ANALYSIS OF TESTS FOR TWO FORMS OF SPECIFICATION ERROR IN LINEAR REGRESSION ANALYSIS

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ABSTRACT

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By

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In this study two new specification error tests based on a Power Series Expansion Model (POSEX) are developed. The first test is designed to detect a misspecified conditional mean of the dependent variable and the second to detect heteroskedastic disturbance terms.

Two versions of the test for a misspecified conditional mean are presented. One of these versions is shown to yield the same results as the procedures currently in use yet offers the advantage of being easier to implement. The two versions of the test are then compared on six misspecified models using a sample experiment. It was found that both tests have an extremely high probability of correctly rejecting the null hypothesis if the misspecified conditional mean is caused by using the wrong functional form of either the regressand or regressors. In contrast, when the specification error is caused by omitting a variable, the power of the test is a function of the relation between the omitted variable and those included in the model.

Four versions of the test for heteroskedastic disturbance terms are presented. These four tests are then compared with various versions of Goldfeld & Quant's parametric and non-parametric test, Glejser's test, Park's test, and Ramsey's test (BAMSET) by using a sample experiment on ten heteroskedastic models. It was discovered that when no information about the form of the heteroskedasticity is available, the most powerful test is BAMSET with the observations reordered by ranking the dependent variable. However, since this is a non-constructive test, if heteroskedasticity is found, no corrective procedure is suggested. Of the constructive tests, two versions of the test formulated in this study were found to be the most powerful.

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By

Ronald L. Tracy

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INTRODUCTION AND REVIEW OF THE LITERATURE

I.1 Introduction

Linear regression analysis is one of many statistical procedures that can be used to indicate a relationship among different variables. This method requires specification of the variable whose conditional mean is to be estimated (the dependent variable), of the variables that affect the mean of the dependent variable (the independent variables), and of the distribution of the unexplained variation (the disturbance term).

One such regression model is

$$\underline{y} = X\underline{\beta} + \underline{u} \tag{1.1}$$

where $\underline{y}$ is the n x 1 vector of observed dependent variables, X is the n x k matrix of nonstochastic independent variables of rank k, $\underline{\beta}$ is the k x 1 vector of parameters to be estimated, and $\underline{u}$ is the n x 1 vector of disturbance terms.

If the method of least squares is employed to estimate the regression model in (1.1), the estimator for the parameter $\underline{\beta}$, $\hat{\underline{\beta}} = (X'X)^{-} X'\underline{y}$, and the model's variance σ^2, $\hat{\sigma}^2 = (\underline{y} - X\hat{\underline{\beta}})'(\underline{y} - X\hat{\underline{\beta}})/(n-k)$ can be obtained. If, however, the statistical properties of these estimators are to be ascertained and tests of significance carried out, the distribution of the disturbance terms must also be known. If, for example, the vector of disturbance terms has a normal distribution with a mean of zero and a covariance matrix of $\sigma^2 I$ (hereafter denoted $N(\emptyset, \sigma^2 I)$), the resulting estimators are unbiased, efficient, and consistent.

Difficulties arise when the disturbance term has a different distribution than that which has been hypothesized. When an incorrect assumption is made about the distribution of the disturbance term, a specification error has been committed. It must be emphasized that a specification error arises only because the exact distribution of the disturbance term is incorrectly assumed, not because it is distributed differently than required by the classical assumption (that $\underline{u} \sim N(\emptyset, \sigma^2 I)$).

Typically there are two major types of specification error. The first type concerns the distributional form of the disturbances and the second deals with the parameters of that distribution. In the first case, a specification error of incorrect distributional form is made when the vector of disturbance terms $\underline{u}$ is actually distributed differently than has been hypothesized. An example of this is if the disturbance terms are assumed to be distributed normally whereas they are actually distributed as log normal.

The second type of specification error is committed if an incorrect assumption is made about the parameters that define the exact distribution of the disturbance terms. In the context of the classical assumptions that $\underline{u} \sim N(\emptyset, \sigma^2 I)$, where only two parameters are needed to define the distribution completely, this second type of specification error can be divided into three types.

The first error arises when an incorrect assumption is made about the population mean. Most commonly, this type of error occurs when the expected value of the vector $\underline{u}$ is assumed to be zero instead of some non-zero vector $\underline{z}$.

The second error occurs when one makes an incorrect assumption about the population variance. The most common form of this error arises when it is incorrectly assumed that the variance of each disturbance term is identical (homoskedastic) whereas the true variances would compose a non-constant vector $\underline{v}$ (heteroskedastic).

The third and last error incurred involves the correlation between the disturbance terms $u_1,\ldots,u_n$. In its most common form, this error occurs when it is assumed that the disturbance terms are independent of one another whereas elements of the disturbance vector that are adjacent are actually correlated (first order autocorrelation).

The purpose of this study is to examine, compare and prepare statistical tests designed to help the researcher determine if a given regression model is misspecified because the vector of disturbance terms has an incorrectly specified mean or variance vector. The remaining two forms of specification errors involving the disturbance terms, incorrect distributional form, and autocorrelation have been studied in great detail by other authors. The reader is referred to Shapiro, Wilk, & Chen [1968] and to Huang & Bolch for more information on distributional form errors and to Kramer [1969], Berenblut & Webb [1973], and Abrahamse & Louter [1971] for more information on autoregressive errors.

Notation

When tests are examined to determine if a model has been misspecified, the null hypothesis (hereafter H_0) is that no specification error exists. This null hypothesis will be tested against two alternative hypotheses. The first alternative (hereafter H_1) is that the disturbance terms have an incorrectly specified mean vector; the second alternative (hereafter H_2) is that the disturbance terms have an incorrectly specified variance vector.

To simplify the complexity of the statistical discussion, certain notational conventions are used throughout this study. First, matrices are always denoted by either upper case Greek or Latin letters. Second, any Greek or Latin letter that is underscored denotes a column vector, (e.g., $\underline{y}$ or $\underline{\beta}$). Third, any lower case Greek or Latin letter not underscored represents a scalar. Fourth, parameters are denoted by Greek letters, whereas random variables are represented by Latin letters.

An estimator of a parameter is signified by that parameter topped by a symbol (for example, $\hat{\beta}$, $\tilde{\beta}$, $\overset{*}{\beta}$, $\dot{\beta}$, $\overset{0}{\beta}$ are all estimators for β). In a like manner, the predictor of a random variable is denoted by a symbol over that random variable. When the inverse of a matrix is required, the symbol $^{-}$ immediately to the right of the matrix is used (for example, the inverse of the matrix A is A^{-}). The operator DIAG denotes that the diagonal elements of the specified matrix are formed into a column vector. The operator E denotes the expected value operator. A prime ' to the right of a vector or a matrix denotes the transpose of that vector or matrix. The capital letter I denotes the identity matrix while the vector $\underline{i}$ denotes a column of ones.

Some standard notation on tests will be reviewed as this notation will be used extensively throughout this study. The probability of incorrectly rejecting the null hypothesis (H_o) (type I error) is denoted as alpha (α) or is referred to as the alpha level of the test. The probability of incorrectly accepting the alternative hypothesis (type II error) is denoted as beta (β). The probability of correctly accepting the alternative hypothesis then becomes $1-\beta$ and is referred to as the power of the test.

Outline

Before the various testing procedures designed to detect an incorrectly specified mean or variance vector are compared, a detailed discussion of each specification error is given. This discussion is followed by a review of the pertinent literature on different predictors of the true disturbance terms, on various tests for detecting an incorrectly specified mean vector, and on various tests for detecting an incorrectly specified variance vector.

In the second chapter, a new test for each of the two forms of specification error under discussion is described. Following a detailed explanation of the new testing procedure, the test is applied to the case of H_o vs. H_1, with careful attention paid to developing the exact distribution theory. The new procedure is applied to testing H_o vs. H_2 with special attention focused on certain aspects of the distribution theory.

The third chapter begins with a restatement of the hypotheses posed in Chapters I and II. A sampling experiment is presented that compares the two new tests with the previously discussed tests for

H_o vs. H_1 and H_o vs. H_2. Since all of the tests presented were designed for particular situations, special attention is given to the experimental design so that all tests can be compared fairly.

In Chapter IV, the experimental results are presented. Comparisons and contrasts between the various tests as well as between the various models tested are made. The hypothesis presented in the previous chapter are examined.

Finally, in Chapter V, a summary of the entire study is given. This is followed by a discussion of the major conclusions of this study and the inferences which can be drawn from them. Some suggestions for further research are given.

I.2 Effects and Causes of a Misspecified Mean Vector

Assume that one hypothesizes the regression model

$$\begin{aligned} & y_i = \beta_1 + x_{i2}\,\beta_2 + \ldots\ldots + x_{ik}\,\beta_k + u_i\ , \ i = 1,\ldots n, \\ & \underline{u} \sim N(\emptyset,\ \sigma^2 I). \end{aligned} \qquad (1.2)$$

If these assumptions are correct, model (1.2) would be the 'true' model; that is, the model which generated each element of the vector of dependent variables y_i. The regression model would thus be correctly specified and the resultant least squares estimators, $\hat{\underline{\beta}}$ and $\hat{\sigma}^2$, would be unbiased, consistent, and efficient.

It is evident that if the hypothesized model had had a disturbance term with a constant mean vector $r = r\underline{i}$, it could be transformed into an hypothesized model with a zero mean vector by subtracting the vector $\underline{r}$ from the dependent variable $\underline{y}$ or by incorporating r into the constant coefficient. Hence, it will be assumed from this point on, and without loss of generality, that the disturbance term in the hypothesized model has a zero mean vector and that the alternate hypothesis (H_1) is that of a non-zero, non-constant mean vector. Therefore, if $\underline{u}$ is actually distributed as $N(\underline{z},\sigma^2 I)$, $\underline{z}$ being a non-constant mean vector, then the model hypothesized in (1.2) is misspecified because of an incorrectly specified mean vector. The effect of this specification error on the least squares estimators $\hat{\underline{\beta}}$ and $\hat{\sigma}^2$ can be demonstrated by examining the effect of regressing the vector $\underline{z}$ on the matrix X. The resulting regression model is

$$\underline{z} = X\underline{\gamma} + \underline{v},\ \underline{v} \sim N(\emptyset,\ \sigma^2 I). \tag{1.3}$$

Thus, the bias in the least squares estimators caused by the misspecification is seen to be

$$E(\hat{\underline{\beta}}) - \underline{\beta} = \underline{\gamma} \text{ and}$$
$$E(\hat{\sigma}^2) - \sigma^2 = E(\underline{y} - X\hat{\underline{\beta}})'\ (\underline{y} - X\hat{\underline{\beta}})/(n-k) - \sigma^2 = \underline{z}'\ \underline{z}/(n-k).$$

Hence, as a result of an incorrectly specified mean vector, $\hat{\sigma}^2$ has an upward bias and hence always causes a loss in efficiency, which in turn causes tests of significance to be unduly conservative. In addition, the extent to which any parameter β_i is biased by the misspecification is directly related to the correlation between the corresponding independent variable $\underline{x}_i$ and the vector $\underline{z}$. Further, the constant vector will always be biased unless all the variation in $\underline{z}$ can be explained by the other independent variables. If the vector $\underline{z}$ is a constant vector, that is, $\underline{z} = z\underline{i}$ where $\underline{i}$ is a column vector of ones, only the constant term is biased and by the amount z. Similarly, if $\underline{z}$ is a non-constant vector and uncorrelated with all the independent variables, $\underline{x}_1,\ldots,\underline{x}_k$, only the intercept term will be biased and by the amount

$$\bar{z} = \sum_i z_i/n.$$

Since the estimators of $\underline{\beta}$ and σ^2 are affected by the error of a non-zero mean vector, it must be determined under what circumstances such an error can occur. One such circumstance is when the original data is collected or transcribed incorrectly. Typically, it is assumed that these errors are distributed normally and have an expected value of zero. If this is not true, however, and, in fact, the data contains an upward (downward) bias, only the intercept term and the variance are affected since the bias will presumably be uncorrelated with the independent variables in the model.

Another situation in which a non-zero mean occurs is when a variable is omitted from the hypothesized model. This may occur if the hypothesized model is given by (1.2),

$$\underline{y} = X\underline{\beta} + \underline{u}, \ \underline{u} \sim N(\emptyset, \sigma^2 I),$$

whereas the true model (the model that actually generated the dependent variable $\underline{y}$) is

$$\underline{y} = X\underline{\beta} + W\underline{\delta} + \underline{e}, \ \underline{e} \sim N(\emptyset, \sigma^2 I), \tag{1.4}$$

where X, $\underline{\beta}$ and $\underline{y}$ are as previously defined, W is an n x m matrix of m additional independent variables, and $\underline{\delta}$ is a column vector (m elements long) of additional parameters. The non-zero mean of $\underline{u}$ in this case is equal to $W\underline{\delta}$. Such an error can be committed if there is no data available on the variable(s) $\underline{w}_1,\ldots,\underline{w}_m$ or if the variable(s) are erroneously excluded from the hypothesized model because the researcher was not aware of their occurence in the true model. Note that the omitted variables cannot be included in the model and have their significance tested because the researcher is either unaware of their occurence in the true model or cannot obtain the necessary data.

One final way that a specification error due to a non-zero mean vector can occur is when the incorrect functional form of the regressors or regressand is used. Given the true model

$$y_i = \gamma_1 + \gamma_2 \ln(x_{i2}) + \ldots + \gamma_k \ln(x_{ik}) + v_i,\ i = 1,\ldots,n, \qquad (1.5)$$
$$\underline{v} \sim N(\emptyset, \sigma^2 I)$$

and the hypothesized model, given in (1.2),

$$\underline{y} = X\underline{\beta} + \underline{u},\ \underline{u} \sim N(\emptyset, \sigma^2 I),$$

it is obvious that the hypothesized model has been misspecified. The independent variables have taken on the wrong functional form. As a result, the vector $\underline{u}$ will have a mean given by

$$E(u_i) = \gamma_1 + \gamma_2 \ln(x_{i2}) + \ldots\ldots + \gamma_k \ln(x_{ik}) - (\beta_1 + \beta_2\, x_{ik} + \ldots\ldots + \beta_k\, x_{ik}) \neq 0,\ i = 1,\ldots,n.$$

Although the mean is non-zero, it may result in a relatively small bias in each of the estimated parameters because of the high correlation between the hypothesized independent variable and the true independent variables.

It is interesting to note that a similar violation is caused when the incorrect form of the regressand is used. (This error can also cause the additional specification error of incorrect distributional form.) If, for example, the true model is

$$\exp(\underline{y}) = X\underline{\gamma} + \underline{v},\ \underline{v} \sim N(\emptyset, \sigma^2 I), \qquad (1.6)$$

whereas the hypothesized model is given by (1.2), then the hypothesized model has been misspecified because the wrong functional form for the dependent variables has been assumed. The mean of $\underline{u}$ would, in this case, be

$$E(u_i) = E(y_i - (\beta_1 + \beta_2 x_{i2} + \ldots + \beta_k x_{ik}))$$

$$= E(\log_e(\gamma_1 + x_{i2}\,\gamma_2 + \ldots + x_{ik}\,\gamma_k + v_i) -$$

$$(\beta_1 + \beta_2 x_{i2} + \ldots + \beta_k x_{ik})),\ i = 1,\ldots,n,$$

which in general is non-zero for any set of x's. Although this non-zero value is different from that which occurs when the misspecification is due to the incorrect functional form of the regressand, the relationship is strikingly similar.

One final point is that though incorrectly including an independent variable in a model is committing a specification error, this error does not affect the mean of the disturbance term; hence, the model is not misspecified because of a non-zero mean vector. This can be demonstrated by hypothesizing the model

$$\underline{y} = X\underline{\beta} + W\underline{\delta} + \underline{u},\ \underline{u} \sim N(\emptyset,\ \sigma^2 I), \tag{1.7}$$

where $\underline{y}$, X, $\underline{\beta}$, W, and $\underline{\delta}$ are as previously defined, whereas the true model is

$$\underline{y} = X\underline{\beta} + \underline{u},\ \underline{u} \sim N(\emptyset,\ \sigma^2 I).$$

The expected value of the hypothesized model would be

$$E(\underline{y}) = X\underline{\beta} + W \cdot 0 = X\underline{\beta}$$

which is exactly the true model; thus, the only cost of this specification error is a loss of efficiency in estimating the vector of parameters $\underline{\beta}$ and the variance σ^2.

I.3 Effects and Causes of a Misspecified Variance Vector

Given the model

$$\underline{y} = X\underline{\beta} + \underline{u},\ \underline{u} \sim N(\emptyset,\ \sigma^2 I),$$

it should be noted that a constant variance vector $\sigma^2\underline{i}$ (=DIAG $(\sigma^2 I)$) is assumed. This does not, however, imply that a specification error is made when a non-constant variance vector ($\underline{v}$) is correctly hypothesized. Rather, just as a method exists of transforming any hypothesized model with a non-zero mean vector into a model with a zero mean vector, a transformation exists that will change any model with a hypothesized non-constant variance vector into a model with a constant variance vector. One simply divides each observed dependent and independent variable by the square root of the corresponding hypothesized variance (v_i). This transforms the model

$$\underline{y} = X\underline{\beta} + \underline{u},\ \underline{u} \sim N(\emptyset,\ \sigma^2 V),$$

where DIAG (V) = $\underline{v}$, into the model

$$\frac{y_i}{\sqrt{v_i}} = \beta_1 \frac{1}{\sqrt{v_i}} + \beta_2 \frac{x_{i2}}{\sqrt{v_i}} + \ldots + \beta_k \frac{x_{ik}}{\sqrt{v_i}} + w_i,\ i=1,\ldots,n,\ \underline{w} \sim N(\emptyset,\ \sigma^2 I).$$

Hence it can now be assumed without loss of generality that the hypothesized model will always have a constant variance vector.

A regression model with constant variance ($\mathrm{var}(u_i) = \sigma^2$, $i=1,\ldots,n$) is said to be homoskedastic. Since estimation using classical least squares requires the assumption that the $E(u_i) = 0$, $i = 1,\ldots,n$, a homoskedastic model conforming to the classical assumptions has $E(u_i^2) = \sigma^2$, $i = 1,\ldots,n$. If the model violates this assumption, it is said to be heteroskedastic (non-constant variance vector).

If a model suffers from heteroskedasticity, it is known that the least squares estimators of $\underline{\beta}$, $\hat{\underline{\beta}}$, are unbiased and consistent but are inefficient and asymptotically inefficient. Further, the least squares estimator of the variance of the model is inappropriate since

$$E(\hat{\sigma}^2) = \frac{1}{n-k} E(\underline{y}-X\hat{\underline{\beta}})'\ (\underline{y} - X\hat{\underline{\beta}})$$

$$= \frac{1}{n-k} (\Sigma\ \sigma_i^2 - E(\underline{u}'(X(X'X)^{-}X')\underline{u})) \neq \Sigma\ \sigma_i^2/n,$$

where σ_i^2 is the variance of the i'th disturbance term.

Heteroskedasticity is generally believed to be a more serious problem when cross-sectional data is used than when time-series data is used. This belief is held because the magnitude of the dependent variable over each observation differs, in general, much more in cross-sectional data than in time-series data. This belief, however, is not always justified. The dependent variable in time-series data can be heteroskedastic if it covers a large number of years or if major changes have occurred since its conception.

I.4 Review of Literature

The following section is divided into four parts. First, the different types of residuals that are currently being used in testing both alternative hypothesis H_0 vs. H_1 and H_0 vs. H_2 are discussed. Second, the testing procedures now being utilized for testing H_0 vs. H_1 are discussed. Third, the different testing procedures now being used to test H_0 vs. H_2 are reviewed. Finally, previous sampling experiments that have compared various tests for H_0 vs. H_2 are discussed.

I.4.1 Different Residuals Being Used in Specification Error Test

If one could observe the vector of disturbances $\underline{u}$, either hypothesis could be easily tested. For example, to test for a non-zero mean vector, only a simple t test of $\bar{u}(=\sum_i u_i)$ about zero is required. In a similar vein, testing for a non-constant variance can be done by stratifying the u_i's and using an F test for equal variances. Unfortunately, however, since the disturbance terms are not observable, another testing procedure must be devised. The procedure that most often suggests itself is to use some predictor of the vector $\underline{u}$ as a proxy for the unobserved disturbance term.

So far, three residuals have been used in the literature. The first of these, which is both the easiest to compute and most frequently used, is the residual obtained from ordinary least squares (hereafter OLS). It is defined as

$$\underline{\hat{u}} = \underline{y} - X\underline{\hat{\beta}}.$$

Under H_o, $\underline{\hat{u}}$ is normally distributed with $E(\underline{\hat{u}}) = \underline{y} - X\underline{\beta} = 0$ and

$$\begin{aligned} \mathrm{Var}\ (\underline{\hat{u}}) &= E(\underline{y} - X\underline{\hat{\beta}})\ (\underline{y} - X\underline{\hat{\beta}})'/(n - k) \\ &= E(\underline{y}\,\underline{y}' - X(X'X)^{-}X'\,\underline{y}\,\underline{y}' - \underline{y}\,\underline{y}'X(X'X)^{-}X' + X(X'X)^{-}\,X'\underline{y}\,\underline{y}X(X'X)^{-}X') \\ &= E(I - X(X'X)^{-}X')\,\underline{y}\,\underline{y}'\,(I - X(X'X)^{-}X') \\ &= \quad M\,E(\underline{y}\,\underline{y}')M \\ &= \quad M\,\sigma^2\,I\,M = \sigma^2 M, \end{aligned}$$

that is, $\underline{\hat{u}} \sim N(\emptyset, \sigma^2 M)$.

The second technique utilized was developed by Theil [1965, 1968] and Koerts [1967]. These residuals, denoted $\underline{u}^*$, are called the Best Linear Unbiased Scalar-covariance (BLUS) predictors of the true disturbance terms $\underline{u}$. They are defined as

$$\underline{u}^* = A'\underline{y},$$

where A is an n x (n - k) dimensional matrix satisfying the conditions

a) $A'X = 0$

b) $A'A = I_{n-k}$, and

c) $AA' = M$.

Under H_o the $\underline{u}^*$'s are normally distributed with

$$\begin{aligned} E(\underline{u}^*) &= A'(E(\underline{y})) \\ &= A'E(\underline{u}) = 0 \\ \mathrm{VAR}(\underline{u}^*) &= E(A'\underline{y}\underline{y}'\,A) \\ &= A'\,E(\underline{y}\underline{y}')A \\ &= A'\sigma^2 IA = \sigma^2 I_{n-k}, \end{aligned}$$

that is $\underline{u}^* \sim N(\emptyset, \sigma^2 I_{n-k})$. It is important to note that although this orthonormalization process ensures that the $\underline{u}^*$'s are independent of one another, it also limits the number of residuals to only (n-k) instead of n.

The third technique was developed by Hedayat & Robson [1970] and is called stepwise or recursive residuals, denoted by $\underline{\tilde{u}}$. The basic idea of this method is to "obtain (residuals) by a stepwise fitting of the linear model to successively more observations" [Hedayat and Robson, 1970, p. 1574]. The first step of the procedure is to estimate the model using OLS and only k+1 observations. The least squares residual that corresponds to the (k+1)'th observation becomes the first stepwise residual, $\tilde{u}_1$. The next step is to reestimate the model using k+2 observations. As before, the stepwise residual is the one that corresponds to the last observation,((k+2) in this case), and is denoted by $\tilde{u}_2$. As this process is continued, n-k independent stepwise residuals are generated, $\tilde{u}_1,\ldots,\tilde{u}_{n-k}$.

These same n-k residuals can be obtained with only a single matrix inversion by using a recursive technique developed by Harvey & Phillips [1973]. The first step of the procedure is to estimate the model using OLS and k+1 observations, just as before, denoting the estimate of the vector $\underline{\beta}$ as $\underline{\tilde{\beta}}_{(1)}$. The least squares residual that corresponds to the (k+1)'th observation becomes the first recursive residual, denoted by $\tilde{u}_1$. The second step is to calculate a new estimate of the vector $\underline{\beta}$. This is done by using the recursive formula (with j=2).

$$\underline{\tilde{\beta}}_{(j)} = \underline{\tilde{\beta}}_{(j-1)} + \frac{(X'_{j-1}X_{j-1})^{-} \underline{x}_j \; (y_j - \underline{x}'_j \underline{\tilde{\beta}}_{j-1})}{1 + \underline{x}'_j \; (X'_{j-1}X_{j-1})^{-} \; \underline{x}_j}$$

where $(X'_{j-1}X_{j-1})^{-}$ denotes the inverse matrix used to calculate $\tilde{\beta}_{(j-1)}$, and $\underline{x}_j'$ is the row vector that corresponds to the (k+j)'th observation. To obtain the next inverse matrix $(X_j X_j)^{-}$, the recursive formula

$$(X_j'X_j)^{-} = (X_{j-1}'X_{j-1})^{-} + \frac{(X'_{j-1}X_{j-1})^{-}\underline{x}_j\underline{x}_j'\ (X_{j-1}'X_{j-1})^{-}}{1 + \underline{x}_j'\ (X_{j-1}'X_{j-1})^{-}\ \underline{x}_j}$$

is used. These n-k residuals are distributed under H_0 as $N(\emptyset,\ \tau^2 I_{n-k})$, where τ^2 is the associated variance. As in the case of the BLUS residuals, the stepwise (recursive) residuals are independent and k observations have been lost.

I.4.2 Present Procedures to Test for the Disturbance Terms Having a Non-Zero Mean

The first test for H_0 vs. H_1 was developed by Ramsey [1969] using BLUS residuals, $\underline{u}^*$. Recall that $\underline{u}^* = A'\underline{y}$ where $A'X = 0$, $A'A = I_{n-k}$ and $AA' = M$. Ramsey hypothesized that if the disturbance terms had an incorrectly specified non-zero mean vector, $\underline{z}$, "then the mean of the i'th disturbance terms z_i can be expressed as a linear function of the moments of $\hat{y}_i$, the least squares estimator of the conditional mean of y_i." [Ramsey, 1968, p. 66]. Stated formally,

$$E(u_i) = z_i = \alpha_0 + \alpha_1\, m_{i10} + \alpha_2\, m_{i20} + \alpha_3\, m_{i30} + \ldots, \qquad i = 1,\ldots,n, \tag{1.8}$$

where m_{ij0} is the j'th moment about the origin of $\hat{y}_i$. Given that BLUS residuals have the property that if $E(\underline{u}) = \underline{z} \neq 0$, then

$$E(\underline{u}^*) = E(A'\underline{y}) = A'E(\underline{y}) = A'E(\underline{u}) = A'\underline{z},$$

he suggested pre-multiplying equation (1.8) by the matrix A'. This yielded the equation

$$E(\underline{u}^*) = A'\underline{z} = A'\alpha_0 + A'\alpha_1\, \underline{m}_{10} + A'\alpha_2\, \underline{m}_{20} + A'\alpha_3\, \underline{m}_{30} + A'\alpha_4\, \underline{m}_{40} + \ldots. \tag{1.8'}$$

Removing the expected value operator from equation (1.8') and noting that $A'\hat{\underline{y}} = A'X\hat{\underline{\beta}} = 0$, Ramsey formulated the errors in variable model

$$\underline{u}^* = \alpha_2 \underline{y}^{*(2)} + \alpha_3 \underline{y}^{*(3)} + \alpha_4 y^{*(4)} + \underline{w} \tag{1.9}$$

where $\underline{w} \sim N(\emptyset, \xi^2 I_{n-k})$ under H_0. In this formulation $\underline{y}^{*(i)} = A'\hat{\underline{y}}^{(i)} = A'\{\hat{y}_1^i, \ldots, \hat{y}_n^i\}$. Given that under H_0, $E(\underline{u}^*) \neq 0$, it follows that under H_0, the $E(\alpha_2) = E(\alpha_3) = E(\alpha_4) = 0$. Hence, an F-test was proposed by Ramsey to test for the joint significance of α_2, α_3, and α_4. This procedure he named RESET (Regression Specification Error Test).

The RESET procedure has been examined by Ramsey and Gilbert [1972] using a monte carlo sampling procedure. Their results (as ammended by unpublished results of this author) have indicated that just as expected, under the null hypothesis, the test was not biased (the percent rejection corresponded to the α level); second that for the alternative model examined, the power was close to 100 percent under the alternative hypothesis of incorrect functional form; third that the test had virtually no power under the alternative hypothesis of an omitted variable for the model examined (the reason for this result will be explained later).

Because BLUS residuals are utilized in this procedure, three difficulties associated with those residuals are inherent in RESET. First, since the A matrix is difficult to calculate, the $\underline{u}^*$'s are not easily computed. Second, the BLUS procedure can be used to generate only n-k residuals from the original observations. Third, because there are only n-k residuals, in order to find a one to one correspondence between the residuals and the n observations, the k observations that are discarded in calculating the matrix A must be carefully noted.

Since all of the problems just outlined are caused by the use of BLUS residuals, Ramsey and Gilbert [1972] suggested substituting the standard least squares residuals,

$$\hat{\underline{u}} = (I - X (X'X)^{-} X') \underline{y} = M\underline{y},$$

for the BLUS residuals in the RESET technique. With this substitution, equation (1.9) becomes

$$\hat{\underline{u}} = M \underline{u} = \alpha_1 \hat{\underline{y}}^{(2)} + \alpha_2 \hat{\underline{y}}^{(3)} + \alpha_3 \hat{\underline{y}}^{(4)} + \underline{w} = Q \underline{\alpha} + \underline{w}, \qquad (1.10)$$

where $\underline{w} \sim N(0,\sigma^2 M)$.

This procedure, however, creates another problem. The standard F-test used to test the hypothesis that $\alpha_1 = \alpha_2 = \alpha_3 = 0$ breaks down because of nonindependence between the numerator and denominator. To show this nonindependence one can express the F statistic as a ratio of quadratics in the disturbance terms $\underline{u}$. The F-statistic in this particular case is

$$F = \frac{\underline{u}' (M'Q(Q'Q)^{-}Q'M) \underline{u}/3}{\underline{u}' (M-M'Q(Q'Q)^{-} Q'M) \underline{u}/(n-k-3)}$$

where Q is defined implicitly as in equation (1.10).

Since

$$(M'Q(Q'Q)^{-} Q'M) (M-M' Q(Q'Q)^{-} Q'M) \neq 0,$$

it follows that the numerator and denominator are not independent as a necessary and sufficient condition for their independence is that the product of the two quadratics be identically zero.

To correct for this non-independence, Ramsey & Schmidt [1974] have suggested pre-multiplying equation (1.10) by the matrix M. This results in:

$$M\hat{\underline{u}} = \hat{\underline{u}} = \alpha_1 M\hat{\underline{y}}^{(2)} + \alpha_2 M\hat{\underline{y}}^{(3)} + \alpha_3 M\hat{\underline{y}}^{(4)} + M\underline{w} = MQ \underline{\alpha} + M\underline{w}$$

where $M \underline{w} \sim N(\emptyset, \sigma^2 M)$. It is easily seen from the quadratic form that the F-statistic has a numerator and denominator which are independent. Writing F once again as a ratio of quadratic forms one gets

$$F = \frac{\underline{u}'\ (M'Q(Q'MQ)^{-}Q'M)\ \underline{u}/3}{\underline{u}'(M-M'Q(Q'MQ)^{-}\ Q'M)\underline{u}/(n-k-3)} .$$

Since independence of two quadratic forms is proven if their product is identically zero and given that M is idempotent (that is, MM=M) one obtains

$$(M'Q(Q'MQ)^{-}Q'M)(M-M'Q(Q'MQ)^{-}Q'M) = M'Q(Q'MQ)^{-}Q'M-M'Q(Q'MQ)^{-}Q'M \equiv 0,$$

thus proving that the numerator and denominator are independent. All the initial problems associated with the original formulation of the RESET technique are thus rectified in this newly defined RESET test. However, this procedure still requires the calculation of the well-defined matrix $M = (I - X\ (X'X)^{-}\ X')$. Although this is not a difficult process, it is a time-consuming and cumbersome one. Moreover, it is important to note that although the BLUS and OLS residuals are unbiased predictors of the error vector $\underline{u}$ under H_0, they are biased under H_1; that is, though the expected value of the residual vector is equal to the expected value of the disturbance term under the null hypothesis, the two sets of expected values are unequal under H_1. This can be clearly shown by defining a general set of residuals $\dot{\underline{u}} = B\ \underline{u}$, where B is a matrix with n columns. Under the alternative hypothesis of non-zero mean $(E(\underline{u}) = \underline{z} \neq \emptyset)$, the expected value of the general set of residuals is

$$E(\dot{\underline{u}}) = B\ E\ (\underline{u}) = B\ \underline{z} \neq \underline{z}.$$

It can thus be inferred that with any test in which a predictor (such as OLS or BLUS residuals) of the true disturbance term $\underline{u}$ is used, an incorrect measure of the non-zero vector $\underline{z}$ is being employed. Hence, a procedure that is unbiased under both H_0 and H_1 and where the calculation of the matrix M is not required would be preferred.

I.4.3 Heteroskedasticity

There are two different types of tests for heteroskedasticity; constructive tests and non-constructive tests. Simply stated, a non-constructive test for heteroskedasticity enables one to test the null hypothesis of homoskedasticity but does not help one to estimate the individual variances if H_0 is rejected. In contrast, a constructive test not only enables one to test for H_0 vs. H_2, but also provides an estimate of σ_i^2, i=1,...,n, (the variance of the i'th disturbance term); if the null hypothesis is rejected. These estimates of the variance can then be used to reestimate the model using Aiken's Generalized Least Squares (hereafter GLS) technique. However, it should be noted that since fewer assumptions about the form of the heteroskedasticity are usually necessary to use non-constructive than constructive tests, the former tend to be more widely applicable.

Non-Constructive Tests

There are three different types of non-constructive tests employed to test H_0 vs. H_2; they are an F-test, a likelihood ratio test, and a non-parametric peak test.

GQP - The first test utilizing the F-test was designed by Goldfeld & Quant [1965]. It can be used by a researcher who knows, or hypothesizes, that the individual variances $\sigma_1^2,\ldots,\sigma_n^2$ are monotonically related to one of the variables, say $\underline{x}_j$, and that the error term is normally distributed. The procedure is first to order the observations of variable $\underline{x}_j$ in increasing magnitude (decreasing magnitude if it is hypothesized that $\underline{x}_j$ is inversely related to the variance) so that $x_{ij} < x_{kj}$ where $i < k$. The remaining variables are reordered to conform to this ordering. Second, the observations are separated into two groups (denoted as group I and II, respectively) omitting the central p, $(\frac{n}{4} < p < \frac{n}{3})$, observations. Each group will have $m = (n-p)/2 > k$ observations. Third, using OLS, the model is estimated using each subset of the data. Fourth, the OLS estimate of the variance of the disturbance term from the first group of data is calculated and denoted as s_1 while the variance from the second group is calculated and denoted as s_2.

The ratio of these two independent, scaled, chi squared variables, denoted by $R_1 = s_2/s_1$, defines a statistic that has an F distribution with m-k and m-k degrees of freedom. Under H_0 of homoskedasticity, s_1 and s_2 have the same scaled chi squared distribution, whereas under H_2 of heteroskedasticity of the form hypothesized, s_1 and s_2 will have different scaled chi squared distributions.

There are, however, two difficulties with this procedure. First, the technique requires knowledge (or at least an hypothesis) about which single independent variable is causing the heteroskedasticity. Although this knowledge is sometimes available, it usually is not. Second, though it has been found that omitting the central p observations increased the power of this test, the technique should prove less powerful (in correctly rejecting H_0) than tests that do not discard information. Finally, in the test's favor, it should be mentioned that the distribution of R_1 is independent of the values of the regression coefficient and, under the null hypothesis, is independent of the value of the variance of the disturbance term.

<u>THEIL</u> - A similar test has been suggested by Theil [1965] using BLUS residuals. He suggested that the (n-k) BLUS residuals be divided into two equal groups of m observations after the central p, $(\frac{n}{4} < p < \frac{n}{3})$, observations have been omitted. Denoting t_1 as the sum of squared residuals from the first group and t_2 as the sum of squared residuals from the second, the statistic $R_2 = \frac{t_2}{t_1}$ is calculated. It is distributed as F with m and m degrees of freedom under the null hypothesis. Under the alternative hypothesis that the heteroskedasticity is a function of the order of the observations (for example, a function of time in time series data), R_2 is distributed as scaled F with m and m degrees of freedom.

The problem of the loss of information associated with the GQP procedure is thus partially solved by using this procedure. If one does not omit the central p observations in both tests, the F-statistic using the GQP procedure has (n-2k)/2 and (n-2k)/2 degrees of freedom, whereas with the Theil procedure, the F-statistic has (n-k)/2 and (n-k)/2 degrees of freedom. The reason for this is that in order to use the GQP procedure, one must calculate the residuals after the observations have been divided into groups. By contrast, since the BLUS residuals are independent of one another, they can be calculated before the data is divided into groups. However, to use this procedure effectively, one must still discard p observations. Finally, it must be recalled that two problems are added because BLUS residuals are used. First, it is difficult to calculate the vector $\underline{u}^*$. Second, it is difficult to reorder the n-k residuals when some variable, say $\underline{x}_j$, is related to the heteroskedastic disturbance terms $u_1,\ldots,u_n$. It can, however, be accomplished by carefully noting the k observations that are discarded in calculating the matrix A. Since the remaining n-k observations correspond to the n-k BLUS residuals, reordering can be done.

RECURSIVE-P - The final technique utilizing an F-statistic was developed by Harvey & Phillips [1973]. In this technique the F-statistic is defined in terms of recursive residuals. The prerequisite for using this procedure, just as for the previous two procedures, is that one have knowledge as to which variable, say $\underline{x}_j$, is monotonically related to the heteroskedastic variances σ_i^2, and that the disturbance terms be normally distributed. If these prerequisites are met, the test can be carried out. First the n-k recursive residuals are calculated. Second, the first k observations of the vector $\underline{x}_j$ are discarded and the remaining n-k observations are reordered in increasing magnitude (decreasing magnitude if $\underline{x}_j$ is inversely related to the variances $\sigma_1^2,\ldots,\sigma_n^2$). Third, the n-k residuals are reordered to conform to this new ordering. Fourth, the residuals are divided into two equal groups of m observations, after the central p observations ($\frac{n}{3} > p > \frac{n}{4}$) have been omitted. Finally, denoting t_1 and t_2 as the sum of the squared residuals from group one and two respectively, the ratio $R_3 = \frac{t_2}{t_1}$ is defined. This ratio has an F distribution with m and m degrees of freedom under H_0, whereas under H_2, R_3 is distributed as scaled F with m and m degrees of freedom.

To use this test, like Theil's, it is not required that the residuals be recalculated. Hence, k degrees of freedom are saved. Also, even though the recursive residuals are easier to calculate and reorder than the BLUS residuals, they are still not as easily manipulated as the OLS residuals. Finally, since the BLUS residuals have the property of having the minimum variance for the class of residuals which have a scalar covariance matrix, the BLUS procedure will probably have more power against H_2 than will the recursive residual technique.

BAMSET - In the next procedure, Bartlet's M statistic is used. Developed by Ramsey [1969], the test, which he named BAMSET (Bartlet's M Specification Error Test), requires use of BLUS residuals as did the Theil procedure. This procedure involves first calculating the n-k BLUS residuals and then separating the residuals into three mutually exclusive and exhaustive groups of approximately equal size (sample size n_1, n_2 and n_3 respectively). Denoting s_1, s_2 and s_3 as the sum of squared residuals from groups one, two, and three respectively, one can form a likelihood ratio test. The ratio used in the test is ℓ^*, defined as

$$\ell^* = \left(\frac{s_1}{n_1}\right)^{n_1/2} \left(\frac{s_2}{n_2}\right)^{n_2/2} \left(\frac{s_3}{n_3}\right)^{n_3/2} \Bigg/ \left(\frac{s_1 + s_2 + s_3}{n_1 + n_2 + n_3}\right)^{\frac{n}{2}}$$

Since ℓ^* is a likelihood ratio, it is well known that $-2 \log_e \ell^*$ is asymptotically distributed as χ^2 with, in this case, 2 degrees of freedom. Under H_0, the values of s_1, s_2 and s_3 are found to be statistically equal, whereas under H_2, they are found to be statistically different from one another.

As an alternative form of this same procedure, Ramsey & Gilbert [1972] have suggested that OLS residuals instead of BLUS residuals be used. They have, however, pointed out that since under H_0 the OLS residuals are heteroskedastic and not independent (recall that $E(\underline{\hat{u}\hat{u}}') = \sigma^2 M$), the asymptotic distribution of the resulting ratio cannot be determined.

At this point, some remarks about this test must be made. First, since the observations are not reordered, the three groups are a function of the index i. Hence, the test should prove most powerful against the alternative hypothesis when the heteroskedasticity is a function of the observation number i. Nevertheless, this form of heteroskedasticity was not what the test was specifically designed for. Rather, it was designed as a general test to detect any form of heteroskedasticity. Because in using this procedure, one makes no assumption as to the form of the heteroskedasticity, it should be expected that BAMSET will prove less powerful against H_2 than tests that utilize knowledge as to the form of the heteroskedasticity. However, when knowledge as to the form of the heteroskedasticity does not exist, the BAMSET test is the only one that can be used. To increase the power when knowledge of the variable (say $\underline{x}_j$) that is related to the heteroskedastic disturbances is known, it has been suggested by Sutcliff [1972] that the residuals should be reordered by the variable $\underline{x}_j$ before the grouping is made. Recall that this can be done with BLUS residuals if one discards the observations of the vector $\underline{x}_j$ that correspond to the observations omitted in calculating the A matrix.

<u>GQN</u> - The last group of tests are two non-parametric tests. The first of these was developed by Goldfeld & Quant [1965] for cases in which no assumptions about the distribution of the disturbance term can be made. However, this test still requires knowledge that a variable, say $\underline{x}_j$, is monotonically related to the heteroskedastic disturbance terms $u_1,\ldots,u_n$. The procedure requires first that the regression model be estimated using OLS. Second, the variable $\underline{x}_j$ is

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ordered by increasing magnitude (decreasing magnitude if $\underline{x}_j$ is inversely related to the heteroskedastic disturbance terms) and the OLS residuals are reordered to conform to this ordering. Third, the number of peaks (a peak is defined as $|\hat{u}_j|<|\hat{u}_{j+1}|$) occurring in the reordered residuals are counted. By using a table provided by Goldfeld & Quant [1967], the cumulative probability that heteroskedasticity is present can be determined. Under the null hypothesis, there will be a small number of peaks, whereas under H_2, the number will be large.

Some observations of this technique are in order. First, it has been found [Goldfeld & Quant, 1967] that for small sample sizes, $n < 10$, the procedure is biased because the OLS residuals are not mutually independent. Second, just as with all the other tests (including BAMSET with the reordering procedure), it is necessary to know which variable is monotonically related to the heteroskedastic disturbances. Third, given that OLS residuals are themselves heteroskedastic under H_0, it is surprising that for larger sample sizes, $n > 10$, the test is not biased. Finally, while it would rarely be inappropriate to use this test for heteroskedasticity, it should be selected only when the disturbance terms are not distributed normally. Since if the disturbance terms are normally distributed, other tests exist which prove more powerful at correctly rejecting the null hypothesis.

<u>RECURSIVE-N</u> - The last non-constructive test to be discussed was designed by Hedayat & Robson [1970]. With this non-parametric test, the peak tables provided by Goldfeld & Quant are also used. This test is exactly the same as the GQN test which was just reviewed with the exception that recursive residuals are used instead of OLS residuals.

This test offers the advantage of not being biased even for small sample sizes because the n-k recursive residuals are mutually independent.

It must once again be stressed, however, that this test, just as the GQN test, is a non-parametric test and hence should be used when the distribution of the disturbance term is unknown.

Constructive Tests

As previously mentioned, constructive tests for heteroskedasticity are most often viewed as being less general than non-constructive tests because they usually require more precise a priori information about the functional form of the heteroskedastic disturbances. For example, some of the most popular assumptions about the functional form of constructive tests are:

$$E(u_i^2) = \sigma^2 x_{ij} , \qquad (1.11a)$$

$$E(u_i^2) = \sigma^2 x_{ij}^2 , \qquad (1.11b)$$

$$E(u_i^2) = \sigma^2 (a + b\, x_{ij}^2) , \qquad (1.11c)$$

$$E(u_i^2) = \sigma^2 (a + b\, x_{ij})^2 , \qquad (1.11d)$$

$$E(u_i^2) = \sigma^2 E(y_i) \text{ , and} \qquad (1.11e)$$

$$E(u_i^2) = \sigma^2 E(y_i^2), \; i=1,\ldots,n. \qquad (1.11f)$$

Glejser [1969] divided these assumptions into two types of heteroskedasticity, pure and mixed. Pure heteroskedasticity is defined as $E(u_i^2) = \sigma^2 f(z_i)$, $i=1,\ldots,n$, where $f(z_i)$ represents a function in some variable z_i which passes through the origin, whereas mixed heteroskedasticity is defined as $E(u_i^2) = \sigma^2(f(z_i) + a), i=1,\ldots,n,$ that is, the heteroskedastic disturbance term has an intercept term.

According to this convention, only equations (1.11a) and (1.11b) represent pure heteroskedasticity.

Though the assumptions are more rigid, constructive tests do offer two advantages over non-constructive tests. First, the relation between a single independent variable and the disturbance term need not be monotonic. Second, since in constructive tests an estimator of the heteroskedastic variances (call it $\overset{0}{\sigma}_i^2$) is defined, the heteroskedasticity can be corrected either by dividing the model by $\overset{0}{\sigma}_i$ and reestimating using OLS or by reestimating the model using GLS (generalized least squares) and employing the values of $\overset{0}{\sigma}_i^2$ on the diagonal of the estimated variance covariance matrix.

Three constructive tests, all formulated in terms of a basic regression model, are described in this study. Ordinary least squares estimators for the model's parameters are used in two of the tests, while in the third maximum likelihood estimators are used. The estimates obtained from all the tests are then tested either individually or in a group.

PARK - The first estimation technique (that has since been used as a testing procedure) was developed by Park [1966]. Before that time, it was assumed that if the variable $\underline{x}_j$ were related to the heteroskedastic disturbances, $u_1, \ldots, u_n$, the relation was specified by $E(u_i^2) = \sigma^2 x_{ij}$, $i=1,\ldots,n$. In order to ease the restrictiveness of this assumption, Park suggested that when $\underline{x}_j$ is known to be the cause of the heteroskedasticity, it should be assumed that the

$$E(u_i^2) = \sigma^2 x_{ij}^{\alpha}, \quad i = 1,\ldots,n. \tag{1.12}$$

Park then posited that the value of α could be estimated by formulating a regression model. By taking natural logs and removing the expected value operator, he obtained the model

$$\ln u_i^2 = \ln \sigma^2 + \alpha \ln x_{ij} + \ln v_i, \; i=1,\ldots,n, \tag{1.13}$$

where v_i is distributed as χ^2 with one degree of freedom. Park then suggested replacing the unobserved dependent variable $\ln u_i^2$ by its OLS predictor $\ln \hat{u}_i^2$. When this proxy is used, model (1.13) becomes

$$\ln \hat{u}_i^2 = \ln \sigma^2 + \alpha \ln x_{ij} + \ln w_i, \; i=1,\ldots n, \tag{1.14}$$

where w_i is distributed as scaled χ^2 with one degree of freedom where the scaling factor is

$$E(w_i)\Big/\frac{E(\Sigma w_i)}{n} = \frac{m_{ii}\sigma^2}{\left(\frac{n-k}{n}\sigma^2\right)} = \frac{m_{ii}}{\left(\frac{n-k}{n}\right)},$$

and where m_{ii} is the i'th diagonal element of the matrix $M(=I - X(X'X)^{-1}X')$. Estimating the model using least squares, Park obtained estimators of $\ln \sigma^2$ and α. These estimators would then enable the researcher to correct the heteroskedastic model.

In carrying this technique one step further, others (for instance Goldfeld & Quant [1972]) have indicated that if one denotes $\hat{\alpha}$ as the OLS estimate of α and $\hat{\sigma}_{\hat{\alpha}}$ as the estimated standard error of $\hat{\alpha}$, the ratio R_y could be defined as

$$R_y = \frac{\hat{\alpha}}{\hat{\sigma}_{\hat{\alpha}}}.$$

This ratio is approximately distributed as student's t with n-2 degrees of freedom. Under H_0, $\alpha = 0$, whereas under H_2 of the type hypothesized, $\alpha \neq 0$.

Three points must be made. First, this process still requires knowledge of the single variable causing the heteroskedasticity. The test does not, however, require that a monotonic relation exist between the variable and the disturbance terms.

Second, $\ln \hat{u}_i^2$ is a biased predictor of $\ln u_i^2$. Recalling that $E(\underline{\hat{u}}\,\underline{\hat{u}}') = \sigma^2 M$ and denoting $\underline{m}_i'$ as the i'th row of the matrix M, one finds that

$$E(\ln \hat{u}_i^2) = E(\ln(\underline{m}_i'\underline{u})^2) \neq E(\ln u_i^2)$$

Third, it must be pointed out that when one estimates model (1.14) by the method of least squares and assumes, as Park did, the classical assumptions that the disturbance terms are distributed $N(\emptyset, \sigma^2 I)$, four specification errors are committed.

The first of these errors is that of incorrectly assuming a normally distributed disturbance term (recall that the disturbance terms are distributed as $\log_e$ scaled χ^2 with one degree of freedom). This error, however, does not affect the properties of the estimators of the $\ln\sigma^2$ or α, but rather affects the tests of significance (that is a t-test or an F-test). Hence, the t-test proposed to test H_0 vs. H_2 could be biased. It has been found by Srivastava [1958], however, that a t-test is robust against considerable non-normality; therefore, the procedure might prove reliable. This is especially true since the disturbance terms are distributed as $\log_e$ of a scaled χ^2 with one degree of freedom which is a two-tailed distribution.

The second specification error is that of a non-zero mean vector. The expected value of the i'th element of this vector is

$$E(\ln w_i) = E(\ln \underline{m}_i' \underline{u}) \neq 0$$

where $\underline{m}'_i$ is the i'th row of the M matrix. Since w_i is based on the matrix M $(=I - X(X'X)^{-}X')$, $\ln (w_i)$ is not independent of the variable $\ln (x_i)$ and hence the estimate of α will be biased. In addition, the estimate of $\ln \sigma^2$ will be biased unless all of the non-zero variation in $\ln (w_i)$ can be absorbed by the estimate of α.

The third specification error is that of heteroskedasticity. This error will cause the estimated variances to be biased and hence make the estimators of ln σ^2 and α inefficient. Therefore, the proposed t-test will prove more conservative than it would otherwise be. Also, it should be noted that since the dependent variable is heteroskedastic under H_0, the null hypothesis will be rejected by the test a disproportionate number of times.

The last specification error is non-independence. Like the misspecification of heteroskedasticity, non-independence causes the estimated variance to be biased; hence, the estimators of ln σ^2 and α are inefficient and the t-test is unduly conservative. Worse yet, however, is the fact that the non-independence in the disturbance terms adversely affects the t-test procedure another way. If the ratio calculated is to be distributed as student's t, the numerator and denominator must be independent. Unfortunately, when the disturbance terms are not independent, the numerator and denominator of this ratio are not independent; thus, the t-test procedure must again be questioned. Since there is no evidence that the t-statistic is robust against non-independence, the question arises as to whether this procedure is valid. The question is considered further on in this study.

FIML - In this procedure, suggested by Rutemuller & Bowers [1968], a likelihood ratio test is utilized. It has the advantage, unlike the previous procedure, of having an asymptotic distribution theory that is well defined.

Given the heteroskedastic model

$$y_i = \beta_1 + \beta_2\, x_{i2} + \dots + \beta_k\, x_{ik} + v_i,\ i=1,\dots,n,\ \underline{v} \sim N(\emptyset, V) \quad (1.15)$$

where $V = \begin{bmatrix} \sigma_1^2 & & & \emptyset \\ & \ddots & & \\ & & \ddots & \\ \emptyset & & & \sigma_n^2 \end{bmatrix}$, Rutemuller & Bowers proposed an estimation method whereby $\sigma_1^2,\dots,\sigma_n^2$ and $\beta_1,\dots,\beta_n$ could be jointly determined. They posited that if the variances were a function of some variables $\underline{z}_1,\dots,\underline{z}_n$ (typically these variables would be independent variables from the model 1.15) whose exact functional form was known (say $f(\underline{z}_1,\dots,\underline{z}_n)$), the parameters in the function $f(\cdot)$ and parameters $\underline{\beta}$ could be jointly determined.

Because Rutemuller & Bower's procedure requires knowledge about the function $f(\cdot)$, it will be assumed, for illustrative purposes, that $f(\cdot)$ is a quadratic in a single variable, that is

$$E(v_i^2) = \sigma^2(\alpha_0 + \alpha_1 x_{ij} + \alpha_2 x_{ij}^2),\ i=1,\dots,n. \quad (1.16)$$

They then suggested transforming model (1.15) into the homoskedastic model

$$\frac{y_i}{\sqrt{\alpha_0 + \alpha_1 x_{ij} + \alpha_2 x_{ij}^2}} = \frac{\beta_1 + \beta_2 x_{i2} + \dots + \beta_k x_{ik}}{\sqrt{\alpha_0 + \alpha_1 x_{ij} + \alpha_2 x_{ij}^2}} + u_i,$$

$$i = 1,\dots,n, \quad (1.17)$$

where $\underline{u} \sim N(\emptyset, \sigma^2 I)$, under H_0.

Since this model cannot be estimated using ordinary least squares, they recommended using maximum likelihood. Setting up the likelihood function

$$L_1 = \prod_{i=1}^{n} \frac{1}{\sqrt{\pi}\sqrt{\alpha_0 + \alpha_1\, x_{ij} + \alpha_2\, x_{ij}^2}} \exp\left(-\frac{1}{2}\frac{(y_i - \beta_1 - \beta_2\, x_{i2} \dots - \beta_x x_{i1})^2}{\alpha_0 + \alpha_1 x_{ij} + \alpha_2 x_{ij}^2}\right)$$

$$(1.18)$$

they defined $\underline{\alpha}^{\dagger}$ and $\underline{\beta}^{\dagger}$ as the estimators that maximize L_1. Likewise, they denoted L_0 as the equation (1.18) when α_1 and α_2 are constrained to equal zero (this is equivalent to OLS estimation of model 1.15) and defined $\hat{\alpha}_0$ and $\hat{\underline{\beta}}$ as the estimators that maximize L_0. Finally, they defined the likelihood ratio $\ell^{\dagger}$ as

$$\ell^{\dagger} = \frac{L_1}{L_0} .$$

Being a likelihood ratio, $-2 \log_e \ell^{\dagger}$ is asymptotically distributed as χ^2 with, in this case, 2 degrees of freedom (the number of degrees of freedom always equals the number of extra parameters included in L_1). Under the null hypothesis of homoskedasticity, the additional parameters in model (1.17), α_1 and α_2, are equal to zero and $\alpha_0 = \sigma^2$ (the model's variance), whereas under the alternative hypothesis of heteroskedasticity of the form hypothesized, α_2 and α_3 are not equal to zero. Therefore, including the polynomial is found to increase the model's efficiency.

An alternative test formulation of this same test has been suggested by Goldfeld & Quant [1972]. They hypothesized that the estimators $\underline{\alpha}^{\dagger}$ could be tested individually by using a t-test. This procedure would, of course, enable an experimentor to differentiate between pure and mixed heteroskedasticity. It must be realized, however, that since $\underline{\alpha}^{\dagger}$ is only asymptotically distributed normally, the test proposed would not have a student's t distribution; hence, the test statistic would not be exact for small sample sizes. This revised procedure might, nonetheless, pose only minimum difficulties under H_0 since there is evidence [Srivastava, 1958] that a t-test is robust against considerable non-normality.

Two final points concerning this test must be made. First, Rutemuller & Bowers suggested that if the exact functional form $f(\cdot)$ is not known, one should use the regression model itself as a proxy for the unknown function. This procedure would, using the likelihood ratio test, result in $-2 \log_e \ell^{\dagger}$ being distributed as χ^2 with k-1 degrees of freedom. Also, the t-test procedure (Goldfeld & Quant's suggestion), although only an asymptotic test, might be useful in determining which variable is causing the heteroskedastic disturbances.

Second, Rutemuller & Bowers' procedure, though well defined, tends to be more difficult to implement than any other test for heteroskedasticity. There are two reasons for this; first, a good maximum likelihood (hill climbing) computer program is needed, and second, since the estimation is accomplished through an iterative procedure, the process is more costly and time consuming than are other testing procedures.

<u>GLEJSER</u> - In this test, the last constructive test to be examined, OLS is used to estimate the parameters in the heteroskedastic model. The test, put forth by Glejser [1969], was designed to detect and correct for heteroskedasticity that is a polynomial in some variable. It should, however, be noted that prior knowledge about the degree of the polynomial and about the identity of the variable is required before the test can be used.

For illustrative purposes, the form of the heteroskedasticity will be postulated as

$$E(u_i^2) = \sigma^2(\alpha_1 + \alpha_2 x_{ij} + \alpha_3 x_{ij}^2)^2, \; i=1,\ldots,n. \qquad (1.19)$$

With the disturbance terms taking on this form, Glejser suggested that a regression model be used so that α_1, α_2, and α_3 can be estimated.

Using $\hat{u}_i^2$ as a proxy for u_i^2 and taking the positive square root of equation (1.19), he formulated the model

$$|\hat{u}_i| = (\alpha_1 + \alpha_2 x_{ij} + \alpha_3 x_{ij}^2)\ v_i,\ i=1,\ldots,n, \tag{1.20}$$

where v_i, $i=1,\ldots,n$, are distributed are scaled χ with one degree of freedom with the scale factor equal to $\sqrt{m_{ii}\sigma^2}$ where m_{ii} is the i'th diagonal element of the matrix M. He then suggested estimating the model using OLS. Finally, he suggested calculating a set of t ratios (t_2 and t_3) defined as

$$t_i = \frac{\hat{\alpha}_i}{\hat{\sigma}_{\hat{\alpha}_i}}\ ,\ i=2,3,$$

where $\hat{\alpha}_i$ is the OLS estimate of α_i and $\hat{\sigma}_{\hat{\alpha}_i}$ is the OLS estimate of the standard error of $\hat{\alpha}_i$. Although he indicated that the exact distribution of these ratios was unknown, he suggested that they might be approximately distributed as student's t (with n-3 degrees of freedom in this case). Assuming that his suggestion is true, a standard t-test could be performed. Under the null hypothesis of homoskedasticity, α_2 and α_3 are each equal to zero and $\alpha_1 = \sigma$ (the standard deviation of the disturbances), whereas under H_2 of the form hypothesized, α_2 and α_3 are different from zero.

Glejser's model and testing procedure also enabled him to determine easily whether the heteroskedasticity was of the pure or of the mixed variety. He suggested that if H_0 was rejected, the type of heteroskedasticity could be determined by testing the additional hypothesis of whether α_1 is equal to zero (pure heteroskedasticity). To test this hypothesis, a t ratio (similar to t_2 and t_3 above) would be calculated and if it is again assumed that the ratio is approximately distributed as student's t (with n-3 degrees of freedom in this case), a standard t-test can be performed.

A number of observations can now be made. First, it must be remembered that one must have a priori knowledge about the degree of the polynomial and about the identity of the variable causing the heteroskedasticity in order to use the test. According to Glejser, however, using the wrong degree of the polynomial presents little difficulty as this error has only a small affect on the test's power.

Second, because Glejser uses a t-test in each coefficient to test for H_0 vs. H_2, the correct α level is difficult to obtain. The reason for this is that since the probability of a type I error in using individual t-tests is the union of the probability of committing a type I error in testing each coefficient, the correct alpha level is difficult to obtain. However, when an F-test procedure is instead used, this problem is circumvented since when more coefficients are being tested for significance, the degrees of freedom are correctly varied.

Third, since $|\hat{u}_i|$ is used as the dependent variable, a biased predictor of the heteroskedastic disturbance is being used. This is easily perceived by once again recalling that $E(\underline{\hat{u}}\,\underline{\hat{u}}) = \sigma^2 M$, and m_{ii} is the i'th diagonal element of the M matrix,

$$E(|\hat{u}_i|) = \sqrt{\sigma^2 m_{ii}} \neq E(|u_i|), \; i=1,\ldots,n.$$

The fourth point is that just as with PARK's test, Glejser's estimation of model (1.19) using OLS while assuming the classical assumptions causes him to commit four specification errors. The first error is that of a non-zero mean vector. In model (1.19), the expected value of the i'th element of $\underline{v}$ is

$$E(v_i) = \sqrt{m_{ii}\sigma_i^2} \; .$$

Since this vector will probably be uncorrelated with $\underline{x}_j$ and $\underline{x}_j^2 (=[x_{ij}^2])$, only the constant term will be affected. Under H_0, its expected value will be

$$E(\alpha_1) = \sum_{i=1}^{n} (m_{ii}\,\sigma^2)^{1/2} \Big/ n$$

$$= \sigma\,[(m_{11})^{1/2} + \ldots + (m_{nn})^{1/2}]/n$$

Because $m_{ii} < 1$ and $m_{11} + \ldots + m_{nn} = n\text{-}k$, one can say that $\sigma > E(\alpha_1) > \frac{n-k}{n}\,\sigma$. This bias will, of course, affect any test of significance on the constant term.

The second specification error is that of incorrectly assuming a normally distributed disturbance term. As mentioned in the section on Park's test, this will cause the tests of significance to be biased. However, as previously mentioned because of Srivastava's [1958] findings that the t-test is robust against considerable non-normality, this specification error might cause only minor difficulties. It should, however, be further noted since these disturbance terms are distributed as scaled χ with 1 degree of freedom which is only a one-tailed distribution, it must be considered more "non-normal" than two-tailed distributions. Hence, one should expect the Park testing procedure (disturbance terms having a 2-tailed distribution) to be a more exact test under H_0.

The third and fourth specification errors are those of heteroskedastic and nonindependent disturbance terms. As is true in the case of the Park procedure, these errors cause considerable difficulty. First, both errors will cause a loss of efficiency thus making both the t and the F tests proposed too conservative. Second, the fact that the dependent variable is heteroskedastic under H_0 (recall OLS residuals,

$E(\underline{\hat{u}\hat{u}}') = \sigma^2 M$) causes a disproportionate number of rejections under the null hypothesis. Third, because the dependent variables are not mutually independent, the t tests break down for lack of independence between their numerators and denominators. Hence, the validity of this testing procedure must be carefully checked.

Summary

All tests for heteroskedasticity require either some knowledge of or an hypothesis regarding the form taken by the heteroskedastic disturbances. The amount of information required differs drastically, however, ranging from knowledge of a monotonic relationship to the exact form of the relationship. Table 1 summarizes these assumptions, indicates for which tests the assumptions of normality can be dropped, and restates all the relevant observations.

As can be seen from Table 1, non-constructive tests offer the advantage of not requiring as much *a priori* information as constructive tests. The latter, however, have the advantage of providing a corrective procedure for the problem of heteroskedasticity. What would be optimal is a test which would combine the advantages of both sets of tests. Since preliminary evidence exists [Glejser, 1969; Ramsey & Gilbert, 1972] which indicates the exact functional form of the heteroskedastic disturbance need not be specified for a constructive test to detect the presence of heteroskedasticity, a very general constructive test might be formed.

In addition, any such general test might very well not require the specification of a single variable which is causing the disturbances to be heteroskedastic. Rather, the test might only require that the

Table 1: Summary of Tests for Heteroskedasticity

Tests	Assumptions Concerning Heteroskedasticity							Observations							
	1	2	3	4	5	6	7	A	B	C	D	E	F	G	H
Nonconstructive															
GQP			X						X					X	O
THEIL			+	X					X					X	B
RECURSIVE-P			X						X					X	R
BAMSET	X		+	+						X					B/O
GQN			X					X		X					O
RECURSIVE			X					X	X						R
Constructive															
PARK					X						X	X			O
FIML		X					X			X			X		N
GLEJSER						X					X	X			O

X Applicable to test
\+ Applicable, though not originally suggested

1. No assumptions according to original formulation.
2. Any linear function of the independent variables.
3. The disturbances are monotonically related to a single known variable.
4. The disturbances are monotonically related to the order in which the observations are taken.
5. The disturbances are a function in some power of a known variable.
6. The disturbances are a quadratic in some known variable.
7. The exact form taken by the heteroskedastic disturbances is known.

A. Normality of disturbance terms not required (nonparametric test).
B. Exact test.
C. Asymptotically exact test.
D. Testing procedure is not exact because OLS residuals are used.
E. Biased estimate of σ_1^2 is used in test.
F. Time consuming alternative procedure.
G. Discarded p observations resulting in loss of information.
H. Residuals used : O-OLS, R-Recursive, B-BLUS, N-None.

variable(s) causing the heteroskedasticity are present in the model. The test itself might then approximate the correct functional form taken by those variables if the disturbance terms are in fact heteroskedastic. Such a procedure would not require the vast amount of information now needed and hence would be of tremendous use to the average researcher.

One final point must be made. Although there is often a lack of knowledge about which variable is causing the heteroskedasticity in a specific situation, none of the currently available testing procedures is designed to deal with such a situation. Unfortunately, researchers have incorrectly devised a way to circumvent the problem. When the cause of the suspected heteroskedasticity is not known, it is not uncommon for the researcher to select some test and to use this test with first one independent variable, then another and another until the test indicates the presence of heteroskedasticity. It cannot be emphasized enough that this technique is entirely incorrect. First, this _ad hoc_ technique actually violates the assumption that the variable causing the heteroskedastic disturbances is known. However, more importantly, this technique usually will lead to an incorrect conclusion.

To illustrate this, one could take the example of the hypothesized model

$$\underline{y} = \beta_0 + \beta_1\underline{x}_1 + \beta_2\underline{x}_2 + \underline{u},$$

$\underline{u} \sim N(\emptyset, V)$, where V indicates an unknown variance-covariance matrix, and where $\underline{x}_1$ and $\underline{x}_2$ are mutually independent vectors. Having no preconceived hypothesis as to the cause of the suspected heteroskedastic disturbances, the researcher decides to use one of the standard tests for heteroskedasticity to determine which variable is causing the problem. Carefully setting the probability of incorrectly rejecting the null hypothesis (type I error) of homoskedasticity at .05 (=α level), the researcher is ready to begin testing H_0 vs. H_2. Using the test, first with $\underline{x}_1$ and then with $\underline{x}_2$, the researcher concludes that the heteroskedasticity is caused by variable $\underline{x}_2$ and that the probability of type I error is .05. Using this procedure has led the researcher, as it usually does, to an incorrect conclusion. The probability of type I error occurring is the probability of its occurring when $\underline{x}_1$ was tested plus the probability of its occurring when $\underline{x}_2$ was tested minus the probability of its occurring in both of the tests. Hence,

$$\begin{aligned}
\Pr(\text{Type I}) &= \Pr(\text{Type I}_{\underline{x}_1} + \text{Type I}_{\underline{x}_2}) \\
&= \Pr(\text{Type I}_{\underline{x}_1}) + \Pr(\text{Type I}_{\underline{x}_2}) - \Pr(\text{Type I}_{\underline{x}_1\underline{x}_2}) \\
&= .05 + .05 - .0025 \\
&= .0975
\end{aligned}$$

(The independence of $\underline{x}_1$ and $\underline{x}_2$ was assumed so that the calculation of the intersection would be possible.) Therefore, if one wants to have a probability of type I error equal to .05, the α level for each test must be set at about .025. Hence, if a homoskedastic model had 20 independent variables and each were tested to ascertain whether or not it was causing heteroskedasticity, the probability of incorrectly rejecting H_0 would be very, very high.

I.4.4 Studies Comparing Tests for Heteroskedasticity

Since there are nine tests designed to detect heteroskedasticity, a researcher is faced with a difficult choice as to which test he should use in any particular situation. As has already been mentioned, the amount of apriori information possessed by the researcher determines to some extent which test(s) he can use. However, in many cases after this first elimination process has been gone through, there still remain a number of different tests from which to choose. The researcher must thus use another criterion on which to base his test decision. That criterion might be that the most desirable test is the one which has the highest probability of correctly rejecting H_0 (power) given a specified alpha level (the probability of incorrectly rejecting H_0).[1] In five studies, this criterion has been used to compare various tests for heteroskedasticity with one another. Since, however, no two regression models are exactly alike, no comparative study can furnish the researcher with the complete solution appropriate to his particular problem. In these studies, however, the various tests have been compared under different conditions, that is, with different sample sizes, alpha levels, and forms of heteroskedasticity. By making conclusions regarding the performance of specific tests under general categories of conditions, the experimentors proposed to establish certain broad criteria for the researcher to use in choosing a test for his particular situation.

There are two basic types of study that compare tests for heteroskedasticity. The first and most common type of study uses a sampling experiment. In such an experiment, the probability of correctly rejecting H_0 (the test's power) is determined through the use of a repetitive sampling process. This procedure is analogous to

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[1]Still another criterion might be the robustness of the remaining tests to other specification errors. However, only the robustness of the BAMSET test has been analyzed [Ramsey & Gilbert, 1972], and hence such a criterion cannot be made.

determining the probability of selecting a blue ball out of a box with three blue balls and two red balls by repeatedly selecting a ball out of the box, recording its color, and returning it before the next ball is drawn. The probability of drawing a blue ball would then be the ratio of

$$\frac{\text{number of times a blue ball was selected}}{\text{number of times the experiment was repeated}} .$$

To use this procedure in discriminating among the various tests for heteroskedasticity, the person conducting the experiment (hereafter referred to as the experimentor to differentiate him from the researcher who will use his findings) formulates a regression model, such as

$$E(y_i) = \alpha + \beta x_i \quad i=1,\ldots,n, \tag{1.21}$$

where the value of the vector $\underline{x}$ and the value of the parameters α and β have been previously specified by the experimentor. Model (1.21) is used to generate the expected value of a vector of dependent variables $E(\underline{y})$. Next, the experimentor specifies a population from which to select randomly the disturbances v_i, $i=1,\ldots,n$. Since the experimentor wants the regression model to be heteroskedastic, he specifies that v_i, $i=1,\ldots,n$ are independently and identically distributed as $N(0, \sigma^2_{x_i})$, where $\sigma^2_{x_i}$ denotes that the i'th population variance is related in some way, specified by the experimentor, to the i'th observation of the independent variable x. The experimentor then selects n random observations from this population. Defining

$$y_i = E\,(y_i) + v_i, \; i=1,\ldots,n$$

he generates n observations of independent variables $y_1,\ldots,y_n$. Following this, he applies each of the tests for heteroskedasticity that is to be compared to this vector of observed dependent variables,

$\underline{y}$. The experimentor next randomly selects another sample of n observations of v_i from the specified population and again calculates y_i. Once again he applies all the tests to the new vector of independent variables, $\underline{y}$. Repeating this procedure N times, he can determine the power of each test by calculating the ratio of $(\frac{\text{number of times that the test rejected } H_0}{N})$. Given that the alpha level of all these tests is the same, the test with the greatest power would be selected as being the best test to use given heteroskedasticity of the form hypothesized.

The difficulty with this procedure is that, of course, the power that is calculated for each test is often dependent on the specific model which the experimentor formulated and on the values of $\underline{x}$ and α and β which he chose. Even more importantly, this procedure requires a very large number of replications so that the probability of choosing an unrepresentative set of samples is very low.

An alternative way of calculating a test's power and a way that eliminates the repetitive sampling procedure has been suggested by Imhof [1961]. This method requires that the disturbance terms be independently and identically distributed as normal with a specified mean and variance. Also, it requires either the specification of the distribution from which the values of the vector $\underline{x}$ can be drawn or for the values of the vector $\underline{x}$ to be exactly specified. After the experimentor has satisfied these conditions, the exact probability of correctly accepting H_2 is calculated for each test by computing the probability that each quadratic form will occur.

Although this procedure eliminates the need to sample repeatedly from the population of the disturbance terms and hence the possibility of drawing a sample that is biased (that is, that the sample could have been drawn from a distribution other than the one specified), it still requires that a sample be drawn for the vector $\underline{x}$. Also, this technique cannot be applied to all the tests reviewed, but rather only to those tests that define a statistic that can be expressed as a convolusion of independent quadratic forms in normal variables. Only three of the tests presented meet this requirement.

Of the five studies comparing the various tests for heteroskedasticity,a sampling procedure is used in four while the direct calculation of the power by Imhof's method is used in the fifth. Unfortunately, in only one of these five studies are more than three tests compared. Each of these studies will be reviewed in the chronological order in which they were undertaken. This section will then conclude with a series of remarks which can be applied to all of the comparative studies.

Goldfeld & Quant I - The first comparative study was undertaken by Goldfeld & Quant in 1965. In that study, they compared the two tests for heteroskedasticity which they had developed (referred to in this study as GQP and GQN) by using a sampling experiment. In this experiment they generated their dependent variables y_i by the regression model

$$y_i = \alpha_0 + \alpha_1 x_i + u_i, \; i = 1,\dots,n, \tag{1.22}$$

where the disturbance terms u_i were independently and identically distributed as $N(0, 1)$. The x_i's were drawn from a uniform distribution with a mean μ_x and a standard deviation of σ_x. They used their two tests to discriminate between the null hypothesis that the dependent variables were generated by the model

$$\frac{y_i}{x_i} = \frac{\alpha_0 + \alpha_1 x_i}{x_i} + u_i, \ i = 1,\ldots,n,$$

versus the alternative hypothesis that they were generated by the model

$$y_i = \alpha_0 + \alpha_1 x_i + u_i \quad i = 1,\ldots,n.$$

Since the true model was the alternative hypothesis, it followed that the null hypothesis should be rejected. To compare their two tests for a variety of situations, Goldfeld & Quant generated the dependent variable using model (1.22) and two different sample sizes, n = 30 and n = 60. They also used 15 different combinations of values for μ_x and σ_x. For each sample of the x's, 100 replications of the experiment were made. In addition, since a central number of observations are omitted in the GQP procedures, each hypothesis was tested by using the GQP procedure five times. No observations were omitted the first time, but four additional observations were omitted each subsequent time the test was used. The power of each test was then calculated for each experiment.

Goldfeld & Quant's results indicated that the power of both of their tests increased as the sample size increased and as the ratio of $\frac{\sigma_x}{\mu_x}$ increased. They also found that the power of their parametric test (GQP) increased and then decreased as an increasing number of central observations were omitted; they concluded that the optimum number of observations to omit, p, was between one-third and one-quarter of the sample size. Finally, as one would expect, it was found that the nonparametric test (GQN) had less power than the parametric test (GQP) for any particular experiment. However, it was also observed that as the ratio $\frac{\sigma_x}{\mu_x}$ increased, the nonparametric test's power increased relative to the parametric test's.

Glejser - The next study was reported in 1969 by Glejser. After proposing a new test for heteroskedasticity (referred to in this study as GLEJSER), he felt that a comparison should be made between his test and the popular parametric test of Goldfeld & Quant. To make the comparisons he used a sampling experiment.

In his experiment, a vector of dependent variables was generated by the model

$$y_i = \beta_0 + \beta_1 x_i + u_i f(x_i), \; i = 1,\ldots,n, \qquad (1.23)$$

where the u_i's were independently and identically distributed as N(0, 1). In his study, eight functional forms, $f(\cdot)$'s, were used to generate the heteroskedastic disturbances. The values of x_i were chosen from three different normal distributions with a mean of 50 and standard deviation of 5, 10, and 30 respectively. Finally, each model was tested using three different sample sizes; they were n = 20, 30 and 60. Thus, 72 cases (8 x 3 x 3) were studied by Glejser. 100 replications of each case were used to determine the power of each test under the various alternative forms of heteroskedasticity.

Since Glejser's test is a constructive test for heteroskedasticity, Glejser had to specify the functional form taken by the heteroskedastic disturbance. He decided to hypothesize that the heteroskedasticity was a linear function of either $x_i^{1/2}$ and x_i or $x^{-1/2}$ and x^{-1} depending on whether $f(x_i)$ is a function of a power in x_i or in $\frac{1}{x_i}$ respectively. Of course, he pointed out that generally, in practice, this information would not be known.

After thus specifying the functional form used in his test, Glejser was able to test the significance of each of the estimated parameters by using a two-tailed t-test. Since, however, Glejser's testing procedure is not exact, he found that in using a two-tailed t-test on a homoskedastic model, a nominal alpha level (= probability of type I error) of 11% was needed to reject the null hypothesis 5% of the time. Hence all 72 cases were examined using his test with a nominal alpha level of 11% so that the probability of type I error would be .05.

After Glejser completed his study, he made some observations about his findings. First he concluded that generally, his test compared favorably with the parametric test of Goldfeld & Quant's. He also concurred with Goldfeld & Quant's findings that the power of both tests increased with sample size. Next, he discovered that his test could not detect the presence of mixed heteroskedasticity when it in fact existed. Finally, he found that because his two regressors (x_i and $x_i^{1/2}$ or x_i^{-1} and $x_i^{-1/2}$) were highly correlated, the test's power was generally unaffected by using just a single regressor.

Ramsey & Gilbert - The third study is similar to that of Goldfeld & Quant's in that the experimentors, Ramsey & Gilbert [1972], compared two of their own tests with one another. They compared the BAMSET procedure using first BLUS and then OLS residuals under the null hypothesis and under the alternative hypothesis of heteroskedasticity. A sampling experiment was used to compare the two procedures.

To generate the vector of dependent variables under the alternative hypothesis, the model

$$y_i = 1.0 + 2.0\ x_{i'} - .8\ x_{i2} + u_i \sqrt{i/25},\ i = 1,\ldots,n, \tag{1.24}$$

where the u_i's are independently and identically distributed as N(0, 1), was used. Ten values of $\underline{x}_1$ and $\underline{x}_2$ were obtained from a table of random numbers. These ten numbers were then replicated two, three, and five times to generate sample sizes of n = 20, 30 and 50 respectively. In a basically similar way, a homoskedastic model was generated. Realizing that a sample of disturbance terms, unrepresentative of the population from which they were drawn, would adversely affect the results, Ramsey & Gilbert replicated each experiment 1000 times.

Two surprising results were obtained. First, since it is well known that OLS residuals are heteroskedastic under the null hypothesis, Ramsey & Gilbert were surprised to find that with the BAMSET procedure the residuals were found to be homoskedastic. This meant that the percentage of times that H_0 was incorrectly rejected corresponded to the alpha level. Secondly, they were surprised to find that when the alternative hypothesis was correct, using OLS residuals in the BAMSET procedure always proved more powerful than when the procedure was applied using BLUS residuals. They offered no explanation for either of these results. A possible explanation for both of these findings will, however, be offered by this author later on in this study.

Goldfeld & Quant II - The final comparative study using the sampling experiment approach was again conducted by Goldfeld & Quant [1972]. This is, to date, the most extensive comparison of tests for heteroskedasticity made. Goldfeld & Quant compared four different tests for heteroskedasticity (PARK, GLEJSER, GQP, and FIML).

They generated the vector of dependent variables by using the model

$$y_i = 2 + 2\,x_i + u_i\,\sqrt{a+b\,x_i + c\,x_i^2},\ i = 1,\ldots,n,$$

where the u_i's are independently and identically distributed as N(0, 1). The parameters a, b, and c are given various combinations of values (7 combinations in all); the x_i's are independently distributed as either uniform or log normal. (All seven combinations of values a, b, and c were tested using the uniformally distributed x_i's while only two cases were examined using the log normally distributed x_i's.) All nine cases were then compared using three sample sizes n = 30, 60 and 90. Finally, each experiment, 21 in all, was replicated either 50 or 100 times.

After carrying out this elaborate study, Goldfeld & Quant drew three major conclusions. First, they concluded that the FIML method appeared "to be the most powerful test for detecting heteroskedasticity." [Goldfeld & Quant, 1972, p. 118]. Tangentially, they found that their suggested asymptotic t-test on the coefficients obtained from the FIML technique was inferior to the likelihood ratio test originally posed by Rutemuller & Bowers. This result, they asserted, was due to the high intercorrelation between the parameters b and c.

Goldfeld & Quant's second conclusion was that the power of each test increased with the number of observations; in this finding, they concurred with all previous experimentors. Finally, using four different tests, they were able to substantiate Glejser's finding that mixed heteroskedasticity is more difficult to detect than pure homoskedasticity.

Harvey & Phillips - In the final study, Harvey & Phillips compared the three exact tests for heteroskedasticity (GQP, THEIL, and RECURSIVE-P). Rather than use a sampling experiment, they calculated the probability of correctly accepting the alternative hypothesis of heteroskedasticity by the method suggested by Imhof. That is, they calculated the probability of the quadratic form's occurring.

Harvey & Phillips compared the three tests for two types of heteroskedasticity. They assumed that the variances of the disturbance terms u_i were either $E(u_i^2) = \sigma^2 x_{ij}$ or else $E(u_i^2) = \sigma^2 x_{ij}^2$. Noting that these variances critically depend on the distribution of $\underline{x}_j$, they assumed that $\underline{x}_j$ would take on four distributional forms. They first assumed the $\underline{x}_j$'s were distributed normally, then log normally, uniformly and finally equally spaced. They then made their comparisons using three sample sizes of (n=) 10, 20, or 30 observations, in either 2, 3, or 4 regressors and omitting varying numbers of central observations.

In computing the powers of the different rests under varying situations, they observed that Imhof's method seemed erratic in the widely varying amounts of time that it took for the different calculations. When the study was fully completed, however, they were nevertheless able to make a number of observations. First, as expected, it was found that the power of all three testing procedures increased with the number of sample observations (n), and decreased with the number of regressors (k). Second, they were able to substantiate Goldfeld & Quant's findings that omitting a number of central observations increases the power of the testing procedure. In conjunction with this, they also observed that the number seemed to differ depending on the distribution of the $\underline{x}_j$'s. However, since omitting any number within the vicinity of the optimum number resulted in very little loss of power, they felt that the difference due to the distribution of the $\underline{x}_j$'s could be ignored. Third, they found virtually no difference in power among the three tests though the THEIL test (using BLUS residuals) usually out-performed the RECURSIVE-P test (using recursive residuals). Finally, and most interestingly, they found that the power of all their tests varied considerably with the distribution of the $\underline{x}_j$'s with the highest power typically occurring when the $\underline{x}_j$'s were distributed log normally and the lowest power when they were distributed uniformally.

A number of observations can be made on the comparative studies undertaken to date. First, with the single exception of Ramsey & Gilbert's study, all of the sampling experiments used a small number of replications (50 or 100). By using such a small number, the probability of drawing an unrepresentative sample is much higher than it would be if a much larger number of replications were made. This is especially true for Goldfeld & Quant's most comprehensive study [1972] as they occasionally repeated the experiment only 50 times.

Second, the point has been made by Goldfeld & Quant [1972, p. 90] that the power of the BAMSET procedure, reported by Ramsey & Gilbert, was calculated using a form of heteroskedasticity that the test could best detect. Although this is true, and was mistakenly not pointed out by Ramsey & Gilbert, Goldfeld & Quant's point is equally valid when applied to each of the other comparative studies. For example, though Glejser used seven different heteroskedastic models when the heteroskedasticity was generated by x_{ij}^{-1} instead of x_{ij}, the knowledge he incorporated into this test likewise changed. Similarly, in Goldfeld & Quant's own two studies, the power of the different tests is reported as if the researcher knew the variable that is causing the disturbance terms to be heteroskedastic. Finally, Harvey & Phillips' study makes the identical assumption. What must be shown is what the power of each of the different tests is when the wrong variable is thought to be causing the heteroskedastic disturbances and when the wrong functional form is used.

Third, the preliminary findings given by Harvey & Phillips indicated that the distribution of the variable causing the heteroskedastic disturbances affects the power of various testing procedures requires further study. It could well be that the power of the tests is affected not so much by the distributional forms of the dependent variable as by the parameters that exactly specified the range of those variables.

Finally, all of the testing procedures should be compared, unless they can be shown equivalent, under the same conditions. In this way, firmer conclusions can hopefully be drawn as to which test should be used, given a particular situation.

I.5 Summary

In this chapter of the study, a vast amount of information on the occurrence of a non-zero mean vector and heteroskedasticity in the regression model has been drawn together. In an attempt to clarify these two problems, a detailed discussion was given as to when and how both difficulties arise and what the effects will be. To further illuminate this area, an in depth review of the tests that have been proposed, and are now being used to detect each error, was given. Finally, different attempts at comparing the various tests for heteroskedasticity were presented.

It is apparent that though a tremendous amount of effort has been put forth to test for the presence of these two specification errors, further attempts must be made. Two such attempts might be a more general test for heteroskedasticity and a simpler formulation of the test for a non-zero mean vector. It is hoped this study will contribute to this goal.

CHAPTER II

A NEW APPROACH

In this chapter two new specification error tests will be presented. Both of these tests are based on the ability of a Power Series Expansion Model to estimate the conditional mean of the dependent variable. The first of these tests is used to discriminate between the null hypothesis of a zero mean vector for the disturbances and the alternative hypothesis of a non-zero mean vector. Similarly, the second test is used to discriminate between the null hypothesis of homoskedasticity (constant variance vector) and the alternative hypothesis of heteroskedasticity (non-constant variance vector). Both tests are being proposed in response to the objections raised earlier in this study with the current testing procedures.

Because of the central importance of a Power Series Expansion Model to both testing procedures, the concept of a Power Series Expansion model will be introduced first. After this discussion, the test designed to determine if the disturbance terms have a non-zero mean will be presented. This will be followed by a discussion of the second testing procedure, a test for heteroskedasticity.

II.1 Estimation Using a Power Series Expansion Model

In this section, the concept of a Power Series Expansion (POSEX) model will be introduced. It will be derived from both a univariate and multivariate Taylor series expansion. The similarities between this model and Ramsey's RESET model will then be shown. Finally, an instrument will be suggested to replace the cumbersome expansion terms that appear in any multivariate POSEX model.

II.1.1 Development of a Power Series Expansion Model

A Power Series Expansion (POSEX) model is an expansion of the hypothesized model in powers of the independent variables. This model is applicable in those situations in which the conditional mean is an analytic function in the independent variables. Suppose the regression model is given by

$$y_i = f(x_i) + u_i, \; E(u_i) = 0. \tag{2.1}$$

Consider first using a Taylor series expansion in the variable x to approximate the conditional mean expressed by the function f(x). In this case, the function f(x) is approximated by

$$f(x) = f(a) + f'(a)\,(x-a) + \frac{f''(a)}{2!}\,(x-a)^2 + \frac{f'''(a)}{3!}\,(x-a)^3 + \ldots, \tag{2.2}$$

where $f^{(n)}$ denotes the n'th derivative of the function $f(\cdot)$ and <u>a</u> is chosen for the ease of calculating f(a) and so that the function is continuous between a and x. If, for example, the function $f(\cdot)$ were unknown, but n values of x and f(x) were observed, the POSEX model would be

$$y_i = f(x_i) + u_i \doteq \beta_0 + \beta_1 x_i + \beta_2 x_i^2 + \ldots + \beta_h x_i^h + u_i,$$
$$i = 1,\ldots,n \tag{2.3}$$

where $a = 0$, $f^{(i)}(0) = \beta_i$ and $E(u_i) = 0$. This model will yield a good approximation if f(x) can be expressed by a low series expansion.

Estimating this model by the method of least squares, one would obtain unbiased estimators of the true coefficients. That is,

$$E(\hat{\beta}_j) = \frac{f^{(j)}(0)}{j!}, \; j = 0,\ldots,h,$$

where $f^{(j)}(0)$ is the j'th derivative of the function $f(\cdot)$ evaluated at zero, and j! is j factorial. Hence, given any value x_0, an estimate of $f(x_0)$, is

$$\hat{y}_0 = \hat{\beta}_0 + \hat{\beta}_1 x_0 + \hat{\beta}_2 x_0^2 + \ldots + \hat{\beta}_k x_0^h = \hat{f(x_0)}.$$

Although $\hat{y}_0$ is an unbiased predictor of $f(x_0)$, the variance of this predictor will increase to the extent to which x_0 lies outside the observed sample points $x_1,\ldots,x_n$.

Unfortunately, though this procedure is quite simple, it is not always applicable. Often, the function which is to be approximated is not a function in a single variable but rather is multivariate. To analyze the multivariate case is conceptually no different from analyzing the single variate case. The Taylor series expansion of the m variate function $f(x_1,\ldots,x_m)$ is written as

$$f(x_1,\ldots,x_m) = f(a,\ldots,a_m) + [(x_1-u_1)\frac{\partial}{\partial x_1} + \ldots + (x_m-a_m)\frac{\partial}{\partial x_m}] f_{a_1..a_m} + \ldots$$

$$\frac{1}{k!}[(x_1-a_1)\frac{\partial}{\partial x_1} + \ldots + (x_m-a_m)\frac{\partial}{\partial x_m}]^k f_{a_1 \ldots a_m} + \ldots,$$

where $\frac{\partial}{\partial x_j}$ represents the partial derivative operator with respect to x_j, and $f_{a_1 \ldots a_m}$ denotes the evaluation of the function $f(\cdot)$ after the partial derivatives have been taken at the points $a_1 \ldots a_m$. Expanding this Taylor series (for the bivariate case, that is, m = 2), one obtains

$$f(x_1, x_2) = f(a_1, a_2) + \frac{\partial f_{a_1\ a_2}}{\partial x_1}(x_1 - a_1) +$$

$$\frac{\partial f_{a_1\ a_2}}{\partial x_2}(x_2 - a_2) + \frac{\partial^2 f_{a_1\ a_2}}{\partial x_1^2}(x_1 - a_1)^2 +$$

$$2\,\frac{\partial^2 f_{a_1\ a_2}}{\partial x_1\,\partial x_2}(x_1 - a_1)(x_2 - a_2) +$$

$$\frac{\partial^2 f_{a_1\ a_2}}{\partial x_2^2}(x_2 - a_2)^2 + \ldots$$

Reformulating this expansion into a POSEX model, as was done in the single variate case (model 2.3), one obtains the model

$$y_i = f(x_{i1}, x_{i2}) + u_i \doteq \beta_0 + \beta_{1,10}\, x_{i1} + \beta_{1,01}\, x_{i2} +$$

$$\beta_{2,20}\, x_{i1}^2 + \beta_{2,11}\, 2x_{i1}\, x_{i2} + \beta_{2,02}\, x_{i2}^2 + \ldots +$$

$$\beta_{h,h0}\, x_{i1}^h + \beta_{h,(h-1)1}\, 2x_{i1}^{h-1}\, x_{i2}^1 + \ldots +$$

$$\beta_{h,1(h-1)}\, 2x_{i1}^1\, x_{i2}^{h-1} + \beta_{h,0h}\, x_{i2}^h + u_i,\ i=1,\ldots,n \qquad (2.4)$$

where $\beta_{j,ik} = \dfrac{\partial^j f_{a_1\ a_2}}{\partial x_1^i\, \partial x_2^k}$, $a_1 = a_2 = 0$, $f(0, 0) = \beta_0$ and $E(u_i) = 0$.

Unfortunately, there are $(h+1)(h+2)/2 = t$ parameters to be estimated in the above model. Hence, unless one has more observations than parameters $(n > t)$, the procedure breaks down.

One possible solution to this problem is to assume that $\dfrac{\partial^i f}{\partial x_1^i}\dfrac{\partial^j f}{\partial x_2^j} = b_{i+j}\,\dfrac{\partial^{i+j} f}{\partial x_1^i\, \partial x_2^j}$. When this assumption is made, the implication is that $\beta_{2,20} = b_2\, \beta_{1,10}^2$, $\beta_{2,11} = b_2\, \beta_{1,10}\, \beta_{1,01}$, and that $\beta_{2,02} = b_2\, \beta_{1,01}^2$. Using this assumption and denoting β_{11} as $\beta_{1,10}$, β_{12} as $\beta_{1,01}$ and α_i as b_i, one can transform model (2.4) into

$$y_i = f(x_{1i}, x_{2i}) + u_i \doteq \beta_0 + \beta_{11} x_{i1} + \beta_{12} x_{i2} + \alpha_2 (\beta_{11} x_{i1} + \beta_{12} x_{i2})^2 + \dots + \alpha_h (\beta_{11} x_{i1} + \beta_{12} x_{i2})^h + u_i, \; 1 = 1,\dots,n. \quad (2.5)$$

This POSEX model now has only h+2 parameters to estimate in the bivariate case and only h+m in the m-variate case.

Note, however, that although model (2.5) involves very few parameters,to obtain estimates of those parameters,one must use a non-linear estimation process. To surmount this inconvenience one could use a two-stage procedure. The first stage would specify the linear combination of the x_i's to be used for each term that is of the form $(\beta_{11} x_{i1} + \dots + \beta_{1k} x_{ik})^j$, where j is greater than one. This first stage would provide an instrument for the non-linear terms so that a non-linear estimation technique is not needed. The second stage could then provide estimates of the h+2 parameters (in the bivariate case). When this procedure is used, the model to be estimated in the second stage would be (once again for the bivariate case)

$$f(x_{i1}, x_{i2}) \doteq \beta_0 + \beta_{11} x_{i1} + \beta_{12} x_{i2} + \alpha_1 q_i^2 + \dots + \alpha_h q_i^h + u_i, \; i = 1,\dots,n, \quad (2.6)$$

where q_i^j represents a linear combination of the x_i's raised to the j'th power. It might be mentioned, however, that this simplified POSEX model could instead be formulated as

$$f(x_{i1}, x_{i2}) \doteq \beta_0 + \alpha_1 q_i + \alpha_2 q_i^2 + \dots + \alpha_h q_i^h + u_i, \; i = 1,\dots,n. \quad (2.7)$$

by using the linear combination of the x_i's specified in the first stage for the linear as well as the non-linear terms involving the x_i's.

However, this formulation was rejected. Although both models (2.6) and (2.7) are simplified versions of the more complicated POSEX model (2.5), model (2.6) was chosen since it maintained more of the essence of model (2.5) than did model (2.7).

II.1.2 Similarities to Ramsey's RESET Model

This model (2.6) is strikingly similar to the model Ramsey [1966] used in his RESET test to determine if the disturbance term has a non-zero mean. Recall that Ramsey felt that if the vector of disturbance terms in model (2.1) were hypothesized to be distributed as $N(\emptyset, \sigma^2 I)$, whereas they were actually distributed as $N(\underline{z}, \sigma^2 I)$ then the mean vector $\underline{z}$ could be expressed as a linear function in the moments about the origin of $\hat{\underline{y}}$. The equation he suggested was

$$E(\underline{u}) = \underline{z} = \alpha_0 + \alpha_1 \hat{\underline{y}} + \alpha_2 \hat{\underline{y}}^{(2)} + \alpha_3 \hat{\underline{y}}^{(3)} + \alpha_4 \hat{\underline{y}}^{(4)} + \dots \tag{2.8}$$

where $\hat{\underline{y}}^{(j)} = \{\hat{y}_1^j, \dots, \hat{y}_n^j\}^1$. Premultiplying equation (2.8) by the matrix A' (recall that BLUS residuals $\underline{u}^* = A'y$), limiting the expansion to four terms, and removing the expected value operator, he obtained

$$\underline{u}^* = A'\underline{y} = A'\underline{z} = \alpha_2 A'\hat{\underline{y}}^{(2)} + \alpha_3 A'\hat{\underline{y}}^{(3)} + \alpha_4 A'\hat{\underline{y}}^{(4)} + \dots \underline{w} \tag{2.9}$$

where $\underline{w} \sim N(\emptyset, \sigma^2 I_{n-k})$ under H_0. This model is a power series expansion in the OLS predictor of the dependent variable, $\hat{\underline{y}}$. That is, $\hat{y}_i$ is the q_i in model (2.6).

To show still more clearly the similarities between Ramsey's model (2.9) and the POSEX model (2.6) Ramsey's model (2.9) will be reformulated using the POSEX technique. When model (2.1) is rewritten as

$$u_i = y_i - \beta_1 - \beta_2 x_{i2} - \dots - \beta_k x_{ik}, \; i=1,\dots,n,$$

it is clear that the disturbance terms u_i are a function of the dependent variable y_i and the hypothesized independent variables $x_{i1},\dots,x_{ik}$. Under the null hypothesis that model (2.1) generated the dependent variable y_i, the y_i's are a linear function of $x_{i1},\dots,x_{ik}$, hence, the u_i's can be written as a linear function in the variables $x_{i1},\dots,x_{ik}$. However, under the alternative hypothesis that the $E(\underline{u}) = \underline{z} = 0$, the y_i's may be any (generally non-linear) function of both the k variables $x_{i1},\dots,x_{ik}$ that the researcher hypothesized in model (2.1) and of a set of m variables $z_{i1},\dots,z_{im}$ that the researcher mistakenly did not hypothesize as being part of model (2.1). Hence, the vector $\underline{u}$ must be written as a non-linear function both of the k hypothesized variables $\underline{x}_1,\dots,\underline{x}_k$ and of the m erroneously excluded variables $\underline{z}_1,\dots\underline{z}_m$.

Because the m variables $z_{i1},\dots,z_{im}$ are erroneously excluded from the hypothesized model (2.1), they cannot be identified. Hence, the u_i's must be approximated by a function in the variables $x_{i1},\dots,x_{ik}$. This function in $x_{i1},\dots,x_{ik}$, if it is analytic, can itself be approximated by a power series expansion mode in $x_{i1},\dots,x_{ik}$. This series of two approximations yields the POSEX model

$$\begin{aligned} u_i \doteq f(x_{i1},\dots,x_{ik}) \doteq \beta_0 + \beta_{11} x_{i1} + \dots + \beta_{1k} x_{ik} + \\ \alpha_2(\beta_{11} x_{i1} + \dots + \beta_{1k} x_{ik})^2 + \dots + \\ \alpha_h (\beta_{11} x_{i1} + \dots + \beta_{1k} x_{ik})^h + v_i, \; i=1,\dots,n \end{aligned}$$

where $E(v_i) = 0$, which is a k variate extension of the bivariate model posed in equation (2.5). Once again this model requires a non-linear estimation process to estimate the h+k parameters. To solve this estimation problem Ramsey suggested using $\hat{y}_i^j$ as an instrumental variable for each term that is of the form $(\beta_{11} x_{i1} + \dots + \beta_{1k} x_{ik})^j$, where j is greater than one. The variable $\hat{y}_i$ was chosen since it provided a linear combination of $x_{i1},\dots,x_{ik}$ based on the relation between the dependent variable y_i and the independent variables $x_{i1},\dots,x_{ik}$.

When this instrument is used, the model becomes

$$u_i \doteq \beta_0 + \beta_{11} x_{i1} + \dots + \beta_{1k} x_{ik} + \alpha_2 \hat{y}_i^2 + \dots + \alpha_h \hat{y}_i^h + v_i,\ i = 1,\dots,n, \tag{2.10}$$

where v_i, $i=1,\dots,n$, are independently and identically distributed as $N(0, \sigma^2)$ under H_0. Multiplying model (2.10) by A', one derives the model

$$\underline{u}^* = A'\underline{u} = \alpha_2 A'\hat{\underline{y}}^{(2)} + \alpha_3 A'\hat{\underline{y}}^{(3)} + \alpha_4 A'\hat{\underline{y}}^{(4)} + \underline{w}, \tag{2.11}$$

where $\underline{w} \sim N(\emptyset, \sigma^2 I_{n-k})$ under H_0 and h is set equal to 4. Hence, Ramsey's model (2.9) has been obtained by using a POSEX model formulation.

II.1.3 A Suggested Instrument

In using the method of instrumental variables to simplify the POSEX model so that the linear estimation techniques can be used, the researcher must choose an instrument which is highly correlated with the term that it is replacing. However, unless the correlation between the two variables is exactly one, using the instrument reduces the accuracy of the approximation. Hence, since the vector $\hat{\underline{y}}$ used by Ramsey is

$$\hat{\underline{y}} = \hat{\beta}_0 + \hat{\beta}_1 \underline{x}_1 + \dots + \hat{\beta}_k \underline{x}_k = X\hat{\underline{\beta}},$$

if the vector of estimated parameters is not a multiple of the vector of parameters $\{\beta_{11},\ldots,\beta_{1k}\}'$, that occur in the expansion term $(\beta_{11}\underline{x}_1 + \ldots + \beta_{1k}\underline{x}_k)^{(j)}$, then using the instrument $\hat{\underline{y}}^{(j)}$ will reduce the accurate of the POSEX model.

As an alternative to using the instrument $\hat{\underline{y}}$, one could choose $(e_1\underline{x}_1 + \ldots + e_k\underline{x}_k)$, which is another linear combination of $\underline{x}_1,\ldots,\underline{x}_n$, which might, in general, be more highly correlated with the term $(\beta_{11}\ \underline{x}_1 + \ldots + \beta_{1k}\ \underline{x}_k)$. Since correlation is a measure of how two groups of variables vary with respect to one another, the coefficients $e_1,\ldots,e_k$ should be chosen by examining the variance within each of the vectors $\underline{x}_1,\ldots,\underline{x}_k$. However, the variance within a vector is not the only important characteristic to be taken into consideration. The coefficients $e_1\ldots e_k$ must also reflect the scale of each of the vectors $\underline{x}_1,\ldots,\underline{x}_k$. For example, if the sample variances of each vector are identical, the vector that has the smaller elements (the smallest mean) should be given more weight. The rationale for this might not be immediately apparent, but an example will clarify the point. Three observations are drawn from two populations resulting in the samples (990, 1000, 1010) and (10, 20, 30). The sample variance is 100 in both cases. However, the variance of 100 results in a 2% variation $(=\frac{20}{1000}\cdot 100)$ in the sample points in the first sample and a 100% variation $(=\frac{20}{20}\cdot 100)$ in the sample points in the second sample. Hence, since the variation in a variable, not the variance of a variable, is the important characteristic, the coefficients $e_1\ldots e_k$ must be chosen by a method that takes into account both the variance and the mean of each vector of independent variables.

One such technique is the method of principal components.[2] To use this technique, one first forms the k x k matrix of squares and cross-productions (Σ) from the vectors $\underline{x}_1,\ldots,\underline{x}_k$, that is

$$\Sigma = \begin{pmatrix} \underline{x}_1' \\ \vdots \\ \underline{x}_k' \end{pmatrix} (\underline{x}_1 \ldots \underline{x}_k)$$

Second, one finds the eigenvalues and eigenvectors of the matrix Σ. Selecting the largest of these eigenvalues and denoting the eigenvector associated with that eigenvalue as $\begin{pmatrix} e_1 \\ \vdots \\ e_k \end{pmatrix}$, one can define the vector $\underline{p}$ (the first principal component of the matrix X) as

$$\underline{p} = e_1\, \underline{x}_1 + \ldots + e_k\, \underline{x}_k.$$

This vector $\underline{p}$ is calculated in such a way that whichever vector $\underline{x}_1,\ldots,\underline{x}_k$ has the most variation (reflecting both mean and variance) has the largest coefficient. The one with the second most variation has the second largest coefficient, etc.

Finally, this vector possesses the statistical property of being the best linear predictor of the vectors $\underline{x}_1,\ldots,\underline{x}_k$. This is easily shown by noting that no other normalized, linear combination of the variables $\underline{x}_1,\ldots,\underline{x}_k$ has a greater variance than does the vector $\underline{p}$. Therefore, no other normalized, linear combination of the $\underline{x}$'s contains as much of the variability that is in the vectors $\underline{x}_1,\ldots,\underline{x}_k$ than does the vector $\underline{p}$. Hence, no other combination can

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[2] It was first suggested that principal component analysis be used in conjunction with Ramsey's RESET test by Professor Dudley Wallace. His suggestion is greatly appreciated.

predict the vectors $\underline{x}_1,\ldots,\underline{x}_k$ better than the vector $\underline{p}$. This is not to say, however, that the coefficients $e_1,\ldots,e_k$ have any greater probability of equalling the parameters $\beta_{11},\ldots,\beta_{1k}$ (parameters from the term to be replaced) than did the estimates $\hat{\beta}_1,\ldots,\hat{\beta}_k$ (the estimates used to calculate Ramsey's instrument $\hat{\underline{y}}$). It only says that given no knowledge as to the vector $\{\beta_{11},\ldots,\beta_{1k}\}'$ no vector of weights $\{w_1,\ldots,w_k\}' = \underline{w}$ will produce a vector $X\ \underline{w}$ that contains more variation than does the vector $X\ \underline{e} = \underline{p}$. Therefore, since the unknown variability of the dependent variable is what is trying to be captured, no other vector can do a better job than the vector $\underline{p}$.

Hence, if a POSEX model is used to approximate an analytic multivariate function, no single instrumental variable should, in general, provide as good an approximation as that obtained by using the vector $\underline{p}$. However, since the vector $\underline{p}$ is more difficult to calculate than the vector $\hat{\underline{y}}$, any decision as to which should, in general, be used becomes more difficult. A sampling experiment will be conducted later in this study to provide some insight into whatever trade-offs might exist between the two instruments. It is, however, evident that a POSEX model can be formulated to approximate, to varying degrees of accuracy, any analytic univariate or multivariate function. Hence, besides providing the foundation for the two specification error tests which will be next presented in this study, one hopes that this technique might be adapted to further uses by other researchers.

II.2 POSEX Test for a Non-zero Mean

If one hypothesizes the model

$$y_i = \beta_1 + \beta_2 x_{i2} + \dots + \beta_k x_{ik} + u_i, \quad i=1,\dots,n, \tag{2.12}$$

where it is supposed that the vector of disturbance terms $\underline{u} = (u_1,\dots,u_n)'$ is distributed as $N(\emptyset, \sigma^2 I)$, the null hypothesis of a zero mean vector is that the $E(\underline{u}) = \emptyset$ whereas the alternative hypothesis is that the $E(\underline{u}) = \underline{z} \neq \emptyset$. This formulation is used in three tests which have been developed to test for the null versus the alternative hypothesis (recall the tests developed by Ramsey, Ramsey & Gilbert, and Ramsey & Schmidt referred to earlier). However, the hypothesis space can be similarly divided by yet another criterion. Rewriting model (2.12) in matrix notation, one obtains

$$\underline{y} = X\,\underline{\beta} + \underline{u}, \quad \underline{u} \sim N(\emptyset, \sigma^2 I) \tag{2.13}$$

where $\underline{y}$ and $\underline{u}$ are (n x 1) column vectors, $\underline{\beta}$ is a (k x 1) column vector, and X is an (n x k) matrix of rank k. If X is independent of $\underline{u}$, $\underline{y}$ is distributed as $\underline{u}$ with a mean of $X\underline{\beta}$; that is, $\underline{y}$ is distributed as $N(X\underline{\beta}, \sigma^2 I)$. Given this formulation, the hypothesis space can be divided exactly as before by basing the division on the mean of the vector $\underline{y}$. The null hypothesis then would be that the $E(\underline{y}) = X\underline{\beta}$ (referred to in this section as H_0), instead of $E(\underline{u}) = 0$, and the alternative hypothesis would be $E(\underline{y}) \neq X\beta$ instead of $E(\underline{u}) \neq 0$.

Using this formulation of the hypothesis space is convenient since the $\underline{y}$'s, unlike the $\underline{u}$'s, are observable. This procedure totally eliminates the need to select a predictor for the disturbance terms. Using this formulation of the hypothesis space, a POSEX model will be developed which will estimate the conditional mean of the dependent variables $y_1,\dots,y_n$.

II.2.1 Formulating the POSEX Model and Testing Procedure

Under the null hypothesis, the $\underline{y}$'s are generated by model (2.13), whereas under the alternative hypothesis, the $\underline{y}$'s are generated either by some other function (non-linear) of the hypothesized variables $\underline{x}_1,\ldots,\underline{x}_k$ or by some function (maybe linear) of the hypothesized variables $\underline{x}_1,\ldots,\underline{x}_k$ and of the erroneously excluded variables $\underline{z}_1,\ldots,\underline{z}_m$. Hence, under H_0, $\underline{y}$ is a simple linear function of $\underline{x}_1,\ldots,\underline{x}_k$ while under H_1, $\underline{y}$ is some unknown function of $\underline{x}_1,\ldots,\underline{x}_k$ and $\underline{z}_1,\ldots,\underline{z}_m$. Under H_1, depending on the number of omitted variables m ($\geq$ 0) and on their relation to $\underline{x}_1,\ldots,\underline{x}_k$ (the necessary relation will be investigated later in this section), the vector $\underline{y}$ can be approximated by the variables $\underline{x}_1,\ldots,\underline{x}_k$. Also, under H_1, the unknown function can be approximated by formulating a POSEX model in the variables $\underline{x}_1,\ldots,\underline{x}_k$. Using the POSEX technique, which was previously described and illustrated in model (2.5), one formulates the model

$$\begin{aligned} \underline{y} = f(\underline{x}_1,\ldots,\underline{x}_k) + \underline{u} &\doteq \beta_{11} + \beta_{12}\,\underline{x}_2 + \ldots + \\ &\beta_{1k}\,\underline{x}_k + \alpha_2\,\underline{q}^{(2)} + \alpha_3\,\underline{q}^{(3)} + \alpha_4\,\underline{q}^{(4)} + \underline{u} \end{aligned} \tag{2.14}$$

where $E(\underline{u}) = 0$, and where a four-term expansion is used (same as in Ramsey's RESET model). Because two instruments have been proposed as the vector $\underline{q}$, $\hat{\underline{y}}$ (the OLS predictor of $\underline{y}$) by Ramsey and $\underline{p}$ (the first principal component of the matrix X) by Wallace, both instruments in turn will be used. Later on in this study, they will be compared to determine which provides the better instrument.

Model (2.14) must now be examined under H_0 so that a test for discriminating between H_0 and H_1 can be formulated. Under H_0 that model (2.13) generated and dependent variable $\underline{y}$, model (2.14) becomes

$$E(\underline{y}) = \beta_1 + \beta_2 \underline{x}_2 + \dots + \beta_k \underline{x}_k + 0 \cdot \underline{q}^{(2)} + 0 \cdot \underline{q}^{(3)} + 0 \cdot \underline{q}^{(4)}$$

$$= x\underline{\beta} + Q \cdot \underline{0}.$$

Hence, to test the null hypothesis that (2.13) generated the vector $\underline{y}$, one only need test the hypothesis that $\alpha_2 = \alpha_3 = \alpha_4 = 0$. If the parameters α_2, α_3 and α_4 are found to be jointly equal to zero, H_0 is not rejected, whereas if α_2, α_3 and α_4 are found to be jointly different from zero, H_0 is rejected. This hypothesis is easily tested by using an F-test for the included variables $\underline{q}^{(2)}$, $\underline{q}^{(3)}$, and $\underline{q}^{(4)}$ [Goldberger, 1964, pp. 174-175].

The procedure is to estimate model (2.13) and model (2.14) by the method of least squares. Denoting $\hat{\underline{y}}_1$ and $\hat{\underline{y}}_2$ as the OLS predictors of the vector $\underline{y}$ from model (2.13) and model (2.14) respectively and $\hat{\underline{u}}$ as the OLS residuals from model (2.14), one calculates the ratio

$$\left(\frac{\hat{\underline{y}}_2' \hat{\underline{y}}_2 - \hat{\underline{y}}_1' \hat{\underline{y}}_1}{\hat{\underline{u}}' \hat{\underline{u}}}\right) \Big/ \left(\frac{n - k - 3}{3}\right).$$

This statistic is distributed as F with 3 and n-k-3 degrees of freedom under H_0 because the ratio can be rewritten in terms of two independent quadratics in the normally distributed disturbance term $\underline{u}$.

In examining the F statistic under the alternative hypothesis an interesting observation can be made. Denoting w_i as the portion of the 'true' model that remains unexplained by the hypothesized model, the quadratic ratio becomes

$$\frac{(\underline{u}' + \underline{w}') Q_1 (\underline{u} + \underline{w})}{(\underline{u}' + \underline{w}') Q_2 (\underline{u} + \underline{w})},$$

where $\underline{u}$ is the disturbance term and Q_1 and Q_2 are the appropriate quadrations. Since this is a ratio of two non-central χ^2's, this ratio can be greater than or less than one. Therefore, a two-tailed F-test should be used.

A well defined test statistic has been developed using this formulation. Hence, no matter what the sample size, the distribution of the statistic is known. Also, unlike Ramsey's and Ramsey & Schmidt's test procedures,with this formulation, the use of predictors of the disturbance term $\underline{u}$ and the calculation of the matrix M or A' are avoided. Therefore, if a researcher uses the new formulation of setting up a POSEX model to explain the vector $\underline{y}$, he avoids both of the difficulties associated with Ramsey's and Ramsey & Schmidt's testing procedures. Finally, it has been pointed out that the appropriate test is a two tailed F test and not a one tailed test as was mistakenly used by the previous authors.

II.2.2 Comparison With Previous Testing Procedures

It is interesting to note that formulating a POSEX model to improve the estimate of the conditional mean of $\underline{y}$ can, under certain conditions, be shown equivalent to Ramsey's and Ramsey & Schmidt's testing procedures which determine whether a disturbance term has a non-zero mean. Of course, as has previously been shown, the hypothesis space can be equivalently divided by setting up the null and alternative hypotheses in terms of the vector of disturbance terms $\underline{u}$ or vector of dependent variables $\underline{y}$.

Assume that the hypothesized model is

$$\underline{y} = \beta_1 + \beta_2 \underline{x}_2 + \dots + \beta_k \underline{x}_k + \underline{u} = X\underline{\beta} + \underline{u}$$

where $\underline{u}$ is assumed to be distributed $N(\emptyset, \sigma^2 I)$ under H_0. Setting up a POSEX model to estimate the conditional mean of $\underline{y}$, and using $\hat{\underline{y}}$

(the OLS predictor of $\underline{y}$ obtained after the hypothesized model is estimated as the instrument, one obtains

$$\begin{aligned}\underline{y} &= \beta_{11} + \beta_{12}\,\underline{x}_2 + \cdots + \beta_{1k}\,\underline{x}_k + \alpha_2\,\hat{\underline{y}}^{(2)} + \alpha_3\,\hat{\underline{y}}^{(3)} + \alpha_4\,\hat{\underline{y}}^{(4)} + \underline{w}, \\ &= X\underline{\beta} + \alpha_2\,\hat{\underline{y}}^{(2)} + \alpha_3\,\hat{\underline{y}}^{(3)} + \alpha_4\,\hat{\underline{y}}^{(4)} + \underline{w}, \qquad (2.15)\end{aligned}$$

where $\underline{w}$ is assumed to be distributed $N(\emptyset, \sigma^2 I)$ under H_0.

If model (2.15) is premultiplied by the matrix A' (recall that the BLUS residual vector $\underline{u}^* = A'\underline{y}$, where $A'X = 0$, $A'A = I_{n-k}$ and $AA' = M = (I-X(X'X)^{-}X'))$, the model becomes

$$\begin{aligned}A'\underline{y} = \underline{u}^* &= A'\,X\underline{\beta} + A'\hat{\underline{y}}^{(2)}\alpha_2 + A'\hat{\underline{y}}^{(3)}\alpha_3 + A'\hat{\underline{y}}^{(4)}\alpha_4 + A'\underline{w} \\ &= \tilde{\underline{y}}^{(2)}\alpha_2 + \tilde{\underline{y}}^{(3)}\alpha_3 + \tilde{\underline{y}}^{(4)}\alpha_4 + A'\underline{w} \qquad (2.16)\end{aligned}$$

where $A'\underline{w}$ is distributed as $N(\emptyset, \sigma^2 I_{n-k})$ under H_0 and where $A'\hat{\underline{y}}^{(j)} = \tilde{\underline{y}}^{(j)}$. Model (2.16) is the model in which Ramsey tested $\alpha_2 = \alpha_3 = \alpha_4 = 0$ and hence obtained his RESET test for the disturbance term $\underline{u}$'s having a non-zero mean.

Likewise, if model (2.15) is premultiplied by the matrix M (recall that the OLS residual vector $\hat{\underline{u}} = M\underline{y} = (I-X(X'X)^{-}X')\underline{y})$, the model becomes

$$\begin{aligned}M\underline{y} = \hat{\underline{u}} &= MX\underline{\beta} + M\hat{\underline{y}}^{(2)}\alpha_2 + M\hat{\underline{y}}^{(3)}\alpha_3 + M\hat{\underline{y}}^{(4)}\alpha_4 + M\underline{w} \\ &= M\hat{\underline{y}}^{(2)}\alpha_2 + M\hat{\underline{y}}^{(3)}\alpha_3 + M\hat{\underline{y}}^{(4)} + M\underline{w} \qquad (2.17)\end{aligned}$$

where $M\underline{w}$ is distributed $N(\emptyset, \sigma^2 M)$ under H_0. Model (2.17) is Ramsey & Schmidt's model whereby they were able to test for the disturbance term $\underline{u}$'s having a non-zero mean by testing if $\alpha_2 = \alpha_3 = \alpha_4 = 0$.

Hence, since Ramsey's and Ramsey & Schmidt's models can be obtained from the POSEX model by premultiplying the POSEX model by either the matrix A' or the matrix M, respectively, all three models are mathematically equivalent. Furthermore, since all three tests use an F-test to determine if the parameters α_2, α_3 and α_4 are different from zero, all three tests are likewise mathematically equivalent. Therefore, since neither of the previously reviewed tests offer any advantage over formulating a POSEX model and yet both offer the disadvantage of compelling the researcher to calculate either the matrix A' or the matrix M, there is no apparent reason to use either Ramsey's or Ramsey & Schmidt's testing procedure.

II.2.3 Examination of the New Testing Procedure Under H_1

As has been mentioned, two basic errors can cause the vector of disturbance terms $\underline{u}$ to have a non-zero mean; likewise, the same two basic errors can cause the vector of dependent variables $\underline{y}$ to have a conditional mean other than $X\underline{\beta}$. The first error occurs when the wrong functional form of the regressors or regressand is used in the hypothesized model. The second error occurs when a number of relevant independent variables are omitted in the hypothesized model. The new testing procedure will be examined under both these errors.

Incorrect Functional Form of Either the Regressors or Regressand

If a researcher hypothesizes model (2.12) whereas the dependent variable $\underline{y}$ is actually generated by the model

$$\underline{y} = f(\underline{x}_1, \ldots, \underline{x}_k) + \underline{v} \tag{2.18}$$

where $\underline{v}$ is distributed as $N(\emptyset, \sigma^2 I)$ and $f(\cdot)$ is some function other than the one hypothesized in model (2.12), a specification error has been committed. Note that although this specification error is caused by incorrectly hypothesizing the functional form of the regressors, a similar error can be caused by incorrectly hypothesizing the functional form of the regressand (see section I.2). Hence, only the former error will be examined in this study.

Therefore, when a researcher hypothesizes model (2.12) while the dependent variable actually has been generated by model (2.18), the specification error which has been committed is usually that of incorrectly specifying the functional form of either the regressors or regressand. If the researcher suspects this error, he may want to examine whether a POSEX model would better explain the conditional mean of $\underline{y}$ than would model (2.12). If the POSEX model does better explain the conditional mean of $\underline{y}$, the researcher knows that model (2.12) was misspecified. The POSEX model that the researcher would formulate is

$$\begin{aligned}\underline{y} &= \beta_{11} + \beta_{12}\,\underline{x}_2 + \cdots \pm \beta_{1k}\,\underline{x}_k + \alpha_2\,\underline{q}^{(2)} + \alpha_3\,\underline{q}^{(3)} + \alpha_4\,\underline{q}^{(4)} + \underline{w} \\ &= X\beta + Q\underline{\alpha} + \underline{w} \qquad (2.19)\end{aligned}$$

where $\underline{w}$ was assumed to be distributed $N(\emptyset, \sigma^2 I)$ and where $\underline{q}$ is used to represent either the instrument $\hat{\underline{y}}$ (OLS predictor of $\underline{y}$ obtained after estimation of model (2.12)) or the instrument $\underline{p}$ (the first principal component of the matrix X). If the estimate of $\underline{\alpha}$ is statistically different from zero, the null hypothesis that model (2.12) generated the vector $\underline{y}$ will be correctly rejected.

The probability of this test's correctly rejecting H_0 depends largely on the function $f(\cdot)$ and on the instrument $\underline{q}$ chosen. First, as previously stated, $f(\cdot)$ must be analytic, since a non-analytic function cannot be expressed as a power series expansion. Second, since the POSEX model proposed involves a four term expansion, one must be able to approximate $f(\cdot)$ using only a four term expansion in the variables $\underline{x}_1,\ldots,\underline{x}_k$.[3] If $f(\cdot)$ can only be approximated using

[3]Although four terms has been suggested in this study, any number of terms may be used. There is, however, a trade off since adding more terms changes the number of degrees of freedom involved in the proposed test.

more than a four term expansion in $\underline{x}_1,\ldots,\underline{x}_k$, the POSEX model which is given in the equation (2.19) will provide a poor approximation; hence the testing procedure suggested will prove unreliable. However, since most of the standard non-linear functions are analytic, and since a good approximation of most standard analytic functions can be obtained using as few as two or three expansion terms (for example, the exponential, logorithmic, and sinosoidal functions are all approximated in three or fewer expansion terms [Thomas, 1966]), these conditions should generally cause no difficulty.

Finally, the probability of correctly rejecting H_0 will also vary in accordance with the correlation between the instrument $\underline{q}$ and the expansion terms which it replaces. That is, since $\underline{q}$ is an instrument (representing either $\hat{\underline{y}}$, the OLS predictor of $\underline{y}$ obtained from the hypothesized model, or $\underline{p}$, the first principal component of the matrix X), this statement simply means that the test's power varies with the quality of the instrument used.

In summary, if a model is misspecified because the functional form of $\underline{x}_1,\ldots,\underline{x}_k$ is incorrectly hypothesized, the power of the suggested test depends on two factors. The first factor, the functional form of $f(\cdot)$ which generates the vector $\underline{y}$, does not generally cause difficulties. The reason for this is that the functional forms of $f(\cdot)$ generally thought probable are both analytic and easily approximated by using a power series expansion (two examples are the exponential function and the logorithmic function). The other factor responsible for causing a loss in the test's power is the instrument chosen to replace the expansion terms. It is felt by this investigator that the first principal component $\underline{p}$

will, however, in general, provide a reliable instrument for the expansion terms. It must be recalled, nevertheless, that the OLS predictor of $\underline{y}$, $\hat{\underline{y}}$, has been successfully used as an instrument for the expansion terms by Ramsey. This instrument has the advantage of being more easily obtained than $\underline{p}$. This investigator feels, however, that $\hat{\underline{y}}$ will, in general, be less highly correlated with the expansion terms than will $\underline{p}$, and hence be less reliable. Any final conclusion as to which of the two instruments is the more reliable must of course be postponed until they are actual compared in a sampling experiment.

Omitted Variables

Assume that once again a researcher hypothesizes model (2.12)

$$\underline{y} = \beta_1 + \beta_2 \underline{x}_2 + \dots + \beta_k \underline{x}_k + \underline{u} = X \underline{\beta} + \underline{u},$$

where it is assumed that $\underline{u} \sim N(\emptyset, \sigma^2 I)$, whereas the model that actually generated the dependent variable $\underline{y}$ is

$$\underline{y} = X \underline{\beta} + \underline{z}_1 \delta_1 + \dots + \underline{z}_m \delta_m + \underline{v} = X \underline{\beta} + z \underline{\delta} + \underline{v}, \qquad (2.20)$$

where $\underline{v} \sim N(\emptyset, \sigma^2 I)$. Model (2.12) is misspecified because m independent variables, $\underline{z}_1,\dots,\underline{z}_m$, have been omitted. If the researcher suspects that he has inadvertently omitted some variables, he can formulate a POSEX model to explain the conditional mean of $\underline{y}$. If, in a statistical sense, the POSEX model explains the conditional mean of $\underline{y}$ better than does model (2.12), the indication is that the model (2.12) is misspecified.

In the POSEX model which the researcher would use to explain the condition mean of $\underline{y}$, the variables $\underline{x}_1,\dots,\underline{x}_k$ would be used in the expansion. It must be remembered that the researcher suspects that he may have erroneously omitted some variables; however, he does not know the identity of the variables which he may have omitted. The POSEX model thus formulated would be

$$\underline{y} = \beta_{11} + \beta_{12}\,\underline{x}_2 + \cdots + \beta_{1k}\,\underline{x}_k + \alpha_2\,\underline{q}^{(2)} + \alpha_3\,\underline{q}^{(3)} + \alpha_4\,\underline{q}^{(4)} + \underline{w}$$

$$= x\,\underline{\beta} + Q\,\underline{\alpha} + \underline{w} \tag{2.21}$$

where $\underline{w}$ is assumed to distributed $N(\emptyset,\ \sigma^2 I)$ and where $\underline{q}$ represents either the instrument $\hat{\underline{y}}$ or the instrument $\underline{p}$. If the estimates of $\underline{\alpha}$ are found to be statistically different from zero, the model hypothesized as generating the vector $\underline{y}$ (model (2.12)) is found to be misspecified.

Needless to say, the probability of this test's correctly rejecting H_0 depends on the relationship between the variables erroneously omitted and the instruments used in place of the expansion terms. Since the idea that the power of the test depends on the instrument chosen has already been discussed, further elaboration is not needed here. Rather, this section will focus on how the test's power is affected by the characteristics of the variables omitted.

In order that the analysis which follows will not be unnecessarily complicated, it will be assumed that only one variable is omitted erroneously from the hypothesized model (2.12). Assume that the model which actually generated the vector of dependent variables $\underline{y}$ is

$$\underline{y} = \beta_1 + \beta_2\,\underline{x}_2 + \cdots + \beta_k\,\underline{x}_k + \delta\,\underline{z} + \underline{v}$$

$$= x\,\underline{\beta} + \underline{z}\,\delta + \underline{v} \tag{2.22}$$

where $\underline{v} \sim N(\emptyset,\ \sigma^2 I)$, and $\underline{z}$ is a non-stochastic vector.

Since model (2.22) and model (2.21), used to test whether the null hypothesis is misspecified, differ only in their second terms, the second model's ability to discriminate between H_0 ($\underline{\alpha} = 0$) and H_1 ($\underline{\alpha} \neq 0$) is directly related to the proportion of the vector $\underline{z}$ that lies in the space spanned by the matrix Q. Although this cursory observation is somewhat illuminating, a more indepth analysis is required.

Since the instrument $\underline{q}$ (either $\underline{p}$ or $\hat{\underline{y}}$) is a function of $\underline{x}_1,\ldots,\underline{x}_k$, the omitted variable can be characterized as one of three types, depending on the omitted variable's relation to the variables $\underline{x}_1,\ldots,\underline{x}_k$. The first type of omitted variable is highly correlated with the variables $\underline{x}_1,\ldots,\underline{x}_k$, the next type is uncorrelated with them, and the final type is moderately correlated with them.

To simplify the analysis of each type of omitted variable, all of the variables $\underline{x}_1,\ldots,\underline{x}_k$ and $\underline{z}$ will be orthogonalized. This linear transformation yields the k + 1 vectors $\underline{a}_1,\ldots,\underline{a}_k$, $\underline{a}_{k+1}$ corresponding respectively to the vectors $\underline{x}_1,\ldots,\underline{x}_k$, $\underline{z}$. Thus, the vector $\underline{a}_{k+1}$ contains only that part of the vector $\underline{z}$ which is not already explained by the variables $\underline{x}_1,\ldots,\underline{x}_k$. The three cases of $\underline{z}$ to be analyzed, having either high, low, or medium correlation with $\underline{x}_1,\ldots,\underline{x}_k$ will correspond directly to the vector $\underline{a}_{k+1}$ containing either little, a great deal, or moderate amounts of additional information.

The reason for this inverse relation between the amount of information contained in the vector $\underline{a}_{k+1}$ and the correlation between $\underline{z}$ and $\underline{x}_1,\ldots,\underline{x}_k$ is that the latter measures the amount of linear relation between $\underline{z}$ and $\underline{x}_1,\ldots,\underline{x}_k$ while the former contains the amount of information remaining after any linear relation has been removed. For example, when the vector $\underline{z}$ is highly correlated with $\underline{x}_1,\ldots,\underline{x}_k$

(that is, a large portion of the information embodied in $\underline{z}$ is also contained in $\underline{x}_1,\ldots,\underline{x}_k$), and the linear information is removed through orthogonalization, the vector $\underline{a}_{k+1}$ will contain very little information. Hence, high correlation between z and $\underline{x}_1,\ldots,\underline{x}_k$ will imply very little additional information in the vector $\underline{a}_{k+1}$.

Finally, before each type of omitted variable is analyzed in turn, it is important to stress that only the linear relation between $\underline{z}$ and $\underline{x}_1,\ldots,\underline{x}_k$ has been eliminated. Hence, there is no implication that vector $\underline{a}_{k+1}$ is independent of the vectors $\underline{a}_1,\ldots,\underline{a}_k$, but only that $\underline{a}_{k+1}$ is uncorrelated with the vectors $\underline{a}_1,\ldots,\underline{a}_k$.

Omitted Variable Highly Correlated with the Matrix X

In the case of this type of omitted variable, if $\underline{z}$ were correctly added to the hypothesized model (2.12), the model would be highly multicolinear. When the variable $\underline{z}$, however, is erroneously omitted, efficiency will be lost, but the loss will be small. Unfortunately, though, there is always a cost involved when a specification error is made. In this case, the estimates of the parameters $\beta_1,\ldots,\beta_k$, in the hypothesized model (2.12) will be biased. As previously mentioned, the amount of the bias associated with each estimate depends on the correlation between the variable associated with that parameters and the variable $\underline{z}$.

If a POSEX model is used to determine whether model (2.12) is misspecified when the omitted variable $\underline{z}$ is highly correlated with the variables $\underline{x}_1,\ldots,\underline{x}_k$, the probability that the POSEX model will better explain the conditional mean of $\underline{y}$ is very small. The reason for this is that so little information is left in the vector $\underline{a}_{k+1}$

that even if it were explained by the POSEX model, it still may not provide a statistical improvement over the hypothesized model.

Finally, even though the testing technique being suggested does not offer a very high probability of correctly rejecting H_0 when the omitted variable is highly correlated with the included variables $\underline{x}_1,\ldots,\underline{x}_k$, the cost of such an error is low. A small loss in efficiency will occur and biased estimates of $\beta_1,\ldots,\beta_k$ will be obtained. However, even if the omitted variable $\underline{z}$ had been correctly included, the model would have been multicollinear; hence, the matrix X'X would be ill-conditioned, so that the estimates of $\beta_1,\ldots,\beta_k$ and σ^2 (the model's variance) have relatively large standard errors and the estimates are very sensitive to small perterbations in the values of the regressors. Therefore, the incorrect omission of the variable $\underline{z}$ is relatively inconsequential even though the omission cannot be detected by the POSEX test.

Omitted Variable Uncorrelated with the Matrix X

In the case of this type of omitted variable, the vector $\underline{z}$ is virtually identical to the vector $\underline{a}_{k+1}$. When the variable $\underline{z}$, which is uncorrelated with $\underline{x}_1,\ldots,\underline{x}_k$, is erroneously omitted, two difficulties arise. First, since $\underline{z}$ is uncorrelated with $\underline{x}_1,\ldots,\underline{x}_k$, only the constant term $\hat{\beta}_1$ will be biased. The amount of the bias will equal $\overline{z} = \sum_{i=1}^{n} z_i/n$; hence, the expected value of the estimator $\hat{\beta}_1$ will be $E(\hat{\beta}_1) = \beta_1 + \overline{z}$. Also, since none of the variation embodied in $\underline{z}$ is used to explain the conditional mean of $\underline{y}$, the hypothesized model (2.12) will be inefficient.

If a POSEX model were able to explain the conditional mean of $\underline{y}$ better than the hypothesized model (2.12), one of two things could occur. First, if the vector $\underline{z}$ is independent of as well as uncorrelated with the vectors $\underline{x}_1,\ldots,\underline{x}_k$, the POSEX model will have virtually no power. The reason for this is that since $\underline{z}$ is independent of $\underline{x}_1,\ldots,\underline{x}_k$, and as $\underline{q}$ is a linear combination of $\underline{x}_1,\ldots,\underline{x}_k$, the vector $\underline{z}$ is independent of the vector $\underline{q}$ as well as of $\underline{q}^{(2)}$, $\underline{q}^{(3)}$, and $\underline{q}^{(4)}$. Hence, in the POSEX model (2.21), since the vector $\underline{z}$ is independent of the vectors $\underline{x}_1,\ldots,\underline{x}_k$, $\underline{q}^{(2)}$, $\underline{q}^{(3)}$, $\underline{q}^{(4)}$, the POSEX model adds nothing to the hypothesized model.

In the second case, however, the POSEX model will improve upon the hypothesized model if $\underline{z}$ is not independent of $\underline{x}_1,\ldots,\underline{x}_k$. Generally, however, in economic data, if the variable $\underline{z}$ is uncorrelated with the variables $\underline{x}_1,\ldots,\underline{x}_k$, it is also independent of $\underline{x}_1,\ldots,\underline{x}_k$. Hence, the analysis of this case will be postponed until the next section.

Therefore, when $\underline{z}$ is uncorrelated with the variables $\underline{x}_1,\ldots,\underline{x}_k$, the POSEX model again proves to be of little use in detecting the error. However, once again, some consolation can be taken in the fact that when an uncorrelated variable is omitted, only the constant term and the estimate of the variance will be biased.

Omitted Variable Somewhat Correlated With the Matrix X

In the last case, which is the most common, the vector $\underline{z}$ is neither uncorrelated nor highly correlated with the vectors $\underline{x}_1,\ldots,\underline{x}_k$. Hence, in this case, because $\underline{z}$ is correlated with $\underline{x}_1,\ldots,\underline{x}_k$, the estimates of $\beta_1,\ldots,\beta_k$ are biased, however, since $\underline{z}$ is not highly correlated with $\underline{x}_1,\ldots,\underline{x}_k$, the estimators of $\beta_1,\ldots,\beta_k$ are not efficient. Therefore, this type of omitted variable can cause all of the estimators in the hypothesized model, $\hat{\beta}_1,\ldots,\hat{\beta}_k$, $\hat{\sigma}^2$ (model's variance) to be biased.

This most troublesome type of omitted variable, however, is the case where a POSEX model might better (in a statistical sense) estimate the conditional mean of y_i than did the hypothesized model. In this case, unlike that in which $\underline{z}$ was highly correlated with $\underline{x}_1,\ldots,\underline{x}_k$, the vector $\underline{a}_{k+1}$ still contains some information; hence, a POSEX model can improve on the hypothesized model by estimating the variation in the vector $\underline{a}_{k+1}$. Also, unlike the case in which $\underline{z}$ was uncorrelated with $\underline{x}_1,\ldots,\underline{x}_k$ (and hence maybe independent), $\underline{a}_{k+1}$ is not necessarily independent of the vectors $\underline{x}_1,\ldots,\underline{x}_k$ either squared, cubed, or quadrupled. Therefore, the ability of the POSEX model (2.21) to provide a better estimate of the conditional mean of $\underline{y}$ than did the hypothesized model (2.12) depends on how great a portion of $\underline{z}$ and hence $\underline{a}_{k+1}$ lies in the space spanned by $\underline{q}^{(2)}$, $\underline{q}^{(3)}$, and $\underline{q}^{(4)}$. First, since $\underline{q}$ is a linear combination of the vectors $\underline{x}_1,\ldots,\underline{x}_k$; then $\underline{q}^{(2)}$, $\underline{q}^{(3)}$, and $\underline{q}^{(4)}$ are functions of the vectors $\underline{x}_1,\ldots,\underline{x}_k$ squared, cubed, and quadrupled, respectively.

In addition, since only the linear relation between the vectors $\underline{x}_1,\ldots,\underline{x}_k$ and the vector $\underline{z}$ has been removed from the vector $\underline{z}$ (resulting in the vector $\underline{a}_{k+1}$), it is not unreasonable to expect that $\underline{q}^{(2)}$, $\underline{q}^{(3)}$, and $\underline{q}^{(4)}$ might be able to explain still more of the variation given in the vector $\underline{a}_{k+1}$. The reason for this is the point that has been stressed over and over again; since only the linear relation between the vector $\underline{z}$ and the vectors $\underline{x}_1,\ldots,\underline{x}_k$ has already been explained by the hypothesized model (resulting in the vector $\underline{a}_{k+1}$), there is no reason to assume that a relation between $\underline{a}_{k+1}$(or $\underline{z}$) and the vectors $\underline{x}_1^{(2)},\ldots,\underline{x}_k^{(2)}$, or $\underline{x}_1^{(3)},\ldots,\underline{x}_k^{(3)}$, or $\underline{x}_1^{(4)},\ldots,\underline{x}_k^{(4)}$ does not exist. If such a relation does exist, given that $\underline{q}^{(2)}$, $\underline{q}^{(3)}$, and $\underline{q}^{(4)}$ are combinations of these vectors, the variation in the vector $\underline{a}_{k+1}$ might be better explained. If the POSEX model provides a better estimate of the conditional mean of $\underline{y}$ because it uses part of the variation in the vector $\underline{a}_{k+1}$, then the estimate of the vector of parameters $\underline{\alpha}$ will be statistically different from zero. Hence, model (2.12) will be found to be misspecified because the POSEX model better explained the conditional mean of $\underline{y}$.

Even though it at first appeared as if the third type of omitted variable would cause the most difficulties, it has been demonstrated that a POSEX model can be used more effectively in this case than in the other cases. Of course, the probability of correctly rejecting H_0 depends heavily on that portion of $\underline{z}$ which is spanned by the vectors $\underline{x}_1,\ldots,\underline{x}_k$ squared, cubed, and quadrupled.

It has been shown that the probability of correctly rejecting H_0, when a variable $\underline{z}$ has been omitted by using an F-test and a POSEX model most certainly depends on the relationship between the variables $\underline{x}_1,\ldots,\underline{x}_k$ and variable $\underline{z}$. It appears as if the power of the procedure is the highest when $\underline{z}$ is moderately correlated with the hypothesized variables $\underline{x}_1,\ldots,\underline{x}_k$. If $\underline{z}$ is uncorrelated with the hypothesized variables, $\underline{z}$ is most probably independent of them and hence independent of any linear combination of them. If $\underline{z}$ is too highly correlated with $\underline{x}_1,\ldots,\underline{x}_k$, then little improvement in explaining the conditional mean of $\underline{y}$ can be ascertained by using a POSEX model.

II.2.4 Summary

In this section, a new testing procedure for determining whether a model has been misspecified has been obtained. It is shown to be equivalent to two current testing procedures, but is also shown to offer the advantage of being more easily formulated and carried out.

When the new test was examined under both common causes of the specification error, it was suggested that the test would be more powerful when an instrument highly correlated with the non-linear term is used. In addition, it was discovered that when the error is caused by incorrectly formulating the functional form of either the regressors or regressand, the power of the test increases if the correct function is analytic and can be approximated easily. Also, finally, when the error is caused by omitting a variable from the hypothesized model, the power is related to the correlation between the omitted variable and the hypothesized variables, the highest power being obtained when the correlation was moderate.

II.3 POSEX Test to Determine if the Disturbance Terms Are Heteroskedastic

Given the hypothesized model

$$\underline{y} = \beta_1 + \beta_2 \underline{x}_2 + \dots + \beta_k \underline{x}_k + \underline{u} = X \underline{\beta} + \underline{u} \qquad (2.23)$$

where all the vectors are n x 1 and where $\underline{u}$ is assumed to be distributed $N(\emptyset, \sigma^2 I)$, a number of tests exist that will compare the null hypothesis (H_0) of homoskedasticity with the alternative hypothesis (H_2) of heteroskedasticity. All of these tests, however, require a great deal of a priori information regarding the variable, presumably $\underline{x}_1,\dots,\underline{x}_k$, that is related to the heteroskedastic disturbances. Since, however, a POSEX model can approximate any analytic function, it is possible to use a general POSEX model to estimate the variance of each of the n disturbance terms. A test can be developed which will determine whether the POSEX model is better able to explain the conditional variance of the vector of disturbance terms $\underline{u}$.

Under the null hypothesis of homoskedasticity, no group of variables (or model) will be able to explain the constant variance of the disturbance terms. Hence, if a model is able to explain the variances of the disturbance terms, the variances are not homoskedastic and the null hypothesis should thus be rejected.

Finally, since a model to explain the variances is used as the basis of the proposed test, this test will be a constructive test. That is, in the case in which the null hypothesis of homoskedasticity in model (2.23) is rejected, a procedure will be offered which will enable the researcher to transform model (2.23) into a homoskedastic model.

Before the model and test are developed, however, an estimator of the unobserved variance of $u_1, \ldots, u_n$ must be selected. The POSEX model will then be developed. Next, it will be shown how the POSEX model can be used to reestimate the parameters in the hypothesized model. Also, it will be demonstrated how a priori information can be included in the POSEX model. Finally, a number of different ways of estimating the POSEX model will be suggested. Included with each of these suggestions will be a test to determine if the disturbance terms are either homoskedastic or heteroskedastic.

II.3.1 Estimators of the Variance of u_i

Unfortunately, the variance of each disturbance term, $\sigma_1^2,\ldots,\sigma_n^2$, is not observable. Hence, a number of estimators of $\sigma_1^2,\ldots,\sigma_n^2$ have been obtained. Denoting $\hat{u}_i$ as the i'th least squares residual obtained from model (2.23), the first estimate of σ_i^2 used (by Park and by Glejser, by Goldfeld & Quant, and by Ramsey & Gilbert) was $\hat{u}_i^2$. Unfortunately, under the null hypothesis of homoskedasticity, $E(\underline{u}\,\underline{u}') = \sigma^2 I$, the least squares residuals are heteroskedastic, $E(\underline{\hat{u}\hat{u}}') = \sigma^2 M = \sigma^2(I-X(X'X)^{-}X')$. Hence, since the diagonal terms of $E(\underline{\hat{u}}\,\underline{\hat{u}}')$ are $E(\underline{\hat{u}}_i^2)$, one finds that the expected value of the n estimates, $\hat{u}_1^2,\ldots,\hat{u}_n^2$ are $\sigma^2 m_{11},\ldots,\sigma^2 m_{nn}$, where m_{ii} is the i'th diagonal of the matrix M. Therefore, $\hat{u}_i^2$ is a biased estimate of σ_i^2 even under H_0.

Under the alternative hypothesis that model (2.27) is heteroskedastic, however, the estimates $\hat{u}_i^2$ become weighted averages of the true variances. Since the diagonal elements of the matrix $\underline{\hat{u}\hat{u}}'$ are $\hat{u}_1^2,\ldots,\hat{u}_k^2$, this weighting scheme is most clearly demonstrated by taking the expected value of the matrix $\underline{\hat{u}\hat{u}}'$ under the alternative hypothesis that the

$$E(\underline{uu}') = \begin{bmatrix} \sigma_1^2 & & \emptyset \\ & \ddots & \\ \emptyset & & \sigma_n^2 \end{bmatrix} = V.$$

One finds that

$$\begin{aligned} \text{DIAG}\,[E(\underline{\hat{u}\hat{u}}')] &= \text{DIAG}\,[E(M\underline{uu}'M)] \\ &= \text{DIAG}\,[ME(\underline{uu}')M] \\ &= \text{DIAG}\,[MVM] \end{aligned}$$

$$= \text{DIAG} \begin{bmatrix} m_{11} \cdots m_{1n} \\ \vdots \quad \vdots \\ m_{n1} \cdots m_{nn} \end{bmatrix} \begin{bmatrix} \sigma_1^2 & & \emptyset \\ & \ddots & \\ \emptyset & & \sigma_n^2 \end{bmatrix} \begin{bmatrix} m_{11} \cdots m_{1n} \\ \vdots \quad \vdots \\ m_{n1} \cdots m_{nn} \end{bmatrix}$$

$$= \text{DIAG} \begin{bmatrix} m_{11}\ \sigma_1^2 \cdots m_{1n}\ \sigma_n^2 \\ \vdots \qquad \vdots \\ m_{n1}\ \sigma_1^2 \cdots m_{nn}\ \sigma_n^2 \end{bmatrix} \begin{bmatrix} m_{11} \cdots m_{1n} \\ \vdots \quad \vdots \\ m_{n1} \cdots m_{nn} \end{bmatrix}$$

$$= \text{DIAG} \begin{bmatrix} \sum_i m_{1i}\ m_{i1}\ \sigma_i^2 \cdots \sum_i m_{1i}\ m_{in}\ \sigma_i^2 \\ \vdots \qquad \vdots \\ \sum_i m_{ni}\ m_{i1}\ \sigma_i^2 \cdots \sum_i m_{ni}\ m_{in}\ \sigma_i^2 \end{bmatrix}$$

Since M is symmetric ($m_{ij} = m_{ji}$), the j'th diagonal element becomes $\sum_{i=1}^{n} m_{ji}^2\ \sigma_i^2$. Hence, if one defines $M^{(2)}$ as being the squared elements of the matrix M (i.e., $\{m_{ij}{}^2\}$), then

$$\text{DIAG } E(\hat{\underline{u}}\hat{\underline{u}}') = M^{(2)} \text{ DIAG } [V] = M^{(2)}\ \{\sigma_1^2,\ldots,\sigma_n^2\}'. \qquad (2.24)$$

Therefore, since under H_2, each estimate $\hat{u}_1{}^2,\ldots,\hat{u}_n{}^2$ is a weighted sum of $\sigma_1^2,\ldots,\sigma_n^2$; $\hat{u}_1{}^2,\ldots,\hat{u}_n{}^2$ are biased estimates of $\sigma_1^2,\ldots,\sigma_n^2$.

Hence, since $\hat{u}_1{}^2,\ldots,\hat{u}_n{}^2$ are biased estimates of $\sigma_1^2,\ldots,\sigma_n^2$ under both H_0 and H_2, it is perplexing to account for the findings of Goldfeld & Quant (using their non-parametric test) and Ramsey & Gilbert (using the BAMSET procedure with OLS residuals). They found, by using sampling procedures, that the probability of type I error corresponded to what was theoretically expected and that the probability of type II error was modest. Of course, the results could have been due to the specific models used and hence to the structure of the matrix X. However, this investigator does not find this explanation at all adequate. Rather a theorem based on the matrix M, will be stated and proven (in Appendix A), and another explanation offered in place of the one just mentioned. (In addition, three interesting corollaries to this theorem are also stated and proven in Appendix A but will not be used in this study.)

Theorem: Regardless of how the vectors $\underline{x}_1,\ldots,\underline{x}_k$ are obtained (stochastic or non-stochastic) the diagonal elements of the matrix M will have a maximum squared variation of $\frac{k(n-k)}{n(n-1)} < \frac{k}{n}$, where squared variation of $t_1,\ldots,t_n$ is defined as $\Sigma(t_i - \bar{t})^2/n\text{-}1$.

This theorem provides a vehicle for understanding the findings of Goldfeld & Quant and of Ramsey & Gilbert. They both observed that under H_0, when OLS estimates of $\sigma_1^2,\ldots,\sigma_n^2$ were used to test H_0 versus H_2, the probability of type I error corresponded to the nominal alpha level at which the test was used. This finding implied that the OLS estimates were homoskedastic under H_0. It has, however, always been assumed that the matrix M has unequal diagonal elements since $E(\hat{u}_i^2) = \sigma^2 m_{ii}$, $i=1,\ldots,n$. Hence the implication that the estimates $\hat{u}_1{}^2,\ldots,\hat{u}_n{}^2$ were homoskedastic seemed difficult to accept; however, the theorem proven in this study provides a plausible explanation for this finding.

It indicates that regardless of how the variables are chosen, the maximum squared variation of the diagonal elements of M is never greater than $\frac{k}{n}$. Therefore as $n \to \infty$ the squared variation $\to$ zero regardless of the matrix X. Further, even with small sample sizes, the variation is minimal if the number of parameters is small. Hence, although OLS residuals may not be homoskedastic when the disturbance terms are homoskedastic, they may appear to be, especially if n is large or k is small.

As previously mentioned, their second finding was that under H_2, the probability of type II error was reasonably small. This implied that the OLS estimates are heteroskedastic even though each estimator is a weighted sum of each true variance, $\sigma_1^2,\ldots,\sigma_n^2$. The type II error which was found is consistent with the theorem and corolaries proven in this study. Even though each of the terms, $\hat{u}_i{}^2, i=1,\ldots,n$, is a weighted sum of $\sigma_1^2,\ldots,\sigma_n^2$, the weights are such that the greatest weight given $\hat{u}_i$ is that associated with σ_i^2. This is evident if one recalls that

$$E(\hat{u}_i^2) = m_{i1}^2\,\sigma_1^2 + \ldots + m_{ii}^2\,\sigma_i^2 + \ldots + m_{in}^2\,\sigma_n^2. \qquad (2.25)$$

Since, however, $m_{i1}^2 + \ldots + m_{in}^2 = m_{ii} \le 1$, because M is idempotent, the portion of the weight given to each variance is

$$\frac{m_{i1}^2}{m_{ii}}, \ldots, \frac{m_{ii}^2}{m_{ii}}, \ldots, \frac{m_{in}^2}{m_{ii}}. \qquad (2.26)$$

Since this series consists only of positive numbers which sum to one, and since the mean of the diagonal elements of the matrix M is $\frac{n-k}{n}$, the series in equation (2.26) is dominated by $\frac{m_{ii}^2}{m_{ii}} = m_{ii}$. Therefore, the weighting scheme given in equation (2.25) favors the term σ_i^2. Hence, when the variances $\sigma_1^2,\ldots,\sigma_n^2$ are unequal (heteroskedastic), the estimates $\hat{u}_1^2,\ldots,\hat{u}_n^2$ are also unequal. Therefore, the finding of Goldfeld & Quant and of Ramsey & Gilbert that the OLS estimates are heteroskedastic when $\sigma_1{}^2,\ldots,\sigma_n^2$ are unequal is correct. As a result, even though $\hat{u}_1{}^2,\ldots,\hat{u}_n{}^2$ are biased estimators for $\sigma_1^2,\ldots,\sigma_n{}^2$ under both H_0 and H_2; under H_0 they are generally homoskedastic while under H_2 they are generally heteroskedastic.

One final difficulty regarding the OLS estimators of $\sigma_1^2,\ldots,\sigma_n^2$ still exists. Since $\hat{u}_1,\ldots,\hat{u}_n$ are not mutually independent, $\hat{u}_1^2,\ldots,\hat{u}_n^2$ are not mutually independent. This lack of independence, it will be recalled, caused certain difficulties in Glejser's and Park's testing procedures.

In large part to solve this problem of independence, Ramsey [1969] suggested another estimator. Since the BLUS residuals, $u_1^*,\ldots,u_{n-k}^*$ are mutually independent, he suggested that the n-k mutually independent estimates $u_1^{*2},\ldots,u_{n-k}^{*2}$ be used to test model (2.23).

These estimates also have the desirable property of being unbiased under H_0. When it is recalled that $\underline{u}^* = A'\underline{u}$, where A' is chosen such that $A'X = 0$, $A'A = I_{n-k}$, $AA' = M$, and that the diagonal elements of $\underline{u}^*\underline{u}^{*\prime}$ are $u_1^{*2},\ldots,u_n^{*2}$, it follows that

$$\begin{aligned}
\text{DIAG}\ [E(\underline{u}^*\underline{u}^{*\prime}) &= \text{DIAG}\ [E(A'\underline{uu}'\ A)] \\
&= \text{DIAG}\ (A'\ \sigma^2 IA) \\
&= \text{DIAG}\ (\sigma^2\ A'A) \\
&= \text{DIAG}\ (\sigma^2\ I_{n-k}) \\
&= \begin{pmatrix} \sigma^2 \\ \vdots \\ \sigma^2 \end{pmatrix}_{n-k}
\end{aligned}$$

However, under H_2, $u_1^{*2},\ldots,u_n^{*2}$ are biased estimators of $\sigma_1^2,\ldots,\sigma_n^2$. In fact, given that

$$E(uu') = \begin{bmatrix} \sigma_1^2 & & \emptyset \\ & \ddots & \\ \emptyset & & \sigma_n^2 \end{bmatrix} = V,$$

it is found that

$$
\begin{aligned}
\mathrm{DIAG}\,[E(\underline{u}^*\underline{u}^{*\prime})] &= \mathrm{DIAG}\,[E(A'\underline{u}\,\underline{u}'A)] \\
&= \mathrm{DIAG}\,[A'VA] \\
&= \mathrm{DIAG} \begin{bmatrix} a_{11} \cdots\cdots a_{1n} \\ \vdots \qquad\quad \vdots \\ a_{1,n-k} \cdots a_{n-k,n} \end{bmatrix} \begin{bmatrix} \sigma_1^2 & & \emptyset \\ & \ddots & \\ \emptyset & & \sigma_n^2 \end{bmatrix} \begin{bmatrix} a_{i1} \cdots a_{1,n-k} \\ \vdots \qquad \vdots \\ a_{1n} \cdots a_{n\ n-k} \end{bmatrix} \\
&= \mathrm{DIAG} \begin{bmatrix} a_{11}\ \sigma_1^2 \cdots a_{1n}\ \sigma_n^2 \\ \vdots \qquad\quad \vdots \\ a_{n-k\ 1}\sigma_1^2 \cdot a_{n-k\ n}\ \sigma_n^2 \end{bmatrix} \begin{bmatrix} a_{11} \cdots a_{1\ n-k} \\ \vdots \qquad \vdots \\ a_{1n} \cdots a_{n\ n-k} \end{bmatrix} \\
&= \mathrm{DIAG} \begin{bmatrix} a_{11}^2\ \sigma_1^2 + \cdots\cdots + a_{1n}^2\ \sigma_n^2 \\ \vdots \qquad\qquad\qquad \vdots \\ a_{n-k\ 1}^2\ \sigma_1^2 + \cdots + a_{n-k\ n}^2\ \sigma_n^2 \end{bmatrix} = (A^{(2)})'\ \mathrm{DIAG}[V]
\end{aligned}
$$

if $A^{(2)}$ is defined as $\{a_{ij}{}^2\}$.

Hence, under H_2, the squared BLUS residuals are a weighted sum of the true unobserved variances $\sigma_1^2,\ldots,\sigma_n^2$. The weights $a_{i1}{}^2,\ldots,a_{in}{}^2$, which are associated with each squared residual $\overset{*}{u}_i{}^2$, sum to one since $A'A = I_{n-k}$. However, unlike the squared OLS residuals, none of these weights is dominant. Hence, no squared BLUS residual actually estimates any one of the variances $\sigma_1^2,\ldots,\sigma_n^2$. Therefore, under H_2, it is conceivable for the squared BLUS residuals to be homoskedastic.

Ramsey & Gilbert's observation that the BAMSET procedure used with OLS residuals was more powerful against H_2 than was the same procedure using BLUS residuals can now be explained. Since the squared BLUS residuals, under H_2, are each an apparently equally weighted sum of the true variances $\sigma_1^2,\ldots,\sigma_n^2$, the extent of the heteroskedasticity is masked. Consequently, OLS estimates of $\sigma_1^2,\ldots,\sigma_n^2$ are, under H_2, more heteroskedastic than are the BLUS estimates of $\sigma_1^2,\ldots,\sigma_n^2$. Therefore, the BAMSET procedure can more easily detect heteroskedasticity when OLS estimates are used.

Although the OLS estimates of $\sigma_1^2,\ldots,\sigma_n^2$ are less biased, under H_2, than are the BLUS estimates, they are still biased. To offer a solution for this problem, Rao [1970] and Chew [1970] independently developed Minimum Norm Quadratic Estimators (MINQUE). Given that $M^{(2)}$ is defined as $\{m_{ij}{}^2\}$, both Rao and Chew suggested that when $M^{(2)}$ is non-singular, the MINQUE estimator $\ddot{\sigma}^2$ can be defined. The vector of estimators $\underline{\ddot{\sigma}}^2$ is defined as $(M^2)^- \, \mathrm{DIAG}(\underline{\hat{u}\hat{u}'})$, where $\underline{\hat{u}}$ is the vector of OLS residuals. These estimators are unbiased under H_0 and H_2. This will first be shown under H_2. As stated before (2.24), under H_2,

$$E(\underline{uu'}) = V = \begin{bmatrix} \sigma_1^2 & & \emptyset \\ \vdots & \ddots & \\ \emptyset & & \sigma_n^2 \end{bmatrix} \quad \text{and } E[\mathrm{DIAG}(\underline{\hat{u}}\,\underline{\hat{u}'})] = M^2\,\mathrm{DIAG}(\underline{u}\,\underline{u}').$$

Hence, one obtains

$$\begin{aligned} E(\underline{\ddot{\sigma}}^2) &= (M^2)^- \, E[\mathrm{DIAG}(\hat{u}\,\hat{u}')] \\ &= (M^2)^- \, (M^2) \, E(\mathrm{DIAG}\,(\underline{u}\,\underline{u}')) \\ &= \mathrm{DIAG}\,(E(u\,u')) = \begin{pmatrix} \sigma_1^2 \\ \vdots \\ \sigma_n^2 \end{pmatrix} . \end{aligned}$$

The estimator $\underline{\ddot{\sigma}}^2$ can similarly be shown to be unbiased under H_0 by just replacing $\sigma_1^2,\ldots,\sigma_n^2$ by the constant variance σ^2.

Although this procedure offers unbiased estimates of $\sigma_1^2,\ldots,\sigma_n^2$ under H_0 and H_2, it does have three drawbacks. First, the n estimates $\ddot{\sigma}_1^2,\ldots,\ddot{\sigma}_n^2$ are not independent. This is obvious since n estimates are obtained by using a linear transformation of the OLS residuals which have a rank of only (n-k). Second, though the

MINQU estimators are unbiased under H_0 and H_2, they also may be negative. To solve this problem, Rao & Subrahaniam [1971] have suggested that when $\ddot{\sigma}_i^2$ is negative, either a small number or a different estimate of σ_i^2 be used in place of $\ddot{\sigma}_i^2$. Although this is a solution to the problem of negative estimates, the resulting estimates are now neither unbiased (under H_0 or H_2) nor MINQU. Hence, this investigator feels that the cost of correcting the negative MINQU estimates is greater than the cost of leaving the estimates negative.

The third problem with this procedure is that $M^{(2)}$ is not always non-singular. Mallela [1972] has, however, found a necessary and sufficient condition for the matrix $M^{(2)}$ to be non-singular.

The last set of estimates of the variances $\sigma_1^2,\ldots,\sigma_n^2$ are obtained from studentized residuals. Define the i'th studentized residual is $\dot{u}_i = \hat{u}_i/\sqrt{m_{ii}}$, where $\hat{u}_i$ is the i'th OLS residual obtained after estimation of model (2.23) and where $m_{ii}(\neq 0)$ is again the i'th diagonal element of the matrix M. The studentized estimator of the variance of the i'th disturbance term σ_i^2, $\dot{\sigma}_i^2$ is defined as $\dot{\sigma}_i^2 = \dot{u}_i^{\,2}$; this estimate is unbiased under H_0. When $\underline{m}_i$ is defined as the i'th column of the matrix M and it is recalled that $\underline{m}_i'\underline{m}_i = m_{ii}$, because M is idempotent, one obtains

$$\begin{aligned}
E(\dot{\sigma}_i^2) &= E(\dot{u}_i^{\,2}) \\
&= E(\hat{u}_i^2/m_{ii}) \\
&= \frac{1}{m_{ii}}\ E(\hat{u}_i^2) \\
&= \frac{1}{m_{ii}}\ E(\underline{m}_i'\ \underline{u}\ \underline{u}'\ \underline{m}_i) \\
&= \frac{1}{m_{ii}}\ \underline{m}_i'\ \sigma^2 I\ \underline{m}_i \\
&= \sigma^2\ \frac{1}{m_{ii}}\ \underline{m}_i'\ \underline{m}_i = \sigma^2\ \frac{m_{ii}}{m_{ii}} = \sigma^2
\end{aligned}$$

Also, since the i'th studentized estimate can be written in quadratic form in the normally distributed disturbance terms $\underline{u}$,

$$\dot{\sigma}_i^2 = \frac{\hat{u}^2}{m_{ii}} = \underline{u}' \, \underline{m}_i \, \underline{m}_i' \, \frac{1}{m_{ii}} \, \underline{u} = \underline{u}' \, Q_i \, \underline{u}$$

$\dot{\sigma}_i^2$ is distributed as χ^2 with one (=trace Q_i) degree of freedom.

However, there are two problems with the studentized estimates of $\sigma_1^2,\ldots,\sigma_n^2$. First, the n estimates $\dot{\sigma}_1^2,\ldots,\dot{\sigma}_n^2$ are not distributed independently. Since $\dot{\sigma}_i^2$ can be expressed as a quadratic form in the normally distributed disturbance terms, $\dot{\sigma}_i^2$ is independently distributed of $\dot{\sigma}_j^2 (i \neq j)$ if and only if the products of the two quadratics are identically zero. Hence, if $Q_i Q_j \neq 0$, $\dot{\sigma}_i^2$ is not independently distributed of $\dot{\sigma}_j^2$,

$$\begin{aligned} Q_i Q_j &= \frac{1}{m_{ii}} \, \underline{m}_i \, \underline{m}_i' \, \frac{1}{m_{jj}} \, \underline{m}_j \, \underline{m}_j' \\ &= \frac{1}{m_{ii} \, m_{jj}} \, \underline{m}_i' \, \underline{m}_i \, \underline{m}_j \, \underline{m}_j' \\ &= \frac{m_{ij}}{m_{ii} \, m_{jj}} \, \underline{m}_i \, \underline{m}_j \neq 0. \end{aligned}$$

Therefore, $\dot{\sigma}_i^2$ and $\dot{\sigma}_j^2$ $(i \neq j)$ are not independently distributed of one another.

Second, just as with the OLS estimator, the studentized estimates are biased under H_2. However, also, just as with the OLS estimates, the weighting scheme is such that the expected value of the i'th studentized estimate is

$$E(\dot{\sigma}_i^2) = \frac{m_{i1}^2}{m_{ii}} \, \sigma_1^2 + \ldots + \sigma_i^2 \, \frac{m_{ii}^2}{m_{ii}} + \ldots + \frac{m_{in}^2}{m_{ii}} \, \sigma_n^2. \qquad (2.27)$$

The weights are:

$$\frac{m_{i1}^2}{m_{ii}}, \ldots, \frac{m_{ii}^2}{m_{ii}}, \ldots, \frac{m_{in}^2}{m_{ii}} .$$

Since, as before, the sum of these weights, which are all positive, is one, and the i'th weight $\frac{m_{ii}^2}{m_{ii}} = m_{ii}$ has an expected value of $\frac{n-k}{n}$, the i'th weight dominates the series. Thus, if the variances $\sigma_1^2,\ldots,\sigma_n^2$ are unequal (heteroskedastic), the estimates $\dot{\sigma}_1^2,\ldots,\dot{\sigma}_n^2$ will be unequal, though not unbiased.

Four estimates of the variances of the n disturbance terms have been suggested. They all have some disadvantages. The MINQU estimates are the only ones unbiased under both H_0 and H_2. The OLS and studentized estimates are similar to one another, except that the studentized estimates are unbiased under H_0 whereas the OLS estimates are only homoskedastic. The BLUS estimates are the only ones that are mutually independent.

This investigator has decided to use either studentized or MINQU estimates because they are both unbiased under H_0. BLUS estimates of $\sigma_1^2,\ldots,\sigma_n^2$ were not chosen, though they are unbiased under H_0, because of the bias that they contain under H_2. Finally, OLS estimates were not chosen because they have no apparent advantage over studentized estimates.

III.3.2 The POSEX Model

As has been previously mentioned, either some assumption or *a priori* knowledge about the heteroskedastic error terms is necessary for all the variances $\sigma_1^2,\ldots,\sigma_n^2$ to be estimated. If estimation were to be attempted without such knowledge or assumptions, the estimation process would break down. The researcher would be attempting to estimate n + k parameters ($\beta_1,\ldots, \beta_k, \sigma_1^2,\ldots,\sigma_n^2$) with only n observations. Obviously, if one has a choice, it is more desirable to incorporate *a priori* knowledge about the variances $\sigma_1^2,\ldots,\sigma_n^2$ than make assumptions that may or may not be correct. However, it is not unusual for a researcher to be confronted with a model that is suspected of being heteroskedastic although no knowledge is available as to which variable is causing the heteroskedasticity. In this case, some assumption is necessary if estimation is to be made.

Present methods of estimation require that one make an assumption about the variable(s) that are causing the disturbances to be heteroskedastic and about the functional form that these variables take on. In contrast, if a POSEX model can be used, many of these assumptions can be dropped since a POSEX model estimates any analytic function in a known set of variables. Hence, in using a POSEX model, the only assumption necessary, if no knowledge exists, is that the heteroskedastic disturbances be an analytic function of the independent variables specified in the model. In developing a test based on the POSEX model, this assumption is less restrictive than any constructive or non-constructive (with one exception) test now being used. Of course, if knowledge does exist, the POSEX model should be changed to reflect that knowledge. This process will be examined later in this section.

In order to develop a POSEX model without much *a priori* information, one must assume that the variances are some analytic function of the independent variables from the hypothesized model (2.23), $\underline{x}_1,\ldots,\underline{x}_k$. The variance of the i'th disturbance term can be written as

$$E(u_i^2) = \sigma^2 f(x_{i1},\ldots,x_{ik}),\ i=1,\ldots,n. \tag{2.28}$$

When this assumption is used, the POSEX model must approximate the analytic function $f(\cdot)$ in a four term power series expansion. The equation formulated would be

$$E(\underline{u}^2) = \sigma^2 f(\underline{x}_1,\ldots,\underline{x}_k) \doteq \beta_{11} + \beta_{12}\,\underline{x}_2 + \ldots + \beta_{1k}\,\underline{x}_k +$$
$$\alpha_2\,\underline{q}^{(2)} + \alpha_3\,\underline{q}^{(3)} + \alpha_4\,\underline{q}^{(4)} \qquad (2.29)$$

where $\underline{q}$ denotes either the instrument $\hat{\underline{y}}$ (OLS predictor of $\underline{y}$ obtained after estimating model (2.23)) of $\underline{p}$ (the first principal component of the matrix X). Finally, when the expected value operator is removed and either the studentized estimates $\dot{\sigma}_1^2,\ldots,\dot{\sigma}_n^2$ or the MINQU estimates $\ddot{\sigma}_1^2,\ldots,\ddot{\sigma}_n^2$ are used as the instrument for the unobserved variances $\sigma_1^2,\ldots,\sigma_n^2$, equation (2.29) becomes

$$\overset{\circ}{\sigma}{}_i^2 = (\beta_{11} + \beta_{12}\,x_{i2} + \ldots + \beta_{1k}\,x_{ik} + \alpha_2\,q_i^2 + \alpha_3\,q_i^3 + \alpha_4\,q_i^4)w_i,$$
$$i = 1,\ldots,n,$$

where $\overset{\circ}{\underline{\sigma}}{}^2$ denotes either the vector $\dot{\underline{\sigma}}^2$ or the vector $\ddot{\underline{\sigma}}^2$, and where $w_1,\ldots,w_n$ are identically distributed as χ^2 with one degree of freedom under H_0. Note that the disturbance term is not added onto equation (2.29) but is multiplied by the model. Recalling that under H_0, $\overset{\circ}{\sigma}{}_1^2,\ldots,\overset{\circ}{\sigma}{}_n^2$ are each distributed as scaled χ^2 with one degree of freedom, model (2.29) is used to estimate the scale factors.

Under H_0 that $E(u_i{}^2) = \sigma^2$ for $i=1,\ldots,n$, only β_{11} will be significantly different from zero. The null hypothesis of homoskedasticity will be accepted if

$$\beta_{12} = \ldots = \beta_{1k} = \alpha_2 = \alpha_3 = \alpha_4 = 0,$$

whereas if any of the estimates are statistically different from zero, then H_0 will be rejected.

Under H_2 that $E(u_i^2) = \sigma_1^2 f(x_{i1},\ldots,x_{ik})$, for $i=1,\ldots,n$, the coefficients $\beta_{12},\ldots,\beta_{1k}$, α_2, α_3, and α_4 should be jointly different from zero. Of course, the probability that the estimates will be statistically different from zero depends on a number of factors. Three factors have been mentioned previously; they are whether the instrument $\underline{q}$ is correlated with the expansion terms it replaces; whether $f(\cdot)$ is analytic; and whether $f(\cdot)$ is approximated by a low order expansion. One other factor which will influence the probability that the coefficients will be statistically equal to zero is how well the estimators $\dot{\underline{\sigma}}^2$ or $\ddot{\underline{\sigma}}^2$ approximate the unobserved variances, $\sigma_1^2,\ldots,\sigma_n^2$.

II.3.3 Estimation of the POSEX Model and Testing for Heteroskedasticity

The conditional variance of the i'th disturbance term is given by the POSEX model

$$\overset{\circ}{\sigma}{}_i^2 = (\beta_{11} + \beta_{12}\, x_{i2} + \ldots + \beta_{1k}\, x_{ik} + \alpha_2\, q_i^2 + \alpha_3\, q_i^3 + \alpha_4\, q_i^4)\, w_i$$

$$i = 1,\ldots,n, \tag{2.30}$$

where w_i, $i=1,\ldots,n$, are identically distributed as χ^2 with one degree of freedom under H_0. The parameters β_{11}, $\beta_{12},\ldots,$ β_{1k}, α_2, α_3 and α_4 must now be estimated to determine if heteroskedasticity of the form hypothesized, is present.

Maximum Likelihood Estimation

The first estimation procedure to suggest itself is maximum likelihood. However, since the disturbance terms, w_i, $i=1,\ldots,n$, are identically distributed as χ^2 with one degree of freedom, under H_0, this procedure breaks down. The reason for this is that a χ^2

distribution with one degree of freedom is an unbounded function; thus, no maximum exists.

Estimation Using Ordinary Least Squares

The second method to suggest itself is the method of least squares. Denoting the estimates of the parameters as $\beta_{11}, \ldots, \beta_{1k}$, α_2, α_3 and α_4, one finds that under H_0,

$$E(\hat{\beta}_{11}) = \sigma^2$$

$$E(\hat{\beta}_{12}) = \ldots = E(\hat{\beta}_{1k}) = E(\hat{\alpha}_2) = E(\hat{\alpha}_3) = E(\hat{\alpha}_4) = 0,$$

whereas under H_2, the expected value of the estimates of $\beta_{12}, \ldots, \beta_{ik}$ α_2, α_3, and α_4 are jointly non-zero. The $E(\hat{\beta}_{11})$ under H_2 depends on whether heteroskedasticity is mixed, $E(\hat{\beta}_{11}) \neq 0$, or whether the heteroskedasticity is pure, $E(\hat{\beta}_{11}) = 0$. Because the hypothesis space is divided depending on whether $\beta_{12}, \ldots, \beta_{1k}$, α_2, α_3, and α_4 are different from zero or not, an F test for the included variables $\underline{x}_2, \ldots, \underline{x}_k$, $\underline{q}^2$, $\underline{q}^3$, and $\underline{q}^4$ is suggested.

However, two difficulties exist with the suggested F test. The first difficulty is that the dependent variables $\overset{\circ}{\sigma}{}^2_1, \ldots, \overset{\circ}{\sigma}{}^2_n$ are each distributed as $\sigma^2 \chi^2$ with one degree of freedom under H_0. Hence, the statistic calculated by using the F test procedure is a ratio of quadratic forms in <u>non</u>-normally distributed variables. Therefore, the statistic is not distributed as F. Research carried out by Donaldson [1968], however, indicates that an F distribution appears to be robust against non-normality. He discovered, by using a sampling experiment, that statistics which are a ratio of quadratics in variables distributed as either log normal or exponential (Pearson type III distributions) are approximately distributed as an

F distribution. This finding was true for sample sizes greater than 4. Of course, the approximation became less and less accurate the farther out on the tails the comparison was made. Since a distribution with one degree of freedom is a Pearson type III distribution, it would not be surprising if the statistic calculated using the F test procedure were approximately distributed as F.

Second, since neither dependent variable (MINQU estimates or studentized estimates of $\sigma_1^2,\ldots,\sigma_n^2$) is composed of elements that are mutually independent, the disturbance terms $w_1,\ldots,w_n$ are not mutually independent. Once again, Donaldson's findings can cast some light on the problem. He discovered that non-independence between the numerator and denominator of his quadratic forms helped to explain why the statistics, which he calculated using variables distributed other than normal, were distributed as F. To apply Donaldsom's findings to the current situation, it should be noted that the n non-independent estimates, $\mathring{\sigma}_1^2,\ldots,\mathring{\sigma}_n^2$ can theoretically be expressed as n-k independent estimates by some linear transformation of the n estimates. Denoting this transformation by the (n-k) x n matrix B, the (n-k) independent estimates $\overset{+}{\sigma}{}_1^2,\ldots,\overset{+}{\sigma}{}_{n-k}^2$ are defined as $\overset{+}{\underline{\sigma}}{}^2 = B\,\mathring{\underline{\sigma}}{}^2$. Using this formulation, the statistic calculated by using the F test process can be expressed as a quadratic in n-k independently distributed variables $\overset{+}{\sigma}{}_1^2,\ldots,\overset{+}{\sigma}{}_{n-k}^2$. However, when so expressed, the quadratic forms are no longer independent. Hence, the findings of Donaldsom are now applicable. Given those findings, the lack of independence between $w_1,\ldots,w_n$ might enhance the robustness of the statistic, defined by the F test procedure, to non-normality.

Therefore, even though unbiased estimates of the parameters in model (2.31) can be obtained using ordinary least squares (regardless of the fact that the disturbance terms are distributed asymmetrically), the normal tests of significance break down. However, given the findings of Donaldson, the statistics calculated might still be distributed approximately as F.

Indirect Maximum Likelihood Estimation

The final estimation procedure suggested circumvents the problem that the disturbance terms, $w_1, \ldots, w_n$'s, are not mutually independent. This is accomplished by formulating a model which uses both the k parameters $\beta_1, \ldots, \beta_k$ and k + 3 parameters, $\beta_{11}, \ldots, \beta_{1k}$, α_2, α_3 and α_4. The model to be estimated is:

$$\frac{y_i}{\overset{+}{\sigma}_i} = \beta_1 \frac{1}{\overset{+}{\sigma}_i} + \beta_2 \frac{x_{i2}}{\overset{+}{\sigma}_i} + \ldots + \beta_k \frac{x_{i2}}{\overset{+}{\sigma}_i} + u_i, \; i=1,\ldots,n, \qquad (2.31)$$

where $u_1, \ldots, u_n$ are independently and identically distributed $N(0, \sigma^2 I)$, under H_0, and where

$$\overset{+}{\sigma}_i = \sqrt{\beta_{11} + \beta_{12}\, x_{i2} + \ldots + \beta_{ik}\, x_{ik} + \alpha_2\, q_i^2 + \alpha_3\, q_i^3 + \alpha_4\, q_i^4} \;.$$

To estimate this model, a maximum likelihood procedure must be used. The maximum of the likelihood function L, under H_2, is defined as L_2,

$$L_2 = \prod_{i=1}^{n} \sqrt{2\Pi\, \overset{+}{\sigma}_i} \quad e^{-1/2\, u_i^2}$$

where $\overset{+}{\sigma}_i$ is as defined above and u_i is obtained from model (2.31). The estimates of the parameters that maximize L_2 will be denoted as $\hat{\hat{\beta}}_1, \ldots, \hat{\hat{\beta}}_k$, $\hat{\hat{\beta}}_{11}, \ldots, \hat{\hat{\beta}}_{1k}$, $\hat{\hat{\alpha}}_2$, $\hat{\hat{\alpha}}_3$, and $\hat{\hat{\alpha}}_4$. Under H_0,

$\beta_{12} = \cdots = \beta_{1k} = \alpha_2 = \alpha_3 = \alpha_4 = 0$ whereas under H_2, $\beta_{12},\ldots,\beta_{1k}$, α_2, α_3 and α_4 are jointly non-zero.

To test the hypothesis of H_0 vs. H_2, a likelihood ratio statistic can be used. Under H_0 that the variances are homoskedastic, the maximum of the likelihood function L is defined as L_0:

$$L_0 = \prod_{i=1}^{n} \frac{1}{\sqrt{\Pi\ \sigma^2}} \exp\left(-\frac{1}{2}\ \frac{y_i - \beta_1 - \beta_2\ x_{i2},\ldots,\beta_k\ x_{ik})^2}{\sigma^2}\right).$$

If one then defines the likelihood ratio statistic $\hat{\hat{\ell}}$ as

$$\hat{\hat{\ell}} = \frac{L_0}{L_2},$$

it follows that $-2 \log_e \hat{\hat{\ell}}$ is distributed as χ^2 with k + 2 degrees of freedom.

This testing procedure is basically the one suggested by Rutemuller & Bower. However, a POSEX model is used to explain the variance rather than a model composed of the independent variables $\underline{x}_1,\ldots,\underline{x}_k$. This last estimation and testing procedure does not contain any of the problems which were associated with the previous two procedures. However, this estimation procedure is more easily implemented in theory than in practice.

III.3.4 Further Observations on the POSEX Procedure

POSEX Model and a Reestimation Procedure

If it is found that the estimate of the parameters $\beta_{12},\ldots,\beta_{1k}$, α_2, α_3 and α_4 in the model (2.30) are statistically different from zero, H_0 is rejected. Since model (2.30) estimates the conditional mean of $\sigma_1^2,\ldots,\sigma_n^2$, n estimates of $\sigma_1^2,\ldots,\sigma_n^2$ can be obtained. The estimates, denoted as $\hat{\sigma}_1^2,\ldots,\hat{\sigma}_n^2$ can be used to reestimate model (2.30) and thereby increase the efficiency of the estimates of the regression parameters. Two methods of reestimation exist. The first method is to transform model (2.23) into the model

$$\frac{y_i}{\hat{\sigma}_i} = \beta_1 \frac{1}{\hat{\sigma}_i} + 2 \frac{x_{i2}}{\hat{\sigma}_i} + \ldots + \beta_k \frac{x_{ik}}{\hat{\sigma}_i} + u_i \qquad (2.32)$$

where u_i, $i=1,\ldots,n$ are assumed to be independently and identically distributed $N(0, \sigma^2 I)$. Ordinary least squares can be used to reestimate model (2.32).

The second method is to use Aiken's Method of Generalized Least Squares. Using this method and denoting

$$\Omega = \begin{bmatrix} \hat{\sigma}_1^2 & & \emptyset \\ & \ddots & \\ \emptyset & & \hat{\sigma}_n^2 \end{bmatrix},$$

the new estimator for $\underline{\beta}$,

$$\hat{\hat{\underline{\beta}}} = (X' \, \Omega^{-} X)^{-} X' \, \Omega^{-} \underline{y}.$$

These two methods will yield identical estimates of the parameters $\beta_1,\ldots,\beta_k$ and σ^2.

However, it should be pointed out that since $\hat{\sigma}_1^2,\ldots,\hat{\sigma}_n^2$ are estimates, it is possible for them to be negative. If this is the case, model (2.32) cannot be used to reestimate model (2.23) unless the negative estimate is removed. This investigator suggests using the absolute value of the estimate when the estimate is negative; when this is done, the magnitude of the estimate variance remains the same and the square root can be taken. However, when this is done the estimates $\beta_1,\ldots,\beta_k$ and σ^2 will no longer be identical to those obtained by the method of Generalized Least Squares.

<u>Incorporating a priori Information into the POSEX Model</u>

Depending on the <u>a priori</u> knowledge about the heteroskedastic disturbances, the POSEX model (2.30) can be varied in many ways. Three different types of <u>a priori</u> information will be presented here. First, a set of $m(\geq 1)$ variables $\underline{z}_1,\ldots,\underline{z}_m$ is thought to be included with the variables $\underline{x}_1,\ldots,\underline{x}_k$ in the unknown analytic function, $f(\cdot)$. Second, only a set of $m(\geq 1)$ variables $\underline{z}_1,\ldots,\underline{z}_m$ is thought to be in the unknown function, $f(\cdot)$. Third, only a set of $m(\geq 1)$ variables $\underline{z}_1,\ldots,\underline{z}_m$ is known to be causing the heteroskedastic disturbances in some known way.

In the first case, the POSEX model could be formulated to include the variables $\underline{z}_1,\ldots,\underline{z}_m$. This would change model (2.30) to

$$\overset{\circ}{\sigma}{}_1^2 = (\beta_{11} + \beta_{12}\, x_{i2} + \cdots + \beta_{1k}\, x_{1k} + \delta_1\, z_{i1} + \cdots + \delta_m\, z_{im} + \alpha_2\, q_i^2 + \alpha_3\, q_i^3 + \alpha_4\, q_i^4)\, w_i \tag{2.33}$$

where $w_1,\ldots,w_n$ are each distributed as χ^2 with one degree of freedom under H_0, and where the instrument q_i is a linear combination of $\underline{z}_1,\ldots,\underline{z}_m$ as well as $\underline{x}_1,\ldots,\underline{x}_k$.

In the second case, the POSEX model could be formulated to include the variables $z_{i1},\ldots,z_{im}$ but not to include $x_{i1},\ldots,x_{ik}$. Hence, the POSEX model would be

$$\overset{\circ}{\sigma}{}_i^2 = (z_{i1}\,\delta_1 + \cdots + z_{im}\,\delta_{im} + \alpha_2\, q_i^2 + \alpha_3\, q_i^3 + \alpha_4\, q_i^4)\, w_i \tag{2.34}$$

where $w_1,\ldots,w_n$ are identically distributed as χ^2 with one degree of freedom under H_0, and where q_i is a linear combination of the variables $z_{i1},\ldots,z_{im}$ and not the variables $x_{i1},\ldots,x_{ik}$.

In the third case, a POSEX model will not be formed since the exact functional form involving the variables $\underline{z}_1,\ldots,\underline{z}_m$ is known.

To illustrate this, it will be assumed that the function involving the variables $\underline{z}_1, \ldots, \underline{z}_m$ is a quadratic of the second degree. The model to be examined would be:

$$\overset{\circ}{\sigma}{}_i^2 = (\delta_{11} z_{i1} + \ldots + \delta_m z_{im} + \delta_{21} z_{i1}^2 + \ldots + \delta_m z_{im}^2) w_i \tag{2.35}$$

where $w_1, \ldots, w_n$ are identically distributed as χ^2 with one degree of freedom under H_0. If m is large, the squared terms could be replaced by an instrument; however, if the knowledge embodied in model (2.35) is correct, introducing the instrument will reduce the probability that the model will be able to estimate the heteroskedastic disturbances.

Similarities Between the POSEX Procedure and Other Constructive Testing Procedures

Using the POSEX model building technique presented in this study, any of the current constructive testing procedures can be deduced. To illustrate this contention, assume that it is known that a single variable $\underline{x}_j$ in the form of a second degree quadratic is causing the heteroskedasticity. Using this information, Glejser's model can be obtained. If the model is estimated using OLS and either a t or F test issued to test if the coefficients are statistically significant from zero, Glejser's testing procedure has been obtained. Similarly, Rutemuller & Bower's model and Park's model can be deduced using the concept of a POSEX model, *a priori* information, and the different estimation procedures suggested. Thus, using the POSEX formulation and *a priori* knowledge as to the heteroskedastic disturbances, one can deduce all of the constructive testing procedures.

II.3.5 Summary

In this section, a POSEX model was suggested to explain the variance of the disturbance terms when heteroskedasticity is presumed present. It was required only that the unobserved variances $\sigma_1^2,\ldots,\sigma_n^2$ be a function of the independent variables from the hypothesized model.

Since the variances $\sigma_1^2,\ldots,\sigma_n^2$ are unobserved, four different estimators of the variances were discussed. It was shown that although squared OLS estimates are biased, they are, nevertheless, homoskedastic under very non-restrictive conditions. Two estimators were then chosen to estimate the unobserved variances.

Two possible ways in which to estimate the POSEX model were suggested. A testing procedure for distinguishing between H_0 and H_2 was associated with each of these estimation procedures. Finally, some extensions of the POSEX procedure were suggested.

CHAPTER III

HYPOTHESES AND EXPERIMENTAL DESIGN

A large number of hypotheses have been made in the two previous chapters of this study. Unfortunately, since there are an infinite number of different models that can be specified, none of these hypotheses can, in general, be proven correct. Rather, each hypothesis must be carefully examined using a very carefully selected subset of model specifications. If an hypothesis is not refuted in the models chosen, it will then be assumed that it can be generalized as being valid for other similarly specified models. However, as the new models differ more and more from the models that were chosen for examination, the probability that the generalization will be invalid increases. In contrast, it should be noted that if an hypothesis is shown invalid for the models specified, the hypothesis has been proven invalid in general.

Another difficulty still remains in testing the hypotheses made in this study. Since all of the hypotheses concern various test statistics, a method must be found whereby the probability of type I and II errors can be determined for each test. However, since the distribution of most of the test statistics discussed in this study is not known, a sampling experiment, similar to others that have been discussed, will be used to analyze the various statistics.

This chapter will be divided into two sections. First, all of the hypotheses made in this study will be restated and briefly explained. Second, a sampling experiment will be presented that examines various tests for a misspecified mean and heteroskedastic disturbance term.

III.1 Hypotheses

It is most convenient to divide these hypotheses into two groups. The first group contains hypotheses that are applicable to tests for a misspecified conditional mean. The second group comprises those hypotheses that are applicable to tests for heteroskedastic disturbance terms.

III.1.1 Misspecified Conditional Mean

Five broad hypotheses are made in this study regarding tests designed to determine if a model has a misspecified conditional mean. Since the reasoning behind each hypothesis has been previously given, each of the five hypotheses will only be stated in this section of the study.

1. Under the null hypothesis, Ramsey's test, Ramsey & Schmidt's test, and the proposed test will each have a probability of type I error equal to the alpha level at which each test is conducted.
2. When a variable is omitted from the hypothesized model, the probability that Ramsey's test, Ramsey & Schmidt's test and the proposed test will each correctly reject H_0 will increase as the correlation between the omitted variable and the included variables increases. At some point, however, this trend will reverse itself and as the correlation increases past this point, the probability of correctly rejecting H_0 will decrease.

3. When the wrong functional form of either the regressors or regressand is used, the power of all three tests will be an increasing function of two factors. The first factor is whether the correct functional form is analytic. The second factor, which only becomes important if the function is analytic, is the accuracy with which a Taylor expansion in four terms can approximate the correct function.
4. Under the alternative hypothesis that the conditional mean of the vector $\underline{y}$ is misspecified, the power of all three testing procedures will be an increasing function of the number of sample observations (n).
5. Under the alternative hypotheses of a misspecified conditional mean, the power of the proposed test will be greater than the power of either Ramsey's test or Ramsey & Schmidt's test.

III.1.2 Heteroskedastic Disturbance Terms

Ten broad hypotheses are made in this study regarding tests designed to determine if an hypothesized model is heteroskedastic. Since, as before, the reasoning behind each hypothesis has been previously given, each of the ten hypotheses will only be stated in this section of the study.

1. Under H_0 of homoskedasticity, the only tests that will have a probability of type I error equal to the alpha level will be those tests that define a statistic whose exact distributional form is known. However, all other tests will have a probability of type I error approximately equal to the alpha level at which those tests are examined. Furthermore, that approximation will become increasingly accurate as the alpha level increases.

2. The probability of any test's correctly rejecting H_0 will be an increasing function of the amount of correct a priori information available.
3. The power of all the tests for heteroskedasticity will increase as the number of sample observations (n) increases.
4. The power of the various tests for heteroskedasticity will, in general, be independent of the distributional form of the variable causing the disturbances to be heteroskedastic.
5. The tests for heteroskedasticity will not display an increased probability of type I error when the independent variables are not drawn from a fixed distribution even though this choice of independent variables insures that the diagonal elements of the matrix M are not equal.
6. The power of the POSEX model and testing procedures to determine if a model is heteroskedastic will be a decreasing function of the number of terms needed by a Taylor series expansion to approximate the functional form (taken by the disturbance terms) to some level of accuracy.
7. The power of the POSEX model and testing procedures will be, in general, increased if the instrument $\underline{p}$ (first principal component of the matrix X) is used for the expansion terms versus the instrument $\hat{\underline{y}}$ (the OLS predictor of y).
8. The power of the POSEX model and testing procedures will, in general, be increased when $\ddot{\underline{\sigma}}^2$ (MINQU estimates of $\underline{\sigma}^2$) is used as the predictor of $\underline{\sigma}^2$ versus when $\dot{\underline{\sigma}}^2$ (studentized estimates of $\underline{\sigma}^2$) is used.

9. The BAMSET tests with OLS residuals will have a higher probability of correctly rejecting H_0 than the same tests using BLUS residuals.
10. If the same amount of a priori information is incorporated into all of the testing procedures, the POSEX model and tests will have the highest probability of correctly rejecting H_0.

III.2 Sampling Experiment

In order to test these hypotheses, the probability of type I and type II errors must be calculated under various model specifications. Because the finite distribution of most of the test statistics is not known, these probabilities are most easily calculated by using a sampling experiment. Hence, in the first part of this section, a general sampling experiment will be outlined. Each of the following two parts of this section will, in turn, be concerned with using this experiment to examine various alternative tests for either a misspecified conditional mean of the vector $\underline{y}$ or heteroskedastic disturbance terms. Both of these parts will have the same format. First, each of the alternative hypotheses to be generated will be discussed with their relationships to one another expressly pointed out. Next, the various tests to be examined under the null and various alternative hypotheses will be selected with special attention paid to justifying this selection. After these two parts, a final summary of the experiment and of all the models that are examined will be given.

III.2.1 General Design of the Sampling Experiment

In conducting this experiment, the basic procedure will be to test if a model that is hypothesized to estimate the conditional mean of a sample of variables, $y_1,\ldots,y_n$, is misspecified because either the conditional mean has been misspecified or the disturbance terms are heteroskedastic. This procedure will be repeated on 1000 independently drawn samples of first 30, then 60, and finally 90 dependent variables, $y_1, \ldots, y_n$, (n = 30, 60 or 90). The percentage of times that each specification error test rejects H_0, that the hypothesized model is correctly specified, will then be recorded for nominal alpha levels of .01, .05, and .10. The first four sample moments of each test statistic will also be calculated. In this way, by defining 17 different populations of dependent variables from which the 1000 samples of $y_1, \ldots, y_n$, (n = 30, 60 or 90) are chosen, the testing procedures under examination can be compared.

Each of the seventeen populations is defined by specifying the conditional mean of the dependent variable and by adding on a disturbance term that has a mean of zero and a specified variance. These population definitions will be referred to as the 'true' models. Sixteen of these 'true' models are specified differently than is the hypothesized model. Hence, it can be observed how the power of the various testing procedures varies under different specification errors. These sixteen models will be explained and examined, in turn, later in this section of the study.

At this point, only the first 'true' model will be examined. It is

$$y_i = 50 + 5x_{i1} + 5x_{i2} + u_i, \quad i = 1, \ldots, n \tag{3.1}$$

where $u_1, \ldots, u_n$ are independently and identically distributed as N(0, 2500). The variables $x_{11}, \ldots, x_{n1}$, are independently drawn from a uniform distribution with end points of 0 and 100 (mean of 50 and population variance of 833.33). In contrast, variables, $x_{12}, \ldots, x_{n2}$, are independently drawn from a log normal distribution with a mean of 20.327 and population variance of 413.197. The population parameters of the second variable guarantee that the Pr $(0 \leq x_{i2} \leq 100) > .99$ for $i = 1,\ldots,n$. Hence, with a probability of .99, both variables cover the range, 0 to 100. Since, however, the two variables come from two different independent populations, they are independent of one another. One drawing of 90 observations was made for each of the two variables. These observations are divided into 3 groups of 30 observations each. Hence, when $n = 30$, the first group will be used; when $n = 60$, the first and second are used; and when $n = 90$, all three are used. All 90 of these observations together with various sample statistics for either $n = 30$, or 60, or 90 are given in Appendix B.

The conditional mean of the dependent variables obtained from all seventeen 'true' models will be estimated using the hypothesized model

$$y_i = \beta_0 + \beta_1 x_{i1} + \beta_2 x_{i2} + v_i, \; i = 1,\ldots,n. \tag{3.2}$$

where $v_1,\ldots,v_n$ are assumed to be independently and identically distributed as $N(0, \sigma^2)$. Hence, when the 'true' model is model (3.1), the hypothesized model will be a correctly specified model. In this way, the probability of type I error can be calculated for each of the tests examined. Similarly, since each of the specification error tests is also used when the hypothesized model (3.2) is misspecified, the probability of type II error can be calculated.

Only one difference exists in the basic procedure just outlined when any of the sixteen remaining 'true' models are used. When the 'true' model has a conditional mean other than that specified by the hypothesized model, only tests for a misspecified conditional mean will be examined. Likewise, when the 'true' model generates dependent variables that are heteroskedastic, only tests for heteroskedasticity will be examined. Although in using this procedure, the interrelation between the various specification errors is not brought out (a study by Ramsey & Gilbert, 1972, does make this comparison), this procedure was necessary to save computer time and money.

Finally, in order to simplify the discussion of the sixteen remaining models, three new variables will be defined. They will be denoted by the vectors $\underline{x}_3$, $\underline{x}_4$, and $\underline{x}_5$, respectively. The variables $x_{13},\ldots,x_{n3}$ will be drawn from a normal population with a mean of 50 and a variance of 400. These population parameters ensure with a probability of .99 that $x_{13},\ldots,x_{n3}$ will lie in the range of 0 to 100. Once the sampling is made, the observations $x_{13},\ldots,x_{n3}$ are never redrawn. Since $\underline{x}_3$ is drawn from a population independent of the populations from which $\underline{x}_1$ and $\underline{x}_2$ are drawn, $\underline{x}_3$ is independent of both $\underline{x}_1$ and $\underline{x}_2$.

The second set of variables $x_{14},\ldots,x_{n4}$, is a sum of the first three variables. The i'th observation of $\underline{x}_4$ is defined by

$$x_{i4} = 5.428 \log_e x_{i1} + 7.71 \log_e x_{i2} + \frac{3(x_{i3} - 50)}{20}. \qquad (3.3)$$

This variable is defined in such a way as to have a moderate correlation with either $\underline{x}_1$ or $\underline{x}_2$. Note that $x_{14},\ldots,x_{n4}$ will also be correlated with various powers of either $x_{11},\ldots,x_{n1}$ or $x_{12},\ldots,x_{n2}$.

The third and final additional variable, $x_{15},\ldots,x_{n5}$, is defined to be highly correlated with either $\underline{x}_1$ or $\underline{x}_2$. The i'th observation of $\underline{x}_5$, defined in terms of $\underline{x}_1$, $\underline{x}_2$, and $\underline{x}_3$ is

$$x_{i5} = .5428\, x_{i1} + .771\, x_{i2} + \frac{3(x_{i3} - 50)}{20}\ . \qquad (3.4)$$

The $E(x_{i5})$ is 42.81 and the variance of x_{i5} is 500.148. The population correlation coefficient between $\underline{x}_5$ and $\underline{x}_1$, and between $\underline{x}_5$ and $\underline{x}_2$ is .70. Also, because of the way $\underline{x}_5$ is defined, the coefficient of determination obtained by regressing $\underline{x}_5$ on $\underline{x}_1$ and $\underline{x}_2$ is 0.98.

A listing of all three variables appears in Appendix B. Also, in Appendix B, corresponding to the sample sizes of 30, 60, and 90 are the sample means, variance covariance matrix and correlation matrix of the variables, $\underline{x}_1$, $\underline{x}_2$, $\underline{x}_3$, $\underline{x}_4$ and $\underline{x}_5$.

III.2.2 Sampling Experiment to Examine Tests that Discriminate Between $\underline{H}_0$ Versus $\underline{H}_1$

Of the sixteen remaining 'true' models (that is, the models that actually generate the dependent variable) to be used in this experiment, six were generated so that the hypothesized model (3.2) will misspecify the conditional mean of the dependent variable. These six 'true' models are divided into two categories. The first category consists of three models designed so that the hypothesized model (3.2) mistakenly omits a relevant variable. The second category, consisting of the remaining three models, is designed so that either the regressors or the regressand of the hypothesized model has the wrong functional form. These two groups of models will be discussed in turn.

Variable Omitted from the Hypothesized Model

To generate a population of dependent variables that omits a relevant variable from the hypothesized model (3.2), a model

$$y_i = \beta_0 + \beta_1 x_{i1} + \beta_2 x_{i2} + \beta_3 z_i + u_i, \; i = 1,\ldots,u, \qquad (3.5)$$

where $u_1,\ldots,u_n$ are independently and identically distributed as $N(0, \sigma^2)$, is used. In using this 'true' model, both sets of variables

$x_{11},\ldots,x_{1n}$ and $x_{12},\ldots,x_{n2}$ are defined as before. In each of the three 'true' models that use this basic form, a different set of variables $z_1,\ldots,z_n$ is used.

These different variables are the variables $\underline{x}_3$, $\underline{x}_4$, and $\underline{x}_5$ that were previously defined. The three 'true' models will then be

$$y_i = 50 + 5x_{i1} + 5x_{i2} + 5x_{13} + u_i,\ i = 1,\ldots,n, \tag{3.5a}$$

$$y_i = 50 + 5x_{i1} + 5x_{12} + 5x_{14} + u_i,\ i = 1,\ldots,n,\ \text{and} \tag{3.5b}$$

$$y_i = 50 + 5x_{i1} + 5x_{12} + 5x_{i5} + u_i,\ i = 1,\ldots,n, \tag{3.5c}$$

where in each model $u_1,\ldots,u_n$ are independently and identically distributed as N(0, 2500).

These three models (3.5a), (3.5b), and (3.5c) each generates a dependent variable that causes the hypothesized model to be misspecified because of an omitted variable. However, the omitted variables are related to the included variables in different ways. The first omitted variable is independent of the included variables, the second is moderately correlated with the included variables, and the third is highly correlated with the included variables. Hence, a relation between correlation and the power of the various tests can be obtained.

Incorrect Functional Form of the Hypothesized Model

Three models are designed to cause the hypothesized model to be misspecified because an incorrect functional form is used. The models are designed so that the correct functional forms are increasingly difficult to approximate with a four-term Taylor series expansion. The three 'true' models will be defined as:

$$y_i = \exp\,(2 + .05x_{i1} + .05x_{i2} + 2u_i),\ i = 1,\ldots,n, \qquad (3.6a)$$

$$y_i = e^{1.0}\, x_{i1}^{1.0}\, x_{i2}^{1.0}\, e^{u_i},\ i = 1,\ldots,n, \text{ and} \qquad (3.6b)$$

$$y_i = \exp\,(-(-.25 + .02x_{i1} - .05x_{i2} + .5u_i)^{-2}),\ i=1,\ldots,n, \qquad (3.6c)$$

where in each model $u_1,\ldots,u_n$ are independently and identically distributed as N(0, 1), and $x_{11},\ldots,x_{n1}$ and $x_{12},\ldots,x_{n2}$ are as previously defined.

Because model (3.6a) is an exponential model, it will be the most accurately approximated (of the three models) by a Taylor series expansion. The second model (3.6b) is analytic; however, since it is a multiplicative function, it is less accurately approximated than model (3.6a). Finally, since in the neighborhood of zero the last function is discontinuous,model 7 is a non-analytic function in x_{i1} and x_{i2} and hence cannot be approximated using a Taylor series expansion.

Note that each of these three models is written such that the hypothesized model has the incorrect functional form of the regressors. However, the first two of these models can be reformulated so that the hypothesized models will have the incorrect functional form of the regressand. Written in this way, the two models become

$$\log_e y_i = 2 + .05x_{i1} + .05x_{i2} + 2u_i,\ i = 1,\ldots,n, \text{ and} \qquad (3.6a')$$

$$\log_e y_i = 1 + x_{i1} + x_{i2} + u_i,\ i = 1,\ldots,n, \qquad (3.6b')$$

where in each case $u_1,\ldots,u_n$ are independently and identically distributed as N(0, 1). Since an incorrect functional form of the regressand can equivalently be expressed as an incorrect functional form of the regressors, only one of the errors need be examined in this study.

The three models (3.6a), (3.6b), and (3.6c) each causes the hypothesized model (3.2) to be misspecified because of an incorrect functional form. However, the functional forms are chosen so that they are not approximated equally accurately by a Taylor series expansion in four terms. Hence, a relation can be determined between the power of the various tests and the degree of accuracy by which a Taylor series expansion of four terms can approximate the 'true' functional form.

The Tests Compared

Three testing procedures (Ramsey's test, Ramsey & Gilbert's test and Ramsey & Schmidt's test) have been used in the literature to determine if the conditional mean of the disturbance terms has been misspecified. The distributions of two of the resulting test statistics are known (Ramsey's statistic and Ramsey & Schmidt's statistic), while the distribution of the third is unknown. Hence, since Ramsey & Gilbert's testing procedure offers no advantage over the other two tests and offers the disadvantage of defining a test statistic that has an unknown distribution, their test will not be examined in this study.

Using a POSEX model, two additional tests have been developed. They both determine if the conditional mean of the dependent variable has been misspecified. The two tests differ, however, in the instrument used to replace the expansion terms in the POSEX model. In one version, the vector $\underline{p}$ (obtained from the first principal component of the matrix X) is used as the instrument, while in the other version, the vector $\hat{\underline{y}}$ (the OLS predictor of the dependent variable $\underline{y}$) is used.

This implies that four tests should be examined in this study. However, it should be recalled that Ramsey's and Ramsey & Schmidt's models and tests have been shown to be mathematically equivalent to using a POSEX model and test with the instrument $\hat{\underline{y}}$. Hence, only one of the three tests should be used. Because it is mathematically easier to formulate and calculate the test statistic, the POSEX test, with the instrument $\hat{\underline{y}}$, has been chosen. Therefore, the hypothesized model will be tested for a misspecified conditional mean of the dependent variable only by using both POSEX testing procedures.

III.2.3 Sampling Experiment to Examine Tests that Discriminate Between H_0 vs. H_2.

Ten 'true' models remain to be defined. All of these models are used to examine the different tests designed to determine if an hypothesized model is heteroskedastic. Hence, each of these 'true' models is designed so that the hypothesized model will be misspecified because it was incorrectly assumed to be homoskedastic. Since in each of the models the heteroskedastic disturbance terms are generated in a different way, a relation can be found between the power of the various tests and the form taken by the heteroskedastic disturbance terms.

These ten models can be divided into three groups. In the first group, the heteroskedastic disturbance terms are a simple function of one variable. In the second group, the disturbance terms are a non-linear function of one variable. Finally, in the third group, the disturbance terms are a function of a variable whose mean and variance is conditional on some other variable.

Heteroskedastic Disturbance Terms are a Simple Function of One Variable

Six 'true' models are generated which have the disturbance term as a function of a single variable. All of these models are of the form

$$y_i = 50 + 5x_{i1} + 5x_{i2} + z_i\, u_i,\ i = 1,\ldots,n, \tag{3.7}$$

where $u_1,\ldots,u$ are independently and identically distributed as N (0,1) and $x_{11},\ldots,x_{n1}$ and $x_{12},\ldots,x_{n2}$ are as previously defined. The variable z_i represents one of six variables that will cause the hypothesized model (3.2) to be heteroskedastic. These six variables will be chosen for their relationship to the hypothesized model.

In the first two models, the variables used for z_i are x_{i1} and x_{i2} respectively. Since both of these variables are included in the hypothesized model, the heteroskedastic disturbances (generated by the first two 'true' models) are a function of a variable that the researcher can identify. However, the variables differ from one another since they are drawn from different distributions.

The third and fourth 'true' models are generated when either the variable x_{i3} or the variable x_{i4} is used as z_i. Recall that x_{i3} is drawn from a normal population that is independent of x_{i1} or x_{i2}, whereas x_{i4} is generated so that it is partially correlated with x_{i1} and x_{i2}. Hence, while the first two models generate heteroskedastic disturbances that are a function of a variable that the researcher can identify, the third and fourth models generate disturbances that are either independent of those known variables or are only partially correlated with them.

In the fifth model, z_i will be replaced with a function of the index i. The particular function is $\frac{100(i)}{n}$. Although this particular variable will be (like x_{i3}) independent of x_{i1} and x_{i2}, it represents the type of variance that increases over time. Note, however, that normally when the heteroskedasticity is generated by a function of time, the independent variables are also highly correlated with a time index. Since in this case, i is independent of x_{i1} and x_{i2}, this particular form of heteroskedasticity will be a more difficult type to detect than the normal type. Rather, the model generated using x_{i4} conforms to the more typical occurence of the variance's increasing over time since it is partially correlated with x_{i1} and x_{i2}.

The sixth and last model of the group replaces z_i with $E(y_i)$. This form of heteroskedasticity has been suggested by Theil [1951]. The population correlation coefficient between $E(y_i)$ and x_{i1} is .817 while between $E(y_i)$ and x_{i2} it is .576.

All six of these models correspond to various types of heteroskedasticity. The first two represent heteroskedastic disturbances caused by a variable included in the hypothesized model. The third represents heteroskedastic disturbances that are generated independently of the model's variables, while the fourth represents disturbances that are partially correlated with those variables. The fifth represents disturbances that are related to some indexing scheme that cannot be identified. Finally, the sixth represents the case where the heteroskedastic disturbances are generated by the dependent variable.

Heteroskedastic Disturbances that are a Non-Linear Function of One Variable

Two of the 'true' models to be generated have heteroskedastic disturbances that involve a non-linear function. Both of these models are of the form

$$y_i = 50 + 5x_{i1} + 5x_{i2} + f(z_i)u_i, \; i = 1,\ldots,n, \tag{3.8}$$

where $u_1,\ldots,u_n$ are independently and identically distributed as $N(0\ ,\ 1)$ and $x_{11},\ldots,x_{n1}$ and $x_{12},\ldots,x_{n2}$ have been previously defined. The function and variable $f(z_i)$ represent two different non-linear functions in a variable denoted as z_i. The two functions, both analytic, can be approximated using a Taylor series expansion with different degrees of accuracy.

The first analytic function, $f(z_i)$, has been suggested by Goldfeld & Quant [1972]. It is a second degree quadratic in the variable x_{i1}. This function can be quite accurately approximated with a Taylor series expansion of just two terms. The function will be

$$(500 + 10x_{i1} + x_{i1}^2)^{1/2}.$$

The function $f(z_i)$ to be used in the second model is also an analytic function; hence it can be approximated with a Taylor series expansion. However, this approximation requires more expansion terms in the Taylor series to achieve the same accuracy as is achieved with the first function. The function is

$$75 + 50 \text{ SIN } E(y_i).$$

<u>Heteroskedastic Disturbances that are a Function of a Variable with a Non-Constant Mean</u>

This last group of 'true' models are quite different from any of the previously defined models. First a new variable, denoted as the vector $\underline{x}_6$, must be generated. The i'th observation of this variable is drawn from a uniform distribution with end points of 0 and 1.5i (population mean of .75i and variance of $.1878i^2$). Since each observation is drawn from a population with a different mean and variance, the vector $\underline{x}_6$ has a non-constant mean. The sample drawn appears in Appendix B together with various sample statistics.

The first model generated with the variable $x_{16},\ldots,x_{n6}$ is

$$y_i = 50 + 5x_{i6} + 5x_{i2} + u_i, \; i = 1,\ldots,n \tag{3.9}$$

where $u_1,\ldots,u_n$ are independently and identically distributed as N(0, 2500). In contrast, the other 'true' model in this group is generated with a non-constant variance. It is

$$y_i = 50 + 5x_{i6} + 5x_{i2} + x_{i6}\, u_i, \; i = 1,\ldots,n, \tag{3.10}$$

where $u_1,\ldots,u_n$ are independently and identically distributed as N(0, 1).

In testing both of these models, the hypothesized model is

$$y_i = \beta_0 + \beta_1 x_{i6} + \beta_2 x_{i2} + v_i, \; i = 1,\ldots,n, \tag{3.11}$$

where $v_1,\ldots,v_n$ is assumed to be independently and identically distributed as $N(0, \sigma^2)$. Hence, model (3.11) is correctly specified when model (3.9) is generated and is incorrectly specified when model (3.10) is generated.

Both of these 'true' models represent forms of models previously examined; a homoskedastic model and a heteroskedastic model in an identifiable variable. However, since the variable x_{i6} is used in both of these models, there is a significant difference between these two models and any of the previously generated models. In the case of these two models, one of the independent variables is drawn from a population with non-constant mean and variance. Under the null hypothesis this will cause the diagonal elements of the matrix M to be more unequal than previously and hence cause the OLS residuals to be more heteroskedastic than previously. However, if the theorem proven in this study is correct, the OLS residuals should still appear to be homoskedastic under H_0, and heteroskedastic only under H_2.

Tests Examined that Discriminate Between H_0 and H_2

One final decision must still be made before the experiment is examined; that is, which of the tests for heteroskedasticity reviewed and suggested is to be used to test if either of the two hypothesized models (3.2 and 3.11) is misspecified.

A total of nine tests are currently being used in the literature to discriminate between H_0 and H_2. They have been denoted in this study as GQP, THEIL, RECURSIVE-P, GQN, RECURSIVE-N, BAMSET, PARK, GLEJSER, and FIML. However, all of the tests do not have to be examined if the findings of Harvey & Phillips are referred to. It should be recalled that they found that the tests denoted as GQP, THEIL and RECURSIVE-P (recall that these test procedures were identical except for the predictors of $\sigma_1^2,\ldots,\sigma_n^2$ used) had virtually identical power under a large number of alternative hypotheses. Hence, there seems to be no reason to compare the three tests again. Consequently, only the more commonly used test, GQP, will be examined in this experiment. Similarly, since the two testing procedures GQN and RECURSIVE-N are identical except for the prediction of $\sigma_1^2,\ldots,\sigma_n^2$ that are used, only the more widely accepted procedure, GQN will be used in this experiment.

However, in contrast, since Ramsey & Gilbert [1972] found indications that the BAMSET testing procedure can be used even more successfully with OLS than with BLUS residuals, this procedure will be examined using both sets of residuals. The two tests will be differentiated by suffixing BAMSET with O for OLS residuals and T for BLUS residuals (developed by Theil). Hence, seven of the current testing procedures will be examined in this experiment.

In this study, two different testing procedures have been suggested to discriminate between H_0 versus H_2. Both of these procedures used a POSEX model to explain the unobserved variances $\sigma_1^2,\ldots,\sigma_n^2$. However, because the model could be estimated in two different ways, two different testing procedures were suggested. It should be recalled that when the POSEX model was estimated with OLS, an F-test was suggested to test H_0 versus H_2, whereas, when full information maximum likelihood (FIML) was used to estimate the model, a likelihood ratio test was suggested. These two tests bring the number of tests to be examined in this study to nine.

When these tests were used in a cursory examination, it was discovered that the theoretically expected results were not being obtained with some of the tests. All of these tests were formulated with models that were estimated using a maximum likelihood procedure. Since estimation by maximum likelihood requires an iteration convergence procedure, it was found that the theoretically expected results could only be obtained by increasing the number of iterations. This result was not entirely unexpected since Rutemuller & Bowers [1968] found they needed 15 iterations to converge using the FIML technique. However, since in this experiment the hypothesized model is examined for heteroskedasticity 33,000 times (11 different populations of dependent variables are estimated with 3 different sample sizes and each is replicated 1000 times - 11 x 3 x 1000), increasing the number iterations needed for each examination becomes very costly. For example, it was found in the preliminary study that the 2 tests that use iterative estimation (FIML, POSEX using FIML) required 6 times the amount of computer time than the 6 tests that do not use iterative estimation. Hence, it was decided that neither of the tests which use a maximum likelihood procedure would be examined in this experiment.

The information lost by not examining these two tests could prove to be very small. One of the tests that was dropped from the experiment was based on the POSEX model. However, since one test still remains that is based on the POSEX model, the POSEX procedure can still be carefully examined.

The second test that was dropped is the procedure developed by Rutemuller & Bower [1968] and denoted as FIML in this study. Much evidence already exists on this technique. For example, Goldfeld & Quant [1972] discovered that when the correct form of the heteroskedasticity was known, the FIML testing procedure had a higher probability of correctly rejecting H_0 than any other test. In contrast, they also found that when the form of the heteroskedasticity was not known, the FIML testing procedure seemed to lose this advantage. Therefore, since the FIML testing technique takes much more computer time than other testing procedures (in the preliminary examination it took 15 times as long as the other tests) yet offers no gain in power when the form of the heteroskedasticity is not known, it appears as if the test has a comparative disadvantage to other tests when *a priori* information does not exist. Hence, since in this experiment it is assumed that no *a priori* information exists as to the form of the heteroskedasticity, very little information will be lost by dropping the FIML testing technique.

Seven tests remain to be examined in this experiment. Since it is assumed that no a priori information exists as to the form of the heteroskedastic disturbances, many versions of the different tests are used. In the four non-constructive tests (BAMSETT, BAMSETO, GQP, and GQN), for example, the observations can be reordered by a variable that is suspected of causing the heteroskedasticity. Since, however, no information is available, each of the tests will be reordered in turn by using one of the independent variables of the hypothesized model or by using $\hat{\underline{y}}$ (the OLS predictor of $\underline{y}$). In addition to these three versions, each of the tests will also be used without reordering. In this way, four different assumptions as to the form of the heteroskedasticity are being made. These different tests will be designated by suffixing the test's name with the variable that was used for reordering or by N for no reordering. Thus GQP becomes GQPX1 if the test is reordered by the vector $\underline{x}_1$; GQPX2 if reordered by $\underline{x}_2$; GQPY if reordered by $\hat{\underline{y}}$; and GQPN if no reordering occurs.

Likewise, in the two constructive tests (PARK and GLEJSER) that are currently used in the literature, assumptions will also have to be made. Since no information exists as to the form of the heteroskedasticity, it will be assumed in Park's test (denoted as PARK) that the disturbances are of the form

$$E(u_i^2) = z_i^{\alpha} \sigma^2.$$

Since the variable z_i is unknown, this variable will be assumed to be, in turn, one of the independent variables of the hypothesized set or $\hat{y}_i$ (the OLS predictor of y_i). These three versions will be denoted as PARKX1 (PARKX6 when $\underline{x}_6$ is used instead of $\underline{x}_1$), PARKX2, and PARKY.

In Glejser's test (denoted as GLEJSER) it will be assumed that the heteroskedastic disturbances are of the form

$$E(u_i^2) = (\beta_0 + \beta_1 z_i + \beta_2 z_i^2)^2 \sigma^2.$$

Since once again z_i is not known, it will be assumed, in turn, that z_i is one of the independent variables in the hypothesized model or $\hat{y}_i$ (the OLS predictor of y_i). Each of these different versions will be denoted as GLEJSERX1 (GLEJSERX6 when $\underline{x}_6$ is used in the hypothesized model instead of $\underline{x}_1$), GLEJSERX2, and GLEJSERY. The estimated coefficients $\hat{\beta}_1$ and $\hat{\beta}_2$ will be tested for significance using an F-test as was suggested earlier in this study.

These assumptions together with the assumption necessary for the non-constructive tests expand these six tests into 22 tests. It is important to note that the 'true' models were designed so that each of the assumptions made in the 22 tests would be exactly correct in at least one instance. In this way it can be determined how the power of each of these tests varies when a correct versus an incorrect assumption is made as to the form of heteroskedasticity.

Although it has been shown how assumptions can be incorporated into a POSEX model, in using the model in this experiment it will only be assumed that the heteroskedastic disturbances are a function of the independent variables in the hypothesized model. Hence, when the model is hypothesized to be a function of $\underline{x}_1$ and $\underline{x}_2$ the POSEX model designed to test for heteroskedasticity becomes

$$\underline{\hat{\sigma}}^2 = \beta_0 + \beta_{11} \underline{x}_1 + \beta_{12} \underline{x}_2 + \gamma_2 \underline{q}^{(2)} + \gamma_3 \underline{q}^{(3)} + \gamma_4 \underline{q}^{(4)} + \underline{v}, \tag{3.12}$$

where $\overset{\circ}{\underline{\sigma}}^2$ is a predictor of $\sigma_1^2,\ldots,\sigma_n^2$, $\underline{q}$ is an instrument for the expansion terms and $\underline{v}$ is assumed to be distributed as N $(\emptyset, \sigma^2 I)$. A similar model could, of course, be formulated when the hypothesized model is a function of $\underline{x}_6$ and $\underline{x}_2$. Since OLS is being used to estimate this model (3.12), an F-test will be used to determine if the parameters β_{11}, β_{12}, γ_2, γ_3 and γ_4 are significantly different from zero. This test will be denoted as POSEXH1 to indicate that it is a POSEX model designed to test heteroskedasticity and estimated using OLS (the first of the two estimation procedures earlier suggested).

Since it has been suggested that either $\hat{\underline{y}}$ (the OLS predictor of $\underline{y}$) or $\underline{p}$ (which uses the first principle component of the matrix X) to be used as the instruments for the expansion terms, each will be used in turn To differentiate between the two instruments, the acronym POSEXH1 will be suffixed by Y if $\hat{\underline{y}}$ is used as the instrument or P if $\underline{p}$ is used. Similarly, since it has been suggested that either $\dot{\underline{\sigma}}^2$ (studentized predictors of $\sigma_1^2,\ldots,\sigma_n^2$) or $\ddot{\underline{\sigma}}^2$ (MINQU predictors of $\sigma_1^2,\ldots,\sigma_n^2$) be used as the dependent variable in model (3.12), each will be used in turn. As before, to differentiate between their use, either an S (studentized) or M (MINQU) will suffix the acronyms POSEXH1P and POSEXH1Y. In this way, four versions of the POSEX test designed for heteroskedasticity and estimated using OLS (POSEXH1) will be examined. They will be denoted as

POSEXH1PS - POSEXH1 using the instrument $\underline{p}$ and studentized predictor,

POSEXH1PM - POSEXH1 using the instrument $\underline{p}$ and MINQU predictors,

POSEXH1YS - POSEXH1 using the instrument $\hat{\underline{y}}$ and studentized predictors, POSEXH1YM - POSEXH1 using the instrument $\hat{\underline{y}}$ and MINQUE predictors.

Hence, 26 different versions of the 7 different tests are to be examined in this experiment. Through the use of the different versions of each test, it will be possible to determine the relationship between the power of each test and the version of each test used. This will be especially enlightening when the different versions of each test are the result of different assumptions as to the form of the heteroskedasticity.

III.2.4 Summary

In this section, a sampling experiment has been designed to examine tests that determine if a model has a misspecified conditional mean or has heteroskedastic disturbance terms. The basic procedure used in the experiment was then presented: First, a population of dependent variables is defined. Second, a sample consisting of n (set first at 30, then at 60 and finally at 90) observations, is drawn from this population. Third, the hypothesized model is estimated with the first sample of n observations. Fourth, specified tests are used to determine if the hypothesized model is misspecified and the results are recorded. By repeating this process 1000 times, one can determine the percentage of times that a given test indicates that a model is misspecified. This percentage then corresponds to either the probability of type I error (if the hypothesized model is correctly specified) or the power of the test (if the hypothesized model is misspecified).

In the second and third parts of this section, 17 different populations of dependent variables are defined. The models that generate each of these populations are summarized in Table 2 below.

TABLE 2: Models that Generate the Dependent Variable

Dependent Variable	
1	$y_i = 50 + 5x_{i1} + 5x_{i2} + 50u_i$
2	$y_i = 50 + 5x_{i1} + 5x_{i2} + 5x_{i3} + 50u_i$
3	$y_i = 50 + 5x_{i1} + 5x_{i2} + 5x_{i4} + 50u_i$
4	$y_i = 50 + 5x_{i1} + 5x_{i2} + 5x_{i5} + 50u_i$
5	$y_i = \exp(2 + .05x_{i1} + .05x_{i2} + 2u_i)$
6	$y_i = e^{1.0} x_{i1}^{1.0} x_{i2}^{1.0} e^{u_i}$
7	$y_i = \exp(-(-.25 + .02x_{i1} - .05x_{i2} + .5u_i)^{-2})$
8	$y_i = 50 + 5x_{i1} + 5x_{i2} + x_{i1}u_i$
9	$y_i = 50 + 5x_{i1} + 5x_{i2} + x_{i2}u_i$
10	$y_i = 50 + 5x_{i1} + 5x_{i2} + x_{i3}u_i$
11	$y_i = 50 + 5x_{i1} + 5x_{i2} + x_{i4}u_i$
12	$y_i = 50 + 5x_{i1} + 5x_{i2} + E(y_i)u_i$
13	$y_i = 50 + 5x_{i1} + 5x_{i2} + \frac{100i}{n} u_i$
14	$y_i = 50 + 5x_{i1} + 5x_{i2} + (500 + 10x_{i1} + x_{i1}^2)^{1/2}u_i$
15	$y_i = 50 + 5x_{i1} + 5x_{i2} + (75 + 50 \text{ Sin}(E(y_i)))u_i$
16	$y_i = 50 + 5x_{i6} + 5x_{i2} + 50u_i$
17	$y_i = 50 + 5x_{i6} + 5x_{i2} + x_{i6}u_i$

$u_1, \ldots, u_n$ are independently and identically distributed as N(0, 1). The variables $\underline{x}_1$, $\underline{x}_2$, $\underline{x}_3$, $\underline{x}_4$, $\underline{x}_5$, and $\underline{x}_6$ are listed in Appendix B together with the relevant sample statistics.

The conditional mean of the dependent variables generated by the 17 models will be estimated using two hypothesized models. The first 15 populations of dependent variables will be estimated using the hypothesized model

$$y_i = \beta_0 + \beta_1 x_{i1} + \beta_2 x_{i2} + v_i, \; i = 1,\ldots,n, \qquad (3.13)$$

where $v_1,\ldots,v_n$ are assumed to be independently and identically distributed as N $(0, \sigma^2)$. The conditional mean of the remaining two populations of dependent variables will be estimated using the hypothesized model

$$y_i = \beta_0 + \beta_1 x_{i6} + \beta_2 x_{i2} + v_i, \; i = 1,\ldots,n \qquad (3.14)$$

where $v_1,\ldots,v_n$ are again assumed to be independently and identically distributed as N $(0, \sigma^2)$.

Each of these hypothesized models will be tested to see if it is misspecified. The hypothesized model used for the first 7 populations of dependent variables will be tested for a misspecified conditional mean. The first population should prove to be the only set of dependent variables that is correctly specified. Two tests will be used to determine this. The first is denoted as POSEXMP (POSEX test for a misspecified conditional mean using the vector $\underline{p}$ as the instrument) and the second as POSEXMY (same test as before except that the vector $\hat{\underline{y}}$ is used as the instrument). The hypothesized models used for the remaining 10 populations of dependent variables, together with the population defined by model 1, will be tested for heteroskedastic disturbance terms. The first and fifteenth populations should prove to be the only sets of dependent variables that are correctly specified. This will be done by using the 26 different tests which are listed in Table C1 (the first table in Appendix C).

CHAPTER IV

RESULTS OF SAMPLING EXPERIMENT AND OBSERVATIONS ON THE MAINTAINED HYPOTHESES

In this chapter, the results of the sampling experiment outlined in the last chapter will be given. These results consist of reporting the estimated parameters of the hypothesized model, examining the percentage of times the various tests reject the null hypothesis (power), comparing and contrasting the experimental results between models in the same group, and commenting on the hypotheses stated in the previous chapter.

To facilitate the discussion of these results, the two hypothesized models will be restated and the groupings of the six 'true' models reviewed. For models 1 through 15, the hypothesized model is

$$y_i = \beta_0 + \beta_1 x_{ii} + \beta_2 x_{i2} + v_i, \; i = 1,\ldots,n, \tag{4.1}$$

while for models 16 and 17, the hypothesized model is

$$y_i = \beta_0 + \beta_6 x_{i6} + \beta_2 x_{i2} + v_i, \; i = 1,\ldots,n. \tag{4.2}$$

In both cases, it is assumed that v_i, i=1,...,n, are independently and identically distributed as N $(\emptyset, \sigma^2)$ and that n is equal to first 30, next 60, then 90. The 'true' models were divided into six groups for convenience. They were (1) a model that corresponded to the hypothesized model (4.1), (2) models that included a variable not in model (4.1), (3) models that had a different functional form than

(4.1), (4) models that were heteroskedastic due to a simple function of one variable, (5) models that were heteroskedastic due to a non-linear function of one variable, and (6) models that included a variable which had a conditional mean. Each of these six groups of models will be discussed in one of the three sections of this chapter. The first section will consider the correctly specified model (model 1); the second, the models with a misspecified conditional mean; and the third, models that are heteroskedastic. In analyzing the results of the experiment on each group of models, the estimates of the parameters of the hypothesized model are given first. Following this are the results of the specification error tests applied to each of the models within the group and a summary of these results.

So as to avoid needless repetition, some standardized notation will be introduced at this time. 1000 estimates of the parameters β_0, β_1 (β_6), β_2 and σ^2 are obtained for each of the seventeen models. For each of these models, the arithmetic average of the estimates of each of the four parameters is denoted as $\overline{\beta}_0$, $\overline{\beta}_1$ ($\overline{\beta}_6$), $\overline{\beta}_2$ and $\overline{\sigma}^2$. The variance of each of the estimates of β_0, β_1 (β_6) and σ_2 is denoted by $V(\beta_0)$, $V(\beta_1)$, $(V(\beta_6))$, $V(\beta_2)$. These variances are calculated using the standard algorithm,

$$V(\beta) = \sum_{i=1}^{1000} (\beta_i - \overline{\beta})^2/999.$$

Also, since the hypothesized model is estimated using OLS, an estimate of the variance of $\hat{\beta}_0$, $\hat{\beta}_1$ ($\hat{\beta}_6$) and $\hat{\beta}_2$ is obtained for each of the 1000 times the model is estimated. The average of each of these estimated variances is denoted as $\overline{\sigma}^2$ $(\hat{\beta}_0)$, $\overline{\sigma}^2$ $(\hat{\beta}_1)$ $(\overline{\sigma}^2(\hat{\beta}_6))$ and $\overline{\sigma}^2$ $(\hat{\beta}_2)$ respectively. In addition, and F statistic is calculated to determine if the hypothesized model explains the conditional mean of y_i, i=1,...,n better than does the sample mean $\bar{y} = \sum_{i=1}^{n} y_i/n$. The average of these F statistics for each of the seventeen models is denoted as $\bar{F}$.

IV.1 Hypothesized Model is Correctly Specified

The first group of models consists only of one model. In this case, the hypothesized model is correctly specified. The 'true' model is

$$y_i = 50.0 + 5.0x_{i1} + 5.0x_{i2} + 50u_i, \quad i=1,\ldots,n, \tag{4.3}$$

where $u_1,\ldots,u_n$ are distributed independently and identically as N(0, 1). The estimates of the parameters of the hypothesized model are shown in Table 3 for samples of 30, 60, and 90. It is evident from this table that the estimated parameters become increasingly accurate as the sample size increases from 30 to 90. This is especially true for the estimate of the variance of the disturbance term. It should also be noted that the estimated variance of each of the parameters $(\bar{\sigma}^2(\beta_i))$ decreases as sample size increases, that is, there is a gain in efficiency with increasing sample size. This gain in efficiency is also clear from the fact that the sample variance of each parameter $(V(\beta_i))$ decreases as the sample size increases. Finally, it should be stressed that the estimated variance of the estimates of each of the parameters is extremely close to the sample variance of each of the parameters. In addition, with only one exception (sample size 90, parameter β_0), the difference between the estimated variance and sample variance becomes smaller as the sample size increases.

TABLE 3: Estimates of Parameters in a Correctly Specified Model

Sample Size	30			60			90		
Parameters	0	1	2	0	1	2	0	1	2
$\bar{\beta}_i$	49.437	4.990	5.046	49.747	4.996	5.006	50.162	4.995	4.999
$\bar{\sigma}^2\ (\hat{\beta}_i)$	627.5744	.09683	.7740	193.6708	.05099	.08030	131.3647	.03270	.03613
$V\ (\hat{\beta}_i)$	641.6177	.1027	.7924	194.9459	.05297	.07451	127.5776	.03140	.03586
$\bar{\sigma}^2$, $\bar{F}$	2463.820	152.206		2484.122	482.204		2494.440	552.735	

Twenty-eight specification error tests were applied to model 1. Two of these tests were designed to detect a misspecified conditional mean while the other twenty-six were designed to detect heteroskedastic disturbance terms. The results of these tests appear graphically in Figure 1. The actual number of times out of 1000 that a test rejected H_0 appears in Appendix C with the relevant test statistics.

Figure 1 is designed to enable the reader to make a general comparison of the various tests among the three sample sizes and among the three alpha levels; it is not meant to be used for determining specific percentages of rejection. If the reader wants this specific sort of information, he should use the tables in Appendix C. In Figure 1, the test acronyms appear on the left side of the page (a list of these acronyms appears in Appendix C). These are grouped in terms of the knowledge utilized in each of the tests. First are the POSEX tests, which require limited knowledge. These are followed by tests assuming that $\underline{x}_1$ is causing the heteroskedasticity, next by tests assuming $\underline{x}_2$, then by those assuming $\underline{y}$, and last by those assuming the order of observations. Each test has three lines associated with it. Each of the three lines indicates one of the three different sample sizes used, the upper line representing 30, the middle line 60, and the lower line 90. The length of the line up to the first letter (o for sample size 30, x for sample size 60, and m for sample size 90) represents the percentage of times the test rejected the null hypothesis at the .01 alpha level. The length of the line up to the second and third letters represents the percentage of times the test rejected H_0 at the .05 and .10 alpha levels respectively (the line is continuous, the starting point for

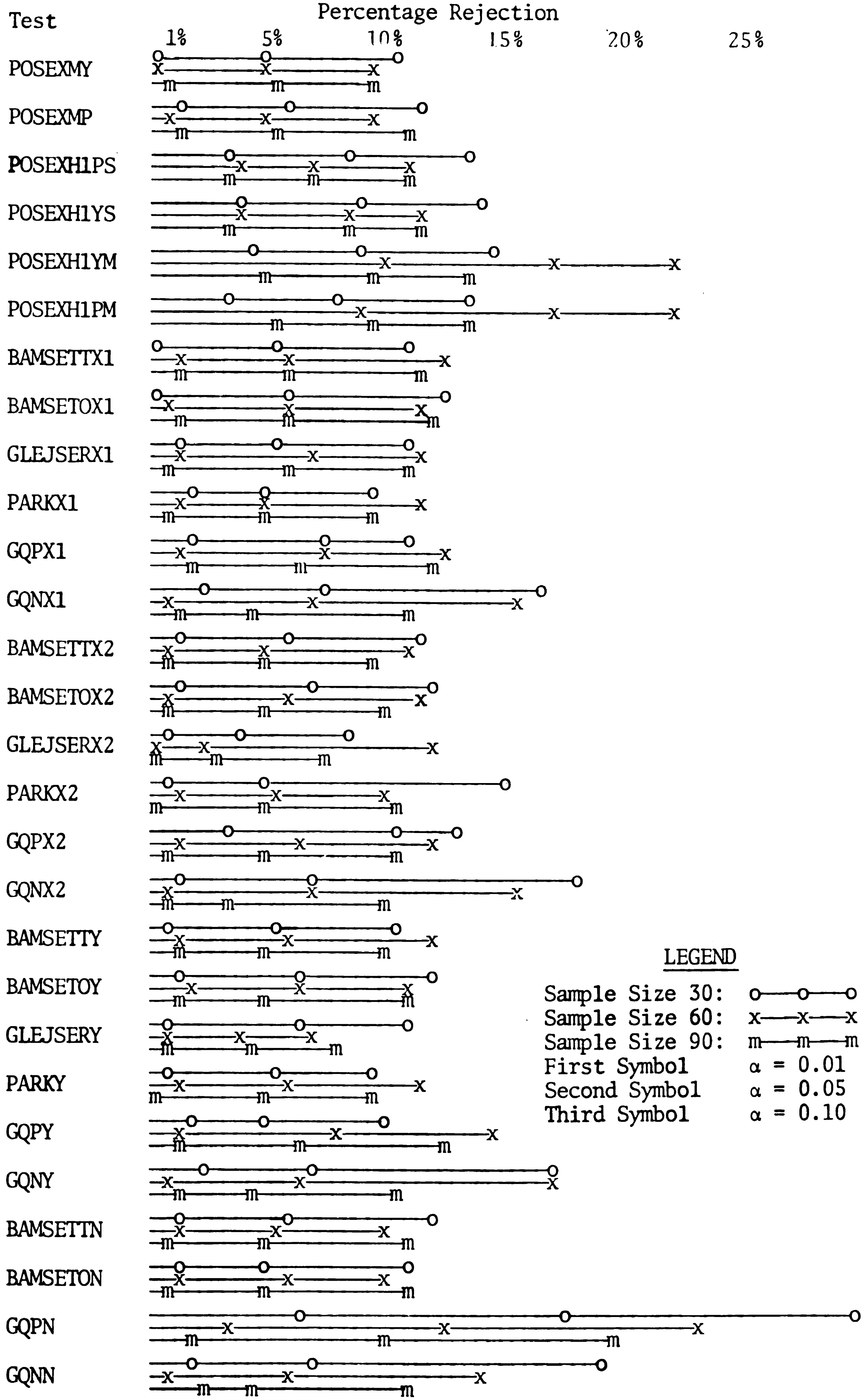

FIGURE 1: A Schematic Diagram of Test Results for Model 1

each of the 3 percentage levels being the same). The results shown in Figure 1 can now be analyzed.

Since the hypothesized model is correctly specified, the estimated alpha level (the percentage of times each test was observed to reject H_0) should correspond to the nominal alpha level at which the test was made. Hence, the first o, x, and m for each test should be approximately aligned with the 1% rejection level; the second o, x, and m with the 5% rejection level; and the third o, x, and m with the 10% rejection level.

Both tests for a misspecified conditional mean (POSEXMY and POSEXMP) conform to these criteria. The largest deviation from the expected result occurs with the test POSEXMP at the .10 alpha level. In this case, the percentage of rejections is approximately 11%, a deviation of 1% from the expected result.

The results for the tests for heteroskedasticity are much more varied. In order to analyze the results, it is useful to set up a confidence interval about each of the nominal alpha levels. In doing this, one presumes that the nominal alpha levels are correct so that the probability of a rejection is known. Using the binomial distribution, one obtains the standard deviation of the number of rejections at each nominal alpha level. The standard deviation is 3 for the .01 alpha level, 7 for the .05 and 9.5 for the .10. Since a binomial is approximated by a normal distribution, ± 2 standard deviations from the nominal alpha level will be used as a 95% confidence interval.

When this procedure is followed, the tests whose estimated alpha levels lie outside the confidence intervals with the most regularity are the POSEX tests for heteroskedasticity and both of Goldfeld &

Quant's testing procedures. Of these, the tests that lie the furthest from the nominal alpha levels are POSEXH1YM, POSEXH1PM and GQPN. With all three tests, the estimated alpha levels average over 10 standard deviations away from the nominal alpha levels. This difference is large enough to cast serious doubts on the tests' validity. Interestingly, the GQPN test procedure (Goldfeld & Quant's Parametric test with no reordering) displays the greatest amount of divergence from the expected result. This is surprising since the test defines a statistic with a known distribution and hence the estimated alpha level should approximate the nominal alpha level at which the test was made.

For samples of 60 and 90, the estimated alpha levels of the remaining two POSEX tests are within 2 standard deviations of the .10 nominal alpha level. However, as the nominal alpha level decreases to .05 and to .01, the number of standard deviations between the estimated alpha level and the nominal alpha level increases. This result, although unfortunate, was not unexpected since the testing procedure used defines a test statistic that is only approximately distributed as F. It was also known [Donaldson, 1968] that this approximation becomes less accurate the farther out on the tail the comparison is made.

In contrast, estimates for two of the three tests based on Glejser's method are not within a 95% confidence region for a nominal alpha level of .10, while for the lower nominal alpha levels, the estimates are within the region. Since Glejser indicated that a nominal alpha level of .11 should be used to obtain a 5% rejection level, it is surprising that a relatively high degree of accuracy is obtained when a nominal alpha level of .05 is used. No explanation can be given for this result although it should be pointed out that while Glejser used a t test on each included variable, an F test on the joint effect of the variables was used in this study.

The estimates of the alpha levels of the BAMSET tests (eight of them) were never more than three standard deviations from the nominal alpha levels used. Since half of the eight tests were defined using OLS residuals and the other half using Theil's BLUS residuals, this agreement between the estimated and nominal alpha levels confirms the findings of Ramsey & Gilbert [1971] that the test can be used with either set of residuals.

In contrast, it is extremely surprising that the estimates of the alpha levels obtained for both the Goldfeld & Quant parametric and non-parametric testing procedures were so frequently outside of the 95% confidence interval about each alpha level (44 out of 72 times). Since both of these procedures define a statistic with a known distribution, it was expected that these results would always lie within the confidence limit.

Equally surprising is the small number of times the three Park testing procedures lay outside of the confidence regions (1 out of 27 times). The estimated alpha levels diverged from the nominal alpha levels less frequently in this test than did any other test examined. Since the statistic is only approximately distributed as t, this accuracy was unexpected. However, as previously mentioned, it was anticipated that the estimated alpha levels in the PARK procedure would agree with the nominal alpha levels more frequently than would the estimates obtained from using the GLEJSER procedure.

The only general comment that can be made applies to the sample size used in each test. It appears that as the sample size increases, the percentage of rejections generally approaches the alpha level at which the test was made. However, there were exceptions even to this, most notably POSEXH1PM, POSEXH1YM, GLEJSERY2, and GQPY.

In general, it appears that if the three tests that lie the furthest outside of the confidence interval are discarded (POSEXH1PM, POSEXH1YM, and GQPN), the overall results are reasonable. When the sample size is small and the alpha level is large, the estimates of the alpha levels obtained by using Goldfeld & Quant's testing procedures lie the furthest outside a 95% confidence interval about .10. However, as the sample size increases, the difference between the nominal and estimated alpha levels decreases. At the lowest alpha level examined, .01, the estimates of the alpha level obtained using the POSEX procedures lie the furthest from the nominal alpha level of .01. The nearest agreement between the nominal and estimated alpha levels were obtained by using either the PARK or BAMSET testing procedures.

IV.2 Hypothesized Models with a Misspecified Conditional Mean

Two of the model groups are examined in this section; the group that includes a variable not in the hypothesized model and the group that has a different functional form than the hypothesized model. After each of these groups has been analyzed, the section will end with a discussion of the hypotheses made in Chapter III that pertain to tests for a misspecified conditional mean.

IV.2.1 Misspecified Conditional Mean Due to an Omitted Variable

There are three hypothesized models that omit a relevant variable. In each case, the 'true' model is

$$y_i = 50.0 + 5.0x_{i1} + 5.0x_{i2} + 5.0z_i + 50u_i, \quad i=1,\ldots,n, \tag{4.4}$$

where $u_1,\ldots,u_n$ are independently distributed as N(0, 1) and z_i denotes the variable omitted from the hypothesized model (4.1). It should be recalled, however, that the three models differ in the degree of correlation between the variable omitted from the hypothesized model and the variables included in the hypothesized model. In the first case, the omitted variable is independent of the included variables (drawn from an independent normal distribution with a mean of 50 and variance of 400); in the second case, it is moderately correlated with each included variable (x_{i4} is defined as

$$x_{i4} = 5.428 \log_e x_{i1} + 7.711 \log_e x_{i2} + \frac{3(x_{i1}-50)}{20},$$

$i=1,\ldots,n$);

and in the third, it is highly correlated (.7) with each included variable (x_{i5} is defined as

$$x_{i5} = .5428\, x_{i1} + .771\, x_{i2} + \frac{3(x_{i3}-50)}{20},$$

$i=1,\ldots,n$,

hence the coefficient of determination between x_{i5} and both x_{i1} and x_{i2} is .98). The estimates of the parameters of the hypothesized models are shown in Table 4 for each of the three sample sizes.

It is obvious from Table 4 that the estimates of β_0, β_1, β_2 and σ^2 obtained do not correspond to the parameters in the 'true' model (4.4). The reason for this is that when an hypothesized model that

TABLE 4: Estimates of Parameters in Models with an Omitted Variable

Sample Size	30			60			90		
Parameters	0	1	2	0	1	2	0	1	2
				Model 2					
$\bar{\beta}_i$	305.544	4.426	7.718	342.371	4.730	4.529	335.186	4.613	5.037
$\bar{\sigma}^2 (\hat{\beta}_i)$	3925.953	.6057	4.842	1021.484	.2689	.4235	641.6897	.1597	.1765
$V (\hat{\beta}_i)$	641.6177	.1027	.7924	194.9459	.05297	.07451	127.5776	.03140	.03586
$\bar{\sigma}^2, \bar{F}$	15413.06	14.705		13102.07	51.641		12184.83	103.793	
				Model 3					
$\bar{\beta}_i$	165.085	5.760	7.927	191.990	5.784	6.048	190.604	5.766	6.079
$\bar{\sigma}^2 (\hat{\beta}_i)$	781.1044	.1205	.9634	261.9452	.0690	.1086	178.1539	.0443	.0490
$V(\hat{\beta}_i)$	641.6177	.1027	.7924	194.9459	.05297	.07451	127.5776	.03140	.03586
$\bar{\sigma}^2, \bar{F}$	3066.570	177.863		3359.842	494.391		3382.904	855.319	
				Model 4					
$\bar{\beta}_i$	50.353	7.619	9.301	56.141	7.670	8.789	55.416	7.652	8.860
$\bar{\sigma}^2(\hat{\beta}_i)$	697.873	.1077	.8607	212.4531	.05593	.08809	142.7931	.03554	.03927
$V(\hat{\beta}_i)$	641.6177	.1027	.7924	194.9459	.05297	.07451	122.5776	.03140	.03586
$\bar{\sigma}^2, \bar{F}$	2739.807	222.832		2725.031	776.047		2711.450	1383.849	

omits a relevant variable is estimated, the estimates of β_0, β_1, β_2 and σ^2 obtained include that part of the omitted variable that each of the variables explains. A detailed discussion of this identification problem is given in section I.2 of this study. Since, however, the population correlation between the omitted variable and the included variable is known in models 2 and 4, the expected values of $\hat{\beta}_0$, $\hat{\beta}_1$, $\hat{\beta}2$ and $\hat{\sigma}^2$ can be obtained. These are given in Table 5 below.

TABLE 5: Expected Value of β_0, β_1, β_2 and σ^2 in Models 2 & 4

Parameter	Model 2	Model 4
$E(\beta_0)$	300	50.0
$E(\beta_1)$	5	7.7140
$E(\beta_2)$	5	8.855
$E(\sigma^2)$	12500	2725

The estimates in Table 4 are very close to these expected values. In addition, generally as the sample size increases, the estimates converge on these expected values (two exceptions are $\overline{\beta}_0$ for sample size 30, which is extremely close to begin with, and $\overline{\beta}_1$ for sample size 90).

Unfortunately, the expected values for the parameters in the hypothesized model cannot be calculated for model 3 because the amount of x_{i1} and x_{i2} in x_{i4} is not known. However, since in model 2 the omitted variable is independent of the included variables while in model 4 it is highly correlated with the included variables, the estimated parameters for model 3, being moderately correlated, should lie between those of model 2 and model 4. This observation is confirmed by the results shown in Table 4. Moreover, since the correlation between $\underline{x}_2$ and $\underline{x}_4$ is always greater than that between $\underline{x}_1$ and $\underline{x}_4$, the estimates of β_2 (associated with $\underline{x}_2$) differs from 5.0 (the true value of β_1 and β_2) more than the estimate of β_1 (associated with $\underline{x}_1$).

All three of the models were tested for a misspecified conditional mean by using the POSEXMY and POSEXMP tests. The results are shown in Table 6.

In model 2, since the omitted variable is independent of the included variables, the test is expected to have very little power. This expectation is confirmed in the test results for model 2 in Table 6. It should, however, be noted that for a sample size of 30, the POSEXMP test, and, for sample size 90, the POSEXMY test rejected the null hypothesis much too infrequently (this is especially obvious at the 10% α level). Since these low rejections are not observed for each sample size, however, it does not appear as if the tests are biased.

In model 3 the test results also confirmed earlier expectations. It appears that if the omitted variable has non-linear components of the included variable, the test shows substantial power. This is especially true for sample sizes of 60 and 90. It should also be noted that the POSEXMP test shows a marked power advantage over the POSEXMY test.

The test results on the last model in this group (model 4) also provided the expected results. Since the omitted variable is highly correlated with the included variables, very little of the omitted variable is not explained by the hypothesized model; hence, the tests for a misspecified model should have very little power. In fact, all

TABLE 6: Percentage of Rejections of Models
With an Omitted Variable

Sample Size		30			60			90	
α (100%) Level	1%	5%	10%	1%	5%	10%	1%	5%	10%
Test									
					Model 2				
POSEXMY	0.8	3.1	8.6	1.9	7.6	15.6	0.1	0.7	1.7
POSEXMP	0.4	1.2	1.7	1.0	7.3	15.5	0.3	3.2	6.9
					Model 3				
POSEXMY	1.5	6.3	12.2	19.9	44.7	54.9	45.6	68.0	80.5
POSEXMP	2.2	10.0	17.5	30.2	55.5	66.8	62.6	81.4	90.6
					Model 4				
POSEXMY	0.5	4.3	9.4	0.8	5.3	9.9	0.6	4.1	9.7
POSEXMP	0.4	5.3	9.9	0.8	3.9	8.8	1.1	4.6	10.6

of the test results for model 4 in Table 6 are within 2 standard deviations of the nominal alpha level used for the test. Thus, the test showed no gain in power over testing a correctly specified model.

IV.2.2 Misspecified Conditional Mean Due to an Incorrect Functional Form

There are three hypothesized models that have the wrong functional form. The 'true' models are:

$$\text{model 5} \quad y_i = \exp(2 + .05x_{i1} + .05x_{i2} + 2u_i), \tag{4.5}$$

$$\text{model 6} \quad y_i = e^{1.0}\, x_{i1}^{\;1.0}\, x_{i2}^{\;1.0}\, e^{u_i}, \text{ and} \tag{4.6}$$

$$\text{model 7} \quad y_i = \exp(-(-.25 + .02x_{i1} - .05x_{i2} + .5_{u_i})^{-2}) \tag{4.7}$$

These 'true' models, it should be recalled, differ in the degree of accuracy with which the correct functional form can be approximated using a four-term Taylor series expansion.

The estimates of the parameters of the hypothesized models are shown in Table 7 for each of the three sample sizes. It is immediately obvious that the estimates given in the table do not correspond to the parameters given in models (4.5), (4.6) and (4.7). This is due to an identification problem caused by using the wrong functional form.

Each of the three models was tested for a misspecified conditional mean by using the POSEXMY and POSEXMP tests. The results are given in Table 8. Generally, the results are as expected. There was a marked increase in power in both models 5 and 6 as the sample size was increased from 30 to 60. In contrast, the power stayed relatively constant as the sample size was increased from 60 to 90.

TABLE 7 : Estimates of Parameters in Models with an Incorrect Functional Form

Sample Size	30			60			90		
Parameters	0	1	2	0	1	2	0	1	2
Model 5									
$\bar{\beta}_i$	-5148.78	142.812	105.024	-522013.8	2099.281	24771.41	-340047.	1885.605	14333.63
$\bar{\sigma}^2(\hat{\beta}_i)$	$287x10^4$	443.293	3543.725	$316x10^9$	$831x10^5$	$131x10^6$	$1594x10^8$	$397x10^5$	$438x10^5$
$V(\hat{\beta}_i)$	$1476x10^5$	72174.39	14155.0	$3379x10^9$	$4761x10^5$	$7768x10^6$	$6499x10^8$	$2743x10^5$	$9858x10^5$
$\bar{\sigma}^2, \bar{F}$	$1128x10^4$	2.6544		$405x10^{10}$	34.26		$3027x10^9$	25.326	
Model 6									
$\bar{\beta}_i$	-3092.50	70.080	195.956	-5808.16	85.607	311.82	-4964.69	82.903	274.360
$\bar{\sigma}^2(\hat{\beta}_i)$	10970469.	1692.585	13530.69	7967874	2097.655	3303.717	$783x10^4$	1947.715	2152.233
$V(\hat{\beta}_i)$	6741709.	1645.323	20531.11	30780259	1540.998	68727.13	$1125x10^4$	1230.913	22939.09
$\bar{\sigma}^2, \bar{F}$	43069414	3.716		$1022x10^5$	23.316		$1486x10^5$	22.950	
Model 7									
$\bar{\beta}_i$	.0854	-.00023	.00849	.1461	-.00071	.00742	.1160	.00035	.00673
$\bar{\sigma}^2(\hat{\beta}_i)$	.01638	$253x10^{-8}$	$202x10^{-7}$	.00493	$130x10^{-8}$	$204x10^{-8}$	.00343	$854x10^{-9}$	$943x10^{-9}$
$V(\hat{\beta}_i)$	.00786	$170x10^{-8}$	$970x10^{-8}$	.00290	$101x10^{-8}$	$402x10^{-9}$	.00193	$629x10^{-9}$	$152x10^{-9}$
$\bar{\sigma}^2, \bar{F}$	.06430	2.620		.06323	14.405		.06512	23.960	

TABLE 8: Percentage of Rejections of Models that have an Incorrect Functional Form

Sample Size		30			60			90	
α (100%) Level	1%	5%	10%	1%	5%	10%	1%	5%	10%
Test									
				Model 5					
POSEXMY	2.5	7.9	12.9	85.4	89.1	90.9	66.7	72.3	77.9
POSEXMP	13.8	21.1	25.3	90.5	93.0	94.5	92.3	93.9	95.2
				Model 6					
POSEXMY	1.5	5.7	12.3	39.8	51.4	57.6	34.2	43.8	52.6
POSEXMP	8.6	14.5	20.6	55.6	66.1	70.7	51.7	61.3	69.7
				Model 7					
POSEXMY	1.6	5.2	9.8	4.8	19.1	29.7	1.0	4.9	10.0
POSEXMP	16.7	33.6	41.0	43.3	55.5	60.5	16.3	23.8	29.3

It thus appears as if the power function rises very quickly with respect to sample size and flattens out soon thereafter. Also, in models 5 and 6, it should be noted that the test POSEXMP was more powerful than POSEXMY for every sample size and alpha level examined.

Although there was generally a decrease in power going from models 5 to 6 to 7, the decrease in going from 6 to 7 was not as marked as expected when the POSEXMP was used. This was especially true for a sample size of 30 where the power actually increased substantially. Since model 7 is a non-analytic function (the function is not continuous) in the neighborhood of 0.0 and since a Taylor series expansion is not able to approximate a non-analytic function, it was expected that the percentage of rejections would correspond to the alpha level at which the test was made. These expected results were obtained for sample size of 30 and 90 when the POSEXMY test was used but were never obtained when the POSEXMP test was used.

IV.2.3 Examination of Hypotheses on Tests Designed to Detect a Misspecified Conditional Mean

Five hypotheses were stated in section III.1 relating to tests designed to detect a misspecified conditional mean vector. Observations on each of these hypotheses will be stated in turn.

Hypothesis 1 - In both the POSEXMY (equivalent to Ramsey's and Ramsey & Schmidt's) and POSEXMP tests, the estimated alpha levels were within 2 standard deviations (95% confidence region) of the nominal levels at which the tests were made, as illustrated in Figure 1.

Hypothesis 2 - It was also observed, as hypothesized, that the power of each test increased as the correlation between the omitted variable and the included variable would increase. However, it was also observed that this power function would decrease (as hypothesized) when the correlation increased past some point. Unfortunately, since only three points are observed on the power function, a more precise statement cannot be made.

Hypothesis 3 - The third hypothesis was not completely verified by the experiment although it was observed that the power of both tests to detect the misspecified model decreased from model 5 to model 6, and to a lesser extent, to model 7. (Recall that a four-term Taylor series expansion became less accurate at approximating the correct functional form of the model as the model numbers increased from 5 to 7.) The reason that this acceptance is only partial is that for the POSEXMP test for sample size 30, the power calculated in model 7 is greater than that in either model 5 or 6; this finding is contrary to the hypothesis since model 7 is a non-analytic function.

Hypothesis 4 - It was also observed that the power of both tests did not always increase as the sample size increased from 30 to 60 to 90 observations. Rather, the power increased as hypothesized only when the misspecified model either had an omitted variable with medium correlation or when the correct functional form was easily approximated by a Taylor series expansion. That is, when the theory behind the POSEX tests indicates that the tests would have little power, the power is not increased by increasing the sample size.

Hypothesis 5 - The last hypothesis was maintained for every alpha level, sample size, and model examined. It was continually observed that the POSEXMP test was more powerful than the POSEXMY test was more powerful than the POSEXMY test (recall that this is equivalent to Ramsey's and Ramsey & Schmidt's test). However, it should be pointed out that some might find the POSEXMY test more appealing because of its simplicity.

Summary

Hence, while the first, second, and fifth maintained hypotheses were conclusively supported, the experimental results did not completely substantiate the third and fourth hypotheses. However, the findings did indicate that the fourth hypothesis was true in certain important cases and that the third hypothesis seemed always to be true for large sample sizes.

IV.3 Hypothesized Models with Heteroskedastic Disturbance Terms

The remaining three model groups are analyzed in this section; the group of models that are heteroskedastic due to a simple function of one variable, the group that is heteroskedastic due to a non-linear function of one variable, and the group that includes a variable with a conditional mean. As before, after each group of models has been analyzed, a discussion of the hypotheses made in Section III.1 that pertain to tests for a misspecified conditional mean will be given.

IV.3.1 Heteroskedasticity due to a Simple Function of One Variable

There are six hypothesized models that are heteroskedastic because the disturbance terms are multiplied by a single variable. In each case, the 'true' model is

$$y_i = 50.0 + 5.0x_{i1} + 5.0x_2 + z_i u_i, \; i=1,\ldots,n. \tag{4.8}$$

However, the variable z_i differs for each model; it is x_{i1} in model 8,

x_{i2} in model 9, x_{i3} in model 10, x_{i4} in model 11, $E(y_i)$ in model 12, and 100i/n in model 13. Because of these differences, the six models differ in the form of heteroskedasticity and the relation between the disturbance terms and the included variables in the model. The estimate of the parameters of the hypothesized model is shown in Table 9 for each of the sample sizes examined.

In every case, the estimates of β_0, β_1, and β_2 are statistically equal to the true values of the parameters. Of the divergences from the true values, the greatest is about 11% and occurs in model 12 for sample size 90. This unbiasedness is even more evident if model 12 is discarded since the largest bias in the remaining models is less than 2%. It should also be noted that in all but one case ($\bar{\beta}_2$, model 9), the bias in parameters β_1 and β_2 becomes smaller (or shows a negligible increase) as the sample size increases from 30 to 60 to 90. This does not appear to be true for β_0. This result, however, is not entirely surprising since the estimates of the intercept terms have such large variances associated with them.

Next, it should be noted that the estimated variance of all the parameters and the sample variance of the parameters decreases as the sample size increases; that is, there is an increase in efficiency as the sample size increases. Also, it should be noted that with a few exceptions (most notably β_2 in model 12 and in model 9), the average estimates of the variance of each parameter $(\bar{\sigma}^2(\hat{\beta}_i))$ are extremely close to the observed variance in the parameter estimates $(V(\hat{\beta}_i))$.

The last estimated parameter to be examined is the variance of the disturbance term $z_i u_i$. If the averages of the estimates of the variance $(\bar{\sigma}^2)$ for the different sample sizes are simply compared,

TABLE 9: Estimation of Simple Heteroskedastic Models

Sample Space	30			60			90		
Parameters	0	1	2	0	1	2	0	1	2
				Model 8					
$\hat{\beta}_i$	49.722	4.983	5.038	50.273	4.986	5.002	50.410	4.990	4.996
$\bar{\sigma}^2(\hat{\beta}_i)$	916.19	.1414	1.130	266.68	.0702	.1106	193.30	.0481	.0531
$V(\hat{\beta}_i)$	380.05	.1683	.7760	152.32	.0869	.1804	80.994	.0546	.0667
$\bar{\sigma}^2, \bar{F}$		3596.9	109.90		3420.6	359.45		3670.5	572.22
				Model 9					
$\hat{\beta}_i$	50.120	4.995	5.008	50.442	4.997	4.982	50.762	4.996	4.975
$\bar{\sigma}^2(\hat{\beta}_i)$	103.94	.0160	.1282	57.971	.0152	.0240	61.047	.0152	.0167
$V(\hat{\beta}_i)$	84.734	.0129	.3322	88.003	.0120	.3856	86.915	.0085	.1941
$\bar{\sigma}^2, \bar{F}$		408.06	987.59		743.56	1924.2		1159.2	2076.4
				Model 10					
$\hat{\beta}_i$	49.174	4.989	5.055	49.666	4.997	5.007	50.032	4.997	5.000
$\bar{\sigma}^2(\hat{\beta}_i)$	869.12	.1341	1.072	257.872	.0679	.1069	167.811	.0418	.0462
$V(\hat{\beta}_i)$	915.01	.1495	1.229	236.115	.0689	.0572	162.448	.0415	.0450
$\bar{\sigma}^2, \bar{F}$		3412.1	76.51		3307.6	368.21		3186.5	656.24
				Model 11					
$\hat{\beta}_i$	49.710	4.990	5.034	49.923	4.995	5.002	50.313	4.995	4.995
$\bar{\sigma}^2(\hat{\beta}_i)$	452.25	.0690	.5570	134.88	.0355	.0559	94.109	.0234	.0258
$V(\hat{\beta}_i)$	329.62	.0670	.6424	114.82	.0341	.0886	68.273	.0192	.0457
$\bar{\sigma}^2, \bar{F}$		1775.5	213.35		1730.1	695.42		1787.0	1159.7
				Model 12					
$\hat{\beta}_i$	48.649	4.881	5.275	53.323	4.908	4.925	55.775	4.929	4.854
$\bar{\sigma}^2(\hat{\beta}_i)$	46735.	7.721	57.64	14929.	3.930	6.190	11798.	2.936	3.244
$V(\hat{\beta}_i)$	24684.	7.777	58.99	17756.	4.267	28.97	8445.2	2.653	12.80
$\bar{\sigma}^2, \bar{F}$		183478	2.080		191486	9.151		224025	11.578
				Model 13					
$\hat{\beta}_i$	49.615	4.985	5.061	49.585	5.003	5.005	50.240	4.994	4.996
$\bar{\sigma}^2(\hat{\beta}_i)$	894.64	.1380	1.103	264.72	.0696	.1098	178.26	.0443	.0490
$V(\hat{\beta}_i)$	789.46	.1269	.8713	254.92	.0621	.1227	155.50	.0415	.0652
$\bar{\sigma}^2, \bar{F}$		3512.3	75.591		3395.4	363.75		3385.0	620.86

extremely misleading information will result. This becomes evident if one notes that for any model and sample size, the variance of the disturbance term is

$$\mathrm{Var}(z_i u_i) = E(z_i u_i - E(z_i u_i))^2 = E(z_i^2 u_i^2) = E(z_i^2)\ E(u_i^2)$$

$$= E(z_i)^2 = \mathrm{Var}(z_i) + (\mathrm{Mean}(z_i))^2$$

since the $E(u_i^2)=1$ and where Mean and Var denote the sample mean and variance of z_i. Hence, the variance of the disturbance term depends on the sample of z_i used. The expected variance of the disturbance term for each model and sample size appears in Table 10. Whereas the

TABLE 10: Variance of Disturbance Terms In Simple Heteroskedastic Models

Sample Size	30	60	90
Model 8	3677.922	3516.607	3694.970
Model 9	432.890	978.705	1334.608
Model 10	3055.475	2936.759	3204.806
Model 11	1810.687	1764.628	1806.472
Model 12	188204.8	205926.9	229139.5
Model 13	3530.487	3431.250	3398.457

superficial examination of the estimated variances in Table 9 could lead one to conclude that the estimate of the variance was biased (especially model 9), one now finds that the estimates are unbiased with the divergencies from the true values generally decreasing slightly as the sample size increases.

Next, each of the six models was tested to determine if the disturbance terms are heteroskedastic. The results for each model appear on a separate figure and will be examined in turn. The reader is referred to section IV.1 for a basic explanation of all the following figures.

The test results for model 8 appear in Figure 2. In this model, the heteroskedasticity is caused by the variable $\underline{x}_1$. The most obvious result which can be inferred from Figure 2 is that the group of tests that assume that $\underline{x}_2$ is causing the disturbances to be heteroskedastic together with those tests that did not reorder the observations have comparatively little power. Noteworthy for their slight differences are the BAMSET tests when the observations have been reordered by $\underline{x}_2$. Also strikingly obvious is the fact that the tests with the greatest power are those that assume (correctly) that the heteroskedasticity is caused by $\underline{x}_1$. Next most powerful are the tests that use the predicted value of $\underline{y}$ which are closely followed by the POSEX tests. The difference in power among these tests appears to be very small for a sample size of 90 and increases as the sample size decreases.

Generally, the results are as expected. Since the POSEX tests require less _a priori_ information, it was expected that they would have less power than the tests which correctly assumed that $\underline{x}_1$ was causing the disturbances to be heteroskedastic. One rather surprising finding is the extremely good results obtained by the tests that assumed that the predicted values of y were causing the heteroskedasticity. Another somewhat surprising result was how well the BAMSET tests did when the observations were incorrectly reordered by $\underline{x}_2$.

The results for the next model, appearing in Figure 3, are unfortunately not as definitive. The heteroskedasticity is caused by $\underline{x}_2$ in this model. Unsurprisingly, the most powerful tests overall seem to be those that assumed that the variable $\underline{x}_2$ was causing the disturbance terms to be heteroskedastic. Although these tests as well as all the others seem to show a marked loss in power for a

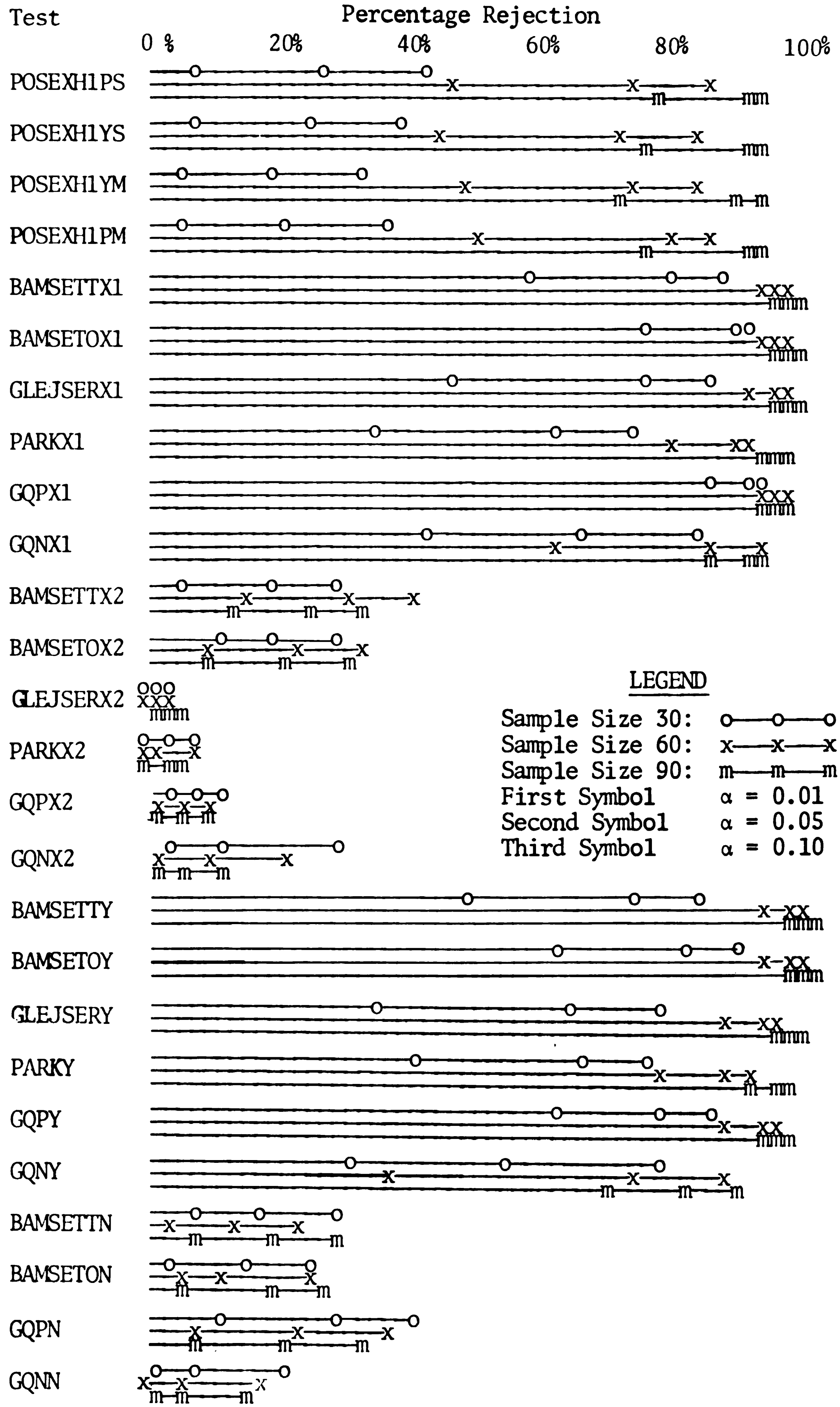

FIGURE 2: A Schematic Diagram of Test Results for Model 8

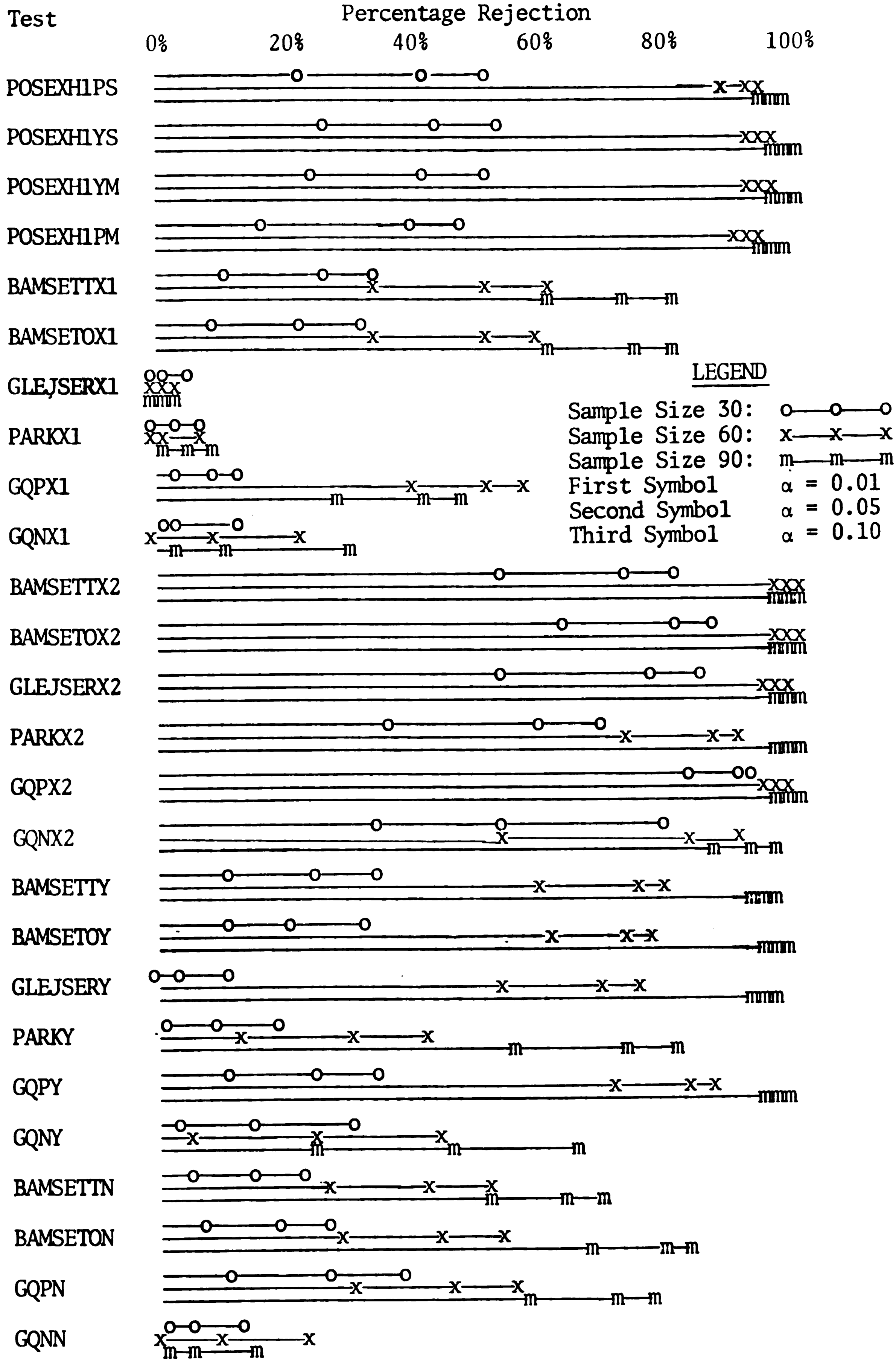

FIGURE 3: A Schematic Diagram of Test Results for Model 9

sample size of 30, the power loss does seem to be less for this group of tests. The next most powerful set of tests seem to be the POSEX tests. This superiority over the remaining tests is most evident for sample size 60 and, to a slightly lesser degree, for sample size 30. The tests that reordered the observations by using the predicted values of $\underline{y}$ did comparatively worse, particularly for sample size 30, in correctly rejecting the null hypothesis in this model than they did in the previous model. Surprisingly, the tests that did not reorder the observations showed a considerable increase in power over that displayed in the previous model. Similarly, the Goldfeld & Quant and BAMSET tests that reorder the observations by the wrong variable ($\underline{x}_1$ in this case) showed a marked increase in power over the last model.

In both models, the tests that correctly assume the variable which is causing the heteroskedasticity seem to display the greatest power. However, since a test that incorrectly assumes that the variable causing the heteroskedasticity has low power, the tests that display the greatest overall power are the POSEX tests. Nevertheless, these are closely followed by the BAMSET tests that reorder the observations by the predicted value of $\underline{y}$.

In contrast to the overall excellent results obtained in the last model, the results of Model 10, appearing in Figure 4, are extremely poor. It should be stressed that this was expected since the variable causing the heteroskedasticity is independent of the variables in the hypothesized model. One surprising result is the slight power advantage displayed by all of the BAMSET tests. Equally notable was the tremendous power displayed by the GQPN. However,

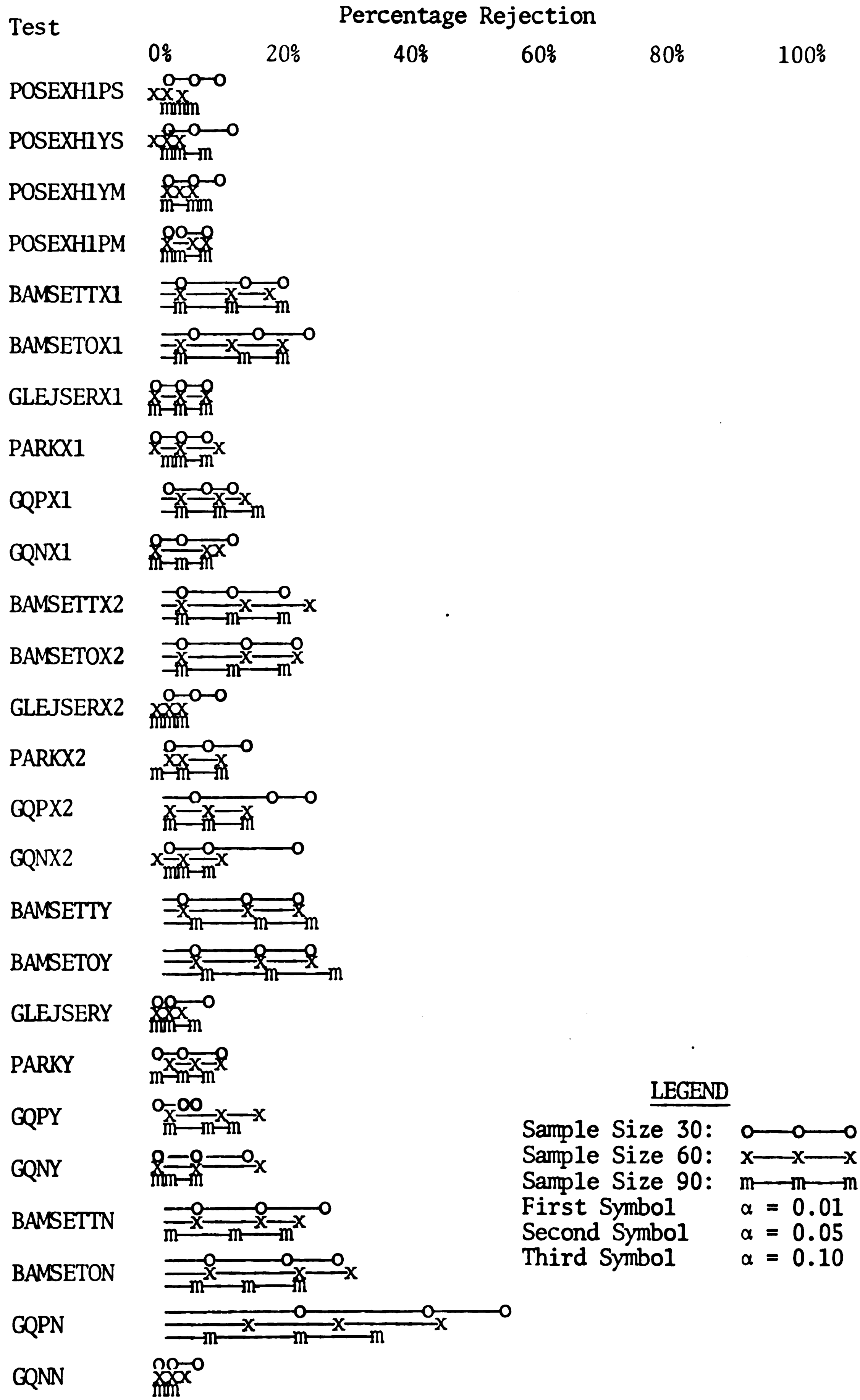

FIGURE 4: A Schematic Diagram of Test Results for Model 10

since the alpha level under H_0 could not be determined, this result loses much of its significance. For all of the tests it was suspected that the percentage of rejections would correspond to the alpha level at which the tests were made.

The results of model 11, appearing in Figure 5, were even more consistent with the expected results than those obtained from the previous model. Recall that in this model the variable causing this model to be heteroskedastic is a weighted sum of non-linear functions of $\underline{x}_1$, $\underline{x}_2$, and $\underline{x}_3$, two of which are variables in the hypothesized model. With the exception of the tests that did not reorder the observations, all of the tests displayed extremely similar power. Nevertheless, the GQP tests seem to show a slight overall advantage closely followed by the POSEX tests, BAMSET tests, GLEJSER and PARK tests.

The results of model 12 are given in Figure 6. In this model, the E(y) is causing the disturbances to be heteroskedastic. The tests that assumed $\underline{x}_1$ was causing the heteroskedasticity seem to display the greatest power. Next most powerful are the tests that assume $\hat{\underline{y}}$ is causing the problems which are closely followed by the POSEX tests. This ranking is most evident if one makes the comparison with a sample size of 30; at the other two sample sizes, 60 and 90, the differences appear to be negligible. Although the E(y) is composed both of $\underline{x}_1$ and $\underline{x}_2$, it appears as if the tests that rank $\underline{x}_1$ display an advantage because the mean and variance of $\underline{x}_2$ is less than those of $\underline{x}_1$.

In the last model examined in this section, the variable causing the disturbances to be heteroskedastic is 100i/n, where i is the

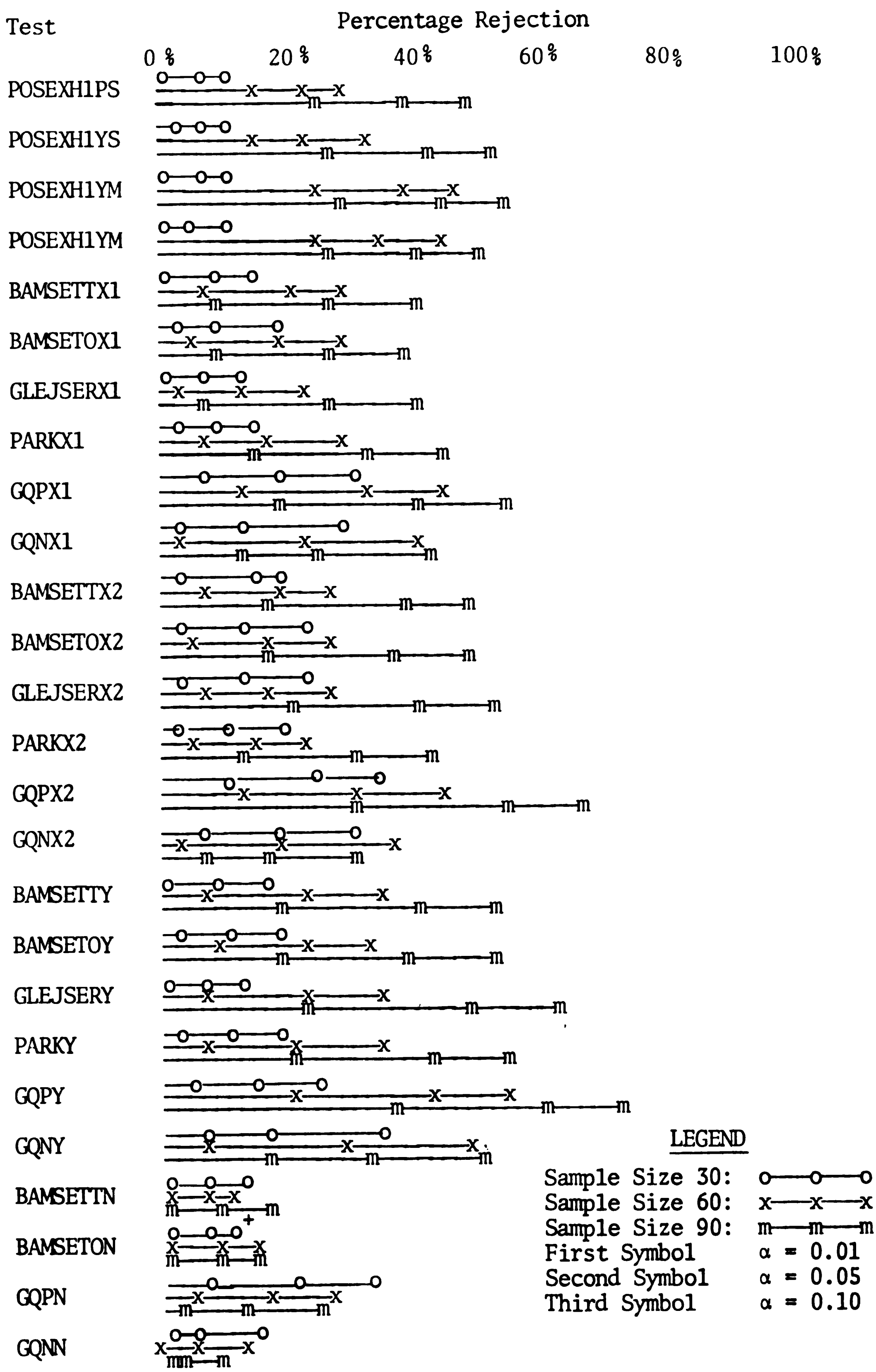

FIGURE 5: A Schematic Diagram of Test Results for Model 11

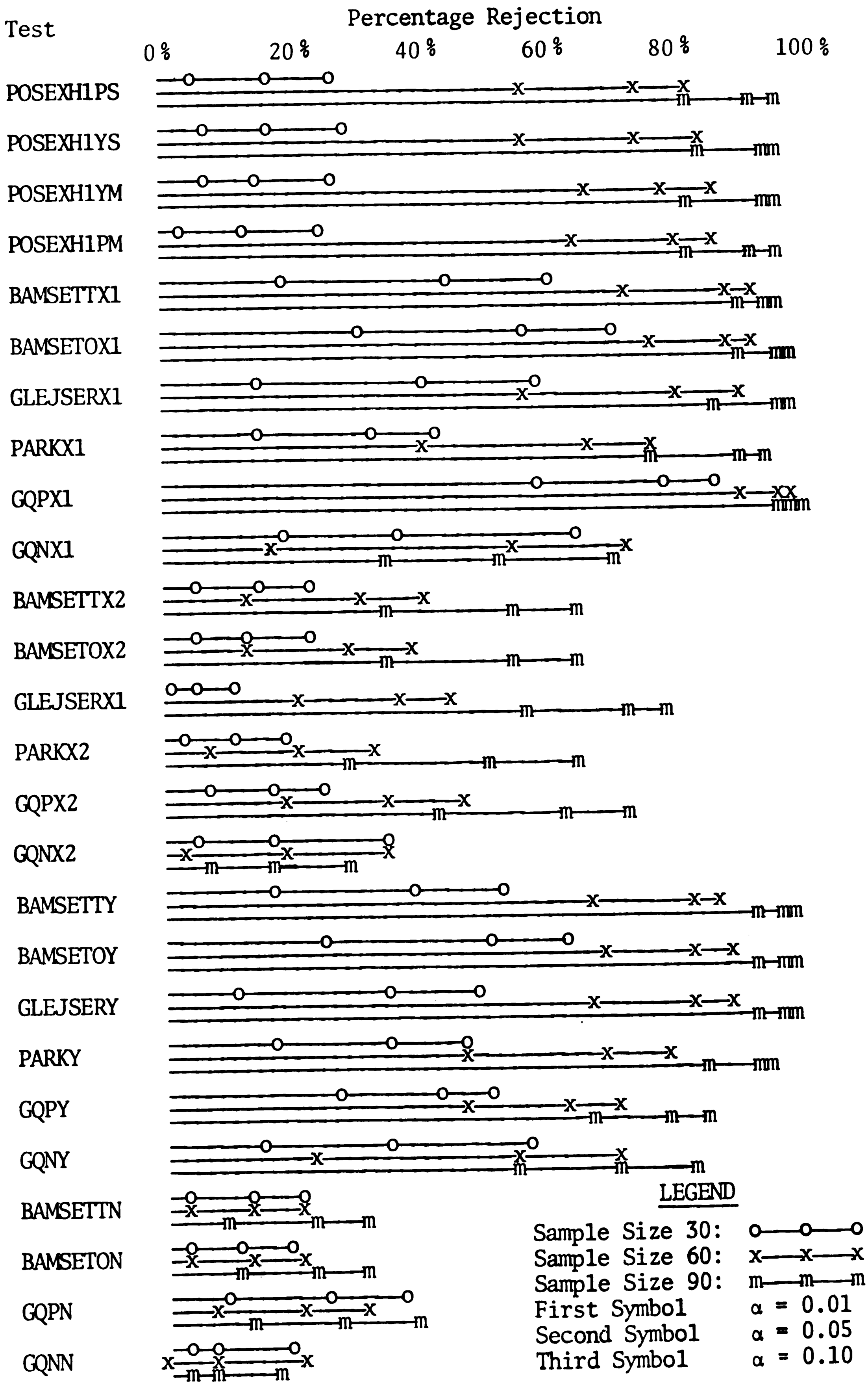

FIGURE 6: A Schematic Diagram of Test Results for Model 12

observation number and n is the number of observations. The results of this model appear in Figure 7. Since the variance of the disturbance term increases with the observation number, it was not surprising that the most powerful tests were those that did not reorder the observations. Once again, however, the other BAMSET tests displayed a much greater power than was expected. This is true to a lesser degree with respect to the GQP tests and the POSEX tests. Also, unlike most other models, only a small increase in the power was observed in all the tests as sample size increased from 30 to 90.

The results of this group of models seem to indicate that if the variable that is causing the heteroskedastic disturbance is known, the test used should reflect this knowledge. Under these conditions, the GQP test seems to be the most powerful followed closely by the BAMSET tests and the GLEJSER and PARK tests. Although the latter two are not as powerful, they have the advantage of being constructive tests. If, however, knowledge about the variable causing the problem is unknown, it appears that the tests with the greatest power are BAMSET tests with the observations reordered by the predicted value of $\underline{y}$. This is followed by GQPY, the POSEX tests, GLEJSERY and PARKY. The last three tests have the advantage of being constructive tests. Three surprising results were observed in this group of models. First was the generally high power displayed by the BAMSET tests. Second was the typically large gain in power observed as the sample size was increased from 30 to 60 observations. Third was the unexpectedly high power displayed when tests based on the $E(\underline{y})$ were used when the correct knowledge was unavailable.

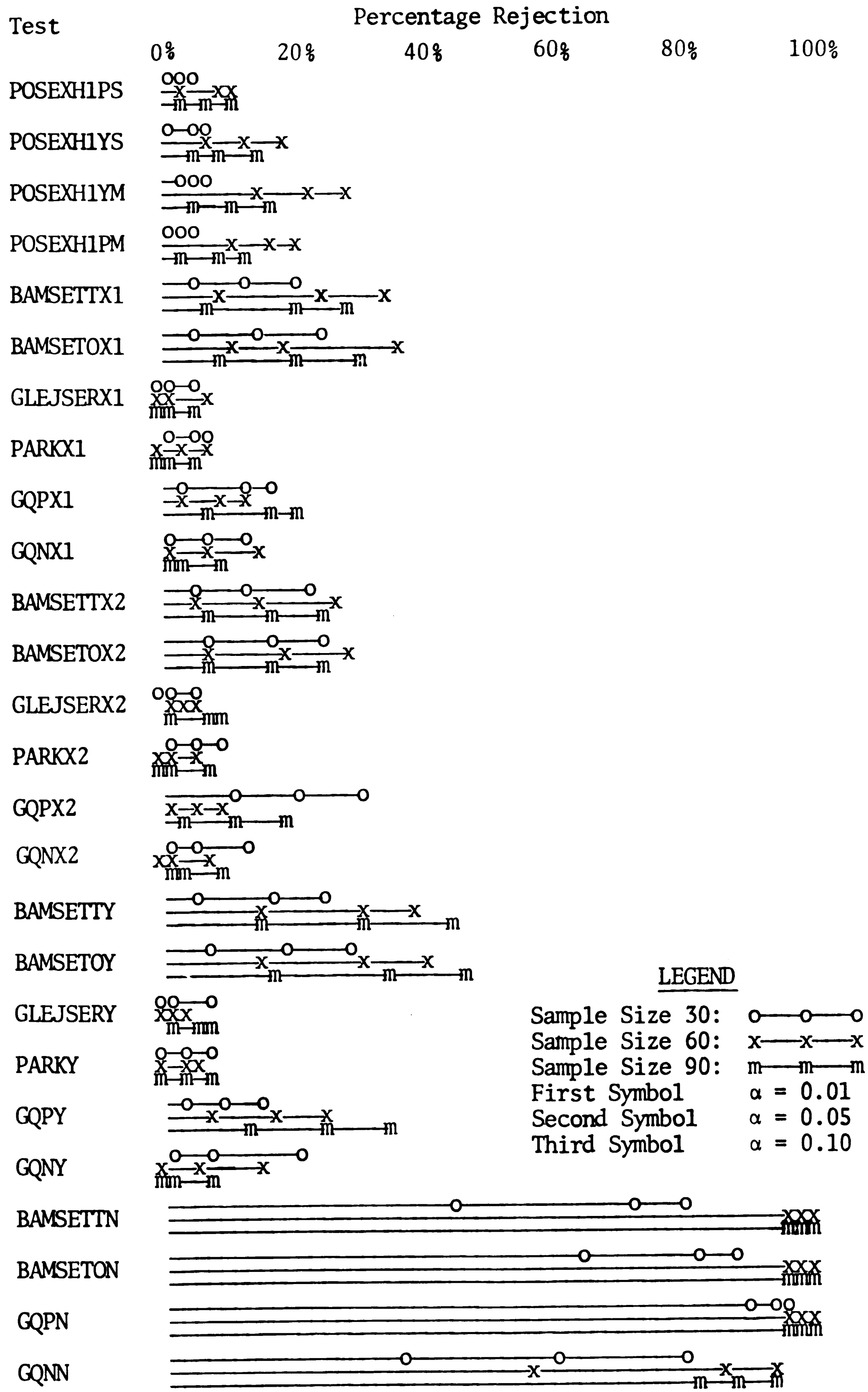

FIGURE 7: A Schematic Diagram of Test Results for Model 13

IV.3.2 Heteroskedasticity Due to a Non-linear Function

Two models are examined which are heteroskedastic because the disturbance terms are multiplied by a non-linear function of a single variable. The basic model is given in equation (4.8); however, in this group of models, z_i is a non-linear function. In model 14, the disturbance term is multiplied by $(500 + 10x_{i1} + x_{i1}^2)^{1/2}$ while in model 15, the disturbance term is multiplied by $75 + 50 \sin (E(y_i))$. These two functions differ from the last group of models in two ways. First, the heteroskedasticity generated in these models is mixed (has a non-zero intercept) while in the previous group it was pure. Second, the heteroskedastic disturbances generated in this group are more complex than in the previous group since more Taylor series expansion terms are needed to correctly identify the function in this latter group.

The results obtained from estimating the hypothesized model appear in Table 11. As with the previous group of models, the estimates of β_0, β_1, and β_2 obtained are statistically equal to the parameters given in model (4.8). However, as with previous models, while the estimates of β_1 and β_2 converge to the true values as the sample size increases, the estimate of β_0 does not. In contrast, all three parameters show a significant gain in efficiency as the sample size increases from 30 to 60 to 90 observations. Finally, it should be noted that although the estimated variances seem to be very volatile, this is once again due to the samples of $\underline{x}_1$ and $\underline{x}_2$ used in this study. When these differences are taken into account, the variance converges to the expected variance as the sample size increases.

TABLE 11: Estimation of Non-Linear Heteroskedastic Models

Sample Size	30			60			90		
Parameters	0	1	2	0	1	2	0	1	2
				Model 14					
$\bar{\beta}_i$	49.569	4.982	5.048	50.178	4.986	5.003	50.042	4.990	4.996
$\bar{\sigma}^2(\beta_i)$	1174.793	.1813	1.4490	345.1756	.0908	.1431	247.5318	.06161	.06808
$V(\beta_i)$	582.7640	.2109	1.0726	217.8450	.1088	.2170	119.5675	.06686	.08319
$\bar{\sigma}^2, \bar{F}$	4612.170	56.423		4427.398	275.811		4700.299	444.937	
				Model 15					
β_i	49.610	4.988	5.038	49.444	4.996	5.012	50.476	4.990	5.002
$\bar{\sigma}^2(\beta_i)$	1498.404	.2312	1.8481	533.8733	.1405	.2214	367.2037	.09139	.1010
$V(\beta_i)$	1391.667	.2084	2.939	373.7813	.1230	.09549	274.4053	.0840	.06768
$\bar{\sigma}^2, \bar{F}$	5882.6470	45.379		6847.731	178.7311		6972.707	300.886	

The results of testing model 14 for a heteroskedastic disturbance term are given in Figure 8. The results of testing this model are similar to the results obtained for the last group of models. The most powerful tests are again the ones that used correct knowledge as to the variable that is causing the disturbances to be heteroskedastic. Also, it is interesting to note that of the tests using $\underline{x}_1$, the PARK test uses the least correct a priori information (it assumes that $E(u_i^2) = x_{i1}^{\alpha} \sigma^2$, whereas GLEJSER correctly assumes that the $E(u_i^2) = (\beta_0 + \beta_1 x_{i1} + \beta_2 x_{i2}) \sigma^2$ and the other tests correctly assume that the heteroskedastic function is monotonic in the range examined) and hence shows the least power of all the parametric tests compared.

Also, as before, there seems to be a large gain in power as the sample size increases from 30 to 60 and less of a gain as the number of observations are increased from 60 to 90. It should also be noted that the BAMSET tests that incorrectly assumed the wrong variable as well as the tests that did not reorder the observations all displayed a greater power than one would expect.

Finally, it must be pointed out that the POSEX tests, though displaying a respectable amount of power, were generally less powerful than the tests that assumed that the heteroskedastic disturbances were caused either by variable $\underline{x}_1$ or by the $E(\underline{y})$. At first glance, this result seems unexpected since the POSEX tests were to have an advantage when the heteroskedastic disturbances were non-linear. However, because the tests examine a predictor of the squared disturbance term, the function that is examined by the tests is

$$((500 + 10x_{i1} + x_{i1}^2)^{1/2})^2 = 500 + 10x_{i1} + x_{i1}^2.$$

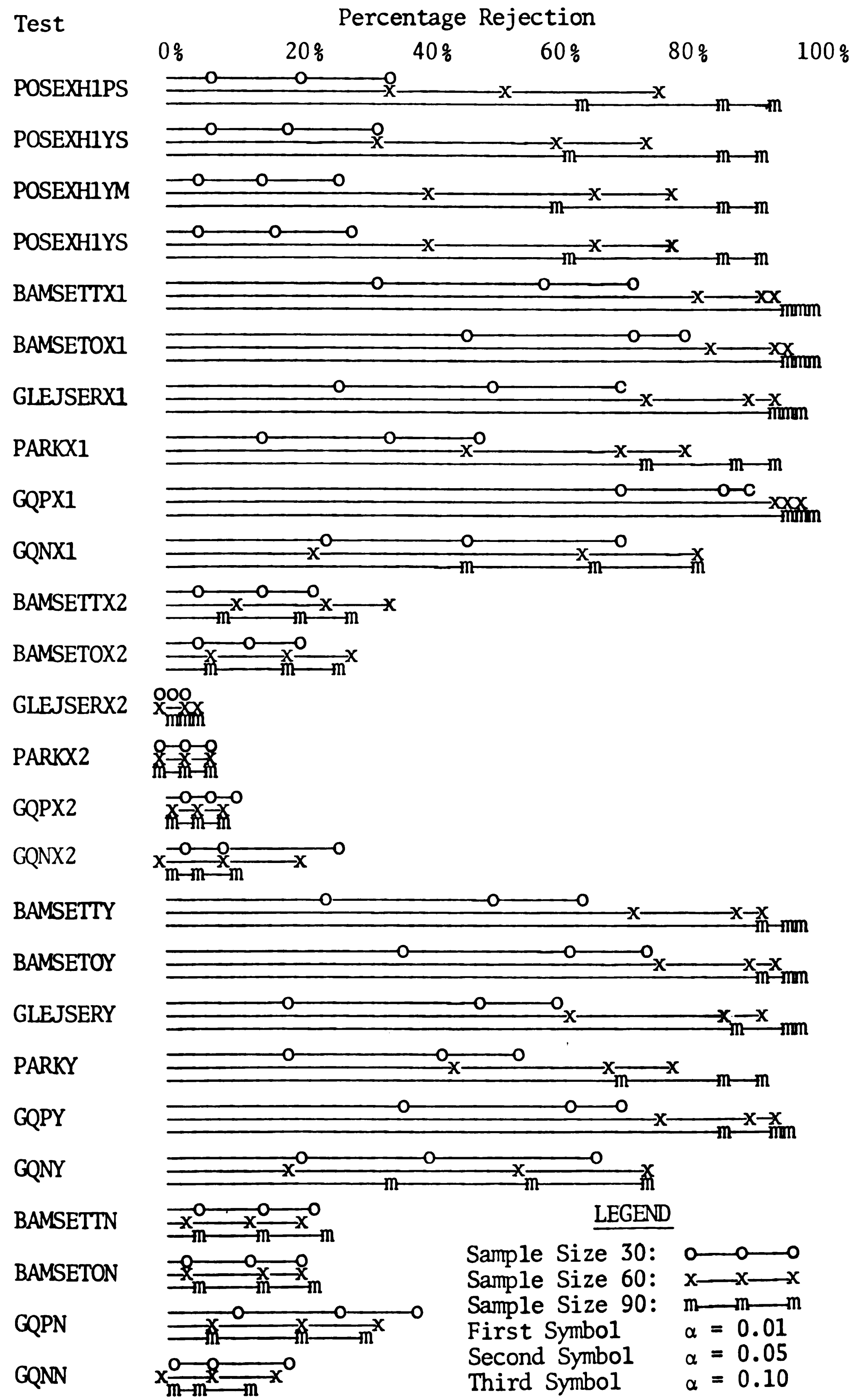

FIGURE 8: A Schematic Diagram of Test Results for Model 14

The slope of this function is $2x_{i1} + 10$ which is a monotonic function in the region in which x_{i1} is restricted ($0 \leq x_{i1} \leq 100$). Therefore, the advantage the POSEX test appeared to have is minimized and in some cases removed altogether (the Goldfeld & Quant tests and the BAMSET tests). In conclusion, it should be recalled that this model was included in this study because it was a more complex function (involving two terms and a non-zero intercept), not because it was thought to be non-monotonic in the area of interest. Instead, the following model was designed to fill this gap.

In Figure 9, the test results for model 15 are reported. In this model, the POSEX tests have the lowest power closely followed by the other constructive tests (GLEJSER and PARK). Although this result was suspected for the latter two tests, it was unexpected for the POSEX tests. However, in retrospect, it should have been expected. To understand why this is, one must carefully examine the function used; it is: $75 + 50\,\mathrm{Sin}(E(y))$. Since Sin x ranges between -1 and +1, it was expected that this function would range between 125 and 25. However, because $E(y_i)$ is conditional on x_{i1} and x_{i2}, both of which range between 0 and 100 with a probability greater than .99, the $E(y_i)$ can range from 0 to 1050. This means that the function oscillates between 25 and 125 a total of 167 times. Therefore, since only a maximum of 90 points are observed on this function, the function is not clearly defined and the points appear to be random. Thus, the POSEX tests, as well as all the other tests, do not reject the null hypothesis of homoskedasticity as often as would be expected.

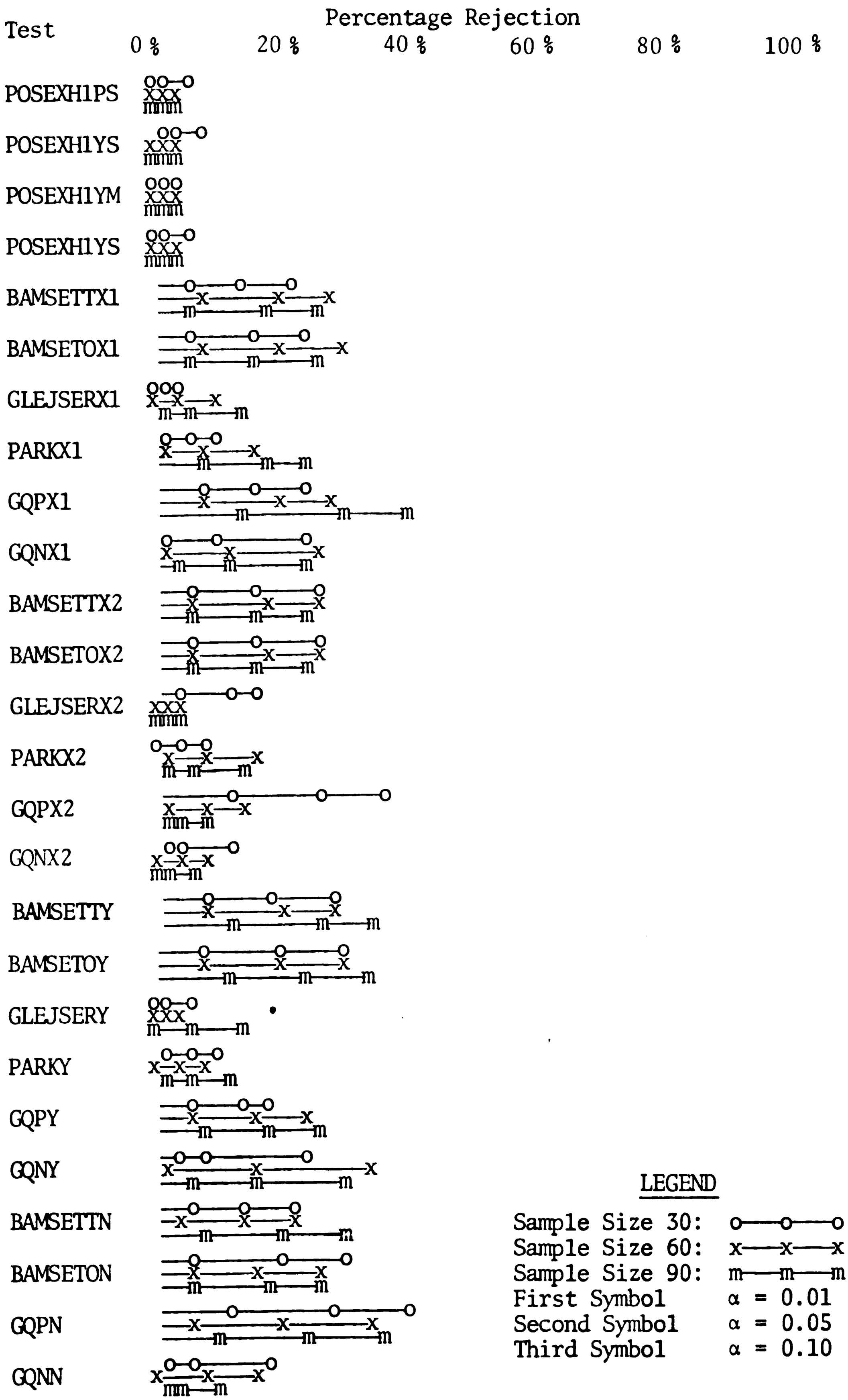

FIGURE 9: A Schematic Diagram of Test Results for Model 15

Finally, since the constructive tests reject the null hypothesis of homoskedasticity less often than do the non-constructive tests, it appears as if the non-constructive tests are slightly more sensitive to the alternative hypothesis than are the constructive tests. This is probably because the distribution of the non-constructive test statistics is known exactly (or asymptotically) while the distribution of the constructive test statistics is known only approximately. However, it should be emphasized that while the loss in power incurred in using the constructive tests is slight, these tests possess the great advantage of providing the researcher with estimates of the heteroskedastic variances.

IV.3.3 Models that Involve the Variable $\underline{x}_6$

There are two models which involve the variable $\underline{x}_6$. One of these models is homoskedastic while the other is heteroskedastic because the variable $\underline{x}_6$ is multiplied by the disturbance term (see Table 2). In both cases, the model hypothesized is

$$y_i = \beta_0 + \beta_6 x_{i6} + \beta_2 x_{i2} + v_i, \; i=1,\ldots,n.$$

These models differ from all of the previous models in that the variable $\underline{x}_6$ is drawn from a uniform population conditional on the observation index i. The distribution which x_{i6} (i'th observation) is drawn from is (0, i.5i). Hence, each observation of $\underline{x}_6$ is drawn from a different distribution.

The results of estimating the hypothesized model appear in Table 12. It is once again obvious that the estimates of β_0, β_6, and β_2 are statistically equal to the 'true' parameters' values of 50, 5, and 5 in both models. Also, a gain in efficiency is noted for the

TABLE 12: Estimation of Models Involving Variable $\underline{X}_6$

Sample Size		30			60			90	
Parameters	0	6	2	0	6	2	0	6	2
					Model 16				
$\bar{\beta}_i$	48.658	5.014	5.049	49.237	5.013	5.003	49.945	4.999	4.999
$\bar{\sigma}^2(\hat{\beta}_i)$	530.5287	.7933	.7924	128.9696	.1018	.08149	82.1195	.03671	.03597
$V(\hat{\beta}_i)$	523.7870	.7453	.8096	129.0531	.1048	.07775	89.4585	.03633	.03544
$\bar{\sigma}^2, \bar{F}$	2474.531	30.934		2484.465	352.106		2493.629	765.799	
					Model 17				
$\bar{\beta}_i$	49.607	5.004	5.016	49.685	5.011	4.998	50.011	5.000	4.996
$\bar{\sigma}^2(\hat{\beta}_i)$	51.1984	.08553	.08543	54.8857	.0433	.03468	57.5410	.0257	.02521
$V(\hat{\beta}_i)$	81.7112	.2162	.09036	44.2165	.1026	.07069	51.4249	.1067	.03303
$\bar{\sigma}^2, \bar{F}$	266.7890	203.618		1057.315	875.671		1747.281	1155.445	

estimated parameters β_0, β_6 and β_2 as the sample size increases. Finally, it should be observed that the average of the estimated variance of each parameter, $\bar{\sigma}^2(\hat{\beta}_i)$, is different from the observed variance, $V(\hat{\beta}_i)$ for the parameter β_6 in model 17 for all three sample sizes.

The results of testing model 16 for heteroskedasticity appear in Figure 10. Since this is a homoskedastic model, the percentage of rejections for all the tests should correspond to the alpha level at which the test was made. However, since in this case, the variables $x_{16},\ldots,x_{n6}$ are drawn from n different populations, the diagonal elements of the matrix M will vary more than they will in the other homoskedastic model examined. Hence, the expected value of the OLS predictors, $\hat{u}_1^2,\ldots,\hat{u}_n^2$, of the time variance σ^2 will vary more than in the other homoskedastic model examined. Therefore, on the basis of this information, it would seem reasonable to suspect that tests for heteroskedasticity which use OLS residuals will incorrectly reject the null hypothesis a disproportionate number of times. However, as was shown earlier in this study, since the maximum squared variation in the diagonal elements of the matrix M is $\frac{k(n-k)}{n(n-1)} \le \frac{k}{n}$, the OLS predictors of the variance (which are a function of the diagonal elements of the matrix M), although not constant, actually display little variation under the null hypothesis of homoskedasticity. Thus, it should instead be expected that all of the tests for heteroskedasticity will reject this model as often as they rejected the other correctly specified model (model 1, Figure 1).

The test results substantiate these expectations. The single exception is the test GQPY which rejected the null hypothesis a much greater percentage of times than it did in testing model 1 (the null

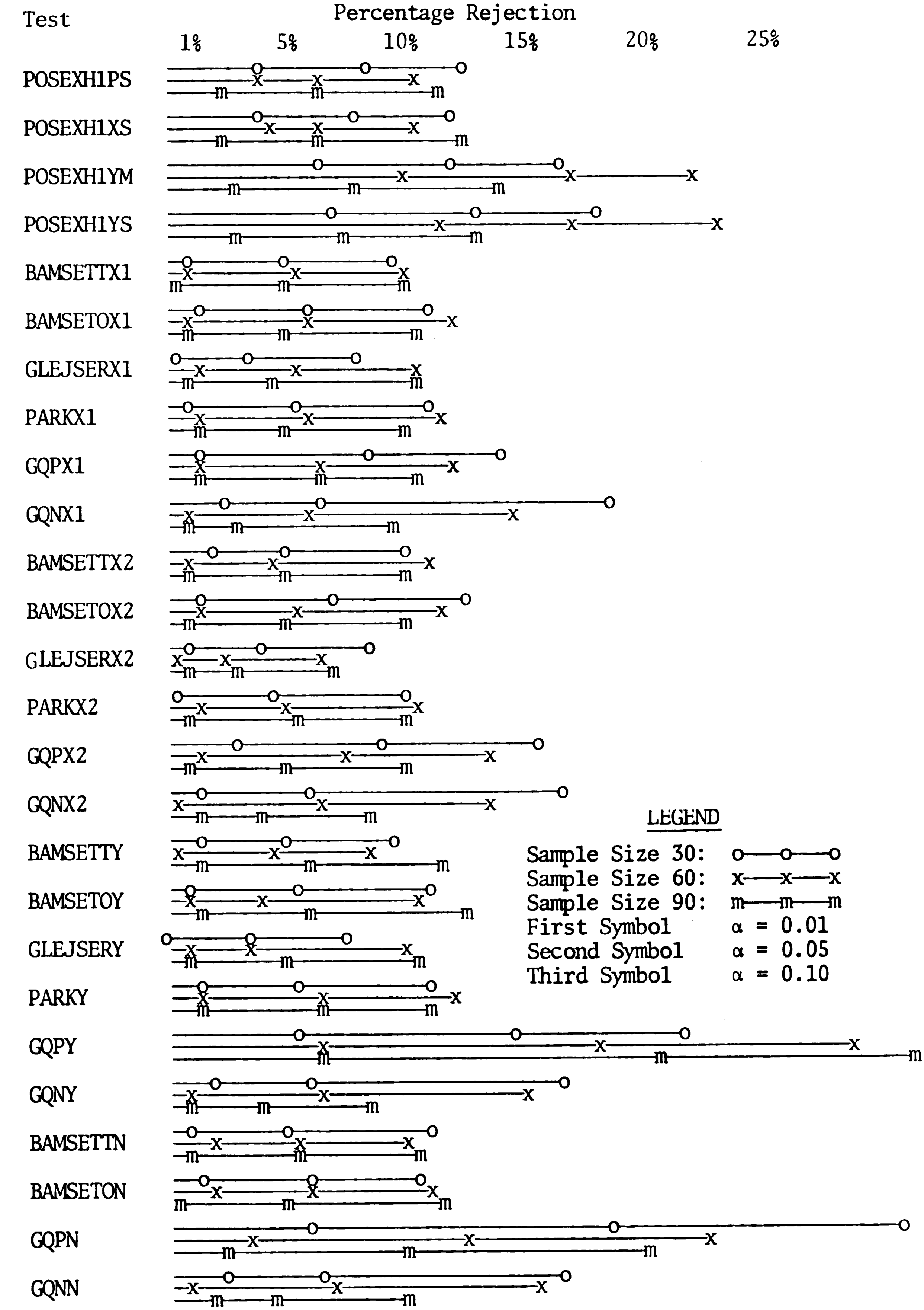

FIGURE 10: A Schematic Diagram of Test Results for Model 16

model). The rest of the tests generally rejected the null hypothesis about the same percentage of times as they did when model 1 was tested. There are, of course, some occasions where, for a specific alpha level and sample size, different results are obtained (for example, GLEJSER2, sample size 60, alpha level .10; and PARKX2, sample size 30, alpha level .10), but no general pattern was visible. Also, because the tests suffixed by X1 are now using a different variable, $\underline{x}_6$, there were some minor differences in the percentage of rejections for sample size 30, but by sample size 90, these differences had vanished.

The results of the heteroskedastic model involving $\underline{x}_6$ appear in Figure 11. There are many marked differences between these results and the results of model 8 (heteroskedastic in $\underline{x}_1$) which appeared in Figure 2. The most striking difference is that the tests which do not reorder the observations show an extremely large gain in power with the percentage of rejections about tripling. The only other major increase in power is observed at all three alpha levels for the POSEX tests when only 30 observations were used. Interestingly, the PARK, GLEJSER, and GQN tests all show a decrease in power for all three alpha levels when the 30 observations category is used. This decrease is especially acute for the tests when the expected value of $\underline{y}$ is thought to be causing the heteroskedasticity. In general, none of the tests, except those without reordering, showed any change in power for either sample size 60 or 90.

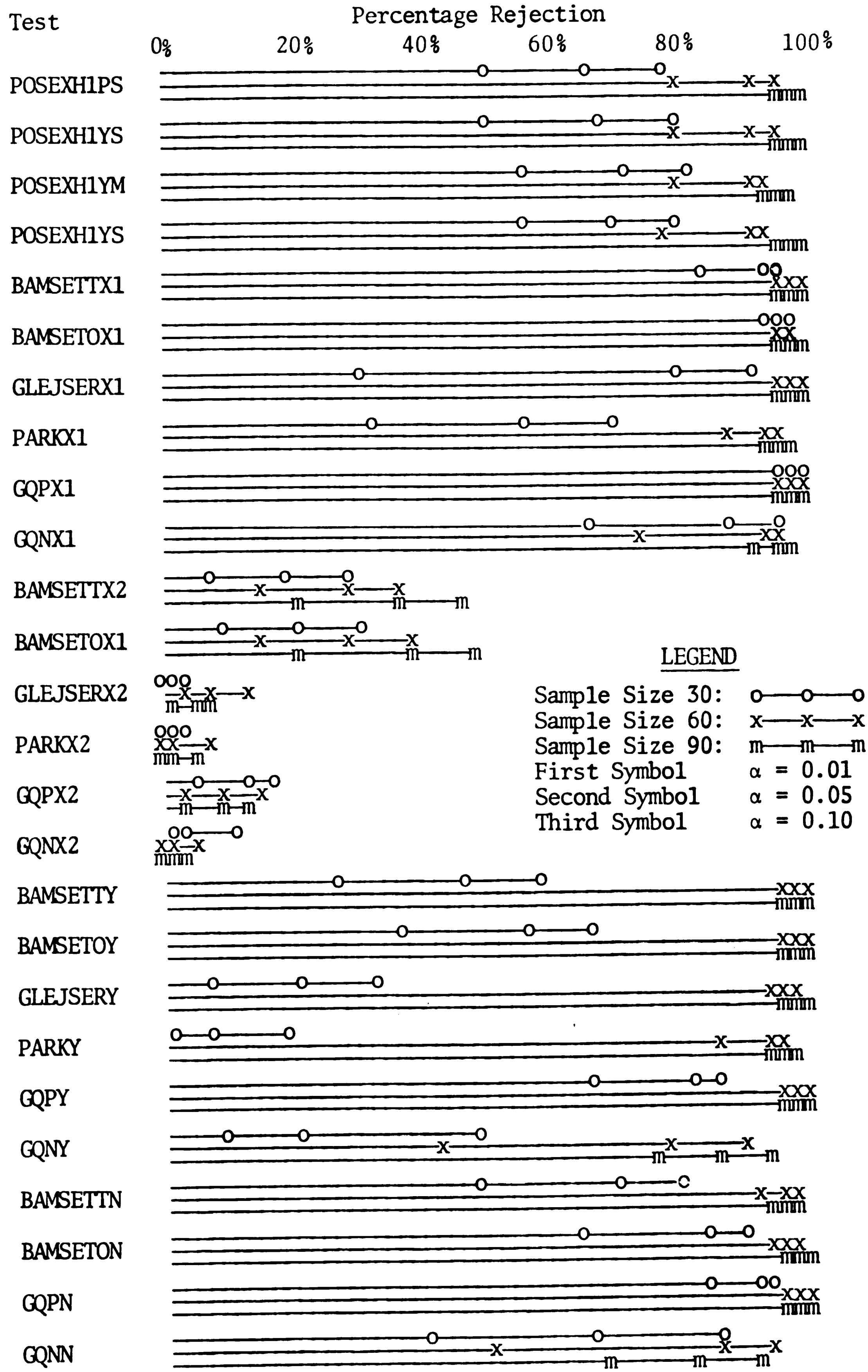

FIGURE 11: A Schematic Diagram of Test Results for Model 17

IV.3.4 Examination of Hypotheses on Tests Designed to Detect Heteroskedasticity

Nine hypotheses were stated in section III.1 relating to tests designed to detect heteroskedastic disturbance terms. A number of comments, observations, and findings pertaining to those hypotheses will now be given.

Hypothesis 1 - In testing the correctly specified model (model 1), it was observed (from Figure 1) that the tests POSEXH1YM, POSEXH1PM, and GQPN rejected the null hypothesis many more times than hypothesized. Although this finding was contrary to the hypothesis, it was especially unexpected in the case of Goldfeld & Quant's parametric test. Since the distribution of the GQPN test statistic is known, it was expected that the estimates of the alpha levels would be very close to the nominal alpha levels at which the tests were made. Instead, it averaged over 10 standard deviations away from the nominal alpha levels. In general, it was not observed, as hypothesized, that the tests which were within a 95% confidence region ($\pm$ two standard deviations) about the nominal alpha levels were those with a test statistic with a known distribution. Rather, the tests that were within the confidence region most regularly were the PARK and the BAMSET (asymptotic distribution of the test statistic is known) testing procedures. The tests that were outside the confidence limits most regularly (if the three extremely inaccurate tests are discarded) for high alpha levels were both of the Goldfeld & Quant procedures and for small alpha levels, the two remaining POSEX procedures. However, it was observed, as hypothesized, that the estimated alpha levels, in general, converged toward the nominal alpha levels at which the tests were made.

Hypothesis 2 - The experimental results substantiated the hypothesis that the probability of any test's correctly rejecting H_0 is an increasing function of the amount of a priori information available. It was further observed that when a simple function of some variable was causing the heteroskedasticity (models 8 through 13), the tests that used this information were the most powerful (the results can be seen in Figures 2 through 7). It was, however, also observed that when the heteroskedasticity was caused by either $\underline{x}_1$ or $\underline{x}_2$, only a small decrease in power resulted from using the same tests with $\underline{y}$ instead of either $\underline{x}_1$ or $\underline{x}_2$. This observation was predictable since $\underline{y}$ is a weighted sum of $\underline{x}_1$ and $\underline{x}_2$ and therefore embodies both correct and incorrect information.

It was further noted that when the a priori information also concerned the functional form of the heteroskedastic disturbances, a notable increase in power was observable. This observation was made on model 14 since it is heteroskedastic because of a quadratic function of $\underline{x}_1$. In this model, the tests that used $\underline{x}_1$ still showed the highest power. However, when the test results that used $\underline{x}_1$ were compared with those of model 8 (simple function of $\underline{x}_1$), a marked decrease in power was observed. It should also be noted that since the GLEJSERX1 test assumes (correctly in this case) that the $E(u_i^2) = (\beta_0 + \beta_1 x_{i1} + \beta_2 x_{i1}^2)\sigma^2$, this test correctly showed the smallest decrease in power while the PARKX1 test showed the largest decrease in power because it incorrectly (in this case) assumed that the $E(u_i^2) = x_{i1}^{\alpha} \sigma^2$. Therefore, without exception, the results indicate that the most powerful test for heteroskedasticity is the one incorporating the most correct information about the heteroskedastic function.

Hypothesis 3 - The experimental results also indicated (as hypothesized) that if a test was observed to have any notable power when a sample size of 30 was used, this power increased as the sample size was increased. However, in many cases, it was also noted that the gain in power was minimal as the number of observations was increased from 60 to 90. Presumably, this was because the power was already approaching 100% and hence only a small increase could be made.

Hypothesis 4 - Recalling that since the basic difference between model 8 and 9 is the distribution of the variable causing the heteroskedasticity (the variables cover the same range with probability of .99), any noticeable differences in the percentage of rejections should be primarily due to the different distributional forms. In comparing the two models (Figures 2 and 3), the tests using $\underline{x}_1$ in model 8 must be compared with the tests using $\underline{x}_2$ in model 9 since these are the variables causing the heteroskedasticity in each model. However, when this comparison is made, no appreciable differences can be observed (the tests on model 8 seem to have a slight edge for sample size 30 but for sample sizes 60 and 90, the tests on model 9 show more power). Nonetheless, a comparison of the POSEX tests in two models seems to indicate clearly that these tests have more power when $\underline{x}_2$ is causing the heteroskedasticity. A comparison of the tests that do not reorder the sample observations reinforces this result. However, since these tests are not expected to have any power, this increase in power is itself surprising. It should also be noted that the tests which assume that $E(\underline{y})$ is causing the disturbances to be heteroskedastic are more powerful when $\underline{x}_1$ is causing the disturbances to be heteroskedastic. This seems to be because the E(y) is more dependent on $\underline{x}_1$ since it has a larger mean and variance than $\underline{x}_2$. However, these differences in the mean and variance are caused by the fact that the variables have different distributions and hence must be considered. Therefore, no clear pattern emerges. Also, since the differences in power disappear as the sample size increases, this investigator feels that the distributional form of the variable causing the heteroskedasticity is not as important as are the parameters of that distribution (e.g., mean, variance, or range over which the variables vary).

Hypothesis 5 - The next hypothesis concerns the probability of type I error when the model includes a variable that is drawn from a distribution with a non-constant mean and variance. In this case, it should be recalled that since all the variables are not drawn from fixed distributions, the diagonal elements of the matrix M vary more than in the previous models examined. Hence, the OLS residuals will be more heteroskedastic than those residuals obtained from the other homoskedastic model. However, since it has been shown that the squared variation of the diagonal elements of the matrix M is always small, the OLS residuals should appear to be homoskedastic. The three tests which would be affected if this hypothesis is wrong are the GLEJSER, PARK and BAMSET tests.

The test results of model 16 appear in Figure 10. If these results are compared with the results obtained from testing model 1 (the homoskedastic model consisting of variables drawn from fixed distributions whose results appear in Figure 1), one sees that all of the tests (with the exception of the test GQPY) reject the null hypothesis approximately the same percentage of times in both models. It was further observed that any divergences that do exist become insignificant as the sample size is increased from 30 to 60 and then to 90 observations. Hence, the experimental results substantiate the claim that the OLS residuals are nearly homoskedastic even when a variable in the model is drawn from a non-constant distribution.

Hypothesis 6 - Unfortunately, since the non-monotonic function used in this study (model 15) could not be properly defined by the small number of observations available, the power of the POSEX tests could not be determined for a non-monotonic function. Therefore, it could not be determined how the power of the POSEX tests relates to the complexity of the function causing the heteroskedastic disturbances.

Hypotheses 7 & 8 - The next two hypotheses predicted the probable relationship between the four POSEX tests. However, it was observed that all four tests had virtually identical power. In some cases, one of the tests would show a slight advantage, but no general pattern could be detected. However, it must be remembered that since the tests POSEXH1PM and POSEXH1YM were found to give poor estimates of the nominal alpha level under the null hypothesis, the other two tests are recommended. Also, since the test POSEXH1PS requires the calculation of principal components and yet offers no power advantage over POSEXH1YS, it appears as if one should use the POSEXH1YS test for its simplicity. It is interesting to note that it was hypothesized that this test would have the lowest power.

Hypothesis 9 - In contrast to the last findings, the test results substantiated this hypothesis. It was observed, with rare exception, that the BAMSET tests were more powerful when OLS rather than BLUS residuals were used. The few exceptions occurred when the wrong variable was used to order the observations. Although in no case was the difference in power very great, the result nevertheless substantiates the claim that the BLUS residuals to some extent mask the heteroskedasticity. It should also be noted that regardless of the residuals used, the test estimates of the alpha level under the null hypothesis were within a 95% confidence limit of the nominal alpha levels used.

Hypothesis 10 - The last hypothesis indicated that if the same amount of *a priori* information is built into all the tests, the POSEX tests would have the most power. Although the POSEX tests can be altered to include *a priori* information, this was not done in this experiment. Therefore, this hypothesis really states that given no information, the POSEX tests will be the most powerful. One version of each test will be compared to the POSEX tests. The version used will assume that $\hat{\underline{y}}$ (or $E(\underline{y})$) is causing the heteroskedasticity. $\hat{\underline{y}}$ was chosen rather than $\underline{x}_1$ or $\underline{x}_2$ since it is a linear combination of $\underline{x}_1$ and $\underline{x}_2$ and hence is more general than either $\underline{x}_1$ or $\underline{x}_2$. The one other alternative would be to use each test first with $\underline{x}_1$ and then with $\underline{x}_2$; however, since the correct alpha level cannot be determined, this procedure was not undertaken (presumably an alpha level of .10 could be obtained by using each test at approximately the .05 alpha level).

First, the tests POSEXH1PM, POSEXH1YM, and GQNY will be discarded from the comparison because of the large biases displayed in testing the null model. Of the remaining tests, the largest bias is by GQPY for high alpha levels (distribution of the test statistic known) and by the two POSEX tests for low alpha levels. In comparing the remaining tests (Figures 2 through 9 and 11), it was discovered that the most powerful tests were the BAMSET tests. These were followed by the GQPY test, the POSEX tests, and the GLEJSER and PARK tests. Although it was not found that the POSEX tests were the most powerful given no information, they are the most powerful of all the constructive tests.

Summary

From the above results, it appears as if only the second, third, fourth, fifth, and ninth hypotheses were strongly substantiated. The rest of the hypotheses (with the exception of the sixth which was not adequately tested) were only shown to be true in certain cases and not in general. In the first hypothesis although it was found that the estimates of the alpha levels for all of the tests become more accurate as the sample size increases it was not found that the tests with the smallest divergence between the estimated and nominal alpha levels were those with a test statistic that has a known distribution. In the seventh and eighth hypotheses, although it was found that all of the POSEX tests had reasonable power, none of the four tests could be found superior. Lastly, in the tenth hypothesis, although the POSEX tests were observed to have the most power out of the class of constructive tests, it was not observed that they were the most powerful in general.

CHAPTER V

SUMMARY, CONCLUSIONS, AND SUGGESTIONS FOR FURTHER RESEARCH

The disturbance terms in a linear regression model must have an expected value of zero and be homoskedastic if one is to obtain estimates using ordinary least squares which have certain desirable properties. Several tests are currently being used to detect a non-zero mean in the disturbance term and others to detect heteroskedasticity.

A major problem with the current testing procedures for disturbance terms with a non-zero mean is that these tests have not achieved the simplicity necessary for popular acceptance. The first procedure which was developed is based on BLUS residuals and hence is computationally difficult. While a revised version of the test uses OLS residuals, which are more easily calculated, the procedure is still somewhat cumbersome since the calculation of the matrix M is necessary.

In contrast, the current tests for heteroskedasticity have achieved the necessary simplicity for popularity; however, they are not always applicable since they all require some a priori knowledge about the variable causing the heteroskedastic disturbances. One group of current tests requires further that the variable be monotonically related to the disturbance terms, while the other group requires that the researcher hypothesize the functional form taken by the disturbance terms.

Two new testing procedures, one for each specification error, are suggested in this study. Both tests are based on a Power Series Expansion (POSEX) Model which has the advantage of approximating any analytic function by using a linear combination of known variables raised to various powers. Two linear combinations are used in this study; the first principal component of the known variables and the least squares predictors based on a regression model in those variables.

The proposed procedure for a misspecified conditional mean is to transform the hypothesized model into a POSEX model by adding three power series expansion terms. If this model explains the conditional mean of the vector $\underline{y}$ statistically better than does the hypothesized model, then the hypothesized model is misspecified. This procedure achieves the simplicity previously lacking since it does not require the calculation of any special matrix and since the test statistic can be obtained by using any existing least squares program. In addition, it was proven that when this procedure is used, with the expansion term being the OLS predictor of $\underline{y}$ from the hypothesized model, the test statistic is mathematically equivalent to the statistic obtained from the existing testing procedures for disturbance terms with a non-zero mean.

The testing procedure proposed for heteroskedasticity uses the same POSEX model as above to explain the conditional mean of an instrument for the unobserved variances. The instrument used is either studentized or Minimum Norm Quadratic (MINQU) predictors of the unobserved variances because they are both unbiased under the null hypothesis. If this model explains the conditional mean of the vector of predicted variances statistically better than does the sample mean of those variances, then the hypothesized model is heteroskedastic. This procedure achieves the generality previously lacking since it uses a POSEX model which approximates any analytic function and hence does not require that the functional form be hypothesized. In addition, knowledge of the variable causing the heteroskedasticity is not required since all of the variables from the hypothesized model are included in the POSEX model.

Moreover, it was proven in this study that regardless of how the matrix X is chosen (stochastic or non-stochastic), the diagonal elements of the matrix M display a minimal squared variation. Hence, the squared OLS residuals, which are a function of the diagonal elements of the matrix M, will be approximately homoskedastic provided that the disturbances are homoskedastic.

In order to compare the new tests with the current tests, a sampling experiment was used. Seventeen definitions of the conditional mean were used to compare the tests under various null and alternative models. 1000 samples of the conditional mean of the vector $\underline{y}$ were examined so as to ensure that the samples would reflect the population from which the vector $\underline{y}$ was drawn.

It was found that although both versions of the POSEX test for a misspecified conditional mean are exact, the test using principal components is more powerful. However, since the POSEX test using $\underline{y}$ is less complicated to use, a trade-off exists between the test's simplicity and its power.

It was also discovered that the power of the POSEX procedures for a misspecified conditional mean varied depending on the number of observations used, the correlation that an omitted variable has with the variables included in the hypothesized model, and the correct functional form of the variable which is used in the hypothesized model. The power was found to increase substantially as the sample size was increased from 30 to 60 but only moderately as the sample size was further increased from 60 to 90. If an omitted variable was causing the conditional mean to be misspecified, it was discovered that when the omitted variable was moderately correlated with the variables included in the hypothesized model, the test had the most power. Finally, as the functional form used to define the conditional mean of the vector $\underline{y}$ becomes more complex, the power of the POSEX procedure to reject correctly the hypothesized linear model decreases.

The results of the tests for heteroskedasticity were more varied. Although most of the tests were found to be relatively exact under H_0, both POSEX procedures using MINQU predictors of the variance and, in some cases, Goldfeld & Quant's parametric test were not exact. It was also noted that the estimated alpha levels using Park's procedure were, as expected, closer to the nominal alpha levels than the estimates obtained using Glejser's procedure. Finally, it was noted that the BAMSET procedure was always exact when either OLS or BLUS residuals were used.

With striking uniformity, the power of the testing procedures increased as more correct knowledge was incorporated into them. In a parallel fashion, as the tests became more general, they also showed a marked decrease in power. An exception to this was the small decrease in power observed in general when $\hat{y}$, rather than the correct variable, was used as the variable causing the heteroskedasticity.

Although the power varied depending on the sample size, it did not vary according to the distribution of the variable causing the disturbance to be heteroskedastic. Again, the greatest gains in power caused by increasing the sample size were made between 30 and 60, not between 60 and 90 observations.

Of the POSEX procedures for heteroskedasticity, the most useful seems to be the one that uses studentized predictors of the variance for the dependent variable and $\hat{y}$'s for the expansion variables. As previously mentioned, the two POSEX procedures that use MINQU predictors for the dependent variable are biased and hence cannot be considered. Also, since the remaining two tests have approximately the same power under the alternatives examined, the less complicated test was chosen as the more useful.

Finally, it was observed that the BAMSET procedure which reordered the variables by $\hat{y}$ had the greatest overall power. However, of the procedures that offer a corrective procedure, the POSEX test generally had the most power. Since the POSEX procedure is more general than the BAMSET procedure, this result should have been expected. Both tests require that the variable causing the heteroskedasticity be in the hypothesized model; however, the BAMSET procedure also requires that the functional form taken by that variable be monotonic whereas the POSEX procedures does not.

This study has offered solutions to the problems posed at the beginning of this study. In the first instance, it has provided a procedure to test for a misspecified conditional mean that is mathematically identical to the current procedures yet much less complicated to use. In addition, it has proposed a different version of the same test, based on principal components, which, although more complicated, has a higher probability of correctly rejecting the alternative models.

In the second instance, it has provided a general constructive test for heteroskedasticity that is more powerful than the current constructive procedures. In addition, it has also offered a procedure to ease the restrictive knowledge requirement that had been previously demanded of all current tests. Applying this new procedure to the BAMSET test proved to be the most powerful procedure overall under the alternatives examined. However, since the BAMSET test does not provide a corrective procedure if heteroskedasticity is present, a trade-off exists between power and being able to correct for the heteroskedastic disturbances. It should, nonetheless, be reiterated that if knowledge about the variable causing the heteroskedasticity is available, it should be incorporated into either the BAMSET or the POSEX procedure. When this is done, the power of both tests increases substantially.

In carrying out this study, further questions which require research have been generated. In examining the tests for a misspecified conditional mean, it was observed that the omitted variable's relation to the included variables is of paramount importance. A study that examined this relation in more detail would be of great use. A useful way to perform this comparison might be to calculate the probability of the quadratic's occurring in normally distributed variables.

Similarly, in analyzing the tests for heteroskedasticity, two areas for additional research became clear. First, since heteroskedastic disturbances need not be monotonically related to the variable causing the difficulty, various non-monotonic forms should be examined. Second, since the POSEX procedure was found to be the most powerful of the constructive tests examined, the gains in efficiency made from using this procedure should be examined.

APPENDICES

APPENDIX A

THEOREM AND COROLLARIES REGARDING THE MATRIX M

Theorem: Regardless of how the vectors $\underline{x}_1,\ldots,\underline{x}_k$ are obtained (stochastic or non-stochastic) the diagonal elements of the matrix M will have a maximum squared variation of $\frac{k(n-k)}{n(n-1)} \leq \frac{k}{n}$ where squared variation of $t_1,\ldots,t_n$ is defined as $\Sigma(t_i - \bar{t})^2/(n-1)$.

Proof:

Defining m_{ii} as the i'th diagonal element of the matrix M, the squared variation (s^2) of the diagonal elements is

$s^2 = \dfrac{\Sigma m_{ii}^2 - \frac{1}{n}(\Sigma m_{ii})^2}{n-1}$. Recalling that M is idempotent and denoting n as the number of observations and k as the number of parameters,

$\sum_i m_{ii} = n - k$, since the trace of an idempotent matrix equals its trace. Also, since M is idempotent, no diagonal element can be greater than one ($m_{ii} \leq 1$ for all i), hence $\Sigma m_{ii}^2 \leq n - k$. Since s^2 is maximized if Σm_{ii}^2 is as large as possible, the maximum value taken by s^2 is

$$\max s^2 = \frac{(n-k) - \frac{1}{n}(n-k)^2}{(n-1)} = \frac{n(n-k) - (n-k)^2}{n(n-1)}$$

$$= \frac{(n-k)\ (n-n+k)}{n(n-1)}$$

$$= \frac{k(n-k)}{n(n-1)} \leq \frac{k}{n}$$

QED

Corollary 1: Defining the coefficient of variation (V) as

$$V(x) = \frac{s}{\bar{x}} ,$$

where s^2 is the squared variation of x, and $\bar{x}$ denotes the sample mean of x, the maximum coefficient of variation of the diagonal elements of the matrix M is $\sqrt{\frac{k}{n-1}}$.

Proof.

Since the maximum squared deviation of the diagonal elements of the matrix M is

$$\frac{k(n-k)}{n(n-1)} ,$$

and since the mean of $m_{11},\ldots,m_{nn}$ is $\frac{n-k}{n}$, one finds that the maximum coefficient of determination (V) is

$$\text{maximum } V^2 = \frac{k(n-k)}{n(n-1)} \qquad \frac{n-k}{n}$$

$$= \frac{k}{n-1}$$

which implies that maximum V is $\sqrt{\frac{k}{n-1}}$.

QED

Corollary 2: The sum of the squared diagonal elements of the matrix M is less than $(n-k)^2/n$ and greater than (n-k).

Proof:

From the theorem, the maximum of Σm_{ii}^2 is n-k. This occurs when (n-k) of the elements are equal to one and k elements are equal to zero. In contrast, the minimum occurs when all the elements are equal to one another. Hence minimum $\Sigma m_{ii}^2 = n(\frac{n-k}{n})^2 = \frac{(n-k)^2}{n}$.

QED

Corollary 3: The sum of the squared off-diagonal elements of the matrix M is less than $\frac{(n-k)}{n} < k$.

Proof:

Since M is a symmetric, idempotent matrix, the sum of the squared elements of any row or column equals the diagonal element that appears in that row or column, $\Sigma_i \Sigma_j m_{ij}^2 = n-k$. Also, since the squared off-diagonal elements are maximized when the diagonal elements are minimized, one obtains:

$$\begin{aligned}
\text{maximum} \sum_{\substack{i \\ i \neq j}} \sum_{j} m_{ij}^2 &= (n-k) - \frac{(n-k)^2}{n} \\
&= \frac{n(n-k) - (n-k)^2}{n} \\
&= \frac{(n-k)\ (n-n+k)}{n} \\
&= \frac{(n-k)k}{n} < k
\end{aligned}$$

QED

This theorem and corollaries are especially interesting since they indicate very strongly that although the matrix M will not equal the identity matrix, it approaches the identity matrix as n gets large or as k gets small. Turning to the last two corollaries, it is especially interesting to note that regardless of the matrix X, bounds can be put on both the diagonal and off-diagonal elements of the matrix M. An example will illustrate the significance of all of these statements. If one has a moderate number of observations, say 30, and 4 regressors, the following statements can now be made about the matrix M.

1. $\Sigma\, m_{ii} = 27$ implying that $\bar{m} = \frac{27}{30} = .9$
2. maximum variance of $m_{11},\ldots,m_{nn} = \frac{(3)(27)}{(30)(29)} < \frac{3}{30} = \frac{1}{10}$
3. maximum coefficient of variation $= \sqrt{\frac{3}{29}} \doteq \sqrt{\frac{1}{10}}$

4. maximum $\Sigma m_{ii}^2 = 27$

minimum $\Sigma m_{ii}^2 = \frac{(27)(27)}{30} = 24.3$

5. maximum of $\sum_{\substack{i \\ i \neq j}} \sum_{j} m_{ij}^2 = \frac{3(27)}{30} < 3$

implying that the average off diagonal element $(\bar{m}_{ij})$ equals $\frac{3}{n^2-n} = \frac{3}{270} = .0111.$

Therefore, in this modest example, the average diagonal element equals .9 and the average off-diagonal element = .0111.

APPENDIX B

ANALYSIS AND LIST OF EXPERIMENTAL DATA

TABLE B1. SAMPLE OF VARIABLES X1, X2, X3, X4, X5, AND X6 USED IN EXPERIMENT

X1	X2	X3	X4	X5	X6	OBS.
6.78473	20.2887	102.940	41.5415	27.2664	.598-12	1
39.8675	14.2427	65.5400	42.0153	34.9522	.534-24	2
90.5878	25.0247	59.00[illegible]	50.6354	69.5151	3.94385	3
17.8008	5.60120	94.9400	35.5557	20.7113	2.65260	4
81.0749	10.2171	73.0200	45.2276	55.[illegible]375	3.57397	5
87.6414	5.86363	16.4800	32.8900	47.0646	7.14848	6
67.9641	26.4822	37.6800	49.3142	58.4617	7.88100	7
43.8766	49.4454	72.6600	54.0000	65.3376	3.22353	8
98.5444	10.8302	77.8600	47.4641	66.0190	11.9168	9
51.6529	27.7230	66.2800	49.4676	51.8536	.136-26	10
44.0934	9.52702	57.9600	39.1254	52.4752	13.5526	11
79.4276	19.4615	56.3600	47.5873	59.0721	4.45-14	12
31.4788	5.13236	31.4400	28.5491	18.2[illegible]97	10.1407	13
75.0572	13.6567	38.3200	50.9555	58.0726	9.38199	14
28.2434	13.1049	31.2800	35.1649	22.8201	12.3251	15
26.8628	12.4681	70.7200	40.1724	26.9930	21.5999	16
12.4840	31.2020	50.4800	40.3708	30.9151	21.9035	17
88.2872	24.0044	38.6000	47.1449	64.7497	24.2999	18
78.8047	10.5017	54.0600	42.4433	51.4[illegible]	11.2197	19
.906842	10.0233	32.5300	14.6271	5.60723	23.5550	20
23.8821	34.3942	66.9200	47.0389	42.0191	1.58974	21
82.1371	14.8233	56.0200	42.6175	53.9156	15.5091	22
51.7585	27.0392	42.64[illegible]	45.7402	47.3378	12.1299	23
24.7230	23.8649	56.3800	42.9028	32.8515	.456480	24
23.8966	2.62350	31.4800	21.0857	12.2159	20.1510	25
93.0598	11.2956	34.[illegible]60	40.9042	56.8259	25.0806	26
94.0784	20.1205	21.9200	43.5970	62.3567	23.1[illegible]08	27
44.6762	9.14620	20.5600	33.2722	26.3859	28.0790	28
40.3735	21.5421	47.6400	43.3894	30.1697	23.7062	29
57.0002	29.6323	79.1200	52.4419	58.1542	41.8095	30
33.4147	19.1880	63.0800	43.7864	34.8935	40.5456	31
89.9997	149.874	80.9000	60.1855	161.540	45.9021	32
75.6220	9.76796	106.140	49.6276	56.2141	13.7629	33
85.8959	8.49301	35.7600	40.7202	54.2364	20.2916	34
36.9247	3.74930	65.0400	32.0206	25.1355	5[illegible]	35
89.9995	19.5916	41.3200	46.0610	60.6546	47.7881	36
35.9522	13.5149	64.9200	41.9065	32.7158	34.5708	37
39.3672	6.70468	52.9800	35.0545	26.9848	37.0093	38
9.42564	42.9201	46.6000	40.6519	37.6976	49.0038	39

TABLE B1. (CONTIUED)

X1	X2	X3	X4	X5	X6	OBS.
79.5166	13.9261	40.4200	42.0221	32.4616	56.0559	40
14.5101	24.4277	60.4400	40.7241	28.2759	23.0957	41
5.04679	22.9682	54.0200	33.6436	21.1409	21.0733	42
1.53453	13.1487	37.6200	20.3300	9.11516	35.0077	43
46.9975	16.0300	44.7000	41.4944	37.0742	13.0870	44
12.7189	3.19055	58.2800	24.0760	10.6207	60.3704	45
35.1592	5.62925	61.1600	34.3190	25.0906	.619785	46
55.9033	19.7061	39.0200	43.1762	43.3007	24.3873	47
1.26800	23.3848	53.9400	28.2335	13.6794	12.0060	48
99.4071	10.6150	43.3200	42.1764	41.1411	30.0521	49
53.8159	1.61470	77.4600	29.0072	34.5703	53.5297	50
64.8811	87.9684	24.2400	53.0021	99.1771	69.3220	51
64.3092	19.9371	59.2200	44.0564	45.6616	67.1474	52
32.6936	6.66017	33.5200	31.0757	20.4051	2.93391	53
57.2367	70.5525	54.3800	55.5167	86.1961	50.0122	54
74.2425	6.04700	28.6000	54.0449	41.7511	46.0085	55
63.0451	5.45581	50.2000	55.0012	41.1509	35.4109	56
42.7801	21.0241	59.6400	45.0973	41.2622	62.2565	57
54.7038	15.3125	40.6200	41.3536	40.0922	55.0457	58
63.2810	7.57753	48.2000	37.8573	39.9212	80.9242	59
92.2433	54.7770	73.4200	58.9363	95.8157	26.9234	60
.632860	5.35942	77.4400	14.5766	9.59163	43.1387	61
87.1970	6.55020	35.3800	44.0511	57.6877	1.69306	62
90.5968	3.78115	28.8600	31.5444	48.9202	47.0609	63
95.6273	15.7671	32.9200	51.4300	59.7730	26.5576	64
38.4896	11.5754	59.6200	40.1515	31.2752	11.5383	65
27.8038	11.1970	38.0000	34.8736	21.9248	11.9983	66
6.56683	8.77791	38.1600	25.1964	8.56395	4.75614	67
49.5913	9.15098	62.2000	40.0717	35.7682	62.7189	68
22.3634	14.5907	48.0800	37.2449	23.1063	42.2369	69
26.3540	9.59867	22.5000	35.1074	55.5765	7.94323	70
25.2983	32.2854	67.0800	46.6877	41.1860	10.2461	71
88.4594	59.6314	39.3000	54.2526	92.4251	10.1903	72
54.1052	134.053	32.1400	64.2492	137.544	39.1100	73
62.2352	149.500	58.5600	63.3127	150.330	43.6766	74
93.1901	4.01807	37.7000	33.4921	51.8365	27.6103	75
66.1216	9.55580	56.6200	41.1495	44.2536	58.7477	76
31.7865	80.6051	43.2800	51.6112	78.3022	61.4312	77
84.7244	47.8259	26.9600	50.4600	79.4062	12.7636	78
4.08369	5.79036	60.6600	22.7771	82.5038	44.9469	79
98.2598	21.3991	33.5400	46.0210	67.3351	47.0220	80
47.5557	39.3603	47.0400	48.3354	55.7160	43.3618	81
38.8548	18.3598	27.1200	38.3740	31.8138	71.1222	82
52.1354	11.6923	68.2600	45.1588	40.0529	55.6825	83
34.0846	26.1536	63.6800	46.3719	40.7175	18.2928	84
70.8496	36.0757	70.8600	53.8855	69.3465	80.5955	85
55.0507	35.6181	61.0800	50.9655	59.0051	98.1317	86
86.5052	17.1482	48.9600	45.9680	60.0233	17.3490	87
21.1924	5.35942	31.1200	26.6872	12.8034	126.544	88
64.7906	10.8483	41.2000	39.7017	42.2123	110.070	89
89.4377	5.59188	45.7600	37.0263	52.2221	125.505	90

TABLE B2: Means, Variances, Covariances, and Correlations of Variables $\underline{x}_1$, $\underline{x}_2$, $\underline{x}_3$, $\underline{x}_4$, $\underline{x}_5$, and $\underline{x}_6$ for Sample Size 30.

Upper Triangle, Covariance; Lower Triangle, Correlations; and Diagonal Variances

	$\underline{x}_1$	$\underline{x}_2$	$\underline{x}_3$	$\underline{x}_4$	$\underline{x}_5$	$\underline{x}_6$
$\underline{x}_1$	879.406	-14.668	-107.068	126.942	449.972	---
$\underline{x}_2$	-.047	110.005	60.440	61.132	85.918	-15.980
$\underline{x}_3$	-.157	.250	531.116	102.758	68.150	-102.338
$\underline{x}_4$	.486	.661	.506	77.713	131.451	---
$\underline{x}_5$	.847	.457	.165	.833	320.711	---
$\underline{x}_6$	---	-.145	-.423	---	---	109.88
Means	52.901	17.969	50.243	41.629	43.297	13.171

TABLE B3: Means, Variances, Covariances, and Correlations of Variables $\underline{x}_1$, $\underline{x}_2$, $\underline{x}_3$, $\underline{x}_4$, $\underline{x}_5$, and $\underline{x}_6$ for Sample Size 60.

Upper Triangle, Covariances; Lower Triangle, Correlations; and Diagonal, Variances.

	$\underline{x}_1$	$\underline{x}_2$	$\underline{x}_3$	$\underline{x}_4$	$\underline{x}_5$	$\underline{x}_6$
$\underline{x}_1$	842.269	94.956	-53.896	152.717	522.506	---
$\underline{x}_2$	.141	535.013	-56.046	126.489	455.630	88.495
$\underline{x}_3$	-.091	-.119	417.924	55.904	-97.784	-114.586
$\underline{x}_4$	.574	.597	.299	83.874	188.804	---
$\underline{x}_5$	.715	.783	-.010	.819	633.440	---
$\underline{x}_6$	---	.185	-.271	---	---	428.413
Means	51.714	21.064	50.188	40.997	44.875	26.228

TABLE B4: Means, Variances, Covariances, and Correlations of Variables $\underline{x}_1$, $\underline{x}_2$, $\underline{x}_3$, $\underline{x}_4$, $\underline{x}_5$, and $\underline{x}_6$ for Sample Size 90.

Upper Triangle, Covariances; Lower Triangle, Correlations; and Diagonal, Variances.

	$\underline{x}_1$	$\underline{x}_2$	$\underline{x}_3$	$\underline{x}_4$	$\underline{x}_5$	$\underline{x}_6$
$\underline{x}_1$	866.219	83.888	-65.669	151.622	525.011	---
$\underline{x}_2$	.102	783.908	- .585	182.181	649.839	62.241
$\underline{x}_3$	- .114	- .001	384.240	2.628	21.540	-105.188
$\underline{x}_4$	.524	.662	.273	96.484	230.648	---
$\underline{x}_5$	.635	.826	.039	.835	789.233	---
$\underline{x}_6$	---	.802	- .194	---	---	768.222
Means	53.186	23.467	53.109	41.352	47.429	32.622

APPENDIX C

TEST RESULTS

TABLE C1: Acronyms Used to Designate the Specification Error Tests Being Examined

Acronym	Test
POSEXMY	Power Series Expansion (POSEX) model for a non-zero Mean using $\hat{Y}$ as the instrument
POSEXMP	POSEX model for a non-zero Mean using P as the instrument
POSEXH1PS	POSEX model for Heteroskedasticity using method 1 to estimate the model, P as the instrument and studentized predictors (S) of the variances
POSEXH1YS	POSEXH1, $\hat{Y}$ as the instrument and studentized predictors (S) of the variances
POSEXH1YM	POSEXH1, $\hat{Y}$ as the instrument and MINQU predictors of the variance
POSEXH1PM	POSEXH1, P as the instrument and MINQU predictors of the variance
BAMSETTN	BAMSET testing procedure using Theil's BLUS residuals (BAMSETT) and Not reordering the observations
BAMSETTX1	BAMSETT, reordering the observations by the variable $\underline{X}_1$.
BAMSETTX2	BAMSETT, reordering the observations by the variable $\underline{X}_2$.
BAMSETTY	BAMSETT, reordering the observations by $\hat{Y}$.
BAMSETON	BAMSET testing procedure using OLS residuals (BAMSETO) and Not reordering the observations
BAMSETOX1	BAMSETO, reordering the observations by the variable, $\underline{X}_1$.
BAMSETOX2	BAMSETO, reordering the observations by the variable, $\underline{X}_2$.
BAMSETOY	BAMSETO, reordering the observations by $\hat{Y}$.

TABLE C1 (cont'd)

Acronym	Test
GLEJSERX1	GLEJSER's test using $\underline{X}_1$ as the independent variable
GLEJSERX2	GLEJSER's test using $\underline{X}_2$ as the independent variable
GLEJSERY	GLEJSER's test using $\hat{Y}$ as the independent variable
PARKX1	PARK's test using $\underline{X}_1$ as the independent variable
PARKX2	PARK's test using $\underline{X}_2$ as the independent variable
PARKY	PARK's test using $\hat{Y}$ as the independent variable
GQPN	Goldfeld & Quant's Parametric test with No reordering
GQPX1	GQP, reordering the observations by the variable $\underline{X}_1$
GQPX2	GQP, reordering the observations by the variable $\underline{X}_2$
GQPY	GQP, reordering the observations by $\hat{Y}$
GQNN	Goldfeld & Quant's Non-parametric test with No Reordering
GQNX1	GQN, reordering the observations by the variable $\underline{X}_1$
GQNX2	GQN, reordering the observations by the variable $\underline{X}_2$
GQNY	GQN, reordering the observations by $\hat{Y}$

TABLE C2: Test Results - Model 1, Sample Size 30

TEST	ALPHA LEVEL .01	.05	.10	MEAN	VARIANCE	SKEWNESS	KURTOSIS
POSEXMY	4.	49.	103.	.110261E+01	.957248E+00	.189819E+01	.912479E+01
POSEXMP	14.	59.	113.	.115224E+01	.112727E+01	.226669E+01	.118041E+02
POSEXH1PS	35.	86.	135.	.125266E+01	.183520E+01	.575683E+01	.635644E+02
POSEXH1YS	42.	89.	142.	.128139E+01	.234950E+01	.682536E+01	.841451E+02
POSEXH1YM	43.	90.	145.	.130702E+01	.263083E+01	.647156E+01	.726195E+02
POSEXH1PM	36.	82.	134.	.125542E+01	.200652E+01	.576790E+01	.610992E+02
BAMSETTN	16.	59.	119.	.213828E+01	.476064E+01	.204268E+01	.906295E+01
BAMSETTX1	7.	57.	110.	.202610E+01	.397230E+01	.163004E+01	.569917E+01
BAMSETTX2	16.	59.	113.	.212522E+01	.464457E+01	.199547E+01	.858576E+01
BAMSETTY	9.	53.	107.	.203569E+01	.385308E+01	.178457E+01	.727055E+01
BAMSEOTN	17.	51.	110.	.208717E+01	.476783E+01	.239099E+01	.124885E+02
BAMSEOTX1	7.	58.	124.	.208288E+01	.412365E+01	.160443E+01	.605888E+01
BAMSEOTX2	15.	71.	121.	.221356E+01	.494449E+01	.190439E+01	.752773E+01
BAMSEOTY	16.	65.	118.	.208506E+01	.465833E+01	.192478E+01	.755073E+01
GLEJSERX1	14.	57.	111.	.114974E+01	.164346E+01	.312279E+01	.200617E+02
GLEJSERX2	9.	39.	83.	.104611E+01	.125251E+01	.281583E+01	.158908E+02
GLEJSERY	12.	64.	110.	.111343E+01	.139385E+01	.204753E+01	.823512E+01
PARKX1	21.	49.	94.	.113376E+01	.590016E+01	.706867E+01	.726735E+02
PARKX2	9.	52.	111.	.113377E+01	.275930E+01	.351603E+01	.236650E+02
PARKY	12.	57.	96.	.107164E+01	.338653E+01	.436233E+01	.319048E+02
GQPN	66.	175.	294.	.216487E+01	.271406E+01	.434133E+01	.307091E+02
GQPX1	33.	105.	149.	.138263E+01	.168140E+01	.299154E+01	.161206E+02
GQPX2	18.	76.	130.	.131283E+01	.140735E+01	.395711E+01	.299798E+02
GQPY	18.	52.	102.	.112590E+01	.107533E+01	.314252E+01	.182599E+02
GQNN	22.	70.	191.	.306600E+01	.254018E+01	.390472E+00	.289583E+01
GQNX1	27.	73.	166.	.301400E+01	.259640E+01	.448155E+00	.313459E+01
GQNX2	16.	70.	178.	.303400E+01	.244329E+01	.332084E+00	.282254E+01
GQNY	26.	69.	170.	.302200E+01	.262414E+01	.471659E+00	.326579E+01

TABLE C2: Test Results - Model 1, Sample Size 60

TEST	ALPHA LEVEL .01	.05	.10	MEAN	VARIANCE	SKEWNESS	KURTOSIS
POSEXMY	6.	49.	93.	.102321E+01	.726989E+00	.158228E+01	.582291E+01
POSEXMP	11.	48.	94.	.104819E+01	.769942E+00	.189365E+01	.855660E+01
POSEXH1PS	41.	72.	110.	.111935E+01	.138995E+01	.461198E+01	.367069E+02
POSEXH1YS	38.	83.	116.	.113361E+01	.168661E+01	.552479E+01	.518137E+02
POSEXH1YM	101.	172.	222.	.198511E+01	.175439E+02	.877774E+01	.998812E+02
POSEXH1PM	91.	169.	217.	.192409E+01	.151637E+02	.917454E+01	.115022E+03
BAMSETTN	14.	53.	101.	.201781E+01	.468441E+01	.282220E+01	.174910E+02
BAMSETTX1	14.	62.	127.	.216619E+01	.454987E+01	.185439E+01	.735105E+01
BAMSETTX2	11.	50.	108.	.206806E+01	.414177E+01	.193303E+01	.875310E+01
BAMSETTY	14.	61.	119.	.214139E+01	.442872E+01	.175324E+01	.670771E+01
BAMSEOTN	16.	59.	125.	.213745E+01	.496562E+01	.236048E+01	.119703E+02
BAMSEOTX1	12.	58.	116.	.216246E+01	.432296E+01	.188777E+01	.810819E+01
BAMSEOTX2	12.	59.	109.	.210974E+01	.466038E+01	.221140E+01	.109379E+02
BAMSEOTY	18.	63.	111.	.220035E+01	.454487E+01	.187671E+01	.753693E+01
GLEJSERX1	17.	68.	114.	.114578E+01	.144794E+01	.221109E+01	.992167E+01
GLEJSERX2	5.	24.	69.	.906958E+00	.777948E+00	.245280E+01	.130773E+02
GLEJSERY	8.	39.	70.	.981386E+00	.940569E+00	.246070E+01	.125809E+02
PARKX1	16.	49.	113.	.113254E+01	.320761E+01	.480757E+01	.458832E+02
PARKX2	13.	54.	101.	.107402E+01	.228812E+01	.277431E+01	.134100E+02
PARKY	15.	62.	116.	.113373E+01	.309189E+01	.313142E+01	.165220E+02
GQPN	35.	126.	231.	.151717E+01	.269629E+00	.213166E+01	.944238E+01
GQPX1	16.	67.	119.	.111680E+01	.318335E+00	.154139E+01	.671777E+01
GQPX2	17.	75.	123.	.112133E+01	.330192E+00	.151861E+01	.640200E+01
GQPY	17.	81.	144.	.115940E+01	.359980E+00	.167213E+01	.794150E+01
GQNN	9.	62.	142.	.372700E+01	.296143E+01	.425115E+00	.314199E+01
GQNX1	8.	69.	155.	.374500E+01	.320118E+01	.433089E+00	.316868E+01
GQNX2	9.	61.	137.	.364500E+01	.323221E+01	.465551E+00	.336842E+01
GQNY	8.	66.	172.	.367000E+01	.323033E+01	.421504E+00	.287806E+01

TABLE C2: Test Results - Model 1, Sample Size 90

TEST	ALPHA LEVEL .01	.05	.10	MEAN	VARIANCE	SKEWNESS	KURTOSIS
POSEXMY	11.	57.	97.	.103059E+01	.772162E+00	.180167E+01	.836836E+01
POSEXMP	13.	56.	111.	.104566E+01	.311592E+00	.183351E+01	.854533E+01
POSEXH1PS	36.	71.	112.	.103475E+01	.129785E+01	.531086E+01	.495267E+02
POSEXH1YS	34.	83.	116.	.108903E+01	.124801E+01	.461991E+01	.377303E+02
POSEXH1YM	52.	97.	136.	.122906E+01	.199899E+01	.550757E+01	.539959E+02
POSEXH1PM	56.	93.	133.	.124322E+01	.232402E+01	.660038E+01	.712480E+02
BAMSETTN	9.	51.	108.	.203031E+01	.417968E+01	.206348E+01	.948885E+01
BAMSETTX1	14.	60.	114.	.211343E+01	.464333E+01	.219530E+01	.992810E+01
BAMSETTX2	8.	50.	97.	.197669E+01	.375536E+01	.191809E+01	.778156E+01
BAMSETTY	13.	47.	98.	.205012E+01	.415405E+01	.195059E+01	.793889E+01
BAMSEOTN	10.	51.	111.	.203201E+01	.413615E+01	.180595E+01	.712991E+01
BAMSEOTX1	14.	59.	120.	.213505E+01	.487013E+01	.231830E+01	.111375E+02
BAMSEOTX2	11.	52.	98.	.204496E+01	.431479E+01	.222729E+01	.100867E+02
BAMSEOTY	14.	49.	111.	.213712E+01	.435177E+01	.195140E+01	.860480E+01
GLEJSERX1	7.	58.	112.	.106388E+01	.120660E+01	.222196E+01	.108937E+02
GLEJSERX2	5.	31.	76.	.999167E+00	.855170E+00	.200639E+01	.986476E+01
GLEJSERY	12.	43.	81.	.103825E+01	.108236E+01	.245887E+01	.125292E+02
PARKX1	11.	48.	95.	.103135E+01	.233405E+01	.330118E+01	.194969E+02
PARKX2	5.	49.	104.	.106664E+01	.231100E+01	.246694E+01	.116289E+02
PARKY	7.	49.	94.	.100727E+01	.193368E+01	.260156E+01	.120033E+02
GQPN	22.	99.	155.	.137396E+01	.116345E+00	.197243E+01	.929952E+01
GQPX1	10.	49.	104.	.105971E+01	.162255E+00	.121052E+01	.622023E+01
GQPX2	18.	66.	121.	.109606E+01	.197911E+00	.144247E+01	.697965E+01
GQPY	15.	64.	124.	.112080E+01	.180885E+00	.112198E+01	.517247E+01
GQNN	25.	45.	108.	.410900E+01	.366879E+01	.424092E+00	.302037E+01
GQNX1	16.	47.	109.	.414900E+01	.345756E+01	.410921E+00	.302673E+01
GQNX2	11.	34.	98.	.403200E+01	.338436E+01	.408272E+00	.331714E+01
GQNY	14.	43.	105.	.412406E+01	.349812E+01	.332017E+00	.285645E+01

TABLE C5: Test Results - Models 2, 3 and 4

Test	% Rejection at α .01	.05	.10	Mean	Variance	Skewness	Kurtosis
				Model 2			
				Sample Size 30			
POSEXMY	0.8	3.1	8.6	.433063	.078897	1.20911	5.01158
POSEXMP	0.4	1.2	1.7	.788937	.202364	.971755	4.20716
				Sample Size 60			
POSEXMY	1.9	7.6	15.6	.367349	.071884	1.20513	4.58735
POSEXMP	1.0	7.3	15.5	.496730	.100863	.913994	.375351
				Sample Size 90			
POSEXMY	0.1	0.7	1.7	.798915	.191194	.911975	4.28510
POSEXMP	0.3	3.2	6.9	.518366	.112849	1.08354	4.34822
				Model 3			
				Sample Size 30			
POSEXMY	1.5	6.3	12.2	1.41422	1.39668	1.62245	6.29281
POSEXMP	2.2	10.0	17.5	1.70927	1.94973	1.71405	7.29449
				Sample Size 60			
POSEXMY	19.9	44.7	54.9	3.45077	4.37355	1.28624	5.74488
POSEXMP	30.2	55.5	66.8	3.97375	5.02404	1.00875	4.43899
				Sample Size 90			
POSEXMY	45.6	68.0	80.5	4.76302	5.83823	.974417	4.13101
POSEXMP	62.6	81.4	90.6	5.80468	7.10041	.777068	3.62015
				Model 4			
				Sample Size 30			
POSEXMY	0.5	4.3	9.4	1.01705	.816278	1.83557	8.09556
POSEXMP	0.4	5.3	9.9	1.12064	.958701	1.85398	8.29186
				Sample Size 60			
POSEXMY	0.8	5.3	9.9	.962272	.649272	1.57627	5.79053
POSEXMP	0.8	3.9	8.8	.998673	.677300	1.73510	7.15953
				Sample Size 90			
POSEXMY	0.6	4.1	9.7	1.03153	.744858	1.79539	8.04375
POSEXMP	1.1	4.6	10.6	1.01445	.738918	1.63736	6.77984

TABLE C6: Test Results - Models 5, 6 and 7

Test	% Rejection at α .01	.05	.10	Mean	Variance	Skewness	Kurtosis
				Model 5			
				Sample Size 30			
POSEXMY	2.5	7.9	12.9	1.13959	1.74522	2.69912	11.6914
POSEXMP	13.8	21.1	25.3	5.38091	1158.26	15.6630	285.515
				Sample Size 60			
POSEXMY	85.4	89.1	90.9	453.589	159614.	.519493	2.97602
POSEXMP	90.5	93.0	94.5	12039.5	526541000.	4.77357	46.0781
				Sample Size 90			
POSEXMY	66.7	72.3	77.9	296.825	165941.	1.43958	4.52777
POSEXMP	92.3	93.9	95.2	48.5093	4991.84	4.58076	29.7045
				Model 6			
				Sample Size 30			
POSEXMY	1.5	5.2	12.3	.845045	.794669	2.95879	17.6064
POSEXMP	8.6	14.5	20.6	2.83147	75.6904	9.68327	120.388
				Sample Size 60			
POSEXMY	39.8	51.4	57.6	21.3371	5034.05	6.51388	55.2004
POSEXMP	55.6	66.1	20.2	34.5887	16791.2	8.73581	97.2244
				Sample Size 90			
POSEXMY	34.2	43.8	52.6	18.0231	3290.12	5.85384	43.4622
POSEXMP	51.7	61.3	69.7	10.6566	286.901	5.23155	52.3475
				Model 7			
				Sample Size 30			
POSEXMY	1.6	5.2	9.8	1.44189	1.32832	2.52995	14.8763
POSEXMP	16.7	33.6	41.0	3.32074	10.9710	3.52249	29.5735
				Sample Size 60			
POSEXMY	4.8	19.1	29.7	2.34178	1.67149	1.27134	5.68311
POSEXMP	43.3	55.5	60.5	5.33288	23.5468	1.67051	6.69945
				Sample Size 90			
POSEXMY	1.0	4.9	10.0	1.50024	.850417	1.59515	6.70044
POSEXMP	16.3	23.8	79.3	2.71246	8.79796	2.36190	9.23780

TABLE C7: Test Results - Model 8, Sample Size 30

TEST	ALPHA LEVEL .01	.05	.10	MEAN	VARIANCE	SKEWNESS	KURTOSIS
POSEXMY	22.	65.	102.	.102394E+01	.166779E+01	.331538E+01	.189844E+02
POSEXMP	18.	51.	83.	.947367E+00	.151176E+01	.392462E+01	.264265E+02
POSEXH1PS	107.	307.	469.	.237458E+01	.254072E+01	.317209E+01	.198194E+02
POSEXH1YS	105.	284.	422.	.228981E+01	.248113E+01	.284541E+01	.156544E+02
POSEXH1YM	87.	222.	362.	.209107E+01	.212065E+01	.291336E+01	.163254E+02
POSEXH1PM	83.	245.	394.	.217043E+01	.226893E+01	.325143E+01	.206503E+02
BAMSETTN	95.	208.	314.	.391158E+01	.128040E+02	.161461E+01	.628253E+01
BAMSETTX1	625.	837.	914.	.114777E+02	.319116E+02	.680358E+00	.340433E+01
BAMSETTX2	88.	211.	321.	.389356E+01	.129674E+02	.172223E+01	.719598E+01
BAMSETTY	504.	763.	860.	.100752E+02	.292352E+02	.832771E+00	.360774E+01
BAMSEOTN	65.	174.	275.	.341522E+01	.103966E+02	.175233E+01	.739409E+01
BAMSEOTX1	790.	930.	964.	.144523E+02	.394877E+02	.526245E+00	.327517E+01
BAMSEOTX2	70.	190.	272.	.343434E+01	.110205E+02	.179092E+01	.760111E+01
BAMSEOTY	662.	866.	932.	.125283E+02	.378959E+02	.699133E+00	.349926E+01
GLEJSERX1	504.	798.	904.	.632699E+01	.154153E+02	.213448E+01	.125580E+02
GLEJSERX2	.	7.	22.	.764928E+00	.424894E+00	.163272E+01	.655696E+01
GLEJSERY	389.	688.	822.	.532093E+01	.115094E+02	.188850E+01	.916900E+01
PARKX1	371.	652.	772.	.804429E+01	.597092E+02	.233681E+01	.103274E+02
PARKX2	4.	38.	86.	.939210E+00	.191260E+01	.269052E+01	.118172E+02
PARKY	435.	700.	793.	.857752E+01	.550449E+02	.198849E+01	.902932E+01
GQPN	135.	324.	436.	.284193E+01	.692107E+01	.428839E+01	.325771E+02
GQPX1	35.	83.	122.	.122730E+01	.260900E+01	.608349E+01	.661515E+02
GQPX2	901.	963.	987.	.184611E+02	.355432E+03	.482853E+01	.470581E+02
GQPY	667.	829.	895.	.939659E+01	.131247E+03	.652969E+01	.785258E+02
GQNN	27.	83.	234.	.361400E+01	.176076E+01	.546868E+00	.324039E+01
GQNX1	463.	699.	874.	.644300E+01	.289565E+01	.266450E+00	.285757E+01
GQNX2	43.	123.	293.	.383500E+01	.210788E+01	.484696E+00	.333751E+01
GQNY	338.	583.	810.	.592800E+01	.267950E+01	.300467E+00	.297328E+01

TABLE C8: Test Results - Model 8, Sample Size 60

TEST	ALPHA LEVEL .01	.05	.10	MEAN	VARIANCE	SKEWNESS	KURTOSIS
POSEXMY	21.	79.	145.	.114732E+01	.127505E+01	.222537E+01	.102672E+02
POSEXMP	27.	78.	126.	.112450E+01	.124283E+01	.224789E+01	.963721E+01
POSEXH1PS	497.	775.	890.	.404231E+01	.841476E+01	.589943E+01	.679623E+02
POSEXH1YS	471.	760.	879.	.398029E+01	.978502E+01	.613891E+01	.637762E+02
POSEXH1YM	529.	785.	886.	.554857E+01	.117598E+03	.103235E+02	.135354E+03
POSEXH1PM	549.	810.	900.	.545173E+01	.907616E+02	.106271E+02	.156607E+03
BAMSETTN	67.	167.	256.	.330110E+01	.116162E+02	.195489E+01	.864325E+01
BAMSETTX1	974.	993.	998.	.259265E+02	.852245E+02	.453889E+00	.349667E+01
BAMSETTX2	181.	331.	436.	.531344E+01	.244151E+02	.169599E+01	.691178E+01
BAMSETTY	952.	998.	994.	.225673E+02	.790984E+02	.553944E+00	.345284E+01
BAMSEOTN	77.	187.	276.	.352244E+01	.130689E+02	.217292E+01	.108211E+02
BAMSEOTX1	981.	998.	999.	.277394E+02	.947347E+02	.423715E+00	.325179E+01
BAMSEOTX2	123.	262.	357.	.446920E+01	.197091E+02	.209236E+01	.923364E+01
BAMSEOTY	963.	992.	997.	.241577E+02	.877744E+02	.487292E+00	.321366E+01
GLEJSERX1	967.	997.	999.	.117274E+02	.252891E+02	.151670E+01	.732911E+01
GLEJSERX2	9.	29.	47.	.735137E+00	.953183E+00	.417224E+01	.292732E+02
GLEJSERY	912.	982.	996.	.939386E+01	.140323E+02	.111140E+01	.572619E+01
PARKX1	849.	940.	969.	.214383E+02	.258597E+03	.158014E+01	.650946E+01
PARKX2	5.	30.	72.	.847561E+00	.150431E+01	.292808E+01	.152525E+02
PARKY	829.	923.	959.	.167246E+02	.117164E+03	.134391E+01	.624060E+01
GQPN	101.	255.	391.	.181543E+01	.841418E+00	.296249E+01	.170770E+02
GQPX1	20.	59.	90.	.946402E+00	.383717E+00	.184226E+01	.822716E+01
GQPX2	995.	1000.	1000.	.140879E+02	.719536E+02	.183058E+01	.804330E+01
GQPY	920.	975.	989.	.795205E+01	.257343E+02	.171518E+01	.664535E+01
GQNN	9.	75.	192.	.424700E+01	.216816E+01	.532689E+00	.327410E+01
GQNX1	650.	909.	978.	.939500E+01	.450958E+01	.363561E+00	.347399E+01
GQNX2	16.	101.	220.	.416100E+01	.323431E+01	.431645E+00	.323689E+01
GQNY	402.	784.	916.	.813000E+01	.414725E+01	.372629E+00	.295515E+01

TABLE C9: Test Results - Model 8, Sample Size 90

TEST	ALPHA LEVEL .01	.05	.10	MEAN	VARIANCE	SKEWNESS	KURTOSIS
POSEXMY	35.	81.	138.	.112073E+01	.131962E+01	.224277E+01	.994395E+01
POSEXMP	27.	72.	119.	.105928E+01	.128432E+01	.272427E+01	.133699E+02
POSEXH1PS	812.	959.	983.	.522180E+01	.927188E+01	.431940E+01	.318980E+02
POSEXH1YS	800.	950.	980.	.504301E+01	.805634E+01	.421537E+01	.311904E+02
POSEXH1YM	784.	945.	978.	.515170E+01	.130092E+02	.530438E+01	.435414E+02
POSEXH1PM	798.	957.	981.	.538341E+01	.165779E+02	.535401E+01	.407963E+02
BAMSETTN	95.	210.	312.	.375830E+01	.139839E+02	.170860E+01	.680990E+01
BAMSETTX1	1000.	1000.	1000.	.407986E+02	.134994E+03	.474537E+00	.350159E+01
BAMSETTX2	151.	279.	369.	.473786E+01	.205544E+02	.167519E+01	.626956E+01
BAMSETTY	995.	999.	1000.	.332850E+02	.121665E+03	.392907E+00	.294106E+01
BAMSEOTN	89.	211.	296.	.362446E+01	.130164E+02	.164823E+01	.638817E+01
BAMSEOTX1	1000.	1000.	1000.	.430743E+02	.150452E+03	.481478E+00	.340391E+01
BAMSEOTX2	121.	242.	335.	.415398E+01	.174077E+02	.179880E+01	.726374E+01
BAMSEOTY	997.	999.	1000.	.353509E+02	.135690E+03	.442824E+00	.316881E+01
GLEJSERX1	1000.	1000.	1000.	.179954E+02	.341167E+02	.994951E+00	.493632E+01
GLEJSERX2	13.	42.	64.	.876405E+00	.115150E+01	.363535E+01	.223371E+02
GLEJSERY	996.	1000.	1000.	.132573E+02	.157827E+02	.769243E+00	.474499E+01
PARKX1	973.	996.	999.	.368212E+02	.528263E+03	.145011E+01	.667698E+01
PARKX2	4.	35.	91.	.989347E+00	.180527E+01	.366894E+01	.321816E+02
PARKY	959.	990.	997.	.285477E+02	.223529E+03	.925656E+00	.477046E+01
GQPN	99.	249.	351.	.157308E+01	.348427E+00	.219379E+01	.986399E+01
GQPX1	23.	63.	96.	.958764E+00	.237564E+00	.165871E+01	.772895E+01
GQPX2	1000.	1000.	1000.	.137294E+02	.452112E+02	.206989E+01	.116174E+02
GQPY	970.	990.	994.	.678194E+01	.134047E+02	.165943E+01	.774031E+01
GQNN	20.	67.	172.	.488100E+01	.287371E+01	.397311E+00	.302223E+01
GQNX1	899.	965.	988.	.115350E+02	.608286E+01	.276275E+00	.276246E+01
GQNX2	20.	51.	110.	.419700E+01	.341160E+01	.464029E+00	.294770E+01
GQNY	746.	867.	942.	.995200E+01	.489659E+01	.195346E+00	.297438E+01

TABLE C10: Test Results - Model 9, Sample Size 30

TEST	ALPHA LEVEL .01	.05	.10	MEAN	VARIANCE	SKEWNESS	KURTOSIS
POSEXMY	3.	22.	50.	.815991E+00	.603627E+00	.211297E+01	.969115E+01
POSEXMP	5.	44.	83.	.100795E+01	.840248E+00	.197277E+01	.883511E+01
POSEXH1PS	247.	446.	535.	.277602E+01	.332157E+01	.116224E+01	.440323E+01
POSEXH1YS	281.	455.	557.	.299811E+01	.448912E+01	.145446E+01	.606298E+01
POSEXH1YM	261.	441.	530.	.278317E+01	.384600E+01	.134502E+01	.602570E+01
POSEXH1PM	188.	413.	508.	.250184E+01	.271092E+01	.104612E+01	.409454E+01
BAMSETTN	59.	156.	244.	.321264E+01	.993483E+01	.188421E+01	.777178E+01
BAMSETTX1	112.	271.	364.	.434509E+01	.151496E+02	.143340E+01	.528979E+01
BAMSETTX2	557.	758.	844.	.105695E+02	.356451E+02	.583351E+00	.306275E+01
BAMSETTY	120.	267.	362.	.436436E+01	.157319E+02	.158212E+01	.636092E+01
BAMSEOTN	77.	195.	284.	.365271E+01	.127027E+02	.229449E+01	.129825E+02
BAMSEOTX1	106.	243.	342.	.413842E+01	.141458E+02	.153910E+01	.579006E+01
BAMSEOTX2	653.	846.	903.	.121641E+02	.362535E+02	.544469E+00	.306231E+01
BAMSEOTY	115.	225.	342.	.419414E+01	.152994E+02	.177647E+01	.749518E+01
GLEJSERX1	2.	28.	51.	.952631E+00	.798462E+00	.202250E+01	.103905E+02
GLEJSERX2	551.	794.	882.	.758079E+01	.404758E+02	.351440E+01	.259507E+02
GLEJSERY	4.	36.	112.	.122390E+01	.100361E+01	.187871E+01	.108531E+02
PARKX1	7.	37.	75.	.919832E+00	.242649E+01	.499535E+01	.441536E+02
PARKX2	370.	616.	719.	.741138E+01	.487363E+02	.223106E+01	.109062E+02
PARKY	27.	108.	191.	.165745E+01	.513971E+01	.298996E+01	.178789E+02
GQPN	110.	285.	405.	.270458E+01	.768769E+01	.577965E+01	.528543E+02
GQPX1	858.	945.	962.	.149376E+02	.212242E+03	.288675E+01	.141806E+02
GQPX2	34.	96.	146.	.130629E+01	.220645E+01	.472391E+01	.453172E+02
GQPY	115.	253.	355.	.243672E+01	.100155E+02	.560572E+01	.501005E+02
GQNN	13.	56.	133.	.294900E+01	.209049E+01	.472479E+00	.298170E+01
GQNX1	10.	38.	136.	.308100E+01	.157201E+01	.758172E+00	.385436E+01
GQNX2	352.	568.	913.	.591490E+01	.280341E+01	.283034E+00	.318017E+01
GQNY	46.	154.	326.	.389000E+01	.233423E+01	.480211E+00	.313700E+01

TABLE C11: Test Results - Model 9, Sample Size 60

TEST	ALPHA LEVEL .01	.05	.10	MEAN	VARIANCE	SKEWNESS	KURTOSIS
POSEXMY	507.	618.	668.	.881998E+01	.148988E+03	.332861E+01	.196762E+02
POSEXMP	599.	739.	765.	.104693E+02	.180523E+03	.323278E+01	.187864E+02
POSEXH1PS	927.	961.	978.	.283948E+02	.200254E+04	.426016E+01	.279549E+02
POSEXH1YS	951.	981.	989.	.364927E+02	.425545E+04	.557858E+01	.484471E+02
POSEXH1YM	963.	982.	990.	.130884E+03	.329195E+06	.182618E+02	.429206E+03
POSEXH1PM	939.	967.	980.	.757646E+02	.234242E+05	.436244E+01	.288495E+02
BAMSETTN	276.	446.	531.	.679184E+01	.391694E+02	.149963E+01	.552599E+01
BAMSETTX1	363.	540.	640.	.674774E+01	.597552E+02	.156483E+01	.600645E+01
BAMSETTX2	994.	1000.	1000.	.403239E+02	.259115E+03	.657765E+00	.364270E+01
BAMSETTY	624.	774.	821.	.154886E+02	.150663E+03	.127007E+01	.489408E+01
BAMSEOTN	299.	461.	558.	.720070E+01	.426820E+02	.144500E+01	.516279E+01
BAMSEOTX1	369.	546.	629.	.882712E+01	.530560E+02	.160982E+01	.623240E+01
BAMSEOTX2	997.	1000.	1000.	.419277E+02	.257519E+03	.693790E+00	.379861E+01
BAMSEOTY	633.	756.	809.	.156936E+02	.159204E+03	.130654E+01	.508356E+01
GLEJSERX1	2.	9.	21.	.655221E+00	.454536E+00	.245092E+01	.126381E+02
GLEJSERX2	985.	998.	1000.	.287621E+02	.327425E+03	.160123E+01	.694736E+01
GLEJSERY	564.	713.	780.	.957644E+01	.924892E+02	.174351E+01	.653095E+01
PARKX1	2.	27.	75.	.838495E+00	.137110E+01	.242113E+01	.102501E+02
PARKX2	756.	893.	939.	.174795E+02	.179961E+03	.136094E+01	.524224E+01
PARKY	142.	310.	439.	.334563E+01	.135035E+02	.193502E+01	.806146E+01
GQPN	324.	475.	570.	.266479E+01	.398378E+01	.250362E+01	.111802E+02
GQPX1	1000.	1000.	1000.	.363119E+02	.706899E+03	.220620E+01	.105201E+02
GQPX2	410.	538.	602.	.353473E+01	.182807E+02	.441003E+01	.352198E+02
GQPY	733.	858.	895.	.654766E+01	.403995E+02	.309699E+01	.179211E+02
GQNN	7.	75.	191.	.412800E+01	.262624E+01	.440391E+00	.315854E+01
GQNX1	6.	98.	237.	.444500E+01	.231729E+01	.351415E+00	.291528E+01
GQNX2	576.	888.	966.	.896200E+01	.431287E+01	.220304E+00	.270453E+01
GQNY	58.	266.	454.	.542300E+01	.329937E+01	.382593E+00	.297277E+01

TABLE C12: Test Results - Model 9, Sample Size 90

TEST	ALPHA LEVEL .01	.05	.10	MEAN	VARIANCE	SKEWNESS	KURTOSIS
POSEXMY	480.	633.	661.	.819753E+01	.131017E+03	.286944E+01	.137334E+02
POSEXMP	583.	698.	743.	.944223E+01	.137832E+03	.282541E+01	.142418E+02
POSEXH1PS	983.	991.	997.	.430778E+02	.824218E+04	.637871E+01	.547459E+02
POSEXH1YS	992.	1000.	1000.	.443163E+02	.598085E+04	.628556E+01	.558625E+02
POSEXH1YM	990.	1000.	1000.	.376834E+02	.428255E+04	.506729E+01	.359205E+02
POSEXH1PM	982.	992.	997.	.413925E+02	.853720E+04	.617334E+01	.511445E+02
BAMSETTN	536.	668.	726.	.130134E+02	.137341E+03	.156273E+01	.599229E+01
BAMSETTX1	638.	768.	836.	.165295E+02	.180820E+03	.133432E+01	.506555E+01
BAMSETTX2	1000.	1000.	1000.	.907213E+02	.638147E+03	.264453E+00	.315278E+01
BAMSETTY	969.	983.	987.	.446667E+02	.495190E+03	.509531E+00	.301924E+01
BAMSEOTN	704.	821.	860.	.180346E+02	.167897E+03	.965358E+00	.386347E+01
BAMSEOTX1	632.	779.	831.	.167796E+02	.190359E+03	.135762E+01	.516428E+01
BAMSEOTX2	1000.	1000.	1000.	.939406E+02	.626225E+03	.388947E+00	.354727E+01
BAMSEOTY	970.	984.	968.	.453679E+02	.519025E+03	.540903E+00	.305809E+01
GLEJSERX1	.	8.	37.	.998885E+00	.436944E+00	.112619E+01	.551889E+01
GLEJSERX2	1000.	1000.	1000.	.606884E+02	.159538E+04	.248637E+01	.127010E+02
GLEJSERY	956.	982.	969.	.318534E+02	.891621E+03	.243079E+01	.115525E+02
PARKX1	17.	61.	109.	.119131E+01	.310334E+01	.355173E+01	.217984E+02
PARKX2	993.	999.	1000.	.394061E+02	.400380E+03	.981234E+00	.443753E+01
PARKY	570.	767.	849.	.935753E+01	.462273E+02	.122093E+01	.513584E+01
GQPN	593.	747.	801.	.358728E+01	.796898E+01	.247071E+01	.115424E+02
GQPX1	1000.	1000.	1000.	.597386E+02	.168141E+04	.456618E+01	.509562E+02
GQPX2	363.	434.	497.	.210307E+01	.344070E+01	.281606E+01	.138709E+02
GQPY	981.	994.	997.	.126443E+02	.943850E+02	.264743E+01	.158652E+02
GQNN	24.	69.	152.	.474400E+01	.293940E+01	.461216E+00	.306794E+01
GQNX1	47.	119.	317.	.577800E+01	.245717E+01	.249952E+00	.315962E+01
GQNX2	909.	961.	991.	.115720E+02	.600482E+01	.341014E+00	.321808E+01
GQNY	264.	472.	684.	.741400E+01	.369230E+01	.257033E+00	.328975E+01

TABLE C13: Test Results - Model 10, Sample Size 30

TEST	ALPHA LEVEL .01	.05	.10	MEAN	VARIANCE	SKEWNESS	KURTOSIS
POSEXMY	4.	33.	79.	.980537E+00	.759776E+00	.178366E+01	.724102E+01
POSEXMP	3.	34.	60.	.928650E+00	.685952E+00	.191326E+01	.812503E+01
POSEXH1PS	18.	50.	94.	.110574E+01	.748206E+00	.259365E+01	.139151E+02
POSEXH1YS	17.	63.	111.	.109074E+01	.849487E+00	.278681E+01	.151778E+02
POSEXH1YM	19.	61.	109.	.107098E+01	.878750E+00	.266103E+01	.138304E+02
POSEXH1PM	17.	44.	85.	.106239E+01	.588302E+00	.254134E+01	.131435E+02
BAMSETTN	68.	167.	259.	.342550E+01	.107100E+02	.172796E+01	.664802E+01
BAMSETTX1	48.	137.	199.	.294544E+01	.872984E+01	.187085E+01	.727822E+01
BAMSETTX2	40.	122.	200.	.284772E+01	.730033E+01	.166124E+01	.623043E+01
BAMSETTY	40.	131.	214.	.295836E+01	.816881E+01	.174381E+01	.675804E+01
BAMSEOTN	75.	190.	282.	.363389E+01	.112827E+02	.169453E+01	.698737E+01
BAMSEOTX1	56.	159.	238.	.318373E+01	.999558E+01	.189882E+01	.821700E+01
BAMSEOTX2	45.	137.	214.	.301885E+01	.831549E+01	.168411E+01	.640371E+01
BAMSEOTY	52.	152.	237.	.320137E+01	.978693E+01	.191223E+01	.812199E+01
GLEJSERX1	7.	44.	87.	.105279E+01	.143526E+01	.475510E+01	.554070E+02
GLEJSERX2	21.	56.	102.	.112031E+01	.193735E+01	.437995E+01	.402753E+02
GLEJSERY	3.	23.	70.	.922034E+00	.951455E+00	.283465E+01	.181133E+02
PARKX1	8.	35.	78.	.957907E+00	.228075E+01	.420362E+01	.320384E+02
PARKX2	17.	74.	130.	.135560E+01	.392225E+01	.309845E+01	.173355E+02
PARKY	8.	38.	97.	.100699E+01	.232251E+01	.382016E+01	.278059E+02
GQPN	220.	416.	537.	.346200E+01	.149836E+02	.814023E+01	.116561E+03
GQPX1	64.	173.	247.	.179136E+01	.379693E+01	.460918E+01	.448600E+02
GQPX2	25.	73.	110.	.113435E+01	.139285E+01	.336544E+01	.207179E+02
GQPY	20.	50.	81.	.101851E+01	.126345E+01	.434504E+01	.330645E+02
GQNN	6.	15.	69.	.208900E+01	.226134E+01	.528420E+00	.293557E+01
GQNX1	9.	33.	122.	.275300E+01	.191390E+01	.622600E+00	.294599E+01
GQNX2	17.	80.	226.	.342800E+01	.200682E+01	.334292E+00	.263503E+01
GQNY	8.	55.	137.	.283600E+01	.205316E+01	.685117E+00	.302504E+01

TABLE C14: Test Results - Model 10, Sample Size 60

TEST	ALPHA LEVEL .01	.05	.10	MEAN	VARIANCE	SKEWNESS	KURTOSIS
POSEXMY	.	14.	41.	.757862E+00	.402189E+00	.165680E+01	.619704E+01
POSEXMP	4.	20.	48.	.823970E+00	.487456E+00	.235830E+01	.133143E+02
POSEXH1PS	1.	12.	27.	.672351E+00	.237647E+00	.213285E+01	.118084E+02
POSEXH1YS	5.	17.	35.	.704122E+00	.300008E+00	.238638E+01	.117587E+02
POSEXH1YM	16.	44.	67.	.901231E+00	.709767E+00	.554650E+01	.639708E+02
POSEXH1PM	12.	29.	52.	.855861E+00	.598458E+00	.555603E+01	.598686E+02
BAMSETTN	55.	166.	221.	.317874E+01	.105220E+02	.209011E+01	.928648E+01
BAMSETTX1	37.	121.	181.	.281379E+01	.792116E+01	.195071E+01	.810168E+01
BAMSETTX2	45.	135.	241.	.311108E+01	.826717E+01	.168140E+01	.684676E+01
BAMSETTY	49.	138.	214.	.297171E+01	.927220E+01	.203635E+01	.879421E+01
BAMSEOTN	80.	213.	294.	.380059E+01	.130076E+02	.176332E+01	.724944E+01
BAMSEOTX1	37.	129.	201.	.289540E+01	.851795E+01	.203214E+01	.891175E+01
BAMSEOTX2	40.	143.	225.	.297342E+01	.816132E+01	.163099E+01	.619350E+01
BAMSEOTY	66.	162.	240.	.328767E+01	.113562E+02	.199923E+01	.826226E+01
GLEJSERX1	5.	34.	79.	.925915E+00	.875288E+00	.182783E+01	.710347E+01
GLEJSERX2	4.	23.	40.	.783960E+00	.583111E+00	.258911E+01	.135706E+02
GLEJSERY	1.	14.	38.	.809568E+00	.518812E+00	.188457E+01	.818086E+01
PARKX1	7.	39.	92.	.978267E+00	.201452E+01	.348616E+01	.255182E+02
PARKX2	10.	43.	97.	.100748E+01	.222601E+01	.323142E+01	.182481E+02
PARKY	11.	57.	108.	.107621E+01	.234217E+01	.250204E+01	.107917E+02
GQPN	134.	289.	439.	.189749E+01	.851459E+00	.213656E+01	.932509E+01
GQPX1	28.	85.	140.	.113161E+01	.471822E+00	.223352E+01	.130341E+02
GQPX2	33.	96.	144.	.114917E+01	.492338E+00	.194527E+01	.902509E+01
GQPY	28.	97.	159.	.114076E+01	.503675E+00	.193961E+01	.910690E+01
GQNN	1.	10.	39.	.258300E+01	.254766E+01	.393232E+00	.292747E+01
GQNX1	8.	79.	193.	.414800E+01	.257067E+01	.583004E+00	.371402E+01
GQNX2	6.	44.	99.	.316400E+01	.318829E+01	.513168E+00	.310572E+01
GQNY	7.	64.	165.	.400700E+01	.252147E+01	.473040E+00	.310311E+01

TABLE C15: Test Results - Model 10, Sample Size 90

TEST	ALPHA LEVEL .01	.05	.10	MEAN	VARIANCE	SKEWNESS	KURTOSIS
POSEXMY	6.	29.	58.	.974839E+00	.572303E+00	.195104E+01	.897305E+01
POSEXMP	8.	40.	79.	.923062E+00	.655980E+00	.187555E+01	.775282E+01
POSEXH1PS	17.	43.	62.	.858299E+00	.519986E+00	.274833E+01	.146493E+02
POSEXH1YS	19.	42.	70.	.839246E+00	.625910E+00	.325297E+01	.184804E+02
POSEXH1YM	27.	51.	81.	.897640E+00	.733858E+00	.328634E+01	.188741E+02
POSEXH1PM	22.	48.	76.	.910679E+00	.592605E+00	.270012E+01	.134854E+02
BAMSETTN	29.	124.	199.	.282217E+01	.755424E+01	.178173E+01	.715097E+01
BAMSETTX1	37.	119.	195.	.284428E+01	.917672E+01	.270804E+01	.154377E+02
BAMSETTX2	45.	124.	198.	.291679E+01	.821012E+01	.186017E+01	.734378E+01
BAMSETTY	57.	156.	235.	.322038E+01	.934381E+01	.174566E+01	.689912E+01
BAMSEOTN	55.	134.	219.	.305295E+01	.849062E+01	.159247E+01	.552991E+01
BAMSEOTX1	40.	134.	226.	.295493E+01	.101032E+02	.258334E+01	.136579E+02
BAMSEOTX2	41.	128.	227.	.292468E+01	.903040E+01	.216493E+01	.974808E+01
BAMSEOTY	70.	189.	275.	.358496E+01	.112098E+02	.166239E+01	.640601E+01
GLEJSEPX1	5.	41.	81.	.955730E+00	.901939E+00	.201943E+01	.897091E+01
GLEJSEPX2	2.	21.	46.	.845191E+00	.590638E+00	.185920E+01	.783805E+01
GLEJSERY	1.	21.	54.	.869824E+00	.634103E+00	.167893E+01	.659956E+01
PARKX1	10.	42.	89.	.993517E+00	.188033E+01	.269400E+01	.129970E+02
PARKX2	8.	38.	92.	.944861E+00	.169220E+01	.250035E+01	.111792E+02
PARKY	7.	45.	86.	.973499E+00	.188632E+01	.293036E+01	.150584E+02
GQPN	93.	229.	342.	.154512E+01	.267193E+00	.199093E+01	.879676E+01
GQPX1	22.	78.	136.	.109463E+01	.235547E+00	.137569E+01	.542837E+01
GQPX2	34.	109.	154.	.112945E+01	.293143E+00	.164446E+01	.770242E+01
GQPY	25.	71.	116.	.105165E+01	.223803E+00	.132021E+01	.614922E+01
GQNN	2.	6.	22.	.279200E+01	.300574E+01	.389696E+00	.278729E+01
GQNX1	8.	40.	84.	.383000E+01	.373684E+01	.258410E+00	.280100E+01
GQNX2	11.	36.	79.	.369100E+01	.379732E+01	.345371E+00	.276333E+01
GQNY	8.	20.	63.	.357900E+01	.341117E+01	.350437E+00	.292933E+01

TABLE C16: Test Results - Model 11, Sample Size 30

TEST	ALPHA LEVEL .01	.05	.10	MEAN	VARIANCE	SKEWNESS	KURTOSIS
POSEXMY	5.	28.	60.	.873117E+00	.716424E+00	.215438E+01	.948897E+01
POSEXMP	5.	39.	63.	.898607E+00	.755546E+00	.217666E+01	.942428E+01
POSEXH1PS	25.	72.	127.	.120694E+01	.836488E+00	.210435E+01	.942376E+01
POSEXH1YS	30.	77.	129.	.122492E+01	.120111E+01	.365900E+01	.273597E+02
POSEXH1YM	29.	83.	129.	.120229E+01	.116243E+01	.344675E+01	.249565E+02
POSEXH1PM	19.	66.	118.	.115703E+01	.768567E+00	.200541E+01	.868166E+01
BAMSETTN	24.	75.	142.	.236147E+01	.582776E+01	.203634E+01	.855794E+01
BAMSETTX1	26.	97.	159.	.258758E+01	.625311E+01	.180995E+01	.710220E+01
BAMSETTX2	38.	116.	213.	.286910E+01	.724698E+01	.169176E+01	.649825E+01
BAMSETTY	20.	97.	172.	.267310E+01	.608556E+01	.165151E+01	.647371E+01
BAMSEOTN	21.	73.	128.	.229354E+01	.577938E+01	.253048E+01	.141618E+02
BAMSEOTX1	35.	106.	194.	.277864E+01	.664290E+01	.159592E+01	.582000E+01
BAMSEOTX2	39.	145.	241.	.305237E+01	.803001E+01	.164915E+01	.725103E+01
BAMSEOTY	39.	117.	201.	.283583E+01	.717344E+01	.161365E+01	.607615E+01
GLEJSERX1	10.	62.	131.	.128764E+01	.147478E+01	.211340E+01	.105248E+02
GLEJSERX2	31.	107.	183.	.155486E+01	.275952E+01	.282607E+01	.158657E+02
GLEJSERY	15.	72.	137.	.139154E+01	.154463E+01	.196251E+01	.863035E+01
PARKX1	39.	108.	168.	.160730E+01	.983527E+01	.767873E+01	.110180E+03
PARKX2	40.	111.	195.	.174275E+01	.570087E+01	.252608E+01	.109260E+02
PARKY	31.	114.	200.	.177080E+01	.730414E+01	.454137E+01	.399047E+02
GQPN	73.	210.	332.	.231563E+01	.361720E+01	.488264E+01	.402564E+02
GQPX1	110.	260.	356.	.236441E+01	.623724E+01	.545901E+01	.628887E+02
GQPX2	72.	197.	310.	.197710E+01	.280819E+01	.278814E+01	.156585E+02
GQPY	60.	164.	256.	.184207E+01	.272732E+01	.345075E+01	.248019E+02
GQNN	23.	66.	168.	.307100E+01	.234430E+01	.486308E+00	.309583E+01
GQNX1	43.	138.	305.	.387500E+01	.217955E+01	.529306E+00	.338988E+01
GQNX2	65.	183.	367.	.408600E+01	.224285E+01	.257271E+00	.257964E+01
GQNY	79.	179.	362.	.412800E+01	.234796E+01	.468862E+00	.301054E+01

TABLE C17: Test Results - Model 11, Sample Size 60

TEST	ALPHA LEVEL .01	.05	.10	MEAN	VARIANCE	SKEWNESS	KURTOSIS
POSEXMY	23.	81.	139.	.117030E+01	.111662E+01	.193077E+01	.801343E+01
POSEXMP	31.	107.	161.	.129054E+01	.137028E+01	.201193E+01	.860479E+01
POSEXH1PS	155.	230.	309.	.211570E+01	.833540E+01	.614916E+01	.674787E+02
POSEXH1YS	157.	248.	332.	.231528E+01	.104676E+02	.565728E+01	.520112E+02
POSEXH1YM	268.	396.	481.	.435989E+01	.112606E+03	.987773E+01	.145094E+03
POSEXH1PM	258.	368.	460.	.397720E+01	.902343E+02	.105607E+02	.172403E+03
BAMSETTN	23.	73.	129.	.234223E+01	.610268E+01	.250636E+01	.130686E+02
BAMSETTX1	75.	221.	302.	.372492E+01	.107516E+02	.145280E+01	.555605E+01
BAMSETTX2	70.	195.	286.	.361511E+01	.104737E+02	.152956E+01	.596726E+01
BAMSETTY	84.	235.	352.	.411243E+01	.132427E+02	.176493E+01	.772525E+01
BAMSECTN	27.	90.	158.	.245995E+01	.665877E+01	.225678E+01	.110445E+02
BAMSEOTX1	69.	207.	305.	.374866E+01	.105341E+02	.149992E+01	.590143E+01
BAMSEOTX2	66.	188.	274.	.357128E+01	.103843E+02	.152951E+01	.576190E+01
BAMSEOTY	92.	231.	334.	.414018E+01	.129307E+02	.171477E+01	.744001E+01
GLEJSERX1	34.	133.	243.	.171153E+01	.234257E+01	.222043E+01	.118647E+02
GLEJSERX2	72.	187.	280.	.192981E+01	.429524E+01	.246694E+01	.125078E+02
GLEJSERY	82.	233.	350.	.226798E+01	.389297E+01	.234618E+01	.134699E+02
PARKX1	75.	183.	290.	.239100E+01	.104281E+02	.343275E+01	.249790E+02
PARKX2	58.	154.	245.	.196135E+01	.646787E+01	.227618E+01	.977882E+01
PARKY	85.	228.	351.	.272528E+01	.102499E+02	.250060E+01	.125633E+02
GQPN	51.	171.	285.	.160399E+01	.367344E+00	.208127E+01	.926179E+01
GQPX1	144.	328.	452.	.186235E+01	.949440E+00	.145653E+01	.585036E+01
GQPX2	143.	346.	468.	.192013E+01	.107404E+01	.191409E+01	.915870E+01
GQPY	218.	441.	563.	.215603E+01	.133984E+01	.165038E+01	.699907E+01
GQNN	7.	53.	134.	.361400E+01	.285386E+01	.530069E+00	.331289E+01
GQNX1	34.	237.	423.	.526500E+01	.300178E+01	.340360E+00	.296857E+01
GQNX2	32.	199.	373.	.496500E+01	.334112E+01	.222833E+00	.286567E+01
GQNY	70.	302.	499.	.561500E+01	.342220E+01	.307813E+00	.283123E+01

TABLE C18: Test Results: Model 11, Sample Size 90

TEST	ALPHA LEVEL .01	.05	.10	MEAN	VARIANCE	SKEWNESS	KURTOSIS
POSEXMY	34.	104.	163.	.125160E+01	.139810E+01	.217361E+01	.102965E+02
POSEXMP	39.	120.	194.	.137892E+01	.159005E+01	.231163E+01	.117290E+02
POSEXH1PS	254.	406.	495.	.291482E+01	.130046E+02	.554376E+01	.503203E+02
POSEXH1YS	281.	437.	537.	.307566E+01	.122269E+02	.461493E+01	.350034E+02
POSEXH1YM	304.	465.	560.	.337160E+01	.169456E+02	.466212E+01	.329823E+02
POSEXH1PM	277.	417.	513.	.325494E+01	.203570E+02	.566926E+01	.484968E+02
BAMSETTN	24.	90.	173.	.250810E+01	.616316E+01	.171757E+01	.661559E+01
BAMSETTX1	102.	285.	411.	.455315E+01	.141739E+02	.158672E+01	.716556E+01
BAMSETTX2	171.	393.	509.	.551052E+01	.162930E+02	.106927E+01	.444087E+01
BAMSETTY	199.	411.	543.	.593605E+01	.197090E+02	.128115E+01	.543086E+01
BAMSEOTN	20.	94.	158.	.239295E+01	.574675E+01	.166563E+01	.608319E+01
BAMSEOTX1	109.	285.	403.	.458005E+01	.146703E+02	.164689E+01	.743015E+01
BAMSEOTX2	174.	378.	505.	.550185E+01	.176248E+02	.125120E+01	.530682E+01
BAMSEOTY	200.	394.	531.	.592629E+01	.202997E+02	.136364E+01	.570438E+01
GLEJSERX1	73.	270.	429.	.238775E+01	.270037E+01	.142944E+01	.686551E+01
GLEJSERX2	215.	427.	542.	.334643E+01	.834271E+01	.195306E+01	.964128E+01
GLEJSERY	242.	503.	639.	.366656E+01	.737180E+01	.217793E+01	.118427E+02
PARKX1	154.	341.	465.	.367846E+01	.160919E+02	.214471E+01	.100416E+02
PARKX2	135.	319.	431.	.327008E+01	.120814E+02	.161466E+01	.586111E+01
PARKY	218.	435.	564.	.451663E+01	.184267E+02	.153635E+01	.615576E+01
GQPN	31.	138.	251.	.141164E+01	.144614E+00	.181511E+01	.792353E+01
GQPX1	316.	559.	688.	.204033E+01	.656353E+00	.129042E+01	.693703E+01
GQPX2	204.	420.	555.	.180761E+01	.606499E+00	.162934E+01	.815912E+01
GQPY	372.	624.	746.	.219932E+01	.837181E+00	.136946E+01	.654210E+01
GQNN	22.	47.	105.	.412200E+01	.347659E+01	.501406E+00	.317998E+01
GQNX1	132.	259.	432.	.631800E+01	.382070E+01	.433276E+00	.316970E+01
GQNX2	89.	174.	325.	.569300E+01	.416692E+01	.264530E+00	.325694E+01
GQNY	177.	331.	520.	.669000E+01	.398388E+01	.209359E+00	.293995E+01

TABLE C19: Test Results, Model 12, Sample Size 30

TEST	ALPHA LEVEL .01	.05	.10	MEAN	VARIANCE	SKEWNESS	KURTOSIS
POSEXMY	12.	47.	82.	.938872E+00	.112897E+01	.296181E+01	.159295E+02
POSEXMP	14.	52.	93.	.102033E+01	.127014E+01	.314266E+01	.190625E+02
POSEXH1PS	59.	178.	289.	.184347E+01	.152164E+01	.278682E+01	.164427E+02
POSEXH1YS	79.	184.	296.	.198491E+01	.336876E+01	.502724E+01	.431335E+02
POSEXH1YM	75.	167.	277.	.198655E+01	.614855E+01	.907456E+01	.129525E+03
POSEXH1PM	43.	138.	255.	.170500E+01	.139259E+01	.279102E+01	.166409E+02
BAMSETTN	45.	143.	215.	.309803E+01	.874665E+01	.184122E+01	.735435E+01
BAMSETTX1	195.	464.	616.	.630350E+01	.157345E+02	.107405E+01	.446963E+01
BAMSETTX2	56.	158.	234.	.329111E+01	.938574E+01	.174125E+01	.688255E+01
BAMSETTY	177.	408.	548.	.570134E+01	.158768E+02	.100850E+01	.398602E+01
BAMSEOTN	36.	114.	204.	.286470E+01	.777957E+01	.209434E+01	.100282E+02
BAMSEOTX1	317.	588.	722.	.765329E+01	.207513E+02	.862217E+00	.404870E+01
BAMSEOTX2	55.	149.	242.	.321841E+01	.940247E+01	.193191E+01	.899508E+01
BAMSEOTY	262.	511.	641.	.686920E+01	.208103E+02	.964615E+00	.404294E+01
GLEJSERX1	168.	428.	598.	.353681E+01	.660884E+01	.209429E+01	.125418E+02
GLEJSERX2	15.	63.	110.	.122416E+01	.152471E+01	.264479E+01	.133923E+02
GLEJSERY	124.	363.	494.	.306520E+01	.589633E+01	.198177E+01	.100607E+02
PARKX1	162.	335.	448.	.422538E+01	.304192E+02	.391991E+01	.280080E+02
PARKX2	33.	113.	195.	.165325E+01	.498004E+01	.245019E+01	.106953E+02
PARKY	182.	358.	482.	.427321E+01	.235728E+02	.236177E+01	.125125E+02
GQPN	94.	254.	385.	.250011E+01	.458536E+01	.526544E+01	.533809E+02
GQPX1	81.	188.	257.	.194654E+01	.499619E+01	.487486E+01	.452409E+02
GQPX2	599.	797.	866.	.701607E+01	.371003E+02	.293015E+01	.166023E+02
GQPY	277.	445.	519.	.380560E+01	.255426E+02	.632790E+01	.696092E+02
GQNN	33.	81.	207.	.347300E+01	.197725E+01	.622234E+00	.358140E+01
GQNX1	204.	388.	653.	.519100E+01	.253115E+01	.380867E+00	.299069E+01
GQNX2	54.	170.	353.	.403300E+01	.226017E+01	.447614E+00	.293766E+01
GQNY	167.	361.	581.	.488900E+01	.313782E+01	.176889E+00	.322479E+01

TABLE C20: Test Results, Model 12, Sample Size 60

TEST	ALPHA LEVEL .01	.05	.10	MEAN	VARIANCE	SKEWNESS	KURTOSIS
POSEXMY	169.	286.	384.	.236303E+01	.728411E+01	.293604E+01	.167952E+02
POSEXMP	162.	233.	355.	.244192E+01	.877155E+01	.321476E+01	.185626E+02
POSEXH1PS	581.	753.	841.	.745960E+01	.195305E+03	.105207E+02	.169238E+03
POSEXH1YS	582.	761.	852.	.832527E+01	.238532E+03	.793754E+01	.100657E+03
POSEXH1YM	654.	799.	873.	.183295E+02	.332809E+04	.126820E+02	.239908E+03
POSEXH1PM	667.	822.	982.	.174733E+02	.220784E+04	.102090E+02	.166176E+03
BAMSETTN	45.	137.	210.	.281925E+01	.828710E+01	.182306E+01	.730199E+01
BAMSETTX1	738.	892.	945.	.136236E+02	.427089E+02	.716592E+00	.383768E+01
BAMSETTX2	148.	315.	429.	.501074E+01	.183844E+02	.164877E+01	.733079E+01
BAMSETTY	678.	831.	877.	.134212E+02	.583512E+02	.698810E+00	.366148E+01
BAMSEOTN	42.	137.	215.	.265795E+01	.863150E+01	.194068E+01	.841462E+01
BAMSEOTX1	772.	893.	944.	.142950E+02	.447883E+02	.673236E+00	.376968E+01
BAMSEOTX2	147.	295.	404.	.485195E+01	.184367E+02	.179372E+01	.819686E+01
BAMSEOTY	707.	845.	892.	.142060E+02	.638159E+02	.598267E+00	.329858E+01
GLEJSERX1	579.	829.	914.	.616499E+01	.126147E+02	.207488E+01	.131215E+02
GLEJSERX2	224.	379.	458.	.355380E+01	.194522E+02	.293135E+01	.150634E+02
GLEJSERY	665.	837.	986.	.748004E+01	.243847E+02	.162092E+01	.764789E+01
PARKX1	428.	675.	789.	.769979E+01	.403404E+02	.185348E+01	.863353E+01
PARKX2	70.	224.	330.	.257017E+01	.858618E+01	.223390E+01	.108136E+02
PARKY	486.	695.	794.	.959869E+01	.506564E+02	.138965E+01	.552346E+01
GQPN	76.	213.	320.	.169063E+01	.545715E+0[illegible]	.265642E+01	.146850E+02
GQPX1	195.	359.	473.	.198025E+01	.147545E+01	.164840E+01	.683266E+01
GQPX2	925.	983.	998.	.674533E+01	.153396E+02	.220763E+01	.112874E+02
GQPY	485.	637.	713.	.350965E+01	.311186E+01	.292427E+01	.158637E+02
GQNN	8.	85.	220.	.429800E+01	.239963E+01	.391135E+00	.292014E+01
GQNX1	182.	551.	743.	.694000E+01	.389630E+01	.413426E+00	.326787E+01
GONX2	36.	200.	357.	.493300E+01	.360612E+01	.394295E+00	.314499E+01
GONY	237.	563.	718.	.676300E+01	.653336E+01	-.188403E+00	.308391E+01

TABLE C21: Test Results, Model 12, Sample Size 90

TEST	ALPHA LEVEL .01	.05	.10	MEAN	VARIANCE	SKEWNESS	KURTOSIS
POSEXMY	203.	318.	388.	.267953E+01	.106398E+02	.275793E+01	.132190E+02
POSEXMP	193.	317.	382.	.252570E+01	.838548E+01	.256528E+01	.117168E+02
POSEXH1PS	846.	943.	977.	.129132E+02	.528348E+03	.605167E+01	.500768E+02
POSEXH1YS	869.	958.	983.	.119802E+02	.418070E+03	.873575E+01	.125176E+03
POSEXH1YM	845.	976.	980.	.123019E+02	.503947E+03	.816387E+01	.106156E+03
POSEXH1PM	830.	942.	971.	.142184E+02	.902313E+03	.638110E+01	.560964E+02
BAMSETTN	107.	232.	340.	.456012E+01	.150185E+02	.154520E+01	.568827E+01
BAMSETTX1	910.	965.	989.	.202985E+02	.732080E+02	.588323E+00	.380672E+01
BAMSETTX2	352.	557.	657.	.809948E+01	.398596E+02	.153073E+01	.772632E+01
BAMSETTY	937.	971.	978.	.249936E+02	.112339E+03	.329761E+00	.322822E+01
BAMSEOTN	112.	237.	327.	.433175E+01	.148983E+02	.146879E+01	.522239E+01
BAMSEOTX1	920.	973.	994.	.211319E+02	.796557E+02	.603917E+00	.371237E+01
BAMSEOTX2	366.	558.	658.	.828655E+01	.435231E+02	.161212E+01	.824639E+01
BAMSEOTY	941.	970.	979.	.259808E+02	.122035E+03	.311035E+00	.313656E+01
GLEJSERX1	884.	981.	997.	.863164E+01	.130775E+02	.138316E+01	.707912E+01
GLEJSERX2	575.	730.	812.	.812453E+01	.597871E+02	.235406E+01	.113732E+02
GLEJSERY	939.	975.	985.	.142773E+02	.789325E+02	.260025E+01	.163618E+02
PARKX1	780.	927.	961.	.134826E+02	.679299E+02	.125202E+01	.544786E+01
PARKX2	291.	521.	654.	.519313E+01	.180052E+02	.121811E+01	.467605E+01
PARKY	855.	936.	96[illegible].	.171710E+02	.970890E+02	.901445E+00	.449872E+01
GQPN	141.	288.	409.	.165658E+01	.425265E+00	.198253E+01	.855831E+01
GQPX1	439.	647.	743.	.239507E+01	.155757E+01	.255739E+01	.182860E+02
GQPX2	981.	995.	998.	.595254E+01	.769685E+01	.181556E+01	.891958E+01
GQPY	675.	8[illegible]1.	855.	.353448E+01	.500815E+01	.164823E+01	.651243E+01
GQNN	30.	77.	179.	.498700E+01	.297781E+01	.299412E+00	.304160E+01
GQNX1	352.	535.	728.	.785200E+01	.457867E+01	.509640E+00	.358794E+01
GQNX2	79.	175.	300.	.549700E+01	.443843E+01	.397925E+00	.305767E+01
GQNY	569.	727.	849.	.688800E+01	.630977E+01	-.224496E+00	.345490E+01

TABLE C22: Test Results, Model 13, Sample Size 30

TEST	ALPHA LEVEL .01	.05	.10	MEAN	VARIANCE	SKEWNESS	KURTOSIS
POSEXMY	10.	55.	118.	.115654E+01	.104230E+01	.194869E+01	.900585E+01
POSEXMP	12.	54.	112.	.118984E+01	.116705E+01	.254195E+01	.146346E+02
POSEXH1PS	16.	42.	64.	.102277E+01	.109293E+01	.607087E+01	.616883E+02
POSEXH1YS	25.	59.	88.	.110025E+01	.171421E+01	.627445E+01	.606530E+02
POSEXH1YM	31.	63.	89.	.111317E+01	.137102E+01	.622150E+01	.606381E+02
POSEXH1PM	19.	48.	68.	.100380E+01	.123551E+01	.618522E+01	.621516E+02
BAMSETTN	467.	731.	825.	.963316E+01	.293126E+02	.871273E+00	.393393E+01
BAMSETTX1	54.	137.	221.	.316673E+01	.912078E+01	.185972E+01	.778205E+01
BAMSETTX2	65.	179.	249.	.327685E+01	.108925E+02	.174272E+01	.640846E+01
BAMSETTY	65.	178.	256.	.342880E+01	.977366E+01	.160777E+01	.619914E+01
BAMSEOTN	663.	849.	908.	.124236E+02	.404083E+02	.773583E+00	.367490E+01
BAMSEOTX1	58.	163.	254.	.339425E+01	.103158E+02	.186240E+01	.787252E+01
BAMSEOTX2	74.	186.	265.	.341711E+01	.114251E+02	.170793E+01	.643713E+01
BAMSEOTY	74.	206.	296.	.370135E+01	.114215E+02	.162233E+01	.645077E+01
GLEJSERX1	4.	29.	60.	.987694E+00	.857450E+00	.178910E+01	.749924E+01
GLEJSERX2	5.	19.	50.	.811106E+00	.784710E+00	.263046E+01	.135093E+02
GLEJSERY	3.	28.	83.	.999850E+00	.918332E+00	.188439E+01	.793413E+01
PARKX1	11.	52.	85.	.100045E+01	.273495E+01	.367696E+01	.216112E+02
PARKX2	12.	60.	96.	.101552E+01	.249512E+01	.293763E+01	.138323E+02
PARKY	8.	47.	84.	.101384E+01	.230170E+01	.298211E+01	.142197E+02
GQPN	910.	960.	982.	.221263E+02	.800575E+03	.658454E+01	.801055E+02
GQPX1	111.	213.	313.	.221356E+01	.781192E+01	.473664E+01	.381912E+02
GQPX2	48.	133.	189.	.155223E+01	.297248E+01	.373574E+01	.233861E+02
GQPY	36.	94.	153.	.130542E+01	.205417E+01	.361415E+01	.246438E+02
GQNN	373.	619.	813.	.608100E+01	.295339E+01	.357268E+00	.309829E+01
GQNX1	19.	73.	184.	.305900E+01	.258811E+01	.390231E+00	.306353E+01
GQNX2	25.	65.	147.	.273900E+01	.283571E+01	.627807E+00	.346774E+01
GQNY	23.	87.	225.	.310500E+01	.285083E+01	.330391E+00	.261596E+01

TABLE C23: Test Results, Model 13, Sample Size 60

TEST	ALPHA LEVEL .01	.05	.10	MEAN	VARIANCE	SKEWNESS	KURTOSIS
POSEXMY	20.	74.	132.	.121661E+01	.109863E+01	.199649E+01	.917731E+01
POSEXMP	34.	98.	168.	.132258E+01	.141019E+01	.231714E+01	.126703E+02
POSEXH1YS	81.	136.	197.	.141070E+01	.440973E+01	.617459E+01	.664594E+02
POSEXH1YM	158.	239.	298.	.230861E+01	.225471E+02	.100802E+02	.153486E+03
POSEXH1PM	119.	182.	229.	.197416E+01	.159211E+02	.102662E+02	.145596E+03
BAMSETTN	982.	997.	999.	.252383E+02	.762748E+02	.431600E+00	.319641E+01
BAMSETTX1	107.	257.	355.	.422935E+01	.159499E+02	.169570E+01	.718331E+01
BAMSETTX2	64.	176.	271.	.343694E+01	.108805E+02	.183489E+01	.788787E+01
BAMSETTY	154.	311.	407.	.499180E+01	.207291E+02	.168287E+01	.701356E+01
BAMSEOTN	994.	1000.	1000.	.299361E+02	.967031E+02	.360667E+00	.304044E+01
BAMSEOTX1	117.	276.	371.	.440552E+01	.172384E+02	.167763E+01	.704273E+01
BAMSEOTX2	71.	194.	296.	.361345E+01	.119020E+02	.179678E+01	.763614E+01
BAMSEOTY	165.	323.	422.	.519761E+01	.226054E+02	.169311E+01	.703423E+01
GLEJSERX1	4.	23.	74.	.913638E+00	.778216E+00	.175266E+01	.694318E+01
GLEJSERX2	15.	38.	64.	.836187E+00	.103584E+01	.291308E+01	.145450E+02
GLEJSERY	6.	27.	46.	.744979E+00	.762550E+00	.348282E+01	.240981E+02
PARKX1	4.	30.	75.	.879319E+00	.149177E+01	.271793E+01	.139416E+02
PARKX2	3.	26.	57.	.785969E+00	.136378E+01	.308108E+01	.166557E+02
PARKY	7.	32.	68.	.865776E+00	.151127E+01	.295756E+01	.144775E+02
GQPN	999.	1000.	1000.	.173682E+02	.105668E+03	.208600E+01	.107021E+02
GQPX1	28.	60.	105.	.101346E+01	.514724E+00	.289649E+01	.181607E+02
GQPX2	35.	92.	143.	.114451E+01	.571640E+00	.251420E+01	.149043E+02
GQPY	79.	188.	265.	.141848E+01	.772915E+00	.155643E+01	.641906E+01
GQNN	582.	833.	956.	.905900E+01	.494847E+01	.199128E+00	.272621E+01
GQNX1	10.	74.	166.	.395400E+01	.275063E+01	.637934E+00	.343538E+01
GQNX2	2.	26.	77.	.305200E+01	.273003E+01	.388438E+00	.306135E+01
GQNY	7.	51.	151.	.389000E+01	.240230E+01	.549857E+00	.358101E+01

TABLE C24: Test Results, Model 13, Sample Size 90

TEST	ALPHA LEVEL .01	.05	.10	MEAN	VARIANCE	SKEWNESS	KURTOSIS
POSEXMY	11.	56.	96.	.101531E+01	.788613E+00	.183612E+01	.739981E+01
POSEXMP	17.	75.	133.	.115854E+01	.970123E+00	.166441E+01	.677288E+01
POSEXH1PS	43.	83.	124.	.108166E+01	.128775E+01	.353766E+01	.213876E+02
POSEXH1YS	56.	109.	159.	.118863E+01	.191652E+01	.446246E+01	.341566E+02
POSEXH1YM	64.	123.	179.	.127991E+01	.220671E+01	.423817E+01	.302600E+02
POSEXH1PM	48.	99.	145.	.115833E+01	.153018E+01	.355921E+01	.209956E+02
BAMSETTN	999.	1000.	1000.	.406534E+02	.126518E+03	.361146E+00	.309428E+01
BAMSETTX1	89.	218.	304.	.386535E+01	.138335E+02	.173291E+01	.688744E+01
BAMSETTX2	76.	186.	257.	.348917E+01	.117370E+02	.162592E+01	.585383E+01
BAMSETTY	167.	328.	451.	.521242E+01	.216589E+02	.161985E+01	.661871E+01
BAMSECTN	1000.	1000.	1000.	.462015E+02	.152755E+03	.287413E+00	.301878E+01
BAMSEOTX1	94.	226.	314.	.398092E+01	.146643E+02	.174294E+01	.695626E+01
BAMSEOTX2	83.	199.	267.	.357966E+01	.123720E+02	.163412E+01	.591146E+01
BAMSEOTY	178.	355.	470.	.545780E+01	.236148E+02	.161372E+01	.656240E+01
GLEJSERX1	6.	19.	52.	.839876E+00	.668230E+00	.204150E+01	.932677E+01
GLEJSEPX2	18.	72.	109.	.109871E+01	.166364E+01	.291873E+01	.154668E+02
GLEJSERY	13.	51.	89.	.998077E+00	.121061E+01	.261182E+01	.131616E+02
PARKX1	3.	26.	63.	.810433E+00	.137915E+01	.344179E+01	.238291E+02
PARKX2	2.	25.	74.	.845497E+00	.122273E+01	.211077E+01	.830504E+01
PARKY	6.	35.	82.	.933861E+00	.170354E+01	.302357E+01	.173791E+02
GQPN	1000.	1000.	1000.	.173273E+02	.629171E+02	.174014E+01	.830858E+01
GQPX1	47.	125.	196.	.117924E+01	.373372E+00	.182093E+01	.999156E+01
GQPX2	71.	170.	226.	.126012E+01	.423666E+00	.165480E+01	.739163E+01
GQPY	138.	255.	359.	.150373E+01	.598048E+00	.137479E+01	.535994E+01
GQNN	839.	908.	954.	.108630E+02	.624882E+01	.797413E-01	.302240E+01
GQNX1	15.	39.	106.	.412500E+01	.338476E+01	.395603E+00	.301658E+01
GQNX2	14.	37.	101.	.430400E+01	.299858E+01	.395185E+00	.319484E+01
GQNY	6.	22.	71.	.375300E+01	.306105E+01	.441859E+00	.284078E+01

TABLE C25: Test Results, Model 14, Sample Size 30

TEST	ALPHA LEVEL .01	.05	.10	MEAN	VARIANCE	SKEWNESS	KURTOSIS
POSEXMY	19.	59.	95.	.101316E+01	.141379E+01	.314426E+01	.174068E+02
POSEXMP	16.	46.	81.	.954067E+00	.125957E+01	.341050E+01	.203116E+02
POSEXH1PS	79.	221.	351.	.204265E+01	.210486E+01	.302648E+01	.185674E+02
POSEXH1YS	76.	203.	331.	.198208E+01	.215588E+01	.321701E+01	.200476E+02
POSEXH1YM	51.	167.	272.	.182682E+01	.186264E+01	.328044E+01	.206507E+02
POSEXH1PM	61.	183.	293.	.188153E+01	.181665E+01	.310311E+01	.192442E+02
BAMSETTN	58.	160.	243.	.332297E+01	.956273E+01	.169845E+01	.656503E+01
BAMSETTX1	335.	603.	730.	.781532E+01	.217759E+02	.924617E+00	.395490E+01
BAMSETTX2	56.	160.	248.	.333145E+01	.100452E+02	.184951E+01	.799286E+01
BAMSETTY	254.	523.	653.	.698034E+01	.202156E+02	.102925E+01	.415380E+01
BAMSEOTN	39.	133.	216.	.298487E+01	.823809E+01	.189979E+01	.842471E+01
BAMSEOTX1	480.	735.	925.	.951432E+01	.273251E+02	.718975E+00	.353524E+01
BAMSEOTX2	51.	136.	222.	.305812E+01	.890264E+01	.186175E+01	.793984E+01
BAMSEOTY	379.	630.	752.	.845561E+01	.261547E+02	.919660E+00	.382398E+01
GLEJSERX1	268.	572.	725.	.452331E+01	.102149E+02	.205453E+01	.118208E+02
GLEJSERX2	.	12.	37.	.822142E+00	.522048E+00	.172965E+01	.698461E+01
GLEJSERY	207.	494.	624.	.384127E+01	.811910E+01	.196037E+01	.936298E+01
PARKX1	167.	366.	494.	.447989E+01	.282364E+02	.321761E+01	.197241E+02
PARKX2	6.	35.	74.	.987876E+00	.195171E+01	.285252E+01	.147408E+02
PARKY	196.	443.	562.	.492480E+01	.287618E+02	.291154E+01	.174205E+02
GQPN	110.	281.	405.	.262823E+01	.509554E+01	.405589E+01	.300701E+02
GQPX1	33.	80.	122.	.122886E+01	.209753E+01	.521944E+01	.525183E+02
GQPX2	712.	870.	918.	.901502E+01	.629844E+02	.314802E+01	.195875E+02
GQPY	384.	630.	721.	.496758E+01	.271901E+02	.532173E+01	.568009E+02
GQNN	25.	83.	203.	.350200E+01	.183183E+01	.601387E+00	.344860E+01
GQNX1	251.	474.	718.	.545200E+01	.290260E+01	.288584E+00	.323812E+01
GQNX2	31.	108.	271.	.359700E+01	.200520E+01	.470884E+00	.303476E+01
GQNY	219.	411.	670.	.521600E+01	.268603E+01	.169885E+00	.277531E+01

TABLE C26: Test Results, Model 14, Sample Size 60

TEST	ALPHA LEVEL .01	.05	.10	MEAN	VARIANCE	SKEWNESS	KURTOSIS
POSEXMY	17.	72.	128.	.111136E+01	.111902E+01	.211432E+01	.945033E+01
POSEXMP	22.	70.	126.	.111204E+01	.109532E+01	.214548E+01	.938050E+01
POSEXH1FS	358.	638.	773.	.340236E+01	.676975E+01	.561733E+01	.611728E+02
POSEXH1YS	333.	612.	753.	.336902E+01	.792188E+01	.583210E+01	.564130E+02
POSEXH1YM	436.	672.	793.	.487838E+01	.971532E+02	.100159E+02	.129665E+03
POSEXH1PM	420.	599.	809.	.476762E+01	.760954E+02	.104775E+02	.154904E+03
BAMSETTN	44.	139.	214.	.290560E+01	.919344E+01	.198915E+01	.871800E+01
BAMSETTX1	841.	937.	967.	.163595E+02	.551225E+02	.624817E+00	.347919E+01
BAMSETTX2	117.	254.	356.	.431782E+01	.171487E+02	.176348E+01	.716978E+01
BAMSETTY	749.	902.	948.	.144964E+02	.509226E+02	.772822E+00	.382103E+01
BAMSEOTN	47.	158.	227.	.310265E+01	.102800E+02	.217789E+01	.106499E+02
BAMSEOTX1	861.	950.	973.	.173410E+02	.587098E+02	.549883E+00	.326059E+01
BAMSEOTX2	81.	195.	290.	.376104E+01	.144348E+02	.220149E+01	.100572E+02
BAMSEOTY	781.	916.	958.	.153557E+02	.546272E+02	.676628E+00	.350021E+01
GLEJSERX1	750.	928.	967.	.778690E+01	.166202E+02	.161913E+01	.739588E+01
GLEJSERX2	9.	31.	49.	.785875E+00	.943114E+00	.371751E+01	.239453E+02
GLEJSERY	645.	872.	941.	.632806E+01	.937615E+01	.110949E+01	.545460E+01
PARKX1	489.	720.	820.	.862474E+01	.497836E+02	.204235E+01	.109582E+02
PARKX2	4.	37.	79.	.928745E+00	.186988E+01	.376818E+01	.279561E+02
PARKY	467.	695.	800.	.785346E+01	.374416E+02	.166788E+01	.870787E+01
GOPN	84.	215.	344.	.172266E+01	.615595E+00	.272270E+01	.148040E+02
GQPX1	16.	53.	95.	.960007E+00	.341679E+00	.167503E+01	.697497E+01
GQPX2	953.	986.	992.	.730416E+01	.156419E+02	.163686E+01	.697698E+01
GQPY	777.	910.	958.	.484275E+01	.756356E+01	.160632E+01	.639689E+01
GQNN	4.	79.	165.	.415700E+01	.228063E+01	.440614E+00	.282316E+01
GQNX1	245.	651.	841.	.733200E+01	.381559E+01	.326747E+00	.323718E+01
GQNX2	9.	97.	222.	.415700E+01	.312748E+01	.275018E+00	.303009E+01
GQNY	197.	552.	759.	.589700E+01	.400440E+01	.416008E+00	.321917E+01

TABLE C27: Test Results, Model 14, Sample Size 90

TEST	ALPHA LEVEL .01	.05	.10	MEAN	VARIANCE	SKEWNESS	KURTOSIS
POSEXMY	32.	71.	131.	.109710E+01	.116116E+01	.217775E+01	.977605E+01
POSEXMP	23.	62.	116.	.105302E+01	.113698E+01	.262704E+01	.132022E+02
POSEXH1PS	651.	885.	954.	.438968E+01	.749626E+01	.433143E+01	.318142E+02
POSEXH1YS	636.	880.	944.	.425353E+01	.654451E+01	.418129E+01	.303025E+02
POSEXH1YM	624.	877.	944.	.438548E+01	.107403E+02	.518901E+01	.407394E+02
POSEXH1PM	635.	881.	949.	.456891E+01	.135873E+02	.530192E+01	.397424E+02
BAMSETTN	68.	161.	266.	.327555E+01	.106751E+02	.170953E+01	.661625E+01
BAMSETTX1	980.	997.	1000.	.258479E+02	.884439E+02	.610019E+00	.375609E+01
BAMSETTX2	96.	223.	301.	.394399E+01	.150366E+02	.175193E+01	.660410E+01
BAMSETTY	941.	982.	993.	.215250E+02	.754043E+02	.542837E+00	.323725E+01
BAMSEOTN	60.	167.	249.	.317801E+01	.100499E+02	.163500E+01	.618183E+01
BAMSEOTX1	981.	998.	1000.	.271015E+02	.968404E+02	.624758E+00	.372449E+01
BAMSEOTX2	86.	195.	270.	.355039E+01	.130771E+02	.188169E+01	.777168E+01
BAMSEOTY	949.	983.	996.	.226167E+02	.832479E+02	.602072E+00	.349456E+01
GLEJSERX1	967.	997.	999.	.119316E+02	.228421E+02	.104606E+01	.508732E+01
GLEJSERX2	15.	34.	68.	.912822E+00	.112107E+01	.351894E+01	.220227E+02
GLEJSERY	909.	989.	996.	.890423E+01	.110492E+02	.954001E+00	.550278E+01
PARKX1	764.	904.	957.	.134401E+02	.798131E+02	.170275E+01	.832974E+01
PARKX2	4.	44.	78.	.956441E+00	.175228E+01	.262190E+01	.126555E+02
PARKY	720.	877.	930.	.120656E+02	.586222E+02	.109474E+01	.480996E+01
GQPN	83.	215.	326.	.152121E+01	.274554E+00	.205659E+01	.880174E+01
GQPX1	15.	53.	95.	.969553E+00	.209234E+00	.150684E+01	.695118E+01
GQPX2	997.	1000.	1000.	.729409E+01	.101952E+02	.180673E+01	.986227E+01
GQPY	884.	965.	978.	.422073E+01	.345589E+01	.127841E+01	.563263E+01
GQNN	18.	64.	145.	.465900E+01	.296158E+01	.376176E+00	.293855E+01
GQNX1	476.	670.	834.	.855400E+01	.499408E+01	.354650E+00	.327920E+01
GQNX2	20.	60.	123.	.430400E+01	.350519E+01	.426889E+00	.296745E+01
GQNY	377.	584.	761.	.799300E+01	.431126E+01	.230068E+00	.297965E+01

TABLE C28: Test Results, Model 15, Sample Size 30

TEST	ALPHA LEVEL .01	.05	.10	MEAN	VARIANCE	SKEWNESS	KURTOSIS
POSEXMY	2.	4.	27.	.728765E+00	.397350E+00	.195352E+01	.971021E+01
POSEXMP	5.	25.	57.	.872704E+00	.685303E+00	.235989E+01	.124966E+02
POSEXH1PS	5.	28.	55.	.892050E+00	.453229E+00	.222509E+01	.111695E+02
POSEXH1YS	14.	48.	78.	.972460E+00	.684514E+00	.270193E+01	.139779E+02
POSEXH1YM	11.	51.	81.	.971498E+00	.663724E+00	.244356E+01	.119655E+02
POSEXH1PM	2.	27.	56.	.874686E+00	.425132E+00	.194767E+01	.898906E+01
BAMSETTN	50.	142.	226.	.308283E+01	.882690E+01	.183477E+01	.771744E+01
BAMSETTX1	53.	148.	228.	.310196E+01	.884709E+01	.180517E+01	.752423E+01
BAMSETTX2	55.	160.	251.	.331450E+01	.939841E+01	.169605E+01	.682642E+01
BAMSETTY	75.	189.	281.	.357790E+01	.107198E+02	.158594E+01	.594607E+01
BAMSEOTN	64.	198.	294.	.360895E+01	.115660E+02	.179117E+01	.808829E+01
BAMSEOTX1	53.	153.	235.	.321158E+01	.977497E+01	.179796E+01	.708992E+01
BAMSEOTX2	61.	158.	253.	.332851E+01	.101919E+02	.173017E+01	.672547E+01
BAMSEOTY	78.	204.	309.	.371511E+01	.118893E+02	.163373E+01	.629164E+01
GLEJSERX1	4.	18.	47.	.894152E+00	.831071E+00	.396482E+01	.409793E+02
GLEJSERX2	45.	119.	169.	.153082E+01	.452250E+01	.380951E+01	.241241E+02
GLEJSERY	2.	26.	53.	.913336E+00	.775055E+00	.208885E+01	.911213E+01
PARKX1	23.	51.	98.	.115038E+01	.462580E+01	.430175E+01	.270078E+02
PARKX2	7.	37.	82.	.990795E+00	.282516E+01	.549218E+01	.516890E+02
PARKY	18.	59.	104.	.114224E+01	.321513E+01	.330907E+01	.178143E+02
GQPN	119.	278.	400.	.262793E+01	.585117E+01	.509298E+01	.465533E+02
GQPX1	127.	262.	360.	.240018E+01	.701987E+01	.331691E+01	.186974E+02
GQPX2	72.	159.	235.	.173575E+01	.308445E+01	.405013E+01	.386867E+02
GQPY	49.	130.	188.	.157193E+01	.329739E+01	.487159E+01	.416114E+02
GQNN	21.	65.	174.	.311800E+01	.226234E+01	.428985E+00	.308713E+01
GQNX1	26.	94.	238.	.356700E+01	.194746E+01	.460924E+00	.301855E+01
GQNX2	17.	45.	119.	.267600E+01	.233536E+01	.643904E+00	.338399E+01
GQNY	30.	83.	236.	.359600E+01	.193672E+01	.575993E+00	.363021E+01

TABLE C29: Test Results, Model 15, Sample Size 60

TEST	ALPHA LEVEL .01	.05	.10	MEAN	VARIANCE	SKEWNESS	KURTOSIS
POSEXMY	1.	4.	20.	.661551E+00	.303350E+00	.166241E+01	.693411E+01
POSEXMP	2.	13.	25.	.657548E+00	.370735E+00	.279004E+01	.173607E+02
POSEXH1PS	.	3.	12.	.716470E+00	.178834E+00	.117230E+01	.492548E+01
POSEXH1YS	.	3.	10.	.704754E+00	.186209E+00	.125477E+01	.531606E+01
POSEXH1YM	9.	16.	31.	.827782E+00	.361151E+00	.368676E+01	.279587E+02
POSEXH1PM	7.	16.	29.	.836452E+00	.343894E+00	.372648E+01	.300429E+02
BAMSETTN	39.	143.	212.	.304414E+01	.897592E+01	.215630E+01	.104358E+02
BAMSETTX1	76.	193.	289.	.355868E+01	.118369E+02	.172574E+01	.693387E+01
BAMSETTX2	67.	176.	252.	.335295E+01	.103183E+02	.165428E+01	.633998E+01
BAMSETTY	74.	202.	276.	.362181E+01	.121669E+02	.172316E+01	.673081E+01
BAMSEOTN	62.	165.	258.	.337494E+01	.112505E+02	.208833E+01	.978748E+01
BAMSEOTX1	95.	205.	310.	.368058E+01	.124690E+02	.164820E+01	.636094E+01
BAMSEOTX2	65.	175.	255.	.340717E+01	.107885E+02	.172948E+01	.691233E+01
BAMSEOTY	86.	195.	293.	.372159E+01	.125180E+02	.161646E+01	.575626E+01
GLEJSERX1	3.	37.	97.	.105020E+01	.864752E+00	.146873E+01	.561131E+01
GLEJSERX2	2.	14.	32.	.775121E+00	.466873E+00	.244072E+01	.134116E+02
GLEJSERY	2.	9.	42.	.898218E+00	.484965E+00	.174527E+01	.797253E+01
PARKX1	26.	75.	124.	.124748E+01	.414867E+01	.458912E+01	.427904E+02
PARKX2	26.	83.	159.	.140559E+01	.352994E+01	.241918E+01	.102662E+02
PARKY	5.	33.	74.	.895161E+00	.170272E+01	.295193E+01	.145222E+02
GQPN	65.	209.	332.	.158572E+01	.508721E+00	.253212E+01	.129138E+02
GQPX1	27.	71.	131.	.107887E+01	.443648E+00	.202126E+01	.995929E+01
GQPX2	78.	190.	277.	.144049E+01	.864051E+00	.178331E+01	.734987E+01
GQPY	60.	169.	243.	.138413E+01	.806314E+00	.297782E+01	.247525E+02
GQNN	5.	75.	157.	.394400E+01	.267754E+01	.346259E+00	.280522E+01
GQNX1	19.	115.	280.	.445400E+01	.300489E+01	.570341E+00	.349782E+01
GQNX2	4.	32.	86.	.332900E+01	.258534E+01	.488392E+00	.341646E+01
GQNY	24.	169.	302.	.495500E+01	.277975E+01	.426352E+00	.313766E+01

TABLE C30: Test Results, Model 15, Sample Size 90

TEST	ALPHA LEVEL .01	.05	.10	MEAN	VARIANCE	SKEWNESS	KURTOSIS
POSEXMY	1.	18.	46.	.690184E+00	.378787E+00	.175846E+01	.677373E+01
POSEXMF	2.	13.	36.	.681715E+00	.395848E+00	.191476E+01	.818320E+01
POSEXH1PS	.	8.	27.	.795800E+00	.217613E+00	.125737E+01	.530980E+01
POSEXH1YS	.	12.	30.	.774977E+00	.229789E+00	.135867E+01	.537775E+01
POSEXH1YM	1.	13.	33.	.788752E+00	.231436E+00	.138538E+01	.550843E+01
POSEXH1PM	.	8.	29.	.807585E+00	.214513E+00	.124783E+01	.529990E+01
BAMSETTN	78.	[illegible]	303.	.381698E+01	.136000E+02	.207968E+01	.101569E+02
BAMSETTX1	63.	170.	263.	.338190E+01	.103747E+02	.166237E+01	.622886E+01
BAMSETTX2	66.	161.	248.	.326952E+01	.127132E+02	.248405E+01	.134523E+02
BAMSETTY	110.	251.	345.	.414735E+01	.145258E+02	.147493E+01	.542267E+01
BAMSECTN	62.	185.	264.	.352936E+01	.117326E+02	.197719E+01	.888076E+01
BAMSEOTX1	69.	137.	266.	.341130E+01	.108747E+02	.165971E+01	.609623E+01
BAMSEOTX2	61.	191.	243.	.329642E+01	.123888E+02	.233673E+01	.121153E+02
BAMSEOTY	114.	243.	337.	.416675E+01	.152243E+02	.151996E+01	.563539E+01
GLEJSERX1	12.	54.	131.	.119156E+01	.114200E+01	.165178E+01	.666249E+01
GLEJSERX2	.	[illegible]	29.	.825433E+00	.439714E+00	.146341E+01	.618357E+01
GLEJSERY	6.	59.	147.	.128655E+01	.102954E+01	.150803E+01	.652884E+01
PARKX1	55.	153.	224.	.194010E+01	.770026E+01	.356232E+01	.251855E+02
PARKX2	10.	67.	137.	.116588E+01	.242485E+01	.229305E+01	.959253E+01
PARKY	17.	65.	114.	.119916E+01	.281196E+01	.285261E+01	.147099E+02
GQPN	100.	238.	359.	.156578E+01	.307072E+00	.185969E+01	.738375E+01
GQPX1	13.	49.	84.	.982414E+00	.194563E+00	.126505E+01	.669272E+01
GQPX2	144.	295.	395.	.156113E+01	.596357E+00	.163914E+01	.745521E+01
GQPY	75.	184.	283.	.130944E+01	.357772E+00	.114816E+01	.444555E+01
GQNN	13.	35.	106.	.433400E+01	.281926E+01	.371889E+00	.298035E+01
GQNX1	47.	124.	244.	.530700E+01	.325801E+01	.412343E+00	.291294E+01
GQNX2	6.	16.	59.	.370500E+01	.276874E+01	.401646E+00	.307047E+01
GQNY	59.	[illegible]	[illegible]	.557700E+01	.348155E+01	.246600E+00	.278426E+01

TABLE C31: Test Results, Model 16, Sample Size 30

TEST	ALPHA LEVEL .01	.05	.10	MEAN	VARIANCE	SKEWNESS	KURTOSIS
POSEXMY	15.	51.	110.	.113778E+01	.107884E+01	.217208E+01	.108252E+02
POSEXMP	13.	52.	107.	.112117E+01	.104872E+01	.206763E+01	.939904E+01
POSEXH1PS	43.	85.	125.	.121242E+01	.194418E+01	.413467E+01	.278104E+02
POSEXH1YS	40.	80.	121.	.118752E+01	.197655E+01	.458631E+01	.336197E+02
POSEXH1YM	68.	120.	165.	.152919E+01	.526154E+01	.607956E+01	.568568E+02
POSEXH1PM	70.	132.	178.	.155148E+01	.452075E+01	.493471E+01	.367150E+02
BAMSETTN	10.	47.	109.	.205905E+01	.420468E+01	.204073E+01	.966430E+01
BAMSETTX1	12.	52.	96.	.201392E+01	.405801E+01	.195477E+01	.809478E+01
BAMSETTX2	18.	50.	101.	.212116E+01	.455432E+01	.217990E+01	.100199E+02
BAMSETTY	13.	51.	95.	.199833E+01	.406894E+01	.197148E+01	.800974E+01
BAMSEOTN	17.	59.	105.	.213612E+01	.472410E+01	.215957E+01	.103506E+02
BAMSEOTX1	14.	58.	111.	.211781E+01	.434279E+01	.189573E+01	.770872E+01
BAMSEOTX2	16.	68.	123.	.219049E+01	.492062E+01	.209212E+01	.966630E+01
BAMSEOTY	12.	54.	108.	.204055E+01	.426111E+01	.188530E+01	.776814E+01
GLEJSERX1	7.	37.	82.	.103347E+01	.107454E+01	.232687E+01	.116161E+02
GLEJSERX2	9.	40.	84.	.103387E+01	.139223E+01	.374026E+01	.299215E+02
GLEJSERY	2.	36.	74.	.985382E+00	.932099E+00	.203105E+01	.873403E+01
PARKX1	10.	56.	109.	.110605E+01	.261567E+01	.263166E+01	.114868E+02
PARKX2	6.	46.	102.	.108333E+01	.245441E+01	.337828E+01	.214049E+02
PARKY	15.	55.	109.	.109999E+01	.306889E+01	.336408E+01	.180049E+02
GQPN	62.	185.	306.	.216776E+01	.309822E+01	.576897E+01	.554079E+02
GQPX1	32.	90.	157.	.137168E+01	.193924E+01	.499197E+01	.501984E+02
GQPX2	28.	87.	141.	.135182E+01	.136965E+01	.298392E+01	.182661E+02
GQPY	57.	144.	216.	.174935E+01	.338586E+01	.428929E+01	.324062E+02
GQNN	24.	66.	164.	.304800E+01	.243813E+01	.412584E+00	.302953E+01
GQNX1	27.	64.	185.	.310800E+01	.252486E+01	.427527E+00	.314439E+01
GQNX2	16.	59.	163.	.300300E+01	.239138E+01	.435123E+00	.319790E+01
GQNY	19.	62.	166.	.300500E+01	.231129E+01	.471857E+00	.305048E+01

TABLE C32: Test Results, Model 16, Sample Size 60

TEST	ALPHA LEVEL .01	.05	.10	MEAN	VARIANCE	SKEWNESS	KURTOSIS
POSEXMY	8.	52.	103.	.101350E+01	.772148E+00	.193154E+01	.895751E+01
POSEXMP	8.	52.	100.	.101443E+01	.786867E+00	.195141E+01	.891255E+01
POSEXH1PS	42.	76.	106.	.114065E+01	.291184E+01	.981522E+01	.158222E+03
POSEXH1YS	43.	77.	104.	.113912E+01	.293851E+01	.982799E+01	.158716E+03
POSEXH1YM	99.	170.	222.	.200916E+01	.215283E+02	.105343E+02	.157195E+03
POSEXH1PM	97.	169.	228.	.200048E+01	.216454E+02	.108969E+02	.168824E+03
BAMSETTN	18.	56.	98.	.205277E+01	.454886E+01	.217385E+01	.961160E+01
BAMSETTX1	12.	56.	101.	.209548E+01	.459313E+01	.218517E+01	.103957E+02
BAMSETTX2	11.	47.	110.	.206380E+01	.431678E+01	.214845E+01	.108514E+02
BAMSETTY	7.	44.	87.	.193506E+01	.344527E+01	.169516E+01	.657896E+01
BAMSEOTN	18.	61.	111.	.213807E+01	.496704E+01	.231875E+01	.115474E+02
BAMSEOTX1	12.	61.	119.	.213799E+01	.488851E+01	.236063E+01	.133726E+02
BAMSEOTX2	14.	53.	117.	.212789E+01	.479144E+01	.229108E+01	.119508E+02
BAMSEOTY	9.	41.	104.	.202173E+01	.394850E+01	.193762E+01	.867117E+01
GLEJSERX1	13.	55.	106.	.110447E+01	.132431E+01	.251261E+01	.137293E+02
GLEJSERX2	7.	26.	63.	.907934E+00	.781054E+00	.247883E+01	.132995E+02
GLEJSERY	8.	36.	98.	.104746E+01	.106317E+01	.270339E+01	.167411E+02
PARKX1	14.	61.	116.	.112426E+01	.279297E+01	.298635E+01	.159057E+02
PARKX2	13.	52.	104.	.108453E+01	.254453E+01	.285767E+01	.141308E+02
PARKY	16.	67.	121.	.118579E+01	.310874E+01	.312235E+01	.170420E+02
GQPN	34.	124.	224.	.151304E+01	.266571E+00	.218121E+01	.105585E+02
GQPX1	15.	75.	136.	.113025E+01	.332382E+00	.148732E+01	.618668E+01
GQPX2	17.	67.	122.	.113430E+01	.342438E+00	.169595E+01	.764759E+01
GQPY	63.	180.	283.	.149566E+01	.636120E+00	.218209E+01	.130005E+02
GQNN	10.	69.	157.	.378300E+01	.307499E+01	.433102E+00	.325997E+01
GQNX1	9.	61.	144.	.369500E+01	.301499E+01	.517663E+00	.346899E+01
GQNX2	7.	64.	137.	.370900E+01	.309141E+01	.418737E+00	.342835E+01
GQNY	10.	67.	149.	.372200E+01	.311984E+01	.579252E+00	.345516E+01

TABLE C33: Test Results, Model 16, Sample Size 90

TEST	ALPHA LEVEL .01	.05	.10	MEAN	VARIANCE	SKEWNESS	KURTOSIS
POSEXMY	9.	42.	95.	.980346E+00	.719059E+00	.191397E+01	.998713E+01
POSEXMP	6.	41.	91.	.995690E+00	.699520E+00	.154281E+01	.616395E+01
POSEXH1PS	25.	67.	113.	.107710E+01	.745908E+00	.258159E+01	.140697E+02
POSEXH1YS	26.	63.	123.	.108737E+01	.773726E+00	.262233E+01	.142858E+02
POSEXH1YM	28.	79.	138.	.114030E+01	.848575E+00	.269973E+01	.154548E+02
POSEXH1PM	29.	76.	132.	.113199E+01	.827484E+00	.262262E+01	.145303E+02
BAMSETTN	11.	53.	105.	.199542E+01	.400131E+01	.197287E+01	.859271E+01
BAMSETTX1	4.	52.	99.	.198022E+01	.376314E+01	.166237E+01	.636130E+01
BAMSETTX2	10.	52.	98.	.197796E+01	.383341E+01	.180753E+01	.695187E+01
BAMSETTY	14.	61.	114.	.210698E+01	.432890E+01	.178822E+01	.690899E+01
BAMSEOTN	7.	51.	117.	.200319E+01	.391039E+01	.170560E+01	.674530E+01
BAMSEOTX1	8.	49.	106.	.204565E+01	.399659E+01	.187235E+01	.798043E+01
BAMSEOTX2	12.	49.	98.	.202422E+01	.419021E+01	.209093E+01	.934777E+01
BAMSEOTY	14.	61.	127.	.217184E+01	.468949E+01	.204465E+01	.955115E+01
GLEJSERX1	10.	47.	105.	.100776E+01	.103172E+01	.185305E+01	.739392E+01
GLEJSERX2	8.	31.	72.	.992760E+00	.846340E+00	.206475E+01	.181725E+02
GLEJSERY	10.	49.	104.	.107219E+01	.108339E+01	.186117E+01	.831640E+01
PARKX1	13.	52.	100.	.104787E+01	.248983E+01	.336522E+01	.214759E+02
PARKX2	8.	56.	102.	.106136E+01	.219752E+01	.249618E+01	.107855E+02
PARKY	17.	63.	109.	.109923E+01	.259165E+01	.274257E+01	.129623E+02
GQPN	26.	99.	201.	.137103E+01	.121666E+00	.215869E+01	.101272E+02
GQPX1	11.	52.	103.	.106052E+01	.162617E+00	.127473E+01	.658997E+01
GQPX2	15.	65.	104.	.106287E+01	.180922E+00	.136481E+01	.620987E+01
GQPY	67.	207.	321.	.141264E+01	.294090E+00	.104085E+01	.448942E+01
GQNN	18.	43.	98.	.413100E+01	.350134E+01	.348673E+00	.299162E+01
GQNX1	12.	32.	97.	.407900E+01	.325001E+01	.312484E+00	.293266E+01
GQNX2	13.	35.	86.	.401000E+01	.326517E+01	.409581E+00	.308784E+01
GQNY	10.	39.	87.	.399800E+01	.336536E+01	.280204E+00	.279608E+01

TABLE C34: Test Results, Model 17, Sample Size 30

TEST	ALPHA LEVEL .01	.05	.10	MEAN	VARIANCE	SKEWNESS	KURTOSIS
POSEXMY	134.	238.	323.	.243880E+01	.986076E+01	.331777E+01	.188124E+02
POSEXMP	150.	267.	348.	.266537E+01	.140767E+02	.463664E+01	.369264E+02
POSEXH1PS	306.	450.	572.	.495437E+01	.897518E+02	.107274E+02	.191489E+03
POSEXH1YS	303.	464.	583.	.568391E+01	.188067E+03	.113546E+02	.189516E+03
POSEXH1YM	342.	490.	599.	.899080E+01	.588471E+03	.908662E+01	.120409E+03
POSEXH1PM	339.	486.	587.	.695646E+01	.173098E+03	.640237E+01	.736286E+02
BAMSETTN	494.	722.	812.	.966924E+01	.300096E+02	.783630E+00	.388507E+01
BAMSETTX1	868.	967.	988.	.170875E+02	.465420E+02	.387557E+00	.293713E+01
BAMSETTX2	81.	207.	302.	.376556E+01	.137288E+02	.187330E+01	.783036E+01
BAMSETTY	281.	482.	597.	.689779E+01	.290999E+02	.118850E+01	.475836E+01
BAMSEOTN	664.	860.	911.	.124141E+02	.377359E+02	.634316E+00	.345816E+01
BAMSEOTX1	955.	993.	999.	.215965E+02	.691553E+02	.734640E+00	.426830E+01
BAMSEOTX2	99.	226.	321.	.400740E+01	.150639E+02	.196759E+01	.915925E+01
BAMSEOTY	377.	580.	681.	.820210E+01	.348023E+02	.908531E+00	.368071E+01
GLEJSERX1	528.	825.	934.	.656620E+01	.154463E+02	.196636E+01	.876565E+01
GLEJSERX2	0.	2.	16.	.498167E+00	.291838E+00	.236983E+01	.181210E+02
GLEJSERY	88.	218.	334.	.246489E+01	.553503E+01	.242339E+01	.186964E+02
PARKX1	340.	576.	715.	.715422E+01	.467339E+02	.229103E+01	.104437E+02
PARKX2	0.	12.	33.	.709535E+00	.851132E+00	.252539E+01	.116332E+02
PARKY	14.	89.	192.	.174205E+01	.289435E+01	.187688E+01	.842293E+01
GQPN	851.	931.	957.	.238891E+02	.161851E+04	.641666E+01	.587965E+02
GQPX1	54.	131.	187.	.152588E+01	.454959E+01	.587033E+01	.611304E+02
GQPX2	998.	1000.	1000.	.105373E+03	.201670E+05	.503454E+01	.375858E+02
GQPY	682.	835.	887.	.116489E+02	.269157E+03	.546238E+01	.416680E+02
GQNN	428.	680.	884.	.626400E+01	.235866E+01	.351334E+00	.344983E+01
GQNX1	684.	881.	967.	.737700E+01	.285773E+01	.335376E+00	.326502E+01
GQNX2	13.	36.	122.	.267200E+01	.219661E+01	.553630E+00	.318328E+01
GQNY	93.	225.	501.	.453900E+01	.218466E+01	.352572E+00	.316028E+01

TABLE C35: Test Results, Model 17, Sample Size 60

TEST	ALPHA LEVEL .01	.05	.10	MEAN	VARIANCE	SKEWNESS	KURTOSIS
POSEXMY	110.	215.	302.	.195845E+01	.412433E+01	.268692E+01	.152623E+02
POSEXMP	112.	218.	308.	.198651E+01	.432616E+01	.262279E+01	.139548E+02
POSEXH1PS	811.	940.	973.	.762325E+01	.475000E+02	.355011E+01	.206220E+02
POSEXH1YS	816.	938.	974.	.766811E+01	.500618E+02	.362743E+01	.210098E+02
POSEXH1YM	811.	941.	967.	.895921E+01	.116556E+03	.495099E+01	.394894E+02
POSEXH1PM	809.	937.	966.	.886164E+01	.112896E+03	.519342E+01	.447315E+02
BAMSETTN	949.	986.	995.	.227110E+02	.895565E+02	.541213E+00	.304533E+01
BAMSETTX1	1000.	1000.	1000.	.434848E+02	.153789E+03	.282669E+00	.310506E+01
BAMSETTX2	151.	290.	387.	.476176E+01	.210668E+02	.181521E+01	.764837E+01
BAMSETTY	995.	1000.	1000.	.330189E+02	.140171E+03	.483559E+00	.315910E+01
BAMSEOTN	963.	990.	996.	.246233E+02	.101982E+03	.569004E+00	.314088E+01
BAMSEOTX1	1000.	1000.	1000.	.493150E+02	.180822E+03	.248526E+00	.303024E+01
BAMSEOTX2	167.	302.	405.	.496574E+01	.226752E+02	.179506E+01	.751504E+01
BAMSEOTY	999.	1000.	1000.	.384135E+02	.150049E+03	.510028E+00	.333247E+01
GLEJSERX1	999.	1000.	1000.	.199488E+02	.954747E+02	.176415E+01	.779809E+01
GLEJSERX2	36.	89.	132.	.120042E+01	.251879E+01	.296131E+01	.146247E+02
GLEJSERY	969.	996.	999.	.129014E+02	.315809E+02	.112875E+01	.556632E+01
PARKX1	903.	968.	979.	.270623E+02	.434001E+03	.186698E+01	.822643E+01
PARKX2	2.	29.	71.	.866615E+00	.140391E+01	.248095E+01	.110525E+02
PARKY	886.	962.	983.	.193193E+02	.132968E+03	.100485E+01	.400155E+01
GQPN	993.	999.	1000.	.164969E+02	.173430E+03	.463945E+01	.510545E+02
GQPX1	48.	103.	151.	.107321E+01	.852903E+00	.284412E+01	.157016E+02
GQPX2	1000.	1000.	1000.	.519362E+02	.145455E+04	.261302E+01	.145560E+02
GQPY	996.	998.	999.	.183750E+02	.213989E+03	.303259E+01	.199515E+02
GQNN	527.	873.	957.	.873100E+01	.392456E+01	.202964E+00	.273337E+01
GQNX1	758.	962.	990.	.103110E+02	.585213E+01	.306610E+00	.280776E+01
GQNX2	1.	17.	56.	.271700E+01	.248339E+01	.571913E+00	.322825E+01
GQNY	443.	804.	928.	.826800E+01	.387805E+01	.220454E+00	.276275E+01

TABLE C36: Test Results; Model 17, Sample Size 90

TEST	ALPHA LEVEL .01	.05	.10	MEAN	VARIANCE	SKEWNESS	KURTOSIS
POSEXMY	110.	197.	200.	.177751E+01	.373513E+01	.247148E+01	.116059E+02
POSEXMP	63.	140.	219.	.147636E+01	.231868E+01	.242483E+01	.111497E+02
POSEXH1PS	980.	992.	998.	.101731E+02	.289325E+02	.155549E+01	.656750E+01
POSEXH1YS	970.	990.	996.	.958299E+01	.228143E+02	.135590E+01	.583536E+01
POSEXH1YM	964.	989.	995.	.874772E+01	.193380E+02	.141314E+01	.591261E+01
POSEXH1PM	974.	990.	997.	.933298E+01	.247667E+02	.160447E+01	.662268E+01
BAMSETTN	956.	989.	993.	.277591E+02	.146783E+03	.630313E+00	.376101E+01
BAMSETTX1	1000.	1000.	1000.	.757499E+02	.293265E+03	.333391E+00	.289404E+01
BAMSETTX2	218.	387.	488.	.627319E+01	.383419E+02	.221919E+01	.120483E+02
BAMSETTY	1000.	1000.	1000.	.564423E+02	.278728E+03	.312102E+00	.296262E+01
BAMSEOTN	988.	997.	1000.	.353708E+02	.180525E+03	.495715E+00	.332502E+01
BAMSEOTX1	1000.	1000.	1000.	.807518E+02	.349098E+03	.247086E+00	.269198E+01
BAMSEOTX2	219.	397.	496.	.640344E+01	.398215E+02	.221612E+01	.120041E+02
BAMSEOTY	1000.	1000.	1000.	.602061E+02	.305561E+03	.274735E+00	.281433E+01
GLEJSERX1	1000.	1000.	1000.	.350716E+02	.268477E+03	.189879E+01	.858832E+01
GLEJSERX2	13.	53.	78.	.918165E+00	.115117E+01	.278975E+01	.142698E+02
GLEJSERY	1000.	1000.	1000.	.189096E+02	.361915E+02	.714250E+00	.396733E+01
PARKX1	950.	982.	989.	.421043E+02	.909286E+03	.124580E+01	.470157E+01
PARKX2	2.	21.	57.	.761430E+00	.114571E+01	.286937E+01	.162018E+02
PARKY	967.	991.	994.	.313589E+02	.266471E+03	.658376E+00	.313640E+01
GQPN	994.	998.	998.	.117740E+02	.653419E+02	.299625E+01	.206257E+02
GQPX1	42.	98.	130.	.914119E+00	.387510E+00	.168573E+01	.655584E+01
GQPX2	1000.	1000.	1000.	.539988E+02	.826900E+03	.150706E+01	.599420E+01
GQPY	998.	1000.	1000.	.229996E+02	.208877E+03	.194158E+01	.904522E+01
GQNN	709.	847.	941.	.980306E+01	.489890E+01	.315396E+00	.310366E+01
GQNX1	944.	982.	998.	.126910E+02	.793145E+01	.339218E+00	.288384E+01
GQNX2	.	9.	31.	.331200E+01	.243106E+01	.522623E+00	.297420E+01
GQNY	788.	897.	959.	.104240E+02	.565988E+01	.261945E+00	.307374E+01

BIBLIOGRAPHY

BIBLIOGRAPHY

Abrahamse, A.P.J. and Louter, A.A. [1971]. One new test for autocorrelation in least squares regression. Biometrika. 58:53-60.

Anscombe, F.J. and Tukey, J.W. [1963]. The examination and analysis of residuals. Technometrics. 5:141-160.

Beckman, R.J. and Trussel, H.J. [1974]. The distribution of an arbitrary studentized residual and the effects of updating in multiple regression. Journal of the American Statistical Association. 69:199-201

Bement, T.R. and Williams, J.S. [1969]. Variance of weighted regression estimation when sampling errors are independent and heteroskedastic. Journal of the American Statistical Association. 64:1369-1382.

Berenblut, I.I. and Webb, G.I. [1973]. A new test for autocorrelated errors in the linear regression model. Journal of the Royal Statistical Society, Series B. 35:33-51.

Chew, V. [1970]. Covariance matrix estimation in linear models. Journal of the American Statistical Association. 65:173-181.

Cochran, W.G. [1934]. The distribution of quadratic forms. Proceedings of the Cambridge Philosophical Society. 30:178-191.

Donaldson, T.S. [1968]. Robustness of the F-test to errors of both kinds and the correlation between the numerator and denominator of the F ratio. Journal of the American Statistical Association. 63:660-676.

Frame, J.S. [1972]. Matrix theory and linear algebra with applications. East Lansing, Michigan. (mimeographed lecture notes).

Glejser, H. [1969]. A new test for heteroskedasticity. Journal of the American Statistical Association. 64:316-323.

Goldberger, A.S. [1964]. Econometric theory. New York: John Wiley and Sons.

Goldfeld, S.M. and Quant, R.E. [1965]. Some tests for homoskedasticity. Journal of the American Statistical Association. 60:539-547.

Goldfeld, S.M. and Quant, R.E. [1967]. Corrigenda. Journal of the American Statistical Association. 63:1518-1519.

Goldfeld, S.M. and Quant, R.E. [1972]. Nonlinear methods in econometrics. Amsterdam: North-Holland.

Graybill, F.A. and Marsaglia, G. [1957]. Idempotent matrices and quadratic forms in the general linear hypothesis. Annals of Mathematical Statistics. 28:678-686.

Grossman, S.I. and Stayan, G.P.H. [1972]. Optimality properties of Theil's BLUS residuals. Journal of the American Statistical Association. 67:672-673.

Hartley, H.O. and Jaytellake, K.S.E. [1973]. Estimation for linear models with unequal variances. Journal of the American Statistical Association. 68:189-192.

Harvey, A.C. and Phillips, G.D.A. [1973]. A comparison of the power of some tests for heteroskedasticity in the general linear model. University of Kent Discussion Paper Number 3. Kent, England.

Hedayat, A. and Robson, D.S. [1970]. Independent stepwise residuals for testing homoskedasticity. Journal of the American Statistical Association. 65:1573-1581.

Hogg, R.V. and Craig, A.T. [1958]. On the decomposition of certain χ^2 variables. Annals of Mathematical Statistics. 29:608-610.

Huang, C.J. and Bolch, B.W. [1973]. On the testing of regression disturbances for normality. Unpublished workshop paper.

Imhof, J.P. [1961]. Computing the distribution of quadratic forms in normal variables. Biometrica. 48:419-426.

Kendall, M.G. and Stuart, A. [1958]. The advanced theory of statistics, vol. I; Distribution. London: C. Griffin.

Kendall, M.G. and Stuart, A. [1961]. The advanced theory of statistics, Vol. II: Inference and relationship. London: C. Griffin.

Kmenta, J. [1971]. Elements of econometrics. New York: MacMillan.

Koerts, J. [1967]. Some further notes of disturbances estimates in regression analysis. Journal of the American Statistical Association. 62:169-183.

Koerts, J. and Abrahamse, A.P.J. [1968]. On the power of the BLUS procedure. Journal of the American Statistical Association. 63:1227-1236.

Kramer, G. [1969]. On the power of several tests for serial correlation in the classical linear regression model. Proceeding of the European Meetings of the Econometric Society. Brussels, September, 1969.

Lancaster, T. [1968]. Grouping estimators on heteroskedastic data. Journal of the American Statistical Association. 63:182-191.

Layard, M.W.J. [1973]. Robust large-sample tests for homogeneity of variances. Journal of the American Statistical Association. 68:195-198.

Levenbach, H. [1973]. The estimation of heteroskedasticity from a marginal likelihood function. Journal of the American Statistical Association. 68:436-469.

Maddala, G.S. [1971]. Generalized least squares with an estimated variance covariance matrix. Econometrica. 39:23-33.

Mallela, P. [1972]. Necessary and sufficient conditions for MINQU estimation of heteroskedastic variances in linear models. Journal of the American Statistical Association. 67:486-487.

Meyer, H.A., ed. [1956]. Symposium on monte carlo methods. New York: John Wiley and Sons.

Neudecker, H. [1969]. A note on BLUS estimation. Journal of the American Statistical Association. 64:949-952.

Park, R.E. [1966]. Estimation with heteroskedastic error terms. Econometrica. 34:888.

Pearson, E.S. and Hartley, H.O. [1955]. Biometrica tables for statisticians, Vol. I. Cambridge: Cambridge University Press.

Prais, S.J. and Houthakker, H.S. [1955]. The analysis of family budgets. Cambridge: Cambridge University Press.

Putter, J. [1967]. Orthonormal bases of error spaces and their use for investigating the normality and variances of residuals. Journal of the American Statistical Association. 62:1022-1036.

Ramsey, J.B. [1968]. Tests for specification errors in classical linear least squares regression analysis. Unpublished Ph.D. dissertation, University of Wisconsin.

Ramsey, J.B. [1969]. Tests for specification errors in classical least squares regression analysis. Journal of the Royal Statistical Society, Series B. 31:350-371.

Ramsey, J.B. [1973]. Graduate econometric theory. Birmingham, England. (mimeographed lecture notes).

Ramsey, J.B. and Gilbert, R.F. [1969]. A monte carlo study of some small sample properties of tests for specification error. Econometrics Workshop Paper Number 6813, Michigan State University. East Lansing, Michigan.

Ramsey, J.B. and Gilbert, R.F. [1972]. A monte carlo study of some small sample properties of tests for specification error. Journal of the American Statistical Association. 67:180-186.

Ramsey, J.B. and Schmidt, P. [1974]. Some further exact results on the use of OLS and BLUS residuals in specification error tests. East Lansing, Michigan. (mimeographed).

RAND Corporation. [1950]. A million random digits and one hundred thousand deviates. Santa Monica: Rand Corporation.

Rao, C.R. [1965]. Linear statistical inference and its applications. New York: John Wiley and Sons.

Rao, C.R. [1970]. Estimation of heteroskedastic variances in linear models. Journal of the American Statistical Association. 65:161-172.

Rao, J.N.K. and Subrahmaniam, K. [1971]. Combining independent estimators and estimation in linear regression with unequal variance. Biometrics. 27:971-990.

Rutemuller, H.C. and Bowers, D.A. [1968]. Estimation in a heteroskedastic regression model. Journal of the American Statistical Association. 63:552-557.

Shapiro, S.S., M.B. Wilk, and H.J. Chen. [1968]. A comparison study of various tests for normality. Journal of the American Statistical Association. 63:1343-1373.

Srivastava, A.B.L. [1958]. Effect of non-normality on the power function of T-test. Biometrika. 45:421-429.

Sutcliffe, C.M.S. [1972]. Heteroskedasticity its diagnosis and treatment. Unilever Ltd.

Theil, H. [1951]. Estimates and their sampling variance of parameters of certain heteroskedastic disturbances. Review of the International Statistical Institute. 19:141-147.

Theil, H. [1965]. The analysis of disturbances in regression analysis. Journal of the American Statistical Association. 60:1067-1079.

Theil, H. [1968]. A simplification of the BLUS procedure for analyzing regression disturbances. Journal of the American Statistical Association. 63:242-251.

Thomas, G.B. [1962]. Calculus and analytic geometry. Reading: Addison Wesley.