ESSAYS ON TRADE, IMMIGRATION, AND LABOR MARKET SHOCKS

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ABSTRACT

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This dissertation comprises three essays on trade, immigration, and labor market shocks. Its ultimate focus is to understand how domestic workers respond to and/or are affected by trade liberalization, immigration, and shocks to foreign labor markets. Depending on the purpose of each essay, workers are characterized as being identical, or as having different ages and abilities, or as possessing different skills. Each essay builds a theoretical model and undertakes its analysis by elucidating how asset values, wages, and/or unemployment change in general equilibrium.

The first essay "Trade, Welfare, and International Labor Market Spillovers" examines welfare implications of trade liberalization and spillover effects of labor market shocks in a global economy. It proposes a two-country new trade theory framework with two types of labor (skilled and unskilled) combind with an imperfect labor market arising from country-specific real minimum wages. The model identifies two key forces that shape the results: i) external scale effects generating employment expansion among unskilled workers, and ii) wage effects arising from intensive use of skilled workers. The model provides three clear predictions. First, the introduction of trade results in a rise of wage inequality in both countries. Second, trade enhances welfare by lowering unemployment across countries only when external scale effects dominate. Third, when wage effects outweigh external scale effects, a country will be better off if the other country raises its minimum wage, but both countries will be worse off under unduly strong external scale effects in the global economy.

The second essay "Immigration, Firm Heterogeneity, and Welfare" studies the impact of immigration on firm adjustment and on native workers' welfare. Heterogeneous immigrants are added to the standard monopolistic competition model with heterogeneous firms. Based on interaction

between idiosyncratic firm productivity and immigrants who differ in terms of ability (or skill) and legality, the model shows: (i) under some reasonable conditions, the highest-productivity firms prefer highly-skilled immigrants relative to native workers despite high fixed cost, whereas the lowest-productivity firms are willing to employ illegal immigrants despite risk of being fined; (ii) illegal immigration lowers the productivity cutoff and, at the same time, highly-skilled immigration raises it via the exit/entry process of firms; (iii) immigration alters the equilibrium distribution of firms and average productivity, thereby influencing revenues and profits of average productivity firms; and (iv) average productivity affected by both types of immigration determines native workers' welfare in the long-run.

Finally, the third essay "Worker Heterogeneity and Adjustment to Trade Liberalization" investigates trade liberalization's impact on heterogeneous workers. This essay focuses on the role of age together with ability of workers in a search and matching model of the labor market. In essence, the model shows that age and ability are substitutes. As a result, the asset value of an unemployed worker decreases as he/she ages, but increases in ability. When an economy enjoys a comparative advantage in the high-tech sector and protects the low-tech sector by tariff, trade liberalization reduces an unemployed worker's asset value in the low-tech sector and raises her/his asset value in the high-tech sector. The model explains that trade liberalization leads younger and/or abler workers who otherwise are stuck in the low-tech sector to switch sectors, although not all movers become winners, while simultaneously it forces older and/or less able workers to exit the labor force.

To my family

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## 1.1 Introduction

Much attention has been paid to the impact of trade liberalization on wages and unemployment, yet relatively little is known about cross-country adjustments caused by shocks to labor markets linked by trade for goods. This paper examines how trade interacts with labor market institutions to produce effects on distribution, employment and aggregate welfare.[1] The main question is: How does variation of labor market institution, *e.g.* minimum wage, in one country affect its trading partners? Consider a world comprising Europe and the U.S. Well known is that Europe has a more rigid labor market than has the U.S. Davis (1998) and Meckl (2006) introduce minimum wages into the Heckscher-Ohlin model and shed light on insights that European minimum wages prop up U.S. wages. In other words, tightening labor market constraints in one country results in transmission of a positive labor market effect to the other country.[2] This result has been the conventional view among scholars and politicians.

Recently, the same question has received attention in the monopolistic competition model with heterogeneous firms and country-specific minimum wages. Egger, Egger, and Markusen (2012) overturn the results by the aforementioned models and conclude that European minimum wages

[1]See also Krugman (1995) for detailed discussion of labor market institutions and trade.

[2]In the earlier work, many models are built on the classic Heckscher-Ohlin model. For instance, Davidson *et al*. (1988, 1999) introduce search frictions into that model, which implies that if labor market frictions tighten in one country, then the unemployment rate rises. However, the trading partner benefits by a lower unemployment rate due to the terms-of-trade appreciation. In the spirit of Brecher (1974), Davis (1998) and Meckl (2006) develop a trade model with a flexible wage U.S. and a rigid wage Europe; they conclude that if Europe raises the minimum wage, it benefits the U.S. by increasing the U.S. wage. The key insight is that Europe determines U.S. wages *via* factor price equalization.

prop up U.S. unemployment. That is, stronger labor market institution in a country results in a negative labor market spillover effect on its trading partners. However, the model by Egger *et al*. (2012) hinges on the assumption of a single factor in which the real wage is subject to an exogenously specified minimum-wage. Hence, the assumption fails to address an adjustment mechanism *via* factor prices on external shocks. One may conjecture that when both worker types are indispensable in production, but skilled workers are used intensively, variations in the minimum wage would impact firms' variable costs relatively lightly, while variations in the stock of skilled workers would impact heavily.

To explore this issue, this paper develops a two-country new trade theory framework with heterogeneous workers and labor market frictions arising from country-specific real minimum wages.[3] The model's novelty is the addition of two types of labor, skilled and unskilled, into the work by Egger *et al*. (2012) who emphasize external scale effects generating employment expansion. In this extension, this paper finds that their main result is not necessarily robust.[4] The particular model considered here is where, in each country, the final output producers assemble various intermediate inputs according to the generalized Constant Elasticity of Substitution (CES) production technology which captures external scale effects.[5] Each intermediate input firm with some market power has the ability to produce a unique variety valued by final output producers. Production by intermediate input firms involves both fixed and variable costs. Fixed costs are incurred as investment in units of the final good; variable costs use skilled and unskilled workers. Unskilled workers face the threat of unemployment due to a country-specific minimum wage, whereas skilled workers with flexible real wages are fully employed. Wage effects derive from intensive use of skilled workers. It then is possible to solve explicitly for equilibrium assuming a Pareto distribution from which a firm draws random productivity.

[3]Melitz (2003) relies on a single factor with a flexible real wage among symmetric countries. Accordingly, the model developed herein may be viewed as a variant of Melitz (2003), incorporating flexible love-of-variety and multiple factors with an imperfect labor market.

[4]External scale effects (following Ethier 1982) are known also as love-of-variety, variety gains, increasing returns to variety, and returns to specialization.

[5]As Ethier (1982) stresses, trade in intermediate inputs is important (see Yi 2003; Amiti and Konings 2007; and more recently, Amiti and Davis 2011). It occurs mostly among developed countries and represents respectively 56% and 73% of overall trade flows in goods and services (see Miroudot *et al*. 2009; Sturgeon and Memedovic 2010).

By identifying two counteracting forces, namely wage effects and external scale effects, this paper shows that shocks to a country's labor markets exert either positive or negative spillover effects on its trading partners. When wage effects dominate external scale effects, the U.S. will be better off if Europe raises its minimum wage. The intuition is as follows: Suppose Europe raises its real minimum wage. An increase in the minimum wage directly increases all intermediate input firms' variable costs. Due to worsened profitability, marginal firms begin to exit the market. Then, a reduced number of active firms lead to a fall in skilled workers' wage, thus, European unemployment increases and welfare falls. Its trading partner the U.S. is impacted indirectly by a rise in European minimum wage. Two effects compete in this scenario. On one hand, potential entrants would benefit from positive wage effects or a lowered skilled workers' wage. Lower variable cost enables those firms to enter the market. On the other hand, those firms also encounter negative external scale effects or decreased demand for inputs by final output producers. If wage effects dominate, a rise in the minimum wage leads potential firms in the U.S. to enter the market. Entry of these firms expands employment (thus lowers unemployment for unskilled workers) and results in enhancing the U.S. welfare. When external scale effects dominate wage effects, the model has qualitatively the same prediction as in Egger *et al*. (2012).

In addition to focusing on how domestic labor market outcomes are affected by foreign labor market institutions, this paper also studies welfare implication of trade liberalization. It shows that the introduction of trade increases wage inequality between different skill-type workers in both countries. Intuitively, without trade impediments, opening to trade expands market size for all intermediate input firms, compared to autarky. Those firms' increased aggregate demand for skilled labor results in a rise of skilled workers' wages since the stock of skilled workers is exogenous. In the model, an increased wage for skilled workers implies a rise of wage inequality since the real minimum wage for the unskilled is binding. The result of widened wage inequality is consistent with the literature on trade liberalization and wage inequality (see, *e.g.*, Yeaple 2005; Helpman *et al*. 2010; Harrigan and Reshef 2015; Burstein and Vogel 2010).

The other main result is that trade yields two opposite forces: positive external scale effects generating employment expansion, but negative wage effects arising from intensive use of skilled workers. Gains from trade materialize only when the former effects dominate. From the perspective of intermediate input firms, market demand and variable production costs are critical to the determination of their profitability. With strong external scale effects, liberalized trade leads to entry of potential firms. An increased aggregate demand for unskilled workers reduces the unemployment rate. Accordingly, total output and total labor income increase, thereby improving welfare across countries. Although average productivity falls in the product market with heterogeneous firms, gains from trade rise *via* positive employment expansion. Adjustment at the extensive margin of firms is crucial in the present paper, since selection effects are neutralized by the assumption of no trade cost. Interestingly, this paper shows a possibility that trade worsens both economies if negative wage effects outweigh positive employment effects. With the wage inequality result, we can say that by the opening to trade, skilled workers always become winners whereas unskilled workers may be losers due to unemployment or the threat thereof.

A long line of literature exists on trade and unemployment. Traditional international trade theory uses full-employment conditions in its simple and elegant Heckscher-Ohlin (HO) model. Although contributing to our understanding of trade patterns together with two important theorems (Stolper-Samuelson and Rybczynski), the model cannot be relied upon once attention is directed to unemployment issues. Many scholars extended the HO model by employing minimum wages (Brecher 1974; Davis 1998; Oslington 2002; Meckl 2006), implicit contracts (Matusz 1985, 1986), search frictions (Davidson *et al.* 1988, 1999), and fair wages (Kreickemeier and Nelson 2006). However, many economists have made the point of claiming that the HO model provides no explanation towards intra-industry trade as under the assumptions countries with identical factor endowments would not trade and produce goods domestically. Krugman's new trade theory (1980) -and its generalized version by Melitz (2003)- successfully explain intra-industry trade patterns by emphasizing love-of-variety. In their models' structure, their choice of consumer preferences is the standard Dixit-Stiglitz (1977) specification. As in the HO model, these papers assume perfectly competitive labor markets. Using various sources, many economists began to develop intra-industry trade models with equilibrium unemployment. Matusz (1996) merges efficiency wage theory into Krugman's model (1980) with homogeneous firms and homogeneous workers. Davidson *et al*. (2008) introduce search frictions as in Albrecht and Vroman (2002) with (*ex ante*) homogeneous firms and heterogeneous workers. Based on Melitz (2003), Helpman and Itskhoki (2010), Helpman *et al*. (2010), and Felbermayr *et al.* (2011) employ search frictions. Davis and Harrigan (2011) and Amiti and Davis (2011) embed efficiency wages. Egger and Kreickemeier (2009) incorporate fair wages. Egger *et al*. (2012) use minimum wages. The above models, using Melitz's framework, assume either symmetric countries or homogeneous workers or both. This paper employs heterogeneous workers and heterogeneous firms with flexible external scale economies to consider labor market linkages between asymmetric countries.

This paper contributes to the literature on labor market interdependence between asymmetric countries. In the new trade theory, Matusz (1996) has shed light on the possibility of positive correlation of labor market outcomes. He constructs an intra-industry trade framework in the Ethier (1982) type model with homogeneous firms and an efficiency wage. His conclusion implies that relaxing constraints on the efficiency wage permits employment expansion in one country, boosting the world economy through trade, thereby expanding employment in the other. Recently, Egger *et al*. (2012) extends Matusz's (1996) arguments in a variant of the Melitz (2003) model with country-specific minimum wages; they conclude that a fall in a country's minimum wage has positive spillover effects for the trading partner *via* external scale effects. The model developed herein identifies wage effects and scale effects. In theory, one country's labor market shocks can have both positive and negative spillover effects on the other country, and their possible coexistence is demonstrated in this paper's framework.

The present paper relates to earlier models examining the link between welfare and trade liberalization. A theoretical possibility of negative welfare effects due to trade liberalization has been raised by Montagna (2001) who develops a monopolistic competition model with technical heterogeneity among firms and countries. She shows that trade reduces the minimum efficiency to survive in the more efficient country and argues that adverse welfare effects may prevail in an advanced technology economy if love-of-variety is sufficiently low. The present paper raises such a possibility of losses of trade if external scale effects are sufficiently low relative to wage effects.

This paper is complementary to Egger and Kreickemeier (2009) who examine the effect of trade liberalization on a labor market in the presence of positive variable trade costs and external scale effects. They conclude that a negative employment effect is triggered if variable trade costs are not too low and the external scale effects are moderate, whereas a positive employment effect can be expected if variable trade costs are negligible and the external scale effect is strong. This also is well in line with Matusz's (1996) conclusions assuming full external scale effects and no trade cost.

It also closely relates to Egger *et al*. (2012) predicting that without trade impediments, trade liberalization always leads both economies to higher levels of welfare, reducing the unemployment rate due to positive employment effects *via* external scale economies. In addition, a country's lowered minimum wage always yields positive spillover effects to its trading partner *via* the channel of external scale effects. While insightful, their model is an incomplete analysis. First, they rely on a single factor whose real wage is subject to an institutionally-set minimum, which shuts down the channel of wage effects. Second, it is impossible to consider effects of variations in the stock of skilled workers. Thus, introducing a second factor with flexible real wages complements many parts of the results of Egger *et al*. (2012). A key finding of the present paper is that the second factor revives the force of wage effects which acts to offset the force of external scale effects on external shocks. Eventually, *only* when external scale effects dominate, do gains from trade materialize and stronger foreign labor market institutions harm domestic workers. In addition, variations in foreign labor stock with flexible wages also would impact domestic workers.

In the following section, I develop a baseline model in autarky. Section 3 characterizes autarky equilibrium in both the product market and the labor market. Section 4 discusses trade equilibrium with asymmetric labor market institutions. Section 5 examines the impact on its trading partners of shocks to a country's minimum wage or factor supplies. Section 6 concludes the paper.

## 1.2 The Model: Autarky

Consider a world economy comprising two asymmetric countries indexed by $i$ and $j$. In each, two types of goods are produced: homogeneous final output and differentiated intermediate inputs. Assume that both countries share final and input production technology, but differ from each other in factor endowments (skilled and unskilled workers), size of real minimum wages, and mass of potential entrants. All notation is written in terms of country $i$. For country $j$, the subscript changes from $i$ to $j$.

### 1.2.1 Production technology and firm behavior

In the spirit of Dixit-Stiglitz (1975) and Ethier (1982), the final output used for consumption as well as investment is produced by assembling (without the use of labor) various intermediate inputs according to the generalized CES production technology[6]

$$Y_i = M_i^{-\frac{\eta}{\sigma-1}} \left( \int_{v \in V_i} [z_i(v)]^{\frac{\sigma-1}{\sigma}} dv \right)^{\frac{\sigma}{\sigma-1}} \tag{1.1}$$

where $V_i$ denotes the set of all available input varieties with measure $M_i$, $z_i(v)$ is the quantity of input variety $v$ employed in the production of $Y_i$.

Worth noting are a number of features of final goods technology in (1.1). First, $\sigma > 1$ is the constant elasticity of substitution between varieties. Second, $\rho = \frac{\sigma-1}{\sigma} < 1$ captures the degree of complementarity between input varieties. Third, two independent parameters, $\eta \in [0,1]$ together with $\sigma$, measure the degree of external scale effects. To understand the third feature, consider the case where $z_i(v) = z_i$, $\forall\, v \in V_i$. Final output becomes $Y_i = M_i^{\frac{\sigma-\eta}{\sigma-1}} z_i$. Define $y_i = \frac{Y_i}{M_i z_i}$, which measures the final output gain derived from spreading a certain amount of production among $M_i$ differentiated inputs instead of concentrating it on a single input (see Benassy 1996). Thus, the

[6] In the unpublished version by Dixit-Stiglitz (1975), they include the product diversity gain (loss) with measure $M$, interpreting as public good (bad). Later, Benassy (1996) and Montagna (2001) rediscover it in a consumption context. See also Neary (2004), Eckel (2008), Acemoglu, Antras, and Helpman (2007), Egger and Kreickemeier (2009), and recently, Egger *et al*. (2012).

final output gain is $y_i = M_i^{\frac{1-\eta}{\sigma-1}}$ with an elasticity $\frac{1-\eta}{\sigma-1} \equiv \chi(\eta,\sigma)$. A marginal final output gain for additional input variety is called the ***external scale effect***.[7] In the borderline case of $\eta = 0$, external scale effects are full, *i.e.*, $\chi(0,\sigma) = \frac{1}{\sigma-1}$ and the expression in (1.1) is equivalent to the standard Dixit-Stiglitz (1977) specification. As $\eta$ rises, external scale effects become weaker, which is interpreted as an intermediate input *per se* becoming less important.[8] If we set $\eta = 1$, final output is produced under constant returns to scale in the measure of inputs, thereby showing no external scale effects, *i.e.*, $\chi(1,\sigma) = 0$.

Let $Y_i$ be the numeraire. Taking input price $p_i(v)$ as given, final goods producers choose input quantity $z_i(v)$ in order to maximize their profits: $max_{z_i(v)}\, P_iY_i - \int_{v\in V_i} p_i(v)z_i(v)dv$. We observe that $P_iY_i = \int_{v\in V_i} p_i(v)z_i(v)dv$ in equilibrium since, in a perfectly competitive final output market, free entry drives profits of final goods producers to zero.[9] Using $P_i = 1$ due to the choice of numeraire, the optimal input demand for variety $v$ is

$$z_i(v) = \frac{Y_i}{M_i^{\eta}} p_i(v)^{-\sigma}. \tag{1.2}$$

Each intermediate input firm with some market power has the ability to produce a unique variety valued by final output producers. Production by intermediate input firms involves fixed and variable cost. To operate, all intermediate input firms spend the same fixed investment cost $f$ normalized to one. Variable costs use skilled and unskilled workers with skill intensity $\beta \in [0,1]$, but vary with a firm's random productivity $\phi$. The variable cost function for a firm with productivity $\phi$ to produce $z_i$ amount of variety $v$ assumes the Cobb-Douglas form:

$$c_i(\phi) = \frac{z_i}{\phi} s_i^{\beta} w_i^{1-\beta} \tag{1.3}$$

[7]If we treat the final output technology as a utility function, $\chi$ is similarly interpreted as a marginal utility gain from additional consumption variety, so called love-of-variety.

[8]Using data, Haveman and Hummels (2004) state that complete specialization model considerably overstates either the extent of specialization (the degree to which goods are differentiated) or the degree to which consumers value that differentiation. In the same vein, Ardelean (2009) reports that love-of-variety is forty two percent lower than is assumed in Krugman's model, which implies existing studies may overstate the variety gains from trade liberalization.

[9]Thus, the index of intermediate goods prices is given as $P_i = M_i^{\frac{\eta}{\sigma-1}} \left(\int_{v\in V_i} p_i(v)^{1-\sigma} dv\right)^{\frac{1}{1-\sigma}}$.

where $s_i$ is the wage rate for skilled workers and $w_i$ is the minimum wage for unskilled workers in country $i$. The cost function in (1.3) has the convenience of generating unit labor demand for skilled and unskilled workers, respectively, as in Harrigan and Reshef (2015).[10]

Original work by Egger *et al*. (2012) considers only a minimum-waged worker, *i.e.*, $\beta = 0$. The homogeneous worker assumption, in particular one such as binding to a minimum wage, fails to address an adjustment mechanism *via* factor prices. It naturally is conjectured that depending on skill intensity in production, heterogeneous firms may be influenced differently by exogenous shocks, such as variations in minimum wage or in factor supply. With a high skill intensity (high $\beta$), variations in the minimum wage would have relatively lighter impact on firms' variable costs, while variations in the stock of skilled workers would impact heavily.

Taking the isoelastic demand by final goods producers in (1.2) and aggregate variables, an intermediate input firm maximizes its profit by setting its optimal price

$$p_i(\phi) = \frac{s_i^{\beta} w_i^{1-\beta}}{\rho \phi}. \tag{1.4}$$

The revenue and profit generated by an input firm with productivity $\phi$ are automatically calculated. Firm profit is then $\pi_i(\phi) = \frac{r_i(\phi)}{\sigma} - 1$ where $r_i(\phi)$ is firm revenue and $\frac{r_i(\phi)}{\sigma}$ is variable profit.

### 1.2.2 Input firm entry and aggregate variables

A firm draws a random productivity $\phi$ from Pareto distribution function with shape parameter $\kappa$.[11] Productivity is distributed over $[1, \infty)$ according to

$$G(\phi) = 1 - \phi^{-\kappa}$$

[10] See Romalis (2004) who uses Krugman's model in explaining how factor proportions determine the structure of commodity trade. See also Bernard *et al*. (2007).

[11] See Helpman, Melitz, and Yeaple (2004) and Chaney (2008).

where its corresponding density function is $g(\phi) = \kappa\phi^{-\kappa-1}$. The shape parameter $\kappa$ measures the concentration of the firm's productivity distribution. With a high $\kappa$, the input sector becomes more homogeneous in the sense that more input firms are located among the least productive firms. The Pareto distribution not only generates a good approximation of the distribution of firm size (see, *e.g.*, Axtell 2001), but also provides analytical tractability, enabling a comparison of results in the closed economy with those from the open economy.

Denote $M_i$ as the mass of active firms and $\phi_i^*$ as a cutoff productivity. Assume that the total mass of potential entrants in the differentiated input sector is given by exogenous $\bar{N}_i$.[12] Thus, only firms drawing productivity above $\phi_i^*$ engage in production. The cutoff productivity condition (CPC) yields

$$M_i = \bar{N}_i[1 - G(\phi_i^*)] = \bar{N}_i[\phi_i^*]^{-\kappa}. \tag{1.5}$$

By setting $\pi_i(\phi_i^*) = 0$ ($\Leftrightarrow r_i(\phi_i^*) = \sigma$), the zero-profit cutoff productivity (ZCP) indicates

$$\frac{Y_i}{M_i^\eta}\left(\frac{s_i^\beta w_i^{1-\beta}}{\rho\phi_i^*}\right)^{1-\sigma} = \sigma. \tag{1.6}$$

Next, define the (weighted) average productivity as $\tilde{\phi}_i = \left(\int_{\phi_i^*}^\infty \phi^{\sigma-1}\mu_i(\phi)d\phi\right)^{\frac{1}{\sigma-1}}$ where $\mu_i(\phi) = \frac{g(\phi)}{1-G(\phi_i^*)}$ is the productivity distribution of firms in equilibrium. Using the Pareto distribution, the average productivity condition (APC) is[13]

$$\frac{\tilde{\phi}_i}{\phi_i^*} = \left[\frac{\kappa}{\kappa-\sigma+1}\right]^{\frac{1}{\sigma-1}} \tag{1.7}$$

where $\kappa > \sigma - 1$ is assumed.

[12]The present paper can be seen as a steady-state model or a static variant model of the dynamic version in Melitz (2003). In Melitz (2003), there is an unbounded mass of potential entrants and free entry, resulting in expected zero profit in equilibrium. In the present paper, however, the assumption of exogenous $\bar{N}_i$ is adopted to simplify the analysis throughout without losing the main insights from Melitz (2003). See also Do and Levchenko (2009) and Chaney (2008).

[13]Revenues of an average firm are proportional to revenues of a marginal firm, $\frac{r_i(\tilde{\phi}_i)}{r_i(\phi_i^*)} = \frac{\kappa}{\kappa-\sigma+1}$. This fact will be used later in the welfare analysis.

Using the index of intermediate goods prices, the profit maximization condition (PMC) is

$$M_i^{\chi} = p_i(\tilde{\phi}_i). \tag{1.8}$$

Other aggregate variables are obtained easily. For example, $R_i = M_i r_i(\tilde{\phi}_i)$ and $\Pi_i = M_i \pi_i(\tilde{\phi}_i)$ where $R_i = \int_{\phi_i^*}^{\infty} r_i(\phi) M_i \mu_i(\phi) d\phi$ and $\Pi_i = \int_{\phi_i^*}^{\infty} \pi_i(\phi) M_i \mu_i(\phi) d\phi$ represent aggregate revenue and profit, respectively.

### 1.2.3 Labor market

Country $i$ is endowed with $\bar{H}_i$ of skilled workers and $\bar{L}_i$ of unskilled workers. Labor markets for both types of worker are perfectly competitive. In the spirit of Brecher (1974), we consider here a very stark labor-market institution. Assume that the government sets a minimum wage $w_i$ above the market clearing level for unskilled workers. Let $U_i$ denote unemployment rate for unskilled workers.

Let us find the labor demand for each type of worker to produce one unit of output within a firm. Using (1.3), the marginal cost of a firm with productivity $\phi$ is then

$$mc_i(\phi) = \frac{s_i^{\beta} w_i^{1-\beta}}{\phi}. \tag{1.9}$$

Denote $h_i(\phi)$ and $l_i(\phi)$ as the unit labor demand for skilled and unskilled workers, respectively. Similarly to Harrigan and Reshef (2015), applying Shephard's lemma yields

$$h_i(\phi) = \frac{\beta}{\phi} \left( \frac{s_i}{w_i} \right)^{\beta - 1}, \tag{1.10}$$

$$l_i(\phi) = \frac{1-\beta}{\phi} \left( \frac{s_i}{w_i} \right)^{\beta}. \tag{1.11}$$

Within a firm with productivity $\phi$, the unit labor cost $s_i h_i(\phi) = \frac{\beta}{\phi} s_i^{\beta} w_i^{1-\beta}$ is incurred when hiring skilled workers and $w_i l_i(\phi) = \frac{(1-\beta)}{\phi} s_i^{\beta} w_i^{1-\beta}$ when employing unskilled workers.[14] The total unit labor cost a firm with productivity $\phi$ spends is just $mc_i(\phi)$ in (1.9). For a firm to produce $z_i(\phi)$ amount of goods, it spends $mc_i(\phi) z_i(\phi)$ which equals to $\rho r_i(\phi)$. Hence,

$$c_i(\phi) = \rho r_i(\phi). \tag{1.12}$$

Denote $W_i$ as the total labor cost expenditures: $W_i = \int_{\phi_i^*}^{\infty} c_i(\phi) M_i \mu_i(\phi) d\phi$ from the sum of wage payments by all intermediate input firms. Integrating both sides in (1.12) over $\phi \in [\phi_i^*, \infty)$,

$$\int_{\phi_i^*}^{\infty} c_i(\phi) M_i \mu_i(\phi) d\phi = \rho \int_{\phi_i^*}^{\infty} r_i(\phi) M_i \mu_i(\phi) d\phi. \tag{1.13}$$

The relation in (1.13) implies that $W_i = \rho M_i r_i(\tilde{\phi}_i) = \rho R_i$. Due to the choice of final output as numeraire,

$$W_i = \rho Y_i. \tag{1.14}$$

In equilibrium, $(1-U_i)\bar{L}_i$ unskilled workers will be employed, whereas $\bar{H}_i$ skilled workers will be fully employed since the skilled wage is assumed to be flexible. The aggregate labor demand for skilled workers is obtained by integrating over the total labor demand of firms with productivity $\phi \in [\phi_i^*, \infty)$: $\int_{\phi_i^*}^{\infty} h_i(\phi) z_i(\phi) M_i \mu_i(\phi) d\phi$. Similarly, the aggregate labor demand for unskilled workers is $\int_{\phi_i^*}^{\infty} l_i(\phi) z_i(\phi) M_i \mu_i(\phi) d\phi$.

Using input demand in (1.2), demand for skilled workers in (1.10), and demand for unskilled workers in (1.11), the two labor market equilibrium conditions then are

$$\bar{H}_i = \int_{\phi_i^*}^{\infty} \left( \frac{\beta}{\phi} \left( \frac{s_i}{w_i} \right)^{\beta-1} \right) \frac{Y_i}{M_i^{\eta}} p_i(\phi)^{-\sigma} M_i \mu_i(\phi) d\phi, \tag{1.15}$$

[14] The demand ratio of skilled to unskilled workers to produce a unit of input variety is inversely related to the relative wage of skilled workers, *i.e.*, $\frac{h_i(\phi)}{l_i(\phi)} = \frac{\beta}{1-\beta} \frac{w_i}{s_i}$.

$$(1-U_i)\bar{L}_i = \int_{\phi_i^*}^{\infty} \left(\frac{1-\beta}{\phi}\left(\frac{s_i}{w_i}\right)^{\beta}\right)\frac{Y_i}{M_i^{\eta}} p_i(\phi)^{-\sigma} M_i \mu_i(\phi) d\phi. \tag{1.16}$$

The sum of wage incomes for unskilled and skilled workers is equal to total labor cost expenditures, *i.e.*, $W_i = (1-U_i)w_i\bar{L}_i + s_i\bar{H}_i$.

Manipulating (1.14)-(1.16), the unemployment rate is

$$U_i = 1 - \frac{(1-\beta)W_i}{w_i\bar{L}_i} \tag{1.17}$$

and one simple relation is obtained

$$\beta = \frac{s_i\bar{H}_i}{\rho Y_i}. \tag{1.18}$$

Equation (1.18) states that a fraction $\beta$ of the total labor cost payments $\rho Y_i$ is given to skilled workers, which must be equal to the total labor income for skilled workers, $s_i\bar{H}_i$. As $\beta$ rises, wage payments to skilled workers out of total labor costs rise accordingly. In the present paper, ***wage effects*** arise from intensive use of skilled workers. The magnitude of the effects will be determined by the parameter $\beta$. Together with external scale effects $\chi(\eta,\sigma)$, $\beta$ will play a significant role throughout the paper.

## 1.3 Analysis of Autarky Equilibrium

Equilibrium variables in autarky are characterized completely by deriving the solution for cutoff productivity and skilled workers' wage $(\phi_i^*, s_i)$. Given the solution, all the other aggregate variables $(\tilde{\phi}_i, M_i, Y_i, W_i)$ in the product market and $U_i$ in the labor market are determined endogenously.

Substituting CPC (1.5) and APC (1.7) into PMC (1.8), cutoff productivity is increasing in skilled workers' wage with an assumption of $\theta \equiv 1 - \kappa\chi > 0$[15]

$$\phi_i^* = \lambda \left[ \frac{w_i^{1-\beta}}{\bar{N}_i^{\chi}} \right]^{\frac{1}{\theta}} s_i^{\frac{\beta}{\theta}}, \tag{1.19}$$

where $\lambda = \left[ \frac{1}{\rho} \left( \frac{\kappa - \sigma + 1}{\kappa} \right)^{\frac{1}{\sigma - 1}} \right]^{\frac{1}{\theta}} > 0$.

Plugging CPC (1.5) and $Y_i$ of (1.18) into ZCP (1.6), cutoff productivity is decreasing in skilled workers' wage with an assumption of $\beta \leq \frac{1}{\sigma-1}$[16]

$$\phi_i^* = \mu \left[ \frac{w_i^{(1-\beta)(\sigma-1)} \bar{N}_i^{\eta}}{\bar{H}_i} \right]^{\frac{1}{\vartheta}} s_i^{\frac{\sigma-1}{\vartheta}\left(\beta - \frac{1}{\sigma-1}\right)} \tag{1.20}$$

with $\mu = \left[ \frac{\beta(\sigma-1)}{\rho^{\sigma-1}} \right]^{\frac{1}{\vartheta}} \geq 0$ and $\vartheta = \sigma - 1 + \kappa\eta > 0$.

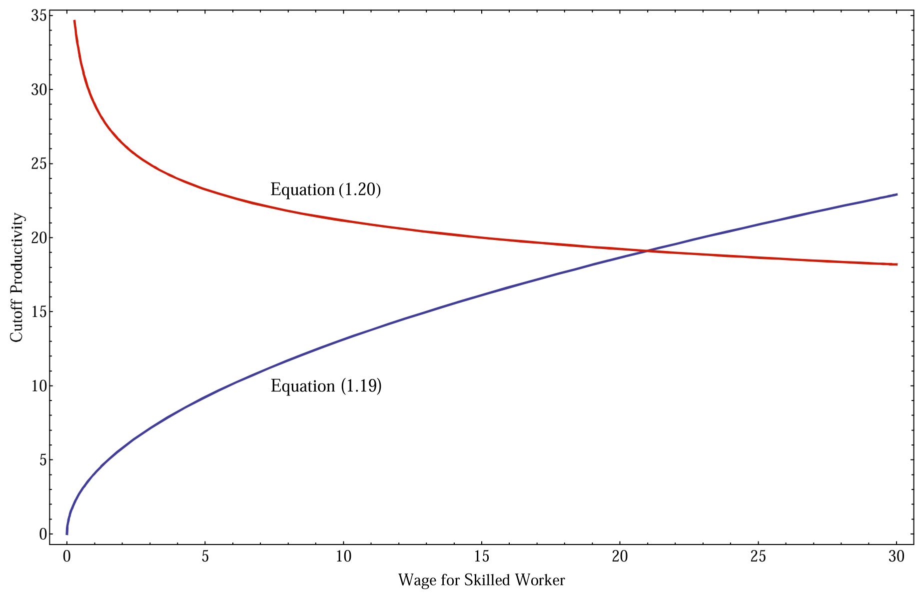
**Proposition 1**. *A unique autarky equilibrium exists in which cutoff productivity and skilled workers' wage are determined.*

$$\phi_i^* = \psi \left[ \bar{N}_i^{\beta - \chi(\eta,\sigma)} \left( \frac{w_i^{1-\beta}}{\bar{H}_i^{\beta}} \right) \right]^{\frac{1}{\theta + \beta\kappa}} \tag{1.21}$$

[15] The condition $1 > \kappa\chi$ is $0 \leq \frac{1-\eta}{\sigma-1} < \frac{1}{\kappa}$. That is, the elasticity of final output gain with respect to the measure of variety $\chi(\eta,\sigma) = \frac{1-\eta}{\sigma-1}$ has a upper bound $\frac{1}{\kappa}$. Technically, it is known that if $\phi$ is Pareto distributed, then $ln\phi$ is exponentially distributed with standard deviation of $\frac{1}{\kappa}$. Recall that $\kappa > \sigma - 1$ due to the assumption of Pareto distribution. Thus, $0 \leq \chi(\eta,\sigma) < \frac{1}{\kappa} < \frac{1}{\sigma-1}$ which completes the restrictions of the elasticity.

[16] Factor intensity $\beta$ should be in $[0,1]$ and elasticity of substitution $\sigma$ is larger than one. In a general form, $\beta \leq min\{\frac{1}{\sigma-1}, 1\}$ should be satisfied. When $\sigma > 2$, factor intensity $\beta$ should decrease to keep $0 \leq \beta \leq \frac{1}{\sigma-1}$. Depending on $\sigma$, factor intensity has a upper bound $\frac{1}{\sigma-1}$. If $\beta(\sigma-1) > 1$, we are no longer able to consider the case of $\beta = 0$ because this would imply that $0 > 1$ (contradiction). Lastly, at $\beta(\sigma-1) = 1$, the curve in (1.20) becomes horizontal.

Figure 1.1 Cutoff Productivity and Wage for Skilled Worker in Autarky Equilibrium




Note: Following Bernard et al. (2007) and Harrigan and Reshef (2015), it is assumed that $\kappa = 3.4$ for Pareto shape parameter, $\sigma = 3.8$ for elasticity of substitution. From Ardelean (2009), I adopt $\eta = 0.42$ for external scale parameter. For the rest of parameters, I set $w_i = 7.25$ for minimum wage, $\bar{N}_i = 100$ for the mass of potential entrants, and $\beta = 0.15$ for factor intensity. The mass of skilled workers, $\bar{H}_i$, is assumed to be sufficiently small.

with $\psi = \left(\lambda^{\theta(1-\beta(\sigma-1))}\mu^{\beta\vartheta}\right)^{\frac{1}{\theta+\beta\kappa}} > 0$ and

$$s_i = \left(\frac{\mu}{\lambda}\right)^{\frac{\theta\vartheta}{\theta+\beta\kappa}} \left[\frac{\bar{N}_i}{w_i^{(1-\beta)\kappa}\bar{H}_i^{\theta}}\right]^{\frac{1}{\theta+\beta\kappa}}. \tag{1.22}$$

**Proof**. See Appendix and Figure 1.1.

An increase in the stock of skilled workers $\bar{H}_i$ reduces both skilled workers' wage $s_i$ in (1.22) and cutoff productivity $\phi_i^*$ in (1.21).[17] A rise in minimum wage $w_i$ decreases skilled workers' wage $s_i$ in (1.22) and raises cutoff productivity $\phi_i^*$ in (1.21). Clearly, an increase in the exogenous mass of potential firms $\bar{N}_i$ raises skilled workers' wage $s_i$ in (1.22).

How does an increase in the exogenous mass of potential firms $\bar{N}_i$ affect the cutoff productivity level? To answer, we carefully should look at the solution in (1.21). An increased mass of potential firms $\bar{N}_i$ exhibits two adjustment channels: ***wage effects*** $\beta$ and ***external scale effects*** $\chi(\eta,\sigma)$. Relative forces between them determine sign$\{\beta - \chi(\eta,\sigma)\}$ which points to the direction of the

[17] A necessary and sufficient condition for $\phi_i^* \geq 1$ is $N_i \leq \left\{\psi^{\theta+\beta\kappa}\frac{w_i^{1-\beta}}{\bar{H}_i^{\beta}}\right\}^{\frac{1}{\chi-\beta}}$.

change in cutoff productivity. If wage effects dominate external scale effects, *i.e.*, $\beta > \chi(\eta, \sigma)$, cutoff productivity rises as $\bar{N}_i$ rises. To understand this result, we focus on a marginal firm. An increased mass of potential firms implies a higher demand for skilled workers in the labor market, thereby raising the wage for skilled workers. That increased wage means higher variable costs, forcing marginal firms to exit the market. Thus, wage effects drive cutoff productivity upward. Contrarily, an increased mass of potential firms raises $M_i$, thereby expanding final output through scale economies. Expanded final output indirectly raises demand for intermediate inputs, resulting in an increase in the returns to all intermediate input firms. Thus, external scale effects drive cutoff productivity downward. However, if final output firms use production technology that generates weak external scale effects and intermediate input firms use skill-intensive technology, *i.e.*, a sufficiently small $\chi$ relative to $\beta$, marginal firms in the differentiated input sector cannot offset increased variable costs by larger market demand, thus *exit* the market. If $\beta < \chi(\eta, \sigma)$, then potential firms *enter* the market.

Egger *et al*. (2012) show that as $\bar{N}_i$ increases, cutoff productivity would not change at all if there is no external scale effect. In contrast, cutoff productivity increases due to prevailing wage effects even without external scale effect, as long as $\beta > 0$. In Egger *et al*. (2012), the effect of an increase in $\bar{N}_i$ depends only on external scale effects, driving cutoff productivity downward. This results because they only consider minimum-wage workers. In general, though, an additional stabilizing force exists by means of increasing factor prices in response to a larger number of potential entrants.[18]

In the next section, to facilitate comparison between autarky and trade equilibrium, it is useful to explicitly solve for aggregate product market variables. In the goods market, average productivity is derived by plugging (1.21) into APC (1.7),

$$\tilde{\phi}_i = \frac{\psi}{\rho\lambda^{\theta}} \left[ \bar{N}_i^{\beta - \chi(\eta,\sigma)} \left( \frac{w_i^{1-\beta}}{\bar{H}_i^{\beta}} \right) \right]^{\frac{1}{\theta+\beta\kappa}}. \tag{1.23}$$

[18] Egger *et al*. (2012) point out this matter in footnote 7 on page 777 but they seem to believe that the assumption of a single factor is innocent but helps keep the analysis tractable as stated in footnote 3 on page 772.

Plugging (1.21) into CPC (1.5), the mass of active firms is

$$M_i = \frac{\bar{N}_i^{\frac{1}{\theta+\beta\kappa}}}{\psi^\kappa}\left(\frac{\bar{H}_i^\beta}{w_i^{1-\beta}}\right)^{\frac{\kappa}{\theta+\beta\kappa}}. \tag{1.24}$$

Since $Y_i = M_i r_i(\tilde{\phi}_i) = M_i \frac{\kappa\sigma}{\kappa-\sigma+1}$, the total final output produced $Y_i$ becomes

$$Y_i = \left(\frac{\kappa\sigma}{\kappa-\sigma+1}\right)\frac{\bar{N}_i^{\frac{1}{\theta+\beta\kappa}}}{\psi^\kappa}\left(\frac{\bar{H}_i^\beta}{w_i^{1-\beta}}\right)^{\frac{\kappa}{\theta+\beta\kappa}}. \tag{1.25}$$

Next, the labor market equilibrium in autarky is briefly discussed. Using $W_i = \rho M_i r_i(\tilde{\phi}_i)$ and (1.25), the total labor costs $W_i$ are

$$W_i = \left(\frac{\kappa(\sigma-1)}{\kappa-\sigma+1}\right)\frac{\bar{N}_i^{\frac{1}{\theta+\beta\kappa}}}{\psi^\kappa}\left(\frac{\bar{H}_i^\beta}{w_i^{1-\beta}}\right)^{\frac{\kappa}{\theta+\beta\kappa}}. \tag{1.26}$$

Using (1.17), the rate of unemployment is

$$U_i = 1 - \frac{(1-\beta)\kappa(\sigma-1)}{(\kappa-\sigma+1)\psi^\kappa \bar{L}_i}\left[\frac{\bar{N}_i\bar{H}_i^{\beta\kappa}}{w_i^{\theta+\kappa}}\right]^{\frac{1}{\theta+\beta\kappa}}. \tag{1.27}$$

Obviously, the unemployment rate is one when $\beta = 1$. The Egger *et al*. (2012) model is a special case of the present paper when $\beta = 0$. A higher minimum wage $w_i$ (or a lower stock of skilled workers $\bar{H}_i$) lowers the total mass of input firms $M_i$, total output $Y_i$, and total labor income $W_i$ from (1.24)-(1.26) with a rise in the rate of unemployment $U_i$ from (1.27). This completes the analysis in autarky.[19]

[19] Examining the effect of a rise in $\bar{N}_i$ is not the primary focus of our analysis. However, it is interesting to know that a larger $\bar{N}_i$ stimulates entry everywhere in the productivity distribution and thus, a larger number of high productive firms operate in the market. If $\beta > \chi(\eta,\sigma)$, then $\phi_i^*$ in (1.21) rises. However, it is clear to predict that as $\bar{N}_i$ rises, total mass of input firms, total output, and total labor income increases from (1.24)-(1.26). Unemployment rate clearly falls in (1.27). Consequently, a rise in $\bar{N}_i$ will have a positive impact on welfare in the long-run.

## 1.4 Introduction of Trade

Consider free international trade between country $i$ and $j$. Assume both are in all respects identical except in the minimum wage level, *i.e.*, $w_i \neq w_j$, $\bar{L}_i = \bar{L}_j$, $\bar{H}_i = \bar{H}_j$, and $\bar{N}_i = \bar{N}_j$.[20] This assumption will be relaxed in the next section. Assume throughout that no trade costs are present, so that all firms are exporters and charge the same price in both domestic and foreign markets.[21] In each country, final goods producers do not discriminate among intermediate inputs produced in different countries. Last, all notation is written in terms of country $i$. For country $j$, the subscript changes from $i$ to $j$ and from $j$ to $i$.

### 1.4.1 Firm behavior between asymmetric countries

Denote $M_{it}$ as the mass of active firms in country $i$ where the subscript $t$ refers to free trade. Final output is produced by assembling both domestic and foreign input varieties according to the CES production technology,[22]

$$Y_{it} = Y_{jt} = M_t^{-\frac{\eta}{\sigma-1}} \left( \int_{v \in V_i} [z_{it}(v)]^{\frac{\sigma-1}{\sigma}} dv + \int_{v_* \in V_j} [z_{jt}(v_*)]^{\frac{\sigma-1}{\sigma}} dv_* \right)^{\frac{\sigma}{\sigma-1}} \tag{1.28}$$

where $M_t \equiv M_{it} + M_{jt}$, $V_j$ denotes the set of all available varieties with measure $M_{jt}$, and $z_{jt}(v_*)$ is the quantity of variety $v_*$ produced by country $j$.

Taking intermediate input demands by final goods producers and other aggregate variables, intermediate goods firms solve their profit maximization problem. Each firm with different productivity $\phi$ follows the constant mark-up pricing rule:

$$p_{it}(\phi) = \frac{s_{it}^{\beta} w_i^{1-\beta}}{\rho \phi}, \quad p_{jt}(\phi) = \frac{s_{jt}^{\beta} w_j^{1-\beta}}{\rho \phi}. \tag{1.29}$$

[20] Egger *et al*. (2012) point out that Krugman's type model with homogeneous firm fails to have binding minimum wages in both countries after trade. The assumption of heterogeneous firms, however, enables both countries to have different minimum wages.

[21] By the assumption of no trade cost, selection effects in Melitz (2003) are neutralized in this paper. It is crucial to focus on the adjustment at the extensive margin of firms throughout the paper.

[22] The price indices are equalized across countries. Due to the choice of $Y$ as numeraire, we set $P = 1$.

Similar to the closed economy, cutoff productivity conditions (CPCs) yield

$$M_{it} = \bar{N}_i[\phi_{it}^*]^{-\kappa}, \quad M_{jt} = \bar{N}_j[\phi_{jt}^*]^{-\kappa}. \tag{1.30}$$

Two zero-profit cutoff productivity conditions (ZCPs) are

$$r_{it}(\phi_{it}^*) = \sigma, \quad r_{jt}(\phi_{jt}^*) = \sigma. \tag{1.31}$$

Using the common index of intermediate goods prices, profit maximization conditions (PMCs) yield

$$M_t^{\chi} = p_{it}(\tilde{\phi}_{it}), \quad M_t^{\chi} = p_{jt}(\tilde{\phi}_{jt}) \tag{1.32}$$

where $\tilde{\phi}_{it}$ and $\tilde{\phi}_{jt}$ (APCs) are defined similarly to in the closed economy case.[23]

Equation (1.31) implies that the ratio of two cutoff productivities can be expressed as the ratio of variable costs from both countries: $\phi_{jt}^*/\phi_{it}^* = s_{jt}^{\beta} w_j^{1-\beta}/s_{it}^{\beta} w_i^{1-\beta}$.[24] In each country, the government imposes a different minimum wage so that it affects a firm's price setting by influencing variable costs. In Egger *et al*. (2012), the ratio of two cutoff productivities equals the ratio of two minimum wages, *i.e.*, $\phi_{jt}^*/\phi_{it}^* = w_j/w_i$. Unlike the Egger *et al*. (2012) model, the generalized version of the model here proposed entails skilled workers' wages. In this case, we take account of endogenously-determined skilled workers' wage of each country in computing the ratio of two cutoff productivities. An interesting finding is it is unnecessary to do so in the present model because skilled workers' wages in open economies are equalized. Intuitively, this result makes sense. Suppose country $j$ sets a higher minimum wage than that of country $i$. As discussed in autarky, country $j$ has a smaller goods market than has country $i$. In the autarky equilibrium, $s_i > s_j$ (1.22).

[23] As Egger *et al*. (2012) point out, in the open economy it is necessary to distinguish between the average productivity of domestic firms, $\tilde{\phi}_{it}$, and the average productivity in the market, $\hat{\phi}_{it}$, with domestic production as well as imports from foreign firms. $\tilde{\phi}_{it}$ and $\hat{\phi}_{it}$ are identical only if the negative lost-in-transit effect and the positive export-selection effect are of the same size. Since we abstract from any trade impediments and all firms export, $\tilde{\phi}_{it} = \hat{\phi}_{it}$. For more details, see Egger and Kreickemeier (2009).

[24] Due to the constant relationship between cutoff and average productivity, $\tilde{\phi}_{jt}/\tilde{\phi}_{it} = s_{jt}^{\beta} w_j^{1-\beta}/s_{it}^{\beta} w_i^{1-\beta}$ should be satisfied.

Upon opening to trade, firms in country $j$ encounter a larger foreign market than that in country $i$. Given the same use of skill intensity in production across countries, the increase in the magnitude of labor demand for skilled workers in country $j$ should be higher proportionally than in country $i$. Due to the perfectly competitive labor market for skilled workers, wages for skilled workers adjust to clear the labor markets and turn out to be equal between the two countries. Indeed, we can derive this result using two zero profit conditions (ZPCs) and two profit maximization conditions (PMCs).

**Lemma 1**. *Skilled workers' wages are equalized in open economies, $s_{it} = s_{jt}$.*[25]

**Lemma 2**. $M_t = \left[1+\left(\frac{w_i}{w_j}\right)^{(1-\beta)\kappa}\right] M_{it}$

**Proof**. See Appendix.

### 1.4.2 Trade equilibrium

Equilibrium variables in free trade between asymmetric countries are characterized completely by deriving the solution for cutoff productivity and skilled workers' wage in each country. Given the solution, all the other aggregate variables $(\tilde{\phi}_{it}, M_{it}, Y_{it}, W_{it}, \tilde{\phi}_{jt}, M_{jt}, Y_{jt}, W_{jt})$ in product markets and $(U_{it}, U_{jt})$ in labor markets are determined across countries.

Using CPCs (1.30), ZCPs (1.31), PMCs (1.32), APCs, and Lemma 1-2, two key equations are derived for each country.

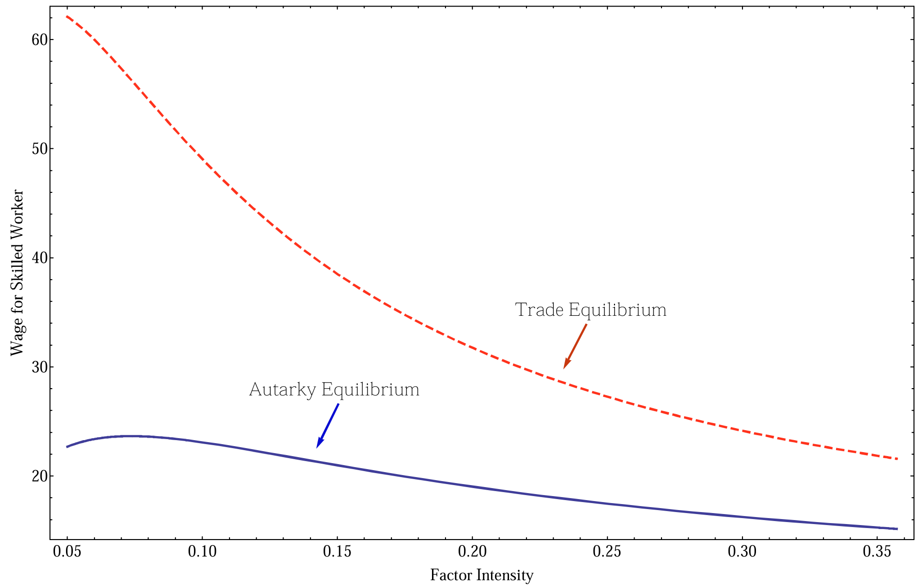
$$\phi_{it}^* = \lambda \left[\frac{w_i^{1-\beta}}{\bar{N}_i^{\chi}}\right]^{\frac{1}{\theta}} \left[1+\left(\frac{w_i}{w_j}\right)^{(1-\beta)\kappa}\right]^{\frac{-\chi}{\theta}} s_{it}^{\frac{\beta}{\theta}}, \tag{1.33}$$

$$\phi_{it}^* = \mu \left[\frac{w_i^{(1-\beta)(\sigma-1)}\bar{N}_i^{\eta}}{\bar{H}_i}\right]^{\frac{1}{\vartheta}} \left[1+\left(\frac{w_i}{w_j}\right)^{(1-\beta)\kappa}\right]^{\frac{\eta}{\vartheta}} s_{it}^{\frac{\sigma-1}{\vartheta}\left(\beta-\frac{1}{\sigma-1}\right)}. \tag{1.34}$$

**Proposition 2**. *A unique trade equilibrium exists in which cutoff productivity and skilled workers' wage are determined in each country,*

[25] However, if one country has a higher mass of skilled workers than the other, factor price equalization fails. See Section 5.

Figure 1.2 Wage for Skilled Worker and Factor Intensity in Trade Equilibrium




Note: The same parameter values are imposed as in Figure 1.1. Solid and dashed lines refer respectively to autarky and trade equilibrium.

$$\phi_{it}^{*} = \psi\left(\frac{w_i^{1-\beta}}{\bar{H}_i^{\beta}}\right)^{\frac{1}{\theta+\beta\kappa}}\left[\bar{N}_i\left(1+\left(\frac{w_i}{w_j}\right)^{(1-\beta)\kappa}\right)\right]^{\frac{1}{\theta+\beta\kappa}\left\{\beta-\chi(\eta,\sigma)\right\}} \tag{1.35}$$

with $\psi > 0$ as defined in Proposition 1 and

$$s_{it} = \left(\frac{\mu}{\lambda}\right)^{\frac{\theta\vartheta}{\theta+\beta\kappa}}\left[\frac{\bar{N}_i}{\bar{H}_i^{\theta}}\left(\left(\frac{1}{w_i}\right)^{(1-\beta)\kappa}+\left(\frac{1}{w_j}\right)^{(1-\beta)\kappa}\right)\right]^{\frac{1}{\theta+\beta\kappa}}. \tag{1.36}$$

**Corollary 1**. Using (1.21) in Proposition 1 and (1.35) in Proposition 2,

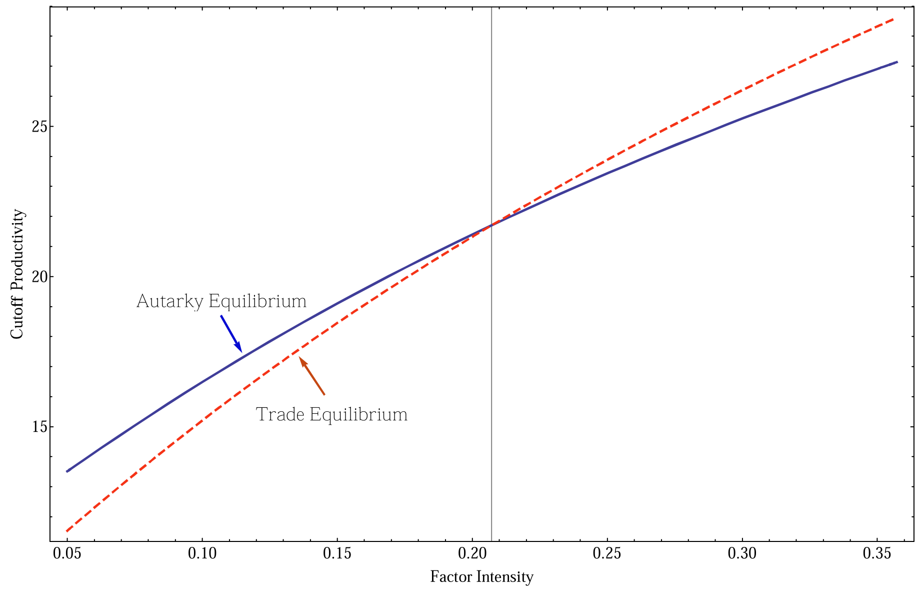
$$\frac{\phi_{it}^{*}}{\phi_i^{*}} = \left[1+\left(\frac{w_i}{w_j}\right)^{(1-\beta)\kappa}\right]^{\frac{1}{\theta+\beta\kappa}\left\{\beta-\chi(\eta,\sigma)\right\}}.$$

**Corollary 2**. Using (1.22) in Proposition 1 and (1.36) in Proposition 2,

$$\frac{s_{it}}{s_i} = \left[1+\left(\frac{w_i}{w_j}\right)^{(1-\beta)\kappa}\right]^{\frac{1}{\theta+\beta\kappa}} \equiv \Phi_i.$$

Trade liberalization induces an increase of skilled workers' wage due to an increase of aggregate labor demand for skilled workers (Corollary 2). As illustrated in Figure 1.2, with constant

Figure 1.3 Cutoff Productivity and Factor Intensity in Trade Equilibrium




Note: Parameter values are as noted in Figure 1.1. Vertical line is given at $\beta = \chi = \frac{1-\eta}{\sigma-1} = \frac{1-0.42}{3.8-1}$ (approximately, 0.207). Solid and dashed lines refer respectively to autarky and trade equilibrium.

external scale effects $\chi$, the dotted line of skilled workers' wage in trade equilibrium is always above the solid line in autarky equilibrium regardless of $\beta$. Moreover, wage inequality between different skill-typed workers grows after trade liberalization at any given $\beta$. Due to the assumption of binding minimum wage, there is no change in (real) wage for unskilled workers so that a larger market size ought to tie with an increased wage inequality. It is easy to show the ratio of wage inequality before and after free trade: $\frac{s_{it}}{w_i} / \frac{s_i}{w_i} = \Phi_i > 1$ where $\Phi_i$ is defined in Corollary 2. This result is consistent with the literature on trade liberalization and wage inequality (see, *e.g.*, Yeaple 2005; Helpman *et al*. 2010; Harrigan and Reshef 2015; Burstein and Vogel 2010). Last, one can compare wage inequality before and after free trade between country $i$ and $j$. Given an assumption that both countries are identical in all respects other than the minimum wage level, wage inequality in country $i$ in free trade compared with one in autarky is less severe than one in country $j$ if and only if $w_i < w_j$. This result is straightforward in Corollary 2 because $w_i < w_j$ implies $\Phi_i < \Phi_j$.

Figure 1.3 illustrates Corollary 1 in which with constant external scale effects $\chi$, the dotted line of cutoff productivity in trade equilibrium crosses from below the solid line in autarky equilibrium as $\beta$ rises. Based on parameter values as noted in Figure 1.1, one additional parameter is imposed

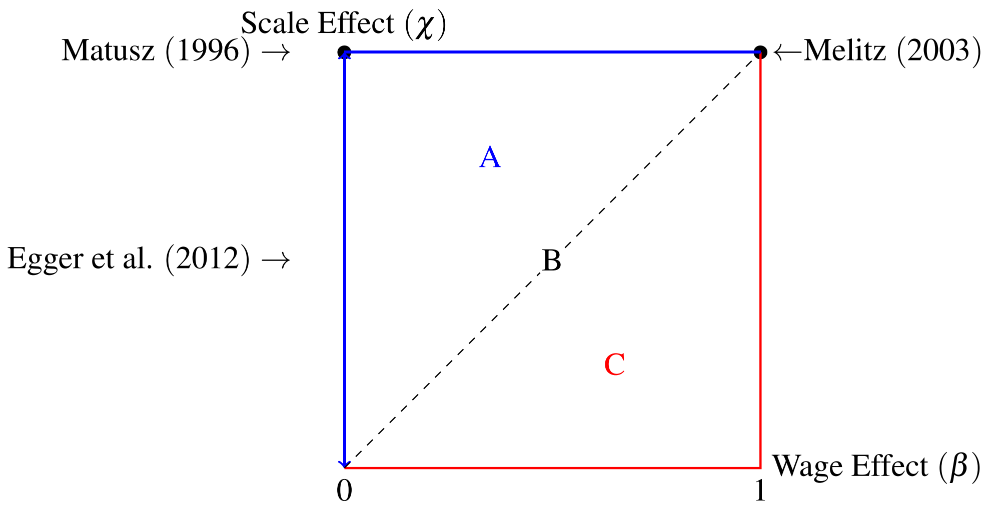
$w_j = 8.5$ for minimum wage in country $j$. In Corollary 1, sign$\{\beta - \chi(\eta, \sigma)\}$ is important and governs the scale of $\phi_{it}^*/\phi_i^*$. Clearly, $\phi_{it}^*/\phi_i^* = 1$ at $\beta = \chi(\eta, \sigma)$. If $\beta > \chi(\eta, \sigma)$, then $\phi_{it}^*/\phi_i^* > 1$ and *vice versa*. As noted in autarky, positive sign$\{\beta - \chi(\eta, \sigma)\}$ is interpreted similarly on the opening of trade. On one hand, trade liberalization expands the goods market *via* external scale economies, thus leads potential entrants into the market. On the other, it raises labor demand for both skilled and unskilled workers. Firms have to bear higher payments to workers, resulting in higher variable costs (negative wage effect). If the latter effects always dominate the former, *i.e.*, $\beta > \chi(\eta, \sigma)$, liberalized trade induces marginal firms to exit the market, resulting in $\phi_{it}^*/\phi_i^* > 1$. If $\beta < \chi(\eta, \sigma)$, then potential firms enter the market, $\phi_{it}^*/\phi_i^* < 1$.

All other aggregate variables in the trade equilibrium are provided in Appendix. Using $\Phi_i$ in Corollary 2, it is convenient to deliver the ratio of equilibrium variables before and after free trade: $\frac{\phi_{it}^*}{\phi_i^*} = \frac{\tilde{\phi}_{it}}{\tilde{\phi}_i} = \Phi_i^{\beta - \chi}$ and $\frac{M_{it}}{M_i} = \frac{Y_{it}}{Y_i} = \frac{W_{it}}{W_i} = \Phi_i^{\kappa(\chi - \beta)}$. When wage effects outweigh external scale effects, *i.e.*, $\beta > \chi(\eta, \sigma)$, an increased cutoff productivity $\phi_{it}^*$ triggers the mass of active firms, total output, and total labor income to fall. In contrast, $\beta < \chi(\eta, \sigma)$ implies that potential firms would not worry much about having to pay skilled-workers' wages since production technology for intermediate input firms less intensively uses skilled workers. Thus, potential firms enter the market, thereby raising the mass of active firms, total outputs, and total labor costs.

Figure 1.4 illustrates how the current paper relates to several others in terms of firm exit/entry, depending on wage effects and external scale effects. Melitz (2003) assumes both a full love-of-variety ($\chi = 1/(\sigma - 1)$) and a perfectly competitive labor market ($\beta = 1$), and predicts that, absent trade costs, cutoff productivity does not change at all after trade liberalization. At $\beta = 1$, there is no employment for unskilled workers in the world, regardless the existence of minimum wages, because input firms' production technology obviates hiring any. The two countries become symmetric, thereby producing equal mass of varieties in each.[26] Thus, his model corresponds to the upper-right corner in Figure 1.4. Matusz (1996) considers a trade model with efficiency wage and full external scale effects ($1/(\sigma - 1)$). He predicts that trade liberalization relaxes constraints on

[26]For example, $M_t = 2M_{it} = 2M_{jt}$.

Figure 1.4 Trade Liberalization and Change in Cutoff Productivity




Note: To have the interval of skill intensity $\beta \in [0,1]$, elasticity of substitution between variety should be $\sigma \leq 2$ which satisfies one of the stability conditions (see footnote 15). Region A shows $\phi_{it} < \phi_i$ if $\beta < \chi(\eta,\sigma)$ while Region C shows $\phi_{it} > \phi_i$ if $\beta > \chi(\eta,\sigma)$. Line B indicates $\phi_{it} = \phi_i$ if $\beta = \chi(\eta,\sigma)$.

efficiency wage in the labor market and permits employment expansion in both countries. Although his model deals with homogenous firms, entry of more firms after free trade can be interpreted as a reduction in the cutoff productivity in the framework of the present paper. Since his model has labor market frictions from efficiency wage, it corresponds to $\beta = 0$ and $1/(\sigma-1)$ in the upper-left corner in Figure 1.4. Egger *et al.* (2012) explicitly consider flexible external scale effects but minimum-wage workers. They predict that potential firms always enter the market after trade liberalization, $\phi_{it}^* < \phi_i^*$. Thus, their model can be located on the vertical line at $\beta = 0$. The model developed herein illuminates the rest of the area in the parameter space $(\chi(\eta,\sigma),\beta)$ of Figure 1.4.

### 1.4.3 Unemployment and welfare

From the discussion in the previous subsection, it is clear that sign$\{\beta - \chi(\eta,\sigma)\}$ determines firm performance and firm exit/entry. In particular, $\beta > \chi(\eta,\sigma)$ implies that the most productive firms in the interval $(\tilde{\phi}_{it},\infty)$ increase its production volume by serving both domestic and foreign markets, resulting in higher profits. Relatively less productive firms in $(\phi_{it}^*,\tilde{\phi}_{it})$ serve both markets but have lower profits compared with ones in autarky, while all input firms in $(\phi_i^*,\phi_{it}^*)$ exit the market

(Corollary 1). The two types of worker are unevenly affected by trade liberalization. Regardless of sign$\{\beta - \chi(\eta,\sigma)\}$, also clear is that the transition from autarky to free trade results in increased returns to skilled workers in both countries (Corollary 2). In contrast to the model by Egger *et al*. (2012), some unskilled workers may face unemployment despite liberalized trade's positive employment effects.

Denote $U_{it}$ and $E_{it}$ as the unemployment and employment rate for unskilled workers of country $i$ in free trade $t$. By construction, $E_{it} = 1 - U_{it}$ ($E_i$ is defined similarly in autarky). Using the definition of $\Phi_i$, the following ratio holds:

$$\frac{E_{it}}{E_i} = \Phi_i^{-\kappa\{\beta - \chi(\eta,\sigma)\}} \tag{1.37}$$

Positive sign$\{\beta - \chi(\eta,\sigma)\}$ implies that negative wage effects, deriving from an increased wage for skilled workers, outweigh positive employment effects to unskilled workers. Free trade shrinks employment of unskilled workers (1.37), resulting in a higher rate of unemployment in each country. Thus, free trade harms some unskilled workers who have worked in less productive firms.[27]If $\beta < \chi(\eta,\sigma)$, then both skilled and unskilled workers become winners.

The model presented herein accords with the efficiency wage model developed by Matusz (1996), analyzing labor market effects of trade liberalization without variable costs in an Ethier-type (1982) framework. He predicts that introduction of trade results in establishment of more firms in each country and commensurately-higher employment levels, thereby lowering unemployment. That is consistent with the result under full external scale effects, $\eta = 0$ (see also Figure 4). The present paper is complementary to the model by Egger and Kreickemeier (2009). Respective findings by Egger and Kreickemeier (2009) derive from consideration of the effect of trade

[27]In the borderline case, $\eta = 1$ shows no external scale effects, $\chi = 0$. Then, $P_t = 1$ implies $p_{it}(\tilde{\phi}_{it}) = 1 = s_{it}^{\beta} w_i^{1-\beta}/\rho\tilde{\phi}_{it}$. Since there is no change in price index, no change is made in nominal wages in both countries. But there is a larger demand for skilled workers in both countries, which drives skilled workers' wage upward. This implies that variable cost $s_{it}^{\beta}\bar{w}_i^{1-\beta}/\phi$ increases. In order for average firm's price to be 1, average productivity level should rise compared with the one in autarky. This can be done by the exit of marginal firms. From the perspective of marginal firms, the unit cost $s_{it}^{\beta}\bar{w}_i^{1-\beta}/\phi$ increases so that it cannot bear the costs anymore. A slightly higher productive firm is in the same position. The marginal firm's productivity increases up to a certain level at which average productivity satisfies $p_{it}(\tilde{\phi}_{it}) = 1$. Hence, cutoff and average productivity rises. This leads $Y_{it}$ and $M_{it}$ to falls and consequently unemployment $U_{it}$ rises. The same logic applies to the case in country $j$.

liberalization on a labor market in the presence of positive variable trade costs and external scale effects. They conclude that a negative employment effect is triggered if variable trade costs are not too low and external scale effects are moderate, whereas a positive employment effect can be expected if variable trade costs are negligible and external scale effects are strong.

Next is economy-wide welfare. Define welfare measure $\Omega_{it}$ as the sum of aggregate labor income $W_{it}$ and aggregate profits of domestic firms $\Pi_{it}$.[28] That is, $\Omega_{it} = W_{it} + \Pi_{it} = \rho M_{it} r_{it}(\tilde{\phi}_{it}) + M_{it}[r_{it}(\tilde{\phi}_{it})/\sigma - 1]$. Hence,

$$\Omega_{it} = M_{it}[r_{it}(\tilde{\phi}_{it}) - 1]. \tag{1.38}$$

The welfare measure in (1.38) is unlike Melitz (2003) which focuses mainly on welfare per worker, represented by the number of varieties consumed and the average productivity. The reason is that his model assumes free entry conditions so that the present value of aggregate profits equals total entry costs of firms in the productivity draw. Hence, only wage income is available for consumption. Meanwhile, the present paper assumes exogenous $\bar{N}$ potential firms so that the welfare measure also includes aggregate profits. Since an average productivity firm's revenue is constant, *i.e.*, $r_{it}(\tilde{\phi}_{it}) = \frac{\kappa\sigma}{\kappa-\sigma+1}$, the mass of active firms $M_{it}$ in (1.38) determines economy-wide welfare level in country $i$. The following proposition summarizes the results.

**Proposition 3**. ***(Economy-wide Welfare)*** *Trade welfare is improving if and only if $\beta < \chi(\eta, \sigma)$.*

This paper shows that when $\beta > \chi(\eta, \sigma)$, the possibility exists that welfare worsens due to trade liberalization (*i.e.*, $M_{it}/M_i < 1$). Noteworthy here is that Neary's statement (2004) that "Dixit-Stiglitz specification imposes too benign a view of product diversity. It clearly fails to capture one of the concerns of anti-globalization protesters: that liberalizing trade may reduce rather than increase variety." In addition, a theoretical possibility of negative welfare effects due to trade liberalization has been raised by Montagna (2001) who develops a monopolistic competition model with technical heterogeneity among firms and countries. She shows that adverse welfare

[28]The welfare definition follows Egger *et al*. (2012) who explain that total income can be used as a utilitarian welfare measure in each country, due to assuming a single homogeneous final good.

effects may prevail in an advanced technology economy if love-of-variety is sufficiently low. In contrast, Egger *et al*. (2012) conclude that trade liberalization always leads both economies to higher levels of welfare once external scale effects, *i.e.*, $\eta < 1$, are in play. In their model, workers always benefit from free trade because of positive external scale effects (thus positive employment effects) and economy-wide welfare always improves. Their analysis is incomplete in the presence of heterogeneous workers.

## 1.5 Labor Market Shocks

This paper is motivated by the increasing interest in potential labor market spillovers between asymmetric countries. It is particularly relevant for the U.S. and Europe, or for nations within Europe, closely linked by trade. As long as policymakers in one country concern themselves with labor market reforms intended to lower unemployment, abundant debates easily occur between in-country proponents and opponents in their country and among their trading partners. Recently, labor market reforms -the so-called Hartz IV reforms in Germany- which aimed at lowering unemployment rates, have been criticized by other countries as being in effect *beggar-thy-neighbor* policies. However, recent empirical papers do not support this viewpoint (see, *e.g.*, Felbermayr *et al*. 2012). Two possible questions of particular interest are (i) how variations in country $i$'s minimum wage affect country $j$ and (ii) how variations in country $i$'s factor endowment affect country $j$.

In what follows, both countries are asymmetric such as $w_i \neq w_j$, $\bar{L}_i \neq \bar{L}_j$, $\bar{H}_i \neq \bar{H}_j$, and $\bar{N}_i \neq \bar{N}_j$. Thus, Lemma 1 and 2 should be revised thus:

**Generalized Lemma** 1. *Skilled workers' wages are relatively equalized,* $\frac{s_{it}}{s_{jt}} = \frac{\bar{H}_j}{\bar{H}_i}$.

**Generalized Lemma** 2. $M_t = \left[1 + \left(\frac{\bar{N}_j}{\bar{N}_i}\right)\left(\frac{\bar{H}_j}{\bar{H}_i}\right)^{\beta\kappa}\left(\frac{w_i}{w_j}\right)^{(1-\beta)\kappa}\right] M_{it}$.

The ratio of skilled workers' wages between asymmetric countries is inversely related to the ratio of stocks of skilled workers. In addition, the total mass $M_t$ of firms in the world is characterized using relative minimum wages, relative endowments of skilled workers, relative mass of potential entrants between two asymmetric countries, and the mass of active firms in one country.

For later use, one additional Lemma is useful. We define $\Gamma_i$ as country $i$'s input share $(\frac{M_{it}}{M_t})$ among total input varieties used in final output production.

**Generalized Lemma 3**. $\Gamma_i + \Gamma_j = 1$.

$\Gamma_i = \left(\frac{\bar{N}_i}{\bar{N}_j}\right)\left(\frac{\bar{H}_i}{\bar{H}_j}\right)^{\beta\kappa}\left(\frac{w_j}{w_i}\right)^{(1-\beta)\kappa} \Big/ \left(1 + \left(\frac{\bar{N}_i}{\bar{N}_j}\right)\left(\frac{\bar{H}_i}{\bar{H}_j}\right)^{\beta\kappa}\left(\frac{w_j}{w_i}\right)^{(1-\beta)\kappa}\right)$.

$\Gamma_j = \left(\frac{\bar{N}_j}{\bar{N}_i}\right)\left(\frac{\bar{H}_j}{\bar{H}_i}\right)^{\beta\kappa}\left(\frac{w_i}{w_j}\right)^{(1-\beta)\kappa} \Big/ \left(1 + \left(\frac{\bar{N}_j}{\bar{N}_i}\right)\left(\frac{\bar{H}_j}{\bar{H}_i}\right)^{\beta\kappa}\left(\frac{w_i}{w_j}\right)^{(1-\beta)\kappa}\right)$.

**Proof**. See Appendix.

By virtue of Generalized Lemma 1-2, equilibrium cutoff productivity and skilled workers' wage in each country are produced as follows: For country $i$,

$$\phi_{it}^{*} = \psi \left( \frac{w_i^{1-\beta}}{\bar{H}_i^{\beta}} \right)^{\frac{1}{\theta+\beta\kappa}} \left[ \bar{N}_i \left( 1 + \left( \frac{\bar{N}_j}{\bar{N}_i} \right) \left( \frac{\bar{H}_j}{\bar{H}_i} \right)^{\beta\kappa} \left( \frac{w_i}{w_j} \right)^{(1-\beta)\kappa} \right) \right]^{\frac{1}{\theta+\beta\kappa}\left\{\beta-\chi(\eta,\sigma)\right\}}, \tag{1.39}$$

$$s_{it} = \left( \frac{\mu}{\lambda} \right)^{\frac{\theta\vartheta}{\theta+\beta\kappa}} \left[ \frac{\bar{N}_i}{\bar{H}_i^{\theta}} \left( \left( \frac{1}{w_i} \right)^{(1-\beta)\kappa} + \left( \frac{\bar{N}_j}{\bar{N}_i} \right) \left( \frac{\bar{H}_j}{\bar{H}_i} \right)^{\beta\kappa} \left( \frac{1}{w_j} \right)^{(1-\beta)\kappa} \right) \right]^{\frac{1}{\theta+\beta\kappa}}. \tag{1.40}$$

For country $j$, the subscript changes from $i$ to $j$ and from $j$ to $i$.

In the remainder of this paper, it should suffice to focus on cutoff productivity and skilled workers' wage in each country. All other equilibrium variables responding to external shocks in labor markets can be determined accordingly. It is crucial to focus on adjustment at the extensive margin of firms since selection effects are neutralized by assuming no trade cost. Equipped with (1.39), (1.40), and Generalized Lemma 3, a trade equilibrium as the starting point is ready to be examined.

### 1.5.1 Change in minimum wage policy

We arrive at discussing how variations in one country's minimum wage affect the other economy. In open economies, do bad labor market institutions in one country positively or negatively affect its trading partners? This is tackled in the seminal paper by Davis (1998) who explicitly considers international labor market linkages based on the Heckscher-Ohlin model. Davis (1998) concludes that high European minimum wages prop up U.S. wages. In contrast, Egger *et al*. (2012) raise a theoretical possibility of international negative spillover based on the intraindustry trade model. They conclude that high European minimum wages prop up the U.S. unemployment rate. Seemingly, high European minimum wages may be positively *or* negatively correlated with labor market outcomes in the U.S. Who wins, is the question. This paper's answer is "It depends."

Define $\varepsilon^i_{w_i}$ as an elasticity of cutoff productivity in country $i$ with respect to minimum wage in country $i$.

$$\varepsilon^i_{w_i} = \left(\frac{1-\beta}{\theta+\beta\kappa}\right)\left[1+\kappa(\beta-\chi)\Gamma_j\right] > 0. \tag{1.41}$$

Define $\varepsilon^j_{w_i}$ as an elasticity of cutoff productivity in country $j$ with respect to minimum wage in country $i$.

$$\varepsilon^j_{w_i} = -\left(\frac{1-\beta}{\theta+\beta\kappa}\right)\kappa(\beta-\chi)\Gamma_i \gtreqless 0 \;\;\Leftrightarrow\;\; \beta \lesseqgtr \chi(\eta,\sigma). \tag{1.42}$$

Suppose country $i$ raises its real minimum wage. An increase in $w_i$ directly increases all intermediate input firms' variable costs. Due to worsened profitability, marginal firms exit the market in (1.41). A reduced number of input varieties $M_{it}$ leads to a fall in skilled workers' wage in (1.40). Consequently, unemployment increases and welfare falls in country $i$. Elsewhere, country $j$ is indirectly impacted by a rise in $w_i$ since there is no change of minimum wage policy in country $j$. Potential entrants in country $j$ would benefit from positive wages effects from a rise in $w_i$ (a fall in $s_{jt}$ in (1.40)). Lower variable cost enables those firms to enter the market. At the same time, however, those firms also encounter decreased demand for inputs by final output producers (negative external scale effects). If $\beta > \chi(\eta,\sigma)$, then a rise in $w_i$ leads potential firms in country $j$ to enter the market due to prevailing positive wage effects in (1.42). Entry of these firms, a rise in $M_{jt}$, expands employment (thus lowers unemployment for unskilled workers) and results in enhancing welfare in country $j$. Positive spillover effects of bad labor market institutions to its trading partners are a possible channel under positive sign$\{\beta-\chi(\eta,\sigma)\}$ which supports the theoretical prediction by Davidson *et al*. (1988, 1999) and Davis (1998). If negative external scale effects dominate positive wage effects, the fall in $M_{it}$ (due to a rise in $w_i$) leads marginal firms to exit the market (1.42), shrinks employment, thus worsens country $j$'s economy. Negative sign$\{\beta-\chi(\eta,\sigma)\}$ supports the theoretical prediction by Matusz (1997) and Egger *et al*. (2012). The following proposition summarizes possible spillover effects of a change in minimum wage.

**Proposition 4**. *(**Factor Price Shock**) A rise in one country's minimum wage harms its own economy. If $\beta > \chi(\eta,\sigma)$, it benefits its trading partners by expanding employment (lowering unemployment rate) and improving welfare and vice versa.*

### 1.5.2 Change in factor endowments

Suppose country $i$ receives unskilled immigrants. Due to the existence of binding real minimum wages, no impact on trade equilibrium outcomes is observed except for a proportional increase in the unemployment rate within country $i$. When there are unskilled immigrants in country $i$, country $j$ is wholly insulated from the shock and *vice versa*. With respect to unskilled immigration in the present paper, Davis' (1998) insulation hypothesis holds in *both* countries.

How would countries respond to skilled emigrants in either country? Define $\varepsilon^i_{\bar{H}_i}$ as an elasticity of cutoff productivity in country $i$ with respect to the mass of skilled workers in country $i$.

$$\varepsilon^i_{\bar{H}_i} = -\left(\frac{\beta}{\theta+\beta\kappa}\right)\left[1+\kappa(\beta-\chi)\Gamma_j\right] < 0. \tag{1.43}$$

Define $\varepsilon^j_{\bar{H}_i}$ as an elasticity of cutoff productivity in country $j$ with respect to the mass of skilled workers in country $i$.

$$\varepsilon^j_{\bar{H}_i} = \left(\frac{\beta}{\theta+\beta\kappa}\right)\kappa(\beta-\chi)\Gamma_i \lesseqqgtr 0 \;\;\Leftrightarrow\;\; \beta \lesseqqgtr \chi(\eta,\sigma). \tag{1.44}$$

A fall in $\bar{H}_i$ directly and negatively impacts its economy. It raises skilled workers' wage (1.40) thus induces an increase in variable costs throughout all intermediate input firms, resulting in marginal firms exiting the market (1.43). Thus, total mass of varieties declines, the unemployment rate rises, and welfare worsens in country $i$. Elsewhere, a fall in $\bar{H}_i$ leads skilled workers' wage in country $j$ to fall (Generalized Lemma 1 and Equation (1.40)). Intermediate input firms in country $j$ observe a decreased variable cost (positive wage effects). At the same time, those same firms encounter decreased demand for inputs by final output producers. If $\beta > \chi(\eta,\sigma)$, then a fall in $\bar{H}_i$ leads potential entrants in country $j$ to enter its intermediate input market in (1.44). The entry of firms stimulates country $j$'s economy by increasing the total mass of variety and lowering unemployment. In this way, country $j$ benefits from a decrease in the mass of skilled workers in country $i$.

**Proposition 5**. (***Factor Supply Shock***) *A fall in the mass of skilled workers in country i raises its skilled workers' wage, while it lowers country j's. The economy in country i is always harmed. If $\beta > \chi(\eta, \sigma)$, then a fall in $\bar{H}_i$ benefits country j by lowering unemployment rate and thus enhancing welfare and vice versa.*

### 1.5.3 Magnitude of impact from shocks

The direction of labor market spillover from one country to another is now clear. If $\beta > \chi(\eta, \sigma)$, then country $i$ will benefit if country $j$ either increases its minimum wage or has skilled emigrants (and *vice versa*). It is as if both countries are on a teeter-totter in which one country benefits from the other country's bad shocks in its labor market. If country $i$ (or $j$) either raises its minimum wage or has skilled emigrants, then both countries are worse off when $\beta < \chi(\eta, \sigma)$. It is as if both countries are in the same boat in which one country is also harmed by the other country's bad shocks in its labor market. However, it is less clear to understand the magnitude of impact from labor market shocks. For the sake of argument, assume that the minimum wage in country $j$ is larger than the one in country $i$ and both countries have similar endowments: $w_i < w_j$, $\bar{H}_i \simeq \bar{H}_j$, and $\bar{N}_i \simeq \bar{N}_j$. From Generalized Lemma 3, it is easy to show that $\Gamma_i > \Gamma_j$.

#### 1.5.3.1 Direct impact within country

Which country will be most heavily impacted by labor market shocks within its own economy? Using (1.41)-(1.44), the following ratio is obtained

$$\frac{\varepsilon^j_{w_j}}{\varepsilon^i_{w_i}} = \frac{\varepsilon^j_{\bar{H}_j}}{\varepsilon^i_{\bar{H}_i}} = \frac{1 + \kappa(\beta - \chi)\Gamma_i}{1 + \kappa(\beta - \chi)\Gamma_j}, \tag{1.45}$$

Depending on sign$\{\beta - \chi\}$, two cases can be detected. If $\beta > \chi$, then $\varepsilon^j_{w_j} > \varepsilon^i_{w_i}$ and $\varepsilon^j_{\bar{H}_j} > \varepsilon^i_{\bar{H}_i}$

since $\Gamma_i > \Gamma_j$. This result implies that country $j$ with a high minimum wage has a strong incentive to reform its labor market, as any variations in the labor market to reduce unemployment will stimulate heavily its own economy. However, country $i$ will face negative spillover effects as examined earlier. Consequently, there is a higher probability for country $j$ with high minimum wages to implement a *beggar-thy-neighbor* policy when $\beta > \chi$. Contrarily, $\varepsilon^j_{w_j} < \varepsilon^i_{w_i}$ and $\varepsilon^j_{\bar{H}_j} < \varepsilon^i_{\bar{H}_i}$ when $\beta < \chi$. In this case, country $i$ with low minimum wage has a strong incentive to reform the labor market. Unlike the earlier case, there is no conflict with country $j$ in which positive spillover effects are present. Consequently, there is a higher probability for country $i$ with low minimum wages to implement a *love-thy-neighbor* policy when $\beta < \chi$.

#### 1.5.3.2 Indirect impact into country

Compared with direct impact within country $i$, how heavily will country $i$ be impacted by labor market shocks from country $j$? Using (1.41)-(1.44), the following ratio is relevant

$$\left|\frac{\varepsilon^i_{w_j}}{\varepsilon^i_{w_i}}\right| = \left|\frac{\varepsilon^i_{\bar{H}_j}}{\varepsilon^i_{\bar{H}_i}}\right| = \left|\frac{-\kappa(\beta-\chi)\Gamma_j}{1+\kappa(\beta-\chi)\Gamma_j}\right| \in [0,1). \tag{1.46}$$

As indicated by (1.46), the magnitude of impacts of labor market shocks from country $j$ to $i$ cannot be larger than one within country $i$. Using data from 20 OECD countries, Felbermayr *et al*. (2012) suggest that, on average, the effect of foreign institutions on domestic unemployment amounts to about 10% of the effect of domestic institutions. Similarly, Heid and Larch (2013) show that using data from 28 OECD countries labor market reforms in one country have small spillover effects on trading partners.

#### 1.5.3.3 Relative magnitude of spillover effects

The next question, then, is which country will have a heavier impact on its trading partner? Using (1.41)-(1.44), the following ratio is obtained:

$$\frac{\varepsilon^{j}_{w_i}}{\varepsilon^{i}_{w_j}} = \frac{\varepsilon^{j}_{\bar{H}_i}}{\varepsilon^{i}_{\bar{H}_j}} = \frac{\Gamma_i}{\Gamma_j}. \tag{1.47}$$

From (1.47), it is clear that any variations in labor market in country $i$ with a low minimum wage will heavily impact on country $j$ since $\Gamma_i > \Gamma_j$. This result is reminiscent of a striking result from Davis (1998), who maintains the U.S. is wholly insulated from factor supply shocks in Europe, whereas factor supply shocks in the U.S. powerfully affect Europe.

## 1.6 Conclusion

This paper essentially extends the paper by Egger *et al*. (2012) to incorporate two types of labor in a two-country new trade theory framework with heterogeneous firms and country-specific minimum wages. It highlights the role of a second factor with flexible real wages when one factor faces risk of unemployment. This paper finds that upon external shocks, wage effects arising from intensive use of skilled workers counteract external scale effects generating employment expansion. With respect to welfare implications of trade liberalization, gains from trade arise *only* when external scale effects dominate wage effects. This paper raises the possibility of losses of trade under strong wage effects relative to external scale effects in the absence of trade cost. Regarding spillover effects of labor market shocks, this paper further predicts that labor market shocks in a country can exert either positive or negative spillover effects on the trading partners. Whereas a higher foreign minimum wage harms domestic workers under strong external scale effects, it otherwise can benefit domestic workers under strong wage effects.

Although empirical evidence is hitherto scarce, in future empirical studies it should not be surprising to find both positive and negative estimates of labor market spillover effects. Last, the model developed in this paper can be extended to studies of other issues such as entry of newly industrializing countries, international factor mobility, outsourcing, etc., all of which will be of particular interest.

APPENDIX

# Appendix

1. Proof of Proposition 1

We can solve for skilled workers' wage, using (1.19) and (1.20). Let me write down the two equations:

$$\phi_* = \lambda \left[\frac{w^{1-\beta}}{N\chi}\right]^{\frac{1}{1-\kappa\chi}} s^{\frac{\beta}{1-\kappa\chi}}$$

and

$$\phi_* = \zeta \left[\frac{w^{1-\beta} N^{\frac{\eta}{\sigma-1}}}{\bar{H}^{\frac{1}{\sigma-1}}}\right]^{\frac{1}{1+\frac{\kappa\eta}{\sigma-1}}} s^{\frac{\beta-\frac{1}{\sigma-1}}{1+\frac{\kappa\eta}{\sigma-1}}}.$$

By equating two cutoff productivities,

$$s^{\frac{\beta(\sigma-1+\kappa\eta)-(1-\kappa\chi)(\beta(\sigma-1)-1)}{(1-\kappa\chi)(\sigma-1+\kappa\eta)}} = \left(\frac{\zeta}{\lambda}\right)\left(\frac{1}{\bar{H}}\right)^{\frac{1}{\sigma-1+\kappa\eta}} \left(w^{\frac{(1-\beta)(\sigma-1)}{\sigma-1+\kappa\eta}+\frac{1-\beta}{\kappa\chi-1}}\right) N^{\frac{\eta}{\sigma-1+\kappa\eta}+\frac{\chi}{1-\kappa\chi}}.$$

After some algebra,

$$s = \left(\frac{\zeta}{\lambda}\right)^{\frac{(1-\kappa\chi)(\sigma-1+\kappa\eta)}{1+\kappa(\beta-\chi)}} \left[\frac{N}{w^{\kappa(1-\beta)}\bar{H}^{1-\kappa\chi}}\right]^{\frac{1}{1+\kappa(\beta-\chi)}}.$$

Similarly, we can solve for $\phi_*$. Next, write down (1.19) and (1.20) as follows.

$$s = \left(\frac{1}{\lambda}\right)^{\frac{1-\kappa\chi}{\beta}} \left(\frac{N\chi}{w^{1-\beta}}\right)^{\frac{1}{\beta}} \phi_*^{\frac{1-\kappa\chi}{\beta}}$$

and

$$s = \left(\frac{1}{\zeta}\right)^{\frac{\sigma-1+\kappa\eta}{\beta(\sigma-1)-1}} \left[\frac{\bar{H}}{w^{(1-\beta)(\sigma-1)}N^{\eta}}\right]^{\frac{1}{\beta(\sigma-1)-1}} \phi_*^{\frac{\sigma-1+\kappa\eta}{\beta(\sigma-1)-1}}.$$

By equating two wages for skilled workers,

$$\phi_*^{\frac{\beta(\sigma-1+\kappa\eta)-(1-\kappa\chi)(\beta(\sigma-1)-1)}{\beta(\sigma-1)-1}} = \Big(\frac{1}{\lambda}\Big)^{1-\kappa\chi} \zeta^{\frac{\beta(\sigma-1+\kappa\eta)}{\beta(\sigma-1)-1}} \Big(\frac{N^\chi}{w^{1-\beta}}\Big)\Big(\frac{w^{(1-\beta)(\sigma-1)}N^\eta}{\bar{H}}\Big)^{\frac{\beta}{\beta(\sigma-1)-1}}$$

After some algebra,

$$\phi_* = \psi \Big[N^{\beta-\chi(\eta,\sigma)}\Big(\frac{w^{1-\beta}}{\bar{H}^\beta}\Big)\Big]^{\frac{1}{1+\kappa(\beta-\chi)}}$$

with $\psi = \Big(\lambda^{(1-\kappa\chi)(\frac{1}{\sigma-1}-\beta)}\zeta^{\beta(1+\frac{\kappa\eta}{\sigma-1})}\Big)^{\frac{\sigma-1}{1+\kappa(\beta-\chi)}} > 0$. We arrive at the results in Proposition 1.

2. Trade Equilibrium: Aggregate Variables

I show the case in country $i$. The average productivity is derived by plugging (1.35) into APC,

$$\tilde{\phi_{it}} = \frac{\psi}{\rho\lambda^\theta}\Big[\bar{N}_i^{\beta-\chi(\eta,\sigma)}\Big(\frac{w_i^{1-\beta}}{\bar{H}_i^\beta}\Big)\Big]^{\frac{1}{\theta+\beta\kappa}}\Big[1+\Big(\frac{w_i}{w_j}\Big)^{\kappa(1-\beta)}\Big]^{\frac{\beta-\chi}{\theta+\beta\kappa}}.$$

Plugging (1.35) into CPC, the mass of active firms is

$$M_{it} = \frac{\bar{N}_i^{\frac{1}{\theta+\beta\kappa}}}{\psi^\kappa}\Big(\frac{\bar{H}_i^\beta}{w_i^{1-\beta}}\Big)^{\frac{\kappa}{\theta+\beta\kappa}}\Big[1+\Big(\frac{w_i}{w_j}\Big)^{\kappa(1-\beta)}\Big]^{\frac{\kappa(\chi-\beta)}{\theta+\beta\kappa}}.$$

Since $Y_{it} = M_{it} r_{it}(\tilde{\phi}_{it}) = M_{it}\frac{\kappa\sigma}{\kappa-\sigma+1}$, total final output produced $Y_{it}$ becomes

$$Y_{it} = \Big(\frac{\kappa\sigma}{\kappa-\sigma+1}\Big)\frac{\bar{N}_i^{\frac{1}{\theta+\beta\kappa}}}{\psi^\kappa}\Big(\frac{\bar{H}_i^\beta}{w_i^{1-\beta}}\Big)^{\frac{\kappa}{\theta+\beta\kappa}}\Big[1+\Big(\frac{w_i}{w_j}\Big)^{\kappa(1-\beta)}\Big]^{\frac{\kappa(\chi-\beta)}{\theta+\beta\kappa}}.$$

Using $W_{it} = \rho M_{it} r_{it}(\tilde{\phi}_{it})$, total labor costs $W_{it}$ are

$$W_{it} = \Big(\frac{\kappa(\sigma-1)}{\kappa-\sigma+1}\Big)\frac{\bar{N}_i^{\frac{1}{\theta+\beta\kappa}}}{\psi^\kappa}\Big(\frac{\bar{H}_i^\beta}{w_i^{1-\beta}}\Big)^{\frac{\kappa}{\theta+\beta\kappa}}\Big[1+\Big(\frac{w_i}{w_j}\Big)^{\kappa(1-\beta)}\Big]^{\frac{\kappa(\chi-\beta)}{\theta+\beta\kappa}}.$$

Unemployment rate, $U_{it} = 1-(1-\beta)\frac{W_{it}}{w_i\bar{L}_i}$, is

$$U_{it} = 1 - \frac{\kappa(1-\beta)(\sigma-1)}{(\kappa-\sigma+1)\psi^{\kappa}\bar{L}_i}\left[\frac{\bar{N}_i\bar{H}_i^{\beta\kappa}}{w_i^{\theta+\kappa}}\right]^{\frac{1}{\theta+\beta\kappa}}\left[1+\left(\frac{w_i}{w_j}\right)^{\kappa(1-\beta)}\right]^{\frac{\kappa(\chi-\beta)}{\theta+\beta\kappa}}.$$

For country $j$, the other two equations are derived by changing notations from $i$ to $j$ and from $j$ to $i$.

3. Generalized Lemma 1

The model is built on the two country case. Two lemmas can be further generalized in many country case. In the case of two country case, write down four equations: two PMCs and two ZCPs.

$$M_t^{\frac{1-\eta}{\sigma-1}} = \frac{w_i^{1-\beta}}{\rho}\left(\frac{\kappa}{\kappa-\sigma+1}\right)^{\frac{1}{1-\sigma}}\frac{s_{it}^{\beta}}{\phi_{it}^{*}} \qquad (a)$$

$$M_t^{\frac{1-\eta}{\sigma-1}} = \frac{w_j^{1-\beta}}{\rho}\left(\frac{\kappa}{\kappa-\sigma+1}\right)^{\frac{1}{1-\sigma}}\frac{s_j^{\beta}}{\phi_{jt}^{*}} \qquad (b)$$

$$s_{it}^{\beta} = \left(\frac{\rho}{w_i^{1-\beta}}\right)^{\frac{\beta(\sigma-1)}{\beta(\sigma-1)-1}}\left(\frac{\beta\sigma\rho}{\bar{H}_i}\right)^{\frac{\beta}{1-\beta(\sigma-1)}}M_t^{\frac{-\eta\beta}{\beta(\sigma-1)-1}}[\phi_{it}^{*}]^{\frac{\beta(\sigma-1)}{\beta(\sigma-1)-1}} \qquad (c)$$

$$s_{jt}^{\beta} = \left(\frac{\rho}{w_j^{1-\beta}}\right)^{\frac{\beta(\sigma-1)}{\beta(\sigma-1)-1}}\left(\frac{\beta\sigma\rho}{\bar{H}_j}\right)^{\frac{\beta}{1-\beta(\sigma-1)}}M_t^{\frac{-\eta\beta}{\beta(\sigma-1)-1}}[\phi_{jt}^{*}]^{\frac{\beta(\sigma-1)}{\beta(\sigma-1)-1}} \qquad (d)$$

Plugging (c) into (a),

$$M_t^{\frac{1-\eta}{\sigma-1}+\frac{\eta\beta}{\beta(\sigma-1)-1}} = \left(\frac{w_i^{1-\beta}}{\rho}\right)^{\frac{1}{1-\beta(\sigma-1)}}\left(\frac{\beta\sigma\rho}{\bar{H}_i}\right)^{\frac{\beta}{1-\beta(\sigma-1)}}\left(\frac{\kappa}{\kappa-\sigma+1}\right)^{\frac{1}{1-\sigma}}[\phi_{it}^{*}]^{\frac{1}{\beta(\sigma-1)-1}}.$$

Similarly, plugging (d) into (b),

$$M_t^{\frac{1-\eta}{\sigma-1}+\frac{\eta\beta}{\beta(\sigma-1)-1}} = \left(\frac{w_j^{1-\beta}}{\rho}\right)^{\frac{1}{1-\beta(\sigma-1)}}\left(\frac{\beta\sigma\rho}{\bar{H}_j}\right)^{\frac{\beta}{1-\beta(\sigma-1)}}\left(\frac{\kappa}{\kappa-\sigma+1}\right)^{\frac{1}{1-\sigma}}[\phi_{jt}^{*}]^{\frac{1}{\beta(\sigma-1)-1}}.$$

By equating both equations right above,

$$1 = \left(\frac{\bar{H}_j}{\bar{H}_i}\right)^{\frac{\beta}{1-\beta(\sigma-1)}} \left(\frac{w_i^{1-\beta}}{w_j^{1-\beta}}\right)^{\frac{1}{1-\beta(\sigma-1)}} \left(\frac{\phi_{it}^*}{\phi_{jt}^*}\right)^{\frac{1}{\beta(\sigma-1)-1}}.$$

That is,

$$\frac{\phi_{it}^*}{\phi_{jt}^*} = \left(\frac{\bar{H}_j}{\bar{H}_i}\right)^{\beta} \left(\frac{w_i}{w_j}\right)^{1-\beta}.$$

The ratio of two cutoff productivities can be expressed as the ratio of variable costs from both countries: $\phi_{jt}^*/\phi_{it}^* = s_{jt}^{\beta} w_j^{1-\beta} / s_{it}^{\beta} w_i^{1-\beta}$. This implies relative skilled workers' wage equalization which is Generalized Lemma 1.

Using Equation (1.40),

$$s_{it} = \left(\frac{\mu}{\lambda}\right)^{\frac{\theta\vartheta}{\theta+\beta\kappa}} \left[\frac{\bar{N}_i}{\bar{H}_i^{\theta}}\left(\left(\frac{1}{w_i}\right)^{\kappa(1-\beta)} + \left(\frac{\bar{N}_j}{\bar{N}_i}\right)\left(\frac{\bar{H}_j}{\bar{H}_i}\right)^{\beta\kappa}\left(\frac{1}{w_j}\right)^{(1-\beta)\kappa}\right)\right]^{\frac{\mathbf{1}}{\theta+\beta\kappa}}$$

Taking the ratio of skilled worker's wages between two countries,

$$\frac{s_{it}}{s_{jt}} = \frac{\left[\bar{N}_i\left(\frac{1}{w_i}\right)^{\kappa(1-\beta)}\left(\frac{1}{\bar{H}_i}\right)^{1-\kappa\chi} + \bar{N}_i\left(\frac{\bar{H}_j}{\bar{H}_i}\right)^{\kappa\beta}\left(\frac{1}{w_i}\right)^{\kappa(1-\beta)}\left(\frac{1}{\bar{H}_i}\right)^{1-\kappa\chi}\right]^{\frac{1}{\theta+\beta\kappa}}}{\left[\bar{N}_j\left(\frac{1}{w_j}\right)^{\kappa(1-\beta)}\left(\frac{1}{\bar{H}_j}\right)^{1-\kappa\chi} + \bar{N}_i\left(\frac{\bar{H}_i}{\bar{H}_j}\right)^{\kappa\beta}\left(\frac{1}{w_i}\right)^{\kappa(1-\beta)}\left(\frac{1}{\bar{H}_j}\right)^{1-\kappa\chi}\right]^{\frac{1}{\theta+\beta\kappa}}}.$$

Factoring $\left(\frac{1}{\bar{H}_i}\right)^{1-\kappa\chi}\left(\frac{\bar{H}_j}{\bar{H}_i}\right)^{\kappa\beta} / \left(\frac{1}{\bar{H}_j}\right)^{1-\kappa\chi}$ out inside the bracket,

$$\frac{s_{it}}{s_{jt}} = \left\{\frac{\left(\frac{1}{\bar{H}_i}\right)^{1-\kappa\chi}\left(\frac{\bar{H}_j}{\bar{H}_i}\right)^{\kappa\beta}}{\left(\frac{1}{\bar{H}_j}\right)^{1-\kappa\chi}}\right\}^{\frac{1}{\theta+\beta\kappa}} \left\{\frac{\bar{N}_i\left(\frac{\bar{H}_i}{\bar{H}_j}\right)^{\kappa\beta}\left(\frac{1}{w_i}\right)^{\kappa(1-\beta)} + N_j\left(\frac{1}{w_j}\right)^{\kappa(1-\beta)}}{\bar{N}_j\left(\frac{1}{w_j}\right)^{\kappa(1-\beta)} + \bar{N}_i\left(\frac{\bar{H}_i}{\bar{H}_j}\right)^{\kappa\beta}\left(\frac{1}{w_i}\right)^{\kappa(1-\beta)}}\right\}^{\frac{1}{\theta+\beta\kappa}}.$$

Canceling the second bracket,

$$\frac{s_{it}}{s_{jt}} = \left\{\frac{\left(\frac{1}{\bar{H}_i}\right)^{1-\kappa\chi}\left(\frac{\bar{H}_j}{\bar{H}_i}\right)^{\kappa\beta}}{\left(\frac{1}{\bar{H}_j}\right)^{1-\kappa\chi}}\right\}^{\frac{1}{1+\kappa(\beta-\chi)}}$$

which is $\frac{s_{it}}{s_{jt}} = \frac{\bar{H}_j}{\bar{H}_i}$. When $\frac{\bar{H}_j}{\bar{H}_i} = 1$, it indicates Lemma 1.

4. Generalized Lemma 2

Using $M_t = \left[1 + \left(\frac{\bar{N}_j}{\bar{N}_i}\right)\left(\frac{\phi_{it}^*}{\phi_{jt}^*}\right)^{\kappa}\right] M_{it}$ and Generalized Lemma 1,

$$M_t = \left[1 + \left(\frac{\bar{N}_j}{\bar{N}_i}\right)\left(\frac{\bar{H}_j}{\bar{H}_i}\right)^{\kappa\beta}\left(\frac{w_i}{w_j}\right)^{\kappa(1-\beta)}\right] M_{it}.$$

Similarly, the case in country $j$ is derived. If we assume $\frac{\bar{H}_j}{\bar{H}_i} = 1$ and $\frac{\bar{N}_j}{\bar{N}_i} = 1$, it indicates Lemma 2.

5. Generalized Lemma 3

$\Gamma_i + \Gamma_j = 1$. So, $\Gamma_i = 1 - \frac{M_{jt}}{M_t} = \frac{M_t - M_{jt}}{M_t}$. That is, $\Gamma_i = \frac{\frac{M_t}{M_{jt}} - 1}{\frac{M_t}{M_{jt}}}$. Using the result in Generalized Lemma 2, $\frac{M_t}{M_{jt}} = 1 + \left(\frac{\bar{N}_i}{\bar{N}_j}\right)\left(\frac{\bar{H}_i}{\bar{H}_j}\right)^{\kappa\beta}\left(\frac{w_j}{w_i}\right)^{\kappa(1-\beta)}$. Therefore, $\Gamma_i = \frac{\left(\frac{\bar{N}_i}{\bar{N}_j}\right)\left(\frac{\bar{H}_i}{\bar{H}_j}\right)^{\kappa\beta}\left(\frac{w_j}{w_i}\right)^{\kappa(1-\beta)}}{1 + \left(\frac{\bar{N}_i}{\bar{N}_j}\right)\left(\frac{\bar{H}_i}{\bar{H}_j}\right)^{\kappa\beta}\left(\frac{w_j}{w_i}\right)^{\kappa(1-\beta)}}$. $\Gamma_j$ is similarly derived.

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# CHAPTER 2
# Immigration, Firm Heterogeneity, and Welfare

## 2.1 Introduction

Over the last three decades, a large literature has emerged investigating immigration's impact on host countries, particularly on native workers' wages. The public believes that immigrants depress wages by competing with native workers. This claim can be understood by assuming that immigration shifts the labor supply for a *given* labor demand and a *given* labor supply of native workers in the canonical model of labor supply and demand. A majority of studies, however, fail to support the claim. According to Peri (2014) who reviews 27 empirical studies published 1982-2013, the largest concentration of estimated effects clusters around zero. To understand why so many empirical studies have found very small to no effect of immigration on native worerks' wages, several explanations have appeared, focusing on natives: complementarities between natives and immigrants (Ottaviano and Peri 2007) and specialization of natives (Peri and Sparber 2009; Peri 2012).[1] Several authors provide explanations by focusing on firms: exit/entry (Dustman and Glitz 2011; Olney 2012 ) and technology adoption (Lewis 2011). While insightful and persuasive, those studies assume that firms are either absent or homogeneous, and provide no rigorous theoretical model in a general equilibrium.

Different types of immigration and governmental immigration policies also motivate this paper. Immigration can be classified by two key characteristics. The first is whether the immigration involves skilled or unskilled labor. The second is the legality of the immigration. With respect

[1]The pertinent literature being so voluminous, it is not feasible to review it in its entirety. A few empirical studies have enjoyed much attention. In particular, Borjas (2003) and Borjas and Katz (2007), focusing on native workers' responses, emphasize that as new immigrants arrive into a local labor market, native workers move out. Because of this effect, native wages are unaffected. However, Card (2001) and Card and Lewis (2007) failed to find evidence that natives respond to immigration by moving to areas with fewer immigrants.

to immigration policies, for example, the United States allows highly-skilled immigrants to enter the country on firm-sponsored H-1B visas established by the Immigration Act of 1990, while it bans firms from hiring illegal immigrants by the Immigration Reform and Control Act of 1986. Immigration types and policies are related closely to firm-level decisions. Given immigration policies, a firm's willingness to hire immigrants will be a function of the expected benefit from hiring *versus* the expected cost; these benefits and costs are likely to vary by firm productivity. Profit-maximizing firms encounter a nontrivial decision problem in response to immigration. The present paper places heterogeneous firms at the center of analysis and examines firm adjustment in response to immigration. The contribution of this paper is to shed light on the channel of average productivity effects, taking into account firm adjustment, in understanding immigration's welfare effects.

This paper adds heterogeneous immigrants to the standard monopolistic competition model with heterogeneous firms by Melitz (2003). It commences by analyzing equilibrium outcomes in the absence of immigration as a benchmark model. Introducing immigration into the benchmark model creates new problems for heterogeneous firms because immigrants who are heterogeneous in terms of ability and legality can influence output and alter cost structure at the firm-level. A firm hiring highly-skilled immigrants can produce a larger output but has to spend an additional fixed hiring cost and may have to pay them higher wages compared to native workers. In contrast, a firm hiring illegal immigrants can produce a smaller output but save some fixed costs for production and pay lesser wages to them than to native workers. The present model considers a firm with employment choices among native workers, highly-skilled immigrants, and illegal immigrants and assumes all workers are perfectly substitutable in production. This paper's interest is on a possible equilibrium that high-productivity firms hire highly-skilled immigrants, middle-productivity firms demand native workers, and low-productivity firms employ illegal immigrants. This equilibrium arises if highly-skilled immigrants are the most competitive workers, and illegal immigrants are the least competitive in production, whereas illegal immigrants have their advantage in fixed cost compared with other workers. Intuitively, these conditions make sense because, on one hand, marginal cost plays a larger role than fixed cost as firms need to produce more output, but, on the other hand, fixed cost also plays a role in determining profitability.

This paper's main results are summarized as follows: First, highly-skilled immigration forces the lowest-productivity firms to exit the market, while illegal immigration leads low-productivity firms to enter the market. Second, average productivity firms benefit from highly-skilled immigration but are harmed by illegal immigration in terms of profits. Last, native workers' welfare depends on average productivity effects driven by relative forces from the two immigration types. The underlying logic of the main results is a firm's exit and entry in the long-run, which affects variations in average productivity of active firms in the market. Intuitively, it makes sense that highly-skilled immigration raises average productivity of active firms. Firms that do not hire highly-skilled immigrants are hit by positive demand shock on one hand and by negative wage shock on the other, which forces the least productive of them to exit the market. Thus, productivity cutoff increases, as does average productivity. Owing to the larger total output and the fewer active firms in the market, firms hiring native workers can benefit from highly-skilled immigration. In contrast, illegal immigration impacts in the opposite direction. The least-productivity firms have a stronger incentive to employ illegal immigrants in production. Even if a small number of those firms begin to hire illegal immigrants, illegal immigration worsens average productivity of firms in the product market due to the entry of low-productivity firms. A change in average productivity alters aggregate labor demand for native workers. Under an inelastic labor supply of native workers, aggregate labor demand for native workers would determine their equilibrium wage. As a result, highly-skilled immigration would benefit native workers yet, simultaneously, illegal immigration would harm them.

This paper closely relates to a small but growing literature on firm heterogeneity and immigration.[2] Existing papers rely on Melitz (2003) who considers heterogeneous firms in the model of

[2]To the best of this author's knowledge, only one paper. Giovanni *et al*. (2014) considers both immigration and emigration in the world economy. They use a multi-sector Melitz model of the world economy calibrated aggregate and firm-level data and evaluate the global welfare impact of observed levels of migration. Their main finding is that the long-run impact of observed levels of migration is large and positive for the remaining natives of both the main sending-countries and the main receiving-ones. Main insight of the result is the so-called love-of-variety, *i.e.*, an increased number of varieties consumed across countries.

Krugman (1980). In the spirit of Peri (2009) who considers task specialization, Haas and Lucht (2013) study imperfect substitutability between immigrants and native workers at the firm-level. They show that only firms hiring above a certain immigrant share can survive in the market, due to wage cost advantages. However, their finding is useful only in explaining wage disparities of natives across states having different proportions of immigrants. Rather than focusing on low-skilled immigrants, Strezhnev (2014) considers highly-skilled immigration quotas and heterogeneous firms, concluding that only a fraction of firms hire immigrant workers, and that only high-productivity firms will benefit from an increase in quota numbers. In fact, his paper lacks the insight of a positive productivity effect that high productivity firms hiring high-skilled immigrants would generate. Unlike Haas and Lucht (2013), this present paper assumes perfect substitution and focuses on variations in average productivity of active firms. And unlike Strezhnev (2014), this paper suggests a unified framework by incorporating highly-skilled and low-skilled immigrants, in particular, illegal immigrants. In so doing, this paper delineates the whole picture of firms' adjustment in a monopolistic competition model. It shows that a country's exposure to immigration induces resource reallocations among firms and generates variations in average productivity. These results cannot be explained by representative firm models where the average productivity level is exogenously given as the productivity level common to all firms.

One thread of immigration literature relates to highly-skilled immigration. Many authors focus mainly on the link between highly-skilled immigration and innovation (see, *e.g.*, Kerr 2008, 2013 among many others), or the supply-side of highly-skilled immigration (see, *e.g.*, Kerr and Lincoln 2010). Such immigrants bring to the host economy new skills that may spur innovation, ultimately increasing productivity -hence wages also- of native workers. With respect to firm sorting, Gandal, Hanson, and Slaughter (2004) find that highly-skilled immigrants from Russia into Israel were sorted towards more skill-intensive firms. With respect to wage, Peri, Shih, and Sparber (2011), using data over the period 1990-2000 in the U.S., find that a one-percent increase of highly-skilled workers in the total employment increased native workers' wages by 4-6 percent. Consistent with these results, the present paper can provide a theoretical framework to analyze the impact of highly-skilled immigrants on firms and on native workers.

Another thread of the literature is on illegal immigration. Most studies of illegal immigration have focused on policy effectiveness to control illegal immigrants (see, *e.g.*, Ethier 1986; Bond and Chen 1987) or on the supply-side of illegal immigrants (see, *e.g.*, Bandyopadhyay and Bandyopadhyay 1998, Woodland and Yoshida 2006, Hanson 2006). Without exception, however, all authors assume a representative firm in perfect competition. Regarding the labor market, many models concerning illegal immigration use full-employment or minimum wages. When an increase in the number of illegal immigrants is interpreted as an increase in the supply of labor in the economy, it always either presses down wages in models with perfect labor markets, or reduces the probability of being hired among native workers in models with minimum wage. In this way, many authors set up models in which illegal immigrants always harm the welfare of native workers! The present paper is, however, the first to introduce heterogeneous firms in this thread. Nevertheless, it confirms that a negative welfare effect of illegal immigration can be robust in a monopolistic competition model with heterogeneous firms.

The following section presents a benchmark model and characterizes the model's equilibrium outcomes in the absence of immigration. Section 3 introduces immigration and analyzes firms' new problems. Section 4 characterizes product and labor market equilibrium, examines firms' adjustments in response to immigration, and discusses immigration's welfare effect. Section 5 discusses this paper's several assumptions. The final section concludes.

## 2.2 Benchmark Model

The benchmark model builds on the standard monopolistic competition model with heterogeneous firms by Melitz (2003). Consider an economy that is endowed with an exogenous mass of native workers, each supplying one unit of labor in a perfectly competitive labor market. The economy comprises two sectors: a final good sector and a differentiated goods sector. The final good sector produces a homogeneous good $Y$ by assembling all intermediate input varieties without the use of labor under perfect competition. The monopolistic competition sector with $M$ firms produces a continuum of differentiated intermediate goods. The homogeneous final good is chosen as a numeraire.

### 2.2.1 Final output

Final output $Y$, used for consumption as well as investment, is produced according to the CES-production technology:

$$Y = \left( M^{-(1-\rho)} \int_{v \in V} [y(v)]^{\rho} dv \right)^{\frac{1}{\rho}} \tag{2.1}$$

where $V$ denotes the set of all available input varieties with measure $M$, $y(v)$ is the quantity of input variety $v$ employed in the production of $Y$, $\rho \in (0,1)$ controls the constant elasticity of substitution between varieties, $\sigma \equiv 1/(1-\rho)$. Without the term $M^{-(1-\rho)}$ in (2.1), the specification for final output is the standard Dixit-Stiglitz (1977). The inclusion of the term $M^{-(1-\rho)}$ enables the final output technology to exhibit constant returns to scale in the level and the number of varieties. In other words, the specification for final output (2.1) closes down, so-called variety or external-scale effects, an increase in aggregate output due to an increase in the number of input varieties.[3] Closing this channel of influence allows us to evaluate as clearly as possible the welfare of native workers.

Final output producers choose $y(v)$, for all $v$, to maximize their profit under perfect competition:

[3] As discussed in Blanchard and Giavazzi (2003) and Egger and Kreickemeier (2009), if equal amounts of every input $y(v) = \bar{y}$ were to be used in the production of final output, then the production implies $Y/M = \bar{y}$. In a consumption context, external scale effects are translated as love-of-variety as in Krugman (1980) and Melitz (2003).

$PY - \int_{v \in V} p(v)y(v)dv$. Due to free entry, profits of final output producers are driven down to zero. The price index would satisfy $P = \left(M^{-1} \int_{v \in V} p(v)^{1-\sigma} dv\right)^{1/(1-\sigma)}$ in equilibrium. The demand function of intermediate inputs for a variety $v$ is, with $P = 1$ (due to the choice of numeraire),

$$y(v) = \frac{Y}{M} p(v)^{-\sigma} \tag{2.2}$$

where $p(v)$ is the price of variety $v$ and the elasticity of demand for variety $v$ with respect to its price is equal to $\sigma$ in absolute value. Although aggregate output $Y$ is endogenous, it will be treated as exogenous by intermediate input firms since every firm is of negligible size relative to the size of the market. Moreover, every intermediate input firm faces a demand equal to one $M$th of the aggregate output.

### 2.2.2 Intermediate input firms

Firms are identical prior to entry. A firm has to bear an entry cost to enter the differentiated sector. After paying the entry cost, it realizes its productivity $\phi$ from a given distribution $G(\phi)$. Each firm with the same productivity behaves the same and can be distinguished by its idiosyncratic productivity. This paper indexes intermediate input firms solely in terms of their productivity $\phi$ as in Melitz (2003). Once it knows its productivity, a firm chooses whether to exit the market or produce a unique variety under monopolistic competition.

Each firm has to invest $f$ units of final output to start production.[4] In production, labor is the only input. Firms demand native workers with identical skill level or ability. This paper uses skill level and ability interchangeably. Firm production technology $y$ is linear in the amount of labor $l$ and depends on firm productivity level $\phi$ and worker ability level $a$:

$$y = \phi a l$$

[4]In the monopolistic competition model, Krugman (1980) and Melitz (2003) assume that production technology requires a fixed number of workers and some other authors, for example, Helpman and Itskohoki (2010) and Egger *et al.* (2012), assume that production needs a fixed amount of the numeraire good as in the present paper.

where $\phi a$ represents the amount of an intermediate good a native worker can produce in a firm with productivity $\phi$. Also, $\phi a$ can be viewed as the *total* productivity of a firm with productivity $\phi$ hiring native workers with ability $a$.

A firm's total cost involves fixed cost and variable cost in production. The start-up fixed investment cost is $f$ and it is assumed all firms face the same fixed cost. The variable cost includes only wage payment. To hire $l$ number of native workers, a firm has to spend $wl$ where $w$ is the common wage determined in a perfectly competitive labor market. Hence, total cost function of a firm with productivity $\phi$ is

$$c(\phi) = f + wl(\phi)$$

where marginal cost function is $mc(\phi) = w/\phi a$. Marginal cost is constant, but varies by a firm's productivity $\phi$.

Inverse marginal cost is $[mc(\phi)]^{-1} = \phi a/w$, which measures competitiveness in production.[5] Competitiveness in production can be explained by two ways. First, it increases in firm's total productivity $\phi a$, but decreases in unit labor cost $w$. Second, it increases in idiosyncratic firm productivity $\phi$ and worker-competitiveness $a/w$ (defined as worker ability divided by unit labor cost). The second interpretation is useful in the next section. In the benchmark model, worker-competitiveness is unimportant since all firms demand identical native workers and face the same wage rate.

Taking demand (2.2) and wage $w$ as given, intermediate input firms solve the profit maximization problem: $max\, \pi(\phi) = p(\phi)y(\phi) - wl(\phi) - f$. Under monopolistic competition, all firms choose the same mark-up $1/\rho$ over their constant marginal cost. The optimal pricing strategy of a firm with productivity $\phi$ is,

$$p(\phi) = \frac{w}{\rho \phi a}. \tag{2.3}$$

[5]The term *competitiveness* is borrowed from Harrigan and Reshef (2015).

Using (2.2) and (2.3) yields firm revenues

$$r(\phi) = \rho^{\sigma-1}\phi^{\sigma-1}\left(\frac{a}{w}\right)^{\sigma-1}\frac{Y}{M} \tag{2.4}$$

which depend on many variables. First, revenues increase in firm productivity level $\phi$ and worker-competitiveness in production $a/w$. Second, aggregate output influences the demand for variety. Higher total ouput results in higher demand for each variety, thus higher revenues for a firm. Third, the number of input varieties in the market influences the demand for a variety. Thus, a larger number of active firms reduce the demand for each variety, and the revenues of a firm accordingly.

Plugging (2.2)-(2.4) into the profit schedule yields:

$$\pi(\phi) = \frac{r(\phi)}{\sigma} - f \tag{2.5}$$

where $r(\phi)/\sigma$ is variable profit of a firm with productivity $\phi$.

### 2.2.3 Equilibrium

Consider first the determination of productivity cutoff that requires two equilibrium conditions: zero-profit condition and free entry condition. Denote $\phi_E$ as productivity cutoff at which firms initiate production. Using (2.5), the zero-profit condition is $\pi(\phi_E) = 0$. So, any entering firms drawing a productivity level $\phi < \phi_E$ will exit the market and never produce, and active firms with $\phi > \phi_E$ will generate positive profits. To enter the differentiated sector, however, firms must pay entry cost $f_e$ measured in units of final output. There would be no entry of firms if sunk entry cost exceeds expected value of entry, and an infinite entry if the cost is dominated. Thus, sunk entry cost equals expected value of entry in the long-run. Using notations, the free entry condition is $f_e = \int_{\phi_E}^{\infty} \pi(\phi) dG(\phi)$. For analytical tractability, assume a Pareto distribution $G(\phi) = 1 - \phi^{-z}$ where $z > 0$ is a shape parameter and the lower bound of productivity levels is normalized to unity.[6] The corresponding density distribution is provided by $g(\phi) = z\phi^{-z-1}$over $[1, \infty)$. Under

[6]The Pareto distribution generates a good approximation of the distribution of firms' sizes. With a high $z$, the intermediate input sector becomes more homogeneous in the sense that more intermediate input firms are located

the Pareto distribution, the zero-profit condition and the free entry condition determine equilibrium productivity cutoff

$$\phi_E = \left( \frac{\sigma - 1}{z - \sigma + 1} \frac{f}{f_e} \right)^{\frac{1}{z}} \tag{2.6}$$

where the condition $z > \sigma - 1$ is assumed to ensure that the firm size distribution has a finite mean (see Appendix for derivation of (2.6)). Productivity cutoff $\phi_E$ is endogenously determined by economic fundamentals: fixed amount $f_e$ of the numeraire good for entry, start-up fixed investment $f$ for intermediate good production, and constant elasticity of substitution $\sigma$ from final output technology together with the Pareto shape parameter $z$ that measures the concentration of the firm's productivity distribution.

With productivity cutoff $\phi_E$ in (2.6), the *ex post* equilibrium productivity distribution is defined as

$$\mu(\phi) = \begin{cases} \frac{g(\phi)}{1-G(\phi_E)} & if \ \phi > \phi_E \\ 0 & otherwise \end{cases}$$

where the *ex ante* probability of successful entry is $1 - G(\phi_E)$.

Average productivity level

$$\phi_A \equiv \left( \int_{\phi_E}^{\infty} \phi^{\sigma - 1} \mu(\phi) d\phi \right)^{\frac{1}{\sigma - 1}} \tag{2.7}$$

can completely characterize aggregate variables such as price index, aggregate output, aggregate revenues and aggregate profits, $P = p(\phi_A) = 1$, $Y = My(\phi_A)$, $R = Mr(\phi_A)$, and $\Pi = M\pi(\phi_A)$, where aggregate revenue $R$ is equal to aggregate final output $Y$ due to choice of numéraire.

An average productivity firm's revenue and profit can be calculated as

$$r(\phi_A) = \frac{z\sigma f}{z - \sigma + 1}, \quad \pi(\phi_A) = \frac{(\sigma - 1)f}{z - \sigma + 1} \tag{2.8}$$

among least-productive firms.

where the derivation of (2.8) makes use of the Pareto distribution, (2.6) and (2.7).

The labor market clears by equating aggregate labor supply and demand. Assume an exogenous mass $\bar{N}$ of native workers supply their labor inelastically. Firms' labor demands are determined by profit maximization, $l(\phi) = \rho^{\sigma} w^{-\sigma} a^{\sigma-1} \phi^{\sigma-1} Y/M$ from (2.2)-(2.4). The labor market equilibrium condition is

$$\bar{N} = \int_{\phi_E}^{\infty} l(\phi) M \mu(\phi) d\phi.$$

To derive native workers' wage, let us choose the labor market equilibrium as a starting point. The condition provides that total labor income $w\bar{N}$ is a constant share $\rho$ of aggregate revenue $R$, *i.e.*, $w\bar{N} = \rho R$. This result is partly due to monopolistic competition pricing by the constant mark-up $1/\rho$. Then feeding $w\bar{N} = \rho R$ back to the labor market equilibrium condition and using the Pareto distribution arrives at the equilibrium wage of native workers:

$$w = \rho a \phi_A = \rho a \left( \frac{z}{z-\sigma+1} \right)^{\frac{1}{\sigma-1}} \left( \frac{\sigma-1}{z-\sigma+1} \frac{f}{f_e} \right)^{\frac{1}{z}} \tag{2.9}$$

where see Appendix for the derivation. Another way to derive native workers' wage is to choose the price index as a starting point. Using the price index, $P = p(\phi_A) = 1$, native workers' wage can be derived as in (2.9).

To complete the characterization of equilibrium outcomes, return to aggregate output $Y$. As mentioned above, final output is used for consumption and investment. In equilibrium, final output producers yield aggregate output $Y = Mz\sigma f/(z-\sigma+1)$ from (2.8). Workers receive $w\bar{N}$ as total labor income and consume a fraction $\rho$ of aggregate output $Y$. All active firms use $Mf$ units of final output as a start-up fixed investment cost. As a result, the remaining aggregate output $M(\sigma-1)f/(z-\sigma+1)$ $(= Y - \rho Y - Mf)$ equals $\Pi$, which is equivalent to all sunk entry cost spent.

This section is closed by a simple comparative statistic analysis. An increase in the number of native workers has no impact on the distribution of firm productivity level (2.6)-(2.7), thus no impact on native workers' wage as evident in (2.9). This result hinges on the structure of final output technology. In other words, it is due only to no external scale effect. An increased number of native workers not only increases aggregate output, but also raises the mass of active firms due to entry of new firms. At the end of this process, it only raises proportionally the mass of active intermediate input firms as well as the aggregate revenue (2.8), thus $Y/M$ in final output firm's demand for variety would not change. If an external scale effect were allowed (technically, $M^{-(1-\rho)}$ absent in the structure of final output technology), native worker's wage could be higher in a larger country due only to increased number of input variety. The increased welfare implication in a larger country can be troublesome in examining the welfare effect of immigration, because an increased number of native workers can be thought of as an influx of immigrants, in particular, who are identical to native workers in all aspects. That is, an influx of immigrants *per se* can result in improving native worker's welfare via an increase in input variety. This paper closes down this channel of influence to analyze as clearly as possible the welfare effect of immigration.

## 2.3 Immigration and Firm Heterogeneity

This section adds immigration to the benchmark model. The introduction of immigration is not a simple shift of labor supply in the economy. Immigration can be classified by the two key characteristics. The first is whether the immigration involves skilled or unskilled labor. The second is the immigration's legality. The classification relates closely to governmental immigration policies. For example, the United States allows highly-skilled immigrants to enter the country on firm-sponsored H-1B visas established by the Immigration Act of 1990, while, by the Immigration Reform and Control Act of 1986, it bans firms from hiring illegal immigrants. This paper pays particular attention to the two distinct immigration types: legal immigration of highly-skilled workers and illegal immigration of unskilled workers (in short, highly-skilled immigration and illegal immigration).[7]

### 2.3.1 Immigration and cost structure

Immigration has a significant impact on individual firms in many respects, in particular cost structure. First, the legality of immigration distorts start-up fixed investment cost. To begin production, a firm must invest a fixed amount of final output. Consider a firm's decision to hire highly-skilled immigrants. Since highly-skilled immigrants are required to hold firm-sponsored H-1B visas, the firm should apply to the U.S. government to obtain the visas. In the process, the firm has to bear extra fixed hiring costs, raising start-up fixed investment cost.[8] In contrast, consider the firm decides to hire illegal immigrants. Due to the illegal status, the firm can have a strong incentive to cut some start-up fixed investment cost. The under-investment in fixed cost can be thought of as providing unpleasant working conditions, unsafe production facilities, etc, to illegal immigrants. Hence, im-

[7]With respect to the legal immigration of unskilled labor, many aspects of such immigration can be captured by studying illegal immigration in the model of this paper. This paper does not directly deal with the illegal immigration of highly-skilled workers, but can view that illegal immigrants are, on average, unskilled, although they include highly-skilled immigrants.

[8]The plain filing fee for a H-1B visa is high and a firm has to bear the associated legal fees (retention of immigration-specialist attorney, investment of existing firm-employees' time and effort), onerous requirements and substantial paper-work, huge delays, etc (see also Ruiz, Wilson, and Choudhury 2012 and Kerr and Lincoln 2010 for further details).

migration leads the fixed cost structure to satisfy $f_L < f_N < f_H$ where $L$, $N$, and $H$ respectively denote low-skilled illegal immigrant, native worker, highly-skilled legal immigrant. Assume that all firms share the same fixed cost structure. If $f_L$ is close to $f_N$, it can be interpreted as no degree of exploitation of illegal immigrants, while, if $f_H$ is close to $f_N$, it translates that hiring highly-skilled immigrants incurs no additional fixed hiring cost.

Second, skill-level pertaining to immigration affects the firm's total productivity. As explained in the previous section, the firm's total productivity is measured by the product of firm productivity and worker ability. With respect to skill-level, native workers are sharply contrasted with low-skilled illegal immigrants and highly-skilled immigrants, *i.e.*, $a_L < a_N < a_H$.[9] This paper uses skill level as a measure of worker quality or ability and views that ability as identical in each group. Depending on employment strategy, the firm's total productivity differs satisfying $\phi a_L < \phi a_N < \phi a_H$ for all $\phi$. Hence, the specification for firm technology can be written as

$$y_i = \phi a_i l_i$$

where, $i \in \{L, N, H\}$, $l_i$ represents the number of $i$-type workers employed, and $y_i$ gives the amount of a variety that $i$-type workers can produce. Assume that all workers are perfect substitutes.[10]

Third, ability and wage pertaining to immigration affects marginal cost function, thus competitiveness in production (inverse marginal cost). In production, variable costs arise only due to wage payments. Depending on employment strategy, marginal cost structure would differ. Using notations, $mc_i(\phi) = w_i/\phi a_i$ for $i \in \{L, N, H\}$, where the common wage rates, $w_L$, $w_N$, and $w_H$, for illegal immigrants, native workers, and highly-skilled immigrants, respectively. Those wages are determined endogenously in each perfectly competitive labor market.[11] From inverse marginal

[9]According to Peri (2014), immigrants are over-represented among very high and very low levels of education. He also notes that the group of immigrants with very low education includes most of the undocumented workers. In part, this is because very few legal ways of entry exist for foreign workers with low schooling levels.

[10]Many authors believe that highly-skilled immigrants possess complementarities with native workers, while illegal immigrants substitute them. Complementarity acts as a positive contributor to the welfare effect of native workers. This paper holds the conservative view by taking the perfect substitution assumption. If a native worker's welfare improves in the model of this paper, then it must not be due to a complementarity effect.

[11]Under current law in the United States, a highly-skilled immigrant with a H-1B visa must be paid the higher of the prevailing wage or the actual wage paid to similarly employed U.S. citizens. The present paper considers that, in

cost, competitiveness in production differs, across firms, by two factors: idiosyncratic firm productivity $\phi$ and worker-competitiveness $a_i/w_i$. Unlike the benchmark case, worker-competitiveness plays an important role.

In sum, immigration can alter total cost function. Depending on a choice $i \in \{L, N, H\}$, an intermediate input firm with productivity $\phi$ can have total cost function

$$c_i(\phi) = f_i + w_i l_i(\phi).$$

In the benchmark model, a firm cannot choose a particular cost function because the cost structure is unique and is controlled by only its idiosyncratic productivity from the lottery draw.

### 2.3.2 New maximization problems

Now upon entry, and after knowing its productivity, the intermediate input firm can choose to hire workers among illegal immigrants, native workers, and highly-skilled immigrants. Given immigration policies, a firm's willingness to hire immigrants will be a function of the expected benefit from hiring *versus* the expected cost. Intuitively, these benefits and costs are likely to vary by firm productivity. Assume that firms have full information about workers. That is, all firms recognize worker ability and know if a worker is legal or not with no uncertainty.[12] Immigration creates the following two maximization problems for heterogeneous firms.

**Hiring illegal immigrants** is against the law in the U.S. Assume that a firm that wants to use illegal immigrants faces a probability $\delta$ of being detected by the authorities, which may lead to the partial loss of revenues.[13] That is, the firm hiring illegal immigrants expects to have $(1-\delta)r_L(\phi)$,

equilibrium, the highly-skilled immigrant's wage is higher than the native worker's wage, $w_N < w_H$. Among empirical studies, Rivera-Batiz (1999) and Borjas (2005) report that wages for legal workers in the United States is higher than those for illegal workers.

[12]The U.S government has mandated all federal agencies to use so-called E-Verify, an internet-based system comparing information from an employee's Form I-9 and governmental data, to check employment eligibility (see Stark and Jakubek 2012 for more details). If illegal immigrants are perfectly indistinguishable from native workers, it would be the case that all types of workers are paid the same wage (see Ethier 1986 for further discussion).

[13]Many studies, including Ethier (1986), Bond and Chen (1987), and Woodland and Yoshida (2006), consider the detection rate which is increasing in the effort the authorities devote to internal investigation. However, this paper's ultimate focus is not on policy effectiveness to control the number of illegal immigrants. This paper takes the detection rate as given.

instead of $r_L(\phi)$. Taking demand and factor prices as given, intermediate input firms solve the following maximization problem: $max\ \pi_L(\phi) = (1-\delta)p_L(\phi)y_L(\phi) - w_L l_L(\phi) - f_L$.

Profit-maximizing pricing strategy of a firm that hires illegal immigrants is

$$p_L(\phi) = \frac{w_L}{\rho(1-\delta)\phi a_L}. \tag{2.10}$$

Due to the risk of being detected by the authorities, the firm sets a slightly higher mark-up, $1/\rho(1-\delta)$, over their constant marginal cost, $w_L/\phi a_L$.

Using (2.2) and (2.10) yields expected firm revenues,

$$(1-\delta)r_L(\phi) = \rho^{\sigma-1}(1-\delta)^{\sigma}\phi^{\sigma-1}\left(\frac{a_L}{w_L}\right)^{\sigma-1}\frac{\tilde{Y}}{\tilde{M}}$$

where a tilde stands for variables in the presence of immigration to distinguish variables in the benchmark model.

Firms that hire illegal immigrants expect to generate expected variable profits $(1-\delta)r_L(\phi)/\sigma$ and expected profits:

$$\pi_L(\phi) = \frac{(1-\delta)r_L(\phi)}{\sigma} - f_L. \tag{2.11}$$

**Hiring highly-skilled immigrants** leads firms to solve the following maximization problem: $max\, \pi_H(\phi) = \{p_H(\phi)y_H(\phi) - w_H l_H - f_H\}$. The profit-maximizing strategy is then,

$$p_H(\phi) = \frac{w_H}{\rho\phi a_H}, \tag{2.12}$$

where the marginal cost of output for a firm with productivity $\phi$ that hires highly-skilled immigrants is $w_H/\phi a_H$.

Firms that hire highly-skilled immigrants generate revenues,

$$r_H(\phi) = \rho^{\sigma-1}\phi^{\sigma-1}\left(\frac{a_H}{w_H}\right)^{\sigma-1}\frac{\tilde{Y}}{\tilde{M}}$$

and profits

$$\pi_H(\phi) = \frac{r_H(\phi)}{\sigma} - f_H. \tag{2.13}$$

In sum, firms solve a nontrivial decision problem $max\{0, \pi_L(\phi), \pi_N(\phi), \pi_H(\phi)\}$ using profit functions (2.5), (2.11) and (2.13) given immigration policies. In the economy with heterogeneous firms, it would be clear that some firms find highly-skilled immigrants profitable, while some other firms find illegal immigrants profitable. Any model with representative firms fail to explain different incentives in employment decisions across firms. Firm heterogeneity plays a critical role in investigating the optimal firm's responses to immigration.

### 2.3.3 Productivity cutoffs

This paper focuses on equilibrium in which all native workers and two types of immigrants are fully-employed. In equilibrium, firms that hire illegal immigrants could have the lowest productivity, firms that employ highly-skilled immigrants could have the highest productivity, and firms that choose native workers could have productivity in the middle of the distribution. To understand a firm's employment strategy, it would be necessary to investigate conditions from several productivity cutoffs. Since immigrants impact cost structure of firms, marginal and fixed cost would play a critical role in the determination of productivity cutoffs.

Denote $\tilde{\phi}_E$ as the least productivity level with which firms can survive in the market. Firms with a productivity level below $\tilde{\phi}_E$ choose not to produce because, for these firms, variable profits do not cover their fixed cost. Since the lowest fixed cost takes place when hiring illegal immigrants, the **Zero-Profit Condition** can be imposed on equation (2.11) by setting $\pi_L(\tilde{\phi}_E) = 0$,

$$\sigma f_L = \rho^{\sigma-1}(1-\delta)^{\sigma}\phi_E^{\sigma-1}\left(\frac{a_L}{w_L}\right)^{\sigma-1}\frac{\tilde{Y}}{\tilde{M}}. \tag{2.14}$$

Denote $\tilde{\phi}_N$ as the productivity cutoff at which firms are indifferent to hiring between illegal immigrants and native workers in production. This productivity cutoff indicates that firms with productivity below $\tilde{\phi}_N$ would demand illegal immigrants only, but firms with productivity above $\tilde{\phi}_N$ would begin to employ native workers only.[14] From (2.5) and (2.11), marginal firms with productivity $\tilde{\phi}_N$ satisfy $\pi_L(\tilde{\phi}_N) = \pi_N(\tilde{\phi}_N)$ (refer this equality to **Indifference Condition I**). Indifference Condition I means that fixed cost difference, $f_N - f_L$, should be equal to variable profit difference, $[r_N(\tilde{\phi}_N) - (1-\delta)r_L(\tilde{\phi}_N)]/\sigma$, among firms with productivity $\tilde{\phi}_N$ hiring between native workers and illegal immigrants.

Using the Zero-Profit Condition and Indifference Condition I generates productivity ratio of $\tilde{\phi}_N$ and $\tilde{\phi}_E$

$$\frac{\tilde{\phi}_N}{\tilde{\phi}_E} = \left(\frac{f_N - f_L}{f_L}\right)^{\frac{1}{\sigma-1}} \left[\frac{\chi_N}{(1-\delta)^\sigma} - 1\right]^{\frac{1}{1-\sigma}} \tag{2.15}$$

where $\chi_N = \left(\frac{a_N}{w_N}\right)^{\sigma-1} / \left(\frac{a_L}{w_L}\right)^{\sigma-1}$ that measures the competitiveness of a native worker relative to an illegal immigrant. Also, $\chi_N$ is a decreasing function of relative wage of native worker to illegal immigrant. Thus, the productivity ratio $\tilde{\phi}_N/\tilde{\phi}_E$ is not constant, but decreases in $\chi_N$ (or increases in $w_N/w_L$). To pin down the productivity ratio, it is necessary to know the equilibrium relative wage which is determined endogenously in labor markets.

For some firms to find the attractiveness of hiring illegal immigrants, a necessary condition is $\tilde{\phi}_E < \tilde{\phi}_N$.

**Lemma 1**. $\tilde{\phi}_E < \tilde{\phi}_N$ *implies* $1 < \chi_N/(1-\delta)^\sigma < f_N/f_L$.

The first inequality in Lemma 1 is $\left(\frac{a_N}{w_N}\right)^{\sigma-1} > (1-\delta)^\sigma \left(\frac{a_L}{w_L}\right)^{\sigma-1}$. It is required to satisfy the fact that variable profit when hiring native workers is higher than that when hiring illegal immigrants. This condition implies that native worker competitiveness $a_N/w_N$ need not be higher than illegal immigrant competitiveness $a_L/w_L$ due to the risk $\delta$ of being detected by the authorities. The non-negativity of detection rate plays a role as lowering illegal immigrant competitiveness. The second inequality in Lemma 1 can be written as $\left(\frac{a_N}{w_N}\right)^{\sigma-1}/f_N < (1-\delta)^\sigma \left(\frac{a_L}{w_L}\right)^{\sigma-1}/f_L$, providing that native worker competitiveness per fixed cost $f_N$ is less than adjusted illegal immigrant competitiveness

[14]Strictly speaking, firms with a very high productivity are likely to hire highly-skilled immigrants only, instead of native workers.

per fixed cost $f_L$.

Similarly, $\tilde{\phi}_H$ denotes a productivity cutoff at which firms are indifferent to hiring between native workers and highly-skilled immigrants. It means that firms with productivity above $\tilde{\phi}_H$ would begin to employ highly-skilled immigrants only. From (2.5) and (2.13), marginal firms with productivity $\tilde{\phi}_H$ satisfy $\pi_N(\tilde{\phi}_H) = \pi_H(\tilde{\phi}_H)$ (refer this equality to **Indifference Condition II**). Indifference Condition II means that fixed cost difference, $f_H - f_N$, should be equal to variable profit difference, $[r_H(\tilde{\phi}_H) - r_N(\tilde{\phi}_H)]/\sigma$, among firms with productivity $\tilde{\phi}_H$ hiring between highly-skilled immigrants and native workers.

Using the Zero-Profit Condition and Indifference Condition II generates productivity ratio of $\tilde{\phi}_H$ and $\tilde{\phi}_E$

$$\frac{\tilde{\phi}_H}{\tilde{\phi}_E} = \left(\frac{\chi_N}{(1-\delta)^{\sigma}}\right)^{\frac{1}{1-\sigma}} \left(\frac{f_H - f_N}{f_L}\right)^{\frac{1}{\sigma-1}} (\chi_H - 1)^{\frac{1}{1-\sigma}} \tag{2.16}$$

where $\chi_H = \left(\frac{a_H}{w_H}\right)^{\sigma-1} / \left(\frac{a_N}{w_N}\right)^{\sigma-1}$ that measures the competitiveness of a highly-skilled immigrant relative to a native worker. Also, $\chi_H$ is a decreasing function of relative wage of highly-skilled immigrant to native worker. Thus, the productivity ratio $\tilde{\phi}_H/\tilde{\phi}_E$ is not constant, but depends on $\chi_N$ and $\chi_H$. To pin down the productivity ratio, relative wages should be evaluated in equilibrium relative wages determined in labor markets.

For some firms to find the attractiveness of hiring highly-skilled immigrants, a necessary condition is $\tilde{\phi}_E < \tilde{\phi}_H$.

**Lemma 2**. $\tilde{\phi}_E < \tilde{\phi}_H$ *implies* $1 < \chi_H < f_H/f_N$.

The first inequality in Lemma 2 is $\left(\frac{a_H}{w_H}\right)^{\sigma-1} > \left(\frac{a_N}{w_N}\right)^{\sigma-1}$ . It is required to satisfy the fact that variable profit when hiring highly-skilled immigrants is higher than that when hiring native workers. This condition implies that highly-skilled immigrant competitiveness $a_H/w_H$ must be higher than native worker competitiveness $a_N/w_N$. The second inequality in Lemma 2 can be written as $\left(\frac{a_H}{w_H}\right)^{\sigma-1}/f_H < \left(\frac{a_N}{w_N}\right)^{\sigma-1}/f_N$, providing that highly-skilled immigrant competitiveness per fixed cost $f_H$ is less than native worker competitiveness per fixed cost $f_N$.

Lemmas 1 and 2 provide that highly-skilled immigrants are the most competitive workers but illegal immigrants are the least competitive in production, *i.e.*, $\left(\frac{a_H}{w_H}\right)^{\sigma-1} > \left(\frac{a_N}{w_N}\right)^{\sigma-1} > (1-\delta)^{\sigma}\left(\frac{a_L}{w_L}\right)^{\sigma-1}$. This condition makes sense because as firms need to produce more outputs, marginal cost plays a larger role than fixed cost. Illegal immigrants, however, have their advantage in fixed cost compared with other workers, *i.e.*, $\left(\frac{a_H}{w_H}\right)^{\sigma-1}/f_H < \left(\frac{a_N}{w_N}\right)^{\sigma-1}/f_N < (1-\delta)^{\sigma}\left(\frac{a_L}{w_L}\right)^{\sigma-1}/f_L$.[15] The following proposition has been established.

**Proposition 1**. *Under the conditions of Lemmas 1-2, the least productive firms hire illegal immigrants, firms in the middle range of the productivity distribution hire native workers, and the most productive firms hire highly-skilled immigrants.*

Employment strategy is endogenously determined by the initial productivity draw. Highest-productive firms prefer to employ highly-skilled immigrants despite high fixed costs for production, whereas least-productive firms have the incentive to employ illegal immigrants due to low fixed costs for production. Proposition 1 implies that firms are heterogeneous in terms of fixed costs as well as marginal costs in the presence of immigrants. If there is no difference among fixed costs, then at most there is only one type of firms that will choose workers who are most competitive in production. If there is no fixed hiring cost ($f_N = f_H$), then firms with highly-skilled immigrants always generate a higher profit than firms with native workers under the condition. If there is no degree of exploitation ($f_L = f_N$), then firms with illegal immigrants would disappear as long as the worker-competitiveness of native workers is stronger than that of illegal immigrants.

Firm-specific variables can be affected by relative worker-competitiveness in the presence of immigration. For example, a firm's revenue can be written as relative to revenue of the least productivity firm. For $\phi' \in [\tilde{\phi}_E, \tilde{\phi}_N)$, $\phi'' \in [\tilde{\phi}_N, \tilde{\phi}_H)$, and $\phi''' \in [\tilde{\phi}_H, \infty)$,

$$\frac{(1-\delta)r_L(\phi')}{(1-\delta)r_L(\tilde{\phi}_E)} = \left(\frac{\phi'}{\tilde{\phi}_E}\right)^{\sigma-1}, \quad \frac{r_N(\phi'')}{(1-\delta)r_L(\tilde{\phi}_E)} = \left(\frac{\phi''}{\tilde{\phi}_E}\right)^{\sigma-1}\frac{\chi_N}{(1-\delta)^{\sigma}},$$

[15] As long as Lemmas 1-2 hold, $\tilde{\phi}_N < \tilde{\phi}_H$ satisfies.

$$\frac{r_H(\phi''')}{(1-\delta)r_L(\tilde{\phi}_E)} = \left(\frac{\phi'''}{\tilde{\phi}_E}\right)^{\sigma-1} \frac{\chi_N \chi_H}{(1-\delta)^{\sigma}}.$$

Among firms that demand illegal immigrants, productivity difference measures revenue difference. However, productivity difference alone is insufficient to explain revenue difference across firms choosing a worker group other than illegal immigrants. Productivity difference together with worker-competitiveness difference measures revenue difference between firms that hire illegal immigrants and firms that employ native workers, or between firms that hire illegal immigrants and firms that employ highly-skilled immigrants. This revenue difference enables firms to afford higher fixed costs for production depending on their employment policy.

## 2.4 Equilibrium with Immigration

This section characterizes equilibrium outcomes in the presence of immigration by clearing both product market and labor markets, and compares results from the benchmark model.

### 2.4.1 Product market

Immigration alters expected value of entry, thus the equilibrium number of firms changes in response to immigration to dissipate net profit by firms' exits and entries in the long-run. The free entry condition in the presence of immigration is imposed such that sunk entry cost equals expected value of entry.[16] Under the Pareto distribution, zero profit condition (2.14), two productivity ratios (2.15) and (2.16), and free entry condition, determine zero-profit productivity cutoff

$$\tilde{\phi}_E = \left( \frac{\sigma - 1}{z - \sigma + 1} \frac{\Phi(\chi_N, \chi_H)}{f_e} \right)^{\frac{1}{z}} \tag{2.17}$$

where $\Phi(\chi_N, \chi_H) = \sum_i \theta_i f_i$ is expected fixed cost, for $i \in \{L, N, H\}$, $\theta_i$ is the *ex ante* probability of being a firm that hires $i$-type workers given the *ex ante* probability of successful entry.[17] Productivity cutoff $\tilde{\phi}_E$ is no longer constant and is proportional to expected fixed cost that depends on relative worker-competitiveness $\chi_N$ and $\chi_H$.

Recall that, in the absence of immigration, the expected fixed cost is a constant $f_N$. By comparing zero-profit productivity cutoff in the presence of immigration (2.17) with one in the absence of immigration (2.6), the following lemma is immediate.

**Lemma 3.** *Productivity cutoff ratio $\tilde{\phi}_E / \phi_E$ is proportional to expected fixed cost ratio, i.e., $\frac{\tilde{\phi}_E}{\phi_E} = \left[ \frac{\Phi(\chi_N, \chi_H)}{f_N} \right]^{\frac{1}{z}}$.*

In the product market, equilibrium mass $\tilde{M}$ of total active intermediate input firms is the sum of mass $M_L$ of firms hiring illegal immigrants, mass $M_N$ of firms hiring native workers, and mass

[16] That is, $f_e = \int_{\tilde{\phi}_E}^{\tilde{\phi}_N} \pi_L(\phi) dG(\phi) + \int_{\tilde{\phi}_N}^{\tilde{\phi}_H} \pi_N(\phi) dG(\phi) + \int_{\tilde{\phi}_H}^{\infty} \pi_H(\phi) dG(\phi)$.

[17] In equilibrium, $\theta_i$ is not constant, but depends on $\chi_N$ and $\chi_H$. See also Appendix for derivation of (2.17).

$M_H$ of firms hiring high-skilled immigrants, *i.e.*, $\tilde{M} = M_L + M_N + M_H$.

Average productivity augmented with worker-competitiveness (in short, average productivity) can be defined as

$$\tilde{\phi}_A = \left\{ \frac{1}{\tilde{M}} \left( M_L \frac{(1-\delta)^{\sigma}}{\chi_N} \tilde{\phi}_L^{\sigma-1} + M_N \tilde{\phi}_N^{\sigma-1} + M_H \chi_H \tilde{\phi}_H^{\sigma-1} \right) \right\}^{\frac{1}{\sigma-1}}$$

where $\tilde{\phi}_i^{\sigma-1}$, for $i \in \{L,N,H\}$, is defined as average productivity of all intermediate input firms with $i$-type workers. Unlike the benchmark model, worker-competitiveness augments average productivity which reflects the market shares of all intermediate input firms producing with illegal immigrants, native workers, and highly-skilled immigrants. By the virtue of $\tilde{\phi}_A$, aggregate variables are completely characterized: $\tilde{P} = p_N(\tilde{\phi}_A) = 1$, $\tilde{Y} = \tilde{M} y_N(\tilde{\phi}_A)$, $\tilde{R} = \tilde{M} r_N(\tilde{\phi}_A)$ and $\tilde{\Pi} = \tilde{M} \pi_N(\tilde{\phi}_A)$ where aggregate revenue $\tilde{R}$ is equal to aggregate final output $\tilde{Y}$ due to the choice of num*é*raire.[18]

After some algebra, an average productivity firm's revenue and profit can be calculated as $r_N(\tilde{\phi}_A) = \frac{z\sigma\Phi(\chi_N,\chi_H)}{z-\sigma+1}$ and $\pi_N(\tilde{\phi}_A) = \frac{(\sigma-1)\Phi(\chi_N,\chi_H)}{z-\sigma+1}$.[19] Of interest is a comparison of an average productivity firm's revenue and profit in the presence of immigration, with others in which immigration is absent. The following lemma is immediate.

**Lemma 4.** *Revenue and profit ratio of average productivity firms are proportional to expected fixed cost ratio, i.e.,* $\frac{r_N(\tilde{\phi}_A)}{r(\phi_A)} = \frac{\Phi(\chi_N,\chi_H)}{f_N}$ *and* $\frac{\pi_N(\tilde{\phi}_A)}{\pi(\phi_A)} = \frac{\Phi(\chi_N,\chi_H)}{f_N}$.

High expected fixed costs reflect high probability of being a firm that hires native workers or highly-skilled immigrants. Hence, expected fixed costs are positively related to expected profits from (2.17). In equilibrium, revenue and profit would be proportional to expected fixed cost. As can be seen in (2.17), expected fixed cost depends on two relative worker-competitivenesses,

18 With the solution for $\tilde{\phi}_E$ at hand, the *ex post* distribution of firm productivity can be described thus: $\tilde{\mu}(\phi) = g(\phi)/[1-G(\tilde{\phi}_H)]$ if $\tilde{\phi}_H \leq \phi$, $= g(\phi)/[G(\tilde{\phi}_H)-G(\tilde{\phi}_N)]$ if $\tilde{\phi}_N \leq \phi < \tilde{\phi}_H$, $= g(\phi)/[G(\tilde{\phi}_N)-G(\tilde{\phi}_E)]$ if $\tilde{\phi}_E \leq \phi < \tilde{\phi}_N$, $=0$ otherwise. For example, aggregate revenue is defined as $\tilde{R} = \int_{\tilde{\phi}_E}^{\tilde{\phi}_N} (1-\delta) r_L(\phi) M_L \tilde{\mu}(\phi) d\phi + \int_{\tilde{\phi}_N}^{\tilde{\phi}_H} r_N(\phi) M_N \tilde{\mu}(\phi) d\phi + \int_{\tilde{\phi}_H}^{\infty} r_H(\phi) M_H \tilde{\mu}(\phi) d\phi$.

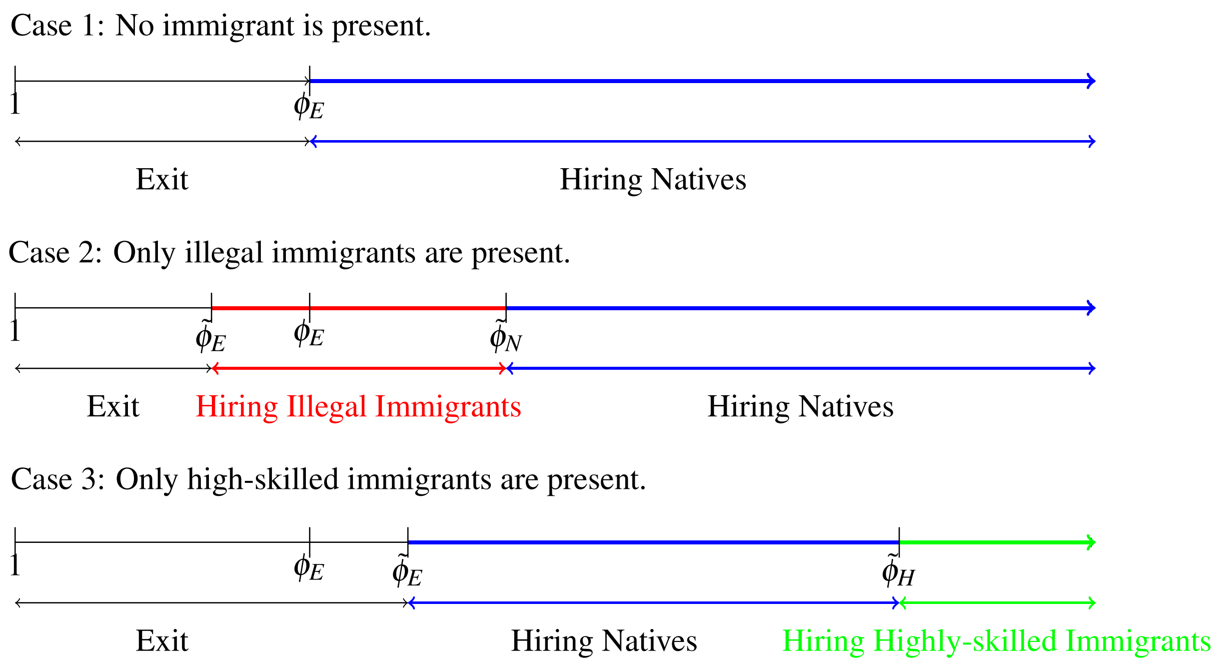
19 The derivation makes use of the Pareto distribution, the *ex post* distribution of firm productivity $\tilde{\mu}(\phi)$, and adding-up conditions for $\tilde{R}$ and $\tilde{\Pi}$. It is realized that average revenue and profit per firm are identical to an average-productivity firm's revenue and profit in equilibrium, *i.e.*, $\bar{r} = r_N(\tilde{\phi}_A)$ and $\bar{\pi} = \pi_N(\tilde{\phi}_A)$.

$\chi_N$ and $\chi_H$, more fundamentally on governmental immigration policies. Consider an economy that disfavors highly-skilled immigrants (say, high $f_H$) and favors illegal immigrants (say, low $\delta$). This economy would have a lower productivity cutoff than in the absence of immigration. On the other hand, an economy that favors highly-skilled immigrants (say, low $f_H$) and disfavors illegal immigrants (say, high $\delta$) would have a higher productivity cutoff than in the absence of immigration. The underlying logic of this result relies on the adjustment of firms via exit/entry. The results from Lemmas 3-4 can be better understood by examining highly-skilled immigration and illegal immigration separately.

Consider an economy that has no illegal immigrants, but native workers and highly-skilled immigrants (see Figure 2.1). No illegal immigration means that either there is no illegal immigrant in the economy, or that even if there are illegal immigrants, no firm hires them. The latter instance takes place when the conditions in Lemma 1 are violated. That is, firms have no incentive to hire illegal immigrants if (i) the competitiveness of illegal immigrants is weaker than that of native workers and (ii) illegal immigrant (adjusted) competitiveness per fixed cost $f_L$ is less than native worker competitiveness per fixed cost $f_N$, $\chi_N/(1-\delta)^\sigma > f_N/f_L$. In that economy, $\tilde{\phi}_E > \phi_E$ from Lemma 3 (see Appendix). In equilibrium, highly-skilled immigration leads low-productivity firms with productivity $\phi \in [\phi_E, \tilde{\phi}_E)$ to exit the market, while high-productivity firms with $\phi \in [\tilde{\phi}_H, \infty)$ begin to employ highly-skilled immigrants. Firms with $\phi \in [\tilde{\phi}_E, \tilde{\phi}_H)$ remain hiring native workers only. Although a small fraction of firms in the product market employ highly-skilled immigrants substituting native workers, the remaining firms can gain extra profits. Intuitively, it makes sense that highly-skilled immigration increases average productivity. Highly-skilled immigration raises the profits of high productivity firms. Firms that do not hire highly-skilled immigrants are hit by positive demand shock on one hand, and by negative wage shock on the other, which forces the least productive of them to exit the market. Thus, productivity cutoff increases, as does average productivity. Due to the larger total output and the less active firms, firms hiring native workers benefit from highly-skilled immigration.

In contrast, consider an economy that has no highly-skilled immigrants, but native workers

Figure 2.1 Immigration and Productivity Cutoffs




Note: $\tilde{\phi}_E$ is least productivity level with which firms can survive in the market. $\tilde{\phi}_N$ is the productivity cutoff at which firms are indifferent to hiring between illegal immigrants and native workers in production. $\tilde{\phi}_H$ is the productivity cutoff at which firms are indifferent to hiring between highly-skilled immigrants and native workers in production. Last, $\phi_E$ is least productivity level with which firms can survive in the absence of immigration.

and illegal immigrants (see Figure 1). No highly-skilled immigration means that either there is no highly-skilled immigrant who wants to work for the economy, or that even if there are highly-skilled immigrants, no firm wants to employ them. In particular, the latter instance arises when the conditions in Lemma 2 are violated. That is, firms have no incentive to hire highly-skilled immigrants if highly-skilled immigrants are not competitive enough relative to native workers. In that economy, $\tilde{\phi}_E < \phi_E$ from Lemma 3 (see Appendix). From (2.15) and (2.17), $\tilde{\phi}_E < \tilde{\phi}_N$.[20] When putting together, $\tilde{\phi}_E < \phi_E < \tilde{\phi}_N$. In equilibrium, low-productivity firms with $\phi \in [\tilde{\phi}_E, \phi_E)$ enter the market and hire illegal immigrants in production, while some existing firms with $\phi \in [\phi_E, \tilde{\phi}_N)$ begin to replace native workers with illegal immigrants. Of course, firms with $\phi \in [\tilde{\phi}_N, \infty)$ remain to employ native workers. In this paper's model, small firms with productivity near $\phi_E$ have stronger incentive to employ illegal immigrants in production.[21] Even if a small fraction of firms

[20]From (2.15) and (2.17), calculation is straightforward regarding the productivity cutoff $\tilde{\phi}_N$ versus $\phi_E$.
[21]Brown *et al.* (2013) report that employing illegal immigrants reduces by 19% a firm's exit hazard.

with productivity near $\phi_E$ begin to hire illegal immigrants, illegal immigration worsens average productivity of firms in the product market due to entry of low-productivity firms. As a result, firms hiring native workers are harmed.

**Proposition 2**. *Illegal immigration lowers productivity cutoff and average productivity, thereby decreasing revenues and profits of average productivity firms, whereas highly-skilled immigration raises productivity cutoff and average productivity, thereby increasing revenues and profits of average productivity firms.*

Firm-level adjustment has gained little attention in discussion of the impact of immigration. Moreover, it will be shown that average productivity effects, taking into account firm adjustment, contribute to explanation of the welfare of native workers.

### 2.4.2 Labor market

There are an exogenous mass $\bar{L}$ of illegal immigrants, $\bar{N}$ of native workers, and $\bar{H}$ of highly-skilled immigrants, each supplying one unit of labor in the economy.[22] Firms' labor demands are determined by profit maximization. By equating aggregate supply and demand, labor market equilibrium conditions are $\bar{L} = \int_{\tilde{\phi}_E}^{\tilde{\phi}_N} l_L(\phi) M_L \tilde{\mu}(\phi) d\phi$ for illegal immigrants, $\bar{N} = \int_{\tilde{\phi}_N}^{\tilde{\phi}_H} l_N(\phi) M_N \tilde{\mu}(\phi) d\phi$ for native workers, and $\bar{H} = \int_{\tilde{\phi}_H}^{\infty} l_H(\phi) M_H \tilde{\mu}(\phi) d\phi$ for highly-skilled immigrants.[23] The labor market equilibrium conditions yield that total labor incomes are a constant share of aggregate revenue, similar to the benchmark model. Feeding $w_L\bar{L} + w_N\bar{N} + w_H\bar{H} = \rho\tilde{R}$ back to the labor market equilibrium condition for native workers and using the Pareto distribution, arrives at the equilibrium wage of native workers.[24] The primary interest of this paper is to understand the welfare effect of immigration. The following Proposition summarizes the wage ratio of native workers before and after immigration.

**Proposition 3.** *Expected fixed cost ratio summarizes wage ratio of native workers in the long-*

[22] A primary entry route for highly-skilled immigrants is via the H-1B program which has an annual quota. For illegal immigrants, the exact number is unknown, but its estimated number is provided (see Passel and Cohn 2011).

[23] See the definition of $\tilde{\mu}$ on footnote 17.

[24] See Appendix for derivation of native worker wage and for the determination of relative wages.

*run, i.e.,* $\frac{w_N}{w} = \left[\frac{\Phi(\chi_N,\chi_H)}{f_N}\right]^{\frac{1}{z}} \left[\frac{\Phi(\chi_N,\chi_H)}{f_L\chi_N/(1-\delta)^{\sigma}}\right]^{\frac{1}{\sigma-1}}$.

**Proof**. See Appendix.

**Corollary**. From Proposition 3, it can be shown that $w_N > w$ under no illegal immigration and $w_N < w$ under no highly-skilled immigration.

**Proof**. The definition of $\Phi$ is $\Phi = \sum_{i\in\{L,N,H\}} \theta_i f_i$ . That is, $\Phi(\chi_N,\chi_H) = f_L[1-(\tilde{\phi}_N/\tilde{\phi}_E)^{-z}] + f_N[(\tilde{\phi}_N/\tilde{\phi}_E)^{-z} - (\tilde{\phi}_H/\tilde{\phi}_E)^{-z}] + f_H(\tilde{\phi}_H/\tilde{\phi}_E)^{-z}$. Consider there is no immigration ($\tilde{\phi}_N = \tilde{\phi}_E$ and $\tilde{\phi}_H \to \infty$). That is, $\Phi = f_N$ which shows the baseline case $w_N = w$. Consider there is no illegal immigration ($\tilde{\phi}_N = \tilde{\phi}_E$). From the calculation in Proposition 3, $w_N/w = (\Phi/f_N)^{\frac{1}{z}+\frac{1}{\sigma-1}}$ under no illegal immigration. In this case, $\Phi(\chi_H) = f_N + (f_H - f_N)(\tilde{\phi}_H/\tilde{\phi}_E)^{-z} > f_N$ where the inequality comes from the fact that $f_H > f_N$. Thus, no illegal immigration implies that $w_N > w$. In contrast, consider there is no highly-skilled immigration ($\tilde{\phi}_H \to \infty$). From the calculation in Proposition 3, $w_N/w = (\Phi/f_N)^{\frac{1}{z}}[\Phi(1-\delta)^{\sigma}/f_L\chi_N]^{\frac{1}{\sigma-1}}$. The first bracket shows $\Phi(\chi_N) = f_L + (f_N - f_L)(\tilde{\phi}_N/\tilde{\phi}_E)^{-z} < f_N$, where the inequality comes from the fact that $\tilde{\phi}_N > \tilde{\phi}_E$ and $f_N > f_L$. In the second bracket, the condition $\Phi(\chi_N) < f_L\chi_N/(1-\delta)^{\sigma}$ is satisfied as long as Lemma 1 holds. Thus, no highly-skilled immigration implies that $w_N < w$.

Return to the instance that there is no illegal immigration. Wage ratio for native workers in Proposition 3 shows $w_N > w$ (See Corollary). To understand this, remember the results from Proposition 2. Highly-skilled immigration raises both zero-profit productivity cutoff and average productivity, and expands total output. Although some native workers who used to work in highest-productivity firms are replaced, an increased total output leads middle-sized firms to demand more native workers. In the labor market, equilibrium wage for native workers increases due to a larger aggregate demand for them. Hence, native workers benefit from highly-skilled immigration.

In contrast, consider no high-skilled immigration. Wage ratio for native workers in Proposition 3 provides $w_N < w$ (See Corollary). Although some small firms hire illegal immigrants and generate higher revenues and profits, newly-entered low productivity firms lower average productivity in the long-run (Proposition 2). Lowered average productivity reflects lower total output, thus lower demand for native workers. Hence, native workers receive a lower wage. In the literature on illegal immigration, without exception, all authors assume a representative firm in perfect competition. With respect to the labor market, many models concerning illegal immigration use full-employment or minimum wages. When an increase in the number of illegal immigrants is interpreted as an increase in the supply of labor in the economy, it always either presses down wages in models with perfect labor markets, or reduces the probability of being hired among native workers in models with minimum wage. In this way, many authors set up models in which illegal immigrants always harm the welfare of native workers! However, the present paper is the first to introduce heterogeneous firms in this thread of literature. Nevertheless, it confirms that a negative welfare effect of illegal immigration can be robust in a monopolistic competition model with heterogeneous firms.

Whenever an economy has both highly-skilled immigration and illegal immigration, the net effect of immigration on native workers' welfare can be ambiguous. It depends on relative forces from the two immigration types, thus $w_N \gtreqless w$ as stated in Proposition 3, since highly-skilled immigration benefits native workers and, at the same time, illegal immigration harms them. Accordingly, it can be predicted that an economy favoring highly-skilled immigrants and disfavoring illegal immigrants would improve native workers' welfare.

What does empirical evidence say about the impact of immigration on native workers' welfare in the United States? Immigration literature presents a wide range of estimates of immigration's effects on native workers' wages. Recent evidence from Ottaviano and Peri (2008, 2012), using Census data 1990-2006, shows that, in the long-run, immigration had small positive effects, on average 0.6 percent, on native workers' wages. Peri (2014) further notes that some of the estimated effects on the wages of less-educated native workers in the U.S. are negative; this holds true only for the 1990s, a particularly low-skill-intensive time; during 2000-2010, net immigration was highly-skilled-intensive. Were net immigration highly-skilled-intensive, then immigration would generate a positive welfare effect on native workers as summarized in Proposition 3. In contrast, Hotchkiss *et al*. (2012), using administrative data from the Georgia Department of Labor, report that illegal immigrants have a negligible impact on native workers' wages. In using the latter paper, a caution: it should not be used as evidence for supporting illegal immigration; native workers' welfare can be affected indirectly by positive effects from highly-skilled immigrants thus counteracting negative effects from illegal immigrants.

## 2.5 Discussion

Many studies explain why immigration has a negligible impact on native wages by complementarities between natives and immigrants, specialization of natives, exit/entry of firms, or technology adoption. It is worth discussing the present paper's modeling perspectives, its corresponding assumptions, and related main results.

First, the present paper assumes that all workers are perfectly substitutable. Many authors pay attention to the complementarity between native workers and immigrants. In principle, the property of complementarity positively contributes to native workers' welfare. Among empirical studies on immigration, Ottaviano and Peri (2007) conclude that immigrants and native workers are imperfect substitutes, whereas Borjas, Grogger, and Hanson (2008) reject that hypothesis and conclude that they are perfect substitutes. By looking at only highly-skilled immigrants, Peri and Sparber (2011) conclude that highly-skilled immigrants and native workers are imperfect substitutes. Campos-Vazquez (2008) provides, at the level of firms, an elasticity of substitution between immigrants and natives at between 10 and 15, consistent with estimates based on local and national labor market comparisons. To avoid this controversy and to render the analyses clear, this present paper's stance is conservative, by taking a perfect substitution assumption: a negative welfare effect is attributed to both negative productivity effects and a substitution effect from illegal immigration; however, a positive welfare effect is driven by highly-skilled immigration, although substitutable.

Second, specialization of native workers due to the introduction of immigration is ignored by assuming perfect substitution between native workers and immigrants. The possibility of specialization is ignored because of analytic clarity, not because of little importance. Peri and Sparber (2009) find that native workers specialize in different tasks demanding communications and language skills, while immigrants gravitate to manual work. In this way, task specialization helps natives upgrade their jobs and protect their wages from immigrant competition. Assuming that native workers have comparative advantages in a task would help enhancing their welfare when immigrants are allowed to work. In essence, the property of specialization also acts as a positive contributor to native workers' welfare. Thus, this paper decides to close down this channel of influence in firm production.

Third, exit/entry of firms is essential to explain firm adjustment in response to immigration. Recently, Olney (2012) uses data on firms in U.S. Metropolitan Statistical Areas 1998-2008, and data on immigration from the Current Population Survey, to study immigration's impact on firms' expansions. His finding is that low-skilled immigration has significant positive impact on the number of small firms, yet no impact on the number of medium or large firms. Dustman and Glitz (2011) use administrative data, comprising the entirety of firms in Germany 1985-1995. In contrast to Olney (2012) who has only low-skilled immigrants, their data includes net overall inflow of immigration by different educational attainment, thus considers both low-skilled and highly-skilled immigration. They emphasize that the creation and destruction of firms are an important channel in the overall adjustment to local labor supply shock. Both those papers are well understood in a simple structure of the present model in the sense that low-skilled immigration (in particular illegal immigration) causes entry of lower productivity firms, thereby increasing the number of small firms. Both highly-skilled and low-skilled immigration would yield creation and destruction of firms during transition to a new equilibrium because highly-skilled immigration forces small firms to exit the market while low-skilled immigration (in particular illegal immigration) leads small firms to enter the market. Beyond these empirical studies, the present paper identifies an additional channel, namely average productivity effects, which contribute to predicting an average-productivity firm's performances. In the setting of the present model, it is predicted that an economy that favors highly-skilled immigrants and disfavors illegal immigrants is likely to benefit average-productivity firms by increasing their revenue and profit.

Fourth, in this paper, the firm's problem can be viewed as a technology choice problem. For example, Yeaple (2005) considers that homogeneous firms can choose to produce with two different technologies that feature a constant marginal cost and a fixed cost. High-technology is more skill-intensive than is low-technology. Acquisition of high-technology requires higher fixed cost in payments for technology adoption and capital goods that embody new technologies, but guarantees lower marginal cost (see also Busto 2011). In the model of this present paper, employing highly-skilled immigrants or illegal immigrants can be translated into, for example, that an increase in the supply of low-skilled workers, due to entry of illegal immigrants, may push firms to adopt more low-technology in place of more high-technology. On the other hand, an increase in the supply of highly-skilled workers, due to entry of highly-skilled immigrants, may encourage firms to adopt more high-technology in place of more low-technology. The key difference between Yeaple (2005) and the present paper is that cost structure (fixed cost vs. marginal cost) is altered by immigration in the present paper, while his paper assumes that heterogeneous technologies that feature a marginal cost and a fixed cost are exogenously given.

From the perspective of employment policy, the present paper closely relates to Davidson and Matusz (2014) who study, depending on firm productivity, searching strategy toward a worker (as a fixed unit in production) with heterogeneous abilities. Their model shows that higher productivity firms find it profitable to incur higher fixed costs associated with higher-quality fixed inputs due to variable cost reduction and the added fixed costs may not be worth it to low-productivity firms that produce only a small output. Although results from their model may seem to resemble mine, the ultimate focus of each differs: their model is set up to study the impact of trade liberalization on searching strategy, while mine is constructed to explain the welfare effect of immigration.

This paper closes by discussing an essential assumption: the production function for the final output exhibits no external scale effects. External scale effects mean that an increase in the number of intermediate input varieties raises aggregate output in a production context. In a consumption context, consumers can enjoy higher welfare, love-of-variety, due to immigration. Since immigrants are not simply workers but consumers, they can increase the host country's demand for goods and services and lead to a greater demand, via love-of-variety, for labor, thus increased wages and employment in the economy. This may be true but, to make the analysis as clear as possible, the present paper closes the channel of external scale effects (or love-of-variety). In so doing, this paper shows that a positive welfare effect of immigration is attributed to positive average productivity effects driven by highly-skilled immigration, rather than to external scale effects.

## 2.6 Conclusion

This paper poses and responds to the following questions: How do firms respond to immigration? Taking firms' adjustments into account, how does immigration affect native workers' welfare? This paper added highly-skilled immigration and illegal immigration to the monopolistic competition model with heterogeneous firms. It shows that highly-skilled immigrants work for high-productivity firms, whereas illegal immigrants are employed by low-productivity firms. This equilibrium arises when highly-skilled immigrants have comparative advantages in reducing marginal cost in production and illegal immigrants have comparative advantages in reducing fixed cost for production. Under the conditions, highly-skilled immigration drives low-productivity firms out of the market, thereby *increasing* average productivity, while illegal immigration leads potential low-productivity firms to enter the market, thereby *decreasing* average productivity in the long-run. A change in average productivity alters aggregate output, which in turn determines native workers' equilibrium wage. This paper shows that highly-skilled immigration benefits native workers, and that illegal immigration harms them. In sum, the net effect of immigration on native workers' welfare depends on relative forces from two immigration types.

In many respects, this paper's analysis is parsimonious. First, this paper assumes that native workers are identical in terms of ability. It would be interesting to extend the model to the case that there are distributions of worker abilities rather than a single ability. Second, a full employment condition in the labor market, although convenient from the perspective of analytical tractability, is simplistic. In reality, one could think of many possible sources of unemployment, such as search and matching, minimum wages, etc. With respect to welfare analysis for native workers, constructing unemployment for native workers would be an interesting avenue. Third, firms decide within the parameters of the government's immigration policy. While this paper did not attempt to build government objectives, it is in principle possible to use this paper's model to analyze policy efficacy between a) promoting highly-skilled immigration and b) banning illegal immigration, in one framework. Finally, it also is possible to extend the model to one with multiple sectors, to study how immigrants within an economy react to government's immigration-policy shocks. Variations in governmental immigration policy affect immigrants in switching among sectors and, in turn, also affect native workers. It is neither this paper's intention, nor within its scope, to provide a formal discussion of such effects.

APPENDIX

# Appendix

1. Productivity cutoff: derivation of (2.6)

Use the Pareto distribution $G(\phi) = 1 - \phi^{-z}$ and $g(\phi) = z\phi^{-z-1}$. From the free entry condition,

$$\begin{aligned} f_e &= \int_{\phi_E}^{\infty} \Big\{ \frac{r(\phi)}{\sigma} - f \Big\} dG(\phi) \\ &= \int_{\phi_E}^{\infty} f \Big\{ \Big( \frac{\phi}{\phi_E} \Big)^{\sigma-1} - 1 \Big\} dG(\phi) \\ &= \frac{f}{\phi_E^{\sigma-1}} \frac{z}{\sigma - z - 1} \phi^{\sigma-z-1} \mid_{\phi_E}^{\infty} + \phi^{-z} \mid_{\phi_E}^{\infty} \\ &= \frac{(\sigma-1)f}{z - \sigma + 1} \phi_E^{-z}. \end{aligned}$$

In the second equality, I use the zero-profit condition. The equilibrium productivity cutoff is $\phi_E^z = \frac{\sigma-1}{z-\sigma+1} \frac{f}{f_e}$.

2. Native workers' wage: derivation of (2.9)

Step1) From the labor market equilibrium condition,

$$\begin{aligned} \bar{N} &= \int_{\phi_E}^{\infty} x(\phi) M \mu(\phi) d\phi \\ &= \int_{\phi_E}^{\infty} \Big\{ \frac{1}{\phi a} \Big( \frac{w}{\rho \phi a} \Big)^{-\sigma} \frac{Y}{M} \Big\} M \mu(\phi) d\phi \\ &= \frac{z}{z - \sigma + 1} \rho^{\sigma} w^{-\sigma} Y a^{\sigma-1} \phi_E^{\sigma-1}. \end{aligned}$$

Multiplying $w$ on both sides, $w\bar{N} = \frac{z}{z-\sigma+1}\rho^{\sigma}w^{1-\sigma}Ya^{\sigma-1}\phi_E^{\sigma-1}$.

Step2) Use zero-profit condition $\frac{Y}{M}\left(\frac{w}{\rho\phi_E a}\right)^{1-\sigma} = \sigma$ yield

$$w\bar{N} = \frac{z\rho}{z-\sigma+1}\sigma M.$$

Step3) Use $R = \int_{\phi_E}^{\infty} r(\phi)M\mu(\phi)d\phi$.

$$\begin{aligned} R &= \int_{\phi_E}^{\infty}\left(\frac{\phi}{\phi_E}\right)^{\sigma-1}\sigma M\mu(\phi)d\phi \\ &= \sigma M\frac{z}{\phi_o^{\sigma-z-1}}\int_{\phi_o}^{\infty}\phi^{\sigma-z-2}d\phi \\ &= \frac{\sigma Mz}{z-\sigma+1}. \end{aligned}$$

Inserting $\sigma M$ to $w\bar{N}$,

$$w\bar{N} = \frac{z\rho}{z-\sigma+1}\sigma M = \frac{z\rho}{z-\sigma+1}Y\frac{z-\sigma+1}{z} = \rho Y.$$

Step 4) Use results step 1) and 3). Then,

$$\begin{aligned} w^{\sigma} &= \frac{z}{z-\sigma+1}\rho^{\sigma}\frac{Y}{\bar{N}}a^{\sigma-1}\phi_E^{\sigma-1} \\ &= \frac{z}{z-\sigma+1}\rho^{\sigma}\left(\frac{w}{\rho}\right)a^{\sigma-1}\phi_E^{\sigma-1}. \end{aligned}$$

Thus,

$$w = \rho a\Big(\frac{z}{z-\sigma+1}\Big)^{\frac{1}{\sigma-1}}\phi_E.$$

Plugging (2.6) arrives at the solution for native workers' wage.

3. Productivity cutoff in the presence of immigration: derivation of (2.17)

From the free entry condition, $f_e = \int_{\tilde{\phi}_E}^{\tilde{\phi}_N} \pi_L(\phi)dG(\phi) + \int_{\tilde{\phi}_N}^{\tilde{\phi}_H} \pi_N(\phi)dG(\phi) + \int_{\tilde{\phi}_H}^{\infty} \pi_H(\phi)dG(\phi)$.

$$\begin{aligned}
\int_{\tilde{\phi}_E}^{\tilde{\phi}_N} \pi_L(\phi)dG(\phi) &= \int_{\tilde{\phi}_E}^{\tilde{\phi}_N}\Big\{\Big(\frac{\phi}{\tilde{\phi}_E}\Big)^{\sigma-1} f_L - f_L\Big\}dG(\phi) \\
&= \frac{f_L}{\tilde{\phi}_E^z}\frac{z}{z-\sigma+1}\Big[1-\Big(\frac{\tilde{\phi}_N}{\tilde{\phi}_E}\Big)^{\sigma-z-1}\Big] - \frac{f_L}{\tilde{\phi}_E^z}\Big[1-\Big(\frac{\tilde{\phi}_N}{\tilde{\phi}_E}\Big)^{-z}\Big] \\
\int_{\tilde{\phi}_N}^{\tilde{\phi}_H} \pi_N(\phi)dG(\phi) &= \int_{\tilde{\phi}_N}^{\tilde{\phi}_H}\Big\{\Big(\frac{\phi}{\tilde{\phi}_E}\Big)^{\sigma-1}\frac{f_L\chi_N}{(1-\delta)^\sigma} - f_N\Big\}dG(\phi) \\
&= \frac{f_L\chi_N}{\tilde{\phi}_E^z(1-\delta)^\sigma}\frac{z}{z-\sigma+1}\Big[\Big(\frac{\tilde{\phi}_N}{\tilde{\phi}_E}\Big)^{\sigma-z-1} - \Big(\frac{\tilde{\phi}_H}{\tilde{\phi}_E}\Big)^{\sigma-z-1}\Big] + \frac{f_N}{\tilde{\phi}_E^z}\Big[\Big(\frac{\tilde{\phi}_H}{\tilde{\phi}_E}\Big)^{-z} - \Big(\frac{\tilde{\phi}_N}{\tilde{\phi}_E}\Big)^{-z}\Big] \\
\int_{\tilde{\phi}_H}^{\infty} \pi_H(\phi)dG(\phi) &= \int_{\tilde{\phi}_H}^{\infty}\Big\{\Big(\frac{\phi}{\tilde{\phi}_E}\Big)^{\sigma-1}\frac{f_L\chi_N\chi_H}{(1-\delta)^\sigma} - f_H\Big\}dG(\phi) \\
&= \frac{f_L\chi_N\chi_H}{\tilde{\phi}_E^z(1-\delta)^\sigma}\frac{z}{z-\sigma+1}\Big(\frac{\tilde{\phi}_H}{\tilde{\phi}_E}\Big)^{\sigma-z-1} - \frac{f_H}{\tilde{\phi}_E^z}\Big(\frac{\tilde{\phi}_H}{\tilde{\phi}_E}\Big)^{-z}.
\end{aligned}$$

After some algebra,

$$f_e = \frac{1}{\tilde{\phi}_E^z}\frac{\sigma-1}{z-\sigma+1}\Big\{f_L\Big[1-\Big(\frac{\tilde{\phi}_N}{\tilde{\phi}_E}\Big)^{-z}\Big] + f_N\Big[\Big(\frac{\tilde{\phi}_N}{\tilde{\phi}_E}\Big)^{-z} - \Big(\frac{\tilde{\phi}_H}{\tilde{\phi}_E}\Big)^{-z}\Big] + f_H\Big(\frac{\tilde{\phi}_H}{\tilde{\phi}_E}\Big)^{-z}\Big\}.$$

Define $\theta_L = 1-\Big(\frac{\tilde{\phi}_N}{\tilde{\phi}_E}\Big)^{-z}$, $\theta_N = \Big(\frac{\tilde{\phi}_N}{\tilde{\phi}_E}\Big)^{-z} - \Big(\frac{\tilde{\phi}_H}{\tilde{\phi}_E}\Big)^{-z}$, and $\theta_H = \Big(\frac{\tilde{\phi}_H}{\tilde{\phi}_E}\Big)^{-z}$. Then,

$$f_e = \frac{\Phi}{\tilde{\phi}_E^z} \frac{\sigma - 1}{z - \sigma + 1}$$

where $\Phi = \sum_{i \in \{L,N,H\}} \theta_i f_i$ .

Zero-profit productivity cutoff in presence of immigration is then,

$$\tilde{\phi}_E = \left( \frac{\sigma - 1}{z - \sigma + 1} \frac{\Phi(\chi_N, \chi_H)}{f_e} \right)^{\frac{1}{z}}.$$

4. Proof of Proposition 3

Step 1) Show that $w_L \bar{L} + w_N \bar{N} + w_H \bar{H} = \rho \tilde{R}$.

From $\bar{L} = \int_{\tilde{\phi}_E}^{\tilde{\phi}_N} x_L(\phi) M_L \mu^*(\phi) d\phi$,

$$\bar{L} \quad = \quad \rho^\sigma w_L^{-\sigma} (1-\delta)^\sigma a_L^{\sigma-1} \tilde{Y} \frac{z}{\sigma - z - 1} \tilde{\phi}_E^{\sigma-1} \left[ 1 - \left( \frac{\tilde{\phi}_N}{\tilde{\phi}_E} \right)^{\sigma - z - 1} \right].$$

Multiplying $w_L$ on both sides,

$$w_L \bar{L} \quad = \quad \frac{\rho \sigma z f_L \tilde{M}}{z - \sigma + 1} \left[ 1 - \left( \frac{\tilde{\phi}_N}{\tilde{\phi}_E} \right)^{\sigma - z - 1} \right].$$

I use zero-profit cutoff condition, $\sigma f_L = \rho^{\sigma-1} \tilde{\phi}_E^{\sigma-1} {w_L}^{1-\sigma} a_L^{\sigma-1} (1-\delta)^\sigma \tilde{Y} / \tilde{M}$.

From $\bar{N} = \int_{\tilde{\phi}_N}^{\tilde{\phi}_H} x_N(\phi) M_N \mu^*(\phi) d\phi$ and $\bar{H} = \int_{\tilde{\phi}_H}^{\infty} x_H(\phi) M_H \mu^*(\phi) d\phi$, similar exercises provide

$$
\begin{aligned}
w_N\bar{N} &= \frac{\rho\sigma z f_L\tilde{M}}{z-\sigma+1}\frac{\chi_N}{(1-\delta)^\sigma}\Big[\Big(\frac{\tilde{\phi}_N}{\tilde{\phi}_E}\Big)^{\sigma-z-1}-\Big(\frac{\tilde{\phi}_H}{\tilde{\phi}_E}\Big)^{\sigma-z-1}\Big] \\
w_H\bar{H} &= \frac{\rho\sigma z f_L\tilde{M}}{z-\sigma+1}\frac{\chi_N\chi_H}{(1-\delta)^\sigma}\Big(\frac{\tilde{\phi}_H}{\tilde{\phi}_E}\Big)^{\sigma-z-1}.
\end{aligned}
$$

In sum,

$$
\begin{aligned}
w_L\bar{L}+w_N\bar{N}+w_H\bar{H} &= \frac{z(\sigma-1)}{z-\sigma+1}\tilde{M}\Phi(\chi_N,\chi_H) \\
&= \rho\tilde{Y}.
\end{aligned}
$$

Step 2) Use the labor market equilibrium condition for native workers and the result from Step 1).

$$
\bar{N}=\rho^\sigma w_N^{-\sigma}a_N^{\sigma-1}\tilde{Y}\frac{z}{\sigma-z-1}\tilde{\phi}_E^{\sigma-1}\Big[\Big(\frac{\tilde{\phi}_N}{\tilde{\phi}_E}\Big)^{\sigma-z-1}-\Big(\frac{\tilde{\phi}_H}{\tilde{\phi}_E}\Big)^{\sigma-z-1}\Big].
$$

Inserting $w_L\bar{L}+w_N\bar{N}+w_H\bar{H}=\rho\tilde{R}$ to the above condition and after some algebra,

$$
w_N=\rho a_N\left(\frac{z}{z-\sigma+1}\right)^{\frac{1}{\sigma-1}}\left(\frac{\sigma-1}{z-\sigma+1}\frac{\Phi(\chi_N,\chi_H)}{f_e}\right)^{\frac{1}{z}}\left[\frac{\Phi(\chi_N,\chi_H)}{f_L\chi_N/(1-\delta)^\sigma}\right]^{\frac{1}{\sigma-1}}.
$$

Step 3) Recall that $w=\rho a\left(\frac{z}{z-\sigma+1}\right)^{\frac{1}{\sigma-1}}\left(\frac{\sigma-1}{z-\sigma+1}\frac{f}{f_e}\right)^{\frac{1}{z}}$. Therefore,

$$
\frac{w_N}{w}=\left[\frac{\Phi(\chi_N,\chi_H)}{f_N}\right]^{\frac{1}{z}}\left[\frac{\Phi(\chi_N,\chi_H)}{f_L\chi_N/(1-\delta)^\sigma}\right]^{\frac{1}{\sigma-1}}.
$$

5. Equilibrium relative wages

Relative wage of native worker to illegal immigrant is

$$\frac{w_N}{w_L} = \frac{\frac{\chi_N}{(1-\delta)^\sigma}\left[\left(\frac{\tilde{\phi}_N}{\tilde{\phi}_E}\right)^{\sigma-z-1} - \left(\frac{\tilde{\phi}_H}{\tilde{\phi}_E}\right)^{\sigma-z-1}\right]}{\left[1-\left(\frac{\tilde{\phi}_N}{\tilde{\phi}_E}\right)^{\sigma-z-1}\right]}\frac{\bar{L}}{\bar{N}}.$$

Relative wage of highly-skilled immigrant to native worker is

$$\frac{w_H}{w_N} = \frac{\chi_H\left(\frac{\tilde{\phi}_H}{\tilde{\phi}_E}\right)^{\sigma-z-1}}{\left[\left(\frac{\tilde{\phi}_N}{\tilde{\phi}_E}\right)^{\sigma-z-1} - \left(\frac{\tilde{\phi}_H}{\tilde{\phi}_E}\right)^{\sigma-z-1}\right]}\frac{\bar{N}}{\bar{H}}.$$

From (2.15) and (2.16),

$$\begin{aligned}
\frac{\tilde{\phi}_N}{\tilde{\phi}_E} &= \left(\frac{f_N-f_L}{f_L}\right)^{\frac{1}{\sigma-1}}\left[\frac{\chi_N}{(1-\delta)^\sigma}-1\right]^{\frac{1}{1-\sigma}} \\
\frac{\tilde{\phi}_H}{\tilde{\phi}_E} &= \left(\frac{\chi_N}{(1-\delta)^\sigma}\right)^{\frac{1}{1-\sigma}}\left(\frac{f_H-f_N}{f_L}\right)^{\frac{1}{\sigma-1}}(\chi_H-1)^{\frac{1}{1-\sigma}}.
\end{aligned}$$

where $\chi_N = \left(\frac{a_N}{w_N}\right)^{\sigma-1}/\left(\frac{a_L}{w_L}\right)^{\sigma-1}$ and $\chi_H = \left(\frac{a_H}{w_H}\right)^{\sigma-1}/\left(\frac{a_N}{w_N}\right)^{\sigma-1}$.

Equilibrium relative wages can be determined endogenously two equations and two unknowns ($\chi_N$ and $\chi_H$).

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# CHAPTER 3

# Worker Heterogeneity and Adjustment to Trade Liberalization

## 3.1 Introduction

Trade liberalization impacts an economy's welfare, elevating it to a higher level compared to pre-liberalization. Artuc *et al*. (2010) and Artuc and McLaren (2010) estimate 8-10 years to approach a new steady state. During transition to that new steady state, welfare changes accompany reallocation of production factors within and between sectors, incurring adjustment losses.[1] Davidson and Matusz (2004) estimate that adjustment costs could be as high as 80% of gross benefit of trade liberalization. Those costs appear to be unavoidable, borne by individuals, and typically unevenly distributed. Intuitively, workers are likely to experience unemployment and bear a significant amount of earning variations depending on worker characteristics.

Related empirical studies support the claim on worker adjustment to trade liberalization. Kletzer (2001) and the OECD Employment Outlook (2005) report that trade-related workers tend to have been displaced in import-competing industries at 3.7%-8.3% per annum across countries. A substantial fraction of workers displaced by trade liberalization, about 37%-43% across countries, remains unemployed or exits the labor force (Menezes-Filho and Muendler 2011). Re-employed workers experience differently across age groups or education groups. For example, Kletzer (2001) and Kuhn (2002) provide evidence that U.S workers aged 35-44 are about 11% likelier to be reem-

[1] Aggregate adjustment costs consist of private adjustment costs and public-sector adjustment costs. Private adjustment costs can be divided into two cases: labor adjustment costs and capital adjustment costs. The former includes unemployment, obsolescence of skills, training costs, personal mental costs and so on. The latter includes underutilized capital, obsolete machines or buildings, transition cost of shifting capital to other activities and so forth. See Francois *et al*. (2011) for further details.

ployed than workers aged 45 or older at time of displacement; moreover, in comparison with the reference group of high-school dropouts, workers with a college degree (or higher) are 25% likelier to be reemployed. The survey paper of Matusz and Tarr (1999) indicates that post re-employed earnings vary from serious losses to positive gains.

This paper focuses on the hitherto largely ignored role of workers' ages together with ability and examines the impact of trade liberalization on heterogeneous workers in a search and matching model of the labor market. The present model consists of overlapping generations of finite-lived workers and infinitely-lived firms. Due to the finite nature of a worker's time horizon, the asset value of an unemployed worker decreases as he/she ages, but increases in ability. The shorter horizon of older workers renders them less valuable to firms, thus, in this paper, no match arises between old workers and firms. In essence, the definition of oldness depends on not only age *per se* but also ability, with higher ability substituting for age.

Given the substitution property between age and ability at hand, consider that an economy has a comparative advantage in high-tech sector production, and protects the low-tech sector by tariff. After tariff removal, the asset value of an unemployed worker in the low-tech sector decreases, while the asset value of the same worker in the high-tech sector increases. Depending on a worker's age and ability, this paper identifies three distinct groups impacted by trade liberalization. The first group comprises those who exit the labor force. Due to the shortened age-threshold in the low-tech sector, relatively older workers in the low-tech sector have no hope of being matched with firms, thereby exit the labor force. The second group comprises stayers, *i.e.*, those who remain both in the low-tech sector and in the high-tech sector. Those stuck in the low-tech sector are relatively older and/or less able and bear asset value losses due to trade liberalization. However, stayers who remain in the high-tech sector are youngest and most able, and experience asset value gains. The last group comprises movers who switch sectors due to tariff removal. Some unemployed workers in the low-tech sector are able to switch sectors and search for jobs in the high-tech sector although not all movers become winners. Relatively older and less able workers are harmed, but becoming movers still is better than deciding to stay in the low-tech sector.

The present paper's results are analogous to the work of Falvey *et al.* (2010) who employed an educational sector in a Heckscher-Ohlin model and considered how the characteristics of unskilled workers impact when and whether they opt for skill upgrading in response to trade liberalization in a skill-abundant country. Both their work and herein describe the adjustment by reallocation of workers between sectors. Their paper considers the full-employment condition whereas this present paper assumes imperfect labor markets. Both works show that age and ability are substitutes in the sense that older and less able workers are harmed, and that younger and more able workers are benefited, by liberalization. However, Falvey *et al.* (2010) assumed fixed wages, and deviated from the complexity of adjustment in a labor market. In this paper, broadly seen, skill upgrading is basically equivalent to sector shifting.

Classical international trade theories offer several determinants in identifying winners and losers after trade liberalization. For instances, the Ricardo-Viner model stresses factor specificity in a sector while the Heckscher-Ohlin model emphasizes factor abundance in a country. Both models shed light on the distributional effect of trade liberalization. Both models, however, predict that workers are always better off or worse off due to trade liberalization, regardless of worker characteristics. Recent theoretical trade models neglect either labor market frictions or personal characteristics, in particular, age. Models with labor market frictions assume infinite-lived workers, thus fail to describe differing-age workers' adjustments in response to free trade, for example, Helpman *et al.* (2008) among many others. Models with differing-age workers use full-employment, see Falvey *et al.* (2010) and Dix-Carneiro (2010). Models with imperfect labor market and workers' ages easily are complicated and intractable, for instance, see Cosar (2013). The model developed herein is relatively tractable to many aspects of adjusting workers described by empirical evidence as indicated above. The present paper seeks to contribute to the literature by focusing on the role of age together with ability of workers in a search and matching model of the labor market.

The model developed in this paper modifies and extends earlier work in two papers, those of Davidson and Matusz (2006) and of Hahn (2009). Both introduce search and matching process. Whereas Davidson and Matusz (2006) assume infinitely-lived workers with different abilities in a two-sector model, Hahn (2009) explicitly considers different-aged workers with uniform ability in a one-sector model.[2] The paper by Davidson and Matusz (2006) shows that high-ability workers are attracted to the high-tech sector while low-ability workers are attracted to the low-tech sector. After tariff removal, they identify two groups of workers: those who remain trapped in the low-tech sector have low-ability (the so-called stayers) and those who switch sectors from the low-tech to the high-tech sector have relatively high-ability (the so-called movers). Despite these insights, the paper remains mute in describing adjustment of differing-age workers. Hahn (2009) uses total training cost and total surplus to determine oldness. An age threshold can be derived determining a worker's choice in the labor market. If a worker is older than the threshold, then he/she exits the labor force. In Hahn's paper, however, the asset value of the unemployed is not derived and trade consideration is absent.

The remainder of this paper is structured as follows: Section 2 outlines a one sector model and examines its equilibrium and the effect of free trade. Section 3 extends the one-sector to a two-sector model and analyzes its equilibrium after trade liberalization. Section 4 concludes the paper.

[2]Hahn's (2009) paper focuses on examining the existence, uniqueness, and efficiency of steady state.

## 3.2 One-sector model

Consider a single sector model with overlapping generations of workers. Assume that each generation comprises the same number of workers normalized as unity, and the total size of the worker population is $T$. Firms are homogeneous and infinite-lived in the sector. Both workers and firms are risk neutral. Time is continuous.

### 3.2.1 Value function of workers

Workers are characterized by ability $a$ and age $t$ denoted as a pair $(a,t)$. Assume that the probability distribution of ability is the same across generations, and each worker lives for a finite period $T$.[3] Then $T-t$ means that a worker with age $t$ has $T-t$ years of life remaining.[4] A worker whose age is closer to $T$ may be considered as being old, but "how old is old?"

This paper will use total surplus $S(a,t)$ and total training costs $z$ to define *oldness* in the spirit of Hahn (2009). Define $z/P$ as total training costs in real terms where $P$ is the price index. Total training cost corresponds to the lump-sum training cost once a match forms in Mortensen and Pissarides (1999). Assume that both a worker and a firm share the exogenous total training costs $z$. Let $x/P$ be training costs a worker pays and $y/P$ be training costs a firm pays, thus, $z = x + y$. A worker can contribute to create positive total surplus $S$ via production activity. At a certain age approaching to $T$, a match between a worker and a firm may not be mutually beneficial if the worker is no longer able to contribute to create total surplus more than total training costs in the lifetime remaining, *i.e.*, $S(a,t) < z/P$.

Denote $\eta$ as an age threshold determining young and old, $S(a,\eta) = z/P$, in this paper. You will see later that $\eta$ is increasing in ability: $\eta = \eta(a)$ with $\eta' > 0$. The present paper calls workers "young" if they are in $t \leq \eta(a)$, and "old" if they are in $t > \eta(a)$.[5]

[3] The choice of probability density function changes none of the results, qualitatively.

[4] Hahn (2009) interprets age zero as after education has been completed. So, age zero is the (normalized) age of entry into the labor market.

[5] Note that young is not entirely young in a literal sense. As $t$ increases, workers with ability $a$ in period $t < \eta(a)$ are becoming older, too.

Unemployed young workers always have two choices: search for a job or drop out of the labor force. The decision criterion is based on two values between the asset value when unemployed $U(a,t)$ and the outside option $W$ which is assumed to be zero across all age groups. An unemployed young worker keeps searching for a job as long as $U(a,t) > 0$. Otherwise, she/he simply drops out of the labor force. If an unemployed young worker chooses to search, then he/she has to spend time to meet a firm at matching rate $m$. Once a new match is formed, she/he has to pay training costs $x/P$ as an upfront fee.[6] How about unemployed old workers? Since we define old workers as those who cannot create total surplus over total training costs in the lifetime remaining, no further matching with firms arises.

The asset value function for an unemployed worker with $(a,t)$ is

$$\rho U(a,t) = \begin{cases} m[E(a,t) - U(a,t) - x/P] + \dot{U}(a,t) & for \quad t \in [0, \eta(a)] \\ \dot{U}(a,t) & for \quad t \in (\eta(a), T] \end{cases} \tag{3.1}$$

where $\rho$ denotes the discount rate, $E(a,t)$ is the asset value function for an employed worker with $(a,t)$, and a dot represents time derivative. The value function for an unemployed young worker in (3.1) equals matching rate multiplied by capital gains plus the asset's depreciation rate $\dot{U}$. Note that capital gains include training costs $x/P$ a worker pays. Unemployed old workers in (3.1) have no asset gains because no matching arises.

After match, every worker engages in production activity and is paid real wage $w(a,t)/P$.[7] Every time, wage is determined via the Nash bargaining process. An employed worker separates

[6] A worker and a firm together can pay either the one-time total training cost or the amortized per-period portion of the total training cost in every period. Firms are imperfectly informed about workers with $(a,t)$ before meeting workers. Once they meet, they recognize immediately the others' type. Hence, even if firms cannot select their workers ex ante, they can do so ex post. After a firm meets a worker and knows his/her characteristics, the firm is indifferent to paying by one of the two ways as long as the worker is young. As in the paper, once the one-time training cost as an upfront fee is adopted for simplicity, the interpretation of the cost is similar to sunk training cost or transaction cost to form a match. Moreover, we can understand total training cost as an irreversible contract between a firm and a worker. Once a new match arises and both parties agree and sign on spending total training costs, all the costs will incur for sure. For example, training instructor's salary, training-related equipment packages, etc "per training course". Once hired and once ordered, it is irreversible.

[7] Wage also depends on a worker's ability and age and is determined endogenously on the equilibrium (see Appendix).

from a firm at break-up rate $b$.

The asset value function for an employed worker with $(a,t)$ is

$$\rho E(a,t) = w(a,t)/P + b[U(a,t) - E(a,t)] + \dot{E}(a,t) \quad for \quad t \in [0,T] \tag{3.2}$$

The second term on the right-hand side in (3.2) is the break-up rate times the capital loss. $\dot{E}(a,t)$ is the asset's rate of depreciation of a worker with $(a,t)$. If a worker is separated from a firm, she/he returns to the labor market as an unemployed.

### 3.2.2 Value function of firms

Firms are imperfectly informed about workers before meeting them and are not allowed to discriminate against workers by age and ability.[8] All firms share the same expected value before matching with workers. They can meet a worker at matching rate $v$.

The asset value for a vacancy that a firm maintains is

$$\rho V = -c/P + v[\mathbf{E}J(a,t) - V - y/P] \tag{3.3}$$

where $\mathbf{E}$ is an expectation operator with respect to $(a,t)$. The first term $c/P$ on the right hand side in (3.3) is the real costs of maintaining a vacancy. $\mathbf{E}J(a,t)$ is the expected value of a new match for a firm.[9] When a job is vacant, a firm's asset gains are the expected asset gains minus the training costs a firm pays. A free entry condition can be imposed so that firms will enter the labor market as long as that is profitable.

Let $p$ be the price of a good. Via production activity, a worker with ability $a$ can produce $a$ units of a good. Total revenue is then $pa/P$. After the firm pays wages $w(a,t)/P$ after the Nash

[8] Hairault *et al*. (2007) use a direct search model so that firms can discriminate workers in the labor market.

[9] Let $a_{min}$ and $a_{max}$ be the minimum and maximum ability, respectively. Denote $u$ as the total number of unemployed workers, calculated as $u = \int_{a_{min}}^{a_{max}} u(a)da = \int_{a_{min}}^{a_{max}} \left[\int_0^{\eta(a)} \mu(a,t)dt\right] da$, where $\mu(a,t)$ is the number of unemployed workers with $(a,t)$. Then the expected of a new match is defined as $\mathbf{E}J = \int_{a_{min}}^{a_{max}} \left[\frac{1}{u}\int_0^{\eta(a)} J(a,t)\mu(a,t)dt\right] f(a)da$ where $f(a)$ is the probability density function of ability.

bargaining process, current profit is calculated as $[pa - w(a,t)]/P$.

The asset value function for a firm when filling a vacancy is

$$\rho J(a,t) = [pa - w(a,t)]/P + b[V - J(a,t)] + \dot{J}(a,t) \tag{3.4}$$

where the second term on the right hand side in (3.4) is the break-up rate times the asset loss. The last term captures the asset's rate of depreciation of a firm.

If separated from a worker, a firm goes to the labor market and again a job is vacant. Firms and workers with age $T$ are separated automatically, and those firms immediately go to the labor market. As Hahn (2009) stresses, a firm attaches no value to a match at age $T$ because the match is dissolved with certainty. Of course, a worker's asset value is zero because of death.

With respect to search and matching, this paper uses a constant returns to scale matching function. There are two arrival rates, $m(\theta)$ and $v(\theta)$, where $\theta$ is the labor market tightness which measures the number of vacancies per unemployed worker. It becomes easier for workers to find jobs and more difficult for firms to fill their vacancies as labor market tightness $\theta$ increases: $v'(\theta) < 0 < m'(\theta)$.

As explained earlier, wage is determined via Nash bargaining process every time. Assume that a worker has an exogenous bargaining power $\beta \in [0,1]$ whereas a firm has the fraction $1-\beta$. Then the following rule is always satisfied

$$\beta S(a,t) = E(a,t) - U(a,t) \tag{3.5}$$

where $S(a,t) = E(a,t) - U(a,t) + J(a,t) - V$ is the total surplus created by a firm and a worker with $(a,t)$.

This completes the one-sector model and we are ready to characterize the equilibrium.

### 3.2.3 Equilibrium

#### 3.2.3.1 Age threshold and labor market tightness

On the equilibrium path, total training costs are sunk in every new match. The net total surplus of a new match is thus $S(a,t) - z/P$. From a worker's perspective, his/her claim on the net surplus becomes $\beta[S(a,t) - z/P]$, which equals the increase of the worker's lifetime utility $E(a,t) - U(a,t) - x/P$. Via the rule specified in (3.5), the amount of the training costs a worker pays is determined by the exogenous bargaining power of a worker multiplied by total training costs:

$$x = \beta z, \; y = (1-\beta)z. \tag{3.6}$$

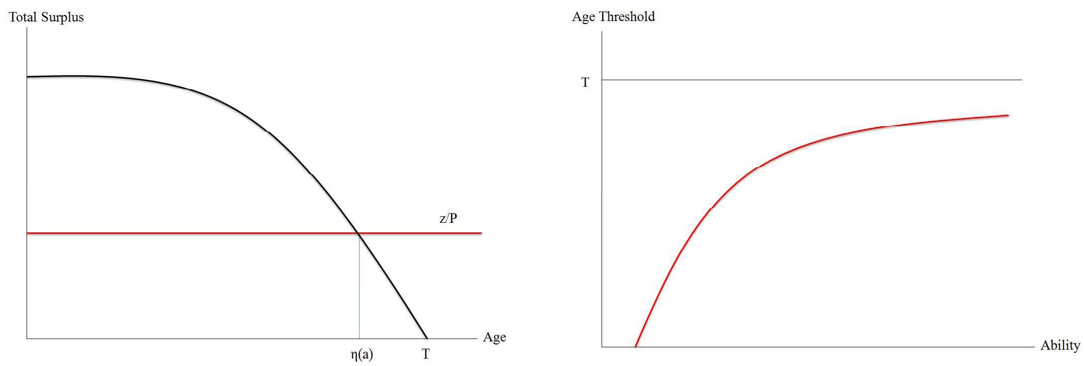
Equation (3.6) provides that all workers pay a constant fraction $\beta$ of total training costs $z$ to a firm, while all firms pay a constant fraction $1-\beta$ of total training costs $z$ irrespective of worker types. In a special case, if all firms have full bargaining power, they can extract the whole surplus, which implies that, once a match is formed, $z$ should be paid only by the firm.

Using (3.1)-(3.5), the total surplus is described as follows:

$$S(a,t) = \begin{cases} [pa/P]\chi_y(t) + [z/P][1-(\rho+b)\chi_y(t)] & for \quad t \in [0, \eta(a)] \\ [pa/P]\chi_o(t) & for \quad t \in (\eta(a), T] \end{cases} \tag{3.7}$$

where $\chi_y(t) = [1-e^{-(\rho+b+\beta m)[\eta(a)-t]}]/(\rho+b+\beta m)$ and $\chi_o(t) = [1-e^{-(\rho+b)[T-t]}]/(\rho+b)$, both of which act as an age-discounting operator. The total surplus created by both a firm and a worker with $(a,t)$ is increasing in ability $a$, but decreasing in age $t$. Since there is no match with unemployed old workers, the matching rate $m$ does not appear for an old worker's generating surplus. Of interest is to find that total surplus is decreasing faster as a worker gets older in a literal sense.

Figure 3.1 Total Surplus and Age Threshold




Note: The left graph provides that total surplus decreases as age increases at a given ability in (3.7). The right graph shows that age threshold is increasing in ability in (3.8).

When age $t$ is $T$, the total surplus should be zero $S(a,T)=0$ because $\chi_o(T)=0$ (see Figure 3.1).[10]

Until now, this paper treats an age threshold $\eta(a)$ as given. As explained, the age threshold determines how to divide a worker's time line as either young or old. From (3.7) and the boundary condition $S(a,\eta(a))=z/P$, age threshold is determined as a function of ability $a$:

$$\eta(a)=[1/(\rho+b)]ln\left[1-\frac{(\rho+b)z}{pa}\right]+T \tag{3.8}$$

which shows the positive relationship between age threshold and ability (see Figure 3.1). The value of the logarithm in (3.8) is negative, thus $\eta(a)<T$ for any abilities. The existence of total training costs $z$ is important to generate heterogeneous age thresholds depending on ability. If there is no such cost in (3.8), $\eta(a)$ is $T$ regardless of differing ability levels.

To understand the positive relationship in (3.8), consider two types of young workers: low ability and high ability. An unemployed high-ability worker can create higher value of surplus than can an unemployed low-ability worker because total surplus is increasing in ability from (3.7). That is, when the unemployed low-ability worker could create only just $z/P$, the unemployed high-ability worker can create more than $z/P$. By definition of oldness, it must be the case that high-ability workers have a higher age threshold than have low-ability workers.

[10]If the total training costs go to zero, the total surplus in equation (3.7) is identical to one equation, $S(a,t)=[pa/P][1-e^{-(\rho+b)[T-t]}]/(\rho+b)$ for $t\in[0,T]$.

In the steady state, the total number of unemployed and the total number of vacancies do not change over time. In particular, the number of unemployed workers with $(a,t)$ must be constant over time. In addition, free entry requires that all rents from new vacancy creation are exhausted, *i.e.*, $V = 0$ in (3.3).

Labor market tightness $\theta$ is derived from free entry condition:

$$c/P = v(\theta)[\mathbf{E}J - y/P] \tag{3.9}$$

which the real costs of maintaining a vacancy $c/P$ are equal to the arrival rate for firms $v(\theta)$ multiplied by expected asset value of a new match for a firm $\mathbf{E}J$ minus the training costs $y/P$ a firm pays.

#### 3.2.3.2 Asset values of unemployed workers

To describe a worker's behavior, it is important to have the asset value for unemployed workers. Unemployed old workers for $t \in (\eta(a), T]$ have zero asset value on the equilibrium.

Using (3.1)-(3.5) and (3.8), the asset value for unemployed young workers for $t \in [0, \eta(a)]$ is derived

$$U(a,t) = \frac{m}{\rho\phi P}[\kappa pa - x(\rho+b)]\chi(t) \tag{3.10}$$

where $\chi(t) = [\phi e^{-\rho[\eta(a)-t]} - \rho e^{-\phi[\eta(a)-t]}]/(b+m) \geq 0$ and $\phi = \rho + b + m$. Similar to the case of total surplus in (3.7), $\chi(t)$ plays a role as an age-discounting factor. Note that $\kappa$ is a function of $\beta$ with $\kappa(1) = 1$ and $\kappa \in [0,1]$.

Equation (3.10) provides that an unemployed worker's asset value is increasing in ability $a$ but decreasing in age $t$ (see Appendix). Asset value $U(a,t)$ falls as age $t$ increases up to $\eta(a)$.[11] At age $\eta(a)$, the asset value in (3.10) is zero because of $\chi(\eta) = 0$. The zero asset value continues for

[11]Consider the case that there is no labor market friction and the asset value for a worker is $\rho Y(t) = w + \dot{Y}$, where $Y$ is assumed independent of ability. If we think about the steady state in the infinite-lived worker model, then the term $\frac{dY}{dt}$ is assumed 0. In the finite-lived worker model, however, the asset value must be decreasing, $\frac{dY}{dt} < 0$. This implies that $Y$ is a function of $t$ in the finite-lived worker case.

$\eta(a) < t \leq T$. As a worker ages, she/he loses asset values because of the finite time horizon left to create surplus. Accordingly, in this paper, a worker's age negatively affects her/his asset value.[12] If we fix $a = \bar{a}$, then $U(\bar{a},t)$ represents a lifetime asset value schedule of a worker with ability $\bar{a}$. Instead of fixing $a$, if we fix $t = \bar{t}$, then $U(a,\bar{t})$ shows all asset values of workers with age $\bar{t}$.

**Proposition 1.** *An unemployed worker's asset value decreases in age, but increases in ability.*

At first glance, the expression in (3.10) looks unfamiliar, but contains a richer functional form compared to the (usual) infinite-lived worker model. To mimic the infinite-lived worker, let $T$ go to infinity. For any age $t$,

$$U(a) = \frac{m}{\rho\phi P}[\kappa pa - x(\rho + b)]$$

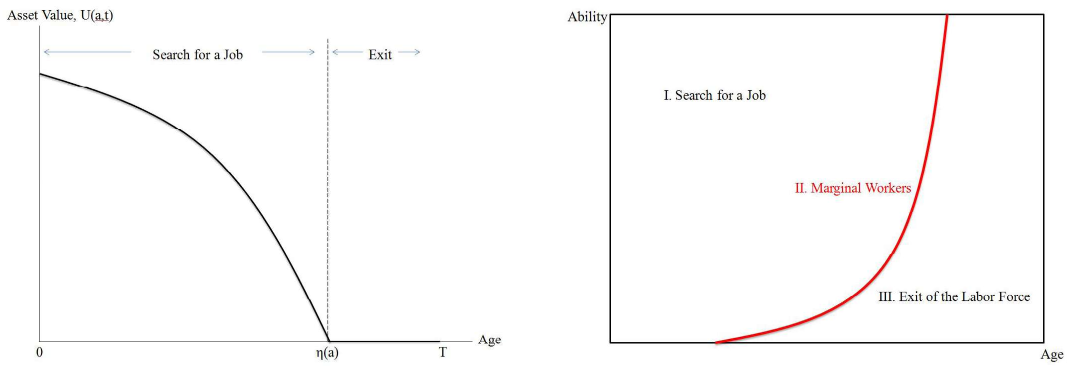
where $\chi(t)$ disappears compared to the expression in (3.10). A similar equation can be found in Davidson and Matusz (2006) who use the infinite-lived worker model.

#### 3.2.3.3 Worker choice

To understand a worker's choice between searching for a job and dropping out of the labor force, focus on workers with a set of abilities and ages $(a,t)$ such that $U(a,t) = 0$. Let us call them marginal workers. To characterize the relationship between ability and age among marginal workers, consider a worker with age $\bar{t}$ and ability $\bar{a}$ who has chosen to search for a job. That is, searching for a job is much more attractive than being out of the labor force, $U(\bar{a},\bar{t}) > 0$. Then we can detect different worker types among marginal workers: Workers surely exist who have the same age $\bar{t}$, but have lower ability $a < \bar{a}$ among marginal workers, $U(a,\bar{t}) = 0$. Also workers exist who have the same ability $\bar{a}$, but have higher age $t > \bar{t}$ among marginal workers, $U(\bar{a},t) = 0$. Hence, if a marginal worker (call this worker $A$) has a higher ability than any other marginal workers (call these workers $B$), then worker $A$ has always a higher age threshold than has $B$. This argument is

[12] We can find a similar result in the model of Hairault *et al* (2007). The finite nature of a worker's time horizon reduces the expected value of hiring older workers from the firm's perspective. The shorter the horizon of older workers the less value they have, unless they have high productivity levels. This is referred to as "horizon effect."

Figure 3.2 Asset Value of Unemployed Workers and Worker Choice




Note: The left graph provides the asset value of an unemployed worker with a given ability. The right graph shows a set of ability and age to determine worker choice in the labor market.

summarized in Proposition 2.

**Proposition 2.** *Under Proposition 1, any two marginal workers, $(a',t')$ and $(a'',t'')$, satisfy the following relationship: if $a' > a''$, then $t' > t''$.*

Denote $a_m(t)$ as a marginal worker's ability at age $t$ and $Q$ as a set of ability and age of searchers pertaining to the sector:

$$Q = \{(a,t)|a_m(t) < a \leq a_{max} \text{ and } 0 \leq t < \eta(a)\}. \tag{3.11}$$

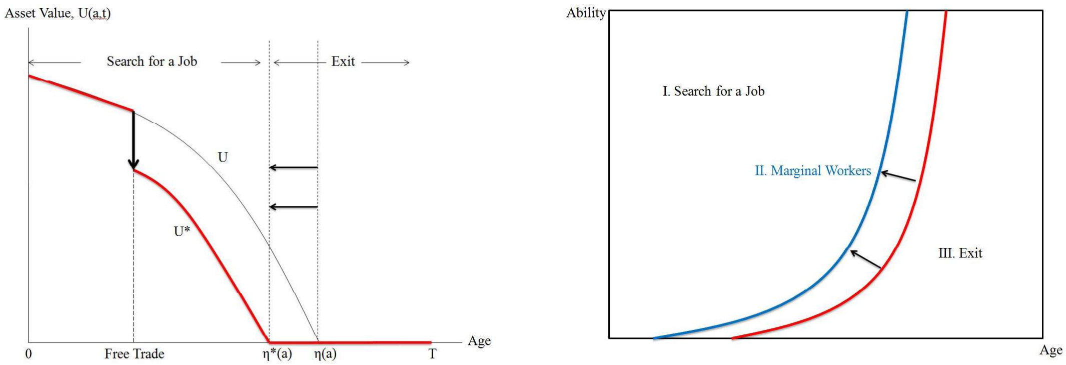
In the labor market, unemployed workers with $(a,t) \in Q$ search for jobs $(U(a,t) > 0)$ and workers with $(a,t) \notin Q$ exit the labor force $(U(a,t) = 0)$ (see also Figure 3.2).

### 3.2.4 Trade liberalization in one-sector model

In a one-sector model, this paper has not yet specified the sector. Depending on which sector we have in mind, the analysis of trade liberalization goes either way. Consider an import-competeting sector.[13] Due to the removal of tariff, output price $p$ falls, which leads the price index $P$ to fall. Denote $P^*$ as the changed price index, where the asterisk represents free trade. A decreased output

[13]Many empirical studies provide that after trade liberalization, the manufacturing sector shrinks, employment decreases, and output falls (see, for example, Trefler 2004).

Figure 3.3 Asset Value and Worker Choice after Trade Liberalization




Note: The left graph shows how the asset value of an unemployed worker with a given ability changes due to trade liberalization. The right graph illustrates how the set of ability and age of marginal workers change due to trade liberalization.

price induces real total training costs to rise. Hence, age threshold falls from $\eta(a)$ to $\eta^*(a)$ from (3.8). From Propositions 1-2, the decreased age threshold implies that marginal workers after free trade are abler and younger than previous marginal workers before free trade: $Q^* = \{(a,t)|a_m^*(t) < a \; and \; t < \eta^*(a)\}$. Accordingly, the asset value for unemployed workers (3.10) decreases from $U(a,t)$ to $U^*(a,t)$ (see Figure 3.3).[14] At a given age, less able workers may exit of the labor force. At given ability $a$, workers aged 0 to $\eta^*(a)$ experience asset value loss. Trade liberalization directly and heavily impacts workers on the verge of age thresholds. That is, workers aged from $\eta^*(a)$ to $\eta(a)$ who could have looked for jobs, now are forced to stay out of the labor force. In sum, all workers in the import-competing sector are harmed by trade liberalization. In turn, consider an export sector. If price $p$ increases, then the story exactly reverses. All workers have extended age thresholds and benefit from free trade.

What happens to labor market tightness? To answer, we should understand the expected value of a new match for a firm in advance. On one hand, the composition of working forces is abler and younger due to the exit. Firms then can expect to meet abler and younger workers on average. This force raises $\mathbf{E}J$. On the other hand, the decrease in output price directly impacts a firm's revenue causing it to fall. In addition, both maintaining vacancies and training workers become much more

[14]Consequently, both total surplus and real wage decrease, too.

costly due to the decreased price index. This force reduces $\mathbf{E}J$. However, decrease in the total surplus in (3.7) implies that the latter force dominates the former. Thus, $\mathbf{E}J$ falls to $\mathbf{E}J^*$. From free entry condition, $c/P^* = v(\theta)[\mathbf{E}J^* - y/P^*]$. The value of maintaining a vacancy rises, $c/P < c/P^*$ and the training costs a firm pays increases, $y/P < y/P^*$. It is clear that $\mathbf{E}J^* - y/P^* < \mathbf{E}J - y/P$. Hence, the matching rate $v(\theta)$ should increase. After free trade, it is easy for firms to find workers while it is difficult for workers to find firms. In the exporting sector, labor market tightness rises.

This section has shown how finite-lived workers respond to trade liberalization. However, two limitations in the one-sector model can be pointed out. First, workers between sectors have no choice; in reality, economies reallocate factors of production not only within sectors, but also between sectors. Second, workers in the import-competing sector always are harmed, whereas workers in the export sector always benefit. Such results have little appeal. In reality, at least, some movers may benefit. These limitations lead to construction of a two-sector model.

## 3.3 Extension to two-sector model

Consider a small open economy that consists of two sectors: the low-tech and the high-tech, denoted by the subscript $l$ and $h$, respectively. World prices are given as $p_l^*$ and $p_h^*$, where the asterisk represents free trade. Initially, the economy is distorted by import tariffs. To be specific, the country has a comparative advantage in production of the high-tech sector and protects the low-tech sector by tariff, $\tau > 0$. In the presence of trade barriers, the domestic price of the low-tech good is $p_l = (1+\tau)p_l^*$ and that of the high-tech good equals the world price, $p_h = p_h^*$. A price index is chosen to be set as $P = \left[(1+\tau)p_l^* p_h^*\right]^{1/2}$. In both sectors, firms are free to enter the labor market as long as it is profitable. Last, workers are free to move between sectors.

In the extension of the one-sector model, this paper assumes that all aspects are identical between the two sectors, except total training costs. In particular, total training costs of the low-tech sector relative to the price of the low-tech good is less than those of the high-tech sector relative to the price of the high-tech good, $z_l/p_l < z_h/p_h$ (see survey by Hammerash 1993).[15] In the presence of sector-specific total training costs, a worker can have a different age threshold in each sector; that is, the worker can be considered as young in one-sector yet old in the other.

Analogous to the derivation as in (3.8), age threshold for a worker with ability $a$ in sector $j \in \{l, h\}$ is

$$\eta_j(a) = [1/(\rho+b)]ln\left[1 - \frac{(\rho+b)z_j}{p_j a}\right] + T \tag{3.12}$$

where $\eta_h(a) < \eta_l(a)$ always is satisfied.

[15] The assumption $z_l/p_l < z_h/p_h$ implies $x_l/p_l < x_h/p_h$, which means that workers have to pay higher training costs in the high-tech sector than in the low-tech sector. Automatically, firms in the high-tech sector have to spend higher training costs than those in the low-tech sector, $y_l/p_l < y_h/p_h$. Of note, Kletzer (2001) points out that, without training, a new job in a new sector will pay less than the old job.

### 3.3.1 Worker's behavior

For the baseline equilibrium, this paper focuses on the scenario that less able workers have no sector choice, but are stuck in the low-tech sector in the first place. Instead, abler workers can decide in which sector to work, holding the option to drop out of the labor force. The equilibrium can be generated easily by introducing an ability threshold $\hat{a} \in [a_{min}, a_{max}]$ such that $\eta_h(\hat{a}) = 0$, where $a_{min}$ and $a_{max}$ are denoted as the minimum and maximum ability level, respectively. Workers with $a \in [a_{min}, \hat{a}]$ have no age threshold in the high-tech sector, while workers with $a \in (\hat{a}, a_{max}]$ have age threshold in each sector with $\eta_h(a) < \eta_l(a)$. Unemployed old workers for $t \in (\eta_j(a), T]$ in sector $j$ have zero asset value on the equilibrium.

The asset value function for an unemployed young worker for $t \in [0, \eta_j(a)]$ in sector $j \in \{l, h\}$ is

$$U_j(a,t) = \frac{m_j}{\rho \phi_j P}[\kappa_j p_j a - x_j(\rho + b)]\chi_j(t) \tag{3.13}$$

where $\chi_j(t)$ and $\phi_j$ similarly are defined as in (3.10). Similarly to Proposition 1, an unemployed worker's asset value in sector $j$ decreases in age but increases in ability.

An unemployed worker with $(a,t)$ chooses a sector to work based on two values between $U_l(a,t)$ and $U_h(a,t)$. She/he decides to search for a job in the low-tech sector if $U_l(a,t) \geq U_h(a,t)$ or to search in the high-tech sector if $U_l(a,t) < U_h(a,t)$. Marginal workers in the two-sector model must distribute themselves such that the expected lifetime return from the search is equal across sectors. That is, we can find marginal workers such that $U_l(a,t) = U_h(a,t)$. Consider that a worker with $(\bar{a}, \bar{t})$ chooses to search in the high-skilled sector, $U_l(\bar{a}, \bar{t}) < U_h(\bar{a}, \bar{t})$. Then, at least, must exist a worker with the same age but lower ability who is indifferent to choosing a sector between high-skilled and low-skilled, $U_l(\bar{a}, \bar{t}) = U_h(a, \bar{t})$. A marginal worker has $(a, \bar{t})$ with $a < \bar{a}$. Also, a worker must exist with $\bar{a}$ and age $t > \bar{t}$ who is indifferent to choosing a sector between the two, $U_l(\bar{a}, t) = U_h(\bar{a}, t)$. A marginal worker has $(\bar{a}, t)$ with $t > \bar{t}$. Hence, Proposition 2 still holds in the two sector model. Denote $a_m(t)$ as a marginal worker's ability at age $t$ and $\eta_m(a)$ as a marginal worker's age at the given ability $a$.[16]

Define $H$ as a set of ability and age of searchers pertaining to the high-tech sector:

$$H = \{(a,t)|a_m(0) < a \leq a_{max} \ and \ 0 \leq t < \eta_m(a)\}$$

where $a_m(0)$ is the marginal worker's ability at age zero. Unemployed workers with $(a,t) \in H$ search for jobs in the high-tech sector, $U_l(a,t) < U_h(a,t)$.

Define $L = L_h \cup L_l$ as a set of ability and age of searchers pertaining to the low-tech sector:

$$L_h = \{(a,t)|a_m(0) < a \leq a_{max} \ and \ \eta_m(a) < t < \eta_l(a)\}$$

where workers with $(a,t) \in L_h$ may include those who had worked in the high-tech sector and

$$L_l = \{(a,t)|a_{min} \leq a < a_m(0) \ and \ 0 \leq t \leq \eta_l(a)\}.$$

Unemployed workers with $(a,t) \in L$ search for jobs in the low-tech sector, $U_l(a,t) > U_h(a,t)$. Last, workers with $(a,t) \notin H \cup L$ should be out of the labor force.

### 3.3.2 Trade liberalization in two-sector model

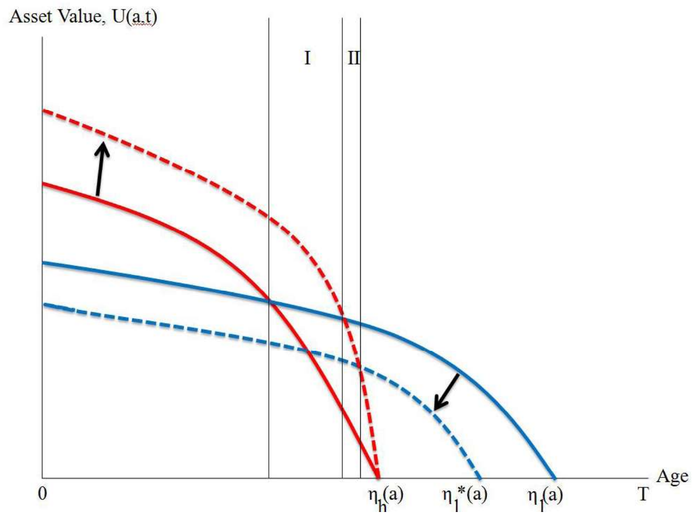
Of primary interest is study of the impact of trade liberalization on heterogeneous workers in the two-sector model. As protection in the low-tech sector ceases, domestic price in that sector falls from $p_l$ to $p_l^*$, leading to a fall in the price index, $P^* < P$.[17] Due to the change in $p_l$, the age threshold in the low-tech sector decreases from $\eta_l^*(a)$ to $\eta_l(a)$ in (3.13).[18] Both $p_l^*$ and $P^*$ contribute to changes in the asset values of workers. An unemployed worker's asset value in the low-tech sector declines ($U_l^*(a,t) < U_l(a,t)$) while his/her asset value in the high-tech sector rises

[16]Using the equality condition for marginal workers, two boundary conditions are obtained: $\hat{a} < a_m(0)$ and $\eta_m(a_{max}) < \eta_h(a_{max})$.

[17]We can think of the case that the economy is engaged in free trade and then the price of high-skilled good rises. The price index then rises. Still, the import-competing sector shrinks and the export sector expands. Qualitatively, it is the same story as shown in this paper.

[18]Despite the change in $p_l$, no change in $\eta_h(a)$ takes place. For a given ability, the following relation holds: $\eta_h = \eta_h^* < \eta_l^* < \eta_l$.

Figure 3.4 Movers after Trade Liberalization




Note: At given ability $a$, the solid red line and the solid blue line represent the asset value of an unemployed worker in the high-tech sector, and one in the low-tech sector, respectively. The dashed red line and the dashed blue line represent asset values after trade liberalization. Interval I indicates workers who become better-off movers while Interval II indicates workers who become worse-off movers.

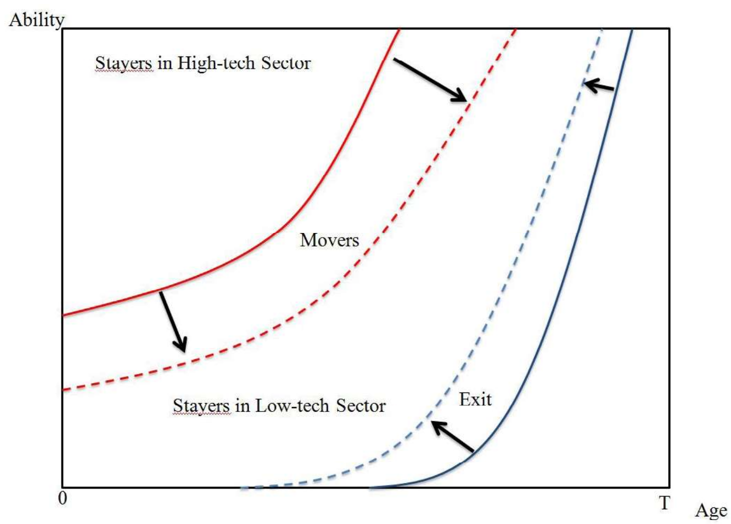
$(U_h(a,t) < U_h^*(a,t))$ in (3.13) (see Figure 3.4).

Trade liberalization causes redistribution of workers within and across sectors, thus changes the set of ability and age of marginal workers indifferent between job search in the low-tech sector and job search in the high-tech sector. An important observation is that marginal workers become less able and older after free trade.[19]

Three groups of workers are impacted by trade liberalization. The first consists of those who exit the labor force. The exit group is created due to the shortened age threshold in the low-tech sector, $\eta_l^*(a) < \eta_l(a)$. in the sense that those workers have no opportunities to obtain jobs, it is the weakest group. The second group can be called stayers and are detected in both sectors. Stayers in the high-tech sector are winners who experience asset value gains, while those in the low-tech sector are losers who are harmed by asset value losses. As illustrated in Figure 3.5, stayers in the high-tech sector include youngest and ablest workers with $(a,t) \in H$, while those in the low-tech sector include relatively less able and/or older workers. Some stayers in the low-tech

[19]Two boundary conditions are $\eta_m(a_{max}) < \eta_m^*(a_{max})$ and $a_m^*(0) < a_m(0)$.

Figure 3.5 Trade Liberalization and Worker Responses




Note: The solid red line represents the set of ability and age of marginal workers before trade liberalization, whereas the dashed red line does so after trade liberalization. The solid blue line represents the set of ability and age of workers who are indifferent between searching for jobs in the low-tech sector and dropping out of the labor force before trade liberalization, whereas the dashed blue line does so after trade liberalization.

sector may include workers who moved from the high-tech sector and they have with $(a,t) \in L_h^* = \{(a,t)|a_m^*(0) < a \leq a_{max} \textit{ and } \eta_m^*(a) < t < \eta_l^*(a)\}$. The remainder of stayers in that sector have been trapped mainly due to a lack of ability with $(a,t) \in L_l^* = \{(a,t)|a_{min} \leq a < a_m^*(0) \textit{ and } 0 \leq t \leq \eta_l^*(a)\}$. The third group can be called movers. It consists of relatively more able and younger workers compared to stayers stuck in the low-tech sector. Movers have the set of ability and age, $(a,t) \in H^* \setminus H$ where $H^* = \{(a,t)|a_m^*(0) < a \leq a_{max} \textit{ and } 0 \leq t < \eta_m^*(a)\}$. Not all movers benefit. Among movers, relatively younger and abler workers experience an increase in their asset values (see Interval I in Figure 3.4). Relatively older and less able movers suffer losses in their asset values (see Interval II in Figure 3.4). Although relatively older and less able workers are harmed, they are better off than deciding to stay in the low-tech sector. Proposition 3 summarizes the results discussed above.

**Proposition 3.** *Trade liberalization offers new opportunities to abler and younger workers who otherwise are stuck in the low-tech sector but have the potential to become movers: simultaneously it forces less able and/or older workers to exit the labor force. And not all movers benefit.*

The process of adjusting to free trade somewhat resembles Melitz (2003) who shows that those firms which are most productive begin to export while least productive firms exit due to trade liberalization. Analogous to firms' adjustment behaviors, trade liberalization induces relatively younger and/or abler workers in the low-tech sector to enter the high-tech sector, while older and/or less able workers in the low-tech sector exit the labor force.

Results from the present paper are analogous to the work of Falvey *et al.* (2010) who consider a two factor (skilled and unskilled) and a two sector Heckscher-Ohlin model with an education sector. They investigate how characteristics of unskilled workers, age and ability, affect when and whether they opt for skill upgrading in response to trade liberalization. They show that younger and abler unskilled workers are most likely to upgrade, while older and less able upgraders are likely to lose. In their paper, not all upgraders are better off as a result of trade liberalization. Falvey *et al.* (2010) avoid the complexity of adjustment via labor market, although their model considers skill upgrading as a worker's choice. In addition, their model assumes fixed wages. In contrast, the present paper derives the asset value of an unemployed worker of different age and ability and examines adjustment of heterogeneous workers via an imperfect labor market. The key insight of both papers is that age and ability are substitutes. In the broad viewpoint, though, it can be understood that skill upgrading of unskilled workers in the work of Falvey *et al.* (2010) is basically equivalent to sector shifting choices of movers in the present model.

Comments on the mobility assumption between sectors in the present model close this section. Return to the two classical trade models: Usually, the Ricardo-Viner model is considered as the short-run one in which specific-factor in each sector can be interpreted as having infinite mobility costs in switching sectors. On the other hand, the Heckscher-Ohlin model takes the long-run view. Two factors, for example high-ability and low-ability workers, have no restriction of movement between two sectors, for example high-tech and low-tech. In reality, worker mobility probably lies between these two extremes and likely varies by worker heterogeneity. The results of this paper can be robust although mobility cost is an increasing function of age as in Artuc (2009) who studies the relation between worker mobility, age, and welfare effects of free trade.[20] Even if the mobility cost is granted, the present model would predict that youngest and ablest workers who have maximum ability to be mobile between sectors would be winners, while oldest and least able workers who have no ability to switch sectors would be losers. A middle range of workers (in terms of age and/or ability) who may have conditional sectoral mobility would be either winners or losers, depending on exogenous change in output price due to trade liberalization.

[20] Mobility cost is related to many papers that deal with the amount and the speed of resource allocation between sectors. See, for example, Wacziarg and Wallack (2004) and Menezes-Filho and Muendler (2011).

## 3.4 Conclusion

Trade liberalization causes resource reallocations towards export sectors that have comparative advantages in an economy. These reallocations are an important source of gains from trade. However, the transition to trade liberalization involves adjustment costs. Although a number of empirical studies have emphasized both unemployment and personal characteristics in understanding adjustments of trade-related workers impacted by trade liberalization, many recent theoretical trade models neglect either labor market frictions or worker heterogeneity.

This paper focuses on the hitherto largely ignored role of age together with ability of workers in a search and matching model of the labor market. An important component of the model herein was that older workers are defined as those who cannot create total surplus over total training costs in their remaining lifetimes. Hence, no matching arises between old workers and firms. The model has shown that (i) age threshold to determine oldness of a worker in a sector positively depends on his/her ability and (ii) the asset value of an unemployed worker decreases in age, but increases in ability.

When a country enjoys a comparative advantage in the production of the high-tech sector, and protects the low-tech sector by tariff, an interesting feature of the model is that one-and-the-same worker can be regarded as *young* in the low-tech sector, yet as *old* in the high-tech sector. Tariff removal causes change of age threshold of workers in the low-tech sector due to change in real training costs. It also leads the asset value of an unemployed worker in the low-tech sector to fall and that in the high-tech sector to rise. The present paper concludes that trade liberalization forces some younger and/or abler workers who otherwise are stuck in the low-tech sector to switch sectors and offers new opportunities to be winners; simultaneously it forces older and/or less able workers to exit the labor force. This author closes by pointing out the key insight of this paper, namely that age and ability are substitutes.

The model developed herein uses representative firms in the analysis. For future research, it would be interesting to see what happens to the model with heterogeneous firms.

APPENDIX

# Appendix

1. Derivation of Equation (3.7) and (3.8)

**Case 1.** The total surplus created by the old with ability $a$, $\eta(a) < t \leq T$.

From (3.1), (3.2), and (3.4), $\rho S = pa/P - bS + \dot{S}$. From (5), $J = (1-\beta)S$. The surplus is

$$S(a,t) = pa[1 - e^{-(\rho+b)(T-t)}]/(\rho+b)P$$

Using the boundary condition, $S(a,\eta(a)) = z/P$. Equation (3.8) is derived.[21]

**Case 2.** The total surplus created by the young with ability $a$, $t < \eta(a)$.

From (3.1), (3.2), and (3.4), $\rho S = pa/P - bS - m(E-U) + mx/P + \dot{S}$. In equilibrium, we know that $x = \beta z$ is satisfied in (3.6). Using (3.5), $E - U = \beta S$, $(\rho + b + m\beta)S = pa/P + mx/P + \dot{S}$.

Using $S(a,\eta(a)) = z/P$, the total surplus created by the young is
$S(a,t) = \int_t^{\eta(a)} (pa/P + m\beta z/P)e^{-(\rho+b+m\beta)(\eta(a)-s)}ds + e^{-(\rho+b+m\beta)(\eta(a)-t)}S(a,\eta(a))$.

The value of a new match is

$$S - z/P = \frac{pa/P - (\rho+b)z/P}{(\rho+b+m\beta)}\left[1 - e^{-(\rho+b+m\beta)[\eta(a)-t]}\right]$$

Equation (3.7) is derived.[22]

2. Wage schedule

For the old, $w(a,t)/P = pa/P - (b+r)J + \dot{J}$. Hence, $w(a,t)/P = pa/P - (\rho+b)(1-\beta)S + (1-\beta)[(\rho+b)S - pa/P] = pa/P - (1-\beta)pa/P$. That is,

[21] Let us check that $E(a,\eta(a)) - U(a,\eta(a)) = (1+m/b)(c_1+A)e^{\phi\eta(a)} = x/P = \beta S(a,\eta(a))$. Hence, the boundary condition is confirmed. I used the fact, $A(a,\eta(a)) = 0$.

[22] As a worker ages, the surplus declines, $\frac{dS}{dt} = \frac{1}{P}e^{-\phi(R(a)-t)}[\sigma(\rho+b) - pa] < 0$.

$$w(a,t)/P = \beta pa/P.$$

For the young, use $J = (1-\beta)S$, $(\rho+b)S - \dot{S} = pa/P - m\beta S + mz/P$, where $w(a,t)/P = pa/P + (1-\beta)[m\beta S - (pa+mx)/P]$.

After some algebra, the wage schedule for the young is

$$w(a,t) = pa - (1-\beta)\left[(\rho+b)(pa+m\beta z) + m\beta[pa-(\rho+b)z]e^{-(\rho+b+m\beta)(\eta(a)-t)}\right]/(\rho+b+m\beta)$$

with $\chi_y(t) = [1 - e^{-(\rho+b+\beta m)[\eta(a)-t]}]/(\rho+b+\beta m)$,

$$w(a,t)/P = \begin{cases} \beta[pa/P] + (1-\beta)[\beta m/P][pa - z(\rho+b)]\chi_y(t) & for \quad t \in [0, \eta(a)] \\ \beta[pa/P] & for \quad t \in (\eta(a), T] \end{cases}$$

The old are paid constant wage in proportion to their abilities. Wages for the young decrease in age.

3. Derivation of Equation (3.10)

For the young, rearrange Equation (3.1) and (3.2) as follows.

$$\begin{bmatrix} \dot{E} \\ \dot{U} \end{bmatrix} = \begin{bmatrix} \rho+b & -b \\ -m & \rho+m \end{bmatrix} \begin{bmatrix} E \\ U \end{bmatrix} + \begin{bmatrix} -w(a,t)/P \\ mx/P \end{bmatrix}$$

**Step 1**) Calculate the eigenvalues of the first term of RHS, using $(\rho+m-\lambda)(\rho+b-\lambda) - mb = 0$. Two real eigenvalues are $\lambda_1 = \rho+b+m$ and $\lambda_2 = \rho$.

**Step 2**) Calculate the corresponding eigenvectors. Denote $\lambda_1$ for $\begin{bmatrix} 1 \\ -m/b \end{bmatrix}$ and $\lambda_2$ for $\begin{bmatrix} 1 \\ 1 \end{bmatrix}$. The general solutions, $E_g$ and $U_g$ are expressed as

$$\begin{bmatrix} E \\ U \end{bmatrix}_g = c_1 \begin{bmatrix} 1 \\ -m/b \end{bmatrix} e^{\phi t} + c_2 \begin{bmatrix} 1 \\ 1 \end{bmatrix} e^{\rho t}$$

where the characteristic matrix becomes $\Phi = \begin{bmatrix} e^{\phi t} & e^{\rho t} \\ -\frac{m}{b} e^{\phi t} & e^{\rho t} \end{bmatrix}$ and $\phi = \rho + b + m$.

Use the fact that $E(a,t)$ and $U(a,t)$ should be continuous at age $\eta(a)$. From (3.7), $e^{-(\rho+b)(T-\eta(a))} = 1-(\rho+b)z/pa$. Check that $E(a,\eta(a)) = \frac{w}{(\rho+b)P}(1-e^{(\rho+b)(T-\eta(a))}) = \frac{w}{(\rho+b)P}[1-(1-(\rho+b)z/pa)] = \beta z/P$.

**Step 3**) To calculate $c_1$ and $c_2$, use $E(a,\eta(a)) = c_1 e^{\phi\eta(a)} + c_2 e^{\rho\eta(a)} = \beta z/P$. And use $U(a,\eta(a)) = -\frac{mc_1}{b} e^{\phi\eta(a)} + c_2 e^{\rho\eta(a)} = 0$.[23] By substitution, $c_1 = \frac{b}{m+b}\frac{\beta z}{P} e^{-\phi\eta(a)}$ and $c_2 = \frac{m}{m+b}\frac{\beta z}{P} e^{-\rho\eta(a)}$.

Then $U_g = -\frac{m}{b+m}\frac{\beta z}{P} e^{-\phi\eta(a)} + \frac{m}{b+m}\frac{\beta z}{P} e^{-\rho\eta(a)} = \frac{\beta m z}{(b+m)P}\left(e^{-\rho[\eta(a)-t]} - e^{-\phi[\eta(a)-t]}\right)$.

**Step 4**) Let us find out particular solution, $\begin{bmatrix} E \\ U \end{bmatrix}_p = \Phi \int_{\eta(a)}^{t} \Phi^{-1} b(s) ds$ where $b(t) = \begin{bmatrix} -\frac{w(a,t)}{P} \\ \frac{mx}{P} \end{bmatrix}$.

After some algebra, we get $\begin{bmatrix} E \\ U \end{bmatrix}_p = \Phi \begin{bmatrix} A \\ B \end{bmatrix}$ where $A = \int_{\eta(a)}^{t} -\frac{b}{(b+m)P}(w+mx)e^{-\phi t} dt$ and $B = \int_{\eta(a)}^{t} \frac{b}{(b+m)P}(-\frac{m}{b}w + mx)e^{-\rho t} dt$.

**Step 5**) $\begin{bmatrix} E \\ U \end{bmatrix} = \begin{bmatrix} E \\ U \end{bmatrix}_g + \begin{bmatrix} E \\ U \end{bmatrix}_p$ is the sum of the general solution and the particular solution.

**Step 6**) The asset value of an unemployed young worker is

[23] Ending values are $U(a,T) = 0$ and $E(a,T) = 0$

$$U(a,t) = \frac{m}{(b+m)P}\left[pa(C-D) + \Theta(\beta) - \beta z(\rho+b)(C-D)\right]$$

where $C = [1 - e^{-\rho[\eta(a)-t]}]/\rho$,

$D = [1 - e^{-(\rho+b+m)[\eta(a)-t]}]/(\rho+b+m)$,

$E = [1 - e^{-(\rho+b+m\beta)[\eta(a)-t]}]/(\rho+b+m\beta)$ and

$\Theta(\beta) = \Big\{\beta[pa - z(\rho+b)][\frac{(1-\beta)m}{b+m\beta}(E-C) + (D-E)] - (1-\beta)(\rho+b)(pa+m\beta z)(C-D)\Big\}/(\rho+b+m\beta)$.

In the above equation, $\Theta(0) = -pa(C-D)$ and $\Theta(1) = 0$. Let us use one-to-one function $\kappa(\beta)$ such that $\kappa pa(C-D) = pa(C-D) + \Theta(\beta)$. Hence, Equation (3.10) is derived.

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