A CHECK ANALYSIS OF A CONTINOUS SPAN REINFORCED CONCRETE BRIDGE

Thesis for the Degree of B. S.

MICHIGAN STATE COLLEGE

R. C. McLravy

1949

A Check Analysis of a Continuous Span Reinforced Concrete Bridge

A Thesis Submitted to

The Faculty of

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of

AGRICULTURE AND APPLIED SCIENCE

by

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Candidate for the Degree of

Bachelor of Science

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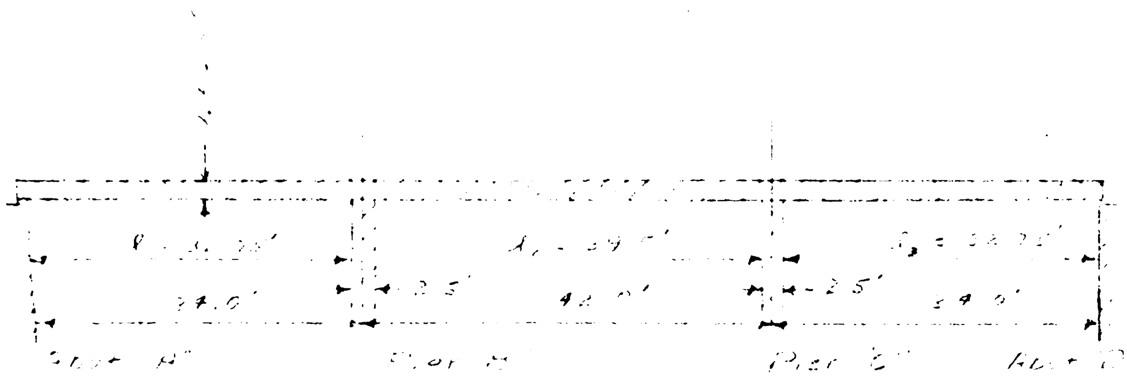
INTRODUCTION

The methods used to determine maximum stresses in this analysis are those given in the American Association of State Highway Officials titled, "Standard Specifications for Highway Bridges", the 1944 edition. The maximum permissable stresses are also those given in the above mentioned book.

Due to the shortness of this term, only the structural design of the superstructure and abutments was analyzed. For a check analysis of the piers of this structure see the thesis of T. Nakamura, Spring 1949.

SUPERSTRUCTURE

[illegible]



l = Unsupported Length

Curb to Curb width = [illegible]

Over all width of Superstructure = [illegible].25'

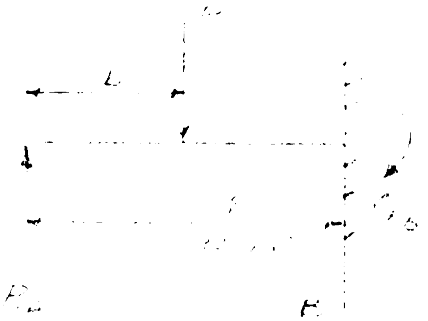
Over all Length of Superstructure = [illegible].0'

[illegible] Length of Abutment [illegible]

SLAB

Case I

To find: Position of a concentrated load F to make M_{FBA} maximum.



[illegible] $M_{FBA} = \frac{Fa}{[illegible]}$ [illegible]

M_{FBA} [illegible]

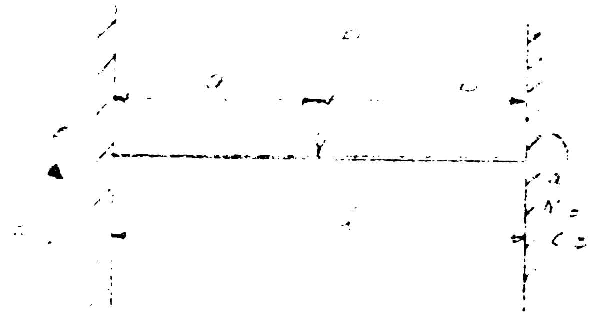
[illegible]

[illegible]

[illegible]

[illegible] $= \frac{[illegible]}{1732}$ [illegible]

Case II Both ends fixed



$M = \frac{Fab^2}{l^2}$; $b = l - a$

$M = Pa - \frac{2Pa^2}{l} + \frac{Pa^3}{l^2}$

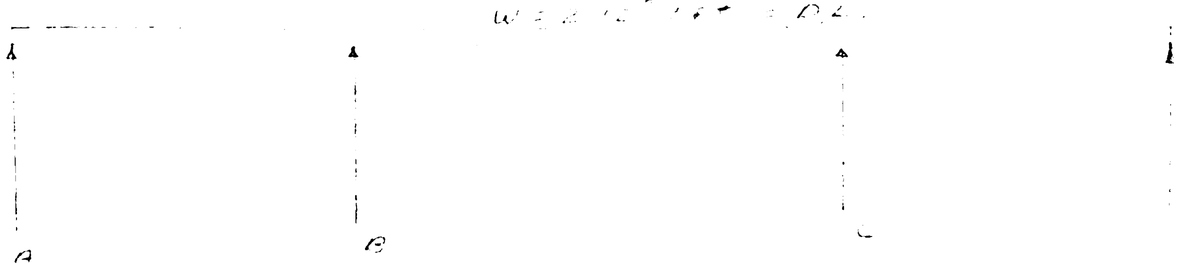
$\frac{dM}{da} = P - \frac{4Pa}{l} + 3\frac{Pa^2}{l^2}$

[illegible]

[illegible]

[illegible]

$w = [illegible]$



$M_{FAB} = M_{FDC} = 0$

$M_{FBA} = M_{FCD} = \frac{1}{8} w l^2 = \frac{1}{8} \times 2.12 \times 32.75^2 = [illegible]$

$M_{FBC} = M_{FCB} = \frac{1}{12} w l^2 = \frac{1}{12} \times 2.12 \times 37.5^2 = [illegible]$

$K = \frac{I}{l}$; $K_{BA} = K_{CD} = \frac{3 \times I_o}{32.75}$; $r_{BA} = \frac{K}{\Sigma K} = \frac{\frac{3I}{32.75}}{\frac{3I_o}{32.75} + \frac{4I_o}{37.5}} =$

$K_{BC} = \frac{4 I_o}{37.5}$

$[illegible] = 0.477$; $r_{BC} = 0.1067 I_o$ / $0.1927 I_o$ $[illegible]$

M_{FAB}		M_{BA}	M_{BC}	M_{CB}	M_{CD}	M_{DC}
1		0.477	0.523	0.523	0.477	
FE1.1	0	−264	+276.0	−276	+264.0	
		+3.22	+4.18	−4.[illegible]	−3.[illegible]	
			−2.[illegible]	+2.0[illegible]		
		+1.0	+1.0[illegible]	−1.0[illegible]	[illegible]	
			−0.[illegible]	+0.[illegible]		
		+0.[illegible]	+0.2[illegible]	−0.[illegible]	−0.26	
			−0.14	+0.14		
		.07	+.07	−.07	−[illegible]	
		−278.8[illegible]	+27[illegible]	−278.85	+2[illegible]	

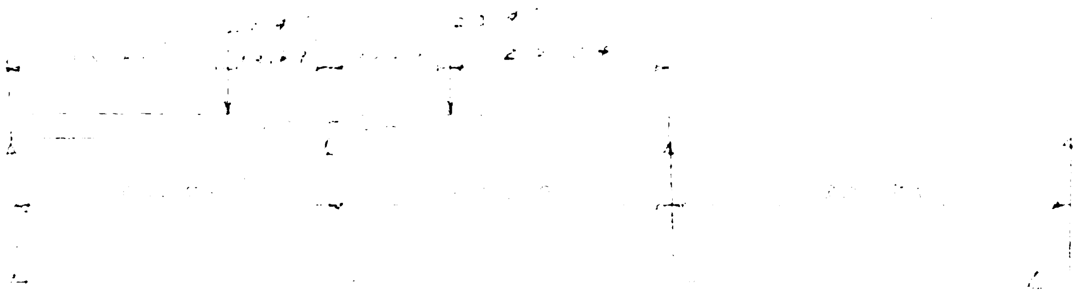
(In kip feet)

(H) Max. Neg. [illegible]

1) By Equiv. Lane Loading

$w = 640^{\#}$ [illegible] $+ 30\%$ Imp. [illegible]

$P =$ [illegible] $= 23.4^{k}$



$M =$ [illegible] 18 [illegible]

$M_{DEAD} =$ [illegible] 25 [illegible]

$M =$ [illegible] P [illegible] $= 23.4$ [illegible]

$M_{LIVE} =$ [illegible] 24 [illegible]

[illegible]

	M_{BA}	M_{BC}	M_{CB}	M_{CD}
r	0.477	0.[illegible]3	.523	.477
FEM 0	-254.[illegible]	+24[illegible]	-177.5	0
	+6.67	+7.73	+70.4	+8[illegible]
		+46.4	+3.7	
	-22.[illegible]	-[illegible]4.2	-1[illegible]	-1[illegible]
		-0.11	-12.1	
	+0.43	+0.45	+6.33	+5.7[illegible]
		+3.16	+0.24	
	-1.51	-1.65	-.13	-[illegible]
		-.06	-.52	
	+.03	+.021	+.43	+.24
		+.21	0	
	-.[illegible]0	-.[illegible]	-.05	
			+.03	.06
	-275.5	+275.5 ''k	-59.1	+59.1 ''k

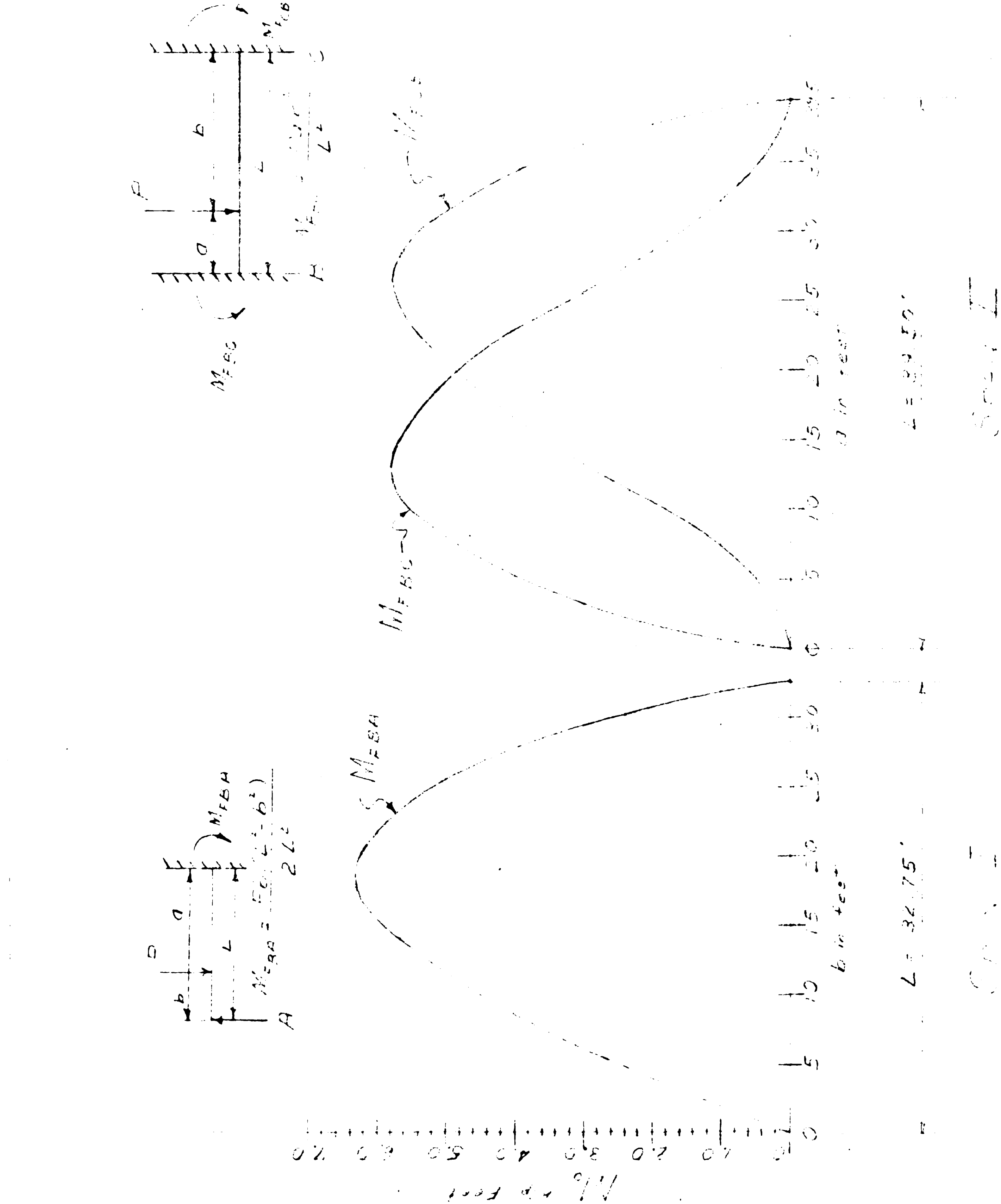
Max. Neg. M. (Eq. 20) = 275.5 + [illegible] =

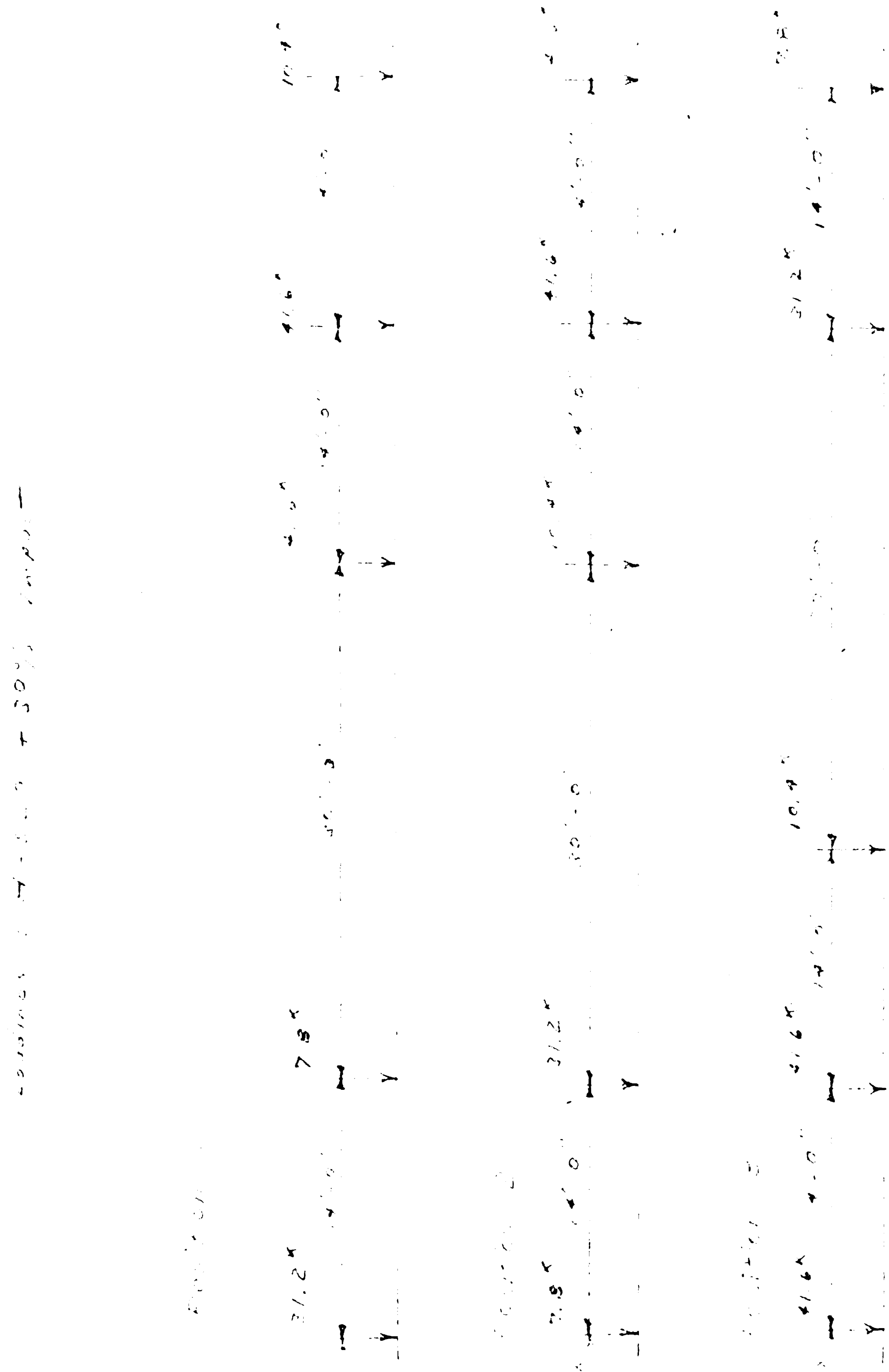
254.25 ''k = [illegible]

SLAB

C. Max. Negative Moment by Influence Lines (H-S20) Loading

1. Influence Lines





Trial (1) $x = -6'$

$M_{CA} + M_{CB} = 41.6 \times 3.8 + 1?.4 \times 5.0 + \ldots + 31.2 \times 5.4 + 7.5 \times ?$

$= 396.2$ T ft

Trial (2) $x = -2'$

$M_{CA} + M_{CB} = 41.6 \times 3.2 + ?.4 \times 4.8 + \ldots 31.2 \times 4.6 + 7.5 \times 5.4$

$= 414.17$ ft

Trial (3) $x = +11'$

$M_{CA} + M_{CB} = 41.6 (? + 5.0) + \ldots + ?.4 \times 3.? + ?$

$= 4??$ ft

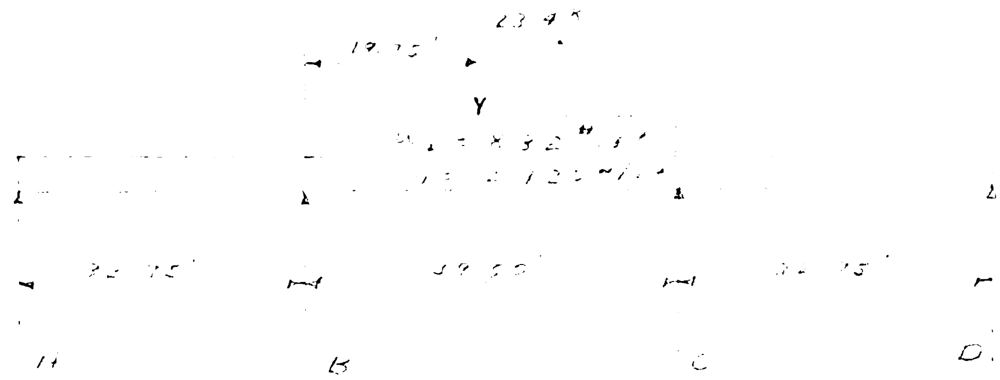
Trial 4 $x = +3$

Mom... $= 41.6 (5.4 + 4.4) + \ldots + ?.4 \times 4.4 = 455$ T ft

... will not produce Max. M ...

SLAB

D. Max. Positive Moment by Equiv. Line Loads
1. Middle Span



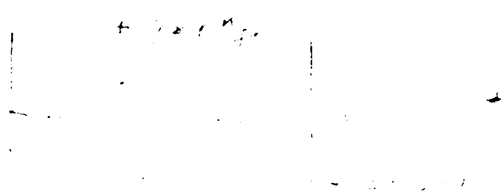
$M_{FBA} = M_{FCD} = 224^{k}$ ft (same)

$M_{FBC} = -M_{FCB} = \frac{1}{12} \times 2.432^{k} \times 39.5^{2} + 23.4 \times [illegible] \times 19.75$ [illegible] 39.5^{2}

= [illegible]

	M_{BA}	M_{BC}	M_{CB}	M_{CD}
	.477	.523	.523	.477
FEM	−224.0	+499.2	−415.2	+254.0
	−[illegible]2.2	−112.0	−112.8	+102.6
		+56.0	−56.0	
	−26.6	−29.4	+29.4	+26.6
		+15.0	−15.0	
	−7.2	−7.8	+7.8	+7.2
		+3.9	−3.9	
	−1.86	−2.04	+2.04	+1.86
		+1.02	−1.02	
	−0.49	−0.53	+0.53	+0.49
	−322.31	+322.31	−322.3	+322.31

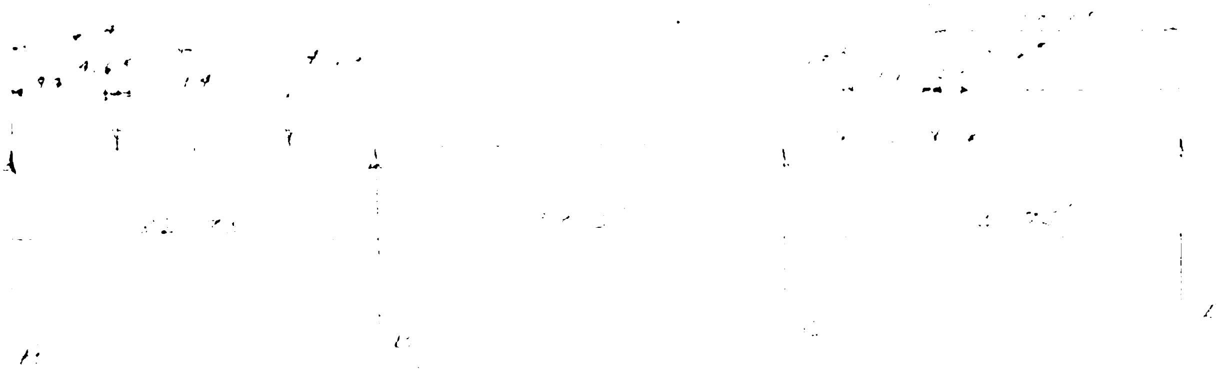
Simply Supported
Positive M = [illegible] = 224 + 768
= [illegible]



+ M = [illegible] = 784 − 322.3
= [illegible]

372.3 372.3 372.3 372

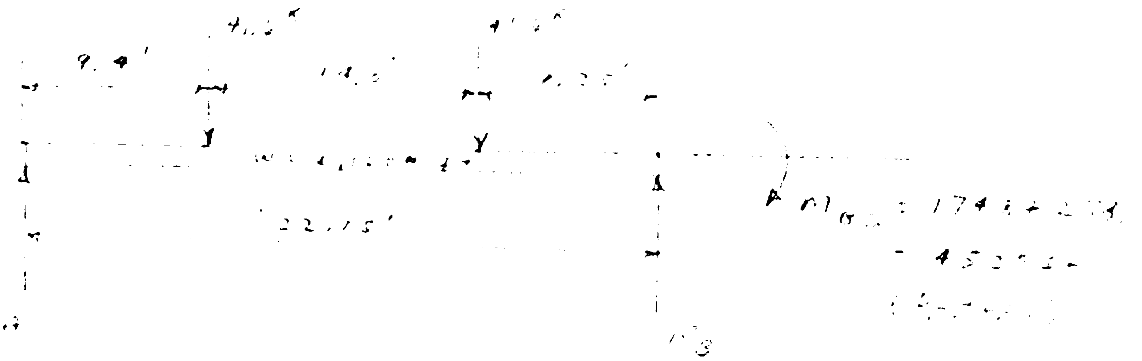
[illegible]



[illegible]

	M_{BA}	M_{BC}	M_{CB}	M_{CD}
r	.477	.523	.523	.477
FEM	+420	0	0	-218.4
	-202	-218	+114	+104
		+57	-105	
	-27	-30	+55	+50
		+28	-15	
	-13.4	-14.6	+7.8	+7.2
		+3.9	-7.3	
	-1.9	-2.0	+3.8	+3.5
		+1.9	-1.0	
	-.9	-1	+0.6	+0.5
	+174.8	-174.8	+5[illegible]	-[illegible]

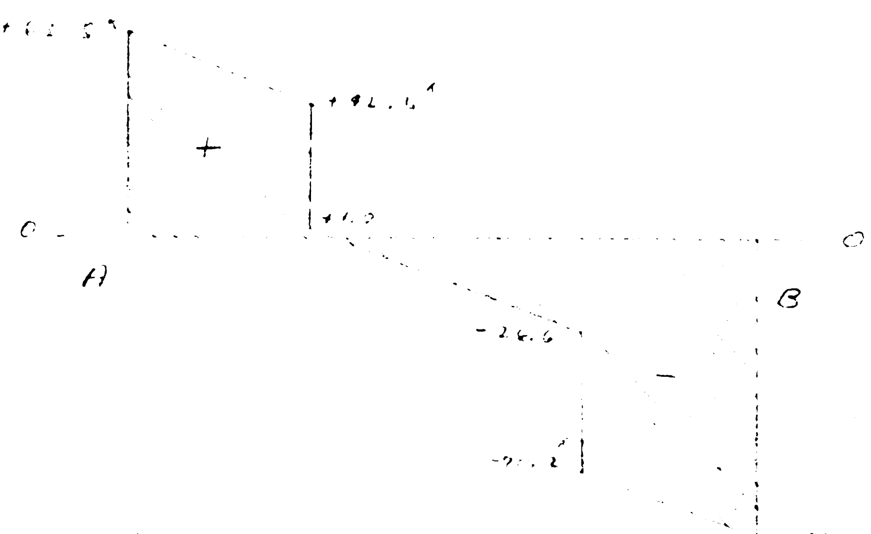
E Cont.
2. ...



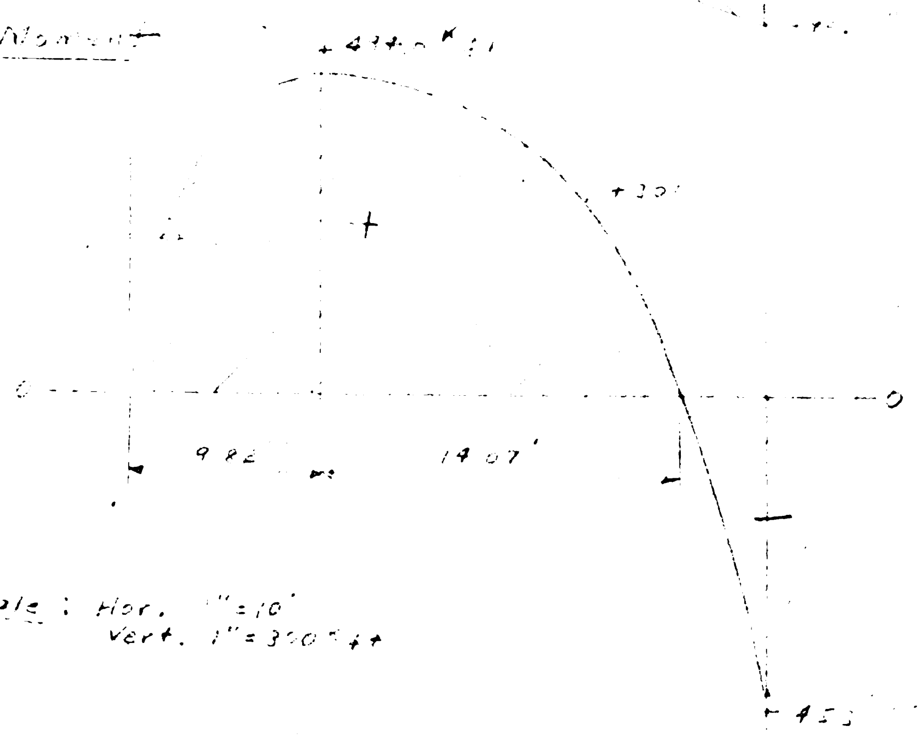
$32.75\,R_A = 41.6 \times ... + 41.6 \times 23. ... + 2.1 ... \times \frac{32.75^2}{2} - 453$

$R_A = \frac{204...}{32.75} = 62.5^k$

3 Shear



4 Moment



Scale : Hor. 1" = 10'
Vert. 1" = 300'k

SLAB

F. [illegible] Diagram [illegible]

1. [illegible]

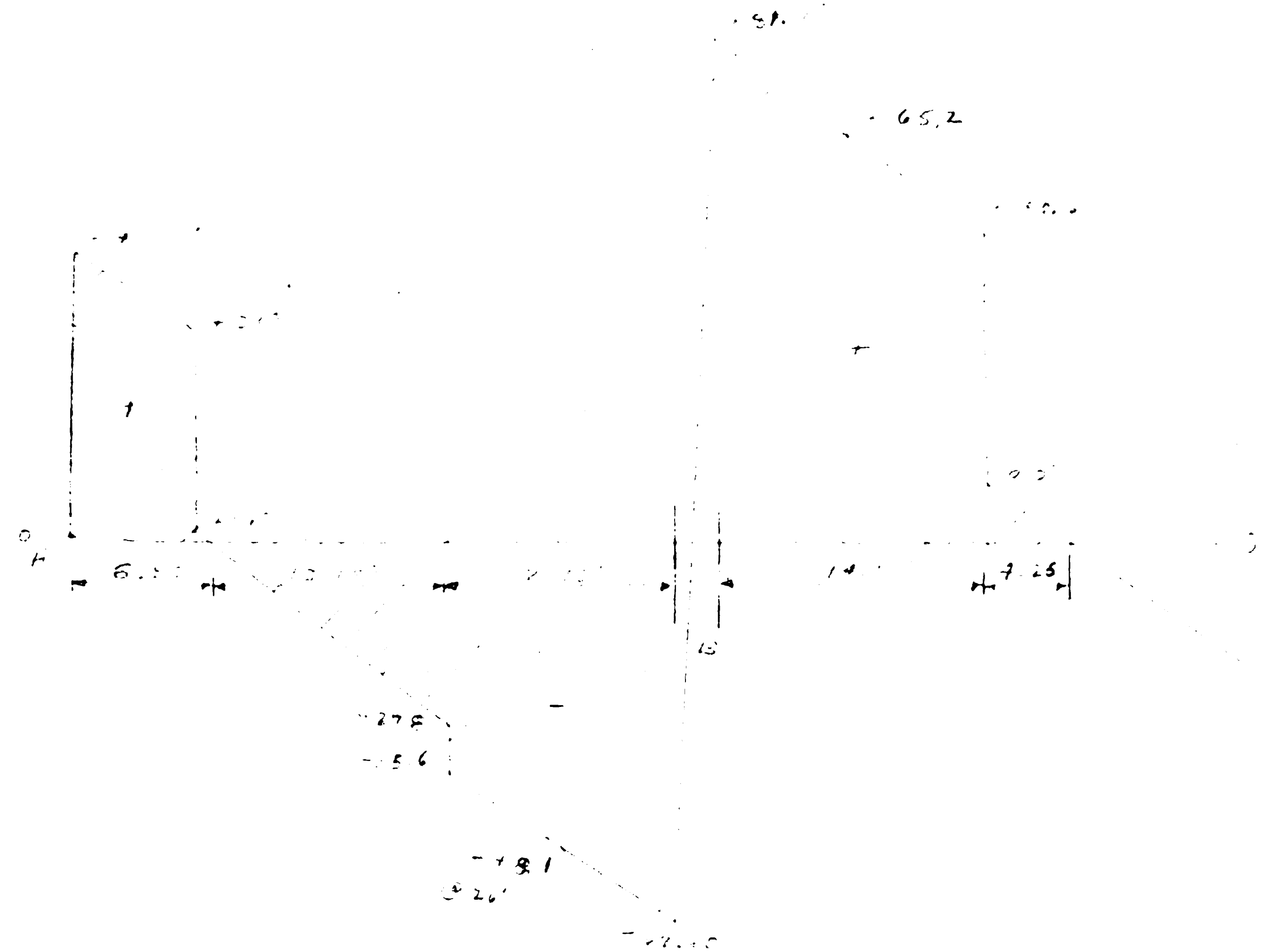
2. [illegible]

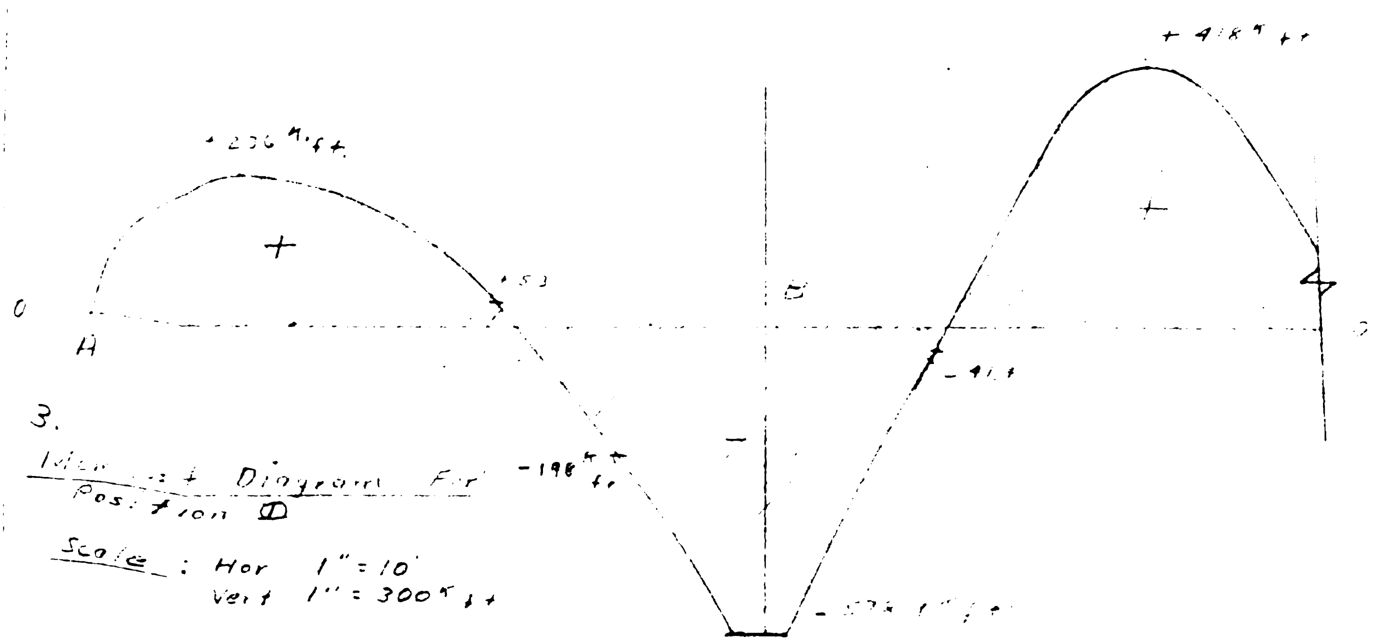
[illegible]

$\Sigma M_C = 0$ [illegible] R_{BC} + 4.6 [illegible] + 24 [illegible]

+ 2.12 [illegible]

[illegible] = [illegible] 144. [illegible] per [illegible]



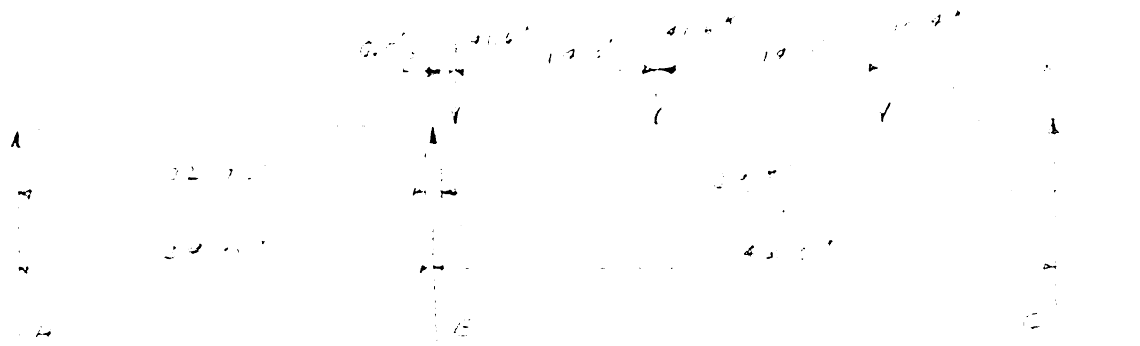


3.

Moment Diagram For Position D

Scale: Hor 1" = 10'
Vert 1" = 300k ft

6. Maximum Shear ([illegible])



L.L.

FEM_{BC} = [illegible] = 311.6 [illegible]

FEM_{CB} = [illegible] = [illegible]

	M_{AB}	M_{BA}	M_{BC}	M_{CB}
r	[illegible]	.52 [illegible]	.5 [illegible]	[illegible]
FEM	0	−311.6	+208	0
	+147	+10 [illegible]	−1 [illegible]	+ [illegible]
		−5 [illegible]	+42	
	+26.2	+2 [illegible]	−43	− [illegible]
		−21	+144	
	+1 [illegible]	+ [illegible]	−7 [illegible]	−6 [illegible]
		−3.7	+5.5	
	+1 [illegible]	+1 [illegible]	[illegible]	−2.6
	+ [illegible]	−[illegible]2.2	+11[illegible]	[illegible]

[illegible]

ΣM_2 = +25.9 +6 [illegible] 316.3 [illegible]

R_A = [illegible] − +56 [illegible] = [illegible] = [illegible]

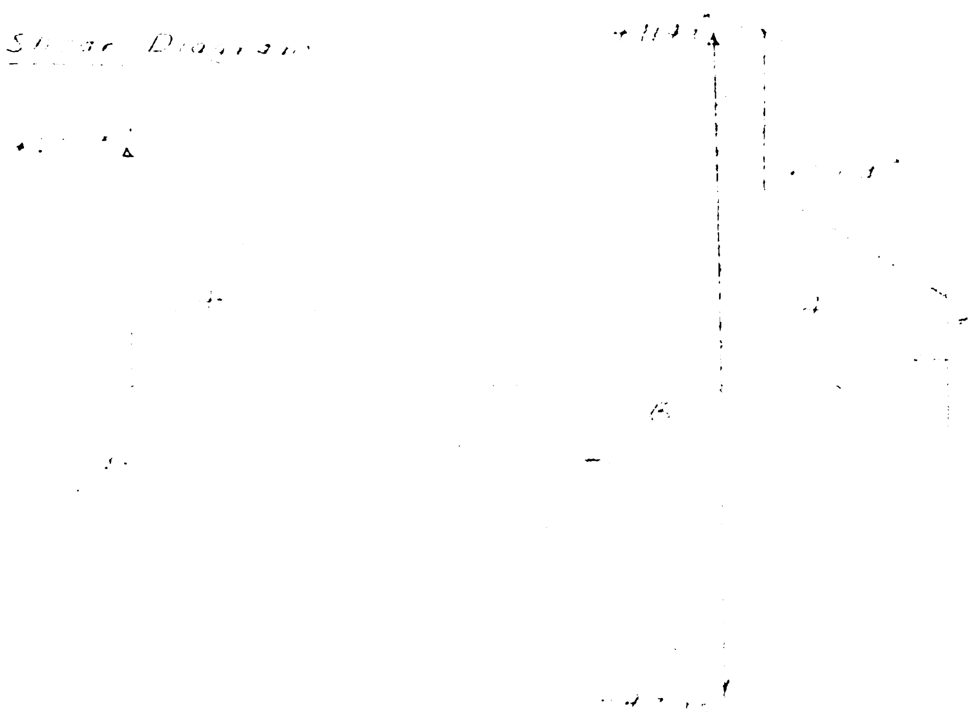
R_{BA} = [illegible] + [illegible] = 16 [illegible] = 47.2 [illegible]

[illegible]

[illegible]

R_B = [illegible]

1. Shear Diagram



Max. Possible Shear = 14.0^k per Lane

H. Distribution Reinforcing Steel

Unit Req'd. = % of Positive Reinforcing Steel

(by AASHO 1944) $\% = \frac{100}{\sqrt{S}} = \frac{100}{\sqrt{[illegible]}} = 15.7\%$ for middle span

% End Span $= \frac{100}{\sqrt{[illegible]}} = 17.5\%$ for End spans

1. Area Req'd — Middle Span = .157 × 2.54 = 0.41 □"/ft

(In Middle Half) Area furn. = 0.60 □"/ft > 0.41 □"/ft ok.

2. Area Req'd — End Span = 0.175 × 2.5[illegible] = 0.45 □"/ft

Area furn = 0.60 □"/ft > 0.45 □"/ft [illegible]

3. [illegible] Middle [illegible]

$[illegible] .15 \times [illegible] = 0.14 [illegible]$

$A [illegible] = [illegible]$

$0.28 > 0.14$ [illegible]

4. [illegible]

$A_{reqd} = .113 \times .88 = 0.1 [illegible]$

$A [illegible] > 0.1 [illegible]$

I Temperature Steel

1. Area [illegible] = 0.2 [illegible]

$= .002 \times 12 \times 15. [illegible]$

[illegible]

$A [illegible] \frac{3}{4} [illegible] 12 [illegible] = 0.3 [illegible]$

$[illegible] > 0. [illegible]$ O.K.

I. Width of Slab over which wheel loads are distributed

$$E = \frac{10N + W}{4N}$$

E = width of slab over which wheel load is distributed

N = Max. no. of lanes permissible

W = Curb to curb width in ft.

$$E = \frac{10 \times [illegible]}{4 \times [illegible]} = \frac{172}{32} = 5.36'$$ per wheel

10.72' per axle

SLAB

[illegible]. Stresses

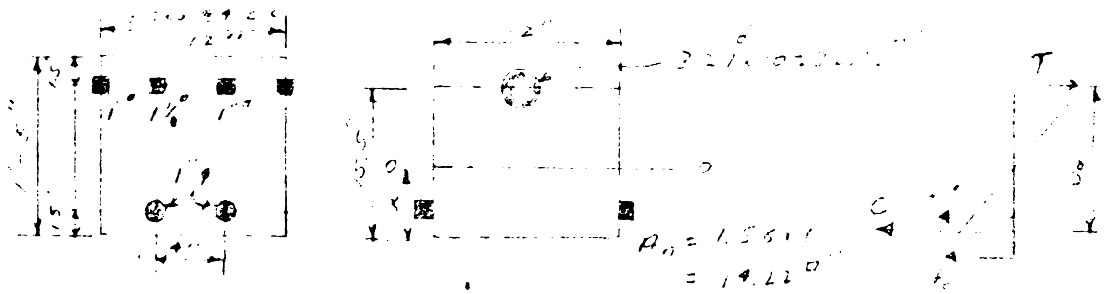
1. At point of Max. Neg. M.

At Section 3-3

L.L. max neg M = $\frac{300.07^{k\text{-}ft}}{10.72'}$ = 28.00^{k} ft/ft

D.L. " " " = $\frac{278.?}{10'}$ = $27.8?^{k}$ ft/ft

Total max neg. M = 55.89^{k} ft/ft



$$\frac{12x^2}{2} + 14.22(x-1.5) = 32.75(15.5-x)$$

$$6x^2 + 14.22x - 21.33 = 507 - 32.75x$$

$$x^2 + 7.82x = 88.15$$

$$(x+3.91)^2 = \sqrt{88.15 + 15.3} = 10.16$$

$$x = 10.16 - 3.91 = 6.25$$

$C_1 = f_c(6.25)\frac{1}{2} \times 12 = 37.5 f_c$; $C_1 y_1 = 37.5 f_c \times \frac{2}{3} \times 6.25$
$= 156.5 f_c$

$C_2 = 14.22 f_c \times \frac{4.75}{6.25} = 10.81 f_c$; $C_2 y_2 = 10.81 \times 4.75 =$
$= 51.5 f_c$

$C = 48.31 f_c$; $C y = 208.0 f_c$

$y = \frac{208.0 f_c}{48.31 f_c} = 4.32$

$jd = 4.32 + (\overbrace{15.5 - 6.25}^{9.25}) = 13.57''$

(Cont)

[illegible]

[illegible] [illegible] [illegible] Center [illegible]

[illegible]

[illegible]

<u>11.5 [illegible]</u>

$C = T =$ [illegible] $= 2[illegible],700^{\#}$

$C =$ [illegible] $\frac{1}{2}$ [illegible] $= 24$ [illegible]

$f_c =$ [illegible] / 24 [illegible] $=$ <u>810 [illegible]</u> [illegible]

b. Section [illegible]

[illegible] $=$ [illegible] $= 2$ [illegible]

[illegible] $= 1.5$ [illegible]

$f_s = \frac{2 \ldots 12}{1.5 \ldots 13.5} =$ <u>1,12[illegible]</u> [illegible]

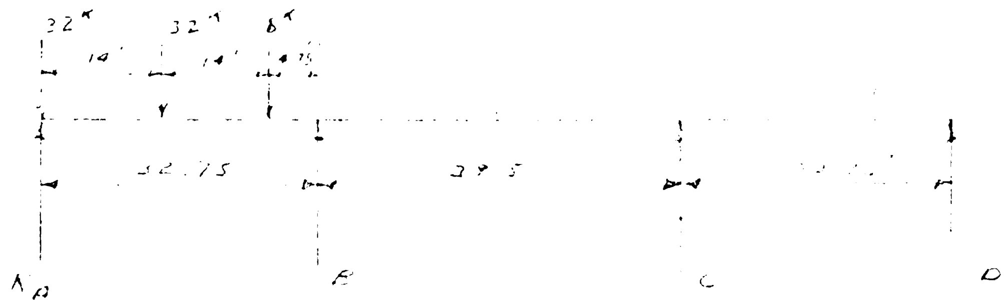
c. Max. Shear

$v = \frac{V}{bjd} = \frac{9,170}{12 \times .875 \times 13.5} =$ <u>[illegible]</u> < 160

Allow [illegible] $\times$ [illegible] $=$ [illegible]
([illegible])

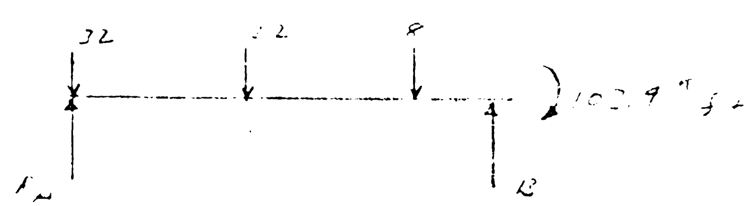
Abutment

a. Reaction of S.S. [illegible]



L.L. M_{FBA} = 32 × 5.6 + 8 × 4 = 211^k ft per 10.00 ft
(USE 10' Lane width = 211^k ft per 10')

	.477	.523	.523	.477
F.E.M.	+211	0	0	0
	−101	−110		
			−55	
			+28.8	+26.2
		+14.4		
	−6.9	−7.5		
			−3.8	
			+2.0	+1.8
		+1.0		
	−.48	−.52		
	+103.6	−103.4	−[illegible]	+26.0



$R_A(L.L.) = \dfrac{8 \times 4.75 + 32 \times 16.75 + 32 \times 32.75 - 103.4}{32.75}$

$R_A(L.L.)$ = 49.3^k [illegible]

R_A(L.L. + Imp.) = 49.3 × 8.[illegible] × 1.15 = [illegible]

Abut.

A. (Cont)

$$R_2(D.L.) = \frac{2.12 \times 32.75^2}{2} - 278.9 = \frac{926}{32.75}$$

$= \underline{28.2^k}$ per [illegible] lane $= \frac{28.2 \times 92}{10} = 250^k$

Wt. of Bridge Seat $= 92 \times 2.04 \times 0.83 \times 150 = 23.5^k$

" " " Curbs $= 2 \times 32.75 \times 0.83 \times 3.5 \times 150$

$= 27.24^k \times 0.398$ (Carry over Factor) $= 10.8^k$

Wt. of Parting Strip $= 4 \times 32.75 \times 0.5 \times 150 \times .368 = \underline{3.6^k}$

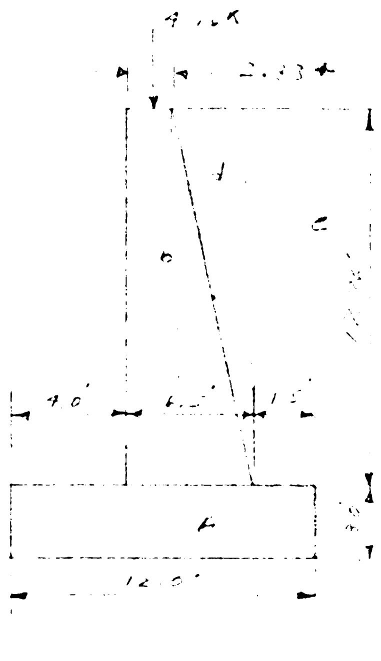
37.9^k

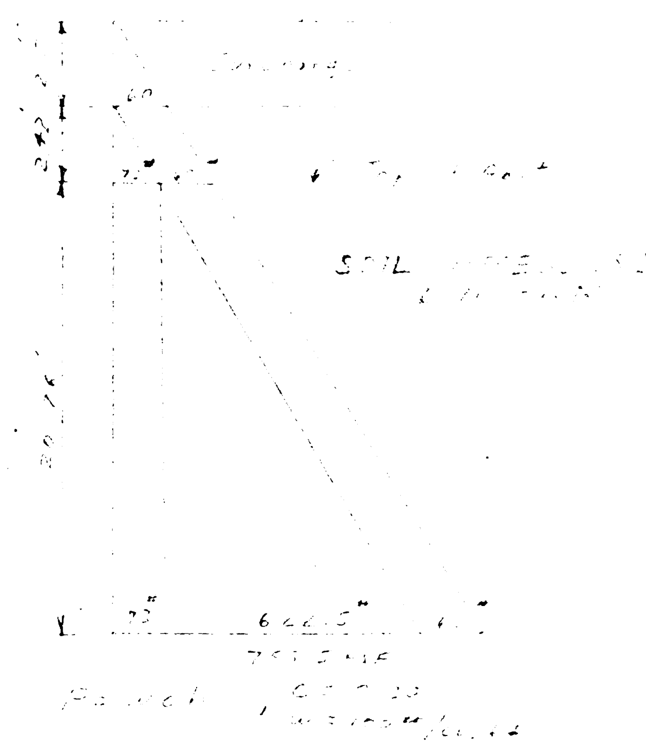
R_2 (D.L. of S.S.) $= 250 + 37.9 = 287.9^k$

R_2 (D.L.) over Length of Abut. $= \frac{287.9}{120} = \underline{2.40^k/ft}$

$R_{2\,total}$ over length of Abut. $= \frac{287.9^k + 290^k}{120} = 4.70^k/ft$

B. Abut. Soil Pressures





Case I — [illegible]

[illegible] 662.5 [illegible] = 44.[illegible]

			$\bar{Y}$	M
[illegible]	[illegible]	[illegible]	6 [illegible]	[illegible]
[illegible]	2,224 [illegible]	[illegible]	[illegible]	14.[illegible]
[illegible]	4 [illegible]	[illegible]	1.2	42.[illegible]
[illegible]	4 [illegible]	[illegible]	[illegible]	33.[illegible]
[illegible]	1.5 [illegible]	[illegible]	[illegible]	3[illegible]

W [illegible] — [illegible] 20 [illegible] M_y

[illegible] / 126.[illegible]

$\bar{x}$ = [illegible]

[illegible]

[illegible] (126.[illegible]) [illegible] (1.[illegible]) = [illegible]

([illegible])

[illegible] (0.1) = [illegible]

Alt.

B cont.

3. Case III S.S. DL + LL. — No Surcharge

$$M_{o_3} = 44{,}600 + \frac{73 \times 20.16^2}{2} = 6[illegible] \text{ \#-ft}$$

$$M_{RT_3} = 171{,}2[illegible]0 + 24{,}600 + 242 \times [illegible] \times 100 \times 9.6$$

$$= 218{,}320 \text{ \#-ft}$$

$M_R = 171{,}120$
$24{,}[illegible]$
$12{,}[illegible]$
$2[illegible],[illegible]2[illegible]$

$R = 23{,}540$
$4{,}760$
$1{,}370$
$29{,}710 \text{ \#}$

$$\bar{x} = \frac{15[illegible],[illegible]20}{29{,}710} = \underline{5.28'} \text{ O.K.}$$

$e = 0.72'$

$$P_{Toe} = \frac{29{,}710}{12}(1.36) = \underline{3{,}[illegible]} \text{ PSF} < 5{,}560 \text{ OK.}$$

$$P_{Heel} = \frac{29{,}710}{12}(0.64) = \underline{1{,}570} \text{ PSF}$$

4. Case IV S.S. DL. only - No overturning Mo

$M_o = 0$

$$M_{RT_4} = 171{,}120 + 2.42 \times 3.67 \times 100 \times 14.16 + 2{,}340 \times 5.17$$

$$= 171{,}120 + 12{,}570 + 12{,}[illegible] = \underline{196{,}200} \text{ \#-ft}$$

$$R_4 = 23{,}540 + 1{,}370 + 2{,}340 = 27{,}240 \text{ \#}$$

$$\bar{x} = \frac{196{,}200}{27{,}340} = \underline{7.18'} < 8.0 \text{ O.K.}$$

$e = 6 - 7.18 = 1.18'$

$$P_{Heel} = \frac{27{,}240}{12}(1.59) = \underline{3{,}620} \text{ PSF}$$

$$P_{Toe} = \frac{27{,}240}{12}(0.41) = \underline{935} \text{ PSF}$$

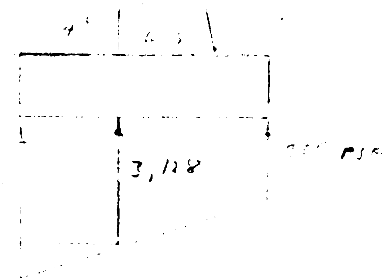
B. cont.

Summation of Cases & values

Max. Toe Pressure = 4,240 psf (Case II)
Max. Heel " = 3,630 " (Case IV)
Max. M_{over} = 74,610 #ft (Case II)

C. Stresses in toe

1. f_s in toe (Case II)



$s = \dfrac{4,240 - 900}{12} = 278$ psf/ft

$4 \times 278 = 1112$

$M_o\,T = 3,128 \times 4 \times 2 + 1112 \times \frac{4}{2} \times 4 \times \frac{2}{3} = 30,980$ #ft

$M_o\,\downarrow = 4 \times 3 \times 150 \times 2 = 3,600$ #ft

M_o net = $\underline{27,380}$ #ft ; $V = 12,500 + 2,224 - 1800$
$V = \underline{12,920}$ #

$f_s = \dfrac{27,380 \times 12}{.6 \times .875 \times 33} = \dfrac{18,800 \text{ psi}}{\text{by } 4.2\%} > 18,000$

2. Shear

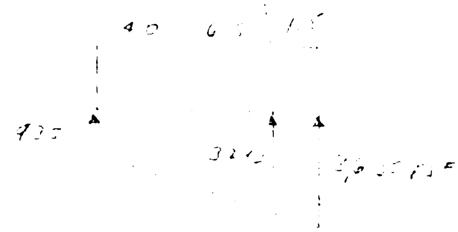
$v = \dfrac{12,920}{12 \times .875 \times 33} = 37.5$ psi < 60 O.K.

3. Bond

$u = \dfrac{12,920}{2.7 \times .875 \times 33} = \dfrac{126 \text{ psi}}{\text{(use S.A.)}} < 150$

D. Stresses in Heel

1. Bottom of Heel – Case II



$S = \frac{[illegible]}{[illegible]} = [illegible]$

$224 [illegible] = 337 [illegible]$

$M_1 = [illegible] \times \frac{1.5^2}{2} + 3[illegible] \times \frac{[illegible]}{2} \times [illegible] = 4{,}074$ #-ft

$M_2 = \frac{1.5^2}{2} \times 2[illegible] \times 100 + 1.5^2 \times 3[illegible] = 3{,}281$ #-ft

$M = 793$ #-ft

$A_{s\,req'd} = \frac{793 \times 12}{18{,}000 \times .875 \times [illegible]} = \underline{0.0152\ \text{in}^2/\text{ft}}$

<u>No Reinf. Req'd</u>

2. Top of Heel – Case II

$M_o = 3{,}281$ #-ft

$M_o = [illegible] \times \frac{1.5^2}{2} + 4[illegible] \times \frac{1.5^2}{2} \times \frac{1}{3} = 1{,}17[illegible]$ #-ft

$M_{o\,net} = 2{,}104$ #-ft

$f_c = \frac{[illegible],104 \times 12 \times 6}{12 \times 33^2} = 11.7$ p.s.i (Tension) < [illegible]

Allow. in tension [illegible]

[illegible] ([illegible])

1. [illegible]

$M_1 =$ [illegible] $\frac{}{2}$ [illegible]

$+$ [illegible] 41, [illegible]

$f_s = \frac{41, \text{[illegible]}}{\text{[illegible]}}$ 15,2[illegible] [illegible]

[illegible] f_s [illegible])

$M =$ [illegible] $\frac{}{2}$ [illegible] $+ 1(\text{[illegible]})$ [illegible] $\frac{\text{[illegible]}}{}-6^2 +$ [illegible]

$= \underline{13,610}$ #-ft

$f_s = \frac{\text{[illegible]}}{\text{[illegible]}} = \underline{\text{[illegible]}}$ [illegible]

[illegible] ([illegible])

$V =$ [illegible] $+ 1\text{[illegible]}$ [illegible] $= 7,$[illegible]

$=$ [illegible] #

$v = \frac{\text{[illegible]}}{12 \times \text{[illegible]}} = \underline{10.7}$ [illegible]

F. Factor [illegible] Sliding

$P_{\text{[illegible]}} =$ [illegible] $= 7,$[illegible] #

$P_{friction} =$ [illegible] $12,$[illegible] #

$F.S. = \frac{12,\text{[illegible]}}{\text{[illegible]}} = \underline{1.27} < 2$ ([illegible] factor)

[illegible]

F. [illegible]

1. Stem

A_s reqd = [illegible] height

= 2.22 [illegible]

A_s furn. = [illegible]

= [illegible]

[illegible] OK

2. Base

A_s reqd = [illegible] per [illegible]

[illegible]

A_s furn. = [illegible]

= 1.85 [illegible]

[illegible] OK

CONCLUSION

The steel and concrete stresses within the superstructure were found to conform closely to the maximum allowable stresses. The only discrepancy noted was the maximum concrete stress at the section of maximum negative moment, which was slightly high. The resulting stress was 1,018 psi as compared to an allowable 1,000 psi. The allowable stress was thus exceeded by 1.8%. Overstresses of 1% are commonly accepted, but 1.8% is high. If one inch of concrete were added to the depth of the slab five feet each side of the pier, this overstress would be eliminated.

The superstructure reinforcing steel at the supports is provided with a clear covering of 0.94" of concrete. This wearing surface would seem thin for a structure which is expected to carry heavy truck traffic. No requirements, however, as to the depth of wearing surface were given by the AASHO.

The stresses within the abutments, the maximum soil pressures and the position of the resultant soil pressure were found to be within the allowable limits specified. The factor of safety against sliding for the abutment was found to be 1.34, which is less than the usually accepted desireable value of 2.0. No minimum allowable factor of safety against sliding was given by the AASHO.

[redacted]

[redacted]

Pocket has:
1 Suppl.