A STUDY OF THE EFFECT OF MATHEMATICS ACTIVITY MATERIALS UPON CERTAIN ASPECTS OF CREATIVE THINKING ABILITY OF PROSPECTIVE ELEMENTARY SCHOOL TEACHERS

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WALDECK ERNEST MAINVILLE, JR.

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Calhoun C. Collier
Major professor

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ABSTRACT

A STUDY OF THE EFFECT OF MATHEMATICS ACTIVITY MATERIALS UPON CERTAIN ASPECTS OF CREATIVE THINKING ABILITY OF PROSPECTIVE ELEMENTARY SCHOOL TEACHERS

By

Waldeck Ernest Mainville, Jr.

Chairman: Dr. Calhoun C. Collier

The recent trend in elementary school mathematics towards an active learning technique has prompted new mathematics programs for prospective elementary school teachers.

The present direction of these programs, while not uniform, is towards an integrated sequence which relates the development of mathematical concepts, skills, and problem solving techniques and the methodological aspects of these areas with an increased emphasis on activity materials.

This study investigated one aspect of these emerging programs: the use of mathematics activity materials.

See following [illegible]

activity sheets were used; many supplemented with concrete materials.

Both groups were responsible for the same material and received the same homework assignments, course guides, quizzes and final examination. Both groups were taught by the researcher.

Three tests were constructed by the researcher for the study. Forms A and B of the Mathematical Creativity Test provided pre- and posttest measures of mathematical creativity. Each form contained five divergent thinking items. Students were permitted seven minutes on each item. The tests were scored for fluency and originality. A 30-item final examination provided a measure of mathematical achievement.

Analysis of covariance was used to determine whether differences in average measures of mathematical creativity occurred from pre- to posttest between the two groups. Fisher's t tests were used to determine: whether differences in average measures of mathematical achievement occurred between the two groups; and, whether differences in average measures of mathematical creativity occurred between the pre- and posttest for each group. A Pearson product-moment correlation coefficient was computed between the posttest in mathematical creativity and the final examination.

Additional data for the study originated from observational notes, student information sheets, and an instructional rating system form.

Waldeck Ernest Mainville, Jr.

Purpose

The purpose of the study was to compare the mathematical creativity of two classes of prospective elementary teachers in relation to a preservice content course in mathematics, one section of which was exposed to mathematics activity materials related to the topics in the course. A second purpose of the study was to compare the mathematical achievement of these two classes at the end of the course.

Procedure

Thirty students enrolled in Ms 100, Elements of Mathematics I, at the Portland Campus of the University of Maine for the fall semester, 1971, comprised the experimental group for the study. Twenty-four students enrolled in another section of the course were the control group.

The control group received a lecture-textbook presentation. The instructional technique was informal; open-ended questions were used, student responses were elicited, and methodological aspects and historical anecdotes were used.

The same informal instructional technique was used with the experimental group. In addition, mathematics activity materials were used. The instructor's lectures averaged less than one-half of each session. Within this time activity materials were introduced, either through teacher-class activities or, in conjunction with prepared overhead transparencies. The remaining time was devoted to individual and small group activities. Thirty-three

Findings and Conclusions

No significant differences were found between the average measure of mathematical creativity or the average measure of mathematical achievement for either group. The use of mathematics activity materials did not appear to increase the mathematical creativity of the experimental group nor did it appear to have a detrimental effect on the mathematical achievement of this group.

Significant differences for both groups were found between the pre- and posttest in mathematical creativity; the posttest means were higher. A preservice content course taught in an informal style appeared to have increased the mathematical creativity of both groups.

The correlation between measures of mathematical achievement and measures of mathematical creativity was not significant. These instruments were apparently measuring different aspects of mathematical ability. It may be that mathematical achievement tests discriminate against students highly creative in mathematics.

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To

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Celine, Robbie, and Lisa

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CHAPTER I

INTRODUCTION

> Communication is of the essence in mathematics, and prospective teachers must pay special attention to all of the ways in which mathematics is most effectively communicated. They should be led to regard mathematics as a creative activity--something which one <u>does</u> rather than merely something which one learns. The active participation of the student in the process of discovering and communicating mathematical ideas is crucial for his real understanding. Courses should be taught in ways that foster active student involvement in the development and presentation of mathematical ideas.
>
> (MAA, 1971, p. 20)

Need for the Study

The center stage in mathematics education, particularly at the elementary level, is now dominated by what might be called an "active" philosophy. This philosophy stresses the child's experiences in the real world as the basis for understanding. It places emphasis on the use of activity materials, encourages a multi-approach to concept learning, develops pattern searching, and uses lessons which integrate mathematics with other disciplines. Biggs and MacLean (1969) provide these remarks in the forward of their book *Freedom to Learn*:

The phrases active learning, discovery methods, and laboratory approach have become part of our educational jargon for the past few years. What do these phrases mean?

For children, these phrases mean an approach to learning that presents a wide variety of opportunities, an approach that encourages them to ask questions and find the answers, an approach that fosters the use of physical materials, an approach that gives experience designed to help them analyze and abstract, and an approach that provides a chance to develop their individual potential.

For teachers, these phrases mean an opportunity to explore and discover new and better ways of teaching mathematics, an opportunity to develop an awareness of mathematical possibilities in the environment, and an opportunity to use a highly motivated approach for more efficient education.

This active learning technique is not a recent idea in mathematics. Such an approach has been suggested at various periods in this century by Moore, Perry, McLellan and Dewey (Fitzgerald, 1970, pp. 6-8) and was evident during the progressive education movement. What is new is both the current emphasis on the child in relation to the newer curriculums and the increased use of activity materials.

Active learning concerns itself more with the processes learned by the child than the specific subject matter products he may acquire (Shulman, 1970, p. 34). Accordingly, a basic tenet of active learning is the provisition of "meaningful" experiences to children which will foster creative mathematical activities, often of their own choosing. Such an approach requires two considerations: (a) knowledge of the child's developmental level and (b) rich materials, tools, and teaching aids to stimulate and facilitate those creative mathematical activities.

A teacher who has had no personal experiences in some sort of creative mathematical work, however, can scarcely expect to be able to inspire, to lead, to help, to stimulate, or even to recognize the creative ability of his students (Polya, 1962, p. 209). If children are to engage in creative mathematical activities, then teachers must feel at ease with the techniques as well as the activity materials. This suggests new teacher education programs to give teachers plenty of opportunity for original creative work in mathematics so that they can know from their own experiences that original creative work is possible (Cockran, Barson & Davis, 1970, p. 215).

The present direction of these teacher education programs, while not uniform, is towards an integrated sequence which relates the development of mathematical concepts, skills, and problem solving techniques and the methodological aspects of these areas with an increased emphasis on activity materials. Fitzgerald (1970, p. 26) mentions five institutions, including Michigan State University, employing aspects of this approach. Clarkson (1970), Kipps (1970), Neatrour (1971), Spitzer (1969), and Springer (1968) describe five additional programs.

While these teacher education programs reflect a growing trend, related research on their effectiveness is scarce. One reason is because of the newness of these programs. Another reason lies with the current time lag between ongoing research and publication of that research for general consumption. Still another reason lies in the omission of pure research by the eminent leaders of this approach, such as Biggs, Davis, and Dienes. At any result, recent articles on active learning, the laboratory approach, elementary school mathematics, or teacher preparation which have included reviews of research (Fey, 1969; Fitzgerald, 1970; Kieren, 1969, 1971; Riedesel, 1970; Vance & Kieren, 1971) cite few studies related to active learning and the use of mathematics activity materials with pre- or inservice teachers.

Statement of the Problem

Does the use of mathematics activity materials in a preservice content course for elementary teachers increase these prospective teachers' mathematical creativity?

Purpose of the Study

The purpose of the study was to compare the mathematical creativity of two classes of prospective elementary teachers in relation to a preservice content course (Elements of Mathematics I), one section of which was exposed to mathematics activity materials germane to the concepts and skills contained in the course. A second purpose of the study was to compare the mathematical achievement of these two classes at the conclusion of the course.

Hypotheses

For the purpose of the study two hypotheses will be tested.

Major hypothesis: No difference exists in the average measure of mathematical creativity between the experimental group and the control group.

Minor hypothesis: No difference exists in the average measure of mathematical achievement between the experimental group and the control group.

Background of the Study

The reasons for the renewed interest in active learning and the increased use of mathematics activity materials in the elementary schools are both complex and varied. A brief overview of the history and trends in mathematics education during the past two decades will aid in understanding some of these reasons.

The formulation and practical implementation of the experimental programs in school mathematics during this period has had a significant effect upon the mathematics curriculum in the American schools. Much has been written regarding the factors underlying the implementation of these experimental programs and the ensuing reform movement (Osborne & Crosswhite, 1970; Weaver, 1970). Wooton (1965) provided this historical perspective.

> In the eyes of many thoughtful members of the mathematical community, the picture of mathematics education in American high schools in 1950 was not a pretty one. In particular, they were dissatisfied both with the content of the course offerings and with the spirit in which the material was presented. They were convinced that the traditional subject matter was inappropriate to the times. Worse, they were alarmed at what they felt were the implications for the future. In their opinion there was

> undue emphasis being placed on skills, an unnecessary preoccupation with the immediate usefulness of what was taught, and an unfortunate distortion of the students' ideas as to the nature of mathematics. They believed that these things were actually dangerous to the future welfare of the country (p. 5).

In addition, the vast amount of mathematics that had been created within the past century and the increasing need in our society for mathematical competence due to scientific and technological advances certainly contributed. These latter causes dictated, to a large extent, the initial direction undertaken by these experimental projects; a direction from the polar position of mathematics in terms of social utility, advocated during the first half of this century, to a position of sound mathematics, characterized by increased use of axiom systems, rigorous proofs, precise terminology, set theory, abstractions, structure, and the separation of mathematics from physical experiences and scientific applications.

While many of these programs have since been regarded as research efforts, such research initially extended to the creation and evaluation of sound mathematics curriculums in the secondary schools. It was assumed that secondary teachers, with a stronger subject matter preparation in mathematics, could be retrained easier than elementary teachers. This is not to say that effective methods of teaching and developments in learning theory and educational psychology were totally ignored, for a great many of the experimental programs enlisted the resources of pure mathematicians, educators and educational psychologists. However, the initial commitment was directed towards the content of the various programs (DeVault & Weaver, 1970, p. 143).

When mathematics educators turned to the elementary schools, their major goal was also the improvement of content (Houston, 1967, p. 5). However, the emphasis which had been given to abstract manipulations and relationships and the theoretical approach employed in the secondary schools have not proved as successful with elementary children (Scott, 1966, p. 20). Reflecting on this period in the curriculum reform, Beberman (1971) wrote:

> During the 1960s, I became increasingly disturbed by what I saw happening in elementary school mathematics . . . . Children and teachers were struggling with a new vocabulary . . . . and were being drilled in the manipulation of trivial abstractions . . . . The results hardly seemed worth the effort. Children certainly did no better, and perhaps they did worse, on tests of computation than they would have under the traditional programme . . . . What was even worse, children did not seem able to apply mathematics in new science programmes in the elementary and junior high school (p. 26).

Lloyd Scott (1966), in a penetrating analysis of the curriculum changes in the elementary schools, suggested one reason for this trend.

> The new written materials principally have included changes in content. They have not included a modernization of the predagogy to any extent, and they have not dealt with the various instructional materials which a teacher utilizes in her direct appeal to children's senses. In other words, the program has been modernized, but with some few exceptions, the instruction has not (p. 139).

The emphasis on mathematics and its structures at the expense of other goals of instruction in the experimental programs generated reactions, comments and criticisms from other mathematicians, educators and committees.

In 1962 a group of mathematicians (On the Mathematics Curriculum) published a lengthly list of objectives for newer programs, several of which related to pedagogical issues. Since then, three reports of the Cambridge Conferences (1963, 1967, 1969) have offered further suggestions and criticisms.

The most outspoken critic, however, was Morris Kline. Kline stood "as the spokesman for hosts of doubters--superintendents, parents, and others (DeMott, 1962, p. 298)." Kline (1961) criticized many aspects of the newer programs in mathematics. In his opinion, the problem of mathematics education was not an outmoded curriculum but the poor presentation of the materials. There was, he argued, little motivation, little intuitive development before generalization, no inclusion of applications, and little participation on the part of the student in creating the material he was to learn. Kline (1970) viewed mathematics as primarily a creative activity calling for ". . . imagination, geometric intuition, experimentation, judicious guessing, trial and error, the use of analogies of the vaguest sort, blundering and fumbling (p. 271)."

The emerging work of learning theorists focused on another weakness in the elementary mathematics programs, a lack of emphasis given to the intellectual characteristics of children. Lovell (1971) noted:

> In the 1950s changes began to take place in the teaching of mathematics. From that time children were increasingly expected to look at familiar mathematical ideas in new ways and learn about new ideas. Unfortunately those who were responsible for suggesting these changes often failed to realize, sufficiently, that the development of children's thinking must also be considered at the same time. In other words, they overlooked the fact that there must be some kind of match between the quality of the thinking skills of the child and the complexity of the mathematical ideas to which he is introduced (p. 1).

Travers (1969), in discussing the current emphasis on active learning, also noted this shortcoming. He identified a major reason for the trend toward active learning; the influence of Jean Piaget.

> Another observation to be made about the mathematics laboratory movement concerns the influence of Piaget. This famous psychologist's emphasis on studying the child's patterns of thought and the development of mental abilities as the child grows, has given rise to attempts to devise learning experiences in mathematics (such as the use of physical models) which will best account for the child's patterns of thought at his particular developmental level. But the curriculum reform movement on this side of the Atlantic seems to have gone in quite the opposite direction--looking first at the mathematics that is to be taught, and then devising learning experiences that are dictated by the subject matter at hand with little regard for the learning patterns of the child (p. 524).

Piaget's theory of intellectual development has captured the interest of modern psychologists and educators for several reasons: (a) He has introduced a score of new and interesting problems which previously have gone unnoticed; (b) He has reoriented current conceptions of the child's development with novel, imaginative and comprehensive ideas; (c) Finally, his theory, of all such theories of intellectual development, seems most securely founded upon the child (Ginsburg & Opper, 1969, pp. ix-x).

Although Piaget has been writing for several decades and has personally conducted experiments in the area of learning of mathematical concepts by children, it was not until the early sixties that his work was recognized by portions of the mathematics education community in this country (Rosskopf, Steffe & Taback, 1971, p. vii). They found, within his theory, several implications for the teaching of elementary school mathematics; implications which are being tried in several experimental programs.

Bärbel Inhelder, one of Piaget's closest collaborators, discussed the current research being conducted by Piaget and how it related to mathematics education in an interview with Suydam and Riedesel (1969). In response to a question concerning the number of such projects in Europe, Inhelder answered:

> There are in Europe a number of projects concerning the teaching of mathematics which are linked, to varying degrees, with Piagetian research on the development of number and with both our studies on that of geometric concepts.
>
> The most promising projects are, in my opinion, those where mathematicians and psychologists are closely collaborating in the preliminary research concerning the information and succession of mathematical structures in the child's thought and where the educationists then transpose these psychogenetic findings according to the requirements and possibilities of the schools.

> The fertility of Genevan developmental research as regards the teaching of mathematics is, I think, mainly due to the fact that our studies have been directed toward an epistemology of mathematics (p. 1).

The Nuffield Mathematics Project in England has used portions of Piaget's study of growth and development in determining the sequential order of their curriculum for children from five to thirteen. Within this child-centered program, as with many schools in England, a great emphasis has been placed on mathematics activity materials. This emphasis has also been attributed to Piaget, who believes that formulation of certain concepts proceeds by stages, and that one of these stages involves concrete manipulative experiences with physical materials (Davis, 1966, p. 357).

On this continent the Madison Project, under the direction of Robert Davis, and the Centre de Recherches en Psycho-Mathématique, under the direction of Zolton Dienes, have both taken into consideration Piaget's stages of learning. Each of these programs uses a child-centered approach emphasizing mathematics activity materials.

Importance of Mathematical Creativity

Identification and encouragement of individual creativity in mathematics is of great importance to mathematicians, mathematics educators, students and their teachers. One indication of this importance can be inferred from the proceedings of the first Cambridge Conference. In their report (Goals for School Mathematics, 1963) the participants suggested pedagogical principles and techniques for "Fostering Independent and Creative Thinking," including: (a) directed discovery with students working singly or in small groups, (b) aids and innovations, such as a mathematics laboratory or reference library of suitable extra projects, and (c) a restructuring of examinations to reflect the emphasis on understanding and creativity rather than responses which can be of a rote or mechanical nature (pp. 17-20).

Another indication may be found in the recent SMAC Newsletter (1971, p. 2) which listed eight areas in mathematics education where planning and preliminary work is underway for future SMAC reports. One of these areas is creativity in mathematics.

R. Davis (1966) listed four needs as "most urgent" in elementary school mathematics: (a) a greater use of physical materials, (b) a greater diversity of types of experiences for children, (c) the identification and early introduction of basic mathematical ideas, and (d) more emphasis on student originality and creativity within the school mathematics program.

Kidd, Myers and Cilley (1970), in citing goals that teachers should strive to attain, included creativity in mathematics.

> To many people, mathematics appears to be a rigid system consisting only of symbols and a set of rules for manipulating them. Actually, mathematics has a great deal of room for creativity. In fact, the learning of this subject is greatly facilitated when students are challenged to use their ingenuity to discover its many uses

> and properties. They should be encouraged to give illustrations, formulate hypotheses, make guesses, construct logical arguments, relate different mathematical ideas, play games of strategy, work puzzles and solve problems (pp. 5-6).

Significance of the Study

This study was primarily concerned with examining the performance of prospective elementary school teachers' mathematical creativity in relation to a preservice content course employing mathematics activity materials. Although the primary objective in introducing the materials was not to nurture creativity, the use of such materials has been suggested by several authorities for accomplishing that objective. Torrance (1964), for example, compiled a list of 20 suggestions for nurturing creativity in schools. Included in that list were suggestions "encourage manipulation of objects and ideas" and "encourage and evaluate self-initiated learning (pp. 92-93)." Hallman (1967) compiled a similar list of 12 ways for nurturing creativity, including the suggestion to "provide opportunities for students to manipulate materials, ideas, concepts, tools, and structures (p. 329)."

A recent publication by Parnes (1967) listed 27 programs or techniques which were designed to nurture creative behavior. Several of these programs included the use of activity materials, although none were specifically designed for a mathematics class.

The key to nurturing creativity, however, still lies with the teacher, as Reed (1957) noted in an article in The Arithmetic Teacher.

> The key to creative thinking is the teacher herself and her attitude towards arithmetic. Unless the teacher is a creative person in her approach to problems and in developing a stimulating environment for quantitative thinking there will be few opportunities for children to think creatively (pp. 11-12).

Recently, Laycock (1970), in describing a creative mathematics program at Nueva, California, observed that:

> The teacher is the key. He must not be afraid of this kind of mathematics . . . . All the materials and programs and gadgets cannot replace the teacher! Gene Watson's famous phrase is: "tool in the hands of a fool is nothing but a tool. No material is any better than the person who presents it (p. 328)."

As yet, however, creativity in mathematics classes is still low. Williams (1968), who has developed new teaching materials and techniques to inform teachers how to particular subject matter areas can be taught more creatively, reports that "activities for stimulating creative thinking are carried out in the language arts far more often than in arithmetic (p. 203)." Of the ideas suggested for varying grades in language arts, science, social studies, art-music, and arithmetic by elementary teachers, only about five per cent, the smallest number, came from arithmetic (p. 204).

In mathematics, the number of activity materials has increased significantly. This increase was noted a decade ago by Sudduth (1962) in her doctoral study; it is no understatement to say that this increase continues today.

And while a large portion of the research relating to these activity materials is still theoretical, increasingly studies are suggesting that, for certain children under certain situations, mathematics activity materials or an active learning program can increase both concept development and skills in mathematics and perhaps also increase areas of the affective domain, such as creativity.

However, the single most important variable in the success of many of these programs, as with programs designed to nurture creativity, may be the teacher. Brownell (1966), in a study to evaluate the effectiveness of materials in programs either new or familiar to teachers, found that the quality of teaching was a significant variable. Fitzgerald (1970) noted in his recent article on the laboratory approach that "There is considerable agreement that a teacher needs to have the experience of learning in a laboratory setting if she is going to be effective in directing a laboratory (p. 26)." Such a viewpoint has also been suggested by Dienes (1970), LeBlanc (1970) and Morley (1969). Dienes stated:

> If we wish teachers to be able to set up concrete problem situations that the children can manipulate, then they must also learn to set up such concrete situations for themselves and to manipulate them themselves (p. 265).

And Lola May (1971), in an article directed towards elementary teachers, said:

> You need the first-hand experiences, and this means you <u>must</u> work with the materials and learn the same way the students learn . . . .

> Teachers become creative after working with materials prepared by someone else. You learn to create your own materials, ones that will help your students. Only after real involvement will most teachers change their methods of teaching (p. 79).

Wilkinson (1970), however, found that laboratory methods of teaching sixth grade mathematics can be used by teachers without prior inservice or preservice training.

The elementary teacher, then, has been suggested as the key variable in both the nurturing of creativity in the classroom and the "successful" employment of mathematics activity materials in an active learning classroom. In addition, the use of these mathematics activity materials may be one way to nurture this creativity in mathematics.

Such suggestions and conclusions, however, need further answers from research. As Kieren (1971) noted in his recent article on "Manipulative Activity in Mathematics Learning":

> It is obvious from reading the articles or the advertisements in any recent mathematics teachers' journal on this continent or across the Atlantic that the use of manipulative activities in the teaching and learning of mathematics is in vogue . . . . Nonetheless it is an understatement to say that research is needed into the role and effects of manipulative activity in mathematics (p. 228).

Fehr (1970) had these comments at a recent Triple-T (Training of Teachers of Teachers) colloquium at the University of Illinois:

> Bluntly, little is still known about the many basic questions of mathematics education today, the training of teachers, the teaching-learning situation, the ideal type of mathematics (p. 17).

Selection of the Sample

The accessible population for the study consisted of all students enrolled in Ms 100 (Elements of Mathematics I) during the fall semester of 1971 at the University of Maine at Portland-Gorham.

Students who had enrolled in the two sections of MS 100 given on the Portland Campus comprised the sample subjects for the study. One of the sections, selected by a toss of a coin, constituted the experimental group while the remaining section was designated as the control group. The experimental group met from 11:00 to 11:50 a.m. and the control group met from 1:00 to 1:50 p.m.; each group met on Monday, Wednesday, and Friday, and were taught by the researcher.

Description of the Course

Ms 100 is a three-credit-hour course of special interest to prospective primary and elementary teachers with major emphasis on an intuitive approach to the real number system. The course is normally taught by the Department of Mathematics using a lecture-textbook technique. The course meets for approximately forty-five 50-minute sessions, not counting a two-hour final examination. The text for this course during the past two years, selected by the staff, has been the Second Edition of Modern Mathematics: An Elementary Approach (Wheeler, 1970). A general course outline was also drawn up by the staff.

The major topics studied in Ms 100 include logic, sets and relations, and the systems of whole numbers and integers. Additional topics include equations, inequalities, elementary number theory, and numeration systems.

Modification of the Ms 100 topics occurred with the deletion of the majority of the material on logic in favor of the inclusion of material on patterns and functions.

Ms 101 (Elements of Mathematics II), a continuation of Ms 100, emphasizes the rational and real number systems and introduces the students to concepts in geometry, probability, and statistics. Ms 101 was not involved in the study.

Class Meetings

Each class in the study met for forty-two 50-minute sessions over a period of 16 weeks. Two of these meetings were devoted to pre- and posttesting and five 35-minute quizzes were given.

The control group received a traditional lecture-textbook presentation with the researcher lecturing approximately 40 minutes each session; the remaining time was devoted to student questions and homework assignments.

The experimental group received a minimum number of lectures; overall it averaged less than 25 minutes per session. The remaining time was devoted to: (a) the use of mathematics activity materials which had been selected for their relevance to the concepts or skills under consideration, (b) student questions, and (c) the homework assignment. The instructional approach varied daily but included teacher-class activities (e.g., "guess my rule" and bingo games, pattern searching and generalization), individual and small group activity sheets (e.g., mini-computers, puzzles, Cuisenaire rods, chip trading, attribute games), and informal large and small group discussions. When individual or small group activities were in progress. the researcher circulated among the students to provide assistance or guidance. A more detailed description of the procedure is given in section two of Chapter III. A list of the materials and activity sheets plus some sample activity sheets appears in Appendix A.

Both groups received the same homework assignments, course guides, quizzes, and final examination. These items appear in Appendix B.

Instruments and Procedures

Statistical data for the major and minor hypotheses were obtained from three sets of scores. During the third meeting Form A of the Mathematical Creativity Test was administered as a pretest to both groups. Form B of the Mathematical Creativity Test was administered to both groups as a posttest during the final week of classes on the same day of the week as the pretest had been given. Both forms required a full 50-minute class to administer.

A two-hour final examination was administered to both groups at the same time and in the same room during examination week. All three tests were scored by the researcher.

The experimental design consisted of a comparison of pretest and posttest scores in mathematical creativity and a comparison of the mean final examination scores for experimental and control groups.

To test the major null hypothesis the dependent variable was the student's posttest in mathematical creativity minus his pretest through an analysis of covariance. This analysis controls for differences which may be present in the group prior to treatment while assessing differences between the groups following treatment.

To test the minor null hypothesis the dependent variable was the mean scores of both groups based on their final examination scores. The test statistic employed was a Fisher's t test for uncorrelated group means.

Significant differences between the experimental and control groups were conceded for measures which exceeded the .05 level of confidence. These statistical tests were run on the University of Maine's IBM 360 computer.

Information sheets were compiled on each student, observational notes were maintained throughout the study, and a student instructional rating system form was completed. Data from these sources were reported only when they related to or supported conclusions extracted from the statistical analysis.

Mathematical Creativity Test

The construction of both forms of the Mathematical Creativity Test by the researcher and used in the study was guided by the three known written tests in mathematical creativity (Buckeye, 1968, 1970a; Evans, 1964; Prouse, 1964), plus a review of the research on psychometric measurement of general creativity. The theoretical model guiding the construction of the tests was based partly on Guilford's (1959a) Structure of Intellect, which is reviewed in section two of Chapter II. A description of the construction of the test, the scoring procedure used, and relevant validity and reliability estimates are provided in section three of Chapter III. The tests appear in Appendix C.

Definitions for the Study

The following are definitions of terms which are used frequently in the thesis.

Creativity.--The ability to combine ideas, things, techniques or approaches in a new way. This ability must be thought of from the point of view of the person who is actually doing the creating (Romey, 1970, p. 4). When applied to teaching it includes the teacher who: (a) uses teaching aids to add meaning and interest to verbal instruction, (b) knows which aids will add to his lesson and has them at hand when he needs them, and (c) gives careful attention to discovering what works for him and uses it effectively (Johnson, 1967, p. 39).

Convergent thinking (Convergent production).--The class of abilities dealing with the production of correct responses which are generally closely determined by the information given.

Divergent thinking (Divergent production).--The class of abilities dealing with the production of a diversity of responses in situations where more than one response may be acceptable.

Fluency.--The ability to generate many responses to a problem or stimulus. The quantity of output is important even when the quality is disregarded, although the response to the problem or stimulus must be appropriate.

Flexibility.--The capacity to bring about change in meaning, interpretation or use; the ability to change a strategy, or a way of doing a task. It implies the ability to develop a new interpretation of a goal through understanding or a change in direction of thinking.

Originality.--The production of unusual, far-fetched, remote, or clever responses. It includes the ability to develop novel ideas, particularly those new to the individual concerned.

Measure of fluency.--The fluency score obtained by the subject of the Mathematical Creativity Test.

Measure of originality.--The originality score obtained by the subject on the Mathematical Creativity Test.

Measure of mathematical creativity.--The total score (fluency score plus originality score) obtained by the subject on the Mathematical Creativity Test.

Measure of mathematical achievement.--The total score obtained by the subject on the final examination.

Active learning.--The teaching technique discussed in Chapter I under "Need for the Study."

Mathematics activity materials.--The variety of models and manipulative materials, literature, tools and teaching aids which are frequently used in active learning situations. These materials vary in cost, purpose and appropriateness. Some of them lend themselves to group work in classes while others are intended for individual work. Some of them are purchased commercially, others are teacher or student made (see Appendix A).

Davidson (1968) has illustrated the diversity of some of these materials by using fifteen classifications, such as Blocks, Cards, Measuring Devices, Calculators/Computers, Models, Numerical Games and Puzzles. Other descriptions of both activity materials and their uses can be found in Davidson and Fair (1970), Fitzgerald (1968, 1970), Hillman (1968), Johnson (1967), Kalman (1970), Phillips (1967), and Rosskopf and Kaplan (1968).

Assumptions of the Study

For the purposes of the study the following assumptions have been applied:

a. That the mathematics activity materials were appropriate for the students at this level and were germane to the topics discussed in the course.

b. That the change in student performance revealed by the Mathematical Creativity Test was a result of treatment.

c. That the Mathematical Creativity Test, used to obtain a pre- and posttest measure of mathematical creativity, provided valid measures.

d. That the final examination was a valid measure of the student's achievement in mathematics.

e. That the variables not controlled in the study had a random effect on the results and did not produce erroneous conclusions.

f. That the setting and population in which the study was conducted was not so unusual that the outcomes within limitation could not be generalized to other similar populations.

Limitations of the Study

The study was designed and undertaken within the limits stated below:

a. Only students in two sections of Ms 100 (Elements of Mathematics I) at the University of Maine at Portland-Gorham during the fall semester of 1971 were used. Any generalization of the results is limited to populations similar to the experimental group.

b. The Mathematical Creativity Test sampled only a portion of the 24 types of divergent thinking operations. The remainder of the operations remained untested and, therefore, statements could not be made about them.

c. The limitations inherent in any statistical study.

Organization of the Thesis

The thesis is composed of five chapters: an introduction, a review of the related literature, the implementation of the study, an analysis of the data, and a summary and conclusions.

In the introductory chapter the background, need, and significance of the study is discussed, together with the definitions, assumptions and limitations. Two hypotheses are listed and a discussion of the experimental design is given.

The second chapter contains a review of the related literature in both activity learning and creativity. The chapter is divided into four sections. The first section contains a review of the accomplishments of the major research centers in creativity and a brief overview of general creativity. The second section contains a discussion of the five major areas of general creativity. In-depth reviews of studies in mathematical creativity appear in the third section. The final section contains a review of the literature on active learning relative to teacher training.

Chapter III, the implementation of the study, is divided into five sections. The first section contains a description of the sample and four between-group comparisons. The classroom environment for both groups is explained in section two. Test construction and instrumentation along with estimates of test validity and reliability appear in the third section. The fourth section contains a discussion of the experimental design. Two null and alternate hypotheses and the corresponding statistic to test each hypothesis is explained and analyzed in the final section.

In Chapter IV, an analysis and discussion of the data with respect to the two null hypotheses is provided.

The last chapter includes a summary of the thesis and a discussion of the conclusions and recommendations. The results are also discussed and implications for future research are considered.

CHAPTER II

REVIEW OF THE RELATED LITERATURE

Introduction

Research and theory related to the study has been examined from the literature on both creativity and active learning. Of these two areas, literature related to creativity has been the more plentiful. Parnes and Brunelle (1967) have reported that the number of titles relating to creativity were appearing in professional literature with increasing frequency. For example, from January, 1965, to June, 1966, the quantity of research published equaled that of the preceeding five years, which equaled that of the preceeding ten years, and that again equaled the quantity published during the one hundred year period between 1850 and 1950 (p. 52).

Since 1950 much of this research on creativity has been conducted at university research centers. The most influencial of these centers, with their major scholars in creativity, are the subject of the first section of this chapter, together with an historical overview of creativity.

Five areas of creativity have received considerable attention in the past two decades: (a) definitions and theories of creativity, (b) characteristics of creative persons, (c) conditions which influence creativity, (d) techniques for nurturing creativity, and (e) measuring creativity. Each of these areas will be reviewed in the second section of this chapter.

Despite the voluminous nature and research on creativity, little has been conducted in the area of mathematical creativity. Studies in mathematical creativity are reviewed in depth in the third section of this chapter.

The final section of this chapter reviews the limited but growing body of literature on active learning, including mathematics activity materials, in relation to pre- and inservice training of elementary teachers.

In summary, the sections of this chapter are: (a) Creativity: An Overview, (b) Areas of Creativity, (c) Mathematical Creativity, and (d) Active Learning.

Creativity: An Overview

Human beings have always been intrigued by their own creativity and the creativity of their species. They have always been puzzled by the forces which lie behind a work of art, a new idea, a scientific theory, or an invention. Such curiousity early resulted in a culturally inherited conception of creativity as being that property of the genius which mysteriously accounts for his uncommon ability and which normally the common man cannot understand or possess (Razik, 1967, p. 301). Such a misconception influenced the early investigations of creativity. Biographical authors of persons often characterized as geniuses were forced to attempt subjective explanations of the creative process by imputing certain effects to the person's temperment, cognitive abilities and environmental influences (Razik, 1966a, p. 147).

The pioneering research in creativity has been traced to Francis Galton, now recognized as the founder of psychological and mathematical studies of individual differences. Galton's first attempts at empirical investigations of creative genius and creative production appeared in his Heridity Genius (1869). It was here that Galton viewed men of genius, not as a kind of race apart, but as the extreme top end of a continuous distribution (Vernon, 1970, p. 10).

Freud's psychoanalytic probing into the unconscious also affected investigation in this area. Razik (1966a) noted:

> It was Freud who offered an escape from such subjective limitations by systematically observing, isolating, and defining regularities which appeared as the data of his psychoanalytic probing into the unconscious. Freud discovered much which sheds light on the many facets of creativity emerging as functions of psychological abnormality and, though his interest in creativity apart from this relation was only slight, he made it seem possible to isolate the variables of creativity empirically and to systematize behavioral regularities inductively (p. 147).

Coupled with the work of Galton and Freud was the pioneering work in the basic principles of measuring mental abilities by Charles Spearman and Alfred Binet in the first decade of the twentieth century (Vernon, 1970, p. 10).

During the first-half of the twentieth century attention shifted from creativity mainly as an ability to a concern for the personality characteristics and emotional drives of creative individuals (Vernon, 1970, p. 13). Studies were conducted (Cox, 1926; Ellis, 1904; Patrick, 1935, 1937; Roe, 1946, 1952a,b) relating to psychological aspects of creativity with special attention to the personalities, interests, and aversions of creative individuals or their creative production. Many of these studies were restricted to special groups and occupations, such as poets and artists.

Also significant in this period was the lifelong studies of eminent historical figures and high IQ children by Louis M. Terman and his associates (Guilford, 1967, p. 4).

The present era of research on creativity began around mid-century. Guilford (1967) suggested that the turning point resulted from a number of causes including the recent scientific and technological gains: the pressures brought on society by World War II, the cold war and the space age; and the concomitant demands these conditions made on creative imagination (p. 6). Razik (1967) supported Guildford's remarks but justifiably credited Guilford's presidential address to the American Psychological Association as a major cause. In that address Guilford (1950) indicated that most tests and achievement examinations used by Americah psychologists and educationists were "convergent." He cited the neglect of creativity and outlined a research program to explore the subject based on the factorial conception of personality.

Since 1950 the research on creativity has been voluminous. Much of this recent research has been conducted at university research centers. The activities of the most influencial of these centers, with their major scholars in creativity, were identified by Torrance (1959).

The Aptitude Research Project at the University of Southern California (J. P. Guilford) has studied individual differences in the performances of the general run of educated individuals, assuming that whatever the essential mental functions of creative thinkers are, they are shared to some degree by most of mankind (Guilford, 1970, p. 150). Guilford's most significant contribution has been the discovery of the nature of some of man's creative thinking abilities and the development of tests to measure them, at least in adults (Torrance, 1959, p. 309).

The Institute of Personality Assessment and Research at the University of California at Berkeley (Donald W. MacKinnon and Frank Barron) has attempted to determine what traits or qualities set recognized creative producers, in several fields, apart from educated humanity in general. Highly creative professional men were given extensive assessments. Psychoanalytic, psychological and humanistic assessment devices were used in interpreting the data (MacKinnon, 1965, 1966).

The Institute has found that our present identification methods may be keeping many of our potentially creative producers out of colleges and graduate schools and, among those admitted, grading practices may well be failing or discouraging many so that, though admitted, they do not graduate (Razik, 1966b, p. 161).

The Laboratory School of the University of Chicago (Philip W. Jackson and J. W. Getzels) has conducted studies on the relations between aptitude for creativity and the traditional variable of intelligence as measured by an IQ. They have made other contributions showing the importance of creative thinking abilities in school achievement at the secondary school level (Getzels & Jackson, 1963). Their work has shown that not only are intelligence tests biased against the highly creative adolescent, so too are the teachers.

The Creative Education Foundation at the University of Buffalo (Sidney J. Parnes and the late Alex F. Osborn) has concentrated on the improvement of adult creative production through special courses or programs (Osborn, 1963). Activities of the Foundation include research, teaching, publication, and distribution of information regarding the nature and nurture of creativity. They have also initiated and sponsored the Creative Problem Solving Institute, which has had 17 annual meetings. The Foundation, in turn, established the Journal of Creative Behavior, the only periodical devoted exclusively to creativity. Osborn also devised a technique for nurturing creative thinking called "brainstorming" (Osborn, 1963).

The University of Utah (Calvin W. Taylor) has been concerned with the development of criteria, the effects of organizational factors, and education (Torrance, 1959, p. 310). In addition to these research efforts, special conferences on creativity were initiated in 1955. There have been eight of these conferences, each concentrating on the most recent creativity research findings in relation to industry, education, technology, science, art, and other areas (Taylor & Parnes, 1970, p. 169).

Torrance (1959), in citing these research centers, also indicated the work he and his colleagues began in 1958 at the Bureau of Educational Research at the University of Minnesota. Concluded in 1966, they studied more than 15,000 children from nursery school through sixth grade. Torrance and his associates engaged in a continuing program of development and research related to the identification, development and utilization of creative talent in children and on performances of teachers who attempted to teach creative thinking (Torrance, 1965a; Razik, 1966b). Their work has created measures and methods that are usable by teachers in classroom settings. Since 1966, Torrance has been associated with the University of Georgia.

Other significant developments within the past 20 years have been summarized in recent writings by Guilford (1970) and Razik (1966b).

Areas of Creativity

The review of the literature on general creativity in this section has been organized into five areas and was limited to studies and opinions related to this study.

Definitions and Theories of Creativity

In a recent article in the New York State Mathematics Teachers Journal, Rising (1966) observed:

> Educators have a way of hitting upon words or phrases as scapegoats or catch words to represent NEW thinking. In just this way, the work creativity has been taken up and banded about in the educational literature. It is virtually impossible to define this word, because each individual who talks about it or listens to others talk about it has his own personal definition (p. 99).

This viewpoint is shared by many educators and psychologists. Vernon (1970), as editor of the Penguin Modern Psychology Readings on Creativity, stated in the introduction that "there are many kinds, as well as degrees, of creativeness (p. 12)." He noted that "To the psychologist, however, creative thinking is merely one of the many kinds of thinking which range from autistic fantasy and dreaming to logical reasoning. Indeed to some extent it seems to partake of both extremes (p. 12)."

The British psychologist Hudson (1966) provided the comment that:

> 'Creative,' it must first be established, is an adjective with widespread connotations . . . .
>
> In some circles 'creative' does duty as a word of general approbation--meaning approximately, good It is rather the same with its derivative noun, 'creativity'. This odd word is now part of psychological jargon, and covers everything from the answers to a particular kind of psychological test, to forming a good relationship with one's wife. 'Creativity', in other words, applies to all those qualities of which psychologists approve. And like so many other virtues--justice, for example--it is as difficult to disapprove of as to say what it means (pp. 100-101).

American psychologists (Gallagher, 1964; Gowan, 1965; Kneeler, 1965; Wilson, 1965), greatly influenced by Guilford's theoretical view of creativity, are prone to view creativity in terms of divergent thinking. Wilson (1965) observed:

> In recent writings the tendency is to equate creative thinking with divergent thinking . . ., i.e., thinking which may proceed by a variety of paths to a diversity of possible answers (p. 31).

Guilford (1965) sees creative thinking as clearly involving what he categorized as "divergent-productive thinking and the abilities to effect transformations of information, with the abilities of fluency, flexibility, elaboration and redefinition playing significant roles (p. 18)."

According to Getzels and Jackson (1963), divergent thinking tends to be stimulus-free, while convergent thinking is stimulus-bound. They felt that the less inhibited, stimulus-free students were more creative (pp. 171-172).

Torrance (1966b) views creativity as:

> A process of becoming sensitive to problems, deficiencies, gaps in knowledge, missing elements, disharmonies, and so on: identifying the difficulty; searching for solutions, making guesses, or formulating hypotheses about the deficiencies; testing and retesting these hypotheses and possibly modifying and retesting them; and finally communicating the results (p. 6).

Torrance suggested that this describes a natural process with strong human needs involved at each stage. With this description one could begin defining operationally the kinds of abilities, mental functioning and personality characteristics that facilitate or inhibit the process (pp. 6-7). Torrance (1962b, 1963a) also views the abilities of evaluation, a high degree of sensitivity, a capacity to be disturbed, elaboration, redefinition and divergent thinking as essentials of the creative personality.

Some authorities (Ausubel, 1963; Kreuter & Kreuter, 1964; Kubie, 1958; Mueller, 1964) insist that the term creative be reserved for some very rare and particularized kind of ability, such as in the fields of art, music and writing. Ausubel (1963) felt that "Creative achievement . . . reflects a rare capacity for developing insights, sensitivities, and appreciations in a circumscribed content area of intellectual or artistic ability (p. 100)." According to Ausubel, the creative individual who embodies this capacity is, by definition, an uncommon individual, much rarer than the intelligent person. Ausubel does not deny the existance of general creative abilities, but he claims that such abilities do not constitute the essence of creativity (pp. 99-100).

Other authorities, on the other hand, apply the term creative to a general creative ability possessed to some degree by all essentially healthy individuals. They argue that to some extent everyone has the capacity for creative behavior even though few individuals will make scientific and artistic contributions which will achieve historical distinctions.

Thurstone (1952) maintained that an act is creative if the thinker reaches a solution that is unique for him whether that idea be artistic, mechanical, or theoretical (p. 22). Fleigler (1959) agreed. He said that when a man creates he ". . . manipulates external symbols or objects to produce an unusual event uncommon to himself and/or his environment."

In summary, various definitions and theories of creativity have appeared in the literature, usually emphasizing novel combinations or unusual associations of ideas, although no agreement has been reached regarding the degree of "unusualness." Some authorities require that such combinations or unusual associations have social or theoretical value; at least they must have some emotional impact on other people. Others do not emphasize this criteria but are content if the creative product is "new" to the individual doing the creating.

Characteristics of Creative Persons

In the literature on creativity, many studies are reported which have identified various personal characteristics, or traits, of creative persons. As could be anticipated, traits of creative people frequently appear in more than one study. In this review general characteristics are first identified from a sampling of the more recent articles. Later, traits of creative teachers are considered. Finally, some surprising results of studies dealing with creative children will be reviewed.

The creative person.--In an exploratory study of creative adolescents, Drews (1963) indicated that one outstanding characteristic of creative students is an openness that fosters keen awareness and sensitivity to experiences both within and without themselves. This openness makes them consider many alternatives. Drews found that along with this open, searching behavior, creative students show a growing trust in their own perceptions and an unwillingness to accept authority without critical examination. If popular ideas make sense to them, they are willing to accept them. Creative students were unwilling to adopt their ideas and behavior to the demands of the group. Their imagination and sensitivity help put themselves in others shoes--to see another's world as if it were their own. In addition, Drews found that creative students are not cynical and nihilistic, but seem to have kept a sense of awe and wonder and hope.

Givens (1962) suggested that the creative person sees ordinary happenings in a new light. Ideas that seem commonplace to other people hold deep meanings because they have the ability to put them together into a significant whole, or to synthesize.

MacKinnon (1961, 1962a, 1965, 1966) and his colleagues have carried out a series of studies which were concerned with traits of creative persons. They sampled from the fields of artistic creativity, architecture, industrial research, enginnering, physical science and mathematics.

MacKinnon's findings suggest that artistic creativity reveals itself as an expression of the creator's needs, perceptions and motivations. Through his products the artist externalized something of himself into the public field. What seemed to characterize particularly the artistically creative was the relative absence of repression and supression as mechanisms for the control of impulse and imagery.

In scientific creativity the creator worked largely as a mediator between externally defined needs and goals. He simply operated on some aspect of his environment so as to produce a novel or appropriate product.

Architects were examined because they represented both artist creativity and scientific creativity. MacKinnon found that creative architects more often viewed themselves as being inventive, independent, enthusiastic, determined, and industrious than did the less creative members of their profession.

Summarizing the likenesses of the more creative individuals from all these groups, MacKinnon found that they were above average in intelligence, more fluent, more alert, more independent in thought and action, more discerning, relatively free from conventional restraints and inhibitions, more open to perception of complex equivalents in experience, and inclined to recognize and admit unusual and unconventional self views. They also had an intense commitment to what they choose to do.

Although the studies were from various career fields, two values--aesthetic and theoretical--that seem to be conflicting were both high ranking and of almost the same strengths for all groups. One does not usually think of the mathematician in terms of the aesthetic (emotional concern for beauty) or of the writer or artist in terms of the theoretical (rational concern for knowledge and the search for truth). Yet MacKinnon found that scientists, writers and architects show almost the same concern for both values (E. Brown, 1962, p. 28).

MacKinnon's studies showed that the grades achieved in school for these creative people ranged around "B" average for architects and between a "C" and a "B" for research scientists. Many did not have academic grades that would admit them to graduate school today (Razik, 1966b, p. 161).

Hughes (1969) assembled a similar list by surveying mature creative scientists. He observed that they were distinguished from their less creative peers by "their good but selective memory, openness to new experience, self-discipline, introversion, divergent thinking, attraction to resolve disorder, insistence on free time, and their need for a supportive climate (p. 82)."

Roe (1946, 1952a, 1952b) conducted studies of both artists and research scientists. She found that artists in general were above average in intelligence with a sensitive nonaggressive personality. They also tended towards abstract thinking. Roe's findings of creative research scientists agreed with MacKinnon to the extent that their scientific process of investigation was motivated by curiousity and was fostered only in a free and unrestricted environment divorced from authority.

Stein (1956) conducted a study to determine levels of creativity in industrial scientists. Using psychological tasks and biographical questionnaires, Stein found that the less creative scientists were more submissive to authority and more acceptant of tradition while the more creative scientists viewed themselves as more different from their work groups and from the general population than did the less creative scientists.

Eiduson (1958), studying personality, thinking and perceptual differences between artists and non-artists, found that artists "look for ways of thinking that are original and unusual (p. 25)." The artists also showed a tolerance for ambiguity and desire for personal recognition and self-expression.

The characteristics or traits found above were identified largely from information from interviews, biographical inventories and introspective comments of creative people. Another way to isolate those traits which distinguish creative people is through factor analysis. This method has been used by Guilford (1959b) who has identified certain primary aptitude traits which may bear directly on creativity. These include the ability to see problems (a generalized sensitivity to problems), fluency, flexibility, originality, redefinition (the ability to improvise) and elaboration. Nonaptitude traits, including motivation and temperament, were not clearly identified.

Barron (1955) and Getzels and Jackson (1962a) have also employed Guilford's tests in their study of creative people.

Several summaries (Cattell, 1959; MacKinnon, 1965; Razik, 1966b; Torrance, 1965b) of traits of creative persons have been assembled. Several traits which reoccur in the summaries seem pertinent to the educational process. They indicate that the creative person: (a) was less repressed, less inhibited, less formal, less conventional, and showed less authoritarian values, (b) was less impressed by what others think, (c) was more intuitive and perceptive with an open, searching behavior, (d) shows greater sensitivity to certain types of experiences, (e) was highly motivated to achieve in situations where independence of thought and action were required, (f) produced novel and unconventional solutions to problems, (g) showed flexibility in his tolerance for ambiguity, (h) has a strong sense of humor and playfulness, (i) is not afraid of failure, or being laughed at, (j) is more dedicated when solving problems, and (k) finds an original kind of order in disorder.

The creative teacher.--When we look at specific traits of teachers who are creative, the literature is much less plentiful. Stiles (1959) indicated that a creative teacher must be an educated person, curious, adept at applying knowledge, have a sound liberal preparation, be a specialist in his field, possess a reliable knowledge of how people learn, and be able to devise alternate ways to reach students (pp. 355-356).

Cropley (1967) provided some additional traits:

> Creative teachers are 'resourceful, flexible, and willing to "get off the beaten track"'. In particular, they display a very high level of ability to form good relationships with highly creative students in their classes, although they usually enjoy good relations with other children as well (pp. 96-97).

Cropley also noted that they may be nonconforming, discontented and fault-finding in their relationships to their colleagues and in their out-of-school life. Frequently frustrated by failure to complete difficult problems they willingly undertake, they may seem short-tempered and even boorish at times.

Tan (1967) found that teachers rated as high-creative or low-creative differed on the basis of observed originality in the classroom but did not differ as to fluency and flexibility.

Using a modification of the Flanders Interaction Analysis technique to record observed teacher behavior along with a creativity self-rating scale, Morgan (1967) worked with social studies student teachers and found that creativity seemed to be strongly related to the extent to which individual teachers rated themselves as creative, their masculinity, and their sociability.

Gensemer (1967), working with 66 seniors majoring in secondary education during their practice teaching, found that creative student teachers are more interested in using their hands as a means of expressive and emotional outlet, are more achievement oriented, are less compliant in interpersonal relations, enjoy exploring and experimenting with the environment, have less respect for conventional rules and are less concerned with certainty.

Hansen (1967), studying the behavior of creative and less creative basic business teachers, found that creative teachers exhibited significantly higher number of behaviors in the categories: (a) accepts or uses ideas of students, (b) lecturing, and (c) demonstrating through examples. Less creative teachers exhibited a significantly higher number of behaviors in the categories: (a) gives directions and (b) accepts silence or confusion. There was, however, no significant differences in the number of questions asked by the creative teacher and the number of questions asked by the less creative teacher.

Bruch (1967), following a study of 22 student teachers and 22 master teachers, concluded that the "effectively creative teacher" in verbal interaction with children, as measured by the Aschner-Gallagher-Interaction System, was spontaneously more flexible, displayed a higher proportion of divergent thinking and a lower proportion of evaluative thinking, elicited from children more divergent thinking, and tolerated a longer delay between classroom management procedures than do less creative teachers.

Creativity and intelligence.--There has always been considerable interest in the relation between creativity and intelligence, particularly the extent to which the latter can account for the former. MacKinnon (1966) stated:

> It will come as no surprise that highly creative persons have been found to be, in the main, well above average. But the relation between intelligence and creativity is not as clear-cut as this would suggest, if for no other reason than that intelligence is a many-faceted thing. There is no single psychological process to which the term "intelligence" applies; rather there are many types of intellective functionings (p. 153).

The recent interest in creativity and intelligence can probably be traced to Guilford's 1950 presidential address, in which he predicted that the correlations between scores on tests of intelligence and on tests of creativity would be moderate or low. The highly intelligent would probably not be highly creative and the highly creative would not be highly intelligent (p. 447).

Several studies conducted during this past decade reached conclusions which tend to support this prophecy. Furthermore, those creative aspects of thinking which are not commonly related to intelligence tests are related to performance in the classroom and are, therefore, of special interest to teachers (Cropley, 1967, p. 1).

MacKinnon (1962a), reporting on his earlier studies, noted that:

> As for the relation between intelligence and creativity, save for the mathematicians where there is a low positive correlation between intelligence and the level of creativeness, we have found within our creative samples essentially zero relationship between the two variables, and this is not due to a narrow restriction in range of intelligence (p. 487).

Getzels and Jackson (1962a) published a study in which 533 boys and girls in a private school in Chicago were administered five creativity measures. They had an average IQ of 132 on previous intelligence tests. Getzels and Jackson selected the top 20 percent on creative measures who were below the top 20 percent in IQ (High Creative Group) and the top 20 percent in IQ who were below the top 20 percent on the creativity measures (High Intelligence Group). Those who were in the upper 20 percent on both measures were not studied.

These two groups were then compared on total scholastic achievement, motivation for achievement, perception by teachers, personal values, imaginative production, career aspirations, and family background.

Getzels and Jackson found that a difference of 23 IQ points between the High Creative Group and the High Intelligence Group was not reflected in the average school achievement of the two groups. They concluded that intelligence was not a reliable predictor of creativity and demonstrated that creativity and high intelligence tend to correlate only up to a certain point.

The results obtained by Getzels and Jackson have not been accepted by all researchers (McNemar, 1964; Wallach and Kogan, 1965). At issue are several criticisms, including a faulty research design, the failure to administer an IQ test at the time they administered the creativity tests, the deliberate exclusion in their study of the 20 percent high creative and high intelligent (implying that both these qualities could not be found in the same group), the omission of the basic correlations between creativity, intelligence and achievement, and the criticism that the kind of ability measured by their open-ended tests is not closely linked with IQ.

Furthermore, since the total mean IQ was 132 and the High Creative and High Intelligence Groups had mean IQ's of 127 and 150, there was some doubt concerning the extent to which these findings can be taken to reflect the state of affairs in school children as a whole.

Torrance (1962a) and his coworkers have conducted no fewer than eight replication studies which have avoided some of Getzel and Jackson's shortcomings. The result of these studies, with children of various levels of ability at the University of Minnesota Laboratory Elementary School and the Minneapolis School System and with university graduate students, suggest there are no significant differences in overall academic achievement between the High Intelligence Groups and the High Creative Groups. The results of their correlational studies supported the conclusion by Getzels and Jackson. Torrance (1962a) proposes about 120 as the IQ threshold beyond which creativity bears a relationship to classroom performance which is independent of IQ (p. 63). Torrance estimates that by depending solely on IQ tests, about 70 percent of the top fifth of the creative school children will be neglected. He noted that "This percentage seems to hold fairly well, no matter what educational level we study, from kindergarten through graduate school (p. 4)." Although a certain level of intelligence is needed to be creative, beyond that level, they found, there is small relationship between intelligence and creativity, at least in the way "intelligence" is now used. Torrance believes that although outstanding creativity is seldom found among children of below average

IQ, some type of creative talent may be found anywhere along the scale, except perhaps, at the bottom.

Yamamoto (1964c) studied the relationship among groups of highly creative high school students and highly creative elementary students who had each been divided into three groups according to the level of their intelligence; High Intelligence Group (IQ above 135), Middle Intelligence Group (IQ between 120 and 135), and Low Intelligence Group (IQ below 120). Yamamoto found that, among high school groups, an increment in intelligence beyond an IQ of 120 had little effect on academic achievement of these highly creative students. This increment did not occur with the highly creative elementary students.

In one study, Yamamoto (1964a) used a design similar to the one used by Getzels and Jackson with the corrections suggested above. Using 272 high school students, he identified three groups: (a) High Intelligence Group, in upper 20 percent on IQ but not on creativity; (b) High Creativity Group, in upper 20 percent on test of creativity but not on IQ; and (c) High Intelligence--High Creativity Group. Although the High Intelligence Group averaged 20 points higher in IQ than the High Creativity Group and 7 points higher in IQ than the High Intelligence--High Creativity Groups, he found no statistically significant differences among these three groups on various achievement measures.

In another study Yamamoto (1964b) corrected for the effects of intelligency by analysis of covariance and found that the high creative thinkers surpassed the low creative students. He concluded that there were differences in achievement between the highly divergent (creative) students and the uncreative students which were not due to differences in IQ. These differences led Yamamoto to the notion that there is a distinct relationship between performance on creativity tests and success in school learning.

Wallach and Kogan (1965) studied 151 fifth grade students. Defining creativity as "the ability to generate unique and plentiful associations, in a generally task-appropriate manner, and in a relatively playful context (p. 353)," they proceeded to study intelligence and creativity as two dimensions relatively independent of each other. Intelligence was measured using indices from WISC, SCAT, and STEP tests. Creativity was measured with ten subtests constructed by the researchers. The results of their statistical analysis (average correlation among the ten creativity measures was 0.4, among the ten intelligence indicators was 0.5, and between the two sets of measures was 0.1) led Wallach and Kogan to conclude that there is a unified dimension of creativity which exists apart from a unified dimension of intelligence (p. 242).

In summary, it appears that some children can be both intellectually gifted and outstanding in certain creative abilities although "creativity is not necessarily an attribute for the gifted, nor, as some researchers have suggested, a taboo for them (Schmadel, 1960)." MacKinnon (1962b) remarked:

> A certain amount of intelligence is required for creativity, but beyond that point being more intelligent or less intelligent is not crucially determinative of the level of . . . creativity (p. 18).

The teacher and the creative student.--To nurture learning and to create learning tasks that will be effective for students with widely divergent abilities, interests, and traits requires an ever deepening understanding of those students. Recent studies, however, indicate that teachers recognize and understand the highly intelligent student better than the highly creative ones. Following are some of the findings in this area.

Getzels and Jackson (1962b) found that the creative students in written responses to six stimulus pictures showed a degree of imagination and originality unmatched by the high IQ student. Stories by creative students made abundant use of humor, unusual situations and unexpected endings.

Torrance (1962b) found that highly creative students more often than other students produced imaginative work judged to be off the beaten track and that included fantastic ideas.

Barron found that creative students seem to thrive on disorder and responded by creating new and, for them, superior arrangements out of the confusion (Razik, 1966b, p. 163).

Also investigated in the study by Getzels and Jackson (1962a) was the question of teacher preference. They found that teachers have a decided preference for the student with high IQ. When asked, teachers clearly preferred the high IQ over the highly creative student in spite of the fact that, in this particular experiment, the high IQ student and the highly creative student were equally superior to other students in school achievement. Getzels and Jackson also found that the high IQ student tends to hold a self-image consistent with what he feels the teacher would approve. The creative student was more inclined not to conform to this model. The creative student considers high marks and goals that lead to adult success in life less important than does a member of the high IQ group. The creative student has much greater interest in unconventional careers than his peers (pp. 38-39). Getzels and Jackson (1962b) also found that the creative student rated a sense of humor, along with a wide range of interests and emotional stability, much higher than did members of the high IQ group.

Torrance (1963b) found that teachers rate the highly intelligent student as more desirable students; more ambitious and hardworking, more friendly and less unruly.

In a study using 66 characteristics as measuring factors to obtain teachers' and parents' conceptions of the ideal pupil, Torrance (1963c) found that teachers and parents indicated great ambivalence towards the kind of pupil who could be described as highly creative. Independence in thinking, for example, ranked in second place while independence of judgment ranked only nineteenth and being courageous only twenty-ninth. Teachers felt it far more important for students to be courteous than courageous. It was also more important that children do their work on time, be industrious, be obedient, and be popular among their peers.

Conditions Which Influence Creativity

One theme having implications for creativity concerns the extent to which an individual's personal characteristics interrelate with the effects of his environment. Meyer (1970) remarked that within some societies there are areas where creativity is encouraged and areas where it is discouraged. Within such a cultural milieu, she noted, there are influences protecting many elements from change (pp. 15-16). Taylor (1964) suggested that these include facts involved in educational settings, working conditions and climate, and training programs. A person's home environment may also be regarded as an external influence (p. 29).

Such influences may be a factor in stifling creativity because persons who are especially susceptible to conformity pressures from these influences tend to have other personality characteristics that are deleterous for creative thinking. Crutchfield (1967) wrote:

> Conformity pressures may be expected to be injurious to creative thinking . . . because . . . outer pressures and inner compulsion to conform arouse extrinsic, ego involved motives in the problem solver (p. 125).

When we turn to the creative teacher, one environmental influence which has an effect is the social structure of the school system. Chesler (1966) related elements of the social structure of the school system to the extent to which teachers were ranked as innovative, or creative. He found significant relationships between perceived support and inter-communication patterns, and the extent to which teachers tried new things.

Otte (1964), studying the reactions of elementary school teachers and principals to various factors that promote and hamper creativity in teaching, identified 13 factors which foster creativity in teaching and 9 factors which hamper creativity in teaching. Otte concluded that principals should develop skill in releasing creative potential from teachers and decrease the pressure for conformity and rigid daily schedules. In addition, Otte observed that school principals should develop a helpful attitude towards creativity, eliminate restrictive administrative policies, depart from routine procedures, and experiment with novel teaching methods. Otte also noted a need for teachers to be free from traditional textbook coverage initiative, to develop empathy and rapport with the principal and to work to eliminate negative attitudes by other teachers toward creative thinking.

Lindgren (1967), in observing the relationship between teachers and administrators, indicated that even though a teacher may be quite creative, he does not necessarily enjoy support from those to whom he is responsible. He stated:

> Probably more teachers would be creative if they received encouragement and support from administrators. Unfortunately, such support is often lacking. One survey of school principals' ratings of teacher effectiveness showed that teachers who showed more ingenuity (one aspect of creativeness or divergent thinking) tended to get lower ratings. Everyone seems to be in favor of creativity, but not of creative people (p. 496).

Techniques for Nurturing Creativity

Evidence suggests that children's creativity in mathematics and in other subjects diminishes during school years and often much of it is lost by the time they reach fourth grade (Torrance, 1963a, p. 83).

The exact reasons for this decline have not been identified, but increasingly the major blame is directed towards the traditional educational system in America. Vaughan (1969) noted that one thing made clear through investigation is that:

> Our traditional programs in education are effective instruments of our authoritarian society and antithetical to the development of creativity, and . . . they have been effective and efficient in producing quiet, orderly, and courteous children, rather than flexible, sensitive, and courageous individuals (p. 230).

Such traditional practices in the schools have been criticized on the grounds that both teachers and parents want to produce the conventional socially well-adjusted child and viewed the unusually talented or creative child with suspicion (Vernon, 1970, p. 11). Razik (1966a) cited a growing body of evidence and suggested that:

> Our educational structure itself discourages the development of creative potential, for highly creative children are not the most satisfactory students. They resist group work, are stubborn, often embarrass teachers with wild questions and off-beat ideas. Their humor and playfulness are often unappreciated in the classroom (p. 148).

Razik's remarks specifically implicate the classroom teacher as a factor contributing to the decline of creativity and creative thinking in children. Such an implication has been supported by the research of Getzels and Jackson (1962b) and Torrance (1963b), cited earlier in this thesis, and in other studies. Williams (1966) reported on a study of statements made on a questionnaire administered to more than 500 teachers across the country just beginning training. He found that these teachers did not understand what is meant by the term creativity, and they had difficulty in identifying creative talent in classroom students. Eberle replicated this study and again found that teachers were unable to identify their most creative students (Williams, 1968, p. 199).

Nevertheless, several authorities do feel that creative learning can be taught. Wallen and Travers (1963) stated:

> Although it is true that both theory and research in learning support the notion that creative, novel, insightful behavior cannot be rigidly controlled or predicted at this time . . ., present evidence suggests rather strongly that insight is more likely to result when certain appropriate responses have been

> previously acquired and that the development of such responses may be taught directly (p. 489).

"Almost any penetrating analysis of what is required for successful nurturance of creative talent leads to a recognition of the needs for helping teachers improve certain skills (Torrance, 1966a, p. 170)." Several suggestions for accomplishing that objective appear in the literature.

MacKinnon (1962a), commenting on the implications of his research, suggested that activities which stressed searching for common principles, the use of analogies, similes, metaphors, symbolic equivalents of experience in a number of sensory and imaginal experiences, would promote development of creative thinking (p. 494).

Torrance (1961a) listed five principles which he believed to be important in developing creative thinking. He suggested that teachers and parents who follow these principles would assist in the development of creative potential in their children. The principles were: (a) be respectful of unusual questions, (b) be respectful of unusual ideas of children, (c) show children that their ideas have value, (d) provide opportunities for self-initiated learning and give credit for it, and (e) provide for periods of nonevaluated practice or learning.

A subsequent study with fifth grade children by Enochs (1964) examined the efficacy of these principles and found that creative thinking can be nurtured by applying these principles.

Later, Torrance (1964) compiled a more extensive list of 20 suggestions.

Hallman (1967) stated that "Creative teaching was the best way, and perhaps the only way, to promote creative behavior in pupils (p. 327)." His 12 ways to nurture creativity in the classroom being: (a) Provide for self-initiated learning on the part of the pupils; (b) Develop nonauthoritarian learning environments; (c) Encourage pupils to over-learn; to saturate themselves with information, imagery, and meanings; (d) Encourage creative thought processes (to seek new connections among data, to associate, imagine, think up hypotheses, make wild guesses and to build on the ideas of others); (e) Defer judgment of students' efforts; (f) Promote intellectual flexibility among the students; (g) Encourage students to evaluate their progress; (h) Encourage students to become more sensitive persons; (i) Make effective use of questions; (j) Provide opportunities for students to manipulate materials, ideas, concepts, tools, and structures; (k) Assist the students in coping with frustration and failure; and (l) Urge pupils to consider problems as wholes.

Durr (1964) cautioned that: "Most suggested procedures for cultivating creativity are grounded in studies of the characteristics of those who are creative or studies of environmental factors which have influenced them (p. 179)." Durr then provided the suggestions to: (a) foster academic attainments; (b) develop self-discipline;

(c) promote individuality; (d) build self-confidence; (e) increase sensory awareness; and (f) promote flexible approaches to problems.

Hughes (1969) provided suggestions for nurturing creativity covering a wider range of issues.

> To maximize student potential . . . in any field, we must optimize the flexibility and humaneness of prospective teachers. They must be educated to be socially conscious, individual-concerned human beings first and educators second. An implication of the 'adaptability' characteristic for the educational process is that there should be plenty of variety in course content and meeting formats. We should use audio and visual aids, reports, debates, outside speakers, instructors from other disciplines, visits to industry, and different seating arrangements and room assignments (p. 79).

Several authorities (Torrance, 1966a; Williams, 1968) have suggested inservice creativity workshops for teachers. Thompson (1968) found that a two-week workshop (Exploring Creativity) was effective in changing teachers' attitudes towards pupils, but did not serve to increase creative thinking scores. The participants, however, did feel the workshop was helpful in stimulating creative teaching ideas. Pugh (1968) also studied the effects of a creative teaching workshop and his results were similar to Thompson.

Williams (1964) designed a study based on the premise that ability to identify creative students can be developed if teachers are exposed to what is known about the creative child and the creative process. Teachers were given training to help them understand the creative process and the creative individual. When teachers' selections before training were compared with their selections after training, a statistically significant difference between the choices was found.

However Duplisea (1969) found that preservice elementary students, when presented with a treatment specifically designed to provide information about creative thinking, did not recognize creative thinking more significantly than similar subjects who were not presented with this treatment.

In a study with prospective teachers Holman (1968) tried to determine whether their creativity could be increased through integrating creative teaching methods into an already existing course in elementary school curriculum. A second purpose of the study was to determine if prospective elementary school teachers could help elementary students increase their creative potential through the use of creativity exercises. Two experimental and two control groups participated. Holman found that on four of the seven creativity measures used there was a significant difference in favor of the experimental group, however, this was not correlated with the amount of growth made by the elementary students they taught.

Holman's latter findings are somewhat typical of the conflicting reports that have emerged as a result of studies with elementary students.

In a study with primary school children Torrance (1961b) set out to show whether children in the first three grades could be taught to produce ideas by the use of appropriate teaching methods; he found that in the second and third grades, trained children consistently surpassed untrained on all measures of creativity employed in the study.

In another study, however, Torrance (1965b) demonstrated that creative thinking scores increase sharply, even without specific training, when the teacher is himself interested in and aware of creativity.

Along the same lines, Weber (1967) studied the effects of indirect versus direct teacher behavior with the assumption that indirect teacher behavior fosters pupil creativity more than does direct teacher behavior. He concluded that, if teachers value, as one of their instructional goals, the fostering of creative potentials of pupils, then it becomes their responsibility to instruct their pupils through a consistent use of indirect teaching behavior beginning with the earliest schooling experiences of those pupils.

However, when Broome (1967) compared children's growth in creative thinking, vocabulary development, reading comprehension and arithmetic reasoning, he could find no significant difference between adjusted scores on any of the measures employed when children were taught by low-creative or high-creative teachers.

Gallagher (1964), summarizing an extensive discussion on teaching creativity, had these remarks:

> First of all, it is not useful to talk about teaching for productive thinking or creative thinking. The terms are too broad and too inclusive to allow the teacher or educator to develop anything very specific in the way of curriculum. On the other hand, it does seem feasible for the teacher to develop certain kinds of specific intellectual skills which cover a narrower range of activities (p. 206).

Much more research on the examination of the techniques by which creative learning can be nurtured is needed (Cropley, 1967, p. 88).

Measuring Creativity

The problem of learning about creativity has been complicated by several factors, not the least of which is the multiplicity of meanings attached to the term "creativity" (Wilson, 1958, p. 109). A second factor, obviously related to the first, is the development of instruments to assess this creativity. Cropley (1967) summarizes quite well the present position in creativity relative to these two factors when he stated:

> The term 'creativity' is coming to have a highly circumscribed meaning in the field of psychological measurement, although it is still used in a very loose way by some psychologists. In its strictly psychometric sense the word is emptied of the social, aesthetic, and professional connotations which are connected with its everyday use, although, of course, the 'scientific' use of the term does not preclude the possibility that creativity tests are related to creative behaviour in later life. Hence, although the concept of creativity is a difficult one to employ with precision because of its impreciseness, the term is coming to be accepted by many psychologists and

> educators as referring to an intellective mode characterised by thinking of the divergent kind . . . . creativity means something very like what Guilford refers to as 'divergent thinking' . . ., rather than what the layman has in mind when he uses the term (pp. 7-8).

Guilford (1950) had noted that the kinds of items usually included on general intelligence tests concentrate heavily on items which required thinking of the convergent kind and neglected the divergent kind.

Guilford's (1959a) approach to the measurement of creativity has centered around three main dimensions of intellect, which he labels "operations," "products," and "contents." Guilford argues that one needs to know (a) what kind of material is being processed (contents), (b) what is being done to it (operations), and (c) what kinds of results this leads to (products). Hence, any intellectual task will elicit particular kinds of operations which are carried out on the contents of the task and lead to a certain kind of product. Guilford has identified five kinds of operations which are carried out on four possible kinds of contents and may lead to one of six kinds of product. This has resulted in 120 factors of intellect, 24 of which were in a category described as divergent thinking (see Figure 1). Ninety-eight of these factors have been empirically verified by factor analysis; 23 in the divergent thinking category (Guilford, 1970, pp. 157, 161).

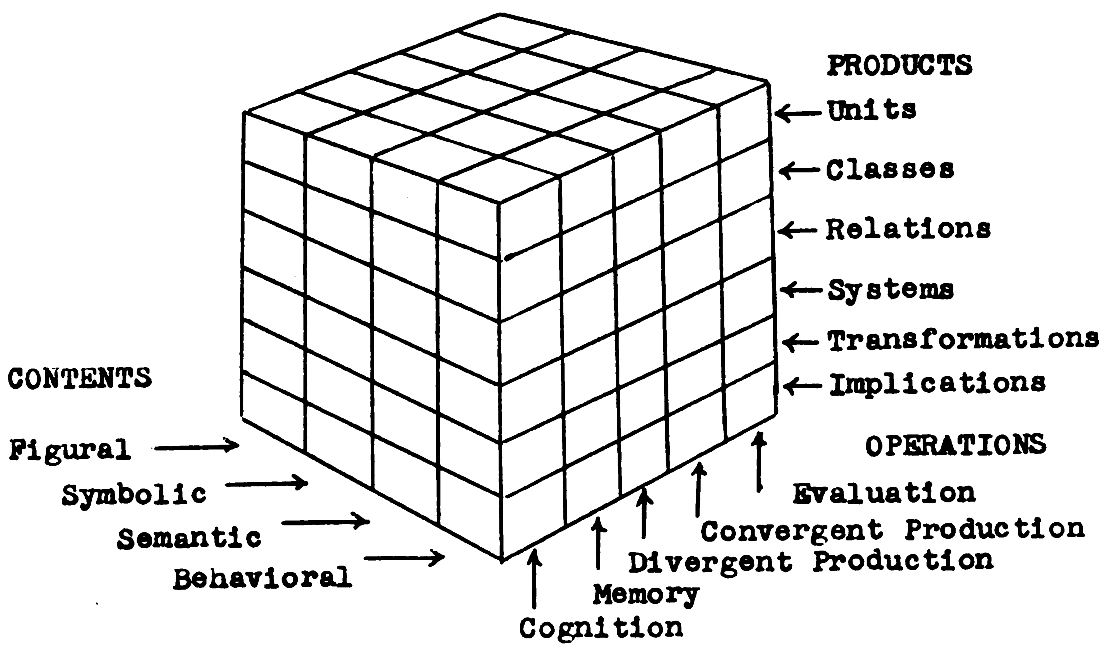

Figure 1.--Guilford's Theoretical Model for the Complete Structure of Intellect.

Guilford's distinction between abilities for divergent thinking and abilities for convergent thinking have been especially useful in studying creativity (Razik, 1967, p. 305). Guilford (1970) cautioned, however, that "Each person is probably uneven with respect to his skills in those different modes of intellectual functioning. Furthermore, the divergent-production functions are not the only ones that make significant contributions to creative output (pp. 157-158)." Nevertheless, many researchers feel that the similarity between the concept of creativity and divergent thinking is close enough for the two terms to be used almost interchangably. Cropley (1967) points out, however, that "this is a result of limitations placed on the use of creativity, rather than an assumption that divergent thinking is necessarily a predictor of later creativeness (p. 8)."

At any result, experimental tests now in existence which are used to measure creativity almost exclusively emphasize aspects of divergent thinking (Guilford, 1966, pp. 186-189).

Guilford (1959a) has constructed an elaborate battery of tasks (e.g., Unusual Uses, Plot Tiles, Impossible Consequences) to test the factors of intellect which he regards as important in creativity. These tasks have been used largely with college and professional people, but because of their length, have rarely been applied in their entirety.

In education the most often used tests have been the various versions of Torrance's tests. After nine years of preliminary experimentation Torrance (1966b) made available the Torrance Test of Creative Thinking in the form of a Research Edition, together with the extensive technical data which described the current level of the instruments' development (p. 1). Torrance followed Guilford in emphasizing complexity in the construction of his tests, which like those developed by Guilford, include a wide variety of tasks (e.g., Ask-and-Guess, Product Improvement, Just Suppose).

Torrance (1962a) had reported that among the advantages of these tests is the fact that they have been found suitable for use with subjects ranging from kindergarten to post graduate studies.

Getzels and Jackson's (1962a) attempt to measure creativity in their studies was limited to testing four adapted tasks: (a) Word Associations, (b) Uses of Things, (c) Hidden Shapes, and (d) Fables.

Attempts, similar to the above, to measure the kinds of skills important in divergent thinking have been going on for some time. Cropley (1967) cited five such tests, one used as early as 1922 (p. 102). Buckeye (1968) provided a more extensive list of ten tests used by various researchers from 1916 to the present (pp. 19-21). More recently, Kaltsounis (1971) and G. Davis (1971) have listed commercial and noncommercial instruments in creativity in related articles.

These tests have been developed in an attempt to measure characteristics that are important to the creative process; both verbal and nonverbal tasks are used. The tests are generally multi-dimensional and have attempted to assess sensitivity to problems, fluency, and flexibility in thinking by formulation of divergent alternates in contrast to engaging in conformity thinking; and the ability to redefine situations and make mental examinations of consequences. Responses may be scored for sensitivity to problems, word and ideational fluency, flexibility, originality, and a variety of causes and consequences proposed (Razik, 1966b, p. 162).

Numerous authorities (V. Brown & Harvey, 1968; Harvey, Hoffmeister, Coates & White, 1970; Klein, 1967; Tryon & Bailey, 1966) have been critical of aspects of these creativity tests, such as their reliability, validity, predictability, and their relationship to conventional IQ tests. For the most part these criticisms have been well-grounded. Mackler (1962) and Wodtke (1964) have both shown that creativity tests have unsatisfactorily low reliability. With respect to validity, Mehrens and Lehman (1969) had these comments:

> It is hard to agree on constructual definitions of creativity, let alone operational definitions. Even if we could agree on an operational definition, it would be hard to indicate validity for the measure because of the lack of an adequate criterion measure . . . . Is there a quality criterion or simply a quantity criterion for judging creativity (pp. 117-118)?

At the present time researchers feel that there is not enough correlation between the quality criterion and the quantity criterion to assume that the scores measure the same thing (Skager, Schultz & Klein, 1965, p. 38).

Another related criticism has resulted from attempts to show that these tests measure something which is not measured by IQ tests. These attempts have not met with much success. Wallach and Kogan (1965) suggested that relationships among creativity tests, which are supposed to be measuring something common to them but distinct from IQ tests, are weaker than relationships between creativity tests and IQ tests. DeCecco (1968), citing previous evidence, suggested that "we have little reason to believe that intelligence tests, as imperfect as they are, are les predictive of creativity tests than are current creativity tests (p. 121)."

Wallach and Kogan (1965) did devise a battery of three verbal tests (Instances, Alternate Uses, Similarities) and two visual tests (Pattern Meaning, Line Meaning) which were administered individually and scored for number of responses (fluency) and uniqueness (originality). They reported important differences in some of the properties of scores they yielded than Torrance and others. These results, reported earlier in this thesis, yielded increased factorial validity and reliability (as high as .93).

Despite these criticisms, and the occational overall criticism of all creativity tests (Klein, 1967), the limits and shortcomings of these tests have been assumed *a priori* by the researchers who use them. Most researchers in education who employ these creativity tests, it would seem, share Torrance's (1966b) philosophy. Torrance stated that the major reason for his interest in developing measures of creative thinking ability is that he believes that such instruments can yield one useful basis for making instruction more nearly in consonance with the growth characteristics and behavioral reaction of a particular pupil or group of pupils (p. 9).

Mehrens and Lehmann (1969), in discussing creativity tests, stated:

> There are many potential benefits available if one could effectively isolate and measure the construct of creativity . . . .
>
> At the present time there are some interesting creativity tests on the market . . . . These, however, are only research instruments, and much more work is needed in the area before we can really feel comfortable with the results these tests give us (p. 118).

Mathematical Creativity

Studies concerned with creativity in mathematics, either with students or with teachers, are extremely scarce. This section contains a review of these studies.

In his doctoral thesis Buckeye (1968) designed a classroom environment specifically to encourage and develop creativity in prospective elementary mathematics teachers. This was done by encouraging and respecting students' questions, their imaginative and unusual ideas, and demonstrating that their ideas have value. Opportunities for practice and experimentation without evaluation, evaluating self-initiated learning, and associating evaluation with causes and consequences were encouraged. Creative thinking was also developed through assignments and challenging enrichment problems. Little use, however, was made of mathematics activity materials.

Female students in six intact sections of a General Mathematics for Elementary Teachers course at Indiana University participated in the study; four of these sections comprised the experimental group. The remaining two sections were taught by the conventional lecture method. Four different instructors were involved.

Measures of general creativity were obtained by the AC Test of Creativity, a test designed for engineers and supervisors in business. Attitude was measured by the Dutton Attitude Scale. Mathematical creativity was measured by a test designed by Buckeye.

Buckeye was able to show a significant increase in both mathematical achievement and general creativity. The lecture method, on the other hand, appeared to have a detrimental effect on creativity.

Several correlational studies were run between achievement, creativity, attitude, and mathematical creativity. Buckeye found a significant correlation between achievement and attitude, but attitude was not significantly related to creativity.

In an unpublished study Buckeye (1970a) considered the effects of achievement, attitude and general creativity in a mathematics methods course at Eastern Michigan University. Two groups participated in the experiment in the fall and two groups during the spring. Buckeye taught all four groups.

Each group was given the same assignments, enrichment problems, unit tests and achievement test in mathematics. In addition, one group each semester used a laboratory approach to the teaching of mathematics. Students in these groups were given opportunities for discovery through use of various concrete experiences and activities. The activities were not necessarily related to the concepts being studied at that time since some reviewed earlier materials, some previewed new concepts, and some were recreational. Although the activities were not specifically stated, it is assumed that they were taken from Buckeye's numerous publications, such as A Downpour of Math Lab Experiments (Ewbank, Buckeye, & Ginther, 1970).

Data used in the study were derived from the AC Test of Creative Ability, the Dutton Attitude Scale and an achievement test formulated by Buckeye.

Using analysis of covariance Buckeye found that the laboratory approach to the teaching of mathematics methods appeared to increase general creativity and attitude of prospective elementary teachers. Buckeye also found that the laboratory approach had no detrimental effect on mathematical achievement.

Buckeye's study differs from the study reported in this thesis since it concerned a mathematics methods course and not a course emphasizing content. In addition, the analysis of covariance statistic used data from Forms A and B of the AC Test, a test measuring general creative ability.

It is rather puzzling that this measure of general creativity was used since Buckeye has designed two forms of a Test of Creative Ability in Mathematics (Buckeye, 1968, 1970b). Through successive modifications, these instruments have been tested on several hundred pre- and inservice elementary teachers as well as junior high students (1970b).

Buckeye's tests consist of six divergent thinking items which are scored for fluency only. Several approaches have been taken to estimate the reliability and validity of these tests. These estimates are given in section three of Chapter III.

No correlation coefficient was given between Buckeye's revised test and the AC Test, however, the earlier version did not correlate with the AC Test. Referring to this lack of correlation Buckeye (1968) stated:

> The AC Test indicated general area of creative ability where the DBL Test [of Creative Ability in Mathematics] was supposed to indicate creative ability in mathematics. This could have been expected since research has indicated that individuals differ in their degrees of creative potential in various fields (pp. 55-56).

For purposes of comparison with the present study it would have been interesting to see the results of these tests relative to his latter study, however, some implications may be gained from Baur's doctoral study.

Baur (1970), who was one of Buckeye's validators in the 1968 study, conducted a study at Indiana University to determine if the creative ability in mathematics of prospective elementary teachers could be changed when exposed to different combinations of three variables: (a) classroom environment; (b) creative problems; and (c) type of instructor. All three of these variables had been confounded in Buckeye's earlier study.

Eight sections of Mathematics for Elementary Teachers participated in the study. The three variables, each with two levels, occurred in all possible combinations. The classroom variable contained a creative (Torrance, 1961a, 1964) and traditional classroom; the problems variable contained creative problems and no creative problems; and the instructor variable contained a mathematics educator and a pre-mathematician. Three mathematics educators and two pre-mathematicians were randomly assigned, when possible, to the groups. Each instructor followed the same course syllabus but was responsible for his own assignments, tests, and quizzes.

Forms A and B of Buckeye's Test of Creative Ability in Mathematics and an achievement test formulated by Baur were used to obtain data for the study.

Using a factorial analysis of variance Baur found that significant differences in mean change in mathematical creativity favored the creative classroom, the creative problem, and the mathematics educator groups. In achievement, significant differences in mean change favored the mathematics educator group. The interaction effect in mean change in achievement between individuals who were taught by a mathematics educator or pre-mathematician favored those individuals who received creative problems.

Baur's study, however, did not test for an increase in creativity following a treatment involving mathematics activity materials. Only one other study of this type has been found. This recent doctoral study, conducted at the Wisconsin Research and Development Center for Cognitive Learning by Rochelle Meyer (1970), concerned the development and testing of a program to encourage individual creative mathematical activity in first-grade students. Fifteen lessons each of a 20-minute duration were constructed. The lessons involved eight open-ended geometric problems using triangles. They were given to a group of six subjects. Concrete materials were provided for the activities. All lessons were conducted by a certified primary teacher employed by the Center.

The major working hypotheses of Meyer's experiment, tested through a Solomon Four-Group Design employing analysis of variance (Campbell & Stanley, 1966, pp. 24-25), were that participation by the subjects would increase their observable mathematical creativity but not affect their general creativity. The effects of general creativity were measured by the Torrance Tests of Creative Thinking, Figural Forms A and B. A unique test instrument, involving videotapes, was developed by Meyer to measure six observable aspects of mathematical creativity. Five of these aspects related to the activities which the first-graders were engaged in when they pursued one of the two test items; the sixth aspect described the results of these activities.

Meyer found no significant effect of treatment on either general creativity or mathematical creativity. One could suggest several reasons for the latter result: (a) the small number of subjects; (b) the possibility that first-grade students may exhibit such a high degree of observable mathematical creativity that only a minimal increase could be expected from fifteen 20-minute lessons; (c) the test instrument itself or the observer; or (d) the materials employed in the lessons.

Torrance Tests of Creative Thinking were also used in a doctoral study involving general creativity, intelligence and discovery learning in mathematics by Lanier (1967).

Lanier found a significant relationship between the performance (as rated by teachers employing the discovery teaching approach) of 69 fourth, fifth and sixth-grade students and intelligence in the categories of reasoning, computation, structure, numeration, and composite performance. Within these categories, only structure was significantly related to general creativity.

Assessing mathematical performance through the Iowa Tests of Basic Skills, Lanier found a significant relationship between the problem solving category and general creativity.

Lanier concluded that teachers oriented to discovery teaching may still evalute student performance from a basis of intelligent behavior, however certain mathematical topics seem more readily grasped by students who are more creatively endowed than by those who are more intelligent. It is possible that the discovery teaching approach could be the variable responsible for change in mathematical performance in this study.

Borgen (1970) also found that creativity was related to arithmetic achievement (arithmetic concepts, problem solving and total arithmetic achievement), as measured by the Iowa Test of Basic Skills administered to 483 students enrolled in grades four, five, and six.

Borgen investigated the effect of creativity, dogmatism, and achievement in arithmetic with two intergrade groups, one of which had been enrolled in New School classrooms for one year. Such schools are designed to create independent, courageous people able to face and deal with the shifting complexities of the modern world (Stretch, 1970, p. 76).

Borgen found that grade level was an important factor in creativity. Sixth-grade students scored significantly higher than fifth-grade students and fifth-grade students scored significantly higher than fourth-grade students on some of Guilford's tests of creativity. No significant differences, however, were found between New School and control groups.

In England, Richards and Bolton (1971) administered tests of intelligence, mathematical ability, and general creativity to 265 students in their final year at three junior schools (mean age 11 years). The three groups were matched for social class, intelligence and time devoted to mathematics teaching; the major difference between them was that one group was in a Nuffield Project pilot area and committed to a discovery approach, one group received largely traditional teaching, and in the third group a conscious attempt was made to "keep a balance" between traditional and discovery approaches.

Five creativity tests were used in the study and came from batteries developed by Torrance (1966b), Guilford (1959a), Wallach and Kogan (1965) and Getzels and Jackson (1962a). The remaining tests were constructed in England.

Richards and Bolton found that performance on tests of intelligence, mathematical ability, and general creativity was largely determined by a common factor identified as general ability. They also found that the most important determinant on the performance in mathematics was general ability and that divergent thinking plays only a minor role.

A comparison of the three groups showed that, in general, the performance of the Nuffield-based group on the mathematics tests was significantly below that of the other two groups although they did better on the creative thinking tests.

Richards and Bolton concluded that teaching procedures which encourage divergent modes of thinking will produce minimal effects on student's performance on tests of mathematical ability.

McCormack (1969) conducted a study in science education which has certain implications for mathematics education. Using 69 upperclassmen enrolled in a methods course at Colorado State College, McCormack added creativity-training activities to one of the two groups' laboratory sessions. They were: (a) brainstorming; (b) inquiry development sessions; (c) morphological analysis of problems; and (d) written creative thinking exercises.

Pre- and posttest data were obtained from the Torrance Test of Creative Thinking and the Science Education Achievement Test. Using analysis of covariance McCormack found that the experimental group was statistically superior to the control group in general creativity. No difference was found in achievement gain.

McCormack concluded that creativity training may be included as a portion of an existing elementary methods course and effect gratifying creativity improvement with no loss in subject matter achievement. Furthermore, such methods courses may be improved by including creativity training as a regular course of action.

McCormack obtained significant negative correlation between creativity and achievement test scores which suggested to him that highly creative methods course students may be discriminated against by standard subject matter evaluation instrument orientation. He implied that evaluation instruments should probably involve higher levels of thought, including creativity.

Two independent doctoral studies by Prouse (1964) and Evans (1964), published at almost the same period, were concerned with the specific development and scoring of mathematical creativity tests with upper elementary or junior high school children. Neither of these studies, however, attempted to use these instruments to measure increase in mathematical creativity as a result of specific treatments.

Evans evaluated his creativity test, consisting of 16 items, on above average students in grades five through eight. Each item on his test represented some mathematical situation and the students were asked to respond in as many different ways as they could. Test items were scored with respect to fluency, flexibility, and originality.

Evans found that individual students at each grade level scored better on individual tests than the majority of people at the other grade levels, so that, relative to his test, performance was not dependent upon grade level or age.

Evans also found significant positive correlation between scores on his creativity test and intelligence, arithmetic achievement, grades in mathematics, attitude towards mathematics, and general creativity. He concluded that above-average intelligence is necessary, but not sufficient, for a high degree of performance on his test.

Prouse (1964) employed 14 seventh-grade mathematics classes enrolling 312 students to evaluate his creativity test, as well as four other instruments: (a) A Subject Preference Survey; (b) a Teacher Rating Form of the student's creativity; and (c) and (d) two Structure of Intellect Models designed by Guilford--Number Rules and Match Problems V.

Prouse's creativity test consisted of 10 items based on certain characteristics attributed to potentially creative students in mathematics (Carlton, 1959). Of these, seven items were hypothesized to belong to the divergent thinking category (scoring: a fluency score and an originality score) while the remaining three were of the convergent thinking category (scoring: one point for each correct response).

Prouse concluded that the more prominent mathematical creative abilities appear to be concentrated in the divergent thinking categories, particularly associated with the fluency score; yet an analysis of the discrimination indices for both divergent thinking items and convergent thinking items suggested an overemphasis by teachers on the latter approach in the classroom.

Unlike Evans, a low correlation (.48) between intelligence scores and creativity scores was found, suggesting that Prouse's mathematical creativity test was measuring other aspects besides general mathematical ability. This may also have been attributed to the variables used in scoring the test instrument.

In England, Foster (1970) examined the nature of creativity and the relationship between personality qualities and qualities of creativity. As part of this scheme, assessments of children's creative ability were made over the complete range of activities normally associated with junior school education. The assessment of creative ability in mathematics centered around the construction of two tests designed by Foster. The tests were administered to 265 children in five primary schools (ages 9 to 11 years).

Test A, an individual test, involved a pack of playing cards, randomly faced upwards. Students were to select any six cards having some common attribute. Students were scored for the number of different sets constructed within a five-minute time limit. Replacement of cards to the deck was permitted after each set was constructed.

Test B was administered as a group test. Students in each group were instructed to find as many "sums" (arithmetical processes) as possible using the numbers 2, 3, and 6 and the four fundamental operation signs. No time limit was imposed. Two points were given for each "sum" completed and answered, one point for each "sum" made up but not answered, and three points for each "sum" correctly answered which was made up of more than three figures or symbols. This latter test was similar to one of Evans' items.

Test-retest reliability coefficients for both tests were extremely high (.88 for test A and .85 for test B) while between test correlation was low (.31). Foster concluded that: "Although both tests call for a creative use of mathematics they seem to involve different aspects of what appears to be a very side frame of reference (p. 6)."

Active Learning

There is a limited but growing body of experimental research on active learning, including mathematics activity materials. These studies, dealing with various age and intellectual levels of children, appear to be scattered across several areas of theoretical concern, such as the relative merits of unimodel versus multimodel environments, the role of mathematics activity materials in the classroom, and the place of these materials in the instructional sequence (e.g., macro-instructional or micro-instructional; Kieren, 1971, p. 229). Vance and Kieren (1971) prefaced an examination of recent literature related to the organization and evaluation of elementary school mathematics laboratories with these suggestions:

> The teacher, in examining any research or evaluation reports, might ask three questions:
> 1. What good ideas, evidences, and inspriation can I draw from this research or the evaluation of this program?
> 2. How can I adapt and use the ideas in the curriculum of my school or in my classroom?
> 3. How can I best evaluate the effectiveness of the new procedures (mathematics laboratories) as I use them with my students (pp. 585-586)?

This scattering of studies has hindered specific conclusions and recommendations. A possible exception to the above remarks may be the numerous studies concerned with the effectiveness of the Cuisenaire rods in mathematics classrooms, although, even here, the results are not conclusive (Fitzgerald, 1970, p. 18).

Added to the inconclusiveness of these studies is the on-going theoretical discussion of active learning and mathematics activity materials for which no answers have yet been provided (Kieren, 1971, p. 232).

Studies related to active learning and mathematics activity materials with elementary teachers in either pre- or inservice classes are still quite rare. This section contains a review of these studies.

At the University of Minnesota, Hendrickson (1969a) compared the relative effectiveness of a teaching method incorporating some experiences in a mathematics laboratory, an approach incorporating enrichment problems, and a conventional lecture technique with respect to student achievement in mathematics and change in student attitude toward mathematics.

Within two independent levels (determined by scores on Factor IV of the Cattell 16 PF Inventory), 90 students were randomly assigned to one of the three groups. One teacher was involved in the study which ran for 10 weeks.

The control and enrichment-treatment groups attended four 50-minute lectures per week with the latter group receiving exercises that involved pattern discovery, mathematical creativity, and unique problem solving situations in place of text exercises. The laboratory-treatment group received three 50-minute lectures per week plus a two-and-one-half-hour laboratory session in which they pursued materials (e.g., Multibase Arithmetic Blocks, Attribute Games, Cuisenaire rods) leading to the discovery of mathematical ideas. All groups used the same text.

Data used in the study were derived from the pre- and posttests: (a) Structure of the Number System, (b) The Arithmetic Attitude Scale, (c) The Math Attitude Scale, and (d) The Mathematics Semantic Differential. A final examination was also administered.

Hendrickson found that there were no significant differences among groups or between levels on mathematical achievement and the only significant differences in attitude towards mathematics, based on The Math Attitude Scale, favored the control group. The poorest attitudes shown were in the laboratory group.

All treatments showed significant gains from pre-test to posttest in achievement and gains in attitude.

One wonders if the additional time required in the classroom by the laboratory group may have biased the attitudinal results.

Bluman (1971) conducted a study at the Community College of Allegheny County using a course (Introduction to College Mathematics) taken by liberal arts and education majors. Surveyed were topics from logic, sets, the real number system, algebra, geometry, statistics, and computers. Bluman implemented a laboratory method with two groups using filmstrips, experiments, demonstrations, and problem sessions. Instructor lectures were kept to a minimum and used only when audio-visual materials were not available. Supplementary books and programmed materials were placed in the learning center for student use. In addition, each student in the laboratory group received a manual which divided the course into 24 units. Each unit consisted of: (a) an overview, (b) the topic, (c) suggested textbooks, (d) topic objectives, (e) methods of instruction, (f) procedures, (g) evaluation activities, (h) homework assignments, and (i) suggestions for further study.

Two control groups were taught by the traditional lecture method. Two instructors were used; each taught a laboratory and control group.

Bluman used a two-way analysis of variance (treatment, instructor) to measure achievement for the groups. He found no significant differences for treatment or instructor but did find an unexplained significant interaction. Measuring attitude by Harshbarger's Instrument and Seager's Diagnostic Instrument of Supervision, Bluman found that students using the laboratory method had a more favorable attitude towards mathematics. In addition, they did more self-initiated study than was required.

In another doctoral study Smith (1970) investigated the effects of laboratory experiences in a course (Introduction to Mathematics) at West Virginia University which, although not specifically designed for prospective elementary teachers, did include content on decimal and nondecimal numeration systems.

Forty-eight students from two sections were partitioned evenly into a control group and three experimental groups. The control group received four lectures. The three experimental groups received varying treatments: (a) one lecture and three laboratory sessions, (b) two lectures and two laboratory sessions, and (c) three lectures and one laboratory session.

A traditional lecture technique was used. The laboratory sessions utilized individual stations; each station equipped with an open-end variable-base abacus (bases 2,3,4,5) which the student constructed, adjusted, counted with, and demonstrated notions.

Using analysis of covariance Smith found that the laboratory experience of the three experimental groups did facilitate learning of concepts and improve the retention of those concepts significantly. He concluded that students who are neophytes in a discipline may learn better when they proceed from concrete to abstract by means of models.

Implementers of laboratory programs have assumed that elementary teachers who receive laboratory experiences will use the approach subsequently when teaching. Two doctoral studies investigated this assumption.

Boonstra's (1970) recent clinical study, conducted at Michigan State University, investigated the effects of laboratory experiences on the behavior of prospective elementary teachers who were engaged in student teaching. However, Boonstra was only able to give them two two-hour laboratory experiences using the activity materials in the Madison shoeboxes (Tower Puzzle, Peg Game, Geoboard, and Centimeter Blocks).

Boonstra observed their teaching only once using a tape recorder and movie camera which exposed a single frame of film once every three seconds. Verbal behavior in the classroom was analyzed by means of the Flanders Interaction Analysis.

Boonstra found that two experiences were not sufficient to cause these students to adopt a student-centered approach in their student teaching nor were they sufficient to cause these students to adopt a teaching technique in which children learn through the use of manipulative activity materials.

More recently Postman (1971) tested the assumption by analyzing videotapes of lessons taught before and after preservice teachers received laboratory experiences.

Postman constructed an observational schedule to measure the extent to which four components of the laboratory approach were present in instruction. The components were: (a) active use of concrete materials by students, (b) the guided discovery approach, (c) students working independently as individuals or in small groups, and (d) the teacher directing her comments to individuals or small groups.

In each of two studies, preservice teachers presented specified topics to a class of well-trained role-playing peers. The role-players' "knowledge" of the topic was also specified.

In the first study each teacher was videotaped before and after treatment. Fourteen members of the experimental group then received laboratory experiences for six weeks as part of a methods course; four control members received no laboratory experiences. Members of the experimental group were not encouraged to use a laboratory method during the second videotaping.

Postman found that gains on indices of the laboratory approach seemed generally unrelated to the laboratory experience. Some gains in the percentage of the teacher's comments directed to small groups may have been related to the laboratory experiences. No statistical tests were run and lessons taught prior to and following treatment were not always the same.

In the second study 20 preservice teachers using three different lessons were videotaped only once. Postman found significant between-topic differences in teaching behavior with these groups and cautioned researchers that topics can make big differences in teaching behavior.

Postman, like Boonstra, found that mere involvement in laboratory experiences is not sufficient to cause teachers to use the laboratory approach subsequently when teaching.

These latter two studies both tried to assess the effectiveness of laboratory experiences using film or video techniques. Hendrickson (1969b) reported on another assessment technique, asking the teachers' students.

At the University of Minnesota an Experienced Teacher Fellowship Program was conducted for elementary teachers in mathematics and science. One of the primary objectives of the program was to encourage these teachers to use more open-ended activities and inquiry oriented procedures in their teaching.

The 22 participating teachers were visited in their classrooms during the spring of the following year. Students in nine of these classrooms were given a short questionnaire whose items provided some indications of the kinds of activities the class engaged in, the teaching techniques used, and the attitudes of teachers and students. Nine comparison classrooms were also given questionnaires.

Feedback from the students indicated that the teachers who had participated in the Fellowship Program did more demonstrating and experimenting, gave students more opportunity to pursue open-ended activities, relied less on textbooks, but also tended to share their newly acquired knowledge in a "tell'em" way (Hendrickson, 1969b, pp. 773-774). Special training seemed to predispose a teacher to seek more suggestions from students for class activity of an experimental or problem solving nature.

Several recent reports concerning action research do exist, both in this country (Clarkson, 1970; Howell, 1972; Kipps, 1970; Spitzer, 1969) and abroad (Biggs, 1968; Matthews, 1968). These reports suggest numerous positive effects with pre- and inservice elementary teacher method and content courses when mathematics activity materials are used. Indeed, it is difficult to find unsatisfied researchers who have employed these materials. However, the conclusions of many of these reports lack experimental documentation.

CHAPTER III

IMPLEMENTATION OF THE STUDY

Introduction

The purpose of the study was to compare the mathematical creativity of two classes of prospective elementary teachers in relation to a preservice content course in mathematics, one section of which was exposed to mathematics activity materials. A second purpose was to compare the mathematical achievement of these two classes at the end of the course.

Fifty-four students were involved in the study composing the experimental and control groups. Group descriptions and between-group comparisons are the subject of the first section of this chapter.

The major factor which differentiated the treatments for the two groups was the use of mathematics activity materials with the experimental group. The classroom environment for this group and the control group is described in the second section of this chapter.

Two equivalent forms of a creativity test and an achievement test were administered to each student. The construction and instrumentation of these tests and relevant information concerning validity and reliability estimates are the subject of the third section of this chapter.

The experimental design consisted of a comparison of pretest and posttest scores in mathematical creativity and a comparison of mean achievement scores for the experimental and control groups. This design is discussed in the fourth section of this chapter.

Two hypotheses were tested during the study. In the final section of this chapter both hypotheses and their alternates are cited and the corresponding statistic to test each hypothesis is explained and analyzed.

In summary, the sections of this chapter are: (a) The Sample; (b) Classroom Environment; (c) Instruments; (d) Experimental Design; and (e) Hypotheses.

The Sample

The population for the study consisted of 54 students who had enrolled in the two sections of Ms 100 (Elements of Mathematics I) given on the Portland Campus of the University of Maine at Portland-Gorham during the fall semester of 1971.

The experimental group, which met from 11:00 to 11:50 a.m., consisted of 7 males and 23 females. The remaining 10 males and 14 females comprised the control group, which met from 1:00 to 1:50 p.m. Each group met on Monday, Wednesday and Friday over a period of 16 weeks.

All 42 sessions for both groups were taught by the researcher.

The experimental and control groups were compared through various information sheets. Previous mathematical background, college classification, and grade point averages were found for both groups. This information is summarized in Tables 1, 2, 3, and 4.

The average number of high school courses in mathematics completed by the experimental and control groups was 3.3 and 3.0, respectively. Four students in the experimental group took a total of 15 semesters of mathematics in college. Seven students in the control group took a total of 14 semesters of mathematics in college.

TABLE 1.--Years of High School Mathematics Courses Completed by the Experimental and Control Groups.

	Experimental		Control	
Years	Number	Percent	Number	Percent
1	0	.00	0	.00
2	3	.10	3	.13
3	15	.50	18	.75
4	11	.37	3	.13
5	1	.03	0	.00

TABLE 2.--High School Mathematics Courses Completed by the Experimental and Control Groups.

Courses	Experimental		Control	
	Number	Percent	Number	Percent
Algebra I	28	.93	20	.83
Algebra II	24	.80	22	.92
Geometry	29	.97	22	.92
General or Business math	5	.17	1	.04
Senior math	14	.47	7	.29

TABLE 3.--Experimental and Control Group Student Classification.

Class	Experimental		Control	
	Number	Percent	Number	Percent
Freshman	7	.23	12	.50
Sophomore	14	.46	5	.21
Junior	6	.20	5	.21
Senior	3	.10	2	.08

TABLE 4.--Experimental and Control Group Grade Point Averages.

Range	Experimental		Control	
	Number	Percent	Number	Percent
0.00 - 1.95	3	.10	0	.00
1.95 - 2.25	5	.17	1	.04
2.25 - 2.75	10	.33	11	.46
2.75 - 3.35	8	.27	10	.42
3.35 - 4.00	1	.03	1	.04
Unknown	3	.10	1	.04

Note: Based on the rating scale A = 4.0, B = 3.0, C = 2.0, D = 1.0, and F = 0.0.

The coding system at the University of Maine at Portland-Gorham categorizes Ms 100 as an introductory level course; such courses are usually taken by freshmen and sophomores. Seventy percent of the experimental group and 71 percent of the control group had these class standings. The breakdown of these two classes, seen in Table 3, indicated a larger percentage of freshmen in the control group than the experimental group.

Classroom Environment

The students in the control group received a traditional lecture-textbook presentation. On the average, the first 10 minutes of each class were concerned with either student questions or homework assignments. The remaining 40 minutes were devoted to newer materials. Extensive use was made of the blackboard. The researcher employed a very informal teaching style and tried to elicit student responses. Open-ended questions were used whenever possible. Methodological aspects and historical anecdotes related to the concepts or skills being studied were frequently given. The mathematics activity materials which were illustrated in the textbook (e.g., number line, Sieve of Eratosthenes) were used.

The informal teaching style employed with the control group was also used with the experimental group. There was, however, additional use of mathematics activity materials. On the average, less than 25 minutes per session were devoted to lecture techniques. Within this

25 minute period numerous mathematics activity materials were used. Frequently these materials were used with an overhead projector. For example, order properties of the whole numbers were introduced on the overhead projector using sets of chips, the centimeter rods, and a transparent number line. During the lecture period 45 overhead transparencies were used. Neither the transparencies nor the overhead projector were used with the control group. Appendix D contains sample transparencies.

The experimental group saw two films: Piaget's Development Theory: Classification, and The Whole Number System: Key Ideas. Eight films had been requested from the Audio-Visual Center at the University of Maine at Orono; only these two were delivered.

Also used during this lecture period were teacher-class activities, such as INTEO, a bingo game using positive and negative integers. INTEO was used to reinforce the four operations of arithmetic on the set of integers. The researcher acted as caller and each student played on his own INTEO card.

The remaining class time, which average 25 minutes per session, was devoted to student questions, homework assignments, and individual and small group activities.

Thirty-three activity sheets were prepared and distributed to each student. Most of these sheets were completed and discussed in class. Concrete materials, used with many of the activity sheets, could be borrowed by students who wished to complete activity sheets outside of class. Appendix A contains a list of these activity sheets with their area of emphasis. Sample activity sheets and a list of concrete materials are also included.

During this latter period the researcher circulated among the students to provide assistance or guidance, to ask other questions related to the materials being used, to offer further activities, or to participate with students in an activity.

A student instructional rating system form, adapted from a form used at Michigan State University, was distributed during the last class meeting to determine the quality of instruction to both groups. The results have been tabulated in Appendix E.

Both groups were responsible for the same material; they received the same homework assignments, course guides, quizzes, and final examination. The quizzes and final examination were given on the same days to both groups. The same classroom was used for both groups. The room contained eight tables and had a seating capacity of 32 students.

Instruments

Three general indices, developed by the researcher, were obtained for each student. Two of these indices concerned measures of creative thinking in mathematics; the third measured mathematical achievement.

Mathematical Creativity Test

Experimental tests now in existence which are used to measure creativity almost exclusively emphasize aspects of divergent thinking (Guilford, 1966, pp. 186-189). The construction of the two forms of the Mathematical Creativity Test used in the study reflect this experimental trend.

The evolution of the Mathematical Creativity Test began with the examination of the three known written tests in mathematical creativity (Buckeye, 1968, 1970b; Evans, 1964; Prouse, 1964) and several tests of general creativity. Each test was examined relative to preliminary objectives formulated by the researcher: (a) the test must be oriented towards mathematics; (b) the test must be designed for elementary teachers or prospective elementary teachers; (c) the test must be a written test which could be administered within a 50-minute session; (d) the test must be easily scored and yield a single raw score; (e) finally, the test must conform to the recent research on psychometric measurement of creativity. In particular, it must measure aspects of divergent thinking as envisioned in Guilford's Structure of Intellect (Guilford, 1970, pp. 157, 161).

A preliminary version of the test was constructed and contained six items. Two of the items were taken from Evans' test, two from Buckeye's tests and two were written by the researchers.[1] The latter two items were similar in

[1]Permission to use items from their tests was given by Evans and Buckeye.

format to Torrance's (1966b) Just Suppose and Unusual Uses Tests. All six items on the test contained two illustrative examples. A five-minute time limit was imposed on each item.

The preliminary version was piloted on 30 elementary teachers enrolled in a summer methods course in mathematics taught by the researcher. The test was scored for fluency and originality. The fluency score was obtained by allowing one point for each correct response. Originality was obtained by allowing two, one, or zero points depending on the relative merits of a response or its creative strength (Torrance, 1966b). The total score was the sum of the fluency and originality scores. The scoring procedure followed Prouse's method and was related to the scoring procedure employed by Wallach and Kogan (1965).

The results of the scoring, together with the thoughtful suggestions and criticisms from these teachers, resulted in the first revision of the test. Four of the original six items were reworded and the five-minute time limit was extended to seven minutes. This revised version was labeled Form A.

A second form of the test was constructed and labeled Form B. Of the six items included on Form B, three were taken from Evans' test, one from Buckeye's test, and two were written by the researcher.

Forms A and B of the creativity test were administered to 27 Ms 100 students who were not involved in the study. Fourteen students took Form A first while the remainder took Form B. The other form of the test was administered one week later. An analysis of the scores on these tests resulted in the removal of one item from both forms. This item, if included, would have reduced the Pearson product-moment correlation between the two forms from .73 to .43. The final version of Forms A and B are included in Appendix C.

Validity.--Attempts to establish some type of validity on creativity tests generally fall into one of three approaches: (a) identifying high and low groups on some measure and then determining whether or not they can be regarded as "creative"; (b) identifying criterion groups on some behavior regarded as creative and determining whether or not they can be differentiated from their peers by test scores; or (c) basing test results on research concerning the lives and personalities of eminent creative people, the nature of creative performances, and the functioning of the human mind (Torrance, 1962b, pp. 9-10, 1966b, p. 24). The first procedure relates to construct validity, the second concurrent validity, and the third content validity. Such procedures, however, have not been entirely successful (Cropley, 1967, p. 110). It was reported in section two of Chapter II that numerous authorities have been critical about this aspect of creativity tests. Such standards fall short of validity standards generally agreed upon by psychometricians (Razik, 1966a, p. 149).

Of the mathematical creativity tests reviewed, only validity estimates for Buckeye's tests could be obtained. Buckeye (1970b) reported construct validity by considering the difference between observed test scores and instructor categorization of each student's creative ability in mathematics. Students who scored higher than one standard deviation above the test mean were considered to be above average in creative ability, and students who scored lower than one standard deviation below the test mean were considered to be below average in creative ability in mathematics. Of the 254 students considered by Buckeye, 227 were rated similarly by the test and their instructors with respect to creative ability in mathematics.

The instructors in Baur's (1970, p. 28) study, employing Buckeye's construct validity procedure, rated 135 of 161 students on Form A and 138 of 161 students on Form B of Buckeye's tests as similar. Both studies, however, fail to state what criteria were used in the instructor's evaluation of the students.

Content validity, usually based on how adequately the content of the test samples the domain of behavior about which inferences are to be made, has been reported by Buckeye (1968, 1970b) on the basis of the careful construction of the instrument and the several revisions.

Although validity reports were not available for Evans' test, the content validity may be high due to the careful construction of the instrument and the pilot testing which preceeded the final version of the test.

The only estimate of validity that can be reported for the Mathematical Creativity Test concerns content validity. Of the ten items used on both forms of the Mathematical Creativity Test, six of the items had received extensive piloting and revision by either Buckeye or Evans. The remaining four items, developed by the researcher, were specifically chosen for their similarity to items employed on tests designed by Torrance. All ten items were checked against the researcher's objectives and received preliminary examination and, in some cases, revision during the two phases of pilot testing.

Reliability.--Most of the customary concepts of test reliability are relevant to the measurement of creative thinking abilities, however, the very nature of these abilities creates a number of problems. Torrance (1966b), in discussing reliability estimates, cautions that:

> Because . . . emotional, physical, motivational, and mental health factors affect creative functioning and development and may contribute to a lowering of . . . reliability as traditionally estimated, it should not be assumed that the measuring instruments are unreliable or lacking in usefulness (p. 18).

It was reported in section two of Chapter II that numerous authorities have shown that creativity tests have unsatisfactorily low reliabilities.

Of the mathematical creativity tests reviewed, only reliability coefficients for Buckeye's tests could be obtained. Buckeye (1970b) reports a reliability coefficient of .62 and .58 for his Forms A and B, based on Rulon's (1939) method. The equivalent form reliability coefficient, using Form A as one half and Form B as the second half with Rulon's method, was reported as .73. Similar results were obtained when Buckeye employed an analysis of variance procedure. Baur (1970, p. 29) reported a Pearson product-moment correlational coefficient of .72 for the equivalent forms of Buckeye's tests.

The reliability coefficients for the separate forms of the Mathematical Creativity Test were computed by Hoyt's Analysis of variance method (Myers, 1966, pp. 294-299). This method was used rather than a split-half or Rulon method because each form had an odd number of items, each of which had the same time limit. The test could not be split and still retain the needed assumptions for the split-half or Rulon method. Furthermore, the items differed in levels of difficulty which, unless adjusted, would tend to inflate error estimates and lower estimates of internal consistency (Myers, 1966, pp. 297-298).

Reliability coefficients for Forms A and B, based on the 27 subjects who piloted the tests, were .24 and .13, respectively. These coefficients are measures of internal consistency. When tests have high internal consistency, they can be expected to measure the same thing, or things, and they are generally regarded as functionally homogeneous (Guilford, 1956, p. 446). This was not one of the objectives in constructing these tests (and it was one reason Buckeye's tests were not used). Guilford (1970) has identified 23 of 24 factors of intellect in the divergent thinking categories of his Structure of Intellect model. In the selection of items for the Mathematical Creativity Test, heterogeneity of divergent thinking items, rather than homogeneity, was a desired goal.

Another desired goal, however, was a relatively high measure of equivalence between the two forms. A Pearson product-moment correlation coefficient was used to obtain a measure of equivalence between Forms A and B of the Mathematical Creativity Test. This coefficient was .73 for the pilot group, which was significant at the .05 level.

Table 5 summarizes the reliability coefficients obtained for the pilot group, the experimental group, the control group, and both groups during the study.

Final Examination

The final examination in Ms 100 served as an index for both groups for measuring mathematical achievement. The examination was constructed with the aid of a 4 x 5 matrix model whose rows differentiated the cognitive levels of comprehension, computation, applications and analysis. The columns were partitioned according to chapters in

TABLE 5.--Reliability Coefficients for the Mathematical Creativity Test.

Groups	Form A[a]	Form B[a]	Equivalent Form[b]
Pilot	.24	.13	.73[c,e]
Experimental	.17	.32	.57[d]
Control	.09	.14	.68[d,e]
Experimental and Control	.18	.21	.61[d,e]

[a]Computed by analysis of variance.

[b]Pearson product-moment correlation coefficient.

[c]Between-test time interval was one week.

[d]Between-test time interval was 16 weeks.

[e]Significant at the .05 level.

Wheeler (1970), the textbook used in the course. The matrix model and its interpretation were adapted from the NLSMA studies (Romberg & Wilson, 1969). Test entries for the matrix model were selected from a data bank of test items compiled from six years of teaching similar classes by the researcher. Not each of the twenty entries in the model are represented by a specific question; the length of time spent on a particular topic or its importance governed the proportional number of test items chosen.

Thirty multiple-choice items were selected; each item had four answer choices. One point was awarded for each correct response. The total score was the number of correct responses.

The examination was checked for readibility and mathematical content by two colleagues in the Department of Mathematics who taught similar classes on either the Portland or Gorham Campus. Their suggestions resulted in minor revisions of the examination.

The examination was administered during a two-hour examination period on a Saturday morning, three days after the last class meeting. One of the students completed the examination in 47 minutes and two students remained for the entire two-hour period. The average time spent on the examination by the 54 students was 80 minutes.

Validity.--A measure of concurrent validity was found by computing a Pearson product-moment correlational coefficient between each student's final examination score and his total performance on the five quizzes. This coefficient was .67 for the experimental group, .42 for the control group, and .55 for both groups. All three coefficients are significant at the .05 level.

The examination was carefully constructed using an established matrix model based on the contents of Ms 100. Test items had been used with similar students during the past six years. Two colleagues proofread the examination and revisions were made. Under such procedures, the researcher assumes that the test has a high degree of content validity.

Reliability.--Two reliability coefficients were computed for the control group, the experimental group, and both groups. These coefficients were determined by using a split-half (odd-even) method and the Kuder-Richardson method of rational equivalence (K-R 20). The results appear in Table 6.

TABLE 6.--Reliability Coefficients for the Final Examination.

Groups	Split-half Method[a]	K-R 20 Method
Experimental	.53	.68
Control	.64	.68
Experimental and Control	.58	.65

[a]Test items were placed on the final examination using a table of random numbers.

Inter-correlational Measure

A Pearson product-moment correlational coefficient was computed between Form B of the Mathematical Creativity Test and the final examination. The tests had been administered to both groups within a six-day period. The correlation between the two tests was .13 for the experimental group, .15 for the control group, and .14 for both groups. None of these correlations were significant at the .05 level.

This result would suggest that, relative to these groups, the Mathematical Creativity Test was measuring other aspects of mathematics besides mathematical achievement.

Experimental Design

The principal question regarding the study was: Does the use of mathematics activity materials in a preservice content course for elementary teachers increase these prospective teachers' mathematical creativity? A secondary question regarding the study was: Does the use of mathematics activity materials in a preservice course for elementary teachers decrease these prospective teachers' mathematical achievement?

Mathematical Creativity

The data required to answer the principal question were sought from the administration of a Mathematical Creativity Test. The experimental design for this portion of the study consisted of a comparison of pretest and posttest scores in mathematical creativity for experimental and control groups using an analysis of covariance.

During the third class meeting Form A of the Mathematical Creativity Test was administered as a pretest. Two absent students took the test outside of class two days later.

Although analysis of covariance controls for differences which may be present in the groups prior to treatment, initial comparisons for both groups on the pretest were conducted. After computing group means and standard deviations, a two-tailed Fisher's t test was employed to determine preliminary comparisons relative to the uncorrelated means of both groups. The result of this procedure is summarized in Table 7.

TABLE 7.--t Test Between Means of the Experimental and Control Groups on the Pretest in Mathematical Creativity.

Group	N	Mean	SD	df	t
Experimental	30	35.97	11.84	29	1.001*
Control	24	36.21	11.31	23	

*Not significant.

A t value of 2.008 was needed at the .05 level of significance; therefore, the obtained value of t, 1.001, was not significant when 52 degrees of freedom were used.

Form B of the Mathematical Creativity Test was administered as a posttest during the final week of classes on the same day of the week as the pretest had been given. All students were present for this test.

Mathematical Achievement

The data required to answer the secondary question were sought from the administration of a final examination. The experimental design for this portion of the study consisted of a comparison of uncorrelated group means for the experimental and control groups using a one-tailed Fisher's t test.

Hypotheses

Two hypotheses were tested in an effort to determine the use of mathematics activity materials in a pre service content course in mathematics.

The major null and alternate hypotheses were:

Ho: No difference exists in the average measure of mathematical creativity between the experimental and control group.

H_1: The average measure of mathematical creativity for the experimental group will exceed that of the control group.

The minor null and alternate hypotheses were:

Ho: No difference exists in the average measure of mathematical achievement between the experimental and control group.

H_1: The average measure of mathematical achievement for the control group will exceed that of the experimental group.

Statistics

The major hypothesis was tested by means of analysis of covariance. The dependent variable was the subject's posttest in mathematical creativity minus his pretest through analysis of covariance.

Elashoff's (1969) discussion was accepted as a basis for using this statistic. She indicated that analysis of covariance is widely used to adjust criterion scores such as a posttest for the effects of a covariate such as a pretest in order to compare treatments. In general, analysis of covariance controls for differences which may be present in the groups prior to treatment while assessing differences between the groups following treatment. Elashoff noted that one research question that could be answered by analysis of covariance is: "are the average criterion scores significantly different for the . . . treatments (p. 384)?"

Elashoff also discussed the conditions the data must satisfy for covariance analysis to be a valid technique.

> The analysis of covariance is a valid technique for testing for differences in average criterion scores among treatments if we can assume:
>
> a) random assignment of individuals to treatments,
> b) within each treatment, criterion scores have a linear regression on x [covariate] scores,
> c) the slope of the regression line is the same for each treatment (there is no slope-treatment interaction),
> d) for individuals with the same score x, in the same treatment, criterion scores, y, have a normal distribution,
> e) the variance of the distribution of y scores for all students with the same x score in a particular treatment is the same for all treatments and x scores.
> f) criterion scores are a linear combination of independent components; an overall mean, a treatment effect, a linear regression on x, and an error term (p. 385).

Assumptions (a), (d), and (e) parallel the assumptions for analysis of variance. That is, the residuals or error terms are assumed to be normally and independently distributed with zero means and equal variances for each population group (Myers, 1966, p. 302). Assumptions (b) and (c) are peculiar to the covariance analysis and assumption (f) describes the components of the statistical model for analysis of covariance. In using analysis of covariance in the study these assumptions have been considered and regarded.

Also considered were the effects on the analysis of covariance when as assumption was not satisfied. Such a case occurs with assumption (a). In this study students were not randomly assigned to experimental and control groups. The design employed intact groups and the treatment was randomly assigned to one of the groups. However, the covariance analysis can still be used when assignment to groups is not random. Elashoff (1969) described the two major difficulties, involving interpretation of the covariance analysis, in this situation:

> First, we can never be sure that the covariance adjustment has removed all bias--some bias may still be present from a disturbing variable which was overlooked. Secondly, when the x variable [the covariate] shows real differences among groups covariance adjustments may involve extrapolation (p. 387).

The two assumptions peculiar to analysis of covariance, (b) and (c) are considered in conjunction with Table 8 in Chapter IV. The regression lines have been shown to be parallel with the proper form of the regression equation fitted to the line.

It is assumed that the covariate measure, the pretest, is independent of the treatment effect since it was administered prior to the treatment.

The minor null hypothesis was tested by means of a one-tailed Fisher's t test. This statistic utilized the small sample uncorrelated data formula with the means and standard deviations on the final examination for both groups. The rationale for using this statistic was found in Guilford (1956, p. 220).

The small sample uncorrelated data formula requires four assumptions: (a) random sampling (independence between and within groups), (b) normality of the populations (or normality of the sampling distributions of the sample means by virtue of the Central Limit Theorem), (c) equality of population variances, and (d) the number of subjects in both groups should not both exceed thirty (McSweeney, 1970, pp. 5-6). These assumptions have been considered and regarded in the study.

The previous remarks relative to random assignment of treatment to intact groups also apply with this statistic. The Fisher t test, however, can still be used when assignment to groups is not random if group means are used as the basic observations, and treatment effects are tested against variations in these means (Campbell & Stanley, 1966, p. 23). In this case there are no formal means of certifying that the two groups would have been equivalent had it not been for the treatment. Furthermore, if the means on the final examination of the two groups do differ, this difference could well have come about through the differential recruitment of subjects making up the groups.

The groups might have differed anyway, without the occurrance of treatment (Campbell & Stanley, 1966, p. 12).

The assumptions relative to homogeneity of variance are considered in conjunction with Table 11 in Chapter IV. The population variances for the two groups has been shown to be the same.

In testing both hypotheses significant differences between the experimental and control groups were conceded for measures which exceeded the .05 level of confidence. The University of Maine's IBM 360 computer was used in testing the hypotheses.

CHAPTER IV

ANALYSIS OF DATA

Introduction

Data for the study were gathered from five sources: observational notes, student information sheets, an instructional rating system form, Forms A and B of the Mathematical Creativity Test, and a final examination.

To obtain the first set of data, observational notes recording students use of activity materials, attitudinal responses, and attendance records were maintained. Additional notes were compiled during testing sessions. These data are contained in the first section of this chapter.

Some results from the student information sheets which provided vital demographic data on the experimental and control groups have been tabulated in section one of Chapter III. Further discussion of that data and additional unreported data from these sheets appear in the second section of this chapter.

Following the treatment period a student instructional rating system form was given to the experimental and control groups. Summarized data from this rating form are reported in the third section of this chapter.

Two null hypotheses were tested during the study. In the final section of this chapter these hypotheses are restated and tests of significance are presented and interpreted with respect to the data from the mathematical creativity tests and the final examination.

In summary, the sections of this chapter are: (a) Observational Notes; (b) Student Information Sheets; (c) Student Instructional Rating System Form; and (d) Test Data.

Observational Notes

Excerpts from the observational notes were reported to illustrate and describe relevant phenomenon apparent during the treatment and testing sessions.

The notes provided only a minor portion of the data collected during the study. They indicated that both groups were informed during the first class session that they were involved in an experimental study dealing with mathematics activity materials and that only the experimental group would be using these materials in class. They were not told the purpose of the study. It was observed that initial reaction from both groups was favorable although the control group was a bit disappointed with their role. No students attempted to change groups.

During the entire treatment period students were extremely cooperative and interested. After the study was completed several students requested information on the purpose of the study and the results of testing. Group pride was noted throughout the treatment period, especially following written quizzes.

Initial enrollment figures indicated that 32 students enrolled in the experimental group and 26 students enrolled in the control group. The notes showed that two students in each group dropped the course. Three of these students withdrew from the university and the fourth student changed her program.

Classroom attendance was optional. The notes indicated that no student in either group attended all 42 sessions. Four students in the experimental group were absent 10 or more times each with a maximum of 12 absences for two students. Seven students in the control group were absent 10 or more times, one student had 15 absences, another 17. The average number of absences for the experimental and control groups was 4.7 and 6.2, respectively.

In the experimental group student response to the mathematics activity sheets and materials was observed to be good. Students were quick to express apparent disappointment on days when activity materials were not being used. On the average, however, responses by these prospective teachers appeared to be less favorable than responses made by inservice teachers who had piloted many of the activity sheets and materials in two prior courses.

Students in the experimental group preferred to work on the activity sheets in small groups rather than individually. These small groups were observed to remain almost the same throughout the treatment period although students could select their own seats.

During those class sessions when more than one activity sheet was available, most students first chose those sheets which used concrete materials to those not needing concrete materials. The latter activity sheets were often completed outside of class if classtime elapsed. Students were permitted to borrow concrete materials outside of class to complete activities or pursue related activities. The notes indicated that this occurred only five times. In addition, two students borrowed concrete materials to present individual reports in other courses they were taking. This additional use of the activity materials, however, was offset by the number of students who left partially completed activity sheets on their tables at the end of a class session.

The notes showed that no student absent for a class ever requested activity sheets used during his absence. Some students collected activity sheets for absent friends but no records were kept on the frequency of this practice.

Notes related to the testing sessions indicated that the posttest in mathematical creativity was administered on the same day of the week and during the same times during the day as was the creativity pretest. The pretest and posttest sessions were similar and the testing and scoring was conducted under analogous circumstances by the researcher.

The final examination was administered to both groups at the same time and in the same room. The final examination was scored by the researcher first, then sent to the Testing and Research Center at the University of Maine at Orono for a complete data screening.

Student Information Sheets

Two information sheets were given to the experimental and control groups which requested data on previous mathematical training, college classification and major, and grade point averages. Written responses were obtained from all students in both groups.

A comparison of the experimental and control groups' mathematical background, summarized earlier in Tables 1 and 2, indicated that the groups appear to have similar high school mathematics backgrounds with at least 80 percent of the students in both groups completing two years of algebra and one year of plane geometry.

More students in the control group had taken other mathematics courses in college but more college mathematics courses had been completed by students in the experimental group.

One student in the experimental group had received prior experiences with mathematics activity materials. No student in the control group had indicated prior use with these materials although a few had heard about some of the more popular materials, such as the Cusineaire rods.

A comparison of the experimental and control class standings was summarized earlier in Table 3. It indicated that one-half of the students in the control group were freshmen; only one-fourth of the students in the experimental group had this class standing. Group percentages were reversed for sophomore students.

The course used during the study is designed for elementary education majors. Only 15 of the 30 students in the experimental group and 13 of the 24 students in the control groups reported that their major was elementary education. Students at the university need not declare majors until the end of their sophomore year and five students in each group left this question blank. It appeared that most of the remaining students were taking the course to complete the mathematics and science requirements, a portion of the general education requirements imposed on all university students. This condition may have been a significant factor in the outcome of the study.

Overall academic ability, measured by grade point averages, appeared to favor students in the control group. The grade point range, reported earlier in Table 4, indicated that 88 percent of the students in the control group had grade point averages between 2.25 and 3.35; only 60 percent of the students in the experimental group reported grade point averages in this range. The 19 freshmen in the study, however, had received no official grades from the registrar when this question was answered.

<u>Student Instructional Rating System Form</u>

Both experimental and control groups were taught by the researcher. The instructional technique, described in section two of Chapter III, was characterized as informal. Open-ended questions were used whenever possible and student responses were elicited. Methodological aspects and historical anecdotes related to course topics were used.

During the last class session a student instructional rating system form was distributed to the experimental and control groups. Written responses were obtained from 24 of the 30 students in the experimental group and 23 of the 24 students in the control group. An average group response was determined for each item on the form. Data from the rating form were used to determine the quality of group instruction. These data are reported in Appendix E.

From the data reported it appeared that the control group found the instructor more enthusiastic and more interested in teaching. His use of examples or personal experiences appeared to help them more with topics in class. They also found it easier to take notes.

Both groups were responsible for the same material; they received the same assignment sheets and tests. In addition, mathematics activity materials were used with the experimental group. Data from the rating form were also used to determine an evaluation of the course design and topics.

From the data reported it appeared that the experimental group found that the instructor covered too much material. They also found that the homework assignments were too time-consuming relative to their contribution to understanding of course material.

The control group found the course more challenging, yet better organized. They considered themselves more attentive in class.

Any of these conditions may have been a significant factor in the outcome of the study.

Test Data

Two null hypotheses were tested in an effort to determine the use of mathematics activity materials in a preservice content course in mathematics for elementary teachers. Data which related to these two hypotheses were provided from the administration of three tests. Preceding the treatment period Form A of the Mathematical Creativity Test was administered to the students in the experimental and control groups. At the conclusion of the treatment period Form B of the Mathematical Creativity Test and a final examination were administered to the same students.

Mathematical Creativity

The major null hypothesis was: No difference exists in the average measure of mathematical creativity between the experimental and control group. The statistic used to test this hypothesis was analysis of covariance. This statistic adjusted the posttest data in mathematical creativity (Form B) for the effects of the pretest data (Form A) to assess differences in the experimental and control groups due to treatment. The data given in Table 8 related to the testing of that hypothesis.

TABLE 8.--Analysis of Covariance Table.

Source	df	SS	MS	F
Treatments (Adj) (Between)	1	15.418	15.418	0.113*
Error (Adj) (Within)	51	6945.297	136.182	

Note: Homogeneity of regression F = 1.013 not significant.

*Not significant.

A test of homogeneity of regression must be met in order to justify the use of the analysis of covariance procedure. This test was considered to be met at the .05 level when 1 and 50 degrees of freedom were used if the F value for homogeneity of regression was below 4.030. The F value of 1.013 was well below the prescribed limit and hence the test for homogeneity of regression was met.

An F value of 4.028 was required to demonstrate significance at the .05 level for this hypothesis when 1 and 51 degrees of freedom were used. It is clear that the F value of 0.113 fell short of the value required to demonstrate significance on this measure. Average measures of mathematical creativity were not significantly different for the experimental and control groups.

Although the treatment did not result in significant differences for the experimental and control groups, differences between pre- and posttest data on the Mathematical Creativity Test indicated that further comparisons were in order. These were accomplished by using a one-tailed Fisher's t test on the correlated pre- and posttest means for each group. The result of this procedure is summarized in Tables 9 and 10.

TABLE 9.--t Test Between Means on the Pretest and Posttest in Mathematical Creativity for the Experimental Group.

Test	N	Mean	SD	df	t
Pretest: Form A	30	35.97	11.84	29	7.844*
Posttest: Form B	30	53.07	13.21	29	

*Significant at the .05 level.

A t value of 1.699 was needed at the .05 level of significance; therefore, the obtained value of t, 7.844, was significant when 29 degrees of freedom were used. The significant differences between the means indicated that it appeared probable the experimental group did differ in its mathematical creative ability on the pre- and posttest.

TABLE 10.--t Test Between Means on the Pretest and Posttest in Mathematical Creativity for the Control Group.

Test	N	Mean	SD	df	t
Pretest: Form A	24	36.21	11.31	23	8.534*
Posttest: Form B	24	54.29	13.56	23	

*Significant at the .05 level.

A t value of 1.714 was needed at the .05 level of significance; therefore, the obtained value of t, 8.534, was significant when 23 degrees of freedom were used. The significant difference between the means indicated that it appeared probable the control group did differ in its mathematical creative ability on the pre- and posttest.

Mathematical Achievement

The minor null hypothesis was: No difference exists in the average measure of mathematical achievement between the experimental and control group. This hypothesis was tested by means of a one-tailed Fisher's t test. This statistic used the small sample uncorrelated data formula with the means and standard deviations on the final examination for both groups. The data given in Table 11 related to the testing of that hypothesis.

TABLE 11.--t Test Between Means of the Experimental and Control Groups on the Final Examination.

Group	N	Mean	SD	df	t
Experimental	30	21.53	3.66	29	0.206*
Control	24	22.54	3.53	23	

Note: Homogeneity of variance F = 1.080 not significant.

*Not significant.

A test of homogeneity of variance was required in order to justify the use of the Fisher's t test. This test was considered to be met at the .05 level when 29 and 23 degrees of freedom were used if the F value for homogeneity of variance was below 1.967. The obtained F value of 1.080 was less than the prescribed limit and hence the test for homogeneity of variance was met.

A t value of 1.676 was required to demonstrate significance at the .05 level for this hypothesis when 52 degrees of freedom were used. The obtained t value of 0.206 fell below the value required to demonstrate significance on this measure. Average measures of mathematical achievement were <u>not</u> significantly different for the experimental and control groups.

CHAPTER V

SUMMARY AND CONCLUSIONS

Summary

This thesis reports the results of an experimental study to determine the use of mathematics activity materials in a preservice content course in mathematics for elementary teachers. This section contains a summary of that study.

The Problem

The recent trend in elementary school mathematics towards an active learning technique has been characterized by both an increased emphasis on the child in relation to contemporary mathematics curriculums and an increased use of activity materials in the classroom. As such, active learning concerns itself more with the processes learned by the child than the subject matter products he may acquire. Accordingly, a basic tenet of this approach is the provision of "meaningful" experiences to children which will foster creative mathematical activities, often of their own choosing. Such an approach requires both a knowledge of the child's developmental level and rich materials, tools, and teaching aids to stimulate and facilitiate these creative mathematical activities.

Elementary teachers, with no personal experiences in some sort of creative mathematical work, can scarcely expect to be able to inspire, to lead, to help, to stimulate, or even to recognize the creative abilities of their students. If teachers are to feel at ease with the techniques as well as the activity materials, then new teacher education programs must be designed to give these needed experiences. Such programs are emerging in colleges and universities. Research investigating their effectiveness, however, is scarce and few studies related to active learning and the use of mathematics activity materials have been conducted.

The Purpose

This study attempted to investigate one aspect of these emerging programs specifically, the use of mathematics activity materials by prospective elementary teachers. The purpose of the study was to compare the mathematical creativity of two classes of prospective elementary teachers in relation to a preservice content course in mathematics, one section of which was exposed to mathematics activity materials related to the topics in the course. A second purpose of the study was to compare the mathematical achievement of these two classes at the conclusion of the course.

The Literature

Research and theory related to the study was examined from the literature on both creativity and active learning. Of these two areas, literature related to creativity has been the more plentiful. Since 1950 much of the research on creativity has been conducted at six university centers. Studies at these centers have concentrated on five aspects of creativity: definitions and theories of creativity, characteristics of creative persons, conditions which influence creativity, techniques for nurturing creativity, and measuring creativity.

Definitions and theories of creativity usually emphasize novel combinations or unusual associations of ideas although no agreement has been reached regarding the degree of "unusualness." Some authorities require that such combinations or unusual associations have social or theoretical value; others are content if the creative product is "new" to the individual doing the creating.

Summaries of traits of creative individuals indicate that these individuals tended to be less repressed, less inhibited, less formal, less conventional and showed less authoritarian values. They tended to be more independent, more sensitive to certain experiences, more intuitive, and more perceptive. Creative individuals tended to be intrinsically motivated; they produced novel and unconventional solutions to problems and reinforcement resulted from satisfactorily completing tasks.

A number of studies have attempted to compare creativity and intelligence. The findings indicated that some individuals can be both intellectually gifted and highly creative. A certain amount of intelligence is required for creativity, but beyond that point being more intelligent or less intelligent does not appear to be a determinative of creative ability. Several authorities felt that all individuals are to some extent creative with considerable variation in ability and motivation existing. Studies in this area also indicated that teachers recognize and understand the highly intelligent student better than the highly creative one. Teachers have a decided preference for the highly intelligent student.

Most authorities believed that creativity could be nurtured although they expressed reservations about the direct teaching of creativity. Various suggestions have been compiled to nurture creative thinking. Validation studies which have used these suggestions have produced contradictory findings. Other studies have shown that the environment, including the social structure of the schools, had a marked effect on creativity.

Tests have been developed in an attempt to measure characteristics that are important to the creative process. These tests are generally multi-dimensional and have emphasized aspects of divergent thinking. Numerous authorities have been critical of aspects of these tests, such as their low reliability, questionable validity, and predictability. Much more research is needed in this area.

Despite the voluminous nature of research on creativity, little has been conducted in the area of mathematical creativity. Studies with prospective elementary teachers have found that classroom environments could be designed in content or methods courses in mathematics which appear to increase either general or mathematical creativity without having detrimental effects on subject matter achievement. Similar attempts at nurturing general or mathematical creativity in mathematics courses for students have produced contradictory findings.

Five noncommercial mathematical creativity tests have been constructed and tested on subjects from elementary to graduate school. These tests, like instruments designed to measure general creativity, are generally multi-dimensional and have emphasized aspects of divergent thinking. Only one of these tests has two equivalent forms available.

There is a limited but growing body of experimental research on active learning, including mathematics activity materials. These studies, dealing with various age and intellectual levels of children, are scattered across several areas of theoretical concern, which has hindered specific conclusions and recommendations.

Studies investigating the effects of active learning and mathematics activity materials in content or methods courses for pre- or inservice elementary teachers report contradictory findings in the areas of mathematical achievement and attitude towards mathematics. One study found an increase in general creativity due to the use of mathematics activity materials in a methods class.

Three studies have investigated the assumption that elementary teachers who receive laboratory experiences will use the approach subsequently when teaching. The findings of one study support this assumption.

The Study

The scope of the study was restricted to 54 prospective elementary teachers at the Portland Campus of the University of Maine. The 30 students who served as the experimental group and the 24 students who served as the control group were enrolled in two Ms 100 (Elements of Mathematics I) sections, a three-credit-hour course of special interest to prospective elementary teachers with major emphasis on an intuitive approach to the real number system. Both groups were taught by the researcher.

The control group received a traditional lecture-textbook presentation with the researcher lecturing approximately 40 minutes each session. The instructional technique was informal; open-ended questions were used whenever possible and student responses were elicited. Methodological aspects and historical anecdotes related to course topics were used. The remaining period, approximately 10 minutes each session, was devoted to student questions and homework assignments.

The same informal instructional technique was used with the experimental group. In addition, mathematics activity materials were used. Less than 25 minutes per session were devoted to lectures; within this period numerous mathematics activity materials were introduced, either through teacher-class activities or in conjunction with prepared overhead transparencies. Two films were shown. The remaining class time, approximately 25 minutes per session, was devoted primarily to individual and small group activities. Thirty-three activity sheets were distributed; concrete materials were used with many of the activity sheets. During this period the researcher circulated among the students to provide assistance or guidance.

Both groups were responsible for the same material. They received the same homework assignments, course guides, quizzes and final examination.

The Tests

Three tests were constructed by the researcher for the study. Two equivalent tests measured mathematical creativity, the third test measured mathematical achievement.

The construction of Forms A and B of the Mathematical Creativity Test was guided by the three known written tests in mathematical creativity and several tests of general creativity. Items on the test were selected relative to preliminary objectives formulated by the researcher.

Two pilot testing sessions were conducted; items were revised or eliminated following these testing sessions.

Each form of the creativity test contained five items; students were permitted seven minutes on each item. The test was scored by the researcher for fluency and originality. An estimate of content validity was based on the careful construction of the test. Internal consistency reliability coefficients were low (.13 and .24 for the pilot group on Forms A and B); the two forms of the test had a relatively high measure of equivalence (.73 for the pilot group).

The final examination was constructed with the aid of a 4 x 5 matrix model whose rows differentiated cognitive levels and whose columns were partitioned according to mathematical topics. Thirty multiple-choice items were selected. The examination was checked for readibility and mathematical content by mathematics educators and revisions were made. Measures of concurrent validity were found between the final examination and the five quizzes given during the semester (.42 to .67). A high degree of content validity was assumed. Several reliability coefficients were computed (.53 to .68).

Analysis and Results

Data concerning the major and minor hypotheses tested in the study originated from five sources; observational notes, student information sheets, an instructional rating system form, and the creativity and achievement tests.

The student information sheets indicated that the experimental and control groups had similar high school and college mathematics backgrounds. Approximately 70 percent of the students in both groups were freshman or sophomores. Overall academic ability, measured by grade point averages, appeared to favor the control group.

The observational notes and instructional rating system form indicated that, within the experimental group, student response to the mathematics activity sheets and materials was good. The students preferred to work in small groups rather than individually during these activities. These students, however, felt that the instructor covered too much material. They also felt that the course was less organized and expressed more difficulty in taking notes. The control group found the course more challenging. They also found the instructor more enthusiastic and more interested in teaching.

Data obtained from the mathematical creativity tests and final examination were analyzed statistically. Fisher's t tests were used to compare the following differences:

a. Means on the pretest in mathematical creativity for the experimental and control groups.

b. Means on the final examination for the experimental and control groups.

c. Means on the pre- and posttest in mathematical creativity for the experimental group.

d. Means on the pre- and posttest in mathematical creativity for the control group.

An analysis of covariance was used to adjust the posttest data in mathematical creativity for the effects of the pretest data to compare differences in the experimental and control groups due to treatment.

Pearson product-moment correlations were computed to determine the relationship between:

a. Forms A and B of the Mathematical Creativity Test for the experimental group, the control group, and both groups.

b. Form B of the Mathematical Creativity Test and the final examination for the experimental group, the control group, and both groups.

c. The final examination and the five quizzes for the experimental group, the control group, and both groups.

Hypotheses

The major aspects of the study were tested by the following null hypotheses:

Major hypothesis: No difference exists in the average measure of mathematical creativity between the experimental and control group.

Minor hypothesis: No difference exists in the average measure of mathematical achievement between the experimental and control group.

The major hypothesis was tested by means of analysis of covariance. The minor hypothesis was tested by means of a one-tailed Fisher's t test. Significant differences between the experimental and control groups were conceded for measures which exceeded the .05 level of confidence.

Conclusions

Neither of the two hypotheses were rejected. No significant differences were found between the average measure of mathematical creativity or the average measure of mathematical achievement for either group. The use of mathematics activity materials did not appear to increase the mathematical creativity of the experimental group nor did it appear to have a detrimental effect on the mathematical achievement of this group.

Significant differences for both groups between the pre- and posttest in mathematical creativity were found; the posttest means were higher. A preservice content course taught in an informal style appeared to have increased the mathematical creativity of both groups.

The correlation between measures of mathematical achievement and measures of mathematical creativity was not significant. These instruments were apparently measuring difference aspects of mathematical ability. It may be that mathematical achievement tests discriminate against students highly creative in mathematics.

Discussion

Several limitations must be kept in mind when interpreting the results of the study. There were only 54 students and one instructor involved in the study. Not all of the students were prospective elementary teachers. It was observed, however, that this latter group had mathematical backgrounds similar to the remaining students and they were evenly divided between groups.

Another more serious limitation was the restrictive nature of the three tests constructed for the study, together with their validity and reliability estimates. The mathematical creativity tests measured only a portion of Guilford's 24 divergent thinking factors of intellect and, like other measures of creativity, must be considered experimental in nature.

Several reasons were suggested for the significant differences found for both groups between the pre- and posttest in mathematical creativity. One reason may have been due to the researcher's awareness of and interest in creativity. A more plausible explanation, suggested by student responses, was linked to the indirect teaching technique used with both groups. Data from the pilot study suggested that scores in mathematical creativity may increase as a result of students taking creativity tests. This was offered as a partial explanation. However, the experimental and control groups had a much longer delay between tests than the pilot group.

The use of mathematics activity materials did not increase mathematical creativity. It was suggested that potential increases in mathematical creativity due to the use of mathematics activity materials may have been offset by losses in mathematical creativity due to the more disorganized experimental class whose students expressed more difficulty in taking class notes.

Further Research

Larger studies using other samples, carefully chosen variables, carefully designed treatments, and more sophisticated statistical measures are needed before generalizations can be made concerning the use of mathematics activity materials with prospective elementary teachers.

In addition, much more research is needed on developing and analyzing instruments which will accurately measure creative ability in mathematics.

Conclusions

The major aspects of the study were tested by two hypotheses. The following findings were derived.

a. Average measures of mathematical creativity were not significantly difference for the experimental and control groups.

b. Average measures of mathematical achievement were not significantly different for the experimental and control groups.

Neither of the two hypotheses were rejected. From these findings it appeared that the use of mathematics activity materials in a preservice content course for elementary teachers did not increase the mathematical creativity of the students using those materials. The use of those materials, however, did not appear to have a detrimental effect on the students' mathematical achievement.

Although the treatment did not result in significant differences for the experimental and control groups on either mathematical achievement or mathematical creativity, differences in the pretest and posttest means in mathematical creativity for each group were observed. Fisher's t tests were conducted to compare these pre- and posttest means for each group. The following findings were derived from these tests.

c. The mean pre- and posttest measures of mathematical creativity were significantly different for the experimental group. The posttest mean was higher than the pretest mean.

d. The mean pre- and posttest measures of mathematical creativity were significantly different for the control group. The posttest mean was higher than the pretest mean.

From these findings, it appeared that a preservice content course in mathematics, taught by an informal teaching style using open-ended questions and eliciting student responses, may have increased students' mathematical creativity.

A test of significance was conducted on the Pearson product-moment correlation between the posttest in mathematical creativity and the final examination. It was found that:

e. The correlation between measures of mathematical achievement and measures of mathematical creativity was not significant for the experimental group, the control group, or both groups.

This finding, coupled with the apparent increase from pre- to posttest in mathematical creativity, suggested that the mathematical creativity tests and the final examination, an achievement test, were measuring different aspects of mathematical ability. It may be that mathematical achievement tests discriminate against students highly creative in mathematics.

Discussion

The present study contains several limitations which must be kept in mind when interpreting the results. The number of students was small and at least 16 of the 54 students in the study reported that they were not prospective elementary teachers. The majority of these 16 students were taking the course only to complete general education requirements. This factor may not have been crucial in testing differences between groups since:

(a) these students were evenly divided between the two groups; (b) they had mathematical backgrounds similar to the remaining students; and (c) a trend favoring one of the groups was not observed.

Only one instructor, a mathematics educator, was involved in the study. As Baur (1970) observed, the results are certainly restricted in terms of generalizing to other classes with different instructors.

One of the more serious limitations of the study was the restrictive nature of the tests used and their validity and reliability estimates. All three tests were constructed specifically for the study and estimates of validity and reliability were a function of the tests and the students sampled. In addition, measures of creativity are still experimental in nature with many conceptual and technical difficulties (Yamamoto, 1966). The two creativity tests measured only a portion of Guilford's (1959a) 24 divergent thinking factors of intellect and a number of creative thinking processes remained untested.

Another limitation of the study would certainly be the use of intact groups rather than students randomly assigned to groups. From preliminary comparisons these groups appeared similar in all areas except general academic ability (i.e., by grade point averages) with slight gains favoring the control group. The groups, however, were randomly selected to treatments and the statistical tests used to test the hypotheses permitted this procedure.

All but one student in the study had a posttest score in mathematical creativity greater than his pretest score. Average measures of mathematical creativity for both groups were significantly higher on the posttest than the pretest. There may have been several reasons for this result. The researcher was interested in and aware of creativity and the creative process. Torrance (1965b) and Williams (1964) have both demonstrated that, under these conditions, even without specific techniques for nurturing creativity, creative thinking scores increase sharply.

A more plausible explanation was suggested by student responses on the student instructional rating system form which indicated that an informal teaching technique was used, student responses were elicited, students were allowed to ask questions, and student opinion was encouraged. Such indirect teaching strategies have been found to increase creativity (Enochs, 1964) and are frequently included in lists to nurture creativity.

A portion of the explanation for the apparent gain may lie in the test instrumentation. Both forms of the mathematical creativity test were piloted on a group of students taking the same course but not involved in the study. Half of these students took Form A first, the remainder took Form B. The second form was administered one week later. Although the mean scores on these two forms were not significantly different for the pilot group, 81 percent of the pilot group showed some gain in mathematical creativity from the first test to the second test. These data suggested that scores on a creativity test may increase as a result of having taken a similar test. This explanation, however, seemed questionable in light of the sixteen-week time interval between tests for the experimental and control groups.

The analysis of covariance data revealed that the use of mathematics activity materials did not increase mathematical creativity. This result seemed to contradict the remarks by several authorities (Hallman, 1967; Parnes, 1967; Torrance, 1964) that the use of such materials would nurture creativity. What cannot be overlooked, however, is that these authorities have not suggested that creativity could be nurtured solely by using activity materials. Research findings in creativity support Gallagher's (1964) contention that nurturing creativity is too broad a concept to allow specific techniques to be identified or developed.

Furthermore, studies in creativity suggest that the acquisition of knowledge may have a far more powerful effect on creativity than any creative training procedures which could be devised (Williams, 1966, p. 77). Range of subject matter information and breadth of knowledge seem to be the crux for creative performance. Supportive research findings have indicated that individuals differ in their degree of creative potential in various fields. Whatever potential increases in mathematical creativity that may have resulted from the use of mathematics activity materials in the experimental group may have been offset by their losses in mathematical creativity due to the more disorganized experimental class whose students expressed more difficulty in taking class notes.

The latter data also ran counter to Buckeye's (1970a) findings that the use of a mathematics laboratory in a methods class significantly increased creativity. However, Buckeye did not report data from his own mathematical creativity test but chose to measure general creative ability. His mathematical creativity test did not correlate with the general creativity test he used and, in light of Williams' (1966) remarks, the researcher suspects that both sets of data were available but contradictory.

No significant correlation for either group was observed between the posttest in mathematical creativity and the final examination. Fourteen of the 54 students in the study scored higher than one standard deviation above their respective group means in mathematical creativity. Of these 14 students, only 4 also scored higher than one standard deviation above their respective group means on the final examination. It appeared that 10 of the 14 highly creative students, or 71 percent of this group, may have been discriminated against by the final examination. McCormack (1969), who obtained similar results, suggests that standard subject matter instruments should probably involve higher levels of thought, including creativity.

The results of the study were consistent with previous findings from several studies (Buckeye, 1970a; Bluman, 1971; Hendrickson, 1969a) that the use of mathematics activity materials does not have a detrimental effect on mathematical achievement.

Further Research

Several limitations of the study discussed in the previous section resulted from circumstances beyond the researcher's control. Although Ms 100, Elements of Mathematics I, is specifically designed for prospective elementary teachers, other students are not prohibited from enrolling in the course. As a result, a large number of non-elementary education majors participated in the study. Because of the special nature of the students used in the study, further research concerning the use of mathematics activity materials should be considered for other samples of prospective elementary teachers.

Four groups of randomly selected Ms 100 students had been requested for the study so that a Solomon Four-Group Design (Campbell & Stanley, 1966) could be used. This design would have enabled the researcher to consider not only the effect of the treatment but also the effect that the pretest in mathematical creativity had on the posttest. The latter effect remains untested and further research in this area is needed.

Cronbach (1966) suggests that interaction of variables such as subject matter, quality of experience, student characteristics, and particular outcome variables are certainly valid objects of study in determining for whom, for which topics, and with what materials are manipulative activities valuable. Baur (1970) considered three of these variables (i.e., creative problems, creative classroom, and instruction) when studying mathematical creativity in a course similar to the one used in this study and found significant and nonsignificant interaction effects. One additional question which remains unanswered in this study is the interaction of classroom instruction and the use of mathematics activity materials. Other formative studies using carefully chosen variables and carefully defined treatments should be conducted. Statistical measures, such as those suggested by Kieren (1971, p. 233) for investigating interaction effects, should be used.

Much more research is needed on developing instruments which will accurately measure creative abilities in mathematics. These instruments, together with statistical techniques similar to those used by Richards and Bolton

(1971), may provide additional answers to the theoretical questions posed by Kieren (1971).

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APPENDICES

APPENDIX A

LIST OF ACTIVITY MATERIALS WITH SAMPLE ACTIVITY SHEETS

LIST OF ACTIVITY MATERIALS WITH SAMPLE

ACTIVITY SHEETS

Concrete Materials

A Blocks

braid boards

centimeter rods (Houghton Mifflin)

cards (3x5)

chips

coins

Color Cubes

colored blocks

colored paper

counters

Creature Cards

Cuisenaire rods

dice

Dienes' Multibase Blocks

graph paper

INTEO (bingo game)

lattices

minicomputers

Napier's bones

number lines

one-inch blocks

overhead projector

peg games

People Pieces

popsicle sticks

puzzle games (miscellaneous types)

scissors

Sieve of Eratosthenes

soma cubes

string

tangrams

tower puzzles

Activity Sheets

	Area of Emphasis	Title of Activity	Pages
1.	Logic	Tangrams	5
2.	Logic	Magic Square	1
3.	Logic	Stick Puzzles	1
4.	Logic	Soma Cube	2
5.	Set Theory	Attribute Games and Problems	6
6.	Pattern Recognition	Providing Drill and Practice Using a Pattern Problem	1
7.	Pattern Recognition	Regions in a Circle	1
8.	Pattern Recognition	Interior Angles of any Polygon	1
9.	Pattern Recognition	Sum of the Numbers $1 + 3 + 5 + \ldots + 25$	1
10.	Pattern Recognition	Number Series	1
11.	Pattern Recognition	Continued Dividing and Summing	1
12.	Relations	Finding the Relation	4
13.	Properties of Relations	Special Relations	1
14.	Functions	Surface Area	1
15.	Functions	Peg Game	1
16.	Functions	Tower of Hanoi Puzzle	1
17.	Functions: f(x) Notation	Function Notation	3
18.	Mathematical Systems and their Properties	Flips and Turns of a Rectangle	3
19.	Mathematical Systems and their Properties	Braid Arithmetic	3

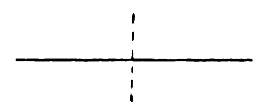
	Area of Emphasis	Title of Activity	Pages
20.	Whole Numbers: Basic Facts	Basic Facts (Cuisenaire rods)	3
21.	Whole Numbers: Operations and Order	Number Lines	2
22.	Whole Numbers: Algorithms	Napier's Bones	2
23.	Numeration: Place Value	Chip Trading	2
24.	Numeration: Place Value	Multibase Arithmetic Blocks	3
25.	Numeration: Place Value	Minicomputers	3
26.	Integers	Activities with Integers (Puzzles)	1
27.	Integers	Point Plotting with Integers	2
28.	Integers: Four Fundamental Operations	INTEO (bingo game)	1
29.	Integers: Operations and Order	Greatest or Least Number (Dice Game	2
30.	Integers: Operations and Order	The Celsius Thermometer	2
31.	Integers: Operations and Order	Postman Stories	2
32.	Primes and Composites	Sieve of Eratosthenes	3
33.	Factoring	Prime Factorization	1

Sample Activity Sheets

PATTERNS: Continued Dividing and Summing

The exercise below illustrates very nicely how a generalization of a pattern provides us with an answer that would be almost impossible to physically perform.

1. Take a piece of string 12 inches long. Cut it in half and drop one of the halves on the table.



2. Take the remaining half and cut it in half. Drop one of these pieces on the table.

3. What is the total length of the string that is on the table?

   _____ inches + _____ inches = _____ inches

4. Now take the piece of string in your hand and cut it in half. Place one of these pieces along with the others on the table.

   What is the total length of the string that is now on the table?

   _____ inches + _____ inches + _____ inches = _____ inches

5. Now suppose this process of halving could be continued five more times. What would be the total length of string on the table?

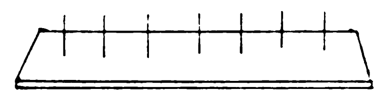
   Could you do the problem more quickly by arithmetic. What pattern were you following above?

NUMBERATION: PLACE VALUE Chip Trading

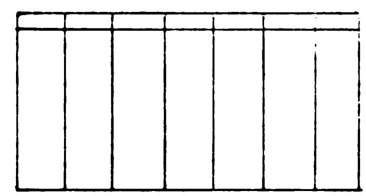
Activities designed to give students an understanding of place value concepts have always been used by elementary teachers. Such activities have included a multitude of concrete and semi-concrete materials, such as (a) place value charts and tables, (b) bundles of pencils, straws, popsicle sticks, (c) counting men, (d) open and closed abaci, and (e) more recently, the Cuisenaire rods and the Dienes' Multibase Arithemetic System.

The "Chip Trading" activities were developed at the Cambridge Conference workshops. They represent a series of games in exchanging colored plastic chips which are designed to give students a basic understanding of the meaning of position and the value of a number, based on its position in a numeral. Each game sets up a fixed rate of exchange between chips of various colors, and the students physically carry out "exchanges" by trading chips back and forth.

Although not essential, it is possible to use colored chips which have holes in them to allow them to be placed on nailboards or open abaci. The nailboards should be about 14 inches long and 2 inches wide with at least 7 nails.



An alternate technique is to use a "game board" constructed from a piece of paper having equally spaced vertical lines. Children enjoy coloring the top portion of each column according to the exchange rate for a game.



So that students can make exchanges readily in many different contexts without getting involved in large quantities, it is best to begin with small exchange rates, such as three. Let the students make their own arbitrary decision as to the hierarchy of colors. One possible exchange rate for the colors white, yellow, blue, red, and black is:

3 whites = 1 yellow	3 blues = 1 red
3 yellows = 1 blue	3 reds = 1 black

1. To begin a game collect 4 or 5 of your classmates; designate one student as banker whose initial responsibility is to order the chips by color on the nailboard or game board.

2. Each player in turn tosses a die which tells him how many chips of the first color the banker must give him (with the color scheme above that would be white). Whenever a player receives three chips of the same color (using 3 as the exchange rate), he requests the banker to exchange them for 1 chip of the next higher value. Each player must keep his collection of chips on his own game board. The player who is first to get a black chip wins the game and becomes banker for the next game.

3. Variations of the game can be developed by using a different coloring scheme or a different exchange rate. For example, you may wish to play a few games using 3

as an exchange rate and then move the game to the country of "Pento" which has an exchange rate of 5 to 1.

4. Another variation of this game is to request the banker to give you the least number of chips equivalent to the number you have rolled on the die. Additional exchanges must still be made by players.

5. Have one player select some chips and hide them behind his back. If he tells you his exchange rate and the value of his chips in terms of whites, can you guess exactly which chips he has chosen. For example, using an exchange rate of 3, if I had chips totaling 11 white chips I could have:

   (a) 11 white chips
   (b) 8 white chips and 1 yellow chip
   (c) 5 white chips and 2 yellow chips
   (d) 2 white chips and 3 yellow chips
   (e) 2 white chips and 1 blue chip

6. Select an exchange rate and a handful of white chips. What is the smallest number of colored chips another player could have which would be equivalent to your handful of white chips?

7. For a given colored chip, perhaps red, and a given exchange rate what is the value of the red chip in terms of white chips?

8. For a given exchange rate suppose two players each took a handful of various colors. What are some methods that could be used to determine who had a higher value (relative to white)?

9. If two players each had a handful of various colors and these handfuls were combined how might the total value of the chips (relative to white) be determined?

10. For a given exchange rate if you select a handful of various colored chips and a classmate then requests a specific number of chips from your handful, how could you determine if that request could be granted?

11. What other activities could you invent with chips?

APPENDIX B

Ms 100 ASSIGNMENTS, QUIZZES, AND FINAL EXAMINATION

Ms 100 ASSIGNMENTS, QUIZZES, AND FINAL EXAMINATION

Ms 100 Assignemnts

Textbook: Modern Mathematics: An Elementary Approach. Second Edition by Ruric E. Wheeler (Brooks-Cole Publishing Co., 1970).

Class Meetings	Area of Emphasis in Class	Suggested Assignments After Class
1 Sept 8	Preliminaries	Read Sections 1.1-1.2.
2 Sept 10	The Two Roles of Logic in Mathematics 1.1 - 1.2	pp. 4-5; 1b, 2,7,8.
3 Sept 13	Experimental Testing	
4 Sept 15	Sets and Relations on Sets 2.1	pp. 36-37: 1-7,11,12.
5 Sept 17	Venn Diagrams Operations on Sets 1.5, 2.2 - 2.3	p. 45: 2,3; p. 53: 2; p. 66: 9.
6 Sept 20	Operations on Sets 2.2 - 2.3	p. 45: 1,4-8; p. 53: 1,3,6,10,14.
7 Sept. 22	Operations on Sets 2.4	p. 59: 1,3-5.
8 Sept 24	Patterns, Relations and Functions 2.4 - 2.6	p. 5: 4-6; p. 60: 7,8.
9 Sept 27	Patterns, Relations and Functions 2.4 - 2.6	p. 63: 1,2,7,8.
10 Sept 29	Patterns, Relations and Functions 2.4 - 2.6, 14.4	p. 63: 3-5; p. 66: 11; p. 520: 1,3,4.

Class Meetings	Area of Emphasis in Class	Suggested Assignments After Class
11 Oct 1	Patterns, Relations and Functions 2.4 - 2.6; 14.4	p. 65: 7; p. 546: 2a,b.
	Quiz I	
12 Oct 6	Introduction to Mathematical Systems 1.6 - 1.7	p. 29: 1-4.
13 Oct	Properties of Mathematical Systems 1.6 - 1.7	p. 92: 11; p. 93; 12; p. 106: 9.
14 Oct 8	The Set of Whole Numbers 3.1 - 3.3	pp. 73-74: 1-3,5-7,11.
15 Oct 11	The System of Whole Numbers 3.4 - 3.5	pp. 80-81: 1,2,4,7; p. 106: 8.
16 Oct 13	The System of Whole Numbers 3.4 - 3.5	p. 92: 1-5, 8,9.
17 Oct 15	Order Relations on the System of Whole Numbers 3.6 - 3.7	pp. 96-97: 1-5,7.
18 Oct 18	Additional Properties of The System of Whole Numbers: Subtraction and Division 3.8 - 3.9	pp. 103-104: 1-8.
19 Oct 20	Summarizing the Properties of the System of Whole Numbers 3.9	p. 105: 3.
	Quiz II	
20 Oct 22	Introduction to Numeration Systems	p. 144: 1; p. 145: 3; p. 117: 1,2,4,8,10.
21 Oct 25	Decimal Numeration System Place Value 4.1 - 4.2	p. 117: 3,5,6.
22 Oct 27	Decimal Numeration System Algorithms 4.3 - 4.4	p. 124: 1,2,4,6.

Class Meetings	Area of Emphasis in Class	Suggested Assignments After Class
23 Oct 29	Decimal Numeration System: Algorithms 4.3 - 4.4	p. 128: 1,4a,c.
24 Nov 1	Decimal Numeration System: Algorithms 4.3 - 4.4	p. 128: 2a,b,3.
25 Nov 3	Nondecimal Numeration Systems 4.5 - 4.7	p. 135: 1-4,10.
26 Nov 5	Nondecimal Numeration Systems 4.5 - 4.7	p. 145: 7; p. 140: 1b,c,e,f,2a,c, e,f,3; p. 144: 1a-c,3.
	Quiz III	
27 Nov 8	The Set of Integers 5.1 - 5.2	p. 150: 1-5,7; p. 180: 2a,b,d,g,h,i; p. 195: 9a-e.
28 Nov 10	The System of Integers 5.3 - 5.4	p. 157: 1,2,4.
29 Nov 12	The System of Integers 5.3 - 5.4	p. 162: 1,3,5.
30 Nov 15	The System of Integers 5.3 - 5.4	p. 151: 8,10; p. 162: 4; p. 163: 12.
31 Nov 17	Order Relations on The System of Integers 5.5	p. 117: 9; p. 168: 1,6.
32 Nov 19	Additional Properties of The System of Integers: Subtraction and Division 5.6	pp. 174-175: 1-5,8,9.
33 Nov 22	Open Sentences and Inequalities 5.7	p. 178: 1-3.
34 Nov 24	Summarizing the Properties of the System of Integers 5.8	p. 180: 2c,e,f,6.

Class Meetings	Area of Emphasis in Class	Suggested Assignments After Class
	Quiz IV	
35 Nov 29	Elementary Number Theory 6.1 - 6.2	p. 189: 1-5.
36 Dec 1	Primes and Composites 6.3	p. 195: 1-4,6,10; p. 216: 3.
37 Dec 3	G.C.D. and L.C.M. 6.4 - 6.5	p. 199: 1-3,7.
38 Dec 6	G.C.D. and L.C.M. 6.4 - 6.5	p. 203: 1,2,5,6,8; p. 216: 4,5,7.
39 Dec 8	Division and Euclidean Algorithm 6.6	p. 199: 5; p. 209: 1-3.
40 Dec 10	Catch Up Day	
	Quiz V	
41 Dec 13	Experimental Testing	
42 Dec 15	Some Final Remarks	

Ms 100 Quizzes

Ms 100 Quiz I October 1, 1971 Name__________

I. (6 points) Consider the following pattern, defined on the set of natural numbers {1,2,3,4, . . . }.

$$(2 \times 2) - 1 = 3 \times 1$$
$$(3 \times 3) - 1 = 4 \times 2$$
$$(4 \times 4) - 1 = 5 \times 3$$
$$(5 \times 5) - 1 = 6 \times 4$$

$$(\square \times \square) - 1 = \bigcirc \times \triangle$$

1. Can you complete the next two lines of the pattern?

______________________ ______________________

2. Does this pattern continue indefinitely?
Yes No

3. Consider the line containing the frames □ , ○ , and △ . If □ is replaced by 21, what should △ be replaced by? __________

4. Find a relation between ○ and △ . Is this relation also a function? Yes No ______________

II. (14 points) For the following questions the universe set is the set U = {b,e,m,n,p,r,s,t,}. Sets A,B and C, identified below, are subsets of this universe set.

A = {m,n,p}   B is the set of vowels.

C = {□|□ is a letter in the word "Tennessee"}.

1. Tabulate the following sets:

__________ B   __________ C   __________ $\bar{A}$

__________ $A \cap C$ __________ $B \cup A$ __________ $B \times C$

2. Are any of the sets that you tabulated above equivalent? Yes No

3. Are sets A and C disjoint? Yes No

4. Tabulate any proper subset of A. ______________

5. Can you find any sets above which contain the member b? ________________________________

Ms 100   Quiz II   October 20, 1971   Name_______

I. (10 points) Identify the property which makes each of the following statements true.

______________ 1. (3 + 1) + 7 = 7 + (3 + 1)

______________ 2. 0 + 8 = 8 + 0

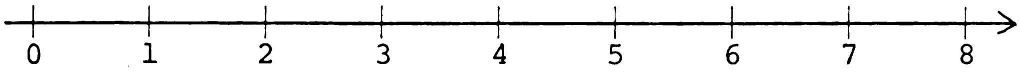
______________ 3. (2 x 9) x 8 = 2 x (9 x 8)

______________ 4. 3 + (8 + 0) = 3 + 8

______________ 5. 5 x (3 + 2) = 5 x (2 + 3)

II. (3 points) On the number line described below circle all whole numbers simultaneously satisfying the inequalities:

$$2 + \square < 9 \text{ and } 3 + \square \geqslant 7$$



III. (7 points) Complete the following short-answer items.

1. If $n(A) + n(B) \neq n(A \cup B)$ for sets A and B, what can you conclude about A and B?

   ____

2. In the sentence "I ranked number four in my class." How is the whole number <u>four</u> being used?

   ____

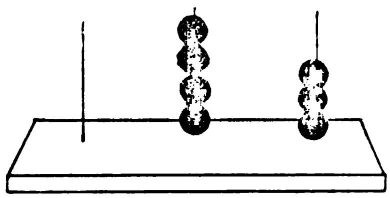
3. Apply the distributive property to rename $2a + a$. What other property in the system of whole numbers would be needed?

   ____

Ms 100 Quiz III November 5, 1971 Name__________

I. (8 points) The open-end abacus illustrated below, consisting of a base, rods and counters, shows the number 43 in base 10.

1. If the counters on the abacus were representing a number expressed in base 5, what would that number be when re-expressed in base 10?

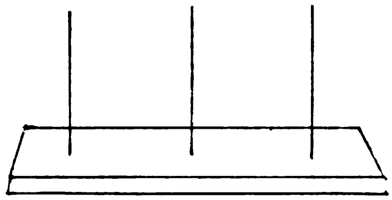


2. In a base 7 system of numeration, what place-value would be associated with the rod on the abacus in the left-most position?

3. In a base 5 system, how many counters would you need to add to the abacus above before you arrived at the number represented by 100_{five}?

4. What does 100_{five} represent in the base 10 system?

II. (5 points) Draw the counters on the base 10 abacus below to show 276.



1. What is the expanded form (expanded numeral) for 276?

2. What position does the numeral "7" occupy relative to the usual base 10 place-value chart?

III. (7 points) Complete both of the following items.

1. $6 \times 10^3 + 3 \times 10^2 + 4 \times 10^0$ equals ______________

2. Divide 1101 by 37 using any version of the subtractive algorithm.

Ms 100 Quiz IV November 24, 1971 Name____________

INSTRUCTIONS: When used below the letters x,y and z represent integers. As such, they may be positive or negative, or they may represent 0. You may interpret ^-x, ^-y, and ^-z as the additive inverses of x,y and z. Values for x,y and z vary throughout the quiz items.

1. (2 points) Identify the additive inverses of $^-5$ and $^-(^+2 - ^-7)$.

2. (2 points) Find y if $^+3(^-2 + y) = ^-9$.

3. (2 points) If x represents a positive integer is $0 - ^-x$ positive or negative?

4. (2 points) Find x if $(^-3)(x) = ^+12$.

5. (2 points) If $x|y$ (e.g., x divides y exactly) and $y|x$ at the same time, what conclusion(s) can you draw about the relationship between x and y?

6. (3 points) Suppose $x + ^-y = z$ with both y and z representing positive integers:

   a. Can x ever represent a negative integer?
   b. Which is greater, x or y?

7. (4 points) If $x < y - z$ and all three letters represent integers, which of the following conclusions, if any, are incorrect?

$x < y$ $\quad$ $x + z < y$ $\quad$ $y < z$ $\quad$ $x < y + z$

8. (3 points) What possible advantage(s) does the system of integers have over the system of whole numbers?

Ms 100 Quiz V December 10, 1971 Name______________

INSTRUCTIONS: When used below the letters m and n represent positive integers; the letters p,q,t and s represent positive prime numbers.

1. (2 points) Is there a prime number between each positive integer n; $1 < n < 10$; and its double $2 \cdot n$?

2. (2 points) Can the sum of two positive primes ever be prime?

3. (2 points) If $m = q \cdot n + r$ with $0 \leqslant r < n$, can $n | m$?

4. (2 points) Express 273 as a product of primes.

5. (2 points) Find the g.c.d. (m,n) if $m = p^4 . q^1 . t^3$ and $n = p^1 . t^3 . s^3$.

6. (4 points) Find the l.c.m. (n, n + 2). Two answers are possible.

7. (3 points) Replace $\triangle$ and $\bigcirc$ so that $2^2 . 3^1 . 5^1$ will be a multiple of $2^{\triangle} . 3^{\bigcirc} . 5^1$.

8. (3 points) If the g.c.d. (m,n) = 6 and the l.c.m. (m,n) = 120, what is $m \cdot n$?

Ms 100 Final Examination

Ms 100 Elements of Mathematics Name ________________

DIRECTIONS: Select the best answer in each of the following thirty items. Each item is worth one point. Fill in the space with the corresponding number on the answer sheet provided. This examination will be machine scored.

1. There are as many prime numbers between 1 and 10 as there are between:

   1. 10 and 20   2. 20 and 30

   3. 30 and 40   4. 40 and 50

2. "Things equal to the same thing are equal to each other." describes which property of the equivalence relation "equals"?

   1. the reflexive property   2. the trichotomy property

   2. the symmetric property   4. the transitive property

3. If $a = n(A)$ and $b = n(B)$, then all of the following statements are true <u>except</u>:

   1. $a + b \leq n(A \cup B)$ 2. $a + b \leq n(A) + n(B)$

   3. $a \cdot b \leq n(A \times B)$ 4. $a \cdot b \leq n(A) \cdot n(B)$

4. Which property in the system of whole numbers is illustrated by the statement $(9 \cdot 0) \cdot 1 = 9.0$?

   1. the zero property of multiplication   2. the commutative property of multiplication

   3. the identity property of multiplication   4. the associative property of multiplication

5. If ^{+}m represents a positive integer and ^{-}n represents a negative integer, then the sum $m + {}^{-}n$ represents a positive integer <u>unless</u>:

   1. $^{+}m > {}^{+}n$ 2. $^{+}m - {}^{+}n > {}^{+}m$

   3. $^{+}m - {}^{+}n < {}^{-}n$ 4. $^{-}n < {}^{-}m$

6. Let P,Q,R,S, and T represent points on a number line whose coordinates are $^{+}2$, $^{+}5$, $^{-}3$, $^{+}6$, and $^{-}8$, respectively. If one begins at P and moves in succession to Q,R,S, and T, what is the total distance moved?

   1. 10   2. 22

   3. 24   4. 34

7. Systems of numeration always possess the following characteristics, with the possible exception of:

   1. a finite collection of symbols   2. a symbol for one

   3. a symbol for zero   4. a set of rules for combining symbols

8. Which number underlined below is used as an ordinal number?

   1. I have <u>2</u> cars.
   2. My telephone number is <u>773-2981</u>.
   3. Today is December <u>18</u>, 1971.
   4. On our last vacation we traveled <u>478</u> miles.

9. Tabulate the following subset of natural numbers.

   $\{n \mid 6 \geq n, n \neq 4\}$

   1. {0, 1, 2, 3, 4, 5}
   2. {1, 2, 3, 4, 5, 6}
   3. {0, 1, 2, 3, 5, 6}
   4. {1, 2, 3, 5, 6}

10. The letters x,y, and z represent whole numbers. In the expression $x \cdot y = z$, by how much <u>more</u> will the product be increased if y is increased by 1?

    1. 1
    2. x
    3. y
    4. z

11. If $A \subsetneq B$, $n(A) = 7$, $n(\overline{B}) = 15$, and $n(B \cup \overline{B}) = 29$, find $n(A \cup \overline{B})$.

    1. 7
    2. 14
    3. 15
    4. 22

12. To find the sum of 23, 370, and 16 by the columnar addition algorithm, as illustrated below, which properties of the whole number system are used in justifying the column set-up?

$$\begin{array}{r} 23 \\ 370 \\ +\ 16 \\ \hline \end{array}$$

    1. Closure, commutative, and associative properties of addition.
    2. Commutative and associative properties of addition.
    3. Commutative and associative properties of addition and the distributive property.
    4. Closure, commutative, and associative properties of addition and the distributive property.

13. $^{-}3(^{+}2 + {}^{-}9)$ equals:

    1. $^{-}21$
    2. $^{+}21$
    3. $^{-}33$
    4. $^{+}33$

14. Sets A, B, and C are non-empty subsets of a universe set U = {a, b, c, d, e}. A = {a, c}; B C = {c}; B ∪ C = {b, c, d }; and A ∪ C = {a, b, c,} . Set B equals:

1. {c}   2. {c, d}

3. {b, c}   4. {b, c, d}

15. The table illustrated below describes a relation between □ and △ . Under this relation the number 5 corresponds to:

□	△
1	1
2	5
3	14
4	30
5	?

1. 46

2. 55

3. 66

4. 71

16. If x represents a whole number between 4,000 and 5,000 and y represents a whole number between 2,000 and 3,000, then their difference x - y will lie:

1. between 0 and 2,000   2. between 2,000 and 4,000

3. between 1,000 and 3,000   4. between 3,000 and 5,000

17. The numeral "1011" in base 2 represents the same number as:

1. "23" in base 4   2. "101" in base 10

3. "201" in base 5   4. "11" in base 12

18. For which pair of prime numbers (m,n) are the following all composite numbers?

$$m + 1$$
$$n + 2$$
$$m \cdot n$$
$$m \cdot n + 1$$

1. (2,3)   2. (3,5)

3. (5,7)   4. (7,11)

19. If x and y represent different integers and x·y = x, which of the following statements would be impossible?

1. $y = {}^{+}1$ 2. $x = {}^{-}1$

3. $x = 0$ 4. $x = {}^{+}1$

20. The table illustrated below describes a basic fact chart for a binary operation * defined on numbers represented by x, y, and z. Which of the following properties is illustrated in the chart?

1. * is closed
2. * is commutative
3. * is associative
4. * is distributive

*	x	y	z
x	x	x	z
y	x	y	z
z	y	z	x

21. If f is a function described by the rule $\square \xrightarrow{f} 2\cdot\square + 1$, then, f(3) equals:

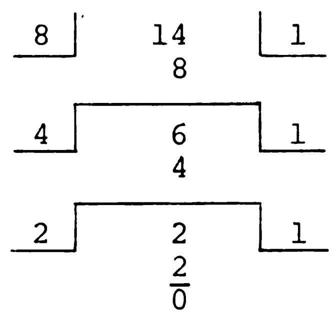
1. 3
2. 4
3. 7
4. 8

22. If A = {p, q, r} and B = {p, s}, which one of the following ordered pairs is not a member of A x B?

1. (p, p)
2. (r, s)
3. (q, r)
4. (q, p)

23. The sequence illustrated below represents an algorithm for:

1. finding the g.c.d. (8,14)
2. factoring 14 into prime factors
3. expressing "14" as a base two numeral
4. dividing 14 by 8



24. In writing the numerals in the Hindu Arabic decimal numeration system from 0 to 100, how many times do you write "9"?

1. 10
2. 11
3. 19
4. 20

25. Which integers satisfy the inequality $\square^2 < 3 \cdot \square$?

1. only 0 and $^+1$ 2. only $^+1$ and $^+2$

3. 0, $^+1$ and $^+2$ 4. $^-2$, $^-1$, 0, $^+1$, $^+2$

26. A portion of a number array is illustrated below. $8\searrow$ is 13; $9\leftarrow$ is 8; and $(5\searrow)\leftarrow$ is 8. What is $(25\longrightarrow)\nearrow$?

1. 19
2. 20
3. 26
4. 34

```
            1
          2   3
        4   5   6
      7   8   9   10
   11  12  13  14   15
            .
            .
            .
```

27. Let x and y represent positive integers. If 7 divided into x produces a remainder of 4 and 7 divided into y produces a remainder of 5, what remainder do you get when 7 is divided into the sum x + y?

1. 2   2. 6

3. 9   4. 20

28. The prime factorization of the g.c.d. (24,16) is:

1. 2^4 2. 6

3. $2^4 \cdot 3^1$ 4. $2^3 \cdot 3^1$

29. All of the following statements about sets A = {2,4,7,8}, B = {1,3,5,7}, and C = {2,6} are true <u>except</u>:

1. B and C are disjoint sets   2. A is the compliment of B

3. C is a proper subset of A   4. Sets B and A are equivalent

30. The l.c.m. (a,b) = a·b unless:

1. g.c.d. (a,b) = 1   2. $a \mid b$ or $b \mid a$

3. a and b are relatively prime   4. a and b are both prime and $a \neq b$

APPENDIX C

MATHEMATICAL CREATIVITY TESTS

MATHEMATICAL CREATIVITY TESTS

Form A

Name________________________

Sex: M_____ F _____

Do Not Write in this Box		
F _ _ _ _ _ _____		
O _ _ _ _ _ _____		
T _____	A _____	B ___

THIS IS A TEST OF YOUR ABILITY TO THINK OF A LARGE NUMBER OF CREATIVE IDEAS IN CONNECTION WITH MATHEMATICS.

Look at the following sample item:

Express 24 using three equal numbers or four equal numbers.

Examples: $24 = 6 + 6 + 6 + 6$
$24 = (24 - 24) + 24$
$24 = 8 + 8 + 8$
$24 = 23 + \frac{23}{23}$

There are, of course, many more possible responses that could have been given.

There will be five different items in this test, somewhat like the one above. Two examples will be included for each item. You will be given seven minutes on each item to write down other possible results. Write as many different possible results as you can think of. Your answers need not be complete sentences.

If you run out of spaces on any item you may use the other side of the paper.

You will be scored for fluency (number of correct responses) and originality (novelty of your responses).

Are there any questions?

STOP HERE. WAIT FOR FURTHER INSTRUCTIONS BEFORE TURNING THIS PAGE.

1. Examine the following sets of three numbers to find out in what way or ways any one of the numbers differs from the other two in the set.

Example: 1, 2, 7 — 2 and 7 are prime, 1 is not.
2 is even, 1 and 7 are not.

a. 2, 4, 6

b. 3, 5, 10

c. 10,15,17

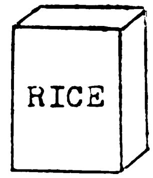
d. 5,12,15

e. 0, 1, 2

2. In the spaces below, list as many possible ways you can think of to find the number of grains of rice there are in a pound of rice.

   Example: Count them one-by-one.

   Cook ten grains each day and keep track of the number of days.



a. ______________________________

b. ______________________________

c. ______________________________

d. ______________________________

e. ______________________________

f. ______________________________

g. ______________________________

h. ______________________________

i. ______________________________

j. ______________________________

k. ______________________________

l. ______________________________

m. ______________________________

n. ______________________________

3. In the spaces below, list as many possibilities as you can of what might happen if numbers were "written" from right-to-left instead of from left-to-right (e.g., "36" meant sixty-three and not thirty-six).

Example: Left-handed people might be overjoyed.

"99" + "3" $\neq$ "102"

a. ______________________________

b. ______________________________

c. ______________________________

d. ______________________________

e. ______________________________

f. ______________________________

g. ______________________________

h. ______________________________

i. ______________________________

j. ______________________________

k. ______________________________

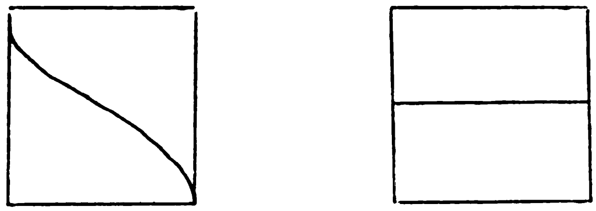
l. ______________________________

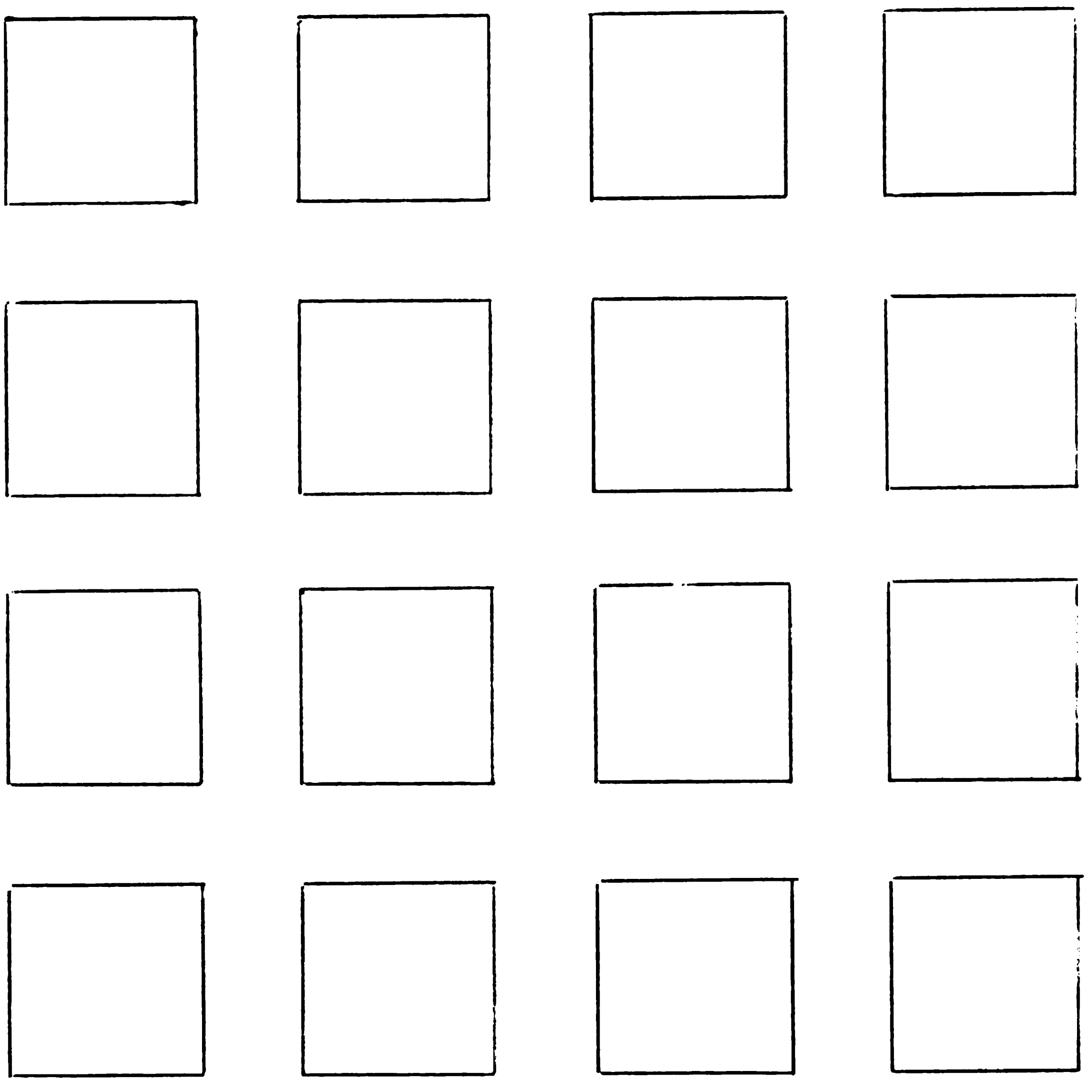
m. ______________________________

n. ______________________________

4. Divide a square in half using only one line in as many different ways as you can.

Example:





5. Find as many different ways as you can to express the number one using exactly four nines. You may use any of the operation symbols +, −, x, ÷; and, if necessary, the grouping symbols ( ).

Example: $1 = \frac{99}{99}$ $1 = \frac{(9 + 9) - 9}{9}$

1 = ________ 1 = ________

1 = ________ 1 = ________

1 = ________ 1 = ________

1 = ________ 1 = ________

1 = ________ 1 = ________

1 = ________ 1 = ________

1 = ________ 1 = ________

1 = ________ 1 = ________

1 = ________ 1 = ________

1 = ________ 1 = ________

1 = ________ 1 = ________

Form B

(First sheet identical to Form A)

1. Examine the following sets of three numbers to find out what common properties exist among the three numbers in each set.

Example: 2, 5, 7 — All three numbers are whole numbers. All three numbers are prime numbers.

a. 3, 11, 19 ____________________

b. 4, 12, 20 ____________________

c. 11, 55, 77 ____________________

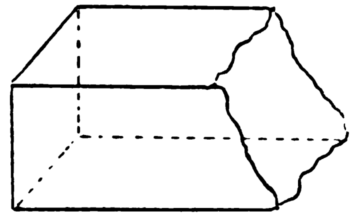
d. $\frac{1}{8}, \frac{1}{4}, \frac{1}{2}$ ____________________

e. 9, 16, 25 ____________________

2. In the spaces below, list as many possible ways you can think of to find the weight of the "block" of butter illustrated below.

   Example: Weigh yourself before and after eating it and subtract the difference.

   Place it on a scale, read the weight.



a. ______________________________

b. ______________________________

c. ______________________________

d. ______________________________

e. ______________________________

f. ______________________________

g. ______________________________

h. ______________________________

i. ______________________________

j. ______________________________

k. ______________________________

l. ______________________________

m. ______________________________

n. ______________________________

3. In the spaces below, list as many possibilities as you can of what might happen if every person owned and wore his own "computer" watch which could add, subtract, multiply and divide any numbers.

   Example: Stock in computer watch companies would be a good financial investment.

   We would probably not have to memorize the basic facts of addition and multiplication.

a. ______________________________

b. ______________________________

c. ______________________________

d. ______________________________

e. ______________________________

f. ______________________________

g. ______________________________

h. ______________________________

i. ______________________________

j. ______________________________

k. ______________________________

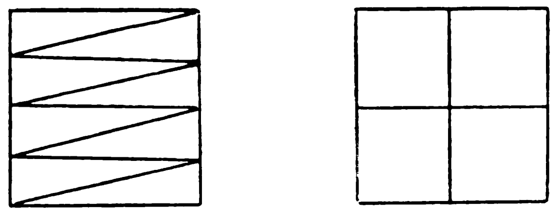
l. ______________________________

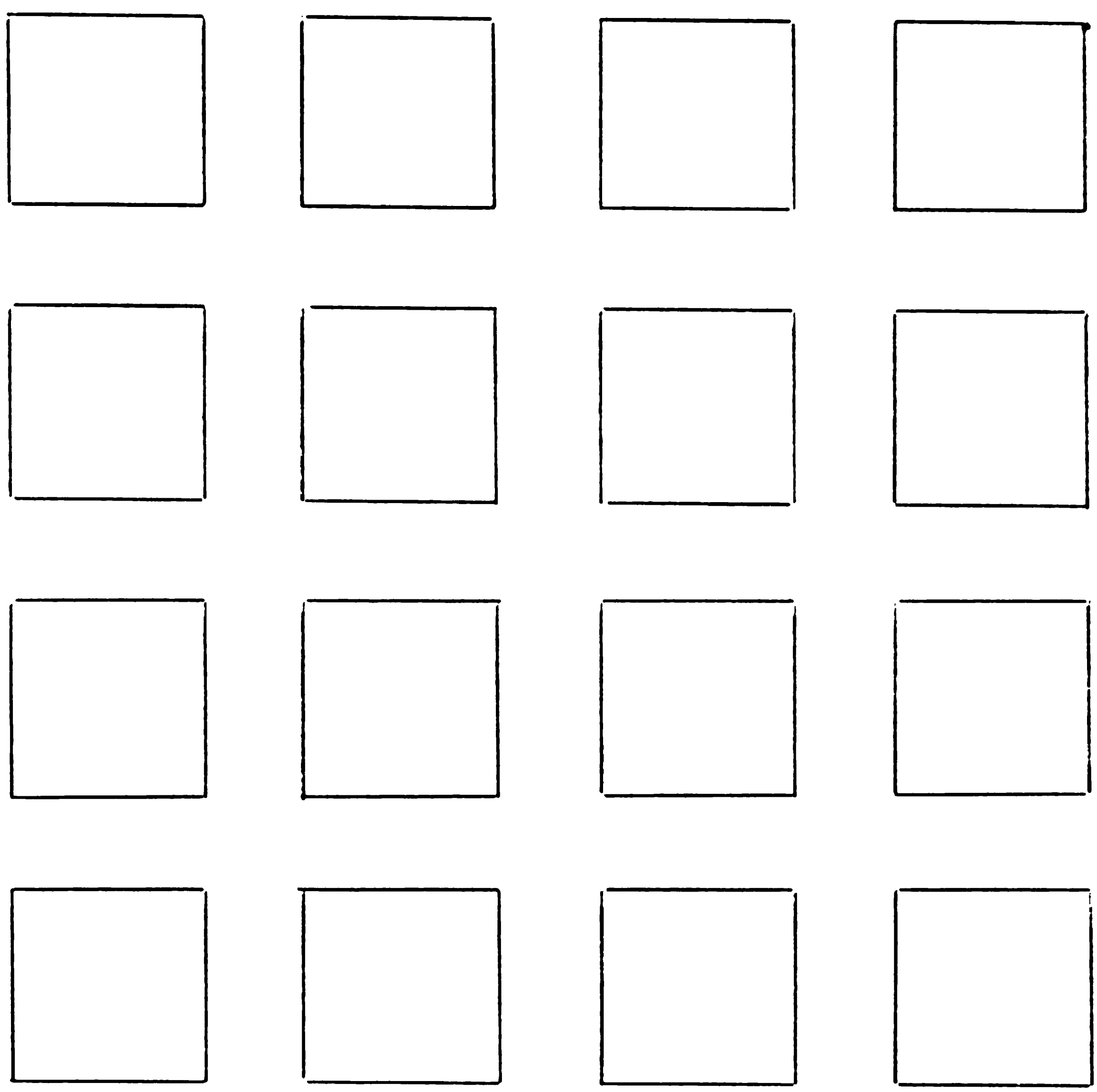
m. ______________________________

n. ______________________________

4. Divide a square into 2, 4, or 8 parts using only straight lines in as many different ways as you can. Each of the parts of the square must be the same shape and the same size; that is, congruent.

Example:





5. How many different whole numbers can you find which can be expressed using exactly <u>three fours</u>. You may use any of the operations +, −, x, ÷; and, if necessary, the grouping symbol ( ). The set of whole numbers consists of:

$\{0, 1, 2, 3, 4, 5, \ldots\}$

Example: $4 + 4 + 4 = 12$ $\frac{44}{4} = 11$

APPENDIX D

SAMPLE OVERHEAD TRANSPARENCIES

SAMPLE OVERHEAD TRANSPARENCIES

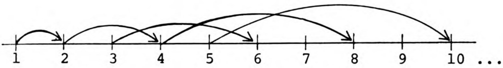
Techniques of Describing a Relation

STATEMENT "This is the doubles relation."

EQUATION $\triangle = 2 \times \square$

SET OF ORDERED PAIRS $\{(1,2), (2,4), (3,6), (4,8), (5,10), \ldots\}$

ARROWS $1 \longrightarrow 2 \quad 2 \longrightarrow 4 \quad 3 \longrightarrow 6 \quad 4 \longrightarrow 8 \quad \ldots$



$\square \longrightarrow \triangle$

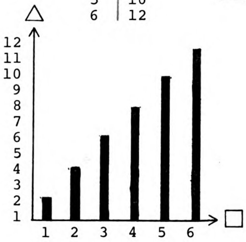
$\square \longrightarrow 2 \times \square$

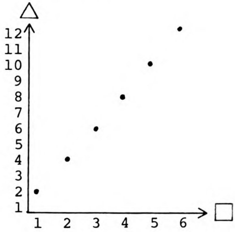
TABLE

$\square$	$\triangle$
1	2
2	4
3	6
4	8
5	10
6	12

$\square$	1	2	3	4	5	6
$\triangle$	2	4	6	8	10	12

GRAPHS





The Division Algorithm

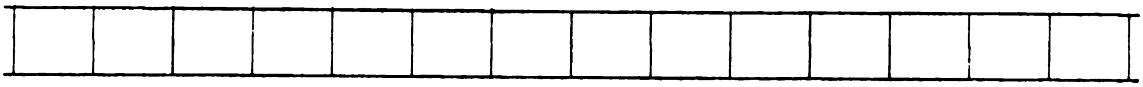
If x and y are any two positive integers, then there exist unique nonnegative integers q and r such that

$$x = q \cdot y + r \qquad \text{and} \qquad 0 \quad r \quad y.$$

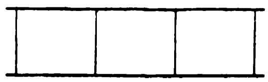
This theorem can be illustrated quite nicely using Cuisenaire rods:

consider the pair of positive integers 3 and 14;

A 14-rod looks like this.



and a 3-rod like



Notice now how 4 3-rods plus two more equals a 14-rod:

(Illustrated with the rods)

Thus, $14 = 4 \cdot 3 + 2$.

(Other examples provided)

(This portion hidden until the conclusion of the above discussion.)

In a branch of mathematics called number theory this is a powerful theorem. To illustrate how it might be used in number theory consider the following numbers:

1 25 64 81 144 324

What common property do these numbers share?

What remainder(s) do you get when these numbers are divided by 4?

(Hidden until students respond)

Can anyone think of a perfect square which, when divided by 4 leaves a remainder different from 0 or 1?

Now lets see why this happens. (Discussion uses the division algorithm to partition integers into odd and even sets. The two cases are then squared and divided by 4.)

APPENDIX E

STUDENT INSTRUCTIONAL RATING SYSTEM FORM

STUDENT INSTRUCTIONAL RATING SYSTEM FORM

For each item below respond by marking the space with one of the following categories:

2 If you strongly agree with the statement.
1 If you agree with the statement.
0 If you neither agree nor disagree with the statement.
-1 If you disagree with the statement.
-2 If you strongly disagree with the statement.

Average Experimental Response	Average Control Response	Item
1.39	1.67	1. The instructor was enthusiastic when presenting course material.
1.61	1.79	2. The instructor seemed to be interested in teaching.
1.30	1.58	3. The instructor's use of examples or personal experiences helped to get points across in class.
1.09	1.13	4. The instructor seemed to be concerned with whether the students learned the material.
0.84	0.84	5. You were interested in learning the course material.
0.78	1.00	6. You were generally attentive in class.
0.09	0.54	7. You felt that this course challenged you intellectually.
0.74	0.92	8. You have become more competent in this area due to this course.

Average Experimental Response	Average Control Response		Item
0.84	0.92	9.	The instructor encouraged students to express opinions.
1.04	1.13	10.	The instructor appeared receptive to new ideas and others' viewpoints.
1.52	1.58	11.	The student had an opportunity to ask questions.
0.65	0.71	12.	The instructor generally stimulated class discussion.
-1.22	-1.41	13.	The instructor attempted to cover too much material.
-1.26	-1.20	14.	The instructor generally presented the material too rapidly.
-1.52	-1.34	15.	The homework assignments were too time consuming relative to their contribution to your understanding of the course material.
-1.18	-1.29	16.	You generally found the coverage of topics in the assigned readings too difficult.
1.22	1.34	17.	The instructor appeared to relate the course concepts in a systematic manner.
1.26	1.58	18.	The course was well organized.
1.22	1.41	19.	The instructor's class presentation made for easy note taking.
1.30	1.41	20.	The direction of the course was adequately outlined.
1.09	1.00	21.	You generally enjoyed going to class.