from triaxial tests, reported lateral pressure increases of 23 to 50% above static pressure due to a drop of 60 F for a typical steel silo. He pointed out that these results depend to a great extent on the modulus of elasticity of wheat, which is a function of the magnitude of pressure on the grain mass.

Thompson and Ross (1982), using equation II.21 and considering the tangent modulus of soft red Winter wheat as it is affected by internal pressure and moisture content, reported a lateral pressure increase of 15% above static pressure caused by a drop of 55 F for steel silos.

The investigations summarized above have helped to identify the variables that must be considered when computing thermal forces on silos, and provide estimates of those forces. However, comparison of the various equations reveals the following deficiencies:

(i) Expressions II.20 and II.22 do not consider mechanical or thermal properties for the stored material.

(ii) Equation II.23 considers the restraint offered by the stored material but relies on an empirical constant of 1/3 to account for unspecified properties of the grain. Furthermore, both the thermal gradient through the bin wall and the friction of grain against the wall are ignored, and no consideration is given to restraints at the bin ends.

Britton specifically noted the potential effect of end restraints but as a simplification assumed free ends. His work also ignores thermal gradient in the wall and grain-wall friction, but does examine the case where the foundation modulus varies with depth.

2.3 Grain Properties

Properties of stored materials and their variation directly affect the pressure applied by these materials on retaining structures. Substantial research effort has been devoted to the study of behavior of individual grains as well as of grain mass. Researchers agree that grain, being a biological material, has some specific properties that distinguish it from other bulk materials. Grain properties can vary in both time and space, the variation being caused by numerous factors (Pamelard 1959). However, in order to propose some answer to the problem of estimating grain pressure, most researchers have assumed grain to be an elastic, homogeneous, isotropic medium, for which the Mohr-Coulomb theory of failure applies.

2.3.1 Physical Properties

In 1943 a commission on silos in France initiated studies to determine the properties of a number of pulverized materials and cereals in order to provide silo designers with some data.
A. Reimbert (1956) reported the findings. The angle of internal friction, determined using a torsion shearing apparatus, was found to depend on shear deformation created at the perimeter of the tested sample and on the level of

pressure applied to it. For constant pressure the value of the angle of internal friction reaches a maximum at a very small deformation (a few millimeters) and decreases toward a minimum as the deformation continues to rise. Therefore two values, a maximum and a minimum, were specified for the angle of internal friction. Two groups of granular materials were distinguished based on how the angle of internal friction varies with pressure.

Despeyroux (1958) noted that the first group, for which the angle of internal friction decreases with pressure at low values and increases at high values of pressure, includes wheat and most other grains. The second group comprises a few materials such as millet and superphosphates. For the first group, the values of pressure occurring in silos correspond to the range for which the angle of internal friction decreases with increasing pressure. He pointed out that the apparent density of stored grains varies with pressure and that for a pressure of about .05 MPa, which commonly occurs in grain silos, this variation is in the form of a 3% to 5% increase due to the compaction of grain under its own weight.

Zakerzewski (1959) summarized results obtained by A. Reimbert, Litwienko, Airy and Theimer regarding unit weight, angle of friction between granular material and bin wall, and angle of natural repose, which was assumed to approximate the angle of internal friction. He noted the variation of the latter with pressure and considered average values.

Stewart (1966), in a study of the variation of the angle of internal friction with moisture content for sorghum grain, concluded that the variation is linear. Stewart added that the usual practice of using the angle of repose as an approximation of the angle of internal friction is very likely to produce errors in bin design.

Gazi (1976) reviewed and evaluated work on the determination of the angle of repose of granular materials. He noted that the angle of repose increases with moisture content, reaches a maximum at a moisture content of 12% to 14% and then decreases. He reported that this effect is due to the surface layer of moisture on the granules, and that surface tension holds the granules together. Britton (1973) reported that if grain with an acceptable moisture content is stored and is not subjected to rewetting or drying operations during the storage period, then major changes in moisture content are unlikely to occur.

Lawton (1980) published the results of a study aimed at determining the coefficients of friction of different bin wall materials against wheat and barley for moisture contents varying from 10% to 20%. He concluded that, in general, the coefficient of friction increased with moisture content of the stored material. Among the wall materials tested, concrete exhibited the highest coefficient of friction.

2.3.2 Mechanical Properties

Grain was characterized by various investigators as an unpredictable material. Mohsenin (1970) stated that: "..none of the biological materials tested so far show perfect elasticity." Behavior of grain in mass was found to be more significant in determining grain pressure than is the behavior of individual grains. A review follows of selected works on the determination of mechanical properties of wheat, which is considered a typical biological particulate material (Manbeck and Nelson 1970).

Arnolds and Roberts (1966), using the theory of elasticity to study the stress distribution thoughout grain kernels, reported that: "...variations in Poisson's ratio have relatively little effect on the load-deformation curves and the assumption that μ_g=.3 was probably satisfactory."

Mohsenin et al. (1967) studied bulk modulus and compressibility of biological materials. They reported that bulk compression tests offer a means for accurate determination of Poisson's ratio for biological materials.

Manbeck and Nelson (1970) published a paper on experimental evaluation of the three dimensional stress-strain response of wheat in mass. They concluded that the behavior of samples of wheat in mass obtained from grain deposited by gravity flow is anisotropic and that there does exist a plane of mechanical symmetry. The anisotropy was thought to be caused by the shape of the kernels. In another paper, Manbeck and Nelson (1974) suggested an empirical stress-strain law which characterizes wheat in mass as an orthotropic material with large, irrecoverable plastic strains.

Narayan and Bilanski (1970) found that bulk modulus, apparent elastic modulus and Poisson's ratio are relatively constant for moisture contents ranging from 9% to 14%. Values between .38 and .40 were obtained for Poisson's ratio.

Mahmoud and Sayed (1979) in their work on flexible shallow bins used a hyperbola to represent the stress-strain relationship of granular material.

Marchant (1980) reviewed existing models that use incremental elasticity in the stress-strain relationship of granular material. He questioned the validity of using constitutive laws developed for soils in the study of pressures of agricultural products in silos. He specifically noted that previous models did not predict the shear dilatance effect. Marchant formulated a stress-strain law for cereal grains and small seeds. He found good correspondence between predicted and observed strains for various cases of loading in a sample of wheat.

Bishara et al. (1983), assuming isotropic behavior, attempted to obtain some general forms of constitutive equations that could be used for all granular materials including wheat, sand, cement and gravel. The effect of moisture content was ignored. Using a non-linear curve fitting program, they developed expressions for bulk modulus and shear modulus. Two different groups of materials were distinguished according to whether the grain size is less than or greater than 0.1 in. For each group, bulk and shear moduli were functions of the angle of internal friction, the initial and current densities and the volumetric pressure. Bishara et al. noted that the behavior of granular material can be considered time independent.

2.3.3 Thermal Properties

Babbitt (1945) carried out experiments on thermal properties of wheat in bulk at 9.2% moisture content of dry weight with a bulk density of 53 lb/cu ft. The temperature of wheat was varied from 79 F to as high as 120 F during the experiment. Using steady state heat flow across a cylinder of grain heated by means of a wire stretched along the axis, he obtained the following values: 0.00036 cal/(cm sec C) for thermal conductivity, 0.37 cal/(g C) for specifc heat and 0.00115 cm^2/s for thermal diffusivity.

Pfalzner (1951) studied specific heat of hard wheat at different moisture contents ranging from 0 to 16% of wet weight. Specific heat values were found to vary linearly with moisture content for three tested samples of wheat.

Disney (1954) measured specific heat of Bersee wheat at twenty different moisture contents ranging from 0.14% to 33.6% of wet weight. Specific heat values were reported to vary between 0.307 and 0.582 Btu/(lb F).

Hall (1957) reported the value 0.187×10^{-4} per degree F for the coefficient of thermal expansion of corn. No value of coefficient of thermal expansion for wheat was found in the literature.

Griffiths (1964) measured the variation in moisture content through a mass of wheat subjected to a temperature gradient. Wheat initially at a moisture content of 13.44% of dry weight was placed between two aluminium plates 0.203 m apart held at two different temperatures, 25 C and 35 C. After 7 months, Griffiths detected no further change in moisture content of wheat. At equilibrium, the grain near the plate at 25 C and the grain near the one at 35 C were respectively at 16.8% and 11.2% moisture content.

Kazarian and Hall (1965) noted that most of the values of thermal properties of grains found in the literature were determined by steady state heat flow across the grain, which requires a long time to reach equilibrium and provokes moisture migration in the grain. Using transient heat flow methods, they carried out experiments to determine thermal conductivity of soft white wheat. The value of thermal conductivity was found to vary from 0.070 Btu/(hr ft F) at 5% moisture content to 0.08 at 20%. Thermal diffusivity was reported to decrease and specific heat to increase with increasing moisture content for wheat and other crops.

Kirshan and Singh (1970) found that thermal properties of grain depend on grain composition, such as fat and protein content, which differ with the variety of grain considered. They stated "It is thus apparent that any reference to thermal properties must specify the variety and moisture content."

Thorpe (1982) proposed an analysis based on the findings of Griffiths, postulating that moisture migration through non-isothermal grain masses occurs through the mechanism of vapor diffusion. Reasonable agreement was reported between Thorpe's predicted values and Griffiths' measurements.

The ASAE 1983 yearbook lists values for specific heat, thermal conductivity and thermal diffusivity for various types of grain.

2.4 Heat Transfer in Grain Bins

Heat transfer in grain storage bins has been the subject for many publications due to the important effect of high temperatures on grain quality. In most early works, the following assumptions were made:

(i) temperature variation occurs only in radial direction.

(ii) grain properties are not affected by temperature change.

(iii) internal heat generated by respiration of the grain and by insect and fungal activity is negligible.

The phenomenon of heat conduction in bins is governed by Fourier's equation, which in polar coordinates is:

$$\Phi(\partial^2 T/\partial r^2 + (\partial T/\partial r)/r) = \partial T/\partial t \qquad \text{II.24}$$

Assuming a uniform initial temperature distribution for the grain, Babbitt (1945) studied the effect on grain temperature of daily and seasonal atmospheric temperature variations which he represnted as a periodic function of time. Using an analytical solution to equation II.24 for semi-infinite solids, Babbitt concluded that changes in outside temperature do not penetrate far into the grain mass. Diurnal external temperature variations were found to have lesser effect on wheat temperature than did seasonal variations.

Similar conclusions were drawn by Converse et al. (1973), who carried out an experimental study on wheat stored in an upright concrete bin, and developed an analytical solution to equation II.24 using Bessel functions and the same assumptions as Babbitt for initial grain temperature and seasonal and diurnal temperature variations.

Considering the same temperature profile and thermal properties as did Converse et al., Carson et al. (1979) used Galerkin's approximation to predict temperature distribution in grain. A trial solution of three terms was found to yield results close to those obtained from Bessel functions.

Yaciuk (1973), assuming a non-uniform initial grain temperature and a non-periodic function for external temperature, attempted a solution to equation II.24 using a finite difference approximation. The resistance of the bin wall to heat transfer was considered and solar radiation was taken as variable.

Recent studies have addressed the development of two-dimensional models. Muir et al. (1980) developed a conduction heat model to predict temperature distribution along the radial and axial directions of unventilated grain bins using a finite difference method.

Metzger and Muir (1983) proposed a mathematical model to simulate intermittent aeration of grain bins in response to seasonal weather variations. The model considers temperature distribution in radial and axial directions of a bin subjected to simultaneous conduction and forced convection heat transfer. The conduction component of the model is based on the model proposed by Muir et al. (1980). Heat and moisture generation were neglected. Metzger and Muir noted: "Heat and moisture generation might be expected from respiration of the grain, and insect and fungal activity, but these are probably negligible until the rate of deterioration increases to an unacceptable level."

CHAPTER 3

ANALYSIS

3.1 Introduction

3.1.1 Problem Definition

Existing methods for evaluating stresses in grain silos are usually based on membrane theory, thus ignoring potential effects of restraints at ends of silos. Furthermore, the additional stresses caused by thermal contraction of the bin typically are ignored. In order to achieve the objectives of this study, it was necessary to develop a method:

- to determine stresses in bin walls due to static grain pressure, and

- to predict the incremental pressures and stresses on bin wall that would result from a drop in temperature of air surrounding the wall. The aim is to develop a formulation that can be adapted to any granular material for which a constitutive law is available.

3.1.1.1 Static Grain Pressures

Static grain pressure is caused by the action of gravity on the grain mass. In addition to causing stresses in silo walls, static pressures are commonly taken as a reference to which are compared other types of grain pressures such as dynamic pressures and the so-called secondary pressures that include temperature induced and moisture induced pressures. Dynamic grain pressure, for example, is usually evaluated by applying a magnification factor to static pressure. The variation of static pressure with depth affects temperature induced stresses because the resistance of the grain mass to compression is a function of the confining pressure.

3.1.1.2 Temperature Change

Changes in the temperature of air surrounding a structure affect the temperature distribution within that structure. Babbitt (1945) experimentally determined the thermal diffusivity, Φ, of bulk wheat to be .00115 cm^2/s. In order to predict analytically the heat flow in a cylindrical grain bin, he adopted the following assumptions:

- The grain bulk has uniform thermal properties and is initially at a uniform temperature throughout.

- Ambient air temperature is adequately approximated by the summation of two harmonic terms , one for daily variation and one for annual.
- Heat flow is in the radial direction only.
- There is no internal heat source.
- The bin walls offer no resistance to heat flow.

Under these conditions, the temperature at a point within the grain is

$$T = T_d e^{-\sqrt{\Omega_d/2\Phi}\, z} \cos(\Omega_d t - \sqrt{\Omega_d/2\Phi}\, z - \epsilon_d) + T_a e^{-\sqrt{\Omega_a/2\Phi}\, z} \cos(\Omega_a t - \sqrt{\Omega_a/2\Phi}\, z - \epsilon_a) \qquad \text{III.1}$$

From this expression, it is seen that decreasing thermal diffusivity both slows and damps the penetration of temperature change. For example, Babbitt calculated that a daily temperature amplitude (peak to peak) of 20 F damps to a 2 F amplitude at only 5 inches into the mass. An annual amplitude of 77 F damps to 2 F at about 13 feet within the mass, and requires half a year to reach that point. Babbitt's observations lead to the realization that a sudden drop in ambient temperature will not be accompanied by immediate thermal contraction of the grain. Therefore that effect cannot be relied on to reduce the magnitude of thermal stress in the wall, and it is reasonable and conservative to adopt as an assumption Babbitt's conclusion: " wheat remains at the temperature at which it was put into the bin."

Given that assumption, there are three bounding possibilities for temperature distribution through the bin wall:

- the wall temperature is uniform throughout and is equal to the grain temperature
- the wall temperature is uniform throughout and is equal to the ambient air temperature
- there is a temperature gradient through the wall, with the temperatures at the inner and outer surface being equal to the temperatures of grain and ambient air respectively.

The first possibility would predict no thermally induced stresses and therefore is rejected as unrealistic and unsafe for use in design. The second possibility would predict the largest magnitude for the circumferential tension force resultant, and is adopted as an assumption for use in estimating that resultant. This assumption is realistic for large silos, particularly if constructed of steel. The third possibility would predict the largest magnitude for moment resultants and is adopted as an assumption for use in estimating those resultants.

3.1.2 Physical Description

3.1.2.1 Stresses Caused by Static Pressures

Circumferential tension and axial compression forces in a cylindrical silo may be computed using the membrane theory of shells. However, with that theory, shear forces and bending moments are neglected. Grain silos are often constructed with some type of connection at their base whereby displacements are more or less inhibited. Restraining either translation or rotation at the silo base will cause stress resultants which may reach significant magnitudes. Furthermore, static grain pressure is maximum at the base, where bin wall displacements are most likely to be restrained. A comprehensive analysis of force distribution near the bottom is formulated. It aims at investigating the effects of varying edge restraint and mechanical properties of grain.

3.1.2.2 Temperature-induced Pressures and Stresses

When a silo full of grain is subject to a temperature drop, its wall tends to contract. The grain enclosed in the silo partially prevents that inward displacement of the wall since the confined mass behaves like an elastic solid. The bin wall is therefore subject to incremental lateral pressure and hoop stress. If temperature varies through the wall thickness, so will thermal strains. These differential strains are the source of significant bending moments and stresses.

3.1.3 Assumptions

3.1.3.1 Assumptions for Both Gravity and Temperature Effects

- The silo is full of grain.
- Janssen's formula for pressure distribution is valid.
- The silo is a hollow right cylinder of uniform thickness with an upright axis.
- The wall material is homogeneous, isotropic and linearly elastic, and both wall thickness and lateral displacement are small compared to radius.

A consequence of these assumptions is that displacement will be axisymmetric. Therefore the problem reduces to an analysis in two dimensions. Furthermore, the assumption of linear elasticity permits the use of superposition so that the actions of gravity and temperature may be considered separately.

3.1.3.2 Additional Assumptions for the Effect of Temperature

-Grain temperature is constant in time and space.

-The experimental observations of Manbeck and Nelson correctly characterize the constitutive behavior of wheat. These results were the only ones found in the literature that describe the response of wheat to horizontal forces. Wheat comprises a major proportion of the agricultural products that are stored in silos, and its mechanical properties are typical of several other grains. It should be noted that the technique developed in this thesis permits other constitutive models to be substituted for that of Manbeck and Nelson.

-The horizontal and vertical stresses given by Manbeck and Nelson are principal stresses. As a consequence, there will be no frictional shearing force along the vertical bin wall.

-Vertical strain in the grain remains constant during application of incremental horizontal strain and pressure. This assumption means that grain will not move upward when the bin wall compresses it and leads to maximum horizontal pressure on the grain. It is a realistic hypothesis for behavior near the bottom of the silo where vertical restraint is provided by the bottom and by the overlying mass of grain.

-Grain is linearly elastic for sufficiently small horizontal displacements.

Two alternative assumptions are necessary for temperature distribution. They are:

-Following a drop in air temperature, the temperature of the bin wall will be equal to the reduced air temperature and will be constant through the thickness.

-Or, there will be a linear variation of temperature through the wall, with the inside surface at the grain temperature and the outside surface at the air temperature.

3.2 Wall Stresses Induced by Gravity

3.2.1 Formulation

Figure 1 represents an element of the silo wall and the stress resultants acting on it. Since the problem is axisymmetric, twisting moments, shears on vertical faces and circumferential shear are not present. The formulation and solution of the governing differential equation follow Timoshenko (1959), with addition of non-linear variation of pressure with depth, vertical tractive load and silo dead weight.

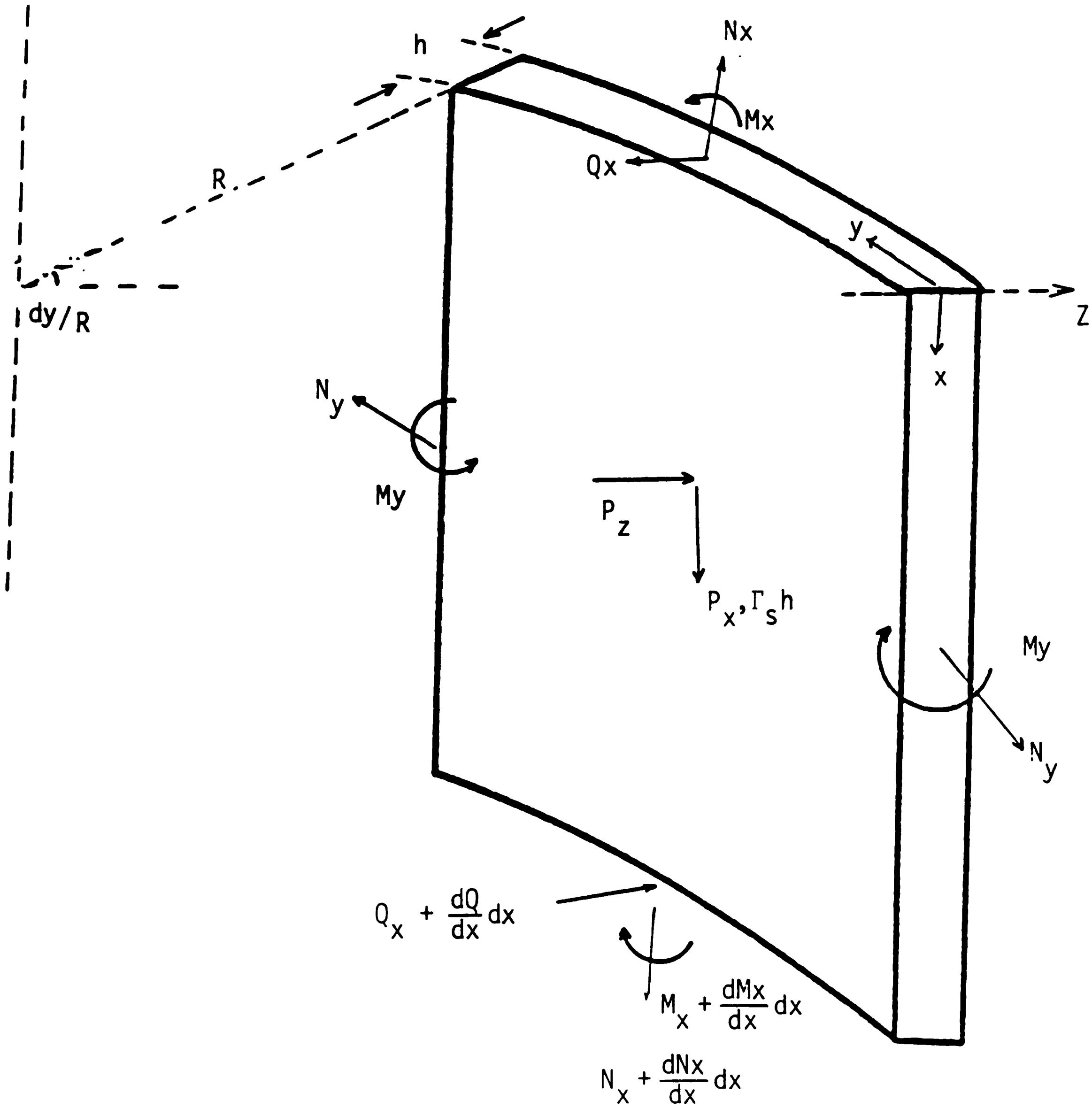

Figure 1. Element of Bin Wall

Equilibrium of vertical forces is expressed as

$$N_x = -\int_0^x \beta_1 p_z \, dx - \Gamma_s hx \qquad \text{III.2}$$

Equilibrium of radial forces is given by

$$p_z dxdy + (dQ_x/dx)dydx - N_y dxdy/R = 0 \qquad \text{III.3}$$

Rotational equilibrium about the circumferential axis is

$$(dM_x/dx)dxdy - Q_x dxdy = 0 \qquad \text{III.4}$$

Equations III.3 and III.4 may be written

$$p_z + dQ_x/dx - N_y/R = 0 \qquad \text{III.5}$$

$$dM_x/dx - Q_x = 0 \qquad \text{III.6}$$

Strains in the silo wall are

$$\epsilon_x = du/dx \qquad \text{III.7}$$

$$\epsilon_y = w/R \qquad \text{III.8}$$

Hooke's law leads to expressions for the resultant force

$$N_x = Eh(\epsilon_x + \mu\epsilon_y)/(1-\mu^2) = Eh(du/dx + \mu w/R)/(1-\mu^2) \qquad \text{III.9}$$

$N_y = Eh(\epsilon_y + \mu\epsilon_x)/(1-\mu^2) = Eh(w/R+\mu du/dx)/(1-\mu^2)$ III.10

A simplification of equation III.10 is achieved by eliminating du/dx:

$N_y = \mu N_x + Ehw/R$ III.11

Since axisymmetric loading of an axisymmetric shell produces axisymmetric deformation, all derivatives of w with respect to y disappear and moments are simply

$M_x = -Dd^2w/dx^2$ III.12

$M_y = \mu M_x$ III.13

where

$D = Eh^3/12(1-\mu^2)$ III.13 1/2

The preceding equations enable formulation of the governing differential equation. Specifically, equation III.12 is differentiated and substituted into equation III.6. The derivative of the resulting expression, together with equation III.11 are substituted into equation III.5 to produce

$d^4w/dx^4 + 4\beta^4 w = p_z/D - \mu N_x/RD$ III.14

where

$$\beta^4 = Eh/4R^2D \qquad \text{III.15}$$

From equilibrium of grain mass in x direction

$$\pi R^2 \Gamma_g x - 2\pi R \int_0^x \beta_1 p_z \, dx - \pi R^2 p_z/k = 0 \qquad \text{III.16}$$

or

$$\int_0^x \beta_1 p_z \, dx = R(\Gamma_g x - p_z/k)/2 = 0 \qquad \text{III.16 1/2}$$

Substituting in III.2

$$N_x = R(p_z/k - \Gamma_g x)/2 - \Gamma_s h x \qquad \text{III.17}$$

Equation III.17 and Janssen's formula II.1

$$p_z = \Gamma_g R(1 - e^{-2\beta_1 kx/R})/2\beta_1$$

are substituted into III.14 to give

$$d^4w/dx^4 + 4\beta^4 w = B_1(1 - e^{-2\beta_1 kx/R}) + B_2 x \qquad \text{III.18}$$

where

$$B_1 = \Gamma_g R(k - \mu/2)/2\beta_1 kD \qquad \text{III.19}$$

$$B_2 = \mu(\Gamma_s h + \Gamma_g R/2)/RD \qquad \text{III.20}$$

The solution is

$$w(x)=e^{(-\beta x)}(C_1\cos\beta x+C_2\sin\beta x)+e^{(-\beta(H-x))}(C_3\cos\beta(H-x)+C_4\sin\beta(H-x)) + (B_1+B_2x)/4\beta^4 - B_1e^{(-2k\beta_1 x/R)}/(4\beta^4+(2\beta_1 k/R)^4) \qquad \text{III.21}$$

The constants C_1, C_2, C_3, C_4 are determined from two boundary conditions at each end. Translation of either edge may be fixed or free, that is:

$$w = 0 \qquad \text{III.22}$$

or

$$Q_x = 0 \qquad \text{III.23}$$

Similarly, rotation of either edge may be fixed or free

$$dw/dx = 0 \qquad \text{III.24}$$

or

$$M_x = 0 \qquad \text{III.25}$$

After the appropriate boundary conditions are imposed, equation III.21 and its derivatives are used to determine displacement, slope and curvature. Then resultants are computed from equations III.17, III.11, III.12, III.13, and III.6.

Maximum stresses at any height x are given by

axial:

$$\sigma_x = N_x/h \pm 6M_x/h^2 \qquad \text{III.26}$$

circumferential

$\sigma_y = N_y/h \pm 6M_y/h^2$ III.27

shear

$s = 1.5Q_x/h$ III.28

3.2.2 Examples

Two representative grain silos will serve as basis for comparison. Grain properties, silo dimensions and wall material properties are given in Table 1. For both silos the top edge is free and the bottom edge is restrained against translation and rotation. Figure 3 shows the variation of wall forces and moment with vertical position for the shorter silo. The first and fifth lines of Table 2 contain extreme values of forces and moment for the two silos. These values are not directly proportional to height because of the non-linearity of equation II.1. According to sign convention being adopted, positive wall strain is elongation and positive wall stress is tension.

The silos were modified and reanalyzed to investigate effects of the four possible combinations of boundary conditions at the bottom edge. Extreme values of moments are compared in Table 2. Figure 2 shows important cases for bottom edge restraints.

Properties of grain that will be stored in a silo may be difficult to estimate a priori. Therefore, it is instructive to investigate the sensitivities of computed

Table 1 Base Cases for Gravity Pressures

Quantity	Value	Unit
Γ_g	8.0	KN/m^3
β_1	0.4	none
θ_2	32	degrees
R	3.0	m
h	0.155	m
H	15 or 30.0	m
Γ_s	24	KN/m^3
E	25	GPa
μ	0.2	none
Boundary conditions	Top edge free to translate and rotate, bottom edge restrained against translation and rotation	

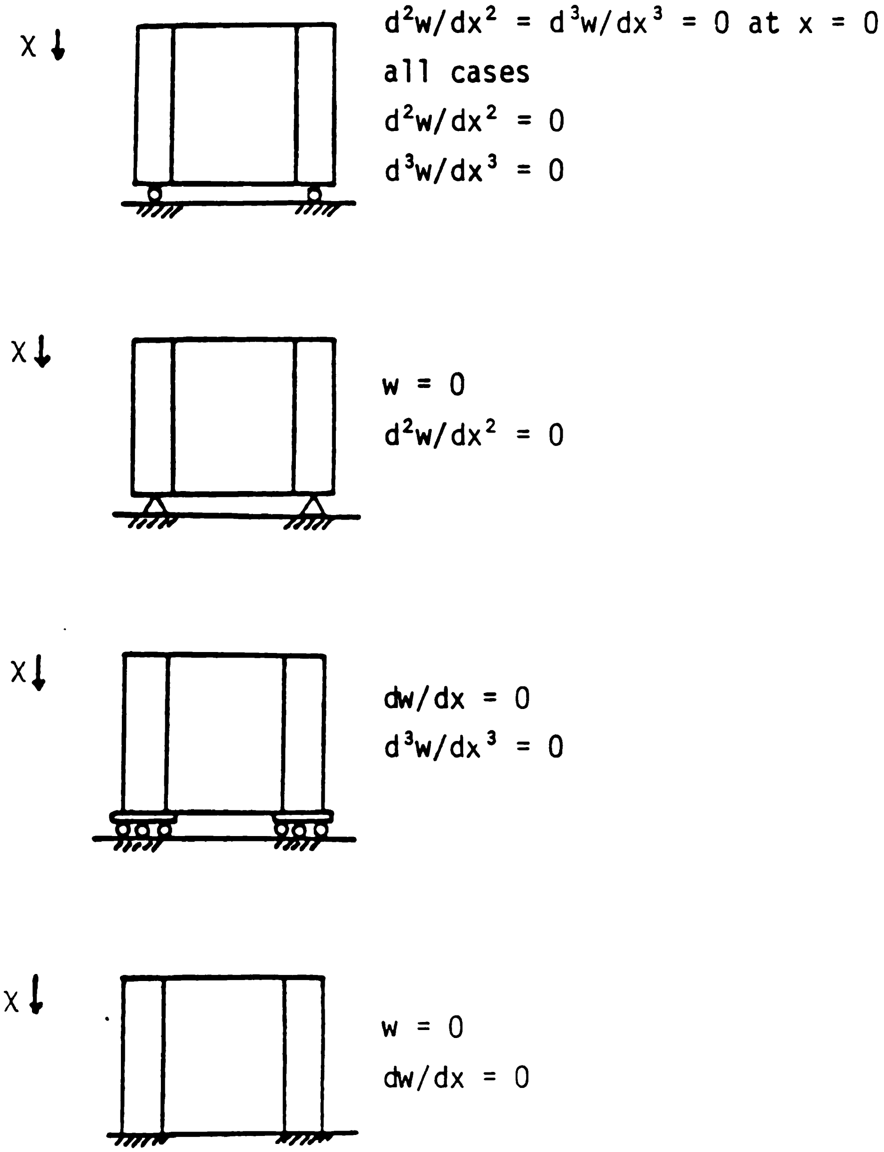

Figure 2. Type of Bottom Edge Restraints

Table 2 Extreme Values of Wall Forces and Moments for Gravity Pressures

F: Free, R: Restrained
Trans: Translation, Rot: Rotation

H, m	Bottom edge Trans, Rot	N_x, KN/m	N_y, KN/m	M_x, KN.m/m	Q_x, KN/m
15	R, R	-132,0	-39.7,64.3	-4.70,1.01	-18.0,1.2
15	R, F	-132,0	-39.7,67.7	-0.07,1.56	-9.1,1.9
15	F, R	-132,0	0,62.7	0,0.07	0,0.1
15	F, F	-132,0	0,63.7	0,0	0,0
30	R, R	-338,0	-101,89.1	-8.49,1.79	-32.2,2.2
30	R, F	-338,0	-101,93.8	-0.12,2.78	-16.2,3.4
30	F, R	-338,0	0,82.1	0,0.06	0,0.1
30	F, F	-338,0	0,82.3	0,0	0,0

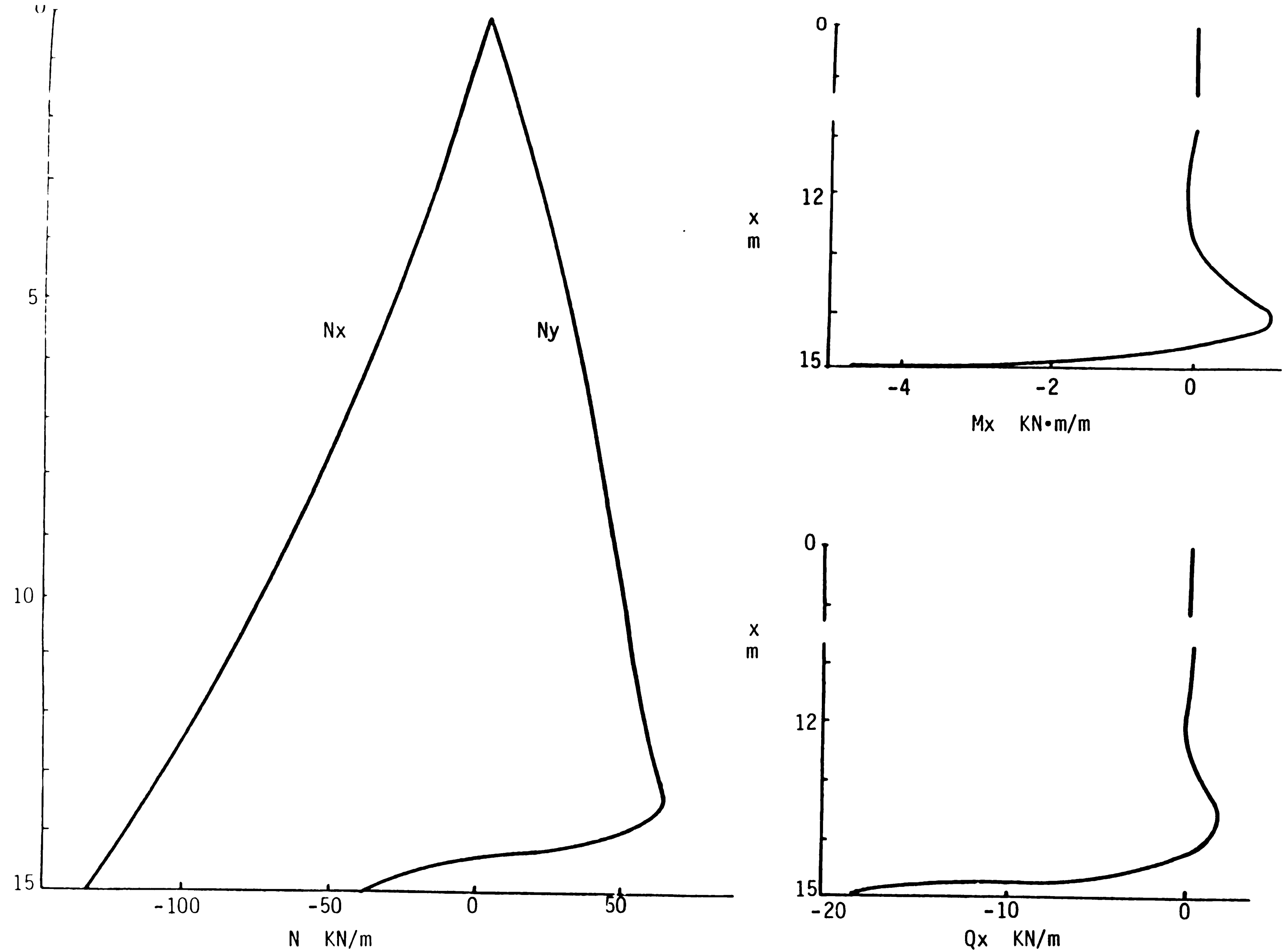

Figure 3. Stress Resultants in Silo Wall Caused by Gravity

wall forces and moments to variations in estimated grain properties. Sensitivity is defined as:

$$P_i(F_j - F_i)/(P_j - P_i)F_i \qquad \text{III.29}$$

where

P : Grain property

F : Wall force or moment

i : Base case

j : Case with variation

The silos, with bottom edge restrained in both translation and rotation, were analyzed again with each of the pertinent grain properties separately reduced by 12.5%. Table 3 presents the sensitivities computed by comparing extreme values given by those analyses to the base cases. None of the sensitivities exceeds unity.

3.2.3 Discussion

Shell theory has been used to compute shear forces and bending moments as well as circumferential and axial forces resulting from horizontal pressure and vertical friction in a cylindrical grain silo. Shear and moment are significant in a relatively small region (of length $2\pi/\beta$, about 3 m for the examples considered) near the bottom of the silo and can be nearly eliminated by providing a sliding bearing at the edge.

Table 3 Sensitivities of Extreme Wall Forces and Moments to Variations in Grain Properties for Gravity Pressures.

Grain property	H m	$-N_x$	$-N_y$	$+N_y$	$-M_x$	$-Q_x$
Γ_g	15	0.55	0.58	0.98	0.83	0.84
Γ_g	30	0.69	0.65	0.99	0.82	0.82
β_1	15	0.36	0.40	-0.46	-0.15	-0.18
β_1	30	0.33	0.31	-0.75	-0.19	-0.20
θ_2	15	-0.55	-0.50	-0.70	-0.61	-0.58
θ_2	30	-0.38	-0.40	-0.30	-0.33	-0.35

Table 4 Values of Bin Wall and Grain Properties used to Illustrate Temperature Effects.

Quantity	values	units
Γ_g	0.008	MN/m^3
θ_2	25	degrees
β_1	0.34	none
k	1.61	none
R	3	m
h	0.15	m
H	36	m
E	12	GPa
μ	0.2	none
μ_g	0.4	none
α	1.2 E-5	m/m.C
T or ΔT	40	C

Boundary conditions: Top edge free to translate and rotate, bottom edge restrained against translation and rotation

Away from ends, hoop tension and vertical stress resultants can be adequately approximated by membrane theory. If mechanical properties of the grain are uncertain, the designer should use an upper limit estimate for unit weight and a lower limit estimate for angle of internal friction.

3.3 Pressures and Wall Stresses Induced by Temperature Drop

3.3.1 Grain Stiffness

3.3.1.1 Experimental Observations

Wheat in mass was studied by Manbeck and Nelson (1975) who conducted triaxial tests and carried out a dimensionless analysis. They characterized wheat as an orthotropic material and their experiments showed that strain depends non-linearly on the level of stress and the ratio of horizontal to vertical stress in the grain mass. The range of the latter ratio considered in the experiments was .5 to 1.61. It was found that beyond the interval of these two values flow conditions occur. k=.5 corresponds to the lower limit equilibrium of grain and is in agreement with the theoretical value indicated for this situation by expression II.18 (k=.494). k=1.61 corresponds to the case of upper limit equilibrium of grain and confirms the value indicated by expression II.18 1/2 (k=1.63) for this case. Manbeck and

Nelson assumed the horizontal and vertical stresses applied to the grain mass by triaxial tests to be principal stresses. This assumption is also used in this study where the evaluation of stresses inside the stored grain mass is based on Janssen's formula.

Horizontal stress-strain relations are given by

$$\epsilon_h = .01(-4.92+12.2k-4.71k^2)(\sigma_h/.28)^{.454} \qquad \text{III.30}$$

and

$$\epsilon_v = .01(10-12.2k+3.72k^2)(\sigma_h/.28)^{.52} \qquad \text{III.31}$$

where

ϵ_h: horizontal strain in the grain mass

ϵ_v: vertical strain in the grain mass

σ_h: horizontal stress in the grain mass, expressed in megaPascals

3.3.1.2 Grain Response to Horizontal Pressure

A grain stiffness at a given point of contact between grain and bin wall is defined as the ratio of a small change in lateral grain pressure against bin wall to the corresponding change in horizontal displacement of the bin wall at this given point.

$$K_x = -\triangle p_z / \triangle w \qquad \text{III.32}$$

where the subscript x indicates that k_x is a function of vertical position.

Prior to any change in wall temperature, the horizontal stress σ_h inside the grain mass at any point near the bin wall is equal to the lateral grain pressure p_z given by Janssen's formula (Equation II.1) at that point.

Compression and contraction are counted positive for grain mass. At the point of their contact, grain and bin wall undergo equal horizontal displacements during contraction of the wall. If strain in the grain does not vary with radial position, then the horizontal strains of grain and wall also are equal in absolute value.

$$\epsilon_y = - \epsilon_h \qquad \text{III.33}$$

The minus sign is due to the sign conventions used for bin wall and grain mass.

From shell theory

$$\epsilon_y = w/R \qquad \text{III.34}$$

Therefore

$$d\epsilon_y = dw/R = -d\epsilon_h \qquad \text{III.35}$$

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$$K_x = -dp_z/dw = d\sigma_h/Rd\epsilon_h = E_h/R \qquad \text{III.36}$$

with

E_h : modulus of elasticity of grain in horizontal direction.

An expression of the ratio of horizontal to vertical stresses is then derived in terms of ϵ_h and σ_h as follows

From III.31

$$3.72k^2 - 12.2k + 10 = 51.6\epsilon_v/(\sigma_h^{.52}) \qquad \text{III.37}$$

Solving for k

$$k = (12.2 \pm \sqrt{\Delta_1})/7.44 \qquad \text{III.38}$$

where

$$\Delta_1 = (767.8\epsilon_v)/(\sigma_h^{.52}) \qquad \text{III.39}$$

Manbeck's results were obtained for

$$k < 1.61 \qquad \text{III.40}$$

so the realistic value of the ratio is

$$k = 1.64 - 3.72\sqrt{\epsilon_v/(\sigma_h^{.52})} \qquad \text{III.41}$$

By substituting k into the expression of horizontal strain

III.30 and differentiating with respect to horizontal stress, the expression of the modulus of elasticity is obtained, from which the formula of grain stiffness is

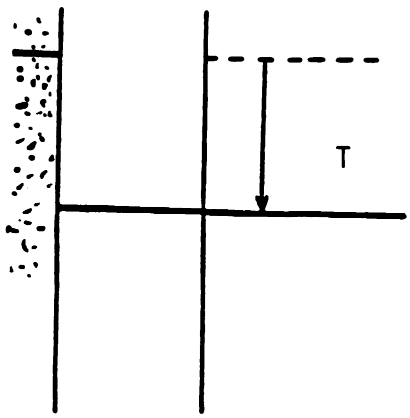
$$K_x=(56.1\sigma_h^{.546}/R)/(1.1+2.34\epsilon_v^{.5}/(\sigma_h^{.26})+4.3\epsilon_v/(\sigma_h^{.52})) \qquad \text{III.42}$$

3.3.2 Formulation

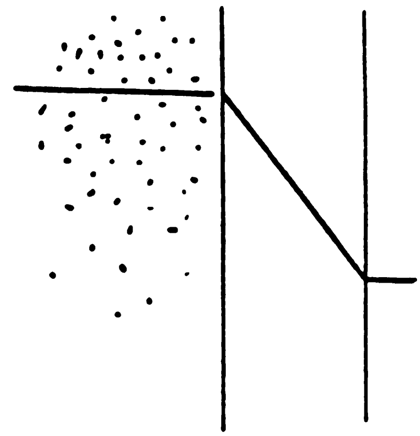
Following a drop in outside air temperature, the temperature distribution through the wall will be intermediate to the two cases shown in Figures 4a and 4b. The uniform distribution, Figure 4a, produces large hoop tension, and the gradient, Figure 4b, produces large bending moments. The latter distribution actually is a linear combination of a uniform temperature drop (Figure 4a) and a pure gradient (Figure 4c). The stresses in both the limiting cases, shown in Figures 4a and b, and any intermediate cases, can be computed as combinations of stresses for the uniform distribution (Figure 4a) and the pure gradient (Figure 4c).

3.3.2.1 Uniform Drop in Wall Temperature

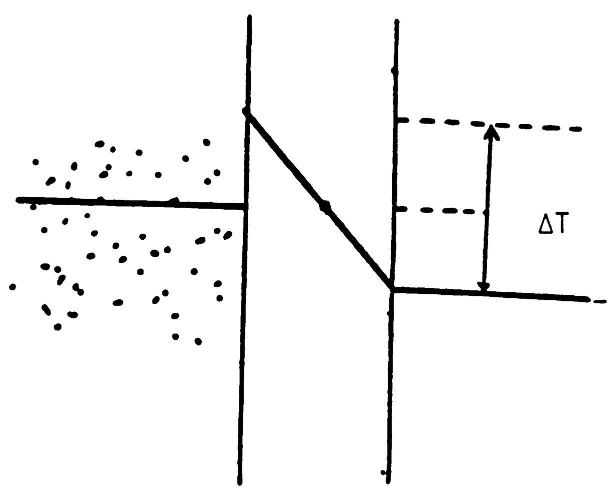
The action of the drop, T, in outside temperature on the bin wall is considered alone in this section.
Displacement of the wall is resisted by its stiffness and by



a. Uniform Drop



b. Gradient



c. Pure Gradient

Figure 4. Temperature Distribution

that of the grain.

The circumferential strain in the bin wall created by the temperature drop for an empty bin free of any end restraints is

$$\epsilon_y = -\alpha T \qquad \text{III.43}$$

The grain mass will react as an elastic foundation to any inward lateral displacement of bin wall by applying a radial pressure in the opposite direction of the displacement given as

$$p_z = -K_z w \qquad \text{III.44}$$

This radial pressure is constant around the circumference for a given depth. However it does vary with depth because the grain stiffness varies with the confining pressure. Neglecting the effect of temperature in the axial direction of the bin wall and applying Hooke's law gives the stress resultant in hoop direction

$$N_y = Eh\alpha T + Ehw/R \qquad \text{III.45}$$

The contraction of the bin wall being partially prevented by grain creates a tension in the wall. Any inward movement of the wall reduces this tension.

From equilibrium of forces in radial direction given by Equation III.5

$$N_y = R(p_z + dQ_x/dx) \qquad \text{III.46}$$

From III.45 and III.46

$$-dQ_x/dx - p_z + Ehw/R^2 + Eh\alpha T/R = 0 \qquad \text{III.47}$$

By recalling expressions III.6 and III.12

$$Q_x = dM_x/dx = -Dd^3w/dx^3 \qquad \text{III.48}$$

From III.44, III.47 and III.48

$$d^4w/dx^4 + 4\beta^4 w + Eh\alpha T/DR = 0 \qquad \text{III.49}$$

where

$$4\beta^4 = Eh/DR^2 + K_x/D \qquad \text{III.49 1/2}$$

3.3.2.2 Temperature Gradient Through the Wall

Let ΔT represent the temperature gradient through the bin wall, that is the difference between the temperatures of the inside and outside faces of the wall. Assume that the grain bin is subject only to the action of temperature gradient. The effect of a temperature gradient on an unrestrained shell element, as shown in Figure 1, is to induce curvatures along the axial and circumferential directions

$$\chi_y = \chi_x = \alpha \Delta T/h \qquad \text{III.50}$$

The bending moments taking into account the changes in curvatures due to temperature gradient are:

$$M_x = -D(d^2w/dx^2 - \alpha \Delta T(1 + \mu)/h)$$
$$M_y = -D(\mu d^2w/dx^2 \quad -\alpha \Delta T(1 + \mu)/h) \qquad \text{III.51}$$

The hoop stress resultant is

$$N_y = Ehw/R \qquad \text{III.52}$$

Recalling III.51, III.52, III.6, III.5 and III.44, the governing equation of the deflection of a cylinder subject to a temperature gradient is

$$d^4w/dx^4 + 4\beta^4 w = 0 \qquad \text{III.53}$$

where $4\beta^4$ is given by expression III.49 1/2.

3.3.3 Solution Technique

3.3.3.1 Introduction

The differential equations III.49 and III.53 contain a variable coefficient which is the grain stiffness. This quantity is a function not only of the independent variable x but also of the dependent variable w. This is due to the fact that grain stiffness varies with the level of pressure in the grain which, in turn, depends upon location along the height of the silo and changes with radial displacement of the bin wall.

The equations are therefore quasi-linear. Attempts to obtain a closed form solution were made difficult by the form of the available constitutive law of grain. Specifically, approaches based on power series and minimium potential energy methods were attempted. In the first approach, a polynomial expression was sought for the deflection w of bin wall. For that purpose grain stiffness K_x was expanded into a power series. The method was abandoned because of convergence difficulties. In the second, with w and K_x also approximated by power series, the expression of strain energy for the shell was a quadratic.

Therefore, it did not allow superposition of separately computed effects of gravity and temperature loading. This would have forced the use of a power series approximation of combined gravity and temperature loadings despite the fact that an exact solution had been found for gravity loading, which is the dominant effect. Therefore the energy approach also was abandoned.

3.3.3.2 Structural Analysis Model

a. Approach

An approach termed incremental structural analysis was then considered. It consists of subdividing the grain silo into ring elements (Figure 5) to which are applied incremental changes in outside temperature. In each ring element, grain is incorporated as a spring having the same stiffness as grain at the point considered (Figure 6). For small element heights and small increments in temperature it is reasonable to assume that K_x, and therefore the coefficient of w in equations III.49 and III.53, are constant.

The respective solutions to III.49 and III.53 are

$$w(x) = w_h(x) + Eh\alpha T/4\beta^4 RD \qquad \text{III.54}$$

and

$$w(x) = w_h(x) \qquad \text{III.54 1/2}$$

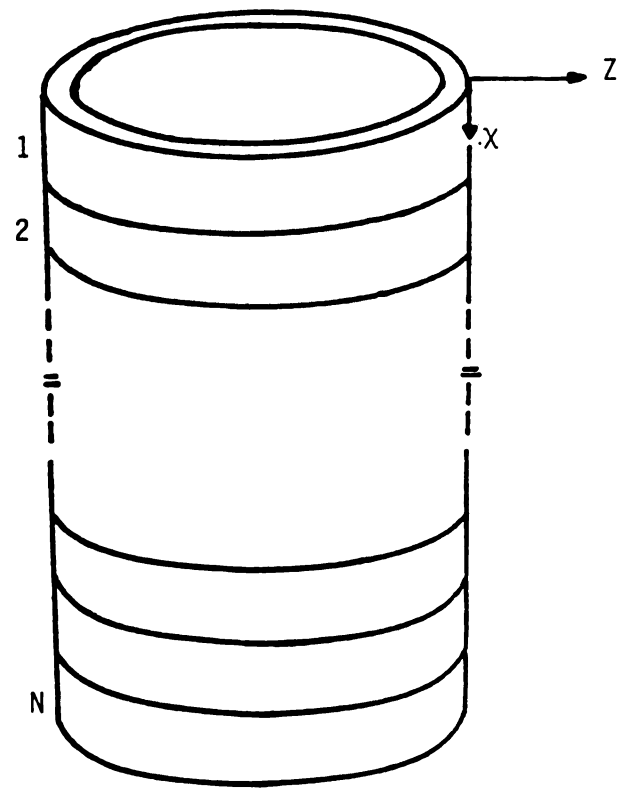


Figure 5. Subdivision of Silo into Small Ring Elements

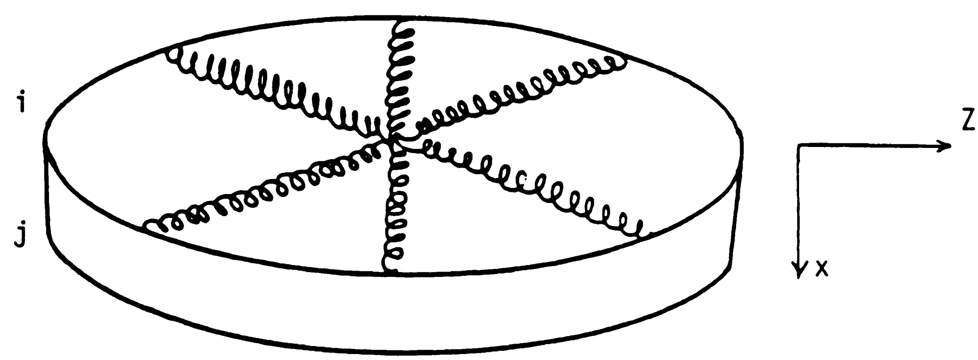


Figure 6. Ring Element

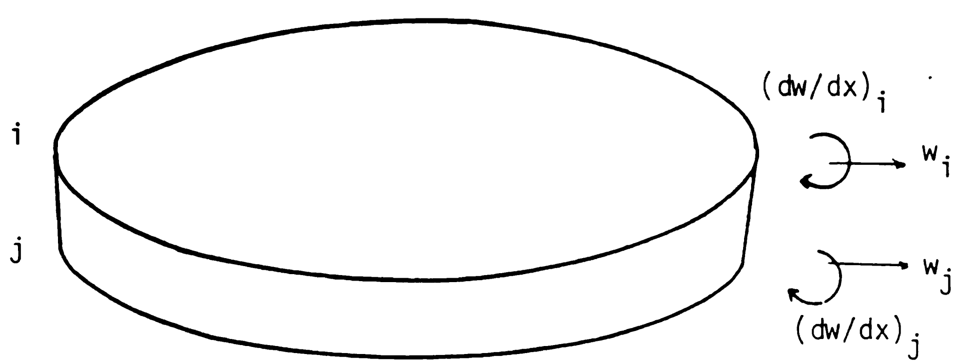


Figure 7. Degrees of Freedom per Element

where

$$w_h(x) = e^{(-\beta x)}(C_1\cos\beta x + C_2\sin\beta x) + e^{(\beta x)}(C_3\cos\beta x + C_4\sin\beta x) \qquad \text{III.55}$$

b. Element Stiffness Matrix K_{ij}

b1. Definition of K_{ij}

A member, k_{ij}, of an element stiffness matrix is the magnitude of force or moment at degree of freedom i necessary to impose a unit translation or rotation at degree of freedom j and zero translations and rotations at other degrees of freedom.

b2. Dimension of K_{ij}

Since each element has two edges with two degrees of freedom at each edge as shown by Figures 6 and 7, the element stiffness matrix is 4 by 4.

b3. Numbering of Degrees of Freedom

The degrees of freedom are numbered as follows:

1: translation at x=0

2: rotation at x=0

3. translation at $x=H_e$

4. rotation at $x=H_e$

b4. Displacement Function

(i). Translation is given by expression III.55
where $4\beta^4$ is given by expression III.49 1/2
Rotation, force and moment are computed as follows

(ii). Rotation is dw/dx
(iii). Force from expression III.48
(iv). Moment from equation III.12

b5. Solving for the C Coefficients

A column j of the element stiffness matrix corresponds to a unit translation or rotation at degree of freedom j and zero translation or rotation at the other three degrees of freedom. Figure 8 shows a unit translation at the upper node of an element. These four conditions are used as four simultaneous equations based on (i) and (ii), which are solved for the four unknown C coefficients.

b6. Evaluation of k_{ij}

Then each member k_{ij} of column j is computed as indicated by (iii) and (iv) using those values of C.

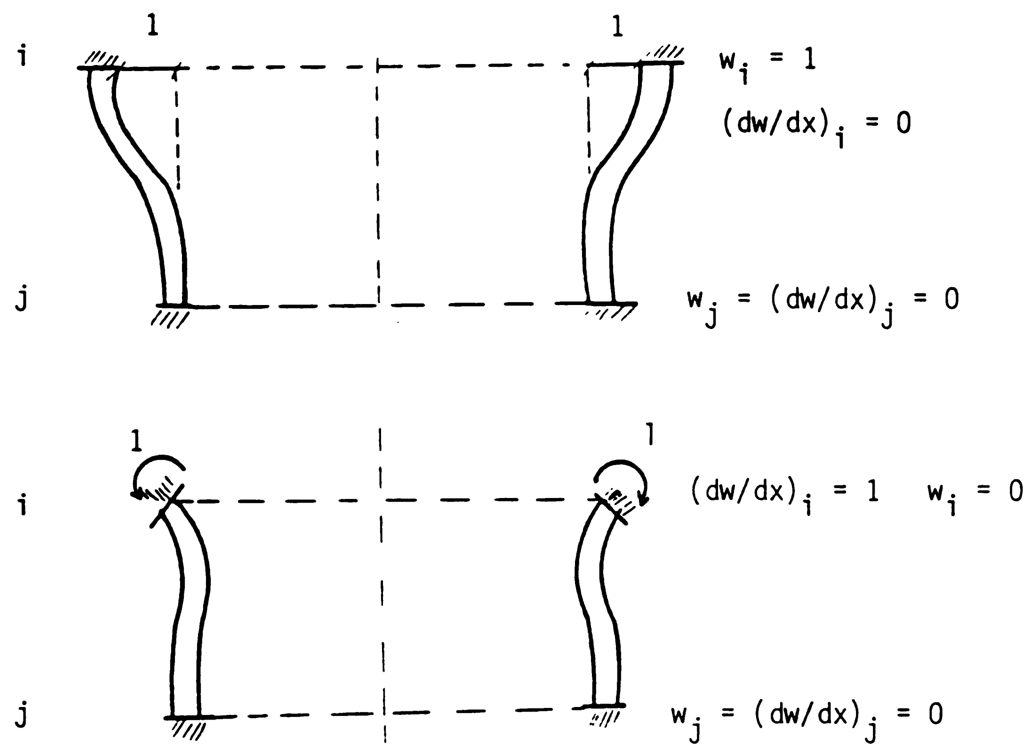


Figure 8. Unit Displacements Load Application to Element Edges

c. Load Vector

For each ring element, the equivalent edge loads caused by a change in temperature are found by assuming the element clamped at both edges. The expression of deflection of the element is given by equation III.54 or III.54 1/2 as appropriate involving four unknown constants, C. Four simultaneous equations are established by expressing the condition that the element is clamped at both edges, and these are solved for the four constants. Moment and shear at element edges are evaluated from III.12 or III.51 and III.6.

d. Equation for Assembled Silo

For each element, the element load vector is equal to the matrix product of the element stiffness matrix and the element displacement vector. However, the deflection and slope at adjoining edges of ring elements must be numerically identical. Therefore the assembled displacement vector is simply the set of all element displacements and, by extension, the assembled stiffness matrix and load vector are generated by combination of the element stiffness matrices and load vectors, respectively. Taking into account restraint conditions assigned to the silo at its edges, the assembled expression, from which the unknown displacements for a temperature increment may be determined, is

$$[K]\{U\} = \{F\} \qquad \text{III.56}$$

where

$[K]$ is the assembled stiffness matrix

$\{U\}$ is the vector of unknown displacements

$\{F\}$ is the assembled load vector corresponding to the action of temperature

e. Stress Resultants and Lateral Pressure.

After displacements at element junctions are computed, the stress resultants and changes in magnitude of lateral grain pressure may be evaluated at each temperature step. The procedure is described first for a uniform distribution of bin wall temperature.

1. After solution of the assembled equations, four displacements (translation d_1, rotation d_2 at edge i; translation d_3, rotation d_4 at edge j) are known and the updated value of grain stiffness is also determined for each element.

2. A system of four simultaneous equations is set up and solved for the four unknown constants C that appear in expression III.54. For edge i, x=0 and for edge j, $x=H_e$, which represents the height of the ring element.

$w_i = d_1$

$(dw/dx)_i = d_2$

$w_j = d_3$ III.57

$(dw/dx)_j = d_4$

3. The four coefficients are substituted into Equation III.54 which then gives the lateral deflection at any point within the ring element.

4. Moment, shear and hoop tension may then be calculated by differentiation of w and substitution in Equations III.12 III.6 and III.45.

5. The change in lateral pressure at a given point on the bin wall and for the temperature step considered is calculated from Equation III.44 using w obtained in step 3.

Similarly, this procedure is implemented for a temperature gradient through the wall. In steps 2. and 3., the expression of w that should be used is III.54 1/2. Moment and shear in step 4 are given by Equations III.51 and III.6. Hoop tension is obtained from III.52.

f. Temperature Increments

Initial values for quantities are first calculated for each element:

f.1. Expression II.1 gives lateral pressure using the initial value assigned to the pressures ratio k.

f.2. Expression III.31 yields vertical strain in the grain using the value of lateral pressure from f.1 and the initial value assigned to the pressures ratio k.

f.3. Expression III.42 gives the value of the grain stiffness K_x using lateral pressure from f.1. and vertical strain from f.2.

Then for each temperature increment and for each element, the calculations are carried out according to the following pattern:

1. Wall radial displacements w are determined from assembled stiffness matrix and load vector as indicated by Equation III.56

2. Increase in lateral pressure is obtained from the product $K_x w$. Lateral pressure is then updated.

3. The grain stiffness K_z is updated by using the value of vertical strain from f.2 and the updated value of lateral pressure from 2.

4. Pressure ratio k may be updated from III.41 using the updated value of lateral pressure from 2. and the constant value of vertical strain from f.2.

3.3.4 Model Validation

The incremental model was implemented on a microcomputer (Said and Bartali 1986). The model was validated in two ways.

First, the case of zero grain stiffness was considered, which corresponds to an empty silo. The results for both a uniform temperature drop and a temperature gradient were compared to those given by analytical solutions (developed in section 3.2) for a cylindrical shell subject to the same temperature loadings. Complete agreement was found between the two approaches concerning deformations and stress resultants of the shell wall.

The second validation was conducted by using Andersen's equation II.21, with the stress-dependent modulus of elasticity of grain in the horizontal direction developed in this study. At locations remote from bin ends, the results given by the model for increases in lateral pressure due to a uniform temperature drop were in very good agreement with Andersen's equation. It is to be noted that the latter equation was developed using membrane theory assumptions.

3.3.5 Examples

Base cases used in the examples are shown in Table 4. Grain characteristics used are the same as those used by Manbeck and Nelson to determine the constitutive law of wheat.

3.3.5.1 Effect of a Uniform Drop in Temperature

a. Lateral Pressure

Several example silos were analyzed in order to investigate the effect of various factors on thermally induced lateral pressure. These factors include silo dimensions, mechanical properties of bin material, temperature incrementation, number of elements used for silo discretization and initial value of the ratio of lateral to vertical pressure in the grain. Two factors were found to significantly affect the change in magnitude of lateral pressure. They are bin radius and initial value of the pressures ratio. Results are summarized in Figure 9 and Table 5. At edges where the translation of bin wall is inhibited there is no increase in lateral pressure. This

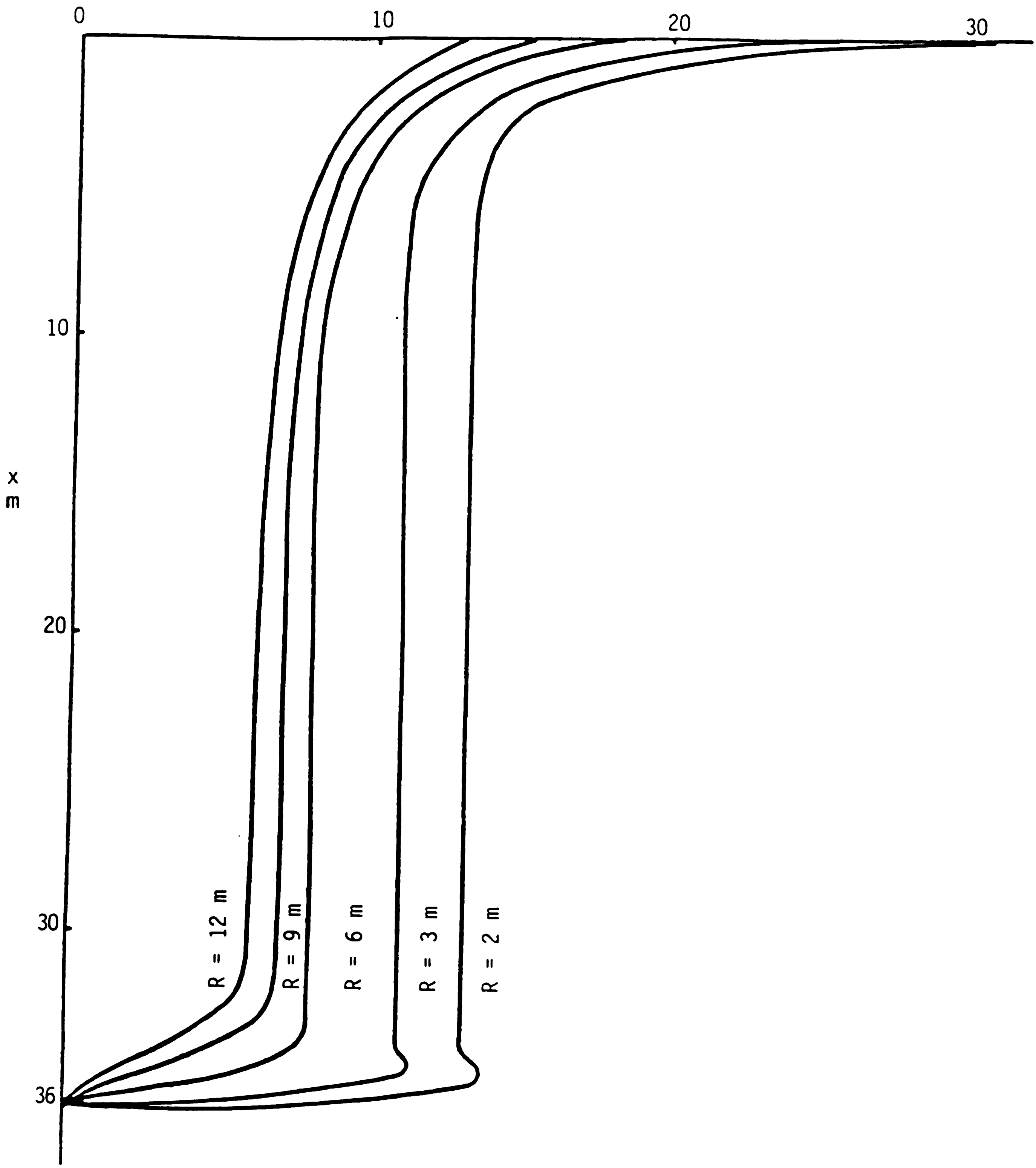


Figure 9. Variation of Percentage Increase in Lateral Pressure due to a Uniform Temperature Drop

Table 5 Variation of Percentage Increase with Respect to Static Pressure of Thermally Induced Lateral Pressure with Pressure Ratio. Uniform Drop in Temperature 40 C Conditions Other than k Summarized in Table 4.

Depth	Values of ratio k				
m	0.5	0.56	1	1.3	1.61
0.00	21.27	20.97	21.37	23.05	25.60
0.45	15.82	15.62	16.07	17.44	19.49
1.35	11.85	11.73	12.29	13.50	15.26
2.70	9.51	9.45	10.15	11.33	13.02
5.40	7.65	7.65	8.60	9.85	11.58
9.51	6.57	6.63	7.86	9.23	11.07
13.62	6.10	6.20	7.61	9.07	10.96
17.74	5.85	5.98	7.53	9.02	10.94
21.85	5.71	5.86	7.49	9.01	10.93
25.97	5.63	5.79	7.48	9.00	10.93
30.08	5.58	5.75	7.47	9.00	10.93
34.20	5.76	5.95	7.76	9.35	11.35
34.74	5.61	5.97	7.56	9.11	11.06
35.19	4.40	4.55	5.93	7.15	8.69
35.55	2.28	2.36	3.09	3.72	4.52
36.00	0.00	0.00	0.00	0.00	0.00

edge effect damps out quickly with distance from the restrained edges, and the pressure increase becomes almost constant and is in agreement with Andersen's equation. If the edge is free the increase in the pressure agrees with Andersen's formula through the entire depth.

b. Stress Resultants

The circumferential force caused by a uniform drop in temperature in a silo wall is higher when the silo is loaded (Table 7).
The examples confirm the prediction that uniform drop in the temperature of the bin wall leads to magnitudes of hoop tension higher than those for temperature gradient except at the top edge (Table 8).
Free and clamped bottom edges for top-free silos were considered. Extreme values of wall forces and moment due to a uniform temperature drop are shown in Table 11. The hoop stress resultant reaches a maximum near the bottom edge, and is higher when the bottom edge translation is restrained. Providing translational freedom eliminates positive and negative moments and shear and significantly reduces positive (tensile) circumferential force. Table 10 shows that for a silo with a top free edge and a bottom fixed edge, a uniform drop of temperature leads to a higher shear force at the bottom than does a gradient of temperature. Figure 10 and Figure 11 show the localized

Table 6 Percentage Increase in Lateral Pressure with Respect to Static Pressure due to a Uniform Temperature Drop and a Temperature Gradient. Conditions Summarized in Table 4

Depth m	Uniform drop 40 C	Gradient 40 C
0.00	25.60	-9.08
0.45	19.49	0.36
1.35	15.26	5.35
2.70	13.02	-0.04
5.40	11.58	0.00
9.51	11.07	0.00
13.62	10.96	0.00
17.74	10.94	0.00
21.85	10.93	0.00
25.97	10.93	0.00
30.08	10.93	0.00
34.20	11.35	0.00
34.74	11.06	0.00
35.19	8.69	0.00
35.55	4.52	0.00
36.00	0.00	0.00

Table 7 Circumferential Wall Force due to a Uniform Temperature Drop in an Empty Silo and a Loaded Silo.
Conditions Summarized in Table 4

Depth m	N_y (KN/m) empty silo	full silo
0.00	-0.000	4.797
0.45	-0.000	5.907
1.35	0.000	7.771
2.70	0.000	9.454
5.40	-0.000	10.640
9.51	-0.000	11.310
13.62	-0.000	11.530
17.74	-0.000	11.580
21.85	-0.000	11.590
25.97	-0.000	11.600
30.08	-0.004	11.590
34.20	-33.600	-21.400
34.74	-9.570	1.370
35.19	178.000	186.100
35.55	507.000	511.000
36.00	864.000	864.000

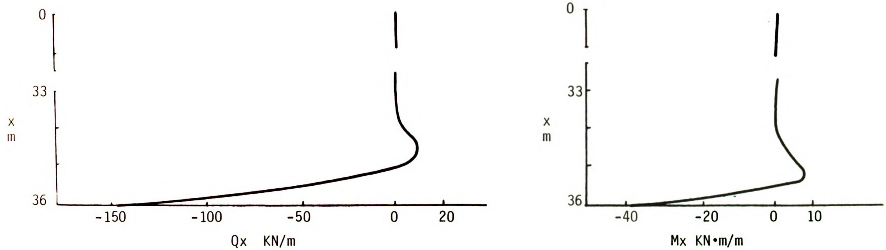


Figure 10. Moment and Shear in Silo Wall Caused by a Uniform Temperature Drop

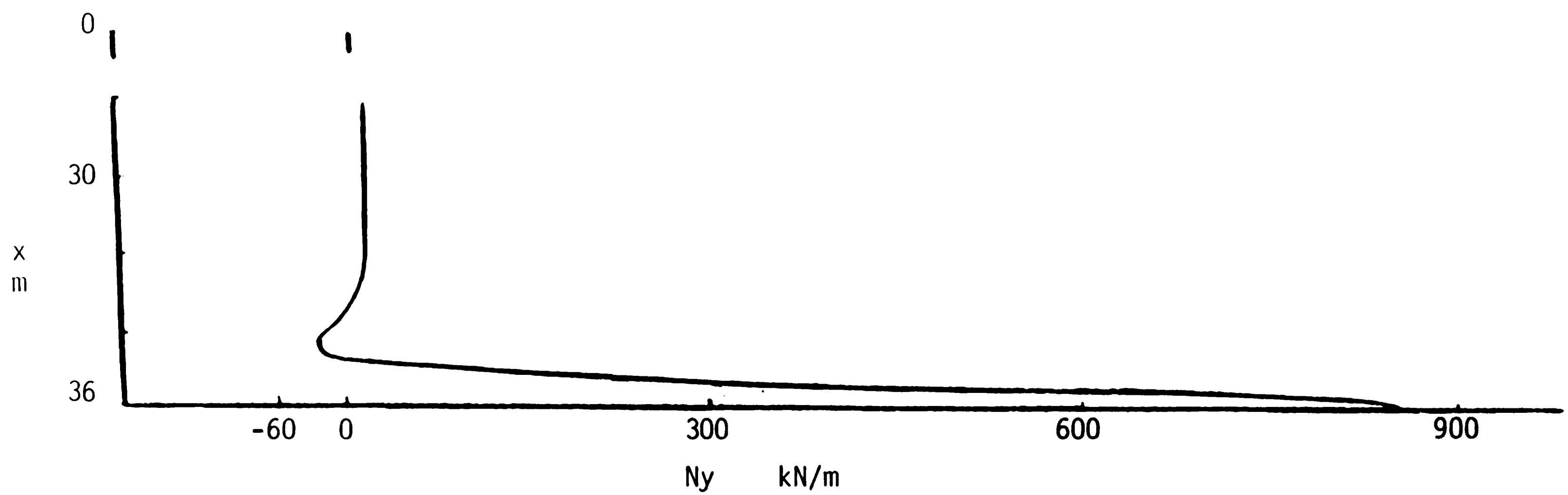


Figure 11. Circumferential Force in Silo Wall Caused by a Uniform Temperature Drop

Table 8 Circumferential Wall Force due to a Uniform Temperature Drop and a Gradient of Temperature Conditions Summarized in Table 4

Depth m	N_y (KN/m) uniform drop	gradient
0.00	4.797	304.600
0.45	5.907	-16.400
1.35	7.770	-29.980
2.70	9.454	2.180
5.40	10.640	0.003
9.51	11.310	0.000
13.62	11.530	0.000
17.74	11.580	0.000
21.85	11.590	0.000
25.97	11.600	0.000
30.08	11.590	0.000
34.20	-2.140	0.000
34.74	137.000	0.000
35.19	186.100	0.000
35.55	511.100	0.000
36.00	864.000	0.000

effect of edge restraints. Edge effects disappear within a distance that can be approximated by one wave length, $2\pi/\beta$ (about 2.3 m for the examples considered), in the expression of lateral deflection, given by equation III.54.

The membrane theory approximation

$$N_y = p_z R \qquad \text{III.58}$$

is valid at points remote from restrained edges, and at all points for silos with free edges. For points near restrained edges, moments are about the same as those given by the analytic method (i.e. grain stiffness is irrelevant since there is no deflection).

3.3.5.2 Effect of a Temperature Gradient

In this section, the effects of a pure gradient through the bin wall such as defined in Figure 4.c are discussed

a. Lateral Pressure

Changes in lateral pressure caused by a temperature gradient are extremely small compared to those caused by a uniform drop in temperature, as shown in Table 6. This is due to the fact that a temperature gradient causes mainly changes in internal bending moment and practically no

Table 9 Bending Moment Caused by a Uniform Temperature Drop and a Gradient of Temperature Conditions Summarized in Table 4

Depth m	Mx (KN.m/m) Uniform drop	Gradient
0.00	0.000	0.000
0.45	-0.003	-5.575
1.35	-0.005	-13.870
2.70	-0.001	-13.520
5.40	0.000	-13.500
9.51	-0.000	-13.500
13.62	0.000	-13.500
17.74	0.000	-13.500
21.85	0.000	-13.500
25.97	0.000	-13.500
30.08	-0.000	-13.500
34.20	0.655	-13.500
34.74	4.580	-13.500
35.19	7.880	-13.500
35.55	2.030	-13.500
36.00	-37.930	-13.500

Table 10 Shear Force due to a Uniform Temperature Drop and a Temperature Gradient Conditions Summarized in Table 4

Depth m	Q_x (KN/m) Uniform drop	Gradient
0.00	0.000	0.000
0.45	-0.043	-16.800
1.35	-0.052	-1.875
2.70	-0.058	0.235
5.40	-0.043	0.001
9.51	-0.015	0.000
13.62	-0.003	0.000
17.74	-0.000	0.000
21.85	-0.000	0.000
25.97	-0.000	0.000
30.08	-0.000	0.000
34.20	4.130	0.000
34.74	9.815	0.000
35.19	0.229	0.000
35.55	-39.330	0.000
36.00	-147.800	0.000

displacement. These displacements are null for a top and bottom clamped silo; the bin wall is highly stressed but it does not deform.

b. Stress Resultants

The examples show that a temperature gradient through the wall generates significant bending moments along most of the silo height (Table 9), which are approximated by:

$$M_x = M_y = D(1 + \mu)\alpha \Delta T/h \qquad \text{III.59}$$

except at free edges.

At free edges, a temperature gradient causes large hoop tension (Table 8) that can possibly be a serious cause of failure as stated by Kent (1931) and also large shear forces (Table 10). Figure 12 demonstrates the localized effect of top end conditions.

Free and restrained bottom edges for top-free silos were considered to study the effect of edge restraints. The effect on forces and moment near the bottom was significant, but the maximum values there are comparable to extremes elsewhere along the height so that there is little effect on the overall extreme values shown in Table 12. However, the shear forces associated with free top and bottom edges have opposite signs.

Table 11 Extreme Values of Wall Forces and Moments Caused by a Uniform Temperature Drop of 40 Conditions Other than Bottom Edge Connections Summarized in Table 4

Bottom edge Trans, Rot	N_y KN/m	M_x KN.m/m	Q_x KN/m
R, R	-24.7,864	-37.93,7.87	-147.8,9.89
R, F	-44.98,864	-0.529,12.2	-73.91,15.35
F, R	4.79,12	-.005,0	-.058,.01
F, F	4.79,12.02	0,0	-.058,0

Table 12 Extreme Values of Wall Forces and Moments Caused by a Temperature Gradient of 40 Conditions Other than Bottom Edge Connections Summarized in Table 4

Bottom edge Trans, Rot	N_y KN/m	M_x KN.m/m	Q_x KN/m
R, R	-63.1,304.6	-13.5,0	-16.8,0
R, F	-97.78,304.6	-14.40,0	-16.8,26.3
F, R	-63.2,304.6	-13.5,0	-16.8,0
F, F	-63.2,304.6	-14.08,0	-16.8,16.96

R : Restrained
F : Free
Trans : translation
Rot : Rotation

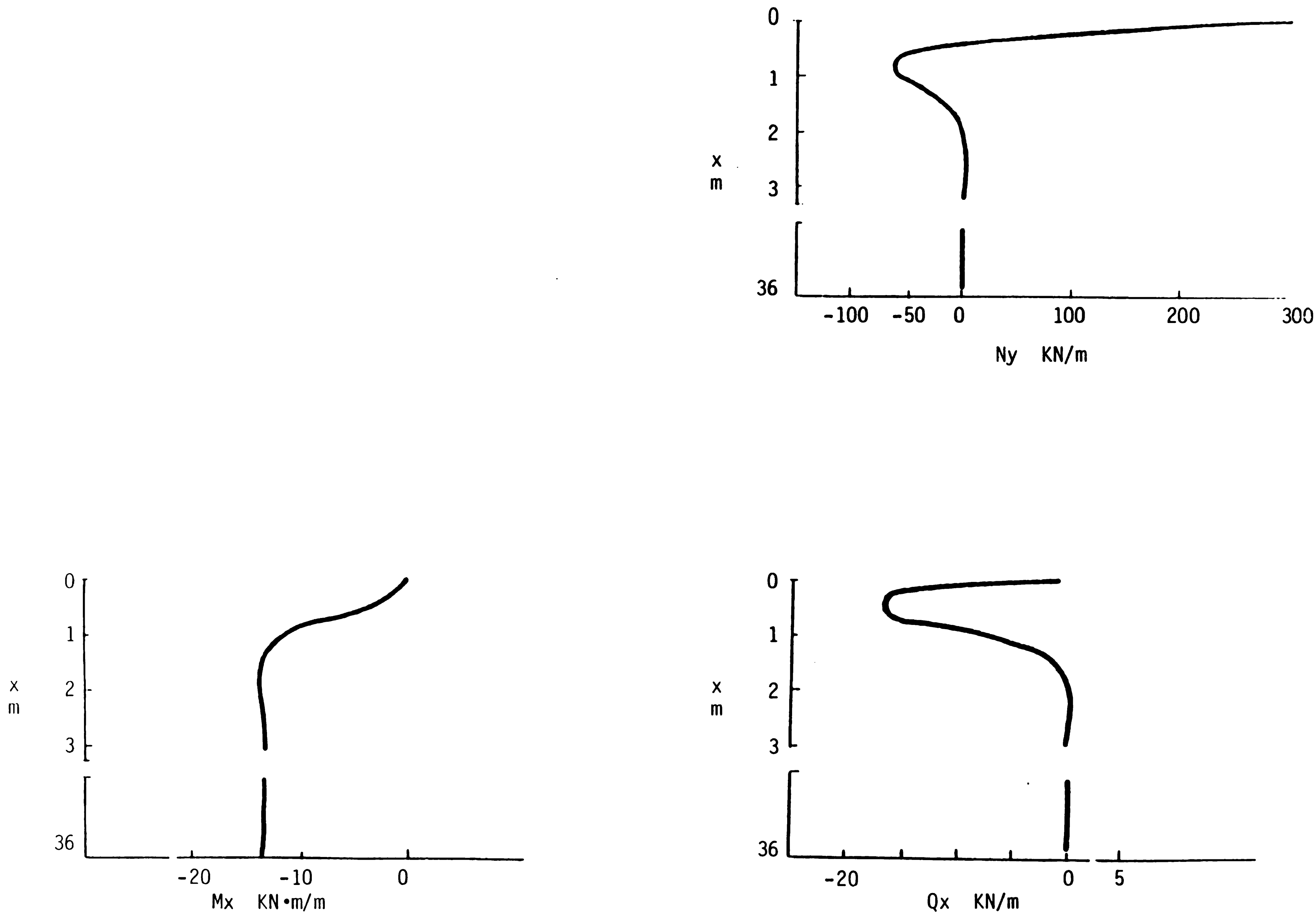


Figure 12. Stress Resultants in Silo Wall Caused by a Temperature Gradient

3.3.6 Discussion

The presence of grain in a silo is a source of magnification of thermally induced pressure and hoop stress. These are mostly affected by a uniform drop in temperature of bin wall rather than by a temperature gradient through the wall. In the first situation the bin wall tends to deflect appreciably but is partially restrained by the stored grain, whereas in the second, the bin wall exhibits little or no displacement toward the grain. The parameters that affect the magnitude of the increase in lateral pressure are the bin radius and the initial ratio of lateral to vertical pressure of the grain. The magnitude of the increase in lateral pressure decreases as the radius of the bin increases whereas it increases with the initial value of the pressures ratio. The effect of the radius is consistent with what Andersen's equation predicts. However, the latter does not include any information about the pressures ratio. As the wall moves radially inward, the lateral pressure in the grain increases and may become higher than the vertical pressure inside the grain depending on the magnitude of wall displacement. That is an upper limit equilibrium state for the granular material is approached. Lumbroso's expression II.18 1/2 gives an accurate estimation of the value of k corresponding to the upper limit equilibrium of a stored grain mass and can give an indication to the designer to estimate the first value to be assigned to the ratio k. Since it is difficult to assess the value of the pressure ratio a priori, it is safer from a design point of view to assume a relatively high value when investigating the thermally induced pressures in a grain bin. For the different ratios k examined, final values were less than 2 % higher than initial ones. In this study, the conservative value of 1.61 was used as initial value for k. The presence of grain inside a bin represents a thermal mass and therefore a source for a temperature gradient through the wall. This gradient does not have any apparent effect on lateral pressure but it does cause large bending moments in the bin wall.

The effect of restraints is localized near the edges and damps out within a distance corresponding to one wave length, $2\pi/\beta$,from these edges. When an edge is restrained against translation, it is the bin wall that absorbs the effect of a temperature drop. The restraint relieves the grain from any incremental compression. This results in a zero increase in lateral pressure at the location of restraint.

The grain stiffness used in the model exhibits little sensitivity to the degree of refinement of temperature increments and subdivision of silo into small elements. A one-step temperature drop is sufficient to predict pressure changes caused by a drop in outside temperature, if the Manbeck and Nelson results are used.

Due to the small wall thickness of steel grain silos, these structures are unlikely to experience temperature gradient through the wall. Reinforced concrete grain silos may experience either a uniform distribution, or a combination of uniform distribution and pure gradient.

CHAPTER 4

CONCLUSIONS AND RECOMMENDATIONS

4.1 Conclusions

4.1.1 Stresses Caused by Gravity

1. Bending theory of thin shells may be used to compute shear forces, bending moments and circumferential and axial forces in grain silos where internal pressure is given by Janssen's formula, and vertical forces due to friction and dead weight are included.

2. Hoop tension and vertical stress resultants can be adequately approximated by membrane theory at distances exceeding one wave length $2\pi/\beta$ from a fixed edge.

3. Shear and moment are important in a relatively small region near the bottom edge of a silo if it is restrained.

4. Providing a sliding bearing edge nearly eliminates shear and moment resultants.

5. When mechanical properties of the grain are not certain the designer should use an upper limit estimate of unit weight and a lower limit estimate of angle of internal friction in order to achieve a safe design.

4.1.2 Thermally-induced Pressures and Stresses

1. Grain stiffness concept and thin shell theory are applicable to studies of thermal pressures and stresses in cylindrical grain bins.

2. The incremental model that has been developed offers a rational method to predict temperature effect on silos containing granular material for which a constitutive law is available.

3. A linearized grain stiffness is sufficiently accurate for predicting increases in lateral pressures caused by a temperature change.

4. Membrane theory provides an adequate approximation of hoop tension resultant caused by a uniform temperature drop except near restrained edges.

5. In regions near restrained edges and for thermal gradient alone, grain stiffness is irrelevant because displacements are very small. Therefore the analytical solution is applicable.

6. For a grain bin, a uniform drop in wall temperature is critical for hoop tension whereas a temperature gradient generates large bending moments on most of the silo height.

7. Large shear and tension forces are created by a temperature gradient at free edges of bin walls.

8. At locations away from restrained edges, the presence of grain in a silo causes increase in hoop tension when the temperature of the wall is reduced.

9. Bin radius and initial value of ratio of lateral to vertical pressure in grain are the two factors that affect significantly the change in lateral pressure induced by a uniform drop in wall temperature.

10. Effects of uniform temperature drop and of temperature gradient can combine with other factors to cause maximum pressures and stresses on bin wall.

4.2 Recommendations for Further Studies

1. Further experimental research is needed to determine the constitutive laws of granular materials stored in silos so that incremental hoop tension due to thermal contraction of the bin can be estimated more accurately.

2. Further research, both experimental and analytical, on temperature variation in stored grain and bin walls would eliminate the need for necessarily conservative assumptions and therefore might permit savings in material for silos designed to withstand temperature change.

LIST OF REFERENCES

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ACI Committee 313, 1977. Recommended Practice for Design and Construction of Concrete Bins and Bunkers for Storing Granular Materials, American Concrete Institute, Detroit, Michigan, 38 pp.

Airy, W., 1897. "The Pressure of Grain, "Proceedings, Institution of Civil Engineers, Vol. 131, pp. 347-358.

Amundson, L. R., 1945. "Determination of Band Stresses and Lateral Wheat Pressures for a Cylindrical Grain Bin, " Agricultural Engineering, 26 (8): pp. 321-324.

Andersen, P., 1966. "Temperature Stresses in Steel Grain Storage Tanks," Civil Engineering, January, p. 74.

ASAE., 1983. Agricultural Engineering yearbook, 30th edition. American Society of Agricultural Engineers, St. Joseph Michigan.

Babbitt, J. D., 1945. "The Thermal Properties of Wheat in Bulk," Canadian Journal of Research, 23 (F): pp. 388-401.

Bishara, A. G., 1976. "Silage Constitutive Laws and Finite Element Analysis of Silage-Silo Interaction," Proceedings, TASS World Congress on Space Enclosures, Concord University, Montreal, July, pp. 867-877.

Bishara, A. G. and others., 1983. "Static Pressures in Concrete Circular Silos Storing Granular Materials," " American Concrete Institute Journal, May-June, pp. 201-216

Bovey, H. T., 1903. "Discussion on Grain Pressures in Deep Bins," Trans. CSCE, Vol. 17, pp. 608-654.

Bovey, H. T., 1903. "Experiments on Grain Pressures in Deep Bins and the Strength of Wooden Bins," Engineering News, Vol. 2, pp. 32-34.

Briassoulis D. and Curtis J. O., 1985. "Effect of End Restraints and Forces on the State of Stress in Circular silos," Trans. ASAE, pp. 222-231.

Britton, M. G., 1973. Strains in Deep Bin Walls due to Ambient Temperature Decrease, Unpublished Ph. D. thesis, Texas A and M, College Station.

Caquot, A., 1957. "la Pression dans les Silos, " Proceedings of the 4th International Conference on Soil Mechanics and Foundation Engineering, London, Vol. 2, pp. 191-195.

Carson, W. M. and others., 1979. "Galerkin's Method in Agricultural Engineering," Canadian Agricultural Engineering Journal, Vol.21, pp. 125-130.

Converse, H. H. and others., 1969. "Transient Heat Transfer within Wheat Stored in a Cylindrical Bin," ASAE Paper No. 69-855 presented at 1969 Winter Meeting of ASAE, Chicago, Ill.

Despeyroux, J., 1958. "Efforts Exerces sur les Parois par la Matiere Ensilee," Annales de l'Institut Technique du Batiment et des Travaux Publics, Vol. 131, pp. 1216-1224.

Fordham, A. A., 1937. "The Direct Measurement of Lateral Pressure on Walls and Bins," Engineering, Vol. 143, pp. 561-562.

Gazi, M. A., 1976. "Flow Properties of Granular Masses: A Review of the Angle of Repose," The Arabian Journal for Science and Engineering, 1(12), pp. 69-77.

Hall, C. W., 1957. Drying Farm Crops, Edwards Brothers, Inc. Ann Arbor, Michigan

Hamilton, M. E., 1964. An Analysis of Lateral Strain in Cylindrical Grain Bin Walls Subjected to Active and Passive Pressures, Unpublished M. S. thesis, Oklahoma State University, Stillwater.

Hay, W. W., 1920. "Design of Circular Reinforced Concrete Bins," Concrete, 17(3), pp. 73-76.

Hay, W. W., 1928. "Design of Deep Circular Bins," Concrete 32(6), pp. 43-44.

Issacson, J. D. and Boyd, J. S., 1965. "Mathematical Analysis of Lateral Pressures in Flat Bottomed, Deep Grain Bins," Trans. ASAE, 8(3), pp. 358-360, 364.

Jakobson, B., 1958. "On Pressure in Silos," Proceedings, 1958 Brussels Conference on Soil Mechanics and Foundation Engineering, Rotterdam Vol. 1, pp. 103-107.

Jamieson, J. A., 1903. "Grain Pressure in Deep Bins," Trans. CSCE, Vol. 17, pp. 554-607.

Janssen, H. A., 1895. "Versuche uber Getreiderdruck in Silozellen," Zeitschrift des VDI, Vol.39, pp. 1045-1049.

Jenike, A. W. and Johanson, J. R., 1968. "Bin Loads," Proceedings, American Society of Civil Engineers, Journal of the Structural Engineering Division, (ST4), pp. 1011-1041.

Jenike, A. W., 1954. "Flow of Bulk Solids," Bulletin No. 64, Utah Engineeriing Experiment Station, University of Utah, Salt Lake City, Utah.

Jenike, A. W., 1962. "Gravity Flow of Solids," Trans. Instn. of Chem. Engrs, Vol. 40, pp. 264-271.

Kazarian, E. A. and Hall, C. W., 1965. "Thermal Properties of Grain," Trans. ASAE, 8(1), pp. 33-37, 48.

Kent, C. H., 1931. "Thermal Stresses in Thin Walled Cylinders," Paper APM 53-13. National Applied Mechanics Meeting ASME, Lafayette, Indiana, June 15-16.

Ketchum, M. S., 1919. Design of Walls, Bins and Grain Elevators, 3rd ed., Mac Graw Hill Book Co., New York.

Kirshan, A. and Singh, R., 1970. "Influence of Moisture and Gravity on Selected Thermal Properties of Gravity," " Journal of Agricultural Engineering, X(5,6), pp. 1-7.

Lawton, P. J., "Physical Properties of Agriculture Produce Stored Materials," Journal Agricultural Engineering Research, Vol. 25, pp. 75-86.

Lencenzer, D., 1963. "An Investigation into the Behavior of Sand in a Model Silo," Structural Engineerig, 41(12), pp. 389-398.

Long, J. D., 1931."The Design of Grain Storage Structures," Agricultural Engineering, 12(7), pp. 274-275.

Lumbroso, A., 1977. "Bases Scientifiques du Calcul des Enceintes Renfermant des Massifs Pulverulents et du Calcul des Silos," Annales de l'Institut Technique du Batiment et des Travaux Publics, Juin 77, p. 21.

Lvin, J. B., 1970. "Analytical Evaluation of Pressures of Granular Materials on Silo Walls," Powder Technology, Vol. 4:280-285.

Mahmoud, A. A., 1979. Shallow Flexible Cylindrical Grain Bins, Ph. D. thesis, University of Windsor, Windsor, Canada.

Mac Calmont J. R. and Ashby, W., 1934. "Pressure and Loads of Ear Corn in Cribs," Agricultural Engineering, 15(4), pp. 123-125, 128.

Manbeck, H. B. and Nelson, G. L., 1972. "Methods and Instrumentation for Evaluating the Stress-Strain Behavior of Wheat in Mass," Trans. ASAE, 15(5), pp. 919-923.

Manbeck, H. B. and Nelson, G. L., 1975. "Three Dimensional Constitutive Equations for Wheat in Mass," Trans. ASAE, 18(6), pp. 1122-1127.

Manbeck, H. B., 1982. "Predicting Thermally Induced Pressures in Grain Bins," Paper No. 82-4070, ASAE.

Marchant, J. A. and Westgate, G. R., 1982. "A Technique for Measuring the Stress Exerted by Granular Materials on Retaining Structures," Journal of the Agricultural Engineering Research, 27(2), pp. 93-100.

Metzger, J. F. and Muir, W. E., 1983. "Computer Model for Two-dimensional Conduction and Forced Convection in Stored Grain," Canadian Agricultural Engineering Journal, Vol. 25, pp. 119-125.

Mohsenin, N. N., 1970. Physical Properties of Plant and Animal Materials, vol. 1 Gordon and Breach Science publishers, New York. 734 p.

Moysey, E. B., 1979. "Active and Passive Pressures in Deep Grain Bins," Trans. ASAE, Vol. 22, pp. 1409-1413.

Muir, W. E., 1970. "Temperatures in Grain Bins," Canadian Agricultural Engineering Journal, 12(1), pp. 21-24.

MWPS-1 1983. Structures and Environment Handbook, Midwest Plan Service. 11th Ed. Iowa State University, Ames, Iowa.

Myers, K. F., 1974. Overpressures in Bolted Steel Grain Bins due to Ambient Temperature Fluctuations. Unpublished M. S. thesis, Ohio State University, Columbus, Ohio.

Narayan, C. V. and Bilanski, W. K., 1970. "Behavior under High Pressure of Wheat Grains in Bulk," Trans. ASAE, 13(3), pp. 298-320.

NRCC. 1977. Canadian Farm Building Code, NRCC. No. 15564. National Research Council of Canada, Ottawa.

Pamelard, H., 1959. "Remarques sur le Calcul des Silos," Genie Civil, No. 136, pp. 490-492.

Pfalzner, P. M., 1951. "The Specific Heat of Wheat," Canadian Journal of Technology, Vol. 29, pp. 261-268.

Reimbert, M. and Reimbert, L., 1976. Silos Theory and Practice, Trans Tech Publications, Clausthal.

Roberts, I., 1884. "Determination of the Vertical and Lateral Pressures of Granular Substances," Proceedings of the Royal Society of London, Vol. 36, pp. 225-240.

Said, H., 1985. Contribution à l'Etude des Pressions Engendrees par une Baisse de Temperature dans les Silos, Memoire de 3 eme Cycle Genie Rural Encadre par Bartali, H. Institut Agronomique et Veterinaire Hassan II, Rabat.

Saxena, N. C., 1970. Effect of Grain Temperature Changes on Pressures in Storage Bins, Unpublished M. S. thesis, The Ohio State University, Columbus.

Stewart, B. R., 1968. "Effect of Moisture Content and Specific Weight on Internal Friction Properties of Sorghum Grain," Trans. ASAE, 11(2), pp. 260-262.

Thompson, S. A., 1982. "Thermal Stresses in Grain Bins using the Tangent Modulus of Grain," Paper No. 82-4069. ASAE.

Thorpe, G. R., 1982. "Moisture Diffusion through Bulk Grain Subjected to a Tempertaure Gradient," Journal of the Stored Products Research, 18(1), pp. 9-11.

Timoshenko, S. P. and Woinovsky-Krieger S., 1959. Theory of Plates and Shells, 2nd edition. Mc Graw-Hill Book Co. New York. 580 p.

Toltz, M., 1903. "Discussion of Grain Pressure in Deep Bins," Trans. CSCE, Vol. 17, pp. 607-654.

Turitzin, A. M., 1963. "Dynamic Pressure of Granular Material in Deep Bins," Proceedings, ASCE, Journal of the Structural Engineering Division, 89 (ST2):49-73.

Weiland, W. F., 1964. "Structural Failures of Grain Storage Facilities," Trans. ASAE, pp. 130-133.

Yaciuk, G., 1973. Temperatures in Grain Storage Systems, Ph. D. thesis. University of Manitoba, Winnipeg, 171 p.

Zakerzewski, M. S., 1959. "Design of Silos for Grain Storage," Die Sivielle Ingenieur in Suidechike Africa, Vol. 4, pp. 69-89.