COMPARISON OF THE SAMPLING METHODS USED IN FORESTRY

Thesis for the Degree of Ph. D.

MICHIGAN STATE UNIVERSITY

Don L. Kulow

1963

THESIS

This is to certify that the

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Don Lee Kulow

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COMPARISON OF THE SAMPLING METHODS

USED IN FORESTRY

by

Don L. Kulow

AN ABSTRACT

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ABSTRACT

COMPARISON OF THE SAMPLING METHODS USED IN FORESTRY

by Don L. Kulow

In testing the accuracy of most of the field sampling techniques used in forestry, a large number of each kind of sample is needed for a thorough statistical analysis. This requires considerable time and expense for the field work and is further complicated by changing stand conditions. To overcome these difficulties, three forest areas of 10.4 acres each were mapped to a scale of 1:120, and samples were drawn from these maps. These areas, of oak-hickory, sugar maple-beech, and ash-elm-red maple, represent the most common forest types found in southern Michigan.

A series of 144 sampling methods was tested in each forest. These consisted of six areal plot shapes: (1) circular, (2) triangular, (3) square, (4) rectangular 1:2, (5) rectangular 1:4, and (6) rectangular 1:8. These were each sampled by a series of 1/5-, 1/10-, 1/20-, 1/40-, 1/80-, and 1/160-acre plot sizes. Two types of Bitterlich point-samples were examined. One type was adjusted for edge bias, during the sampling process, according to Grosenbaugh's peripheral zone scheme, while the "unadjusted" point-sample was corrected by a formula adjustment during analysis. Both types of samples were tested for basal area factors of 5, 10, 20, 30, 40, and 50. Each of these samples was distributed throughout the forest populations by random, systematic, and multiple-random-start techniques.

An arbitrary number of sampling units was assigned to each forest, and sampled by basal area. The expected value, standard deviation,

standard error, coefficient of variation, and the deviation or difference between the expected value and the true population mean were computed for each sample. To find the most precise sampling method, the coefficient of variation of all the samples was subjected to analysis of variance. As a check on the sampling ability of the various distributions, the deviation of the expected value from the known mean was also analysed. No attempt was made to evaluate the efficiency of any of the methods.

The Lansing Woods is characteristic of an upland oak-hickory forest. The samples that gave the most accurate results are: (1) BAF-5 point-sample with an average coefficient of variation of 5.87 percent; (2) the one-fifth acre plots with an average CV of 6.11 percent; (3) the BAF-10 with an average of 7.28 percent; and (4) the one-tenth acre plot with 7.56 percent. The differences among these are insignificant. For unadjusted point-sampling, the random, systematic, and multiple-random start distributions had an average of 11.12, 10.93, and 12.33 percent, respectively. For plot samples, in the same order, this was 13.44, 14.16, and 14.39 percent. All of the differences among these could be attributed to chance. However, an analysis of the difference between the expected value and the true mean showed that random distribution was significantly more precise than the other two.

The Toumey Woods has the characteristics of a sugar maple-beech type. The most accurate samples with no important differences among them are: (1) BAF-5 point-sample with an average coefficient of variation of 4.58 percent; (2) BAF-10 with a CV of 5.66 percent; (3) one-fifth acre plot with 5.74 percent; and (4) one-tenth acre with 7.67 percent. The average coefficient of variation for the unadjusted point-samples are: 6.60, 7.55, and 8.70 percent, respectively, for the random, systematic, and multiple-random-start distributions. For plot samples this is 18.79, 20.60, and 18.97 percent, in the same sequence. None of the differences within point-sample distributions or within plot-sample distributions is significant, however, the systematic arrangement was highly superior to the others in the analysis of the difference between the expected and the true mean.

The Red Cedar Woods is similar in characteristics to the ash-elm-red maple type. The samples that gave the most accurate results are: (1) BAF-10 point-sample with an average coefficient of variation of 5.38 percent; (2) BAF-5 with 7.33 percent; (3) the one-fifth acre plot with 8.57 percent; and (4) the one-tenth acre with 10.19 percent. The differences among these were insignificant. For the unadjusted point-samples, the random, systematic, and multiple-random-start distributions had average coefficients of variation of 10.26, 13.43, and 15.56 percent, respectively. The random method proved to be highly superior to either the systematic or the multiple-random-start system.

The adjusted point-samples gave significantly less accurate results in all cases except on the Lansing Woods where the timber was relatively small, compared to the other two forests.

Differences in the sampling accuracy due to sample unit size were very important. The inverse relationship between the coefficient of variation and the sampling unit size is illustrated by several regression equations. Plot shape was of no consequence in this study.

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CHAPTER I

INTRODUCTION

The choice of sampling designs used for estimating timber is usually based more on personal preference than on the precision of the method involved. Various studies have compared two or more of the favored sampling designs, but no attempt has been made to compare all of the major sampling systems on the same forest.

In order to compare the precisions of the various sampling techniques on an equivalent basis, a large number of each kind of sample is required for thorough statistical analysis. The use of an actual forest stand requires a great deal of field work and is complicated by changes in the stand condition with time. These factors make the determination of the most accurate means of forest sampling by field measurements a very costly procedure.

This study provides a comparison among the most important sampling techniques used in forestry by mapping the position of each tree in a given population and drawing samples of basal area, (cross-sectional area of the tree at 4.5 feet in square feet), from this map. Several forest areas, representing the most common combinations of species and site conditions in the southern lower peninsula, are studied. A number of sampling units are assigned to each area, and the most frequently used forest sampling procedures are applied to these units. The results are analyzed with respect to the accuracy of the sampling method involved. No attempt is made to ascertain the efficiency of any of these methods through cost or time studies. The project is limited to the statistical precision of forest sampling.

CHAPTER II

REVIEW OF LITERATURE

Studies designed to determine the best method of sampling forest populations have been conducted early in forest management history. With the use of statistical analysis applied to sampling error, tests could be made on the precisions of plots of various sizes and shapes; on systems using different numbers and distributions of sampling units; and on the efficiency of various sampling designs.

Early Studies

Early studies were confined largely to comparisons between two or more of the most popular types of forest sampling units. These were usually strip samples and plot samples. Wright (1925) compared these on an actual forest stand. He used strips ten chains long and one chain wide. Tally was kept by two-chain intervals, and these segments were treated as the line plots. Statistically, the line plots gave the best results. Candy (1927) used systematic strips and line plots, and concluded that only line plots could give sampling with a determined level of accuracy. Robertson (1927) made the same type of test and also discarded strip sampling as the least precise. He favored a large number of small plots, evenly spaced over the area rather than a few large plots. From time to time during the sampling process, he would analyse the results to see if a sufficient number of units had been taken to obtain his estimates within the required limit of accuracy. This is essentially the same system presented by Freese (1962) based upon a formula by

Schumacher and Chapman (1948). In New England, Goodspeed (1934) found that the two methods of sampling gave essentially the same results.

By 1932, it was apparent that size and shape of the sampling unit were also important to the results of the sample. Experiments with agriculture crops indicated that the long narrow plots tended to be the most effective for reducing the variability of yields among samples (Clapham, 1932; Justesen, 1932; Kalamkar, 1932).

In California, Hasel (1938) took a 100 percent inventory of nine sections of redwood forest by two and one-half acre plots (2.5 x 10 chains). He combined these into different "universes" of plots[1/] of various shapes and sizes, and found that the long narrow plots (2.5 x 40 chains) gave the most accurate results. In a later article (1942), he stated that strip plots would give the best results only when these were of maximum length for the area sampled. Both Hasel (1938) and Yates and Zacopanay (1935) stated that an increase in plot size results in an increase in the variance when total plot area is fixed. This has been supported in later studies where variance decreased as plot size increased (Barton, 1956; Freese, 1961, 1962; Johnson and Hixon, 1952; Mesavage and Grosenbaugh, 1956; Meyer, 1948).

Ecologists were also interested in the best way to sample forest populations.

> Although the objectives of ecologists and foresters differ, both methodology and research results overlap to a considerable degree and there is an exchange of ideas in both directions (Lindsey, 1956).

1/ "Universe of plots" comes from Palley and O'Regan's (1961) conceptional forest population in which a forest is thought to be made up of two distinct components: 1), The physical population of trees growing on a particular forest; and 2), The partitioning of these trees into elementary units according to the system of sampling used. The conceptional population or universe of plots is the aggregate of all these elementary units.

Many ecological methods of sampling refer only to tree-area relationships (Cottam, 1947; Cottam and Curtis, 1949). However, some of these studies did make use of standard forest sampling methods for measuring basal area.

Borman (1953) took a complete census of 4.3 acres of the Duke University forest by squares of two by two meters. He arranged these into 28 other universes of plots of different shapes and sizes for comparison. Analysis showed that the longest plots (10 x 140 meters) gave the best results providing the long axis crosses the population contours. However, he pointed out that the greater periphery in narrow plots would tend to produce a greater edge-error[2/] than square plots of equal area.

In California, Johnson and Hixon (1952) inventoried a forty-acre tract of Douglas-fir by plots measuring one-half chain by one-half chain. These were then combined into seven other universes of plots varying from one-half to one chain wide, and from one to six chains in length. Long narrow plots proved to have the lowest coefficient of variation. This parameter decreased as the plot size increased. Freese (1961) also points out the usefulness of the long narrow plots for reducing variability. Meyer (1948) strongly recommended long narrow plots placed at right angles to the contour of the land.

2/ Errors of omission or inclusion of individuals occurring near the edge of a sampling unit of fixed area.

Point-Sampling

For the first twenty-five years of forest sampling, the use of the fixed area plot (i.e. circles, squares, rectangles, strips) was firmly fixed in the minds of American foresters. In 1947, a new concept in forest sampling was introduced in Germany by an Austrian forester named Bitterlich (1947, 1948). This method did away with the necessity of determining plot boundaries. The average basal area of a forest could be obtained by simply counting the number of trees whose bole diameter exceeded a selected angle. The method became known as the Bitterlich system or the point-sampling system, and was introduced into this country by Grosenbaugh (1952). He expanded the use of the method to measure other forest characteristics such as board- and cubic-foot volumes (Grosenbaugh, 1955), and later derived its probability theory (Grosenbaugh, 1958).

Various types of instruments have been devised to expedite the use of the system (Bitterlich, 1948; Bruce, 1955; Lemmon, 1958; Stage, 1958). The literature is replete with explanations concerning the mechanics and theory of the system (Afanasiev, 1957; Grosenbaugh, 1958; Lemmon, 1958; Palley, 1962), so a complete description will not be given here.

With the acceptance of point-sampling as a valid means of estimating forest parameters, a renewed interest in comparative studies was revived.

In ecology, Shanks (1954) found the Bitterlich method superior to standard plots for classifying vegetation. Rice and Penford (1955) compared point-sampling with the ecologist's paired-tree method and found it to be the most precise means of measuring average basal area. Barton (1956) mapped a twenty-acre forest to scale, and sampled it with point-sampling, 1/5-, 1/10-, and 1/40-acre plots and strips. Point-sampling proved to be the most accurate. Sudia (1954) found that the method ranked third in a comparison with five other methods. He pointed out, however, that the system that works best in one area may not work in another. Lindsey *et al*., (1958) mapped 20 acres of Indiana hardwoods, and found point-sampling the fourth best in a comparison of nine methods.

In forest research, Husch (1955) tested the accuracy of point-sampling of three different angles of inclusion, namely, 52.09 minutes, 104.18 minutes, and 208.38 minutes for basal area factors of 2.5, 10, and 40, respectively. He arbitrarily selected one hundred points in a forty-acre tract for study. However, he did not allow any tree to appear more than once in the sample, so as the angle decreased -- thus including a larger number of trees -- he reduced the number of sampling points to prevent duplication. This resulted in a biased estimate of the error, for which he did not make corrections (Grosenbaugh, 1955). Under the circumstances, his results showed the 104.18-minute angle (BAF-10) to be the most accurate.

A study in southern Illinois inventoried two forests of eight and 23 acres by plots two by two chains in size (Deitschman, 1956). Basal area was recorded by small and large poles and as sawtimber trees. Deviations from the known parameter varied from 20 to 50 percent. This may have been caused by failure to check boundary trees carefully. Avery (1955) in Arkansas found that estimations in volume by the Bitterlich system ranged from a negative two percent to a positive 20 percent. Afanasiev (1958) also made point-sampling volume comparisons against one-chain strips and one-fifth acre plots. Deviations from the known volumes for point sampling ranged from -22 to 104 percent in individual compartments. For the entire 296-acre forest, the difference between the estimated volume and the actual value was only 1.5 percent. Borgeson et al., (1958) found differences between individual point-samples and one-fifth acre plots of -25 to 88 percent for basal area. Trappe (1957) also found the Bitterlich system to be much more variable than fixed area sampling. However, Afanasiev (1958) and Borgeson et al., (1958) concluded that point-sampling compares favorably with conventional methods when the time required for sampling is considered.

Grosenbaugh and Stover (1957) have attested to the ease of taking Bitterlich plots. On a continuous forest inventory program in southeast Texas, they compared the method with concentric one-quarter acre circular plots. Statistical analysis showed point-sampling to be inferior to plot-sampling, as far as accuracy is concerned, but required much less time to execute. Cox (1961), in testing three basal area factors (10, 18, and 27), found that the BAF-27 had the same sampling error as one-fifth acre plots, but required the measurements of only 58 trees as compared to 235 trees on the fixed radius system.

A large body of empirical evidence has shown point-sampling to be a valid means of estimating basal area (Bell, 1957; Ker, 1957; Malain, 1961; Stage, 1962; Lindsey, 1956; Warren, 1960). This has led to the increased use of the method at all levels of forest sampling (Bryan, 1959).

In applying point-sampling, some procedure had to be used to correct for the bias created by the edge of the forest. The point-sampling procedure assumes that each tree in the forest is surrounded by an imaginary ring equal to the product of its diameter and some constant K called the plot radius factor. Grosenbaugh (1958) says that the probability of any tree being included in the sample is equal to the ratio of its K x diameter-area to the total area of the forest. He calls this "probability proportional to size." In cases where the tree is closer to the edge of the forest than its imaginary plot radius, a proportion of its circle will lie outside the forest. Thus:

> . . . bias arises because random or systematic location of sample points . . . is limited by the tract boundaries, so that the trees with enlarged rings or diameters projecting beyond the boundary have less chance of being sampled than their size . . . . would indicate.

To eliminate this bias, the portion of each sample tree ring that lies outside the forest must be added to the sample. Grosenbaugh specifies peripheral zones a little wider than the radius of the maximum tree ring expected. These zones are laid out prior to sampling and are illustrated in Figure 1.

Trees in the interior zone will receive a unit weight or will be counted once; trees in the size zones twice; and trees in the corner zones four times.

Palley (1960) recognized this bias and attempted to correct for it by simply moving the forest boundaries out to include all of the tree rings. In a later study (1961), he used the weighting system mentioned above. Haga and Maezawa (1959) took exception to both of these procedures and outlined a technique for obtaining an approximate value of the bias. According to them, this bias is a function of the average tree radius ($\underline{r}$) on the forest, the plot radius factor ($\underline{p}$) used, and the area and perimeter of the forest. This bias is negligible on a forest of more than 25 acres. Using a plot radius factor of 50 and an average tree radius of 3.94 inches they constructed a graph showing the relative bias ($\underline{e}$) for small tracts of land having various areas and perimeters. The percent bias for other values of $\underline{p}$ and $\underline{r}$ can be computed with the following equation:

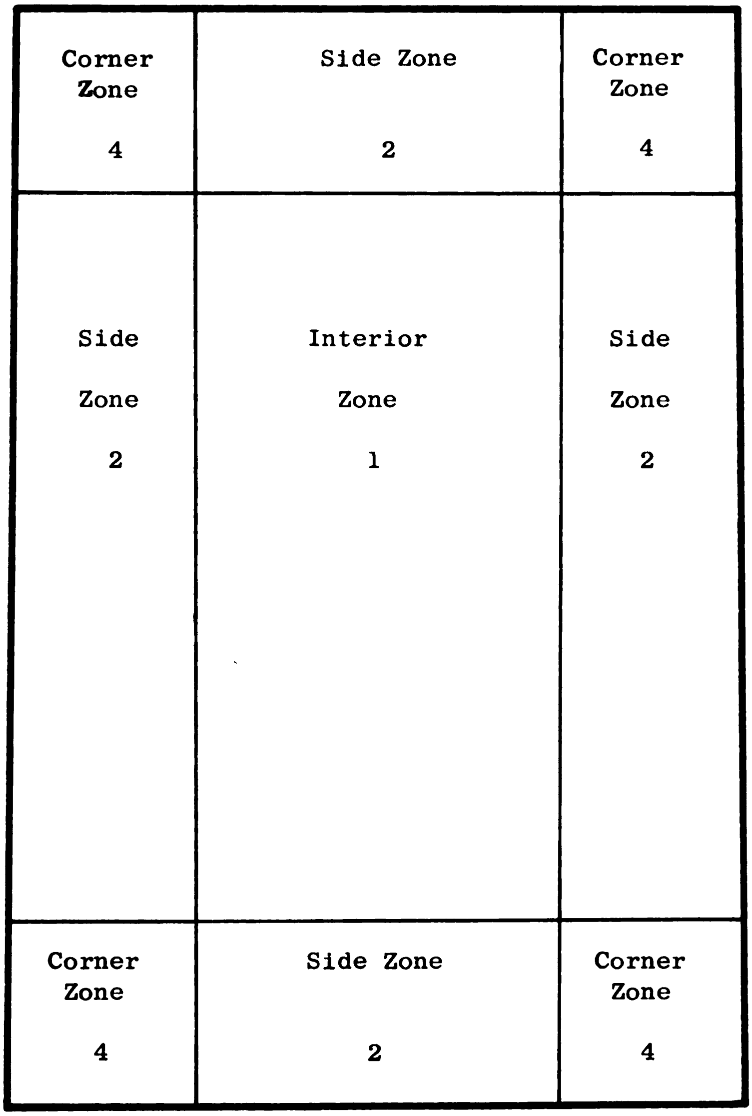

Figure 1. -- Grosenbaugh's peripheral zones for adjusting edge-bias in point-sampling. Trees falling in these zones received the indicated weight.

$$\text{Percent Bias} = (e)\ (^{P}/50)\ (^{r}/3.94) \qquad (1)$$

where: e = the relative bias

P = the plot radius factor

r = the average tree radius in inches

Distribution of the Sampling Units

In distributing the sampling units or points within the forest population, foresters have persisted in using systematic sampling designs. Accurate estimates of error are based upon purely random samples and no one today has managed to come up with a mathematically sound formula for applying this estimate to systematic techniques (Palley, 1961; Shiue, 1960). When a forester attempts to sample a population systematically, he mistakenly assumes that nature has been kind enough to randomize for him. The main argument for systematic sampling is that the results are more accurate than random methods; however, Finney (1949) showed that when there is a strong marked pattern of variation in the population, systematic sampling may give much less accurate results than random methods. Smith and Ker (1957) state that it is impossible to predict this pattern from a sample.

Candy (1927) pointed out the importance of a random sample when he said:

> Any method of survey for which it is possible to calculate the accuracy of the estimate obtained is very much superior to methods in which the accuracy of the estimate is doubtful and not at all calculable.

Wright (1925) said that the most important use of statistical techniques in cruising was to estimate the error of the cruise. The use of random as against systematic distribution has been urged by many investigators in sampling (Borman, 1953; Cochran, 1953; Finney, 1948; Hasel, 1938; Meyer, 1948).

In spite of this, many researchers believe that random samples are less accurate and more expensive than systematic samples. To support this belief, a number of reports have shown that systematic samples are superior to the random technique with respect to precision. Osborne (1942) found that systematic sampling was six times more accurate than random sampling. Meyer (1948) reported that a systematic arrangement of plots would give better coverage of the population, and Hasel (1938) said that the estimation of the mean would also be much more precise. Mesavage and Grosenbaugh (1956) studied three plot arrangements, (4 x 4 and 2 x 8 systematic and random), on twelve forest tracts of 25 acres each. They found that the systematic design was the most precise on nine of the areas, while the random was best on only three of the twelve. Bourdeau (1953), on the other hand, said that the loss in precision from random samples is slight, and more than offset by the fact that estimates of error can be made. He also believes that random sampling requires little more in the way of field work.

It has been pointed out that differences in the precisions of various sampling designs can be attributed to the erroneous assumption that all universes of plots of a given population have a "normal" distribution (Smith and Ker, 1957). To illustrate this, they arranged the data from a study in California (Johnson and Hixon, 1952) by volume classes for each plot size and showed that the distributions ranged from a "Poisson" for very small plots, to a "rectangular" for plots as large as one acre. Only samples with plot sizes of about one-fifth acre could be applied to "normal" analysis techniques.

Several ways have been suggested for obtaining unbiased estimates of the sampling error when systematic sampling is used. Osborne (1942) mentioned the mean-squared-successive-differences, where the difference between successive sampling units are squared, and the estimate of the standard error is made by the following equation (Moore, 1955):

$$\text{Estimated Standard Error} = \left[(\text{Successive differences})^2/n-1/n\right]^{\frac{1}{2}} \qquad (2)$$

where: n is the number of sampling units.

However, Meyer (1956) has pointed out that the exact mathematical distribution of this parameter is not known, so tests of significance are not possible.

Another possible compromise between random and systematic sampling has been suggested by Schumacher and Chapman (1948) and called "proportional sampling of blocks of diverse but unknown areas", or stratification. With this method, the forest is stratified or divided up into rather homogeneous blocks according to some characteristic such as forest condition, age class, volume class, etc. Each block is treated as a separate population from which at least two random samples are drawn. Mudgett and Gevorkiantz (1934) used this method in studying the reliability of early forest surveys. Recently, stratification has been advocated by Bickford (1961), Freese (1962), and Orr (1959) as a means of increasing the accuracy of sample estimates and decreasing the number of sampling units required.

The multiple-random-start method offered by Shiue (1960) is still another way of solving the random vs. systematic problem. A set of systematic samples with one random start is considered to be a cluster of plots. Two or more such sets of plots can be analyzed by cluster techniques, and the problem of estimating the error is solved while keeping some resemblance of a systematic cruise. But, the exact number of random starts to use is not known. Too many can be very inefficient, and too few may lower the precision of the estimate by cluster analysis. Also, the distribution of the individuals within the population so affects the sampling error that the results may be inferior to either systematic or random procedures (Smith and Ker, 1957).

Number of Sampling Units Required

Early sampling problems in forestry consisted of measuring some arbitrary fixed percentage of the forest area. The most common proportion was the ten-percent cruise. With the institution of the nationwide continuous forest inventory program, and the initiation of management on large forest areas, it became apparent that the "proportion of the total" sample was no longer feasible.

Preston (1934) objected to the use of a preconceived intensity of cruise and urged the use of statistics to determine the size of the sample. In an attempt to base the number of units needed upon some characteristic of the population, Girard and Gevorkiantz (1939) published a table of coefficients of variation that varied with stand density and uniformity. Barton and Stott (1946) drew up a set of pictorial charts giving the proportion of the area needed in a sample for stands of different uniformity and density at several levels of accuracy. It was based on the equation:

$$\text{Percent Cruise} = \frac{400\ (\text{plot size})\ (\text{CV})}{(\text{total area})\ (\text{percent accuracy}) + 4(\text{plot size})\ (\text{CV})} \quad (3)$$

where a coefficient of variation (CV) came from the tables of Girard and Gevorkiantz (1939). Meyer (1949) said that the variation in timber volume is affected by volume per acre, size and shape of the sampling units, and the efficiency of the cruising design. He measured 68 one-fifth acre plots on a 54-acre forest, and produced a table giving the number of units needed for six different accuracy levels on areas from 50 to 10,000 acres, and coefficients of variation of 15, 30, and 60 percent. A young dense stand was measured at the CV level of 15 percent while an old growth stand with a few large scattered trees used the 60 percent value.

The subjectiveness of the foregoing methods is apparent. As early as 1935, the number of sampling units needed was thought to be a function of the sampling error (Schumacher and Bull, 1935). Sampling error itself is a function of variations in the population, size and shape of the sampling unit, and the number of sampling units used. According to Schumacher and Chapman (1948) this number of plots may be found by:

$$n = (S/S_{\bar{x}})^2 \qquad (4)$$

where: n = the number of sampling units required

S = the standard deviation of the population (usually) estimated from a pre-sample)

$S_{\bar{x}}$ = the accuracy desired (frequently expressed as a plus or minus some proportion of the sample mean)

Borman (1953) for example, used a plus or minus ten percent of the mean, with a certainty of being right two out of three times. Freese (1962) has expanded the use of formula (4) with respect to the use of a "t" value for various limits of precision. This system is commonly accepted by biometricians and statisticians as a valid means of arriving at n.

Other methods designed to estimate the number of sampling units are the ratio of sample variance to sample range (Snedecor, 1959; Freese,

1962) and the range-mean ratio (Allen and Mongren, 1960). The usefulness of the latter has been demonstrated in using the Bitterlich system, but its mathematical theory has not been proven.

The most common procedure in comparative analysis research has simply been to select an arbitrary number of samples from a population and base all calculations on this fixed number (Barton, 1956; Grosenbaugh and Stover, 1957; Husch, 1955; Johnson and Hixon, 1952; Lindsey et al., 1958).

Sampling Efficiency

Efficiency of sampling may be defined as getting the greatest amount of information for the least expenditure of time and effort (Lindsey et al., 1958). Finney (1948) stated that, the efficiency of one sample with respect to another is expressed as a ratio of their respective precisions. According to Cochran (1953), it is a function of the squared sampling errors expressed as a percent. Thus:

$$\text{Efficiency (percent)} = \frac{(\text{Standard survey error in percent})^2}{(\text{Other survey error in percent})^2} \qquad (5)$$

When time has been measured or cost of sampling has been computed, the formula becomes:

$$\text{Efficiency (5)} = \frac{(\text{Standard survey error in percent})^2\ (\text{Standard survey cost})^2}{(\text{Other survey error in percent})^2\ (\text{Other survey cost})^2} \qquad (6)$$

The choice of which survey data should be in the denominator is purely arbitrary (Grosenbaugh and Stover, 1957).

Using formula 6, Lindsey *et al.*, (1958) found that the Bitterlich method was fourth in efficiency for sampling basal area in a study with eight other methods. Methods called the rangefinder-circle-Bitterlich, the tenth-acre-rangefinder-circle, and the one-fifth acre strip, all ranked ahead of the full Bitterlich method. Johnson and Hixon (1952) in comparing only circular and rectangular plots, found that the one- by three-chain unit was the most efficient. Among only the circular plots, the one-quarter-acre plot was best. Mesavage and Grosenbaugh (1956) tested four rectangular plots of 0.1, 0.2, 0.4 and 0.8 acres,respectively. They found that coefficient of variation decreased as plot size increased, but that the cost also increased. They concluded that in dense homogeneous stands, the one-tenth acre or smaller plots would be the most efficient, while in sparse stands of large timber, plots larger than 0.8 acre would be best.

In studying the efficiencies of sampling distributions, Mesavage and Grosenbaugh (1956) showed the 1:4 systematic grid is more efficient than either the 1:8 grid or the random method. Johnson (1949) tested a series of cluster arrangements and found that taking more than three plots in a cluster was inefficient, and that fewer than this would have given the same precision.

Grosenbaugh and Stover (1957) stated that because of the small amount of work required to use point-sampling, it is more efficient than plot sampling. Husch (1955), in evaluating a number of point-sampling angles, found the 208.38-minute angle to be more efficient than the more precise 104.18-minute angle.

Methods of Study

Most studies of forest sampling techniques have been carried out on an actual forest. The problems inherent in this method have been stated earlier. To avoid this difficulty, several researchers inventoried the entire forest by some basic unit of sampling, such as a rectangle or a square plot. The precision of various sampling distributions, plot sizes, and shapes could then be studied by combining these basic units into universes of plots with other dimensions. Grosenbaugh and Stover (1957) used one-tenth acre squares; Deitschaman (1956) used 2- by 2-chain squares; Hasel (1938) used 2.5- by 10-chain rectangles; and Borman (1953) and Bordeau (1953) used 2-meter squares.

An approach that offers much in that it allows the study of all types of plots including the Bitterlich system, is the one that maps each tree in the forest to scale. Samples are then drawn from this fixed population and analysed. Cottam (1953) constructed an artificial population on paper with a table of random numbers. He later mapped three small forest areas of less than six acres in order to study sampling efficiency for ecological purposes. The method worked so well that Barton (1956) mapped a 20-acre forest to a scale of 1:36. He used all trees over four inches in diameter and measured their coordinates to the nearest foot. In mapping, however, he grouped the trees by 3-inch diameter classes. Lindsey _et al._, (1958) used this same map in their work in ecology.

Palley and O'Regan (1961) have recently applied this mapping technique to the speed of electronic computers. They obtained the coordinate measure to the nearest meter for every tree on a two and one-half acre forest. These were then committed to the memory of the IBM-701 Computer. By inserting a formula for the plot radius factor and the coordinates of the plot centers, they could check the accuracy of any basal area factor. The advantage of this system is that all possible points in the forest can be sampled, and the method is without computing error. Memory systems of these machines are so limited, however, that only very small forests may be used.

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CHAPTER III

METHODS AND PROCEDURE

The Study Areas

Three forest areas were picked for study, to determine the most accurate method of sampling to be applied in each one. These are characteristic of the most common timber and site conditions in the southern half of Michigan's Lower Peninsula. Each area contains 10.4 acres, measuring eight by thirteen chains (528 by 858 feet), and is centered in a larger tract of timber. Table 1 shows the frequency of trees by two-inch diameter classes.

The Lansing Woods. -- This study area is located at Maple Rapids, Michigan, and is owned by the Lansing Company. Characterized as a white oak-red oak-hickory type, the stand occupies a well-drained upland site. Associate species are sugar maple, American elm, red maple, basswood, and beech. The average basal area is 78.10 square feet per acre, and the average tree diameter is 11.3 inches. Fires have not occurred in the stand in the past ten years and grazing has been very light. Selection cuttings have been made from time to time. The last cut was made in April, 1962, and removed some of the white oak on a small part of the study area.

The Toumey Woods. -- This relatively undisturbed area is located in the east-central part of the campus. It is an old-growth stand of beech-sugar maple on the moist fertile soils of a morainal hill. Other species in this stand are red and white oaks, American elm, basswood, and black

Table 1. -- Distribution of trees by two-inch diameter classes.

Diameter Class inches	Lansing Woods	Toumey Woods	Red Cedar Woods
6	291	163	136
8	247	124	138
10	192	88	129
12	132	73	113
14	118	85	107
16	84	80	85
18	51	89	65
20	37	70	49
22	10	56	31
24	4	38	11
26	2	27	15
28	1	21	11
30	1	9	3
32		6	1
34		7	2
36		2	1
38		2	3
40		2	3
42			3
Total	1170	942	906

cherry. The average basal area per acre is 116.92 square feet, and the average tree diameter is 15.3 inches. Fires, grazing and logging have not occurred in this forest.

The Red Cedar Woods. -- This is a timbered area along the Red Cedar River on the western boundary of the Michigan State University campus. The stand is a variant of the SAF type 39 and is characterized principally by green ash-elm-red maple. Associate species are basswood, sycamore, cottonwood, and slippery elm. The site is moist to wet, and is an elongated area bordering a stream that frequently floods. Drainage is poor. The average basal area of the study tract is 91.60 square feet per acre, and the average tree diameter is 13.8 inches. Tree distribution shows a distinctive clumping characteristic that does not exist in the other two tracts. Fires, grazing and cuttings have not occurred during the known history of the stand.

Field Work

The field work consisted of measuring the rectangular coordinates of each tree to the nearest foot and tree diameters to the nearest one-tenth inch. In the Lansing and Red Cedar areas, a base line 13 chains (858 feet) long was run along the northern boundary. Stakes were set at one-chain intervals, and strings were run from these markers, across the forest, perpendicular to the base line. The distance between the strings was checked frequently with a compass and tape. In the Lansing Woods, this job was facilitated by stakes from a previous study marking the corners of each one-chain square. These squares were numbered in 13 rows with eight blocks per row as shown in Figure 2.

In order to obtain the rectangular coordinates of each tree, a steel tape was laid out between successive rows, and the abscissa (a) of each tree was read on the tape, to the nearest foot, at a point where a line from the center of the tree was perpendicular to the tape (Figure 3). The ordinate (b), or the distance from the tree to the tape was measured with a 6-inch base rangefinder. This distance was measured tangent to the circumference of the tree. Where underbrush or other trees obscured the view, the ordinate was obtained with a cloth tape. As each tree location was recorded, the tree was assigned a number for that row and block, and its diameter was measured to the nearest one-tenth inch with a diameter tape.

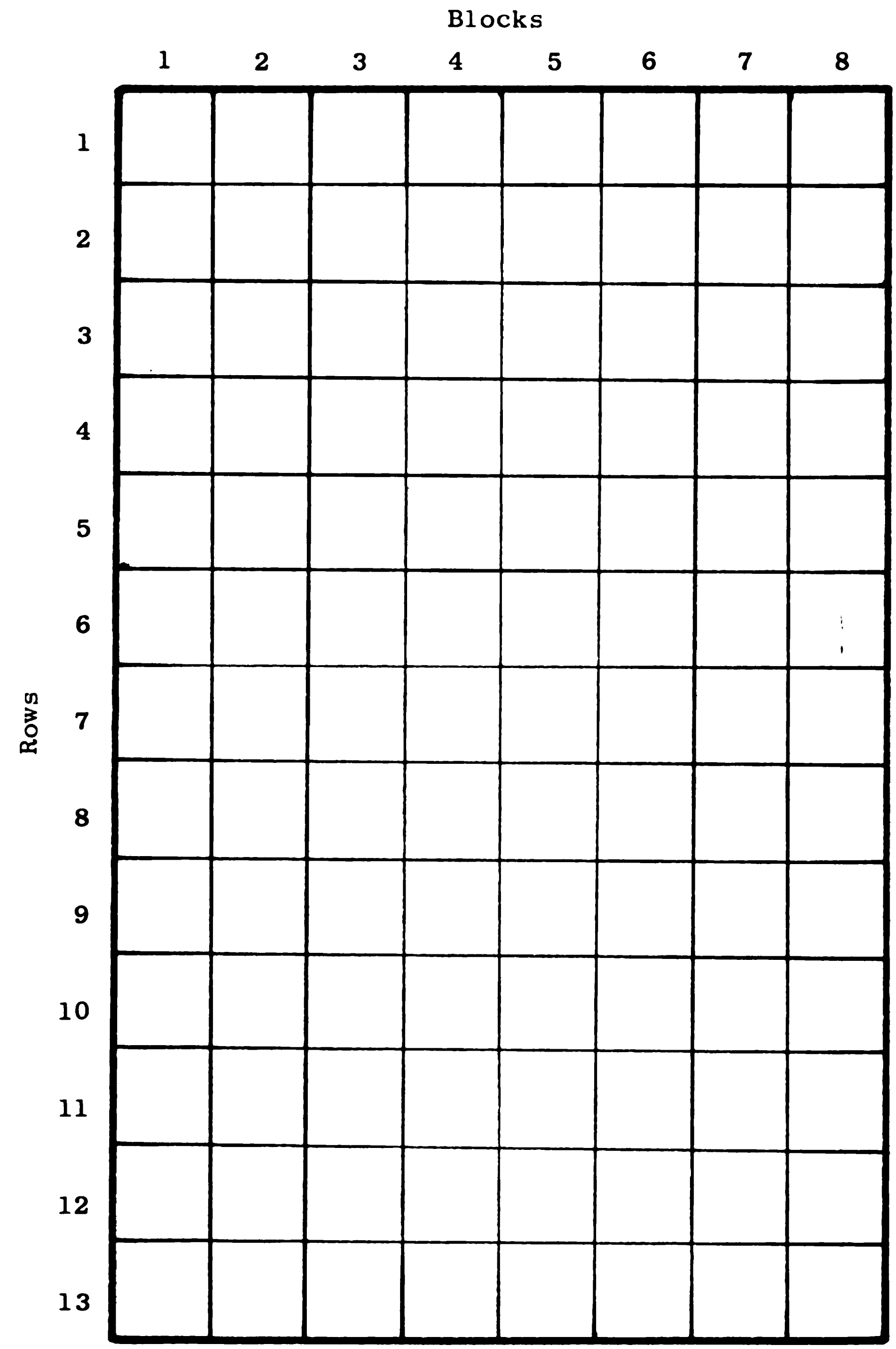

Figure 2. -- Diagram showing row and block arrangement of the one-chain square plots.

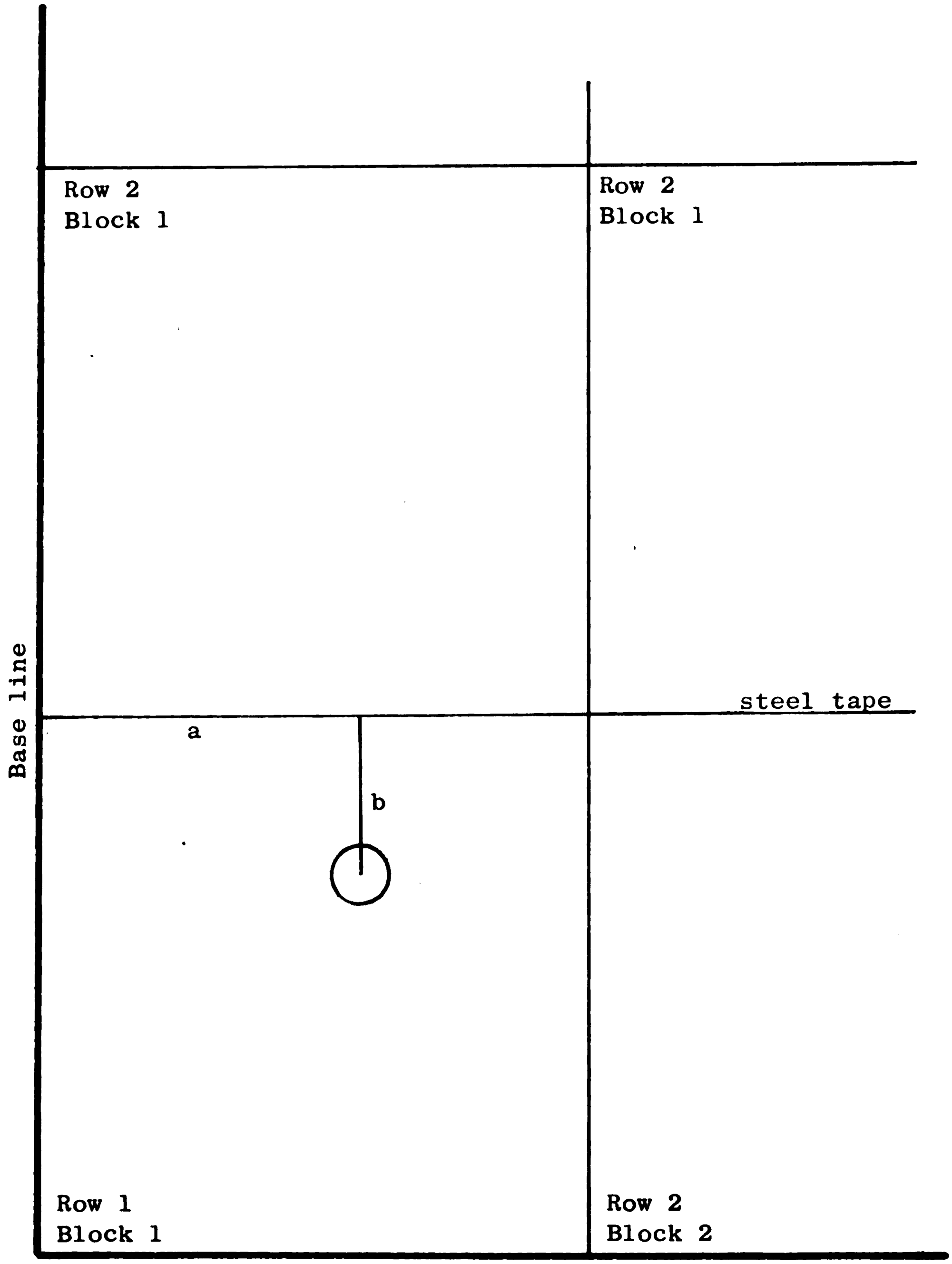

Figure 3. -- Diagram showing the method of obtaining the rectangular coordinates for each tree. The abscissa a is read from the steel tape, starting at the base line. The ordinate b is measured with a rangefinder.

The same procedure was used for Toumey Woods, except the blocks were only sixty-foot squares. In a previous study, iron stakes had been set at these intervals, and since any subdivision scheme is only an aid in measuring and mapping tree locations, this size block was used.

Mapping

The forest areas were mapped on graph paper (ten squares per inch) to a scale of 1:120. The location of each tree center was circled in ink and the tree number, diameter, and basal area to three decimal places were placed adjacent to the circle (Figure in Appendix A). Every other tree located on the boundary of the forest was discarded. Each completed map measured 4.4 by 7.5 feet.

Sampling Methods

Number of Sampling Units. -- Since comparisons among the study areas were not made, an arbitrary number of sampling units was assigned to each forest. As a guide, the number of points used had to give an estimate of the population mean of, at least, a plus or minus ten percent of the sampling mean at the 68 percent level of accuracy. This was applied to the one-fifth acre[3/] circular plots on a random design.

Twenty-five plots were drawn at random from the Lansing Woods. The estimate was well within the required limits; however, only the last plot drawn was discarded to make it easier to apply the number of plots to a systematic grid. Twenty plots were selected at random in Toumey Woods. The results were well within the required limits, so the number was

3/ Correspondence with Mr. Harold W. Kollmeyer of the Michigan Department of Conservation indicated that this is the most common size and shape of plot used in state forest inventory work.

reduced to 16 for the Red Cedar Woods. The resulting calculations failed to meet the standard of accuracy, so two additional plots were drawn. The results are presented in Table 2. The same number of plots was used for each distribution under investigation in this study.

Table 2. -- Statistics for the random, one-fifth acre, circular plots for an arbitrary number of sampling units.

Statistics	Lansing Woods	Toumey Woods	Red Cedar Woods
Number of Samples	24	20	18
Mean per Acre (sq. ft.)	77.16	117.34	92.71
Standard Deviation (sq. ft.)	17.35	35.88	29.14
Standard Error (sq. ft.)	3.54	8.02	6.87
Level of Accuracy (percent)	5	7	7

Distribution of the Sample Unit. -- Several methods of distributing the sample units were mentioned in the literature review. Of primary importance, of course, is the comparison between the random and systematic distributions. The multiple-random-start (MRS) method is a logical compromise between the two, and was also used. Cluster arrangements, such as those suggested by Johnson (1948), or stratification, as proposed by Schumacher and Chapman (1948), are not included in this study, because of the small size of the forest.

In selecting the location of the random samples, each forest is considered to be made up of a population of plot centers equal to the products of the sides of the forest (Palley and O'Regan, 1961). In this study, each woods measures 528 feet by 858 feet for a total of 453,024 possible plot centers. The ordinate and abscissa of the center of each sample was chosen by drawing numbers, with replacement, from a table of random digits. If a point fell closer to the plot boundary than the radius of the one-fifth acre plot (52.7 feet), that point was moved far enough to include all of this unit within the forest.4/ These points were numbered and marked on the maps with ink (see Appendix A).

In the systematic distribution, the distance between samples was computed by the equation:

$$d^2 = A/n \tag{7}$$

where: d = grid distance between plot centers in feet

A = total forest area in square feet

n = number of units in the sample

The grid distances for the Lansing Woods, Toumey Woods, and Red Cedar Woods is 138, 150, and 159 feet, respectively. The grid was placed on the map in a random manner, by drawing the coordinates of one of the sample units from a table of random numbers. The other units were located systematically from this first point according to the prescribed grid. This randomization of the first point conforms with the considerations of the multiple-random-starts as suggested by Shiue (1957).

The multiple-random start distribution was designed, and analysed, according to Shiue (1957), on the basis of a conceptional forest made up of 104 one-tenth acre square plots. The center of these units serve as the center locations for all sampling units. The same number of samples was measured in this distribution as in the random and systematic systems. Four random-starts were used in the Lansing and Toumey Woods and three in the Red Cedar.

4/ This is in accordance with the field sampling procedures used by the U. S. Forest Service in sampling timber on the nationwide forest survey.

Size and Types of Sampling Units

The size, shape, and dimensions of the sample units used in this study are given in Tables 3 and 4. These sampling units include those most frequently used in forest sampling and inventory work. No attempt was made to evaluate those units designed primarily for ecological or botanical research or to test the accuracies of methods like the Bitterlich line-sampling system, since it has not been used in forestry work. Strip samples were also omitted, because of the small size of the forests, and past studies indicate their lack of precision. In place of strip samples, rectangular plots with various short-to-long side ratios were tested.

In order to analyse certain sampling arrangements by regression techniques, as suggested by Palley and O'Regan (1961), the areal plot sizes were decreased in a reciprocal logarithmic fashion. These are 1/5, 1/10, 1/20, 1/40, 1/80, and 1/160 of an acre. All plots of a given shape are concentric for these sizes.

In point-sampling, the six basal area factors used were 5, 10, 20, 30, 40, and 50. Plot radius distances for each of these are given in Appendix B. Both Grosenbaugh's and Haga and Maezawa's peripheral adjustment systems were tested (see page 8 of the Literature Review). To weight the trees, according to Grosenbaugh, the peripheral zones of Table 5 were used. These are based on the large tree diameters of 25", 42", and 42" for the Lansing Woods, Toumey Woods, and Red Cedar Woods, respectively. The corners of the zones were marked on the maps with pins and moved for successive basal area factors. These samples are referred to as the adjusted point-samples.

Table 3. -- Dimensions of the circular, triangular, and square plots by plot size.

Plot Shape	Plot Size in Acres					
	1/5	1/10	1/20	1/40	1/80	1/160
	Plot Dimensions in Feet					
Circular (radius)	53	37	26	19	13	9
Triangular (side)	142	100	71	50	36	25
Square (side)	93	66	47	33	23	17

Table 4. -- Dimensions of rectangular plots by plot size.

Short to Long Side Ratio	Plot Size in Acres					
	1/5	1/10	1/20	1/40	1/80	1/160
1:2	66:132	47:93	33:66	23:47	16:33	12:23
1:4	47:187	33:132	23:93	16:66	12:47	8:33
1:8	33:264	23:187	16:132	12:93	8:66	6:47

Table 5. -- Peripheral zone widths for correcting edge-bias in point-sampling (after Grosenbaugh, 1958).

Basal Area Factor	Lansing Woods	Toumey and Red Cedar Woods
	(feet)	(feet)
5	97	184
10	69	115
20	49	82
30	40	67
40	34	58
50	31	52

At the completion of the sampling by weighting, the process was repeated, giving each tree a count of one, regardless of its position in the forest. The percent bias was then calculated according to Haga and Maezawa. Average tree radii for the Lansing Woods, Toumey Woods, and Red Cedar Woods are 5.65, 7.65, and 6.90 inches, respectively. The resulting bias is given in Table 6 and is always positive. These values were applied directly to all estimates in this method of sampling. These samples are referred to as the unadjusted point-samples.

Table 6. -- Calculated bias based upon forest area, perimeter, and average tree radius (after Haga and Maezawa, 1959)[a].

Basal Area Factor	Plot Radius Factor[b]	Lansing Woods	Toumey Woods	Red Cedar Woods
	feet/inches	(percent)	(percent)	(percent)
5	3.889	.22	.30	.27
10	2.750	.16	.21	.19
20	1.944	.11	.15	.14
30	1.588	.09	.12	.11
40	1.375	.08	.11	.10
50	1.230	.07	.10	.09

[a]These values were obtained from equation (1) with an "e" value of 2.0 based upon a forest area of 4.209 hectares and a perimeter of 844.9 meters (10.4 acres, with 2,772 feet of perimeter).

[b]Radius of the tree ring in feet per inch of tree diameter.

The Sampling Process

In drawing the plot samples from the maps, successive areal plot sizes were drawn to scale on an acetate sheet to form a template. The center of the template was placed over each sample center, and the basal area within each plot size outline was recorded as in Appendix A.

For drawing point-samples from the maps, the plot radius distances from Appendix B were drawn on acetate strips. A pin secured the zero end of the strip to the sample point being measured. By rotating the strip about the pin and observing the relationship of each tree's diameter to the diameter and plot radius factors on the acetate strip, the tree was either counted "in" or "out." Tree diameters on the acetate strips were interpolated to the nearest one-tenth inch. If there was any doubt concerning the count of an individual, the distance between it and the sample point was checked by the Pythagorean theorem. Approximately fifty such checks were required in this study.

Within each forest, three distributions were tested. Six plot shapes and two types of point-samples were sampled in each distribution. These were: circular, triangular, square, rectangular 1:2, rectangular 1:4, rectangular 1:8, and the adjusted and unadjusted point-samples. Six plot sizes were applied to each plot shape and six basal area factors were tested in each type of point-sample. The combination of these items yields 144 separate samples (3 x 8 x 6) in each forest.

CHAPTER IV

RESULTS AND DISCUSSION

One hundred and forty-four methods of sampling involving six plot sizes, eight plot types or shapes and three cruising designs were measured and analysed on each forest. The basic analysis involved the use of five parameters for each sampling method. These are:

The Expected Value. -- The average basal area per acre in square feet. This is not used in the strict statistical sense.

Standard Deviation. -- Standard deviation of the sample expressed in square feet per acre.

Standard Error. -- Standard error of the mean expressed in square feet per acre.

The Coefficient of Variation. -- This is the standard error expressed as a percent of the mean and is not a strict statistical parameter. It may be referred to as sampling error (after Grosenbaugh and Stover, 1957). The accuracy of a sampling system is inversely proportional to its CV. Comparative analysis among the sampling methods used is carried out by an analysis of variance on this parameter.

Deviation from the True Mean. -- This is the deviation of the expected value from the true mean, expressed as a percent of the true mean. It is recorded as either a positive or negative number and is of use only where the population is completely inventoried.

Lansing Woods

Plot Samples. -- The parameter summaries for all samples taken in this forest are presented in Table 7 to Table 14. The most accurate sampling method is the one with the lowest coefficient of variation (CV). In

Table 7. -- Parameter summary, Lansing Woods, circular plots.

Method of Distribution	Parameter	Sampling Unit Size in Acres 1/160	1/80	1/40	1/20	1/10	1/5
Random	Expected Value Sq. Ft.	86.56	92.32	85.44	71.08	79.88	77.16
	Standard Deviation Sq. Ft.	113.44	80.16	46.20	22.02	25.26	17.35
	Standard Error Sq. Ft.	23.15	16.36	9.43	4.49	5.16	3.54
	Coefficient of Variation Percent	26.74	17.72	11.04	6.32	6.46	4.59
	Deviation from True Mean Percent	+ .83	+18.21	+9.40	-8.99	+2.28	-1.20
Systematic	Expected Value Sq. Ft.	120.96	108.56	96.36	84.18	86.20	77.21
	Standard Deviation Sq. Ft.	163.36	109.12	59.48	36.82	32.64	22.61
	Standard Error Sq. Ft.	33.34	22.27	12.14	7.51	6.66	4.61
	Coefficient of Variation Percent	27.56	20.51	12.60	8.92	7.73	5.97
	Deviation from True Mean Percent	+54.88	+39.00	+23.38	+7.78	+10.37	-1.14
Multiple Random Starts	Expected Value Sq. Ft.	74.40	58.80	61.20	69.00	82.03	80.02
	Standard Deviation Sq. Ft.	84.32	54.16	45.36	37.54	36.13	33.14
	Standard Error Sq. Ft.	17.21	11.06	9.26	7.66	7.38	6.76
	Coefficient of Variation Percent	23.13	18.81	15.13	11.10	9.00	8.45
	Deviation from True Mean Percent	-4.74	-24.71	-21.64	-11.65	+5.03	+2.46

Table 8. -- Parameter summary, Lansing Woods, triangular plots.

Method of Distribution	Parameter	Sampling Unit Size in Acres 1/160	1/80	1/40	1/20	1/10	1/5
Random	Expected Value Sq. Ft.	62.08	84.80	87.68	78.28	78.20	75.33
	Standard Deviation Sq. Ft.	106.08	81.76	55.56	32.74	12.71	14.65
	Standard Error Sq. Ft.	21.65	16.69	11.34	6.68	2.59	2.99
	Coefficient of Variation Percent	34.87	19.68	12.94	8.53	3.31	3.97
	Deviation from True Mean Percent	-20.51	+8.58	+12.27	+ .23	+.13	-3.55
Systematic	Expected Value Sq. Ft.	119.36	92.40	96.68	96.96	91.52	80.94
	Standard Deviation Sq. Ft.	188.80	86.80	65.52	43.42	32.54	25.88
	Standard Error Sq. Ft.	38.54	17.72	13.37	8.86	6.64	5.28
	Coefficient of Variation Percent	32.29	19.18	13.83	9.14	7.26	6.53
	Deviation from True Mean Percent	+52.83	+18.31	+23.79	+24.15	+17.18	+3.64
Multiple Random Starts	Expected Value Sq. Ft.	91.20	63.04	58.36	72.08	79.62	79.33
	Standard Deviation Sq. Ft.	103.84	50.00	37.92	44.60	44.45	38.38
	Standard Error Sq. Ft.	21.20	10.21	7.74	9.10	9.07	7.83
	Coefficient of Variation Percent	23.24	17.62	13.26	12.62	11.39	9.87
	Deviation from True Mean Percent	+16.77	-19.28	-25.27	-7.71	+1.95	+1.57

Table 9. Parameter summary, Lansing Woods, square plots.

Method of Distribution	Parameter	Sampling Unit Size in Acres 1/160	1/80	1/40	1/20	1/10	1/5
Random	Expected Value Sq. Ft.	85.12	98.40	84.92	79.12	76.52	76.39
	Standard Deviation Sq. Ft.	113.12	92.00	61.84	28.44	24.71	18.89
	Standard Error Sq. Ft.	23.09	18.78	12.62	5.80	5.04	3.86
	Coefficient of Variation Percent	27.13	19.08	14.86	7.33	6.59	5.05
	Deviation from True Mean Percent	+8.99	+25.99	+8.73	+1.31	-2.02	-2.19
Systematic	Expected Value Sq. Ft.	120.96	93.92	95.08	88.30	85.38	76.62
	Standard Deviation Sq. Ft.	163.36	92.20	66.12	45.28	33.66	22.84
	Standard Error Sq. Ft.	33.34	18.82	13.50	9.24	6.87	4.66
	Coefficient of Variation Percent	27.56	20.04	14.20	10.46	8.05	6.08
	Deviation from True Mean Percent	+54.88	+20.26	+21.74	+13.06	+9.32	-1.89
Multiple Random Starts	Expected Value Sq. Ft.	75.68	59.44	68.12	68.10	72.30	82.12
	Standard Deviation Sq. Ft.	81.60	49.76	45.96	35.28	30.89	33.85
	Standard Error Sq. Ft.	16.66	10.16	9.38	7.20	6.30	6.91
	Coefficient of Variation Percent	22.01	17.08	13.77	10.57	8.71	8.41
	Deviation from True Mean Percent	-3.10	-23.89	-12.78	-12.80	-7.43	+5.15

Table 10. -- Parameter summary, Lansing Woods, rectangular, 1:2.

Method of Distribution	Parameter	Sampling Unit Size in Acres 1/160	1/80	1/40	1/20	1/10	1/5
Random	Expected Value Sq. Ft.	73.60	85.12	83.56	70.74	71.46	78.00
	Standard Deviation Sq. Ft.	107.68	75.76	53.08	35.68	19.19	21.07
	Standard Error Sq. Ft.	21.98	15.46	10.84	7.28	3.92	4.30
	Coefficient of Variation Percent	29.86	18.16	12.97	10.29	5.49	5.51
	Deviation from True Mean Percent	-5.76	+8.99	+6.99	-9.42	-8.50	-.13
Systematic	Expected Value Sq. Ft.	113.28	104.32	92.52	90.92	79.69	79.35
	Standard Deviation Sq. Ft.	132.64	106.48	66.72	40.80	29.88	24.02
	Standard Error Sq. Ft.	27.08	21.74	13.62	8.33	6.10	4.90
	Coefficient of Variation Percent	23.90	20.84	14.72	9.16	7.65	6.17
	Deviation from True Mean Percent	+45.04	+33.57	+18.46	+16.41	+2.03	+1.60
Multiple Random Starts	Expected Value Sq. Ft.	59.52	78.32	68.16	75.60	83.84	79.10
	Standard Deviation Sq. Ft.	89.12	63.28	47.68	36.60	35.72	25.62
	Standard Error Sq. Ft.	18.19	12.92	9.73	7.47	7.29	5.23
	Coefficient of Variation Percent	30.56	16.50	14.27	9.88	8.69	6.61
	Deviation from True Mean Percent	-23.79	+.28	-12.73	-3.20	+7.35	+1.28

Table 11. -- Parameter summary, Lansing Woods, rectangular 1:4.

Method of Distribution	Parameter	Sampling Unit Size in Acres 1/160	1/80	1/40	1/20	1/10	1/5
Random	Expected Value Sq. Ft.	114.08	84.64	67.08	72.28	82.16	77.63
	Standard Deviation Sq. Ft.	144.96	87.28	46.68	25.48	29.21	23.04
	Standard Error Sq. Ft.	29.59	17.81	9.53	5.20	5.96	4.70
	Coefficient of Variation Percent	25.94	21.04	14.21	7.19	7.25	6.05
	Deviation from True Mean Percent	+46.07	+8.37	-14.11	-7.45	+5.20	-.60
Systematic	Expected Value Sq. Ft.	88.64	94.48	94.00	80.02	82.00	72.76
	Standard Deviation Sq. Ft.	113.76	99.28	64.20	42.02	32.86	22.40
	Standard Error Sq. Ft.	23.22	20.26	13.10	8.58	6.71	4.57
	Coefficient of Variation Percent	26.19	21.44	13.94	10.72	8.18	6.28
	Deviation from True Mean Percent	+13.50	+20.97	+20.36	+24.58	+4.99	-6.84
Multiple Random Starts	Expected Value Sq. Ft.	72.37	79.04	98.52	99.18	84.10	75.50
	Standard Deviation Sq. Ft.	96.32	83.20	64.08	55.74	27.47	20.02
	Standard Error Sq. Ft.	19.66	16.98	13.08	11.38	5.61	4.09
	Coefficient of Variation Percent	27.16	21.48	13.28	11.47	6.67	5.42
	Deviation from True Mean Percent	-7.34	+1.20	+26.14	+26.99	+7.68	-3.33

Table 12. -- Parameter summary, Lansing Woods, rectangular, 1:8.

Method of Distribution	Parameter	Sampling Unit Size in Acres 1/160	1/80	1/40	1/20	1/10	1/5
Random	Expected Value Sq. Ft.	93.28	90.40	82.64	84.82	82.40	77.12
	Standard Deviation Sq. Ft.	128.48	99.52	49.20	41.66	25.48	17.16
	Standard Error Sq. Ft.	26.23	20.31	10.04	8.50	5.20	3.50
	Coefficient of Variation Percent	28.12	22.47	12.15	10.02	6.31	4.54
	Deviation from True Mean Percent	+19.44	+15.75	+5.81	+8.60	+5.50	-1.25
Systematic	Expected Value Sq. Ft.	64.96	82.48	89.64	87.14	84.75	81.16
	Standard Deviation Sq. Ft.	89.44	64.00	61.56	44.96	35.37	22.52
	Standard Error Sq. Ft.	18.26	13.06	12.57	9.18	7.22	4.60
	Coefficient of Variation Percent	28.11	15.83	14.02	10.53	8.52	5.67
	Deviation from True Mean Percent	-16.81	+5.61	+14.77	+11.57	+8.51	+3.92
Multiple Random Starts	Expected Value Sq. Ft.	68.48	93.20	110.44	105.54	86.32	77.44
	Standard Deviation Sq. Ft.	121.92	88.40	64.44	55.32	36.97	18.58
	Standard Error Sq. Ft.	24.89	18.04	13.15	11.29	7.55	3.79
	Coefficient of Variation Percent	36.35	19.36	11.91	10.70	8.75	4.89
	Deviation from True Mean Percent	-12.32	+19.33	+41.41	+35.13	+10.52	-.84

Table 13. -- Parameter summary, Lansing Woods, adjusted point-sampling.

Method of Distribution	Parameter	Basal Area Factor (BAF)* 50	40	30	20	10	5
Random	Expected Value Sq. Ft.	83.50	90.00	97.50	100.00	122.50	151.25
	Standard Deviation Sq. Ft.	62.00	60.40	58.80	49.40	69.90	69.80
	Standard Error Sq. Ft.	12.66	12.33	12.00	10.08	14.27	14.25
	Coefficient of Variation Percent	15.16	13.70	12.31	10.08	11.65	9.42
	Deviation from True Mean Percent	6.91	15.24	24.84	28.04	56.85	93.66
Systematic	Expected Value Sq. Ft.	98.00	101.60	101.40	103.40	115.80	133.40
	Standard Deviation Sq. Ft.	70.00	63.60	57.60	52.40	47.60	61.90
	Standard Error Sq. Ft.	14.29	12.98	11.76	10.70	9.72	12.63
	Coefficient of Variation Percent	14.58	12.77	11.60	10.35	8.39	9.47
	Deviation from True Mean Percent	+25.48	+30.09	+29.83	+32.39	+48.27	+70.81
Multiple Random Starts	Expected Value Sq. Ft.	66.50	73.20	80.10	82.40	104.20	113.60
	Standard Deviation Sq. Ft.	65.50	48.00	46.50	40.60	45.60	38.80
	Standard Error Sq. Ft.	13.37	9.80	9.49	8.29	9.31	7.92
	Coefficient of Variation percent	20.10	13.39	11.82	10.06	8.93	6.97
	Deviation from True Mean Percent	-14.85	-6.27	+2.56	+5.50	+33.42	+45.45

*The number associated with the basal area factor (BAF) indicates the square feet of basal area per acre for every tree counted.

Table 14. -- Parameter summary, Lansing Woods, unadjusted point-sampling.

Method of Distribution	Parameter	Basal Area Factor (BAF) 50	40	30	20	10	5
Random	Expected Value Sq. Ft.	83.50	85.00	88.80	77.60	75.80	77.50
	Standard Deviation Sq. Ft.	62.00	58.00	48.90	36.00	23.60	18.80
	Standard Error Sq. Ft.	12.66	11.84	9.98	7.35	4.88	3.84
	Coefficient of Variation Percent	15.16	13.93	11.24	9.47	6.44	4.95
	Deviation from True Mean Percent	+6.91	+8.83	+13.70	- .64	-2.94	-.77
Systematic	Expected Value Sq. Ft.	98.00	101.60	70.10	11.60	81.70	76.40
	Standard Deviation Sq. Ft.	70.00	63.60	57.60	50.40	33.20	18.45
	Standard Error Sq. Ft.	14.29	12.98	11.76	10.29	6.78	3.77
	Coefficient of Variation Percent	14.58	12.77	11.91	11.23	8.30	4.93
	Deviation from True Mean Percent	+25.48	+30.09	+26.38	+17.28	+4.61	-2.18
Multiple Random Starts	Expected Value Sq. Ft.	66.50	68.40	77.40	73.40	77.90	76.90
	Standard Deviation Sq. Ft.	62.00	48.00	45.90	58.80	27.20	29.15
	Standard Error Sq. Ft.	12.67	9.80	9.37	12.00	5.55	5.95
	Coefficient of Variation Percent	19.05	14.33	12.10	16.35	7.12	7.74
	Deviation from True Mean Percent	-8.51	-12.42	-.90	-6.08	-.26	-1.54

this woods, it is a one-tenth acre, triangular plot on the random distribution, and has a CV of 3.31 percent. The causes of variation in CV are plot distribution, treatment (plot shape, size, and interaction), and an undefined quantity called error, so it is possible that this sample is the best because of chance. To analyse this possibility, the CV of all samples were summarized in Table 15 and subjected to analysis of variance techniques. The sums of squares (SS), degrees of freedom (df) and the mean square, associated with each source of variation are given in Table 16.

According to the "F" test, there is no significant difference among the random, systematic ,or multiple-random-start (MRS) distributions except that arising by pure chance. Any one of the three methods yields approximately the same results. Plot shape also has little influence on the magnitude of the sampling error. The preference for one plot shape over another would have to be a matter of efficiency -- i.e., circular plots are easier to establish than angular plots (Lindsey *et al*., 1958), and have less perimeter for a given area. The interaction between plot shape and plot size is also negligible.

Plot size, on the other hand, is a very strong source of variation and is highly significant. The sampling error increases sharply as the plot size decreases. The average sampling errors by plot sizes are shown in Tables 17 and 18. The five degrees of freedom and the associate sums of squares for plot size are partitioned into certain comparisons among the various plot sizes (Appendix C). The results showed that each succeeding increase in plot size significantly decreased the coefficient of variation, except for the one-fifth and one-tenth acre units; either of these could have been used with the same results.

Table 15. -- Summary of the coefficient of variation data for the Lansing Woods plot samples.

Plot Shape	Plot Size*	Random	Systematic	MRS	Plot Size Total	Plot Shape Total
Circular	160	26.74	27.56	23.13	77.43	
	80	17.72	20.51	18.81	57.04	
	40	11.04	12.60	15.13	38.77	
	20	06.32	08.92	11.10	26.34	
	10	06.46	07.73	09.00	23.19	
	5	04.59	05.97	08.45	19.01	
	Total	72.87	83.29	85.62		241.78
Triangular	160	34.87	32.29	23.24	90.40	
	80	19.68	19.18	17.62	56.48	
	40	12.94	13.83	13.26	40.03	
	20	08.53	09.14	12.62	30.29	
	10	03.31	07.26	11.39	21.96	
	5	03.97	06.53	09.87	20.37	
	Total	83.30	88.23	88.00		259.53
Square	160	27.13	27.56	22.01	76.70	
	80	19.08	20.04	17.09	56.21	
	40	14.86	14.20	13.77	42.83	
	20	07.33	10.46	10.57	28.36	
	10	06.59	08.05	08.71	23.35	
	5	05.05	06.08	08.41	19.54	
	Total	80.04	86.39	80.56		246.99
Rect. 1:2	160	29.86	23.90	30.56	84.32	
	80	18.16	20.84	16.50	55.50	
	40	12.97	14.72	14.27	41.96	
	20	10.29	09.16	09.88	29.33	
	10	05.49	07.65	08.69	21.83	
	5	05.51	06.17	06.61	18.29	
	Total	82.28	82.44	86.51		251.23
Rect. 1:4	160	25.94	26.19	27.16	79.29	
	80	21.04	21.44	21.48	63.96	
	40	14.21	13.94	13.28	41.43	
	20	07.19	10.72	11.47	29.38	
	10	07.25	08.18	06.67	22.10	
	5	06.05	06.28	05.42	17.75	
	Total	81.68	86.75	85.48		253.91
Rect. 1:8	160	28.12	28.11	36.35	92.58	
	80	22.47	15.83	19.36	57.66	
	40	12.15	14.02	11.91	38.08	
	20	10.02	10.53	10.70	31.25	
	10	06.31	08.52	08.75	23.58	
	5	04.54	05.67	04.89	15.10	
	Total	83.61	82.68	91.96		258.25
All Plot Shapes	160	172.66	165.61	162.45	500.72	
	80	118.15	117.84	110.86	346.85	
	40	78.17	83.31	81.62	243.10	
	20	49.68	58.93	66.34	174.95	
	10	35.41	47.39	53.21	136.01	
	5	29.71	36.70	43.65	110.06	
	Total	483.78	509.78	518.13		1,511.69

*Plot size in acres, denominator only.

Table 16. -- Analysis of variance, Lansing Woods, plot samples.

Source	df	SS	MS	F
Distribution	2	17.8300	8.9150	1.87 N.S.
Plot Shape	5	12.7520	2.5504	- N.S.
Plot Size	5	6137.4834	1227.4969	254.91 * *
Interaction	25	97.7112	3.9084	- N.S.
Error	70	337.0884	4.8155	
Total	107	6602.8650		

N.S. Non-significant
* * Highly significant (99 percent level)

Table 17. -- Average coefficient of variation, Lansing Woods.

Plot Samples				Point-Samples		
Plot Shape	CV (percent)	Plot Size (acres)	CV (percent)	BAF	Adjusted CV (percent)	Unadjusted CV (percent)
Circular	13.43	1/160	27.82	50	16.61	16.35
Triangular	14.42	1/80	19.27	40	13.28	13.67
Square	13.72	1/40	13.51	30	11.91	11.75
Rect. 1:2	13.96	1/20	9.72	20	10.16	12.35
Rect. 1:4	14.11	1/10	7.56	10	9.66	7.28
Rect. 1:8	14.35	1/5	6.11	5	8.65	5.87

Table 18. -- Average coefficient of variation by distributions, Lansing Woods.

Samples	Random (percent)	Systematic (percent)	MRS (percent)
Plot	13.44	14.16	14.39
Adjusted Point	12.05	11.19	11.88
Unadjusted Point	10.20	10.67	12.78

The relationship between plot size and the corresponding coefficient of variation is shown in Figure 4. The CV from Table 17 was plotted over the reciprocal of the square root of the plot size. The resulting pattern of points can be described by the equation:

$$CV = 457.570(P)^{-\frac{1}{2}} \tag{8}$$

where: CV = the coefficient of variation in percent

P = plot size in square feet

457.57 = the regression coefficient derived from the data

The correlation coefficient between the independent and dependent variables is 99.92 percent and is highly significant. Under conditions similar to this study, an increase in the plot size will increase the accuracy of the estimate. The limitation on this would be the loss in efficiency for large plots.

<u>Point-Sampling</u>.-- The summary of the CV data for point-samples is given in Table 19. The analysis of variance of these data is given in Table 20. Here, the manner of distributing the sample points, and the method of sampling, either with or without peripheral adjustments (type), is of no consequence in this forest. However, the choice of basal area factors used, strongly influences the magnitude of the CV. Table 17 gives the average CV for each basal area factor. Comparison among these factors show that all differences are highly significant except between BAF-5 and BAF-10. The trend of the CV with respect to the basal area factors may be shown by plotting CV over the plot radius factor for each BAF. The resulting pattern of points in Figure 5 may be described by the equation:

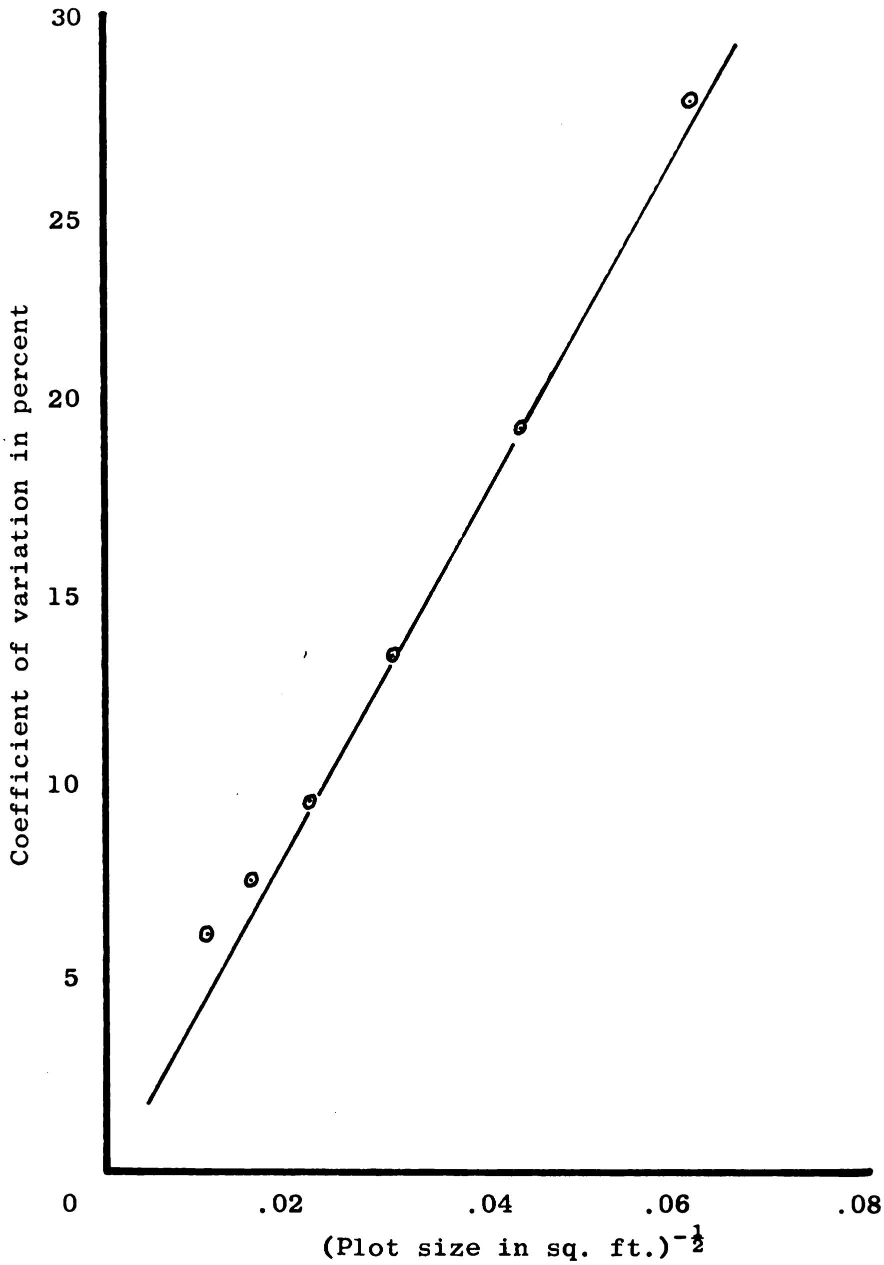

Figure 4. -- The relationship between the coefficient of variation in percent and the reciprocal of the square root of the plot size in square feet. From the equation: $CV = 457.57 \text{ (plot size)}^{-\frac{1}{2}}$, Lansing Woods.

Table 19. -- Summary of the coefficient of variation data for the Lansing Woods, point-sampling.

Plot Type	BAF	Distribution Random	Systematic	MRS	BAF Total	Plot Type Total
Adjusted Point- Samples	50	15.16	14.58	20.10	49.84	
	40	13.70	12.77	13.39	39.86	
	30	12.31	11.60	11.82	35.73	
	20	10.08	10.35	10.06	30.49	
	10	11.65	08.39	08.93	28.97	
	5	09.42	09.47	06.97	25.86	
	Total	72.32	67.16	71.27		210.75
Unadjusted Point- Samples	50	15.16	14.85	19.05	49.06	
	40	13.93	12.77	14.33	41.03	
	30	11.24	11.91	12.10	35.25	
	20	09.47	11.23	16.35	37.05	
	10	06.44	08.30	07.12	21.86	
	5	04.95	04.93	07.74	17.62	
	Total	61.19	63.99	76.69		201.87
All Plot Types	50	30.32	29.43	39.15	98.90	
	40	27.63	25.54	27.72	80.89	
	30	23.55	23.51	23.92	70.98	
	20	19.55	21.58	26.41	67.54	
	10	18.09	16.69	16.05	50.83	
	5	14.37	14.40	14.71	43.48	
	Total	133.51	131.15	147.96		412.62

Table 20. -- Analysis of variance, Lansing Woods, point-sampling, coefficient of variation.

Source	df	SS	MS	F
Distribution	2	13.8040	6.9020	- N.S.
Treatment	11	364.3711	33.1246	11.99 * *
Type	1	2.1903	2.1903	- N.S.
BAF	5	337.0892	67.4178	24.40 * *
Interaction	5	25.0916	5.0183	- N.S.
Error	22	60.7799	2.7627	
Total	35	438.9550		

N.S. Non-significant.
* * Highly significant.

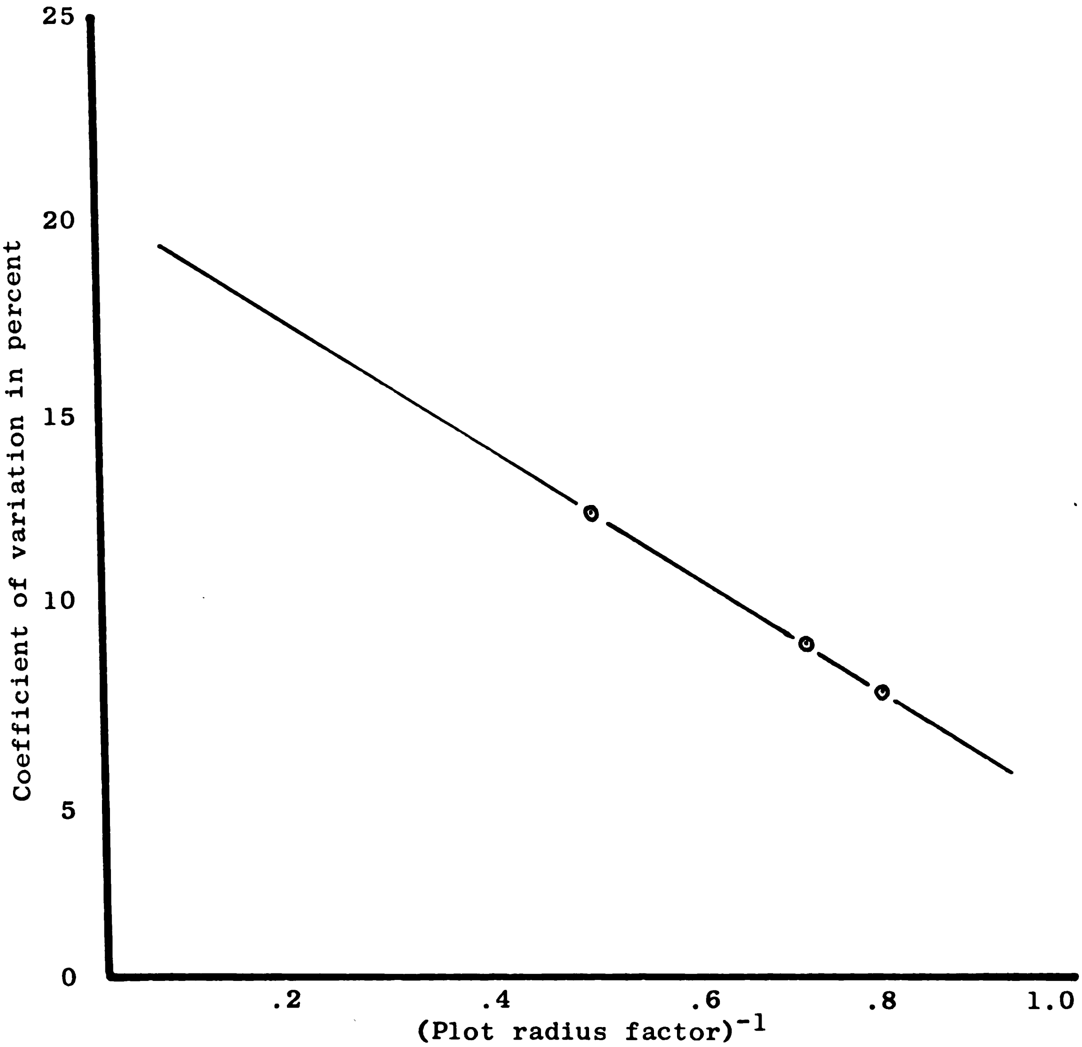

Figure 5. -- The relationship between the coefficient of variation in percent and the reciprocal of the plot radius factor. From the equation: CV = 19.905 - 15.351(Prf)$^{-1}$, Lansing Woods.

$$CV = a + b(Prf)^{-1} \tag{9}$$

where: CV = coefficient of variation in percent

Prf = plot radius factor in feet per inch of tree diameter

a = constant

b = regression coefficient

Following solution, the equation becomes:

$$CV = 19.905 - 15.351\ (Prf)^{-1} \tag{10}$$

with a highly significant correlation coefficient of 97.89 percent. A decrease in the BAF will result in significant decreases in the sampling error.

Comparison of Plot-Samples with Point-Samples. -- The most accurate areal plot is the one-fifth acre size with an average coefficient of variation of 6.11 percent (Table 17). Plot shape and method of distribution is of no importance, in that any sampling method would have given essentially the same results with this size plot. The most accurate point-sampling factor to use is the BAF-5 with an average CV of 7.25 percent (Table 17). The "t" test on the pooled variances is employed to analyse the difference between these two samples (Table 21).

The calculated "t" value of 1.48 on 22 degrees of freedom does not exceed the tabular value at the 95 percent level of accuracy (Freese, 1963), so the difference between these two types of samples may be attributed to chance and either may be used with the same expected precision. The difference between the one-tenth acre plot (CV = 7.56) and the BAF-10 (CV = 8.47) is also unimportant. The calculated "t" is 1.39 on 22 degrees of freedom.

Table 21. -- Analysis of the difference between the most accurate plot sample and point-sample, Lansing Woods.

Method	No. of Observations	df	Ave. CV (percent)	SS
One-fifth acre	18	17	6.11	38.38
BAF-5	6	5	7.25	20.63
Sum and differences		22	1.14	59.01

Deviation from the True Population Mean. -- The possibility exists that comparisons among distributions may not be valid. All sample estimates of error are based upon an infinite population, while in systematic sampling, this idea is not entirely true. Although the location of the first sample has a very large number of possible centers, the fixed grid forces the establishment of all other units throughout the population, and tends to sample the full range of the variation. Grouping of the plots does not occur. In random sampling, on the other hand, the plots can, and often do, sample certain sections of the forest much more intensely than the total number of plots warrants. Perhaps random samples are as good as or better than systematic or MRS because they measure only a part of the population variance. To check this, the deviation of the expected values from the true population mean can be examined. Table 22 lists the averages for each plot shape and plot size. These averages were compiled from the absolute values of Tables 7 to 14. Average deviations for the random, systematic and MRS distributions are 8.70 percent, 18.53 percent, and 12.47 percent, respectively, (from Tables 7 to 14). The closeness of the random estimate to the true value indicates that this method did an adequate job of sampling the population. Table 23 shows that these differences are highly significant.

Table 22. -- Average deviations of the expected value from the known population parameter in percent, Lansing Woods.

Plot Shape	Mean Deviation (percent)	Plot Size (area)	Mean Deviation (percent)	BAF	Adjusted Point-Sample Deviation (percent)	Unadjusted Point-Sample Deviation (percent)
Circular	13.76	1/160	22.64	50	15.75	13.63
Triangular	14.32	1/80	17.35	40	17.20	17.11
Square	13.08	1/40	17.77	30	19.08	13.66
Rect. 1:2	11.42	1/20	13.34	20	21.98	8.00
Rect. 1:4	13.65	1/10	6.44	10	46.18	2.60
Rect. 1:8	13.17	1/5	2.37	5	69.97	1.50

Table 23. -- Analysis of variance, deviation of the expected value from the true mean in percent, Lansing Woods, plot samples.

Source	df	SS	MS	F
Distribution	2	1770.2601	885.1300	7.84 * *
Treatment	35	6749.6629	1928.4751	17.09 * *
Plot Shape	5	89.0740	17.8148	- N.S.
Plot Size	5	5227.5647	1045.5129	9.27 * *
Interaction	25	1433.0242	57.3210	- N.S.
Error	70	7899.0903	112.8441	
Total	107	16,419.0133		

N.S. Non-significant

* * Highly significant

A cursory inspection of the values in Table 22 indicates a lack of correlation between plot shape and the amount of deviations from the true parameter. This is to be expected, as is the fact that plot size increases result in closer estimates of the true mean. However, the differences between the two methods of sampling by Bitterlich are startling. By using Grosenbaugh's peripheral zone technique, the error from the true value ranges from about 16 percent for a BAF-50 to nearly 70 percent for BAF-5. Review of these values in Tables 7 and 14 shows that this is a positive situation. The unadjusted technique shows the same trend as the areal plot sizes, and gives the closest estimate of any method when a BAF-5 is employed. Table 24, however, shows that there is no significance between either sample unit types or among the BA factors. Treatment is highly significant, but the interaction took so much of the sums of squares, that partitioning resulted in non-significance of the three sources under treatment. This is to be expected since variation increased in the adjusted plots as the BAF decreased. Interaction would normally claim a large portion of the sum of squares in this case. Under these circumstances, it is not advisable to use the peripheral method of adjustment.

Table 24. Analysis of variance, deviation of the expected value from the true mean in percent, Lansing Woods, point-samples.

Source	df	SS	MS	F
Distribution	2	1773.3293	886.6646	- N.S.
Treatment	11	27,473.6989	2,4976.0899	13.13 * *
Type	1	4465.3579	4465.3579	2.35 N.S.
BAF	5	2037.6916	407.5383	- N.S.
Interaction	5	5760.3535	1152.0707	- N.S.
Error	22	41,844.1632	1902.0438	
Total	35	55,881.6975		

N.S. Non-significant
* * Highly significant

Toumey Woods

Plot-Samples. -- The parameter summaries for all samples taken in this forest are presented in Tables 25 to 32. To analyse the differences between sampling methods, the coefficient of variations are listed in Table 33, and subjected to the analysis of variance in Table 34. Average values for the coefficient of variation for sample size, shapes, and distributions are given in Tables 35 and 36. These were compiled directly from Table 33.

According to the analysis of variance, plot size is the only factor that contributes significantly to the variation in sampling error. One method of distribution is as good as another and, as far as accuracy is concerned, plot shape also has no significant influence on the variation. The five degrees of freedom allocated for plot size may be partitioned into five comparisons as in the Lansing Woods. However, it is enough to show that the difference between the one-fifth and one-tenth acre plots is insignificant, and the difference between these two and the one-twentieth acre plot is significant.

$$\begin{aligned} 1/5 \text{ vs } 1/10 &= (138.09 - 103.49)^2/2(18) = 33.2544 \\ F &= 33.2544/16.2348 = 2.05 \text{ (not significant)} \end{aligned} \tag{11}$$

$$\begin{aligned} (1/5 + 1/10) \text{ vs } 1/20 &= [2(216.28)-(138.09+103.49)]^2/6(18) = 337.7163 \\ F &= 337.7163/16.2348 = 20.80 \text{ (Highly significant)} \end{aligned} \tag{12}$$

The "F" test is based upon one and 70 degrees of freedom. Either of the two largest plots may be used for the best results.

The relationship between plot size and CV may be shown by plotting the CV over the reciprocal of the square root of the plot size (Fig. 6). Following regression, the relationship may be expressed as:

Table 25. -- Parameter summary, Toumey Woods, circular plots.

Method of Distribution	Parameter	Sampling Unit Size in Acres 1/160	1/80	1/40	1/20	1/10	1/5
Random	Expected Value Sq. Ft.	76.96	126.72	89.84	104.86	114.70	117.34
	Standard Deviation Sq. Ft.	153.44	145.68	77.80	56.10	33.95	35.88
	Standard Error Sq. Ft.	34.31	32.58	17.40	12.54	7.59	8.02
	Coefficient of Variation Percent	44.58	25.71	19.37	11.96	6.62	6.83
	Deviation from True Mean Percent	-34.18	+8.38	-23.16	-10.31	-1.90	+ .36
Systematic	Expected Value Sq. Ft.	114.22	103.31	112.97	124.80	136.56	124.28
	Standard Deviation Sq. Ft.	225.60	127.04	108.88	67.38	35.10	28.38
	Standard Error Sq. Ft.	50.45	28.41	24.35	15.07	7.85	6.35
	Coefficient of Variation Percent	44.17	27.50	21.55	12.07	5.75	5.11
	Deviation from True Mean Percent	-2.31	-11.64	-3.38	+ 6.74	+16.80	+6.29
Multiple Random Starts	Expected Value Sq. Ft.	92.63	209.92	146.79	120.19	132.37	124.23
	Standard Deviation Sq. Ft.	176.80	242.40	140.20	78.38	45.42	35.41
	Standard Error Sq. Ft.	39.53	54.20	31.35	17.53	10.16	7.92
	Coefficient of Variation Percent	42.67	25.82	21.36	14.58	7.68	6.37
	Deviation from True Mean Percent	-20.77	+79.54	+25.55	+2.80	+13.21	+6.25

Table 26. Parameter summary, Toumey Woods, triangular plots.

Method of Distribution	Parameter	Sampling Unit Size in Acres 1/160	1/80	1/40	1/20	1/10	1/5
Random	Expected Value Sq. Ft.	140.64	104.40	88.56	84.88	111.49	118.71
	Standard Deviation Sq. Ft.	231.68	134.64	63.32	41.32	27.28	29.37
	Standard Error Sq. Ft.	51.81	30.11	14.16	9.24	6.08	6.57
	Coefficient of Variation Percent	36.83	28.84	15.99	10.88	5.45	5.53
	Deviation from True Mean Percent	20.29	-10.71	-24.25	-27.40	-4.64	1.53
Systematic	Expected Value Sq. Ft.	110.15	96.64	104.34	123.53	133.84	123.29
	Standard Deviation Sq. Ft.	227.86	123.84	114.92	53.60	41.34	33.68
	Standard Error Sq. Ft.	50.84	27.69	25.70	11.98	9.24	7.53
	Coefficient of Variation Percent	46.15	28.65	24.63	9.70	6.90	6.11
	Deviation from True Mean Percent	-5.79	-17.34	-10.76	5.65	14.47	5.45
Multiple Random Starts	Expected Value Sq. Ft.	124.77	183.60	169.70	127.61	132.78	80.24
	Standard Deviation Sq. Ft.	327.68	199.44	140.16	70.28	46.18	31.16
	Standard Error Sq. Ft.	73.27	44.60	31.34	15.71	10.33	6.97
	Coefficient of Variation	58.72	24.29	18.47	12.31	7.78	8.69
	Deviation from True Mean Percent	+6.71	57.03	45.14	9.14	13.56	-31.37

Table 27. Parameter summary, Toumey Woods, square plots.

Method of Distribution	Parameter	Sampling Unit Size in Acres 1/160	1/80	1/40	1/20	1/10	1/5
Random	Expected Value Sq. Ft.	119.84	141.52	95.64	115.26	114.42	119.35
	Standard Deviation Sq. Ft.	194.88	142.16	75.00	48.18	35.20	21.87
	Standard Error Sq. Ft.	43.58	31.79	16.77	10.77	7.87	4.89
	Coefficient of Variation Percent	36.36	22.46	17.53	9.34	6.88	4.10
	Deviation from True Mean Percent	+2.50	+21.04	-18.20	-.56	-2.14	2.08
Systematic	Expected Value Sq. Ft.	113.28	88.00	128.00	130.96	128.29	130.07
	Standard Deviation Sq. Ft.	226.24	131.52	118.08	63.86	33.12	31.13
	Standard Error Sq. Ft.	50.59	29.41	26.41	14.28	7.41	6.96
	Coefficient of Variation Percent	44.66	33.42	20.63	10.90	5.77	5.35
	Deviation from True Mean Percent	3.11	24.73	9.48	12.01	9.72	11.25
Multiple Random Starts	Expected Value Sq. Ft.	110.40	186.24	144.32	119.74	137.44	127.64
	Standard Deviation Sq. Ft.	193.92	207.72	134.48	65.32	54.43	40.73
	Standard Error Sq. Ft.	43.36	46.45	30.07	14.61	12.17	9.11
	Coefficient of Variation Percent	39.27	24.94	20.83	12.20	8.85	7.14
	Deviation from True Mean Percent	5.58	59.29	23.43	24.11	17.55	9.17

Table 28. -- Parameter summary, Toumey Woods, rectangular 1:2.

Method of Distribution	Parameter	Sampling Unit Size in Acres 1/160	1/80	1/40	1/20	1/10	1/5
Random	Expected Value Sq. Ft.	35.04	119.28	104.40	100.70	111.93	116.01
	Standard Deviation Sq. Ft.	81.92	155.28	76.68	22.48	26.31	31.74
	Standard Error Sq. Ft.	18.32	34.72	17.15	5.03	5.88	7.10
	Coefficient of Variation Percent	52.28	29.11	16.43	4.99	5.25	6.12
	Deviation from True Mean Percent	70.03	2.02	10.71	13.87	4.27	.78
Systematic	Expected Value Sq. Ft.	81.92	120.56	103.52	128.98	121.38	122.22
	Standard Deviation Sq. Ft.	197.92	188.80	109.56	63.12	48.52	27.01
	Standard Error Sq. Ft.	44.26	42.22	24.50	14.11	10.85	6.04
	Coefficient of Variation Percent	54.03	35.02	24.54	10.94	8.94	4.94
	Deviation from True Mean Percent	29.94	3.11	11.46	10.31	3.81	4.53
Multiple Random Starts	Expected Value Sq. Ft.	128.33	202.56	130.11	128.57	130.89	122.00
	Standard Deviation Sq. Ft.	197.92	255.28	114.88	91.06	61.55	35.25
	Standard Error Sq. Ft.	44.26	57.08	25.69	20.36	13.76	7.88
	Coefficient of Variation Percent	34.49	28.18	19.74	15.83	10.51	6.46
	Deviation from True Mean Percent	9.76	73.25	11.28	9.96	11.95	4.34

Table 29. -- Parameter summary, Toumey Woods, rectangular 1:4.

Method of Distribution	Parameter	Sampling Unit Size in Acres 1/160	1/80	1/40	1/20	1/10	1/5
Random	Expected Value Sq. Ft.	61.84	76.23	102.65	107.02	113.60	110.53
	Standard Deviation Sq. Ft.	114.72	109.52	93.48	50.02	45.35	20.20
	Standard Error Sq. Ft.	25.65	24.49	20.90	11.18	10.14	4.52
	Coefficient of Variation Percent	41.48	32.13	20.36	10.45	9.81	4.09
	Deviation from True Mean Percent	-47.11	-34.80	-12.20	- 8.47	-2.84	-5.46
Systematic	Expected Value Sq. Ft.	98.42	119.52	118.40	114.86	117.01	123.26
	Standard Deviation Sq. Ft.	170.88	170.64	104.92	72.96	35.80	25.55
	Standard Error Sq. Ft.	38.21	38.16	23.46	16.31	8.00	5.71
	Coefficient of Variation Percent	38.82	31.93	19.81	14.20	7.61	4.63
	Deviation from True Mean Percent	-15.82	+2.22	+1.26	-1.76	+.08	+5.42
Multiple Random Starts	Expected Value Sq. Ft.	109.92	121.84	150.12	123.00	110.09	122.28
	Standard Deviation Sq. Ft.	160.96	105.76	148.44	82.32	40.36	36.06
	Standard Error Sq. Ft.	35.99	23.65	33.19	18.41	9.02	8.06
	Coefficient of Variation Percent	32.74	19.41	22.11	14.97	8.19	6.59
	Deviation from True Mean Percent	-5.99	+4.21	+28.39	+5.20	-5.84	+4.58

Table 30. -- Parameter summary, Toumey Woods, rectangular 1:8.

Method of Distribution	Parameter	Sampling Unit Size in Acres 1/160	1/80	1/40	1/20	1/10	1/5
Random	Expected Value Sq. Ft.	52.82	81.21	77.52	109.77	109.70	101.06
	Standard Deviation Sq. Ft.	108.48	113.20	77.44	76.56	40.44	25.14
	Standard Error Sq. Ft.	24.26	25.31	17.32	17.12	9.04	5.62
	Coefficient of Variation Percent	45.93	31.17	22.34	15.60	8.24	5.56
	Deviation from True Mean Percent	54.82	30.54	33.70	6.11	6.17	13.56
Systematic	Expected Value Sq. Ft.	71.08	94.02	105.02	111.65	111.83	116.49
	Standard Deviation Sq. Ft.	148.64	112.32	123.96	62.06	48.94	25.93
	Standard Error Sq. Ft.	33.24	25.12	27.72	13.88	10.94	5.80
	Coefficient of Variation Percent	46.76	26.72	26.39	12.43	9.78	4.98
	Deviation from True Mean Percent	39.21	19.59	10.18	4.51	4.35	.37
Multiple Random Starts	Expected Value Sq. Ft.	104.16	128.48	168.56	147.70	129.86	111.50
	Standard Deviation Sq. Ft.	140.80	158.48	135.16	85.40	46.97	24.39
	Standard Error Sq. Ft.	31.48	35.44	30.22	19.10	10.50	5.45
	Coefficient of Variation Percent	30.22	27.58	17.93	12.93	8.08	4.89
	Deviation from True Mean Percent	10.91	9.89	44.17	26.32	11.07	4.63

Table 31. -- Parameter summary, Toumey Woods, adjusted point-sampling.

Method of Distribution	Parameter	Basal Area Factor (BAF) 50	40	30	20	10	5
	Expected Value Sq. Ft.	120.00	126.00	142.50	145.00	168.50	235.75
	Standard Deviation Sq. Ft.	49.50	70.00	64.50	62.80	80.00	81.00
Random	Standard Error Sq. Ft.	11.07	15.66	14.42	14.04	17.89	18.11
	Coefficient of Variation Percent	9.22	12.42	10.11	9.68	10.61	7.68
	Deviation from True Mean Percent	+2.63	+7.76	+21.88	+24.02	+44.11	+101.63
	Expected Value Sq. Ft.	152.50	164.00	165.00	178.00	202.00	274.75
	Standard Deviation Sq. Ft.	104.50	89.20	82.80	92.60	106.80	111.35
Systematic	Standard Error Sq. Ft.	23.37	19.95	18.51	20.71	23.88	24.90
	Coefficient of Variation Percent	15.32	12.16	11.21	11.63	11.82	9.06
	Deviation from True Mean Percent	30.43	40.27	41.12	52.24	72.77	134.99
	Expected Value Sq. Ft.	137.50	174.00	180.00	186.00	207.50	256.75
Multiple Random Starts	Standard Deviation Sq. Ft.	107.50	150.40	137.40	132.60	127.10	121.10
	Standard Error Sq. Ft.	24.04	33.63	30.72	29.65	28.42	27.08
	Coefficient of Variation Percent	17.48	19.32	17.06	15.94	13.69	10.54
	Deviation from True Mean Percent	17.60	48.82	53.95	59.08	77.47	119.59

Table 32. Parameter summary, Toumey Woods, unadjusted point-sampling.

Method of Distribution	Parameter	Basal Area Factor (BAF) 50	40	30	20	10	5
Random	Expected Value Sq. Ft.	105.00	110.00	117.00	116.00	121.00	119.00
	Standard Deviation Sq. Ft.	36.00	40.80	38.70	35.80	27.30	22.90
	Standard Error Sq. Ft.	8.05	9.12	8.65	8.00	6.10	5.12
	Coefficient of Variation Percent	7.67	8.29	7.39	6.90	5.04	4.30
	Deviation from True Mean Percent	10.19	5.92	.07	.79	3.49	1.78
Systematic	Expected Value Sq. Ft.	115.00	122.00	115.50	115.00	119.00	119.25
	Standard Deviation Sq. Ft.	58.50	52.80	39.30	38.40	28.20	20.75
	Standard Error Sq. Ft.	13.08	11.81	8.79	8.59	6.30	4.64
	Coefficient of Variation Percent	11.37	9.68	7.61	7.47	5.29	3.89
	Deviation from True Mean Percent	1.64	4.34	1.21	1.64	1.78	1.99
Multiple Random Starts	Expected Value Sq. Ft.	110.00	124.00	126.00	127.00	121.00	112.50
	Standard Deviation Sq. Ft.	57.50	58.00	57.30	43.60	36.00	27.95
	Standard Error Sq. Ft.	12.86	12.97	12.81	9.75	8.05	6.25
	Coefficient of Variation Percent	11.69	10.46	10.17	7.68	6.65	5.55
	Deviation from True Mean Percent	-5.92	+6.05	+7.76	+8.62	+3.49	-3.78

Table 33. -- Summary of the coefficient of variation data, Toumey Woods, plot-samples.

Plot Shape	Plot Size*	Distribution: Random	Systematic	MRS	Plot Size Total	Plot Shape Total
Circular	160	44.58	44.17	42.67	131.42	
	80	25.71	27.50	25.82	79.03	
	40	19.37	21.55	21.36	62.28	
	20	11.96	12.07	14.58	38.61	
	10	06.62	05.75	07.68	20.05	
	5	06.83	05.11	06.37	18.31	
	Total	115.07	116.15	118.48		349.70
Triangular	160	36.83	46.15	58.72	141.70	
	80	28.84	28.65	24.29	81.78	
	40	15.99	24.63	18.47	59.09	
	20	10.88	09.70	12.31	32.89	
	10	05.45	06.90	07.78	20.13	
	5	05.53	06.11	08.69	20.33	
	Total	103.52	122.14	130.26		355.92
Square	160	36.36	44.66	39.27	120.29	
	80	22.46	33.42	24.94	80.82	
	40	17.53	20.63	20.83	58.99	
	20	09.34	10.90	12.20	32.44	
	10	06.88	05.77	08.85	21.50	
	5	04.10	05.35	07.14	16.59	
	Total	96.67	120.73	113.23		330.63
Rect. 1:2	160	52.28	54.03	34.49	140.80	
	80	29.11	35.02	28.18	92.31	
	40	16.43	24.54	19.74	60.71	
	20	04.99	10.94	15.83	31.76	
	10	05.25	08.94	10.51	24.70	
	5	06.12	04.94	06.46	17.52	
	Total	114.18	138.41	115.21		367.80
Rect. 1:4	160	41.48	38.82	32.74	113.04	
	80	32.13	31.93	19.41	83.47	
	40	20.36	19.81	22.11	62.28	
	20	10.45	14.20	14.97	39.62	
	10	09.81	07.61	08.19	25.61	
	5	04.09	04.63	06.59	15.31	
	Total	118.32	117.00	104.01		339.33
Rect. 1:8	160	45.93	46.76	30.22	122.91	
	80	31.17	26.72	27.58	85.47	
	40	22.34	26.39	17.93	66.66	
	20	15.60	12.43	12.93	40.96	
	10	08.24	09.78	08.08	26.10	
	5	05.56	04.98	04.89	15.43	
	Total	128.84	127.06	101.63		357.53
All Plot Shapes	160	257.46	274.59	238.11	770.16	
	80	169.42	183.24	150.22	502.88	
	40	112.04	137.55	120.44	370.01	
	20	63.22	70.24	82.82	216.28	
	10	42.25	44.75	51.09	138.09	
	5	32.23	31.12	40.14	103.49	
	Total	676.60	741.49	682.82		2100.91

*Plot size in acres, denominator only.

Table 34. -- Analysis of variance, Toumey Woods, plot samples.

Source	df	SS	MS	F
Distribution	2	71.2182	35.6091	2.19 N. S.
Treatment	35	18,313.8007	523.2514	32.23 * *
Plot Shape	5	49.8658	9.9732	- N. S.
Plot Size	5	17,992.2804	3598.4561	221.65 * *
Interaction	25	271.6545	10.8662	- N. S.
Error	70	1136.4365	16.2348	
Total	107	19,521.4554		

N. S. Non-significant
* * Highly significant

Table 35. -- Average coefficient of variation, Toumey Woods.

Plot Sampling				Point-Sampling		
Plot Shape	CV	Plot Size	CV	BAF	Adjusted CV	Unadjusted CV
	(percent)	(acres)	(percent)		(percent)	(percent)
Circular	19.43	1/160	42.79	50	14.01	10.24
Triangular	19.77	1/80	27.94	40	14.63	9.48
Square	18.37	1/40	20.56	30	12.79	8.33
Rect. 1:2	20.43	1/20	12.02	20	12.42	7.35
Rect. 1:4	18.85	1/10	7.67	10	12.04	5.66
Rect. 1:8	19.86	1/5	5.74	5	9.09	4.58

Table 36. -- Average coefficient of variation data by distributions, Toumey Woods.

Samples	Random	Systematic	MRS
	(percent)	(percent)	(percent)
Plot	18.79	20.60	18.97
Adjusted Point	9.95	11.87	15.67
Unadjusted Point	6.60	7.55	8.70

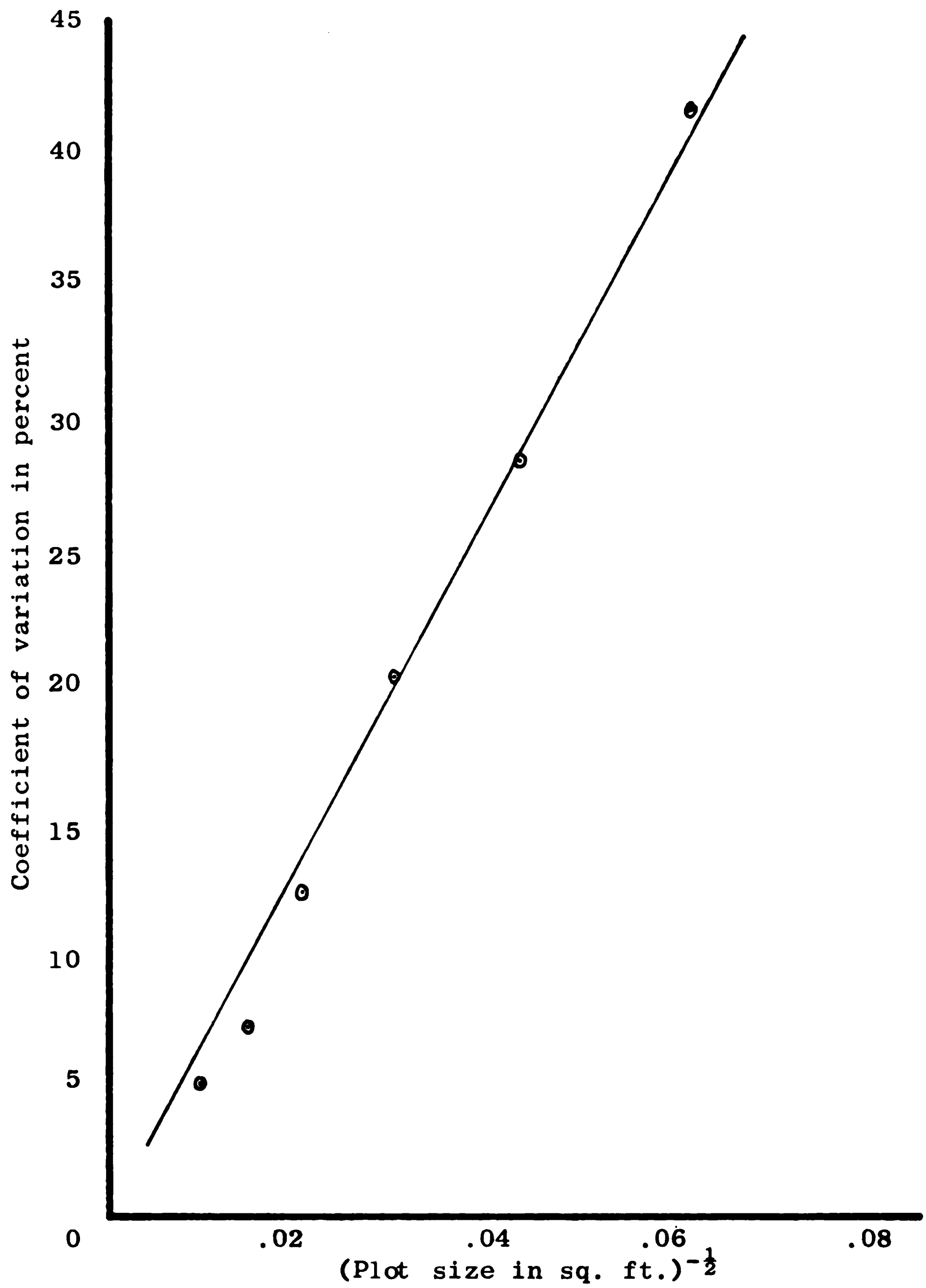

Figure 6. -- The relationship between the coefficient of variation in percent and the reciprocal of the square root of the plot size in square feet. From the equation: CV = 670.399 (Plot size)$^{-\frac{1}{2}}$, Toumey Woods.

$$CV = 670.399(P)^{-\frac{1}{2}} \tag{13}$$

where: CV = coefficient of variation in percent

P = plot size in square feet

670.399 = calculated regression coefficient

The correlation coefficient between the two variables is 99.70 percent. It may be concluded that an increase in plot size will significantly increase the accuracy of the estimate according to Equation 13. Due considerations must be given to time and cost, however.

Point-Sampling. -- A summary of the coefficient of variation data for point-sampling is given in Table 37. The average CV for each sampling method are listed in Table 35 and 36.

Table 37.--Summary of the coefficient of variation data, Toumey Woods.

Plot Type	BAF	Distribution: Random	Systematic	MRS	Total	Plot Type Total
Adjusted Point-Sampling	50	09.22	15.32	17.48	42.02	
	40	12.42	12.16	19.32	43.90	
	30	10.11	11.21	17.06	38.38	
	20	09.68	11.63	15.94	37.25	
	10	10.61	11.82	13.69	36.12	
	5	07.68	09.06	10.54	27.38	
	Total	59.72	71.20	94.03		224.95
Unadjusted Point-Sampling	50	07.67	11.37	11.69	30.73	
	40	08.29	09.68	10.46	28.43	
	30	07.39	07.61	10.17	25.17	
	20	06.90	07.47	07.68	22.05	
	10	05.04	05.29	06.65	16.98	
	5	04.30	03.89	05.55	13.74	
	Total	39.59	45.31	52.20		137.10
All Plot Types	50	16.89	26.69	29.17	72.75	
	40	20.71	21.84	29.78	72.33	
	30	17.50	18.82	27.23	63.55	
	20	16.58	19.10	23.62	59.30	
	10	15.65	17.11	20.34	53.10	
	5	11.98	12.95	16.09	41.02	
	Total	99.31	116.51	146.23		362.05

Analysis of variance in Table 38 shows the method of plot distribution, as well as the treatment, to be highly significant. Separation of the two degrees of freedom into comparisons among distributions shows the random method to be superior to either of the other systems on one and 22 degrees of freedom (Equations 14 and 15).

Table 38 . -- Analysis of variance, Toumey Woods, point-sampling, coefficient of variation.

Source	df	SS	MS	F
Distribution	2	93.9057	46.9529	19.27 * *
Treatment	11	342.8057	31.1642	12.79 * *
Error	22	53.6176	2.4372	
Total	35	490.3290		

* * Highly significant.

$$\text{MRS vs (R+S)} = [2(146.23)-(99.31+116.51)]^2/6(12) = 81.5790 \quad (14)$$

$$F = 81.5790/2.4372 = 33.47^{**} \text{ (Highly significant, 99 percent level)}$$

$$\text{R vs S} = (116.51 - 99.31)^2/2(12) = 12.3267 \quad (15)$$

$$F = 12.3267/2.4372 = 5.06^{*} \text{ (Significant, 95 percent level)}$$

To compare the differences in treatment for plot type and BAF size, the analysis of variance is applied only to the random distribution (Table 39). The other two methods are considered statistically inaccurate.

In analysing the variance for the random distribution, the basal area factor proved to be insignificant in its contribution to the variation, so a regression between BAF and CV was not calculated. In timber

Table 39. -- Analysis of variance for the random distribution of point-samples, Toumey Woods.

Source	df	SS	MS	F	
Type	1	33.7681	33.7681	35.55	* *
BAF	5	20.0098	4.0020	4.21	N.S.
Error	5	4.7496	.9499		
Total	11	58.5275			

N.S. Non-significant
* * Highly significant

as large as that in the Toumey Woods, any BAF would have produced essentially the same results. Plot type, on the other hand, with its difference between Grosenbaugh's peripheral adjustment and the unadjusted sampling, shows high significance. The average CV for the adjusted method is 59.72 percent, while for the unadjusted, it is only 39.59 percent. Sampling by the adjusted method is not recommended.

Comparison of Plot Samples with Point-Samples. -- The most precise plot sample is the one-fifth acre unit, (shape is of no consequence) with an average CV of 5.74 percent (Table 35). In the unadjusted point-sampling method, the BAF-5 has the lowest CV with 4.58 percent (Table 35). To test the difference between these two, the data of Table 40 are subjected to the "t" test by pooling the variance. The calculated "t" value is only 1.639, and does not exceed the tabular value of 2.093 at the 95 percent level for 19 degrees of freedom. It is concluded that the difference between the accuracy of the one-fifth acre areal plot and the BAF-5 point-sample is insignificant, and either may be used with the same results. This, of course, ignores efficiency.

Table 40. -- Analysis of the difference between the most accurate plot and point-samples, Toumey Woods.

Method	Observations	df	Ave. CV (percent)	SS
1/5 acre	18	17	5.74	22.9779
BAF-5	3	2	4.58	1.4954
		19	1.16	24.4733

<u>Deviations of the Expected Value from the True Mean</u>. -- To see if each distribution adequately sampled the population, the deviation of the expected value from the known population mean was subjected to the analysis of variance in the same manner as the sampling error. Table 41 shows the averages of these deviations.

Table 41. -- Average deviation of the expected value from the true population mean, Toumey Woods.

Plot Sampling				Point-Sampling		
Plot Shape	Deviation (percent)	Plot Size (acres)	Deviation (percent)	BAF	Adjusted Deviation (percent)	Unadjusted Deviation (percent)
Circular	15.20	1/160	20.81	50	16.89	5.92
Triangular	17.29	1/80	26.07	40	32.28	5.44
Square	14.22	1/40	19.76	30	38.98	3.01
Rect. 1:2	15.85	1/20	10.29	20	45.11	3.68
Rect. 1:4	10.65	1/10	8.55	10	64.78	2.92
Rect. 1:8	18.34	1/5	6.52	5	118.74	2.52

From Tables 25 to 32 the average deviation of the expected values from the known population mean is 15.85, 9.57, and 20.33 percent for the random, systematic, and MRS distributions, respectively.

The larger deviation by the random method indicates that perhaps this distribution of plots did not sample the full variation within the forest. This question is purely academic, since, under normal circumstances, the true population parameter would not be known, and any guesses as to its magnitude would be no better than a random estimate. In the present case, this phenomenon can be attributed to chance. Another random sample, or the same samples applied to another population, would produce different results.

Table 42 presents the analysis of variance for areal plots, and proves the differences in distributions to be significant at the 95

Table 42. -- Analysis of variance of the deviation of the expected value from the known parameter, Toumey Woods, plot samples.

Source	df	SS	MS	F
Distribution	2	2102.3787	1051.1894	4.63 *
Treatment	35	10,639.6805	303.9909	- N.S.
Plot Shape	5	673.2927	134.6585	- N.S.
Plot Size	5	5577.0439	1115.4088	4.92 * *
Error	70	15,885.1831	226.9312	
Total	35	43,403.9729		

N.S. Non-significant
* Significant (95 percent level)
* * Highly significant

percent level. Inspection of the values in Table 41 shows that the deviations decrease as plot size increases. This is highly significant according to the "F" test. Table 43 presents the analysis for point-sampling. With the large variances introduced by the peripheral adjustments, all aspects are important at the 99 percent level.

Table 43. -- Analysis of variance of the deviation of the expected values from the true parameter, Toumey Woods, point-sampling.

Source	df		SS	MS	F
Distribution	2		1714.0902	857.0451	* *
Treatment	11		40,992.4155	3726.5832	* *
Type		1	21,506.2225	21,506.2225	* *
BAF		5	9114.1206	1822.8241	* *
Interaction		5	10,372.0724	2074.4145	* *
Error	22		697.4672	31.7031	* *
Total	35		43,403.9729		

* * Highly significant

In spite of the differences, the best procedure would call for the use of a random distribution of either one-fifth acre plot samples, or BAF-5 point-samples, since these are statistically the most accurate, according to the analysis of the coefficient of variation data. The use of peripheral adjustments is not recommended.

Red Cedar Woods

Plot Samples.-- The parameter summaries for all samples taken in this forest are presented in Tables 44 to 51. The coefficients of variation are listed in Table 52 and subjected to the analysis of variance in Table 53. According to this analysis, the differences in methods of distribution are highly significant. From Table 52 the differences between distributions produced a highly significant "F" value of 31.27.

The same type of test between the systematic and the MRS shows no important differences. Therefore, the random distribution with the lowest CV of 20.07 percent, will be used for the test on the bases of plot shape and size. The others are discarded as statistically inaccurate. From Table 54, the analysis of variance, it is evident that the plot shape has no bearing on the variation in sampling error. A plot of any shape will give essentially the same results. The size of the plots, on the other hand, is very important. Comparisons among plot sizes were carried out in the same manner as for the Lansing Woods (Appendix C) and the results showed that the one-fifth, one-tenth, and one-twentieth acre plots would give approximately the same accuracy. Plots smaller than these were significantly less precise.

The mathematical relationship between plot size and the coefficient of variation is illustrated in Figure 7. The equation was derived from a regression of the data of the two variables from Table 55. The resulting equation is:

Table 44. -- Parameter summary, Red Cedar Woods, circular plots.

Method of Distribution	Parameter	Sampling Unit Size in Acres 1/160	1/80	1/40	1/20	1/10	1/5
Random	Expected Value Sq. Ft.	181.44	117.92	108.68	89.72	87.76	92.71
	Standard Deviation Sq. Ft.	277.12	166.08	89.88	41.90	30.26	29.14
	Standard Error Sq. Ft.	65.31	39.14	21.18	9.87	7.13	6.87
	Coefficient of Variation Percent	36.00	33.20	19.49	11.01	8.13	7.41
	Deviation from True Mean Percent	+98.08	+28.73	+18.65	-2.05	-4.19	+1.21
Systematic	Expected Value Sq. Ft.	146.88	90.56	81.44	86.38	91.46	81.94
	Standard Deviation Sq. Ft.	344.80	174.72	100.76	88.06	49.44	36.64
	Standard Error Sq. Ft.	81.26	41.18	23.75	20.75	11.65	8.63
	Coefficient of Variation Percent	55.32	45.47	29.16	24.02	12.74	10.53
	Deviation from True Mean Percent	+60.35	-1.13	-11.09	-5.70	-.15	-10.54
Multiple Random Starts	Expected Value Sq. Ft.	140.80	127.68	98.80	110.88	102.96	90.54
	Standard Deviation Sq. Ft.	265.92	165.04	97.64	92.94	56.83	44.94
	Standard Error Sq. Ft.	62.67	38.90	23.01	21.90	13.39	10.59
	Coefficient of Variation Percent	44.51	30.47	23.29	19.75	13.00	11.70
	Deviation from True Mean Percent	+5.37	+39.39	+7.86	-21.05	+12.40	-1.16

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Table 45. -- Parameter summary, Red Cedar Woods, triangular plots.

Method of Distribution	Parameter	Sampling Unit Size in Acres 1/160	1/80	1/40	1/20	1/10	1/5
Random	Expected Value Sq. Ft.	134.72	128.48	106.08	101.38	88.29	90.12
	Standard Deviation Sq. Ft.	191.84	137.60	99.40	47.30	25.53	31.16
	Standard Error Sq. Ft.	45.21	32.43	23.43	11.15	6.02	7.34
	Coefficient of Variation Percent	33.56	25.24	22.09	11.00	6.82	8.14
	Deviation from True Mean Percent	+47.07	+40.04	+15.81	+10.68	-3.61	-1.61
Systematic	Expected Value Sq. Ft.	115.20	99.52	88.00	87.86	82.62	88.52
	Standard Deviation Sq. Ft.	181.60	175.68	116.00	70.76	52.89	36.96
	Standard Error Sq. Ft.	42.80	41.40	27.34	16.68	12.47	8.71
	Coefficient of Variation Percent	37.15	41.60	31.07	18.98	15.09	9.84
	Deviation from True Mean Percent	+25.76	+8.65	-3.93	-4.08	-9.80	-3.36
Multiple Random Starts	Expected Value Sq. Ft.	122.08	97.20	143.24	105.60	97.02	94.66
	Standard Deviation Sq. Ft.	248.32	140.32	143.00	92.40	73.78	44.71
	Standard Error Sq. Ft.	58.52	33.07	33.70	21.78	17.39	10.54
	Coefficient of Variation Percent	47.93	34.02	23.53	20.63	17.92	11.13
	Deviation from True Mean Percent	+33.27	+6.11	+56.37	+15.28	+5.92	+3.34

Table 46. -- Parameter summary, Red Cedar Woods, square plots.

Method of Distribution	Parameter	Sampling Unit Size in Acres 1/160	1/80	1/40	1/20	1/10	1/5
Random	Expected Value Sq. Ft.	171.84	112.32	118.44	94.98	89.49	92.84
	Standard Deviation Sq. Ft.	271.52	161.60	76.72	42.90	31.24	33.02
	Standard Error Sq. Ft.	63.99	38.09	18.08	10.11	7.36	7.78
	Coefficient of Variation Percent	37.24	33.91	15.27	10.65	8.23	8.38
	Deviation from True Mean Percent	+87.60	+22.64	+29.30	+3.69	-2.30	+1.35
Systematic	Expected Value Sq. Ft.	162.40	102.72	82.76	85.12	90.74	85.90
	Standard Deviation Sq. Ft.	349.44	175.92	107.36	75.26	50.12	34.32
	Standard Error Sq. Ft.	82.36	41.46	25.30	17.74	11.81	8.09
	Coefficient of Variation Percent	50.72	40.37	30.58	20.84	13.02	9.42
	Deviation from True Mean Percent	+77.29	+12.14	-9.65	-7.07	-0.94	-6.22
Multiple Random Starts	Expected Value Sq. Ft.	106.72	111.04	98.08	95.94	97.69	85.71
	Standard Deviation Sq. Ft.	226.88	150.00	99.68	79.18	52.93	34.23
	Standard Error Sq. Ft.	53.47	35.35	23.49	18.66	12.47	8.07
	Coefficient of Variation Percent	50.11	31.84	23.95	19.45	12.77	9.41
	Deviation from True Mean Percent	+16.51	+21.22	+7.07	+4.74	+6.65	-6.43

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Table 47. -- Parameter summary, Red Cedar Woods, rectangular 1:2.

Method of Distribution	Parameter	Sampling Unit Size in Acres 1/160	1/80	1/40	1/20	1/10	1/5
Random	Expected Value Sq. Ft.	175.36	130.48	89.08	93.14	88.24	92.32
	Standard Deviation Sq. Ft.	116.96	168.08	88.40	42.78	33.08	31.86
	Standard Error Sq. Ft.	27.56	39.61	20.83	10.08	7.80	7.51
	Coefficient of Variation Percent	15.72	30.36	23.38	10.82	9.97	8.13
	Deviation from True Mean Percent	+91.44	+42.44	-2.75	+1.68	-3.67	+.79
Systematic	Expected Value Sq. Ft.	78.72	111.36	85.40	96.52	87.83	89.03
	Standard Deviation Sq. Ft.	180.64	178.56	96.24	83.48	45.98	36.11
	Standard Error Sq. Ft.	42.57	42.08	22.68	19.67	10.84	8.51
	Coefficient of Variation Percent	54.08	37.79	26.56	20.38	12.34	9.56
	Deviation from True Mean Percent	-14.06	+21.57	-6.77	+5.37	-4.11	-2.80
Multiple Random Starts	Expected Value Sq. Ft.	120.96	93.60	113.48	109.76	99.56	97.16
	Standard Deviation Sq. Ft.	238.72	136.40	96.96	76.68	46.06	48.89
	Standard Error Sq. Ft.	56.26	32.15	22.85	18.07	10.85	11.52
	Coefficient of Variation Percent	46.52	34.35	20.14	16.41	10.90	11.86
	Deviation from True Mean Percent	+32.05	+2.18	+23.89	+19.82	+8.69	+6.07

Table 48. -- Parameter summary, Red Cedar Woods, rectangular 1:4.

Method of Distribution	Parameter	Sampling Unit Size in Acres 1/160	1/80	1/40	1/20	1/10	1/5
Random	Expected Value Sq. Ft.	148.96	123.68	96.04	97.94	107.10	100.10
	Standard Deviation Sq. Ft.	304.48	167.12	88.96	59.46	44.55	35.31
	Standard Error Sq. Ft.	71.76	39.39	20.97	14.01	10.50	8.32
	Coefficient of Variation Percent	48.17	31.85	21.83	14.30	9.80	8.31
	Deviation from True Mean Percent	+62.62	+35.02	+4.85	+6.92	+16.92	+9.28
Systematic	Expected Value Sq. Ft.	34.72	81.44	91.56	84.20	99.22	97.90
	Standard Deviation Sq. Ft.	52.80	132.16	116.56	70.30	57.29	42.53
	Standard Error Sq. Ft.	12.44	31.15	27.47	16.57	13.50	10.02
	Coefficient of Variation Percent	35.83	38.29	30.00	19.68	13.61	10.24
	Deviation from True Mean Percent	-62.10	-11.09	- .04	-8.08	+8.32	+6.88
Multiple Random Starts	Expected Value Sq. Ft.	90.40	81.92	134.56	112.96	100.74	89.84
	Standard Deviation Sq. Ft.	185.44	126.48	152.20	80.52	60.10	40.94
	Standard Error Sq. Ft.	43.71	29.81	35.87	18.98	14.16	9.65
	Coefficient of Variation Percent	48.35	36.39	26.66	16.80	14.05	10.74
	Deviation from True Mean Percent	-1.31	-10.57	+46.90	+23.32	+9.98	-1.92

Table 49. -- Parameter summary, Red Cedar Woods, rectangular 1:8.

Method of Distribution	Parameter	Sampling Unit Size in Acres 1/160	1/80	1/40	1/20	1/10	1/5
Random	Expected Value Sq. Ft.	116.48	106.72	103.52	114.08	91.57	87.82
	Standard Deviation Sq. Ft.	219.36	175.44	90.92	95.52	66.87	41.16
	Standard Error Sq. Ft.	51.70	41.35	21.43	23.22	15.76	9.70
	Coefficient of Variation Percent	44.38	38.75	20.70	21.14	17.21	11.04
	Deviation from True Mean Percent	+27.16	+16.51	+13.01	+24.54	-.03	-4.13
Systematic	Expected Value Sq. Ft.	21.44	19.12	64.08	87.92	88.74	90.54
	Standard Deviation Sq. Ft.	41.44	28.16	79.28	66.98	45.41	43.72
	Standard Error Sq. Ft.	9.77	6.64	18.68	15.79	10.70	10.30
	Coefficient of Variation Percent	45.57	34.73	29.15	17.96	12.06	11.38
	Deviation from True Mean Percent	-76.59	-79.13	-30.04	-4.02	-3.12	-1.16
Multiple Random Starts	Expected Value Sq. Ft.	59.20	92.24	92.68	96.94	90.36	92.60
	Standard Deviation Sq. Ft.	160.00	139.84	86.44	93.52	68.04	38.94
	Standard Error Sq. Ft.	37.71	32.96	20.37	22.04	16.03	9.18
	Coefficient of Variation Percent	63.70	35.73	21.98	22.73	17.74	9.91
	Deviation from True Mean Percent	-35.37	+.70	+1.18	+5.83	-1.35	+1.09

Table 50. -- Parameter summary, Red Cedar Woods, adjusted point-samples.

Method of Distribution	Parameter	Basal Area Factor (BAF) 50	40	30	20	10	5
Random	Expected Value Sq. Ft.	133.50	148.80	138.30	145.60	169.40	249.20
	Standard Deviation Sq. Ft.	92.50	98.80	99.30	76.60	62.60	113.70
	Standard Error Sq. Ft.	21.80	23.28	23.40	18.05	14.75	26.80
	Coefficient of Variation Percent	16.33	15.65	16.92	12.40	8.71	10.75
	Deviation from True Mean Percent	+45.74	+62.44	+50.98	+58.95	+84.93	+172.05
Systematic	Expected Value Sq. Ft.	125.00	122.40	123.30	133.40	150.60	205.00
	Standard Deviation Sq. Ft.	113.00	100.40	89.70	95.00	79.70	91.25
	Standard Error Sq. Ft.	26.63	23.66	21.14	22.39	18.78	21.51
	Coefficient of Variation Percent	21.31	19.33	17.15	16.79	12.47	10.49
	Deviation from True Mean Percent	+36.46	+33.62	+34.61	+45.63	+64.41	+123.80
Multiple Random Starts	Expected Value Sq. Ft.	119.50	133.20	143.40	140.00	152.20	211.40
	Standard Deviation Sq. Ft.	116.50	128.00	127.80	96.80	102.20	146.00
	Standard Error Sq. Ft.	27.46	30.17	30.12	22.82	24.09	34.41
	Coefficient of Variation Percent	22.98	22.65	21.01	16.30	15.83	16.28
	Deviation from True Mean Percent	+30.46	+45.41	+56.66	+52.84	+66.16	+130.79

Table 51. -- Parameter summary, Red Cedar Woods, unadjusted point-samples.

Method of Distribution	Parameter	Basal Area Factor (BAF)					
		50	40	30	20	10	5
Random	Expected Value Sq. Ft.	111.00	115.60	101.40	98.80	97.20	98.40
	Standard Deviation Sq. Ft.	67.50	61.20	54.60	37.80	22.20	30.60
	Standard Error Sq. Ft.	15.91	14.42	12.87	8.91	5.23	7.21
	Coefficient of Variation Percent	14.33	12.78	12.69	9.02	5.38	7.33
	Deviation from True Mean Percent	+21.18	+26.20	+10.70	+7.86	+6.11	+7.42
Systematic	Expected Value Sq. Ft.	100.00	100.00	95.10	92.20	82.80	87.80
	Standard Deviation Sq. Ft.	76.50	70.40	61.20	50.40	33.70	30.95
	Standard Error Sq. Ft.	18.03	16.59	14.42	11.88	7.94	7.29
	Coefficient of Variation Percent	18.03	16.59	15.17	12.88	9.59	8.31
	Deviation from True Mean Percent	+9.17	+9.17	+3.82	+.65	-9.61	-4.15
Multiple Random Starts	Expected Value Sq. Ft.	94.50	97.60	99.90	98.80	85.00	81.40
	Standard Deviation Sq. Ft.	84.00	78.00	78.30	58.40	40.40	34.40
	Standard Error Sq. Ft.	19.80	18.38	18.45	13.76	9.52	8.11
	Coefficient of Variation Percent	20.95	18.84	18.47	13.93	11.20	9.96
	Deviation from True Mean Percent	+3.16	+6.55	+9.06	+7.86	-7.20	-11.13

Table 52. -- Summary of the sampling error data for Red Cedar Woods, plot samples.

Plot Shape	Plot Size *	Distribution: Random	Systematic	MRS	Plot Size Total	Plot Shape Total
Circular	160	36.00	55.32	44.51	135.83	
	80	33.20	45.47	30.47	109.14	
	40	19.49	29.16	23.29	071.94	
	20	11.01	24.02	19.75	054.78	
	10	08.13	12.74	13.00	033.87	
	5	07.41	10.53	11.70	029.64	
	Total	115.24	177.24	142.72		435.20
Triangular	160	33.56	37.15	47.93	118.64	
	80	25.24	41.60	34.02	100.86	
	40	22.09	31.07	23.53	076.69	
	20	11.00	18.98	20.63	050.61	
	10	06.82	15.09	17.92	039.83	
	5	08.14	09.84	11.13	029.11	
	Total	106.85	153.73	155.16		415.74
Square	160	37.24	50.72	50.11	138.07	
	80	33.91	40.37	31.84	106.12	
	40	15.27	30.58	23.95	069.80	
	20	10.65	20.84	19.45	050.94	
	10	08.23	13.02	12.77	034.02	
	5	08.38	09.42	09.41	027.21	
	Total	113.68	164.95	147.53		426.16
Rect. 1:2	160	15.72	54.08	46.52	116.32	
	80	30.36	37.79	34.35	102.50	
	40	23.38	26.56	20.14	070.08	
	20	10.82	20.38	16.41	047.61	
	10	09.97	12.34	10.90	033.21	
	5	08.13	09.56	11.86	029.55	
	Total	98.38	160.71	140.18	399.27	
Rect. 1:4	160	48.17	35.83	48.35	132.35	
	80	31.85	38.29	36.39	106.53	
	40	21.83	30.00	26.66	078.49	
	20	14.30	19.68	16.80	050.78	
	10	09.80	13.61	14.05	037.46	
	5	08.31	10.24	10.74	29.29	
	Total	134.26	147.65	152.99		434.90
Rect. 1:8	160	44.38	45.57	63.70	153.65	
	80	38.75	34.73	35.73	109.21	
	40	20.70	29.15	21.98	071.83	
	20	21.14	17.96	22.73	061.83	
	10	17.21	12.06	17.74	047.01	
	5	11.04	11.38	09.91	032.33	
	Total	153.22	150.85	171.79		475.86
All Plot Shapes	160	215.07	278.67	301.12	794.86	
	80	193.31	238.25	202.80	634.36	
	40	122.76	176.52	139.55	438.83	
	20	78.92	121.86	115.77	316.55	
	10	61.12	78.86	86.38	226.36	
	5	51.41	60.97	64.75	177.13	
	Total	722.59	955.13	910.37		2588.09

*Plot size in acres, denominator only.

Table 53. -- Analysis of variance, Red Cedar Woods, plot-samples.

Source	df	SS	MS	F
Distribution	2	845.7373	422.8686	16.17 * *
Treatment	35	16,715.0288	477.5723	18.26 * *
Error	70	1830.8146	26.1545	
Total	107	19,391.5807		

* * Highly significant

Table 54. -- Analysis of variance, random distribution, Red Cedar Woods, plot-samples.

Source	df	SS	MS	F
Plot Shape	5	296.3909	59.2782	2.38 N. S.
Plot Size	5	4046.3616	809.2723	32.44 * *
Error	25	623.6091	24.9444	
Total	35	4966.4198		

N.S. Non-significant

* * Highly significant

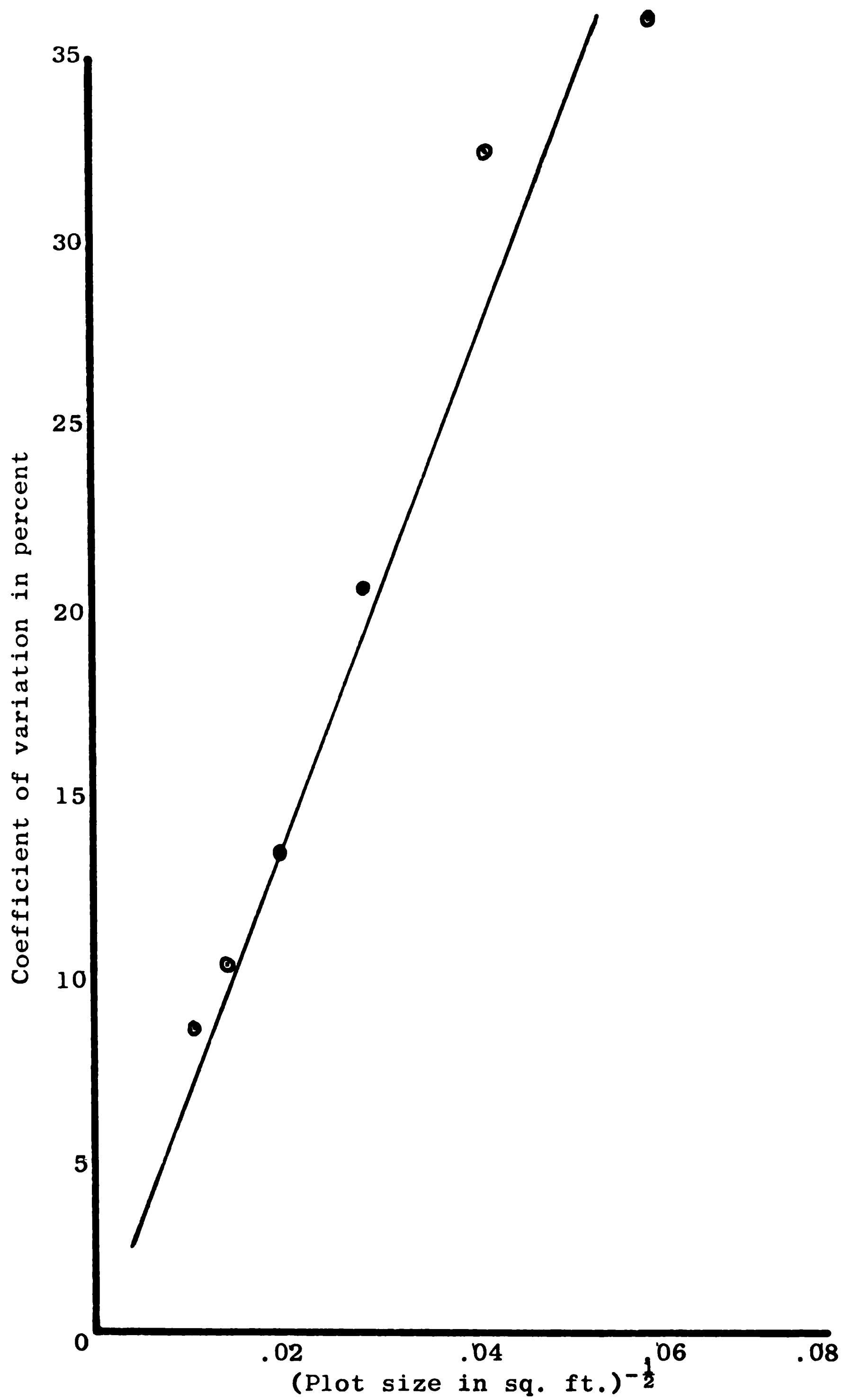

Figure 7. -- The relationship between the coefficient of variation in percent and the reciprocal of the square root of the plot size in square feet. From the equation: CV = 649.965 (Plot size)$^{-\frac{1}{2}}$, Red Cedar Woods.

Table 55. -- Average coefficient of variation for all plot samples in the random distribution, Red Cedar Woods.

Plot Samples				Point-Samples		
Plot Shape	CV	Plot Size	CV	BAF	Adjusted CV	Unadjusted CV
	(percent)	(acres)	(percent)		(percent)	(percent)
Circular	19.21	1/160	35.84	50	16.33	14.33
Triangular	17.80	1/80	32.22	40	15.65	12.78
Square	18.95	1/40	20.46	30	16.92	12.69
Rect. 1:2	16.40	1/20	13.15	20	12.40	9.02
Rect. 1:4	22.38	1/10	10.19	10	8.71	5.38
Rect. 1:8	25.54	1/5	8.57	5	10.75	7.33

$$CV = 649.965(P)^{-\frac{1}{2}} \qquad (16)$$

where: CV = the coefficient of variation in percent

P = the plot area in square feet

649.965 = the regression coefficient derived from the data

The relationship between the two variables is highly significant as shown by a correlation coefficient of 99.48 percent. An increase in the plot size results in a corresponding decrease in the CV. The question of efficiency would, of course, limit the maximum size of the sample plots.

Point-Sampling. -- The CV data are summarized in Table 56. Analysis of variance (Table 57) shows the differences in sample distribution to be highly significant. Comparisons among distributions indicate that the random method is best. Analysis is carried out in the same manner as on the plot sample (Table 58). Plot type, consisting of samples with or without peripheral adjustments, is , of course, highly significant. Plots

Table 56. -- Summary of the sampling error, Red Cedar Woods, point-sampling.

Plot Type	BAF	Distribution: Random	Systematic	MRS	BAF Total	Plot Type Total
Adjusted Point-Samples	50	16.33	21.31	22.98	60.62	
	40	15.65	19.33	22.65	57.63	
	30	16.92	17.15	21.01	55.08	
	20	12.40	16.79	16.30	45.49	
	10	08.71	12.47	15.83	37.01	
	5	10.75	10.49	16.28	37.52	
	Total	80.76	97.54	115.05		293.35
Unadjusted Point-Samples	50	14.33	18.03	20.95	53.31	
	40	12.78	16.59	18.84	48.21	
	30	12.69	15.17	18.47	46.33	
	20	09.02	12.88	13.93	35.83	
	10	05.38	09.59	11.20	26.17	
	5	07.33	08.31	09.96	25.60	
	Total	61.53	80.57	93.35		235.45
All Plot Types	50	30.66	39.34	43.93	113.93	
	40	28.43	35.92	41.49	105.84	
	30	29.61	32.32	39.48	101.41	
	20	21.42	29.67	30.23	81.32	
	10	14.09	22.06	27.03	63.18	
	5	18.08	18.80	26.24	63.12	
	Total	142.29	178.11	208.40		528.80

Table 57. -- Analysis of variance, Red Cedar Woods, point-sampling.

Source	df	SS	MF	F
Distribution	2	182.5303	91.2652	* *
Treatment	11	503.6107	45.7828	* *
Error	22	27.3910	1.2450	
Total	35	713.5320		

* * Highly significant.

Table 58. -- Analysis of variance, random distribution, Red Cedar Woods, point-samples.

Source	df	SS	MS	F
Plot Type	1	30.8161	30.8161	* *
BAF	5	117.4380	23.4876	* *
Error	5	1.3537	.2707	
Total	11	149.6078		

* * Highly significant

with adjustments have an average sampling error of 16.30 percent while those without the adjustment are in error by 13.08 percent (Table 56).

An inspection of the data in Table 55 shows approximately the same variation in CV among adjusted point-samples and among unadjusted point-samples for the different basal area factors. Comparative tests show no significant differences between the BAF-5 and BAF-10 ($t = .0003$). Either will produce the same precision. The difference between the BAF-20 and the combined BAF-5 and BAF-10 is highly significant with a "t" of 135.

The relationship between the basal area factor used and the accuracy of the estimate may be shown by plotting the CV over the plot radius factor for each BAF. The resulting pattern of points is illustrated in Figure 8 and may be described mathematically by the regression equation:

$$CV = a + b(Prf)^{-1} \qquad (17)$$

where: CV = the coefficient of variation in percent

Prf = the plot radius factor in feet per inch of tree diameter

a = a constant derived from the data

b = regression coefficient derived from the data

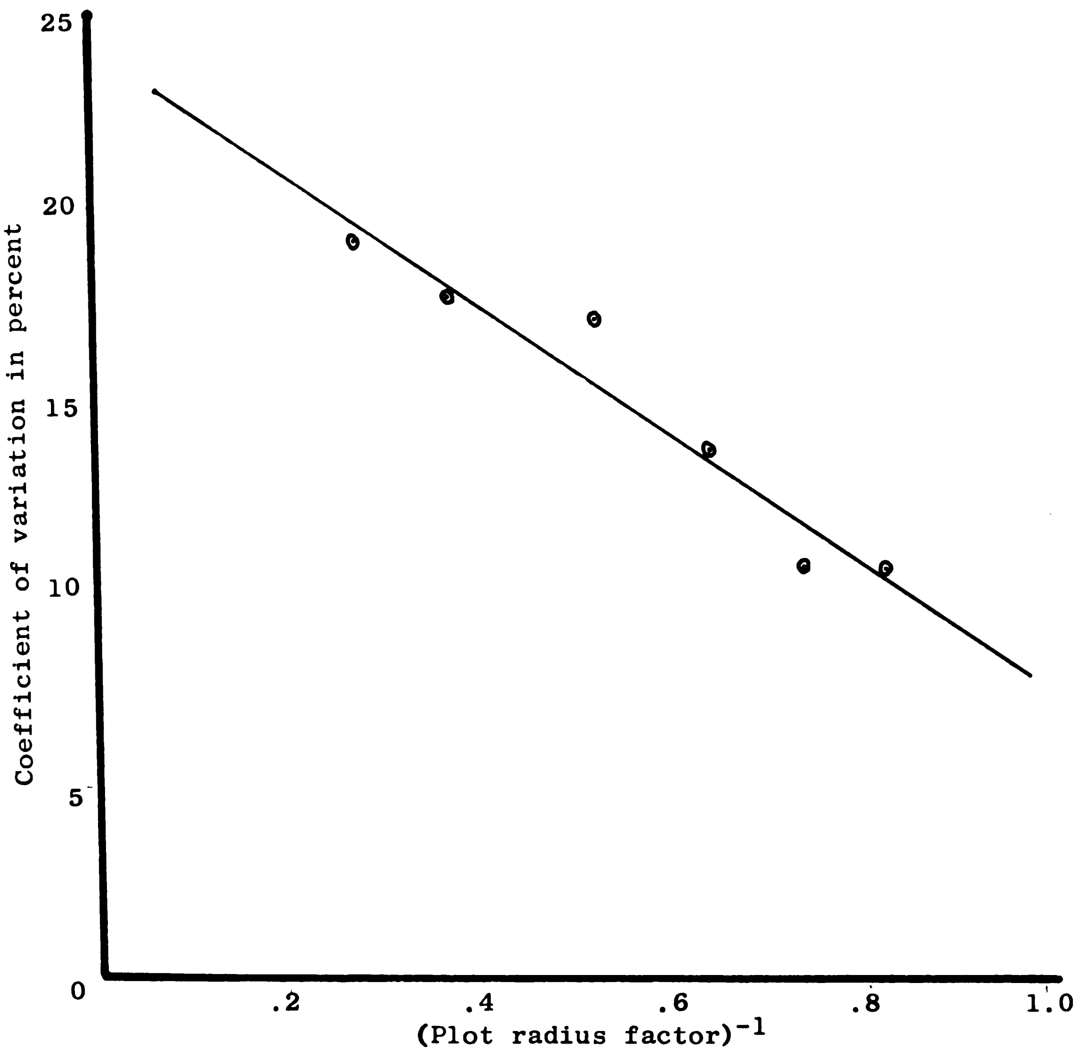

Figure 8. -- The relationship between the coefficient of variation in percent and the reciprocal of the plot radius factor. From the equation: $CV = 23.895 - 16.74(Prf)^{-1}$, Red Cedar Woods.

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Following solution by regression techniques, the equation becomes:

$$CV = 23.895 - 16.74\ (Prf)^{-1} \qquad (18)$$

The constant, a, and the regression coefficient, b, both had an "F" value greater than the tabular value at the 99 percent level (63 and 1200, respectively). The correlation coefficient between the two variables is 96.92 percent and is highly significant. Equation 18 may be used to predict the coefficient of variation for various BAF sizes.

Comparison of Plot-Samples with Point-Samples. -- According to Table 52, the plot system with the smallest average coefficient of variation is the one-fifth acre plot on the random distribution. This has a total CV of 51.41 percent for an average of 8.57 percent. From Table 56, the most accurate point-sample is the BAF-5 on the random distribution, with a total CV of 18.08 percent for an average CV of 3.01 percent. The difference between these is highly significant according to the calculated "t" value of 4.49 on the pooled variances of Table 59. For the best results in sampling this type of forest, a Bitterlich sample, with a BAF-5, on a random distribution should be used.

Table 59. -- Analysis of the differences between the most accurate plot sample and point-sample, Red Cedar Woods.

Sampling Method	Observations	df	Ave. CV (percent)	SS
One-fifth acre	6	5	8.57	7.93
BAF-5 point-sample	2	1	3.01	5.85
Sum and difference		6	5.56	13.78

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<u>Deviation of the Expected Value from the True Parameter</u>. -- To determine how well a sampling method measured the variance in this forest, the average deviation of the expected value from the known mean is analysed. Table 60 lists these averages in percent as compiled from the absolute values of Tables 44 to 51.

Table 60. -- Average deviation of the expected value from the true population parameter in percent, Red Cedar Woods.

Plot Samples				Point-Samples		
Plot Shape	Deviation (percent)	Plot Size (acres)	Deviation (percent)	BAF	Adjusted Deviation (percent)	Unadjusted Deviation (percent)
Circular	18.28	1/160	47.44	50	37.55	11.17
Triangular	16.37	1/80	22.18	40	47.16	13.97
Square	17.93	1/40	16.06	30	47.71	7.86
Rect. 1:2	16.12	1/20	9.66	20	52.47	5.46
Rect. 1:4	18.12	1/10	5.68	10	71.83	7.97
Rect. 1:8	18.05	1/5	3.85	5	142.21	7.90

From Table 60, it is clear that the shape of the plot has little influence on the deviations from the known mean. However, the deviations do vary inversely with sample unit size; going from nearly 50 percent for the smallest plots to less than four percent for the largest. From Tables 44 to 51 these averages by distribution are: random, 21.78 percent; systematic, 16.75 percent; and MRS, 13.95 percent. The random distribution was the least accurate of all, while the systematic was the best. This illustrates the tendency for random samples to sample some areas more than others, while the systematic grid, scatters the plots evenly throughout the population. Analysis of variance (Table 61) shows that the

Table 61. -- Analysis of variance of the deviation of the expected value from the true parameter, Red Cedar Woods, plot samples.

Source	df	SS	MS	F
Distribution	2	1117.5174	558.7587	- N.S.
Treatment	35	25,854.8409	738.7097	- N.S.
Plot Shape	5	83.9942	16.7988	- N.S.
Plot Size	5	23,546.9551	4709.3910	6.11 * *
Interaction	25	2223.8916	88.9557	- N.S.
Error	70	59,911.4276	770.1633	
Total	108	80,883.7859		

N.S. Non-significant
* * Highly significant

difference in distribution is not significant. Plot size is the only important influence in areal samples.

In point-sampling, the unadjusted type shows the same trend as the areal plots, while the adjusted points are just the opposite, with losses in accuracy of over 100 percent in going from the BAF-50 to the BAF-5. Analysis of variance (Table 62) shows that all sources of variation are significant, including the differences in distributions. It should be noted that the unadjusted point-samples with a BAF-5 compares favorably to one-fifth acre plot-sample (7.90 percent vs 3.85 percent, Table 60). Consideration of time and cost would probably dictate the use of point-samples, and for precise estimations of the CV, the random distribution should be used.

Table 62. -- Analysis of the variance of the deviation of the expected value from the true parameter, Red Cedar Woods, point-sampling.

Source	df	SS	MS	F
Distribution	2	1450.7081	725.3540	* *
Treatment	11	52,361.9979	4760.1816	* *
Type	1	29,614.4767	29,614.4767	* *
BAF	5	10,771.4065	2154.2813	* *
Interaction	5	11,976.1147	2395.2230	* *
Error	22	1544.5294	70.2059	
Total	35	55,357.2354		

* * Highly significant.

CHAPTER V

SUMMARY AND CONCLUSIONS

Forest sampling methods used in this country are usually the ones that have empirically given the best results. Comparative studies among a few of the more popular designs have led to the use of some in preference to others. Investigation of a large number of methods is limited by cost and time for field work, and the problem of changing forest conditions.

To overcome these difficulties, three forest areas were mapped to scale on large size graph paper and measured by a series of 144 sampling designs. These were a combination of six areal plot shapes with six plot sizes; two point-sampling or Bitterlich procedures with six basal area factors; and three methods of plot distributions -- random, systematic, and multiple-random-starts (MRS). The precision of each design was tested on the basis of its coefficient of variation by analysis of variance techniques. No attempt was made to determine sampling efficiency with respect to time and cost.

The Lansing Woods

This tract of timber has the characteristics of an upland oak-hickory type. The average tree diameter is 11.3 inches. Some fires, grazing, and cuttings have occurred in the stand. The individual tree distribution is quite uniform.

Plot Size. -- Analysis of variance on the coefficient of variation statistics showed that the one-fifth and one-tenth acre plots were the most accurate by a significant margin of any plots tested (6.11 percent and 7.56 percent, respectively). The relationship between plot size and its coefficient of variation can be accurately expressed by the function:

$$CV = 457.570(P)^{-\frac{1}{2}} \tag{8}$$

where: CV = coefficient of variation in percent

P = plot size in square feet

457.570 = the regression coefficient derived from the data

Plot Shape. -- This factor had no more influence on the variation in coefficient of variation than could be attributed to chance. A plot of any shape would have produced the same results.

Point-Sampling. -- Coefficient of variation for the Bitterlich procedures varied directly with the basal area factor used. Following Palley and O'Regan (1961) this relationship is expressed by:

$$CV = 19.905 - 15.351\ (Prf)^{-1} \tag{10}$$

where: CV = coefficient of variation in percent

Prf = plot radius factor in feet per inch of tree diameter

19.905 = derived regression constant

-15.351 = derived regression coefficient

According to the formula, the smallest possible basal area factors will give the lowest error. On this forest, the BAF-5 and BAF-10 gave the best results with coefficients of variation of 7.25 percent and 8.47 percent,respectively. The difference between these is of no importance. A "t" test between plot and point-samples showed that either could be used with the assurance of the same expected precision. Point-sampling with or without the peripheral adjustments for edge bias showed the same level of sampling error.

Distributions. -- The average coefficient of variation for the random, systematic, and MRS distributions are 13.44 percent, 14.16 percent, and 14.39 percent, respectively. None of these differences is significant.

To test the reliability of a distribution to sample the population completely, the differences between the expected values and the true population mean were analysed. The random distribution showed an average difference of only 8.70 percent as compared to 18.53 percent and 12.47 percent for the systematic and the multiple-random-start methods. This was highly significant and indicates that the random distribution was superior to either of the others. For point-sampling, the differences among distributions were insignificant as far as the difference between the expected and the true population was concerned.

Toumey Woods

This woodlot is a relatively undisturbed area of old-growth timber on a moist fertile morainal hill. The species composition is comparable to the sugar maple-beech type of southern Michigan. The average tree diameter is 15.3 inches. The distribution of the individual trees in the forest is uniform.

Plot Size. -- The two larger plot sizes, (one-fifth and one-tenth acre) gave the most precise estimates of the mean with coefficients of variation of 5.74 and 7.67 percent. Smaller plots produced significantly less accurate results. The relationship between plot size and the coefficient of variation can be expressed by the equation:

$$CV = 670.399\ (P)^{-\frac{1}{2}} \tag{13}$$

where: CV = coefficient of variation in percent

P = plot size in square feet

670.399 = regression coefficient derived from the data

Plot Shape. -- No significant differences were observed among the six plot shapes tested. Neglecting the questions of efficiency, a plot of any shape yields the same results.

Point-Sampling. -- The range between point-sampling with peripheral adjustments (12.49 percent), and without this adjustment (7.62 percent) is highly significant. The large trees in this forest and the wide peripheral zones used, over-adjusted the estimate for each successively smaller basal area factor. Variations among basal area factors were not important. For all practical purposes, a BAF of any size would have resulted in estimations of the same precision. Differences between all point-samples and the best plot samples were also small enough to be ignored. The choice between these two methods would be a matter of efficiency.

Distributions. -- Among distributions, the random arrangement gave the lowest coefficient of variation (18.79 percent vs 20.60 and 18.97 percent for the systematic and MRS, respectively. However, none of these differences were significant.

Analysis of the differences between the estimated and the true mean for both the random and the systematic distribution showed that the systematic was significantly better than the random for both points and plots (Points: 2.10 percent vs 3.71 percent; and plots: 9.57 percent vs 15.85

percent). This may result from the tendency of random samples to cluster in certain areas as compared to the rigid dispersal of the systematic method.

The Red Cedar Woods

This study area occupies the low, poorly drained sites characteristic of the forest type ash-elm-red maple. It is a relatively undisturbed area with an average tree diameter of 13.8 inches. Unlike the Lansing and Toumey Woods, the trees in this forest show a tendency for clustering.

Plot Size. -- The one-fifth acre and the one-tenth acre plots gave the significantly lowest coefficient of variation (9.84 and 12.58 percent), of the six plot sizes tested. Unless considerations of efficiency are important, the use of smaller plots is not recommended as indicated by the equation:

$$CV = 649.965\ (P)^{-\frac{1}{2}} \qquad (16)$$

where: CV = the coefficient of variation in percent

P = plot size in square feet

649.965 = derived regression coefficient

Plot Shape. -- This factor did not contribute an appreciable amount to the size of the coefficient of variation.

Point-Sampling. -- The unadjusted point-samples result in much more accurate estimates than the adjusted point-samples (13.08 percent vs 16.30 percent). Differences in basal area factors also produce significant differences in the sampling error as expressed in the formula:

$$SE = 23.895 - 16.740 \, (Prf)^{-1} \tag{18}$$

where: CV = the coefficient of variation in percent

Prf = the plot radius factor in feet per inch of tree diameter

23.895 = regression constant

-16.740 = regression coefficient

The BAF-5 with an average sampling error of 8.54 percent and the BAF-10 with an error of 8.72 percent are the most accurate Bitterlich point samples tested.

The "t" test between the best point-samples and best plot samples, showed the Bitterlich system to be the more precise of the two (3.01 percent vs 8.57 percent).

Distribution. -- The random distribution of sampling units produced the lowest average significant coefficient of variation of all three methods (20.07 percent vs 26.53 and 25.29 percent for the systematic and MRS, respectively).

A test of the three distribution methods, based on the differences between the expected value and the true mean, showed no significant differences among methods. One method of sample unit distribution estimated the true mean as well as any other, for all practical purposes.

Conclusions

In each study area, the shape of the sample unit had no significant effect on the magnitude of the coefficient of variation. A plot of any shape, within a given size, would yield approximately the same results. Where the forest terrain is steep, or the changes in the forest condition are abrupt, plot shape may become important. If efficiency is considered, the use of circular plots would perhaps be preferred, because of their minimum perimeter for a given plot size, and ease of establishment (Johnson and Hixon, 1952; Lindsey *et al.*, 1958).

The unadjusted point-samples gave more precise results than the adjusted point-samples, except in one forest. In the Lansing Woods, with its comparatively small tree diameter and relatively narrow peripheral zones, the two methods of point-sampling were approximately equal. In analysing the differences between the expected value and the true mean, the adjusted point-samples gave estimates that were as much as 100 percent too high. Until the relationship between tree size, BAF, and edge-bias is thoroughly investigated, the adjustment point-sampling method has little to recommend it, unless the trees are small.

As might be expected, the larger the size of the sample units, the more precise the estimate. In all three forests, the one-fifth and one-tenth acre plots were significantly more accurate than the smaller units. The difference between these two is negligible. On two of the areas, either the BAF-5 or the BAF-10 would give more accurate estimates than basal area factors in which fewer trees were sampled. In the Toumey Woods, with its even distribution of large trees, all basal area factors gave approximately the same results. Several regression equations showed the inverse relationships between sample unit size and the coefficient of variation. The limit on the maximum sampling unit to use, would be a matter of efficiency.

According to Table 63, the most precise point-samples were more accurate than the comparable plot samples in each forest. This difference was of significance in the Red Cedar Woods. Various studies (Bell, 1957; Bergeson, 1958; Deitschman, 1956; Grosenbaugh and Stover, 1957) have demonstrated the effectiveness and ease of using point-samples. Under the assumption that point-samples require less time to measure, and considering their precision in this study, they should be employed for the most efficient results.

Table 63. -- Summary of the coefficient of variation parameters for the most accurate plot samples and point-samples.

Sample	Lansing Woods (percent)	Toumey Woods (percent)	Red Cedar Woods* (percent)
BAF-5	5.87	4.58	7.33
BAF-10	7.28	5.66	5.38
1/5-Acre	6.11	5.74	8.57
1/10-Acre	7.56	7.67	10.19

*Random distribution only.

In the three study areas, the random distribution proved to be statistically equal to, or better than, the systematic and multiple-random-start methods, and, in the case of the Red Cedar Woods, this superiority was highly significant. Analysis of the difference between the expected mean and the true mean showed the random method to be significantly best on the Lansing Woods, while the systematic was superior on the Toumey area. On the Red Cedar Woods, these differences could be attributed to chance. The multiple-random-start arrangement offered little to recommend it in either analysis.

Because it is impossible to determine variation patterns in botanical populations, and because nature fails to randomize properly, the random sample distribution should be used at every opportunity. It is the only arrangement of plot that affords a precise estimate of the sampling error. In this study, it yielded results as good or better than the systematically placed samples.

Although the three forests are distinctive with respect to species, site, tree size, and distribution, nearly the same sampling methods are applicable to each area. Considering the work of other investigatiors on sampling efficiency, and the accuracy of the method involved in this study, the unadjusted BAF-5 or BAF-10 point-sample should be used on a random distribution in forests with these characteristics.

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APPENDIX

APPENDIX A.

Example 1. -- Record Sheet for Sampling Basal Area on Red Cedar Woods.

Circular Plots on a Random Distribution								
Plot 1 Size	BA	Acc. BA	Plot 2 Size	BA	Acc. BA	Plot 3 Size	BA	Acc. BA
1/160	--	0	1/160	--	0	1/160	--	0
1/80	--	0	1/80	--	0	1/80	1379	
1/40	245	245	1/40	1344			184	1563
1/20	894			852	2196	1/40	1467	3030
	3409	4548	1/20	1396	3592	1/20	307	
1/10	979		1/10	625			1163	4500
	1227	6754		442		1/10	2664	
1/5	734			1948			238	
	1131			1069			568	
	672			894	8570		3947	
	601		1/5	1485			908	
	1414	11306		275			2181	
				2248			1277	16283
				721		1/5	275	
				291			1362	
				413			291	18211
				223				
				1344				
				1310				
				245				
				2053				
				1558				
				648	21384			

Plot 4 Size	BA	Acc. BA	Plot 5 Size	BA	Acc. BA	Plot 6 Size	BA	Acc. BA
1/160	1006							
	965		1/160	--	0	1/160	1948	1948
	432	2403	1/80	--	0	1/80	--	1948
1/80	524	2927	1/40	1227	1227	1/40	1614	3562
1/40	812		1/20	1748		1/20	1632	
	772	4511		1651	4626		1485	6679
1/20	307	4818	1/10	210		1/10	--	6679
1/10	747			190		1/5	367	
	950			825			153	
	1227			1147			1379	
	472			442			1211	
	524			482	7922		503	
	601		1/5	625			825	
	535			462			315	
	1211	11135		245			760	
1/5	472			349			315	
	672			260			3168	
	1131			2712			203	

APPENDIX A., Example 1, Con't.

Plot 4 Size	BA	Acc. BA	Plot 5 Size	BA	Acc. BA	Plot 6 Size	BA	Acc. BA
(1/5)	1558		(1/5)	1244		(1/5)	1227	
	147			216			9079	26184
	1867			852				
	2688			299				
	2475			825				
	2337			216				
	1709			1728				
	799			432	18387			
	965							
	1948	29903						

Plot 7 Size	BA	Acc. BA	Plot 8 Size	BA	Acc. BA	Plot 9 Size	BA	Acc. BA
1/160	672		1/160	--	0	1/160	--	0
	1131		1/80	--	0	1/80	--	0
	1277	3080	1/40	601	601	1/40	1748	
1/80	747		1/20	3142	3743		432	
	472	4299	1/10	2181			1179	3359
1/40	636			1709	7633	1/20	601	3960
	1024	5959	1/5	184		1/10	1054	
1/20	950			908			2117	
	747			734			147	7278
	472	8128		299		1/5	323	
1/10	579			772			1100	
	1558			171	11701		2292	
	812						2712	
	524						1244	
	432						2270	
	238	12271					556	
1/5	636						965	18740
	1039							
	1344							
	184							
	147							
	524							
	965							
	1006							
	994							
	1115							
	403	20628						

Plot 10

Size	BA	Acc. BA
1/160	472	
	482	
	1651	2605
1/80	--	2605
1/40	--	2605
1/20	601	3206
1/10	568	
	1990	
	452	6216
1/5	2688	
	1521	
	1310	
	462	
	413	12610

Plot 11

Size	BA	Acc. BA
1/160	994	994
1/80	394	
	340	
	165	1893
1/40	1260	
	2810	5963
1/20	299	6262
1/10	936	7198
1/5	442	
	2074	9714

Plot 12

Size	BA	Acc. BA
1/160	852	853
1/80	--	852
1/40	2181	3033
1/20	238	
	852	
	2074	6197
1/10	323	
	299	6819
1/5	3631	
	299	
	1260	
	994	
	165	
	340	
	936	
	442	14886

Plot 13

Size	BA	Acc. BA
1/160	291	291
1/80	--	291
1/40	--	291
1/20	625	
	545	1461
1/10	171	
	1689	
	385	
	394	
	184	
	1632	
	260	
	697	
	513	7386
1/5	4276	
	2475	
	613	
	223	
	190	
	238	
	648	
	196	
	1131	
	340	17716

Plot 14

Size	BA	Acc. BA
1/160	--	0
1/80	252	252
1/40	--	252
1/20	3801	4053
1/10	1084	
	697	
	394	
	965	7193
1/5	4876	
	613	
	4337	
	503	
	908	
	579	
	799	
	1576	
	1115	
	880	
	825	
	2451	
	153	
	190	26998

Plot 15

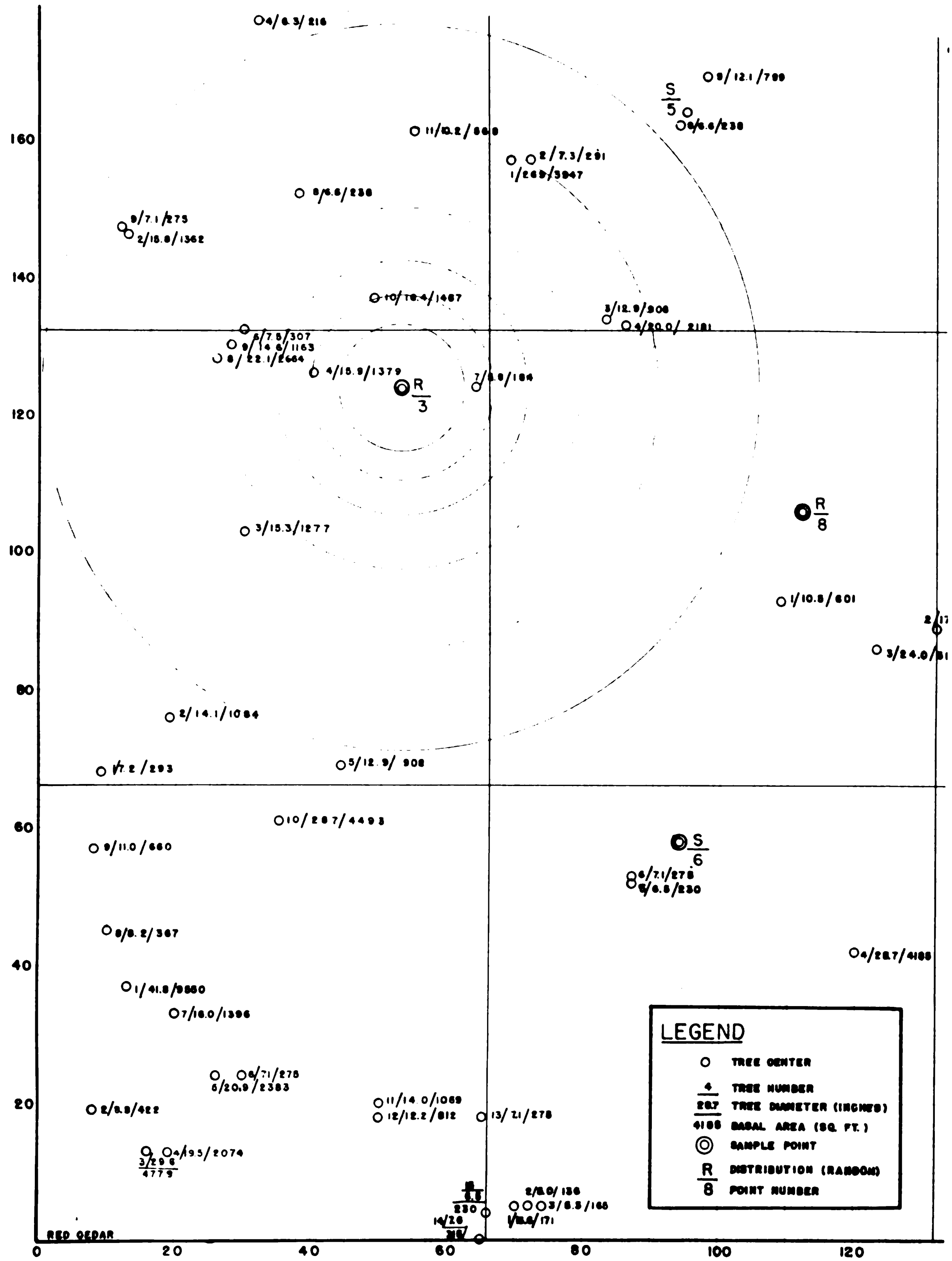
Size	BA	Acc. BA
1/160	3888	
	2737	6625
1/80	1449	8074
1/40	--	8074
1/20	332	
	482	8888
1/10	223	
	1100	
	367	
	4035	14613
1/5	545	
	6305	
	545	
	1195	
	648	
	1115	
	165	25131

Plot 16 Size	BA	Acc. BA	Plot 17 Size	BA	Acc. BA	Plot 18 Size	BA	Acc. BA
1/160	--	0	1/160	1179		1/160	--	0
1/80	--	0		432	1611	1/80	223	223
1/40	697		1/80	--	1611	1/40	--	223
	203		1/40	601	2212	1/20	1414	1637
	1179	2079	1/20	--	2212	1/10	165	
1/20	159	2238	1/10	1748			1163	
1/10	1344			1244			1260	
	203			2117			1115	
	1039			1054			1195	
	1100			147	8522		210	6745
	291		1/5	556		1/5	2885	
	1867			2270			684	
	672	8754		965			349	
1/5	1503			590			307	
	697			852	13755		1748	
	880			852	13755		376	
	1244						590	
	492						332	
	1163						358	
	291						349	14723
	3355							
	1887							
	908							
	323	21497						

APPENDIX A. Example 2.

Plot summary sheet and parameter computation table for circular plots on the random distribution, Red Cedar.

	Plot Size in Acres					
Plot No.	1/160	1/80	1/40	1/20	1/10	1/5
1	0	0	0.245	4.548	6.754	11.306
2	0	0	2.196	3.592	8.570	21.384
3	0	1.563	3.303	4.500	16.283	18.211
4	2.403	2.927	4.511	4.818	11.135	29.903
5	0	0	1.227	4.626	7.922	18.387
6	1.948	1.948	3.562	6.629	6.679	26.184
7	3.080	4.299	5.959	8.128	12.271	20.628
8	0	0	0.601	3.743	7.633	11.701
9	0	0	3.359	3.960	7.278	18.740
10	2.605	2.605	2.605	3.206	6.216	12.610
11	0.994	1.893	5.693	6.262	7.198	09.714
12	0.852	0.852	3.033	6.197	6.819	14.886
13	0.291	0.291	0.291	1.461	7.386	17.716
14	0	0.252	0	4.053	7.193	26.998
15	6.625	8.074	8.074	8.888	14.613	25.131
16	0	0	2.079	2.238	8.754	21.497
17	1.611	1.611	2.212	2.212	8.522	13.755
18	0	0.223	0.223	1.637	6.745	14.723
Total	20.409	26.538	48.900	80.748	157.971	333.747
Plot Mean	1.134	1.474	2.717	4.486	8.776	18.542
Mean per Acre	181.44	117.92	108.68	89.72	87.76	92.71
$\Sigma(\text{Plots})^2$	74.126	112.364	218.694	436.848	1541.995	6765.601
Variance	2.999	4.308	5.050	4.389	9.154	33.967
Standard Deviation	1.732	2.076	2.242	2.095	3.026	5.828
Standard Deviation Per Acre	277.12	166.08	89.88	41.90	30.26	29.14



APPENDIX A. -- Section of the Red Cedar Woods with the circular plot system on random sampling unit three. Basal area values contain three decimal places.

APPENDIX B. -- Plot Radius in Feet by Tree Diameters and Basal Area Factors (BAF)

Tree Diameter Inches	Basal Area Factors					
	BAF-5	BAF-10	BAF-20	BAF-30	BAF-40	BAF-50
	- - - - - - - - - - - - plot radius feet - - - - - - - - - - - -					
5	19.4	13.8	9.7	7.9	6.9	6.2
6	23.3	16.5	11.7	9.5	8.3	7.4
7	27.2	19.3	13.6	11.1	9.6	8.6
8	31.1	22.0	15.6	12.7	11.0	9.8
9	35.0	24.8	17.5	14.3	12.4	11.1
10	38.9	27.5	19.4	15.9	13.8	12.3
11	42.8	30.3	21.4	17.5	15.1	13.5
12	46.7	33.0	23.3	19.1	16.5	14.8
13	50.5	35.8	25.3	20.6	17.9	16.0
14	54.4	38.5	27.2	22.2	19.3	17.2
15	58.3	41.3	29.2	23.8	20.6	18.5
16	62.2	44.0	31.1	25.4	22.0	19.7
17	66.1	46.8	33.0	27.0	23.4	20.9
18	70.0	49.5	35.0	28.6	24.8	22.1
19	73.9	52.3	36.9	90.2	21.1	23.4
20	77.8	55.0	38.9	31.8	27.5	24.6
21	81.7	57.8	40.8	33.3	28.9	25.8
22	85.5	60.5	42.8	34.9	30.3	27.1
23	89.4	63.3	44.7	36.5	31.6	28.3
24	93.3	66.0	46.7	31.1	33.0	29.5
25	97.2	68.8	48.6	39.7	34.4	30.8
26	101.1	71.5	50.5	41.3	35.8	32.0
27	105.0	74.3	52.5	42.9	37.1	33.2
28	108.9	77.0	54.4	44.5	38.5	34.4
29	112.8	79.8	56.4	46.1	39.9	35.7
30	116.7	82.5	58.3	47.6	41.3	36.9
31	120.5	85.3	60.3	49.2	42.6	38.1
32	124.4	88.0	62.2	50.8	44.0	39.4
33	128.3	90.8	64.2	52.4	45.4	40.6
34	132.2	93.5	66.1	54.0	46.8	41.8
35	136.1	96.3	68.0	55.6	48.1	43.1
36	140.0	99.0	70.0	57.2	49.5	44.3
37	143.9	101.8	72.0	58.8	50.9	45.5
38	147.8	104.5	73.9	60.3	52.3	46.7
39	151.7	107.3	75.8	61.9	53.6	48.0
40	155.5	110.0	77.8	63.5	55.0	49.2
41	159.4	112.8	79.7	65.1	56.4	50.4
42	163.3	115.5	81.6	66.7	57.8	51.7

APPENDIX C

Example of the Analysis of Variance and Plot Comparisons

Table 15 on page 41 gives the summary of the coefficient of variation data for the Lansing Woods. Handling of these data by the analysis of variance is as follows:

$$\text{Correction term} = \frac{\left(\sum^{108} x\right)^2}{108}$$

$$\text{C.T.} = \frac{1511.69^2}{108}$$

Total sum of squares

$$= \sum^{108} x^2 - \text{C. T.}$$

$$= (26.74^2 + 17.72^2 + \ldots + 4.89^2) - \text{C.T.}$$

$$= 27{,}762.1859 - \text{C. T.}$$

$= 6602.8650$ on 107 degrees of freedom (df)

$$\text{Distribution SS} = \frac{\sum^{3} (\text{distribution totals}^2)}{\text{Obs. per distribution}} - \text{C. T.}$$

$$= \frac{(483.78^2 + 509.78^2 + 518.13^2)}{36} - \text{C.T.}$$

$= 17.8300$ on 2 df.

$$\text{Treatment SS} = \frac{\sum^{36} (\text{Plot size totals}^2)}{\text{Obs. per total}} - \text{C.T.}$$

$$= \frac{(77.43^2 + 57.04^2 + \ldots + 15.10^2)}{3} - \text{C.T.}$$

$= 6247.9466$ on 35 df.

Error SS = Total SS - Distribution SS - treatment SS

$= 337.0884$ on 70 df.

APPENDIX C. -- Analysis of variance

Source	df	SS	MS	F
Distribution	2	17.8300	8.9150	1.85 N. S.
Treatment	35	6247.9466	178.5128	37.07 * *
Error	70	337.0884	4.8155	
Total	107	6602.8650		

N.S. Non-significant * * Highly significant

Treatment is significant, so the sum of squares and degrees of freedom making up this source may be partitioned into the respective factors: plot size, plot shape, and size-shape interaction. This is carried out in the following way:

$$\text{Plot type SS} = \sum^{6} \frac{(241.78^2 + 259.53^2 + \ldots + 258.25^2)}{18} - \text{C.T.}$$

$$= 12.7520 \text{ on 5 df.}$$

$$\text{Plot size SS} = \sum^{6} \frac{(500.72^2 + 346.85^2 + \ldots + 110.06^2)}{18} - \text{C.T.}$$

$$= 6137.4834 \text{ on 5 df.}$$

Interaction = Treatment SS - Plot type SS - Plot size SS

= 97.7112 on 25 df.

APPENDIX C. -- Analysis of variance.

Source	df	SS	MS	F
Distribution	2	17.8300	8.9150	1.85 N.S.
Treatment	35	6247.9466	178.5138	37.07 * *
Plot type	2	12.7520	2.5504	- N.S.
Plot size	5	6137.4834	1227.4969	254.91 * *
Interaction	25	92.7112	3.9084	- N.S.
Error	70	337.0884	4.8155	
Total	107	6602.8650		

N.S. Non-significant * * Highly significant

The five degrees of freedom for plot type and their associate sums of squares may be partitioned evenly among several orthogonal comparisons according to:

APPENDIX C. -- Orthogonal comparisons of plot sizes.

	Plot Size - Acres						Algebraic
Plot Comparisons	1/160	1/80	1/40	1/20	1/10	1/5	Sum of Coefficients
1/5 vs 1/10	0	0	0	0	+	-	0
1/5 + 1/10 vs 1/20	0	0	0	+2	-	-	0
1/5 + 1/10 + 1/20 vs 1/40	0	0	+3	-	-	-	0
1/5 + 1/10 + 1/20 + 1/40 vs 1/80	0	+4	-	-	-	-	0
Others vs 1/160	+5	-	-	-	-	-	0
Product of Coefficients	0	0	0	0	+	-	= 0

If the comparisons check orthogonally then the sum of squares of the comparisons will equal the sum of squares for the plot size in the analysis of variance, and the test of significance can be based on the error term with one and 70 degrees of freedom.

APPENDIX C. -- Areal Plot Comparisons for the Lansing Woods.

Plot Comparison		Procedure		SS	F
1/5 vs 1/10	=	$(136.01-110.06)^2/2(18)$	=	18.7056	3.88 N.S.
1/5 + 1/10 vs 1/20	=	$[2(174.95)-(246.07)]^2/6(18)$	=	99.8210	* *
1/5 + 1/10 + 1/20 vs 1/40	=	$[3(243.10)-(421.02)]^2/12(18)$	=	439.9841	* *
1/5 + 1/10 + 1/20 + 1/40 vs 1/80	=	$[4(346.85)-(564.12)]^2/20(18)$	=	1453.1499	* *
Others vs 1/160	=	$[5(500.72)-(910.97)]^2/30(18)$	=	4125.8228	* *
		Total sum of squares		6137.4834	

N.S. Non-significant
* * Highly significant

The distributions totals from Table 18 are random, 483.78; systematic, 509.78; MRS, 518.13. The possibility exists that the random distribution may be significantly smaller than the other two. The two degrees of freedom and the sums of squares for distribution may be partitioned into two comparisons -- each with one and seventy degrees of freedom.

$$\text{Systematic vs MRS} = \frac{(518.13-509.78)^2}{72}$$

$$F = .9684 \text{ N.S.}$$

Random vs (Systematic + MRS)

$$= \left|\frac{2(483.78-(509.78 + 518.13)}{6(36}\right|^2$$

$$= 16.8617$$

$$F = \frac{16.8617}{4.8155} = 3.50 \text{ N.S.}$$

There are no significant differences among these distributions.

Point-Sampling. -- The same type of analysis was applied to the point-sampling data from Table 19. There were no significant differences.