AN ECONOMIC ANALYSIS OF SOME CONTROLLED FERTILIZER INPUT-OUTPUT EXPERIMENTS IN MICHIGAN

Thesis for the Degree of Ph. D.

MICHIGAN STATE UNIVERSITY

Wesley Burton Sundquist

1957

This is to certify that the

thesis entitled

An Economic Analysis of Some Controlled Fertilizer Input-Output Experiments in Michigan

presented by

Wesley B. Sundquist

has been accepted towards fulfillment
of the requirements for

PhD degree in Agricultural Economics

Major professor

Date 8/21/57

O-169

AN ECONOMIC ANALYSIS OF SOME CONTROLLED FERTILIZER INPUT-OUTPUT EXPERIMENTS IN MICHIGAN

By

Wesley Burton Sundquist

A THESIS

Submitted to the School of Advanced Graduate Studies of Michigan State University of Agriculture and Applied Science in partial fulfillment of the requirements for the degree of

DOCTOR OF PHILOSOPHY

Department of Agricultural Economics

1957

ACKNOWLEDGMENTS

The author is indebted to several members of the departments of Agricultural Economics and Soil Science of Michigan State University who aided in organizing and conducting the research reported in this thesis. Dr. R. L. Cook, Head of the Department of Soil Science, Dr. J. F. Davis, L. N. Shepard and J. C. Shickluna of the Department of Soil Science and C. R. Hoglund of the Department of Agricultural Economics aided in numerous ways in conducting the research reported here.

Dr. L. L. Boger, Head of the Department of Agricultural Economics at Michigan State University and C. W. Crickman and H. L. Stewart of the Farm Economics Research Division, Agricultural Research Service, United States Department of Agriculture, provided financial aid without which the author could not have conducted this study.

Much of the computational work was performed by the girls in the statistical pool of the Department of Agricultural Economics under the direction of Mrs. Iantha Perfect. Beverly Hamilton deserves a vote of thanks for typing an earlier draft of this thesis.

The author's sincerest appreciation is due to Jack L. Knetsch, formerly of Michigan State University, now with the Tennessee Valley Authority with whom much of the experimental work was planned and conducted and to Dr. Lynn S. Robertson Jr. of the Department of Soil Science who aided in planning the experimental work and who has supervised the field work. Without a tremendous amount of work on his part, the data utilized in this thesis would not have been produced.

Above all, the author appreciates the encouragement and guidance of Dr. Glenn L. Johnson, not only while writing this thesis but throughout the course of my graduate studies. Association with him has helped to make graduate work a really enjoyable and profitable experience.

****

AN ECONOMIC ANALYSIS OF SOME CONTROLLED FERTILIZER INPUT-OUTPUT EXPERIMENTS IN MICHIGAN

By

Wesley Burton Sundquist

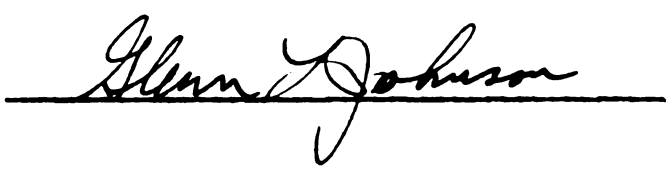
AN ABSTRACT

Submitted to the School of Advanced Graduate Studies of Michigan State University of Agriculture and Applied Science in partial fulfillment of the requirements for the degree of

DOCTOR OF PHILOSOPHY

Department of Agricultural Economics

Year 1957

Approved 

ABSTRACT

The three primary plant nutrients, nitrogen, phosphoric acid and potash are major farm resource inputs. In 1954, farmers in the United States paid over a billion dollars for various commercial forms of these plant nutrients. In order to allocate optimally their resources, farmers need information as to the productivity of expenditures made for various production inputs including the three primary plant nutrients.

In the spring of 1954 the Michigan Agricultural Experiment Station, aided in part by resources contributed by other interested agencies, initiated a series of plant nutrient input-crop yield output experiments which has been expanded in each succeeding year. Experimental input-output information analyzed in this thesis included data for the following crops: (1) a rotation of oats, wheat, alfalfa and corn on a Kalamazoo sandy loam soil in Calhoun and Kalamazoo counties (2) a rotation of corn, field beans and wheat on a Simms loam soil in Gratiot county (3) corn produced in continuous culture on a Wisner clay loam soil in Tuscola county and (4) potatoes grown on a Houghton muck soil at the Experiment Station muck farm near East Lansing. In total, over 1150 individual experimental plots were contained in these experiments in 1956.

The primary objectives of this thesis are (1) to estimate plant nutrient input-crop yield output production surfaces and then (2) to provide an economic analysis of the physical input-output relationships

derived. Continuous function analysis is utilized to estimate the input-output relationships of interest to researchers and farmers.

Two general formulations of the production functions for plant nutrients are fitted for most crops. These are a polynomial of the type:

$$Y = a + b_1N + b_2N^2 + b_3P + b_4P^2 + b_5K + b_6K^2 + b_7NP + b_8NK + b_9PK$$

where N, P and K represent pound per acre inputs of nitrogen, phosphoric acid and potash. The second production function formulation is an exponential of the Carter-Halter type:

$$Y = aN^{b_1c_1}N_P^{b_2c_2}P_K^{b_3c_3}K$$

Both equations are fitted by least squares techniques, the latter being first converted to logarithms.

Significant yield response to applied nitrogen was found for corn, wheat, oats and field beans. Corn, wheat and field beans showed a significant yield response to applications of phosphoric acid. Only potatoes showed a significant response to applied potash for crops produced during the growing seasons for which experimental data were analyzed.

Despite statistically significant response to applied plant nutrients for several crops, applications of plant nutrients were profitable for only two crops assuming current crop and fertilizer prices. Nitrogen applications were profitable for corn produced on a Kalamazoo sandy loam soil in 1955 and for field beans produced on a Simms loam soil in 1956. In computing high-profit plant nutrient inputs, however, no credit was

made for residual fertility or benefits derived from seedings in the small grain crops. Mid and late summer drouths in 1955 and 1956 very probably reduced the crop yield benefits which might have been derived from applied plant nutrients particularly on the lighter soils. Further information on input-output relationships over time and with varying weather conditions is needed to establish a probability distribution of these relationships.

The experimental results analyzed in this thesis are from a very limited number of soil types. These soils tend to be either very fertile or very unproductive. One might expect the largest yield responses to applied plant nutrients on soils with a high production potential but depleted in fertility; such soils are not included in the experiments analyzed here. However, as individual low-treatment plots in the experiments become depleted and if treatments are rerandomized, a wide range of combinations of residual fertility and applied nutrients should be observed.

The adjusted coefficients of multiple correlation between applied plant nutrients and crop yields ranged from .28 to .78 for the various production function formulations for the different crops studied. Further analysis indicated substantial amounts of yield variance not associated with regression were due to experimental error and inability to control entirely unstudied variables. Limited analysis to relate residual fertility, as measured by soil tests, to the deviations of predicted from observed yields ($Y_i - \hat{Y}_i$), was relatively unsuccessful.

However, further extension of this type is needed in order to provide conclusive results.

TABLE OF CONTENTS

TABLE OF CONTENTS - continued

LIST OF TABLES

LIST OF TABLES - continued

TABLE Page

LIST OF FIGURES

CHAPTER I

THE NATURE AND MAGNITUDE OF FERTILIZER USE PROBLEMS

Current Research Problems in Fertilizer Use

Recently much attention has been devoted to the economics of fertilizer use in the United States. Research receiving increased emphasis includes various attempts to determine the most efficient forms and carriers of the three primary plant nutrients: nitrogen, phosphorus and potassium.

A second important area of research is that of attempting to determine the relative effectiveness of alternative methods of fertilizer application. One such alternative is broadcasting the fertilizer and plowing it down prior to planting the crop. A second alternative is placing all or a portion of the fertilizer in bands of varying depths and distances from the seed. A third method is that of applying all or a portion of the fertilizer by top dressing the growing crop. Other alternatives include combinations of the above listed procedures.

A third major area of fertilizer research, which is interrelated with the previously mentioned two, is that of deriving fertilizer input-crop output ratios and relationships. Current research includes deriving such input-output relationships for the three primary plant nutrients

for a number of crops on a variety of soil types and for differing management practices.

This by no means exhausts the list of fertilizer-oriented research work currently being conducted. However, it indicates three of the major areas in which fertilizer research is being conducted and illustrates the diversity of current fertilizer research. The latter research area, that of deriving input-output relationships plus an economic interpretation of these relationships, is the primary concern in this thesis. Derivation of physical input-output ratios or physical production functions is only the first step in an economic analysis designed to determine optimal fertilizer use. Once such physical relationships have been empirically established, profit maximization principles can be employed to determine optimal fertilizer use with a given set of crop and fertilizer prices and given the earning power or marginal value productivity of other farm expenditure or investment categories.

The Importance of Fertilizer as an Agricultural Production Factor

The expanded interest and resources currently being allocated to obtaining more detailed and reliable information about the economics of fertilizer use appears to be warranted by (1) the importance of fertilizer as production factor in United States and Michigan agriculture and (2) the need for greater production from American agriculture in the years ahead. The latter can be obtained only by the use of more production resources and/or a more efficient combination of production factors.

Fertilizer consumption in the United States has increased rapidly over the past several decades as indicated by the data shown in Table 1. Consumption of the primary plant nutrients in 1910 totaled 46,000 tons of nitrogen, 499,000 tons of P_2O_5, the common fertilizer form of phosphorus, and 211,000 tons of K_2O, the common fertilizer form of potassium. By 1954 these totals had increased to 1,868,000 tons of nitrogen, 2,228,000 tons of P_2O_5 and 1,868,000 tons of K_2O. Recently particularly large increases have occurred in the consumption of nitrogen and potassium with nitrogen consumption more than doubling from 1949 to 1954. Preliminary estimates indicate further substantial increases for 1955 with a slight decline in 1956. The decline in consumption in 1956 was accompanied by a decrease in total crop acreage for the United States as a whole during that year.

Increases in fertilizer consumption have occurred in Michigan with even greater relative increases in recent years than for the United States as a whole. In contrast to total United States consumption which declined slightly in 1956, Michigan consumption increased slightly over that of 1955. The annual consumption of the primary plant nutrients for Michigan during the period 1939 to 1955 is indicated in Table 2. During this period total nitrogen consumption increased over 11 fold from 3,314 to 37,184 tons while consumption of P_2O_5 increased from 18,016 to 88,228 tons. Consumption of K_2O, which was only 9,974 tons in 1939, increased to 85,343 tons in 1955. Assuming a price of $.15 per pound for elemental nitrogen, $.10 per pound for P_2O_5 and $.11 per pound for K_2O,[1] the total

[1]These are the prices currently being used by fertilizer experts as being typical of prices paid by Michigan farmers.

TABLE 1

UNITED STATES FERTILIZER CONSUMPTION 1910-1955

Year	Primary Plant Nutrients in Thousands of Tons[1]		
	N	P_2O_5	K_2O
1910	46	499	211
1920	228	660	257
1925	279	680	282
1930	377	793	354
1940	419	912	435
1941	458	993	467
1942	399	1,131	546
1943	508	1,238	643
1944	635	1,405	649
1945	641	1,435	753
1946	759	1,671	854
1947	835	1,775	878
1948	841	1,842	956
1949	911	1,884	1,065
1950	1,126	2,073	1,215
1951	1,265	2,091	1,413
1952	1,484	2,218	1,607
1953	1,648	2,209	1,720
1954	1,868	2,228	1,868

[1]Source: Agricultural Statistics 1955, U. S. Department of Agriculture (Washington: U. S. Government Printing Office, 1956).

TABLE 2

MICHIGAN FERTILIZER CONSUMPTION 1939-1955

Year	Primary Plant Nutrients in Tons[1]		
	N	P_2O_5	K_2O
1939	3,314	18,016	9,974
1940	3,931	19,672	11,074
1941	4,548	21,328	12,175
1942	4,991	34,730	19,303
1943	4,651	39,967	19,280
1944	7,223	36,647	19,628
1945	7,995	37,074	24,909
1946	9,235	51,291	26,096
1947	9,821	47,823	27,946
1948	9,498	56,361	32,186
1949	12,078	59,923	37,498
1950	14,494	66,746	45,171
1951	16,941	70,002	56,272
1952	21,794	75,937	66,513
1953	23,847	75,117	70,253
1954	30,190	76,277	74,172
1955	37,184	88,228	85,343

[1]Source: Michigan Agricultural Statistics, (Michigan Department of Agriculture, July, 1956). These estimates were made by the Soil Science Department at Michigan State University.

cash expenditure for Michigan would have been $11,155,200 for nitrogen, $17,645,600 for phosphorus, and $18,775,460 for potassium in 1955. The total cost for all three of the primary plant nutrients would have been $40,630,240 in 1954 and $47,576,260 in 1955.

Although not all fertilizer is used in production of agricultural crops, non-agricultural uses in Michigan were estimated[1] to be only about 5.3 percent of the total nitrogen, 2.1 percent of the total P_2O_5 and 0.9 percent of the total K_2O consumed. Estimates made in The 1954 Census of Agriculture[2] indicate the total expenditure for fertilizer for farm use in Michigan was only $31,163,000 in 1954. Consequently, at least a portion of the plant nutrients were purchased at prices less than those listed as typical. The estimated cost for the total on-farm consumption of the three primary plant nutrients for the entire United States was $1,024,105,000 in 1954.[3] Thus, farm expenditures on fertilizer exceeded a billion dollars in 1954 and was still increasing.

Reasons for Increased Fertilizer Use

Several reasons exist for increased use of commercial fertilizer by farmers. Plant nutrients have become much cheaper relative to most other farm inputs due primarily to a reduction in bulk and utilization

---

[1]Estimates made by W. H. Heneberry, Department of Agricultural Economics, Michigan State University.

[2]"Use and Expenditures for Fertilizer and Lime," adapted from The 1954 Census of Agriculture, (Washington: U. S. Government Printing Office, 1956).

[3]Ibid.

of more efficient manufacturing processes. Excluding transportation costs, the 1954-55 price of a unit of nitrogen was only about one-third of the adjusted 1920 price.[1] A unit of K_2O was only one-fifth of the adjusted 1920 price in 1954-55 while the adjusted price of a unit of P_2O_5 decreased about 27 percent during this 35-year period.

A second important reason for increased fertilizer use is the availability of more information concerning the yield benefits realized by various crops from application of the primary plant nutrients. This information has been forthcoming in increasing quantities from numerous sources. Experimental results from Agricultural Experiment Stations and private fertilizer companies have been utilized by farmers. Agencies such as the Federal Extension Service, the Tennessee Valley Authority and others have aided in providing farmers with educational materials and demonstrations of the effects of fertilizer on crop yields. In addition, farmers personal experiences with plant nutrients together with those of their neighbors are the basis for increased fertilizer use by many farmers.

It seems that we can validly conclude that commercial fertilizer is an important agricultural production factor as indicated by the fact that the value of the three primary plant nutrients used exceeded a billion dollars in 1954. It is a productive input used by a great number

---

[1]T. P. Hignett, "Our Changing Technology," Methodological Procedures in the Economic Analyses of Fertilizer Data, Edited by E. L. Baum, Earl O. Heady and John Blackmore (Ames: Iowa State College Press, 1956) p. 205.

of farmers producing a variety of crops. Farmers have greatly expanded fertilizer use in the past decade. They need additional information as to what expenditures for fertilizer are yielding in dollar returns. Such information is necessary if farmers are to allocate optimally their capital resources between alternative farm investments and expenditures.

Long-Run Agricultural Production Needs

One of the major problems currently facing American Agriculture is that of surpluses for some of the major farm crops. In view of this problem, a question arises as to the logic of engaging in research which could result in recommendations indicating greater use of commercial fertilizer, larger crop yields and greater total production.

Several studies have been made in which attempts have been made to forecast future needs for farm products in the United States. Predictions of future potential demands for agricultural products are all considerably higher than quantities supplied by current production. Two factors seem to be of primary importance in these higher predictions. First, large population increases have been predicted. Using the period 1951-53 as a base, predictions made by the Bureau of the Census in 1955[1] are for a population increase of 11 percent by 1960 and an increase of more than one-third of the base period population by 1975.

Secondly, large increases in consumer income accompanying an expanding economy have been predicted. Estimates made by the United States

---

[1]Bureau of the Census, Current Population Reports, Series P-25 No. 123 (Washington: U. S. Government Printing Office, October 20, 1955).

Department of Agriculture indicate an increase of real per capital consumer income of almost two-thirds greater than the 1951-53 base period by 1975.[1] Estimates of total crop production needs for 1975 are about 25 percent above actual 1951-53 production.[2] This overall increase is not uniformly distributed over all crops, however. For example, more than proportional increases are predicted for pasture and feed grain crops since a needed increase of 45 percent in livestock production is forecast. The needed average yearly increase in production of feed grains from the 1951-53 base period to 1975 is 5 1/2 times the historical average annual long-term increase.

No attempt will be made here to provide a comprehensive analysis of future agricultural production needs. Rather, the point being made here is that agricultural production needs will be much higher in the years ahead. This greater production must come from use of more resources, more productive resources and/or a more productive combination of resources.[3]

---

[1]H. H. Wooten and J. R. Anderson, "Agricultural Land Resources in the United States--with Special Reference to Present and Potential Cropland and Pasture," Agricultural Information Bulletin 140 (Washington: U. S. Department of Agriculture, June, 1955).

[2]G. T. Barton and R. O. Rogers, "Farm Output, Projected Changes and Projected Needs," Agricultural Information Bulletin No. 162 (Washington: Agricultural Research Service, August, 1956).

[3]A particularly critical problem currently faced by farmers and by farm management researchersis that of finding combinations and quantities of other resources which will increase the marginal value productivity of labor. Numerous farm management studies have indicated an extremely low marginal value product for this extremely important farm resource.

A discussion of the low marginal value productivity of labor as well as a bibliography of other work on this subject may be found in, E. I. Fuller, "Michigan Dairy Farm Organizations Designed to Use Labor Efficiently," Unpublished Masters Thesis, Department of Agricultural Economics, Michigan State University, 1957.

Research workers in the Agricultural Research Service[1] have made projections of probable increases in pasture and cropland acreage of 25 million acres by 1975 or an increase of about one million acres per year. If the projected increase in cropland occurs, the necessary annual increase in crop production per acre will still be about 50 percent larger than that occurring in the post World War II period.

In view of the long-run needs for farm products it is apparent that there will be a need for improved or increased use of farm resources in the next two decades. Improved information about the productivity of various resources, including fertilizer, will help farmers make the necessary production adjustments on an economical basis.

The Type of Information Needed by Farmers

In order to make economically sound decisions regarding how much and what analysis of fertilizer to use, farmers need rather specialized information. First, they need information on yield response to the three primary plant nutrients of the various crops which they produce. They need information about the affects of different forms of fertilizer and different application methods. In addition, this information must be applicable to their particular type of soil, the soil management practices which they use or should use, and the weather conditions which they encounter. Differences in the fertility level of the soil will influence the yields obtained by various amounts of applied plant nutrients; thus,

---

[1]H. H. Wooten and J. R. Anderson, op. cit.

effects of residual fertility need to be known. Finally, the price of fertilizer and the price of the crop produced will influence the high profit combination of plant nutrients to apply.

Specification of the type of information needed by farmers is a guide in determining what research is needed and what research procedures may be followed in obtaining this information. For example, the effects of different variables such as the effects of the various plant nutrients on crop yields, the effects of weather on yield responses and the significance of crop and fertilizer prices on optimal fertilizer use will be treated quite differently in the analysis. Applied plant nutrients can be measured and controlled and their effect on crop yields determined by statistical estimation. Weather cannot be controlled but if experimentation is carried out over a number of years and a variety of weather conditions, yield responses for several sets of weather conditions and a probability distribution of responses with respect to weather can be acquired. In the case of crops and fertilizers, various prices may be applied to the physical input-output relations to correspond with expected farm conditions.

Currently, researchers are attempting to devise methods for incorporating information about soil fertility acquired by chemical soil tests into their predictions of yield responses to fertilization. An attempt will be made in this thesis to reduce unexplained variances in crop yields by taking into account soil test data.

Succeeding chapters of the thesis will pertain to alternative analytical procedures, specification of the experimental work being carried on at the Michigan Agricultural Experiment Station, analysis of the experimental data and, finally, evaluation of the results.

CHAPTER II

METHODS OF ANALYSIS

It is the generally recognized task of scientific endeavor to establish and verify relationships which are universal to some population.[1] When relating various phenomena in the real world we find two dimensions of such relationships subject to variance. First, the relationships may vary with respect to the reliability of the empirical estimates which we can derive or establish for them, i.e., variance in the reliability dimension. Secondly, the size of the population to which such relationships are universal may vary considerably, i.e., variance in the application dimension. One would not expect, for example, to establish relationships between plant nutrients and crop yields as accurate or as general as those which have been established between the volume and pressure of gas as Boyle's law. However, if we believe that there are logical, systematic and describable relationships existing between plant nutrients and crop yields, it seems to be our task as scientists to attempt to quantify such relationships to the best of our ability. This is true particularly in view of the need for such information indicated in Chapter I. In so doing, an optimum level of

---

[1]Most of these relationships will of course be probability statements about relationships. Thus the universality referred to here does not imply absoluteness of the relationships specified, but rather implies universal applicability to some population of the deductions and inferences made.

accuracy of quantitative estimates can be defined by equating the cost of additional accuracy with its value. Failure to structure and quantify relationships systematically, when such action is possible, is likely to result in a failure to make optimum use of scientific procedure in developing a body of interpersonal information useful to researchers working on this and related problems of soil fertility and/or farm management.

Methods of Collecting Data

Two methods of securing data for use in determining the relationships existing between variables are generally recognized as being valid forms of scientific methodology. These are (1) controlled experimentation and measurement of relationships and (2) collection of non-controlled observations, as in astronomy, which typify the population being studied and to which relational inferences are to be made. Both of these two methods have advantages as well as some disadvantages which vary somewhat with the nature of the specific problem being studied. The discussion which follows is an attempt to evaluate the two procedures in the context where determination of plant nutrient input-crop yield output relationships is the problem being investigated.

The former method, controlled experimentation, has the relative advantage of lending itself to more precise estimation of relationships between relevant variables. Greater accuracy is usually obtained in controlled experimentation for two reasons. First, variables can be measured more accurately. Fertilizer applications and crop yields,

for example, can be measured quite accurately on experimental plots. Secondly, controls can be enforced quite rigorously; for example, tillage practices, insect infestations, soil characteristics, etc., can be controlled better on experimental plots than under farm conditions. Such controls, though facilitating accurate estimation of relationships between studied variables have an accompanying disadvantage. This disadvantage is that there is a possibility that no population other than the experimental one may have exactly the same combination of controlled and uncontrolled variables interacting in the production processes being studied. It follows that one may not be able to draw inferences from the experimental results and apply them validly to any given farm population. The alternative method, that of collecting non-controlled observations by a sample survey procedure, has proven effective in numerous types of research. It is difficult, however, to utilize this method when acquiring fertilizer response information because (1) differences in numerous uncontrollable factors such as insect damage, weather, tillage and harvesting methods are apt to bias the results or introduce excessive unexplained variance and (2) studied inputs are difficult to measure accurately. Another shortcoming of using the sample survey method in estimating fertilizer response surfaces is the difficulty of acquiring observations dispersed over the range and combination of plant nutrients necessary to obtain a statistically reliable estimate of the yield response surface. These and other problems have been

encountered by researchers working with non-experimental data.[1]

It is the opinion of most soil scientists and other researchers that the controlled experiment method of obtaining data is not only the more scientific method but the only one producing reliable estimates of fertilizer input-output functions. As experimental input-output data become available, a logical follow-up stage of analysis would be to test the applicability of these results under farm conditions. This procedure should indicate whether or not results obtained from the experimental sample may be validly inferred to some farm population.

The Concept of Functional Relationships

The principles utilized by economists in determining various optimal conditions of resource use and production output are stated in numerous publications by numerous authors. However, it seems desirable to outline briefly some of the principles of economic theory which can readily be applied to the production relationships of interest in agronomic-economic work. In order to apply effectively the deductive principles of economic theory, the relevant production relationships need to be specified rather systematically or formally.

---

[1]For a discussion of problems encountered and results obtained using non-experimental data in fertilizer input-crop output determinations see E. W. Kehrberg, "Some Problems Involved in Fitting Production Functions to Data Recorded by Soil-Testing Laboratories," Methodological Procedures in the Economic Analyses of Fertilizer Data, Edited by E. L. Baum, Earl O. Heady and John Blackmore (Ames: Iowa State College Press, 1956) pp. 134-140 and H. H. Yeh, "Estimating Input-Output Relationships for Wheat in Michigan Using Sampling Data, 1952-54, Unpublished Masters Thesis, Michigan State University, 1955.

Agronomists have hypothesized for years that plant nutrients and crop yields are functionally related. Numerous attempts have been made to specify these relationships in equation form for various crops and plant nutrients. In its simplest form, this functional relationship may be written

$$Y = f(X)$$

where Y is the crop yield and X the plant nutrient, in this example nitrogen. Recognizing that other factors interact with nitrogen, X_1, and are necessary for crop production, we write:

$$Y = f(X_1, X_2, \ldots, X_i, \ldots, X_n)$$

where X_1 represents nitrogen and X_2 to X_n are other factors such as P_2O_5, K_2O, water, temperature etc. To symbolize that all factors except nitrogen are fixed at some constant level, we write

$$Y = f(X_1/X_2, \ldots, X_i, \ldots, X_n).$$

Furthermore, if all factors affecting crop yields cannot be isolated and specified, we say

$$Y = f(X_1/X_2, \ldots, X_i, \ldots, X_n) + U$$

where U is an error term representing the unexplained variance of $\hat{Y}$ (predicted yield) from Y (observed yield).[1] If it can be validly

---

[1]If unexplained variance is to be validly attributed solely to components of the error term, U, the specified functional relationship must be the right one, i.e., it must be the real world functional relationship.

assumed that: (1) factors which contribute to U, i.e., unspecified factors, are normally and randomly distributed with respect to the measured variables (in this case X_1) and (2) that the expected value of U is zero, the existence of this unspecified source of yield variance does not bias statistical estimates of the influence of the observed variables on Y.

The specification of the functional relationship between plant nutrients and crop yields, commonly called a production function, has taken different forms over a period of years. Justice Von Liebig's "Law of the Minimum" was an early attempt to specify the form of fertilizer production functions. This formulation postulated that crop yields increased in direct proportion to additions of the nutrient which was limiting plant growth. Thus, other production factors were assumed to be perfect complements of the limiting factor. This formulation of the fertilizer-crop yield production function has been rejected because researchers have observed that: (1) production factors are not perfect complements, i.e., a given crop yield may be produced with varying quantities and combinations of applied N, P_2O_5, K_2O, water etc. and (2) additional inputs of a factor limiting crop yields does not typically result in linear additions to crop yields but rather it results in diminishing additions to crop yields for a time and eventually further additions of the factor cause an actual decrease in total yield.

Since Von Liebig's early formulation, numerous attempts have been made to use different forms of production functions to describe these input-output relationships. Although numerous types of functions have

been formulated, none has been accepted as "best." These various functions have received adequate discussion in other literature[1] and will not be analyzed here. There are, however, several criteria which must be satisfied by a particular function if it is to provide a realistic formulation of the input-output relationships between fertilizer inputs and crop outputs. The function should be capable of reflecting successively the following yield responses to added inputs of plant nutrients: (1) yields increasing at a diminishing rate and (2) decreasing total yields. If the soil is relatively low in initial fertility, an earlier stage of input-output relationship may be present. This is the stage where yields increase at an increasing rate in response to additional inputs of plant nutrients. In addition, if interaction between plant nutrients is expected, the formulation should include equational variables to specify this interaction.[2]

If the characteristics of plant growth and crop yields could be formulated theoretically to the extent that a proper equational form

---

[1]Historical description of use of production functions in estimating fertilizer-crop yield relations may be found in the following publications: John C. Redman and Stephen Q. Allen "Some Interrelationships of Economic and Agronomic Concepts," Journal of Farm Economics, Vol. XXXVI (August, 1954), pp. 453-465 and Earl O. Heady, John T. Pesek and William Brown, Crop Response Surfaces and Economic Optima in Fertilizer Use, Research Bulletin 424, (Ames: Agricultural Experiment Station, Iowa State College, 1955).

[2]Such interaction may be incorporated into the functional relationship in several ways. It is in a sense automatically included in a production function of product form such as an exponential. Special cross product terms may be included in a polynomial type equation. The point of importance is that it be included so that partial derivatives of yield with respect to individual plant nutrients reflects the level at which other interacting nutrients are considered.

could be deduced, statistical estimation of the production function would be greatly simplified. The statistical task would then be only that of estimating parameters for the variables in the functional relationship and obtaining reliability measures for these parameters. However, lacking a precise theory as to the proper functional form, it is necessary to compare various equations to see which "best" describes the observed relationships. Various problems of design and alternative analysis necessitated by lack of knowledge about the appropriate functional form will be developed later in this chapter.

Determining Economic Optima

After obtaining an estimate of the production function for plant nutrients, various optimal combinations of plant nutrients may be determined. If for example, the following equation:

$$Y = a + b_1X_1 + b_2X_1^2 + b_3X_2 + b_4X_2^2 + b_5X_3 + b_6X_3^2$$

describes the relation of yield to the three plant nutrients X_1, X_2, and X_3, then the following procedure is used to find the combination of plant nutrients producing the maximum yield.

Taking the partial derivatives of the three nutrients with respect to yield gives:

(1) $\frac{\partial Y}{\partial X_1} = b_1 + 2b_2X_1$

(2) $\frac{\partial Y}{\partial X_2} = b_3 + 2b_4X_2$

(3) $\frac{\partial Y}{\partial X_3} = b_5 + 2b_6X_3$.

Setting each partial derivative equal to zero and solving the three equations simultaneously gives the combination of plant nutrients producing the maximum crop yield. To obtain the economically optimal combination of plant nutrients the prices of plant nutrients, Px_i, where $i = 1, 2, 3$, and the product price, Py, need to be considered. These are considered in the profit equation, where π indicates profit, which follows:

$$\pi = Y\,Py - X_1\,Px_1 - X_2\,Px_2 - X_3\,Px_3 - FC.$$

This equation sets profit equal to the value of the product less the cost of the plant nutrients less fixed costs.

When utilizing unlimited resources, the high-profit combination of plant nutrients occurs where the marginal value product[1] of each, which is the value of the product produced by an additional unit of the input factor, is just equal to the cost of the nutrient input, i.e.,

$$\frac{\partial \pi}{\partial X_i} = 0.$$

This occurs where $$\frac{\partial Y}{\partial X_i} \cdot P_y = Px_i.$$

Dividing by Py gives $$\frac{\partial Y}{\partial X_i} = \frac{Px_i}{Py}$$

---

[1]Marginal value products presented later in this thesis do not include a value for residual fertility resulting from applied plant nutrients. The value of residual fertility should be included in the marginal value product, however, problems of measurement prohibit estimating such values at present.

which is the equational form of the partial derivatives which can be readily used in solving for high-profit plant nutrient inputs. Utilizing the partial derivatives of the previous example gives:

$$(1)\ b_1 + 2b_2X_1 = \frac{Px_1}{Py}$$

$$(2)\ b_3 + 2b_4X_2 = \frac{Px_2}{Py}$$

$$(3)\ b_5 + 2b_6X = \frac{Px_3}{Py}$$

Solving these three equations simultaneously gives the optimal combination of plant nutrients for a given set of product and factor prices. A second order condition is necessary to insure that the combination of nutrients is indeed an optimal one, i.e., one maximizing profits. The second partial derivatives of yield with respect to the various nutrients, $\frac{\partial^2 Y}{\partial X_1^2}$, must be negative, indicating that the marginal value productivity of each of the nutrients was decreasing at the point of optimal combination. Attainment of this second order condition is assured by the law of diminishing returns.

Alternative Types of Analysis

Several methods have been used by agronomists and economists to utilize experimental input-output data in analyses designed to predict optimal rates and combinations of plant nutrients. The two most common methods of analysis are (1) continuous function analysis and (2) analysis of variance. A recently developed method termed "discrete point" or

"form free" analysis has not been used extensively as yet.[1]

The existence of several analytical procedures provides two important implications to researchers. First it poses a problem of experimental design. An experimental design which provides satisfactory data for analysis of variance does not necessarily provide satisfactory nor even adequate data for continuous function analysis and conversely so. Secondly, the type of profit maximizing principles that can be applied and the type of inferences which can be made about the existence of optimum plant nutrient combinations vary considerably depending on the analysis used. The important alternative types of analysis will be developed briefly in the following paragraphs. Under ideal circumstances, the various types of analysis should provide similar estimates of high-profit applications of plant nutrients, i.e., the logic of alternative procedures is not conflicting nor inconsistent. However, the reliability and preciseness with which such optima may be specified varies considerably depending on the analysis used. This is particularly true when data are characterized by large amounts of unexplained variance and/or some of the functional forms used are inappropriate.

Continuous Function Analysis

The use of continuous function analysis assumes, essentially, that by using statistical estimating procedures one can obtain a sufficiently

---

[1]For a discussion of this general procedure, see C. Hildreth, "Point Estimates of Ordinates of Concave Functions," Journal of the American Statistical Association, Vol. 49 (September, 1954) pp. 598-619.

reliable estimate of the economically relevant portions of fertilizer-crop yield production surface to be able to predict input-output relationships at any relevant point on the surface. This assumption is made not only for the yield of a crop resulting from one plant nutrient variable with others fixed, but also for several plant nutrients in varying combinations. If the functional relationship between applied nitrogen and crop yields is specified as:

$$Y = a + b_1N + b_2N^2,$$

the parameters b_1 and b_2 are estimated statistically and are assumed to be valid over the range of observations from which they were estimated.

Two important problems involved in this procedure are those of: (1) specifying the correct form of the functional relationship and (2) acquiring a sufficient number of strategically located observations to permit reliable estimation of the parameters. As previously mentioned, selection of the proper functional form is a problem of considerable importance. Currently the consensus of opinion on this problem seems to be that: the function must allow for at least the latter two of the three stages of production i.e., yields (1) increasing at an increasing rate, (2) increasing at a decreasing rate, and (3) decreasing with additional fertilizer inputs. In some cases, the initial stage of production must also be described. Final selection of the proper functional form can be facilitated by statistical measures of the goodness of fit of the various functions to the observed data. Such tests are essentially of two types: (1) coefficients of multiple correlation and

multiple determination or other measures which compare the amount of variance explained by regression with the total amount present in the yield data and (2) standard errors of the parameters and of the prediction equation. These measures not only provide a measure of reliability of these statistics but also provide some insights as to the reliability of derivatives of the function. Earlier in this chapter it was pointed out that these derivatives are necessary in estimating optimum and maximum quantities of plant nutrient inputs. These objective tests may be supplemented by the researchers examination of the magnitude and distribution of residuals of observed from predicted yield values and his general familiarity with the data. Some statisticians would argue that statistical estimating procedures are being improperly used when the statistics derived are used to compare two or more functions in order to choose the best alternative. They would argue that the proper functional form should be established a priori to the fitting by utilizing theory, logic and experience, and the statistical estimation should only be used to estimate the parameters of the equation of proper form. However, lacking sufficient knowledge about the functional form of fertilizer-yield relationships it seems not only justifiable but also necessary to utilize statistical measures as aids in choosing the appropriate functions.

A second major task involved in continuous function analysis is that of designing the experiment to provide enough strategically located observations to permit reliable estimation of the parameters in the equation. The question, of, "when is a production surface adequately specified?" is a nebulous one to which no absolute answer is available.

However, some general bench marks may be established as to what constitutes adequate sampling of the production surface. Sampling only four points on a nitrogen-yield production surface without replication and fitting a function of the form

$$Y = a + b_1 N + b_2 N^2 + b_3 N^3$$

to the data from the four points may result in a good fit for locating the four points, i.e., variance of observations about the estimated surface may be small and the function may appear to fit quite well. However, we know that such estimations are not very reliable because: (1) as many parameters have been estimated as there are observations and consequently no degrees of freedom exist and (2) we realize intuitively from our empirical association with fertilizer-crop yield data that such a functional relationship is much too complex to be validly estimated from only four observations. Although no absolute criteria can be established for determining the experimental design which provides adequate data for continuous function analysis, some criteria can be set up relative to the design needs of alternative analytical procedures.

Relative to the data needed for analysis of variance, the design for functional analysis should include a more complete specification of the production surface, i.e., observations need to be spread more completely over the entire production surface. Regions of the production surface where one expects changing productivity of plant nutrients should be adequately sampled to lend sufficient reliability to estimates of the surface and its derivatives in these critical regions.

Particularly critical regions of the production surface are the origin and points of inflection of the function. Since reliability measures can be calculated for the estimate of the entire surface, replications of individual observations are not as valuable in the case of continuous functions as in the case of analysis of variance, i.e., we are not as interested in measuring significant differences between points on the surface, which requires replicating these points, as we are in obtaining a measure of the reliability of our estimate of the complete production surface and its derivatives. Omitting individual surface points from the design does not affect the reliability of the estimates appreciably. Thus the experimental design used can be flexible to the extent of allowing the use of incomplete factorials or other incompletely specified designs. Very complex functions may be fitted to the data since each added parameter uses only one degree of freedom which is of little consequence in any experiment containing numerous observations. Complications in calculations, however, impose practical limits on the complexity of functions which can be used. In addition, the more complex the function, the more difficult it is to approximate a reliability measure of the partial derivative of the yield estimate with respect to individual plant nutrients,[1] i.e., $\sigma \frac{\partial Y}{\partial X_i}$.

---

[1]The general problem of acquiring estimates of the reliability of yield derivatives is discussed in more detail in Chapter IV.

Analysis of Variance

Historically, the procedure most commonly used by agronomists in analyzing fertilizer input-output data has been analysis of variance. Essentially, this procedure treats individual points on the production surface independently, the objective being to determine whether or not the yield differences between points are statistically significant. Typically no assumption is made as to the shape of the surface between points, nor is any such inference valid from the analytical procedures used. However, one can test for significant differences between surface points assuming different functional forms. For example, a test may be made for significant differences between production surface points under the hypothesis that the surface is flat i.e., that relationships between the yield variable and plant nutrient variables are linear or, alternatively, that they are of quadratic or cubic form. Profit maximization principles can only be applied to selection of the most profitable combination of plant nutrients observed, whereas the optimal combination may fall somewhere between two observed points. One can assume a linear relation between sampled points and interpolate on this basis, however, such interpolation has only subjective justification. Linear interpolation for short distances on the production surface, i.e., between closely spaced points, may be justifiable. However, in order to have points closely spaced and, simultaneously, to have individual point estimates of sufficient reliability requires numerous points and numerous replications.

Experimental designs which provide satisfactory data for use of analysis of variance procedures are less flexible than those satisfactory for continuous function analysis. A complete factorial design is necessary for a comprehensive analysis of variance study. Replications of individual surface points are almost a necessity for testing for significant differences involving within treatment variance for most fertilizer input-output determinations. As in the case of functional analysis, interaction between nutrients can be tested as well as the individual effects of single plant nutrients. For example, with three plant nutrients and assuming linear relationships, seven parameters can be tested for significance. These seven parameters are indicated by the b_i's in the following equation in which the mean of the observation is represented by ξ and the three plant nutrients by N, P and K:

$$\xi_{NPK} = \xi + b_1N + b_2P + b_3K + b_4NP + b_5NK + b_6PN + b_7NPK.$$

Other data such as quality determinations, nutrient deficiency determinations, etc., may be a desired by-product of the input-output study. Such factors may be characterized by different relationships than the curvilinear relations expected for plant nutrient input-crop yield output relations. In this case, replication within the overall continuous function design, of a factorial, to provide data suitable for analysis of variance treatment of these factors on a limited scale may be desirable. It is the conclusion of the author that when the data produced from an experiment are to be used in estimating production surfaces, the advantages gained from sampling additional surface points outweighs that

of obtaining more than two or three replications at any given point. However, once committed to a non-factorial experiment with few replications one has difficulty in utilizing analysis of variance.

Discrete Point Analysis

A method has recently been developed[1] to predict optimal fertilization rates without imposing a specified functional form to the plant nutrient-crop yield relationships. This procedure is a form of discrete point analysis referred to as form free functional analysis. This type of analysis has the advantage of not necessitating as many a priori assumptions as to the form of the production function and suffering the consequences of being wrong. There is, however, no assurance that this method of analysis will provide as good a characterization of the actual production surface as would a functional form with more a priori assumptions. When evaluating the relative merits of discrete point analysis vs those of continuous function analysis, an important factor to consider is the extent to which a theory of yield response to plant nutrient inputs has been formalized. In general, the more developed the theory, the greater would appear to be the justification for imposing additional restrictions on the functional form. The procedure of form free analysis consists of making simple logical assumptions about the input-output relationships, such as diminishing returns to inputs, i.e., concavity of the production surface, and then solving for the optimal surface point, i.e., the optimal fertilizer treatment.

---

[1] Ibid.

Discrete point analysis has the limitation, as do analysis of variance procedures, in limiting optimal points to observed points with no basis for interpolation in between. This is to say the analysis provides a choice of the optimal fertilizer treatment from those included in the experiment with a given set of restrictions as to shape of the surface, crop and fertilizer prices, etc. Rather complete specification of the surface overcomes the interpolation difficulty to some extent but requires experiments with a large number of observations which may become impractical because of the land and labor requirements necessary to conduct them. In addition, reliance on the validity of estimates of individual surface points almost necessitates replication of these points to insure the accuracy of their observed values.

Additional elucidation and empirical testing of this procedure may result in its wider application in the future. However, firstly, because of the limitations and difficulties of alternative procedures, and, secondly, because the experiments were designed specifically for this type of analysis, the analysis in this thesis will be limited to that of continuous functions.

CHAPTER III

EXPERIMENTAL WORK CONDUCTED BY THE MICHIGAN AGRICULTURAL EXPERIMENT STATION

Several experiments are currently being conducted by the Michigan Experiment Station to determine fertilizer input-crop output relationships. The first of these experiments was initiated in the spring of 1954 and additional experiments have been added since. Currently, input-output experiments are being conducted for potatoes, corn, wheat, oats, alfalfa and field beans. In addition, an input-output study for sugar beets was initiated in 1957. Experimental work is conducted by the Department of Soil Science with the Department of Agricultural Economics cooperating on design of the experiments and analysis of the data.[1] This chapter of the dissertation will outline and describe the experimental work being conducted in Michigan.

Characteristics of Experimental Designs

The fertilizer input-output experiments being conducted in Michigan have been designed primarily with the objective of providing adequate data for estimation of the respective crop yield surfaces for different

---

[1]Several interested agencies have aided the experimental work by contributing funds or other resources, these include: The National Plant Food Council, The Davison Chemical Corporation and The Tennessee Valley Authority.

crops. This objective, together with the restriction of limited funds for experimentation, provide the main restrictions for the framework within which the experimental designs were developed. In conforming to these restrictions the designs have the following general characteristics: (1) individual observations cover those portions of the production surfaces of interest to researchers, (2) the experiments contain a minimum number of replicated plots since the objective is to estimate the entire surface over the range in which it is of economic importance thus minimizing the need for establishing accurate measurements of individual surface points (3) the designs involve numerous check plots (plots to which no fertilizer is applied) to establish the origin of fitted functions, i.e., the yield value with no plant nutrients applied and (4) to the extent possible, intercorrelations among the amounts of nutrients applied have been minimized to facilitate estimation of the equational parameters with greater reliability than would be the case if such intercorrelations were high. The designs vary somewhat for different experiments but may be broadly classified as incomplete factorials.

The Oats, Wheat, Alfalfa and Corn Rotation

In the spring of 1955 an experiment was initiated for a rotation of oats, wheat, alfalfa and corn. This experiment is located at two sites in Kalamazoo and Calhoun counties on a Kalamazoo sandy loam soil. This is a light upland soil having a tendency to be somewhat drouthy and of relatively low natural fertility. Each crop of the rotation is

grown each year; thus there are four fields each having the same experimental design. The experiment includes all three of the primary plant nutrients, nitrogen, phosphoric acid and potash in varying combinations. Six treatment levels, including the zero application level, are included in the experiment for each of the plant nutrients. These treatment levels measured in pounds per acre are:

N	-	0	20	40	80	160	240
P_2O_5	-	0	40	80	160	320	480
K_2O	-	0	20	40	80	160	240

Ninety-one individual surface points are sampled, twenty-seven of which are replicated twice in a 3 x 3 x 3 factorial at the 2nd, 4th and 6th treatment levels. There are eleven replications of the check (0,0,0,) treatment.

There are one hundred and thirty plots in each of the four fields in the experiment.

Individual plots are 50 x 14 feet in size, making a total area per plot of about 1/62.5 of an acre. The 14 foot width facilitates use of a 7 foot grain drill for fertilizer and seed application and a 7 foot self propelled combine for harvesting operations. Almost all fertilizer applications are made by broadcasting the fertilizer, either mechanically or by hand, prior to plowing the ground and preparatory to planting the crop. Two notable exceptions are: (1) the first level of applied P_2O_5 (40 pounds per acre) is applied in the row at planting time as a starter

fertilizer and (2) the alfalfa crop is fertilized by top-dressing in the spring. The design for this experiment is shown in detail in Table 3.

Continuous Corn

An experiment in which corn is grown in continuous culture was initiated in Tuscola County in 1956. This experiment is located on a Wisner clay loam soil which is one of the heavier, more productive soils occurring in the state. The experiment contains 204 individual plots representing 139 surface points. Included in the design is a 3 x 3 x 3 factorial replicated three times including observations at the 2nd, 4th, and 6th treatment levels. In addition, there are eight check plots. Inclusion of the triplicated factorial allows limited study of yields and other experimental data by analysis of variance techniques. The seven treatment levels in pounds per acre for the three plant nutrients in this experiment are as follows:

N	-	0	20	40	80	160	240	320
P_2O_5	-	0	40	80	160	320	480	640
K_2O	-	0	20	40	80	160	240	320

Individual plots are 55 x 14 feet in size allowing 4 rows of corn spaced 42 inches apart to be grown on each plot. The design for this experiment is shown in detail in Table 4.

TABLE 3

EXPERIMENTAL DESIGN FOR THE OATS, WHEAT, ALFALFA AND CORN ROTATION

Plant Nutrients (Pounds Per Acre)			No. of	Plant Nutrients (Pounds Per Acre)			No. of
N	P_2O_5	K_2O	Plots	N	P_2O_5	K_2O	Plots
0	0	0	11	80	160	80	2
0	0	0	1	80	160	240	2
0	40	20	1	80	320	40	1
0	160	0	1	80	320	160	1
0	160	80	1	80	320	240	1
0	160	240	1	80	480	0	1
0	480	80	1	80	480	20	2
0	480	240	1	80	480	80	2
20	0	20	1	80	480	160	1
20	40	0	1	80	480	240	2
20	40	20	2	160	40	40	1
20	40	80	2	160	40	160	1
20	40	240	2	160	80	20	1
20	80	40	1	160	80	40	1
20	80	160	1	160	80	80	1
20	160	20	2	160	80	240	1
20	160	80	2	160	160	40	1
20	160	240	2	160	160	160	1
20	320	40	1	160	160	240	1
20	320	160	1	160	320	20	1
20	480	20	2	160	320	80	1
20	480	80	2	160	320	160	2
20	480	240	2	160	320	240	1
40	40	40	1	160	320	40	1
40	40	160	1	160	480	80	1
40	80	20	1	160	480	160	1
40	80	40	2	160	480	240	1
40	80	80	1	240	0	80	1
40	80	240	1	240	0	240	1
40	160	40	1	240	40	20	2
40	160	160	1	240	40	80	2
40	320	20	1	240	40	240	2
40	320	80	1	240	80	160	1
40	320	240	1	240	160	20	2
40	480	40	1	240	160	80	2
40	480	160	1	240	160	240	2
80	0	0	1	240	320	0	1
80	0	80	1	240	320	40	1
80	0	240	1	240	320	160	1
80	40	20	2	240	320	240	1
80	40	80	2	240	480	0	1
80	40	240	2	240	480	20	2
80	80	40	1	240	480	80	2
80	80	160	1	240	480	160	1
80	160	0	1	240	480	240	2
80	160	20	2				

TABLE 4

EXPERIMENTAL DESIGN FOR THE CONTINUOUS CORN EXPERIMENT

Plant Nutrients (Pounds Per Acre)			No. of	Plant Nutrients (Pounds Per Acre)			No. of
N	P_2O_5	K_2O	Plots	N	P_2O_5	K_2O	Plots
0	0	0	0	40	320	80	1
0	0	40	1	40	320	160	1
0	40	240	1	40	320	320	1
0	80	0	1	40	480	40	1
0	80	40	1	40	480	80	1
0	160	320	1	40	480	240	1
0	320	160	1	40	640	20	1
0	480	20	1	40	640	160	1
0	640	80	1	40	640	320	1
0	640	320	1	80	0	0	1
20	40	20	3	80	0	160	1
20	40	80	3	80	40	20	3
20	40	160	1	80	40	80	3
20	40	240	3	80	40	240	3
20	80	20	1	80	80	40	1
20	80	80	1	80	80	160	1
20	80	240	1	80	80	240	1
20	160	20	3	80	160	20	3
20	160	40	1	80	160	80	3
20	160	80	3	80	160	240	3
20	160	240	3	80	160	320	1
20	320	20	1	80	320	0	1
20	320	160	1	80	320	40	1
20	320	320	1	80	320	160	1
20	480	20	3	80	480	20	3
20	480	40	1	80	480	40	1
20	480	80	3	80	480	80	3
20	480	240	3	80	480	240	3
20	640	160	1	80	640	80	1
20	640	320	1	80	640	320	1
40	0	0	1	160	0	20	1
40	0	40	1	160	0	80	1
40	40	20	1	160	40	0	1
40	40	40	1	160	40	80	1
40	40	80	1	160	40	240	1
40	40	160	1	160	80	40	1
40	40	320	1	160	80	160	1
40	80	0	1	160	160	0	1
40	80	40	2	160	160	20	1
40	80	240	1	160	160	80	1
40	160	20	1	160	160	160	1
40	160	80	1	160	160	240	1
40	160	160	1	160	160	320	1
40	160	240	1	160	320	40	1
40	320	20	1	160	320	80	1
40	320	40	1	160	320	160	2

Continued

Table 4 concluded

Plant Nutrients (Pounds Per Acre) N	P_2O_5	K_2O	No. of Plots	Plant Nutrients (Pounds Per Acre) N	P_2O_5	K_2O	No. of Plots
160	480	20	1	240	640	160	1
160	480	80	1	240	640	320	1
160	480	320	1	320	0	80	1
160	640	80	1	320	40	20	1
160	640	240	1	320	40	240	1
240	0	0	1	320	80	40	1
240	0	40	1	320	80	80	1
240	40	20	3	320	80	160	1
240	40	80	3	320	80	240	1
240	40	240	3	320	80	320	1
240	80	0	1	320	160	0	1
240	80	40	1	320	160	40	1
240	80	160	1	320	160	320	1
240	80	320	1	320	320	40	1
240	160	20	3	320	320	80	1
240	160	80	3	320	320	160	1
240	160	240	3	320	320	240	1
240	320	40	1	320	480	20	1
240	320	160	1	320	480	160	1
240	320	320	1	320	480	320	1
240	480	20	3	320	640	40	1
240	480	80	3	320	640	80	1
240	480	240	3	320	640	240	1
240	640	40	1	320	640	320	2

Field Beans, Wheat and Corn Rotation

An intensive rotation of field beans, wheat and corn was initiated in Gratiot county in 1955. Corn was produced on these plots in 1955 and field beans in 1956. The experiment is located on a Simms loam soil, a heavy productive soil which can be cropped quite intensively without hazard of erosion damage. The seven treatment levels for the three plant nutrients are identical to those in the continuous corn experiment.

The treatments in pounds per acre of applied plant nutrients are:

N -	0	20	40	80	160	240	320
P_2O_5 -	0	40	80	160	320	480	640
K_2O -	0	20	40	80	160	240	320

The experiment is an incomplete factorial consisting of 193 individual surface points of which twenty-seven are replicated twice in a 3 x 3 x 3 factorial at the 1st, 4th, and 6th treatment levels. There are 11 check plots in the basic experimental design which contains a total of 233 individual plots. Extra plots were included in the experiment for purposes of other analyses bringing the total number of plots to 258. Individual plots in this experiment are 50 x 14 feet in size.

The design for this experiment includes a more complete specification of the production surface than any of the other experiments with the total of 193 different surface points exceeding that of any other experiment currently being conducted. The experimental design for this experiment is shown in Table 5.

Potatoes

Two experiments have been established to measure the response of potatoes to variable quantities of applied P_2O_5 and K_2O. The first of these experiments was initiated in 1954 and the second in 1956. Both experiments are being conducted on a Houghton muck soil on the Experiment Station muck farm near East Lansing. Only two plant nutrients, P_2O_5 and K_2O are treated as variables in this experiment. The muck soil is high in organic matter content and consequently high in nitrogen. Available

TABLE 5

EXPERIMENTAL DESIGN FOR THE BEANS, WHEAT AND CORN ROTATION

Plant Nutrients (Pounds Per Acre)			No. of	Plant Nutrients (Pounds Per Acre)			No. of
N	P_2O_5	K_2O	Plots	N	P_2O_5	K_2O	Plots
0	0	0	11	20	320	160	2
0	0	20	1	20	320	320	2
0	0	40	1	20	480	20	1
0	0	80	1	20	480	40	1
0	0	160	1	20	480	160	1
0	0	240	1	20	480	320	1
0	0	320	1	20	640	20	2
0	40	0	1	20	640	40	1
0	40	240	1	20	640	80	1
0	80	0	1	20	640	160	2
0	80	40	1	20	640	240	1
0	160	0	1	20	640	320	2
0	160	320	1	40	0	0	1
0	320	0	1	40	0	40	1
0	320	160	1	40	40	20	1
0	480	0	1	40	40	80	1
0	480	20	1	40	40	160	1
0	640	0	1	40	40	320	1
0	640	80	1	40	80	0	1
0	640	320	1	40	80	40	2
20	0	0	1	40	80	240	1
20	40	20	2	40	80	320	1
20	40	40	1	40	160	20	1
20	40	80	1	40	160	80	1
20	40	160	2	40	160	160	1
20	40	240	1	40	160	320	1
20	40	320	2	40	320	20	1
20	80	20	1	40	320	80	1
20	80	80	1	40	320	160	1
20	80	160	1	40	320	320	1
20	80	240	1	40	480	40	1
20	80	320	1	40	480	80	1
20	160	20	1	40	480	240	1
20	160	40	1	40	640	20	1
20	160	160	1	40	640	80	1
20	160	240	1	40	640	160	1
20	160	320	1	40	640	320	1
20	320	20	2	80	0	0	1
20	320	40	1	80	0	320	1
20	320	80	1	80	40	20	1

Continued

Table 5 continued

Plant Nutrients (Pounds Per Acre) N	P_2O_5	K_2O	No. of Plots	Plant Nutrients (Pounds Per Acre) N	P_2O_5	K_2O	No. of Plots
80	40	80	1	160	320	20	2
80	40	160	1	160	320	40	1
80	40	320	1	160	320	80	1
80	80	40	1	160	320	160	2
80	80	240	1	160	320	240	1
80	80	320	1	160	320	320	2
80	160	20	1	160	480	20	1
80	160	80	2	160	480	40	1
80	160	160	1	160	480	160	1
80	320	20	1	160	480	320	1
80	320	80	1	160	640	20	2
80	320	160	1	160	640	40	1
80	320	320	1	160	640	80	1
80	480	40	1	160	640	160	2
80	480	240	1	160	640	240	1
80	480	320	1	160	640	320	2
80	640	0	1	240	0	0	1
80	640	20	1	240	40	20	1
80	640	40	1	240	40	80	1
80	640	160	1	240	40	160	1
80	640	240	1	240	40	320	1
80	640	320	1	240	80	40	1
160	0	0	1	240	80	320	1
160	0	160	1	240	160	20	1
160	40	20	2	240	160	80	1
160	40	40	1	240	160	160	1
160	40	80	1	240	160	240	1
160	40	160	2	240	320	20	1
160	40	240	1	240	320	80	1
160	40	320	2	240	480	160	1
160	80	20	1	240	320	640	1
160	80	80	1	240	480	40	1
160	80	160	1	240	480	80	1
160	80	240	1	240	480	240	2
160	80	320	1	240	640	20	1
160	160	20	1	240	640	80	1
160	160	40	1	240	640	160	1
160	160	160	1	240	640	320	1
160	160	320	1	320	0	0	1
160	320	0	1	320	0	80	1

Continued

Table 5 concluded

Plant Nutrients (Pounds Per Acre) N	P_2O_5	K_2O	No. of Plots	Plant Nutrients (Pounds Per Acre) N	P_2O_5	K_2O	No. of Plots
320	0	320	1	320	320	40	1
320	40	20	2	320	320	80	1
320	40	40	1	320	320	160	2
320	40	80	1	320	320	240	1
320	40	160	2	320	320	320	2
320	40	240	1	320	480	20	1
320	40	320	2	320	480	40	1
320	80	20	1	320	480	160	1
320	80	80	1	320	480	320	1
320	80	160	1	320	640	0	1
320	80	240	1	320	640	20	2
320	160	0	1	320	640	40	1
320	160	20	1	320	640	80	1
320	160	40	1	320	640	160	2
320	160	160	1	320	640	240	1
320	160	320	1	320	640	320	2
320	320	20	2				

nitrogen may be in short supply at a given time but largely because of weather conditions not conducive to sufficiently rapid nitrification. Thus, it usually does not pay to apply commercial nitrogen fertilizers unless temporary nitrogen deficiencies are evident. Furthermore, if nitrogen applications are made, the amount of applied nitrogen is, at best, a poor indicator of the amount of nitrogen available for plant use.

The 1954 Potato Experiment

The experiment initiated in 1954[1] consists of an incomplete factorial

[1]The experimental design and treatment rates used in this experiment were established by Professors G. L. Johnson and J. F. Davis of the Michigan Agricultural Experiment Station.

of seven levels of P_2O_5 and nine levels of K_2O. The treatment rates in pounds per acre are as follows:

P_2O_5 -	1	25	50	100	200	300	450		
K_2O -	1	25	50	100	200	350	550	750	900

The design includes forty-seven surface points, twenty-nine of which are replicated twice for a total of seventy-six plots in the experiment. Individual plots are 49 by 11 feet in size.

Application of a single pound of a plant nutrient to some plots constituted a substitution for the zero treatment level. Utilizing a one pound treatment alleviated the problem of having to use negative logarithms when fitting exponential functions to zero treatments. The design for this experiment is presented in detail in Table 6.

After the first potato crop was produced on these plots, the plots were split and half of the plot continued to receive the original fertilizer treatment while the other half of the plot received no fertilizer in subsequent years. This procedure was practiced because of complications due to the high fertility level of the land at the time the experiment was initiated. This original high fertility level resulted in negligible yield changes with additional applications of plant nutrients. Continuous soil testing of these experimental plots in succeeding years should provide valuable information with respect to residual fertility values since over time a wide range of fertility levels should develop on these plots.

TABLE 6

EXPERIMENTAL DESIGN FOR THE 1954 POTATO EXPERIMENT

K_2O Treatment (pounds per acre)	P_2O_5 Treatment[1] (pounds per acre)						
	1	25	50	100	200	300	450
1	X			X			X
25		XX	XX	XX	XX	XX	X
50		XX	X	XX	X	XX	
100		XX	X	XX	X	XX	
200	X	XX	XX	XX	XX	XX	X
350		XX	X	XX	X	XX	
550		XX	X	XX	X	XX	
750		XX	XX	XX	XX	XX	XX
900	X	X		X		XX	X

[1]Each X represents one experimental plot.

The 1956 Potato Experiment

A new potato experiment was initiated in 1956. This experiment was established on a newly cleared muck soil which had not been previously farmed and which had not received previous applications of plant nutrients. The experimental design includes 46 different surface points. Replications and check plots bring the total number of plots to 114. All surface points are replicated twice with the exception of a 4 x 4 triplicated factorial and 4 check plots.

The design utilized in this experiment is basically an incomplete factorial but it also includes several additional features. Included in the design are:

(1) A 4 x 4 triplicated factorial. The treatment levels included in this factorial are the following in pounds of plant nutrients per acre:

P_2O_5	-	100	200	300	400
K_2O	-	200	400	600	800

Inclusion of this triplicated factorial allows limited study by analysis of variance procedures. The experiment produces a large amount of useful agronomic data as a by-product of the basic input-output study. Such data includes information on the quality and chemical compositions of the product, plant characteristics, residual fertility, etc. Analysis of these data by continuous function analysis may not be feasible or appropriate.[1] By including a triplicated factorial in the experimental design, analysis of variance treatment of such data is facilitated at a small additional cost.

A 3 x 3 composite design[2] is included in the experiment for the

---

[1]For example, protein content of wheat may increase linearly or curvilinearly with additional nitrogen inputs up to some maximum value and then remain unchanged with additional nitrogen inputs. In such an event, continuous function analysis might not be the appropriate means of analyzing data to acquire determinations of quality differences.

[2]This design is described by R. L. Anderson in "A Comparison of Discrete and Continuous Models in Agricultural Production Analysis," Methodological Procedures in the Economic Analyses of Fertilizer Data, Edited by E. L. Baum, Earl O. Heady and John Blackmore (Ames: Iowa State College Press, 1956) p. 49.

purpose of comparing it with a 3 x 3 factorial design as to its effectiveness as a basis for estimating the response surface by least squares techniques. The treatments used for this comparison are shown in Table 7.

TABLE 7

EXPERIMENTAL DESIGN FOR THE 1956 POTATO EXPERIMENT

Plant Nutrients (Pounds Per Acre) P_2O_5	K_2O	No. of Plots	Plant Nutrients (Pounds Per Acre) P_2O_5	K_2O	No. of Plots
0	0	4	200	600	3
0	200	2	200	700	2
25	25	2	200	800	3
25	75	2	250	250	2
50	50	2	250	400	2
50	100	2	250	500	2
50	150	2	250	600	2
50	400	2	250	750	2
75	225	2	300	200	3
100	0	2	300	300	2
100	100	2	300	400	3
100	200	3	300	600	3
100	300	2	300	700	2
100	400	3	300	800	3
100	600	3	300	900	2
100	800	3	350	350	2
150	150	2	350	400	2
150	300	2	350	500	2
150	450	2	350	700	2
150	600	2	400	200	3
200	100	2	400	400	3
200	200	3	400	600	3
200	400	3	400	800	3
200	500	2	400	900	2

(3) Points on three constant-proportion P_2O_5-K_2O diagonals are sufficiently sampled to permit estimation of these diagonals individually. These are the diagonals in the experimental design in which P_2O_5 and K_2O are applied in 1:3, 1:2 and 1:1 proportions respectively. Estimation of yield response along these constant proportion of P_2O_5 and K_2O diagonals allows comparison of these estimates with those derived from estimates of the entire surface.

The Total Experimental Program

Exclusive of the sugar beet experiment initiated in 1957, the nutrient level experiments described in this chapter contain about 1150 individual plots.[1] Relative to experimental work undertaken elsewhere, this is an elaborate project. Over 20 acres of land are required for the experimental work. Acquiring soil samples, plant tissue samples, crop yields and quality determinations for the various crops are tasks entailing large labor inputs. In addition, chemical analysis as well as tabulation and computational analysis of these data are time consuming undertakings.

In addition to the basic input-output determinations, the experiments produce a large amount of by-product data of interest to agronomists and economists. For example, data acquired from these experiments are being utilized to compare alternative methods of testing soil for

---

[1]Numerous other experiments are conducted by the Michigan Agricultural Experiment Station, many of which also provide data for fertilizer input-output determinations.

residual quantities of the three plant nutrients. The influence of various plant nutrient treatments on the quality of crops produced is being studied utilizing data from these experiments. A comparison of experimental results from field and greenhouse experiments is being conducted in conjunction with the basic input-output studies. In brief, the experiments described in this chapter produce a wealth of data which is being used for a diversity of research projects.

Data produced from the experiments described in this chapter from 1954-56 were used for the analysis conducted in Chapter IV.

CHAPTER IV

ANALYSIS OF THE DATA

The Oats, Wheat, Alfalfa and Corn Rotation

The oats, wheat, alfalfa and corn rotation experiment was initiated in 1955 and data have been collected for two years. Only two harvested crops were produced in 1955 as alfalfa and wheat stands could not be established in time for harvest during the 1955 crop year. Yield data were acquired for both corn and oats in 1955 and all four crops were produced in 1956. Due to a heterogeneous stand of alfalfa, no data were acquired for that crop in 1956.

Analysis of the Oats Data

Oats were produced on two of the experimental sites in Calhoun and Kalamazoo counties in 1955. Preliminary graphic analysis of these data indicated that the variance present in the yield data was not associated with different quantities of applied plant nutrients. This hypothesis was further substantiated by fitting a polynomial equation to the data. The equation fitted was of the type

$$Y = a + b_1 N + b_2 N^2 + b_3 P + b_4 P^2 + b_5 K + b_6 K^2 + b_7 NP + b_8 NK + b_9 PK.$$

The variables N, P and K represent per acre applications of N, P_2O_5

and K_2O, respectively. None of the variables in this equation had estimated parameters significantly different from zero. Apparently weather conditions were the main determinants limiting crop yields during the 1955 crop growing season. Unfavorable weather conditions, largely the result of a late summer drouth, prevented crop yield increases which might have occurred with increased applications of plant nutrients under more favorable weather conditions.

Yield data for oats were acquired again in 1956. Preliminary graphic analysis of these data indicated that positive relationships existed between oat yields and applied N and P_2O_5. Furthermore, these relationships appeared to be curvilinear, reflecting diminishing returns to plant nutrient inputs.

The first formulation of the functional relationship which was attempted for the 1956 data was a polynominal equation identical to the one fitted to the 1955 data. This polynomial contains first and second degree terms for N, P_2O_5 and K_2O and first degree, cross-product terms for all nutrients taken two at a time. This formulation containing the estimated parameters is shown in equation I. Values listed below the estimated parameters and included in parentheses are standard errors of the respective parameters. N, P and K represent per acre applications of N, P_2O_5 and K_2O respectively as is the case in all equations unless otherwise indicated.

<u>Equation (I)</u>: $\hat{Y}_o = 43.326378 + .40112190\ N - .00130761\ N^2 - .00650205\ P$
$(.05115313) \quad (.0019075) \quad (.02579697)$

$+ .00000534\ P^2 + .06186818\ K - .00010387\ K^2 + .00000068\ NP - .00010905\ NK$
$(.00004775) \quad (.0519654\text{8}) \quad (.00019148) \quad (.00006495) \quad (.0001302\text{C})$

$+ .00007542\ PK$
$(.00006430)$

The adjusted coefficient of multiple correlation for this equation was .690. The coefficient of multiple determination indicated that about 48 per cent of the variance in crop yields was associated with variance explained by the regression equation. Estimated coefficients for the nitrogen variables were significant at the one per cent probability level. None of the coefficients for other variables were significant at the ten per cent level of probability.

Because of the large amount of variance not associated with regression and the non-significant coefficients which were estimated for several variables, a second formulation of the production function relationship was attempted. This formulation was an exponential equation of the Carter-Halter[1] type. This exponential equation is quite flexible depending on the magnitude of parameters estimated for the variables. In addition to retaining the curvilinear properties postulated to exist in fertilizer input-crop output relationships, use of this equation facilitates estimation of input-output relationships ranging over all three stages of production, i.e., returns to additional plant nutrients which (1) increase at an increasing rate (2) increase at a decreasing rate and (3) become negative. This formulation in equational form is:

$$Y = aN^{b_1} c_1^N P^{b_2 c_2^P} K^{b_3 c_3^K}$$

By taking the logarithm of this equation, we can acquire an equational form of this relationship for which the parameters can be estimated by

---

[1]The usefulness of this equation as a production function formulation was first noted by H. O. Carter and A. N. Halter.

the technique of least squares. The form in which this equation is fitted statistically is:

$$\text{Log } Y = \log a + b_1 \log N + N \log c_1 + b_2 \log P + P \log c_2 + b_3 \log K + K \log c_3.$$

The equation with estimated parameters is shown in Equation II.

Equation (II): $$\text{Log } \hat{Y}_0 = 1.57315152 + \underset{(.02022815)}{.16475028} \log N - \underset{(.00015046)}{.00057687} N -$$

$$\underset{(.01648010)}{.02441092} \log P + \underset{(.00006694)}{.000096095838} P + \underset{(.02017332)}{.063434487} \log K + \underset{(.00014714)}{.000217567362} K$$

The coefficient of multiple correlation for this equation was .760. The coefficient of multiple determination indicated that about 58 per cent of the variance in oat yields was associated with regression.

In this equation, coefficients for nitrogen variables were significant at the five per cent probability level as was the coefficient for the first phosphorus variable.

Testing the significance of coefficients for individual variables in an equation which contains more than one variable for a given plant nutrient is a practice of limited usefulness. The related variables in an equation such as N, N^2, log N, etc., are obviously highly correlated. Estimates of individual parameters may be subject to large standard errors reflecting these high intercorrelations. One might conclude that since individual parameters are not statistically significant, no significant effects are present. This conclusion might well be fallacious.

If the aggregate effect of all variables representing a particular plant nutrient could be tested for significance, the test might indicate a significant aggregate effect. This situation illustrates an inadequacy of current statistical testing procedures. In cases where (1) two or more independent variables in a production function occur in product form or (2) more than one variable is used to measure the effects of a particular plant nutrient, it would be desirable to obtain a reliability measure on the derivative of crop yield with respect to individual plant nutrients. Such derivatives are necessarily utilized in determining marginal nutrient effects and consequently optimal applications of plant nutrients. A satisfactory procedure for computing reliability measures for such derivatives has not yet been developed but is a critical need in much analytical production economics work.

Interpretation of the Statistical Results

It appears desirable to investigate several aspects of the two alternative production function formulations presented here. A comparison of the production surfaces generated by the two functions is of particular interest. In addition, it is interesting to compare the combinations of plant nutrients which (1) maximize yields and (2) maximize profits under various plant nutrient and crop prices. A comparison of predicted oat yields using the two functions and selected combinations of applied plant nutrients is shown in Table 8. Observations from twenty-eight combinations of plant nutrients are included in Table 8. These include observations from all 27 plots in the 3 x 3 x 3 replicated factorial in addition to the average yield from all check plots.

TABLE 8

OBSERVED AND ESTIMATED OAT YIELDS, 1956

Treatment (Pounds per Acre)			Predicted Yield (Bu. per Acre)		Observed Yield[1] (Bu. per Acre)	Residual[2] $(Y_i - \hat{Y}_i)$	
N	P_2O_5	K_2O	Exp.[3]	Poly.		Exp.	Poly
0	0	0	37.4	43.3	38.7	1.3	-4.6
20	40	20	56.7	51.8	67.5	10.8	15.7
20	40	80	58.9	54.9	55.1	-3.8	0.2
20	40	240	64.3	59.6	63.5	-0.8	3.9
20	160	20	56.3	51.3	51.0	-5.3	-0.3
20	160	80	58.5	54.9	70.4	11.9	15.5
20	160	240	63.8	61.2	57.9	-5.9	-3.3
20	480	20	58.8	50.8	56.4	-2.4	5.6
20	480	80	61.1	55.9	60.0	-1.1	0.1
20	480	240	6617	66.0	60.6	-6.1	-5.4
80	40	20	65.7	67.9	75.8	10.1	7.9
80	40	80	68.4	70.6	72.1	3.7	1.5
80	40	240	74.6	74.3	84.2	9.6	9.9
80	160	20	65.3	67.4	76.9	11.6	9.5
80	160	80	67.9	70.7	49.4	-18.5	-21.3
80	160	240	74.0	75.8	71.0	-1.8	-3.0
80	480	20	68.2	66.9	61.2	-7.0	-5.7
80	480	80	70.9	71.7	72.3	1.4	0.6
80	480	240	77.4	80.6	84.3	6.9	3.7
240	40	20	63.7	64.8	71.7	8.0	6.9
240	40	80	66.2	66.5	66.6	0.4	0.1
240	40	240	72.3	67.3	61.7	-10.6	-5.6
240	160	20	63.2	64.3	57.2	-6.0	-7.1
240	160	80	65.8	66.6	66.2	0.4	-0.4
240	160	240	71.7	68.9	69.2	-2.5	0.3
240	480	20	66.1	63.9	76.2	10.1	12.3
240	480	80	68.7	67.6	72.3	3.6	4.7
240	480	240	75.0	73.7	80.6	5.6	6.9

[1]The observed yield for the 0-0-0 treatment is an average of yields from 11 plots, all other observed yields are averages of two plots.

[2]Residuals are deviations of predicted yields from average observed yields.

[3]In computing $\hat{Y}_i$ for zero treatments of plant nutrients using the exponential equation, inputs of a single pound of N, P_2O_5 and K_2O were used. This introduces a slight upward bias in the predicted yield but overcomes the problem of having $Y_i = 0$ when any of the treatments is zero. This procedure is utilized throughout this chapter when computing Y_i from exponential equations.

Statistical measures derived for the equations, including the coefficient of multiple correlation and standard errors of the regression coefficients, indicate that the exponential is a slightly, but not significantly, more appropriate formulation than the polynomial. Measures such as correlation coefficients and standard errors of regression coefficients and equations are not without some limitations in comparing these two functions. The observations, and hence the variances, of the variables are not readily comparable since in one instance they are in real numbers and in the other in logarithms. The real numbers and logarithms, although bearing a consistent monotonic relationship to each other, do not maintain a relationship of equivalence or of constant ratios. Hence, the listed statistical measures should not be given an absolute interpretation for comparative purposes, i.e., they should, instead, serve as a basis for a rough comparison. Inspection of the residual values, $(Y_i - \hat{Y}_i)$ for both functions provides little basis for choice between functions since the individual residual values about the two functions are about equally dispersed with respect to magnitude and direction.[1]

Some additional insight into the appropriateness of the two alternative functions may be gained by comparing the derivatives of these functions with respect to their correspondence to input-output relationships postulated to exist in accordance with currently held theory. In addition, the derivatives are used to calculate plant nutrient combinations which produce (1) maximum yields and (2) maximum profits.

---

[1]These residuals are shown in columns 7 and 8 in Table 8.

Maximum yields occur where the first order partial derivatives of the functions are equal to zero. Maximum profits occur where the partial derivatives with respect to individual plant nutrients are equal to the plant nutrient-crop price ratios.

Since most of the variance explained by regression is associated with the nitrogen variable, the derivatives of the functions with respect to nitrogen are of particular interest. The partial derivatives of the two functions with respect to nitrogen, $\frac{\partial Y_0}{\partial N}$, are represented by the following equations III and IV. All derivatives are taken for a unit (one pound) change in plant nutrients.

Equation (III) - polynomial: $\frac{\partial Y_0}{\partial N} = b_1 - 2b_2 N + b_7 P + b_8 K$

Substituting in the estimated parameters from equation (I) gives

$$\frac{\partial Y_0}{\partial N} = .40112190 - 2(.00130761)N + .00000068 P - .00010905 K$$

Equation (IV) - exponential: $\frac{\partial Y_0}{\partial N} = R(N^{b_1} c_1^N \ln c_1 + c_1^N b_1 N^{b_1-1})$

Where R = antilog $(a + b_2 \log P + + \log C_2 + b_3 \log K + K \log C_3)$.

The expression of the partial derivative of the exponential may be simplified by factoring out Y_0 which leaves

$$\frac{\partial Y_0}{\partial N} = Y_0 (\ln c_1 + \frac{b_1}{N}).$$

Substituting in the estimated parameters from equation (II) gives

$$\frac{\partial Y_0}{\partial N} = Y_0 (-.00132853 + \frac{.16475028}{N})$$

The partial derivatives of the two functions with respect to N are shown in Table 9 with P_2O_5 and K_2O fixed at three different levels, 20-40, 80-160 and 240-480 pounds per acre respectively. These derivatives are also shown in Figure I. Derivatives of the exponential function are larger at small nitrogen inputs than is the case for the polynomial function. It is the opinion of the author that the exponential generates a production surface rising too rapidly with small nitrogen inputs. If this is true, the derivatives are probably too responsive to input changes, i.e., they probably are too large with small inputs and change rapidly to become too small with larger inputs. This phenomenon is due in part to the fact that when $X_i = 0$, $Y = 0$. The function may still be quite reliable over the range of moderate inputs.

The derivative of the exponential function is 1.456 bushels per pound of nitrogen with a nitrogen input of 5 pounds and decreases to .777 bushels when 10 pounds are applied. These values of the derivative seem excessively high from a viewpoint of plant physiology, i.e., it is difficult to visualize how one pound of additional nitrogen could result in the production of 1.456 bushels of additional wheat. However, the derivatives of the exponential type function are not restricted to a linear function of plant nutrient inputs as is the case with a polynomial with only first and second degree terms. This linear restriction on the derivatives of a polynomial can be overcome by modifying the formulation to include variables raised to fractional powers, e.g., powers such as 3/2, 1/2 etc. and/or by adding variables involving powers higher than 2.

TABLE 9

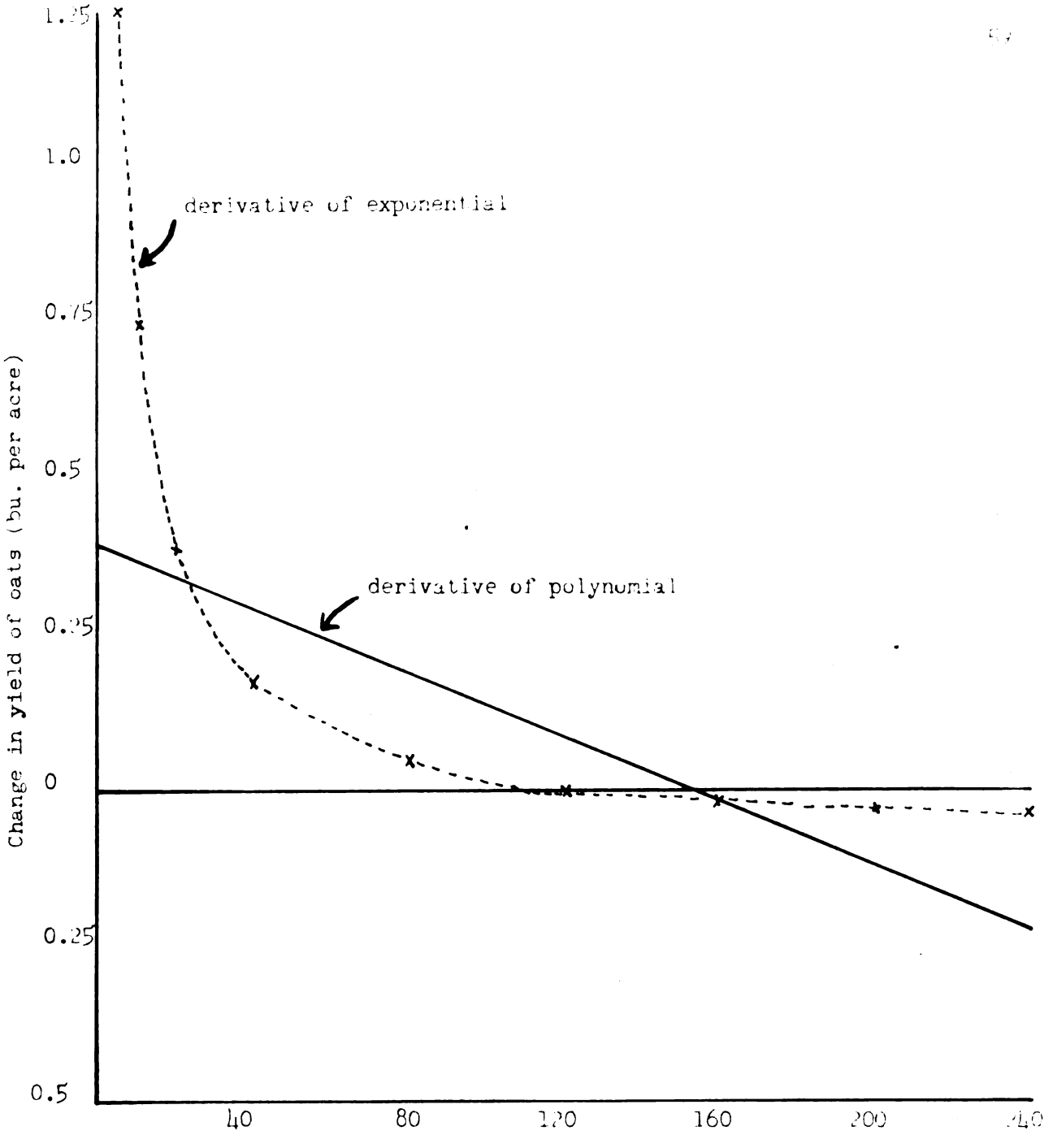
CHANGES IN OATS YIELDS RESULTING FROM UNIT CHANGES IN NITROGEN APPLICATIONS

Treatment Level of P_2O_5 & K_2O[1]	Nitrogen Treatment Level (pounds per acre)	Derivative of Polynomial[2] (bu. per acre)	Derivative of Exponential[2] (bu. per acre)
1	20	.347	.393
1	40	.294	.174
1	80	.190	.050
1	120	.085	.005
1	160	-.019	-.018
1	200	-.124	-.031
1	240	-.229	-.039
2	20	.341	.406
2	40	.288	.180
2	80	.184	.051
2	120	.079	.005
2	160	-.013	-.018
2	200	-.130	-.032
2	240	-.235	-.040
3	20	.323	.462
3	40	.270	.205
3	80	.166	.059
3	120	.061	.006
3	160	-.043	-.021
3	200	-.148	-.037
3	240	-.253	-.046

[1]Nitrogen is varied with P_2O_5 and K_2O fixed at three levels: (1) 40-20, (2) 160-80 and (3) 480-240 pounds per acre respectively.

[2]The derivatives are those resulting from an additional pound of nitrogen.

The statistical fit might not be improved by such a modification but derivatives would be allowed to become a curvilinear function of additional plant nutrients. Further experimentation with the use of fractional powered and more complex polynomials and additional



Applications of P_2O_5 and K_2O are fixed at 160 and 80 pounds per acre respectively.

Fig. 1. Partial derivatives of polynomial and exponential functions for oats with respect to nitrogen.

inspection of the derivatives of these functions is needed.

The partial derivatives of yield with respect to P_2O_5 and K_2O are shown in Tables 10 and 11 and Figures 2 and 3 respectively. As in the case for the derivatives with respect to nitrogen, the derivatives of the exponentials take more extreme values than those of the polynomial. However, the derivatives of both functions are small and may be non-significant and the absolute value of the difference between the two derivatives is not large.

High Profit Combinations of Plant Nutrients

The optimal amount of plant nutrients to apply, as has been previously stated, is a function not only of the productivity of applied nutrients but also of plant nutrient and crop prices. To solve for the combination of applied plant nutrients which will maximize yields, the partial derivatives of yield with respect to all plant nutrients are set equal to zero and solved simultaneously. For the polynomial equation, the maximum estimated yield is obtained with 153 pounds of N, a slightly negative quantity of P_2O_5 and .1 pound of K_2O. The estimated amounts of P_2O_5 and K_2O resulting in maximum yields are neither statistically nor economically significant, i.e., they are not significantly different from zero. It becomes profitable to apply plant nutrients to oats only when the price of oats is in excess of $1.00 per bushel and even then only nitrogen applications are profitable.[1]

---

[1]Plant nutrient prices used in computing the high profit inputs were $0.15 per pound for nitrogen, $0.10 for phosphoric acid and $0.11 for potash.

TABLE 10

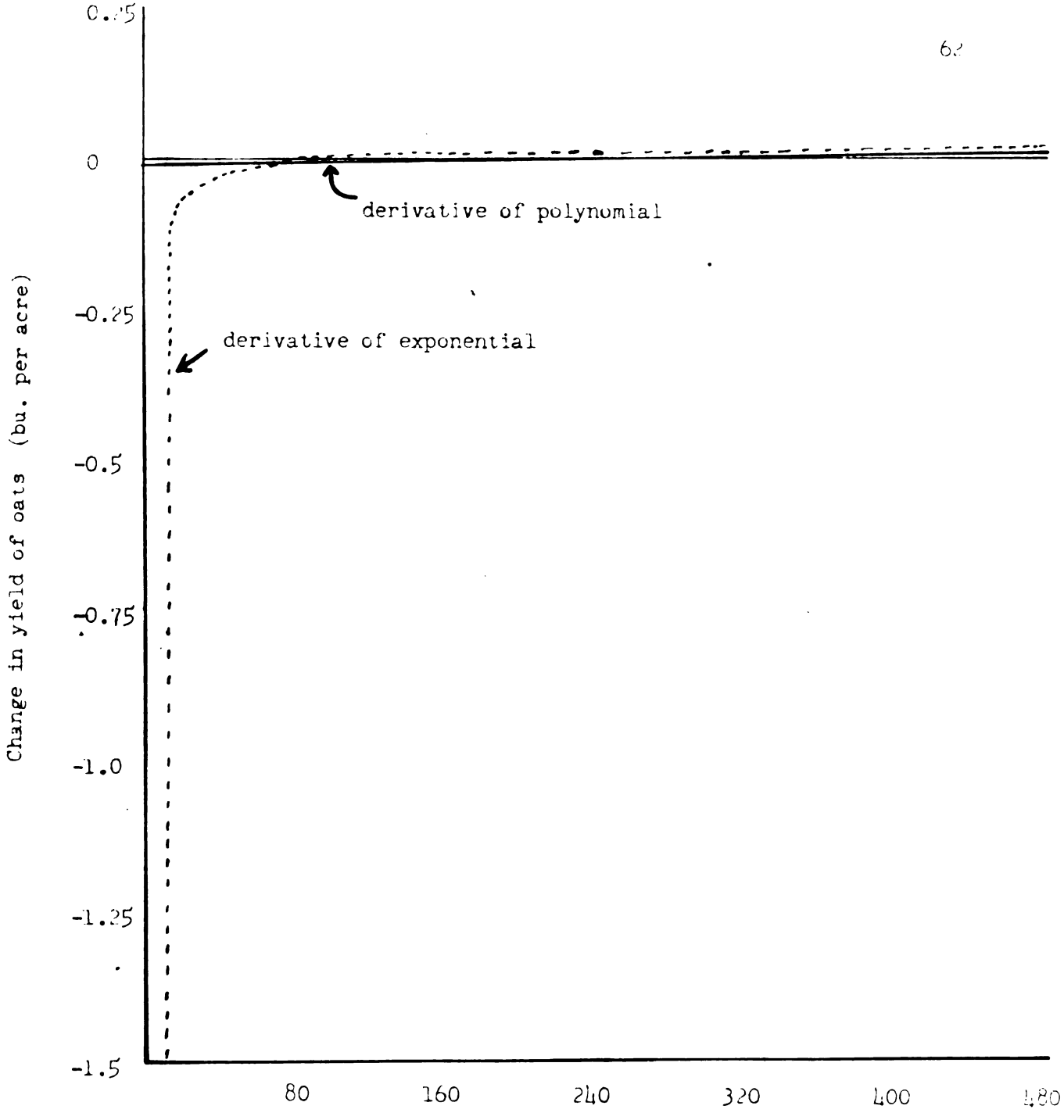
CHANGES IN OATS YIELDS RESULTING FROM UNIT CHANGES IN P_2O_5 APPLICATIONS

Treatment Level of N and K_2O[1]	P_2O_5 Treatment Level (pounds per acre)	Derivative of Polynomial[2] (bu. per acre)	Derivative of Exponential[3] (bu. per acre)
1	20	-.005	-.022
1	40	-.004	-.004
1	80	-.003	.003
1	160	-.002	.007
1	240	-.002	.008
1	320	-.001	.009
1	480	.000	.010
2	20	.000	-.027
2	40	.000	-.006
2	80	.001	.005
2	160	.002	.008
2	240	.003	.010
2	320	.004	.011
2	480	.005	.012
3	20	.012	-.028
3	40	.013	-.006
3	80	.013	.005
3	160	.014	.009
3	240	.015	.011
3	320	.016	.012
3	480	.017	.013

[1]Phosphoric acid is varied with N and K_2O fixed at three levels: (1) 20-20 (2) 80-80 and (3) 240-240 pounds per acre respectively.

[2]Regression coefficients for phosphoric acid variables were not significant.

[3]The regression coefficient for only one phosphoric acid variable was significant.



Applications of N and K_2O are fixed at 80 pounds per acre.

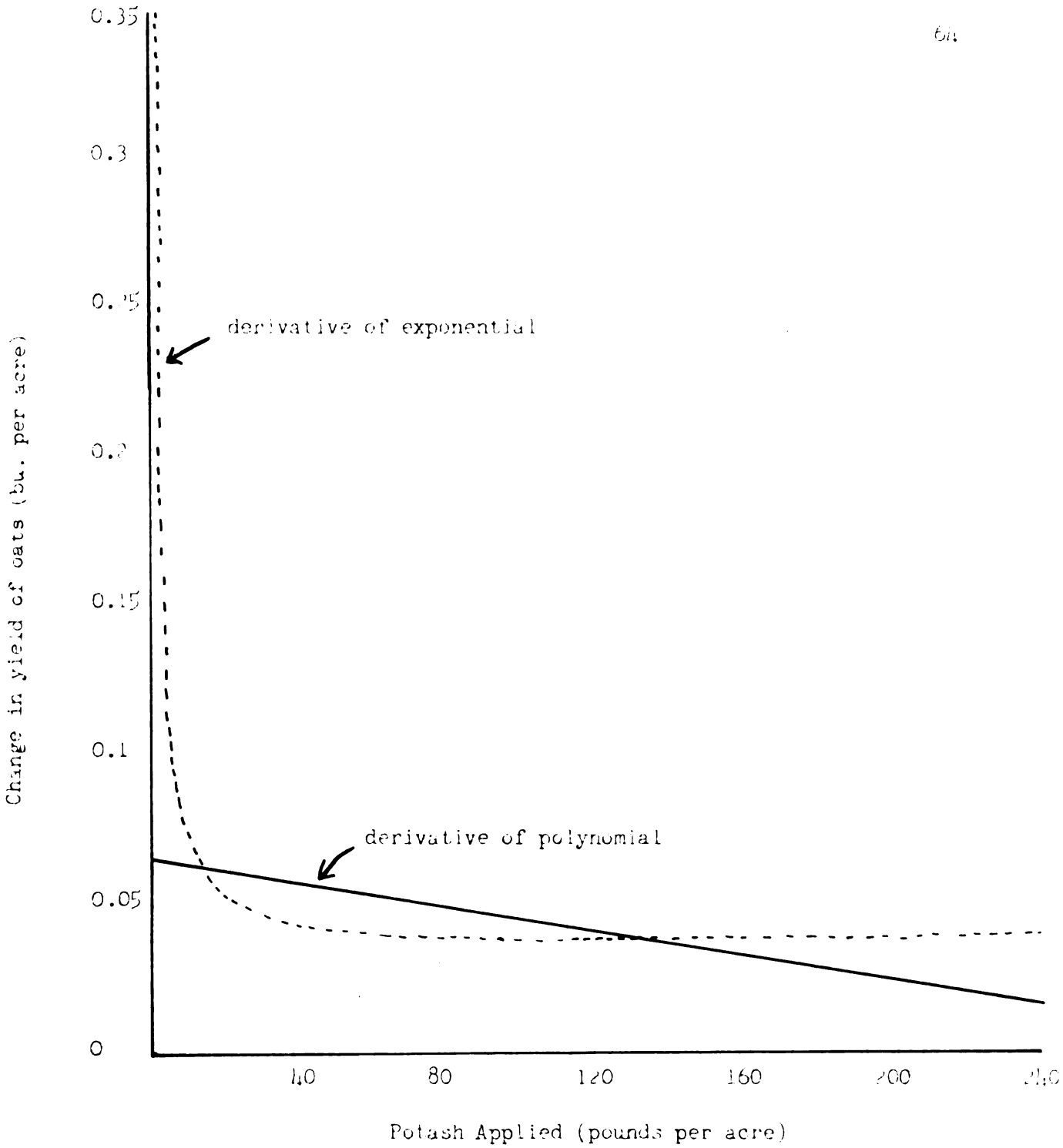
Fig. 2. Partial derivatives of the polynomial and exponential functions for oats with respect to phosphoric acid.

TABLE 11

CHANGES IN OATS YIELDS RESULTING FROM UNIT CHANGES IN K_2O APPLICATIONS

Treatment Level of N and P_2O_5[1]	K_2O Treatment Level (pounds per acre)	Derivative of Polynomial (bu. per acre)	Derivative of Exponential (bu. per acre)
1	20	.059	.046
1	40	.054	.038
1	80	.046	.034
1	120	.037	.033
1	160	.029	.033
1	200	.021	.034
1	240	.012	.034
2	20	.061	.053
2	40	.057	.044
2	80	.049	.039
2	120	.040	.038
2	160	.032	.038
2	200	.024	.039
2	240	.015	.039
3	20	.067	.054
3	40	.064	.044
3	80	.055	.040
3	120	.047	.039
3	160	.039	.039
3	200	.030	.039
3	240	.022	.040

[1]Potash is varied with N and P_2O_5 fixed at three levels: (1) 20-40 (2) 80-160 and (3) 240-480.



Applications of N and P_2O_5 are fixed at 80 and 160 pounds per acre respectively.

Fig. 3. Partial derivatives of polynomial and exponential functions for oats with respect to potash.

On the basis of yield response measured for 1955 and 1956, fertilizing of oats was not a profitable practice. The only possible justifications for application of plant nutrients to the oats crop appear to be when a seeding is being established with the oats and/or the benefits derived from residual plant nutrients by other crops in the rotation.

Analysis of the Wheat Data

Wheat was produced on the Kalamazoo County experimental site in 1956. The yield data produced in this experiment were analyzed in the same manner as the oats data. The original function fitted to the wheat data was a nine variable polynomial. This formulation with estimated parameters is shown in Equation V.

Equation (V): $\hat{Y}_w = 28.538730321 + .085984694l\ N - .0002208456\ N^2 +$
$(.0169599064) \quad (.0000632418)$

$.0163750688\ P - .0000351154\ P^2 + .0085708071\ K + .000021323l\ K^2 +$
$(.0085530282) \quad (.0000158325) \quad (.0172292424) \quad (.0000634867)$

$.00001902l6\ P - .0000799431\ NK + .0000151210\ PK$
$(.0000215352) \quad (.0000445667) \quad (.709260110)$

The adjusted coefficient of multiple correlation for this equation was .66 and the coefficient of multiple determination indicated that about 44 per cent of the variance in yield was associated with variance in applied plant nutrients. As was the case for oats, only the estimated parameters for the nitrogen variables were statistically significant at the one per cent probability level. However, the phosphoric acid variables, P and P^2, were significant at the five per cent probability level.

A Carter-Halter type exponential function was also fitted to the wheat data. The results of this fit are shown in Equation VI.

Equation (VI): $\log \hat{Y}_w = 1.445047179 + .0226358023 \log N + .000172688248 N +$
$(.0117451331) \qquad (.00008736048)$

$.0165763652 \log P - .0000173738\underline{41} P + .0012796889 \log K + .000108232188 K$
$(.0095688935) \qquad (.000038867869) \quad (.0117132969) \qquad (.00008543699\underline{8})$

The adjusted coefficient of multiple correlation for this equation was .65. The adjusted coefficient of multiple determination indicated that about 43 per cent of the variance in crop yields was associated with variance in the amounts of applied plant nutrients. The first three estimated coefficients in this equation were significant at the 10 per cent probability level and were almost significant at the five per cent probability level. The last three coefficients in the equation were not statistically significant. A comparison of observed yields with yields estimated by using the two functions is shown in Table 12. As was the case for the oats data, the tabular comparison includes observations and predictions for 28 combinations of applied N, P_2O_5 and K_2O. The observed yield values are averages of two replications for all treatments except the check (0,0,0) treatment which is an average of eleven replications.

The coefficients of multiple correlation and determination indicated that the two functions were about equally effective in explaining variance in wheat yields. Inspection of the residuals for the two functions, $(Y_i - \hat{Y}_i)$, further substantiates the conclusion that the two functions produce about equally good fits. These residuals are shown in columns 7 and 8 of Table 12.

TABLE 12

OBSERVED AND ESTIMATED WHEAT YIELDS, 1956

Treatment (pounds per acre)			Predicted Yield (bu. per acre)		Observed Yield[1] (bu. per acre)	Residual[2] ($Y_i - \hat{Y}_i$)	
N	P_2O_5	K_2O	Exp.	Poly.		Exp.	Poly.
0	0	0	27.9	28.5	28.2	0.3	-0.3
20	40	20	32.2	30.9	29.5	-2.7	-1.4
20	40	80	32.7	31.5	29.3	-3.4	-2.2
20	40	240	34.1	33.8	34.6	0.5	0.8
20	160	20	32.8	32.1	31.2	-1.6	-0.9
20	160	80	33.3	32.8	30.4	-2.9	-2.4
20	160	240	32.7	35.4	35.1	0.4	-0.3
20	480	20	33.0	30.4	32.1	-0.9	1.7
20	480	80	33.5	31.4	31.9	-1.6	0.5
20	480	240	34.9	34.8	34.9	0.0	0.1
80	40	20	34.0	34.7	37.5	3.5	2.8
80	40	80	34.6	35.0	34.9	0.3	-0.1
80	40	240	36.1	36.6	37.2	1.1	0.6
80	160	20	34.6	36.1	40.9	6.3	4.8
80	160	80	35.2	36.5	36.7	1.5	0.2
80	160	240	36.7	38.3	36.5	-0.2	-1.8
80	480	20	34.8	34.7	28.7	-6.1	-6.0
80	480	80	35.4	35.4	39.0	3.6	3.6
80	480	240	36.9	38.0	38.6	1.7	0.6
240	40	20	37.2	37.0	36.6	-0.6	-0.4
240	40	80	37.8	36.6	35.1	-2.7	-1.5
240	40	240	39.4	36.1	35.1	-3.3	-1.0
240	160	20	37.8	38.8	42.1	4.3	3.3
240	160	80	38.5	38.4	39.3	0.8	0.9
240	160	240	40.1	38.2	38.4	-1.7	0.2
240	460	20	38.1	38.4	42.9	4.8	4.5
240	480	80	38.7	38.3	38.2	-0.5	-0.1
240	480	240	40.3	38.8	38.8	1.5	0.0

[1]The observed yield is the average of two replications except for the check (0,0,0) treatment which is the average of 11 replications.

[2]Residuals are the difference between average observed yields and estimated yields.

Derivatives of the two functions with respect to all three of the plant nutrients are presented in Tables 13-15 and in Figures 4-6. The derivatives of the two functions produce different estimates of the productivity of the various plant nutrients. For example, the derivative of the polynomial indicates that the marginal productivity of nitrogen over the range of 30 to 100 pounds, which is a common range of application, is almost double the marginal productivity schedule generated by the derivative of the exponential. Derivatives of the two functions with respect to P_2O_5 also exhibit substantial differences over the range of usual applications. However, the marginal productivity of phosphorus is low and the absolute value of the differences between the two derivatives is small as is shown in Table 14 and Figure 5.

Estimated derivatives of the two functions with respect to K_2O also differ widely as indicated in Table 15 and Figure 6. The derivative of the polynomial with respect to K_2O exhibits increasing returns to additional applications of K_2O which is not a logical phenomenon. The derivative of the exponential exhibits only slightly diminishing returns. As previously indicated, however, the K_2O variables in both equations lack statistical significance at any acceptable probability level.

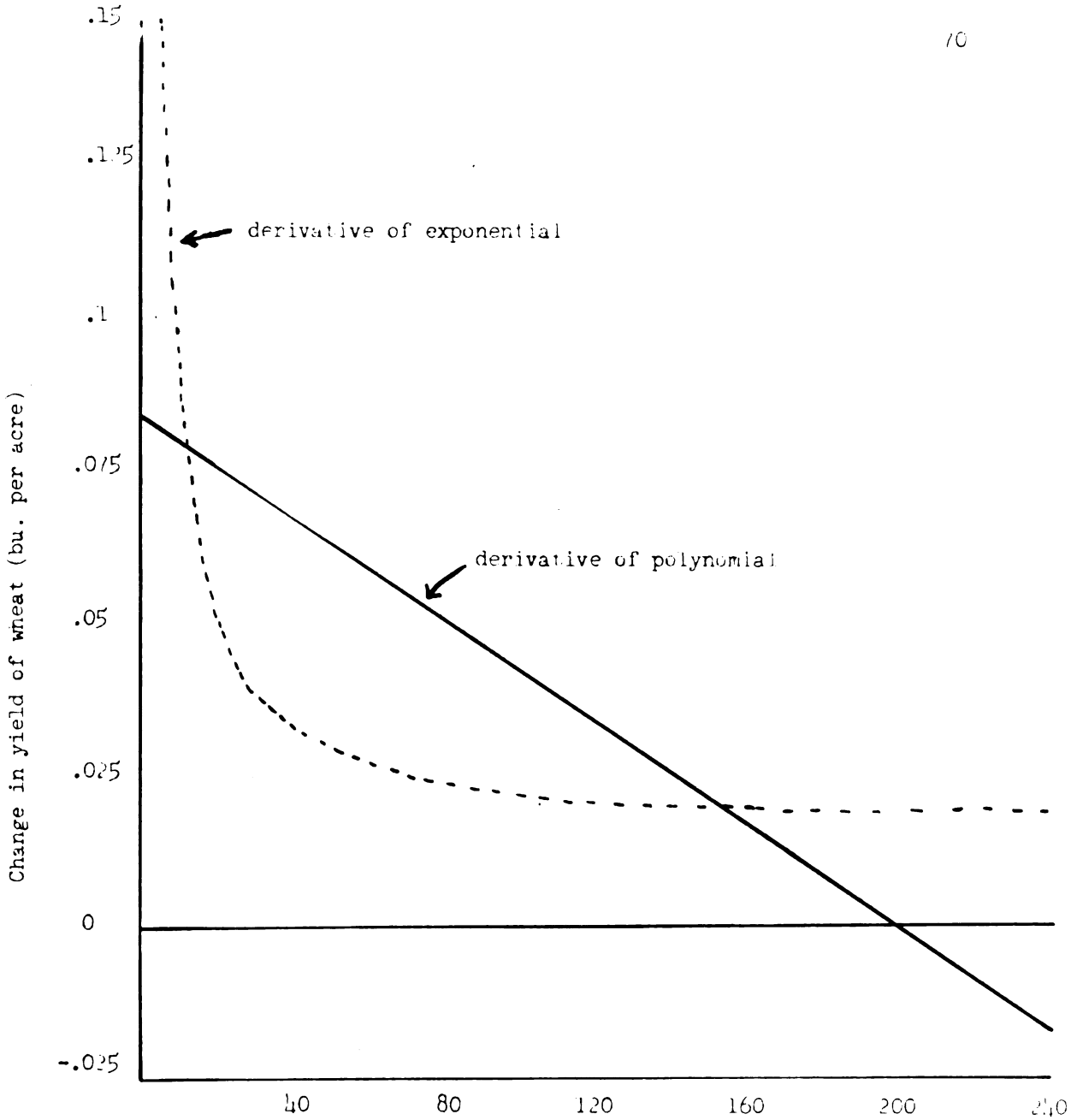
Inferences made about the productivity of all three plant nutrients will vary considerably depending on which function is chosen as "best". Inspection of residuals of the two functions does not provide any satisfactory basis for choosing between the two functions.

TABLE 13

CHANGES IN WHEAT YIELDS RESULTING FROM UNIT CHANGES IN NITROGEN APPLICATIONS

Treatment Level of P_2O_5 & K_2O[1]	Nitrogen Treatment Level (pounds per acre)	Derivative of Polynomial (bu. per acre)	Derivative of Exponential (bu. per acre)
1	20	.076	.049
1	40	.067	.032
1	80	.030	.023
1	120	.032	.021
1	160	.014	.019
1	200	-.003	.019
1	240	-.021	.018
2	20	.074	.051
2	40	.065	.033
2	80	.047	.024
2	120	.030	.021
2	160	.012	.020
2	200	-.006	.019
2	240	-.023	.019
3	20	.067	.053
3	40	.058	.035
3	80	.041	.025
3	120	.023	.022
3	160	.005	.021
3	200	-.012	.020
3	240	-.030	.020

[1]Nitrogen is varied with P_2O_5 and K_2O fixed at three levels; (1) 40-20 (2) 160-180 and (3) 480-240 pounds per acre respectively.



Applications of P_2O_5 and K_2O are fixed at 160 and 80 pounds per acre respectively.

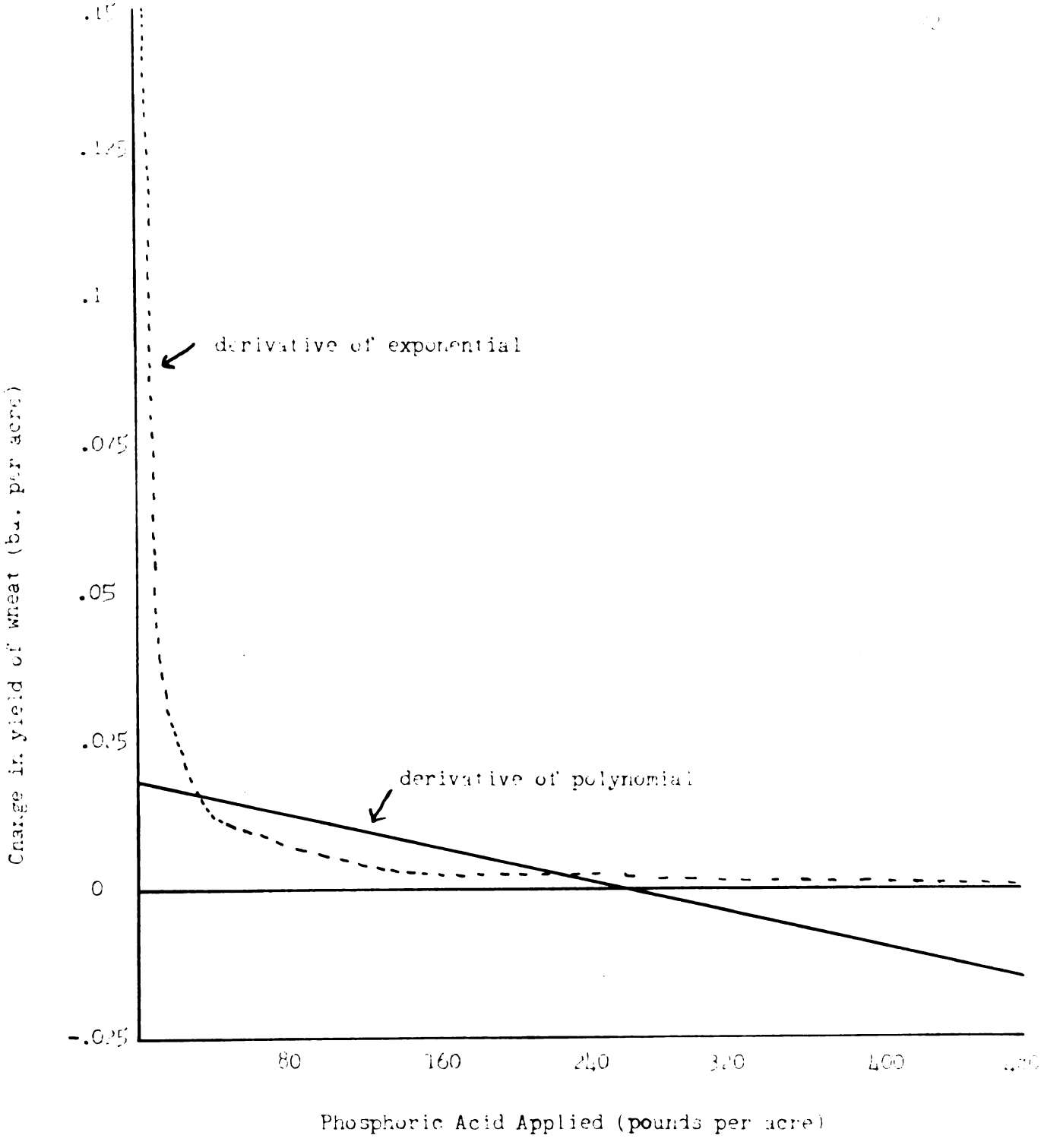
Fig. 4. Partial derivatives of the polynomial and exponential functions for wheat with respect to nitrogen.

TABLE 14

CHANGES IN WHEAT YIELDS RESULTING FROM UNIT CHANGES IN P_2O_5 APPLICATIONS

Treatment Level of N and K_2O [1]	P_2O_5 Treatment Level (pounds per acre)	Derivative of Polynomial (bu per acre)	Derivative of Exponential (bu. per acre)
1	40	.014	.012
1	80	.011	.006
1	160	.006	.002
1	240	.000	.001
1	320	-.005	.001
1	400	-.011	.000
1	480	-.017	.000
2	40	.016	.013
2	80	.013	.006
2	160	.008	.003
2	240	.002	.001
2	320	-.003	.001
2	400	-.009	.000
2	480	-.015	.000
3	40	.022	.015
3	80	.019	.007
3	160	.013	.003
3	240	.008	.002
3	320	.002	.001
3	400	-.004	.000
3	480	-.009	.000

[1] P_2O_5 is varied with N and K_2O fixed at three levels: (1) 20-20 (2) 80-80 and (3) 240-240 pounds per acre respectively.



N and K_2O are fixed at 80 pounds per acre

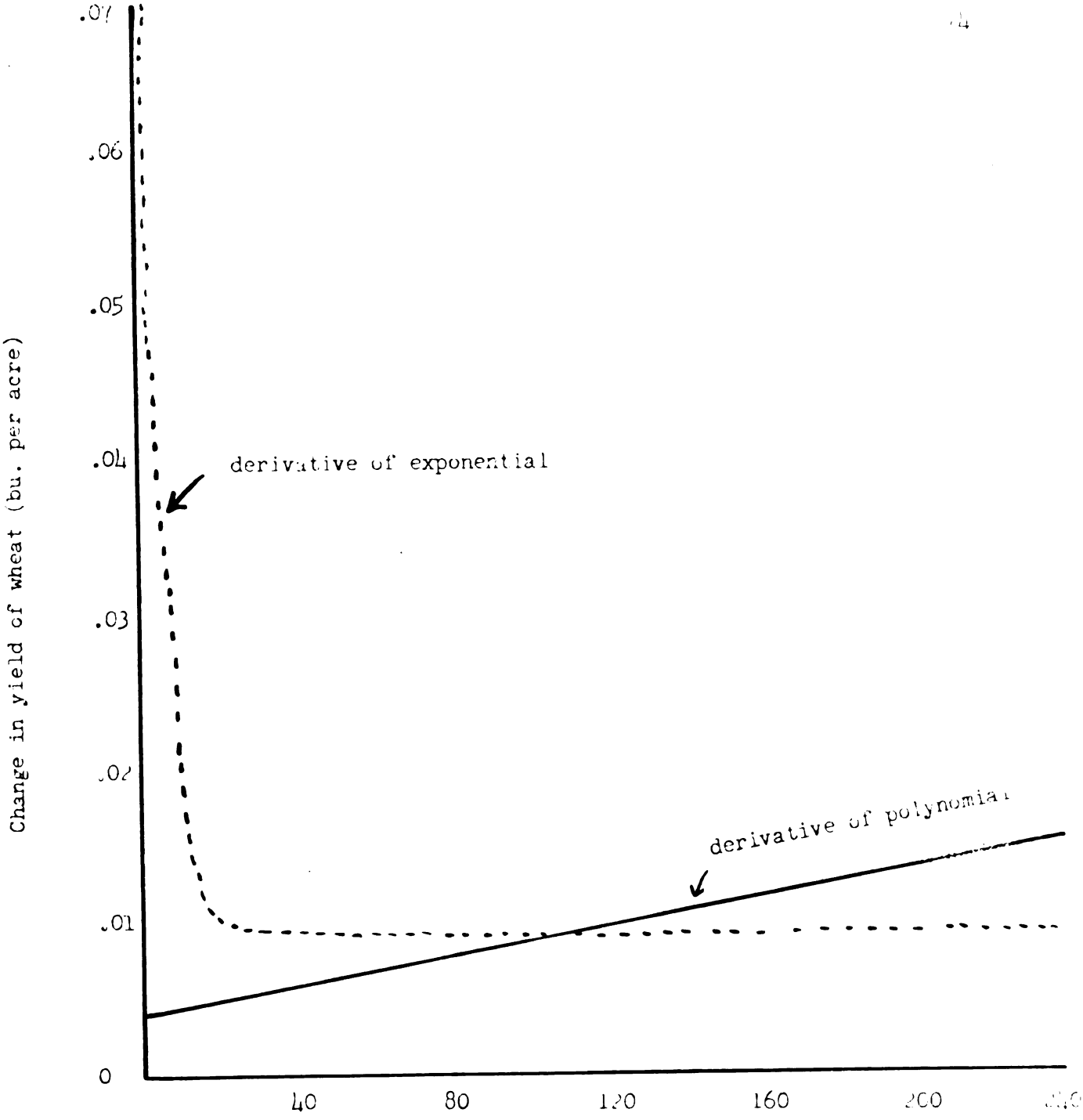
Fig. 5. Partial derivatives of the polynomial and exponential functions for wheat with respect to phosphoric acid.

TABLE 15

CHANGES IN WHEAT YIELDS RESULTING FROM UNIT CHANGES IN K_2O APPLICATIONS

Treatment Level of N and P_2O_5[1]	Potash Treatment Level (pounds per acre)	Derivative of Polynomial (bu. per acre)	Derivative of Exponential (bu. per acre)
1	20	.008	.010
1	40	.009	.009
1	80	.011	.009
1	120	.013	.009
1	160	.014	.009
1	200	.016	.009
1	240	.018	.009
2	20	.005	.010
2	40	.006	.009
2	80	.008	.009
2	120	.010	.009
2	160	.011	.009
2	200	.013	.009
2	240	.015	.009
3	20	-.003	.011
3	40	-.002	.011
3	80	.000	.010
3	120	.002	.010
3	160	.003	.010
3	200	.005	.010
3	240	.007	.010

[1]Potash is varied with N and P_2O_5 fixed at three levels: (1) 20-40 (2) 80-160 and (3) 240-480 pounds per acre respectively.



Applications of N and P_2O_5 are fixed at 80 and 160 pounds per acre respectively.

Fig. 6. Partial derivatives of the polynomial and exponential functions for wheat with respect to potash.

Maximum Yields and High Profit Plant Nutrient Applications

Maximum yields of about 39 bushels per acre are predicted using the polynomial equation. This yield occurs with plant nutrient applications of about 196 pounds of N, 300 pounds of P_2O_5 and 61 pounds of K_2O. The maximum yield predicted using the exponential is in excess of any yield observed in the experiment and requires plant nutrient applications in excess of any quantities applied in the experiment. Since the predicted maximum yield and the plant nutrient inputs producing this yield lie beyond the range of observed values, they are probably invalid inferences.

Both functions generated response surfaces which illustrated substantial positive yield response to N and P_2O_5. However, because of the moderate slopes of the response surfaces, the value of additional production was less than the cost of plant nutrients necessary to obtain the increases in yields.[1]

Applications of nitrogen and phosphoric acid would have been profitable only at wheat prices of about $3.00 per bushel. Such wheat prices appear extremely unlikely. It is important to note that although the derivatives of the two functions differ considerably, the same conclusion, that no fertilizer applications were profitable at typical prices, would be reached using either function as a basis for computing high-profit fertilizer inputs.

---

[1]No credit was given for possible residual fertility. With large fertilizer applications some carryover fertility would be expected, however, the magnitude and value of this carryover can only be determined over time.

Analysis of the Corn Data

Two corn crops have been produced and harvested in the rotation experiment. The corn plots were located at the Calhoun county site in 1955. A severe summer drouth reduced corn yields in this area, particularly on the lighter upland soils. An extensive analysis of the 1955 corn data was conducted by Jack Knetsch and has previously been reported.[1] Knetsch found that a Carter-Halter type exponential provided the best statistical fit to the data. Significant response was found to exist only for applied nitrogen. The fitted function was

$$Y_c = 39.71(N + .01)^{.18627} \quad .96230^{(N + 0.1)},$$

where N was measured in 20-pound units. The addition of .1 of a unit alleviated the problem of forcing the function to have a value of zero when any one of the plant nutrient inputs was zero. The coefficient of multiple correlation for this equation was .69. The high profit nitrogen application varied from 29 to 54 pounds per acre as the price of corn was varied from $.80 to $2.00 per bushel with nitrogen priced at $.15 per pound. A comparison of observed and predicted yields is shown in Table 16.

---

[1]The results of this analysis are contained in: Jack L. Knetsch, "Methodological Procedures and Applications for Incorporating Economic Considerations into Fertilizer Recommendations," Unpublished Master's Thesis, Michigan State University, 1956, and Jack L. Knetsch, L. S. Robertson, Jr., and W. B. Sundquist, "Economic Considerations in Soil Fertility Research," Michigan Agricultural Experiment Station, Quarterly Bulletin, August, 1956, pp. 10-16.

TABLE 16

COMPARISON OF OBSERVED AND PREDICTED CORN YIELDS ON A KALAMAZOO SANDY LOAM SOIL, 1955

N Per Acre (pounds)	Number of Plots	Average of Actual Yields (bu. per acre)	Predicted Yield of Corn (bu. per acre)	Marginal Product of 20-pound Units of N (bu. per acre)
0	18	26.3	25.8	0
20	24	40.6	38.2	12.4
40	14	43.5	41.8	3.6
80	29	43.4	44.1	1.15[1]
160	18	42.8	43.0	-0.50[1]
240	27	40.7	39.8	-0.85[1]

[1]Average marginal product of 20-pound units of nitrogen for the 40-pound incremental intervals shown in column 1.

Corn was produced on the Kalamazoo county site in 1956. Once again the crop was damaged by a severe late summer drouth. Check plot yields were not significantly different from those receiving applied plant nutrients. Preliminary tabulations indicated very little association of yield variance with variance in any of the three applied nutrients. This lack of relationship was further substantiated by functional analysis. A nine-term polynomial was fitted to the data with the estimated parameters shown in Equation VII.

Equation (VII): $\hat{Y}_c = 54.8040409 - .0045761891\ N + .000113767680\ N^2 -$
$(.0391267523) \quad (.000145895854)$

$.0141889261\ P + .000093102508\ P^2 + .0269173492\ K - .000092531450\ K^2 +$
$(.019643161) \quad (.000036524729) \quad (.039750401) \quad (.00146457332)$

$.0000669774119\ NP + .0000357228950\ NK + .000056027392\ PK$
$(.000049677358) \quad (.000099544345) \quad (.000049163862 9)$

None of the parameters in this equation are significantly different from zero. This lack of significance is not surprising since the adjusted coefficient of multiple correlation for the equation is only .23 and the adjusted coefficient of multiple determination is only .05 a value which is not significantly different from zero.

A Carter-Halter type equation which was fitted to the data is shown in Equation VIII.

<u>Equation (VIII):</u>

$$\text{Log } \hat{Y}_c = 1.719957560 + \underset{(.016317686)}{.012753109} \log N - \underset{(.000120965)}{.000018940}\, N$$

$$+ \underset{(.013694926)}{.005789778} \log P - \underset{(.000055033)}{.000110611}\, P - \underset{(.016274205)}{.005317996} \log K + \underset{(.000118306)}{.000185876}\, K$$

Only the fourth term in this equation, P, is statistically significant. Since the phosphoric acid variable is represented by two terms, one of which is not significant, no very valid inferences can be made about the aggregate influence of phosphoric acid. None of the terms representing nitrogen or potash are significantly different from zero. About the same amount of total variance in yield is associated with regression as was the case for the polynomial. The adjusted coefficients of multiple correlation and multiple determination are .24 and .06 respectively. Neither is statistically significant. The only inference which seems to be warranted by these analyses is that no significant part of variance in yield was associated with applied plant nutrients.

In summary, only moderate applications of nitrogen, 30-40 pounds per acre, were profitable on corn on Kalamazoo sandy loam in 1955 and no

plant nutrient applications were profitable in 1956 with the particular weather conditions which occurred in these two years.

Alfalfa

No alfalfa was grown in the first year of the experiment because of the inability to establish any harvestable growth in the first year of the experiment. In 1956 the stand of alfalfa was very heterogeneous. Large contiguous areas in the field had moderately good stands while other areas had almost no alfalfa growing on them. No attempt was made to collect and analyze yield data because yield differences were obviously a function of differences in stand not associated with applied plant nutrients.

Analysis of the Continuous Corn Data

The initial corn crop in a continuous corn rotation was produced on a Wisner clay-loam soil in Tuscola county in 1956. Preliminary inspection of the data indicated small and heterogeneous yield responses to applied plant nutrients. The eight check plots in this experiment had an average yield of 100.6 bushels per acre, while the average of all 210 plots in the experiment was 109.7 bushels per acre.

A nine variable polynomial was fitted to the data and the results of this formulation are shown in Equation IX.

Equation (IX): $\hat{Y}_c = 104.565510278 + .069914346\ N + .050754850\ P -$
$(.034704916) \quad (.017279507)$

$.001629512\ K - .000356932\ N^2 - .000068956\ P^2 - .000053579\ K^2 -$
$(.034855587) \quad (.000108282) \quad (.000028743) \quad (.000116055)$

$.000039695\ NP + .000112058\ NK + .000060759\ PK$
$(.000039782) \quad (.000079363) \quad (.000043830)$

Coefficients of four of the variables N, N^2, P, and P^2 were significant at the one per cent probability level, whereas none of the potash variables were statistically significant. Only a small portion of yield variance was associated with applied plant nutrients as the coefficients of multiple correlation and multiple determination were only .40 and .16 respectively.

Since none of the independent variables containing a potash term were statistically significant, the polynomial was reformulated, dropping the variables containing a potash term. The shortened polynominal is shown in Equation X.

Equation X: $\hat{Y}_c + 104.082698823 + .073704546\ N + .050022736\ P -$
$(.034298683) \quad (.017116212)$

$.000331599\ N^2 - .000056021\ P^2 - .000025460\ NP$
$(.000107266) \quad (.000027334) \quad (.000038963)$

In equation X the first four coefficients are significant at the one per cent probability level. The fifth term, a cross product, was not significant at any acceptable significance level. The coefficients of multiple correlation and multiple determination for the shortened polynomial were .39 and .16 respectively.

Due to the small portion of yield variance associated with applied plant nutrients as indicated by inspection and the fitted polynomials, no attempt was made to fit an exponential type equation to the data.

Maximum Yield and High Profit Combinations of Plant Nutrients

Coefficients for the nitrogen and phosphoric acid variables were similar for the two polynomials fitted to the data. Since the potash

coefficients were not significant, the plant nutrient combination providing maximum yields was restricted to N and P_2O_5 and was calculated from Equation X. The maximum predicted yield, 123.4 bushels per acre, was obtained using 95 pounds of N and 425 pounds of P_2O_5. The cost of using any amount of applied plant nutrients exceeded the returns unless corn prices exceeded $2.00 per bushel. The latter corn price situation is, of course, an unlikely phenomenon.

The high check plot yields, in excess of 100 bushels per acre, probably indicates the soil was quite fertile prior to additional applications of plant nutrients although soil tests indicate only a moderate fertility level. Other possible sources of yield variance were present in the experimental field, including differences in previous cropping history. Although yields from the plot areas with different cropping histories were not statistically different, this factor of heterogenity may have contributed some variance to crop yields.

Analysis of the Bean Data from the Corn, Beans and Wheat Rotation

Field beans were produced on a Simms loam soil in Gratiot county in 1956. The bean crop is part of an intensive cash crop rotation of corn, beans and wheat. Experimental plots had received plant nutrient treatments in 1955 identical to the 1956 treatments. Thus, some residual fertility might have been expected to be present in 1956, particularly on plots receiving heavy fertilizer applications the previous year. Preliminary tabulation of the data indicated a substantial response to

nitrogen applications, a smaller response to phosphoric acid, and no appreciable response to applied potash.

Three functions were fitted to the bean data. The first two functions are exponential type formulations and the third a five variable polynomial. The original production function formulation is a six variable exponential of the Carter-Halter type. Although preliminary analysis had indicated no response to potash, variables containing potash terms were included in this original exponential which is shown in Equation XI.

Equation (XI): $\text{Log } \hat{Y}_b = 1.2034797 + \underset{(.017529035)}{.032812261 \log N} + \underset{(.0010894)}{.000398971 N} +$

$\underset{(.015589387)}{.019527434 \log P} + \underset{(.00004950\text{4})}{.000062271 P} + \underset{(.018591118)}{.001880612 \log K} + \underset{(.000068525)}{.000050911 K}$

The adjusted coefficient of multiple correlation for this equation was .605 and the coefficient of multiple determination was .366. This indicates that about 37 per cent of the variance in bean yields was associated with regression. Because of the large standard errors for the potash coefficients, a second formulation of the exponential was made dropping the potash terms. This exponential is shown in Equation XII.

Equation (XII): $\text{Log } \hat{Y}_b = 1.2074135791 + \underset{(.016676657)}{.0347393520 \log N} + \underset{(.0001065019\text{5})}{.0003965964 N}$

$+ \underset{(.014609617)}{.0214607700 \log P} + \underset{(.00004835223\text{1})}{.0000597327 P}$

The adjusted coefficient of multiple correlation for the shortened exponential was .607 and the coefficient of multiple determination was

.369. Coefficients of the nitrogen and phosphoric acid variables were not changed appreciably by omitting the non-significant potash terms. Phosphoric acid terms were not significant at the 10 per cent probability level as the size of the estimated coefficients for these terms exceeded their respective standard errors. Finally, a five variable polynomial was fitted to the bean data. The results of this fit are shown in Equation XIII.

Equation (XIII): $\hat{Y}_b = 17.6023144 + .0636878985\ N - .00010708441\ N^2 +$
$(.0114222) \qquad (.0000354159)$

$.0127499698\ P - .00001.05617\ P^2 + .00000.63492\ NP$
$(.00580265) \quad (.00000073620) \quad (.0000130957)$

The adjusted coefficients of multiple correlation and determination for this equation were .646 and .417, respectively.

A comparison of observed and predicted yields using the three functions fitted to the data are presented in Table 17. As in previous cases, inspection of the residual quantities, i.e., differences between predicted and observed values, of the three functions provides little basis for choosing any one function over the others. This is true because of the relative uniformity of the magnitude and direction of the residuals. Partial derivatives of the three functions with respect to nitrogen are shown in Table 18 and Figure 7. Partial derivatives with respect to phosphoric acid are presented in Table 19 and Figure 8.

Maximum Yields and Optimum Inputs of Plant Nutrients

Derivatives of the two exponential equations with respect to nitrogen are characterized by properties which are unusual for marginal product

TABLE 17

OBSERVED AND ESTIMATED BEAN YIELDS, 1956

Treatment (pounds per acre)			Predicted Yield[1]			Observed Yield[2]	Residual[3] ($Y_i - \hat{Y}_i$)		
N	P_2O_5	K_2O	Poly	Exp(1)	Exp(2)		Poly	Exp(1)	Exp(2)
0	0	0	17.6	16.1	16.0	17.4	-0.2	1.3	1.4
20	40	20	19.3	19.8	19.6	25.4	6.1	5.6	5.8
20	40	160	19.3	19.8	20.0	25.9	6.1	6.1	5.9
20	40	320	19.3	19.8	20.4	19.6	0.3	-0.2	-0.8
20	320	20	21.9	21.5	21.2	15.4	-6.5	-6.1	-5.8
20	320	160	21.9	21.5	21.6	24.5	2.6	3.0	2.9
20	320	320	21.9	21.5	22.1	21.4	-0.5	-0.1	-0.7
20	640	20	22.7	22.9	22.5	25.8	3.1	2.9	3.3
20	640	160	22.7	22.9	23.0	21.0	-1.7	-1.9	-2.0
20	640	320	22.7	22.9	23.4	14.8	-7.9	-8.1	-8.6
40	80	40	21.0	21.1	20.9	21.9	0.9	0.8	1.0
80	160	80	23.9	23.0	22.8	31.1	7.2	8.1	8.3
160	40	20	25.6	24.2	23.8	23.8	-1.8	-0.4	0.0
160	40	160	25.6	24.2	24.3	26.9	1.3	2.7	2.6
160	40	320	25.6	24.2	24.8	27.8	2.2	3.6	3.0
160	320	20	28.4	26.3	25.8	31.6	3.2	5.3	5.8
160	320	160	28.4	26.3	26.4	33.8	5.4	7.5	7.4
160	320	320	28.4	26.3	26.9	25.6	-2.8	-0.7	-1.3
160	640	20	29.5	27.9	27.4	24.7	-4.8	-3.2	-2.7
160	640	160	29.5	27.9	28.0	33.5	4.0	5.6	5.5
160	640	320	29.5	27.9	28.5	29.3	-0.2	1.4	0.8
240	480	240	31.1	29.6	29.9	29.5	-1.6	-0.1	-0.4
320	40	20	27.6	28.7	28.2	32.4	4.8	3.7	4.2
320	40	160	27.6	28.7	28.8	27.5	-0.1	-1.2	-1.3
320	40	320	27.6	28.7	29.4	26.4	-1.2	-2.3	-3.0
320	320	20	30.7	31.2	30.6	34.1	3.4	2.9	3.5
320	320	160	30.7	31.2	31.2	29.7	-1.0	-1.5	-1.5
320	320	320	30.7	31.2	31.9	27.8	-2.9	-3.4	-4.1
320	640	20	32.2	33.1	32.5	34.8	2.6	1.7	2.3
320	640	160	32.2	33.1	33.1	33.7	1.5	0.6	0.6
320	640	320	32.2	33.1	33.8	30.6	-1.6	-2.5	-3.2

[1]Exp (1) is the four term exponential and Exp (2) is the six term exponential.

[2]The observed yield for the 0-0-0 treatment is an average of yields from 11 plots, all other observed yields are averages of two plots.

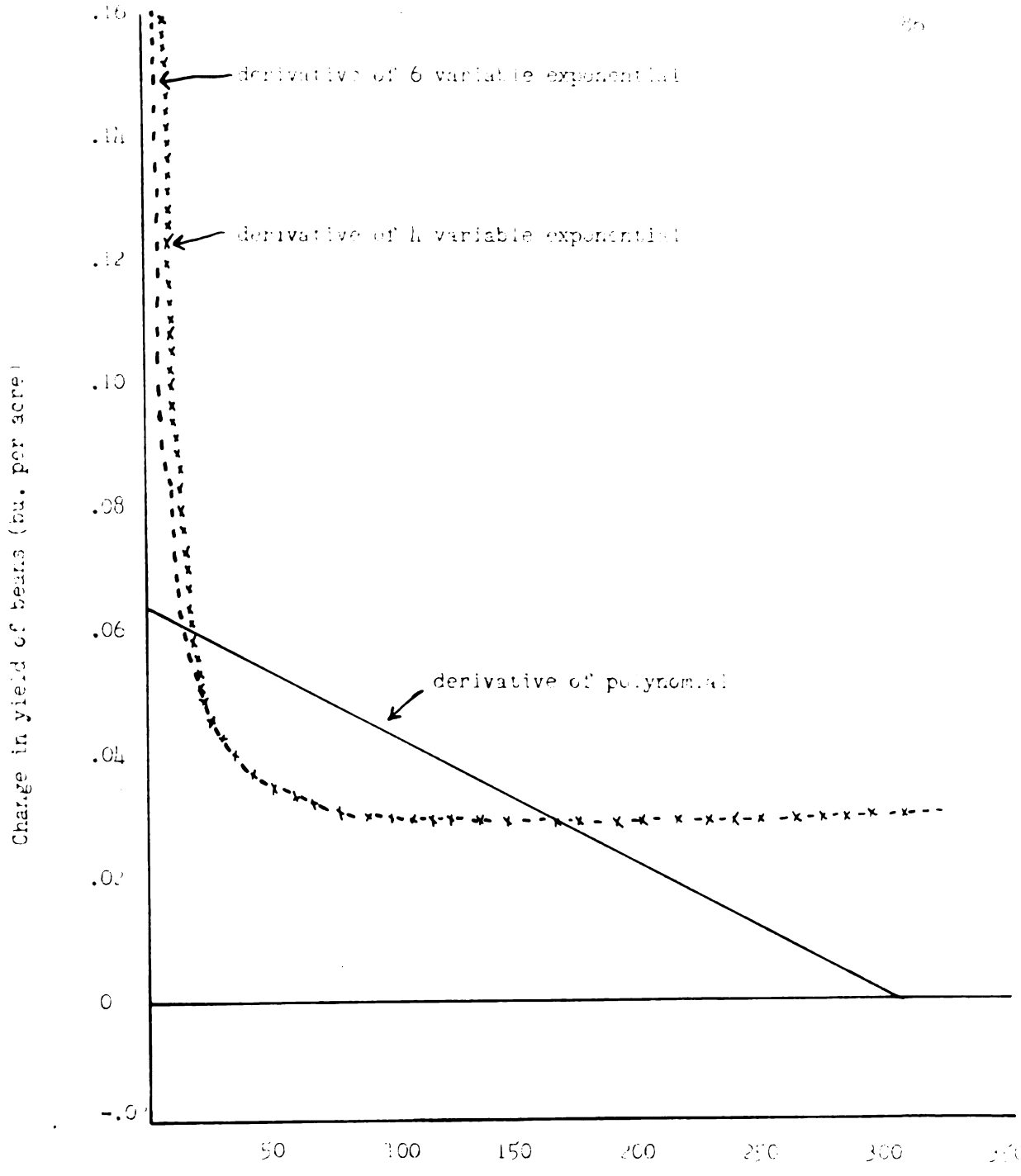
[3]Residuals are deviations of predicted yields from average observed yields.

TABLE 18

CHANGES IN BEAN YIELDS RESULTING FROM UNIT CHANGES IN APPLIED NITROGEN

Treatment Level of P_2O_5 and K_2O[1]	Nitrogen Treatment Level (pounds per acre)	Derivative of Polynomial (bu.per acre)	Derivative of Exp. (1) (bu.per acre)	Derivative of Exp. (2) (bu.per acre)
1	20	.060	.052	.050
1	40	.055	.037	.035
1	80	.047	.030	.029
1	120	.038	.028	.027
1	160	.030	.027	.027
1	200	.021	.027	.027
1	240	.013	.027	.027
1	320	.005	.029	.029
2	20	.060	.055	.052
2	40	.056	.039	.037
2	80	.048	.031	.030
2	120	.039	.029	.029
2	160	.030	.029	.028
2	200	.022	.029	.028
2	240	.013	.029	.029
2	320	.004	.031	.031
3	20	.062	.057	.055
3	40	.058	.040	.039
3	80	.050	.032	.032
3	120	.041	.030	.030
3	160	.032	.030	.030
3	200	.024	.030	.030
3	240	.015	.030	.030
3	320	.002	.032	.032
4	20	.063	.060	.060
4	40	.059	.042	.042
4	80	.051	.034	.034
4	120	.042	.032	.033
4	160	.033	.031	.032
4	200	.025	.032	.032
4	240	.016	.032	.033
4	320	.001	.033	.035

[1]Nitrogen is varied with P_2O_5 and K_2O fixed at: (1) 40-20 (2) 160-80 (3) 320-160 and (4) 640-320 respectively. Derivatives of the polynomial and Exp. (1) are independent of applied K_2O since there were no K_2O variables in the functions for which these derivatives were taken.



P_2O_5 and K_2O are fixed at 160 and 80 pounds per acre respectively.

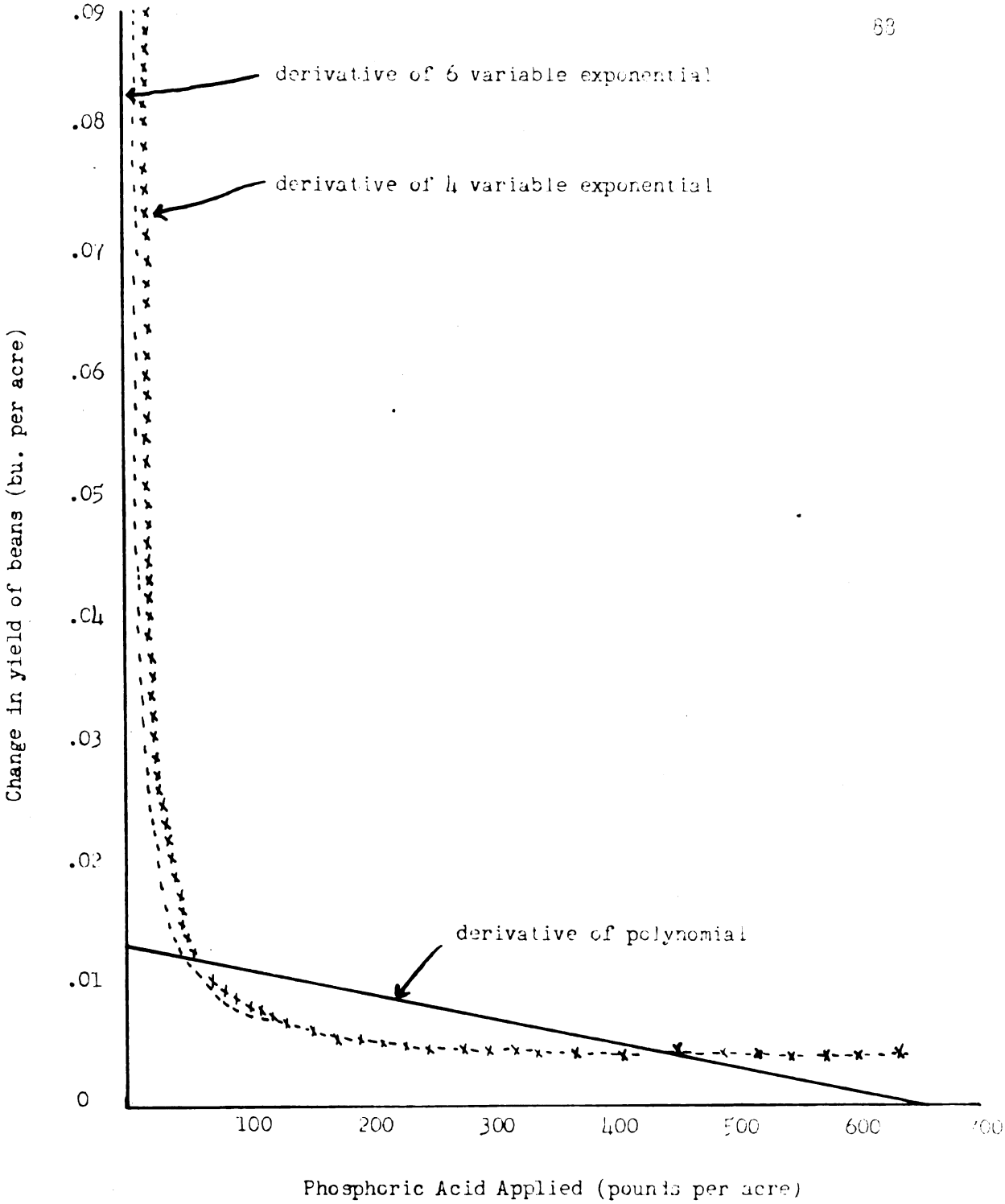
Fig. 7. Partial derivatives of a polynomial and two exponential functions for beans with respect to nitrogen.

TABLE 19

CHANGES IN BEAN YIELDS RESULTING FROM UNIT CHANGES IN APPLIED PHOSPHORIC ACID

Treatment Level of N and K_2O	P_2O_5 Treatment Level (pounds per acre)	Derivative of Polynomial (bu.per acre)	Derivative of Exp. (1) (bu.per acre)	Derivative of Exp. (2) (bu.per acre)
1	40	.012	.013	.012
1	80	.011	.008	.008
1	160	.009	.006	.005
1	240	.008	.005	.005
1	320	.006	.004	.004
1	400	.004	.004	.004
1	480	.003	.004	.004
1	640	.001	.004	.004
2	40	.012	.015	.015
2	80	.012	.009	.009
2	160	.010	.006	.006
2	240	.008	.005	.005
2	320	.006	.005	.005
2	400	.005	.005	.005
2	480	.003	.005	.004
2	640	.000	.004	.004
3	40	.013	.016	.015
3	80	.013	.010	.010
3	160	.011	.007	.007
3	240	.009	.006	.006
3	320	.008	.005	.005
3	400	.006	.005	.005
3	480	.004	.005	.005
3	640	.008	.005	.005
4	40	.014	.019	.018
4	80	.013	.012	.012
4	160	.011	.008	.008
4	240	.010	.007	.007
4	320	.008	.006	.006
4	400	.006	.006	.006
4	480	.005	.005	.006
4	640	.001	.006	.006

[1] P_2O_5 is varied with N and K_2O fixed at (1) 20-20 (2) 80-80 (3) 160-160 and (4) 320-320 respectively. Derivatives of the polynomial and Exp. (1) are independent of applied K_2O since there were no K_2O variables in the functions for which these derivatives were taken.



N and K_2O are fixed at 80 pounds per acre

Fig. 8. Partial derivatives of a polynomial and two exponential functions for beans with respect to phosphoric acid.

schedules. These derivatives, shown in Table 18 and Figure 7, first exhibit a range of diminishing values and then acquire values of increasing magnitude. Because the derivatives of the exponential exhibit this rather illogical property of diminishing returns followed by increasing returns to successive nitrogen inputs, the polynomial equation is probably a more appropriate approximation to the fertilizer response surface.

Because of the phenomenon of increasing returns to nitrogen inputs exhibited by the exponential functions, maximum yields and high profit plant nutrient inputs lie beyond the range of experimental inputs. The maximum yield as calculated from the polynomial equation is 32.2 bushels per acre. This maximum is achieved using slightly less than 318 pounds of nitrogen and about 629 pounds of P_2O_5. The quantities of N and P_2O_5 producing the maximum bean yield are almost identical with those of the highest treatment level in the experiment.

Despite the large phosphoric acid inputs which produced maximum yields, applications of phosphoric acid were not profitable at typical crop and fertilizer prices. Assuming a price of \$0.10 per pound for P_2O_5, use of P_2O_5 became profitable only with bean prices in excess of \$7.00 per bushel. Nitrogen inputs, on the other hand, were profitable over a wide range of bean and nitrogen prices. Assuming a price of \$0.15 per pound for nitrogen, the high profit quantity of nitrogen ranged from about 76 pounds with bean prices at \$3.50 per bushel, to 205 pounds at \$7.50 per bushel. Estimated high profit nutrient inputs for various bean prices are shown in Table 20.

TABLE 20

HIGH PROFIT FERTILIZER INPUTS FOR FIELD BEANS WITH VARYING BEAN PRICES

Bean Price (per bushel)	High Profit Plant Nutrient Input[1]		Predicted Yield (bu.per acre)
	N	P_2O_5	
$3.00	35	0	19.70
3.50	76	0	21.82
4.00	106	0	23.15
4.50	130	0	24.07
5.00	148	0	24.68
5.50	164	0	25.16
6.00	177	0	25.52
6.50	188	0	25.79
7.00	197	0	25.99
7.50	205	35	26.60

[1]N and P_2O_5 were priced at $0.15 and 0.10 per pound respectively.

Analysis of the Potato Data

The original potato experiment was initiated in 1954. Data have been collected for three successive years. Only P_2O_5 and K_2O were varied in this experiment. The response surface estimated for the 1954 data was one of diminishing absolute yields with additional inputs of P_2O_5 and K_2O. The pre-treatment fertility level of the plots was such that, given the weather conditions existing in 1954, the portion of the response

surface characterized by the experiment was that of stage three in the input-output dimension, i.e., negative marginal returns to additional plant nutrient inputs.

Preliminary analysis of data collected from the two succeeding years, 1955 and 1956, indicated no significant change in potato yields associated with applied plant nutrients. Further analysis of the soil test data and associated changes in yields over time may provide useful information as to depletion rates and residual fertility as well as yield response to applied plant nutrients. However, data collected to date from the original potato experiment do not indicate P_2O_5 and K_2O responses of economic consequence.

Data were also collected in 1956 for the 114 plot potato experiment initiated on a previously unfarmed parcel of muck soil. Preliminary analysis of these data indicated that much of the variance in potato yields was not associated with variance in applied plant nutrients. However, as some discernible relationships were evident in the data, a functional analysis was conducted and is presented in Equations XIV and XV.

The first formulation attempted was a five-variable polynomial which is shown in Equation XIV.

$$\underline{\text{Equation (XIV)}}^{1}: \quad \hat{Y}_p = 38.48031437 - \underset{(.04313059)}{.06692653}\ P + \underset{(.01979247)}{.12066566}\ K +$$

$$\underset{(.00009859)}{.00003556}\ P^2 - \underset{(.00002250)}{.00013183}\ K^2 + \underset{(.00003975)}{.00022502}\ PK$$

---

[1]Yields expressed in Equations XIV and XV are pounds per plot. Multiplying pounds per plot by a conversion factor of 6.8062 gives the potato yield in bushels per acre.

The adjusted coefficients of multiple correlation and multiple determination for this equation are .501 and .251 respectively. The second, fourth and fifth variables of this equation have coefficients which are statistically significant at the one per cent probability level.

The second formulation of the functional relationship was an exponential type equation as shown in Equation XV.

Equation (XV): $$\text{Log } \hat{Y}_p = 1.42130959 - \underset{(.03407468)}{.05509970} \log P - \underset{(.00014119)}{.00013065} P + \underset{(.02894742)}{.20502089} \log K - \underset{(.00006317)}{.00021112} K$$

The adjusted coefficients of multiple correlation and multiple determination for Equation XV are .585 and .342 respectively. In the latter formulation, coefficients estimated for both K_2O variables are significant at the one percent probability level.

Maximum Yields and High Profit Plant Nutrient Applications

Because of the complex nature of the P_2O_5-K_2O yield relationship, it is extremely difficult to determine the amounts of plant nutrient inputs (1) which maximize yields or (2) which maximize profits. The complexity of these relationships is further exemplified by the derivatives of the functions. For example, the partial derivative of yield with respect to phosphoric acid for the polynomial is negative for almost any quantity of P_2O_5 unless K_2O is fixed at a level of at least 250 pounds per acre. In the case of the exponential, the partial derivative of yield with respect to phosphoric acid is always negative. Because of these unusual

phenomena, at ordinary potato prices the calculated high-profit quantity of P_2O_5 is negative and consequently outside of the range of observed values.

It is the expressed opinion of soil scientists that the interaction between P_2O_5 and K_2O is an important complementary relationship for potato production on muck soils. This interaction effect may exceed in importance the individual effects of either plant nutrient. There is, in particular, a commonly held belief that applied P_2O_5 will cause significant increases in potato yields only if adequate amounts of K_2O are concurrently present in the soil. In view of this, it may not be illogical to assume that P_2O_5 applications had a non-significant effect on yields at low K_2O treatment levels. Both equations contain at least one nonsignificant P_2O_5 coefficient which may bias the estimate of the production surface and consequently the derivatives of the function.

Further detailed analysis of these data is needed; however, it appears that, given the weather conditions of 1956, no substantial applications of plant nutrients were profitable. If any plant nutrient applications were profitable at all in 1956 they were only moderate applications of potash.

CHAPTER V

SOURCES OF UNEXPLAINED VARIANCE IN YIELDS AND BIAS OF REGRESSION COEFFICIENTS

The analysis presented in Chapter IV was designed primarily to explain variance in crop yields with logically formulated functional relationships between quantities of applied plant nutrients and crop yields. Statistical estimates of the parameters of the plant nutrient variables were made using alternative production function formulations. On the basis of these estimates, inferences were made as to the shape of the plant nutrient-crop yield production surface, plant nutrient combinations producing maximum crop yields, plant nutrient combinations producing maximum dollar profits, etc.

Variance in crop yields was not solely a function of variance in the quantities of applied plant nutrients. The adjusted coefficients of multiple determination ranged from a high of .58 to a low of .05 for the crops analyzed. Lacking knowledge of the exact form of the functional relationship between applied plant nutrients and crop yields and, furthermore, lacking completely effective control over unstudied variables, one should not expect 100 per cent of the variance in crop yields to be associated with regression. One might, however, expect a greater proportion of yield variance to be associated with regression than was found to be the case in the analysis of the preceding chapter.

Failure to characterize the major portion of yield variance by functional analysis raises questions as to whether or not experimental controls were rigidly enforced.

This chapter will be directed first towards an explanation of variance in crop yields not explained by the regression of applied plant nutrients. An additional problem deals with whether or not unspecified variables were randomly and normally distributed with respect to the independent variables studied.

Sources of Unexplained Variance in Yields

Sources of unexplained variance in yield can be broadly classified as being due to (1) experimental error with respect to variables specified and measured and (2) inadequate control over unspecified and unmeasured variables. Since these two sources of yield variance should be normally and randomly distributed with respect to treatment variables, they may be viewed as being sources of within treatment yield variance.

Experimental Error

Some portion of the unexplained variance in crop yields is undoubtedly due to experimental error. Such errors are made by not applying the specified amounts of plant nutrients on individual plots or errors made in acquiring yield measurements from the plots. Other sources of experimental error are uneven seed and fertilizer distribution on plots to mention only a few. In general, however, these errors are expected to be somewhat normally and randomly distributed with respect to

treatments and should be averaged out in the statistical estimating process. Researchers should recognize that this component of variance is present even in rigorously controlled experiments. Competent researchers should attempt to minimize such errors subject to the condition that the cost of reducing the errors is not in excess of the value of the gain in accuracy resulting from their reduction. For example, mechanization of controlled experiments may introduce experimental error in excess of that occurring with the use of hand-labor methods. However, minor increases in experimental error may be more than offset by the acquisition of additional information and better functional analysis resulting from additional plots and/or larger plots. Thus, reduction of experimental error should not be established as an absolute goal but rather one subject to economic considerations.

It is the opinion of the author that the data analyzed in the preceding chapter did not, in general, have excessive experimental error. Some experimental error, however, was present. In particular, the continuous corn experiment was characterized by a considerable amount of such error. Due to unfavorable weather conditions it was necessary to harvest the continuous corn plots by hand. Only a subsample from each plot was harvested; consequently, due to the smaller harvested sample a larger experimental error would be expected. Furthermore, the previous cropping history varied for some of the plots in this experiment. Although corn yields from plots on the two areas with different cropping histories were not significantly different statistically, this heterogeneity of previous land use probably contributed to a minor amount of

variance in yields. Since the total yield variance was small originally, the existence of experimental error made it difficult to isolate the effects on yield variance due to variance in the quantity of applied plant nutrients.

Uncontrolled and Unmeasured Variables

Numerous factors such as weather, insects, bacterial action in the soil, etc.,are possible sources of variance in crop yields not explained by the functional analysis in Chapter IV. The field bean input-output experiment was duplicated in the greenhouse. Results of the greenhouse experimentation are presented here to substantiate the hypothesis that yield variance could be explained by functional analysis given adequate control of unmeasured variables affecting yield and/or specification and measurement of these variables.

A nine variable polynomial was fitted to data produced in the greenhouse. The soil contained in individual greenhouse pots was acquired from the corresponding field plots. The same number of observations were acquired using the same treatment levels as in the field experiment. Yields acquired in the greenhouse were for bean numbers per pot since the beans could not be allowed to mature under greenhouse conditions.[1] The results of this regression analysis are presented in the following equation:

---

[1]Bean count and bean yields are not perfectly correlated, however, the two measures should be sufficiently correlated to allow valid inferences to be made from one to the other quantity-(yield) wise. Bean count might, however, be a considerably less valid measure of the quality of the crop.

$$\hat{Y}_{gh} = 9.22679560 + \underset{(.03784008)}{.45406108}\ N + \underset{(.01919871)}{.0203075}\ P + \underset{(.03779040)}{.07473918}\ K - \underset{(.00011092)}{.00081362}\ N^2 -$$

$$\underset{(.00019855)}{.00001986}\ P^2 - \underset{(.00011181)}{.00258338}\ K^2 + \underset{(.00003765)}{.00197867}\ NP + \underset{(.00007785)}{.00012933}\ NK - \underset{(.00003762)}{.00003616}\ PK$$

$\overline{R} = .91$

$\overline{R}^2 = .828$

These results indicate that about 83 per cent of the variance in bean count for the greenhouse pots was associated with regression. In the functional analysis of the field data, however, only 42 per cent or about one-half as much of the variance in bean yields was associated with regression. The inference suggested by this comparison of analyses is that explanation of more of the variance in yield under field conditions would be possible if variables affecting yield could be better controlled and/or measured and specified in the functional relationship.

Effects of Within Treatment Variance on Statistical Estimates

The presence of within treatment variance should be noted when evaluating the relative success of particular functional forms in characterizing input-output relationships. If there is a difference in the yields from plots receiving the same plant nutrient applications, any function fitted to these data by least squares techniques, or any valid estimating procedure, will miss one or both yield observations. The greater the difference in yields between replicated plots, the greater will be the variance which cannot be explained by the function. Failure to explain this within treatment variance is not therefore a valid criticism of a particular functional form.

The effects of within treatment variance on the amount of total variance explained by regression may be exemplified by use of data from the 1956 wheat experiment. Data from all 130 plots were used in acquiring the statistical estimates made for the nine variable polynomial function presented in Equation V. A second polymial equation was fitted to the average yields of the plots which had a minimum of two replications for a given treatment. These observations include yields from the 3 x 3 x 3 factorial which was replicated twice and the 11 check plots for a total of 65 plots averaged into 28 observations. Statistical results for the function fitted to the average yields from replicated treatments is shown in Equation XVII.

Equation XVII: $\hat{Y}_w = 27.87287 + \underset{(.02923984)}{.11222667}\ N - \underset{(.00010520)}{.00028877}\ N^2 + \underset{(.01461992)}{.02207660}\ P -$

$\underset{(.00002630)}{.0004605}\ P^2 - \underset{(.2923984)}{.01407013}\ K + \underset{(.00010520)}{.00011011}\ K^2 + \underset{(.00002736)}{.00003065}\ NP - \underset{(.00005472)}{.00013398}\ NK +$

$\underset{(.00002736)}{.00001889}\ PK$

The adjusted coefficients of multiple correlation and multiple determination for this equation were .79 and .62 respectively as compared to .66 and .44 for the function fitted to all 130 individual observations. This sizeable increase in the amount of yield variance explained by regression illustrates that within treatment variance was an important component of total yield variance. The parameters of a function fitted to the average value of replicated plots, if all plots are replicated an equal number of

times, should be the same as those for a function fitted to the non-averaged observations, however, averaging values for replicated observations discards part of the information provided by the experimental data.

Factors Related to the Independent Variables Used in Regression Analysis

Visual observation of the experimental plots indicates that there were yield variance creating components which were associated with plant nutrients and which therefore were either (1) sources of biases in the estimated effects of plant nutrients on yields or (2) sources of yield variance which should be considered when evaluating the aggregate effects of applied plant nutrients.

Incidence of Weeds, Lodging and Plant Disease

Observational data collected for oats in 1956 indicated significant differences in weed growth and plant lodging which were associated with nitrogen applications. Prior to harvesting the oats crop, individual plots were ranked as to the degree of weed infestation and plant lodging and then these ranks were tabulated against nitrogen applications. The results of this classification are shown in Table 21. The incidence of weeds in plots was ranked from 0 to 3 with an increase in number rank indicating an increase in weed infestation. Lodging was ranked similarly from 0 to 8. Weeds and lodging not only affected the absolute crop yields produced on some plots but the harvestability of the crop as well. The ratio of the amount of grain produced to the amount of grain

TABLE 21

INCIDENCE OF WEED INFESTATION AND PLANT LODGING ON OAT PLOTS AS RELATED TO NITROGEN APPLICATIONS

Nitrogen Application (pounds per acre)	Number of Plots	Average Lodging Score[1]	Standard Deviation of Lodging Score	Average Weed[2] Score	Standard Deviation of Weed Score
0	18	.889	.741	0	0
20	24	2.250	1.561	.167	.340
40	14	2.143	1.187	.143	.324
80	29	4.690	1.511	.828	.457
160	18	6.444	1.257	1.556	.889

[1]Plots were ranked from 0 to 8 according to the extent of lodging present.

[2]Weed incidence was ranked from 0 to 3.

harvested was probably significantly different for badly lodged plots as compared to non-lodged plots. No statistical measures of these differences were made, however.

Incidence of plant disease as well as weed growth varied with plant nutrient applications on the field bean plots. Particularly, quack grass infestations were more pronounced on high nitrogen plots than on plots receiving smaller applications of nitrogen. Plots with a large amount of plant foliage tended to have more shading of lower leaves and bean pods and consequently more disease infestation. The quantity of foliage on plots was, in turn, associated with the quantity of applied nitrogen.

These and other variance generating factors which are not independent of quantities of applied plant nutrients, but which may influence yields in a manner other than that specified in the production function formulation are sources of bias, i.e., they distort the absolute crop yield producing effects of plant nutrients. If such distortion or bias is a necessary consequence of applying plant nutrients it should be measured and considered when evaluating the effects of plant nutrient applications. In some instances, however, utilization of improved crop management practices may eliminate such effects. For example, if weeds could be adequately controlled and if a sufficiently strong strawed variety of wheat were available for planting, the potential effects of applied plant nutrients on crop yields might be realized. The effects of factors which are sources of bias in plant nutrient input-crop yield output estimates as well as a discussion of other factors interacting with plant nutrients in the production of crops are discussed adequately in other literature[1] and will not be enlarged upon here.

Relationships Between Residual Fertility and Crop Yields

All plots in the two rotation experiments received the same plant nutrient applications in 1955 and in 1956. It seemed logical to expect

---

[1]For a discussion of these factors see L. S. Robertson Jr., G. L. Johnson and J. F. Davis, "Problems Involved in the Integration of Agronomic and Economic Methodologies in Economic Optima Experiments," Fertilizer Innovations and Resource Use, Edited by E. L. Baum, E. O. Heady, J. T. Pesck and C. G. Hildreth (Ames: Iowa State College Press, 1956) pp. 226-242, and L. S. Robertson Jr., W. B. Sundquist and L. N. Shepherd "A Progress Report of the Studies on the Economics of Fertilizer Use on Beans and Potatoes," Mimeographed Report presented at a T.V.A. sponsored symposium on the economics of fertilizer use at Knoxville, Tennessee, March 1957.

some carryover or residual effects in 1956 from plant nutrient applications made in 1955. This was particularly true for the plots receiving heavy plant nutrient applications in the preceding year. Soil tests for P_2O_5 and K_2O were taken preceding and following every crop produced. Consequently, differences in fertility between plots prior to plant nutrient applications made in 1956 would be expected to be related to these soil test measures. The method utilized in attempting to relate variance in crop yields to soil fertility as measured by soil tests will be summarized briefly. Soil test measures were first correlated with the applied amounts of the same nutrient for the individual plots. If this correlation was very high it would indicate that (1) variance in yield could probably be explained as well by the original functional analysis using only applied plant nutrients and/or (2) it would be difficult to include both soil test and applied plant nutrient measures in a functional analysis since the presence of high intercorrelations[1] would reduce the reliability of estimated parameters of a function containing both measures as variables. If, on the other hand, the correlation between quantities of applied plant nutrients and soil tests was low, indicating some independence of the two measures, soil tests might successfully be used to explain a portion of the variance not associated with regression. The procedure used in relating soil test data to unexplained variances was to correlate the soil test data with the

---

[1]The distinction between high and low correlations is quite arbitrary, however, as intercorrelations approach .70 the reliability of estimated parameters probably begins to decrease quite rapidly.

residuals computed from the original functional analysis. The results obtained by this method of analysis are shown for one crop from each of the rotation experiments, namely, the wheat and bean crops produced in 1956.

Effects of Residual Fertility on Wheat Yields

Soil test data were collected for P_2O_5 and K_2O preceding the wheat crop grown in 1956. No analysis of residual nitrogen has been completed to date.[1] The first analysis conducted was that of correlating pre-1956 crop soil test measures[2] with the applications of P_2O_5 and K_2O made in 1955. In the following discussion, soil test values of P_2O_5 and K_2O are designated as P_{st} and K_{st} respectively. Applied P_2O_5 and K_2O are designated P_a and K_a.

A regression analysis was conducted using P_{st} as the dependent variable and P_a as the independent variable. The resulting regression equation is as follows:

$$\hat{P}_{st} = 34.9760 + \underset{(.016565)}{.166659}\ P_a$$

The coefficient of correlation for this equation was .662 and the coefficient of determination .438. A similar regression analysis was

---

[1]Several nitrogen tests determinations have been made for soil samples from these plots. No nitrogen soil tests have as yet been generally accepted as satisfactory. A statistical comparison of the effectiveness of alternative nitrogen soil tests for residual nitrogen is currently in process using soil samples from this experiment.

[2]These soil samples were actually acquired in September of 1955 immediately preceding seeding of the wheat crop which was harvested in 1956.

conducted relating soil test measures and applied quantities of K_2O. The resulting regression equation is as follows:

$$\hat{K}_{st} = 75.1084 + \underset{(.039190)}{.276689}\ K_a$$

The coefficients of correlation and determination for this equation were .526 and .277 respectively. A preliminary inspection of the residuals of these functions indicates that little, if any, improvement could be made by changing the formulation, i.e., by fitting a curvilinear form such as $P_{st} = a + P_a + P_a^2$ to the P_2O_5 variables.

As evidenced by the preceding analysis, applied and residual plant nutrients show a moderate amount of interdependence or correlation. A correlation as high as .66, as was found between P_{st} and P_a, might indicate that the effects of residual and applied P_2O_5 could not be easily separated. The correlation between K_{st} and K_a, .53, does not, however, appear to be prohibitively high.

On the basis of the preceding exploratory analysis it was decided that some reduction in unexplained yield variance might be accomplished by incorporating the residual fertility measures into the analysis. Simple correlation analysis was conducted using P_{st} and K_{st} as separate independent variables and the residuals from the nine variable polynomial (Equation V) as the dependent variable. Designating the residuals or deviations from the polynomial $(Y_i - \hat{Y}_i)$ as D, the results of these simple correlation analyses are as follows:

$$\hat{D} = -.159696 + \underset{(.006831)}{.002106}\ P_{st}$$

$$\bar{r} = .031$$

$$\bar{r}^2 = .00095$$

$$\hat{D} = -1.458583 + \underset{(.006440)}{.014232}\ K_{st}$$

$$\bar{r} = .190$$

$$\bar{r}^2 = .0362$$

Phosphoric acid soil test measures appear to bear no relation to the unexplained residuals of the original functional analysis, whereas, potash soil tests are slightly, but not significantly, related to these residuals. The inference suggested by this analysis appears to be that soil test measures do not provide an aid in reducing unexplained yield variance in the case of wheat, at least not in the simple relational form analyzed here. It should be remembered, however, that soil test measures were correlated with quantities of applied plant nutrients and that most of their effects on yields are probably incorporated in the original functional analysis. Additional work is currently in progress[1] evaluating soil test procedures and relating these measures to quantities of applied plant nutrients.

---

[1]Gordon Anderson of the Department of Agricultural Economics and Arthur Wolcott of the Department of Soil Science of the Michigan Agricultural Experiment Station are cooperating on this phase of research work.

Effects of Residual Fertility on Bean Yields

As in the case of wheat, soil test data were collected prior to fertilization and planting of the 1956 bean crop. P_2O_5 soil test observations ranged in values from 48 to 504, however, only one observation was in excess of 400. K_2O soil tests ranged from a low of 84 to a high of 400. Application rates in the bean experiments ranged from 0 to 320 for K_2O and from 0 to 640 for P_2O_5. A regression analysis was conducted using P_{st} as the dependent variable and P_a as the independent variable. The results of this regression were as follows:

$$\hat{P}_{st} = -97.9670078 + \underset{(.116164)}{2.253587}\ P_a$$

$$\bar{r} = .770$$

$$\bar{r}^2 = .593$$

The same analysis was conducted using potash soil test measures and treatment rates as variables. The results of this regression are shown in the following equation:

$$\hat{K}_{st} = -74.970613 + \underset{(.091887)}{1.245960}\ K_a$$

$$\bar{r} = .645$$

$$\bar{r}^2 = .416$$

As in the case of the wheat experiment, a greater portion of the variance in P_2O_5 soil tests was associated with variance in P_2O_5 applications of the preceding year than was the case for K_2O. Correlations as large as these, .770 and .645 respectively, indicate that the effects

of residual fertility on bean yields might well have been explained at least in part by the original functional analysis in which yield variance was formulated as a function of variance in applied plant nutrients alone.

Despite the high correlation between soil test measures and quantities of applied P_2O_5 and K_2O, a multiple regression analysis was conducted using P_{st} and K_{st} as dependent variables and the residuals, $(Y_i - \hat{Y}_i)$, from Equation XI, the six variable exponential equation, as the dependent variable. The results of this multiple regression analysis are as follows:

$$\hat{D} = 2.0157 - \underset{(.00579)}{.07973}\ P_{st} - \underset{(.00448)}{.02042}\ K_{st}$$

The parameters of this equation are highly significant, however, the adjusted coefficient of multiple correlation is only .008. This value of $\bar{R}$ is not significantly different from zero. As in the wheat experiment, no significant amount of the variance in yields not explained by the original regression analysis with applied plant nutrients as independent variables can be attributed to residual fertility as measured by K_2O and P_2O_5 soil tests.

There are several reasons why failure to relate unexplained yield variance to soil test measures should not be interpreted as meaning that crop yields are not a function of residual fertility. P_2O_5 and K_2O soil tests were found to be significantly related to amounts of applied nutrients, hence, a portion of their effect on yield variance would be expected to be characterized by the original functional analysis.

In addition, soil test measures for nitrogen, which in almost all experiments had the predominant effect on crop yields, were not included in the analysis. Soil test measures are themselves subject to considerable variance because of errors in sampling and in testing the samples.

Additional research needs to be undertaken in calculating sampling and testing variances for soil test procedures. Such research would provide an aid in evaluating the accuracy and adequacy of soil testing procedures currently being used. Another possible explanation of the low correlation between soil tests and residuals is that a more complex formulation of the relationship between soil test measures and unexplained residuals would have been more appropriate, i.e., the linear relationship assumed in simple correlation analysis may be an oversimplification of the relationship between these variables.

Because of the importance of soil test data in making current fertilizer recommendations, additional work needs to be done relating alternative soil test measures to: (1) variance in crop yields (2) quantities of applied plant nutrients to establish substitution ratios between applied and residual plant nutrients and (3) other soil testing methods to determine the most effective soil test procedures available.

The plant nutrient input-output experiments described earlier should provide data well adapted to an analysis of soil testing procedures. The experiments contain extremely high and extremely low levels of plant nutrient applications and consequently a wide range of residual fertility values is developing on the plots. As individual plots become extremely depleted of plant nutrients or extremely fertile they will provide a

wide range of soil test observations. If plant nutrient applications were to be rerandomized on the plots, a wide range of residual and applied nutrient combinations could be observed. Thus the effects of residual fertility might be studied without the complicating influence of highly correlated plant nutrient applications. Furthermore, substitution ratios between residual and applied nutrients could be estimated from a wide range in the combinations of the two.

Conclusions

Several sub-inferences may be drawn from the analysis presented in this chapter. The important conclusion, however, seems to be simply this: Given (1) adequate control over specified factors affecting crop yields, and (2) a random and normal distribution of other factors affecting crop yields, functional analysis should provide an adequate representation of plant nutrient input-crop yield output relationships. The relatively small amount of total variance in crop yields explained by functional analysis is not an inherent characteristic of the analysis and/or the functional forms used but rather is largely a function of the uncontrolled factors enumerated in this chapter.

CHAPTER VI

EVALUATION OF PROCEDURES AND RESULTS

Evaluation of Experimental Designs

The experimental designs used in the several experiments described in this work were formulated with several restrictions and objectives in view. Prior to designing the experiments, it was decided that continuous function analysis of the experimental data would provide a better basis for (1) estimating plant nutrient input-crop yield output coefficients and (2) facilitating an economic analysis to determine optimal plant nutrient applications, than would alternative methods of analysis. Thus the experiments were designed to provide data suitable for continuous function analysis. Restrictions on funds, labor and equipment limited the number and/or size of the experimental plots. Individual treatments or cells in the experimental designs were selected to: (1) describe the economically relevant portion of the production surface sufficiently to obtain reliable estimates of parameters of the production functions (2) establish with adequacy input-output measures for critical points on the production surfaces, e.g., origin of the functions and their inflection points and (3) minimize intercorrelations among treatment variables.

It is the opinion of the author that the experimental designs were quite satisfactory as a basis for providing data for continuous functions

analysis. The experimental designs utilized in the two original rotation experiments are not highly efficient in providing data which readily facilitates estimation of (1) within treatment yield variance[1] (2) crop quality differences associated with treatments, (3) differences in plant nutrient content of plant tissue and (4) differences in other plant and soil characteristics associated with plant nutrients but not in the manner prescribed for the basic input-output relationship.

Once committed to an incomplete factorial design with a minimum number of replications, analysis of such factors as those listed above may be quite difficult.[2] However, the designs which were used are adequate for these determinations if (1) the determinations can be made by correlation analysis or (2) if the determinations for one plant nutrient can be assumed to be independent of the treatment level of other plant nutrients. In the latter case this means that all observations for which the treatment level of the studied variable are constant can be considered as replications of that treatment.

A modification of the incomplete factorial--minimum replication design used in the rotation experiments was incorporated into the continuous corn, the 1956 potato and the new sugar beet experiments. These designs include a triplicated factorial in addition to other treatments which were replicated twice. This modification was incorporated into

---

[1]Inability to specify within treatment variance is not considered to be an important criticism of the experimental design.

[2]The inference being made here is that some of the determinations listed above can best be acquired by analysis of variance techniques.

the designs to facilitate analysis of by-product data produced in the experiment. The experimental designs, as modified, still provide numerous non-replicated treatments in order to specify the production surface adequately for continuous function analysis. Inclusion of a factorial into the experimental design facilitates utilization of analysis of variance techniques on a limited basis at little additional cost.

A possible criticism of the experimental designs which were used might be the large spacing between treatment levels of the various plant nutrients. Obviously, it would be desirable to have observations at treatment levels intermediate to those contained in the experiment, however, the experiments already were large and required a considerable amount of land, labor, machinery, equipment and supervision. Larger experiments would have created problems in conducting experimental work, such as seeding, harvesting etc., with appropriate timeliness. The primary consideration in not enlarging the experiments by including intermediate treatment levels was that of the additional time and cost which would be necessitated by such an expansion.

The correlation between applied and residual plant nutrients is relatively high in these experiments since individual plots receive the same treatment in successive years. A more comprehensive analysis of residual and applied plant nutrient relationships would be facilitated by rerandomizing treatments on the experimental fields. Such a modification of the experimental design would provide observations over a much wider range of combinations of residual and applied nutrients. This is a modification of the experimental design currently being contemplated.

Evaluation of Experimental Procedures

To the extent feasible, the experimental work was conducted utilizing mechanized procedures. When soil conditions allowed, plant nutrient applications were made with a 7 ft. tractor drawn drill. Small grain seedings were also made with a 7 ft. drill which required one round on the plots which were 14 feet wide. Wheat and oats crops were harvested with a 7 ft. self propelled combine. A portion of the corn crop was harvested by using an especially constructed single-row corn picker. In instances where weather conditions prevented fertilizer application and corn harvest by machine, this work was accomplished by use of hand labor.

Some amount of additional experimental error undoubtedly occurs due to use of machinery as compared to hand labor; for example, plant nutrient applications are not precisely weighed out and delivered in exact amounts to individual plots. Small amounts of grain remain in the combine from one plot to another when harvesting etc. and introduce some small experimental error. These errors should, for the most part, however, average out and not bias the plant nutrient input-crop yield output estimates made.

Mechanization of experimental work provides some interesting and important implications particularly with respect to the number and size of individual plots which can be satisfactorily included in an experiment. Two objectives of plant nutrient input-crop yield output research appear to be of relevance here. First, we want research results to be validly

inferrable to some farm population. Farmers typically operate as units fields of a minimum of several acres in size. The larger the experimental plots, the more nearly they represent the conditions actually existing on farms. Farmers, and consequently researchers whose objective is to make input-output estimates applicable to farm conditions, are not particularly interested in measuring within treatment yield variance. Rather, they are interested in determining the variance in yield which can be attributed to variance in plant nutrient applications under farm conditions e.g., the change in yield resulting from application of an additional 20 pounds of nitrogen etc. Researchers are interested, however, in having some assurance that within treatment yield variance is not prohibitively large so as to constitute a large portion of total yield variance. Within treatment variance is reduced by increasing the size of individual experimental plots and the harvested portion of these plots. Increases in plot size are facilitated by mechanizing the experimental procedures used. Errors of inference due to excessive within treatment yield variance can be eliminated alternatively by replicating a given treatment several times and averaging the yields of the several replications. Additional replications of a treatment require more labor and have a higher cost than is true for a comparable enlargement of a given plot. It is the opinion of the author that when the main objective of experimentation is to estimate plant nutrient response surfaces, increasing plot size is a more efficient alternative.

A second objective of our research, that of estimating input-output coefficients to which we can attach acceptable reliability measures,

is aided by increasing the number of individual plots in an experiment. The standard error of estimate for parameters in a functional equation diminishes as the number of observations increases. Attainment of both accurate and applicable research results is, therefore, enhanced by increasing the size and number of experimental plots.

It is the opinion of the author that within treatment variance in the experiments was probably higher than was necessary. Use of larger plots and/or harvesting a larger portion of individual plots would probably have provided results the additional accuracy of which would have been worth the cost of obtaining this accuracy.

Evaluation of Analytical Procedures

The Continuous Function Analysis

A brief justification for utilizing continuous function analysis was presented in Chapter II and will not be repeated or expanded here. Rather, a brief a posteriori evaluation of the effectiveness of the continuous function analysis used will be attempted here. Both polynomial and exponential type formulations of the respective production functions were fitted for all crops for which preliminary analysis indicated that an appreciable amount of variance in yield was associated with variance in applied plant nutrients. No criteria are available which provide a basis for saying one formulation is "absolutely" more appropriate than the other; however, some measures which provide somewhat of a quantitative basis for comparison are available. Furthermore, logic and theory provide

a basis for selecting one formulation in preference to the other in at least two instances.

As previously mentioned, comparison of the coefficients of multiple correlation for the two functions provides a guide as to the relative amount of yield variance associated with regression. This comparison is somewhat subjective, however, since: (1) in the case of the exponentials, variance is measured in logarithms and in the polynomials it is measured in real numerical values. Although the logarithms and real numbers bear a consistent monotonic relationship to each other over the range of the values which they take in the data, they do not retain a relationship of constant ratios. (2) The two formulations differ as to the number of variables in the respective equations, hence there is a small difference in the number of degrees of freedom used in the two analyses. The latter difficulty is not an important one, however, because of the large number of observations and, hence, degrees of freedom, present in the analysis. A comparison of the coefficients of multiple correlation and determination for the functions fitted is shown in Table 22. In three of the six comparisons a larger amount of yield variance is explained by regression for the exponential equations than for the polynomials. In one case, that of the field beans, the polynomial equation has larger values of $\bar{R}$ and $\bar{R}^2$, whereas, in the remaining two comparisons values of $\bar{R}$ and $\bar{R}^2$ for the two equations are almost identical. This comparison provides no very conclusive indication as to the superiority of either type of formulation.

TABLE 22

COMPARISON OF AMOUNTS OF YIELD VARIANCE ASSOCIATED WITH ALTERNATIVE PRODUCTION FUNCTION FORMULATIONS

Crop	Function	Number of Variables	$\bar{R}$	$\bar{R}^2$
Oats, 1956	Polynomial	9	.69	.48
	Exponential	6	.76	.58
Wheat, 1956	Polynomial	9	.66	.44
	Exponential	6	.65	.42
Corn, 1955	Exponential	6	.70	.47
	Polynomial[1]	9	.64	.41
Corn, 1956	Polynomial	9	.23	.05
	Exponential	6	.24	.06
Cont. Corn, 1956	Polynomial	9	.40	.16
	Polynomial	5	.39	.16
Beans, 1956	Polynomial	5	.65	.42
	Exponential	6	.61	.37
	Exponential	4	.61	.37
Potatoes, 1956	Polynomial	5	.50	.25
	Exponential	4	.59	.34

[1]The polynomial used on the 1955 Corn data was a square root polynomial of the form $Y = a + b_1N + b_2\sqrt{N} + b_3P + b_4\sqrt{P} + b_5K + b_6\sqrt{K} + b_7\sqrt{NP} + b_8\sqrt{NK} + b_9\sqrt{PK}$

A second comparison of the two types of functions was included in the analysis. Residual measures, $(Y_i - \hat{Y}_i)$, were computed for both types of functions. These residuals are measures of the deviation of predicted yields from observed yields. The residuals are almost identical for both types of functions for all crops. This is true for the magnitude of residuals as well as for their sign or direction. In summary, inspection and measurement of the residuals provides no discernable basis for choosing one function in preference to the other.

A third comparison of the polynomial and exponential functions which might provide some basis for choosing the more appropriate one is an inspection of the derivatives of these functions. Inspection of the partial derivatives of the exponential functions with respect to individual plant nutrients shows that the derivatives are usually of extreme magnitude (negative or positive) for small inputs of the plant nutrients and then become extremely small quite rapidly.[1] Extremely large derivatives, $\frac{\partial Y}{\partial X_i}$, with small inputs of the X_i are a consequence of the yield being zero when any of the $X_i = 0$. Derivatives of the polynomials in comparison usually take less extreme values.

It is the opinion of the author that over moderate plant nutrient input ranges for most crops, generally in the range of 20 lbs. to 200 lbs., the exponential is probably a satisfactory formulation of most of the

---

[1]There are exceptions to this statement. For example, the partial derivatives of bean yields with respect to nitrogen decreases at first and then increase with additional nitrogen inputs. There are other exceptions to this statement as well.

input-output relationships. Derivatives of the exponentials for field beans and potatoes, however, are contradictory to the usually accepted concept of diminishing returns. Maximum yields predicted using the exponential functions were outside of the range of observed inputs for the bean and potato crops.[1] However, the maximum potato yield predicted was secured using quantities of plant nutrients outside of the range of observed inputs using the polynomial as well.

Calculation of the quantities of plant nutrients which result in maximum profits is a much more complex procedure using the exponential type formulation than using a polynomial. Solving the exponential for optimal inputs requires use of a series of successive approximations known as Newton's method.[2] This method requires in part a graphic approximation refined by solving a series of equations. Statistical estimates of the parameters of both types of equations are rather easily acquired by methods of least squares.

The primary advantage of the exponential type formulation as compared to the particular polynomial used is that it permits derivatives, $\frac{\partial Y}{\partial X_i}$, to take on non-linear forms. Derivatives of a polynomial containing

---

[1]The phenomena of maximum predicted yields being outside of the range of observed inputs is a criticism of the function only if in reality the maximum yield does occur within the range of observed inputs and is fallaciously predicted to be outside. If, indeed, the true maximum yield exists beyond the range of observed inputs it is the experimental design, not the function, which should be criticized.

[2]A complete explanation of the method of solving a Carter-Halter type exponential is explained in a forthcoming article in the Journal of Farm Economics by A. N. Halter, H. O. Carter and J. G. Hocking. The authors also discuss in some detail the properties of this family of functions.

only first and second degree terms will necessarily be restricted to linear form. It is the opinion of the author that until easier computing procedures for solving a Carter-Halter type exponential for optimal plant nutrient inputs are available, that modifications of the polynomial type formulation might be more desirable. Incorporating variables of degree 1/2 or 3/2 or of a degree $\gtrless 2$ and not equal to 1 will result in non-linear derivatives.

Utilization of Soil Test Measures

No significant amount of variance not explained by regression was explained by use of P_2O_5 and K_2O soil test measures. Soil tests for residual quantities of P_2O_5 and K_2O were not significantly correlated with residuals $(Y_i - \hat{Y}_i)$ from the functions fitted for wheat and beans. The analysis presented here does not provide a very comprehensive exploitation of the possibilities of using soil test measures in supplementing functional analysis of applied plant nutrients. The Federal Extension Service, as well as Agricultural Experiment Stations and private fertilizer companies rely heavily on soil test data as a basis for making fertilizer recommendations. Because of the widespread use of these soil test procedures, any additional information relating variance in soil test measures to crop yields would be a very valuable contribution.

Economic Interpretation and Evaluation of Results

The most profitable amounts of plant nutrients to apply were computed for all crops except alfalfa. The analysis presented in Chapter IV

indicated a significant response to nitrogen for corn produced on a Kalamazoo sandy loam soil in 1955. Significant yield responses to applied nitrogen were recorded for oats, wheat, field beans and corn produced on a Wismer clay loam soil in 1956. The only crop not showing a significant response to nitrogen in 1956 was the corn produced on a Kalamazoo sandy loam soil.

Statistically significant response to applied phosphoric acid was recorded for wheat, field beans and the corn produced on a Wismer clay loam soil in 1956. Oats and corn produced on a Kalamazoo sandy loam soil in 1956 did not show significant yield response to applied phosphoric acid. The only crop showing significant yield response to applied potash was the potato crop produced in 1956 on a Houghton muck soil.

Despite the several significant responses recorded, only small amounts of plant nutrient applications were indicated to be profitable. Predicted high-profit plant nutrient inputs for the various crops are shown in Table 23. No applications of P_2O_5 and K_2O were indicated to be profitable for any of the crops produced at typical crop and fertilizer prices. Nitrogen applications were profitable for five of the crops produced if crop prices were sufficiently high. Assuming typical prices, however, nitrogen applications were profitable only for corn produced in 1955 and field beans produced in 1956.

Some qualification of these results seems to be warranted. First, the 1955 and 1956 growing seasons were characterized by severe summer drouths. Thus the responses recorded may not typify the long-run expected responses to applied plant nutrients. Additional data collected

TABLE 23

ESTIMATED HIGH-PROFIT PLANT NUTRIENT APPLICATIONS FOR VARIOUS CROPS

Crop	Estimated High Profit Plant Nutrient Inputs[1]		
	N	P_2O_5	K_2O
Oats, 1955	None	None	None
Oats, 1956	Only if the price of oats > $1.00	None	None
Wheat, 1956	Only if the price of wheat > $3.00	None	None
Corn, 1955 (on Kalamazoo Sandy loam soil)	About 40 lbs. at Corn prices of $1.20-1.50	None	None
Corn, 1956 (on Kalamazoo Sandy loam soil)	None	None	None
Corn, 1956 (on Wisner Clay loam soil)	Only if the price of corn > 2.00	None	None
Field Beans, 1956	75 lbs. with beans $3.50, 150 lbs. with beans at $5.00 and 200 lbs. with beans at $7.00	Only if bean prices are > $7.00	None
Potatoes, 1956	Not varied in experiment	None	None at ordinary prices

[1]Computed with N at $0.15 per lb., P_2O_5 at $0.10 per lb. and K_2O at $0.11 per lb.

over time are needed to obtain a probability distribution of yield responses over the range of existing weather conditions.

As a further qualification, it should be noted that the experimental results reported in the preceding analysis were obtained from soils either (1) relatively unproductive, as in the case of the Kalamazoo sandy loam soil or (2) relatively heavy and productive in the cases of the Simms loam and Wisner clay loam soils. One might expect, a priori, to obtain the greatest yield response to applied plant nutrients from soils with a high productive potential but with low fertility levels. Greater yield response may be noted in future years on low nutrient level plots as residual fertility is depleted.

Concluding Remarks

The analysis of experimental work presented here is rather limited in scope with respect to number of soils, crops and growing seasons. Additional work is needed before the optimal plant nutrient treatments estimated here can be substantiated or invalidated as long-run optimal applications. The distribution of yield responses over time is likely to be characterized by wide dispersions, particularly in the case of the lighter soils which are frequently subject to damaging drouth periods. However, some interesting questions and implications are posed by the results of the analyses presented here.

No significant response was obtained from applied potash for the several crops grown on mineral soils during a two-year period of experimentation. This lack of response poses a question as to the

validity of recommending a program of "balanced" plant nutrient applications. Rather, the general responses recorded from these experiments indicate that nitrogen was the primary source of crop yield response. On the basis of these results it appears that a plant nutrient combination weighted more heavily with nitrogen relative to potash might be optimal at least until residual potash is depleted somewhat.

A second general implication posed by the experimental results is, "despite statistically significant yield responses, in most cases the cost of applying additional plant nutrients exceeded the value of the additional crop produced." This general result would indicate that analysis which only detects significant yield differences which are associated with plant nutrient applications is not an adequate procedure for determining the most profitable application rates. This result in itself would seem to validate or at least vindicate the general type of analysis used in this dissertation, i.e., that of continuous function analysis to which economizing principles may be applied.

In conclusion, at the farm management application level of fertilization practices, these practices cannot be considered independent of other alternative farm business expenditures nor can they be considered independent of the numerous factors with which they interact. For example, a livestock farmer may find it profitable to fertilize oats, not for the oat yield benefits, but in order to establish a clover or grass seeding which is essential to his livestock enterprise. However, if a farm manager is to intelligently and economically synthesize the costs and

benefits of the numerous components of his farm business he needs information as to the productivity of expenditures made for plant nutrients for the various crops he produces. Additional plant nutrient input-crop yield output estimates will help to provide this information.

BIBLIOGRAPHY

Barton, G. T. and Rogers, R. O. "Farm Output, Projected Changes and Projected Needs," Agricultural Information Bulletin No. 162, Washington: Agricultural Research Service, August, 1956.

Baum, E. L., Heady, Earl O. and Blackmore, John. Economic Analysis of Fertilizer Use Data, Ames: Iowa State College Press, 1956.

Baum, E. L., Heady, Earl O., Pesek, John T. and Hildreth, Clifford G. Fertilizer Innovations and Resource Use, Ames: Iowa State College Press, 1957.

Bureau of the Census. Current Population Reports, Series P-25 No. 123, Washington: U. S. Government Printing Office, October 20, 1955.

Bureau of the Census. "Use and Expenditures for Fertilizer and Lime," Adapted from the 1954 Census of Agriculture, Washington: U. S. Government Printing Office, 1956.

Fuller, E. I. "Michigan Dairy Farm Organizations Designed to Use Labor Efficiently," Unpublished Masters Thesis, Department of Agricultural Economics, Michigan State University, 1957.

Heady, Earl O., Pesek, John and Brown, William. "Crop Response Surfaces and Economic Optima In Fertilizer Use," Research Bulletin 424, Ames: Agricultural Experiment Station, Iowa State College, 1955.

Hildreth, C. "Point Estimates of Ordinates of Concave Functions," Journal of the American Statistical Association, Vol. 49, September, 1954, pp. 598-619.

Hutton, Robert F. "An Appraisal of Research on the Economics of Fertilizer Use." Agricultural Economics Branch, Division of Agricultural Relations, Tennessee Authority Report No. T 55-1, Knoxville: Tennessee Valley Authority, 1955.

Johnson, Glenn L. "A Critical Evaluation of Fertilization Research," Farm Management in the West Problems In Resource Use, Report No. 1, The Economics of Fertilizer Application. Conference Proceedings of the Farm Management Research Committee of the Western Agricultural Economics Research Council, Corvallis: 1956, pp. 33-45.

Johnson, Paul R. "Alternative Functions for Analyzing a Fertilizer Yield Relationship," Journal of Farm Economics XXXV November, 1953. pp. 519-529.

Ibach, D. B., and Mendum, S. W. "Determining Profitable Use of Fertilizer," U. S. Department of Agriculture, F. M. 105 Washington: U. S. Government Printing Office, 1953.

Knetsch, Jack L. "Methodological Procedures and Applications for Incorporating Economic Considerations into Fertilizer Recommendations," Unpublished Masters Thesis, Department of Agricultural Economics, Michigan State University, 1956.

Knetsch, Jack L., Robertson Lynn. S., and Sundquist, W. B. "Economic Considerations In Soil Fertility Research, Michigan Agricultural Experiment Station Quarterly Bulletin, August, 1956. pp. 10-16.

Michigan Department of Agriculture. Michigan Agricultural Statistics, July, 1956.

Redman, John C. and Allen, Stephen Q. "Some Interrelationships of Economic and Agronomic Concepts," Journal of Farm Economics, Vol. XXXVI, August, 1954. pp. 453-465.

Robertson, L. S., Sundquist, W. B. and Shepherd, L. N. "A Progress Report of the Studies on the Economics of Fertilizer Use on Beans and Potatoes," Mimeographed Report of the Michigan Agricultural Experiment Station, March, 1957.

U. S. Department of Agriculture. Agricultural Statistics, 1955, Washington: U. S. Government Printing Office, 1955.

Wooten, H. H. and Anderson, J. R. "Agricultural Land Resources in The United States with Special Reference to Present and Potential Cropland and Pasture," Agricultural Information Bulletin 140, Washington: U. S. Department of Agriculture, June, 1955.

Yeh, H. H. "Estimating Input-Output Relationships for Wheat in Michigan Using Sampling Data, 1952-54," Unpublished Masters Thesis, Department of Agricultural Economics, Michigan State University, 1955.