THESIS

This is to certify that the

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THE EFFECT OF CURVATURE AND TORSION ON THE

TEMPERATURE DISTRIBUTION IN A HELIX

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David D. Sayers

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Merle C. Potter
Major professor

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THE EFFECT OF CURVATURE AND TORSION ON THE TEMPERATURE DISTRIBUTION IN A HELIX

By

David D. Sayers

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ABSTRACT

THE EFFECT OF CURVATURE AND TORSION ON THE

TEMPERATURE DISTRIBUTION IN A HELIX

By

David D. Sayers

Traditional analysis treats the helix as a straight wire with the effects of nonuniform heating, torsion and large curvature ignored. Using a helical coordinate system the governing partial differential equation including these effects is derived. The equation is then solved numerically using the finite element method.

The results indicate a strong dependence of the temperature on the torsion parameter when the curvature parameter is significant. As the curvature parameter increases the temperature distribution becomes skew-symmetric and the maximum temperature in the helix increases. Non-uniform heating influences the temperature distribution independent of the curvature and torsion.

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NOMENCLATURE

Letters

a	Wire radius [m]
b	Proportional to the distance between loops [m]
c	Radius of the helix [m]
$g_{ij} = g_{ij}$	Metric tensors
k	Helix material thermal conductivity [W/m·°C]
N	Shape function
q_o	Heat generation per unit volume [W/m^3]
r	Radius coordinate [m]
s	Arc length [m]
T	Temperature [°C]
T_o	Helix surface temperature [°C]
$\overline{X}$	Global coordinate [m]
x	Local coordinate [m]
X	Dimensionless local coordinate
$\overline{Y}$	Global coordinate [m]
y	Local coordinate [m]
Y	Dimensionless local coordinate
$\overline{Z}$	Global coordinate [m]

Greek Symbols

ϕ	Dimensionless temperature
δ	Nonuniform heating parameter [$°C^{-1}$]

λ	Dimensionless nonuniform heating parameter
τ	Torsion $[m^{-1}]$
β	Dimensionless torsion parameter
α	Curvature $[m^{-1}]$
ε	Dimensionless curvature parameter
θ	Angle coordinate
Γ^{k}_{ij}	Christoffel symbol
ξ	Local element coordinate
η	Local element coordinate

<u>Vectors</u>

$\hat{B}$	Unit binormal vector in the helical system
$\hat{i}$	Unit vector in the global system
$\hat{j}$	Unit vector in the global system
$\hat{k}$	Unit vector in the global system
$\hat{N}$	Unit normal in the helical system
$[N]$	The eight shape functions
$\vec{P}$	Position vector of a point in the helix [m]
$\vec{R}$	Radius vector to the helix centerline [m]
$\hat{T}$	Unit tangent vector in the helical system
$\{X\}$	The eight nodal coordinates
$\{Y\}$	The eight nodal coordinates
$\{\phi\}$	The eight nodal temperatures

CHAPTER I

INTRODUCTION

1.1 REVIEW OF LITERATURE

The temperature distribution and the maximum temperature in a helix subject to internal heat generation is the topic of this study. A situation in which this information would be necessary would be in the design of an electrical space heater. Traditional analysis often treats the helix as a straight cylinder with the effects of curvature,[1/] torsion,[2/] and nonuniform heating ignored. Jakob [1] was the first to deal with the effect of variable heat flux on the temperature distribution in a solid, straight cylinder. All effects relating to curvature and torsion were ignored because they were believed to be negligible. Jakob's solution to the governing differential equation involves a combination of zeroth order Bessel functions of the first kind. It was observed that a temperature of infinity is predicted at the first zero of the zeroth order Bessel function. Physically, heat is being generated so fast near the center of the coil that conduction can no longer carry it away and the coil melts. Jakob also noted that this occurs at a surprisingly low current.

[1/] The radius of curvature is the minimum radius of a segment of the coil; curvature is the inverse of this radius of curvature.

[2/] Torsion is related to the distance between the loops measured along the axis of the coil.

In his second work, Jakob [2] was the first to discover that as much as 40 percent more heat per unit volume is generated at the warmest spot in the coil as compared to a location just below the surface, this is due to the increase in electrical resistance with temperature. His work also showed that nonuniform generation of heat does not significantly affect the shape of the temperature profile in the coil, however, it does affect the maximum temperature generated in the coil.

Nicholson [3] was the first to attempt a definition of a helical coordinate system (see Figure 1). The coordinate system was erroneously thought to be orthogonal and was subsequently published as such (see reference [4]).

The helical coordinate system was first correctly defined by Wang [5]. Wang used this coordinate system to solve for the effect of curvature and nonuniform heat generation on the temperature distribution in a helix for small curvatures. The basic solution was found to be a combination of Bessel functions of the first kind of the zeroth order; the first and second correction terms for curvature and nonuniform heating involved Bessel functions of the first kind of zeroth, first and second order. Wang also correctly noted that torsion was insignificant for small curvatures.

1.2 OBJECTIVE OF THE PRESENT STUDY

The objective of the present study is to investigate the effect of curvature and torsion on the temperature distribution in a helix. The helix is assumed to be subjected to internal heat generation due to Joule heating (I^2R). Allowance is made for the variation of the electrical resistance with temperature; this leads to nonuniform heat generation. It is assumed that the outer surface of the helix is

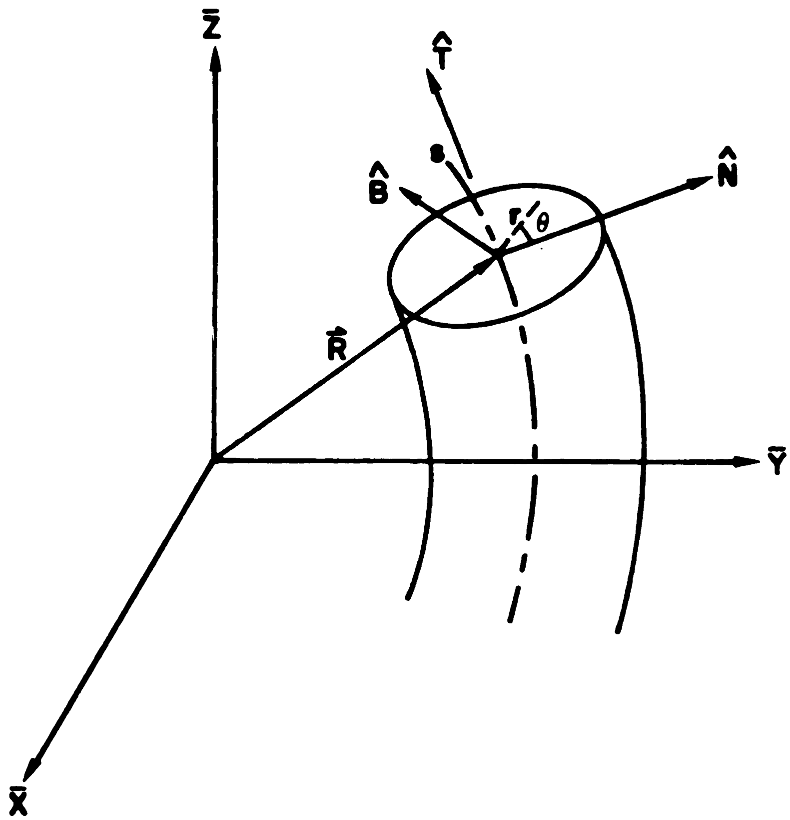


Figure 1.--The Helical Coordinate System

maintained at a uniform temperature T_0 by forced convection. The difference between this study and previous work is that the effect of torsion is included along with the effects of curvature and nonuniform heating. Previously, nonuniform heating had been modeled, neglecting the effects of torsion and large curvature; Wang [5] did include the effect of small curvature. When torsion is neglected and curvature is assumed small, the solution to the problem can be determined analytically. The present study does not neglect torsion and large curvature and as a result the governing equation becomes more complicated and must be solved numerically. The finite element method was selected as the method of solution. Finite elements were used because grids can be changed rapidly and variable coefficients do not make the solution significantly more difficult.

CHAPTER II

FORMULATION OF THE PROBLEM

The mathematical formulation of the problem is discussed in this Chapter. The partial differential equation which describes the problem is presented in cylindrical coordinates. Since the finite element method is usually formulated in rectangular coordinates, a transformation is made from cylindrical to rectangular coordinates. The dependent and independent variables are then expressed in dimensionless form in order to identify the governing parameters and to generalize the results.

2.1 COORDINATE SYSTEM

The radius vector locating a point on the centerline is described by the space curve

$$\vec{R}(s) = \overline{X}(s)\hat{i} + \overline{Y}(s)\hat{j} + \overline{Z}(s)\hat{k} \tag{1}$$

The helical coordinate system with unit vectors $(\hat{T}, \hat{N}, \hat{B})$ can be used to describe the radius vector of Equation (1), as shown in Figure 1. The vectors $\hat{T}$, $\hat{N}$ and $\hat{B}$ are mathematically defined by

$$\hat{T} = \frac{d\vec{R}}{ds} \tag{2}$$

$$\hat{N} = \frac{1}{\alpha}\frac{d\hat{T}}{ds} \tag{3}$$

$$\hat{B} = \hat{T} \times \hat{N} \tag{4}$$

Here α is the curvature and $\hat{T}$, $\hat{N}$ and $\hat{B}$ are orthogonal unit vectors. Using the Frenet formulas, the following relationships result:

$$\frac{d\hat{N}}{ds} = \tau\hat{B} - \alpha\hat{T} \tag{5}$$

$$\frac{d\hat{B}}{ds} = -\tau\hat{N} \tag{6}$$

where τ is the torsion. The coordinate system (r, θ, s), shown in Figure 1, is constructed such that the Cartesian position vector $\vec{P}$ can be expressed as

$$\vec{P}(s) = \vec{R}(s) + r\cos\theta\ \hat{N}(s) + r\sin\theta\ \hat{B}(s) \tag{7}$$

Note that when θ is zero, $\hat{N}$ and r are colinear. Using Equations (2) through (7) there results

$$\begin{aligned} d\vec{P}\cdot d\vec{P} = (dr)^2 + r^2(d\theta)^2 + [(1-\alpha r\cos\theta)^2 + \tau^2 r^2]\ (ds)^2 \\ + 2\tau r^2 ds\ d\theta \end{aligned} \tag{8}$$

The metric tensors g_{ij} and g^{ij} and the Christoffel symbols Γ^k_{ij} are necessary when expressing the tensorial heat transfer equation in the chosen coordinate system. The metric tensors g_{ij} and g^{ij} are expressed, referring to Equation (8), as

$$\begin{aligned} &g_{11} = g^{11} = 1, \quad g_{22} = r^2, \quad g^{22} = G/r^2M \\ &g_{33} = G, \quad g^{33} = \frac{1}{M}, \quad g_{23} = \tau r^2, \quad g^{23} = -\tau/M \\ &g_{12} = g^{12} = g_{13} = g^{13} = 0 \end{aligned} \tag{9}$$

where

$$G = (1 - \alpha r\cos\theta)^2 + \tau^2 r^2 \tag{10}$$

$$M = (1 - \alpha r\cos\theta)^2 \tag{11}$$

The nonzero Christoffel symbols are

$$\Gamma^1_{22} = -r, \quad \Gamma^1_{23} = -\tau r, \quad \Gamma^1_{33} = -\frac{1}{2}\frac{\partial G}{\partial r}, \quad \Gamma^2_{21} = \frac{1}{r}$$

$$\Gamma^2_{13} = \frac{\tau}{M}\left(\frac{G}{r} - \frac{1}{2}\frac{\partial G}{\partial r}\right), \quad \Gamma^2_{23} = \frac{-\tau}{2M}\frac{\partial G}{\partial \theta}, \quad \Gamma^2_{33} = \frac{-G}{2r^2 M}\frac{\partial G}{\partial \theta} \tag{12}$$

$$\Gamma^3_{13} = \frac{-\tau^2 r}{M} + \frac{1}{2M}\frac{\partial G}{\partial r}, \quad \Gamma^3_{23} = \frac{1}{2M}\frac{\partial G}{\partial \theta}, \quad \Gamma^3_{33} = \frac{\tau}{2M}\frac{\partial G}{\partial \theta}$$

These will now be used to write the governing equation in the coordinate system chosen.

2.2 GOVERNING EQUATION

The steady-state, tensorial heat transfer equation for conduction in a solid with heat generation [5] is

$$g^{ij}\left[\frac{\partial^2 T}{\partial x^i \partial x^j} - \Gamma^k_{ij}\frac{\partial T}{\partial x^k}\right] = \frac{-q_o}{k} \tag{13}$$

where q_o accounts for the heat generation per unit volume. Substituting Equations (9) through (12) into Equation (13) results in the partial differential equation which describes the temperature field, namely,

$$T_{rr} + \left(1 - \frac{\alpha r\cos\theta}{1 - \alpha r\cos\theta}\right)\frac{1}{r}T_r + \left(\frac{\alpha\sin\theta}{1 - \alpha r\cos\theta}\right)\left[1 - \frac{\tau^2 r^2}{(1 - \alpha r\cos\theta)^2}\right]\frac{1}{r}T_\theta$$
$$+ \left[1 + \frac{\tau^2 r^2}{(1 - \alpha r\cos\theta)^2}\right]\frac{1}{r^2}T_{\theta\theta} = \frac{-q_o}{k}\left[1 + \delta(T - T_o)\right] \tag{14}$$

where the curvature is expressed as

$$\alpha = \frac{c}{b^2+c^2} \tag{15}$$

and the torsion is

$$\tau = \frac{b}{b^2+c^2} \tag{16}$$

The parameter δ is a measure of the nonuniform heating, a uniform heat flux occurring when $\delta = 0$. The heat flux is assumed to be produced by electrical resistance (Joule) heating. The resistance of a material is not constant but varies with temperature; δ is a means to provide for this variation. As an example, copper has a value of δ equal to $0.004°C^{-1}$.

Equation (14) is linear, nonhomogeneous and has variable coefficients. The boundary conditions are

$$T \text{ is finite at } r = 0 \tag{17}$$

$$T = T_o \text{ at } r = a \tag{18}$$

This last condition is assumed to be maintained by forced convection.

2.3 TRANSFORMATION OF THE GOVERNING EQUATION

Equation (14) was developed in the helical coordinate system. A rectangular coordinate system (x, y) is generally used with the finite element method; therefore, Equation (14) was transformed to the rectangular coordinate system. The standard transformation relations, shown in Figure 2, are

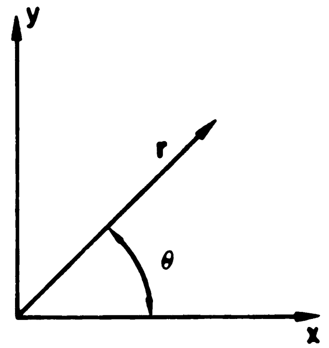


Figure 2.--The Cylindrical Coordinate System

$$x = r\cos\theta \tag{19}$$

$$y = r\sin\theta \text{ and} \tag{20}$$

$$x^2 + y^2 = r^2 \tag{21}$$

Using the chain rule

$$T_r = T_x x_r + T_y y_r \tag{22}$$

$$T_\theta = T_x x_\theta + T_y y_\theta \tag{23}$$

$$T_{rr} = T_{xx}(x_r)^2 + 2T_{xy}x_r y_r + T_{yy}(y_r)^2 \text{ and} \tag{24}$$

$$T_{\theta\theta} = T_{xx}(x_\theta)^2 + 2T_{xy}x_\theta y_\theta + T_{yy}(y_\theta)^2 \tag{25}$$

Substituting Equation (19) through (25) into Equation (14) results in

$$T_{xx}[x^2+y^2(1 + \frac{\tau^2 r^2}{(1-\alpha x)^2})] + T_{yy}[y^2+x^2(1 + \frac{\tau^2 r^2}{(1-\alpha x)^2})]$$

$$- 2T_{xy}\frac{xy\tau^2 r^2}{(1-\alpha x)^2} + T_x[\frac{-\alpha x^2}{1-\alpha x} - \frac{\alpha y^2}{1-\alpha x}(1 - \frac{\tau^2 r^2}{(1-\alpha x)^2}) - \frac{x\tau^2 r^2}{(1-\alpha x)^2}] \tag{26}$$

$$+ T_y[\frac{xy\alpha}{1-\alpha x}(\frac{-\tau^2 r^2}{(1-\alpha x)^2}) - \frac{y\tau^2 r^2}{(1-\alpha x)^2})] =$$

$$\frac{-q_o}{k}[1 + \delta(T-T_o)](x^2+y^2)$$

The quantities in this equation are dimensional; it is more convenient to work with normalized, dimensionless quantities.

2.4 DIMENSIONLESS PARAMETERS

The transformed Equation (26), can be placed in a more general form by introducing dimensionless parameters. Choose as reference quantities the helix radius (a), the thermal conductivity (k) and the heat flux (q_o). The dimensionless variables are then

$$\varepsilon = \alpha a \tag{27}$$

$$\beta = \tau a/\varepsilon = b/c \tag{28}$$

$$Y = y/a \tag{29}$$

$$X = x/a \tag{30}$$

$$\phi = (T-T_o)k/q_o a^2 \tag{31}$$

$$\lambda^2 = a^2 q_o \delta/k \quad \text{and} \tag{32}$$

$$D = \frac{\beta^2\varepsilon^2(X^2+Y^2)}{(1-\varepsilon X)^2} \tag{33}$$

Substituting these dimensionless groups into Equation (26) the partial differential equation that will be solved is

$$\phi_{XX}[X^2 + Y^2(1+D)] + \phi_{YY}[Y^2+X^2(1+D)] + \phi_{XY}[-2XYD]$$

$$+ \phi_X[\frac{-\varepsilon X^2}{1-\varepsilon X} - \frac{\varepsilon Y^2}{1-\varepsilon X}(1-D) - XD] + \phi_Y[\frac{-\varepsilon X}{1-\varepsilon X} - 1]\ YD \tag{34}$$

$$+ [1+\lambda^2\phi]\ (X^2+Y^2) = 0$$

with the boundary conditions

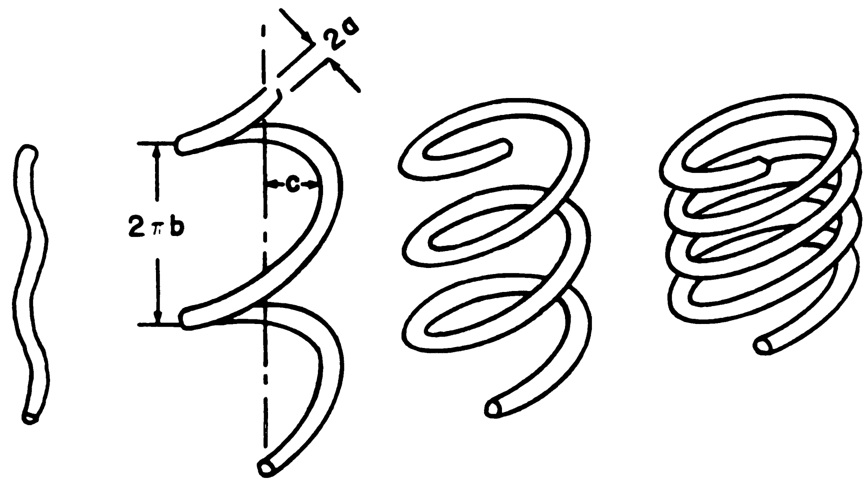
$$\phi = 0 \text{ at } X^2 + Y^2 = 1 \tag{35}$$

$$\phi \text{ is finite at } X^2 + Y^2 = 0 \tag{36}$$

All of the quantities in Equation (34) are dimensionless and ϕ is of order one.

2.5 LIMITS ON PARAMETERS

Several of the dimensionless parameters have bounds which are not apparent from their definitions. For example, ε is bounded by the fact that loops of the helix can at most touch each other; it is not physically possible for different coil loops to occupy the same physical space. This limits ε to a maximum of one. A zero value for ε corresponds to a straight wire and the solution is independent of β. Conversely, if β goes to infinity the solution also approaches the straight wire case (see Figure 3). Also, at λ equal 2.4048 a singularity exists such that λ is limited by this value.



(a) (b) (c) (d)

Figure 3.--Helical coils with ε=0.1 (a) β=3.0, (b) β=0.5, (c) β=0.2 and (d) β=0.1

CHAPTER III

FINITE ELEMENT METHOD

The finite element method is a numerical procedure with the following two primary characteristics:

1. The problem is expressed in integral form.
2. The solution is approximated by a piecewise smooth continuous function.

It utilizes elements that are not necessarily of the same shape but must contain the same number of nodes. For a given number of elements, as the number of nodes per element is increased the accuracy of the solution increases, as does the time required by the numerical procedure.

3.1 THE QUADRILATERAL ELEMENT

The two-dimensional, eight node, quadratic, quadrilateral element was used in this study; it is shown in Figure 4.

The nodes were numbered starting in the lower left corner and proceeding counterclockwise around the element. The eight shape functions are:

$$\begin{aligned}
N_1 &= -\frac{1}{4}(1-\xi)(1-\eta)(\xi+\eta+1) \\
N_2 &= \frac{1}{2}(1-\xi^2)(1-\eta) \\
N_3 &= \frac{1}{4}(1+\xi)(1-\eta)(\xi-\eta-1) \\
N_4 &= \frac{1}{2}(1-\eta^2)(1+\xi)
\end{aligned} \tag{37}$$

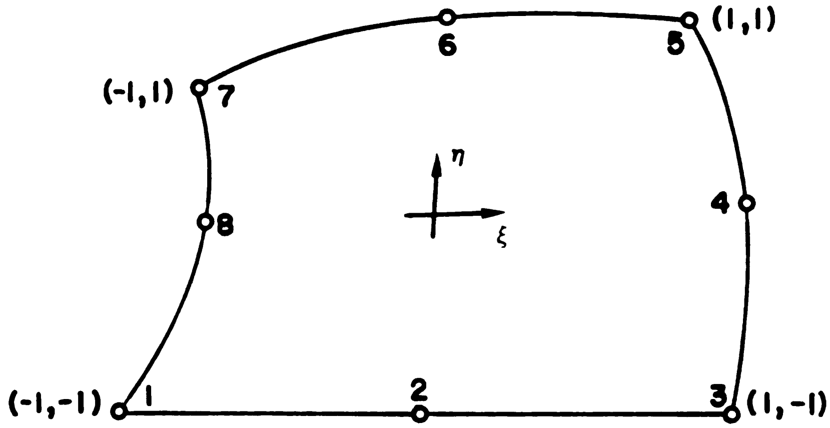


Figure 4.--A Typical Finite Element with Nodes

$$
\begin{aligned}
N_5 &= \frac{1}{4}(1+\xi)(1+\eta)(\xi+\eta-1) \\
N_6 &= \frac{1}{2}(1-\xi^2)(1+\eta) \\
N_7 &= -\frac{1}{4}(1-\xi)(1+\eta)(\xi-\eta+1) \text{ and} \\
N_8 &= \frac{1}{2}(1-\eta^2)(1-\xi)
\end{aligned}
\tag{37}
$$

The shape functions have the following properties:

1. They have a value of one at their node.
2. They have a value of zero at every other node in the element.
3. They sum to one at any point.

A local (ξ, η) coordinate system is defined for the element to ease the task of numerical integration. This local coordinate system is curvilinear and is oriented so that the corners of the element have the values shown in Figure 4. The X and Y coordinates at any point in the element are given by

$$X = [N]\{X\} \quad \text{and} \tag{38}$$

$$Y = [N]\{Y\} \tag{39}$$

where the components of the vector [N] are the eight shape functions evaluated at the desired point (ξ, η) and the components of the vectors {X} and {Y} are the eight values of the X and Y coordinates of the element nodes. Similarly, the temperature at any point is

$$\phi = [N]\{\phi\} \tag{40}$$

where the components of the vector $\{\phi\}$ are the eight nodal temperatures.

3.2 GALERKIN'S METHOD

Galerkin's method is a weighted residual method used to solve a partial differential equation. When Galerkin's method is used in conjunction with finite elements the formulation of the problem, from [6], demands that

$$\begin{aligned}\int_A [N]^T \{\phi_{XX}[X^2+Y^2(1+D)] &+ \phi_{YY}[Y^2+X^2(1+D)] + \phi_{XY}[-2XYD] \\ &+ \phi_X[\frac{-\varepsilon X^2}{1-\varepsilon X} - \frac{\varepsilon Y^2}{1-\varepsilon X}(1-D) - XD] + \phi_Y[\frac{-\varepsilon X}{1-\varepsilon X} - 1]\ YD \\ &+ [1+\lambda^2\phi](X^2+Y^2)\}dA = 0\end{aligned} \tag{41}$$

where the shape functions serve as weighting functions. Equation (41) is used to determine the element matrices which are then summed to determine the global matrices. The global matrices are then solved for the nodal values.

3.3 DEVELOPMENT OF EQUATIONS

The element matrices are calculated from the integral formulation of Galerkin's method using Equation (41). Integrals involving first order derivatives or no derivatives can be evaluated directly. Integrals involving second order derivatives must be expressed in terms of first order terms because the temperature equation does not have a continuous first derivative between elements.

This is accomplished by using the product rule,

$$\int_A \frac{\partial}{\partial X}(f\psi)dA = \int_A \frac{\partial f}{\partial X}\,\psi\, dA + \int_A f\,\frac{\partial \psi}{\partial X}\, dA \tag{42}$$

For example, the integral

$$\int_A \frac{\partial}{\partial X}[[N]^T \frac{\partial \phi}{\partial X}]\, dA \tag{43}$$

can be expressed using Equation (42) as

$$\int_A \frac{\partial}{\partial X}\,[[N]^T \frac{\partial \phi}{\partial X}]\, dA = \int_A \frac{\partial [N]^T}{\partial X}\,\frac{\partial \phi}{\partial X}\, dA + \int_A [N]^T\,\frac{\partial^2 \phi}{\partial X^2} = dA \tag{44}$$

Green's theorem can then be used to simplify the formulation to

$$\int_L [N]^T\,\frac{\partial \phi}{\partial X}\, dL = \int_A \frac{\partial [N]^T}{\partial X}\,\frac{\partial \phi}{\partial X}\, dA + \int_A [N]^T\,\frac{\partial^2 \phi}{\partial X^2}\, dA \tag{45}$$

where L is a closed loop surrounding the area A. Deleting the line integral term introduces a negligibly small error when quadratic elements are used since slopes at the intersection of two elements are nearly identical. Thus, the integral on the element boundary is deleted. The form then reduces to

$$\int_A [N]^T\,\frac{\partial^2 \phi}{\partial X^2}\, dA = -\int_A \frac{\partial [N]^T}{\partial X}\,\frac{\partial \phi}{\partial X}\, dA \tag{46}$$

This method, applied to all higher order terms which appear in Equation (41), produces

$$\int_A \{[(X^2+Y^2(1+D))(\frac{\partial N_i}{\partial X}\frac{\partial N_j}{\partial X}) + (Y^2+X^2(1+D))(\frac{\partial N_i}{\partial Y}\frac{\partial N_j}{\partial Y})$$

$$- XYD(\frac{\partial N_i}{\partial X}\frac{\partial N_j}{\partial Y} + \frac{\partial N_i}{\partial Y}\frac{\partial N_j}{\partial X}) + (\frac{\varepsilon X^2}{1-\varepsilon X} + \frac{\varepsilon Y^2}{1-\varepsilon X}(1-D) + XD)\cdot \tag{47}$$

$$(N_i\frac{\partial N_j}{\partial X}) + YD(1 + \frac{\varepsilon X}{1-\varepsilon X})(N_i\frac{\partial N_j}{\partial Y}) + \lambda^2(X^2+Y^2)N_iN_j]\{\phi\}$$

$$+ (X^2+Y^2)N_j\}\ dA = 0$$

Equation (47) is integrated numerically for each element to produce the desired solution.

3.4 NUMERICAL INTEGRATION

The critical term in Equation (47), when determining the order of numerical integration, is the term containing N_iN_j. This term is sixth order in ξ and η. All other terms in Equation (47) are of a lower order. The required sampling points and weighting coefficients, as given in [7], are presented in Table 1 and shown in Figure 5.

3.5 GRIDS

The temperature distribution was calculated for three different grid configurations. The first grid consisted of eight elements. The resulting nodal values were within 2 percent of the analytical solution [8], for the case $\varepsilon = \lambda = 0$. The second grid consisted of twelve elements. The resulting nodal values were within 1 percent of the analytical values. The final grid configuration consists of twelve elements, as shown in Figure 6; this grid results in nodal values

Table 1.--Numerical Integration Points and Weighting Coefficients

ξ_i, η_i	WC
-0.861136	0.347855
-0.339981	0.652145
0.339981	0.652145
0.861136	0.347855

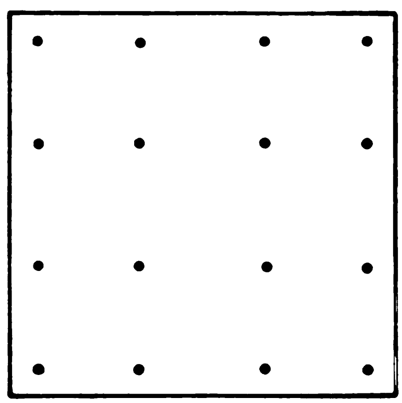


Figure 5.--Numerical Integration Points on an Interior Element

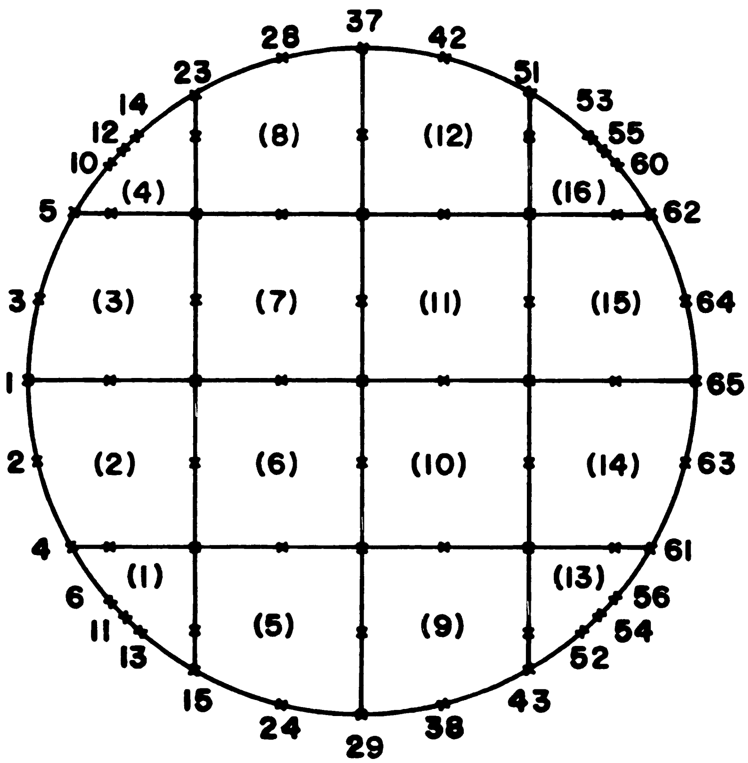


Figure 6.--The Element and Node Numbering Scheme

which are accurate to within 0.01 percent. This is the grid used to obtain the results presented in Chapter IV.

3.6 COMPUTER PROGRAM

The computer program used is a modification of TDHEAT given in [9]. A schematic of the program is shown in Figure 7. Subroutine VALUE evaluates the shape functions and their first derivatives on ξ and η at each numerical integration point (ξ, η). Subroutine JMAT then assembles the Jacobian or J matrix, given by

$$[J] = \begin{bmatrix} \frac{\partial X}{\partial \xi} & \frac{\partial Y}{\partial \xi} \\ \frac{\partial X}{\partial \eta} & \frac{\partial Y}{\partial \eta} \end{bmatrix} \tag{48}$$

This is accomplished by differentiating Equation (38) and (39) with respect to ξ and η. Subroutine PARXY then uses the J matrix to convert from partial derivatives on ξ and η to X and Y using the chain rule since

$$\begin{Bmatrix} \frac{\partial N_i}{\partial X} \\ \frac{\partial N_i}{\partial Y} \end{Bmatrix} = [J]^{-1} \begin{Bmatrix} \frac{\partial N_i}{\partial \xi} \\ \frac{\partial N_i}{\partial \eta} \end{Bmatrix} \tag{49}$$

Subroutine COEF evaluates the coefficients in Equation (47) at each numerical integration point (ξ, η). Subroutine BDYVAL incorporates the boundary condition Equation (35) on the global stiffness matrix and force vector using the method of rows and columns. Subroutine LEQT1F is an IMSL (International Mathematical and Statistical Libraries [101] subroutine that was used to solve for the nodal values from the

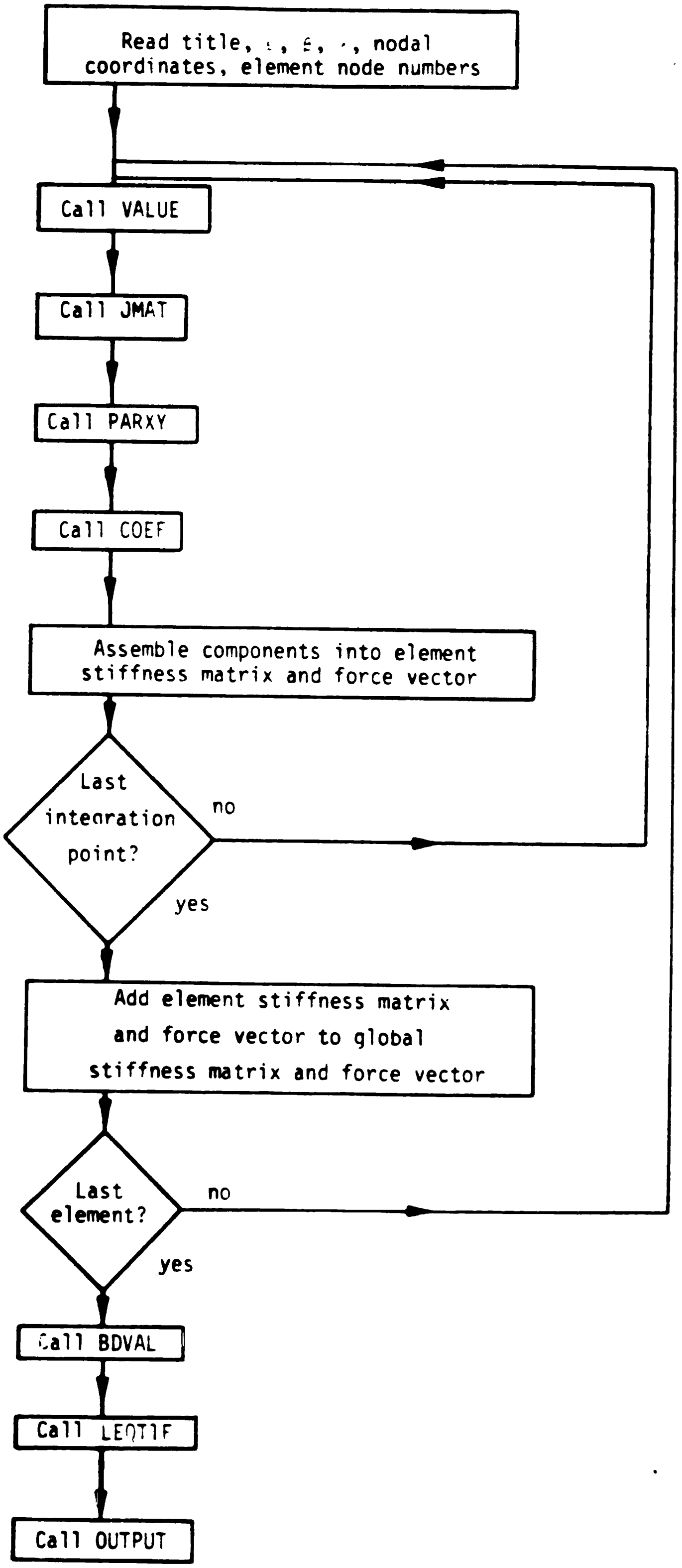


Figure 7.--Computer Flowchart

global stiffness matrix and force vector. Subroutine OUTPUT uses the nodal values to determine the maximum temperature and its location. OUTPUT then uses the nodal values to calculate the temperatures at various locations on the X- and Y- axis. A program listing and sample output is found in the Appendix.

CHAPTER IV

RESULTS AND CONCLUSIONS

4.1 NUMERICAL RESULTS

Temperature distributions and maximum temperatures were calculated for selected values of ε, β and λ using the finite element method, expressed by Equation (47).

Tables 2 through 13 present the values of the dimensionless temperature at selected positions for various values of ε, β and λ. It is evident that increasing ε, β or λ results in an increase in the dimensionless temperature at any point. This is shown graphically in Figures 8 to 13. Note that as ε increases, the temperature distribution is no longer symmetric on the X-axis but remains symmetric with respect to the Y-axis. The actual temperature distribution, however, remains nearly parabolic across either the X- or Y-axis. The maximum temperature increases whenever ε, β or λ increases. The maximum temperature always occurs on the X-axis. It is apparent that β has a very minimal effect on the temperature distribution when ε is small (less than approximately 0.2).

Tables 14 to 17 contain the location and value of the maximum dimensionless temperature for various values of ε, β and λ. As expected, an increase in the maximum temperature is realized whenever ε, β or λ is increased. As ε approaches one, the mathematical model breaks down. This is particularly evident for data presented at λ equal 2.3. This is not unexpected and was first predicted by Jakob [1], a singularity due to the first zero of the zeroth order Bessel function of the first kind exists at λ equal 2.4048. Thus, at λ equal 2.3 the solution is strongly influenced by this singularity.

Figures 14 through 17 show the effect of changing β and λ on the percent increase in the maximum dimensionless temperature. Increasing β or λ intensifies the effect of ε on the maximum temperature. Figure 18 is from the paper by Wang [5]; note the similarity to Figures 14 to 17. Wang assumed small values for ε and as a result underestimates the effect of increasing β since for small ε the solution is essentially independent of β. Wang let ε attain values greater than one when it is limited physically to a maximum of one. At $\varepsilon = 1.0$ the coil is wrapped so tightly that it is continually in contact with itself (see Figure 19). Figure 18 does show that as the first zero of the zeroth order Bessel function is approached the maximum temperature becomes unbounded, independent of the value of ε or β. It is apparent from Tables 2 to 13 that as ε approaches one the numerical scheme used to solve the problem breaks down. This is due to the terms involving $1/(1-\varepsilon X)$ in the governing equations becoming very large, i.e., $1-\varepsilon X$ approaches zero. Just before this occurs the condition of a double maximum occurs. Of these two maxima the larger maximum always occurs at the larger value of X. Since ε has a value near one the isothermal boundary condition should be replaced by a convection boundary condition. Forced convection is ineffective where loops of the coil are very close and as a result the temperature of the fluid surrounding the coil increases above T_o. This could result in the need to use a different numerical method to solve the problem.

4.2 CONCLUSIONS

Based on the results obtained and discussion of the previous section it can be concluded that:

1. As ε increases ϕ_{max} increases and ϕ_{max} occurs at an ever increasing value of X. For $\varepsilon = 0$, $\phi_{max} = .2506$ at $X = 0$; for $\varepsilon = 0.5$, $\phi_{max} = .2644$ at $X = 0.070$ ($\beta = 1.0$ and $\lambda = 0$).

2. As β increases ϕ_{max} increases and the effect of ε is amplified. For $\beta = 0.5$, $\phi_{max} = 0.3163$; for $\beta = 1.0$, $\phi_{max} = 0.3288$ ($\varepsilon = 0.5$ and $\lambda = 1.0$).

3. Increasing λ results in an increase in ϕ_{max}. For $\lambda = 1.0$, $\phi_{max} = 0.3099$; for $\lambda = 2.0$, $\phi_{max} = 0.8824$ ($\varepsilon = 0.2$ and $\beta = 1.0$).

4. The temperature profile is not affected by changes in ε, β or λ to the extent that ϕ_{max} is affected.

5. For small values of ε (less than approximately 0.2) β does not have a significant influence on the maximum temperature, ϕ_{max}, or the temperature distribution. For $\beta = 1.0$, $\phi_{max} = 0.3099$; for $\beta = 2.0$, $\phi_{max} = 0.3163$, a change of less than 2 percent ($\varepsilon = 0.2$ and $\lambda = 1.0$).

6. Maximum temperatures always lie on the X-axis.

7. The temperature distribution is always symmetrical about the Y-axis.

It is evident that the torsion parameter β cannot be neglected if $\varepsilon > 0.2$. Also, when $\varepsilon > 0.2$ the higher order terms in the asymptotic expansion technique used by Wang [5] become significant and must be included.

Table 2.--Dimensionless Temperatures for β=0.5, λ=0.0

X	ϵ=0	ϵ=.2	ϵ=.4	ϵ=.5	ϵ=.7	ϵ=.8	ϵ=.9
-1.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
-0.9375	0.0308	0.0296	0.0288	0.0285	0.0281	0.0280	0.0281
-0.8750	0.0595	0.0573	0.0559	0.0553	0.0546	0.0545	0.0546
-0.8125	0.0860	0.0830	0.0811	0.0804	0.0794	0.0793	0.0797
-0.7500	0.1103	0.1068	0.1045	0.1036	0.1026	0.1026	0.1032
-0.6875	0.1325	0.1287	0.1261	0.1252	0.1242	0.1242	0.1253
-0.6250	0.1526	0.1486	0.1459	0.1450	0.1441	0.1443	0.1458
-0.5625	0.1705	0.1665	0.1639	0.1630	0.1624	0.1628	0.1647
-0.5000	0.1862	0.1825	0.1801	0.1793	0.1790	0.1797	0.1822
-0.4375	0.2011	0.1976	0.1955	0.1949	0.1950	0.1961	0.1993
-0.3750	0.2141	0.2109	0.2091	0.2087	0.2093	0.2109	0.2151
-0.3125	0.2251	0.2224	0.2210	0.2208	0.2221	0.2242	0.2294
-0.2500	0.2341	0.2319	0.2311	0.2312	0.2333	0.2360	0.2425
-0.1875	0.2412	0.2396	0.2393	0.2400	0.2428	0.2462	0.2542
-0.1250	0.2463	0.2454	0.2459	0.2468	0.2508	0.2550	0.2645
-0.0625	0.2495	0.2493	0.2506	0.2520	0.2571	0.2623	0.2735
0.0000	0.2506	0.2513	0.2535	0.2554	0.2619	0.2680	0.2811
0.0625	0.2495	0.2510	0.2541	0.2565	0.2642	0.2711	0.2851
0.1250	0.2463	0.2486	0.2526	0.2555	0.2645	0.2721	0.2870
0.1875	0.2412	0.2442	0.2491	0.2525	0.2626	0.2711	0.2869
0.2500	0.2341	0.2378	0.2434	0.2473	0.2587	0.2680	0.2847
0.3125	0.2251	0.2293	0.2357	0.2400	0.2527	0.2629	0.2805
0.3750	0.2141	0.2189	0.2259	0.2306	0.2446	0.2557	0.2742
0.4375	0.2011	0.2063	0.2139	0.2191	0.2344	0.2465	0.2659
0.5000	0.1862	0.1918	0.1999	0.2055	0.2221	0.2352	0.2555
0.5625	0.1705	0.1765	0.1856	0.1922	0.2135	0.2338	0.2797
0.6250	0.1526	0.1587	0.1682	0.1753	0.1995	0.2243	0.2879
0.6875	0.1326	0.1384	0.1478	0.1549	0.1800	0.2069	0.2800
0.7500	0.1104	0.1157	0.1243	0.1309	0.1549	0.1815	0.2561
0.8125	0.0860	0.0905	0.0978	0.1135	0.1244	0.1481	0.2161
0.8750	0.0595	0.0628	0.0683	0.0725	0.0884	0.1067	0.1601
0.9375	0.0308	0.0326	0.0357	0.0380	0.0470	0.0573	0.0881
1.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000

Table 3.--Dimensionless Temperatures for β=1.0, λ=0.0

X	ε=0	ε=.2	ε=.4	ε=.5	ε=.7	ε=.8	ε=.9
-1.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
-0.9375	0.0308	0.0298	0.0294	0.0294	0.0297	0.0304	0.0346
-0.8750	0.0595	0.0576	0.0569	0.0569	0.0577	0.0591	0.0679
-0.8125	0.0860	0.0835	0.0826	0.0826	0.0840	0.0862	0.0999
-0.7500	0.1103	0.1074	0.1064	0.1065	0.1086	0.1116	0.1307
-0.6875	0.1325	0.1293	0.1283	0.1284	0.1314	0.1355	0.1603
-0.6250	0.1526	0.1492	0.1484	0.1489	0.1526	0.1576	0.1885
-0.5625	0.1705	0.1672	0.1667	0.1674	0.1720	0.1781	0.2156
-0.5000	0.1862	0.1832	0.1830	0.1840	0.1896	0.1970	0.2413
-0.4375	0.2011	0.1984	0.1987	0.2000	0.2068	0.2157	0.2715
-0.3750	0.2141	0.2118	0.2125	0.2142	0.2224	0.2331	0.3013
-0.3125	0.2251	0.2232	0.2245	0.2266	0.2364	0.2490	0.3306
-0.2500	0.2341	0.2328	0.2348	0.2374	0.2488	0.2635	0.3595
-0.1875	0.2412	0.2405	0.2433	0.2464	0.2597	0.2767	0.3879
-0.1250	0.2463	0.2463	0.2500	0.2537	0.2690	0.2884	0.4159
-0.0625	0.2495	0.2502	0.2549	0.2592	0.2768	0.2988	0.4433
0.0000	0.2506	0.2523	0.2580	0.2631	0.2834	0.3078	0.4704
0.0625	0.2495	0.2520	0.2587	0.2644	0.2859	0.3116	0.4746
0.1250	0.2463	0.2496	0.2573	0.2636	0.2867	0.3133	0.4726
0.1875	0.2412	0.2452	0.2538	0.2607	0.2854	0.3130	0.4643
0.2500	0.2341	0.2388	0.2482	0.2557	0.2821	0.3106	0.4496
0.3125	0.2251	0.2304	0.2406	0.2486	0.2768	0.3061	0.4287
0.3750	0.2141	0.2199	0.2308	0.2394	0.2694	0.2995	0.4015
0.4375	0.2011	0.2073	0.2190	0.2281	0.2600	0.2909	0.3679
0.5000	0.1862	0.1928	0.2050	0.2148	0.2485	0.2801	0.3281
0.5625	0.1705	0.1775	0.1911	0.2026	0.2483	0.3069	0.7056
0.6250	0.1526	0.1597	0.1739	0.1862	0.2392	0.3161	0.9636
0.6875	0.1326	0.1394	0.1533	0.1656	0.2214	0.3075	1.1019
0.7500	0.1104	0.1166	0.1293	0.1409	0.1947	0.2813	1.1207
0.8125	0.0860	0.0912	0.1020	0.1119	0.1592	0.2375	1.0199
0.8750	0.0595	0.0633	0.0714	0.0788	0.1150	0.1760	0.7995
0.9375	0.0308	0.0329	0.0374	0.0415	0.0619	0.0968	0.4595
1.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000

Table 4.--Dimensionless Temperatures for β=2.0, λ=0.0

X	ε=0	ε=.2	ε=.4	ε=.5	ε=.7	ε=.8
-1.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
-0.9375	0.0308	0.0305	0.0318	0.0333	0.0410	0.0670
-0.8750	0.0595	0.0589	0.0616	0.0645	0.0797	0.1307
-0.8125	0.0860	0.0853	0.0893	0.0935	0.1161	0.1912
-0.7500	0.1103	0.1096	0.1148	0.1203	0.1502	0.2485
-0.6875	0.1325	0.1319	0.1383	0.1451	0.1821	0.3025
-0.6250	0.1526	0.1521	0.1597	0.1677	0.2116	0.3533
-0.5625	0.1705	0.1703	0.1789	0.1881	0.2389	0.4009
-0.5000	0.1862	0.1864	0.1961	0.2064	0.2638	0.4453
-0.4375	0.2011	0.2017	0.2126	0.2241	0.2900	0.5089
-0.3750	0.2141	0.2152	0.2273	0.2401	0.3147	0.5709
-0.3125	0.2251	0.2268	0.2402	0.2543	0.3378	0.6316
-0.2500	0.2341	0.2365	0.2513	0.2668	0.3594	0.6908
-0.1875	0.2412	0.2443	0.2606	0.2776	0.3795	0.7485
-0.1250	0.2463	0.2502	0.2681	0.2866	0.3980	0.8048
-0.0625	0.2495	0.2543	0.2738	0.2939	0.4150	0.8597
0.0000	0.2506	0.2564	0.2777	0.2995	0.4305	0.9131
0.0625	0.2495	0.2561	0.2786	0.3019	0.4316	0.8839
0.1250	0.2463	0.2538	0.2775	0.3009	0.4307	0.8416
0.1875	0.2412	0.2494	0.2743	0.2985	0.4279	0.7861
0.2500	0.2341	0.2430	0.2690	0.2941	0.4232	0.7175
0.3125	0.2251	0.2345	0.2616	0.2877	0.4166	0.6356
0.3750	0.2141	0.2240	0.2521	0.2792	0.4081	0.5406
0.4375	0.2011	0.2115	0.2406	0.2687	0.3977	0.4325
0.5000	0.1862	0.1969	0.2270	0.2562	0.3855	0.3111
0.5625	0.1705	0.1818	0.2152	0.2506	0.4688	1.2009
0.6250	0.1526	0.1639	0.1987	0.2375	0.5145	1.8253
0.6875	0.1326	0.1433	0.1774	0.2168	0.5227	2.1844
0.7500	0.1104	0.1201	0.1514	0.1886	0.4933	2.2782
0.8125	0.0860	0.0941	0.1207	0.1528	0.4263	2.1067
0.8750	0.0595	0.0654	0.0852	0.1094	0.3128	1.6698
0.9375	0.0308	0.0341	0.0450	0.0585	0.1797	0.9676
1.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000

Table 5.--Dimensionless Temperatures for β=0.5, λ=1.0

X	ε=0	ε=.2	ε=.4	ε=.5	ε=.7	ε=.8	ε=.9
-1.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
-0.9375	0.0358	0.0343	0.0333	0.0329	0.0325	0.0324	0.0327
-0.8750	0.0693	0.0666	0.0648	0.0641	0.0634	0.0634	0.0641
-0.8125	0.1008	0.0970	0.0945	0.0937	0.0928	0.0930	0.0941
-0.7500	0.1300	0.1255	0.1226	0.1261	0.1207	0.1210	0.1227
-0.6875	0.1570	0.1520	0.1489	0.1478	0.1470	0.1476	0.1501
-0.6250	0.1819	0.1767	0.1734	0.1724	0.1718	0.1727	0.1760
-0.5625	0.2046	0.1994	0.1962	0.1953	0.1951	0.1964	0.2007
-0.5000	0.2251	0.2202	0.2172	0.2165	0.2169	0.2187	0.2240
-0.4375	0.2442	0.2395	0.2370	0.2364	0.2375	0.2399	0.2465
-0.3750	0.2607	0.2565	0.2544	0.2542	0.2560	0.2591	0.2673
-0.3125	0.2747	0.2711	0.2696	0.2697	0.2725	0.2765	0.2864
-0.2500	0.2862	0.2833	0.2825	0.2831	0.2870	0.2119	0.3038
-0.1875	0.2952	0.2931	0.2932	0.2942	0.2995	0.3055	0.3195
-0.1250	0.3016	0.3005	0.3016	0.3032	0.3100	0.3171	0.3336
-0.0625	0.3056	0.3055	0.3077	0.3100	0.3184	0.3269	0.3459
0.0000	0.3071	0.3081	0.3116	0.3145	0.3248	0.3347	0.3566
0.0625	0.3056	0.3079	0.3126	0.3163	0.3283	0.3393	0.3627
0.1250	0.3017	0.3049	0.3109	0.3153	0.3290	0.3411	0.3661
0.1875	0.2952	0.2994	0.3065	0.3115	0.3270	0.3402	0.3667
0.2500	0.2862	0.2913	0.2994	0.3051	0.3222	0.3366	0.3645
0.3125	0.2747	0.2806	0.2896	0.2958	0.3146	0.3303	0.3596
0.3750	0.2607	0.2672	0.2771	0.2839	0.3044	0.3213	0.3518
0.4375	0.2442	0.2512	0.2619	0.2692	0.2913	0.3095	0.3413
0.5000	0.2252	0.2326	0.2439	0.2518	0.2755	0.2950	0.3280
0.5625	0.2046	0.2126	0.2250	0.2339	0.2634	0.2917	0.3579
0.6250	0.1819	0.1900	0.2027	0.2122	0.2449	0.2789	0.3676
0.6875	0.1571	0.1647	0.1771	0.1865	0.2200	0.2564	0.3570
0.7500	0.1300	0.1370	0.1483	0.1570	0.1888	0.2243	0.3261
0.8125	0.1008	0.1066	0.1162	0.1236	0.1512	0.1826	0.2750
0.8750	0.0694	0.0736	0.0807	0.0863	0.1071	0.1313	0.2036
0.9375	0.0358	0.0381	0.0420	0.0451	0.0568	0.0705	0.1119
1.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000

Table 6.--Dimensionless Temperatures for β=1.0, λ=1.0

X	ε=0	ε=.2	ε=.4	ε=.5	ε=.7	ε=.8	ε=.9
-1.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
-0.9375	0.0358	0.0345	0.0340	0.0341	0.0349	0.0363	0.0493
-0.8750	0.0693	0.0670	0.0663	0.0664	0.0682	0.0711	0.0981
-0.8125	0.1008	0.0976	0.0967	0.0970	0.1000	0.1045	0.1466
-0.7500	0.1300	0.1262	0.1254	0.1259	0.1301	0.1363	0.1947
-0.6875	0.1570	0.1529	0.1522	0.1530	0.1586	0.1668	0.2424
-0.6250	0.1819	0.1777	0.1772	0.1784	0.1855	0.1957	0.2897
-0.5625	0.2046	0.2005	0.2004	0.2020	0.2108	0.2232	0.3367
-0.5000	0.2251	0.2214	0.2218	0.2239	0.2345	0.2493	0.3832
-0.4375	0.2442	0.2408	0.2419	0.2445	0.2572	0.2747	0.4381
-0.3750	0.2607	0.2578	0.2597	0.2628	0.2778	0.2984	0.4930
-0.3125	0.2747	0.2725	0.2752	0.2790	0.2964	0.3202	0.5478
-0.2500	0.2862	0.2847	0.2884	0.2929	0.3130	0.3403	0.6026
-0.1875	0.2952	0.2945	0.2994	0.3047	0.3276	0.3586	0.6574
-0.1250	0.3016	0.3020	0.3081	0.3142	0.3402	0.3751	0.7122
-0.0625	0.3056	0.3070	0.3145	0.3216	0.3508	0.3898	0.7669
0.0000	0.3071	0.3097	0.3186	0.3267	0.3594	0.4028	0.8216
0.0625	0.3056	0.3094	0.3198	0.3287	0.3639	0.4090	0.8339
0.1250	0.3017	0.3065	0.3182	0.3280	0.3655	0.4124	0.8345
0.1875	0.2952	0.3010	0.3139	0.3245	0.3644	0.4129	0.8235
0.2500	0.2862	0.2929	0.3069	0.3182	0.3605	0.4104	0.8009
0.3125	0.2747	0.2821	0.2971	0.3092	0.3538	0.4051	0.7666
0.3750	0.2607	0.2688	0.2847	0.2975	0.3443	0.3968	0.7207
0.4375	0.2442	0.2528	0.2695	0.2829	0.3321	0.3855	0.6632
0.5000	0.2252	0.2341	0.2515	0.2656	0.3171	0.3714	0.5940
0.5625	0.2046	0.2141	0.2330	0.2490	0.3152	0.4056	1.3087
0.6250	0.1819	0.1914	0.2107	0.2277	0.3026	0.4168	1.7980
0.6875	0.1571	0.1661	0.1848	0.2016	0.2791	0.4049	2.0619
0.7500	0.1300	0.1381	0.1552	0.1708	0.2449	0.3700	2.1003
0.8125	0.1008	0.1076	0.1219	0.1352	0.1999	0.3120	1.9134
0.8750	0.0694	0.0743	0.0850	0.0949	0.1440	0.2310	1.5010
0.9375	0.0358	0.0385	0.0443	0.0498	0.0774	0.1270	0.8632
1.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000

Table 7.--Dimensionless Temperatures for β=2.0, λ=1.0

X	ε=0	ε=.2	ε=.4	ε=.5	ε=.7	ε=.8
-1.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
-0.9375	0.0358	0.0354	0.0376	0.0401	0.0565	0.3495
-0.8750	0.0693	0.0688	0.0732	0.0780	0.1108	0.6916
-0.8125	0.1008	0.1001	0.1066	0.1138	0.1628	1.0263
-0.7500	0.1300	0.1294	0.1380	0.1475	0.2126	1.3535
-0.6875	0.1570	0.1567	0.1673	0.1790	0.2601	1.6733
-0.6250	0.1819	0.1819	0.1945	0.2084	0.3053	1.9856
-0.5625	0.2046	0.2050	0.2196	0.2357	0.3483	2.2905
-0.5000	0.2251	0.2262	0.2426	0.2609	0.3891	2.5879
-0.4375	0.2442	0.2459	0.2643	0.2847	0.4312	3.0143
-0.3750	0.2607	0.2631	0.2836	0.3063	0.4710	3.4343
-0.3125	0.2747	0.2780	0.3006	0.3256	0.5085	3.8482
-0.2500	0.2862	0.2905	0.3152	0.3425	0.5438	4.2557
-0.1875	0.2952	0.3005	0.3275	0.3572	0.5768	4.6570
-0.1250	0.3016	0.3081	0.3375	0.3695	0.6051	5.0521
-0.0625	0.3056	0.3133	0.3451	0.3796	0.6359	5.4409
0.0000	0.3071	0.3161	0.3504	0.3874	0.6621	5.8234
0.0625	0.3056	0.3159	0.3520	0.3903	0.6659	5.6441
0.1250	0.3017	0.3130	0.3508	0.3904	0.6666	5.3776
0.1875	0.2952	0.3075	0.3469	0.3877	0.6641	5.0239
0.2500	0.2862	0.2994	0.3401	0.3821	0.6584	4.5832
0.3125	0.2747	0.2886	0.3306	0.3738	0.6495	4.0553
0.3750	0.2607	0.2751	0.3183	0.3626	0.6374	3.4402
0.4375	0.2442	0.2590	0.3031	0.3486	0.6221	2.7380
0.5000	0.2252	0.2403	0.2852	0.3317	0.6036	1.9487
0.5625	0.2046	0.2203	0.2688	0.3228	0.7331	8.0091
0.6250	0.1819	0.1974	0.2469	0.3046	0.8041	12.268
0.6875	0.1571	0.1717	0.2195	0.2770	0.8164	14.726
0.7500	0.1300	0.1431	0.1866	0.2402	0.7703	15.383
0.8125	0.1008	0.1116	0.1482	0.1941	0.6655	14.239
0.8750	0.0694	0.0773	0.1043	0.1387	0.5022	11.294
0.9375	0.0358	0.0401	0.0549	0.0740	0.2804	6.5475
1.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000

Table 8.--Dimensionless Temperatures for β=0.5, λ=2.0

X	ε=0	ε=.2	ε=.4	ε=.5	ε=.7	ε=.8	ε=.9
-1.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
-0.9375	0.0821	0.0775	0.0754	0.0753	0.0779	0.0825	0.0993
-0.8750	0.1622	0.1538	0.1502	0.1502	0.1560	0.1658	0.2010
-0.8125	0.2402	0.2288	0.2243	0.2247	0.2344	0.2499	0.3050
-0.7500	0.3163	0.3025	0.2977	0.2988	0.3131	0.3349	0.4114
-0.6875	0.3903	0.3750	0.3704	0.3725	0.3921	0.4207	0.5202
-0.6250	0.4622	0.4462	0.4424	0.4458	0.4713	0.5073	0.6314
-0.5625	0.5322	0.5161	0.5138	0.5187	0.5508	0.5948	0.7449
-0.5000	0.6001	0.5848	0.5844	0.5912	0.6306	0.6831	0.8608
-0.4375	0.6602	0.6463	0.6485	0.6573	0.7042	0.7653	0.9708
-0.3750	0.7123	0.7004	0.7055	0.7164	0.7712	0.8410	1.0745
-0.3125	0.7565	0.7469	0.7553	0.7687	0.8316	0.9102	1.1718
-0.2500	0.7928	0.7860	0.7981	0.8140	0.8854	0.9731	1.2629
-0.1875	0.8210	0.8175	0.8338	0.8524	0.9326	1.0295	1.3477
-0.1250	0.8414	0.8416	0.8624	0.8839	0.9732	1.0795	1.4262
-0.0625	0.8537	0.8582	0.8839	0.9085	1.0073	1.1230	1.4984
0.0000	0.8581	0.8673	0.8982	0.9261	1.0347	1.1601	1.5643
0.0625	0.8537	0.8676	0.9040	0.9352	1.0531	1.1866	1.6116
0.1250	0.8414	0.8594	0.9006	0.9347	1.0608	1.2015	1.6439
0.1875	0.8211	0.8427	0.8880	0.9246	1.0579	1.2047	1.6610
0.2500	0.7928	0.8175	0.8663	0.9050	1.0444	1.1964	1.6631
0.3125	0.7566	0.7838	0.8354	0.8758	1.0202	1.1764	1.6500
0.3750	0.7124	0.7417	0.7953	0.8371	0.9853	1.1448	1.6219
0.4375	0.6602	0.6910	0.7461	0.7887	0.9399	1.1016	1.5787
0.5000	0.6001	0.6318	0.6877	0.7309	0.8838	1.0467	1.5204
0.5625	0.5322	0.5641	0.6208	0.6653	0.8306	1.0202	1.6457
0.6250	0.4623	0.4932	0.5484	0.5924	0.7610	0.9639	1.6809
0.6875	0.3903	0.4190	0.4706	0.5121	0.6751	0.8778	1.6260
0.7500	0.3163	0.3417	0.3873	0.4245	0.5728	0.7618	1.4810
0.8125	0.2403	0.2611	0.2986	0.3294	0.4542	0.6161	1.2459
0.8750	0.1622	0.1773	0.2045	0.2270	0.3191	0.4405	0.9207
0.9375	0.0821	0.0903	0.1050	0.1172	0.1677	0.2352	0.5054
1.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000

Table 9.--Dimensionless Temperatures for β=1.0, λ=2.0

X	ε=0	ε=.2	ε=.4	ε=.5	ε=.7	ε=.8
-1.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
-0.9375	0.0821	0.0789	0.0810	0.0849	0.1122	0.2260
-0.8750	0.1622	0.1564	0.1613	0.1695	0.2257	0.4593
-0.8125	0.2402	0.2327	0.2408	0.2536	0.3404	0.7001
-0.7500	0.3163	0.3076	0.3195	0.3373	0.4564	0.9482
-0.6875	0.3903	0.3812	0.3975	0.4206	0.5735	1.2037
-0.6250	0.4622	0.4535	0.4748	0.5035	0.6920	1.4667
-0.5625	0.5322	0.5245	0.5513	0.5860	0.8116	1.7370
-0.5000	0.6001	0.5942	0.6270	0.6681	0.9325	2.0147
-0.4375	0.6602	0.6566	0.6957	0.7431	1.0452	2.2822
-0.3750	0.7123	0.7115	0.7569	0.8104	1.1493	2.5357
-0.3125	0.7565	0.7587	0.8105	0.8701	1.2446	2.7754
-0.2500	0.7928	0.7984	0.8567	0.9222	1.3312	3.0011
-0.1875	0.8210	0.8305	0.8954	0.9667	1.4091	3.2129
-0.1250	0.8414	0.8549	0.9265	1.0036	1.4783	3.4109
-0.0625	0.8537	0.8718	0.9502	1.0330	1.5388	3.5949
0.0000	0.8581	0.8811	0.9663	1.0547	1.5906	3.7651
0.0625	0.8537	0.8815	0.9732	1.0665	1.6253	3.8762
0.1250	0.8414	0.8733	0.9703	1.0676	1.6646	3.9525
0.1875	0.8211	0.8564	0.9577	1.0581	1.6486	3.9941
0.2500	0.7928	0.8310	0.9353	1.0378	1.6372	4.0010
0.3125	0.7566	0.7969	0.9032	1.0069	1.6104	3.9730
0.3750	0.7124	0.7542	0.8612	0.9653	1.5682	3.9104
0.4375	0.6602	0.7029	0.8096	0.9129	1.5106	3.8130
0.5000	0.6001	0.6430	0.7481	0.8499	1.4377	3.6808
0.5625	0.5322	0.5745	0.6785	0.7815	1.4091	3.9879
0.6250	0.4623	0.5026	0.6020	0.7022	1.3373	4.0757
0.6875	0.3903	0.4273	0.5188	0.6122	1.2223	3.9444
0.7500	0.3163	0.3487	0.4287	0.5113	1.0642	3.5939
0.8125	0.2403	0.2666	0.3317	0.3997	0.8629	3.0242
0.8750	0.1622	0.1811	0.2280	0.2772	0.6184	2.2354
0.9375	0.0821	0.0923	0.1174	0.1440	0.3308	1.2273
1.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000

Table 10.--Dimensionless Temperatures for β=2.0, λ=2.0

X	ε=0	ε=.2	ε=.4	ε=.5
-1.0000	0.0000	0.0000	0.0000	0.0000
-0.9375	0.0821	0.0848	0.1160	0.1896
-0.8750	0.1622	0.1680	0.2308	0.3787
-0.8125	0.2402	0.2497	0.3444	0.5674
-0.7500	0.3163	0.3299	0.4568	0.7558
-0.6875	0.3903	0.4085	0.5679	0.9436
-0.6250	0.4622	0.4856	0.6778	1.1311
-0.5625	0.5322	0.5611	0.7865	1.3182
-0.5000	0.6001	0.6351	0.8940	1.5048
-0.4375	0.6602	0.7014	0.9916	1.6764
-0.3750	0.7123	0.7597	1.0792	1.8328
-0.3125	0.7565	0.8100	1.1566	1.9738
-0.2500	0.7928	0.8523	1.2240	2.0996
-0.1875	0.8210	0.8865	1.2813	2.2101
-0.1250	0.8414	0.9127	1.3284	2.3053
-0.0625	0.8537	0.9309	1.3655	2.3852
0.0000	0.8581	0.9411	1.3926	2.4498
0.0625	0.8537	0.9419	1.4058	2.4867
0.1250	0.8414	0.9335	1.4057	2.5012
0.1875	0.8211	0.9161	1.3921	2.4933
0.2500	0.7928	0.8894	1.3651	2.4631
0.3125	0.7566	0.8537	1.3247	2.4105
0.3750	0.7124	0.8088	1.2709	2.3355
0.4375	0.6602	0.7548	1.2036	2.2382
0.5000	0.6001	0.6916	1.1229	2.1185
0.5625	0.5322	0.6197	1.0380	2.0279
0.6250	0.4623	0.5436	0.9372	1.8876
0.6875	0.3903	0.4634	0.8205	1.6974
0.7500	0.3163	0.3790	0.6881	1.4575
0.8125	0.2403	0.2904	0.5398	1.1678
0.8750	0.1622	0.1978	0.3757	0.8283
0.9375	0.0821	0.1010	0.1958	0.4390
1.0000	0.0000	0.0000	0.0000	0.0000

Table 11.--Dimensionless Temperatures for β=0.5, λ=2.3

X	ε=0	ε=.2	ε=.4	ε=.5	ε=.7
-1.0000	0.0000	0.0000	0.0000	0.0000	0.0000
-0.9375	0.2659	0.2545	0.2713	0.2991	0.5511
-0.8750	0.5310	0.5113	0.5480	0.6059	1.1245
-0.8125	0.7954	0.7704	0.8301	0.9203	1.7201
-0.7500	1.0591	1.0317	1.1176	1.2424	2.3381
-0.6875	1.3221	1.2954	1.4104	1.5721	2.9782
-0.6250	1.5843	1.5613	1.7086	1.9095	3.6407
-0.5625	1.8458	1.8294	2.0121	2.2545	4.3255
-0.5000	2.1065	2.0998	2.3210	2.6071	5.0325
-0.4375	2.3347	2.3403	2.5997	2.9274	5.6842
-0.3750	2.5326	2.5518	2.8482	3.2153	6.2812
-0.3125	2.7003	2.7343	3.0666	3.4708	6.8236
-0.2500	2.8379	2.8879	3.2549	3.6938	7.3114
-0.1875	2.9452	3.0124	3.4131	3.8845	7.7444
-0.1250	3.0223	3.1080	3.5412	4.0427	8.1228
-0.0625	3.0692	3.1746	3.6391	4.1682	8.4466
0.0000	3.0858	3.2122	3.7069	4.2619	8.7156
0.0625	3.0692	3.2158	3.7378	4.3147	8.9084
0.1250	3.0224	3.1859	3.7280	4.3199	9.0030
0.1875	2.9453	3.1224	3.6774	4.2774	8.9994
0.2500	2.8380	3.0252	3.5861	4.1873	8.8977
0.3125	2.7005	2.8945	3.4541	4.0496	8.6978
0.3750	2.5328	2.7302	3.2814	3.8642	8.3997
0.4375	2.3349	2.5324	3.0679	3.6312	8.0035
0.5000	2.1068	2.3009	2.8137	3.3506	7.5091
0.5625	1.8460	2.0318	2.5148	3.0224	7.0067
0.6250	1.5845	1.7574	2.2009	2.6684	6.3798
0.6875	1.3223	1.4778	1.8719	2.2884	5.6281
0.7500	1.0593	1.1928	1.5277	1.8826	4.7518
0.8125	0.7956	0.9025	1.1684	1.4508	3.7508
0.8750	0.5311	0.6070	0.7941	0.9931	2.6252
0.9375	0.2659	0.3061	0.4046	0.5095	1.3749
1.0000	0.0000	0.0000	0.0000	0.0000	0.0000

Table 12.--Dimensionless Temperatures for β=1.0, λ=2.3

X	ε=0	ε=.2	ε=.4	ε=.5
-1.0000	0.0000	0.0000	0.0000	0.0000
-0.9375	0.2659	0.2713	0.3884	0.7165
-0.8750	0.5310	0.5450	0.7845	1.4527
-0.8125	0.7954	0.8209	1.1884	2.2086
-0.7500	1.0591	1.0992	1.6000	2.9841
-0.6875	1.3221	1.3798	2.0194	3.7792
-0.6250	1.5843	1.6628	2.4464	4.5939
-0.5625	1.8458	1.9480	2.8813	5.4283
-0.5000	2.1065	2.2355	3.3238	6.2823
-0.4375	2.3347	2.4912	3.7231	7.0593
-0.3750	2.5326	2.7161	4.0800	7.7604
-0.3125	2.7003	2.9102	4.3943	8.3856
-0.2500	2.8379	3.0736	4.6661	8.9350
-0.1875	2.9452	3.2062	4.8954	9.4085
-0.1250	3.0223	3.3080	5.0822	9.8063
-0.0625	3.0692	3.3791	5.2265	10.128
0.0000	3.0858	3.4194	5.3283	10.374
0.0625	3.0692	3.4237	5.3778	10.522
0.1250	3.0224	3.3923	5.3691	10.555
0.1875	2.9453	3.3252	5.3023	10.474
0.2500	2.8380	3.2224	5.1773	10.279
0.3125	2.7005	3.0839	4.9941	9.9691
0.3750	2.5328	2.9098	4.7529	9.5449
0.4375	2.3349	2.6999	4.4534	9.0064
0.5000	2.1068	2.4543	4.0959	8.3535
0.5625	1.8460	2.1688	3.6793	7.6146
0.6250	1.5845	1.8773	3.2353	6.7884
0.6875	1.3223	1.5796	2.7642	5.8751
0.7500	1.0593	1.2759	2.2658	4.8745
0.8125	0.7956	0.9661	1.7402	3.7867
0.8750	0.5311	0.6501	1.1874	2.6117
0.9375	0.2659	0.3281	0.6073	1.3495
1.0000	0.0000	0.0000	0.0000	0.0000

Table 13.--Dimensionless Temperatures for β=2.0, λ=2.3

X	ε=0	ε=.2
-1.0000	0.0000	0.0000
-0.9375	0.2659	0.3679
-0.8750	0.5310	0.7384
-0.8125	0.7954	1.1115
-0.7500	1.0591	1.4871
-0.6875	1.3221	1.8654
-0.6250	1.5843	2.2461
-0.5625	1.8458	2.6295
-0.5000	2.1065	3.0154
-0.4375	2.3347	3.3584
-0.3750	2.5326	3.6603
-0.3125	2.7003	3.9212
-0.2500	2.8379	4.1410
-0.1875	2.9452	4.3199
-0.1250	3.0223	4.4577
-0.0625	3.0692	4.5544
0.0000	3.0858	4.6102
0.0625	3.0692	4.6182
0.1250	3,0224	4.5785
0.1875	2.9453	4.4909
0.2500	2.8380	4.3555
0.3125	2.7005	4.1722
0.3750	2.5328	3.9411
0.4375	2.3349	3.6621
0.5000	2.1068	3.3353
0.5625	1.8460	2.9560
0.6250	1.5845	2.5659
0.6875	1.3223	2.1651
0.7500	1.0593	1.7536
0.8125	0.7956	1.3313
0.8750	0.5311	0.8983
0.9375	0.2659	0.4545
1.0000	0.0000	0.0000

Table 14.--Maximum Dimensionless Temperatures for λ=0.0

ε	β	ϕ_{max}	X
0.0	--	0.2506	0.000
0.2	0.5	0.2514	0.020
0.4	0.5	0.2542	0.050
0.5	0.5	0.2565	0.065
0.7	0.5	0.2646	0.100
0.8	0.5	0.2721	0.125
0.9	0.5	0.2872	0.150
1.0	0.5	0.2968	0.500
0.2	1.5	0.2524	0.020
0.4	1.0	0.2587	0.050
0.5	1.0	0.2644	0.070
0.7	1.0	0.2867	0.120
0.8	1.0	0.3134	0.145
0.9	1.0	0.4747	0.075
1.0	1.0	0.5579	0.500
0.2	2.0	0.2565	0.020
0.4	2.0	0.2786	0.060
0.5	2.0	0.3013	0.085
0.7	2.0	0.4316	0.065

Table 15.--Maximum Dimensionless Temperatures for λ=1.0

ε	β	ϕ_{max}	X
0.0	--	0.3071	0.000
0.2	0.5	0.3083	0.025
0.4	0.5	0.3126	0.055
0.5	0.5	0.3163	0.070
0.7	0.5	0.3291	0.110
0.8	0.5	0.3412	0.135
0.9	0.5	0.3668	0.170
0.2	1.0	0.3099	0.025
0.4	1.0	0.3198	0.060
0.5	1.0	0.3288	0.075
0.7	1.0	0.3656	0.130
0.8	1.0	0.4131	0.165
0.9	1.0	0.8356	0.095
1.0	1.0	0.7668	0.500
0.2	2.0	0.3163	0.025
0.4	2.0	0.3520	0.070
0.5	2.0	0.3907	0.095
0.7	2.0	0.6667	0.105

Table 16.--Maximum Dimensionless Temperatures for λ=2.0

ε	β	ϕ_{max}	X
0.0	--	0.8581	0.000
0.2	0.5	0.8685	0.035
0.4	0.5	0.9041	0.070
0.5	0.5	0.9361	0.090
0.7	0.5	1.0611	0.140
0.8	0.5	1.2050	0.175
0.9	0.5	1.6640	0.225
0.2	1.0	0.8824	0.035
0.4	1.0	0.9734	0.075
0.5	1.0	1.0685	0.100
0.7	1.0	1.6490	0.170
0.8	1.0	4.0026	0.230
0.2	2.0	0.9426	0.035
0.4	2.0	1.4074	0.095
0.5	2.0	2.5014	0.135

Table 17.--Maximum Dimensionless Temperatures for λ=2.3

ε	β	ϕ_{max}	X
0.0	--	3.0858	0.000
0.2	0.5	3.2184	0.040
0.4	0.5	3.7392	0.080
0.5	0.5	4.3235	0.100
0.7	0.5	9.0135	0.155
0.2	1.0	3.4263	0.040
0.4	1.0	5.3814	0.085
0.5	1.0	10.5580	0.110
0.2	2.0	4.6208	0.040

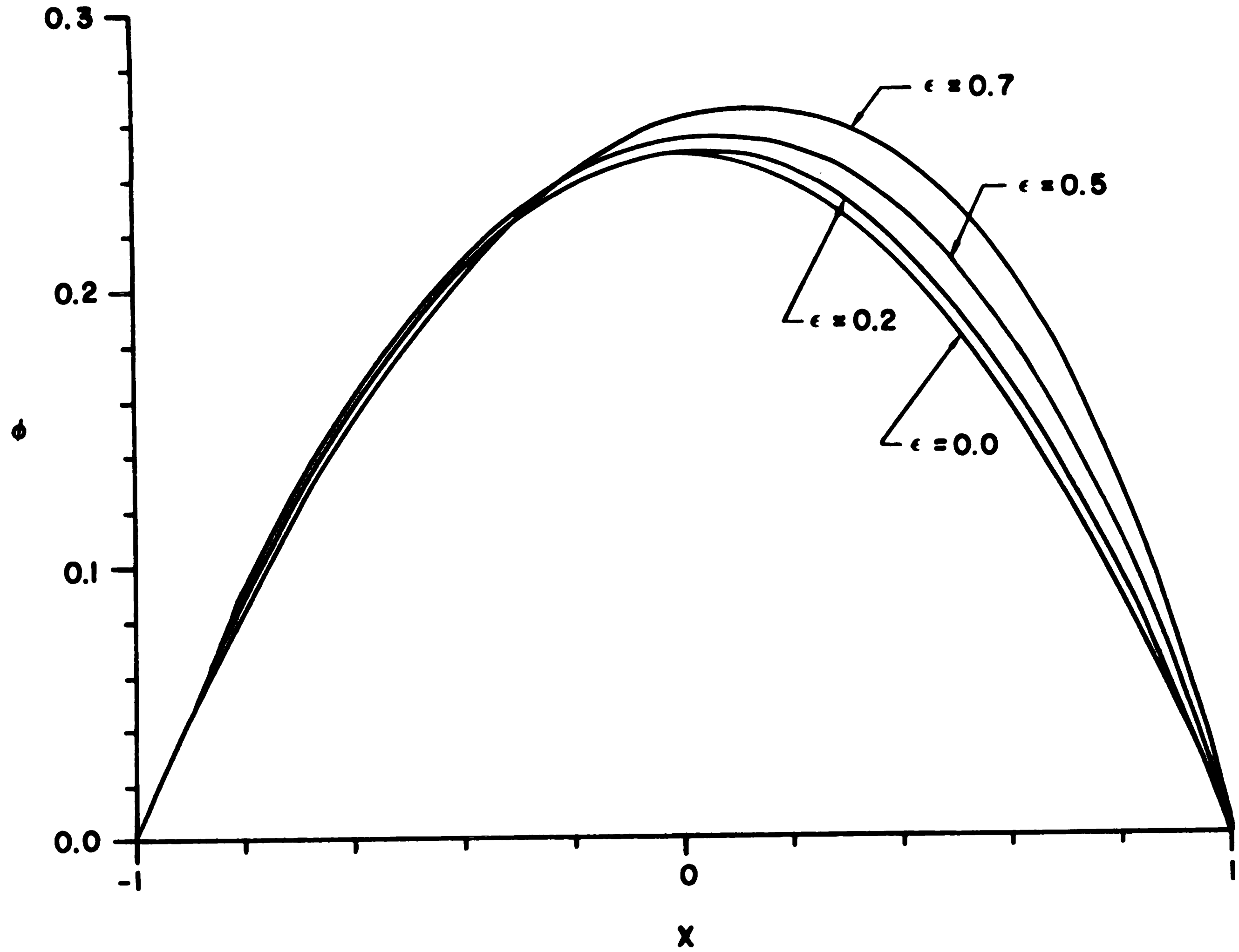


Figure 8.--Dimensionless Temperature Profiles for λ=0.0, β=0.5

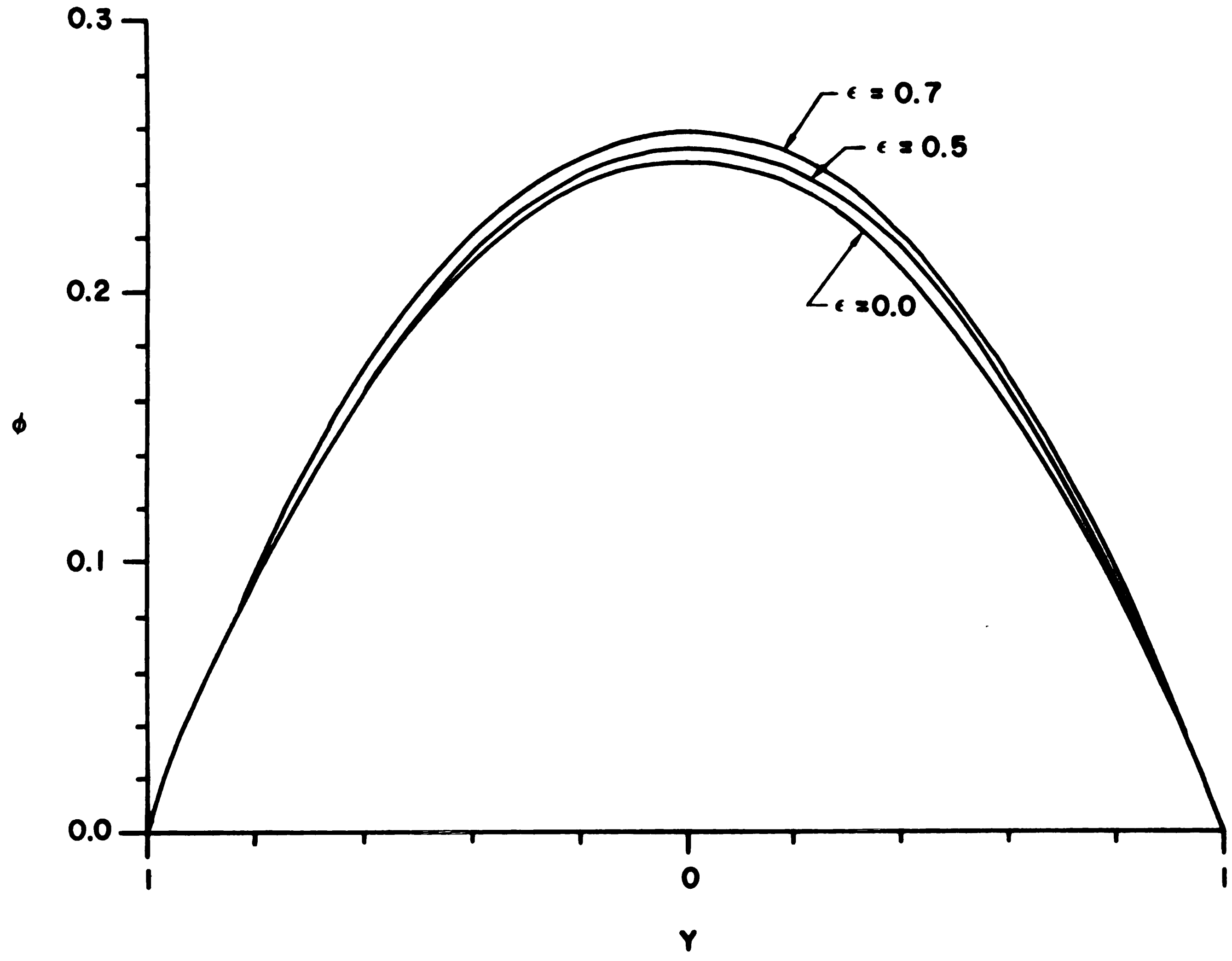


Figure 9.--Dimensionless Temperature Profiles for λ=0.0, β=0.5

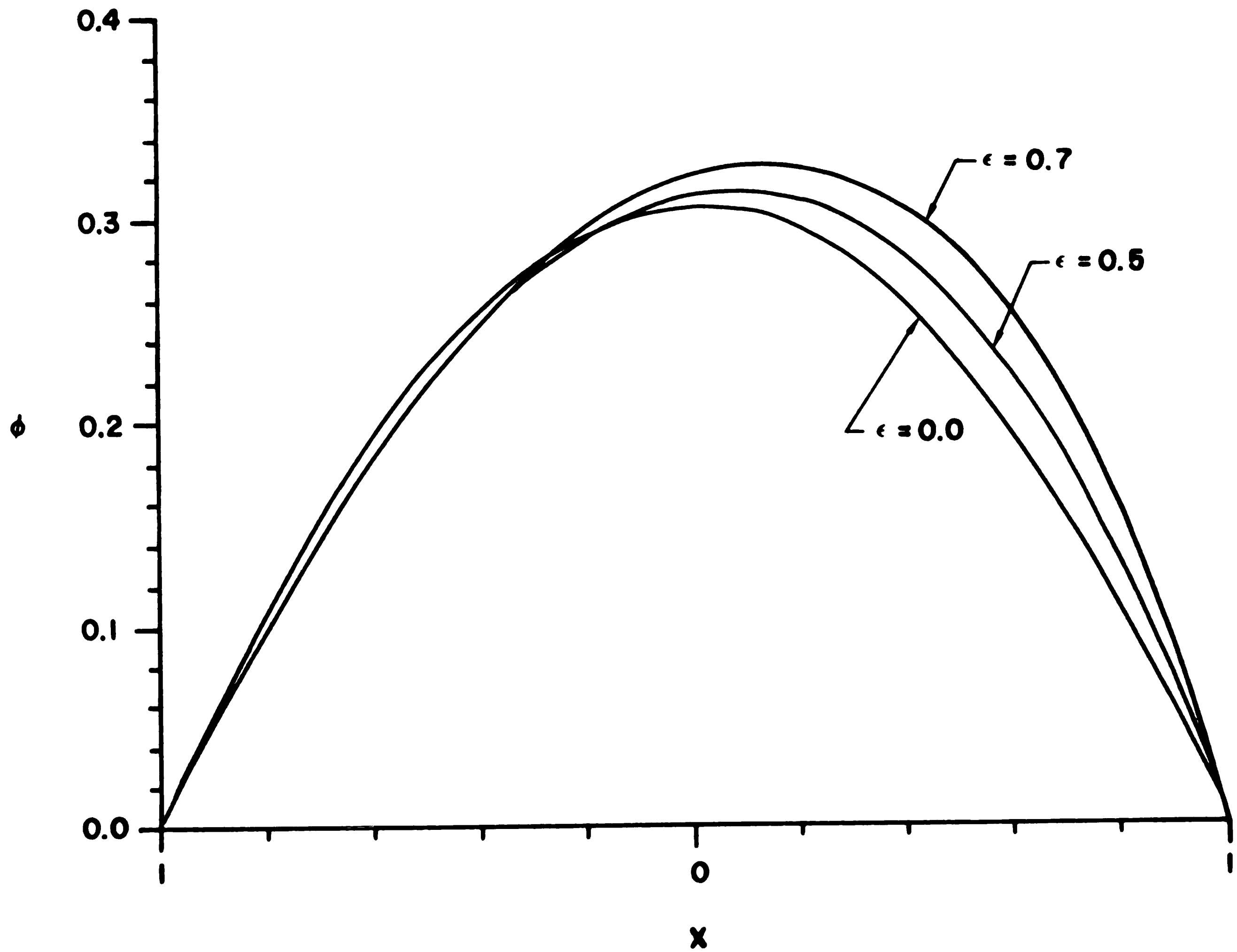


Figure 10.--Dimensionless Temperature Profiles for $\lambda=1.0$, $\beta=0.5$

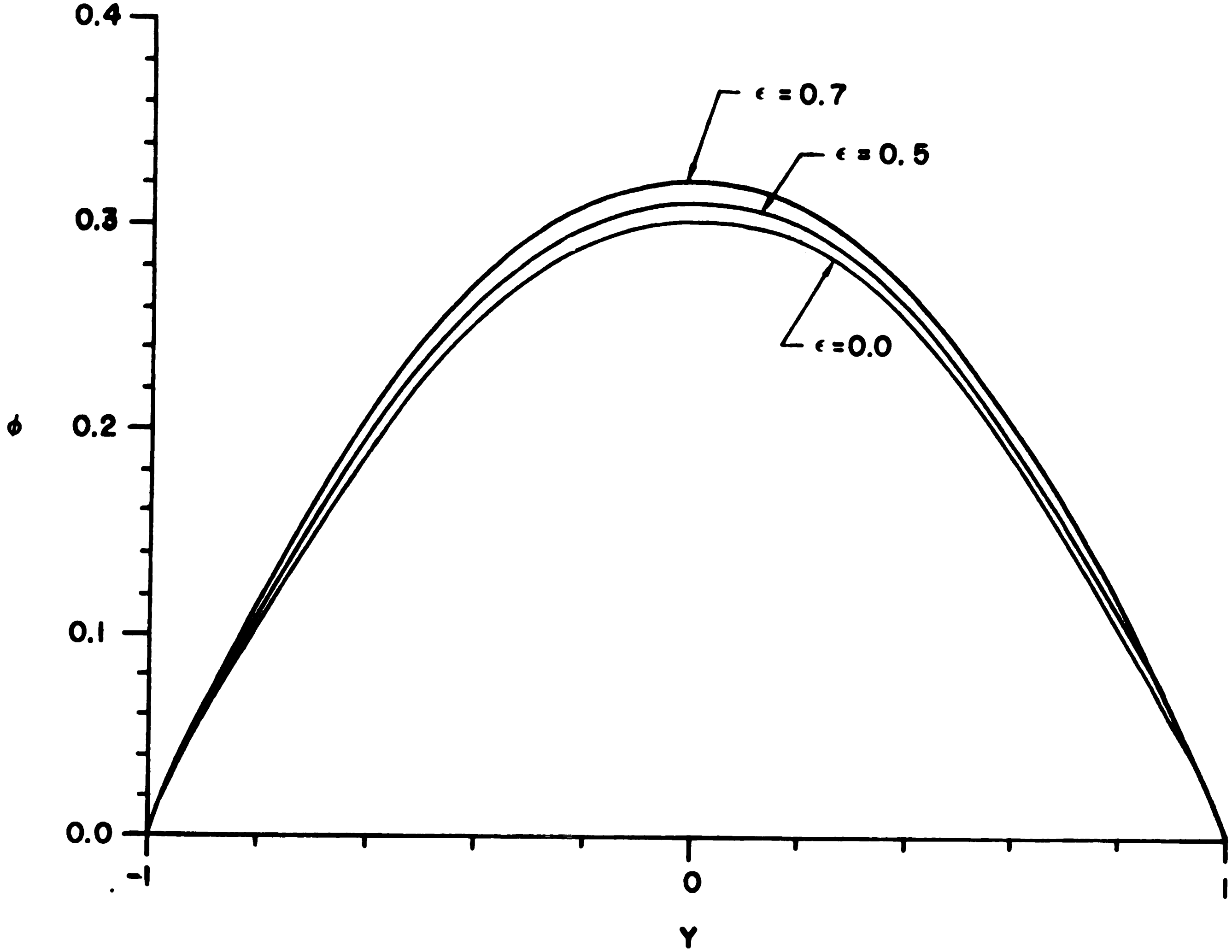


Figure 11.--Dimensionless Temperature Profiles for λ=1.0, β=0.5

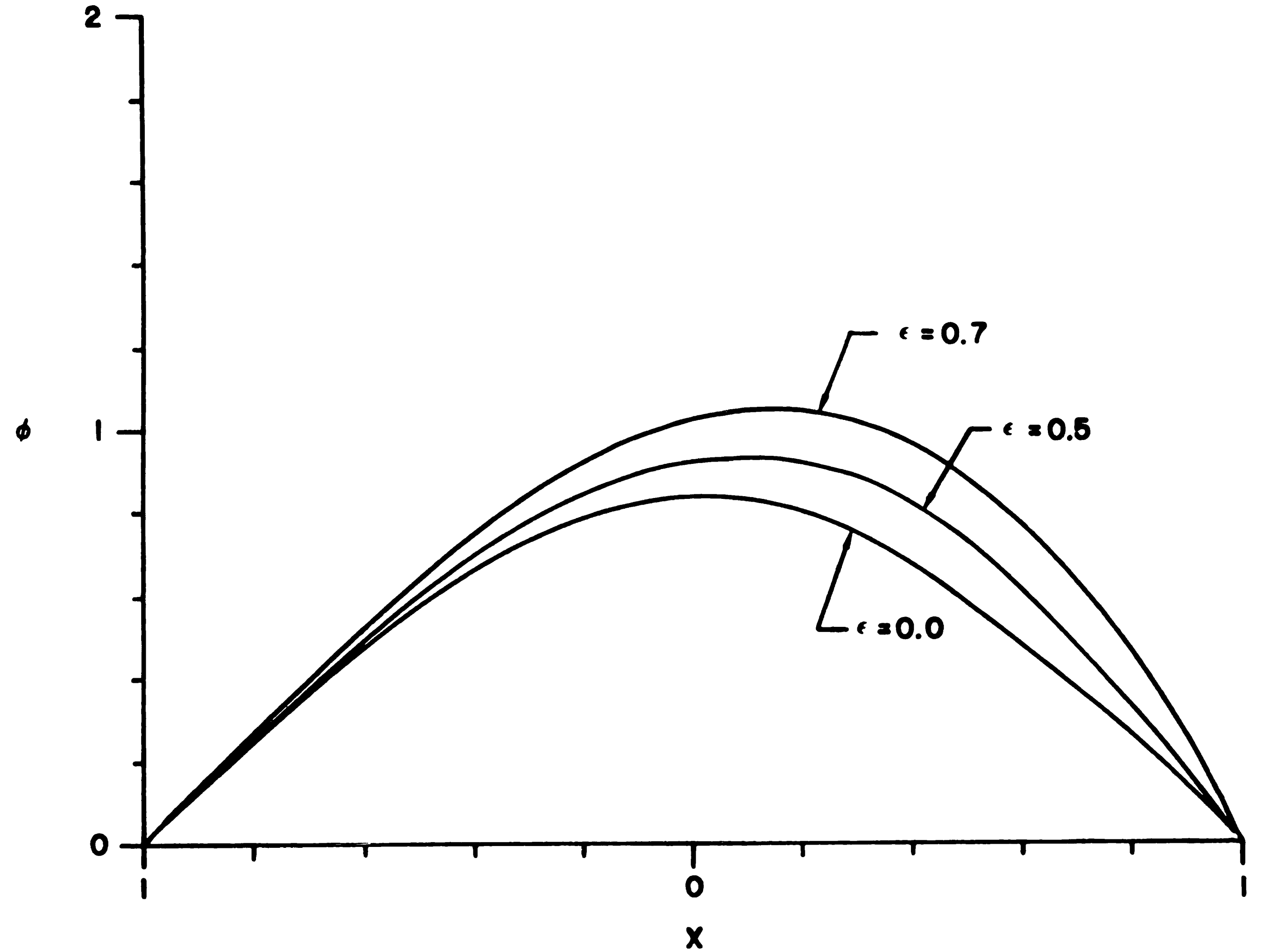


Figure 12.--Dimensionless Temperature Profiles for λ=2.0, β=0.5

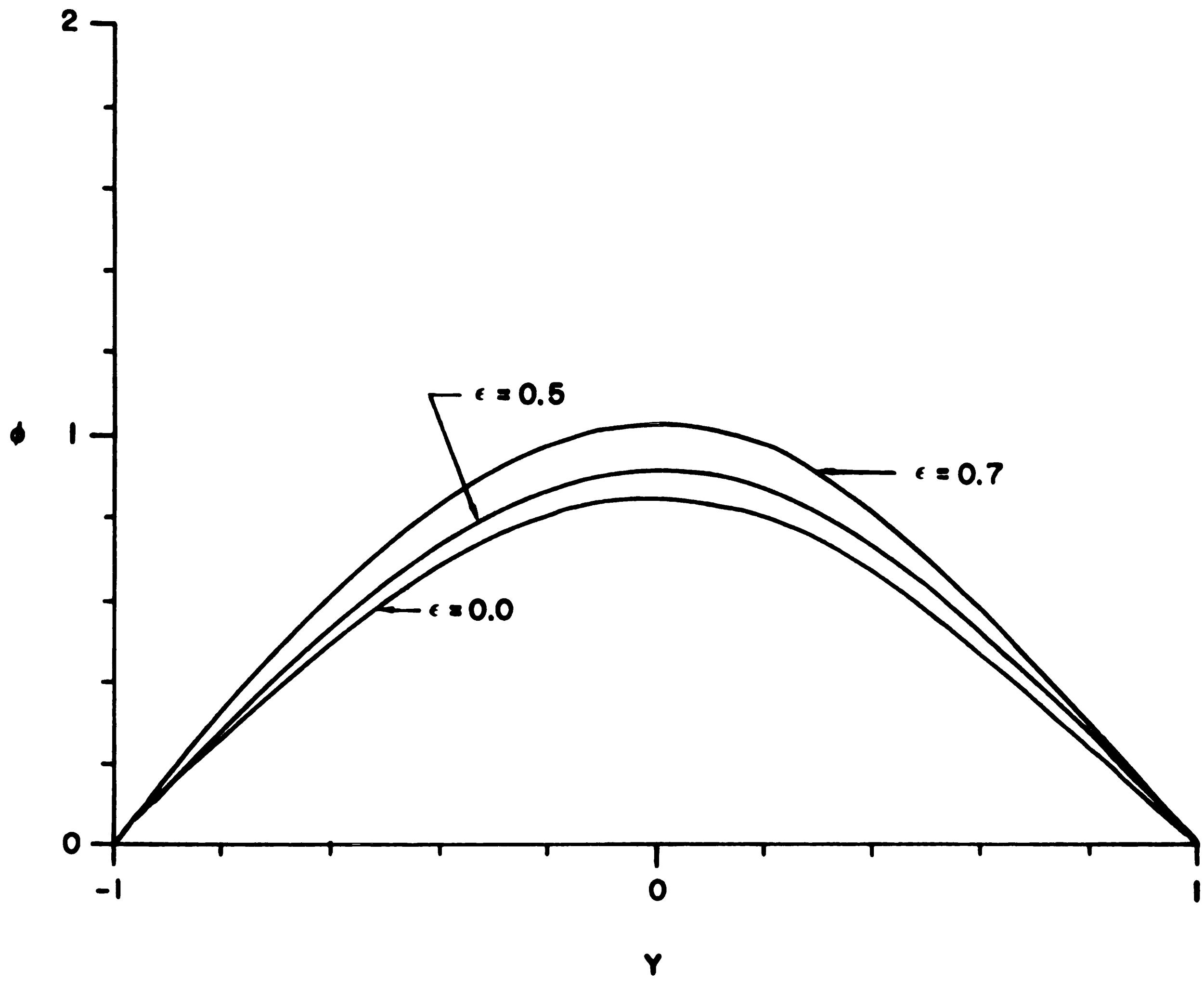


Figure 13.--Dimensionless Temperature Profiles for λ=2.0, β=0.5

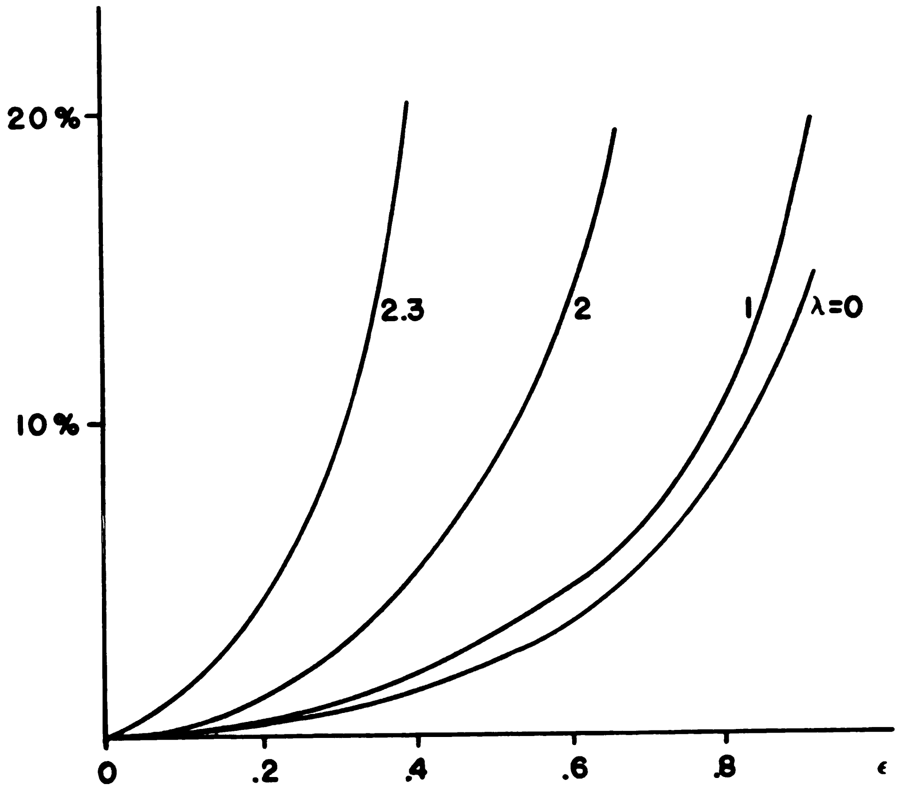


Figure 14.--Percent Increase in Dimensionless Temperature for β=0.5

$(\phi_{max}/\phi_{max}|_{\varepsilon=0.0})$

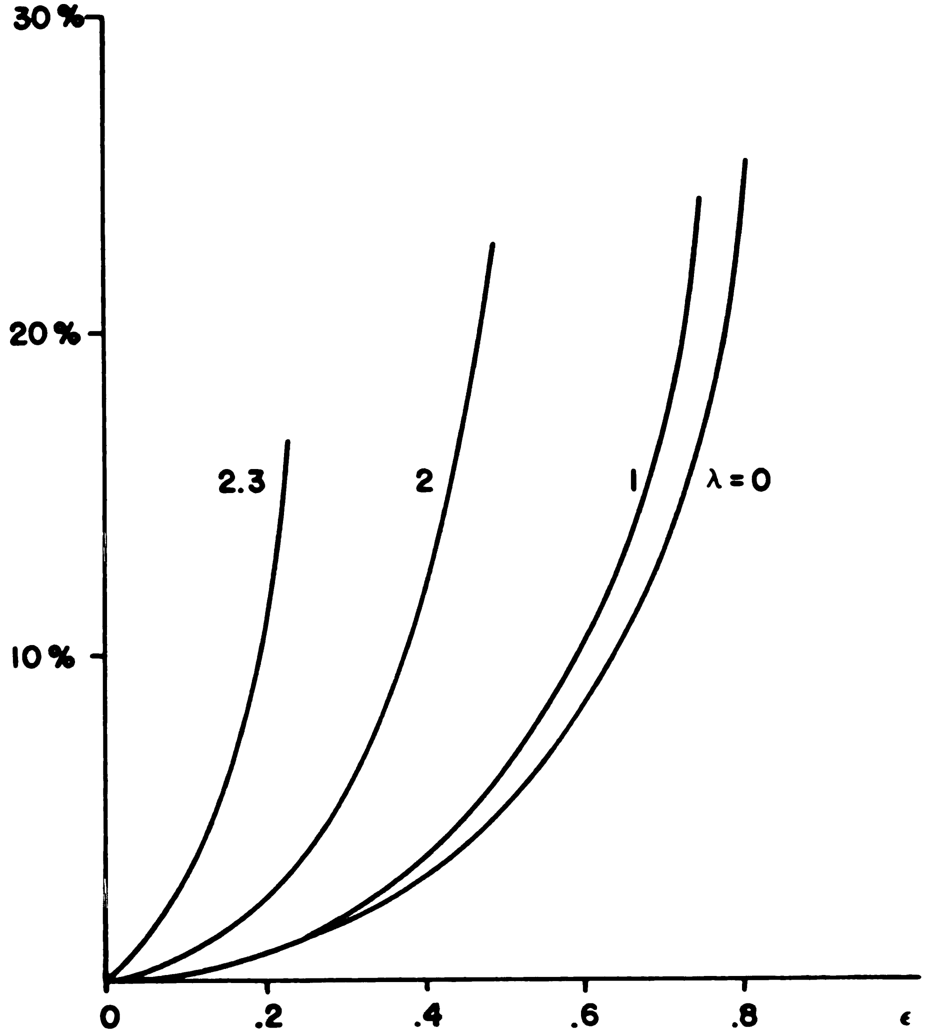


Figure 15.--Percent Increase in Dimensionless Temperature for $\beta=1.0$

$(\phi_{max}/\phi_{max}|_{\varepsilon=0.0})$

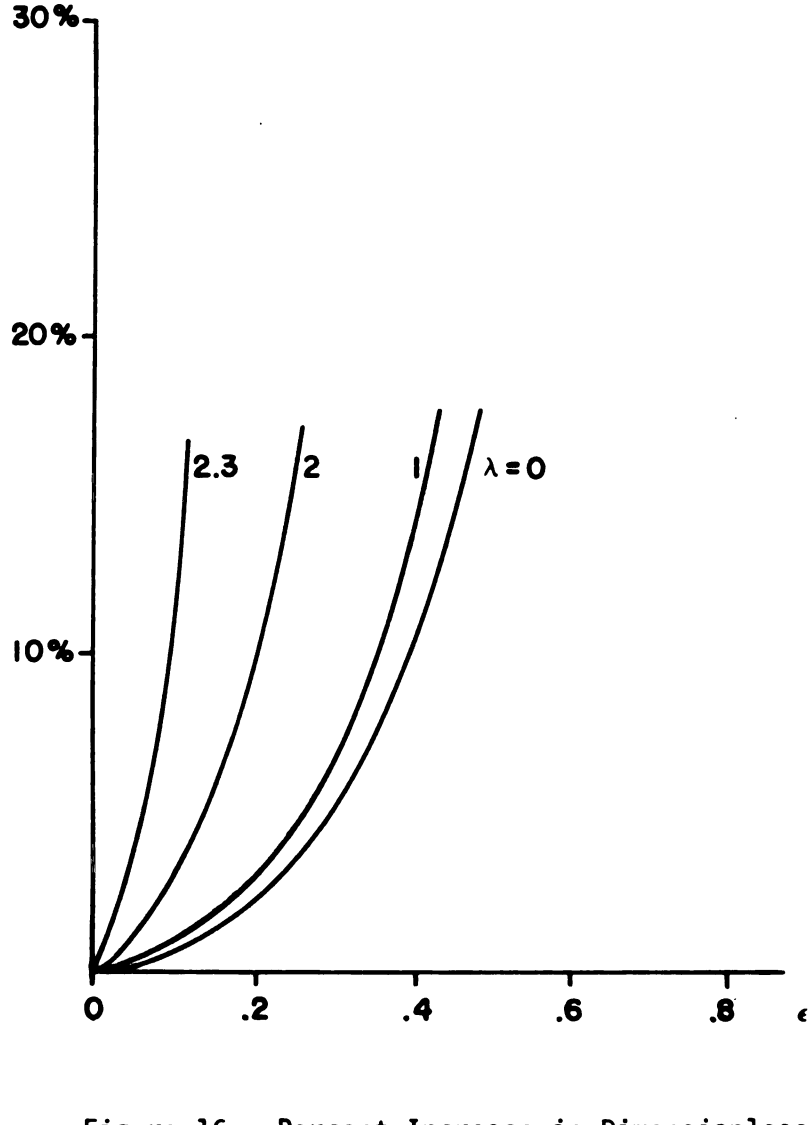


Figure 16.--Percent Increase in Dimensionless Temperature for β=2.0

$(\phi_{max}/\phi_{max}|_{\varepsilon=0.0})$

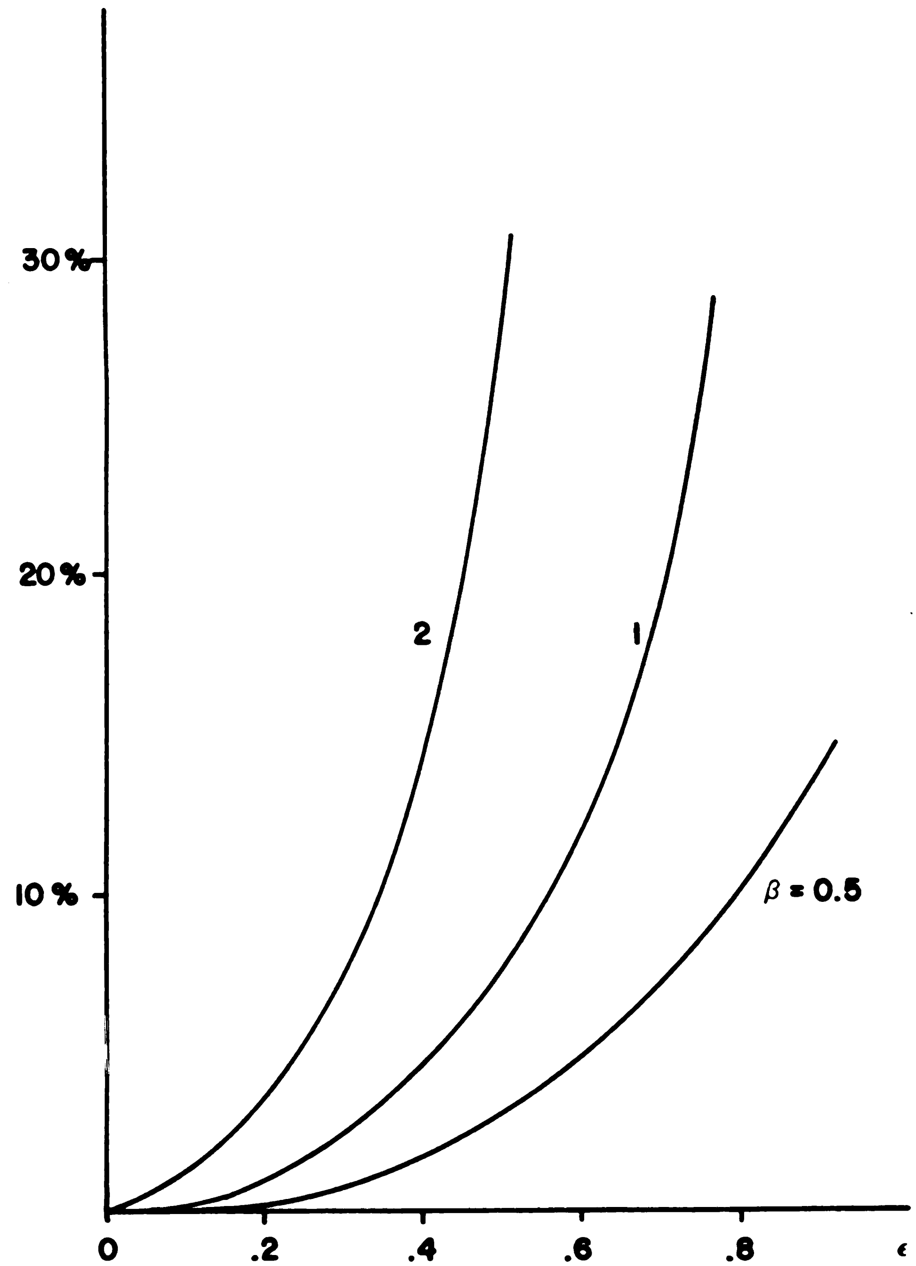


Figure 17.--Percent Increase in Dimensionless Temperature for λ=1.0

$(\phi_{max}/\phi_{max}|_{\varepsilon=0.0})$

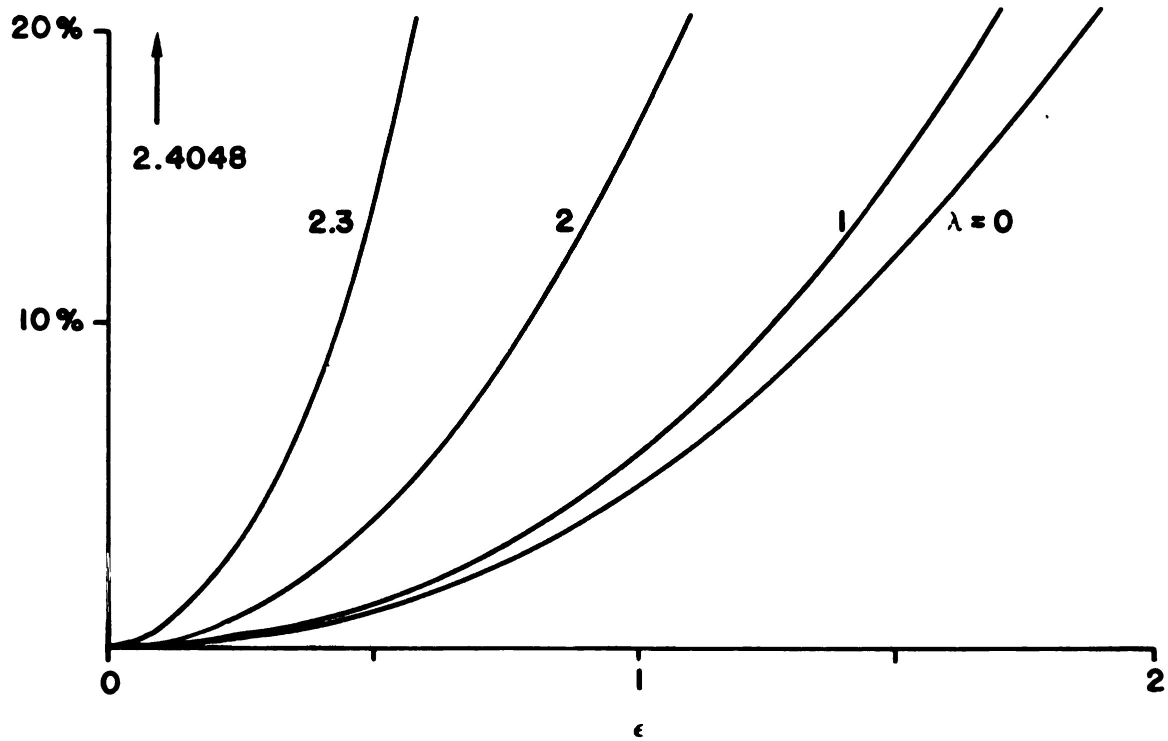


Figure 18.--Percent Increase in Dimensionless Temperature

$(\phi_{max}/\phi_{max}|_{\varepsilon=0.0})$

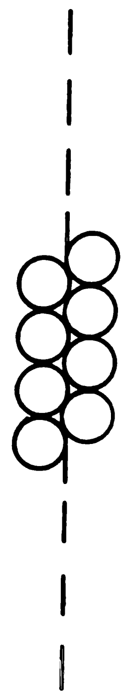


Figure 19.--Helix Configuration for ε=1.0

APPENDIX

```
      PROGRAM ISOPAR (INPUT,OUTPUT,TAPE60=INPUT,TAPE61=OUTPUT)
      DIMENSION VN(8),PNN(8),PNE(8)
      DIMENSION VJ(2,2),X(8),Y(8),PNX(8),PNY(8)
      DIMENSION XC(65),YC(65),COEFA(4),COEFB(4)
      DIMENSION NS(8),ESM(8,8),EF(8),A(65,65),F(65),B(50),IB(50),WK(100)
      COMMON/TLE/TITLE(20)                                              130
      REAL KXX,KYY,KXY,KX,KY,N
      DATA COEFA/-.861136,-.339981,.339981,.861136/
      DATA COEFB/.347855,.652145,.652145,.347855/
      DATA IN/60/,IO/61/,NCL/1/,ID1/0/,KL/8/
      DATA A/4225*0./,F/65*0./
C
C   DEFINITION OF THE INPUT PARAMETERS                                  180
C     TITLE-A DESCRIPTIVE STATEMENT OF THE PROBLEM BEING SOLVED
C     NP-NUMBER OF NODES AND EQUATIONS
C     NE-NUMBER OF ELEMENTS                                             210
C     KXX-MATERIAL PROPERTY IN X DIRECTION                              230
C     KYY-MATERIAL PROPERTY IN Y DIRECTION                              240
C     G-COEFFICIENT MULTIPLYING PHI IN THE DIFF. EQ.                    250
C     Q-SOURCE TERM                                                     260
C     X-X COORDINATES OF THE NODES                                      270
C     Y-Y COORDINATES OF THE NODES                                      280
C     NEL-ELEMENT NUMBER                                                290
C     NS-NODE NUMBERS FOR ELEMENT NODES 1 TO 8
C     VN CONTAINS THE SHAPE FUNCTION VALUES AT E AND N
C     E IS THE VALUE OF SIGH
C     N IS THE VALUE OF NETA
C     PNE CONTAINS THE SHAPE FUNCTION PARTIALS ON E
C     PNN CONTAINS THE SHAPE FUNCTION PARTIALS ON N
C     PNX CONTAINS THE PARTIALS ON X
C     PNY CONTAINS THE PARTIALS ON Y
C     VJ CONTAINS THE JACOBIAN OR J MATRIX
C     VJI CONTAINS THE INVERSE OF J
C     VDJ IS THE VALUE OF THE DETERMINENT OF J
C     A IS THE GLOBAL STIFFNESS MATRIX
C     F IS THE GLOBAL FORCE VECTOR
C                                                                       320
C                                                                       310
C   INPUT OF THE TITLE CARD, CONTROL PARAMETERS AND NODAL COORDINATES   330
      READ(IN,3)TITLE                                                   340
    3 FORMAT(20A4)                                                      350
      READ(IN,*) EE,BE,BL
      READ(IN,*)NP,NE
      READ(IN,*) (XC(I),I=1,NP)                                         370
      READ(IN,*) (YC(I),I=1,NP)                                         380
C                                                                       390
C   OUTPUT OF TITLE, PARAMETER VALUES AND NODAL COORDINATES             540
C                                                                       550
    6 WRITE(IO,4)TITLE,NP,NE,EE,BE,BL
    4 FORMAT(1H1///1X,20A4//1X,5HNP   =I6/1X,5HNE   =I6/1X,5HEE   =E15.5/1
     SX,5HBE   =E15.5/1X,5HBL   =E15.5)
      WRITE(IO,11)                                                      590
   11 FORMAT(//1X,17HNODAL COORDINATES/1X,4HNODE,4X,1HX,14X,1HY)        600
      WRITE(IO,12) (I,XC(I),YC(I),I=1,NP)                               610
   12 FORMAT(I4,2E15.5)                                                 620
C*********                                                              630
C     ASSEMBLYING OF THE GLOBAL STIFFNESS MATRIX AND GLOBAL FORCE MATRIX640
C*********                                                              650
C                                                                       660
C   INPUT AND ECHO PRINT OF ELEMENT DATA                                670
C                                                                       680
      WRITE(IO,8)                                                       690
    8 FORMAT(///1X,12HELEMENT DATA/1X,17HNEL  NODE NUMBERS)             700
      DO 7 KK=1,NE                                                      710
      READ(IN,*) NEL,NS                                                 720
      WRITE(IO,23) NEL,NS                                               730
   23 FORMAT(1X,I3,2X,8I4)
C                                                                       750
C   CALCULATION OF THE ELEMENT STIFFNESS MATRIX AND FORCE VECTOR        760
C                                                                       770
      DO9I=1,8
      II=NS(I)                                                          790
      X(I)=XC(II)                                                       800
    9 Y(I)=YC(II)                                                       810
      DO 99 KI=1,8
      DO 98 KJ=1,8
      ESM(KI,KJ)=0.
```

```
   98 CONTINUE
      EF(KI)=0.
   99 CONTINUE
      DO 30 J=1,4
      E=COEFA(J)
      DO 29 K=1,4
      N=COEFA(K)
      CALL VALUE(E,N,VN,PNN,PNE)
      CALL JMAT(VJ,X,Y,PNN,PNE)
      CALL PARXY (VDJ,PNE,PNN,PNX,PNY,VJ)
      CALL COEF(X,Y,VN,KXX,KYY,KXY,KX,KY,Q,G,EE,BE,BL)
      WC=COEFB(J)*COEFB(K)
      DO 95 L=1,8
      DO 94 M=1,8
      ESM(L,M)=ESM(L,M)+(KXX*PNX(L)*PNX(M)+KYY*PNY(L)*PNY(M)-G*VN(L)*VN
     C(M)-KX*VN(L)*PNX(M)-KY*VN(L)*PNY(M)+KXY*(PNX(L)*PNY(M)+PNY(L)*PNX
     C(M))*.5)*VDJ*WC
   94 CONTINUE
      EF(L)=EF(L) + VN(L)*Q*WC*VDJ
   95 CONTINUE
   29 CONTINUE
   30 CONTINUE
C
C     ASSEMBLE A AND F FROM ESM AND EF
C
      DO 41 IC=1,8
      DO 40 JC=1,8
      II=NS(IC)
      JJ=NS(JC)
      A(II,JJ)=A(II,JJ)+ESM(IC,JC)
   40 CONTINUE
      F(II)=F(II)+EF(IC)
   41 CONTINUE
    7 CONTINUE
C                                                                       950
C  MODIFICATION AND SOLUTION OF THE STSTEM OF EQUATIONS                 960
C                                                                       970
      CALL BDYVAL(A,F,NP,B,IB)
      M=1
      IDGT=4
      CALL LEQT1F (A,M,NP,NP,F,IDGT,WK,IER)
      WRITE(IO,100)
  100 FORMAT(////,1X,* NODAL VALUES*)
      DO 200 I=1,NP
      WRITE(IO,201) I,F(I)
  200 CONTINUE
  201 FORMAT (        I4,E15.5)
      CALL OUTPUT(F,XC,YC)
C*********                                                              1010
      STOP                                                              1400
      END                                                               1410
```

```
      SUBROUTINE VALUE (E,N,VN,PNN,PNE)
      DIMENSION VN(8),PNN(8),PNE(8)
      REAL N
      VN(1)=-.25*(1.-E)*(1.-N)*(E+N+1.)
      VN(2)= .5*(1.-E**2)*(1.-N)
      VN(3)= .25*(1.+E)*(1.-N)*(E-N-1.)
      VN(4)= .5*(1.-N**2)*(1.+E)
      VN(5)= .25*(1.+E)*(1.+N)*(E+N-1.)
      VN(6)= .5*(1.-E**2)*(1.+N)
      VN(7)= -.25*(1.-E)*(1.+N)*(E-N+1.)
      VN(8)= .5*(1.-N**2)*(1.-E)
      PNN(1)= .25*( -E**2+E-2.*E*N+2.*N)
      PNN(2)= -.5*(1.-E**2)
      PNN(3)= .25*(-E-E**2+2.*N+2.*N*E)
      PNN(4)= -N*(1.+E)
      PNN(5)= .25*(E**2+E+2*N+2.*E*N)
      PNN(6)= .5*(1.-E**2)
      PNN(7)= -.25*(E-E**2-2.*N+2.*N*E)
      PNN(8)= -N*(1.-E)
      PNE(1)= .25*(2.*E-2.*E*N+N-N**2)
      PNE(2) = -E*(1.-N)
      PNE(3)= .25*(-N+2.*E-2.*E*N+N**2)
      PNE(4)= .5*(1.-N**2)
      PNE(5)= .25*(2.*E+2.*E*N+N+N**2)
      PNE(6)= -E*(1.+N)
      PNE(7)= -.25*(-2.*E+N-2.*E*N+N**2)
      PNE(8)= -.5*(1.-N**2)
      RETURN
      END
```

```
      SUBROUTINE JMAT (VJ,X,Y,PNN,PNE)
      DIMENSION VJ(2,2),X(8),Y(8),PNN(8),PNE(8)
      VJ(1,1)=0.
      VJ(1,2)=0.
      VJ(2,1)=0.
      VJ(2,2)=0.
      DO 1 I=1,8
      VJ(1,1)=VJ(1,1)+PNE(I)*X(I)
      VJ(1,2)=VJ(1,2)+PNE(I)*Y(I)
      VJ(2,1)=VJ(2,1)+PNN(I)*X(I)
      VJ(2,2)=VJ(2,2)+PNN(I)*Y(I)
    1 CONTINUE
      RETURN
      END
```

```
      SUBROUTINE PARXY (VDJ,PNE,PNN,PNX,PNY,VJ)
      DIMENSION PNX(8),PNY(8),PNE(8),PNN(8),VIJ(2,2),VJ(2,2)
      VDJ=VJ(1,1)*VJ(2,2)-VJ(1,2)*VJ(2,1)
      VDJ=ABS(VDJ)
      VIJ(1,1)=VJ(2,2)/VDJ
      VIJ(1,2)=-VJ(1,2)/VDJ
      VIJ(2,1)=-VJ(2,1)/VDJ
      VIJ(2,2)=VJ(1,1)/VDJ
      DO 1 K=1,8
      PNX(K)= PNE(K)*VIJ(1,1)+ PNN(K)*VIJ(1,2)
      PNY(K)= PNE(K)*VIJ(2,1)+ PNN(K)*VIJ(2,2)
    1 CONTINUE
      RETURN
      END
```

```
      SUBROUTINE COEF(X,Y,VN,KXX,KYY,KXY,KX,KY,Q,G,EE,BE,BL)
      REAL KXX,KYY,KXY,KX,KY
      DIMENSION X(8),Y(8),VN(8)
      XX=0.
      YY=0.
      DO 1 I=1,8
      XX=XX+X(I)*VN(I)
      YY=YY+Y(I)*VN(I)
    1 CONTINUE
      D=BE**2*EE**2*(XX**2+YY**2)/((1.-EE*XX)**2)
      R=XX**2+YY**2
      KXX=(XX**2+YY**2*(1.+D))/R
      KYY=(YY**2+XX**2*(1.+D))/R
      KXY=-2*XX*YY*D/R
      KX=-((EE*XX**2)/(1.-EE*XX)+(EE*YY**2)*(1.-D)/(1.-EE*XX) +XX*D)/R
      KY=-( (EE*XX)/(1.-EE*XX) +1.)*YY*D/R
      G=BL**2
      Q=1.
      RETURN
      END
```

```
      SUBROUTINE BDYVAL(A,F,NP,B,IB)
      DIMENSION A(NP,NP),F(NP),B(NP),IB(NP)
      COMMON/TLE/ TITLE(20)
      DATA IN/60/,IO/61/
      WRITE(IO,200) TITLE
  200 FORMAT(1H1,//////,1X,20A4)
C
C     INPUT OF THE PRESCRIBED NODAL VALUES
C
      N=0
      DO 100 I=1,NP
      READ(IN,*) IB(I)
      IF ( IB(I).EQ.0) GO TO 101
      READ(IN,*) B(I)
      N=N+1
  100 CONTINUE
  101 IF(N.EQ.0) GO TO 1000
      WRITE(IO,208)
  208 FORMAT(////,1X,* PRESCRIBED NODAL VALUES*)
      WRITE(IO,207)      ( IB(M),B(M), M=1,N)
  207 FORMAT (1X,6(I3,E14.5,2X))
C
C     MOD. OF THE MATR. USING THE METHOD OF ROWS AND COLMS.
C
      DO 2 J=1,N
      DO 1 K=1,NP
      A( IB(J),K) =0.
      F(K)=F(K)- A(K,IB(J)) *B(J)
      A(K,IB(J) )=0.
    1 CONTINUE
      A(IB(J),IB(J)) =1.
      F(IB(J)) = B(J)
    2 CONTINUE
 1000 RETURN
      END
```

```
      SUBROUTINE OUTPUT(F,XC,YC)
      DIMENSION VN(8),F(65),XC(65),YC(65),NS(8),C(8),T(8),PNX(8),PNY(8)
      DIMENSION X(8),Y(8)
      REAL N
      DATA C/-1.,-.75,-.5,-.25,0.,.25,.5,.75/
      DATA IN/60/,IO/61/
      WRITE (IO,1)
    1 FORMAT(//1X,7HX-COORD,12X,4HTEMP)
    2 FORMAT(//1X,7HY-COORD,12X,4HTEMP)
    3 FORMAT(//1X,9HMAX TEMP=E15.5,1X,5HAT X=E15.5)
    4 FORMAT(2E15.5)
      DO 19 I=1,4
      READ(IN,*) NEL,NS
      DO 18 J=1,8
      II=NS(J)
      T(J)= F(II)
   18 X(J)=XC(II)
      DO 17 K=1,8
      N=-1.
      E=C(K)
      CALL VALUE(E,N,VN,PNX,PNY)
      TT=0.
      XX=0.
      DO 16 L=1,8
      TT=TT+VN(L)*T(L)
   16 XX=XX+VN(L)*X(L)
   17 WRITE(IO,4) XX,TT
   19 CONTINUE
      XX=1.
      TT=0.
      WRITE(IO,4) XX,TT
      WRITE(IO,2)
      DO 29 I=1,4
      READ(IN,*) NEL,NS
      DO 28 J=1,8
      II=NS(J)
      T(J)=F(II)
   28 Y(J)=YC(II)
      DO 27 K=1,8
      E=-1.
      N=C(K)
      CALL VALUE(E,N,VN,PNX,PNY)
      TT=0.
      YY=0.
      DO 26 L=1,8
      TT=TT+VN(L)*T(L)
   26 YY=YY+VN(L)*Y(L)
   27 WRITE(IO,4) YY,TT
   29 CONTINUE
      YY=1.
      TT=0.
      WRITE(IO,4) YY,TT
      TSAV=0.
      XSAV=0.
      DO 39 I=1,2
      READ(IN,*) NEL,NS
      DO 38 J=1,8
      II=NS(J)
      T(J)=F(II)
   38 X(J)=XC(II)
      N=-1.
      E=-1.
      DO 37 K=1,100
      CALL VALUE(E,N,VN,PNX,PNY)
      TT=0.
      XX=0.
      DO 36 L=1,8
      TT=TT+VN(L)*T(L)
   36 XX=XX+VN(L)*X(L)
      E=E+.02
      IF(TSAV.GE.TT) GO TO 40
      TSAV=TT
   37 XSAV=XX
   39 CONTINUE
   40 WRITE(IO,3) TSAV,XSAV
      RETURN
      END
```

THESIS PROGRAM

```
NP  =     65
NE  =     16
EE  =      .50000E+00
BE  =      .50000E+00
BL  =      .10000E+01
```

NODAL COORDINATES

NODE	X	Y
1	-.10000E+01	0.
2	-.96825E+00	-.25000E+00
3	-.96825E+00	.25000E+00
4	-.86603E+00	-.50000E+00
5	-.86603E+00	.50000E+00
6	-.75000E+00	-.66144E+00
7	-.75000E+00	-.50000E+00
8	-.75000E+00	0.
9	-.75000E+00	.50000E+00
10	-.75000E+00	.66144E+00
11	-.70711E+00	-.70711E+00
12	-.70711E+00	.70711E+00
13	-.66144E+00	-.75000E+00
14	-.66144E+00	.75000E+00
15	-.50000E+00	-.86603E+00
16	-.50000E+00	-.75000E+00
17	-.50000E+00	-.50000E+00
18	-.50000E+00	-.25000E+00
19	-.50000E+00	0.
20	-.50000E+00	.25000E+00
21	-.50000E+00	.50000E+00
22	-.50000E+00	.75000E+00
23	-.50000E+00	.86603E+00
24	-.25000E+00	-.96825E+00
25	-.25000E+00	-.50000E+00
26	-.25000E+00	0.
27	-.25000E+00	.50000E+00
28	-.25000E+00	.96825E+00
29	0.	-.10000E+01
30	0.	-.75000E+00
31	0.	-.50000E+00
32	0.	-.25000E+00
33	0.	0.
34	0.	.25000E+00
35	0.	.50000E+00
36	0.	.75000E+00
37	0.	.10000E+01
38	.25000E+00	-.96825E+00
39	.25000E+00	-.50000E+00
40	.25000E+00	0.
41	.25000E+00	.50000E+00
42	.25000E+00	.96825E+00
43	.50000E+00	-.86603E+00
44	.50000E+00	-.75000E+00
45	.50000E+00	-.50000E+00
46	.50000E+00	-.25000E+00

47	.50000E+00	0.
48	.50000E+00	.25000E+00
49	.50000E+00	.50000E+00
50	.50000E+00	.75000E+00
51	.50000E+00	.86603E+00
52	.66144E+00	-.75000E+00
53	.66144E+00	.75000E+00
54	.70711E+00	-.70711E+00
55	.70711E+00	.70711E+00
56	.75000E+00	-.66144E+00
57	.75000E+00	-.50000E+00
58	.75000E+00	0.
59	.75000E+00	.50000E+00
60	.75000E+00	.66144E+00
61	.86603E+00	-.50000E+00
62	.86603E+00	.50000E+00
63	.96825E+00	-.25000E+00
64	.96852E+00	.25000E+00
65	.10000E+01	0.

ELEMENT DATA

NEL	NODE NUMBERS							
1	11	13	15	16	17	7	4	6
2	4	7	17	18	19	8	1	2
3	1	8	19	20	21	9	5	3
4	5	9	21	22	23	14	12	10
5	15	24	29	30	31	25	17	16
6	17	25	31	32	33	26	19	18
7	19	26	33	34	35	27	21	20
8	21	27	35	36	37	28	23	22
9	29	38	43	44	45	39	31	30
10	31	39	45	46	47	40	33	32
11	33	40	47	48	49	41	35	34
12	35	41	49	50	51	42	37	36
13	43	52	54	56	61	57	45	44
14	45	57	61	63	65	58	47	46
15	47	58	65	64	62	59	49	48
16	49	59	62	60	55	53	51	50

```
THESIS PROGRAM


 PRESCRIBED NODAL VALUES
  1    O.                2    O.                3    O.                4    O.                5    O.                6    O.
 10    O.               11    O.               12    O.               13    O.               14    O.               15    O.
 23    O.               24    O.               28    O.               29    O.               37    O.               38    O.
 42    O.               43    O.               51    O.               52    O.               53    O.               54    O.
 55    O.               56    O.               60    O.               61    O.               62    O.               63    O.
 64    O.               65    O.


 NODAL VALUES
  1      O.
  2      O.
  3      O.
  4      O.
  5      O.
  6      O.
  7       .49527E-01
  8       .12161E+00
  9       .49527E-01
 10      O.
 11      O.
 12      O.
 13      O.
 14      O.
 15      O.
 16       .50293E-01
 17       .14394E+00
 18       .19887E+00
 19       .21651E+00
 20       .19887E+00
 21       .14394E+00
 22       .50293E-01
 23      O.
 24      O.
 25       .20477E+00
 26       .28306E+00
 27       .20477E+00
 28      O.
 29      O.
 30       .13368E+00
 31       .23064E+00
 32       .29288E+00
 33       .31455E+00
 34       .29288E+00
 35       .23064E+00
 36       .13369E+00
 37      O.
 38      O.
```

39	.22214E+00
40	.30506E+00
41	.22215E+00
42	0.
43	0.
44	.58478E-01
45	.16968E+00
46	.23359E+00
47	.25181E+00
48	.23359E+00
49	.16970E+00
50	.58483E-01
51	0.
52	0.
53	0.
54	0.
55	0.
56	0.
57	.61290E-01
58	.15700E+00
59	.61299E-01
60	0.
61	0.
62	0.
63	0.
64	0.
65	0.

X-COORD	TEMP
-.10000E+01	0.
-.93750E+00	.32905E-01
-.87500E+00	.64140E-01
-.81250E+00	.93707E-01
-.75000E+00	.12161E+00
-.68750E+00	.14783E+00
-.62500E+00	.17239E+00
-.56250E+00	.19529E+00
-.50000E+00	.21651E+00
-.43750E+00	.23644E+00
-.37500E+00	.25418E+00
-.31250E+00	.26973E+00
-.25000E+00	.28308E+00
-.18750E+00	.29424E+00
-.12500E+00	.30320E+00
-.62500E-01	.30997E+00
0.	.31455E+00
.62500E-01	.31628E+00
.12500E+00	.31528E+00
.18750E+00	.31153E+00
.25000E+00	.30506E+00
.31250E+00	.29585E+00
.37500E+00	.28390E+00
.43750E+00	.26922E+00
.50000E+00	.25181E+00
.56250E+00	.23394E+00
.62500E+00	.21218E+00
.68750E+00	.18653E+00
.75000E+00	.15700E+00
.81250E+00	.12358E+00
.87500E+00	.86273E-01

.93750E+00	.45080E-01
.10000E+01	0.

Y-COORD	TEMP
-.10000E+01	0.
-.93750E+00	.36865E-01
-.87500E+00	.71434E-01
-.81250E+00	.10371E+00
-.75000E+00	.13368E+00
-.68750E+00	.16137E+00
-.62500E+00	.18675E+00
-.56250E+00	.20984E+00
-.50000E+00	.23064E+00
-.43750E+00	.25000E+00
-.37500E+00	.26683E+00
-.31250E+00	.28112E+00
-.25000E+00	.29288E+00
-.18750E+00	.30210E+00
-.12500E+00	.30879E+00
-.62500E-01	.31294E+00
0.	.31455E+00
.62500E-01	.31294E+00
.12500E+00	.30879E+00
.18750E+00	.30210E+00
.25000E+00	.29288E+00
.31250E+00	.28112E+00
.37500E+00	.26683E+00
.43750E+00	.25000E+00
.50000E+00	.23064E+00
.56250E+00	.20984E+00
.62500E+00	.18675E+00
.68750E+00	.16137E+00
.75000E+00	.13369E+00
.81250E+00	.10371E+00
.87500E+00	.71435E-01
.93750E+00	.36865E-01
.10000E+01	0.

MAX TEMP= .31630E+00 AT X= .70000E-01

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