ABSTRACT

STOCHASTIC MODELS OF SYSTEM OCCUPATIONAL MOBILITY

By

Shelby Stewman

To delineate the underlying structural dynamics which govern opportunities in one's job was a primary aim of this study. The conceptualizations utilized depict mechanisms and limiting contexts operative upon an individual's occupational movement, whether knowingly or not.

The study was longitudinal, extending over forty-five years. Its focus was system occupational mobility. The system of occupations was the Michigan Department of State Police. Since jobs entered by persons leaving the State Police organization were not considered, the system of jobs or labor market under consideration was organizational or formally bounded. Moreover, the examination of the mobility processes was primarily through the use of mathematical models--the central ones being Markov chain models. While the central analysis focused upon the mobility processes, the sequential nature of the longitudinal data (1927-1971) enabled analysis of the relationships of other system dynamics and exogenous forces to the mobility processes. Included were state economics, organizational growth and differentiation, policies concerning recruitment and retirement processes and historical forces such as the Great Depression, World War II, the Korean War, the Vietnam War and the institution of the forty-hour work week.

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The principal substantive thrusts of the study have been the following:

1. An explication and comparison of two Markov chain models of occupational mobility,

2. A critical test of each of these two models, and

3. Specific extensions of the two models including the construction of a system growth model and the initial analytical steps in the construction of a manpower loss rate model.

Two major types of Markov chain models were utilized--those concerned with the path of job vacancies which result from the movement of men and the creation of new job vacancies and those which view mobility in terms of movement of men through strata. Each of the two models is an open system model conceptualizing three types of flows--inflows, thru flows and outflows.

Data were intra-organizational and covered each year from 1927 to 1971. The data also included all moves related to vertical mobility throughout the system.

From 1950 through 1969 the estimated transition probabilities for both models remained relatively stable. Substantively, this means that there existed a stability underlying the dynamics of the mobility processes despite considerable changes in structure observable at the surface level. Substantive examination of other assumptions of the models than that of stationarity were also undertaken.

The predictions from both models were quite accurate for this twenty year period. Two types of tests were undertaken with respect to the demographic (manflow) model. The first involved predictions for short term periods (five years) whereas the second test extended the time period for predictions (nine and fifteen years). For the vacancy chain (interrelated job vacancy flow) model, two basic types of test were made, each of which involved more than one type of time period. The first basic type of test involved job vacancy chain length distribution and the second type pertained to the number of moves from each stratum. As for the first test, new combination or "lumping" schemas were utilized and were productive.

The principal extensions of the occupational mobility models were attempts to derive certain terms assumed as given in the models. Other less direct extensions included a descriptive and supplementary analysis of organizational processes and individual careers.

The present research has utilized an organizational approach to occupational mobility. The conceptualizations by the occupational system models are seen to be applicable to occupational sets within which mobility is limited, whether or not the boundedness of movement is formally organized. Hence, the models are viewed as generalized formulations of intragenerational occupational mobility.

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A THESIS

Submitted to

Michigan State University

in partial fulfillment of the requirements

for the degree of

DOCTOR OF PHILOSOPHY

Department of Sociology

1973

ACKNOWLEDGMENTS

The acknowledgments below are certainly not fully expressive of my indebtedness to those who enabled me to conduct this research, but they are at least a beginning. I should like to thank Thomas L. Conner, who provided principal guidance of the study, for his many hours spent with me long before this study began as well as throughout its duration encouraging my curiosity concerning social structure, theory, mathematics and their interrelations; Santo F. Camilleri for the special critical and theoretical approach to sociology which I have attempted to use; Hans E. Lee for his prophetic pronouncements and continual encouragement of my training in theory, mathematics and computers; William H. Form for providing the basis for my structural view of the world; and Harry Perlstadt whose insight for discovery of data from an unobtrusive Civil Service form was the initial impetus for this research.

I should also like to express my appreciation to the many anonymous State Policemen who made the study possible. Specifically, I could not have conducted this research without cooperation from Col. John R. Plants, Lt.Col. Forrest J. Jacob, Lt.Col. John N. Brown, Capt. Edward A. Lenon, Lt. Harold D. Teddy, Walter L. Riley, Ethel Aldrich and all other members of the Personnel Division. More specifically, however, I am most appreciative to Pearl Petit for her interest, cooperation and detailed knowledge of the system records.

To friends who have actively helped, Chuck Lazer, Pat Herbert and Joe Downs, I am grateful. There is one person who has been of continuous assistance throughout this project--my wife, Cherry. She has provided continuous encouragement, typographical skills, critical editorial skills and somehow endured the highs and lows of an involved research sociologist at work.

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CHAPTER 1

THE PROBLEM

> The powers of ordinary men are circumscribed by the everyday worlds in which they live, yet even in these rounds of job, family, and neighborhood they often seem driven by forces they can neither understand nor govern. (Mills, 1956)

To delineate the underlying structural dynamics which govern opportunities for movement in one's job is a primary aim of this study. While there has been considerable study of national patterns of occupational mobility, these have largely been addressed to occupational distributions. The present study goes beyond the focus upon careers and aggregations of occupations to a conceptualization of the mechanisms and limiting contexts of occupational mobility. Not only will this conceptualization provide a more thorough explanation of the mobility processes, but its focus upon occupational movement within a system context also provides the basis for an evaluation of the effect of operative policies upon general system dynamics and upon the lives of individuals within the system relative to movement or nonmovement. In short, it is an examination of the operation of cumulative structural constraints upon individuals' work biographies.

While this study's scope is restricted to one particular type of occupational grouping, a hierarchical promotion system, the conceptualization is seen to be applicable to other occupational sets within which mobility is limited. More specifically, the conceptualization is applicable whether or not the limiting restrictions or boundedness of movement is formally organized. It is thus seen as a generalized conceptualization of intragenerational occupational mobility. Moreover, while it is not contradictory but rather complementary to conceptualization emphasizing individual motivation and choice, the aspect which provides additional explanatory power is precisely that there exists structural boundaries within which the actual individual choices are limited, whether knowingly or not.

Let us examine the idea of limiting boundaries of occupational mobility. One characteristic of a highly industrialized society is that it has a highly differentiated occupational structure. Hence, there are not only more occupations in which a new entrant to the labor force may begin his career, but there are more moves possible between occupations for an individual already in the labor force. Nevertheless, there are restrictions operative upon the set of all possible moves. The restrictions upon such possible movement depend to a large extent upon the type of occupational grouping to which an individual belongs. For instance, lower level blue collar and white collar jobs provide rather limited opportunities for upward occupational mobility. On the other hand, a rather extended set of opportunities for mobility may exist in an occupational grouping such as fire departments, police forces and other civil service organizations in which there exists a hierarchical promotion system.

In general, it would appear as Theodore Caplow has suggested (1954) that there exists several distinct species of occupations, each having its own restrictions for movement. That this is true and yet has been largely unexplored by sociologists is somewhat ironic since the aspect omitted from the perspective has been that of structure. Most sociologists, by analyzing intragenerational occupational mobility primarily in terms of individual careers, have omitted the organizational nature of such mobility. This study differs in that I will utilize an organizational approach. Mobility processes will be conceptualized in terms of a system of jobs and a system of men. For such systems the distinction between hierarchical promotion systems and other occupational groupings is not in their being bounded but rather in the extent to which these boundaries are formally defined. Nevertheless, the context in which the system framework is perhaps most easily applied is that of a large stratified system of jobs which is formally organized, as for example, a bureaucracy. Moreover, when such a bureaucracy has vertical movement throughout its hierarchy,[†] it is of particular interest for an initial study such as this one. The system chosen was but one of nineteen Civil Service departments in the State of Michigan. It was the Michigan Department of State Police which has a clearly defined and occupational interrelated hierarchical structure. The labor market for this study will also, therefore, be formally defined. That is, jobs entered by persons leaving the State Police organization will not be considered. Should the scope of the study have been larger to include such jobs, interrelations or movement between systems could likewise have been conceptualized within a system or organizational framework.

[†]In other words, the entire system is a continuous hierarchical promotion system, as distinct from an industrial firm with production workers and administrative staff forming two separate spheres of movement.

Also of significance regarding this study is the fact that while much sociological research has focused upon blue collar factory workers and their immediate supervisors, little sociological research input has been toward occupations involving hierarchical promotion systems. Caplow's comment (1954) on the dire lack of analyses involving mobility processes within such systems is still applicable. Perhaps one of the better studies involving a bureaucratic system of this type is Herbert Kaufman's The Forest Ranger. In the study he states the mechanisms of occupational mobility operative in the United States Forest Service. However, Kaufman gives us no more than a qualitative statement regarding policy and its activation. For instance, he does not provide any sort of numerical information as to how the system operates over time with respect to actual flows of men. This study will examine the latter as well as the effects of policy decisions.

An import of another kind for this study may be seen by observing the increasing significance of public service systems in our society. The examination of systems which are governmental serves to point out an important trend of the industrialization process within capitalistic societies--the increasing internal participation of the governmental sector beyond the local level and thus the increasing importance of political processes for structuring society. An indication of their particular importance may be seen in their increase in number and size. To emphasize this point, I refer to a fact mentioned above--the Michigan State Police is but one of nineteen Civil Service departments in the State of Michigan.

Two additional facets of this study should perhaps be stressed. First, the analysis is of a police system. The current exploratory level of sociological research on these systems needs extending to include in depth analyses of the police, the courts and the penal systems and their interrelations with each other and other sectors of society. Perhaps an initial step in this direction would be an analysis of the internal dynamics of each system such that the relevance of the points of linkage are more clearly understood. In any event, it seems to me more appropriate to analyze either of these aspects by means of a processual conceptualization of the problem. It is hardly sufficient to substitute a general historical background and a static study for an examination of a system over time. Neither are cross-classificational studies of many systems at one point in time sufficient. Processual conceptualizations necessitate processual data. This brings us to the other aspect of the study which I feel needs mentioning--this study is longitudinal, extending over forty-five years (1927-1971).

Since there exists no lateral entry for police manpower, occupational mobility is a fundamental and central process which extends from the top to the bottom stratum of the State Police system. The focus upon these mobility processes will, therefore, also serve as a means for investigating the internal dynamics of the system. Moreover, due to the longitudinal nature of the study, the conditions under which internal dynamics themselves change may be investigated. In this regard exogenous as well as endogenous forces affecting internal processes will be taken into account, thus providing an initial base for extending the formulation into a more general theory of system dynamics.

System Characteristics and General Dimensions of the Mobility Processes

Before introducing the general dimensions of the mobility processes, a brief description of the characteristics of the State Police system seems in order. The organizational characteristics which are most pertinent to occupational mobility are the following:

1. It is self-contained in terms of mobility (i.e., no exchange of personnel between the State Police and other Civil Service departments).

2. It has a highly formalized authority structure and a clearly defined stratification system.

3. The authority-status structure is diagrammed as follows:

Director
(Colonel)
|
Assistant Directors
(Lieutenant Colonel, Majors)
|
District and Division Commanders
(Captains)
|
Lieutenants, Detective Lieutenants
|
Staff Sergeants, Detective Staff Sergeants
|
Sergeants, Detective Sergeants
|
Corporals, Detectives
|
Troopers

4. As may be noted from the above authority-status structure, the system has three principal occupations--general policeman, specialized criminal investigators and administrators.

5. Even though it is a state government department, it is highly autonomous relative to internal government; therefore, there

exists a high degree of endogenous determination of the mobility processes.

6. Mobility is almost entirely over a distance of one stratum only (i.e., if movement is upward, a man moves only to the stratum immediately above the one he presently occupies).

7. All men entering the system enter at the bottom stratum (i.e., there is no lateral entry); and entrance varies greatly in size, dependent upon demand, loss rate and state economics. (The range thus far has been from 6 to 150.)

8. In age, the system is quite young; it was created in 1917 and at the end of the period of this study was but 55 years old.

9. The system has generally been expanding from its inception to the present.

Having now in mind the more important characteristics of the system for occupational mobility, we may proceed to describe the general dimensions of the mobility processes. Four general dimensions seem most crucial--the different types of mobility processes, the size of the system, the more powerful exogenous forces affecting mobility and the endogenous control whether directly or indirectly over mobilization.

First, let us consider the types of mobility processes being studied. For an open system (i.e., open to exogenous forces), three types of flows may be said to characterize the mobility processes--flows into, flows through and flows out of the system. For instance, if one is referring to a system of men, the three flows would consist of:

1. Inflows: recruitment, reinstatement, return from leave of absence

2. Flows through: promotion, transfer, demotion

3. Outflows: retirement, leave of absence, leaving for other reasons, dismissal and death.

Secondly, the effect upon these flows by changes in system size must be taken into account. Whether the system is expanding, stable or declining in size has important consequences for conceptualizing the three types of flows mentioned above.

Thirdly, there are certain exogenous forces which greatly affect the size and hence the flow processes. Certainly the demand for original services must be considered. If the demand is in the direction of greater manpower, which has generally been the case for the State Police from its inception in 1917 to the present, there will possibly be the addition of new supervisory or administrative positions whenever growth at the lowest stratum occurs. Moreover, in an expanding system such as this one, one might also expect greater specialization resulting in internal reorganization. This reorganization might involve questions about the newly differentiated function such as geographical location, rate of expansion, type of requirement for entry, necessary training and perhaps the addition of an entirely new supervisory component. If the demand is in the direction of less manpower or the elimination of a specific function as has occurred for minor segments of the State Police, the opposite of the questions above, as well as problems of relocation or dismissal of personnel, become relevant.

Other exogenous forces which should be taken into consideration include expansion into additional services, the condition of state economics and major historical events such as the Great Depression, World War II and the forty-hour work week.

The endogenous forces which seem of most import in terms of internal control over the flow processes may be stated in terms of policies operative in the system. Five such policies or sets of policies seem particularly relevant. First, policies affecting the administrative-staff ratio will determine the actual number of new administrative positions to be added when lower stratum growth and/or specialization occurs. This will, therefore, affect internal flows. Secondly, policies affecting the type of new recruit and time of recruitment (whether continuous or in discrete units of time) set limits on the inflow process. Third, policies toward training procedures affect both inflow and outflow rates. Fourth, policies concerning promotions and demotions affect the flows through and out of the system. Specifically, the "waiting time" period before promotion and the types of internal processes such as reorganization generating possibilities for promotion seem important. With regard to demotion, the effect upon future promotion and "strictness" of enforcement of policy require consideration. Finally, policies for retirement are certainly important for the outflow process.

Introduction to Two Stochastic Models of Occupational Mobility

A question of import for this study arises from the continuous interrelation of theory and method. While I hold that theory and method are at each stage of a study inseparable, this position is perhaps most easily seen in a study in which substantive phenomenon are analytically conceptualized in a formal language--mathematics. In general, this study is one in mathematical sociology. That is, the conceptualization of occupational mobility is in terms of mathematical models.

As stated in the previous section, the mobility processes will be characterized by three types of flows--those into, through and out of the system. At present, these types of flows have been conceptualized in two distinct ways, each a simple Markov chain model. This study will attempt to examine each type of model. The older system approach has been most adequately developed by David J. Bartholomew (1967). This approach views flows in reference to populations of men. Hence, flows are characterized by such processes as recruitment, promotion, demotion and leaving, as well as horizontal movement within strata. Since this type of theory is based upon flows of populations of men in the system, I will refer to it as a demographic approach to occupational mobility.

The demographic model is stated in discrete rather than continuous time. That is, moment-by-moment movement is not recorded. Rather, at discrete points in time the location of each of the personnel is noted. Hence, mobility is inferred from a person occupying different locations at two points in time. At most, the distance of these points will be in the neighborhood of one year. At best, they will be monthly (for this organization only, of course). Utilizing the proportion of movers to each stratum's population allows one to estimate the transition probabilities for the next year. Multiplying the transition probabilities estimated from year t by the size of the appropriate strata at year t+1 generates the predicted flow of personnel across strata for year t+1. The model predicts strata sizes across years. The time unit for analysis is one year. In addition to transition probabilities, the model also includes transitions for new men and men leaving. The relative importance of these parameters in the model will become evident in the development of the model per se. It should perhaps be noted that this model's conceptualization includes all three types of flows mentioned initially as characterizing mobility. The significance of this fact will become obvious in the discussion below.

In the second type of model developed by Harrison White (1970), the flows refer to a structural aspect of the system--job vacancies. The flows of job vacancies are encaptured by focusing upon the following: the stratum at which a job vacancy enters, the path of the vacancy through the system in terms of both intra- and inter-stratum moves and the stratum from which a vacancy leaves. White's theory will be referred to as a vacancy chain approach to occupational mobility.

The vacancy chain model conceives of movement within the opportunity structure--that is, with reference to job vacancies. The flows in this model refer to cohorts of job vacancies. However, all cohorts are treated as if they began at the beginning of the time period (i.e., imbedded in time). For this model timing is not at issue. Rather, it is the successive or sequential interrelations of the moves which are the crucial point. The combination of such chains of movement is the opportunity structure. The unit of time for this model may be yearly or longer periods. Another difference in the models is that for the vacancy chain model there are two predictions rather than one. The time period from which transition probabilities are to be estimated is dependent upon which of the two predictions is to be made. In either case initial vacancies are assumed to be given (i.e., observed or derived). Thus, by being given the number of job vacancies entering each stratum from the outside and the transition probabilities for vacancies, predictions are generated for the length of the job vacancy chain and the total number of job vacancy moves. An obvious difference from the demographic model is that this model's transition probabilities refer to job vacancies not men. Another difference is that the mobility parameters for this model refer only to internal and outgoing flows. The third type of flow characterizing mobility (incoming) has been assumed as given.

Of particular significance for our substantive problem will be the question of possible stability in the underlying dynamics of the system. For both models and hence both types of movement the examination of the estimated transition probabilities will provide the answer to this question. Should the transition probabilities remain stable over several decades, a major substantive finding will have been discovered as well as the utility of the models for long term prediction being enhanced. Substantively, stability of the transition probabilities means there exists a stability underlying the dynamics of the mobility process in spite of observable super-structural change. In case of such long term stability, the form of the stochastic model may allow one to predict the system's behavior over distant periods of time in the future (i.e., the system's limiting structure and structural dynamics). Should the transition probabilities be stable over shorter time periods, the model's effectiveness will obviously be more limited, yet still serve effectively for intermediate periods of time and more short run predictions.

Extensions of the Stochastic Models of Occupational Mobility

Three attempts to extend the two stochastic models will be made. The three extensions involve preliminary conceptualizations to account for elements of the models which were taken as given or derived.

These were the entrants of job vacancies (inflow parameter) in the vacancy chain model and entrants of new recruits (the population component to be multiplied by the inflow parameter) in the demographic model. The extensions are attempts to account for total system growth, growth per stratum and factors potentially affecting loss rate of men. Each extension is of a quite different nature.

The first extension, in which an attempt is made to explain system growth, utilizes a regression model. The model was constructed as a description of the political bargaining process between the Legislature and the State Police. In the model system growth is viewed as a linear function of four exogenous variables--density of highway traffic, traffic deaths, crime rate and the state economy. The first three factors are viewed as manpower demand components and the latter as a necessary condition.

The second effort to extend the models involves the construction of a method to account for arrival of new jobs by stratum. Such a derivation is necessary if the vacancy chain model is to have predictive power for projected time periods. Since I was aware of the fact that new job creations were not constant given considerable economic and political fluctuations, a realistic projection of system evolution necessitated this type of extension. Sociological theory was not sufficiently developed to be of great use in this derivation. Thus, two temporary methods were constructed having little theoretical or substantive basis. In both cases, the immediately prior two years are used as an estimate for deriving the expected number of new jobs per stratum. The first method is based upon a further extension of the regression model. The second merely utilizes average growth of jobs per stratum.

The final extension involves the first step in developing a mathematical model to describe the loss rate of men as a function of the age and seniority structure. Its import for extension purposes lies in the fact that manpower losses are one mode of job vacancy creations. However, the utility of such a model, once constructed, would be considerably more important than this in that it would allow for a much more efficient planning of incoming manpower needs.

Limitations

Since the models and their extensions have been briefly described, perhaps several limitations concerning the scope of this study should be delineated. For this analysis the "civilian" segment of the Michigan Department of State Police will not be considered. The stratification system and mobility processes for this segment are of a different type than that of the "enlisted" segment consisting only of professional policemen. To develop theories for both types of personnel is beyond the scope of this study. Therefore, the theories utilized pertain only to personnel involved directly in police activities and who are members of the police occupational hierarchy.

A second limitation involves the omission of a type of mobility operative only at the Trooper or bottom stratum. Within this stratum a large number of geographical moves occur. Yet, this mobility is of a different type. Whenever a Trooper moves from one post to another, his job moves with him. Hence, the entire Trooper stratum is viewed from within the organization as a "labor pool," and thereby the movement by one man may not necessarily be for the purpose of replacing another man. His movement may simply be a result of changing seasonal demands in certain geographical areas. Also, geographical movement from an urban area to a rural one and vice versa has been, at least during certain periods, an internal policy assuring men of a greater breadth of experience. Whereas those Troopers located at urban posts handle a greater volume of activities, those activities are also of a more limited scope. On the other hand, a Trooper located at a rural post will not handle as large a volume but will be involved in a greater variety of Trooper functions. Since the mobility described above is of a different variety from mobility denoting a major change in function (as opposed to emphasis) and perhaps occupation, it will not be considered in this study. Only those Trooper moves which indicate a change in function, such as a move to a detective unit or to the district or state administrative unit, will be included in the current conceptualization of occupational mobility.

A final limitation involves the lack of conceptualizing the abolition of jobs as a distinct process. Harrison White's vacancy model conceptualizes the probability of a job vacancy leaving according to stratum. Yet, in the State Police system a vacancy may leave either by the job being filled by a new recruit or by the job being abolished. White's model does not distinguish between these two distinct processes. Two comments seem necessary. First, the number of jobs abolished in the State Police is rather small. For this reason, if it becomes necessary to distinguish between the two means by which a vacancy leaves the system, such processes may be observed. Secondly, the notion of system evolution and its accuracy for the vacancy chain conceptualization will be affected by the frequency per year at which positions are abolished. Since the decrease in existing jobs was not considered in the extension of the vacancy chain model, the accuracy of generating system evolution will necessarily be off by the amount that jobs are abolished per year.

Supplementary Analysis of Organizational Structure and Individual Careers

Since the principal thrust of the study is of an organizational or system perspective and is of a mathematical nature, I have placed this particular facet of the study as an appendix. It is, in fact, one of several directions, most of which are not a part of this study, for extension toward a more general theory of organizational dynamics. As noted, this study has not focused upon individual careers. There are two basic reasons for including an analysis of career patterns. First, the phenomenon of occupational mobility when conceptualized in terms of careers is important in its own right. As previously mentioned, most sociological analyses of intragenerational occupational mobility have focused upon some aspect of an individual career. In addition, there has been little empirical investigation of the type of occupational grouping being considered here--a set of occupations restricted to a hierarchical promotion system. Hence, this supplementary analysis will involve investigating a rather unexplored dimension of occupational mobility from a perspective generally utilized by sociologists. A second reason for career level analysis is that additional factors important at the system level may be explored prior to their inclusion into a system conceptualization. Moreover, not only do certain facets of career mobility seem important for inclusion into a systems approach, but it also seems reasonable to investigate these facets descriptively prior to attempting a formal conceptualization.

Two primary aspects of career mobility seem most important based on the above rationale. First investigation of individual probabilities for upward movement per stratum over time will be considered.

Structural variables of system size and internal differentiation will be taken into account in this problem. In particular, the work of Peter M. Blau (1970) will be re-examined empirically and extended analytically with an application to occupational mobility.

The second aspect of individual career patterns to be analyzed includes the type of movement and time prior to movement from each stratum. The type of movement refers to two hierarchical ladders for movement, whereas time will involve comparing required "waiting time" and mean time actually taken before moving. Moreover, comparisons in terms of time will be made according to the last stratum reached in an individual's career. The latter comparisons will be for the purpose of establishing cutoff times, if they exist for nonmovers at each stratum. Analysis of the time element may be particularly important for a system level analysis in which seniority is conceptualized as an important dimension of mobility.

Contributions

A basic contribution, I think, lies in the particular type of conceptualization of occupational mobility utilized. It provides an alternative framework to that generally utilized by sociologists—a career perspective. (For example, see Blau and Duncan, 1967.) In this study, the conceptualization is organizational referring to occupational groupings or sets within which mobility is generally bounded. It, therefore, enables us to investigate the cumulative structural constraints operative upon individuals' work biographies. Moreover, this alternative conceptualization is such that more explanatory power is obtained since the limiting boundaries of the processes are delineated as well as the mechanisms underlying the system processes. In this regard, it also makes available increased information for manpower management purposes.

A second contribution also involves a relatively new focus. The study is of a hierarchical promotion system about which we know little as far as ongoing system dynamics. Thus, this study in conjunction with that of White's (1970), concerning an occupational grouping of clergymen, may be seen as the initial steps in a series of studies applying the organizational or system framework to occupations. The cumulative series of all types of occupational groupings would then (1) depict national subsets of occupations in terms of their restrictions for inter-group movement and (2) delineate the distinct national processes operative upon an individual's choice range.

A third contribution concerns the explication and comparison of the basic conceptualizations underlying the two stochastic models. In brief, during a separate discussion, mathematics are omitted and the distinct mode of conceptualizing the mobility process by each type of model is described. Thus a bounded occupational structure defined by manpower flows is compared with a bounded occupational structure defined by interrelated job vacancy flows. Of special note for this comparison will be their differences concerning degree of structural conceptualization and the adequacy for representing movement in various types of systems.

A fourth contribution lies in the processual nature of the conceptualization. While it is not necessary to mathematize the system conceptualization, as may be seen in the chapter to follow, the original theoretical construction was mathematical. It was, in fact, its mathematical nature which necessitated the gathering of processual data.

Not only are the models processual in the sense of conceptualizing relations in terms of movement, but the application of these conceptualizations are over extended time or processual in a different sense. One of the advantages of certain mathematical formulations lies in the fact that they provide a framework for analyzing data ranging over extended periods of time. The longitudinal nature of the data and the facilitating processual conceptualization of the problem provide information on possible long-trend processes.

Fifthly, the data are continuously sequential, as well as covering an extended time period (45 years). Consequently, we may investigate both exogenous and endogenous conditions under which the internal dynamics themselves may possibly change. Hence, an initial basis will be established for extending the formulation into a more general theory of system dynamics. Only the very elementary steps for constructing such a theory are undertaken within this study. The continuous, sequential nature of the data and the necessity for incorporating "externals" in terms of mobility for a poignant analysis allow these initial steps. Although not within the scope of this particular study, data was gathered which would allow a much more detailed mathematical-historical synthesis once this first step is complete.

A sixth contribution is the <u>critical</u> test of two stochastic models. The tests are not to distinguish between the two types of models; for at this stage of theory construction, it seems premature.†

†For a more detailed rationale for not attempting a test to distinguish between the two types of models, see Chapter 2.

Very few tests of the demographic type of open system model have involved an extended period of time. [For example, see Gani (1963), six years; Sales (1971), three years; Forbes (1971), four and eight years; and Mahoney and Milkovich (1971), three and ten years.] The vacancy chain model has been tested for but one species of occupational grouping--professional. This analysis examines a different species--a hierarchical promotion system. It is, however, the continuously sequential, extended time period which provides the basis for a quite strenuous (and in this sense, <u>critical</u>) test of each type of model. In addition, with the continuous sequential aspect of the data, I have constructed new types of tests for the vacancy chain model suggesting the import for continuous time data for extended theory construction, whether or not the conceptualization is in continuous time.

Seventhly, I have extended the original open system models in terms of deriving those terms assumed as given. One extension involves the interrelation of exogenous forces and internal dynamics. The other two extensions are primarily concerned with more general internal system dynamics and system policies.

An eighth distinct contribution involves the supplementary analysis of organizational structure and individual careers. The import of this is seen in its own light as well as for more general theory construction purposes.

Ninthly, there is throughout the study a careful interrelation of theory and method: the mathematical conceptualization and assumptions are interpreted in terms of a particular substantive manpower system. The explication of mathematical assumptions as well as the logic of decision making in terms of their substantive interpretation or consequences hopefully will allow a reader not trained in mathematics to follow the substantive issues throughout.[†]

Tenthly, this study is an analysis of a governmental system. With their increased growth rate at present being the fastest of any occupational sector in the labor force (Hall, 1969), it is becoming increasingly important as it structures an increasing number of men's work biographies. At a different level it is also becoming increasingly important as a force structuring society.

A final contribution lies in the fact that the system is a specifically important governmental one--a segment of the American police system. Due to their general lack of civilian review boards and the nature of their work, it is significant, first, that full cooperation was given at each step of the study by the personnel of the State Police, and secondly, that public knowledge is, therefore, gained of segments of this particular system's operation.

To summarize, the problem being investigated is that of occupational mobility. The context for this mobility is a set of occupations restricted to one formal organization. In particular it involves a hierarchical promotion system--the Michigan Department of State Police. The dimensions of the mobility processes which seem most important for this system are the following: the different types of mobility flows, system size, exogenous forces greatly affecting mobility and endogenous control, whether directly or indirectly, over mobility. Finally, this study includes two different types of levels of analysis.

[†]At points where more thorough understanding of the mathematics is desired, there is but one "satisficing" solution: to learn more mathematics!

First the study takes into account both system and individual levels of analysis. Secondly, the study contains descriptive as well as formal levels of conceptualization. Moreover, in terms of emphasis on each, the major focus of the study will be upon formal theories at the system level.

CHAPTER 2

CONCEPTUALIZATIONS UNDERLYING THE SYSTEM MODELS OF OCCUPATIONAL MOBILITY

Two distinct system or organizational conceptualizations of occupational mobility will be described below. Since the discussion will be centered around the two representations of mobility processes and system studies may be conducted utilizing these representations without their initial mathematical underpinnings, the mathematics will presently be omitted. The mathematical formulations and the assumptions involved for substantive interpretation of our particular system will be given in the chapters to follow.

The characteristics of the system most pertinent to occupational mobility and the general dimensions of the mobility flows were described in Chapter 1. The manner in which a flow is generated and the representation of the movement once a flow has begun remain to be explicated. Moreover, since two distinct types of models are being utilized, a comparison of their distinctness of conceptualizing occupational mobility seems in order.

Description of the Conceptualizations Underlying the Two System Models

Perhaps a good way to introduce the basic underlying conceptualizations would be to think of past sociological studies concerning occupational structure and mobility. Two primary foci come to mind. First is net redistribution of occupations over time. It is obvious that changes in the distribution of occupations affect mobility of manpower--whether of first entrants to the labor force or of men moving who are already part of the labor force. Yet, as demonstrated by Peter M. Blau and Otis D. Duncan (1967:81 ff.), no one has been able to explicate the specific effects upon mobility processes of historical trends of occupational redistribution.

Choosing to differ somewhat from Blau and Duncan, I propose an alternative conceptualization of the problem. The initial focus would be upon historical trends of job vacancies rather than historical trends of occupational distribution. More specifically, it seems to me that it is not the number of occupations per stratum but rather the number of vacancies within each occupational stratum that has a decisive effect upon mobility. Secondly, it seems reasonable to assume the dynamics of job vacancies operate according to specific labor markets. The focus, therefore, changes to one of a particular labor market within which job vacancies arise. It is within this context that the question of major forces generating job vacancies, whether exogenous or endogenous to the system, becomes meaningful. The variable of age suggested by Blau and Duncan is certainly one of several important forces affecting job vacancies. For instance, mobility is itself a factor since one way in which job vacancies are initiated is by men leaving the particular occupational system. These initial moves may themselves initiate chains of movement. Thus, job vacancies generated by men leaving the system may themselves generate further mobility. Alternately, job vacancies may result from the creation of new jobs. These initial vacancies may likewise initiate chains of movement. Therefore, forces affecting changing demand and generating new positions would also need to be taken into account. Irrespective of mode of a job vacancy's entrance to the system, the major part of mobility is viewed as an effect of the initial entry. Hence, the second aspect to be conceptualized is the internal dynamics of mobility per se. Finally, net redistribution of occupations is viewed as the effect of both job vacancy creations and men moving to fill them.

A second focus suggested by sociological studies of occupational structure and mobility brings us to a principal component of the demographic conceptualization--the flow of manpower between occupations. Sociologists typically examine intragenerational mobility by analyzing net flows of persons from their first job to their current job at the time of the study. Let us imagine a system having three strata. Manflows could be diagrammed as in Figure 1 below. In this conceptualization, structure is defined processually in terms of relations between occupational strata. The particular relations are manpower flows. I will return to this type of conceptualization, place it within a system framework and describe the type of models to be developed after discussing what I refer to as the vacancy chain model.

I would now like to suggest a third way of examining the problem of occupational mobility--a focus which generally has not been utilized. The way in which mobility has usually been conceptualized by sociologists is with reference to populations of men not populations of jobs, i.e., the labor market. The alternative perspective being suggested here is essentially a more complex structural one viewing not merely flows between occupations but rather the <u>flow structure between occupations</u>.

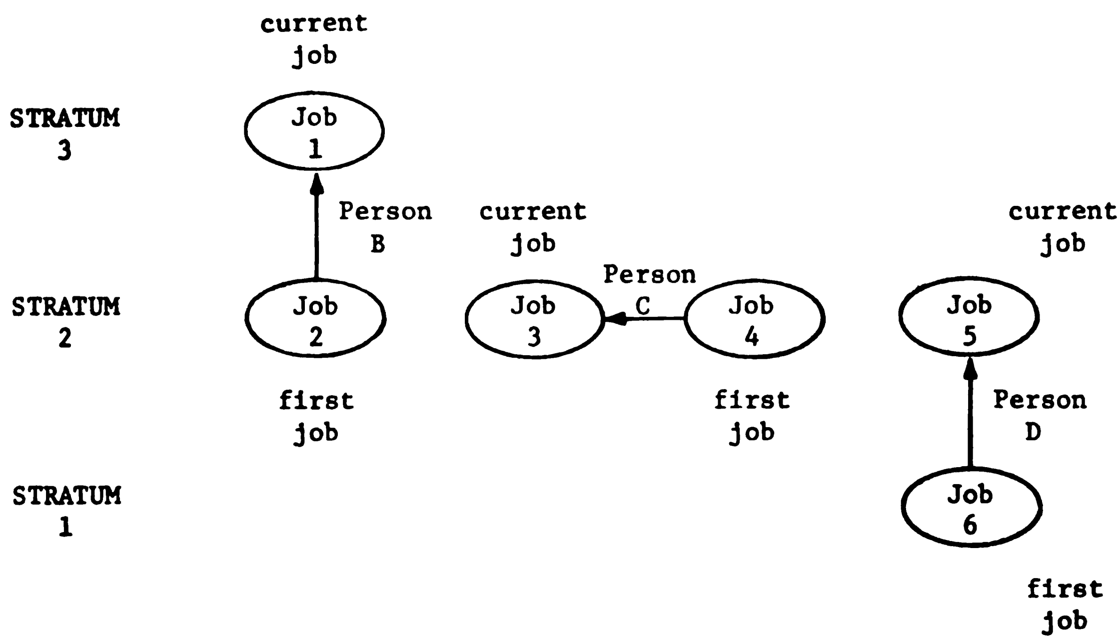

Figure 1. Occupational Structure Defined by Manpower Flows

For the time being, let us forget whether the person's movement refers to first job, second job, et cetera since these can later be obtained from the history of all moves if we desire. Let us examine the way in which the flows between occupations are themselves related. Once again we will consider a three strata system. As one may see in Figure 2, the conceptualization of interrelated movement is quite different from that conceptualized in Figure 1.

The movement diagrammed in Figure 2 may refer to the same movement as that in Figure 1 if Job 2 in the above schema refers to both Jobs 2 and 3 of Figure 1 and both Jobs 4 and 5 of Figure 1 refer to the same job, Job 3, in the above schema. That is, it may be that the job Person C enters is the same job as that left by Person B. Likewise, the job Person C leaves may be the same job Person D enters. The vacancy model conceptualizes just such movement. Singular movement not relating to other moves is also conceptualized by considering creations and abolitions of jobs.

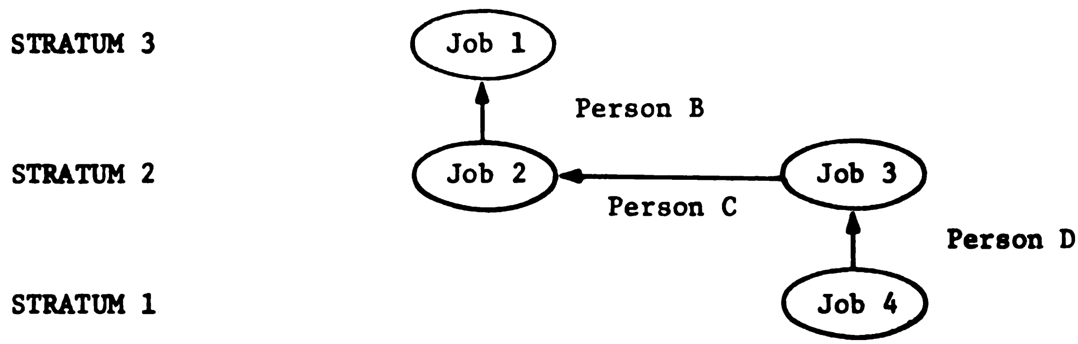

Figure 2. Occupational Structure Defined by Interrelated Manpower Flows.

The conceptualization of a flow structure between occupations depicts relations between relations or the relations between manpower flows. From Figure 2 we may note the length (total number) of related movements as well as the direction (whether horizontal or vertical) of these movements at each stage. Both length and direction when thought of together represent what will be referred to as a <u>chain of movement</u>. Within these chains of movement, each particular individual's intra-generational movement occurs.

By again referring to Figure 2, another question comes to mind. How do these chains of movement begin and end? Two possibilities occur in each case. Chains of movement begin either by a person leaving the system of jobs or by the creation of new jobs within that system. A person may leave a particular system's job market by various modes: retiring, dying, being layed off and quitting. In the case of the termination of a chain of movement, either a person enters from outside the system (a new recruit) or the position which in this case Person D leaves is abolished.

Two additional dimensions have now been added. First, we have denoted the alternative means of initiating and terminating chain movement. Secondly, the notion of a system boundary was implied. That is, the notions of entry and exit imply that the chain of movement occurs within a certain boundary which defines the system or the labor market being analyzed. Let us add a final dimension to this process. Note that movement may refer to movement of job vacancies as easily as to movement of men. If we wish to focus upon jobs, as is the case with the vacancy chain model, the appropriate population is that of jobs not men. Movement, therefore, refers to job vacancies. For instance, in Figure 2 the job vacancy moves from Job 1 to Job 2, then to Jobs 3 and 4. By including the modes of beginning and ending chains as described above, we would have the job vacancy moving from outside to Job 1, then to Jobs 2, 3, 4 and finally outside once more. Note that with the exception of two cases job vacancy movement is simultaneous with man movement. The two exceptions are the creation and abolition of jobs. We have now described the system depicted in Figure 3.

By conceptualizing the movement in terms of job vacancies we also conceptualize the mechanism generating movement. A man moves because a vacancy exists. In other words, the job vacancy operates in terms of a pull. Once we view the entire set of vacancy chains within a system, we are viewing an opportunity structure as well as the mechanisms for movement--job vacancies. Stated another way, this conceptualization of job vacancy chains represents not only a causal structure but the set of relations between opportunities which is an opportunity structure. The above conceptualization takes into account the basic elements of the vacancy chain model.

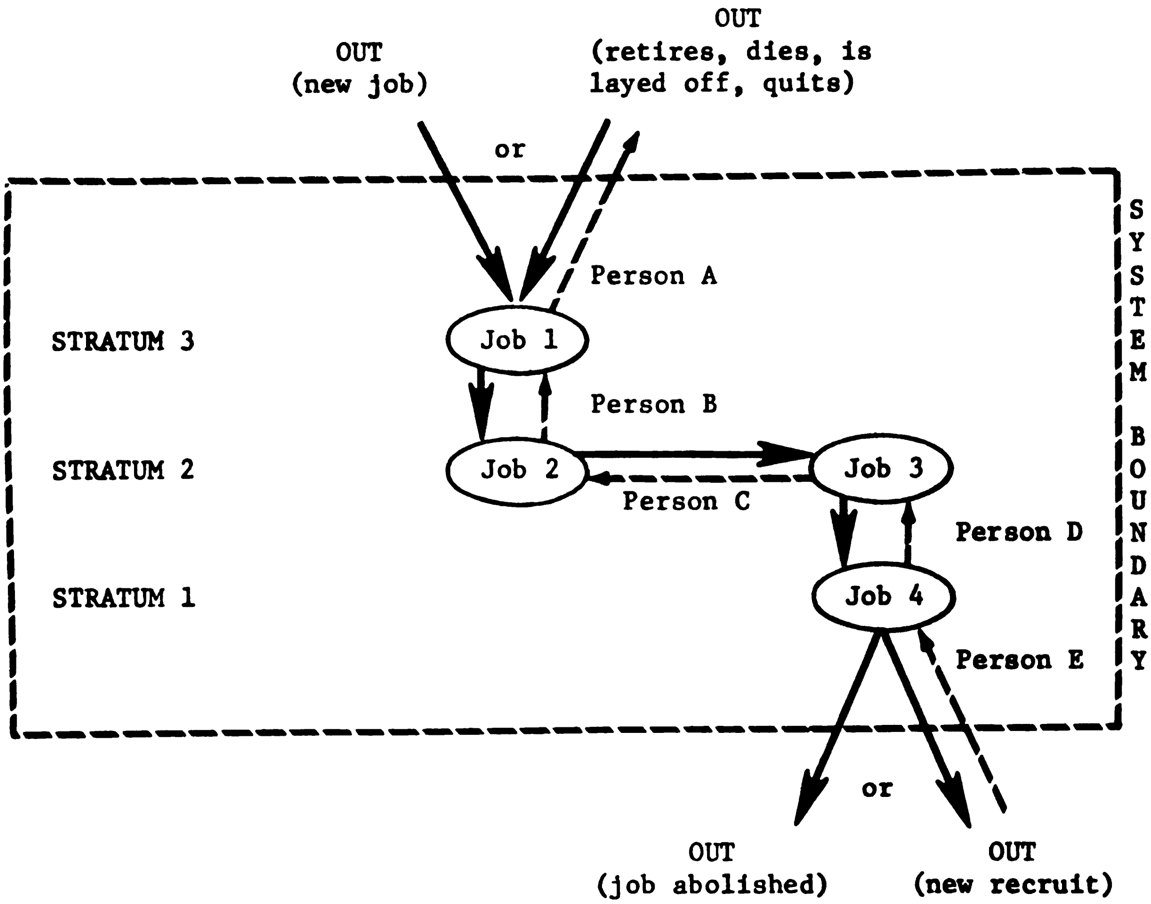

Figure 3. Bounded Occupational Structure as Conceptualized within the Vacancy Chain Model.

Let us now return to the demographic or manflow conceptualization. As in the vacancy chain conceptualization, a system will be depicted by means of distinct boundaries. The boundary limits are once more set by the job market. However, flows now refer not to job vacancies but to populations of men. In addition, there is neither a conceptualization of sequences of moves nor a mechanism prompting movement. Rather, mobility is described in terms of "isolated" manpower flows between strata with the result of this mobility being net distribution of men among the strata. Figure 4 depicts this conceptualization.

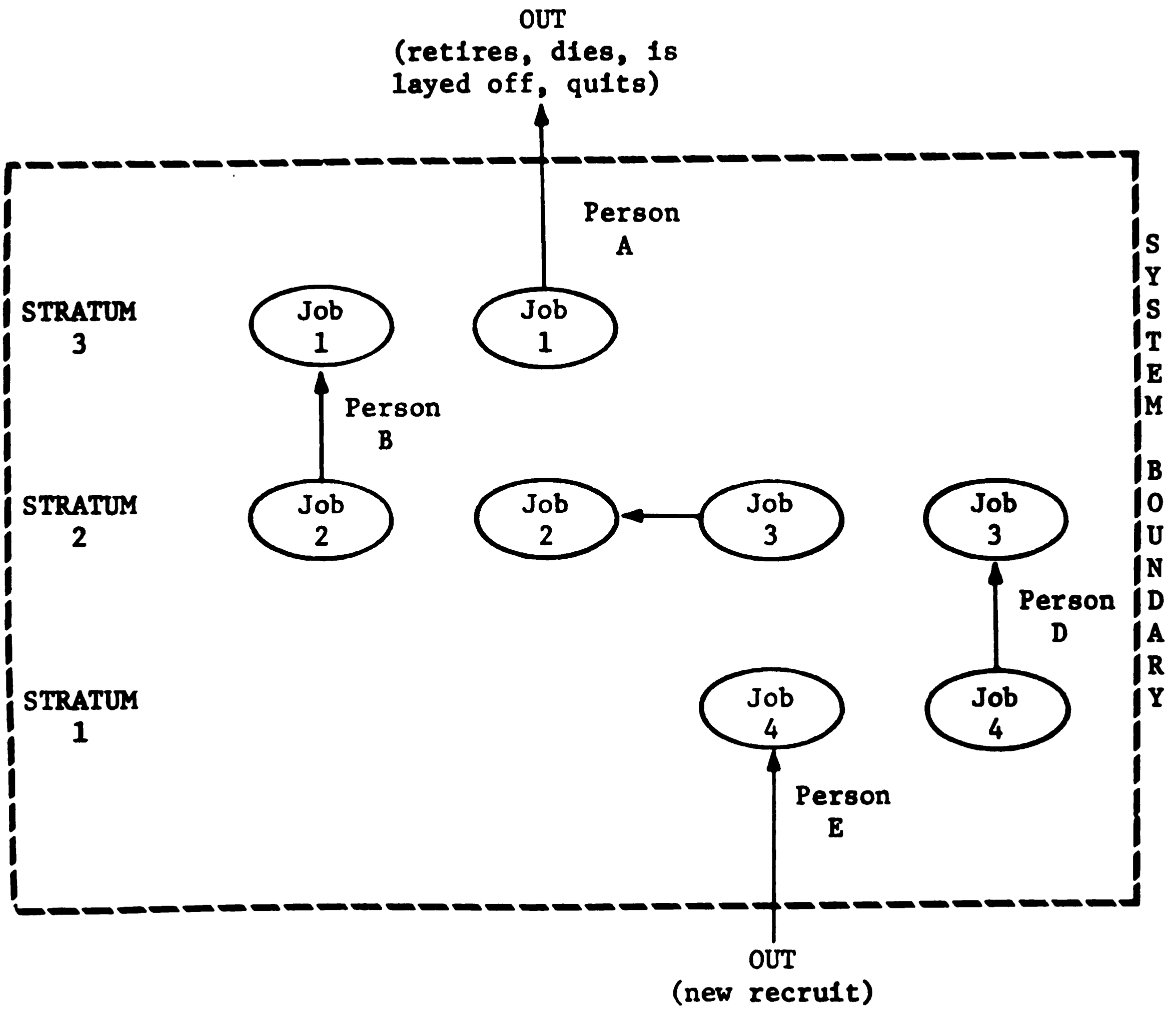

Figure 4. Bounded Occupational Structure as Conceptualized within A Demographic Model.

Comparison of the Basic Distinctions Between the Two System Models

Since each type of conceptualization of mobility has been described, we may now summarize the basic distinctions between them. One obvious difference between the two types of models lies in the object that is moving. It is job vacancies that are moving in the vacancy chain model; whereas, it is men that are moving in a demographic model. A second difference lies in the time span for which movement applies. Within the vacancy chain model, transition probabilities may apply to a time span greater than one year. In fact, the conceptualization is such that with the sequential data being used in this study, the entire time span could be analyzed as though it were one year. I mention this to point out the arbitrariness of a one year period when utilizing this model. This is not the case for the demographic conceptualization. Since more than one move by the initial population of men is not taken into account by a demographic model, a time span such that only one move may occur is the maximum allowed. In the State Police system moves from the Sergeant stratum and above are possible after a waiting period of one year from the date of entrance. It, therefore, seems reasonable to limit the demographic model's time span to periods of one year only.

A third distinction also relates to transition probabilities. The diagonal entries of the transition matrix of the two types of models refer to separate events. The transition matrix for a three strata system would be as follows:

$$\begin{pmatrix} p_{11} & p_{12} & p_{13} \\ p_{21} & p_{22} & p_{23} \\ p_{31} & p_{32} & p_{33} \end{pmatrix} .$$

In the demographic type of model the p_{ii} are the result of two things--either a person remaining in his job or a person changing jobs but remaining in the same stratum. No conceptual distinction is made for these two different events. On the other hand, in the vacancy chain model, the diagonal entries denote <u>only movement</u> within strata. Since a vacancy chain pertains only to movement, persons who remain in their particular jobs are not included in such a chain.

A fourth difference is that the mobility parameters for the vacancy chain model are limited to internal and outgoing flows. The third type of flow characterizing mobility (incoming) has been assumed as given. The demographic type of model conceptualizes all three types of flows.

A fifth difference is that a demographic model conceptualizes no mechanism for movement whatsoever while the vacancy chain model not only conceptualizes a mechanism for movement but one which is appropriate to the State Police. Within the State Police a man does not move unless a job vacancy exists. This type of mobility differs from that of an occupational grouping such as physicians or university professors where status change does not necessarily mean a change in function. Nor does movement within a university, as for example, from assistant to associate professor, open a vacancy at the assistant level. Rather, the job moves with the man. In the State Police system, the opposite holds. Hence, the vacancy chain conceptualization seems particularly apt.

A sixth distinction between the two types of models may be seen in their conceptualization of structure. The demographic type of model conceptualizes structure as the flows of manpower between occupations. The vacancy chain model, however, conceptualizes a second degree structure--the relations between the flows themselves or the flow structure between occupations.

The final distinction which will be drawn between the two types of models lies in the nature of their predictions. While the demographic model's conceptualization of mobility for the State Police system is not as complete as that of the vacancy chain model, one of its major strengths conceptually lies in its end result: its prediction of strata size over time. Within a demographic model, system evolution or the relative size of each stratum over time and mobility are explained at the same time. On the other hand, in the vacancy chain model the generation of job vacancies is taken as given and predictions refer to specific aspects of the mobility processes per se--the length of vacancy chains per stratum of origin and the total number of moves from each stratum. Unless linked with another model, system evolution is not conceptualized within the vacancy chain approach. In fact, even if linked to another model, it would be the other model which explained system evolution while the vacancy chain model explained the effects of such system change.

Use of the Two Models in the Study

Prior to the development and analysis of the models, we may ask how the two types of models will be utilized in the study. We should first observe that each model's strength seems to be the other's main weakness. The demographic model's main contribution seems to be in its accounting for relative strata size over time. Its primary weaknesses are in its inadequacy to conceptualize a mechanism prompting movement (a major fault of many demographic conceptualizations) and its more simple conceptualization of structure resulting in a much less comprehensive description of the processes of system mobility. Conversely, the vacancy chain model's major strengths are the two primary weaknesses of the demographic model (mechanism and more comprehensive structural description). One of the principal weaknesses of the vacancy chain conceptualization lies in the fact that its description of mobility processes begins once a job vacancy has entered the system. Therefore, unless the inflow process is constant, the vacancy model's utility for projection purposes is very limited. In addition to the strengths and weaknesses, neither model has been severely tested with sequential, longitudinal data. The demographic system model has been tested in a few instances. [See Chapter 1.] As for the vacancy chain model, tests on three systems of clergy have been made. However, none of the three tests involved continuously sequential years. Rather, a sampling of one year from each of five decades was chosen. There exists no way in which to thoroughly examine the interrelations of other major system dynamics or perturbations without continuous yearly data. Thus, an examination of the relationship of these factors to each model's conceptualization of mobility was considered to be more important at this stage of model building than an a priori decision in favor of either model. The same logic also suggested that a crucial test to choose one model over the other was premature.

CHAPTER 3

DEMOGRAPHIC MODELS OF OCCUPATIONAL MOBILITY

Having discussed the underlying conceptualization of occupational mobility as formulated within the demographic type of model, the substantive meaning of the model's mathematics should be more easily grasped. The following discussion will involve the mathematical conceptualization of system manflows and the analysis of demographic, mathematical models. Especially of interest will be the interpretation of the model for a particular substantive manpower system, an empirical examination of the mathematical assumptions and determination of the adequacy of this type of model for explaining occupational mobility within the State Police system. With respect to the latter interest, two types of tests will be undertaken. The first concerns predictions of stratum size (or system evolution) over short time periods. The second extends the predictions to much longer periods of time. Throughout the discussion explication of mathematical assumptions as well as the logic of decision making will be emphasized such that hopefully a reader not trained in mathematics may follow the substantive issues.

Introduction to Mathematical Theories of System Manflows

The mathematical theories below refer to flows of populations of men. They were developed by J. Gani (1963) and Andrew Young and Gwen Almond (1961). Extensions of each in terms of the limiting structure of the process and continuous time analogues of the models have been made by David J. Bartholomew (1967). Each of the two demographic theories to be presented pertains to a discrete state, Markov chain model for an open system.† Moreover, each model may be described as a simple Markov chain model. That is, the assumptions include stationarity, a Markovian nature and homogeneity of population. Stationarity means that the transition probabilities remain constant over time. The Markovian assumption is that the probability of moving to another state within the next time unit (t,t+1) depends only on one's position at time t not t-x where x=1, 2, . . . (i.e., not on one's history of previous moves). The homogeneity of population assumption refers to the fact that all members occupying a given state are considered to have identical transition probabilities (McFarland, 1970).

The demographic approach to model building has been utilized for a much longer period of time than that of vacancy chains. As a result much more analytical work exists for this type of model. For instance, certain modifications of the assumptions of a simple Markov chain model have been formulated. (See McFarland, 1970; Mayer, 1972.)

Although McFarland is quite correct when he argues for careful theoretical analysis of a model's assumptions and/or its logical consequences prior to application of the model, the dearth of sociological

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†Two demographic models will be presented in this chapter, one somewhat more complex than the other. In the chapter to follow, an entirely different type of model (vacancy chain) will be examined.

information upon hierarchical promotion systems and upon the changing nature of systems over somewhat extended time periods makes such careful scrutiny most difficult in the present case. That is, a lack of sociological knowledge of important aspects of such systems precludes one being able initially at least to discard the simple Markov chain model as inadequate and construct a more adequate one. Preliminary examination suggests the model may be applicable; but more careful scrutiny of the assumptions must be examined after the data have been collected. The mathematical models will therefore be described; then the initial steps of analysis, which will include a careful examination of the adequacy of the assumptions, will be carried out. It is only at that time that a decision point regarding whether or not to proceed seems appropriate.

Prior to the development and analysis of the manflow models, an additional comment is pertinent. It is the rationale for the decision to conceptualize the system models in discrete rather than continuous time. The demographic type of model does not represent interrelated movement, yet we know that the flows through this system are often dependent ones. If a man leaves the system, a set of moves may be generated. Similarly, if a new job is created above the Trooper stratum, several internal moves may result. An important question for the conceptualization of the problem in time is, therefore, whether loss rate and rate of creation of new jobs are discrete or continuous events. For the leaving rate process a continuous time model is appropriate. In other words, it seems reasonable to assume that the process of men leaving the system is a random one. By assuming that the leaving process is random, one is permitted the use of continuous time and hence differential calculus. For creation of new jobs, however, such an assumption is not valid. For clerical records a position may be established once the State Legislature provides the allocation and the Civil Service Commission approves. For our purposes, it seems more appropriate to assume that a new job is created when a man moves to fill the job. In either the clerical description or the manner I have suggested for this study, the creation process is not random but occurs at discrete intervals in time. That is, generally an entire set or group of positions are created at the same time irrespective of stratum. We may take the easiest case for illustrative purposes--that of the new entrant and the simultaneous creation of a new job at the Trooper stratum. Since all new entrants must graduate from a recruit school and these schools occur at discrete intervals of time, the inflow of new recruits is a discrete time phenomenon. If we take the creation dates as determined either by the Legislature or the Civil Service Commission, they, too, occur in "batches" or discrete intervals of time. Using either description, the process of initial movement as a result of new jobs should therefore be described in terms of discrete time. Moreover, since a considerable amount of initial movement (new jobs) occurs at discrete time points and the consequent or dependent movement thereafter is generally immediate, the conceptualization of those dependent processes should also be one of discrete time. Finally, we may observe that it is generally true that a move from a stratum above the Trooper or bottom stratum will generate a move from this lower stratum. This results from the fact that there is no lateral entry. Thus, unless an upper level position is abolished, a Trooper will move upward and an opening will result in the Trooper stratum. Whether or not the initial internal movement is a result of a man leaving or a new job being created, the inflow of men at the bottom stratum is a discrete time occurrence due to the training schools. In sum, several major flows occur at discrete points in time. Only the leaving process and, with the exception of the inflow process, the moves generated throughout the system by a man leaving are continuous time processes. Therefore, I decided the conceptualization most accurately representing the mobility processes of this system was one of discrete time.

Demographic Model with Inflow Given[†]

Consider a population of men divided into k strata. Let $n_j(t)$ denote the number of people in stratum j between time t and t+1 (t=0, 1, 2, . . .). The initial stratum sizes $n_j(0)$ (j=1, 2, . . ., k) are assumed to be given and we define the total organizational size at time t as

$$N(t)=\sum_{j=1}^{k} n_j(t). \tag{3.1}$$

Since the initial (t=0) stratum sizes are assumed to be given, the initial total system size is also given [Equation (3.1)]. The primary task of a demographic model is to predict the size of each stratum for t>0. That is, the model predicts the end result of movement by populations of men. The manner in which these predictions are derived should become more clear as the model is explicated below. For now, let us recognize that for t>0 the stratum sizes are random variables to be accounted for by this stochastic model. Thus, the accuracy of the derived expected number of people in stratum j at time t will be

[†]The mathematical notation for the model will closely follow that of Bartholomew (1967). The original model was developed by J. Gani (1963).

a principal determinant of the utility of this model. This expected number will be denoted by $\bar{n}_j(t)$.

Let the probability that an individual in grade i will move to grade j in one discrete time period be p_{ij}. For this model, probabilities are assumed to be time-homogeneous and, therefore, a time notation argument is unnecessary. Since in an open system transitions to the outside are possible,

$$\sum_{j=1}^{k} p_{ij} \leq 1. \tag{3.2}$$

In general, of course,

$$\sum_{j=1}^{k} p_{ij} < 1. \tag{3.3}$$

In matrix notation P' will denote the matrix with elements $\{p_{ij}\}$ where i, j=1, . . ., k and the p_{ij}th element lies in the ith row and jth column. For a three strata system (i.e., k=3), the entire possible set of transition probabilities for internal manflows would be of the following form where the i denotes the stratum of origin and the j the destination stratum:

$$P' = \begin{pmatrix} p_{11} & p_{12} & p_{13} \\ p_{21} & p_{22} & p_{23} \\ p_{31} & p_{32} & p_{33} \end{pmatrix} .$$

In the State Police two system characteristics affect the transition matrix. We may recall from the description of general system characteristics in Chapter 1 that virtually all moves are across only one stratum boundary and are thus one step in length. Since vertical movement may occur in two directions--up or down--we must now be more specific. In terms of total movement, demotions are rare. Indeed, even in number, downward movement is rare. The decline in dismissals (a form of demotion at the Trooper level) was most significant during the initial years from 1917 to 1927. The decline in demotions from strata above the Trooper level has continued since the early, more erratic years of the system. Throughout the history they have, however, been very few in number. In general, therefore, $p_{ij}=0$ for $i>j$. As for upward movement, the general statement as to its one step nature is sufficient information. That is, promotion is usually a one step process or from current grade or stratum to the one immediately above it. Hence, we will generally have $p_{ij}=0$ for $j>i+1$. Therefore, our P' will generally have the following form where 1 designates the highest stratum:

$$P' = \begin{pmatrix} p_{11} & 0 & 0 \\ p_{21} & p_{22} & 0 \\ 0 & p_{32} & p_{33} \end{pmatrix} .$$

The transpose of P', call it P, will be the matrix of most use for our calculations. For the same three strata system, we have

$$P = \begin{pmatrix} p_{11} & p_{21} & 0 \\ 0 & p_{22} & p_{32} \\ 0 & 0 & p_{33} \end{pmatrix} .$$

The probability of an individual leaving the system from the ith stratum may be denoted $p_{i,k+1}$ and thus, we have

$$p_{i,k+1}=1-\sum_{j=1}^{k} p_{ij}. \tag{3.4}$$

All new recruits enter the lowest stratum. Moreover, the fact that virtually all men entering (including re-entrances) the system are new recruits or Troopers, we usually have $p_{oj}=0$ for $j\neq 1$ where o designates outside. In general, therefore, the vector of entrance will be the following one:

$$P_o = \begin{pmatrix} 0 \\ 0 \\ \cdot \\ \cdot \\ \cdot \\ 1 \end{pmatrix} .$$

To allow for all possibilities for the model, we may assume that proportion p_{oj} enter the jth stratum, j=1, . . ., k. Hence,

$$\sum_{j=1}^{k} p_{oj}=1. \tag{3.5}$$

As illustrated above, the matrix notation for a recruitment distribution $\{p_{oj}\}$ will be P_o.

We have now accounted for all three types of manpower flows. The p_{oj}'s represent the inflows, the p_{ij}'s the internal or thru flows and the $p_{i,k+1}$'s the outflows. Initially in this section, I introduced one type of population unit which will either be multiplied by these manflow terms to derive stratum redistribution or will be the stratum size derived. The population unit was $\bar{n}_j(t)$ representing the expected number of people in stratum j at time t>0. For t=0 the stratum size is observed and we have $n_j(0)$. One additional type of population unit needs introduction before formulating our basic equation. The remaining unit is that of the entrants or "recruits" to the system. All such recruits to the system within year t [i.e., (t,t+1)] will be denoted by R(t). In all cases R(t) will consist of either all new entrants (i.e., new Troopers) or a large proportion of actual new entrants. In other words, men returning to the system who are in a stratum above that of Trooper are very few in number.

All elements necessary for the equations relating the expected stratum sizes at successive points in time have now been introduced. The equation to be presented describes the size of each of the system's strata as the result of varying types of manflows. In brief, the derivation of expected manpower distribution at time t is produced by multiplying appropriate time homogeneous transition probabilities by the actual or expected manpower distribution at time t-1 and by the size of the recruit cohort entering during (t,t+1). We, therefore, have

$$\bar{n}_j(t)=\sum_{i=1}^{k} p_{ij}\bar{n}_i(t-1)+R(t)p_{oj} \tag{3.6a}$$

where

$$t=1,\ 2,\ .\ .\ .$$

$$j=1,\ .\ .\ .,\ k$$

$$\bar{n}_i(0)=n_i(0).$$

If we denote as P the transpose of the matrix with elements $\{p_{ij}\}$ where the p_{ij}th element is in the ith row and the jth column, P_o, the column vector of probabilities of entrance to the system, and $\bar{N}_t$, the vector of expected stratum sizes at time t, we may write Equation (3.6a) in matrix form as

$$\bar{N}_t=P\bar{N}_{t-1}+R(t)P_o. \tag{3.6b}$$

For this particular model R(t) is either observed or derived and is therefore given for all t. As a result, the solution for the expected stratum sizes may be found recursively from Equation (3.6). Hence, we have

$$\bar{N}_t=P^tN_0+\{\sum_{\tau=0}^{t-1} R(t-\tau)P^\tau\}P_o, \tag{3.7}$$

where P^0 is the unit matrix I. Note that the leaving probabilities

$\{p_{i,k+1}\}$ are not explicit functions in the equations. They are nevertheless significant in that the $\bar{N}_t$ are dependent upon their values as a result of their being complements of the column sums of P and P^T (Bartholomew, 1967).

The equations for estimating the parameters of this model will be designated following the definition of terms. These terms are as follows:

1. $n_{ij}(t)$: The observed number of men moving from stratum i to stratum j during the time period (t,t+1);
2. $n_i(t)$: The observed number of men in stratum i at the beginning of the time period (t,t+1);
3. $n_{io}(t)$: The observed number of men leaving the system from stratum i during the time period (t,t+1);
4. $n_{oi}(t)$: The observed number of men entering the system at stratum i during the time period (t,t+1);
5. $n_{o\cdot}(t)$: The observed number of men entering the entire system during the period (t,t+1);
6. $\hat{p}_{ij}$: Estimated value of p_{ij};
7. $\hat{p}_{oj}$: Estimated value of p_{oj};
8. $\hat{p}_{i,k+1}$: Estimated value of $p_{i,k+1}$.

The following equations give us the desired estimated values:

$$\hat{p}_{ij}=\frac{n_{ij}(t)}{n_i(t)}, \quad i\neq j \tag{3.8}$$

$$\hat{p}_{i,k+1}=\frac{n_{io}(t)}{n_i(t)} \tag{3.9}$$

$$\hat{p}_{ii}=1-\{\sum_{i=1}^{k}\hat{p}_{i,k+1}+\sum_{\substack{i=1\\ i\neq j}}^{k}\hat{p}_{ij}\} \tag{3.10}$$

$$\hat{p}_{oj}=\frac{n_{oi}(t)}{n_{o\cdot}(t)}. \tag{3.11}$$

As already stated, the facet of occupational mobility which this demographic model predicts is the outcome of mobility or the resultant stratum sizes at the end of each time period. In the section immediately following, a slightly more complex demographic model will be developed.

Demographic Model with Total System Size Given

This model differs from the first demographic model in but one respect--the total size rather than the input is given or fixed. In other words, rather than being given a sequence of entrants or "recruits", $\{R(t)\}$, we are now given a sequence of total system sizes, $\{N(t)\}$. Moreover, only two cases of net change in size will be considered--expansion or zero growth. The primary substantive difference in the two models lies in the fact that more detailed information is necessitated by the model to be presented in this section. As may be seen shortly, the "recruitment" process is itself determined by growth and leaving processes within this demographic model, whereas no such relations were conceptualized by the previous demographic model. The model to follow was originally developed by Young and Almond (1961), but Bartholomew's description (1967) will be utilized.

The notation used in the previous section will apply herein as well. In addition, terms defined there will not be repeatedly defined. Where changes are introduced, the new terms will be defined.

Let $M(t)$ denote the increase or zero growth, whichever is the case, between t-1 and t. Hence, $M(t)=N(t)-N(t-1)$ where $t=1, 2, \ldots$ and $M(t)\geq 0$.

Equation (3.6) still holds but must be altered somewhat since $\{R(t)\}$ is now an unknown. In order to maintain zero growth, the system must obviously replace members who have left. Similarly, if the system is to expand, the system must not only replace leaving members but recruit additional entrants to fill new positions. In either case, the expected number of recruits required at time t is

$$\bar{R}(t)=M(t)+\sum_{i=1}^{k} p_{i,k+1}\bar{n}_i(t-1), \quad t=1, 2, \ldots . \tag{3.12}$$

If we now substitute $\bar{R}(t)$ of Equation (3.12) for R(t) in Equation (3.6), the expected stratum sizes become

$$\bar{n}_j(t)=\sum_{i=1}^{k} (p_{ij}+p_{i,k+1}p_{oj})\bar{n}_j(t-1)+M(t)p_{oj}, \quad j=1, \ldots, k. \tag{3.13a}$$

Furthermore, if we denote $\{p_{ij}+p_{i,k+1}p_{oj}\}$ by $\{q_{ij}\}$ and Q as the transpose of the matrix with elements $\{q_{ij}\}$, then Equation (3.13a) may be written in matrix form as

$$\bar{N}_t=Q\bar{N}_{t-1}+M(t)P_o. \tag{3.13b}$$

Similarly, as was the case with Equation (3.6), Equation (3.13) has a recursive solution:

$$\bar{N}_t=Q^tN_0+\{\sum_{\tau=0}^{t-1} M(t-\tau)Q^\tau\}P_o, \tag{3.14}$$

where Q^0 is the unit matrix I. More explicitly, if

$$\bar{N}_1=QN_0+M(1)P_o,$$

then

$$\begin{aligned}\bar{N}_2&=Q\bar{N}_1+M(2)P_o,\\ &=Q^2N_0+M(1)QP_o+M(2)P_o\\ &=Q^2N_0+\{\sum_{\tau=0}^{1} M(2-\tau)Q^\tau\}P_o.\end{aligned}$$

The above equations provide all the necessary information for the derivation of the expected size of each stratum given the size of the total system. Like the more simple demographic model, this model's predictions are also in terms of the end result of movement by populations of men. In particular, the predictions are of stratum size.

Estimations for the model just described are the same as those for the previous demographic model. The formulae to be used for estimating transition parameters are, therefore, Equations (3.8), (3.9), (3.10) and (3.11). The formula from which predictions will be derived is Equation (3.13b).[†]

Even though Bartholomew develops further analytic work concerning the limiting structure (1967), it is generally inapplicable to this particular system. While the Q matrix is stochastic, it is neither triangular nor regular. Moreover, for a governmental system, such as the State Police, political and economic fluctuations are too great to permit constant or geometric growth rates. In short, the "mathematically tractable world" does not easily map onto the empirical reality of this system.

One further comment needs to be made before beginning the analysis. In view of the fact that the demographic model, which assumes total system size as given, is a more complex model and utilizes more detailed information, it will be analyzed first. Should it provide an adequate explanation of the mobility process, analysis utilizing the

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[†]The reasons for using Equation (3.13b) rather than Equation (3.14) are two. First, Equation (3.14) is extremely useful when growth patterns have a form which lend the entire equation toward mathematical ease of solution. This is not the case for the growth of the State Police. Secondly, as a consequence of the first reason, calculation of the predictions over time was much more efficient using Equation (3.13b). These calculations were made by means of an interactive APL/360 System.

more simple demographic model will be unnecessary. In the event, however, that the more complex model is inadequate, the alternative and more simple demographic version will be examined.

Initial Substantive Decisions Concerning Demographic Model with Total System Size Given

This demographic model has three basic elements: system "expansion", stratum size and transition probabilities. For each type of element certain initial decisions are necessary before continuing the analysis. For instance, in the previous section I stated that only two of three possible cases of net change in system size would be considered--expansion or zero growth. Decreases in total size are, therefore, omitted. Explication for this decision and its consequences will be given below. With regard to stratum size, the small number of positions at the highest strata suggested the necessity for combining or lumping some of the hierarchical ranks. Which particular ranks will be lumped and the rationale underlying this lumping process will be described. Finally, it has already been stated that the transition probabilities are assumed to be time homogeneous and that the model is one of discrete time. Nevertheless, the length of discrete time intervals has yet to be decided. It is possible that the discrete time intervals for which the model is appropriate are duration specific. Should this possibility be actualized the substantive meaning needs clarification.

"Expansion" for this model will refer to net system size change which is either "zero" or greater than zero. For years in which system size decreased, "growth" or "expansion" will be treated as though it were zero. There are two primary reasons for this decision. The first reason is that I had previously decided to limit the complexity of the model at this stage of analyzing the model's utility. The second reason lies in the nature of the recruitment process. The basis for both reasons may be seen by a brief examination of the interrelations between the attrition, recruitment and growth processes. We may, first of all, observe that growth occurs only after the persons leaving the system are replaced. The State Police do not allow entrances continuously over time. Rather, all new entrants are trained in a recruit school for approximately ten weeks. In addition, there must generally be at least twenty vacancies for a school to begin. In short, whereas attrition is continuous, replacement is not. There is a lag effect such that only cumulative job vacancies allow for entries. The model, at this point, cannot account for this lag. Obviously, growth is not independent of attrition. This is always true, of course; but in the State Police, growth is more complex since the processes of persons leaving and their being replaced are not similar--one is somewhat continuous, the other periodic. This added complexity of system dynamics will not be conceptualized. Rather, we will assume that all men leaving the system in a certain time period will be replaced in the same time period. Moreover, since the system has generally been expanding, this decision to treat time periods in which system size declined as though they were zero does not seem unreasonable.

Let us examine how seriously this "expansion" assumption violates reality. From 1927 to 1970 the years in which there were net losses and the actual size of these losses are as follows: 1928: -1, 1932: -9, 1933: -44, 1936: -7, 1938: -4, 1942: -50, 1943: -31, 1944: -24, 1953: -20, 1958: -23, 1959: -23, 1960: -13, 1962: -6.

For 13 of the 43 years there was a net loss. In only 8 years was the net loss greater than 10. These years include the Great Depression (1933), World War II (1942-1944), recessions (1953, 1958, 1960) or years immediately following recessions (1959).† Moreover, for the Great Depression and World War II, we would hardly expect either system growth or stability of transition probabilities. For the other years the error of assuming "growth" to be zero may be taken into account in the predictions. That is, the expectation would be that the model's predictions will be greater than actual size, at least for the Trooper stratum which serves as the entry point for virtually all replacements. In these cases the model's predictions will be off because the model assumes that all persons leaving will be replaced.

Stratum size refers either to the observed or expected stratum population of men. In any case, the initial time period (t,t+1) where t=0 will include observed stratum populations. In the following years (t>0), whether or not the term $\bar{N}_{t-1}$ in Equation (3.13b) is observed or derived will depend upon the particular analysis. Ideally, only the population at time point t=0 would be observed and all populations for t>0 would be derived. However, since an error in prediction is cumulative in this model, corrections may be necessary. In which case observed stratum sizes for certain "corrective" time periods or new starting points would also be included for t>0.

While the decision to "correct" or not at a specific time interval must wait until the predictive analysis, a decision on the specific number of strata must be made presently. The State Police system,

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†Harold G. Vatter (1963:63 ff.) notes three recessions between 1950 and 1962.

organized along a paramilitary hierarchy, has a clearly defined authority and status structure. With each position we may associate a degree of both authority and status. Although authority and status are analytically separable structures which may not vary together, within the State Police system they do vary simultaneously. That is, the relation between a position's authority and a position's status is one-to-one. The higher the status, the higher the authority and vice versa.

The ranks within the State Police are Colonel (Col.), Lieutenant Colonel (Lt.Col.), Major (Maj.), Captain (Capt.), Lieutenant (Lt.) or Detective Lieutenant (D/Lt.), Staff Sergeant (S/Sgt.) or Detective Staff Sergeant (D/S/Sgt.), Sergeant (Sgt.) or Detective Sergeant (D/Sgt.), Corporal (Cpl.) or Detective (Det.) and Trooper (Tpr.).[†] Further breakdown within each rank, such as Tpr. 07, Tpr. 09 or Capt. 15, Capt. 16, will not be made. Thus, vertical movement will follow the nine strata stated above.

Since there have never been more than four Col., Lt.Col. and Maj. positions, these strata have been included in a collapsed "stratum" consisting of these three ranks and that of Capt. An additional lumping process consisted of collapsing the S/Sgt. and Sgt. positions into one "stratum". Logically, this seemed reasonable since the S/Sgt. rank is but a half-step above that of Sgt. While perception of the half-step difference may be notable if one is below, the distance would still probably be less than that perceived between Lt. and S/Sgt. Also, and more importantly for purposes of this study, considerable movement

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[†]The names of strata from Cpl., Det. through Lt., D/Lt. and in some cases their Civil Service classification were changed effective August 1, 1972. For the nature of this change and its relation to this study, see Appendix B.

occurred from Sgt. to Lt. making transitions to S/Sgt. unnecessary in most cases. These criteria as well as the small number of S/Sgts. resulted in the combined S/Sgt., Sgt. stratum. The initial strata, therefore, that I have selected are five. The estimated transition probabilities for manflows will be of the matrix form shown in Table 1.

Table 1. Transition Probabilities for Manflows[a]

	Destination Stratum					
Origin Stratum	Col., Capt.	Lt.	S/Sgt., Sgt.	Cpl., Det.	Tpr.	Out
Out	p_{01}	p_{02}	p_{03}	p_{04}	p_{05}	
Col., Capt.	p_{11}	p_{12}	p_{13}	p_{14}	p_{15}	p_{16}
Lt.	p_{21}	p_{22}	p_{23}	p_{24}	p_{25}	p_{26}
S/Sgt., Sgt.	p_{31}	p_{32}	p_{33}	p_{34}	p_{35}	p_{36}
Cpl., Det.	p_{41}	p_{42}	p_{43}	p_{44}	p_{45}	p_{46}
Tpr.	p_{51}	p_{52}	p_{53}	p_{54}	p_{55}	p_{56}

[a]For convenience the D/Lt., D/S/Sgt. and D/Sgt. titles have been omitted. It will be assumed throughout this Chapter that unless stated otherwise Lt. also denotes D/Lt. The same assumption also holds for the other two ranks.

From previous discussion, we know that in general all moves are across only one stratum boundary and are thus one step in length. We also know that demotions are rare. Generally, the matrix for internal transition probabilities will have the form of Table 2.

Table 2. Internal Transition Probabilities for Manflows

Origin Stratum	Destination Stratum				
	Col., Capt.	Lt.	S/Sgt., Sgt.	Cpl., Det.	Tpr.
Col., Capt.	p_{11}	0	0	0	0
Lt.	p_{21}	p_{22}	0	0	0
S/Sgt., Sgt.	0	p_{32}	p_{33}	0	0
Cpl., Det.	0	0	p_{43}	p_{44}	0
Tpr.	0	0	0	p_{54}	p_{55}

Finally, we must examine the system for possible time intervals in which this demographic model is applicable. More specifically, our question is whether or not the transition probabilities differ by type of manflow. For example, in Equation (3.13a) we may identify the inflow parameter, p_{oj}; the parameter indicating the internal distribution of flows, p_{ij} and the outflow parameter, $p_{i,k+1}$. The model generates the expected stratum sizes by multiplying these transition probabilities by the appropriate population. These populations are the stratum sizes at year (t-1) or (t=0) and the given or expected system "growth". By multiplying the transition probabilities by stratum sizes, we generate the internal redistribution of the existing populations per stratum. In addition, we also account for the expected number of recruits necessary to replace members expected to leave the system. On the other hand, by multiplying the size of "expansion" by the inflow parameter, the distribution of new recruits to strata is derived. These two multiplication procedures, when summed, generate the expected stratum sizes.

With this brief description of the manner in which expected stratum sizes are generated, we may more easily understand the specific time interval for which the model is applicable. First of all, we may eliminate the inflow and outflow parameters. These are certainly not time specific durations concerning these types of man movement. What is pertinent for the occurrence of outflows is their continuous nature and the policy of mandatory retirement at age 56, neither of which affects the present decision. What is pertinent for the inflow process is its discrete nature resulting from the necessary cumulation of a cohort of job vacancies. Thus, this time aspect is not of interest at this point since these discrete processes are not constrained otherwise with respect to time. One type of inflow not considered is that of reinstatement. It is the system policy to allow only one year for a man who resigns to return without again entering recruit school. These inflows are few and generally occur at the Tpr. stratum. Moreover, the yearly time constraint does not affect the present decision. Our last consideration is that of internal manflows. It is, in fact, this type of flow which constrains the time interval to a specific duration. To be explicit, more than one move per time interval by the initial population of men is not taken into account by the demographic model. Therefore, a time span such that only one move per man from his stratum of origin is the maximum allowed. In the State Police system, moves from the Sgt. stratum and above are possible after a waiting period of one year from the date of entrance. Hence, for this particular manpower system, the time interval must be limited to periods of one year.

With these initial decisions having been made we may now turn to the basic assumptions underlying the simple Markov chain model of manpower flows.

Examination of the Assumptions Underlying the Simple Markov Chain Model of Manflows

Recall from earlier statements of this Chapter that three basic assumptions are made for this type of model: stationarity, a Markovian nature and homogeneity of population. An examination of the stationarity or constancy of transition probabilities may be made from Tables 3 through 7. The formulae for these transition probabilities were Equations (3.8), (3.9) and (3.10).

Let us arbitarily set a .125 difference as a maximum criteria for acceptance of "constancy". The Tpr. stratum has but two deviations from 1950 to 1970. They occur in 1956 and 1966. There are five additional "off years" if we begin at 1932 and continue to use the .125 criteria. In the Cpl., Det. stratum there are again but two unacceptable years from 1950 to 1970--1965, 1966. If, however, we begin at 1932 there are numerous deviations on both sides of the "stable" range which existed from 1950 to 1970. For the S/Sgt., Sgt. stratum there are three "off years" from 1950 to 1970--1955, 1965 and 1966. There are eight additional, unacceptable years if we extend the period backwards 18 years to 1932. In the Lt. stratum there is much less stability of parameters from 1950 to 1970. Only 10 of the 20 years are "acceptable" by the .125 criteria. Similarly, from 1932 through 1948, 12 of 17 are "acceptable". This earlier "period" had more stability

Table 3. Transitions of Men from Trooper Stratum by Year (Proportions)

Year	Destination			
	Trooper	Cpl., Det.	S/Sgt., Sgt.	Outside
1927	.3384	.2154	.0308	.4154
1928	.5469	.0625	.0	.3906
1929	.5516	.0328	.0	.4262
1930	.7272	.0390	.0	.2338
1931	.7757	.1589	.0	.0654
1932	.9226	.0070	.0	.0704
1933	.6492	.0075	.0	.3433
1934	.8833	.0492	.0082	.0656
1935	.8733	.0915	.0	.0352
1936	.9527	.0059	.0059	.0355
1937	.8271	.0556	.0	.1173
1938	.9534	.0169	.0	.0297
1939	.9563	.0262	.0	.0175
1940	.9258	.0078	.0039	.0625
1941	.9127	.0028	.0225	.0620
1942	.7082	.0203	.0025	.2690
1943	.8653	.0	.0	.1347
1944	.9118	.0098	.0	.0784
1945	.8078	.1139	.0	.0783
1946	.8919	.0077	.0	.1004
1947	.8411	.0767	.0	.0822
1948	.7690	.1522	.0	.0788

Table 3 (cont'd.)

Year	Destination			
	Trooper	Cpl., Det.	S/Sgt., Sgt.	Outside
1949	.8593	.0829	.0	.0578
1950	.8593	.0678	.0025	.0704
1951	.8387	.0562	.0	.1051
1952	.9528	.0323	.0	.0149
1953	.9369	.0257	.0	.0374
1954	.9282	.0446	.0	.0272
1955	.9314	.0465	.0	.0221
1956	.8074	.1222	.0	.0704
1957	.9160	.0470	.0	.0370
1958	.9594	.0209	.0	.0197
1959	.9414	.0293	.0	.0293
1960	.9050	.0495	.0	.0455
1961	.9523	.0168	.0	.0309
1962	.9395	.0289	.0	.0316
1963	.9405	.0332	.0	.0263
1964	.9530	.0207	.0	.0263
1965	.9071	.0600	.0	.0329
1966	.7998	.1296	.0	.0706
1967	.8701	.0873	.0	.0426
1968	.9092	.0449	.0	.0459
1969	.9013	.0619	.0	.0368

Table 4. Transitions of Men from Cpl., Det. Stratum by Year (Proportions)

Year	Destination				
	Trooper	Cpl., Det.	S/Sgt., Sgt.	Lt.	
1927	.0	.5000	.0500	.0	.4500
1928	.0	.9000	.0333	.0	.0667
1929	.0	.9062	.0938	.0	.0
1930	.0	.9375	.0625	.0	.0
1931	.0882	.3530	.3235	.1765	.0588
1932	.0	1.0000	.0	.0	.0
1933	.6111	.3333	.0	.0	.0556
1934	.0	.5000	.5000	.0	.0
1935	.0	1.0000	.0	.0	.0
1936	.0	1.0000	.0	.0	.0
1937	.0	.8333	.1667	.0	.0
1938	.0	.6316	.3684	.0	.0
1939	.0	.6667	.3333	.0	.0
1940	.0	.8824	.1176	.0	.0
1941	.0	.6250	.3125	.0	.0625
1942	.0	.6429	.3571	.0	.0
1943	.0	.9545	.0455	.0	.0
1944	.0	.9524	.0476	.0	.0
1945	.0	.7000	.2000	.0	.1000
1946	.0	.8936	.0851	.0	.0213
1947	.0	.7021	.2766	.0	.0213

Table 4 (cont'd.)

Year	Destination				
	Trooper	Cpl., Det.	S/Sgt., Sgt.	Lt.	Outside
1948	.0	.8621	.1207	.0	.0172
1949	.0	.7692	.2198	.0	.0110
1950	.0085	.8633	.1026	.0	.0256
1951	.0	.9141	.0625	.0	.0234
1952	.0	.9000	.0857	.0	.0143
1953	.0	.9208	.0504	.0	.0288
1954	.0	.8489	.1151	.0	.0360
1955	.0	.8406	.1304	.0	.0290
1956	.0	.8394	.1606	.0	.0
1957	.0	.8515	.1314	.0	.0171
1958	.0	.9557	.0489	.0	.0054
1959	.0	.9011	.0833	.0	.0156
1960	.0051	.8730	.0914	.0	.0305
1961	.0048	.9330	.0478	.0	.0144
1962	.0	.8798	.0865	.0	.0337
1963	.0	.8878	.0683	.0	.0439
1964	.0	.9130	.0628	.0	.0242
1965	.0	.7598	.1373	.0	.1029
1966	.0	7826	.1546	.0	.0628
1967	.0073	.8321	.1022	.0	.0584
1968	.0	.9097	.0645	.0	.0258
1969	.0	.9111	.0736	.0	.0153

Table 5. Transitions of Men from S/Sgt., Sgt. Stratum by Year (Proportions)

Year	Destination				
	Demotion	S/Sgt., Sgt.	Lt.	Col., Capt.	Outside
1927	.0	.6875	.1875	.0	.1250
1928	.0	.8000	.1333	.0	.0667
1929	.0	.8462	.1538	.0	.0
1930	.0	.9286	.0	.0	.0714
1931	.0	1.0000	.0	.0	.0
1932	.0	.7692	.1923	.0	.0385
1933	.2500	.7000	.0	.0	.0500
1934	.0	.9630	.0	.0	.0370
1935	.0	1.0000	.0	.0	.0
1936	.0	.8824	.0882	.0	.0294
1937	.0	1.0000	.0	.0	.0
1938	.0	.9211	.0526	.0	.0263
1939	.0	1.0000	.0	.0	.0
1940	.0	.9792	.0208	.0	.0
1941	.0	.9423	.0	.0	.0577
1942	.0	.9323	.0169	.0	.0508
1943	.0	.9365	.0	.0	.0635
1944	.0164	.9672	.0	.0	.0164
1945	.0	.9841	.0	.0159	.0
1946	.0	.8696	.0	.0290	.1014
1947	.0	.9206	.0159	.0	.0635
1948	.0	.8714	.0857	.0	.0429

Table 5 (cont'd.)

Year	Destination				
	Demotion	S/Sgt., Sgt.	Lt.	Col., Capt.	Outside
1949	.0141	.7606	.1408	.0	.0645
1950	.0	.8405	.0725	.0	.0870
1951	.0	.9014	.0423	.0	.0563
1952	.0	.8612	.0694	.0	.0694
1953	.0	.9054	.0541	.0	.0405
1954	.0	.8514	.1081	.0	.0405
1955	.0	.8102	.0759	.0	.1139
1956	.0	.8414	.0854	.0	.0732
1957	.0	.8571	.0989	.0	.0440
1958	.0	.9109	.0594	.0	.0297
1959	.0	.8812	.0198	.0	.0990
1960	.0	.8572	.0476	.0	.0952
1961	.0	.9355	.0275	.0	.0370
1962	.0	.8839	.0179	.0	.0982
1963	.0	.8975	.0256	.0	.0769
1964	.0	.9076	.0252	.0	.0672
1965	.0	.7273	.0909	.0	.1818
1966	.0	.7931	.0603	.0	.1466
1967	.0	.8388	.0806	.0	.0806
1968	.0	.9091	.0227	.0	.0682
1969	.0	.8929	.0571	.0	.0500

Table 6. Transitions of Men from Lt. Stratum by Year (Proportions)

Year	Destination			
	Demotion	Lt.	Col., Capt.	Outside
1927	.0	.6924	.1538	.1538
1928	.0	.8462	.1538	.0
1929	.0	.8462	.0769	.0769
1930	.0	.9231	.0769	.0
1931	.0	.9167	.0	.0833
1932	.0	.7777	.1667	.0556
1933	.7368	.1579	.0	.1053
1934	.0	.3000	.7000	.0
1935	.0	1.0000	.0	.0
1936	.0	.6667	.3333	.0
1937	.0	1.0000	.0	.0
1938	.0	1.0000	.0	.0
1939	.0	1.0000	.0	.0
1940	.0	.8750	.0	.1250
1941	.0	1.0000	.0	.0
1942	.0	.7500	.1250	.1250
1943	.0	1.0000	.0	.0
1944	.0	1.0000	.0	.0
1945	.0	1.0000	.0	.0
1946	.0	1.0000	.0	.0
1947	.0	1.0000	.0	.0
1948	.0	.5714	.2857	.2857

Table 6 (cont'd.)

Year	Destination			
	Demotion	Lt.	Col., Capt.	Outside
1949[a]				
1950	.0	.8333	.1667	.0
1951	.0	1.0000	.0	.0
1952	.0	.7777	.1667	.0556
1953	.0	.7895	.0526	.1579
1954	.0	.5790	.2105	.2105
1955	.0	.7368	.2632	.0
1956	.0	.7000	.2500	.0500
1957	.0	.8095	.0	.1905
1958	.0	.5769	.3077	.1154
1959	.0	1.0000	.0	.0
1960	.0	.8695	.0435	.0870
1961	.0	.8000	.0800	.1200
1962	.0	.7916	.1667	.0417
1963	.0	.7619	.0	.2381
1964	.0	.8421	.1579	.0
1965	.0	.6315	.2632	.1053
1966	.0	.5217	.3913	.0870
1967	.0	.8421	.1053	.0526
1968	.0	.8846	.0769	.0385
1969	.0	.6923	.1923	.1154

[a]There exists no p_{ij} for Lt. stratum for 1949 since men moved more than once.

Table 7. Transitions of Men from Col., Capt. Stratum by Year (Proportions)

Year	Destination		
	Demotion	Col., Capt.	Outside
1927	.0	.8000	.2000
1928	.0	1.0000	.0
1929	.0	1.0000	.0
1930	.0	.8889	.1111
1931	.0	1.0000	.0
1932	.0	1.0000	.0
1933	.5833	.3334	.0833
1934	.0	.7500	.2500
1935	.0	1.0000	.0
1936	.0	1.0000	.0
1937	.0	1.0000	.0
1938	.0	1.0000	.0
1939	.0	1.0000	.0
1940	.0	1.0000	.0
1941	.0	1.0000	.0
1942	.0	.9167	.0833
1943	.0	1.0000	.0
1944	.0	1.0000	.0
1945	.0	.7500	.2500
1946	.0	1.0000	.0
1947	.0	.7500	.2500
1948	.0	1.0000	.0

Table 7 (cont'd.)

Year	Destination		
	Demotion	Col., Capt.	Outside
1949	.0	.7778	.2222
1950	.0	.8333	.1667
1951	.0	1.0000	.0
1952	.0	.8333	.1667
1953	.0	.9286	.0714
1954	.0	.7143	.2857
1955	.0	.6424	.3571
1956	.0	.8571	.1429
1957	.0	1.0000	.0
1958	.0	.6471	.3529
1959	.0	1.0000	.0
1960	.0	.9474	.0526
1961	.0	.8947	.1053
1962	.0	.8947	.1053
1963	.0	.8571	.1429
1964	.0	.8889	.1111
1965	.0	.6842	.3158
1966	.0	.7778	.2222
1967	.0	.9130	.0870
1968	.0	.9130	.0870
1969	.0	.7826	.2174

than that from 1950 to 1970 for the Lt. stratum; a reversal of the trend of the three strata discussed previously. Finally, in the Col., Capt. "stratum" there is even greater stability in the 1932-1950 "period". In 13 of 18 years the stability of transition parameters is "acceptable", whereas for the 1950-1970 time span in only 11 of 20 years is "acceptability" attained. In sum, the two upper strata have much greater stability in the earlier period, while the three other strata demonstrate the opposite tendency.

In either the earlier or the latter period, it would seem that three strata outflows are quite constant. This is because transitions involving the Tpr. stratum seem sufficiently stable for either "period", even though the stability is greater in the latter one. However, there exists a virtual total lack of movement by the upper two strata in the years for which their "transitions" are stable. Recall that p_{ii} within the demographic conceptualization may refer to two possibilities--movement within the stratum or nonmovement. The model does not conceptualize a distinction between the two. Hence, to analyze this "period" because of stability of transitions is misleading since there is virtually no movement within this time span. Moreover, since the more numerous lower strata are not as constant in terms of manflows, analysis of this period utilizing the model would provide little useful information. First, since the p_{ii}'s for both upper strata are generally 1.0000, these stratum sizes would remain the same. Secondly, since the lower strata do not have rather constant transitions, the assumption of stationarity is not met; and the predictions pertaining at least to the S/Sgt., Sgt. and Cpl., Det. strata would not be expected to be very accurate. For these two reasons, no analysis prior to 1950 utilizing this demographic model will be undertaken.

May a similar conclusion also be made concerning the 1950-1970 time span? Hardly. First, the "stable" transitions do actually refer to movement. Secondly, while there are notable fluctuations in reference to transitions from the two upper strata, there is also some degree of constancy in that at least half of the years meet the criteria. For predictions over two year spans, we might expect reasonably accurate predictions for these strata sizes. Yet for long term predictions, we would expect increasing inaccuracy because the error of each year becomes cumulative. For the other three strata, however, the stationarity assumption generally appears valid.

The second assumption refers to the process being a Markov chain or a discrete state, time homogeneous Markov process. It is the Markovian aspect with which we are concerned here. In mathematical notation this means that

$$
\begin{aligned}
p_{i_n, i_{n+1}} &= \Pr(X_{n+1}=i_{n+1} \mid X_o=i_o, X_1=i_1, \ldots, X_n=i_n) \\
&= \Pr(X_{n+1}=i_{n+1} \mid X_n=i_n) \\
&= p_{ij}, \quad i=i_n, \quad j=i_{n+1}.
\end{aligned}
$$

In other words, the probability that a member of stratum i will move to stratum j is conditional only upon his present state or location, not the history of his locations. Although there is no immediate way to check this assumption as there was with the stationarity assumption, there is reason to believe that perhaps this assumption is valid. Promotions are of a one step nature. The principal deviations from this one step process occurred in the initial stages of the system's existence, 1917 to 1927. After 1927 the transition route was almost always a one step process. Moreover, the movement was almost entirely in an upward direction. Thus, many of the two step possibilities simply do not occur. For instance, it is generally true that $p_{ij}=0$ for $i>j$ and also $j>i+1$. The remaining possibilities are such that either $p_{ij}=p_{ii}$ or $p_{ij}=p_{i,i+1}$. In terms of movement, only one step possibilities are realistic. In short, one's stratum position serves as the principal determinant of movement to another stratum rather than one's historical path. Therefore, it seems reasonable to assume that during the 1950-1970 "period" the process is Markovian.

The final assumption to be examined relates to homogeneity of stratum populations. The assumption is that all members of each stratum are subject to identical sets of transition probabilities.

> It is this assumption which permits one to use the <u>proportion</u> of persons making a particular transition as an estimate of the corresponding transition probability to which any <u>particular</u> person is subjected. (McFarland, 1970:464)

The assumption is probably not entirely correct since certain selective factors are operative. For instance, it has generally been the case that a one year waiting period was required within any stratum before further promotions were possible. In addition, seniority in terms of duration of service in the stratum has had an important influence. (See <u>Rules and Regulations of the Michigan State Police</u>, 1936 and 1945.) As for changes in procedure for promotions since 1945, the greatest apparent change has been the inclusion by the Civil Service Commission of written examinations for several of the promotions. While these tests may possibly nullify to some extent the effect of seniority, they by no means have made it unimportant. Although the general nature of the promotional criteria and decisions is known, the exact weighting of specific criteria over time is not. Hence, from an administrative point of view, data is lacking to test the import of seniority. An inexact method does exist. We might examine the effects of seniority upon promotion by looking at career histories of certain cohorts. This, however, will not be undertaken in this particular study. Some insight may be cast upon this problem in the descriptive analysis concerning career data, although it will not be directed to specifically answer this question. For the moment, logical criteria and the general understanding of the system will have to suffice. Several factors seem important--the waiting period, the effect of seniority beyond this waiting period, the effect of written examinations, the geographical region which is expanding at the fastest rate at particular time periods, previous interactions with those being promoted at the fastest rates and rate of expansion of the administrative versus the detective "wing" of the hierarchy. The list could obviously be extended much further. One important criteria precludes such an extension, however. This is the stationarity of transition probabilities from 1950 to 1970. First, while it may indeed be the case that some selective factors are operative, we are after all dealing with mobility processes. That is, some persons must necessarily move and others stay. The essential element, it seems to me, is that the proportion of men moving from these strata is quite constant. Moreover, since we are not attempting to determine the particular persons who move but rather the result of aggregate flows of populations of men, these selective factors do not seem especially crucial. Were the stationarity assumption not true, modifications would seem necessary and certain of these factors might become very important. Since this is not the case, it would seem that the extent to which this assumption is being violated is not immediately determinable.

More importantly, such violations do not appear to be consequential for this particular model given the validity of stationarity. In short, a test does seem to be a worthwhile enterprise based upon the initial examination of the underlying assumptions.

"Predictions" of Stratum Size for Year Zero†

The basic utility of the more complex demographic model may be seen by comparing the expected or predicted stratum sizes with those which have been observed. An initial discussion of the use of different data sources and the consequences seems in order prior to the analysis of predictions for either short or more extended time periods. Such a discussion will provide a basis for checking the adequacy of genuine predictions.

There are two possible sources from which one may observe stratum sizes. The numbers may be taken from official records of stratum sizes (such as 1956-1970) or from counts of stratum sizes from personnel rosters (pre-1956). An alternative method is simply to take the initial year (t=0) as observed in the above fashion and thereafter to take the net redistribution resulting from differences in manpower inflows and outflows. For instance, if we take the 1950 stratum sizes as given, they are Col., Capt.: 12; Lt.: 12; S/Sgt., Sgt.: 69; Cpl., Det.: 117; and Tpr.: 398. From the S/Sgt., Sgt. stratum there were 5 men moving to the Lt. stratum and 6 men leaving the system. On the other hand, there were 12 Cpls. and 1 Tpr. moving into the S/Sgt., Sgt. stratum. Hence, the outflow was 11 and the inflow 13 with a net stratum

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†Year zero refers to the year immediately following the one from which estimations were made. This makes the accuracy of the prediction a necessity should the same data sources be used for all terms.

size change of +2. This stratum at the beginning of 1951, therefore, has 71 men.

Those who have dealt with large numbers of personnel records can appreciate the fact that within historical record systems the exact matching of the numbers of the two types of observations is most difficult. For the present study, a net difference in total system size of 5 was deemed acceptable. This difference was generally less than or equal to 3. For each particular stratum, however, the differences were in some cases greater. For example, the greatest differences occurred with respect to the Cpl., Det. stratum in 1951 and 1957. It would appear that from approximately 5 to 10 moves were missed in each case. For no other stratum, nor for other years with the Cpl., Det. stratum, were such inequalities found. The reason "approximately" was used is that fluctuations in the adjacent years suggest that part of the inequality, at least for 1951, may be an artifact of dating of records rather than actual movements having been overlooked. However, this cannot be definitely asserted since the figures refer to redistributions of aggregations possibly affected by several types of movement.

In any case, the inequalities mentioned affect the results in that not official stratum sizes but turnover stratum sizes were used to calculate the estimated transition probabilities. By using these figures actual checks could be performed on initial predictions for each year. That is, the assumption of constancy of transition probabilities for at least two years is necessary since estimations taken from, say 1951, must necessarily be "correct" or exact for predicting 1951 stratum sizes. While the above statement does hold true for the more simple demographic model, it is not totally correct for the more complex model being examined. For instance, the assumption of either zero or positive net total system size change is inaccurate for the five years in which the system actually declined in overall size. The years and losses are: 1953: -20, 1958: -23, 1959: -23, 1960: -13 and 1962: -6. The effect of the assumption is that for these years net "growth" was set at zero. The expectation is that for the years 1954, 1959, 1960, 1961 and 1963, the predictions will be larger than the observations. This should particularly hold true for the Tpr. stratum since virtually all replacements enter at this stratum.

In Table 8 one may examine the discrepancies between predicted and observed figures. As expected, the years cited above contain major errors principally occurring within the Tpr. stratum. An additional factor affecting these "predictions" is that the model assumes all men leaving the system will be replaced.

Table 8. Differences between Predicted and Observed System and Stratum Sizes for Year Zero[a]

	Difference					
Year	Col., Capt.	Lt.	S/Sgt., Sgt.	Cpl., Det.	Tpr.	Total System
1951	0	0	0	0	+3	+3
1952	0	0	+5	-1	-3	+1
1953	0	0	+3	-11	0	-8
1954	0	0	0	0	+25	+25
1955	0	0	0	0	+2	+2
1956	0	0	0	0	+1	+1
1957	0	0	0	+2	+5	+7
1958	0	0	0	0	0	0
1959	0	0	0	+6	+19	+25
1960	0	0	0	+2	+23	+25
1961	0	0	0	0	+7	+7
1962	0	0	0	0	+5	+5
1963	0	0	0	0	+4	+4
1964	0	0	0	0	+2	+2
1965	0	0	0	0	+1	+1
1966	0	0	0	0	+1	+1
1967	0	0	0	0	-3	-3
1968	0	0	0	0	+2	+2
1969	0	0	0	0	0	0

[a]Year zero is the prediction for the year immediately following the one from which estimations were made. This makes the accuracy of the prediction a necessity should the same data sources be used for all terms. The differences are in reference to whether or not the predicted sizes are less than or greater than those observed.

Predictions of Stratum Size Over Short Time Periods

With the qualifications in the previous section for the model's adequacy, we may now proceed with the actual tests. Five year time spans will be presented, not in terms of historical or substantively related periods but rather to provide opportunity for comments upon the predictions and observations as the data is presented. This alleviates relating the entire time span in one extended discussion and will hopefully facilitate the somewhat massive amount of data covering the twenty year period. Table 9 provides the information for the years 1950 through 1954. For each transition matrix six predictions will be made, the first of which refers to the zero year or year for which predictions and observations _must necessarily_ be close. It is the following five predictions for which the model's adequacy may be judged. For instance, in reference to predictions using the 1950 transition matrix, only the predictions from 1952 through 1956 are valid. The 1952 prediction is the first valid one since parameters were estimated from data in 1950 and the model assumes these parameters will also hold for 1951 in order to predict the stratum sizes for January 1, 1952. Perhaps it should be noted at this point that these predictions utilize expected or predicted stratum sizes from each previous year. Thus, the only stratum size given or observed is that for the year in which estimations are made.

Table 9. Predicted (P) and Observed (O) Stratum Sizes by Year Based upon Estimated Transition Matrices from 1950 through 1954.

	Stratum Sizes				
	Col., Capt.	Lt.	S/Sgt., Sgt.	Cpl., Det.	Tpr.
			1950 Transition Matrix		
			1951		
P	12	15	71	128	411
O	12	15	71	128	408
			1952		
P	13	18	74	138	406
O	12	18	72	140	404
			1953		
P	14	20	77	147	419
O	14	19	74	139	429
			1954		
P	15	22	81	155	406
O	14	19	74	139	404
			1955		
P	16	24	85	161	446
O	14	19	79	138	452
			1956		
P	17	26	88	169	463
O	14	20	82	137	483

Table 9 (cont'd.)

	Stratum Sizes				
	Col., Capt.	Lt.	S/Sgt., Sgt.	Cpl., Det.	Tpr.
	1951 Transition Matrix				
	1952				
P	12	18	72	140	405
O	12	18	72	140	404
	1953				
P	12	21	74	151	417
O	14	19	74	139	428
	1954				
P	12	24	76	161	401
O	14	19	74	139	404
	1955				
P	12	27	79	170	440
O	14	19	79	138	452
	1956				
P	12	30	82	180	456
O	14	20	82	137	483
	1957				
P	12	33	85	190	714
O	17	21	91	175	702

Table 9 (cont'd.)

	Stratum Sizes				
	Col., Capt.	Lt.	S/Sgt., Sgt.	Cpl., Det.	Tpr.
	1952 Transition Matrix				
	1953				
P	14	19	74	139	428
O	14	19	74	139	428
	1954				
P	15	20	76	139	424
O	14	19	74	139	404
	1955				
P	17	21	77	139	478
O	14	19	79	138	452
	1956				
P	19	22	78	141	506
O	14	20	82	137	483
	1957				
P	26	23	79	143	770
O	17	21	91	175	702
	1958				
P	29	23	80	154	891
O	17	26	101	184	814

Table 9 (cont'd.)

	Stratum Sizes				
	Col., Capt.	Lt.	S/Sgt., Sgt.	Cpl., Det.	Tpr.
	1953 Transition Matrix				
	1954				
P	14	19	74	139	429
O	14	19	74	139	404
	1955				
P	14	19	74	139	482
O	14	19	79	138	452
	1956				
P	14	19	74	140	514
O	14	20	82	137	483
	1957				
P	14	19	74	142	787
O	17	21	91	175	702
	1958				
P	14	19	74	151	914
O	17	26	101	184	814
	1959				
P	14	19	75	163	902
O	19	21	101	192	784

Table 9 (cont'd.)

	Stratum Sizes				
	Col., Capt.	Lt.	S/Sgt., Sgt.	Cpl., Det.	Tpr.
	1954 Transition Matrix				
	1955				
P	14	19	79	138	454
O	14	19	79	138	452
	1956				
P	14	20	83	139	481
O	14	20	82	137	483
	1957				
P	14	21	87	147	744
O	17	21	91	175	702
	1958				
P	14	22	91	162	860
O	17	26	101	184	814
	1959				
P	15	23	96	177	839
O	19	21	101	192	784
	1960				
P	16	24	102	189	820
O	19	23	105	197	748

The predictions from the 1950 transition matrix are rather close to the observed stratum sizes. The largest error occurs in the Cpl., Det. stratum. Moreover, the error is not only cumulative in that an error for a previous year remains thereafter, but the error also increases for predictions from 1953 through 1956. The entire set of differences between predictions and observations may be seen in Table 10.

Table 10. Differences between Predicted and Observed Stratum Sizes Based upon Predictions from Estimated Transition Probabilities from 1950[a] (Numbers)

	Stratum Size Differences				
Year	Col., Capt.	Lt.	S/Sgt., Sgt.	Cpl., Det.	Tpr.
1952	+1	0	+2	-2	+2
1953	0	+1	+3	+8	-10
1954	+1	+3	+7	+16	+2
1955	+2	+5	+6	+23	-6
1956	+3	+6	+6	+32	-20

[a]The sign before the number indicates that the predicted value was either greater or less than the observed one.

A more detailed look at the exact nature of deviations for this Cpl., Det. stratum will perhaps be informative. The transition probabilities of interest are the outflows from the Cpl., Det. stratum and the inflow from the Tpr. stratum. For 1950 (1953) the transition probabilities are as follows: Cpl., Det. to S/Sgt., Sgt. or outside--.1367 (.0759) and Tpr. to Cpl.--.0678 (.0258). The difference between the error in 1953 and 1954 is a result of the differences in these transition probabilities.

That is, while the error due to the cumulative nature of the model would remain at +8, an additional error of another +8 is a result of differing transition flows. For instance, the observed Cpl., Det. stratum size does not change between 1953 and 1954. Utilizing the 1953 probabilities there is no predicted change. However, by assuming that the 1950 probabilities are sufficiently similar to those of 1953, we have the following type of flows: 147.x.1367=20.09 and 419.x.0678=28.4. There are 20 men leaving the stratum and 28 men entering for a net gain of 8 men. Hence, for the larger strata small differences in transition probabilities have considerable effect. In this case the difference in outflow between 1950 and 1953 was .0575 while for inflow, the difference was .0421. The net effect is an error of 8 for the prediction for 1954. Moreover, since there was a similar type of error in the previous year, the total error is cumulative in this instance and by 1954 is +16.

Several aspects of these yearly transitions and their effects might be noted. First, the changes for both types of probabilities were in the same direction. This is not always true. Secondly, had the yearly transitions for 1953 been of a different nature, for instance characterizing greater transitions, the initial error could possibly have been negated. An example of this is the S/Sgt., Sgt. stratum predictions for 1957 and 1958 based upon estimations from 1954 data. The 1954 inflow probabilities are .1151 and the outflow probabilities are .1486. In 1957 (1958) they are .1318 (.0489) and .1429 (.0891) respectively. As would be expected, the predictions are slightly above the observations for 1957 and below them for 1958.

Other types of inaccurate predictions mentioned earlier in this discussion may also be seen from Table 9. For instance, the error in "zero year" predictions for 1951 (from the 1950 Q) and for 1952 (from the 1951 Q) would appear to be a result of the assumption that all men leaving the system will be replaced. On the other hand, the error for 1954 (from the 1953 Q) is a result of the assumption of zero or positive growth. In 1953 there was actually a net total system size loss of 20 men.†

If we briefly focus upon the set of predictions from each Q, a more complete understanding of this model's utility may be seen. While the predictions from the 1950 Q are rather good with the exception of the Cpl., Det. stratum after 1952, the 1951 Q is hardly as accurate. There is a cumulative yet increasing error in the Lt. stratum similar to that of the Cpl., Det. stratum for the 1950 Q matrix. The principal reason is that for 1951 there were no moves of any kind from the Lt. stratum. Therefore, the outflow was .0000 and the stratum could only grow given any inflow at all. Of course, this lack of movement is not typical of the majority of years which accounts for the error. The Cpl., Det. and Tpr. strata data are interesting, but a more clear indication of the dynamics may, I think, be seen from the 1952 Q based predictions.

Predictions from the 1952 Q matrix are not very good for 1957 and 1958. Prior to this they are rather close to the observations. The only exception is for the Tpr. stratum in which the error of +20 in 1954 is a result of the "growth" or expansion assumption. For 1957 and 1958, however, the predictions are rather bad with the exception of the Lt. stratum. The error of the Col., Capt. stratum is rather easily detected.

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†This count is from the official records rather than the turnover resource.

It is principally a result of a re-entry in 1952 of a Capt. on leave of absence. Since no other entries for other years occurred in this level yet the model assumes a constant p_{ij}, a discrepancy in predictions and observations is expected.

A more important occurrence for the overall system dynamics occurred in 1956, the year of greatest expansion in State Police history. The Legislature authorized 257 new positions during 1956 primarily as a result of increasing highway deaths. The inflows and outflows of S/Sgt., Sgt., Cpl., Det. and Tpr. strata were affected.

For the Cpl., Det. stratum there were the following dynamics:

Year	Inflow[†]	Outflow
1952	.0323	.10000
1953	.0357	.0792
1954	≈.0453[§]	.1502
1955	.0465	.1594
1956	≈.1224[§]	.1506
1957	≈.0473[§]	.1483

The probability of most interest is the .1224 inflow occurring in 1956 obviously resulting in a considerable increase for this stratum since the outflow did not change greatly. This would largely account for the discrepancy between predicted and observed values for 1957. However, the 1957 inflow once again normalized. What accounts for the 1958 discrepancy? The answer lies in the increase in size of the Tpr. stratum--an increase from 483 men to 702 men. Thus, while the outflow from the Tpr. to the Cpl., Det. stratum may be the same proportionally,

[†]"Inflow" refers to Tpr. to Cpl., Det. transitions not the proportion of men received into the Cpl., Det. stratum.

[§]The 1954, 1956 and 1957 inflows are too complicated for exactness within this column arrangement as a result of the outside Cpl., Det. inflow; therefore, approximations slightly below actual inflows are given.

its actual number is increased considerably and the small divergence in transition probabilities will still have a much greater effect.

As for the S/Sgt., Sgt. stratum, a similar phenomenon occurs in 1956 with increased "inflow" accounting for the discrepancy between the 1957 predicted and observed stratum size. Finally, the Tpr. errors for 1957 and 1958 are the result of underpredictions for both Cpl., Det. and S/Sgt., Sgt. strata.

The 1953 Q based predictions have deviations from the observed values resulting from the increased dynamics in 1956 for the strata discussed above. There are two other deviations, both of which are the result of the "expansion" assumption. The effect is seen in the 1954 and 1959 Tpr. predictions resulting from actual total system losses of 20 and 23 in 1953 and 1958 respectively. Other than for these perturbations, the predictions are rather good. Of course, the cumulative effect once a predictive error is made generally continues.

The best set of predictions for these years is that from the 1954 Q matrix. Once again the 1956 expansion affected the deviations, but the effect is seen as temporary as the predictions for the Cpl., Det. and S/Sgt., Sgt. strata are rather accurate by 1960. The "expansion" assumption continues to cause errors in predictions for the Tpr. stratum as 1958 and 1959 system losses apparently account for this stratum's lack of return to normalcy.

Overall, the predictions for the initial two or three years appear rather good. However, the 1951 and 1953 Q based predictions are unsatisfactory on the whole. The 1950, 1952 and 1954 Q matrices are generally accurate, given the "growth" assumption and with the exception of the 1956 perturbation. The predictions from the 1954 Q matrix suggest the 1956 effects are temporary and that perhaps the model's predictions later will not continue to err as a result of the cumulative phenomenon. Rather, other factors seem to have balanced the initial 1956 effect.

The predictions and observations for the second set of yearly estimated transition probabilities are provided below in Table 11. Q matrices were estimated from 1955 through 1959 data.

The predictions from the 1955 transition matrix are quite close to the observed stratum sizes for the two upper strata but not for the lower three. The accelerated expansion of 1956 is too close in temporal proximity for even the early years to be accurate. However, as seems to be the case for the 1954 Q, years following the 1956 expansion again resumed a more stable pattern and other factors seem to negate the cumulative effect. By 1961 for instance, the error at the Cpl., Det. stratum is half that produced by the changed dynamics of 1956 (i.e., the 1957 stratum sizes).

The 1956 transition matrix produces predictions far greater than the actual observations. This is as we might expect since 1956 had an extremely accelerated expansion rate at the bottom stratum, seemingly affecting administrative expansion throughout the system. In the Cpl. rank alone, there were 19 new positions filled as compared with 3 new positions for 1955. Comparable relative expansions appear to have occurred throughout the system.

Table 11. Predicted (P) and Observed (O) Stratum Sizes by Year Based upon Estimated Transition Matrices from 1955 through 1959

	Stratum Sizes				
	Col., Capt.	Lt.	S/Sgt., Sgt.	Cpl., Det.	Tpr.
			1955 Transition Matrix		
			1956		
P	14	20	82	137	482
O	14	20	82	137	481
			1957		
P	14	21	84	138	753
O	17	21	91	175	702
			1958		
P	15	22	86	151	873
O	17	26	101	184	814
			1959		
P	15	23	89	168	852
O	19	21	101	192	784
			1960		
P	16	24	94	181	833
O	19	23	105	197	748
			1961		
P	17	25	100	191	836
O	19	25	109	209	713

Table 11 (cont'd.)

	Stratum Sizes				
	Col., Capt.	Lt.	S/Sgt., Sgt.	Cpl., Det.	Tpr.
			1956 Transition Matrix		
			1957		
P	17	21	91	177	707
O	17	21	91	175	702
			1958		
P	20	22	105	237	767
O	17	26	101	184	814
			1959		
P	23	24	126	295	685
O	19	21	101	192	784
			1960		
P	26	26	126	295	685
O	19	23	105	197	748
			1961		
P	29	29	153	334	635
O	19	25	109	209	713
			1962		
P	32	33	182	361	574
O	19	22	112	208	727

Table 11.(cont'd.)

	Stratum Sizes				
	Col., Capt.	Lt.	S/Sgt., Sgt.	Cpl., Det.	Tpr.
	1957 Transition Matrix				
	1958				
P	17	26	101	184	814
0	17	26	101	184	814
	1959				
P	17	31	111	195	788
0	19	21	101	192	784
	1960				
P	17	36	121	203	765
0	19	23	105	197	748
	1961				
P	17	41	130	209	744
0	19	25	109	209	713
	1962				
P	17	46	139	213	746
O	19	24	112	208	727
	1963				
P	17	50	147	217	729
O	21	21	117	205	722

Table 11 (cont'd.)

	Stratum Sizes				
	Col., Capt.	Lt.	S/Sgt., Sgt.	Cpl., Det.	Tpr.
	1958 Transition Matrix				
	1959				
P	19	21	101	198	803
O	19	21	101	192	784
	1960				
P	19	18	102	211	792
O	19	23	105	197	748
	1961				
P	18	16	103	223	781
O	19	25	109	209	713
	1962				
P	17	15	105	239	785
O	19	24	112	208	727
	1963				
P	16	15	107	249	774
O	21	21	117	205	722
	1964				
P	15	15	110	258	765
O	18	19	119	207	723

Table 11 (cont'd.)

	Stratum Sizes				
	Col., Capt.	Lt.	S/Sgt., Sgt.	Cpl., Det.	Tpr.
	1959 Transition Matrix				
	1960				
P	19	23	105	199	771
0	19	23	105	197	748
	1961				
P	19	25	109	205	759
0	19	25	109	209	713
	1962				
P	19	27	113	212	765
0	19	24	112	208	727
	1963				
P	19	29	117	217	754
0	21	21	117	205	722
	1964				
P	19	31	121	221	745
O	18	19	119	207	723
	1965				
P	19	33	125	236	851
O	19	19	121	204	850

Look at the inflow and outflow dynamics of the S/Sgt., Sgt. and Cpl., Det. strata which provide information to explain the excessive predictions. They are:

	S/SGT., SGT.		CPL., DET.	
YEAR	Inflow	Outflow	Inflow	Outflow
1956	.1606	.1566	≈.1244†	.1507
1957	.1304	.1429	≈.1473†	.1483
1958	.0489	.0891	≈.0258†	.0529
1959	.0833	.1188	≈.0320†	.0975
1960	≈.0921†	.1406	.0495	.1270
1961	.0478	.0645	≈.0174†	.0667

From these data one would expect predictions greater than observations for the S/Sgt., Sgt. stratum size for each year. However, the prediction for 1958 should be rather close to the observed stratum size. The error is but four for this year. For the Cpl., Det. stratum the predictions should be much greater for 1959 through 1961. This is actually the case. The Tpr. predictions would be even more inaccurate were it not for the effect of the "growth" assumption. Predictions from years in which there is unusual perturbations throughout the system are heuristically valuable but very inaccurate should the system's accelerated dynamics not continue.

As might be expected from the inflow-outflow data cited above for the years 1956-1961, the predictions of the 1957 Q matrix will also be off for the S/Sgt., Sgt. stratum. The Lt. stratum's predictions are also excessive for each year as well as cumulative. This is a result of the inflow from the S/Sgt., Sgt. stratum being rather high for 1957.

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†These figures are slightly lower than the actual inflows but should be sufficiently adequate for present purposes. The actual predicted inflows are affected by each stratum's outflow outside the system since $P_{oj}>0$ for these strata in these years.

For instance, the S/Sgt., Sgt. to Lt. transitions for these years are as follows: 1957 (.0989), 1958 (.0594), 1959 (.0198), 1960 (.0476), 1961 (:.0282) and 1962 (.0179). We would obviously expect larger predictions.

The predictions from the 1958 Q matrix have the greatest inaccuracies with the Cpl., Det. and Tpr. strata. The problem with the Cpl., Det. stratum is a result of too few new entrants in 1958 and 1959 (15). In 1958 there were but four entrants, one of which was a re-entering Cpl. Thus, the $p_{o,Cpl.}$ is .25, a far greater magnitude than is ever the case when there exists numerous entrants. The Tpr. predictions are off primarily as a result of the "expansion" assumption since 1958, 1959, 1960 and 1962 were years in which the system actually declined in overall system size. This was a result of the State Department of Administration's ruling that positions vacated could not be filled due to the recession's effect upon the State's economic receipts (Annual Report (AR), 1958, 1959).

The 1959 Q matrix's predictions are extremely accurate for the first two of the five years and accurate throughout for the Col., Capt. and S/Sgt., Sgt. strata. The initial errors in the Tpr. stratum are once again due to the "expansion" assumption. The error for the Cpl., Det. stratum is obviously a result of the .0909 $p_{o,Cpl.,Det.}$ inflow since all other transitions are extremely close. Only eleven men entered the system in 1959, one of whom was a re-entering Cpl. Thus, small entering "cohorts" tend to distort future predictions whenever a re-entering member above the rank of Tpr. is involved.

The predictions for this set of Q matrices are considerably more inaccurate than those of the 1950-1954 years. The 1955 Q matrix is too close to 1956 for initial accurate predictions which was the rule for the 1950 to 1954 Q matrices. The 1956 Q matrix is not at all "normal" due to the short time span, extreme accelerated growth and accommodating internal system readjustments. The 1957 and 1958 Q matrices have rather accurate predictions for some but not all strata for any year, even given the effects of the "growth" assumption. It is only in 1959 that accurate predictions for the entire system are once again reached. Moreover, even in 1959 it is only the initial two of the five years in which predictions for the general system are accurate. In sum, the model's utility, at least for this second time span, is not very great. The volatile effects of politics and the economy upon the system and the model's cumulative error characteristic make predictions in proximity of short term, accelerated change very inaccurate.

A detailed examination of the predictions and observations from 1960—1969 Q matrices will not be made. Rather the data is provided in Table 12. A few comments will be made concerning the general adequacy of the model for these years, after which those Q matrices with the greatest accuracy will be projected further with appropriate "corrections" from observed data for the highly volatile years.

The predictions from the transition probabilities estimated from 1960 data are not very close to the observed stratum sizes after the first of the five years. In the Lt., S/Sgt., Sgt. and Cpl., Det. strata the model's predictions are greater than the observations. Once again the predictions are generally unacceptable. However, the predictions from both the 1961 and 1962 transition matrices are extremely accurate until 1967, another year like 1956, at least for the Cpl., Det. stratum and subsequently involving further expansion in other higher strata in the following years.

Table 12. Predicted (P) and Observed (O) Stratum Sizes by Year Based upon Estimated Transition Matrices from 1960 through 1968

	Stratum Sizes				
	Col., Capt.	Lt.	S/Sgt., Sgt.	Cpl., Det.	Tpr.
			1960 Transition Matrix		
			1961		
P	19	25	109	209	730
O	19	25	109	209	713
			1962		
P	19	27	114	219	733
O	19	24	112	208	727
			1963		
P	19	29	119	227	717
O	21	21	117	205	722
			1964		
P	19	31	127	234	705
O	18	19	119	207	723
			1965		
P	19	33	129	239	818
O	19	19	121	204	850
			1966		
P	19	35	134	249	864
O	18	23	116	207	864

Table 12 (cont'd.)

	Stratum Sizes				
	Col., Capt.	Lt.	S/Sgt., Sgt.	Cpl., Det.	Tpr.
	1961 Transition Matrix				
	1962				
P	19	24	112	208	732
O	19	24	112	208	727
	1963				
P	19	23	115	207	731
O	21	21	117	205	722
	1964				
P	19	22	117	206	732
O	18	19	119	207	723
	1965				
P	19	21	119	208	854
O	19	19	121	204	850
	1966				
P	19	23	121	209	866
O	18	23	116	207	864
	1967				
P	19	25	123	213	1004
O	23	19	124	274	939

Table 12 (cont'd.)

	Stratum Sizes				
	Col., Capt.	Lt.	S/Sgt., Sgt.	Cpl., Det.	Tpr.
	1962 Transition Matrix				
	1963				
P	21	21	117	205	726
O	21	21	117	205	722
	1964				
P	22	19	121	202	727
O	18	19	119	207	723
	1965				
P	23	17	124	203	852
O	19	19	121	204	850
	1966				
P	23	16	127	204	848
O	18	23	116	207	864
	1967				
P	23	15	130	210	1004
O	23	19	124	274	939
	1968				
P	23	14	133	217	1090
O	23	26	132	310	981

Table 12 (cont'd.)

	Stratum Sizes				
	Col., Capt.	Lt.	S/Sgt., Sgt.	Cpl., Det.	Tpr.
	1963 Transition Matrix				
	1964				
P	18	19	119	207	725
O	18	19	119	207	723
	1965				
P	15	18	121	212	850
O	19	19	121	204	850
	1966				
P	13	17	123	218	862
O	18	23	116	207	864
	1967				
P	11	16	125	226	1002
O	23	19	124	274	939
	1968				
P	9	15	128	237	1086
O	23	26	132	310	981
	1969				
P	8	15	131	252	1228
O	23	26	140	326	1115

Table 12 (cont'd.)

	Stratum Sizes				
	Col., Capt.	Lt.	S/Sgt., Sgt.	Cpl., Det.	Tpr.
	1964 Transition Matrix				
	1965				
P	19	19	121	204	851
O	19	19	121	204	850
	1966				
P	20	19	123	204	865
O	18	23	116	207	864
	1967				
P	21	19	124	204	1011
O	23	19	124	274	939
	1968				
P	22	19	125	207	1101
O	23	26	132	310	981
	1969				
P	23	19	126	212	1252
O	23	26	140	326	1115
	1970				
P	23	19	128	219	1309
O	23	26	149	366	1133

Table 12 (cont'd.)

	Stratum Sizes				
	Col., Capt.	Lt.	S/Sgt., Sgt.	Cpl., Det.	Tpr.
	1965 Transition Matrix				
	1966				
P	18	23	116	207	865
O	18	23	116	207	864
	1967				
P	18	25	113	212	1009
O	23	19	124	274	939
	1968				
P	19	26	111	223	1092
O	23	26	132	310	981
	1969				
P	20	27	111	238	1233
O	23	26	140	326	1115
	1970				
P	21	27	113	257	1278
O	23	26	149	366	1133
	1966 Transition Matrix				
	1967				
P	23	19	124	274	936
O	23	19	124	274	939

Table 12 (cont'd.)

	Stratum Sizes				
	Col., Capt.	Lt.	S/Sgt., Sgt.	Cpl., Det.	Tpr.
			1968		
P	25	17	141	336	952
O	23	26	132	310	981
			1969		
P	27	15	166	409	1037
O	22	24	148	355	1105
			1970		
P	28	16	190	462	1024
O	23	23	157	391	1125
			1967 Transition Matrix		
			1968		
P	22	24	137	332	981
O	22	24	138	335	977
			1969		
P	23	30	147	366	1087
O	22]	24	148	355	1105
			1970		
P	24	36	159	404	1096
O	23	23	157	391	1125

Table 12 (cont'd.)

	Stratum Sizes				
	Col., Capt.	Lt.	S/Sgt., Sgt.	Cpl., Det.	Tpr.
			1968 Transition Matrix		
			1969		
P	22	24	146	351	1011
O	22	24	148	355	1108
			1970		
P	22	24	154	367	1053
O	23	23	157	391	1125

The 1967 "deviancy" of transition probabilities was the result of two Civil Service rulings affecting the work week. The first ruling in 1963 instigated the 48-hour work week. The average work week had been more than 9 hours a day, 6 days a week (AR, 1960). The second ruling, operative in 1966, instituted a 40-hour, 5-day work week with overtime (AR, 1966). The effect of these policies was to increase manpower considerably in order to maintain the same amount of service. Moreover, it generated a policy of a minimum of three Cpls. per post. The major Cpl. increases occurred in 1967 and 1968. In terms of effect upon the model's predictions, the 1967 accelerated activity within this one stratum affected the predictions for this stratum considerably. Whereas previously the predictions (from 1961 and 1962 matrices) had been extremely accurate, they were off in 1967 by -61 and -64, respectively. Moreover, since the increased dynamics continued somewhat into 1968, the prediction from the 1962 Q was below the observed Cpl., Det. stratum size by 93.

The predictions from the 1963 Q were also rather good until 1967 although they are not as accurate as those from the 1961 and 1962 Q's. The predictions from the 1964 and 1965 Q's are analogous to those from the 1955 Q. The temporal proximity to 1967 and the cumulative effect characteristic of the model make their utility virtually nil. Similarly, the 1966 Q matrix and its generally much larger predictions are analogous to the 1956 Q matrix and its predictions. The 1967 Q matrix predictions, however, appear rather close to the observed values with the exception of overpredictions for the Lt. stratum. The one set of predictions from the 1968 Q is also rather close to the observations. In general, therefore, the 1960-1969 Q matrices fare much better than those from 1950-1959. The major perturbation in the sixties apparently only affected one stratum above that of Tpr. to any great extent--that of Cpl., Det.

To summarize thus far, the short run predictions of one or two years appear very good generally throughout the 20 year time span. Moreover, the predictions based upon estimates from data in 1950 and 1954 are rather good throughout if one takes into account the effects of the "expansion" assumption. The predictions based upon estimates from 1961, 1962 and 1967 are extremely accurate with one exception--the year 1967 for the former two years and also 1968 for the 1962 Q.

Two primary perturbations to the system's dynamics occurred in these 20 years--1956 and 1966. The former perturbation affected the entire system immediately, whereas the latter's immediate effect took place only in the lower level administrative stratum. In both instances, however, for both years in close proximity to these years as well as for these years, predictions were extremely inaccurate. Moreover, slight fluctuations of from .02 to .06 seemed to affect the predictions for the lowest level strata while fluctuations of up to at least .10 hardly affected the accuracy of predictions for the upper strata at all. The obvious reason for this is stratum size per se. Should these raw numbers be translated into proportions, the difference would be rather great. Nevertheless, for many of the years (1951, 1953, 1955, 1956, 1964, 1965, 1966) the predictions are far too inaccurate to "satisfice" the present researcher. While the model's utility is considerable so also are its deficiencies. Modifications of a variety suggested by other researchers (e.g., Blumen et al, 1955; McGuinnes, 1968; Mayer, 1972 or McFarland, 1970) do not appear to be helpful since it is doubtful such modifications could reduce transition fluctuations greatly, if at all. Moreover, system size precludes greater breakdowns within the upper two strata. If would seem, therefore, that one alternative is to readjust the stratum sizes at appropriate points and see if this alteration improves extended predictions. A second alternative is to examine the more simple demographic model.

Predictions of Stratum Size over Extended Time Periods

For the first of the two alternatives mentioned in the conclusion of the previous section, I will utilize the 1954 and 1961 Q matrices. Initial corrections will be made where it seems necessary. The data for the predictions from estimates based on 1954 data are provided in Table 13.

Since 1955 is not a valid prediction but rather used as a check on the correctness of the computer's calculation procedure, there are 15 years for which we may compare the predicted or expected values with those observed. The initial 5 years from 1956 through 1960 were also given in Table 9 and are provided again here for continuity of values for the entire 15 year time span. As stated previously in the initial discussion of the 5 year predictions from the 1954 Q matrix, the predictions from 1956 through 1960 are quite close to the observed stratum sizes. The exceptions are in the Cpl., Det. stratum for 1957 and 1958 and the Tpr. stratum from 1957 through 1960. However, it must be remembered that the 1959 and 1960 predictions must also be qualified since the "growth" assumption accounts for ≈ 23 in 1959 and ≈ 47 by 1960. With this in mind, it would seem that the model still overpredicts the Tpr. stratum by approximately 40, 45, 30 and 25 for years 1957, 1958, 1959

Table 13. Extended Predictions (P) and Observations(O) of System Size by Stratum and Year Based upon Estimated Transition Probabilities from 1954 Data

	Stratum Sizes				
	Col., Capt.	Lt.	S/Sgt., Sgt.	Cpl., Det.	Tpr.
			1955		
P	14	19	79	138	454
O	14	19	79	138	452
			1956		
P	14	20	83	139	481
O	14	20	82	137	482
			1957		
P	14	21	87	147	744
O	17	21	91	175	702
			1958		
P	14	22	91	162	860
O	17	26	101	184	814
			1959		
P	15	23	96	177	839
O	19	21	101	192	784
			1960		
P	16	24	102	189	820
O	19	23	105	197	748
			1961		
P	16	25	109	198	803
O	19	25	109	209	713

Table 13 (cont'd.)

	Stratum Sizes				
	Col., Capt.	Lt.	S/Sgt., Sgt.	Cpl., Det.	Tpr.
			1962		
P	17	26	116	206	807
O	19	24	112	208	727
			1963		
P	18	28	122	212	793
O	21	21	117	205	722
			1964		
P	19	29	128	216	782
O	18	19	119	207	723
			1965		
P	20	31	134	222	896
O	19	19	121	204	850
			1966[a]		
P	18	24	126	212	849
O	18	23	116	207	864
			1967		
P	18	27	132	223	977
O	23	19	124	274	939

[a]The predictions for 1966 used observed stratum sizes from 1965 rather than the expected or derived ones.

Table 13 (cont'd.)

	Stratum Sizes				
	Col., Capt.	Lt.	S/Sgt., Sgt.	Cpl., Det.	Tpr.
			1968[b]		
P	20	24	137	278	1014
O	23	26	132	310	981
			1969[c]		
P	19	29	149	287	1148
	(22)	(29)	(148)	(312)	(1119)
O	23	26	140	326	1115
			1970		
P	20	33	160	298	1188
	(22)	(33)	(162)	(318)	(1162)
O	23	26	149	366	1133

[b]The predictions for 1968 used observed stratum sizes from 1967 rather than the expected or derived ones.

[c]The predictions for 1969 which are within parentheses used observed stratum sizes from 1968 rather than the derived ones. The predictions for 1969 not contained within parentheses were based on expected stratum sizes from 1968. For 1970 the values in parentheses are predictions based on the 1969 expected values within parentheses. Similarly the 1970 predictions not contained within parentheses were derived from the expected values not contained within parentheses in 1969.

and 1960 respectively. The fit is improving as it also improves for the Cpl., Det. stratum by 1959 and 1960. The principal factor accounting for the inaccurate predictions of these two strata for 1957 and 1958 was the accelerated expansion in 1956.

From 1961 through 1964 the fit is very close for all but the Tpr. stratum. Recall that net system losses for 1958, 1959 and 1960 were 60. Hence, as a result of the assumption of "growth" as zero or positive and the cumulative nature of error built into the model, the error in 1961 generated by other factors is approximately 30. The word "approximately" is used since we are dealing with aggregate redistributions; thus the exact figure resulting from any one factor over several years is not determinable. Since the error is cumulative, an error of 60 is to be expected although not necessary. Given this margin of error due to the "growth" assumption, the predictions are rather good even for this stratum from 1961 through 1964.

By 1965, however, the errors in the Lt., S/Sgt., Sgt. and Cpl., Det. strata are increasing such that a correction seemed advisable for the 1966 prediction. With these corrections the 1966 predictions are very accurate and the 1967 predictions are relatively so with the exception of the Cpl., Det. stratum; however, it again seemed advisable to correct the stratum sizes to those observed for 1967 in order to once again have accurate predictions for 1968. The fit is once again very good except that the correction did not improve the fit for the Cpl., Det. stratum to an acceptable degree. For the remaining years, the predictions for all but the Cpl., Det. stratum are relatively accurate. Even with an additional correction in 1969, however, the Cpl., Det. stratum predictions are not improved sufficiently to be acceptable.

It would appear that the 1966 shift in Cpl. positions to maintain at least three Cpls. per post due to the new work week and man hour rulings has had a permanent effect in transition change from the Tpr. stratum. Since these positions were permanent and further expansion followed in the strata above the Cpl., Det. stratum, a somewhat higher inflow of men to this stratum would be expected. However, the 1954 Q cannot adequately handle this change. Thus, we must either change the entire Q or at a minimum re-estimate the Cpl., Det. and Tpr. transitions if further accurate predictions are to be expected.

The estimates from data in 1961 should also provide accurate predictions when extended beyond the 5 year time span. The predictions and observations are given in Table 14.

Once again the initial 5 years already reported in Table 12 are reported here for ease of perceptual continuity for the more extended time span. The predictions from 1963 through 1966 are extremely accurate. However, the 1966 accelerated expansion makes the 1967 predictions for Cpl., Det. and Tpr. stratum sizes very inaccurate. Thus, observed strata sizes for 1967 were used to make predictions for 1968. There is considerable improvement, but a fairly large error in the same two strata still exists. As may be seen from the 1969 and 1970 predictions, even further corrections do not improve the fit between predicted and observed stratum sizes for these two strata to a sufficient degree. Therefore, the same conclusion as was made for the 1954 Q is applicable here. Either we must make new estimates for the entire system, or at a minimum new estimates must be made for the Cpl., Det. and Tpr. strata.

Table 14. Extended Predictions (P) and Observations (O) of System Size by Stratum and Year Based upon Estimated Transition Probabilities from 1961 Data

	Stratum Sizes				
	Col., Capt.	Lt.	S/Sgt., Sgt.	Cpl., Det.	Tpr.
			1962		
P	19	24	112	208	732
O	19	24	112	208	727
			1963		
P	19	23	115	207	731
O	21	21	117	205	722
			1964		
P	19	22	117	206	732
O	18	19	119	207	723
			1965		
P	19	21	119	208	854
O	19	19	121	204	850
			1966		
P	19	23	121	209	866
O	18	23	116	207	864
			1967		
P	19	25	123	213	1004
O	23	19	124	274	939

Table 14.(cont'd.)

	Stratum Sizes				
	Col., Capt.	Lt.	S/Sgt., Sgt.	Cpl., Det.	Tpr.
			1968[a]		
P	22	21	129	274	1027
O	23	26	132	310	981
			1969[b]		
P	21	24	134	277	1174
	(23)	(27)	(138)	(309)	(1070)
O	23	26	140	326	1027
			1970		
P	21	25	139	280	1232
	(23)	(27)	(144)	(308)	(1131)
O	23	26	149	366	1133

[a]The predictions for 1968 are from 1967 observed stratum sizes rather than expected ones.

[b]The predictions in parentheses are based upon observed 1968 stratum sizes while those not in parentheses were based on expected ones. The 1970 predictions within parentheses were based upon the expected predictions in parentheses in 1969 and those not in parentheses in 1970 were based on expected values not in parentheses in 1969.

To summarize this more extended type of predictions, it seems that stability of dynamics is operative for the extended time span upon two criteria--stable transition probabilities and the predictions based upon the p_{ij} and $p_{i,k+1}$ parameters. For extended predictions the cumulative error aspect of this model does, however, necessitate corrections in the sense of using observed rather than expected values for the predictions based upon years which have very unusual dynamics as occurred in this system in 1966. In some cases the model may be self-corrective after a few years as the 1954 Q seems to quickly readjust predictions to accuracy after the 1956 perturbations. Yet, this may be a pecularity of this particular set of transition parameters as opposed to the usual post-perturbation predictions. The 1966 perturbations do not seem to be temporary ones. Rather, the 1966 structural changes seem to have established different flow densities on a more permanent basis. Thus, no self-corrective possibility seems likely. Rather, a new set of estimations seems necessary to improve predictive accuracy. On the whole, the predictive accuracy for the more extended time span is very good, and the utility of the model seems quite high. This conclusion must be qualified somewhat in that it does not seem that one may arbitrarily take any Q and have accurate predictions for extended periods. For example, some Q's were not even adequate for the 5 year span. A tentative check against short run accuracy, say for 5 years, seems necessary before long term accuracy can be assumed. It would seem that the particular nature of the entire set of transitions must be sufficiently between the range of fluctuations for long term predictions to hold. Once such Q's are found, however, the model's adequacy for explaining mobility seems very good. One finding not expected was a relatively high degree of accuracy in predicting the upper two stratum sizes. As already mentioned at the onset of this discussion, the relative small size of these strata allow much greater fluctuations with less noticeable error. In sum, given the above qualifications, this demographic model seems to be quite adequate in its extended predictions and hence its explanation of mobility. As a result no analysis of the more simple demographic model seems necessary.

Summary

From the substantive interpretation of the models to the extended predictions, the demographic model with system size given has generally been quite adequate for representing occupational mobility processes in the State Police. From 1927 through 1949 two major perturbations and the necessary restabilization processes occurred making movement nonstationary processes. An additional disrupting element may have been the system's age since it was but 10 years old in 1927. Thus, for this 23 year period, the assumption of time-homogeneous transition probabilities is not realistic. However, from 1950 to 1970 the mobility processes seem to have reached a sufficient degree of stability to be considered stationary or time-homogeneous. As explained in the discussion, the assumptions of the processes' Markovian nature and of the homogeneity of stratum populations also seemed reasonable. While the latter assumption is not entirely correct, the stationarity of estimated transition probabilities or the proportion of men moving from each stratum remaining "constant" allows us to maintain this assumption. The primary reason for this is that we are not attempting to determine the particular persons who move but rather the result upon stratum size of aggregate flows of populations of men. Therefore, selective and motivational factors do not seem especially relevant. Given the adequacy of the assumptions, a test seemed worthwhile.

Two types of tests were undertaken. The first involved predictions for short time periods (5 years) while the second test extended the time period for predictions (15 and 9 years). Given the knowledge of the consequences of the "expansion" assumption from the analysis of stratum size "predictions" for zero years, the short term predictions were extremely accurate for 2-3 year predictions with the exception of 1955, 1956, 1965 and 1966. The major overall system expansion in 1956 and the 1966 Cpl. expansion for all posts with less than 3 Cpls. presented unusual system perturbations such that for years immediately preceding 1956 and 1966, predictions were less than observed and for the predictions of these years, the expected sizes were greater than those observed. For the remaining 16 years, however, the immediate 1-3 year predictions seem extremely accurate. For the more intermediate length predictions (4-5 years) the cumulative error aspect of the model, together with the assumed error of "expansion", made predictions somewhat less accurate. This was, of course, true of predictions for years 1956 and 1966. Of interest was the return to "normalcy" or stationarity of the transition probabilities after 1956 and also the return to accurate predictions despite the expected continuation of the "cumulative error". The basis for this occurrence seemed to have been in the system's extreme degree of stability for the years 1958 through 1963. System growth and hence consequent movement were at a standstill. In fact, the State Department of Administration had ruled that vacant positions could not be filled as a result of the 1958 depression's or so-called recession's effect upon the State economy. In short a reversal of the 1955-1956 expansion occurred thus negating the expected "cumulative error".

For the more extended time period tests, the model's utility seems quite good. This conclusion, as the former ones, is impressionistic. It takes into account the "expansion" assumption and emphasizes the general long run accuracy rather than extreme accuracy for each intermediate year. The conclusion is also based upon predictions from Q's which generally provided accurate predictions for the 5 year time spans. Thus, it does not seem that one may take any arbitrary Q and expect accurate predictions for extended time periods. A tentative check for short term accuracy seems necessary before long term predictions can be worthwhile. Of particular interest for these predictions was the return to accurate predictions by the 1954 Q in spite of the 1956 system perturbation. That is, the predictions hold even after the 1958-1963 nongrowth period. This finding lends credence to the conclusion of general adequacy of the model's utility for representing occupational system dynamics and for its predictive accuracy in the long run. Also of interest was the finding that perhaps the Cpl., Det. stratum parameters were in need of re-estimation after 1966. This finding is tentative in that only 3 more years' data were available. However, it would seem from this short period that a one stratum re-estimation would be profitable.

The problems for predictions over more intermediate periods of time would seem to result from extreme fluctuations in general system dynamics. Especially affected were the predictions from Q's in the immediate proximity of the fluctuations.

It would seem that the utility of the aggregate manflow model is quite high. This conclusion is based upon the 20 year time period from 1950 to 1970. Based upon analyses of stability of transition probabilities, predictions of short time spans and predictions of extended time spans, the demographic model's adequacy for representing occupational mobility processes seems quite good. The stationarity of transition probabilities represents a stability in the underlying dynamics of the system for 20 years in spite of major surface-level changes in the system. In addition, the stationarity of transition probabilities provided the basis for accurate predictions. This, of course, also necessitated accuracy of representation in the Markovian nature of the mobility processes. It would seem the system's movement is represented by a Markov process. More specifically, it is represented by a simple Markov chain model.

In addition, the model's utility is considerable for extension purposes. Either for administrative management or the Troopers Association "management" purposes, projected simulations of the effects of certain decisions and/or structural changes upon transition probabilities and consequently short run and long run stratum sizes may be made. The major problem for such projections lies in the fact that system size is not a derived term but rather is given or observed. Therefore, until a model generating system size is constructed, simulation would be limited to two primary uses. The first possible use would be for the investigation of immediate policy effects prior to the policy's implementation. Similarly, once legislative authorizations of manpower are given, the immediate results may also be simulated prior to implementation. The second possible use of simulated manpower process would be to generate alternative "expansions" and investigate possible organizational consequences.

Three other directions for research also seem in order. The first, and perhaps most important one, would be an attempt to construct a model to generate system size. This extension will be attempted in a chapter to follow. The model represents the basic components thought to be most important for State Police--Legislature political bargaining over manpower demands. An alternative extension effort would be the construction of a manpower loss rate model. Such a model would provide much more adequate information for relating attrition and replacement processes, as well as coordinating these processes with general system growth or change. Initial steps are also taken in this direction in the chapter involving extensions to the two principal types of stochastic models being examined. A model for this process is, however, not attempted at this stage. A final extension effort which would seem worthwhile, given the general structural background of the present analysis, would involve the construction of an even more complex model to account for individual selectivity from each stratum. It would seem that the present model could be extended for these purposes due to its general adequacy for deriving aggregate manpower redistributions. In particular, such a model would modify the probabilities of movement for certain population segments of each stratum.

A final comment upon the importance of continuous, sequential data seems appropriate. Without this characteristic of the data, the analytical possibilities for relating general system dynamics to the model's dynamics could not have been undertaken. More specifically, the continuous, sequential nature of the data allowed us to relate general system dynamics to specific occupational dynamics. A more penetrating analysis was therefore possible. Of more import is the possibility of further extending this analysis to a mathematical-historical synthesis in order to construct a more general theory of system processes.

CHAPTER 4

VACANCY CHAIN MODEL OF OCCUPATIONAL MOBILITY

Let us, for the moment, stand the demographic or manflow approach "on its head". We may then visualize a different mobile unit--a job vacancy. Moreover, if we imagine that the movement of job vacancies within a system is interrelated, we may visualize a set of connected job vacancies. This set will be called a job vacancy chain and it is upon such chains of movement that the present discussion is focused. In a previous chapter the basic underlying conceptualization of job vacancy chains was discussed. The present discussion pertains to the mathematics of the vacancy chain conceptualization and an analysis of the model's adequacy for explaining occupational mobility with the State Police system. As for the mathematics, a particular concern is the substantive interpretation of the model for a specific type of manpower system. In brief, we may observe the degree to which the model adequately represents the system's occupational movement. Therefore, the analysis begins with the presentation of the model. The other principal analytic concern is with the accuracy of the model's predictions. As we will see, several types of predictions may be generated from the vacancy chain model. Most predictions will refer to chain length distribution, but another kind of prediction--number of moves--will also be analyzed. For the chain length distribution predictions, we will examine the fit between expected and observed distributions for both yearly and combined yearly time periods. With respect to the latter, several schemas for combining "yearly" data will be investigated. For the predictions of total number of moves from each stratum, only yearly and decade time spans will be utilized. As was the case with the last chapter, which focused upon mathematical models, the discussion to follow will involve an explication of mathematical assumptions as well as the logic of decision making, so that hopefully, a reader not trained in mathematics may at any point understand the substantive issues.

Introduction to the Mathematical Theory of System Vacancy Chains

The mathematical theory of job vacancy chains was developed by Harrison White (1970). The basic differences between the demographic and vacancy chain conceptualizations of occupational mobility was described in a separate discussion in Chapter 2. The differences of both of these two approaches to the type of approach generally utilized in sociology was briefly stated there. The extent of divergence of the vacancy chain approach is perhaps worth noting again for it appears to be a radical reconceptualization of the problem for sociologists. In general, I think it fair to say, sociologists have conceptualized intra-generational occupational movement in terms of two primary aspects--individual careers and occupational redistribution. The latter was generally stated as background for the discussion of careers. Thus, comparisons were made of net redistributions over time, but little effort was spent to explain or derive these redistributions. The demographic model attempts this type of derivation. The vacancy chain model explains yet another aspect of these occupational processes. The reason for my choice of "radical" in describing the nature of the reconceptualization by the vacancy chain approach may perhaps be seen by examining its initial focus and its additional representation. First, the foci is upon dual systems--populations of jobs and populations of men. One thing new at the onset, therefore, is the direct representation of a labor market and, more especially, of a set of jobs. It is this delineation of a set of jobs that gives the approach its systemic nature. That is, the set of jobs have a boundary or limiting context within which movement generally occurs. Moreover, the object conceptualized as moving is not men but job vacancies. Thus, by means of a job vacancy's existence, there is operative a pull to generate man movement. In addition to its principal focus upon job vacancies and its systemic nature, the vacancy chain approach is novel in that it conceptualizes the interrelated movement of job vacancies. In this respect it is an approach conceptualizing a second degree structure and thus representing much more of the dynamics of occupational movement. It is the redefinition of the problem in terms of jobs and job vacancies which frees the mental set for visualizing interrelated movement without a necessary particularistic influence by the participants. The resultant representation of the processes as occurring within limited contexts and as being distinctly interrelated is a much more thorough conceptualization than that utilized heretofore.[†]

The vacancy chain model is formulated as a discrete state, imbedded Markov chain model with absorbing states. The three

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[†]It should also be noted that the vacancy chain model is a more general model than of occupational processes. Although it was originally constructed to represent occupational movement, it is also currently being applied to housing markets. For an application of the theory to explain residential mobility, which is being carried out at Michigan State University, see S. Charles Lazer, "A Vacancy Chain Model of Residential Mobility," unpublished manuscript, June, 1972.

constraints of a simple Markov model are once again assumed. They are stationarity of transition probabilities, a Markovian nature of movement and homogeneity of population per stratum or state. In addition, once movement occurs to certain states (absorbing ones), no further movement from these states can occur. The idea that the processes are imbedded in time means that timing of movement is not specified.

Since the conceptualization of the time dimension within this model is rather unusual, perhaps we should examine the basis for the lack of specification of movement in time and the manner in which time is pertinent. In the vacancy chain model all initial job vacancies (the beginnings of each chain) for a given time period are viewed as a cohort and, therefore, treated as if they began at the same discrete moment in time, say the beginning of the time period. Moreover, the remaining movement within any chain having an initial vacancy in this time period is treated as if it occurred in the same time period. In other words, the entire chain is treated as if it has left the system before the cohort for the next time period enters. Exact timing is thus essentially irrelevant. What is at issue in the model is the successive or sequential interrelatedness of the moves (White, 1970:25 ff. and 180 ff.). Therefore, since the principal interest is upon the set of interrelated moves, we will treat a cohort of initial job vacancies as if they began at the same discrete moment in time. We will then examine all directly related job vacancy movement until the job vacancy chain leaves the system, irrespective of time of exit. All initial job vacancies beginning in a certain time period and all resultant job vacancy movement, irrespective of time of movement, will be treated as if it occurred within the given time period.

Let us now consider the meaning and validity of the three assumptions of a simple Markov chain somewhat further. Recall that the population conceptualized as moving in this model is one of job vacancies not men. The assumption of homogeneity of population presents much less of a problem than it potentially did in the manflow model. Job vacancies per job do not have long histories, and hence there does not exist a "cumulative inertia" effect of seniority per stratum. Job vacancies obviously occupy specific jobs. It is also true that all jobs within a particular stratum are hardly the same in all respects. There is then some heterogeneity of jobs within strata. The important question for us it seems is whether or not a job vacancy's occupancy of job A affects its probability for movement any differently than had the vacancy occupied job B within the same stratum, where jobs A and B are any two jobs. Stated somewhat differently, is job A likely to be filled by the same population of men as job B? In virtually all cases this is true with perhaps the exception of certain of the higher level communication and detective positions. Moreover, not only are these positions few in number but the transition probabilities are not necessarily different, given the possible exceptional nature of movement. With respect to the latter point, we are once again dealing with aggregate flows, and, therefore, it is the transition parameter which is of import rather than the distinct job entered. It does seem reasonable, therefore, to view all job vacancies which exist within a stratum as homogeneous for movement purposes.

The second assumption states that the job vacancy movements are Markov processes. That is, movement at time t is said to be dependent only upon the job vacancy's location at time t-1 and not upon locations at times t-2, t-3, ... This Markov assumption, meaning that job vacancy history before the job presently occupied is not influential, seems easily acceptable.

The main assumption remaining to be examined is the stationarity or constancy of transition probabilities. Two basic types of predictions will be generated. Which of the two predictions being utilized will determine the meaning of stationarity. For predictions of number of moves, estimation of parameters must be made from a time period different from and generally previous to the time period for which predictions are made. This is analogous to the manflow or demographic model. The minimum period of constancy is therefore two although stability over extended time periods is obviously advantageous for potential accuracy in long range predictions. The degree to which such stationarity holds will be examined later in the analysis.

For the chain length distribution predictions, estimations may be made from the same time period as the predictions. Stationarity in this case means that transition probabilities are stable throughout the entire time period. For yearly periods stationarity means there are no changes in job vacancy flows within the year. For periods in which job vacancy chains originating in several years are combined, stationarity means that transition probabilities are constant both within yearly periods and across all "yearly" periods combined. Certainly the assumption of stationarity within yearly periods seems reasonable. The validity of the stability of transition probabilities for combined periods may be examined after predictions are made from the yearly data. In this way, the presentation for this type of prediction will be consistent throughout and transition matrices will be presented in close proximity of the predictions. Also, repetition of transition matrices will be avoided. Thus, for the more complex periods, we will withhold judgment on the stationarity assumption.

The nature of the vacancy chain model and its basic underlying assumptions have now been briefly examined. This introduction, combined with the previous discussion in Chapter 2 on the model's conceptualization of the occupational processes, provides, I think, a sufficient basis for the presentation of the mathematical model and its substantive interpretation.

Vacancy Chain Model for A System of Occupations

The vacancy chain model was originally applied by Harrison White (1970) to three American churches to explain the internal mobility of their clergy. Thus, although White's focus was upon occupational mobility, the movement occurred within only one type of occupation--clergy. In the present study we will once again interpret the vacancy chain theory substantively with reference to a system of occupations. However, the set of occupations differs from that of White in two respects. The function is that of police work rather than of a religious nature and the set of occupations includes several different occupational roles rather than only one. In the State Police system there are distinct types of occupations each differing in function--general policeman, specialized criminal investigator and administrator. One of the rather unique aspects of this system is the necessary movement across these occupations due to no lateral entry. All new members enter at the bottom stratum necessitating the selection of administrators and detectives from the police patrol occupation. Moreover, there are also moves across detective and administrative occupations at the lower levels. Once higher level administrative positions are reached, the movement is essentially one way--from administration of detective work to administration of patrol work. The significant point at this time, however, is that there are movements interrelating all three different occupations.

Recall once again that the population conceptualized by the vacancy chain model is one of jobs not men. Let us divide these jobs into k strata. We will assume that a job vacancy's entrance into a particular stratum i (q_{0i}) is given. Therefore, the conceptualized probabilities for job vacancy movement will refer only to movement within or between strata (q_{ij}) and movement to the outside (q_{i0}). If we also assume a vacancy must leave the job which it occupies within a maximum span of time, we have

$$\sum_{j=0}^{k} q_{ij}=1. \tag{4.1}$$

Since the inflow process is either observed or derived, the stochastic model's range of possible moves is exhausted by describing the flows within and out of the system. For this reason the transition probabilities describing the flows within and out of the system must add to one.

Perhaps a comment should be made concerning the assumption of a "maximum time" for a job vacancy's occupancy. This assumption means that if a job vacancy has not left the job which it has been occupying by a certain span of time, for practical purposes the job is considered abolished. Under these conditions the model will regard this job as having left the system, thus ending the job vacancy chain.

Within the State Police organization, a job vacancy of one year will be assumed to be maximum before the job will be "abolished".

The official civil service policy for the State Police is that after a six month vacancy, a job is "abolished" and must be "re-established" before being filled. A one year vacancy maximum has been chosen rather than a six month period for several reasons. Clearly, a set length of time before job abolition is partially an artifact of record keeping. To set the policy for a shorter period of time than many vacant jobs are filled means excessive record preparation and filing. Also, to set the policy for a longer period means excess work at particular intervals for such cutoff points. Thus, I suggest the six month policy is largely a clerical artifact. However, from preliminary observation it seems true that almost all vacant positions are filled within a year. To extend the period longer than one year would have little effect. Finally, for some years there is only one personnel roster per year, making a six month check impossible.

Let us resume the description of the vacancy chain model. In matrix notation, let Q denote transition probabilities between strata where q_{ij} lies in the ith row and jth column; the column vector Q_0 denote $\{q_{i0}\}$; and L represent a column vector of k components, each unity. From Equation (4.1), it follows that

$$QL+Q_0=L \quad \text{where each } q_{i0}>0. \tag{4.2}$$

A vacancy chain is simply the path of a job vacancy once it enters the system. Its length is the number of moves after its entrance, including the terminal move to the outside. The length of a chain will be designated by n. Since the vacancy chain model conceptualizes both horizontal and vertical moves (i.e., only movement), we may note that n equals the number of positions a vacancy has occupied. Let us now denote the probability that a chain begun in stratum i will end in n moves by p_{in}. Within each stratum we may arrange the probabilities for a chain of length n into a column vector called P_n. In the matrix Q^{n-1}, the ith row and jth column element is the sum of the probabilities that in n-1 moves a vacancy beginning in stratum i is currently in stratum j. In other words, if we are given that the job vacancy began in stratum i, this q_{ij} element is the probability that the job vacancy is in stratum j after n-1 moves. By multiplying Q^{n-1} by Q_0, we, therefore, have the probability that a chain begun in stratum i will end or leave the system in n moves. It is

$$P_n=Q^{n-1}Q_0. \tag{4.3}$$

If we now let the mean length of a chain which began in stratum i be λ_i and arrange the λ_i's into a column vector Λ according to stratum, we know that

$$\Lambda=\Sigma nP_n. \tag{4.4}$$

From the theory of absorbing Markov chains, we know that $\Lambda=NL=(I-Q)^{-1}L$ where I is the unity matrix. (See Kemeny and Snell, 1960.) Thus,

$$\Lambda=\Sigma nP_n=(I-Q)^{-1}L. \tag{4.5}$$

By viewing vacancies in terms of successive yearly cohorts, we may represent the number of vacancies arriving in the cohort for year t as F(t) with the row vector F_t giving the number arriving per stratum. Let f_t represent the row vector of proportions of arrivals in the various strata, $f_i(t)=F_i(t)/F(t)$. Further, assume the transition parameters q_{ij} are constant within a year thus applying to all moves in the chains per cohort.

From Equation (4.3) we may compute the prediction of overall length distributions of chains as f_tP_n. Denote the overall mean length predicted n(t). Thus,

$$n(t)=f_t\Lambda=f_t(I-Q)^{-1}L \tag{4.6}$$

by Equation (4.5).

In addition to the probability distribution of vacancy chains beginning in each stratum and the mean length of overall vacancy chains, the total number of moves ever made from stratum i by a cohort of vacancies is an important aspect of mobility.

Let $M_i(t)$ represent the predicted number of moves from stratum i in year t; M_t the vector array by strata; and M(t) the number predicted for all strata. Since all vacancies must make at least one move, let F_t also denote the first moves by stratum. Vacancies initially moving to another stratum are counted by stratum of destination from F_tQ, which serves as the stratum from which the second move is made. Thus, the total predicted moves are

$$M_t=\sum_{h=0}^{\infty} F_tQ^h=F_t(I-Q)^{-1}. \tag{4.7}$$

The total predicted number of moves from all strata M(t) should equal the product of the observed number of chains times the overall average length of the predicted chain. Thus,

$$F(t)n(t)=M(t) \tag{4.8}$$

or

$$\begin{aligned} F(t)n(t)&=F(t)f_t(I-Q)^{-1}L \\ &=F(t)\frac{F_1(t)}{F(t)}\,\frac{F_2(t)}{F(t)}\cdots\frac{F_k(t)}{F(t)}(I-Q)^{-1}L \\ &=F_t(I-Q)^{-1}L=M_tL=M(t). \end{aligned}$$

The parameters for this model have been estimated in the following manner. Let us first designate terms:

1. $a_{ij}(t)$: The total number of observed moves by vacancies from stratum i to stratum j in the time period t;

2. $a_{i0}(t)$: The total number of observed terminal moves from stratum i in time period t;

3. $a_{i.}(t)$: The sum of all observed moves from stratum i, including the terminal moves from time period t;

4. $a_{0j}(t)$: The observed number of vacancy creations in stratum j in time period t;

5. $a_{0.}(t)$: The sum of all observed vacancy creations for the time period t;

6. $\hat{q}_{ij}$: Estimated value of q_{ij} for time period t;

7. $\hat{f}_j(t)$: Estimated vacancy creations in stratum j for time period t.

Two equations give us the desired estimation procedure. They are

$$\hat{q}_{ij}=\frac{a_{ij}(t)}{a_{i.}(t)} \tag{4.9}$$

and

$$\hat{f}_j(t)=\frac{a_{0j}(t)}{a_{0.}(t)}.^{\dagger} \tag{4.10}$$

The two primary aspects of mobility which the vacancy model predicts are chain length distribution and total number of moves from each stratum. As White (1970:33) notes, the same yearly data of chains can yield both the observed length distribution and the transition probabilities estimated in Equation (4.9). Since the transition probabilities cannot be deduced from the observed length distribution, the empirical test of this prediction may be made using data from the same

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†Although this particular estimation will not be used for predictive purposes in this study, it will serve the descriptive purpose of characterizing the relative number of entering vacancies across strata. Its value for predictive purposes becomes relevant when one is analyzing bumper chains rather than vacancy chains. In bumper chains the $f_j(t)$ occupy the same position as the q_{i0} in vacancy chains (White, 1970:32 ff.).

time period. The second prediction--total number of moves--cannot be deduced from the predicted chain length distributions and is a different type of test for the model. (See White, 1970:34.) That is, even though mean chain lengths and number of moves are derived from the multiplier $(I-Q)^{-1}$, the former are row sums [Equation (4.5)] and the latter column sums [Equation (4.7)]. Hence, we have a different aspect of internal mobility: total number of moves from each stratum. For this prediction data for parameter estimation must be from a preceding time span. The reason is that the number of moves were themselves used to estimate the q_{ij} transition probabilities. Therefore, if we multiply some form of Q and the F_t from the same sample, the predicted and observed numbers of moves must be equal. A test may be made only by using the Q matrix from from, say year t, and the number of entering vacancies from a different year, say F(t+x), where x=1, 2, 3, ... This type of test assumes the transition probabilities are constant for the two time periods t and t+x. The first type of prediction, distribution of chain lengths, may be made utilizing data from the same time period since parameter estimates are based on individual job vacancy moves while observations are on the chains as entities (White, 1970:105 ff.).

Preliminary Considerations

Recall from the chapter describing the conceptualization of demographic and vacancy chain models that the demographic model conceptualizes individual moves while the vacancy chain model conceptualizes interrelated moves. Let us now probe somewhat further into those aspects of the vacancy chain theory most important for testing the model's predictive accuracy. An observed population of vacancy chains is the result of an underlying structure or process. In other words, the observed chains have been generated by this process. The vacancy chain theory describes a probabilistic or stochastic process of the generation of vacancy chains, not a particular population of chains (White, 1970:89). It is the nature of the individual movement which the theory explains by means of its conceptualization of interrelations. What we must now determine is the adequacy of this theory to explain occupational mobility within the State Police.

The analysis will cover a forty-three year period with data having been collected for each year.[†] In other words, there are data for all forty-three years rather than, for example, a sample year or two from each decade. Also, the data represent all the moves related to any vertical occupational movement for the enlisted (police) segment of the State Police system for each year. Hence, all years as well as all "relevant" moves occurring within each year are included. As a result, neither selectivity of years nor sampling of the total movement may be said to account for part of the variation between predicted and observed values.

The reason all police movement is not "relevant" for vertical occupational mobility may be seen by a closer look at movement within the Tpr. stratum. Moves of Tpr. are generally of a geographical, not a functional nature. That is, since the majority of Tpr. movement is of a labor pool rather than a changing functional nature, most Tpr. movement is not relevant for our analysis. However, when Tpr. movement does denote a change in function, as for example, from patrol work at a post to full time special investigation or detective work from a district or the State Headquarters, this movement will be taken into account as

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[†]For details of the data collection process, see Appendix A.

"relevant" horizontal movement within the Tpr. stratum. In short, any movement within the Tpr. stratum denoting a change in function is a move relevant to the occupational processes depicted by the model and has been included in this analysis. Also, all movement within strata above the rank of Tpr. is considered to be relevant for occupational mobility purposes and has been included. Thus, with the exception of the geographical or labor pool movement at the Tpr. stratum, <u>all</u> movement within the occupational system of police has been included and observed for <u>all</u> forty-three continuously sequential years (1927-1970).

The choice of strata for this model is the same as that for the demographic model. Moreover, the nature of the one-to-one correspondence between the status and authority structures and the rationale for lumping or combining certain of the original nine ranks into a five strata system were discussed in Chapter 3. To refresh our memory, the specific ranks within the five strata are as follows: Stratum One--Col., Lt.Col., Maj., Capt; Stratum Two--Lt. and D/Lt.; Stratum Three--S/Sgt., D/S/Sgt., Sgt., D/Sgt.; Stratum Four--Cpl. and Det. and Stratum Five--Tpr. The estimated transition probabilities for and creation of job vacancies will be of the matrix form shown in Table 15.[†]

[†]The names of strata from Cpl., Det. through Lt., D/Lt. and in some cases their Civil Service classification were changed effective August 1, 1972. For the nature of this change and its relation to this study, see Appendix B.

Table 15. Transition Probabilities for and Creations of Job Vacancies[a]

Origin Stratum	Destination Stratum: Col., Capt.	Lt.	S/Sgt., Sgt.	Cpl., Det.	Tpr.	Out
Out	$f_1(t)$	$f_2(t)$	$f_3(t)$	$f_4(t)$	$f_5(t)$	
Col., Capt.	q_{11}	q_{12}	q_{13}	q_{14}	q_{15}	q_{10}
Lt.	q_{21}	q_{22}	q_{23}	q_{24}	q_{25}	q_{20}
S/Sgt., Sgt.	q_{31}	q_{32}	q_{33}	q_{34}	q_{35}	q_{30}
Cpl., Det.	q_{41}	q_{42}	q_{43}	q_{44}	q_{45}	q_{40}
Tpr.	q_{51}	q_{52}	q_{53}	q_{54}	q_{55}	q_{51}

[a]For convenience the D/Lt., D/S/Sgt., and D/Sgt. titles have been omitted. It will be assumed throughout this Chapter that unless stated otherwise Lt. also denotes D/Lt. The same assumption also holds for the other two ranks.

Once again, we may note that virtually all moves are across only one stratum boundary and are thus one step in length. Since demotions are also rare, job vacancy movement upward in the system is rare. In general, therefore, the internal transition probabilities for job vacancies will have the form of Table 16.

The time period from which the transition probabilities are estimated depends upon the type of predictions being generated. For both types of predictions, however, the same formula for the estimations is used. This formula is given in Equation (4.9).[†]

[†]For treatment of a special type of "transition" called reallocation, see Appendix C.

Table 16. Expected Internal Transition Probabilities for Job Vacancies

Origin Stratum	Destination Stratum				
	Col., Capt.	Lt.	S/Sgt., Sgt.	Cpl., Det.	Tpr.
Col., Capt.	q_{11}	q_{12}	0	0	0
Lt.	0	q_{22}	q_{23}	0	0
S/Sgt., Sgt.	0	0	q_{33}	q_{34}	0
Cpl., Det.	0	0	0	q_{44}	q_{45}
Tpr.	0	0	0	0	q_{55}

Before beginning to report the data, a few comments are in order. To quote a fellow mathematical sociologist: "In the first place we must recognize that when we have longitudinal data on large numbers of people, the amount of information we have is simply staggering." (Ginsberg, 1971:237). In the present study, the population was not extremely large but the period was an extended one. The State Police varied in size from 119 to 1,721 between the 43 year period, 1927-1970. The amount of information is somewhat voluminous. Therefore, data will be reported which is representative and which will be most helpful in understanding the gamut of dynamics being analyzed from several predictive viewpoints.

Predictions of Chain Length Distribution for Yearly Data

As mentioned earlier concerning the vacancy chain model, the estimation of transition probabilities and predictions of chain length distribution may both be made utilizing data from the same time period. In this section the time period will be one year. The reason for this dual function of yearly data is that transitions refer to individual moves rather than chains as entities. Moreover, one cannot deduce the transition probabilities from the observed length distribution (White, 1970:34). There is, as White notes, some degree of dependence. However, as yet there is no method to specify the degree of dependence (White, 1970:103). The test does seem to be a valid one. Equation (4.3) will be the formula used for deriving predicted chain length distribution.

The estimated transition probabilities for several years are reported in Table 17. The first five columns and last five rows contain the Q matrix. The termination probabilities are in the sixth column. In the first row and first five columns are the percentages of vacancies arriving per stratum. The last column contains the numbers from which the rows were calculated. The first row of this column is the total number of vacancies entering (moves into) the system. The remaining rows of the column refer to the total number of moves from and within each stratum.

From these transition matrices a rather important substantive conclusion may be drawn. It is only in 1948 that the system's internal dynamics seem to have reached some degree of long term stability. Of course, two major perturbations disrupted the system for short periods in each of the 1930 and 1940 decades. The late twenties seem to have been similar to the post-1947 period in the upper two strata but not the

Table 17. Estimated[a] Transition Probabilities for and Creations of Job Vacancies by Year

Origin Stratum	Destination Stratum						Total Number of Moves
	Col., Capt.	Lt.	S/Sgt., Sgt.	Cpl., Det.	Tpr.	Out	(N)
			1927				
Out	.0494	.0617	.0988	.3951	.3951		81
Col., Capt.	.2000	.6000	.0000	.0000	.0000	.0000	5
Lt.	.0000	.2000	.4000	.0000	.0000	.0000	10
S/Sgt., Sgt.	.0000	.0000	.2778	.2222	.1111	.3889	18
Cpl., Det.	.0000	.0000	.0167	.4000	.2667	.3167	60
Tpr.	.0000	.0000	.0000	.0000	.1667	.8333	60
			1930				
Out	.0588	.0000	.0588	.1176	.7794		68
Col., Capt.	.3333	.1667	.0000	.0000	.0000	.0000	6
Lt.	.0000	.0000	.0000	.0000	.0000	1.0000	1
S/Sgt., Sgt.	.0000	.0000	.3333	.5000	.0000	.1667	6
Cpl., Det.	.0000	.0000	.0000	.3684	.1579	.4737	19
Tpr.	.0000	.0000	.0000	.0000	.1270	.8730	63

[a]Estimations are made using Equation (4.9). The Col., Capt. stratum includes Col., Lt.Col., Maj. and Capt.

Table 17 (cont'd.)

Origin Stratum	Destination Stratum: Col., Capt.	Lt.	S/Sgt., Sgt.	Cpl., Det.	Tpr.	Out	Total Number of Moves (N)
			1938				
Out	.0000	.0741	.3333	.1481	.4444		27
Col., Capt.	.0000	.0000	.0000	.0000	.0000	.0000	0
Lt.	.0000	.0000	1.0000	.0000	.0000	.0000	2
S/Sgt., Sgt.	.0000	.0000	.1428	.6428	.0000	.2142	14
Cpl., Det.	.0000	.0000	.0434	.4347	.0869	.4347	23
Tpr.	.0000	.0000	.0000	.0000	.3000	.7000	20
			1948				
Out	.0172	.0517	.0690	.4483	.4138		116
Col., Capt.	.0000	1.0000	.0000	.0000	.0000	.0000	2
Lt.	.0000	.0000	.7500	.0000	.0000	.2500	8
S/Sgt., Sgt.	.0000	.0000	.1250	.5625	.0000	.3125	16
Cpl., Det.	.0000	.0000	.0000	.1268	.8169	.0563	71
Tpr.	.0000	.0000	.0000	.0093	.0450	.9459	111

Table 17 (cont'd.)

Origin Stratum	Destination Stratum: Col., Capt.	Lt.	S/Sgt., Sgt.	Cpl., Det.	Tpr.	Out	Total Number of Moves (N)
			1955				
Out	.0658	.0132	.2105	.1974	.5132		76
Col., Capt.	.1667	.8333	.0000	.0000	.0000	.0000	6
Lt.	.0000	.2500	.7500	.0000	.0000	.0000	8
S/Sgt., Sgt.	.0000	.0000	.1852	.7037	.0000	.1111	27
Cpl., Det.	.0000	.0000	.0000	.3125	.5417	.1458	48
Tpr.	.0000	.0000	.0000	.0000	.0294	.9705	68
			1965				
Out	.0536	.0536	.2411	.2589	.3929		112
Col., Capt.	.2500	.7500	.0000	.0000	.0000	.0000	8
Lt.	.0000	.0769	.8462	.0000	.0000	.0769	13
S/Sgt., Sgt.	.0000	.0000	.4154	.5385	.0000	.0462	65
Cpl., Det.	.0000	.0000	.0000	.2381	.7024	.0595	84
Tpr.	.0000	.0000	.0000	.0000	.0373	.9626	107

S/Sgt., Sgt. or Cpl., Det. ones. The system was apparently beginning to expand considerably and would probably have reached the 1948 stage much more quickly had it not been for the Great Depression. Net growth of the enlisted (police) segment of the State Police was +9, -1, +18, +33, +53, -9, -44, +15 for the years 1927 through 1934 respectively. It was not until 1932 that no recruit schools were conducted, the result being the system lost 9 persons. When there were few jobs on the outside, fewer men left the system. For instance, from 1927 through 1932 the number of men leaving each consecutive year was 41, 28, 27, 20, 10 and 12. There is no doubt but that loss rate was greatly curtailed by increasingly less job opportunity outside the system. Even though the Depression was greatly affecting job opportunities and probably promotions outside governmental systems, within the State Police system there was little effect until 1933. For instance, in 1931 a district administrative system was instituted at the Governor's persuasion and in 1931 and 1932 several Capt. and First Sgt. positions were created and filled (AR 1931, 1932). Vacancy chains extended throughout the system. In 1933 the Depression's effects were extensive as 36 Tprs. were "honorably discharged" and "voluntary demotions" were taken by the majority of all officers above the Tpr. stratum. A new rank of Senior Tpr. was even established for demoted Cpls. and some Sgts. and Lts. The remainder of the thirties were growth years for the system but once the "voluntary demotions" were once again reversed, there was little activity in the upper two strata until 1944. This may be seen in the 1938 transition matrix which is representative of years on either side of it with one exception. The S/Sgt., Sgt. row is reversed in terms of relative transitions for the years 1936, 1937 and 1943 through 1945. That is, the probability was greater for a job vacancy to move within the "Sgt." stratum than to move to the Cpl., Det. stratum. Perhaps the primary reason for this was the virtual total lack of activity in the strata above S/Sgt., Sgt. With no promotion possibilities, greater movement within strata at both the S/Sgt., Sgt. and Cpl., Det. strata occurred. World War II was, of course, another disruptive year, the effect being primarily continued inactivity in upper stratum movement. It cannot be discerned whether the general period's dynamics are a result of reactions and recovery to these two perturbations or whether they are a result of an emergent structure with little seniority by its incumbents which would have to expand considerably or await increasing seniority before further mobility in the upper strata occurred. Nevertheless, the two events no doubt did have considerable effect in terms of temporarily changing the internal dynamics.

After 1947 it appears from the 1948, 1955 and 1965 transition matrices that the system has generally stabilized. The transitions of greatest variance, when the transition matrices not reported are taken into account as well, are the 1948 Cpl., Det. moves. The within stratum moves are too few and, therefore, the corresponding Cpl. to Tpr. moves are too great in proportion. Yet, with minor exceptions the underlying dynamics of the period from 1948 to 1970 appear quite stable.

The second rather significant substantive notion which the transition matrices provide is the rather small number of vacancies entering particularly in the upper two strata. We may see this more clearly by translating the proportion of entering vacancies back into the actual number of entering vacancies per stratum. For the years initially reported, the data are given in Table 18. For no year reported is the number of vacancy chains entering each of the upper two strata greater than six. This is generally true for all 43 years, although less so from 1956-1958 and 1966-1969. The result is most significant for predictive purposes since the number is far too low to expect reliable predictions from a stochastic model. Moreover, in 1930 and 1938 the number of vacancies entering four of the five strata are too low for predictive purposes. Predictions for these two years will, therefore, not be reported. The remaining predictions are presented primarily for heuristic purposes. They provide at least an initial indication of the model's predictive accuracy for those strata having a sufficiently large number of entering vacancies. The appropriate strata are the Cpl., Det. and Tpr. strata for 1927, 1948 and perhaps 1965. Very tentative evidence may be seen from the S/Sgt., Sgt. stratum for 1955; for 1965 I would expect more reliability. The predictions are reported in Tables 19 through 22.

Table 18. Number of Vacancies Entering the System at Each Stratum by Year

	Number of Entering Vacancies					
Year	Col., Capt.	Lt.	S/Sgt., Sgt.	Cpl., Det.	Tpr.	Total Number
1927	4	5	8	32	32	81
1930	4	0	4	8	53	68
1938	0	2	9	4	12	27
1948	2	6	8	52	48	116
1955	5	1	16	15	39	76
1965	6	6	27	29	44	112

Table 19. Predicted (P) and Observed (O) Distribution of Vacancy Chain Length by Stratum of Origin for 1927 (Percent)

	Stratum of Origin									
	Col., Capt.		Lt.		S/Sgt., Sgt.		Cpl., Det.		Tpr.	
j[a]	P	O	P	O	P	O	P	O	P	O
1	20.0	25.0	40.0	.0	38.9	25.0	31.7	12.5	83.3	71.9
2	28.0	50.0	23.6	60.0	27.1	37.5	35.5	71.9	13.9	25.0
3	19.7	25.0	15.6	20.0	17.0	25.0	18.4	9.4	2.3	3.1
4	13.3	0.0	9.9	0.0	9.1	12.5	8.3	6.3	0.4	0.0
5	8.6	.0	5.6	.0	4.4	.0	3.6	.0	.1	.0
6	5.1	.0	2.9	20.0	2.0	.0	1.5	.0	.0	.0
7	2.7	.0	1.4	.0	.9	.0	.6	.0		
8	1.4	.0	.6	.0	.4	.0	.3	.0		
9	.7	.0	.3	.0	.2	.0	.1	.0		
10	.3	.0	.1	.0	.1	.0	.1	.0		
11	.1	.0	.1	.0	.0	.0	.0	.0		
12	.1	.0	.0	.0						
13	.0	.0								
14										
15										
16										
(N)	4		5		8		32		32	

[a] j denotes chain length distribution.

Table 20. Predicted (P) and Observed (O) Distribution of Vacancy Chain Length by Stratum of Origin for 1948 (Percent)

	Stratum of Origin									
	Col., Capt.		Lt.		S/Sgt., Sgt.		Cpl., Det.		Tpr.	
j[a]	P	O	P	O	P	O	P	O	P	O
1	.0	.0	25.0	.0	31.3	.0	5.6	.0	94.6	93.8
2	25.0	100.0	23.4	83.3	7.1	.0	78.0	92.3	4.3	.0
3	23.4	.0	5.3	.0	44.8	37.5	13.4	3.8	.9	6.3
4	5.3	.0	33.6	16.7	13.1	25.0	2.5	1.9	.2	.0
5	33.6	.0	9.9	.0	3.0	37.5	.5	.0	.0	.0
6	9.9	.0	2.3	.0	.6	.0	.1	1.9		
7	2.3	.0	.5	.0	.1	.0	.0	.0		
8	.5	.0	.1	.0	.0	.0				
9	.1	.0	.0	.0	.0	.0				
10	.0	.0								
11										
12										
13										
14										
15										
16										
(N)	2		6		8		52		48	

[a]j denotes chain length distribution.

Table 21. Predicted (P) and Observed (O) Distribution of Vacancy Chain Length by Stratum of Origin for 1955 (Percent)

	Stratum of Origin									
	Col., Capt.		Lt.		S/Sgt., Sgt.		Cpl., Det.		Tpr.	
j[a]	P	O	P	O	P	O	P	O	P	O
1	.0	.0	.0	.0	11.1	6.3	14.6	.0	97.1	97.4
2	.0	.0	8.3	100.0	12.3	31.3	57.1	66.7	2.9	.0
3	6.9	.0	11.3	.0	42.5	37.5	19.4	13.3	.1	2.6
4	10.6	.0	34.7	.0	21.5	12.5	6.1	13.3	.0	.0
5	30.7	40.0	24.8	.0	8.3	12.5	1.9	6.7		
6	25.8	40.0	12.4	.0	2.9	.0	.6	.0		
7	14.6	.0	5.3	.0	1.0	.0	.2	.0		
8	6.8	20.0	2.0	.0	.3	.0	.1	.0		
9	2.8	.0	.7	.0	.1	.0	.0	.0		
10	1.1	.0	.3	.0	.0	.0				
11	.4	.0	.1	.0						
12	.1	.0	.0	.0						
13	.1	.0								
14	.0	.0								
15										
16										
(N)	5		1		16		15		39	

[a]j denotes chain length distribution.

Table 22. Predicted (P) and Observed (O) Distribution of Vacancy Chain Length by Stratum of Origin for 1965 (Percent)

	Stratum of Origin									
	Col., Capt.		Lt.		S/Sgt., Sgt.		Cpl., Det.		Tpr.	
j[a]	P	O	P	O	P	O	P	O	P	O
1	.0	.0	7.7	.0	4.6	.0	6.0	3.4	96.3	95.5
2	5.8	16.7	4.5	16.7	5.1	7.4	69.0	72.4	3.6	4.5
3	4.8	.0	4.7	.0	39.3	37.0	19.0	17.2	.1	.0
4	4.7	.0	33.6	.0	26.5	44.4	4.6	3.4	.0	.0
5	26.4	.0	25.0	66.7	13.5	3.7	1.1	3.4		
6	25.4	16.7	13.4	16.7	6.2	7.4	.1	.0		
7	16.4	50.0	6.3	.0	2.7	.0	.1	.0		
8	8.8	16.7	2.8	.0	1.2	.0	.0	.0		
9	4.3	.0	1.2	.0	.5	.0				
10	2.0	.0	.5	.0	.2	.0				
11	.9	.0	.2	.0	.1	.0				
12	.4	.0	.1	.0	.0	.0				
13	.2	.0	.0	.0						
14	.1	.0								
15	.0	.0								
16										
(N)	6		6		27		29		44	

[a]j denotes chain length distribution.

For 1927 the Cpl., Det. predictions are considerably off. Chains of length 1 and 2 have a 19.2 and 58.0 per cent error respectively. Similarly, the Tpr. predictions are not quite within a 10 per cent range for the two lengths within which most chains are located. With only 32 entering vacancies per stratum judgment must be withheld.

The 1948 prediction is much more accurate for both strata but not entirely satisfactory for the Cpl., Det. stratum in which the error for chains of length 2 is 14.3. The 1955 predictions are even more accurate with the greatest error for the Cpl., Det. stratum being 9.6. However, the increased accuracy is not a function of the number of entering vacancy chains since only 15 enter the Cpl., Det. stratum. The 1965 predictions confirm the expectation of greater reliability yet and are inaccurate for the three pertinent strata in only one case--chains of length 4 for the S/Sgt., Sgt. stratum. The error in this case is 17.9. The increasing accuracy is encouraging, particularly so for the 1965 predictions. For a genuine test of the model, however, we must be able to examine the predictions for each strata.

There are two possible approaches to the problem of an inadequate number of entering vacancies. There is first of all the possibility of collapsing even more strata. For instance, the two upper strata, if combined, might provide sufficient numbers. An alternative approach to the problem would be to combine yearly data. Let us consider the first of the approaches suggested. To decide to combined, for instance, the two upper strata of a five strata system means that we have also decided the five state model is not a Markov chain. The reason for this is that both a five strata system and a system with less strata from "lumping" together two or more of these five cannot be a Markov chain.

(For criteria, see Kemeny and Snell, 1960.) Let us re-examine Table 16 to support the above statement. Were we to lump the two upper strata together, then q_{23} must equal 0, if both the original five strata and the new four strata systems are Markov chains. Since $q_{23} \neq 0$, one of the two stratified systems cannot be a simple Markov chain (MC). The logic is as follows: if the five strata system is a MC then, in the present study, since $q_{23} \neq 0$, the four strata system is not a MC. Alternately, if the data do not conform to the five strata MC model's predictions, this does not imply the system dynamics may necessarily be inadequately represented by a MC model but rather that the current stratified system is not a MC when the states are defined as such (McFarland, 1970). First, can we conclude that the State Police system processes conform to a five state MC? I think not. The information is far too inadequate. Secondly, can we conclude that the five state MC is an inadequate description of the processes being analyzed? Again the answer is no due to an insufficient test. Since there is an alternative method of examining the five strata model, it seems totally unreasonable to discard it without testing it as thoroughly as possible. Another perhaps even better reason to continue with the five strata model is that a collapse of the upper two strata would lose much substantive information about the system. It is substantively important to distinguish Lts. from Capts., Majs., LtCols. and the Col. Similarly, much information would be lost by collapsing any other two strata. The five state description seems to be the only practical and meaningful one. It is practical since more strata than five certainly cannot be tested. It is substantively meaningful because less strata than five provide far too little important information. Thus, I will not take the "lumping solution" but rather the one which seems most productive, the combining of yearly data. Even should the model fail, an attempt to find the reasons for the failure for substantively meaningful strata is an important task.

Alternative Methods of Combining Yearly Data

We have, at least initially, chosen to solve the problem of insufficient numbers of entering vacancy chains per stratum by combining yearly data. The usual approach to this combining process is to aggregate those yearly data having similar q_{ij}. The basis for this method is the assumption that the q_{ij} must be constant within the time period of the model's predictions. There are, however, additional alternatives available for this study as a result of the data being sequential. Recall that "yearly" vacancy chains in this study refer to all vacancy chains which enter the system during a year. If the chains extend to later years, the moves for these years are also counted in the calculation of the yearly (year entered) chains.

In the White study, entering job vacancies did not necessarily refer to yearly entrances but rather to whether or not any portion of a vacancy chain occurred during the so-called "entry" year (White, 1970: 355 ff.). Thus, the assumption of stability across <u>actual</u> yearly entering cohorts could not be tested. Rather, the assumption of stability for the so-called "yearly" entering vacancy was examined. This is important in the present study, as well as in White's study, since it points to the lack of thorough examination of the actual chains, of possible effects upon individual transitions of chains beginning in differing strata or of being initiated by different means. Rather, in the

White study, a general examination using transition probabilities was the basis for combining "yearly" cohorts of vacancy chains.

Several important points of the discussion above are relevant to the present study. First, we may examine the q_{ij} and combine actual yearly entering cohorts of vacancy chains on the basis of the similarity of the q_{ij}. The fact that a standardized procedure has been used, such as including only vacancy chains whose origin occurred in a certain year as though the entire chain occurred within that year, provides a common basis for comparison of "yearly" data. Secondly, we should be aware that this standardization is an artifact for examining the data, especially if it is sequential. By thinking of the process sequentially one can readily see that processual occurrences such as movement of job vacancies have no necessary beginning or ending points once the process is in motion. Therefore, any standardized method of breaking up the process is likely to have overlapping segments. That is, if we trace a vacancy chain which has begun at time 1 until its end at time 12 (for example, a 12 month period), other vacancy chains have likely begun at times 2, 3, ... Moreover, no matter where chronologically one begins and ends, there will be overlaps. In short, the processes of mobility are continuous. Hence, the chronological breaking points at time periods which we are accustomed to thinking about, such as years, are purely arbitrary breaking points. The system's processes do not stop, accelerate, decelerate, et cetera according to calendar years but according to internal policies, political pressures both from within and without, economic fluctuations, turnover of high level administrators, stages of structural differentiation, amount of man hours worked per week, technological innovations and international wars.

It seems clear, therefore, that when we are dealing with longitudinal, sequential data, an arbitrary yearly criteria, whether as in the present study's illustration in Table 17 or as in White's type of "yearly" construct, should not necessarily limit the aggregation schemas. More importantly, I am suggesting that until we have more general theories, mathematical analyses should be extended to include additional system dimensions such as structural adjustments to external forces and internal policy changes or differentiation. In other words, what is required at this stage of theory building is a thorough interrelating of the mathematical and descriptive data such that a more integrated mathematical-historical analysis is obtained. A thorough effort in this direction would extend the scope of this study considerably. Although the scope of the present study will not include such an extensive interrelating process, an initial step in this direction will be made by including pertinent substantive dimensions where possible as criteria for setting limits or bounds on the aggregation process. Totally arbitrary criteria such as decades will also be used to provide a contrast with the above method. The only way to determine whether such combined data meet the assumption of constant q_{ij} is to test the model.

Predictions of Chain Length Distribution for Combined Data Based upon Similar Transition Probabilities

Let us first examine data which has been combined using "yearly" transition matrices with similar q_{ij} as the basis for aggregation. As noted earlier, in virtually none of the pre-1945 years were the number of vacancies entering at the upper two strata large enough for stochastic analysis. Also noted was the virtual lack of movement within these upper strata for the majority of years. Where movement did occur, it was erratic as for instance promotions in 1931-1932, demotions in 1933 and promotions in 1934 to once again regain the former rank. For these reasons no aggregation will be attempted prior to 1945. This leaves twenty-five years for possible aggregation.

When an extremely critical limit such as less than a .10 difference between each cell is set, there are very few combinations possible. This is not terribly unexpected since the possibilities for variation when comparing merely three years is 90 cells. Of combinations requiring less stringent requirements for each cell but less than a .20 difference for any cell and generally less than a .10 difference for most cells, the aggregations once more have not included many years. Therefore, the result is the same as that with single "yearly" data--the number of entering vacancies at the upper two strata is too low for a reliable prediction of distribution of vacancy chain lengths.

We are left with criteria not as rigorous as desired but still rather stringent. We will examine two q_{ij} based transition matrices which have combined data. In this discussion, only the combined transition matrices will be reported. For the yearly matrices from which the first one of these were derived, see Appendix E.

The first set of combined yearly cohorts of job vacancy moves which we will examine includes data from the following "years": 1952, 1954, 1957-1960, 1967-1969. For the q_{i0} cells there are 3 out of 35 which differ from any other cell by greater than .15. The differences are .23, .25, .40. Most of the cells have less than a .10 difference. Of the q_{ij} cells with at least more than 3 moves for the entire row vector, there are 31 of a possible 205 deviation greater than .15, 20>.18, 13>.20, 5>.25; these 5 are of a magnitude of .26, .27, .29, .34 and .38 difference from <u>any</u> of the appropriately compared cells. The transition matrix is reported in Table 23 and the predicted and observed distributions of chain length are reported in Table 24. The predictions agree fairly well with the observations with the exceptions of chains of length 2 for the three upper strata. In all three cases the predictions for chains of lengths 1 and 3 were greater than the observation.

Table 23. Estimated[a] Transition Probabilities for and Creations of Job Vacancies for q_{ij} Based Combined Data[b]

	Destination Stratum						Total Number of Moves
Origin Stratum	Col., Capt.	Lt.	S/Sgt., Sgt.	Cpl., Det.	Tpr.	Out	(N)
Out	.0324	.0374	.1239	.2271	.5792		1017
Col., Capt.	.2500	.6136	.0000	.0000	.0000	.1364	44
Lt.	.0000	.0972	.7778	.0139	.0000	.1111	72
S/Sgt., Sgt.	.0000	.0000	.3309	5993	.0000	.0699	272
Cpl., Det.	.0000	.0000	.0000	.3065	.5972	.0963	571
Tpr.	.0000	.0000	.0000	.0050	.0669	.9281	1001

[a]Estimations are made using Equation (4.9). The Col., Capt. stratum includes Col., Lt.Col., Maj., and Capt.

[b]The years for which entering cohorts of job vacancies were combined are 1952, 1954, 1957-1960, 1967-1969.

Table 24. Predicted (P) and Observed (O) Distribution of Vacancy Chain Length by Stratum of Origin for q_{ij} Based Combined Data[a] (Percent)

	Stratum of Origin									
	Col., Capt.		Lt.		S/Sgt., Sgt.		Cpl., Det.		Tpr.	
j^b	P	O	P	O	P	O	P	O	P	O
1	13.6	6.1	11.1	2.6	7.0	2.4	9.6	3.5	92.8	91.9
2	10.2	30.3	6.7	26.3	8.1	31.7	58.4	61.9	6.3	6.1
3	6.6	3.0	7.7	2.6	37.7	25.4	21.6	23.8	.7	1.7
4	6.4	.0	30.3	15.8	25.4	23.8	7.1	8.2	.2	.2
5	20.2	3.0	22.8	21.1	12.6	10.3	2.3	2.2	.1	.0
6	19.1	24.2	12.1	15.8	5.5	4.8	.7	.0	.0	.2
7	12.2	21.2	5.5	13.2	2.3	.8	.2	.4	.0	.0
8	6.4	3.0	2.3	2.6	.9	.8	.1	.0		
9	3.0	9.1	.9	.0	.3	.0	.0	.0		
10	1.3	.0	.4	.0	.1	.0				
11	.5	.0	.1	.0	.1	.0				
12	.2	.0	.1	.0	.0	.0				
13	.1	.0	.0	.0						
14	.0	.0								
15										
16										
(N)	33		38		126		231		589	

[a]The years for which entering cohorts of job vacancies were combined are 1952, 1954, 1957-1960, 1967-1969.

[b]j denotes chain length distribution.

Initially, the explanation for at least the missed predictions for chain length 1 seems rather obvious. The chains first entering the system at each strata are different than those arriving from an upper stratum. This logic is, however, inapplicable to the highest stratum. Its prediction is also greater than that observed. The explanation is not readily apparent. The predictions of chain length 1 are simply the proportion of all moves within the q_{ij} row vector which are to the outside. This holds true because $P_n=Q^{n-1}Q_0$ and Q^{n-1} for n=1 is I. In view of the fact that $P_1=Q_0$, we note that each prediction of chain length 1 is the appropriate q_{i0}. This suggests a relation between individual moves and distribution of chain length which we may pursue. Let us take a rather simple population of vacancies and examine it more closely. The population chosen was the Col., Capt. stratum. As already noted for this stratum, there are no alternative vacancy entrances other than from outside. The analysis is, therefore, a more controlled one. Let us take the case of three entering chains. The possibilities most likely to occur once the vacancy has entered this stratum are shown in Figure 5. Of the three possibilities only Chain 3 has length 1. Chain 2 has at least one intra-stratum move; if it were to leave the stratum and on the next move go to a Lt. position, the prediction (q_{i0}) would be .25 with .33 of the actual (observed) chains of length 1. This is the opposite of the situation reported in Table 24. If, however, the chain were to move outside on its second move, then the predicted distribution would be .50 and the observed .33. This is a situation comparable to that of Table 24, at least in predicted-observed proportions. It would seem, therefore, that any moves to the outside, which are not immediate ones, bias the predictions such that they are higher than actually is the case.

This is because the estimations are based on individual moves. A counting procedure which views only "isolated" moves does not taken into account the place of those moves in terms of interrelated or sequential movement.

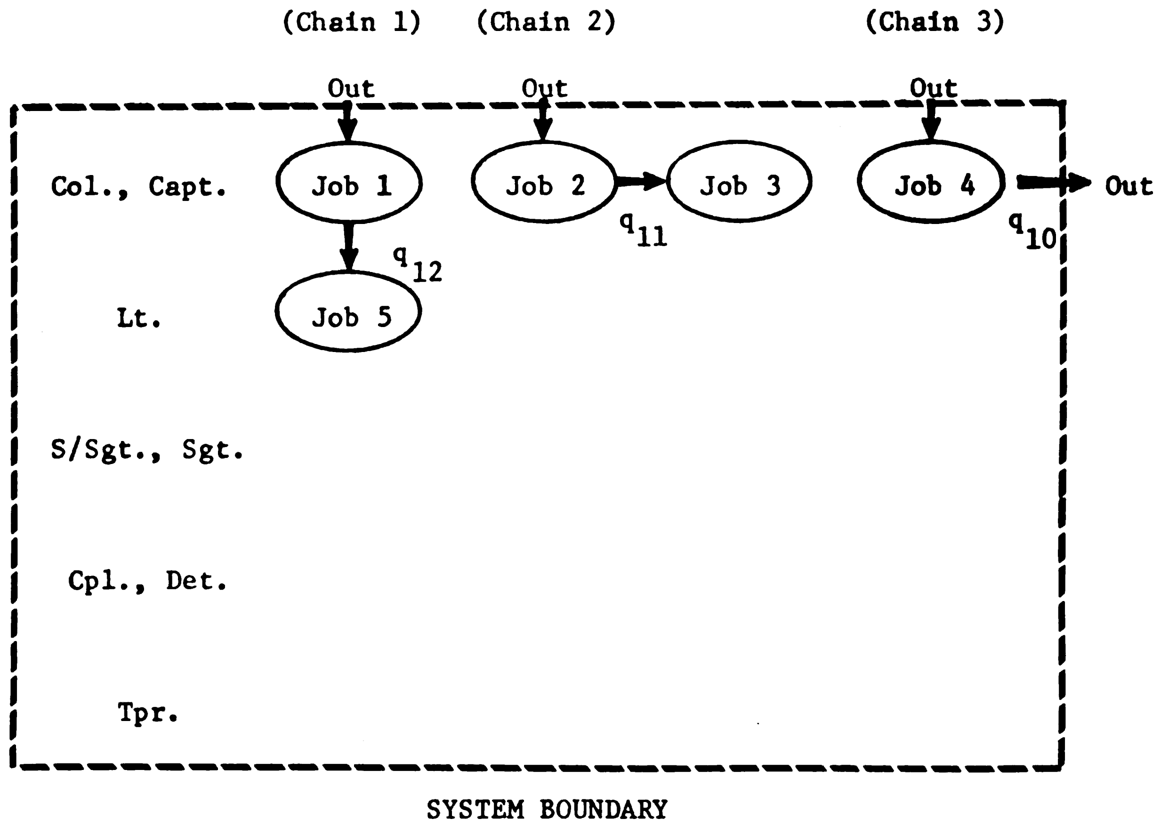

Figure 5. Example of First Moves by a Population of Three Job Vacancy Chains Entering at the Upper Stratum

The above account suggests that constant q_{ij}, while necessary, are not sufficient to produce accurate predictions. The form of the chain, particularly if it involves a sequential movement within the stratum and then to the outside, also has important implications for the model's adequacy. Since the above example was only one of five possible chains of entrance and one of the more controlled ones, it is now much easier to accept White's statement concerning the difficulty in specifying the degree of relation between individual job vacancy moves and job vacancy chains as entities. However, it is no longer the viability of the test which seems important but rather the substantive limitations or scope conditions of the theory. The question now is whether the model has limitations for handling certain chain configurations. The difficulty of explicating this problem should by now be obvious. However, prior to beginning thorough probes as to the scope conditions of the theory, we must first test to see to what extent the theory is adequate at all. It is to provide tests of the generally adequacy of the theory that the present study will concern itself.

A second aggregate matrix was also created using similar q_{ij} as the criteria for aggregation. The years for which data were included in this combined transition matrix are 1961-1966. Actually, the more rigorous criteria initially combined 1961-1964. Once again, however, insufficient numbers of entering job vacancies occurred in the two upper strata. They were 12 for the Col., Capt. stratum and 10 for the Lt. stratum. I, therefore, extended the matrices allowing for somewhat less rigor in the choice of "years" to provide more chains in these two strata. The 1961-1966 combined data transition matrix has the following characteristics: The distribution of q_{i0} is (14/26)<.15; (15/26)<.20; (19/26)<.25; (21/26)<.30; (25/26)<.34; (26/26)<.36; for the q_{ij} there were 20 cells such that their respective rows had less than 5 total moves; of the remaining 130 cells the distribution of the q_{ij} is (41/130)>.15; (31/130)>.20; (20/130)>.25; (14/130)>.30; and 7 cells ranged between .35 and .47 in magnitude of difference from <u>any</u> cell of the proper row and column. These are by no means ideal "yearly" combinations and one might question the earlier statement about stable dynamics since these particular years are sequential. The question seems valid enough. Thus, what criteria have I used? First, the statement was made in a comparative context examining data over an extended period of time. The contextual relative stability was, therefore, one factor. Moreover, even a few irregular cells will generally be inconsistent from the remaining ones thus causing a rather high number of differences. Finally, the numbers which are used as the base for estimations are very small in many cases which undoubtedly affects these fluctuations considerably. However, since the system's entire population of enlisted positions were considered, there exists no possibility of testing the stability notion further if we base our logic on q_{ij} stability alone. Two comments now seem appropriate. First, systems much larger than the present one being studied should be examined if we expect a rigorous test in terms of "yearly", sequential data. Secondly, it does not seem appropriate to limit the discussion of stability within the q_{ij} frames. Rather, historically separable periods as well as more general periods seem appropriate criteria for examining the notion of stability in sequential data.

The information reporting the above discussed q_{ij} based aggregated transition matrix and predicted-observed job vacancy chain lengths is provided in Tables 25 and 26. The compared predictions and observations appear quite good throughout. There are but three differences greater than 12.0 per cent, the largest being for chains of length 2 within the S/Sgt., Sgt. stratum. Moreover, only for this stratum is the error greater than 15.0. A similar symptom as in the previous q_{ij} based combined predictions seems operative in this stratum. Both chains of length 1 and of length 3 are overestimated. An examination of the actual chain configurations also taking into account the method of their origination suggests a very strong reason for the difference. In 1964 there were 19 reallocations from Sgt. to S/Sgt. making the combination of within stratum outside moves extremely high. The q_{ij} also confirm this rather extreme case. This may be seen in Table 27. An additional comment which is pertinent here is that the 1966 reversal of proportions is atypical in comparison with the transition probabilities of 1967-1969. The 1967-1969 data are much like that of 1961-1965 with the one exception which has been pointed out for 1964. The 1967-1969 transition probabilities may be seen in the matrices in Appendix E. Similarly, the corresponding 1957-1960 S/Sgt., Sgt. transition probabilities also in Appendix E provide even further evidence of the stable dynamics of this particular stratum.

Table 25. Estimated[a] Transition Probabilities for and Creations of Job Vacancies for q_{ij} Based Combined Data: 1961-1966

Origin Stratum	Destination Stratum						Total Number of Moves
	Col., Capt.	Lt.	S/Sgt., Sgt.	Cpl., Det.	Tpr.	Out	(N)
Out	.0404	.0274	.1715	.2305	.5303		694
Col., Capt.	.2000	.6857	.0000	.0000	.0000	.1143	35
Lt.	.0000	.2075	.5283	.0189	.0000	.2453	53
S/Sgt., Sgt.	.0000	.0000	.4246	.4683	.0000	.1071	252
Cpl., Det.	.0000	.0000	.0026	.2737	.6113	.1125	391
Tpr.	.0000	.0000	.0000	.0016	.0486	.9498	638

[a]Estimations are made using Equation (4.9). The Col., Capt. stratum include Col., Lt.Col., Maj., and Capt.

Table 26. Predicted (P) and Observed (O) Distribution of Vacancy Chain Length by Stratum of Origin for q_{ij} Based Combined Data[a] (Percent)

	Stratum of Origin									
	Col., Capt.		Lt.		S/Sgt., Sgt.		Cpl., Det.		Tpr.	
j^b	P	O	P	O	P	O	P	O	P	O
1	11.4	10.7	24.5	21.1	10.7	1.7	11.3	5.0	95.0	93.8
2	19.1	32.1	11.0	5.3	9.8	28.6	61.2	73.8	4.6	5.2
3	11.3	.0	8.6	5.3	32.8	26.1	19.6	16.3	.3	.1
4	8.2	.0	19.5	5.3	23.1	25.2	5.6	3.8	.1	.0
5	15.0	3.6	16.4	21.1	12.5	12.6	1.6	.6	.0	.0
6	14.2	21.4	10.0	21.1	6.1	5.0	.5	.6		
7	9.7	21.4	5.3	15.8	2.8	.0	.2	.0		
8	5.6	7.1	2.6	5.3	.13	.0	.1	.0		
9	2.9	.0	1.2	.0	.6	.0	.0	.0		
10	1.4	3.6	.5	.0	.2	.8				
11	.7	.0	.2	.0	.1	.0				
12	.3	.0	.1	.0	.1	.0				
13	. .1	.0	.1	.0	.0	.0				
14	.1	.0	.0	.0						
15	.0	.0								
16										
(N)	28		19		119		160		368	

[a]The time span is from 1961 through 1966.

[b]j denotes chain length distribution.

Table 27. Estimated[a] Transition Probabilities for Job Vacancies from or within S/Sgt., Sgt. Stratum for Years 1961-1966

Year	S/Sgt., Sgt.	Cpl., Det.	Out	Total No. Moves
1961	.4118	.5882	.0000	17
1962	.3333	.6000	.0667	30
1963	.4074	.5185	.0741	27
1964	.4074	.2407	.3519	54
1965	.4154	.5385	.0462	65
1966	.5085	.4746	.0169	59

Predictions of Chain Length Distribution for Combined Data Based upon Substantive Criteria other than Similar Transition Probabilities

It was argued earlier that chronological periods such as a year which we are accustomed to thinking in terms of are not the determinants of system stability or change but rather other factors. It has also been argued that other criteria taking into account mechanisms affecting change should also be used for combining "yearly" data. I will now examine four "historical periods", the boundaries of which were determined by the use of substantive criteria other than similar "yearly" transition probabilities.

The first "period" includes the years 1950-1957. These were very active and visible years for the State Police. In 1952 there was a major riot at Jackson at which 264 State Police were on duty at one time. Others were also mobilized for potential disturbances at Ionia and Marquette. These, however, never materialized and attention was focused upon Jackson. Although the riot and consequent negotiations lasted but six days, a large force of State Troopers remained to guard the prison for a duration of four months (Biennial Report, 1951-1952).

Another major event causing a large mobilization of State Police occurred in 1953. This was the Flint-Beecher tornado which killed 115 persons, injured over 900 others and left over 300 families homeless. The response of the State Police both to the original communications of the disaster and to action thereafter was commendable in several respects. First, men on local patrol at Flint reacted rather quickly to the totally unexpected event. Secondly, the entire system's mobilization was extremely rapid. And more importantly, the organizational, communication and leadership capabilities of the State Police were demonstrated in a critical emergency (cf. Form and Nosow, 1958; AR, 1953). It is my opinion that this demonstration was crucial in the eventual inclusion of the State's Civil Defense operations as a division within the State Police rather than under the directorship of the Commissioner or of equal status with other systems in a coordinating organizational role. The distinction is important and since all promotions within the State Police are from within, its later inclusion opened additional opportunities for occupational mobility. The disaster's immediate effect, however, was as a major mobilization effort and a mechanism to heighten both public and State Police administrative personnel awareness to the increased utility of effective organization and increased planning or routinization of such events as much as possible. In addition, 1953 was a high traffic mortality year with 1,905 deaths. Further heightened activity occurred in the form of a carefully supervised policy of continuous strict traffic law enforcement (AR, 1953).

In 1955 traffic deaths would pass 2,000 for the third time in the State's history until that time. The Legislature reacted before the end of the year as it had in 1937 with a large increase of positions. Earlier in the year, 57 additional positions had been authorized. The special legislative session in November of 1955 authorized 200 more. In addition, in 1956 even more positions were authorized--168. In two years 425 new positions had been created, obviously indicative of the increased activity to follow not only in recruitment and training but also administrative planning and supervision, as well as geographical or spatial dispersion in the number of posts (AR, 1937, 1955, 1956).

All these events happened between 1950 and 1957. Even though a recession was affecting the economy and the Legislature would make no additional authorizations for the fiscal year beginning June 1, 1957, the prior authorizations were still being filled in 1957 in the State Police system. This is the justification for the extension through 1957 of this "period".

Extending the rationale somewhat further for the aggregation of data within this "period", Table 28 reports the organization's actual size from 1950 to 1958 by stratum. The numbers refer to positions which were filled at that time. Each year's size and strata have been readjusted to January 1 of the appropriate year.

Table 28. Actual Organizational Size by Stratum for 1950-1958

Year	Col., Capt.	Lt.	S/Sgt., Sgt.	Cpl., Det.	Tpr.	Total Move
1950	12	12	69	117	398	608
1951	13	15	66	129	413	636
1952	12	18	69	151	397	647
1953	14	18	72	145	426	675
1954	14	17	72	145	407	655
1955	14	17	76	146	456	709
1956	14	16	83	146	483	742
1957	17	19	92	185	704	1017

It appears from Table 28 that a general growth was occurring throughout each stratum as the years progressed. Very slight decreases are probably due to vacant positions not being filled, although turnover in the form of different combinations could also account for some of the fluctuations. The general picture, nevertheless, is of a moderate increase extending to each stratum over subsequent years.

The second period, 1958-1963, was a very different "period". Its most outstanding common dimension was the lack of heightened activity and growth. One facet of this may be seen in the legislative authorizations: 1957: 0, 1958: 0, 1959: 0, 1960: 0, 1961: 0, 1962: 0, 1963: 0. Two recessions greatly affected the policy of the expansion of state services. In the State Police, expansion was at a standstill for seven years. The third "period", 1964-1969, was again one of increased expansion. It included the reorganization according to the new State Constitution, the system's reaction to two policy changes affecting work week man hours and further geographical or spatial expansion--the first since 1957.

The fourth historical period to be included will be the years from 1949 through 1969, a twenty-one year time span. Historically, it was about 1949 that the system dynamics stabilized. The first few post-war years were ones of re-establishing "normalcy" for the system, other systems with which the State Police cooperated and public traffic patterns (cf. Biennial Reports, 1949-1950, 1951-1952; ARs, 1953-1969; Michigan Traffic Accident Facts, 1970). Particularly when compared to the previous twenty-one year period, the relative stabilization of this "period" seems evident. There were no economic fluctuations nor international wars of the magnitude of the Great Depression and World War II. Obviously, depressions or so-called recessions have occurred as in 1957-1958 and 1960-1961. Yet these events slowed expansion rather than reversed it. Similarly, the Korean and Vietnam Wars have not affected the public to the extent of World War II in which gasoline and tires were rationed and speed limits lowered to thirty-five miles an hour (at least in Michigan). The effect of these wars may have been more pervasive in other ways as evidenced by the mass demonstrations for the termination of the Vietnam War; but in terms of greatly affecting the routinization of the entire national system, their disruptions were minor. The effect of the post-1945 perturbations upon the economy, the industrial production plants, state services or "the everyday working man's" style of life has not been nearly so objectively obvious. In conjunction with the historical reasons for setting this extended time span as a unit, the initial observation about relative stablized q_{ij}'s may also be added. We, therefore, have two separate criteria pointing to the same time span.

The tables for these substantively based aggregations are presented below in Tables 29 through 33. The predictions are generally very good. The 1950-1957 predictions are under the observed lengths for chains of length 2 and thus are over the observed for other lengths. This holds for three strata: Col., Capt., Lt. and S/Sgt., Sgt. In the case of the Col., Capt. stratum where the error is 21.8, there is a similarity to the predictions of the S/Sgt., Sgt. stratum for the 1961-1966 q_{ij} based combined data. That is, several within strata outside sequential job vacancy moves were made in 1952 and 1954. Part of these movements involved reallocations within the study's so-called "stratum" which contains four vertically different ranks. The model's scope limitations are again questioned as to their capability of predicting movement involving this type of configuration. Part of the "miss" in prediction for this stratum, as well as the other two strata, is a result of reallocations across ranks involving a downward movement of one rank followed immediately by an exit move. These types of chains have no other interrelations. To the extent that other chains are not of this configuration, the predictions based on individual moves may generally be expected to be higher for the stratum in which the chain originates and lower for the stratum which the chain moved into immediately prior to its exit from the system. Part of the limitation of the White theory seems to be in its inability to distinguish chains started by differing means such as reallocation, new job or retiree. Although documentation for necessary distinctions between the latter three has not been provided, evidence for the case of reallocation does exist and

Table 29. Estimated[a] Transition Probabilities for and Creations of Job Vacancies for General System Based Combined Data

Origin Stratum	Destination Stratum: Col., Capt.	Lt.	S/Sgt., Sgt.	Cpl., Det.	Tpr.	Out	Total Number of Moves (N)
			1950-1957				
Out	.0331	.0365	.1039	.1564	.6701		876
Col., Capt.	.2162	.6216	.0000	.0000	.0000	.1622	37
Lt.	.0000	.1538	.7385	.0154	.0000	.0923	65
S/Sgt., Sgt.	.0000	.0000	.2265	.6519	.0055	.1160	181
Cpl., Det.	.0000	.0000	.0000	.2335	.6407	.1257	334
Tpr.	.0000	.0000	.0012	.0012	.0267	.9709	825
			1958-1963				
Out	.0536	.0595	.2113	.2411	.4345		336
Col., Capt.	.2800	.6000	.0000	.0000	.0000	.1200	25
Lt.	.0000	.1250	.5750	.0500	.0000	.2500	40
S/Sgt., Sgt.	.0000	.0000	.3448	.6000	.0000	.0552	145
Cpl., Det.	.0000	.0000	.0038	.3308	.5154	.1500	260
Tpr.	.0000	.0000	.0000	.0133	.0698	.9169	301

[a]Estimations are made using Equation (4.9). The Col., Capt. stratum includes Col., Lt.Col., Maj., and Capt.

Table 29 (cont'd.)

Origin Stratum	Destination Stratum						Total Number of Moves
	Col., Capt.	Lt.	S/Sgt., Sgt.	Cpl., Det.	Tpr.	Out	(N)
			1964-1969				
Out	.0279	.0205	.1293	.2447	.5777		1075
Col., Capt.	.1765	.7647	.0000	.0000	.0000	.0588	34
Lt.	.0000	.1404	.7368	.0000	.0000	.1228	57
S/Sgt., Sgt.	.0000	.0000	.4183	.4804	.0000	.1013	306
Cpl., Det.	.0000	.0000	.0000	.2884	.6425	.0691	579
Tpr.	.0000	.0000	.0000	.0019	.0630	.9352	1064
			1949-1969				
Out	.0339	.0356	.1344	.2118	.5843		2389
Col., Capt.	.2115	.6635	.0000	.0000	.0000	.1250	104
Lt.	.0056	.1404	.7022	.0169	.0000	.1348	178
S/Sgt., Sgt.	.0000	.0000	.3443	.5539	.0015	.1003	668
Cpl., Det.	.0000	.0000	.0024	.2811	6125	.1040	1231
Tpr.	.0000	.0000	.0004	.0031	.0494	.9471	2268

Table 30. Predicted (P) and Observed (O) Distribution of Vacancy Chain Length by Stratum of Origin for General System Based Combined Data[a] (Percent)

	Stratum of Origin									
	Col., Capt.		Lt.		S/Sgt., Sgt.		Cpl., Det.		Tpr.	
j[b]	P	O	P	O	P	O	P	O	P	O
1	16.2	6.9	9.2	3.1	11.6	3.3	12.6	4.4	97.1	97.6
2	9.2	31.0	10.2	40.6	11.4	35.2	65.1	71.5	2.6	1.9
3	8.3	3.4	11.0	3.1	45.1	28.6	16.9	14.6	.2	.5
4	8.6	3.4	35.2	12.5	21.2	22.0	4.0	7.3	.1	.0
5	23.8	20.7	21.1	25.0	7.4	9.9	1.0	2.2	.0	.0
6	18.3	20.7	8.8	9.4	2.3	1.1	.3	.0		
7	9.4	3.4	3.1	6.3	.7	1.1	.7	.0		
8	3.9	6.9	1.0	.0	.2	1.1	.0	.0		
9	1.5	3.4	.3	.0	.1	.0				
10	.5	.0	.1	.0	.0	.0				
11	.2	.0	.0	.0						
12	.1	.0								
13	.0	.0								
14										
15										
16										
(N)	29		32		91		137		587	

[a]The years for which entering cohorts of job vacancies were combined are 1950-1957.

[b]j denotes chain length distribution.

Table 31. Predicted (P) and Observed (O) Distribution of Vacancy Chain Length by Stratum of Origin for General System Based Combined Data[a] (Percent)

	Stratum of Origin									
	Col., Capt.		Lt.		S/Sgt., Sgt.		Cpl., Det.		Tpr.	
j[b]	P	O	P	O	P	O	P	O	P	O
1	12.0	11.1	25.0	20.0	5.5	5.6	15.0	11.1	91.7	87.7
2	18.4	27.8	7.0	5.0	10.9	16.9	52.2	61.7	6.6	7.5
3	9.4	5.6	9.8	10.0	35.1	32.4	20.7	18.5	1.2	4.1
4	8.5	.0	22.4	20.0	24.5	21.1	7.6	6.2	.4	.7
5	15.8	5.6	17.3	10.0	13.0	18.3	2.8	1.2	.1	.0
6	14.8	27.8	9.8	20.0	6.2	4.2	1.0	1.2	.1	.0
7	10.0	5.6	4.8	15.0	2.7	1.4	.4	.0	.0	.0
8	5.7	5.6	2.2	.0	1.2	.0	.2	.0		
9	2.9	5.6	1.0	.0	.5	.0	.1	.0		
10	1.4	5.6	.4	.0	.2	.0	.0	.0		
11	.6	.0	.2	.0	.1	.0				
12	.3	.0	.1	.0	.0	.0				
13	.1	.0	.0	.0						
14	.1	.0								
15	.0	.0								
16										
(N)	18		20		71		81		146	

[a]The years for which entering cohorts of job vacancies were combined are 1958-1963.

[b]j denotes chain length distribution.

Table 32. Predicted (P) and Observed (O) Distribution of Vacancy Chain Length by Stratum of Origin for General System Based Combined Data[a] (Percent)

	Stratum of Origin									
	Col., Capt.		Lt.		S/Sgt., Sgt.		Cpl., Det.		Tpr.	
j^b	P	O	P	O	P	O	P	O	P	O
1	5.9	3.3	12.3	.0	10.1	.0	6.9	1.5	93.5	92.4
2	10.4	26.7	9.2	27.3	7.6	31.7	62.1	69.6	5.9	6.3
3	8.9	.0	6.9	.0	33.0	23.7	21.7	20.9	.5	1.1
4	6.8	.0	25.3	.0	24.2	28.8	6.6	5.7	.1	.0
5	20.5	.0	21.4	27.3	13.3	8.6	1.9	1.9	.0	.0
6	20.0	26.7	12.8	13.6	6.5	6.5	.6	.0	.0	.2
7	13.3	36.7	6.6	22.7	3.0	.0	.2	.4	.0	.0
8	7.4	3.3	3.1	9.1	1.3	.0	.1	.0		
9	3.7	3.3	1.4	.0	.6	.0	.0	.0		
10	1.7	.0	.6	.0	.3	.7				
11	.8	.0	.3	.0	.1	.0				
12	.4	.0	.1	.0	.1	.0				
13	.2	.0	.1	.0	.0	.0				
14	.1	.0	.0	.0						
15	.0	.0								
16										
(N)	30		22		139		263		621	

[a]The years for which entering cohorts of job vacancies were combined are 1964-1969.

[b]j denotes chain length distribution.

Table 33. Predicted (P) and Observed (O) Distribution of Vacancy Chain Length by Stratum of Origin for General System Based Combined Data[a] (Percent)

	Stratum of Origin									
	Col., Capt.		Lt.		S/Sgt., Sgt.		Cpl., Det.		Tpr.	
j[b]	P	O	P	O	P	O	P	O	P	O
1	12.5	7.4	13.5	5.9	10.0	2.9	10.4	3.7	94.7	94.2
2	11.6	28.4	9.2	28.2	9.4	28.8	61.0	68.9	4.7	4.5
3	8.5	2.5	9.0	4.7	37.0	27.9	20.0	18.5	.4	1.1
4	7.7	1.2	27.6	12.9	23.8	23.7	6.0	6.1	.1	.1
5	20.0	8.6	20.8	20.0	11.5	11.2	1.8	1.8	.0	.0
6	18.0	23.5	11.1	14.1	5.0	4.2	.6	.4	.0	.1
7	11.2	17.3	5.2	11.8	2.0	.6	.2	.2	.0	.0
8	5.8	4.9	2.2	2.4	.8	.3	.1	.0		
9	2.7	3.7	.9	.0	.3	.0	.0	.0		
10	1.2	1.2	.4	.0	.1	.3				
11	.5	1.2	.1	.0	.0	.0				
12	.2	.0	.1	.0						
13	.1	.0	.0	.0						
14	.0	.0								
15										
16										
(N)	81		85		312		508		1396	

[a]The years for which entering cohorts of job vacancies were combined are 1949-1969.

[b]j denotes chain length distribution.

has been pointed out. The error for chains of length 2 for the other two upper strata were even higher--33.8 and 30.4. Two other errors between predicted and observed chain lengths were greater than 15.0. They occurred in the S/Sgt., Sgt. stratum (16.5) for chains of length 3 and within the Lt. stratum (22.7) for chains of length 4.

The 1958-1963 predictions are the best yet. They are off by 13 per cent or .13 for only one prediction and by .10 for another three. Otherwise, they are almost invariably well below the .10 mark. This aggregated data was not a result of q_{ij} based criteria but criteria using other observable system dynamics which included taking into account exogenous forces. The finding substantiates my earlier claim for the use of criteria other than accustomed time periods or arbitrarily standardized ones. The finding also demonstrates that the processual nature of chains and the boundary limits for stability of processes are not necessarily to be found using so-called yearly q_{ij}'s. This seems particularly significant in terms of suggesting both sequential data collection and the inclusion of overall system dynamics in future mathematical analyses. The inference which can be made from the predictions is that the dynamics are extremely stable in this somewhat "holding" period.

The 1964-1969 predictions are again off to some extent, and with two exceptions, the instances are the same as previous errors. That is, the principal errors occur in the upper three strata. The exceptions occur in the Col., Capt. and Lt. strata. Predictions for chains of length 7 are far below (.23 and .18) the observed percentages. Initial examination of the actual chain configurations does not help in this instance. This suggests the need, if the theory seems generally adequate, for more intensive work on the relationship between the mathematical conceptualization, the counting or estimation procedure and the chain configurations. We are once again back to the difficult problem of specifying the meaning of differences between predicted and observed. White does not confront this problem. He only notes its existence. However, for future modifications to the theory or actual applications of it in which engineering adjustments might need to be made, these specifications must be explicated. Moreover, the specifications need to consider the substantive meanings of the chain relations. I have already suggested one substantive problem which the model seems incapable of treating in spite of theoretically constant q_{ij}'s.

For the other inaccurate predictions in the upper three strata, the previous comments for the 1950-1957 and 1961-1966 "periods" are applicable. For instance, in both previous discussions reference was made to the 1964 reallocations. The 1964 population of chains are also in this aggregate "period", at least partially accounting for the inaccurate S/Sgt., Sgt. prediction for chains of length 2. For the other two upper strata there is, once again, no obvious reason for the inaccuracies in chains of length 2.

The predictions for the twenty-one year time span are much improved over the 1950-1957 and 1964-1969 "periods" but less accurate than the 1958-1963 "period". The predictions are extremely accurate (.10) with the exception of chains of length 2 in the upper three strata and chains of length 4 and 5 for the Lt. and Col., Capt. stratum respectively. The inaccuracies for chains of length 2 are .17, .19 and .19 for the Col., Capt., Lt. and S/Sgt., Sgt. strata respectively. For the other errors, the magnitudes were 11.4 and 14.7. Overall the predictions are very close to the observations of actual chains. With the three exceptions for chains of length 2 (which in my opinion are the result of chain configuration rather than stability of dynamics), the assertion of generally stable dynamics for the twenty-one year period is supported. Even with the variations between the 1950-1957, 1958-1963 and 1964-1969 "periods", the predictions are very close. Nevertheless, one noticeable problem with the theory lies in our incomplete understanding of its interrelating individual moves and entire chains. It is a case of a theory having been constructed which at present we do not thoroughly understand. Substantive specification where possible is greatly needed to explicate the model's limitations.[†]

A comparison of the utility of the q_{ij} based criteria as well as the more general system based criteria may be made by contrasting the predictions derived using substantive criteria with those which are arbitrarily chosen with no substantive basis. The arbitrary periods chosen were decades or combined decades. In each case a comparison is possible giving the decade predictions an added advantage of larger numbers of entering vacancies. The transition matrices and predictions are reported in Appendix F. The predictions are generally not as accurate as those aggregations derived from substantive bases. This suggests the predictive utility of such criteria. Moreover, beyond the accuracy of prediction is the possible extension of the theory by relating the underlying dynamics to other (substantive) changing processes.

[†]It has been pointed out by Professor Thomas Conner, Department of Sociology, Michigan State University, that for certain inaccurate predictions there may be no analytic or substantive specification possible.

A final comment concerning the chain length tests will be made. The predictions are still rather good using large arbitrarily aggregated data such as decades. This again, it seems to me, points to the relative high degree of stability of the underlying dynamics of the mobilization processes from 1949-1969. For increased stability one may examine those shorter time spans in which greater predictive accuracy was achieved. That is, there are shorter time spans within this twenty-one year period which have even greater stability. The 1958-1963 "period" is a case in point. For the State Police system I would expect variations within more extended time spans due to the somewhat erratic behavior of the economy, politicians and flows of traffic and their counterpart, traffic deaths.

Predictions of Amount of Movement

A different type of test of the vacancy chain theory involves predicted number of moves per stratum. Since the divisor in the equation for estimated transition probabilities refers to the total number of moves per stratum, one cannot use data from the same year for both estimation and prediction of the number of moves. The predicted and observed number of moves for such a year must be exactly equal. Hence, these predictions have been used as a check on the correctness of the Q and F_t.

The formula for total predicted moves per stratum is Equation (4.7). It is $M_t = F_t(I-Q)^{-1}$. The assumption of constant q_{ij} is once again made. In view of the fact that transition probabilities must be estimated from a preceding time period, there is an actual check, if desired, on the constancy of these parameters. This was not the case for the transition probabilities used for predicting vacancy chain length.[†] The aggregation of data is also permissible for this prediction if the prediction refers to a comparable time period. The transition matrices which will be used in this section of the discussion and have not previously been reported appear in Appendix G. They are for the "years" 1933, 1944 and 1949. Transition matrices previously reported are in Appendices E and F.

Where possible predictions of total number of moves by stratum will be made for each of the five years subsequent to the year of estimation. The predictions and observations are reported in Table 34. The years 1933 and 1944 were chosen to demonstrate the effect of violating the assumption of constant q_{ij}. Substantively, this assumption means that the underlying dynamics of the system are stable. For the years 1933 and 1944 we know this is not likely to hold true. Hence, these two sets of predictions were included purely for heuristic purposes, not to test the theory. For the years 1949-1969, I have previously argued that a large degree of stability exists. I would therefore expect accurate predictions for matrices from these years. From the data reported in Table 34, one can see that expectation is confirmed. The predictions are extremely accurate.

[†]Nevertheless, projected predictions could have been made for chain length predictions had I desired.

Table 34. Predicted (P) and Observed (O) Total Number of Moves within and from Each Stratum by Yearly Probability Transition Matrix and Year of Prediction

	Total Number of Moves within and from Each Stratum				
	Col., Capt.	Lt.	S/Sgt., Sgt.	Cpl., Det.	Tpr.
			1933 Transition Matrix		
			1934		
P	10	8	14	10	24
O	9	7	14	14	45
			1935		
P	1	3	4	29	44
O	0	2	0	27	74
			1936		
P	2	5	3	5	9
O	1	4	7	9	17
			1937		
P	4	14	26	33	117
O	0	0	34	24	123
			1938		
P	2	8	12	7	14
O	0	2	14	23	20

Table 34 (cont'd.)

	Total Number of Moves within and from Each Stratum				
	Col., Capt.	Lt.	S/Sgt., Sgt.	Cpl., Det.	Tpr.
			1944 Transition Matrix		
			1945		
P	12	0	20	99	105
O	4	0	21	57	66
			1946		
P	9	0	30	32	261
O	0	0	17	18	155
			1947		
P	17	0	39	68	114
O	5	2	30	40	79
			1948		
P	11	6	23	129	156
O	2	8	16	71	111
			1949		
P	16	11	30	74	117
O	6	16	36	58	78

Table 34 (cont'd.)

	Total Number of Moves within and from Each Stratum				
	Col., Capt.	Lt.	S/Sgt., Sgt.	Cpl., Det.	Tpr.
			1949 Transition Matrix		
			1950		
P	4	5	20	36	64
O	4	5	19	28	67
			1951		
P	4	6	8	28	63
O	1	5	16	29	70
			1952		
P	7	7	25	23	42
O	5	6	20	23	44
			1953		
P	2	5	13	17	26
O	1	7	11	21	30
			1954		
P	10	11	26	31	57
O	13	12	24	31	59

Table 34 (cont'd.)

	Total Number of Moves within and from Each Stratum				
	Col., Capt.	Lt.	S/Sgt., Sgt.	Cpl., Det.	Tpr.
			1952 Transition Matrix		
			1953		
P	1	5	9	16	27
O	1	7	11	21	30
			1954		
P	8	9	20	30	59
O	13	12	24	31	59
			1955		
P	5	4	25	40	68
O	6	8	27	48	68
			1956		
P	6	11	36	83	333
O	7	12	35	85	316
			1957		
P	0	8	27	65	179
O	0	10	29	69	171

Table 34 (cont'd.)

	Total Number of Moves within and from Each Stratum				
	Col., Capt.	Lt.	S/Sgt., Sgt.	Cpl., Det.	Tpr.
			1958 Transition Matrix		
			1959		
P	0	2	32	35	51
O	0	2	26	42	57
			1960		
P	1	5	30	53	73
O	1	6	26	57	78
			1961		
P	3	5	18	32	45
O	2	7	17	40	41
			1962		
P	5	5	35	43	53
O	7	5	30	56	44
			1963		
P	4	9	33	47	52
O	4	8	27	39	46

Table 34 (cont'd.)

	Total Number of Moves within and from Each Stratum				
	Col., Capt.	Lt.	S/Sgt., Sgt.	Cpl., Det.	Tpr.
			1967 Transition Matrix		
			1968		
P	2	3	32	74	237
O	3	3	33	87	238
			1969		
P	7	12	44	110	137
O	9	10	48	112	155
			1950-1959 Transition Matrix		
			1960-1969		
P	52	80	332	725	1222
O	50	83	406	771	1273

It may have been noted that there is some overlap between the five years predicted from transition matrices for the years 1949 and 1952. These matrices were chosen to provide an alternative check on the effect of changes in transition probabilities. The greatest difference in predictions for 1953 and 1954 occurs in the S/Sgt., Sgt. stratum. From an examination of the matrices and the number of entering vacancies it would seem to be the latter which is largely responsible for this larger difference since a greater difference in q_{ij} for years 1949 and 1952 actually occurs in the Col., Capt. stratum. The number of vacancies entering this stratum for 1953 and 1954 is 5 and 1 while the corresponding number for the S/Sgt., Sgt. stratum is 11 and 4. Thus, as is clear from the equation and the above example, not only the extent of difference between q_{ij} but also the number of vacancies entering per stratum must be taken into account. We would expect, therefore, less agreement between the predictions and observations of the more numerous strata than for the upper two in spite of greater differences between the q_{ij} in the two upper strata. It is only when the number of vacancies entering two strata are comparable that the effect of changes in transition probabilities may be thoroughly seen.

The closeness of fit between the predictions and observations of number of moves is further evidence of both the stability of the underlying dynamics of the mobility processes and of the adequacy of the theory for explaining State Police occupational mobility. The final data given in Table 34 using ten year time spans adds even more credence to the assertion above.

Summary

It is my conclusion with respect to both substantive interpretation and predictive accuracy that the vacancy chain model is generally quite adequate as an explanation of occupational mobility within the State Police system. In terms of substantive interpretation of the hierarchical promotion system we may now assess with more cogency an earlier assertion in Chapter 2 that the vacancy chain model provides a more complete representation of movement than the manflow model. It is clear from the policy of no lateral entry that vertical movement is interrelated. However, the manner in which horizontal and vertical moves are interrelated cannot be logically determined a priori. These two directional, interrelated flows are taken into account by the vacancy chain model. In fact, a vacancy chain is precisely a representation of two directional and interrelated vacancy flows. Hence, the vacancy chain model not only depicts a mechanism for individual man movement which is appropriate to the State Police, but it also provides a thorough description of manner in which man movement is interrelated in this system. This description is seen in the job vacancy chain length distributions. Neither interrelated movement nor specific differentiation of movement from nonmovement within one's stratum is accounted for by the manflow model. The vacancy chain model's adequacy for representing this type of system's occupational mobility is judged to be quite thorough.

The extent of predictive, as well as substantive, accuracy of the model is, of course, dependent upon the extent of validity of the underlying assumptions. For job vacancies the assumption of homogeneity of population seemed quite reasonable. This conclusion was based upon the lack of any significant seniority of job vacancies, the general selection of men from the entire population available, and the aggregate nature of the representation of flows. An even more readily accepted assumption pertained to the representation of job vacancy movement as a Markov process. That is, history of job vacancies prior to the existing job occupancy is not relevant. The final basic assumption regarded the stationarity of transition probabilities. Stationarity had two meanings depending upon which predictions were to be derived. In one case, stationarity referred to the stability of transition parameters within one year. In other words, the q_{ij}'s were constant for all moves in the chains belonging to the year's cohort of entering vacancies. This assumption seemed quite acceptable for all years. In the alternative case stationarity referred to stability or constancy of transition parameters both within and across years. Since there were two major system perturbations between 1927 and 1944, job vacancy movement did not approach stability across years until several years after World War II--1949. Yet from 1949 to 1970 the occupational dynamics did appear to stabilize.[†] Moreover, from 1949 to 1970 the predictive accuracy for predictions having sufficient members of entering chains per stratum was generally high. For this latter time period, especially, I think the model's predictive and substantive accuracy is quite high. Therefore, I

[†]The discrepancy between vacancy chain and demographic models for the year in which stability was ascertained to have been reached is due to the 1949 P matrix for the demographic model. Some men originally in the Lt. stratum in 1949 moved more than once during the year. However, one of the model's constraints is that only one move per man per year is allowed. Hence, there does not exist a 1949 P matrix for the demographic model. The earliest possible year for the 1949-1970 time period for the demographic model's inclusion would be 1950.

have concluded the vacancy chain theory is generally a very adequate model for explaining occupational dynamics of the State Police system.

Two kinds of tests were undertaken. The first involved predictions of chain length distribution. The second was in regard to numbers of moves from each stratum. Since estimates of transition probabilities are based upon individual moves rather than the chains as entities, data within the same time period could be used for both estimation and predictive purposes for the derivation of chain length distribution. Initial tests were attempted using yearly data from 1927 to 1970. However, the lack of sufficient numbers of entering vacancies (and hence chains) in the upper three strata suggested the inappropriateness of considering the predictions based upon so few numbers a genuine test.

The problem of an insufficient number of chains per stratum lead to a consideration of two possible solutions. The first solution considered involved the combining or "lumping" of strata. The alternative was to combine yearly data. Based upon the lack of a serious test of the five strata system and the amount of substantive information which would be lost in combining additional strata, the decision was made to combine yearly data. Alternative methods of combining the yearly data were then examined. More than one alternative for combining data was possible due to the continuous sequential nature of the data. Of special import was the idea that the "yearly" construct was actually a device for standardization purposes rather than a theoretically significant construct. The empirical basis for this idea is clear if we consider the processual nature of job vacancy movement. Hence, the evaluation of similar q_{ij}'s for the standardized "yearly" periods was not necessarily the only available criteria for combining "yearly" data.

An alternative was to consider general system dynamics, exogenous forces, internal policy decisions, et cetera. In brief, we may also combine the "yearly" data on substantively based criteria other than similar Q matrices. For comparative purposes, arbitrary time periods such as decades and combined decades were also utilized.

It is my opinion from the total evidence presented for combined "yearly" data that predictions from the vacancy chain theory were generally quite accurate. This conclusion holds within the time span 1949-1969 whether arbitrary, similar q_{ij} or general system dynamics are chosen as the criteria for aggregating "yearly" data. The accuracy seems improved for the latter two, however, suggesting the import of utilizing substantively meaningful criteria both from an accuracy of prediction and from a theory extension point of view. Related to the latter notion was the finding that while constant q_{ij}'s are necessary, they are not always sufficient for accurate predictions by the theory. Rather, the configuration of vacancy chains may itself affect the degree of accuracy. Also established was the utility of extending criteria for aggregation of data beyond the q_{ij} comparison process to include important exogenous as well as endogenous forces. The latter criteria were, in fact, slightly more effective for defining boundedness of time periods from a predictive accuracy viewpoint.

The second basic type of prediction relates to the number of moves from each stratum. I think it important to first point out the significance of the breakdown by strata rather than general system moves or two strata breakdown such as administrative-worker. By distinguishing the number of moves made from each stratum, a much more complete representation of the process is generated. Secondly, we may once again judge the accuracy of predictions. Within the time period 1949-1970, it is my judgment that the predictions are highly accurate. These final predictions, for both "yearly" and decade time spans, add even more credence to my assertion of the stability of the underlying dynamics of the mobility processes and the adequacy of the theory for explaining State Police occupational mobility.

Given the general adequacy of the theory, the utility would seem to be considerable for extension and/or modification. One general direction in need of pursuit is additional investigation of possible substantive relations between the more general system processes and the direction and length of chains. Moreover, whether the above relations are affected by stratum at which the initial job vacancy entered the system might be important. For example, we might compare the configurations of chains per stratum of entrance as an initial step. Perhaps there are substantive differences in related movement arising from different causes, for example, reallocation as opposed to new jobs. In particular, there is a need to explicate more thoroughly the cases in which the model's predictions are inaccurate given constant q_{ij}'s.

An additional extension of considerable import is the derivation of the flow assumed as given in the vacancy chain model--the number of entering job vacancies. We may recall from the discussion on the model's conceptualization of the process that there are two modes of generating job vacancies. A job vacancy is created when a man leaves the system. Also, a job vacancy enters the system when a new job is created. These are obviously two distinct processes and require two distinct models. For the first type of initial job vacancy, what is needed is a model which will derive manpower loss rate. In the following chapter on extensions of the vacancy chain and demographic models, I will make an initial step in the construction of a loss rate model. No model will, however, be built. Also, should the model of system growth which is designed to generate the given for the demographic or manflow model prove productive, it will be one step removed from what is required for the alternative mode of generating job vacancies--new jobs. What would need to be added to the model for system growth would be a delineation of growth per stratum. Exploration of the sociological literature for aid in this delineation process will be made as well as an attempt to derive the number of new jobs or system growth per stratum.

Finally, the thrust of this chapter also strongly suggests the need for gathering continuously sequential data over extended time periods. The amount of information is, indeed, somewhat staggering; but the possibilities for probes, treatment and discovery of the relations of the processes of a given year to those surrounding it, as well as to other time spans, is much more extended. What now seems called for is a detailed interrelation of mathematical-historical data for this system so that a more general theory of system dynamics may be constructed. Such a synthesizing effort will be undertaken in a study to follow-up the present one. In brief, what is needed is an explication of the system conditions under which occupational dynamics change and the more general exogenous conditions under which general system dynamics change. The extensions in the following chapter and the planned mathematical-historical syntheses are elementary steps in this direction.

CHAPTER 5

INITIAL EXTENSIONS OF THE VACANCY CHAIN AND DEMOGRAPHIC MODELS†

Three initial attempts will be made in order to extend the models analyzed in Chapters 3 and 4. The rationale for each will be given as the individual discussion is undertaken. The three extensions involve preliminary investigation to account for elements of the models which were taken either as given or derived. These were the entrants of job vacancies in the vacancy chain model and entrants of new recruits in the demographic model. The extensions will attempt to account for total system growth, growth at each stratum and factors potentially affecting loss rate of men.

Mathematical Model of System Growth

Total system growth is a significant element in both the vacancy chain and demographic models. In the demographic model, "growth" was assumed either to be zero or positive and was a term ($M(t)$) in the actual equations for expected stratum sizes. To account for changes in

†The more direct extensions of the system occupational models undertaken in this Chapter are not the only extensions undertaken in the study. Since the principal thrusts of the study pertained to formal theories at the system level and linkages to them, I have placed a descriptive and supplementary analysis of organizational structure and individual careers as an appendix [Appendix D].

system size by means of another model would greatly extend the utility of this model for projection purposes. Without such a model the projections are limited to heuristic simulations. As for the import of system growth for the vacancy chain model, a model of total system growth would be but a first step in relation to extension. That is, total system expansion is not directly conceptualized by the model. Rather, expansion by stratum is the factor taken as given in the vacancy chain model. Thus, once a model for system growth is developed, further work remains before the vacancy chain model is directly affected. Since stratum growth when combined or summed accounts for system growth, perhaps system growth could be further broken down into its component parts. It is in this indirect way that an explanation of system growth will perhaps extend the vacancy chain model since it too is quite limited for projection purposes without some means of deriving entering vacancy chains.

The current effort is uninformed by either Harrison White or David Bartholomew. White assumes that rates of job creations ($f_{job}(t)$) are constant (White, 1970:147 ff.) Understandably, he acknowledges that this assumption is too simple for long range system behavior. However, from the analysis in Chapter 3 involving "expansion" fluctuations, it is clear that such an assumption is also inaccurate even in the short run for the State Police system. Thus, White's forays into system evolution, based upon an assumption allowing mathematical tractability, are of little use for this study. Generally, Bartholomew also makes very simple assumptions to extend the demographic models. He provides further analytic work assuming that system growth is either constant or geometric. In addition to the analysis in Chapter 3, a more direct examination of system growth increments may be made from Figure 10 which follows later in this Chapter. From either source, it is obvious that the assumption of either constant or geometric growth does not approach reality for the State Police system. Thus, armchair mathematics leaves the present study uninformed.

Rather than attempt a curve fitting procedure without substantive basis, initial conceptualization concerned the derivation of system growth as a function of changes in system externals or exogenous variables. More explicitly, we may divide the process of increasing the manpower of this system into two parts: first, additional manpower must be authorized or allocated by the Legislature; second, it is necessary to recruit and train men to fill authorized positions. At this point in conceptualizing system growth, we will be dealing with the first of these two processes--allocation of additional manpower. Further specification of the distinctness between allocated and actual growth in manpower will be made below. Let us now, however, pursue the development of a model of allocated system growth.

As a first attempt to construct a conceptualization of the generation of increased demand for manpower, a regression model seemed most feasible. The model was constructed as a description of the political bargaining process between the Legislature and the State Police. In the model, allocated system growth is viewed as a linear function of four exogenous variables--density of highway traffic, traffic deaths, nontraffic crimes and the state economy. The first three are viewed as manpower demand components and the latter as a necessary condition. These four exogenous variables seem to be particularly dominant in the determination of fluctuations in demand for additional manpower. Let us examine possible avenues for conceptualizing the relations between manpower demand and each of these four forces.

First, we need to consider time intervals for both independent and dependent variables. The number of positions in the system during the fiscal year July 1, X through June 30, X+1 is allocated during the first six months of year X. It seems reasonable to assume that these allocations are based on changes in exogenous factors which occurred in the immediate past. Hence, for allocations for year t the (t-1,t-2) time period seemed the obvious one from which the Legislature could note change. Thus, if $M(t)=N(t)-N(t-1)$, the variables affecting $M(t)$ will occur in (t-1,t-2). Given this time framework, let us now consider specific relationships between the exogenous forces and manpower demand.

It is obvious that automobiles are constructed not solely on the basis of transportation and safety. Moreover, at least some humans seem likewise "constructed" to sometimes drive maximizing less than their potential for maintaining everyone's safety. Hence, with the increase in traffic flow, we might expect an increase in accidents and/or fatalities. If such an increase continued over time, we might expect a proportionate increase in demand for State Troopers. Thus, demand for increased manpower might be looked at in terms of the change in traffic flow over the prior two years. If we let $X(t)$ denote a measure of estimated traffic density or estimated vehicle miles traveled (EVMT), we might expect $M(t)=\alpha(X(t-1)-X(t-2))$ where α represents an appropriate constant. Designating that $X(t-1)-X(t-2)=\Delta X(t-1)$, we have $\bar{M}(t)=\alpha\cdot\Delta X(t-1)$.

In addition, it would seem reasonable to expect even greater demand as a result of an "unusual" or "abnormal" increase in traffic accidents and/or fatalities. Originally, it was thought that the "unusual" years might be calculated on a notion of accident miles. A ratio of accidents (if necessary, weighted by per cent of fatalities) to estimated miles driven, could be calculated. For years when such ratios are disproportionately "off-normal", an increased demand for manpower would certainly be expected. From a historical examination of State Police records and annual reports, the traffic fatality rate has been declining steadily since the early 1930's, the first reporting of such figures. There are exceptions, but in general, a steady decline is observed. It does not seem reasonable to continue the logic that additional personnel increasingly decrease the fatality rate. Rather, other factors such as expressways and actual density of traffic seem to be more significant. Where size of State Police force does seem crucial is in terms of effective decrease of numbers of deaths when certain "unacceptable" levels are reached. The Flying Squadron experiment of 1936 which demonstrated, at least to the Legislature, the effectiveness of increased manpower to lower fatalities indicates several important factors. The experiment was one of selective enforcement in which the 13 areas having the most frequent fatalities were patrolled around the clock from noon Saturday until 3:00 a.m. Monday. This concentrated patrol system was instituted on July 11 and continued until September 14. For the same areas and same comparable time periods (weekends), the pre-enforcement period had 55 accidents and 14 fatalities while the enforcement period had 37 accidents and 4 fatalities (AR 1936, 1937).

It seems likely that the following logic, expressed in the 1936 Annual Report, was also presented to the 1937 Legislature.

> There is, of course, no way of recording prevented accidents. But the fact that these patrols made some 300 arrests for violations of the traffic laws that most commonly result in collisions gives reason to believe that the toll would have been much higher than the 4 recorded had the intensified patrol not been operative.
>
> While these results were not unexpected, they do serve as documentary evidence that sufficient personnel and equipment to properly police the highways is to a considerable extent the answer to the State's very serious traffic problem. (p. 16)

The Legislature authorized an additional 100 positions, a considerable expansion in size.

Again in 1956, with traffic fatalities having gone over 2,000 in 1955 and an increasing trend of deaths continuing, the Legislature allocated 257 new positions (AR, 1956). In sum, it would seem that increases in fatalities rather than death rate per mileage traveled is the significant factor to be included. If we express the number of deaths as Y we now have

$$\bar{M}(t)=\alpha\cdot\Delta X(t-1)+\beta\cdot\Delta Y(t-1).$$

The derivation of M(t) is not yet complete since a third exogenous force, nontraffic crimes, was also suggested as an important determinant for manpower needs. If we denote by V those crimes in which State Police are involved but which are not of a traffic law violation nature and designate their occurrence in year t by V(t), then we might expect that demand for this type of function would increase proportionately with increased crime rates or $\bar{M}(t)=\rho(V(t-1)-V(t-2))$ where ρ is an appropriate constant.†

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†The changing nature and accuracy of reporting crimes is not at issue here. Rather, in spite of the changes in the nature of accuracy of reporting crimes, the reports which are filed may serve as a basis for political bargaining with the executive and the legislative state units.

One final addition is necessary before a means for deriving M(t) is complete. Changes in the state economy affect the legislative authorizations as well as the State Department of Administration's approval for filling vacated and unfilled authorized positions. (See AR, 1958, 1959 for the latter organizational interrelationship.) Thus, it seems reasonable to expect that M(t) would be dependent upon changes in state economic receipts. If we denote this economic variable by U, we have $\bar{M}(t)=\theta(U(t-1)-U(t-2))$. With all four variables combined, we have the following equation:

$$\bar{M}(t)=\alpha\cdot\Delta X(t-1)+\beta\cdot\Delta Y(t-1)+\rho\cdot\Delta V(t-1)+\theta\cdot\Delta U(t-1). \quad (5.1)$$

This linear model will be interpreted stochastically and analyzed by means of regression analysis.

Initially, population increase was considered for inclusion as an important system external. While the importance of population change is obvious, it is not population per se but rather the population's use of automobiles, the population involved in injuries and fatalities related to automobile traffic and nonhighway crimes perpetrated by persons whether upon property or other individuals that would seem important for demand of State Police manpower. In short, the specifics in which the population was involved go beyond a superficial conceptualization relating population and manpower. There would seem to be no effect from population except through indirect variables. Moreover, one could hardly expect legislative authorization for additional State Police manpower merely because additional people lived in the State. The exogenous forces utilized not only are more specific in terms of the Michigan population's mode of relating to system manpower but also include those populations from outside Michigan who are involved in activities pertinent to State Police operations. To summarize, the process which was conceptualized was one of allocation by the Legislature, the population's (Michigan or otherwise) involvement in activities pertinent to State Police operations and the state of the State economy. Hence, population as a potential variable was considered and those specific aspects where population was pertinent were conceptualized. Since specific relevant activities in which population was involved were utilized, population per se was omitted.

Before presenting the data, a qualification concerning the meaning of growth in Equation (5.1) should perhaps be made. From the argument it is not real but allocated growth which is the principal referent. While it might be expected that there is a virtual one-to-one relation between allocated size and actual size, this is not the case. There seems to be two major reasons for this. First, the organization has only <u>crude indicators</u> of losses from the system. Even these are limited to retirement rather than all modes of leaving. For instance, there are aggregate data on potential and mandatory retirement per year. The system does not have a model to predict estimated losses from retirement. As far as I could ascertain, neither are there systematized procedures for expected loss rate from other sources. In short, the organization does not seem to have a very accurate image of its expected losses and cannot anticipate the necessary recruitment and training schedules to replace those men leaving the system.

A second factor affecting the system's ability to either replace or fill new positions rather quickly is the nature of entry to the system. To be more explicit, since an initial training period is required, individual men cannot enter the system at different points in time. Rather, training schools are conducted for all new recruits; and, therefore, new entrants are taken in cohorts. This obviously affects the immediacy with which vacant positions can be replaced since a cohort of vacant positions rather than one is required.

While the above system characteristics affect the relation of allocations to actual numbers of men in the system, an even more difficult methodological problem exists should the relationship between allocated and actual size have been one-to-one. The legislative allocations are made for the fiscal year from July 1 through June 30. All other data are for the calendar year. While these problems could probably be dealth with adequately, it is unnecessary to do so for our purposes since we are currently dealing with the mechanisms operative on the demand side of manpower resources. We are, therefore, one step removed from deriving actual system growth, which from the above description of system characteristics would seem to be a difficult task even should the model for allocated growth be adequate.

Let us return to the regression model as expressed in Equation (5.1). In brief, the model states that allocated system "growth" is a stochastic function of the changes in the four exogenous factors over a two year period. In mathematical notation we may write allocated system growth as

$$\bar{M}(t)=\bar{M}(Z(t)) \quad \text{where } Z(t)=[\Delta X(t-1),\Delta Y(t-1),\Delta U(t-1),\Delta V(t-1)].$$

Although a regression model does not take into account time, the manner in which the variables are related in the equation makes them time dependent. The data are from 1946 through 1969, a 24 year time span. Allocated system growth for earlier years was not available since there was no way in which to distinguish the enlisted segment of the

State Police from its civilian segment. The correlation matrix for the five variables is supplied in Table 35.

Table 35. Simple Correlations for Allocated System Growth and Four Exogenous Variables

		Variable				
	Variable	G	X	Y	V	U
G:	System Growth	1.0				
X:	EVMT	.084	1.0			
Y:	No. Deaths	- .081	.731	1.0		
V:	Criminal Arrests	- .352	.140	- .102	1.0	
U:	State Economy	.007	.382	.234	.237	1.0

As may be easily seen, the correlation of the exogenous variables are generally much higher between themselves than that between each one and allocated system growth (ASG). The highest and perhaps most interesting correlations are between the EVMT and the number of traffic deaths. The correlation, .731, suggests that if we were to write an equation making traffic fatalities a linear function of EVMT, the amount of variance which the EVMT could explain would be .534, a considerable amount. Another interesting correlation is between Δ allocated system size and Δ number of deaths. This correlation is not only essentially zero but not positive. There also exists a relatively strong correlation between EVMT and state economics. Economic expansion is positively correlated (.382) with greater EVMT. No doubt the economy acts somewhat as a catalyst.

Nevertheless, the purpose of this aspect of the study was to generate an empirical procedure for expected ASG. The correlations which are important for this purpose are in column one of the correlation matrix. They are extremely low with the exception of Δ criminal arrests. Moreover, should any of the variables be suspect, it is this one for it does not take into account that new men generate more arrests. To conceptualize this aspect would necessarily mean that if there existed an average number of criminal arrests per system member, then only those arrests beyond such a number for each new man are indicative of increased crime. Since there is no way of knowing all the crimes which occur, to attempt to find such an average is substantively problematical. It would also be contrary to the logic of reasoning used for introducing exogenous variables. That is, when the system presents its case to the Legislature, the crime figures are not presented according to available manpower. Thus, the present formula expression still adequately represents the operative mechanisms for generating additional manpower.

As might be expected with such low simple correlations between the "independent" and "dependent" variables, the multiple correlation coefficient is not extremely large--.3834. Therefore, the amount of variance explained by the four exogenous variables (R^2) is but .1470. The partial correlation coefficients are also quite low. They are as follows:

Δ Estimated Vehicle Mileage Traveled:	0.1373
Δ Number of Deaths:	0.1194
Δ Number of Criminal Arrests:	0.3161
Δ State Economic Receipts:	0.1070

Perhaps a more thorough view of the ability of the exogenous variables to account for ASG may be seen from Figures 6, 7, 8 and 9. Two years

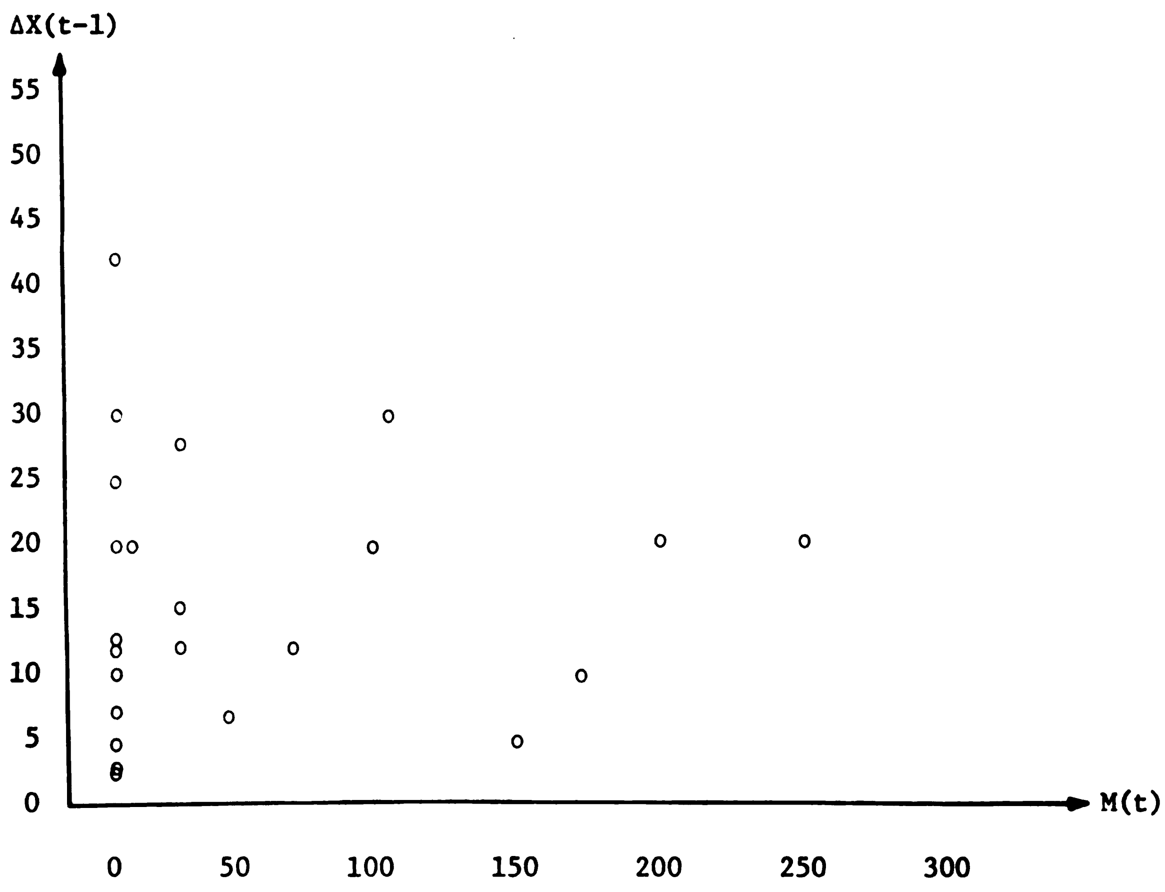

Figure 6. Relationship of Change in EVMT (ΔX(t-1)) to ASG (M(t))

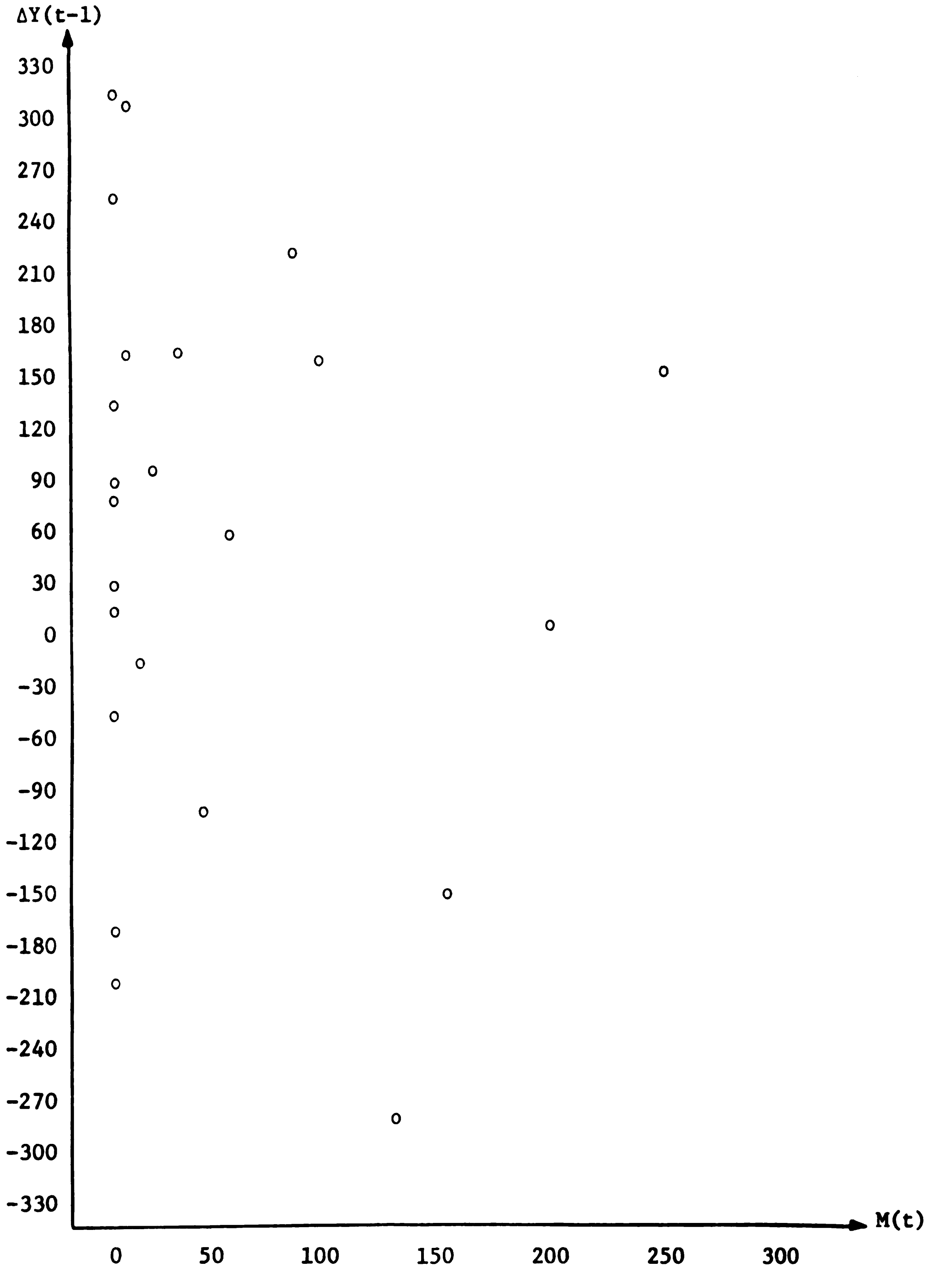

Figure 7. Relationship of Change in Number of Deaths (ΔY(t-1)) to ASG (M(t))

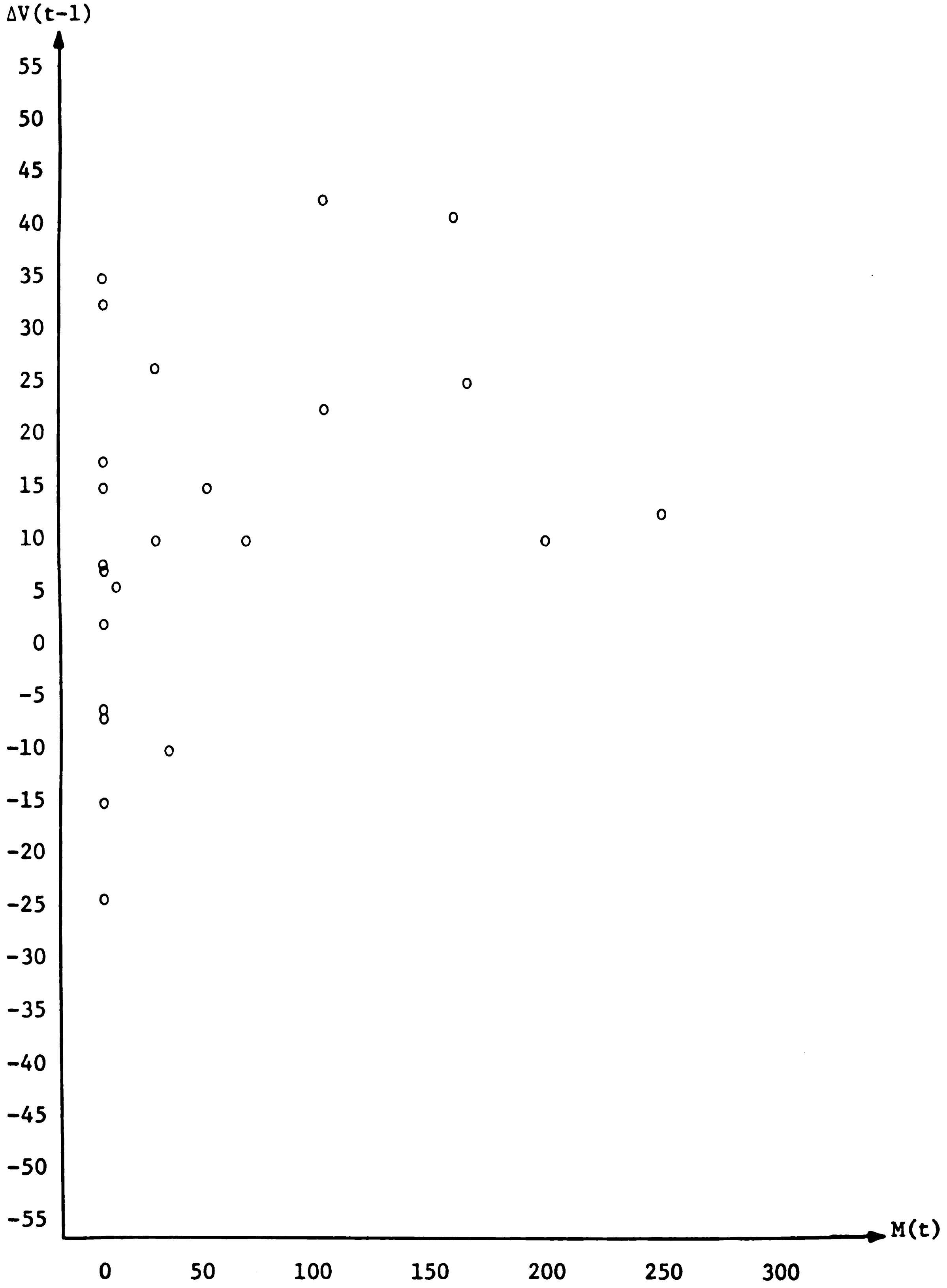

Figure 8. Relationship of Change in Criminal Arrests (ΔV(t-1)) to ASG (M(t))

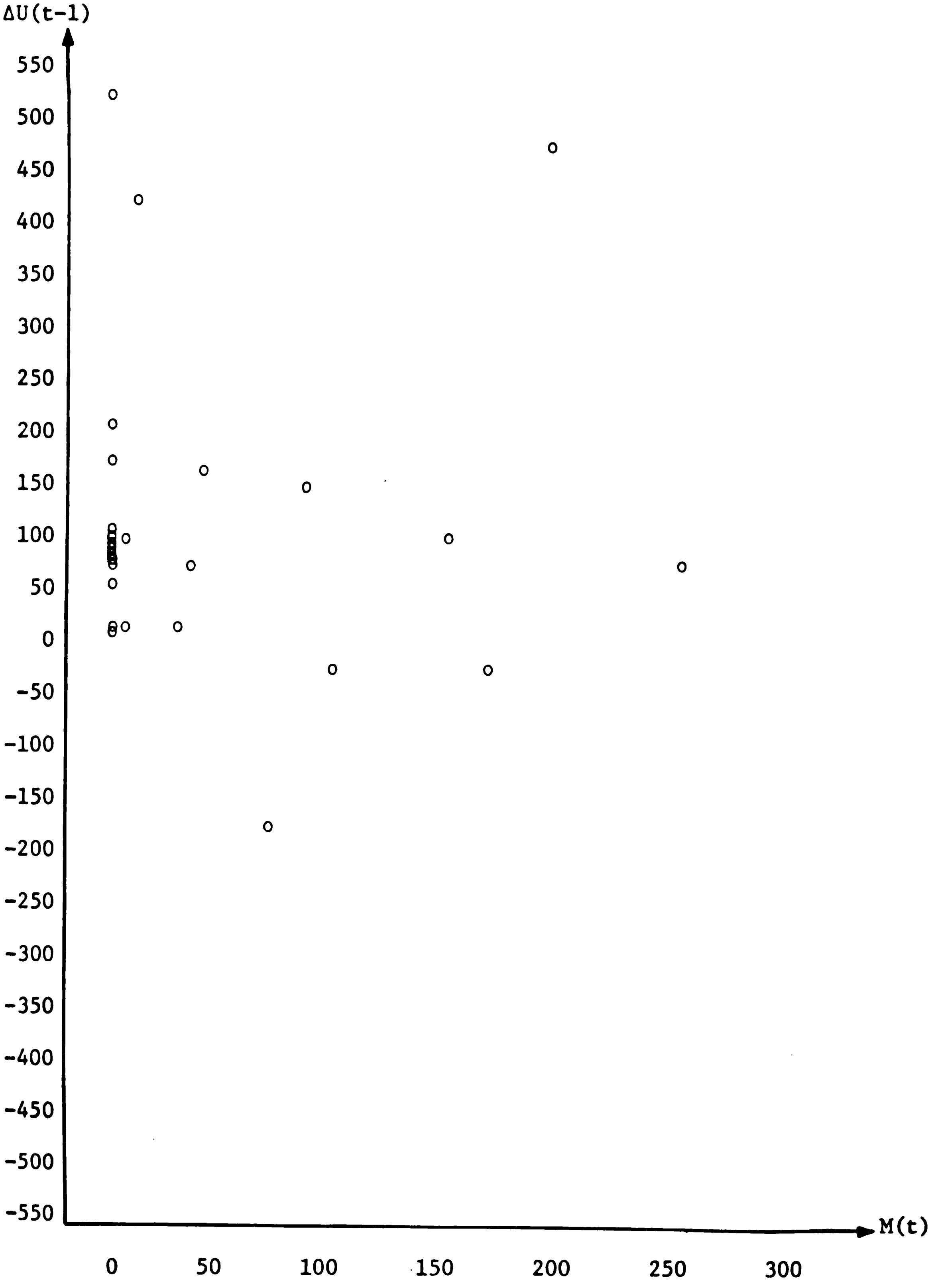

Figure 9. Relationship of Change in State Economy (ΔU(t-1)) to ASG (M(t))

have been altered slightly. In 1951 and 1968 there were -1 and -4 system allocations. They were treated as though zero. Also 1949 was omitted since the allocations were -79 and adding further complexity to the Figure was thought to be too confusing.

From the correlations (simple, partial and multiple), the R^2 and the Figures, it seems rather obvious that ASG is <u>not</u> a linear function of these four exogenous variables. At least for the present purposes--to predict system growth to a sufficient degree of precision to link with another model--the linear formulation is most inadequate. Moreover, no improvement is gained by attempting to artifically adjust variables such as Δ number of deaths and Δ state economy.† In sum, the model fails in the task for which I initially set for it to handle.

Does the model's inadequacy refute the logic used to construct the regression model? The answer is yes to that part which suggested the relationship to be linear. However, the lack of the linear expression to account for a large amount of variance of ASG <u>does not</u> imply that these variables are not the more important determinants of ASG. It merely demonstrates lack of a particular form of relating the five variables--the relationship is not linear. In fact, let us consider a graphic representation of the variables which have the <u>least</u> linear relation to manpower allocations. The three exogenous variables are traffic density, traffic deaths and state economy. Although state economy is not portrayed in the graph, its effects will be obvious.

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†An attempt was made setting the Δ state economy at zero for years in which there was deficit spending, setting Δ number of deaths at zero if there existed a decline in deaths and also adding 100 additional deaths should the actual number of deaths for year t-1 be greater than or equal to 1,900. Annual Reports suggested that whenever the number of deaths approached 2,000 much pressure was generated for additional manpower.

Prior to comparing the curves in Figure 10, perhaps we should attempt to clarify the meaning of the variables in terms of the hypothesized time dimension. For illustrative purposes, let us consider only ASG or M(t) and EVMT or X(t). Recall that in relating these two variables we hypothesized the following:

$$\begin{aligned} M(t) &\equiv N(t)-N(t-1) \\ &= \alpha \cdot \Delta X(t-1) \\ &= \alpha X(t-1)-\alpha X(t-2). \end{aligned}$$

Hence, for graphic representation, the same time (t) will represent for example 1960 EVMT and 1961 allocated system size. The same time relationship also holds between number of traffic deaths and allocated system size. In brief, while an exact Δ allocated system size, Δ EVMT and Δ number of traffic deaths may be seen in the graph, a rather quick image of the relationship may be obtained by comparing the slope of the curve between any two points in time.

Let us examine the relationship between allocated system size (S) and number of traffic deaths (Y). A certain state of the economy was a necessary condition for this relationship to be operative. There were four recessions between 1946 and 1961. These occurred in 1948-1949, 1953-1954, 1957-1958 and 1960-1961 (Vatter, 1963). Moreover, in all but the first of these there were clear-cut effects. There were no allocations of additional manpower. In fact, no allocations were made in 1953-1954 and 1957-1964. Thus, in the Figure, the time intervals (7, 8) and (11-18) are inapplicable due to the overpowering effect of the state economy. Eight of the 23 intervals are therefore accounted for. Of the remaining 15 intervals, in 3 of them allocations were rather obviously based upon the occurrence of a lag period in which zero

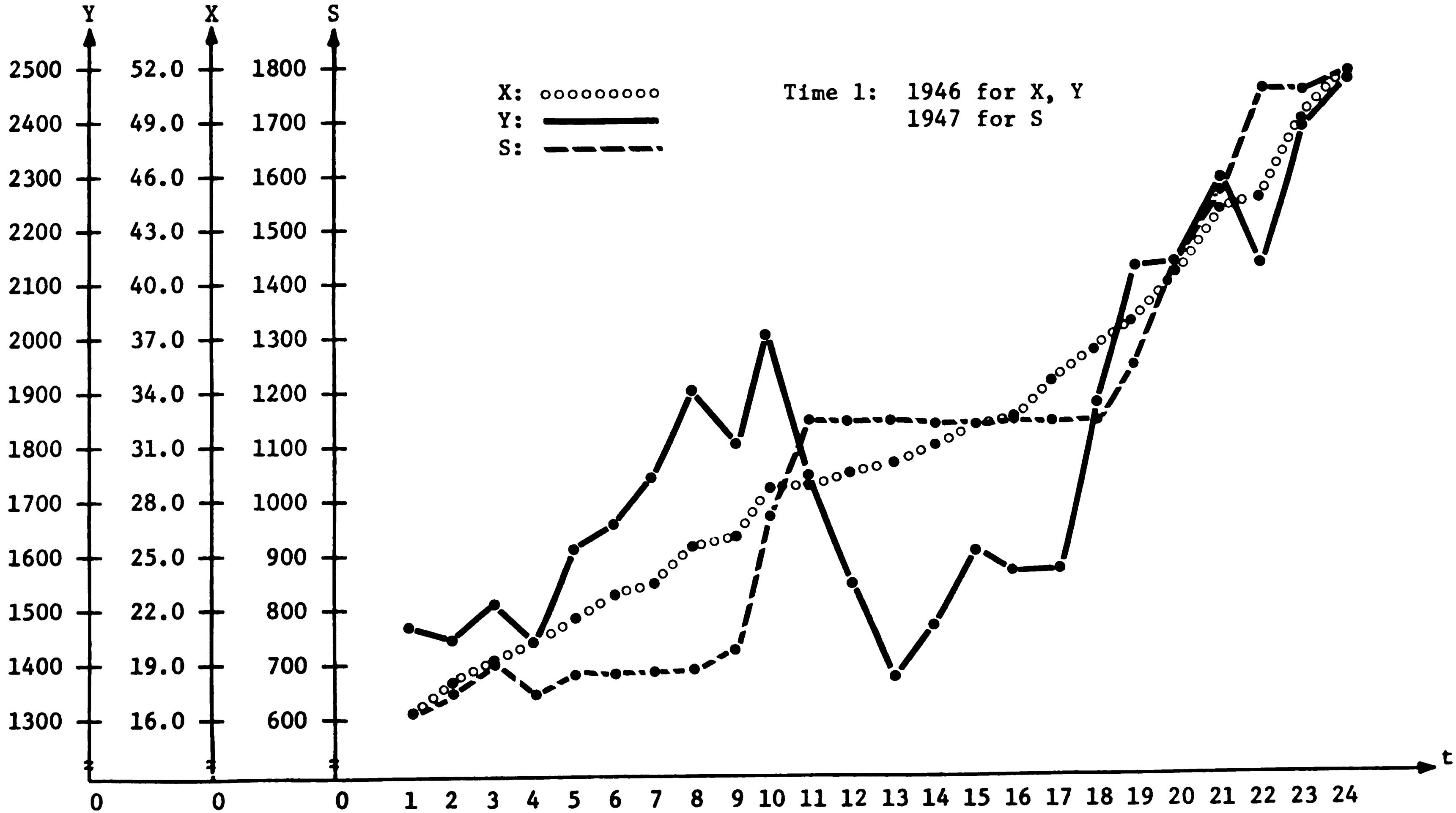

Figure 10. The Relationship of Allocated System Size (S), EVMT (X) and Number of Traffic Deaths (Y) to Time (t)

allocations were made in spite of "demands" in terms of EVMT and deaths continuously increasing. Thus, in the intervals after the recessions, a lag in allocations was being overcome despite short term decreases in deaths. Appropriate intervals suggesting post-recession or lag allocations while decreases in deaths were also occurring would seem to be (8, 9), (10, 11) and (21, 22). The lag effect between allocated system size and EVMT is clear from the graph. Although EVMT is continuously increasing, allocations fluctuate between zero (constant size) and large increases. At each point (1947, 1956) where possible simultaneous increases could occur, an economic recession caused a lag in manpower allocations which thus far at least was subsequently overcome after the recession(s). Subtracting the above three intervals of post-recession allocations, we have 12 remaining intervals in which the condition of the state economy should permit the hypothesized relationship between legislative manpower allocations and traffic deaths to operate, if it in fact is operative. In 8 of the 12 intervals, the relationship is as expected. That is, the slope of the lines are in the same direction indicating in 7 of 8 cases that if deaths increased in year X, then additional manpower was allocated in year X+1. The intervals are (2, 3), (3, 4), (4, 5), (9, 10), (18, 19), (19, 20), (20, 21) and (23, 24). Examining the same 12 intervals for the hypothesized relationship between legislative manpower allocations and EVMT, there are once again 8 of 12 intervals supporting the hypothesis. These intervals are (1, 2), (2, 3), (4, 5), (9, 10), (18, 19), (19, 20), (20, 21) and (23, 24). Of course, the high degree of relationship between traffic density and traffic deaths (correlation coefficient of .731) is a large factor making the relation between EVMT and manpower allocations operative.

It seems clear to me that there are determinative relations between traffic density, traffic deaths, state economy and manpower allocations. Three primary considerations remain. First, have principal exogenous variables been omitted? Perhaps; however, it would seem to me that either they are historical such as state emergencies including civil unrest, prison riots, tornadoes and other natural disasters or they are other major state services suggesting the necessity of a general state wide system allocation model. The first problem is not amenable to modeling. The second certainly is but is far beyond the scope of this study. A second consideration relates to whether the relationships discovered can be adequately expressed mathematically. The primary problem seems to involve economic factors. It would seem that while economic recessions bring additional manpower allocations to a standstill, it is not true that economic expansion is necessarily accompanied by accelerated ASG. The relationship involves a potentially variable lag time and seems to be a most difficult one to explicate.

A final consideration involving the relations between exogenous forces and manpower allocations is whether or not the logic utilized concerning for example deaths and allocations can be demonstrated. The hypothesis is the following--If the Legislature allocates additional manpower, deaths decrease. Alternately, if no additional manpower is allocated, the increase in deaths goes unabated.† For these relations, Figure 11 is applicable. There is one basic difference between

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†The relationships are obviously directionally specific, since manpower has not been seriously cut in the system's history with the exception of the Great Depression and World War II, at which times the "demands" of a traffic nature were also decreased.

Figures 10 and 11. This difference involves the relationship between allocated system size and time. In the bargaining hypotheses, allocated system size differed in actual time from EVMT or number of traffic deaths. Thus, for example, at time 10 EVMT referred to 1955 while allocated system size referred to 1956. The difference in time for the variables was due to the dependence of ASG at time X upon EVMT at time X-1. The task is now different. To see if it is true,that increased allocations reduce deaths, we must examine allocated system size and number of traffic deaths for the same year. We are, therefore, assuming that the system fills the positions allocated. This relationship, as well as EVMT, is shown in Figure 11.

The hypothesized relationships are amply demonstrated. In the interval (3, 4) or 1948-1949, there is an increase in manpower and a comparable decrease in traffic deaths. In time 5 or 1950, the converse (less men, more deaths) occurred. From 1950 through 1953 there is essentially no additional manpower and deaths are continuously rising. Exceptions to the relationship occur in 1954 and 1955. In 1954, deaths decrease in spite of no change in manpower. Also in 1955, although there are 50 additional men, deaths continue to rise. However, perhaps the most dramatic illustrations of manpower-traffic death relations are shown in the time interval (10, 13) or 1956 through 1958. In the first two years there was a steep increase in the number of men and a sharp decline in the number of traffic deaths. Although there are no additional men in the third year (1958), there is a reduction in deaths. Whereas this violates the hypothesized relation, it seems quite reasonable in terms of a lag effect for additional manpower, especially in view of the rate of increase in the prior 3 years. From 1959 through

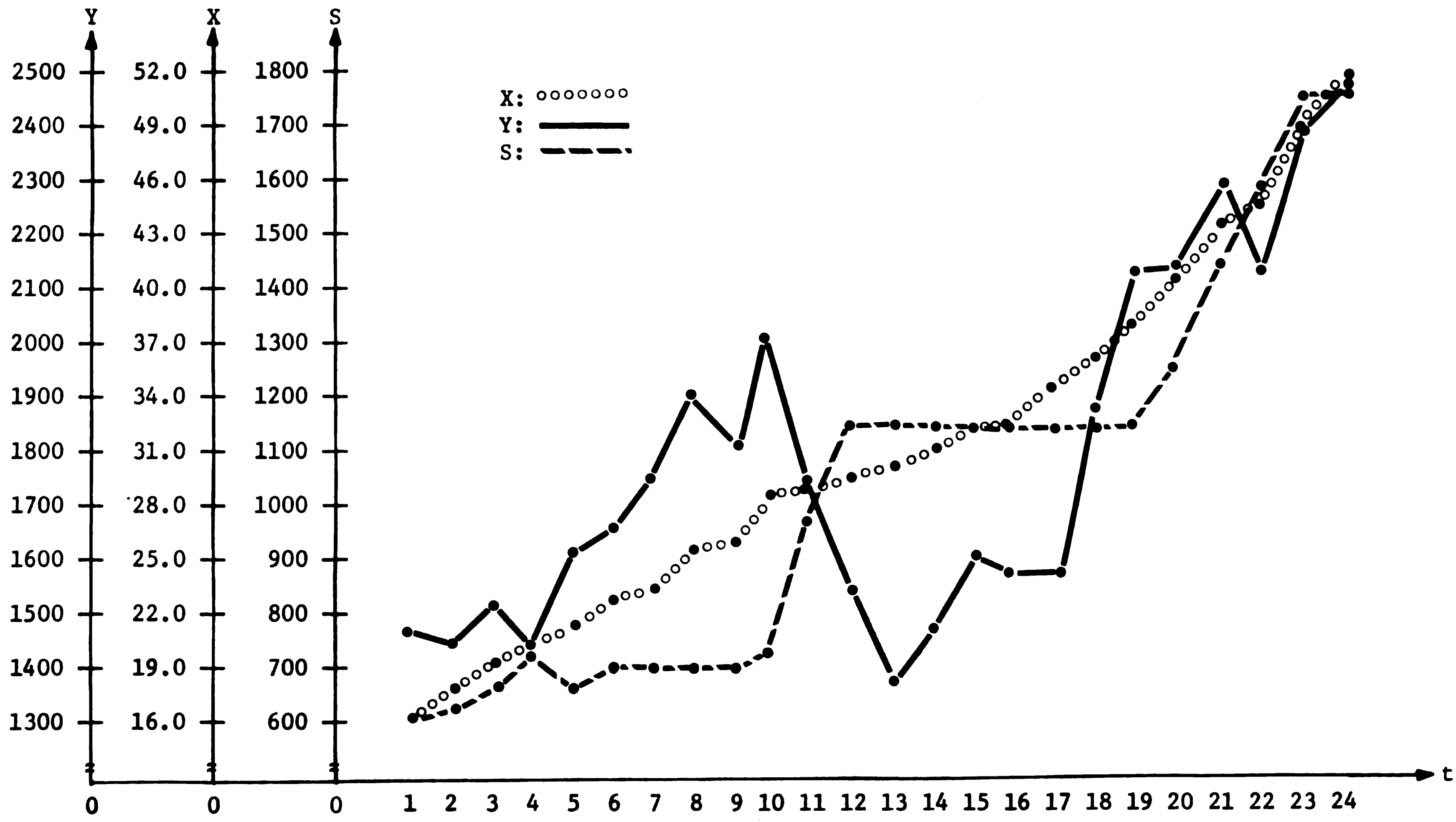

Figure 11. The Relationship of Assumed Realized Allocated System Size (S), EVMT (X) and Number of Traffic Deaths (Y) to Time (t)

1964 (13, 19) there were no additional men added, and deaths increased in all but one year--1961 (16). Thus, 1961 is once again an exception. From 1965 through 1968 (19-23), deaths decrease but once while additional manpower increases continuously. There are, therefore, three more exceptions. In 1969 (24) no additional manpower is achieved and deaths continue to increase, as expected. In 14 of 23 time intervals, the relationship is factually supportive of the logic that additional manpower reduces deaths and lack of additional manpower allows deaths to continue to increase. In 9 of the 23 intervals, the relationship was not upheld. Two of these involve the intervals (8, 9) and (15, 16), recession years 1953 and 1961 in which the rate of increased traffic density also decreased. Moreover, from inspection of the 1955-1958 (10-13) and 1964-1969 (19-24) time spans, additional factors possibly affecting the inability of additional manpower to continuously reduce deaths in the latter period is that it was preceded by a longer period of no increased manpower and the difference between traffic density (EVMT) and hence manpower was not sufficiently overcome in the 1964-1969 period. For example, the manpower curve crosses the EVMT curve in the interval (11, 12) or prior to 1957 in the first time span while it crosses the EVMT curve in (21, 22) or prior to 1967 in the latter period. In any event, the evidence from the most recent period is inconclusive. Overall, however, the evidence offers factual evidence that the logic was quite reasonable. The 1955-1958 time span is particularly convincing when one realizes that one of the variables in the bargaining process is human lives.

In sum, the regression model is hardly adequate for predicting ASG. Nevertheless, corroborative evidence for a nonlinear relationship supports my opinion that the exogenous variables are largely determinant of ASG. Yet, the extreme volatile nature of politics and the economy make explicit mathematical expression of these relationships a difficult task for which no immediate solution is apparent.

Method to Account for Arrival of New Jobs by Stratum

To extend the vacancy chain model such that the number of new jobs per stratum is a derived rather than observed input would increase the utility of the model considerably. The model could then be utilized for projection purposes.

As stated in the introduction to the previous section, White assumes that rates of job creations ($f_{job}(t)$) are constant. Such an assumption is invalid for the State Police system. As demonstrated in the previous section, political and economic change greatly affects this system's expansion. Neither political nor economic change has a constant rate. Rather, they are subject to extreme fluctuations. It appears from Figure 10 of the previous section that these variables could be held as largely responsible for the sporadic behavior of system size fluctuating between rather extreme accelerated growth and none at all. As previously noted, the recessions of the 1957-1963 time span held system growth at a standstill for seven years. Moreover, the political solution to the situation where traffic deaths approached or surpassed the 2,000 figure was one of immediately allocating large increases in manpower to be followed shortly by either an extreme decrease in allocations or no new allocations at all. This type of reaction politics was operative rather than a mechanism which allocated increased manpower at a continuous if not constant rate.

Recall that two distinct processes account for the creations of job vacancies--men leaving the system and new positions being created. In Chapter 4, F_t denoted a row vector of the number of jobs arriving per stratum and $F_i(t)$ represented an element in that array. If we denote, as does White, the arrival of new jobs per stratum by $F_{i,\ job}(t)$ and the creation of job vacancies by men leaving by stratum as $F_{i,man}(t)$, then we have

$$F_i(t)=F_{i,job}(t)+F_{i,man}(t) \quad \text{where } F_{i,man}(t)=n_{i,k+1}(t). \tag{5.2}$$

To account for $F_{i,man}(t)$ is the task of a manpower loss rate model, the initial beginnings for which will be initiated in the section to follow. The present task is to attempt to develop a method for generating $F_{i,job}(t)$.

In the discussion of a growth model, system evolution was held to be a function of exogenous forces. The same argument will, in part, be made here. An additional element which must be specified, however, is the growth rate by stratum. Thus, while certain components of the system may be responsive almost entirely to changes exogenous to the system, the administrative sections of the department must obviously be both responsive and directive of the remaining manpower. This suggests that a reasonable approach would be to relate stratum evolution to overall system evolution and, in particular, to focus upon internal structural changes related to system growth.

Several major organizational changes have been observed. Whereas, in the forties the majority of local posts were two stratum units, they now all have three strata. In addition, at many of the local posts an additional specialized function (Det.) has been added. This extension is now pervasive at all levels--local posts, district headquarters and state headquarters. It is not only a form of specialization but also of vertical differentiation. That is, while the Tpr. function has now been specialized to include separate Tpr. and Det. roles, the Det. function does not have an equivalent rank as that of Tpr. Rather, it is equivalent in rank with the first line supervisor, the Cpl., thus making this type of differentiation also a form of job evolution. The impact of the increased number of these specialized roles is that many Tprs. may now move upward in two directions--Cpl. or Det. Moreover, the volume of this increased specialization should affect the probability for upward movement from the bottom stratum.

A third major organizational change is that additional local posts have been included. These posts have exactly the same structure as other local posts, and as such, there is simply a proportional addition to each stratum. The one factor of significance is that local post structure is cumulatively larger for the entire organization. Offsetting the influence of additional posts is the inclusion of additional specialized functions such as civil defense, investigative services and enlarged communication networks cooperative with local city and county police units. In addition, there has been various restructuring of the state headquarters to accommodate these additional specialized functions as well as the increasing role of detective work and overall system growth.

An important factor for our purposes concerning structural change and system evolution is the proportional size of <u>each</u> stratum relative to the total system size. Let us examine the organizational literature for suggestions as to the changes in strata size which we might expect. The organizational research on the question of stratum size for differentiation greater than two strata is practically void. There are a few particular studies using two strata--administrative, nonadministrative--which may offer some suggestions. One such study by F. W. Terrien and D. L. Mills (1955), examines school districts in California. They find that the larger the size of the organizational unit (school district), the greater the proportion of the administrative component. Unfortunately, for our purposes, the authors specify no more concretely than "the larger the. . ., the greater the. . . ." In short, the form of the relation is left almost totally unspecified. Is it linear, curvilinear or of what form? Secondly, both components or "strata" include nonprofessional personnel. The present study does not utilize both components. Neither do we know the proportion of nonprofessionals in the Terrien-Mills study. A factor complicating the Terrien-Mills finding is that another study found the opposite relation. The Anderson-Warkov (1961) study focused upon Veterans Administration hospitals. They found that the larger the organization (hospital), the smaller the relative size of the administrative component. They further specified that the relation was not linear but more on the order of an exponential, i.e., the administrative component decreases at a decelerating rate. As in the case with the former study, only two "strata" were used and nonprofessionals were included.

Two interpretations of these incongruous findings have been suggested. Anderson and Warkov (1961) suggest three propositions: The relative size of the administrative component (1) decreases as the number of persons performing identical tasks in the same location increases, (2) increases as the number of places at which work is performed increases and (3) increases as the number of tasks performed at the same location increases (i.e., specialization of roles and/or differentiation through the addition of functions). They further suggest that since the Terrien-Mills study focused on organizational units with more than one location whereas their own concentrated on a unit in a single location, Proposition 2 is supported by the Terrien-Mills study and Propositions 1 and 3 by their own.

A second interpretation is given by Stanley H. Udy, Jr. (1965). He suggests that external social pressure which is more prevalent in school systems than hospitals would increase the import of administrative salience and hence administrative size. Udy virtually discounts the import of size and suggests that technological complexity might lead to the Anderson-Warkov finding. It is my contention that while there may be some credence in Udy's suggestion, it is largely an attempt to bring the findings within his own organizational framework. This assertion is based on Udy's easy dismissal of the impact of organizational size and handy pursual of the divergent findings within his framework.† The Anderson-Warkov suggestion seems particularly more plausible for our organizational case since neither social pressure nor technological complexity would seem to have the effect Udy purports. Certainly the internal government of the State Police is responsive to public pressure. However, it is not of the form of "packing in organizational supporters". Also, technological complexity would seem to extend specialization and increase the supervisory ratio of the State Police by providing more specialized crime detection laboratories and a shorter span of immediate control within them. Peter Blau's findings (1968)

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†The one area where Udy's notion of technology might find support is in the State Police's extensive communication network.

support the notion that greater qualifications of the personnel increase the ratio of supervisors.

Perhaps by pointing out Peter Blau's (1970) more comprehensive, theoretical framework at this point, some guidance on the above findings will be found. Blau suggests that expanding size reduces the administrative component because of an economy of scale in supervision and raises it indirectly because of the differentiation in large organizations. Put another way, the proposition asserts that the proportion of managerial and staff component decreases at a decelerating rate as organizational sizes increase.

Let us now relate Blau's proposition to the Anderson-Warkov interpretation and the discussion above. First, Anderson and Warkov imply that an organization which is spatially differentiated needs a higher administrative component for coordination purposes. The State Police is certainly spatially differentiated; however, it also utilizes one of the most comprehensive communication networks in the United States, alleviating much of the coordination problem. Secondly, they suggest that the administrative component increases as specialization and differentiation increase. This is the opposite of Blau's proposition. Perhaps a more congruous proposition for the Anderson and Warkov study would be the following: The relative size of the administrative component increases as the number of tasks requiring high expertise at the same location increases. This is not from Blau's theory; but it is consistent with a finding of Blau (1968) which we referred to earlier--that is, greater qualifications of personnel increase the ratio of supervision. This idea would be applicable to the Investigative Services Section of our organization. However, this

Section's role is a relatively small one and its effect would not be very large when combined with the other structural changes mentioned earlier. One final proposition remains. It is consistent with and contained within Blau's proposition. In short, the organizational literature points to the expectation that all strata but the lowest one will decrease in relative size at a decelerating rate as system size increases. Nevertheless, in its present form, this proposition is still too inexact for specifying an expected number of new jobs per stratum, i.e., $\bar{F}_{i,job}(t)$.

One remaining approach would be to utilize the proportion of jobs in each stratum for the preceding two years to derive a mean proportion of jobs per stratum. Let us denote this estimate according to stratum by ε_i. Thus, multiplying ε_i by the expected growth, $\bar{M}(Z(t))$, generates the expected number of new jobs per stratum:

$$\bar{F}_{i,job}(t)=\varepsilon_i\bar{M}(Z(t)). \tag{5.3}$$

An alternative approach would be to estimate $F_{i,job}(t)$ from the average number of new jobs arriving per stratum for the preceding two years. The rationale for this approach is that the internal dynamics producing the preceding change would remain relatively stable and thus their effect in the immediately preceding years would serve as an indicator for the next year. In this case, we would have a rather direct approach for generating $\bar{F}_{i,job}(t)$ which would not utilize $\bar{M}(Z(t))$. It would be

$$\bar{F}_{i,job}(t)=[F_{i,job}(t-1)+F_{i,job}(t-2)]/2. \tag{5.4}$$

Since the attempt to construct a growth model was not successful, there exists no $\bar{M}(Z(t))$. The use of Equation (5.3) must wait upon further analytic work on system growth. We are, therefore, left with

Equation (5.4), a less "satisfying" formula. The argument for this equation is not a strong one since only the average change over the previous two years is considered. Fluctuations in important exogenous variables are omitted. Similarly omitted are internal perturbations of short term duration. For such events, Equation (5.4) is most likely to be inaccurate. Hence, even should the equation generate accurate predictions, additional theoretical work is necessary to explain these dynamics. In short, even were this effort to be successful, it should be considered but a temporary solution until more substantively informing hypotheses are formulated to explain the processes.

The number of new job vacancies predicted from Equation (5.4) and the number of new job vacancies observed are reported in Table 36. Included in the count of newly created jobs were those Tpr. jobs of a different nature, such as specialized juvenile work or administrative work. Also included were all reallocations irrespective of stratum. While strictly speaking a reallocation refers to job evolution rather than a new job, it has been treated in the table as though it were, in fact, a new job. In many cases it is purely an arbitrary decision as to whether or not the job is treated as reallocated or whether a new position is created and an old position abolished. (See Appendix B). One further methodological aspect needs to be explicated--whenever the derived $F_{i,job}(t)$ had an X.5 value, it was rounded off to X+1; X=1, 2, ...

Table 36. Number of Predicted (P) and Observed (O) Job Vacancies Resulting from the Creation of New Jobs by Stratum and Year

	Stratum									
	Col., Capt.		Lt.		S/Sgt., Sgt.		Cpl., Det.		Tpr.	
Year	P	O	P	O	P	O	P	O	P	O
1939	0	0	5	0	8	8	5	2	43	15
1940	0	0	1	0	8	6	5	1	10	101
1941	0	0	0	0	7	14	2	1	60	41
1942	0	0	0	0	10	11	1	12	71	10
1943	0	0	0	0	13	1	7	0	26	0
1944	0	1	0	1	6	1	6	3	5	7
1945	1	0	1	0	1	7	2	36	4	7
1946	1	0	1	0	4	3	20	5	7	104
1947	0	0	0	0	5	12	21	19	61	12
1948	0	2	0	2	8	5	12	51	58	19
1949	1	1	1	1	9	5	35	24	16	19
1950	2	1	2	1	5	3	38	15	19	13
1951	1	0	1	0	4	2	20	9	16	2
1952	1	3	1	3	3	6	12	3	8	21
1953	2	0	2	0	4	1	6	2	12	0
1954	2	4	2	4	4	6	3	5	11	26
1955	2	0	2	0	4	7	4	11	13	29
1956	2	4	2	4	7	14	8	40	33	222
1957	2	0	2	0	11	11	26	29	126	100
1958	2	2	2	2	13	0	40	7	161	0
1959	1	0	1	0	6	7	18	6	50	4
1960	1	0	1	0	4	4	7	16	2	2
1961	0	0	0	0	6	3	11	10	3	1
1962	0	2	0	2	4	6	13	6	2	0
1963	1	0	1	0	5	4	8	7	1	1
1964	1	0	1	0	5	2	7	3	1	94
1965	0	1	0	4	3	5	5	13	48	16
1966	1	6	2	1	4	9	8	68	55	83
1967	4	0	3	7	7	11	41	47	50	36
1968	3	0	4	0	10	9	58	23	60	135
1969	0	2	4	1	10	10	35	46	86	22

Predictions and observations for the upper two strata are very close. In the Col., Capt. stratum it is not until 1966 that an error greater than 2 arises. For the Lt. stratum an error of 5 existed in 1939 and it was not until 1965 that the error again surpassed 2. Of the entire 31 years there were errors greater than 2 in only 3 and 5 years for the Col., Capt. and Lt. strata respectively. The size of the strata, the lack of large expansion in terms of numbers and the already rather developed administrative hierarchy (by 1939) contribute to this closeness of fit.

In the S/Sgt., Sgt. stratum there is much more error than in the upper two strata, but the inaccuracies are not terribly high nor the number of years too great to discontinue using the method. In 1941, 1943-1945, 1947, 1956, 1958 and 1966 the errors were of a magnitude of 5 or more. In 1939, 1940, 1942, 1948-1949, 1957, 1959 and 1967-1969, there were sufficiently large numbers of entering vacancies to allow "large" error; yet, in general, the error was less than or equal to 1. If we include both sets of years just cited, in 8 or 18 cases the error was less than or equal to 2. Of the total 31 years the following breakdowns occurred with respect to discrepancy between predicted (P) and observed (O) values--$15/31 \leq 2$; $21/31 \leq 3$ and $25/31 \leq 5$. The largest error was 13 and the largest possible error was 14.

In the Cpl., Det. stratum the agreement between P and O values is hardly so close in terms of either distance between the values or of the number of years for which "large" errors exist. For 13 of or almost one half of the 31 years the error was greater than 10. In 7 of these 13 years the error was greater than 20 and in 6 it was greater than 30. For a stratum ranging in size from 15 to 355, this error seems excessive both in terms of magnitude and frequency. In general, the method expressed by Equation (5.4) cannot account for the unusual range of fluctuations for job vacancy creations in this system. For instance, from 1944 through 1952, the changes were as follows: +33, -31, -14, +32, -27, -9, -6, -6. Again, from 1953 through 1958, the differences between years were +3, +6, +29, -11, -22. Finally, from 1964 through 1969, the fluctuations were +10, +55, -21, -24 and +23. During these 3 time spans covering 21 years, in 11 of the years the error was greater than 10. A great amount of fluctuation is apparent and the rather simple method of Equation (5.4) is incapable of handling such changes.

In the Tpr. stratum, as might be expected from the discussion in the previous section concerning overall system growth, the errors are again quite excessive. These are far too excessive in the two lower strata to link with the vacancy chain model for projection purposes. Perhaps no simple method will be capable of accurately predicting these occurrences. In fact, it seems unlikely that extremely close fits will be obtained until much more is known about the system, its relations with other systems and with exogenous forces of significance to the system's principal functions. More detailed empirical information as well as more thorough theoretical development seems necessary to expect more accurate predictions. As was the case with the growth model, the present hypothesized method is inadequate.

Analytic Beginnings of A Loss Rate Model

The following discussion includes the first step in developing a model to describe the loss rate of men as a function of the age and seniority structure. The reason for coupling age and seniority together is that the State Police system has internal policies which set limits on both the recruitment and retirement processes. The import of endogenous control for conceptualization of the outflow process may be seen by stating three policies of the State Police: (1) allowing age of entrance to vary across 10 years (age 21 through 30), (2) permitting retirement before the fifty-sixth birthday and (3) mandatory retirement at age 56. A man entering at age 21 may retire at age 46 or if he chooses, 10 years later shortly before age 56. On the other hand, had the entrant's age been 30, then the time at which he might retire is also approximately the time he must retire.

There is no question as to the importance of an entrant's age in terms of when he may and must retire. Thus, I feel that it will probably be necessary to couple age and seniority into one variable in order to adequately conceptualize the effect of these policies upon the leaving process. Some preliminary investigation as to how to couple age and seniority is currently underway. In fact, several tentative efforts have been made at conceptualizing the entire process. The main problem with these efforts to date has been the overlooking of the potential effect of age during the first years of service. That is, although we know that age and seniority are important for retirement, we do not know if age itself is important for persons leaving before 25 years of service. Hence, perhaps this problem should be investigated first in order not to build in an error focusing too heavily upon leaving after 25 years in the organization.[†]

[†]This fact was pointed out to me by Professor Martin Fox, Department of Statistics and Probability, Michigan State University.

The first problem for investigation is, therefore, the development of a method for testing the notion which I inadvertently assumed true when initially thinking of the leaving process. It must be determined if age affects the leaving rate prior to possible retirement. First, let us think of the leaving rate as a stochastic process. The hypothesis we want to test is that age is not a factor affecting the population of men leaving prior to 25 years of service. One qualifying remark needs mentioning here. It would appear that the first 12 years of service are an adequate sample for investigating this hypothesis. First, from preliminary observations the largest attrition rate occurs during the initial training period. The next highest attrition period prior to 25 years of service would seem to be the first 2 years after the recruit school has ended. For some time a policy requiring a probationary period of 2 years was in effect. That trial period has now been lowered to 1 year. However, for this probationary period and perhaps longer, certain processes of adjustment seem necessary and a higher attrition rate than persons already "adjusted" to the new job would be expected. Finally, while initially it appeared that most Tprs. who move up to the next stratum either move before or around the tenth year, more thorough examination of the timing of Cpl. moves suggested a 12 year end point as more appropriate. Our hypothesis may then be stated as follows--the probability for leaving during the first 12 years of service is not dependent upon age of entry; or

$$H_o: p_{21,j} = p_{22,j} = \cdots = p_{30,j} = p_j, \quad (j=1, \ldots, 12).$$

The hypothesis will be tested by simply comparing the proportions of men of different (21, . . ., 30) ages who leave the system with the same yearly amount of seniority. Initially a modified Chi square test was proposed with the distribution to be tested being the expected frequency of men leaving according to age of entry. Since this frequency depends upon an unknown parameter, p_j, the statistic to be used was

$$\chi^2 = \sum_{i=21}^{30} \sum_{j=1}^{12} \frac{(n_{ij}-n_i\hat{p}_j)^2}{n_i\hat{p}_j}, \quad \text{where } \hat{p}_j = \sum_{i=21}^{30} \frac{n_{ij}}{n}.^{\dagger}$$

By using the "thirteenth year plus" category for any individual not leaving, there is residual information in the sample with which to test goodness of fit after the parameters are estimated.

As will be seen shortly even using all training school graduates from 1930 through 1959, there are far too few numbers of men for this type of yearly loss rate for virtually all of the leaving cells. While the total number of men approximates 1,200, the residual 12+ category maintains over 50 per cent of the graduates after 12 years. In one instance the proportion is less than .50--it is .49. The test suggested is hardly appropriate since to collapse many of the yearly seniority categories would lose valuable information. Moreover, it is yearly losses which we must derive in order to link the results with the demographic model. Since the time span may be greater than one year for the vacancy chain model, a combined set of years might be applicable for seniority loss rates. However, the latter type of application seems problematic in that the time span might easily overlap some two stages of seniority. The yearly rate of $F_{i,man}$ seems to present the fewest problems. As a result a more simple test of the hypothesis will be carried out. The data are reported in Table 37.

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†See Lindgren (1968) for a discussion of this test.

Table 37. **Proportion** of Men Leaving the System by Number of Years in System (Seniority) and Age of Entrance

Age	Seniority (Years) of Men Leaving System (Proportion)														Total
	0	1	2	3	4	5	6	7	8	9	10	11	12	12+	
21	.10	.04	.01	.02	.03	.03	.02	.06	.03	.03	.02	.0	.0	.59	175
22	.09	.05	.03	.05	.03	.02	.01	.02	.02	.03	.01	.01	.01	.57	234
23	.08	.05	.04	.01	.04	.01	.02	.01	.03	.01	.02	.0	.0	.64	231
24	.06	.06	.07	.03	.04	.03	.02	.01	.04	.03	.03	.01	.02	.49	174
25	.07	.02	.06	.04	.02	.04	.04	.02	.02	.04	.02	.01	.01	.53	124
26	.07	.02	.01	.01	.05	.0	.02	.01	.03	.03	.02	.0	.04	.65	100
27	.03	.03	.03	.03	.02	.10	.03	.03	.06	.07	.0	.03	.0	.50	60
28	.10	.10	.05	.05	.0	.0	.0	.03	.03	.0	.03	.0	.0	.59	39
29	.08	.06	.0	.03	.03	.03	.0	.03	.0	.0	.03	.0	.03	.59	34
30	.13	.07	.0	.0	.07	.03	.0	.0	.0	.03	.0	.0	.0	.57	28

The data show a remarkable degree of constancy across ages suggesting age is not a very significant variable for loss rate when organizational policy does not directly affect it. The greatest magnitude of difference occurs, as expected, in the residual category. Yet even for this category the largest difference is .16. For 7 of 10 years the difference is less than .10 and for 5 of these years the difference is less than 3. In the 130 loss rate cells the difference is always less than or equal to .10. In general, it is less than .05. For construction of a loss rate model, the assumption of no age effect until the possible mandatory retirement level seems very reasonable.

A further characteristic of manpower losses may be seen by comparing the trends of such losses across decades. Such comparisons may be made from Figure 12 in which frequency of manpower losses and seniority in the system are graphed. The 1950 decade comparison is valid since no new recruits entered the system in 1958 and 1959 thus allowing 12 years for leaving.

For all three curves the shape is quite similar. We would expect the frequency of losses to generally be greater the more recent the decade purely as a result of enlarged cohorts of entrants. Therefore, it is only the shape of the curves for which we are concerned. In each decade, there is some erratic behavior--in other words, fluctuations both upward and downward. Nevertheless, in general there is a very rapid decrease during the first 2 years (for 0 and 1 of the graph) followed by a decreasing decrease as seniority rises. These findings are as might be expected in that adjustments to new jobs occur initially after which the effects of seniority to remain within the job system are increasingly operative. After 12 years in the system the number of men

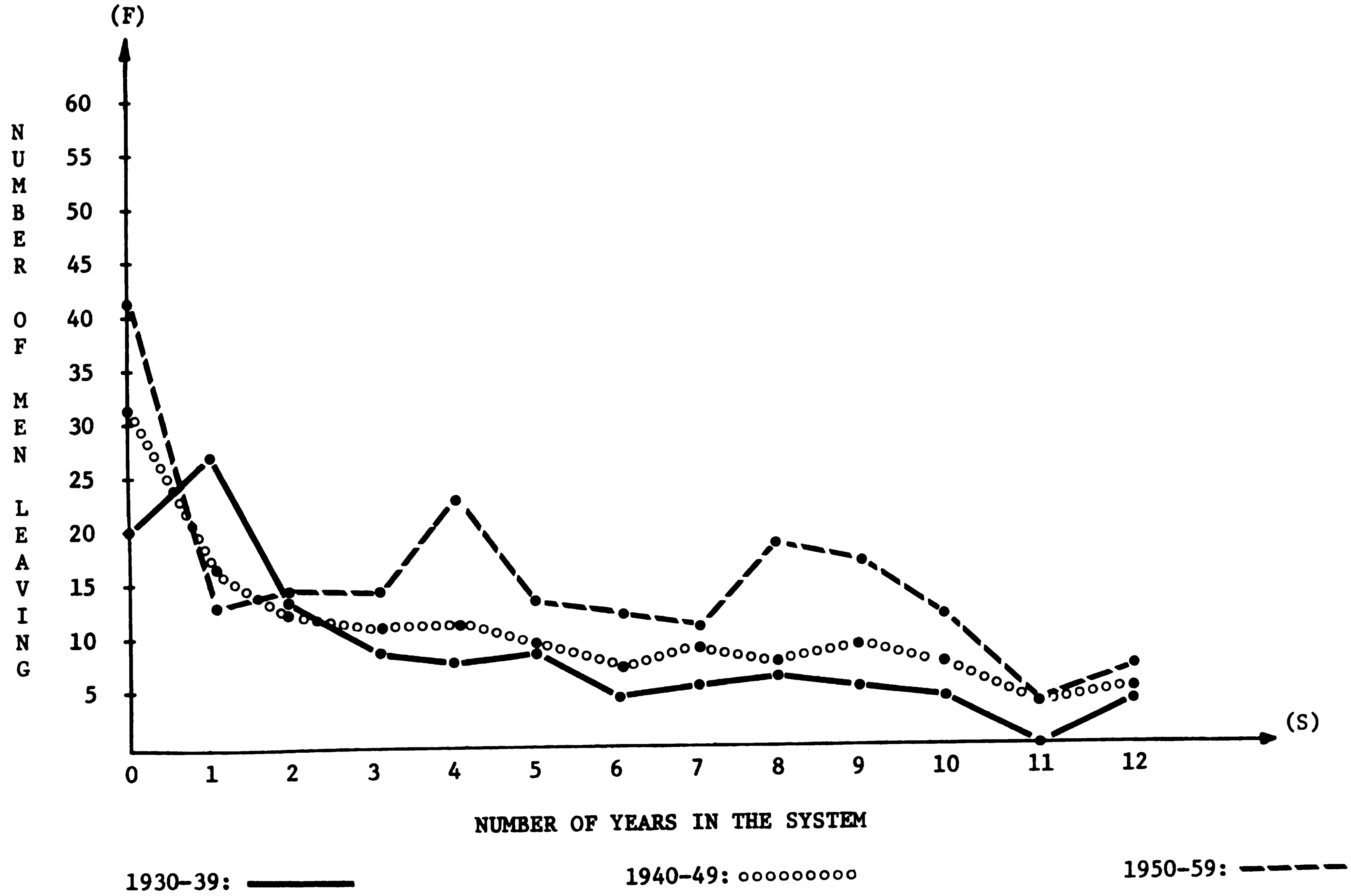

Figure 12. Frequency of Manpower Losses (F) by Seniority (S) in the System and Decade

leaving is extremely low. This trend is expected to continue until the possible retirement date (a seniority of 25 years). Thus, the first impressions of the form of loss rate are supported. In addition, it does not seem necessary to incorporate the age factor in the model's conceptualization until the 25 year or more seniority level. At this point (25 years of service), system policy is operative to move men of certain ages.

Summary

Three attempts to extend the vacancy chain and demographic models were made. The three extensions involved conceptualizations or preliminary conceptualizations to account for elements of the two occupational mobility models which were assumed as given, i.e., observed or derived. The first extension concerned the derivation of ASG by means of system externals. In the other two extensions, I was primarily interested in more specific internal system dynamics and system policies.

In the first effort, the extension relates to the demographic model. A major limitation of the model for projection is the fact that system size is assumed as either observed or derived. Since there was no existing model in which system size was the derived term, we were left to observations, hardly a satisfactory solution if projections are desired. Hence, there existed a need for a model generating system size. The term to be derived was, therefore, M(t). For purposes of constructing the system growth model, I assumed that all men leaving the system would be replaced. Therefore, only entering men who expanded system size were taken into account. Moreover, system growth may be divided into two parts--allocation of additional manpower and entrance into the system of additional manpower. I felt it was the former process which was primarily at issue in the derivation of system growth. Hence, I chose to conceptualize the process of authorization of additional manpower.

Four exogenous variables or system externals were seen to be dominant in the determination of fluctuations in the demand for additional manpower and hence in their authorization. The four exogenous variables were density of highway traffic, traffic accidents and/or fatalities, nontraffic crimes and the state economy. In the model the allocations of system growth were viewed as a linear function of the four exogenous variables. Moreover, the linear model was interpreted stochastically and analyzed by means of regression analysis.

From the correlations (simple, partial and multiple) and the R^2, it is my conclusion that the allocation of system growth is not a linear function of the four exogenous variables. More to the point, the aim of the model was to accurately predict system growth allocations such that they might be linked with another model. For instance, were we to assume total efficiency from allocations to actual added manpower, this linkage would be to the demographic model. In any case, the linear formulation is most inadequate. While additional analysis indicated that the hypothesized system externals were important determinants of allocations of additional manpower, no solution for the explicit mathematical expression of these relationships was apparent. The volatile nature of politics and the economy make the task quite difficult. One possible approach might be to break the time period into several shorter intervals based upon substantive criteria similar to that utilized for the combined "yearly" data in the analysis of the vacancy chain model.

In this manner perhaps theoretical and formal conceptual developments could occur somewhat simultaneously.

The second and third extensions also relate to projections involving an occupational mobility model. In these cases, however, the model was that of job vacancy chains. To extend the model such that the number of entering job vacancies per stratum is a derived rather than an observed input would increase the utility of the model considerably. More specifically, such a derivation would generate the inflow; thus, we would have accounted for all three types of flows within the vacancy chain model. We may once again recall from the discussion on the models' conceptualization that for the job vacancy model there are two modes of entrance to the system. A job vacancy enters the system of jobs when a new job is created. Also, a job vacancy is created when a man leaves the system. Obviously, the two modes of entry involve distinct processes and hence necessitate distinct conceptualizations. The second extension effort attempted to account for new job creations, and the third extension related to manpower loss rate.

In an attempt to account for yearly arrival of new jobs by stratum, sociological knowledge was examined for use and/or insight. Harrison White's assumption of a constant rate of new job creations was hardly valid. Moreover, organizational literature pertaining to this issue was generally limited to two strata breakdown (administrative, nonadministrative). It was also insufficiently precise to be of great use for our purposes. Hence, two methods were constructed, each of which had little theoretical or substantive basis. In the first case, the method hypothesized was based upon a further extension of the regression model of system growth. However, since the growth model was found to be inadequate, its extended usage would also have been inadequate. A second method of even less satisfactory substantive basis was utilized. Expected new jobs were based upon the average of new jobs arriving per stratum for the preceding two years. The rationale was that the internal and external dynamics producing the preceding change would remain relatively stable and thus their effect in the immediately preceding years would serve as a basis for future predictions. The argument admittedly is weak in basis since we already are aware of the volatile nature of fluctuations in system externals.

The findings pertaining to the above method were that the errors for the two lower strata were too great to link these derivations with the vacancy chain model for projection purposes. In short, the hypothesized method is not only theoretically weak but also inadequate in predictive accuracy. It would seem that much more detailed theoretical work will be necessary before accurate predictions can be expected. In particular, more detail and precision will be required both for general internal system dynamics and for the nature of dependence of these dynamics upon change in specific system externals.

As for the third extension effort involving the alternative mode of a job vacancy's entrance to the system, the major concern was upon preliminary steps for the construction of a loss rate model. Of special importance to the processes of men leaving the system are certain system policies relating to age of entrance, seniority and retirement, and mandatory age of retirement. However, before attempting to conceptualize the more general processes of loss rate, I decided to first investigate the effect of age of entrance upon loss rate during the earlier years in the system. A sample of years, the first twelve, was analyzed. In my opinion, age was not an important factor operative upon loss rate during these "earlier" years. Hence, we may now proceed as initially assumed by eliminating from consideration age of entrance as an important determinant of loss rate prior to 25 years of service. It would appear that seniority per se is the most relevant force operative until 25 years of service, at which point both seniority and age of entrance are important. These ideas now appear sufficiently firm to continue to use them as a basis for the more laborious task of building a loss rate model for the system.

CHAPTER 6

CONCLUSION

This study was longitudinal extending over 45 years (1927-1971). Its focus was system occupational mobility. The system of occupations was the Michigan State Police. Since jobs entered by persons leaving the State Police organization were not considered, the system of jobs or labor market under consideration was organizational or formally bounded. Moreover, the examination of the mobility processes was primarily through the use of mathematical models--the central ones being simple Markov chain models. While the central analysis focused upon the mobility processes, the sequential nature of the longitudinal data enabled analysis of the relationships of other system dynamics to such mobility processes. That is to say, I could examine the "big picture" and relate factors which themselves directly affected the mobility processes such as state economics, organizational growth and differentiation, policies concerning recruitment processes and historical forces such as the Great Depression, World War II, the Korean War, the Vietnam War and the institution of the 40-hour work week.

The principal substantive thrusts of this study have been the following:

1. An explication and comparison of two stochastic models of occupational mobility,

2. A critical test of each of these two models, and

3. Specific extensions of the models including the construction of a system growth model and the initial analytical steps in the construction of a manpower loss rate model.

The use of individual careers as an organizing framework was not made. Rather, the principal focus was at the system level--a system of jobs and a system of men. Most sociologists by analyzing intragenerational occupational mobility in terms of an individual career perspective have omitted the organizational or bounded nature of such mobility. That the boundedness or restrictions for movement have been largely unexplored by sociologists is somewhat ironic since the aspect omitted from the perspective has been the scope conditions or limiting ranges within which the set of relations are operative: that is, the ranges of structuring occupational mobility. This study's use of a system framework delineates a job market within which mobility is generally restricted. Moreover, while such a conceptualization is not contradictory but rather complementary to those emphasizing individual motivation and choice, the aspect providing additional explanatory power is precisely that there exists structural boundaries within which the actual individual choices are limited, whether knowingly or not.

The Two Stochastic Models of Occupational Mobility

The central part of the study was the analysis of the two simple Markov chain models of occupational mobility. The initial step in this analysis was the explication and comparison of the two types of models [Chapter 2]. This step omitted the mathematics of the models and focused upon the two distinct underlying conceptualizations of bounded occupational mobility. Seven important distinctions between the two conceptualizations were explicated. The most obvious one, of course, was the object conceptualized as moving. In the demographic model it is men who move. In the vacancy chain model, however, the object visualized as moving is a job vacancy. Although both models depict a system boundary, there is a second major difference in conceptualizing this restricted movement. This difference is once again with respect to each's conceptualization of structure. The demographic model conceptualizes structure as the flows of manpower between occupations. No interrelations between man movement are represented. The vacancy chain model, on the other hand, conceptualizes the relations between the job vacancy flows themselves. Since I have defined structure in terms of relations or processes, two types of relations are defined by the vacancy chain model. First, there are the relations between occupational strata. These relations or processes are defined in terms of job vacancy flows. Secondly, there are the interrelations between these job vacancy flows. These relations or processes are defined in terms of a chain of movement, the initial movements generating subsequent movement within the chain. Thus, the initially conceptualized relations or job vacancy flows are themselves interrelated by this model. It is in this sense that I have spoken of a second degree conceptualization of structure. The vacancy chain model is thus a more complete representation of occupational mobility. Moreover, since either of the two models depicts a job system or labor market, either model's representation of movement is more complete than those conceptualizations merely describing relations in terms of flows.

The second step of the analysis of the two models of occupational mobility was a substantive interpretation of the models for a specific manpower system--the State Police--and an empirical evaluation of their utility [Chapters 3 and 4]. My decision to neither make an a priori choice favoring either model nor to attempt a crucial test to choose one model over the other was based upon two factors--first, it was my assessment that each model's strength was the other's main weakness; second, few empirical tests of the models had been made and very few of the tests took into account other major system dynamics and system externals. In brief, either an a priori choice or a crucial test seemed premature at this stage of model building. Hence, each model was tested separately. Moreover, the data covering 45 continuously sequential years enabled a critical test of each model.

Several criteria may be used to determine each model's adequacy: the representation of the processes in terms of thoroughness, the validity of the basic assumptions, especially the assumption of stationarity or stability of the transition probabilities, the accuracy of predictions and the limitations set for new research directions and/or practical application. Comment has already been made above with respect to the thoroughness of representation of each model. We may, therefore, proceed to the validity of the models' assumptions.

Three basic assumptions are made in a simple Markov chain model. The three assumptions are time-homogeneous or stationary transition probabilities, homogeneity of stratum populations, and the probability that the object moving from stratum i to stratum j is conditional only upon its present state or location and not the history of its locations.

In the demographic model it was generally true that $p_{ij}=0$ for $i>j$ and $j>i+1$. That is, man movement was generally only to the adjacent, higher stratum. This seemed to be a reasonable basis for the last assumption--that the occupational process was Markovian. In the vacancy chain model the Markovian assumption means that a job vacancy's history prior to the job presently occupied is not influential upon the next move. This assumption appeared most reasonable.

For the demographic model, the assumption of population homogeneity within strata means that all members of each stratum are subject to identical sets of transition probabilities. However, from the analysis of organizational structure and individual careers in Appendix D and from general knowledge of the system, we know that this assumption is not true for the demographic model. The essential point, however, is that the proportion of men moving from each stratum remained quite constant. The model does not attempt to determine the specific persons who move but rather the stratum size resulting from net aggregate flows of populations of men. The constancy of proportionality seemed sufficient to continue to use this assumption. Such violations did not appear to be consequential for this model. As for the vacancy chain model, the assumption of homogeneity of population presented much less of a problem. Since job vacancies per job did not have long histories, the "cumulative inertia" effect of seniority per stratum did not exist. The most serious question was the possible effect upon a job vacancy's movement given some heterogeneity of jobs within strata. Stated another way, does a job vacancy's occupancy of Job A rather than Job B affect its subsequent movement if both jobs are within the same stratum? As was the case with the demographic model, we are describing aggregate flows. Hence, it is the transition probability which is significant rather than the specific job entered. The assumption that we view all job vacancies which exist within a stratum as homogeneous for movement within and across strata seemed to appear very reasonable.

The final assumption, that of stationarity of transition probabilities, was generally investigated by means of a direct examination of the estimated parameters. The meaning of stationarity depends upon the model and specific test. In all cases for this system, stationarity in the demographic model referred to the stability of yearly transition probabilities. The yearly period was the necessary time span due to the assumption within the model that only one move per man of the initial population was allowed within each time interval. Stationarity in this model, therefore, referred to constancy across yearly time periods which at a minimum must hold for two year periods. In the vacancy chain model there is no constraint of a maximum time span since all job vacancies entering the system are given and treated as a cohort. However, there are different premises for stationarity depending on which specific test is being conducted. For the prediction of number of moves, estimation of transition parameters must be made from a different (generally previous) time period. Thus, at a minimum, stationarity refers to constancy of transition probabilities across two comparable time periods--the period from which estimations are taken and the period for which predictions are generated. For the prediction of distribution of chain length, the assumption of stationarity is a much more difficult premise to examine. This is due to the fact that only one time period is involved. The reason for this is that estimation of transition probabilities are based upon individual moves, whereas predictions utilizing such estimations refer to interrelated sets or chains of moves. Hence, the assumption of stationarity means that transition probabilities are constant within the time period. Yet, as discussed in the analysis per se, the periods which we normally consider as bases for combining data--years--are totally arbitrary time spans and the continuous occurrence of newly entering job vacancies, as well as other system dynamics, once such processes are set in motion, makes this point clear. To be more specific, the job vacancy processes do not start and stop by calendar years but are "continuous" once set in motion. Moreover, fluctuations within these processes are dependent upon changes in system dynamics, system policies and system externals, all of which are also "continuously" operative. Hence, criteria for stationarity may vary. Three criteria were used in the analysis--similar "yearly" q_{ij}'s, comparison of substantive information of system dynamics other than the q_{ij}'s and decades or combined decades.

The assessment of stationarity for all but the test involving distribution of chain length was made by comparing estimated transition probabilities for appropriate time spans. In both the demographic and vacancy chain models, sufficient stability for application was not reached until 1950. The effects of the Great Depression and World War II, in conjunction with a newly emergent system, were far too great for stability to be maintained. Major system adjustments were necessary at the beginning as well as during and after each of the two perturbations. In relative terms, however, the stability of the demographic model's transition probabilities was somewhat greater than that of the vacancy chain model.

From 1950 through 1969 the estimated transition probabilities for both models remained relatively stable. Substantively, this stability of transition probabilities means that there existed a stability underlying the dynamics of the mobility processes despite considerable change in structure observable at the surface level. The importance of such-long term stability is considerable. First, it demonstrates the potential power of mathematical formulations in terms of generating nonobvious substantive findings. Secondly, the formulations were constructed in terms of a processual construct--the movement of an object between certain states (strata). Thirdly, the formulation is processual in another sense--that of conceptualizing possible change in long term processes. The finding of such long term stability in the system's occupational dynamics is not only of great substantive import, but it also establishes the basis for the stochastic models' further explanation of these dynamics by means of prediction. In other words, the assumptions appear valid and tests are appropriate. Should the models also accurately predict the various outcomes of occupational mobility, we may further claim their utility by virtue of explanatory power.

Two types of tests were undertaken with respect to the demographic model. The first involved predictions for short time periods (5 years) whereas the second test extended the time period for predictions (9 and 15 years). While short term predictions of 1-3 years were generally extremely accurate, the predictions for more intermediate time spans (4-5 years) were somewhat less accurate. This was primarily a result of two things--the cumulative error inherent in the model's formulation and the assumption of continual "expansion". For the more extended time period predictions, the model's accuracy appeared to be quite good. This conclusion took into account the "expansion" assumption and an emphasis upon general long term accuracy rather than extreme accuracy for each intermediate year. Two perturbations during the 20 year period were noteworthy--the general system expansion in 1956 and the system readjustment in policy in 1966 as a result of the institution of the 40-hour work week. Neither perturbation, however, negates the findings of general stability of system dynamics and of predictive accuracy of the demographic model.

For the vacancy chain model several tests were conducted. The lack of sufficient numbers of "yearly" vacancy chains required a decision to either combine strata or combine "yearly" data. In order not to lose valuable substantive information, the latter combinational schema was chosen. As stated above, however, the possible criteria for combining so-called yearly data is numerous. The criteria of similar "yearly" q_{ij} and substantive based (other than q_{ij}) combinations proved to be quite productive modes for data "lumping". In both cases, predictions were generally quite accurate. Of import was the use of continuously sequential data such that substantively meaningful criteria other than similar q_{ij}'s could be utilized. Not only was this done, but the predictions from the exogenous-endogenous substantive based time periods were slightly more accurate adding further credence to the use of these more general substantive based criteria. The second type of vacancy chain model prediction related to the number of moves from each stratum. Two predictions pertaining to "yearly" and decade time spans were made. Once again my judgment was that the predictions were most accurate. These predictions add additional evidence to the predictive accuracy of the model.

In both models, the initial evaluation of a reasonable amount of validity to the basic assumptions, especially that of stationarity of transition probabilities, provided the basis for accurate predictions. Moreover, the extent of accuracy of the predictions lend further credence to the accuracy of representation of these mobility processes by the simple Markov chain models. Based upon the evaluated validity of assumptions of the models and their predictive accuracy, it would seem that each is an adequate theory for explaining State Police occupational mobility.

While the more immediate payoff for applied purposes would seem to be with the demographic model, the more thorough representation of the process--both structurally and in terms of a mechanism generating movement--is provided by the vacancy chain model. It may be that the reason for visualizing a more immediate payoff from the demographic model is a result of its less complex structural representation and its representation of people as the objects moving. To imagine the movement of job vacancies is not only more abstract in that a job vacancy is a theoretical construct but also more removed from out propensity to think in terms of individual persons. It is, however, no less personally oriented in that either model depicts cumulative structural constraints operative upon individuals' work biographies. It would seem for the moment more appropriate to withhold judgment concerning the limitations set for new research directions and/or practical application. This is not to say that we have not begun to work on this problem since the initial extensions of the two models were obviously efforts in this direction.

Extensions of the Two Stochastic Models of Occupational Mobility

Three attempts to extend the occupational mobility models were made [Chapter 5]. The two of these which were most productive related to the construction of mathematical models of system processes. The first extension was the formulation of system growth as a linear function of system externals. It was observed that system growth had two parts--the allocation by the Legislature of additional manpower and the actual recruitment and training of such newly authorized manpower. The principal process in need of derivation was that of allocations of manpower since a model generating the subsequent process would hardly enable projection of future manpower increases or change. Hence, I chose to conceptualize the process of legislative allocation of additional manpower. The focus was, therefore, upon exogenous forces which would be seen by the State Police organization and the State Legislature as increasing the demand for State Police manpower. Four system externals were seen to determine this demand function. They were density of highway traffic, traffic accidents and/or fatalities, nontraffic crimes involving the State Police and the state economy. The first three variables were visualized as demand components and the fourth variable as a necessary condition. In the model the allocations of system growth were formulated as a linear function of the four exogenous variables. In addition the linear model was interpreted stochastically and analyzed by means of regression analysis. While a regression model can theoretically interchange variables since time is not conceptualized, in this particular formulation the independent variables are by definition prior in time to the dependent variable. Hence, the time constraint was built in.

From the analysis it was clear that the allocation process was not a linear function of the four variables. More importantly, the aim of the model--to predict allocations of system growth sufficiently accurate such that this model might be linked with the demographic model--was not achieved. With an assumption that all allocated manpower would be added during the year, a model for deriving actual added manpower could, at least for the time being, be bypassed. Still, the assumption was unnecessary since the growth model was inadequate. The construction of the model was not unproductive. To conclude that authorization of manpower is not a linear function is not to conclude that the exogenous variables are not important determinants of the allocation process. Rather, the conclusion bears only on the linear nature of the relations. More to the point, additional analyses of a descriptive sort indicated that the hypothesized system externals were indeed important determinative variables in the allocation of additional manpower. No solution for alternative mathematical expression of these relationships was found.

An additional extension was productive in that it laid the initial analytic foundation for the construction of a manpower loss rate model. It was known that specific system policies were operative with respect to age of entrance, seniority and retirement, and mandatory age of retirement. All three were definitely important policies determinative of loss rate after 25 years of service. What was unknown to me, however, was the effect of age of entrance upon loss rate during the earlier years in the system. I decided that the elimination of a possible error relating to this unknown, which might have been built into a loss rate model, was a necessary step in the construction of such a model. An analysis of loss rate up to the first 12 years in the system was conducted. Age of entrance did not appear to be a factor affecting loss rate during these years. It would seem that seniority is the most important force operative until 25 years of service, at which point both seniority and age of entrance are important due to their being included in system policies for leaving. These ideas now appear sufficiently firm to initially use them as the basic components of a manpower loss rate model for this system.

The construction of a manpower loss rate model would be one half of the task of deriving entering job vacancies. This would quite obviously be those jobs left vacant by men leaving the system. Should the stratum held by the man prove to be less important than seniority and age of entrance, the knowledge of the latter two variables would be sufficient for deriving loss rate by stratum as well as general loss rate. The only necessary additional information would be an appropriate updating of these variables by stratum. In any event, the model is not yet constructed. The other "half" or mode of generating initial job vacancies is by means of new jobs being created. It was this part of the entrance process to which the third extension effort was directed. In general, the attempt to derive the number of new jobs per stratum was unproductive except to point out possible deficiencies in sociological knowledge of system dynamics. With respect to this latter point, organizational literature pertaining to growth was generally limited to two strata breakdown (administrative, nonadministrative). Sociological information to date was also insufficiently precise to be of great use for our purposes. Perhaps this is too much to expect at this stage of development of sociological theory in that much more detailed as well as precise formulations would be necessary to have appropriate parameters for organizational differentiation. Alternately, perhaps the detail and precision would have been more readily available had analytical work and research been directed more toward construction of theory and prediction.

General Research Directions

The formulation of the problems, the analysis of two quite different stochastic models of occupational mobility, the extension of these models including development or initial development of models for linkage and supplementary analysis of organizational structure and individual careers--these have been undertaken in this study. Conclusions have been drawn. A question of import still remaining is where do we go from here? Three general research directions are apparent to me. They include a mathematical-historical synthesis, comparative studies of other hierarchical occupational systems and other systems of occupations and the accelerated use of processual conceptualizations and continuously sequential, longitudinal data.

The need for a more general theory of system dynamics is clear from the analysis of the occupational mobility models and from the additional analyses of this study. First, the general system conditions under which occupational dynamics are operative need further specification. Secondly, there is also a need to further explicate the more general exogenous conditions under which the specified general system dynamics occur. A follow-up study to the present research in which a mathematical-historical synthesis will be attempted is an elementary step in this direction. The research would include a detailed historical analysis of the system and (subsystems) from its inception in 1917 to 1971, a critical test and extension of Blau's theory of organizational differentiation and a synthesis of these two analyses with the present study.

In the historical overview of the system, an analysis of general system and subsystem dynamics may be conducted. Included would be an analysis of the effects of (1) being a newly created expanding system, (2) the Great Depression, (3) post-Depression recovery and expansion, (4) World War II, (5) the Korean War, (6) "recessions" and (7) the institution of the 48- and then the 40-hour work week, with particular emphasis upon the number of men and years required to regain the equivalent amount of manpower and service as that prior to such change. In addition, the relations between other systems, both police and non-police, would be examined. The emphasis of the investigation would be upon the effects of these major events on organizational policies, organizational structure and the manpower distribution and flow. The data would include job histories, career histories, personnel rosters, payroll rosters, detailed annual reports and other system records.

The second stage of this type of study would pertain to a critical test and extension of the Blau theory of organizational differentiation. From the supplementary analysis in the present study [Appendix D], it appears that at least the basic axiom of Blau's theory is generally correct. The planned study would provide two new aspects to testing the theory that have not as yet been attempted. First, the analysis would include an extended, continuous time period: 1927-1971. (The detail of the data precluded 1917-1927 analysis.) Secondly, there is considerable data to test the theory and to frame it in an open system framework such that mechanisms generating system change may themselves be explored and possibly provide clues to the dynamics underlying the system's structural differentiation. While Blau's reasoning suggests some rationale, it is neither systematic nor is the theory conceptualized as an open system problem. The adequacy of these limitations may, therefore, be investigated.

The third and most important stage of such a proposed study would be an attempt to synthesize the first two stages with the present mathematical study as the first step in the development of a more general theory of organizational processes. This part of the study is an effort to extend the two highly predictive stochastic models of occupational mobility by means of a somewhat new approach--the detailed interrelating of mathematical and historical analyses. More specifically, the historical overview may be continuously related over a 45-year time span (1927-1971) to the findings derived from the analyses of the mathematical models in order to further explicate the relations between those immediately observable organizational dynamics and the underlying dynamics which have been conceptualized mathematically. In addition, the synthesis would involve interrelating the formal conceptualization of organizationally bounded occupational mobility with the verbal axiomatic conceptualization of organizational differentiation. These interrelationships, as well as those suggested by the first two stages of analysis, would provide a more comprehensive basis for the construction of a more general theory of organizational processes. The bases would particularly relate to the mechanisms generating organizational change and the specifics and interrelations of the dynamics themselves.

A rather thorough, though brief, statement of this more general research direction was given for two reasons--first, the study had been planned in general form as a direct result of the present study; second, the stages for such analysis seemed to need explication rather than a mere suggestion that a setup, follow-through process was in order. As yet, there is no super model, nor can I foresee one in the immediate future. Instead, I visualize the eventual linkage of several models, some more general than others. Before such models could have general support, however, a second general direction would necessarily be underway. This is the comparative research of processual models with longitudinal data.

Perhaps a more appropriate way to introduce what I see as the second major research direction implied by the present research would be to return to the idea of an organizational approach to occupational mobility. In general, it appears to me as Theodore Caplow has suggested (1954) that there exists several distinct species of occupations, each having its own restrictions for movement. Moreover, it is the combination of these bounded occupational systems which comprises the set of occupations normally thought of as a national occupational system. While the present study's scope has been restricted to one particular type of occupational grouping, a hierarchical promotion system, the conceptualization by either occupational mobility model is seen to be applicable to other occupational groupings within which mobility is limited. More importantly, the conceptualizations are seen to be applicable whether or not the limiting restrictions or boundedness of movement is formally organized. The conceptualizations are thus viewed as generalized formulations of intragenerational occupational mobility. In short, what I am suggesting is that the distinction between hierarchical promotion bureaucracies and other occupational groupings is not so much in their being bounded but in the extent to which these boundaries are formally defined. In addition, the models are sufficiently general in formulation to encompass all such groupings.[†]

It is my belief that we need to go beyond the demographic type of structural analysis such as that of Blau and Duncan (1967) to organizational analyses of systems of jobs and men. The present study has demonstrated, as did that of White (1970), the possibilities of an organizational approach to occupational mobility. The next step is for this approach to be applied to other hierarchical promotion systems (police and nonpolice), professional systems other than clergy and other occupational groupings in general.

The most serious problems for general comparative application would be the problem of defining the boundaries for systems not formally bounded, the difficulty of data collection for systems which cross city, state and national boundaries and the multiple subsystem operations crossing formally defined organizations. The first problem is perhaps the easier one to solve. While social science theory does not appear to have conceptualized occupational groupings sufficiently for the purposes delineated above, there are many studies of occupational mobility which could serve as initial indicators. The second problem, data collection, may possibly be quite complicated, especially for a multiple organizational system of occupations. Nevertheless, there are undoubtedly great quantities of historical data available for our usage should we investigate the matter more thoroughly. A basic problem is our ignorance of

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[†]There are advantages for each type of model for particular occupational groupings. Yet, it would appear that for most occupational dynamics, the models' scope conditions are sufficiently flexible for general application. Certainly, should one model prove to be inapplicable, the other should apply.

record systems which have improved astronomically in the last 50 years. In any case, the total cooperation and trust that was established between the State Police personnel and me is extremely encouraging. As for the third problem, the tedious nature of collection of the data necessary for these models and their extension will be even more complicated once multiple subsystems encompassing several distinct organizations are included. The extension of scope in this sense, largely unnecessary in the present study, will compound the data collection process several fold. Still, the task seems necessary for qualitative and quantitative extensions of our present theoretical knowledge. Methodological problems of difficult data collection should not define the substantive problems upon which we focus our attention and research. To conclude, the present step of research has been only one piece of a much more extended research process in which comparisons must be made with other types of occupational systems.

The final general research direction strongly suggested by the present study is an accelerated use of processual conceptualizations and continuously sequential, longitudinal data. One of the most advantageous aspects of mathematical models is that they generally require longitudinal data. Moreover, they also engender processual thinking, a necessary step to advance structural sociology beyond its current preoccupation with static conceptualizations. It is time for processual thinking to come of age in sociological research. It is also time that we took seriously the role of testing processual conceptualizations and spend the necessary effort to gather longitudinal data. The present study has provided an example of both. In addition, it has provided an illustration of their usage in conjunction with mathematical models of social processes. Finally, in my opinion, the study has not only demonstrated certain interrelations between descriptive and mathematical formulations of substantive problems; but more importantly, it has shown the power of mathematical models in explaining complex social processes.

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APPENDICES

APPENDIX A

DATA COLLECTION PROCESS

Initial interest in an organizational mobility project was generated from the collection of data in the summer of 1970 by Professors Thomas Conner and Harry Perlstadt in a study of the Michigan Civil Service System. Their data consisted of several pieces of information contained in the Civil Service Requisition for Employees Form CS-150, Rev. 1265, for the period of one year (January 1, 1969 to December 31, 1969). There were approximately 24,000 forms on file in the Civil Service offices for the period indicated. Information from the form enabled the researchers involved to trace vacancy chains through the organizations.

The intent for my own project was to reformulate and/or extend recently proposed mathematical models of labor mobility and apply them to the nineteen Civil Service departments. Upon closer examination, it became clear that data for one year was inadequate. In fact, thorough application required data over extended time periods. Thus, I decided to attempt to gather data over several decades necessitating a limitation of scope from comparative to that of one organization. The Michigan Department of State Police was chosen. With the possibility in mind of both financial support and access to the necessary information, Thomas Conner and I arranged a meeting in October, 1970 with Mr. Bernard Winckoski, Administrator of the Office of Criminal Justice Programs for the State of Michigan, and Captain John R. Plants, Head of the Michigan State Police Executive Division which includes the Planning and Research Section. The request for financial support was rejected but that for access to records--in particular to complete the incomplete vacancy chains begun in 1969--was temporarily accepted. That is, we were referred to the Director of the Personnel Division, Captain Forrest Jacob.

A few weeks later, Professor Conner and I discussed the intention of gathering more data with Captain Jacob. We had two things in mind. First, it was necessary to gather data for the incomplete vacancy chains begun in 1969. That is, some chains were not complete at the end of 1969, and it was necessary to examine selected information from 1970. Secondly, I had examined the annual reports of the State Police in the Michigan State University Library and discovered that before the organization's size was quite large, personnel rosters were listed each year giving the rank of each member. Hence, movement across strata could be traced by comparing yearly rosters. Thus, we asked if we might use personnel rosters for collecting mobility data for the following years: 1938-1939, 1948-1949, 1958-1959 and 1966-1969. Permission was granted that I be allowed to do this if I would collect the data from 8:00 a.m. to 5:00 p.m. Monday through Friday when an extra desk was available in the Division. Thus, data collection initially began in November and continued through mid-December.

Having worked with the people in the Personnel Division on occasions when the personnel rosters were not sufficient and other sources were necessary such as the personnel card file, the abolished position file, or the records containing a position's history and also in conversations involving the department's history, I became aware of other information which seemed valuable for a more thorough understanding of State Police mobility. Perhaps the more important of these data included the notion of careers and age seniority. Thus, shortly before Christmas, I asked permission to extend my data collection in several directions--to collect mobility data over the entire span of time from 1927 to 1970 and to gather information about careers (extending that gathered from annual reports where major headquarters and district commanders are listed), age and seniority structures and overall system growth. Once again full cooperation was given to me under the same arrangements as before. Data collection was again resumed in March, 1971 with the cooperation of a new Personnel Director, Captain Edward A. Lenon. This collection period extended into the fall of 1971.

Finally, the last data collection was made during the late spring and early summer of 1972. This was to fill in missing information and also to gather additional data on the recruitment and promotion processes and their relation to the Civil Service Commission.

The data cover five decades of this organization's existence (1927-1971) and include the following:

1. Historical data accounting for the growth of various functions of the organization and in particular, explicating those factors most important for manpower usage and change

2. Growth rates by stratum for:

   a) the total organization

   b) headquarters or central administrative unit

c) districts

d) individual units or posts located throughout the State

3. Job vacancy chain movement

4. Recruitment population for each year and rate of attrition during the training periods

5. Career data for all strata of major supervisory import in terms of length of time from entering the organization until either leaving or current position as of March, 1971

6. Seniority and age distributions per year

7. Traffic Density Data: estimated miles traveled, death and accident rates and manhours per type of work

8. Crime Rate Data: number of arrests, convictions and manhours.

A vacancy chain was recorded according to the year it was initiated. In most cases a chain was completed within one year. Where this was not true, the chain's path was traced either until it was completed or until a job was vacant for one year. In the latter case, the job was said to be "abolished" and therefore left the system. Vacancy chain movement was primarily traced from personnel rosters. Where necessary, personnel cards, files and position history was also utilized.

The actual tracing of job vacancy chains was made by comparing two personnel rosters for each position. The positions were generally recognizable due to the fact that each person's location was explicit in three ways in these rosters. That is, the person was listed under a specific structural segment of the organization and his rank was given. Moreover, each segment had a specific geographical location.

Hence, when a man stayed at the same geographical location but changed in either rank or structural location, a job vacancy move was determinable. Also, if a man changed geographical locations, it was necessary to trace his new location or his exit from the system to determine the two jobs or locations between which the job vacancy moved.

When there were several men at the same geographical location, in the same structural segment and having the same rank, there generally was a problem only if two or more such men moved during the interval between the two rosters. In such cases, additional information was necessary involving the possibility of several search procedures. In most instances, however, the search procedure most productive was an examination of the active files in which job histories rather than men were given. These files were extremely useful in numerous instances, and assistance in their use was always available. Moreover, such files were perhaps one of the filing segments most actively utilized by member members of the Personnel Division in their everyday work. When a job history had been removed from the file (i.e., the job had been abolished) or a change in job number had occurred sometime in the job's history, additional problems of search were created and other sources had to be examined in a somewhat trial and error fashion. Indicative of the excellence of the record system and the proficiency of the work force in the Personnel Division, every job vacancy move until the late 1920's and early 1930's was always eventually found, given I had not missed it in some fashion in the initial personnel roster investigation. Moreover, there were very few instances even in the earlier years which were nontraceable. Microfilm records, termination or transfer records, general organizational records, the abolished job file, personnel files and much knowledge of the men, the police system and the record system by members of the Personnel Division were invaluable upon such detailed searches. It should, I think, be stressed that all of the above applies to each job vacancy move.

Job vacancy chains were formulated by recording from the appropriate source (generally the personnel roster) the geographical and structural location of each of these moves and where necessary interrelating the specific job numbers. The latter was often necessary for various detective positions and for Cpl. positions, both of which often had multiple positions of the same rank, same geographical location and initially, at least, what appeared to be an undifferentiable structural position. Also, there were frequently simultaneous moves from these similar positions necessitating the specific job history check in order to identify appropriate interrelated moves or job vacancy chains. Generally, inferences of job vacancy moves were based upon the observation of man movement. However, as is stated above, in all cases of recording movement, the interrelation of geographical location, structural position, name of the individual and associated rank were simultaneously involved. Moreover, in some cases the same detail given to the individual person was also focused upon the specific job involved, as when job history traces occurred. The formulation of the actual job vacancy chains was both time consuming and mentally laborious work requiring attention to several facets of information at the same time and requiring perseverance to see that each link of each chain was recorded accurately. My recording of individual movement and the interrelations of specific individual moves occurred as a continuous process and often involved tracing links of several chains at the same time.

While somewhat complicated, it is a task which can easily be learned and efficiency in the tracing procedure increases quite rapidly with practice. Perhaps one qualification should be mentioned--the tracing of many chains over several years involves changes in record systems and is a continuously learning exercise.

Perhaps an illustration of recording a job vacancy chain would be helpful. Let us hypothesize our system has only three strata as conceptualized by Figure 3 in Chapter 2. We may, therefore, use a case illustrating Figure 3. Let us designate Stratum 3 as consisting of S/Sgts., D/S/Sgts., Sgts., and D/Sgts.; Stratum 2 as consisting of Cpls. and Dets.; and Stratum 1 as consisting of Tprs. In the examination of personnel rosters, the initial move could occur at any level. For instance, let us suppose the first move recorded was by Person C who moved from a Cpl. job at Post 1 to a Cpl. job at Post 20. The job vacancy has, therefore, moved from the job at Post 20 (Job 2) to the job at Post 1 (Job 3). To discover this move we examine the same job location and personnel from two rosters. Obviously, for a Cpl. position at Post 1, all Cpls. are not the same. For ease, let us say only one Cpl. position was involved in movement at this post. Hence, all Cpl. names are the same with the exception of two names, one in each roster. To have discovered the Cpl. move from Post 1 to Post 20 meant we traced Person C or the existing Cpl. to his new location at Post 20. We must then compare the two rosters to find what man (Cpl.) left Post 20. We find in the roster having a more recent date that Person B is no longer there. We then look for Person B's new location. We find him at a Sgt. position (Job 1) at Post 5. Hence, the job vacancy moved from Post 5 to Post 20 and from the rank of Sgt. to that of Cpl. Now, we once again inquire who left Job 1. Let us suppose no one did. Rather, it was a new position. No man movement occurred. To corroborate that a new position was actually created, we may check the job history file at Post 5. If this fact is confirmed, then a job vacancy is recorded as moving from the outside to Job 1 in Stratum 3 by means of a new job creation. Alternately, had Job 1 already been in existence, then we would have found that Person A left (retired, quit, died, dismissed) the system creating a vacancy in the Sgt. position (Job 1). In either of the two alternative and mutually exclusive (i.e., both could not occur) cases, one end of the job vacancy chain has been found--the initiation of the chain. It remains for us to complete the remaining end of the chain. That is, what happened to the Cpl. position (Job 3) at Post 1 vacated by Person C? We now examine the other person of the two initially discovered at Post 1 at two different times (two different rosters). We discover from the roster with the earlier date that this man (Person D) had been a Tpr. at Post 10. The job vacancy, therefore, moves from the Cpl. position (Job 3) to the Tpr. position (Job 4). Moreover, since Job 4 at Post 10 is a general Tpr. position or in the labor pool, this Tpr. will be replaced by a new recruit (Person E). Our job vacancy chain is complete.

Perhaps I should mention that several pairs of rosters may have been examined to trace all interrelated moves depicted in Figure 3. Since the above case was merely illustrative, it did not seem necessary to mention possible change of the two personnel rosters being compared in the actual illustration. Also, the only facet necessary to change in Figure 3 to make the figure applicable to the analyses of this study would be to add Stratum 4 (Lts.) and Stratum 5 (Col., Capts.). That is, we would enlarge the system's boundary to include two additional strata. For our particular chain, however, these additional system strata are not pertinent. Even had they been included in the Figure, the initial job vacancy move would have been from the outside directly to the Sgt. position in Stratum 3. It should now be somewhat clear that individual moves must be examined one at a time and in terms of the sequential order of any other related individual movement. Thus, from individual moves an entire job vacancy chain is constructed.

From organizational strata size and the movement recorded according to vacancy chains, one may derive the necessary man movement for the demographic model. This was possible since modes of entry and exit for the vacancy chains were recorded and since job vacancy movement was generally inferred from man movement. When man movement was not involved, it was certainly not pertinent to the demographic model. In addition, attention was simply not paid to the interrelated aspect of the movement. The data for organizational size were taken from two sources--organizational charts where available (1956-1970) and "counting heads" from personnel rosters where records were not available (pre-1956).

The age and seniority data come from several sources: retirement files, personnel cards and folders, microfilm and recruit school records.

The career data were derived from annual reports and personnel rosters, files and folders.

The crime rate, traffic density and accident-fatality data were found in the annual reports. In addition, the annual reports also contain information on vital historical events as well as general background.

In sum, data for the mobility processes were gathered from several sources: personnel files, annual reports, historical information in the headquarters library, personnel cards, microfilm and most importantly, personnel rosters from 1927 to 1970.

Throughout the entire data collection process the cooperation by members of the Personnel Division was complete. No doubt my presence and many questions were often an inconvenience to these persons. Their knowledge of both the current system and detailed historical events, the method in which they were recorded and the filing location of such events was necessary for my work. On very, very few occasions was the trace procedure unproductive. I cannot stress enough that the active recording in these persons' minds of the data or at least location of data was a vital element in my data resources.

APPENDIX B

COMPARISON OF PRE-1971 AND POST-1971
STATUS-AUTHORITY HIERARCHIES

A Civil Service reclassification and/or title change for State Police positions from Cpl., Det. to Lt. became effective August 1, 1971. The actual change occurred October 21, 1971 but was retroactive in pay and seniority to August 1. The new hierarchy is diagrammed below with the old hierarchy in brackets. Deviations from this are noted by an asterick and will be explained in the discussion.

Director
(Col., unclassified)
[same]

Assistant Directors
(Lt.Col. 19, Maj. 17)
[same]

District and Division Commanders
(Capt. 15 and 16)
[same]

First Lt. 14 [Lt. 13]	Det. First Lt. 14 [D/Lt. 13]
Lt. 12 [S/Sgt. 12]*	D/Lt. 12 [D/S/Sgt. 12]
Sgt. 11 [same]	D/Sgt. 11 [same]
Sgt. 10 [Cpl. 10]*	D/Sgt. 10 [Det. 10]

Tpr. 07 and 09
[same]

In terms of hierarchical authority, the diagram shows no changes. Status change took place in three cases: Lt., D/Lt. to First Lt., D/First Lt.; S/Sgt., D/S/Sgt. to Lt., D/Lt.; and Cpl., Det. to Sgt., D/Sgt. Pay change occurred once: Lt. 13, D/Lt. 13 to First Lt. 14, D/First Lt. 14. The only qualifications which need to be made to the above statements are in reference to those cases where there are astericks. Former S/Sgt. 12's who were working at a post of size 35 or greater became Lt. 13's while those at posts with 34 or less men became Lt. 12's. There were 12 of 61 posts with 35+ men on August 1, 1971. There are now 13 such posts. Similarly, Cpl. 10's located at the larger posts become Sgt. 11's while those at the smaller posts became Sgt. 10's.

Not stated or qualified in the diagram are changed titles such as D/Sgt. 11 FM to D/Sgt. 11 Specialist. The FM denoted Fire Marshall Division work; the Specialist denotes a more general title for the same work. Likewise, D/Sgt. 11's in Intelligence or Safety and Traffic formerly denoted by D/Sgt. 11 and D/Sgt. 11 S/T are now denoted by D/Sgt. 11 Specialist.

One additional change has occurred since these 1971 changes were made. There are now two Lt.Col.'s rather than one. This resulted in there now being one Maj. rather than two.

In what way have these changes affected the applicability of this study to the present State Police system? Hardly any at all! The relative position of all but a small number of positions remained the same in spite of many status (title) changes and a few pay changes. The only effect of these changes in terms of the present study would be the decision on strata location of the new Sgt. and Lt. positions at the larger posts, i.e., involving actual number changes from 10 to 11 and 12 to 13 respectively. The Lt. 13 position seems to pose no problems. Consistency with the current study would probably mean combining Lt. 12's and 13's into one "stratum" analogous to the S/Sgt., Sgt. stratum of this study.

In the case of the new Sgt. 11 position, formerly a Cpl. 10, there is more to consider. The criteria for deciding which stratum in which to locate the position would rest on (1) relative position at the post or (2) relative position in the total hierarchy. These are not the first changes of this nature. My own decision would favor the latter criteria for two reasons. First, this would be consistent with my decisions on similar job evolution changes in the past. For instance, those changes from Sgt. to First Sgt. in the 1930's and from Sgt. to S/Sgt. at various points in time were placed in the "Sgt." stratum because of the "half step" rather than full step change. The argument based upon perception and actual movement is in the main body of the text and will not be restated here. As for changes in status from Cpl. to Sgt., however, involving a "full step", these have been recorded as jobs evolving across strata. They connote not only a status change but also a much more complete change in administrative responsibility. The Civil Service logic, it seems to me, was that Cpl.'s at large posts had sufficiently similar responsibility to Sgt.'s at smaller posts to equate their authority status standings. Perhaps this is true. In any event, the policy change's effect upon occupational mobility is quite real, and the decision would be to consider the consequences of this reality in terms of further lateral as well as horizontal movement. Thus, there has been one change which in this study, were it to be continued, would be treated as a job vacancy move across strata. It is the Cpl. 10 to the Sgt. 11 move. The effect would, therefore, be to slightly increase the Sgt. 11 (S/Sgt., Sgt.) stratum, and depending on whether moves are then allowed for Tpr. 09 to Sgt. 11 positions, perhaps change one cell from a 0 to a slightly larger number. Were this to be the case, the transition probabilities between S/Sgt., Sgt. and Cpl., Det. would be slightly different. The consequence for the vacancy chain model, for example, would be a few shorter chains. Overall, however, the effect would be very slight, probably affecting the stability of the 1949-1969 dynamics very little, if at all.

One further comment needs stating. The data gathering process, as a result of these moves, would be increased considerably in spite of their small effect on the study per se. One reason for this lies in the omission of specialist designations such as S/T or FM. They have been replaced by the designation Specialist. The "Specialist" category because of its generality loses its specification of actual function. Of course, S/T and FM men were specialists! The S/T and FM designation made that clear. It was also clear as to what type of specialization. In my opinion the new title loses much information and unnecessarily so. Another reason for increased data processing time lies in the decision of the Civil Service personnel to treat the above moves such that new jobs were created and old ones abolished. The effect of this ridiculous ruling is that all historical continuity of a job is lost. In short, no new functions were created. Rather, the same jobs received new titles, pay or relations to other jobs. However, they were the same jobs. The Civil Service treatment of them as though they were not suggests a lack of familiarity with this system's on going processes over time and their relations to the present processes. That is, it not only amounts to a historical ignorance but a probable historical ignorance for everyone in the future. It is certainly the case that current efficiency in record keeping and tracing has been violated greatly by this set of decisions. Perhaps personnel of the two systems should share more information on these types of effects. It seems to me that a major share of the responsibility for initiating such exchanges lies with those who administer the rules of change--the Civil Service.

As a sociologist, the interrelations between systems are themselves important objects of study. The relationship of the State Civil Service Commission to the State Police seems particularly interesting since it is the only major system, other than the Legislature, which greatly affects the internal dynamics of the State Police. This 1971 ruling is but one instance.

In sum, I will state that it seems as though the 1971 organizational shake-up was more apparent than real for purposes of mobility. It, therefore, has little consequence for the applicability of the present study for the current system. The study's findings are indeed applicable to the present system.

APPENDIX C

REALLOCATIONS AS A TYPE OF MOVEMENT

The method by which I have coded a special type of transition, reallocations, within the State Police will be made explicit below. All reallocations have been coded as job vacancy moves for the vacancy chain model's transition probabilities. Only those reallocations actually involving simultaneous man movement have been coded as movement for the demographic models' transition matrices. The following diagram in Figure 13 should help to make the discussion somewhat more clear.

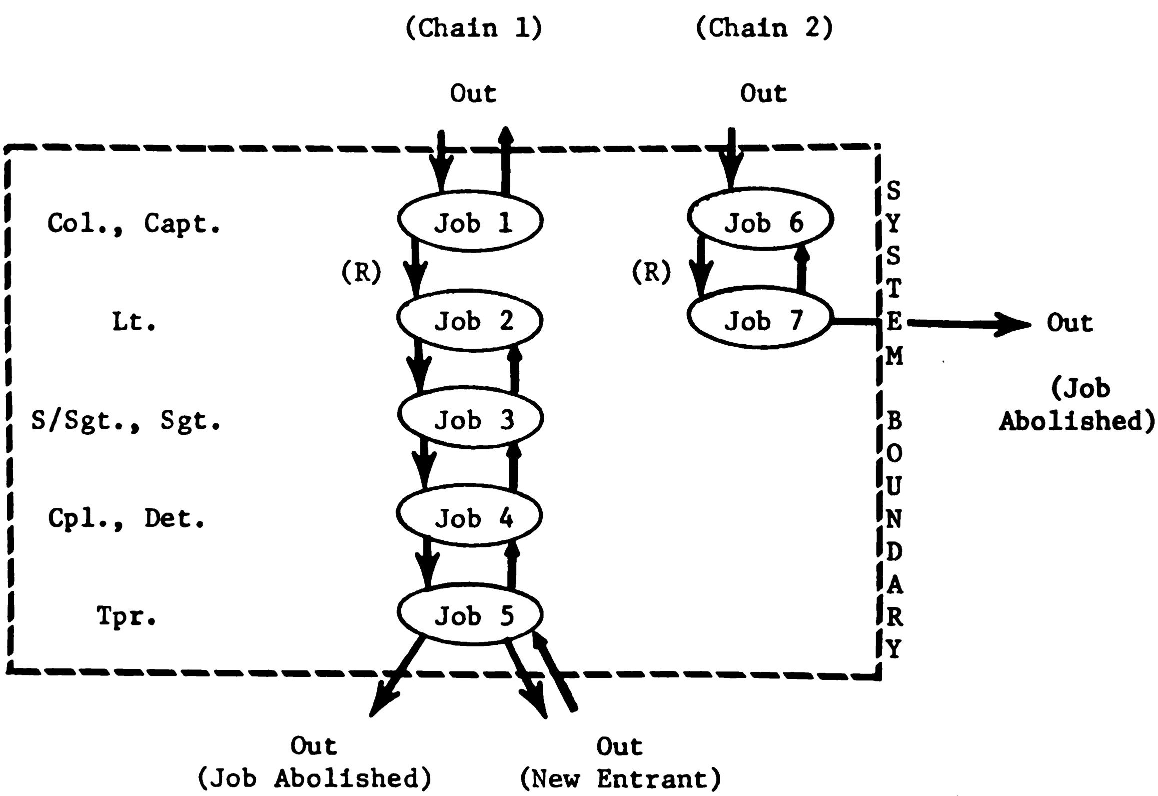

Figure 13. Bounded Occupational Structure with Reallocation (R) as a Type of Job Vacancy Movement

In Chain 1 the reallocation is downward involving no actual man movement between Job 1 and Job 2. For instance, a Capt. retires. The person who is most capable of filling the position may actually be a S/Sgt. who has worked under the retired Capt. The Capt. position is then reallocated downward to a Lt. position and the S/Sgt. promoted to the rank of Lt., leaving his S/Sgt. position vacant to be filled by someone else, in this case a Cpl., et cetera. Should the person most capable of filling the position have been a Lt. with less than one year seniority as a Lt., then the reallocation would still have been downward to Lt. and the person already occupying a Lt. position would move horizontally to the reallocated Lt. position leaving his former Lt. position vacant for someone else to fill.

In the case of the position being filled from below (e.g., the S/Sgt. to Lt. move) for the demographic model, a man is recorded as moving from Job 3 to Job 2 and a man is recorded as leaving the system from Job 1. However, no man movement is recorded from Job 2 to Job 1. On the other hand, for the vacancy chain model, a move by the job vacancy is recorded as occurring from Job 1 to Job 2 and from Job 2 to Job 3, et cetera. No move is recorded for the job vacancy's entrance to the system since this move is assumed as given or derived and not conceptualized by the model.

In the case of the position being filled horizontally (i.e., from within the stratum), the demographic model is recorded as having but one move--the Capt. move from the system. Once again for the vacancy chain model, no move is recorded for the vacancy's entrance. However, a move is recorded both for when the position is reallocated downward (Job 1 to Job 2) and for when the position is filled, i.e., the job vacancy moves on to Job 3.

For Chain 2 a different type of reallocation is occurring involving job evolution or upward reallocation. In this case for the demographic model, one move is recorded--from Job 7 to Job 6. For the vacancy chain model, however, two moves are recorded--from Job 6 to Job 7 and from Job 7 to Out. Although the job vacancy enters at Job 6 by means of reallocation, the model assumes this entry as given and consequently no move is recorded.

Obviously, the exact moves illustrated in Figure 13 are not the only ones for which reallocations are recorded. That is, reallocations of either an upward or downward type have been recorded irrespective of the stratum in which it occurs.

APPENDIX D

DESCRIPTIVE ANALYSIS OF SYSTEM DYNAMICS AND THE INDIVIDUAL'S CAREER

The rather extensive analysis at the system level in the main body of the text has left one facet of occupational mobility unexamined. This particular area involves the relationships between system dynamics and an individual's career. These relationships will be briefly analyzed in two parts--the relationships of internal system change and individual movement and the relationships of general system mobility to length of time per stratum and age of entrance.

Internal System Change and Individual Movement

Obviously, the structural dynamics of a system set boundaries for the individual member's career. Since the occupational system being analyzed is formally bounded, a proposition of significance for us and also a principal one within Peter Blau's theory of differentiation in organizations (1970) is the following:

> HYPOTHESIS 1: Increasing organizational size generates structural differentiation along various dimensions at decelerating rates.

If Blau's reasoning is correct, we may extend the application of the theory to mobility processes. Blau argues that the administrative and supervisory component should decrease proportionately with increasing size. Yet, since the differentiation process is a decelerating one, the rate of decrease of proportionality should also diminish. Relating these statements to the individual's probabilities for upward movement, one would expect the following:

> HYPOTHESIS 2: The probability of upward movement from the lowest stratum decreases with increasing organizational size but decreases at a decelerating rate.

Several elements of the above logic need to be more thoroughly examined in general, as well as in their specific applicability to the State Police occupational system. First, Blau's hypothesis (HYPOTHESIS 1) is stated in causal terms. In this hypothesis size is the causal variable and structural differentiation the affected variable. The key term is generates. In a more recent discussion (1972), Blau uses the term promotes, which is still causal in syntax. In short, there is an implied time sequence involved. For size to generate something, it must precede that something in time. While there is nothing in error about the statement of Blau's hypothesis, there is a significant error in his purported test. First, Blau uses multiple regression or correlational analysis. This type of analysis does not incorporate the time dimension as any variable may be changed, if we like, to be the dependent variable. Moreover, Blau ignores the nature of the hypothesis once he attempts to test it. He never examines data which is actually generative of differentiation. Nor does he examine the conditions which might affect these generative relationships. The reason for such omissions is that Blau has no data to test a time sequential hypothesis since his data are cross-classificational--in other words, his data relate to but one point in time. Of course, he allows size and structural differentiation to vary. Nevertheless, insofar as one is said to cause the other, his data and hence his test are inadequate.

Longitudinal data are required. Moreover, to extend the logic such that the system is conceptualized as an open one which Blau has briefly alluded to (1972), it seems to me that actual empirical investigations are required. It is my contention that we know very little about the relationships between internal system change and external change. A principal reason for this is that, as a general rule, sociologists have not examined longitudinal data, the analysis of which is greatly informed by taking into account exogenous forces. The principal point, however, is that Blau's hypothesis is causal-temporal; his data are not. Therefore, he has yet to actually test the hypothesis.

A second aspect of Blau's logic which is in need of closer scrutiny is the relationship between the administrative-supervisory component and the major work force of the system. According to Blau, not only is the administrative-supervisory component declining in proportionate size but the larger the system, the wider the supervisory span of control. This, Blau argues, is primarily a result of large-scale operations making it possible to economize in administrative-supervisory (A-S) manpower. While Blau also provides a logic for the decreasing rate component of the hypothesis, it is the main argument above with which I am most concerned. More specifically, structural differentiation may be occurring at various levels of the A-S hierarchy and also among non-A-S segments of the population. The important facet for us is that the logic involving structural differentiation provides little information about the hierarchical level of the segmentation. It may, in fact, be that status structures for non-A-S personnel and A-S personnel overlap considerably. That is, if differentiation occurs in a major functioning (as opposed to A-S or support services) section of the system at a level above the lowest one, the new jobs may or may not be administrative. If they are not, what does Blau's theory tell us? Little it seems, other than it is a function of size. Yet, it is specifically at this point that significant substantive implications exist for coordination and mobility of manpower. This is precisely a situation applicable to the State Police. Recall that the Det. specialization from the Tpr. function involved a job evolution such that Dets. were equivalent in status and authority to the first line Tpr. supervisor, the Cpl. Thus, to reason from Hypothesis 1 to Hypothesis 2 without taking the above situation into account is inappropriate. Two hierarchical administrative ladders exist involving administration over detective work and administration over the more general police function--Tpr. work. Moreover, there is no simple administrative hierarchical arrangement within the detective segment as there is among the more general police segment. Since Blau's theory does not adequately inform us of what to expect for the overall system's mobility processes, even if the theory is supported, Hypothesis 2 will be used primarily for heuristic purposes to begin analysis of more complex system processes.

A final point to be discussed prior to the analysis is the manner in which Blau examines the dynamics of the system. They are treated in a closed system frame of reference. Exogenous forces are not taken into account. At most, Blau recognizes that "The larger the volume of work of a certain kind, the larger is the number of persons needed to perform it" (1972:22). The lack of longitudinal data is obviously the principal reason for maintaining a closed system approach. Should an extended time period have been analyzed, exogenous forces too crucial to omit would undoubtedly have been introduced for more powerful explanatory purposes. However, when the data are cross-classificational the effects of a system operative in more enlarged dynamics cannot be examined. In short, a distorted empirical image is provided and actual system dynamics are unobservable.

With these comments in mind, let us begin to analyze Hypothesis 1 and 2. For the present purposes, Hypothesis 1 will be rephrased in order to test it as Blau does. Correlational relationships in this new hypothesis are sufficient to examine the consequential effects of changing system dimensions upon mobility.

HYPOTHESIS 1A: The greater the organizational size, the greater the amount of structural differentiation along various dimensions.

Hypothesis 1A omits one element from Hypothesis 1--the form of the relationship. Thus, it may be further specified that this relationship has the following form--greater increases in size correlate with less proportional increases in structural differentiation as size continues to increase. With this extension of Hypothesis 1A, we have, in fact, the hypothesis which Blau has repeatedly tested. An actual examination of Hypothesis 1 could be carried out with the data base used for this study since it is longitudinal over 43 continuous years. However, such an analysis is a rather detailed study in itself. I will, therefore, limit the analysis to Hypotheses 1A and 2.

Hypothesis 1A will be tested using a subunit of the Michigan State Police--its professional personnel, not the civilian staff, since it is only to this segment of the population which Hypothesis 2 refers. Size will be denoted by the number of employees. Following Blau, structural differentiation will be indexed along several dimensions:

number of functional divisions at headquarters, number of sections per division and number of local posts (geographical differentiation).

The data for these four variables are reported in Table 38. Size of system refers to actual size. The data were taken from two sources--official records of system size (1956-1970) and personnel rosters (pre-1956). For the latter, hand counts were made of the personnel by location and function. The data for number of local posts were taken from annual reports. Data for number of divisions at headquarters and number of sections per division were recorded from official organizational charts. The number of hierarchical levels was taken from personnel rosters (pre-1956) and official system records thereafter. Since there were not consistent organizational charts prior to October, 1952, no further information was recorded prior to "1953" for this form of differentiation. Moreover, the system's organization is not that used by Blau prior to 1965 for extended differentiation within divisions. All divisions are applicable with the exception of the Uniform Division. However, within this Division, prior to the 1965 reorganization there were large bureaus which contained sections and units. The remaining divisions merely had the section-unit breakdown. Due to this complication no data are reported prior to 1965. In addition, since there were some difficulties in determining actual enlisted personnel prior to 1936, years prior to this were omitted.

Table 38. System Size and Internal Structural Differentiation by Year

Year	Size	Number of Posts	Number of Divisions	Number of Sections/Division
1970	1721	59	16	4.1875
1969	1603	59	16	4.1250
1968	1496	59	16	4.0625
1967	1401	59	16	4.0000
1966	1253	56	15	
1965	1237	54	--	
1964	1109	54	8	
1963	1107	54	7	
1962	1113	54	6	
1961	1093	54	6	
1960	1106	54	6	
1959	1130	54	6	
1958	1153	54	6	
1957	1017	51	6	
1956	742	46	6	
1955	709	45	6	
1954	655	45	7	
1953	675	45	7	
1952	647	45		
1951	636	45		
1950	608	45		
1949	577	45		
1948	512	45		
1947	493	45		
1946	391	45		
1945	382	45		
1944	406	45		
1943	437	45		
1942	487	42		
1941	443	39		
1940	341	39		
1939	306	36		
1938	310	36		
1937	225	36		
1936	232	34		

From the data in Table 38, Hypothesis 1A is generally supported. The years for which it is not correct includes only World War II for the relationship between size and posts. For all but years 1953 and 1954, the relationship between size and divisions is upheld. A reorganization placing the Special Investigation and Security Investigation Divisions under the Uniform Division and making them sections within a Detective Bureau decreased the number of divisions by two. On the other hand, a new division--Personnel and Training--made the total system decrease in number of divisions by only one. Nevertheless, the system's size was growing and was likely one of the precipitating changes bringing about this reorganization. The proposition is, therefore, not totally correct. It cannot necessarily account for all types of reorganizations accompanying an expanding system. More thorough examination of the conditions affecting such reorganizations is needed. Since data were not collected with this in mind, such an extension will not be attempted in this study.

In terms of the few years reported for number of sections per division, the hypothesis is correct. Additional subsystem differentiation also occurs.

A simple correlation matrix for the variables for which there are data over several years is reported in Table 39 below.

Table 39. Simple Correlation Matrix for Size, Divisions and Local Posts

	Size	Divisions	Local Posts
Size	1.0		
Divisions	.96	1.0	
Local Posts	.79	.70	1.0

The data again add credence to Hypothesis 1A. The residuals for each of these correlations, however, also suggest the relationship is not perfectly linear.

Data to test Hypothesis 2 were reported in Table 3 of Chapter 3. Several factors stand out in this data. In general, the probability for promotion is very low, ranging between .01 and .06. For the 43 years presented there are 11 years for which the probabilities are greater than .06 and only 6 years for which they are greater than .09. These 6 years are 1927, 1931, 1945, 1948, 1956 and 1966. The first 2 years were ones of reorganization and expansion. A high rate of internal activity is apparent from an examination of the data for actual movement per se. In 1945 and 1948 men were returning from the war. Also, the new vacancies created prior to and during the war which had not been filled, and new vacancies now occurring to fill the lag brought about by the war, accelerated internal movement. The causes for the divergence in 1956 and 1966 were discussed in Chapters 3 and 4. The data do not confirm the hypothesis whatsoever. The probabilities for upward movement are generally very low and constant. When they do vary, the magnitude is considerable suggesting a massive reactionary aspect of this system's on going operations. Intermittently there is a major new input or expansion accelerating the transition rates considerably. Shortly following these perturbations, however, there is a return to a low and relatively constant rate of upward movement. Two factors may account for this constancy--a continued expansion of the detective segment of the system following World War II and a system policy which is operative to maintain a constant proportion of administrative personnel. Since the proportion of Tprs. has remained approximately the same throughout at least the last 20 years, the latter explanation is unlikely. Rather, expansion of a job evolutionary nature appears to have accounted for the transition probabilities not declining further. Whether or not the transition probabilities remain somewhat higher following the last stratum expansion at the Cpl. level remains to be seen. The data following 1956 do not support the notion of an increased relative flow.

To summarize, while the modified Blau hypothesis is generally supported, neither it nor the theory inform our expectations greatly regarding mobility processes. Hypothesis 2 is not supported as expected, contrary to the input of the Blau theory. Were I to have analyzed only those Tprs. moving to the administrative segment of the system perhaps the theory's input would have been more accurate. In spite of this possibility, the fact remains that the theory is most inadequate to inform us of the relationships between structural differentiation of all types, irrespective of the level of the hierarchy, and manpower mobility.

The Effect of Age of Entrance and Seniority within Stratum upon An Individual's Career

This discussion will be centered around two questions: (1) What is the mean length of time per stratum for upward movers? and (2) How does age of entrance affect the probability of an individual's vertical movement?

Of most interest in relation to the first question is an additional question--how far upward? In other words, the analysis will examine the relative time of seniority accumulation before moving upward for members whose movement terminates at each level. The question at issue is whether or not there is a difference in mean time spent per stratum for those who vary in level of hierarchical achievement. For instance, is there a difference in mean time spent per stratum prior to movement to Cpl. and Sgt. for men whose final promotion is to Lt. as compared to those whose final promotion is to Sgt.?

The present analysis is restricted in two senses. First, only those men who have terminated their service with the system are included. In order to contrast those whose final achievement was a given stratum, this limitation was necessary. Thus, if career history is greatly affected by the age of the system, the data will be somewhat biased to former stages since more recent movement generally involves those members still in the system. A second restriction is even more limiting. Unusual movement at any point in a man's career is not as easily analyzed as the more general patterns of movement. Analysis of such movement seems to require a more detailed qualitative analysis extending this study even further. Careers involving movements downward to "District Detective" rather than Det., or exit and re-entry will not be considered. The analysis is, therefore, restricted to those members having had continuous service, no demotions and not having been in a "transient stratum" such as the "District Detective" position during and immediately following World War II. The "District Detective" officially held the rank of Tpr. not Det. and performed a specialized detective function. The stimulus for this type of position was World War II. The tasks were primarily that of special and security investigation. The adjective "District" connoted his location since they were dispersed throughout the entire range of State Police Districts. Following the war some returned to Tpr. roles while others remained in a Det. role, thus enlarging the detective segment of the system on a permanent basis.

It was for these reasons that movement to this position was referred to as movement to a "transient stratum". The title "District Detective" was discarded after the war for the rank and title of Det. irrespective of location.

Data for actual number of years before moving to a given stratum according to last stratum reached are provided in Tables 40, 41, 42 and 43. While I could comment on the modal range of each table and make comparisons, I think the mean number of years spent per stratum is a much easier comparison to grasp. These are presented in Table 44.

Table 40. Frequency Distribution of Years of Seniority in Previous Stratum Prior to Moving to Destination Stratum or Outside for Men Whose Mobility Ended at the Capt. Stratum

Number of Years	Destination				
	Out	Capt.	Lt.	S/Sgt., Sgt.	Cpl., Det.
0	1	3	2	2	2
1	4	2	2	1	
2	3	11	3	5	
3	7	2	8	8	1
4	3	4	4	3	1
5	3	5	2	3	1
6	1	1	4	4	1
7	4	1		2	3
8	2		1	1	1
9			1		5
10			1		8
11					4
12	1				2
13					
14					1
15					
16			1		

Table 41. Frequency Distribution of Years of Seniority in Previous Stratum Prior to Moving to Destination Stratum or Outside for Men Whose Mobility Ended at the Lt. Stratum

Number of Years	Destination			
	Out	Lt.	S/Sgt., Sgt.	Cpl., Det.
0	4	1	4	3
1	6			1
2	8	3	6	1
3	3	6	4	
4	4	4	5	1
5	4	6	8	
6	3	5	2	
7	1	4	2	
8		1	2	2
9	1			3
10		1		3
11				7
12				6
13		1	1	4
14				3
15		1		
16				
17		1		

Table 42. Frequency Distribution of Years of Seniority in Previous Stratum Prior to Moving to Destination Stratum or Outside for Men Whose Mobility Ended at the S/Sgt., Sgt. Stratum

Number of Years	Destination		
	Out	S/Sgt., Sgt.	Cpl., Det.
0	3	5	5
1	8	2	1
2	6	9	1
3	15	11	3
4	12	19	3
5	13	15	
6	10	21	6
7	16	12	4
8	28	17	5
9	8	13	17
10	8	6	16
11	8	6	23
12	1	1	23
13	2	3	13
14	1		18
15	1		5
16	1		
17	2	1	1
18			
19			
20			
21			1

Table 43. Frequency Distribution of Years of Seniority in Previous Stratum Prior to Moving to Destination Stratum or Outside for Men Whose Mobility Ended at the Cpl., Det. Stratum

	Destination	
Number of Years	Out	Cpl., Det.
0	12	7
1	8	3
2	1	5
3	9	1
4	4	3
5		1
6	3	3
7	1	
8	5	5
9	11	6
10	12	7
11	14	14
12	6	7
13	12	18
14	5	16
15	2	12
16	8	6
17		1
18	3	2
19		
20		1

Table 44. Mean Number of Years per Stratum before Moving Upward by Stratum at which Individual Career Mobility Ended

	Stratum at which Mobility Ended			
Destination	Cpl., Det.	S/Sgt., Sgt.	Lt.	Capt.
Cpl., Det.	11.0	10.0	9.5	8.6
S/Sgt., Sgt.		5.7	4.1	3.7
Lt.			5.6	4.3
Capt.				3.9
Out	8.6	6.3	3.0	4.1

By comparing column entries for each row we may note that there is perfect rank ordering of movement between strata. Moreover, the data clearly show that those moving the greatest distance upward before ending their career, move on the average <u>at each transition</u> to the next stratum one year before those immediately below them in final status do. There is an effect of cumulative seniority. However, there does not appear to be a clear cutoff time to distinguish movers from nonmovers at each stratum. From Tables 40-43 it would appear that the upper limit or maximum time for movement from Tpr. to Cpl. for those ending in the Capt. rank is 12 years, those ending in that of Lt., 14 years, those those ending in the S/Sgt., Sgt. stratum, 15 years and those whose final move is to Cpl., 18 years. With the exception of this last case, however, the modal range for the upper three strata are Capt.: 10, Lt: 11 and S/Sgt., Sgt.: 11 and 12. The one year difference is even apparent here. In any case, I repeat that there is no clear cutoff range for nonmovers at any stratum, even though these important differences exist. At most, the significance appears in the mean and modal times for movement upward. To attempt to specify individuals as movers, nonmovers does not appear possible from these rather simple distinctions.

To answer the second question--the effect of age of entrance upon career mobility--let us examine the data in Table 45.

Table 45. Age of Entrance by Stratum at which Individual Career Mobility Ended (Percentage)

Stratum at which Mobility Ended	Age of Entrance										Total Pop.
	21	22	23	24	25	26	27	28	29	30	
Cpl., Det.	.21	.38	.38	.36	.36	.44	.36	.50	.36	.54	.35
S/Sgt., Sgt.	.48	.38	.41	.43	.49	.48	.46	.50	.57	.31	.46
Lt.	.23	.12	.09	.14	.03	.04	.14	.00	.07	.00	.10
Capt.	.08	.12	.12	.07	.12	.04	.05	.00	.00	.15	.09
N	(39)	(56)	(68)	(44)	(33)	(25)	(22)	(8)	(14)	(13)	(322)

The general finding is that those entering at ages 21-25 are more likely to have lower proportions ending their careers in the two lower strata and higher proportions ending their careers in the two upper strata (within the Table) than those entering at ages 26-30. This same finding holds true in each case of several methods of examining the data. For instance, one may compare the distribution of the total population's career mobility (Column 11) with that of the career mobility (Columns 1-10) for each age of entrance aggregate. Only 6 of 40 cells deviate from the finding stated above. Three of the 6 occur at the S/Sgt., Sgt. stratum. They include values higher than the stated finding indicates for ages 21 and 25 and a value lower for age 30. Two other exceptions occur in the Lt. stratum--for ages 25 and 27. The remaining exception is that of individuals entering at age 30 who achieve the rank of Capt. The percentage (15) is the highest of all ages. In general, however, in 34 of 40 cells or 85 per cent the finding holds true. Three other methods of analyzing the data also bring me to the same conclusion. These methods and the findings are reported below.

First, I compared relative percentages (higher, intermediate, lower) within stratum at which career mobility ended. I found only 5 of 40 cells had higher or lower values outside the range of the general finding. Secondly, I calculated the difference between the highest and lowest percentages within each stratum and divided by 2 such that the mean of the two extreme percentages might be used as a cutoff for hi-lo scores. For example, in the Cpl., Det. stratum .54-.21=.33 having a 16.5 per cent midpoint which is 37.5 per cent. The data may be seen in Table 46 below.

Table 46. Number of Cells from Table 45 which Are Higher or Lower than the "Mean" Percentage within Each Stratum by Age of Entrance and Stratum at which Career Mobility Ended

	Stratum at which Career Mobility Ended							
	Cpl., Det.		S/Sgt., Sgt.		Lt.		Capt.	
Age of Entrance	Hi	Lo	Hi	Lo	Hi	Lo	Hi	Lo
21-25	2	3	2	3	2	3	4	1
26-30	3	2	4	1	1	4	1	4

Once again, the initial general finding is upheld. By this method only 2 of 16 cells differ from the general direction expected. These 2 cells are for the 21-25 age grouping within the Lt. stratum. However, 14 of 16 cases (87.5 per cent) are in the direction of the initial findings.

A final alternative method which was utilized was to take the highest percentage per stratum, divide by 3 to generate higher, intermediate and lower ranges and compare cells from Table 45 once again. For instance, the highest percentage in the Cpl., Det. stratum was 54. Hence, the ranges are .00-.18, .19-.36 and .37-.54; the findings are reported below in Table 47. Once again, there is but one set of exceptions to the initial finding. In this case, the percentages for the 21-25 age group are too high for the S/Sgt., Sgt. stratum.

Table 47. Number of Cells from Table 45 According to Three Equal Percentage Intervals by Age of Entrance and Stratum at Which Career Mobility Ended

	Stratum at which Career Mobility Ended											
	Cpl., Det.			S/Sgt., Sgt.			Lt.			Capt.		
Age of Entrance	Lo	Med	Hi	Lo	Med	Hi	Lo	Med	Hi	Lo	Med	Hi
21-25	0	3	2	0	1	4	1	3	1	0	2	3
26-30	0	1	4	0	1	4	4	1	0	4	0	1

Although different breakdowns of the data shift the exceptions somewhat, the general finding that younger entrants (ages 21-25) achieve higher ranks in greater proportions to older entrants (ages 26-30) holds throughout. Moreover, I also examined additional careers in which all criteria except for termination of service were met and in which the rank of Capt. or greater was achieved. The number of careers was 13 for these criteria and it is noteworthy that all 13 entered before age 26 or within the 21-25 year age span. Also, 9 of the 13 entered at ages 21 or 22! In general, therefore, for all data examined, a younger age of entrance (ages 21-25) is more advantageous than an older one (26-30). This finding is in contrast to possible expectations of the effect of maturity, other job experience, et cetera upon subsequent careers. The distribution of age of entrance of the total population of upward movers is extremely close to that of all entrants from 1930-1959, lending further credence to the above assertions.

To summarize this section, there are clearly discernible effects resulting from seniority within a given stratum and age of entrance. Those who move the greatest distance in their career do so "on the average" more quickly <u>at each transition</u> than those moving lesser distances during their entire career. Also, those who enter at younger ages have more representation in the upper strata than those who enter later in the 10 year span allowed, suggesting an advantage for entrance in the earlier 5 years of the system's set of entrance policies.

Summary

Since the principal thrusts of the study pertained to formal theories at the system level and their extensions, I placed this descriptive and supplementary analysis of organizational structure and individual careers as an appendix. The pursual of the interrelations of organizational dynamics and individual careers was one of several directions for additional extensions of the formal models. The interrelation of system policy and individual careers was another direction which seemed important for formal conceptualization at the system level.

For instance, the analysis of the time element for movers and nonmovers may be particularly important at the system level in determining the more specific relationships between seniority per stratum and individual careers. More to the point perhaps, it seemed reasonable to investigate certain of these relations at the descriptive level prior to their formal conceptualization.

The first supplementary analysis focused upon the relations between individual probabilities for upward movement and general system dynamics. As reasoned throughout this study, the structural dynamics of a system set boundaries or parameters for individuals' careers. The structural variables of system size and internal differentiation were taken into account. Since the occupational system being analyzed is formally bounded, a basic proposition of Peter Blau's theory of organizational differentiation (1970, 1972) seemed particularly apt. It specifically related organizational size and internal differentiation. Since the rate of differentiation is not as rapid as the rate of system growth according to this theory, it seemed that a logical expectation for individual movement upward would be that the probabilities for upward flows would decrease as the system expanded. The Blau hypothesis was generally supported, although major perturbations to the system from exogenous forces or certain types of reorganization accompanying an expanding system were not accounted for. This suggests that a more thorough examination of the conditions affecting restructuring is needed. Perhaps more importantly for this supplementary analysis was the finding that neither the system hypothesis nor the theory in general were adequate to inform us on the relationships between structural differentiation and manpower mobility. This finding suggests weaknesses in the theory's thoroughness of representing internal differentiation. The former finding suggested weaknesses in "predictive" accuracy.[†]

A very important observation relative to the Blau hypothesis was that Blau's proposition was stated in causal syntax and was time sequential in nature; yet his data were cross-classificational and therefore relating to but one point in time. Given these facts, I concluded that an actual test of the hypothesis had not been made by Blau. Moreover, the test undertaken in my analysis was actually for a modified Blau hypothesis--the one which I think Blau himself tested. More to the point is the fact that cross-classiciational data, as utilized by Blau, virtually necessitate a closed system frame of reference. Exogenous forces cannot be appropriately taken into account. Neither can crucially important system dynamics. Rather, since actual system dynamics are unobservable using cross-classificational data, a distorted empirical image is provided.

The second supplementary analysis focused upon effects of seniority per stratum and age of entrance upon subsequent upward movement in an individual's career. Two basic findings were made. First, those who move greater distances in their careers "on the average" move upward more quickly <u>at each transition</u> than those moving lesser distances during their entire career. In addition, those who enter from ages 21-25 have higher proportions in the upper strata than those who enter from ages 26-30. Hence, there seems to be an advantage for subsequent career mobility for those entering in the first five years of the

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[†]The reason for placing predictive in quotes is that Blau's hypothesis is not very precise (although more so than most sociological hypotheses) and hence is predictive only in terms of the general form of the relationship.

system's age of entry range. In short, the effects of seniority within stratum and age of entry are quite discernible. However, the aggregate type of analysis from which these findings were derived do not allow closer investigation of conditions under which certain men actually move. In this sense, the findings are clearly lacking and a longitudinal investigation seems in order.

APPENDIX E

ADDITIONAL YEARLY TRANSITION MATRICES AND VECTORS OF JOB VACANCY ENTRANCES FOR q_{ij} BASED COMBINED DATA

Table 48. Estimated[a] Transition Probabilities for and Creations of Job Vacancies by Year

Origin Stratum	Destination Stratum: Col., Capt.	Lt.	S/Sgt., Sgt.	Cpl., Det.	Tpr.	Out	Total Number of Moves (N)
			1952				
Out	.1020	.0612	.2245	.1020	.5510		49
Col., Capt	.0000	.6000	.0000	.0000	.0000	.4000	5
Lt.	.0000	.0000	.6667	.0000	.0000	.3333	6
S/Sgt., Sgt.	.0000	.0000	.2500	.6000	.0000	.1500	20
Cpl., Det.	.0000	.0000	.0000	.2609	.5652	.1739	23
Tpr.	.0000	.0000	.0000	.0000	.0909	.9090	44

[a]Estimations are made using Equation (4.9). The Col., Capt. stratum includes Col., Lt.Col., Maj., and Capt.

Table 48 (cont'd.)

Origin Stratum	Destination Stratum: Col., Capt.	Lt.	S/Sgt., Sgt.	Cpl., Det.	Tpr.	Out	Total Number of Moves (N)
			1954				
Out	.1176	.0588	.1324	.1471	.5441		68
Col., Capt.	.3846	.3846]	.0000	.0000	.0000	.2308	13
Lt.	.0000	.2500	.6667	.0000	.0000	.0833	12
S/Sgt., Sgt.	.0000	.0000	.2917	.6667	.0000	.0417	24
Cpl., Det.	.0000	.0000	.0000	.1613	.5806	.2581	31
Tpr.	.0000	.0000	.0000	.0000	.0678	.9322	59
			1957				
Out	.0000	.0452	.0847	.1808	.7119		177
Col., Capt.	.0000	.0000	.0000	.0000	.0000	.0000	0
Lt.	.0000	.2000	.7000	.0000	.0000	.1000	10
S/Sgt., Sgt.	.0000	.0000	.2414	.6552	.0000	.1034	29
Cpl., Det.	.0000	.0000	.0000	.2609	.6232	.1159	69
Tpr.	.0000	.0000	.0000	.0000	.0117	.9883	171

Table 48 (cont'd.)

Origin Stratum	Destination Stratum: Col., Capt.	Lt.	S/Sgt., Sgt.	Cpl., Det.	Tpr.	Out	Total Number of Moves (N)
			1958				
Out	.2000	.1000	.0750	.2000	.4250		40
Col., Capt.	.2727	.7272	.0000	.0000	.0000	.0000	11
Lt.	.0000	.0000	.6667	.0833	.0000	.2500	12
S/Sgt., Sgt.	.0000	.0000	.4211	.4737	.0000	.1053	19
Cpl., Det.	.0000	.0000	.0000	.3077	.6538	.0385	26
Tpr.	.0000	.0000	.0000	.0000	.0286	.9714	35
			1959				
Out	.0000	.0364	.3091	.1636	.4909		55
Col., Capt.	.0000	.0000	.0000	.0000	.0000	.0000	0
Lt.	.0000	.0000	1.0000	.0000	.0000	.0000	2
S/Sgt., Sgt.	.0000	.0000	.2692	.6923	.0000	.0385	26
Cpl., Det.	.0000	.0000	.0000	.3333	.5238	.1429	42
Tpr.	.0000	.0000	.0000	.0175	.1403	.8421	57

Table 48 (cont'd.)

Origin Stratum	Destination Stratum: Col., Capt.	Lt.	S/Sgt., Sgt.	Cpl., Det.	Tpr.	Out	Total Number of Moves (N)
			1960				
Out	.0130	.0519	.1818	.2857	.4675		77
Col., Capt.	.0000	1.0000	.0000	.0000	.0000	.0000	1
Lt.	.0000	.1667	.8333	.0000	.0000	.0000	6
S/Sgt., Sgt.	.0000	.0000	.2692	.6923	.0000	.0385	26
Cpl., Det.	.0000	.0000	.0000	.2632	.6667	.0702	57
Tpr.	.0000	.0000	.0000	.0256	.0512	.9230	78
			1967				
Out	.0118	.0471	.1235	.3706	.4471		170
Col., Capt.	.0000	1.0000	.0000	.0000	.0000	.0000	2
Lt.	.0000	.0909	.9091	.0000	.0000	.0000	11
S/Sgt., Sgt.	.0000	.0000	.3404	.5745	.0000	.0851	47
Cpl., Det.	.0000	.0000	.0000	.2903	.6129	.0968	124
Tpr.	.0000	.0000	.0000	.0121	.0487	.9390	164

Table 48 (cont'd.)

Origin Stratum	Destination Stratum: Col., Capt.	Lt.	S/Sgt., Sgt.	Cpl., Det.	Tpr.	Out	Total Number of Moves (N)
			1968				
Out	.0086	.0043	.0776	.1336	.7759		232
Col., Capt.	.3333	.6667	.0000	.0000	.0000	.0000	3
Lt.	.0000	.0000	1.0000	.0000	.0000	.0000	3
S/Sgt., Sgt.	.0000	.0000	.3636	.6061	.0000	.0303	33
Cpl., Det.	.0000	.0000	.0000	.4138	.5172	.0690	87
Tpr.	.0000	.0000	.0000	.0000	.0546	.9453	238
			1969				
Out	.0490	.0280	.1259	.3566	.4406		143
Col., Capt.	.2222	.6667	.0000	.0000	.0000	.1111	9
Lt.	.0000	.0000	.9000	.0000	.0000	.1000	10
S/Sgt., Sgt.	.0000	.0000	.4375	.5000	.0000	.0625	48
Cpl., Det.	.0000	.0000	.0000	.3304	.6161	.0536	112
Tpr.	.0000	.0000	.0000	.0000	.1483	.8516	155

APPENDIX F

ADDITIONAL TABLES FOR DATA COMBINED FROM DECADES

Table 49. Estimated[a] Transition Probabilities for and Creations of Job Vacancies by Decade(s)

Origin Stratum	Destination Stratum: Col., Capt.	Lt.	S/Sgt., Sgt.	Cpl., Det.	Tpr.	Out	Total Number of Moves (N)
	1950 Decade						
Out	.0378	.0388	.1224	.1571	.6439		980
Col., Capt.	.2292	.6458	.0000	.0000	.0000	.1250	48
Lt.	.0000	.1266	.7342	.0253	.0000	.1139	79
S/Sgt., Sgt.	.0000	.0000	.2478	.6416	.0044	.1062	226
Cpl., Det.	.0000	.0000	.0000	.2488	.6294	.1219	402
Tpr.	.0000	.0000	.0011	.0022	.0338	.9629	917

Table 49 (cont'd.)

Origin Stratum	Destination Stratum						Total Number of Moves
	Col., Capt.	Lt.	S/Sgt., Sgt.	Cpl., Det.	Tpr.	Out	(N)
			1960 Decade				
Out	.0304	.0274	.1444	.2485	.5494		1316
Col., Capt.	.2000	.7000	.0000	.0000	.0000	.1000	50
Lt.	.0000	.1566	.6627	.0120	.0000	.1687	83
S/Sgt., Sgt.	.0000	.0000	.4015	.5099	.0000	.0887	406
Cpl., Det.	.0000	.0000	.0013	.2996	.6057	.0934	771
Tpr.	.0000	.0000	.0000	.0039	.0621	.9340	1273
			1940-1969				
Out	.0278	.0284	.1248	.1958	.6231		3269
Col., Capt.	.2250	.6167	.0333	.0000	.0000	.1250	120
Lt.	.0052	.1309	.7068	.0157	.0000	.1414	191
S/Sgt., Sgt.	.0012	.0000	.3512	.5143	.0155	.1179	840
Cpl., Det.	.0000	.0000	.0040	.2770	.6008	.1182	1498
Tpr.	.0000	.0000	.0012	.0028	.0812	.9148	3216

Table 50. Predicted (P) and Observed (O) Distribution of Vacancy Chain Length by Stratum of Origin for 1950 Decade (Percent)

	Stratum of Origin									
	Col., Capt.		Lt.		S/Sgt., Sgt.		Cpl., Det.		Tpr.	
j[a]	P	O	P	O	P	O	P	O	P	O
1	12.5	5.4	11.4	2.6	10.6	3.6	12.2	5.8	96.3	96.5
2	10.2	29.7	9.5	36.8	10.9	32.4	63.6	68.2	3.3	2.5
3	8.5	5.4	10.8	5.3	43.5	29.7	17.9	16.2	.3	.8
4	8.9	2.7	33.8	15.8	22.3	18.9	4.6	7.8	.1	.2
5	23.9	6.2	20.8	21.1	8.5	11.7	1.2	1.9	.0	.0
6	18.9	27.0	8.9	10.5	2.9	1.8	.3	.0		
7	10.1	2.7	3.2	7.9	.9	.9	.1	.0		
8	4.4	5.4	1.1	.0	.3	.9	.0	.0		
9	1.7	5.4	.4	.0	.1	.0				
10	.6	.0	.1	.0	.0	.0				
11	.2	.0	.0	.0						
12	.1	.0								
13	.0	.0								
14										
15										
16										
(N)	37		38		111		154		631	

[a]j denotes chain length distribution.

Table 51. Predicted (P) and Observed (O) Distribution of Vacancy Chain Length by Stratum of Origin for 1960 Decade (Percent)

	Stratum of Origin									
	Col., Capt.		Lt.		S/Sgt., Sgt.		Cpl., Det.		Tpr.	
j[a]	P	O	P	O	P	O	P	O	P	O
1	10.0	7.5	16.9	11.1	8.9	1.6	9.3	3.0	93.4	92.1
2	13.8	27.5	8.6	16.7	8.3	27.4	59.4	68.7	5.8	6.2
3	8.8	.0	7.6	2.8	33.6	25.8	21.3	19.8	.6	1.5
4	7.1	.0	23.7	5.6	24.4	27.4	6.8	5.5	.1	.0
5	18.0	2.5	20.0	22.2	13.3	11.1	2.1	1.8	.0	.0
6	17.6	22.5	11.9	16.7	6.4	5.8	.7	.3		
7	11.9	30.0	6.1	19.4	2.9	.5	.2	.3		
8	6.7	5.0	2.9	5.6	1.3	.0	.1	.0		
9	3.4	2.5	1.3	.0	.6	.0	.0	.0		
10	1.6	2.5	.6	.0	.2	.5				
11	.7	.0	.2	.0	.1	.0				
12	.3	.0	.1	.0	.0	.0				
13	.1	.0	.0	.0						
14	.1	.0								
15	.0	.0								
16										
(N)	40		36		190		329		723	

[a]j denotes chain length distribution.

Table 52. Predicted (P) and Observed (O) Distribution of Vacancy Chain Length by Stratum of Origin for 1940-1969 Combined Data (Percent)

	Stratum of Origin									
	Col., Capt.		Lt.		S/Sgt., Sgt.		Cpl., Det.		Tpr.	
j[a]	P	O	P	O	P	O	P	O	P	O
1	12.5	7.7	14.1	5.4	11.8	5.3	11.8	3.9	91.5	89.9
2	11.9	28.6	10.4	32.3	11.7	31.9	58.3	71.5	7.5	8.3
3	9.5	2.2	10.6	4.3	34.2	24.3	20.7	16.5	.8	1.5
4	9.8	2.2	25.9	12.9	22.7	20.8	6.3	5.6	.2	.1
5	19.0	7.7	19.6	19.4	11.2	10.9	1.9	1.7	.1	.0
6	16.7	20.9	10.6	12.9	5.0	5.1	.6	.5	.0	.1
7	10.5	18.7	5.0	10.8	2.1	.8	.2	.2	.0	.0
8	5.5	5.5	2.2	2.2	.9	.5	.1	.0	.0	.0
9	2.6	4.4	.9	.0	.3	.0	.0	.0	.0	.1
10	1.2	1.1	.4	.0	.1	.5	.0	.2	.0	.0
11	.5	1.1	.2	.0	.1	.0	.0	.0		
12	.2	.0	.1	.0	.0	.0				
13	.1	.0	.0	.0						
14	.0	.0								
15										
16										
(N)	91		93		395		642		2037	

[a] j denotes chain length distribution.

APPENDIX G

ADDITIONAL TABLE FOR PREDICTIONS OF

TOTAL NUMBER OF JOB VACANCY MOVES

WITHIN AND FROM EACH STRATUM

Table 53. Estimated[a] Transition Probabilities for and Creations of Job Vacancies by Year

Origin Stratum	Destination Stratum: Col., Capt.	Lt.	S/Sgt., Sgt.	Cpl., Det.	Tpr.	Out	Total Number of Moves (N)
			1933				
Out	.0179	.0804	.1607	.0179	.7232		112
Col., Capt.	.0000	.1111	.0000	.0000	.0000	.8889	9
Lt.	.2800	.0400	.0000	.0000	.0000	.6800	25
S/Sgt., Sgt.	.0000	.4483	.2069	.0000	.0000	.3448	29
Cpl., Det.	.0000	.0000	.0417	.0000	.0000	.9583	24
Tpr.	.0000	.0104	.0417	.2292	.1563	.5625	96

Table 53 (cont'd.)

Origin Stratum	Destination Stratum						Total Number of Moves
	Col., Capt.	Lt.	S/Sgt., Sgt.	Cpl., Det.	Tpr.	Out	(N)
	1944						
Out	.0270	.0000	.0541	.0811	.8378		37
Col., Capt.	.5000	.0000	.2500	.0000	.0000	.2500	4
Lt.	.0000	.0000	.0000	.0000	.0000	.0000	0
S/Sgt., Sgt.	.1426	.0000	.4286	.2857	.0000	.1426	7
Cpl., Det.	.0000	.0000	.0000	.5455	.2727	.1010	11
Tpr.	.0000	.0000	.0156	.0000	.4688	.5156	64
	1949						
Out	.0430	.1183	.1183	.2688	.4516		93
Col., Capt.	.1667	.5000	.0000	.0000	.0000	.3333	6
Lt.	.0625	.1250	.7500	.0000	.0000	.0625	16
S/Sgt., Sgt.	.0000	.0000	.3056	.5000	.0000	.1944	36
Cpl., Det.	.0000	.0000	.0345	.2586	.5862	.1207	58
Tpr.	.0000	.0000	.0000	.0000	.0256	.9744	78