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THE RELATIONSHIP BETWEEN LAND VALUES AND FACTOR COSTS

By

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ABSTRACT

THE RELATIONSHIP BETWEEN LAND VALUES AND FACTOR COSTS

by

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The purchase of farmland can be one of the most difficult investment decisions confronting farm operators. Compared with other production inputs, land is purchased infrequently and usually in a discrete size unit and involve long term financial obligation. When a tract of land is available for sale in the market, knowledge of how much a farmer can afford to pay for that tract of land is very important.

Land values are based on the incomes it can earn, now and in the future, that is the present value of all future benefits. In other words, land value today equals the present value of the income expected from land plus the present value of the price of land he receives when the land is sold at the end of the planning period. Expectation about future incomes stream is estimated using the immediate past incomes stream assuming that the best clue about the future is the immediate past. Since land is a durable that provides services to the production process for more than one

period, the return to land input is determined as the value of the portion of output attributed to land.

The purposes of this study were: First to review the theory of partitioning output among inputs. Second to compare and contrast land values under alternative acquisition assumptions of land and machinery. Third, to study the effects of increases in factor costs on land values.

A new theory of partitioning output among inputs is introduced in this dissertation. This theory states that the portion of output attributed to any input is the integral of long run marginal value products along the firm's expansion path. It is graphically and analytically illustrated that the firm's expansion path and hence the share of output attributed to land changes under alternative assumptions of land and machinery acquisition. Next the effect of increasing factor cost on the value of such inputs as land using a Cobb-Douglas production function is illustrated.

The theory is implemented using linear programming (LP). The LP model maximizes profits subject to resource constraints in each time period for three consecutive periods: 1977, 1978, and 1979. The shadow prices, equal to long run marginal value products, were summed to determine the value of output attributed to land.

The results indicate that the value of the output attributed to land, expected average income, and land values for the current period are greater under the assumption of land ownership than those estimated under the assumption that land is rented. The value of the output attributed to land under the assumption of machinery ownership was greater than that estimated under the assumption of custom hiring machinery services. Land value for machinery ownership was greater than land value under the assumption of custom hiring machinery services. Lastly, the results indicated that as costs of such factors as labor, fuel, and fertilizer increase, the value of land would decrease.

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CHAPTER I

INTRODUCTION

The purchase of farmland can be one of the most difficult investment decisions confronting farm operators. Compared with other production inputs, land is purchased infrequently and usually in a discrete sized units and involves a long term financial obligation. The decision to purchase a parcel of farmland is crucial since only about three percent of all the farmland in the U. S. is transferred from one owner to another each year (Scott).

Land is different than man-made capital assets in that there is a limit to the total supply and each parcel has a locational monopoly. Therefore, when a parcel of land is available for sale in the market, knowing, how much a farmer can afford to pay that parcel of land is very important. If his bid price is significantly below the asking price then he might lose the opportunity to purchase. On the other hand, if it is significantly above the asking price, his purchase price might put him in a difficult financial position leading to financial loss. Therefore, knowing how much the farmland is worth is important for the farm operator.

Factors Effecting Land Values:

The price that farmers are willing to pay is affected

by several factors. One basic factor is the current returns, and expected increase in net returns due to expected increase in physical production over time due to improved varieties and other technological improvements. Scott indicated that the average yield increase in corn production, has approximately doubled in the last 40 years.

People are willing to pay extra for land because of expectations of further inflation. This, of course, depends on how important inflationary forces in the economy seem to be to the purchaser. Prices reflect this to the extent that part of the expected future increase is captured now by the seller, and the holding period required by the buyer to realize this increase in value is lengthened.

Another factor is job security. A farmer buying land assures himself of longer tenure than he could if he was renting. Many people have pride of ownership or ego satisfaction in being able to say they own farmland. This ego satisfaction in owning farmland is certainly not confined only to rural people (Scott).

In most cases, an expansion buyer with excess machinery capacity can often afford to pay a higher than average price for additional farmland. When technology creates a situation in which a farm operation becomes land deficient in relation to other inputs such as labor and machinery, the farmer needs to increase the land input. Thus, it may be economically

feasible for him, if necessary, to pay a higher price for land to increase the total land input of the farm and spread his fixed costs over a larger land area, instead of increasing only certain variable inputs, such as fertilizer and pesticides (Kooti).

Availability and cost of credit also influence the amount the farmer is willing to pay for a parcel of land. As credit becomes easier to obtain, the number of potential buyers for a tract of land increases, and so does the demand for land. As a result, its price increases. The cost of credit adds to the cost of land purchase; therefore, if he pays less for the cost of credit, he may be willing to pay more for the farmland (Kooti).

Government programs also influence the farmland prices. Commodity price support programs insure farm owners a certain minimum price for their crop (at least in the short run), even if production increases. Therefore, it becomes profitable to increase production because by so doing, the net income to land will increase. Acreage allotments, which restrict output in order to increase total revenue, may increase land prices for those fortunate enough to have an allotment. Expectations about the continuance of government programs may affect land prices. With acreage allotments it is essential that the owner be relatively certain the program will be continued, if he is to pay the higher

prices for farmland with non-transferable allotments(Kooti).

DETERMINING LAND VALUES

The previous method of calculating the price of farmland, called the capitalization formulas, suggested that land prices could be determined by dividing the average net after-tax income per acre by the discount rate or return from the next best alternative. For example, if after-tax net return from land is $50 per acre and the rate of return on the best alternative is 10 percent, dividing $50 by 0.10 would price the land at $500 an acre.

The formula, unfortunately, has become hopelessly outdated, yet it continues in use for lack of accurate and easy to apply alternatives. It is outdated because it assumes constant income from land and that land will be valued the same after some planning period as it was at the beginning. Historical data has indicated that neither land values nor net return to land have remained the same. During the last ten years land values have increased at an average rate of 9.5 percent while income to farmland has increased at an average rate of 7 percent (Kooti). Therefore, if the capitalization formula was to be used to estimate land values using today's earnings, they would likely fall below current land prices.

An alternative more realistic method to determine land

prices is to sum the discounted value of the after-tax net income to land plus the discounted value of the after tax capital gains.

Kooti analyzed several methods of determining land values including the "Capital Budgeting Model" which determines the land value given the expected net income to land, marginal income tax rate, rate of pure time preference, planning horizon, absolute risk aversion, and the variation of annual income to land. He concluded that land value is very sensitive to change in expected annual net income to land. An increase in productivity of land increases the price of one acre of land. Absolute risk aversion had a negative impact on land values. An increase in the variation of income decreases the value of one acre of land. Any increase in risk decreases the value of one acre of land. Finally, there was an inverse relationship between the land value and the rate of pure time preference.

Still another capital budgeting model included financial terms and risk. This model was developed by Lee and Rask and adapted by the author. This model determines the maximum bid price of land given the expected annual net income to land, marginal tax rate, tax rate applied to capital gain income, interest rate on mortgage loan, opportunity cost of capital, inflation rate, growth rate of income, planning period, amortization period of the loan, absolute risk aversion and the variation in income.

He concluded that the longer the amortization period of the loan the greater the value of one acre of land. The higher the interest rate on mortgage loan, the smaller the value of one acre of land. The larger the down payment the smaller the value of one acre of land.

The common feature of the capital budgeting models is they assumed income for a lumpy asset was known and the effect of factor costs already accounted for. As a result the effects of factor costs such as machinery, fuel, and fertilizer on land values was not explicitly explained.

Statement of the Problem

The question of how to determine the price of land has received considerable attention (Harris and Nehring, Lee and Rask, and Kooti). But the question of how to identify net return attributable to the capital asset has not received much attention. Also the question of how factor costs affect returns to land and hence land values has not received much attention.

Net income to land is based on the farm receipts and expenses. Cash farm receipts are based on the farm's productive capacity--the crop mix, the typical prospective acreage yield per acre of various crops planted, and the prices of agricultural products. Expenses should represent what

the typical farmer will spend in order to maintain and run the business. Included are: seeds, fertilizer, chemical, irrigation costs, harvesting expenses, fuel, machinery expenses, hired labor, insurance, real estate taxes, and depreciation. Increase in the price of energy related agricultural inputs in particular and other inputs in general without a comparable increase in output prices, decreases the farm income and hence land value. When a farm operator has excess machinery at hand, return to land can increase if more land is available to him. Thus, land value to such a farm operator might be higher than a farm operator who has no machinery at hand. Therefore, considering agricultural factors are important in accurate estimation of return to land and hence land value.

The analysis of output share attributed to each factor of production such as land, machinery, labor, etc. usually is found in the marginal productivity theory of factor demand. According to this theory a factor share of output is determined by multiplying the number of units employed by its marginal value productivity. This is called the Euler's theorem. However, Euler's theorem holds only if all factors are assumed variable and homogeneous function is assumed (Robison, 1980). Durables such as land and machinery are usually acquired in a discrete sized unit, the services they generate in each time period are divisible.

Therefore, a new approach of partitioning output among inputs and therefore the return to each input needs to be developed.

The Purpose of this Study

It is the intent of this study to find more accurate ways of estimating the return to a durable in general and farmland in particular. Once the return to the durable is identified, we are then in a position to apply our capital budgeting models. Robison's theorem of partitioning output among inputs will be reviewed and applied. The application in this study will be to measure returns to the durable under two assumptions 1) asset is indivisible in purchase but divisible in use, and 2) asset is divisible in purchase and use.

Objectives

More specifically, the objectives of this study are:

1) To a new theory of durable asset valuation in general and land valuation in particular.
2) To determine the portion of output attributed to durable asset under the assumptions that:
   a) Asset is indivisible in purchase but divisible in use.
   b) Asset is divisible in purchase and use.
3) Compare and contrast land values under:

a) Complete ownership of land.

b) Land rented.

Land values are determined using linear programming technique and Michigan data for the assumed activities.

4) To compare and contrast return to land and hence land values under the assumptions of

a) Exess machinery ownership.

b) Custom hiring machinery services.

5) To study the effects of costs of factors of production such as fuel, fertilizer, and labor on the net income and hence on land value.

Procedure

The remainder of this dissertation is organized in the following manner. Chapter II is an explanation of the theory of durable evaluation and the developments of the new theory of asset evaluation. Chapter III is a graphical and mathematical illustration of the theory in which output attributed to land is measured. Chapter IV implements the theory using a linear programming technique. Chapter V presents the results of the linear programming model under constant input prices. Chapter VI presents the results of the sensitivity analysis. Finally, conclusions of this study are given in Chapter VII.

CHAPTER II

THEORETICAL FRAMEWORK FOR DURABLE ASSET VALUATION

Durables are defined as factors of production that supply services to the production process for more than one period. While the durable is acquired in a discrete sized unit, the services it generates in each time period are variable. For example, tractors are available in finite sizes, but once acquired the firm can use them up to some maximum capacity determined by the size and reliability of the tractor (Robison and Black).

If durables such as farmland and machinery are used in the production process, the value of their services needs to be determined. Note, that those durables are usually available in discrete sized units, but the services generated from those durables are variable. Neverthless, the net returns attributed to a durable is determined by measuring the portion of output attributed to the services generated by the durable.

The analysis of the output share attributed to each factor of production, usually is found in the marginal productivity theory of factor demand. According to this theory, a firm's demand for an input purchased in a purely competitive market depends on its marginal value product--the marginal physical products of the input multiplied by the output price. The equality between marginal value product and the factor price determines the units of input employed. The units of input used multiplied by the input price divided by the value of total output equalled the input's share.

To illustrate, suppose x units of homogeneous labor are employed at wage rate W. Also let total output Q multiplied by output price P equal the value of total output which when divided into W_x equals the share of output received by labor. If the share of each input used in the production are summed, there is no assurance they would sum to one hundred percent, leaving unanswered the question of which factor gets the surplus or pays the deficit (Robison).

A special case, however, does exist under which, if each input factor is paid its marginal value product, the total product will be exactly exhausted. This, of course, holds for homogeneous function of degree one as Euler's theorem demonstrate. Economically speaking, this implied that pure economic profits are zero which describes the long-run equilibrium under pure competition. Chiang (1974, p. 407) makes the point that because linearly homogeneous functions guarantee zero profit, it was at one time thought only linearly homogeneous production functions would be reasonable, which of course, is not the case.

Clark, Wicksteed and others proved that the long-run competitive equilibrium, rewarding each input according to its marginal product precisely exhausts the total product if there are constant returns to scale which is, after all, what was required for long-run equilibrium (Ferguson, 1972, p. 410-413).

Suppose the firm produces with a variable input x for labor and fixed capital C in the form of land. In the long run, all factors are variable so that each receives their marginal value product. But in the short run, the fixed factor C cannot be varied, or at least we assume it cannot, so that a marginal product cannot be derived for its services. So the return to the fixed factor in the short run is determined using a residual approach.

According to rules of marginal analysis, the variable input is employed until its marginal value product equals its cost; then, if total profits exceed total variable costs, the residual can be attributed to the fixed factor--a difference Marshal called "quasi rent". It is useful to note that quasi rents are never negative because if profits are less than total variable costs, the firm would not produce even in the short run. This residual approach is popular in the farm management literature where it ascribes to agricultural land and other capital inputs the difference between total profits and variable costs. But why give all the residual to the fixed factor--why not some to the variable factor?

Robison argues that the short run residual approach for allocating output among the inputs is not appropriate, primarily because there is rarely an input that is fixed in the production process. That is, even those capital assets are purchased in discrete indivisible units, their services are usually available in divisible units. To measure the returns to durables which are indivisible but have divisible services, a new theorem proved by Robison is now introduced. The new theorem achieves for any function what Euler's theorem did for linear homogeneous functions.

A New Theorem:

The new theorem, the output exhaustion theorem, states that the value of any general process can be divided among the inputs in such a way that the output is just exhausted assuring the inputs each receiving the value of their long run marginal value product.

According to this theorem if $f(x,y)$ describes a continuous production process which depends on two inputs x, and y, where x and y are related by the function $y = l(x)$ for all x and y along the expansion path; then, the sum of the integrals f_x plus f_y equal $f(x,y)$ for all x and y. The proof of the theorem follows:

Since $y = l(x)$, $f(x,y)$ and the derivative of f with respect to x equals: $df(x,l(x))$ which equals

$f_x dx + f_y l'(x)dx$. But $dy = l'(x)dx$ and $x = l^{-1}(y)$, so that $f_y l'(x)dx = f_y dy$ and $df(x,l(x)) = f_x dx + f_y dy$. By the fundamental law of calculus the anti-derivative of $f_x dx + f_y l'(x)dx$ equals f. Since $f_x dx + f_y l'(x)dx$ equals $f_x dx + f_y dy$ where $y = l(x)$, then the anti-derivative of f_x plus the anti-derivative of f_y must also equal f.

The theorem states that if in a two argument input/output function, the relationship between the inputs is specified, then the sum of the integrated partials with respect to the inputs equal the output. The results of this theorem can be easily illustrated. Let output q equals f(x,y) where f is the Cobb-Douglas function $x^a y^b$. In addition let the relationship between x and y be $y = cx$. The partial derivatives of f with respect to x and y can be written as:

(II.1) $$f_x = ax^{a-1}y^b dx$$

and

(II.2) $$f_y = bx^a y^{b-1} dy.$$

Substituting cx for y in (II.1) and y/c for x in (II.2) and integrating allows us to write the sum as:

(II.3) $$\begin{aligned} f(x,y) &= \int_x ac^b x^{a+b-1} dx + \int_y bc^{-a} y^{a+b-1} \\ &= a\, c^b x^{a+b}/(a+b) + by^{a+b}/c^a(a+b) \\ &= x^a y^b. \end{aligned}$$

Euler's theorem meanwhile, is written as:

$$(\text{II}.4) \quad f(x,y) = xf_x + yf_y$$
$$= (a+b)x^a y^b.$$

So that, the inputs only exhaust the output if a+b were equal to one. Robison's more general theorem requires no such restriction.

The theorem has been proven for three or more inputs. For the three inputs; suppose a production process is a continuous process defined over arguments x, y and z. That is let q equals f(x,y,z). Then, if the following binary relationships exist: $y = l(x)$, $y = g(z)$ and $z = h(x)$, then:

$$(\text{II}.5) \quad f(x,y,z) = \int_x f_x(x,\ l\ (x),\ h(x))dx \ +$$
$$\int_y f_y(y,\ \overset{-1}{l}(y),\ \bar{g}^1(y))dy \ +$$
$$\int_z f_z(z,\ g(z),\ \bar{h}^1(z))dz$$

Substituting l(x) for y and h(x) for z into f(x,y,z); then, the derivative of f with respect to x after the substitution equals:

$$(\text{II}.6) \quad f_x = f_x dx + f_y l'(x)dx + f_z h'(x)dx$$

By the fundamental law of calculus, the sum of the anti-derivatives of the partials equals the original function f. That is:

$$(\text{II}.7) \quad \int_x f_x dx + \int_x f_y l'(x)dx + \int_x f_z h'(x)dx = f$$

By the change in variable technique, the following equalities hold:

(II.8) $$\int_x f_x(x,l(x),h(x))l'(x)dx$$

$$= \int_y f\ (\bar{l}^1(y),y,\bar{g}^1(y))dy$$

and

(II.9) $$\int f_x(x,l(x),h(x))h'(x)dx$$

$$= \int f_z(h^{-1}(z),g(z),z)dz.$$

Then, after making the required substitutions, it can be written:

(II.10) $$\int_x f_x dx + \int_y f_y dy + \int_z f_z dz$$

Finally, if the results hold for both the case of two and three inputs, by mathematical induction it holds for functions of any number of inputs.

The Model

Now after having established a methodology for assigning the output to the inputs, the next step is to establish a proper relationship among the inputs. The relationship among inputs depends on the marginal value product of the inputs and their marginal costs. At a point in time, this relationship depends only on those costs which vary with use as opposed to costs which vary with time. Economists prescribe that the relationship among the inputs should be such that, the ratios of their marginal value products divided by their prices be equal for all inputs at any point in time. Thus, to determine the relationship among inputs along the least cost combination curve, the physical production function must be specified. The production function specifies such that, at any time period the output is a function of inputs such as labor, fertilizer, machinery capital, land services and other factors at each time period, assuming that all factors are divisible in use. That is:

$$\text{(II.11)} \qquad q_t = f(x_{1t}, \ldots, x_{nt})$$

where q_t is the output at time period t and $x_{1t}, \ldots, x_{nt}$ are the inputs at time period t. The firm determines the relationship among the inputs by maximizing the objective function as shown below at each time period.

$$\text{(II.12)} \qquad \pi_t = pq_t - \sum_{j=1}^{n} p_j x_{jt} \qquad j = 1, \ldots, n.$$

where π_t is profit at time period t, p is the price of output q and p_j is the price of the j-th input. By matching added return to cost for each input, the optimal level of that input will be determined. After solving, each input expressed as a function of the input costs at each period of time. To demonstrate, if output q valued at price p produced from two inputs x and y which costs the producer p_x and p_y; then, the optimal relationship between x and y would be such that:

$$(II.13) \qquad pf_x/p_x = pf_y/p_y.$$

This relatioship obviously results from maximizing $\pi = pf - p_x x - p_y y$ with respect to both x and y. Then, from (II.13) an expression such as y equals $h(x,p_x,p_y)$ could be found. This expression is the relationship between x and y along the expansion path for x and y.

The maximum units of input x may be given by a firm budget:

$$M = p_x x + p_y y$$

from which x is determined to equal:

$$(M - p_y y)/p_x$$

where M is the total amount of budget available for the firm to spend on the two inputs x and y. Substituting the function $h(p_x,p_y,x)$ for y in $(M - p_y y)/p_x$ and solving for x gives x as a function of the parameters p_x, p_y and M, that is:

$$x = l(p_x,p_y,M).$$

Now that, the relationship between x and y and also the maximum units of input x are determined, the portion of output attributable to theinput x given a continuous production function f(x,y) is examined. The portion of output attributed to input x is the integral of the long-run marginal value product--$pf_x(x,y)$, where y equals $h(p_x,p_y,x)$, that is:

$$\text{(II.14)} \qquad R = \int_{0}^{x = l(p_x, p_y, M)} pf_x(x, h(p_x, p_y, x))dx$$

or

$$\text{(II.15)} \qquad R = pf(l(p_x, p_y, M),\ h(p_x, p_y, l(p_x, p_y, M)))$$

where R is the portion of output attributed to the input x.

Let us illustrate that with an example. Assume that the production function f(x,y) is a Cobb-Douglas function $x^{.8}y^{.5}$ and a budget level of $1,000 with the input price x equal to $5, while the input price of y equals $8, and output price equals $10.

The firm determines the relationship between x and y by maximizing the objective function π, at each time period.

$$\text{(II. 6)} \qquad \pi = px^{.8}y^{.5} - p_x x - p_y y$$

The marginal products of the input x, and input y can be written as:

$$f_x = .8px^{-.2}y^{.5}$$

and

$$f_y = .5px^{.8}y^{-.5}$$

The optimal relationship from (II.13) would be such that as:

$$\text{(II.17)} \qquad .8x^{-.2}y^{.5}/p_x = .5x^{.8}y^{-.5}/p_y \ .$$

This relationship obviously, results from maximizing (II.16) with respect to input x, and input y. Then, from (II.17) an expression such as y equals $0.625 p_x p_y^{-1} x$ is found. This expression simply is the relationship between x and y along the expansion path for x and y. This line shows that, there is a positive relationship between x and y.

The maximum units of input x given a firm budget M of \$1,000 is determined from:

(II.18) $M = p_x x + p_y y$

Solving for x after substituting \$1,000 for M gives:

(II.19) $x = (1000 - p_y y)/p_x$.

Substituting $0.625\ p_x p_y^{-1} x$ for y, \$8 for p_y, \$5 for p_x into (II.19) gives:

$$x = 123.00$$

Then, the portion of output attributed to input x given the continuous production function $x^{.8} y^{.5}$ is determined by integrating the long-run marginal value product of the production function where y equals $.625 p_x p_y^{-1} x$. That is:

$$R = \int 8(.625 p_x p_y^{-1} x)^{.5} x^{-.2} dx$$

where p_x equals \$5, and p_y equals \$8. Therefore, the portion of output attributed to input x is:

$$R = \int_0^{123} 5x^{.3} dx = 50x^{1.3} \Big|_0^{123}$$

or $$R = \$2,004.00$$

Chapter Summary:

In this chapter the new theory of partitioning output among inputs was introduced. According to this theory, the durable asset share of the output is the integral sum of the long-run marginal value product. The relationship among inputs along the expansion path needs to be specified first prior to the determination of the long-run marginal value product and hence the portion of output attributed to the durable asset. The relationship among inputs simply is determined by equating the ratios of marginal value products to their prices.

CHAPTER III

ILLUSTRATIONS OF THE THEORY OF PARTITIONING OUTPUTS TO FACTORS OF PRODUCTION

In Chapter II, the general theory of partitioning output among inputs was introduced. This chapter contains graphical and mathematical explanations of that theory. The first part of this chapter graphically illustrates the partitions of output attributed to land under two different assumptions about asset divisibility: 1) asset divisibility in purchase and use, and 2) asset indivisibility in purchase and divisibility in use. Asset divisibility in purchase makes it possible for the farm operator to acquire this resource in the quantity desired. Indivisibility in the purchase of the asset limits the choice to acquiring the asset in the amount available, even though the services generated are divisible. For example, a farm operator may have to decide to acquire 100 acres of land or none at all. If he acquires the 100 acres then the use decision is divisible, allowing the farm to use any amount of land up to 100 acres.

In the second section of this chapter, the portion of output attributed to land under the assumption of machinery ownership will be examined using a Cobb-Douglas production function. That is, we allow the farm operator to own machinery services γ. The impact of this owned machinery services γ on the portion of output attributed to the input land is explored.

The third and last section of this chapter will examine the change in the portion of output attributed to land as the result of change in factor prices, again illustrated using a Cobb-Douglas production function.

Asset Divisibility in Acquisition and in Use

Assume an asset divisible in purchase and use is used in the production process of a firm. Under this assumption the optimal combinations of inputs used by a firm is found by equating the ratios of marginal value products to their respective prices. Consider Figure 1. The curve Q_1 and Q_2 are isoquants depicting various combinations of inputs, e.g., labor represented by the variable x, and land input represented by the variable y. The inputs x and y are combined to produce outputs Q_1 and Q_2, etc. Moreover, assume that x and y are divisible in purchase and use. Lines B_1B_1' and B_2B_2' represent different budget lines, where the budget line shows different combinations of inputs x and y which have the same cost--an isocost curve. Given the input prices, the output corresponding to isoquant Q_1 can be produced at least-cost at point A, where the isoquant is tangent to isocost curve B_1B_1'. At point A, the ratios of marginal value products of each input to their respective prices are equal, i.e. MVP_x / p_x equals MVP_y / p_y, where MVP_x is the marginal value product of input x, MVP_y is the marginal value product of input y, p_x is the price of input x, and p_y is the price of input y. When this condition is met, the farm operator is in equilibrium.

Now, suppose the farm operator faces an increased budget corresponding to budget line B_2B_2'. Since factor prices remain constant, the slope of the budget line does not change. Hence, it shifts from B_1B_1' to B_2B_2'. The new equilibrium is found by shifting the isoquant curve until it is tangent to B_2B_2'.

Connecting points A and K generates line OE, which is the optimal combination of inputs with increasing resources accompanied by constant input prices; meanwhile at each resource level, the equilibrium point is defined by the equality between the ratios of marginal products to the ratios of input prices. Since the input price ratios have remained constant, the equality forces the ratios of marginal products to remain constant. Therefore, OE is an isocline, a locus of points along which the marginal rate of technical substitution is constant. The expansion path is the particular isocline along which output will expand when factor prices remain constant. The expansion path thus shows how factor proportions change when output or

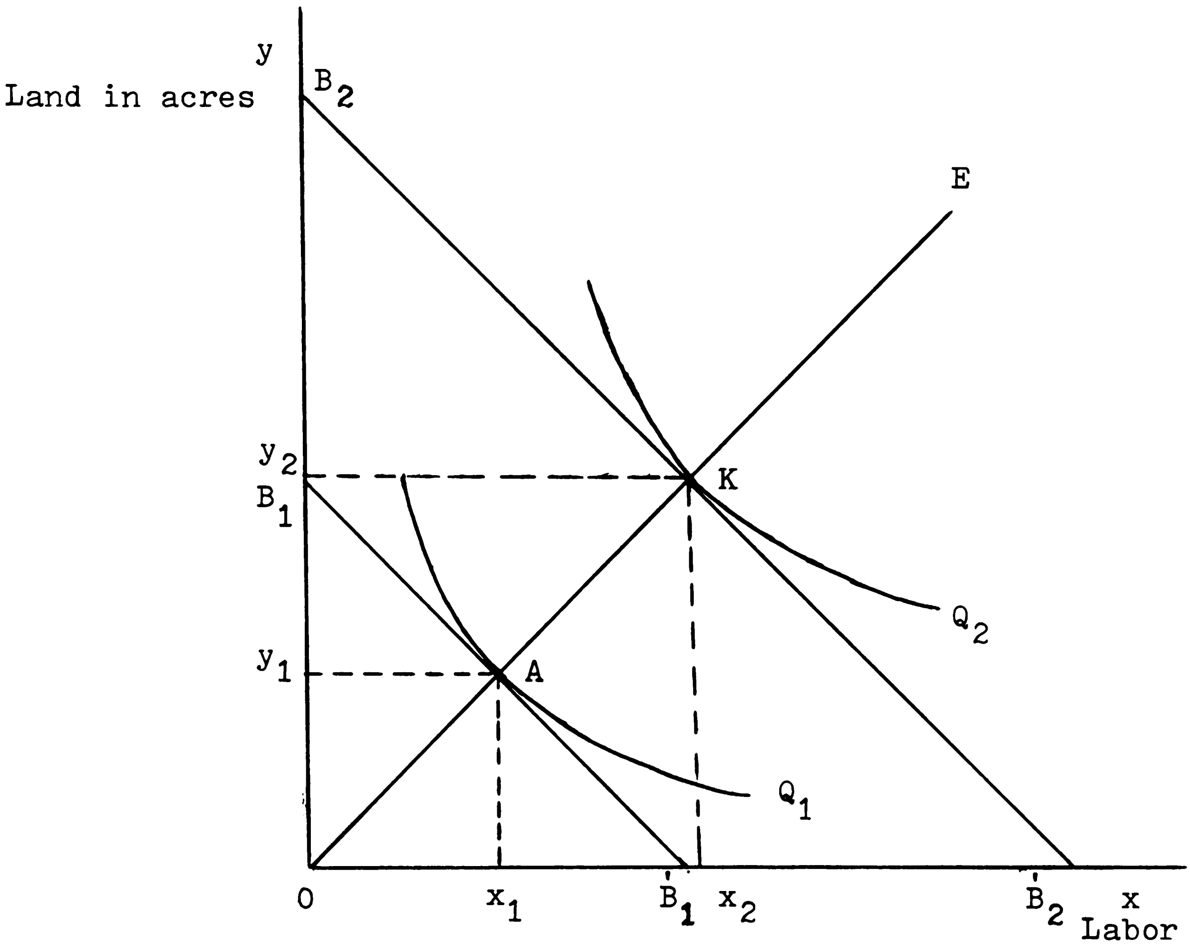

Figure 1: Optimal Combinations of Inputs for Different Levels of Outputs Assuming x and y are Divisible in purchase and Use. OE is the expansion path while lines B_1B_1' and B_2B_2' are budget lines(iso-cost).

expansion changes, input prices remaining constant throughout. The theory of partitioning output among inputs, which was explained in Chapter II, suggested that the portion of output attributed to the respective input is determined by integrating the long-run marginal value products along the expansion path OE.

Next, we will examine whether the assumption of asset indivisibility in acquisition would change the firm's expansion path. If the firm's expansion path changes, so does the portion of output attributed to y.

Asset Indivisibility in Acquisition and Divisibility in Use

Assume an asset used in the production process by the firm is indivisible in purchase but divisible in use. Land may be an example. It is generally acquired in discrete size units but can be used in any amount up to the amount acquired. This manner of acquiring may alter the expansion path from the one derived under the assumption that the asset is divisible in acquision. If this is the case, then the portion of output attributed to the asset, determined by integrating its long-run marginal value products along the expansion path, will change.

Consider now the effects on the expansion path of acquiring an asset in an indivisible quantity. For example, let the firm acquire asset y in indivisible amount F but allow it to utilize the services from F in any amount less than or equal to F, which is the maximum capacity of the asset. Now y represents the amount of services actually used, while F equals the amount of services acquired. The firm's expansion path under this assumption is now determined.

Consider Figure 2. The effects of limiting the use of y to values less than or equal to F requires a vertical constraint beginning from the horizontal axis at point F to the budget line BB'. The area $OFDD'$ is the feasible area, i.e. all input combinations in $OFDD'$ are possible choices. The curves OR and OR' are the ridge lines. Along the ridge line OR the marginal physical product of input x is zero, while along the ridge line OR' the marginal physical product of input y is zero. The area between the ridge lines OR and OR', the marginal products of x and y are positive. The OCDH is both economically and physically feasible, because the marginal products of input x and input y are positive and because the resource requirements do not exceed resources availability. Since the indivisible asset is already acquired in amount F, increasing y incurs no additional opportunity cost. As a result, input y is employed up to the point where marginal value product equals marginal cost, which is zero. Therefore, y is used in the production process is described by the ridge line OC, along which the marginal product of y is zero. However, when the use of the asset services y reaches the maximum amount of services available, F the firm will invest in other variable input x. So the expansion path becomes OCD.

The portion of output attributed to input y, then is found by integrating its marginal value products along the expansion path, OCD. In contrast, the value of the portion of output attributed to y when y is divisible in acquisition is the integral of the marginal value products of y along OE. Since the expansion paths are different, the output attributed to y along OE and OCD may also be different.

A special case is the Cobb-Douglas production function, where the marginal value products are never negative. In this case the expansion path becomes FD instead of OCD. As a result the portion of output attributed to input y is determined as a residual after subtracting all the variable costs from the farm income.

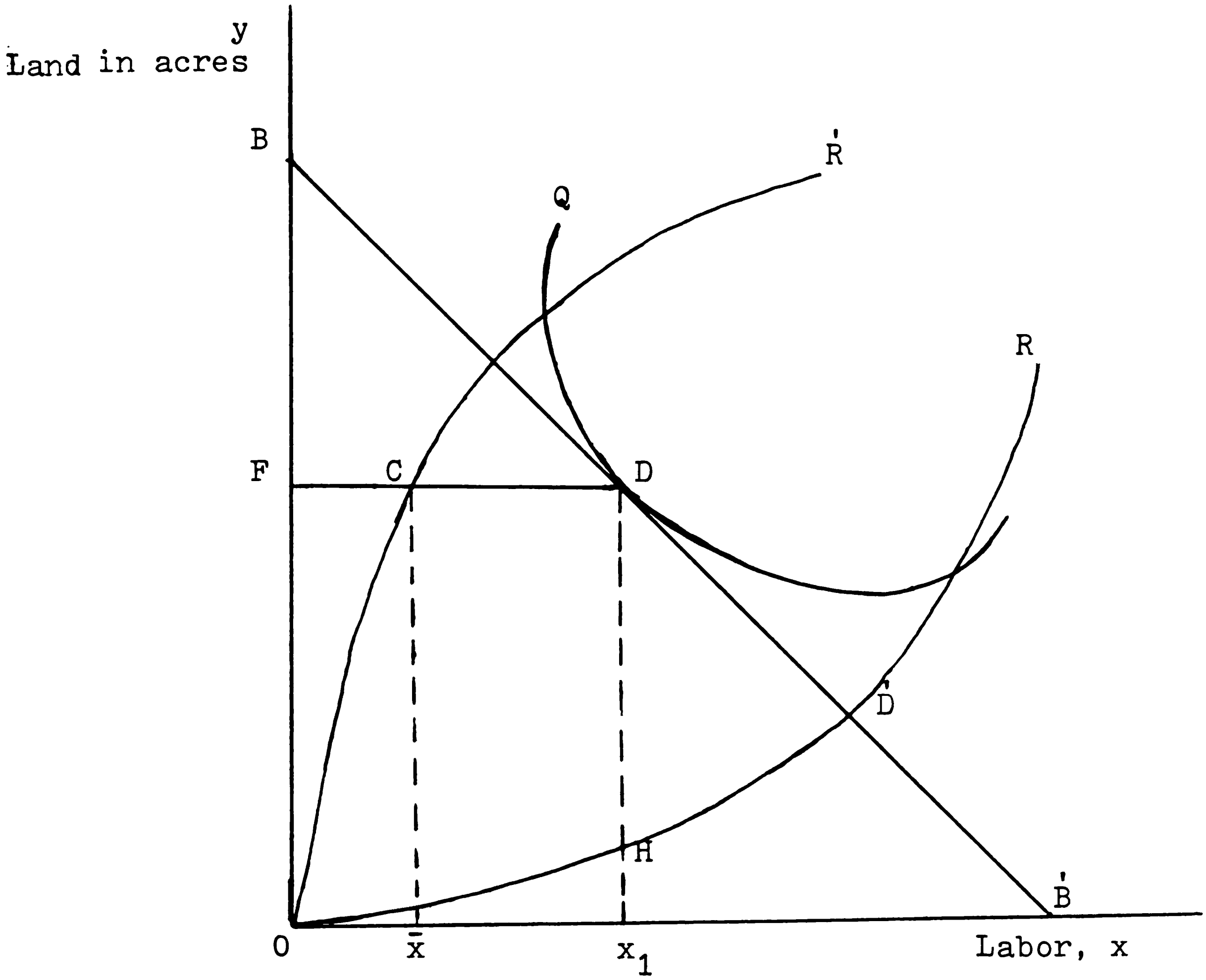

Figure 2: Isoquant and Ridge Lines and the Expansion Path of a Firm Under the Assumption of Asset Indivisibility in Acquisition and Divisibility in Use.

Output Attributed to Land with Machinery Ownership

In the first part of this chapter we showed that the firm's expansion path changes under the assumptions of: 1) asset divisibility in acquisition and use, and 2) asset indivisibility in acquisition and divisibility in use. As a result; we inferred that the portion of output attributed to land may be different under the two assumptions. Now another factor that may change the portion of output attributed to the asset land is explained.

This factor is the availability and manner of acquisition of inputs complimentary to y. Suppose for example, the farm operator has machinery which can generates services up to an amount γ, in addition, assume the farm operator can custom hire machinery services at the rate p_x per unit of x. Since the marginal cost of the owned machinery is small or zero, as long as the services have positive marginal product, γ amount of services will be used with the total services equal the amount purchase plus the amount owned. If x is the total amount of machinery services used in the production process, then z is the amount of machinery services purchased equal to x minus γ. Also, assume that the firm is engaged in production process described by the function f(x,y) that uses the machinery services x and the divisible input land, y. Moreover, let f(x,y) be a

Cobb-Douglas production function of the form $x^{\alpha}y^{\beta}$.

The marginal products of each input are determined by taking the partial derivative of f with respect to x and y. These marginal products f_x and f_y can be written as:

$$f_x = \alpha x^{\alpha-1} y^{\beta} \tag{III.1}$$

and

$$f_y = \beta x^{\alpha} y^{\beta-1} \tag{III.2}$$

Those marginal value products are needed to determine the portion of output attributed to input land, since the portion of output attributed to land is the integral of the long-run marginal value products along the expansion path. However, before the portion of output attributed to land can be determined, the relationship among the inputs x and y need to be specified. This relationship is derived by maximizing the firm's profit. After multiplying f by the price of the output p and subtracting the cost of the inputs, the profit function π can be written as:

$$\pi = px^{\alpha}y^{\beta} - p_x(x-\gamma) - p_y y \tag{III.3}$$

where p_x and p_y are the cost of input x and input y.

Next, the profit function (III.3) is maximized, subject to budget function M equals $p_x(x-\gamma)$ plus $p_y y$ and with respect to x and y gives the following relationship between x and y. That is:

$$\alpha y/p_x = \beta x/p_y$$

or

(III.4) $$x = \alpha p_y y \, / \beta p_x$$

The amount of input y that can be used given a budget level M is:

(III.5) $$y = (M - p_x(x-\gamma))/p_y$$

where γ is the amount of machinery services owned by the farm operator. Substituting $\alpha p_y y/\beta p_x$ for x in (III.5) and solving for y gives:

(III.6) $$\bar{y} = \beta(M + p_x \gamma)/p_y(\alpha+\beta)$$

Equation (III.6) indicates that y is only a function of the parameters. Then the portion of output attributed to input y is determined by integrating the long-run marginal value product along the expansion path. That is:

(III.7) $$R = \int_0^{\bar{y}} pf_y dy = \int_0^{\bar{y}} p\beta x^{\alpha} y^{\beta-1} dy$$

where R is the portion of output attributed to input y. Substituting $\alpha p_y y/\beta p_x$ for x in (III.7) and integrating along y gives:

(III.8) $$R = p\beta(\alpha p_y)^{\alpha}\bar{y}^{\alpha+\beta}/(\beta p_x)^{\alpha}(\alpha+\beta)$$

Equation (III.8) indicates that the portion of output attributed to y is a function of y, p_x, p_y and the parameters α and β. Now the effect of machinery services ownership on the portion of output attributed to input y, can be illustrated.

The effect of machinery ownership on the value of the portion of output attributed to input y is determined by taking the partial derivative of R with respect to γ. That is:

$$\partial R/\partial\gamma = (\partial R/\partial\bar{y}).(\partial\bar{y}/\partial\gamma)d\gamma > 0$$

It is positive, since $\partial R/\partial\bar{y}$ from (III.8) is positive, and $\partial\bar{y}/\partial\gamma$ from (III.6) is also positive. Therefore, the portion of output attributed to y increased as γ increased.

The Effect of Increase in Factor Costs on the Portion of Output Attributed to Input Land

Thus far, in the analysis, the price of factors of production have remained constant. In this section of the chapter, we would like to illustrate the changes in the portion of output attributed to y as the price of the factor x changes.

As input prices increase, the farm operator must adjust his cropping mix and his input combinations so that profit is maximized. However, with increased input costs profit may not reach the previous level, especially if crop prices remain unchanged. If total profit is reduced then it follows that output and revenue attributed to the input must also change. Thus increasing costs of production may reduce income attributed to agricultural inputs such as land.

Consider Figure 3. The curves Q_1, and Q_2 are isoquants, and points A, and K, are profit maximizing combination of x and y determined by input price P_x and P_y. B_1B_1' and B_2B_2', meanwhile are isocost lines associated with outputs Q_1 and Q_2. Connecting the tangency points A, and K generates the expansion path OD. Again the theory dictates that the portion of output attributed to y is determined by integrating its long run marginal value products along the expansion path OE.

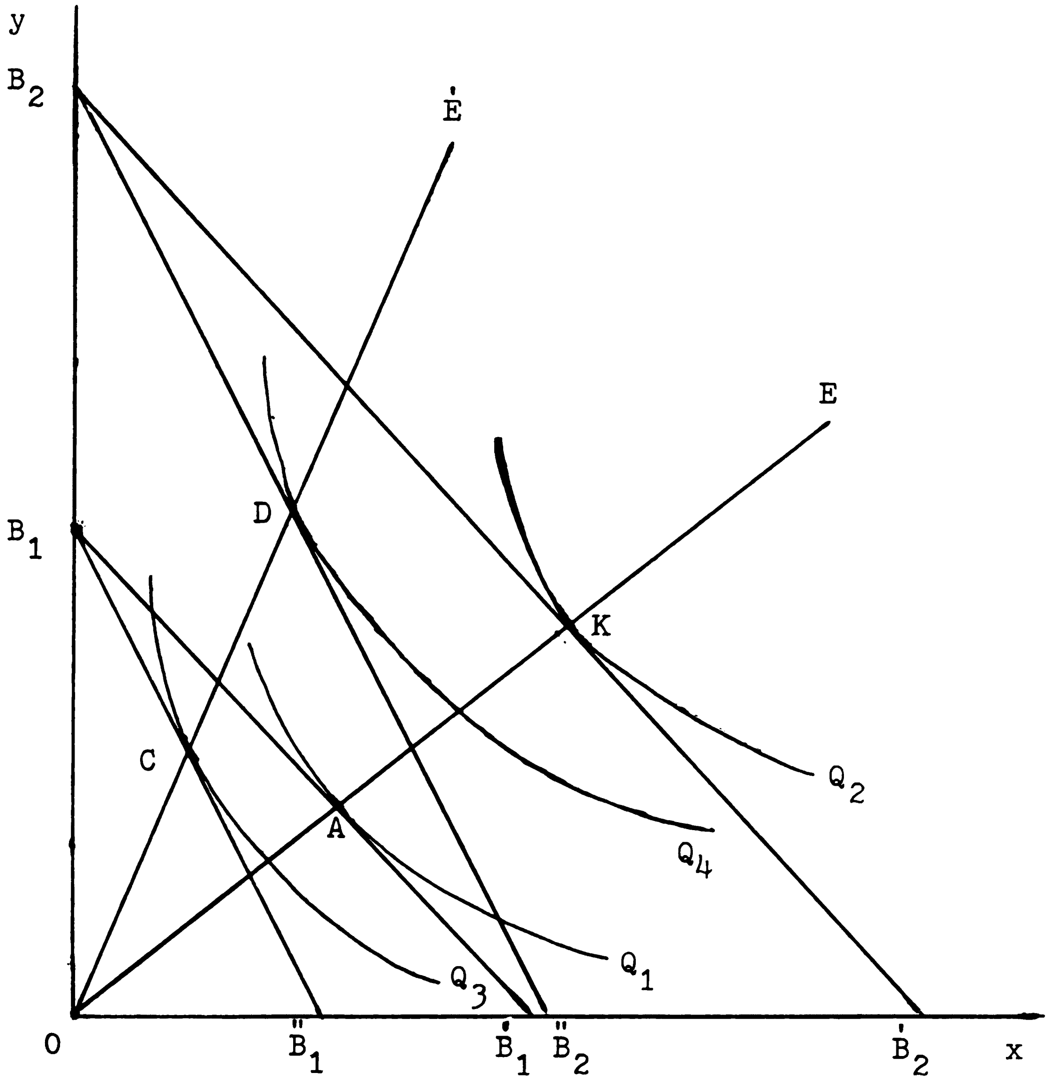

Figure 3: Shifting the Expansion Path as Price of Input x Changes.

Now assume that the price of input x increases while the price of land's services remain unchanged. As a result of increase in the price of input x, the budget line B_1B_1' to B_1B_1'' producing a new profit maximizing tangency point at C with a reduced output of Q_3. Assume that the factor price ratio remains constant at the new level, and the farm operator wishes to expand output to level corresponding to the isoquant Q_4, the new equilibrium, D is found by shifting the budget line until it is tangent to Q_4. Since the new factor price ratio remains constant, the slope of the budget line does not change. The budget line shifts from B_1B_1'' to B_2B_2''.

Connecting points C, and D generates a new expansion path OE'. It is possible that the integral of the long run marginal value products along the expansion path OE' is different from the portion of output attributed to input y determined by integrating the long run marginal value products along the expansion path OE. That is, as price of input x changes, the portion of output attributed to y may change.

Let us illustrate with an example. Assume the Cobb Douglas described earlier, where the optimal amount of x is determined by (III.4), and the optimal amount of y by (III.6) where γ is zero. The portion of output attributed to y is determined by (III.8).

The effect of change in price of input x, p_x on the portion of output attributed to input y is determined by taking the partial derivative of R in (III.8) with respect to p_x, while holding other inputs constant. That is:

$$\text{(III.9)} \quad \partial R/\partial P_x = -P\beta \, (\alpha P_y)^{\alpha} \, \bar{y}^{\alpha+\beta} / (\alpha + \beta)\alpha^{\alpha-1} P_x^{\alpha+1} \, dP_x < 0$$

where $\bar{y} = \beta M/P_y(\alpha + \beta)$.

Equation (III.9) is negative. That indicates, as price of input x increases, the portion of output attributed to y would decrease.

Chapter Summary

The first part of this chapter illustrated how different ways of acquiring the asset affected the value of the input--measured as the value of output attributed to the input. The two ways of acquiring the asset are: 1) asset is divisible in purchase and use, and 2) asset is indivisible in purchase but divisible in use. It was illustrated that firm's expansion path is different under the two assumptions. Since the portion of output attributed to land is determined by integrating the long-run marginal value products along the their respective expansion paths, their values may be different.

The second section of this chapter examined the portion of output attributed to land under the assumption of machinery ownership. This was accomplished by using a Cobb-Douglas production function. It was concluded that as the farm operator owns more machinery services, the value of the portion of output attributed to land increased.

The third and last part of this chapter examined the changes in the value of the portion of output attributed to y as price of input x changed. It was concluded that increase in the price of factor x would have an adverse effect on the value of the share of input y.

CHAPTER IV

IMPLEMENTATION OF THE THEORY

In Chapter II a theory of partitioning outputs among inputs was introduced. In Chapter III the theory was described graphically and analytically using a Cobb-Douglas production function. Now a method is derived to implement the theory. For this purpose linear programming (LP) is used. Linear programming is ideally suited to implement the theory since this technique efficiently selectes the strategy that maximize profit given various prices and resource constraints for an individual farm. Moreover, the solution of an LP Model gives the shadow prices for different resources such as farmland which are in fact the marginal value products of land (Heady and Candler, P. 85) given optimal levels of other inputs. Positive shadow prices for land means that a one unit increase in the amount of land available would increase the potential income by the amount of the shadow price. A set of shadow prices are derived for farmland for different levels of land constraints and for each time period. Summing the shadow prices at each level of land (allowing other inputs to optimally adjust) is equivalent to integrating the long-run marginal value product along the expansion path as demonstrated in Chapter II and Chapter III. The portion of output attributed to land is the sum of the shadow prices for each level of land used. This approach to measure land values will now be discussed in more detail.

LP As A Research Tool

Linear programming was chosen to implement the partitioning theory because of its ability to maximize income and , because of its potential to do sensitivity analysis, allowing us to measure long run marginal value products along the expansion path. There are two requirements for using linear programming technique. The requirements are:

1) A linear objective function: LP maximize the objective function $\sum_{j=1}^{n} C_j x_j$, where x_j represents the activity level and C_j represents the net revenues of the activities.
2) Linear resource constraints such that:

   $\sum_{j=1}^{n} a_{ij} x_j \leq b_i$, and $x_j \geq 0$ for all j

   where a_{ij} represents the technical input-output coefficients for each activity and b_i represents the availability of the resources. For example, if one resource is a monetary budget used to purchase other inputs, varying the amount available b_k and resolving the LP optimal combinations of inputs on different budget lines would be determined.

For binding constraints, linear programming obtains shadow prices--the amount of increase in the profit per unit increase in the constraint given optimal levels of other inputs. The shadow price is equivalent to the marginal value product of the constrained input. By varying the level of land services a set of shadow prices associated with different levels of land input constraint is derived. Because we are dealing with linear programming, shadow prices remain constant between the points corresponding to successive levels of the continuous solution. To demonstrate, let l_i represents increments in the land input constraint. Moreover, assume MVP_{l_i} as the shadow price related to the i-th increment in land input constraint. Then, according to the theory of partitioning output among inputs; the share of land input from output can be written as:

$$R = \sum_{i=1}^{m} l_i MVP_{l_i}$$

where R is the portion of revenue attributed to land, and m is the total number of increments. The above procedure is used to determine the value of the portion of output attributed to land and hence the value for land under the earlier divisibility and divisibility assumptions, namely:

1) Renting Land--under this assumption land is acquired at any quantity desired, that is,

land is divisible in acquisition and use. Under this assumption land is rented at the market rate. Labor is provided by the farm operator as available and additional labor is hired on an hourly basis as needed. Operating capital services are acquired by borrowing. The available capital is set as a limiting factor. The operator owns his machinery.

2) Complete Ownership of land--under this assumption land is purchased in discrete size units but it is divisible in use. It is assumed that the farm operator owns the same number of acres as under the assumption of rented land. All other assumptions are the same as the case of rented land.

3) Custom hiring machinery services--under this assumption, all machinery services are acquired through custom hiring. It is assumed that the farm operator owns the land. That is, land that's indivisible in acquisition but divisible in use. Labor is provided by the farm operator as available and additional labor is hired on an hourly basis as needed. Operating capital services are acquired by borrowing.

4) Machinery ownership--under this assumption the

machinery services are acquired through operator ownership. All other assumptions are the same as custom hiring assumptions.

After completing the above analysis, the change in the portion of output attributed to land and land value as a result of changes in factor prices will be examined. This analysis will be accomplished by changing prices of factors of production one at a time. It is assumed that land services are acquired through operator ownership for this analysis. Labor is provided by the operator as available and additional labor is hired on an hourly basis as needed. Operating capital services are acquired by borrowing. Finally, machinery services are acquired through operator ownership.

Linear Programming Tableau Description

The linear programming tableau for this study has 21 activities and 21 restrictions and accounting equations in each of three periods. The activities and resource constraints and accounting equations are listed in Table 4.1 and Table 4.2, respectively. The activities are production and sale of corn and soybean, hiring labor for different months, fuel buying, fertilizer buying, corn and soybean seeds buying, capital borrowing, custom hiring machinery services, and land renting as indicated in Tableau 4.3 and Tableau 4.4. The resource constraints include maximum amount of operator labor available, maximum operating capital, land and machinery services. Now the activities and the resources are discussed in more detail.

The income earning enterprises are the production and sale of corn and soybean. The input-output for the assumed level of production are given in Tableau 4.3 and Tableau 4.4.

The operator is considered as the major source of labor. The total operator labor available is 3,000 hours per year. The monthly operator labor available is given in Table A:1 of the Appendix. It is assumed that family labor will be depleted before other labor is hired. There is no restriction on the amount of labor hired. Labor requirement per acre for corn and soybean are presented in Table A:2 of the Appendix and Table A:3 of the Appendix respectively.

Crop variable costs for corn and soybean production are given in Table A:4 of the Appendix. These values do not include the cost of fertilizer, fuel, and seeds. The costs of fertilizer, fuel and seeds are calculated by the LP Model. This is accomplished by including the costs of fertilizer, fuel, and seeds in the objective function of the linear programming model as shown in Tableau 4.3 and Tableau 4.4. This will enable us to make sensitivity analysis for such factors as labor, fertilizer, and fuel.

Table 4.1 Linear Programming Model Activities Considered In Each Period

Activity Number	Activity Name	Activity Description
1	CRN	Corn Production
2	SOY	Soybean Production
3	SLCRN	Corn Selling
4	SLSOY	Soybean Selling
5	HLRNM	Labor Hiring for November - March
6	HLRAP	Labor Hiring for April
7	HLRMY	Labor Hiring for May
8	HLRJE	Labor Hiring for June
9	HLRJY	Labor Hiring for July
10	HLRAT	Labor Hiring for August
11	HLRSR	Labor Hiring for September
12	HLROR	Labor Hiring for October
13	FUEL	Diesel Fuel Buying
14	N	Nitrogen Buying
15	PH	Phosphorous Buying
16	POTS	Potash Buying
17	CRNSD	Corn Seed
18	SOYSD	Soybean Seed
19	CAPBW	Capital Borrowing
20	RENTLD	Land Renting
21	MACH	Machinery Activity in Dollars

Table 4.2 Resource Constraints And Accounting Equations

Row Number	Row Name	Row Description
1	LANDT	Total Land Utilized
2	RTLT	Maximum Acreage Available for Rent
3	LRONM	Maximum Operator Labor Available in November - March
4	LROAP	Maximum Operator Labor Available in April
5	LROMY	Maximum Operator Labor Available in May
6	LROJE	Maximum Operator Labor Available in June
7	LROJY	Maximum Operator Labor Available in July
8	LROAT	Maximum Operator Labor Available in August
9	LROSR	Maximum Operator Labor Available in September
10	LROOR	Maximum Operator Labor Available in October
11	SLTCRN	Corn Transfer From Production to Sale Activity
12	SLTSOY	Soybean TransferFrom Production to Sale Activity
13	TFUEL	Total Fuel Utilized
14	TNITGN	Total Nitrogen Utilized
15	TPH	Total Phosphorous Utilized
16	TPOTS	Total Potassium Utilized
17	TCRNSD	Total Corn Seed Utilized
18	TSOYSD	Total Soybean Seed Utilized
19	CAP	Total Capital Borrowed
20	MACHS	Maximum Machinery Services Owned
21	CAPM	Maximum Available Capital

Diesel fuel for corn activity is 7.07 gallon per acre and for soybean activity is 5.68 gallon per acre. Table A:5 of the Appendix presents fuel requirements for corn and soybean production.

The amount of nitrogen applied to corn and soybeans depends on the economic returns of application. Nitrogen should be applied up to the point where marginal value cost of the fertilizer equals marginal revenue of the increased production. The optimal application rate of nitrogen for corn is 120 pounds per acre and for soybeans is 10 pounds per acre.

Application rate per acre for corn is 75 pounds of phosphorus and 100 pounds of potash. A standard rate of 50 pounds of phosphorus and 25 pounds of potash is applied to each acre for soybeans.

Corn is assumed to be planted at a rate of 13 pounds per acre for each corn activity. Soybeans are planted at 50 pounds per acre.

Operating capital is assumed to be equal to the value of machinery investment only. Table A:6 of the Appendix presents the maximum amount of operating capital that can be borrowed at each period of time. The model is constructed so as first to determine resource service requirements and optimal organization based on the annual operating capital. Total operating capital was chosen as the limiting factor because when borrowing the total amount borrowed at any one period of time determines what can be purchased. In other words, borrowed capital is in fact the farm firm budget level at each time period. For example, the limiting budget level for 1977 period is $57,000 under renting land assumption. This budget will be used to pay for: hired labor, land rental charges, fertilizer costs, seed costs, and other costs as presented in Tableau 4.4. The limiting budget level for 1977 period under the assumption of land ownership is $33,300 as indicated in Tableau 4.3. This budget will be used to pay for: hired labor, fertilizer cost, seed costs, fuel cost, and crop variable costs as shown in Tableau 4.3.

Prices for the initial models (1 through 4) are given in Table A:7 of the Appendix. These prices are chosen which should approximate prices for the various inputs and outputs in 1977, 1978, and 1979 crop year. These prices include the price of corn and soybeans, fertilizers, seeds, fuel, labor, and capital. For example, the prices of the above variables for 1977 period are included in the profit row of the tableaus as shown in Tables 4.3 and 4.4.

Table 4.3 An Example of a Linear Programming Tableau Used in This Study: Complete Ownership of Land and Custom Hiring Machinery Services.

	Item	Unit	RHS	CRN	SOY	SLCRN	SLSOY	HLRNM	HLRAP	HLRMY
1	PROFIT	$		-46.50	-28.50	2.00	6.00	-3.80	-3.80	-3.80
2	LANDT	ACRE		1.00	1.00					
3	LRONM	HOUR	1,160	1.155				-1.00		
4	LROAP	HOUR	260	1.043	0.434				-1.00	
5	LROMY	HOUR	270	1.760	0.899					-1.00
6	LROJE	HOUR	250	0.77	0.403					
7	LROJY	HOUR	270		0.496					
8	LROAT	HOUR	270							
9	LROSR	HOUR	250		0.434					
10	LROOR	HOUR	270	0.77	0.434					
11	SLTCRN	BUSHEL		-85.00		1.00				
12	SLTSOY	BUSHEL		-30.00			1.00			
13	TFUEL	GALLON		7.07	5.68					
14	TNITGN	POUND		120.00	10.00					
15	TPH	POUND		75.00	50.00					
16	TPOTS	POUND		100.00	25.00					
17	TCRNSD	POUND		13.00						
18	TSOYSD	POUND			50.00					
19	CAPT	$		46.50	28.50			3.80	3.80	3.80
20	CAPM	$	33,300							
21	MACHS	ACRE		1.00	1.00					

Table 4.3 (Continued)

	Item	Unit	HLRJE	HLRJY	HLRAT	HLRSR	HLROR	FUEL	N	PH
1	PROFIT	$	-3.80	-3.80	-3.80	-3.80	-3.80	-.468	-.14	-.18
2	LANDT	ACRE								
3	LRONM	HOUR								
4	LROAP	HOUR								
5	LROMY	HOUR								
6	LROJE	HOUR	-1.00							
7	LROJY	HOUR		-1.00						
8	LROAT	HOUR			-1.00					
9	LROSR	HOUR				-1.00				
10	LROOR	HOUR					-1.00			
11	SLTCRN	BUSHEL								
12	SLTSOY	BUSHEL								
13	TFUEL	GALLON						-1.00		
14	TNITGN	POUND							-1.00	
15	TPH	POUND								-1.00
16	TPOT	POUND								
17	TCRNSD	POUND								
18	TSOYSD	POUND								
19	CAPT	$	3.80	3.80	3.80	3.80	3.80	.468	.14	.18
20	CAPM	$								
21	MACHS	ACRE								

Table 4.3 (Continued)

	Item	Unit	POTS	CRNSD	SOYSD	CAPBW	MACH
1	PROFIT	$	-.09	-.75	-.17	-.085	-44.00
2	LANDT	ACRE					
3	LRONM	HOUR					
4	LROAP	HOUR					
5	LROMY	HOUR					
6	LROJE	HOUR					
7	LTOJY	HOUR					
8	LROAT	HOUR					
9	LROSR	HOUR					
10	LROOR	HOUR					
11	SLTCRN	BUSHEL					
12	SLTSOY	BUSHEL					
13	TFUEL	GALLON					
14	TNITGN	POUND					
15	TPH	POUND					
16	TPOTS	POUND	-1.00				
17	TCRNSD	POUND		-1.00			
18	TSOYSD	POUND			-1.00		
19	CAPT	$	.09	.75	.17	-1.00	44.00
20	CAPM	$					
21	MACHS	ACRE					-1.00

Table 4.4 An Example of a Linear Programming Tableau Used in This Study: Renting Land and Machinery Ownership.

	Item	Unit	RHS	CRN	SOY	SLCRN	SLSOY	HLRNM	HLRAP	HLRMY
1	PROFIT	$		-46.50	-28.50	2.00	6.00	-3.80	-3.80	-3.80
2	LANDT	ACRE		1.00	1.00					
3	LRONM	HOUR	1,160	1.155				-1.00		
4	LROAP	HOUR	260	1.043	0.434				-1.00	
5	LROMY	HOUR	270	1.760	0.899					-1.00
6	LROJE	HOUR	250	0.770	0.403					
7	LROJY	HOUR	270		0.496					
8	LROAT	HOUR	270							
9	LROSR	HOUR	250		0.434					
10	LROOR	HOUR	270	0.770	0.434					
11	SLTCRN	BUSHEL		-85.000		1.00				
12	SLTSOY	BUSHEL		-30.000			1.00			
13	TFUEL	GALLON		7.070	5.680					
14	TNITGN	POUND		120.000	10.000					
15	TPH	POUND		75.000	50.000					
16	TPOTS	POUND		100.000	25.000					
17	TCRNSD	POUND		13.000						
18	TSOYSD	POUND			50.000					
19	CAPT	$		46.50	28.500			3.80	3.80	3.80
20	CAPM	$	57,000							
21	RTLD	ACRE								

Table 4.4 (Continued)

	Item	Unit	HLRJE	HLRJY	HLRAT	HLRSR	HLROR	FUEL	N	PH
1	PROFIT	$	-3.80	-3.80	-3.80	-3.80	-3.80	-.468	-.14	-.18
2	LANDT	ACRE								
3	LRONM	HOUR								
4	LROAP	HOUR								
5	LROMY	HOUR								
6	LROJE	HOUR	-1.00							
7	LROJY	HOUR		-1.00						
8	LROAT	HOUR			-1.00					
9	LROSR	HOUR				-1.00				
10	LROOR	HOUR					-1.00			
11	SLTCRN	BUSHEL								
12	SLTSOY	BUSHEL								
13	TFUEL	GALLON						-1.00		
14	TNITGN	POUND							-1.00	
15	TPH	POUND								-1.00
16	TPOTS	POUND								
17	TCRNSD	POUND								
18	TSOYSD	POUND								
19	CAPT	$	3.80	3.80	3.80	3.80	3.80	0.468	0.14	0.18
20	CAPM	$								
21	RTLD	ACRE								

Table 4.4 (Continued)

	Item	Unit	POTS	CRNSD	SOYSD	CAPBW	RENTLD
1	PROFIT	$	-.09	-.75	-.17	-.085	-39.00
2	LANDT	ACRE					-1.00
3	LRONM	HOUR					
4	LROAP	HOUR					
5	LROMY	HOUR					
6	LROJE	HOUR					
7	LROJY	HOUR					
8	LROAT	HOUR					
9	LROSR	HOUR					
10	LROOR	HOUR					
11	SLTCRN	BUSHEL					
12	SLTSOY	BUSHEL					
13	TFUEL	GALLON					
14	TNITGN	POUND					
15	TPH	POUND					
16	TPOTS	POUND	-1.00				
17	TCRNSD	POUND		-1.00			
18	TSOYSD	POUND			-1.00		
19	CAPT	$	.09	.75	.17	-1.00	
20	CAPM	$				1.00	
21	RTLD	ACRE					1.00

Sources of Data

Dry Corn Yield--Corn yields are drawn from information presented in Michigan Agricultural Statistics published by the Michigan Department of Agriculture, July 1979, also from unpublished Michigan agricultural data. The average yield per acre are 85 bushels, 81 bushels, and 90 bushels for 1977, 1978, and 1979 period respectively.

Soybean Yield--The information on soybean yields are also drawn from information given in Michigan Agricultural Statistics, July 1979. The average yield per acre are 30 bushels, 24 bushels, and 29 bushels for 1977, 1978, and 1979 periods, respectively.

Labor Requirements--This information is drawn from the user mannual for telplan program 65:0 (FO), and Agricultural Economics Report No. 350 both published by the Department of Agricultural Economics, Michigan State University.

Variable Costs--Variable costs per acre for production of corn and soybeans which include repairs and maintenance, limestone, herbicide, utilities, harvest trucking, corn drying and marketing and other expenses are drawn from Agricultural Economics Report No. 314 and No. 350 published by the Department of Agricultural Economics, Michigan State University.

Diesel Fuel Requirements--Diesel fuel requirements for production of one acre of corn and soybeans are given in Table A.5 of the Appendix. These datas are drawn from James Allen Lehrman M. S. Thesis, M.S.U.

Fertilizer requirements--nitrogen, phosphorous, and potassium for production of corn and soybeans are drawn from Agricultural Economics Report No. 350 published by the Department of Agricultural Economics, Michigan State University.

In the next chapter the results of the analysis will be presented.

Chapter Summary

In this chapter, implimentation of the theory of partitioning output among inputs was discussed. A linear programming technique(LP) was considered as one way of implementing the theory. Activities, resource constraints and accounting equations for the linear programming model was set and discussed. Also example of the tableaus used in the study were given. Finally, sources of data for such inputs as crop yields, operating capital, operator labor, diesel fuel, and fertilizer were discussed.

CHAPTER V

MODEL RESULTS UNDER CONSTANT INPUT PRICES

In Chapter IV we described the linear programming model and the representative farm firm. This description explained the farm firm's inputs, costs, and expected outputs. In this chapter the model results are given under alternative manners of acquisition and use of services from land and machinery. The model results include shadow prices and operating incomes for different levels of land services. The shadow price for land is the additional revenue that can be generated by increasing the availability of land services one more unit. If one unit of land service can be purchased for less than the shadow price, then the operating income increases with an increase in land services. The series of shadow prices for different levels of acres are summed to determine the portion of output attributed to land. This is equivalent to integrating the long run marginal value products along the expansion path as the theory of partitioning output among inputs indicated. Finally, we estimate lifetime returns required for the estimation of land value in the capital budgeting model based on three period LP Models.

In the first part of this chapter, we compare land values based on two alternative assumptions of land

acquisition, namely renting land, and land ownership. The assumption of renting land is equivalent to the assumption of asset divisibility in acquisition and use. While the assumption of land ownership is equivalent to the assumption of asset indivisibility in acquisition but divisibility in use.

Simultaneously in this chapter, we compare land values based on two alternative assumptions of machinery services acquisitions, namely custom hiring and ownership. The assumption of custom hiring is equivalent to the assumption of asset divisibility in acquisition and use. While machinery ownership is equivalent to the assumption that asset is indivisible in acquisition but divisible in use.

Acquiring Land Services through Renting

In this model we assume that land is divisible in acquisition and use. That is, the farm operator acquires land services by renting. In this model, machinery services are acquired through ownership; that is, machinery services are indivisible in acquisition but divisible in use. The operator is the major source of labor with any additional labor being hired as the operator's labor supply deleted. Debt capital is the limiting factor. The borrowing limits are $57,000, $71,000, and $78,000 for the periods 1977, 1978, and 1979 respectively. The limits on debt capital

limit the number of acres planted in each period. Then, the budget availability constraint is set at different levels to determine shadow prices associated with various levels of land services. Since the budget is used to rent land and to purchase other inputs in each time period, therefore, by changing the budget constraint, the number of acres planted also change. Figure 4, graphically illustrates different budget levels considered and the expansion path for 1977 period under the assumption of land rental arrangement. Note, that this procedure is repeated for each of the three periods: 1977, 1978, and 1979. The shadow prices for each period are equivalent to the long run marginal value products along the expansion path for that particular period. Now the results are discussed in more detail.

Table 5.1 presents the shadow prices for land and the associated profits under the assumption that land is acquired by renting.

First the results for 1977 period are discussed. The maximum number of acres planted is 610 for this period. The shadow price for the 610-th acre of land is $71.60. The return for the entire farm over cash cost is $48,005. The shadow price is $76.90, when land services are 110 acres associated with $10,000 budget level and remains constant up to 219 acres associated with $20,000. When land services are between 220 and 539 acres, the shadow price becomes $74.

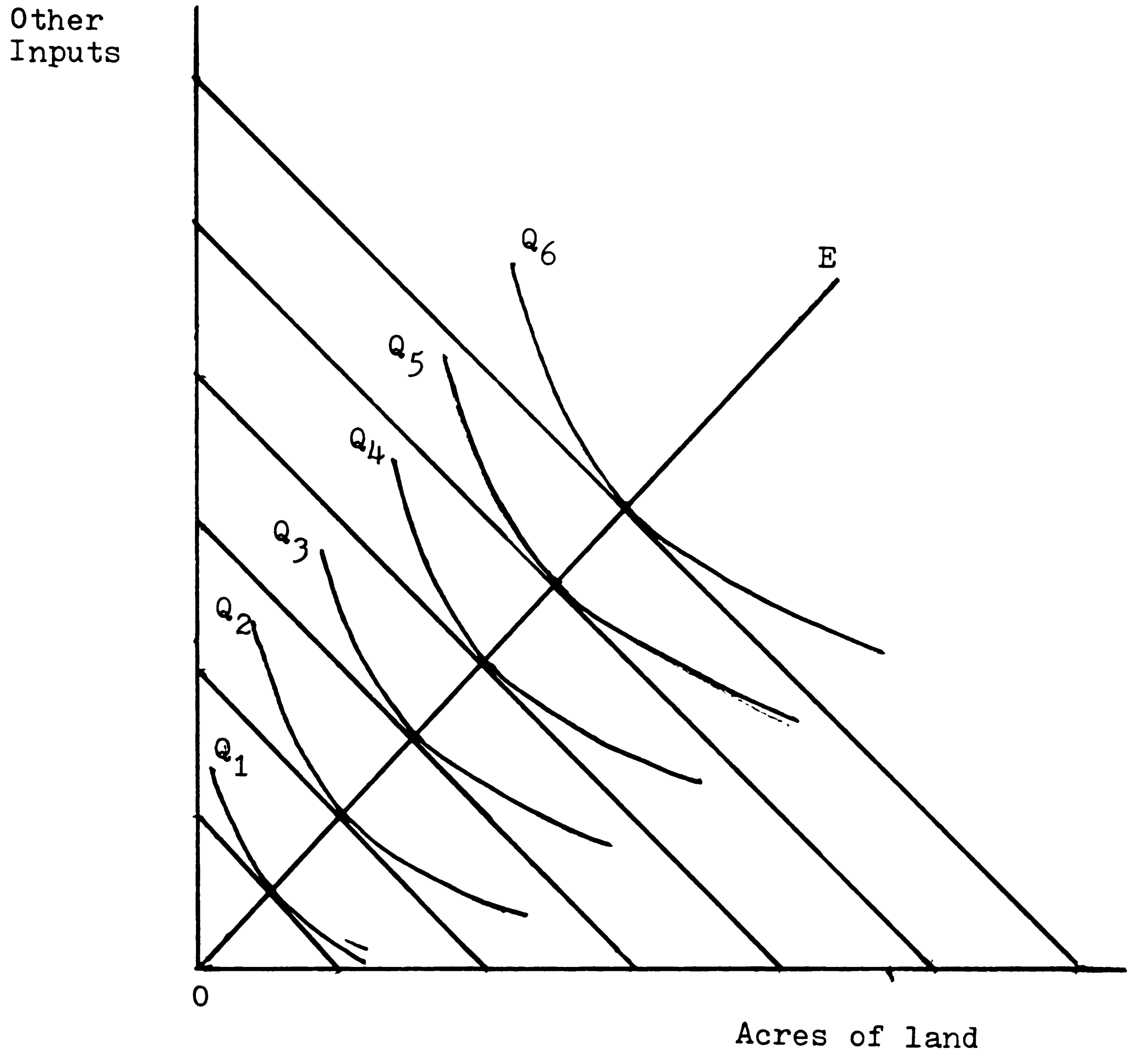

Figure 4: Different Budget Levels and the Associated Expansion Path for 1977 period. The different budget levels are those shown in table 5.1.

When land services are between 540 and 610 acres, the shadow price becomes $71.60. These shadow prices for different levels of land services associated with different budget levels are equivalent to marginal value products along the expansion path for 1977 period. These marginal value products are summed to determine the value of the land share from the output for 1977 period.

The maximum number of acres is 730 for the period 1978. The shadow price at the level of 730 acres, is $54.50. Marginal value products increased from $54.50 at 730 level to $62 at the level of 108 acres associated with budget level of $10,000. It is obvious from Table 5.1 that marginal value products of land for 1978 period are less than those estimated for 1977 period, because the costs of production are relatively greater for the former period than the latter.

The maximum number of acres is 730 for the period 1979. Shadow price at this level of land services associated with $78,000 budget, is $67. Marginal value products increased from $67 at 730 level to $78 at 98 acres associated with $10,000 budget level. The return for the entire farm over cash costs is $46,501. Next the portion of output attributed to land under the assumption land renting for the periods 1977, 1978, and 1979 is determined.

Table 5.1 Optimal Number of Acres, Operating Income and Marginal Value Products (MVP) of Land for Different Levels of Assumed Budgets for:

1) 1977 Period.

Budget Level, $	Optimal Number of Acres	MVP of Land, $	Operating Income, $
10,000.00	110	76.90	8863.00
20,000.00	219	76.90	17727.00
30,000.00	328	74.00	26407.00
40,000.00	433	74.00	34560.00
50,000.00	539	74.00	42712.00
57,000.00	610	71.60	48005.00

2) 1978 Period.

Budget Level, $	Optimal Number of Acres	MVP of Land, $	Operating Income, $
10,000.00	108	62.00	5757.00
20,000.00	215	62.00	11514.00
30,000.00	322	59.60	17139.00
40,000.00	426	59.60	22266.00
50,000.00	529	59.60	27382.00
60,000.00	629	54.50	31951.00
71,000.00	730	54.50	39785.00

Table 5.1 (Continued).

3) 1979 Period.

Budget Level, $	Optimal Number of Acres	MVP of Land, $	Operating Income, $
10,000.00	98	78.00	6800.00
20,000.00	196	78.00	13599.00
30,000.00	295	78.00	20399.00
40,000.00	389	74.80	26449.00
50,000.00	482	74.80	32453.00
60,000.00	576	73.00	38318.00
70,000.00	662	67.00	43040.00
78,000.00	730	67.00	46501.00

Portion of Output Attributed to Land

Thus far, the shadow prices have been determined for different levels of land services associated with different budget levels allowing other inputs to optimally adjust. Now the value of the portion of output attributed to land input is determined.

The theory of partitioning output among inputs suggests that the portion of output attributed to land services is determined by integrating the long run marginal value product (LRMVP) for land. Shadow prices or marginal value products are integrated along the expansion path over the range from zero to the maximum number of acres planted given the available capital for the period considered. With linear programming, the marginal value products may be constant between the different levels of land services. Therefore, the portion of output attributed to land services is the sum of the shadow price multiplied by the quantity of land services over which the LRMVP is constant. For example, consider the results of 1977 period, the shadow prices are $76.90 between 0 and 219 acres, $74 between 220 and 539 acres and $71.60 between 540 and 610 acres. Figure 5 depicts graphically the same results for the 1977 period. As Figure 5, demonstrates the portion of revenue attributed to land services, is the sum of the multiplication of different increments of number of acres by their respective marginal value products.

That is: RT = (219 x 76.90)+(320 x 74)+(71 x 71.60)= \$45,605 where RT is the portion of revenue attributed to land is the portion of output attributed to 610 acres of land services.1/

The net income to one acre of land is estimated by dividing the value of the portion of output attributed to land services by the maximum number of acres given the available capital for each period. For example, the return to one acre of land is determined by dividing the portion of revenue attributed to land input of \$45,605 by the maximum number of acres of 610 for 1977 period. Therefore, the average income to one acre of land is \$74.75 at 1977 period. That is, if the farm operator must decide to rent 610 acres or nothing, he could afford to pay the \$74.75 per acre in, 1977. But if the land services were acquired individually -- one unit at a time, the farm operator could pay the marginal value product for each acre. For example, the farm operator in 1977 could afford to pay up to \$76.90 to acquire the services from the first 219 acres. For the next 320 acres he could pay \$74.75 and for the next 71 acres he could pay \$71.60. This emphasizes the importance of the nature of the acquisition. If divisible in acquisition-- the farm operator pays the marginal value product, given in Table 5.2.

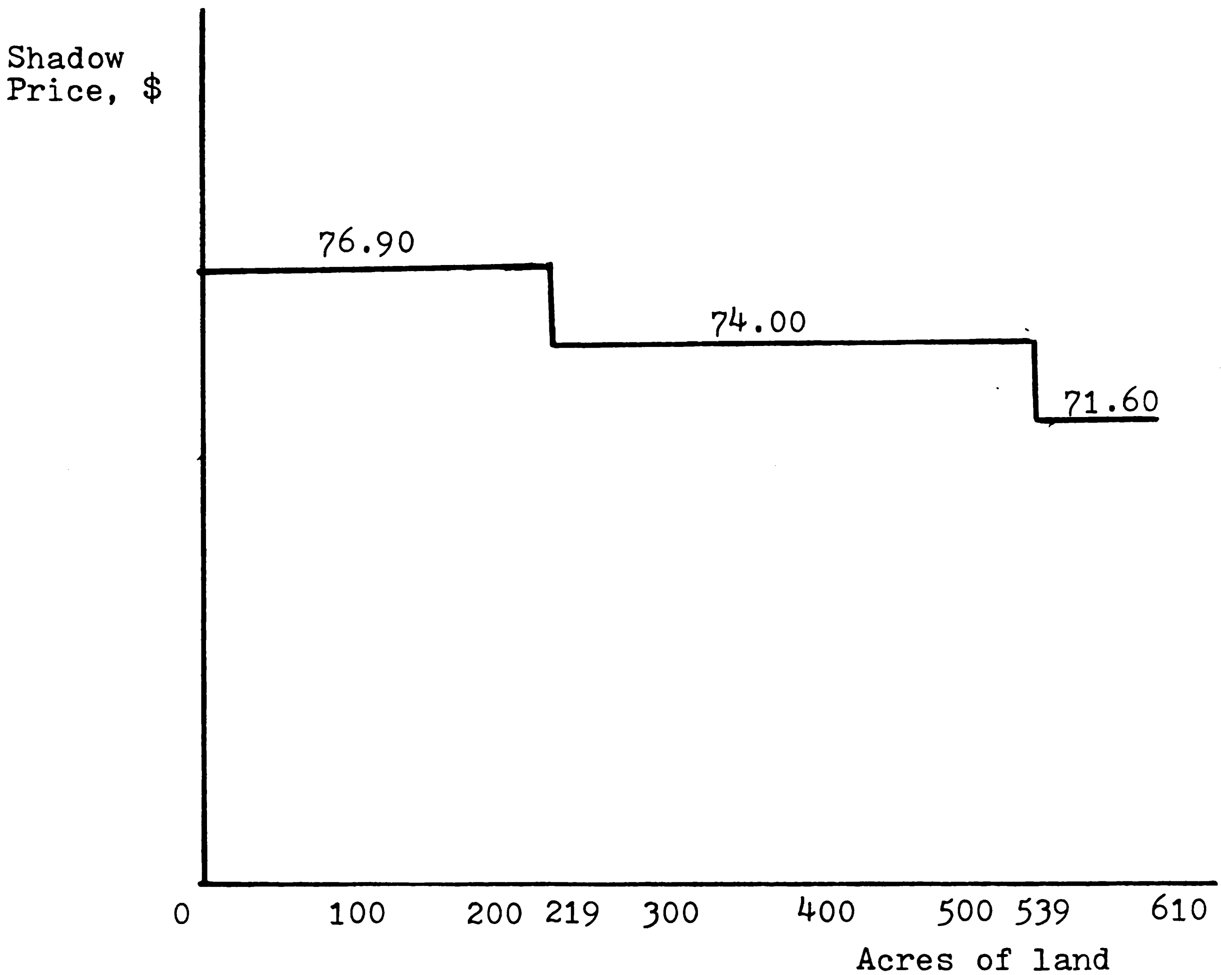

Figure 5: Graphical Representation of Marginal Value Products Associated with Various Levels of Acres for 1977 period.

Table 5.2 Estimated Portion of Output and Annual Income to Land, Given that the Land Services are Acquired through Renting, that is Land Services are Divisible in Acquisition.

Period	Optimal Number of Acres	Total Portion of Output, $	Net Income to Land, $
1977	610	45,605.00	74.75
1978	730	42,999.00	59.00
1979	730	54,176.00	74.00

Average Expected Income

In order to develop agricultural land prices, the net income to land is capitalized. However, before any capitalization take place, the expected income has to be calculated. A three year average is used under the assumption that the best clue to the future is the immediate past. That is, expectation about the future income streams to land under this assumption are based on the average of the income streams reported in Table 5.2, for the past three years. The formula for the average net income is:

$$\text{(V.2)} \qquad R = (R(0)(1+g)^{3} + R(1)(1+g)^{2} + R(2)(1+g))/3$$

where R is the average expected income to land, R(1), R(1), and R(2) are the annual net income to land for the period 1977 through 1979, and g is the growth rate in income. The growth rate in income simply is determined as:

$$\text{(V.3)} \qquad g = \sum_{t=1}^{n} (((R(t) - R(t-1))/R(t-1))/n$$

In this case n equals 2, and g the average growth rate for the annual incomes, when solved for, equals 6 percent. The expected income calculated from (V.2) is $78. This expected income is used to determine the agricultural land value, based on a capital budgeting formula.

Land Value

A Capital Budgeting Formula under inflationary period

was derived in this author's master thesis. The value of as asset under inflation is derived as current year's income R, divided by the time preference rate r:

$$(V.4) \qquad V = R/r.$$

It was concluded in the master thesis that the simple formula in (V.4) is more accurate in predicting land prices than more complicated ones such as the Lee and Rask model that included productivity and risk. So equation (V.4) is used to estimate agricultural land value V given the expected income to land R, and the rate of pure time preference r. This author estimated an average time preference rate of 5.67 percent by dividing returns to land by actual land prices for 1960-1977 periods(Kooti, P. 23). Hence, farmland price for the current period (1980) simply is $78/0.0567 equals $1,376. This is the land value estimated under the assumption that land services are acquired through renting. This land value will be compared to land value under the assumption of land ownership, which it will be discussed next.

Acquiring Land Input through Ownership

In this model, we assume that land is indivisible in acquisition but divisible in use. That is, the operator acquired land services by ownership. In this model, machinery services are acquired through ownership also. That is, machinery services are indivisible in acquisition but divisible in use. The operator is the main supply of labor with outside labor being hired when the labor requirements exceed the capacity of the operator to supply them. Operating capital is borrowed at the beginning of the production period. Debt capital is the limiting factor. The limits of debt capital are $33,300, $44,000, and $46,000 for the periods of 1977, 1978, and 1979 respectively. By limiting the debt capital, we limit the number of acres planted in each period. Then, shadow prices are determined at various levels of land constraints, allowing for other inputs to optimally adjust. Since, in this case land is owned by the operator, we change the land constraint instead of the budget constraint. Because the farm operator already owns the land, and if land is idle because of lack of available capital its shadow price becomes zero. Therefore, we change the land constraint such that all land will be used in the production process. Figure 6 graphically illustrates those constraints for 1977 period. Since the prices remain constant, the shadow prices for various levels of acres in each period are equivalent to the marginal value products along the expansion path OCD.

Table 5.3 presents the shadow prices for land and the associated profits given that land is owned by the farm operator. The maximum number of acres planted, is 610 for

1977 period. With shadow price of $113.90. Operating income of $73,817 is generated from the operation. Shadow price is $123 at 100 acres level, and it remains constant along the range of 100 to 300 acres. Shadow price becomes $119.50 at 400 acres, and it remains constant along the range of 301 to 500 acres. Shadow price changes from $113.90 at 600 acres, and it remains constant up to 610 acres.

The number of acres planted in 1978 is 730 acres which is the same number of acres rented under the assumption of renting land. The shadow price at 730 acres is $80. The shadow price of land increased from $80 at 730 acres to $94 at 100 acres constraint.

The number of acres owned in 1979 period, is 730, which is the same number of acres rented under the earlier assumption that land was acquired through rental arrangements. The shadow price at 730 acres is $99.60 which increased from $99.60 at 730 acres to $117.60 at 100 acres constraint.

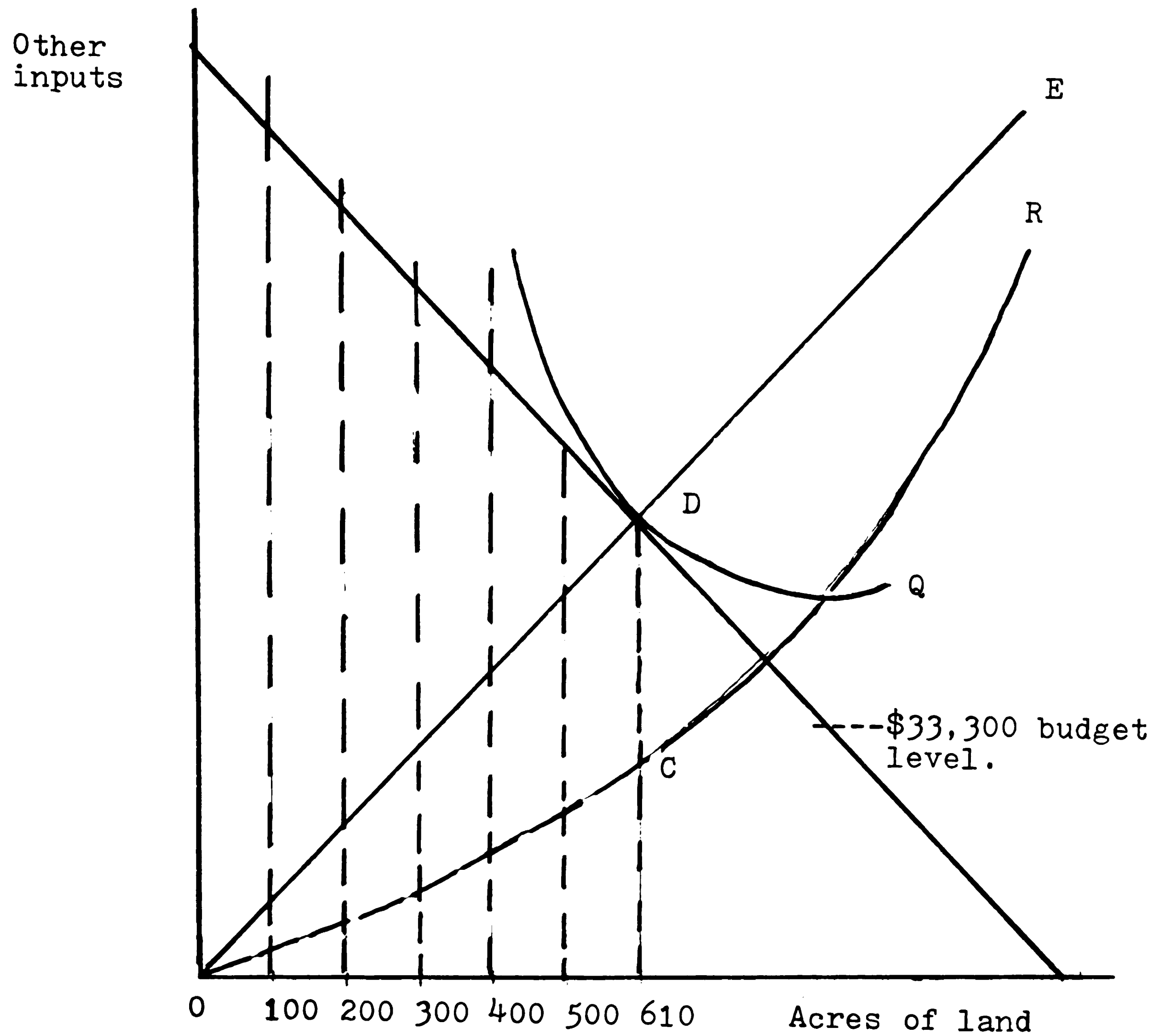

Figure 6: Different Levels of Land Constraints and Expansion Path for Land Ownership, for 1977 period.

Table 5.3 Marginal Value Products (MVP) and Operating Incomes for Different Levels of Acres Under the Assumption of Land Ownership, that is, Land is Indivisible in Acquisition but Divisible in Use for 1977-1979 periods.

Number of Acres	MVP $	Operating Income, $	MVP $	Operating Income, $	MVP $	Operating Income, $
100	123.00	12325.00	94.00	9405.00	117.60	11764.00
200	123.00	24649.00	94.00	18810.00	117.60	23528.00
300	123.00	36974.00	94.00	28215.00	117.60	35292.00
400	119.50	48929.00	90.00	37220.00	112.00	46533.00
500	119.50	60883.00	90.00	46223.00	112.00	57773.00
600	113.90	72678.00	84.00	55055.00	104.00	68789.00
610 1/	113.90	73817.00	84.00	55895.00	104.00	69829.00
700	------	---------	80.00	63154.00	99.60	78849.00
730 2/	------	---------	80.00	65560.00	99.60	81836.00

1/ Optimal number of acres determined under land renting assumption for 1977 period. This number of acres is assumed owned by the farm operator.

2/ This indicates the optimal number of acres for 1978 and 1979 periods.

Now the value of the portion of output attributed to land given that the operator owns his land, is determined. The value of the portion of output attributed to land under the assumption that land is acquired by ownership, is determined as the sum of marginal value products for different levels of land constraints over which the MVP is constant. Table 5.4 presents the portion of revenue attributed to land services, the number of acres planted by the farm operator, and the annual average income to one acre of land for 1977, 1978, and 1979 periods.

The expected average income is calculated in the same procedure described earlier assuming the same growth rate in income. The expected average income for the current period is $120.

Farmland value under the assumption that land services are acquired through ownership, is determined using (V.4), where R is $120 and r is 0.0567. Therefore, the farmland value is estimated as $2,116, given the assumption that land services are acquired through ownership.

Comparing the results of Table 5.1 and Table 5.3 indicate that:

1) Shadow prices for land obtained under the assumption that land services are acquired through ownership are generally greater than those shadow prices obtained under the assumption that all land services are acquired by renting,

because marginal cost of using additional units of land under the assumption of land ownership is small or zero, while under the assumption of rented land, it is the rental charge.

2) Operating incomes associated with land ownership are greater than those associated with rented land.

The results given in Table 5.2 and Table 5.4 indicate that the value of the portion of output attributed to land services under the assumption of land ownership is greater than those associated with rented land. Recall from Chapter III, that the firm's expansion path under the assumption of asset divisibility in acquisition (land rental) is OE in Figure 6. While the expansion path under the assumption of asset indivisibility in acquisition (land ownership) is OCD in Figure 6. As a result, the portion of output attributed to land found by integrating the long run marginal value products along the firm's expansion paths OE, and OCD must be different.

The results given in Table 5.2 and Table 5.4 indicate that the annual average income for one acre of land is greater under the assumption of land ownership than that of rented land. The average annual income is the value of the portion of output attributed to land divided by the maximum number of acres planted in each period. Since the value of the portion of output attributed to land under the assumption of land

ownership is greater than that of rented land; and the maximum number of acres planted is the same, hence annual average income of one acre of land under land ownership is greater than that of rented land.

Finally, land values associated with land ownership is greater than that of rented land, since annual average income for land ownership is greater than that of rented land at each period. It is concluded that farmland is worth more to farmers who own their land than those who rent their land.

Table 5.4 Estimated Portion of Revenue Attributed to Land and Annual Average Income to Land, Given that Land Services are Acquired through Ownership.

Period	Maximum Number of Acres	Total Portion of Revenue, $	Annual Income to Land, $
1977	610	73329.00	120.00
1978	730	65000.00	89.00
1979	730	81028.00	111.00

Acquiring Machinery Services through Custom Hiring

In this model, we assume that machinery services instead of being divisible in use and indivisible in acquisition (ownership), they are divisible in acquisition and use--that is, the operator acquires machinery services by custom hiring. The rates of custom hiring are $44, $53, and $64 for 1977, 1978, and 1979 periods. In addition, we assume the operator acquires land services through ownership. The operator is the main labor supply with outside labor being hired when the labor requirements exceed the capacity of the operator to supply them. Debt capital is set as the limiting factor. The limits of debt capital are $33,300, $44,000, and $46,000 for 1977-1979 periods. Limiting the debt capital limits the number of acres planted in each time period. Then, shadow prices associated with various levels of acres are determined for each time period. As mentioned earlier, the shadow prices for various levels of acres are equivalent to the marginal value products along the expansion path.

Table 5.5 presents the shadow prices for land and operating incomes associated with different levels of acres for the periods 1977 through 1979. Maximum number of acres planted are 344, 401, and 374 for 1977, 1978, and 1979 periods, under the assumption that the operator acquires machinery services by custom hiring.

Table 5.5 Estimated Marginal Value Product(MVP), and Operating Income under the Assumption that Machinery Services are Acquired through Custom Hiring for 1977-1979 periods.

Number of acres	1977 period MVP $	1977 period Operating income, $	1978 period MVP $	1978 period Operating income, $	1979 period MVP $	1979 period Operating income, $
100	75.50	7550.00	36.30	3628.00	47.20	4724.00
200	75.50	15101.00	36.30	7256.00	47.20	9448.00
300	75.50	22652.00	36.30	10884.00	47.20	14172.00
344 1/	75.00	25827.00	32.30	12305.00	46.00	16196.00
374 2/	-	-	32.30	13274.00	46.00	17310.00
400	-	-	32.30	14112.00	-	-
401 3/	-	-	32.30	14152.00	-	-

1/ The optimal number of acres planted given 1977 budget level of $33,300.

2/ The optimal number of acres planted given 1979 budget level of $46,000.

3/ The optimal number of acres planted given 1978 budget level of $44,000.

To determine the portion of output attributed to land services, shadow prices or marginal value products are integrated along the expansion path over the range zero to the maximum number of acres. However, in linear programming the marginal value products are constant between the points corresponding to successive levels of land services supply as discussed earlier. Therefore, the value of the portion of output attributed to land is the sum of marginal value products multiplied by the number of acres along the expansion path over which the LPMVP is constant. Table 5.6 presents the value of the portion of output attributed to land, the number of acres planted and the annual average income to one acre of land for 1977-1979 periods.

Comparing the results given in Table 5.4 and Table 5.6 indicate that the portion of output attributed to land under assumption that machinery services are acquired by ownership is greater than that estimated under the assumption that machinery services are acquired by custom hiring. The maximum number of acres planted for the former is greater than the maximum number of acres of the latter. Finally, the annual average income for one acre of land under the assumption that machinery services are acquired by ownership is more than that of custom hiring.

Table 5.6 Estimated Portion of Output and Annual Average Income to Land Given Complete Custom Hiring Machinery Services.

Period	Maximum Number of Acres	Total Portion of Revenue, $	Annual Average Income to Land, $/ac.
1977	344	25950.00	75.40
1978	401	14148.00	35.00
1979	374	17564.00	47.00

The average expected income using equation (V.2), is $59.60 per acre given that the machinery services are acquired by custom hiring. Farmland value estimated using (V.4), is $1,051 given 5.67 percent rate of pure time preference. This value is less than that estimated under the assumption that all machinery services are acquired through ownership. It was concluded that land is worth more to farmers who own their machinery than those who custom hire their machinery services.

Acquiring Machinery Services through Combination of Ownership and Custom Hiring

In this model, we assume that machinery services are partly divisible and partly indivisible in acquisition. That is, the operator acquires machinery services through ownership and custom hiring. The operator acquires land services by ownership. It is assumed first that the owner has enough machinery services to cover the first one hundred acres. Any additional machinery services are custom hired as needed at the rates of $44, $53, and $64 per acre for 1977, 1978, and 1979 periods respectively. Next we assume the operator has enough machinery for the first two hundred acres, and additional machinery services are custom hired as needed at the rates given earlier. Lastly, we assume the operator has enough machinery for the first three hundred acres and additional machinery services are custom hired as needed.

The operator is assumed to provide labor services for operations of the farm up to 3,000 hours. Any additional labor needed is hired as the operator's labor is exhausted. As mentioned previously, operating capital is borrowed at the beginning of the production periods.

The purpose of comparing land values under the above assumptions is to show that farmers are willing to pay higher prices for land, if they have excess machinery

services on hand. That is, if the farmers have excess machinery services that they own they will pay more for one acre of land. This was illustrated by an example using a Cobb-Douglas production function in Chapter III. Now it is empirically tested using linear programming technique as described in Chapter IV. The results are followed.

Table 5.7a presents the marginal value products (shadow prices), and associated operating income given that the operator has enough machinery for the first one hundred acres. Any additional machinery services are custom hired as needed at the rates specified earlier in this section. As mentioned earlier, shadow prices remain constant between two successive levels of acres in linear programming. The successive increments in number of acres multiplied by the associated shadow prices are summed to determine the portion of revenue attributed to land services in each of the three periods: 1977, 1978, and 1979 one at a time. Then, the annual average income to one acre of land is determined by dividing the revenue share to land by the maximum number of acres planted in each time period. The value of the portion of output attributed to land, maximum number of acres planted and annual average income to land are presented in Table 5.8a.

Table 5.7b and Table 5.7c present the shadow prices, and the associated operating incomes given that the operator

has enough machinery services to cover for the first two hundred acres and three hundred acres. Any additional machinery services are custom hired as needed. The value of the portion of output attributed to land services, maximum number of acres planted, and the annual average income to one acre of land are presented in Table 5.8b, and 5.8c.

Comparing the results of Tables 5.8a, 5.8b, and 5.8c indicate that as number of machinery services owned by the operator increase, maximum number of acres, portion of revenue attributed to land, and annual average income to land increase at each time period.

Table 5.9 presents the estimated expected average incomes to one acre of land, and land values associated with various levels of machinery services ownership. Expected average income to one acre of land is calculated using the annual average income to land reported in Table 5.8, and equation (V.2). Land values are determined by dividing expected average income to one acre of land by the rate of pure time preference of 5.67 percent.

Table 5.7 Estimated Marginal Value Product(MVP), and Operating Income under Complete Ownership of Land and Combination of Machinery Ownership and Custom Hiring under the following Assumptions:
a) operator furnished machinery for the first one hundred acres, additional machinery services are custom hired as needed.

Number of acres	1977 period		1978 period		1979 period	
	MVP $	Operating Income, $	MVP $	Operating Income, $	MVP $	Operating Income, $
100	123.00	12325.00	94.00	9405.00	117.60	11764.00
200	75.50	19875.00	36.30	13033.00	47.20	16488.00
300	75.50	27426.00	36.30	16661.00	47.20	21212.00
388 1/	75.00	33769.00	32.30	19502.00	42.00	24908.00
400	-	-	32.30	19889.00	42.00	25413.00
425 2/	-	-	32.30	20697.00	42.00	26473.00
448 3/	-	-	32.30	21450.00	-	-

1/ Optimal number of acres for 1977 period.
2/ Optimal number of acres for 1978 period.
3/ Optimal number of acres for 1979 period.

Table 5.7 (Cont.)

b) Operator furnished machinery services for the first two hundred acres, additional machinery services are custom hired as needed.

Number	1977 period		1978 period		1979 period	
of acres	MVP $	Operating Income, $	MVP $	Operating Income, $	MVP $	Operating Income, $
100	123.00	12325.00	94.00	9405.00	117.60	11764.00
200	123.00	24649.00	94.00	18810.00	117.60	23528.00
300	78.00	36950.00	36.30	22438.00	47.20	28252.00
400	71.80	39381.00	32.30	25666.00	42.00	32453.00
432 1/	71.80	41710.00	32.30	26700.00	42.00	33797.00
475 2/	-	-	32.30	28089.00	42.00	35636.00
495 3/	-	-	32.30	28748.00	-	-

1/ Optimal number of acres for 1977 period.
2/ Optimal number of acres for 1979 period.
3/ Optimal number of acres for 1978 period.

Table 5.7 (Cont.)

c) Operator furnished machinery services for the first three hundred acres, additional machinery services are custom hired as needed.

Number of acres	1977 period MVP $	1977 period Operating Income, $	1978 period MVP $	1978 period Operating Income, $	1979 period MVP $	1979 period Operating Income, $
100	123.00	12325.00	94.00	9405.00	117.60	11763.00
200	123.00	24649.00	94.00	18810.00	117.60	23528.00
300	123.00	36950.00	94.00	28215.00	117.60	35292.00
400	71.80	44155.00	32.00	31443.00	42.00	39493.00
476 1/	71.80	49652.00	32.00	33875.00	42.00	42685.00
500	-	-	32.00	34669.00	42.00	43693.00
526 2/	-	-	32.00	35501.00	42.00	44799.00
542 3/	-	-	32.00	36046.00	-	-

1/ Optimal number of acres for 1977 period.
2/ Optimal number of acres for 1979 period.
3/ Optimal number of acres for 1978 period.

Table 5.8 Estimated Portion of Revenue Attributed to Land and Annual Average Income to Land Under Combination of Ownership and Custom Hiring Machinery Services.

a) Operator furnished machinery for the first one hundred acres, additional would be custom hired.

Period	Optimal Number of Acres	Total Portion of Revenue to Land, $	Annual Income to Land, $
1977	388	34000.00	87.60
1978	448	21440.00	47.85
1979	425	26450.00	62.00

b) Operator furnished machinery for the first two hundred acres, additional machinery would be custom hired.

Period	Optimal Number of Acres	Total Portion of Revenue to Land, $	Annual Income to Land, $
1977	432	41878.00	97.00
1978	495	28729.00	58.00
1979	475	35570.00	75.00

c) Operator furnished machinery for the first three hundred acres, additional machinery would be custom hired.

Period	Optimal Number of Acres	Total Portion of Revenue to Land, $	Annual Income to Land, $
1977	476	49537.00	104.00
1978	542	35944.00	66.00
1979	526	44772.00	85.00

Table 5.9 Estimated Expected Average Incomes and Land Values Under Combination of Machinery Ownership and Custom Hiring.

Level of Owned Machinery Services in Acres	Expected Incomes, $	Land Values $
000	59.60	1051.00
100	74.60	1316.00
200	86.70	1529.00
300	96.00	1693.00

The results presented in Table 5.9 indicate that as the operator owns more machinery services, the expected average income and hence land value would increase. This results are consistent with the theory of partitioning output among inputs introduced in Chapter II, and the illustrated examples in Chapter III. In Chapter III, it was shown that as the operator owns more machinery, the value of the portion of output attributed to land input would increase.

Chapter Summary

This chapter contains the empirical results of the theory of partitioning output among inputs using linear programming techniques. First part of this chapter contains model results under two alternative ways of acquiring land services: 1) acquiring land services through renting, and 2) acquiring land services through ownership. The results indicated that the portion of revenue attributed to land services and hence land value were greater under the former than the latter.

The last part of this chapter contains the model results under two alternative ways of acquiring machinery services: 1) acquiring machinery services through custom hiring, and 2) acquiring machinery services through combination of ownership and custom hiring. The results indicated that, first the portion of revenue attributed to land services and land value under combination of ownership and custom hiring were greater than those estimated under custom hiring. Second the portion of revenue attributed to land services and land value increase as the operator has more machinery at hand.

FOOTNOTE

1/ Euler's theorem demonstrates that the marginal value product of any input is constant. Therefore, the output attributed to that input is its marginal value product(MVP) multiplied by the number of units employed. Let us assume a production function f(x,y) of the form Cobb-Douglas $x^a y^b$ Also assume the relationship between x and y is specified as x equals ky. According to the Euler's theorem which is based on production function of degree one, MVP of x and MVP of y are both constant. For example, the marginal value product of y is the partial derivative of f(x,y) with respect to y. That is:

(1) $f_y = bx^a y^{b-1}$

where f_y is the marginal product of y. Substituting ky for x in (1) gives:

(2) $f_y = bk^a y^{a+b-1}$

From (2) f_y is constant only if constant returns to scale exist, which that what the Euler's theory is based on, that is (a+b) has to be equal to one. If there is increasing returns to scale, a+b is greater than one, then the marginal product is increasing. Finally, if a+b is less than one, then marginal product of y is decreasing. Therefore, determining the portion of output attributed to y as the marginal product multiplied by the number of units used, is correct only if constant returns to scale exist.

CHAPTER VI

SENSITIVITY ANALYSIS

As stated previously, the linear programming model is developed to determine the income to land associated with different levels of acres allowing for other inputs to optimally adjust. Given a set of input and output prices along with resource levels, the model generates shadow prices given the limited debt capital. The previous chapter presented model results: under alternative assumptions of machinery and land acquisition given that input prices remain constant.

Now the question becomes: how does income to land and hence land value change as a result of input price changes? By changing input prices in the model we can examine empirically how input price changes affect land values. For the purpose of this analysis we assume that the farm operator owns his land and machinery.

Analysis Procedure

In this model, we assume that land services are indivisible in acquisition and divisible in use -- that is, the operator owns his farmland. Also it is assumed that machinery services are provided by operator ownership. The result of this model under the assumption of constant input prices were given in Chapter V, now the effect of input price changes on land value is considered.

In this study several types of price variation were considered. The sensitivity of the return to land and hence agricultural land value to change in the level of prices for diesel fuel, fertilizer, and hired labor are considered.

The sensitivity of return to land and land vlaue to changes in the cost of diesel fuel while holding other input prices constant is analized first. The following range of price of diesel fuel in terms of $ per gallon for the periods 1977, 1978, and 1979 are assumed:

1) Year 1977: 0.468 -0.94 - 1.87
2) Year 1978: 0.490 -0.98 - 1.96
3) Year 1979: 0.850 -1.70 - 3.40

The second part of the sensitivity analysis includes The sensitivity of the portion of output attributed to land services and land value to change in costs of fertilizer. The following range of fertilizers costs in term of $ per pound for the 1977-1979 period are considered:

I- The price range of Nitrogen:

1) Year 1977: 0.14 - 0.28 - 0.56
2) Year 1978: 0.14 - 0.28 - 0.56
3) Year 1979: 0.15 - 0.30 - 0.60

II - The price ranges of Phosphorus are the following:

1) Year 1977: 0.18 - 0.36 - 0.72
2) Year 1978: 0.19 - 0.38 - 0.76
3) Year 1979: 0.19 - 0.38 - 0.76

III- The price ranges of Potassium are the following:

1) Year 1977: 0.09 - 0.18 - 0.36

2) Year 1978: 0.09 - 0.18 - 0.36

3) Year 1979: 0.10 - 0.20 - 0.40

The last part of the sensitivity analysis includes the sensitivity of the portion of output attributed to land services and land value to change in wage rate of hired labor. The following ranges of wage rate in terms of $ per hour are considered:

1) Year 1977: 3.80 - 7.60 - 15.20

2) Year 1978: 4.10 - 8.20 - 16.40

3) Year 1979: 5.30 - 10.60 - 21.20

Next, the results of the sensitivity analysis are discussed.

Results of Sensitivity Analysis

The first set of sensitivity analysis results deals with the change in: long run marginal value products (shadow prices), and hence agricultural land values when diesel fuel prices increase. As mentioned in Chapter III, as the price of an input increases the firm, wishing to maximize profits, would change the combination of inputs and output levels. Equating the ratio of marginal value products to their input prices. Therefore, as diesel fuel price increases, the firm's expansion path changes from OE to OE'. This is illustrated graphically in Figure 7. Since the value of the portion of output attributed to land is determined by integrating the long run marginal value products along the expansion path, its value is different along $O\dot{E}$ than along OE. In Chapter III, this result was illustrated using a Cobb-Douglas production function, showing analytically how the value of output attributed to land decrease as input price increases.

Now the effect of diesel fuel price increase on land value is empirically tested using an LP Model. By changing diesel fuel price in the objective function of the LP Model a set of shadow prices are derived for each period and at different levels of land constraints. Table 6.1 summarizes the marginal value products associated with this sensitivity analysis. The results given in Table 6.1 indicates that marginal value product at each level of land constraint decreases as diesel fuel price increases. Figure 8, shows that marginal value product at each level of land constraint decrease as diesel fuel price increases from \$.94 to \$1.87 per gallon in 1977.

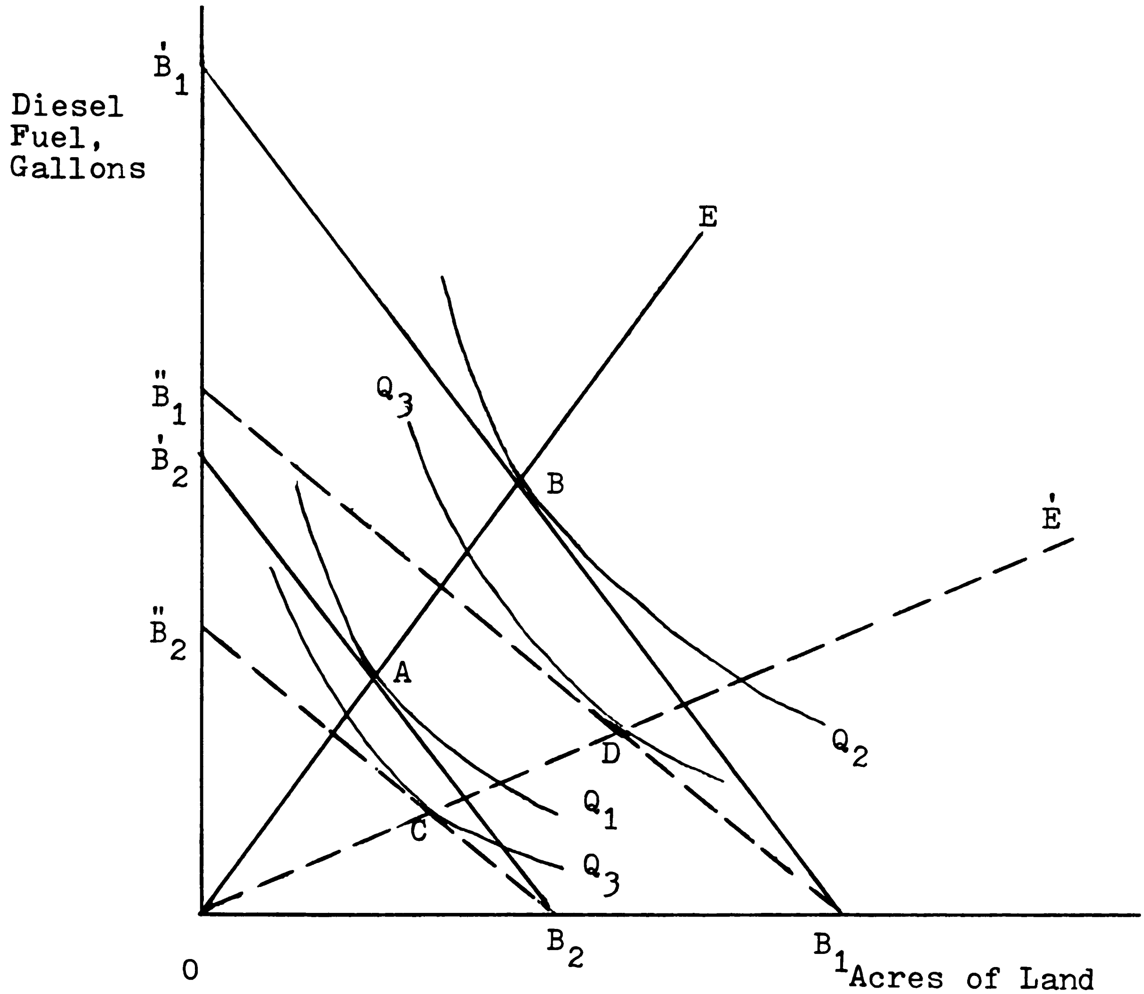

Figure 7: Expansion Path Shifts Resulting from Diesel Fuel Price Increase.

Table 6.1 Sensitivity of Marginal Value Products and Potential Income to change in Diesel Fuel Cost for different level of Acres for 1977 period.

	Diesel Fuel Price Range $0.94-1.87	
Number of Acres	Operating Income Range, $	MVP Range $
100	12034.00-11460.00	120.00-114.60
200	24067.00-22921.00	120.00-114.60
300	36101.00-34382.00	120.00-114.60
400	47765.00-45472.00	116.60-110.90
500	59428.00-56563.00	116.60-110.90
539 1/	63935.00-60885.00	116.60-110.90
586 2/	69366.00- -	116.60- -

1/ This represents the maximum number of acres planted given a budget level of $33,300 and price of diesel fuel of $1.87 per gallon.

2/ This represents the maximum number of acres planted given a budget level of $33,300 and price of diesel fuel of $0.94 per gallon.

Table 6.1 (Cont.) 1978 period

Diesel Fuel Price Range $0.98-1.96		
Number of Acres	Operating Income Range, $	MVP Range $
100	9101.00- 8495.00	91.00-85.00
200	18203.00-16990.00	91.00-85.00
300	27305.00-25485.00	91.00-85.00
400	36006.00-33579.00	87.00-81.00
500	44706.00-41673.00	87.00-81.00
600	53235.00-49594.00	81.00-75.00
656 1/	57600.00-53711.00	77.00-75.00
700	61030.00- -	77.00- -
708 2/	61657.00 -	77.00 -

1/ This is maximum number of acres planted given a budget level of $44,000 and price of diesel fuel of $1.96 per gallon.

2/ This represents the maximum number of acres planted given a budget level of $44,000 and price of diesel fuel of $0.98 per gallon.

Table 6.1 (Cont.) 1979 period.

Diesel Fuel Price Range $1.70-3.40		
Number of Acres	Operating Income Range, $	MVP Range $
100	11233.00-10171.00	112.30-101.70
200	22466.00-20341.00	112.30-101.70
300	33697.00-30512.00	112.30-101.70
400	44409.00-40160.00	107.00- 96.00
500	55118.00-49807.00	107.00- 96.00
600	65603.00-59230.00	99.00- 88.50
611 1/	66663.00-60260.00	99.00- 88.50
686 2/	73893.00 -	99.00 -

1/ This represents the maximum number of acres planted given a budget level of $46,000 and price of diesel fuel of $3.40 per gallon.

2/ This represents the maximum number of acres planted given a budget level of $46,000 and price of diesel fuel of $1.70 per gallon.

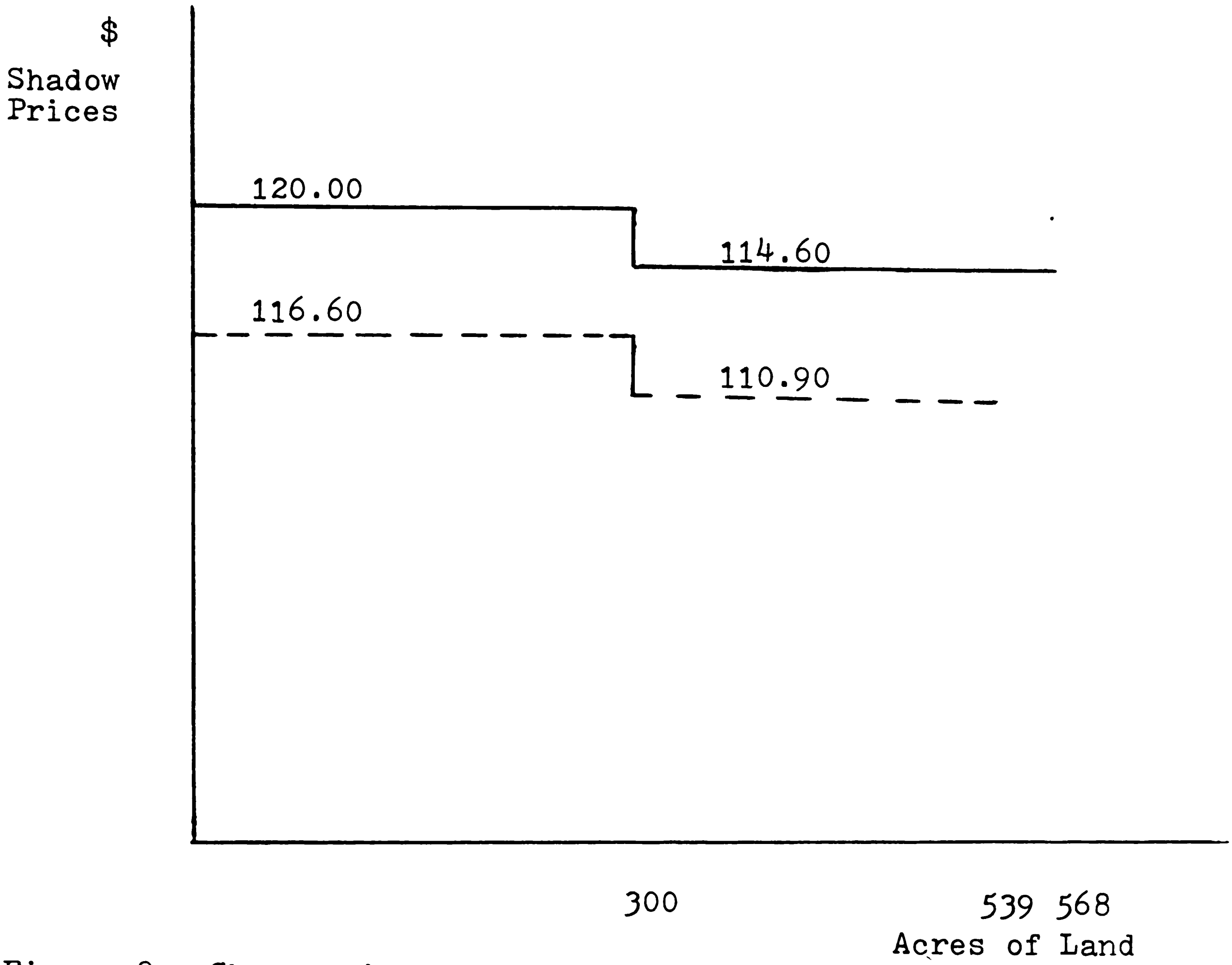

Figure 8: Changes in Marginal Value Products of Land as Diesel Fuel Price Increase for 1977 period.

__ Related to diesel fuel price of $0.94 per gallon.

-- Related to diesel fuel price of $1.87 per gallon.

The value of the portion of output attributed to land then is the sum of marginal value products at various levels of acres. The results of Table 6.2 indicate that as diesel fuel price increases the value of the portion of output attributed to land would decrease. Also the average annual incomes to one acre of land decrease as diesel fuel prices increase, however, the decrease in annual income is not significant. This is because the farm operator use of diesel fuel per acre is relatively small compared to other inputs. Table 6.3 summarizes expected average income which is the average sum of annual income for 1977, 1978, and 1979 period. Also Table 6.3 gives the land values associated with different levels of diesel fuel prices.

Table 6.2 The Sensitivity of Return to Land to Change in Cost of Diesel Fuel, Other Factors Remain Constant.

Assumption A: 100 Percent Increase in Price of Diesel Fuel.

Period	Diesel Fuel Price, $/gal.	Total Portion of Output, $	Optimal Number of Acres	Income to One Acre of Land, $/ac.
1977	0.94	69,348.00	586	118.00
1978	0.98	61,116.00	708	86.00
1979	1.70	73,504.00	686	107.00

Assumption B: 200 Percent Increase in Price of Diesel Fuel.

Period	Diesel Fuel Price, $/gal.	Total Portion of Output, $	Optimal Number of Acres	Income to One Acre of Land, $/ac.
1977	1.87	60,885.00	539	112.95
1978	1.96	53,400.00	656	81.00
1979	3.70	59,533.50	611	97.00

Table 6.3 Expected Average Incomes, and Farmland Values Associated with Changes in Diesel Fuel Price.

Percentage Increase in Diesel Fuel Price	Expected Average Income, $/ac.	Estimated Land Value, $/ac.
000 %	120.00	2116.00
100 %	116.80	2060.00
200 %	109.00	1922.00

The results of Table 6.3 indicate that increase in price of diesel fuel decrease the expected average income for the current period, and the estimated land value. These results are consistent with the theory of partitioning output among inputs introduced in Chapter II. These results are also consistent with the illustrated example in Chapter III. In Chapter III, it was illustrated that as price of any input increases the portion of output attributed to land input decreases. This is in fact what the empirical results are showing. Therefore, it is concluded that as diesel fuel cost increases the value of farmland decreases.

Next, the effect of increase in fertilizer costs on land values are studied. This sensitivity analysis is accomplished by increasing the costs of Nitrogen, Phosphorus and Potassium simultaneously, while holding prices of other inputs constant. Changing the fertilizer costs in the objective function of the LP Model generate new series of shadow prices. It is expected that shadow prices decrease as fertilizer costs increase.

Table A:8 of the Appendix summarizes the model results generated by increase in fertilizer costs for the 1977-1979 period. These results are shadow prices, and profits associated with different levels of acres planted. Table 6.4 summarizes the results generated by increase in fertilizer costs for the 1977-1979 period. This Table presents the portion of revenue attributed to land, maximum number of acres planted, and annual average income associated with the sensitivity analysis. Table 6.5 presents the estimated expected average incomes and farmland values associated with increase in fertilizer costs.

Examination of the results presented in Table A:8 indicate that the shadow prices and profits associated with different levels of acres decrease as fertilizer costs increase.

A few observations can be made from Table 6.4. The portion of revenue attributed to agricultural land services decreases as fertilizer costs increase. Annual average income also decreases as fertilizer costs increase. Finally, maximum number of acres planted decreases as fertilizer costs increase. These results are consistent with the theory of partitioning output among inputs introduced in Chapter II, and the graphical and mathematical illustrations of Chapter III. The derivative of portion of output attributed to land input is negative with respect to price of fertilizer. This is shown in Chapter III, using a Cobb-Douglas production function. The results in Chapter III indicated that the portion of output attributed to land decreases as price of any other input increases.

Table 6.4 The Sensitivity of Return To Land to Change in Fertilizer Costs. Other Factor Prices Remain Constant.

Assumption A: 100 percent increase in fertilizer costs.

Period	Nitrogen Price $/lb.	Phosphorus Price $/lb.	Potassium Price $/lb.	Total Portion of Revenue to Land, $	Optimal Number of Acres	Income to One Acre of Land, $/ac.
1977	0.28	0.36	0.18	54,010.00	500	108.00
1978	0.28	0.38	0.18	57,263.00	618	76.00
1979	0.30	0.38	0.20	60,913.00	618	98.60

Assumption B: 200 percent increase in fertilizer costs.

Period	Nitrogen Price $/lb.	Phosphorus Price $/lb.	Potassium Price $/lb.	Total Portion of Revenue to Land, $	Optimal Number of Acres	Income to One Acre of Land, $/ac.
1977	0.56	0.72	0.36	29,880.00	366	81.60
1978	0.56	0.76	0.36	22,632.00	456	49.60
1979	0.60	0.76	0.40	32,748.00	460	71.00

Table 6.5 Expected Average Incomes, and Farmland Values Associated with Increase in Fertilizer Costs.

Perentage Increase in Fertilizer Costs	Expected Average Income, $/ac.	Estimated Land Values, $/ac.
000 %	120.00	2116.00
100 %	106.00	1869.00
200 %	76.00	1340.00

The results of Table 5.11 indicate that increase in fertilizer costs would decrease the expected average income for the current period, and hence land value significantly, because fertilizer use per acre is relatively greater than diesel fuel use per acre. Land value is estimated as expected average return divided by the pure time preference rate of 5.67 percent. These results are consistent with the theory of partitioning output among inputs introduced in Chapter II. In Chapter III, it was indicated that the portion of output attributed to land input decreases as price of any other input increases.

The last part of sensitivity analysis shows the effect of increase in wage rate of hired labor on land values. This sensitivity analysis is accomplished by increasing the wage rate of hired labor per hour in the objective function of the LP Model. Note that, the prices of other inputs remain constant throughout the sensitivity analysis. Changing the rate of hired labor generates new series of shadow prices only if additional labor is needed over and above the operator supply of labor.

Table A:9 of the Appendix summarizes the model results for shadow prices, and profits associated with different levels of acres and different wage rates of hired labor. Table 6.6 presents the results of portion of output attributed to land services, maximum number of acres planted,

and annual average income to one acre of land associated with different levels of hired labor wage rates. Table 6.7 gives the estimated expected average incomes, and land values associated with increase in wage rate of hired labor.

Examinations of the results presented in Table A:9 of the Appendix indicate that the shadow prices and profits associated with this sensitivity analysis decrease only when additional labor is hired above the amount of labor supplied by the operator.

A few observations can be made from Table 6.6. The portion of revenue attributed to land decreases as wage rate of hired labor increases. Annual average income also decreases as cost of labor increases. Finally, maximum number of acres planted decrease as labor cost increases.

Table 6.6 The Sensitivity of Return to Land to Change in Wage Rate of Hired Labor, Other Factor Prices Remain Constant.

Assumption A: 100 percent increase in wage rate of hired labor.

Period	Wage Rate of Hired Labor $/hour	Total Portion of Revenue to Land, $	Optimal Numner of Acres	Income to One Acre of Land, $/ac.
1977	7.60	70,829.00	593	119.00
1978	8.20	59,801.00	700	85.00
1979	10.60	73,970.00	690	107.00

Assumption B: 200 percent increase in wage rate of hired labor.

Period	Wage Rate of Hired Labor $/hour	Total Portion of Revenue to Land, $	Optimal Number of Acres	Income to One Acre of Land, $/ac.
1977	15.20	65,412.00	564	116.00
1978	16.40	52,054.00	654	79.60
1979	21.20	63,771.00	641	99.00

Table 6.7 Expected Average Incomes, and Farmland Values Associated with Increase in Wage Rate of Hired Labor, while Other Factor Costs Remain Constant.

Percentage Increase in Wage Rate of Hired Labor, $/hour	Expected Average Income, $/ac.	Estimated Land Values, $/ac.
000 %	120.00	2116.00
100 %	116.70	2058.00
200 %	110.70	1953.00

The results of Table 6.7 indicate that increase in wage rate of hired labor would decrease the expected average income to one acre of land and hence land value.

The results of the sensitivity analysis have indicated that the increase in input prices in fact, negatively affected the farmland value. The results of the sensitivity analysis are consistent with the theory of partitioning output among inputs. The results are also consistent with the mathematical illustrations of Chapter III.

Chapter Summary

This chapter contained the results of the change in the value of portion of output attributed to land services and land value as a result of change in diesel fuel costs, fertilizer costs, and wage rate of hired labor. These results indicated that as costs of factors of production increase, land values would decrease. The larger the amount of input used per acre the greater effect that input had on land value.

CHAPTER VII

SUMMARY AND CONCLUSIONS

The purchase of farmland is one of the most difficult investment decision confronting farm operators. Compared with other production inputs, land is purchased infrequently and usually in large size and involves long term financial obligation. When a tract of land is available for sale in the market, knowing, how much a farmer can afford to pay for that tract of land is very important. Valuing land is based on the income it can earn, now and in the future. The land value today reflects the present value of all future benefits. In the other words, land value today equals the present value of the income expected from land plus the present value of the price of land he receives when the land is sold at the end of the planning period. Determining future incomes to land, however, is difficult if not impossible. Expectation about future income stream is estimated using the immidiate past income stream assuming the best clue about the future is the immediate past. Since land is a durable that provides services to the production process for more than one period, the return to land input is determined as the portion of output attributed to it.

The purposes of this study were: first to review the theory of partitioning output among inputs. Second, to determine agricultural land values under the assumptions of 1) acquiring land services through ownership, 2) acquiring land services through renting, 3) acquiring machinery services through custom hiring, 4) acquiring machinery services through combination ownership and custom hiring. Lastly, to study the change in land value resulted from change in costs of factors of production.

Chapter II contained the theoretical framework for durable asset valuation. In this chapter the theory of partitioning output among such inputs as land and machinery was introduced.

The analysis of the output attributed to each factor of production, usually is found in the marginal productivity theory of factor demand. According to this theory, the share of each input is estimated by multiplying the units of input employed by the input price divided by the value of total output. However, the theory of marginal value productivity that stated each input paid its marginal value producd, holds only for homogeneous function of degree one as Euler's theorem demonstrated. Economically speaking, this implies that pure economic profits are zero which describes the long run equilibrium under pure competition.

Robison developed a new theorem on how to value the asset. This theorem stated that the value of any general process can be divided among inputs in such a way that the

output is just exhausted which each input receiving the value of its long run marginal value product. This new theorem stated that the relationship between inputs is specified, then the sum of the integrated partial derivatives of output to each input will equal the output of the firm. The proof of this theory was stated in Chapter II.

The relationship among inputs depends on the marginal value product of the inputs and their marginal value costs. Economists prescribe that the relationship among inputs should be such that, the ratios of their marginal value products-to their prices be equal for all inputs. Thus, to determine the relationship among inputs along the least cost combination curve, a physical production function must be specified. Therefore, the relationship among inputs is determined by maximizing the objective function of the firm.

When, a relationship among inputs and the marginal value product of land along the expansion path is derived, integrating the long run derived demand for land input sums the values of output for each level of the input land and at the optimal levels of other inputs. That is the sum of the integrals of the long-run derived demand curves for all inputs just exhaust the output. The advantage is that no restriction about the production function is imposed. While the Euler's theorem is implied only to homogeneous production of degree one.

Chapter III contains graphical and mathematical illustrations of the theory of determining the portion of output attributed to asset land. The first part of this chapter graphically illustrated the portion of output attributed to land asset under the assumptions of: 1) land is divisible in purchase and use, and 2) land is indivisible in purchase and divisible in use. Asset divisibility made it possible for the farm operator to acquire land in any quantity, and the farm can acquire any desired level of services. Asset indivisibility on the other hand, is available only in discrete size; while the services generated are divisible. It was illustrated that firm's expansion path under the assumption of asset divisibility is different from that under asset indivisibility. The second part of Chapter III indicated that the portion of output attributed to land input is greater under the assumption of machinery ownership than under custom hiring. This was illustrated using a Cobb-Douglas production function. The last part of this chapter examined the change in the portion of output attributed to land input as the result of change in factor prices, using a Cobb-Douglas production function. It was concluded that as price of factors of production increase, the portion of output attributed to land decreases. The theory was implemented using a linear programming technique. The operating characteristics of a representative farm were

incoporated into the model. These characteristics included such factors as land availability, potential crop outputs, labor availability and prices. The LP Model maximizes profits, subject to resource constraints at each time period for the three consequetive periods of 1977, 1978, and 1979. The marginal value products (shadow prices) of land for different levels of acres associated with different budget levels under the assumption that the operator acquired land services by renting was examined first. Then, the marginal value products were integrated along the expansion path to determine the portion of output attributed to land input given the assumption that land services were acquired by renting. The return to one acre of land then is the total portion of output attributed to land divided by the maximum number of acres. The expected average income to land was estimated as the average sum of the average incomes for 1977, 1978, and 1979 periods. This expected average income was used to estimate the agricultural land value for the current period using a simple capital budgeting formula. The estimated value of land under this assumption was 1,376 dollars.

The assumption that the operator acquired land services by ownership resulted in increasing marginal value products (shadow prices) of land associated with different levels of acres and optimal levels of other inputs. Under the same assumption the portion of output attributed to land services

increased. The annual average income to land and expected average income were greater given that land services were acquired by ownership than estimated under the assumption that land services were acquired through renting. Under land ownership, the estimated value of land was $2,116 compared with $1,376 under land rental arrangement.

Chapter V also contains the analysis of variation in output attributed to land and hence its value due to different manner of acquiring machinery services. It was assumed that machinery services were acquired by custom hiring first, and second under combination of ownership and custom hiring. Under the assumption of custom hiring, the expected average income and hence land value were less than those determined under the assumption that machinery services were acquired through combination of ownership and custom hiring.

The results of this analysis indicated that as the operator own more machinery services than custom hire, the expected average income and hence land value would increase. This was consistent with the theory that indicated as operator has more machinery at hand, the portion of output attributed to land input would increase.

The sensitivity analysis was the subject of Chapter VI. The sensitivity analysis was used to study the effect of increasing costs of such factors as: labor, fertilizer, and diesel fuel. This type of analysis was accomplished by changing prices and resource level at each time period in the model. The sensitivity analysis resulted in obtaining new series of marginal value products (shadow prices) of land associated with different levels of acres and optimal levels of other inputs.

The results of the sensitivity analysis indicated the following: 1) An increase in wage rate of hired labor has considerable negative effect on the value of output attributed to land services, maximum number of acres planted, and land value. 2) Increase in the costs of diesel fuel decreased the amount of revenue attributed to land services, maximum number of acres, and land value. 3) Increasing the costs of fertilizer would decrease the value of the output attributed to land services, maximum number of acres, and land value.

In summary it was concluded that:

1) Land is worth more to farmers who own their land than those who rent their land in the farm operation.
2) Land is worth more to farmers who have excess machinery at hand than those who custom hire their machinery services.
3) Land value is affected negatively by increase in costs of such factors as labor, diesel fuel,

and fertilizer.

Areas for Future Research

While the research in this dessertation contributes in its own right to part of the development of the theory of asset valuation, it is also provided the basis for continued development of the theory of investment.

The major areas for future research would involve relaxing the assumption of perfect knowledge. The optimality conditions and the value of agricultural land would be affected by the relaxation of this assumption.

Another area of new research is to build a simultaneous model of supply and demand for durable asset to determine its value at the equilibrium point. Development of simultaneous model for a risky asset is another area of future research.

APPENDIX

Table A.1 Available Operator Labor By Time Period 1/ 2/

Time Period	Available Operator (hours)
November - March	1,160
April	260
May	270
June	250
July	270
August	270
September	250
October	270
	3,000

1/ Kenneth Neal Wegenhoft, "An Economic Comparison of Major Farming Systems on Southern Michigan Cash-Grain Farms", Unpublished M.S. Thesis, M.S.U., 1970.

2/ Assumed that operator works 60 hours per week for 50 weeks per year with two weeks vacation in the November-March time period.

Table A.2 Estimated Labor Requirement Per Acre By Size of Enterprise for Corn.

Time Period	% of Total[1]	50 Acres	100 Acres	200 Acres	500 Acres	1000 Acres
November-March	0.21	1.365	1.260	1.197	1.155	1.134
April	0.19	1.235	1.140	1.083	1.043	1.026
May	0.32	2.080	1.920	1.824	1.760	1.728
June	0.14	0.910	0.840	0.798	0.770	0.756
July	----	-----	-----	-----	-----	-----
August	----	-----	-----	-----	-----	-----
September	----	-----	-----	-----	-----	-----
October	0.14	0.910	0.840	0.798	0.770	0.756
Total[2]	1.00	6.500	6.000	5.700	5.500	5.400

[1]Sherril B. Nott and Stephen B. Harsh. "User's Manual for Telplan Program 65:0(FO), M.S.U.

[2]Agricultural Economics Report No. 350, M.S.U., 1979.

Table A.3 Estimated Labor Requirement Per Acre By Size of Enterprise for Soybeans

Time Period	% of Total[1]	50 Acres	100 Acres	200 Acres	500 Acres	1000 Acres
November-March	----	-----	-----	-----	-----	-----
April	0.14	0.672	0.546	0.434	0.434	0.420
May	0.29	1.392	1.131	0.899	0.899	0.870
June	0.13	0.624	0.507	0.403	0.403	0.390
July	0.16	0.768	0.624	0.496	0.496	0.480
August	----	-----	-----	-----	-----	-----
September	0.14	0.672	0.546	0.434	0.434	0.420
October	0.14	0.672	0.546	0.434	0.434	0.420
Total[2]	1.00	4.800	3.900	3.100	3.100	3.000

[1]Sherril B. Nott and Stephen B. Harsh. "User's Manual for Telplan Program 65:0(FO), M.S.U.

[2]Agricultural Economics Report No. 350, M.S.U., 1979.

Table A.4 Variable Costs Per Acre for Production of Corn and Soybeans

Input	CORN			SOYBEANS		
	1977 1/	1978 2/	1979 3/	1977 1/	1978 2/	1979 3/
Repair and Maintenance	8.10	8.90	9.80	6.10	6.70	6.50
Limestone	0.80	0.88	0.80	0.80	0.88	0.80
Herbicide	10.80	11.90	12.40	13.00	14.30	14.30
Utilities	2.00	2.20	2.20	2.10	2.30	2.30
Harvest Trucking	10.30	11.30	11.30	4.30	4.70	4.70
Corn Drying and Marketing	12.30	13.50	14.00	----	----	----
Other Expenses	2.20	2.40	2.40	2.20	2.40	2.40
Total Variable Costs, $/acre	46.50	51.00	52.00	28.50	31.30	31.00

1/ Agricultural Economics Report No. 314 Enterprise Budgets Michigan, 1977.

2/ These datas are estimated by multiplying 1977 data by 1.1 factor to account for inflation.

3/ Agricultural Economics Report No. 350, Revised Michigan Crops and Livestock Estimated 1979 Budgets, January, 1979.

Table A.5 Diesel Fuel Requirements for Production of Corn and Soybeans.

Operations	Corn (gallon/acre)	Soybeans (gallon/acre)
Discking Stalks	0.56	0.56
Plowing	1.62	1.62
Harrowing	0.38	0.38
Amonia Application	1.15	1.15
Cultivation	0.92	----
Harvesting	0.69	0.69
Hauling	1.47	1.16
Others	0.28	0.12
	7.07	5.68

Source: James Allen Lehrmann, "Direct Economic Effects of Increased Energy Prices on Optimal Corn and Soybeans Production on Cash Grain Farms in Southeastern Michigan", Unpublished M.S. Thesis, M.S.U., 1976.

Table A.6 Estimated Operating Capital 1/

Time Period	Average Size of Farms	Land Rental Rate	Machinery Investment	Annual Operating Capital
1977 2/	490	39.00	57,033.00	57,000.00
1978 2/	460	37.00	71,419.00	71,000.00
1979 3/	600	44.00	78,000.00	78,000.00

1/ Operating capital is determined as equal to the machinery investment.

2/ Agricultural Economics Report No. 350, "Business Analysis Summary for Cash Grain Farms, 1978, Table 1.

3/ Data for 1979 was estimated from 1978 data using inflation rate of 10 percent and the size of farm increased by 8 percent.

Table A.7 Estimated Price Received and Paid by Farmers

	1977	1978	1979
Crops:			
Corn ($/bushel)	2.00	2.05	2.10
Soybean ($/bushel)	6.00	6.45	6.25
Fertilizers:			
Nitrogen ($/lb.)	0.14	0.14	0.15
Phosphorous ($/lb.)	0.18	0.19	0.19
Potassium ($/lb.)	0.09	0.09	0.10
Seeds:			
Corn ($/lb.)	0.75	0.80	0.88
Soybean ($/lb.)	0.17	0.18	0.17
Diesel Fuel ($/gal.):	0.468	0.49	0.85
Labor ($/hour) 1/	3.80	4.10	5.30
Cash Rental of Land $/acre	39.00	37.20	44.00
Capital, %	8.50	9.00	10.00

1/ The wage rates reported are the normal rates multiplied by a factor of 1.33. The factor allows for labor used during repairs, trips between fields, social security, and workmen's compensation costs. The normal wage rates of labor were $2.85, $3, and $4 for 1977, 1978, and 1979 periods, respectively.

Table A.8 Sensitivity of Marginal Value Products of land and Potential Incomes Associated with different levels of acres to change in Fertilizer Costs for 1977 period.

Nitrogen Price Range $0.28-0.56
Phosphorus Price Range $0.36-0.72
Potassium Price Range $0.18-0.36

Number of Acres	Operating Income Range, $	MVP Range $
100	10952.00- 8207.00	109.50-82.00
200	21904.00-16414.00	109.50-82.00
300	32856.00-24495.00	109.50-82.00
366 2/	39840.00-29828.00	105.80-80.00
400	43438.00- -	105.80 -
500 1/	54234.00 -	105.80 -

1/ The maximum number of acres planted given the price of nitrogen of $0.28, price of phosphorus of $0.36, and price of potassium of $0.18 per pound.

2/ The maximum number of acres planted given the price of nitrogen of $0.56, price of phosphorus of $0.72, and price of potassium of $0.36 per pound.

Table A.8 (Cont.) 1978 period.

Number of Acres	Operating Income Range, $	MVP Range $
	Nitrogen Price Range $0.28-0.56 Phosphorus Price Range $0.38-0.76 Potassium Price Range $0.18-0.36	
100	7972.00- 5105.00	79.70-51.00
200	15943.00-10210.00	79.70-51.00
300	23915.00-15315.00	79.70-51.00
400	31486.00-20020.00	75.70-47.00
456 2/	35725.00-22664.00	75.70-47.00
500	39056.00 -	75.70 -
600	46455.00 -	69.60 -
618 1/	47709.00 -	69.60 -

1/ The maximum number of acres planted given the price of nitrogen of $0.28, price of phosphorus of $0.38, and price of potassium of $0.18 per pound.

2/ The maximum number of acres planted given the price of nitrogen of $0.56, price of phosphorus of $0.76, and price of potassium of $0.36 per pound.

Table A.8 (Cont.) 1979 period.

	Nitrogen Price Range $0.30-0.60 Phosphorus Price Range $0.38-0.76 Potassium Price Range $0.20-0.40	
Number of Acres	Operating Income Range, $	MVP Range $
100	10279.00- 7310.00	102.80-73.00
200	20558.00-14617.00	102.80-73.00
300	30837.00-21927.00	102.80-73.00
400	40593.00-28713.00	97.50-67.80
460 2/	46446.00-32789.00	97.50-67.80
500	50348.00 -	97.50 -
600	59879.00 -	89.60 -
618 1/	61569.00 -	89.60 -

1/ This represents the maximum number of acres planted given a budget level of $46,000 and price of nitrogen of $0.30, price of phosphorus of $0.38, and price ofpotassium of $0.20 per pound.

2/ This represents the maximum number of acres planted given a budget level of $46,000 and price of nitrogen of $0.60, price of phosphorus of $0.76, and price of potassium of $0.40 per pound.

Table A. 9 Sensitivity of Marginal Value Products of Land and Potential Incomes Associated with different Level of Number of Acres to change in Costs of Hired Labor for 1977 period.

Wage Rate of Hired Labor Range, $/hr. 7.60-15.20

Number of Acres	Operating Income Range, $	MVP Range $
100	12325.00-12325.00	123.00-123.00
200	24649.00-24649.00	123.00-123.00
300	36974.00-36974.00	123.00-123.00
400	48559.00-47820.00	115.80-108.00
500	60143.00-58662.00	115.80-108.00
564 2/	67553.00-65505.00	115.80-108.00
593 1/	70910.00 -	115.80 -

1/ This represents the maximum number of acres planted given a budget level of $33,300 and wage rate of hired labor of $7.60 per hour.

2/ This represents the maximum number of acres planted given a budget level of $33,300 and wage rate of hired labor of $15.20 per hour.

Table A.9 (Cont.) 1978 period.

Wage Rate of Hired Labor Range, $/hr. 8.20-16.40		
Number of Acres	Operating Income Range, $	MVP Range $
100	9405.00- 9405.00	94.00-94.00
200	18810.00-18810.00	94.00-94.00
300	28215.00-28215.00	94.00-94.00
400	36819.00-36019.00	86.00-78.00
500	45421.00-43817.00	86.00-78.00
600	53679.00-50928.00	73.80-53.60
654 2/	57348.00-53350.00	73.80-53.60
700 1/	60473.00 -	66.00 -

1/ This represents the maximum number of acres planted given a budget level of $44,000 and wage rate of hired labor of $8.20 per hour.

2/ This represents the maximum number of acres planted given a budget level of $44,000 and wage rate of hired labor of $16.40 per hour.

Table A.9 (Cont.) 1979 period.

Wage Rate of Hired Labor Range, 10.60-21.20		
Number of Acres	Operating Income Range, $	MVP Range $
100	11764.00-11764.00	117.60-117.60
200	23528.00-23528.00	117.60-117.60
300	35292.00-35292.00	117.60-117.60
400	46011.00-44966.00	107.00- 96.70
500	56727.00-54634.00	107.00- 96.70
600	66995.00-63406.00	91.00- 64.90
641 2/	70458.00-65713.00	91.00- 64.90
690 1/	74597.00 -	91.00 -

1/ This represents the maximum number of acres planted given a budget level of $46,000 and wage rate of hired labor of $10.60 per hour.

2/ This represents the maximum number of acres planted given a budget level of $46,000 and wage rate of hired labor of $21.20 per hour.

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