ANALYTICAL AND GRAPHICAL ENGINEERING ECONOMIC ANALYSIS AS APPLIED TO THE COMPRESSION REFRIGERATION SYSTEM AND ALLIED COMPONENTS

Thesis for the Degree of Ph. D.

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George E. Sutton

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THESIS

This is to certify that the

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Donald J. Renwick
Major professor

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by

George E. Sutton

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Analytical and Graphical Engineering
Economics as applied to the Compression
Refrigeration System and Allied Components

By George E. Sutton

ABSTRACT

The methods currently available to engineers for selection of mechanical equipment such as air ducts, refrigeration piping, etc. are limited, generally, to published tables of vague origin.

A number of equations are developed relating size to operating and owning costs. These are minimized to produce the optimum size for minimum annual cost.

A systematic approach to graphical solution of equations is developed in order to generalize the solutions at cost equations.

Minimizing of equations in one, two, and three variables is discussed.

Applications of these methods are made to the following: Insulation; Condenser Water Rate; Water piping; Discharge, Suction and Liquid Lines; Air Ducts; and Tubular Heat Exchangers.

The results indicate that the use of generalized tables for pipe sizing, etc. should be discouraged, and that graphical solutions should be used whenever possible in order to produce the most economic selection.

The study of tubular heat exchangers indicates a trend toward a large number of very small tubes, with Reynolds' Numbers in the transition range between laminar and turbulent flow. As a consequence, more study is needed of the character of flow and heat transfer in such small tubes.

Donald J. Rennick
Major Professor

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Analytical and

Graphical Engineering Economic Analysis

as Applied to the Compression Refrigeration System

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Introduction

The sizing and selection of mechanical equipment has long been based upon experience and judgement, neither of which is an easily obtainable commodity. Practice has shown the minimum size consistent with effective operation. Admittedly, this practice, based upon experience, yields the lowest first cost consistent with operable equipment. It can be shown, however, that, in many cases, the operating cost is so high that the total owning and operating cost of the equipment over a period of years is far greater than if the economic size were chosen by analytical means.

There is another strong reason for examining critically present methods of selection. Many references show tabulated sizes, particularly in the case of pipe, which will yield, according to previous practice, the economic system. These data are often based upon prices of some years past.

The cost of electric power has remained essentially constant over many years, while material and installation costs have practically tripled since 1926. These facts would tend to indicate that, in many cases, smaller sizes than previously selected are now justified economically.

The technique of minimizing cost by differentiation of a total cost equation is beset by two disadvantages. First, the equation representing total cost must be derived. Such an equation may, however, be derived approximately, if not exactly, for most applications by use of some ingenuity and reasonable approximations.

The second disadvantage lies in the usual apprehension of engineers toward higher mathematics. This may be due, largely, to the lack of stress placed upon practical aspects of mathematics in scholastic courses and in the practice of engineering.

It is the aim of this treatise to provide a guide to methods of setting up the equations of total cost, and then show how to solve for the economic sizes or quantities. Although the title indicates, correctly, that the apparatus treated relate to the compression refrigeration system, the methods developed may be applied with equal ease to many other engineering systems.

In many cases, assumptions are made necessary. These will be clearly indicated as such. The primary value of the literature search in connection with this work has been in providing bases for these necessary assumptions.

It is believed that graphical solutions, because of their simplicity and generality, offer a valuable contribution. Graphical techniques will be discussed in detail prior to discussion of specific equations.

Chapter I Graphical Techniques

A graph, as used in this work, is defined as a plot of the behavior of some function with variation of a primary variable. The most common graphs encountered in engineering work are linear scale, semi-logarithmic scale, and log-log scale. Nomographs and alignment charts are not graphs as herein defined, but, more properly, form another class of charts.

Linear scales and semi-logarithmic scales have the primary disadvantage that few functions plot as straight lines on them. Log-log scale graphs are advantageous in that any single-termed function, regardless of the number of multiplying and dividing parameters contained and the powers to which the parameters or variables are raised, will plot as a straight line. In addition, multiplication and division may be readily performed. Addition and subtraction of functions may also be performed, but less readily.

Log-log graphs are simply plots on special paper, having a logarithmic scale on both abscissa and ordinate. The simplicity encountered with base ten logarithms will be utilized here throughout.

A change of one cycle upward or to the right represents a multiplication by ten, while a change of one cycle downward or to the left produces division by ten. This decimal characteristic allows much freedom in using such graphs for calculation.

To illustrate the technique of solving an equation of two single-termed functions, consider the following example:

$$\frac{ab}{c} = d^2$$

where a, b, and c are parameters. That is, they may vary with other conditions, but are not functions of the primary variable, d. The equation may be rewritten:

$$\frac{ab}{cd} = d$$

which is of the form:

$$f_1(d) = f_2(d)$$

Thus, for given values of a, b, and c, the solution occurs when $f_1 = f_2$. Figure 1-1 shows a plot of $f_1(d)$ and $f_2(d)$ against values of d. Since f_1 is a function of d raised to the -1 power, the slope is -1 and, since f_2 is a function of d to the +1 power, the slope is +1. The intersection of the lines representing the two functions will occur at the value of d which produces equality of the two functions, which is the solution of the equation. In this case, the values chosen were: a, 10; b, 10; and c, 1; which produces a solution d equal to 10.

In order to generalize, or to provide for other values of a, b and c, the graph may be extended to facilitate calculation of the term $\frac{ab}{c}$, for all values of a, b, and c encountered in the particular problem. Let the ranges of the parameters be chosen as follows:

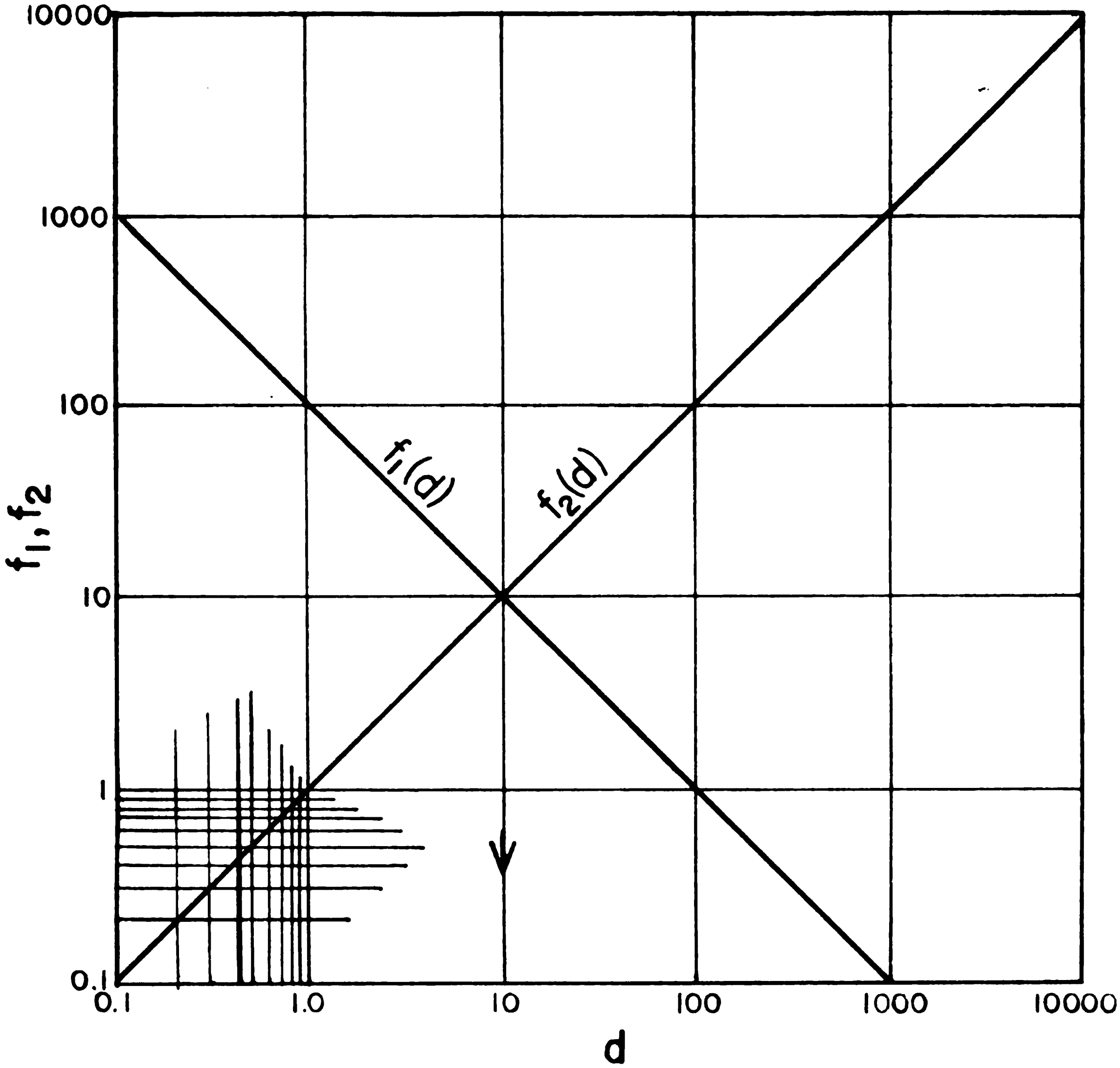

FIGURE 1-1

Solution of the equation: $f_1(d) = f_2(d)$

$$0.1 \leq a \leq 10$$

$$10 \leq b \leq 100$$

$$1 \leq c \leq 100$$

For these ranges of the parameters, the minimum solution for d is 0.1 and the maximum is 31.5. This represents $2\frac{1}{2}$ cycles range for d. Thus a solution area required is then $2\frac{1}{2}$ by $2\frac{1}{2}$ cycles. Prior to demonstration of the method of computing the operating area, that is, the area required to compute the value of $\frac{ab}{c}$, it will be necessary to examine the operations of multiplication and division in more detail.

As previously mentioned, multiplication and division may be performed by vertical movement, which results in addition or subtraction of logarithms. Figure 1-2 demonstrates the operations involved in computing $\frac{ab}{c}$.

Starting with log a, progressing downward, each cycle progressed represents division by 10. Thus division by c may be achieved by progressing downward to log c and then vertically to any chosen unity. The procedure is simplified if all multiplicands are arranged above a certain unity, and all divisors arranged below the same unity as shown in Figure 2. Further multiplication by b is achieved by progressing upward vertically to log b. The equation representing these operations is:

$$\log a - \log c + \log b = \log \frac{ab}{c}$$

Further movement along the lines of slope -1 produces

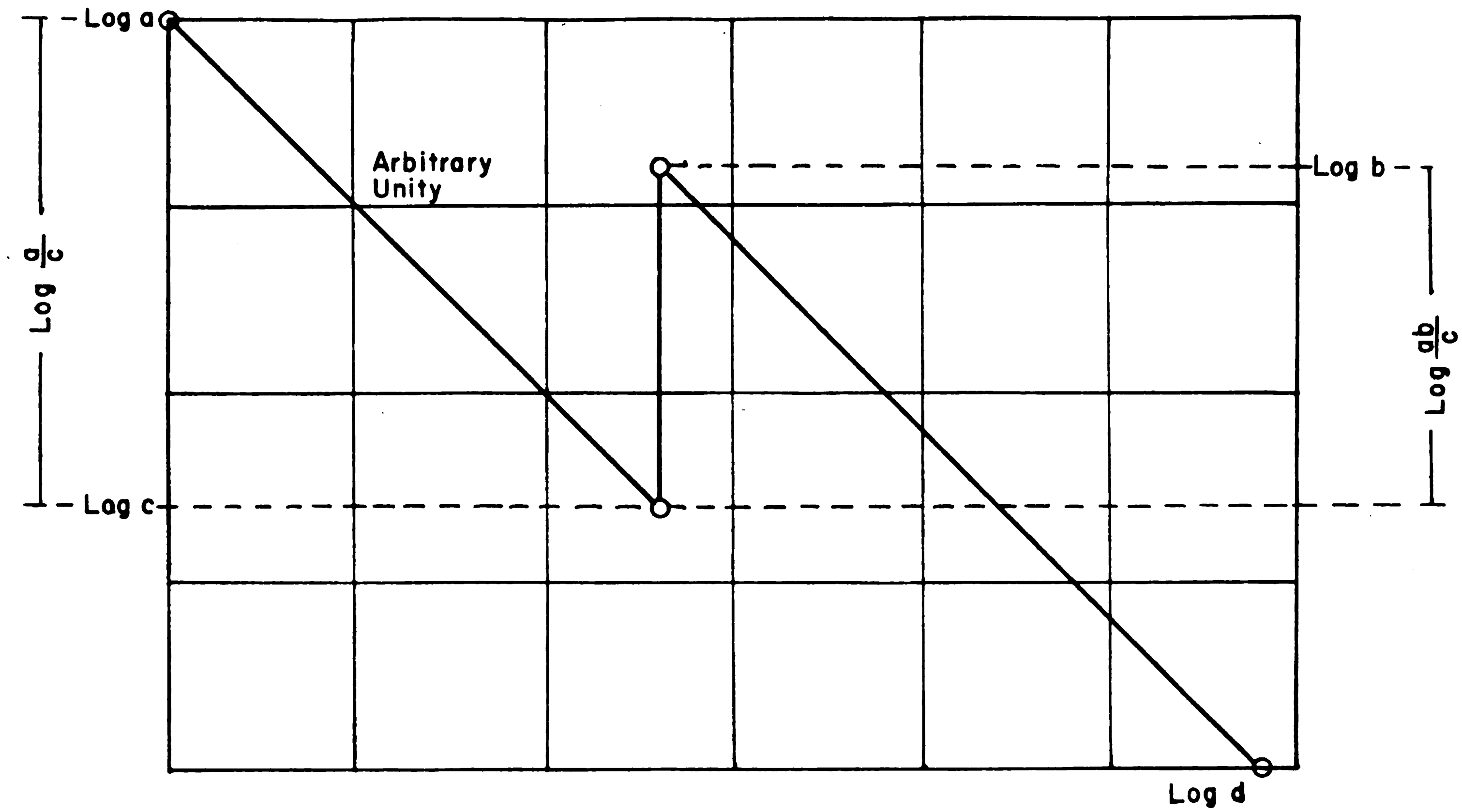

FIGURE 1-2
Solution of the equation $d = \frac{ab}{c}$

division by 10 for each cycle. However, since the decimal is arbitrary, due to the uniformity of the cycles, the value at the right ordinate of Figure 2 may be chosen as $\log \frac{ab}{10{,}000c}$ or $\log \frac{ab}{c}$, or any other decimal value. The only requirement is that the other functions in the equation agree in decimals with the first.

From observation of the operations, an arbitrary rule may be derived for determining the number of cycles, horizontal and vertical, required for a given operation. Assigning the unity scale as shown, multiplication of "a" by 1 requires n_a cycles, vertical and horizontal, where n_a is the range of a in cycles. The division by c requires n_c cycles, horizontal and vertical. Multiplication by b requires no additional cycles vertically, but requires n_b cycles horizontally. It is obvious, then, that the space required will be equal to the sum of the n cycles horizontally, but only the sum of the n's for the maximum range multiplier and maximum range divisor will be required vertically.

For the example chosen, the requirements would be

$$N_a + N_b + N_c = 2 + 1 + 2 = 5 \quad \text{Horizontal}$$

$$N_a + N_c = 2 + 2 = 4 \quad \text{Vertical}$$

The total for the operation and solution becomes $7\frac{1}{2}$ by $6\frac{1}{2}$ cycles, provided the orientation is as shown. That is, if the solution area is not allowed to overlap the operation area. Experience will show that some overlapping may be tolerated, thus reducing the total space requirements. Figure 1-3 shows the total operation and solution.

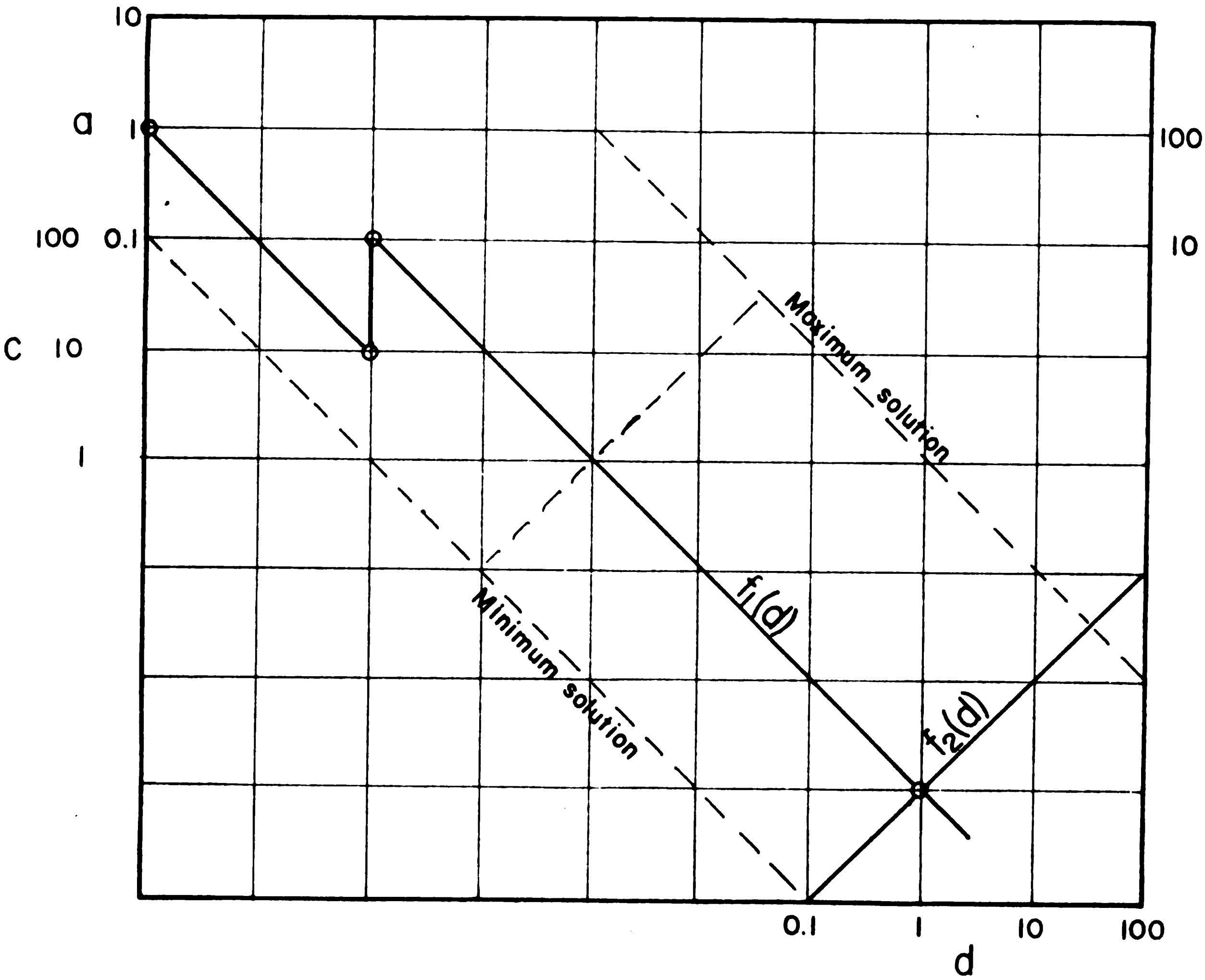

FIGURE 1-3

Solution of the equation: $d = \frac{ab}{cd}$

The example shown is for $a = 1$, $b = 10$, $c = 10$. This illustrates, perhaps, the simplist method of locating the scale in the solution area. These values were chosen such that the solution for d would be 1. Thus the two lines representing the functions must intersect at the ordinate representing 1 as the value of d. The minimum area would be utilized if the function equal to d were moved to the heavy dashed line shown, which represents an overlapping of two cycles. The total area could thus be reduced to $5\frac{1}{2}$ by 5 cycles. This reduction is difficult to predict in advance of actually laying out the graph, however.

A simplification may be made in scale location which will reduce crowding of scales. Since horizontal movement has the same effect as vertical movement, one scale may be placed along the upper abscissa. If the parameter chosen is the primary one, such as flow rate, tonnage, etc., the effect of this parameter upon the solution becomes readily apparent. Figure 1-4 shows the rearrangement of Figure 3 into this pattern.

Such methods result in a graph which may be used to solve the equation for all values of the parameters within the chosen ranges. In the simplicity of such solutions lies the major value of the system.

Solutions which require the use of more complex parameters may be implemented in equally simple ways. The most generally encountered type involves parameters and variables raised to powers other than unity.

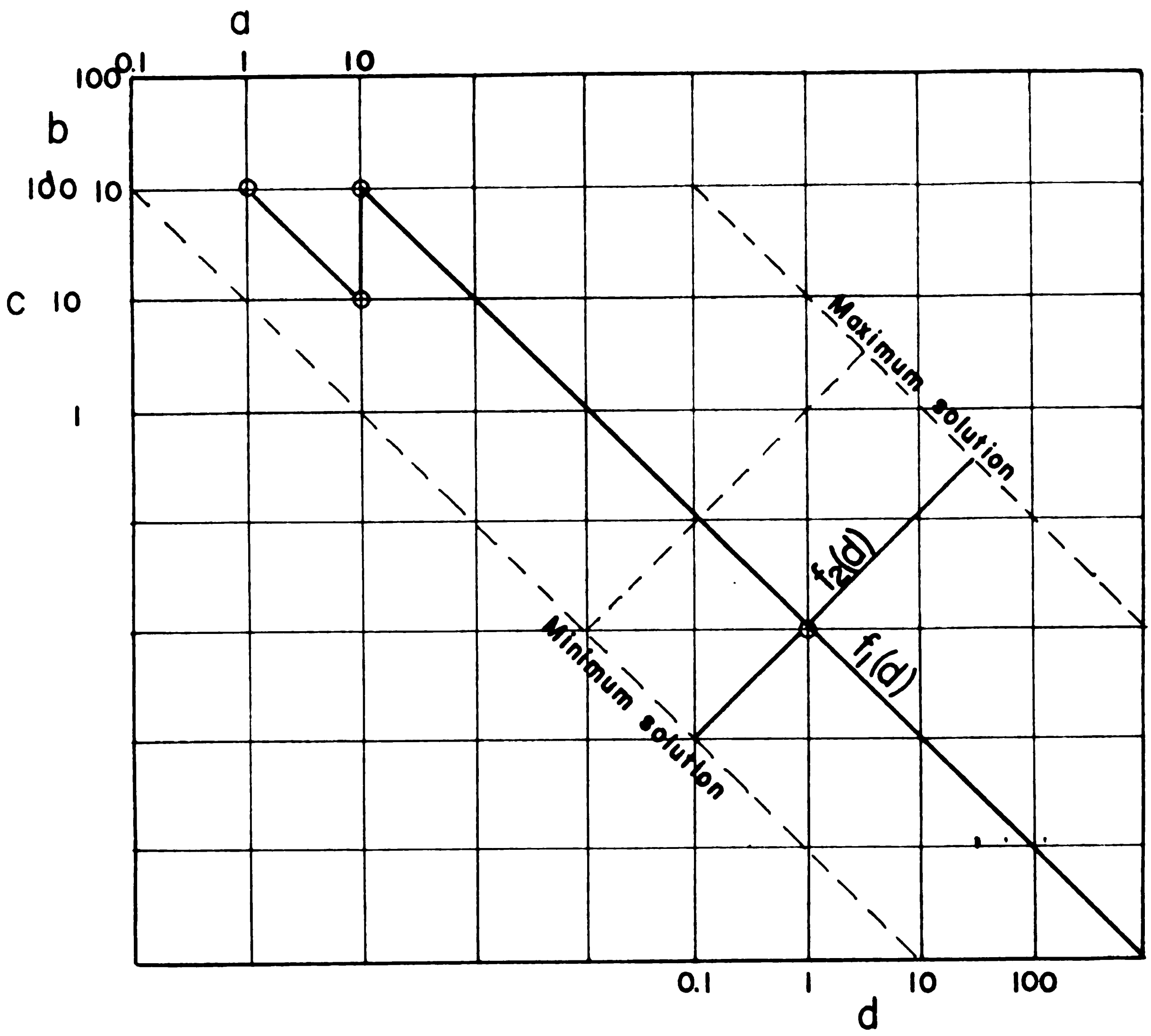

FIGURE 1-4

Solution of the equation: $d = \frac{ab}{cd}$

With such exponential parameters, it is deemed simplest to treat the parameter with its exponent as a new variable raised to unity power. Thus if a^2 were to be a part of the function, a^2 would be plotted on the scale, with values of "a" inscribed so that the location of values of "a" would automatically produce a^2. Since log a^2 is equal to 2 log a, the scale length would be twice the range of "a".

If the primary variable appears at any other power than unity, the solution may be handled in two ways. First, the operation may take place at a slope equal to the power. Second, the variable and its exponent may be treated in the same way as the parameters discussed in the previous paragraph, whereupon the new variable is the variable raised to the appropriate power. The latter system will be used in this treatise.

As an example of this technique, consider the equation:

$$\frac{a^2b}{c} = d^3$$

which may be rewritten as:

$$\frac{a^2b}{cd^{1.5}} = d^{1.5}$$

Let the parameters have the same ranges as previously. The required operations area will now be, using the upper scale for "a":

Horizontal: $(2)+(2)+1+2=7$

Vertical: $2+1=3$

The range of d^3 is from $\frac{(0.1)^2(10)}{100} = 0.001$ to $\frac{(10)^2(100)}{1} = 10{,}000$ which represents values of $d^{1.5}$ of

0.0315 to 100. Thus the solution will require $3\frac{1}{2}$ cycles. The total area will be a maximum of $10\frac{1}{2}$ by $6\frac{1}{2}$ cycles. Figure 1-5 shows the graph for solution of this equation.

It has been shown that any equation consisting of two single-termed functions may be readily handled by graphical means. For three or more terms, the equations are somewhat more difficult to solve. Consider the equation:

$$\frac{ab}{d} + e = cd$$

The solution requires that the sum of two functions of d and a parameter "e" be equal to another function of d. Computation of the two functions is readily carried out by previously demonstrated methods. Determining the sum of function of d and the parameter e, which produces another function of d, is not so readily performed on log-log graphs. Note that the sum is a two-termed function, and does not plot as a straight line.

In general, the equation to be solved is:

$$f_1(d) + f_2(d) = f_3(d)$$

$$\text{or} = \log(f_1 + f_2) = \log f_3$$

but: $$\log(f_1 + f_2) = \log f_1(1 + \frac{f_2}{f_1})$$

$$= \log f_1 + \log(1 + \frac{f_2}{f_1})$$

This operation is easily carried out with a pair of dividers. Figure 1-6 shows the method. The dividers are set between f_2 and f_1, which measures the log $\frac{f_2}{f_1}$. If they are reset with the lower leg on any unity, they may be extended by one unit, which gives $\log(1 + \frac{f_2}{f_1})$. Resetting the lower

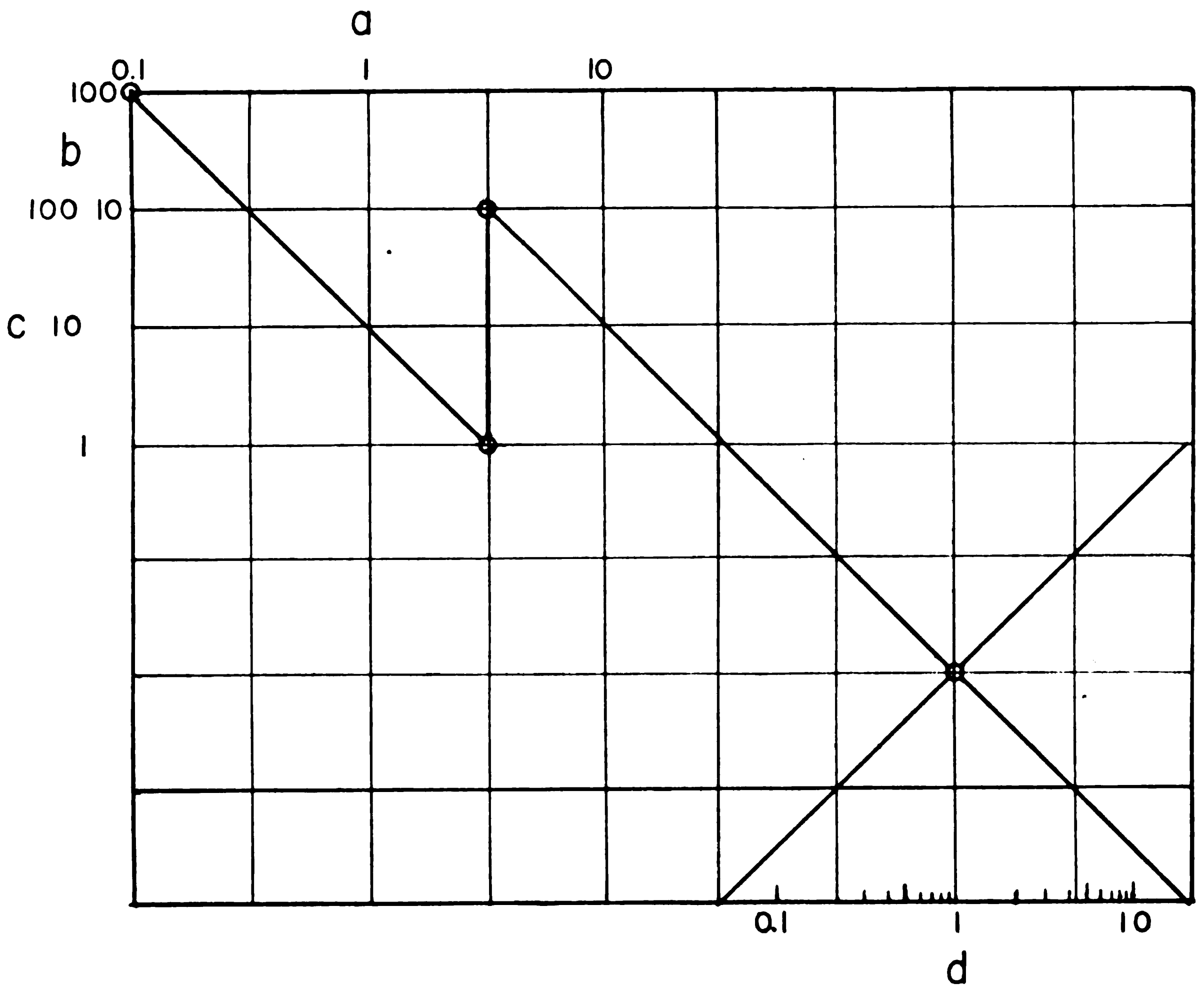

FIGURE 1-5

Solution of the equation:

$$\frac{a^2 b}{c d^{1.5}} = d^{1.5}$$

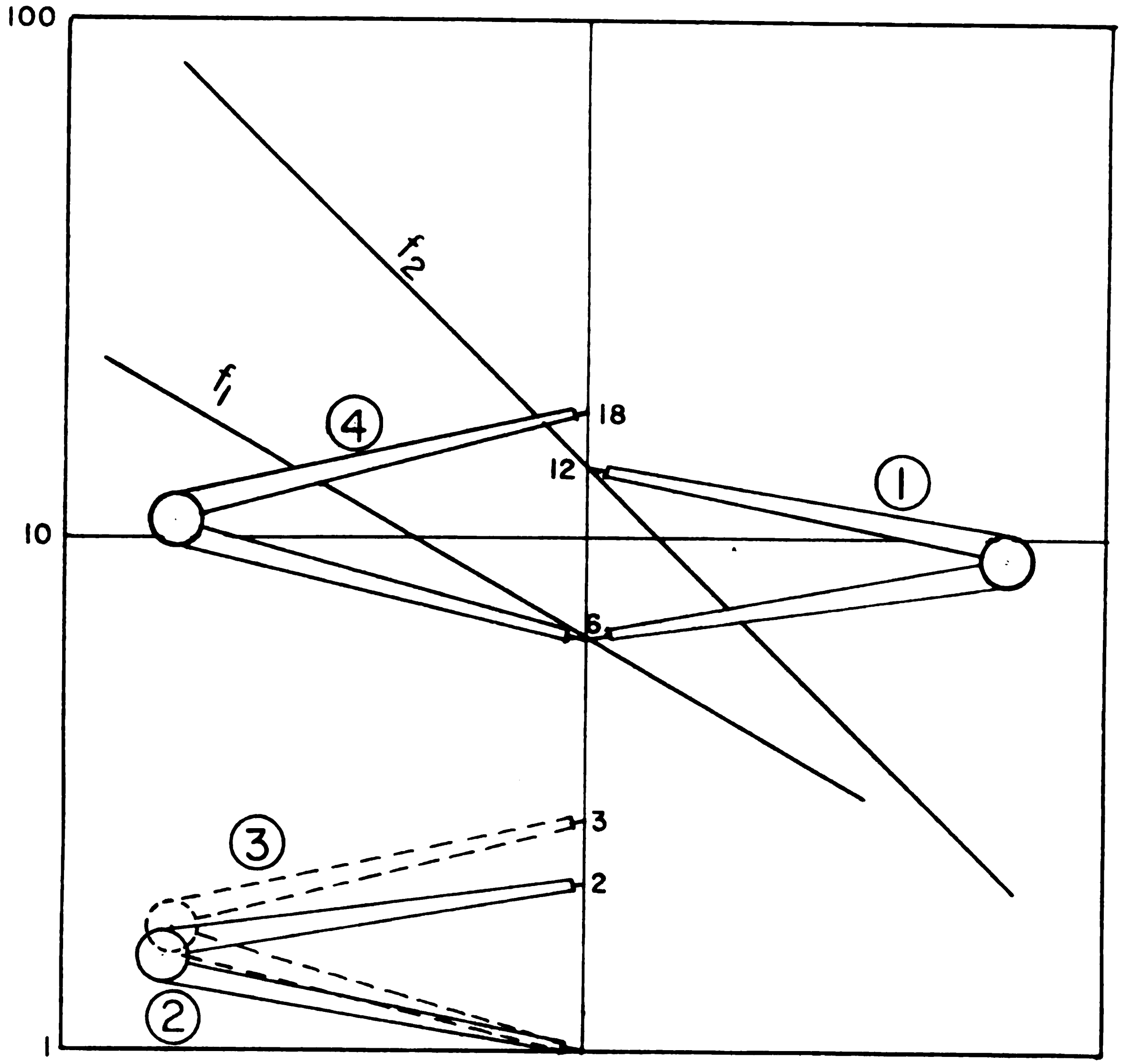

FIGURE 1-6
Addition of functions

Step 1: Set dividers between f_1 and f_2
2: Set lower leg on unity
3: Open upper leg by one unit
4: Reset lower leg on smaller function
Upper leg indicates sum

leg on f_1, the upper point will rest on the point at which the ordinate is $\log (f_1 + f_2)$, as shown in the derivation.

As a numerical example, assume $f_2 = 12$ and $f_1 = 6$. Setting the dividers between them, and then placing the leg on any unity, the span will show an upper reading of 2, which indicates the span to be log 2. Extending the upper leg to 3 will cause the span to be log 3. Adding this to 6 will give log (6)(3) or log 18. Note that the sum of 12 and 6 is 18, so the solution is correct.

For subtraction of two functions, the method is essentially the same, since:

$$\log (f_2 - f_1) = \log f_1 \left[\frac{f_2}{f_1} - 1 \right]$$

$$= \log f_1 + \log \left(\frac{f_2}{f_1} - 1 \right)$$

The calculation is made as before, except that the divider span is reduced by one unit, rather than expanded as before.

The example chosen for graphical solution,

$$\frac{ab}{d} + e = cd$$

is shown on Figure 1-7. The ranges of the parameters are:

$$\begin{aligned} 10 &\le a \le 100 \\ 0.1 &\le b \le 10 \\ 1 &\le e \le 100 \\ 10 &\le c \le 100 \end{aligned}$$

In this case the additive term is a constant for a given solution. Let the values of the parameters be:

$$a = 10, \quad b = 1, \quad c = 10, \quad e = 100$$

for simplicity. The solution for these values will be slightly greater than ten, since: $\frac{(10)(1)}{10+} + 10 \simeq 10^{+}$

In order to insure correct results, the scales of

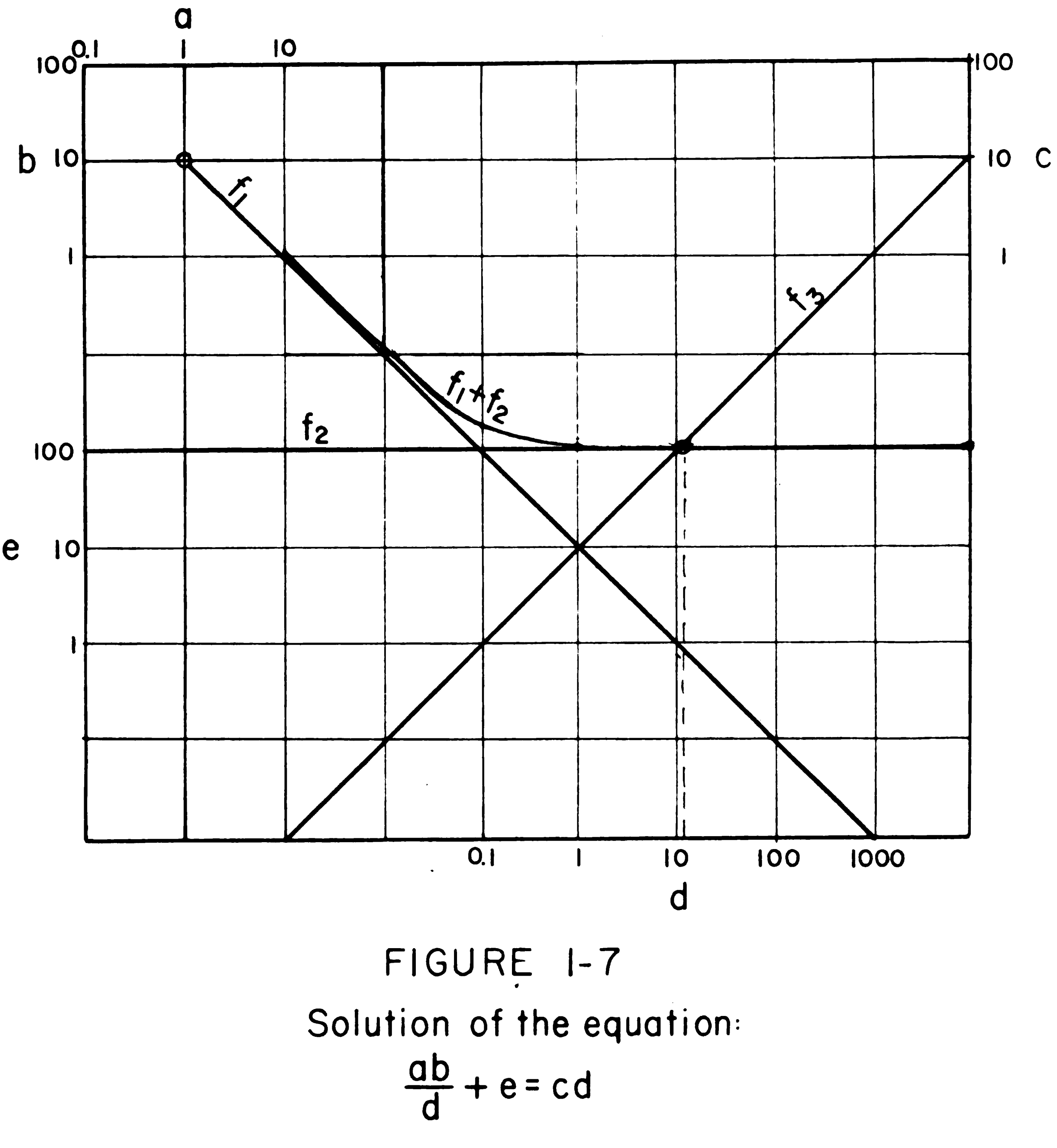

FIGURE 1-7

Solution of the equation:

$$\frac{ab}{d} + e = cd$$

f_1, f_2, and f_3 must be known. These may be arbitrarily chosen, but must be identical. If they are arbitrarily chosen, the location of the scale for the primary variable, d, may be found by taking the case where $e = 0$, whence:

$$\frac{10}{d} = 10d; \quad d = 1$$

This automatically locates the scale for c.

The previous discussion provides ample techniques for the solution of practically any equation of four or less terms. Since no equations have thus far been encountered in this work which necessitated use of more than three terms, no further development is deemed necessary.

Chapter II Minimizing of Equations

In general, differentiation of a function with respect to a variable, and setting this derivative equal to zero will give the value of the variable which will produce either a maximum or a minimum of the function. Equations for total cost of a mechanical component may usually be written as the sum of two functions of the primary variable, one being the annual owning cost and the other being the annual operating cost. These are usually a direct function and a reciprocal function, both of which are smooth continuous functions. The maximums will occur in such cases at infinite and zero values of the primary variable, and there will be only one minimum.

As a simple example of such an equation, consider the following:

$$C_t = Ad + \frac{B}{d}$$

It is desired to compute the value of d which will produce a minimum total cost. Differentiating, and setting the derivative equal to zero:

$$\frac{dC_t}{dd} = A - \frac{B}{d^2} = 0$$

This is the equation which must be solved for the value of d which will produce a minimum cost. The solution is as follows:

$$d = \sqrt{\frac{B}{A}}$$

In the case of two independent variables, the partial derivatives of the function with respect to each of the two variables may be set to zero, and the equations produced thus solved simultaneously to yield the solution for the values of the variables to produce either a maximum or a minimum. The same reasoning as applied to a single variable indicates that the solution will yield a minimum.

As an example, consider an equation:

$$C_t = AL^3d - B(Ld + L^3)$$

The solution is as follows:

$$\left(\frac{\partial C_t}{\partial L}\right)_d = 3AL^2d - B(d + 3L^2) = 0$$

$$\left(\frac{\partial C_t}{\partial d}\right)_L = AL^3 - BL = 0$$

$$AL^3 = BL$$

$$\underline{\underline{L = \sqrt{\frac{B}{A}}}}$$

and: $$3A\left(\sqrt{\frac{B}{A}}\right)^2 = B\left(d + 3\frac{B}{A}\right)$$

$$3\ Bd = Bd + 3\frac{B^2}{A}$$

$$2\ Bd = 3\ \frac{B^2}{A}, \quad \underline{\underline{d = \frac{3}{2}\ \frac{B}{A}}}$$

If the two variables are not independent, the previous method will generally not lead to a solution. In this case the total differential of the function must be set to zero and the differential equation solved. The total differential of a function may be written as follows:

$$dC_t = \left(\frac{\partial C_t}{\partial L}\right)_d dL + \left(\frac{\partial C_t}{\partial d}\right)_L\ dd$$

Consider the general equation:

$$f(L,d) = f_1\ (L,d) + f_2\ (L,d)$$

Differentiating, and setting the total differential equal to zero:

$$df = \left[\left(\frac{\partial f_1}{\partial L}\right)_d + \left(\frac{\partial f_2}{\partial L}\right)_d\right] dL + \left[\left(\frac{\partial f_1}{\partial d}\right)_L + \left(\frac{\partial f_2}{\partial d}\right)_L\right] dd = 0$$

This may be written, symbolically:

$$M\,dL + N\,dd$$

The test for exactness of a differential equation is:

$$\frac{\partial M}{\partial d} = \frac{\partial N}{\partial L}$$

Applying this test:

$$\left[\frac{\partial^2 f_1}{\partial L \partial d} + \frac{\partial^2 f_2}{\partial L \partial d}\right] = \left[\frac{\partial^2 f_1}{\partial d \partial L} + \frac{\partial^2 f_2}{\partial d \partial L}\right]$$

In practically all the cases encountered in Mathematics the order of differentiation has no effect, so that, generally the equality will exist in the previous expression.

The method of solution for an exact differential equation is as follows:*

$$F(L,d) = \int^L M dL + \int^d \left[N - \frac{\partial}{\partial d}\int^L M dL\right] dd = \text{Const.}$$

Applying this to the general equation:

$$F(L,d) = \int^L \left[\frac{\partial f_1}{\partial L} + \frac{\partial f_2}{\partial L}\right] dL + \int^d \left\{\left[\frac{\partial f_1}{\partial d} + \frac{\partial f_2}{\partial d}\right] - \frac{\partial}{\partial d}\int^L \left[\frac{\partial f_1}{\partial L} + \frac{\partial f_2}{\partial L}\right] dL\right\} dd$$

$$= \text{Const.}$$

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* Kells, L. M., Elementary Differential Equations, McGraw-Hill, New York, 1947, pp 44-46.

$$F(d,L) = f_1 + f_2 + \int^d \left\{ \left[\frac{\partial f_1}{\partial d} + \frac{\partial f_2}{\partial d} \right] - \frac{\partial}{\partial d} \left[f_1 + f_2 \right] \right\} dd$$

$$= f_1 + f_2 + \int^d \left[\frac{\partial f_1}{\partial d} + \frac{\partial f_2}{\partial d} - \frac{\partial f_1}{\partial d} - \frac{\partial f_2}{\partial d} \right] dd$$

$$= f_1 + f_2 = \text{Constant}$$

This has the same form as the original equation with the function set equal to some undetermined constant. With no additional conditions, the value of the constant cannot be determined by **elegant** methods, but may be determined by successive approximations if necessary.

If any constraint on the system is known, the method of Lagrange* offers a solution. For two or more variables this method is applicable.

As in the previous case, assume that the function C_t (d,L) is given, and that the constraint is:

$$G(d,L) = \text{constant}$$

Lagrange's method consists of minimizing the combined function:

$$F = C_t(d,L) + \gamma \; G(d,L)$$

where γ is an arbitrary constant, called Lagrange's multiplier.

Differentiation of the function yields two equations, with the constraint furnishing the additional equation necessary to complete the system of simultaneous equations. The system is:

$$\left(\frac{F}{d}\right)_L = 0$$

$$\left(\frac{F}{L}\right)_d = 0$$

$$G(d,L) = \text{Const.}$$

* Advanced Calculus, A.E. Taylor, Ginn and Co., Boston, 1955, pp 198-201

Solution of this system may yield multiple results, which may be maxima, minima, or neither. Classification may be achieved by the following:*

Let:

$$A = \frac{\partial^2 C_t(a,b)}{\partial d^2}, \quad B = \frac{\partial^2 C_t(a,b)}{\partial d \partial L}, \quad C = \frac{\partial^2 C_t(a,b)}{\partial L^2}, \quad D = B^2 - AC$$

where (a,b) are the values of (d,L) found as a solution of the system.

If: $D < 0$ and $A > 0$, C_t is a maximum

$D < 0$ and $A > 0$, C_t is a minimum

$D > 0$ neither a maximum or minimum

$D = 0$, no conclusion may be drawn.

For the case of three variables, consider the case:

$C = C(N,L,d)$

with the constraint:

$G(N,L,d) =$ constant

Let one solution of the system of equations be (a,b,c). The determinant:

$$\begin{vmatrix} \frac{\partial^2 C(a,b,c)}{\partial N^2} - \lambda & \frac{\partial^2 C(a,b,c)}{\partial N \partial L} & \frac{\partial^2 C(a,b,c)}{\partial N \partial d} \\ \frac{\partial^2 C(a,b,c)}{\partial L \partial N} & \frac{\partial^2 C(a,b,c)}{\partial L^2} - \lambda & \frac{\partial^2 C(a,b,c)}{\partial L \partial d} \\ \frac{\partial^2 C(a,b,c)}{\partial d \partial N} & \frac{\partial^2 C(a,b,c)}{\partial d \partial L} & \frac{\partial^2 C(a,b,c)}{\partial d^2} - \lambda \end{vmatrix} = 0$$

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* Advanced Calculus, A. E. Taylor, Ginn and Co., Boston, 1955, pp 232-5

will yield three roots for λ. If all roots are positive, (a,b,c) is a minimum; if all are negative, a maximum. If the signs of the roots are mixed, neither a maximum or minimum exists at (a,b,c).

Similar extensions may be made to a larger number of variables, but, since these seldom appear in engineering economics, they will not be discussed here.

Chapter III Equations not suitable for Graphical Solutions

There arise several situations which produce equations which involve a number of terms, but a relatively few variables, or parameters. Often, these lend themselves better to numerical solution than to graphical, particularly if the solution is to be carried out by successive approximations. An important example of this class of situations is the field of economic insulation thickness.

(a) Insulation of Walls:

The cost of owning and "operating" an ordinary wall may be interpreted as the sum of the cost of the wall and the cost of heating and/or refrigeration. In the case of a wall of known construction, the economics of addition of insulation may be treated mathematically.

The cost per year of the insulation may be determined by dividing the installation and maintenance cost by the number of years representing the expected life of the installation. Let "A" represent the cost per year per board foot of insulation. Let "U" represent the overall coefficient of heat transfer of the wall without insulation; B_h, the cost of heat per 1000 BTU and B_c, the cost of refrigeration per 1000 BTU. Assuming that the addition of insulation will not alter appreciably the heat transfer characteristics of the other materials, the heat transfer per square foot may be written:

$$q = \frac{\Delta t}{\left[\frac{1}{U} + \frac{x}{12K}\right]}$$

where: x = thickness of insulation, inches.

k = conductivity of insulation BTU/hr-sq ft-^{o}F/ft.

Δt = temperature difference between inside air and outside air, ^{o}F

The cost of cooling may be written:

$$C_c = \frac{B_c\ F_c(8760)\,\Delta t_c}{1000\left[\frac{1}{U} + \frac{x}{12k}\right]} \qquad ¢\text{/year-sq ft.}$$

where F_c is the fraction of time that cooling is required, on an annual basis, 8760 being the hours per year.

For heating, the "degree-day" offers a convenient means of computing heating requirements. The number of degree-days is defined as the product of (days/yr.)(65^{o}F - $t_{avg.}$), the 65^{o} reference being chosen as the average temperature at which no heating will be required. Thus the expression:

$$8760\ F_h\,\Delta t_h \approx 24D$$

where D = degree-days. Many publications list representative figures for various localities.*

The heating cost may be expressed as:

$$C_h = \frac{24\ B_h\ D}{1000\left[\frac{1}{U} + \frac{x}{12k}\right]}$$

The total cost per year for insulating one square foot may be written:

$$C_t = Ax + \frac{8760\ B_c\ F_c\ \Delta t_c + 24\ B_h\ D}{1000\left[\frac{1}{U} + \frac{x}{12k}\right]}$$

Note that A is not constant for all values of x.

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* ASHAE Guide, American Society of Heating and Air Conditioning Engineers, Inc., New York, 1956, pp. 451-4.

Differentiating with respect to x, and setting to zero:

$$\frac{dC}{dx} = A - \frac{\left[8760\ B_cF_c\,\Delta t_c + 24\ B_hD\right.}{12{,}000k\left[\frac{1}{U} + \frac{x}{12k}\right]^2}$$

$$\frac{1}{U} + \frac{x}{12k} = \sqrt{\frac{8760\ B_cF_c\,\Delta t_c + 24B_hD}{12000\ Ak}}$$

$$x = 12k\sqrt{\frac{8760\ B_cF_c\,\Delta t_c + 24\ B_hD}{12{,}000\ Ak}} - \frac{12k}{U}$$

From the standpoint of graphical solution, this equation is rather complex. It is not unusually difficult to solve by ordinary algebraic means, however.

It is interesting to note that values of "U" for many composite walls are tabulated in various publications. The 1956 <u>ASHAE Guide</u> pp 190-200 lists many such values for walls, roofs, etc.

As an example of the use of the derived equation, consider an 8 inch concrete block wall, with ½ inch plaster, furred, with metal lath. Table 8, page 191 of the 1956 <u>ASHAE Guide</u> gives $U = 0.34$ BTU/hr-ft^2-$^\circ$F. for this wall. Page 452 of the same reference indicates that $D = 6982$ for Lansing, Michigan. Page 280 also indicates that the design air temperature for this locality is 89°F. Using a design interior temperature of 78°, $\Delta t_c = 11^\circ$F. Cooling costs will be of the order of 1/3¢ per 1000 BTU, and heating costs of the order of 1/4¢ per 1000 BTU. Using glass wool as the insulating material, for which the k value, according to the above reference, is 0.27 BTU/hr-sq ft-$^\circ$F/in. = 12k. The cost of such insulation is approximately 1¢ per year per board foot. Estimating the use factor for air conditioning, F_c, to be 0.2, the solution for the economic thickness of insulation is:

$$x = 0.27\sqrt{\frac{(8760)(1/3)(0.2)(11) + (24)(\frac{1}{4})(6992)}{(12000)(1)(0.0225)}} - \frac{0.27}{0.34}$$

$$= 0.27\sqrt{\frac{6425 + 41,892}{270}} - \frac{0.27}{0.34}$$

$$= 3.615 - 0.795$$

$$= 2.82 \text{ inches}$$

Thus, it is obvious that a structure which is to be "year-round" air conditioned, should be insulated with two to three inches of insulation. Note that a basic structure of higher insulating value, or lower U value, will require less insulation, as shown by the subtracting factor, 12k/U.

If the structure is not to be cooled, this factor may be eliminated from the equation. This would also decrease the economic thickness of insulation.

Low temperature storage units require much more insulation, primarily due to the higher temperature difference, higher use factor, and low refrigeration efficiency. Consider the case of a room, contained within a space at an average temperature of 75°F., which is to be maintained at -10°F. Assuming the structural components to be of negligible thermal resistance; that the use factor is 0.7; and that the refrigeration cost is 2/3¢ per 1000 BTU, the economic thickness will be, for the same insulation used previously:

$$x = 0.27\sqrt{\frac{(8760)(2/3)(0.7)(85)}{(12,000)(1)(0.0225)}}$$

$$= 8.50 \text{ inches}$$

(b) Pipe Insulation.

Pipe insulation is usually carried out by one of two means: First, pre-cast insulation, available in incremental thicknesses; and second, by paste insulation which may be applied in the desired thickness, and allowed to harden in place. The equation for cost may be simplified if the cost may be expressed in terms of the volume of the insulation, say in cents per year per cubic foot. It may be further simplified if the thermal resistance of the pipe or duct is negligible, and only the effect of film resistances and of the insulation need be considered. In most cases, the thermal resistance of the interior film is also negligible with respect to that of the insulation and the exterior film.

Utilizing the above simplifications, the cost of insulation may be expressed as:

$$C_I = \frac{A\ L\ \pi\ (d_o^2 - d_1^2)}{(4)\ 144}$$

where: d_o = the unknown outside diameter of the insulation, inches.

d_1 = the inside diameter of the insulation, inches.

A = cost of insulation, cents per year per cubic foot

L = length of pipe, feet.

The heat transfer may be written, using the above simplifications:

$$q = \frac{\pi L \Delta t_m}{\left[\frac{12}{d_o h_o} + \frac{1}{2k} \ln \frac{d_o}{d_1}\right]}$$

where: t_m = the logarithmic mean temperature difference between the interior fluid and the outside air.

h_o = exterior film conductance, BTU/hr-ft^2-^{o}F.

The cost of heating or cooling may be written as:

$$C_{h,c} \quad \frac{B_{h,c}\, \pi\, L\, \Delta t_m\, (8760)\, F_u}{1000\left[\frac{12}{dh_o} + \frac{1}{2k} \ln \frac{d_o}{h_o}\right]}$$

where: F_u = use factor
and the total cost is:

$$C_t = \frac{A\pi(d_o^2 - d_1^2)\, L}{(4)\, 144} + \frac{8.76\pi B\, F_u\, L\Delta t_m}{\left[\frac{12}{d_o h_o} + \frac{1}{2k} \ln \frac{d_o}{d_1}\right]}$$

Differentiating with respect to d_o, and setting to zero:

$$\frac{dC}{dd_o} = \frac{2\pi A d_o\, L}{(4)(144)} - \frac{8.76\pi B\, F_u\, L\Delta t_m\left[\frac{-12}{d_o^2 h_o} + \frac{d_1}{kd_o}\right]}{\left[\frac{12}{d_o h_o} + \frac{1}{2k} \ln \frac{d_o}{d_1}\right]^2} = 0$$

$$\frac{\left[\frac{12}{d_o h_o} + \frac{1}{2k} \ln \frac{d_c}{d_1}\right]^2}{\left[\frac{d_1}{kd_o} - \frac{12}{d_o^2 h_o}\right]} = \frac{(4)(144)(8.76)\, BF_u\, \Delta tm}{2\, Ad_o}$$

Since the equation contains the logarithm, it can be solved only by some form of successive approximation, and does not lend itself to graphical solution.

Chapter IV: Simple Graphical Solutions

Many cost equations may be written as equations involving one independent variable only, or reduced to such equations by approximations. Such were the equations developed in Chapter III. These may or may not lend themselves to graphical solution. Two such situations, peculiar to refrigeration and air conditioning, are treated in this chapter.

(a) Economic Condenser Water Rate

Temperatures may be measured more easily and less expensively than flow rates. For a given fluid, rate of flow, and rate of heat transfer the temperature rise in the condenser will be constant. Thus, the solution for the economic temperature rise suggests itself.

Assuming a well designed condenser,(see Chap.VI) the only effects produced by varying the condenser water flow rate would be to change the cost of water, and to change the condensing pressure and temperature in the refrigeration cycle. The latter would create a change in compressor work, thus directly affecting the cost of operation of the compressor motor.

Water costs may be simply described as:

$$C_w = \frac{Aw_f}{1000}$$

where:

w_f = rate of flow, lb/hr

A = cost of water, cents/1000 gallons

ρ = density of water, lb/gallon

For any refrigeration cycle, the rate of water flow may be expressed as a function of the operating conditions of the system. The heat dissipated in the condenser, per pound of refrigerant is:

$$H_c = (m/\eta)\ (t_c - t_s) + Q_r$$

where:

m = work per degree temperature difference between condensing temperature and suction temperature, ^oF.

η = overall compressor efficiency for the open system or combined motor and compressor efficiency for a hermetic system.

t_c = condensing temperature, ^oF.

t_s = suction temperature, ^oF.

Q_r = refrigeration effect, the heat absorbed in the evaporator per pound of refrigerant.

The rate of refrigerant flow will be:

$$w_{f_r} = \frac{12000\ T}{Q_r} \quad \text{lb/hr}$$

where T = tons

Since the heat absorbed by water must equal the heat given up by the refrigerant in the condenser; using the average specific heat of water as 1 BTU/lb.-^oF:

$$w_{f_w}\ (t_c - t_w') = w_{f_r}\ H_c = W_{f_w}\ \Delta t_w$$
$$= \frac{12000\ T}{Q_r}\left[\left(\frac{m}{\eta}\right)(t_c - t_s) + Q_r\right]$$

where:

W_{f_w} = weight of flow of water, lb/hr

t_w' = inlet water temperature plus the terminal temperature difference.

Solving for the water flow rate:

$$W_{f_W} = \frac{12000\ T}{Q_r(t_c - t_W')} \left[\left(\frac{m}{\eta}\right)(t_c - t_s) + Q_r\right]$$

Thus, the cost of water is:

$$C_W = \left(\frac{A}{1000\rho}\right) \frac{12000\ T}{Q_r(t_c - t_W')} \left[\left(\frac{m}{\eta}\right)(t_c - t_s) + Q_r\right]$$

The work required is:

$$Wk = W_{f_r}\ m(t_c - t_s)$$

$$= \frac{12000\ T\ m\ (t_c - t_s)}{Q_r}$$

and the cost of work is:

$$C_{Wk} = \frac{B(12000)T\ m(t_c - t_s)}{\eta_o\ (3413)Q_r}$$

where:

B = cost of electricity, cents/KW-hr

3413 = conversion factor, BTU to KW-hr

η_o = overall motor-compressor efficiency

The total operating cost becomes:

$$C_T = \left(\frac{A}{1000\rho}\right) \frac{12000\ T}{Q_r(t_c - t_W')} \left[\left(\frac{m}{\eta}\right)(t_c - t_s) + Q_r\right] + \frac{B(12000T)m(t_c - t_s)}{\eta_o (3413) Q_r}$$

$$= \frac{12000T}{Q_r}\left\{\frac{A}{1000\rho(t_c - t_W')}\left[\left(\frac{m}{\eta}\right)(t_c - t_s) + Q_r\right] + \frac{B\,m(t_c - t_s)}{\eta_o (3413)}\right\}$$

Using methods outlined in Chapter II, the minimum cost condition may be determined as follows:

$$\frac{d\ C_t}{d\ t_c} = \frac{12000\ T}{Q_r}\left\{\frac{A}{1000\rho}\left[\frac{(t_c - t_W')\left(\frac{m}{\eta}\right) - \left[\left(\frac{m}{\eta}\right)(t_c - t_s) + Q_r\right]}{(t_c - t_W')^2}\right.\right.$$

$$\left.\left. + \frac{B\ m}{\eta_o (3413)}\right)\right\} = 0$$

$$\frac{A}{1000\rho}\left\{\frac{\left(\frac{m}{\eta}\right)\left[(t_c - t_W') - (t_c - t_W') - (t_W' - t_s)\right] - Q_r}{(t_c - t_W')^2} + \frac{B\ m}{\eta_o (3413)}\right\} = 0$$

$$\frac{A}{1000\,\rho}\left[\frac{\left(\frac{m}{\eta}\right)(t_w'-t_s) + Q_r}{(\Delta t_w)^2}\right] = \frac{B\,m}{\eta_o\,3413}$$

$$(\Delta t_w)^2 = \frac{A(3413)\,\eta_o}{1000\,B\rho m}\left[\left(\frac{m}{\eta}\right)\quad(t_w'-t_s)+Q_r\right]$$

Using an average density of water as 8.33 lb/gallon, the equation becomes:

$$(\Delta t_w)^2 = \left(\frac{3413}{8330}\right)\quad\frac{A\,\eta_o}{B\,m}\left[\left(\frac{m}{\eta}\right)(t_w'-t_s) + Q_r\right]$$

with Δt_w in ^{o}F.

This equation is completely general, regardless of the refrigerant, for a single stage system. Table 4-1 shows the values of Q_r and m for Dichlorodifluoromethane for various condensing and suction temperatures, assuming 9° superheat at the suction inlet and 9° subcooling at the condenser outlet. Figure 4-1 shows a plot of Q_r with t_c and t_s,

Table 4-1

VARIATION OF REFRIGERATION EFFECT WITH OPERATION CONDITIONS

t_s (°F)	t_c (°F)	W_k (BTU/lb)	m (BTU/lb-°F)	Q_r (BTU/lb)
-40	80	18.47	0.15392	50.59
	90	19.80	.15231	48.15
	100	20.92	.14943	45.76
	110	22.11	.14740	43.31
	120	23.21	.14506	40.82
	130	24.30	.14294	38.31
-20	80	14.61	0.14610	52.96
	90	15.83	.14391	50.52
	100	17.03	.14192	48.13
	110	18.16	.13969	45.66
	120	19.24	.13743	43.19
	130	20.32	.13547	40.68
0	80	11.30	0.14125	55.23
	90	12.49	.13878	52.79
	100	13.69	.13690	50.40
	110	14.79	.13445	47.93
	120	15.85	.13208	45.56
	130	16.90	.13000	42.95
+20	80	8.09	0.13483	57.65
	90	9.29	.13271	55.21
	100	10.46	.13075	52.82
	110	11.56	.12844	50.35
	120	12.60	.12600	47.88
	130	13.65	.12409	45.37
+40	80	5.15	0.12785	59.90
	90	6.34	.12680	57.46
	100	7.49	.12483	55.07
	110	8.59	.12271	52.60
	120	9.63	.12038	50.13
	130	10.66	.11844	47.62

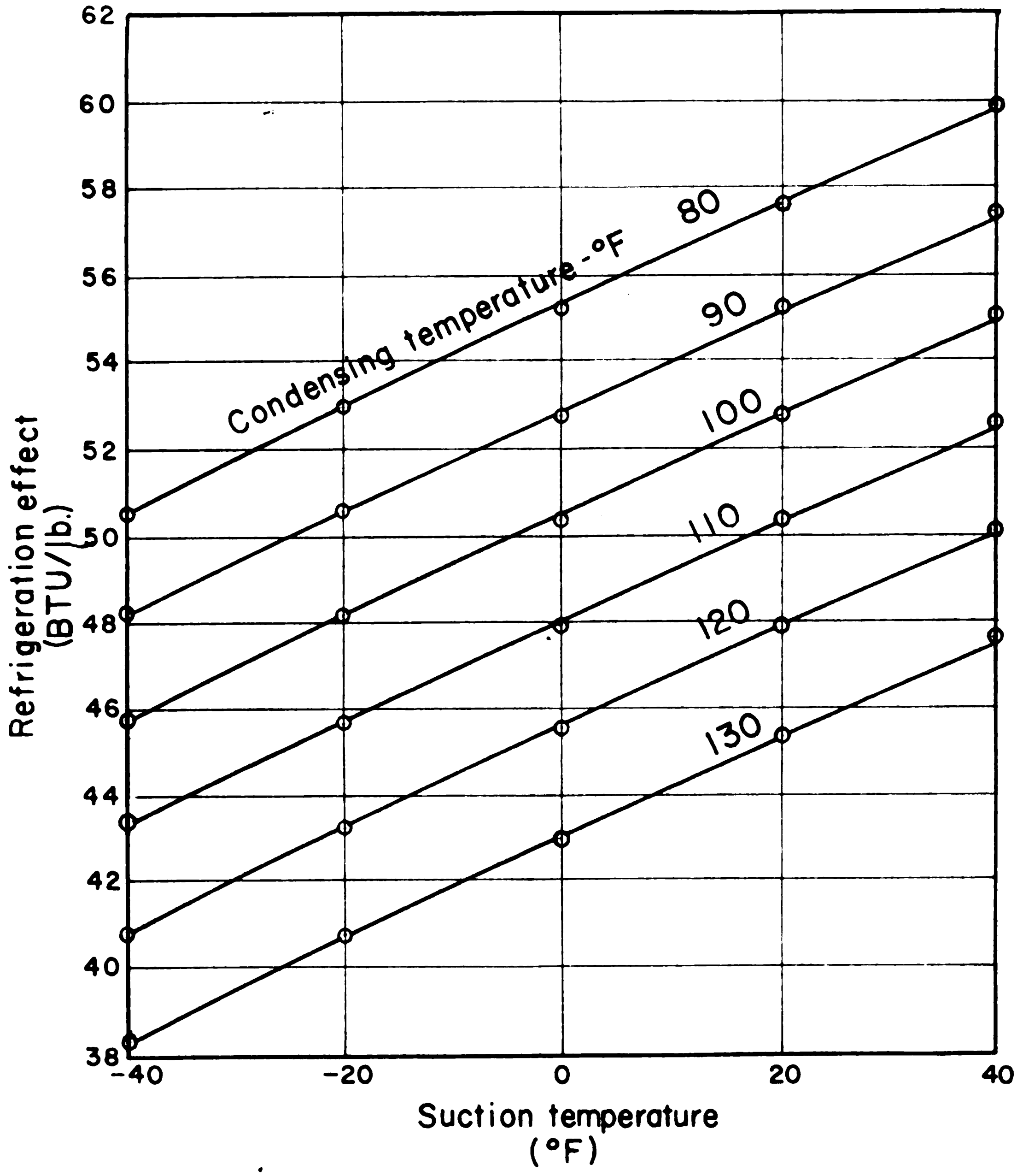

FIGURE 4-1
Refrigeration effect versus operating conditions

The terminal temperature difference will be essentially constant for a given condenser design. Thus, the economic temperature rise of the water appears to be primarily a function of inlet water temperature and suction temperature. It would appear, then, that the water flow rate should be controlled by a valve sensitive to this temperature difference, rather than to condensing pressure.

Unfortunately, the form of the equation renders it difficult to adapt to graphical solution. However, by certain approximations, another form of equation may be derived which may be readily solved by graphical methods.

If all other parts of the cycle are assumed to be unaffected by slight changes in condensing pressure, the part of the work affected is:

$$m\ (\Delta t_w)$$

The water flow rate may be further simplified by using H_c to indicate the amount of heat to be dissipated in the condenser per pound of refrigerant, and treating it as a parameter, rather than a variable. The water cost would then be:

$$C_w = \left(\frac{12000\ T}{Q_r \Delta t_w}\right) \frac{H_c\ \ A}{1000\rho} = \left(\frac{12}{8.33}\right) \frac{A\ T\ H_c}{Q_r \Delta t_w}$$

The cost of the variable part of the work is:

$$C'_{w_k} = \frac{12000\ T\ B\ m \Delta t_w}{Q_r \quad 3413\, \eta_o}$$

Thus:

$$C'_T = \frac{12000\ T\ A\ H_c}{8330\quad Q_r \Delta t_w} + \frac{12000\ T\ B\ m \Delta t_w}{3413\quad Q_r\ \eta_o}$$

$$= \frac{12000\ T}{Q_r} \left[\frac{A\ H_c}{8330 \Delta t_w} + \frac{B\ m\ \Delta t_w}{3413\ \eta_o} \right]$$

Minimizing:

$$\frac{dC_T'}{d \Delta t_w} = 0 = \frac{12000\ T}{Q_r} \left[- \frac{A\ H_c}{8330\ (\Delta t_w)^2} + \frac{B\ m}{3413\ \eta_o} \right]$$

$$(\Delta t_w)^2 = \frac{3413\ \eta_o\ A\ \ H_c}{8330\ B\ \ m} = (0.411)\ \frac{A\ \eta_o}{B} \left(\frac{H_c}{m}\right)$$

In order to use a plot of this equation effectively, values of H_c/m must be known. This ratio is a function of the refrigerant used, the suction and condensing temperatures, and the isentropic compression efficiency. Table 4-2 shows values of this function for various isentropic efficiencies, suction and condensing temperatures for Dichlorodifluoromethane.

It should be noted that the variation within a given isentropic efficiency is slight. Considering the data for η of 0.9, the maximum variation about a mean value of 481.08 is 5.93%. Taking the square root reduces this variation with respect to Δt_w to 2.96%. This is of the order of accuracy of plotting numbers on logarithmic graph paper with $2\frac{1}{2}$ inch cycles. Thus, the final equation to be plotted is:

$$(\Delta t_w)^2 = C \, \frac{A\eta}{B}$$

with the values of C being:

η_I	C
0.7	211.7
0.8	203.4
0.9	197.7

Figure 4-2 shows the solution of this equation for a typical 10 ton system.

Table 4-2

t_s	t_c	H_c/m η_I 0.7	0.8	0.9
-40	30	500.13	478.69	461.99
	90	501.87	478.63	460.57
	100	506.26	481.23	461.75
	110	508.14	481.34	460.52
	120	510.00	481.39	459.19
	130	510.84	481.95	456.90
-20	80	505.34	487.47	473.37
	90	508.16	488.57	473.28
	100	510.57	489.15	472.45
	110	512.56	489.87	471.33
	120	514.30	489.27	469.84
	130	514.58	487.78	466.97
0	80	505.27	491.04	479.93
	90	508.94	492.87	480.40
	100	511.03	493.13	479.25
	110	513.65	494.01	478.69
	120	516.35	494.92	478.27
	130	516.08	492.92	474.84
+ 20	80	513.31	502.56	494.25
	90	516.01	503.50	493.78
	100	518.24	504.02	492.85
	110	520.55	504.52	491.98
	120	522.86	505.00	491.11
	130	522.77	503.10	487.87
+ 40	80	522.41	515.26	509.67
	90	524.61	515.69	508.68
	100	526.88	516.14	507.81
	110	528.64	516.18	506.40
	120	530.74	516.45	505.32
	130	530.65	514.61	503.02
		515.06	496.37	481.08

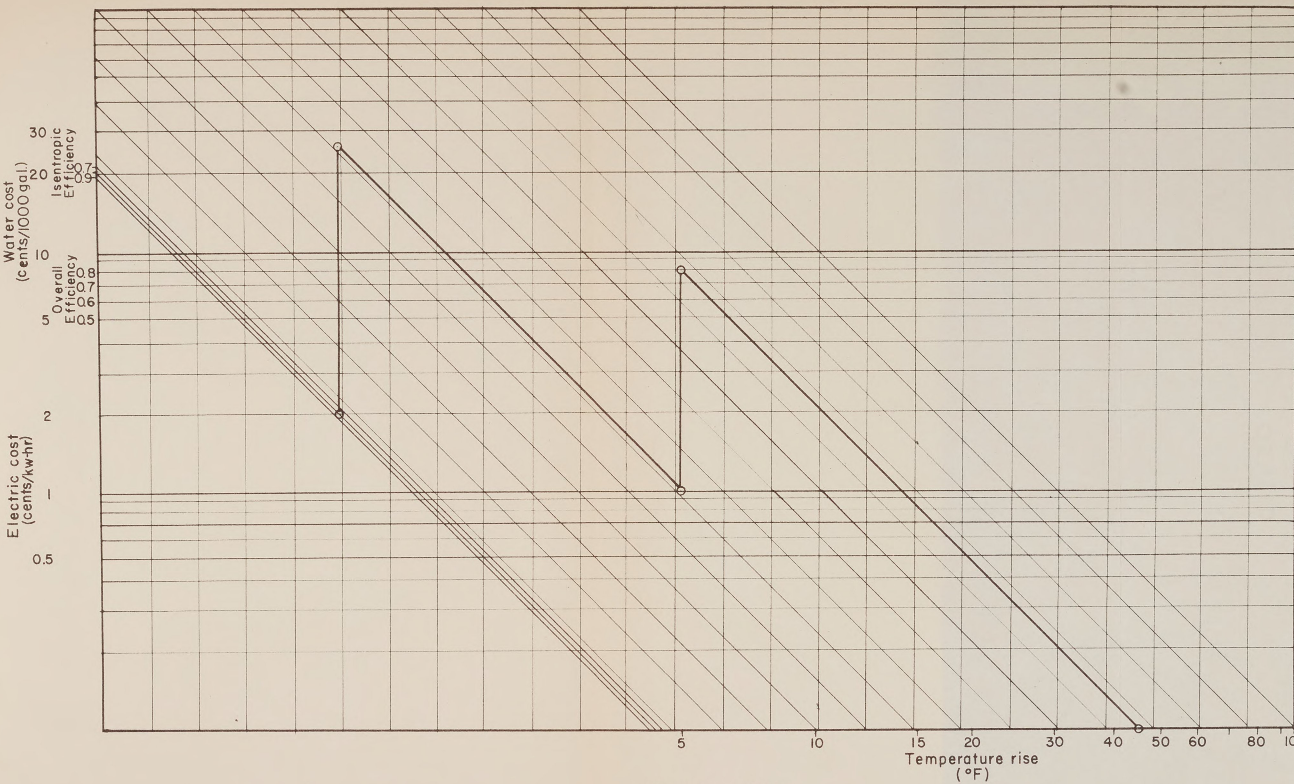

FIGURE 4-2
Economic condenser water temperature rise
(Dichlorodifluoromethane)

The latter simplification leads to slightly higher values of Δt. However, since the total cost equation contains the primary variable to only the first power, the minimum is not sensitive. Figure 4-3 shows a plot of total cost versus temperature rise of the condenser water. It can be seen that there is little difference in total cost for a value ten degrees different from the optimum. However, this difference, of the order of 0.2¢, on an hourly basis will amount to a considerable difference during the course of a year. A variation of 15 - 20° in Δt would add approximately 10% to the total yearly cost.

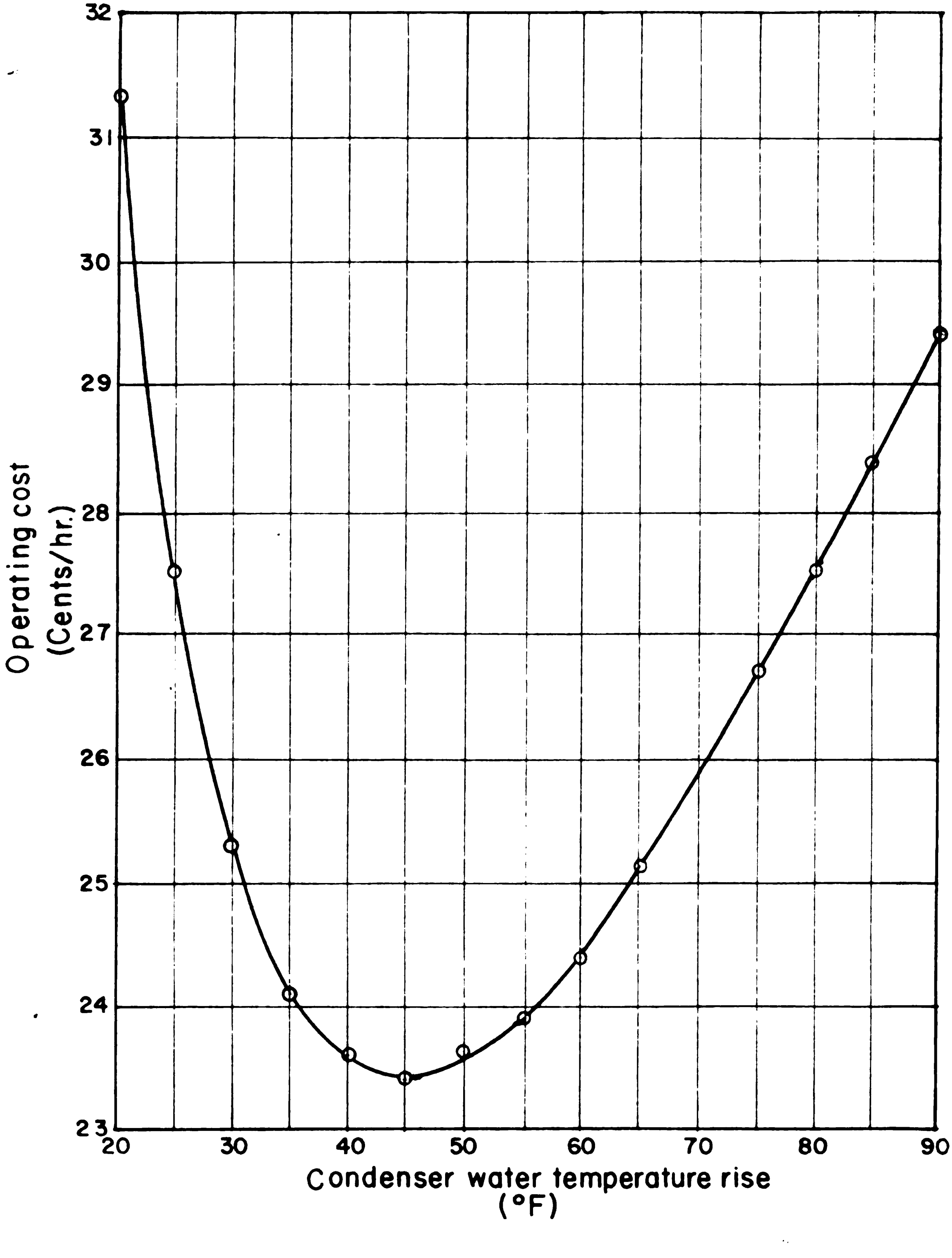

FIGURE 4-3

Variation of operating cost with condenser water temperature rise

(b) Air Ducts

The equation for cost of owning and operating air ducts may be reduced to one in which diameter is the independent variable. Rectangular ducts may be reduced to an equivalent diameter for equal capacity and pressure loss* so that solution for the economic diameter will lead, through proper conversion, to the economic rectangular section.

The cost of material and fabrication may be expressed as a function of the area of the sheet metal involved, say in cents per square foot. The owning cost would be, then:

$$C_o = \frac{A\,L\,\pi\,d}{12} \quad \text{cents/year}$$

where:

A = ¢/sq.ft.-year

L = length, feet

d = diameter, inches

The value of cost, A, is a discontinuous function, since it increases abruptly with a change in allowable minimum gauge. This variation can be expressed, awkwardly, as a function of diameter, but it is so insensitive when so expressed, that it adds little to the accuracy of the calculations. Thus, it will be treated here as a parameter.

The pressure loss in a duct may be expressed as:

$$\Delta P = \frac{f\ L_e\,\rho V^2}{2\ g_c\ D} \quad \text{lb/ft}^2$$

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* A.S.H.A.E. Guide, American Society of Heating and Air Conditioning Engineers, New York, 1956, pp 737-9.

where:

f = friction factor

L_e = equivalent length of straight duct, feet

V = velocity, feet per second

ρ = density, pounds per cubic foot

D = diameter, feet

The velocity may be expressed in terms of diameter, rate of flow, and density, as follows:

$$V = \frac{Q}{60A} \quad \text{feet/second}$$

where:

Q = rate of flow, cubic feet per minute.

A = area of flow, square feet.

Substituting this in the pressure loss equation:

$$\Delta P = \frac{f\, L_e \rho}{2\, g_c D}\left[\frac{Q}{60A}\right]^2 = \frac{f\, L_e \rho\, Q^2}{7200\, g_c D A^2}$$

The friction factor may be accurately approximated, over reasonable ranges, by a function of Reynolds' Number. For Reynolds' Numbers between 10^4 and 10^6, a good approximation is:*

$$f = \frac{0.2}{(R_e)^{0.2}} = \frac{0.2\,\mu^{0.2}}{D^{0.2} V^{0.2} \rho^{0.2}}$$

where:

μ = viscosity, lb./foot-hour

V = velocity, feet per hour

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*Thermodynamics of Fluid Flow, Newman A. Hall, Prentice Hall, New York, 1951, pp 30-1.

D = diameter, feet

ρ = density, pounds per cubic foot

Substituting the value of velocity in terms of area:

$$V = \frac{60Q}{A} \quad \text{feet/hour}$$

$$f = \frac{(0.2)\mu^{0.2}A^{0.2}}{D^{0.2}(60)^{0.2}Q^{0.2}\rho^{0.2}}$$

Substituting this value of f in the pressure loss equation:

$$\Delta P = \frac{(0.2)L_e\,\rho\,Q^{1.8}\rho^{0.8}\mu^{0.2}}{(2)g_c\,(60)^{2.2}\,D^{1.2}\,A^{1.8}}$$

$$= \frac{(0.2)L_e\,Q^{1.8}\rho^{0.9}\mu^{0.2}}{2g_c\,(60)^{2.2}\,D^{1.2}\left[\frac{\pi D^2}{4}\right]^{1.8}}$$

$$= \frac{(0.2)(4)^{1.8}L_e\,\rho^{0.3}\mu^{0.2}\,Q^{1.8}}{2g_c\,(60)^{2.2}\,(\pi)^{1.8}\,D^{4.8}} \quad \text{lb/ft}^2$$

Since the pressure drop is usually quite small in comparison to the total pressure in ducts of reasonable length, the process of flow may be approximated as a constant volume process, so that the work will be:

$$W_k = \frac{\Delta P}{\rho} \quad \text{ft-lb/lb.}$$

The cost of producing flow is then:

$$C_{W_k} = \frac{B\,\Delta P\;W_f}{\eta_o\,\rho(778)(3413)} \quad \text{cents/hour}$$

where:

B = cost of electricity, cents per kilowatt-hour

W_f = flow rate, pounds per hour

η_o = overall fan efficiency

but: $W_f = 60\,Q\rho$

Thus the annual work cost is:

$$C_{wk} = \frac{8760\ BF_u}{(3413)(778)\ \eta}\quad 60Q\ \ P$$

$$= \frac{(0.2)(4)^{1.8}\ L_e\ \rho^{0.8}\mu^{0.2}\ Q^{2.8}\ (8760)}{2g_c\ (60)^{1.2}(\pi)^{1.8}\ (34.3)(778)\eta_o\ D^{4.8}} \text{ cents/year}$$

where:

F_u = use factor, fraction of time system is in use.

Expressing diameter in inches, and flow rate as Q/1000, a number of more convenient magnitude:

$$C_{wk} = \frac{4.422 \times 10^6\ \ BF_u\ \rho^{0.9}\mu^{0.2}\ L_e}{\eta_o\ d^{4.8}}\ \ (Q/1000)^{2.9} \text{ cents/year}$$

The total cost of owning and operating the duct is:

$$C_T = \frac{AL\,\pi d}{12} + \frac{4.422\text{x}10^6\ \ BF_u\ \rho^{0.8}\mu^{0.2}\ L_e}{\eta_o\ d^{4.8}}(Q/1000)^{2.8} \text{ cents/year}$$

Minimizing cost with respect to diameter, in accordance with the methods discussed in Chapter II:

$$\frac{dC_T}{dd} = 0 = \frac{AL\pi}{12} - \frac{2.123 \times 10^7\ BF_u\,\rho^{0.8}\mu^{0.2}\ L_e}{\eta_o\ d^{5.8}}\ (Q/1000)^{2.8}$$

$$d^{5.8} = 8.106\text{x}10^7\ \ \frac{BF_u\,\rho^{0.8}\mu^{0.2}}{A\eta_o}\ \ \left(\frac{L_e}{L}\right)\ \ (Q/1000)^{2.8}$$

The group of factors:

$$8.106 \times 10^7\ \rho^{0.8}\mu^{0.2}$$

is a function of operating conditions only, and primarily of temperature only at low pressures. These may be combined as $\phi(t)$, so that the equation becomes:

$$d^{5.8} = \phi(t)\ \ \frac{B\ F_u}{A\eta_o}\ \ \left(\frac{L_e}{L}\right)(Q/1000)^{2.8}$$

The factor $\phi(t)$ is shown in Table 4-3 for various temperatures at atmospheric pressure.

The cost factor A should include cost of insulation, since the relative cost of heat loss for various size ducts has not been previously considered. As an approximation, a large duct may be treated as a flat surface for purposes of estimating the economic insulation thickness.

For example, consider a duct which is to carry 3000 cubic feet per minute of air at 60°F. in a room at 80°F. Using mineral wool, the economic thickness is approximately:

$$X = 0.27 \sqrt{\frac{(8760)(1/3)(0.2)(20)}{(12000)(0.0225)}} - \frac{0.27}{1.65}$$

$$= 1.19''$$

The same costs were used as in the example of Chapter III(a). Thus the thickness of insulation used would probably be 1 inch. The cost per square foot of duct surface would thus be increased by approximately 1¢ per year over metal and fabricating costs.

The approximate cost of metal and fabrication is 2¢ per year per square foot, so the total cost factor A will be approximately 3¢ per year per square foot. Using B = 2¢ per kilowatt-hour, $F_u = 0.7$, and $\eta = 0.3$, the solution for straight duct ($\frac{L_e}{L} = 1$) is:

$$d = 26.78 \text{ inches.}$$

Figure 4-4 shows the effect of diameter upon total cost for the example chosen. It should be noted that the curve is quite flat for about three inches of diameter, but

Table 4-3

VARIATION OF $\phi(t)$ WITH TEMPERATURE

t (^{o}F)	μ (lb/ft-hr)	ρ (lb/ft^3)	ϕ(t)
40	0.043	0.0793	5.688 x 10^6
60	0.044	0.0763	5.540 x "
80	0.045	0.0734	5.395 x "
100	0.046	0.0708	5.265 x "
120	0.047	0.0684	5.144 x "
140	0.044	0.0661	5.047 x "
160	0.050	0.0640	4.948 x "
180	0.051	0.0620	4.833 x "
200	0.052	0.0601	4.733 x "

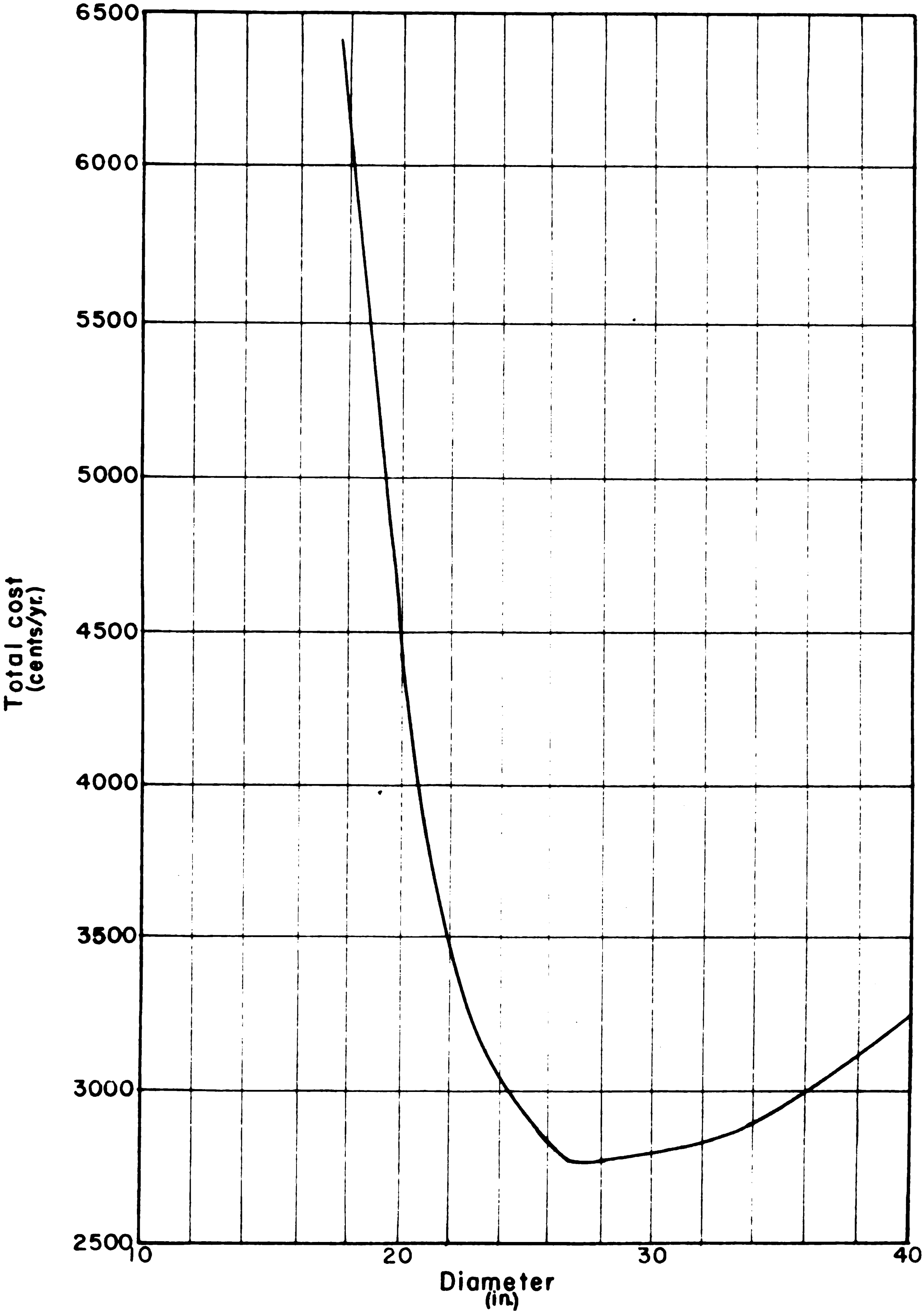

FIGURE 4-4
Variation of cost with duct diameter

increases above and below these values rather rapidly. Thus, the choice of diameter is rather critical. A choice two inches on either side of the economic diameter will result in less than four per cent increase in total cost, while a choice four inches on either side results in approximately ten per cent increase in total cost.

Figure 4-5 shows the solution of the previous example. This offers the general solution for pressures not differing appreciably from atmospheric.

There are many other mechanical systems which may be treated in the manner shown in this chapter, but their presentation would be a repetition of similar procedures. If the equation for total cost can be reduced to a pair of functions of one variable, the methods illustrated should serve to lead to a general graphical solution.

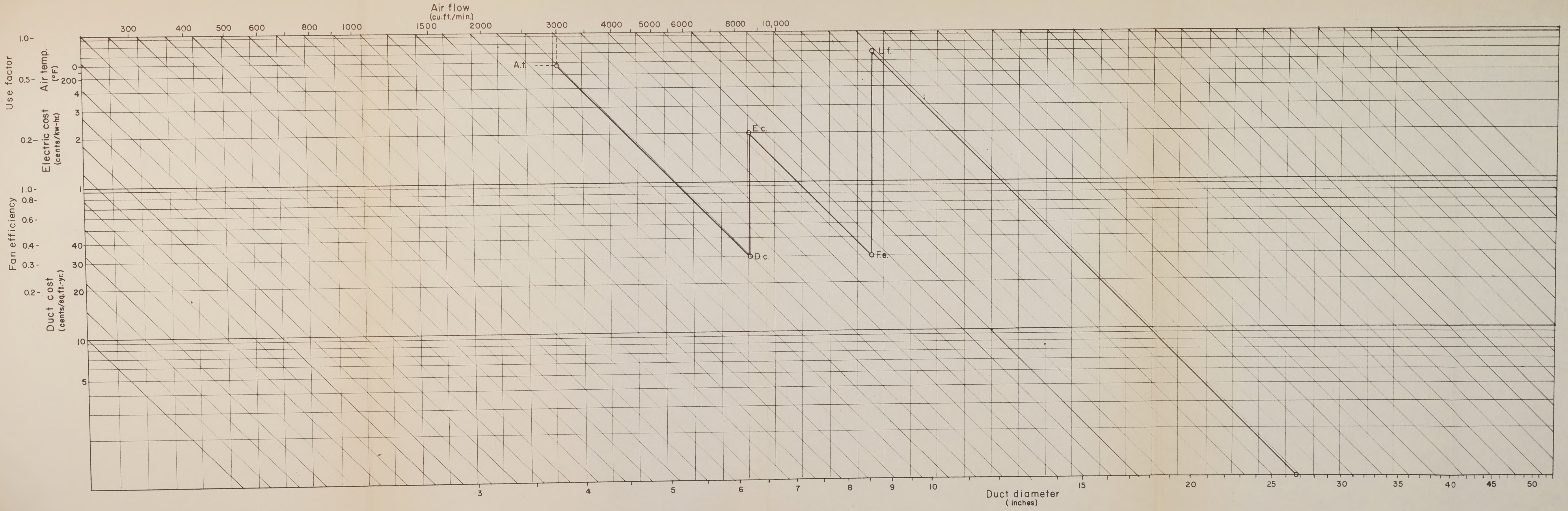

FIGURE 4-5
Air duct sizing

Chapter V. Simple Graphical Solutions with Incremental Variable - Pipe Sizing

Selection of pipe for minimum cost operation does not differ essentially from selection of air duct size, except that pipes are available only in standard incremental sizes.

Some difficulty is encountered in expressing the cost of pipe as a function of diameter. However, a good approximation may be made by assuming that the cost may be expressed as "A" cents per foot per inch in diameter per year.

As before, the cost is the sum of the owning costs and operating costs. The owning cost is:

$$C_p = ALd \quad \text{cents/year}$$

The operating cost is the cost of forcing the fluid through the pipe. The pressure drop due to friction may be expressed as:

$$\Delta P = \frac{fL_e \rho V^2}{2gD} \quad \#/ft^2$$

where:

f = friction factor

L = equivalent length, ft.

ρ = density, lb/ft^3

V = velocity, ft/sec.

D = diameter, ft.

The work required, since the change in fluid volume is small, is very nearly:

$$W_k = \frac{\Delta P}{\rho} = \frac{fLV^2}{2gD} \quad \text{ft-lb/lb}$$

The friction factor, f, may be approximated, as in the case of ducts, by:

$$f = \frac{a}{(R_e)^b}$$

so that the work is:

$$W_k = \frac{a\mu^b \, L V^{2-b}}{2g D^{1+b} (3600)^b \rho^b} \text{ ft - \#/\#}$$

In most cases, the work is done directly or indirectly by electric motors, so that the work cost may be related to electric costs. On a yearly basis, the work cost is:

$$CW_k = \frac{(8760)\ B\ F_u\ W_f\ \mu^b\ L_e\ V^{2-b}}{(3413)(778)(2g)(3600)^b \eta_o D^{1+b} \rho^b}$$

where:

B = electric costs, ¢/kw-hr.

F_u = use factor, fraction of time system is in use.

W_f = weight of flow, pounds/hr.

η_o = overall pumping efficiency

The velocity is related to the weight of flow, flow area and density by:

$$W_f = \rho A V = \frac{\rho \pi\ D^2 V}{4}$$

$$V = \frac{4W_f}{\rho \pi D^2} \text{ ft/hr} = \frac{4W_f}{\rho \pi D^2 (3600)} \text{ ft/sec}$$

Thus:

$$CW_k = \frac{(8760)\ a\ B\ F_u\ W_f \mu^b\ L_e (4)^{2-b}\ W_f^{2-b}}{(3600)^2 (3413)(778)(2g)\ \eta_o\ D^{1+b} \rho^2 \pi^{2-b}\ D^{4-2b}}$$

$$= \frac{(8760)(4)^{2-b}(12)^{5-b}\ aBF_u \mu^b\ L\ W_f^{3-b}}{(3600)^2 (3413)(778)(2g) \pi^{2-b} \eta_o \rho^2\ d^{5-b}}$$

In order to determine the value of a and b, the charts on pages 30 and 31 of Thermodynamics of Fluid Flow, by Newman Hall, Prentice Hall, New York, 1951, were used, as well as other references. The values will be discussed for each case.

The total cost, in cents per year, is as follows:

$$C_t = ALd + \frac{(8760)(4)^{2-b}(12)^{5-b} aBF_u \mu^b L_e W_f^{3-b}}{(3600)^2(3413)(778)(2g)\pi^{2-b}\eta_o \rho^2 d^{5-b}}$$

Minimizing, with respect to diameter:

$$\left(\frac{dC_t}{dd}\right) = AL - \frac{(5-b)(8760)(4)^{2-b}\, aBF_u \mu^b L_e \quad W_f^{3-b}(12)^{5-b}}{(3600)^2(3413)(778)\pi^{2-b}\eta_o \rho^2 \ d^{6-b}} = 0$$

$$Ad^{\frac{6-b}{2}} = \frac{(5-b)(8760)(4)^{2-b}(12)^{5-b} aBF_u \mu^b L_e W_f^{3-b}}{(3600)^2(3413)(778)(2g)(\pi)^{2-b}\rho^2 \ L \ d^{\frac{6-b}{2}}}$$

This may be written:

$$d^{\frac{6-b}{2}} = \phi\left(\frac{L_e}{L}\right) \frac{BF_u}{\eta} \quad \frac{W_f^{3-b}}{d^{\frac{6-b}{2}}} \qquad \text{Equation 5-1}$$

where:

$$\phi = \frac{(5-b)(8760)(4)^{2-b}\ (12)^{5-b}\mu^b}{(8600)^2(3413)(778)(2g)\ \pi^{2-b}\rho^2}$$

Note that ϕ is a constant times a factor which is a function of the operating conditions only, and may be evaluated with respect to temperature and pressure.

This equation is valid for all systems involving only direct work which may be chargeable to electric power, or fuel cost, and in which the work may be accurately approximated as $\frac{\Delta P}{\rho}$. Special cases applying this general equation, as well as other situations will be considered in the remainder of this chapter.

(b). **Water Piping**

Since steel pipe is used extensively for water service, the factors a and b may be approximated as 0.185 and 0.23 respectively. The general equation may be applied, since this is a simple problem of balancing pumping costs against pipe costs. Thus:

$$\phi = \frac{(4.77)(8760)(4)^{1.77}(12)^{4.77}(.185)}{(3600)^2(3413)(778)(64.4)^{1.77}} \mu^{0.23}$$

$$= \frac{\mu^{0.23}}{\rho^2} \quad (7.526 \times 10^{-7})$$

The general equation (5-1) becomes:

$$Ad^{2.885} = \phi \; \frac{BF_u}{\eta} \; \frac{W_f^{2.77}}{d^{2.885}} \; \left(\frac{L_e}{L}\right)$$

Figure 5-1 shows the solution plot for this equation. Approximate values of "A" are shown as a curve on the pipe cost scale. It should be treated as an approximation only, since the values of cost for a given diameter vary from locality to locality and year to year. In addition, variation of the number of fittings for a given length will cause a variation in the unit cost. One method of approximation which yields reasonable results is to assume that the cost of fittings varies directly as the number of equivalent feet for pressure loss calculations. This assumption can be shown to be fairly accurate for an average number of fittings. The result of this assumption is to cause the ratio L_e/L to be equal to unity.

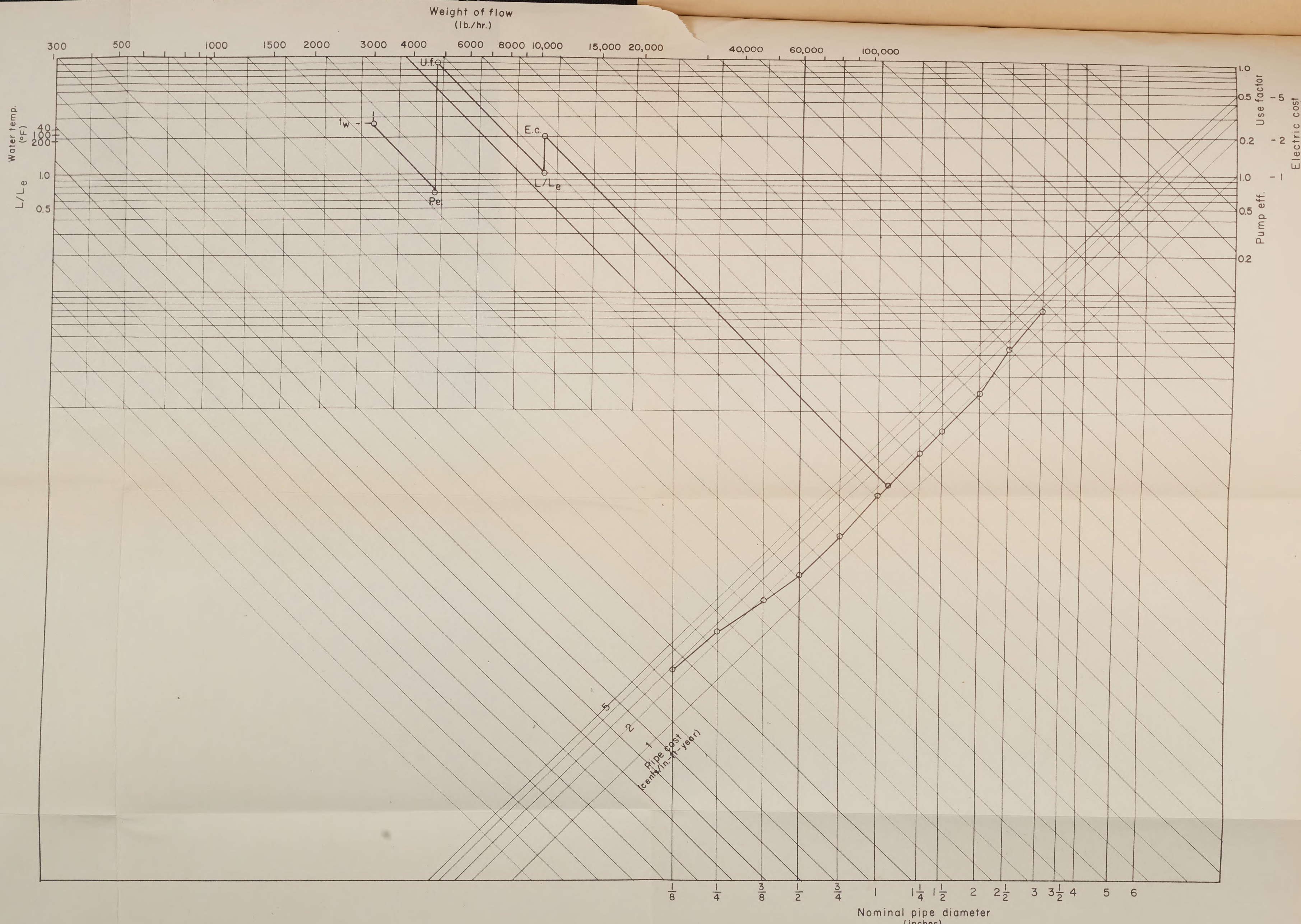

FIGURE 5-1
Economic water pipe size

The value of "A" can, thus, be estimated with good accuracy, by calculating the total cost of fittings and pipe for an average run; adding an installation cost; and dividing by diameter in inches and by the expected life in years.

Table 5-1 shows the calculation of ϕ with temperature, at atmospheric pressure. Since there is only a negligible change in viscosity and density for liquid water under the influence of higher pressures, a liberal extension of the use of this table for higher pressures is justified.

The example shown is for a flow of 3000 pounds per hour of water at 40°F., with an electric cost of 2¢ per kilowatt hour, a pump efficiency of 0.7 and a use factor of 0.9.

Figure 5-2 shows the effect upon total cost of choosing other diameters for this example with an equivalent length of 100 ft. Note the effect of choices smaller than the economic size.

If the water is supplied at an initial pressure, so that pumping is not required, the economic size of pipe is the smallest which will pass the required weight of flow utilizing the available pressure. Using the pressure drop equation:

$$\Delta P = \frac{fL_e \rho V^2}{2\ g\ D} \quad \#/ft^2$$

$$= \frac{fL \rho V^2}{288 g D} \quad \#/in^2$$

$$= \frac{fL_e \rho V^2}{24 g d} \quad \#/in^2$$

TABLE 5-1

$\phi(t)$ for Water Piping

Water Temp. (^oF)	Viscosity (lb-ft/hr)	Density (lb-ft^3)	$\phi(t)$
32	4.33	62.42	2.706×10^{-10}
40	3.75	62.42	2.617
50	3.17	62.38	2.522
60	2.71	62.34	2.436
70	2.37	62.27	2.368
80	2.08	62.17	2.305
90	1.85	62.11	2.247
100	1.65	61.99	2.197
110	1.49	61.94	2.157
120	1.36	61.73	2.119
130	1.24	61.54	2.089
140	1.14	61.39	2.059
150	1.04	61.20	2.027
160	0.97	61.01	2.008
170	0.91	60.79	1.994
180	0.84	60.57	1.971
190	0.79	60.35	1.957
200	0.74	60.13	1.942

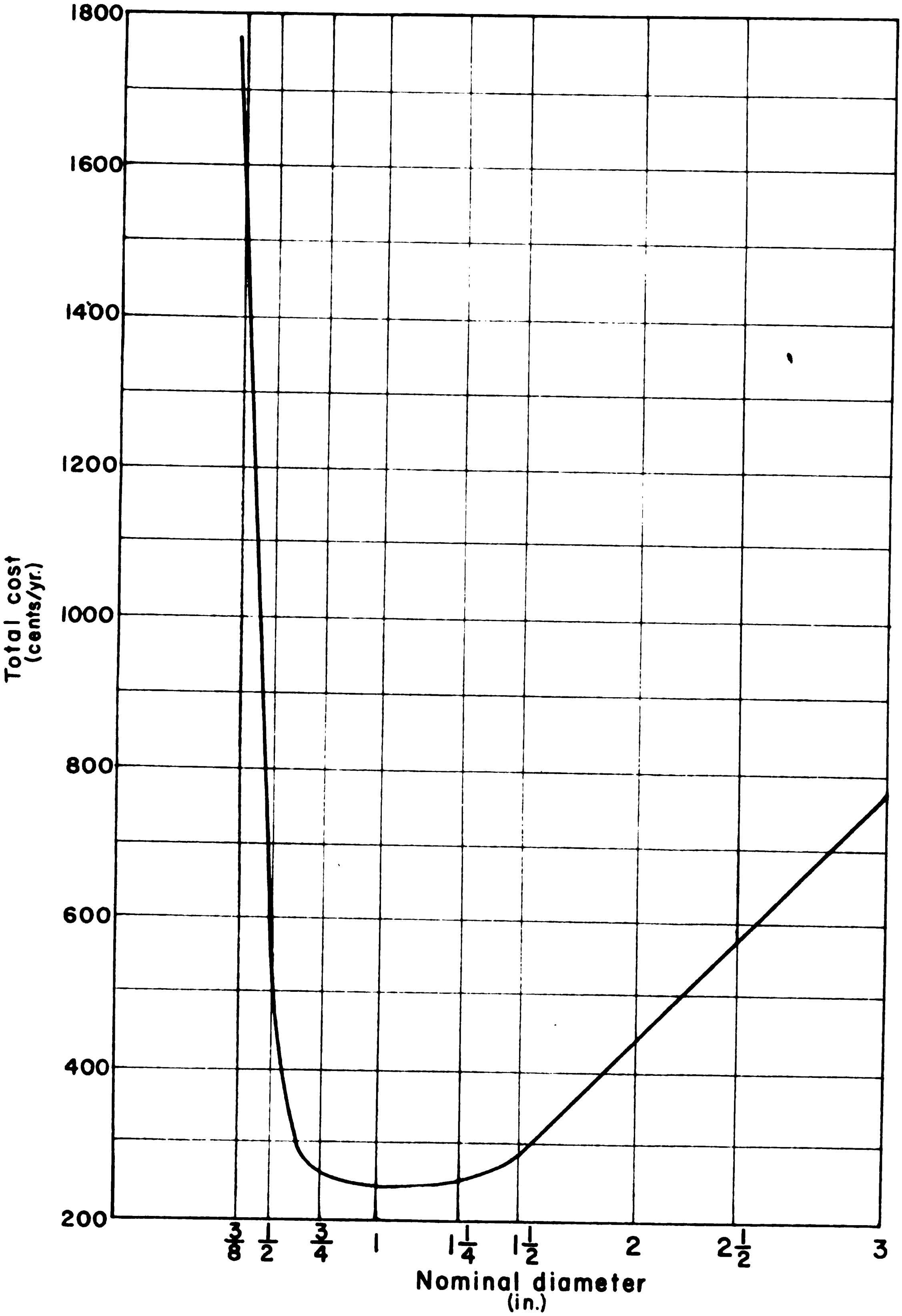

FIGURE 5-2
Variation of cost with diameter

where:

f = friction factor

L_e = length, equivalent

ρ = density, #/ft^3

V = velocity, ft/sec

D = diameter, feet

d = diameter, inches

Using the previous approximation for the friction factor:

$$\Delta P = \frac{0.185\mu^{0.23}\, L_e \rho V^2}{24g\, d\, D^{0.23}\, V^{0.23} \rho^{0.23} (3600)^{0.23}}$$

$$= \frac{(0.185)(12)^{0.23}\, \mu^{0.23}\, L_e \rho^{0.77}\, V^{1.77}}{24g\, (3600)^{0.23}\, d^{1.23}}$$

$$V = \frac{W_f}{\rho A} = \frac{W_f(576)}{\rho \pi d^2 (3600)} \quad \text{ft/sec}$$

$$\Delta P = \frac{(0.185)(576)^{1.77}(12)^{0.23}\mu^{0.23} L_e W_f^{1.77}}{24g\, (3600)^2\, \pi^{1.77}\, \rho\, d^{4.77}}$$

Thus:

$$d^{4.77} = \frac{(0.185)(576)^{1.77}(12)^{0.23}\mu^{0.23} L_e W_f^{1.77}}{24g\, (3600)^2\, \pi^{1.77}\, \rho\, \Delta P}$$

$$= \left[(3.317 \times 10^{-7})\frac{\mu^{0.23}}{\rho}\right] \frac{L_e\, W_f^{1.77}}{\Delta P}$$

$$= \phi\, \frac{L_e\, (W_f)^{1.77}}{\Delta P}$$

Table 5-2 shows the values of ϕ for various temperatures.

Figure 5-3 shows the solution of this equation. The example shown is for 2000 pounds per hour of water at 40°F, with an available pressure loss of 20 pounds per square inch.

TABLE 5-2

ϕ(t) for Water Flow with Given Pressure Loss

Water Temp. (^{o}F)	ϕ(t)
32	8.38×10^{-9}
40	8.12
50	7.82
60	7.56
70	7.34
80	7.11
90	6.93
100	6.78
110	6.62
120	6.51
130	6.40
140	6.29
150	6.18
160	6.10
170	6.03
180	5.95
190	5.88
200	5.80

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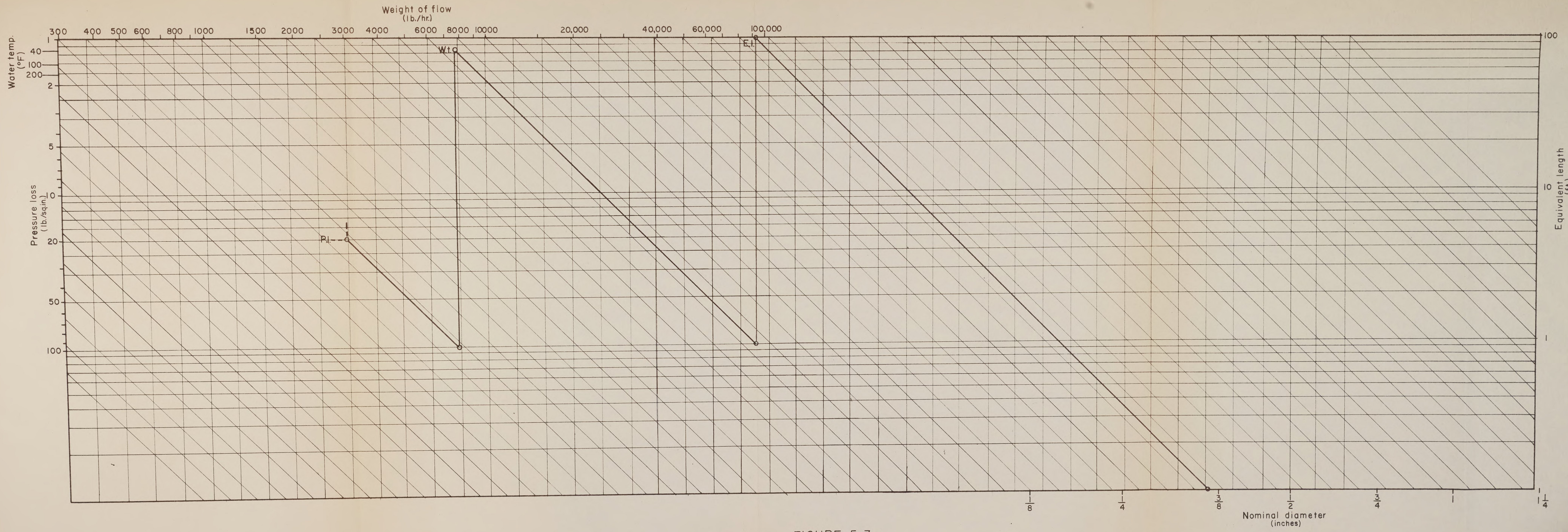

FIGURE 5-3
Water pipe for specified pressure loss

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(c) Compressor Discharge Lines-Dichlorodifluoromethane.

Assuming that the effect of pressure loss in the compressor discharge line is only to increase compressor work, with no appreciable change in efficiency, the general equation may be used.

In this case, copper tubing is in fairly general use, so that the values of a and b used in determining the average value of the friction factor should be 0.0653 and 0.228 respectively*. For these values, the solution becomes:

$$Ad^{2.886} = \frac{(4.772)(8760)(4)^{1.772}(12)^{4.772}(0.0653)BF_u L_e \mu^{.228} W_f^{2.772}}{(3600)^2(3413)(778)(2g)\pi^{1.772}\eta L \rho^2 d^{2.886}}$$

However, the weight of flow may be related to the tonnage of the system by the relationship:

$$W_f = \frac{12{,}000\ rT}{47} = 255.3\ rT$$

The equation, thus becomes:

$$Ad^{2.886} = \frac{(255.3)^{2.772}(4.772)(8760)(4)^{1.772}(12)^{4.772}(.0653)BF_u L_e \overset{2.772}{\vee} T^{2.772}}{(3600)^2(3418)(778)(2g)\pi^{1.772}\eta L \rho^2 d^{2.886}}$$

or:

$$Ad^{2.886} = \phi \frac{L_e}{L} \frac{BF_u}{\eta} \frac{T^{2.772}}{d^{2.886}}$$

where:

$$\phi = (6.2863 \times 10^4) \frac{r^{2.772} \mu^{0.228}}{\rho^2}$$

Values of ϕ are shown in Table 5-3. The solution of the equation is shown in Figure 5-4. The example shown is for

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*Refrigeration and Air Conditioning, R.C. Jordan and G.B. Priester, Prentice-Hall, New York, 1949, p. 151.

TABLE 5-3

$\phi(t)$ for Discharge Lines

Condensing Temp. (°F)	Suction Temp. (°F)	$\phi(t)$
80	-40	1.423×10^{-2}
	-20	1.208×10^{-2}
	0	1.024 "
	20	8.511×10^{-3}
	40	7.394 "
90	-40	1.284×10^{-2}
	-20	1.068 "
	0	8.970×10^{-3}
	20	7.53×10^{-3}
	40	6.491×10^{-3}
100	-40	1.157×10^{-2}
	-20	9.389×10^{-3}
	0	7.772×10^{-3}
	20	6.447×10^{-3}
	40	5.457×10^{-3}
110	-40	1.056×10^{-2}
	-20	8.522×10^{-3}
	0	7.086
	20	5.962
	40	5.023
120	-40	9.823×10^{-3}
	-20	7.992
	0	6.390
	20	5.272
	40	4.451
130	-40	8.836×10^{-3}
	-20	6.830
	0	5.900
	20	4.779
	40	4.007

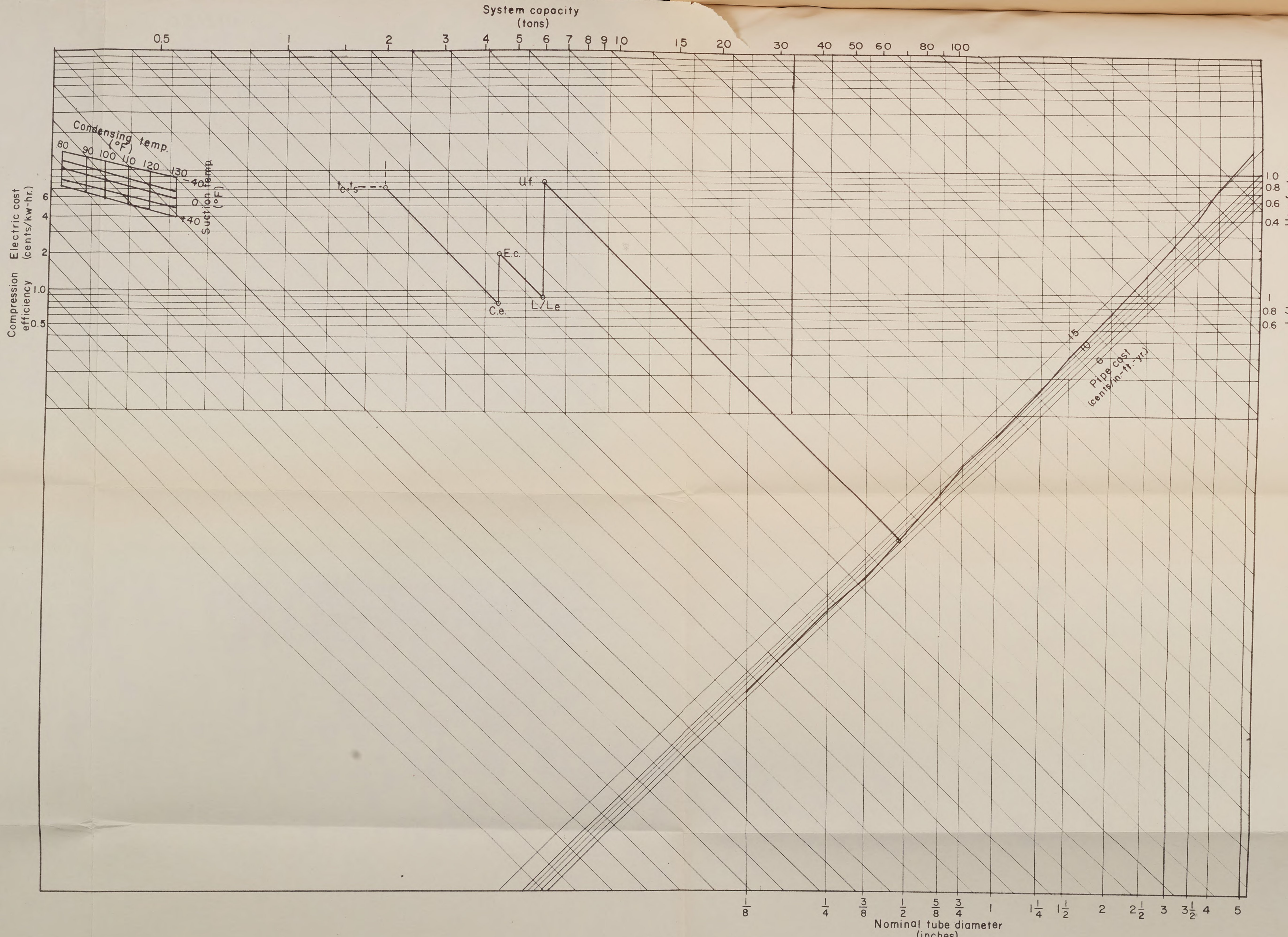

FIGURE 5-4
Discharge line sizing

a condensing temperature of 110°F, a suction temperature of 0°F, a system size of 2 tons, a use factor of 0.8, $B = 2¢/\text{kw-hr}$, $\eta_o = 0.8$ and the curve shown for values of A.

(d) Suction Lines

In the case of suction lines, the general equation is not applicable, since a secondary effect exists. A loss of pressure in the suction line due to friction results in a lower density at the intake of the compressor. Thus, the speed of the compressor must be increased in order to restore the capacity to that of a system with no loss of pressure in the suction line. This results in an increase in friction losses in the moving parts of the compressor.

If it may be assumed, with small changes in speed, that the volumetric efficiency is essentially constant, then:

$$\frac{N_2}{N_1} = \frac{\rho_1}{\rho_2}$$

where:

N = speed, revolutions per minute

ρ = density, pounds per cubic foot

Since the pressure loss is essentially throttling (constant enthalpy), it may be treated as a constant temperature process with the resulting relationship:

$$\frac{\rho_2}{\rho_1} = \frac{P_2}{P_1}$$

Thus:

$$\frac{N_2}{N_1} = \frac{P_1}{P_2} = \frac{P_1}{P_1 - \Delta P}$$

The friction losses in the mechanical parts of the compressor are essentially proportional to the rotative speed. Thus:

$$\frac{W_{f_2}}{W_{f_1}} = \frac{P_1}{P_1 - \Delta P}$$

$$\Delta W_f = W_{f_2} - W_{f_1} = W_{f_1}\left[\frac{P_1}{P_1 - \Delta P} - 1\right] = W_{f_1}\left[\frac{\Delta P}{P_o - \Delta P}\right]$$

where:

W_f = friction work

The mechanical efficiency is defined as:

$$\eta_m = \frac{\text{Indicated work}}{\text{Total work}} = \frac{\text{Indicated work}}{\text{Indicated work} + \text{friction}}$$

thus:

$$\text{Friction work} = (1-\eta_m)\ \text{Indicated work}$$

$$= (1-\eta_m)\ \frac{m(t_c - t_s)}{\eta_I}$$

$$\Delta W_f = (1-\eta_m)\frac{m}{\eta_I}(t_c - t_s)\left[\frac{\Delta P}{P_1 - \Delta P}\right]$$

The total work due to pipe friction is:

$$W_k = \frac{\Delta P}{778\,\rho\,\eta_o} + (1-\eta_m)\,\frac{m}{\eta_I}\,(t_c - t_s)\frac{P}{P_1 - \Delta P}$$

$$= \Delta P\left[\frac{1}{778\,\eta_o\,\rho} + \frac{(1-\eta_m)\ m(t_c - t_s)}{\eta_I\,(P_1 - \Delta P)}\right]$$

where:

η_o = overall efficiency

η_I = isentropic efficiency

but:

$$\Delta P = \frac{f\,L_e\,\rho\,V^2}{2gD}$$

and

$$f \approx \frac{0.0653}{(R_e)^{0.228}}$$

$$\Delta P = \frac{(0.0653)(L_e)\rho V^2 \mu^{0.228}}{2gD\, D^{0.228} V^{0.228} \rho^{0.228} (3600)^{0.228}}$$

$$= \frac{0.0653\, L_e \rho^{0.772} V^{1.772} \mu^{0.228} (12)^{1.228}}{2g\, d^{1.228}\, (3600)^{0.228}}$$

$$V = \frac{W_f}{A} = \frac{(255.3)\ r\ T\ (576)}{\rho \pi d^2\ 3600} \quad \text{ft/sec}$$

$$\Delta P = \frac{3.122\ L_e\ r^{1.772}\ T^{1.772}\ \mu^{0.228}}{\rho\, d^{4.772}} \quad \#/\text{ft}^2$$

where:

r = 47/refrigeration effect (BTU/#)

L_e = equivalent length, feet.

T = tons

μ = viscosity, #/ft-hr

ρ = density, #/ft^3

Thus:

$$W_k = \frac{3.122\ r^{1.772} \mu^{0.228}\ T^{1.772} L_e}{\rho\, d^{4.772}} \left[\frac{1}{778\rho\eta_o} + \frac{(1-\eta_m)m(t_c-t_s)}{(P_1-\Delta P)(\eta_I)}\right]$$

The annual cost is:

$$C_{W_k} = \frac{(255.3)(8760)(3.122)BF_u\ r^{2.772}\mu^{0.228}T^{2.772}L_e}{(3413)\rho\, d^{4.772}}$$

$$\times \quad \frac{1}{778\,\eta_o\rho} + \frac{(1-\eta_m)\ m\ (t_c-t_s)}{\eta_I\left(P_1 - \dfrac{3.122\ r^{1.772}\mu^{0.228}L_eT^{1.772}}{\rho\, d^{4.772}}\right)}$$

The total annual cost is:

$$G = ALd + \left[\frac{(2553)(8760)(3.122)BF_u\ r^{2.772}\mu^{0.228}T^{2.772}L_e}{3413\rho\, d^{4.772}}\right]$$

$$\times \left[\frac{1}{778\,\eta_o\rho} + \frac{(1-\eta_m)\ m\ (t_c-t_s)}{\eta_I\left(P - \dfrac{3.122\ r^{1.772}\mu^{0.228}L_eT^{1.772}}{P\, d^{4.772}}\right)}\right]$$

Minimizing:

$$\frac{dC_t}{dd} = 0 = AL - \left[\frac{(8760)(3.122)(255.3)(4.772)BF_u r^{2.772} \mu^{0.228} T^{2.772} L_e}{3413 \rho d^{5.772}}\right]$$

$$\times \left[\frac{1}{778 \eta_o \rho} + \frac{(1-\ _m)\ m\ (t_c - t_s)}{\eta_I \left[P_1 - \frac{3.122\ r^{1.772} \mu^{0.228} L_e\ T^{1.772}}{\rho d^{4.772}}\right]}\right.$$

$$+ \left[\frac{(8760)(3.122)(255.3)BF_u r^{2.772} L_e\ T^{2.772}}{3413 \rho d^{4.772}}\right]$$

$$\times \left[\frac{\rho d^{3.772}\left[P_1 \rho d^{4.772} - 3.122\ r^{1.772} \mu^{0.228} T^{1.772} L_e\right](1-\eta_m)m(t_c - t_s) - \left[1-\eta_m)\ m\ (t_c - t_s) \rho d^{4.772}\right] P_1 \rho d^{3.772}(4.772)}{\eta_I \left[\rho P_1\ d^{4.772} - 3.122\ r^{1.772} \mu^{0.228}\ T^{1.772}\ L_e\right]^2}\right]$$

$$0 = AL - \left[\frac{(8760)(3.122)(255.3)(4.772)BF_u\ r^{2.772} L_e T^{2.772}}{(3413)\ \rho d^{5.772}}\right]$$

$$\times \left[\frac{1}{778 \eta_o \rho} + \frac{(1-\eta m)\ m\ (t_c - t_s)}{P_1 - \Delta P} - \frac{(4.772)(P_1 - P)(1-\eta_m)m(t_c - t_s)}{(P_1 - \Delta P)^2}\right.$$

$$\left. + \frac{(4.772)(1-\eta_m)\ m\ (t_c - t_s)}{(P_1 - \Delta P)^2}\right]$$

$$Ad^{2.886} = \left[\frac{(8760)(3.122)(255.3)(4.772)BF_u L_e\ r^{2.772} T^{2.772}}{3413\ \rho d^{2.886}}\right]$$

$$\times \left[\frac{1}{778 \eta_o \rho} + \frac{(4.772)(1-\eta_m)\ m\ (t_c - t_s)\ P_1}{(P_1 - \Delta P)^2}\right]$$

If ΔP is small compared to P_1:

$$Ad^{2.886} = \left[\frac{(8760)(3.122)(255.3)(4.772)BF_u L_e \; r^{2.772} T^{2.772}}{3413 \, \rho \, d^{2.886}}\right]$$

$$x\left[\frac{1}{778\,\eta_o\,\rho} + \frac{(4.772)(1-\eta_m)\; m\;(t_c-t_s)}{P_1}\right]$$

$$Ad^{2.886} = \phi \quad \frac{BF_u}{\eta}\left(\frac{L_e}{L}\right) \quad \frac{T^{2.772}}{d^{2.886}}$$

where: $$\phi = \left[\frac{(8760)(3.122)(255.3)(4.772)\,\mu^{0.228}\; r^{2.772}}{3413\,\rho}\right]$$

$$x\left[\frac{1}{778\,\eta_o\,\rho} + \frac{4.772\;(1-\eta_m)\; m\;(t_c-t_s)}{P_1}\right]$$

$$= \frac{9762\,\mu^{0.228}\; r^{\,2.772}}{\rho}\left[\frac{1}{778\,\eta_o\,\rho} + \frac{(4.772)(1-\eta_m) m (t_c-t_s)}{P_1}\right]$$

Note that ϕ is a function only of the operating conditions, and is, thus, a parameter rather than a variable. Table 5-4 shows this parameter for various suction and condensing temperatures.

Figure 5-5 shows the graphical solution of the equation. The example is for a 5 ton system operating at 100°F condensing and 40°F suction, with an electric cost of 2¢/KW-hr., a use factor of 0.8, a compression efficiency of 0.8 and a ratio of length to equivalent length of 0.8. The choice would be 1½ inch nominal tubing.

Note the increased effect of operating conditions on the economic line size, compared to that upon the discharge line selection.

TABLE 5-4

$\phi(t)$ for Suction Lines

Suction Temp. (°F)	Condensing Temp. (°F)	$\phi(t)$
-40	80	3.288
	90	3.950
	100	4.721
	110	5.776
	120	6.970
	130	8.704
-20	80	0.938
	90	1.123
	100	1.345
	110	1.623
	120	1.967
	130	2.390
0	80	0.303
	90	0.362
	100	0.434
	110	0.523
	120	0.628
	130	0.779
20	80	0.0986
	90	0.119
	100	0.146
	110	0.172
	120	0.207
	130	0.252
40	80	0.0372
	90	0.0449
	100	0.0542
	110	0.0655
	120	0.0791
	130	0.0961

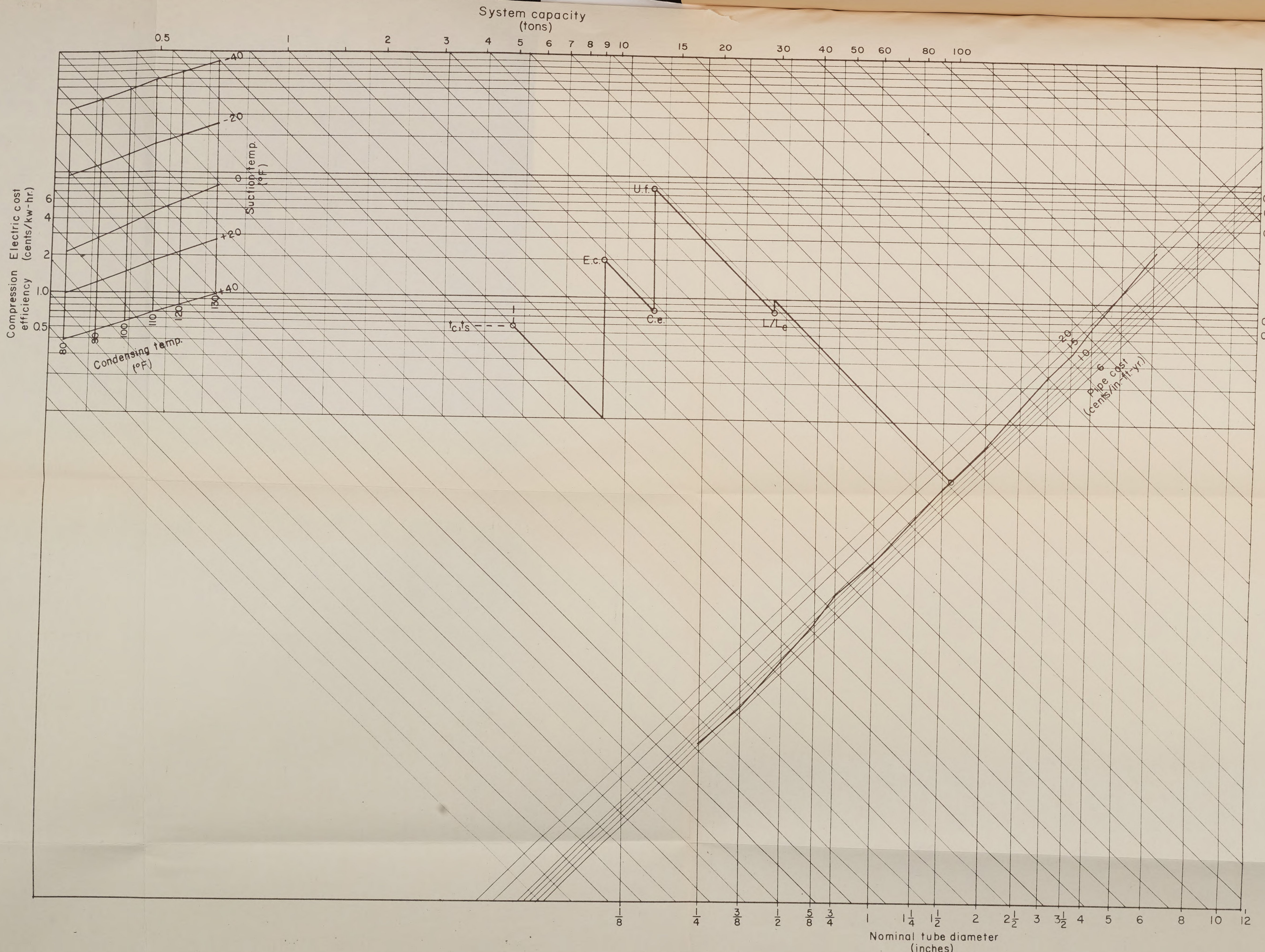

FIGURE 5-5
Suction line sizing

(e) Liquid Line Sizing-Dichlorodifluoromethane

Here, the problem is similar to that of service water piping, where a given pressure may be dissipated. In this case, the amount of subcooling obtained in the condenser or receiver, represents the maximum pressure differential which may be dissipated without obtaining flashing in the expansion valve.

The pressure equivalent of subcooling depends upon the condensing temperature and the number of degrees of subcooling. Figure 5-6 shows this relationship in terms of feet of head. If a vertical rise exists between the condenser and the expansion valve, this must be subtracted from the equivalent rise to give the net head available for pipe friction loss.

The relationship between subcooling and head is sufficiently near linear that only small error is obtained by assuming linearity, that is:

$$\Delta H = A \, \Delta t_{sub}$$

at any given condensing temperature.

Also:

$$\Delta H = \frac{\Delta P}{\rho} = \frac{3.122 \, L_e \, r^{1.772} \, T^{1.772} \mu^{0.228}}{\rho^2 \, d^{4.772}}$$

so that:

$$d^{4.772} = \frac{3.122 \, L_e \, r^{1.772} \, T^{1.772} \mu^{0.228}}{\Delta H \rho^2}$$

$$= \phi \, \frac{L_e \, T^{1.772}}{\Delta H}$$

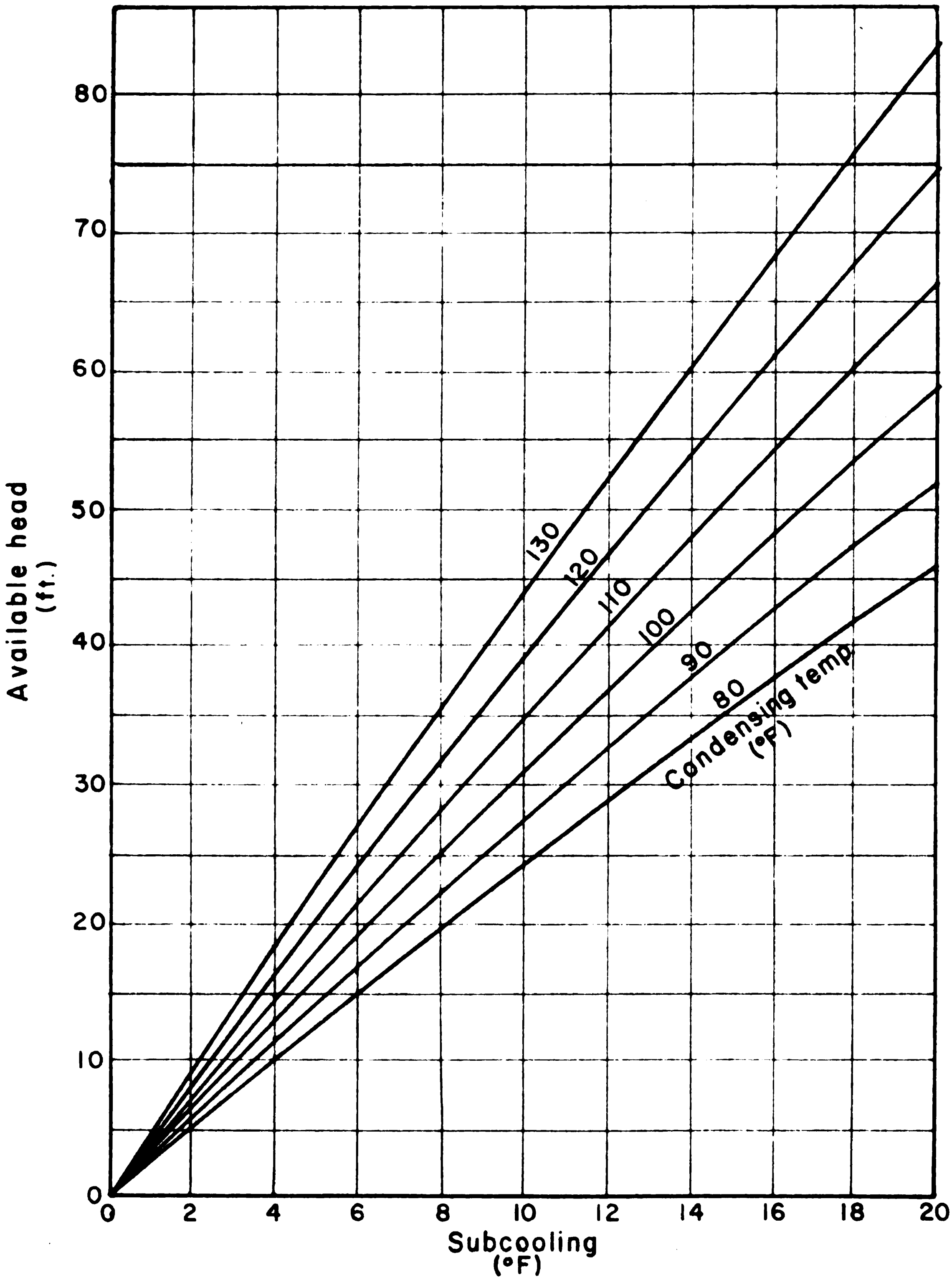

FIGURE 5-6

Head equivalent of subcooling

The values of Φ for various operating conditions are shown in Table 5-5.

Figure 5-7 shows the solution of the equation.

It should be noted that undersizing of the line will produce flashing at the expansion valve. Thus it is advisable to choose the size above the solution if the solution falls between two nominal sizes.

TABLE 5-5

$\phi(t)$ for Liquid Lines

Suction Temp. (°F)	Condensing Temp. (°F)	$\phi(t)$
-40	80	4.046×10^{-4}
	90	4.523
	100	4.692
	110	4.895
	120	5.125
	130	6.826
-20	80	3.731×10^{-4}
	90	4.154
	100	4.291
	110	4.458
	120	4.638
	130	6.044
0	80	3.463×10^{-4}
	90	3.842
	100	3.945
	110	4.091
	120	4.219
	130	5.620
20	80	3.210×10^{-4}
	90	3.550
	100	3.639
	110	3.749
	120	3.863
	130	4.972
40	80	2.999×10^{-4}
	90	3.307
	100	3.380
	110	3.469
	120	3.562
	130	4.643

TABLE 5-5

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	120	3.863
	130	4.972
40	80	2.999×10^{-4}
	90	3.307
	100	3.380
	110	3.469
	120	3.562
	130	4.643


80
100

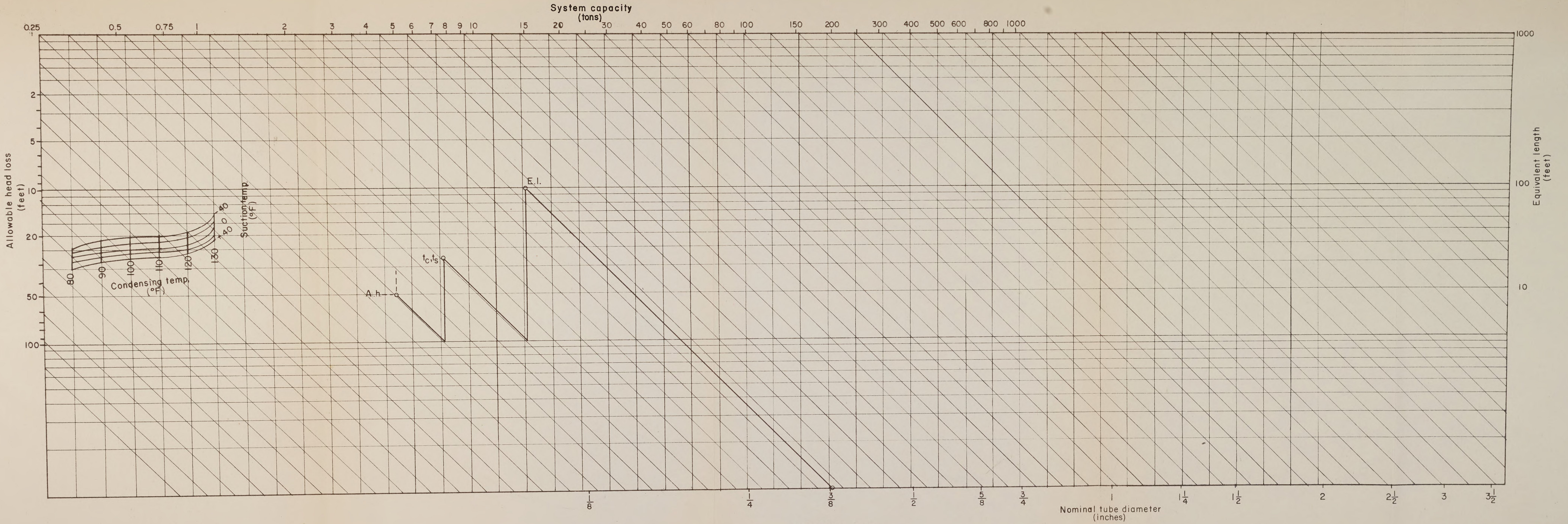

FIGURE 5-7
Liquid line sizing

Chapter VI: Tubular Heat Exchanger

The determination of the length, diameter, and number of tubes for a tubular heat exchanger is somewhat more complex than problems previously discussed, particularly from the difficulty in assessing owning costs.

Assume that the cost of the shell and tubing can be expressed as:

$$C_1 = A_1 NLd$$

and that the cost of making the end connections can be expressed as:

$$C_2 = A_2 N$$

The cost of friction is the same as in previous piping problems, assuming copper tubing, except that the number of tubes is not necessarily one.

As before (see Pages 51-2):

$$W_k = \frac{fLV^2}{2gD} \text{ ft-lb/lb}$$

$$= \frac{fLW_f V^2}{2gD} \text{ ft-lb/hr}$$

$$= \frac{(0.0653) \quad LW_f V^2}{(Re)^{0.228} \quad 2gD}$$

$$= \frac{(0.0653)\mu^{0.228} \, L \, W_f \, V^{1.772}}{2g \, (3600)^{0.228} \pi^{1.772} \rho^{0.228} \, D^{1.228}}$$

but $$V = \frac{W_f}{3600 \rho A} = \frac{W_f \quad 4}{3600 \rho N \pi D^2}$$

Substituting this value of V

$$W_k = \frac{(0.0653)\mu^{0.228} \, L \, W_f^{2.772} \, (4)^{1.772}}{(2g)(3600)^2 \pi^{1.772} \rho^2 \, N^{1.772} \, D^{4.772}}$$

Thus:

$$C_{wk} = \frac{(0.0653)(4)^{1.772}(8760)(12)^{4.772}\mu^{0.228} L W_f^{2.772}}{\eta\,(2g)(3600)^2(3413)(778)\pi^{1.776}\rho^2\, N^{1.772}\, d^{4.772}}$$

For the sake of generality, let:

$$C_{wk} = \phi_1 \frac{L}{d^{4.772} N^{1.772}}$$

Thus, the total cost may be expressed as:

$$C_t = A_1 NLd + A_2 N + \phi_1 \frac{L}{d^{4.772} N^{1.772}} \qquad (6\text{-}1)$$

Two constraints apply to the system. First, all of the variables must have values greater than zero. Second, the heat balance must be satisfied. Writing the heat balance:

$$W_f C \Delta t = UA\Delta t_m = \frac{UN\pi dL}{12} \Delta t_m \qquad (6\text{-}2)$$

where:

W_f = weight of flow, #/hr.

C = specific heat, BTU/lb-°

Δt = temperature change of fluid, °F

U = overall coefficient of heat transfer, BTU/hr-ft^2-°F

Δt_m = logarithmic mean temperature difference, °F.

It is assumed that the ratio U/h_i, where h_i is the heat transfer coefficient at the interior surface of the tubes, can be approximated from knowledge of similar systems, and U is the overall coefficient, defined as:

$$U = \frac{1}{\text{resistances}}$$

U can then be expressed in terms of N, L, and d as follows:

$$U = \frac{U}{h_i} h_i = \frac{U}{h_i}\ 0.023 \frac{K}{D} \left[Re\right]^{0.8} \left[Pr\right]^{0.4}$$

where:

Re = Reynolds Number = $\frac{DV\rho}{\mu}$, dimensionless

Pr = Prandtl Number = $\frac{C\mu}{K}$, dimensionless

Thus:

$$U = \frac{U}{h_1}(0.023)\left[\frac{K}{D}\right]\left[\frac{D^{0.8}V^{0.8}\rho^{0.8}}{\mu^{0.8}}\right]\left[\frac{C^{0.4}\mu^{0.4}}{K^{0.4}}\right]$$

$$= \left[\frac{U}{h_1}(0.023)\frac{K^{0.6}\rho^{0.8}C^{0.4}}{\mu^{0.4}}\right]\left[\frac{V^{0.8}}{D^{0.2}}\right]$$

where:

K = conductivity of fluid, BTU/hr-ft^2-$^\circ$F/ft

C = specific heat, BTU/lb-$^\circ$F

ρ = density, lb/ft^3

V = velocity, ft/hr

μ = viscosity, lb/ft-hr

D = diameter, ft.

but:

$$V = \frac{W_f}{\rho A} = \frac{W_f\ 4}{\rho N \pi D^2} = \frac{W_f(576)}{\rho N \pi d^2}$$

and:

$$U = \frac{U}{h_1}\ \frac{(0.03)(12)^{0.2}(4)^{0.8}C^{0.4}K^{0.6}W_f^{0.8}}{(3600)^{0.8}(25)^{0.8}\pi^{0.8}\ N^{0.8}d^{1.8}\mu^{0.4}}$$

Substituting for U in Equation 6-2 and simplifying:

$$\frac{N^{0.2}\ L}{d^{0.8}} = \phi_2$$

This is the form of the second constraint.

Using Lagrange's method, as given in Chapter II:

$$\left(\frac{\partial F}{\partial N}\right)_{L_1 d} = A_1Ld + A_2 - \frac{1.772\ \phi_1 L}{N^{2.772}d^{4.772}} + \frac{0.2\ \gamma L}{N^{0.8}d^{0.8}} = 0$$

$$(1)\quad A_1N^{2.772}L\,d^{5.776} + A_2N^{2.772}d^{4.772} - 1.772\phi_1 L\; 0.2\gamma N^{1.972}Ld^{3.972} = 0$$

$$\left(\frac{\partial F}{\partial L}\right)_{N_1 d} = A_1Nd + \frac{d_1}{N^{1.772}d^{4.772}} + \frac{\gamma N^{0.2}}{d^{0.8}} = 0$$

$$(2)\quad A_1N^{2.772}d^{5.772} + \phi_1 + \gamma N^{1.972}d^{3.972} = 0$$

$$\left(\frac{\partial F}{\partial d}\right)_{N_1 L} = A_1NL - \frac{4.772\,\phi_1 L}{N^{1.772}d^{5.772}} - \frac{0.8\,\gamma LN^{0.2}}{d^{1.8}}$$

$$(3)\quad A_1N^{2.772}d^{5.772} - 4.772\,\phi_1 - 0.8\,\gamma N^{1.972}d^{3.972} = 0$$

These three equations, and the second constraint:

$$\frac{N^{0.2}L}{d^{0.8}} = \phi_2$$

form a system of four simultaneous equations in N, L, d, and γ. Combining (2) and (3):

$$\phi_1 + \gamma N^{1.972}d^{3.972} = -4.772\,\phi_1 - 0.8N^{1.972}d^{3.972}$$

$$1.8\,\gamma\, N^{1.972}d^{3.772} = -5.772\,\phi_1$$

$$\gamma = \frac{-3.267\,\phi_1}{N^{1.972}d^{3.972}}$$

Substituting this value in (2):

$$A_1N^{2.772}d^{5.772} + \phi_1 - 3.267\,\phi_1 = 0$$

$$N^{2.772}d^{5.772} = 2.267\,\frac{\phi_1}{A_1}$$

Substituting this expression in (1):

$$2.267\,\phi_1 L + 2.267\,\frac{\phi_1 A_2}{dA_1} - 1.772\phi_1 L + (0.2)(-3.267\phi_1)L = 0$$

$$-0.158\,\phi_1 L + 2.267\,\frac{\phi_1\; A_2}{d\; A_1} = 0$$

$$Ld = \frac{2.267\; A_2}{0.153\; A_1} = 14.348\,\frac{A_2}{A_1}$$

Combining this equation, and the constraint:

$$L = \phi_2 \frac{d^{0.8}}{N^{0.2}}$$

$$Ld = 14.348 \frac{A_2}{A_1} = \phi_2 \frac{d^{1.8}}{N^{0.2}}$$

$$N^{0.2} = \frac{\phi_2 A_1}{(14.348) A_2} \quad d^{1.8}$$

$$N = \left(\frac{\phi_2 A_1}{14.348 \ A_2} \right)^5 \quad d^9$$

But:

$$N^{2.772} d^{5.772} = 2.267 \frac{\phi_1}{A_1}$$

$$d^{5.772} = \frac{2.267 \ \phi_1}{A_1 N^{2.772}}$$

$$= \frac{2.267 \ \phi_1}{A_1} \left[\frac{14.348 \ A^2}{\phi_2 A_1} \right]^{13.86} \frac{1}{d^{24.948}}$$

$$d^{30.72} = \frac{2.267 \ \phi_1}{A_1} \left[\frac{14.348 \ A_2}{\phi_2 A_1} \right]^{13.86}$$

$$d = \frac{(2.267)^{0.0326} \ \phi_1^{\ 0.0326}}{A_1^{\ 0.0326}} \left[\frac{19.348 \ A_2}{2 A_1} \right]^{0.4512}$$

$$= \frac{3.416 \ \phi_1^{\ 0.0326} \ A_2^{\ 0.4512}}{A_1^{\ 0.4838} \ \phi_2^{\ 0.4512}}$$

$$L = \frac{14.348 \ A_2}{A_1 d}$$

and

$$N = \left(\frac{\phi_2 A_1}{14.348 \ A_2} \right)^5 \quad d^9$$

$$= 1.650 \times 10^{-6} \ \frac{\phi_2^{\ 5} \ A_1^{\ 5}}{A_2^{\ 5}} \quad d^9$$

These are left in this form by intent, since the solution for d will not necessarily yield a nominal diameter. Thus, the nominal diameter nearest the ideal should be used in the equations for L and N.

In order to examine these results, consider as an example the following:

Water at an average temperature of 60°F.

$W_f = 3600$ lb/hr.

$\frac{\Delta t}{\Delta t_m} = 1$ $\qquad\qquad$ $\eta = 0.8$

$F_u = 0.8$

$A_1 = 2$(tube cost) $\cong$ 13.72¢/in-ft-year

$A_2 = 1$ cent/tube-year

$\frac{h_i}{U} = 2$ ft^{-1}

B = 2¢/Kw-hr

From the previous chapter:

$$\phi_1 = .0653 \frac{(8760)(4)^{1.772}(12)^{4.772} \, BF_u \, \mu^{0.228} W_f^{2.772}}{(3600)^2 \, (3413)(778)(2g)\pi^{1.772} \eta \rho^2}$$

$$= 3.652 \times 10^{-8} \frac{B \, F_u \mu^{0.228} \, W_f^{2.772}}{\eta \rho^2}$$

and

$$\phi_2 = \frac{(3600)^{0.8}(25)^{0.8}\pi^{0.8} \, c^{0.6}\mu^{0.4} \, W_f^{0.2}}{(0.023)(12)^{0.2} \, (4)^{0.8} \, K^{0.6}} \; \frac{h_i}{U} \; \frac{\Delta t}{\Delta t_m}$$

$$= 1.563 \; \frac{c^{0.6} \mu^{0.4}}{K^{0.6}} \; W_f^{0.2} \; \frac{h_i}{U} \; \frac{\Delta t}{\Delta t_m}$$

For water at 60^oF:

$\mu = 2.71$ lb/ft-hr

$\rho = 64.34$ lb/ft^3

$K = 0.344$ BTU/hr-ft^2-^oF/ft

$C = 1.000$ BTU/lb-^oF

For these values:

$\phi_1 = 0.160$

$\phi_2 = 45.43$

$$d = \frac{(3.416)(0.160)^{0.0326}(1)^{0.4512}}{(13.72)^{0.4838}\ (45.43)^{0.4512}}$$

$= 0.1804$ inches

This is smaller than commercially available 1/8 inch nominal copper tubing, which has, for type K, an inside diameter of 0.185 inches. If 1/8 inch were used:

$$L = \frac{(14.348)(1)}{(13.72)(0.186)} = 5.653 \text{ ft.}$$

and

$$N = \frac{(1.65 \times 10^{-6})(45.43)^5(13.72)^5}{(1)^5}\ (0.186)^9$$

$= 42$ tubes

Assuming the same conditions, with

$W_f = 360{,}000$ lb/hr

$\phi_1 = 5.589 \times 10^4$

$\phi_2 = 114.1$

$$d = \frac{(3.416)(8.559 \times 10^5)^{0.0326}\ (1)^{0.4512}}{(13.72)^{0.4838}\ (114.1)^{0.4512}}$$

$= 0.1805"$

This is almost identical to the previous result. Thus L would have the same value, and:

$$N = \left[\frac{(114.1)(13.72)}{(14.348)(1)}\right]^5 (0.185)^9$$

$$= (109.1)^5 (0.185)^9$$

$$= 4{,}120 \text{ tubes}$$

A check of Reynold's numbers indicates, for the first case: Re = 2644, and for the second case: Re = 2693. For both cases, the flow should be laminar, rendering the use of the McAdams equation for heat transfer invalid. In addition, there has been little study of heat transfer in the diameter range encountered here, so that the validity may also be challenged on that count.

The assumption was made that $\frac{h_i}{U}$ may be approximated in liquid-liquid heat exchangers, using larger tubes. In the case of liquid-gas the ratio would be of the order of 100. The effect of a change to this ratio would be to reduce d to approximately one seventh of the values found for liquid-liquid for the same rate of flow. The number of tubes would be increased by a factor of approximately $(\frac{1}{7})^9 (50)^5$, or $7\frac{1}{2}$. The length would be increased by about 7 to 1.

Doubt as to the validity of the assumption of $\frac{h_i}{U}$ notwithstanding, the trend of solutions is obviously toward a large number of very small tubes. Since the problem of support of such tubes is a difficult one, the compact heat exchanger composed of expanded and/or corrugated plates appears to offer a step in the right direction.

Kays and London* have performed studies of compact exchangers, but the tubes used were fairly large compared to the solutions found in the examples previously offered in this chapter.

It is obvious that, until a general equation for corrective heat transfer is evolved, there can be no general solution for the economic configuration of heat exchangers. In each category, the empirical equation which most nearly fits the situation should be used to evaluate the expression:

$$U = \frac{U}{h_i} \quad h_i(N,L,d)$$

The solution for a minimum cost can then be carried out by the methods of this chapter.

---

* Compact Heat Exchangers, W.M. Kays and A.L. London, National Press, Palo Alto, Calif., 1955.

Chapter VII Summary

The preceeding work has opened many opportunities for further study and re-examination of proper choices in such components as ducts, piping, etc. The methods developed are not limited to the problems discussed, but have widespread application in the field of engineering economics. It is hoped that the methods are so presented as to enable engineers to make use of them in their own applications.

A summary of the results is included in this chapter.

(a) Effect of choice other than economic size.

The general behavior of the total cost with the primary variable is easy to deduce from the cost equation in the case of the single variable.

In the case of condenser water, the variable appears as Δt and $\left(\frac{1}{\Delta t}\right)$, so that the variation of cost for choices above and below the optimum will be lines with equal slopes on logarithmic paper. Thus the minimum would not be expected to be sensitive to changes in the variable. Figure 4-3 shows a typical cost curve for this type equation.

The form of the cost equation for planar insulation is of the same form in the variable, thickness, so similar results would be expected. However, for cylindrical insulation, the first term involves the variable to the second power and the second term involves the first power and the logarithm. The generalized equation for the variable part of the cost is:

$$C_{T_V} = \phi_1 \, d_o^2 + \phi_2 \quad \frac{d_o}{C \;\; d_o \ln d_o}$$

The latter term varies approximately as the reciprocal logarithm, so that it would not be sensitive. In this case, oversizing would be more expensive than undersizing, since the first term is in the square of the variable.

Flow problems may be reduced to an equation of the type:

$$C_T = \phi_1 d + \frac{\phi_2}{d^{4.8}}$$

Oversizing will approach a first power increase but undersizing will approach approximately a 4.8 power, or very steep, increase. Figure 5-2 shows a typical variation with an equation of this type.

With a four-dimensional equation such as that for a heat exchanger, it is virtually impossible to determine by inspection the effect of varying any one of the variables, since, in this case, they are not independent. It would be necessary to investigate each by successive trials.

(b) Comparison with published tables.

This is not meant to be a reflection on any individual or individuals, since the tables to which referred are of unknown origin. Many tables have been reproduced so often as to lose all reference to their origin.

Each of the situations discussed previously will be compared to available data.

(1) Insulation.

No tables have been encountered covering economic thickness of insulation. A nomograph for the solution of economic pipe insulation was found *. The form of equation used is unknown, and comparisons have not been made.

(2) Air duct sizing.

There are two possible bases of comparison: pressure loss and velocity.

For the parameters used in the example on Figure 4-4, the following holds:

CFM	Diameter (inches)	Velocity (ft/min.)	Friction loss (in.water/100 ft)
300	9.5	608	0.13
3000	27	754	0.028
10000	53	653	0.008

These velocities are within the ranges recommended for residences.**

(3) Condenser Water.

No tables or charts comparable to that developed have been encountered. However, the equation used is similar to that of Jordan and Priester,*** and gives comparable results.

* Fundamentals of Power Plant Engineering, George E. Remp, National Press, Millbrae, Calif., 1949, p. 197

** ASHAE Guide, 1956, p. 747 and 735.

*** Refrigeration and Air Conditioning, R.C. Jordan and G.B. Priester, Prentice-Hall, New York, 1948, P 244-5.

(4) Water Piping.

Perry* gives a nomograph for generalized flow, derived from a more simplified equation. The results are near those obtained from Figure 5-1, but not so accurate.

(5) Discharge Lines.

Using Figure 5-4, and the parameters used in the example shown, the following is a comparison, for a condensing temperature of 105°F, with Table A.26, Jordan and Priester**:

Line Size (Nominal type K)	Tons Capacity (Fig. 5-4)	Tons Capacity (Jordan & Priester)
3/8	1.3	---
1/2	2.2	1.43
3/4	5.1	2.97
1	8.0	5.05
1 1/4	14.9	7.72
1 1/2	22	10.92
2	39	19.2
2 1/2	61	32.2
3	93	51.5
3 1/2	135	72.0
4	180	95.8

The higher capacity shown from Figure 5-4 may be partially explained by the fact that the additional work imposed upon the compressor is such a small fraction

* Chemical Engineers Handbook, John H. Perry, McGraw-Hill, New York, 1950, pp 384-6.

** Refrigeration and Air Conditioning, R. C. Jordan and G.B. Priester, Prentice-Hall, New York, 1943, p. 491.

of the total work as to be of minor importance compared to the cost of the tubing.

The increased cost of pipe compared to the relatively stable cost of electricity may also be one contributing factor to the increase in apparent capacity.

It should be noted that the cost of electricity used was 2¢/KW-hr. If electricity is more costly, these capacity values will be reduced, and vice versa.

(6) Suction Lines.

Comparing the results of Figure 5-5 with the same table:

Tube Size (nominal)	Tons Capacity (105° Condensing 40° Suction)	Pressure drop per 100 ft.(psi) for comparable capacity, Jordan & Priester
3/8	0.27	2
1/2	0.45	2
3/4	1.0	2
1	1.65	1
1 1/4	3.0	0.6
1 1/2	4.4	0.8
2	7.7	0.7
2 1/2	13	0.6
3	18	<0.5
3 1/2	26	<0.5
4	36	0.6

Thus, the results are comparable. However, for low suction temperatures, the secondary effect considered herein leads to larger sizes. Table 12-5 of Jordan and Priester give factors by which to multiply capacities at + 40° suction to correct for other suction temperatures.

An approximate comparison is shown below:

t_s (^oF)	f (Fig. 5-5)	f (Jordan & Priester)
40	1.00	1.00
20	0.68	0.86
0	0.5	0.74
-20	0.26	0.56
-40	0.16	--

Even though the cost of pipe has increased, consideration of the secondary effect tends to lead to larger pipe sizes.

(7) Liquid Lines.

Since this solution is not an economic one, comparison is only useful to verify validity.

(8) Heat Exchangers.

No data were encountered with which to compare results.

It is important to note that "rules of thumb" should be avoided in choosing piping or tubing sizes. The solutions for economic size are so readily carried out that there should be no good reason for developing any such rules.

(c) Future Work.

In order to utilize this presentation effectively, accurate costs are required. More consideration should be given to this factor than most organizations expend.

The problems associated with flow seem to have been well explored, so that no further work appears necessary in this field.

The problem of heat exchange has by no means been solved, but only an approach to the problem offered. The results of this preliminary study tend to indicate a necessity for considerable study in the realm of very small passages, tubular and otherwise, both with respect to friction and heat exchange.

In addition to the applications considered in this treatise, there are many applications of the graphical methods to solution of equations, as well as the methods of minimizing equations, to other fields of interest. It is hoped that these methods, and their presentation herein, will prove valuable to engineers in all fields, as well as lead to better design for economy in many systems.

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