PRACTICAL TREATMENT OF STRUCTURAL TENSION FIELD WEBS

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Leo Vaughn Nothstine

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This is to certify that the

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presented by

Leo Vaughn Nothstine

has been accepted towards fulfilment of the requirements for

the degree ~~A~~ of Civil Engineer

Chester L. Allen
Major professor

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PRACTICAL TREATMENT OF STRUCTURAL TENSION FIELD WEBS

by

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A THESIS

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THESIS

CONTENTS

Introductory

In structures which are designed for minimum weight, the problem of loading thin sheet material is confronted. Loading thin sheets efficiently becomes a problem of stability. It is not the purpose of this paper to develop new equations of stability but to reduce the practical preliminary equations for use by the engineer.

In Part I of this paper, tension field theory is discussed from the standpoint of development. This material is considered to be in agreement with the leading engineers of today. It is in no manner complete, but it is indicative of the intricacies of the developed theory. Since this work falls beyond the scope of many graduate engineers, it seems expedient to simplify the material and develop a method of design which will be adequate and useable by the engineer.

The second phase of the paper deals with a simplified method for solving the stresses in the tension field as well as the development of a chart for designing the rivet connection of the web to flanges of the beam. An example is made up to demonstrate the use of the design chart which will familiarize the engineer with the simplified method.

The third part of the paper deals with the test of a beaded web beam. This test is in no way a development of the first parts of this report, but it is incorporated to show results obtained by the beaded web in comparison to the web of riveted stiffener angles.

NOTATION
(unless otherwise specified.)

A = Area

a = Long side of a shear panel

α = Angle of web wrinkles

b = Short side of shear panel

br = (subscript) bearing

c = compression (subscript)

cr = critical (subscript)

D = Diameter

d = Vertical stiffener spacing

E = Youngs Modulus

e = strain

F = Total force

f = force per unit area

h = Effective depth of web

I = Moment of inertia

K = A function of the type of support given to the sheet edges

M = Moment

p = rivet spacing, or pitch distance

Q = Static moment of flanges about the neutral axis

q = Shear flow in pounds per inch

r = Thickness of sheet

t = tension (subscript)

tu = tensile yield (subscript)

ty = tensile ultimate (subscript)

μ = Poisson's ratio

V = Total Shear

T = Unit shear stress

x,y = coordinate axis

σ = Normal Stress (unit)

PART I

DISCUSSION OF TENSION FIELD THEORY

PART I

Ultimate strength of flat panels under shear.

In the structural analysis of the wing beams of airplanes, the stress analyst is faced with several problems which, in general, are not faced by the civil engineer. The civil engineer endeavors to make the web sheet of beams thick enough so that they will carry the load before buckling takes place in the web. In this case buckling is considered one form of failure. The shearing stresses which cause this buckling determines the allowable maximum which can be applied. The equation developed will then apply.[12]

$$\tau_{cr} = K \frac{\pi^2 E}{12 (1 - \mu^2)} \left(\frac{t}{b} \right)^2 \qquad \text{Eq. 4}$$

This equation is principally used to determine the critical shear stress allowable in a shear resistant beam (under that of the shear strength of the material) due to the physical dimensions of the panels. The coefficient "K" has a value which is dependent upon the ratio of the dimensions of the panels as well as on the rigidity of the stiffeners and flanges, or it may be considered as the end fixity of the panels. (See figure 9, Part II).

The equation is derived by setting up the differential equation for a loaded plate and considering the edges as being simply supported. Then expressing the boundary conditions where the edges have no deflection, substitute in the first equation analyzing for the critical stress.

For very thin sheets, the buckling stress given by this equation is extremely low and, in the interests of efficient design with regard to weight, the aircraft engineer raises the question of how much addi-

tional shear can be carried by such a buckled plate before some portion of the sheet has a total stress equal to the yield point of the material, which would cause permanent deformations, or the ultimate strength of the structure is reached.

The aircraft engineer would also like to know the effect of this web buckling on the rest of the structure. For instance, the additional load on the flanges and on the web stiffeners if there are any. This problem has been the subject of considerable research the past few years. Currently, there is continuous study to make the designs more accurate and reliable. At the present time, it can be said that most of the tension field design methods used in our modern airplane are too conservative.

In order to take advantage of tension field design, it is necessary to be able to compute the true margin of safety instead of a number which is termed a conservative margin because of assumptions used and certain qualities of design not considered. The fact that the true conditions taking place in the web when in the tension field condition are generally not fully understood has caused ultra conservatism in production methods and repairs designed.

To get a mathematical picture of the tension field theory it is necessary to consider a beam having a relatively thick web under the action of shear and bending. For such a section, the usual bending moment and shear equations of applied mechanics are valid.

$$\sigma_x = \frac{M_x\, y}{I} \qquad \text{and} \qquad \tau = \frac{VQ}{Ib}$$

σ_x = Normal stress

where M_x = bending moment at any value of x

y = distance from neutral axis to fiber considered

I = moment of inertia about the neutral axis of cross section

V = applied shearing force

Q = static moment for which τ is determined about the neutral axis

b = thickness of the section at which τ is to be determined

For the web, $b = t$. This corresponds with the equation on page 1.

Considering an element of the web on the neutral axis, so that bending stresses are absent, take an element whose sides are parallel and perpendicular to the shear force. The stress pattern will then be seen to be uniform shearing stresses on the four faces of the element, see figure 1. However the same result can be represented by an element considered to be at 45° to the direction of the applied shear by pure normal stresses of equal magnitude in which

$$\sigma_c = \sigma_t = \tau$$

as shown in figure 1 also. For web plates which are thick, this stress distribution plus bending stresses, will hold up to stress values approaching the yield point of the material. At that time plastic flow will enter the picture and the mathematics will no longer hold.

The foregoing discussion takes on new significance in regard to principal stress patterns if we assume that the web plate in the beam is very thin. Considering the compressive stress against which thin plates have very little resistance, it is seen that the tendency will be for the web to

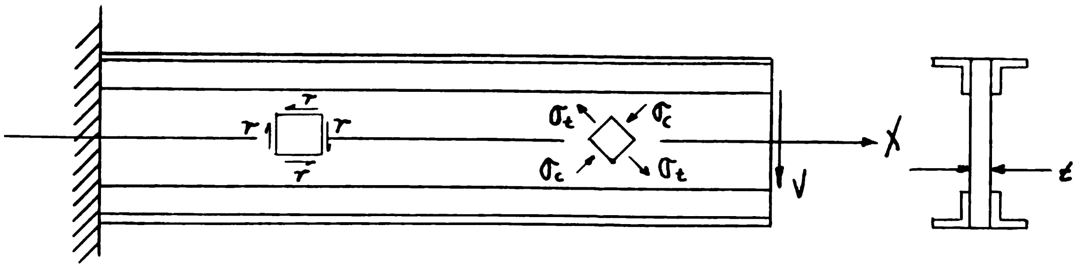

τ
τ
τ
τ
σ_t
σ_c
σ_c
σ_t
X
V
t

Fig. 1

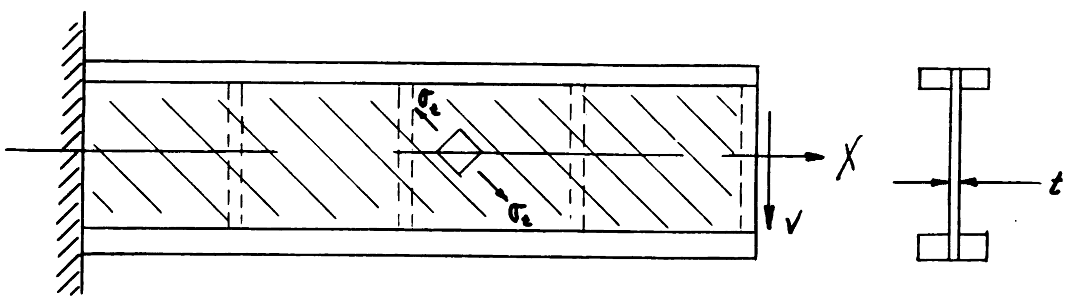

σ_t
σ_t
X
V
t

Fig. 2

buckle in a direction perpendicular to the line of action of the compressive stress. The value of the applied shear will diminish as the web becomes thinner and thinner, the limiting case being for a sheet of zero thickness. In this case, the sheet buckles upon application of any shear load and can only resist shear by means of the tensile stresses at 45°, as shown in figure 2.

It is obvious, with such a stress pattern, that the tensile stresses will tend to pull the two flanges together. Thus it can be seen that some means of vertical stiffening is required to counteract this tendency.

The limiting case of a web having no compressive strength has been treated in detail by Wagner (reference 2) and beams approximating this are known as Wagner beams. It is interesting to point out at this point that the center portions of the wing on the Consolidated Liberator Bomber B-24 is treated as a Wagner beam.

Wagner assumes that the web buckles immediately upon application of the shear load and that the only stresses resisting the shear forces are the tensile stresses set up in the web. These tensile stresses act at approximately 45°. Assuming infinitely rigid parallel span flanges and vertical stiffeners, the following equations for this limiting condition can be shown to apply. Keeping in mind the above assumptions and referring to figure 3, for notation the diagonal tension stress developed in the web will be

$$\sigma_t = \frac{2V}{ht} \frac{1}{\sin 2} \qquad \text{Eq. 5}$$

Total force in the compression flange

$$F_c = -\frac{Vx}{h} - \frac{V}{2} \cot \alpha \qquad \text{Eq. 6}$$

Total force in the tension flange

$$F_t = \frac{Vx}{h} - \frac{V}{2} \cot \alpha \qquad \text{Eq. 7}$$

Axial force in the vertical stiffeners

$$F_v = - V\frac{d}{h} \text{ Tan} \qquad \text{Eq. 8}$$

where V = applied shear load

h = effective depth of web

t = web thickness

d = vertical stiffener spacing

α = angle of web wrinkles, theoretically 45° for this case, but actually somewhat less.

In summary of the pure tension field case, the following assumptions were made: 1) The sheet carries the entire shear load. 2) The flanges are pinned to the verticals and are pin-connected at the fixed end of the beam (in cantilever beam). 3) There is no gusset action at the connections of the vertical flanges. 4) The sheet immediately goes into the wave state and completely supports the applied shear by means of the diagonal tension field set up when the load is applied.

Accuracy of results by the derived equations are not too consistent. For instance the equation for determining the compressive stress in the web stiffeners gives results which have been found to be from two to five times above the actual. The other equations give reasonable approximations for the first analysis. For web thickness over about .030 inch, the results are conservative and become more so as the web thickness and the flange stiffness increases.

For the above reason which causes designs to be too heavy it is desired to analyze the situation with perhaps revised assumptions to obtain a more accurate picture of the actual stress situation.

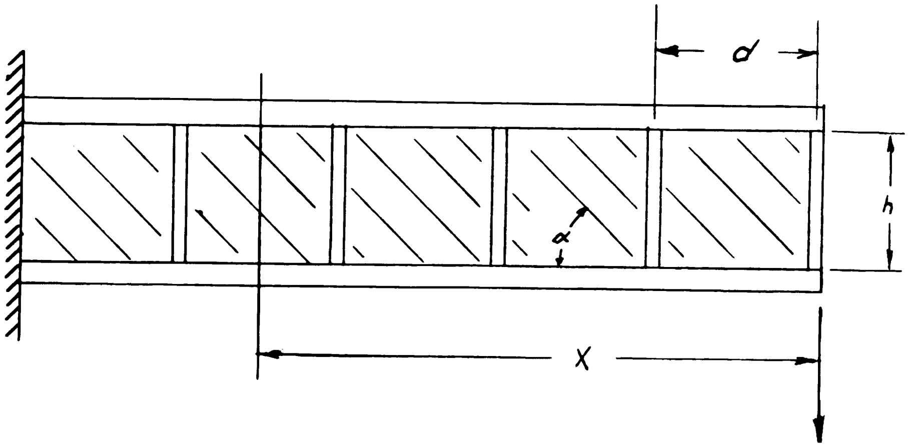


Fig. 3

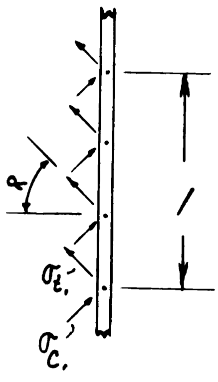


Fig. 4

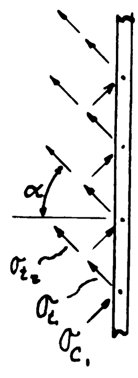


Fig. 5

Investigations of this problem have been made and are for the most part incomplete at this time. Especially as for the experimental data which will be some time in catching up. At the present time there is very little published work to be found on this subject.

A revised set of assumptions will change the picture and divide the shear stress up between the following parts: 1) Shear carried by the flanges due to their small but finite shear stiffness. 2) The shear carried by the web before buckling, as a resistant member. 3) The diagonal tension field in the web, which carries additional shear of the web.

In order to account for the shear carried by the flanges, the shear flow between the flanges is assumed uniform. This shear flow is the value conventionally calculated for such a beam at the flange rivet line by the following equation. This method was suggested by Green of Consolidated Aircraft.

$$q = \frac{VQ}{I} \qquad \text{Eq. 9}$$

where q   shear flow in web in pounds per inch

V   total shear in pounds

Q   statical moment of the flange about the neutral axis

I   moment of inertia of the entire beam cross section when V is not greater than the buckling strength; and equals the moment of inertia of the flanges only when V is shear resisted by a diagonal tension field.

The shear resisted by the web plate only, would then be given by

$$V_w = \frac{V}{I/Qh} \qquad \text{Eq. 10}$$

where h = effective depth of the web, taken from the distance between the centroids of the flange rivets.

At the buckling stress of the web, the shear by the web is equal to

$$V_{cr} = \tau_{cr} ht \qquad \text{Eq. 11}$$

and the stress distribution on a unit element of the end vertical (points of shear application) is as shown in Fig. 5, in which

$$\sigma_{c1} = \sigma_{t1} = \sigma_{cr}$$

The value of τ_{cr} is given by the equation 4 where K is obtained from a table. It is conservatively assumed that the shear panels in such beams correspond to panels simply supported on all four edges.

Above the critical buckling stress, the compressive stress remains constant at its critical value, and any additional shear is carried by an increase in the diagonal tension only. The stress pattern for this case is shown in Fig. 5 and the shear carried by the tension field only is

$$V_t = \sigma_{t_2} \, ht \sin\alpha \cos\alpha \qquad \text{Eq. 12}$$

where α is the angle of plate buckling. The total average tensile stress is then

$$\sigma_{t_{ave}} = \sigma_{t_2} + \sigma_{t_1} = \sigma_{t_2} + \tau_{cr} \qquad \text{Eq. 13}$$

In all of the above it has been assumed that the flanges have been infinitely stiff in bending. But if it is considered that they may deform in the direction of the web, there will be a tendency to unload the sheet between verticals owing to this deflection. This condition is taken into account by a correction factor which is a function of the flange moments of inertia and the beam dimensions. The correction factor is given by the following equations.[3]

$$\frac{1}{R} = \frac{\sigma_{t\,max.}}{\sigma_{t\,ave.}} = \frac{wd}{2} \, \frac{\sinh wd + \sin wd}{\cosh wd - \cos wd} \qquad \text{Eq. 14}$$

where $\quad wd = 1.25d \sqrt[4]{\dfrac{t}{(I_c + I_T)h}} \sin\alpha \qquad \text{Eq. 15}$

and $\quad R$ = correction factor

3. Sechler & Dunn, "Airplane Structural Analysis and Design"(1942) pp 238

$\sigma_{t\ ave.}$ average sheet tensile stress given by equations

$\sigma_{t\ max.}$ maximum tensile stress developed in the sheet

I_c Moment of inertia of the compression flange about it's own neutral axis.

I_r Moment of inertia of inertia of the tension flange about it's own neutral axis.

The correction factor R can be found either from the above equations or by extrapolation from curves which are not developed in this paper. This factor is only applied to the sheet tensile stress and to that portion of the flange load arising from sheet tension. The end load in the verticals, since it is independent of the tensile stress distribution between the panels is not affected.

Because of the deformation of the flanges under the action of the tension field, the correction factor must be applied to $\sigma_{t\ ave}$ to obtain the maximum value. This rivet factor is given by

$$C_r = \frac{\text{rivet spacing} - \text{rivet diameter}}{\text{rivet spacing}} \qquad \text{Eq. 16}$$

The maximum value of the tensile stress is then

$$\sigma_{t\ max} = \left(\frac{\sigma_{t_2}}{R} + \tau_{cr}\right)\frac{1}{C_r} \qquad \text{Eq. 17}$$

from which

$$\sigma_{t_2} = \left(\sigma_{t\ max} - \frac{\tau_{cr}}{C_r}\right) C_r\ R \qquad \text{Eq. 18}$$

and

$$V_t = \left(\sigma_{t\ max} - \frac{\tau_{cr}}{C_r}\right) C_r\ Rht \sin\alpha \cos\alpha \qquad \text{Eq. 19}$$

When the maximum tensile stress equals the tensile yield point of the material, σ_{typ}, the value of the web shear above buckling is given by

$$V_{ty} = \left(\sigma_{typ} - \frac{\tau_{cr}}{C_r}\right) C_r\ Rht \sin\alpha \cos\alpha \qquad \text{Eq. 20}$$

and when it is equal to the ultimate tensile strength, σ_{tu}, the corresponding shear is

$$V_{tu} = \left(\sigma_{uts} - \frac{\tau_{cr}}{C_r}\right) C_r \, Rht \sin \alpha \cos \alpha \qquad \text{Eq. 21}$$

The total shear carried by the beam for $\sigma_{t\,max} = \sigma_{typ}$ and $\sigma_{t\,max} = \sigma_{uts}$, respectively, is then

$$V_y = (V_{cr} + V_{ty}) \frac{I}{Qh} \qquad \text{Eq. 22}$$

and

$$V_u = (V_{cr} + V_{tu}) \frac{I}{Qh} \qquad \text{Eq. 23}$$

The foregoing equations are all based on vertical web-stiffening members. The case of the stiffeners not being at 90° to the flanges will not be considered in this paper. It can be said that the same method is used with an additional correction factor introduced.

PART II

SIMPLIFIED DESIGN OF TENSION FIELD WEBS

PART II

Due to the shortage of trained personnel and the urgency of the production situation in our factories, short cuts for determining approximate design criteria are at a premium. Many aircraft factories are attempting to instruct a portion of their engineering layout men in the fundamentals of structures so as to help relieve the burdens of their structural engineers. It is with this in mind that this treatment of tension field beams is considered as a valuable instrument for determining a preliminary design.

Formation of the tension field may be visualized by considering one panel of the beam shown in figure 1. Under the shear load, V, the panel tends to deform as it shown in figure 2. Evidently the diagonal AD is lengthened and BC is shortened as is evidenced by the dotted lines. The web carries the load, V, as tension along one diagonal and compression along the other. Since the web is of thin sheet it will buckle as an effect of the compressive component. It then is unable to resist any more force in that direction. This leaves the tension component to carry the shear. After the web has buckled it can be pictured as having a washboard appearance with the tensile stresses acting parallel to the wrinkles, as indicated in figure 3.

If an element is considered from the web as in fugure 4a, the stresses τ_v and τ_h are equal. These represent the horizontal and vertical shear for the web in the unbuckled state.

In figure 4b another element from the same web is cut at an angle of 45°. To observe the stresses acting on this element Mohr's circle is constructed in figure 4c. From this it is apparent that these stresses are

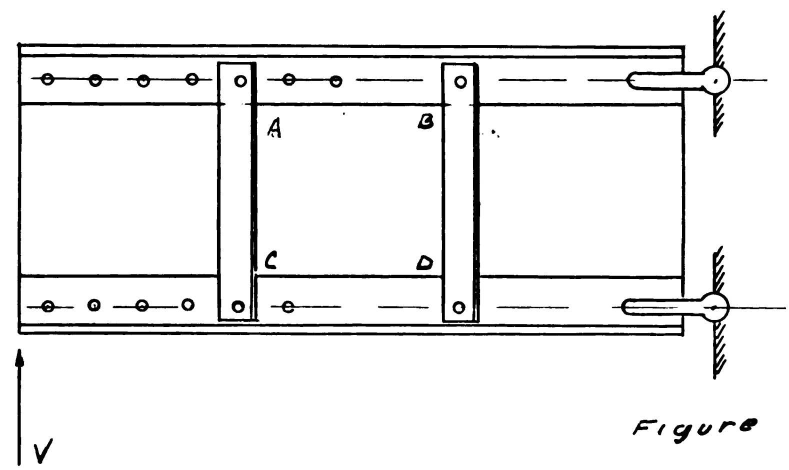


Figure 1.

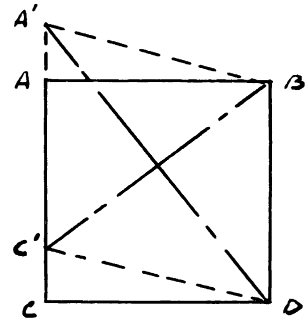


Figure 2

compression and tension stresses of equal magnitude.

After buckling, the web can take no more compression. If the compression that the web has taken is neglected a free body of an element may appear as shown in 5a. Mohr's circle for this case is shown in figure 5b. From this the stresses on the new rectangular element in figure 5c can be obtained. From the solution of Mohr's circle the values for the stresses may be found as follows:

$$\sigma_x = \sigma_t \cos^2 \theta \qquad \text{Eq. 1}$$

$$\sigma_y = \sigma_t \sin^2 \theta \qquad \text{Eq. 2}$$

$$\tau_v = \sigma_t \sin \theta \cos \theta \qquad \text{Eq. 3}$$

$$\tau_h = -\sigma_t \sin \theta \cos \theta \qquad \text{Eq. 4}$$

Solution for f_t is required. Since the vertical shear stress is f_v and the vertical shear is V the following can be written.

$$V = \tau_v \, t \, d$$

$$= \sigma_t (\sin \theta \cos \theta) \, t \, d$$

$$= (\sigma_t / 2) \, t \, d \sin 2\theta \qquad \text{Eq. 5}$$

where t = thickness web

d = depth of web

$$\sigma_t = \frac{2\,V}{t\,d} \times \frac{1}{\sin 2\theta} \qquad \text{Eq. 6}$$

Tests show that the tension field wrinkles occur at an angle near 45°. Then $\sin 2\theta = 1$ and equation 6 becomes

$$\sigma_t = 2 \frac{V}{t\,d} \qquad \text{Eq. 7}$$

This indicates that the tensile stress is twice shear stress found for a shear resistant beam. This equation is an easy means for finding the approximate stress in a tension field web.

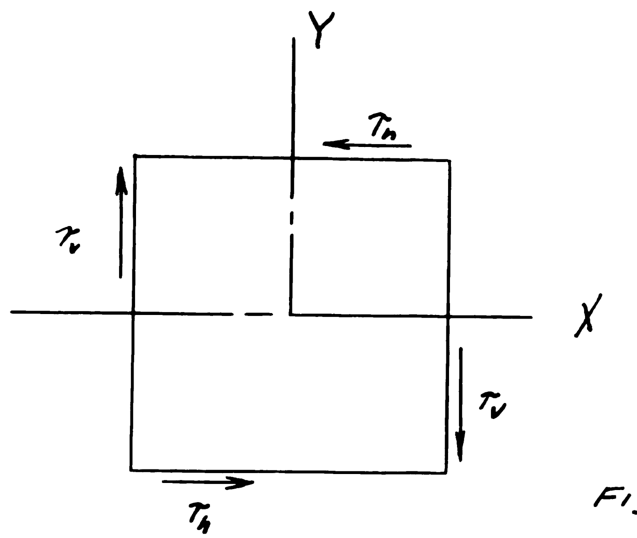


Figure 4a.

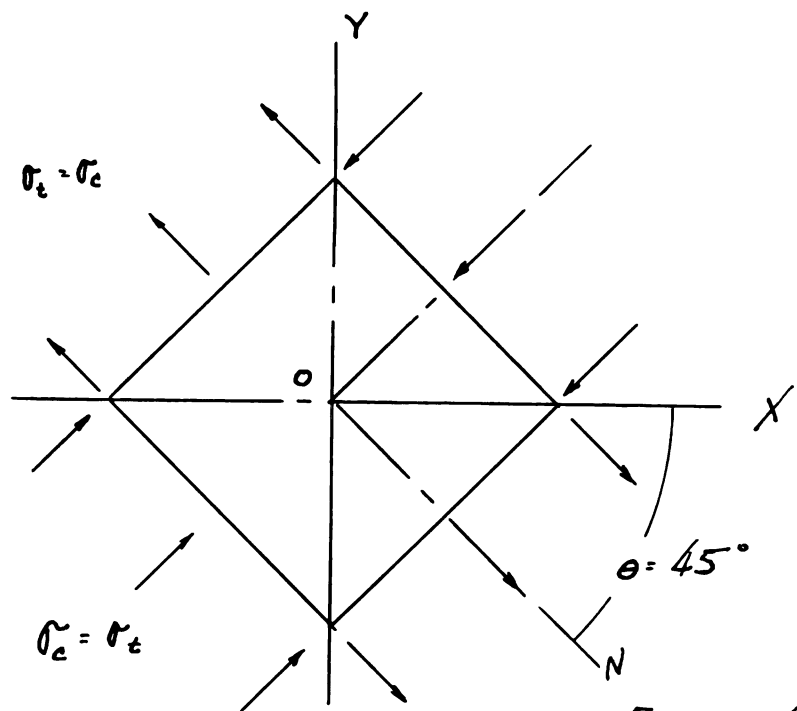


Figure 4b.

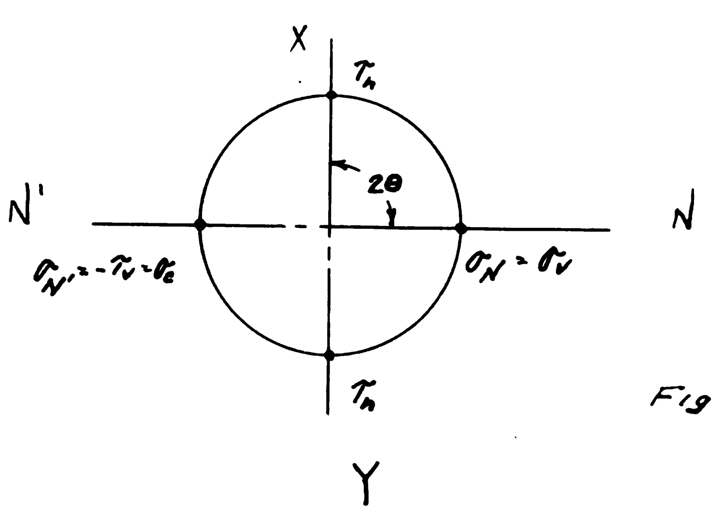


Figure 4c.

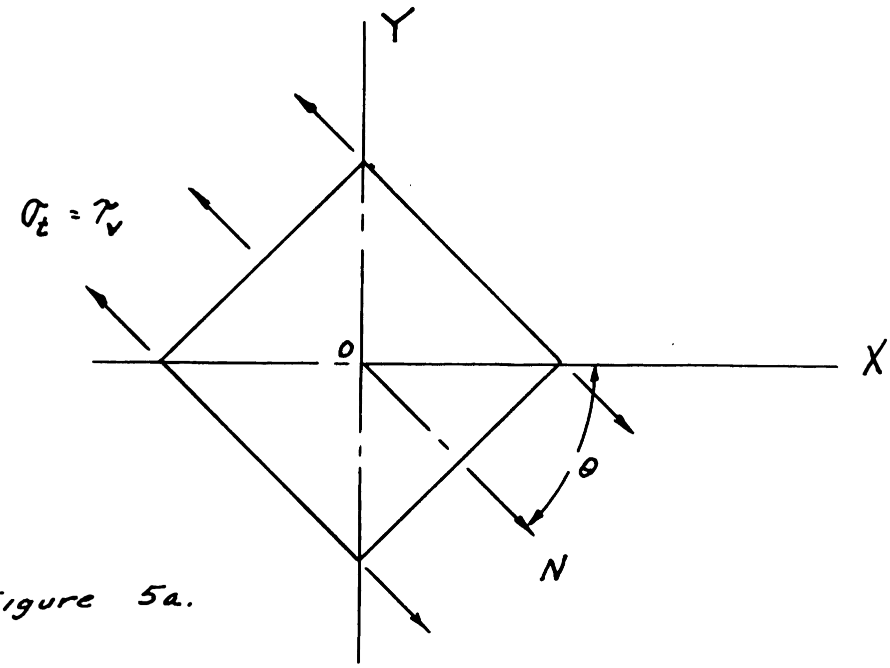


Figure 5a.

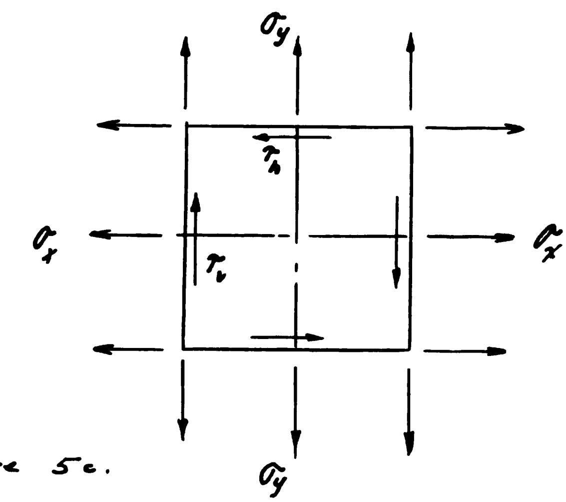


Figure 5c.

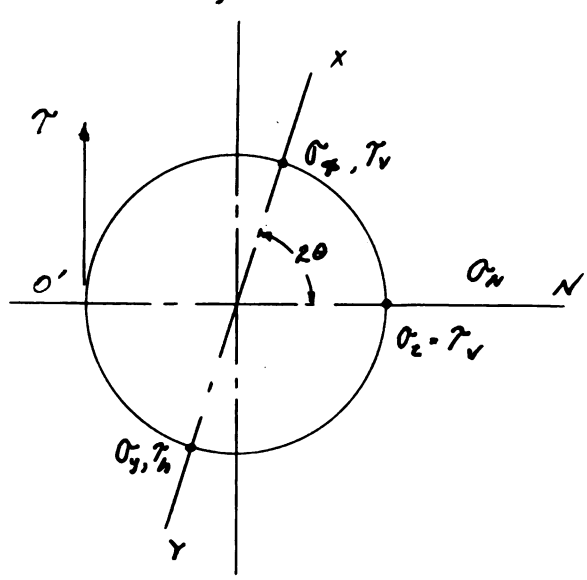


Figure 5b.

The next problem confronting the designer is the size and spacing of the rivets connecting the web to the flanges. By plotting charts from basic equations, this can be determined readily. To find the force acting on any one rivet, note that tension is now responsible for stressing the web flange rivets. Figure 6 shows three common rivet patterns.
The rivet pitch is defined as the distance between two rivets in one of the rows. In the calculations Dural Alclad 24ST will be used for the web. The rivets are A17ST (type AD) for diameters up to and including 5/32". For 3/16" and 1/4" 17ST (type D) are used. The allowable stress values for rivets and sheet are:

A17St (Type AD)	= 27,000 lb. per in.^{2}		Ult. shear str.
17ST (Type D)	= 30,000 lb. per in.^{2}		Ult. shear str.
Alclad 24 ST	= 82,000 lb. per in.^{2}	(F_{br})	Ult. bearing str.
	= 56,000 lb. per in.^{2}	(F_t)	Ult. tensile strength.

When using the design chart, corrections must be made for variation in rivet pattern. Assume one row as in figure 6a for the development of the charts.

Force per rivet = tension in the sheet.

$$F_r = \sigma_t \,(p/\sqrt{2}\,)\, t$$

$$= 2\,\frac{V}{t\,d}\,(p/\sqrt{2}\,)\, t$$

$$= \sqrt{2}(V/d)\, p \qquad \text{Eq. 7}$$

Shear on Rivet type AD

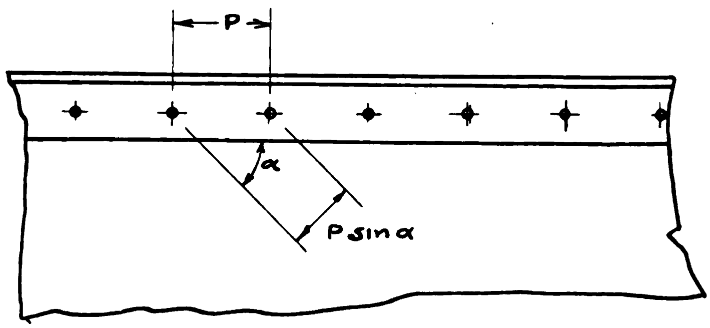
Force on the rivet = Ult. single shear strength of rivet.

$$F_r = \pi\,(D^2/4)\, F_s$$

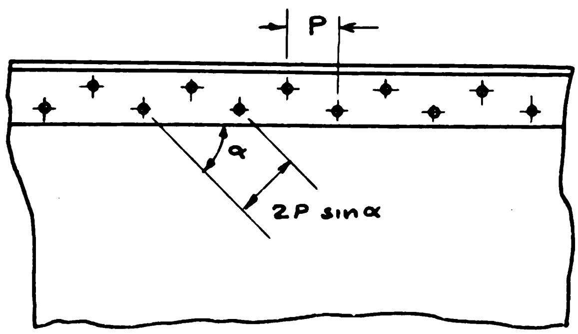
Rewriting and substituting,

$$V/d = 14{,}970\ (D/p)\ D \qquad \text{Eq. 8}$$

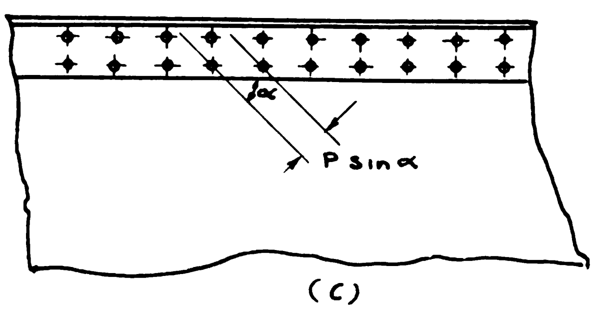
If the rivets are in double shear they will carry twice this loading. Notice that this equation is shear per inch.



(a)



(b)



(c)

Figure 6.

Shear on type D rivets

$$V/d = 16{,}630\ (D/p)\ D \qquad \text{Eq. 9}$$

Bearing on the web or force on the bearing area is equal to the Ultimate bearing strength of 24st alclad.

$$F_r = D\ t\ F_{br}$$

$$V/d = 58{,}000\ (D/p)\ t \qquad \text{Eq. 10}$$

In any of the rivet patterns in figure 6 the full tensile load must be carried across a diagonal strip containing one rivet hole. Therefore, (the force on the sheet is equal to the ultimate tensile strength of the sheet)

$$F_r = \left[(p/2\ ) - D\right] t\ F_t$$

Substituting from the above equations the following is obtained.

$$V/d = 28{,}000\ \left[1 - (D/p)\sqrt{2}\ \right] t \qquad \text{Eq. 11}$$

When equations (8-11) are plotted, a design chart for the web flange rivets of the tension field web is obtained. This has been done in figure 8. Equations 8 and 9 are plotted on the left side of the vertical D/p axis. These lines will all pass thru the origin. The broken lines on this side of the chart represent rivet pitches. They were obtained by selecting a pitch and computing a D/p for the various rivet sizes. The lower lines on the right hand side represent bearing on the web. They were plotted from equation 10, and also pass thru the origin. The upper side represents tension in the web. This was plotted from equation 11.

Solving equations 10 and 11 simultaneously will give a constant D/p value of .287. This is shown as a horizontal broken line in the right hand chart. For D/p values less than .287, the web will be critical in tension. The intersection of the sheet thickness lines with the broken horizontal line at .287 D/p value represents the maximum shear per inch that each particular web can carry. At the web would be on the verge of failure in both tension and bearing.

In the chart the rivets are assumed to be in single shear. If they are in double shear, only half of the design shear per inch is applied to a given shear area. Therefore only half of the design shear is used on the left side of the chart. Since assuming one bearing area between the rivet and the web in drawing the right hand side of the curve, it follows that any variation from this must be compensated for. That is, in a double row of rivets there are two bearing areas available for a given shear per inch. Therefore allow for this by using only one-half the design shear per inch on the right hand side of the chart. Table I outlines the different possibilities and indicates how to use the chart with each pattern.

TABLE I Proper shear per inch to use with design chart for various rivet patterns.

Rivet pattern	Divid shear per inch by	
	L.H. Side	R.H. Side of Fig.8
Single row, single shear	—	—
Single row, double shear	2	—
Double row, single shear	2	2
Double row, double shear	4	2

In order to know whether a web is shear resistant or acting as a tension field it is necessary to calculate the buckling stress. This is

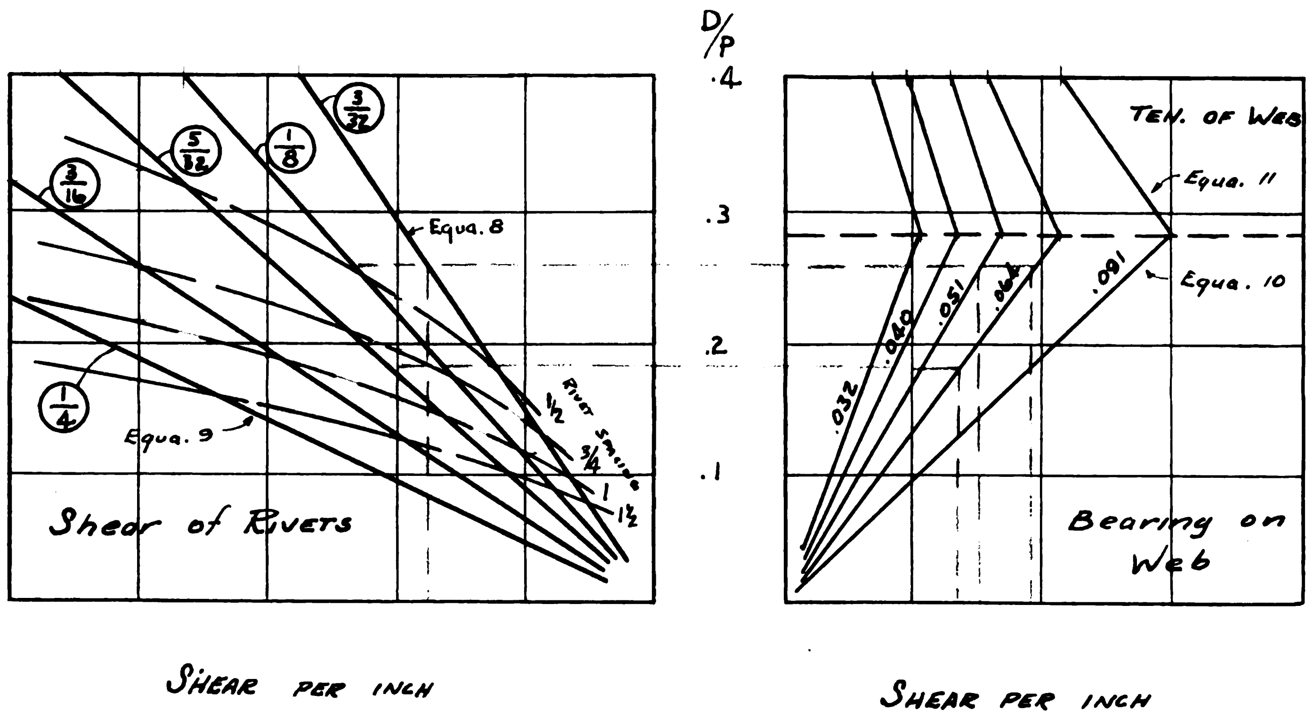

D/P
.4
.3
.2
.1
3/32
1/8
5/32
3/16
1/4
Equa. 8
Equa. 9
Rivet Spacing
1/2
3/4
1
1 1/2
Shear of Rivets
TEN. OF WEB
Equa. 11
Equa. 10
.091
.064
.051
.040
.032
Bearing on Web
SHEAR PER INCH
SHEAR PER INCH

Figure 8

generally done by using the dimensions of the web and Figure 9 which is taken from ANC-5, Strength of Aircraft Elements. The critical stress which must not be exceeded if the web is not to buckle, is also given in ANC-5 as,

$$F_{cr} = K\,E\,(t/b)^2 \qquad \text{Eq. 12}$$

where F_{cr} is the critical shear stress for the web (#/sq. in.)

- K is a constant from figure 9
- E is 10^7 for alclad 24ST
- t is thickness of the web in inches
- b is the short side of the panel in inches

If the shear stress is greater than F_{cr} tension field conditions result.

Illustrative problem:

Assume a Wagner type beam, that is, the rails take the tension and compression. The web is then designed to take this shear.

Let shear, $V = 6{,}000$ lbs.

effective depth, $d = 7.94$ inches

assume a web thickness, $t = .064$

$$\frac{V}{d\,t} = \frac{6000}{7.94 \times .064} = 13{,}720 \text{ lb. per sq. in.}$$

$$F_{cr} = 7.2 \times 10^7\,(.064/5)^2 = 11{,}800 \text{ lb. per inch}^2$$

The shear stress is above F_{cr}, therefore the web will buckle and tension field conditions exist.

$$\text{From Eq. 6,} \quad F_t = 2\,\frac{6000}{.064 \times 7.94} = 23{,}700 \text{ lb. per in.}^2$$

$$\frac{56{,}000}{23{,}700} - 1 = 1.36 \quad \text{Margin in tension.}$$

Design of web flange rivets.

$$\text{Shear per inch,} \quad \frac{6{,}000}{7.94} = 756 \text{ lb. per inch.}$$

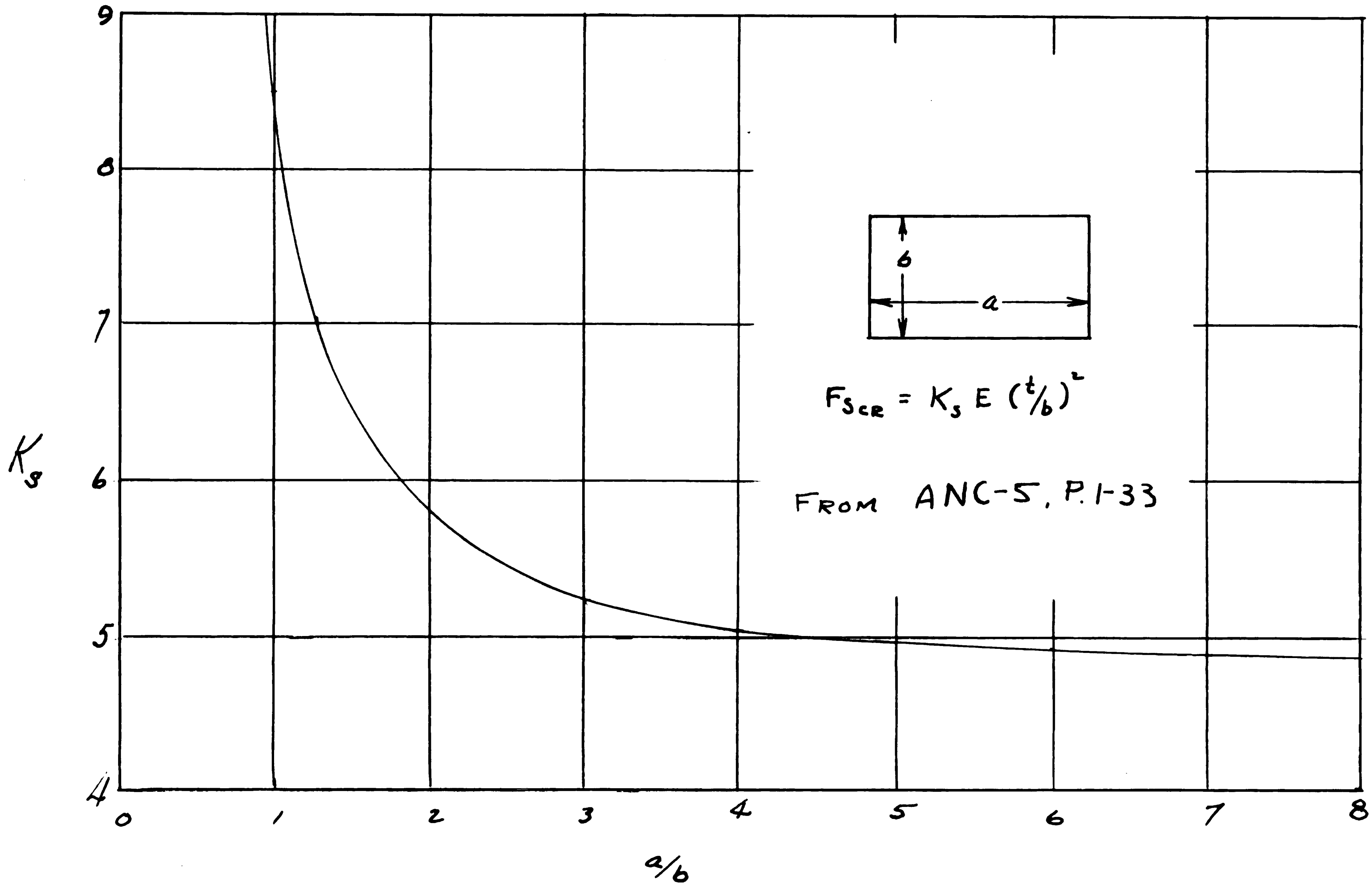


Figure 9

Then from figure 8, (a single row of rivets in double shear) 756/2 = 378 lb. per inch on the left side of the figure and 756 lb. on the right side. Using 5/32" rivets, 7/8" pitch, the allowable bearing is 680 lb. per inch. This will be insufficient. By choosing a 1/8" rivet at a pitch of 1/2", the allowable bearing goes up to 880 lb. per inch, which is satisfactory. It is also observed that with 1/8" rivets at $\frac{1}{2}$ inch spacing, a web of .051 could be used at a lower margin and a better weight efficiency.

PART III

CONSTRUCTION AND TEST OF A BEADED SPAR

PART III

Construction and Test of a Production Beaded Spar

In the manufacture of airplane spars of recent design production men often ask the question, "Could the spars be designed out of a sheet with formed bead stiffeners and thus eliminate the tedious assembly of adding the stiffener angles which are riveted to the web?" Having no figures for calculating exactly what would be the outcome, it was decided to make a test specimen to determine the feasibility of a re-design of the spars for the B24-E which is the Ford Motor Company version of the Consolidated Liberator Bomber.

For a basis of consideration for a new spar, there could be no increase in the total weight allowed. With that in mind, a design with increased web thickness was used which would be of equal weight as the present web plus the weight of the stiffeners and rivet heads.

Introduction to the test

The static test was conducted on a cantilever spar to determine the strength characteristics. The spar was fabricated from aluminum alloy material. The important characteristic of the spar was the .081 thick web which differed from the conventional stiffener web construction in that it had vertically beaded impressions spaced three inches apart and 3/8 inch bead depth. No added stiffeners were used. The web of 24S aluminum alloy material was pressed out, using a forming die, in the "SO" condition, then heat treated to the ST condition. The web was straightened after heat treat by attempting to stretch it, though little straightness was obtained by this method. The web was bolted to the upper and lower flanges. The wrinkles due to heat treat were eliminated after the web

was bolted to the flange. The web appeared uniform in every respect.

Summary and conclusions

The tested beaded web spar is considered unsatisfactory as compared to a stiffener web spar of equal weight.

The beaded web spar has approximately 980% greater buckling strength over the flat panel shear resistant type when subjected to shear stresses.

The beaded web spar failed under a load of 13,600 lbs., by the buckling of the entire web. A stiffener web spar of the same weight failed at 27,500 lbs. However, it is noted that the critical shear resistant load for the stiffener web spar was approximately 8,000 lbs.

A picture of the web tested is shown in figure 1. In order to investigate further this type of web construction, it is recommended that a web with greater bead depth and extension of the beads to the flanges be built.

Test Procedure

The spar was loaded at one end by a 20-ton hydraulic jack. The other end was bolted to an upright steel column. Deflection gages were mounted on the upper flange spaced at equal one-foot intervals along the spar. Electric strain gages were placed at predetermined regions on the web and flanges totaling 46 gages. Gage locations are shown in Figure 2. The gages were connected to a 48 point automatic scanning recording apparatus which determined graphically the stresses in the various regions of the spar. The spar was loaded in increments of 2000 pounds until definite failure occurred.

Reference: Article by William T. Thomson in Aero Digest, May 1943, Airframe Stress Analysis by the Electrical Strain Gage.

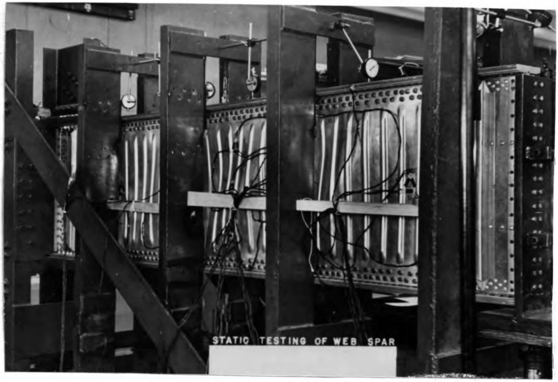


Figure 1

Test results

The spar failed due to buckling of the web under a load of 13,600 lbs. Just prior to this ultimate load, the web appeared perfectly normal and showed no buckling characteristics. Diagonal web waves were produced by the failing load. The waves stretched from end to end on the web.

Discussion

A complete stress analysis was successfully accomplished on the spar by the use of electric strain gages. The direction of the principal stresses was approximately 36° measured from the horizontal base line. Being a single test, it is not possible to predict the possibilities other than they appear to be rather limited as indicated by the conclusions of this report.

Calculation of the critical buckling stress of flat sheet panel subjected to shear stresses

$F_{cr} = K\ E\ (t/b)^2$

a = 70 inches

b = 24 inches

t = .081 inches

$a/b = \frac{70}{24} = 2.92$

$F_{cr} = 5.3 \times 10.3\ .\ 10^6\ (\frac{.081}{24})^2$ K = 5.3 (Ref. ANC-5)

$= 54.6 \times 10^6 \times .0000114 = 620$ psi

Shear stress $= \frac{P}{A} = \frac{13000}{1.95} = \underline{6680}$ psi P = 13000 lb. load just prior to failure

Per cent increase in strength due to the beads

$$\frac{6680 - 620}{620} \times 100 = \frac{6060}{620} \times 100 = 980\%$$

Calculations of the stresses in the chords

The load in the upper and lower chord flanges is given by the equation

$F = \frac{M}{h}$ M is spar moment

h is spar depth

The stress then becomes

$\sigma_c = \frac{M}{A\ h}$ where A = area of chord flange

The per cent difference between the calculated flange stress and the stress obtained from the test data is computed as follows:

The slope of the (42 - 43) curve on page 10 is

$$\frac{3500}{8000} = .437$$

The slope of the calculated curve is

$$\frac{3800}{8000} = .475$$

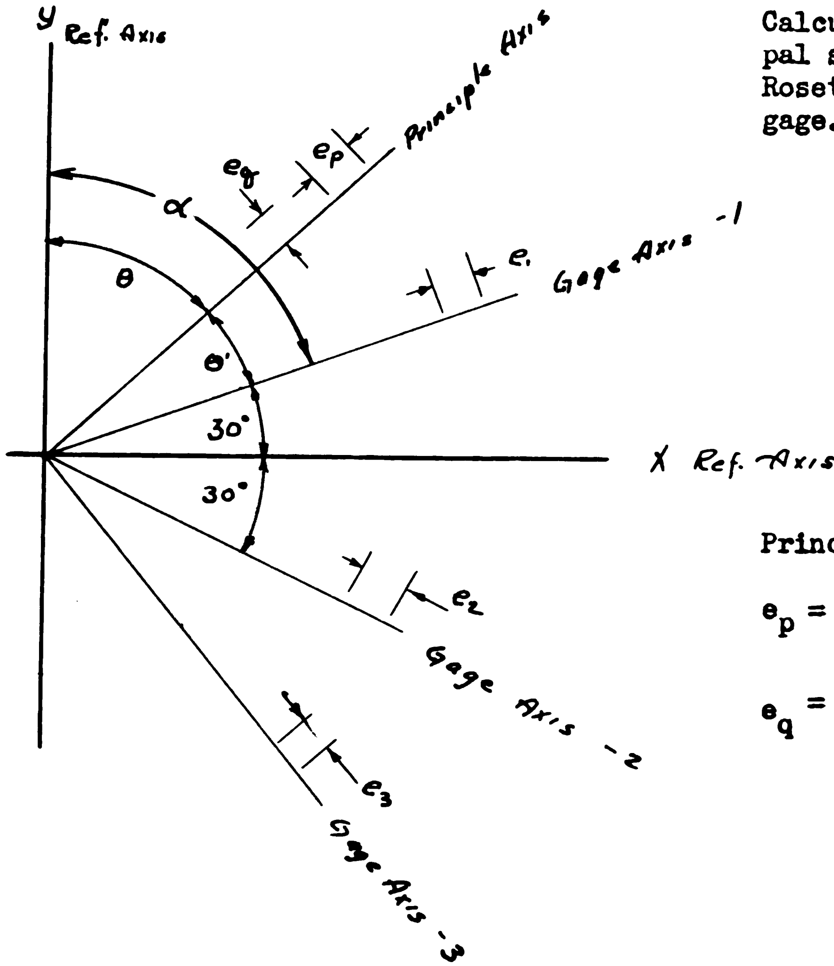
This is a 13.0% difference between the calculated stress and that obtained from the test data.

Computations of the Stresses in the Chords

Load lbs.-V	x span-in	$\frac{x}{h}$	$F = V\frac{x}{h}$ lbs.	$\frac{F}{A} =$ psi	Location gage
2000	86	3.16	6320	1580	#25 & #29
6000	86	3.16	18960	4500	
10000	86	3.16	31600	7900	
2000	54	1.97	3940	985	#42 & #43
6000	54	1.97	11820	2950	
10000	54	1.97	19700	4920	
2000	17.5	.656	1312	325	#18 & #22
4000	17.5	.656	2624	655	
6000	17.5	.656	3936	984	

$A = (.625 \times 5 \quad 2 \times 2 \times 2 \times .09 \quad 2 \times .081) = 4.017 \text{ in}^2$
area of chord including flanges and web portion

$h = 26.7$ in.



Calculations of the principal strains for the 60° Rosette electrical strain gage.

Principal strains:

$e_p = e + \Delta \qquad \theta = \alpha - \theta'$

$e_q = e - \Delta$

e_p = strain corresponding to S_p

e_q = strain corresponding to S_q

Where S_p and S_q are the principal stresses

$$e = \frac{e_p \quad e_q}{2} = \frac{e_1 \quad e_2 \quad e_3}{3} \; ; \quad \Delta = \frac{e_p - e_q}{2} = \frac{e_1 - e}{\cos 2\theta'}$$

$$\tan 2\theta' = \frac{\sqrt{3}}{1 - 2\left[\frac{e_1 - e_2}{e_2 - e_3}\right]}$$

Knowing the principal strains from gages solve for the principal stresses

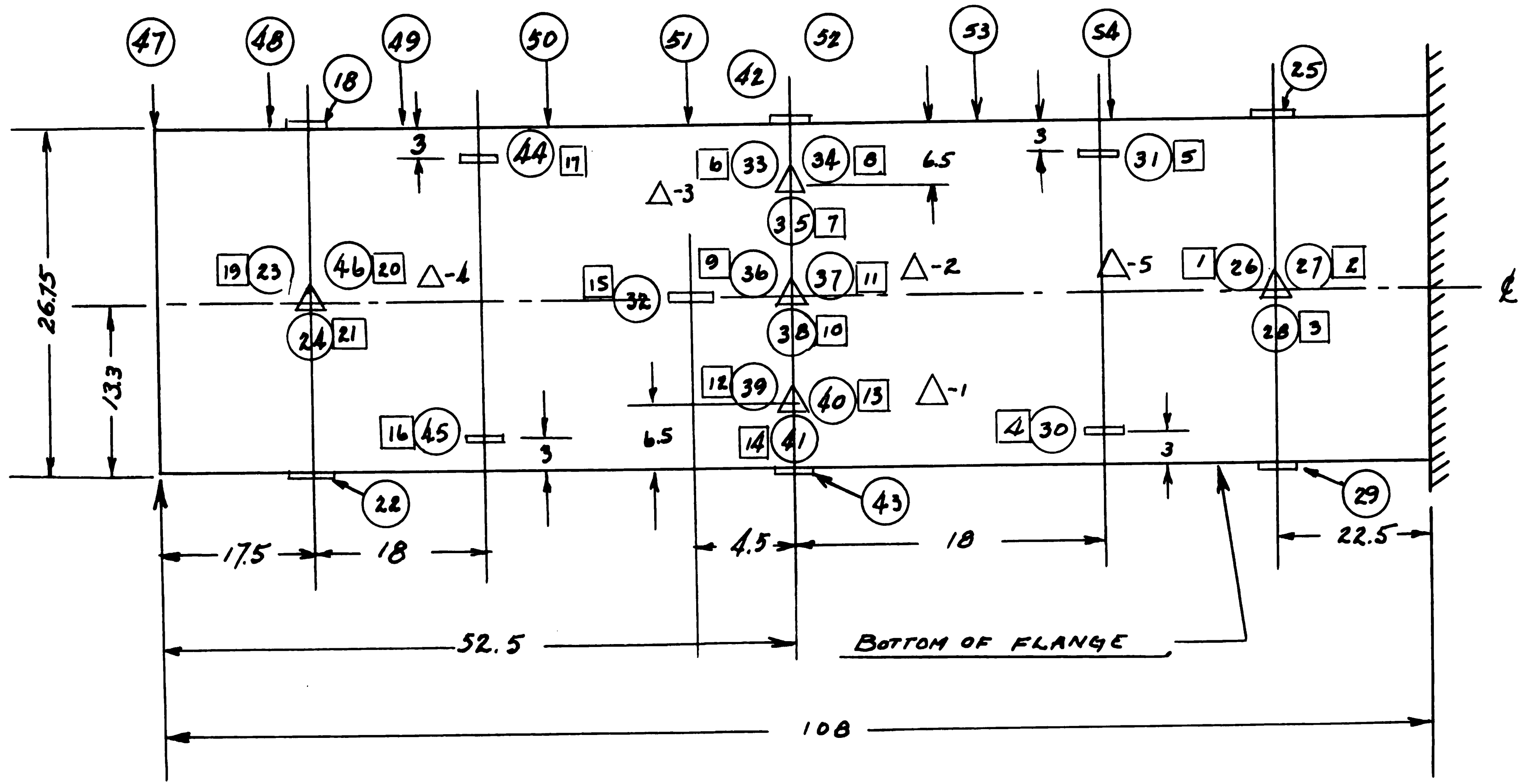
$$\sigma_p = \frac{E}{1 - u^2}(e_p + u e_q), \qquad \sigma_q = \frac{E}{1 - u^2}(e_q + u e_p)$$

u = Poisson's Ratio

E = Modulus of Elasticity

$$\text{Maximum shear} = \frac{\sigma_p - \sigma_q}{2}$$

See numerical computation following tables of average strains.



Indicates concave side of web bead.

Indicates beveled side of web bead.

Figure 2. Beaded Web Spar.

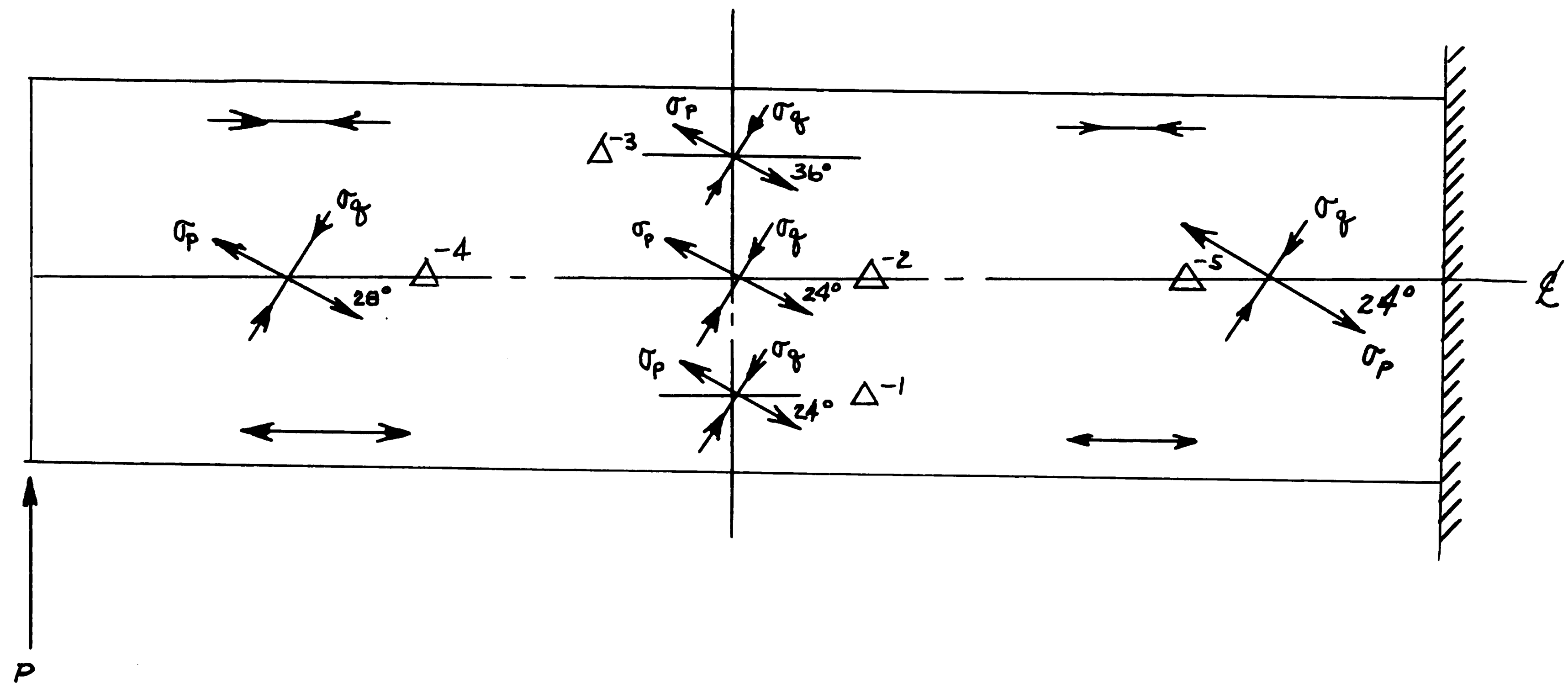
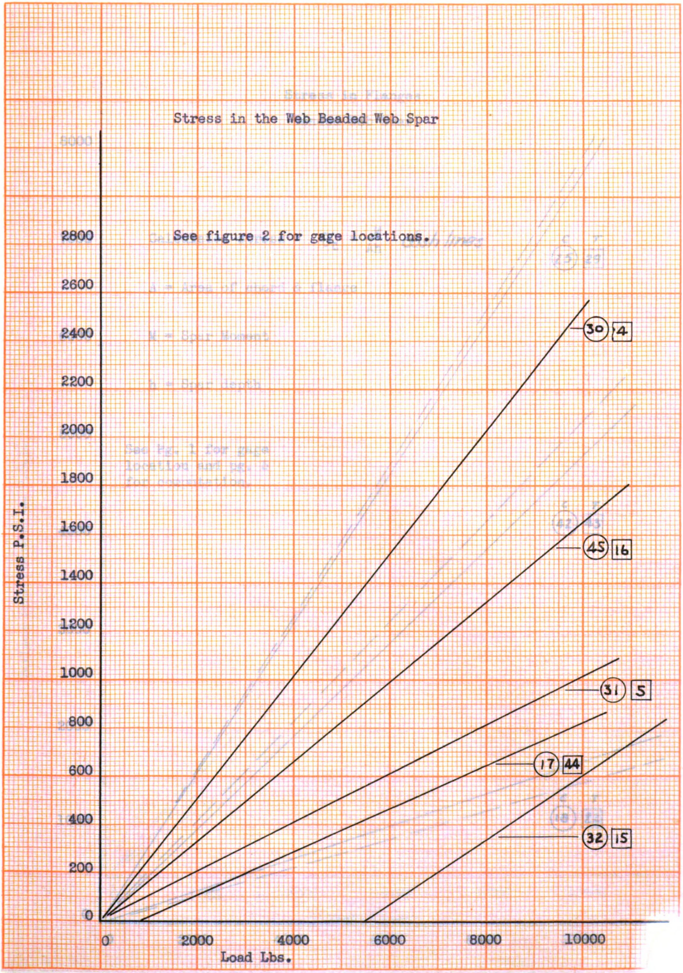


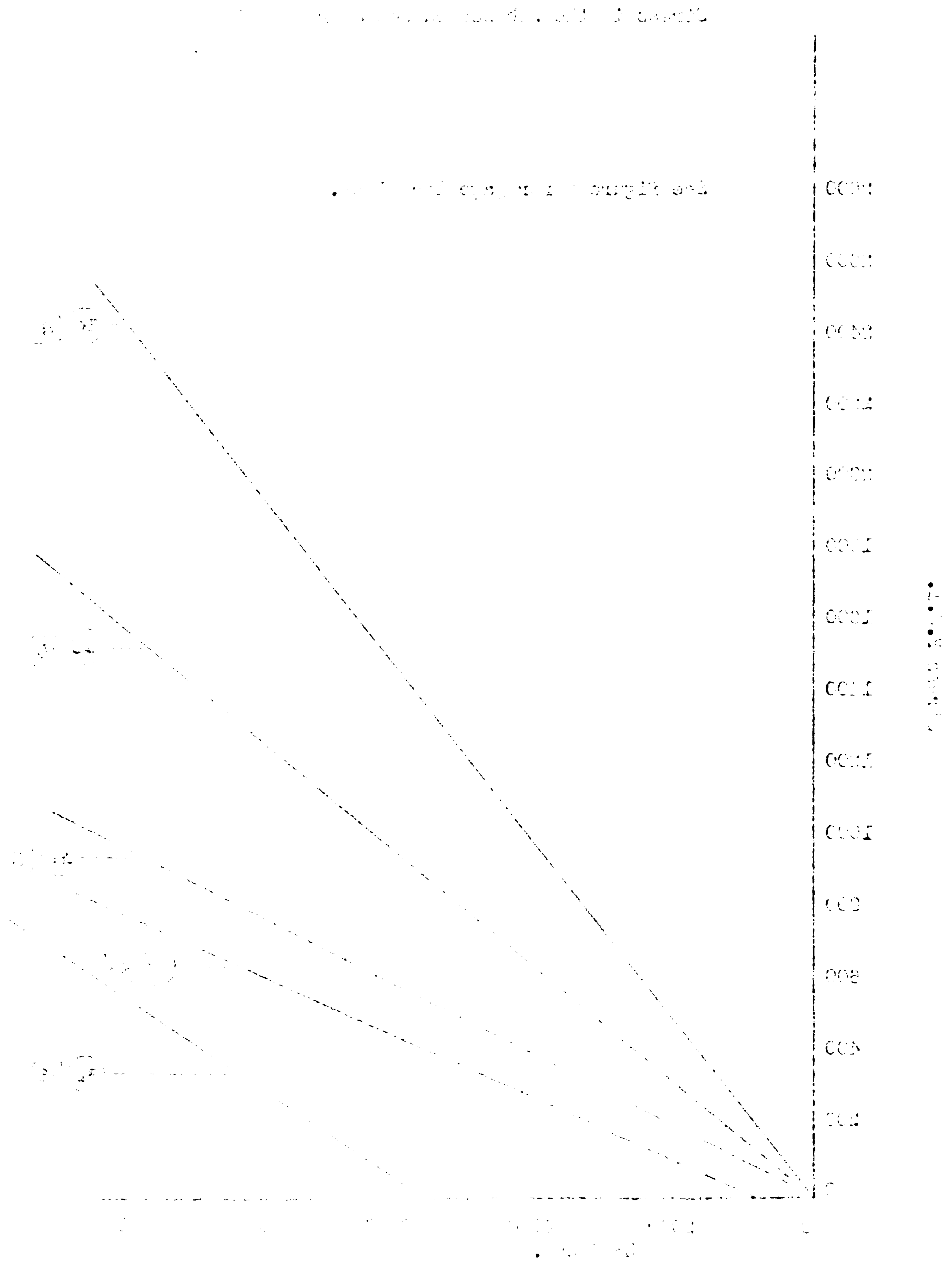
Figure 3. Stresses in Spar and Direction of Principle Stresses.

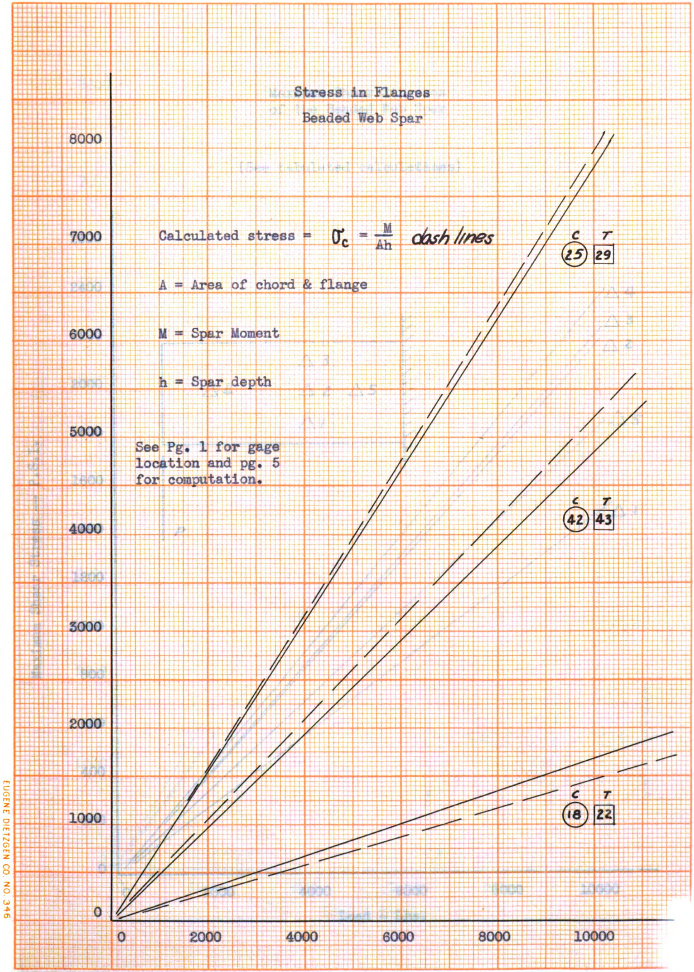
Compression

Tension

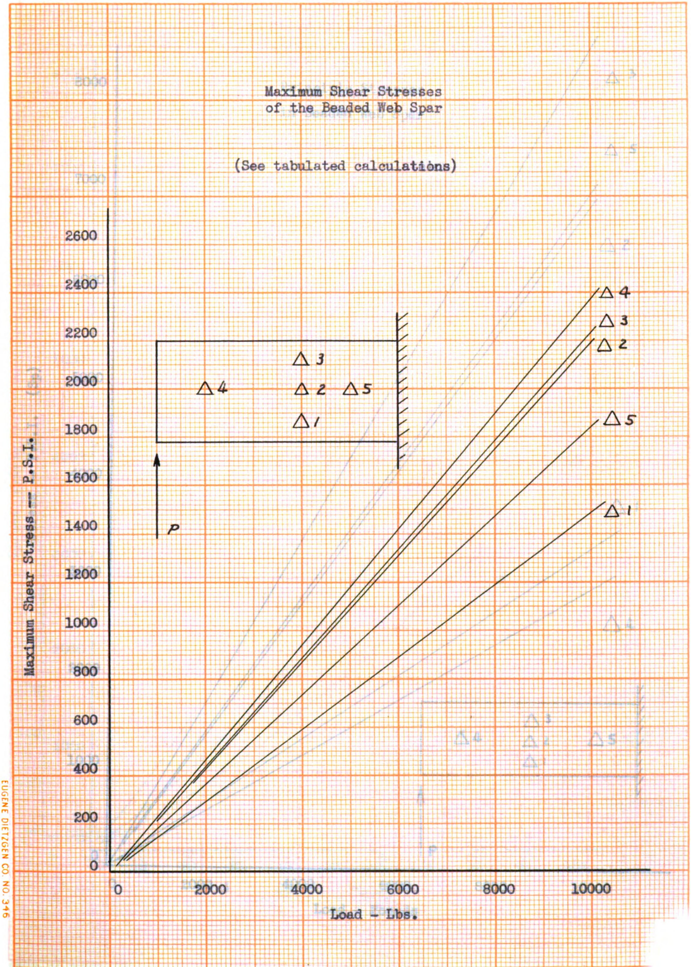
Principle stresses σ_p & σ_q

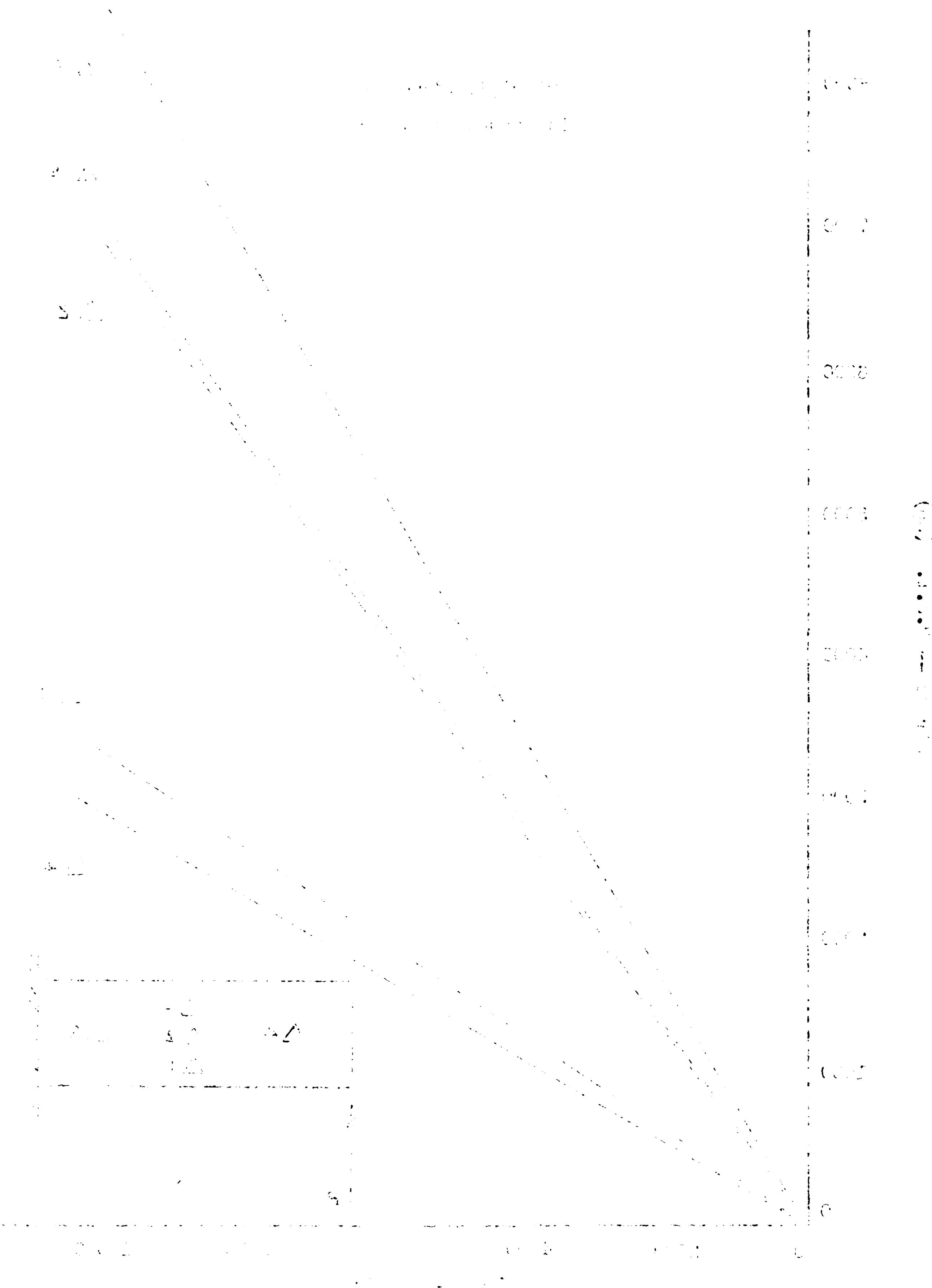

Stress in the Web Beaded Web Spar
See figure 2 for gage locations.
Stress P.S.I.
2800
2600
2400
2200
2000
1800
1600
1400
1200
1000
800
600
400
200
0
0
2000
4000
6000
8000
10000
Load Lbs.
30 4
45 16
31 5
17 44
32 15
EUGENE DIETZGEN CO. NO. 346

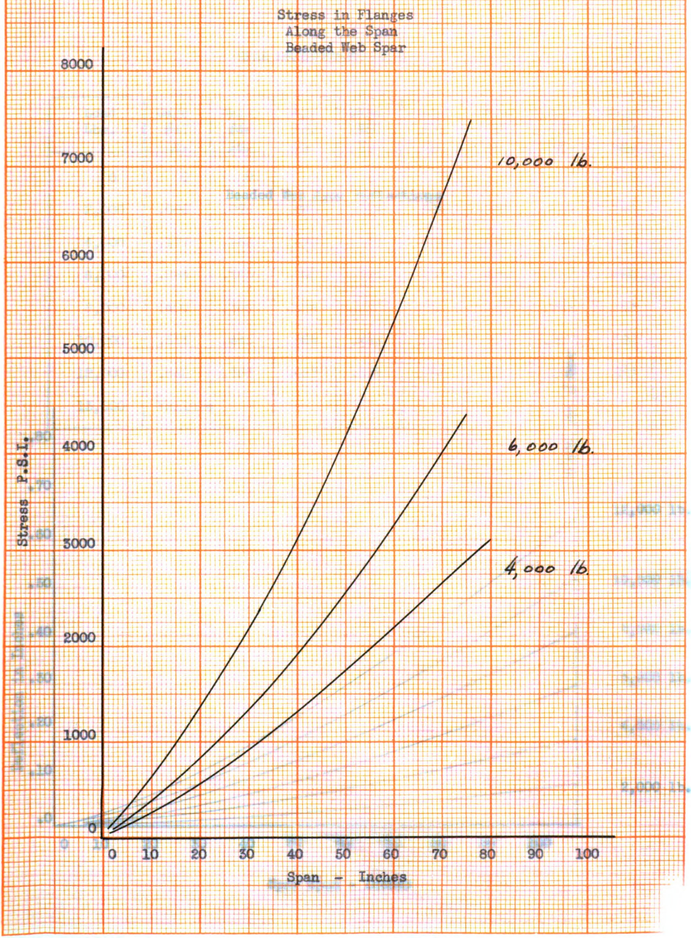


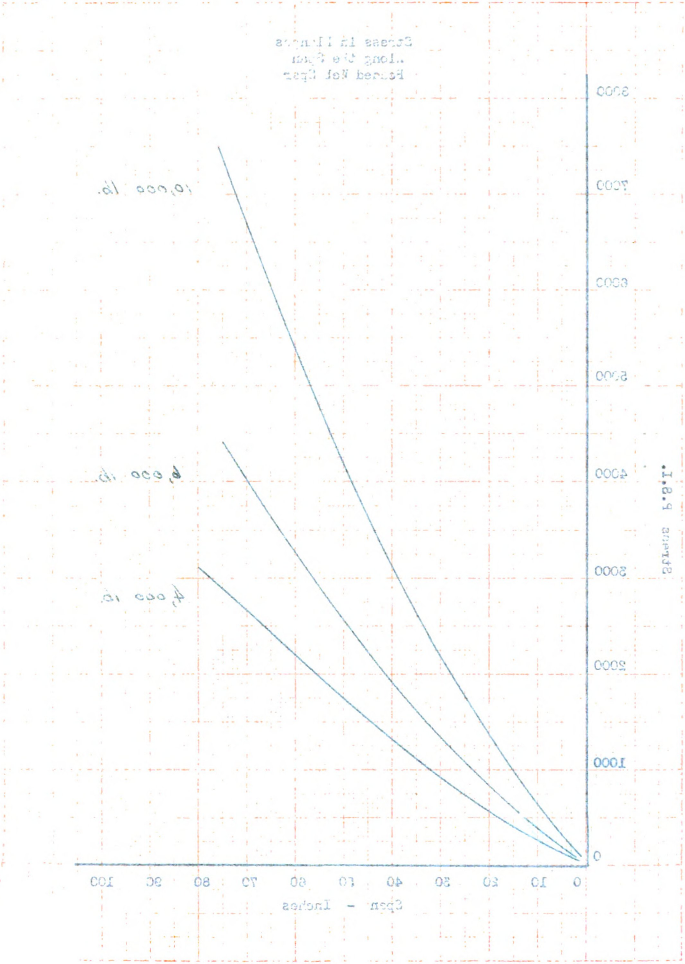


Stress in Flanges
Beaded Web Spar
Calculated stress = $\sigma_c = \frac{M}{Ah}$ dash lines
A = Area of chord & flange
M = Spar Moment
h = Spar depth
See Pg. 1 for gage location and pg. 5 for computation.
C T
25 29
42 43
18 22
8000
7000
6000
5000
4000
3000
2000
1000
0
0
2000
4000
6000
8000
10000
EUGENE DIETZGEN CO. NO. 346


Maximum Shear Stresses
of the Beaded Web Spar
(See tabulated calculations)
2600
2400
2200
2000
1800
1600
1400
1200
1000
800
600
400
200
0
Maximum Shear Stress -- P.S.I.
0
2000
4000
6000
8000
10000
Load - Lbs.
△ 3
△ 4
△ 2
△ 5
△ 1
P
△ 4
△ 3
△ 2
△ 5
△ 1
EUGENE DIETZGEN CO. NO. 346

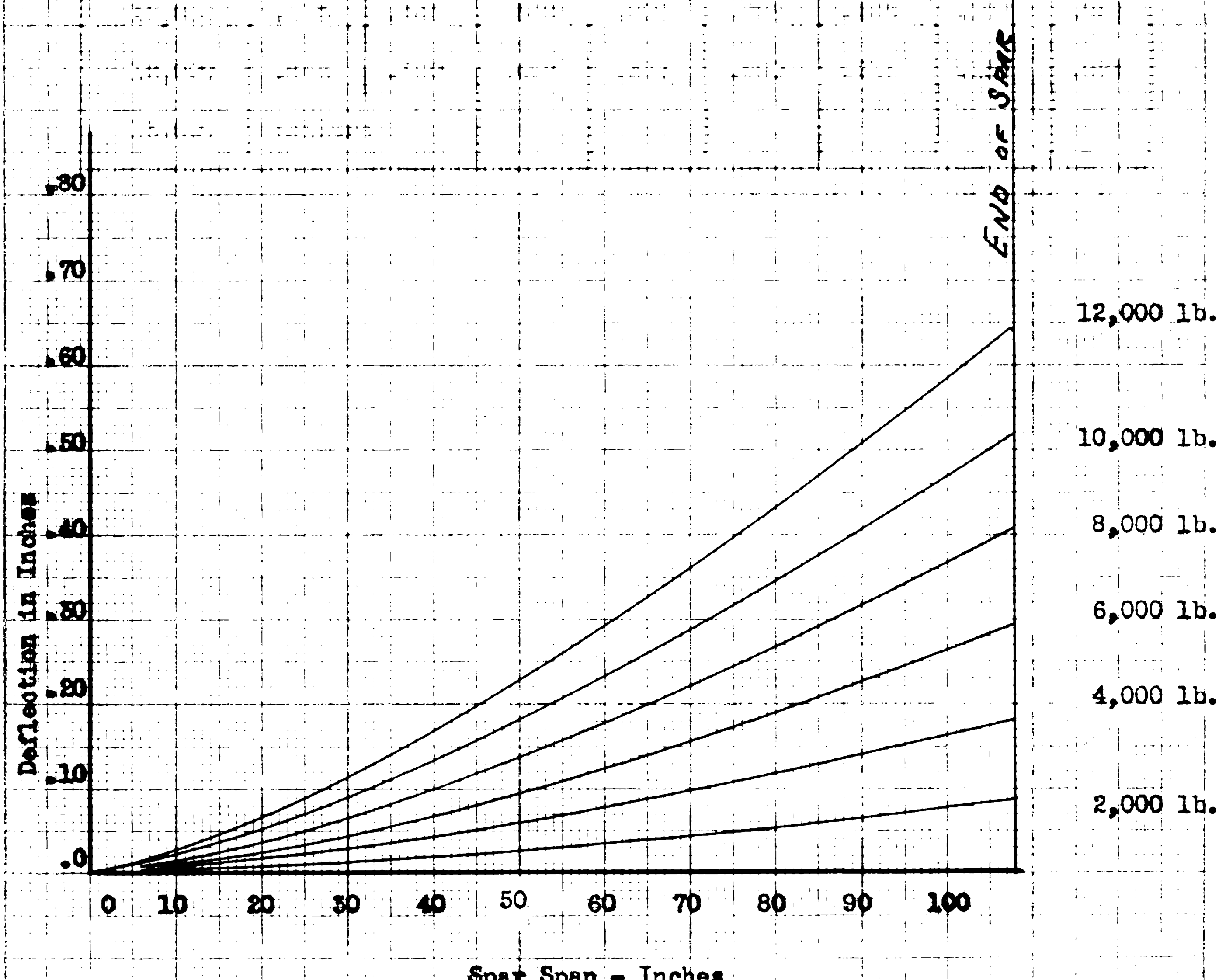


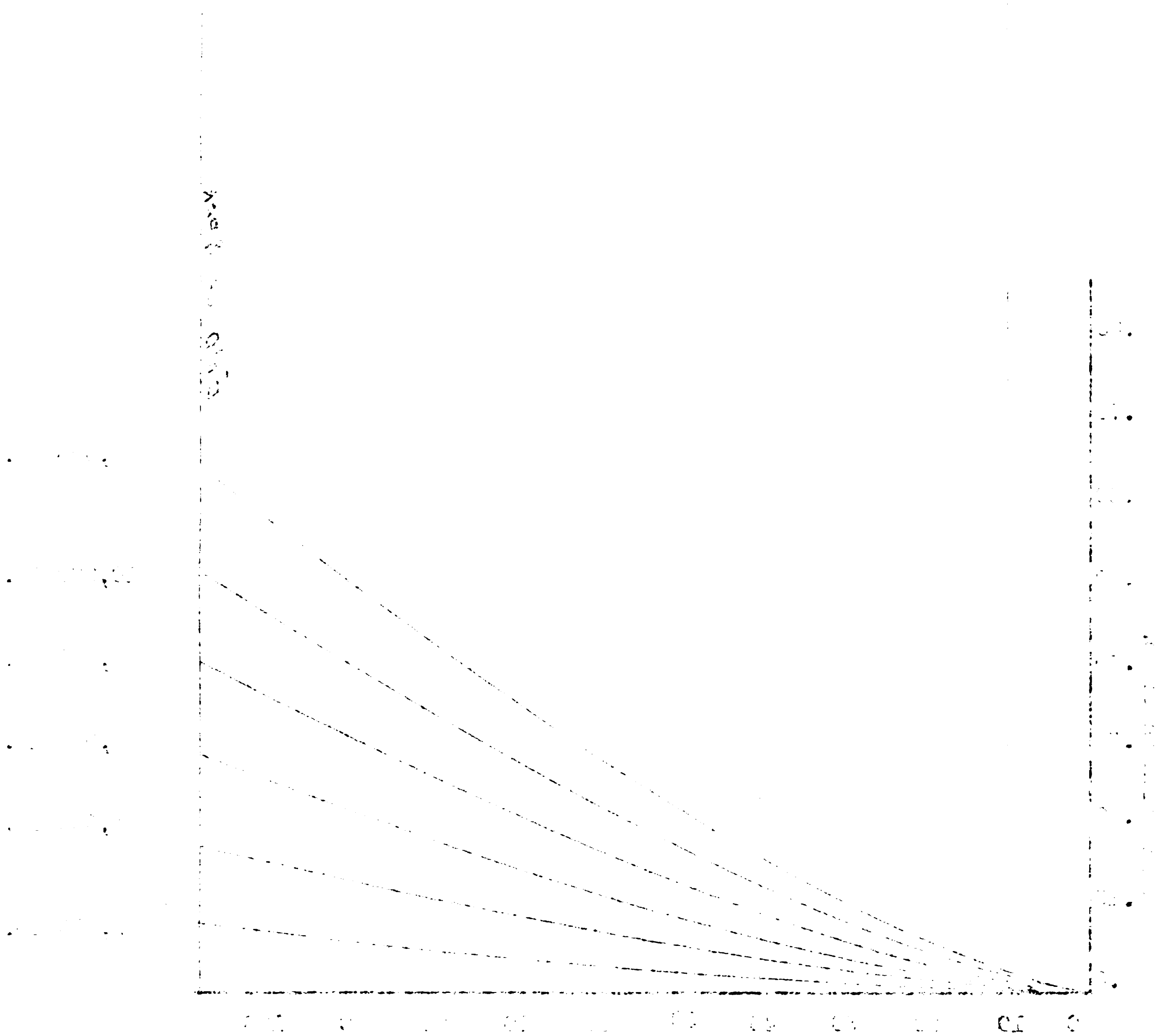

Stress in Flanges
Along the Span
Beaded Web Spar
8000
7000
6000
5000
4000
3000
2000
1000
0
Stress P.S.I.
10,000 lb.
6,000 lb.
4,000 lb.
0
10
20
30
40
50
60
70
80
90
100
Span - Inches



Stress in Flanges
Along the Span
Formed Web Spar
10,000 lb.
6,000 lb.
4,000 lb.
Stress P.S.I.
0
1000
2000
3000
4000
5000
6000
7000
8000
Span - Inches
0
10
20
30
40
50
60
70
80
90
100

Beaded Web Spar Deflections





SPAR DEFLECTION TABLE

Load Lbs.	Gage #47 In.	Gage #48 In.	Gage #49 In.	Gage #50 In.	Gage #51 In.	Gage #52 In.	Gage #53 In.	Gage #54 In.
0	0	0	0	0	0	0	0	0
2,000	.086	.075	.060	.049	.036	.026	.016	.009
4,000	.177	.153	.125	.100	.076	.054	.034	.018
6,000	.284	.245	.202	.162	.124	.089	.057	.031
8,000	.400	.386	.278	.230	.126	.127	.082	.048
10,000	.513	.445	.369	.299	.228	.166	.107	.059
12,000	.642	.560	.467	.378	.290	.211	.134	.074
15,600	Failure							

STRAIN GAGE TEST DATA

Load Lbs.	Gage #1	Gage #2	Gage #3	Gage #4
2,000	90	- 80	15	40
4,000	175	-160	30	85
6,000	250	-250	70	145
8,000	360	-340	120	210
10,000	470	-440	180	280
12,000	520	-520	320	1100
	Gage #5	Gage #6	Gage #7	Gage #8
2,000	- 10	135	- 20	- 140
4,000	- 10	285	- 60	- 280
6,000	- 0	440	- 95	- 440
8,000	30	600	- 165	- 620
10,000	55	790	- 210	- 785
	Gage #9	Gage #10	Gage #11	Gage #12
2,000	110	20	- 80	100
4,000	210	40	- 180	180
6,000	320	55	- 260	260
8,000	420	85	- 360	330
10,000	520	345	- 410	380

tension

- compression

all gage readings measured in micro-inches per inch.

STRAIN GAGE TEST DATA

Load Lbs.	*Gage #13	Gage #14	Gage #15	Gage #16
2,000	- 65	+ 60	0	0
4,000	- 140	+ 100	0	+ 5
6,000	- 200	+ 150	0	+ 20
8,000	- 270	+ 220	- 10	+ 40
10,000	- 320	+ 300	- 40	+ 60
	Gage #17	Gage #18	Gage #19	Gage #20
2,000	0	- 10	+ 90	- 85
4,000	0	- 30	+190	-190
6,000	+ 20	- 50	+280	-280
8,000	+ 20	- 75	+390	-370
10,000	+ 38	- 100	+489	-465
	Gage #21	Gage #22	Gage #23	Gage #24
2,000	+ 20	+ 30	- 100	0
4,000	+ 30	+ 60	- 200	0
6,000	+ 45	+ 80	- 335	- 5
8,000	+ 76	+ 110	- 460	- 20
10,000	+ 115	+ 140	- 600	- 45

*All gage readings measured in micro-inches per inch.

STRAIN GAGE TEST DATA

Load Lbs.	*Gage #25	Gage #26	Gage #27	Gage #28
2,000	- 130	- 90	+ 86	- 5
4,000	- 250	- 200	+ 140	- 20
6,000	- 400	- 320	+ 250	- 45
8,000	- 520	- 440	+ 330	- 85
10,000	- 680	- 590	+ 430	- 145
	Gage #29	Gage #30	Gage #31	Gage #32
2,000	+ 140	- 45	- 30	0
4,000	+ 250	- 90	- 60	0
6,000	+ 400	- 140	- 85	- 20
8,000	+ 530	- 170	- 120	- 40
10,000	+ 670	- 180	- 130	- 80
	Gage #33	Gage #34	Gage #35	Gage #36
2,000	- 60	+ 100	+ 20	- 85
4,000	- 120	+ 180	+ 40	- 180
6,000	- 165	+ 280	+ 60	- 280
8,000	- 235	+ 360	+ 90	- 360
10,000	- 240	+ 470	+ 125	- 480

*All gage readings measured in micro-inches per inch.

STRAIN GAGE TEST DATA

Load Lbs.	*Gage #37	Gage #38	Gage #39	Gage #40
2,000	+ 80	0	- 110	+ 60
4,000	+ 160	0	- 220	+ 120
6,000	+ 250	0	- 350	+ 180
8,000	+ 340	- 5	- 480	+ 235
10,000	+ 430	- 20	- 680	+ 270
	Gage #41	Gage #42	Gage #43	Gage #44
2,000	- 30	- 50	+ 80	- 20
4,000	- 50	- 130	+ 165	- 45
6,000	- 80	- 200	+ 250	- 60
8,000	- 100	- 300	+ 340	- 105
10,000	- 140	- 415	+ 420	- 110
	Gage #45	Gage #46		
2,000	+ 60	+ 100		
4,000	+ 115	+ 190		
6,000	+ 160	+ 290		
8,000	+ 210	+ 390		
10,000	+ 240	+ 485		

COMPUTATIONS OF HORIZONTAL STRESSES

Load Lbs.	*Gage #17	Gage #44	$\frac{e_{17} + e_{14}}{2}$	11.2×10^6 x Strain Stress P.s.i.	Gage #15	Gage #32	$\frac{e_{15} + e_{32}}{2}$	11.2×10^6 x Strain Stress P.s.i.
2,000	0	- 20	10	114.0	0	0	0	0
4,000	0	- 45	22.5	250	0	0	0	0
6,000	+ 20	- 60	40	445	0	- 20	10	114
8,000	+ 20	- 100	60	668	- 15	- 40	27.5	306
10,000	+ 38	- 110	74	824	- 40	- 80	60	667
	Gage #16	Gage #45	$\frac{e_{16} + e_{45}}{2}$		Gage #18	Gage #22	$\frac{e_{18} + e_{22}}{2}$	
2,000	0	+ 60	30	334	- 10	+ 30	20	228
4,000	+ 5	+ 115	60	667	- 30	+ 60	45	500
6,000	+ 20	+ 160	90	1000	- 50	+ 80	65	724
8,000	+ 40	+ 210	125	1390	- 75	+ 110	92.5	1030
10,000	+ 60	+ 240	150	1670	- 100	+ 140	120	1330
	Gage #5	Gage #31	$\frac{e_{5} + e_{31}}{2}$		Gage #42	Gage #43	$\frac{e_{42} + e_{43}}{2}$	
2,000	- 10	- 30	20	228	- 50	+ 80	65	724
4,000	- 10	- 60	35	390	- 130	+ 165	147	1640
6,000	0	- 85	42.5	473	- 200	+ 250	225	2500
8,000	- 30	- 120	75	835	- 300	+ 340	320	3560
10,000	- 55	- 130	92.5	1030	- 415	+ 420	417.5	4650
	Gage #30	Gage #4	$\frac{e_{30} + e_{4}}{2}$		Gage #25	Gage #29	$\frac{e_{25} + e_{29}}{2}$	
2,000	- 45	+ 40	42.5	473	- 130	+ 140	135	1500
4,000	- 90	+ 85	87.5	975	- 250	+ 260	255	2500
6,000	- 140	+ 145	142.5	1580	- 400	+ 400	400	4460
8,000	- 170	+ 210	190	2130	- 520	+ 530	525	5850
10,000	- 180	+ 280	230	2560	- 680	+ 670	675	7500

*All gage readings in micro-inches per inch.

TABLES OF AVERAGE STRAINS

Load Lbs.	*Gage #9	Gage #36	$\frac{e_9 + e_{36}}{2} = e_2$	$e_1 - e_2$	$e_2 - e_3$	$\frac{e_1 - e_2}{e_2 - e_3}$
2,000	+ 110	- 85	97.5	- 17.5	+ 87.5	- .20
4,000	+ 210	- 180	195	- 25	+ 175	- .1428
6,000	+ 320	- 280	300	- 45	+ 272.5	- .165
8,000	+ 420	- 360	390	- 40	+ 345	- .1159
10,000	+ 520	- 480	500	- 80	+ 317.5	- .253

	Gage #10	Gage #38	$\frac{e_{10} + e_{38}}{2} = e_3$
2,000	+ 20	0	10
4,000	+ 40	0	20
6,000	+ 55	0	27.5
8,000	+ 85	- 5	45
10,000	+ 345	- 20	182.5

	Gage #11	Gage #37	$\frac{e_{11} + e_{37}}{2} = e_1$
2,000	- 60	+ 80	80
4,000	- 180	+ 160	170
6,000	- 260	+ 250	255
8,000	- 380	+ 340	350
10,000	- 410	+ 430	420

*All gage readings in micro-inches per inch

TABLES OF AVERAGE STRAINS

Load Lbs.	*Gage #6	Gage #33	$\frac{e_6 + e_{33}}{2} = e_1$	Gage #8	Gage #34	$\frac{e_8 + e_{34}}{2} = e_1$
2,000	+ 135	- 60	97.5	- 140	+ 100	120
4,000	+ 285	- 120	202.5	- 280	+ 180	230
6,000	+ 440	- 165	302.5	- 440	+ 230	380
8,000	+ 600	- 235	417.5	- 620	+ 360	490
10,000	+ 720	- 240	515	- 785	+ 470	627
	Gage #7	Gage #35	$\frac{e_7 + e_{35}}{2} = e_2$	Gage #6	Gage #33	$\frac{e_6 + e_{33}}{2} = e_2$
2,000	- 20	+ 20	20	+ 135	- 60	97
4,000	- 60	+ 40	50	+ 285	- 120	202
6,000	- 95	+ 60	77.5	+ 440	- 165	302
8,000	- 165	+ 90	127.5	+ 600	- 235	417
10,000	- 210	+ 125	167.5	+ 790	- 240	515
	Gage #8	Gage #34	$\frac{e_8 + e_{34}}{2} = e_3$	Gage #7	Gage #35	$\frac{e_7 + e_{35}}{2} = e_3$
2,000	- 140	+ 100	120	- 20	+ 20	20
4,000	- 280	+ 180	230	- 60	+ 40	50
6,000	- 440	+ 280	360	- 95	+ 60	77
8,000	- 620	+ 360	490	- 165	+ 90	127
10,000	- 785	+ 470	627	- 210	+ 125	167

*All gage readings in micro-inches per inch.

TABLES OF AVERAGE STRAINS

Load Lbs.	*Gage #12	Gage #39	$\frac{e_{12} + e_{39}}{2} = e_2$	Gage #1	Gage #26	$\frac{e_1 + e_{26}}{2} = e_2$
2,000	+ 100	- 100	105	+ 90	- 90	90
4,000	+ 180	- 220	200	+ 175	- 200	187.5
6,000	+ 260	- 350	305	+ 280	- 320	300
8,000	+ 330	- 480	405	+ 360	- 440	400
10,000	+ 380	- 680	530	+ 470	- 590	530
	Gage #13	Gage #40	$\frac{e_{13} + e_{40}}{2} = e_1$	Gage #2	Gage #27	$\frac{e_2 + e_{27}}{2} = e_1$
2,000	- 65	+ 60	62.5	- 80	+ 86	83
4,000	- 140	+ 120	130	- 160	+ 140	150
6,000	- 200	+ 180	160	- 250	+ 250	250
8,000	- 270	+ 235	252.5	- 340	+ 330	335
10,000	- 320	+ 270	295	- 440	+ 430	435
	Gage #14	Gage #41	$\frac{e_{14} + e_{41}}{2} = e_3$	Gage #3	Gage #28	$\frac{e_3 + e_{28}}{2} = e_3$
2,000	+ 60	- 30	45	+ 15	- 5	10
4,000	+ 100	- 50	75	+ 30	- 20	25
6,000	+ 150	- 80	125	+ 70	- 45	57.5
8,000	+ 220	- 100	160	+ 120	- 85	102.5
10,000	+ 300	- 140	220	+ 180	- 145	162.5

*All gage readings in micro-inches per inch.

TABLES OF AVERAGE STRAINS

Load Lbs.	*Gage #19	Gage #33	$\frac{e_{19} + e_{23}}{2} = e_2$	Gage #21	Gage #24	$\frac{e_{21} + e_{24}}{2} = e_3$
2,000	+ 90	- 100	95	+ 20	0	10
4,000	+ 190	- 200	195	+ 30	0	15
6,000	+ 280	- 335	307.5	+ 45	0	22.5
8,000	+ 390	- 460	425	+ 76	- 5	40
10,000	+ 480	- 600	540	+ 115	- 45	80
	Gage #20	Gage #46	$\frac{e_{20} + e_{45}}{2} = e_1$			
2,000	- 85	+ 100	92.5			
4,000	- 190	+ 190	190			
6,000	- 280	+ 290	285			
8,000	- 370	+ 340	355			
10,000	- 465	+ 485	475			

*All gage readings measured in micro-inches per inch.

CALCULATION OF THE PRINCIPLE STRESSES

(1) Load Lbs.	(2) $e_1 - e_2$ Strain Micro-In. per In.	(3) $e_2 - e_3$ Strain Micro-In. per In.	(4) $2\frac{e_1 - e_2}{e_2 - e_3}$	(5) $\frac{3}{1 + 2\frac{(e_1 - e_2)}{(e_2 - e_3)}}$ Tan 2θ'	(6) 2θ' Degrees	(7) $e = \frac{e_1 + e_2 + e_3}{3}$ Strain Micro-In. per In.	(8) Cos 2θ'	(9) $e_1 - e$ Strain Micro-In. per In.	(10) $\frac{e_1 - e}{\text{Cos } 2\theta'} =$ Strain Micro-In. per In.
2,000	- 7.0	80	- .112	1.95	62.8	61.0	.456	22	48.1
4,000	- 37.5	162.5	- .456	3.17	72.5	120.8	.30	29.2	97.4
6,000	- 50	242.5	- .412	2.93	71.15	202.5	.323	47.5	148.0
8,000	- 65	297.6	- .440	3.07	71.95	279.2	.309	55.8	180.0
10,000	- 95	367.5	- .516	3.57	74.35	375.8	.269	59.2	224.0

Load Lbs.	(11) e_p Strain Micro-In. per In.	(12) e_q Strain Micro-In. per In.	(13) $e_p + e_q$ Prin. Strain Micro-In. per In.	(14) q $11.2 \times 10^6 \times 13$ Prin. Stress P.s.i.	(15) $e_q + e_p$ Prin. Strain Micro-In. per In.	*(16) q $11.2 \times 10^6 \times 15$ Prin. Stress P.s.i.	(17) p - q P.s.i.	(18) $\frac{p - q}{2}$ Max. Shear Stress P.s.i.
2,000	109.1	12.9	112.9	1310	49	573	737	368
4,000	218.2	23.4	225.1	2640	95.4	1120	1520	760
6,000	350.5	34.5	366.7	4300	170	2000	2300	1150
8,000	459.0	99.2	491.7	5750	249	2920	2830	1415
10,000	599.8	131.8	643.1	7500	329.8	3860	3640	1820

e_1 = #2 and #27 Gages

e_2 = #1 and #26 Gages .33 $e_p = e + \Delta$ $E = 10.3 \times 10^6$

e_3 = #3 and #28 Gages $e_q = e - \Delta$

*Note: All strain gage readings multiplied by 1.09 to find actual strain. This is to compensate for the error in the gage factor control.

CALCULATION OF THE PRINCIPLE STRESSES

(1) Load Lbs.	(2) $e_1 - e_2$ Strain Micro-In. per In.	(3) $e_2 - e_3$ Strain Micro-In. per In.	(4) $2\frac{e_1 - e_2}{e_2 - e_3}$	(5) $\frac{3}{1 + 2\frac{(e_1 - e_2)}{(e_2 - e_3)}}$ Tan 2θ'	(6) 2θ' Degrees	(7) $e = \frac{e_1 + e_2 + e_3}{3}$ Strain Micro-In. per In.	(8) Cos 2θ'	(9) $e_1 - e$ Strain Micro-In. per In.	(10) $\frac{e_1 - e}{\text{Cos } 2\theta'}$ = Strain Micro-In. per In.
2,000	- 2.5	85	- .059	+ 1.83	61.35	65.8	.479	26.7	54.5
4,000	- 5.0	180	- .0555	1.84	61.5	133.0	.477	57	119.0
6,000	- 22.5	285	- .158	2.05	64	205	.438	80	183.0
8,000	- 70.0	385	- .365	2.70	69.6	273	.347	82	237.0
10,000	- 65.0	460	- .282	2.85	70.3	365	.3305	110	334.0

Load Lbs.	(11) e_p Strain Micro-In. per In.	(12) e_q Strain Micro-In. per In.	(13) $e_p + e_q$ Prin. Strain Micro-In. per In.	(14) p $11.2 \times 10^6 \times 13$ Prin. Stress P.s.i.	(15) $e_q + e_p$ Prin. Strain Micro-In. per In.	(16) q $11.2 \times 10^6 \times 15$ Prin. Stress P.s.i.	(17) p - q P.s.i.	(18) $\frac{p - q}{2}$ Max. Shear Stress P.s.i.
2,000	120.3	11.3	124.0	1450	50	586	854	427
4,000	252	14.0	256.6	3000	96	1130	1870	935
6,000	388	22.0	395.2	4640	149	1750	2990	1495
8,000	510	36.0	521.9	6140	204	2380	3760	1880
10,000	699	31.0	709	8320	261	3060	5260	2630

e_1 = #19 and #23 Gages

e_2 = #46 and #20 Gages = .33 $e_p = e + \Delta$

e_3 = #24 and #21 Gages $e_q = e - \Delta$

CALCULATION OF THE PRINCIPLE STRESSES

(1) Load Lbs.	(2) $e_1 - e_2$ Strain Micro-In. per In.	(3) $e_2 - e_3$ Strain Micro-In. per In.	(4) $2\frac{e_1 - e_2}{e_2 - e_3}$	(5) $\frac{3}{1 + 2\frac{(e_1 - e_2)}{(e_2 - e_3)}}$ Tan 2θ'	(6) 2θ' Degrees	(7) $e = \frac{e_1 + e_2 + e_3}{3}$ Strain Micro-In. per In.	(8) Cos 2θ'	(9) $e_1 - e$ Strain Micro-In. per In.	(10) $\frac{e_1 - e}{\text{Cos } 2\theta'} =$ Strain Micro-In.
2,000	- 42.5	+ 65	- 1.31	- 5.6	79.85	70.8	.176	- 8.3	- 47.0
4,000	- 70	75	- 1.87	- 1.98	63.25	135	.450	- 5	- 111.0
6,000	- 145	180	- 1.60	- 2.88	70.9	196.6	.328	- 36.6	- 113
8,000	- 152.5	245	- 1.24	- 7.2	82.1	272.5	.137	- 20	- 161
10,000	- 235	310	- 1.52	- 3.34	73.35	348.3	.286	- 53.3	- 187.0

Load Lbs.	(11) e_p Strain Micro-In. per In.	(12) e_q Strain Micro-In. per In.	(13) $e_p + e_q$ Prin. Strain Micro-In. per In.	(14) p $11.2 \times 10^6 \times 13$ Prin. Stress P.s.i.	(15) $e_q + e_p$ Prin. Strain Micro-In. per In.	(16) q $11.2 \times 10^6 \times 15$ Prin. Stress P.s.i.	(17) p - q P.s.i.	(18) $\frac{p - q}{2}$ Max. Shear Stress P.s.i.
2,000	23.8	+ 68	43.3	489	75.8	880	- 397	- 198.5
4,000	24.0	+ 246.0	205.0	2310	287.0	3230	- 860	- 430
6,000	83.6	309.6	183.6	2140	337.1	3950	- 1810	- 905
8,000	111.5	433.5	254.5	2970	470.4	5500	- 2530	- 1265
10,000	171.3	535.3	347.8	3450	591.7	6900	- 3450	- 1725

e_1 = #13 and #40 Gages

e_2 = #12 and #39 Gages = .33 $e_p = e + \Delta$

e_3 = #14 and #41 Gages $e_q \neq e - \Delta$

CALCULATION OF THE PRINCIPLE STRESSES

(1) Load Lbs.	(2) $e_1 - e_2$ Strain Micro-In. per In.	(3) $e_2 - e_3$ Strain Micro-In. per In.	(4) $2\frac{e_1 - e_2}{e_2 - e_3}$	(5) $\frac{3}{1 + 2\frac{(e_1 - e_2)}{(e_2 - e_3)}}$ Tan 2θ'	(6) 2θ' Degrees	(7) $e = \frac{e_1 + e_2 + e_3}{3}$ Strain Micro-In. per In.	(8) Cos 2θ'	(9) $e_1 - e$ Strain Micro-In. per In.	(10) $\frac{e_1 - e}{\text{Cos } 2\theta'} =$ Strain Micro-In. per In.
2,000	- 17.5	+ 87.5	- .20	2.9	35.	62.7	.326	17.3	53.0
4,000	- 25	175.	- .1428	2.41	33.7	128	.382	58.	110.0
6,000	- 45	272.5	- .165	2.60	34.5	194	.358	61	170.0
8,000	- 46	345.	- .1159	2.25	33.	261	.407	89	218.0
10,000	- 80	317.5	- .253	2.3	33.	366	.400	54	135.0

Load Lbs.	(11) e_p Strain Micro-In. per In.	(12) e_q Strain Micro-In. per In.	(13) $e_p + e_q$ Prin. Strain Micro-In. per In.	(14) p $11.2 \times 10^6 \times 13$ Prin. Stress P.s.i.	(15) $e_q + e_p$ Prin. Strain Micro-In. per In.	(16) q $11.2 \times 10^6 \times 15$ Prin. Stress P.s.i.	(17) p - q P. s. i.	(18) $\frac{p - q}{2}$ Max. Shear Stress P.s.i.
2,000	115	9.7	118	1380	13.5	158	1122	561
4,000	238	18	244	2850	97	1140	1710	855
6,000	364	24	371	4350	144	1670	2680	1340
8,000	479	43	493	5770	201	2350	3420	1710
10,000	501	231	577	6800	397	4660		

e_1 = #37 and #11 Gages

e_2 = #36 and # 9 Gages = .33 $e_p = e + \Delta$

e_3 = #38 and #10 Gages $e_q = e - \Delta$

CALCULATION OF THE PRINCIPLE STRESSES

(1) Load Lbs.	(2) $e_1 - e_2$ Strain Micro-In. per In.	(3) $e_2 - e_3$ Strain Micro-In. per In.	(4) $2\frac{e_1 - e_2}{e_2 - e_3}$	(5) $\frac{3}{1 + 2\frac{(e_1 - e_2)}{(e_2 - e_3)}}$ Tan 2θ'	(6) 2θ' Degrees	(7) $e = \frac{e_1 + e_2 + e_3}{3}$ Strain Micro-In. per In.	(8) Cos 2θ'	(9) $e_1 - e$ Strain Micro-In. per In.	(10) $\frac{e_1 - e}{\text{Cos } 2\theta'} =$ Strain Micro-In.
2,000	23	77	.597	1.08	47.2	79	.678	41	60.5
4,000	28	152	.368	1.27	51.8	160	.618	70	114.0
6,000	58	225	.515	1.14	48.75	246	.659	114	173.0
8,000	73	290	.505	1.15	49.	345	.656	145	223
10,000	112	348	.644	1.05	46.4	436	.689	191	276

Load Lbs.	(11) e_p Strain Micro-In. per In.	(12) e_q Strain Micro-In. per In.	(13) $e_p + e_q$ Prin. Strain Micro-In. per In.	(14) p $11.2 \times 10^6 \times 13$ Prin. Stress P.s.i.	(15) $e_q + e_p$ Prin. Strain Micro-In. per In.	(16) q $11.2 \times 10^6 \times 15$ Prin. Stress P.s.i.	(17) p − q P. s. i.	(18) $\frac{p - q}{2}$ Max. Shear Stress P.s.i.
2,000	169.5	18.5	175.6	2050	74	865	1185	592
4,000	274	46	289	3390	136.5	1600	1790	895
6,000	419	73	443	5200	211	2470	2730	1365
8,000	562	122	608	7100	309	3600	3500	1750
10,000	712	160	765	8950	395	4620	4330	2165

e_1 = #8 and #34 Gages

e_2 = #6 and #33 Gages = .33 $e_p = e + \Delta$

e_3 = #7 and #35 Gages $e_q = e - \Delta$

References Part I

1. Niles and Newell, "Airplane Structures", Wiley.
2. H. Wagner, "Structures of Thin Sheet Metal, Their Design and Construction", N.A.C.A. Tech. Memo. 490.
3. Sechler and Dunn, "Airplane Structural Analysis and Design", Wiley.

References Part II

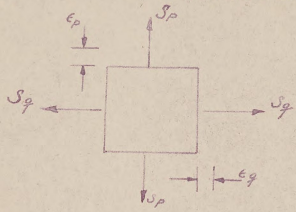
1. ANC-5 Strength of Aircraft Elements, issued by the Army-Navy Committee on Aircraft Design Criteria.

References Part III

1. "Airframe Stress Analysis by the Electrical Strain Gage", by William T. Thomson, Aero Digest, May 1943.

STRESS ANALYSIS BY STRAIN GAGE

Taken from SAE Journal (Transactions) Vol. 50, No. 8 - Aug. 1942
Pg. 349 - by C.R. Strang



$$\epsilon_p = \frac{S_p}{E} - \mu \frac{S_q}{E} \qquad \text{Eq. 1}$$

$$\epsilon_q = \frac{S_q}{E} - \mu \frac{S_p}{E} \qquad \text{Eq. 2}$$

from which,

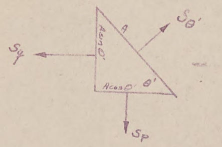
$$S_p = \frac{E}{1-\mu^2} (\epsilon_p + \mu \epsilon_q) \qquad \text{Eq. 3}$$

$$S_q = \frac{E}{1-\mu^2} (\epsilon_q + \mu \epsilon_p \qquad \text{Eq. 4}$$

Thus, if the principal strains are known, the principal stresses can be computed.

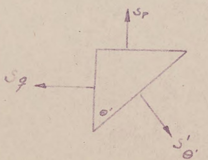
Using 3 known strains in a direction 60° to each other to determine principal stresses.

The stress in any plane is:



$$S_{\theta'} = S_p \cos^2\theta' + S_q \sin^2\theta' \qquad \text{Eq. 5}$$

$$S'_{\theta'} = S_p \sin^2\theta' + S_q \cos^2\theta' \qquad \text{Eq. 6}$$



Sub. strain values in Eq. 1. using S_θ & S'_θ as principal stresses & then substitute the strain relations from Eqs. 3 & 4 for S_p & S_q from Eqs. 5 & 6

L.V.N

From Eq. 1

$$\epsilon_{\theta'} = \frac{(S_p \cos^2\theta' + S_q \sin^2\theta')}{E} - \mu \frac{(S_p \sin^2\theta' + S_q \cos^2\theta')}{E}$$

Subs. for S_p & S_q from Eqs. 3 & 4

$$\epsilon_{\theta'} = \frac{\frac{E}{1-\mu^2}(\epsilon_p + \mu\epsilon_q)\cos^2\theta' + \frac{E}{1-\mu^2}(\epsilon_q + \mu\epsilon_p)\sin^2\theta'}{E}$$

$$- \mu \frac{\frac{E}{1-\mu^2}(\epsilon_p + \mu\epsilon_q)\sin^2\theta' + \frac{E}{1-\mu^2}(\epsilon_q + \mu\epsilon_p)\cos^2\theta'}{E}$$

$$\epsilon_{\theta'} = \epsilon_p \cos^2\theta' + \epsilon_q \sin^2\theta' \qquad \text{Eq. 7}$$

$$= \left[\left(\frac{\epsilon_p + \epsilon_q}{2}\right) + \left(\frac{\epsilon_p - \epsilon_q}{2}\right)\right]\cos^2\theta' + \left[\left(\frac{\epsilon_p + \epsilon_q}{2}\right) - \left(\frac{\epsilon_p - \epsilon_q}{2}\right)\right]\sin^2\theta'$$

$$= \left(\frac{\epsilon_p + \epsilon_q}{2}\right) + \left(\frac{\epsilon_p - \epsilon_q}{2}\right)\cos 2\theta' \qquad \text{Eq. 8}$$

Let $\epsilon = \frac{\epsilon_p + \epsilon_q}{2}$ and $\Delta = \frac{\epsilon_p - \epsilon_q}{2}$

then $\epsilon_{\theta'} = \epsilon + \Delta \cos 2\theta'$

for each gage
$$\epsilon_1 = \epsilon + \Delta \cos 2\theta' \qquad \text{Eq. 9}$$
$$\epsilon_2 = \epsilon + \Delta \cos 2(\theta' + 60) \qquad \text{Eq. 10}$$
$$\epsilon_3 = \epsilon + \Delta \cos 2(\theta' + 120) \qquad \text{Eq. 11}$$

Adding above equations, sum of Δ quantites = 0

$$\therefore \epsilon = \frac{\epsilon_1 + \epsilon_2 + \epsilon_3}{3}$$

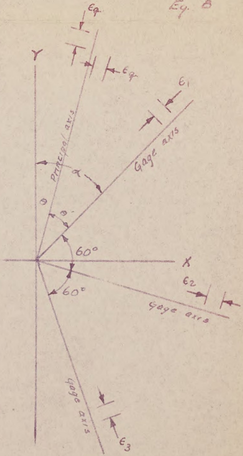
$$\Delta = \frac{\epsilon_1 - \epsilon}{\cos 2\theta'}$$

Eq. 9 ÷ Eq. 10

$$\frac{\epsilon_1 - \epsilon}{\epsilon_2 - \epsilon} = \frac{\Delta \cos 2\theta'}{\Delta \cos 2(\theta' + 60)} = \frac{\cos 2\theta'}{\cos 2\theta' \cos 120^\circ - \sin 2\theta' \sin 120^\circ}$$

$$= \frac{1}{\cos 120 - \tan 2\theta' \sin 120}$$

$$\tan 2\theta' = \frac{\sqrt{3}}{1 + 2\left(\frac{\epsilon_1 - \epsilon_2}{\epsilon_2 - \epsilon_3}\right)}$$



L. V. N.

Symbols:-

$\epsilon_{1,2,3}$ = Strain

E = Young's Modulus

S = Stress (unit)

θ' = angle of gage 1 to principal axis.

μ = Poisson's Ratio.

p, q = direction of principal stresses

$$\left.\begin{aligned} \epsilon &= \frac{\epsilon_p + \epsilon_q}{2} \\ \Delta &= \frac{\epsilon_p - \epsilon_q}{2} \end{aligned}\right\} \text{ by assignment, for simplification only.}$$

Leo V. Nothstine

Pocket has: 1 Suppl.