AN INVENTORY COST-PRICING MODEL OF THE FIRM

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ABSTRACT

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by William H. Parks

The purpose of this study was to develop a model relating inventory and pricing decisions and showing their interdependence. The model assumes that, in order to achieve an optimum strategy, there should be a uniform return on all products. The model, which attempts to quantify these relationships and their implications, is based on observations made in a particular industry--the steel service center industry.

The major hypothesis is that traditional price theory does not relate to the actual problems existing in the industry today. A method of approximating the rewards and benefits of competitive markets in areas where traditional competitive behavior is inappropriate is needed. The model provides a framework within which the firm can operate independently and at the same time maximize profits.

The model constructed for this purpose builds on existing inventory and pricing formulas and combines them into an integrated model of the firm. The model was programmed for Fortran IV to test the effect of changes in costs and demand data on the optimal

strategies of the firm. The results support the hypothesis and indicate that, under certain assumptions, implementing the decision criteria in an actual firm would have beneficial results by returning a measure of competitiveness to the market for metal center industry goods without provoking a disastrous price warfare. In addition, the model promotes an economically tenable equilibrium rather than the essentially arbitrary one resulting from price leadership.

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William H. Parks

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CHAPTER I

INTRODUCTION

The major purpose of this research is to develop a model which relates inventory and pricing decisions. This model simulates a multi-product firm's pricing and inventory decisions in such a way that there is a uniform return on all products. The model links together adaptations of established inventory and pricing formulas so that they are interdependent.

The cost to a firm of not having an item in stock includes loss of markup. Because it affects the expected cost of the stockout, an inventory policy to defend against this loss from being out of stock partially depends upon the price charged. And pricing to achieve a target rate of return includes a return on inventory investment. This establishes substantial interdependence between optimal inventory and price policies.

Because uneven rates of return invite selective market entry, and pricing structures that encourage selective entry are inherently unstable and inhibit natural scale economies, the model, to encourage equilibrium, uses a uniform rate of return. The possibility of entry discourages excessive returns but allows adequate returns to existing firms.

> Each of the . . . (firms) . . . will appraise the condition of entry and, anticipating that entry may occur if price exceeds a given level, will regulate his price policies accordingly.[1]

A reasonable price level will result in industries, such as the one discussed here, in which entry is relatively easy.

The assumptions used in the model relate to an existing industry--the steel service center industry. Though observation of industry pricing practice does not generally reflect it, under these assumptions, including a uniform rate of return, a larger annual demand volume rationally leads to a lower average cost and price for items than a smaller demand. It will also be shown that establishing different relative priorities for the seller, the consumer, and the economy can, ceteris paribus, result in different prices. The carrying cost affects the inventory policy both directly in the inventory formula and indirectly through the investment cost effect in price, which reinforces effects upon inventory policy. Given certain assumptions about the pattern of demand during the period between the placing of an order and its arrival, it can be shown that, contrary to general opinion, larger orders tend to have higher inventory costs.

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[1]Joe S. Bain, Barriers to New Competition (Cambridge: Harvard University Press, 1965), p. 4.

This reduces the size of price differentials for smaller orders that can be justified on the basis of handling costs. When demand is complex, consisting of both large and small orders, the size ratio between large and small orders affects inventory costs and prices.

Steel Service Center Industry

Steel service centers purchase steel in large quantities from steel mills and carry it in inventory until their customers purchase it. Because conditions are about the same for any other metals the service centers may carry, this paper will occasionally group all centers together and refer to them as metal center industries.

The metal center industry provides an excellent basis for constructing a model for inventory-pricing decisions for three reasons:

1. Since over 50 per cent of the total investment in the average metal center is in inventory, inventory policy is of prime importance in determining the firm's overall policy.

2. When service centers are out of stock, they usually purchase material from competitors at retail and resell it to their customers as if it were part of their own stock. This simplifies the calculations necessary to determine the cost of being out of stock. It also justifies the use of the lost margin as the major component

of this cost and hence the reasonableness of the interdependence of price and inventory policy.

3. Because industry demand has such a low price elasticity within a given range, it may reasonably be taken as constant.

Methodology

In order to give the reader sufficient background to judge the reasonableness of the inventory cost-pricing model's simplifying and limiting assumptions, an extensive description and analysis of the metal center industry is included in this paper. The supply and demand characteristics are explored and related to the necessary inputs of the inventory cost-pricing model. The information was gathered by field trips through more than thirty plants across the country plus interviews with the management of these firms.

The model itself has three basic components: (1) a pricing formula which, for a given rate of return, selects the order quantity which will allow the lowest price; (2) a series of formulas to calculate the probability and financial consequences of running out of stock, and to compare them with the costs of avoiding stockouts; and (3) a formula for choosing the inventory policy which will yield the lowest total cost. The three formulas are linked together to compute price and inventory decisions; the output from each formula is used as the input for the next. The cycle is repeated until there are no more changes.

Implicit in the model's character is the assumption that in a multi-product firm it is undesirable to allocate all costs to individual items. Also, several specific inventory assumptions about the probability of receiving orders will be examined in a later chapter. To simplify the model, the price computed is, depending on the input costs, either a base price or a price for some average size order. Price setting by the firm implies a market structure which allows prices that are not competitively determined.

The Non-Competitive Equilibrium

The incidence of administered prices in the United States has received considerable attention.[2] The reader is referred to "Administered Prices: A Compendium on Public Policy,"[3] for a thorough discussion of the problems involved. The theoretical implications of administered prices or "prices resulting from a quasi-agreement"[4]

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[2] For a recent treatise on this subject see: A. A. Berle, "The Impact of the Corporation on Classical Economy," Quarterly Journal of Economics, February, 1965, pp. 25-40.

[3] Subcommittee on Anti-Trust and Monopoly of Committee on the Judiciary, U. S. Senate, 88th Congress, 1963.

[4] William Fellner, Competition Among the Few (New York: Alfred A. Knopf, 1949).

were first systematically tested by Chamberlin[5] and Robinson.[6] A current unresolved conflict has erupted in the clash between the federal government and several industries over pricing policies, the government insisting that it be included among the forces determining the price level. This has been tantamount to a veto power in some industries. Thus the government has implicitly recognized the non-competitive equilibrium and used its extra-legal powers to influence the equilibrium level.

In the steel service center industries prices are not determined competitively for two main reasons: (1) the demand for steel is very inelastic, and (2) marginal cost curves are not upward sloping within the relevant range.

Inelastic Demand

The inelasticity of the demand curve for steel was documented statistically by Theodore Yntema in his study for U. S. Steel presented to the T. N. E. C. The study yielded an:

> . . . elasticity of demand . . . of .3 to .4
> The evidence and argument adduced in the

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[5]Edward Hastings Chamberlin, The Theory of Monopolistic Competition, 7th ed. (Cambridge: Harvard University Press, 1960).

[6]Joan Robinson, The Economics of Imperfect Competition, 13th ed. (London: MacMillan & Co., 1964).

> preceding pages of this paper support the conclusion that such a value--or one even lower--for the elasticity of demand for steel is not a statistical happenstance, but a reality.[7]

This analysis was supported by Mr. Paradiso before the Joint Economic Committee:

> All studies that have been made that I know of starting . . . at the time of the T. N. E. C. hearings . . . showed that demand for steel was inelastic. And in studies made since, we have not found any correlation between demand and price . . . Demand is inelastic within the range of price variation that we have experienced.[8]

If, in fact, demand is price-inelastic (and this is certainly true of the metal center industry) then one can expect to find non-competitive pricing wherever the opportunity presents itself. This is not due to unethical or irrational conduct by the firm, but because, as Hirsh notes, rational conduct of the firm requires abandoning the assumptions of competitive conditions.

> Conventional pricing theory . . . appears to break down under inelastic demand

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[7]U. S. Steel Corporation T. N. E. C. Papers, Vol. 1, Pamplet No. 5, A Statistical Analysis of the Demand for Steel, 1919-38. (Introduced into the record of the T. N. E. C. as Exhibit No. 1411) (New York: United States Steel Corporation, 1949), p. 28.

[8]U. S. Congress, Economic Joint Committee, Hearings, Steel Prices, Unit Costs, Profits, and Foreign Competition, 88th Congress, 1st Session, 1963, p. 296.

> conditions . . . The profit maximizing conditions of conventional pricing theory implicitly exclude pricing under inelastic demand conditions.[9]

Barloon supports the rationality of the firm's conduct:

> The literature dealing with oligopoly in this (steel) and other industries has spelled out the influences deterring quotations below published levels in times of deficient demand. In the iron and steel industry these influences are all present, their strength being heightened by the absence of product differentiation and by the highly inelastic demand for the product.[10]

Zvi Griliches' statement on rationality is also applicable to the firm's conduct:

> In economics it has been found useful to assume that people know what they are doing even if it may seem strange at first sight. If they do not know, they are probably learning.[11]

The factors influencing demand in the metal center industry are generally much like those in the steel industry proper, with consumer sales generally the most price elastic of steel sales. But

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[9] W. Z. Hirsch, "On the Phenomenon of Inelastic Demand," Southern Economic Journal, July, 1951, p. 36.

[10] Marvin J. Barloon, "Institutional Foundations of Pricing Policy in the Steel Industry," Business History Review, September, 1954, p. 230.

[11] Zvi Griliches, "Are Farmers Rational?" Journal of Political Economy, February, 1960, p. 71.

since even relatively fewer service center sales than mill sales are to manufacturers of consumer goods, and the metal center industry supplies most of the maintenance and repair industry which logically can be assumed to have lower price elasticities than steel as a whole, it is reasonable to conclude that, not counting the customers' changes between mill and service center, the market for metal center's products is less price elastic than that for the steel industry as a whole. Although the aggregate demand for steel is extremely price inelastic, individual firms and sectors of the market may face very high price elasticities in the demand for steel.

Declining Marginal Costs

Many economists discuss declining cost curves from the standpoint of welfare economics as a problem of determining the best allocation of subsidies or side payments to achieve maximum welfare. The possibility of continuously declining costs is often dismissed in the way Oort dismisses it:

> Since the marginal equilibrium is determined by demand as well as by cost conditions, a study of the causes of decreasing cost would, except in the implausible event that the cost function is negatively inclined throughout its entire range, require an analysis of demand as much as of cost. Assuming that the average cost has the familiar U shape, decreasing

> costs are due as much to a 'deficiency' of demand as to the behavior of cost.[12]

But this analysis is no longer generally valid because the familiar "U" shape is found in fewer industries today. Recent technological advances and the revolution in information systems and data handling have made these "implausible" cost curves a reality in many industries. Bain, in fact, casually dismisses the familiar "U" shaped cost curve:

> A fourth potential determinant, neglected here, is whether or not established firms will ever encounter diseconomies of large scale; i.e., rising unit costs because the firm exceeds a certain size. We neglect this possibility as improbable, and will assume approximately constant unit costs as firm size exceeds the minimum necessary for lowest costs.[13]

Inventory systems are particularly open to economies of scale as a simple example will illustrate. A firm which has an average demand of one unit per year must inventory at least one unit in order to be prepared for its customers. Its investment is turned over only once a year. A firm which expects 100 orders per year may only need an average inventory of 10 units and thus gets 10 times as many sales dollars for each dollar of inventory investment.

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[12] C. J. Oort, Decreasing Costs as a Problem of Welfare Economics (Amsterdam: Druhkerij, 1948), p. 111 (italics added).

[13] Bain, op. cit., p. 20.

Metal service centers, partially because of their major investment in inventories, are especially open to the economies of scale--usually over 40 per cent of their total investment is in inventories. The large inventory investment tends to override any increasing costs and promote declining total and marginal cost curves. A downward sloping MC, or even one without a pronounced upward slope, promotes non-competitive pricing.

Usefulness of the Model

This paper indicates that use of an inventory-cost-pricing model will return a measure of competitiveness to the market for metal center industry goods without provoking disastrous price warfare, and will promote an economical tenable equilibrium rather than the essentially arbitrary one resulting from price leadership. The author's analysis of the industry indicates that price competition in fast-moving, high-volume goods, through either established or marginal centers, precipitates price disequilibrium or warfare, and that fear of this occurrence promotes price rigidities. Furthermore, marking up all items by the same dollar or percentage amount encourages excessive entry. A fast-moving item has a much higher return on investment than a slow-moving item bearing the same markup, and firms may enter the market at low cost by selling only the fast-moving items. This study also underlines the important interdependence of inventory and pricing policies.

The description of the industry is divided into three main sections. Chapter II discusses the cost characteristics of the firm and the relationship of various costs to volume and size of the firm. The factors involved in firm and industry demand, followed by specific demand examples, are considered in Chapter III with auxiliary descriptions in the appendices. Chapter IV focuses on variations in short-term demand for an individual item and advances some solutions to computing their probability.

Chapter V is a general discussion of inventory models and specific adaptations for the metal center industry. Several firms have instigated quantitative pricing methods; their formulas are presented in Chapter VI.

The heart of the study is Chapter VII. Here special pricing and inventory formulas are integrated into a single complete model. Under its limiting assumptions the model can simultaneously solve the formulas for prices which yield a uniform return on investment and the formulas for the optimal inventory policy associated with that return. Chapter VIII lists some results obtained and discusses a few of their implications. The conclusion, Chapter IX, summarizes the research and its general implications.

CHAPTER II

COST CHARACTERISTICS OF THE FIRM

Introduction

This chapter has three main purposes: (1) to describe in general the cost characteristics of the metals center industry and to analyze those relevant to the inventory cost-pricing model developed in Chapter VII, (2) to show the form of the cost information necessary to the operation of the inventory cost-pricing model, and (3) to demonstrate the absence of an upward sloping marginal or variable cost curve by establishing the predominance of level and declining marginal cost curves. In addition, because the superiority of results produced by the cost-pricing model over non-competitive pricing depends upon increasing economies of scale, relevant scale economies are noted wherever they appear.

Cost incurred by the firm can be classified in several ways, for example, as controllable and non-controllable, fixed and variable, and so on. However, they seldom fit precisely into a single category and must be roughly classified by their dominant characteristics. The separation between controllable costs is somewhat artificial and is used primarily for managerial decision making. Generally, in the final analysis, the only costs really uncontrollable are those which originate outside the firm. Even then, the term "uncontrollable" is misleading because the firm, while not always able to control the purchase cost of an item, may be able to change its utilization within the firm to effect lower costs (this was done by the railroads, for instance, who tore out tracks in order to lower their property taxes).[1] Or, as some firms have done, they may change their purchasing habits to effect savings by ordering in larger quantities. McMullen, in fact, rejects the term "uncontrollable", though this may be partly for morale reasons.

> It should be noted that the word semicontrollable rather than noncontrollable is used. If a company is desirous of an enthusiastic budget program, it will not admit that there are very many expenses that are not controlled. All expenses reflect management decisions to some degree.[2]

The preferred separation for this chapter is into fixed and variable expenses. Whether or not a cost is fixed is determined by its behavior under changing volume conditions within a specified time period. Most costs are fixed in the short run, which can be defined

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[1]"Rail Tax Fight," Wall Street Journal, November, 8, 1963, p. 1.

[2]Kenneth McMullen, "Effective Overhead Budgeting," NAA Bulletin, September, 1961, p. 50.

as encompassing time enough to change all factors of production except plant, but variable in the long run. This type of definition is not useful for the cost analysis used here since this chapter is concerned with how costs vary and the circumstances which change the definition of a cost from fixed to variable and vice versa.

Accountants assume that within the relevant range of firms productive capacity costs remain constant. This is evidenced by the fact that accounting cost curves are straight lines. However, it is argued here that to assume constant variable costs per unit is an over-simplification because variable costs per unit change. The economist shows this with the use of curved total cost lines. The economist's curves have a nearly straight section which corresponds to the accountant's relevant range.

To achieve an intelligent discussion of cost variations two limitations must be observed: (1) the allowable length of time for adjustment of plant, etc.; and (2) the amount of volume variation considered relevant. Consequently, in order that cost variations be discussed intelligently, some limit must be set on the volume changes which cause the cost variability.

Extensive interviews lead the author to believe that, generally speaking, volume increases of 5 to 20 per cent per year over present operations produce little distortion of variable costs because the company has time to adjust to the new volume. Increases over 50 per cent per year, however, almost certainly produce distortion and strains which show in marginal or variable costs. The problem associated with volume changes and time periods are often more closely related to an adaptation process than to changing physical plant characteristics.

Because service centers sell a variety of products in different markets, several external factors can simultaneously influence a firm's total volume. This diffusion of demand makes sales trends more stable than is usual in the capital goods industry. At times demand increases substantially in one product, but this is easily handled by diverting resources from other lines. Internal factors which add additional lines or increase selling effort also influence volume, but they are often preceded by increased capacity. Management is naturally more skilled at predicting internally-caused volume changes but must attempt to foresee external conditions as well.

The response to these forces depends on the manager's assessment of the permanency of these external or internal changes. If he believes the change to be semi-permanent or permanent, he will generally expand fixed assets to the extent practical by adding plant or equipment. When changes promise to be temporary, he is likely to optimize his actions by changing only variable factors.

Additional points pertinent to an understanding of the inventory cost-pricing model are discussed in greater detail in their respective cost groups.

Types of Costs Encountered

The costs of a steel service center may be roughly divided into eight groups:

1. Material costs
2. Costs of physical movement
3. Processing costs
4. Costs of obtaining orders
5. Information and data processing costs
6. Capability costs
7. Costs of possession
8. Unallocated costs

Material Costs

The price of most steel is quoted per hundred pounds (cwt.). Some specialty steels, however, are quoted by the pound. The choice is merely traditional--more often the higher priced steels are quoted by the pound. The mill's prices are built up from a base price by addition of extra charges for various specifications. These charges plus the applicable freight cost constitute the service center's material cost.

Analysis Extra

The charge for varying the alloy and purity percentage of the steel is called an "analysis extra" and varies from a few cents to several dollars per hundred weight. Alloying the steel can impart to the product such characteristics as machinability, strength, resistence to abrasion, etc. The alloy content for a single grade of analysis varies within limits; therefore, each batch or "heat" must be identified.

Size, Shape, and Finish Extras

Size extras are supposedly determined by the cost of changing steel billets or slabs into certain sizes and shapes. The extra varies according to the size, shape, or gauge of the finished steel. The more difficult the shaping or the more time involved, the higher the charge. The final finish extra is charged for the added work or care necessary. The total of all the listed extras make up the base quantity mill price.

Quantity Extras

The quantity extra is charged when the amount of steel ordered is less than the base amount. These classifications vary between products. Smaller quantities purchased at the mill are charged a quantity extra which ranges from a few cents to a dollar or more depending on the weight. This extra is charged per item on the basis of the total item weight ordered for delivery at one time to one destination. In a few instances, customers are allowed to add several items to obtain a higher weight class.

Material costs to the firms are obviously variable according to volume. What is not so obvious is that material costs are not necessarily directly variable with volume. For a single item, the larger the volume (demand), the larger the optimum order quantity. This reduces the quantity extras, and hence lowers the price per unit resulting in a less than 100 per cent variable cost. The actual cost behavior depends upon the cause of volume charges. If the firm expands volume by increasing the number of items sold, those added items which are purchased in smaller quantities could increase the cost of materials more than the increase in volume. Therefore, it is possible for the material cost to increase at an increasing rate. More generally though, due to a stronger competitive position or an expanding economy, there is increased volume without a change in product mix and this increased volume lowers material costs per hundred weight. Material cost is, therefore, subject to economies of scale.

The difference in material costs caused purely by volume are a small portion of the total material costs, but, because the material cost predominates in a total cost array, the impact upon overall cost behavior is considerable.

Freight Charges

The basic mill price, plus the applicable quantity extras, is the charge that the service center pays the steel mill directly. In addition, the service center pays freight charges from the mill to the service center, and so tries to order sufficient quantities from the mill so that the order makes up a complete truck or carload, even though an individual item does not. Since the abandonment of the basing point system, many mills equalize freight charges with the closest producing mill. Freight charges should be included in the material cost. If enough different items are ordered from a single source to receive the lowest freight rate, the freight rate is independent of the amount of any single item ordered, and a constant amount per hundred pounds should be added. If there are not enough items, the freight charge added will vary between quantities. The inventory cost-pricing model utilizes an array of quantities and their corresponding prices, including all the appropriate extras.

Thus there will be an appropriate price if 6,000 pounds of steel are ordered and another, lower price, if 20,000 pounds are ordered.

Costs of Physical Movement

The costs of physical movement are the workers' wages and fringe benefits and use-related expenses of capital equipment. Supervision and space costs are not directly related and thus should not be accounted for in this way. Although all service center labor costs are partially related to volume, it is difficult to accurately classify these costs according to the amount of increase or decrease of varying kinds of volume changes. However, the expenses of moving service center steel can be broken into three cost categories: (a) receiving and stocking; (b) order picking, assembling, and loading; and (c) delivery. Receiving and stocking costs are a component of the total inventory investment because they are incurred when the material arrives in the warehouse. For this reason, they should be added to the total cost of inventory material. The other costs are incurred when the metal is sold, and should, therefore, be included in the cost-pricing, but not the inventory, section of the model.

Receiving and Stocking Costs

Receiving and stocking consist of unloading the trucks or rail-cars and storing the steel which is shipped from the mill in lifts, bundles, skids, and boxes. The actual unloading is usually done with overhead cranes, though at times lift trucks are used. The material is either unloaded and left near the dock or moved directly to a storage area, depending upon the material and the availability of space.

It is more efficient to handle the material in the form that it arrives at the service center, rather than break the bulk package and stock singly. But, since service center customers seldom order in full lifts of 5,000 pounds, the service center must either break the package when stocking or when order picking. In some installations, the material must be stocked by hand. This procedure, which may require unloading the material by crane onto carts, and then by hand into racks, is common for cold finished bar installations.

The cost per hundred weight to unload and stock steel is usually only a few cents. The cost can vary considerably, however, and may range from one cent or less per hundred weight up to twenty cents or more per hundred weight. The objective determinants of receiving and stocking cost differences are wage rates, types of material, and physical layout and equipment of individual plants. Given these, the efficiency of material handling depends upon the scheduling ability of management and the training and cooperation of workers. The receiving and stocking efficiency is measured by comparing the average time necessary to unload and shelve a given amount of identical steel; but since no two installations are identical, one cannot determine the relative influence of physical and human differences.

Though cranes are often designed to lift 5 or 10 tons, most orders are broken into 5,000 pound lifts. Thus the number of lifts is not increased substantially when relatively smaller lots are ordered. Given the existing equipment, the marginal receiving cost may be considered 100 per cent variable because the truck or car is usually loaded to capacity with one or more orders and the only extra cost occurs because the material must be carried to different places. The cost of stock steel varies between lines and warehouses. The responsiveness of costs to changes in volume also varies widely. Short-term increases in volume sometimes increase costs more than proportionately due to the lack of room for all material in the normal storage area. The excess, stored in access or work areas, reduces the efficiency of the work force. Stocking costs may or may not increase proportionately if the average amount purchased is increased from 6,000 to 10,000 pounds. If they do not, the costs are semi-fixed and probably subject to economies of scale. If they do increase, the costs are completely variable and any economies of scale must come from availability or suitability of investment in automated or labor-saving machinery.

Order Picking Costs

Finding the material for a customer's order and bringing it to the required area for shipment or processing is called order picking. Although these costs vary with the amount and diversity of the customer's order and so are of importance in computing quantity extras, for the inventory cost-pricing model, which ignores quantity extras, order picking costs are a set amount per hundred pounds. This set amount varies with the installation's size and efficiency since larger installations are subject to some economies of scale.

Delivery Costs

If the volume of business changes, what are the likely changes in delivery costs? Since delivery costs increase with added volume, the problem is to decide whether the increase is greater than, the same as, or less than the volume increase; and, therefore, whether the per unit costs increase, decrease, or remain the same. When service centers deliver orders with their own or leased trucks, the variance of delivery costs is chiefly determined by three factors:

(1) the traveling time between stops, (2) the weight of the orders, and (3) the number of items in the order.

Traveling time between stops.--The delivery cost per hundred weight is likely to increase if the volume increase is due to an increase in the number of items per order or if added customers in the existing market result in an increase in customer density. However, if volume is increased by cultivating customers beyond the normal territorial range, there is usually a decrease in customer density, resulting in increased basic travel time, and, therefore, an increase in delivery costs per order.

Weight of the orders.--Along with the basic unloading time, the weight of the orders constitutes a factor in the costs of unloading the material. Although the basic unloading time is spent in getting a spot at the dock, presenting bills of lading, etc., the time spent unloading the order is slightly sensitive to weight changes. From 100 pounds, the point at which mechanical aides are likely to be used to unload, up to the load capacity of the crane, the time involved is relatively constant. However, as the tonnage, or the variety of items increases above this point, the time required, and therefore the unloading cost, increases. Relatively homogeneous items such as steel bars may be combined for a single lift by the crane, and hence the time may not change significantly with variety of items.

<u>Number of items in the order.</u>--Increasing volume due to an expanding economy will also result in increased density from either more customer orders or increased number of size of items per order or both which in turn decreases per unit costs. Driver wage expectations may also influence costs.[3]

Generally, decreased volumes will result in higher per unit costs due to less effective utilization of men and equipment. Conversely, increased volume will usually decrease per unit delivery costs. It follows that economies of scale will predominate in the long run.

Though the computation of transportation costs is not directly utilized in the model, it is often a major component in the cost of being out of stock. This is true because the sequence of movements necessary to pick up an item from a computer are just the reverse of the movements necessary to deliver it to a customer. Sufficient experience is not, in most situations, available to compute the pick-up cost, but ample delivery cost records are available, and these can easily be modified to give the needed information.

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[3]At one plant drivers, if not given a full load, were "held up by traffic" until a ten-hour day had been put in. Thus it became necessary to load the trucks for a certain number of overtime hours.

Processing Costs

Processing costs are not an input of the inventory cost-pricing model, but a familiarity with these costs helps the reader to understand the characteristics of metals centers. Furthermore, the scale characteristics of processing costs are important because the costs are a relatively large part of the overall costs. The cost of processing consists of extra movement costs, set up costs, processing labor costs, and cost of machines and supplies.

Material which is to be processed must be handled more often than unprocessed material. Steel is brought to the processing area, processed, and then picked up and taken to the loading area. This clearly doubles the necessary handling from storage to dock. Also, the set up time necessary varies with the job and the operation. Generally, each piece of a different size or shape requires a new set up. This may take from a few seconds to several minutes. Maintenance expenses such as saw blades, resharpening tools, and oxygen which vary directly with the use of the machine, have a constant per unit cost. The actual processing time varies directly with the number of items or cuts of material involved.

Machinery costs, being sunk, are not technically relevant to variable costing decisions. However, a related question is how soon existing equipment will have to be replaced. These fixed prior costs may be charged against current production in which case the cost should be constant per unit with volume changes, or may be charged over a given time period, thus showing a lower cost per unit with fuller utilization of the machinery. These opposing methods really center around a fundamental, but perhaps unanswerable, question: Will the machine wear out or become obsolete first? Since a machine tends to do both at the same time, one can only say that the greater the impact of obsolescence, the more per unit costs will decline with fuller utilization.

Most processing costs are variable, though utilization rates may slightly reduce their variability. In spite of this, economies of scale are not particularly significant.

Costs of Obtaining Orders

The inventory cost-pricing model assumes that the overall costs of obtaining orders are independent of the prices of particular items and, therefore, regards these costs, with one exception, as part of the costs of overhead rather than as inputs of the model itself. Salesmen in service centers are divided into two groups: outside salesmen who call on customers and inside salesmen who receive telephone orders from customers.

Outside salesmen solicit orders and, more importantly, keep the firm's image fresh in the customer's mind. They remind him of the goods and services the firm has available and inform the customer of current developments in operation and techniques. Unlike most salesmen, the service center's outside salesmen take only 10 to 20 per cent of the firm's orders themselves. The service center depends upon the customer to call in the majority of his orders because when the outside salesmen make their calls, they seldom have an order waiting for them. Though he seldom leaves his desk, the inside salesman may write 80 per cent of the orders. During the rush period, other personnel may be pressed into service to take phone orders.

Other things being equal, outside selling costs are insensitive to volume changes, unless the salesmen are paid a commission. With the exception of commissions, which may be a cost input of the model, the outside selling cost is fixed and per unit costs decrease as volume increases. Outside selling effort is, however, one of the factors affecting volume. But market stimulation, by requiring more salesmen, may result in higher per unit selling costs, especially if obtained by pushing sales in areas of lower customer density.

Volume changes may result from changes in: (1) the number of orders, (2) the number of different items on each order, (3) the average order size, or (4) a combination of these three. An increase in the number of orders increases the need for inside salesmen, and an increase in the number of items per order may slightly increase this need, but an increase in order size will not increase the need for inside salesmen. Statistically, higher sales lower the variance around the mean number of telephone calls received. The tendency to staff for peak periods could decrease per unit costs as volume increases. Unless, as previously mentioned, greater than proportional sales effort is used to increase the volume, per unit costs of obtaining orders will decrease or remain constant as volume increases.

Costs of Data Processing and Information

The costs of gathering information and processing data are complex; some variable and others fixed. Although some of the costs may be explicitly identified as variable by item, and so used as an input of the inventory cost-pricing model, the majority must be included in desired contribution to overhead. When an order is received, the salesman writes it down on a form or blank and then sends it to be processed. Although the procedure varies, the following steps must be taken: (1) checking the customer's credit, (2) checking the availability of the material ordered and deducting from inventory records the amount sold, (3) noting on the warehouse's order copy the place where the material is stored, (4) sending a copy of the order to the warehouse, (5) posting to the customer's account, and (6) billing the customer.

If volume changes because of an increase in new customer orders, the cost of checking the customer's credit is likely to increase proportionately. But if the increased volume is caused by an increase in weight per item or an increase in number of items per order, then the cost remains fixed and results in a decline in the cost per unit. The effects on the costs of posting the order to the account and billing the customer are similarly related to the reasons for increased volume, though billing is also partially sensitive to the number of different items per order.

Checking material availability, posting inventory reductions, and noting the storage location on the warehouse copy are all semi-variable costs. The cost of sending orders to the warehouse is usually fixed and not volume sensitive.

The cost of obtaining and processing data depends upon the causes of volume changes. However, knowledge of these causes and of the relative variability of the gathering and processing steps functions, one must also determine the relative importance of each step in the cost structure. It follows that the smaller the percentage of change due to an increase in the number of orders, the less costs will vary and thus the more per unit costs will decrease.

Costs of Capability

A metals center has made certain investments to acquire its capability of operating. These include inventory, warehouse and office space, material handling and processing equipment, delivery trucks, etc. As a consequence of doing business, it has an investment in accounts receivable and, in order to facilitate continuing operation, an investment in cash. The center needs warehouse space and machinery in order to function; and, though many costs cannot be attributed to any individual items, and so are not direct input of the inventory cost-pricing model, and since item stocking choices eventually affect plant investment decisions whether or not the warehouse is presently close to capacity, some space costs may legitimately become inputs of the model.

In metal centers warehouse costs can be related to two main areas: (1) inventory storage, and (2) materials handling.

The level of warehouse investment needed is determined by the level of sales and the necessary inventories. In other words, warehouse investment is a function both of stocks and flow. Machinery and equipment investment is a function of flows. Both warehouse and machinery and equipment are fixed costs and thus are only related to changes in inventory and sales in the long run. In the short run, warehouse costs are not only fixed, but sunk, and any relation to day-to-day costs is not of practical significance. In the long run, warehouse investment, though influenced by sales, is not necessarily proportional, but is, in fact, a step function. Despite the slight dissimilarity of the industries, Alpert's analysis still seems applicable: "In the process industries it has been found that the investment cost per unit of capacity decreases as the size of the plant increases."[4]

It is obvious that a part of warehouse plant costs will be related to inventories directly; this portion is assumed in the long run to vary directly with inventory investment. The area used for material handling can be taken as varying with sales volume, but Mr. Alpert's analysis masks the tendency of large installations to have bigger access lanes due to the longer distances involved. However, these larger lanes may give greater efficiencies for high volume installations.

Office space costs also have some scale economies, but they are overshadowed by the availability of space and management's perquisites. Thus, it is not unusual to find that although two firms do approximately the same volume, one may have two to four times as much office space as the other, so that though it seems certain that the proportionate area needed to generate and process a given amount

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[4]S. B. Alpert, "Economy of Scale in the Metal Removal Industry," Journal of Industrial Economy, July, 1959, p. 175.

of sales decreases as the size of the firm increases, the particular economies have little relevance to actual costs.

The accounts receivable and cash costs seem to be closely related, but where the receivables cost can be directly attributed to a specific sale, the cost of cash cannot readily be so attributed and is only generally related to volume. Therefore, the inventory cost-pricing model specifically includes receivables costs but excludes cash costs.

Cost of Possession or Carrying Cost

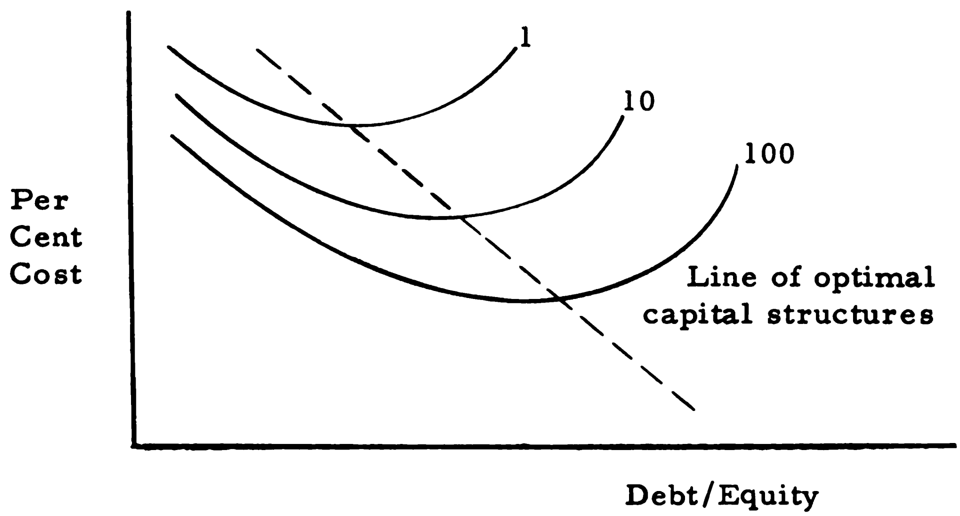
The cost of carrying inventories is perhaps the single most important cost input in the inventory cost-pricing model. It is balanced against the stockout cost, essentially an opportunity cost, which is considered later.

The firm's investment in inventories consists of capital costs and the costs of obsolescence, deterioration, taxes, and insurance. These costs vary with the number of units invested in inventory. The optimal inventory level is a function of the volume of sales. Assuming the sales mix remains constant, the ratio of inventory necessary to sales decreases as sales increase, even though the actual level of inventory increases. Thus, sustained increases in volume reduce the inventory costs per unit of sales.

This is true partly because demand for steel service center products is a form of compound poisson, which has been effectively approximated by the gamma distribution. In both distributions the greater the mean, the higher the concentration around the mean and hence the lower the multiple of that mean necessary to contain an acceptable per cent of the demand. This subject is considered in more detail in Chapter IV.

At first glance, the money cost of inventory investment, or the cost of capital required for inventory, would seem to be completely variable. However, for several reasons larger firms have lower costs of capital. First, the economies of scale apply to financing and so a large firm is likely to be able to issue securities, either debt or equity, at a lower cost. Second, scales alone may lead to greater acceptance of the firm and hence lower costs through stock sales at a higher price earnings ratio or through debt placement at a lower effective interest rate. Third, the greater safety purported to be found in large firms may well allow larger proportions of debt. These factors are represented diagramatically in Figure 1 which shows hypothetical costs of capital for firms with $1 million, $10 million, and $100 million total capitalization. The effect of size on a firm has been discussed by Barges[5] and others. Implicit in the arguments is that, ceteris paribus, larger firms will have an optimal capital structure with a higher debt to equity ratio than smaller firms. Although this higher ratio is not necessary to a decreasing cost of capital as the firm increases, it is certainly consistent with it.

FIGURE 1



Other per unit inventory costs either decrease as inventories increase or are directly related to inventory levels. Therefore, marginal inventory costs decline as sales increase even more than the decreasing ratio of inventories to sales would suggest.

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[5] Alexander Barges, The Effect of Capital Structure on the Cost of Capital (Englewood Cliffs, N. J.: Prentice-Hall, 1963), pp. 24-31.

Unallocated Costs

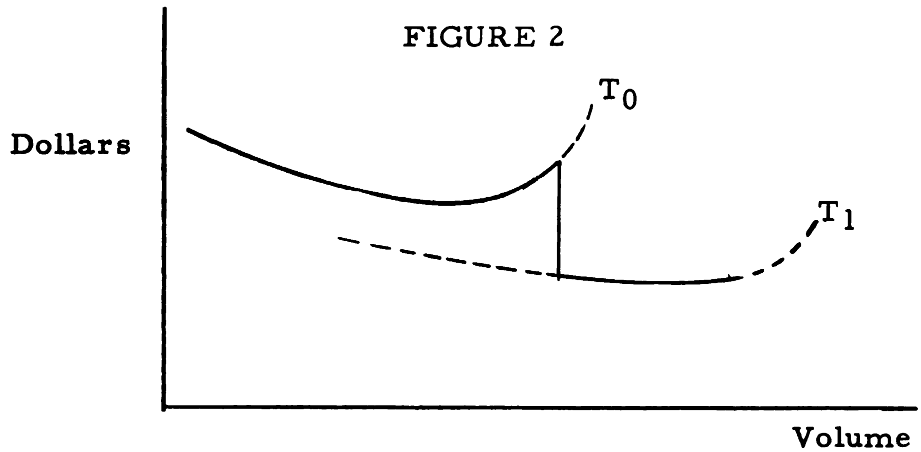
Most unallocated costs, including some of the expenses mentioned earlier, as well as general salaries, seem to be step costs more closely related to scale than to short-run volume changes. Physical plant and executive salaries both tend to exhibit ratchet characteristics. They are easily raised during good times, but remain level when volume falls off. Naturally, in periods of extreme duress, the firm can sell off parts of the plant, dismiss some executives, or substantially lower an existing salary; but these steps are seldom taken.

The Scalloped Marginal Cost Curve

Rising marginal costs are typically encountered when, as a result of inadequate planning and increasing income, some important phase of operation becomes inefficient. Results of several interviews suggest that managers often do not recognize the inefficiency until costs rise with increasing volume, or profit margins decline. Then management belatedly directs special attention to these offending costs, often reducing them to levels lower than those which had previously existed. The old adage of the squeaky wheel certainly applies to cost, especially when profit margins are squeezed.

Though it encompasses a time span, a strong argument may be made for scalloped, but ultimately declining, marginal cost curve. In other words, an upward sloping MC in time T_0 results in a retreat to a lower MC in time T_1. Notice in FIGURE 2 that the drop in cost has generated an entirely new curve.

FIGURE 2



Hypothetical MC Curves Encompassing A Time Span

At times, scalloped curves may result because of the necessity of adding complete production units or even complete work shifts. More often, as some factor costs rise others decline more than enough to offset this increase. Sometimes increases in utilization decrease the costs per unit sold until a factor must be increased because its use is approaching a practical limit. Marginal costs are rising in the warehouse at this point, but the firm cannot effectively utilize an extra shift. However, other factors, such as office costs per unit, may be declining. The direction of the MC then depends on whether office and sales costs decline more than plant costs increase. The answer determines the shape of the marginal cost curve[6] at this point.

It is clear that accountants are becoming increasingly concerned with cost behavior as opposed to cost bookkeeping and that this concern has long existed.[7] Analysis of cost behavior gives stress to two closely related fundamental concepts: (1) cost accountability should not be carried past the point of either cost controllability or cost variability, and (2) the contribution to overhead or fixed

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[6]One of the inherent disadvantages of many large firms is the higher wage scale caused by union activity or the threat of it. This is partly offset by an ability to attract a slightly higher caliber worker. In any case, any meaningful comparison of scale costs should compare working hours rather than wages paid.

[7]It is interesting to note that Sidney Davidson ("Old Wine Into New Bottles," Accounting Review, April, 1963, pp. 278-279) quotes John Maurice Clark in Studies in the Economies of Overhead Costs, 1923, as defining overhead costs as "costs that cannot be traced . . . to particular units of output" or costs which do not vary with output. He cautions that the presence of these largely fixed non-direct items makes a unique definition of cost impossible. He says, "if cost accounting sets out, determined to discover what the cost of everything is and convinced in advance that there is one figure which can be found and which will furnish exactly the information which is desired for every figure . . . a figure which is right for some purpose . . . must necessarily be wrong for others." Davidson says that, "Clark was suggesting that cost accounting turn from a preoccupation with allocation of overhead costs to an analysis of overhead costs, that attention be shifted from the correct cost figure to the correct cost concept . . . that we should be concerned with cost accounting not cost bookkeeping."

cost is an extremely important but often misinterpreted figure for most decision making in the firm.[8]

Meaningful cost figures must be custom produced for any important decision making, with the traditional accounting figures useful mainly for simple bookkeeping and as an occasional benchmark to provide a psychological spur to better performance, or as a whip to measure and chasten laggards. Internal income statements for an individual item in a firm should only show variable costs. These statements should be summed for a product group, and the costs that are variable by group, but not by item, should be subtracted. These group statements in turn should be summed to a plant or division level, and more costs deducted. At each level the most significant figure is the contribution to the next level's overall profitability.

The present practice of arbitrarily adding fixed costs to a product points up the need for a re-education of management and accountants to convince them that contributions to overall profitability and traditional "net income" are entirely separate things and that standards for assessing one are completely inadequate for assessing the other. This confusion often results in management mistakes in evaluating the profitability of specific products. The problem occurs

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[8]See, for instance, James A. Constantin, Principles of Logistics Management (New York: Meredith Publishing Co., 1966), pp. 186-196.

because some accountants apply external reporting statements to internal situations and is aggrevated by the common accounting practice of including some apportionment of fixed costs within product costs.[9] Implementing the inventory cost-pricing model for a firm requires substantially custom produced information in approximately the form suggested above.

The following is a partial list of the cost inputs of the inventory cost-pricing model:

Material cost
Quantity schedule
Direct warehouse costs
Cost of picking up material from competitors
Accounts receivable cost
Cost of possession

Conclusions

To visualize the actual behavior of costs, it may be useful to review the listed cost categories and get some idea of the magnitude of each. TABLE 1 gives an indication of these values for a typical warehouse.

Though these values are only estimates for a typical warehouse, they have a beguiling look of accuracy. In fact, these figures are only used to get some rough idea of the relative magnitude of

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[9]This is recognized by those accountants who favor the direct costing approach.

TABLE 1. Typical Cost Relationships in A Steel Service Center

Cost	Per Cent of Total Cost	Per Cent Variable	Per Cent Fixed
Material	70	70	
Physical Movement	6	6	
Processing	4	4	
Sales	6	1	5
Information	2		2
Capability	3	1	2
Possession	2	2	
Unallocated	7	2	5
TOTALS	100	86	14

costs, not to provide a guide for judgement. In spite of the apparent preponderance of variable costs, if material and processing costs, both of which are directly billed to the customer, are subtracted, most of the remaining costs are fixed. This generally means that short-run fluctuations in volume have a significant effect upon profit.

Over longer periods of time more costs become variable and the natural economies of scale assert themselves in the firm's long-run marginal cost curve which is the aggregate of the LRMC curves for the various cost categories already discussed. A. A. Walters provides an excellent summary and review of the general shape of long-run cost curves.

> The traditional theory of the firm is not so helpful in suggesting the shape of the long-run cost function as for the short-run counterpart. There is general agreement that, with given factor prices, long-run average cost falls for low ranges of output. Economies of scale arise first because of the ease of dealing with large quantities. The second reason is alleged to be the spreading of risks and reduction of the costs of uncertainty. The third, and probably most generally accepted reason for falling costs is the existence of indivisibilities in both men and capital equipment. Large machines are usually more efficient than small ones. The optimum size of machines for each process may differ so that high multiples of machines are required to reduce average costs to a minimum. Certainly average costs at first decline, with size, but there is very little agreement on the shape of the curve as output goes on increasing. E. A. G. Robinson argued that the coordination of management and control becomes increasingly less efficient and so rising cost of management gives rise to increasing long-run average costs. Sargant, Florence, and others have criticized this rationalization on the grounds that the propositions have not been tested in any systematic empirical study. It might also be urged that recent developments in computers and other managerial techniques have increased the relative efficiency of large managements.[10]

[10] A. Walters, "Production and Cost Functions," Econometrica, 1963, p. 40.

It is evident that most of the costs in steel service centers do not conform to the traditional concepts of rising marginal costs. In fact, a survey of the physical characteristics of service centers shows that the larger the firm, the greater the potential economies. Arguments for decreasing returns to scale predicated upon rising cost curves usually are based upon the uniqueness of the entrepreneurial spirit and the limitations of an effective span of control. But, despite the owner-manager's many advantages, today's professional managers are more than capable of meeting the entrepreneur on his own terms. The problem of management in the largest firms has arisen largely because of the impossibility of a single individual retaining the immense and diverse information necessary to effectively control the organization. Traditionally, the multitude of decisions chokes of management's ability to manage, and as a result, the smaller intelligent firms are able to compete very effectively in both decision making and cost comparisons. However, today's tools for quantitative decision making, when combined with computer technology, can give the largest firms lower costs by automatically providing an evaluation of the relevant variables or even a complete decision-making program. At the very least, automatic data processing gives the largest firms the same informational advantages as the small firms. This gives larger firms the flexibility of the small by allowing the natural economies to function unimpeded by human limitations. Although this potential has not been fully utilized, it allows the possibility of continually declining marginal costs despite all foreseeable increases in firm size.

CHAPTER III

THE EFFECT OF PRICES ON FIRM AND INDUSTRY DEMAND

Introduction

Because of the complicated and unusual price-volume or mean demand characteristics of the steel industry, demand merits a more thorough analysis than would otherwise be indicated. Three basic areas of demand can be delineated: demand on the total industry, demand on the steel service center industry, and demand on the individual steel service center. Widely varying elasticities are caused by the different forces which shape demand for each area. To present the subject with a degree of clarity, it is necessary to discuss each area and the particular factors that influence it separately.

Total Industry Demand

After World War II, the steel industry could increase prices more than other industries because: (1) the demand for steel products far exceeded the supply, (2) no foreign mills were competing with the domestic mills, and (3) losses of customers to substitutes were not of immediate concern. Steel production, due partially to record auto sales, reached a high point in 1955 which marked the beginning of the end of the post-war price independence.

Demand Versus Supply

Although steel production slumped after 1955, the industry increased its production capacity by building new mills. The slump occurred partially because the pent-up demand for automobiles and heavy appliances (stemming both from the unavailability of these products during the war years and from consumers' large cash balances) had been largely satisfied, and consumers became interested in other goods which used less steel in their construction (record players, sports equipment, pleasure boats, and such). The government, too, switched expenditures from heavy war equipment to missiles and space vehicles which use less steel. Thus, even though demand for steel remained stable, its percentage of the gross national product declined. Domestic mill production declined from its 1955 peak and did not regain the same level until late 1964 and early 1965 when, not only was there a record demand for cars, but steel was also stockpiled because of a possible strike.

Foreign Mills

After the war domestic mills shipped large tonnages to foreign countries, principally European, since European and Japanese mills, if capable of operation at all, were unable to meet local demand. When foreign steel products expanded operations during the business boom of the fifties, they were not confined by heavy investment in existing facilities and so today have plants that are more modern and up-to-date than all but a few American plants--a contrast accentuated by several significant advances in steel-making techniques. From the mid-1950's on, foreign mills have had the capacity to produce more steel than they need domestically. They, naturally, have turned to the United States market as a profitable outlet for their production. The 1959 steel strike encouraged foreign steel importation and helped establish channels of distribution which have continued in operation. Although imported steel is as yet almost unknown in parts of the central and mountain states, in some areas of the country, primarily port cities, it has become a substantial portion of the total steel business and may even predominate.

American steel mills have often been criticized for not meeting the prices at which imported steel is sold in the United States.

Adams and Dirlam,[1] for instance, are particularly critical of the administered prices of domestic wire rod. They feel that prices were maintained to squeeze the independent converters, and cite relative prices of imported and domestic steel to back this contention. This squeeze is more apparent than real because converters are also free to buy imported steel, and it would be foolish for domestic mills to destroy their price structure to compete in one product.

The service centers have faced similar competition and reacted in a similar way. However, the influx of foreign steel tended to subvert the service center's price structure to a greater extent than the mill's. The initial effect on markets has been the establishment of a two-price system. Most buyers are unwilling to pay substantial premiums for domestic steel and thus its price has usually dropped to compete with foreign steel.

Domestic steel's position is partially protected by American laws and sentiments. The laws in particular have been effective in reserving substantial parts of the market for domestic steel. Federal requirements of domestic origin are particularly important in reserving defense markets for domestic steel producers. In addition, most state and local agencies are required to give preference to domestic

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[1] Walter Adams and Joe Dirlam, "Steel Imports and Vertical Oligopoly Power," American Economic Review, September, 1965.

goods. However, some vendors, to lower their costs, mingled foreign with domestic steel and sold the resulting mix as domestic. They were able to sell the steel at substantial discounts from the prevailing domestic price disrupting the market structure still further. Either openly or covertly more centers are switching to foreign steel all the time thus further subverting the pricing structure. This is unfortunate for the suppliers, but benefits to the consumer should outweigh the harm to the mills and certainly to the service centers.

Substitutes

There are two main points relevant to a discussion of substitutions for steel:

(1) Where the peculiar properties of a given material are unimportant and substantial price differences exist, the material that is less costly in the long run will be used. As an example, plastic pipe has replaced cast iron and copper in many applications because the price saving on the plastic outweighs any physical properties involved. The material that is less costly in the long run will be used. However, because of the fixed start-up costs involved in switching from one metal to another, manufacturers will not switch for probable short-term savings. Generally speaking, the expected price variation in steel is not great enough to substantially change

steel's competitive position. Other factors are usually more important.

(2) Materials such as aluminum are sometimes used, despite higher costs, when they have qualities superior to steel. Steel however, is sometimes used despite its higher cost because of properties peculiar to it.

There are a few direct steel substitutes and these are sometimes used in spite of cost disadvantages rather than because of lower costs. Steel also is sometimes used when it is at a cost disadvantage. The reason for this behavior is that other materials are not perfect substitutes for steel nor is steel a perfect substitute for other materials. Barloon, in analyzing pricing in the steel industry, makes this point quite clearly:

> The inelasticity of demand is not relieved appreciably by the availability of competitive materials. Materials competitive with steel usually compete on a price basis. Aluminum and other non-ferrous metals are examples. Electrical and thermal conductivity, relative tensile strength, workability, corrosion resistence, and other performance factors are the chief foci of competition between rival metals, most of them already priced at low levels five times or more that of carbon steels. Steel price reductions would, therefore have little effect on substitution tonnages and competitive materials do not appreciably

> moderate the inelasticity of demand for the industry product.[2]

Not only are studies of steel industry demand universal in showing a preponderance of inelastic demand situations, but Hirsch has collected results which show this condition exists in many other areas.

> A survey of the results reveals that in most of the empirical studies that we could find, inelastic demands were more often indicated than elastic ones . . . More specifically, of 34 commodities whose demand elasticity was estimated on the producer-manufacturer level in the United States, all but 4 were inelastic.[3]

It would, therefore, seem conclusive that the availability of possible substitute materials has little effect on steel demand despite changes in steel prices, and can, therefore, be ignored for most purposes.

Service Center Industry Demand

Customer Demand Classification

Demand for products of the steel service center industry can be classified into four types: (1) demand for products in smaller

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[2]Marvin J. Barloon, "Institutional Foundations of Pricing Policy in the Steel Industry," Business History Review, September, 1954, p. 220.

[3]W. Z. Hirsch, op. cit., p. 31.

quantities than the mill sells, (2) demand for products on which the customer needs some pre-production, (3) demand for products needed within a few days, and (4) demand related to functional discounts. These types often overlap.

Smaller Than Mill Quantities

The demand for steel in less than mill quantities is very price inelastic if, as is often the case, the customer has only two rational alternatives: to buy from the service center or to buy from the mill and resell excess material, usually at a loss. Prices must, therefore, increase dramatically above a normal base level before these customers desert their only practical supplier, the service center. However, some purchasers buy in smaller than mill quantities to avoid making the investment in and assuming the risk of inventorying their own steel. These price sensitive buyers can easily switch between service centers and mill purchasing because they have storage space and handling equipment on their own premises.

Preproduction Processing

A majority of service center sales include some processing. When buying processed steel, the customer purchases services as well as goods. If the total package price appears excessive to him, his only alternatives are to buy the steel from a mill and process it himself or to have a third party perform the processing. Since these alternatives usually require too much extra investment or handling and planning, the customer's demand for the service center products is very price inelastic.

Immediate Need

Material needed within a few days is usually very insensitive to industry prices. This is particularly true when greater than normal mill lead times prevail. The customer has no reasonable alternate sources of supply.

Functional Discounts

Functional demand only includes mill quantities purchased from service centers because of the discount. The service center receives a functional discount and passes part of it on to the customer while the steel is shipped directly from the mill to the customer. As long as the service centers pass on some of their discounts, the customer's demand is almost completely inelastic. Only when the service centers refuse to pass on part of the functional discount is the customer indifferent to purchase from mill or service center.

Substitution Effect

Though substitution between products is a major concern for steel mills, this subject is of little more than passing interest to most service centers. The price changes possible for a service center are limited and many service center items in fact have no close substitutes. Those substitutions that do occur do not always affect the service centers, since they sometimes sell the substitute as well as the steel.

Import Market

Though there are no significant physical differences between domestic and imported steels, the distribution channels have been differentiated enough so that in some areas imported and domestic steel are competing products. Imported steels are generally sold on the basis of the relationship between their quality and price. Domestic steels are sold primarily on their quality, integrity, and reliability, the dependability of the service center, the mill guarantees and technical assistance, and patriotism. The sale of imported steel is increasing in this country because imported steel is competing with domestic steel on domestic terms but with a price advantage. Importers now stress their full line to counter the domestic sellers' argument that importers have limited stock and a three- to six-month

lead time. The channels of distribution seem to be merging in many important ways, though different firms are involved.

The changing channels of distribution make definitions of the steel service center industry difficult. If domestic and imported steel service centers were separate industries, previous statements would be modified to say that the domestic steel service center industry demand is very price elastic, especially in port cities. However, earlier statements of extreme demand inelasticity are correct because domestic and imported steel are generally considered the same industry.

Mill Competition

The relationship between mill and service center is unusual in that the service center is both the mill's best customer and closest competitor. The mills support the service center's sales efforts because they receive the same price whether the steel is sold directly or through a service center--though individually they are not assured of a service center's sales. Mills even run an active advertising program to further purchases from service centers. Some of this largess can be counted as fostering service center loyalty to an individual mill, but a certain amount of this effort seems to be a part of a complicated informal trade-off in which mills support the service centers, who in turn refrain from "pushing" imported steel.

Service Center Firm Demand

General Differences Between Firm and Industry

The individual service center does not see inelastic industry demand curves but rather customer loyalty as reflected in the customer's choice of vendors at different relative prices. The customer finds no close substitutes for steel, but finds lists of substitute steel vendors as close as the telephone book. Not only may the customer initiate the contact, but he is also besieged by rival concerns attempting to lure away his business. Even if there are no price differentials between firms, the customer's loyalty is tested everyday. When the firm varies its price from the market price, it is trading customers for higher margins, or sacrificing margins for added customers. Those customers who do not follow the lowest price are either influenced by firm and product differentiation or are ignorant of the differentials. In the following sections the factors influencing competitor and customer reactions are discussed.

The Market Area

Excluding direct mill sales, a firm's potential market is generally limited to the area in which the firm can match the prevailing price and still meet all variable costs. This area varies between products, though if shipping one product into an area is profitable, lower incremental costs may allow other product shipments. The actual market area is seldom as large as the potential area for most firms are unable or unwilling to exploit differences among them due to competitive factors or capital constraints. A significant market penetration is necessary to pay the freight costs incurred with a regular delivery schedule, and initial sales effort and other expenses necessary to achieve the market penetration may deter much potential expansion. The discussion will center around the potential market, though most remarks also apply to actual markets.

Mill points and import cities determine market area. To discourage price cutting by mill point service centers, service center prices are seldom higher far from mills than the nearest mill-point prices plus freight. Generally, service centers at mill points are at an advantage because their market areas are less restricted and they can afford to sell as far away as a driver can deliver on a round trip. Some firms have even extended this range by using two drivers--one to drive and return the initial distance and one to deliver the steel to the customers. This extends the market that the service center is able to service to over 300 miles. By adding more drivers, the distance can be extended further. In sections of the West, deliveries over greater distances are common.

The mill point or port city service center enjoys the advantage of shipping into markets which are unable to ship back, but this very advantage crowds the field and tends to limit the market which any one firm may gain. Further, competitive conditions often depress mill and point prices and limit profits. Also, some steel users, who in other circumstances would be service center customers, buy direct because of the mill proximity and because LTL rates for a short haul are not onerous. The natural market may, of course, be limited by the existence of other mill points.

The firm located away from a mill or port finds its market more circumscribed but has the advantage of being in the local area. The non-mill port service center usually has a market area shaped like a comet's tail. The local service center is in the nose of this area with the market extending away from the mill point. Other mill points may displace or indent the area, or in some circumstances cut it off completely. The firm operating within this envelope sells only a few miles towards the mill point because the prices received decline as the freight absorption goes up. In effect, because the market price is usually the mill price plus freight, the absorption increases at twice the freight rate. As an additional deterrent, the mill point service centers solicit more intensely close to their base. Service centers can usually ship more than twice as far to points the same distance from the mill point because they only absorb single freight from the service center to the customers rather than double. The company uses its own trucks whenever practical to save money by reducing freight absorption costs and to improve delivery service.

Customers located further away from the mill than the service center and in the same direction can be serviced up to the logistically practical delivery distance, and even beyond in some cases, with the aid of common carriers. TABLE 2 summarizes these relationships.

TABLE 2. Summary of Transportation Factors Affecting Market

Customers location compared to service center's	closer to mill	same distance from mill	further from mill
Price received by service center compared to local delivery price	lower	the same	higher
Freight absorption	commercial & private truck	private truck	none
Usual limiting factors	commercial freight	out-of-pocket hauling costs	logistics of servicing customers

Influences on Demand Elasticity

Within the market area just discussed, the market, the firm, the customer, and the product are the main factors that influence the rate at which demand changes.

The Market

There are five main market variables which affect the price elasticity of the firm: (1) the size of the market, (2) the number of competitors, (3) the history of market behavior, (4) the present price level, and (5) the present price consistency. The first two are interrelated: The larger the market and the larger the number of competitors, the more price elastic the demand curve faced by the individual firm. The larger the number of firms, the lower the incentive for a single firm to match another firm's price reduction because the price-cutting firms, though it may increase its market share, does not take away sufficient business from any one firm to justify meeting the lowered price. The firm also sees a very elastic demand curve and is more likely to become a price cutter on its own. When the firm charges higher prices than the competition, sales losses are higher in large than in small markets because of the greater price knowledge of customers in larger markets, plus the lower probability of other firms following.

A rapidly growing market, however, or one in which the firms in the market are operating at capacity, tends to be less price elastic above the market price because customers shop for price. The closer the market to capacity, the less the firm's incentive to meet the lower price competition which increases demand selectivity below the market.

The history of market behavior affects the attitudes of customers and competitors whose responses are, therefore, more likely to be in line with past experience. The present price level affects competitors' reactions because firms are less likely to reduce prices on an already unprofitable item. The present price consistency is also important because customers may not shop for price as critically if they find several generally accepted prices in the market.

Firm Characteristics

The size of the vending firm and its size relative to the size of the market also influence price elasticity as well as affecting competitors' responses. The larger the firm, the lower the demand elasticity because a firm which has a larger portion of the market at the going price cannot increase its sales at lower prices as much as a smaller firm can. When a larger firm's prices are higher than market prices, the results are economically indeterminate, but two auxiliary factors may be influential: (1) competitors will not exploit a large firms weakness to the extent that they will a small firm's, partially due to a fear of retaliation; and (2) competitors cannot as easily absorb a given percentage decrease in the large firm's business.

The competitor's view of the firm's place in the industry is very important in assessing the reaction of those competitors to a price change. Other firms frequently follow when an acknowledged leader changes prices. Consequently, the market leader would expect very low price elasticity in its demand curve. When other firms change prices, the price elasticity of their demand is greatly influenced by the market leader's position. If he follows, then demand is very price inelastic. In some cases there is no single market leader, and the probable actions of the competitors are much harder to predict.

Although loyalty to or appreciation of a firm's non-price advantages does exist, and a firm's reputation with certain customers may cause them to continue buying from the firm at higher prices, such loyalty is not as common as some buyers believe. The customer may profess willingness to see the vendor through difficult times, even if this friendship costs him money, but few actually follow their heart with their pocketbook! Some large, multi-plant firms are especially price conscious and they continue substantially to demand price elasticity. Another simple but often overlooked factor is the ease with which price schedules can be read. The more widely circulated the price schedules and the easier they are to read, the more elastic is the demand.

Customer Characteristics

Given other service center's reactions, the steel service center's customers shape the firm's demand curve. When each customer's decision to buy or not to buy at a given price is summed, the total purchases determine a point on the firm's demand curve.

These decisions to buy or not to buy despite higher price represent the value which each customer attaches to his present vendor's higher price and the price that induces the customer to switch is the total value of the non-price attributes to the customer. Two of these attributes, convenience, and habit, disappear for the buyer when he leaves the higher-priced service center for the new vendor. Hence, he may not return to the higher-priced service center until it has substantially reduced price.

The customer's size and the amount of his purchases determine whether or not he can influence the service center to resist price increases or to offer price reductions. The larger firms often influence price levels by offering blanket or bid contracts for a series of service center size orders. Thus the equity in Galbraith's theory of counterveiling power becomes an illusion when large buyers meet small suppliers. The small seller lacks both the help that agriculture receives from government and the protection that the small buyer gets from the Robinson-Patman Act. Service centers and other small suppliers are constantly tempted to cut prices to get large orders, especially in the automotive industry, where the pressure to cut costs is intense. Price solidarity in this and other industries is broken by customers offering to order a full year's requirements from a single firm at a special price. One firm's giving a special price exerts leverage on other firms to meet that price. The service center is sometimes caught between giants, the mills who maintain their price structure, and large buyers who insist upon price concessions. Naturally, any softness occurs in the service center's margins.

The products the customer sells influence his price-demand behavior. Ceteris paribus customers selling products in which the steel is incidental, are not as price sensitive as customers whose steel costs are a larger percentage of the product costs. For instance, steel cost averages 70 per cent of the construction steel fabricator's selling price. Steel fabrication is a very competitive business with the variance in steel prices often meaning the difference between profit and loss. Thus steel fabricators have an almost completely elastic demand curve, and, as a rule, are almost exclusively motivated by price differentials. At the other extreme, firms which operate on a cost-plus basis, such as public utilities, or whose asset values determine rates, may not pay particular attention to price, except to insure that they are not paying more than the market price.

Product Characteristics

The characteristics of the product and the exactness of its specifications influence the customer's valuation of non-price considerations. Certain goods which have no special qualities, such as common structurals in standard lengths or hot rolled secondary sheet, are bought and sold in large quantities almost entirely on a price basis. But goods with special characteristics, such as cold finished steels, tool steels, and processed steels, are likely to be less price elastic. Certain products which require special knowledge of application to sell correctly, such as tubing, also tend to be less price elastic.

In the final analysis, all factors affecting demand elasticity influence either competitor or customer behavior.

The Shape of the Demand Function as Seen by the Firm

General

The firm's ultimate concern may be the marginal revenue line, but it is first necessary to obtain the demand or average revenue line, which depends almost entirely on the reaction of other firms. As previously noted, the industry curve is so inelastic that volume changes from other sources completely obscure it to the average firm. Even in the general case when industry demand is less inelastic, the reactions of other firms seem more important. E. A. G. Robinson, in his book Monopoly, brings out this point.

> Suppose a firm reduces its prices in order to attract more customers, the extent of its success will depend partly, of course, on the power of price differences to induce customers to change their habits, or of cheaper prices to bring in new purchasers, but to an even greater extent its success will depend upon the way in which other firms respond to its actions.[4]

The Time Element

The demand line faced by the firm is determined, other things being equal, by the customer's reactions to price changes. However,

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[4] E. A. G. Robinson, Monopoly (New York: Pitman Publishing Corp., 1949), p. 23.

customers may be willing to maintain relationships at higher prices for a short time but not for an extended period. Hence, if a firm maintains a higher price, it will find its demand decreasing over time; and if maintaining a lower price, it will find demand increasing over time. That is, the longer the differential is maintained, the more elastic the demand curve.

Followership

Firms may be divided into groups according to competitors reactions to their price conduct at any given time. The competitor's tendency to match changes by the firm may be called followership, and hence firms may be classified according to the amount of followership they have.

The Firm With Complete Followership

If a firm had complete followership, its demand curve would equal a certain percentage of the market demand at any given price. This very inelastic demand curve could be more elastic if the market area shrank at higher price levels and expanded at lower price levels.

The Firm With Partial Followership

A firm with partial followership finds some competitors willing to follow its leadership part of the time. In rare instances all firms may follow on a certain product. However, it is probable that across-the-board price changes produce few followers--resulting in very elastic demand curve.

Partial followership may result in differentiated prices for a short time without substantial gains or losses in customers, although in the long run, unless confined to a few products, price differentiation results in elastic demand curves.

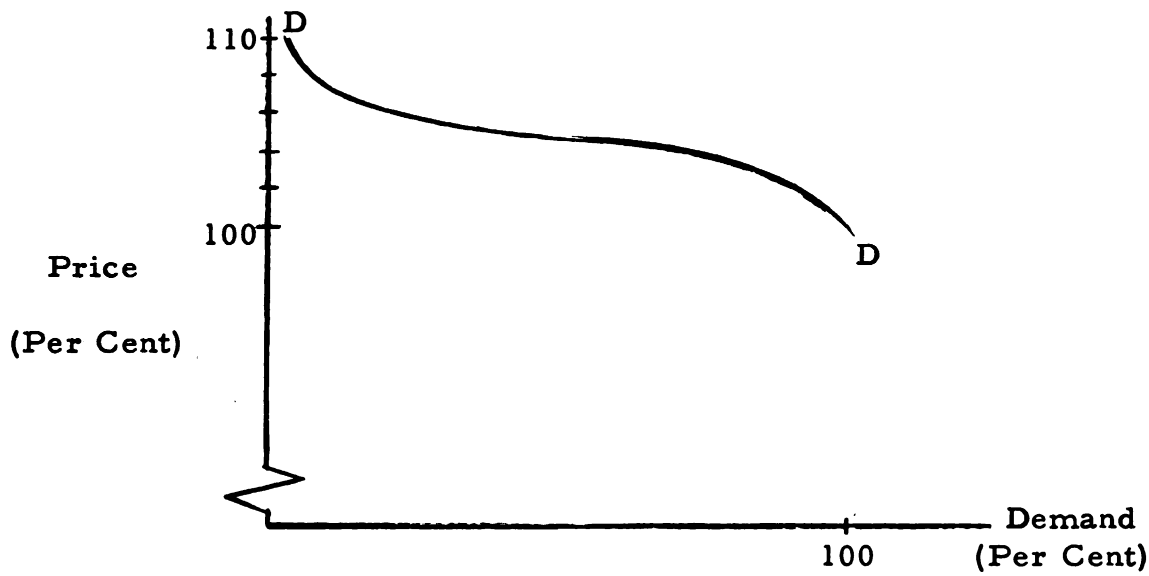
The Firm With No Followership

Firms may change prices unilaterally. Lowering prices means that the firm accepts the risk that other firms will follow and the changes in demand be very slight. Raising prices means risking that other firms will not follow. What happens when other firms don't follow depends upon the non-price values of the firm's customers. Obviously, a recently-added line of steel carried by a junk dealer will be more price elastic upward and less downward than an established line of a firm with a record of service and dependability.

The balance of this chapter is devoted to some specific examples based on interviews with general or marketing managers and other metal center employees. Material used in these examples is based on their opinions supplemented by a few case histories.

Firms which have maintained higher prices alone sometimes find that differentials of 1 per cent or 2 per cent makė little difference in the demand but that 5 per cent means a substantial reduction and that 10 per cent often means the market share approaches zero. The curve is shaped this way for products with differentials maintained for a period of time. FIGURE 3 shows the estimated demand curve for a particular product in a medium-sized market.

FIGURE 3. Typical Firm Demand Curve in a Medium-Sized Market



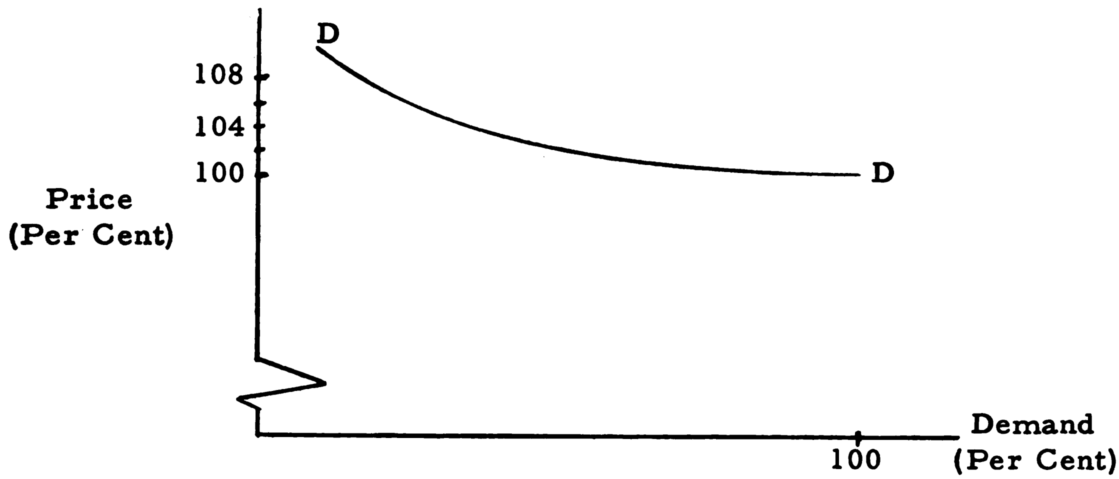
Notice that while demand is substantially unaffected by a 1 per cent or 2 per cent differential in a differential of between 3 per cent and 7 per cent, the demand curve is very elastic. Above 7 per cent, the demand becomes increasingly inelastic because the remaining customers buy for convenience, the sale being an unimportant auxiliary item of a larger order. The customers deserting the firm at differentials of 1 per cent or 2 per cent are large quantity buyers or comparison shoppers searching for the lowest price. Deciding whether the price differential is substantial enough to warrant changing vendors is both a qualitative and quantitative decision.

Those buyers changing firms for a 3 per cent to 7 per cent reduction in the example will pay a slight premium for non-price considerations. Since customers' buying habits can be grouped by attitude, use of steel, etc., a demand curve based on how much business is done with each group may be constructed. For example, of a firm's customers for mild steel plate in lighter sizes:

45 per cent are Structural Fabricators;
35 per cent are Industrial Fabricators, etc.; and
20 per cent are Miscellaneous, placing small orders.

Experience suggests nearly all of the structural fabricators would change suppliers for a 2 per cent price differential, and the majority would change with the slightest differential. Industrial fabricators in this market are not as price conscious, though it seems logical that most of them would switch suppliers rather than pay a 5 per cent premium for service. Miscellaneous customers often pay a substantial premium before changing suppliers. The details of this construction are shown in FIGURE 4.

FIGURE 4. Firm Demand for 1/4" Mild Steel Plate



<u>The Marginal Revenue Curve</u>

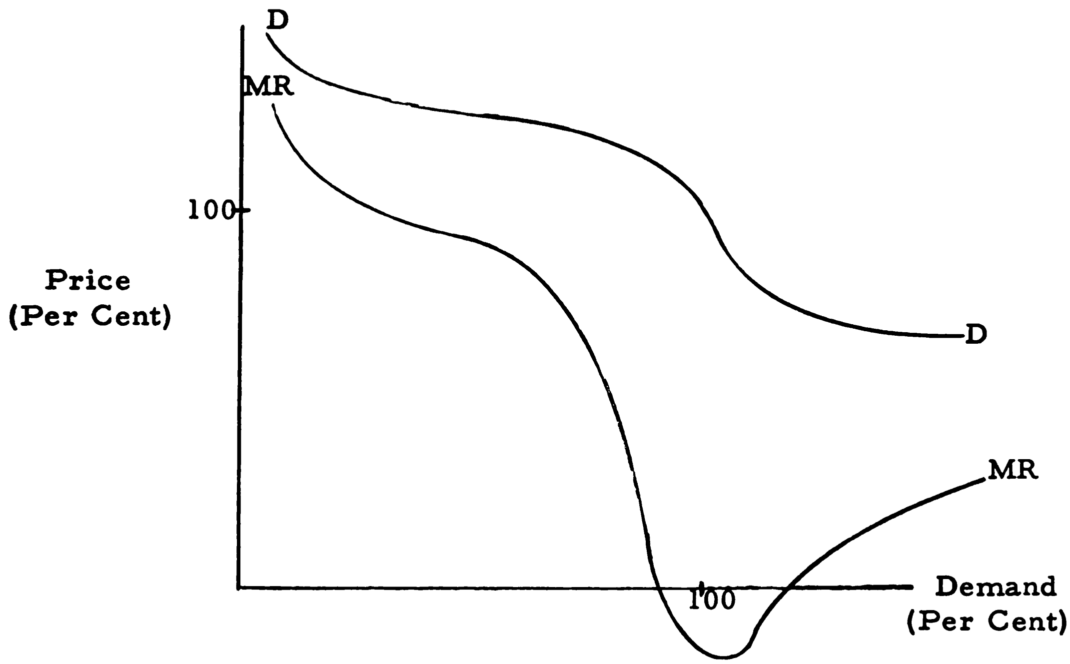
There is only one marginal revenue curve that is consistent with a given average or total revenue curve and vice versa, which means specifying any one of the curves automatically determines the other two. Therefore, given an average revenue curve, any discussion of marginal revenue curves must be purely descriptive. However, this does not negate their importance to a study of demand because the intersection of marginal revenue and marginal cost curves should determine the firm's equilibrium.

When the firm has complete followership, its marginal revenue curve is a percentage of the industry marginal revenue curve. Thus if the industry has an average demand elasticity of, say, .3, the firm's marginal revenue will be negative. In fact, by definition, an average revenue curve elasticity of less than 1.0 will have a negative marginal revenue. Economic theory supports a special case known as the "kinked" demand curve in which the firm has complete followership when it decreases prices but is alone when it increases prices. The usual explanation, however, tends to simplify the situation since it uses straight line demand curves and an assumption of all or no followership.

When the firm has partial followership, the marginal revenue curve is likely to be negative in the vicinity of the market price and positive at prices considerably above or below that price. This is illustrated in FIGURE 5. Naturally, as the firm with partial followership changes prices, the average market price changes, and, if by chance all firms follow the lead, the firm in effect starts again with a new market.

When the firm changes prices alone, the average revenue curve is more elastic than that of the firm with partial followership. Hence, the marginal revenue curve will be positive a smaller percentage of the time than when there is some followership.

FIGURE 5. Marginal Revenue Curve for
A Firm With Partial Followership



CHAPTER IV

VARIATIONS IN SHORT-TERM DEMAND

This chapter focuses upon the demand for a single item. There are two types of variations occurring in this demand. First, changes in the level of demand. That is, in one year, 100 tons of a product may be sold, and in the next year only 65 tons. Second, even if the general level forecast is met, short-run random variations occur around this demand and these variations may be greater than the variations caused by changes in the level of demand. Several methods have been used to forecast changes in demand level. Past demand may be used directly or modified by the use of trend lines or trend analysis. Past experience can be put to use in sophisticated techniques such as the various types of exponential smoothing. These forecasts are based upon information developed within the corporation. Other methods of forecasting are based upon outside information such as forecasts of GNP or industrial production.

The Nature of Short-Term Demand Variation

The demand for any item sold to a non-captive market will vary because of the random arrival of customers. Over a three month period, this variation may not be substantial, however, over one week the variation may be great, and for a one-hour period, the variation may be even greater. In a single moment there will be either zero or one customer arriving. This situation can be seen during the summer months at the ubiquitous Dairy Queens on our highways. At some moments a que forms in front of each window. Only a few minutes later, for no apparent reason, there may be no customers at all at the Dairy Queen. Though somewhat modified and lessened by the propensity of motorists to pass Dairy Queens with long ques, these cycles nevertheless still occur throughout the day. The problem of arrivals and their distribution is spread throughout industry. As another instance, the telephone company is faced with a choice of staffing for peak call arrivals and not burdening the customer with excessive waiting times or staffing for loads only slightly greater than average and forcing the customer to wait during peak load times. For information operators, the telephone company has solved part of this problem by going to area rather than strictly local information operators, a shift which tends to increase the average number of arrivals and therefore reduces the variation.

In these simple demand situations all customers buy one unit at a time and one can also assume exogenously produced estimates of demand. Because each customer's decision to buy is independent of other customers' decisions, the probability of a customer ordering the one item at any given time is constant. The effect is the same as sampling with replacement. There will be some instances when no customers arrive and some periods when more than one customer arrives at the same instant. Since the customers arrive independently and at random, they have the same probability characteristics as the probability of finding randomly spaced defects in a wire. This is a classic description of the poisson distribution. If an arrival distribution with a given mean can be proved independent, it is poisson by definition.[1] Independence means that a single parameter, the mean, uniquely defines the distribution, and, therefore, high demands over short time periods may have the same characteristics as low demands over longer periods.

In some industries demand is more complex because customers arriving randomly order varying amounts. At a drive-in, for instance, some customers order one hamburger, yet once in a while a customer orders for a group and takes forty hamburgers. This depletes the hamburger inventory forty times as much as the arrival which takes one hamburger. This problem in inventory control was recognized over thirty years ago by R. H. Wilson who considered it

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[1]Thomas L. Saaty, Mathematical Methods of Operations Research (New York: McGraw-Hill Book Co., 1959), pp. 335-336.

unsolvable:

> It is recognized that special conditions such as that of an item for which a number of requests for small amounts (4, 3, 6, 5, etc.) and some requests for comparatively large amounts (100, 500, 2000, etc.) are received cannot be controlled by the use of the calculator.[2]

Such demand conditions are common in industry, particularly the metal center industry where it occurs in a modified form.

Metal centers sell for varied uses and in varied amounts. One might expect the order size to be spread randomly from very small up to approximately the smallest mill orders. However, metal centers sell on quantity schedules which add extra charges for ordering smaller quantities. There may be a half dozen or more quantity brackets, and customers tend to group their order sizes at or slightly above each quantity bracket. These various size orders tend to be closely grouped around their means. For instance, between 2,000 and 3,000 pound orders may cluster just above 2,000 pounds with a mean of 2,200 pounds. Characterizing the clusters by their means, therefore, does not conceal essential characteristics. But even this simplification leaves a very complicated demand pattern with each arrival group having its own mean.

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[2] R. H. Wilson, "A Scientific Routine for Stock Control," Harvard Business Review, Vol. 13, No. 1 (October, 1934), p. 128.

Approximation of Demand Variations

Early inventory models used subjectively determined safety stocks to defend against variations in demand. Later models assumed a desired customer satisfaction level and used a normal distribution as an approximation of the variation in demand to be expected during some period of time. The normal distribution allows for a probability of some negative number of arrivals during a certain time period. This is clearly an unrealistic representation.

The poisson distribution which employs arrival probabilities overcame this drawback and worked out very well in certain simple demand situations. However, metal centers, as we mentioned earlier, have the problem of varying sizes of arrivals which are not effectively approximated with the use of a simple poisson because the single parameter left large unexplained variations in the distribution. To overcome this problem, McMillan and Demaree used the gamma distribution which may be called a non-random poisson.[3] It uses both a mean and variance and closely approximates the demand in certain products. Though the gamma distribution meets the test of

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[3]Claude McMillan and John Demaree, The Management of Metal Inventories (Cleveland, Ohio: Steel Service Center Institute, 1966).

predictability,[4] it fails on theoretical grounds for it assumes a non-random arrival. A completely independent random arrival process is uniquely described by one parameter, the mean, since the mean and variance are the same. The addition of a variance separate from the mean removes its independent random quality. Despite its short-comings, the gamma distribution is by far the best distribution presently adapted to steel service center demand fluctuations.

A more accurate method of showing distribution is provided by a mathematical model developed by Professor R. M. Adelson of the Imperial College of Science and Technology which he calls the compound poisson distribution.[5] Going beyond the stuttering poisson demand model developed by Professor H. P. Galliher of MIT,[6] Professor Adelson created a general form of the stuttering poisson. Adelson shows the derivation of the compound poisson by which it is possible to add many distributions together to get the actual total distribution.

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[4]See E. Martin Basic, Development and Application of Gamma-Based Inventory Management Theory (Unpublished Dissertation, Michigan State University, 1965).

[5]R. M. Adelson, "Compound Poisson Distributions," Operational Research Quarterly, Vol. 17, No. 1 (March, 1966), pp. 73-75.

[6]H. P. Galliher, Philip M. Morse and M. Simond, "Dynamics of Two Classes of Continuous Review Inventory Systems," Operations Research, Vol. 7 (1959), p. 362.

The compound poisson is not only the best existing statistical measure of inventory variation, but it is also surprisingly easy to use. The author has, therefore, replaced his earlier mechanical summation with an adaptation of one of Adelson's formulas. The formula finds the density of the poisson at any given level through the use of the previously found densities at lower levels. The probability of zero orders is equal to e to the minus sum of the means power, or:

$$R(0) = e^{-\sum(M)}$$

Each succeeding density is built on the previous densities. Thus it is a simple matter to get the left tail cumulative compound poisson probabilities by adding all computed densities. For inventory control purposes, however, the relevant probability is that of the right tail. Fortunately, merely subtracting the left tail from unity gives the right tail. Appendix D contains a program to compute this distribution. The right tail is useful in comparing the cost of being out of stock with the cost of guarding against that probability.

If there were an infinite number of prospective customers, then there would be complete independence of customer arrivals, and hence the compound poisson would be a complete "fit" for actual demand. Though such is not the case, it is contended that the dependence of arrivals is so slight that it can be ignored. Though there is no way to prove that a demand pattern came from a certain theoretical distribution, it can be said that a demand pattern is consistent with the distribution. The following example shows the compound poisson closely approximates the actual demand.

The data tested is the demand pattern for a certain steel item over a 67 week period. The chi-square test was first used on a simplified compound poisson. This was necessary because the loss of a degree of freedom for each additional parameter used ruled out more than a few streams. In this case the compound poisson used contained only 5 streams rather than the 20 used in a later test. The distribution was divided into 8 cells which yielded a chi-square value of 3.94 which is not significant at the .20 level.

As an additional check, the Kolmogorov-Smirnov test was used. This non-parametric test has the advantage of being exact while the chi-square test is only approximate. However, according to Hoel,[7] little is known about the magnitude of the correction necessary when distribution parameters are estimated from the data. Using a compound poisson with a stream for each arrival size gives cumulative theoretical values very close to the observed. TABLE 3 shows the observed and theoretical values. The original inventory card, stated in pounds, was translated into bars. (One bar weighs

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[7]Paul G. Hoel, Introduction to Mathematical Statistics (New York: John Wiley & Sons, Inc., 1962), p. 349.

TABLE 3. Observed and Theoretical Cumulative Distributions*

	Observed	Theoretical		Observed	Theoretical
0	.2836	.3012	31	.7761	.7827
1	.3582	.3735	32	.7910	.7890
2	.4179	.4484	35	.8060	.8007
3	.4478	.4740	38	.8209	.8100
4	.4627	.4885	51	.8358	.8293
6	.5373	.5556	63	.8657	.8903
7	.5820	.5732	64	.8806	.9030
8	.6119	.5924	65	.8955	.9160
10	.6418	.6186	68	.9254	.9265
13	.6716	.6405	73	.9403	.9467
14	.6866	.6532	78	.9552	.9539
16	.7164	.6725	84	.9701	.9624
20	.7313	.6982	95	.9851	.9775
23	.7463	.7194	97	1.0000	.9790
24	.7612	.7235			

*Only values with changes in observed values are included.

32 pounds.) The greatest difference between the observed and theoretical is .0427. A value as great as .13 is not significant at the .20 level. An additional distribution, subjected to the Kolmogorov-Smirnov test, was not significant at the .20 level. This is particularly rewarding when it is considered that the compound poisson is a "cream of wheat" or lumpy distribution due to the varying arrival sizes.

Adelson's compound poisson accommodates any number of poisson streams and is thus readily adaptable to the clustered streams characteristic of goods sold from a quantity schedule of prices such as steel service center products. Simplifying customer arrivals by grouping around their mean order sizes makes the demand variations a more practical input for the inventory cost-pricing model discussed in Chapter VII, though some of the predictive qualities may be lost. Chapter V discusses inventory models and shows some adaptations necessary for the inventory cost-pricing model.

CHAPTER V

INVENTORY MODELS AND SYSTEMS*

Simple Inventory Systems

Inventory systems or models contain one or more of the following costs:

(1) Order cost--the cost of ordering merchandise or, in a production inventory model, the cost of production. In a linear system this is composed of a fixed order or set up cost, C_1, plus a variable cost component, C_2.

(2) Carrying cost--the per unit cost, C_3, of holding inventory--for instance, the per unit cost of inventory investment, insurance, extra storage, and so forth.

(3) Shortage cost--the cost, C_4, of being out of an item when it is asked for, such as profit foregone and loss of customer.

Though each of the above costs may be present in any inventory system, not all inventory systems will have all costs. For example, one system may involve an order cost and carrying cost

*I am indebted to Professor Claude McMillan for many of the ideas presented in this section.

but no shortage cost (1, 2); another may entail a carrying cost and shortage cost but no order cost (2, 3,); and a third may have order, carrying, and shortage costs (1, 2, 3). A system without a carrying cost is very unlikely (1, 3).[1]

The EOQ (Economic Order Quantity) formula, shown in FIGURE 6, is a well-known system which carries a (1, 2) designation because it has both an order cost and a carrying cost but no shortage cost. As the quantity ordered increases, the cost of ordering over a given time period increases and carrying cost increases.[2]

Adding the order and carrying cost curves together gives a total cost curve as shown in FIGURE 6.

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[1]Eliezor Naddor, Inventory Systems (New York: John Wiley & Sons, 1966), pp. 7, 8.

[2]Given the following definitions:

D = Demand per period
C_1 = Fixed ordering cost
C_2 = Variable ordering cost (material cost)
C_3 = Carrying cost per unit
Q = Order quantity

Let us total cost defined as:

$C(Q)$ = (carrying cost) (average inventory) + (order cost) (number of orders placed)

$$= (C_3)\ (Q/2) + (C_1 + C_2Q)\ (D/Q)$$

$$= C_3/Q2 + C_1D/Q + C_2D).$$

FIGURE 6. Graphical Representation of the E. O. Q. Formula

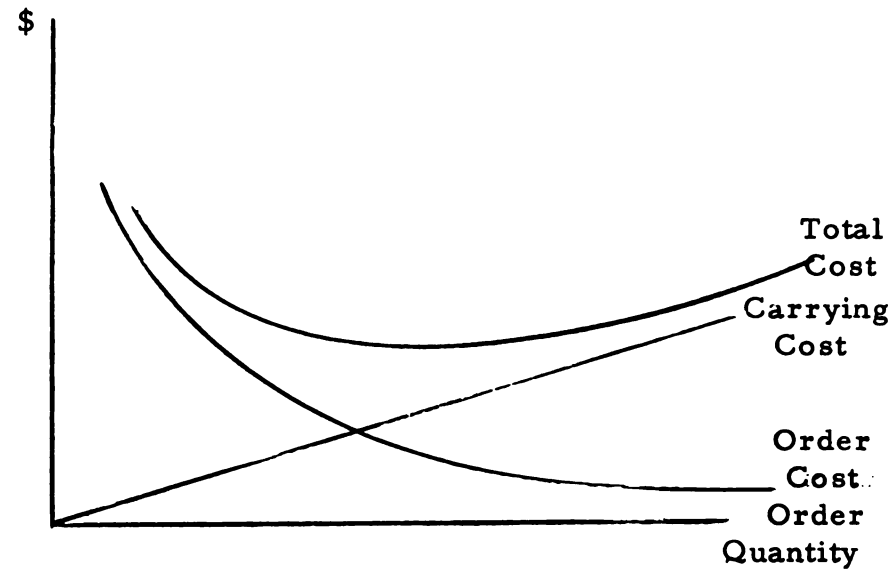

FIGURE 7 illustrates various total cost curves relevant to various quantities when a quantity discount schedule is in effect. Discrete breaks arise in the total cost curve due to variable order

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The E. O. Q. formula is derived by setting the first derivative equal to zero as shown below:

$$C^1(Q) = C_3/2 - C_1\, D/Q^2 = 0$$

$$\text{E. O. Q.} = \sqrt{\frac{2C_1D}{C_3}}$$

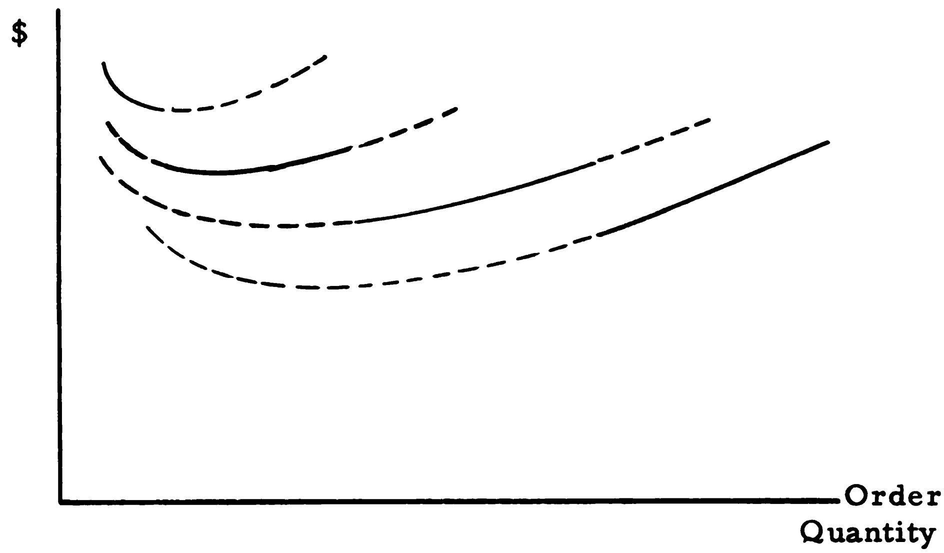
Second order conditions reveal that total cost at E. O. Q. is a global minimum.

The E. O. Q. Formula

$$\text{E. O. Q.} = \sqrt{\frac{2\text{ (demand per unit) (fixed ordering cost)}}{\text{(carrying cost per unit)}}}$$

cost decreases at various quantity levels. The economic order quantity in a discrete case cannot be derived analytically in the usual manner.[3]

FIGURE 7. E. O. Q. For Item With Price Breaks



Inventory Characteristics of Metals Centers

The E. O. Q. formula approach has other serious drawbacks in addition to the problem of price or quantity discounts. If demand is not assumed to be known in advance, the E. O. Q. formula breaks down because the computation of an adequate safety stock to guard against shortage distorts the formula's meaning. Systems have been developed which start with a level of service to be maintained,

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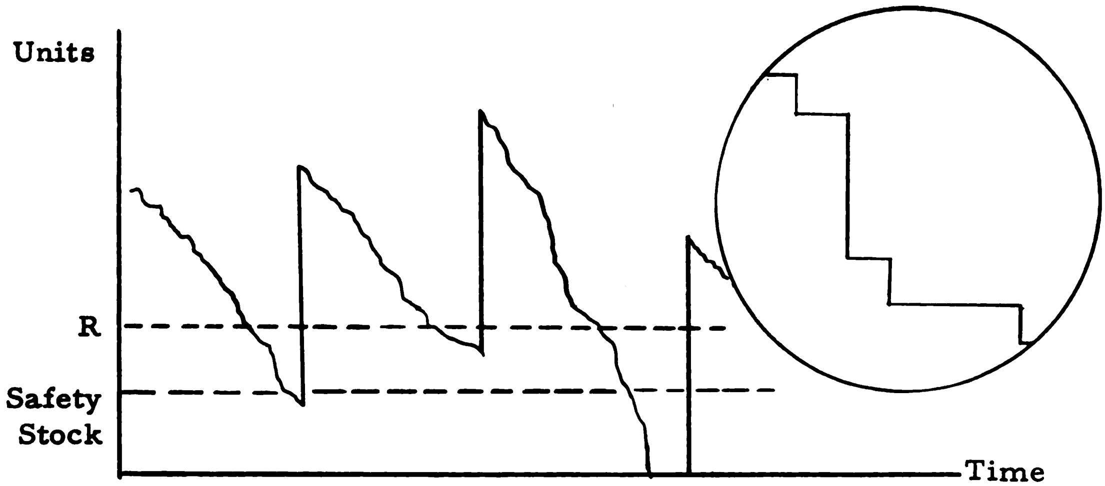
[3]For a solution method see Eliezor Naddor, op. cit., pp. 94-96.

but a rational service level depends partially on the cost of not maintaining that service. Thus an acceptable service level should be an output of the system and not an input. The reorder point, and hence the service level, may be determined by matching the marginal cost of maintaining extra service with the marginal cost of being out of stock. In other words, not only is the quantity ordered, Q, important, but the reorder point, R, which determines the safety stock and service level, is of equal importance. The value of R affects the probability that we will be out of stock during any given time period. Lead time is the period between the reorder point and the point when merchandise is received. Lead time may not only vary between items but also may vary for the same item on different orders. In this study varying lead time is not taken into consideration. Varying demand, however, is considered. FIGURE 8, inventory on hand, is shown in a simulated demand situation. As can be seen by the out-of-stock occurrence toward the end of the graph, demand is not a smooth, determinable function.

The insert in FIGURE 8 shows a small portion of the demand schedule from the main figure. Notice that while demand has been shown as a sloping line, in actual practice it is made up of a series of vertical and horizontal lines. The horizontal lines represent time. The distance between one vertical line and the next is the time between drawings from inventory.

FIGURE 8. Graphical Representation of Inventory Levels Over Time



Although the problem of estimating demand is often very complicated, in constructing a price investment model in this study, the problem is assumed away by taking annual demand as given. However, the probabilities of various demands during the lead time must still be estimated. This problem was discussed in Chapter IV.

The Demaree-McMillan model is a major component of the inventory cost-pricing model and is, therefore, explained in detail in this chapter. As previously noted, the Demaree-McMillan model uses a gamma distribution, but since the inventory cost-pricing model uses a form of poisson distribution, the Demaree-McMillan model will be explained in terms of a poisson distribution.

The Demaree-McMillan Model

The probability of being out of stock during any one inventory cycle is the probability of having a demand that exceeds the amount on hand during the period between the time that more stock is ordered and the time it arrives. The expected demand during lead time is the mean M of the poisson distribution of orders. In constructing the Demaree-McMillan model the first step is to identify a simple poisson distribution. The formula for the density of the poisson at any given point is:

$$P(R) = \frac{e^{-M} M^{R}}{R!}$$

where R is the reorder point and M is the distribution mean and variance.

Demaree and McMillan use two basic formula in their inventory model. The first is for the optimal probability of a stockout; that is, for the point at which the combined total of stockout cost and carrying cost is minimized.[4]

The formula is:

$$\text{Optimal Probability of a Stockout} = \frac{(\text{Material Cost*})\ (\text{Carrying Cost})\ (\text{Quantity Ordered})}{(\text{Shortage Cost})\ (\text{Annual Demand}) + (\text{Material Cost})\ (\text{Carrying Cost})\ (\text{Quantity Ordered})}$$

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[4]For a derivation of this formula see Naddor, *op. cit.*, pp. 235-242.

*Material cost is sometimes called the variable order cost.

The following notation is used through the paper:

C_2 = Annual Carrying Cost (expressed as a decimal)

K = Expected Annual Demand

D = Actual Demand During Lead Time

b_j = Material Cost (varies according to quantity schedule)

R = Reorder Point

Q_j = Quantity Ordered

M = Mean of Demand During Lead Time

P_j = Sales Price Charged by Metals Center

A_2 = Cost of Ordering Material

A_4 = Added Costs Incurred Because of a Stockout

The Demaree-McMillan model utilizes the formula:

$$(V, 1) \qquad P(D>R) = \frac{C_2\, b_j\, Q_j}{(P_j + A_4 - b_j)\, K + C_2\, b_j\, Q_j}$$

to obtain the optimal probability of being out of stock, and so can be compared with an array of stockout probabilities to find an optimal reorder point, R. It is possible to convert the formula to a slightly different form.

$$(V, 2) \qquad P(D>R)\,(P_j+A_4-b_j)\frac{K}{Q_j} = C_2\, b_j - P(D>R)(C_2\, b_j)$$

Here the marginal cost of a stockout is compared directly with the cost of carrying extra inventory to reduce the probability of a stockout. When the two costs are equal, the optimal probability of a stockout has been reached. The section of the formula reading - P(D>R) (C_2 b_j) has been added so that the marginal inventory carrying cost will not be overstated.

The other formula which Demaree and McMillan use is the formula for the total annual cost. The formula is:

Total Cost = (Cost of being out of stock) (The probability of being out of stock) (Annual Demand/Quantity Ordered) + (Average Inventory) (Material Cost) (Carrying Cost) + (Cost of Ordering) (Annual Demand/Quantity Ordered) + (Annual Demand) (Material Cost)

(V, 3) or

$$\text{Total Cost} = P\,(D>R)\,\left(\frac{K}{Q_j}\right)(P_j + A_4 - b_j) + \frac{Q_j}{2} + (R-M)b_jC_2 + \frac{K}{Q_jA_2} + Kb_j$$

The formula for the optimal probability of being out of stock (1) is a marginal formula, whereas the total cost formula obviously is not. The total cost formula is used to determine the best quantity to order. This entails repeating the formula for each price break in a quantity schedule. It could also be used to compute the optimal reorder point, but the process would be very inefficient. Therefore, the Demaree-McMillan model uses probability tables and formula (V, 1) for optimal probability of a stockout to compute the reorder point. Because R and Q are interdependent, the formulas may be repeated alternatley to achieve a higher degree of accuracy.[5]

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[5]For a complete explanation of the Demaree-McMillan model, see Claude McMillan and John Demaree, op. cit.

CHAPTER VI

PRICING FORMULAS

Introduction

This chapter summarizes some attempts made by firms in the industry to alter the traditional price schedules. When traditional price schedules, such as those discussed in Appendix B, were first used, they often bore no relationship to historical costs and they are certainly unrelated to present higher costs. An intensive effort has been made to have processing charges reflect processing costs. Yet the problem of material costs corresponding to prices has received substantially less attention. The pricing portion of the inventory cost-price formula is closely related to the formulas for target rate of return.

The study of small firms' pricing policies by Lanzillotti[1] revealed a marked tendency for small firms to go through the motions of pricing on the basis of internally generated information when they

[1]Robert L. Lanzillotti, Pricing, Production and Marketing Policies of Small Manufacturers (Pullman, Wash.: Bureau of Economic and Business Research, Washington State University, Bulletin No. 40).

actually rely upon larger competitors to set the general price level. Thus the determination of pricing policy for the industry essentially falls on the larger producer.

In 1949 Professor Eiteman[2] advocated a pricing policy which hinted at formula pricing, though he did not explicitly use it. Because his analysis was predicated mainly on the single-product firm, fixed and variable cost allocations were not of particular importance. Since that time, the prevalence of target rates of return in pricing policies has been attested to by several researchers.[3] In fact, pricing on the basis of a target return has been one of the methods widely used by businessmen.

Traditionally, pricing in the metal center industry has been rather haphazard. Prices are often implicitly supported by layers of local trade customs, examples of which are shown in Appendix B, rather than being based on varying associated costs. Books designed to guide the businessman often stress the problem of locating

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[2] Wilfred J. Eiteman, Price Determination: Business Practice Versus Economic Theory (Ann Arbor, Mich.: Bureau of Business Research, University of Michigan, 1949), pp. 28-31.

[3] Robert Lanzillotti, "Pricing Objectives in Large Companies," American Economic Review, December, 1958, pp. 923-932.

profitable and unprofitable products but few have advocated an intensive use of pricing strategies to increase profits.[4]

An American System

A large American service center has done extensive work in arriving at cost-justified pricing. While its main efforts have been directed towards processing charges, the establishment of some cost-justified pricing of merchandise was attempted. Their general system for standard (exclusive of quantity extras) prices is shown in FIGURE 9 (see page 98).

There are certain procedural problems with the formula, but on the whole it represents a significant departure from the traditional pricing schemes. Its principle of determining price on the basis of direct investment and expenses is not merely acceptable but has conceptual advantages over existing methods. The major limitation of the system is not with its conception but with its execution. The system uses "expenses" for one of the base factors, but these "expenses" are probably a conglomeration of direct and indirect expenses. A substantial portion represents arbitrary allocations of

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[4]An exception to this is John Y. D. Tse, Profit Planning Through Volume-Cost Analysis (New York: The MacMillan Co., 1960).

FIGURE 9. Development of Standard Prices
Based on Expenses and Desired R.O.I.

Investment = Inventory + Plant + Accounts Receivable + Cash

Assumed Desired R.O.I. = 20% before taxes

Solve in terms of "price spread" over cost and add back to costs.

Price spread = 20% x sum of following per 100#:
Material Cost/Annual Turnover of Product
Plant Investment/Annual Turnover
Accounts Receivable/Annual Turnover
Cash Requirements/Annual Turnover

Accounts Receivable = Material Cost + Expenses + Price Spread

Cash = Material Cost + Expenses

EXAMPLE

(Plant investment is calculated as 1/3 of
material cost for this product group)

Material cost = $8.00/100#
Expenses (base quantities) = .40 + .10 or .50/100#
Annual turnovers = 4 on material and plant space, 12 on receivables, and 20 on cash

Inventory	$8.00/4	$2.00
Plant (1/3 of inv.)	2.67/4	.67
Accounts Receivable	(8.00+.50+Profit spread)/12	.71+P.S./12
Cash	(8.00+.50)/20	.43
TOTAL		3.82+P.S./12
X Desired Return on Investment		.20
Price Spread =		.76+P.S./60
	=	.77

Price = $8.00 + .50 + .77 = $9.27/100#

plant and general expenses. These allocations inhibit the formation of optimal price decisions.

Another problem is that by grouping inventories, one loses the cost and inventory characterisitics of widely varying items. For instance, there are three main shape categories of carbon steel bars--rounds, flats, and squares. Generally, flats and squares are slower moving than some rounds. The system differentiates by shape, adding an extra charge to flats and squares because their average demand is slower. As a specific example, a one inch round bar may have twenty times the demand of a 1 1/32 inch round bar, but the system groups them all together even though purchase quantities will be larger and cost lower for the one inch bar. Also, the one inch bar will probably have a faster turnover.

The other system discussed in this chapter, from Dominion Steel of Canada, has a slightly different purpose than the American formula.

The Canadian or Dominion Steel Formula

The Dominion Steel formula is essentially a fighting formula in that it is designed for the selfish purpose of maintaining the competitive position of the firm in relation to its rivals. It attempts to relate prices to costs and remove the incentives for selection price cutting by competitors. In this formula, shown in FIGURE 10, some of the drawbacks of the American system are eliminated. Dominion has changed from a group calculation to an item calculation and has dropped out a return on cash investment. However, Dominion adds a profit spread to the factor for accounts receivable, which could be criticized if furnishing credit is part of the expense of the sale.

The use of what is essentially a plant and equipment turnover ratio times the cost of sales for one of the factors is really a combination of two utilization rates--one for plant and equipment necessary to receive, assemble, and ship orders; and another for plant and equipment necessary to store the inventory. The first costs are correctly handled, but the second costs should be related to inventory turnover rates. An adequate theoretical model would separate this cost into its component parts. If this is not done, the model loses some of its crisp, cost-pricing and competitors will concentrate on those products with the greatest return, selectively undercutting prices to gain customers. With optimum pricing, which yields a uniform return, other firms could still undercut on certain items, but the returns would be substandard and yield less profit than following the leader's prices.

Interviews with Dominion Steel management have established that the total costs mentioned in FIGURE 10 are really total variable

FIGURE 10. Dominion Steel Method of Calculating
Standard Prices on an Item Basis

Profit Spread = RR* (material cost/turnover + plant and equipment investment/total annual cost of sales x total cost of sales for each item + accounts receivable factor x (stored costs + item cost + order cost + material cost + profit spread))

= .20 x(material cost/turnover + .5 (stored cost + item cost + order cost + material cost) + .14 (stored cost + item cost + order cost + material cost + profit spread))

= .20 material cost/turnover + .65 stored cost + .64 item cost + .64 order cost + .64 material cost + .14 profit spread

= .20 material cost/turnover + .128 stored cost + .128 item cost + .128 order cost + .128 material cost + .028 profit spread

.972 profit spread = .20 mat. cost/turnover + .128 stored cost + .128 item cost + .128 order cost + .128 mat. cost

Profit spread = .206 mat. cost/turnover + .132 (stored cost + item cost + order cost + mat. cost)

However, stored cost + order cost + item cost + material cost = total cost

Therefore, Profit spread = .206 mat. cost/turnover + .132 total cost

Selling Price = .206 mat. cost/turnover + 1.132 total cost

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*RR = return required

Note: Assume 20 per cent required return, capital turn of .5 and 45 days in accounts receivable.

costs. The 20 per cent desired return includes a portion to cover fixed costs. Other assumptions in the example shown include a "Capital Turnover" of .5 and an average of 45 days in accounts receivable.

The American method is based on some excellent cost monitoring and control systems but stumbles theoretically. The Canadian system, which is an adaptation of the American system, is preferable theoretically, but, at the time the author gathered his information, was deficient from a practical standpoint primarily because of an ineffective inventory system.

The pricing section of the inventory cost-pricing model, shown in FIGURE 11, follows the same general lines as the Dominion Steel model. But the pricing section does not use the profit factor in calculating the desired return on accounts receivable. It splits out a factor for return on inventory plant space investment, but does not use a factor for assembly space, which is a fixed rather than a marginal cost. The desired return must be sufficient to cover those costs not explicitly covered in the section. This actually allocates fixed cost on the basis of investment as a determinant of price.

The following formula (FIGURE 11), as well as the two previously discussed formulas, are based upon an order of a given size. They may be for the largest size order, in which case a quantity

FIGURE 11

DESIRED RETURN (Average inventory)/ Annual demand

+ Direct costs of filling orders

+ Cost of Warehouse space. (Average inventory)/Annual demand

$$+ \frac{\text{Cost of Ordering}}{\text{Quantiry Ordered}} + \text{Material Cost}$$

+ Accounts Receivable Factor . Material Cost +Direct Costs + Cost of Ordering/Quantity Ordered

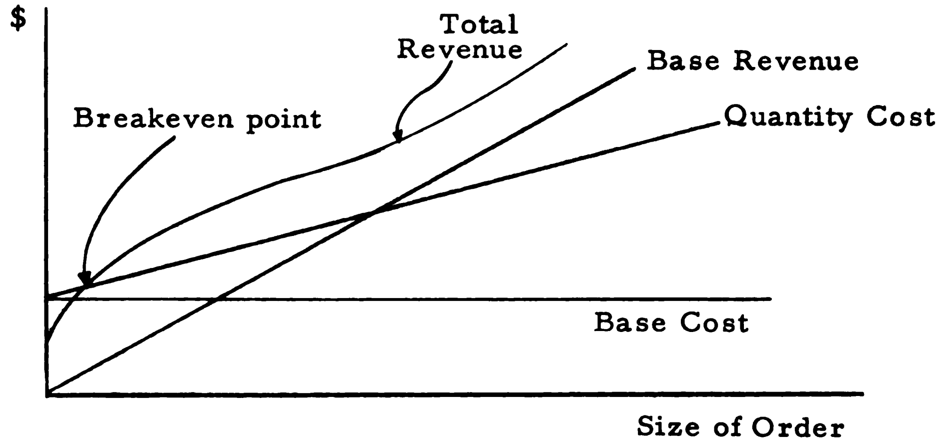
$$+ \text{Cost of Warehouse Space} \ \frac{\text{Average Inventory}}{\text{Annual Demand}}$$

$$+ \text{Material Cost} \ \frac{\text{Average Inventory}}{\text{Average Demand}} \ . \ \text{Carrying Cost}$$

$$+ \text{Carrying Cost} \cdot \text{Material Cost} \cdot \frac{\text{Average Inventory}}{\text{Annual Demand}}$$

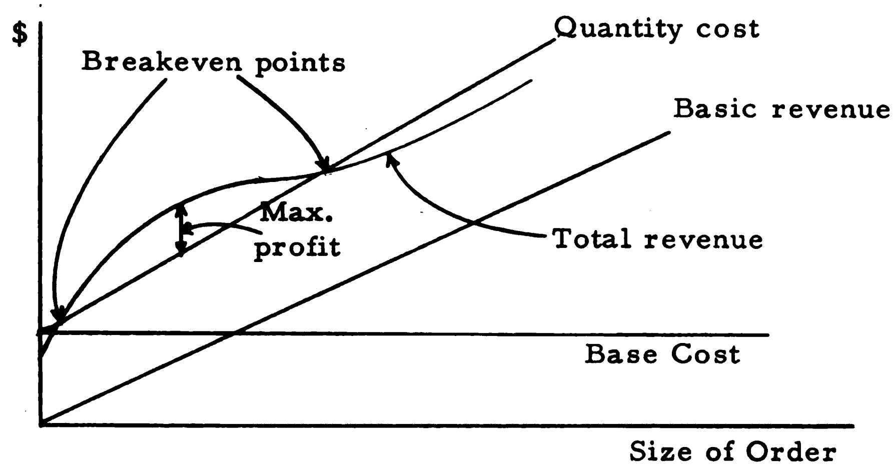
schedule is added for lesser quantities, or it may be based on some other size order which will probably have quantity schedules with additions or subtractions for different sizes. The formulas, however, only give the price for a given quantity. The costs associated with various quantities are possible to graph, thereby enabling their comparison with the base revenue to obtain a general idea of the necessary quantity schedule. FIGURE 12 illustrates a typical situation. (See page 104.)

FIGURE 12



The relationship between costs and revenues will vary between firms, not only due to variations in overall efficiency, but also to variations in the order size for which the company is most efficient. FIGURE 13 illustrates the cost revenue relationship for a firm which is most efficient in small- and medium-sized orders.

FIGURE 13



A company may have more success in imposing a quantity schedule on the industry with item base pricing. As an alternative to price leadership, the differential in cost curves may cause the firm to concentrate on their order sizes which bring it the most profit.

The pricing formulas are only for a certain size order, and whether they are to be for large orders or small should be decided before gathering the cost data necessary to initiate the computations. This is a minor question in a pricing formula, but in the inventory cost-pricing model explained in Chapter VII, the size question must be considered in conjunction with the quantity schedule.

CHAPTER VII

DESCRIPTION OF INVENTORY COST-PRICING MODEL

Introduction

This chapter develops a model linking pricing and inventory models into a unified whole. The theory of competitive pricing assumes an upward sloping long-run marginal cost curve and an optimal size for firms. It will be assumed that in the long-run, marginal costs are flat or downward sloping for all relevant firm sizes in the metal center industry. A second assumption is that if the firm takes the initiative of pricing lower than the market, the price will usually decline equally. This type of pricing may continue until marginal cost equals marginal revenue, the time when firms are likely to incur losses. However, owners or managers can be expected to watch carefully their own effect upon the market. Soon they learn to follow the established market price or a recognized market leader.

The price level bears an indirect relationship to the cost patterns. When the price level is too high, additional firms will be attracted to the industry. If the extra firms entering the industry cause lower than normal profits, some firms may be tempted to discount prices in order to increase volume. A certain amount of price cutting by marginal firms is normal, but excessive discounting will lead to price warfare because the established firms can no longer afford to ignore the discounters. Cyclical variations encourage both the entrance of new firms and price cutting.

The American and Canadian pricing formulas are designed to discourage selective discounting by equalizing the return on all products, but, as mentioned in Chapter VI, each contains a flaw limiting efficient application. The formula advanced in this paper is intended to fulfill this same purpose. Pricing formulas all assume an optimizing inventory policy. If unreasonable amounts are invested in inventory, price formulas, which depend heavily on inventory turnover, would give higher than optimal prices. Assuming quantity discounts, when a firm purchases in small quantities, its material costs will be higher. In order to have the best pricing system, a preferable formula would strike a balance between excessive investment in inventories and excessive material prices between carrying costs and shortage costs.

Chapter V mentioned that one of the major determinates of an optimal inventory policy is shortage cost. In metal centers most shortage costs are composed of two parts--the margin foregone and the cost of acquiring the item from a competitor. The margin is equal to the price minus the material cost. Price, therefore, becomes a major determinate of inventory policy by influencing the equilibrium between the annual cost of carrying inventory and the shortage cost. The comparison of shortage and carrying costs determines the best reorder point R. Thus, price should be considered in any rational inventory policy, and conversely, inventory policy acknowledged as part of determining prices.

Pricing methods described previously implicitly assume the inventory policy to be such that price would be higher if any other policy were chosen. From the standpoint of resource allocation, preference will be given the lowest price consistent with a fair rate of return. Hence, price, subject to the constraint of a stated return, should be minimized.

The Demaree-McMillan inventory formula and the inventory section of the inventory cost-pricing model search for order quantities which minimize total cost. A necessary part of these formulas is a description of demand during lead time for which Demaree and McMillan use the gamma distribution. The Adelson compound poisson formula mentioned in Chapter IV is used in the inventory cost-pricing model. R. M. Adelson's basic formula is:

$$R_{j+1} = \frac{1}{J+1} (a_1R_j + 2a_2R_{j-1} + 3a_3R_{j-2} + \ldots + j\, a_j\, R_0)$$

where $R_0 = e^{-\sum a_i}$

and

i = size of the orders in an arrival stream.

a_1 = mean of the orders of size 1.

a_2 = mean of the orders of size 2.

etc.

R_0 = Probability of zero units.

R_j = Probability of exactly j units of demand.

The formula yields an exact tabulation of the probability of exactly R units of demand for any compound arrival stream. It follows that the sum of the computed R's is the cumulative left tail distribution. Obviously, the right tail is 1 minus the left tail, or

RSUM - (1 - R) = Probability of demand greater than R, or $P(D>R)$.

RSUM is the probability of being out of stock with a given reorder point R_j. In other words, RSUM is the probability that R_j or more units will be demanded during the lead time.

The optimal RSUM is computed in the formula (2) from Chapter V which is:

$$P\,(D>R)\,(Pj+A_4+b_j)\cdot\frac{K}{Q_j} = C_2 b_j - P\,(D>R)\,(C_2 b_j).$$

A routine for computing the RSUM formula for a compound poisson is given in Appendix E.

In this section the optimal reorder point is reached when the expected shortage cost:

$$P\,(D>R)\,(P_j+A_4-b_j)\,\frac{K}{Q_j}$$

is equal to the annual marginal carrying cost:

$$C_2b_j-P(D>R)\,(C_2b_j).$$

The probability of being out of stock--assuming a reorder point R and mean M, can be taken as the probability of demand greater than R.

As shown in Chapter V, the marginal probability of being out of stock (P (D>R)) times the cost of being caught short, should be equal to the marginal cost of carrying inventory to guard against the shortage.

At this time it is necessary to extend and simplify the notation found in Chapter V. Values are added to aid in constructing an example. To review:

		Example Values
K =	expected annual demand	50
R =	reorder point	7.5
M =	mean of the demand during lead time	5
C_2 =	annual carrying cost	.10
D =	actual demand during lead time	
P_j =	price charged by the metals center	

		Example Values
A_2 =	cost of ordering material	\$2.50
A_4 =	added costs incurred because of a stockout	\$2.50
C_1 =	the return on investment above variable costs required by the firm for profit and overhead costs	.40
C_3 =	the annual cost of warehouse space	\$ .50
C_4 =	cost of carrying receivables	.017
A_3 =	the direct cost of filling orders	\$ .75

		Example Values Q_j	b_j
Q_j =	order quantity	10	\$11.50
b_j =	material cost	20	10.40
		40	10.10
		60	9.80
		100	9.60
		200	9.55
		400	9.50

The notation is simplified by designating some values by single capital letters:

A = average inventory $(\frac{Q_j}{2} + R-M)$

B = annual ordering cost $(A_2 \frac{K}{Q_j})$

C = the cost of being out of stock ($P_j + A_4 - b_j$)

F = the annual cost of carrying one unit in inventory ($C_2 b_j$)

The Inventory Cost-Pricing Model

The model consists of three main parts: (1) a section to compute and minimize price; (2) a section to compute the probability of being out of stock, the financial consequences of being out of stock, and to compare them with the cost of preventing the stockout; and (3) a section to compute and minimize total cost.

These sections are interrelated and each depends to some extent upon the solution of the other two for its solution. Using arbitrary values for the outputs of the two sections, one could select any of the sections to start the computations. However, the system seems to reach a final solution faster if minimum price is the first value.

In this section of the model the computations involved in finding a minimum price require substituting various values from a quantity price schedule into a single formula and selecting the combination which yields the lowest price. The formula to compute price is:

Required return times material cost times the average inventory, divided by the estimated yearly demand $(C_1 b_j (A)/K)$

plus the direct costs of filling the order (A_3) plus the space cost times the average inventory divided by the estimated demand ($C_3(A)/K$) plus the order cost divided by the order quantity (A_2/Q_j) plus the material cost (b_j) plus the cost of carrying receivables times the amount invested in receivables divided by the estimated demand per year ($C_4(b_j+A_3+A_2/Q_j+C_3(A)/K+C_2(b_j(A)/K))$) plus the carrying cost times the material cost times the average inventory divided by the estimated annual demand ($C_2b_j(A)/K$). The complete formula is:

$$P_j = C_1b_j(A)/K+C_3(A)/K+A_3/Q_j+b_j+C_4(b_j+A_3+A_2/Q_j$$
$$+C_3(A)/K+C_2(b_j(A)/K))+C_2b_j(A)/K$$

In the example the tentatively optimal price is:

$$A = (\frac{60}{2} + 2.5)$$
$$= 32.5$$

$$P_j = .40 \times 9.80 \times (32.5)/50 + .75 + .50 \times (32.5)/50$$
$$+2.50/60 + 9.80 + .017 (9.80 + .75 + 2.50/60$$
$$+.50 \times (32.5)/50 + .10 (9.80 \times (32.5)/50))$$
$$+.10 \times 9.80 \times (32.5)/50 \text{ or}$$

$$P_j = \$14.298$$

In other words, the price charged is established by adding all the variable costs and attaching a return to cover fixed costs to the average inventory. Purchase quantities and invoice costs are changed and price recomputed. When price has been computed for each order quantity, the minimum price is chosen. This minimum price and its corresponding order quantity and cost are used as inputs in the next section to compute the best reorder policy.

Comparison of Shortage and Carrying Costs

The actual comparison of shortage and carrying cost is relatively simple. The complicated part is the computation of the probability of demand greater than R. This computation was discussed earlier. In the shortage cost, $(P_j+A_4-b_j)$, the term A_4 contains not only the extra costs associated with acquiring material, but the difference between the expected price, which includes quantity extras, and the calculated price (P_j).

The price P_j in the shortage cost is the price calculated in the pricing section of the model. However, the material cost b_j may be from either that section or the section dealing with total cost depending upon which assumptions, explained in Chapter VIII, are used in the model.

The formula used in this section may be either:

$$P(D>R)C\frac{K}{Q_j} = F - P(D>R)F \quad \text{or:}$$

$$P(D>R)C\frac{K}{Q_j} + P(D>R)F = F$$

The formula used in this section of the inventory cost-pricing model is:

$$P(D>R)C\frac{K}{Q_j} = F - P(D>R)F$$

In the example this value is reached using a simple poisson. The formula becomes:

$$C = (14.30 + 2.50 - 9.80) = 7.00$$

$$F = .10\ (9.80) = .98$$

$$P(D>R)\ 7.00 \times \frac{50}{60} = .98 - P(D>R)\ .98$$

So, $P(D>R) = .144$ at the optimal point, and the reorder point which comes closest to having that probability is 8, so R = 8.

In any case, the reorder point R is increased or decreased until the two sides of the formula are approximately balanced. Since the poisson is a discrete distribution, an exact answer is not feasible. At this time the model continues to the computation of the lowest total cost.

Computing the Lowest Total Cost

The computation of total cost is needed to find the optimal reorder quantity for the inventory solution of the inventory cost-pricing model.

The formula for total cost is material cost (b_j) times the expected annual demand (K) plus the average inventory (A) times the annual cost of carrying one unit (F) plus the annual ordering cost (B) plus the cost of warehouse space (C_3) times the average inventory (A) plus probability cost of being caught short $P(D > R)$ x (C) times the expected demand during lead time (M) times the number of order periods in a year (K/Q_j) plus total direct costs (A_3K) plus the cost of carrying receivables (C_4) times all expenses $((b_j+A_3+A_2/Qj)K+b_jAC_2)$. The complete formula is:

$$TC = Kb_j+FA+B+C_3 A +P(D > R)CMK/Q_j+A_3K+C_4((b_j+A_3+A_2/Q_j)K+FA).$$

After substituting various order quantities (Q_j) and prices (b_j) in computing the total cost, a comparison is made between total costs and the lowest is chosen. The corresponding Q_j is the optimal order quantity for the inventory system.

In the example given, the values from the pricing and inventory formulas substituted in the example are:

$$B = (2.50 \times 50/60) = 2.08$$

$$\text{Total cost} = 50 \times 9.80 + .98 \times 33 + 2.08 + .50 \times 33 + .144 \times 7.00 \times 5 \times 50/60 + .75 \times 50 + .017 ((9.80 + .75 + .04) \times 50 + .98 \times 33)$$

$$\text{Total cost} = 592.17$$

After finding the lowest total cost, the cycle is repeated until all values are stable. Practical experience has shown that usually two cycles are sufficient to reach all optimum points when these formulas are used in a computer program.

FIGURE 14 on page 118 is a flow chart showing in simplified form the actual computer program used to compute actual values from the formulas.

General Comments and Assumptions

If all costs were variable, then the carrying cost, which includes cost of capital, could be the same as the return on investment. However, when some costs are fixed, the return on investment really becomes a return on investment plus a contribution to fixed cost. This makes minimizing price and minimizing total cost somewhat incompatible, so a choice must be made as to whose interests are to be served first.[1] Price can be minimized given the lowest total cost, or total cost minimized given the lowest price. A third choice is to solve for lowest cost and price without the constraints of requiring compatibility in the solution.

At first glance, the model should find the lowest cost and then compute the lowest price, given that lowest cost with its corresponding order quantity and material cost. Since between the buyer and

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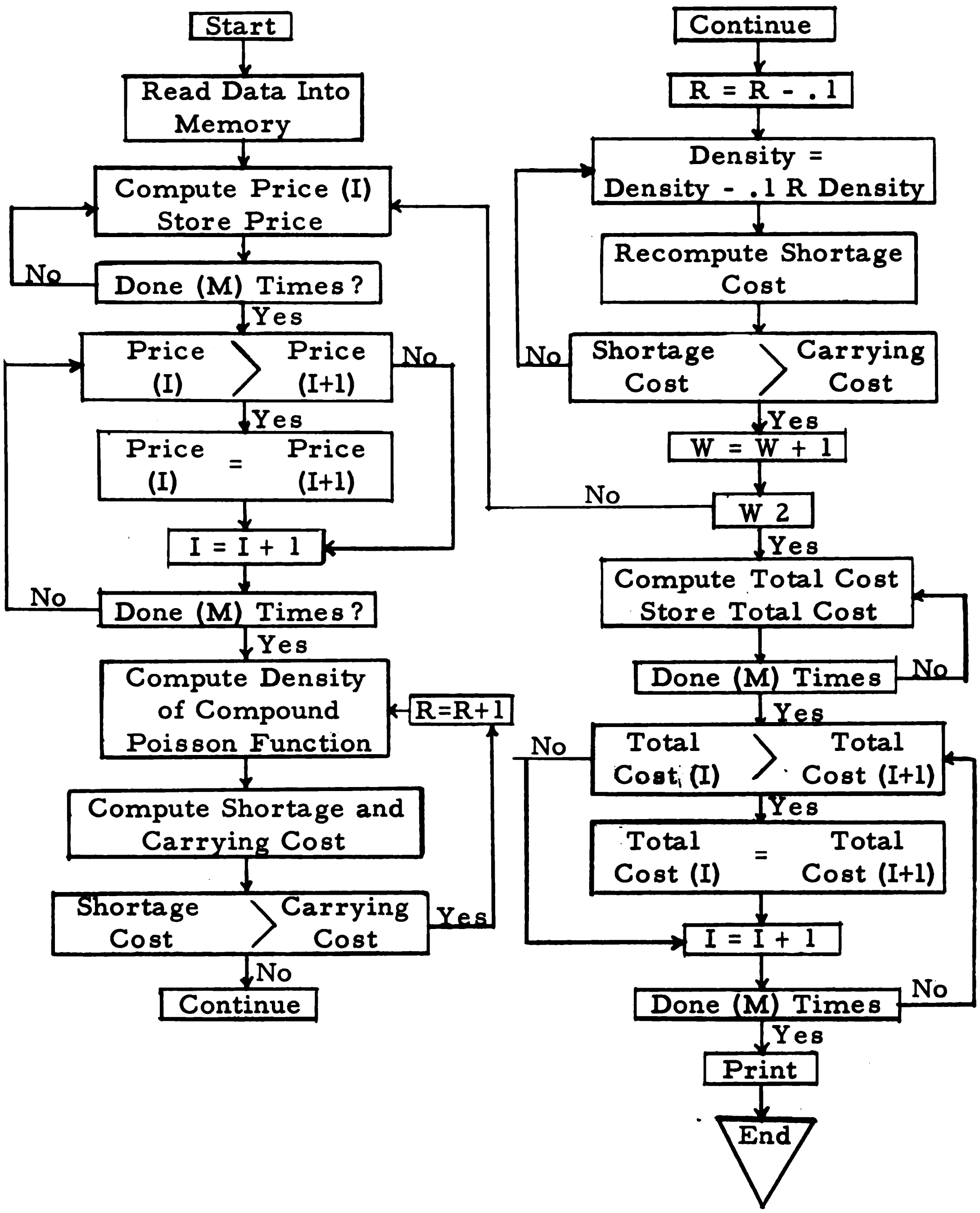
[1]This dilemma is yet another of the conundrums posed by joint costs because in a single-product firm complete allocation could rationally be made.

FIGURE 14. Inventory Cost-Pricing Model Flow Chart



the seller one's loss is the other's gain, there is no particular reason to prefer one to the other, except to the extent that demand is not perfectly inelastic, a higher price results in a restriction of output. On these grounds alone, the price should be set at its minimum value. The company is free to pursue a lowest cost policy partially independent of the pricing mechanism. There is still a link between the formulas through the calculation of shortage cost and its inclusion in the total cost computation. The company will not make its desired rate of return on total investment if it manages inventories to minimize total cost, but it can always manage inventories using the quantity and cost from the pricing formula and be assured of the required rate of return. The company will, however, be better advised to increase quantity until total cost is minimized, because up to this point, increases in quantity result in returns higher than the marginal cost of increasing inventory. Therefore, the company will be better off to increase its inventory until the added returns just balance the added costs. The company receives less than its desired return on investment on the added inventory, but it is still more than the incremental cost of the investment as measured by the carrying cost. With these amended priorities the model strives for an optimal solution within the confines of its constraints.

The model is shown in Appendix D as a computer program. Chapter VIII presents and discusses some results obtained by running the program on an IBM 360-50 Computer.

CHAPTER VIII

RESULTS OF THE MODEL

Introduction

The inventory cost-pricing model has been programmed for an IBM 360-50 and has been run on that machine. Because the model has various conditions which must be fulfilled and these conditions are not always compatible, the model was run several times with minor changes until an output was obtained which seemed to provide the best compromise of the criteria. For instance, it was found that uncoupling the computation of minimum price and minimum cost could be achieved by simultaneously computing the optimal probability of a stockout and the lowest minimum price. These results were then used to find the reorder quantity which would result in the lowest total cost. Each time the model was run the annual expected demand was varied in units of 1,000 pounds from 1,000 to 20,000 pounds per year. Although in some instances the model was run for annual demands up to 40,000 pounds per year, it was found that most patterns emerged clearly within the range of 1,000 to 20,000 pounds per year and, therefore, to save computer time, the model was not used extensively above 20,000 pounds.

Actual Running of the Model

The model was used to simulate expected demands from 1,000 to 20,000 pounds per year. TABLE 4 shows relationships which exist between the various outputs when inputs are at their "normal" levels. Notice that minimum price decreases more rapidly than average cost so predictably the markup decreases for large quantities.

One of the model's unresolved dilemmas has been the different strategies necessary to obtain the lowest price and cost. It is unlikely that the lowest cost computations and the lowest price computations would use the same order quantities and the same order prices. In fact, the order quantities turn out to be critical here and the reorder points are not specifically significant. The pricing formula and the shortage cost formula are determined concurrently and, therefore, the changes in order quantity necessary to achieve the lowest price have an immediate effect upon the computation of the optimal probability of a stockout. Because the actual order quantity is determined by the computation of the lowest total cost, this order quantity becomes a shadow order quantity. Therefore, the shortage cost is computed for a reorder quantity which may not be followed by the firm. It

TABLE 4. Output Results Obtained Using "Normal" Values

Annual Demand K	Minimum Price P_j	Average Total Cost	Mean Demand during LT M	Reorder Point R	Ratio[a] R/M	Prob. of Stockout During Lead Time[b]	Reorder Quantity Q_j Computed from: Price P_j	Cost TC
10	17.10	12.73	1.6	4.01	2.50	.124	10	20
20	15.43	11.79	3.2	6.45	2.16	.121	20	20
30	14.20	11.51	4.8	9.57	1.99	.099	20	20
40	13.70	11.37	6.4	12.54	1.96	.082	20	20
50	13.40	11.17	8.0	15.41	1.93	.070	20	20
60	13.19	11.01	9.6	18.19	1.89	.061	20	20
70	13.03	10.89	11.2	20.90	1.87	.055	20	20
80	12.91	10.80	12.8	23.57	1.84	.049	20	60
90	12.81	10.73	14.4	26.17	1.82	.045	20	60
100	12.73	10.68	16.0	28.74	1.80	.041	20	60
110	12.66	10.63	17.6	31.27	1.77	.038	20	60
120	12.60	10.59	19.2	33.76	1.76	.035	20	60
130	12.55	10.56	20.8	36.23	1.74	.033	20	100
140	12.51	10.51	22.4	38.68	1.73	.031	20	100
150	12.34	10.45	24.0	38.12	1.59	.054	40	100
160	12.22	10.40	25.6	38.79	1.52	.069	60	100
170	12.15	10.37	27.2	41.02	1.51	.067	60	100
180	12.09	10.34	28.8	43.27	1.50	.064	60	100
190	12.04	10.32	30.4	45.5	1.50	.061	60	100
200	11.99	10.30	32.0	47.71	1.49	.059	60	100

[a]Ratio = Reorder Point (R)/Mean Demand During Lead Time (M). [b]Prob. DDLT > R.

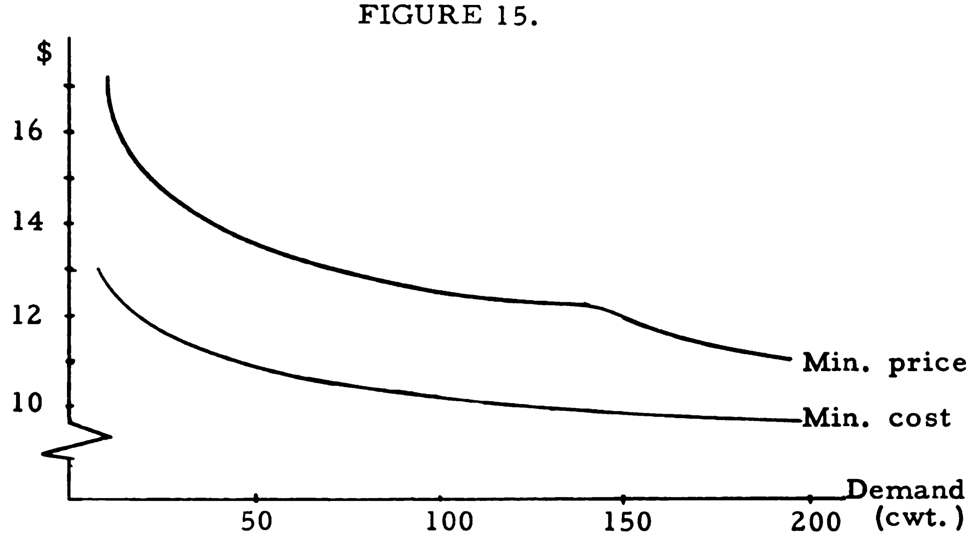
would, of course, be possible to recompute the shortage cost utilizing the actual optimal reorder quantities specified by the lowest cost formula, but then the pricing formula would not correspond to its ingredients. It is suggested that the present situation, while not optimal, represents the best compromise available at this time. Therefore, the reader must keep in mind when scanning the various data that there are certain times when larger quantities will result in higher probabilities of a stockout even though this seems to contradict some of the basic conclusions of the model.

As shown in TABLE 4, a consistent reorder pattern emerges in which the quantity computed from the optimal price is lower than the quantity computed from the minimum total cost. Also, as demand increases, the ratio between the reorder point and the mean expected demand during lead time usually decreases. However, in this particular case the ratio decreases so rapidly between 14,000 and 15,000 and again between 15,000 and 16,000 that the reorder point decreases between 14,000 and 15,000 and only returns to the level of 15,000 at 16,000. Correspondingly, the stockout probability has steadily decreased as volume increased, but at 14,000 the stockout probability jumps upwards and only above 16,000 again begins to decline. Although this does not seem logical at first glance, looking across at the minimum price column one sees that between 14,000 and 15,000 and between 15,000 and 16,000 the minimum price actually decreased by several times the amount it decreased between 13,000 and 14,000.

The drop in minimum price occurred because the optimal reorder quantity increased from 20 to 40 cwt. and then to 60 cwt. which allowed the lower price computation. The increase in reorder quantity also reduces the number of lead time periods per year and hence the shortage cost per lead time period is likely to increase. Despite the changes in reorder quantity, the ratio of the reorder point to the mean demand during lead time continues to drop as demand increases.

FIGURE 15 shows in graphic form the changes in price and cost associated with demand increases. From the precipitous early drops, the price adjustments practically disappear until between 14,000 and 16,000 pounds per year the changes in reorder quantity cause substantial decreases.

FIGURE 15.



Shortage Cost Changes

The model was run for shortage costs, A_4, from \$2.50 to \$10.00 in increments of \$2.50. TABLE 5 shows some results obtained at the usual out-of-pocket shortage costs, A_4, of \$2.50 per hundred weight and then again with an out-of-pocket shortage cost of \$7.50 per cwt. Many of the results are predictable. The increase in reorder point and the associated reduction in stockout probability are to be expected. The minimum price computation (P_j) is most affected with very small annual demands. However, the difference still amounts to 12 cents per 100 pounds when annual demand is 20,000 pounds per year. This is down from 66 cents difference in minimum price when annual demand is only 1,000 per year. Approximately the same relationships apply to the minimum average cost computed. The probability of a stockout calculated has not been included in TABLE 5 because it bears a direct relation to the reorder points shown. For instance, during a particular lead time when the direct costs of being out of stock are \$2.50 and the expected annual demand is 1,000 pounds per year, the optimal probability of being out of stock during that lead time is .124. The optimal probability is reduced to .077 when A_4 is increased to \$7.50. However, the probability of being out of stock any specific number of units is not computed. Thus, the model does not give a direct answer to the question:

TABLE 5. Changes in Output Caused by Changes in Shortage Cost - A_4

	A_4 = \$2.50					A_4 = \$7.50				
Annual Demand K	Q_j from P_j	Q_j from TC	Reorder Point R	Minimum Price P_j	Average Total Cost	Q_j from P_j	Q_j from TC	Reorder Point R	Minimum Price P_j	Average Total Cost
10	10	20	4.0	17.10	12.73	10	20	4.8	17.76	12.86
20	20	20	6.4	15.43	11.79	20	20	7.8	15.89	11.90
30	20	20	9.6	14.20	11.51	20	40	11.2	14.54	11.60
40	20	20	12.5	13.70	11.37	20	60	14.4	14.00	11.45
50	20	20	15.4	13.40	11.17	20	60	17.5	13.66	11.23
60	20	20	18.2	13.19	11.01	20	60	20.5	13.43	11.06
70	20	20	20.9	13.03	10.89	20	60	23.4	13.25	10.94
80	20	60	23.6	12.91	10.80	20	60	26.2	13.11	10.85
90	20	60	26.2	12.81	10.73	20	60	28.9	13.00	10.78
100	20	60	28.7	12.73	10.68	20	60	31.6	12.91	10.72
110	20	60	31.3	12.66	10.63	20	60	34.2	12.83	10.67
120	20	60	33.8	12.60	10.59	20	60	36.8	12.77	10.63
130	20	100	36.2	12.55	10.56	20	100	39.4	12.71	10.60
140	20	100	38.7	12.51	10.51	40	100	39.2	12.66	10.52
150	40	100	38.1	12.34	10.45	60	100	40.0	12.48	10.47
160	60	100	38.8	12.22	10.40	60	100	42.3	12.34	10.43
170	60	100	41.0	12.15	10.37	60	100	44.7	12.28	10.40
180	60	100	43.3	12.09	10.34	60	100	47.0	12.21	10.37
190	60	100	45.5	12.04	10.32	60	100	49.3	12.16	10.35
200	60	100	47.7	11.99	10.30	60	100	51.6	12.11	10.33

How many units will we be out of stock in the course of a year? However, one can obtain a related figure by multiplying the out of stock probability by the mean demand during lead time and dividing through by the quantity ordered. This gives the probability of not having an item in stock, given that customer orders it, and does not indicate how many units the inventory will be short. An interesting change is the emergence of the 4,000 pound order when the demand is 3,000 pounds per year. Again, at a certain demand level, prices fall to the extent that it becomes more advantageous, as shown by the diminished reorder point, to increase the possibility of a stockout. And, although the effect is not as pronounced as it is with lower direct shortage costs, it still occurs when shortage costs are relatively high. These changes occur because the price-determined minimum quantity moves from 20 to 40 and then to 60. Thus, since the number of lead time periods per year is the annual demand, K, divided by the quantity ordered, Q_j or K/Q_j, the overall probability of a stockout decreases though the probability per period increases.

A comparison of the $2.50 shortage cost run with another similar run which did not include stockout cost as part of the total cost computation, showed identical results except for the average cost. The difference within the range tested from 1,000 to 20,000 pounds per year was from 9 cents for 1,000 pounds per year to 31 cents for

20,000 pounds per year. One further difference is that there is a tendency for the order quantity consistent with the lowest total cost to be higher when the shortage cost is included in the computation. This is true because higher order quantities result in fewer stock cycles and hence lower shortage costs. Without this change the minimum total cost calculation is determined primarily by the interplay of quantity extras and carrying costs.

Required Return

The required return, C_1, which includes contributions to fixed costs, was run for rates between 10 per cent and 60 per cent. Due to limitations in the space available in TABLE 6, only information on returns of 10 per cent and 60 per cent are shown. These required returns affect investment and so the model shows the necessity of a reduction in average investment to receive the lowest minimum price as the required return increases.

Given an annual demand of 1,000 pounds, one orders 2,000 pounds at a time when the required return is 10 per cent, but 1,000 pounds with a required return of 20 per cent. Furthermore, at 60 per cent only 1,000 pounds are ordered for both 1,000 and 2,000 pound annual demands. With a 10 per cent return, the model switches to 4,000 pound orders at 7,000 pounds annual demand and then to 6,000

TABLE 6. Model Results for 10 and 60 Per Cent Returns

	$C_1 = .10$				$C_1 = .60$			
Annual Demand K	Q_j from P_j	Q_j from TC	Prob. DDLT>R	Min. Price P_j	Q_j from P_j	Q_j from TC	Prob. DDLT>R	Min. Price P_j
10	20	20	.247	14.22	10	20	.103	19.04
20	20	20	.167	13.06	10	20	.070	
30	20	20	.127	12.66	20	20	.085	15.29
40	20	20	.102	12.45	20	60	.072	14.58
50	20	20	.086	12.32	20	60	.062	14.14
60	20	20	.074	12.23	20	60	.055	13.84
70	40	20	.114	12.06	20	60	.049	13.62
80	60	20	.138	11.89	20	60	.045	13.45
90	60	60	.127	11.79	20	60	.041	13.32
100	60	60	.118	11.71	20	60	.038	13.20
120	60	60	.103	11.58	20	60	.033	13.03
140	60	60	.091	11.49	20	60	.029	12.90
160	60	60	.082	11.42	20	60	.026	12.80
180	60	60	.074	11.37	20	60	.023	12.72
200	60	100	.068	11.32	40	100	.039	12.63
250	100	118	.086	11.18	60	119	.046	12.20
300	100	200	.074	11.10	60	200	.040	12.03
350	100	200	.065	11.03	60	200	.035	11.90
400	100	200	.058	10.99	60	200	.032	11.80

at 8,000 annual demand; but for a 20 per cent return the model moves directly from 2,000 to 6,000 pound orders at 10,000 pounds annual demand. When the required return is 50 per cent, the switch from 2,000 pound orders is made at 17,000 pounds demand per year though only 4,000 pounds are ordered. However, at 18,000 pounds per year the model orders 6,000 pounds at a time. With a still higher required return of 60 per cent per year the model does not switch from ordering 2,000 pounds at a time until it reaches 19,000 pounds per year, and then for both 19,000 and 20,000 pounds per year the model orders 4,000 pounds and only switches to 6,000 pounds between 20,000 and 25,000 pounds demand per year. With very low rates of return, such as 10 per cent, one would order 10,000 pounds at a time if the annual demand were greater than 25,000 pounds a year. With a required return of 20 per cent, one would not order 10,000 pounds unless the estimated annual demand were 30,000 pounds or more per year. And with a required rate of return of 30 per cent, 10,000 pound lots are only ordered if the estimated annual demand is 40,000 pounds per year or more.

When the above decisions are contrasted with the reorder quantities necessary for the lowest total cost, a number of conclusions emerge. First, with a few minor exceptions caused by differences in shortage cost, the reorder quantities to minimize total cost do not vary with the different rates of return. Also, there are no cases in which reordering in 1,000 pound lots gives the minimum total cost. Reordering in 2,000 pound quantities is only advisable up to annual rates of demand of 3,000 pounds; at 4,000 pounds annual demand the model switches to ordering in 6,000 pound lots. At 20,000 pounds annual demand the minimum cost is obtained by ordering in 10,000 lots. Inventory practitioners have generally believed that orders should always be placed at the minimum quantity of a particular price bracket. The model, however, contains a provision for testing whether larger quantities within the price bracket will yield a lower total cost and this proves to be true with annual demands between 20,000 and 25,000 pounds per year. TABLE 6 does not show the demands of 21,000; 22,000; 23,000; and 24,000 pounds per year, but starting with 22,000 pounds per year the model finds a lower cost obtained by ordering in greater than the minimum quantities in the 10,000 pound price bracket. In particular, at 22,000 pounds per year the optimal order quantity is 10,500 pounds. At 23,000 pounds per year 11,000 pounds; at 24,000 pounds per year 11,400 pounds; and at 25,000 pounds per year, as shown in the figure, 11,900 pounds. For 26,000 pounds per year the order quantity is 12,300 pounds and at 27,000 pounds per year the model obtained the lowest cost by ordering 20,000 pounds per year. These do not change for different required rates of return because the average cost is only minimally affected by changes in the required rate of return. In this particular run of the model, an approximation of the shortage opportunity cost was included in the total cost computation with the result that average cost is larger when prices are higher. Consequently, there are small changes in the average cost associated with different rates of return. On the other hand, the minimum price is obviously going to be affected by changes in the required rate of return. The range of prices between the highest and the lowest rates of return, that is, 10 per cent and 60 per cent, is from $4.82 when annual demand is 1,000 pounds per year down to 81 cents when annual demand is 40,000 pounds per year. The markup above cost ranges from $6.16 for a rate of return of 60 per cent on items with an annual demand of 1,000 pounds per year down to 49 cents for a rate of return of 10 per cent and annual demands of 40,000 pounds per year.

Because the opportunity cost of a stockout is considerably lower when a required rate of return is 10 per cent than when it is 60 per cent, the probability of a stockout during lead time for items with an annual demand of 1,000 pounds ranges from .247 with a required return of 10 per cent down to .103 with a required return of 60 per cent. As the reorder quantity jumps and the number of lead time periods per year decrease, there is, in most cases, an increase in the probability of a stockout at any one given period. For instance, with a rate of return of 10 per cent the probability of a stockout jumps from .074 with an annual demand of 6,000 pounds to .114 for 7,000 pounds and .138 for 8,000 pounds. This occurs as the reorder quantity moves from 2,000 pounds to 4,000 pounds and then up to 6,000 pounds. With a required rate of return of 60 per cent the same thing occurs but at much higher annual demands and the effect is not nearly as great. Thus, at 19,000 pounds per year the reorder quantity is 4,000 pounds and the probability of a stockout jumps to .040; at 21,000 pounds per year (not listed in TABLE 6) reorder quantity jumps to 6,000 pounds and the probability of a stockout to .051.

Carrying Costs

Among those input variables of the inventory cost-pricing model which are subject to qualification, the carrying cost, C_2, may be the most important because it affects both inventory policy and price determination. It is certainly the most important determinate of inventory policy and, because inventories are so important to the steel service center industry, special care should be taken in determining carrying cost. The carrying cost is often defined as the cost of capital plus the cost associated with carrying the inventory; that is, insurance costs, obsolescence costs, etc. Many commentaries on the cost of capital lead off with the statement: "Though we may not know what the cost of capital is, we know one thing that it is not--it is not the cost of debt." Yet for inventory decision making there may be instances where the carrying cost will include the cost of debt to represent the cost of capital. What accounts for this difference? While the cost of capital is ordinarily used to determine the acceptability of long-run projects, inventory policy is, by its very nature, short-run. Acceptability of an inventory investment may change from year to year, from month to month, and even from day to day. Thus, in terms of impending shortage, steel service centers may attempt to build their inventories by lowering their carrying costs. Because the financial support for building the inventories may come from short-term bank loans, it seems reasonable for carrying costs to include the cost of short-term debt as the cost of capital. Alternatively, when it is desirable to reduce the overall inventory levels, increasing the carrying cost will promise optimal reduction in each item.

In light of the above discussion, it seemed reasonable to run the inventory cost-pricing model for varying carrying costs from 5 per cent to 30 per cent per year. TABLE 7 shows the results for two of these runs at 5 per cent and 25 per cent carrying costs per year. Each category of the model output is changed. Price is least affected

TABLE 7. The Model With Carrying Costs C_2 at .05 and .25

	$C_2 = .05$					$C_2 = .25$				
Annual Demand K	Q_j from P_j	Q_j from TC	Reorder Point R	Min. Price P_j	Average Total Cost	Q_j from P_j	Q_j from TC	Reorder Point R	Min. Price P_j	Average Total Cost
10	10	20	5.1	17.37	12.15	10	20	2.1	17.03	13.99
20	20	20	8.1	15.23	11.51	10	20	5.4	15.49	12.72
30	20	20	11.4	14.26	11.27	20	20	7.1	14.40	12.06
40	20	20	14.6	13.76	11.00	20	20	9.8	13.86	11.81
50	20	60	17.6	13.44	10.83	20	20	12.4	13.52	11.66
60	20	60	20.5	13.22	10.72	20	20	15.0	13.29	11.55
70	20	60	23.4	13.06	10.64	20	20	17.6	13.12	11.48
80	20	60	26.1	12.94	10.58	20	40	20.1	12.99	11.42
90	20	60	28.8	12.84	10.52	20	60	22.5	12.89	11.31
100	20	60	31.5	12.75	10.46	20	60	25.0	12.81	11.21
110	20	60	34.1	12.68	10.41	20	60	27.4	12.74	11.13
120	20	60	36.7	12.62	10.37	20	60	29.8	12.68	11.05
130	40	100	36.6	12.56	10.32	20	60	32.1	12.63	10.99
140	60	100	37.6	12.39	10.27	20	60	34.5	12.58	10.94
150	60	100	39.9	12.26	10.25	20	60	36.8	12.54	10.89
160	60	100	42.2	12.19	10.23	20	60	39.1	12.50	10.85
170	60	100	44.5	12.12	10.21	20	60	41.4	12.47	10.82
180	60	100	46.8	12.07	10.19	20	60	43.6	12.44	10.78
190	60	100	49.1	12.02	10.17	40	60	42.4	12.27	10.70
200	60	100	51.4	11.97	10.16	60	60	42.7	12.22	10.65

because the price computation depends primarily upon the required return, not particularly on carrying costs. However, the optimal quantity to be ordered to receive the minimum price is significantly affected by the carrying cost. Thus, when the carrying cost is 25 per cent per year, the price determined order quantity does not rise above 2,000 pounds until the annual demand reaches 19,000 pounds per year. At this time, quantity ordered is increased to 4,000 pounds. Then at 20,000 pounds annual demand the quantity ordered reaches 6,000 pounds. Contrast this with a reduction in carrying cost to 5 per cent per year: at 13,000 pounds per year the quantity ordered switches from 2,000 pounds to 4,000 pounds and again to 6,000 pounds when annual demand reaches 14,000 pounds. The minimum price is generally increased by roughly 2 per cent between carrying costs of 5 per cent and 25 per cent. Further comments on price appear later in conjunction with TABLE 8.

The minimum cost computation is affected considerably more than the minimum price when the carrying cost, C_2, is increased. This is particularly true for low annual demands where the difference in minimum total cost reaches $1.84 with annual demand of 1,000 pounds. The difference drops below $1.00 when annual demands reach 3,000 pounds and from there on declines rather slowly so that for demands of 20,000 pounds the difference is only 49 cents. The order quantity determined from total cost is similarly affected. A 5 per cent carrying cost increases the quantity ordered from 2,000 pounds to 6,000 pounds at annual demand of 5,000 pounds and to 10,000 pounds when annual demand reaches 13,000 pounds. With an annual carrying cost of 25 per cent the quantity ordered does not change to 6,000 pounds until annual demand equals 9,000 pounds and even at 20,000 pounds annual demand the quantity ordered is still only 6,000 pounds.

As could be expected, the reorder point which is determined by the balance of shortage and annual carrying cost undergoes drastic changes when the carrying cost is increased 5 to 25 per cent. In particular, for annual demands of 1,000 pounds the reorder point drops from 510 pounds to 210 pounds. And similarly with annual demands of 10,000 pounds per year the reorder point decreases from 3,150 pounds to 2,500 pounds. The same is true with an annual demand of 20,000 pounds where the reorder point drops from 5,140 pounds to 4,270 pounds. Even greater changes result when the carrying cost is increased to 30 per cent, where for an annual demand of 1,000 pounds the reorder point drops to 140 pounds.

TABLE 8 shows the complete range of prices generated by the model for carrying costs from 5 per cent to 30 per cent. Though it is not shown in this table, it should be noted that average total cost

TABLE 8. Prices For Various Carrying Costs

Annual Demand K	$C_2 = .05$	$C_2 = .10$	$C_2 = .15$	$C_2 = .20$	$C_2 = .25$	$C_2 = .30$
10	17.37	17.10	16.94	16.98	17.03	16.99
20	15.23	15.43	15.40	15.45	15.49	15.55
30	14.26	14.20	14.24	14.32	14.40	14.51
40	13.76	13.70	13.73	13.79	13.86	13.93
50	13.44	13.40	13.42	13.46	13.52	13.58
60	13.22	13.19	13.20	13.24	13.29	13.34
70	13.06	13.03	13.04	13.08	13.12	13.17
80	12.94	12.91	12.92	12.95	12.99	13.04
90	12.84	12.81	12.82	12.85	12.89	12.93
100	12.75	12.73	12.74	12.77	12.81	12.85
110	12.68	12.66	12.68	12.70	12.74	12.77
120	12.62	12.60	12.62	12.65	12.68	12.71
130	12.56	12.55	12.57	12.60	12.63	12.66
140	12.39	12.51	12.52	12.55	12.58	12.61
150	12.26	12.39	12.49	12.51	12.54	12.57
160	12.19	12.22	12.44	12.48	12.50	12.54
170	12.12	12.15	12.26	12.44	12.47	12.50
180	12.07	12.09	12.15	12.27	12.44	12.47
190	12.02	12.04	12.09	12.15	12.27	12.45
200	11.97	11.99	12.04	12.10	12.22	12.41

increased consistently in every instance as the carrying costs went up. When annual demand is 1,000 pounds, the minimum price decreases from $17.37 to $17.10 as carrying costs increase from 5 per cent to 10 per cent. The price drop continues for carrying costs at 15 per cent where the minimum price drops to $16.94, then begins a rise at 20 per cent at $16.98 and at 25 per cent is $17.03. However, at 30 per cent the price again drops even though, as previously mentioned, the total cost rises in all cases. Again, the carrying cost directly affects the reorder point determination. Thus, lower average investment due to reorder point decreases may be more than enough to overcome the increase in carrying costs and so force a lower minimum price. With annual demands of 2,000 pounds the minimum price first rises, then falls, then continues to rise again. The fall between 10 per cent and 15 per cent occurs because price determined reorder quantities dropped from 2,000 to 1,000 pounds and, therefore, the average inventory decreased, allowing for a slightly lower minimum price. At 3,000 pounds annual demand the drop, followed by a subsequent rise, is again accounted for by the reorder point change. For an annual demand of 1,000 pounds per year the lowest price occurs with a carrying cost of 15 per cent. But for annual demands from 3,000 pounds to 13,000 pounds the 10 per cent carrying cost provides the lowest prices. Interestingly, for a 30 per cent carrying cost and

1,000 pounds annual demand the reorder point drops to only 140 pounds from 210 pounds when the carrying cost is 25 per cent. This rapid decrease results in a cost minimum price for a 30 per cent carrying cost below that for the identical situation with a 25 per cent carrying cost. High carrying costs do not cause this reaction at any other rate of annual demand.

Changes in Lead Time

The computer program was used to simulate lead times of one month and two months, and so on up to twelve months. The extremely long lead times are unrealistic except in times of an impending strike when the fear of being caught short becomes so great that many firms order far in advance of their actual needs and in greater quantities than they are likely to need. The sudden influx of orders and the resulting increase in lead times results in still greater anticipatory buying and, in turn, even further increases in lead time. TABLE 9 shows some of the results of changes in lead time.

The most significant effect upon both the price and average cost calculations occurs when demand is low. When demand is as great as 20,000 pounds per year, there are very small price differences. An appreciable change in the quantities ordered also occurs when lead times increase and it becomes economical to order in larger

TABLE 9. Effects of Increased Lead Time for Estimated Annual Demands up to 20,000 Pounds Per Year

Annual Demand	Minimum Price	Average Cost	Ratio*
Lead Time One Month			
10	16.53	12.61	2.67
20	15.15	11.80	2.49
30	13.94	11.55	2.33
40	13.46	11.38	2.33
50	13.17	11.17	2.28
60	12.97	11.02	2.25
70	12.81	10.92	2.22
80	12.70	10.85	2.19
90	12.61	10.80	2.16
100	12.53	10.76	2.32
110	12.47	10.71	2.16
120	12.42	10.66	2.08
130	12.37	10.62	2.06
140	12.33	10.58	2.04
150	12.20	10.53	1.83
160	12.09	10.49	1.72
170	12.03	10.47	1.72
180	11.97	10.45	1.71
190	11.92	10.43	1.70
200	11.88	10.42	1.69
Lead Time Two Months			
10	17.10	12.82	2.51
20	15.43	11.95	2.02
30	14.20	11.75	1.99
40	13.70	11.51	1.96
50	13.40	11.30	1.93
60	13.19	11.17	1.89
70	13.03	11.08	1.87
80	12.91	11.02	1.84
90	12.81	10.98	1.82
100	12.73	10.91	1.80

TABLE 9. -- Continued

Annual Demand	Minimum Price	Average Cost	Ratio*
110	12.66	10.85	1.78
120	12.60	10.81	1.76
130	12.55	10.77	1.74
140	12.51	10.74	1.73
150	12.34	10.69	1.59
160	12.22	10.65	1.52
170	12.15	10.63	1.51
180	12.09	10.62	1.50
190	12.04	10.61	1.50
200	11.99	10.61	1.49
Lead Time Six Months			
10	18.11	13.30	1.91
20	15.91	12.24	1.78
30	14.46	11.92	1.79
40	13.90	11.62	1.79
50	13.54	11.40	1.79
60	13.30	11.26	1.78
70	13.13	11.16	1.77
80	12.99	11.10	1.75
90	12.88	11.03	1.73
100	12.79	10.96	1.72
110	12.72	10.90	1.71
120	12.66	10.86	1.70
130	12.60	10.82	1.69
140	12.55	10.79	1.68
150	12.37	10.73	1.55
160	12.24	10.68	1.49
170	12.18	10.67	1.48
180	12.12	10.66	1.48
190	12,06	10,65	1,47
200	12.01	10.65	1.47

TABLE 9. -- Continued

Annual Demand	Minimum Price	Average Cost	Ratio*
Lead Time One Year			
10	19.16	13.92	1.70
20	16.44	12.63	1.62
30	14.76	12.14	1.64
40	14.12	11.77	1.66
50	13.72	11.53	1.67
60	13.45	11.38	1.67
70	13.25	11.28	1.67
80	13.10	11.21	1.66
90	12.98	11.11	1.66
100	12.88	11.03	1.65
110	12.79	10.97	1.64
120	12.72	10.92	1.64
130	12.66	10.89	1.63
140	12.61	10.81	1.52
150	12.42	10.76	1.46
160	12.28	10.74	1.45
170	12.21	10.73	1.45
180	12.15	10.72	1.45
190	12.09	10.71	1.44
200	12.04	10.70	1.44

*Reorder Point/Mean Demand During Lead Time quantities. This would indicate that, though superficially just a matter of hoarding, the shift to large orders which occur when lead times lengthen may be economically justified. This does not mean the buyer realizes the economic justification for his larger orders. As in some earlier cases, the model picks an order quantity above the minimum in a price bracket at annual demands of 20,000 pounds. For example, order quantities are 10,200 pounds with an eight month lead time and 10,600 pounds for a twelve month lead time. At twelve months the order for 19,000 pounds also moves up from 10,000 to 10,200 pounds. The ratio of reorder point to mean demand during lead time not only declines in most instances as demand increases, but also declines as the lead time increases because, though the reorder point increases each time the lead time increases, still the ratio of the reorder point to the mean demand during lead time decreases in most cases. This phenomenon is related to the well-known fact that the standard deviation of a randomly drawn sample from an infinite universe increases as the square root of the mean size. Though the ratio decreases in most cases, the stockout probability during lead time remains relatively constant as the lead time increases. When the annual demand is 10,000 pounds, the stockout probability for the lead times of 2, 4, 6, 8, 10, and 12 months is .0413, .0409, .0408, and .0400, respectively. These small differences result from approximations of the marginal probability of a stockout necessary to balance stockout and carrying costs.

The approximate annual probability of a shortage occurring when a customer arrives may be found by multiplying the probability of a stockout by the mean expected demand during lead time and dividing the sum by the quantity ordered. With a lead time of two months, an expected annual demand of 4,000 pounds, and an order quantity of 6,000 pounds, the annual probability of a shortage is .00874. Other probabilities, not shown in the table, range from .0054 for an item with a one month lead time and an annual demand of 40,000 pounds up to .061 for an item with 1,000 pounds of annual demand and a twelve month lead time. It follows that the probability of fulfilling a customer's order directly from stock ranges from a low of 93.9 per cent to a high of 99.46 per cent. Thus, at least with the figures used, the overall optimum probabilities of a stockout are considerably lower than some people familiar with the industry have assumed. Though the pricing portion of the model tends to reduce differences between items with different annual demands and also tends to compensate for changes in lead time, the optimal probabilities of a stockout for one item in the model may be ten times as great as for another item. Therefore, in actual industry situations, the differences in optimal probabilities of a stockout would probably be considerably greater than those shown in the model.

Changes in Order Size

Because the size of the order has a great effect upon the probable demand distribution, one of the major unsolved problems of the steel warehouse industry is the lack of an effective inventory policy for items having small annual demand and a large average order size. The model was run with a mean demand of 24 per cent of the annual demand. However, this mean demand was made up of a constant size order of 200 pounds equal to 4 per cent of the annual demand and orders of varying sizes which are equal to 20 per cent of the annual demand. The model was run with the same mean and with the variable portion of the mean demand consisting of orders from 100 to 4, 000 pounds. Due to space limitations, TABLE 10 only contains demands up to 30, 000 pounds per year. Even though a large portion of the output from each run was omitted, it was still necessary to limit the table to primary order sizes of 100, 500, and 2, 000 pounds.

An interesting finding in the data is that the ratio of the re-order point to the mean demand during lead time increases substantially as the character of the order changes. When annual demand is only 1, 000 pounds per year and this demand consists of 2, 000 pound

TABLE 10. Changes in Primary Order Size

	100 pound order size						500 pound order size			2000 pound order size		
Annual Demand K	Min Price P_j	Ave. Total Cost	Q_j from TC	Mean DDLT M	Reorder Point R	Ratio R/M	Min. Price P_j	Ave. Total Cost	Ratio R/M	Min. Price P_j	Ave. Total Cost	Ratio R/M
10	16.69	12.71	20	2.40	3.85	1.60	17.39	12.88	2.07	15.46	12.38	0.72
20	15.11	11.95	20	4.80	7.06	1.47	15.81	12.16	2.04	18.61	12.95	4.14
30	13.97	11.75	40	7.20	10.53	1.46	14.72	11.96	2.04	16.07	12.32	3.03
40	13.48	11.49	60	9.60	13.87	1.44	14.18	11.70	2.02	14.88	11.93	2.66
50	13.17	11.30	60	12.00	17.18	1.43	13.75	11.50	1.97	15.74	12.03	3.45
60	12.97	11.19	60	14.40	20.44	1.42	13.57	11.37	1.93	15.01	11.76	3.02
70	12.82	11.11	60	16.80	23.64	1.41	13.42	11.29	1.88	14.48	11.59	2.73
80	12.70	11.07	60	19.20	26.79	1.40	13.27	11.24	1.86	14.18	11.69	3.12
90	12.61	11.00	100	21.60	29.90	1.38	13.17	11.16	1.83	14.62	11.55	2.94
100	12.53	10.93	100	24.00	32.99	1.37	13.07	11.09	1.81	14.30	11.42	2.74
120	12.42	10.85	100	28.80	39.12	1.36	12.93	11.00	1.77	13.87	11.38	2.83
140	12.33	10.81	100	33.60	45.17	1.34	12.82	10.90	1.61	13.88	11.07	2.12
160	12.10	10.74	100	38.40	47.96	1.25	12.44	10.83	1.52	13.23	11.07	2.19
180	11.98	10.73	105	43.20	53.72	1.24	12.31	10.82	1.51	13.00	11.02	2.07
200	11.88	10.73	116	48.00	59.43	1.24	12.20	10.82	1.50	12.77	11.04	2.11
220	11.80	10.72	127	52.80	65.10	1.23	12.12	10.81	1.49	12.81	11.01	2.05
240	11.74	10.72	200	57.60	70.76	1.23	12.04	10.81	1.48	12.66	10.99	1.98
260	11.68	10.70	200	62.40	76.39	1.22	11.98	10.78	1.47	12.55	10.99	2.04
280	11.63	10.68	200	67.20	81.98	1.22	11.92	10.76	1.46	12.55	10.94	1.96
300	11.59	10.67	200	72.00	87.59	1.22	11.87	10.75	1.45	12.43	10.92	1.94

orders, a revealing sidelight occurs. The model "gives up" reacting to very large orders and instead attempts, unsuccessfully, to base its policy only on orders weighing 200 pounds. However, when demand is 2,000 pounds per year, the model attempts to encompass most of the demand though in doing so the reorder point becomes unrealistically high. The reorder point is unrealistically high: 1,985 pounds though the order size is 2,000 pounds. This occurred because the model actually moved to 2,000 pounds but found that it was slightly over-compensating at this point; not realizing that, with this discrete order size distribution, the firm would be unable to satisfy a demand for 2,000 pounds when the amount on hand was less than that, it began splitting the difference between the probability of a stockout with a reorder point of 19 and one of 20 until it found that proportion at which the annual carrying cost and the stockout costs were closest to being equal. At one point when order size is 4,000 pounds, the reorder point jumps from 45 to 79. Here the model suffers at 79 from the same misapplication that it does at 19. It is obvious that at many points the model reorder point is jumping to contain multiples of the large size order. The minimum price computation is likewise adversely affected by the increase in the order size. Not only is the minimum price higher for higher demands in almost every instance, but the gyrations of the reorder point tend at times to give the unreasonable answer that the minimum price will be higher for higher annual demands than for lower. This explains why very high average order sizes, particularly for items of low demand, are extremely disrupting to the steel service center firms. Many firms are unable to find a rational solution for this problem. Its gravity is emphasized when it disrupts the present form of the inventory cost-pricing model by forcing irrational answers in both pricing and inventory policy. Though it is possible to institute special program modifications to handle these problems, the mere fact that such modifications are necessary points up the sophistication of the problem.

Calculation of the approximate stockout as a percentage of annual demand shows that in most cases the stockout probability is less than 2 per cent and in all but one case less than 3 per cent. This calculation is misleading and inaccurate, however, in cases wherein an appreciable portion of the reorder periods there is zero demand during lead time. Then a more accurate calculation of the annual stockout probability is obtained by reducing the denominator by the proportion of lead time in which zero demand during lead time occurs.

Conclusion

The model simulations described here are not by any means comprehensive. However, they have been carefully chosen to include those variations thought to have practical importance in any actual implementation of the model. It is for this reason that real problems such as varying customer order sizes have been thoroughly explored. Some changes, such as variations in material cost schedules, have yielded interesting results but, as a practical matter, the almost infinite combinations available make it unlikely that any meaningful generalizations could be reached, and so they have not been included in this chapter. In summary, the model has explored relevant areas in which the data may have both practical and theoretical application.

In Chapter IX many of the limitations inherent in a model of this sort are discussed along with general conclusions. The chapter also briefly explores some of the areas in which further study seems appropriate.

CHAPTER IX

CONCLUSIONS AND SOME AREAS FOR FURTHER STUDY

This paper has explored the steel service center industry by examination of the significant industry features and quantitatively through the use of an inventory-pricing model. There is substantial evidence that declining or flat marginal costs exist in firms of all sizes throughout the industry. The inventory effect is perhaps the most important factor in declining costs though the revolution in data processing has reduced the informational problems of large firms and opened the way for further economies of scale.

The evidence of inelastic industry demand curves is largely circumstantial and deduced from studies of the steel industry. However, the consistency of the results of studies of steel industry demand is so striking, and the similarities between the markets for steel mill output and steel service center sales so great, that corresponding results can be anticipated in almost every instance. The evidence of market strategies is fragmentary and based upon observations by the author.

There is pervasive evidence that traditional price competition is unlikely to produce an economically acceptable equilibrium in the steel service center industry. The economic need for an optimizing system has led to a determined attack on price leadership or administered prices by both academicians and government intervention. The available alternatives open to wholesaling firms have been sufficiently distasteful, however, to cause the firms to cling to traditional methods.

The Inventory-Pricing Model

An alternative is proposed herein based upon existing quantitative methods used for business decisions. The foundation of the proposal is the contention that pricing and inventory (investment) policies are interdependent. This interdependence has both obvious (prices should be related to inventory turnover) and subtle (quantity discounts may not be justified to the extent usually advocated) ramifications.

Complicated arrival distributions have been used in an attempt to provide a realistic model. This may be useful in straight inventory systems, though associated problems may make some minor revisions necessary.

Limitations of the Inventory Cost-Pricing Model

Because of the complexity of the model, its limitations are not readily apparent. For instance, quantity discounts are very important in the steel service center industry; and though quantity schedules are included as an input, they are not part of the determined price model. Therefore, since the model's price output applies to an exact quantity, a complete set of price additions or discounts must be constructed to give an actual industry model. A quantity schedule was not included because quantity schedules are basically a problem of cost analysis and so, if included, should be an input from outside the system rather than an output generated by the system.

A further limitation of the model is inherent in two alternative implicit assumptions, either (1) the firm's sales closely approximate those of the industry, or (2) the firm has access to industry sales and so can use a given percentage of the industry sales in constructing its own relative demand patterns. Either of these assumptions is necessary if the model is to be relevant as a competitive tool. If the firm's perceived demand does not closely approximate that of the industry, other firms will find it advantageous to selectively cut prices on those items for which the price is too high because demand has been underestimated. The other firms will allow the pricing firm to take most of the business on unprofitable, under-priced items.

Related problems arise because the model is basically static rather than dynamic. That is, the model gives answers based upon the assumption that the past relationships will continue into the future. Obviously, this is not completely true--to the extent that the present conditions change the model will not give optimal answers. Answers closely approximating the optimal would be given if the data were constantly updated. However, constantly updating the data, thereby shifting pricing, ordering, and inventorying policies, would be costly. The requirements for stable market prices, both from the point of view of the customers and of the other firms in the industry, preclude constant price changes. This is true not only because of the time necessary for the other firms to adjust to changes in prices, but also because disorderly markets affect customer relationships.

The need for estimates outside the model creates a rather difficult problem. First, an estimate of the rate of return desired is necessary. This particular rate of return includes not only the usual return on assets but also a return to cover fixed cost. The computation would be difficult enough if only internal factors needed to be considered. However, the rate of return must be calculated at a high enough rate to yield an acceptable return to the majority of industry yet low enough to provide an effective barrier to further market entry. These conflicting requirements make the input of the rate of return the single most crucial variable in the model. Since the estimate must be made without substantial empirical data, a trial and error process of estimation will result in still further revisions of the model's prices. The carrying cost used as a determinant of inventory policy is not as crucial in the external operating results of the model because its effect upon pricing is indirect. Nevertheless, the calculation of the cost of capital is extremely important. Furthermore, as mentioned in Chapter VIII, the cost of capital can be used to regulate the overall inventory investment. Though this is a corruption of the theoretical cost of capital, it is particularly valuable if the firm needs to retrieve funds from inventory. Thus, inventories can be reduced to pay off a bank loan, to reduce the taxable base at a certain time of year, etc.

Further limitations in the model are inherent in the decision to minimize price rather than total cost. This is true because the total cost calculation is made subject to a computed minimum price. Thus certain costs have been decided upon before the computation of total cost and the total cost calculation merely allows the firm, given its previous minimum price computation, to minimize costs. The firm uses one inventory policy in computing its lowest price and, then, using that lowest price, finds the optimal inventory policy to minimize total costs. The firm will make more profit than if it followed the inventory policy consistent with the minimum price. Though it will not earn the overall required rate of return on the additional inventory, it will earn more than the cost of capital.

A further limitation exists because of the ambiguity of the shortage cost. The shortage cost computation is naturally included in computing the price and in computing the inventory policy. However, its inclusion in the computation of total cost is questionable because the shortage cost includes, as an opportunity cost, the amount lost by not having the item in stock. Variations of the program, run without an approximation of shortage opportunity cost in the total cost calculation, have been made without substantial changes in results. There seems to be no clear-cut answer to the question of whether to include shortage cost in the total cost computation; hence the model may be criticized either way. Because the basic distribution computation is incremental, the model can only produce an approximate total shortage cost. To the extent that the total cost calculation determines inventory policy, it should actually represent a type of marginal or incremental analysis and, therefore, the shortage cost, no matter what its nature, should be included.

Despite these limitations, the use of the inventory-pricing model could lead to an economically justifiable industry equilibrium. It is impossible to tell whether such a system will in fact be utilized, but there are signs that some industry leaders are moving toward this type of pricing. Even if inventory and pricing decisions are not explicitly linked, sophisticated inventory systems and individualized pricing based upon target rates of return will produce an implicit link.

Areas for Further Testing

An obvious extension of the present model would be a dynamic simulation of the firm with its various problems associated with changes in lead times, etc. This model, in turn, could become one of a series of models which would interact together to form a complete industry. To find the results of different types of firm behavior the inputs for this industry model would be changed to simulate various markets. Thus existing markets could be programmed and predictions made which could be checked against the actual changes occurring.

Another promising area is that of implementing the existing model in an individual firm. The firm selected would be carefully studied so that the individual firm would then use the output as a basis for its own decision making. Although this is an interesting possibility, it would demand a higher degree of cooperation between the firm and the researchers than seems likely to occur in most situations. Assuming this cooperation, the inventory section of the model would be implemented first to gain the confidence of the firm. If the results were satisfactory, limited application of the pricing section to a particular line of goods might be allowed. Because the amount of computer time necessary would make it imperative that the firm have its own computing facilities, it is likely that only large firms could make adequate use of the model.

Since the model is easily adaptable to various characteristics of a firm, the use of the model in other industries would be fruitful. Either the entire model could be applied or the compound poisson alone could be applied to other inventory situations. The author feels that this distribution offers such wide possibility that a major effort should be made to acquaint industry with its possible applications.

Other uses for the compound poisson, aside from the inventory problem, may be suggested. It seems particularly likely that this distribution could have extensive use in PERT networks which make use of probability distributions. Two types of distributions are primarily used at the present time: the beta distribution and the normal distribution. It may be implied that the poisson is better than the normal because by using the normal distribution there is a positive probability that a given task will take less than zero time. The beta distribution, on the other hand, is used because time estimates have generally been divided into three categories: pessimistic, most likely, and optimistic. Using these limitations of the probable time, researchers soon noted that these three parameters could specify a beta distribution. However, to secure distribution estimates, an alternative series of questions could be asked of the individuals concerned with production. They could be asked to specify a minimum amount of time necessary for completion. A distribution function would be computed with the minimum time as a starting point and the difference between the minimum time and the average time as the mean. With no other parameters specified, this is a poisson distribution. It could easily be changed, as in the inventory cost-pricing model, to yield a right tail cumulative probability distribution. All tasks in that particular path of the PERT network could then be combined to obtain a compound poisson. This would allow a direct calculation of the probabilities associated with a given time for each path in the network. More sophisticated versions of the PERT scheduling program entail the use of cost estimates. The reformulated PERT networks stated in terms of the compound poisson are particularly adaptable to computing trade-offs between costs and time.

There are strong indications the compound poisson distribution is well suited to use in constructing a model of stock price changes. The model postulated has two types of compound poisson distributions:

one for the buyers and another for the sellers. It is the interaction of these two distributions which should determine changes in stock prices. Although an exact method for doing this has not been worked out as yet, a likely method uses a cumulative right tail distribution as a grid against which to place randomly generated numbers and then uses the corresponding demand as a partial output. This demand comprises one-half the system. The other half of the system consists of another compound poisson, a set of random numbers, and, again, a corresponding supply. The interaction of these two sets of cumulative compound poissons would determine the price level of transactions. Changes in demand and supply patterns would be generated and their effect upon price analyzed.

Conclusion

The purpose of this study has been to delineate major problems facing the steel service center industry and to suggest some tentative solutions. The author has made some estimates, backed by analysis, of cost and demand behavior. A quantitative model has been proposed to integrate inventory and pricing policy. Hopefully, these provide some insights into understanding industry behavior.

While the primary purpose was to postulate a model for decision making, the model may also foster rational solutions to some of the complex problems confronting the industry.

APPENDIX A

FIRM DESCRIPTIONS

The Small Competitor

View of the Market

The competitive problems of the small firm are basically different from those of the large firm. In a small firm the manager must make a profit within the framework of the existing market. In large markets, two opposing approaches pull him: on the one hand he desires to make a profit at his present sales level and on the other he desires to cut prices to obtain extra volume and make the firm more efficient. The manager usually specializes in a few related lines to protect his sales and profits and to compete with the larger specialists and general line warehouses in terms of inventory, services, etc. But he is tempted to carry only fast-moving items in several lines, using price as a wedge to gain entry into those markets. Some firms follow one strategy and some follow the other, but most firms seem to balance the two, trying to be "good guys" and not disturb the market even though they feel the need to make every possible sale.

Small firms differ widely, but certain types stand out:

The Ex-Junk Dealer

This firm started out buying and selling scrap. After World War II the firm moved into the service center business. It buys steel from any source and may, at times, intermix imported and domestic steel. Some of the steel, usually unsold structurals, is stored inside. The firm is ready at any time to make a "deal," and has seldom knowingly lost a sale due to price. Although the firm long ago outgrew the junk yard, the same philosophy still holds: "Make the sale, do anything, but MAKE THE SALE" still echoes whenever there is a temptation to stick with a price. One respondent reported that, while his competitors were lucky to turn over their stock once a year, he turned his over ten to twelve times, often selling a whole shipment before it was unloaded. (Customers were persuaded to buy ahead at attractive prices.) The wheeler-dealer is extremely knowledgeable and canny--he has to be in order to survive.

The Small General Line Warehouse

These warehouses compete with large general line warehouses but often enjoy a slight locational market advantage. A small warehouse may exist in large markets, but is more likely to exist in small peripheral markets. When this firm's market is small, it is more likely to protect the market by selling at the traditional "book" price. In larger markets the firm feels less necessity to protect the price structure.

The Small Firm Specialists

Small firms specializing in certain types of steel are often an important factor in the specialty market. Sometimes they are more influential in their special field than are large firms, and because they depend on a single group of products, they are often more cognizant of the need to maintain price stability than are other small firms. Since their specialization allows them to compete with large firms in service and in other non-price ways, they are less likely to need a price differential to compete effectively.

Some small firms are innovators which question the leadership of the larger firms, sometimes, though seldom, setting their own prices. The firm may not have any more advantageous information than other firms, but it adapts available information to the decision-making process. (One firm president, speaking of an item which he had recently raised in price, said: "I don't know how low my costs were, but I knew they weren't that low!")

Other small firms have found that they are not equipped to handle large tonnage items effectively. They consequently have raised prices for larger quantity brackets and have been very pleased with the result. This is not to imply that any single-handed change of schedules is likely to be successful, but rather that the innovative small service center is able to find the small areas in which changes can profitably be made.

Most firms in the market, however, are not innovators but usually follow market leadership if everyone else does. Thus when other firms offer special concessions, they follow the special price to make the sale. In short, they are opportunists willing to go along when necessary but seldom making prices themselves.

Large Service Centers

Large service centers started small and consequently have many of the same characteristics as the small ones. In addition, like all businesses, they are concerned with achieving certain goals, theirs being: (a) making a profit, (b) maintaining or increasing market share, (c) minimizing price cutting, and (d) restricting market entry.

Objectives

Making a Profit

The large firm is usually very concerned with showing a profit on the books because its percentage of the total market may remain constant, so that it must be prepared to make a profit at the prevailing levels. The large firm already has economies of scale and knows that if it can't make money at its present size, then it probably can't make money at any size. The large firm losing money faces a very serious problem: it must change its course from the present base without the help of natural economies.

The firm generally has gone through "volume-itis," and now tries to have every item make some contribution to the profit picture. Some successful firms feel that "it all washes out in the end," but most firms anxiously analyze their operations. "Captive" service centers are sometimes more concerned with profitability because of demands made upon them by the parent company. On the other hand, the parent company may be more concerned with volume if the service center's main purpose is to offer the mill a ready outlet for products.

Maintaining Market Share

Though the large firm is most anxious to maintain or increase its profits, maintaining or increasing its share of the market is also very important. The large firm knows that if its market share is slipping, its profits may soon be adversely affected. While it is difficult to measure an acceptable profit level, market shares can be determined with much greater accuracy. A level market share is no cause for complacency, but a decreasing market share is cause for alarm.

Since most firms interviewed are projecting increasing sales due not only to changes in the economy, but also to an increasing share of the market, the question naturally arises of who will receive the declining share of the market. Whatever the answer, it is clear that any firm regards declining market penetration as a danger signal of as much significance as declining profits. However, the extremely profitable firm is likely to be less concerned than the firm already marginal, for if sales and profits are rising, the declining market share is less noticeable.

Minimizing Price-Cutting

One of the reasons for declining market share may be selective price-cutting by competitors. Price-cutting is endured to some extent in all markets because the profit loss from meeting every shaded price is too great. Most larger firms publish price catalogs which are used by both buyers and sellers in finding the market or established price and as a base for the negotiations which result in price shading.

Restricting Entry Into the Market

Restricting entry into the service center industry has taken two forms: (1) public information programs and news releases showing the sub-normal returns for most firms in the business, and (2) pricing to alleviate the non-rational aspect of the pricing schedules, which have tended to foster selective entry on fast-moving items. This last method is used both to limit price competition, and to restrict the economic returns of entry into the industry.[1]

Alternative Actions

The large service center faces several tests in achieving its objectives. The first is tight cost control. Without this control, gains in any other areas cannot be fully utilized to provide benefits to the company. The details of cost are covered in other sections, but efficient and knowledgeable operation is a prerequisite to other intelligent action.

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[1]For a theoretical treatment of the problems of restricting entry, see Dale K. Osborne, "The Role of Entry in Oligopoly Theory," *Journal of Political Economy*, August, 1964, pp. 396-402.

Having mastered the internal problems, the large firm can turn its attention to the marketplace. The dilemma of the large firm is that while it would like to remain aloof from such crass behavior as price shading or cutting, handling of imported steel, etc., it finds itself constantly forced to protect its interests, and sometimes to take the offensive. Though a certain amount of price shading goes on at all times, the price leaders are not apt to act unless the price cutting becomes intense. If they did, the same thing would start again with lower margins for all firms, until no profits remained. This does not imply that the non-price leading large firms do not cut prices, for, in fact, some of the worst offenders in bargained prices are large firms. In many cases, the large price-cutting firm is in financial difficulty or attempting to penetrate new markets.

The problem of keeping salesmen from price-cutting is immense, for a salesman uses price differentials as a crutch to compensate for his or his firm's deficiencies. Some firms have forbidden competitive pricing and insisted that their salesmen stick by the book, but most firms attempting to limit such price-cutting have removed authority for price cuts to an executive such as the sales or branch manager. This method, along with tying the salesman's compensation to profitability, usually seems to work.

The price leader or leaders try to get conformity from other firms on prices by generally relying on the implied threat of downward revisions in price to keep most others in line. One firm, in serving notice to its competitors that it would no longer tolerate tampering with prices, has devised a system of meeting prices without the expensive and laborious job of reprinting catalogs. The firm's catalog contains slots on the first page for postcards, which tell current prices charged. When a competitor cuts prices, the firm can meet the new price throughout the market within forty-eight hours; thus, by doing away with the lag in reaction time, destroying the competitor's incentive to cut prices. Since no one gains from a "blood bath," the single plant firm least of all, the possibility of prompt downward revision promotes some price adherence.

To achieve their goals, large service centers have several alternative courses of action: (1) follow traditional price (2) meet competition selectively, (3) lower published prices, (4) revise prices relative to demand, and (5) some combination of these.

(1) Following traditional price has an added advantage over some of the other alternatives in that it is legal. In some cases, when price shading is not severe, this may well be the best course of action.

(2) If selectively meeting competition means matching prices on some items, then it is a legal and sometimes intelligent course of action which limits price-cutting incentives. But most price cutting, or price meeting, is not offered to all equally; instead, the cut price is offered to selected customers and/or orders. Although the firm is supposed to treat equally customers who compete with each other, selective pricing is widely practiced and few firms have not indulged in it at some time. Since, however, the lack of a set book price tends to make the market disorderly, the market leader lowers a published price only as a last resort. The basic strategy is to chastise the price cutters so that they don't make any money either and to limit their ability to undercut published prices so that the leader can retain his market share. Although across-the-board cuts in a product line lower prices on items which have little competition as well as on those heavily discounted, the discounter often remains undiscouraged since he often doesn't know his costs and is, therefore, willing to cut prices past the zero profit point.

(4) Revising prices relative to demand is an attempt to follow costs and competition, and thus punish the price cutter and restrain prospective market entrants. Those items with the lowest market demand have the highest markup and those with the highest demand have the lowest markup. However, this limits the effectiveness

of selective price cuts in generating new business because the items usually most open to price cutting already have low margins, and the items with higher margins do not generate enough demand even at lower prices to entice the price cutter. The inventory cost-pricing model in Chapter VII quantifies and elaborates the relationships assumed to exist by some firms adjusting relative prices.

Conclusion

The strategy of the small firm is usually limited to followership. Since its choices are limited, the small firm usually follows the market's leaders, only a few companies following active pricing policies of their own. The large firm, on the other hand, whether or not it takes a leadership position, plays an active role in pricing since its action, or lack of action, has a profound effect upon the market as a whole. In other words, the competitive strategy of the firm is a product not only of its objectively determined industry position, but of its determination to influence its environment, its objectives and the objectives of its competitors.

APPENDIX B

PRESENT PRICING PROCEDURES

Price systems vary between markets and may change in the same market at different times. In the last few years some complex pricing systems, which contain all or most of the following components, have become more rational: (1) basic price, including size and analysis extras; (2) quantity extras; (3) processing charges; and (4) delivery charges.

Basic Price, Including Size and Analysis Extras

The basic service center price charged for a given steel item is really a complex of ingredients called the mill price plus inbound freight, and the service center's markup. The mill price includes various charges by the mill purported to reflect their extra costs for certain steel characteristics at base quantities, primarily size and analysis extras. For example, the pricing procedure for cold finished carbon bars in the Chicago area begins with the mill charging a base price for all cold finished bars; in 1965, $7.90 per cwt. with a base quantity of 10,000 pounds. In addition, because of the costs of rolling and drawing, the mill charges for the size, from $1.37 per cwt.

for a 1 inch round bar to $23.30 per cwt. for a small rectangular bar less than 1/8 inch thick and 4 inches wide (rectangular bars are called flats). Another charge is added according to the analysis: a round, 1 inch diameter C1018, for instance--a common analysis-- will cost $.15 per cwt. extra, and a 1/8 inch diameter Ledloy B, a special free machining steel, will cost $2.60 per cwt. extra. Since the service center base is 2,000 pounds and the mill charges a quantity extra of $.90 (see TABLE 11), with a price $1.35 over the mills, service center markup is $.45 per cwt. TABLE II illustrates the basic pricing system in the Chicago area.

TABLE 11. Factors Combined to Make the Service Center Base Price for Two Cold Finished Bars in the Chicago Area

Cost per cwt.	1" rd. C1018	1/8" rd. Ledloy B
Base price	$ 7.90	$ 7.90
Size extra	1.35	7.90
Analysis extra	.15	2.60
Freight in	.15	.15
Cost to Service Center	$ 9.55	$18.25
Service Center Markup*	1.35	1.35
Base Price to Customer	$11.90	$19.60

*In the Chicago area base markups are the same per hundred pounds for all cold finished items.

This general type of pricing system is standard for cold finished bars in the mid-west. However, in some sections of the mid-west and the east, and most other sections of the country, the service center base markup is a percentage of the mill price, including applicable extras. One firm has pioneered larger markups for such slower moving items as flats and squares, adding a $1.00 charge--called a merchandising extra--which makes the base markup $2.35 per cwt. In some areas, the merchandising extra concept has been refined and based upon demand or turnover rates for individual items. There are also a few systems in use which calculate markups as a per cent return on investment plus some direct costs.

Quantity Extras

Service centers add charges called quantity extras for materials purchased in less than base quantities. The extra charges are based either on the weight of the item, the weight of the total order, or both, and originally represented estimations of item and order costs. Today cost justified quantity schedules are very important for two reasons: (1) justification of cost discrimination is required by law and (2) prices that do not follow costs invite selective price cutting.

There have been several efforts to impose a schedule of extras strictly according to item based on the belief that most costs are item rather than order related. In some areas such schedules are in effect. Most large purchasers, however, prefer order quantity extras so that they can order many separate small items and still pay base prices. As a rule, the customers have the upper hand in this power struggle. Price leaders have initiated several variations of the quantity/item extra schedule. TABLE 12 illustrates a typical schedule. Particularly in the mid-west, customer and competitor resistance has forced combinations of hot rolled products for order quantities, as in TABLE 12, without an item extra. In addition, firms in some parts of the mid-west allow a deduction of $1.00 per cwt. on a single item 2,000 pounds or more ordered in a stock size.

TABLE 12. Order and Item Quantity Schedule for Hot Rolled Products

20,000 lbs. and over	Item Base	Total Order Base
10,000 lbs. to 19,999 lbs.	Base	add 25¢ per cwt.
5,000 lbs. to 9,999 lbs.	Base	add 50¢ per cwt.
2,000 lbs. to 4,999 lbs.	add 15¢ per cwt.	add 75¢ per cwt.
1,000 lbs. to 1,999 lbs.	add 50¢ per cwt.	add $1.25 per cwt.
400 lbs. to 999 lbs.	add $1.50 per cwt.	add $4.00 per cwt.
100 lbs. to 399 lbs.	add $3.00 per cwt.	add $7.00 per cwt.
under 100 lbs.	add $3.00 per cwt.	uniform price of $27 per cwt. plus $3 delivery

TABLE 13. Typical Quantity Schedule
Based Upon Order Size Only

	Order Size
20,000+	base
10 - 20,000	.25
5 - 10,000	.75
2 - 5,000	2.00
1 - 2,000	4.00
400- 1,000	7.00
under 400	12.00

There is general agreement in the industry that most directly variable costs are item, rather than order, related. However, as a warehouse manager states: "No matter how much we would like to, we can never go back to purely item quantity brackets." The only real justification for order extras is that delivery costs are order-related and delivery charges are inadequate to cover costs so that service centers lose less money per hundred pounds delivering large orders than delivering small ones. A pseudo-justification exists in that common accounting practice adds larger amouns of non-specified overhead and fixed costs to get a high cost per order.

The price leader needs to make even orders profitable without running afoul of price discrimination laws or alienating too many customers. These two problems have not been as serious as convincing competitors, who instinctively fear change, that the present price structure is outmoded.

Processing Charges

At one time almost all processing charges were stated in terms of an additional fee per hundred pounds of material. Although charges per piece or per unit have gained acceptance in many areas, under one schedule, still in effect in some areas, material costing $9.00 per cwt. could have a $257.00 per cwt. shearing charge.

The newer schedules always differentiate between the first piece and additional pieces, with the charge for the first piece including setup charges. (Plate cutting has charges for the first three to ten pieces depending upon the size because of multiple head torches.) Charges for common processing operations may be divided into five main groups: (1)plate flame cutting, (2) hack sawing and friction sawing structurals, (3) hack sawing and shearing bars, (4) sheet shearing, and (5) plate shearing. This paper only comments on the first four.

Plate Flame Cutting

Plate flame cutting schedules used around the country are mainly copies of a standard schedule which contains an implicit setup charge and scrap charge by charging for rectangles of metal around irregular shapes, etc. However, these peripheral charges seem insufficient to cover scrap costs on plates 6 or more inches thick. In addition, the schedule does not differentiate between extremely detailed fine specification burning and common shapes. Also, there are no charges for the number of entries necessary to cut a piece, though extra time and effort add to the cost. Some sections of the country have changed the schedule through snipping, i.e., by lowering minimum charges.

Hack and Friction Sawing Structurals

Friction sawing steel is less expensive, but less accurate, than hacksawing and also leaves a burr which must be chipped off. Although most schedules of steel sawing charges are realistic and fairly well thought out, most of them make no provision to minimize the cost of drop-offs, shorts left after cutting, or to charge them to the customer. Also, the difference between friction and hack sawing charges encourages customers to pressure small firms without friction saws to do hack sawing at friction sawing rates; those firms with friction saws find them under-utilized because customers demand hack saw quality work for friction sawing prices. In fact, in some markets, though no structurals seem to be cut with friction saws, all sawing is charged at friction rates. Because few firms have friction saws, using the great differential between rates as a competitive weapon seems foolish in that the low prices drive out the high.

Shearing Bars

Bar shearing charges often do not cover costs. For instance, several pieces of 2 x 2 x 1/4 angle, each under 4 feet long are cut for $.75 for the first cut and $.04 for each additional cut, an unprofitable schedule because the stops must be reset after each cut. In addition, some firms have further reduced the schedule by changing the $2 per section minimum charge to a $2 per order minimum charge. However, sawing adequately reflects the cost differentials between jobs.

Sheet Shearing

In some markets the old sheet shearing schedules, which charge by the cwt., are in evidence. These schedules are hard to use for estimates, make no provision for setup charges, do not allow for economies from high production runs, and encourage price cutting pressure. Charges of over $100 per cwt. are common on material that costs less than $10 per cwt. Revised schedules were published but the graduations between sizes, and to some extent gauges, were so large that they did not reflect cost differentials. Further, without an intermediate price between first cuts and production cutting, the price was either less than full costs for short runs or non-competitive for production runs. Later schedules seem to have eliminated most of these deficiencies.

Delivery Charges

In many market areas, delivery charges to metroplitan and suburban points do not reflect costs. Most delivery costs are directly related to the number of stops made. Charges, however, are usually directly proportional to the order weight. The cost of delivery consists of (1) basic mileage to arrive and return from route, (2) driving time between customers, (3) basic unloading time, and (4) weight-related unloading time.

On an average route the cost of delivery works out somewhat like the example shown in TABLE 14. Most metropolitan areas either have no delivery charge or a \$.15 per cwt. charge regardless of weight. If, as is true in some parts of the country, the average order size is 500 pounds, the problem facing a firm is clear. Prices must reflect costs, and in this particular example there is a \$3.00 minimum average delivery cost. If the firm delivers only 100 orders per day, the loss from insufficient delivery charges may be over \$25,000 per year.

From a strategic standpoint, a \$5.00 minimum delivery charge is not likely to succeed because many specialty houses will not follow it, fearing that the higher charge will cause customers to group their orders with a general line warehouse to avoid paying several high minimum delivery charges. Some areas with a minimum delivery charge only apply it to orders under 100 pounds, and in some markets free delivery is offered. The customer's propensity to pick up his own material is another factor limiting delivery charges, since service centers are not set up to load customers' trucks.

TABLE 14

Order Size	Cost	Delivery Charge @ $0.15 cwt.	Difference
50 lbs.	$3.00	$ 0.08	$ -2.92
100 lbs.	3.05	0.15	-2.90
500 lbs.	3.15	0.75	-2.40
1000 lbs.	3.25	1.50	-1.75
2000 lbs.	3.50	3.00	- .50
5000 lbs.	4.25	7.50	3.25
10000 lbs.	5.50	15.00	9.50

Conclusions

Pricing systems vary widely and this section only touches lightly on a few of the many existing systems. However, so many of the systems studied do not relate to the underlying costs that the problem must also be due to difficulty in making meaningful cost calculations.

APPENDIX C

SINGLE ITEM PRICING STRATEGIES

Except in a purely competitive market, no firm can consider its costs and prices in isolation, but most firms, as first shown by Chamberlin,[1] consider the reaction their conduct produces in the market place. This section is concerned with reactions of competitors and customers, and, given their reactions and their effect upon the industry, with optimizing the firm's long-run profitability. Though not completely accurate, the assumption throughout this section that retail steel demand is constant at varying prices presents a more truthful picture of reality than do alternative assumptions.

Excluding extra marketing efforts, there are only three ways to increase a product's net return: (1) reduce costs, (2) lower prices, and (3) raise prices. Each of these ways works at certain times, but none of them works all of the time. The results of varrying assumptions can be illustrated by using a standard product, 4140 alloy steel shafting, of the X Corporation, whose present situation is shown in FIGURE 16.

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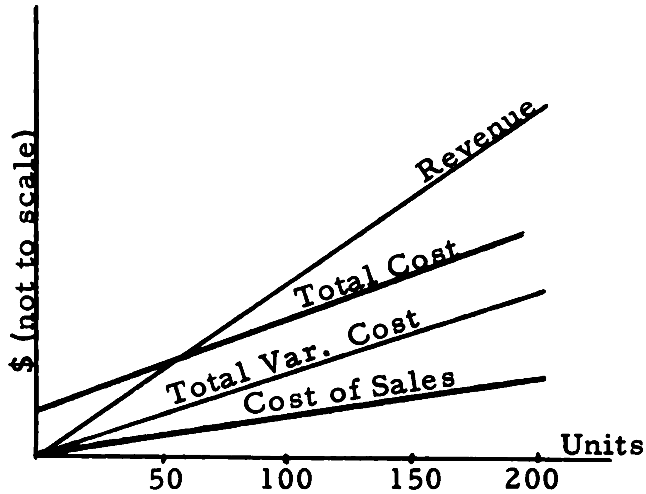
[1]Chamberlin, op. cit.

FIGURE 16

X Corporation Income Statement for 4140 Alloy Steel Shafting

Sales	$38,585	100%
Cost of Sales	29,570	76.6
Gross Profit	9,015	23.4
Variable Costs	6,745	17.5
Net Margin	2,270	5.9
Fixed Costs (Allocated)	3,815	
Net Profit (Loss)	1,545	



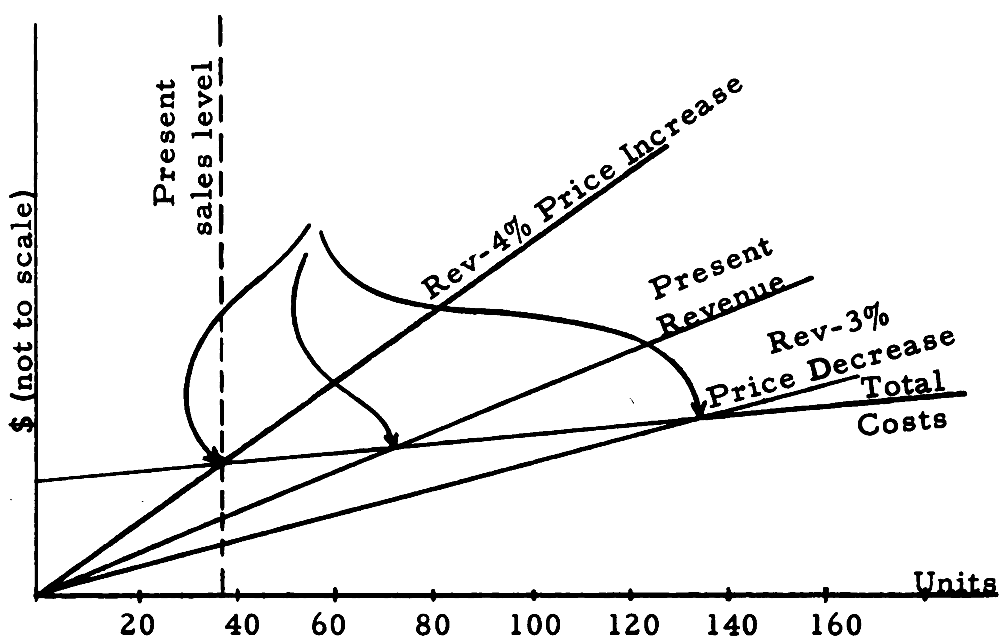
This product is presently sold at 23.4 per cent of gross profit and contributes over $2,000 to fixed cost (marginal pre-tax profit). (Though after fixed costs are allocated, using conventional accounting techniques, the product shows a net loss.) It is interesting to know the volume increase necessary to "break even" on this product.

(1) The decrease in direct costs necessary to raise the net from $2,270 to $3,815 can be calculated. If variable costs are cut from $6,745 to $5,200, or 22.9 per cent, the net margin will be increased enough to cover period and allocated costs without distributing the market. A reduction in costs of nearly 25 per cent is admirable, but unless personnel in responsible positions have been derelict, unrealistic.

(2) If demand is elastic, it may be possible to reduce prices and thereby increase sales enough to "break even". A 3 per cent

decrease in selling price will decrease the net margin to 2.9 per cent. In order to achieve a net margin of $3,815, sales must be $131,552 per month or unit sales equal to $135,621 at the old selling price. This is an increase of over 350 per cent, an elasticity beyond reasonable possibility. FIGURE 17 shows the results of a 3 per cent price decrease on the "G. P. 3 per cent price decrease" line.

FIGURE 17. X Corporation--Influence of Revised Selling Prices for 4140 Alloy Steel Shafting



Total product demand elasticity is close to zero in the relevant range; consequently, this course of action depends on competitors not following suit and an almost complete lack of customer loyalty. To continue to net $2,270, it is necessary to sell $78,276. (See FIGURE 17.)

Also of interest is the increase in present sales necessary to increase the net margin to $3,815. Sales must be increased to $64,661, or by $26,076. This may be the most desirable alternative, but it probably would be necessary to spend an extra amount in order to realize the increased sales, and if this extra cost were only 1 per cent of sales, the sales would have to be increased to $77,857 rather than to $64,661. Note also that this dramatic increase in sales is achieved at the expense of competitors who may attempt to regain their share of the market through price cutting or extra marketing effort, either of which will reduce industry margins.

(3) A price increase of 4 per cent on the same volume of sales will give the required net margin of $3,815. To remain above the old level with a 4 per cent price increase, the volume loss must be less than $2,270 divided by 9.9 per cent, or $22,929. This is shown on the "4 per cent price increase" line in FIGURE 17. Since raising prices is an individualistic approach, the success of which depends on the particular market, the probable market reaction to any change must be diagnosed and the changes occurring in net margin for each price and volume change calculated.

TABLE 15 shows margin and profit changes given various price increases. Former net margins are on the horizontal axis, and price increases are on the vertical axis. The upper section of

TABLE 15. The Relationship Between Price Increases and Margins

Price Increase (%)	2	4	5	6	8	10
2	50 100	67 50	71.4 40	75 33	80 25	83 20
4	33 200	50 100	55.6 80	60 67	67 50	71.4 40
5	28.5 250	44.4 125	50 100	54.5 83	61.5 62.5	67 50
6	25 300	40 150	45.5 120	50 100	57 75	62.5 60
8	20 400	33 200	38.5 160	42.9 133	50 100	55.6 80
10	16.7 500	28.5 250	33 200	37.5 167	44.4 125	50 100

Upper part of box contains the per cent of original sales needed to maintain the same dollar margin.

Lower part of box contains the per cent net margin will increase if unit sales remain the same.

---

each box shows the per cent of original volume necessary to retain the same net margin after a price increase; the lower section shows the percentage increase in pre-tax profit if volume remains the same. This chart gives rough answers to two questions; answers which must be known before changing prices: (1) "What are the penalties if others don't follow?" The upper section shows the maximum volume loss possible while obtaining the same profit. (2) "Are the rewards worth the risk if others follow?" The lower section shows the percentage gain if competitors follow and unit volume remains the same.

By adding the expected frequencies of competitors' actions in each situation, a new matrix can be created. Competitors have only three basic reactions: (1) none follows the increase, (2) some follow the increase, and (3) all follow the increase. Since all reactions will fit one of these three categories, after estimating two probabilities, the third can be found by subtracting from unity.

TABLE 16. Estimated[a] Percentage of Price Changes Followed by Competitors

Price Increase (%)	2	4	6	8	10
1	65 25	55 15	40 10	25 5	15 5
2	60 25	50 15	35 10	17 5	9 5
3	53 20	40 14	30 10	12 5	5 4
4	45 18	35 13	25 10	7 5	3 3
5	40 15	30 12	15 10	3 5	1 3

Upper figure in box is per cent of time that <u>all</u> competitors follow price change and lower figure is per cent of time that <u>some</u> follow.

[a]Estimated from interviews with steel service center managers.

The estimated possibilities of each event under given circumstances are shown in TABLE 16 on the preceding page. The top portion of each box shows a percentage estimate of how much of the time all competitors will follow a given price change. The bottom portion shows the per cent of the time that some will follow. The total of these two subtracted from 100 per cent gives the predicted per cent of the time that no competitors will follow. It should be emphasized that these estimates, although hypothetical, are for a specific firm, market and product--in this case, 4140 steel shafting.

TABLE 16 can be expanded for a 6 per cent net margin closely approximating the 5.9 per cent of the earlier sample, and in TABLE 17 the expected volume loss under the three possibilities is shown. (See page 190.)

TABLE 18 combines information from TABLE 16 and TABLE 17. It shows the firm's highest value strategy. In this case, a 2 per cent to 2.5 per cent price increase yields the greatest expected return since the example had a low net margin which did not even cover the allocation of overhead expenses, and the possibility of attracting more competition into the market was not relevant. If margins were higher, the analysis might have to be modified.

TABLE 17. Estimated Losses in Volume for Certain Price Increases at a 6% Base*
(In Per Cent)

Percentage Increase	.5	1	1.5	2	2.5	3	3.5	4	4.5	5
Loss of volume if none follow	1	4	9	16	25	36	49	64	81	100
Volume loss if some follow	0	1	2	4	6	9	12	16	20	25
Volume loss if all follow		NEGLIGIBLE								
Value if none follow	107	112	114	112	106	96	81	59	33	-0-
Value if some follow	108	116	123	128	133	137	139	140	140	138
Value if all follow	108	117	125	133	141	150	158	167	175	183

*Estimated on the basis of data collected and several interviews.

TABLE 18. Expected Values of Various Price Increases

Percentage Increase	.5	1	1.5	2	2.5	3	3.5	4	4.5	5
% of time none follow	47	50	50.5	50	57	60	62	65	70	75
% of time all follow	43	40	37.5	35	33	30	28	25	20	15
% of time some follow	10	10	10	10	10	10	10	10	10	10
Val. if none follow x prob. that none follow	50	56	60	62	61	58	50	38	23	-0-
Val. if all follow x prob. that all follow	46	47	47	47	48	45	44	42	35	27
Val. if some follow x prob. that some follow	11	12	12	13	14	14	14	14	14	14
Expected Value of Change	107	115	119	122	122	117	108	94	72	41

APPENDIX D

A COMPUTER PROGRAM FOR THE CUMULATIVE COMPOUND POISSON

```
      DIMENSION A(200), B(200), AZ(200)
      DOUBLE PRECISION R(200),XSUM,S,X
      N = 200
      READ (1,2) (A(I),I=1,N)
 2    FORMAT (20F4.3)
      ZSUM=0
      TMEAN=0
      DO 17 I=1,N
      ZSUM=ZSUM+A(I)
      B(I)=A(I)*I
      TMEAN=TMEAN + B(I)
 17   CONTINUE
      R(1)=1/(2.7182818**ZSUM)
      XSUM=R(1)
      WRITE (3,60) R(1),ZSUM,TMEAN
 60   FORMAT ('0',F12.10,4X,2F10.2)
      D0 90 J=1,N
      S=0.0
```

```
      D0 25 I=1,J
25    S=S+B(I)*R(J-I+1)
      R(J+1)=S/J
      XSUM=XSUM+R(J+1)
      X=R(J+1)
      RX=J
      RSUM=(1-XSUM)
41    FORMAT ('0','R',F4.1, '=', F12.10,6X,'SUM OF R=',F12.10)
      WRITE 93,41) RX,X,XSUM
      IF (XSUM-.997)90,90,100
90    CONTINUE
100   STOP
      END
```

APPENDIX E

THE INVENTORY COST-PRICING MODEL
IN G LEVEL FORTRAN IV

```
      DIMENSION BASE(10), TC(10), PRICE(10), OQUAN(10),
     1Y(250), A(300), B(300), AZ(300)
      DOUBLE PRECISION R(300),XSUM,S,X
C  R(J+1)=1/(J+1)*(A(1)*R(J)+2*A(2)*R(J-1)...+(J+1)*A(J+1)*R(0))
      READ (1,4) TRANC, DICOST, CARYC, OCOST, SPACOS,
     1ROI, DEMYR, RECOST, M, N
      WRITE (3,4) TRANC, DICOST, CARYC, OCOST, SPACOS,
     1ROI, DEMYR, RECOST, M, N
      DEMYRX =DEMYR
      TRANCX=TRANC
      CARYCX=CARYC
      OCOSTX=OCOST
      ROIX=ROI
    4 FORMAT (8F7.4,2I4)
      D0 6 I=1,M
      READ (1,5) OQUAN(I), BASE(I)
    6 WRITE (3,5) OQUAN(I), BASE(I)
    5 FORMAT (2F9.2, F8.2)
      READ (1,2) (A(I),I=1,N)
    2 FORMAT (20F4.2)
      D0 11 J=1,N
   11 AZ(J) = A(J)
      NTIMES=29
      D0 92 K=1,NTIMES
      WRITE (3,85)
   85 FORMAT ( '1DEMAND MIN  AVE   PRICE  TC  MEAN '
     1'REORDER'  RATIO  LEADTIME AVECOST  PRICE'/
     1' PER  PRICE  COST  ORDER   ORDER  DDLT  POINT',
     1'          STKOUT STKOUT  STKOUT'/
     1' YEAR                 QUAN QUAN              ',
     1'              PROB   PROB   PROB')
  303 TRANC=0.0
  304 IF (K-10)203,203,204
  203 TRANC=TRANC + 2.50
```

```
      GO TO 13
205   TRANC=TRANCX
      IF (K-11) 405,405,204
405   CARYC=0.0
205   CARYC=CARYC + .05
      IF (K-16) 13,13,206
206   CARYC=CARYCX
      IF (K-23) 207,207,208
207   OCOST=OCOST+ 2.50
      GO TO 13
208   OCOST=OCOSTX
      IF (K-24) 210,209,210
209   ROI =.00
210   IF (K-29) 211,211,212
211   ROI=ROI+.10
      GO TO 13
212   ROI=ROIX
      GO TO 13
  3   CONTINUE
      D0 12 J=1,N
 12   A(J)=A(J)+AZ(J)
 13   CONTINUE
      W=0
      ZSUM=0
      TMEAN=0
      D0 17 I=1,N
      ZSUM=ZSUM+A(I)
      B(I)=A(I)*I
      TMEAN= TMEAN + B(I)
 17   CONTINUE
      RX =TMEAN
  7   D0 10 I=1,M
      PRICE(I)=ROI*BASE(I)*(OQUAN(I)/2+(RX-TMEAN))/DEMYR
     1+DICOST+SPACOS*(OQUAN(I)/2+(RX-TMEAN))/DEMYR
     1+OCOST/OQUAN(I)+BASE(I)
     1+RECOST*(BASE(I)+DICOST+OCOST/OQUAN(I)+SPACOS*
     1(OQUAN(I)/2+(RX-TMEAN))/DEMYR+BASE(I)*(OQUAN(I)/2
     1+(RX-TMEAN))/DEMYR*CARYC)+CARYC*BASE(I)*(OQUAN
     1(I)/2+(RX-TMEAN))/DEMYR
 10   CONTINUE
      PRIMIN=PRICE(1)
      D0 20 I=2,M
      IF (PRIMIN-PRICE(I)) 20,15,15
 15   LOCOS=(I)
```

```
      PRIMIN+PRICE(I)
 20   CONTINUE
      R(1)=1/(2.7182818**ZSUM)
      XSUM=R(1)
      D0 90 J=1,N
      S=0.0
      D0 25 I=1,J
 25   S=S+B(I)*R(J-I+1)
      R(J+1)=S/J
      XSUM=XSUM+R(J+1)
      X=R(J+1)
      RX=J
      RSUM=(1-XSUM)
      SHOCOS=RSUM*(PRIMIN+TRANC-BASE(LOCOS))*DEMYR/
     1OQUAN(LOCOS)+CARYC*BASE(LOCOS)*RSUM
      ANCOS=CARYC*BASE(LOCOS)
      IF(SHOCOS-ANCOS)     100,109,90
 90   CONTINUE
100   RSUM= RSUM +.1*X
      RX=RX-.1
      SHOCOS=RSUM*(PRIMIN+TRANC-BASE(LOCOS))*DEMYR/
     1OQUAN(LOCOS)+CARYC*BASE(LOCOS)*RSUM
      IF(SHOCOS-ANCOS)     100,109,108
108   RX= RX+.01
      RSUM = RSUM - .01*X
      SHOCOS=RSUM*(PRIMIN+TRANC-BASE(LOCOS))*DEMYR/
     1OQUAN(LOCOS)+CARYC*BASE(LOCOS)*RSUM
      IF (SHOCOS-ANCOS) 109,109,108
109   CONTINUE
 38   CONTINUE
      D0 40 I-1,M
      TC(I )=BASE(I)*DEMYR+(OQUAN(I)/2+(RX-TMEAN))*BASE(I)*
     1CARYC+OCOST*DEMYR/OQUAN(I)+SPACOS*(OQUAN(I)/2
     1+(RX-TMEAN))+SHOCOS*TMEAN*(DEMYR/OQUAN(I))
     1+RECOST*((BASE(I)+DICOST+OCOST/OQUAN(I))*DEMYR
     1+DICOST*DEMYR+(BASE(I)*(OQUAN(I)/2 +(RX-TMEAN))*
     1CARYC))
 40   CONTINUE
      TCMIN=TC(1)
      D0 50 I=2,M
      IF (TCMIN-TC(1))50,45,45
 45   IQLOW=(I)
      TCMIN=TC(I)
 50   CONTINUE
```

```
70    CONTINUE
      W=W+1
      IF (W-2) 7,80,80
80    TOTLOW=TCMIN
      FQLOW=OQUAN(IQLOW)
81    FQLOW=FQLOW+1
      AQLOW=FQLOW-1
      TCNEW=TOTLOW
      TOTLOW=BASE(IQLOW)*DEMYR+(FQLOW/2+(RX-TMEAN)*
     1BASE(IQLOW)*CARYC+OCOST*DEMYR/FQLOW+SPACOS*
     1(FQLOW/2+(RX-TMEAN))+SHOCOS*TMEAN*(DEMYR/FQLOW)
     1+RECOST*((BASE(IQLOW)+DICOST+OCOST/FQLOW)*DEMYR
     1+DICOST*DEMYR+(BASE(IQLOW)*(FQLOW/2+(RX-TMEAN))*
     1CARYC))
      IF (TOTLOW-TCNEW) 81,82,82
82    CONTINUE
      PROUT=RSUM*TMEAN/OQUAN(LOCOS)
      RATIO=RX/TMEAN
      TCOUT =RSUM*TMEAN/AQLOW
      TOTREV=PRIMIN*DEMYR
      CTOVER=TOTREV-TOTLOW
      AVCOS=TOTLOW/DEMYR
      AVCTO=CTOVER/DEMYR
      WRITE (3,86) DEMYR,PRIMIN,AVCOS,LOCOS ,AQLOW,
     1TMEAN,RX,RATIO,RSUM,TCOUT,PROUT
86    FORMAT (2X,F4.0,2X,2F6.2,4X,2F5.0,2X,2F8.2,5X,
     1F6.2,F7.4,2F9.4 )
      DEMYR=DEMYR+10
      IF (DEMYR-200) 3,3,91
91    CONTINUE
      DEMYR=DEMYRX
      D0 93 J = 1,N
93    A(J) = AZ(J)
92    CONTINUE
      STOP
      END
```

BIBLIOGRAPHY

Walter Adams and Joe Dirlam, "Steel Imports and Vertical Oligopoly Power," American Economic Review, September, 1964.

M. A. Adelman, "Administered Prices and Inflation," Quarterly Journal of Economics, February, 1961.

R. M. Adelson, "Compound Poisson Distribution," Operational Research Quarterly, Vol. 17, No. 1 (March, 1966).

S. B. Alpert, "Economy of Scale in the Metal Removal Industry," Journal of Industrial Economy, July, 1959.

American Iron and Steel Institute, The Competitive Challenge to Steel, December, 1961.

Kenneth Arrow, Samuel Karlin and Herbert Scarf, Studies in the Mathematical Theory of Inventory and Production (Stanford, Cal.: Stanford University Press, 1958).

Jules Backman, Price Practices and Price Policies (New York: The Ronald Press Co., 1958).

Joe S. Bain, Barriers to New Competition (Cambridge, Mass.: Harvard University Press, 1965).

__________, Industrial Organization (New York: John Wiley & Sons, Inc., 1959).

M. F. Barloon, "Institutional Foundations of Pricing Policy in the Steel Industry," Business History Review, September, 1954.

Alexander Barges, The Effect of Capital Structures on the Cost of Capital (Engelwood Cliffs, N. J.: Prentice-Hall, 1963).

Martin Basic, Development and Application of a Gamma-Based Inventory Management Theory (Unpublished Dissertation, Michigan State University, 1965).

Burnham Putnam Beckwith, Marginal-Cost Price-Output Control,(New York: Columbia University Press, 1955).

A. A. Berle, "The Impact of the Corporation on Classical Economy," Quarterly Journal of Economics, February, 1965.

R. S. Bower, "Decreasing Marginal Cost in Brick Production," Journal of Industrial Economics, November, 1964.

Elwood S. Buffa, Models for Production and Operations Management,(New York: John Wiley & Sons, Inc., 1963).

Duncan Burn, The Steel Industry 1939-1959: A Study in Competition and Planning (Great Britain: Cambridge University Press, 1961).

James P. Cairns, "Benefits from Restrictive Practices: The British Experience," Canadian Journal of Economics and Political Science, May, 1964.

Ralph Cassady, Jr., Price Warfare in Business Competition: A Study of Abnormal Competitive Behavior (East Lansing, Mich.: Bureau of Business and Economic Research, Michigan State University, 1963).

Edward Chamberlin, The Theory of Monopolistic Competition, 7th ed. (Cambridge, Mass.: Harvard University Press, 1960).

James A. Constantin, Principles of Logistics Management (New York: Meredith Publishing Co., 1966).

David C. Coyle, Steel: The Near-Sighted Bellwether (Washington, D. C.: Public Affairs Institute, 1961).

Carrol R. Daugherty, Melvin G. de Chazeau, Samuel S. Stratton, The Economics of the Iron and Steel Industry, Vol. II (New York: McGraw-Hill Book Co., Inc., 1937).

Brian Dixon, Price Discrimination and Marketing Management, Michigan Business Studies, Vol. XV, No. 1 (Ann Arbor, Mich.: Bureau of Business Administration, University of Michigan, 1960).

Wilfred J. Eiteman, Price Determination: Business Practices Versus Economic Theory (Ann Arbor, Mich.: Bureau of Business Research, University of Michigan, 1949).

William Fellner, Competition Among the Few (New York: Alfred A. Knopf, 1949).

Albert Arthur Fitzpatrick, Pricing Methods of Industry (Boulder, Col.: Pruett Press, Inc., 1964).

Joseph W. Ford, The Steel Import Problem, Studies in Industrial Economics, No. 7 (New York: Fordham University Press, 1961).

H. P. Galliher, Philip M. Morse and M. Simons, "Dynamics of Two Classes of Continuous Review Inventory Systems," Operations Research, 7.

Zvi Griliches, "Are Farmers Rational?" Journal of Political Economy, February, 1960.

I. M. Grossak, "Duopoly, Defensive Strategies and the Kinked Demand Curve," Southern Economic Journal, April, 1966.

G. Hadley and T. M. Whitin, Analysis of Inventory Systems (Englewood Cliffs, N. J.: Prentice-Hall, Inc., 1963).

A. J. Harrison, "Economies of Scale and the Structure of the Road Haulage," Oxford Economic Papers, November, 1963.

W. Z. Hirsch, "On the Phenomenon of Inelastic Demand," Southern Economic Journal, July, 1951.

Paul G. Hoel, Introduction to Mathematical Statistics (New York: John Wiley & Sons, Inc., 1962).

Ira Horowitz, "An Econometric Analysis of Supply and Demand in the Synthetic Rubber Industry," International Economic Review, September, 1963.

Robert J. Lanzillotti, "Pricing Objectives in Large Companies," American Economic Review, December, 1958.

__________, Pricing, Production and Marketing Policies of Small Manufacturers (Pullman, Wash.: Washington State University Press, 1964).

Fritz Machlup, The Basing Point System: An Economic Analysis of a Controversial Pricing Practice (Philadelphia, Penn.: The Blakiston Co., 1949).

__________, The Political Economy of Monopoly (Baltimore, Md.: The Johns Hopkins Press, 1952).

John F. Magee, Production Planning and Inventory Control (New York: McGraw-Hill Book Co., Inc., 1958).

Claude McMillan and John Demaree, The Management of Metal Inventories (Cleveland, Ohio: Steel Service Center Institute, 1966).

Kenneth McMullen, "Effective Overhead Budgeting," N. A. A. Bulletin, September, 1961.

Edwin S. Miles, Price, Output and Inventory Policy (New York: John Wiley & Sons, Inc., 1962).

Eliezor Naddor, Inventory Systems (New York: John Wiley & Sons, Inc., 1966).

C. J. Oort, Decreasing Costs as a Problem of Welfare Economics (Amsterdam: Drukkerij, 1958).

Daniel Orr and Paul MacAvoy, "Price Strategies to Promote Cartel Stability," Economica, May, 1965.

Dale K. Osborne, "The Role of Entry in Oligopoly Theory," Journal of Political Economy, August, 1964.

Alfred Oxenfeldt, David Miller, Abraham Shuckman and Charles Winick, "Insights Into Pricing" from Operations Research and Behavioral Science (Belmont, Cal.: Wadsworth Publishing Co., 1961).

"Rail Tax Fight," Wall Street Journal, November 8, 1963.

E. A. G. Robinson, Monopoly (New York: Pitman Publishing Corp., 1949).

Joan Robinson, The Economics of Imperfect Competition (London: Macmillan and Co., Ltd., 1954).

Thomas L. Saaty, Mathematical Methods of Operations Research (New York: McGraw-Hill Book,Co., 1959).

Erich Schneider, Pricing and Equilibrium, English version by Esra Bennathon (New York: The Macmillan Co., 1962).

S. C. Schuman and S. B. Alpert, "Economies of Scale: Some Statistical Evidence: Comment," Quarterly Journal of Economics, August, 1960.

John Scoville and Noel Sargent, Fact and Fancy in the T.N.E.C. Monographs (New York: National Association of Manufacturers of the United States of America, 1942).

Senate Subcommittee on Antitrust and Monopoly, Administered Prices: A Compendium on Public Policy (Washington, D. C.: U. S. Government Printing Office, 1963).

R. L. Sich, "Evidence of Detriment Caused by Restrictive Trading Agreements," Oxford Economic Papers, November, 1965, London.

Martin K. Starr and David W. Miller, Inventory Control: Theory and Practice (Englewood Cliffs, N. J.: Prentice-Hall, 1962).

George J. Stigler, The Theory of Price (New York: The Macmillan Co., 1952).

A. Sutherland, "Economics in the Restrictive Practices Court," Oxford Economic Papers, November, 1965, London.

John Y. D. Tse, Profit Planning Through Volume-Cost Analysis (New York: The Macmillan Co., 1960).

Spencer A. Tucker, The Break-Even System: A Tool for Profit Planning (Englewood Cliffs, N. J.: Prentice-Hall, Inc., 1963).

U. S. Congress Hearings before the Temporary National Economic Committee, Investigation of Concentration of Economic Power, 77th Congress, 2nd Session, Part 16-19 (Washington, D. C.: U. S. Government Printing Office, 1940).

U. S. Congress Hearings before the Temporary National Economic Committee, Investigation of Concentration of Economic Power, 76th Congress, 3rd Session, Monograph #41, Price Discrimination in Steel (Washington, D. C.: U. S. Government Printing Office, 1941).

U. S. Congress Hearings before the Temporary National Economic Committee, 76th Congress, 3rd Session, Monograph #42, The Basing Point Problem (Washington, D.C.: U. S. Government Printing Office, 1941).

U. S. Congress, Economic Joint Committee, Steel Prices, Unit Costs, Profits and Foreign Competition, Hearings, 88th Congress, 1st Session, pursuant to sec. 5 (a) of PL 304, 89th Congress (Washington, D.C.: U.S. Government Printing Office, 1963).

Harvey M. Wagner, Statistical Management of Inventory Systems (New York: John Wiley & Sons, Inc., 1962).

A. A. Walters, "Production and Cost Functions," Econometrica, 1963.

Donald S. Watson, ed., Price Theory in Action: A Book of Readings, (Boston: Houghton, Mifflin Co., 1965).

P. J. D. Wiles, Price Cost and Output (New York: Frederick A. Praeger, 1963).

R. H. Wilson, "A Scientific Routine for Stock Control," Harvard Business Review, Vol. 13, No. 1 (October, 1934).