ECONOMIC WELL SPACING FOR AN INDUSTRIAL WATER SUPPLY

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Economic Well Spacing for an Industrial Water Supply

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INTRODUCTION

The ever increasing use of ground water for municipal and industrial supplies has brought about the need for workable methods for estimating well performance.

In the past the procedure of obtaining a sufficient supply of water from the ground consisted of tapping a water bearing formation and extracting the desired amount of water from it. The growth of communities and industries and hence the increased need for larger quantities of water has brought out the fact that these ground reservoirs are not inexhaustible. The over-development of certain areas has proven costly and could have been avoided if proper spacing of wells had been predetermined.

The problem of economically spacing wells and forecasting their performance will be discussed in this report and a step by step procedure involving a typical problem will be presented.

This problem is intended to illustrate what can be done to utilize data normally available to the designer in well built up districts. It is recognized that in many instances the data may not be as complete as that used here, but on the other hand the information used here is not nearly as complete as it could be.

It has been a common failure of many industries to choose locations for their plants without proper consideration as to where or how they were going to obtain sufficient quantities of water. Many plants have been forced to abandon their sites for more suitable ones, just for that reason. The paper industry of central Michigan is an excellent example of what may happen if proper water supply planning is not utilized. Several plants have chosen sites where apparently there was

water in sufficient quantities to last forever, but an investigation would reveal that the number of abandoned wells far exceeds the number of producing ones. Without further investigation it is evident that any forecast that may have been made in regard to the long-time out put of these wells was expensively inaccurate.

If this example were an outstanding one there would be little need for modification of the present methods used in regard to well field design, but it is not the exception; it is quite general throughout the United States.

While this report does not attempt to solve all the problems, or even most of the problems of the well designer, I believe sincerely that it is a step in the right direction.

The problem presented will attempt to illustrate in general a procedure to use in applying several formulas and basic principles devised by a number of investigators; it will not attempt to derive these formulae or delve into their theories. References will be made for further study, and to substantiate the methods used herein. It is also realized by the writer that many specific problems as to specifications have been omitted in hopes of amplifying the main objective.

PROBLEM

Conditions of problem

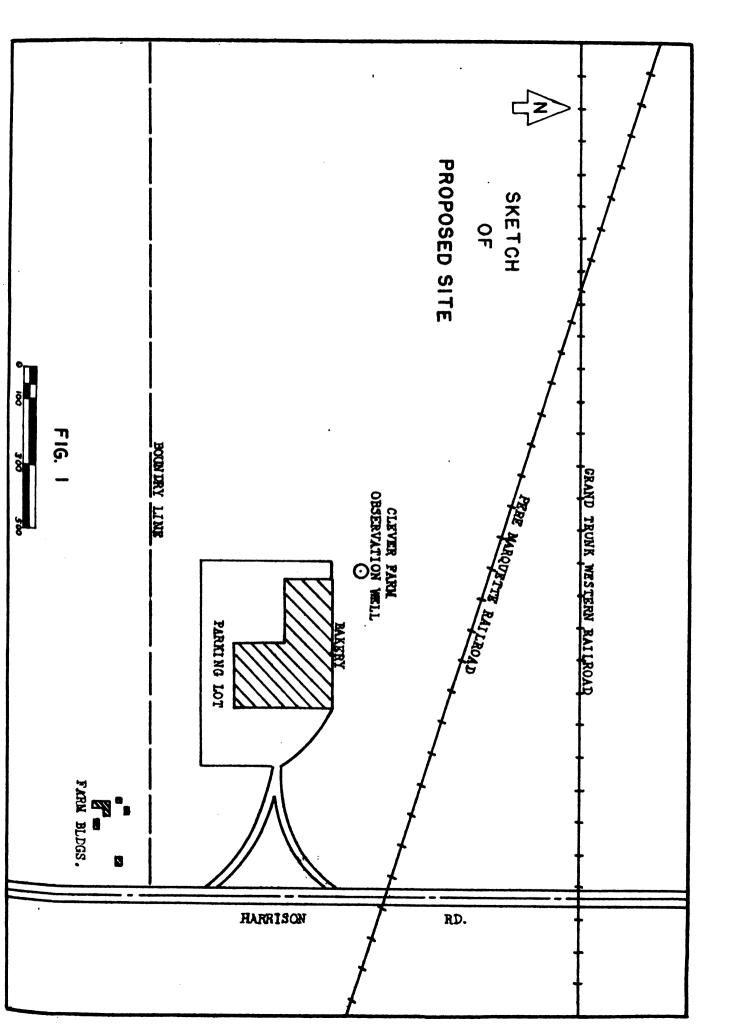
A suitable site for a large bakery is to be investigated to determine whether the bakery may obtain a suitable ground water supply of its own. Also, the cost of obtaining will be compared to purchasing township water. The site chosen is the $N_{\frac{1}{2}}$ of the $SW_{\frac{1}{4}}$ of sec. 24 excluding railroad right of way property, and is as shown by Figure 1. It is assumed that the bakery has an option on the property and is considering construction of their plant here.

Collection of data

By a thorough canvass of the surrounding area, as much information regarding existing wells as possible is obtained. Information as to the log of the materials encountered in drilling these wells is often obtainable from the owner or the driller. The pumping rates of surrounding areas, likely to have effect on the field, are obtainable from the records kept by the municipal and industrial water works operators.

Water level measurements are often kept by Government agencies.

Locations of abandoned wells for possible observation wells may be found by this canvass. A very important place to check is the Ground Water Division of the United States Geological Survey, and also the State Geological Survey, for that area. They collect such data as is mentioned above, including water level measurements, and also have permeability and other figures based on pumping tests conducted in the area. They also offer advice in the method of obtaining these figures by the use of pumping tests.



The small home wells have valuable information too. The amount of casing in the well is indicative of the depth to rock. The quality of the water may be tested to check its contents and uniformity. The ease of obtaining depth to water may be helpful. Core samples from drillings may be used for laboratory tests. Libraries and newspaper files are excellent sources for obtaining records of old wells, and often mentioned are water levels at various times which may lead to an estimate of the rate of decline of the water table. If time is available, talks with old-timers or drillers may add little bits of obscure data that may fit well into the overall picture.

This investigation includes all such bits of information and an attempt to correlate these in a logical manner to their best advantage.

Expected Geologic Conditions to be encountered

A geologie cross section was drawn using the logs of existing wells and projecting them on an assumed plane perpendicular to the earth's surface with its line of intersection as indicated on the map of Figure 2. The wells that indicate only the bedrock surface are wells whose complete logs were not obtainable. The bed rock level was obtained by taking the elevation at which the drive casing was "landed". A method that is not usable in a report of this kind consists of making a 3-dimensional model, using a map of the area for a base and soda straws erected perpendicular to the map's surface at each well location. By using different color paints for different types of strata, and painting the straws using a convenient scale for elevation, the resulting model gives a very enlightening picture of the different stratas.

Analysis of Geology and proof or suggestion of interconnection

The cross-sectional diagram (Figure 7) will be used for estimating the following:

- I The depth to the water-bearing formation.
- II The thickness of the water bearing formation, and hence the total depth of the well.
- III The depth to bedrock, and hence the casing setting.
- IV Hydraulic connection between the various other well fields.

Taking the above items in order, a brief discussion will be made of each to point out the reasons for selecting the estimated values.

The depth to the top of the water bearing sandstone throughout the cross section shows that its elevation varies between 780 M. S. L. and 758 M. S. L. It is quite probable the elevation at the proposed site will fall somewhere in between these values. It is also reasonable to assume that the elevations nearest the site are more apt to be correct. A straight line drawn between the two adjacent wells shown should give a value close enough for estimating purposes, so the estimated elevations will be taken as 770 for all wells in the field.

The water bearing strata which is found in this area is the Saginaw sandstone. It is apparent from the logs shown that there are layers of shale present which do not seem to be continuous; but are found only in lens form. This condition seems to be quite general throughout the entire Saginaw formation, as pointed out by several geologists. The shales are relatively impermeable, and the sandstone is bounded by these shale layers at approximate depths of 770 and 460. The estimated thickness above the limestone of the water bearing formation or aquifer

will be taken as 200, allowing for the shale lenses. The total depth of the well then may be determined by taking the difference between the ground elevation of the estimated bottom of the aquifer, or 400 feet.

The depth to bed rock, or to the base of the drift, is normally where the well casing is set. The well driller is more likely to bid the contract lower if he is sure of approximately how much casing the well will require. The depth that will be estimated will be 60 feet, and was obtained by the same method as was used in estimating the depth of the water-bearing formation.

The fact that the shale formations cannot be traced from wells for only a short distance leads to the conclusion that they "pinch out" and leave the sandstone formation hydraulically connected. It is further apparent from the cross-section that the sandstone may be traced completely through the area at the same elevation. This uninterrupted ground water reservoir is then not only the source of supply for the proposed well field of the problem, but is also the same source for the Industrial and Municipal water supplies of the Greater Lansing Area. This fact must be taken into consideration, as it will have a direct bearing on the static water levels in the entire area. Further proof and a more thorough discussion will be given later concerning the interference to be expected from other well fields when an analysis of the Water Level data is made.

Static Water Level estimates

Information regarding past water levels was collected from various sources, and from these an attempt will be made to predict future expected conditions. Perhaps a brief history of the ground water developments will give a better basis for the reasoning to be used.

Prior to 1885, privately owned shallow wells furnished most of the domestic supply for the Lansing area. At this time the public supply was started using shallow wells drilled in a sand formation. Ten years later the first wells penetrating the sandstone were drilled to a depth of approximately 150 feet. By 1915 various other locations had wells penetrating the sandstone to a maximum total depth of 350 feet. The 1920 average daily withdrawal amounted to about 4.7 M. C. D., and by 1930 the rate had almost doubled. Finally, by 1944, the Lansing water supply system had reached 14 M. G. D. average rate, drawing from 91 wells. East Lansing and Michigan State College's rates also increased rapidly. Michigan State College is using approximately 1.5 M. G. D., and East Lansing ·9 M. G. D. Other considerable withdrawals of water are made by the industries and air-conditioned buildings.

Picture this vast underground reservoir supplying this tremendous amount of water. Its ability to transmit water is limited by the cross-sectional area of the aquifer, its permeability, and its hydraulic gradient. Consequently, when the rate of pumping of an area exceeds its transmitting properties it can only result in a decrease in head or lowering of pumping levels. It is evident from the water level records in the Lansing area that this condition exists. An example can be shown by the use of an old newspaper clipping which gage the static water level of the Michigan State College number 3 well as 1.5 feet from the top of the ground in February, 1905. The water level in this well now is approximately 40 feet. From the same source the Lansing well on Townsend Street was reported as having a static of 2 feet the same year. Its present level is about 35 feet, and it is quite some distance from any

large pumping. Some of the water levels in the Lansing Area nearer the more heavily pumped sectors are over 100 feet from the top of the ground. Recent installations of the Landell Metropolitan Area's wells will probably have increased effects on the lowering of the water table.

By numerous computations using pumping rates of the various fields and expected increases due to new fields, estimates of the proposed bakery site could be made. It is felt by the author that such computations would not be warranted on this particular problem; so estimates will be based on water level records taken in the immediate vicinity of this site.

At present there is a 10-inch well located on the property selected for the investigation which was originally put down as a natural gas test well, and later the casing pulled to the base of the drift and the hole sealed off at a depth of 455 feet. There is a water level recording instrument on this well maintained by the United States Geological Survey since November, 1945. A continuous record has been kept of the fluctuations of the water level in this well since that time. A typical chart showing the fluctuations over a period of a week is shown in Figure 3.

During this period of record the water level in this well has varied between 28 feet and 48 feet. It is apparent from a study of these graphs that the level is most affected by pumping in the vicinity. Positive proof of interference from the College wells will be shown later in the results of the pumping test. To show proof of interference from the nearest Lansing well field (Riverside Well Field) a period is taken when the College wells remain at a comparatively steady rate, and when the next nearest field or the city's Pere Marquette field also

remain at a steady rate. The data is tabulated below:

1948	M. S. C. Pumpage	PM Pumpage	Riverside Pumpage
Jan.	45 M. G. appx.	47.7	0.5
Feb.	45 M. G. appx.	45.0	6.8
Mar.	45 M. G. appx.	51.9	0.7

From these pumping records it is shown that while the College wells remained relatively constant and the PM field's rate decreased 2.7 from January to February, and then again increased their rate by 6.9 million gallons, that the Riverside field had a sizeable increase in pumping. This increase in pumping of 6.3 million gallons from January, 1948 to February, 1948 was accompanied by a lowering of the level in the recording observation well. During the month of January the water level averaged about 45 feet from the measuring point. During February, the average level was about 47 feet, and went as low as 48.1 feet. During the month of March it recovered to an average level of 45 feet again.

It would seem from this analysis that the effects of adjacent pumping should be taken into consideration in estimating a safe assumed static for the proposed well field.

Taking the months when the heaviest pumping occurred in adjacent fields and comparing them to the water level in the observation well, should give an indication as to what the lowest expected level should be. During July and August of 1946, the total pumpage of the Lansing's City wells reached a maximum for the year of 545 million gallons and 526 million gallons. During this period the Riverside field pumped the largest quantity (82 million gallons) for the period of water level recording. At this time the observation well reached an all time low

with a depth to water of 46.5 feet. The heaviest pumpage for 1947 also occurred during the months of July and August, with a total pumpage of 486 million gallons and 579 million gallons, respectively. The high during August, 74 million gallons, which was 8 milli pumpage figure for the Riverside field occurred/less than in 1946. During this period the low water level was recorded as 47.5 feet from the measuring point.

The measuring point in all cases was taken at a point 2.2 feet above the surface of the ground. The ground at this point has an elevation of approximately 865 feet.

From the above observed data it would seem that the maximum expected depth to water during the year would be about 48 to 49 feet, and the average would fall somewhere between 28 and 48 feet. By an inspection of all the data recorded it appears that the average would fall between 35 and 40 feet. In view of the installations of new township and city wells within the Lansing area, and an inspection of the last year's data, an average depth of about 40 feet would appear to be a very reasonable figure. Allowing for slightly increased rates during the next few years a design figure will be placed on the safe side and taken as 45 feet from the ground surface, or at an elevation of 820 feet M. S. L.

Selection of storage and transmissibility coefficients

Essentially, there are two basic methods for determining permeability of ground water formations; the "Equilibrium Method" and the "Nonequilibrium Method. The equilibrium method makes the basic assumption in its formulas that the cone of depression around the discharging well has reached equilibrium (steady-state flow of water). The non-equilibrium method does not depend on the cone of depression reaching equilibrium, and also introduces the factors of time and elasticity. This last method

may also be applied to the recovery of the water level in the vicinity of a discharging well, after the discharge of the well has stopped. For a more complete comparison of the methods, the reader is referred to "Geological Survey Water-Supply Paper 887".

During the month of January, 1948, a controlled pumping test was conducted at Michigan State College under the direction of the United States Geological Survey. With the permission of the Lansing office this data was made available for use in determining the water-bearing characteristics to be used in this problem. The observations made from the water level recorder located on this site (shown in Figure 1) are the water levels to be used in computing the transmissibility and storage coefficients.

The non-equilibrium formula recently developed under the direction of Theis is based on the assumption that Darcy's law is analogous to the law of the flow of heat by conduction, and thus the mathematical theory of heat-conduction is largely applicable to hydraulic theory. The Theis' non-equilibrium formula is:

$$s = \frac{114.6 Q}{T} \qquad \frac{1.84r^2 S}{Tt} \qquad \frac{e^{-u du}}{U}$$

Theis, C. V., the relation between the lowering of the piezometric surface and the rate and duration of discharge of a well using ground-water storage: American Geophys. Union Trans. 1935, pps. 519-524 where s is the drawdown, in feet, at any point in the vicinity of a well pumped at a uniform rate; Q is the discharge of the well, in gallons per minute; T is the coefficient of transmissibility of the aquifer in gallons per day, through each strip extending the height of the aquifer, under a unit

gradient—this is the average field coefficient of permeability multiplied by the thickness of the aquifer; r is the distance from the pumped well to the point of observation, in feet; S is the coefficient of storage of the aquifer; and t is the time the well has been pumped in days.

The coefficient of transmissibility is defined as the number of gallons of water that will move in one day through a vertical strip of the aquifer one foot wide and having the full height of the aquifer with a hydraulic gradient of 100%.

The coefficient of storage is defined as the volume of water, measured in cubic feet, released from storage in each vertical column of the aquifer having a base one foot square when the artesian head is lowered one foot.

The assumptions under which this formula is valid are as follows:

- 1. The water-bearing formation is equally permeable in all directions.
- 2. The formation is of infinite areal extent.
- 3. The wells penetrate the full thickness of the formation.
- 4. The coefficient of transmissibility is constant at all places and at all times.
- 5. The formations release water from storage instantaneously with a decline in water level.

Any divergence of actual field conditions from these idealized assumptions result in variations and inconsistencies in the alignment of the observed data.

The solution of the integral is greatly simplified by the use of graphs, a method devised by Theis, and will be demonstrated in the solution of the equation.

The exponential integral in equation (1) is replaced by the term W(u) which is read "well-function of u" and the equation is re-written as follows:

$$s = \frac{114.6 \, Q}{T} \, W(u)$$

The value of the integral is given by the following series:

$$W(u) = -0.577216 - \log e U + U - U^2 + U^3 - U^4$$

$$2.2! 3.3! 4.4!$$
(2)

Where
$$u = \frac{1.87r^2s}{Tt}$$
 (3)

Values of W(u) for values of u between 10⁻¹⁵ and 9.9 are given in Table 1. To avoid repeated solutions for u a "type curve" of values of u versus W(u) are plotted on logarithmic paper as shown in Fig. 3. By putting equations (1) and (3) in the following form:

$$s = \underbrace{\begin{bmatrix} 114.6 \text{ Q} \end{bmatrix}}_{\text{T}} \text{ W(u)} \tag{4}$$

$$\frac{r^2}{T} = \begin{bmatrix} T \\ 1.875 \end{bmatrix} u \tag{5}$$

it is seen that s is related to r 2/t as W(u) is to u. By plotting the test data values of the drawdown s against the distance from pumped well to observation well squared over the corresponding time (r 2/t) on logarithmic tracing paper, the resulting curve should match the "type curve" where its corresponding values are in proportion. By selecting a "matching point" on the two curves and rewriting the equations as follows:

$$T = \frac{114.6 \ Q \ W(u)}{s}$$
 (6)

and

$$S = \frac{u T}{1.87 r^2/t} \tag{7}$$

the values for transmissibility and storage can be solved for directly.

The controlled pumping test that was run on the college number 12 well was conducted as follows:

- I All wells were held constant prior to test until the recording chart on the observation well showed relative stabilization.

 (No appreciable change in water level.)
- II Daily chart installed on recording instrument to make possible closer check on time factor.
- III Number 12 pump shut off and exact time noted. Other wells held constant.
- IV Recorder changed at 24 hour intervals.
- V After being off for 48 hours approximately, number 12 pump was again started and as before the exact time was noted.
- VI Daily charts were maintained on the observation well until
 the magnitude of the water level changes due to the resumed
 pumping were so small as to be obscured by other fluctuations.
- VII The distance between the two wells was obtained.

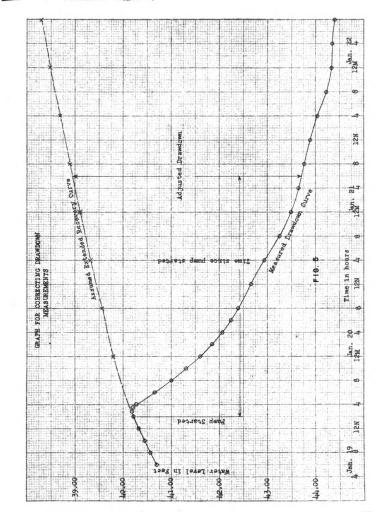
Using the non-equilibrium method the data will be analyzed. Hereafter the data collected during the period that the pump was off will be referred to as the "recovery data" and the data collected during the period after the pump resumed operation will be referred to as the "drawdown data".

The recovery data was first analyzed. The procedure followed was to extract the values of the water levels from the recorder chart and their corresponding times and put them in tabular form, selecting points at various time intervals to give fairly even spacing when plotted on logarithmic paper. The values for s were computed by assuming the depth to water at the start of the test to be static level, s is then

the difference between the water levels during the test and the assumed static level. The time is converted to minutes, using the time the pump went off as zero minutes. Another column is computed for the r 2/t values by squaring the distance in feet from pumped well to observation well and multiplying by 1440 to convert the time from days to minutes, and dividing by each selected time. These values were plotted and values for T and S determined for comparison with the drawdown computations. Of the two sets of data, the greatest weight was assigned to the drawdown data, because the extension of the recovery curve was thought to be more accurate than the assumed static used in the recovery data. The procedure will be outlined for the analysis of the drawdown data in detail as these values will be used in the subsequent design. This method in general is the same used on the recovery data.

The solid line of figure 5 is a reproduction of the observed data on the drawdown test. The dashed line represents the trend of the water level if no change in pumpage had occurred. This extended curve was obtained as follows:

I The plotted curve for s versus r 2/t of the recovery data was superimposed on the type curve, matching the two curves as closely as possible, keeping their coordinate axes parallel. The observed data curve was extended by tracing the type curve to the desired point. Values of s were then extracted from this extended portion and plotted with their corresponding times on figure 5 as shown by the X-marks. Through these points the extended curve was drawn. The values for the adjusted drawdown s are now obtainable by scaling the distance between the two lines. Putting the required data in tabular form for



convenience of plotting we have:

(1)	(2)	(3) t	(4) s	(5)	
DATE	HOUR	MIM.	ADJ. D/D	r 2/t	RATE OF PUMPING
Jan. 19	2:00 p	0	0	<i>∞</i>	200
Jan. 19	2:30 p	30	•02	1.96×10^8	200
Jan. 19	3:30 p	90	•07	6.53×10^{7}	200

From the values of (4) and (5) the graph shown in figure 6 was plotted on transparent logarithmic paper. Superimposing this plotted data on the type curve of figure 4 keeping the coordinate axes parallel the best "fit" was obtained; that is, the spot at which the most points bearing the greatest weight fell on the type curve. The first few points were assigned less weight than the following points because of the lag in reaction of the observation well, a slight error in the time element would be magnified, and the variable rate of pumping that is likely to occur until a relatively constant pumping level is reached. The last few points were also assigned less weight because their decreasing rate of change had diminished to the point where other pumping effects in the area were likely to be of greater magnitude than that of the tested discharging well.

The fitting of the type curve to the plotted points is shown by the dashed line of figure 6. The next step is the selection of a "match" point, or a point that is common to both curves. This point was selected as shown by the cross at a point on the type curve (figure 6) where the value u = 0.1 for convenience in computing. The values of both graphs at this point were noted and recorded for use in solving the following equations for T & S:

Legarithmic Graph of the Adjusted Drawdown Levels in the Clever Farm Observation Well 107

Drawdown or s recovery in feet

$$T = \frac{114.6 \ Q \ W(u)}{s}$$
 (6)

Where Q = 200 GPM, W(u) = 1.82, s = 4.65 feet

$$T = \frac{114.6 \times 200 \times 1.82}{4.65} = 9,000 \text{ G.P.D. feet}$$

and
$$S = \underbrace{u T}_{1.84 r} 2/t$$

Where u = 0.1, T = 9,000, $r = 2/t = 2.4 \times 10^6$

$$s = \frac{0.1 \times 9000}{1.87 \times 2.38 \times 10^{-6}} = 2.02 \times 10^{-4}$$

To relate the transmissibility to the more common term of permeability, the value for T can be divided by the thickness of the formation to give the average permeability.

Pav.
$$=\frac{T}{M}$$

Where T = 9000 and M = 200 feet assumed permeable thickness.

Pav. = $\frac{9000}{200}$ = 45 GPD per square foot per feet of hydraulic gradient.

The value obtained from the recovery data for T and S were 8750 and 1.66×10^{-4} which were reasonably close. If both the recovery and drawdown data were weighted equally, the average values would be used as the accepted ones. For the reasons mentioned previously the values obtained from the drawdown will be used as the accepted ones for further design.

A tabulation of the data thus far obtained will now be made for use in the analysis of the spacing of the wells and selection of the most economical grouping.

Estimated depth of each well 400 feet

(all ground elevations assumed filled in to 855 MSL)

Estimated length of casing required 60 feet

The site for the proposed well field is about 2600 feet long and 1300 feet wide at its widest part. To come well within state health specifications the wells will be located 300 feet away from any boundary and will be kept in a line lengthwise of the field. The spacing between the two outermost wells will be held to a maximum distance of 2000 feet.

Several combinations of well groupings will be investigated and the unit cost of water for each determined. An analysis of these values will be used in selecting the most economical design.

The basic assumptions to be used in the computations of the various well combinations are as follows:

- I All wells drawing from storage for a 6 months period. (No appreciable recharge to the aquifer)
- II Dewatering of the aquifer to be avoided by limiting effective drawdown to 40 feet in all wells.
- Hehavior of the water levels in the wells will corresponde to theoretical conditions of non-equilibrium method.
- IV 8 hour pumping day to furnish total amount.
- V Values for T and S determined will hold true for all wells.

The assumption made of no recharge to the aquifer for 6 months may sound severe, but quite often drought periods have lasted for this length of time.

It has been noted by well operators in the Lansing area that the wells that have pumping levels constantly below the top of the aquifer progressively fall off in yield. It is thought that this is caused partly from the dewatering of the sandstone and a subsequent incrustation of the dewatered portion of the formation decreasing its ability to pass water. For this reason a suggested remedy for this situation will be incorporated in this design. By limiting the effective or daily average drawdown to a point somewhere above the top of the aquifer, the formation should not become dewatered for any long period of time, and hence incrustation and drops in yield due to this should be held at a minimum.

The computed water levels will probably be somewhat in error from what actually would occur; however, their relative behavior should corresponde very closely. The safety factor included throughout should take care of any errors. Computed values for water levels using similar data have been found to agree within 10% in artesian formation near Fulkin, Texas. Other investigations throughout the country have closely approximated this value, through careful analysis of the data available to them.

The proposed bakery is assumed to be in operation for an 8-hour day and consequently their demand will be for that period. Reservoirs could be used for balancing the supply, but for the sake of estimating the 8-hour period will be used. The amount withdrawn during this 8-hour period should be the maximum for the day, regardless of the number of hours of pumping.

In many cases the coefficients of transmissibility and storage may be checked from other tests or more observations of the same test. The values obtained here were the only ones available in the area, so they will be the accepted ones.

Computations:

General formula for computing the drawdown in any one of a group of wells:

$$s_1 = s_{1-1} + s_{2-1} + s_{3-1} + s_{n-1}$$

 $s_2 = s_{1-2} + s_{2-2} + s_{3-2} + s_{n-2}$
 $s_n = s_{1-n} + s_{2-n} + s_{3-n} + s_{n-N}$

which is read as follows: the drawdown in well one is equal to the drawdown effect of pumping of number one on number one, plus the effect of number 2 on number 1, plus the effect of number 3 on number 1, etc.

Where
$$s = \frac{114.6 \text{ Q W(u)}}{T}$$
 (4)

and W(u) varies as before with u

where
$$u = \frac{1.87 \text{ r}^2\text{S}}{\text{Tt}}$$
 (3)

let
$$c = 114.6 \text{ W(u)}$$
 (8)

Now s = c Q

or in the general equation

$$s_{n-N} = C_{n-N} Q_N$$

By the use of equations (3) and (8) c values will be computed for a six months (183 days) period for various radii, using a 14 inch diam. well.

$$u = \frac{1.87 \text{ r}^2\text{S}}{\text{Tt}} = \frac{1.87 \text{ x} \cdot 2.02 \text{ x} \cdot 10^{-4}\text{r}^2}{9000 \text{ x} \cdot 183} = 2.30 \text{ x} \cdot 10^{-10}\text{r}^2$$

$$C = \frac{114.6 \text{ W(u)}}{9000} = 0.0 1273 \text{ W(u)}$$

C Values for Various Radii
Table 2

r	r ²	u	W(u)	С
7/12	. •34.	7.80 x 10-11	22.70	.289
100	1.0 x 10 ⁴	2.30×10^{-6}	12.41	.158
250	6.25×10^4	1.43×10^{-5}	10.58	•135
500	2.5 x 10 ⁵	5.73 x 10 ⁻⁵	9.19	.117
667	4.45 x 10 ⁵	1.02 x 10 ⁻⁴	8.61	.109
750	5.62 x 10 ⁵	1.29×10^{-4}	8.38	.107
1000	1.00 x 10 ⁶	2.30×10^{-4}	7.80	•099
1333	1.78 x 10 ⁶	4.10 x 10 ⁻⁴	7.22	•092
1500	2.25 x 10 ⁶	5.16 x 10 ⁻⁴	6.99	•089
2000	4.00 x 10 ⁶	9.18 x 10 ⁻⁴	6.42	•082

Values of W(u) taken from Table 1.

To find the maximum pumping rate that will produce an effective drawdown of 40 feet at the end of the 6 months design period, the Q rate value will be the rate for a 24-hour pumping period. It is seen, then, that for 8 hours three times this rate could be pumped. The actual drawdown, pumping at this rate, will be different from the design value and will be computed later to determine the pumping level for chossing a pump.

For the purpose of illustration and comparison of rates, the first computation will be for the yeild from one well under these conditions.

Eq. for 24 hour pumping s= CQ

or
$$Q = \frac{s}{c}$$

or for 8 hour pumping

$$1/3 Q = \frac{s}{c}$$
 or $Q = \frac{3 s}{c}$

where s = 40 feet and c from table is .289

$$Q = 3 \times 40 = 414 \text{ G.P.M.}$$

To obtain δ hour rate, 3 x s or 120 will be used for value of s.

Because closer spacing gives lesser yields, the maximum space of 2000 feet between outer wells will be utilized in the following computations:

For 2 wells spaced 2000 feet apart

Because both wells are affected similarly, one equation is needed as the Q values are equal:

$$s_1 = s_{1-1} + s_{2-1}$$

$$s_1 = c_{1-1}Q + c_{2-1}Q$$

$$s_1 = Q (c_{1-1} + c_{2-1})$$

Substituting 120 = Q (.289 + .082)

$$Q = \frac{120}{.371} = 324 \text{ G. P. M. per well}$$
 Field Value $2 \times 324 = 648 \text{ G.P.M.}$

3 Wells 1000 feet apart

In this case the two outside wells will have the same rate, but the inside rate will be less. Therefore, two equations are required.

$$I s_1 = c_{1-1} Q_1 + c_{2-1} Q_2 + c_{3-1} Q_3$$

where $Q_1 = Q_3$

II
$$s_2 = c_{1-2} Q_1 + c_{2-2} Q_2 + c_{3-2} Q_3$$

I 120 = (.289 + .082)
$$Q_1$$
 + .099 Q_2

II 120 = 2(.099)
$$Q_1 + .289 Q_2$$

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I 120 =
$$.371 Q_1 + .099 Q_2$$

2
$$225 = .371 Q_1 + .542 Q_2$$
 II x 1.68

$$Q_2 = \frac{105}{.443} = 237 \text{ G. P. M.}$$

$$Q_1 = Q_3 = 258 \text{ G. P. M.}$$

substituting Field Value

$$2(258) + 237 = 753 \text{ G. P. M.}$$

4 Wells 667 feet apart

Two pairs of similar wells, two unknown Q values, so there are two equations. As before the general equations hold true, but due to similarity of spacing, requires less equations. The method is now made clear so hereafter the equations will be shortened, showing only the direct substituting equations.

I 120 = (.289 + .082)
$$Q_1$$
 + (.109 + .092) Q_2

II 120 =
$$(.109 + .092)$$
 Q₁ + $(.289 + .109)$ Q₂

I 120 = .371
$$Q_1 + .201 Q_2$$

2 222 = .371
$$Q_1 + .735 Q_2$$
 II x 1.85

subtracting I from 2

$$Q_2 = \frac{102}{534} = 191 \text{ G. P. M.}$$

Substituting Field Value
$$Q_1 = 220 \text{ G. P. M.}$$
 $2(191) + 2(220) = 822 \text{ G. P. M.}$

5 Wells 500 feet apart

I 120 = (.289 + .082)
$$Q_1$$
 + (.117 + (.117 + .089) Q_2 + .099 Q_3

II 120 = (.117 + .089)
$$Q_1$$
 + (.289 + .099) Q_2 $\frac{1}{4}$.117 Q_3

III 120 = (2 x .099)
$$Q_1$$
 + (2 x .117) Q_2 + .289 Q_3

I 120 = .371
$$Q_1$$
 + .206 Q_2 + .099 Q_3

2 216 = .371
$$Q_1$$
 + .699 Q_2 + .210 Q_3 II x 1.80

IV 96 =
$$+ .493 Q_2 + .111 Q_3$$
 subtract I from 2

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I 120 = .371
$$Q_1$$
+ .206 Q_2 + .099 Q_3

3
$$225 = .371 Q_1 + .439 Q_2 + .542 Q_3$$
 III x 1.88

$$V = 105 = + .233 Q_2 + .443 Q_3$$
 subtract 1 from 3

IV
$$96 = .493 Q_2 + .111 Q_3$$

5
$$222 = .493Q_2 + .937 Q_3$$
 V x 2.12

$$Q_3 = \frac{126}{826} = 153 \text{ G.P.M.}$$

$$Q_2 = 160 \text{ G. P. M.}$$
 substituting

$$Q_1 = 194 \text{ G. P. M.}$$
 substituting

Field values

$$153 + 2(160) + 2(194) = 861 \text{ G.P.M.}$$

Since the field values or rate obtainable from each well group rapidly approaches a constant yield as the number of wells increases, it would seem advisable to stop at this point, analyze the values obtained, and select the most practical group.

The only apparent method for making an economical selection would be a comparative cost study.

A necessary factor to consider in determining the power required is the pumping level, or the distance the pump is required to lift the water from the ground. Computations for estimating this value for each well group considered will now be made.

The difference between an 8-hour pumping period at the daily rate and the 6 months pumping period at the same rate should give the reduction of the non-pumping water level in the well over the 6 months period.

This reduction of water level should then also be applicable to the pumping level for the 8-hour rate. By adding this difference to the

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pumping level for 8 hours at the 8-hour rate, the pumping level at the end of 6 months is obtained. As the rate of decline after the first few hours approaches a straight line function, the mean of the two levels is expressed in equation form: (the average pumping level)

Average pumping level = Pumping level at 6 months + pumping level at 8 hours, divided by 2.

For 1 well

$$s_1 = c_{1-1} Q_L = .289 \times \frac{414}{3} = 40.0$$
 (6 months period)
 $s_1' = c_{1-1} Q_L = .208 \times \frac{414}{3} = \frac{28.7}{11.3}$ (8 hour period)
 $s_1'' = c_{1-1} Q_L = .208 \times 414 = 86.1$ feet (actual level at 8 hours)
 $s_1'' = c_{1-1} Q_L = .208 \times 414 = 86.1$ feet (actual level at 8 hours)
 $s_1 = 86.1 + 11.3 = 97.4$ (actual level after 6 months)
Average = $97.4 + 86.1 = 91.8$, or approximately 92

So the pumping level below ground surface would be 92 feet plus the original static of 45 feet, or 137 feet.

For 2 wells

Pumping level below ground surface is 78 + 45 = 123 feet

Similarly the pumping levels were obtained for 3, 4, and 5 wells. Since the method used was generally the same, the actual computations

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will be omitted and their results will be tabulated below with those for 1 and 2. The additional computed data obtained thus far will also be included in the following table.

Assuming elevated storage to furnish 30# pressure or 69.30 head (alright for one story construction), the height above the ground to discharge water to storage will be taken as a constant value of 70 feet. Disregarding friction losses in valves and pipe, the total design head will be assumed to be 70 feet, plus the pumping level.

TABLE 3

No. of	Field Value GPM	Field Value GPD	Average Rate per well GPM	Average Appx P. L. feet	Total Head feet
1	414	199,000	414	137	207
2	648	311,000	324	123	193
3	753	362,000	251	115	185
4	822	395,000	205	112	182
5	861	414,000	172	101	171

The values shown in Table 3 are the necessary ones for selection of the pumps to be used. To get present day pump costs, this table was taken to Mr. Worden, the Cooke Company's pump representative for this area, for the selection of pumps required and their prices. From interviews with various people connected with estimating costs of well water, estimates for the rest of the costs involved in the system were obtained. A need for an average rate of 350,000 G. P. D. will arbitrarily

be chosen for use in choosing the most economical combination of wells.

To estimate the total cost of supplying water to the bakery, only the factors related to the transmission of water to the elevated storage will be considered. This is to demonstrate their relative cost to buying the water from the township.

Cost Estimates for Various Well Combinations

Case I - 1 well

Cost of Well

The pump selected for the first combination was a 30 hip., oil lubricated, 4 stage 12 inch pump $1\frac{1}{4}$ inch shaft, 6 inch column.

Unit	Price
Pump and motor	\$2400
Well and pump installed	4000
Well house	500
Meters and electric installations	1000
	\$790 0
Maintenance, 5%	3 95
Engineering, Int. on investment, & Misc. 20%	1580
Total	\$9875.00

The life of the pump, well, etc. is estimated to be 20 years, so the annual depreciation is

$$\frac{9875}{20}$$
 or \$493.75

Using a 300 day working year, the total number of gallons pumped will be

300 x daily rate = 300 x 199,000 = 50.7 million gallons

The depreciation cost per 1,000 gallons is then found by dividing the yearly depreciation by the number of gallons in thousands or:

The power cost was computed by the use of the tables in Anderson's Water Well handbook. Similar tables are available to the well designer in numerous other handbooks. At the same time that the pump selections were made, their efficiencies were noted and will be those used below.

Pump efficiency 79%, Motor efficiency 87%, Wire to water efficiency=
.87 x .79 = .69 or 69% from the table in the handbook.

For efficiency of 69%, 0.4550 KW hours / 1000 gallons is required for a 100 foot head. The total head or field head for Case 1 is 207 feet so,

 $\frac{207}{100}$ x .4550 = 0.942 KW hours / 1000 gallons

Using a unit price of \$0.02 per KW hour,

.02 x .942 = \$0.01884 / 1000 gallons power cost

The total cost per 1000 gallons may then be found by adding the depreciation cost to the power cost.

.9739 + 1.884 = 2.858 cents / 1000 gallons

Using the same method, the costs per 1000 gallons were obtained for the other four combinations of wells. Included in the estimates involving more than 1 well was the added cost of transmission main necessary to bring the water to the reservoir.

As the method is illustrated, only the final values will be presented. These will be found in the following table along with the cost of obtaining the additional water from the township for those combinations not producing the desired quantity. The cost of township water is estimated to be 10.6 cents / 1000 gallons, based on present rates.

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TABLE 4

(1) No. of Wells	(2) Daily Field Rate 1000 GPD	(3) Amt. pur- chased from twp. 1000 GPD	(4) Cost of purchased water/day dollars	(5) Cost of pumped water/1000 gals. cents	(6) Cost of pumped water/da dollars	(7) Total cost/day y dollars
1	199	151	16.00	2.858	5.69	21.69
2	311	39	4.14	3.104	9.65	13.79
3	361	0	0	3•233	11.32	11.32
4	394	0	0	3.695	12.92	12.92
5	414	0	0	3.687	12.90	12.90

Inspection of column (7) of Table 4 indicates the most economical well combination to be the third one from the top. The reason for the decrease in cost of the 5 well group over the 4 well group was caused by the selection of pumps. To fit the 4 well combination, the pumps available were not designed for high efficiency under the given conditions. Other conditions being equal, there would be a gradual increase in the cost per day rate for each additional well over three.

Using the 3 well combination, the possibility of spacing them closer together will be investigated. The equations for the 3 well combination were adjusted to determine the well yields for 500 feet spacing and 100 foot spacing. The computations were carried on in the same manner as previously illustrated and the design value obtained. The comparison in cost by redicing the space between them is as shown in Table 5.

TABLE 5

(1) Well Spacing feet	(2) Daily Meld rate 1000 GPD	(3) Amt. from twp. 1000 GPD	(4) Cost of twp water per day dollars	(5) Cost of pumped water per 1000 gals cents	(6) Cost of To pumped water per day dollars	(7) otal Cost day dollars
1000	361	0	0	3.233	11.32	11.32
500	340	10	1.06	3.229	11.19	12.25
100	292	58	6.16	3.319	9.68	15.84

It is noted, that even though the cost of transmission main is cut in half the increased head and requirement of additional water makes the 1000 foot spacing the cheapest.

It is noted from column (5) of the above table that the cost per 1000 gallons for the 500 foot spacing is the minimum cost. Theoretically, then, spacing of the 3 wells at a distance between 500 and 1000 feet where the daily rate would be exactly 350,000 gallons would be the ideal locations. It is the opinion of the writer that this degree of accuracy is not warranted in this problem, as the difference between actual and assumed conditions are quite likely to be in error.

The final design values for the supply will be as follows:

- I Three 14 inch diameter wells to a depth of 400 feet.
- II feet of 14 inch casing; remainder—open hole.
- III Spaced 1000 feet apart; no well closer than 300 feet to the property line.
- 3 deep well 8 inch turbine pumps, having 1 inch shaft, 5 inch column, 8 stage, and a 15 h.p. electric motor. Pump, 81% efficient, motor, 87% efficient. 3 phase, 440 volts, 60 cycle, A. C. U. S. Motor.

- V 4 inch transmission main to 6 inch main discharging to elevated storage or system.
- VI 6 inch emergency line from township supply for fire protection and major breakdowns.
- VII Capacity of elevated storage tank, 100,000 gallons.

SUMMARY AND CONCLUSIONS

The methods outlined in a publication by the American Water Works Association entitled "Standard Specifications for Deep Wells" will be the basis for a comparison study. The section entitled "Testing for Yield and Drawdown" states, "In most instances the maximum capacity of the test pump should be equal to the maximum quantity of water that it is anticipated the well will produce, and not merely equal to the capacity of the pump that is planned for permanent use."

They indicate that this information may be used in the future for ordering larger pumping equipment, or in the design of adjacent wells. This information could be very misleading to the untrained owner of the well. As was pointed out in the design problem, the presence of adjacent wells, if improperly located, could produce costly results if the producing properties were assumed to be the same. As to pumping the well at its maximum capacity for a basis of what can be expected in the future, no considerations of the lowering in pumping levels has been mentioned. If the formation is unable to obtain sufficient recharge to meet the withdrawals the pumping level declines. Quite often wells that have passed these short term pumping tests have suffered losses in yields. The procedure usually followed is to lower the pump-bowl settings to prevent a loss of yield. Constant recession of the water level finally results in a loss of yield to the point where the withdrawal of water balances the recharge rate.

The preliminary design, as outlined in this report, offers a method of determining the safe yield of a formation, and hence the removal of these undesirable series of events.

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The accuracy to which the values were carried out in this problem may not be used as the absolute expected figures, but should be used with a feeling of security that they are the worst conditions to be expected.

Careful observations during construction of the wells should be made, and corrections made where assumed values differ from actual ones. It would also be desirable upon completion of the wells to run controlled pumping tests on the various wells and observing water levels in the adjacent wells for a check on the preliminary values. After completion of the system, frequent and regular checks of water levels would give valuable information as to the possibility of increasing the capacity of the field.

For economy in selection of proper spacing, it would seem that the set of conditions that most closely approximates the desired amount of water would be the cheapest. This conclusion might be the one made from inspecting columns (2) and (7) of Tables 4 and 5. It is conceivable, though, by the reduction of cost for smaller pumps and the decreased size of well necessary, that a greater number of wells might offer a greater long-time saving. The wells in this problem were all assumed to be 14 inch; however, it is more economical to design the size of the well to fit the size of the pump desired, as the yield is only slightly decreased by a decrease in the diameter of the well.

Comparatively few industries keep complete records of their well installations or operation figures. These records are vitally needed to make an investigation for expansion of the system, tracing contamination, and in making repairs or changes in the present system.

Other problems may be solved using similar methods of attack; among these are locating boundaries of preglacial valleys, locating desirable localaties for well fields, and determining sources of recharge. A bibliography will be included for further study of related subjects.

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7.	1,,,,,,,,	22.000									N×10-4	N×10-4	N×10-3	N×10-1	N×10-1	N
N	N×10-11	N×10-11	N×10-13	N×10-11	N×10-11	N×10-10	N×10-9	N×10-6	N×10-7	N×10 ⁻⁴						
1.0	33. 9616 33. 8662 33. 7792	31, 6590 31, 5637 31, 4767	29.3564 29.2611 29.1741	27, 0538 26, 9585 26, 8715	24, 7512 24, 6559 24, 5689	22, 4486 22, 3533 22, 2663	20, 1460 20, 0507 19, 9637	17, 8435 17, 7482 17, 6611	15.5409 15.4456 15.3586	13, 2383 13, 1430 13, 0560	10. 9357 10. 8404 10. 7534	8. 6332 8. 5379 8. 4509	6. 3315 6. 2363 6. 1494	4. 0379 3. 9436 3. 8576	1. 8229 1. 7371 1. 6595	0.2194 .1860 .1584
1.3	33, 6992 33, 6251	31.3966 31.3225	29.0940 29.0199	26. 7914 26. 7173	24, 4889 24, 4147	22, 1863 22, 1122	19, 8837	17. 5811 17. 5070	15. 2785 15. 2044	12. 9759 12. 9018	10. 6734	8, 3709	6.0695 5.9955	3. 7785	1. 5889	.1355
1.5	33, 5561	31. 2535 31. 1890	28. 9509 28. 8864	26. 6483 26. 5838	24. 3458 24. 2812	22.0432 21,9786	19. 8096 19. 7406 19. 6760	17, 4380 17, 3735	15, 1354	12.8328	10. 5303	8. 2958 8. 2278 8. 1634	5, 9266 5, 8621	3. 6374 3. 5739 3. 5143	1. 4645 1. 4092	. 1000
1.7	33, 4309	31, 1283 31, 0712	28, 8258 28, 7686	26.5232 26.4660	24, 2206 24, 1634	21, 9180 21, 8608	19. 6154 19. 5583	17, 3128 17, 2557	15. 0103 14. 9531	12.7683 12.7077 12.6505	10. 4051 10. 3479 10. 2939	8. 1027 8. 0455	5. 8016 5. 7446	3, 4581	1. 3578 1. 3098	. 07465
2.0	33, 3197 33, 2684	31.0171 30,9658	28. 7145 28. 6632	26.4119 26.3607	24, 1094 24, 0581	21, 8068 21, 7555	19, 5042 19, 4529 19, 4041	17, 2016 17, 1503 17, 1015		12.5964 12.5451	10. 2426	7. 9915 7. 9402	5. 6906 5. 6394	3. 4050 3. 3547	1. 2649 1. 2227	. 05620 . 04890
2.1	33, 2196	30, 9170 30, 8705	28. 6145 28. 5679	26.3119 26.2653	24. 0093 23. 9628	21. 7067 21. 6602	19. 4041 19. 3576 19. 3131	17, 1015 17, 0550 17, 0106	14. 8477 14. 7980 14. 7524 14. 7080	12, 4964 12, 4498	10, 1938 10, 1473 10, 1028	7. 8914 7. 8449	5. 5907 5. 5443	3. 3069 3. 2614 3. 2179	1. 1829 1. 1454	.04261
2.4	33, 1286 33, 0861 33, 0453	30, 8261 30, 7835 30, 7427	28, 5235 28, 4809 28, 4401	26. 2209 26. 1783 26. 1375	23, 9183 23, 8758 23, 8349	21. 6157 21. 5732 21. 5828	19. 3131 19. 2706 19. 2298	16. 9680 16. 9272	14. 7080 14. 6654 14. 6246	12, 4054 12, 3628 12, 3220	10. 1028 10. 0603 10. 0194	7. 8004 7. 7579 7. 7172	5, 4999 5, 4575 5, 4167	3. 1763 3. 1365	1. 1099 1. 0762 1. 0443	. 03250 . 02844 . 02491
2.6	33,0060	30, 7035 30, 6657	28. 4009 28. 3631	26. 0983 26. 0606	23, 7957	21. 4931 21. 4554	19, 1905 19, 1528	16.8880 16.8502	14. 5854 14. 5476	12. 2828 12. 2450	9. 9802 9. 9425	7. 6779 7. 6401	5. 3776 5. 3400	3.0983 3.0615	1.0139	.02185
2.8	32,9683 32,9319 32,8968	30, 6294	28. 3268 28. 3917	26. 0242 25. 9891	23. 7216 23. 6865	21. 4190 21. 3839	19, 1164 19, 0813	16. 8138 16. 7788	14. 5113	12. 2087 12. 1736	9. 9061 9. 8710	7.6038 7.5687	5, 3037 5, 2687	3.0261 2.9920	. 9573	. 01686
3.0	32. 8629 32. 8302	30.5604	28. 2578 28. 2250	25, 9552 25, 9224	23. 6526 23. 6198	21, 3500 21, 3172	19.0474 19.0146	16.7449	14.4423	12, 1397 12, 1069	9. 8371 9. 8043	7, 5348 7, 5020	5, 2349 5, 2022	2.9591 2.9273	. 9057 . 8815	.01305
3,2	32, 7984 32, 7676	30, 4958 30, 4651	28, 1932 28, 1625	25, 8907 25, 8599	23, 5881 23, 5573	21, 2855 21, 2547	18, 9829 18, 9521	16. 6803 16. 6495	14. 3777 14. 3470	12.0751 12.0444	9, 7726 9, 7418	7. 4703 7. 4395	5. 1706 5. 1399	2.8965 2.8668	. 8583 . 8361	.01013
3.4	32, 7378 32, 7088	30, 4352 30, 4062	28. 1326 28. 1036	25, 8300 25, 8010	23, 5274 23, 4985	21, 2249 21, 1959	18, 9223 18, 8933	16. 6197 16. 5907	14. 3171 14. 2881	12.0145 11.9855	9.7120 9.6830 9.6548	7, 4097 7, 3807 7, 3526	5, 1102 5, 0813 5, 0532	2.8379 2.8099 2.7827	. 8147 . 7942 . 7745	.007891 .005970 .006160
3.6	32, 6806 32, 6532 32, 6266	30, 3780 30, 3506 30, 3240	28. 0755 28. 0481 28. 0214	25, 7729 25, 7455 25, 7188	23, 4703 23, 4429 23, 4162	21. 1677 21. 1403 21. 1136	18. 8651 18. 8377 18. 8110	16.5625 16.5851 16.5085	14. 2599 14. 2325 14. 2059	11.9574 11.9300 11.9033	9. 6274	7. 3252 7. 2985	5. 0259 4. 9993	2.7563 2.7506	.7554 .7371	.005448
3.9	32, 6006 32, 5753	30, 2980 30, 2727	27. 9954 27. 9701	25. 6928 25. 6675	23, 3902 23, 3649	21. 0877 21. 0623	18. 7851	16. 4825 16. 4572	14. 1799 14. 1546	11. 8773 11. 8520	9. 6007 9. 5748 9. 5495	7, 2725 7, 2472	4. 9735 4. 9482	2, 7056 2, 6813	.7194	.004267
4.1	32. 5506 32. 5265	30, 2480 30, 2239	27. 9454 27. 9213	25. 6428 25. 6187	23, 3402 23, 3161	21. 0376 21. 0136	18. 7851 18. 7596 18. 7351 18. 7110	16. 4325 16. 4084	14. 1299 14. 1058	11. 8273 11. 8032	9, 5248	7, 2225 7, 1985	4. 9236 4. 8997	2.6576 2.6344	. 6859	.003349
4.3	32, 5029	30, 2004	27. 8978 27. 8748	25.5952 25.5722	23, 2926 23, 2696	20, 9900	18.6874	16.3848 16.3619	14. 0823 14. 0593	11.7797 11.7567	9.4771 9.4541	7, 1749 7, 1520	4. 8762 4. 8533	2.6119 2.5899 2.5894	. 6546 . 6397	. 002633
4.5	32, 4575 32, 4355	30, 1549 30, 1329	27, 8523 27, 8303	25, 5497 25, 5277 25, 5062	23, 2471 23, 2252 23, 2037	20. 9446 20. 9226	18. 6420 18. 6200	16. 3394 16. 3174	14. 0358	11,7342 11,7122 11,6907	9. 4317 9. 4097 9. 3882	7, 1295 7, 1075 7, 0860	4. 8310 4. 8091 4. 7877	2.5474 2.5268	.6253 .6114 .5979	.002073 .001841 .001635
4.7	32. 4140 32. 3929 32. 3723 32. 3521	30.1114 30.0904 30.0607	27, 8088 27, 7878 27, 7672 27, 7470	25, 4852 25, 4646	23. 2037 23. 1826 23. 1620	20, 9011 20, 8800 20, 8594	18. 6200 18. 5965 18. 5774 18. 5568	16, 2959 16, 2748 16, 2542	13, 9933 13, 9723 13, 9516	11, 6697 11, 6491	9.3671 9.3465	7. 0650 7. 0444	4. 7667 4. 7462	2, 5068 2, 4871	. 5848	.001453
5.0	32, 3521 32, 3323	30. 0495 30. 0297	27.7470	25, 4144 25, 4246	23. 1418 23. 1220	20. 8392 20. 8194	18, 5366 18, 5168	16, 2340 16, 2142	13, 9314 13, 9116	11.6289	9, 3263 9, 3065	7. 0242 7. 0044	4.7261 4.7064	2.4679 2.4491	.5598 .5478	.001148
5.2	32, 3129	30.0103	27, 7271 27, 7077 27, 6887	25, 4051	23. 1026	20, 8000	18. 4974 18. 4783 18. 4596	16. 1948 16. 1758	13. 8922 13. 8732	11.5896 11.5706		6. 9850 6. 9659	4. 6871 4. 6681 4. 6495	2. 4306 2. 4125	. 5362 . 5250	.0009086
5.4	32, 2752 32, 2568	29. 9726 29. 9542	27, 6700 27, 6516	25, 3674 25, 3491	23.0648 23.0465	20, 7622 20, 7439	18, 4413	16, 1571 16, 1387	13. 8545 13. 8361	11.5519 11.5336	9, 2681 9, 2494 9, 2310 9, 2130	6. 9473 6. 9289 6. 9109	4, 6313 4, 6134	2,3948 2,3775 2,3604	. 5140 . 5034 . 4930	. 0007198 - 0006409 . 0005708
5.6	32, 2388 32, 2211 32, 2037	29, 9362 29, 9185 29, 9011	27, 6336 27, 6159 27, 5985	25, 3310 25, 3133 25, 2959	23, 0285 23, 0108 22, 9934	20, 7259 20, 7082 20, 6908	18, 4233 18, 4066 18, 3882	16. 1207 16. 1030 16. 0856	13. 8181 13. 8004	11. 5155 11. 4978 11. 4804	9, 1953 9, 1779	6. 8932 6. 8758	4. 5958 4. 5785	2.3437 2.3273	. 4830 . 4732	.0005085
5.9	32, 1866 32, 1698	29. SS40 29. S672	27. 5814 27. 5646	25. 2789 25. 2620	22, 9763 22, 9595	20. 6737 20. 6569		16. 0685 16. 0517	13. 7830 13. 7659 13. 7491	11.4633 11.4465	9, 1808	6,8588	4. 5615 4. 5448	2, 3111 2, 2953	4637	. 0004039
6.1	32, 1583 32, 1370	29. 8507 29. 8344	27, 5481	25, 2455 25, 2293 25, 2133	22, 9429 22, 9267	20.6403	18, 3543 18, 3378 18, 3215	16. 0352 16. 0189	13. 7491 13. 7826 13. 7163	11. 4300 11. 4138	9, 1275 9, 1112	6, 8254 6, 8092	4. 5283 4. 5122	2, 2797 2, 2645 2, 2494	. 4454 . 4366 . 4280	. 0003211 . 0002864 . 0002555
6.4	32, 1210 32, 1053	29. 8184 29. 8027 29. 7872	27, 5158 27, 5001 27, 4846	25, 2133 25, 1975 25, 1820	22.9107	20.6081	18. 3055 18. 2898	16, 0029 15, 9872	13, 7003 13, 6846	11. 3978 11. 3820	9, 0952 9, 0795 9, 0640	6, 7932 6, 7775 6, 7620	4. 4963 4. 4806 4. 4652	2. 2346 2. 2301	.4280 .4197 .4115	.0002279
1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1	32. 0698 32. 0745 32. 0595 32. 0446	29, 7872 29, 7719 29, 7569	27, 4846 27, 4693 27, 4543	25, 1820 25, 1667 25, 1517	22, 8794 22, 8641 22, 8491	20. 5768 20. 5616 20. 5465	18. 2742 18. 2590 18. 2439	15. 9717 15. 9564 15. 9414	13, 6691 13, 6538 13, 6388	11. 3565 11. 3512 11. 3352	9.0487	6,7467	4. 4501	2. 2058 2. 1917	. 4036	.0001816
6.8	32, 0306 32, 0300	29. 7421 29. 7275	27, 4395 27, 4249	25, 1369 25, 1223	22. 8491 22. 8343 22. 8197	20. 5317 20. 5171	18. 2291 18. 2145	15. 9265 15. 9119	13.6240 13.6094	11. 3214 11. 3068 11. 2924	9.0189	6, 7169	4. 4204 4. 4059	2. 1779 2. 1643	.3883	.0001448
7.0	32.0156	29,7131	27. 4105 27. 3963	25, 1079	22, 8053	20, 5027	18, 2001 18, 1860	15, 8976 15, 8834	13 5950	11, 2782	8.9899 8.9757	6. 6879 6. 6737	4.3916 4.3775 4.3636	2. 1508 2. 1376 2. 1246	.3738	.0001155 .0001032 .00009219
7.2	31.9875 31.9737	29, 6849 29, 6711	27, 3823 27, 3685	25, 0797 25, 0659	22, 7771 22, 7633	20. 4746 20. 4608	18.1720 18.1582	15, 8694 15, 8556	13, 5668 13, 5530 13, 5394	11. 2642 11. 2504 11. 2368	8.9617 8.9479 8.9343	6. 6598 6. 6460 6. 6324	4, 3500 4, 3500 4, 3364	2.1118 2.0991	. 3599 . 3532 . 3467	.00009219
7.4	31, 9601 31, 9467	29.6575 29.6441	27. 3549 27. 3415	25, 0523 25, 0389	22.7497 22.7363	20, 4472 20, 4337	18. 1446 18. 1311	15, 8420 15, 8286 15, 8153	13, 5260 13, 5127	11, 2234 11, 2102	8, 9209 8, 9076	6. 6190	4.3231 4.3100	2.0867 2.0744	.3403	.00006583
7.6	31, 9334 31, 9203 31, 9074	29, 6308 29, 6178 29, 6048	27, 3282 27, 3152 27, 3023	25, 0257 25, 0126 24, 9997	22, 7231 22, 7100 22, 6971	20, 4205 20, 4074 20, 3945	18. 1179 18. 1048 18. 0019	15. 8022 15. 7893	13, 4997 13, 4868	11. 1971	8. 8946 8. 8817	6, 5927	4. 2970 4. 2842	2,0623 2,0503	.3280	. 00005263
7.9	31. 8947	29. 5921 29. 5795	27, 2895 27, 2769 27, 2645 27, 2523	24, 9869	22. 6844 22. 6718	20, 3818 20, 3602	18.0792	15, 7766 15, 7640	13, 4740	11. 1714 11. 1589	8, 8689 8, 8563	6, 5671 6, 5545	4. 2716 4. 2591	2, 0396 2, 0269 2, 0155	.3163	.00004210
8.1	31.8697 31.8574	29, 5671	27, 2645 27, 2523	24, 9619	22. 6594 22. 6471	20, 3568 20, 3445	18.0542 18.0419	15, 7516 15, 7393	13, 4490 13, 4367	11.1464 11.1342	8, 8439 8, 8317 8, 8195	6, 5421 6, 5298 6, 5177	4. 2468 4. 2346 4. 2226	2, 0135 2, 0042 1, 9930	. 3050 . 2996 . 2943	.00003370 .00003015 .00002699
8.3	31.8453 31.8333	29. 5427 29. 5307	27, 2401 27, 2282	24, 9375 24, 9256 24, 9137	22, 6350 22, 6230	20, 3324 20, 3204 20, 3066	18.0298 18.0178 18.0060	15, 7272 15, 7152 15, 7034	13. 4246 13. 4126 13. 4008	11. 1220 11. 1101 11. 0982	8, 8076 8, 7957	6, 5057	4. 2107 4. 1990	1.9820	. 2891	.00002415
8.6	31. 8215 31. 8098 31. 7982	29, 5189 29, 5072 29, 4957	27, 2163 27, 2046 27, 1931	24, 9137 24, 9020 24, 8905	22.6112 22.5995 22.5879	20 2060	17. 9943 17. 9827	15, 6917 15, 6801	13, 3891 13, 3776	11. 0865 11. 0750	8. 7840 8. 7725	6. 4822 6. 4707	4. 1874 4. 1759	1, 9604 1, 9498	. 2790 . 2742	. 90001936
8.8	31. 7868 31. 7755	29, 4842 29, 4729	27. 1816 27. 1703	24, 8790 24, 8678	22, 5765	20, 2813 20, 2739 20, 2626	17, 9713 17, 9600	15. 6687 15. 6574	13, 3661 13, 3548	11.0635 11.0523	8.7610 8.7497	6, 4592	4, 1646 4, 1534	1. 9393 1. 9290 1. 9187	. 2694 . 2647 . 2602	.00001552 .00001390 .00001245
9.0	31. 7643 31. 7533	29, 4618 29, 4507	27, 1592 27, 1481	24, 8566 24, 8455	22, 5540 22, 5429	20, 2514	17. 9488 17. 9378 17. 938	15, 6462 15, 6352	13.3437 13.3325	11.0411	8. 7395 8. 7275	6, 4368 6, 4258 6, 4148	4, 1423 4, 1313 4, 1205	1.9087	. 2557 . 2513	.00001245 .00001115 .000009988
		29, 4398 29, 4290	27, 1372 27, 1264	24, 8346 24, 8238 24, 8131	22, 5320 22, 5212	20, 2294 20, 2186	17, 9160	15.6243 15.6135	13.3217	11. 0191 11. 0083 10. 9976	8, 7166 8, 7058 8, 6951	6, 4040	4. 1098 4. 0992	1. 8888 1. 8791	. 2470	.000008948
9.4 9.5 9.6	31,7208	29, 4183 29, 4077 29, 3972	27, 1157 27, 1051 27, 0046		22, 5105 22, 4999 22, 4895	20, 2079 20, 1973 20, 1869	17, 9033 17, 8948 17, 8843	15, 6028 15, 5922 15, 5817	13, 3002 13, 2896 13, 2791	10.9870	8, 6845 8, 6740	6.3828 6.3723	4.0887	1. 8695 1. 8599	. 2387	. 000007185 . 000006439
		29. 3868	27, 0946 27, 0843 27, 0740 27, 0639	24. 7920 24. 7817 24. 7714	22, 4791	20, 1765 20, 1663	17. 8843 17. 8739 17. 8637	15, 5713	13. 2688 13. 2585	10, 9652	8. 6637 8. 6534	6.3620 6.3517	4. 0581 4. 0579 4. 0479	1. 8505 1. 8412 1. 8320	. 2308 . 2269 . 2231	.000005771 .000005173 .000004637
9.8	31.6690	29, 3064	27. 0639	24.7714 24.7613	22. 4587	20. 1561	17. 8535	15. 5509	13. 2483	10.9458	8, 6433	6, 3416	4.0179	1.8520	. 2031	.00004637
							Table	1								

