AN ECONOMETRIC APPROACH TO TECHNOLOGICAL CHANGE AND RETURNS TO SCALE IN STEAM-ELECTRIC GENERATION

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This is to certify that the

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ABSTRACT

AN ECONOMETRIC APPROACH TO TECHNOLOGICAL CHANGE AND RETURNS TO SCALE IN STEAM-ELECTRIC GENERATION

by

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Three null hypotheses were tested concerning steam-electric generation at the plant level, over the period 1948 to 1965. These null hypotheses are that:

(1) Steam-electric generation is subject to constant returns to scale.

(2) Technological change in steam-electric generation did not significantly affect plant efficiency between 1948 and 1965.

(3) The relationship between plant output of electricity and inputs of capital, fuel, and labor is independent of the number of turbogenerator units per plant.

These null hypotheses were tested using a three-stage procedure. First, an appropriate model of the generating plant was selected from a number of alternative models, on the basis of freedom from specification error. Second, using an independent data sample, the appropriateness of the chosen

model was confirmed. Third, the null hypotheses were tested using estimates of the parameters of the chosen model.

The most appropriate model was found to be a fixed-relative-factor-proportions model with output exogenously determined. The generating process was found to be subject to increasing returns to scale. Technological change did not significantly improve plant efficiency, except insofar as it made scale increases possible. Ceteris paribus, increases in the number of turbogenerator units per plant were found to necessitate increases in total installed plant capacity for a given level of output.

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by

Jeffrey A. Roth

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"All, all are gone, and the temples, libraries,
And schools of Castalia are no more. At rest
Amid the ruins, the glass beads in his hand,
Those hieroglyphs once so significant
That now are only colored bits of glass,
He lets them roll until their force is spent,
And silently they vanish in the sand."

--Hermann Hesse,
"The Last Glass Bead Game Player"

At various points in the dissertation process, the student-apprentice may find himself identifying with Sisyphus, the mythical king of Corinth whose sentence was a life of pushing a rock to the top of a hill, only to watch it roll back to the bottom. An important distinction is that while Sisyphus was left to his perpetual task in solitude, the thesis writer is blessed with the support--technical, financial, and spiritual--of others around him.

In particular, I wish to acknowledge the efforts of my guidance committee in my behalf. Professor Bruce Allen graciously accomodated his schedule to my needs, in order to provide helpful comments concerning exposition and the reasons for professional interest in my results. Professor Jan Kmenta initially stimulated my interest in econometrics and has provided helpful advice and encouragement throughout my entire graduate career. More attention to his suggestions at an early stage of the dissertation process would have saved me much unnecessary grief. My chairman, Professor James Ramsey, made me aware of many of the difficulties and limitations of econometric research. Furthermore, he helped me to discover the need for advance planning in a research project. The final product benefitted greatly from the comments of all three men.

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TABLE OF CONTENTS

ACKNOWLEDGMENTS ii

LIST OF TABLES .viii

LIST OF FIGURES ix

CHAPTER

I. INTRODUCTION

A. Scale, Technology, and the Thermal Electric Production Function 1

B. Existing Literature on Thermal Electric Production and Cost Functions 8

1. Direct Methods 9

2. Profit-Maximization Models with Factor Substitution 16

3. Non-Substitution Models 24

4. Other Approaches 42

II. CONSIDERATIONS RELATED TO POSTWAR TECHNOLOGICAL CHANGE 48

A. The Productive Process in Electrical Generation 49

1. The Furnace 50

2. The Steam Generator 51

3. The Turbogenerator 53

4. Steam Cycles 54

B. Postwar Innovations in Steam-Electric Generation 59

1. Innovations in Combustion 59

2. Innovations in Steam Generation 62

TABLE OF CONTENTS (cont'd.)

3. Innovations in Generator Cooling 66

4. Summary of Innovations in Plant Design 68

5. The Methodology of Innovations in Plant Design 69

C. The Stratification of Generating Plants . . . 71

III. THEORETICAL STRUCTURE AND METHODOLOGY 77

A. Recent Results in Production Theory 78

1. Alternative Models of the Firm 79

2. Generalized Production Functions 84

B. Generalizations of the ZKD Results 86

1. The Generalized n-factor Cobb-Douglas Function 87

2. The Generalized n-factor CES Function . . 91

C. The Alternative Models to Be Considered . . . 95

1. Applications of the Generalized ZKD Model 95

a. Cobb-Douglas 95

b. CES 96

c. GPF's based on the Cobb-Douglas and CES functions 98

2. Models Used by Previous Researchers . . . 105

a. The Hart and Chawla Model 105

b. A Modification of the Ling Model . . . 109

3. Models Involving the Number of Units per plant 110

D. Methodology of the Present Study 114

1. Data for the Present Study 114

2. Stage One--Restriction of the Maintained Hypothesis 116

3. Stage Two--Confirmation of the Restricted Maintained Hypothesis . . . 122

4. Stage Three--Inference Concerning Scale, Technology, and the Number of Units 125

IV. EMPIRICAL RESULTS

A. Stage One--Choice of a Model 133

B. Stage Two--Confirmation of the Chosen Model 144

C. Stage Three--Results Concerning Scale, Technology, and Machine Mix 149

1. Scale Effects 152

2. Effects of Technological Change 154

3. Significance of the Number of Units per Plant 161

4. Consistency of the Present Results with Previous Results 165

V. CONCLUSIONS FROM THIS STUDY

A. Summary of the Thesis 171

B. Implications of the Results 175

C. Limitations of the Study and Suggestions for Future Research 177

1. Procedural Limitations 177

2. Limitations of Scope 179

3. Suggested Further Research 182

APPENDICES

I. APPENDIX A: RESULTS OF STAGE ONE 185

II. APPENDIX B: THE RESULTS OF STAGE TWO 193

III. APPENDIX C: DATA USED IN THE STUDY 196

TABLE OF CONTENTS (cont'd.)

IV. APPENDIX D: SUMMARY OF MODELS CONSIDERED 226

BIBLIOGRAPHY 229

LIST OF TABLES

Table

1. Coal Furnace Type Planned in Samples of Plants Under Construction 61

2. Number of Turbine Bleed points for Samples of Units Under Construction 61

3. Generator Cooling Method in Sample of Units Under Construction 68

4. Definition of Cells for Classification of Generating Plants 73

5. Correlation Coefficients Among Factors, by Cell 173

6. Estimation Results for LEHCC/N, LEHCF/N, and LEHCL/N 150

7. Results of Tests of the Constant Returns to Scale Hypothesis 153

8. Results of Tests for Equality of Γ_2 across Cells 156

9. Results of Tests for Equality of Γ_1 across Cells 160

10. Results of Tests that $\delta = 0$ 162

11. Results of Stage One 186

12. Results of Stage Two 194

13. List of Plants in Sample, by File Number 200

14. Data 204

15. Alternative Models of the Steam-Electric Generating Plant 226

LIST OF FIGURES

Figure

1. Schematic Diagram of Steam-Electric Generating Plant 49

2. Schematic Diagram of a Watertube Boiler 52

3. Graphic Representation of the Rankine Steam Cycle 55

4. Graphic Representation of the Reheat-Regenerative Steam Cycle 57

Chapter I. Introduction

This chapter has two major objectives. In Section A., the problem to be considered will be defined and its methodological and empirical importance examined. In Section B., a critical discussion of previous studies of the present problem and related issues will be presented.

I.A. Scale, Technology, and the Thermal Electric Production Function

In recent decades there have been numerous innovations in fossil-fueled steam-electric generation. Examples, to be discussed in detail in Chapter II.A., include additional reheat and regenerative preheat cycles, hollow-conductor generator cooling systems, and use of steam in its supercritical state. Simultaneously, the size of turbogenerator units, as measured by output rating of the generator, has been increasing. For example, according to the Modern Plant Design Surveys compiled by Power magazine, the capacity of the largest central-station turbogenerator unit under construction increased from 150 electrical megawatts (mw) in 1949 to 715 mw in 1965.[1] Thus, the possible causes of any observed improvement in the efficiency of the generation

[1] These Surveys appear on a fairly regular annual basis in special issues of Power magazine. They contain detailed technical information on the characteristics of turbogenerator units under construction at the time of publication.

process include both technological change and changes in scale of plant. The problem addressed in this dissertation is to estimate the separate effects on the steam-electric generation process of changes in technology and scale. This will be done in the following manner.

First, technologically homogeneous populations of turbogenerator units will be defined. Then, using sample production data from each population, the production function for each population will be estimated. By comparing the estimated efficiency parameters of the production functions across populations and testing the hypothesis of constant returns to scale within each population, one has a means of examining the effects of changes in scale and technology on the production function for electricity. The problem is of interest for two reasons, one of which may be classified as "applied," the other as "methodological."

An example of an applied problem stems from a controversy in public utility regulation related to the "fair rate of return principle." A "fair return" has been defined as the "entire excess in operating revenues, over and above current operating deductions, for which a commission will make provision in a rate case as a component of the company's annual revenue requirements."[2] Furthermore, ". . . the allowed-for return is arrived at as a multiple of two factors:

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[2]J. C. Bonbright, Principles of Public Utility Rates (New York: Columbia University Press, 1961), 149.

the rate base, and the . . . 'fair' rate of return thereon. The rate base . . . represents the total quantum of invested capital. . . ."[3]

Briefly, the controversy centers on whether regulation under this principle leads to inefficiency in the regulated industry. Two different arguments have been advanced to show that this is the case. The first is that regulation according to the fair return principle "creates an environment in which incompetence is rewarded and efficiency is penalized because the determination of total revenue requirements on a cost-plus basis assures the company that all expenses will be covered, while at the same time eliminating the possibility that any gains from greater productivity can be retained."[4] Statements alleging that this is in fact the case have been made by economists J. S. Bain, G. C. Means, and H. M. Trebing, and, perhaps not unexpectedly, by American Telephone and Telegraph.[5]

In contrast to this argument that regulation alters the regulated firm's incentive to maximize profits, Averch

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[3]Bonbright, op. cit., 149-150.

[4]H. M. Trebing, "Toward an Incentive System of Regulation," Public Utilities Fortnightly, 72 (July 18, 1963), 22.

[5]J. S. Bain, Industrial Organization (New York: Wiley, 1959), 599; G. C. Means, Pricing Power and the Public Interest (New York: Harper, 1962), 191; H. M. Trebing, "A Critique of the Planning Function in Regulation," Public Utilities Fortnightly, 79 (March 16, 1967), 26ff; American Telephone and Telegraph Co., Profits, Performance, and Progress (1959), 91.

and Johnson[6] concluded that even a profit-maximizing firm, regulated to earn a fair rate of return on capital, has an incentive to adjust to the regulatory constraint by the uneconomic substitution of capital for other factors, so that factors are not combined optimally from the social point of view. Thus, the argument that regulation leads to allocative inefficiency has been made both with and without reference to the assumption of profit-maximization.

On the other hand, J. C. Bonbright claims at least a plausible case for the thesis that "what has saved regulation from being a critical influence in the direction of mediocrity and tardy technological progress has been its very 'deficiencies' in the form of regulatory lags and . . . acquiescence by commissions in fairly prolonged periods of theoretically 'excessive' earnings on the part of companies."[7]

Definitive empirical resolution of this controversy is of course difficult since one cannot observe what a regulated utility's record of innovation would have been in the absence of regulation during a given period. But empirical observation of a series of innovations that did in fact improve the efficiency of steam-electric generation certainly strengthens the presumption against the thesis that regulation stifles technological progress. On the other hand, if one observes

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[6]H. Averch and L. L. Johnson, "Behavior of the Firm under Regulatory Constraints," American Economic Review 52 (Dec. 1962), 1053-1069.

[7]Bonbright, op. cit., 262.

a long series of capital-embodied innovations, each the result of expensive research and development, without a corresponding improvement in generating efficiency, the credibility of the Averch-Johnson hypothesis is strengthened.

A study of the profitability of various innovations by electric utilities is beyond the scope of the present study. However, comparison of the parameters of the production function across cells defined by the innovations embodied in the members of those cells should give a tentative indication whether or not these innovations have contributed to efficiency. This comparison will be made as a part of the present study.

The application in this study of three fairly recent theoretical developments lends methodological interest to the work. The first, a set of tests developed by J. B. Ramsey for specification error in least squares regression analysis, is useful in the problem of choice between alternative forms of a given maintained hypothesis.[8] One may apply the tests to a regression estimated from sample data to determine whether that regression is subject to one or more types of specification error, such as omitted variables, incorrect functional form, simultaneous equation problems, or heteroskedasticity. Previous applications of the tests include a study of the U. S. aggregate production function

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[8] J. B. Ramsey, "Tests for Specification Error in Classical Linear Least-squares Regression Analysis," Journal of the Royal Statistical Society, Ser. B., 31 (1969), 350-371.

by Ramsey and Zarembka, a study of the demand function for money by R. F. Gilbert, a study by T. W. Murray involving international trade data, and a study of the term structure of interest rates by F. Bonello.[9]

A second recent theoretical development being applied in the present study is the notion of the generalized production function (GPF), developed by Zellner and Revankar.[10] Since one feature of the GPF is a returns-to-scale parameter which varies over the range of output, use of such a transformation of the production function should provide an improvement over previous studies of returns to scale in steam-electric generation, which were constrained by constancy of this parameter. In the review of the literature in Section B of this chapter, it will be pointed out that both Nerlove and Dhrymes and Kurz found indication of such variability.[11]

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[9]J. B. Ramsey and P. Zarembka, "Alternative Functional Forms and the Aggregate Production Function," Michigan State University Econometrics Workshop Paper #6705; R. F. Gilbert, The Demand for Money: an Analysis of Specification Error, unpublished Ph.D. dissertation, Michigan State University (1969); T. W. Murray, Data Errors and Economic Parameter Estimation: A Case Study of International Trade Data, unpublished Ph.D. dissertation, Michigan State University (1969); F. Bonello, The Term Structure of Interest Rates, the Expectations Hypothesis, and the Formulation of Expected Interest Rates, unpublished Ph.D. dissertation, Michigan State University (1968).

[10]A. Zellner and N. S. Revankar, "Generalized Production Functions," Review of Economic Studies 36 (2), (April 1969), 241-250.

[11]M. Nerlove, "Returns to Scale in Electricity Supply," ed by C. Christ, Measurement in Economics-Studies in Mathematical Economics and Econometrics in Memory of Yehuda Grunfeld (Stanford: Stanford University Press 1963) 167-198; P. J. Dhrymes and M. Kurz, "Technology and Scale in Electricity Generation," Econometrica 32 (July 1964), 287-315.

However, lacking the means to parameterize it, they were unable to test its significance. Thus, there is prior evidence that use of GPF's will be helpful in examining the steam-electric generation process. To this author's knowledge, the literature contains only two instances of estimation of a GPF, the first being a production function for the U. S. transportation equipment industry in the Zellner and Revankar paper cited above, and the second a U. S. aggregate production function in the Ramsey and Zarembka study, also cited above.

The third recent theoretical result being applied in the present study, due to Zellner, Kmenta, and Drèze (ZKD), is a production model in which ordinary least squares estimates of the parameters of a production function are shown to be consistent.[12] Since use of the specification error tests is limited to the single-equation case, the ZKD results are important to the methodology of the present study.

In turn, use of the tests will be helpful in testing the appropriateness of the ZKD model to the steam-electric generating plant. One of the underlying assumptions of this model is that output is a stochastic function of the inputs employed by a firm attempting to maximize expected profit. As will become clear in the review of the literature in Section B., this assumption is directly incompatible with the assumption, made by many previous investigators of the industry, that output of the generating plant is exogenously determined.

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[12]A. Zellner, J. Kmenta and G. Drèze, "Specification and Estimation of Cobb-Douglas Production Function Models," Econometrica 34 (October 1966) 784-795.

Comparison of the specification error test results for both types of models will assist in determining which assumption is appropriate.

In summary, it is clear that the problem addressed in the present study--separation and quantification of the effects of returns to scale and technological innovation in steam-electric generation--is of interest because of its bearing on the problem of regulation of public utilities. Furthermore, the study is of methodological interest because of the application of several recent theoretical developments to the problem.

I.B. Existing Literature on Thermal Electric Production and Cost Functions

Previous economic studies of thermal electricity generation may be classified into four groups. First, there are those in which no explicit consideration is made of optimization by the firm, so that estimation of a cost or production relation is carried out by ordinary least squares, without benefit of an explicitly formulated model. On the other hand, those studies which do consider the implications of optimization by the firm may be grouped into two sub-classes. In the first sub-class are those using models that allow for factor substitution in the neoclassical tradition, so that simultaneous equation estimation is usually required for consistency of the estimates. In the second sub-class are those in which substitution of factors is not permitted, the production coefficients being either fixed relative proportions of output or functions of variables other than factor prices or output. Estimation of these models has been carried out by ordinary least squares, using inputs as dependent variables, with output and the other variables as regressors. Finally, there are several studies which do not neatly fit into any of the first three types. An example is a study by Ling, in which a synthesis of engineering and economic principles leads to the estimation of a rather unusual cost relation.[13]

Each of these studies will be discussed in turn. Special attention will be given in the following discussion to the problem addressed, the specification of the model, the treatment of the technologically heterogeneous nature of generating plants, and the level of aggregation at which the study is carried out.

Direct Methods

The first postwar economic study of thermal electrical generation is the ordinary least squares estimation by Nordin of the relation between the total fuel cost and output as a percentage of capacity.[14] 541 observations were made on a single turbogenerator unit in 1941. Each observation consists of the approximate output and fuel input during one eight-hour shift. The functional form estimated was quadratic

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[13] S. Ling, Economies of Scale in the Steam-Electric Power Generating Industry (Amsterdam: North-Holland, 1964).

[14] J. A. Nordin, "Note on a Light Plant's Cost Curves," Econometrica 15 (3), (July, 1947), 231-235.

in output; a cubic equation was also estimated, but it did not significantly change the goodness of fit.

Two comments, the first previously pointed out by Galatin, should be made regarding this study.[15] First, Nordin has explicitly considered the instantaneous nature of the generation process, using hourly observations on output as an approximation to instantaneous output, then summing the hourly outputs to approximate the output for an eight-hour shift. Second, the absence of consideration of optimization by the firm was no doubt necessitated by the lack of appropriate econometric techniques in 1947, the year of publication. However, since the observed turbogenerator was the only one in the plant, there was no possibility of short-run reallocation of labor among machines, as would be possible in a multi-unit plant. Therefore, it seems quite plausible that no short-run optimization by the plant management was possible. Thus, this study is an example of the appropriate use of a simple estimation technique.

Another estimation of the cost function at the plant level was published by Lomax in 1952.[16] The data consisted of a cross-section of thermal plants in two regions of Great Britain, each plant having operated more than 6600 hours during a one-year period overlapping 1947-1948. A log-linear

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[15] M. Galatin, Economies of Scale and Technological Change in Thermal Power Generation (Amsterdam: North-Holland, 1968), 53.

[16] K. S. Lomax, "Cost Curves for Electricity Generation," Economica 19 (1952), 193-197.

relationship was estimated between average working costs (total cost excluding cost of land and equipment) as the dependent variable, and installed capacity and load factor as independent variables.[17] No other functional forms were considered.

The estimated results indicate that _ceteris paribus_, average costs fall as either installed capacity or load factor increases. Assuming that capital is the fixed factor, one may draw the inference that plants in the sample are experiencing decreasing average costs. However, unless one assumes that the prices of variable factors are identical for all plants, one does not know whether the source of decreasing costs lies in factor markets or the production process. Further, even if factor prices are assumed to be identical for all plants, the fact that no attempt was made to stratify observed plants on the basis of embodied technology makes it impossible to tell whether the regression results with respect to installed capacity are phenomena of scale or of technology. Presumably the larger plants employ more modern technology, so that stratification by embodied technology is necessary to determine whether, independently of technology, larger plants generate electricity at lower average cost.

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[17] Load factor is a measure of intensity of capital utilization, defined as total actual output for some period divided by the potential output if the turbogenerator had been operated at capacity during the period.

Another attempt to estimate cost functions without explicitly considering the implications of optimization was made by Johnston.[18] The study consists of time-series estimation of the relation between total working costs and output for 17 "short-run firms"--plants whose installed capacity did not change during the period of observation; and for 23 "long-period firms"--plants whose installed capacity did change during the period. Linear, quadratic, and cubic relations were estimated for all plants, the "best" form being chosen on the basis of "fit," as measured by R^2 and $\bar{R}^2$. In no case did the cubic term significantly improve the fit.

Criticisms of this work made by Galatin center on Johnston's supposed confusion about the assumptions necessary to consider the estimated relations as cost functions.[19] Johnston reasons that if a plant with variable capacity size over time is assumed to operate on the long-run total cost envelope to the short-run total cost curves, then a regression of cost on output will be an estimate of the long-run cost function for the plant. Galatin claims that this is so only if ". . . in each year of observation, the firm operates at that output having minimum average cost on the particular short-run total cost function relating to the scale of plant in that year."[20] Since that does not hold, Galatin continues,

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[18]J. Johnston, Statistical Cost Analysis (New York: McGraw-Hill, 1958) 44.74.

[19]Galatin, *op. cit.*, 54-61.

[20]*Ibid.*, 57.

". . . it is difficult to give meaning to Johnston's results. . . ."[21] Galatin's error here has a history dating back to Jacob Viner in 1931.[22] The necessary condition is in fact that the firm choose its scale of plant so that the given output cannot be produced at lower average cost by using a different scale of plant.[23] While the plausibility of this assumption for Johnston's sample is as yet unresolved, this is at least the correct necessary condition.

The same question led Galatin to question why Johnston used "a measure of output rather than capacity of plant as the independent variable for estimating capital costs" in a later part of his study.[24] Although Galatin is correct for the short run in stating that capital costs are independent of the level of plant operation, in the long run they are a function of the scale of plant, which is in turn a function of output for a cost-minimizing firm. Thus, output becomes a valid explanatory variable for capital costs. Regressing capital expenditures on installed capacity as Galatin suggests would have led Johnston into the error of regressing capacity on itself, except for the effect of variations in the price of capital and in the amount of ancillary equipment installed with new turbogenerators.

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[21]Galatin, _op. cit._, 57.

[22]Jacob Viner, "Cost Curves and Supply Curves," _Zeitschrift für Nationalekonomie_, III (1931), 23-46.

[23]See, for example, R. H. Leftwich, _The Price System and Resource Allocation_, 4th Ed. (Hinsdale, Illinois: Dryden Press, 1970), 161ff.

[24]Galatin, _op. cit._, 160.

One of Galatin's objections regarding Johnston's work does seem valid, however. Galatin correctly points out that a long-run total cost function is theoretically derived from a fixed production function.[25] Johnston's time-series includes observations on plants containing units of different vintages. The different technologies embodied in these units may change the form of the production function over time. Therefore, it may be difficult to define Johnston's regression as an estimate of the long-run cost function. Stratification of the plants into technologically homogeneous cells and estimation of a separate function for each cell would alleviate this problem.

A more fundamental question arises from Johnston's behavioral assumptions about the firms in his sample. In order to justify single-equation estimation, Johnston points out that since the firms in the study "were all generating electricity . . . at such times and in such quantities as directed by the British Central Electricity Board, . . . they were in no wise . . . adjusting output in the search for maximum profits. . . ."[26] Leaving aside the question of whether it is reasonable to assume cost-minimization by a firm that is not attempting to maximize profits, the assumption of cost minimization usually indicates the presence of a simultaneous equation problem. It may be that Johnston is

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[25]Galatin, op. cit., 57.

[26]Johnston, op. cit., 54.

using an ordinary least squares estimator where it may not be appropriate.

A second questionable technique is Johnston's reliance on significance tests and goodness-of-fit measures in his choice of functional form.[27] As explained in Chapter III.D., this approach to choosing the specific functional form of a general maintained hypothesis does not lead one to unbiased tests of hypotheses about the parameters of the relationship. A procedure less conducive to pretesting bias is first to choose a functional form using some criterion such as that discussed in Chapter III.D., basing this choice on information gained from one data sample. Then the parameters of the chosen function may be estimated using data from a second sample, drawn independently of the first.

Despite these shortcomings, the equations estimated by Johnston yielded high values of R^2. For twelve of the seventeen "short-period firms," the equation chosen on the basis of $\bar{R}$ by Johnston contained only a linear term in output, indicating a horizontal marginal cost curve. A significantly positive coefficient for an index of time suggested that such factors as plant obsolescence are shifting the cost function upward over time. Serial correlation statistics calculated for the residuals of six of the estimated equation did not indicate significant non-randomness of the disturbances.

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[27]Johnston, op. cit., 54ff.

The cost equations estimated for the 23 "long-period firms" were quite similar to those for the "short-period firms." Again, the values of R^2 were greater than .95, and the total cost functions chosen on the basis of $\overline{R}$ were predominately linear in output. Since the estimated functions had significantly positive intercepts, the indication is that long-run average cost decreases over small values of output, approaching 2 mills per kilowatt-hour asymptotically.

The fourth investigation of influences on the cost of thermal electricity generation without consideration of the problems of optimization was conducted by Iulo.[28] In this purely empirical study, the problem was to describe the relationship between average cost for a firm and various "historical," "operating," and "market" characteristics such as the size of producing units, construction cost, and consumption per residential customer. No attempt is made to explain by means of theory the relations observed in Iulo's sample and no production or cost function is estimated.

Profit-Maximization Models with Factor Substitution

The first attempt to study thermal electricity generation in a neoclassical framework was the estimation by Nerlove of a firm's cost function.[29] His estimated function was derived

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[28]W. Iulo, Electric Utilities--Costs and Performance (Pullman, Washington: Washington State University, 1961).

[29]Nerlove, *op. cit.*

from cost-minimization by a firm generating electricity according to a Cobb-Douglas production function. The study is carried out at the firm level since Nerlove is concerned with the implications of the form of the cost function for the regulation of the firm. No consideration is given to the fact that Nerlove's sample of firms operating in 1955 contains machines of different vintages which are likely to incorporate varying technologies. Thus, it is difficult to separate scale effects from the effects of technological change, particularly since a search of the engineering literature as well as evidence later collected by Dhrymes and Kurz reveals that larger units are likely to be newer and technologically more advanced.[30]

Nerlove's procedure is to solve simultaneously the three-factor Cobb-Douglas production function and the familiar marginal productivity conditions resulting from cost minimization with fixed factor prices. The reduced form thus derived yields a cost function, linear in the logarithms of output and the factor prices:

$$\text{(I-1)} \quad C + K + \frac{1}{r}Y + \frac{a_1}{r}P_1 + \frac{a_2}{r}P_2 + \frac{a_3}{r}P_3 + V$$

where C, Y, P_1, P_2, and P_3 represent logarithms of cost, output, and the prices of labor, capital, and fuel, respectively. K is the constant term and a_i, i = 1,2,3, is the

[30]Dhrymes and Kurz, op. cit.

exponent of each of the respective factors in the Cobb-Douglas function r; the returns-to-scale parameter, is equal to the sum of the a_i, i = 1,2,3. V is assumed to be distributed stochastic disturbance.

In order to incorporate the restriction that the price coefficients must sum to unity, Nerlove divides both sides of the cost function by the price of fuel, pointing out that division by either of the other prices would have been equivalent. This yields one of his models to be estimated, called Model A:

$$\text{(I-2)} \quad C - P_3 = K + \frac{1}{r}Y + \frac{a_1}{r}(P_1 - P_3) + \frac{a_2}{r}(P_2 - P_3) + V$$

where all variables have been defined above.

In order to avoid data problems associated with the price of capital, Nerlove makes the assumption that the price of capital is the same for all firms, and derives an alternative model to be estimated, Model B:

$$\text{(I-3)} \quad C = K' + \frac{1}{r}Y + \frac{a_1}{r}P_1 + \frac{a_3}{r}P_3 + V$$

where $K' = K + (a_2/r)\ P_2$.

Estimation of Models A and B by ordinary least squares gave generally plausible results, with decreasing long-run average costs indicated. Rather than merely accepting these results, Nerlove went on to plot the regression residuals, and found strong evidence that the relationship is not in fact linear in logarithms. Hypothesizing that this result may arise from an inverse relation between the degree of returns to scale and output, he re-arranged his observations by increasing order of output, divided them into quintiles, and estimated a separate cost function for each quintile. This procedure provided strong empirical evidence for the variable returns to scale hypothesis. In order to allow for this cause of non-linearity of the cost function, Nerlove added a quadratic term in Y and re-estimated Models A and B. The most striking result of this modification was an increase in the estimated returns to scale parameter for firms in the three largest size groups.[31]

Nerlove's results have two important implications for the present study. The first is the importance of analysis of residuals to the choice of one model from a number of alternatives. Nerlove's original Models A and B met the criteria employed by most other investigators of steam-electric generation, namely a high proportion of "explained" variance as measured by $\bar{R}^2$ or R^2, and significant coefficient estimates having the theoretically expected signs. Had he not employed additional criteria concerning the distribution of residuals, he would not have become aware of significant non-linearity in the cost-output relationship and taken steps to explain it theoretically. In the present study, advantage is taken of new techniques for the analysis of residuals. The second implication of Nerlove's results for the present study is strong empirical evidence of a variable returns to scale

[31]Nerlove, _op_. _cit_., 184ff.

parameter. Since publication of Nerlove's work, a means of parameterizing variable returns to scale as a function of output in a neoclassical production function has been discovered by Zellner and Revankar.[32] Use of their technique is discussed in Chapter III.A. below.

A second approach in the neo-classical tradition to the study of thermal electricity generation is a 1964 study by Dhrymes and Kurz.[33] In this article, plants were initially assumed to minimize the cost of producing an exogenously determined output, produced according to a three-factor CES production function. The sample consisted of 362 new plants constructed between 1937 and 1959, each plant being observed only once, in its first year of operation. The observations were divided into 16 cells, according to vintage of construction and size, as measured by the nameplate-rated installed capacity. A separate empirical analysis was carried out for each cell.

The first step in the empirical analysis was to estimate the input demand equations resulting from simultaneous solution of the constrained cost-minimization conditions. The explanatory power of the labor equation was quite weak, and the coefficients measuring the sensitivity of the labor input to changes in relative factor prices were for the most part statistically insignificant. Explaining this result by

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[32]Zellner and Revankar, op. cit.

[33]Dhrymes and Kurz, op. cit.

engineering features of the production process that allow one to treat the labor input as a constant proportion of output, Dhrymes and Kurz revised their production function to a mixed Leontief-CES production, given by:

$$(I\text{-}4) \quad Q = \min[g(L), A(\alpha_f F^{\beta_f} + \alpha_K K^{\beta_K})^{\frac{1}{\gamma}}]$$

where Q represents output, and L, F, and K the inputs of labor, capital, and fuel, respectively.

Cost is defined by:

$$(I\text{-}5) \quad C = p_K K + p_F F + p_L L$$

where p_K and p_F represent the prices of capital and fuel, respectively.

Minimization of cost with respect to K and F under the constraint of the production function (I-4) yields input demand functions.

The capital demand function was approximated by:

$$(I\text{-}6) \quad \log K = a_o + a_1 \frac{P_K}{P_F} + a_2 \log Q$$

and estimated by ordinary least squares. Then, as the second step of a two-step least squares procedure, the fitted values of log K obtained from estimation of equation (I-6) were substituted into the fuel demand equation, which in turn was estimated by restricted least squares, where the restriction is that $\alpha_F \alpha_K = 1$. This procedure gave estimates $\hat{\alpha}_F, \hat{\alpha}_K, \hat{\beta}_F$, and $\hat{\beta}_K$ of four of the production function parameters $\alpha_F, \alpha_K, \beta_F$ and β_K. Estimates of the remaining parameters of the production function, A and γ, were obtained by applying ordinary least squares to:

(I-7) $\log Z = A + \gamma \log Q$

where $Z = \hat{\alpha}_F F^{\hat{\beta}_F} + \hat{\alpha}_K K^{\hat{\beta}_K}$.

The results obtained appear quite satisfactory, in the sense that the proportion of variance explained by the regressors in the estimated fuel input equation is high in all cells. With one exception all estimated coefficients are significant, with the theoretically expected signs. Comparison of estimated parameters across cells yields the conclusions that technological progress has operated to increase the efficiency of electricity generation and that increasing returns to scale is the rule in the industry.

For six of the thirteen cells for which the number of observations was sufficient for estimation, the hypothesis that $\beta_K = \beta_F$ was accepted, indicating homogeneity of the production function. Further, for all but one of the remaining six cells, a simulation-estimation experiment provided strong indications that the degree of returns to scale tends to fall with increases in plant size, as measured by nameplate rating of installed capacity. Dhrymes and Kurz point out that no probability content may be attached to these results with regard to the degree of returns to scale.

Two comments are in order regarding the method used by Dhrymes and Kurz to stratify the plants contained in the sample. A two-way classification was employed, based on technological period, i.e., vintage of the plant, and on size of plant as measured by nameplate-rated capacity.[34]

In the first place, as is discussed in Chapter II, inspection of a source of information on technical specifications of turbogenerators such as the Power Modern Plant Design Surveys reveals that even among units of approximately the same size, the technology employed in units constructed during a four-to-five year period is likely to vary considerably.[35] Inferences about the impact of technology on the production function based on technologically heterogeneous cells are not likely to be as precise as they would have been had technologically homogeneous cells been employed.

In the second place, inferences based on linear regression are usually made under the maintained hypothesis that the dependent variable has a conditional mean which is some linear function of the hypothesized regressors and that its conditional distribution is normal with constant variance over the sample. Use of nameplate-rated capacity as both a criterion for stratification of observations and as the dependent variable in the capital input equation will, in general, cause violation of this maintained hypothesis. The conditional mean of the dependent variable within a cell becomes a function of the intervals chosen to define cells

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[34]Dhrymes and Kurz, op. cit., 297.

[35]See Note 1, p. 1.

rather than of the hypothesized regressors. Furthermore, the conditional distribution of the dependent variable cannot be normal since it is truncated at the end points of the cell, while the normal density function extends from $-\infty$ to $+\infty$. Thus, for purposes of inference, an alternative method of stratifying the sample would appear to be preferable.[36]

Another aspect of the work of Dhrymes and Kurz that relates to the present study is that as mentioned above they, like Nerlove, found a strong empirical suggestion that the degree of returns to scale in steam-electric generation tends to fall as output increases. However, Dhrymes and Kurz were not able to make statements with probability content about this phenomenon. Since the publication by Dhrymes and Kurz, the generalized production function developed by Zellner and Revankar has given one a means of parameterizing variability of returns to scale, so that inferences can be made about its significance.

Non-Substitution Models

The first study to make use of a non-substitution model to investigate technological change in steam-electric generation was published by Komiya.[37] His sample of 235 newly-constructed plants was divided into four vintage cells, using

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[36]For a discussion of this issue, see J. B. Ramsey, Tests for Specification Error in Classical Linear Least Squares Regression Analysis, unpublished Ph.D. dissertation, University of Wisconsin, 1968.

[37]R. Komiya, "Technological Progress and the Production Function in the United States Steam Power Industry," Review of Economics and Statistics 44:2 (May, 1962), 156-166.

vintage as a proxy for technology. Each of the cells was further subdivided by the type of fuel used--coal or non-coal. Two models were considered and estimated in this study. The first was a Cobb-Douglas type production model, in which the elasticity of substitution between any pair of three inputs--labor, capital, and fuel--is constrained to unity. This model was rejected because the estimated regression coefficients were generally insignificant and did not have the *a priori* expected signs. The conclusion was that the substitution model did not explain the input-output relationship of power generation satisfactorily, although acknowledgement was made of the possible existence of simultaneous equation problems. Some general criticisms of this use of coefficient estimates as a criterion for the choice between several alternative models are made in Chapter III.D.

Factor substitution was dismissed on the grounds of these empirical results, and attention was focused on a "limitational model," which allowed no such substitution. Following Komiya's notation, the following set of input functions was estimated:

(I-8) $\log Y_f = \log A_f + \beta_f \log X_2$

(I-9) $\log Y_c = \log A_c + \beta_c \log X_1 + \mu_c \log X_2$

(I-10) $\log Y_\ell = \log A_\ell + \beta_\ell \log X_1 + \mu_\ell \log X_2$.

The variables are defined as follows: Y_f is fuel input in Btu's when the unit is operated at capacity. Y_c is the capital cost of equipment per generating unit, measured in constant dollars. Y_ℓ is the average number of employees per generating unit, X_1 the average size of generating unit in megawatts, and X_2 the number of units in the plant. If $\beta_c = \beta_\ell = 1$ and $\mu_c = \mu_\ell = 0$, the model becomes one of fixed proportions.

Using dummy variables, various versions of this model were estimated, some allowing the slope and intercept parameters to vary across the cells in the sample, others incorporating various restrictions on these parameters across cells. Komiya's regression results indicate that with respect to both fuel and labor, economies of scale are important. The results also indicate a long-run trend of substitution of capital for labor.

Several remarks regarding Komiya's results are in order. First, as explained above in the discussion of the work by Dhrymes and Kurz, vintage is a rather poor proxy for technology since at any given period, design engineers may be experimenting with several different technologies. Second, measuring the capital input by capital cost per unit as Komiya did, it seems difficult to impute the "scale effect" with respect to capital entirely to improved efficiency of the productive process. Quantity discounts on turbogenerator units would also give rise to a decrease in per-unit capital costs as the number of machines increased.

Third, rejection of the possibility of factor substitution on the basis of the poor regression results obtained from the Cobb-Douglas model seems a bit premature. Such results could be accounted for by simultaneous equation problems, as suggested by Komiya, or by the restriction of the pair-wise elasticity of substitution to unity implicit in the use of the Cobb-Douglas function. Furthermore, Komiya's inclusion of each plant only once, when new, does not allow one to observe long-run capital-fuel substitution in a single plant, in the form of installation of technologically more advanced machines that make more efficient use of fuel. For these reasons, Komiya's results must be viewed with some reservations. These problems could be corrected by making the following procedural changes: an alternative classification scheme that more precisely reflected technological change; an alternative measure of capital input; experimentation with alternative functional forms that allow factor substitution at non-unitary elasticities of substitution; and incorporation of successive observations on plants in the sample.

Another attempt to examine changes in productivity in steam-electric generation was made by Barzel.[38] He began by stating two difficulties with estimated production functions that cast doubt on their validity in studies of technological change over time. First, one does not usually know the functional form relating inputs to output and is ordinarily compelled to derive it from the same set of data used for purposes of inference. Second, one does not know whether or not shifts in the production function are neutral. Barzel

[38] Y. Barzel, "Productivity in the Electric Power Industry, 1929-1955," Review of Economics and Statistics 44 (2), (May 1962), 156-166.

alone among previous investigators seems to have been aware of the first problem; but unfortunately, he did not have available statistical techniques to solve it correctly.

Instead, he derived an index of productivity change between year 1 and year 2, so that he might examine its path over time. The index was derived under the following assumptions: constant returns to scale, perfect competition in all markets, and constant marginal products of all inputs over the two years being compared. Barzel then tested the appropriateness of these assumptions.

His first test of the hypothesis of constant returns to scale was estimation of a Cobb-Douglas relation between output and the three inputs, labor, capital, and fuel. The sum of the estimated factor exponents was significantly less than unity indicating decreasing returns to scale. Barzel ignored this result as a "statistical twist" and attempted to measure economies of scale using the following approach.

Defining productivity as output per unit of cost, size as the installed generating capacity of each plant, and load factor as the number of Kwh produced during 1959 per installed kw capacity, Barzel estimated a log-linear relation using productivity as the dependent variable, with size and load factor as regressors. This relation may be written:

$$\text{(I-11)} \quad \frac{Q}{C} = AK^{\alpha}(LF)^{\beta},$$

where Q, C, K, and LF represent output, total cost, capital, and load factor, respectively. Barzel did not complete the stochastic specification, but one may assume a multiplicative lognormally distributed random disturbance, e^{u_i}.

Barzel's results from estimation of equation (I-11) on a 1959 cross-section of plants that first operated between 1953 and 1955 were interpreted to mean that a 10% increase in plant size causes a 1.09% increase in productivity; while a 10% increase in load factor causes a 3.73% increase in productivity. Applying these cross-section estimates to time-series observations between 1929 and 1955 on productivity as defined by Barzel, installed capacity, and load factor, Barzel accounts for the 2.83-fold increase in productivity over the period as the product of four factors: 1.48, due to the increase in quantity of electricity purchased annually per customer; 1.23, due to increased average plant size; 1.18, due to higher average load factor; and 1.32, the residual, which Barzel attributes to technological progress during the period.

Several doubts arise when one considers Barzel's methodology carefully. First, load factor is used to mean output per installed kilowatt of capacity, so that the product of load factor and capacity is output. Since the right-hand side of equation (I-11) estimated by Barzel involves that product, one sees that both sides of the equation are functions of output. Thus, the explanatory power of equation (I-11) would not appear to be great.

Second, application of the 1959 estimates of the parameters of equation (I-11) to time-series data on the variables in equation (I-11) is implausible since the true values of the parameters are likely to have varied as technology changed between 1939 and 1955. As explained in Chapter II of the present study, many of the technological innovations observed during the postwar period have operated in the direction of making larger-scale plants technologically feasible. Thus, Barzel's assumption of a stable relation between productivity and scale over more than a quarter-century seems a bit rash. By ignoring the fact that technological change has brought about the use of larger-scale plants, Barzel may have seriously underestimated its impact.

In a second paper Barzel attempted to consider the effect of scale of plant on the quantities used of each of the factors of production in steam-electric generation.[39] Assuming output to be a Cobb-Douglas type function of the inputs, utilization factor, and size of unit, Barzel showed that this formulation led to an identity relation between dependent and independent variables in the function--the identity whose existence he ignored in equation (I-11), which was used in his previous paper. Therefore, he rejected all models incorporating an explicit production function in favor of models examining the demand for factors.

The form selected for estimation of the fuel input function is given by:

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[39] Y. Barzel, "The Production Function and Technical Change in the Steam-Power Industry," Journal of Political Economy 72 (April, 1964).

(I-12) $\log y_f = \sum_i b_i \log X_i$, $i = 0, \ldots, 19$,

where y_f represents fuel input and the independent variables are defined as follows:

x_0 = unity

x_1 = plant size measured by the number of installed kilowatts

x_2 = anticipated average load of plant, measured by the observed load factor in first full year of operation

x_3 = a within-plant index of x_2 over time

x_4 = anticipated average input price ratio measured as the average price of fuel per Btu to the average labor cost per man-year in the first year of operation of the plant

x_5 = a within plant index of x_4 over time

x_6 = age of plant defined as accumulated number of hours of operation

x_7 - x_{19} define a set of dummy variables, where x_7 has a value of 10 for plants that began operation in 1943 and 1 for other years, and the other x_i are similarly defined. The reader will note that the logarithmic transformation converts these binary variables into the familiar values of 1 and 0.

Equation (I-12) was estimated using data on 220 plants first operated between 1941 and 1959. Coefficients of x_1, x_2, and x_3 are .896, .848, and .893, respectively, all significantly different from unity. These results indicate that fuel input rises less than proportionately to changes in those variables. Coefficients of the factor price ratio variables are also significant, with the theoretically expected signs. Coefficients of the year dummies, which represent the effect of technological change if all other variables affecting demand for fuel are included in equation (I-12), indicate sporadic decreases in fuel requirements over the period of the sample.

Labor input was also used in another model, with a vector of independent variables identical to that of equation (I-12). Although serious potential measurement errors are acknowledged in the values of labor input, size, anticipated load factor, and the load index coefficients, the results indicate similar but more pronounced scale and load effects than were evident in the fuel equation. Results similar to those of the fuel equation were obtained with respect to the other independent variables.

An input equation for capital, as measured by the undeflated value of plant and equipment, was also estimated. The estimated coefficients indicated the following: a size effect similar to that obtained in the other regressions, substitutability of capital for the other two factor prices, and a slight positive relation between capital input and load factor. Shifts indicated by the coefficients of the dummy variables indicate a rapid increase in capital input until 1951, with no clear pattern thereafter.

Two comments seem appropriate regarding Barzel's methodology. First, rejection of all explicit production function models because the modified Cobb-Douglas function produced an identity seems a bit premature. Although the particular function used by Barzel produced an identity, other familiar production functions are not subject to that problem.

Second, there is evidence to the effect that newer, more advanced technology makes possible the use of larger scales of plant.[40] Thus, rather than allowing technological change to shift only the intercepts of the input functions, one would be interested in exploring the interaction between technological change and the scale effect. Along the same lines, in order to separate scale and load effects from the effects of technology, one would wish to see whether the coefficients of installed capacity size and load factor vary across technologically homogeneous cells. Finally, one would be interested in seeing whether input functions of the Barzel type give evidence of variable returns to scale similar to that found by Nerlove and by Dhrymes and Kurz.

Another approach to the problem of identifying the effects of economies of scale and technological change in steam-electric generation was employed by Galatin.[41] His discussion emphasized the problems caused by aggregation of instantaneous data on individual turbogenerator units into annual data on entire plants when production is studied at the plant level. He began by developing a model of a multi-unit plant, for which the objective is to minimize the cost

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[40]Ling, op. cit., p. 20.

[41]Galatin, op. cit.

of generating some exogenously determined level of output. Economies of scale and technological change were defined in terms of this model.

Unit heat rate, a measure of efficiency defined as fuel input to a turbogenerator unit in Btu's per kilowatt of output was assumed to be a function of the capacity of the unit and the degree of capacity utilization. For empirical use, Galatin developed a measure of capacity utilization, PF*, which is essentially the published plant factor, corrected for the fact that a given machine may not operate "hot and connected to load"[42] at all times during the year. A similar correction to unit heat rate was also made.

Galatin went on to point out that the instantaneous production period for electricity may be approximated by finite intervals of length t, say one hour, while data are usually collected over longer intervals denoted by T, e.g., one year. Choosing a Cobb-Douglas function as an example, Galatin showed that when a t-period stochastic relation is aggregated into T-period intervals, ordinary least squares estimation is generally not appropriate for estimation of the T-period relation.

However, under certain assumptions, among them that all units within a given plant are the same size, ordinary least squares is appropriate for estimation of the following pair of alternative functions:

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[42]i.e., actually generating electrcity for transmission to customers.

(I-13) $a_{it} = \alpha(PF^*)^{-1}_{it} + \beta X_{iK} + \gamma + v_{it}$

(I-14) $a_{it} = \alpha(PF^*)^{-1}_{it} + \beta(X_{iK})^{-1} + \gamma + v_{it}$,

where a_{it} is Galatin's corrected unit heat rate, PF* is the corrected plant factor, and X_{iK} is the nameplate rating of the i-th turbogenerator unit.[43]

These models were estimated using a sample of 812 observations on 158 different plants in which all units were of the same capacity. The sample data were divided into cells by unit vintage. Equation (I-14) was chosen as the "better fit," presumably on the basis of R^2, significance of estimated coefficients, and consistency of the estimated relationship with _a priori_ expectations. Estimates $\hat{\alpha}$ and $\hat{\beta}$ of coefficients α and β were positive in all cells, indicating that increases in either plant factor or unit size lead to decreases in corrected heat rate. This result was taken as evidence of economies of scale; comparison of parameter estimates across cells provided evidence of decreasing unit heat rates due to improved technology.

To investigate the effects of scale and technological change on the capital input, Galatin assumed capital cost per turbogenerator unit to be a polynomial function of the number of units in the plant and the capacity of the units, given by:

(I-15) $\frac{C_T}{N}$ or $\frac{C_E}{N} = \alpha_1 N + \alpha_2 N^2 + \alpha_3 N^3 + \beta X_K$,

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[43]Galatin, _op. cit._, 103.

where C_T is total capital cost of a plant including land, structures, and equipment; C_E is total equipment cost, N the number of machines, and X_K the size of each machine. The results of estimation suggested that cost per machine falls for plants containing one to three machines and rises thereafter; not surprisingly, per-unit capital cost increased with the size of unit. Galatin acknowledged that unlike the fuel input equation, which he considered a pure *ex post* production function, the capital input equation combined effects from changes in the production process with effects from changes in the price of capital.

Finally, a function was estimated relating labor input to the number and size of machines, as well as intensity of machine use. The estimated function was:

(I-16) $L = \alpha + \beta_1 N = \gamma X_K + \delta PF^*$

where L is the average number of employees of the plant, PF* is the adjusted plant factor discussed above, and the other variables are as defined for equation (I-15). The estimated results indicated that labor input increases with respect to all three independent variables; however, the rate of increase is less than proportional to increases in the number of machines, indicating the presence of economies of scale. Furthermore, comparison of the coefficients across cells indicated that technological innovation has accounted for at least part of a significant reduction in labor requirements over the period covered in this study.

Galatin's methodology gives rise to some doubts about his results. His recognition of the problems inherent in aggregation of hourly data into annual data is commendable. However, his concentration on that one type of mis-specification caused him to ignore other problems in model construction.

For instance, although Galatin presents a model of the multi-unit plant in his Chapter II, that model provides no explicit justification in terms of either the objectives of the plant managers or the technology of the industry for his maintained hypothesis in Chapter III that average fuel input is related to both corrected plant factor and size of machine.[44] This hypothesis is the basis for the estimation of equations (I-13) and (I-14). Similarly, no theoretical justification is presented for the estimated labor and capital input functions.

Even if the maintained hypothesis stated by Galatin in his Chapter III is accepted, the choice of the specific functional form of the relation is undetermined. Equations (I-13) and (I-14) are constructed so as to avoid aggregation problems; but no consideration is given to alternative functional forms that might satisfy that requirement. Furthermore, no attempt is made to determine on grounds other than goodness of fit and significance of the coefficients whether either of these is in fact an appropriate form of the

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[44]Galatin, op. cit. 35.

relation. The reader is referred to the discussion above of the paper by Nerlove, in which analysis of the residuals led to modifications of a model which gave satisfactory results on the same grounds used by Galatin, and to Chapter III.D. below, in which the deficiencies of Galatin's choice of procedure are explained.

Another question raised by Galatin's specification of the model to be estimated arises from his exclusion from the sample of plants which consisted of units of several different capacities. This restriction was made to ensure homoskedasticity, but no test was made to determine whether the disturbances of equations (I-13) and (I-14) as estimated were in fact homoskedastic or whether estimation with an unrestricted sample would have yielded homoskedastic disturbances.

A criticism of the capital input equation (I-15), originally made by Somermeyer, is discussed by Galatin in an appendix.[45] Somermeyer pointed out that since no restrictions were placed on the coefficients, the estimated equation implied negative average capital expenditures for some fairly common combinations of number and size of machines. A functional form was suggested that would avoid this problem. Finally, the comments made above in the discussion of the Dhrymes and Kurz article regarding the use of vintage as a

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[45] Galatin, _op_. _cit_., 139-141.

proxy for technology are also applicable to the same procedure as used by Galatin.

In another study, following similar methodology to that of Komiya, Hart and Chawla recently made a comparison of production functions for steam-electric generation between Britain and the U. S.[46] Under the assumption that two non-substitutable factors are used in the generation of electricity, the production function is given by:

(I-17) $Y = \min\{\kappa_1 X_1, \kappa_2 X_2\}$,

where Y denotes the output of electricity, X_1 the input of capital, and X_2 the input of fuel.

This formulation assumes constant returns to scale and allows for the possibility that one of the factors may not be fully used. However, Hart and Chawla wish to relax the assumption of constant returns to scale. Furthermore, their measure of capital is corrected for unused capacity, and they assume that the fuel input is fully used. Therefore, they restate the production function as a pair of equations, one for each factor:

(I-18) $Y = \kappa_i X_i^{\beta}$, $i = 1, 2$.

The interpretation of this formulation is that the expansion path of the firm is a ray through the origin with slope κ_1/κ_2, and that the function is homogeneous of degree β.

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[46] P. E. Hart and R. K. Chawla, "An International Comparison of Production Functions: The Coal-Fired Electricity Generating Industry," Economica 37 (May, 1970), 164-177.

This model may be termed a "fixed-relative-factor-proportions" model. Hart and Chawla explain that if both capital and fuel are fully used as the firm proceeds along its expansion path, the estimates of β from both component equations of the model should be equal. If the estimates are significantly different, ". . . either . . . the . . . production function . . . is inappropriate, or . . . there is a multiplicative measurement error, presumably in X_1, which takes the value $X_1^{b_1-b_2}$," where b_1 and b_2 are the estimates of β in equation (I-18) obtained when i = 1 and 2, respectively.[47] An example of this type of measurement error, which would make (b_1-b_2) positive, would be failure to correct completely for inaccuracies in the measurement of capital for unused capacity.

Having explained equation (I-18), Hart and Chawla then decide to make their work comparable with that of Komiya. Toward this end, they assume output to be the exogenous variable and rewrite the equation as:

(I-19) $X_i = \kappa_i^* Y^{\beta^*}$, i = 1, 2.

They then proceed to estimate equation (I-19).

Two problems, one notational and the other substantive, arise in the transformation from equation (I-18) to (I-19). Both are overlooked by Hart and Chawla. First, identifying the parameters of equation (I-19) by asterisks, it is clear that $\kappa_i^* = \kappa_i^{-\frac{1}{\beta}}$ and $\beta^* = \frac{1}{\beta}$ for i = 1, 2. This should have been pointed out to avoid confusion.

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[47]Hart and Chawla, op. cit., 171.

Second, Hart and Chawla ignore a substantive problem because their discussion does not include the stochastic specification of their model. If one assumes that equation (I-18) is characterized by a multiplicative disturbance term e^{u_i}, where the u_i are i.i.d. $N(\emptyset,\sigma^2 I)$, then the transformation indicated by equation (I-19) contains a multiplicative disturbance term e^{v_i}, where $v_i = u_i/\beta$. Obviously, the v_i are i.i.d. $N(\emptyset,\frac{\sigma^2}{\beta^2}I)$. But if in fact the v_i are i.i.d. $N(\emptyset,\sigma^2 I)$, then conversely the u_i are i.i.d. $N(\emptyset,\beta^2\sigma^2 I)$. Hart and Chawla apparently implicitly assume the distributions of both u_i and v_i to be $N(\emptyset,\sigma^2 I)$. Since the hypothesis tests employed by Hart and Chawla require certain distributional assumptions about the disturbances, this change in distributional properties caused by the transformation from equation (I-18) to (I-19) should be made clear.

Incorporating the stochastic specifications of equations (I-18) and (I-19), the equations may be written:

(I-18)' $Y_j = \kappa_i X_{ij}^{\beta} e^{u_{ij}}$, $i = 1, 2; j = 1, \ldots, n$

(I-19)' $X_{ij} = \kappa_i^* Y_j^{\beta^*} e^{v_{ij}}$, $i = 1, 2; j = 1, \ldots, n$, where j refers to the observed plant, u_{ij}, v_{ij} are each i.i.d. $N(\emptyset,\sigma^2 I)$, and other terms are defined as above.

The data used by Hart and Chawla were from the five-year period 1959 to 1963 with data from each country being divided into two groups--"old vintage," installed before 1954; and "new vintage," installed during or after that date.

Cross-sectional regressions were run for each of the five years for each group of plants. Estimates of the scale parameter β were uniformly less than unity for all regressions, indicating increasing returns to scale. For the U. K. the new vintage plants were shown to have higher returns to scale than the old. The opposite result, obtained for the U. S. sample, was attributed to measurement errors. While old vintage American plants appeared to be more efficient than their British counterparts, no significant corresponding difference was noted for new vintage plants.

The study might have been improved by the use of available information on technical characteristics of installed generating equipment in order to stratify the plants in the sample. As was pointed out in discussions of other previous studies, technological homogeneity within cells can be guaranteed with more certainty in this way. By so doing, one would know whether the observed differences in efficiency between British and American old vintage plants were due to British delays in implementing new technological advances or to some other factor causing inefficiency in British plants of comparable design to the American plants.

Other Approaches

A substantially different approach to the problem of economies of scale at the firm level in steam-electric generation was employed by Ling.[48] Rejecting a purely econometric approach, Ling used an "analytical" approach, designed to incorporate the principles of economic optimization within the constraints known to engineers. Concerning himself with economies of scale at the firm level, Ling started by stating and demonstrating, with frequent reference to the engineering literature, three principles that govern economic considerations:[49]

(1) The forced outage (breakdown) rate of generating units is independent of size, so that the use of larger generating units implies a larger required reserve capacity to keep the probability of system failure constant.

(2) Larger generating units can be installed at a lower cost per kilowatt of capacity than smaller ones.

(3) Larger generating units are necessary to use effectively the more advanced steam conditions (primarily higher temperatures and pressures), which result in lower fuel cost due to lower heat rates, usually at higher investment cost.

Furthermore, reference was made by Ling to the results of Hegetschweiler and Bartlett on fractional load performance

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[48] Ling, op. cit.

[49] Ibid., 20.

which indicate decreasing heat rate as unit load factor increases.[50] Heat rate is the ratio of fuel input (Btu) to output (mw).

After stating these assumptions, Ling postulated a hypothetical generating plant. Values of parameters such as heat rate, probability of turbine failure, etc., were chosen for the hypothetical plant so as to be plausible from an engineer's point of view. Under these assumptions, Ling simulated long-run plant expansion according to an economically efficient plan first derived by Kirchmayer, et al.[51] Under this plan, the first unit added to a plant facing increasing demand for its output should have a capacity equal to 10% of the initial plant capacity. Subsequent additional units should have capacities equal to successively smaller proportions of existing capacity until the 7% level is reached. The capacity of each unit added after this point should have a capacity equal to 7% of the existing capacity before the addition.

Using load duration curves to explain efficient allocation of load among existing units in the short run, Ling calculated the value of average cost for the hypothetical

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[50]H. Hegetschweiler and R. L. Bartlett, "Predicting Performance of Large Steam Turbine-Generator Units for Central Stations," ASME Transactions 79 (1957) 1085.

[51]L. K. Kirchmayer, A. G. Mellor, J. F. O'Mara and J. R. Stevenson, "An Investigation of the Economic Size of Steam-Electric Generating Units," AIEE Transactions 74 (III), (1955), 600-609.

firm before any additions, assuming efficient allocation of load. A correction was made to the cost data to allow for the effect of different intensities of capacity utilization, as measured by the plant factor. To generate a long-run cost function from this short-run function, Ling calculated average cost for his hypothetical firm after each successive addition made according to the economically efficient plan just explained. This calculation was made for each value of installed capacity, under various assumed values of the system load factor.

The final step in the derivation of an analytical long-run cost function was the choice of a functional form to fit the data generated according to the above process. A Cobb-Douglas type average cost function was rejected due to indications of non-linearity and unsatisfactory marginal cost functions derived from the function.[52] Instead, the following non-linear form was adopted:

(I-19) $C_a = kS^n\eta^m + p \ln \eta$

where C_a represents annual average generating cost in mills per kilowatt-hour, η represents system load factor, S system installed capacity, and k, n, m, and p parameters to be estimated. Fitting equation (I-19) to the data computed from the analytical model yielded a value for $\bar{R}$ of 0.999, and indicated average generating cost to be a decreasing function of system capacity and system load factor.

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[52] Ling, op. cit., 47.

As the final step in his analysis, Ling fitted equation (I-19) to a sample of data on four large independent steam-generating stations, the data having been collected by the Federal Power Commission between 1938 and 1958. When applied to real data, the non-linear form given by equation (I-19) yielded only a slight improvement over the Cobb-Douglas form, in terms of the value of $\overline{R}^2$ and significance of the coefficients. Both functions performed quite well, given various data limitations cited by Ling. Among the limitations was a very narrow range of variation in the value of system load factor, making inferences based on the estimated coefficient of that variable suspect.

A problem cited by Ling points up what may be a significant oversight on his part. All the assumptions about efficiency of components, forced outage rate, etc., underlying the analytical model are themselves based on some implicitly postulated state of technology, while the technology represented in the data varied as new plants were added during the period of the sample.[53] One would be interested in seeing whether stratification to hold the state of technology constant would change the empirical results. Although on the basis of his data, Ling's conclusion that the effect of technological advance on efficiency has been continuous and gradual seems plausible, any conclusion about the relative importance to efficiency of technological change

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[53]Ling, op. cit. 28.

and economies of scale must await such stratification.[54] However, with these reservations, Ling appears to have integrated successfully a number of engineering and economic principles.

This concludes the review of previous studies of the steam-electric generation process. It should be remembered by the reader that most of the criticism of the prior work centered on the implications of the methods used to classify plants or units on the basis of some proxy for technology, the attention paid to the stochastic specifications of the models, and the extent to which possible alternative specifications were considered. It was also pointed out that in the only two studies that considered the possibility of variable returns to scale over various outputs, strong indications thereof were found. However, since previous authors had no means of representing this variability as a production function parameter, no statistical inferences concerning this phenomenon could be made. It is the purpose of the present study to use some recent theoretical developments to solve some of these difficulties.

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[54] Ling, _op_. _cit_., 71.

Chapter II. Considerations Related To Postwar Technological Change

In Chapter I.B. the point was made that proxies used by most investigators for technological change in steam-electric generation had serious deficiencies. Specifically, it was explained that the use of nameplate-rated capacity by Dhrymes and Kurz led to pre-testing bias. Further, the use of machine vintage, it was claimed, did not reflect accurately the pattern of technological change in the industry because at any given time several different technologies are being used in newly constructed plants. Thus, heterogeneous generating equipment may be lumped together within a vintage cell while similar equipment may be treated as being different because of an arbitrary decision based on date of construction.

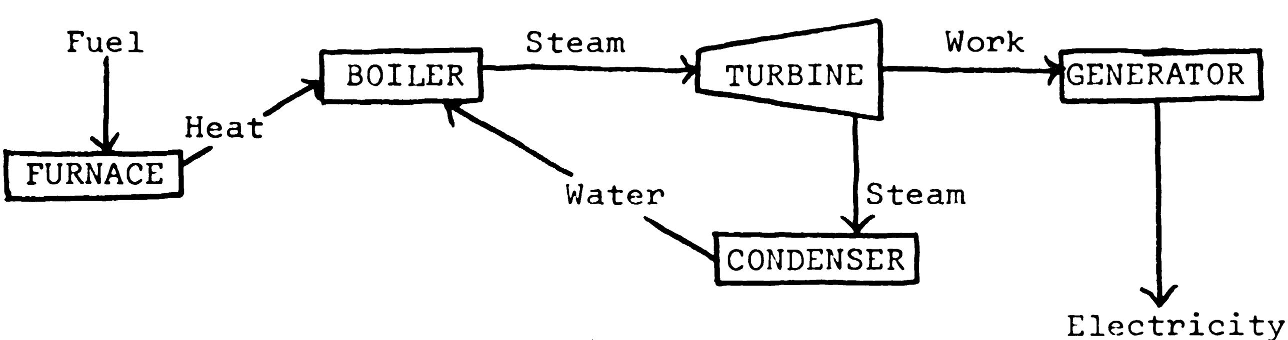
The present chapter is written in the belief that in order to make meaningful statements about the economic effects of technological change in this industry, one must identify the specific technological changes that have actually affected the physical process of generating electricity. Section A. of this chapter is devoted to a brief summary of the generation process and to a consideration of the major innovations in generating equipment during the period of this study, 1948 to 1965. Based on this discussion, a classification of generating plants intended to group together technologically homogeneous plants is proposed in Section B.

II.A. The Productive Process In Electrical Generation

The process of fossil-fueled electricity generation is conveniently broken down into three stages: fuel combustion, which takes place in the furnace; steam generation, which is the function of the boiler and several other heat transfer devices that improve efficiency; and electricity generation, which is accomplished by the turbogenerator unit.

The output of each prior stage is the input to the next stage. Specifically, fuel is the input to the furnace, which produces heat. The heat is transferred to the boiler, where it is used to convert water into steam. The steam is conducted to the turbine, where its expansion causes the turbine to rotate. The work output of the rotating turbine shaft is used to run an alternating current generator. The steam used in the turbine is condensed and re-used in the boiler. The process is described in Figure 1:

Figure 1.--Schematic Diagram of Steam-Electric Generating Plant



Each of the three productive stages--production of heat, production of steam, and generation of electricity--will be discussed in order before turning to an examination of the entire system.

The Furnace

The functions of a furnace may be listed as follows: to provide heat by burning fuel, to transfer the heat to part of the water and steam by radiation, and to facilitate the proper circulation of water and steam within the boiler.[1]

Conventional furnace fuels are vaporized oil, vaporized natural gas, and either crushed or pulverized coal.[2] Modern furnaces may burn all three, either separately or simultaneously, allowing firms to take advantage of fluctuations in relative prices and qualities of the fuels. Other factors which affect the choice of fuel are handling cost, cost of disposal of the by-products of combustion, operating labor required, and maintenance of the fuel-processing equipment and furnace.[3]

Gas, oil, and pulverized coal are burned "in suspension"; that is, air and atomized fuel are mixed and ignited in a burner and blown into the furnace, where the mixture burns

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[1]A. H. Zerban and E. P. Nye, Power Plants (Scranton: International Textbook, 1956), 122.

[2]Pulverized coal has been ground to a dust-like consistency. Crushed coal is in the form of small chunks. Atomic energy is not considered in the present study.

[3]B. G. A. Skrotski and W. A. Vopat, Power Station Engineering and Economy (New York: McGraw-Hill, 1960), 25ff.

suspended in mid-air by large fans. Crushed coal is ordinarily burned lying on a grate called a fuel bed, as air is blown through it to increase the combustion rate and to carry the heat to transfer surfaces, where the heat raises the temperature of the boiler water.

In either case, a technologically efficient furnace provides conditions for continuous combustion by keeping the fuel in constant contact with air after it is ignited and by providing conditions suitable for complete combustion. The latter function requires time and space for the carbon to burn completely in suspension so that the colder surfaces surrounding the furnace don't chill the mixture below kindling temperature, thereby ending combustion.

Artificially induced air turbulence within the furnace increases the available time for combustion, improves fuel-air mixing, and keeps fuel and air passing each other at high speeds, so that combustion products are swept away and unburned carbon exposed.[4] Therefore, induced turbulence in the furnace chamber reduces air pollution and improves plant efficiency since less available heat is lost in the form of unburned carbon, which either goes up the smokestack as flyash or collects and hardens on the sides of the furnace chamber as slag.

The Steam Generator

The function of the steam generator is to provide for

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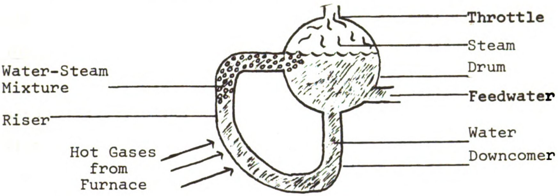
[4] B. G. A. Skrotski, Electric Generation--Steam Stations (New York: McGraw-Hill, 1956), 88ff.

efficient vaporization of water. At one time the only essential component of a steam generator was a drum, in which vaporization took place. But since about 1900, the more complex watertube boiler has been standard for central-station plants.[5] This type contains a large number of water circuits of the type illustrated in Figure 2.

Figure 2--Schematic Diagram of a Watertube Boiler



Feedwater enters the boiler drum below water level and mixes with the water already circulating in the unit. The heat transferred from the furnace to the riser at the left in Figure 2 forms steam bubbles in the riser, which are of course less dense than the water. Therefore, upon re-entering the drum, the steam collects at the top, from where it can be drawn off at the throttle. Since the mixture in the riser is less dense than the water in the downcomer, gravity induces continuous circulation.[6]

The moisture content of steam decreases as the temperature at which water-to-steam conversion takes place increases. This

[5]Zerban and Nye, op. cit., 160.

[6]Skrotski, op. cit., 112.

temperature, called the "saturation temperature," in turn increases as the pressure at conversion increases. Moisture content in steam greater than 12% increases the likelihood of the formation of droplets in the steam at the low-pressure end of the turbine. These droplets cause erosion of the turbine blades. In order not to exceed the 12% limit, steam produced in the boiler may be still further pressurized and heated in a device called a "superheater" before entering the turbine. Superheaters became standard equipment for steam generators before 1948, the first year of this study.[7]

The Turbogenerator

Once the steam has been produced and pressurized, it is used in the turbine to provide work output to the generator. Since the generator is usually mounted directly on the turbine shaft, the turbine and generator are considered a single unit, the "turbogenerator."

Allowing the steam to flow through a nozzle to a lower pressure in the turbine converts part of the internal energy of the steam into kinetic jet energy. Within a large turbine, this process is repeated at multiple stages of nozzles so that the turbine shaft is driven at several points. Of course, due to the expansion of the steam at previous stages, the pressure at each stage is lower than at the preceding stage. Since steam flows from high- to low-pressure locations in the turbine, efficiency is increased by lowering the steam pressure at the exhaust, known as backpressure. Backpressure

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[7]W. J. Kearton, Steam Turbine Theory and Practice (London: Pitman and Sons, 1931) 21.

can be reduced either by increasing the area of the exhaust annulus or by increasing the surface area of the condenser, a device which uses cold water from an external source to condense the steam into reusable water.[8] The purpose of condensation is to remove as much internal energy as possible from the steam.[9]

Each turbine in a plant drives an a-c generator, which actually produces electricity for transmission to consumers. Unfortunately, generation of electricity produces heat as a by-product in the generator. Since the necessary electrical insulation is also effective thermal insulation, this heat must be removed artificially. Consequently, all large generators have some sort of cooling system for the windings.

Steam Cycles

With an understanding of the operation of the individual components of a generating unit, one can discuss the theory of operation of a complete unit. Using this theory to define efficiency in terms of steam cycles, one is then able to assess the contribution to efficiency of various technological innovations.

As an introduction to the theory of steam-electric generation, several energy forms should be defined. Kinetic energy, due to velocity, and potential energy, due to

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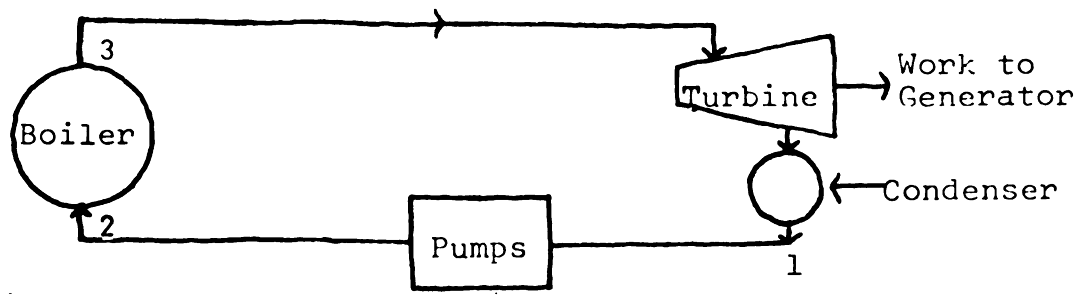
[8]W. A. Wilson and L. G. Malouf, "An Approach to the Economic Problem of Matching Condenser Surface with Exhaust-Annulus Area," ASME Transactions 1956, 73, 135.

[9]Skrotski and Vopat, op. cit., 272.

position, are well known. Internal heat energy of steam is the amount of heat that has been transferred from the furnace to the steam in the boiler. Another energy form, which exists only so long as a flow is present, is called flow energy, and is evaluated as the product of the pressure and volume of a vapor at a given temperature. The sum of flow energy and internal heat energy is defined as enthalpy (h) and is measured in British thermal units per pound (Btu/lb.). Thus, the enthalpy of a given volume of steam may be increased by raising either its temperature or its pressure.

The simplest operational power cycle using vapor as the working medium is the Rankine power cycle, which is illustrated in Figure 3:

Figure 3--Graphic Representation of the Rankine Steam Cycle



Work efficiency is defined as the ratio of net output of work to input of work. In the case of transformation of energy from heat to work, cycle efficiency E_t is defined by:

$$\text{(II-1)} \quad E_t = \frac{\text{work output - work input}}{\text{heat input}}.$$

Due to the "law of conservation of energy," the work output of the turbine is $(h_3 - h_4)$, where h_i represents enthalpy at point i in Figure 3. Since work input to the boiler is $(h_2 - h_1)$ and heat input is $(h_3 - h_2)$, one may write:

(II-2) $$E_t = \frac{(h_3 - h_4) - (h_2 - h_1)}{h_3 - h_2}$$

which is a fundamental definition in thermodynamics.[10]

Prior to 1920, metallurgical conditions limited steam temperatures and pressures to about 600°F and 250 pounds per square inch (psi). As mentioned above, water droplets, which are likely to form from such low-temperature steam when pressure drops over the turbine stages, will damage turbine blades. One method of preventing this is known as the "reheat cycle." In this cycle, instead of passing the steam straight through all stages of the turbine, it is bled off at an intermediate stage, before the pressure drop is sufficient to cause the formation of droplets. The steam is removed, reheated, and returned to the turbine at the following stage. The process of reheating, in addition to raising the maximum tolerable pressure, increases thermal efficiency because the steam is reintroduced at a higher enthalpy than that at which it was removed.[11]

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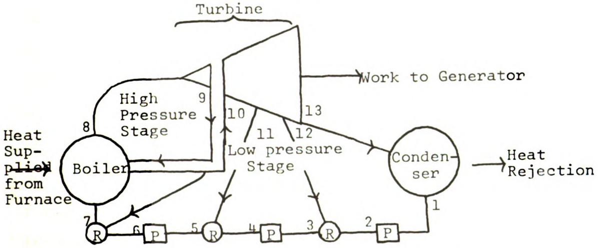
[10]This discussion is summarized from several basic engineering texts. See Zerban and Nye, op. cit., Ch. 2, Skrotski and Vopat, op. cit., Ch. 3, Skrotski, op. cit., Ch. 3, and C. M. Leonard, Fundamentals of Thermodynamics (Englewood Cliffs: Prentice-Hall, 1958).

[11]Skrotski and Vopat, op. cit., 62ff.

Attempts to improve efficiency by raising the average temperature of the medium in the cycle led to "regenerative preheating" of boiler feedwater. In a regenerative cycle steam is bled from the turbine at an intermediate stage as in the reheat cycle; but instead of being reheated, the steam is used to heat feedwater in a feedwater heater. Several regenerative cycles may be combined in a single boiler-turbogenerator system.[12]

The combined reheat-regenerative cycle is described in Figure 4, using three regenerative preheaters as an example.[13]

Figure 4--Graphic Representation of The Reheat-Regenerative Steam Cycle



Each unit marked P in Figure 4 is a feedwater pump. Each unit designated R is a regenerative preheater. The numbers represent points at which enthalpy is different from its value at the immediately preceding point.

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[12]Skrotski and Vopat, *op. cit.*, 64-67.

[13]Figure 4 and the discussion following were adopted from Skrotski and Vopat, *Ibid.*, 67ff.

Using the thermal cycle illustrated in Figure 4, one notes that the formula for thermal efficiency of such a cycle is:

$$(II\text{-}3)\quad E_t = \frac{(h_8-h_9)+(1-m_1)(h_9-h_{10})+(1-m_1-m_2)(h_{10}-h_{11})}{h_8-h_7} + \frac{(1-m_1-m_2-m_3)(h_{11}-h_{12}) - AW}{h_8-h_7},$$

where m_1, m_2, and m_3 are the masses of steam removed at each bleedpoint; and AW is the total energy demanded by all auxiliary power needs.

Equation (II-3) requires some elaboration. Heuristically, (h_8-h_9) represents the work done in the high-pressure section of the turbine by one unit of steam. Then if the mass m_1 of that steam is removed at point 9, $(1-m_1)$ of it loses enthalpy in the amount (h_9-h_{10}), the work output of one unit of steam between points 9 and 10. Similarly, $(1-m_1-m_2)$ units of steam produces work equal to $(h_{10}-h_{11})$ per unit, and so on. Obviously, additional regenerative cycles add positive terms of the form $(1-m_1-\ldots)$ (h_k-h_{k+1}) to the numerator, increasing E_t. Each successive term is smaller than the one preceding it since an additional value of m_i is being subtracted from the multiplier. In other words, regenerative preheaters exhibit diminishing marginal productivity.

Both reheat and regenerative cycles were well known by the beginning of the present survey period, 1948-1965, and regenerative preheaters were standard equipment by 1948.

However, according to Power magazine's annual Modern Plant Design Survey, temperatures and pressures were not generally high enough to justify the additional cost of installing reheat cycles until the early 1950's, at which time they, too, became standard.[14]

II.B. Postwar Innovations In Steam-Electric Generation

The rather tedious discussion just concluded is a necessary prerequisite to understanding the importance of various innovations that were introduced into steam-electric generation during the period of this study, 1948-1965. This understanding, in turn, will form the basis for the classification of turbogenerator units into technologically homogeneous cells. With these considerations in mind, the discussion now turns to the postwar innovations, their contribution to efficiency, and their economic implications.

Innovations in Combustion

Two new types of coal furnaces appeared during the period of the present study, 1948-1965: the cyclone furnace and the pressurized furnace.

In order to burn lower-grade coal, which produces more flyash, plants in some areas built shortly after World War II were equipped with cyclone furnaces.[15] These receive crushed

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[14]See Chapter I, note 1, of this manuscript.

[15]Skrotski and Vopat, *op. cit.*, 144.

coal in a whirling stream of high-velocity air, which throws the coal to the rim of the burner by centrifugal force, where the high-speed air promotes a high combustion rate. The ash melts to form slag, which drains into a secondary furnace carrying burning coal particles with it. Combustion is continued in the secondary furnace so that the amount of flyash is substantially reduced.[16] An additional advantage of the cyclone furnace is that the coal crusher requires less auxiliary power than the pulverizer used in suspension furnaces. Even though cyclone furnaces require additional forced-draft fans, it is often less costly to substitute expenditure on such capital equipment for expenditure on low-grade fuel, even when the cost of auxiliary power to run the fans is taken into account.[17] That is, the potential use of such equipment exemplifies the possibility of capital-fuel substitution in a generating plant at the design stage.

During the late 1940's the American Gas and Electric System was developing the pressurized furnace, in which combustion and heat transfer take place under 40 to 50 psi pressure, provided by forced-draft fans. These units require special airtight gaskets surrounding the burning chamber. However, units with such gaskets can also be used in a conventional induced-draft operation.

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[16]Skrotski, op. cit, 105ff.

[17]Skrotski and Vopat, op. cit., 144ff.

The main advantages of the pressurized furnace are: reduced air leakage, which improves both efficiency and control of the furnace; a reduction in heat loss, because of turbulence; and improved safety, because no adjustment is needed during start-up and low-load periods. Its disadvantages include difficulty in locating leaks in the inner shell and limited access for manual slag removal. Furthermore, since auxiliary induced-draft fans are usually required, the initial cost is greater than that of conventional furnaces. After the first pressurized unit was placed in operation in 1949, there was a "wait-and-see" period of four to five years during which ordinary pulverized coal and cyclone furnaces were used extensively, before pressurized furnaces gained general acceptance for large plants. Table 1 shows the postwar trend in furnace design for plants under construction, as reported in Power magazine's annual Modern Plant Design Surveys of 1948, 1957 and 1966.

Table 1

Coal Furnace Type Planned in Samples of Plants Under Construction[18]

Type	1948	1957	1966
Pulverized coal, induced draft	100%	92%	24%
Cyclone	0	8%	0
Pressurized	0	0	76%

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[18]Entries were calculated from data in the 1948, 1957, and 1966 Modern Plant Design Surveys of Power Magazine (New York: McGraw-Hill, monthly, 1948-1965).

The data in Table 1 are consistent with the hypothesis that rising fuel costs led to use of the more efficient cyclone furnace, which was itself replaced in large plants by the still more efficient pressurized furnace.

Innovations in Steam Generation

The most remarkable improvements in the efficiency of electricity generation have resulted from innovations in steam generators. Innovations have taken place in the technologies of metallurgy, welding, and feedwater pumps, all of which aided engineers in their attempts to raise design temperatures and pressures.

The effect of higher design temperatures and pressures on cycle efficiency may be discussed in the context of equation (II-2). It will be recalled that heat input during the cycle is represented by $(h_3 - h_2)$ in that equation. It may be shown that for any increase ε in $(h_3 - h_2)$,

$$\frac{(h_3-h_4)-(h_2-h_1)}{h_3-h_2} - \frac{(h_3+\varepsilon-h_4)-(h_2-h_1)}{h_3+\varepsilon-h_2} = \frac{-\varepsilon(h_4+h_1)}{(h_3-h_2)^2+\varepsilon(h_3-h_2)}.$$

Since $h_3 > h_2$ for a positive input of heat during the cycle, and since h is non-negative throughout the cycle, this difference must be negative. Therefore, at constant values of h_4, h_2, and h_1, a larger input of heat $(h_3 - h_2)$ during the cycle decreases cycle efficiency, as defined by equation (II-2) One method of decreasing the necessary heat input during the cycle, thereby increasing cycle efficiency, is to raise the average steam temperature over the entire cycle.

This may be accomplished by means of superheating and/or raising boiler pressure, both of which make more efficient use of a given heat input (h_3 - h_2). Consequently, as improved welding and metallurgical techniques and more powerful feedwater pumps permitted, engineers steadily raised design temperatures and pressures between 1948 and 1957.[19] For example, the proportion of central-station plants under construction designed for operation above 1000°F/1500 psi rose from 2% in 1947 to 52% in 1958.[20]

With a furnace of given efficiency, the decrease in heat input for a given work output due to the higher design pressure made possible by metallurgical progress implies a decrease in fuel input for the given work output. These technical advances are clear examples of capital-fuel substitution. Empirical analysis of this substitution is simplified by the fact that the joint effect of the improvements in welding, metallurgy, and feedwater pumps is embodied in higher design temperatures and pressures. Therefore, in lieu of detailed separate studies of these three technologies, one can summarize their effects in terms of temperature and pressure.

Another example of capital-fuel substitution is the addition of regenerative preheaters to turbogenerators. Over

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[19]Skrotski and Vopat, op. cit., 60ff.

[20]See the 1947 and 1948 Modern Plant Designs of Power magazine, op. cit.

the period of the present study, 1948 to 1965, the average number of regenerative preheaters per unit increased steadily, as shown in Table 2. In engineering terminology, the addition of preheaters lowers the heat rate of a turbo-generator unit, i.e., it lowers the fuel input required for a given output.

Table 2

Number of Turbine Bleedpoints for Samples of Units Under Construction[21]

No. Bleedpoints	Percent of Plants Under Construction		
	1948	1957	1966
0	1	1	0
1	8	1	0
2	8	2	3
3	18	6	0
4	38	13	6
5	20	30	6
6	5	20	26
7	2	21	34
8	0	6	22
11	0	0	3
	100%	100%	100%

This particular form of capital-fuel substitution appears particularly well suited to large plants. This becomes apparent when one notes that in 1966 the five units under construction with less than six bleedpoints were the five smallest units, in terms of generator rating in kilowatts. Incidentally, the data in Table 2 offer strong evidence for the contention made in Chapter I.B. that in any given year new plants embody

[21]Power, op. cit.

various technologies, so that vintage of unit is not an adequate proxy for technology.

As mentioned above, the use of higher steam pressures required thicker boiler drum walls made of stronger alloys. Furthermore, units had to be taller to make the gravitational force of falling water sufficient to maintain circulation. These rising capital costs, together with ever increasing fuel costs, spurred the Ohio Power Company to announce, in 1953, plans for the world's first steam generator to produce steam in its supercritical state.[22]

Stated simply, conventional sub-critical boilers heat water under pressure and circulate it through tubes to the boiler drum where the lower pressure allows the water to flash to steam. On the other hand, supercritical generators heat the water at a pressure above 3206 psi, the critical pressure, so that at 705.4°F water vaporizes without boiling. Since the water never flashes to steam, supercritical units eliminate the cost of a drum while taking advantage of the greater efficiency afforded by the higher pressures.[23]

Immediately after plans for Ohio's Philo supercritical unit were announced, four other supercritical units were ordered by other firms. But after this initial flurry other power companies waited for the feedback of experience before

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[22]C. R. Earle, "Power Engineers Take Long Step to 4500-psi, 1150-F Steam," Power Engineering 58 (1), (Jan. 1954), 58-61.

[23]Skrotski and Vopat, op. cit., 391ff.

building supercritical units, so that in 1957, the year the first five began operation, no others were under construction.[24] However, by 1966, 26% of all units under construction incorporated this technological advance,[25] which has been called "the most widely and rapidly accepted innovation in the steam power plant design in the long history of technological development in the industry."[26]

Just as increasing pressures had compelled the introduction of the reheat cycle before World War II, so supercritical pressures demanded the introduction of double-reheat cycles to protect turbine blades. In 1966 all supercritical plants under construction incorporated double reheat and up to eleven regenerative cycles.

Innovations in Generator Cooling

The standard procedure for cooling generators at the beginning of the survey period was to force air or hydrogen through the generator passages in order to absorb the heat losses. In a conventional air-cooled system, the air passed over the tube and fin surfaces of a cooler before returning to the generator passages to be recirculated. However, more efficient means of cooling generators have been developed, the relative advantages and disadvantages of which are

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[24]J. Tillinghast, R. H. Pechstein, and W. S. Morgan, "Design and Operating Experience with a Supercritical Pressure Unit--Part 1," Power Engineering 70 (3), (March 1966), 58-61.

[25]Power Modern Plant Design Survey, 1966.

[26]Tillinghast, Pechstein, and Morgan, op. cit., 58.

briefly discussed in the next few paragraphs.

Since hydrogen is so much less dense than air, energy losses due to friction of the generator rotor against the gas in which it is turning, known as "windage losses," are only 10% as high in hydrogen as they are in air. In addition, the higher thermal conductivity of hydrogen raises the level of potential output for a given amount of heat loss. Finally, insulation is much more durable when it is not in contact with oxygen.[27] Thus, in 1948 half the units under construction listed in the Power survey had hydrogen-cooled generators; ten years later, hydrogen was the planned medium for all units under construction. The fact that hydrogen was not used as a cooling agent before the end of World War II may be due to wartime rationing of hydrogen.

Prior to 1952, the cooling system design required that heat from the conductor pass through the electrical insulation before reaching the cooling medium.[28] Since that year many new generators have been built with hollow conductors, so that the heat is transferred directly from the metal to the cooling medium. Hollow-conductor cooling is so much more efficient than tube-and-fin cooling that the upper limit to maximum generating, which used to be set by thermal stress, is now determined purely by mechanical efficiency considerations.

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[27] Skrotski, op. cit., 351.

[28] Ibid., 351-352.

Since the pumping load for a given amount of heat loss is much less for a liquid than for a gas, so that the heat transfer rate is much higher, modern large plants have begun using liquid hollow-conductor cooling systems.[29] Table 3 does not indicate the full strength of the trend toward liquid hollow-conductor cooling for all plants unless one realizes that the new cooling methods completely predominate among the largest units under construction.

Table 3

Generator Cooling Method in Sample of Units Under Construction

Type	1948	1957	1966
Hydrogen	69%	100%	58%
Air	31	0	0
Liquid Hydrogen	0	0	27
Other Liquid	0	0	3
Liquid Hydrogen, Hollow Conductor	0	0	12

Summary of Innovations in Plant Design

From the discussion just concluded, it is apparent that power plant design between 1948 and 1965 was a dynamic process with frequent changes in the state of technology. Because of these changes, it is to be expected that newer plants are more efficient than older ones. In furnace design, the stoker furnace common in the 1940's was replaced during the 1950's by pressurized and cyclone furnaces designed to burn fuel more efficiently. Metallurgical advances

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[29]Skrotski, *op. cit.*, 352.

allowed rather steady increases in design temperatures and pressures so that the 900 psi - 900°F boiler of 1948 was superseded by giant 2400 psi - 1100°F boilers a decade later. These in turn became obsolete for large plants when the boiler was replaced by the supercritical steam generator, which reduced capital costs as it provided high-temperature, high-pressure steam which is conducive to efficient turbine operation. In turbine technology, the higher temperatures and pressures were accompanied by increases in the number of reheat and regenerative cycles included in new turbogenerators. As pointed out above in the discussion of steam cycles, these improve efficiency by raising the average steam temperature over the whole generating cycle. Finally, as the old forced-draft air cooling systems for generators were replaced with hollow-conductor systems, often with hydrogen as the cooling medium, a given-size turbine was enabled to drive increasingly higher rated generators.

The Methodology of Innovation in Plant Design

Considering these examples of technological innovation, several comments about the nature of technological change in this industry seem appropriate. The approach of plant design engineers appears to be inductive and experimental rather than deductive. For example, it was pointed out above that after supercritical steam generators were first introduced, there was a period of several years during which engineers could observe supercritical units in operation before the method gained widespread acceptance. Even after that time, according to the surveys of modern plant design conducted by Power magazine during the period, there was extensive experimentation with pressures in the neighborhood of 5000 psi. These pressures have been largely abandoned, with 3500 psi being the most common supercritical pressure at present.

Examination of the engineering literature provides additional evidence for the contention that power plant design is an inductive, experimental science. The literature is replete with examples of case studies giving detailed descriptions of innovations incorporated in new units and evaluations of their success.[30] On the other hand, even the few studies that might be termed theoretical in nature rely on polynomial approximations to empirically observed non-linear relations rather than on functional forms deduced from theoretical relations. This is due in part to the experimental nature of technological change in steam-electric generation.[31] However, it should also be noted that the use of steam at high and supercritical pressures is not relevant for low-capacity generating units. This becomes evident from

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[30]See for example: "Etiwanda, Study in Station Economy," Power 96 (6), (June 1952), 100ff.; W. Greacen, III, "Reheat Unit Extends Goudey Capacity," Power 97 (2), (Feb. 1953), 71-75; "Power Takes You Through New Centra Costs Plant," Power 96 (2), (Feb. 1952), 75-79.

[31]See for example, C. W. Elston and P. H. Knowlton, Jr., "Comparative Efficiencies of Central-Station Reheat and

the study of Power magazine's annual survey of modern plant design for any year; the explanation is discussed by Ling and in Chapter IV below.[32] For these reasons the discussion of innovations just concluded will form the basis of the stratification method for turbogenerator units to be explained in Chapter II.C. Hopefully, this method will prove superior to the vintage-size proxies used by previous authors to represent technological change.

II.C. The Stratification Of Generating Plants

The discussion of postwar innovations just concluded emphasized the contribution of each development to the technical efficiency of the generating process. By relating these innovations to observable design characteristics of newly-installed turbogenerators, one can construct a classification system defining technologically homogeneous cells for turbogenerator units. An entire plant may be said to belong to a cell if all its units belong to that cell.

Classes of units were defined with respect to the following physical characteristics: temperature and pressure at the turbine throttle, furnace type, number of bleedpoints for

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Non-Reheat Steam Turbing-Generator Units," ASME Transactions 74 (1952), 1389; H. Hegetschweiler and R. L. Bartlett, op. cit.; G. B. Warren and P. H. Knowlton, Jr., "Relative Engine Efficiencies Realizable from Large Modern Steam Turbing-Generator Units," ASME Transactions 63 (1941), 125.

[32] Ling, op. cit., 20.

regenerative feedwater heaters, number of reheat cycles, and generator cooling apparatus. The definitions were made with the following considerations in mind. First, insofar as possible, each change in any of these characteristics that might be expected to produce an observable improvement in plant efficiency should be recognized in the cell definitions. Second, where a large majority of plants incorporating a given innovation are similar with respect to all other design characteristics, units incorporating the innovation but differing with respect to the other characteristics should not be included in any cell. Third, since rates of progress in the various components of the generating process are not equal, the cell definitions should not preclude the possibility of overlapping with respect to some characteristics. For example 6 or 7 bleedpoints were installed on many turbines using 1800-2050 psi steam, and on many in the 2400-3206 psi range. Therefore, if those pressure ranges are the most important differentiating characteristics between two cells, the acceptable bleedpoint ranges for both cells should include 6 and 7. Fourth, insofar as possible the cell division should provide a number of observations within each cell sufficient for empirical analysis.

The classification resulting from these considerations is presented in Table 4.

The units in Cell I were constructed with generators rated in the 40 to 80 megawatt (mw) range, for the most part.

Table 4

Definition of Cells for Classification of Generating Plants

Cell	Number of Observations	Furnace Type and Fuel	Number of Bleedpoints	Number of Reheat Cycles	Pressure Temperature	Generator Cooling
I	212	Stoker, PC, G &/or O	0-4	0	800-990 psi/ 900-950°F	Air, H_2
II	85	Cy, Press	0-7	0-1	800-1449 psi/ 900-1000°F	Air, H_2
III	138	PC, Press, G &/or O	5-7	1	1450-1550 psi/ 1000°F	H_2
IV	270	PC, Press, Cy, G &/or O	5-7	1	1800-2050 psi/ 1000-1050°F	H_2
V	62	PC, Press, Cy, G &/or O	6-8	1	2100-3260 psi/ 1000-1100°F	H_2, Li
VI	41	PC, Press, Cy, G &/or O	6-8	1	2100-3206 psi/ 1000-1100°F	H-C
VIII	13	PC, Press, G &/or O	4-8	1-2	3206 psi/ 1000°F	H_2, Li, H_2/Li, H-C

Symbols: Cy - Cyclone
G &/or O - Gas and/or Oil
H_2 - Hydrogen
H-C - Hollow Conductor
Press - Pressurized Furnace
Li - Liquid
PC - Pulverized Coal

These are the lowest ratings represented in the data sample for the present study. The majority of the Cell I units in the sample were constructed between 1948 and 1950 although some units of this type were being installed through the mid-1950's. It is anticipated that these units will be among the least efficient in the sample.

The units placed in Cell II differ from those in Cell I primarily in their use of cyclone and pressurized furnaces although some were designed to operate at slightly higher temperatures and pressures. These units were common in plants constructed between 1948 and 1952.

The units in Cells III and IV were the most common type installed during the early 1950's although their construction continued to some extent throughout the decade. The two cells differ from each other primarily in design pressure. The nameplate rating of attached generators varied between 40 and 125 mw in Cell III, and between 100 and 160 mw in Cell IV. It will be noted that the higher design temperatures and pressures of these units made additional regenerative preheaters efficient, as shown by the larger number of bleedpoints. In addition, it can be seen from Table 4 that the higher temperatures and pressures also justified the additional cost of a reheat cycle.

Cells V and VI represent the bulk of generating equipment installed in central-station plants during the late 1950's and early 1960's. The wide range of generator ratings for these units, 130 to 570 mw, indicates a high degree of versatility. They differ from units in the lower-numbered cells primarily in having higher design temperatures and pressures. They differ from each other solely in the type of generator cooling system installed.

Cell VII includes all supercritical units constructed between the year 1957, when the first such unit began operations, and the year 1965. The generator range in this class is 600 to 850 mw. All these units incorporated pressurized furnaces and at least four bleedpoints. Problems of turbine pitting due to condensation of the steam made use of at least one reheat cycle mandatory and designers of a number of these units experimented with a second one. Liquid hydrogen was introduced as a generator cooling medium in some of the units and all such generators employed hollow conductors. Unfortunately, very few new plants were built before 1965 containing only supercritical units. Since a plant is included in a cell only if all units in the plant are in the same cell, only a limited sample of data on supercritical units is available.

The reader will note from the discussion of the preceding few paragraphs that there is a great deal of overlapping of technologies across both time and generator ratings. Thus, it becomes apparent that the common practice of grouping units by either or both of these two criteria conceals great disparities in technology among members of a given group. Specific examples of this practice are cited in Chapter I.B.

Chapter III. Theoretical Structure and Methodology

In Chapter I, the problem being considered was stated to be the separation and quantification of the effects of technological change and returns to scale in steam-electric generation. Previous approaches to the problem were also discussed in that chapter. It was argued that those studies lacked a suitable proxy for technological change, and that insufficient consideration was given to alternative specifications of the production model for electricity generation, particularly to specifications permitting variable returns to scale in the generation process. In Chapter II, an alternative classification system for power plants was proposed, with the objective of defining technologically homogeneous cells.

The focus of this chapter is on the specification of the production model for steam-electric generation. In Part A, summaries of theoretical results of Zellner, Kmenta, and Drèze (ZKD) and of Zellner and Revankar are presented.[1] The former study is a production function model in which ordinary least squares estimates of the parameters of a Cobb-Douglas production function are shown to be consistent. The latter

[1]A. Zellner, J. Kmenta, and J. Drèze, op. cit., 784-795; A. Zellner and N. S. Revankar, op. cit., 241-250.

derives a method of generalizing a neoclassical production function with given values of the elasticity of substitution and returns-to-scale parameters to a function with the same elasticity of substitution, but with returns to scale a prespecified function of output. In Part B, the model will be applied to two generalized production functions (GPF's), one a generalization of an n-factor Cobb-Douglas function, the other a generalization of an n-factor CES function. In Part C, specific models to be considered on the basis of the results of Part B will be presented along with some specific models of previous investigators which were discussed in Chapter I.B. Estimation techniques for the models will be discussed. In Part D. a methodology for testing three hypotheses regarding steam-electric generation is presented. In the next chapter, this method is applied to the models presented in Part C of this chapter.

III.A. Recent Results in Production Theory

This section introduces two recent developments in production theory, both of which are relevant to the present study. First, the ZKD production model of the firm is introduced and compared to the traditional model of Marschak and Andrews.[2] Second, a brief explanation will be given of the notion of GPF's as developed by Zellner and Revanker.

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[2]J. Marschak and W. J. Andrews, "Random Simultaneous Equations and the Theory of Production," Econometrica 12 (1944), 143-205.

Alternative Models of the Firm

Following the notation of Marschak and Andrews, assume that the i-th firm produces output according to a two-factor Cobb-Douglas production function given by:

$$\text{(III-1)}\quad Y_i = AX_{1i}^{\alpha_1}X_{2i}^{\alpha_2},\ i = 1,\ldots, n,$$

where Y, X_1, and X_2 represent flows of output, labor, and capital, respectively, per unit of time.

Profit, denoted by π, is defined by:

$$\text{(III-2)}\quad \pi = pY - w_1X_1 - w_2X_2,$$

where p, w_1, and w_2 represent the prices of output, labor, and capital, respectively.

Conditions (III-3) and (III-4) are necessary for maximization of profits by the i-th firm:

$$\text{(III-3)}\quad \frac{\partial\pi_i}{\partial X_{1i}} = 0,$$

$$\text{(III-4)}\quad \frac{\partial\pi_i}{\partial X_{2i}} = 0.$$

Simultaneous solution of equations (III-3), (III-4), and (III-1), and the addition of error terms to be justified below, yields the following system of equations:

$$\text{(III-5)}\quad \log Y_i - \alpha_1 \log X_{1i} - \alpha_2 \log X_{2i} = \lambda_o + v_{0i},$$

$$\text{(III-6)}\quad \log Y_i - \log X_{1i} = \lambda_1 + v_{1i},$$

$$\text{(III-7)}\quad \log Y_i - \log X_{2i} = \lambda_2 + v_{2i},$$

$i = 1, \ldots n$, where $\lambda_o = \log A$, $\lambda_1 = \log \frac{w_1}{p\alpha_1}$, and $\lambda_2 = \log \frac{w_2}{p\alpha_2}$. Each of v_{0i}, v_{1i}, and v_{2i} is a stochastic disturbance term assumed to be i.i.d. normal with zero mean and equal variance for all firms.[3] Each disturbance is the sum of two random variables, the first of which depends on "the firm's 'economic efficiency' and on the degree of competition between its customers, or workers, or creditors"; while the second "is assumed to be the combined effect of a larger number of causes (not necessarily independent of each other), of which none has an influence considerably surpassing the influence of each of the other causes."[4]

The reduced form of equations (III-5), (III-6), and (III-7) is given by:

$$\text{(III-8)} \quad \log Y_i = \frac{\lambda_0 + v_{0i} - \alpha_1\lambda_1 - \alpha_2\lambda_2}{-(1-\alpha_1-\alpha_2)},$$

$$\text{(III-9)} \quad \log X_{1i} = \frac{\lambda_0 + v_{0i} - \lambda_1 (1-\alpha_1) - \alpha_2\lambda_2}{-(1-\alpha_1-\alpha_2)},$$

$$\text{(III-10)} \quad \log X_{2i} = \frac{\lambda_0 + v_{0i} - \alpha_1\lambda_1 - (1-\alpha_1)\lambda_2}{-(1-\alpha_1-\alpha_2)}.$$

Examination of (III-9) and (III-10) shows immediately that the factor inputs X_{1i} and X_{2i} are functions of v_{0i}, the production function disturbance. Consequently, ordinary least squares estimates of the production function will, in general,

[3] Irving Hoch, in "Simultaneous Equation Bias in the Context of the Cobb-Douglas Production Function," *Econometrica* 26 (1958), 566-578, multiplied the antilogs of λ_1 and λ_2 by parameters R_1 and R_2 to allow for the possibility that firms in the sample may exhibit systematic errors with respect to satisfying the first-order conditions.

[4] Marschak and Andrews, *op. cit.*, 157 and 155.

be biased and inconsistent. This is the well-known problem of "simultaneous equation bias."[5]

However, it has been shown by Zellner, Kmenta, and Drèze (ZKD) that under certain conditions, ordinary least squares estimates of two-factor Cobb-Douglas production function parameters are consistent.[6] Their production model involves the assumption that output is a stochastic function of the inputs, the disturbance being generated by such factors as, "weather, unpredictable variations in machine or labor performance, and so on."[7] Consequently, profit is stochastic, and firms are assumed to maximize its expected value. ZKD used the following method to prove that these assumptions imply that ordinary least squares estimates of the Cobb-Douglas parameters are consistent.

Following the ZKD notation, the production function is written:

$$\text{(III-11)} \quad Y_i = AX_{1i}^{\alpha_1} X_{2i}^{\alpha_2} e^{u_{0i}}$$

where the u_{0i} are assumed to be normally distributed with zero mean and constant variance; other variables are as defined above. Expected profit is defined by:

$$\text{(III-12)} \quad E(\pi) = p^{+}E(Y) - w_1 X_1 - w_2^{+} X_2 = p^{+} A X_1^{\alpha_1} X_2^{\alpha_2} e^{\frac{\sigma_{00}}{2}} - w_1^{+} X_1 - w_2^{+} X_2,$$

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[5] See, for example, A. S. Goldberger, Econometric Theory, (New York: Wiley and Sons, 1964), 288-290.

[6] Zellner, Kmenta, and Drèze, op. cit.

[7] Ibid., 787.

where σ_{00} is the variance of the production function disturbance, and $^{+}$'s indicate the expected values of the prices of output and the inputs. Necessary conditions for the maximization of expected profit are given by:

(III-13) $\frac{\partial E(\pi)}{\partial X_{1i}} = 0,$

(III-14) $\frac{\partial E(\pi)}{\partial X_{2i}} = 0.$

Solving equations (III-13), (III-14), and the production function equation (III-11) simultaneously, the reduced form equations of this model are:

(III-15) $\log Y_i = \frac{[\alpha_0-\alpha_1 k_1-\alpha_2 k_2+(1-\alpha_1-\alpha_2)u_{0i}-\alpha_1 u_{1i}-\alpha_2 u_{2i}]}{(1-\alpha_1-\alpha_2)},$

(III-16) $\log X_{1i} = \frac{[\alpha_0-(\alpha_2-1)k_1-\alpha_2 k_2+(\alpha_2-1)u_{1i}-\alpha_2 u_{2i}]}{(1-\alpha_1-\alpha_2)},$

(III-17) $\log X_{2i} = \frac{[\alpha_0+\alpha_1 k_1-(\alpha_1-1)k_2+\alpha_1 u_{1i}-(\alpha_1-1)u_{2i}]}{(1-\alpha_1-\alpha_2)},$

where $k_1 = \log\left(\frac{w1}{p\alpha_1}\right) - \frac{\sigma_{00}}{2}$ and $k_2 = \log\left(\frac{w2}{p\alpha_2}\right) - \frac{\sigma_{00}}{2}$. u_{1i} and u_{2i} are stochastic disturbances resulting from deviations of factor prices p, w_1, and w_2 from their anticipated values p^+, w_1^+, and w_2^+. From equations (III-16) and (III-17), it is clear that X_{1i} and X_{2i} are independent of u_{0i}, the production function disturbance. Consequently, under the ZKD assumptions, and further assuming that $E(u_{0i}u_{1i}) = E(u_{0i}u_{2i}) = 0$, ordinary least squares (OLS) estimates of the production function parameters are consistent.

Recently, Hodges proved a similar result using a two-factor CES production function:[8]

(III-18) $Y_i = \gamma[\delta X_{1i}^{\rho}+(1-\delta)X_{2i}^{\rho}]^{\frac{\nu}{\rho}}$,

where the variables are as defined above. She pointed out that the generalization of her results to the case of several inputs was feasible.[9] In the original Hodges paper, $-\rho$ was used as the exponent for each factor, and $-\frac{\nu}{\rho}$ as the exponent of the term in brackets. For the sake of consistency and simplicity in the generalization of her results to n factors, the notation used in equation (III-18) has been adopted for the present study.

For the purposes of this thesis, the reader should note the fundamental difference between the ZKD model as applied to steam-electric generation and models used by previous investigators of the industry. In the two studies by Dhrymes and Kurz and by Nerlove, both of which explicitly considered optimization by the producer, output was assumed to be exogenously determined. The cost-minimization conditions necessitated estimation of a simultaneous equation system in each of these studies. Similarly, the functions estimated by Komiya and by Hart and Chawla implicitly treated output as independent of any stochastic disturbance although they did not treat the problem of optimization by the producer. The

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[8]Dorothy J. Hodges, "A Note on Estimation of Cobb-Douglas and CES Production Function Models," *Econometrica* 37, 721-725.

[9]*Ibid.*, Note 2, 721.

generalized ZKD models to be examined in the present study introduce a directly opposing assumption, namely that output is a stochastic function of the inputs. The comparison of the ZKD-type models with the Hart and Chawla functions, to be carried out in the present study, affords an opportunity to compare the appropriateness for the generating plant of these two fundamentally different approaches. Unfortunately, both the lack of good factor-price data and other problems associated with the simultaneous-equation nature of the Dhrymes-Kurz and Nerlove models precluded a direct comparison of those models with the generalized ZKD models on the criteria to be used in this study.

Generalized Production Functions

A second development relevant to the present work is the notion, due to Zellner and Revankar, of the generalized production function (GPF).[10] Quoting those authors, ". . . given a neoclassical production function with a given elasticity of substitution (constant or variable) . . . this function can be transformed to yield a neoclassical GPF with the same elasticity of substitution and with the returns to scale variable and *satisfying a preassigned relationship to the output level*."[11] (Italics theirs).

The importance of this result to the problem at hand is obvious. All previous studies of returns to scale in

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[10] Zellner and Revankar, *op. cit.*

[11] *Ibid.*, 241.

steam-electric generation have been constrained to the assumption of a constant value of returns to scale parameter over all levels of output. For example, within that framework Dhrymes and Kurz concluded that "increasing returns to scale is the prevailing phenomenon in (steam) electric generation."[12] However, as pointed out in the review of the literature in Chapter I.B., Nerlove found some evidence that the degree of returns to scale decreased somewhat as output increased. Due to an inability to parameterize this variability of returns to scale, he was unable to make any statistical inferences about the phenomenon. It is exactly this parameterization that GPF's make possible.

To understand the notion of a GPF, consider a neoclassical production function f, homogeneous of arbitrary degree, such that:

(III-19) $Y = f(X_1, \ldots, X_n)$,

where Y denotes the real output rate, and X_i, $i = 1, \ldots, n$, denote inputs to the production process. A generalization of that production function may be defined by:

(III-20) $V = g(Y) = g[f(X_1, \ldots, X_n)]$,

where g is a transformation function satisfying the conditions $g(0) = 0$ and $\frac{dg}{df} > 0$ for $0 \leq f < \infty$. Under these conditions, as pointed out by Zellner and Revankar, equation (III-20) has all the properties of a neoclassical production function.

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[12] Dhrymes and Kurz, op. cit., 308.

It should be noted that if equation (III-19) is a production function, then equation (III-20) is not itself a production function but a transformation of one since the units in which V is measured will not in general be those in which Y is measured.

The particular form of the transformation function g depends upon the nature of the pre-specified relation between the degree of returns to scale and the value of V. Specifically, Zellner and Revankar prove the following theorem for two inputs: "Let f(L,K) be a neoclassical production function homogeneous of degree α_f, a constant. Then a production function [sic], V = g(f), with preassigned returns to scale function (V), can be obtained by solving the following differential equations:

$$\text{(III-21)} \quad \frac{dV}{df} = \frac{V}{f} \frac{\alpha(V)}{\alpha_f}.\text{"}$$ [13]

They point out that generalization of this theorem to cases involving more than two inputs is direct.[14] Some specific returns-to-scale functions and their corresponding GPF's are discussed in the next section of this chapter.

III.B. Generalizations of the ZKD Results

In this section, the ZKD model is applied to generalized production functions (GPF's) based on n-factor Cobb-Douglas and CES functions. The result that ordinary least squares

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[13]Zellner and Revankar, op. cit., 242.

[14]Ibid., Note 2, 241.

estimates of the parameters of the function being generalized are consistent is shown to hold for these cases. Non-generalized n-factor Cobb-Douglas and CES functions are cited as special cases of these generalizations.

The Generalized n-factor Cobb-Douglas Function

Following the approach of ZKD, assume that a plant is generating electricity under a production function of the form:[15]

$$\text{(III-22)} \quad Y_i = A \prod_{r=1}^{n} X_{ri}^{\alpha_r} e^{u_{0i}}, \quad i = 1, \ldots, m,$$

where Y_i represents output of the i-th plant; X_{ri}, $r = 1, \ldots, n$, represents the rate of use of the r-th input for the i-th plant, and the u_{0i} are i.i.d. normal stochastic disturbances having zero mean. Further, assume that the prices of output and of the n inputs, denoted by p, $w_1, \ldots, w_n$, respectively, are distributed independent of the production function disturbance and have expected values p^+, $w_1^+, \ldots, w_n^+$, respectively.

In the general context of GPF's an alternative expression for equation (III-22) is given by:

$$\text{(III-23)} \quad g^{-1}(V_i) = A \prod_{r=1}^{n} X_{ri}^{\alpha_r} e^{u_{0i}},$$

where V_i is a transformation of the rate of output as defined by Zellner and Revankar, and $g^{-1}(\cdot)$ is the inverse of a transformation function having the properties listed immediately

[15]Zellner, Kmenta, and Drèze, op. cit., 786-789.

after equation (III-20).

The expected value of profit is defined by $E(\pi)$, where:

$$\text{(III-24)} \quad E(\pi) = p^{+}E(Y) - \sum_{r=1}^{n} w_r^{+} X_r .$$

Since the production function disturbance u_{0i} is normally distributed, the expected value of output is given by:

$$\text{(III-25)} \quad E(Y) = E[g^{-1}(V)] = A \prod_{r=1}^{n} X_r^{\alpha_r} e^{\frac{\sigma_{00}}{2}},$$

where σ_{00} is the variance of the production function disturbance u_{0i}. A proof of this result is given in Parzen.[16]

A necessary condition for maximization of $E(\pi)$ is given by:

$$\text{(III-26)} \quad \frac{\partial E(\pi)}{\partial X_r} = 0, \; r = 1, \ldots, n.$$

Two distinct types of stochastic disturbances will keep conditions (III-26) from being fulfilled exactly. The first, denoted by u_{ri}^{*}, is said by ZKD to result from managerial errors. The second, denoted by u_{ri}^{+}, is attributed to deviations between expected and realized prices. Assume that disturbances of the latter type are randomly distributed over plants so that:

$$\text{(III-27)} \quad \log \left(\frac{w_r^{+}}{p^{+}}\right) = \log \left(\frac{w_r}{p}\right) + u_{ri}^{+}.$$

Further, let $u_{ri} = u_{ri}^{*} + u_{ri}^{+}$.

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[16] E. Parzen, Modern Probability Theory and its Applications (New York: Wiley, 1960) 348.

Then, substituting equation (III-25) into equation (III-24), and differentiating with respect to X_{ri}, condition (III-26) implies n equations of the form:

$$\text{(III-28)} \quad \frac{\partial E(\pi)}{\partial X_{ri}} = \frac{p\alpha_r Y_i e^{\frac{\sigma_{00}}{2}}}{X_{ri} e^{u_{0i}}} - w_r = 0.$$

Rearranging terms, taking logarithms, recalling equation (III-27), and using the definition $Y_i \equiv g^{-1}(V_i)$, one obtains n equations of the form:

$$\text{(III-29)} \quad G_i - x_{ri} = k_{ri} + u_{ri} + u_{0i}, \quad r = 1, \ldots, n,$$

where $G_i = \log [g^{-1}(V_i)]$, $x_{ri} = \log X_{ri}$, and $k_{ri} = \log \frac{w_r}{\alpha_r p} - \frac{\sigma_{00}}{2}$.

Further, taking logarithms of both sides of equation (III-23), one obtains:

$$\text{(III-30)} \quad G_i - \sum_{r=1}^{n} \alpha_r x_{ri} = k_{0i} + u_{0i},$$

where $k_{0i} = \log A$.

In matrix notation, the system consisting of the (n+1) equations (III-29) and (III-30) may be written:

$$\text{(III-31)} \quad Ax = k + Bu,$$

where

$$A = \begin{pmatrix} A_{11} & \vdots & A_{12} \\ \cdots & & \cdots \\ A_{21} & \vdots & A_{22} \end{pmatrix}, \quad A_{11} = I_n, \quad A_{12} = \begin{pmatrix} 1 \\ \cdot \\ \cdot \\ \cdot \\ 1 \end{pmatrix}_{(n\times 1)},$$

$$A_{21} = [\alpha_n \alpha_{n-1} \cdots \alpha_2 \alpha_1]_{(1\times n)}, \quad A_{22} = 1,$$

$$X = \begin{pmatrix} -x_{ni} \\ -x_{n-1,i} \\ \cdot \\ \cdot \\ \cdot \\ -x_{2i} \\ -x_{1i} \\ G_i \end{pmatrix}, \quad K = \begin{pmatrix} k_{ni} \\ k_{n-1,i} \\ \cdot \\ \cdot \\ \cdot \\ k_{2i} \\ k_{1i} \\ k_{0i} \end{pmatrix}, \quad B = \left(\begin{array}{c|c} I_n & \begin{matrix} 1 \\ \cdot \\ \cdot \\ \cdot \\ 1 \end{matrix} \\ \hline 0 \; . \; . \; . \; 0 & 1 \end{array}\right), \quad U = \begin{pmatrix} u_{ni} \\ u_{n-1,i} \\ \cdot \\ \cdot \\ \cdot \\ u_{2i} \\ u_{1i} \\ u_{0i} \end{pmatrix}.$$

Applying a well-known formula, one may invert the partitioned matrix A, under the conditions that A_{11} and A_{22} are non-singular, as in the present case.[17] The solution for X is given by:

(III-32) $X = A^{-1}K + A^{-1}Bu.$

The elements of A^{-1} are not of importance to the result being derived. $A^{-1}B$ may be partitioned as:

$$A^{-1}B = \left(\begin{array}{cccc|c} & & I_n & & \emptyset_{(n\times 1)} \\ \hline -\dfrac{\alpha_n}{1-\sum_{r=1}^{n}\alpha_r} & -\dfrac{\alpha_{n-1}}{1-\sum_{r=1}^{n}\alpha_r} & \cdot\;\cdot\;\cdot & -\dfrac{\alpha_1}{1-\sum_{r=1}^{n}\alpha_r} & 1 \end{array}\right)$$

The $(n\times 1)$ null vector in $A^{-1}B$ implies that the x_{ri}, $r = 1, \ldots, n$, are independent of the u_{0i}, the production function

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[17] See, for example, C. G. Cullen, Matrices and Linear Transformations (Reading, Mass.: Addison-Wesley, 1966), 40.

disturbances. Therefore, ordinary least squares estimators for the parameters of the model given in equation (III-22) are unbiased and consistent. For the special case in which $V_i \equiv g(Y_i) = Y_i$, the derivation just completed proves the same result for an n-factor Cobb-Douglas production function which has not been generalizes according to the Zellner and Revankar procedure.

<u>The Generalized n-factor CES Function</u>

Generalization of the Hodges results to a generalized n-factor CES production function is similar to the derivation just completed.[18] To begin, assume that the i-th plant is generating electricity according to the following production function:

$$\text{(III-33)} \quad Y_i = A\left(\sum_{r=1}^{n} \alpha_r X_{ri}^{\rho}\right)^{\frac{\nu}{\rho}} e^{u_{0i}},$$

where Y_i and X_{ri}, $r = 1, \ldots, n$, have been defined above, and α_r, $r=1, \ldots, n$, A, ν, and ρ are the distribution, efficiency, scale, and substitution parameters, respectively. It should be noted that an implicit assumption in this version of the generalized CES function is that the elasticity of substitution between pairs of inputs is equal for all pairs of inputs. This restriction is made to simplify estimation.

In the context of GPF's, equation (III-33) may be

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[18]Hodges, <u>op</u>. <u>cit</u>.

re-written as:

$$\text{(III-34)} \quad g^{-1}(V_i) = A\left(\sum_{r=1}^{n} \alpha_r X_{ri}^{\rho}\right)^{\frac{\nu}{\rho}} e^{u_{0i}},$$

where V_i is a generalized production function as defined by Zellner and Revankar, and $g^{-1}(\cdot)$ is the inverse of a transformation function having the properties listed immediately after equation (III-20).

Under the ZKD assumptions regarding prices of the output and of the n inputs, the expected value of profit $E(\pi)$ is defined by equation (III-24). A necessary condition for the maximization of $E(\pi)$ is given by equation (III-26).

Since the disturbance term u_{0i} in the production function equation (III-33) is assumed to be normally distributed, the expected value of output is given by:

$$\text{(III-35)} \quad E(Y) = E[g^{-1}(V)] = A\left(\sum_{r=1}^{n} \alpha_r X_{ri}^{\rho}\right)^{\frac{\nu}{\rho}} e^{\frac{\sigma_{00}}{2}} = AZ^{\frac{\nu}{\rho}} e^{\frac{\sigma_{00}}{2}},$$

where σ_{00} is the variance of the production function disturbance, and $Z = \sum_{r=1}^{n} \alpha_r X_{ri}^{\rho}$.

Substituting equation (III-34) into equation (III-24) and applying condition (III-26) for maximization of $E(\pi)$, one obtains n equations of the form:

$$\text{(III-36)} \quad \left(\frac{g^{-1}(V_i)}{e^{u_{0i}}}\right)^{\frac{\nu}{\rho}-1} X_{ri}^{\rho-1} = \frac{w_r^{+}\rho A^{(\frac{\nu}{\rho}-2)}}{p^{+}\alpha_r \nu e^{\frac{\sigma_{00}}{2}}}.$$

These equations are first-order conditions for maximization of $E(\pi)$ by the i-th firm with respect to each of the n factors. The reader will note that before substitution, equation (III-34) was divided through by $e^{u_{0i}}$ so that both sides of equation (III-36) would be non-stochastic.

Applying the definitions of u_{ri}^{+}, u_{ri}^{*}, and u_{ri} given above and taking logarithms of both sides of equation (III-36), the conditions for maximization of $E(\pi)$ imply n input demand functions of the form:

$$\text{(III-37)} \quad (\frac{\nu}{\rho}-1)\ln[g^{-1}(V_i)]+(\rho-1)\ln X_{ri} = c'_{ri}+(\frac{\nu}{\rho}-1)u_{0i}+u_{ri},$$

where $c'_{ri} = \ln\left(\frac{w_r\rho A^{(\frac{\nu}{\rho}-2)}}{p\nu\alpha_r}\right) - \frac{\sigma_{00}}{2}.$

So far, the derivation has proceeded in a manner precisely analogous to the derivation of the equation system (III-29) in the proof of the ZKD result for the n-factor Cobb-Douglas production function. However, the non-linear nature of the CES function necessitates a different approach to the solution for the input demand equations. The n equations represented by (III-37) plus the production function given by equation (III-34) form a system of (n+1) equations which may be solved simultaneously in the following manner for the (n+1) unknowns X_{ri}, r = 1, . . ., n, and Y_i.

Any pair of the n input equations (III-37), e.g., the r-th and the s-th, $r \neq s$, may be solved simultaneously for X_{si}

in terms of X_{ri}. In general,

$$\text{(III-38)} \quad X_{si} = e^{\frac{(c'_{ri}-c'_{si}+u_{ri}-u_{si})}{(\rho-1)}} X_{ri}$$

Equation (III-38) may be used to express all inputs except the r-th in terms of the r-th. In this way equation (III-34) may be re-written so that no input except X_{ri} appears in the production function, which is now given by:

$$\text{(III-39)} \quad g^{-1}(V_i) = A\{X_{ri}^{\rho}[\alpha_r + \sum_{s\neq r} \alpha_s e^{\frac{(c'_{ri}-c'_{si}+u_{ri}-u_{si})\rho}{(\rho-1)}}]\}^{\frac{\nu}{\rho}} e^{u_{0i}}.$$

An equivalent logarithmic transformation of the production function is given by:

$$\text{(III-39a)} \quad \ln[g^{-1}(V_i)] = \ln A + \nu \ln X_{ri}$$

$$+\frac{\nu}{\rho}\ln[\alpha_r + \sum_{s\neq r} e^{\frac{(c'_{ri}-c'_{si}+u_{ri}-u_{si})\rho}{(\rho-1)}}]+u_{0i}.$$

Substituting equation (III-39a) into equation (III-37), one obtains the following solution for $\ln X_{ri}$:

$$\text{(III-40)} \quad \ln X_{ri} = [(1-\frac{\nu}{\rho})\nu+(\rho-1)]^{-1}\{c'_{ri}+(1-\frac{\nu}{\rho})\ln A-(\frac{\nu}{\rho}-1)^{\frac{\nu}{\rho}} \times$$

$$\ln[\alpha_r + \sum_{s\neq r} e^{\frac{(c'_{ri}-c'_{si}+u_{ri}-u_{si})\rho}{(\rho-1)}}]+u_{ri}\}.$$

Since the solution for $\ln X_{ri}$ does not involve u_{0i}, factor inputs are seen to be independent of the production function disturbance term, implying consistency of single-equation estimators for the parameters of the generalized n-factor CES function. The special case of this derivation in which $V_i \equiv g(Y_i) = Y_i$ is an application of the ZKD results to the non-generalized n-factor CES function.

III.C. The Alternative Models To Be Considered

In this section the models of steam-electric generation to be considered in this thesis are listed and the techniques to be used for their estimation explained. The models to be considered fall into two groups--those which are applications of the generalized ZKD results presented in the preceding section, and those which have been used by previous investigators of steam-electric generation, including the fixed-relative-factor-proportions model of Hart and Chawla.

Applications of the Generalized ZKD Model

Six specific models are presented as special cases of the results of the preceding section. These include three-factor Cobb-Douglas and CES functions, and two specific GPF-s based on each of these functions.

Cobb-Douglas

The simplest production function to be considered is the three-factor Cobb-Douglas function, which may be denoted by CD. Following the ZKD approach, it may be written as:

(III-41) $$Y_i = \gamma K_i^{\alpha_1} F_i^{\alpha_2} L_i^{\alpha_3} e^{u_i},$$

where Y_i, K_i, F_i, and L_i represent the output of electricity and inputs of capital, fuel, and labor, respectively, for the i-th firm.[19] The u_i are i.i.d. normal random disturbances having zero mean, and γ, α_1, α_2, and α_3 are the production function parameters.

Production function (III-41), the CD, is easily estimated by ordinary least squares (OLS) after a logarithmic transformation has been performed, yielding:

(III-41a) $\ln Y_i = \ln \gamma + \alpha_1 \ln K_i + \alpha_2 \ln F_i + \alpha_3 \ln L_i + u_i$.

According to the generalization of the ZKD model presented in the preceding section of this chapter, OLS estimators of the parameters of the CD function are unbiased and consistent.

CES

Under the generalization of the Hodges result carried out in the preceding section of this chapter, OLS estimates of the parameters of an n-factor CES function are consistent under the ZKD assumptions. Setting n = 3, the generalization implies that OLS estimates of the parameters of the following functions are unbiased and consistent, under the ZKD assumptions:

(III-42) $Y_i = \gamma[\alpha_1 K_i^{\rho} + \alpha_2 F_i^{\rho} + \alpha_3 L_i^{\rho}]^{\frac{\nu}{\rho}} e^{u_{0i}}$,

where the variables are as defined above. γ, α_r, $r = 1,2,3,\rho$, and ν, are the efficiency, distribution, substitution, and scale parameters, respectively. u_{0i} is a stochastic disturbance, i.i.d. as normal with zero mean.

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[19] Discussion of the definitions and measurement techniques for output and the three inputs is contained in the appendix to this chapter.

It is obvious that equation (III-42) is non-linear and therefore difficult to estimate in its present form. Kmenta used a Taylor series expansion around $\rho = 0$ to develop an approximation to a two-factor version of equation (III-42) which is linear in ρ.[20] This approximation can be estimated by ordinary least squares. To generalize the approximation to the three-factor case, one first takes logarithms of both sides of equation (III-42), which yields:

(III-43) $\ln Y_i = \ln \gamma + \frac{\nu}{\rho} A + u_{0i},$

where $A = \ln (\alpha_1 K_i^\rho + \alpha_2 F_i^\rho + \alpha_3 L_i^\rho)$. Approximating A by the first two terms of a Taylor series about $\rho = 0$, and substituting this approximation for A into equation (III-43), one has:

(III-44) $$\ln Y_i = \ln\gamma + \nu(\alpha_1 \ln K_i + \alpha_2 \ln F_i + \alpha_3 \ln L_i) + \frac{1}{2} \rho\nu[\alpha_1\alpha_2(\ln K_i - \ln F_i)^2 + \alpha_1\alpha_3(\ln K_i - \ln L_i)^2 + \alpha_2\alpha_3(\ln F_i - \ln L_i)^2] + u_{0i},$$

which may be estimated by ordinary least squares, in the following form, which will be denoted CES:

(III-45) $$\ln Y_i = \beta_0 + \beta_1 \ln K_i + \beta_2 \ln F_i + \beta_3 \ln L_i + \beta_4(\ln K_i - \ln F_i)^2 + \beta_5(\ln K_i - \ln L_i)^2 + \beta_6(\ln F_i - \ln L_i)^2 + u_i,$$

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[20] J. Kmenta, "On Estimation of the CES Production Function," International Economic Review, 8 (June, 1967), 180-194.

where $\beta_0 = \ln \gamma$

$\beta_1 = \nu\alpha_1$

$\beta_2 = \nu\alpha_2$

$\beta_3 = \nu\alpha_3$

$\beta_4 = \frac{1}{2} \rho\nu\alpha_1\alpha_2$

$\beta_5 = \frac{1}{2} \rho\nu\alpha_1\alpha_3$

$\beta_6 = \frac{1}{2} \rho\nu\alpha_2\alpha_3$

Under the constraint $\sum_{r=1}^{3} \alpha_r = 1$, it is clear that the sum of the estimates of β_1, β_2, and β_3 is a consistent estimate of ν. Using carats to denote estimates of the coefficients of equation (III-45), the variance of $\hat{\nu}$ defined in this way is equal to $\mathrm{Var}(\hat{\beta}_1) + \mathrm{Var}(\hat{\beta}_2) + \mathrm{Var}(\hat{\beta}_3) +$

$$2[\mathrm{Cov}(\hat{\beta}_1,\hat{\beta}_2) + \mathrm{Cov}(\hat{\beta}_1,\hat{\beta}_3) + \mathrm{Cov}(\hat{\beta}_2,\hat{\beta}_3)].$$

Thus, under the assumption of normality of the u_{0i}, statistics are defined to test for a deviation of ν, the returns to scale parameter, from unity. Tests for differences in the value of $\ln \gamma$ across technologically homogeneous cells give an indication of the effect of technological change on the production function, provided inputs and output are measured in the same respective units in all cells. Finally, as in the two-factor case, the approximation may be divided into two groups of terms--the first four, which conform to a Cobb-Douglas function; and the last three, which "correct" the Cobb-Douglas for the effect of non-unitary pairwise elasticities of substitution. An F-test of the joint significance of β_4, β_5, and β_6 is a test of the deviation of the estimated CES function from Cobb-Douglas form.

GPF's Based on Cobb-Douglas and CES Functions

In Section A of this chapter, the notion of the generalized production function as defined by Zellner and Revankar was explained. The reader will recall that using their procedure for generalization, a neoclassical production function may be transformed to make returns to scale a pre-assigned function of the transformed rate of output. Zellner and Revankar considered two specific generalizations of a neoclassical production function f.[21] In this thesis both those generalizations will be applied to the CD and CES models, as given by equations (III-41a) and (III-45), respectively.

The first generalization to be considered is derived from a pre-assigned returns-to-scale function $\alpha(V)$, where

(III-46) $\alpha(V) = \alpha_f + h(\frac{\alpha - V}{\alpha + V}),\ \alpha_f > h \geq 0.$

V is the value of the generalized production function, defined by equation (III-20). α_f is the degree of homogeneity of f, the neoclassical production function to be generalized; a and h are constant parameters. Inspection of equation (III-46) reveals that returns to scale are $(\alpha_f + h)$ when $V = 0$ and approach the limit $(\alpha_f - h)$ as V approaches infinity.

To apply Zellner and Revankar's procedure for defining generalized production functions (GPF's), one substitutes equation (III-46) into the differential equation (III-21). Solution of the resulting equation defines the following function:

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[21]Zellner and Revankar, op. cit., 244 and 246ff.

$$\text{(III-47)} \quad Y^{\frac{1}{1+r}}[(1+r)a+(1-r)Y]^{\frac{2r}{1-r^2}} = kf$$

where k is a constant of integration and $r = \frac{h}{\alpha_f}$. For the CD and CES models, $\alpha_f = \sum_{i=1}^{3} \alpha_i$. As Zellner and Revankar point out, testing the hypothesis that r = 0 is a test of the hypothesis that returns to scale are independent of output.

Applying this transformation to the three-factor version of the stochastic Cobb-Douglas function presented above, one has:

$$\text{(III-48)} \quad Y_i^{\frac{1}{1+r}}[(1+r)a+(1-r)Y_i]^{\frac{2r}{1-r^2}} = \Gamma K_i^{\alpha_1} F_i^{\alpha_2} L_i^{\alpha_3} e^{u_{0i}},$$

where $\Gamma = k\gamma$, and all other variables and parameters are as defined above. Taking natural logs of both sides of equation (III-48), one obtains the function:

$$\text{(III-49)} \quad \frac{1}{1+r} \ln Y_i + \frac{2r}{1-r^2} \ln [(1+r)a+(1-r)Y_i] = \ln \Gamma + \alpha_1 \ln K_i + \alpha_2 \ln F_i + \alpha_3 \ln L_i + u_{0i}.$$

Estimation of equation (III-49), which may be called CD1, is carried out according to the following method, which Ramsey and Zarembka call "two-stage maximum likelihood."[22]

If one knew the values of a and r, one could estimate the remaining parameters of equation (III-49) by using the entire left-hand side as the dependent variable. Although knowledge of a and r is not available, consistent estimates of them may be found using the following approach. These estimates

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[22]Ramsey and Zarembka, op. cit.

will be maximum-likelihood under the usual assumptions about the distribution of the u_{0i}. Substitute for γ, α_1, α_2, and α_3 in the equation (III-49) their maximum-likelihood estimators as functions of the left-hand side, so that the likelihood function appears as a function of only the data and the unknown parameters a and r. One then chooses estimates $\tilde{a}$ and $\tilde{r}$ such that the likelihood function so derived is maximized. Estimates of ln Γ and the α_i, i - 1,2,3, conditional on the choice of $\tilde{a}$ and $\tilde{r}$ are obtained by OLS estimation of the following:

$$\text{(III-49a)} \quad \frac{1}{1+\tilde{r}} \ln Y_i + \frac{2\tilde{r}}{1-\tilde{r}^2} \ln [(1+\tilde{r})a+(1-\tilde{r})Y_i] = \ln \Gamma + \alpha_1 \ln K_i + \alpha_2 \ln F_i + \alpha_3 \ln L_i + u_{0i}.$$

Applying the results of Box and Cox, confidence intervals for $\tilde{a}$ and $\tilde{r}$ may be obtained using the fact that $-2 \ln \ell$, where ℓ is the likelihood function, is asymptotically distributed as Chi-square with k degrees of freedom, where k is the number of parameters to be estimated.[23] This information may be used to test for constancy of the returns to scale parameter over the observed range of output. Comparison of estimates of ln Γ across technologically homogeneous cells will provide an indication of the effects on the production function of technological change. Returns to scale functions may be calculated and compared across cells using equation (III-46) and

[23]G. E. P. Box and D. R. Cox, "An Analysis of Transformations," Journal of the Royal Statistical Society, Ser. B, 26 (1964), 214-219.

the parameter estimates obtained for CD1.

A similar procedure may be used to examine a CES function, generalized by applying the specific transformation defined by equation (III-47). Using the Taylor series approximation discussed above for the three-factor CES function, and Ramsey and Zarembka's "two-stage maximum likelihood" extimation procedure, the stochastic specification to be used is

(III-50)

$$\frac{1}{1+\tilde{r}} \ln Y_i + \frac{2\tilde{r}}{1-\tilde{r}^2} \ln [(1+\tilde{r})a+(1-\tilde{r})Y_i] = \beta_0+\beta_1 \ln K_i+\beta_2 \ln F_i + \beta_3 \ln L_i + \beta_4(\ln K_i - \ln F_i)^2 + \beta_5(\ln K_i - \ln L_i)^2 + \beta_6(\ln F_i - \ln L_i)^2 + u_i,$$

where all variables and parameters are as defined above, and the u_i are assumed to be i.i.d. as $N(\emptyset,\sigma^2 I)$. This model may be called CES1.

A second generalized function examined by Zellner and Revankar is derived from a returns-to-scale function defined by:

(III-51) $$\alpha(V) = \frac{\alpha_f}{1+\theta V},$$

where $\alpha_f > 0$ is the degree of homogeneity of the function being generalized. For $\theta > 0$, returns to scale fall from α_f at $V = 0$, to zero as $V\to\infty$. Applying the Zellner and Revankar method of generalizing neoclassical production functions, denoted by f, the GPF with returns to scale behaving according to equation (III-51) is given by:

(III-52) $$Ye^{\theta Y} = c^h f^h,$$

where c and h are constants.

Applying the transformation defined by equation (III-52) to a three-factor Cobb-Douglas function, adding a multiplicative random disturbance normally distributed with zero mean, and taking logarithms, one obtains:

(III-53) $$\ln Y_i + \theta Y_i = \ln \gamma + \alpha_1 \ln K_i + \alpha_2 \ln F_i + \alpha_3 \ln L_i + u_{0i},$$

where i denotes the number of the observation, and all the other terms are defined as previously. Under the ZKD assumptions the logarithm of the likelihood function, $\ln \ell$, is given by:

(III-54) $$\ln \ell = \text{const.} - \frac{N}{2} \ln \sigma^2 + \sum_{i=1}^{N} \ln (1+\theta Y_i) - \frac{1}{2\sigma^2}[z_i(\theta) - \ln \gamma - \alpha_1 \ln K_i - \alpha_2 \ln F_i - \alpha_3 \ln L_i]^2.$$

where N is the number of observations and $z_i(\theta) = \ln Y_i + \theta Y_i$. Maximizing $\ln \ell$ with respect to σ^2, and substituting the conditional maximum-likelihood estimate of σ^2 thus obtained into equation (III-53), one obtains:

(III-55) $$\ln \ell^* = \text{const.} - \frac{N}{2} \ln \{ \sum_{i=1}^{n} [z_i(\theta) - \ln \gamma - \alpha_1 \ln K_i - \alpha_2 \ln F_i - \alpha_3 \ln L_i]^2 \} + \sum_{i=1}^{N} \ln(1+\theta Y_i).$$

For $\theta = \theta_0$, it is evident from equation (III-55) that maximization of ln ℓ^* is equivalent to estimation of the following regression by ordinary least squares:

$$\text{(III-56)} \quad \ln Y_i + \theta_0 Y_i = \ln \gamma + \alpha_1 \ln K_i + \alpha_2 \ln F_i + \alpha_3 \ln L_i + u_{0i}.$$

This model may be denoted as CD2. By repeating this procedure for various values of θ, one can find the estimates of the parameters θ, γ, α_1, α_2, and α_3 associated with the global maximum of the likelihood function. These estimates will be maximum-likelihood if the $u_{0i} \sim N(\emptyset, \sigma^2 I)$. Zellner and Revankar attribute this "maximum-likelihood search procedure" to Box and Cox.[24]

Similarly, under the ZKD assumptions about the stochastic nature of the productive process, maximum-likelihood estimates are available for the parameters of an approximation to a generalized three-factor CES function, under the same generalization as employed in equation (III-56). This approximation is given by:

$$\text{(III-57)} \quad \ln Y_i + \theta_0 Y_i = \beta_0 + \beta_1 \ln K_i + \beta_2 \ln F_i + \beta_3 \ln L_i + \beta_4 (\ln K_i - \ln F_i)^2 + \beta_5 (\ln K_i - \ln L_i)^2 + \beta_6 (\ln F_i - \ln L_i)^2 + u_i,$$

where all variables and parameters are as defined above. This function may be called CES2.

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[24]Box and Cox, op. cit.

As in the case of the generalization considered in equation (III-49a), a test is available for the hypothesis that the returns to scale parameter is constant over the observed range of output. Such a test may be based on the fact that $-2 \ln \ell$ is distributed as Chi-square, where ℓ is defined by equation (III-54). Degrees of freedom for the Chi-square test are 5 for CD2 and 8 for CES2.

Models Used by Previous Researchers

In addition to the CD, CES, CD1, CES1, CD2, and CES2 models discussed above, alternative versions of the models estimated by Hart and Chawla and by Ling will also be considered in this thesis. A discussion of each of these models follows:

The Hart and Chawla Model

As explained in the review of the literature in Chapter I.B., the model used by Hart and Chawla is a fixed-relative-factor-proportions model. The reader will recall that under the assumptions that all inputs are fully utilized and that inputs are combined in fixed relative proportions, the exponents of all factors are equal and determine the degree of homogeneity of the production function. No matter which component of equation (I-18)' or (I-19)' is considered, the value of β will be the same.

Two characteristics of the three-factor approach employed in the present study preclude use of their model as presented.

First, the measurement of capacity has not been converted for unused capacity. Second, it seems unrealistic to assume that labor is fully utilized. Therefore, the empirical analysis of the fixed-relative-proportions function will be carried out under different assumptions. Assuming a multiplicative stochastic disturbance term and using the notation of the present study, the following equation will be considered:

$$\text{(III-58)} \quad Y = \min[\gamma_1 K_i^{\beta_1} e^{u_{1i}}, \gamma_2 F_i^{\beta_2} e^{u_{2i}}, \gamma_3 L_i^{\beta_3} e^{u_{3i}}].$$

Under this production function the inputs are assumed to be combined in fixed proportions, determined by the proportional values of γ_1, γ_2, and γ_3. If factors are combined in these proportions, Y is given by the minimum of the three components of equation (III-58), which may be called the "relevant component." To examine the degree of homogeneity of equation (III-57), assume that $\beta_2 < \beta_1, \beta_3$, so that for input levels K_{i0}, F_{i0}, and L_{i0}:

$$Y_i^0 = \min[\gamma_1 K_{i0}^{\beta_1} e^{u_{1i}}, \gamma_2 F_{i0}^{\beta_2} e^{u_{2i}}, \gamma_3 L_{i0}^{\beta_3} e^{u_{3i}}] = \gamma_2 F_{i0}^{\beta_2} e^{u_{2i}}.$$

Assume that all inputs are increased by the same proportion t. Then

$$Y_i^1 = \min[\gamma_1 (tK_{i0})^{\beta_1} e^{u_{1i}}, \gamma_2 (tF_{i0})^{\beta_2} e^{u_{2i}}, \gamma_3 (tL_{i0})^{\beta_3} e^{u_{3i}}]$$

$$= \gamma_2 t^{\beta_2} F_{i0}^{\beta_2} e^{u_{2i}} = t^{\beta_2} Y_i^0.$$

Therefore, the degree of homogeneity of this fixed-relative-proportions production function is β_2, the factor exponent in the relevant component term of the equation.

The rate of underutilization of another factor, e.g., capital, may be defined as the ratio of actual output to potential output if capital were the "limiting" component. Algebraically, at input levels K_{i0}, F_{i0}, and L_{i0}, that ratio is given by $U_0 = \frac{\gamma_2 F_{i0}^{\beta_2} e^{u_{2i}}}{\gamma_1 K_{i0}^{\beta_1} e^{u_{1i}}}$. To examine the degree of homogeneity of the rate of underutilization of capital, one may observe that $U_1 = \frac{\gamma_2 t^{\beta_2} F_i^{\beta_2} e^{u_{2i}}}{\gamma_1 t^{\beta_1} K_i^{\beta_1} e^{u_{1i}}} = t^{(\beta_2 - \beta_1)} U_0$. Therefore, the degree of homogeneity of the rate of underutilization of capital is $(\beta_2 - \beta_1)$. Similarly, the rate of underutilization of labor is proportional to $(\beta_3 - \beta_1)$.

The three-factor modification of the Hart and Chawla model given by equation (III-58), with the properties explained immediately above, will be considered further in the empirical procedure of this study. For purposes of estimation, it is convenient to consider it in the following form:

(III-58a) $Y_i = \gamma_1 K_i^{\beta_1} e^{u_{1i}}$ (UHCC)

(III-58b) $Y_i = \gamma_2 F_i^{\beta_2} e^{u_{2i}}$ (UHCF)

(III-58c) $Y_i = \gamma_3 L_i^{\beta_3} e^{u_{3i}}$ (UHCL)

The reader will recall that after developing their model analogously to the development of equation (III-58), Hart and Chawla chose to estimate the model under the assumption that

Y_i was exogenously determined. For purposes of comparison with their results, the corresponding modification of equation (III-58) will also be considered in the present study. In this form the model is given by:

(III-59a) $K_i = \Gamma_1 Y_i^{\alpha_1} e^{v_{1i}}$ (EHCC)

(III-59b) $F_i = \Gamma_2 Y_i^{\alpha_2} e^{v_{2i}}$ (EHCF)

(III-59c) $L_i = \Gamma_3 L_i^{\alpha_3} e^{v_{3i}}$ (EHCL)

where all variables are as defined above. It should be noted that $\Gamma_j = \gamma_j^{-\frac{1}{\beta j}}$ and $\alpha_j = \frac{1}{\beta_j}$, j = 1,2,3. Furthermore, if the u_{ji}, j = 1,2,3, i = 1, . . ., n, are assumed to be i.i.d. as $N(\emptyset,\sigma^2 I)$, then the v_{ji}, j = 1,2,3, i = 1, . . ., n, are i.i.d. as $N(\emptyset,\frac{\sigma^2}{\beta_j^2} I)$. These results are apparent from solution of the j-th equation of (III-58) for the regressor in that equation, which yields the j-th equation of (III-59).

For both these models, OLS estimators are consistent. The estimation may be carried out on logarithmic transformations of the original functions, given by the following:

(III-60a) $\ln Y_i = \ln \gamma_1 + \beta_1 \ln K_i + u_{1i}$ (LUHCC)

(III-60b) $\ln Y_i = \ln \gamma_2 + \beta_2 \ln F_i + u_{2i}$ (LUHCF)

(III-60c) $\ln Y_i = \ln \gamma_3 + \beta_3 \ln L_i + u_{3i}$ (LUHCL)

(III-61a) $\ln K_i = \ln \Gamma_1 + \alpha_1 \ln Y_i + v_{1i}$ (LEHCC)

(III-61b) $\ln F_i = \ln \Gamma_2 + \alpha_2 \ln Y_i + v_{2i}$ (LEHCF)

(III-61c) $\ln L_i = \ln \Gamma_3 + \alpha_3 \ln Y_i + v_{3i}$ (LEHCL)

The reader will recall from Chapter I.B. that according to Hart and Chawla, possible causes of differences between $\hat{\beta}_1$, $\hat{\beta}_2$, and $\hat{\beta}_3$ include measurement errors in one or more of the variables and inappropriateness of the fixed-relative-proportions model. The methods to be explained in Stages One and Two of the empirical procedure are designed to test for specification errors such as these. Therefore, statistical inference in Stage Three may be carried out on the presumption that underutilization of one or more factors accounts for any observed statistically significant differences between $\hat{\beta}_1$, $\hat{\beta}_2$, and $\hat{\beta}_3$.

A Modification of the Ling Model

The final study discussed in Chapter I.B. was the synthesis of engineering and economic principles by Ling that led to estimation of the average cost function reproduced here for the convenience of the reader:

(I-19) $C_a = kS^n PF^{(m+p \ln PF)}$

where C_a represents annual average generating cost in mills per kilowatt-hour, PF represents system plant factor, S system installed capacity, and k, n, m, and p are parameters to be estimated.

The present study is not concerned with cost functions. However, under the assumption that C_a is proportional to plant heat rate, a modification of equation (I-19) may be made. It was explained in Chapter I.B. that Ling treated heat rate as a decreasing function of capacity and load factor because turbogenerators operate more efficiently if they are large units being operated near full capacity. To test both the validity and importance of this assumption, one may consider an alternative to the Ling function, with heat rate, a, replacing C_a in equation (I-19). This replacement yields the following:

(III-62) $a_i = kS_i^n PF_i^{(m+p \ln PF)}$

Taking logarithms of both sides of equation (III-62) and including a disturbance term, one obtains the following function, which may be estimated by ordinary least squares:

(III-63) $\log a_i = \ln k + n \ln S_i + m \ln PF_i + p(\ln PF_i)^2 + u_i$ (LING)

Models Involving the Number of Units per Plant

As was apparent in Chapter I.B., previous economic literature on the subject of steam-electric generation has been concerned with operation of the firm, the generating plant, and the individual unit. Since the present study is not concerned with the effects of managerial decisions on generating efficiency but only with the effects of changes in technology, there is no need to aggregate data to the firm level. Since the generally available operating data are aggregated to the plant level, the focus of this study is at that level. Nevertheless, it was considered desirable to incorporate into the analysis the fact that a given value of plant generating capacity may be embodied in various numbers of turbogenerator units.

Allocation of a given quantity of generating capacity among a larger number of machines may have several effects on plant efficiency. First, an implication of this procedure is that if one of the units is taken out of operation for any reason, a larger percentage of total capacity remains in use. Therefore, over a one-year period as regular preventive maintenance takes place on each unit in turn, a larger percentage of capacity is in use at any point in time. When annual operating data on the plant are considered, the result is an observed increase in plant efficiency. Second, since a malfunction in one unit takes a smaller proportion of total capacity out of commission, the plant may operate with a smaller quantity of generating capacity in reserve against loss of load. This effect also contributes to observed efficiency of plant operation. However, the third effect of installing a given plant capacity among more units is that the average capacity of the units decreases. It was explained in Chapter II.A. that smaller-capacity units are not likely to operate as efficiently as larger ones. For this reason, one would expect plant efficiency to decrease as more units comprise a plant of given installed capacity. In order to examine the joint significance of these three effects on plant efficiency, one may incorporate N_i, the number of units in the i-th plant, into the analysis as an exogenous variable.

One method of accomplishing this is to multiply a production function such as those discussed so far in this chapter by $e^{\delta N_i}$. A test of the statistical significance of δ is a test of the null hypothesis that the net effect of differences in the number of units per plant is insignificant. It was decided to include this term in some of the production functions discussed above. Of course, when logarithmic transformations are made to facilitate estimation, the term enters additively as δN_i. This line of argument justifies consideration of the following group of functions, in addition to those already discussed:

(III-64) $\ln Y_i = \ln \gamma + \alpha_1 \ln K_i + \alpha_2 \ln F_i + \alpha_3 \ln L_i + \delta N_i + u_i$ (CD/N)

(III-65) $\ln Y_i = \beta_0 + \beta_1 \ln K_i + \beta_2 \ln F_i + \beta_3 \ln L_i + \delta N_i + \beta_4(\ln K_i - \ln F_i)^2 + \beta_5(\ln K_i - \ln L_i)^2 + \beta_6(\ln F_i - \ln L_i)^2 + u_i$ (CES/N)

(III-66) $Y_i = \min[\gamma_1 K_i^{\beta_1} e^{\delta_1 N_i} e^{u_{1i}}, \gamma_2 F_i^{\beta_2} e^{\delta_2 N_i} e^{u_{2i}}, \gamma_3 L_i^{\beta_3} e^{\delta_3 N_i} e^{u_{3i}}]$

(III-66a) $\ln K_i = \ln \Gamma_1 + \alpha_1 \ln Y_i + \delta_1 N_i + v_{1i}$ (LEHCC/N)

(III-66b) $\ln F_i = \ln \Gamma_2 + \alpha_2 \ln Y_i + \delta_2 N_i + v_{2i}$ (LEHCF/N)

(III-66c) $\ln L_i = \ln \Gamma_3 + \alpha_3 \ln Y_i + \delta_3 N_i + v_{3i}$ (LEHCL/N)

All variables and parameters are as defined above.

For each of these models it is assumed that the multivariate distribution of the stochastic disturbance term conditional on the observed regressor matrix is $N(\emptyset, \sigma^2 I)$. **The potential models are summarized in Table 15,** Appendix D.

It should be mentioned that the alternatives are not mutually exclusive. For example, a non-stochastic fixed-relative-factor-proportions model is a special case of a non-stochastic CES model, in which all pairwise elasticities of substitution are constrained to zero. Similarly, the CD function given by equation (III-41a) is a special case of the CES function in which all pairwise elasticities of substitution are constrained to unity. The CES function is itself a special case of CES2, in which $\theta_0 = 0$.

Having given these model specifications, one may state the maintained hypothesis under which the remainder of the present study is to be carried out. That maintained hypothesis is that at least one of these models is appropriate for the purpose of statistical inference about the effects of changes in scale and technology in the steam-electric generating plant. A precise definition of "appropriate" is given in the next section of this chapter. Temporarily, an appropriate model may be thought of as one for which the stochastic assumptions are not contradicted by the sample data on which statistical inferences are to be based.

The tasks remaining in this thesis may be broadly grouped into three stages--restriction of the maintained hypothesis to a subset of the alternative models presented above; independent confirmation of the chosen alternative(s) and formulation of testable hypotheses about the effects of changes in scale, technology, and the number of turbogenerator units per steam-electric generating plant; and tests of these hypotheses. Procedures for accomplishing these tasks are discussed in the next section of this chapter.

III.D. Methodology of the Present Study

As pointed out at the end of the preceding section of this chapter, a maintained hypothesis has been defined by listing a set of alternative models of the steam-electric generating process and stating the assumption that at least one of them is appropriate for statistical inference. The procedures for making statistical inferences about steam-electric generation under this maintained hypothesis may be conveniently discussed in the three stages mentioned at the end of the preceding section.

Data for the Present Study

Before discussing the three stages of empirical analysis, it is desirable to discuss briefly the data used for the present study. The data consisted of two parts. The first part was a summary of design characteristics of new turbo-generator units installed in privately-owned central-station power plants between 1948 and 1965, as reported in the "Modern Plant Design Surveys" published during that period.[25] The second part was a collection of annual data on output of electricity and inputs of fuel, capital, and labor for central-station plants which first operated during one of the

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[25] Power, New York: McGraw-Hill, monthly (1948-1965).

years 1948 to 1965. These data were collected from the Federal Power Commission series, Steam-Electric Plant Construction Cost and Annual Production Expenses.[26] Thus, the data included a virtually complete survey of the design characteristics of all turbogenerator units installed between 1948 and 1965, as well as annual production data on the 175 plants in which the units were installed. Care was taken to record additions of turbogenerator units to the plants being observed, or other changes in those plants. A detailed discussion of the data is contained in Appendix C.

If the design characteristics of the original equipment of an observed plant were those of one of the cells defined in Table 4, that plant was placed in the appropriate cell and a time series of annual production data for the plant was compiled. Otherwise, the plant was ignored. The time series for each included plant was extended until 1965 or until a turbogenerator unit was added that did not possess the characteristics defining the cell. In this way, the production data for each cell consisted of a panel of time-series observations on a cross-section of technologically homogeneous central-station plants. The number of observations for each cell appears in Table 4.

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[26] Federal Power Commission, Steam-Electric Plant Construction Costs and Annual Production Expenses (Washington, D. C., published annually).

Stage One--Restriction of the Maintained Hypothesis

Two objectives have been met to this point in the present study. In Chapter II.B. a set of seven cells was defined and used to classify a sample of annual production data on steam-electric generating plants. In Chapter III.C. twelve alternative models of the steam-electric generation process were stated. The problem addressed in Stage One is the selection of one or more of the twelve alternatives as appropriate for statistical inference for each of the seven cells.

All twelve alternatives are to be estimated by ordinary least squares, except the generalized production functions CD1, CES1, CD2, and CES2, given by equations (III-49), (III-50), (III-56), and (III-57), respectively. The latter are to be estimated by a two-step procedure, in which the second step is ordinary least squares. The problem in Stage One is to determine which member or members of the set of alternatives are "appropriate," in a sense to be defined below. This problem has been considered for the OLS case in a paper by Ramsey and in one by Ramsey and Zarembka.[27] One approach to the problem involves the use of tests developed by Ramsey for detecting specification error in ordinary least squares regression analysis.[28]

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[27]J. B. Ramsey, "Models, Specification Error, and Inference: A Discussion of some Problems in Econometric Methodology," Michigan State University Econometrics Workshop Paper #6714 (1967); Ramsey and Zarembka, op. cit.

[28]Ramsey, "Tests for Specification Errors," op. cit., 350-371.

To understand the application of the tests to the problem of choice from a set of alternative proposed structural relationships, consider an ordinary least squares regression model of the form:

(III-67) $y = \beta X + u$,

where y is an $(N \times 1)$ regressand vector, X is an $(N \times K)$ regressor matrix, β is a $(K \times 1)$ vector of regression coefficients, and u is the $(N \times 1)$ vector of stochastic disturbances. To complete the specification of the model, researchers ordinarily assume that the model (III-67) satisfies the full ideal conditions, as defined by Anscombe and Tukey, namely:[29]

i) X is of rank K.

ii) The multivariate distribution of u conditional on the observed regressor matrix X is $N(\emptyset, \sigma^2 I)$.

This assumption is made for each of the twelve alternative models being considered in the present study. If the full ideal conditions are met for model (III-67) in a given data sample, then the model is appropriate for that sample. If not, then at least one of a number of different types of specification error has been committed.

Ramsey has derived the distributions of the vector u under specification errors of the following types: omitted variables, incorrect functional form, omitted relevant

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[29]F. J. Anscombe and J. W. Tukey, "The Examination and Analysis of Residuals," Technometrics 5 (1963), 141-160.

simultaneous equations, and heteroskedasticity.[30] In the same paper, tests for specification error are presented, based on Theil's optimal residual vector.[31] These tests are called RESET, RASET, BAMSET, KOMSET, and WSET. For any linear least-squares model of the form (III-67), the tests may be applied to the null hypothesis that the model satisfies the full ideal conditions, i.e., that the model is appropriate for the given sample. The alternative is that the full ideal conditions are not satisfied by the model, i.e., that at least one type of specification error has been committed.

Unfortunately, a paucity of data in Cells V, VI, and VII precluded execution of Stage One of the analysis to those cells. To carry out Stage One for Cells I-IV, the tests were applied in the following manner. From each panel of data from Cells I-IV, a sample of 50 observations on output of electricity, installed capacity, inputs of fuel and labor, number of units, plant factor, and heat rate was drawn randomly without replacement.[32] The samples drawn for each of

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[30]Ramsey, "Tests for Specification Errors," op. cit.

[31]H. Theil, "The Analysis of Disturbances in Regression Analysis," Journal of the American Statistical Association 60, 1067-1079.

[32]It is well-known from sampling theory (see, for example, T. Yamane, Elementary Sampling Theory, 71-74) that sampling without replacement from a finite population alters the distributions of estimators of population parameters. For this reason, the sampling procedure in Stage One was carried out with replacement.

of the four cells were used separately for OLS estimation of each of the twelve alternative models listed at the end of the preceding section of this chapter. Theil's optimal residual vector was calculated for each estimated regression.

Each vector of Theil residuals was used in three of the specification error test--RESET, WSET, and BAMSET. It has been demonstrated in Monte Carlo experiments by Ramsey and Gilbert that these tests are more powerful against various mis-specifications than are RASET and KOMSET.[33] As explained by Ramsey, the statistics calculated in the three tests are as follows.[34] RESET is based on an F statistic with 3, (N-K-3) degrees of freedom. WSET is based on the W statistic, whose definition and distribution are given by Shapiro and Wilk.[35] BAMSET uses a modification of Bartlett's M statistic, distributed as Chi-square with 2 degrees of freedom under the null hypothesis.

Using each of the three test statistics, the null hypothesis that the full ideal conditions are satisfied was tested for each alternative model at the 1% level. It should

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[33]J. B. Ramsey and R. F. Gilbert, "A monte Carlo Study of Some Small Sample Properties of Tests for Specification Error," Michigan State University Econometrics Workshop Paper #6813, East Lansing (1969) 8 and 26.

[34]Ramsey, op. cit. (1969), 356.

[35]S. S. Shapiro and M. B. Wilk, "An Analysis of Variance Test for Normality (Complete Samples," Biometrika 52 (1965), 591-611.

be noted that the three test statistics are not in general statistically independent, as discovered in Monte Carlo experiments by Ramsey and Gilbert.[36] The joint α-level of the three tests depends on the type of mis-specification that has occurred; but of course, it is higher than if the tests were statistically independent.

After all twelve alternatives are tested in this manner, one is left with zero, one, or more than one model that has not been rejected as inappropriate by at least one of the tests. The implication of each of these three possible outcomes depends on which tests rejected a particular alternative. Specifically, if a model was rejected only for non-normality of the residuals by WSET, the model need not be rejected since non-normality affects only the maximum-likelihood property of OLS estimators. Non-normality becomes important in defining test statistics based on the OLS parameter estimates, however.

If exactly one of the alternatives is not rejected as inappropriate for a sample by RESET or BAMSET, the problem of choice is solved for that sample. If more than one of the alternatives is not rejected, the situation becomes ambiguous. As mentioned above, some of the alternatives being considered in the present study are not mutually exclusive. Therefore, it is entirely conceivable that more than one of the models is appropriate. But on the other

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[36] Ramsey and Gilbert, _op_. _cit_, 31.

hand, it is possible that one or more Type II errors have been committed, i.e., that specification error in one or more alternatives has not been detected by the tests at the α-level being used. Use of the tests on a different sample from the same population at a smaller α-level may be helpful in resolving the ambiguity.

If all alternatives are rejected as inappropriate, the problem is more difficult since it is evident that the maintained hypothesis is incorrect. As pointed out by Ramsey, the specification errors of omitted variables, incorrect functional form, and simultaneous equation problem are all observationally equivalent, so that the tests cannot be used to indicate which type(s) of specification error led to rejection of a given model.[37] Furthermore, there is no indication which model is "closer," in some sense, to an appropriate model than is some other. Thus, one has no indication of how to find a correct maintained hypothesis. Some procedures recently developed by Box and Cox, and Box and Tidwell, may be of help in deriving models that satisfy the full ideal conditions.[38]

Assuming that at least one functional form may be found that is not rejected as inappropriate for a sample,

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[37]Ramsey, op. cit. (1969).

[38]Box and Cox, op. cit.; G. E. P. Box and P. W. Tidwell, "Transformations on the Independent Variables," Technometrics 4 (1962), 531-550.

one has a more specific maintained hypothesis than before for that sample. Under that restricted maintained hypothesis, one may proceed to Stage Two of the analysis.

Stage Two--Confirmation of the Restricted Maintained Hypothesis

Assuming that at least one appropriate model is found for each data sample in Stage One, the original maintained hypothesis has been tentatively restricted to the chosen model(s). Before carrying out statistical inferences under the maintained sub-hypothesis, it is desirable to confirm that sub-hypothesis independently of the procedure used in Stage One. In this way, its credibility is strengthened. This confirmation is the major objective of Stage Two of the empirical analysis.

For each of the four data samples to which Stage One was applied, the following procedures were carried out in Stage Two. First, another data sample was chosen at random from the available data in each cell. This time the sample was drawn without replacement to avoid problems associated with duplicate observations. The model tentatively chosen for each cell was re-estimated using the new data sample, and Theil residual vectors were again calculated for each estimated model. Using these residual vectors, the specification error tests RESET, WSET, and BAMSET were again applied to the chosen models. Failure in Stage Two to reject the null hypothesis of no specification error in the tentatively chosen model for each cell was taken as independent confirmation of its appropriateness for that cell. Models confirmed in this way for Cells I-IV were to be used for statistical inference in Stage Three. As mentioned in the discussion of Stage One, inference in Cells V, VI, and VII was to be carried out using the same model as that used in Cell IV. The decision to generalize the Cell IV results to Cells V-VII reflects the fact that the latter group of cells is technologically more similar to Cell IV than to any other cell.

Another problem considered in Stage Two was the possibility of non-normality of the stochastic disturbances. As explained in the preceding section, non-normality was not considered sufficient grounds for rejection of a model in Stage One of the analysis. A similar view was taken in Stage Two, leaving open the possibility that statistical inference might have to be carried out in Stage Three without the assumption of normality of disturbances. It was thought that inferential techniques based on normality of disturbances would not cause serious errors unless the skewness and kurtosis of the actual distribution departed noticeably from their values for the normal distribution. Therefore, another objective of Stage Two was to test for the significance of skewness and non-normal kurtosis for those chosen models which were rejected for non-normality by WSET.

A measure of the skewness of a population distribution is the third moment about the population mean. To render this measure scale-free, it is divided by σ^3, where σ is the population standard deviation. The resulting coefficient of skewness may be denoted by $\sqrt{\beta_1}$. In the normal distribution, $\sqrt{\beta_1} = 0$. The estimate of this coefficient based on a sample of (N-K) Theil residuals u_i^*, $i = 1, \ldots, (N-K)$, with mean $\bar{u}^*$, as used in the specification error tests, is given by $\sqrt{b_1}$, where $\sqrt{b_1} = \frac{m_3}{(m_2)^{\frac{3}{2}}}$, $m_3 = \frac{\sum_{i=1}^{N-K}(u_i^*-\bar{u}^*)^3}{N-K}$, and $m_2 = \frac{\sum_{i=1}^{N-K}(u_i^*-\bar{u}^*)^2}{N-K}$. Under the null hypothesis that the sample is drawn from a normal population, $\sqrt{b_1}$ is approximately normally distributed with mean zero and variance $\frac{6}{N-K}$.[39] Using this information, a one-tailed test at the 1% level was carried out for skewness in those cases where WSET indicated the presence of non-normal disturbances.

A measure of the kurtosis of a population is given by the fourth moment about the mean divided by the square of the population variance. This coefficient of kurtosis, denoted by γ_2, equals 3 in a normal population. A sample estimate of the value of γ_2 is given by $g_2 = \frac{m_4}{m_2^2}$, where $m_4 = \frac{\sum_{i=1}^{N-K}(u_i^*-\bar{u}^*)^4}{N-K}$ and all other terms are as defined above. Under the null hypothesis that the sample is drawn from a normal population, g_2 is approximately distributed as normal

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[39] G. W. Snedecor and W. G. Cochran, <u>Statistical Methods</u>, 6th ed. (Ames, Iowa: Iowa State University, 1967), 86ff.

with mean 3 and variance $24/(N-K)$.[40] Using this information, a one-tailed test at the 1% level was carried out for non-normal kurtosis in those cases where WSET indicated non-normality of disturbances. Tables for the distributions of $\sqrt{b_1}$ and g_2 are given by Snedecor and Cochran.[41]

In summary, the result of Stage Two is a statement of a maintained hypothesis that includes at least one model, confirmed as suitable for inferences about the effects of changes in scale and technology, for each cell defined in Table 4. Each model takes the form of a stochastic function to be estimated. The distributional assumptions made about the disturbance term are dictated by the results of tests for specification error, skewness, and kurtosis.

Stage Three--Inferences Concerning Scale, Technology, and the Number of Units

Under the maintained hypotheses developed in Stage Two, one is prepared to make inferences about the effects on plant efficiency of changes in scale, technology, and the number of turbogenerator units in the steam-electric plant. A procedure designed for making these inferences is as follows.

For each of the cells to which Stages One and Two had been applied, a third random sample of 50 observations was drawn from the available data, independently of the samples previously drawn. As in Stage Two, these samples were drawn

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[40] Snedecor and Cochran, *op. cit.*

[41] *Ibid.*

without replacement. For the remaining cells, all available data were incorporated in Stage Three. For each cell in the former group, the model confirmed in Stage Two for that cell was estimated using the third sample drawn. As explained in Stage Two, the model selected for Cell IV was also estimated for Cells V, VI, and VII. The estimated models were used as a basis for inferences concerning three general null hypotheses:

(1) Steam-electric generation exhibits constant returns to scale at the plant level.

(2) Technological change in steam-electric generation has not affected the production function for that process at the plant level.

(3) The production function of the steam-electric generating plant is not affected by the number of turbogenerator units in the plant.

Of course, statements of these null hypotheses in testable terms depends on the specific model being estimated as a basis for inference in each cell. These statements, as well as a discussion of appropriate testing procedures for each hypothesis, comprise the remainder of this chapter.

For the first null hypothesis, constant returns to scale, tests are available regardless of whether the distribution of disturbances for any estimated model is assumed to be normal or not. In general, define R as a population returns-to-scale parameter for a cell of generating plants, and let R represent its OLS estimate. Specific definitions of R depend on the production function being considered. For the following models--CD, CES, CD1, CES1, CD2, CES2, CD/N, and CES/N--$R = \sum_{i=1}^{3} \alpha_i$, where α_i is the exponent of the i-th input. For these models $\hat{R} = \sum_{i=1}^{3} \hat{\alpha}_i$, with population variance $\sigma_{\hat{R}}^2$, and sample variance $s_{\hat{R}}^2 = \sum_{i=1}^{3} s_{\hat{\alpha}_i}^2 + 2 \sum_{\substack{i,j=1 \\ i \neq j}}^{3} \mathrm{Cov}(\hat{\alpha}_i, \hat{\alpha}_j)$.

For the models based on the work of Hart and Chawla, one has three groups of equations, {LUHCC, LUHCF, LUHCL}, {LEHCC, LEHCF, LEHCL}, and {LEHCC/N, LEHCF/N, LEHCL/N}: The definition of returns to scale in these situations is more complex. As explained in the preceding section of this chapter, each group of three equations represents the three components of one fixed-relative-proportions model. The reader will recall that at any point the returns-to-scale parameter for the production function, R, may be defined as the degree of homogeneity, i.e., the factor exponent, of the relevant component, β^*. $\hat{R}$ is given by $\hat{\beta}^*$, the OLS estimate of β^*, with population variance $\sigma_{\hat{\beta}^*}^2$ and sample variance $s_{\hat{R}}^2 = s_{\hat{\beta}^*}^2$.

Defining R and $\hat{R}$ in the manner appropriate for the function on which statistical inference is to be based, one may test the null hypothesis of constant returns to scale, i.e., that R = 1. A test statistic T_1 may be defined by $T_1 = \frac{\hat{R}-1}{s_{\hat{R}}}$. If the stochastic disturbance term of the model in question is assumed to be normally distributed, T_1 is distributed as "Student's" t with (N-K) degrees of freedom, where N is the number of observations and K the number of regressors. Using this fact, a test of the null hypothesis is available at any arbitrary level of significance.

If the normality assumption is not warranted by the results of Stage Two of the analysis, a less powerful test of the null hypothesis may be based on the Tchebycheff Inequality, as stated by Kendall and Stuart.[42] Under the null hypothesis that R = 1, the Inequality is given by:

(III-68) $\Pr\{|\hat{R}-1| \geq \lambda\sigma_{\hat{R}}\} \leq \frac{1}{\lambda^2}$,

where λ is an arbitrary constant not less than unity and all other variables are as defined above. Let $\lambda = \frac{ks_{\hat{R}}}{\sigma_{\hat{R}}}$, where k is an arbitrary positive constant. Then inequality (III-68) may be written:

(III-69) $\Pr\{|\hat{R}-1| \geq ks_{\hat{R}}\} \leq \frac{\sigma^2_{\hat{R}}}{k^2 s^2_{\hat{R}}}$.

Since the probability limit (plim) of the right-hand side of inequality (III-69) is $1/k^2$, that inequality may be replaced by:

(III-70) $\text{plim}\ \{|\hat{R}-1| \geq ks_{\hat{R}}\} \leq \frac{1}{k^2}$,

for any arbitrary k. In particular, for $k = \sqrt{20}$, inequality (III-70) provides a large-sample two-tailed test of the null hypothesis at the 5% level. By using the appropriate definition of R and the most powerful test justified by the

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[42]M. G. Kendall and A. Stuart, The Advanced Theory of Statistics, I (New York: Hafner, 1963), 88.

distributional results of Stage Two, one may test the null hypothesis of constant returns to scale, no matter which of the models discussed serves as a maintained hypothesis.

The second null hypothesis to be tested in Stage Three of the present study is that technological change in steam-electric generation has not affected the generating plant production function between 1948 and 1965. To test this null hypothesis, one tests for differences in the production function parameters across the technologically homogeneous cells defined in Table 4. Such a test, to be carried out between pairs of cells, is meaningful only if the same production function is assumed for both members of a pair, while all variables are measured in the same units for both members of a pair. The economic interpretation of any observed differences in parameters across cells will, of course, depend on the directions and magnitudes of the specific differences.

To begin a general discussion of the problem of testing for equality of the same production function parameter across two cells, define α_i and α_j as the values of parameter α in Cells i and j. Further, let $\hat{\alpha}_i$ and $\hat{\alpha}_j$ denote the OLS estimates of α_i and α_j, respectively. Let $s_{\hat{\alpha}_i}$ and $s_{\hat{\alpha}_j}$ denote the sample standard errors of the estimated coefficients, and $\sigma^2_{\alpha_i}$ and $\sigma^2_{\alpha_j}$ their population variances. Assume that $\sigma^2_{\alpha_i} = \sigma^2_{\alpha_j}$, and recall that $\hat{\alpha}_i$ and $\hat{\alpha}_j$ are independent due to the independence of the data samples used in their calculation.

Then a test statistic may be defined as $T_2 = \frac{\hat{\alpha}_i-\hat{\alpha}_j}{\sqrt{s^2_{\alpha_i}+s^2_{\alpha_j}}}$.

This statistic is useful in testing the null hypothesis that $\alpha_i = \alpha_j$.

Under the null hypothesis and the assumption of normality of the stochastic disturbance term in both estimated equations, T_2 is distributed as "Student's" t with 2(N-K) degrees of freedom. If the normality assumption is not warranted by the results of Stage Two of the empirical analysis, a less powerful test of the null hypothesis is available, based on the null hypothesis. Under the null hypothesis that $\alpha_i = \alpha_j$, the Inequality is given by:

(III-71) $\Pr\{|\hat{\alpha}_i-\hat{\alpha}_j| \geq \lambda\sqrt{\sigma^2_{\hat{\alpha}_i}+\sigma^2_{\hat{\alpha}_j}}\} \leq \frac{1}{\lambda^2}$

where λ is an arbitrary constant not less than unity, and all other variables are as defined above. Following a derivation similar to that of inequality (III-70), (III-71) may be replaced by:

(III-72) $\text{plim}\ \{|\hat{\alpha}_i-\hat{\alpha}_j| \geq k\sqrt{s^2_{\hat{\alpha}_i}+s^2_{\hat{\alpha}_j}}\} \leq \frac{1}{k^2}$,

for any arbitrary k. Noting that $|T_2| = \frac{|\hat{\alpha}_i-\hat{\alpha}_j|}{\sqrt{s^2_{\hat{\alpha}_i}+s^2_{\hat{\alpha}_j}}}$, it is apparent that rejecting the null hypothesis if $T_2^2 \geq 20$ provides a large-sample two-tailed test of the null hypothesis at the 5% level, when one makes no distributional assumptions about the stochastic disturbances for the estimated production function models.

The third and final null hypothesis which may be tested in Stage Three is that the value of N_i, the number of turbogenerators in the i-th generating plant, does not affect the plant production function. The reader will note that potentially this null hypothesis may be impossible to test under the maintained hypothesis restricted for the purpose of inference. Only if the maintained hypothesis is given by CD/N, CES/N, or {LEHCC/N, LEHCF/N, LEHCL/N} is this hypothesis relevant. A test of the null hypothesis is simply a test of the significance of δ, the coefficient of N_i under these potential maintained hypotheses.

Let $\hat{\delta}$ denote the OLS estimate of δ, and $s_{\hat{\delta}}$ the standard error of $\hat{\delta}$. Under the assumption that the least squares disturbances for the estimated model satisfy the full ideal conditions, $T_3 = \frac{\hat{\delta}}{s_{\hat{\delta}}}$ is distributed as "Student's" t with (N-K) degrees of freedom. Therefore, under the normality assumption, a test of the null hypothesis may be carried out at any desired level of significance.

If the normality assumption is not made about the least-squares disturbances, a test of the null hypothesis may be based on the Tchebycheff Inequality. Under the null hypothesis that $\delta = 0$, the Inequality states that:

(III-73) $\Pr\{|\hat{\delta}| \geq \lambda\sigma_{\hat{\delta}}\} \leq \frac{1}{\lambda^2}$,

where $\sigma_{\hat{\delta}}$ denotes the population variance of $\hat{\delta}$, and λ is an arbitrary constant not less than unity. Following a derivation similar to the two just completed, inequality (III-73) may be replaced by:

(III-74) $\text{plim}\ \{|\hat{\delta}| \geq ks_{\hat{\delta}}\} \leq \frac{1}{k^2}$

for any arbitrary k. Noting that $|T_3| = \frac{|\hat{\delta}|}{s_{\hat{\delta}}}$, it is clear that rejection of the null hypothesis when $T_3 \geq \sqrt{20}$ provides a distribution-free, large-sample, two-tailed test of the null hypothesis.

In this section of Chapter III, a procedure has been proposed which is carried out in three stages. In the first stage, a maintained hypothesis consisting of a number of potential alternative models of steam-electric generation is restricted to one or more specific models. In the second stage, an independent test of the restricted maintained hypothesis is carried out for the purpose of independent confirmation. Also, tests are used to examine the skewness and kurtosis of stochastic disturbances under the restricted maintained hypothesis. In the third stage, tests are proposed for three null hypotheses concerning steam-electric generation at the plant level. The specific tests to be used depend on the restricted maintained hypothesis employed and on the distributional assumptions warranted by the results of Stage Two. The results of applying all three stages to the data sample used in the present study are discussed in Chapter IV.

Chapter IV. Empirical Results

The objective of Chapter IV is to report the results of the empirical procedure explained in Chapter III. D. The results are reported stage by stage, in the order of their exposition in the previous chapter. The reader is reminded that the alternative models are summarized, by name, in Table 5, page 113.

IV. A. Stage One--Choice of a Model

The reader will recall that at the end of Chapter III. C. 18 equations representing 12 different models of the steam-electric generating plant were listed. It was mentioned in Chapter III. D. that the problem to be addressed in Stage One of the empirical analysis was to choose one or more of the alternatives as a restricted maintained hypothesis under which to carry out statistical inference concerning the generation process. To accomplish this, each equation was tested for specification error using data samples from Cells I through IV. The results of the specification error tests RESET, WSET, and BAMSET, together with the value of R^2, are reported for each model tested in Appendix A.

The most striking result of the specification error tests is that all functions considered were rejected at the 1% level for non-normality of disturbances by WSET.

It is well-known that non-normality of disturbances affects only the maximum-likelihood property of OLS estimates. Therefore, it was decided to use the results of WSET as one of several criteria for the choice of statistical tests to be used in Stage Three, rather than as a criterion for the choice of a model.

Considering only the results of RESET and BAMSET, the three-factor functions based on generalizations of the ZKD model were not found to be appropriate, for the most part. At the 1% level CD was rejected by BAMSET in Cells I and III, while CES was rejected by at least one of the two tests in all cells except IV. The possibility that misspecification associated with these models might be the result of variable returns to scale did not appear plausible, since generalization of these functions according to the Zellner and Revankar procedure did not appreciably affect the test statistics. Further, all the estimates of r, a, and θ, the parameters of the returns to scale functions in CD1, CD2, CES1, and CES2, were not significantly different from zero. Therefore, the hypothesis of a constant returns-to-scale parameter over the observed range of output could not be rejected, even at the 5% level.

The reader will recall that two versions of a fixed-relative-proportions model were considered. The model consisting of the components LUHCC, LUHCF, and LUHCL, in which output was assumed to be a stochastic function of inputs, was not found to be free of specification error. At least one of the components was rejected by at least one of the tests in each cell examined. Due to problems associated with duplicate observations, neither LUHCC nor LUHCL was tested in Cell III, and LUHCL was not tested in Cell IV. Similar test results were obtained for the model consisting of LEHCC, LEHCF, and LEHCL, in which output was assumed to be exogenously determined.

Addition of a term in N, the number of turbogenerator units in the plant, to the CD, CES, and fixed-relative-proportions functions improved the test results somewhat. CD/N was rejected at the 1% level by RESET in Cell II, and CES/N was rejected by BAMSET in Cells I and II. But with these exceptions, the null hypothesis of no specification error except for non-normal disturbances was not rejected for either model in any of the cells examined. Values of R^2 were greater than .99 for both models in all four cells.

Unfortunately, both estimated functions exhibited a number of coefficients with negative signs. Other estimates were not significantly different from zero. Only in Cells III and IV were coefficients of any factors in CD/N except fuel significant at the 5% level, and in both those cells the estimated capital coefficients were negative.

Similar results were obtained for CES/N except in Cell II, where the coefficients of both ln K and N were significantly negative. It was suspected that the large standard errors of the estimated coefficients might result from a high degree of multicollinearity among the factors. The standard error of the i-th coefficient, $s_{\hat{\alpha}_i}$, is given by $\sqrt{s^2 x x^{ii}}$, where s^2 is the mean squared error of the estimated regression and x^{ii} is the i-th diagonal element of $[X'X]^{-1}$, X being the observed regressor matrix. By decreasing the value of $|X'X|$, multicollinearity increases the values of the x^{ii} and, in turn, the values of the $s_{\hat{\alpha}_i}$. To examine the extent to which multicollinearity caused the large observed standard errors, the x^{ii} were compared to the values of s^2 for a few cells. The value of x^{ii}/s^2 ranged from 76.39 for the labor input in Cell I to abour 3.5 billion for the fuel input in Cell III. Even when these ratios are deflated by their respective means, their values are approximately 10.2 and 218 million. These figures provided some support for the presumption of a high degree of multicollinearity among the inputs.

Further evidence that a high degree of multicollinearity was in fact present is apparent from Table 5, in which the pairwise correlation coefficents for the factors are presented.

Table 5

Correlation Coefficients Among Factors, by Cell

Factor Pair \ Cell	I	II	III	IV
F,K	.906	.896	.955	.962
F,L	.829	.609	.788	.755
K,L	.779	.699	.793	.801

The final model to be considered was the group consisting of LEHCC/N, LEHCF/N, and LEHCL/N. These equations are the components of a fixed-relative-proportions model with output exogenous, modified by inclusion of a term in N, the number of units per plant. With the exception of Cell II, this set of equations performed fairly well, disregarding the problems of non-normality. As can be seen from Appendix A, at the 1% level RESET rejected only the fuel input equation in Cell I and the labor input equation in Cell III. But neither of those tests rejected any input function in Cell IV. All slope coefficients, representing elasticities of inputs with respect to output, lay between 0 and 1, as would be expected under the increasing returns to scale hypothesis suggested by most earlier writers discussed in Chapter I. B.

The values of R^2 calculated for each input equation are not surprising to a student of the steam-electric generation process. For the fuel equations, R^2 ranged from .9858 to .9969 across cells, indicating that output is an important determinant of the fuel input. This is in agreement with the findings of many engineers who have examined that relationship under various steam conditions.[1] A smaller proportion of the variation in capital input was explained by output and the number of machines, presumably because differences in plant factor across plants account for some variation across plants in the installed capacity required to generate a given annual output with a given number of units. The value of R^2 for the capital input equation ranged from .8063 to .9300 across cells. Finally, relatively low values of R^2, between .5048 and .7182, were obtained for the labor input equation in the various cells. This is not surprising in light of the observation made by Dhrymes and Kurz that the operations performed by labor "are more or less supplementary to the major operations of turning fuel into electricity."[2]

The fact that the results obtained in Cell II differed so markedly from the results obtained in the other cells led to the suspicion that some plants in the sample from that cell might not be representative of the cell. Therefore, plots of the data for Cell II were made and examined

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[1]Ling, op. cit., pp. 28-35.

[2]Dhrymes and Kurz, op. cit., pp. 287-315.

for the presence of outliers. Three outliers were found. Further investigation of the outlying observations shed some light on the causes. One plant, which produced an outlier in its second year of operation, had a plant factor of only 7% and a heat rate 30 to 40% higher than that of other plants operated by the same firm. The guess was made that as a new plant, it was still being tested and adjusted. The other two outliers were apparently subject to some sort of reporting error, since data for the year of observation of the outlier on average production cost and plant heat rate in each case differed substantially from the corresponding data for the same plant, for preceding and subsequent years. The fact that the sample in Cell II did in fact contain outliers indicated that the specification error tests had successfully identified a sample for which the fixed-relative-proportions model was not appropriate. However, since the inappropriateness could be explained by undesirable properties of the sample, it was impossible to determine whether in addition the model was inappropriate for the population defined by Cell II.

Clearly none of these models, which comprise the set of alternatives under the maintained hypothesis, was found to be appropriate for all cells. However, several of the models could immediately be eliminated from further consideration for some cells. The CES model was strongly rejected in all cells except IV, as was the CD model in all cells except II and IV. Generalization of these models to parameterize variability in returns to scale over the observed range of output did not significantly alter the results. However, inclusion of a term in N to incorporate the effects of variations in machine mix noticeably improved both the specification error tests results and the observed value of R^2 in most cells. Similarly, addition of this term to the equations LEHCC, LEHCF, and LEHCL raised the calculated values of R^2 without introducing specification error. Therefore, several models were immediately eliminated from further consideration, either because of statistically significant specification error, because another alternative produced a higher value of R^2 without statistically significant specification error, or because a model involving fewer parameters yielded both equal values of R^2 and similar specification error test results. The models immediately eliminated were CD in Cells I, III, and IV; CD1 and CD2 in all cells; CES, CES1, and CES2 in all cells; the model consisting of LUHCC, LUHCF, and LUHCL in all cells; the model consisting of LEHCC, LEHCF, and LEHCL in all cells; CD/N in Cell II; and CES/N in Cells I and II. Therefore, the specification error tests were able to eliminate most, but not all, of the potential alternatives.

After these models were rejected, the following possibilities remained:

Cell I--{LEHCC/N, LEHCF/N, LEHCL/N}, CD/N

Cell II--CD, {LEHCC/N, LEHCF/N, LEHCL/N}

Cell III--CD/N, CES/N, {LEHCC/N, LEHCF/N, LEHCL/N}

Cell IV--CD/N, CES/N, {LEHCC/N, LEHCF/N, LEHCL/N}

The reader will note that for each cell, one three-equation model and at least one one-equation model remained. Furthermore, by reference to Appendix A, it is obvious that some of the components of the three-equation model were actually rejected by RESET or BAMSET at the 1% level. Because of the following considerations, the model was not rejected out of hand despite these test results.

To date, no rigorous method is available for comparing alternative models according to the presence of specification error when the alternatives consist of different numbers of equations. Two considerations are relevant to this problem. First, it seems reasonable to assume that if the tests are applied, using the same data, to a one-equation model and to each equation of a three-equation model at the same nominal α-level, the probability of rejecting one component of the three-equation model is greater than the probability of rejecting the one-equation model, under the full ideal conditions. This should be kept in mind in comparing models consisting of different numbers of equations. Second, both a knowledge of the technology of steam-electric generation and an examination of Table 5 indicates high positive pairwise correlations between the factors. It is well-known that this phenomenon is a cause of multicollinearity when all three factors are used as explanatory variables in one regression. In turn, large coefficient standard errors are associated with multicollinearity. Consequently, operationally significant relationships between output and the individual factors may appear statistically insignificant when all three factors appear as regressors in the same equation. *A fortiori*, such a situation will lower the power of the tests. For these reasons, it may be desirable to accept provisionally the hypothesis that the three-equation model is the relevant model, subject to further testing in Stage Two.

Another consideration relates to one of the overall objectives of this study--namely, inference concerning the effects of technological change on the production function, to be carried out in Stage Three. For this objective, it is helpful to be able to express technological change parametrically, a relatively simple task if the same model is used for all cells. Thus, one would like to use the same functions for as many cells as possible without completely violating the criteria for acceptance of a model.

The three-equation model consisting of LEHCC/N, LEHCF/N, and LEHCL/N is helpful in meeting this objective.

Considering all these factors, and recalling that the purpose in Stage One of the analysis is merely to generate a restricted hypothesis, to be tested in Stage Two, it was decided to relax the specification error test criteria somewhat. Specifically, it was decided to accept as a tentative hypothesis the three-equation model even if one of its components was rejected at the 1% level by either RESET or BAMSET but not both. On this modified criterion, the model {LEHCC/N, LEHCF/N, LEHCL/N} is tentatively acceptable for all cells except Cell II. As mentioned above, the fact that this rejection was due at least partially to the presence of outliers in the Cell II sample makes evaluation of the appropriateness of the model to the Cell II population impossible. Use of this model for all cells as a tentatively maintained hypothesis has two advantages over the other models for the inferences to be carried out in Stage Three. First, the problem of multicollinearity due to high pairwise correlation coefficients between factors is avoided. Second, a model with comparable coefficients over all cells is available. Neither of these conditions is met by any other model not rejected immediately on the basis of the RESET and BAMSET results.

Therefore, the following model was tentatively accepted as a maintained hypothesis for all four cells:

(III-66a) $\ln K_i = \ln \Gamma_1 + \alpha_1 \ln Y_i + \delta_1 N_i + v_{1i}$ (LEHCC/N)

(III-66b) $\ln F_i = \ln \Gamma_2 + \alpha_2 \ln Y_i + \delta_2 N_i + v_{2i}$ (LEHCF/N)

(III-66c) $\ln L_i = \ln \Gamma_3 + \alpha_3 \ln Y_i + \delta_3 N_i + v_{3i}$ (LEHCL/N)

The terms v_{1i}, v_{2i}, and v_{3i} are assumed to be i.i.d. stochastic disturbances, with zero means and constant variances. The assumption of normality was not warranted by the results of WSET. Having tentatively chosen this maintained hypothesis, an attempt at independent confirmation of it was made in Stage Two.

IV.B. Stage Two--Confirmation of the Chosen Model

The next task is to use a different data sample to test the model given in equations (III-66) for specification error. In this way one can obtain independent evidence regarding its appropriateness for each of those cells. Another task, as explained in Chapter III.D., is to test whether the distribution of disturbances for this model, despite being non-normal, has skewness and kurtosis coefficients sufficiently close to those of the normal distribution to warrant the assumption of normality for the inferences to be carried out in Stage Three, below. The results of both these tests are discussed in the present section.

For the purposes of Stage Two, a second data sample of 50 was randomly drawn without replacement from the available data in Cells I-IV. Using these samples, the chosen model--LEHCC/N, LEHCF/N, and LEHCL/N--was re-estimated and tested for specification error using RESET, WSET, and BAMSET. Furthermore, skewness and kurtosis statistics were calculated for the distribution of Theil residuals for each equation in each cell. The values of all these statistics, as well as the corresponding values of R^2, are presented in Appendix B. The null hypothesis being tested with the specification error test statistics is that the disturbances are i.i.d. $N(\phi, \sigma^2 I)$. The null hypothesis being tested with the aid of $\sqrt{b_1}$ and g_2 is that the skewness and kurtosis parameters are zero and 3, respectively, as they would be if the true distribution of disturbances were normally distributed. For those equations and cells for which the value of a statistic indicates rejection of the null hypothesis, the level of significance of the rejection is indicated with asterisks--1 for the 10% level, 2 for the 5% level, and 3 for the 1% level. The results are summarized in Appendix B.

The reader will recall that in Stage One, the fixed-relative-proportions model was accepted if no more than one of the two tests RESET and BAMSET rejected no more than one of the three component equations at the 1% level.

Using the same standard in Stage Two, the model was confirmed for Cells I, III, and IV, as is evident from Appendix B. As in Stage One, the rejection in Stage Two of the model for Cell II was apparently due to the presence of outliers in the data sample. The fact that the results of Stage Two so closely duplicated the results of Stage One was taken as independent confirmation of the results obtained in the first stage. However, in basing inferences on this model, the possibility of mis-specification should be borne in mind.

The second problem in Stage Two, investigation of skewness and kurtosis, was less clear-cut. Based on the statistics $\sqrt{b_1}$ and g_2 calculated from the Theil residuals for each equation, the null hypothesis of normal skewness was rejected at the 1% level for all equations in Cell II and for the capital equation in Cell III. Also at the 1% level, the null hypothesis of normal kurtosis was rejected for the fuel equation in Cell I and all equations in Cell II.

The choice of test procedures for Stage Three, based on the results of these skewness and kurtosis tests, depends heavily on the interpretation of the fixed-relative-proportions production model as it was stated in Chapter III.C. For this reason, the reader is reminded that according to that discussion the degree of homogeneity of the production function is given by the smallest of the three factor exponents, i.e., the degree of homogeneity of what what was called the relevant component term in Chapter III.C. Of course, if the three exponents are not significantly different from each other, any of the three may yield the degree of homogeneity of the production function. For the sake of consistency, the statistical tests to be used in Stage Three should be based on the relevant component equation. Therefore, it is evident that if the relevant component of the model in Cell I is LEHCF/N, for which non-normal kurtosis is indicated, the tests for Stage Three involving Cell I should not be based on the normality assumption. Similarly, tests involving Cell II should not be based on normality if LEHCC/N is the relevant component. In both these cases tests should be based on the Tchebycheff Inequality, as explained in Chapter III.D. Similarly, all tests involving Cell II should be based on the Tchebycheff Inequality. In all other situations involving Cells I-IV, tests may be constructed that incorporate the normality assumption for the regression disturbances.

As explained in Chapter III.D., the available numbers of observations for Cells V, VI, and VII were too small to allow the three-stage procedure to be used. Therefore, as was explained earlier, the restricted maintained hypothesis for those cells was to be the same as that for

Cell IV. Extending that rule to include the stochastic part of the maintained hypothesis, tests involving Cells V, VI, and VII are based on the assumption of normality.

Concisely stated, the maintained hypothesis for Stage Three is given as follows:

$$Y_i = \min \{\gamma_1 K_i^{\beta_1} e^{\delta_1 N_i} e^{u_{1i}},\ \gamma_2 K_i^{\beta_2} e^{\delta_2 N_i} e^{u_{2i}},\ \gamma_3 K_i^{\beta_3} e^{\delta_3 N_i} e^{u_{3i}}\}$$

This model is to be estimated in separate components-- (III-66a), (III-66b), and (III-66c), denoted as LEHCC/N, LEHCF/N, and LEHCL/N. respectively.

The u_{ji}, j = 1, 2, 3, i = 1, . . ., n, are assumed to be i.i.d. as follows:

Cell I: $u_{1i}, u_{3i} \sim N(\phi, \sigma^2 I)$

$u_{2i}, \sim (\phi, \sigma^2 I)$

Cell II: $u_{1i}, u_{2i}, u_{3i} \sim (\phi, \sigma^2 I)$

Cell III: $u_{2i}, u_{3i} \sim N(\phi, \sigma^2 I)$

$u_{1i}, \sim (\phi, \sigma^2 I)$

Cells IV-VII: $u_{1i}, u_{2i}, u_{3i} \sim N(\phi, \sigma^2 I)$.

The form of estimation is as separate equations-- (III-66a), (III-66b), and (III-66c). It should be noted that this formulation, also called {LEHCC/N, LEHCF/N, LEHCL/N}, includes the implicit assumption that output is exogenously determined, with inputs used as the dependent variables. Although the model is to be estimated by ordinary least squares, the specification error test results dictate caution in the interpretation of the estimated results, which are presented in the next section of this Chapter.

IV. C. Stage Three--Results Concerning Scale, Technology, and Machine Mix

As the first step in Stage Three of the empirical analysis, a third sample of 50 was drawn from each of the available data pools in Cells I-IV. The data base for Cells V, VI, and VII consisted of all available data from each of these cells. For each cell, the model consisting of the three equations LEHCC/N, LEHCF/N, and LEHCL/N was estimated, using the data sample for that cell. The estimated results are presented in Table 6. For each of the three components for each cell, estimates of the three estimated coefficients Γ, α, and δ appear immediately above their respective standard errors. The value of R^2 for each equation also is given.

The first item of concern in interpreting the results is determination of the relevant component of the three-equation model. The reader will recall from the discussion in Chapter III.C that the relevant component is the one including the factor that "limits" output, in the sense that increases in output require proportionately larger increases in the "limiting" factor than in the others. Estimates of the proportionate increases in each input required for a given change in output, i.e., the output elasticities of the inputs, are given by $\hat{\alpha}_i$, i = 1,...,3. From Table 6, it is evident that the relevant component

Table 6

Estimation Results for LEHCC/N, LEHCF/N, and LEHCL/N

Model	Statistic \ Cell No.	I	II	III	IV	V	VI	VII
LEHCC/N	$\hat{\Gamma}_1$	-0.603	-0.199	0.542	0.038	2.351	2.288	4.545
	$s_{\hat{\Gamma}_1}$	0.304	0.273	0.575	0.410	0.499	0.779	0.534
	$\hat{\alpha}_1$	0.833	0.729	0.621	0.720	0.405	0.418	0.162
	$s_{\hat{\alpha}_1}$	0.059	0.054	0.098	0.061	0.074	0.116	0.069
	$\hat{\delta}_1$	-0.067	0.021	0.169	0.111	0.240	0.193	0.345
	$s_{\hat{\delta}_1}$	0.046	0.069	0.057	0.032	0.047	0.058	0.038
	R^2	.846	.806	.910	.930	.797	.744	.928
LEHCF/N	**Statistic \ Cell No.**	**I**	**II**	**III**	**IV**	**V**	**VI**	**VII**
	$\hat{\Gamma}_2$	9.811	10.221	9.505	9.587	10.798	9.243	9.520
	$s_{\hat{\Gamma}_2}$	0.063	0.079	0.121	0.091	0.427	0.501	0.090
	$\hat{\alpha}_2$	0.922	0.858	0.958	0.940	0.765	0.993	0.947
	$s_{\hat{\alpha}_2}$	0.012	0.015	0.020	0.014	0.063	0.074	0.012
	$\hat{\delta}_2$	0.036	0.022	-0.002	0.026	0.081	-0.026	-0.007
	$s_{\hat{\delta}_2}$	0.010	0.020	0.012	0.007	0.040	0.038	0.006
	R^2	.994	.985	.996	.996	.882	.918	.998

Table 6 (cont'd.)

LEHCL/N	Statistic \ Cell No.	I	II	III	IV	V	VI	VII
	$\hat{\Gamma}_3$	3.204	5.650	5.291	4.047	4.434	5.028	8.453
	$s_{\hat{\Gamma}_3}$	0.395	0.250	1.022	0.997	0.666	0.948	1.079
	$\hat{\alpha}_3$	0.759	0.248	0.299	0.438	0.518	0.359	0.021
	$s_{\hat{\alpha}_3}$	0.076	0.049	0.173	0.148	0.099	0.141	0.140
	$\hat{\delta}_3$	-0.100	0.122	0.260	0.258	-0.200	0.141	-0.401
	$s_{\hat{\delta}_3}$	0.060	0.063	0.101	0.078	0.063	0.071	0.078
	R^2	.718	.427	.683	.684	.328	.556	.762

in each cell over the observed range of output in each cell is the fuel equation, LEHCF/N. In all cells, $\hat{\alpha}_2$ significantly exceeded $\hat{\alpha}_1$ and $\hat{\alpha}_3$ at the 5% level. For this reason, an estimate of the degree of homogeneity of the production function in each cell is given by $\hat{\beta}_2$ in each cell, where $\hat{\beta}_2 = \frac{1}{\hat{\alpha}_2}$.

Scale Effects

Having chosen the relevant component equation, one may proceed to test the hypotheses stated in Chapter III.D. concerning the effects on the production function of changes in scale, technology, and machine mix. The null hypothesis concerning scale is that at the plant level, the steam-electric generation process exhibits constant returns to scale. Under the null hypothesis, $\alpha_2 = 1$. Under the alternative of increasing returns to scale, $\alpha_2 < 1$. The null hypothesis was tested against this one-tailed alternative at the 5% level, using the statistic T_1, as defined in Chapter III.D. For Cells I and II, for which the normality assumption was not warranted for LEHCF/N, the null hypothesis was rejected if $T_1^2 > 20$, as explained by the Tchebycheff Inequality. For the remaining cells, a test at the 5% level was based on the fact that T_1 is distributed as "Student's" t with (N-2) degrees of freedom. The results of these tests are presented in Table 7. The null hypothesis was rejected in favor of the alternative of increasing returns to scale in all cells except Cell VI.

Table 7

Results of Tests of the Constant Returns to Scale Hypothesis*

I:	$T_1^2 = 42.25$*
II:	$T_1^2 = 86.68$*
III:	$T_1 = -2.1$*
IV:	$T_1 = -4.3$*
V:	$T_1 = -3.7$*
VI:	$T_1 = -.095$
VII:	$T_1 = -4.4$*

*indicates increasing returns to scale significant at the 5% level.

It is interesting to note that the plants in Cell VI employed units with the largest nameplate-rated generators and most advanced furnaces, turbines, and generator cooling systems of any subcritical steam generators. Furthermore, the supercritical steam generators in Cell VII exhibited statistically significant increasing returns to scale. It appears reasonable to infer from these results that the introduction of supercritical generators in 1958 was the result of implicit recognition by engineers that the advanced units in Cell VI exhausted all technological scale economies possible using subcritical steam generators.

The introduction of supercritical steam generators removed this barrier to growth of electricity generators, by restoring the condition of increasing returns to scale. In this connection, it is significant that the largest unit installed in a Cell VI plant by 1965 was rated at 570 mw; while the largest unit installed in a supercritical plant by that date was rated at 850 mw. These figures are reported in Chapter II.B. By 1968, according to Power[3] magazine's Modern Plant Design Survey, the corresponding figures were 725 mw and 1300 mw. Apparently, the size of the largest unit in supercritical plants grew by 53% between 1965 and 1968 while the corresponding size grew by only 39% during the same period for Cell VI plants because of the superior potential economies of scale associated with supercritical steam generation.

Effects of Technological Change

The second question to be considered concerning the results presented in Table 6 is whether technological change as represented by the cell classification scheme of Table 4 has affected plant efficiency independently of scale effects. The results presented in Table 7 indicate that variations in output require significantly less than proportional variations in fuel input, one form of economy of

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[3]Power, op. cit., October, 1968.

scale. One method of isolating changes across cells in fuel efficiency independently of scale or output effects is to compare across cells the fuel input necessary to produce one megawatt-hour of electricity with one turbgenerator unit. Examining equation (III-66b), LEHCF/N, it is seen that when $Y_i = N_i = 1$, $F_i = \Gamma_2 e^{\delta_2} e^{u_i}$. As will be explained below, δ_2 was not significantly different from zero for any cell except Cell IV. Therefore, the expected value of F_i when $Y_i = N_i = 1$ is $\Gamma_2 e^{\frac{\sigma_{00}}{2}}$, Therefore, under the assumption that σ_{00} is equal for all cells, and under the null hypothesis that technological change has not affected efficiency independently of scale effects, Γ_2 is equal for all cells. Under the alternative that a given class of plants is mor efficient than another, Γ_2 in the former cell is less than Γ_2 in the latter. This null hypothesis was tested for each pair of cells, using the test statistic T_2, as defined in Chapter III.D. Under the stochastic specification of the maintained hypothesis, the appropriate test when either Cell I or II is a member of the pair being considered is to reject the null hypothesis when $T_2^2 > 20$. For pairs not involving either of those cells, one may construct a test using the fact that T_2 is distributed as "Student's" t with 2(N-K) degrees of freedom. The results of these tests are presented in Table 8. In calculating T_2, Γ_2 for the higher-numbered cell was

Table 8

Results of Tests for Equality of Γ_2 across Cells

Cell No.	I	II	III	IV	V	VI
II	$T_2^2 = 16.48$ (-)					
III	$T_2^2 = 5.03$ (+)	$T_2^2 = 24.65$* (+)				
IV	$T_2^2 = 4.08$ (+)	$T_2^2 = 27.72$* (+)	$T_2 = -.54$			
V	$T_2^2 = 5.23$ (-)	$T_2^2 = 1.77$ (-)	$T_2 = -2.91$*	$T_2 = 2.77$*		
VI	$T_2^2 = 1.26$ (+)	$T_2^2 = 3.72$ (+)	$T_2 = .91$	$T_2 = .68$	$T_2 = 2.36$*	
VII	$T_2^2 = 7.00$ (+)	$T_2^2 = 34.36$* (+)	$T_2 = -.094$	$T_2 = .52$	$T_2 = 2.93$*	$T_2 = -.54$

* indicates significant at $\alpha = .05$

subtracted from Γ_2 for the lower-numbered cell. Therefore, if the latter cell includes more efficient plants than the former, the difference is positive. For those pairs for which the appropriate test was based on the Tchebycheff Inequality, so that the test statistic is T_2^2, the sign of the difference appears in parentheses in the corresponding entry in Table 8.

From Table 8, a mixed impression emerges. As expected, plants in Cell II appear significantly less efficient than plants in Cells III, IV, and VII, independently of scale effects. Similarly, plants in Cell V appear significantly less efficient than plants in Cells VI and VII. These results are in accordance with _a priori_ expectations. However, the result that plants in Cell V are less efficient than plants in Cells III and IV is surprising. The reader will recall that the only technological difference between Cells V and VI is that generators in Cell VI employed conventional liquid and hydrogen cooling systems. Steam generators in both these cells operated at higher temperatures and pressures than those of Cells III and IV; but more importantly, generators in Cells V and VI had higher nameplate ratings than did those in Cells III and IV. The implication of the results in Table 8 concerning Cell V seems to be that heat losses of these large generators using conventional tube-and-fin cooling systems decreased plant efficiency substantially. Only when hollow-conductor cooling was introduced, in Cell VI, did the increase in efficiency from higher temperatures and pressures more than compensate for the heat loss incurred with the higher-rated generators.

Differences in Γ_2 between all other pairs of cells were not significant. For these cells, more advanced technology did not improve plant efficiency at levels of output approaching one megawatt-hour. Therefore, the advantage to technological change in these cells appears to have been to allow the construction of larger plants. These large-scale plants were able to operate more efficiently than their less advanced predecessors only at higher rates of output. This would appear to explain the fact, discussed in Chapter II, that at any point in time, only plants with the most highly-rated generators incorporate the most advanced technology.

In this connection, a comparison of the values of Γ_1, the intercept of the capital component, LEHCC/N, across cells is revealing. This comparison indicated that to generate one megawatt-hour of electricity with one turbogenerator, plants in the technologically more advanced cells require that generator to be of greater installed capacity. Following an argument similar to that which justified comparison of Γ_2 across cells, if Γ_1 increases as technology advances, higher levels of technology require larger turbogenerators to be equally efficient at low outputs. To test whether such a phenomenon actually occurred, the null hypothesis that r_1 is equal for all cells was tested against the alternative that for any pair of cells r_1 is larger in the higher-numbered cell of the pair. As with the test for differences in r_2 across cells, r_1 for the higher-numbered cell was subtracted from r_1 for the lower-numbered cell. Therefore, negative differences support the contention that technological change has made the process more capital-intensive.

As shown in Table 9, with two exceptions--III and IV, and V and VI--the sign of the difference indicated that for each pair of cells, the one involving the more advanced technology required a more highly rated unit to generate one megawatt-hour of electricity than did the other. The difference was statistically significant at the 5% level for the following pairs of cells: I-V, I-VI, I-VII, II-V, II-VII, III-VII, IV-V, IV-VI, IV-VII, V-VII, and VI-VII. Since r_1 and r_2 did not increase similarly, and since in this model inputs are combined proportionately to r_1, r_2 and r_3, technological change appears to have made the industry more capital-intensive. These results provide additional support for the practice of engineers to use the most advanced technology only in construction of the most highly-rated units, at a given point in time.

Table 9

Results of Tests for Equality of Γ_1 across Cells

Cell No.	I	II	III	IV	V	VI
II	$T_2^2 = .977$ (−)					
III	$T_2^2 = 3.10$ (−)	$T_2^2 = 1.355$ (−)				
IV	$T_2 = -1.25$	$T_2^2 = .231$ (−)	$T_2^2 = .509$ (+)			
V	$T_2 = -5.05$*	$T_2^2 = 20.100$* (−)	$T_2^2 = 5.646$ (−)	$T_2 = -3.58$*		
VI	$T_2 = -3.45$*	$T_2^2 = 9.078$ (−)	$T_2^2 = 3.252$ (−)	$T_2 = 2.56$*	$T_2 = .063$	
VII	$T_2 = -8.38$*	$T_2^2 = 62.567$* (−)	$T_2^2 = 26.021$* (−)	$T_2 = -6.69$*	$T_2 = -3.00$*	$T_2 = -2.39$

* indicates significance at the 5% level.

Significance of the Number of Units per Plant

The third area involving the results in Stage 3 is the effect of machine-mix, as represented by N, the number of turbogenerator units in a plant of given installed capacity. The relevant estimated coefficient is δ, and arguments were advanced in Chapter III.D for both positive and negative signs for δ. Only the value of $\hat{\delta}_2$, in LEHCF/N, is relevant from the standpoint of plant efficiency, since fuel is the "limiting" factor. However, the sign and statistical significance of $\hat{\delta}$ were considered for all three equations, to explore possible effects of machine-mix on the inputs of all three factors. The null hypotheses that δ_1, δ_2, δ_3, = 0 were tested against two-tailed alternatives at the 5% level, using the test statistic T_3, defined in Chapter III.D. Depending on the equation and the cell, tests were based on either the Tchebycheff Inequality or "Student's" t-statistic, whichever was appropriate under the maintained hypothesis. The results of these tests are presented in Table 10. For those tests for which the Tchebycheff Inequality was used, so that T_3^2 is the appropriate test statistic, the sign of $\hat{\delta}$ is given in parentheses.

As is clear from examination of Table 10, $\hat{\delta}$ was not significantly non-zero for any equation in Cells I and II. In the capital equation, LEHCC/N, $\hat{\delta}_1$ was significantly positive in Cells IV, V, VI, and VII. This indicates that

Table 10

Results of Tests that $\delta = 0$

Cell No \ Function	LEHCC/N	LEHCF/N	LEHCL/N
I	$T_3 = -1.44$	$T_3^2 = 14.52$ (+)	$T_3 = -1.67$
II	$T_3^2 = .0925$ (+)	$T_3^2 = 1.2633$ (+)	$T_3^2 = 3.71$ (+)
III	$T_3^2 = 8.86$ (+)	$T_3 = -.1894$	$T_3 = 2.58$*
IV	$T_3 = 3.43$*	$T_3 = 3.57$*	$T_3 = 3.30$*
V	$T_3 = 5.07$*	$T_3 = 1.99$	$T_3 = -3.16$*
VI	$T_3 = 3.30$*	$T_3 = -.69$	$T_3 = 1.98$
VII	$T_3 = 8.96$*	$T_3 = -1.12$	$T_3 = -5.15$

for a plant containing more units, a given increase in output requires a proportionately greater increase in installed capacity. Given the previous finding that technologically advanced units are inefficient at low outputs, this result is not surprising. In the fuel equation, LEHCF/N, $\hat{\delta}_2$ was significantly positive in Cell IV, indicating that for that cell, the elasticity of fuel input with respect to output increases as the number of units in the plant increases. For other cells, $\hat{\delta}_2$ was not significant. The results for LEHCL/N are mixed, with $\hat{\delta}_3$ being significantly negative in Cells V and VII, but positive in Cells III and IV.

The only consistent pattern in these results is the group of significantly positive values of $\hat{\delta}_1$ in LEHCC/N for Cells IV-VII. This indicates that for the most technologically advanced plants, the amount of installed plant capacity necessary to generate a given output increases as the plant capacity is divided among more turbogenerator units. The value of N, the number of units, did not significantly affect plant efficiency, which is measured by LEHCF/N. Therefore, it appears that the loss of plant efficiency one would expect from division of a given plant capacity among more units was approximately compensated for by the gains in efficiency from additional possibilities of substitution among units made possible by the addition of units to the plant.

The empirical results reported here suggest the following conclusions:

(a) Fuel is the "limiting" factor in steam-electric generation for all cells in the sense that an increase in output requires a proportionately greater increase in the input of fuel than of the other two factors, capital and labor.

(b) Increasing returns to scale is characteristic of steam-electric generation at the plant level, since an increase in output requires less than proportional increases in all three inputs.

(c) Technological change has not significantly affected plant efficiency independently of scale, since the fuel input function did not shift significantly across cells as defined in Table 4. However, by enabling firms to construct plants incorporating more highly-rated turbogenerators, technological advances have made plants more efficient through economies of scale.

(d) Technological change in the industry has operated in the form of substitution of capital for the other factors, since only the capital input function intercept increased significantly between 1948 and 1965.

(e) The most technologically advanced plants are less suitable than other plants for low levels of output. Evidence for this is that at low levels of output, the newer plants make no more efficient use of fuel than the older plants, while requiring proportionately higher levels of installed plant capacity.

(f) Variability of the returns-to-scale parameter is not statistically significant for any single technologically homogeneous group of generating plants. However, the most modern subcritical plants exhausted virtually all technical potential economies of scale using subcritical generation. Plants incorporating supercritical generators are subject to increasing returns to scale.

(g) The number of turbogenerator units in a plant of given installed capacity is not consistently related to plant efficiency, except in the following manner: for the most modern plants in the sample, allocation of plant capacity among more units raises the plant capacity required to produce any given level of output.

Given these conclusions, it appears that the methodology of the present study, particularly the technolocial classification scheme and the three-equation model, has made possible more precise statements than those of previous authors. A brief discussion of the consistency of the present findings with earlier findings is presented below.

Consistency of the Present Results with Previous Results

For the most part, the results obtained in the present study tend to confirm the results of previous authors. Since Komiya[4] and Hart and Chawla[5] estimated models conceptually similar to the group consisting of LEHCC/N, LEHCF/N, and LEHCL/N, direct numerical comparison with their results is possible. The comparison indicates a fair degree of consistency among the three studies. Less directly, the

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[4]Komiya, R., *op. cit.*, pp. 156-166.

[5]Hart, P.E., and Chawla, R.K., *op. cit.*, pp. 164-177.

present study tends to support and to explain most of the conclusions of Nerlove,[6] Dhrymes and Kurz,[7] and Galatin.[8]

In his fuel input equation, Komiya used average size of generating unit as a measure of the size of plant, as opposed to installed capacity in the present study. Nonetheless, the present results may be consistent with Komiya'a conclusion that the "improvement in the thermal efficiency can be explained by the increase in the scale of production rather than by the shift of the function."[9] In both studies, the conclusions were reached that increases in the scale of plant operations require less than proportional increases in fuel input; while technological change has not significantly shifted the intercept of the fuel input equation. These conclusions were also apparent in the Hart and Chawla results. Although shifts in Komiya's estimated capital equation across vintage cells were not significant, they were of the same direction as those of the present study, indicating increases over time in the amount of capital used in plants of a given size. The lack of significance may result from his use of vintage as a proxy for technology.

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[6]Nerlove, M., op. cit., pp. 167-198.

[7]Dhrymes and Kurz, op. cit.

[8]Galatin, M., op. cit.

[9]Komiya, op. cit., p. 161.

It has been stressed throughout this thesis that this practice groups technologically dissimilar plants together, which tends to conceal differences between the production function parameters of the technologically dissimilar plants. This effect could account for the absence of significant shifts of the production function in both Komiya's study and the present one. Hart and Chawla did not consider the significance of the shift of the capital intercept across their two cells, but it appears consistent with Komiya's results. As in the present study, Komiya's labor input equation performed less satisfactorily, with lower values of R^2 and non-systematic changes in the coefficients across cells. In neither his study nor the present one were any conclusions regarding the labor input justified by the results. Hart and Chawla did not consider the labor input.

Considering studies in other situations, less specific comparisons are possible. The present study tends to confirm the general conclusion of Galatin, Nerlove, and Dhrymes and Kurz that increasing returns to scale is the prevailing condition in steam-electric generation. However, several more specific conclusions of those authors were not directly confirmed. For example, testing of generalized production functions in the present study provided no support for the suggestions by Nerlove and Dhrymes and Kurz that returns to scale are a variable function of output. However, Nerlove's data sample, a cross-section of plants operating in 1955, and Dhrymes and Kurz's vintage samples both include technologically dissimilar plants. Therefore, what both these studies took for decreases in the returns-to-scale parameter due to increases in output may in fact be due to technological change in subcritical generating plants. The reader will recall that for the plants in Cell VI returns to scale were found to be approximately constant in the present study. Any Cell VI plants observed by previous authors producing large outputs under constant returns to scale could be interpreted as a decrease in the returns-to-scale parameter if plants were not carefully differentiated.

Another apparent discrepancy between the present study and previous one concerns the effect of technological change on plant efficiency. Both Galatin and Dhrymes and Kurz claim to have shown that technological change has improved plant efficiency independently of scale effects, while the present results indicate that technological change improved plant efficiency only by allowing firms to achieve economies of scale. The apparent contradiction with Galatin's result is easily resolved when one realizes that his conclusion that technological change has significantly shifted the production is based on the entire period of his study, 1920-1953. However, the results of the present study are comparable with only the last two of Galatin's vintage cells, 1945-50 and 1951-53, between which he reports no significant shift.

The source of the contradiction with the results of Dhrymes and Kurz is less apparent. Their conclusion that technological change has increased plant efficiency independently of scale effects is based on the fact that in their study the estimated CES production function intercept $\hat{A}$ increases monotonically across vintage cells for three of four capacity cells. No standard errors for $\hat{A}$ are presented, so that one is unable to evaluate the significance of this shift. In a footnote, the authors point out that "comparison of A across size groups, for a given technological period, is not very meaningful since we are estimating segments of a production function by a suitable approximation which does not have to yield the same constant, $\hat{A}$, for each segment."[10] However, since technologically more advanced plants are used to produce larger outputs than are less advanced ones, as explained in Chapter II of this thesis, production functions for different cells are also estimated over different segments. Since plants in all cells but one exhibited increasing returns to scale in the

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[10]Khrymes and Kurz, *op. cit.*, p. 309.

present study, the observed differences between cells in plant efficiency observed by Dhrymes and Kurz appear to be the joint result of technology and scale changes. Recognizing that technological change has frequently enabled firms to increase their scales of plant, the present conclusions are not inconsistent with those of Dhrymes and Kurz.

Taken as a whole, the results of the present study appear to confirm the work of previous authors and to make their conclusions somewhat more specific. Some implications of these results are discussed in Chapter V.

Chapter V. Conclusions from this Study

V.A. Summary of the Thesis

The problem explicitly addressed at the outset of this study was to separate the postwar effects of technological change and increases in scale of production on the efficiency of stem-electric generation. It was pointed out in Chapter I that this question might be of interest to both regulatory agencies and regulated electric power companies. Furthermore, it was pointed out that previous studies of this question were generally subject to two methodological problems. First, such proxies as vintage were used for indicating technological change, rather than considering specific innovations. In this way, important technological differences between generating plants constructed contemporaneously were often concealed. Thus, spurious differences were attributed to plants constructed at different periods, even though they may have incorporated identical technology. Second, the production models used as maintained hypotheses under which to study the question were simply assumed, with no attention paid to their appropriateness for the generating plant. For this reason, the models were often unwittingly estimated under undesirable constraints or assumptions, e.g., restrictions on the elasticity of substitution, which destroyed the optimal properties of the estimation technique employed. Furthermore, failure to examine the appropriateness of the stochastic specifications of the estimated models introduced the possibility of unwarranted inferences. Hypothesis tests based on unfulfilled normality assumptions may have raised the probability of Type II errors above the nominal α-levels.

In Chapters II and III, a procedure was proposed to alleviate these problems. It was thought that adequate indicators of technological change could not be defined without a basic understanding of the mechanical process of converting fuel into electricity. Following a discussion of this subject, what appeared to be the most important postwar technological innovations in steam-electric generation were used as defining characteristics for technologically homogeneous cells of generating plants. These cells were thought to represent technological change more adequately than the vintage cells used by previous authors.

To insure that inferences concerning the various cells were carried out under appropriate maintained hypotheses, a three-stage procedure was proposed. Under the assumption that an inappropriate model would be subject to statistically significant specification error, the first stage was to estimate a number of alternative models for each cell and to apply three specification error tests to each estimated model. Among others, the potential alternatives included generalizations of the ZKD model in which output is a stochastic function of the inputs and firms are assumed to maximize expected profit, generalized production functions to allow for variability of returns to scale, and various fixed-relative-factor-proportions functions. The choice of a model for each cell was based on the specification error test results, relative values of R^2 for the alternatives, agreement of signs of estimated coefficients with prior expectations, and freedom from such difficulties as multicollinearity. The model selected in Stage One was a fixed-relative-proportions model, with output and the number of turbogenerators per plant assumed to be exogenous at the plant level. Similar models had been applied previously to this industry by Komiya and by Hart and Chawla.

In Stage Two, the chosen model was re-estimated, using a different data sample for each cell, and tested again for specification error, as independent confirmation of the choice made in Stage One. Furthermore, in each cell a test was made of the null hypothesis that the distribution of disturbances for each component of the three-equation model had skewness and kurtosis parameters equal to those of the normal distribution. For those components and cells for which either hypothesis was rejected, it was decided to use the Tchebycheff Inequality for hypothesis tests in Stage Three, since that inequality requires no distributional assumptions. In this way, it was hoped that "Student's" t-test would be used only in those cases where the normality assumption was warranted.

In Stage Three, the three components of the fixed-relative-factor-proportions model were again estimated, using a third data sample from each cell. The following null hypotheses were tested, using the estimated results:

a. Steam-electric plants are subject to constant returns to scale. This hypothesis was rejected for most cells.

b. Technological innovations have not affected the production function parameters across cells, except as they have allowed generating plants to capture economies of scale. This null hypothesis was not rejected for most cells, with one exception. For the most modern plants in the sample, technological change has taken the form of substitution of capital for the other factors.

c. The number of turbogenerator units in a plant does not significantly affect either factor proportions or returns to scale. This null hypothesis was not rejected for most cells.

In addition to the results of the hypothesis tests, the estimated results indicated some other facts about steam-electric generation. First, in the context of a fixed-relative-proportions model, fuel may be considered the "limiting" factor, in the sense that increases in output require proportionately greater increases in the fuel input than in the other inputs. Since despite this fact, inputs are assumed to be combined in fixed proportions, one concludes that capital and labor are underutilized in the generating process. Second, plants incorporating the most modern technology are least suitable for generating low annual outputs. This helps to explain why all plants being constructed at a given time do not incorporate the most modern technology available. Third, variability of returns to scale does not appear statistically significant within any technological cell. However, the most advanced plants to use subcritical steam generators appear to have exhausted all potential economies of scale.

For the most part, these conclusions seem consistent with the results of previous investigators. In the concluding section of this chapter, a short discussion of the limitations of this study is presented, along with some implications for policy and for further research.

V.B. Implications of the Results

The fundamental conclusion of the present study may be stated as follows. The steam-electric generation process is subject to increasing returns to scale at the plant level. Some postwar technological innovations, notably those that increased turbogenerator temperatures and pressures, have improved plant efficiency by allowing plants to capture the potential economies of scale. No specific postwar innovation appears to have increased plant efficiency independently of scale effects. Moreover, the most modern subcritical plants appear to have exhausted all potential scale economies using that method.

This conclusion speaks well for the influence of regulatory agencies in the power industry. The fact that generating plants have grown larger and more efficient under regulation appears to invalidate the arguments cited in Chapter I contending that regulation leads to inefficiency of the regulated industry. In fact, considering the statement in Chapter II that innovation in this industry appears to be an inductive, trial-and-error process, fair-rate-of-return regulation may have stimulated innovation by relieving firms of part of the potential loss from expensive unsuccessful experiments. Of course, any conclusion of this sort would require more detailed study of the historical patterns of innovation and rate schedules of specific firms.

A second implication of the conclusions of this study is that the trend toward concentration of generating equipment in fewer, larger generating plants is both explicable and desirable economically. As emphasized in the discussion of results, the more advanced levels of technology appear to improve operating efficiency only in plants producing relatively large levels of output. Of course, this is due to the fact that successful innovations have improved efficiency by making economies of scale accessible through higher design temperatures and pressures. It is not surprising that a regulated profit-maximizing firm will increase its scale of plant to achieve economies of scale as these become technologically attainable. Furthermore, the improved efficiency in the use of limited supplies of fossil fuels is undeniably a benefit to society of this concentration.

V.C. Limitations of the Study and Suggestions for Future Research

Procedural Limitations

The results of this study are conditional on several details of procedure, which should be discussed explicitly. The first of these details concerns the relationship between technological change and the capital input. Not only is capital equipment in a given generating plant heterogeneous, but in a time-series study of the present type, the array of potential kinds of capital equipment changes as technology advances. An example of this type of secular change in the concept of capital is the fact that the term "boiler", common in 1948, has since been replaced by the term "steam generator", so that supercritical turbogenerator units may be adequately described. For these reasons, no single physical measure of capital was available for the present study. Measures based on the monetary value of capital, being calculated for the benefit of tax accountants rather than observing economists, appear to be unsatisfactory also.

In the present study, the proxy chosen for capital was installed plant capacity. Within any of the technologically homogeneous cells, this proxy is likely to be fairly satisfactory, since the type and quantity of ancillary equipment--preheaters, superheaters, etc.--is likely to be roughly uniform for all plants in a given cell. However, it should be pointed out that in this framework, such substitution of capital for other factors as plant automation will not be recorded. Therefore, it is entirely possible that with some alternative measure of capital or classification system for generating plants, the fixed-relative-proportions model employed in the present study would have been completely untenable. To the extent allowed by available data, significant examples of this type of factor substitution were incorporated in the cell definitions, with the results that alternative measures of the capital input might have caused more significant changes in the intercept of the capital input equation across cells.

A second procedural limitation of the present study was the limitation of the range of potential alternative models to those that could be estimated by a single-equation technique. This limitation was caused by a lack of adequate factor price data and the fact that the specification error test statistics used in Stages One and Two have been developed only for single-equation ordinary least squares estimation. This limitation precluded direct consideration of models of the Dhrymes and Kurz variety, in which plants are assumed to combine substitutable inputs in an effort to minimize the cost of generating an exogenously determined level of output. However, the effect of failure to incorporate the cost-minimization conditions into an equation system to be estimated, known as simultaneous equation bias, did not appear to be statistically significant in the model estimated for the purpose of statistical inference. The specification error tests are designed to reject models in which the simultaneous-equation problem is statistically significant.

Limitations of Scope

Three limitations were placed on the scope of the present study for reasons of convenience, the time period of the data sample, and the lack of quantifying social costs and benefits. While these limitations in no way invalidate the results, they do suggest caution in their application.

First, for the sake of convenience no consideration was given to the process of transmission of electricity after generation. The present results strongly indicate that productive efficiency is increased if generating facilities are concentrated in large units and plants, which are then able to achieve economies of scale. But it should be borne in mind that in areas of low population density, this concentration of generating capacity is likely to raise transmission costs substantially. Other conditions equal, transmission losses increase with the distance between the source of electricity and the site of consumption. In recent years, transmission losses have been reduced by increasing the voltage at which the electricity is sent from the generating plant. But in turn, high-voltage transmission requires a greater investment in transmission equipment--heavier wires, stronger towers, and larger transformers. Therefore, in any managerial decision concerning the construction of a new plant, the gains in generating efficiency must be weighed against possible resulting increases in transmission costs. To the author's knowledge, no economic investigation of electricity transmission has been carried out to date.

Another limitation in the scope of the present study is exclusive concentration on fossil-fueled thermal electricity generation. Hydroelectric plants were excluded from the study for the sake of convenience, and nuclear plants were excluded due to the fragmentary nature of reported experience with nuclear generation that existed at the time data were collected for the present study. Since that time, production data on nuclear-fueled plants have become available, as have more extensive data on plants using exclusively supercritical steam generators. The present study could be extended fairly easily to include these data.

The third and most implicit limitation on the scope of the present study is the exclusive concentration of the production function analysis on those inputs which are purchased in market transactions--capital, fuel, and labor. In recent years concern has mounted over pollution of air with the by-products of combustion and thermal pollution of the source of cooling water with hot water from steam generators. Looked at another way, these types of pollution may be viewed as the use of two factors not purchased in the market--clean air, and water of a suitable temperature to support aquatic life. Concentration of generating facilities, which the present study has shown to improve the efficiency of fuel use, tends to concentrate the use of these two factors in fewer areas. The implications of this fact should be considered.

On the one hand, this concentration of generating equipment redistributes the cost of generating the nation's electricity from society at large to those living near the large plants. This inequitable redistribution of social cost may be considered undesirable. But on the other hand, the devices used in large plants to improve combustion efficiency simultaneously reduce the emission of unwanted by-products; while sophisticated condensing equipment used in large plants raises steam cycle efficiency as it reduces the temperature of used water returned to its sources. Thus, it is not clear that efficient use of factors purchased in the market is incompatible with general reduction of the social costs of electricity generation. No consideration was given to these questions in the present study, due to a lack of data and adequate techniques for measuring social cost.

Suggested Further Research

Some related research is suggested immediately by a consideration of the limitations of the present study. For example, by simply updating the study from 1965 one could broaden it to include more observations on plants incorporating supercritical steam generators, as well as the class of plants using nuclear fuel.

The functions developed in the present study relate inputs of three factors to the output of electricity, for plants incorporating various levels of generation technology. A similar approach could be used to relate plant output to available electricity for users, under various types of transmission technology. In this way, a link could be established between factor inputs and the usable output of electricity in various technologically homogeneous cells, with the cells defined in terms of both generation and transmission equipment. With this type of link, one would be better equipped to examine the trade-off between gains of generation efficiency and losses of transmission efficiency resulting from concentration of generating equipment in fewer large plants.

In a similar way, the output of undesirable by-products of combustion could be related to the input of fuel in plants of various well-defined technological types. If one had appropriate data, it would then be possible to examine the impact of technological change and economies of scale on air pollution by steam-electric generating plants.

In conclusion, the present study appears to have demonstrated a viable approach to problems of technological change and returns to scale in steam-electric generation.

The results of the study are of course conditional on that approach, but they appear consistent with the results of previous studies carried out using different approaches. It appears that the method may be extended to more recent technological changes, and to a broader concept of plant efficiency, in which transmission costs and the social costs of electricity generation are included.

APPENDICES

Appendix A

Results of Stage One

In this Appendix, the results are presented for Stage One of the empirical analysis, in which a model was tentatively selected as a restricted maintained hypothesis for each of Cells I through IV. The results are presented in Table 12 in the following manner. For each model considered, and for each of the cells used in Stage One, the values of R^2 and three specification error test statistics are presented. The statistic produced by RESET is F, that of WSET is W, and that of BAMSET is M. In addition, if a model was rejected by a test, that is indicated by one or more asterisks following the test statistic. One asterisk represents rejection at the 10% level, two represent rejection at the 5% level, and three represent rejection at the 1% level. All statistics presented are the output of DATGEN, a program designed to carry out the specification error tests.

Table 11

Results of Stage One

Eq. (III-41a): CD

Statistic \ Cell No.	I	II	III	IV
F	1.6071	4.0777**	3.5501**	3.6739*
W	.5711***	.6989***	.0685***	.7464***
M	18.3359***	4.9286*	51.2817***	3.2072
R^2	.9890	.9902	.9589	.9978

Eq. (III-45): CES

Statistic \ Cell No.	I	II	III	IV
F	3.7787**	4.5784***	4.3156***	3.7309**
W	.5233***	.7323***	.0986***	.7655***
M	17.6391***	2.7689	44.4970***	4.2259
R^2	1.0000	.9991	1.0000	.9906

Table 11 (cont'd.)

Eq. (III-49): CD1

Statistic \ Cell No.	I	II	III	IV
F	1.6002	4.0776**	4.0662**	3.6739
W	.5710***	.6989***	.0596***	.7464***
M	18.3313***	4.9296*	56.4919***	3.2067
R^2	.9890	.9902	.9549	.9978

Eq. (III-50): CES1

Statistic \ Cell No.	I	II	III	IV
F	3.4432**	4.5785***	4.7836***	3.7160**
W	.5178***	.7323***	.0934***	.7657***
M	18.1493***	2.7689	48.0752***	4.1975
R^2	1.0000	.9950	.9904	.9853

Eq. (III-56): CD2

Statistic \ Cell No.	I	II	III	IV
F	1.3603	4.0777**	4.0706**	3.6606**
W	.5687***	.6989***	.0596***	.7463***
M	18.1744***	4.9287*	56.5339***	3.1823***
R^2	.9892	.9902	.9550	.9978

Table 11 (cont'd.)

Eq. (III-57): CES2

Cell No. / Statistic	I	II	III	IV
F	3.4432**	4.5785***	4.7836***	3.7160**
W	.5178***	.7323***	.0934***	.7657***
M	18.1493***	2.7689	48.0752***	4.1975
R^2	1.0000	.9950	.9904	.9853

Eq. (III-60a): LUHCC

Cell No. / Statistic	I	II	III	IV
F	.7340	1.3045	N/A	2.2357
W	.7211***	.4752***	N/A	.3959***
M	4.5762	9.9807***	N/A	13.6106***
R^2	.8714	.7946	.8217	.8558

Eq. (III-60b): LUHCF

Cell No. / Statistic	I	II	III	IV
F	1.0353	4.1498**	2.7376*	4.0156**
W	.4365***	.7016***	.0700***	.6946***
M	16.6246***	9.6867***	48.5665	2.9456
R^2	.9861	.9900	.9568	.9966

Table 11 (cont'd.)

Eq. (III-60c): LUHCL

Statistic \ Cell No.	I	II	III	IV
F	5.3996***	.9784	N/A	N/A
W	.7544***	.6540***	N/A	N/A
M	9.0251**	.9663	N/A	N/A
R^2	.4429	.4312	.5619	.7171

Eq. (III-61a): LEHCC

Statistic \ Cell No.	I	II	III	IV
F	.0135	5.2949***	4.0455**	15.4237***
W	.7433***	.7652***	.8138***	.7157***
M	1.0905	1.0280	6.1490**	8.2170
R^2	.8714	.7946	.8217	.8558

Eq. (III-61b): LEHCF

Statistic \ Cell No.	I	II	III	IV
F	5.0075***	5.5063***	125.5156***	3.9265**
W	.4548***	.7064***	.8042***	.6981***
M	20.5522***	2.5924	6.0813**	.9428
R^2	.9861	.9900	.9568	.9966

Table 11 (cont'd.)

Eq. (III-61c): LEHCL

Statistic \ Cell No.	I	II	III	IV
F	1.5355	3.1933**	11.9313***	1.1927
W	.7321***	.71860***	.8151***	.7618***
M	.0539	.5742	1.5208	7.1331**
R^2	.4429	.4312	.5619	.7171

Eq. (III-63): LING

Statistic \ Cell No.	I	II	III	IV
F	4.8002***	2.8242**	.3638	1.2834
W	.6970***	.7296***	.7236***	.7952***
M	2.6565	3.0542	1.6176	.1623
R^2	.3012	.7191	.3750	1.0000

Eq. (III-64): CD/N

Statistic \ Cell No.	I	II	III	IV
F	1.1290	4.5113***	.8350	.9672
W	.6359***	.6769***	.7100***	.6368***
M	8.0869**	2.7919	1.3358	7.3137**
R^2	.9964	.9907	.9948	.9982

Table 11 (cont'd.)

Eq. (III-65): CES/N

Statistic \ Cell No.	I	II	III	IV
F	1.1480	.2323	.9285	1.1515
W	.6154***	.6651***	.6270***	.5829***
M	11.5573***	12.3708***	3.1220	1.5594
R^2	.9956	.9942	.9998	.9968

Eq. (III-66a): LEHCC/N

Statistic \ Cell No.	I	II	III	IV
F	2.1190	6.9671***	1.3297	3.1780**
W	.7888***	.7990***	.7705***	.7514***
M	2.4883	2.8539	5.2865*	1.8188
R^2	.8467	.8063	.9109	.9300

Eq. (III-66b): LEHCF/N

Statistic \ Cell No.	I	II	III	IV
F	7.6798***	2.7097	.2246	1.1163
W	.7654***	.6471***	.7578***	.7597***
M	1.3412	18.8550***	1.1427	4.6826*
R^2	.9946	.9858	.9962	.9969

Table 11 (cont'd.)

Eq. (III-66c): LEHCL/N

Statistic \ Cell No.	I			
F	1.1404	5.1355***	9.2395***	2.3142*
W	.6511***	.7941***	.8308***	.7234***
M	.5419	7.6183**	.4808	7.2864
R^2	.7182	.5048	.6830	.6843

Appendix B

The Results of Stage Two

In Table 13 are the values of R^2, $\sqrt{b_1}$, g_2, and the results of the specification error tests RESET, WSET, and BAMSET obtained when the model {LEHCC/N, LEHCF/N, LEHCL/N} was re-estimated, using a second data sample. The null hypothesis being tested with the specification error test statistics is that the disturbances are i.i.d. $N(\phi, \sigma^2 I)$. The null hypothesis being tested with the aid of $\sqrt{b_1}$ and g_2 is that the skewness and kurtosis parameters are zero and 3, respectively, as they would be if the true distribution of disturbances were normally distributed. For those equations and cells for which the value of a statistic indicates rejection of the null hypothesis, the level of significance of the rejection is indicated with asterisks--1 for the 10% level, 2 for the 5% level, and 3 for the 1% level.

Table 12

Results of Stage Two

Eq. (III-66a): LEHCC/N

Statistic \ Cell No.	I	II	III	IV
F	1.2116	9.1878***	1.2205	1.6070
W	.7578***	.8216***	.7835***	.6390***
M	2.6742	9.5109***	.8732	11.2815***
R^2	.8488	.8660	.8970	.9631
$\sqrt{b_1}$	.7048**	1.0055***	1.0095***	-.3907
g_2	3.4025	5.1739***	3.4476	4.7104**

Eq. (III-66b): LEHCF/N

Statistic \ Cell No.	I	II	III	IV
F	4.1869**	2.7781*	1.1988	3.7444**
W	.3405***	.6333***	.6966***	.7810***
M	15.4880***	5.0655*	1.4300	.4914***
R^2	.9900	.9924	.9957	.9981
$\sqrt{b_1}$	-.2460	2.6979***	.4525	.0164
g_2	1.8786***	15.2708***	2.6582	2.3119

Table 12 (cont'd.)

Eq. (III-66c): LEHCL/N

Statistic \ Cell No. I				
F	.0440	6.3280***	3.6512**	.9876
W	.07577***	.6674***	.8088***	.6933***
M	4.7150*	10.6981***	2.1001	4.1014
R^2	.6356	.6186	.6110	.7937
$\sqrt{b_1}$	.3452	1.5459***	.2870	-.8474
g_2	3.0017	6.0484***	2.7377	2.9765

Appendix C

Data Used in the Study

The data used in the present study were collected for 175 privately-owned central-station steam-generating plants which first began operations during the period 1948 to 1965, inclusively. These data were stored on magnetic tape for computer use, one tape file being allotted to each plant included in the sample. These plants and the firms by which they were owned are listed by file number in Table 14. An attempt was made to include all plants constructed during the 1948-1965 survey period; however, a few plants, for which the necessary data were not complete, were dropped from the sample.

Data on design characteristics of newly installed turbogenerator units were collected from the Modern Plant Design Surveys published annually by Power magazine.[1] Specifically, data were collected for each new unit on the furnace type and fuel used, number of bleedpoints and number of reheat cycles for the turbine, operating temperature and pressure of the steam generator, and both the

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[1] Power, New York: McGraw-Hill. Monthly, (1948-1965).

system and medium used to cool the generator. Data were also collected on the degree of plant automation. Unfortunately, these data were not sufficiently complete to be useful. On the basis of design characteristics of the original plant equipment, each plant was placed in a cell according to the definitions presented in Table 4 of Chapter II. If none of those cell definitions was appropriate, the plant was dropped from the sample.

For each plant in each cell, a time-series of annual production data was begun with the first year of plant operation. The production data were collected from the annual supplements to Steam-Electric Plant Construction Cost and Annual Production Expenses[2], published by the Federal Power Commission. Each time-series was continued until either 1965 or the first year in which a turbo-generator unit was installed whose design characteristics did not match those of the cell in which the plant had been placed originally.

The specific data that formed the sample for this study are listed by cell in Table 14 at the end of this appendix by plant file number. Using Table 13, the file number may be used to identify each plant and the firm by which it is owned. Output data are simply annual observations on net generation of electricity, measured in

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[2]Federal Power Commission, Steam-Electric Plant Construction Costs and Annual Production Expenses. Washington, D. C. Published annually.

thousands of megawatt-hours.[3] The capacity variable is net continuous plant capability when not limited by condenser water. This figure, which is usually slightly larger than the nameplate rating suggested by the manufacturer, is the absolute maximum output at which a turbogenerator may be safely operated. Fuel input, measured in Btu's, is calculated as the sum over all fuel types of the products of the fuel input in tons (coal), barrels (oil), or cubic feet (gas) by the Btu content per ton, barrel, or cubic foot, whichever is appropriate. The labor input, measured in man-hours, is calculated as the product of average number of employees, as reported by the Federal Power Commission for each plant, by the average weekly working hours of power plant employees, as reported by the U.S. Department of Labor. It may be argued correctly that these data do not reflect the number of shifts of operation of the various plants. However, no superior data were available. Plant factor, reported by the Federal Power Commission, is designed to measure the ratio of actual output divided by 8760 times the nameplate rating of installed turbogenerator units. An observed plant

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[3] Ibid.

factor greater than 100 indicates that the plant for which it is observed operated at an output greater than nameplate rating at some period during the year. Plant factor is reported as zero for all plants not in production a full year or those plants adding or deleting a unit during the year. The number of turbogenerator units operated for the entire year is reported for each year of observation. Finally, heat rate, a measure of plant efficiency used by engineers, is calculated as the quotient of fuel input in Btu's divided by output in kilowatt-hours.

Table 13

List of Plants in Sample, by File Number

File	Firm	Plant
1	Alabama Power Co.	Barry
2	Alabama Power Co.	Gadsden
3	Alabama Power Co.	Greene County
4	Southern Electric Generating Co.	Gaston
5	Arizona Public Service Co.	Cholla
6	Arizona Public Service Co.	Four Corners
7	Arizona Public Service Co.	Ocotillo
8	San Diego Gas & Electric Co.	Encina
9	San Diego Gas & Electric Co.	South Bay
10	Southern California Edison	Alamitas
11	Southern California Edison	Cool Water
12	Southern California Edison	El Segundo
13	Southern California Edison	Etiwanda
14	Southern California Edison	Huntington Beach
15	Southern California Edison	Mandalay Beach
16	Southern California Edison	Redondo Beach
17	Southern California Edison	San Bernardino
18	Public Service Co. of Colorado	Arapahoe
19	Public Service Co. of Colorado	Cameo
20	Public Service Co. of Colorado	Cherokee
21	Connecticut Light & Power Co.	Norwalk Harbor
22	Hartford Electric Light Co.	Middletown
23	United Illuminating Co.	Bridgeport Harbor
24	Delaware Power & Light Co.	Edge Moor
25	Potomac Electric Power Co.	Chalk Point
26	Potomac Electric Power Co.	Dickerson, Md.
27	Florida Power Corp.	P.L. Barton
28	Florida Power Corp.	Higgins
29	Florida Power Corp.	Sewannee
30	Florida Power & Light Co.	Cape Kennedy
31	Florida Power & Light Co.	Ft. Meyers
32	Florida Power & Light Co.	Palatka
33	Florida Power & Light Co.	Cutler
34	Florida Power & Light Co.	Port Everglades
35	Gulf Power Co.	Scholz
36	Gulf Power Co.	Lansing Smith
37	Tampa Electric Co.	J.J. Gannon
38	Tampa Electric Co.	Hooker's Point
39	Georgia Power Co.	Harllee Branch
40	Georgia Power Co.	Hammond
41	Georgia Power Co.	Jack McDonough
42	Georgia Power Co.	McManus

Table 13 (cont'd.)

43	Georgia Power Co.	Mitchell
44	Georgia Power Co.	Yates
45	Central Illinois Light Co.	E.D. Edwards
46	Central Illinois Gas & Light Co.	Sabrooke
47	Central Illinois Public Service Co.	Meredosia
48	Commonwealth Edison Co.	Ridgeland
49	Commonwealth Edison Co.	Will County
50	Electric Energy, Inc.	Joppa
51	Illinois Power Co.	Hennepin
52	Illinois Power Co.	Vermillion
53	Indiana-Michigan Electric Co.	Breed
54	Northern Indiana Public Service Co.	Bailly
55	Indianapolis Power & Light Co.	H.T. Pritchard
56	Public Service Co. of Indiana	Gallagher
57	Public Service Co. of Indiana	Wabash River
58	Interstate Power Co.	Fox Lake
59	Interstate Power Co.	Lansing
60	Iowa Power & Light Co.	Council Bluff
61	Iowa Public Service Co.	Neal
62	Iowa Southern Utilities Co.	Bridgeport
63	Central Kansas Power Co.	Colby
64	Kansas Gas & Electric	Gordon Evans
65	Western Power & Gas Co.	Cimarron River
66	Kentucky Utilities Co.	E.M. Brown
67	Kentucky Utilities Co.	Green River
68	Gulf States Utilities Co.	Roy S. Nelson
69	Gulf States Utilities Co.	Sabine
70	Gulf States Utilities Co.	Willow Glen
71	Louisiana Power & Light Co.	Little Gypsy
72	New Orleans Public Service, Inc.	Michaud
73	Southwestern Electric Power Co.	Knox Lee
74	Southwestern Electric Power Co.	Lone Star
75	Southwestern Electric Power Co.	Wilkes
76	Central Maine Power Co.	Walter F. Wyman
77	Baltimore Gas & Electric Co.	Charles F. Crane
78	Holyoke Water Power Co.	Mt. Torn
79	New England Power Co.	Brayton Point
80	Western Massachusetts Electric Co.	West Springfield
81	Consumers Power Co.	J.H. Campbell
82	Consumers Power Co.	B.C. Cobb
83	Consumers Power Co.	Dan E. Karn
84	Consumers Power Co.	J.R. Whiting
85	Detroit Edison	River Rouge
86	Detroit Edison	St. Clair
87	Upper Peninsula Generating Co.	Presque Isle
88	Upper Peninsula Power Co.	Escambia

Table 13 (cont'd.)

89	Upper Peninsula Power Co.	J.H. Warden
90	Minnesota Power & Light Co.	Aurora
91	Minnesota Power & Light Co.	Clay Boswell
92	Northern States Power Co.	Lawrence
93	Northern States Power Co.	Wilmarth
94	Northern States Power Co.	Bison
95	Mississippi Power Co.	Jack Watson
96	Mississippi Power Co.	Sweatt
97	Mississippi Power & Light Co.	Delta
98	Mississippi Power & Light Co.	Natchez
99	Kansas City Power & Light Co.	Hawthorn
100	Kansas City Power & Light Co.	Montrose
101	Missouri Public Service Co.	Ralph Green
102	Missouri Power & Light Co.	Mexico
103	Union Electric Co.	Meramer
104	Nevada Power Co.	Clark
105	Nevada Power Co.	Reid Gardner
106	Nevada Power Co.	Sunrise
107	Sierra Pacific Power Co.	Tracey
108	Public Service Co. of New Hampshire	Merrimack
109	Atlantic City Electric Co.	B.L. England
110	Public Service Electric & Gas Co.	Hudson
111	Public Service Electric & Gas Co.	Mercer
112	Public Service Co. of New Mexico	Reeves
113	Consolidated Edison	Astoria
114	Consolidated Edison	Ravenswood
115	Long Island Lighting	E.F. Barrett
116	Long Island Lighting	Port Jefferson
117	New York State Electric & Gas Corp.	Milliken
118	Niagara Mohawk Power Corp.	Albany
119	Niagara Mohawk Power Corp.	Dunkirk
120	Carolina Power & Light Co.	Ashville
121	Carolina Power & Light Co.	H.B. Robinson
122	Carolina Power & Light Co.	Weatherspoon
123	Duke Power Co.	G.G. Allen
124	Duke Power Co.	Dan River
125	Duke Power Co.	W.S. Lee
126	Duke Power Co.	Marshall
127	Montana-Dakota Utilities Co.	Heskett
128	Otter Tail Power Co.	Crookston
129	Otter Tail Power Co.	Ortenville
130	Cincinnati Gas & Electric Co.	W.J. Beckjard
131	Cleveland Electric Illuminating Co.	Eastlake
132	Ohio Edison Co.	Niles
133	Toledo Edison	Bay Shore
134	Oklahoma Gas & Electric Co.	Arbuckle

Table 13 (cont'd.)

135	Oklahoma Gas & Electric Co.	Mustang
136	Public Service Co. of Oklahoma	Southwestern
137	Metropolitan Edison Co.	Portland (Penn)
138	Pennsylvania Electric Co.	Shawville
139	Pennsylvania Electric Co.	Warren
140	Pennsylvania Power & Light Co.	Brunner Island
141	Philadelphia Electric Co.	Cromby
142	Philadelphia Electric Co.	Eddystone
143	West Penn Power Co.	Armstrong
144	West Penn Power Co.	Mitchell
145	South Carolina Electric & Gas Co.	Canadys
146	South Carolina Generating Co.	Urquhart
147	Central Power & Gas Co.	J.L. Bates
148	Houston Lighting & Power Co.	Sam Bertran
149	Houston Lighting & Power Co.	Webster
150	Houston Lighting & Power Co.	T.H. Wharton
151	Southwestern Public Service Co.	Cuningham
152	Southwestern Public Service Co.	Nichols
153	Southwestern Public Service Co.	Plant "X"
154	Texas Electric Service Co.	Eagle Mountain
155	Texas Electric Service Co.	Morgan Creek
156	Texas Electric Service Co.	Permian Basin
157	Texas Power & Light Co.	Collin County
158	Texas Power & Light Co.	Lake Creek
159	Texas Power & Light Co.	River Crest
160	Texas Power & Light Co.	Stryker Creek
161	Texas Power & Light Co.	Galley
162	West Texas Utilities Co.	Oak Creek
163	West Texas Utilities Co.	Paint Creek
164	Utah Power & Light Co.	Carbon County Section II
165	Utah Power & Light Co.	Gadsby Section I
166	Utah Power & Light Co.	Gadsby Section II
167	Utah Power & Light Co.	Gadsby Section III
168	Utah Power & Light Co.	Naughton
169	Northern Virginia Power Co.	Riverton
170	Virginia Electric & Power Co.	Portsmouth
171	Virginia Electric & Power Co.	Possum Point
172	Appalachian Power Co.	Clinch River
173	Appalachian Power Co.	Kanawha River
174	Winconsin Power & Light Co.	Rock River
175	Wisconsin Power & Light Co.	Nelson Dewey

Table 14: Data

FILE NO.	YEAR	OUTPUT	CAPACITY	FUEL	LABOR	PLANT FACTOR	NO. MACHINES	HEAT RATE
2	1949	396.5	120.0	4800574.0	3952	-0	2	12107
2	1950	1048.6	120.0	12539705.0	4243	100	2	11959
2	1951	1083.1	120.0	12980576.2	4284	103	2	11985
2	1952	1034.0	120.0	12443952.0	4233	98	2	12035
2	1953	1017.4	120.0	12329722.0	4150	97	2	12119
2	1954	1054.6	134.0	12914712.3	3767	100	2	12246
2	1955	806.8	134.0	9751198.0	3841	77	2	12086
2	1956	744.5	137.0	9005186.9	3869	71	2	12096
2	1957	712.0	137.0	8610679.8	3809	68	2	12094
2	1958	803.2	137.0	9673583.1	3772	76	2	12044
2	1959	805.1	137.0	9926645.1	3740	76	2	12330
2	1960	803.7	137.0	9827898.6	3758	66	2	12228
2	1961	680.9	137.0	8267689.8	3690	0	2	12142
2	1962	383.5	137.0	4810302.0	3584	32	2	12543
2	1963	506.6	137.0	6181894.5	3832	42	2	12203
2	1964	390.4	137.0	4861467.9	3800	32	2	12453
2	1965	412.0	137.0	5152030.6	3519	34	2	12505
15	1948	684.3	126.0	8112931.3	2652	0	2	11856
19	1957	20.0	22.0	255300.0	704	-0	1	12765
19	1958	108.6	22.4	1401039.6	697	56	1	12901
19	1959	119.3	22.4	1610278.0	1315	62	1	13498
32	1951	64.4	28.3	789753.5	1218	0	1	12263
32	1952	212.3	28.3	2589887.5	1453	85	1	12199
32	1953	234.4	28.3	2955000.8	1577	98	1	12607
32	1954	247.4	32.9	2978332.8	1573	100	1	12039
32	1955	252.5	33.0	3070753.3	1569	102	1	12161
38	1948	98.4	30.0	1193815.5	1347	0	1	12132
38	1949	239.1	30.0	2841427.2	1872	91	1	11884
43	1949	123.2	45.0	1556214.0	2662	0	2	12632
43	1950	248.3	45.0	2940088.0	3162	63	2	11841
43	1951	251.4	45.0	3040816.2	3198	64	2	12096
43	1952	214.1	45.0	2660578.2	3196	54	2	12427
43	1953	185.8	45.0	2191042.8	3071	47	2	11792
43	1954	151.7	46.0	1951396.2	2939	38	2	12864
43	1955	125.3	45.0	1689139.2	2974	32	2	13481
43	1956	152.2	45.0	1981200.0	3037	39	2	13017
43	1957	268.7	45.0	3434670.0	2981	68	2	12783
43	1958	236.2	52.0	2992336.0	2993	60	2	12669
43	1959	197.6	52.0	2583580.0	3083	50	2	13075
43	1960	152.8	52.0	2002809.0	3015	39	2	13107
43	1961	154.3	52.0	2028774.4	2993	0	2	13148
43	1962	127.4	52.0	1708604.8	2596	30	2	13411
43	1963	116.0	52.0	1640533.2	2596	24	2	14143
47	1948	182.4	100.0	2160828.8	2989	0	2	11847
47	1949	550.9	100.0	6394056.0	3120	63	2	11607
47	1950	760.0	100.0	8763156.6	3245	87	2	11530
47	1951	662.4	100.0	7502150.8	3360	76	2	11326
47	1952	719.7	100.0	8392740.8	3320	82	2	11661
47	1953	732.2	100.0	8551993.2	3362	84	2	11680

FILE NO.	YEAR	OUTPUT	CAPACITY	FUEL	LABOR	PLANT FACTOR	NO.MACHINES	HEAT RATE
47	1954	740.0	120.0	8625136.8	3395	84	2	11656
47	1955	637.6	120.0	7399833.0	3345	73	2	11606
47	1956	615.3	120.0	7166617.4	3203	70	2	11647
47	1957	637.7	120.0	7359201.6	3271	73	2	11540
47	1958	477.5	129.0	5660027.8	3198	55	2	11853
47	1959	464.8	129.0	5460337.8	3165	53	2	11748
59	1949	106.2	26.5	1444249.6	707	0	2	13599
59	1950	137.1	26.5	1785616.8	749	59	2	13024
59	1951	138.0	26.5	1818040.0	756	59	2	13174
59	1952	127.0	26.5	1642599.4	747	55	2	12934
59	1953	96.6	26.5	1343538.0	706	42	2	13908
59	1954	123.7	28.2	1628437.8	704	53	2	13164
59	1955	148.9	28.2	2001657.6	743	64	2	13443
59	1956	154.5	28.2	2012815.0	790	66	2	13028
59	1957	204.0	59.9	2682145.4	787	-0	3	13148
59	1958	311.5	59.9	3919350.0	779	63	3	12582
59	1959	319.3	59.9	4041398.4	822	65	3	12657
59	1960	333.8	62.0	4137129.6	867	64	3	12394
59	1961	336.5	62.0	4247625.2	861	0	3	12623
59	1962	312.2	62.0	3931936.0	865	54	3	12594
59	1963	281.8	62.0	3665459.2	865	49	3	13007
59	1964	297.1	62.0	3731492.8	867	53	3	12560
59	1965	317.1	62.0	4047137.6	911	57	3	12763
67	1950	481.2	60.0	5662058.4	2496	0	2	11767
67	1951	570.4	60.0	6824472.0	2730	108	2	11964
67	1952	561.7	60.0	6907264.0	2739	107	2	12297
67	1953	572.7	60.0	6951673.0	2822	109	2	12138
67	1954	1014.0	148.0	12262850.4	3146	0	3	12094
67	1955	1083.3	140.0	13333811.4	3139	103	3	12309
67	1956	1023.6	140.0	12505068.6	3162	97	3	12217
67	1957	791.0	140.0	9623826.0	3105	75	3	12167
67	1958	755.0	147.0	9087826.4	3075	72	3	12037
73	1951	337.8	60.0	4112486.4	1260	64	2	12174
73	1952	545.9	90.0	6616063.4	1370	-0	3	12120
73	1953	757.0	90.0	9205747.5	1494	96	3	12161
73	1954	799.6	101.0	9856353.3	1449	101	3	12327
73	1955	798.5	102.0	9918812.5	1446	101	3	12422
73	1956	1174.2	174.0	14124600.0	1789	-0	4	12029
73	1957	1165.2	174.0	13353674.9	1780	73	4	12083
73	1958	1172.7	174.0	13870970.1	1763	77	4	11828
73	1959	1041.0	174.0	12359110.5	1767	68	4	11872
73	1960	1176.5	174.0	14378727.2	1776	77	4	11967
73	1961	1206.9	180.0	14381681.6	1763	0	4	11916
73	1962	1417.6	174.0	17184209.6	1772	93	4	12122
73	1963	1329.6	174.0	15996712.0	1772	87	4	12031
73	1964	1356.0	186.0	16221526.2	1776	83	4	11963
73	1965	594.1	186.0	6995671.2	1780	36	4	11775
74	1954	226.0	52.0	2558196.0	994	-0	1	11319
74	1955	326.5	52.0	3432500.0	991	85	1	10513
74	1956	300.2	52.0	3100200.0	998	78	1	10327

FILE NO.	YEAR	OUTPUT	CAPACITY	FUEL	LABOR	PLANT FACTOR	NO.MACHINES	HEAT RATE
74	1957	252.0	52.0	2890382.4	994	65	1	11470
74	1958	204.1	52.0	2328469.6	984	53	1	11408
74	1959	213.7	52.0	2424608.0	1028	55	1	11346
74	1960	227.9	52.0	2594638.2	991	59	1	11385
74	1961	209.6	50.0	2382404.4	984	0	1	11366
74	1962	273.9	52.0	3165133.5	989	71	1	11556
74	1963	300.6	52.0	3514600.0	1112	78	1	11692
74	1964	155.2	52.0	1865463.6	1033	34	1	12020
74	1965	259.6	52.0	3016520.1	994	59	1	11611
80	1950	331.8	46.0	3772364.0	2662	82	1	11369
80	1951	287.5	46.0	3206172.3	2730	71	1	11152
80	1952	364.1	96.0	4093554.7	2698	0	2	11243
80	1953	656.3	96.0	7493605.8	2781	78	2	11418
80	1954	634.9	100.0	7269760.2	2774	75	2	11450
80	1955	667.6	104.0	7648472.0	2767	79	2	11457
80	1956	671.8	104.0	7711488.0	2995	80	2	11479
82	1949	824.3	120.0	9186676.8	2912	78	2	11145
82	1950	895.2	180.0	10123030.8	3453	0	3	11308
82	1951	1196.6	180.0	13426042.8	3738	76	3	11220
82	1952	1196.2	180.0	13342862.4	3320	76	3	11154
82	1953	1177.3	180.0	13247739.0	3362	74	3	11253
82	1954	1198.6	180.0	13544696.2	3519	76	3	11300
82	1955	1336.2	192.0	14891966.0	3717	77	3	11145
87	1955	29.1	22.0	371658.2	785	-0	1	12772
87	1956	122.1	22.5	1509744.0	790	63	1	12365
87	1957	156.5	23.0	1907066.8	787	81	1	12186
87	1958	115.2	22.9	1445734.4	779	60	1	12550
87	1959	138.1	22.9	1698560.0	781	72	1	12299
87	1960	123.1	23.0	1578216.0	785	64	1	12821
87	1961	165.8	23.0	2013498.0	779	0	1	12144
87	1962	266.7	60.5	3218140.9	824	-0	2	12067
87	1963	422.4	60.5	4933128.0	1236	77	2	11679
88	1958	56.7	29.0	794178.0	697	0	1	13529
88	1959	88.6	29.0	1242500.0	699	40	1	14024
88	1960	95.6	29.0	1363780.4	702	43	1	14265
88	1961	129.0	29.0	1773408.0	702	0	1	13747
88	1962	137.6	29.0	1698656.0	700	68	1	13800
88	1963	123.3	28.0	1730832.0	783	61	1	14038
88	1964	90.7	28.0	1295616.0	743	45	1	14285
88	1965	107.8	28.0	1575251.4	745	54	1	14613
89	1960	81.0	16.0	1278458.8	785	59	1	15783
89	1961	94.2	20.0	1688830.4	738	0	1	17928
89	1962	102.7	20.0	1902255.0	824	62	1	18522
89	1963	110.5	20.0	2142644.0	824	67	1	19390
89	1964	97.8	20.0	1974496.8	867	59	1	20189
89	1965	92.2	20.0	1914214.4	828	56	1	20762
137	1964	101.7	53.0	1348090.5	1156	22	1	13256
134	1954	525.4	78.0	6276560.0	1408	91	1	11946
134	1955	369.0	78.0	4456122.0	1280	64	1	12076
134	1956	374.7	78.0	4569211.9	1331	65	1	12248

FILE NO.	YEAR	OUTPUT	CAPACITY	FUEL	LABOR	PLANT FACTOR	NO. MACHINES	HEAT RATE
134	1957	287.9	78.0	3661534.8	1325	50	1	12718
134	1958	238.8	78.0	3056176.2	1230	41	1	12798
134	1959	199.8	78.0	2593139.4	1151	35	1	12979
134	1960	246.4	78.0	3033654.0	1198	43	1	12312
134	1961	197.2	78.0	2578085.7	1271	0	1	13073
134	1962	235.3	78.0	2988841.4	1277	37	1	12702
134	1963	262.9	78.0	3325451.6	1236	41	1	12649
134	1964	307.6	78.0	3685601.7	1280	48	1	11982
134	1965	233.3	78.0	3034987.9	1283	36	1	13009
136	1952	198.4	80.3	2396245.0	747	0	1	12078
136	1953	504.4	80.3	5842978.4	1370	72	1	11584
136	1954	813.6	160.0	9436076.0	1573	0	2	11598
136	1955	1001.6	160.0	11607299.8	1611	71	2	11589
136	1956	1135.7	160.0	13083356.0	1664	80	2	11520
136	1957	808.6	160.0	9526073.9	1739	57	2	11781
136	1958	755.8	160.0	9025640.0	1599	54	2	11942
136	1959	760.9	160.0	8987227.0	1685	54	2	11811
136	1960	627.3	160.0	7975173.2	1776	44	2	12713
136	1961	722.9	160.0	8547333.6	1640	0	2	11824
136	1962	729.9	160.0	8598366.0	1607	52	2	11780
136	1963	742.2	160.0	8774416.4	1524	53	2	11822
136	1964	775.3	160.0	9541387.1	1982	53	2	12307
136	1965	734.0	160.0	6780828.0	1449	50	2	11963
147	1958	211.4	75.0	2544334.4	1312	-0	1	12036
147	1959	330.7	75.0	3933262.4	1356	57	1	11894
153	1952	132.5	40.0	1589000.0	1494	0	1	11992
155	1950	114.6	37.5	1422308.4	1498	0	2	12411
155	1951	264.4	37.5	3163860.0	1680	80	2	11966
155	1952	463.8	77.5	5629036.3	1909	0	3	12137
155	1953	569.0	77.5	7057914.6	2075	84	3	12404
156	1949	324.6	50.0	4362424.0	1914	0	4	13439
156	1950	432.2	50.0	5917025.2	2246	99	4	13690
156	1951	412.6	50.0	5528940.5	1806	94	4	13400
156	1952	426.7	50.0	5966901.5	1743	97	4	13984
156	1953	446.5	50.0	6150702.6	1743	102	4	13775
156	1954	409.5	52.0	5534539.2	1780	93	4	13515
156	1955	430.7	52.0	5778980.0	1693	98	4	13418
156	1956	413.0	52.0	5479994.3	1664	94	4	13269
156	1957	433.9	52.0	5779260.0	1822	99	4	13319
163	1953	67.2	36.0	788657.2	913	0	1	11739
163	1954	172.5	36.0	2028183.3	952	66	1	11758
163	1955	225.8	72.0	2602431.0	991	43	2	11525
163	1956	366.7	72.0	4270044.0	1165	63	2	11645
163	1957	393.7	72.0	4600433.2	1076	68	2	11685
163	1958	421.9	73.0	4879726.2	1230	73	2	11566
169	1950	184.3	34.5	2559922.4	1165	61	1	13890
169	1951	224.1	34.5	2967935.6	1008	74	1	13244
169	1952	219.0	34.5	2963127.6	996	72	1	13530
169	1953	199.2	34.5	2623613.2	1162	66	1	13171
169	1954	187.1	39.0	2426035.2	1201	62	1	12967

FILE	YEAR	OUTPUT	CAPACITY	FUEL	LABOR	PLANT FACTOR	NO. MACHINES	HEAT RATE
169	1955	197.0	39.0	2457302.6	1115	65	1	12474
169	1956	199.6	39.0	2508172.8	1165	66	1	12566
169	1957	201.7	39.0	2615316.4	1159	67	1	12966
169	1958	174.7	39.0	2328107.0	1189	58	1	13326
169	1959	155.2	39.0	1996134.4	1192	51	1	12862
169	1960	151.8	40.0	1995440.8	1198	50	1	13145
169	1961	119.8	40.0	1603406.8	1148	0	1	13384
169	1962	151.6	40.0	1990809.6	1154	50	1	13132
169	1963	142.3	40.0	1440454.0	1154	37	1	12827
169	1964	96.1	40.0	1241665.0	1115	32	1	12921
169	1965	117.5	40.0	1528005.5	1118	39	1	13004
171	1949	451.0	60.0	5210794.6	2954	86	1	11554
171	1950	475.0	60.0	5521204.2	3037	90	1	11624

DATA FOR CELL 2

FILE NO.	YEAR	OUTPUT	CAPACITY	FUEL	LABOR	PLANT FACTOR	NO. MACHINES	HEAT RATE
46	1950	37.5	20.0	548182.3	1123	21	1	14618
46	1951	58.1	20.0	914492.0	1092	33	1	15740
46	1952	138.8	50.0	2016800.1	1785	0	2	14530
46	1953	222.0	50.0	3016041.0	2158	51	2	13586
46	1954	217.9	50.0	2917827.6	1987	0	2	13391
46	1955	246.2	88.0	3294535.8	2272	0	3	13382
46	1956	348.9	88.0	4615005.6	2579	50	3	13227
46	1957	378.5	88.0	4895715.6	2567	54	3	12935
46	1958	361.2	88.0	4554227.2	2501	52	3	12636
46	1959	430.2	88.0	5464996.4	2548	61	3	12703
46	1960	443.7	88.0	5559350.6	2685	63	3	12530
58	1951	94.8	23.0	1194417.6	714	0	2	12599
58	1952	136.6	23.0	1810821.4	736	68	2	13256
58	1953	135.9	23.0	1821925.2	623	67	2	13406
58	1954	134.1	23.5	1856433.6	662	67	2	13844
58	1955	134.7	23.5	1651110.8	702	67	2	13742
58	1956	151.8	23.5	2060956.0	707	75	2	13577
58	1957	160.0	23.5	2152934.5	704	79	2	13456
58	1958	141.9	23.5	1892175.2	697	71	2	13370
58	1959	143.4	23.5	1955338.9	699	71	2	13636
58	1960	123.3	23.5	1689793.8	702	61	2	13705
58	1961	128.5	23.5	1752626.9	738	0	2	13639
60	1954	183.7	47.6	2084224.0	1780	0	1	11346
60	1955	264.8	47.6	2942295.4	1776	69	1	11111
60	1956	279.5	47.6	3120166.5	1789	72	1	11163
60	1957	287.0	47.6	3266351.1	1946	74	1	11381
62	1953	37.5	40.0	499700.0	1204	0	2	13325
62	1954	201.1	41.6	2653981.8	1201	57	2	13197
62	1955	233.9	41.0	3034651.2	1446	67	2	12974
62	1956	273.8	41.0	3550510.0	1290	71	2	12968
62	1957	287.3	57.0	3770198.4	1490	0	3	13123
62	1958	331.8	60.0	4433443.6	1517	57	3	13362
62	1959	371.6	61.1	4863000.0	1521	64	3	13140
62	1960	392.6	61.1	5128345.2	1593	66	3	13404
62	1961	382.4	61.1	5055418.2	1722	0	3	13220
62	1962	378.2	61.1	5021286.4	1730	61	3	13277
62	1963	398.8	61.0	5478713.6	1648	64	3	13738
62	1964	399.2	61.1	5421289.0	1611	64	3	13580
62	1965	398.6	61.1	5487600.4	1615	64	3	13767
76	1958	495.0	96.0	5382512.9	1722	0	2	10874
76	1959	622.3	96.0	6732815.9	1891	81	2	10819
76	1960	581.5	96.0	6323128.6	1900	75	2	10874
76	1961	664.5	96.0	7285750.3	1886	0	2	10964
76	1962	704.8	96.0	7678235.2	1895	91	2	10894
76	1963	654.7	96.0	7101276.1	1895	85	2	10847
76	1964	786.5	96.0	8464341.4	1900	102	2	10762
76	1965	1279.8	216.0	13306892.0	1904	0	3	10398
92	1949	65.7	25.0	958654.4	915	0	2	14591
92	1950	175.7	25.0	2510433.6	390	80	2	14288

FILE NO.	YEAR	OUTPUT	CAPACITY	FUEL	LABOR	PLANT FACTOR	NO. MACHINES	HEAT RATE
92	1951	182,1	45,0	2612439,2	1092	0	3	14346
92	1952	201,8	45,0	2705505,8	1453	51	3	13407
92	1953	264,7	45,0	3589218,8	1370	67	3	13560
92	1954	287,1	49,0	3652587,2	1573	73	3	13419
92	1955	226,1	49,0	3064201,0	1611	57	3	13552
92	1956	210,4	49,0	2823919,2	1622	53	3	13422
92	1957	199,5	49,0	2787060,6	1697	51	3	13970
92	1958	192,9	55,0	2693499,3	1856	49	3	13963
92	1959	211,0	55,0	2925781,2	1891	54	3	13866
92	1960	209,9	55,0	2873806,4	1927	53	3	13691
92	1961	212,9	51,5	2958109,7	1845	0	3	13894
92	1962	245,5	51,5	3343215,6	1772	58	3	13618
92	1963	232,9	51,5	3184391,7	1772	55	3	13656
92	1964	298,5	51,5	2896036,6	1735	49	3	13890
92	1965	171,5	51,5	2455336,8	1739	41	3	14317
93	1949	78,0	12,3	1075102,8	915	71	1	13783
93	1950	77,5	12,5	1078857,4	915	71	1	13921
93	1951	96,9	25,0	1336807,0	1008	0	2	13796
93	1952	136,8	25,0	1813517,8	1038	62	2	13257
93	1953	134,7	25,0	1779800,0	1038	62	2	13213
93	1954	143,6	26,9	1894773,8	1035	66	2	13195
93	1955	95,6	26,9	1303000,5	1033	44	2	13630
93	1956	124,9	26,9	1643793,0	1040	57	2	13161
93	1957	104,5	26,9	1405874,6	1035	48	2	13453
93	1958	110,2	29,0	1478135,2	984	50	2	13413
93	1959	120,3	29,0	1609117,1	986	55	2	13376
93	1960	104,1	29,0	1398292,8	991	48	2	13432
93	1961	108,8	27,9	1451582,8	984	0	2	13342
93	1962	112,6	27,9	1605197,3	989	55	2	13421
93	1963	122,8	27,9	1661264,7	989	56	2	13528
93	1964	109,6	27,9	1494880,4	991	50	2	13639
93	1965	99,4	27,9	1375071,0	994	45	2	13834
94	1950	29,0	4,0	524888,0	582	0	1	18100
94	1951	31,8	4,0	573648,0	546	90	1	18039
94	1952	36,7	10,0	658526,4	706	0	2	17943
94	1953	45,5	10,0	817533,2	706	52	2	17968
94	1954	49,8	10,3	881144,0	704	57	2	17694
94	1955	55,0	10,3	981842,2	702	63	2	17852
94	1956	61,0	10,3	1093034,2	749	69	2	17919
94	1957	50,5	10,3	967091,4	1076	58	2	19150
94	1958	25,4	12,8	576408,0	1025	29	2	22693
94	1959	29,1	12,6	645372,4	1028	33	2	22178
94	1960	50,8	12,8	1014330,6	1074	58	2	19967
94	1961	50,6	12,1	1128018,0	1066	0	2	22293
94	1962	46,7	12,1	935817,0	1071	53	2	20039
94	1963	39,3	12,1	826194,8	1030	45	2	21023
94	1964	66,3	12,1	1326557,6	1033	75	2	20008
94	1965	35,5	12,1	640446,4	1118	30	2	21830
102	1950	60,2	19,0	852251,4	915	0	2	14157
102	1951	111,4	19,0	1650535,0	1260	0	2	14816

FILE NO.	YEAR	OUTPUT	CAPACITY	FUEL	LABOR	PLANT FACTOR	NO.MACHINES	HEAT RATE
102	1952	127.2	19.0	1833480.0	1287	76	2	14414
102	1953	118.4	19.0	1722876.5	996	71	2	14551
102	1954	120.4	22.0	1733445.2	994	72	2	14397
102	1955	127.5	22.0	1776859.6	1074	77	2	13936
102	1956	124.9	22.0	1838965.0	1082	75	2	14724
102	1957	121.9	22.0	1724250.4	1201	73	2	14145
102	1958	124.1	20.8	1723333.8	1230	75	2	13887
102	1959	126.3	20.8	1796660.8	1192	76	2	14225
102	1960	126.6	20.8	1821393.6	1198	76	2	14387
102	1961	121.6	20.8	1754262.6	1189	0	2	14427
102	1962	145.4	20.8	1458838.2	1195	63	2	13841
102	1963	90.6	20.8	1300032.0	1195	54	2	14349
102	1964	114.9	20.8	1575558.6	1156	69	2	13712
102	1965	128.5	21.1	1804509.0	1159	77	2	14043
103	1953	654.6	110.0	6551636.0	4233	0	1	10006
116	1949	316.2	44.0	3512070.8	2080	82	1	11107
116	1950	407.8	88.6	6110308.8	3245	0	2	14984
116	1951	571.3	68.0	6671924.0	3454	74	2	11678
116	1952	658.5	88.0	7496801.0	2573	85	2	11388
116	1953	594.9	88.0	6555312.4	2947	77	2	11019
116	1954	569.8	95.0	6405600.0	3519	74	2	11242
116	1955	522.5	95.0	5750280.6	3263	68	2	11005
116	1956	564.2	95.0	6083978.6	3286	73	2	10783
116	1957	513.2	95.0	5637198.8	3312	67	2	10984
127	1955	60.2	27.6	935514.2	1074	27	1	15540
127	1956	68.9	27.6	1013802.0	1123	39	1	14714
127	1957	97.6	27.6	1513290.0	1118	56	1	15505
127	1958	93.2	27.6	1407000.0	1148	53	1	15097
127	1959	114.4	27.6	1679632.0	1110	52	1	14684
127	1960	136.8	27.6	2017008.0	1115	62	1	14744
127	1961	156.9	27.6	2264310.0	1148	0	1	14432
127	1962	154.6	27.6	2244890.0	1277	71	1	14521
127	1963	204.0	97.6	2834546.0	1854	0	2	13895
127	1964	297.4	97.6	3769808.0	1941	34	2	12676
127	1965	323.6	97.6	4182416.0	1987	37	2	12925
128	1949	33.7	10.0	550440.0	1123	0	2	16334
128	1950	49.7	10.0	822360.0	1373	57	2	16546
128	1951	48.3	10.0	774840.0	1176	0	2	16042
128	1952	48.1	10.0	755040.0	1121	55	2	15697
128	1953	46.6	10.0	703560.0	1038	53	2	15098
128	1954	48.5	11.3	806432.6	1076	55	2	16627
128	1955	52.2	11.3	862048.0	1033	0	2	16514
128	1956	53.3	11.3	868312.0	1040	61	2	16291
128	1957	56.7	11.3	924648.0	994	65	2	16308
128	1958	29.4	11.3	490066.0	1025	34	2	16670
128	1959	26.8	11.3	447952.4	945	31	2	16715
128	1960	15.3	11.3	259778.4	909	17	2	16979
128	1961	22.3	11.3	379728.0	902	0	2	17028
128	1962	24.1	12.0	400156.0	906	28	2	16604
128	1963	24.2	13.0	404741.8	906	28	2	16725

FILE NO.	YEAR	OUTPUT	CAPACITY	FUEL	LABOR	PLANT FACTOR	NO. MACHINES	HEAT RATE
128	1964	19.3	13.0	347491.0	867	22	2	18005
128	1965	12.5	13.0	239329.6	869	14	2	19146
129	1951	86.3	15.0	1210049.4	1092	66	1	14021
129	1952	92.8	15.0	1357106.4	1121	70	1	14624
129	1953	95.6	15.0	1347720.0	1038	73	1	14097
129	1954	83.4	16.7	1117400.0	1035	63	1	13398
129	1955	89.2	16.8	1201481.6	1033	68	1	13470
129	1956	94.7	16.8	1275492.4	1040	72	1	13469
129	1957	96.2	16.8	1205656.0	1035	69	1	13369
129	1958	98.4	16.8	1322520.0	984	75	1	13440
129	1959	95.0	18.5	1287043.2	986	72	1	13548
129	1960	90.1	18.5	1247729.4	991	68	1	13848
129	1961	99.1	21.0	1355321.4	943	0	1	13676
129	1962	120.0	21.0	1625209.2	989	83	1	13543
129	1963	99.2	21.0	1344063.0	989	69	1	13548
129	1964	66.0	21.0	940964.0	991	46	1	14257
129	1965	23.3	21.0	372780.0	994	16	1	15999
165	1952	325.8	60.0	3925119.9	1370	56	1	12048
165	1953	372.7	66.0	4321104.4	1267	71	1	11594
165	1954	435.6	66.0	5163683.9	1822	83	1	11827
165	1955	329.2	66.0	4154743.2	1239	63	1	12621
165	1956	302.0	66.0	3978712.2	1248	57	1	13175
165	1957	337.4	66.0	4931549.6	1118	64	1	14616
165	1958	200.7	66.0	3554160.0	984	38	1	17709
165	1959	265.9	66.0	3656256.4	986	51	1	13758
165	1960	289.5	66.0	3450865.6	1033	55	1	11920
165	1961	269.5	66.0	3772563.2	1666	0	1	13998
165	1962	179.2	66.0	2939247.0	1030	30	1	16402
165	1963	174.7	66.0	2749834.4	1030	29	1	15740
165	1964	182.0	66.0	2615134.1	909	30	1	14369
165	1965	237.6	66.0	2133593.0	994	39	1	8980

DATA FOR CELL 3

FILE NO,	YEAR	OUTPUT	CAPACITY	FUEL	LABOR	PLANT FACTOR	NO.MACHINES	HEAT RATE
8	1955	550.7	96.0	5740707.3	2767	63	1	10424
8	1956	921.9	220.0	9441759.1	3120	0	2	10242
8	1957	1284.3	220.0	13370714.3	3312	74	2	10146
8	1958	1523.5	302.0	15725358.5	4182	0	3	10288
8	1959	1779.2	302.0	18259244.6	3946	68	3	10268
8	1960	1731.8	302.0	17823175.1	3900	66	3	10292
8	1961	1629.2	305.0	16684245.4	3977	0	3	10241
8	1962	1554.4	306.0	15971855.7	3543	59	3	10275
8	1963	1434.9	306.0	14856686.1	3543	55	3	10350
8	1964	1349.7	323.5	14043545.8	3221	46	3	10405
8	1965	1234.6	323.5	12579089.0	2981	43	3	10518
23	1958	471.3	62.0	4628788.0	1681	72	1	9821
23	1959	516.2	62.0	5693593.6	1767	79	1	9867
23	1960	576.5	82.0	5790532.0	1776	88	1	10044
31	1959	755.5	141.3	7516449.0	1767	69	1	9949
31	1960	779.1	141.3	7768867.1	1982	57	1	9972
31	1961	778.6	141.3	7743648.3	1968	0	1	9953
31	1962	702.4	141.3	6991053.0	1936	51	1	9953
31	1963	720.2	141.3	7171054.6	2068	52	1	9957
31	1964	647.0	141.3	6473155.6	2024	47	1	10005
31	1965	457.5	141.3	4725661.2	1967	33	1	10329
37	1958	786.4	260.0	8111853.8	2788	0	2	10315
37	1959	1254.6	260.0	13112325.0	3411	60	2	10450
51	1953	271.9	60.0	2787570.0	2905	0	1	10252
51	1954	446.5	70.0	4540742.0	3105	85	1	10170
51	1955	415.5	70.0	4237790.4	3263	79	1	10199
51	1956	381.6	70.0	3975747.6	3245	72	1	10419
51	1957	340.7	70.0	3619174.8	3022	65	1	10623
51	1958	405.7	77.0	4240296.0	2747	77	1	10452
57	1953	270.8	180.0	2757993.0	1909	0	2	10185
57	1954	1700.2	380.0	17447922.8	3602	0	4	10262
57	1955	2416.8	380.0	24202124.0	4295	77	4	10014
66	1957	421.9	105.0	4140120.8	1573	0	1	9813
66	1958	600.1	105.0	6106233.2	1640	69	1	10175
66	1959	478.6	105.0	4878576.6	1726	60	1	10193
66	1960	495.4	105.0	5058424.0	1735	62	1	10211
66	1961	513.5	105.0	5253767.4	1763	0	1	10231
66	1962	504.4	105.0	1306377.6	2678	51	1	2590
72	1957	520.2	120.0	5620715.1	1490	0	1	10805
72	1958	699.5	120.0	7418851.6	1517	68	1	10760
72	1959	754.2	120.0	8142775.8	1397	75	1	10797
72	1960	699.0	120.0	7724871.0	1693	69	1	11051
72	1961	800.3	120.0	8724900.8	1271	0	1	10902
72	1962	669.9	120.0	7385419.8	1483	66	1	11025
04	1952	359.3	170.0	3449735.2	3486	0	2	9601
84	1953	1597.3	276.0	15253602.0	4150	0	3	9550
84	1954	2212.4	276.0	21191739.8	4181	91	3	9579
84	1955	2250.2	325.0	21525462.4	4337	79	3	9566
84	1956	2316.8	325.0	22143103.6	4534	81	3	9558

FILE NO.	YEAR	OUTPUT	CAPACITY	FUEL	LABOR	PLANT FACTOR	NO. MACHINES	HEAT RATE
34	1957	2113.0	325.0	20214570.0	4595	74	3	9564
34	1958	2075.4	325.0	19866636.0	4551	73	3	9568
34	1959	2068.5	325.0	19751875.0	4480	73	3	9549
34	1960	2026.0	325.0	19384400.0	4460	71	3	9568
34	1961	1825.5	325.0	17619553.4	4469	0	3	9652
34	1962	2128.7	325.0	20480616.8	4367	75	3	9621
34	1963	2372.9	325.0	22988677.2	4367	83	3	9688
34	1964	2386.8	345.0	23124726.9	4708	84	3	9689
34	1965	4153.1	345.0	24134792.6	4595	88	3	3919
91	1958	162.2	70.0	1676496.8	1353	0	1	10336
91	1959	412.0	70.0	4088782.4	1432	73	1	9924
91	1960	762.0	140.0	7457165.0	1511	0	2	9786
91	1961	821.6	140.0	8176035.0	1558	0	2	9951
91	1962	891.0	140.0	8793264.0	1566	68	2	9869
91	1963	895.0	140.0	8906937.6	1566	68	2	9952
91	1964	974.9	140.0	9559564.4	1446	74	2	9806
91	1965	911.0	140.0	9194034.0	1449	69	2	10092
118	1953	1567.0	240.0	14759037.6	3403	0	3	9419
118	1954	2013.6	408.0	19553772.0	4595	0	4	9637
118	1955	2323.3	408.0	21795994.9	5328	83	4	9373
118	1956	2533.8	408.0	24062887.2	5741	90	4	9497
118	1957	2611.5	408.0	24841817.0	5755	93	4	9512
118	1958	2259.0	408.0	24131458.0	5653	81	4	10635
118	1959	1923.6	408.0	18370444.8	5713	69	4	9550
118	1960	1785.6	402.0	16900161.6	5538	64	4	9465
118	1961	1465.8	408.0	14236922.2	5535	0	4	9713
118	1962	1813.5	388.0	17663317.2	4920	52	4	9713
118	1963	2485.1	388.0	23466614.6	4244	71	4	9443
118	1964	2514.7	388.0	23831150.0	4171	72	4	9477
118	1965	2694.6	398.0	25638414.0	4099	77	4	9515
119	1951	1375.7	160.0	13117934.0	3486	98	2	9535
119	1952	1393.4	160.0	13143324.0	3818	99	2	9433
119	1953	1460.6	160.0	13883088.0	4150	104	2	9505
119	1954	1226.7	204.0	11790720.0	4306	0	2	9612
119	1955	1210.1	204.0	11524390.4	4213	85	2	9524
119	1956	1419.6	204.0	13497045.8	4202	101	2	9508
119	1957	1375.7	204.0	13152580.0	4223	98	2	9561
119	1958	1047.9	204.0	10149813.0	4182	75	2	9686
132	1954	1181.7	212.0	11439279.0	2567	0	2	9680
132	1955	1407.0	212.0	13544358.0	3304	76	2	9626
132	1956	1519.4	212.0	14681228.0	3453	82	2	9663
132	1957	1504.0	230.0	14925878.8	3395	81	2	9924
132	1958	1475.4	230.0	14408320.0	3403	79	2	9766
132	1959	1521.3	230.0	15222099.2	3370	82	2	10006
132	1960	1198.3	213.0	12160950.8	3387	54	2	10149
132	1961	1068.9	213.0	10727928.0	3280	0	2	10036
132	1962	1067.8	213.0	10802373.4	3337	49	2	10116
132	1963	1103.7	213.0	11015331.4	3296	50	2	9980
132	1964	1115.6	213.0	11255958.0	3387	51	2	10090
132	1965	1105.3	213.0	11029638.8	3229	50	2	9979

FILE NO.	YEAR	OUTPUT	CAPACITY	FUEL	LABOR	PLANT FACTOR	NO. MACHINES	HEAT RATE
152	1961	351.7	106.0	4052922.0	1353	0	1	10347
152	1962	732.8	212.0	7653682.2	1524	0	2	10444
152	1963	1134.7	212.0	11548454.5	1524	57	2	10178
152	1964	1221.2	212.0	12213611.8	1569	61	2	10001
152	1965	1060.0	212.0	10777648.8	1656	53	2	10168
157	1955	482.0	153.0	5212444.2	1363	0	1	10814
157	1956	919.9	153.0	9646930.8	1373	84	1	10487
157	1957	982.0	153.0	10378905.0	1449	90	1	10569
157	1958	807.3	153.6	8633128.0	1353	74	1	10694
157	1959	569.2	153.0	6263890.2	1644	52	1	11005
157	1960	573.7	153.6	6413176.2	1487	52	1	11179
157	1961	555.6	153.6	6172125.3	1517	0	1	11109
157	1962	711.4	153.0	7730594.1	1360	52	1	10867
157	1963	566.4	153.0	5852614.5	1772	41	1	10333
157	1964	849.2	153.0	8777534.4	1363	62	1	10336
157	1965	628.8	153.0	6787107.6	1325	46	1	10794
164	1957	234.8	100.0	2440478.6	1118	0	1	10394
164	1958	713.0	100.0	7323242.4	1107	81	1	10271
164	1959	612.8	100.0	6302370.0	1110	77	1	10285
164	1960	735.1	100.0	7517518.8	1198	92	1	10227
164	1961	772.5	100.0	7953751.2	1230	0	1	10296
164	1962	633.0	100.0	6521525.4	1154	64	1	10303
164	1963	564.0	100.0	5866361.2	1195	57	1	10401
164	1964	573.3	100.0	5926912.2	1239	57	1	10522
164	1965	554.2	100.0	5855085.6	1201	56	1	10565
174	1954	324.8	75.0	3349206.0	1697	0	1	10312
174	1955	507.6	153.0	505612.8	2024	0	2	996
174	1956	849.3	153.0	8408701.8	2288	64	2	9901
174	1957	904.7	150.0	8886848.6	2277	69	2	9823
174	1958	896.3	164.0	8806212.0	2378	63	2	9825
174	1959	938.8	153.0	9212928.0	2302	71	2	9814
174	1960	749.3	153.0	7483821.6	2272	57	2	9988
174	1961	840.1	150.0	8295555.8	2296	0	2	9874
174	1962	867.0	161.0	8554157.6	2348	62	2	9866
174	1963	911.3	161.0	9017103.8	2307	65	2	9895
174	1964	898.3	161.6	8897130.2	2313	64	2	9904
174	1965	835.6	164.1	8302136.4	2318	60	2	9936
175	1960	588.1	100.0	5731586.8	1941	67	1	9746
175	1961	635.0	100.0	6166917.0	1927	0	1	9712

DATA FOR CELL 4

FILE NO.	YEAR	OUTPUT	CAPACITY	FUEL	LABOR	PLANT FACTOR	NO. MACHINES	HEAT RATE
1	1954	1133.3	261.0	10387602.2	3105	0	2	9763
1	1955	1574.3	261.0	16183186.0	3676	77	2	9552
1	1956	1732.1	261.0	17000792.2	3461	81	2	9540
1	1957	1817.1	261.0	17301713.0	3726	83	2	9574
1	1958	2114.1	270.0	20453858.2	3734	97	2	9675
1	1959	2537.5	503.0	24638941.6	4562	0	3	9710
1	1960	3478.7	512.5	33610885.8	4997	68	3	9662
1	1961	3424.1	515.5	33655874.0	5207	0	3	9829
1	1962	3629.0	515.5	35550949.6	5274	72	3	9796
1	1963	3172.2	515.5	30685132.8	5315	68	3	9673
1	1964	3399.3	515.5	33163355.0	5204	67	3	9756
1	1965	3473.8	515.5	34055260.8	5009	83	3	9803
4	1960	2149.4	485.0	20323034.4	3552	0	2	9455
4	1961	4173.1	727.5	38502496.8	5617	0	3	9226
4	1962	6825.8	1008.0	62819025.0	6592	0	4	9202
4	1963	7285.3	1024.0	66931060.0	6539	78	4	9187
4	1964	7525.2	1024.0	68910102.0	7021	81	4	9157
4	1965	7896.3	1024.0	74345040.0	7079	85	4	9415
5	1962	648.5	119.8	6192401.4	1566	0	1	9549
5	1963	850.8	119.8	8271126.0	1566	85	1	9609
5	1964	931.4	119.8	8946294.4	1569	92	1	9605
5	1965	714.6	119.8	6938860.0	1573	71	1	9710
7	1960	1191.9	229.4	11609136.0	1693	0	2	9740
7	1961	1559.3	229.4	15497613.7	1722	0	2	9939
7	1962	1596.0	229.4	15786341.0	1463	80	2	9891
7	1963	1114.1	229.4	11094723.4	1483	56	2	9958
7	1964	682.8	229.4	7020648.0	1528	34	2	10282
7	1965	405.3	229.4	4538419.8	1449	20	2	11198
9	1961	931.9	137.0	9114666.7	1476	0	1	9781
9	1962	1381.3	273.0	13499936.5	1813	0	2	9773
9	1963	1952.7	273.0	18916109.0	1970	82	2	9687
9	1964	2159.7	510.0	21021268.5	2024	0	3	9733
9	1965	2591.0	510.0	25286975.1	1987	62	3	9760
10	1956	566.2	175.0	2722411.5	957	0	1	4791
10	1957	2343.4	350.0	22329043.0	1035	-0	2	9528
10	1958	1910.9	350.0	18362940.6	1189	67	2	9610
10	1959	2490.0	350.0	2395,662.3	2096	87	2	9619
10	1960	2502.6	350.0	24155414.5	2272	87	2	9651
10	1961	2751.0	691.0	27115766.3	2747	0	3	9857
11	1962	457.0	67.0	36896419.9	1236	0	1	80736
11	1963	428.6	67.0	4928679.5	1236	0	1	11499
11	1964	654.7	146.0	6702955.0	1239	0	2	10238
11	1965	749.9	146.0	7806544.0	1283	58	2	10410
12	1955	814.0	175.0	7665632.4	702	0	1	9417
12	1956	1784.9	350.0	16927347.5	915	0	2	9484
12	1957	2500.6	350.0	23670414.6	1242	86	2	9466
12	1958	2247.7	350.0	21744899.6	1148	77	2	9674
12	1959	2412.9	350.0	23577819.7	2178	84	2	9652
12	1960	2359.8	350.0	22979140.0	2354	81	2	9738

FILE NO.	YEAR	OUTPUT	CAPACITY	FUEL	LABOR	PLANT FACTOR	NO. MACHINES	HEAT RATE
12	1961	2344.8	330.0	22695333.4	2214	0	2	9679
12	1962	1992.0	330.0	19542629.5	2225	68	2	9811
12	1963	1458.2	350.0	14679164.1	3378	50	2	10067
13	1953	654.9	220.0	6608701.5	996	0	2	10091
13	1954	2045.1	280.0	20471875.1	1532	95	2	10010
13	1955	2215.4	280.0	21576794.8	909	103	2	9739
13	1956	1859.6	264.0	18563078.5	957	86	2	9982
13	1957	1721.8	264.0	17151664.9	1449	80	2	9961
13	1958	1834.4	264.0	18161177.6	1394	84	2	10065
13	1959	1836.0	264.0	18118788.9	2384	86	2	9869
13	1960	1812.1	264.0	17616625.1	2437	84	2	9832
13	1961	2017.3	264.0	19613694.8	2337	0	2	9822
13	1962	1634.2	264.0	16380288.5	2760	76	2	10023
17	1958	680.7	125.0	6808476.1	1312	0	2	10002
17	1959	980.6	125.0	9728530.9	1315	0	2	9921
17	1960	933.1	125.0	9367703.2	1280	0	2	10039
17	1961	948.5	133.0	9546153.2	1271	0	2	10064
17	1962	861.3	133.0	8641039.2	1442	0	2	10033
17	1963	886.9	133.0	8861427.8	1368	0	2	9991
17	1964	737.9	126.0	7485633.2	1322	64	2	10145
17	1965	643.3	126.0	6683394.9	1532	56	2	10389
27	1959	619.6	125.5	6349365.6	2178	58	1	10248
27	1960	755.0	125.5	7825933.8	2238	0	1	10365
27	1961	959.4	250.0	9853602.2	2460	0	2	10271
27	1962	1336.2	250.0	13436721.5	2554	60	2	10056
34	1960	695.9	205.2	6802295.2	1817	0	1	9775
34	1961	2254.8	416.4	21596269.2	2091	0	2	9578
34	1962	2707.6	416.4	26461539.9	2101	71	2	9459
34	1963	2956.9	416.4	27775009.8	2307	75	2	9393
35	1965	507.6	142.8	4984804.0	1656	0	1	9820
40	1954	560.5	216.0	5573561.2	2443	0	2	9944
40	1955	1490.3	324.0	15176343.0	3345	0	3	10183
40	1956	1660.7	324.0	16540524.0	4493	63	3	9960
40	1957	1607.5	324.0	15907515.2	4554	61	3	9896
40	1958	1630.1	312.0	16823055.8	4592	64	3	10013
40	1959	1845.4	300.0	18749458.2	4603	70	3	10160
40	1960	1962.1	300.0	19039423.0	4667	72	3	10010
40	1961	1566.3	300.0	15662434.2	4715	0	3	10000
40	1962	1656.8	300.0	16700841.6	4614	50	3	10080
40	1963	1793.0	300.0	18035331.6	4573	55	3	10059
40	1964	1321.5	317.7	13212242.8	4562	40	3	9998
40	1965	1573.1	300.0	15759928.8	0	48	3	10018
45	1960	497.7	125.0	4817926.4	1369	0	1	9680
45	1961	811.0	125.0	7960392.0	1599	0	1	9816
45	1962	916.2	125.0	8903109.6	1607	82	1	9717
45	1963	946.9	134.8	9289210.4	1607	84	1	9810
45	1964	939.4	125.0	9099309.0	1611	84	1	9686
45	1965	928.9	125.0	8981636.4	1515	83	1	9669
49	1955	1428.6	320.0	14101533.6	4832	0	2	9871
49	1956	2061.8	320.0	19725476.4	5990	78	2	9567

FILE NO.	YEAR	OUTPUT	CAPACITY	FUEL	LABOR	PLANT FACTOR	NO.MACHINES	HEAT RATE
49	1957	2986.5	580.0	28565784.0	6872	0	3	9565
49	1958	3845.2	585.0	35928489.6	7216	76	3	9344
49	1959	3821.1	585.0	36105976.8	7316	76	3	9449
49	1960	4120.8	585.0	39126594.4	7847	82	3	9495
49	1961	4065.3	585.0	38663665.6	6036	0	3	9513
49	1962	3977.2	585.0	38246649.6	8158	68	3	9616
49	1963	5479.7	1095.0	52302978.4	9435	0	4	9545
49	1964	7171.0	1168.0	67815224.0	10490	64	4	9457
49	1965	6668.1	1168.0	64244317.6	10971	60	4	9606
50	1954	4153.5	625.0	42279647.2	11344	0	4	10179
50	1955	6694.3	937.5	65799343.2	13051	0	6	9829
50	1956	7581.9	937.5	73601352.0	14976	92	6	9708
50	1957	7993.1	937.5	76229496.0	15235	96	6	9646
50	1958	7658.3	1023.0	75906290.2	15416	86	6	9659
50	1959	8146.7	1023.0	78758248.8	16070	89	6	9665
50	1960	8016.6	1023.0	77819057.4	15777	88	6	9707
50	1961	8106.0	1023.0	78912736.0	15129	0	6	9733
50	1962	8376.2	1023.0	79353649.0	13349	87	6	9474
50	1963	8450.8	1023.0	81439650.0	12896	88	6	9637
50	1964	8316.6	1023.0	80222056.0	13092	86	6	9646
50	1965	8256.0	1023.0	80092323.8	12627	86	6	9699
61	1964	636.9	147.0	6754977.6	1156	0	1	10606
61	1965	776.7	147.0	7548459.5	1242	60	1	9719
64	1961	531.0	160.0	5242759.2	1271	0	1	9873
64	1962	750.2	162.8	7437132.0	1318	57	1	9914
64	1963	995.0	162.8	9772652.0	1524	76	1	9842
64	1964	1034.4	162.8	10226281.4	1528	79	1	9886
64	1965	1030.2	162.8	10164716.0	1573	79	1	9867
68	1959	552.5	222.0	6150249.2	1356	0	2	11132
68	1960	1573.6	384.0	16908754.4	1900	0	3	10745
68	1961	1934.0	384.0	20617424.1	1900	0	3	10661
68	1962	1852.4	384.0	19806337.2	1936	54	3	10692
68	1963	1432.4	384.0	15768413.5	1978	42	3	11008
68	1964	1376.8	384.0	15196698.0	1982	40	3	11038
68	1965	1672.7	384.0	17790126.2	2277	49	3	10636
70	1960	515.6	162.0	4715180.0	991	0	1	9127
70	1961	574.6	162.0	8445811.3	1189	0	1	14699
70	1962	597.8	162.0	6252501.6	1195	42	1	10459
70	1963	1120.8	162.0	11509785.0	1318	78	1	10269
70	1964	2003.5	382.0	20624002.2	1404	0	2	10294
70	1965	2297.9	382.0	23005808.6	1408	65	2	10012
71	1961	1041.2	247.0	10240593.6	1681	0	1	9835
71	1962	1503.1	247.0	14749480.8	1730	69	1	9813
71	1963	1658.6	247.0	16042247.2	1772	77	1	9672
71	1964	1658.4	247.0	15934346.4	1817	76	1	9608
78	1961	1036.1	142.0	10042555.4	1722	0	1	9693
78	1962	1116.8	142.0	10527132.0	1772	94	1	9426
78	1963	1067.9	142.0	10201099.0	1772	90	1	9552
78	1964	1133.2	142.0	10871792.4	1776	95	1	9594
78	1965	1133.8	142.0	10680516.4	1780	95	1	9597

FILE NO.	YEAR	OUTPUT	CAPACITY	FUEL	LABOR	PLANT FACTOR	NO.MACHINES	HEAT RATE
83	1960	1552.2	250.0	14587776.0	2395	68	1	9162
85	1953	364.7	250.0	3707544.0	4441	0	2	9637
85	1954	2557.7	446.0	23627164.0	6624	0	4	9238
85	1955	4128.1	656.0	30043272.0	6980	75	4	9216
85	1956	4287.3	656.0	39442326.0	7114	78	4	9200
85	1957	4617.9	656.0	42551600.0	7121	84	4	9214
85	1958	4331.9	464.0	40523520.0	7052	80	4	9248
95	1957	205.2	81.0	2959223.0	994	-0	1	10201
95	1958	453.1	81.0	4674630.0	1025	0	1	10317
95	1959	526.2	80.0	5434218.9	986	0	1	10327
95	1960	522.3	160.0	5428857.5	1528	0	2	10257
95	1961	561.5	162.0	6062450.0	1558	0	2	10797
95	1962	779.7	274.0	8307702.5	1813	0	3	10655
95	1963	1103.8	274.0	11562361.8	1813	0	3	10475
95	1964	1428.3	274.0	14773729.2	1817	55	3	10344
95	1965	1558.1	274.0	16028565.4	1822	60	3	10287
100	1958	477.3	175.0	4682440.0	1763	0	1	9810
100	1959	1088.7	175.0	11284183.2	2507	80	1	10181
100	1960	1712.9	350.0	17298965.6	3428	0	2	10058
100	1961	2076.8	350.0	21401240.0	3758	0	2	10207
100	1962	1800.1	350.0	18529398.0	4161	55	2	10294
100	1963	2345.5	350.0	23980394.0	4367	71	2	10224
100	1964	2901.2	525.0	29613030.4	4915	0	3	10207
100	1965	3273.6	540.0	33875674.0	5506	66	3	10348
105	1965	462.3	120.0	4582742.4	1325	0	1	9913
105	1964	236.0	85.0	2385199.2	991	0	1	10107
106	1965	422.3	85.0	4209474.5	1035	59	1	9968
108	1961	732.5	113.6	6957477.8	1661	0	1	9498
108	1962	730.1	113.6	7089957.6	1669	73	1	9711
108	1963	724.5	113.6	7038986.4	1730	73	1	9714
108	1964	771.2	113.6	7541209.6	1735	77	1	9779
108	1965	790.5	113.6	7643289.0	1697	79	1	9669
109	1963	805.3	140.0	7858943.0	2307	66	1	9759
109	1964	970.3	300.0	9335212.8	2602	0	2	9621
109	1965	1973.9	300.0	18352326.4	2898	75	2	9297
113	1953	165.7	180.0	1207218.7	1743	-0	1	7286
113	1954	2427.5	364.0	23713172.3	3726	-0	2	9769
113	1955	2719.7	364.0	26772058.8	1168	86	2	9844
113	1956	2673.5	364.0	25773376.3	4451	85	2	9640
113	1957	2600.2	369.0	25268254.3	4471	82	2	9718
113	1958	2997.8	694.0	29355420.6	5125	-0	3	9792
113	1959	4351.8	709.0	41696656.6	7069	71	3	9581
113	1960	4069.9	719.0	39073803.0	8301	67	3	9601
113	1961	5680.0	1096.0	55392034.2	8651	0	4	9752
113	1962	6621.2	1493.0	66767794.6	10836	-0	5	9788
113	1963	7579.5	1493.0	74544730.3	12236	56	5	9835
113	1964	7319.7	1493.0	72887129.4	14785	54	5	9958
113	1965	6612.5	1493.0	66283714.2	15732	49	5	10024
114	1963	3130.9	752.0	30185721.8	2760	-0	2	9641
114	1964	4588.8	752.0	44201025.1	3139	65	2	9632

FILE NO.	YEAR	OUTPUT	CAPACITY	FUEL	LABOR	PLANT FACTOR	NO.MACHINES	HEAT RATE
111	1965	5930.7	1682.0	58380130.0	5465	0	3	9844
115	1958	1036.5	170.0	10062124.8	2501	76	1	9708
115	1959	866.4	170.0	8059800.0	2507	63	1	9303
115	1960	944.4	175.0	9104430.2	2478	69	1	9640
115	1961	936.0	175.0	8574732.8	2460	0	1	9142
115	1962	876.7	187.5	8594775.4	2513	53	1	9804
115	1963	1163.2	350.0	11328816.4	2678	0	2	9739
115	1964	2080.5	350.0	10223592.9	2808	63	2	4914
115	1965	2259.4	350.0	21771941.9	2857	69	2	9636
117	1955	161.2	135.0	1514625.0	2479	0	1	9396
117	1956	1028.8	135.0	9781470.0	2413	87	1	9508
117	1957	970.5	135.0	9094832.0	2443	82	1	9371
117	1958	1174.2	290.0	11010125.0	2788	0	2	9377
117	1959	1816.7	290.0	17028736.0	3083	77	2	9373
117	1960	1626.6	290.0	15896709.6	3098	72	2	9373
117	1961	1463.6	290.0	13681593.2	3034	0	2	9348
117	1962	1439.6	290.0	13347600.0	3008	61	2	9276
117	1963	1649.9	290.0	15239526.0	3008	70	2	9237
117	1964	1987.7	290.0	18274500.0	2808	84	2	9194
117	1965	2021.8	290.0	18749845.4	2608	85	2	9274
131	1954	2223.4	300.0	20869516.8	4347	-0	3	9386
131	1955	2694.7	300.0	25158780.0	4873	103	3	9336
138	1954	1148.1	265.0	10645997.0	2774	-0	2	9273
138	1955	1971.6	265.0	18444969.4	3221	85	2	9355
138	1956	2090.3	265.0	19287340.8	3203	90	2	9227
138	1957	1850.6	276.0	17361669.6	3353	80	2	9382
138	1958	1927.8	266.0	17635241.4	3362	83	2	9148
141	1954	250.7	153.0	2442616.0	1201	-0	1	9743
141	1955	1589.0	385.0	15157966.4	1900	-0	2	9539
141	1956	2509.0	385.0	24048054.3	2163	82	2	9585
141	1957	2607.3	385.0	24867033.7	2153	85	2	9537
141	1958	2787.8	385.0	27172137.7	2214	91	2	9747
141	1959	2728.8	385.0	25927737.0	2219	89	2	9502
141	1960	2593.6	388.0	24478368.0	2230	84	2	9438
141	1961	2633.1	388.0	24821544.0	2255	0	2	9427
141	1962	2438.2	388.0	23165514.0	2266	67	2	9501
141	1963	2481.2	388.0	23562812.0	2307	68	2	9497
141	1964	2634.0	388.0	25146944.0	2272	72	2	9547
141	1965	2394.5	366.0	23108564.0	2318	65	2	9651
143	1958	850.4	175.0	8386922.0	1476	-0	1	9862
143	1959	1845.9	359.0	18786622.0	1891	-0	2	10172
143	1960	2497.9	360.0	24549938.0	2230	87	2	9828
143	1961	2354.8	351.0	22675560.0	2378	0	2	9630
143	1962	2584.0	362.0	24738378.0	2390	90	2	9574
143	1963	2705.8	362.0	26697276.0	2307	95	2	9867
143	1964	2653.1	364.0	26882814.0	2272	93	2	10114
143	1965	2658.4	364.0	26447352.0	2277	93	2	9949
160	1958	598.0	175.0	6147476.4	1435	0	1	10280
160	1959	816.3	175.0	8658520.0	1685	59	1	10607
160	1960	1017.9	175.0	10378056.0	1569	73	1	10196

FILE NO	YEAR	OUTPUT	CAPACITY	FUEL	LABOR	PLANT FACTOR	NO MACHINES	HEAT RATE
169	1962	1254.7	175.0	12674792.0	1360	81	1	10102
169	1963	1253.2	175.0	12770202.6	1195	81	1	10190
169	1964	1150.6	175.0	12236935.4	1404	76	1	10365
169	1963	534.6	160.0	5434286.4	1236	0	1	10165
169	1964	807.7	160.0	8350265.5	1322	56	1	10338
169	1965	809.4	160.0	8226331.8	1242	57	1	10163
172	1959	3437.6	450.0	30955335.8	5384	88	2	9005
172	1960	3561.9	450.0	31955400.0	5328	91	2	8971
173	1953	661.2	351.5	6009699.2	5105	0	2	9089
173	1954	3027.7	430.0	27571163.4	5465	98	2	9106
173	1955	3223.5	430.0	29480619.4	5204	105	2	9146
173	1956	3030.9	430.0	27880499.2	5325	99	2	9109
173	1957	3010.9	430.0	27706872.8	5216	81	2	9111
173	1958	3130.0	430.0	28602828.8	5207	84	2	9138
173	1959	3072.6	430.0	27937790.4	5014	82	2	9093
173	1960	3078.9	430.0	27083232.0	5204	82	2	9056
173	1961	3052.1	430.0	27722150.4	5043	0	2	9083
173	1962	2976.1	430.0	27019781.6	4779	80	2	9079
173	1963	3132.1	430.0	28559748.6	4779	84	2	9118
173	1964	3267.9	430.0	29925357.0	4791	88	2	9102
173	1965	3320.8	430.0	30324161.8	4761	89	2	9132

DATA FOR CELL 5

FILE NO.	YEAR	OUTPUT	CAPACITY	FUEL	LABOR	PLANT FACTOR	NO. MACHINES	HEAT RATE
14	1958	994.3	435.0	9424682.0	1107	0	2	9479
14	1959	2811.6	435.0	26536706.4	2096	74	2	9438
14	1960	3096.1	665.0	29404650.3	3221	0	3	9497
14	1961	5350.2	870.0	51605067.5	3157	0	4	9645
14	1962	5190.9	870.0	49920073.6	3337	68	4	9617
14	1963	5115.0	870.0	49202688.0	3461	67	4	9619
14	1964	5174.8	870.0	49651576.4	4130	68	4	9595
14	1965	4624.6	870.0	44865089.2	3726	61	4	9701
15	1959	1809.5	430.0	17000848.2	1767	0	2	9395
15	1960	3060.5	430.0	28231840.4	2272	80	2	9225
15	1961	3500.8	430.0	32194180.9	2173	0	2	9196
15	1962	3093.1	430.0	28860625.0	2225	0	2	9331
15	1963	2857.2	430.0	26916797.2	2348	75	2	9388
15	1964	2513.0	430.0	23694755.0	2395	66	2	9429
26	1959	705.8	169.0	6358788.0	1850	0	1	9009
26	1960	2274.6	356.0	20506166.4	2478	0	2	9015
26	1961	2593.2	367.0	23620811.2	2706	0	2	9109
26	1962	3416.1	550.5	30622395.0	3337	0	3	8964
26	1963	3808.2	550.5	33915722.0	4244	74	3	8906
26	1964	4149.2	550.5	36766059.2	3469	81	3	8861
26	1965	4001.3	550.5	35932083.2	3478	78	3	8980
79	1964	2632.6	494.0	23078000.0	3758	0	2	8766
79	1965	3822.1	507.4	33650020.0	2525	91	2	8804
81	1963	1595.8	260.0	14127618.0	2802	69	1	8853
81	1964	1663.9	275.0	14771736.0	2891	71	1	8878
81	1965	1953.9	275.0	17389196.4	2981	84	1	8900
85	1956	1443.3	265.0	13294248.0	5075	0	1	9211
85	1957	2068.3	530.0	18987192.0	5216	-0	2	9180
85	1958	3592.3	850.0	33057480.0	5412	-0	3	9202
85	1959	5097.2	860.0	46638092.0	6206	69	3	9150
85	1960	4797.9	860.0	44593472.0	6443	65	3	9294
85	1961	5040.5	860.0	46373796.0	6355	0	3	9200
111	1961	3321.1	654.0	29539942.2	5371	0	2	8895
111	1962	3919.7	645.0	34782657.0	5933	69	2	8874
111	1963	4314.3	644.0	38432114.8	6633	75	2	8908
111	1964	4532.4	666.0	40241269.4	6609	79	2	8879
111	1965	4181.2	658.0	37872277.9	6624	73	2	9058
123	1957	654.2	329.4	6115494.0	2236	-0	2	9348
123	1958	2192.1	329.4	20119519.2	2952	76	2	9178
123	1959	2511.9	590.7	23042278.8	3206	0	3	9173
123	1960	3932.5	851.4	35739446.4	4171	0	4	9086
123	1961	5612.6	1112.1	51073372.0	4879	0	5	9100
123	1962	6688.0	1136.0	60611033.6	1874	66	5	9063
123	1963	7445.5	1154.9	67639172.8	1931	74	5	9085
123	1964	7419.9	1154.9	67280540.2	1959	73	5	9068
123	1965	7442.5	1154.9	67920760.8	1901	74	5	9126
125	1965	285.3	386.0	19284823.8	2732	0	1	67359
137	1959	960.3	165.0	8932576.4	2466	0	1	9302
137	1960	894.3	154.0	8432057.2	2437	0	1	9429

FILE NO.	YEAR	OUTPUT	CAPACITY	FUEL	LABOR	PLANT FACTOR	NO.MACHINES	HEAT RATE
137	1961	871.8	154.2	8277069.0	2870	0	1	9494
145	1962	647.9	139.5	6103919.0	1401	-0	1	9421
145	1963	941.4	139.5	8496110.2	1401	79	2	9025
145	1964	1293.8	260.0	11930267.8	1735	-0	2	9186
145	1965	1536.0	260.0	13769952.0	1822	64	2	8965
146	1958	533.9	275.0	4853492.0	1763	76	2	9091
146	1959	1691.0	275.0	15430269.3	2384	54	2	9125
146	1960	1833.6	275.0	17075342.0	2478	51	2	9312
146	1961	1998.9	275.0	18410725.3	2419	0	2	9210
146	1962	1824.5	234.8	16753753.4	2390	61	2	9183
146	1963	1911.3	234.8	17571459.0	2596	70	2	9193
146	1964	1780.8	258.0	16393935.6	2602	48	2	9206
146	1965	1815.4	258.0	16608629.7	2567	58	2	9149

DATA FOR CELL 6

FILE NO.	YEAR	OUTPUT	CAPACITY	FUEL	LABOR	PLANT FACTOR	NO. MACHINES	HEAT RATE
3	1965	930.8	260.0	8709674.4	2484	0	1	9357
21	1960	658.9	158.0	6293502.0	1776	0	1	9552
21	1961	1082.8	158.0	10281373.2	2378	0	1	9495
21	1962	1234.1	164.0	11694072.0	2431	94	1	9476
21	1963	1850.9	336.0	17543263.0	2513	0	2	9478
21	1964	2103.6	334.0	20214266.4	2808	73	2	9609
21	1965	2379.7	334.0	22605685.4	2815	83	2	9499
33	1965	1572.1	411.0	14992402.1	1780	0	1	9537
39	1965	992.1	244.5	9260197.2	3271	0	1	9334
41	1964	2484.5	512.0	23554459.2	3180	0	2	9481
41	1965	3512.7	504.0	33613038.0	4388	67	2	9569
54	1963	1270.6	183.0	12178224.4	3296	75	1	9585
54	1964	1093.3	183.0	10489456.0	3345	64	1	9594
54	1965	1259.4	183.0	12057035.6	3312	74	1	9574
56	1959	1262.0	286.0	13248829.6	2466	0	2	10498
56	1960	1885.1	437.0	20370144.6	5039	0	3	10806
56	1961	3080.6	588.0	30699091.6	5740	0	4	10030
56	1962	3673.6	588.0	36361463.2	5892	70	4	9898
56	1963	3813.5	588.0	37476400.2	5892	73	4	9827
56	1964	3902.5	588.0	38229598.4	5989	74	4	9796
56	1965	3895.1	588.0	15917763.2	5920	74	4	4087
69	1962	940.5	460.0	9644458.2	1236	0	2	10255
69	1963	1834.7	460.0	18503778.8	1483	44	2	10085
69	1964	1743.4	460.0	17523241.2	0	41	2	10051
69	1965	2021.5	460.0	20437925.4	1863	48	2	10110
77	1962	1254.8	191.0	11686995.0	2513	75	1	9314
77	1963	2360.0	384.0	21984902.4	2637	0	2	9316
77	1964	2519.5	368.0	23713649.0	2602	72	2	9412
77	1965	2210.2	388.0	21212675.4	2732	63	2	9598
120	1964	722.5	200.0	6603790.0	1817	0	1	9685
120	1965	1172.7	200.0	11633234.4	2070	65	1	9408
121	1960	686.0	165.0	6488557.0	1735	0	1	9459
121	1961	1063.6	185.0	10639104.2	1804	0	1	9818
121	1962	1017.1	165.0	9786519.6	1772	56	1	9718
121	1963	1169.8	165.0	11404650.6	1772	65	1	9758
121	1964	1112.2	165.0	10804851.0	1900	61	1	9715
121	1965	1177.3	185.0	11257275.8	1787	65	1	9562
140	1961	814.1	320.0	5665468.0	3075	0	1	6959
140	1962	1728.0	330.0	16164995.2	3172	54	1	9355
140	1963	2374.1	330.0	22957675.6	3296	73	1	9836
140	1964	2305.0	337.0	21232323.4	3387	72	1	9211

DATA FOR CELL 7

FILE NO.	YEAR	OUTPUT	CAPACITY	FUEL	LABOR	PLANT FACTOR	NO. MACHINES	HEAT RATE
23	1965	3580.3	712.0	31208265.6	2691	0	2	8717
53	1961	2953.0	500.0	26038227.6	3485	0	1	8818
53	1962	2989.7	500.0	26428864.0	3378	76	1	8840
53	1963	3134.0	500.0	27882316.8	3420	79	1	8897
53	1964	2700.6	500.0	24265060.8	3552	68	1	8992
53	1965	3141.5	500.0	28053203.8	3602	80	1	8931
119	1965	2065.9	393.0	18152862.0	4968	50	1	9050
142	1960	1823.2	650.0	16678582.6	2065	-0	2	9148
142	1961	3752.8	726.0	32140422.0	2378	0	2	8564
142	1962	4882.4	726.0	41930423.7	2513	79	2	8588
142	1963	3506.5	726.0	30555592.8	2637	57	2	8714
142	1964	4273.5	726.0	37707416.7	2643	69	2	8824
142	1965	4981.1	679.0	43664907.3	2567	80	2	8766

Appendix D

Summary of Models Considered

Table 15

Alternative Models of the
Steam-Electric Generating Plant

Equation Number	Page	Name	Form
(III-41a)	96	CD	$\ln Y_i = \ln \gamma + \alpha_1 \ln K_i + \alpha_2 \ln F_i + \alpha_3 \ln L_i + u_i$
(III-45)	97	CES	$\ln Y_i = \beta_0 + \beta_1 \ln K_i + \beta_2 \ln F_i + \beta_3 \ln L_i + \beta_4 (\ln K_i - \ln F_i)^2$ $+ \beta_5 (\ln K_i - \ln L_i)^2 + \beta_6 (\ln F_i - \ln L_i)^2 + u_i$
(III-49)	100	CD1	$\frac{1}{1+r} \ln Y_i + \frac{2r}{1-r^2} \ln [(1+r)a + (1-r)Y_i] = \ln \Gamma + \alpha_1 \ln K_i +$ $\alpha_2 \ln F_i + \alpha_3 \ln L_i + u_{0i}$
(III-50)	102	CES1	$\frac{1}{1+\tilde{r}} \ln Y_i + \frac{2\tilde{r}}{1-\tilde{r}^2} \ln [(1+\tilde{r})a + (1-\tilde{r})Y_i] = \beta_0 + \beta_1 \ln K_1 + \beta_2 \ln F_i$ $+ \beta_3 \ln L_i$ $+ \beta_4 (\ln K_i - \ln F_i)^2$ $+ \beta_5 (\ln K_1 - \ln L_i)^2$ $+ \beta_6 (\ln F_i - \ln L_i)^2$ $+ u_i$

Table 15--Continued

Alternative Models of the
Steam-Electric Generating Plant

Equation Number	Page	Name	Form
(III-56)	104	CD2	$\ln Y_i+\theta_0 Y_i = \ln \gamma+\alpha_1 \ln K_i+\alpha_2 \ln F_i+\alpha_3 \ln L_i+u_{0i}$
(III-57)	104	CES2	$\ln Y_i+\theta_0 Y_i = \beta_0+\beta_1 \ln K_i+\beta_2 \ln F_i+\beta_3 \ln L_i$ $+\beta_4(\ln K_i - \ln F_i)^2+\beta_5(\ln K_i - \ln L_i)^2$ $+\beta_6(\ln F_i - \ln L_i)^2+u_i$
(III-60a)	108	LUHCC	$\ln Y_i = \ln \gamma_1+\beta_1 \ln K_i+u_{1i}$
(III-60b)	108	LUHCF	$\ln Y_i = \ln \gamma_2+\beta_2 \ln F_i+u_{2i}$
(III-60c)	108	LUHCL	$\ln Y_i = \ln \gamma_3+\beta_3 \ln L_i+u_{3i}$
(III-61a)	108	LEHCC	$\ln K_i = \ln \Gamma_1+\alpha_1 \ln Y_i+v_{1i}$
(III-61b)	108	LEHCF	$\ln F_i = \ln \Gamma_2+\alpha_2 \ln Y_i+v_{2i}$
(III-61c)	108	LEHCL	$\ln L_i = \ln \Gamma_3+\alpha_3 \ln Y_i+v_{3i}$
(III-63)	110	LING	$\log a_i = \ln k+n \ln S_i+m \ln PF_i+p(\ln PF_i)^2+u_i$
(III-64)	112	CD/N	$\ln Y_i = \ln \gamma+\alpha_1 \ln K_i+\alpha_2 \ln F_i+\alpha_3 \ln L_i+\delta N_i+u_i$
(III-65)	112	CES/N	$\ln Y_i = \beta_0+\beta_1 \ln K_i\beta_2 \ln F_i\beta_3 \ln L_i+\delta N_i+\beta_4(\ln K_i - \ln F_i)^2$ $+\beta_5(\ln K_i - \ln L_i)^2+\beta_6(\ln F_i - \ln L_i)^2+u_i$

Table 15--Continued

Alternative Models of the
Steam-Electric Generating Plant

Equation Number	Page	Name	Form
(III-66a)	112	LEHCC/N	$\ln K_i = \ln \Gamma_1 + \alpha_1 \ln Y_i + \delta_1 N_i + v_{1i}$
(III-66b)	112	LEHCF/N	$\ln F_i = \ln \Gamma_2 + \alpha_2 \ln Y_i + \delta_2 N_i + v_{2i}$
(III-66c)	112	LEHCL/N	$\ln L_i = \ln \Gamma_3 + \alpha_3 \ln Y_i + \delta_3 N_i + v_{3i}$

BIBLIOGRAPHY

American Telephone and Telegraph Co. "Profits, Performance, and Progress." New York. (1959).

Anscombe, F., and Tukey, J. "The Examination and Analysis of Residuals." Technometrics. Vol. 5. (1963). pp. 141-160.

Averch, H., and Johnson, L. L. "Behavior of the Firm under Regulatory Constraints." American Economic Review. Vol. 52. (December, 1962). pp. 1052-1069.

Bain, J. S. Industrial Organization. New York: Wiley. (1959).

Barzel, Y. "The Production Function and Technical Change in the Steam-Power Industry." Journal of Political Economy. Vol. 72. (April, 1964). pp. 133-150.

________. "Productivity in the Electric Power Industry, 1929-1955." Review of Economics and Statistics. Vol. 45. (November, 1963). pp. 395-408.

Bonbright, J. C. Principles of Public Utility Rates. New York: Columbia University Press. (1961).

Bonello, F. The Term Structure of Interest Rates, the Expectations Hypothesis, and the Formulation of Expected Interest Rates. Unpublished Ph.D. dissertation. Michigan State University. (1968).

Box, G. E. P., and Cox, D. R. "An Analysis of Transformations." Journal of the Royal Statistical Society. Ser. B. Vol. 26. (1964). pp. 211-243.

________., and Tidwell, P. "Transformations on the Independent Variables." Technometrics. Vol. 4. (1962). pp. 531-550.

Cullen, C. G. Matrices and Linear Transformations. Reading, Mass.: Addison-Wesley. (1966).

Dhrymes, P. J., and Kurz, M. "Technology and Scale in Electricity Generation." Econometrica. Vol. 32. (July, 1964). pp. 287-315.

Earle, C. R. "Power Engineers Take Long Step to 4500-psi, 1150-F Steam." Power Engineering. Vol. 58. (January, 1954). pp. 58-61.

Elston, C. W., and Knowlton, P. H. Jr. "Comparative Efficiencies of Central-Station Reheat and Non-Reheat Steam Turbine-Generator Units." ASME Transactions. Vol. 74. (1952). pp. 1389-1391.

"Etiwanda, Study in Station Economy." Power. Vol. 96. (June, 1952). pp. 100ff.

Federal Power Commission, Steam-Electric Plant Construction Costs and Annual Production Expenses. Washington, D. C. Published annually.

Galatin, M. Economies of Scale and Technological Change in Thermal Power Generation. Amsterdam: North-Holland. (1968).

Gilbert, R. F. The Demand for Money: an Analysis of Specification Error. Unpublished Ph.D. dissertation. Michigan State University. (1969).

Goldberger, A. S. Econometric Theory. New York: Wiley and Sons. (1964).

Greacen, W. III. "Reheat Unit Extends Goudey Capacity." Power. Vol. 97. (February, 1953). pp. 71-75.

Hart, P., and Chawla, R. "An International Comparison of Production Functions: The Coal-Fired Electricity Generating Industry." Economica. Vol. 37. (May, 1970). pp. 164-177.

Hegetschweiler, H. and Bartlett, R. "Predicting Performance of Large Steam Turbine-Generator Units for Central Stations." ASME Transactions. Vol. 79. (1957). pp. 1085-1089.

Hoch, I. "Simultaneous Equation Bias in the Context of the Cobb-Douglas Production Function." Econometrica. Vol. 26. (1958). pp. 566-578.

Hodges, Dorothy J. "A Note on Estimation of Cobb-Douglas and CES Production Function Models." Econometrica. Vol. 37. (July, 1969). pp. 721-725.

Iulo, W. Electric Utilities--Costs and Performance. Pullman, Wash.: Washington State University. (1961).

Johnston, J. Statistical Cost Analysis. New York: McGraw-Hill. (1958).

Kearton, W. J. Steam Turbine Theory and Practice. London: Pitman and Sons. (1931).

Kendall, M. G., and Stuart, A. The Advanced Theory of Statistics. Vol. I. New York: Hafner. (1963).

Kirchmayer, L. K.; Mellor, A. G.; O'Mara, J. F.; Stevenson, J. R. "An Investigation of the Economic Size of Steam-Electric Generating Units." AIEE Transactions. Vol. 74, Part III. (1955). pp. 600-609.

Kmenta, J. "On Estimation of the CES Production Function." International Economic Review. Vol. 8. (June, 1967). pp. 180-194.

Komiya, R. "Technological Progress and the Production Function in the United States Steam Power Industry." Review of Economics and Statistics. Vol. 44. (May, 1962). pp. 156-166.

Leftwich, R. H. The Price System and Resource Allocation. 4th Ed. Hinsdale, Illinois: Dryden Press. (1970).

Leonard, C. M. Fundamentals of Thermodynamics. Englewood Cliffs: Prentice-Hall. (1958).

Ling, S. Economies of Scale in the Steam-Electric Power Industry. Amsterdam: North-Holland. (1964).

Lomax, K. S. "Cost Curves for Electricity Generation." Economica. Vol. 19. (1952). pp. 193-197.

Marschak, J., and Andrews, W. J. "Random Simultaneous Equations and the Theory of Production." Econometrica. Vol. 12. (1944). pp. 143-205.

Means, G. C. Pricing Power and the Public Interest. New York: Harper. (1962).

Murray, T. W. Data Errors and Economic Parameter Estimation: A Case Study of International Trade Data. Unpublished Ph.D. dissertation. Michigan State University. (1969)

Nerlove, M. "Returns to Scale in Electricity Supply." in C. Christ, Ed., Measurement in Economics--Studies in Memory of Yehuda Grunfeld. Stanford: Stanford University Press. (1963). pp. 167-198.

Nordin, J. A. "Note on a Light Plant's Cost Curves." Econometrica. Vol. 15. (July, 1947). pp. 231-235.

Parzen, E. Modern Probability Theory and its Applications. New York: Wiley and Sons. (1960).

Power. New York: McGraw-Hill. Monthly. (1948-1965).

"Power Takes You Through New Centra Costa Plant." Power. Vol. 96. (February, 1952). pp. 75-79.

Ramsey, J. B. "Models, Specification Error, and Inference: A Discussion of Some Problems in Econometric Methodology." Michigan State University Econometrics Workshop Paper No. 6714. East Lansing. (1967).

________. Tests for Specification Error in Classical Linear Least Squares Regression Analysis. Unpublished Ph.D. dissertation. University of Wisconsin. (1968).

________. "Tests for Specification Error in Classical Linear Least-squares Regression Analysis." Journal of the Royal Statistical Society. Ser. B. Vol. 31. (1969). pp. 350-371.

________., and Gilbert, R. F. "A Monte Carlo Study of Some Small Sample Properties of Tests for Specification Error." Michigan State University Econometrics Workshop Paper No. 6813. East Lansing. (1969).

________., and Zarembka, P. "Alternative Functional Forms and the Aggregate Production Function." Michigan State University Econometrics Workshop Paper No. 6705. East Lansing. (1967).

Shapiro, S. S., and Wilk, M. B. "An Analysis of Variance Test for Normality (Complete Samples)." Biometrika. Vol. 52. (1965). pp. 591-611.

Snedecor, G. W., and Cochran, W. G. Statistical Methods, 6th Ed. Ames, Iowa: Iowa State University. (1967).

Theil, H. "The Analysis of Disturbances in Regression Analysis." Journal of the American Statistical Association. V. 60. pp. 1067-1079.

Tillinghast, J.; Pechstein, R. H.; and Morgan, W. S. "Design and Operating Experience with a Supercritical Pressure Unit--Part 1." Power Engineering. Vol. 70. (March, 1966). pp. 58-61.

Trebing, H. "A Critique of the Planning Function in Regulation." Public Utilities Fortnightly. Vol. 79. (March 16, 1967). pp. 26ff.

________. "Toward an Incentive System of Regulation." Public Utilities Fortnightly. Vol. 72. (July 18, 1963). pp. 22ff.

Viner, J. "Cost Curves and Supply Curves." Zeitschrift für Nationalekonomie. Vol. 3. (1931). pp. 23-46.

Warren, G. B., and Knowlton, P. H. Jr. "Relative Engine Efficiencies Realizable from Large Modern Steam Turbine-Generator Units." ASME Transactions. Vol. 63. (1941). pp. 125ff.

Wilson, W. A., and Malouf, L. G. "An Approach to the Economic Problem of Matching Condenser Surface with Exhaust-Annulus Area." ASME Transactions. Vol. 73. (1956). pp. 135ff.

Yamane, T. Elementary Sampling Theory. Englewood Cliffs: Prentice-Hall. (1967).

Zellner, A.; Kmenta, J; and Drèze, J. "Specification and Estimation of Cobb-Douglas Production Function Models." Econometrica. Vol. 34. (October, 1966). pp. 287-315.

________., and Revankar, N. S. "Generalized Production Functions." Review of Economic Studies. Vol. 36. (April, 1969). pp. 241-250.

Zerban, A. H., and Nye, E. P. Power Plants. Scranton: International Textbooks. (1956).