WITHDRAWAL RESISTANCE OF A PILE TYPE FOUNDATION

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This is to certify that the

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ABSTRACT

WITHDRAWAL RESISTANCE OF A
PILE TYPE FOUNDATION

By Neil Franklin Meador

In certain applications, foundations are required to anchor a structure to the soil. The anchoring capacity of a foundation is referred to as its withdrawal resistance. Consideration was limited to circular foundations that have a depth into the soil from 4 to 12 times their diameter. These are the ratios that are commonly used in foundations for light buildings. Only forces acting along the axis of the foundation and consequently only pure uplifting forces were considered.

The objective of the research was to determine the character of the withdrawal resistance of a shallow pile-type foundation and to develop a prediction equation for the withdrawal force.

Research reported in the literature includes tests where full scale, driven piles were pulled from the soil. In these tests the total withdrawal resistance was assumed to be frictional forces on the soil-foundation interface and the pressure normal to this surface is assumed to vary

linearly with depth below the ground surface. With these assumptions, the force required to withdraw the foundations indicated that pressures normal to foundation surface during withdrawal were approximately equal to the passive earth pressures. Other research reports the movement of a shallow pile type foundation that results when withdrawal forces are applied and also the maximum withdrawal force for several depths of embedment.

A mathematical analysis was presented which characterizes the soil as an elastic solid. The approach was to treat the problem as an axially symmetric problem in cylindrical coordinates. Several forms of the Love strain function were assumed and the corresponding stresses and displacements computed. Not all boundary conditions were satisfied by simple selection of constants. One boundary condition was satisfied by a Fourier series and one was satisfied by a numerical point-by-point matching of functions.

The experimental analysis consisted primarily of withdrawal of a model foundation from a dune sand. In these tests the depth of embedment, coefficient of friction between the sand and the foundation, density of the sand, and the withdrawal rate were varied. On some or all of the tests the withdrawal force, bulk density of the sand, variation of horizontal pressure on the foundation during withdrawal, and vertical stress in the foundation at different depths were measured.

Neil Franklin Meador

The results of the experimental analysis show that the withdrawal resistance varies as the square of depth of embedment. This is shown by the total withdrawal force variation in tests at different depths of embedment and also is shown by the variation with depth of the vertical stress in a single foundation. It is also shown that the coefficient of friction on the foundation-soil interface and the density of the sand are linearly related to withdrawal force. The coefficient of earth pressure that was computed from the tests was about 0.92. This value was determined by several different methods and indicates that the coefficient of earth pressure acting during withdrawal is intermediate between the coefficient of earth pressure at rest (K_0 = 0.4 to 0.6) and the passive earth pressure ($K_p \simeq 2.0$).

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WITHDRAWAL RESISTANCE OF A
PILE TYPE FOUNDATION

By

Neil Franklin Meador

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To

Joyce, Michael, and Kelly

Mr. and Mrs. O. S. Meador

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LIST OF SYMBOLS

A,B,C,E	Constants
A_n	$n^{\underline{th}}$ constant of a series
a, a_1, a_2	Constants
B_k	$k^{\underline{th}}$ constant of a series
B_n	$n^{\underline{th}}$ constant of a series
C_i	$i^{\underline{th}}$ constant of a series
c	Cohesion
D	Depth of embedment
d	Diameter of foundation
E	Modulus of Elasticity
E_i	$i^{\underline{th}}$ constant of a series
F(r)	Stress σ_{zz} at the plane z = D
f	Withdrawal resistance as a function of z
G	Shear modulus
G_i	$i^{\underline{th}}$ constant of a series
I_0, I_1	Modified Bessel functions of the first kind of zero and first order
J_0, J_1	Bessel functions of the first kind of zero and first order
K_A	Coefficient of active earth pressure
K_0, K_1	Modified Bessel functions of the second kind of zero and first order
K	Coefficient of earth pressure

K_o	Coefficient of earth pressure at rest
K_p	Coefficient of passive earth pressure
k	Constant
L	Distance along r axis within which boundary conditions are satisfied
ln	Natural logrithm
P	Total withdrawal force
p_{ave}	Average overburden pressure
p	Foundation perimeter
R	Radius of foundation
r	Rate of withdrawal
r,θ,z	Coordinate system (z in direction of gravity)
t	r coordinate of point at which u_z is evaluated
u_r,u_θ,u_z	Displacements in r, θ, and z directions
X_z	Body force per unit volume in z direction
Y_0,Y_1	Bessel functions of the second kind of zero and first order
α_j	J$\underline{th}$ constant in a series
β_k	K$\underline{th}$ constant in a series
γ	Weight per unit volume
∂	Partial differential symbol
θ	Coordinate of r, θ, z system
λ_i	i$\underline{th}$ constant in a series
μ	Friction angle on soil-foundation interface
ν	Poisson's ratio
π	Pi

ρ	Angle between foundation axis and withdrawal force
σ_1	Maximum principal stress
σ_2	Intermediate principal stress
σ_3	Smallest principal stress
σ_A	Active horizontal earth pressure
σ_p	Passive horizontal earth pressure
σ_{rr}	Stress normal to a plane perpendicular to the r direction
$\sigma_{r\theta}$	Shear stress in r direction on plane perpendicular to θ direction
σ_{rz}	Shear stress in r direction on plane perpendicular to z direction
$\sigma_{z\theta}$	Shear stress in z direction on plane perpendicular to θ direction
σ_{zz}	Stress normal to a plane perpendicular to the z direction
$\sigma_{\theta\theta}$	Stress normal to a plane perpendicular to the θ axis
ϕ	Angle of internal friction of soil
ϕ	Love's strain function
ω	u_r at $z = 0$
∇	Del operator
Σ	Summation symbol

I. INTRODUCTION TO THE PROBLEM

A. The Nature of the Problem

The usual function of a foundation is to so distribute the weight of the structure and its contents upon the underlying soil that differential settlements will not cause cracking or tipping of the structure. With less frequency, the function of the foundation is to withstand a pulling away of the structure from the soil. Examples of this latter case would occur when the uplift of a wind exceeds the weight of the building, certain cantilever structures, submerged buoyant structures, guy wire anchors, and others. When this occurs, the foundation must anchor the structure to the soil. This anchoring capacity is herein referred to as the withdrawal resistance of the foundation.

Consideration in this investigation is restricted to foundations such as shown in Figure 1 where the vertical dimension (D) is large compared to the horizontal dimension (2R). This is the type of foundation of most interest when considering withdrawal resistance. For this type foundation, withdrawal resistance would certainly be a function of the angle between the axis of the foundation and the line of action of the force (ρ). Consideration herein is restricted to the case where the line of action of the withdrawal force and the axis of the foundation are coincident and vertical. The question of oblique forces is reserved and recommended for further study.

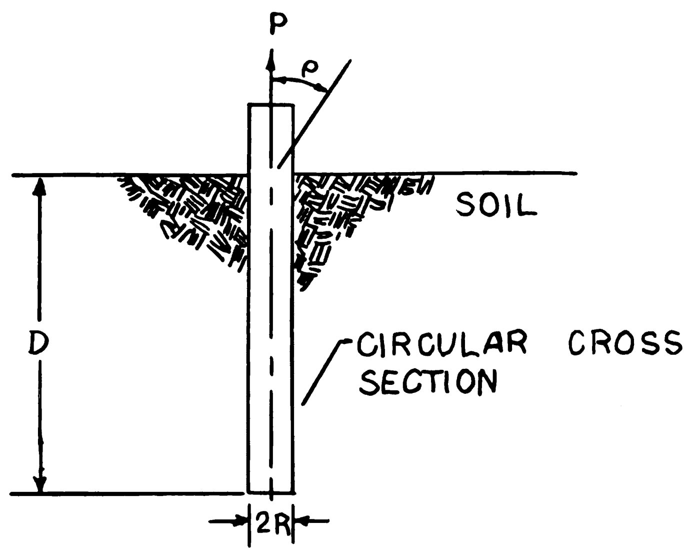

Figure 1. Foundation geometry and nomenclature.

The foundation depth divided by the foundation least dimension (D/2R) was restricted to a minimum of 4 and a maximum of 12. The reason for this is that these ratios correspond to the foundations most commonly used in light buildings. Much research has been done on piling which has a much higher (D/2R) ratio. Most of the research has been concerned with bearing capacity, although some has treated the withdrawal problem. The type of withdrawal failure for piling may differ from that of a shallow foundation, and, if more than one type of failure is present, the percentage of resistance from each type would be expected to be quite different.

B. Objective

The objective of this research was to determine the character of the withdrawal resistance of a shallow pile type foundation and to develop a prediction equation for the withdrawal force.

II. BACKGROUND THEORY AND RESEARCH

A. Soil Characterization and Properties

1. Stress-Strain Characterizations

The usual assumptions as to the rheological properties of soil when solving a problem involving stress and displacement is that the soil is an ideal, isotropic, Hookean solid. Most people who accept this assumption are reluctant to do so because it does not describe the soil's strain dependency upon history of loading, dilatancy, and the magnitude of strains observed in most cases. Scott (1963) indicates that the elasticity assumption is best made on a non-cohesive dry soil which has been subjected to repeated cycles of loading producing stresses that are low compared to failure.

Another assumption used in problems of retaining wall pressure, bearing capacity of deep foundations, and similar problems, is that the soil flows plastically at failure. The plastic regions are considered quite narrow and the resulting problem is the construction of a slip line field that is in equilibrium with the boundary conditions. These methods are described in most soil mechanics texts such as Terzaghi (1943).

2. Horizontal Earth Pressure

For withdrawal of a foundation from the soil the horizontal earth pressure supplies the normal force governing the shearing strength of the soil and the frictional force on the foundation-soil interface. Rankine in 1857 first solved for the passive and active lateral pressures when the soil is in a state of plastic equilibrium. His determination for horizontal pressures acting in cohesionless soils was

$$\sigma_A = \gamma z \tan^2 \left(45 - \frac{\phi}{2} \right) \qquad (2.1)$$

$$\sigma_p = \gamma z \tan^2 \left(45 + \frac{\phi}{2} \right) \qquad (2.2)$$

The horizontal pressure under conditions of stress due only to the weight of the soil mass (earth pressure at rest) has been investigated by a number of people and reported in such books as the <u>Conference On Earth Pressure Problems</u>. A notable work on earth pressure at rest is by Bishop (1958), who states that the horizontal pressure is linear with z and the ratio of horizontal to vertical pressure is

$$K_o = \nu/1-\nu \qquad (2.3)$$

for an ideal elastic solid. This defines a coefficient of earth pressure at rest when the soil is considered to be an ideal elastic solid. From measurements on both cohesive and non-cohesive soils, he found the coefficient of earth pressure best approximated by

$$K_o = 1 - \sin \phi \qquad (2.4)$$

Horizontal pressures occurring on a pile when it was pulled from the soil have been reported by Ireland (1957). He computed the coefficient of lateral pressure by measuring the force required to withdraw the piles and solving for the coefficient in an equation for withdrawal considering only frictional failure along the foundation-soil interface. Coefficients of earth pressure that he computed were near and some were even above the coefficient of passive earth pressure. Ireland concluded that designs should use a value of 1.75 for the earth pressure coefficient. The piles were driven and the high coefficients of lateral pressures he observed are probably a combination of increased lateral pressure due to the driving operation and the shearing of the sand upon pulling.

When a soil is sheared, it will usually exhibit a dilatancy. Sand, for example, will expand upon being sheared if the sand is more dense than the critical density and will contract if less dense. Since withdrawal forces on a foundation are resisted primarily by shear near the foundation surface, it would be expected that dilatancy would cause change in the horizontal pressure during withdrawal. If this is true and this factor is the most significant factor, then the lateral pressure would be expected to increase or decrease depending upon whether the sand is above or below the critical density.

3. Failure Criteria

Failure during withdrawal of a foundation is usually considered to occur at the soil-foundation interface. However, the failure will occur within the soil whenever the friction on the interface is greater than the strength of the soil. This frictional force can be increased by either increasing the foundation surface roughness or by increasing the surface of the foundation without increasing its effective diameter. This latter method could be accomplished by using a corrugated pile surface.

When the entire failure occurs along the interface, Lundgren (1967) estimates the uplift resistance by the following formula:

$$\sigma_{rz} = K\, p_{ave} \tan \mu \qquad (2.5)$$

where σ_{rz} = unit shearing resistance,

K = coefficient of earth pressure at the interface of the pile and the soil,

p_{ave} = average overburden pressure,

$\tan \mu$ = coefficient of friction between pile and soil.

The uncertainties in the equation are whether the value of K is the coefficient of earth pressure at rest or possibly some other value depending upon dilatancy, and when does this type of failure occur.

When the failure occurs in the soil and the soil is a sand, then the condition for failure in sand is given by Kirkpatrick (1957). Starting from the Couloumb equation for failure in sand

$$\sigma_{rz} = c + \sigma_{rr} \tan \phi \tag{2.6}$$

and considering the Mohr theory of strength, the condition for failure in terms of principal stresses is

$$\{(\sigma_1 - \sigma_2)^2 - [(\sigma_1 + \sigma_2)\sin\phi]^2\}\{(\sigma_2 - \sigma_3)^2 - [(\sigma_2 + \sigma_3)\sin\phi]^2\}\{(\sigma_3 - \sigma_1)^2 - [(\sigma_3 + \sigma_1)\sin\phi]^2\} = 0. \tag{2.7}$$

Provided the stresses are known throughout the region, this equation can be used to predict the zone where failure will occur.

Hurst (1959) observed withdrawal failures in a soil that was cohesive and therefore had some tensile strength. Radial tension cracks on the surface and uplifting of large quantities of the soil with the foundation seem to indicate tension failure within the soil mass, and a lifting of a quantity of soil. However, the shape of lifted soil mass (usually assumed to be a cone) and the depth at which this failure will occur rather than a foundation interface failure, is not known. The proportion of the total withdrawal force that may be attributed to tension failure in the soil is much higher in shallow foundations and is the major difference between this research and that concerning the uplifting of piles.

B. Experimental Techniques and Results

Hurst (1959) has investigated the uplifting of pole-type foundations. The procedure was to apply a withdrawal force hydraulically to a number of poles embedded 3.5, 4.5 and 5.5 feet into holes backfilled with earth, crushed stone and concrete. The data recorded was the movement of the foundation at several loading levels. Also, data as to the soil compaction versus water content, bulk density, Atterberg limits, type of soil, and water capacity were reported. The conclusions of this study are concerned mostly with the comparison of various backfill treatments, but of most importance to this present study is the data that can be used to check any prediction equation.

Other research concerned with uplift resistance of piles has been reported by Ireland (1957) and by Yoshimi (1964). Both of these papers consider the uplift resistance to be essentially given by

$$\sigma_{rz} = K\gamma z \tan \mu$$

where σ_{rz} is the frictional resisting force. From the data collected both solved for K and found that the values were near the coefficient of passive earth pressure. Ireland suggests a design value of K = 1.75 and Yoshimi reports a value of about 2.9.

C. Mathematical Analysis

As mentioned in Section A.1 of this chapter, a common assumption as to the nature of the soil, when it is desired to solve for stress and/or strain distribution, is that it is an isotropic Hookean solid. This assumption opens the analysis to the techniques and solutions of elasticity.

Ruderman (1939) started with the assumption of elasticity in order to investigate the stress distribution in the soil in the vicinity of a pile loaded with a gravity load. From elasticity he took the Mindlin (1936) solution for stresses and strains arising from a concentrated force being applied at a point below the surface of a semi-infinite body. An assumption of linear decrease in stress in the pile with z was made and then the Mindlin solution was integrated along the z axis from the z = 0 plane to the bottom of the pile.

This analysis has several faults. The solution is not valid very close to the pile surface because the Mindlin solution assumes a body force, not a surface traction, and also because the pile was assumed to be a line along the z axis. Despite these difficulties, the solution can show the variation in stresses at some distance from the pile when a withdrawal or bearing load is applied to the pile. Using Ruderman's solution and computing the radial stress, σ_{rr}, it is found that withdrawal of the foundation causes a tensile σ_{rr} or a decrease in compressive stress if the lateral pressure, due to the soil weight, is considered. This tensile stress would arise from the attachment of the soil to the foundation sides and also to the bottom.

Although Timoshenko and Goodier (1951) did not consider the withdrawal of a foundation from the soil, they did consider some problems with axially symmetric deformation of a circular cylinder which shows some techniques that could be applied in this problem. In these problems, Love's "strain function" was used. The stresses and deformation are written by Fung (1965) as derivatives of a potential function as follows:

$$\sigma_{rr} = \frac{\partial}{\partial z}\left[\nu\nabla^2\phi - \frac{\partial^2\phi}{\partial r^2}\right] \tag{2.8}$$

$$\sigma_{\theta\theta} = \frac{\partial}{\partial z}\left[\nu\nabla^2\phi - \frac{1}{r}\frac{\partial\phi}{\partial r}\right] \tag{2.9}$$

$$\sigma_{zz} = \frac{\partial}{\partial z}\left[(2 - \nu)\nabla^2\phi - \frac{\partial^2\phi}{\partial z^2}\right] \tag{2.10}$$

$$\sigma_{rz} = \frac{\partial}{\partial r}\left[(1 - \nu)\nabla^2\phi - \frac{\partial^2\phi}{\partial z^2}\right] \tag{2.11}$$

$$2Gu_r = -\frac{\partial^2\phi}{\partial r \partial z} \tag{2.12}$$

$$2Gu_z = 2(1-\nu)\nabla^2\phi - \frac{\partial^2\phi}{\partial z^2} \tag{2.13}$$

where σ_{rr}, $\sigma_{\theta\theta}$, σ_{zz} are stresses normal to planes perpendicular to the r, θ, and z direction respectively

σ_{rz} is the shear stress in the r direction acting on the plane perpendicular to the z direction

u_r, u_z are displacements in the r and z directions respectively.

ϕ is the Love strain function

$$\nabla^2 = \frac{\partial^2}{\partial r^2} + \frac{1}{r}\frac{\partial}{\partial r} + \frac{\partial^2}{\partial z^2}$$

Since the problems considered were axially symmetric, $\sigma_{z\theta}$, $\sigma_{r\theta}$, and u_θ are zero, and ϕ is independent of θ. The restriction on the potential function, as written by Fung (1965), is that it must satisfy the equation

$$\nabla^2\nabla^2\phi = -\frac{X_z}{(1-\nu)} \tag{2.14}$$

where X_z is the body force per unit volume in the z direction called γ in other parts of this thesis.

Timoshenko and Goodier (1951) started by assuming the solution of the equation

$$\nabla^2\phi = 0 \tag{2.15}$$

which is also a solution of equation 2.15 when $X_z = 0$ is of the form

$$\phi = f(r) \sin kz \tag{2.16}$$

or

$$\phi = f(r) \cos kz \tag{2.17}$$

and found that

$$f(r) = AI_0(kr) + BK_0(kr) \tag{2.18}$$

where I_0 is a modified, Bessel function of first kind of zero order

K_0 is a modified Bessel function of the second kind of zero order

A, B, k are constants.

After the form of strain function is selected, the stresses and displacements are computed from equations 2.8 to 2.13 and then the constants are selected such that the stresses and displacements satisfy the boundary conditions. The potential function can be expressed as a sine and/or cosine Fourier series thereby satisfying more complex boundary conditions.

This technique of expressing the Love strain function as a Fourier series was also used extensively by Pickett (1944). He used more than one Fourier series to satisfy the boundary conditions. The constants of each series then depends upon the constants of the other series, and therefore the final solution involves the solution of simultaneous equations giving relations between the coefficients.

III. MATHEMATICAL ANALYSIS

A. Mathematical Statement of the Problem

Starting with the assumption that the soil is an isotropic and homogeneous, Hookean solid, the mathematical analysis of Timoshenko, et al. (1951) and Pickett (1944) described in the last section can be used as a basis for the analysis.

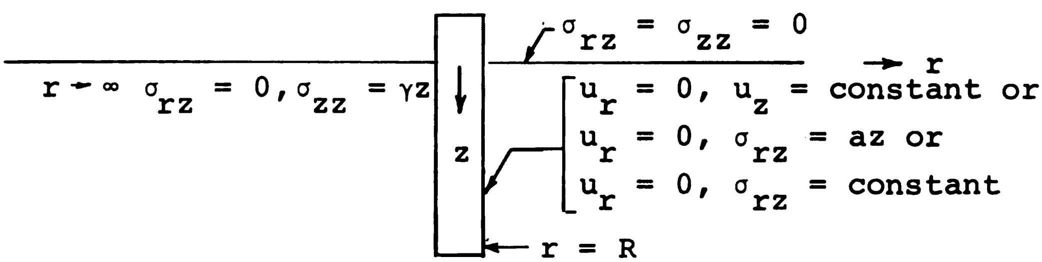

Figure 2. Mathematical description of the problem including boundary conditions.

Focusing attention on the boundary conditions of the problem, consider first the $z = 0$ plane of the semi-infinite solid shown in Figure 2. This plane is stress free, therefore, the normal and shearing stress must be zero. Consider the $r = R$ surface (the surface of the cylindrical foundation), the boundary conditions could be considered to be (1) $u_r = 0$, $u_z =$ constant; (2) $u_r = 0$, $\sigma_{rz} = az$; and (3) $u_r = 0$, $\sigma_{rz} =$ constant. These boundary conditions on the foundation surface either consider the foundation as a rigid body, or make use of experimental information obtained, or consider the soil highly cohesive to arrive at the conditions respectively considered.

Two more boundary conditions must be examined. They are the stresses as r and z become infinite. Considering that the stresses caused by the weight of the soil will be added later, then for the first part of the problem all stresses must vanish as r and z become infinite. This latter condition will, however, be violated in the solution in anticipation of either superimposing another solution which will satisfy that boundary condition or confining attention to the region where z is small compared to D.

The problem is axially symmetric and therefore the stresses and strains as derivatives of the Love strain function will be the same as equations 2.8 through 2.13 in the preceding section. Again $\sigma_{z\theta}$, $\sigma_{r\theta}$, and u_θ are zero becuase of axial symmetry.

The solution of the problem now proceeds by selecting functions of r and z which will satisfy the biharmonic equation (2.14). After differentiating the function in accordance with equations 2.8 through 2.13 to obtain the stresses and displacements that would occur, the constants in the function are selected so that the boundary conditions are satisfied.

B. Possible Forms of the Potential Function

If the function

$$\phi = [\omega G z^2/(1-2\nu)] - [\gamma z^4/24(1-\nu)] \qquad (3.1)$$

is selected it will be found that

$$\nabla^2\nabla^2\phi = -\gamma/(1-\nu) \qquad \sigma_{zz} = -\gamma z$$

$$\sigma_{rr} = \sigma_{\theta\theta} = -\frac{\nu}{(1-\nu)}\gamma z \qquad \sigma_{rz} = 0$$

$$u_r = 0 \qquad 2Gu_z = \frac{G\omega}{2} - \frac{1-2\nu}{2(1-\nu)}\gamma z^2$$

which is the stress and displacement condition caused by the weight of the soil. The remaining functions that may make up the Love strain function can be selected to satisfy the equation

$$\nabla^2\nabla^2\phi = 0. \qquad (3.2)$$

As shown in the last section, if the Love strain function is assumed to be a product of f(r) and f(z) then assuming f(z) to be trigometric functions sin αz or cos αz determines f(r) to be

$$f(r) = [AI_0(\alpha r) + BK_0(\alpha r)] \qquad (3.3)$$

It can also be shown by the same development that if f(z) is assumed to be hyperbolic functions sinh αz or cosh αz then

$$f(r) = CJ_0(\alpha r) + EY_0(\alpha r)] \qquad (3.4)$$

where J_0 is a Bessel function of the first kind of zero order

Y_0 is a Bessel function of the second kind of zero order

C, E, α are constants.

All the $f(z) \cdot f(r)$ combinations given in the preceding paragraph are solutions of the harmonic equation 2.15 and are also solutions of the biharmonic equation 3.2. However, there are more solutions to the biharmonic equation than those indicated. Examine the equation

$$\phi = \alpha r J_1(\alpha r)[\sinh \alpha z] \qquad (3.5)$$

It can be easily determined that

$$\nabla^2 \phi = 2\alpha^2 J_0(\alpha r)[\sinh \alpha z]$$

and since $J_0(\alpha r) \sinh \alpha z$ is a solution to the harmonic equation then $\alpha r J_1(\alpha r) \sinh \alpha z$ must be a solution of the biharmonic equation. This same argument can be advanced to show that the following functions of f(r) and f(z) when multiplied by each other properly will satisfy the biharmonic equation:

$$\alpha r J_1(\alpha r) \qquad \alpha r I_1(\alpha r) \qquad \alpha z \sin \alpha z \qquad \alpha z \sinh \alpha z$$

$$\alpha r Y_1(\alpha r) \qquad \alpha r K_1(\alpha r) \qquad \alpha z \cos \alpha z \qquad \alpha z \cosh \alpha z .$$

By being multiplied properly is meant that the trigometric functions must be multiplied by Bessel functions $J_n(\alpha r)$ and $Y_n(\alpha r)$, hyperbolic functions must be multiplied by $I_n(\alpha r)$ and $K_n(\alpha r)$, and functions f(r) and f(z) which do not satisfy the harmonic equation cannot be multiplied by each other. The solutions to the harmonic and biharmonic equations give 36 different terms that can be used to meet boundary conditions.

Other functions can be found by the trial and error method which will satisfy the biharmonic equation and can also be used to meet the boundary conditions. For this axially symmetric problem functions which contain ln r give some interesting results in terms of stresses. These functions will be used where it is advantageous to do so.

C. Solution of the Problem

As was shown there are a large number of terms that can be combined to make up the potential function. The stresses are required to be finite as r becomes infinite. All Bessel functions except $I_n(\alpha r)$ satisfy this condition therefore the $I_n(\alpha r)$ Bessel functions will not be considered in the solution. The $Y_n(\alpha r)$ and $K_n(\alpha r)$ Bessel functions are infinite at $r = 0$ however this is out of the region of consideration so they are possible solutions.

After combining the possible terms of the potential function in a multitude of ways the function

$$\phi = [\omega G z^2/(1-2\nu)] - [\gamma z^4/24(1-\nu)] + 4A(\ln r+1) + \sum_{k=1}^{k=\infty} [B_k J_0(\beta_k r)][\beta_k z \sinh \beta_k z] \tag{3.6}$$

was selected. Computing the stresses and displacements of interest from formulas 2.10, 2.11, and 2.12 one obtains first

$$\nabla^2\phi = [2\omega G/(1-2\nu)] - [\gamma z/2(1-\nu)] + 4A(\ln r + 1) + \sum_{K=1}^{k=\infty} 2\beta_k^2 [B_k J_0(\beta_k r)] [\cosh \beta_k z] \tag{3.7}$$

and

$$\frac{\partial^2\phi}{\partial z^2} = [2\omega G/(1-2\nu)] - [\gamma z^2/2(1-\nu)] + \sum_{k=1}^{k=\infty} \beta_k^2 [B_k J_0(\beta_k r)][2 \cosh \beta_k z + \beta_k z \sinh \beta_k z] \tag{3.8}$$

then

$$2Gu_r = \sum_{k=1}^{k=\infty} \beta_k^2 [B_k J_1(\beta_k r)][\sinh \beta_k z + \beta_k z \cosh \beta_k z] \tag{3.9}$$

$$\sigma_{rz} = 4A(1-\nu)/r + \sum_{k=1}^{k=\infty} \beta_k^3 [B_k J_1(\beta_k r)] [2\nu \cosh \beta_k z$$

$$+ \beta_k z \sinh \beta_k z] \qquad (3.10)$$

$$\sigma_{zz} = -\gamma z + \sum_{k=1}^{k=\infty} \beta_k^3 [B_k J_0(\beta_k r)] [1-2\nu) \sinh \beta_k z$$

$$- \beta_k z \cosh \beta_k z]. \qquad (3.11)$$

In order to satisfy the boundary condition, $u_r = 0$ when $r = R, \beta_k$ was selected as roots of the equation

$$J_1(\beta_k R) = 0 \qquad (3.12)$$

which causes the first term to become zero when $r = R$. Now examining the boundary condition σ_{rz} = constant when $r = R$. It is found that the condition is already satisfied and

$$\sigma_{rz} = 4A(1-\nu)/R. \qquad (3.13)$$

The constant A can be related to the total withdrawal force P by integrating the shear stress over the foundation surface to obtain

$$A = P/8\pi D(1-\nu). \qquad (3.14)$$

Next investigating the value of σ_{zz} at $z = 0$, it is found that the boundary condition $\sigma_{zz} = 0$ is satisfied because the last term of the equation becomes zero.

The last condition is that $\sigma_{rz} = 0$ when $z = 0$, $r = r$. It can be seen that this condition is not satisfied unless

$$\sum_{k=1}^{k=\infty} 2\nu\beta_k^3[B_k J_1(\beta_k r)] = -\ 4A(1-\nu)/r. \tag{3.15}$$

Since the left side of the equation consists of non-orthogonal functions over the region $r = R$ to $r = L$, the usual method of selecting the constants B_k to be constants of a Fourier-Bessell series does not apply. The constants can be selected, however, by a point-by-point matching of the left and right side of the equation. This would require solving for the constants as coefficients of simultaneous linear equations. The number of equations would be equal to the number of terms retained in the series.

Another solution to the problem can be obtained by selecting the potential function (stress due to soil weight can be added later)

$$\phi = \sum_{n=1}^{n=\infty} [A_n K_0(\alpha_n r) + B_n \alpha_n r K_1(\alpha_r r)] \sin \alpha_n z$$

$$+ \sum_{i=1}^{i=\infty} J_0(\lambda_i r)[C_i \sinh \lambda_i z + E_i \cosh \lambda_i z$$

$$+ G_i \lambda_i z \sinh \lambda_i z]. \qquad (3.16)$$

If the formulas 2.10, 2.11, and 2.12 are used to compute the stresses and displacements one obtains

$$u_r = \sum_{n=1}^{n=\infty} \alpha_n^2 [A_n K_1(\alpha_n r) + B_n \alpha_n r K_0(\alpha_n r)] \cos \alpha_n z$$

$$+ \sum_{i=1}^{i=\infty} \lambda_i^2 J_1(\lambda_i r)[C_i \cosh \lambda_i z + E_i \sinh \lambda_i z$$

$$+ G_i(\sinh \lambda_i z + \lambda_i z \cosh \lambda_i z)], \qquad (3.17)$$

$$\sigma_{rz} = \sum_{n=1}^{n=\infty} \alpha_n^3 [-A_n K_1(\alpha_n r) - B_n \alpha_n r K_0(\alpha_n r)$$

$$+ 2(1-\nu) B_n K_1(\alpha_n r)] \sin \alpha_n z +$$

$$+ \sum_{i=1}^{i=\infty} \lambda_i^3 J_1(\lambda_i r)[C_i \sinh \lambda_i z + E_i \cosh \lambda_i z$$

$$+ G_i(\lambda_i z \sinh \lambda_i z - 2\nu \cosh \lambda_i z)], \text{ and} \tag{3.18}$$

$$\sigma_{zz} = \sum_{n=1}^{n=\infty} \alpha_n^3 [A_n K_0(\alpha_n r) - 2(2-\nu) B_n K_0(\alpha_n r)$$

$$+ B_n \alpha_n r K_1(\alpha_n r)] \cos \alpha_n z$$

$$- \sum_{i=1}^{i=\infty} \lambda_i^3 J_0(\lambda_i r)\{C_i \cosh \lambda_i z + E_i \sinh \lambda_i z$$

$$+ G_i[-(1-2\nu) \sinh \lambda_i z + \lambda_i z \cosh \lambda_i z]\}. \tag{3.19}$$

The condition that $u_r = 0$ at $r = R$ is satisfied if

$$A_n = - B_n \alpha_n R \frac{K_0(\alpha_n R)}{K_1(\alpha_n R)} \tag{3.20}$$

and λ_i are the solutions of the equation

$$J_1(\lambda_i R) = 0. \tag{3.21}$$

The condition that $\sigma_{rz} = 0$ at $z = 0$ is satisfied if

$$E_i = 2\nu G_i.$$

The shear distribution upon the foundation can now be selected to be a function of z, for example, g(z). The equation for shear at r = R is then

$$g(z) = \sum_{n=1}^{n=\infty} 2\alpha_n^3(1-\nu)B_n K_1(\alpha_n R) \sin \alpha_n z. \tag{3.22}$$

The constants B_n can now be solved for by the usual Fourier series method. Then

$$\alpha_n = n\pi/D$$

and

$$B_n = [\alpha_n^3(1-\nu)K_1(\alpha_n R)D]^{-1} \int_0^D g(z)\sin\left(\frac{n\pi z}{D}\right) dz. \tag{3.23}$$

The last condition, that $\sigma_{zz} = 0$ when $z = 0$ is satisfied if

$$\sum_{i=1}^{i=\infty} \lambda_i^3 C_i J_0(\lambda_i r) = \sum_{n=1}^{n=\infty} [A_n K_0(\alpha_n r) - 2\ (2-\nu)B_n K_0(\alpha_n r)$$

$$+ B_n \alpha_n r K_1(\alpha_n r)]. \tag{3.24}$$

Again, as before the left side of the equation is not orothogonal over the region r = R to r = L and equation 3.24 must be satisfied by point-by-point matching as previously indicated.

Consider now the solution as z becomes large. The superposition of another solution in order to satisfy the boundary conditions when z becomes large will be discussed here but will not actually be worked out. If another term can be added to the Love strain function which will meet all boundary conditions and which will have a free constant available, then the free constant can be selected to match one stress or displacement boundary condition at a fictitious boundary where z = D, r = r. The way the constant would be selected would be to first load the portion of the solid below z = D with the σ_{zz} stress obtained from the first solution when z = D. Let that stress determined from the first solution that is acting on the z = D plane be called F(r). The stresses and displacements in the lower portion can be obtained by integrating the Boussinesq solution for a force acting at a point on a semi-infinite body. The displacement in the z direction at point z = D, r = t would be

$$u_z = \int_0^{\infty} \int_0^{2\pi} \frac{(1-\nu^2) F(r) r d\theta dr}{\pi E (r^2 - 2tr \cos\theta + t^2)^{1/2}} . \qquad (3.25)$$

This is now set equal to u_z that arises from the first solution thereby determining the free variable. This same procedure can be followed once more matching u_r and σ_{rz} along the boundary. This latter superposition (matching u_r) will determine all the available constants and the Cerruti solution for a horizontal force on the boundary of a semi-infinite solid would be the function integrated.

IV. EXPERIMENTAL ANALYSIS

A. Experimental Model

1. Soil

The soil selected for the experimental model was a dune sand from the shore of Lake Michigan. This sand was selected because of its rather uniform grain size distribution. It was assumed that a uniform sand would be less sensitive to handling and that the physical characteristics, such as void ratio, could be more easily controlled and held constant throughout the sand. To establish the particle size distribution, a sieve analysis was performed and the results are shown in Figure 3. This analysis shows that the sand is quite uniform for a naturally occurring sand and that it would be classified as a medium to fine sand.

Examination of the sand under a microscope showed that the sand grains had rounded edges. The standard test for determination of the specific gravity of the sand grains as described by Lamb (1951) was performed and indicated a specific gravity of 2.67.

A number of direct shear tests, as described by Lamb (Ibid.), were performed on the sand with normal

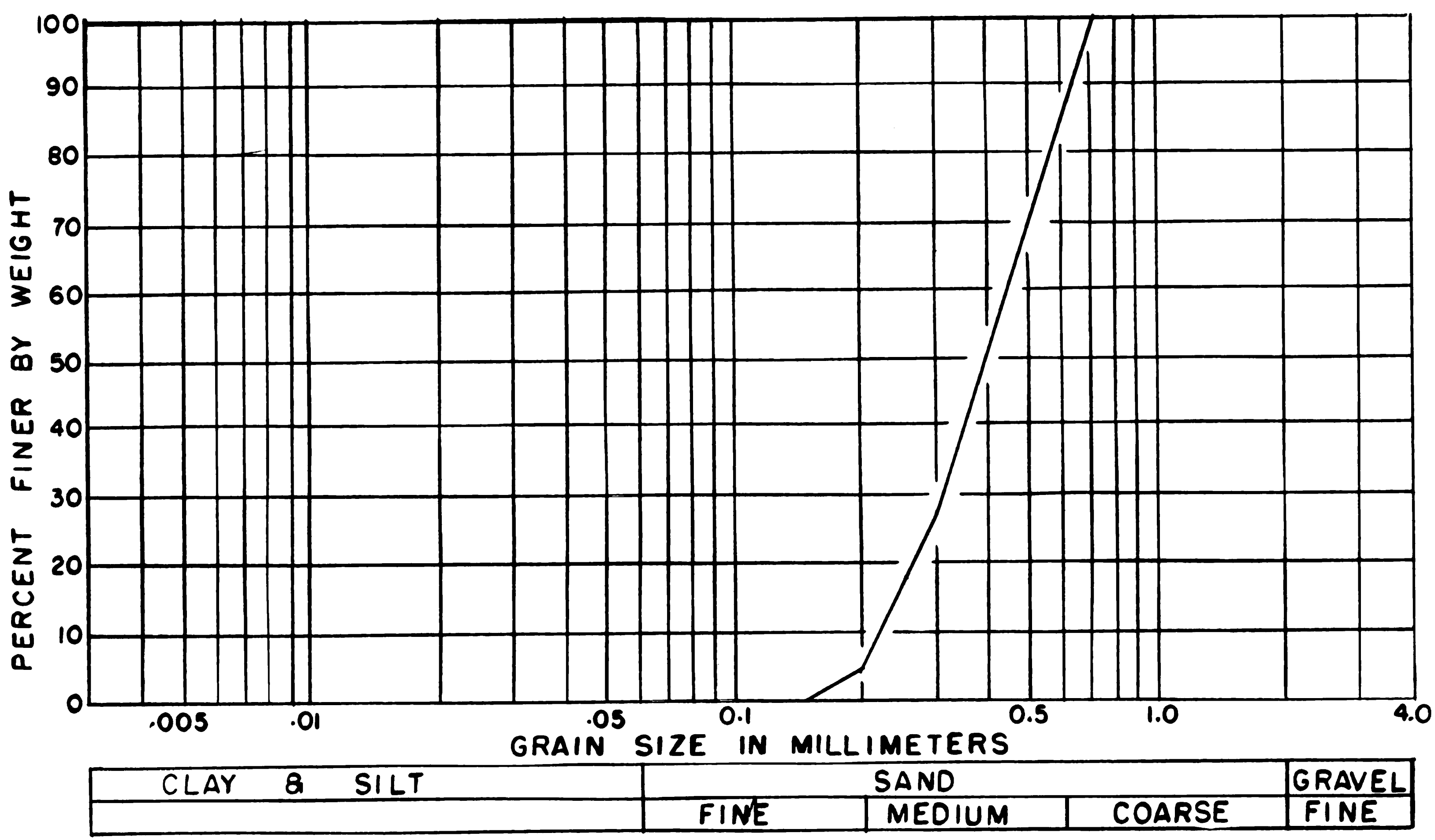

Figure 3. Grain size distribution for the test soil.

pressures of the same magnitude as occurred during the withdrawal tests. These direct shear tests establish the variation of the peak coefficient of internal friction as a function of the normal pressure and of porosity. The data is given in Figure 4 and is of the same nature as was found by Taylor and Leps (1938). It was also observed that the sample expanded during shearing except for one case. This exceptional case was at a void ratio of 0.678, or porosity of 0.404 and establishes an approximate value for the critical void ratio, i.e., the void ratio above which the sample will contract and below which the sample will expand upon being sheared. This critical void ratio was obtained by placing the sand in the container being careful not to vibrate it. It is concluded that this sand *in situ* would be below the critical void ratio (i.e., more dense). This expansion of the sand on being sheared led to the theory that when withdrawing the foundation from the sand, the sand would be subjected to shearing stress and therefore would tend to increase in volume as indicated in the previous chapter.

One other item of interest is the fact that the peak friction angle, a measure of the sand strength, decreased as the normal force and porosity increased. This phenomenon is discussed by Leonards (1962) and he indicates that the change in friction angle is at least partially due to the work done by volume change. This work done

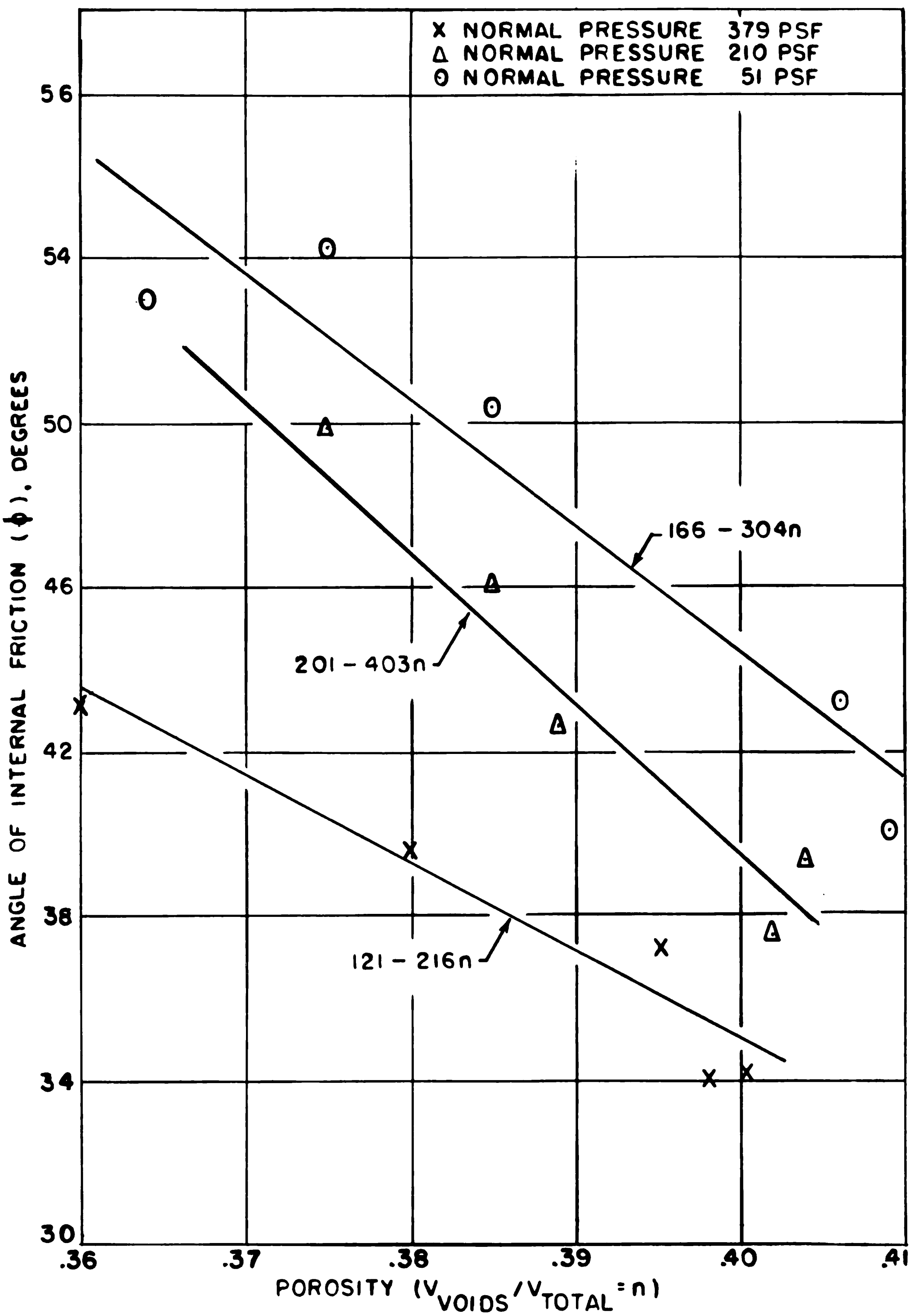

Figure 4. Angle of internal friction versus porosity for different values of normal pressure.

during volume change would have the effect of reducing the friction angle with increasing normal force and porosity. Relating this to the case of withdrawal of the foundation, this means that the "apparent" shear strength of the sand compared to the lateral force acting, decreases with increasing depth below ground surface. This decrease in strength, however, may be offset somewhat by a decrease in void ratio, due to the greater confining pressure.

All tests were conducted with the sand in an air dry state (about .15 percent water content). The angle of repose of the sand was measured by means of a tilt-table shown in Figure 5. The angle of repose was 36°.

2. Foundation

The foundation selected was a three-inch diameter, 28 gauge, galvanized steel cylinder thirty inches long. The cylinder was made in two halves to facilitate the installing of strain gauges on the interior surfaces. The two cylinder halves were soldered together and the entire surface sanded to give a uniform surface roughness. The bottom was sealed with a rubber stopper and a wooden stopper with a hook for pulling was attached to the top. The cylinder embedded in the sand is shown in Figure 6.

3. Foundation Surface Roughness

As already mentioned the cylinder surface was galvanized metal. To obtain a greater value of the

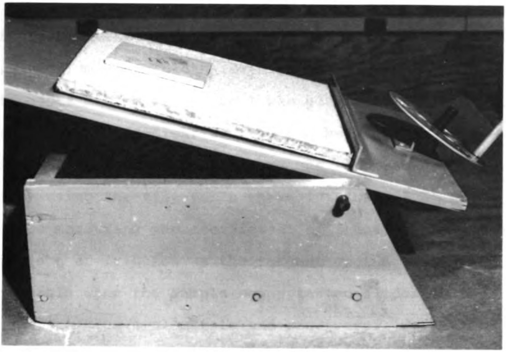

Figure 5. Tilt-table for determination of the coefficient of friction and angle of repose.

Figure 6. Experimental cylinder embedded in the sand tank.

coefficient of friction between the sand and the foundation, a medium emery cloth (Crystal Bay Emery Cloth LN31 by Minnesota Mining & Mfg. Co.) was glued to the foundation surface. The coefficient of friction between the galvanized metal or the emery cloth and the sand was determined by using the tilt-table shown in Figure 5. This method of measuring the coefficient of friction gives the coefficient of friction as the tangent of the tilt angle of the table when the sample being tested slides on the sand surface. Several metals and sand papers were tried and it appeared that a limiting value of the angle was about 33° for the sandpaper and the normal pressures used. It was also observed that the coefficient of friction decreases with normal pressure. This reduction of the coefficient of friction would reduce the maximum transfer of force at greater depths below the sand surface when compared to the lateral pressure.

4. Sand-tank and Foundation Withdrawal Device

The sand-tank was 36" in diameter and 26" deep. This diameter was large enough so that the sand acted as a semi-infinite solid. This was checked by measuring the movements of the surface of the sand at various distances from the foundation. It was found that no measurable (0.001 inch) deflection of the surface occurred six inches from the surface of the foundation. The apparatus for measuring the deflection of the sand surface is shown in Figure 7 and consisted of dial gauges measuring the movement of one inch square blocks on the sand surface. The dial gauges exerted 50 to 200 grams force on the soil surface which will change the stresses in the vicinity of the gauges and thereby give a false reading. The movement, if any, in the vicinity of the gauges could not be large, however, or the gauges would have registered some.

The apparatus for withdrawing the foundation from the soil is shown in Figure 8. It consists of an electric motor coupled to a gear reducer. From the gear reducer, a belt is used to transfer the motion to a horizontal shaft. The foundation was then pulled by the wrapping of a nylon cord around the horizontal shaft. Although the system produces a constant displacement rate under no-load conditions, slip and stretch in the system cause the displacement rate to be reduced during loading. The effect of this system, when withdrawing a foundation, was nearly a constant loading rate (linearly increasing force) until maximum force was reached. After the maximum force was reached, the displacement rate increased substantially and became erratic as the sand alternately held and suddenly failed. A graph of the total withdrawal force versus time in Figure 9 illustrates the described action in a representative test.

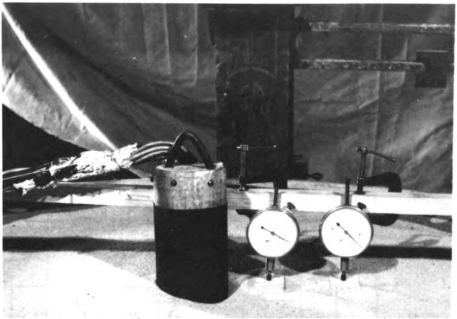

Figure 7. Apparatus for measuring the movement of the sand surface.

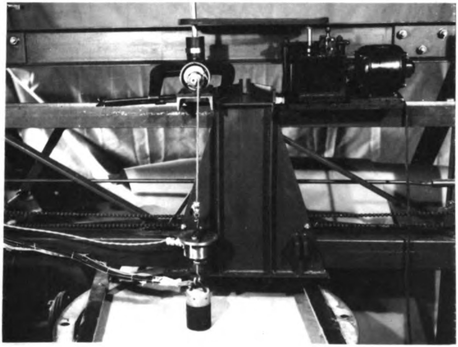

Figure 8. Apparatus used to withdraw the foundation from the soil.

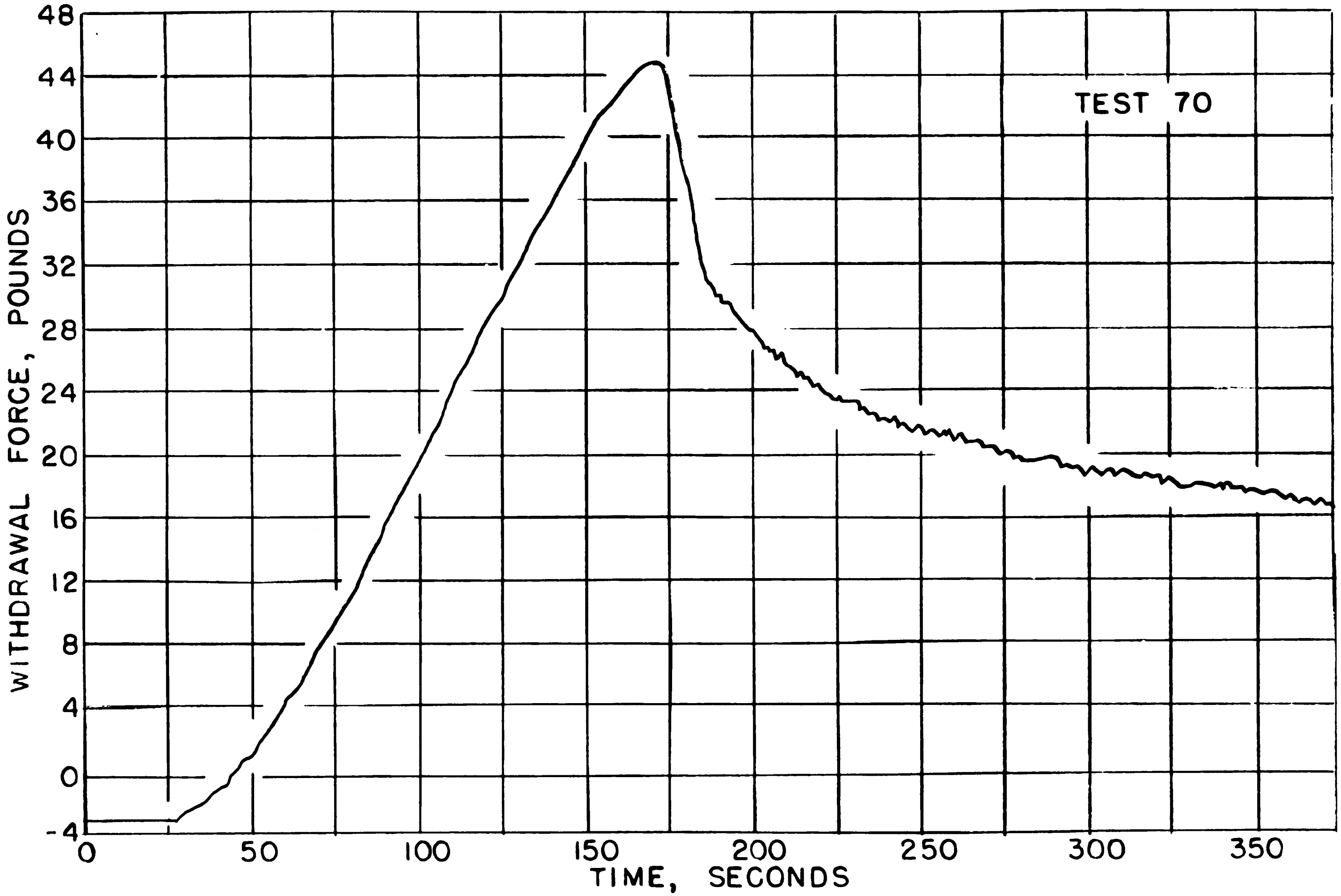

Figure 9. Typical withdrawal force versus time graph showing the loading characteristics of the withdrawal system and the foundation.

Although a constant displacement rate is desirable Taylor and Leps (1938) showed that the shearing strength of dry sand is quite insensitive to the shear strain rate. This was also somewhat substantiated by tests shown in Figure 10. This graph shows the effect of withdrawal rate upon maximum total withdrawal force. It appears from this graph that experimental variation exceeds the effect of a twenty-fold increase in withdrawal rate.

B. Experimental Procedure

1. Experimental Design

The experimentation was to produce a prediction equation for the total withdrawal force. With this as the experimental objective, the tests were organized and the important variables were selected by considering simple theories. The important variables were selected as follows:

1. Withdrawal force, P (pounds)
2. Soil density, γ (pounds/cubic foot)
3. Coefficient of friction of the foundation surface, tan μ (dimensionless)
4. Soil angle of internal friction, ϕ (dimensionless)
5. Depth of foundation embedment, D (feet)
6. Diameter of the foundation, d (feet)
7. Coefficient of earth pressure, K (dimensionless)

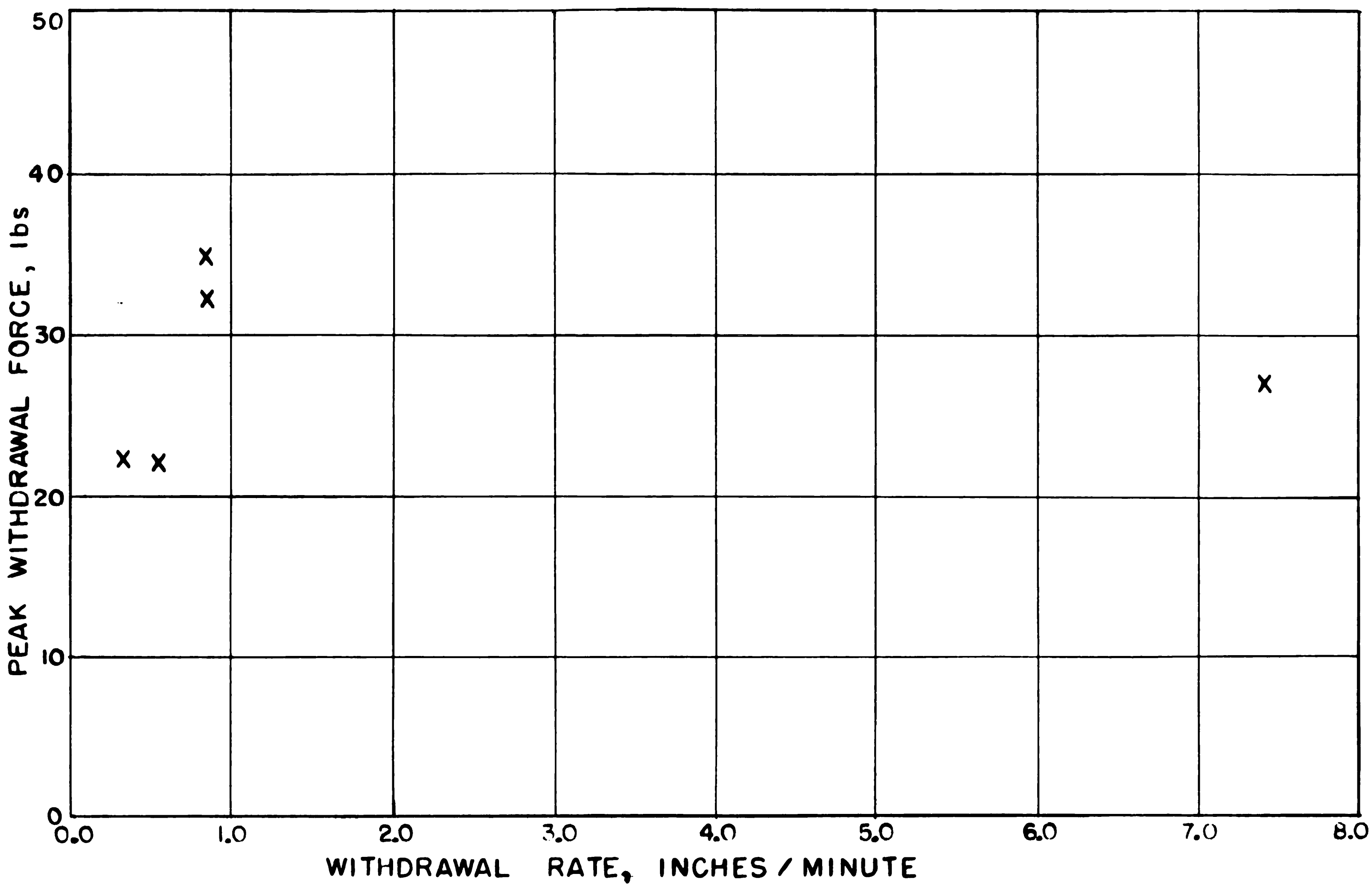

Figure 10. Total withdrawal force shown as a function of withdrawal rate.

The experimentation was organized such that the main emphasis was placed on the relationship between withdrawal force and depth of embedment. The other factors previously listed were tested in such a way that their influence on total withdrawal force could be checked against theory. The tests conducted are listed in Table 1.

The standard dense condition of the sand was achieved by placing the sand in the sand-tank in five-inch layers. After placing each layer, the layer was rodded in a standard manner. To achieve a less dense sand, the sand was not rodded. This procedure produced a maximum variation in density of less than two percent in the standard dense state and a significantly less dense sand in the test where the sand was to be in the loose condition. After each test the sand was entirely removed from the sand-tank and thoroughly mixed before being used for the next test. This procedure eliminated segregation of sand particles. The tests were also somewhat randomized to take care of unknown variations.

The bulk density of the sand was calculated from the weight of the entire sand tank and the volume of sand. The weight of the sand tank was obtained by placing it on a scale throughout the tests and the volume was determined from the diameter of the tank and the depth of sand, minus the volume of the foundation. This method gives an average bulk density and does not show local variations in density.

Table 1. List of withdrawal tests showing how the variables were tested.

Test Number	Soil density	Coeff. of friction	Depth of embedment, inches	Rate of withdrawal, inches/min.
62	Standard	.384	20	0.54*
63	Standard	.384	20	7.44*
64	Standard	.384	20	0.33*
65	Standard	.384	20	0.85*
66	Standard	.384	20	0.85*
67	Standard	.384*	20	0.54*
68	Standard*	.625*	20*	0.54
69	Standard*	.625*	20*	0.54
70	Loose*	.625	20	0.54
71	Standard	.625	10*	0.54
72	Standard	.625	24*	0.54
73	Standard	.625	15*	0.54
74	Standard	.625	24*	0.54

*Variable of primary interest.

The total withdrawal force was measured by the transducer shown in Figure 6. The output from this transducer was amplified and fed into an X-Y recorder to be plotted as the Y-axis and time as the X axis. The output was zeroed with the foundation suspended in air thereby automatically subtracting the weight of it from the withdrawal force. The transducer was calibrated by hanging a known weight from the bottom of the foundation which was in turn, hooked to the transducer.

During one test the strain gauges on the inside surface of the foundation were used to measure the transfer of withdrawal force to the soil as a function of depth in the soil. The strain gauge bridge was arranged with active gauges in opposite arms of the bridge circuit and with temperature compensating gauges in the remaining two arms of the circuit. This arrangement, along with the placement of the active gauges on opposite ends of a diameter of the foundation, gave double sensitivity to axial strains while cancelling out any bending strains. Gauges were placed at five inch intervals along the length of the foundation, thereby giving five vertical strain bridge circuits. Due to trouble with the gauges and with the amplifiers, not all circuits produced useful data. The signal from the strain gauge circuits was amplified by Brush Universal amplifiers and recorded on a Brush oscillograph.

During one other test the same procedure was followed as was described in the preceding paragraph, except that strain gauge circuits were used to measure the horizontal pressure rather than the vertical force. As with the vertical gauges, there were five bridge circuits arranged to give double sensitivity to circumferential strain of the cylinder and to provide temperature compensation. The gauges were calibrated by placing the foundation in water and thereby subjecting it to hydrostatic pressures. This calibration procedure did, however, present some difficulty with temperature compensation due to the fact that the temperature compensating gauges were attached to galvanized metal placed in the center of the cylinder not bonded to the cylinder walls. This resulted in the active and temperature compensating gauges being subjected to different temperatures. This difficulty was overcome by taking the rapid response of the gauges to be from pressure change and the gradual drift of the signal to be from temperature effects.

C. Results

1. Distribution of Withdrawal Force Transmitted to the Soil

A tabular summary of the tests performed and the numerical results obtained is shown in Table 2. These results were used in the determination of a prediction equation for the withdrawal resistance.

Table 2. Withdrawal tests results.

Test No.	Sand Weight, lbs.	Bulk Density, lbs/ft^2	Embedment Depth, in.	Tan μ	Peak Withdrawal Force, lbs.	Time to Peak Force, sec.	Diameter of Found., in.	Comments
62	1090	95.1	20	0.384	22.0	60	3.02	$\mu = 21°$
63	1100	95.8	20	0.384	27.0	12	3.02	
64	1107	96.3	20	0.384	22.2	265	3.02	
65	1112	97.0	20	0.384	34.8	80	3.02	
66	1107	96.3	20	0.384	32.1	80	3.02	
67	1100	95.8	20	0.384	27.8	125	3.02	
68	1100	95.8	20	0.625	----	---	3.06	$\mu = 32°$, Force off scale
69	1096	95.4	20	0.625	54.0	260	3.06	
70	1064	92.8	20	0.625	44.8	145	3.06	
71	----	95.7	10	0.625	18.5	80	3.06	
72	1326	96.2	24	0.625	87.7	225	3.06	Vertical strain measured
73	835	96.9	15	0.625	48.4	205	3.06	
74	1327	96.2	24	0.625	----	---	3.06	Force off scale, Horizontal strain measured

During test number 72 strain gauges were used to measure the vertical force in the foundation at several points below the surface of the sand. This data, along with the total withdrawal force, is shown in Figure 11 as a function of time. Several observations from these graphs are of interest. One observation is that the maximum force transmitted through the foundation occurs later in time as z (the depth below the surface) is increased. The importance of this can be better seen if the withdrawal force transmitted to the soil above the point z is plotted against z for various instants near the time of maximum force. This graph is shown in Figure 12. It can be seen in this graph that before the peak total withdrawal force is reached a larger percentage of the force is resisted near the surface, but as time passes the percentage of force transmitted to the soil near the top decreases. This seems to indicate that failure proceeds from the top to the bottom of the foundation. Also, if a least squares regression of the form $f = az^2$ is fitted to the withdrawal force (f) and the depth (z) at the time of maximum total force, the resulting equation is

$$f = 0.1597z^2 \qquad (4.1)$$

where f is the withdrawal force in pounds and

z is depth below the soil surface in inches

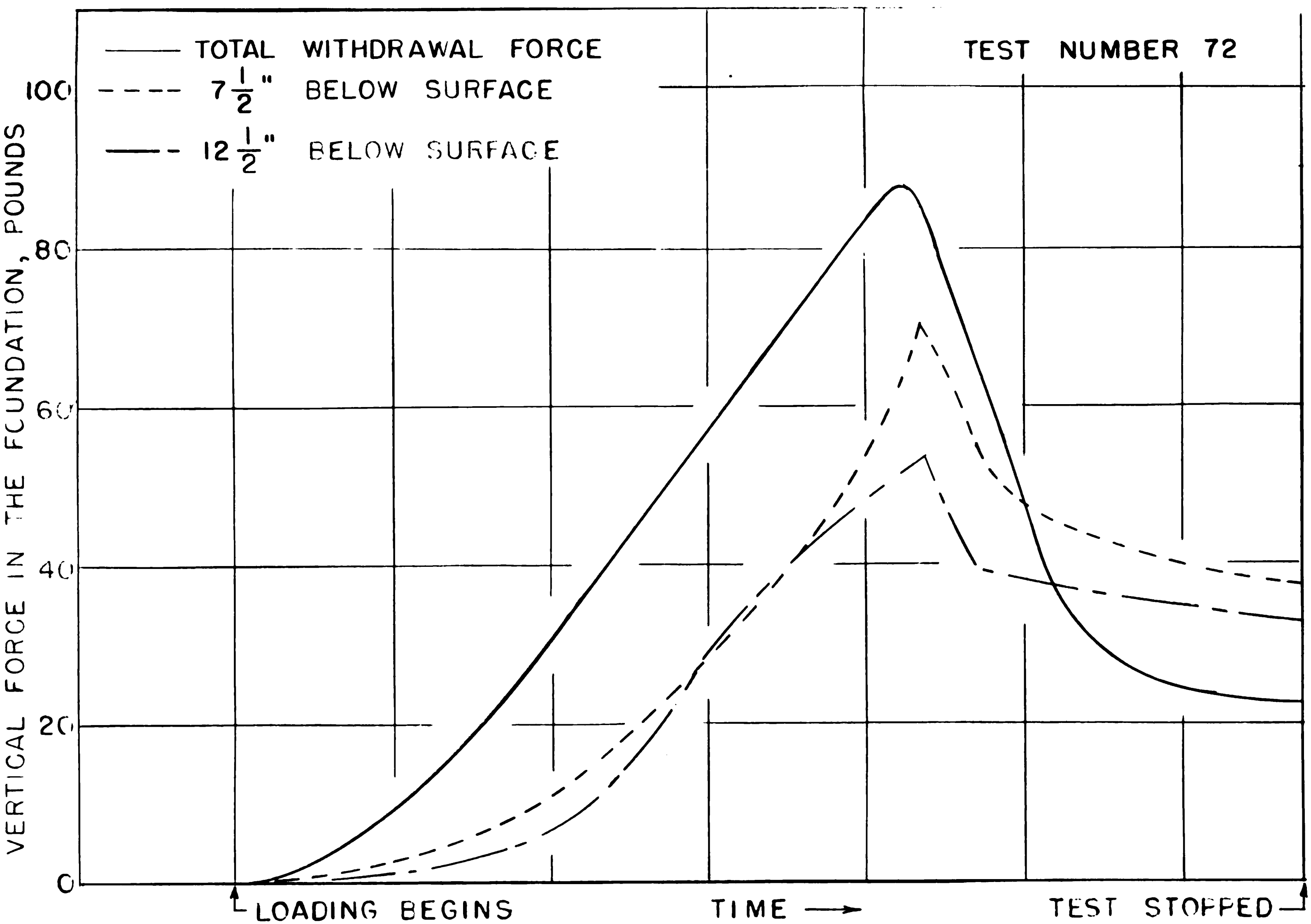

Figure 11. Withdrawal force versus depth at various points below the sand surface.

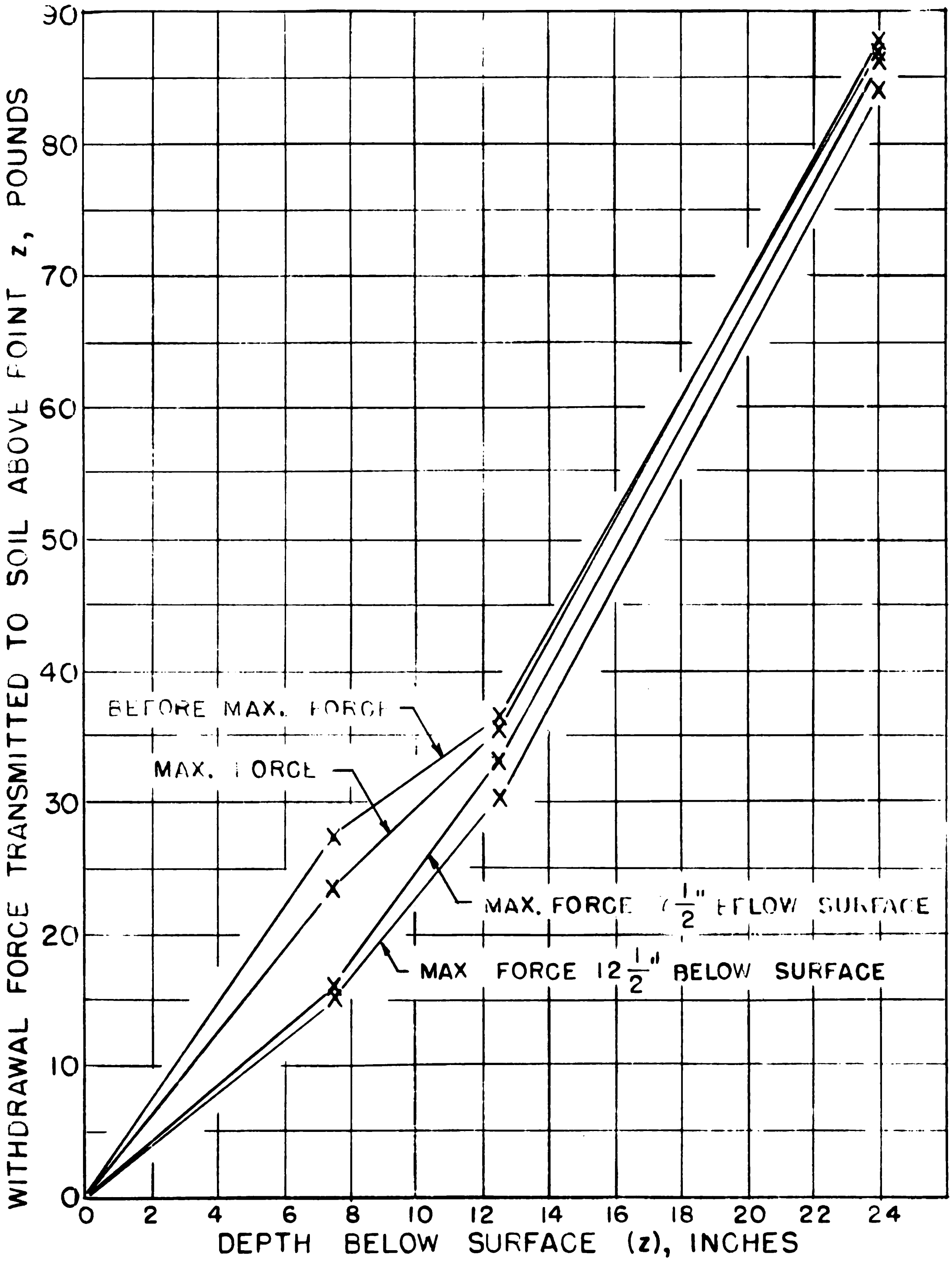

Figure 12. Force transmitted to the soil versus depth of embedment. Forces were selected at several instants of time near the time of maximum withdrawal force.

or

$$f = 23.00z^2 \qquad (4.2)$$

where z is depth below the soil surface in feet.

If the failure occurs as a frictional failure on the foundation-soil interface, then the withdrawal force is given by

$$f = \frac{K\gamma p \tan\mu}{2} z^2 \qquad (4.3)$$

From the above regression line then

$$23.00 = \frac{K\gamma p \tan\mu}{2}$$

All factors except K were measured in test 72, therefore, solving for K one obtains

$$K = 0.948.$$

This value is quite high compared to the coefficient of earth pressure at rest (K_o). For the test sand, using the angle of internal friction, the estimates of K_o would be from 0.4 to 0.6. The difference between the K and K_o values will be discussed more fully in the last part of this section.

2. Horizontal Pressure Variation During Withdrawal

The horizontal pressure acting at two points below the sand was measured during test number 74. The variation of pressure is shown in Figure 13 as a plot of horizontal

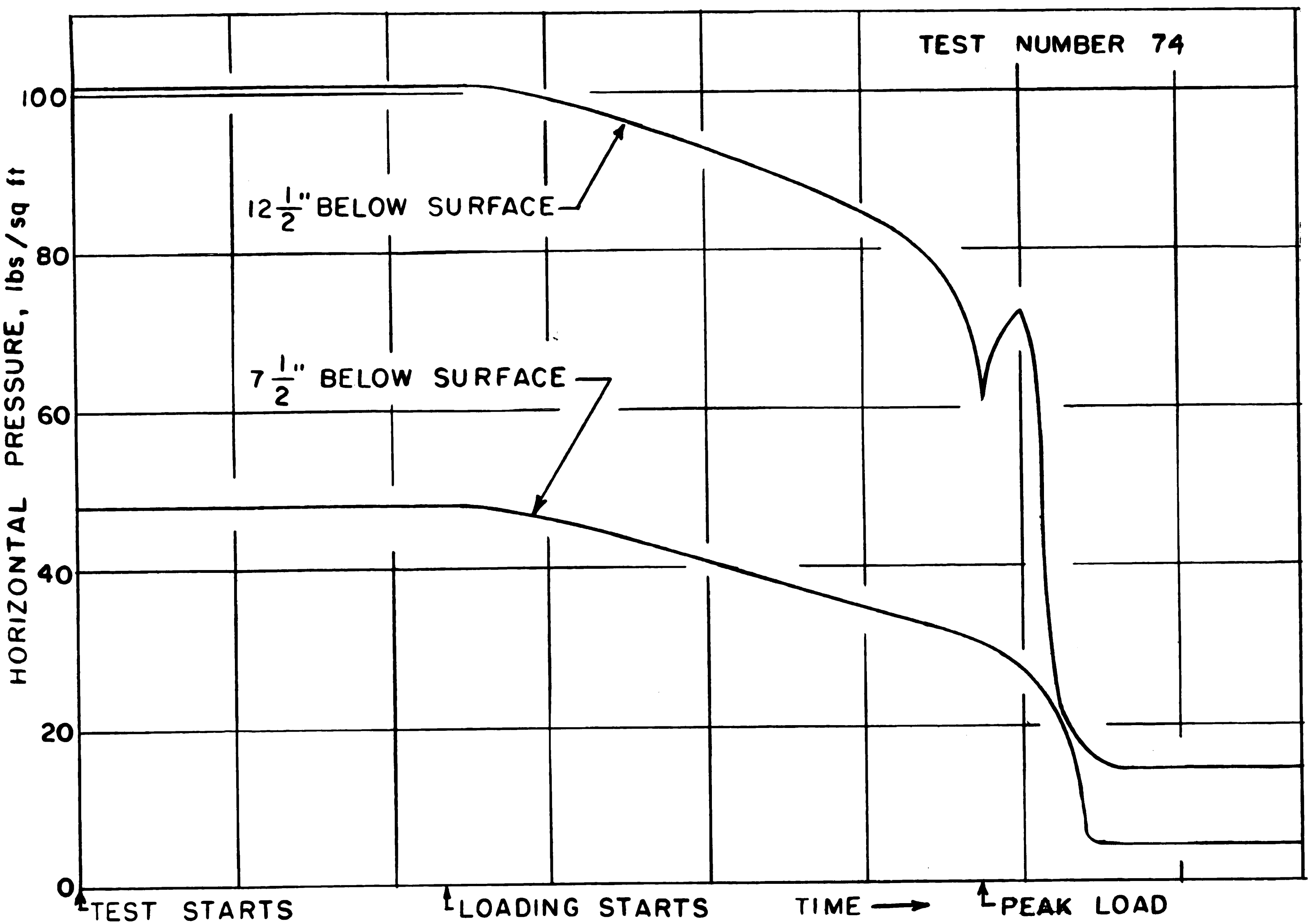

Figure 13. Variation of horizontal pressure during withdrawal.

pressure versus time. These horizontal pressures are not as reliable quantitatively as they are qualitatively. The doubt about their magnitude arises from temperature compensation difficulties during calibration.

Several important observations can be made from the horizontal pressure versus time graph. First, the most evident is that the horizontal pressure decreases during withdrawal. As previously mentioned, this would be expected if the sand was at a density less than the critical density. When comparing the density during this test (96.2 pounds per cubic foot) to the maximum (105.0) and the minimum (94.5) density that was attained in the laboratory, it can be seen that the sand was relatively loose but probably not below the critical density.

One other observation of importance is that the rate of decrease in lateral pressure seems independent of z except after the peak load. After the peak load the pressure at the greater z dropped off most rapidly. This may be due to the sand flowing into the space under the foundation. However, when this is taking place the withdrawal force was less than its maximum value and therefore is of little design importance.

3. Prediction Equation for Withdrawal Forces

A graph of withdrawal force versus depth of embedment is given in Figure 14. Data plotted is from tests

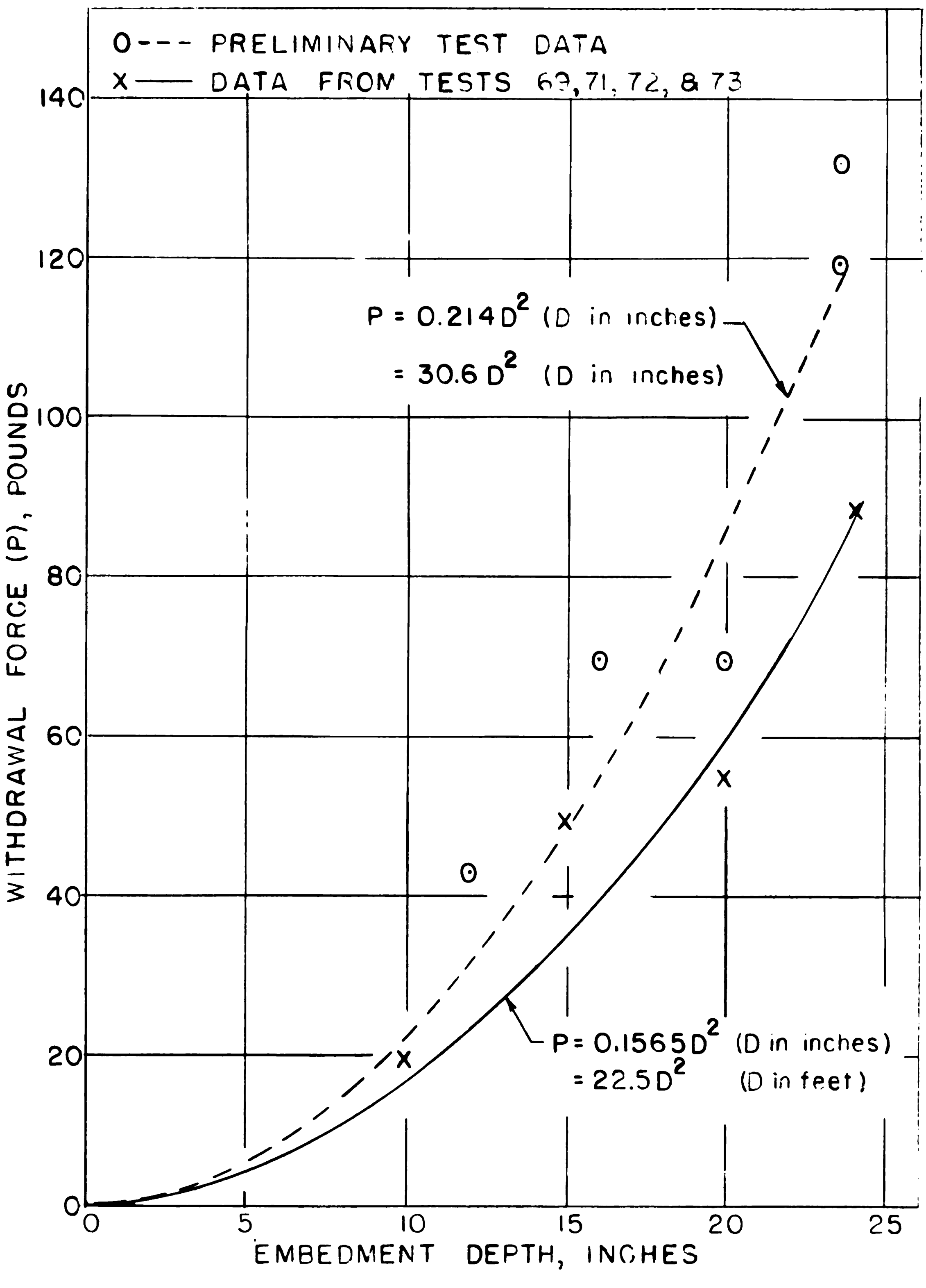

Figure 14. Variation of total withdrawal force with embedment depth.

where only depth was varied and also data from some preliminary tests. A least squares regression line of the form

$$P = aD^2 \tag{4.4}$$

was fitted to the data. The resulting equation was

$$P = 22.5D^2 \tag{4.5}$$

where P is in pounds and D in feet. As was done for the data from the strain gauges measuring vertical force, the withdrawal force equation can be assumed to be

$$P = \frac{K\gamma p \tan\mu}{2} D^2 . \tag{4.6}$$

and the value of K can be determined. For the regression line fitted to the data the calculated value of K would be

$$K = 0.926.$$

Again the value of K is much higher than the coefficient of earth pressure at rest. The K value obtained, however, does agree quite close, 0.948 to 0.926, with the data obtained from the strain gauges. This reinforcement of data taken two different ways suggests that there is not an error in the measurement but either K is really that large or the theory from which K is calculated is not describing the actual phenomenon.

Using the value of K determined from the P versus D data and adjusting to the value of μ used in test number 67 it is found that the equation 3.6 closely predicts the value of total withdrawal force obtained. It therefore is logical to conclude that the equation, as written, can predict the withdrawal force for various values of μ.

As was done in the preceding paragraph one can vary the value of γ as was done in test number 70 and see if the equation derived will predict the total withdrawal force. When this is done, the prediction equation over-estimates the value of the total withdrawal force. This could be expected if one considers that the value of K is altered when one attempts to alter γ.

The fitting of a least squares regression line through the preliminary data also adds evidence that the prediction equation is similar to equation 3.6. In addition to this, the preliminary tests used a different size cylinder (6 inches in diameter), different coefficient of friction, and method of loading (loading with weights to failures). The value of K calculated from this data is

$$K = .893$$

which is approximately the same as previously predicted. The sand was less dense in the preliminary tests and therefore, the lower value of K would be expected.

In Figure 15 is shown some data taken from a publication entitled "Resistance of Steel-and Wood-Pole Foundations to Uplifting and Overturning Forces" by H. T. Hurst and J. P. H. Mason, Jr. (1959). This data is for wood poles being pulled from a hole in soil backfilled with No. 7 gravel. When a regression line, of the form $P = aD^2$, is fitted to this data, it appears that the line predicts very well the total withdrawal force. The value of K was not determined because the value of μ was not known. The value of K tan μ was, however, determined to be

$$K \tan \mu = 3.43.$$

This data reinforces the choice of prediction equation by showing the same type failure for full scale foundations with much larger soil particles but still without cohesive forces.

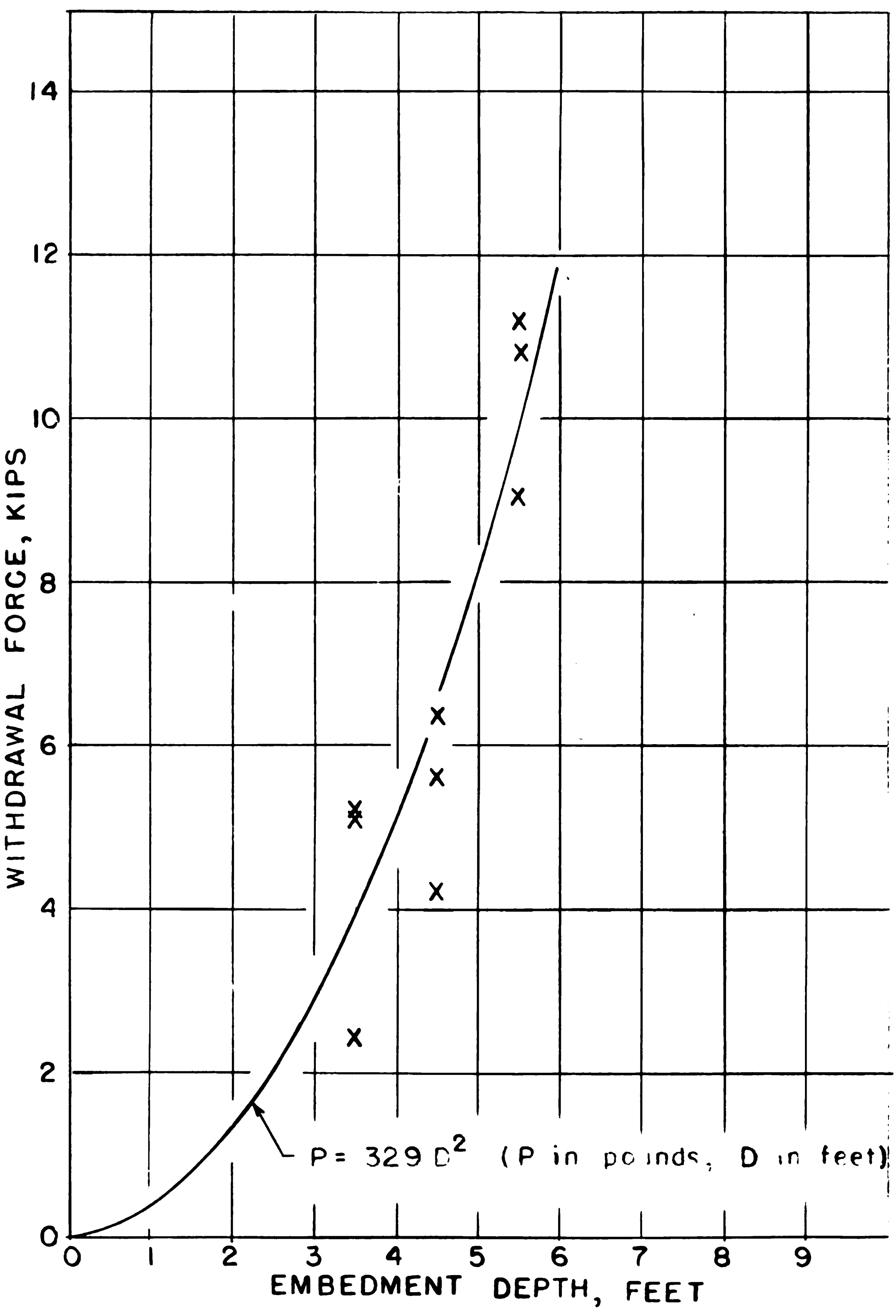

Figure 15. Withdrawal force versus depth for wood poles in gravel.

V. IMPLICATIONS OF THE RESULTS

The prediction equation for withdrawal force as a function of depth has been shown to be of the form

$$P = aD^2 \qquad (5.1)$$

This has been shown by (1) measuring P for various values of D, (2) fitting this form of the equation to data published elsewhere, and (3) measuring the tensile force at several points on an individual foundation being withdrawn from the soil. This form of the equation is what would be expected from theory provided that the following conditions were valid:

1. friction angle (μ) is not a function of confining pressure,
2. horizontal pressure increases linearly with depth, and
3. horizontal pressure is not a function of withdrawal force.

Closer examination of these conditions in this study and by others (1967)* has shown (1) and (3) appearing to be false. The friction angle decreases with greater confining

*Coyle, et al. (1967) reported on these factors in November after this research was completed.

pressure and the horizontal pressure decreases when withdrawal forces are applied. Both of these relationships will tend to decrease the value of "a". The magnitude of the change of the friction angle with various normal pressures can be seen from the data shown in Table 3 of the Appendix.

It was also shown that if the value of the friction angle and/or the diameter of the foundation was changed by a specific amount, the withdrawal force could be predicted by

$$P = (a_1 p \tan \mu) D^2 \qquad (5.2)$$

and with less accuracy the equation

$$P = (a_2 p\gamma \tan \mu) D^2$$

will account for the variation in soil density.

Now the only unknown quantity is the value of a_2 which, according to equation 4.6, is equal to 2K. The value of K was solved for in tests where P was measured for various values of D, in the equation for shear transferred to the soil as a function of depth, and in preliminary tests of P versus D. The value of K determined in each of these three ways was 0.926, 0.948, and 0.893 with an average of 0.92. This value of K is approximately twice as large as K_o given in most references in soil mechanics but less than some values reported in other published research. Most of the other values published are for piles driven in the ground and therefore, should more nearly approximate passive earth pressures. The value of K determined from the tests indicates that withdrawal of a foundation is resisted by pressures greater than earth pressures at rest but less than passive earth pressures or, that failure occurs in another way that has the effect of giving the same form of withdrawal force transfer to the soil as a function of depth.

It was shown that the coefficient of earth pressure decreases when withdrawal forces are applied. In the test soil used this means that the tension applied to the soil in the radial direction by the withdrawal forces and the sand flowing into the vacated space below the foundation combine to more than compensate for any dilatant expansion of the soil.

Two solutions for stresses and displacements in the soil mass have been found. The assumptions of the solution are that:

1. the soil is isotropic, homogeneous and an ideal Hookean solid, and
2. a shear stress on the foundation-soil interface is constant in the first solution and can take any form in the second solution.

The solution must be evaluated by numerical methods. The evaluated solution will give a picture of the stress pattern in the soil at low loads.

VI. RECOMMENDATIONS FOR FURTHER STUDY

The total problem of foundation withdrawal could be thought of as consisting of four parts. They are (1) pure uplift in a cohesionless soil, (2) pure uplift in a cohesive soil, (3) oblique forces on a foundation in a cohesionless soil, and (4) oblique forces on a foundation in a cohesive soil. As was indicated in the first section of this thesis, effort in this study was concentrated on part (1) of the above list. The remaining three parts are of interest, and in fact more nearly represent the actual situation in practically all cases. The interaction of withdrawal and overturning moments should be of major interest and value. The three remaining parts are recommended for future study.

The mathematical analysis herein presented represents a basic approach to the problem which can be refined by meeting the boundary conditions when z becomes infinite and can be numerically evaluated to obtain the qualitative and possible quantitative variation of stresses and displacements. The solutions obtained herein could be applied to many problems and need not be confined to the usual concept of foundations.

SELECTED REFERENCES

Bishop, Alan W. (1958). Test Requirements for Measuring the Coefficient of Earth Pressure at Rest. Volume 1, pp. 2-14 in Brussels Conference 58 on Earth Pressure Problems. Published with financial assistance of UNESCO. Brussels. 158 pp.

Burmister, D. M. (1940). Stress Distribution for Pile Foundations, pp. 339-341, Proceedings of the Purdue Conference on Soil Mechanics and Its Applications, Purdue University, Lafayette, Indiana. 482 pp.

Coyle, Harry M., and Ibrahim H. Sulaiman (1967). Skin Friction for Steel Piles in Sand. Journal of the Soil Mechanics and Foundations Division of ASCE. 93:261-278.

Fung, Y. C. (1965). Foundations of Solid Mechanics. Prentice-Hall, Inc., Englewood Cliffs, New Jersey. 525 pp.

Hildebrand, F. B. (1962). Advanced Calculus for Applications. Prentice-Hall, Inc., Englewood Cliffs, New Jersey. 646 pp.

Hurst, H. T. (1959). Resistance of Steel and Wood-Pole Foundations to Uplifting and Overturning Forces. Virginia Agricultural Experiment Station Research Rept. No. 28, Virginia Polytechnic Institute, Blacksburg, Virginia. 16 pp.

Ireland, H. O. (1957). Pulling Tests on Piles in Sand. Volume 2, pp. 43-45, in Proceedings of the Fourth Conference on Soil Mechanics and Foundation Engineering, Butterworth Scientific Publications, London. 467 pp.

Kirkpatrick, W. M. (1957). The Condition of Failure for Sands. Volume 1, pp. 172-178, in Proceedings of the Fourth International Conference on Soil Mechanics and Foundation Engineering, Butterworth Scientific Publications, London. 466 pp.

Kolbuszewski, J. (1958). Fundamental Factors Affecting Experimental Procedures Dealing with Pressure Distribution in Sands. Volume 1, pp. 71-83, Brussels Conference on Earth Pressure Problems, Published with the financial aid of UNESCO. Brussels. 158 pp.

Kopacsy, J. (1957). Three-dimensional Stress Distribution and Slip Surfaces in Earth Works at Rupture. Vol. 1, pp. 339-342 in Proceedings of the Fourth International Conference on Soil Mechanics and Foundation Engineering. Butterworth Scientific Publications, London. 466 pp.

Lambe, William T. (1951). Soil Testing For Engineers. John Wiley and Sons, Inc., New York. 165 pp.

Leps, T. M. (1939). The Effect of Gradation on the Shearing Properties of a Cohesionless Soil. Thesis for the degree of M.S., Massachusetts Institute of Technology, Cambridge, Mass. (Unpublished).

Leonards, G. A., et al. (1962). Foundation Engineering. McGraw-Hill Book Co., Inc., New York. 1136 pp.

Lundgren, Raymond (1967). How to Design Piles Against Uplift. Wood Preserving News, March, pp. 6-11.

Mindlin, Raymond (1936). Force at a Point in the Interior of a Semi-Infinite Solid. Physics 7: 195-202.

Nishida, Yoshichika (1957). An Estimation of the Point Resistance of a Pile. Journal of Soil Mechanics and Foundation Division of ASCE, 83: SM2, paper 1206.

Pickett, Gerald (1944). Application of the Fourier Method to the Solution of Certain Boundary Problems in the Theory of Elasticity. Journal of Applied Mechanics Division of ASME. 66:A176-A182.

Potyondy, J. G. (1961). Skin Friction Between Various Soils and Construction Materials. Geotechnique 11:339-351.

Rocha, M. (1957). The Possibility of Solving Soil Mechanics Problems by the Use of Models. Volume 1, pp. 183-188, Proceedings of the Fourth International Conference on Soil Mechanics and Foundation Engineering. Butterworth Scientific Publications, London. 466 pp.

Ruderman, J. (1939). Stress Distribution Around a Loaded Pile. Thesis for the degree of M.S. Civil Engineering Dept., Columbia University, New York. (Unpublished).

Scott, Ronald F. (1963). Principles of Soil Mechanics. Addison-Wesley Publishing Company, Inc., Reading, Mass. 550 pp.

Taylor, D. W., and T. M. Leps (1939). Shearing Properties of Ottawa Standard Sand as Determined by the M.I.T. Stain-Control Direct Shearing Machine, Record of Proceedings of Conference on Soils and Foundations, Corps of Engineers, Boston, Massachusetts.

Terzaghi, Karl (1943). Theoretical Soil Mechanics. John Wiley and Sons, Inc., New York. 510 pp.

Timoshenko, S., and J. N. Goodier (1951). Theory of Elasticity. McGraw-Hill Book Company, Inc., New York. 506 pp.

Winslow, A. M. (1950). Differentiation of Fourier Series in Stress Solutions for Rectangular Plates. Quarterly Journal of Mechanics and Applied Mathematics, 4:449-460.

Yoshimi, Yoshiaki (1964). Piles in Cohesionless Soil Subjected to Oblique Pull. Journal of Soil Mechanics and Foundations Division of ASCE. 90:SM6, 11-24.

APPENDIX

Table 3. Coefficient of friction test results.

Test No.	Surface	Normal force lbs/sq. ft.	Friction Angle	Comments
1	Galvanized Metal	1.1	21.0°	Average 21.2°
2	Galvanized Metal	1.1	20.0°	
3	Galvanized Metal	1.1	21.0°	
4	Galvanized Metal	1.1	22.0°	
5	Galvanized Metal	1.1	22.0°	
6	Emery Cloth	1.1	33.0°	Average 32.1°
7	Emery Cloth	1.1	33.0°	
8	Emery Cloth	1.1	32.5°	
9	Emery Cloth	1.1	31.0°	
10	Emery Cloth	1.1	31.0°	
11	Emery Cloth	1.1	31.0°	Average 31.6°
12	Emery Cloth	1.1	32.0°	
13	Emery Cloth	1.1	32.3°	
14	Emery Cloth	1.1	31.0°	
16	Emery Cloth	167	30.0°	Average 29.4°
17	Emery Cloth	167	30.0°	
18	Emery Cloth	167	28.0°	
19	Emery Cloth	167	29.5°	