VALIDITY OF THE MANY-BODY APPROXIMATIONS IN SPHERICAL NUCLEI

Thesis for the Degree of Ph. D.

MICHIGAN STATE UNIVERSITY

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1972

This is to certify that the

thesis entitled

VALIDITY OF THE MANY-BODY APPROXIMATIONS

IN SPHERICAL NUCLEI

presented by

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has been accepted towards fulfillment
of the requirements for

Ph.D. degree in Physics

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Date Feb.22, 1972

ABSTRACT

VALIDITY OF THE MANY-BODY APPROXIMATIONS

IN SPHERICAL NUCLEI

By

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Reduced transition probabilities B(E2), and Spectroscopic factors S_j for stripping (d,p) reactions are calculated in the quasi-boson approximation (QBA), Random Phase approximation (RPA), Tamm-Dancoff approximation (TDA), and improved Random Phase approximation (IRPA). The effect of the Pauli principle on the J=0 pairs is shown to be negligible, while it is not for $J\neq 0$ pairs. The effects of these approximations on the B(E2) and S_j systematics are studied by comparing the averages of the absolute deviations from experimental values for each approximation. These studies have been carried out for a large number of spherical nuclei. The problem is formulated by using double-time Green's function techniques.

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Shakir M. Mustafa

A THESIS

Submitted to
Michigan State University
in partial fulfillment of the requirements
for the degree of

DOCTOR OF PHILOSOPHY

Department of Physics

1972

TO MY WIFE KHALIDA,

DAUGHTER AND SON DINA AND MAZIN

ACKNOWLEDGEMENTS

I like to express my sincere gratitudes to Professor Truman O. Woodruff for his many helps and discussions during this work. Further gratitudes are expressed to the Department of Physics at Michigan State University, and to the Government of Iraq, who made it possible for me to continue this work. Thanks go to Mrs Jean Strachan for typing the first three pages and to Mrs Debbie Bruchmann for typing the last two pages of the references.

I am deeply indebted to my wife for the years of patience and encouragement and thank her for the typing of this thesis.

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I. Introduction

In the fermi gas model of a nucleus, the forces between pairs of nucleons are neglected, and the nucleus assumed to be contained in a sphere of definite volume V with radius $R=r_oA^{1/3}$. While this degenerate gas model give a qualitatively correct picture of the nucleus, actual numerical results for the energy levels are far from accurate.

Progress in understanding the structure of the nucleus began after the conception of the shell model of the nucleus by Mayer, Haxel, Jensen and Suess[15]. The great successes of the shell model, in which the nucleons are assumed to move independently in a certain average potential, showed that the main part of interaction between the nucleons can be reduced to a spherically symetrical, self-consistent field acting on all of them. The next important step was made by Bohr and Mottelson[12,13] who proposed the unified nuclear model, in which an additional self-consistent part of the interaction is extracted which is non-spherical and time dependent. With this model it is possible to explain many of the regularities in the low-lying nuclear levels in the language of collective motions. The pairing-plus-multipole-multipole forces model represents a further development of these models.

The accumulation of data on the transition rates in even-even nuclei, from the first 2^+ state to the 0^+ ground state shows enhancement by about a factor of 40 on the average

over the single particle estimate, which can be explained by assuming a collective motion of the nucleons. Before the unified model was suggested by Bohr and Mottelson, it was believed that the independent particle and the collective models represented two opposite limits for nuclear physics.

The tendency of Fermi particles to pair was recognized many years ago. This idea motivated Racah[45] (1942) to introduce the seniority coupling scheme. Since the J=0 pair is more bounded than the $J\neq 0$ pair, and also J=0 pairs behave like bosons, which makes it possible for these pairs to be close to each other, therefore the ground state of the system can have a minimum energy if all particles are paired. These ideas led Mayer (1950)[42] to explain J=0 in the ground state of even-even nuclei.

In 1956 Cooper[43] showed that two fermions coupled to J=0 form a bound state, and behave like bosons. This explained why a fermion system (electrons) can exhibit superconductivity and a boson system (He^4) superfluidity. In 1957 Bardeen, Cooper, and Schrieffer (BCS)[44] used this idea in establishing their remarkably successful theory of superconductivity.

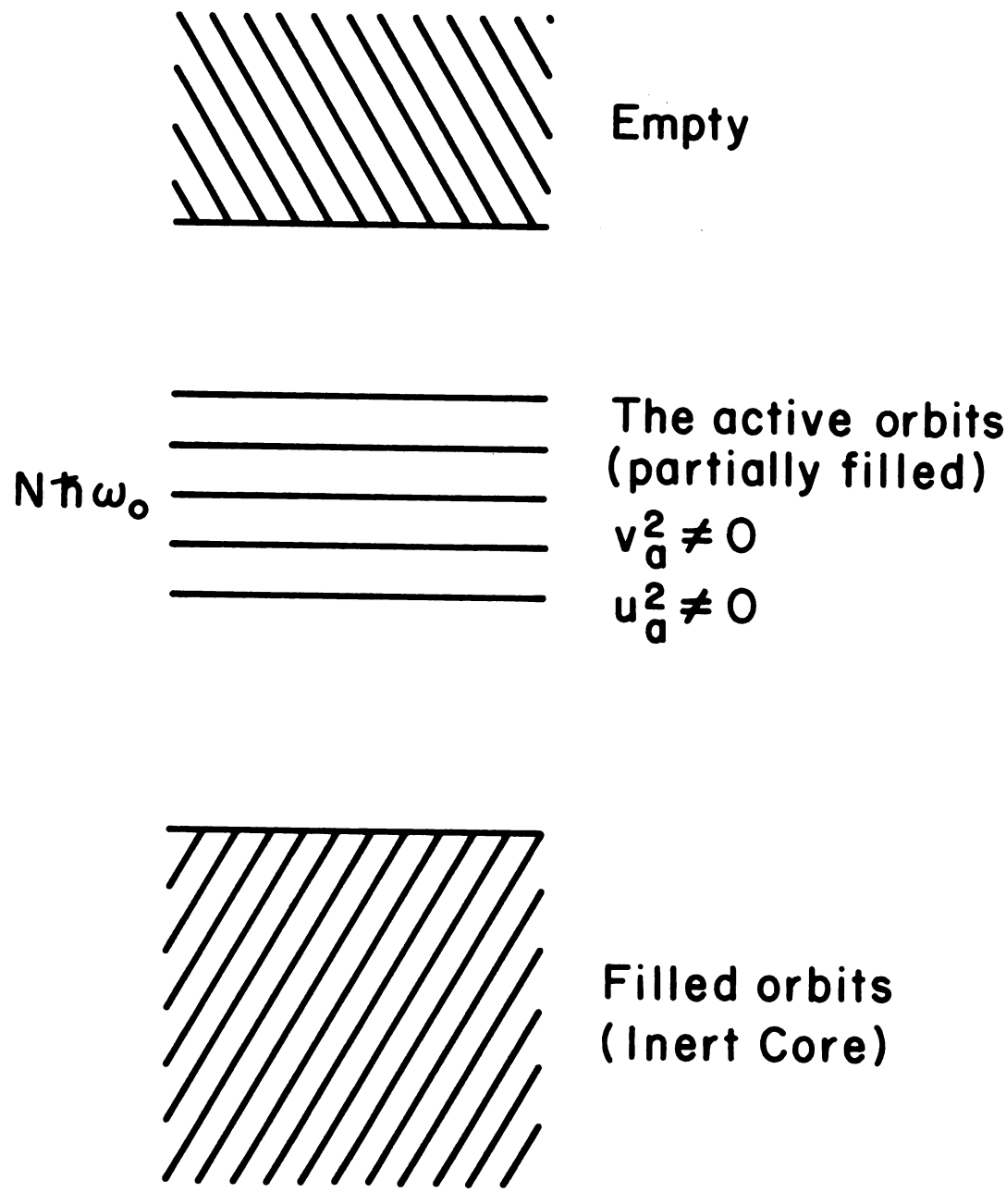
In 1958 Bohr, Mottelson, and Pines introduced for the first time this idea into nuclear physics to explain the energy gap in the spectrum of even-even nuclei. The superconducting solution of the nuclear problem is characterized by a depressed ground state energy and an energy gap in the

spectrum of quasi-particles. The BCS treatment of the pairing part of the Hamiltonian provided a good approximation to the real situation; the other part of the Hamiltonian which contains the long range of the interaction can be expanded in multipole-multipole interactions. This pairing plus quadrupole approximation leads to a fairly good understanding of the properties of the nucleus and its quadrupole vibrations. The descriptions of various properties of the nucleus depends upon the model for the nuclear force and the approximation scheme. Several studies have been done for various approximations used in nuclear physics. In most of these studies solvable models were used. In these models N particles are assumed to occupy single level j, and the single particle energy is taken to be zero. These particles are assumed to interact via a pairing interaction. For this kind of interaction the solvable model turns out to give a fairly good description of this part of interaction. This similarity between the degenerate system and the actual systems will be pointed out, whenever it arises, as we study the actual systems. In spite of this similarity, different correlations have been neglected in such models, and, therefore, a study of various many-body approximations in actual systems might be worthwhile. The pairing-plus-quadrupole model has been used successfully by KS for the collective motion in spherical nuclei. In this study a phenomenological approach is used, in which the single particle

energies are taken from experiment. In this method the nucleons in closed shells form an enert core, and only those nucleons outside this core are assumed to be active nucleons occupying partially filled active orbitals as is shown in figure (1). The effect of faraway orbits is assumed to produce an effective charges and effective interaction. This phenomenological treatment has the advantage of redusing the dimensionality of the problem, and it makes it easy to study the different approximations at once and systimatically for a large number of nuclei. In this study I consider the same spherical nuclei studied by KS and tabulate the average deviations from experiment for each type of approximation. The effect of the Pauli principle is especially noticeable in the random phase approximation. The quantities studied here are the reduced transition probabilities, the spectroscopic factors for the stripping (d,p) reactions on odd mass isotopes, and the energies of the first exited J=0 states, which are calculated in the QBA only, since the numerical calculations showed that the Pauli principle effect is negligible for this part of the interaction. It might be important to notice here that if one does not take the effective charges and instead uses the real charges ($e^p=1$, $e^n=0$), one needs to include the faraway orbitals. This will give the same results for the reduced transition probability but requires more labor[34].

Figure 1. Schematic illustration of the shell-model's basic act of truncation.

Figure 1


Empty
$N\hbar\omega_o$
The active orbits
(partially filled)
$v_a^2 \neq 0$
$u_a^2 \neq 0$
Filled orbits
(Inert Core)

II. a: Double-Time Green's Functions

Various many-body techniques have been applied in efforts to treat the two-body residual interaction between the nucleons in the outer shells of the nucleous, with these nucleons assumed to move in a self-consistent field produced by the core and to have single-particle energies ε_α determined to fit the experimental data.

The Green's function method furnishes justifications and physical insights into these methods; its physical meaning emerges from the fact that it describes the propagation of one or a few particles in a system of many particles. The hierarchy of equations of motion for the Green's functions provides an exact solution to the Hamiltonian for the problem under consideration; the major approximation enters the decoupling of these equations.

Following Zubarev[6] we define double-time Green's functions as follows:

$$G_a(t,t') = i\,\theta(t,t')\langle[A(t),B(t')]\rangle\ , \qquad \text{(II.1)}$$

$$G_r(t,t') = -i\,\theta(t,t')\langle[A(t),B(t')]\rangle\ , \qquad \text{(II.2)}$$

where A and B are two operators of the system at different times, and

$$\theta(\tau) = \begin{cases} 1 & \tau > 0 \\ 0 & \tau < 0 \end{cases}\ ,$$

r and a stand for retarted and advanced Green's functions respectively. The commutator

$$[A,B] = AB-\eta BA \ ,$$

where

$$\eta = \begin{cases} -1 & \text{Fermion} \\ +1 & \text{Boson} \end{cases} \ ,$$

its fourier transform with respect to time $G(\omega)$ is given by:

$$G(\omega) = \frac{1}{2\pi}\int_{-\infty}^{+\infty} d\tau e^{i\omega\tau} G(\tau)$$

$$\equiv << A(t) \mid B(t') >>_{\omega} \ . \qquad \text{(II.3)}$$

A higher order Green's function $\Gamma(\omega)$ defined by

$$\Gamma(\omega) = << C(t)\ A(t) \mid \ B(t') >>_{\omega} \ ,$$

can be reduced to the lower order $G(\omega)$ by means of the RPA decoupling procedure:

$$\Gamma(\omega) = <C(t)>_{\omega} <<A(t) \mid B(t') >>_{\omega} = <C(t)>_{\omega} G(\omega) \ . \qquad \text{(II.4)}$$

This approximation can be improved by evaluating $<C(t)>_{\omega}$ in the first 2^{+} state (IRPA) instead of the ground state (RPA) . The equation for $G(\omega)$ is given by:

$$\omega G(\omega) = \frac{<[A(t),B(t)]>_{\omega}}{2\pi} + <<[A(t),H] \mid \ B(t') >>_{\omega} \ ,$$

where H is the Hamiltonian. Similar equations can be obtained

for all higher order Green's functions from which a complete set of coupled equations is obtained.

The time correlation functions are

$$F_{AB}(t,t') = \int_{-\infty}^{+\infty} J(\omega) e^{\beta\omega} e^{-i\omega(t-t')} d\omega = \langle A(t)B(t')\rangle, \tag{II.5}$$

$$F_{BA}(t,t') = \int_{-\infty}^{+\infty} J(\omega) e^{-i\omega(t-t')} d\omega = \langle B(t')A(t)\rangle.$$

where $J(\omega)$ is the spectral function and $\beta = \frac{1}{KT}$, K is the Boltzmann constant, and T is the absolute temperature.

$$J(\omega) = i \frac{G(\omega + i\gamma) - G(\omega - i\gamma)}{e^{\beta\omega} - \eta}, \quad \gamma=0.$$

Exploiting the analytical property of $G(\omega)$ in the complex plane, we can write[6]

$$G(\omega) = \frac{1}{2\pi} \int_{-\infty}^{+\infty} (e^{\beta E} - \eta)\, J(E) \frac{dE}{\omega - E}. \tag{II.6}$$

In the nucleus the residual interactions are prevented from destroying the independent particle picture by the presence of the energy gap in the spectrum of quasi-particles. Therefore the spectral function $J(\omega)$ can be writen as;

$$J(\omega) = I(\omega_i)\, \delta(\omega - \omega_i) + I(-\omega_i)\, \delta(\omega + \omega_i).$$

Using the above expression for $J(\omega)$ the expression for $G(\omega)$ takes the following form;

$$. \; G(\omega) = \frac{1}{2\pi} \left[\frac{e^{\beta\omega}-\eta}{\omega-\omega_i} I(\omega_i) + \frac{e^{-\beta\omega}-\eta}{\omega+\omega_i} I(-\omega_i) \right] .$$

The Residues (Res.) of $G(\omega)$ at the poles ($\omega=\pm\omega_i$) are given by

$$\begin{aligned} \underset{\omega=\omega_i}{\text{Res.}} G(\omega) &= \underset{\omega\to\omega_i}{\text{Limit}} (\omega-\omega_i) G(\omega) = \frac{1}{2\pi}(e^{\beta\omega_i}-\eta) I(\omega_i) , \\ \underset{\omega=-\omega_i}{\text{Res.}} G(\omega) &= \underset{\omega\to-\omega_i}{\text{Limit}} (\omega+\omega_i) G(\omega) = \frac{1}{2\pi}(e^{-\beta\omega_i}-\eta) I(-\omega_i) . \end{aligned} \tag{II.8}$$

In many cases we need to evaluate the expectation values of the product of two operators at the same time. Setting t=t', equations 7 take the forms:

$$\langle BA \rangle = \int_{-\infty}^{+\infty} J(\omega) d\omega \quad , \quad \langle AB \rangle = \int_{-\infty}^{+\infty} J(\omega) e^{\beta\omega} d\omega . \tag{II.9}$$

Eqns. 9, 10, and 11 then yield:

$$\begin{aligned} \frac{1}{2\pi}\langle BA \rangle &= \frac{\text{Res.}G(\omega_i)}{(e^{\beta\omega_i}-\eta)} + \frac{\text{Res.}G(-\omega_i)}{(e^{-\beta\omega_i}-\eta)} , \\ \frac{1}{2\pi}\langle AB \rangle &= \frac{\text{Res.}G(\omega_i)}{(1-\eta e^{-\beta\omega_i})} + \frac{\text{Res.}G(-\omega_i)}{(1-\eta e^{\beta\omega_i})} . \end{aligned} \tag{II.10}$$

Taking the limit $T \to 0$ (Since the thermal energy is very small compaired to the nuclear energies) we obtain

$$\langle BA \rangle = \frac{-2\pi}{\eta} \operatorname{Res}_{\omega=-\omega_i} G(\omega)$$
$$\langle AB \rangle = +2\pi \operatorname{Res}_{\omega=+\omega_i} G(\omega) \qquad \text{(II.10)}$$

If Green's function is written in the form $G(\omega) = N/D(\omega)$, then the eigen values are given by the poles in the Green's function and the eigenvalue equation is given by $D(\omega_i) = 0$. Expanding $D(\omega)$ around the pole ω_i, the residue of $G(\omega)$ at the pole is given by

$$\operatorname{Res}_{\omega=\pm\omega_i} G(\omega) = \frac{N}{\frac{\partial D(\omega)}{\partial \omega}\Big|_{\omega=\pm\omega_i}}, \qquad \text{(II.11)}$$

where

N= constant.

b: <u>Quasi-Particle Scheme (BCS approximation)</u>

The elementary excitations in a system of non-interacting (or independent) particles are simply desicribed by $\varepsilon(p)=p^2/2m$; as the interaction is turned on, the propagation of a particle in the system is affected by the presence of the other particles, and as the particle moves it will push and pull other particles. Thus the particle plus its surrounding environment will behave like a new particle, characterized by a new normalized

parameter m* (effective mass); the effect of the interaction with other particles is all included in the new effective mass, and the elementary excitations are now described by the free-particle-like expression; $\varepsilon(p)=p^2/2m^*$.
In this manner the original interacting particles are transformed into independent quasi-particles. It is important to notice here that the quasi-particle picture is based on a mathematical device; its main object is to keep the independent particle picture from being destroyed by the interaction. The independent particle idea is very important for the Hartree-Fock treatment of the nucleus in which the nucleons are described by independent single particle wave functions.

The quasi-particles we face in nuclear physics are determined by the short-range interactions (such as the pairing interaction), and in this case the quasi-particle is just a mixture of particle and hole near the Fermi surface. The transformation to a quasi-particle description is effected by means of the Bogoliubov-Valatin (B-V) transformation:

$$\begin{aligned} a_\alpha &= u_a c_\alpha - s_\alpha v_a c^\dagger_{-\alpha} \quad , \qquad \alpha\equiv(a,m_\alpha) \ , \ -\alpha\equiv(a,-m_\alpha), \\ a^\dagger_\alpha &= u_a c^\dagger_\alpha - s\, v_a c_{-\alpha} \quad , \qquad s_\alpha=(-)^{a-m_\alpha} \end{aligned} \tag{II.12}$$

where $a^\dagger_\alpha$, a_α are the quasi-particle creation and annihilation operators, $c^\dagger_\alpha$, c_α ; are those for particles, and α stands for (a,m_α) , and $-\alpha$ for $(a,-m_\alpha)$, with $a=j_a$;

the angular momentum. In order for this transformation to be canonical (i.e. preserves the commutation relation), u_a and v_a must satisfy the following:

$$u_a^2 + v_a^2 = 1; \qquad \text{(II.13)}$$

with this transformation the energies of the elementary excitations or quasi-particles are given by:

$$E_a = [\ (\varepsilon_a - \lambda)^2 + \Delta^2\ ]^{1/2}\ ,$$

where ε_a is the independent-or single-particle energy, λ is the chemical potential (Fermi energy) and Δ is the energy gap.

The point is that the spectrum of elementary excitations of the quasi-particles has an energy gap. This energy gap is very important in determining the validity of the independent-particle picture. The B-V transformation by itself is an exact unitary transformation. The major errors come from the neglect of certain terms in the transformed Hamiltonian, which are assumed to be small, in the approximation usually referred to as the BCS approximation after Bardeen Cooper and Schrefer, who introduced it in the theory of superconductivity. In this approximation, the wave function is not an eigenfunction of the number of particles operator, i.e.,

$$[H,N] \neq 0,$$

with a resulting small effect on the energy eigenvalues of the low-lying states. This can be seen from the exact solution

of the pairing Hamiltonian in the j^n degenerate system, i.e.

$$E_\nu - E_0 = \frac{g}{4}\nu(2\Omega-\nu+2), \qquad \text{(II.15)}$$

which is independent of N. Here ν is the seniority no., Ω is the pair degeneracy, g is the strength of the pairing interaction, E_0 is the ground state energy; $-\Omega g$. The quantity v_a^2 represents the average occupation number of the original particles, which interact with a short-range interaction, which mixes the particle and the hole, so that this distribution has a diffuse Fermi surface in contrast to the situation for independent particles, where the system has a sharp Fermi surface.

c: The Pairing Interaction

After separation of the self-consistent part of the interactions there remains some interaction between the particles; the so called "residual interaction". This residual interaction is rather weak, but it plays an important role in determining various nuclear properties. The pairing interaction is one example of such a residual interaction. In this interaction two particles occupy atates related by time reversal, i.e. (a,m_α) and $(a,-m_\alpha)$, are coupled to angular momentum J=0, since the J=0 state is much more strongly bound than the others, so that the neglect of $J\neq 0$ states is a good approximation for the pairing interaction. The short range forces play an important role in the formation of bound

pairs provided that the interaction is attractive in a sufficiently large neighborhood of the Fermi surface (coherent interaction). To see this, first notice that the energy of a particle close to the Fermi surface is $\varepsilon_F = V_F(p-p_F)$ where V_F and P_F are the velocity and the momentum at the Fermi surface respectivly[46]. Now the Schrodinger equation for two interacting particles,

$$(E-H_0)\psi = V\psi$$

or $$\psi = (E-H_0)^{-1}V\psi$$

where

H_0 = the kinetic energy,

and V the interaction energy, and $E=-\Delta$ is the binding energy of the system. In the momentum representation the wave function $\psi(p)$ for the bound state can be written as

$$\psi(p) = -\int_{-\infty}^{+\infty} \frac{\langle p|V|p'\rangle}{\Delta+\varepsilon(p')}\,\psi(p')dp', \qquad \text{(II.16)}$$

where $\varepsilon(p)$ is the eigenvalue of H_o . Let $V= -g$ = constant in a limited range of p. In this case $\psi(p)$ will be approximately constant, therefore equation (II.16) reduces to

$$g\int \frac{dp}{\Delta+\varepsilon(p)} = 1 \ . \qquad \text{(II.17)}$$

For small values of g; Δ will be small too, and equation (II.17) will have a solution if

$$\int \frac{dp}{\varepsilon(p)}$$

or for three-dimensional case

$$\int \frac{p^2 dp}{\varepsilon(p)}$$

is very large to compensate for the small values of g. Substituting the value of $\varepsilon(p)$ near the Fermi surface we get

$$\int_0^{p_{m.}} \frac{p_F^2 dp'}{V_F p'} = \infty \quad , \quad p_{m.} \equiv p_{max.}$$

where $p' = p - p_F$. This shows that in the presence of the Fermi sea even a very weak attraction will produce bound pairs near the Fermi surface.

In superconductors the interaction transmitted by the phonons is coherent at the Fermi surface, and it produces the phenomenon of superconductivity. In nuclei, however no one has been able to prove the existence of such a coherent interaction. However the following, experimental facts indicate the existence of pairing interaction;

1- All even-even nuclei have J=0 ground states.

2- The energy gap in the elementary excitations is equal to the energy required to break a pair of J=0.

3- In the odd nuclei the odd nucleon is weakly bound while it is strongly bound in the neighboring even-even nuclei where it is assumed to form part of a pair.

Therefore the experimental data on the odd-even mass difference can provide good information about the pairing strength g. One can choose g to satisfy the gap equations and to fit the odd-even mass difference at the same time. The odd-even mass difference is given by:

$$B(N+1) + B(N-1) - 2B(N) = 2E_{\alpha} \ ,$$

where N is the odd mass no., B(N) is the binding energy of N nucleons and E is the quasi-particle energy.

The short-range forces causes two particles which occupy time reversed states to scatter each other twice from their common orbit to another orbit where they still occupy time-reversed states (a,m_{α}) and $(a,-m_{\alpha})$; in this way they scatter to all possible a-levels and hence cause the particle density distribution to be spherical, while the long-range part of the two-body interaction tends to align the nucleon orbitals and produce a deformation. This competition between short range and long range forces determines the nuclear shape. the pairing interaction has to be smaller than the field interaction in order for shell structure to exist, but as we saw, only a small residual interaction is needed in order to create bound pairs near the Fermi surface.

There exit two approaches to the pairing interaction; the first is the general BCS or Hartree-Fock-Bogoliubov treatment in which the single-particle energies and wave functions must include the effect of the residual interaction for the valence nucleons as well as the core nucleons, and ε_{α} is a solution of the general Hartree-Fock(HF) equations. In this case the pairing interaction includes $J\neq 0$, and λ and Δ are not constants but rather they change with the

excitation energy. The second approach which is adopted in this work, is the phenomenological approach in which the single- particle energies, ε_α , and wave functions are assumed to be known in advance. Where ε_α is taken from experiment, and the contibution to the pairing from J=0 pairs is taken to be a constant, g; the contributions from J≠0 are neglected as they are assumed to be small. This method gives good results, since the experimental values of ε_α include all the neglected effects; also λ and Δ are taken to be constant and one uses their ground states values which are solution of the BCS equations:

$$2/g = \sum_b \frac{\Omega_b}{E_b} \quad , \quad \Omega_b = b+1/2 \ ,$$

$$N = \sum_b \Omega_b \{1- \frac{\varepsilon_b - \lambda}{E_b} \} \ , \qquad \text{(II.16)}$$

where the pairing constant g is taken to fit the odd-even mass differences, and N is the number of nucleons outside a closed shell. The above equations are valid for neutrons and protons seperatly.

d: <u>The Hamiltonian</u>

We can seperate the Hamiltonian to three parts as follow:

$$H = H_0 + H_p + H_{Q-Q}$$

where H_0 is the single particle Hamiltonian (kinetic energy) and is given by:

$$H_0 = \sum_{\alpha\xi}\varepsilon_\alpha^\xi c_\alpha^{\xi\dagger} c_\alpha^\xi \ , \qquad \text{(II.17a)}$$

H_p is the pairing interaction and is given by:

$$H_p = -\frac{1}{4}\sum_{\alpha\beta\xi} g^\xi s_\alpha s_\beta c_\alpha^{\xi\dagger} c_{-\alpha}^{\xi\dagger} c_{-\beta}^\xi c_\beta^\xi \ , \qquad \text{(II.17b)}$$

Where the matrix element of the short-range interaction is assumed to be larger in the states coupled to J=0 i.e.

$$|\langle (j_\alpha)^2 J=0|V|(j_\beta)^2 J=0\rangle| >> |\langle (j_\alpha)^2 J\neq 0|V|(j_\beta)^2 J\neq 0\rangle| \ ,$$

and approximately given by:

$$\langle \alpha-\alpha|V|\beta-\beta\rangle \simeq -g \ ,$$

and H_{Q-Q} is the quadrupole-quadrupole interaction. This part of the Hamiltonian which induces the collective vibrations is assumed to have the following form:

$$\begin{aligned} H_{Q-Q} = & -\frac{1}{2}\sum_{\alpha\beta\gamma\delta\xi\mu}\chi^\xi (-)^\mu q_\mu^\xi(\alpha\gamma) q_{-\mu}^\xi(\beta\delta) c_\alpha^{\xi\dagger} c_\beta^\xi \, c_\delta^\xi c_\gamma^\xi \\ & -\frac{1}{2}\chi^{pn}\sum_{\alpha\beta\gamma\delta\mu}(1+R(np))(-)^\mu q_\mu^p(\alpha\gamma) q_{-\mu}^n(\beta\delta) \\ & \qquad \times c_\alpha^{p\dagger} c_\beta^{n\dagger} c_\delta^p c_\gamma^n \ , \qquad \text{(II.17c)} \end{aligned}$$

where R(n,p), interchanges n and p in all the expressions which follow it in the term where it appears. Following KS and others, no pairing interaction between neutron and proton is assumed, the effect of p-n pairing is assumed to be important in light nuclei, A<70, although its effect is not very well understood. There exist no reliable method to treat such

interactions; their effects can be shown to be incuded in the experimental values of ε_α , and hence they are not included in the Hamiltonian. The pairing part of the Hamiltonian is to be diagonalized by means of the BCS approximation after transforming to the quasi-particle scheme by means of the B-V transformation which leads to a new Hamiltonian in terms of the quasi-particle operators a_α and $a_\alpha^\dagger$. After the BCS approximations the number of particles is no longer conserved and therefore a chemical potential λ has to be introduced as a Langrange multipier, its value to be fixed by requiring that $\langle N \rangle$ give the correct mean value of the number of particles. The inverse to the B-V transformation is:

$$c_\alpha^\dagger = u_a a_\alpha^\dagger + s_\alpha v_a a_{-\alpha} ,$$

$$c_\alpha = u_a a_\alpha + s_\alpha v_a a_{-\alpha}^\dagger .$$

We define the seniority zero pair operators for quasi-particle as follow :

$$A_a = \frac{1}{\sqrt{\Omega_a}} \sum_{m_\alpha > 0} s_\alpha a_\alpha^\dagger a_{-\alpha}^\dagger , \qquad (II.18)$$

$$A_a = (A_a^\dagger)^\dagger ;$$

the number operator is;

$$N_a = \sum_{m_\alpha} a_\alpha^\dagger a_\alpha .$$

Their commutation relations are:

$$[A_a, A_b^\dagger] = \delta_{ab}(1 - \frac{N_a}{\Omega_a}) \ ,$$

$$[N_a, A_b^\dagger] = 2\delta_{ab}A_a^\dagger \ , \qquad \text{(II.19)}$$

$$[N_a, A_b] = -2\delta_{ab}A_a \ .$$

In terms of these operators the pairing Hamiltonian takes the form:

$$H_0 + H_p = U + H_1 + H_2 + H_{res.} = H_p'$$

where

$$U = \sum_a [2\Omega_a v_a^2(\varepsilon_a - \lambda) - g\Omega_a v_a^4] - g(\sum_a \Omega_a u_a v_a)^2 \ ,$$

$$H_1 = \sum_a [(\varepsilon_a - \lambda)(u_a^2 - v_a^2) + 2g u_a v_a (\sum_a \Omega_a u_a v_a)] N_a \ , \qquad \text{(II.20)}$$

$$H_2 = \sum_a [2\sqrt{\Omega_a} u_a v_a (\varepsilon_a - \lambda) - g(\sum_b \Omega_b u_b v_b)\sqrt{\Omega_a}(u_a^2 - v_a^2)](A_a^\dagger + A_a) \ ,$$

$$H_{res.} = -g \sum_{ab}\sum \sqrt{\Omega_a \Omega_b}(u_a^2 A_a^\dagger - v_a^2 A_a)(u_b^2 A_b - v_b^2 A_b^\dagger) + g\sum_{ab}\sum \sqrt{\Omega_b} u_a v_a \{N_a(u_b^2 A_b - v_b^2 A_b^\dagger) + (u_b^2 A_b^\dagger - v_b^2 A_b) N_a\} - g \sum_{ab}\sum u_a v_a u_b v_b N_a N_b \ .$$

By minimizing the ground state energy U or equivalently setting $H_2 = 0$, we find the following results:

$$2/g = \sum_a \frac{\Omega_a}{E_a} \ , \qquad N = \sum_a \Omega_a (1 - \frac{\varepsilon_a - \lambda}{E_a}) \ ,$$

where

$$E_a=[(\varepsilon_a-\lambda)^2+\Delta^2]^{1/2} \quad , \quad \Delta = g \sum_a \Omega_a u_a v_a \ ,$$

$$v_a^2= \frac{1}{2}(1- \frac{\varepsilon_a-\lambda}{E_a}) \quad , \quad u_a^2=1-v_a^2 \ .$$

Equations (II.21) are known as the BCS or the gap equations, the first two of which have to be solved samiltaneusly for λ and Δ . In order for a superconducting solution to exist, the pairing constant g must not be very small and the level separations of the single particle energies must not be very large. This does not means that no pairing correlation will exist for small g, but rather that no advantages can be gained by transforming to the quasi-particle picture. The ground state energy of the system is given by:

$$U'= \sum_a [(\varepsilon_a-\lambda) 2\Omega_a v_a^2 - g\Omega_a v_a^4] - \frac{\Delta^2}{g} + \lambda N \ , \qquad \text{(II.22)}$$

where N is the number of particles.

The new Hamiltonian takes the form:

$$H'-U = \sum_a E_a N_a + H_{res.}$$

where the term $H_{res.}$ is usually neglected in the BCS approximation, as it is small compared to the diagonal part of the transformed H, where the diagonal part represent independent quasi-particles with the excitation energies for seniority zero, $J=0^+$ states given by $\omega_a=2E_a$. In this way one can construct n states, where n is the number of levels in the major shell. One of these states is a spurious state introduced by violating the conservation of particle number. It is well-known that each time a conservation rule is violated a spurious excited state will appear.

If the rest of the Hamiltonian is included, the number of particles is again conserved, and the spurious state disappears. The pairing Hamiltonian for both kind of particles is:

$$H'-U = \sum_{a\xi}\sum [E_a^{\xi} N_a^{\xi} + H_{res.}^{\xi}] \ . \qquad \text{(II.23)}$$

After transformation to quasi-particles by the B-V transformation, the quadrupole Hamiltonian can be expressed in terms of the following operators:

$$A^{\dagger}(abJM) = \sum_{m_\alpha}\sum_{m_\beta} (abJ|m_\alpha m_\beta M) a_\alpha^{\dagger} a_\beta^{\dagger} \ , \quad A(abJM) = [A^{\dagger}(abJM)]^{\dagger}, \qquad \text{(II.24)}$$

$$A^{o}(abJM) = \sum_{m_\alpha}\sum_{m_\beta} (-)^{b+m_\beta} (abJ|m_\alpha m_\beta M) a_\alpha^{\dagger} a_{-\beta} \ ,$$

where $(abJ|m_\alpha m_\beta M)$ is the Clebsch-Gordan coefficient. The following recoupling formula will introduce terms in the Hamiltonian which contains Racah coefficients W(abJc;LK):

$$[(a_\alpha \times a_\beta)_\mu^L \times a_\gamma]_M^J = \sum_{Kq}\sum \sqrt{\Omega_K \Omega_L} W(abJc;LK) [a_\alpha \times (a_\beta \times a_\gamma)_q^K]_M^J ; \qquad \text{(II.25)}$$

these terms are neglected as they are small compared to other terms. The final result is

$$H = \sum_a E_a N_a - \sum\sum\sum\sum\sum\sum_{abcd\mu\xi} \frac{\chi^{\xi}}{10} q^{\xi}(bd) q^{\xi}(ac) \{ (-)^{\mu} \frac{1}{4} U_{bd}^{\xi} U_{ac}^{\xi} A^{\xi\dagger}(bd2-\mu)$$

$$\times A^{\xi\dagger}(ac2\mu) + \frac{1}{2} U_{bd}^{\xi} U_{ac}^{\xi} A^{\xi\dagger}(bd2\mu) A^{\xi}(ac2\mu) + \frac{1}{4}(-)^{\mu} U_{bd}^{\xi} U_{ac}^{\xi}$$

$$\times A^{\xi}(bd2\mu) A^{\xi}(ac2\mu) + U_{bd}^{\xi} V_{ac}^{\xi} (-)^{\mu} A^{\xi\dagger}(bd2-\mu) A^{\xi o}(ac2\mu) + V_{bd}^{\xi} U_{ac}^{\xi}$$

$$\times A^{\xi o}(bd2\mu) A^{\xi}(ac2\mu) - 5(-)^{d-a} \delta_{\mu o} V_{bd}^{\xi} V_{ac}^{\xi} \sum\sum_{Kq} (-)^{K} W(bdac;2K)$$

$$\times A^{\xi\dagger}(abKq) A^{\xi}(cdKq) \} - \frac{1}{10} \sum\sum\sum\sum\sum_{abcd\mu} (1+R(pn)) \chi^{np} q^{n}(bd) q^{p}(ac)$$

$$\times \{ (-)^{\mu} \frac{1}{4} U_{bd}^{n} U_{ac}^{p} A^{n\dagger}(bd2-\mu) A^{p\dagger}(ac2\mu) + \frac{1}{2} U_{bd}^{n} U_{ac}^{p} A^{n\dagger}(bd2\mu) A^{p}(ac2\mu)$$

$$+(-)^{\mu}\tfrac{1}{4}U^{n}_{bd}U^{p}_{ac}A^{n}(bd2\mu)A^{p}(ac2\mu)+(-)^{\mu}U^{n}_{bd}V^{p}_{ac}A^{n\dagger}(bd2-\mu)$$

$$\times A^{po}(ac2\mu)+V^{n}_{bd}U^{p}_{ac}A^{no}(bd2\mu)A^{p}(ac2\mu)-5(-)^{d-a}\delta_{\mu o}V^{n}_{bd}V^{p}_{ac}$$

$$\times\Sigma\Sigma_{Kq}(-)^{K}W(bdac;2K)A^{n\dagger}(abKq)A^{p}(cdKq)\}\ , \qquad \text{(II.26)}$$

where

$$U^{\xi}_{ab}= u^{\xi}_{a}v^{\xi}_{b}+ u^{\xi}_{b}v^{\xi}_{a}\ ,$$

$$V^{\xi}_{ab}= u^{\xi}_{a}u^{\xi}_{b}- v^{\xi}_{a}v^{\xi}_{b}\ ,$$

and χ is the strength of the quadrupole force; its value is to be fixed by fiting the energy of the first 2^{+} state with the experimental value, and

$$q(ab) =\langle a||a_{o}^{2}r^{2}Y_{2\mu}||b\rangle = (-)^{a-b}q(ba)\ , \qquad \text{(II.27)}$$

is the non-dimentional matrix element, and a_{o} is given by

$$a_{o}^{2}= \frac{m\omega_{o}}{h}\ ;$$

m is the nucleon mass and ω_{o} is the harmonic oscillator frequency, and α,β,δ, and γ are the harmonic oscillator single-particle wave functions. Applying the Wigner-Eckart theorem to $q(\alpha\beta)$ yields:

$$q(\alpha\beta) = \frac{(b2a|m_{\beta}M\ m_{\alpha})}{\sqrt{2a+1}}(a||a_{o}^{2}r^{2}Y_{2\mu}||b)\ ,\ J=2$$

but $\quad (b2a|m_{\beta}M\ m_{\alpha})=(-)^{c-m_{\beta}}[\frac{2a+1}{5}]^{1/2}(ba2|m_{\beta}-m_{\alpha}-M)\ ,$

therefore

$$q(\alpha\beta) =\frac{1}{\sqrt{5}}\ s_{\beta}(ab2|m_{\alpha}-m_{\beta}M)q(ab)\ . \qquad \text{(II.28)}$$

The quadrupole P-N interaction has the effect of lowring the energy of the 2^+ states; neglecting it will result in increasing the energy due to the increased symmetry energy which is associated with the independent polarization of the neutrons and protons. The exact commutation relation for the pair operators can be obtained by using the recoupling formula (Eqn.II.25); the result is (Appen. B)

$$[A^{\dagger}_{(abJM)},A_{(cdKq)}] = -\delta_{Mq}\delta_{JK}(\delta_{ac}\delta_{bd}-(-)^{a+b-J}\delta_{ad}\delta_{bc})$$
$$+P(ab)P(cd)(-)^{a+d+J+K}\sum_{L\mu}\sqrt{(2K+1)(2L+1)}$$
$$\times\{{Kdb \atop aJL}\}(LKJ|\mu qM)A^{o}_{(adL\mu)}\delta_{cb}\ , \qquad (II.29)$$

where

$$P(ab) = (1 - (-)^{a+b+J}R(ab)),$$

and R(ab) is the exchange operator; $R(ab)\psi_{(ab)}=\psi_{(ba)}$.

e: Collective Operators

The collective vibrations can be described by means of collective operators B_{JM} and $B^{\dagger}_{JM}$, which obey the following equations of motion

$$[B^{\dagger}_{JM},H] = -\omega B^{\dagger}_{JM}\ , \quad [B_{JM},H] = \omega B_{JM}\ , \qquad (II.30)$$

where ω is the energy of the vibrational excited state.The ground state wave function is defined by the set of equations

$$B_{JM}|0> = 0 \qquad (II.31)$$

The vacuum $|0\rangle$ just defined is different from the BCS vacuum and is believed to be better than the BCS vacuum. The excited state wave function of angular momentum J is given by:

$$B^{\dagger}_{JM}|0\rangle = |JM\rangle \ . \qquad \text{(II.32)}$$

This collective operators can be expanded in terms of A(ab) and $A^{+}(ab)$ with real coefficients;

$$B^{\dagger}_{JM} = \frac{1}{2}\Sigma\Sigma\Sigma_{ab\xi}[\psi^{\xi}_{(ab)}A^{\xi\dagger}_{(abJM)} - (-)^{J-M}\phi^{\xi}_{(ab)}A^{\xi}_{(abJ-M)}] \qquad \text{(II.33)}$$

where

$$\psi^{\xi}_{(ab)} = \langle 0|A^{\xi}_{(abJM)}|JM\rangle \ ,$$

$$\phi^{\xi}_{(ab)} = (-)^{J-M}\langle 0|A^{\xi\dagger}_{(abJ-M)}|JM\rangle$$

The amplitudes ψ and ϕ are defined in Ref. 2. The expression for B^{+}_{JM} above is obtained as in Ref.2 with the help of quasi-boson commutation relation; see Eqn.(III.1) If the exact commutation relation had been used (see RPA approximation, Sec. d)the result would be

$$B^{\dagger}_{JM} = \frac{1}{2}\Sigma\Sigma\Sigma_{ab\xi}\frac{1}{\alpha^{\xi}_{ab}}[\psi^{\xi}_{(ab)}A^{\xi\dagger}_{(abJM)} - (-)^{J-M}\phi^{\xi}_{(ab)}A^{\xi}_{(abJ-M)}] \ ,$$

where

$$\alpha^{\xi}_{ab} = 1 - n^{\xi}_{a} - n^{\xi}_{b} \ , \quad n_i = \langle a^{\dagger}_i a_i\rangle \ . \qquad \text{(II.34)}$$

These quantities will be drived later on when we treat the RPA approximation.

f: The Reduced Transition Probability B(E2)

The ground state reduced transition probability B(E2) is given by:

$$B(E2) = \sum_{M\mu}\sum |<0|Q_{2\mu}|2^{+}M>|^{2} \quad , \tag{II.35}$$

where $Q_{2\mu}$ is the quadrupole operator. In terms of the operators A(ab) and A^{+}(ab) the quadrupole operator takes the form:

$$Q_{2\mu} = \sum_{ab\xi}\sum\sum \frac{e_f^{\xi}}{2\sqrt{5}} \frac{q^{\xi}(ab)}{a_o^2} u_{ab}^{\xi}(A^{\xi\dagger}_{(ab2\mu)} + (-)^{\mu} A^{\xi}_{(ab2-\mu)}) \quad , \tag{II.36}$$

for $\mu = M = 0$ equations II.35 and II.36 yeild:

$$B(E2) = \frac{1}{4a_o^4} \left|\sum_{ab\xi}\sum\sum e_f^{\xi} q^{\xi}(ab) u_{ab}^{\xi}(\psi^{\xi}_{(ab)} + \phi^{\xi}_{(ab)})\right|^{2} , \tag{II.37}$$

where e_f is the effective charge (e_f^n=1, e_f^p=2). The above result is obtained after appling the Wigner-Eckart theorem to the matrix element q(ab) and using the definition (II.24) for the operators A(ab) and A^{+}(ab). Terms which contain the scattering operators A^{o}(ab) make zero contribution, since these operators have non-vanishing matrix elements only between states with definite numbers of particles in contrast to A(ab) and A^{+}(ab) which connect states with different number of particles.

III. The Approximation Methods

In the equation-of-motion methods, one calculates the dynamical observables by calculating the appropriate relationships, rather than calculating the absolute wave functions. The most interesting quantities in any nuclear system do not depend on a knowledge of the wave functions themselves, but rather on certain relationships between them. Thus the equation-of-motion methods reduce the labor involved in calculating a certain observable, as calculation of the entire complicated wave function is not required by these methods. To linearize the equations of motion one must depend on some kind of approximation scheme. The approximations QBA, TDA, and RPA which are studied in this work all lead to linearized equations of motion. In particular the equations of motion for the Green's functions can be linearized (decoupled) by means of the above approximations.

a: The Quasi-Boson Approximation QBA

In this approximation a pair of particles (Fermions), their angular momenta coupled to integral J, are treated as bosons; this means the Pauli principle is neglected. Therefore one expects QBA overestimates the number of particles which participate in a certain collective states. As a result of this approximation the enhancement predected for the reduced transition probability B(E2), is larger than

the single particle estimate. The effect of the Pauli principle will be demonstrated in (III.d). The commutation relations used in the QBA are:

$$[A_{(abJM)}, A^{\dagger}_{(cdKq)}] = \delta_{JK}\delta_{Mq}(\delta_{ac}\delta_{bd} - (-)^{a+b+J}\delta_{ad}\delta_{bc}) \ ,$$

$$[A_a, A^{\dagger}_a] = \delta_{ab} \ ,$$

$$[N_a, A^{\dagger}_b] = 2\delta_{ab}A^{\dagger}_a \ , \qquad \text{(III.1)}$$

$$[N_a, A_b] = -2\delta_{ab}A_a \ .$$

For the first two relations to be valid to a good approximation, N_a/Ω_a must be small i.e. the number of particles in level a must be smaller than the number of available states. It is clear from the above commutation relations that the different modes of the system are well separated, i.e. $J=0^+$ and $J=2^+$ modes are independent.

b: <u>The First 0^+, Seniority 0 State</u>

The exited states for this mode of vibration are usually described by the independent quasi-particle Hamiltonian where the excitation energies of the system are given by $\omega = 2E_a$. If only the important part of $H_{res.}$ is included, a dispersion relation for these states can be obtained. A pair is two nucleons with their angular momenta

coupled to a resultant angular momentum J=0. The lowest energy state of the system is that in which as many nucleons as possible are paired. The number of unpaired nucleons is referred to as the seniority of the state. Addition of extra pairs to the system will not change the angular momentum or the seniority of the state. Therefore seniority 0 states can be generated by succesive applications of the pair operator A_a^+ to the vacuum, and are given for the j^n configuration as follow:

$$|0\rangle = \underline{n=0} \quad \underline{n=2} \quad \underline{n=4} \quad \text{---------} \quad \underline{2\Omega-2} \quad \underline{n=2\Omega} \quad \nu=0$$

where n is the number of particles, and 2Ω is the maximum number of nucleons that can occupy the j-level. The Hamiltonian for weakly interacting quasi-particles is given by: (see eqn. II.20)

$$H_p' = \Sigma E_a N_a - g\Sigma\Sigma\sqrt{\Omega_a \Omega_b}\,(u_a^2 A_a^\dagger - v_a^2 A_a)\,(u_b^2 A_b - v_b^2 A_b^\dagger) \quad . \qquad \text{(III.2)}$$

where other terms have been neglected, as they are assumed to be small compared to the second terms in this Hamiltonian H_p'. We define the following Green's functions:

$$G_{ab}^{o}(\omega) = \langle\langle A_a(t) \,|\, A_b^\dagger(t')\rangle\rangle_\omega \quad , \qquad \eta = 1 \quad ,$$
$$G_{ab}^{o\dagger}(\omega) = \langle\langle A_a^\dagger(t) \,|\, A_b^\dagger(t')\rangle\rangle_\omega \quad . \qquad \text{(III.3)}$$

The quasi-boson commutation relations lead to the following motion for G and G^+ (see app.G)

$$(\omega-2E_a)G^o_{ab}(\omega)=\frac{\delta_{ab}}{2\pi}-g\sqrt{\Omega_a}u_a^2\Sigma_c\sqrt{\Omega_c}(u_c^2G^o_{cb}-v_c^2G^{o\dagger}_{cb})$$
$$-g\sqrt{\Omega_a}v_a^2\Sigma_c\sqrt{\Omega_c}(v_c^2G^o_{cb}-u_c^2G^{o\dagger}_{cb}),$$
(III.4)
$$(\omega+2E_a)G^{o\dagger}_{ab}(\omega)=-g\sqrt{\Omega_a}v_a^2\Sigma_c\sqrt{\Omega_c}(u_c^2G^o_{cb}-v_c^2G^{o\dagger}_{cb})$$
$$-g\sqrt{\Omega_a}u_a^2\Sigma_c\sqrt{\Omega_c}(v_c^2G^o_{cb}-u_c^2G^{o\dagger}_{cb}).$$

Now let

$$D_b=\Sigma_c\sqrt{\Omega_c}(u_c^2G^o_{cb}-v_c^2G^{o\dagger}_{cb}), \qquad \text{(III.5a)}$$

and

$$B_b=\Sigma_c\sqrt{\Omega_c}(v_c^2G^o_{cb}-u_c^2G^{o\dagger}_{cb}). \qquad \text{(III.5b)}$$

The equations for $G(\omega)$ and $G^+(\omega)$ then take the form:

$$G^o_{ab}(\omega)=\frac{\delta_{ab}}{2\pi(\omega-2E_a)}-\frac{g\sqrt{\Omega_a}u_a^2}{(\omega-2E_a)}D_b-\frac{g\sqrt{\Omega_a}v_a^2}{(\omega-2E_a)}B_b,$$
(III.6)
$$G^{o\dagger}_{ab}(\omega)=\frac{-g\sqrt{\Omega_a}v_a^2}{(\omega+2E_a)}D_b-\frac{g\sqrt{\Omega_a}u_a^2}{(\omega+2E_a)}B_b.$$

Using the definitions of D_b and B_b we can find two coupled equations for D_b and B_b, namely:

$$1/g\ D_b = C_b u_b^2 - \sum_a \Omega_a \frac{\omega(u_a^2-v_a^2)+2E_a(u_a^4+v_a^4)}{\omega^2-4E_a^2} D_b - \sum_a \Omega_a \frac{4E_a u_a^2 v_a^2}{\omega^2-4E_a^2} B_b \ ,$$

$$1/g\ B_b = C_b v_b^2 - \sum_a \Omega_a \frac{4E_a u_a^2 v_a^2}{\omega^2-4E_a^2} D_b - \sum_a \Omega_a \frac{-\omega(u_a^2-v_a^2)+2E_a(u_a^4+v_a^4)}{\omega^2-4E_a^2} B_b \ . \qquad \text{(III.7)}$$

and by using $\sum_a \Omega_a/2E_a = 1/g$,

the result can be written

$$aD_b + bB_b = C_b u_b^2 \ ,$$

$$bD_b + dB_b = C_b v_b^2 \ , \qquad \text{(III.8)}$$

where

$$a = \sum_a \Omega_a \frac{\omega^2+2\omega E_a(u_a^2-v_a^2)-8E_a^2 u_a^2 v_a^2}{2E_a(\omega^2-4E_a^2)} \ ,$$

$$b = \sum_a \Omega_a \frac{8E_a^2 u_a^2 v_a^2}{2E_a(\omega^2-4E_a^2)} \ ,$$

$$d = \sum_a \Omega_a \frac{^2-2\omega E_a(u_a^2-v_a^2)-8E_a^2 u_a^2 v_a^2}{2E_a(\omega^2-4E_a^2)} \ , \qquad \text{(III.9)}$$

and $$C_b = \frac{\sqrt{\Omega_b}}{2\pi(\omega-2E_b)g} \ .$$

Figure 2. The first excited ($J=0^+$ and seniority zero) states in Sn isotopes plotted against the active number of neutrons N.

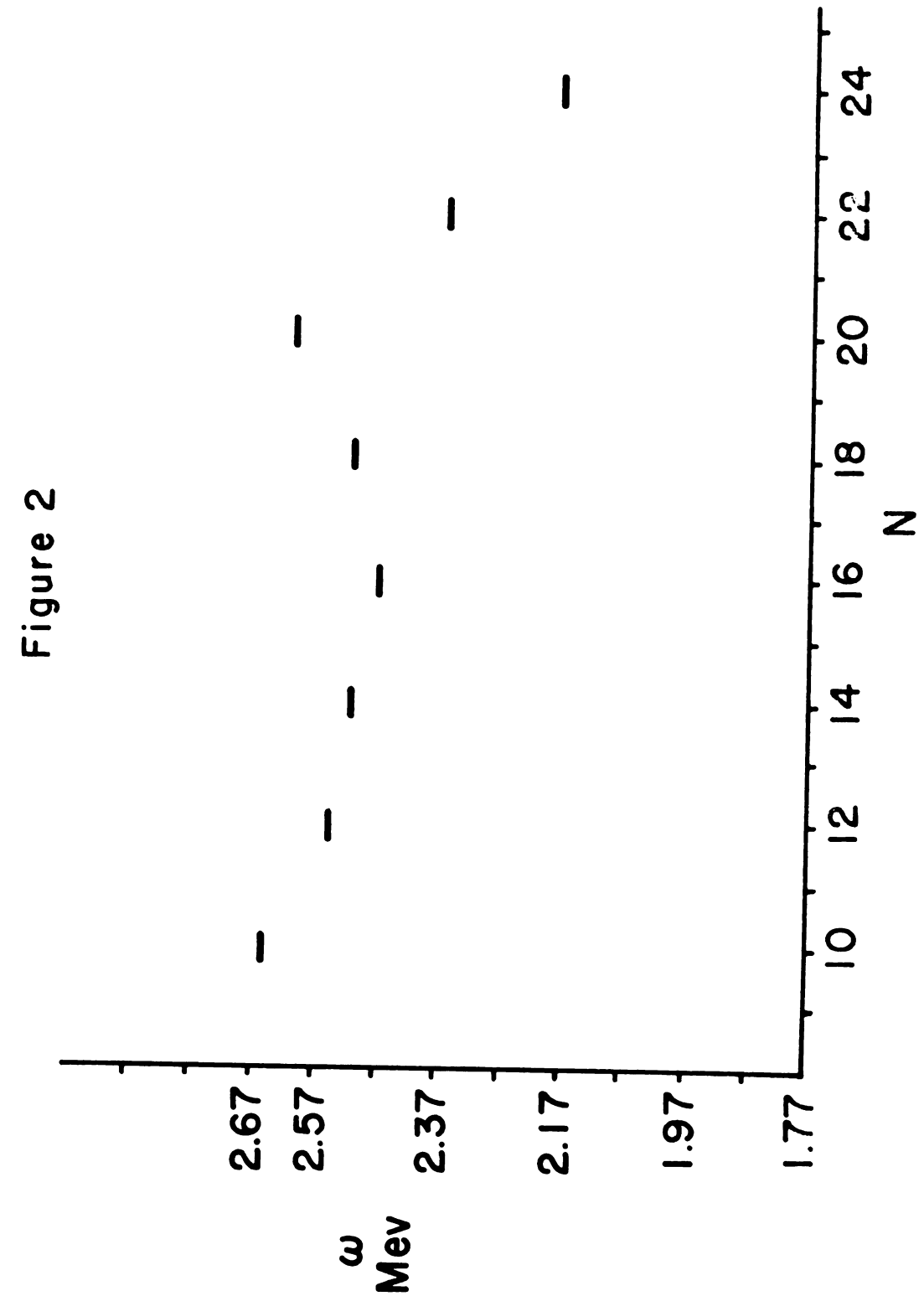


Figure 2

The poles in the Green's function are determined by the solvability condition for the homogeneous equations; i.e.

$$ad=b^2 \qquad \text{(III.10)}$$

Substituting the above values of a,b, and d in equation (III.10) we get the following eigenvalue equation:

$$\sum_{ab} \Omega_a \Omega_b \frac{\omega^2-4(\Delta^2+\eta_a\eta_b)}{4E_aE_b(\omega^2-4E_a^2)(\omega^2-4E_b^2)} = 0 \quad , \qquad \text{(III.11)}$$

where

$$\eta_i = \varepsilon_i - \lambda \quad .$$

From the above eigenvalue equation we see that for each $\omega = + \omega_i$ there exists another solution at $-\omega_i$, and hence Res $G(\omega)_{\omega=-\omega_i}$ is not zero.

c: <u>The Collective Motion</u>

The Y^2_μ $Y^2_{-\mu}$ force is decisive in producing the collective 2^+ states. It has two effects: the first is to introduces a dynamical correlation between the effective nucleons, and the second is to produce the effective charges. Since its action is not limited to those effective orbits, therefore neglecting the contributions from far away shells is assumed to be included in the effective charges. If one tries to use the real charges instead one must include the far away orbits; this increases the dimensionality of the problem, but leads to the same results[34] . The following Green's functions are needed

$$G^{n}_{ab}(\omega) = \langle\langle A^{n}_{(ab)} | A^{n\dagger}_{(ab)} \rangle\rangle_{\omega} \quad ,$$

$$G^{n+}_{ab}(\omega) = \langle\langle A^{n\dagger}_{(ab)} | A^{n\dagger}_{(ab)} \rangle\rangle_{\omega} \quad ,$$

$$G^{pn}_{ab}(\omega) = \langle\langle A^{p}_{(ab)} | A^{n\dagger}_{(ab)} \rangle\rangle_{\omega} \quad , \qquad \text{(III.12)}$$

$$G^{pn\dagger}_{ab}(\omega) = \langle\langle A^{p}_{(ab)} | A^{n\dagger}_{(ab)} \rangle\rangle_{\omega} \quad .$$

Their equations of motion are:

$$(\omega - E^{n}_{ab}) G^{n}_{ab}(\omega) = \frac{\langle [A^{n}_{(ab)}, A^{n\dagger}_{(ab)}] \rangle}{2\pi} - \frac{X^{n\,n}}{10} q(ab) U^{n}_{ab} \sum_{cd} \Sigma q^{n}(cd) U^{n}_{cd}$$

$$\times (G^{n+}_{cd} + G^{n}_{cd}) - \frac{X^{np\,n}}{10} q(ab) U^{n}_{ab} \sum_{cd} \Sigma q^{p}(cd) U^{p}_{cd}$$

$$\times (G^{np+}_{cd} + G^{np}_{cd})$$

$$(\omega + E^{n}_{ab}) G^{n+}_{ab}(\omega) = \frac{X^{n\,n}}{10} q(ab) U^{n}_{ab} \sum_{cd} \Sigma q^{n}(cd) U^{n}_{cd} (G^{n+}_{cd} + G^{n}_{cd})$$

$$+ \frac{X^{np\,n}}{10} q(ab) U^{n}_{ab} \sum_{cd} \Sigma q^{p}(cd) U^{p}_{cd} (G^{pn+}_{cd} + G^{pn}_{cd}) \, .$$

Similar equations can be obtained for $G^{pn}_{ab}(\omega)$ and $G^{pn+}_{ab}(\omega)$.

Now let

$$R^n = \sum_{cd}\sum q^n(cd)\,U^n_{cd}\,(G^{n+}_{cd}(\omega)+G^n_{cd}(\omega)) \quad ,$$
$$R^{pn} = \sum_{cd}\sum q^p(cd)\,U^p_{cd}\,(G^{pn\dagger}_{cd}(\omega)+G^{pn}_{cd}(\omega)) \quad . \qquad \text{(III.14)}$$

The equations of motion then take the form:

$$(\omega-E^n_{ab})\,G^n_{ab}(\omega) = \frac{<[A^n_{(ab)},A^{n\dagger}_{(ab)}]>}{2\pi} - \frac{q^n(ab)\,U^n_{ab}}{10}\,(X^nR^n + X^{pn}R^{pn}) \quad ,$$
$$(\omega+E^n_{ab})\,G^{n+}_{ab}(\omega) = \frac{q^n(ab)\,U^n_{ab}}{10}\,(X^nR^n + X^{pn}R^{pn}) \quad ,$$
$$(\omega-E^p_{ab})\,G^{pn}_{ab}(\omega) = -\frac{q^p(ab)\,U^p_{ab}}{10}\,(X^{pn}R^n + X^pR^{pn}) \quad , \qquad \text{(III.15)}$$
$$(\omega+E^p_{ab})\,G^{pn\dagger}_{ab}(\omega) = \frac{q^p(ab)\,U^p_{ab}}{10}\,(X^{pn}R^n + X^pR^{pn}) \quad .$$

These equations of motion are obtained with the help of boson commutation relations. The first two equations reduce to the following equation

$$R^n(1 - \frac{X^n}{5}f^n) - \frac{X^{pn}}{5}f^nR^{pn} = F^n \quad ,$$

and the last two equations reduce to the following equation

$$R^n\,\frac{X^{pn}}{5}f^p - (1 - \frac{X^p}{5}f^p)R^{pn} = 0 \quad ,$$

where $E_{ab} = E_a + E_b$,

$$f^{\xi}=\sum_{ab}\sum\frac{q(ab)^{\xi 2}U_{ab}^{\xi 2}E_{ab}^{\xi}}{(E_{ab}^{\xi 2}-\omega^2)} \quad , \quad F^n=\frac{q(ab)^{n2}U_{ab}^{n}}{\pi(\omega-E_{ab}^{n})} \quad , \quad \xi\equiv(p,n) \; . \qquad \text{(III.15)}$$

The poles in the Green's function corresponding to the collective motion are given by the condition for the solvability of the homogeneous equations i.e.

$$(1-\frac{X^n}{5}f^n)(1-\frac{X^p}{5}f^p)-\frac{X^{pn2}}{25}f^nf^p=0 \; . \qquad \text{(III.16)}$$

Following KS the strength of the quadrupole-quadrupole forces between neutrons, protons, and proton and neutron are taken to be equal; i.e. $X^n = X^p = X^{pn} = X$. If X^{pn} is taken to be different from the other and such that it is much smaller than they are, then the system of neutrons and protons tends to polarize independently causing or increase in the energy of the 2^+ state above the observed value; also B(E2) is then not sensitive to X^{pn}. Therefore X^{pn} has to be of the same size of X^n and X^p. The eigenvalue equation for $X^{pn}=0$, seperates into two equations each for each kind of nucleons i.e.

$$\frac{5}{X}=\sum_{ab}\sum\frac{q(ab)^{\xi 2}U_{ab}^{\xi 2}E_{ab}^{\xi}}{(E_{ab}^{\xi 2}-\omega^2)} \quad ,$$

and for $X^n = X^p = X^{pn} = X$, we get

$$\frac{5}{X} = \sum_{ab\xi} \frac{q^{\xi 2}_{(ab)} U^{\xi 2}_{ab} E^{\xi}_{ab}}{(E^{\xi 2}_{ab} - \omega^2)} \quad . \qquad \text{(III.17)}$$

The collective solution is that value of ω which is smaller than the minimum E_{ab}. In the deformed region where X is large no solution exists.

To evaluate the amplitudes $\psi_{(ab)}$ and $\phi_{(ab)}$ we start as follows:

$$\langle A^{\xi}_{(ab)} A^{\xi\dagger}_{(ab)} \rangle = 2\pi \text{Res.} G^{\xi}_{ab}(\omega)_{\omega=+\omega_i} \quad .$$

Now

$$\langle 0|A^{\xi}_{(ab)} A^{\xi\dagger}_{(ab)}|0\rangle = \sum_{Kq}\sum \langle 0|A^{\xi}_{(abJM)}|Kq\rangle\langle Kq|A^{\xi\dagger}_{(abJM)}|0\rangle \qquad \text{(III.18)}$$

Applying the Wigner-Eckart theorem we get

$$\langle Kq|A^{\xi\dagger}_{(abJM)}|0\rangle = \frac{(0JK|0Mq)}{\sqrt{2K+1}} (K||A^{\xi\dagger}_{(abJ)}||0) \quad ,$$

$$(0JK|0Mq) = (-)^{J+M} \sqrt{2K+1}\ (KJ0|-qM0)$$

$$= (-)^{J+M}\sqrt{2K+1}\left[(-)^{J-M} \frac{\delta_{JK}\delta_{Mq}}{\sqrt{2K+1}}\right] \quad ,$$

therefore

$$\langle Kq|A^{\xi\dagger}_{(abJM)}|0\rangle = (-)^{2J} \frac{\delta_{JK}\delta_{Mq}}{\sqrt{2J+1}} (K||A^{\xi\dagger}_{(abJ)}||0) \quad ,$$

hence

$$\langle 0|A^{\xi}_{(abJM)} A^{\xi\dagger}_{(abJM)}|0\rangle = \langle 0|A^{\xi}_{(abJM)}|JM\rangle\langle JM|A^{\xi\dagger}_{(abJM)}|0\rangle \quad ,$$

now

$$\langle JM|A^{\xi\dagger}_{(abJM)}|0\rangle = \langle [B_J, A^{\xi\dagger}_{(abJM)}]\rangle = \psi^{\xi}_{(ab)} .$$

Therefore we arrive to the following results

$$\psi^{\xi 2}_{(ab)} = 2\pi \mathrm{Res.} G^{\xi}_{ab}(\omega)_{\omega=+\omega_i} ,$$
$$\phi^{\xi 2}_{(ab)} = -\frac{2\pi}{\eta} \mathrm{Res.} G^{\xi}_{ab}(\omega)_{\omega=-\omega_i} . \qquad \text{(III.19)}$$

To find Res.G(ω), we solve for R^n and R^{pn} i.e.

$$R^n = \frac{1-\frac{X}{5}f^p}{1-\frac{X}{5}(f^n+f^p)} ,$$

$$R^{pn} = \frac{\frac{X}{5}f^p F^n}{1-\frac{X}{5}(f^n+f^p)} .$$

Substituting these values of R^n and R^{pn} in equations (III.15) we get

$$G^n_{ab}(\omega) = \frac{(1+\delta_{ab})}{2\pi(\omega-E^n_{ab})} - \frac{q^{n2}_{(ab)}U^{n2}_{ab}}{10\pi(\omega-E^n_{ab})^2}\frac{X}{1-\frac{X}{5}(f^n+f^p)} . \qquad \text{(III.20)}$$

Now

$$\mathrm{Res.}\ G^n_{ab}(\omega)_{\omega=\pm\omega_i} = \frac{q^{n2}_{(ab)}U^{n2}_{ab}}{2\pi(\omega-E^n_{ab})^2 \frac{\partial}{\partial\omega}(f^n+f^p)}\Bigg|_{\omega=\pm\omega_i} ,$$

where ω_i is the solution of the eigenvalue equation, and from now on we will use $\pm\omega$ instead of $\pm\omega_i$. The results are:

$$\left.\begin{matrix}\mathrm{Res.}\,G^{n}_{ab}(\omega) \\ +\omega\end{matrix}\right\} = \frac{q(ab)\,U^{n2}_{ab}\,U^{n2}_{ab}}{\pm 4\pi\omega\,(\omega \mp E^{n}_{ab})^{2}}\,[\Sigma_{\xi} P^{\xi}(\omega)]^{-1} \quad , \qquad \text{(III.21)}$$

$$\mathrm{Res.}\,G^{n}_{ab}(\omega)_{-\omega}$$

where

$$P^{\xi}(\omega) = \Sigma_{ab}\Sigma \frac{q(ab)\,U^{\xi 2}_{ab}\,U^{\xi 2}_{ab}\,E^{\xi}_{ab}}{(E^{\xi 2}_{ab} - \omega^{2})^{2}} \quad .$$

d: Random Phase Approximation (RPA)

Linearizing the equations of motion by means of the RPA is much better than linearizing it by means of QBA. In QBA extra terms in the commutation relation have been neglected on the assumption that they are small. They consist of terms like $a_\alpha^{\xi\dagger} a_\beta^{\xi}$. If these terms are included, higher order Green's functions will be generated which have the form

$$\Gamma^{\xi}_{(\omega)} = \langle\langle a_\alpha^{\xi\dagger} a_\beta^{\xi} A^{\xi}_{(ab)} | A^{\xi\dagger}_{(ab)} \rangle\rangle_\omega \ ,$$

these can be reduced to lower order Green's functions by means of the RPA:

$$\Gamma^{\xi}_{(\omega)} = \langle a_\alpha^{\xi\dagger} a_\beta^{\xi} \rangle G^{\xi}_{ab}(\omega) \quad . \qquad \text{(III.22)}$$

The extra terms in the equation of motion of a Green's function are similar to the original terms obtained in QBA except for a multiplicative factor $\langle a_\alpha^{\xi\dagger} a_\beta^{\xi} \rangle$. After collecting terms we get a set of equations similar to equations (III.15) except that now the right hand side is multiplied by $(1-n_a-n_b)$, where $n_i^{\xi} = \langle a_i^{\xi\dagger} a_i^{\xi} \rangle$ and where the average is taken in the ground state. This can be improved by evaluating n_i in the first 2^+ state which leads to the improved random phase approximation (IRPA). In this approximation the effect of the Pauli principle is included; therefore this method will predict a smaller number of particles is participating in any collective motion than is predicted by QBA. This has

an effect on the theoretical prediction of the enhancement over the single particle estimate for the reduced transition probability from the 2^+ state to the ground state.

The relations needed here are (see Appendix D)

$$\langle a_\alpha^{\xi\dagger} a_\beta^{\xi} \rangle = n_a^{\xi}\, \delta_{\alpha\beta}$$

$$\langle [A^{\xi}_{(ab)}, A^{\xi\dagger}_{(ab)}] \rangle = (\delta_{ac}\delta_{bd} - (-)^{a+b}\, \delta_{ad}\delta_{bc})(1-n_a^{\xi}-n_b^{\xi}) \qquad \text{(III.23)}$$

The quantity n_a represents the quasi-particle average which can be calculated (as in Appendix Ea) from $\langle A^{\xi\dagger}_{(ab)} A^{\xi}_{(ab)} \rangle$. After applying the Hartree-Fock factorization to the product of four operators the result is

$$n_a^{\xi} = \frac{1}{N_o} \sum_b \phi^{\xi 2}_{(ab)} D_{ab} \quad , \qquad D_{ab} = \frac{1}{1+\delta_{ab}} \, . \qquad \text{(III.24)}$$

The amplitudes $\psi'^{\xi}_{(ab)}$ and $\phi'^{\xi}_{(ab)}$ for this approximation will be given by

$$\left.\begin{matrix}\psi'^{\xi 2}_{(ab)} \\ \phi'^{\xi 2}_{(ab)}\end{matrix}\right\} = \frac{q^{\xi 2}_{(ab)} U^{\xi 2}_{ab} \alpha^{\xi 2}_{ab}}{2\omega(\omega \mp E^{\xi}_{ab})^2} \, [\, P'^{\xi}(\omega)]^{-1} \quad , \qquad \text{(III.25)}$$

where

$$P'^{\xi}(\omega) = \sum_a \sum_b \frac{q^{\xi 2}_{(ab)} U^{\xi 2}_{ab} E^{\xi}_{ab} \alpha^{\xi}_{ab}}{(\omega^2 - E^{2}_{ab})^2} \quad ,$$

$$\alpha^{\xi}_{ab} = 1 - n_a^{\xi} - n_b^{\xi} \quad ,$$

$$N_o^{\xi 2} = \sum_{ab}\sum \phi_{(ab)}^{\xi 2} D_{ab} \quad .$$

In IRPA we need to evaluate $n_a'^{\xi} = \langle 2 | a_\alpha^{\xi\dagger} a_\alpha^{\xi} | 2 \rangle$ which is obtained from $\langle 2 | A_{(ab)}^{\xi\dagger} A_{(ab)}^{\xi} | 2 \rangle$. The results are (Appendix-Ea)

$$n_a'^{\xi} = \frac{\sum_b [(\psi_{(ab)}^{\xi 2} + \phi_{(ab)}^{\xi 2})(1-n_b^{\xi}) + \phi_{(ab)}^{\xi 2}] D_{ab}}{N_o'^{\xi} + \sum_b (\psi_{(ab)}^{\xi 2} + \phi_{(ab)}^{\xi 2}) D_{ab}} \quad , \qquad \text{(III.26)}$$

where

$$N_o'^{\xi 2} = \sum_{ab}\sum [(\psi_{(ab)}^{\xi 2} + \phi_{(ab)}^{\xi 2}) \alpha_{ab}'^{\xi} + \phi_{(ab)}^{\xi 2}] D_{ab} \quad .$$

The validity of QBA depends on the smallness of n_a . As an example, the values of n_a are listed in table 1. for Hg^{202} and Sn^{124}. The values of n_a are plotted againest $(\varepsilon_a - \lambda)$ in Figures 3(a,b). For comparison with particle distribution v_a^2, the later is plotted in Figures 3(c,d)

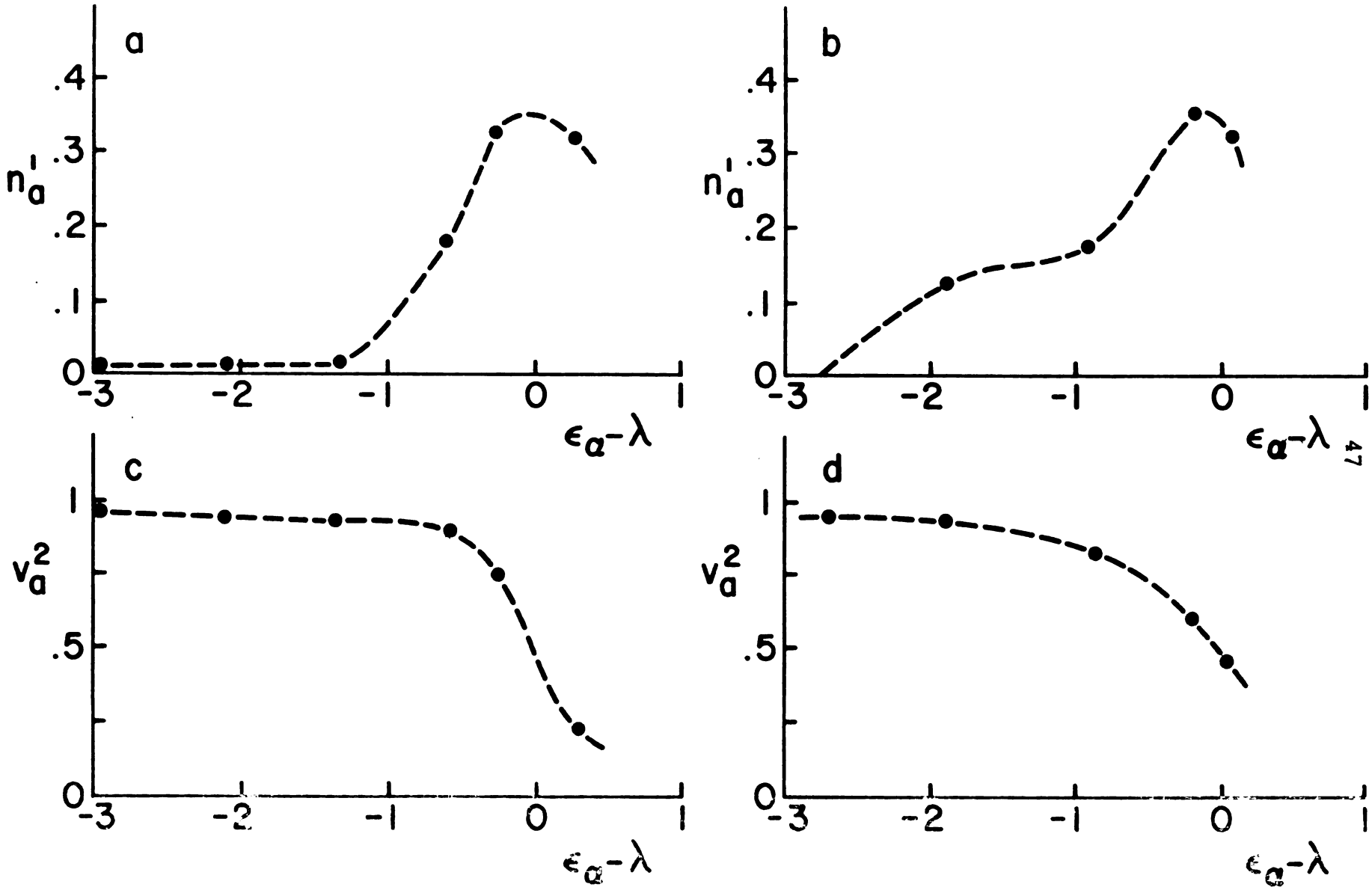
Table 1.

The values of n'_a for Sn^{124} and Hg^{202} isotopes.

Hg^{202}	a	$h_{9/2}$	$f_{7/2}$	$i_{13/2}$	$p_{3/2}$	$f_{5/2}$	$p_{1/2}$
	n'_a	.011	.015	.007	.171	.319	.296
Sn^{124}	a	$d_{5/2}$	$g_{7/2}$	$s_{1/2}$	$h_{11/2}$	$d_{3/2}$	
	n'_a	.05	.122	.173	.347	.309	

Figure 3. The values of n_a' and v_a^2 are plotted against $(\varepsilon_a - \lambda)$ for Sn^{124} in (b,d) and for Hg^{202} in (a,c).

Figure 3



If the exact commutation relations are used for the J=0 pairs the equations of motion of Green's function will contain a factor $(1-N_a/\Omega_a)$ on the right hand side. This factor will comes in as a result of the RPA decoupling (see appendix G). To examin the validity of the QBA in the pairing part of the Hamiltonian, we need to calculate $N_a=\langle a_\alpha^\dagger a_\alpha \rangle$. This can be done (see appendix E) by applying the Hartree-Fock factorizationa to $\langle A_a^\dagger A_a \rangle$, the result is given by:

$$N_a^2 = \frac{1}{2}\langle A_a^\dagger A_a \rangle . \qquad \text{(III.27)}$$

As an example the values of N_a/Ω_a for Sn^{110} are listed in table 2 .

Table 2 .

The values of N_j and N_j/Ω_j for Sn^{110} isotope.

j	$d_{5/2}$	$g_{7/2}$	$s_{1/2}$	$h_{11/2}$	$d_{3/2}$
N_j	1.50×10^{-2}	1.10×10^{-2}	0.36×10^{-2}	0.50×10^{-2}	0.26×10^{-2}
N_j/Ω_j	0.50×10^{-2}	0.28×10^{-2}	0.36×10^{-2}	0.08×10^{-2}	0.13×10^{-2}

Similar results are obtained for the rest which indicate that QBA is a very good approximation for J=0 Hamiltonian. This also shows that J=0 pairs are strongly bound, therefore they behave mor like Bosons. Similar calculations for J=2 pairs show that QBA is less valid.

e: Tamm-Dancoff Approximation (TDA)

This method of linearizing the equations of motion can be classified as a higher order phase approximation. In this approximation the Hamiltonian is diagonalized in the space spanned by a limited number of shell model states. It underestimates the reduced transition probabilities for the low-lying collective states. For this approximation the collective operator

$$B^{T\dagger}_{JM} = 1/2 \sum_{ab\xi} \psi^{T\xi}_{(ab)} A^{\xi\dagger}_{(ab)} \quad ; \qquad \text{(III.28)}$$

where T refers to TDA and

$$\phi^{\xi}_{(ab)} = 0 \ .$$

The excited state $|\, J=2 >$ is therefore given by

$$|JM> = B^{T\dagger}_{JM}|0> \ .$$

Thus we obtain Tamm-Dancoff results if we set $\phi^{\xi}_{(ab)}=0$ or equivalently $G^{+}=0$ in the previous formulas (Eqns. III.15), since the solution $\omega=+\omega_i$ for the eigenvalue equation is associated with A^{+} and the negative solution with A in the expression for B^{+} in the RPA. In this approximation one does not expect the resulting eigenvalue equation to admit negative solutions which implies Res. $G^{T}=0$. Hence setting $G^{+}=0$ we get

$$(\omega-E^{n}_{ab})G^{nT}_{ab}(\omega)= \frac{<[A^{n}_{(ab)},A^{n\dagger}_{(ab)}]>}{2\pi} - \frac{q^{n}(ab)U^{n}_{ab}}{10}(X^{n}R'_{n}+X^{np}R'_{np}),$$

$$(\omega-E^{p}_{ab})G^{npT}_{ab}(\omega)= - \frac{q^{p}(ab)U^{p}_{ab}}{10}(X^{np}R'_{n}+X^{p}R'_{np}),$$

where

$$R'_{n} = \sum_{cd}\sum q^{n}(cd)U^{n}_{cd}G^{nT}_{cd}(\omega) \quad ,$$

$$R'_{np}= \sum_{cd}\sum q^{p}(cd)U^{p}_{cd}G^{npT}_{cd}(\omega) \quad .$$

From the first equation we get (for $X^{n}= X^{p} = X^{pn} =X$)

$$R'_{n} = \frac{F^{n}(1-b^{n}X)}{1-b^{p}X-b^{n}X} \quad ,$$

and from the second equation we get

$$R'_{np} = \frac{b^{p}XF^{n}}{1-b^{n}X-b^{p}X} \quad ,$$

where

$$b^{\xi} = \sum_{ab}\sum \frac{q^{\xi 2}(ab)U^{\xi 2}_{ab}}{(E^{\xi}_{ab}-\omega)} \quad ,$$

$$F^{n} = \frac{q^{n}(ab)U^{n}_{ab}}{\pi(\omega-E^{n}_{ab})} \quad .$$

Solving for $G^{n}_{ab}(\omega)$ we get

$$G^{nT}_{ab}(\omega) = \frac{1+\delta_{ab}}{2\pi(\omega - E^{n}_{ab})} - \frac{Xq(ab)^{n2}U^{n2}_{ab}}{10\pi(\omega - E^{n}_{ab})^{2}} \frac{1}{1-b^{n}X-b^{p}X} ,$$

therefore

$$\mathrm{Res.}G^{nT}_{ab}(+\omega) = \frac{q(ab)^{n2}U^{n2}_{ab}}{\pi(\omega - E^{n}_{ab})^{2}} [\Sigma_{\xi} P^{\xi T}(\omega)]^{-1} ,$$

$$\mathrm{Res.}G^{nT}_{ab}(-\omega) = 0 ,$$

where

$$P^{\xi T}(\omega) = \Sigma\Sigma_{ab} \frac{q(ab)^{\xi 2}U^{\xi 2}_{ab}}{(\omega - E^{\xi}_{ab})^{2}} .$$

Therefore

$$\psi^{\xi T2}_{(ab)} = \frac{2q(ab)^{\xi 2}U^{\xi 2}_{ab}}{(\omega - E^{\xi}_{ab})^{2}} [\Sigma_{\xi} P^{\xi T}(\omega)]^{-1} .$$

The expression for B(E2) is the same as before except now we use $\psi^{\xi T}_{(ab)}$ instead of $\psi^{\xi}_{(ab)}$ and put $\phi^{\xi}_{(ab)}=0$.
The eigenvalue equation corresponding to the collective motions, for this case is given by:

$$1- (b^{n} + b^{p})X = 0 ,$$

substituting for b we get the following results:

$$\frac{10}{X} = \Sigma\Sigma\Sigma_{ab\xi} \frac{q(ab)^{\xi 2}U^{\xi 2}_{ab}}{E^{\xi}_{ab} - \omega} .$$

From this eigenvalue equation we see that the negative values

of ω are not solutions. Therefore Res.$^{nT}G(-\omega)= 0$, which implies that $\phi^{\xi}(ab)=0$. The difference between this TDA and RPA is this: the collective operator B^{+} is expanded in terms of A^{+} only in TDA, which is equivalent to expanding the wave function of a certain excited state in terms of two quasi-particles states only neglecting the two quasi-hole states, while these two quasi-hole states are included in RPA where the two kinds of states are treated symetrically. Thus the negative solutions are associated with the operator A(ab) while the positive solutions are associated with the operator $A^{+}(ab)$.

Table 3. B(E2→0) values in units of $10^{-48}cm^4$, calculated for each approximation and compared with the experimental results (the last column). These results are calculated for the parameters given in tables 11-14.

$\omega_{exp.}$	Isotope	A	QBA	RPA	IRPA	TDA	Exp.
1.450	28Ni30	58	.033	.021	.012	.014	.072
1.333	28Ni32	60	.064	.039	.023	.025	.091
1.172	28Ni34	62	.096	.054	.032	.032	.083
1.340	28Ni36	64	.094	.056	.031	.034	.087
0.992	30Zn34	64	.636	.439	.332	.158	.170
1.039	30Zn36	66	.604	.418	.311	.159	.145
1.078	30Zn38	68	.551	.381	.278	.154	.125
1.040	32Ge38	70	.665	.456	.329	.189	.172
0.835	32Ge40	72	.864	.529	.377	.199	.230
0.596	32Ge42	74	1.092	.638	.463	.194	.317
0.563	32Ge44	76	1.010	.765	.507	.184	.263
0.635	34Se40	74	1.213	.617	.436	.217	.210
0.559	34Se42	76	1.225	.680	.489	.210	.480
0.614	34Se44	78	.970	.689	.494	.098	.385
0.666	34Se46	80	.804	.613	.465	.188	.283
0.655	34Se48	82	.811	.551	.421	.189	.213
0.450	36Kr42	78	1.440	.750	.492	.202	.510
0.618	36Kr44	80	.900	.666	.468	.189	.340
0.777	36Kr46	82	.638	.499	.398	.177	.180
0.880	36Kr48	84	.545	.408	.331	.178	.150
1.078	38Sr48	86	.337	.283	.266	.135	
1.836	38Sr50	88	.313	.262	.160	.185	.130
2.180	40Zr50	90	.110	.098	.056	.080	
0.934	40Zr52	92	.165	.151	.179	.071	.790
0.920	40Zr54	94	.295	.255	.234	.113	.790
0.871	42Mo52	94	.208	.160	.139	.087	.270
0.778	42Mo54	96	.379	.273	.214	.129	.300
0.787	42Mo56	98	.565	.382	.291	.171	.270
0.536	42Mo58	100	1.125	.653	.495	.212	.610
0.833	44Ru52	96	.250	.157	.106	.105	.250
0.660	44Ru54	98	.476	.262	.195	.146	.480
0.540	44Ru56	100	.807	.397	.327	.183	.570
0.475	44Ru58	102	1.150	.599	.474	.215	.730
0.556	46Pd58	104	.866	.479	.370	.202	.550
0.512	46Pd60	106	1.077	.620	.473	.225	.650
0.434	46Pd62	108	1.400	.794	.586	.244	.740
0.374	46Pd64	110	1.730	.960	.671	.258	.860

Table 3.(Continued)

$\omega_{exp.}$	Isotope	A	QBA	RPA	IRPA	TDA	Exp.
0.633	48Cd58	106	.631	.379	.275	.168	.470
0.633	48Cd60	108	.718	.453	.332	.187	.540
0.658	48Cd62	110	.752	.497	.370	.200	.500
0.617	48Cd64	112	.853	.555	.413	.211	.540
0.558	48Cd66	114	.976	.605	.444	.217	.580
0.513	48Cd68	116	1.060	.622	.452	.219	.600
0.562	52Te68	120	1.162	.752	.564	.269	.550
0.564	52Te70	122	1.080	.702	.530	.264	.650
0.603	52Te72	124	.921	.617	.466	.253	.390
0.667	52Te74	126	.736	.516	.386	.236	.530
0.743	52Te76	128	.554	.407	.295	.211	.410
0.840	52Te78	130	.395	.306	.214	.181	.340
0.441	54Xe74	128	1.610	.929	.730	.339	
0.538	54Xe76	130	1.165	.736	.566	.314	.480
0.668	54Xe78	132	.797	.550	.405	.281	.320
0.850	54Xe80	134	.497	.375	.255	.238	.300
1.320	54Xe82	136	.164	.154	.073	.143	
0.464	56Ba76	132	1.750	1.040	.800	.390	.730
0.605	56Ba78	134	1.179	.783	.578	.356	
0.818	56Ba80	136	.732	.542	.371	.313	
1.426	56Ba82	138	.218	.205	.122	.185	.300
1.257	50Sn62	112	.328	.249	.168	.135	.180
1.299	50Sn64	114	.351	.267	.179	.145	.200
1.293	50Sn66	116	.364	.275	.181	.149	.210
1.230	50Sn68	118	.357	.268	.178	.145	.230
1.171	50Sn70	120	.333	.250	.169	.138	.220
1.140	50Sn72	122	.294	.225	.152	.127	.250
1.131	50Sn74	124	.240	.188	.127	.111	.210
0.790	58Ce80	138	.954	.702	.488	.368	
1.596	58Ce82	140	.279	.267	.111	.242	.360
0.650	58Ce84	142	1.250	.872	.652	.400	.590
1.570	60Nd82	142	.170	.166	.193	.150	.340
0.695	60Nd84	144	1.283	.965	.761	.426	.440
0.453	60Nd86	146	2.300	1.590	1.260	.493	.840
0.747	62Sm84	146	1.270	1.018	.858	.436	
0.551	62Sm86	148	1.990	1.540	1.271	.502	.890
0.334	62Sm88	150	3.810	2.820	1.977	.574	1.320
0.637	64Gd86	150	1.870	1.570	1.304	.515	
0.155	76Os112	188	10.550	5.210	3.640	.910	2.800
0.187	76Os114	190	8.270	4.480	3.270	.885	2.550
0.329	78Pt116	194	3.000	2.180	1.790	.701	1.940
0.356	78Pt118	196	2.470	1.790	1.430	.656	1.270
0.408	78Pt120	198	1.820	1.360	1.042	.592	1.350

Table 3.(COntinued)

$\omega_{exp.}$	Isotope	A	QBA	RPA	IRPA	TDA	Exp.
0.426	80Hg116	196	1.290	1.047	.952	.469	
0.412	80Hg118	198	1.270	.983	.820	.458	1.130
0.368	80Hg120	200	1.312	.948	.722	.442	.850
0.439	80Hg122	202	.854	.657	.447	.376	.590
0.430	80Hg124	204	.621	.482	.297	.297	
0.960	82Pb120	202	.267	.239	.151	.186	
0.899	82Pb122	204	.193	.175	.087	.148	.170
0.803	82Pb124	206	.081	.077	.025	.072	.130

Table 4. B(E2) values in units of $10^{-48}cm^4$, are listed for each one of the approximations QBA, RPA, IRPA and TDA. The experimental B(E2) in units of $e^2 10^{-48}cm^4$, are listed in the last column. These values are calculated for the parameters given in tables 15-20.

Isotope	A	QBA	RPA	IRPA	TDA	Exp.
28Ni30	58	.114	.066	.041	.037	.072
28Ni32	60	.130	.082	.054	.042	.091
28Ni34	62	.146	.087	.057	.044	.083
28Ni36	64	.112	.070	.043	.039	.037
30Zn34	64	.682	.452	.343	.068	.170
30Zn36	66	.622	.414	.308	.165	.145
30Zn38	68	.496	.336	.249	.145	.125
32Ge38	70	.619	.433	.321	.177	.172
32Ge40	72	.910	.515	.360	.200	.230
32Ge42	74	.997	.686	.480	.082	.317
32Ge44	76	.872	.946	.562	.169	.263
34Se40	74	1.290	.617	.436	.221	.210
34Se42	76	1.149	.752	.522	.201	.480
34Se44	78	.857	.746	.531	.184	.385
34Se46	80	.698	.592	.467	.174	.283
34Se48	82	.667	.457	.351	.173	.213

Table 4.(Continued)

Isotope	A	QBA	RPA	IRPA	TDA	Exp.
36Kr42	78	1.404	1.010	.571	.202	.510
36Kr44	80	.814	.719	.511	.179	.340
36Kr46	82	.547	.460	.388	.161	.180
36Kr48	84	.444	.347	.294	.151	.150
38Sr48	86	.302	.265	.288	.125	
38Sr50	88	.313	.262	.160	.185	.130
40Zr50	90	.227	.203	.120	.156	
40Zr52	92	.256	.245	.312	.103	.790
40Zr54	94	.403	.369	.356	.146	.790
42Mo52	94	.289	.254	.270	.112	.270
42Mo54	96	.497	.410	.350	.157	.300
42Mo56	98	.741	.529	.405	.207	.270
42Mo58	100	1.351	.802	.600	.242	.610
44Ru52	96	.306	.232	.210	.117	.250
44Ru54	98	.580	.398	.325	.165	.480
44Ru56	100	.992	.568	.456	.206	.570
44Ru58	102	1.340	.734	.578	.236	.730
46Pd58	104	.975	.560	.438	.213	.550
46Pd60	106	1.167	.677	.525	.233	.650
46Pd62	108	1.478	.835	.628	.250	.740
46Pd64	110	1.788	.998	.704	.262	.860
48Cd58	106	.673	.408	.300	.170	.470
48Cd60	108	.747	.467	.345	.186	.540
48Cd62	110	.772	.503	.375	.198	.500
48Cd64	112	.866	.562	.417	.208	.540
48Cd66	114	.974	.614	.450	.215	.580
48Cd68	116	1.030	.627	.457	.216	.600
50Sn62	112	.328	.249	.170	.134	.180
50Sn64	114	.358	.272	.182	.146	.200
50Sn66	116	.371	.282	.186	.152	.210
50Sn68	118	.365	.277	.185	.149	.230
50Sn70	120	.343	.260	.176	.142	.220
50Sn72	122	.306	.233	.157	.132	.250
50Sn74	124	.257	.198	.132	.116	.210
52Te68	120	1.210	.800	.598	.280	.550
52Te70	122	1.140	.753	.567	.275	.650
52Te72	124	.975	.663	.500	.264	.390
52Te74	126	.782	.555	.413	.247	.530
52Te76	128	.596	.443	.323	.223	.410
52Te78	130	.405	.323	.228	.189	.340
54Xe74	128	1.720	1.o10	.782	.356	
54Xe76	130	1.230	.794	.604	.329	.480
54Xe78	132	.788	.569	.426	.284	.320
54Xe80	134	.411	.326	.227	.209	
54Xe82	136	.164	.154	.073	.143	

Table 4.(Continued)

Isotope	A	QBA	RPA	IRPA	TDA	Exp.
56Ba76	132	1.830	1.117	.851	.406	.730
56Ba78	134	1.130	.799	.608	.353	
56Ba80	136	.561	.447	.344	.260	
56Ba82	138	.218	.205	.122	.185	.300
58Ce80	138	.682	.557	.467	.292	
58Ce82	140	.279	.267	.111	.242	.360
58Ce84	142	.934	.754	.628	.338	.590
60Nd82	142	.170	.166	.193	.150	.340
60Nd84	144	.898	.772	.710	.343	.440
60Nd86	146	1.840	1.500	1.192	.452	.840
62Nd88	146	.804	.727	.764	.327	
62Nd90	148	1.500	1.300	1.150	.443	.690
62Nd92	150	3.195	2.730	1.980	.550	1.320
64Gd84	150	1.270	1.177	1.156	.426	
76Os112	188	11.360	5.320	3.670	.960	2.800
76Os114	190	8.980	4.681	3.340	.938	2.550
78Pt116	194	3.530	2.480	1.960	.761	1.940
78Pt118	196	2.920	2.070	1.630	.714	1.270
78Pt120	198	2.173	1.600	1.230	.648	1.350
80Hg116	196	1.580	1.254	1.115	.521	
80Hg118	198	1.530	1.166	.975	.502	1.130
80Hg120	200	1.552	1.105	.861	.476	.850
80Hg122	202	1.008	.761	.543	.402	.590
80Hg124	204	.742	.561	.370	.319	

Table 5 .

The average of the absolute deviations from the experimental B(E2) values for the corresponding approx.

The Approximations	QBA	RPA	IRPA	TDA	$B(E2)^{exp.}_{av.}$
$\lvert\delta B(E2)\rvert_{av.}$	.68	.28	.157	.286	.537
$\frac{\lvert\delta B(E2)\rvert_{av.}}{B(E2)^{exp.}_{av.}} \times 100$	127%	53%	29%	53%	

These deviations are for B(E2) given in table 4.

Table 6 .

The average of the absolute deviations from the experimental B(E2) values for the corresponding approx.

The Approximations	QBA	RPA	IRPA	TDA	$B(E2)^{exp.}_{av.}$
$\lvert\delta B(E2)\rvert_{av.}$	.611	.254	.178	.287	.527
$\frac{\lvert\delta B(E2)\rvert_{av.}}{B(E2)^{exp.}_{av.}} \times 100$	115%	50%	36%	54%	

These deviations are for B(E2) given in table 3.

IV. Deuteron Stripping Reaction

This reaction is considered to be a very important tool in nuclear structure studies. It provides good information about the single particle energies in the residual nucleus A+1, their relative positions, and their variations with the mass number A. The spectroscopic factors for these reactions provide direct information about the occupations number v_a^2. Cohen and Price (1960) performed experiments with (d,p) and (d,t) on Sn isotopes; they measured u_a, and v_a directly, from which they obtained good information about the unperturbed single particle energies.

The spectroscopic factors of the single particle transfer reaction represent the probability of the appearance of the single particle state in the target nuclear wave function. Therefore one expects this probability to decreases when the effect of the Pauli principle is included. The spectroscopic factors provide a good basis for comparisons between experimental results and the predictions of nuclear models.

The cross section for this reaction can be written as follows

$$\frac{d\sigma}{d\Omega} = \frac{2J_f+1}{2J_i+1} \sum_{\ell} S_{\ell} \Phi(\ell, Q, \theta) \quad ,$$

where J_f and J_i are the spin of the residual nucleus and

target respectively, and $\Phi(\ell,Q,\theta)$ is the single particle cross section, which is a function of the orbital angular momentum ℓ, the Q-value of the reaction and the proton angle θ. The quantity S_ℓ is the specroscopic factor which is a sum of overlap integrals; it is defined as follows

$$S_\ell = \sum_{j=\ell\pm 1/2} S_j \ ,$$

where

$$S_j = |\langle JM|(j,J_o),JM\rangle|^2 \ ,$$

and $|JM\rangle$ is the wave function for the even-even nucleus (the wave function of the excited state in the residual nucleus), and $|(jJ_o),JM\rangle$ is the wave function of the ground state of the odd target coupled to the free neutron wave function, and is given by:

$$|(jJ_o),JM\rangle = \sum_{mM_o} (jJ_oJ|mM_oM)C^{n\dagger}_{jm}|J_oM_o\rangle \ ,$$

where $|J_oM_o\rangle$ is the wave function of the target, which can be expanded in terms of the seniority one and three states as follows

$$|J_oM_o\rangle = C^{J_o}_{J_o} a^\dagger_{J_oM_o}|0\rangle + \sum_{j'm'} C^{J_o}_{j'} (2j'J_o|0m'M_o) B^\dagger_{2\nu} a^\dagger_{j'm'}|0\rangle \ ,$$

where $|0\rangle$ is the quasi-particle vacuum. The coeffients $C^{J_o}_{J_o}$ and $C^{J_o}_{j'}$ can be obtained by diagonalizing the Hamiltonian H in the space of one quasi-particles with zero and one

phonon. These amplitudes are tabulated in KS . The particle operator $C^{n\dagger}_{jm}$ can be expressed in terms of quasi-particle operators by means of B-V transformation

$$C^{n\dagger}_{jm} = u^n_j \, a^{n\dagger}_{jm} + s_j v^n_j a^n_{j-m} \ .$$

Straightforward calculations (see Appendix F) yeild the following result for S_j

$$S_j = |u^n_j C^{J_o}_{J_o} \psi'^n_{(jJ_o)} - v^n_j C^{J_o}_j \sqrt{(2J_o+1)/5}|^2 \ .$$

This formula is similar to that obtained by Yoshida and Sorensen except for the amplitude $\psi'^n_{(jJ)}$ the value of which depends on the approximatio used. The phase of $\psi'_{(jJ)}$ is chosen to agree with that of $C^{J_o}_j$ (see Appendix F).

Table 7 .

S_ℓ values, are listed for each one of the approximations QBA, RPA, IRPA and TDA. The experimental S_ℓ, are listed in column 7.

ℓ	Reaction	QBA	RPA	IRPA	TDA	Exp.	J_o^π
1	$Ni^{61}(d,p)Ni^{62}$	0.47	0.32	0.18	0.37	0.26±.04	$3/2^-$
1	$Zn^{67}(d,p)Zn^{68}$	0.25	0.20	0.19	0.21	0.11±.02	$5/2^-$
1	$Se^{77}(d,p)Se^{78}$	0.07	0.07	0.08	0.08	0.029±.005	$1/2^-$
2	$Zr^{91}(d,p)Zr^{92}$	1.64	0.72	0.36	1.44	1.33±.2	$5/2^+$
2	$Mo^{95}(d,p)Mo^{96}$	0.87	0.27	0.40	0.70	0.30±.05	$5/2^+$
2	$Ru^{101}(d,p)Ru^{102}$	0.13	0.20	0.18	0.16	0.032±.008	$5/2^+$
2	$Pd^{105}(d,p)Pd^{106}$	0.18	0.20	0.23	0.20	0.068±.03	$5/2^+$
2	$Sn^{115}(d,p)Sn^{116}$	0.18	0.14	0.11	0.16	0.10±.015	$1/2^+$
2	$Sn^{117}(d,p)Sn^{118}$	0.19	0.14	0.12	0.17	0.16±.025	1/2+
2	$Sn^{119}(d,p)Sn^{120}$	0.16	0.13	0.11	0.14	0.06±.01	$1/2^+$
2	$Te^{125}(d,p)Te^{126}$	0.07	0.07	0.07	0.07	0.027±.004	$1/2^+$
2	$Ba^{135}(d,p)Ba^{136}$	0.01	0.01	0.02	0.01	0.32±.04	$3/2^+$

Table 8.

B(E2) values in units of $10^{-48}cm^4$ from KS.

Isotope	$B(E2)_{theor.}$	$B(E2)_{exp.}$
28Ni30	0.017	0.072
28Ni32	0.051	0.091
28Ni34	0.100	0.083
28Ni36	0.092	0.087
30Zn34	0.264	0.170
30Zn36	0.245	0.145
30Zn38	0.164	0.125
32Ge38	0.458	0.172
32Ge40	0.476	0.230
32Ge42	0.609	0.317
32Ge44	0.729	0.263
32Ge46	0.451	
34Se40	0.696	0.210
34Se42	0.919	0.480
34Se44	0.770	0.385
34Se46	0.594	0.283
34Se48	0.327	0.213
36Kr42	1.784	0.510
36Kr44	0.812	0.340
36Kr46	0.550	0.180
36Kr48	0.313	0.150
38Sr48	0.205	
38Sr50	0.143	0.130
40Zr50	0.141	
40Zr52	0.080 }	0.790
40Zr54	0.216 }	
42Mo52	0.166	0.270
42Mo54	0.360	0.300
42Mo56	0.683	0.270
42Mo58	0.915	0.610
44Ru52	0.279	0.250
44Ru54	0.563	0.480
44Ru56	0.947	0.570
44Ru58	1.424	0.730

Table 8.(Continued)

Isotope	$B(E2)_{theor.}$	$B(E2)_{exp.}$
46Pd58	1.006	0.550
46Pd60	1.261	0.650
46Pd62	1.603	o.740
46Pd64	2.009	0.860
48Cd58	0.447	0.470
48Cd60	0.571	0.540
48Cd62	0.687	0.500
48Cd64	0.758	0.540
48Cd66	0.799	0.580
48Cd68	0.809	0.600
50Sn62	0.350	0.180
50Sn64	0.381	0.200
50Sn66	0.399	0.210
50Sn68	0.414	0.230
50Sn70	0.416	0.220
50Sn72	0.365	0.250
50Sn74	0.273	0.210
52Te68	1.183	0.550
52Te70	1.307	0.650
52Te72	1.o80	0.390
52Te74	0.729	0.530
52Te76	0.468	0.410
52Te78	0.289	0.340
54Xe74	1.654	
54Xe76	1.174	0.480
54Xe78	0.592	0.320
54Xe80	0.344	
54Xe82	0.198	
56Ba76	1.814	0.730
56Ba78	0.929	
56Ba80	0.509	
56Ba82	0.294	0.300

Table 8.(Continued)

Isotope	$B(E2)_{theor.}$	$B(E2)_{exp.}$
58Ce80	o.631	
58Ce82	0.392	0.360
58Ce84	0.828	0.590
60Nd82	0.361	0.340
60Nd84	0.908	0.440
60Nd86	2.101	0.840
62Sm84	0.900	
62Sm86	2.189	0.890
62Sm88	4.000	1.320
64Gd84	0.974	
64Gd86	1.872	
76Osll2	11.800	2.800
76Osll4	9.300	2.550
78Ptll6	5.200	1.940
78Ptll8	4.086	1.270
78Ptl20	3.060	1.350
80Hgll6	1.250	
80Hgll8	1.355	1.130
80Hgl20	0.982	o.850
80Hgl22	0.749	0.590
80Hgl24	0.461	
82Pbll8	0.337	
82Pbl20	0.280	
82Pbl22	0.216	0.170
82Pbl24	0.101	0.130

Table 9.

The values of S_ℓ from ref. 19.

ω	ℓ	Reaction	$S_{theor.}$	$S_{exp.}$	J_o^π
1.18	1	$Ni^{61}(d,p)Ni^{62}$	0.320	0.26±.04	$3/2^-$
1.08	1	$Zn^{67}(d,p)Zn^{68}$	0.210	0.11±.02	$5/2^-$
0.62	1	$Se^{77}(d,p)Se^{78}$	0.070	0.029±.005	$1/2^-$
0.94	2	$Zr^{91}(d,p)Zr^{92}$	1.500	1.330±.2	$5/2^+$
0.81	2	$Mo^{95}(d,p)Mo^{96}$	0.680	0.30±.05	$5/2^+$
0.48	2	$Ru^{101}(d,p)Ru^{102}$	0.11	0.032±.008	$5/2^+$
0.52	2	$Pd^{105}(d,p)Pd^{106}$	0.14	0.068±.03	$5/2^+$
1.30	2	$Sn^{115}(d,p)Sn^{116}$	0.18	0.10±.015	$1/2^+$
1.22	2	$Sn^{117}(d,p)Sn^{118}$	0.19	0.16±.025	$1/2^+$
1.17	2	$Sn^{119}(d,p)Sn^{120}$	0.15	0.06±.01	$1/2^+$
0.69	2	$Te^{125}(d,p)Te^{126}$	0.06	0.027±.004	$1/2^-$
0.83	2	$Ba^{135}(d,p)Ba^{136}$	0.02	0.32±.04	$3/2^+$

V. Numerical Results

In this study we considered the same spherical nuclei studied by KS, the single particle energies are those given in References 1 and 10; the values of λ^{ξ} and Δ^{ξ} are extrapolated from KS (rough estimate) and then used in iterations to satisfy the BCS equations (gap equation). These values of λ^{ξ} and Δ^{ξ} are the values which minimize the ground state energy, and their dependence on the excitation energies is assumed to be small and is neglected. The values of the theoretical B(E2) are calculated for two sets of single particle energies: the first set are those given in KS, the second set are those given in Reference 10. The following A-dependences are used:

$$a_o^4 = 10.746\times10^{51}A^{-2/3} ,$$

$$\varepsilon_j(A)=\varepsilon_j(A_o)(A_o/A)^{1/3}+\alpha_j(A_o/A)^{1/3}[1-(A/A_o)^{1/3}]+\Delta\varepsilon_j(Z,N).$$

If both $j=\ell\pm1/2$ are present in the major shell, α_j is given by

$$\alpha_{\ell+1/2}=-(\varepsilon_{\ell-1/2}(A_o)-\varepsilon_{\ell+1/2}(A_o))\frac{\ell}{2\ell+1} ,$$

$$\alpha_{\ell-1/2}=(\varepsilon_{\ell-1/2}(A_o)-\varepsilon_{\ell+1/2}(A_o))\frac{\ell+1}{2\ell+1} .$$

If only one of them is present, then α_j is given by:

$$\alpha_{\ell+1/2} = -\frac{7\ell}{A_o^{2/3}} ,$$

$$\alpha_{\ell-1/2} = \frac{7(\ell+1)}{A_o^{2/3}} ,$$

where $\Delta\varepsilon_j(Z,N)$ is a special shift in the single particle energy, and A is the mass number. The numerical calculations of B(E2) and Sj were performed on the CDC 6500 computer at Michigan State University. The energies of the 2^+ states are the experimental values taken from the table of isotopes (Ref.48). The results for B(E2) are listed in Tables 3&4 for each approximation. In table 7 the values of S_ℓ are listed, while the averages of the absolute deviations from experimental values $|\delta B(E2)|_{av}$ for B(E2) are tabulated along with the corresponding averages of the experimental values of B(E2) in tables 5&6 . The quantities $|B(E2)|_{av}$ and $B(E2)^{exp.}_{av}$ are defined as follows

$$|\delta B(E2)|_{av} = \frac{1}{N}\sum_{i=1}^{N} |B(E2)^{th.}_i - B(E2)^{exp.}_i| ,$$

$$B(E2)^{exp.}_{av} = \frac{1}{N}\sum_{i=1}^{N} B(E2)^{exp}_i ,$$

where $B(E2)^{th}$, and $B(E2)^{exp}$ are the theoretical and experimental values of B(E2) respectively, and N is the total number of cases. The values of g are those given in KS and in Ref. 10, where those values are taken to fit the experimental

odd-even mass differences. To convey an idea of the dependence of Δ and λ on the number of particles; these values are plotted against the number of particles for Sn in figure 4. From tables 5 and 6 it is clear that (for this set of parameters) the IRPA gives the best result, while TDA does not differ much from RPA as far as the absolute diviations from experiment are concerned. Indeed the TDA underestimates the B(E2), on the other hand the RPA overestimates it; therefore, including the Pauli principle in TDA will increase the deviations from experimental values. In this work the Pauli principle is not included for TDA. In Figure 4 the quantities λ and Δ are plotted against the neutron number for Sn isotopes. We have to know the N-dependence of λ and Δ in order to obtain new values by extrapolation.

Figure 4. The parameters λ_n and Δ_n are plotted against the mass number A for Sn isotopes.

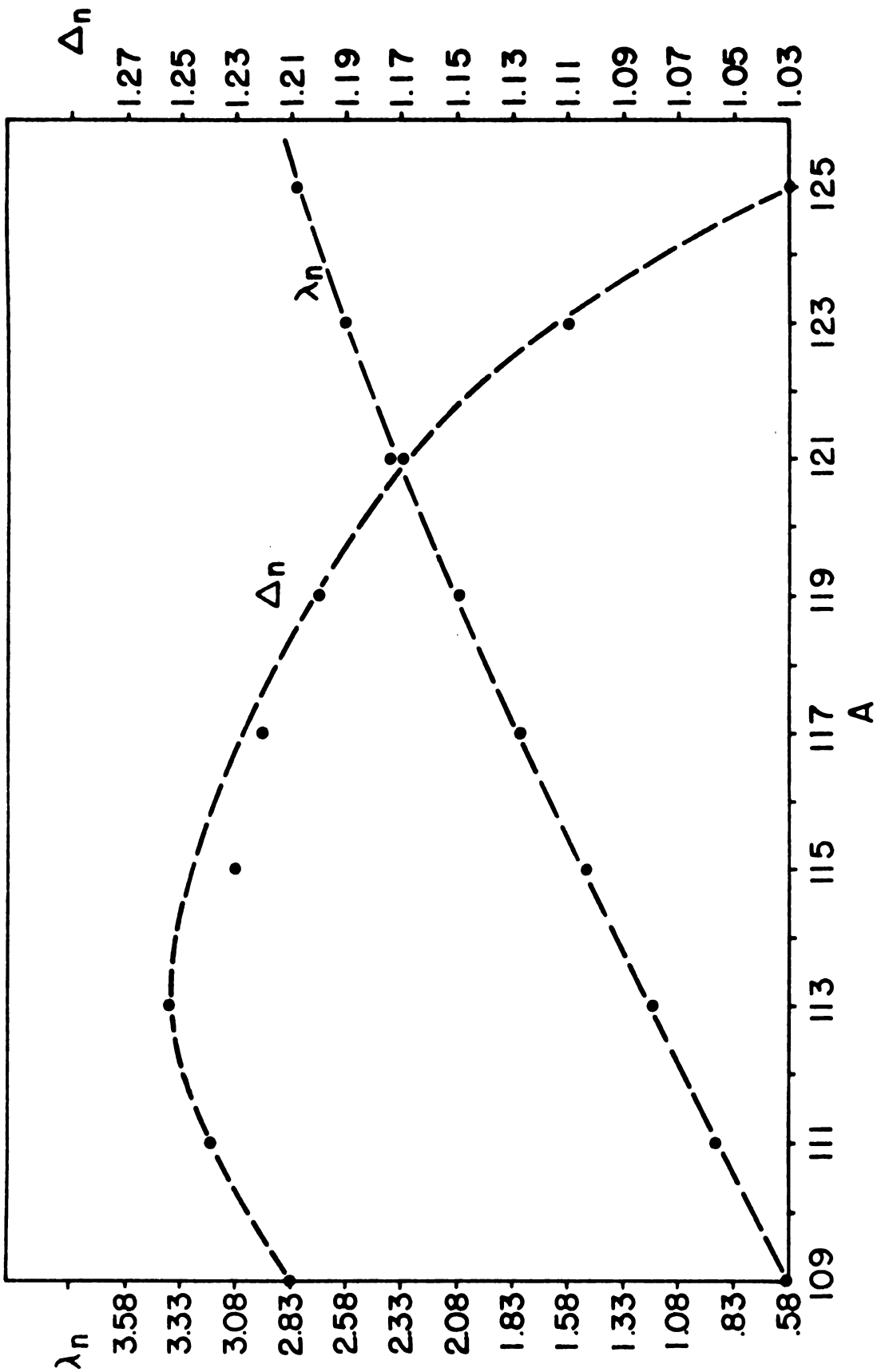

Δn
1.27
1.25
1.23
1.21
1.19
1.17
1.15
1.13
1.11
1.09
1.07
1.05
1.03
λn
3.58
3.33
3.08
2.83
2.58
2.33
2.08
1.83
1.58
1.33
1.08
.83
.58
λn
Δn
109
111
113
115
117
119
121
123
125
A

Figure 4

APPENDICES

Appendix A

Useful Relationships

$$\sum_{m_\gamma} (abc|\, m_\alpha m_\beta m_\gamma)(cde|m_\gamma m_\delta m_\eta) = (-)^{a+b+d+e} \sum_{g m_\lambda} \sqrt{(2c+1)(2g+1)}$$

$$\times \begin{Bmatrix} abc \\ deg \end{Bmatrix} (age|m_\alpha m_\lambda m_\eta)(bdg|m_\beta m_\delta m_\lambda) \ , \tag{A1}$$

where $\begin{Bmatrix} abc \\ deg \end{Bmatrix}$ is the six j-symbol.

$$(abJ|m_\alpha m_\beta M) = (-)^{a+b-J}(abJ|-m_\alpha -m_\beta -M) \tag{A2}$$

$$= (-)^{a+b-J}(baJ|m_\beta m_\alpha M) \tag{A3}$$

$$= \sqrt{(2J+1)/(2b+1)}\,(-)^{a-m_\alpha}(aJb|m_\alpha -M-m_\beta) \tag{A4}$$

$$= \sqrt{(2J+1)/(2a+1)}\,(-)^{b+m_\beta}(Jba|-Mm_\beta -m_\alpha) \tag{A5}$$

$$= \sqrt{(2J+1)/(2b+1)}\,(-)^{a-m_\alpha}(Jab|M-m_\alpha m_\beta) \tag{A6}$$

$$= \sqrt{(2J+1)/(2a+1)}\,(-)^{b+m_\beta}(bJa|-m_\beta M m_\alpha) \tag{A7}$$

$$\sum_{m_\alpha}\sum_{m_\beta} (abJ|m_\alpha m_\beta M)(abJ'|m_\alpha m_\beta M') = \delta_{JJ'}\delta_{MM'} \tag{A8}$$

$$\sum_J (abJ|m_\alpha m_\beta M)(abJ|m_\alpha' m_\beta' M') = \delta_{m_\alpha m_\alpha'}\delta_{MM'} \tag{A9}$$

In the following commutation relations the collective operator B_{JM} is given by:

$$B_{JM} = \frac{1}{2}\sum_{ab\xi} \frac{1}{\alpha^\xi_{ab}} \{\psi^\xi_{(ab)} A^\xi_{(abJM)} - (-)^{J-M}\phi_{(ab)} A^{\xi\dagger}_{(abJM)}\} \tag{A10}$$

$$[B_{JM}, A^{\xi\dagger}_{(cdKq)}] = \delta_{JK}\delta_{Mq}\psi^\xi_{(cd)} \tag{A11}$$

$$[B_{JM}, A^{\xi}_{(cdKq)}] = \delta_{JK}\delta_{Mq}(-)^{J-M}\,\phi^\xi_{(cd)} \tag{A12}$$

$$[B^\dagger_{JM}, A^{\xi\dagger}_{(cdKq)}] = -\delta_{JK}\delta_{Mq}(-)^{J-M}\,\phi^\xi_{(cd)} \tag{A13}$$

$$[B^{\dagger}_{JM}, A^{\xi}_{(cdKq)}] = -\delta_{JK}\delta_{Mq}\,\psi^{\xi}_{(cd)} \tag{A14}$$

$$[B^{\dagger}_{JM},\ a^{\xi\dagger}_{\beta}] = \sum_{\alpha}(-)^{J-M}\,\frac{\phi^{\xi}_{(ab)}}{\alpha^{\xi}_{ab}}(abJ|m_{\alpha}m_{\beta}M)\,a^{\xi}_{\alpha} \tag{A15}$$

$$[B^{\dagger}_{JM},\ a^{\xi}_{\beta}\ ] = \sum_{\alpha}\frac{\psi^{\xi}_{(ab)}}{\alpha^{\xi}_{ab}}(abJ|m_{\alpha}m_{\beta}M)\ a^{\dagger}_{\alpha} \tag{A16}$$

$$[A^{\dagger}_{(abJM)},\ a_{\gamma}] = \sum_{m_{\alpha}}\delta_{cb}(acJ|m_{\alpha}m_{\gamma}M)\ a^{\dagger}_{\alpha} - \sum_{m_{\beta}}\delta_{ca}(bcJ|m_{\beta}m_{\gamma}M)\,a^{\dagger}_{\beta} \tag{A17}$$

Appendix B

The Exact Evaluation of $[A^{\dagger}_{(abJM)}, A_{(cdKq)}]$

Starting from the definition of the operators $A_{(abJM)}$ and $A^{\dagger}_{(abJM)}$, we can write the commutation relation for these operators as follow

$$[A^{\dagger}_{(abJM)}, A_{(cdKq)}] = \sum_{m_{\alpha}}\sum_{m_{\beta}}\sum_{m_{\gamma}}\sum_{m_{\delta}}(abJ|\ m_{\alpha}m_{\beta}M)(cdK|m_{\gamma}m_{\delta}q)$$

$$\times[a_{\alpha}a_{\beta}, a_{\delta}a_{\gamma}],$$

by using the fermion commutation relations;

$$\{a^{\dagger}_{\alpha}, a_{\beta}\} = \delta_{\alpha\beta}\ ,\quad \{a^{\dagger}_{\alpha}, a^{\dagger}_{\beta}\} = \{a_{\alpha}, a_{\beta}\} = 0$$

we arrive to

$$[A^{\dagger}_{(abJM)}, A_{(cdKq)}] = \sum_{m_{\alpha}}\sum_{m_{\beta}}\sum_{m_{\gamma}}\sum_{m_{\delta}}(abJ|m_{\alpha}m_{\beta}M)(cdK|m_{\gamma}m_{\delta}q)\{-(\delta_{\alpha\gamma}\delta_{\beta\delta}$$

$$-\delta_{\alpha\delta}\delta_{\beta\gamma}) + \delta_{\alpha\gamma}a^{\dagger}_{\beta}a_{\delta} - \delta_{\alpha\delta}a^{\dagger}_{\beta}a_{\gamma} - \delta_{\beta\gamma}a^{\dagger}_{\alpha}a_{\delta} + \delta_{\beta\delta}a^{\dagger}_{\alpha}a_{\gamma}\} \quad (B1)$$

where

$$\alpha \equiv (a, m_{\alpha}) .$$

Let us call the first two terms in (B1) by T;

$$T = \sum_{m_{\alpha}} \sum_{m_{\beta}} -\{(abJ|m_{\alpha}m_{\beta}M)(abK|m_{\alpha}m_{\beta}q)\delta_{ac}\delta_{bd} - (abJ|m_{\alpha}m_{\beta}M)$$

$$\times (baK|m_{\beta}m_{\alpha}q)\delta_{ad}\delta_{bc}\},$$

and by using relations (A3,A8) we get

$$T = -\delta_{Mq}\delta_{JK}(\delta_{ac}\delta_{bd} - (-)^{a+b-J}\delta_{ad}\delta_{bc}) .$$

The last four terms can be writen in a compact form by using interchange operator R(ab) which is defined by

$$R(ab)\psi_{(ab)} = \psi_{(ba)} ,$$

Its effect on the Clebsch-Gordan coefficients is to multiply them by a phase factor i.e.

$$R(ab)(abJ|m_{\alpha}m_{\beta}M) = (baJ|m_{\beta}m_{\alpha}M) = (-)^{a+b-J}(abJ|m_{\alpha}m_{\beta}M) .$$

Therefore to keep the terms containing Clebsch-Gordan coefficients unchanged the operator $(-)^{a+b-J}R(ab)$ must be used. Let the last four terms be T' which can be writen as follow:

$$T' = -P(ab)P(cd)\sum_{m_{\alpha}} \sum_{m_{\beta}} \sum_{m_{\gamma}} \sum_{m_{\delta}} (abJ\ m\ m\ M)(cdK\ m\ m\ q)\delta_{\gamma\beta}a^{\dagger}_{\alpha}a_{\delta},$$

where

$$P(ab) = (1 - (-)^{a+b-J}R(ab))$$

Now by applying relations (A3,A5,A1) we get the following

$$\sum_{m_\beta}(bdK|m_\beta m_\delta q)(abJ|m_\alpha m_\beta M) = (-)^{d+m_\delta}(-)^{a+d+J+K}\sqrt{(2K+1)/(2b+1)}$$

$$\times\sum_{m_\beta}(Kdb|q-m_\delta m_\beta)(baJ|m_\beta m_\alpha M) = (-)^{d+m_\delta}(-)^{a+d+J+K}\sum_{L\mu}\sqrt{(2K+1)}\times$$

$$\sqrt{(2L+1)}\begin{Bmatrix}Kdb\\aJL\end{Bmatrix}(LKJ|\mu qM)(adL|m_\alpha -m_\delta\mu),$$

hence T' become

$$T'=-P(ab)P(cd)(-)^{a+d+J+K}\sum_{L\mu}\sqrt{(2K+1)(2L+1)}\begin{Bmatrix}Kdb\\aJL\end{Bmatrix}(LKJ|\mu qM)$$

$$\times\sum_{m_\alpha}\sum_{m_\delta}(-)^{d+m_\delta}(adL|m_\alpha -m_\delta\mu)a^\dagger_\alpha a_\delta .$$

Now set $m_\delta\to -m_\delta$, and since d and hence m_δ are half integers;

$$(-)^{d-m_\delta}=-(-)^{d+m_\delta},$$

therefore T' will have the following form:

$$T' = P(ab)P(cd)(-)^{a+d+J+K}\sum_{L\mu}\sqrt{(2K+1)(2L+1)}\begin{Bmatrix}Kdb\\aJL\end{Bmatrix}(LKJ|\mu qM)$$

$$\times A^{o}_{(adK\mu)}{}^{\delta}{}_{cb},$$

where

$$A^{o}_{(adL\mu)}=\sum_{m_\alpha}\sum_{m_\delta}(-)^{d+m_\delta}(adL|m_\alpha m_\delta\mu)a^\dagger_\alpha a_{-\delta},$$

$$-\delta\equiv(d,-m_\delta) .$$

Finally we get:

$$[A^\dagger_{(abJM)},A_{(cdKq)}] = -\delta_{Mq}\delta_{JK}(\delta_{ac}\delta_{bd} - (-)^{a+b-J}\delta_{ad}\delta_{bc})$$

$$+P(ab)P(cd)(-)^{a+d+J+K}\sum_{L\mu}\sqrt{(2K+1)(2L+1)}\begin{Bmatrix}Kdb\\aJL\end{Bmatrix}(LKJ|\mu qM)$$

$$\times A^{o}_{(adL\mu)}{}^{\delta}{}_{cb} . \qquad (B2)$$

Appendix C

The reduced matrix element q(ab)

The reduced matrix element of the nondimensional quadrupole transition operator is given by

$$q(ab) = (N'\ell'a||\rho^2 Y_2||N\ell b) = \frac{(-)^{a-b}}{\sqrt{4\pi}}\sqrt{5(2a+1)}$$

$$\times (a2b|1/2\ 0\ 1/2)\frac{1+(-)^{\ell+\ell'}}{2} R_{\alpha\beta} \qquad (C1)$$

where

$$R_{\alpha\beta} = \int_0^\infty R_\alpha(\rho) R_\beta(\rho)\rho^4 d\rho \quad , \quad \alpha\equiv(N\ell) \ , \ \beta\equiv(N'\ell') ,$$

$$\rho = a_o r = (m\omega_o/\hbar)^{1/2} r$$

$R_\alpha(\rho)$ is the radial part of the Harmonic oscilator wave function, m; is the nucleon mass , ω_o is the oscilator frequency and N is the principal quantum number of the harmonic oscilator wave function, such that the energy is $h\omega_o(N+3/2)$. The radial integrals $R_{\alpha\beta}$ are given in Table 10.

Table 10. The radial integrals $R_{\alpha\beta}$

N'	ℓ'	$R_{\alpha\beta}$
N	ℓ	$N+\frac{3}{2}$
N±2	ℓ	$-\frac{1}{2}[(N+\ell+2\pm1)(N-\ell+1\pm1)]^{\frac{1}{2}}$
N±2	$\ell\pm2$	$\frac{1}{2}[(N+\ell+1\pm2)(N+\ell+3\pm2)]^{\frac{1}{2}}$

Table 10.(Continued)

N	$\ell\pm2$	$-[(N+\ell+2\pm1)(N-\ell+1\mp1)]^{\frac{1}{2}}$
N±2	$\ell\mp2$	$\frac{1}{2}[(N-\ell\pm2)(N-\ell+2\pm2)]^{\frac{1}{2}}$

Appendix D

Evaluation of $<[A_{(abJM)}, A^{\dagger}_{(cdKq)}]>$

Using the following vacum expectation value

$$<a_i^{\dagger}a_j> = n_i\delta_{ij} \ , \qquad (D1)$$

we get

$$<[A_{(abJM)}, A^{\dagger}_{(cdKq)}]> = \sum_{m_\alpha}\sum_{m_\beta}\sum_{m_\gamma}\sum_{m_\delta}(abJ|m_\alpha m_\beta M)(cdK|m_\gamma m_\delta q)$$
$$\times\{\delta_{\alpha\gamma}\delta_{\beta\delta} - \delta_{\alpha\delta}\delta_{\beta\gamma} - <(\delta_{\alpha\gamma}a^{\dagger}_{\delta}a_{\beta} - \delta_{\alpha\delta}a^{\dagger}_{\gamma}a_{\beta} - \delta_{\beta\gamma}a^{\dagger}_{\delta}a_{\alpha} + \delta_{\beta\delta}a^{\dagger}_{\gamma}a_{\alpha})>\}$$

and by using relations(A3,A8) we get

$$<[A_{(abJM)}, A^{\dagger}_{(cdKq)}]> = \delta_{JK}\delta_{Mq}\ (\delta_{ac}\delta_{bd} - (-)^{a+b-J}\ \delta_{ad}\delta_{bc})$$
$$\times(1-n_a-n_b) \qquad (D2)$$

Appendix E

a: Evaluation of n_a and n'_a

To evaluate n_a we start with

$$<A^{\dagger}_{(ab)}A_{(ab)}> = \phi^2_{(ab)} = \sum_{m_1}\sum_{m_2}\sum_{m_3}\sum_{m_4}(abJ|m_1m_2M)(abJ|m_3m_4M)$$
$$\times<a^{\dagger}_{\alpha}a^{\dagger}_{\beta}a'_{\beta}a'_{\alpha}> \ . \qquad (E1)$$

where

$\alpha=(a,m_1)$, $\beta=(b,m_2)$, $\alpha'=(a,m_3)$, $\beta'=(b,m_4)$.

Applying the usual factorizations to the product of four operators and using Eqn.(D1),we get

$$\langle a^\dagger_\alpha a^\dagger_\beta a'_\beta a'_\alpha\rangle = \langle a^\dagger_\alpha a'_\alpha\rangle\langle a^\dagger_\beta a'_\beta\rangle - \langle a^\dagger_\alpha a'_\beta\rangle\langle a^\dagger_\beta a'_\alpha\rangle \ .$$

$$= n_a n_b \delta_{\alpha\alpha'}\delta_{\beta\beta'} - n_a n_b \delta_{\alpha\beta'}\delta_{\beta\alpha'} \qquad \text{(E2)}$$

Substituting(E2)in(E1) and using relations(A3,A8) we get

$$\langle A^\dagger_{(ab)} A_{(ab)}\rangle = n_a n_b (1-(-)^{2a}\delta_{ab}) \ , \quad (-)^{2a}=-1$$

therefore

$$n_a n_b = \phi^2_{(ab)} D_{ab} \qquad \text{(E3)}$$

where

$$D_{ab} = \frac{1}{1+\delta_{ab}} \ .$$

In RPA the amplitudes $\phi_{(ab)}$ are given by

$$\phi^2_{(ab)} = \frac{q^2_{(ab)} U^2_{ab} \alpha^2_{ab}}{2\omega(\omega+E_{ab})^2}\left[\sum_\xi P^\xi(\omega)\right]^{-1}$$

where

$$P^\xi(\omega) = \sum_{ab} \frac{q^{\xi 2}_{(ab)} U^{\xi 2}_{ab} E^\xi_{ab} \alpha^\xi_{ab}}{(E^{\xi 2}_{ab}-\omega^2)^2}$$

where

$$\alpha_{ab}=1-n_a-n_b .$$

Since α_{ab} appears in both the denominator and nominator of $\phi_{(ab)}$, the later is less sensetive to the choice of α_{ab}, therefore it is a very good approximations to set $\alpha_{ab}=1$ in evaluating n_a. If α_{ab} is included, n_a can be calculated by iterations.

Now let

$$N_o = \sum_a n_a = \sum_b n_b \ ,$$

then

$$N_o^2 = \sum_{ab} n_a n_b = \sum_{ab} \phi^2_{(ab)} D_{ab} \ ,$$

therefore

$$n_a = \frac{1}{N_o} \sum_b \phi^2_{(ab)} D_{ab} \ . \qquad \text{(E4)}$$

To evaluate n_a' we need to evaluate $\langle 2 | A^\dagger_{(abJM)} A_{(abJM)} | 2 \rangle$, where

$$|2\rangle = B_2^\dagger |0\rangle \ ,$$

$$B_2 = \frac{1}{2} \sum_{ba} (\psi_{(ab)} A^\dagger_{(ab)} - \phi_{(ab)} A_{(ab)}) \ .$$

Using the following commutation relations:

$$[B_{2\nu}, B^\dagger_{2\nu'}] = \delta_{\nu\nu'} \ ,$$

and relations (A11,A12,A13,A14), we get

$$\langle 2 | A^\dagger_{(abJM)} A_{(abJM)} | 2 \rangle = (\psi^2_{(ab)} + \phi^2_{(ab)}) \alpha_{ab} + \phi^2_{(ab)})$$

$$= n_a' n_b' / D_{ab} \ .$$

Following the same procedure in evaluating n_a we get

$$n_b' = \frac{\sum_a ((\psi^2_{(ab)} + \phi^2_{(ab)})(1 - n_a) + \phi^2_{(ab)}) D_{ab}}{N_o' + \sum_a (\psi^2_{(ab)} + \phi^2_{(ab)}) D_{ab}} \ , \qquad \text{(E5)}$$

where n_a is given by (E4), and

$$N_o'^2 = \sum_{ab} ((\psi^2_{(ab)}+\phi^2_{(ab)})\alpha_{ab}+\phi^2_{(ab)})D_{ab} \cdot$$

b: The Collective Operators

The collective operator $B^\dagger_{JM}$ can be expanded in terms of $A^\dagger_{(abJM)}$ and $A_{(abJM)}$ with real coeffecients

$$B^\dagger_{JM} = \sum_{ab}[x_{abJ}A^\dagger_{(abJM)} - (-)^{J-M}y_{abJ}A_{(abJM)}] \cdot$$

From the definetions of $\psi_{(ab)}$ and $\phi_{(ab)}$ we get

$$\psi_{(ab)} = \langle 0|A_{(abJM)}|2\rangle = \langle 0|[A_{(abJM)}, B^\dagger_{JM}]|0\rangle$$

$$= 2x_{abJ}\alpha_{ab} ,$$

similarly

$$\phi_{(ab)} = 2y_{abJ}\alpha_{ab} \cdot$$

Therefore the expression for $B^\dagger_{JM}$ in RPA takes the following form

$$B^\dagger_{JM} = \frac{1}{2}\sum_{ab}\frac{1}{\alpha_{ab}}[\psi_{(ab)}A^\dagger_{(abJM)} - \phi_{(ab)}A_{(abJM)}] \cdot$$

c: Evaluation of N_a/Ω_a

Using similar procedure that used in evaluating n_a i.e.

$$\langle A^\dagger_a A_a\rangle = \frac{1}{\Omega_a}\sum_{m_\alpha>0}\sum_{m_{\alpha'}} S_\alpha S_{\alpha'}\langle a^\dagger_\alpha a^\dagger_{-\alpha} a_{-\alpha'} a_{\alpha'}\rangle$$

where

$$\alpha=(a,m_\alpha) \quad , \quad \alpha'=(a,m_{\alpha'}) \quad , \quad \Omega_a = a+1/2$$

Now applying the factorizations (E2) and (D1) we get

$$\langle A_a^\dagger A_a \rangle = \frac{1}{\Omega_a} \sum_{m_\alpha>0} 2N_a^2 \qquad , \quad N_a=\langle a_\alpha^\dagger a_\alpha \rangle \ .$$

Using the following

$$\sum_{m_\alpha>0} 1 = \Omega_a$$

we get

$$N_a/\Omega_a = \sqrt{\langle A_a^\dagger A_a \rangle / 2\Omega_a^2} \qquad \text{(E6)}$$

d: The Factorization

In the Hartree-Fock method; it is assumed that the ground state may be represented by an independent particle state vector such that the average in that state of products of single particle operators takes the form

$$\langle c_\alpha^\dagger c_\beta^\dagger c_\beta, c_\alpha, \rangle = \langle c_\alpha^\dagger c_\alpha \rangle \langle c_\beta^\dagger c_\beta, \rangle - \langle c_\alpha^\dagger c_\beta, \rangle \langle c_\beta^\dagger c_\alpha, \rangle + \langle c_\alpha^\dagger c_\beta^\dagger \rangle \langle c_\beta, c_\alpha, \rangle . \qquad \text{(E7)}$$

This expression differs from (E2) in the presence of the last term, which is zero unless the particles participate in pairing interaction.

In the quasi-particles scheme the interacting particles transformed to independent quasi-particles by means of B-V transformations, where the pairing interaction between the original particles is now absorbed by the quasi-particle energy; E_a . Therefore the product of four quasi-particles

operators takes the following form

$$\langle a^{\dagger}_{\alpha} a^{\dagger}_{\beta} a_{\beta'} a_{\alpha'} \rangle = \langle a^{\dagger}_{\alpha} a_{\alpha'} \rangle \langle a^{\dagger}_{\beta} a_{\beta'} \rangle - \langle a^{\dagger}_{\alpha} a_{\beta'} \rangle \langle a^{\dagger}_{\beta} a_{\alpha'} \rangle , \quad \text{(E8)}$$

where the third term which describes the pairing sets equal to zero.

Appendix F

The S_j for (d,p) stripping on odd mass target

The quantity S_j is defined by

$$S_j = |\langle JM | (jJ_o), JM \rangle|^2 ,$$

where

$$|JM\rangle = B^{\dagger}_{JM} |0\rangle ,$$

$$|(jJ_o), JM\rangle = \sum_{M_o m} \Sigma(jJ_o J | mM_o M) C^{\dagger}_{jm} |J_o M_o\rangle ,$$

where $J_o M_o$ is the wave function of the target wgich can be expanded in terms of seniority one and three states;

$$|J_o M_o\rangle = C_{Jo} a^{\dagger}_{J_o M_o} |0\rangle + \sum_{j'm'\nu} \Sigma \, C^{Jo}_{j'} (j' 2 J_o | m' \nu M_o) a^{\dagger}_{j',m'} B^{\dagger}_{2\nu} |0\rangle$$

where $|0\rangle$ is the quasi-particle vacume~. The coefficients C^{Jo}_{j} can be obtained by diagonalizing the Hamiltonian in the space of one quasi-particle with zero, one, and two phonons , these coefficients are tabulated by KS. The single particle creation operator $C^{\dagger}_{jm}$ can be expressed

in terms of quasi-particle operators $a^{\dagger}_{jm}$ and a_{jm} by means of B-V transformations;

$$C^{\dagger}_{jm} = u_j a^{\dagger}_{jm} + S_j v_j a_{j-m}$$

Now for J=2 we get

$$\langle 2|(jJ_o),2\rangle = C_{Jo}u_j\langle 2|A^{\dagger}_{(jJ_o)}|0\rangle - (-)^{j+Jo}v_j C_{Jo}\langle 2|A^{o}_{(Joj)}|0\rangle$$

$$+ (-)^{j+Jo}v_j C^{Jo}_j\sqrt{(2J_o+1)/5} + v_j\sqrt{5(2J_o+1)}$$

$$\times \sum_{j'a}\Sigma(-)^{j'+J_o}C^{Jo}_{j'}\,\psi''_{(ja)}\psi''_{(j'a)}\begin{Bmatrix} j & a & 2\\ j' & J_o & 2\end{Bmatrix} \qquad (F1)$$

the last result is obtained with the help of relations (A1,A10-A13). The first two terms in the last Eqn. reduce to $C_{Jo}u_j\psi''_{(jJ_o)}$ after using the following results

$$\langle 2|A_{(jJ_o)}|0\rangle = \langle 0|[B_2,A^{\dagger}_{(jJ_o)}]|0\rangle = \psi_{(jJ_o)}$$

$$\langle 2|A^{o}_{(J_oj)}|0\rangle = 0$$

where

$$\psi''_{(ab)} = (-)^{a-b}\psi'_{(ab)}$$

$$\psi'_{(ab)} = -(-)^{a-b}\,\mathrm{sgn}(a2b|1/2\ 0\ 1/2)\,\psi_{(ab)}$$

where the phase of $\psi''_{(ab)}$ is that of C^{Jo}_j. Neglecting the last term in (F1) as it is small compared to the other terms, we get

$$S_j = |u_j C_{J_o}\psi'_{(jJ_o)} - v_j C^{Jo}_j\sqrt{(2J_o+1)/5}|^2 \qquad (F2)$$

Appendix G

a: Calculation of G^{o}_{ab}

The Green's function G^{o}_{ab} provide the solution to the pairing part of the Hamiltonian with small residual interaction and hence describes the behaviour of the 0^{+}, seniority zero states in even-even nucleus. If the exact commutation relations of A_a and $A^{\dagger}_a$ are used, the following commutation relations are obtained:

$$[A_c, H'_p] = 2E_c A_c - g\{\Sigma_b \sqrt{\Omega_c \Omega_b}\, u_b^2 u_c^2 (1-\frac{N_c}{\Omega_c}) A_b - \Sigma_b \sqrt{\Omega_c \Omega_b}\, u_c^2 v_b^2 (1-\frac{N_c}{\Omega_c}) A^{\dagger}_b$$
$$- \Sigma_a \sqrt{\Omega_c \Omega_a}\, u_a^2 v_c^2 A^{\dagger}_a (1-\frac{N_c}{\Omega_c}) + \Sigma_a \sqrt{\Omega_a \Omega_c}\, v_a^2 v_c^2 A_a (1-\frac{N_c}{\Omega_c})\}.$$

Let $(1-\frac{N_c}{\Omega_c}) = L_c$,

collecting similar terms we get:

$$[A_c, H'_p] = 2E_c A_c - g\sqrt{\Omega_c}\, u_c^2 L_c \; \Sigma_b \sqrt{\Omega_b}\,(u_b^2 A_b - v_b^2 A^{\dagger}_b)$$
$$- g\sqrt{\Omega_c}\, v_c^2 \; \Sigma_a \sqrt{\Omega_a}\,(v_a^2 A_a - u_a^2 A^{\dagger}_a) L_c \quad . \qquad (G1)$$

Similarly

$$[A^{\dagger}_c, H'_p] = -2E_c A_c - g\sqrt{\Omega_c}\, v_c^2 L_c \; \Sigma_b \sqrt{\Omega_b}\,(u_b^2 A_b - v_b^2 A^{\dagger}_b)$$
$$- g\sqrt{\Omega_c}\, u_c^2 \; \Sigma_a \sqrt{\Omega_a}\,(v_a^2 A_a - u_a^2 A^{\dagger}_a) L_c \qquad (G2)$$

Substituting (G2) in the equations of motion for $G^{o}(\omega)$ we get equatins (III.4), where L_c set equal to one.

Table 11.

$G_p = 24/A$ | $G_n = 24/A$

Proton levels $29 < Z < 39, A_o = 90$ | Neutron levels $31 < N < 49, A_o = 58$

$f_{7/2}$	$f_{5/2}$	$p_{3/2}$	$p_{1/2}$	$g_{9/2}$	$f_{7/2}$	$p_{3/2}$	$f_{5/2}$	$p_{1/2}$	$g_{9/2}$
-4.6	-.6	0	1.8	2.8	-4.0	0	0	3.0	4.0

$\Delta\varepsilon_{7/2} = \Delta\varepsilon_{5/2} = .11(N-40)$

Isotope	A	λ_p	Δ_p	λ_n	Δ_n
28Ni30	58			-0.980	1.190
28Ni32	60			-0.420	1.475
28Ni34	62			0.142	1.537
28Ni36	64			0.752	1.420
30Zn34	64	-1.697	1.209	0.212	1.718
30Zn36	66	-1.526	1.187	0.835	1.543
30Zn38	68	-1.361	1.167	1.712	1.236
32Ge38	70	-0.797	1.388	1.688	1.134
32Ge40	72	-0.663	1.370	2.650	1.248
32Ge42	74	-0.532	1.351	3.248	1.309
32Ge44	76	-0.405	1.329	3.714	1.264
32Ge46	78	-0.282	1.305	4.108	1.116
34Se40	74	-0.149	1.406	2.644	1.182
34Se42	76	-0.039	1.391	3.233	1.255
34Se44	78	0.073	1.375	3.689	1.219
34Se46	80	0.185	1.355	4.074	1.074
34Se48	82	0.299	1.333	4.414	0.812
36Kr42	78	0.480	1.306	3.219	1.204
36Kr44	80	0.572	1.301	3.666	1.177
36Kr46	82	0.670	1.295	4.042	1.047
36Kr48	84	0.772	1.289	4.374	0.788
38Sr48	86	1.317	1.194	4.336	0.766
38Sr50	88	1.386	1.211		

Table 12.

$G_p = 26/A$ | $G_n = 23/A$

Proton levels $37<Z<49, A_o=90$ | Neutron levels $51<N<75, A_o=120$

$f_{5/2}$	$p_{3/2}$	$p_{1/2}$	$g_{9/2}$	$d_{5/2}$	$g_{7/2}$	$s_{1/2}$	$h_{11/2}$	$d_{3/2}$
0	.6	1.8	3.4	0	.8	1.3	2.5	2.8

$\Delta\varepsilon_{9/2} = -.055(N-50)$ | $\Delta\varepsilon_{7/2} = .14(50-Z)$

Isotope	A	λ_p	Δ_p	λ_n	Δ_n
40Zr50	90	2.241	0.827		
40Zr52	92	2.169	0.843	-0.481	0.714
40Zr54	94	2.096	0.860	-0.044	0.907
42Mo52	94	2.745	0.951	-0.476	0.709
42Mo54	96	2.638	0.948	-0.054	0.909
42M056	98	2.533	0.947	0.426	1.049
42Mo58	100	2.429	0.947	0.862	1.220
44Ru52	96	3.138	0.964	-0.477	0.707
44Ru54	98	3.021	0.954	-0.073	0.915
44Ru56	100	2.906	0.945	0.368	1.062
44Ru58	102	2.791	0.937	0.772	1.215
46Pd58	104	3.098	0.842	0.668	1.208
46Pd60	106	2.978	0.833	0.993	1.306
46Pd62	108	2.859	0.825	1.288	1.364
46Pd64	110	2.741	0.819	1.566	1.389
48Cd58	106	3.373	0.640	0.552	1.197
48Cd60	108	3.247	0.632	0.866	1.272
48Cd62	110	3.123	0.625	1.163	1.312
48Cd64	112	3.000	0.619	1.451	1.324
48Cd66	114	2.878	0.613	1.733	1.316
48Cd68	116	2.757	0.607	2.006	1.293
50Sn62	112			1.030	1.252
50Sn64	114			1.333	1.247
50Sn66	116			1.640	1.231
50Sn68	118			1.938	1.214
50Sn70	120			2.210	1.190
50Sn72	122			2.457	1.146
50Sn74	124			2.683	1.075

Table 13.

G_p=23/A — Proton levels 51<Z<82, A_o=207

$g_{7/2}$	$d_{5/2}$	$h_{11/2}$	$d_{3/2}$	$s_{1/2}$
0	.8	2.1	2.6	2.95

G_n=23/A — Neutron levels 68<N<81, A_o=120

$d_{5/2}$	$g_{7/2}$	$s_{1/2}$	$h_{11/2}$	$d_{3/2}$
0	.8	1.3	2.5	2.8

Isotope	A	λ_p	Δ_p	λ_n	Δ_n
52Te68	120	-0.287	0.615	1.930	1.185
52Te70	122	-0.280	0.602	2.201	1.164
52Te72	124	-0.273	0.589	2.445	1.123
52Te74	126	-0.266	0.577	2.667	1.055
52Te76	128	-0.260	0.565	2.873	0.938
52Te78	130	-0.254	0.554	3.066	0.808
54Xe74	128	-0.026	0.754	2.652	1.036
54Xe76	130	-0.023	0.738	2.855	0.937
54Xe78	132	-0.020	0.723	3.045	0.795
54Xe80	134	-0.018	0.707	3.224	0.581
54Xe82	136	-0.015	0.693		
56Ba76	132	0.221	0.829	2.837	0.921
56Ba78	134	0.222	0.811	3.024	0.782
56Ba80	136	0.222	0.793	3.201	0.572
56Ba82	138	0.222	0.775		
58Ce80	138	0.473	0.831	3.179	0.563

Table 14.

G_p=23/A — Proton levels 51<Z<82, A_o=207

$g_{7/2}$	$d_{5/2}$	$h_{11/2}$	$d_{3/2}$	$s_{1/2}$
0	.8	2.1	2.6	2.95

G_n=22/A — Neutron levels 83<N<125, A_o=207

$h_{9/2}$	$f_{7/2}$	$i_{13/2}$	$p_{3/2}$	$f_{5/2}$	$p_{1/2}$
-.9	0	.72	1.45	1.78	2.35

Isotope	A	λ_p	Δ_p	λ_n	Δ_n
58Ce82	140	0.472	0.812		
58Ce84	142	0.471	0.793	-1.512	0.614
60Nd82	142	0.737	0.815		
60Nd84	144	0.735	0.796	-1.498	0.603
60Nd86	146	0.733	0.777	-1.293	0.812
62Sm84	146	1.025	0.770	-1.483	0.592

Table 14.(Continued)

Isotope	A	λ_p	Δ_p	λ_n	Δ_n
62Sm86	148	1.021	0.750	-1.281	0.797
62Sm88	150	1.018	0.731	-1.078	0.928
64Gd84	148	1.373	0.744	-1.470	0.582
64Gd86	150	1.371	0.722	-1.269	0.783
76Osl12	188	2.640	0.539	1.201	0.898
76Osl14	190	2.629	0.533	1.380	0.835
78Ptl16	194	2.798	0.435	1.555	0.755
78Ptl18	196	2.786	0.430	1.735	0.680
78Ptl20	198	2.773	0.425	1.910	0.592
80Hgl16	196	2.987	0.305	1.548	0.746
80Hgl18	198	2.973	0.300	1.726	0.672
80Hgl20	200	2.960	0.296	1.900	0.585
80Hgl22	202	2.947	0.291	2.072	0.468
80Hgl24	204	2.934	0.287	2.280	0.294
82Pbl16	198			1.541	0.737
82Pbl18	200			1.718	0.664
82Pbl20	202			1.890	0.578
82Pbl22	204			2.060	0.462
82Pbl24	206			2.268	0.289

Table 15.

$G_p = 24/A$ | $G_n = 23/A$

Proton levels $20<Z<50, A_o=90$

$f_{7/2}$	$f_{5/2}$	$p_{3/2}$	$p_{1/2}$	$g_{9/2}$
-3.5	.5	0	1.8	2.8

$\Delta\varepsilon_{7/2}=\Delta\varepsilon_{5/2}=-.11(N-40), 20<Z<38$

$\Delta\varepsilon_{9/2}=-.055(Z-40), 38<Z<50$

Neutron levels $20<N<50, A_o=58$

$f_{7/2}$	$p_{3/2}$	$f_{5/2}$	$p_{1/2}$	$g_{9/2}$
-4.0	0	0	3.0	4.0

$\Delta\varepsilon_{9/2}=-.055(Z-40)$

Isotope	A	λ_p	Δ_p	λ_n	Δ_n
28Ni30	58			-1.015	1.500
28Ni32	60			-0.345	1.645
28Ni34	62			0.246	1.597
28Ni36	64			0.843	1.371
30Zn34	64	-0.552	1.466	0.227	1.538
30Zn36	66	-0.574	1.382	0.806	1.319
30Zn38	68	-0.604	1.309	1.795	0.723
32Ge38	70	0.021	1.550	1.749	0.670
32Ge40	72	-0.063	1.485	2.975	0.953
32Ge42	74	-0.156	1.421	3.647	1.112
32Ge44	76	-0.257	1.357	4.109	1.126
32Ge46	78	-0.366	1.290	4.489	1.016
34Se40	74	0.458	1.567	2.897	0.938
34Se42	76	0.341	1.490	3.537	1.088
34Se44	78	0.219	1.412	3.985	1.101
34Se46	80	0.093	1.333	4.355	0.993
34Se48	82	-0.034	1.251	4.681	0.753
36Kr42	78	0.836	1.460	3.428	1.068
36Kr44	80	0.707	1.359	3.862	1.077
36Kr46	82	0.576	1.257	4.223	0.971
36Kr48	84	0.445	1.156	4.541	0.737
38Sr48	86	1.405	1.267	4.402	0.722
38Sr50	88	1.389	1.214		
40Zr50	90	1.912	1.172		

Table 16.

$G_p=24/A$					$G_n=23/A$				
Proton levels $37<Z<49, A_o=90$					Neutron levels $50<N<78, A_o=120$				
$f_{7/2}$	$f_{5/2}$	$p_{3/2}$	$p_{1/2}$	$g_{9/2}$	$d_{5/2}$	$g_{7/2}$	$s_{1/2}$	$h_{11/2}$	$d_{3/2}$
-3.5	.5	0	1.8	2.8	0	.8	2.0	2.5	2.8
$\Delta\varepsilon_{7/2}=\Delta\varepsilon_{5/2}=-.11(N-40), 20<Z<38$					$\Delta\varepsilon_{7/2}=.14(48-Z), Z<48$				
$\Delta\varepsilon_{9/2}=-.055(Z-40), 38<Z<50$					$\Delta\varepsilon_{7/2}=-.1, Z=48$				
					$\Delta\varepsilon_{7/2}=\Delta\varepsilon_{11/2}=.15(50-Z), Z>50$				
					$\Delta\varepsilon_{1/2}=-.2, Z=50$				
					$\Delta\varepsilon_{3/2}=.05(50-Z), Z>50$				

Isotope	A	λ_p	Δ_p	λ_n	Δ_n
40Zr52	92	1.902	1.124	-0.480	0.716
40Zr54	94	1.893	1.078	-0.041	0.910
42Mo52	94	2.270	1.137	-0.481	0.713
42Mo54	96	2.258	1.100	-0.060	0.917
42Mo56	98	2.248	1.065	0.414	1.063
42Mo58	100	2.238	1.031	0.839	1.235
44Ru52	96	2.543	1.087	-0.489	0.714
44Ru54	98	2.526	1.056	-0.091	0.929
44Ru56	100	2.510	1.027	0.334	1.078
44Ru58	102	2.494	0.999	0.726	1.221
46Pd58	104	2.698	0.887	0.600	1.205
46Pd60	106	2.677	0.865	0.929	1.287
46Pd62	108	2.658	0.845	1.237	1.334
46Pd64	110	2.639	0.826	1.532	1.355
48Cd58	106	2.869	0.668	0.416	1.187
48Cd60	108	2.844	0.653	0.734	1.231
48Cd62	110	2.820	0.639	1.056	1.245
48Cd64	112	2.797	0.625	1.386	1.246
48Cd66	114	2.775	0.612	1.708	1.249
48Cd68	116	2.753	0.599	2.002	1.247
50Sn62	112			1.083	1.237
50Sn64	114			1.395	1.241
50Sn66	116			1.702	1.240
50Sn68	118			1.987	1.233
50Sn70	120			2.247	1.209
50Sn72	122			2.485	1.162
50Sn74	124			2.705	1.088

Table 17.

G_p=23/A					G_n=23/A				
Proton levels 50<Z<76, A_o=207					Neutron levels 50<N<78, A_o=120				
$g_{7/2}$	$d_{5/2}$	$h_{11/2}$	$d_{3/2}$	$s_{1/2}$	Same as in Table				
0	.8	2.1	2.6	2.95					

Isotope	A	λ_p	Δ_p	λ_n	Δ_n
52Te68	120	-0.287	0.615	1.775	1.217
52Te70	122	-0.280	0.602	2.028	1.195
52Te72	124	-0.273	0.589	2.260	1.148
52Te74	126	-0.266	0.577	2.475	1.073
52Te76	128	-0.260	0.565	2.677	0.965
54Xe74	128	-0.026	0.754	2.237	1.047
54Xe76	130	-0.023	0.738	2.442	0.938
56Ba76	132	0.221	0.829	2.216	0.903

Table 18 .

G_p=23/A					G_n=23/A				
Proton levels 50<Z<76, A_o=207					Neutron levels 78<N<82, A_o=139				
Same as in Table					$g_{7/2}$	$d_{5/2}$	$h_{11/2}$	$s_{1/2}$	$d_{3/2}$
					-1.2	0	.8	1.33	1.6

Isotope	A	λ_p	Δ_p	λ_n	Δ_n
52Te78	130	-0.254	0.554	1.762	0.785
54Xe78	132	-0.020	0.723	1.747	0.772
54Xe80	134	-0.018	0.707	1.961	0.557
54Xe82	136	-0.015	0.693		
56Ba78	134	0.222	0.811	1.732	0.760
56Ba80	136	0.222	0.793	1.943	0.548
56Ba82	138	0.222	0.775		
58Ce80	138	0.473	0.831	1.926	0.540
58Ce82	140	0.472	0.812		
60Nd82	142	0.737	0.815		

Table 19.

$G_p=23/A$	$G_n=23/A$
Proton levels $50<Z<76, A_o=207$	Neutron levels $82<N<90, A_o=141$
Same as in Table	$f_{7/2}$ $p_{3/2}$ $h_{9/2}$ $f_{5/2}$ $p_{1/2}$ $i_{13/2}$
	0 .83 1.55 1.88 2.25 2.8

Isotope	A	λ_p	Δ_p	λ_n	Δ_n
58Ce84	142	0.471	0.793	-0.392	0.534
60Nd84	144	0.735	0.796	-0.381	0.525
60Nd86	146	0.733	0.777	-0.128	0.688
62Sm84	146	1.025	0.770	-0.370	0.516
62Sm86	148	1.021	0.750	-0.120	0.676
62Sm88	150	1.018	0.731	0.155	0.767
64Gd84	148	1.373	0.744	-0.359	0.508
64Gd86	150	1.371	0.722	-0.112	0.665

Table 20.

$G_p=25/A$					$G_n=23/A$					
Proton levels $76<Z<82, A_o=207$					Neutron levels $112<N<126, A_o=207$					
$g_{7/2}$	$d_{5/2}$	$h_{11/2}$	$d_{3/2}$	$s_{1/2}$	$h_{9/2}$	$f_{7/2}$	$i_{13/2}$	$p_{3/2}$	$f_{5/2}$	$p_{1/2}$
0	1.81	2.14	3.13	3.48	-1.09	0	.7	1.45	1.77	2.34
$\Delta\varepsilon_{11/2}=.5$, $\Delta\varepsilon_{7/2}=-.1$					$\Delta\varepsilon_{13/2}=.2$ (except for Z=82)					

Isotope	A	λ_p	Δ_p	λ_n	Δ_n
76Os112	188	3.194	0.636	1.303	0.972
76Os114	190	3.182	0.629	1.473	0.916
78Pt116	194	3.342	0.512	1.634	0.838
78Pt118	196	3.328	0.506	1.800	0.758
78Pt120	198	3.315	0.501	1.966	0.659
80Hg116	196	3.533	0.352	1.627	0.828
80Hg118	198	3.518	0.347	1.791	0.749
80Hg120	200	3.504	0.342	1.955	0.652
80Hg122	202	3.489	0.338	2.122	0.524
80Hg124	204	3.475	0.333	2.316	0.345
82Pb116	198			1.543	0.783
82Pb118	200			1.723	0.707
82Pb120	202			1.899	0.616
82Pb122	204			2.073	0.495
82Pb124	206			2.279	0.319

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