THESIS

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FUNDAMENTALS OF AN AUTOMATED PROCEDURE
FOR THE SYNTHESIS OF PLANAR MECHANISMS

By

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ABSTRACT

FUNDAMENTALS OF AN AUTOMATED PROCEDURE FOR THE SYNTHESIS OF PLANAR MECHANISMS

By

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The goal of this work is to identify the basic steps and examine the feasibility of a unified procedure for synthesis of planar mechanisms. The basic steps are: to disconnect a mechanism to arrive at a set of compound pendulums; to locate the input and output links at the desired positions in order to satisfy the motion requirement; to form the objective function as the sum of squares of the distance between the ends of the disconnected joints, and to conduct a systematic search to find the optimum of the objective function using nonlinear programming techniques.

The applicability of this procedure was tested by solving four example problems. The example problems were chosen from different class of synthesis problems, they are a function generation problem, two path generation problems, and an angle coordination problem. In each case a satisfactory solution was obtained.

TO MY PARENTS

ACKNOWLEDGMENTS

I wish to give my best appreciation to Dr. B. Fallahi, my major advisor, for his guidance and assistance throughout this research program. I would also like to thank the other members on my Master committee, Dr. B. S. Thompson and Dr. C. J. Radcliffe, for their time and advise.

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CHAPTER 1

A SURVEY ON THE OPTIMIZATION METHODS AND THE OBJECTIVE FUNCTION FORMULATION FOR THE MECHANISM SYNTHESIS PROBLEMS

Mechanism synthesis using nonlinear programming consists of two elements, an optimizer and a modeller. The optimizer is an algorithm for finding the optimum of an objective function. The modeller is an algorithm to calculate the measure of goodness. In the mechanism synthesis problem, this measure is related to the closeness of the generated motion to the desired motion. In the following, a survey of the optimization techniques, application of the optimization techniques to the mechanism synthesis problems, and objective function formulation for the synthesis problems are discussed.

1.1 A Survey of Optimization Techniques

A general nonlinear programming has the following form[1]:

$$\text{minimize } F(X)$$

subject to:

inequality constraints: $g_i(X) \geq 0$, $i=1,2,\ldots,n$

equality constraints: $h_j(X) = 0$, $j=1,2,\ldots,m$

where $F(X)$ is the objective function to be minimized by an optimal

Unconstrained Search Technique	Class zero Method
	Class One Method
	Class Two Method
Constrained Search Technique	Penalty Function Method
	Reduced Gradient Method
	Stochastic Programming Method

Figure 1.1 The Classification of the Optimization Methods

choice of X(i.e. the design variables $[x_1, x_2, \ldots, x_p]^T$).

Fig.1-1 shows the classification of the optimization techniques.

1.1.1 The Unconstrained Search Technique

Because many of the available methods treat the constrained case as a variation of the unconstrained one, the unconstrained search technique still plays an important role irrespective of the fact that the realistic design problems are constrained problems.

The unconstrained search technique can be classified into three groups according to the order of the derivative of the objective function used.

A. Class Zero Method

Class zero methods use only function to perform the search, the simplest one of these methods is represented by the Random Walk Method[2]. Hooke and Jeeves[3] introduced another class zero method. This method moves toward the minimum by making a series of unidimensional moves followed by a pattern move.

A.1 Application of the Class Zero Method

Kramer and Schaefer[4] used the Hooke and Jeeves method to find the minimum of an objective function. This objective function was formed as the error between position and velocity of the actual mechanism and that defined by the desired motion. In 1974, a sequential unconstrained optimization program PATSH was developed by Daniel [5]. Rao[6] adopted this method in solving his consideration of the structural and mechanical error together. The mechanical error, due to manufacturing tolerances on the link dimensions, is assumed random in nature. Pazouki[7] also used PATSH to get a set of single infinity of solutions. There were infinite sets of mechanism with objective function of constant value which is an optimum.

B. Class One Method

In this group, the local gradient of the objective function is used to identify the search direction. The simplest one is the method of the steepest descent. Cauchy's method[8] improved the steepest descent method by introducing a flexible step size. The value of the step size is chosen to achieve the maximum decrease of the objective function in the step direction. The Fletcher-Reeves method[9] exhibits a quality known as quadratic convergence is also another one belongs to this group. Another widely used class one method is the Variable Metric Method[10] which also converges quadratically.

B.1 Application of the Class One Method

Tull and Lewis[11] used Cauchy's method to minimize the error between the generated coupler curve and the desired path. Suh and Mecklenburg[12] introduced the Fletcher-Reeves method in their work. For specified number of precision points, the error between the desired motion and the generated one was emphasized to be as small as possible. Bagci[13] synthesized a mechanism of which a link was guided through a specified curve. The objective function in this work was minimized by Variable Metric Method. Variable Metric Method was also used by Mabie and Mitchiner[14]. In their work, a mechanism that generates an approximately straight coupler path was synthesized.

C. Class Two Method

Class two methods use function, gradient and second derivative information to perform optimization. The most common class two method is Newton's Quasilinearization Method[15]. However, difficulities arising in the evaluation of the second partial derivatives and the large computional efforts required make for major limitations in the usefulness of this method.

1.1.2 The Constrained Search Technique

A. Penalty Function Method

In the penalty function method, the constrained problem is tranformed to an unconstrained problem by adding a penalty term to the objective function[16]. One of the most widely used penalty functions is SUMT[17].

A.1 Application of the Penalty function Method

Kimbrell[18] used the SUMT penalty function method to solve problems concerning mechanical clearance and tolerance in mechanism. Fox and Willmert[19] formed an objective function as the least square of the deviation of the crank and rocker angle from their desired position. To find the minimum, Fox and Willmert used the Fiacco and McCormick Penalty function method[20]. Kramer[21] introduced the Selective Precision Synthesis in its objective function formulation, and the minimum was found also by the Fiacco and McCormick Penalty Method. Ragsdell and Root developed BIAS[22] based on multiplier method.

B. The Reduced Gradient Method

This method was first given by Wolfe[23] for a nonlinear objective function with linear constraints. The reduced gradient is the rate of change of the objective function with respect to the decision variables while the state variables were adjusted to maintain feasibility.

B.1 Application of the Reduced Gradient Method

Wilde and Beightler[24] developed an algorithm based on the constrained derivative. Ragsdell[25,26] implemented the reduced grident method in a computer code called OPT[27]. OPT was extensively used in this work.

C. Stochastic Programming Method

Clearance and tolerance in the mechanisms by nature is stochastic[28]. The stochastic programming method is an efficient tool to analyze the effect of clearances and tolerances on the performance of a mechanism. The essence of the stochastic programming method is to convert the probabilistic nature of the problems into an equivelent deterministic model.

C.1 Application of the Stochastic Programming Method

Dhande and Chakraborty[29] developed a stochastis model for handling tolerance and clearance in four bar linkage using dynamic programming to perform the optimization. Rao and Reddy[30] also presented a work in which the nominal link lengths, the tolerance on the link lengths and also the clearance in joints were considered as design variables. The constraints were also stated in probabilistic terms which are satisfied with certain minimum specified probability. Lee[31] introduced the Heuristic Optimization Technique[32] to minimize the stress and load on the pin in a Geneva mechanism.

1.2 Review of The Objective Function Formulation

In mechanism synthesis problems, an objective function is needed to quantitify how good a design is. Then an optimization technique is used for a systematic search for a better design. In this section, a review of the different ways of formulating the objective function for the synthesis of mechanisms is presented. This survey is focused on the developement of the precision point error which can be categorized into exact synthesis method and approximate synthesis method.

1.2.1 Exact Synthesis Method

Approximately before 1960, the mechanism synthesis problems were solved analytically. The analytical approach requires the number of chosen precision points be equal to that of unknown parameters in the mechanism system. Hence the motion generated by the synthesized mechanism has exact zero error at the selected precision points. In this approach, the mechanism synthesis problem can be solved without any optimization techniques.

Roth and Freudenstein[33] used a modified Newton-Raphson procedure to solve a nine precision point path synthesis problem. Suh and Radcliffe[34] used displacement metric to yield a set of simultaneous nonlinear equations which were solved for the unknown parameters. The spacing between the precision points on the structural error was studied by Maclarnan[35]. Han[36] formulated the objective function by summing over the error at all the precision points whose number is equal to that of the unknowns. Then the derivatives of the objective function, with respect to the unknown parameters, were set to zero. The resulting set of nonlinear equations were solved by the Newton method. Bagci[37] formed the design equations by partitioning Freudenstein's displacement equations into dyadic loop equations which were then solved using the linear superposition technique.

1.2.2 Approximate Synthesis Method

The approximate synthesis method requires the errors at the chosen precision points be as small as possible. The number of the precision points is independent of that of the unknown parameters in the mechanism. Therefore, an optimization technique is needed to minimize the errors at the chosen precision points.

The least square error formulation was first introduced by Levitskii and Shakvazian[38] in 1954. Notable examples using the least square error were the works of Eschenbach and Tesar[39], Garrett and Hall[40], Tomas[41], and Tull and Lewis[9]. Suh and Mecklenburg[42] used the Powell algorithm to minimize the error between the generated and the desired motion. Bagci [13] summed the square errors in the generated coordinates of two body points which are two specified points on a link to be guided through a desired path. Sutherland and Roth[43] synthesized a function generating mechanism using an improved least square error method in which weighting factors were introduced, Rao and Amekar[44] also used the least square error to synthesize a spherical 4-R function generating mechanism.

Generally, in the works mentioned above, the structural error was formed as the difference between the desired motion and the generated motion. However,there is an alternative. Suh and Mecklenburg[11] had syntheized a four bar function generator by assigning the input and output links at the desired position. The distance between the two end points of crank and rocker was compared with a desired coupler length. This difference was minimized to arrive at a desired function generator. The same approach was also used by Root[45]. In 1975, Kramer[46, 47] introduced the selective synthesis method. In this approach, the accuracy neighborhoods of varying size were constructed around each prescribed performance point. Then a mechanism was sought whose generated output goes through each accuracy neighborhood.

CHAPTER 2

A UNIFIED PROCEDURE
FOR THE DIMENSIONAL SYNTHESIS OF MECHANISMS

2.1 Introduction

The process of synthesizing a mechanism has three interrelated phases. The first phase is called type synthesis. In the type synthesis, the type of links or constructional units, such as linkworks, gears, cams, belts, etc., is determined. The second phase, called number synthesis, deals with the number of links and the number of pairs of a given type required to obtain a given number of degree of freedom. The third phase is called dimensional synthesis. By dimensional synthesis it is understood the determination of the dimension of parts - lengths and angles - necessary to create a mechanism that will effect a desired motion transformation. It is clear that intuition and past experierence with mechanism are invaluable in the first two phases. However in the dimensional synthesis phase, a more rational approach, as mentioned in chapter one, has been developed.

From a functional point of view the basic types of synthesis problems are as follows: Function-generation is concerned with the coordination of the positioning of output and input links. Path-generation involves the calculation of the dimensions of a mechanism, the coupler point of which is to describe a desired path. There are two types of the path-generation problems. The first simply requires the path to be generated, while the second requires the crank angle to be a specific value at each point on the curve. Motion-generation is the third type with two sub-categories. The first concerns the design of a mechanism to guide a body through a series of prescribed positions and orientations, while the second involves the design of a mechanism to produce specified instantaneous motion characteristics.

In the following sections, these three types of mechanism synthesis problems will be solved through a newly developed approach.

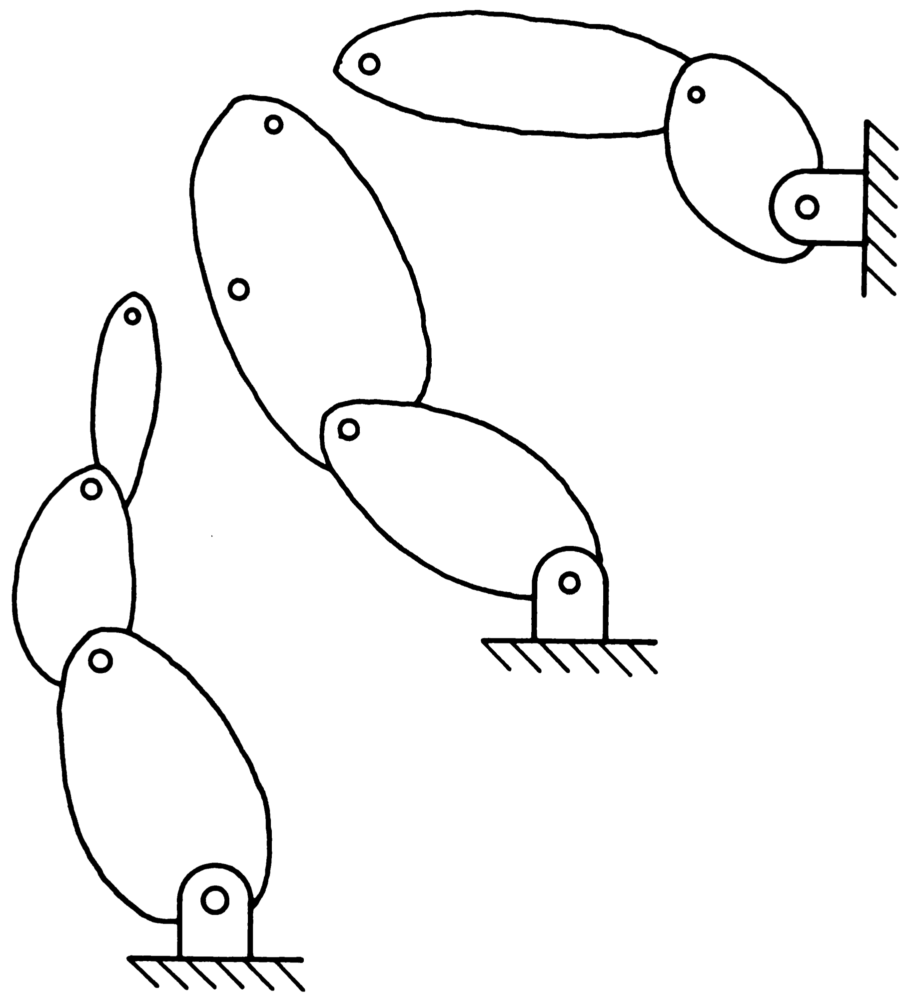

Figure 2.1 A Compound Pendulum

2.2 Discussion of The Objective Function Formulation

The steps in formulating the objective function in the direct method is to develop the input and output relationship through the analytical methods as discussed in section 1.2.1. Next the input link is positioned at the desired position. The generated output is compared with the desired values. Finally the sum of the square of the difference between the actual and desired values becomes the objective function.

An alternative to the direct method, the indirect method is introduced in this study. The procedure of formulating objective function by the indirect method is as follow: Once the mechanism is defined, the mechanism will be broken at a specified number of joints to arrive at a set of compound pendulums as shown in Fig.2-1. Having generated the compound pendulum, the input link and output link are moved to the desired locations and orientations to satisfy the motion requirements. Since the initial design does not produce the desired motion, there would be a separation between the end points of the broken joints. Therefore, an appropriate objective function to measure the degree of approximation of the actual motion to the desired motion would be the sum of the square of the distance of the end points of the broken joints.

Conclusively, there are two main steps in carrying out the proposed procedure for calculating the objective function. The first step is the identification of a set of joints to be broken. The second step is the identification of the sequence of links and joints in each chain of the compound pendulum to calculate the coordinates of the end points of the broken joints.

Once the objective function is formed, any nonlinear programming technique can be used to find its optimum. However, the generalized reduced gradient method will be used in this study, because of its general superiority in solving engineering problems[47].

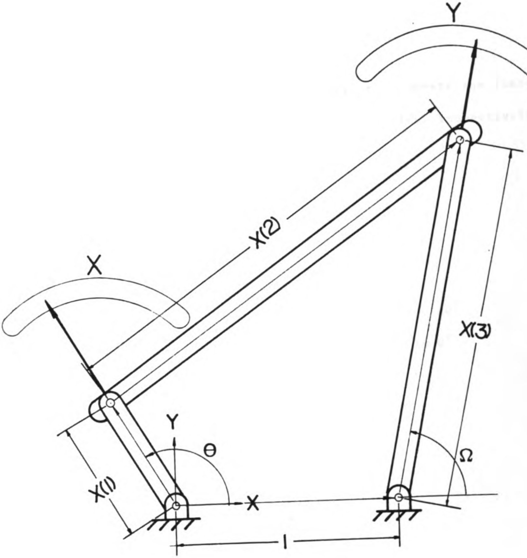

Figure 2.2 A Function Generator - Direct Method

2.3 Problem of Correlation of Crank and Rocker Angle – A Four Bar Function Generator

A four bar mechanism as shown in Fig.2-2 is used to generate the function $Y=X^{1.8}$. X and Y are related to crank and rocker angles, respectively as in Table.2-1.

Table 2-1

range of variables	range of angles
X : 1 to 5	crank angle : 150° to 60°
Y : 1 to $5^{1.8}$	rocker angle : 90° to 0°

Ten precision points are selected according to

$$\theta_i = \theta_0 - (i-1)\Delta\theta, \; i=1,2,\ldots,10$$

where

$$\theta_0=150^\circ$$

$$\Delta\theta=9^\circ.$$

Knowing the desired crank angle θ_i, the corresponding desired rocker position is calculated as

$$(\Omega_d)_i=\Omega_0[1-(Y_i-1)/(5^{1.8}-1)], \; i=1,2,\ldots,10$$

where

$$\Omega_0=90^\circ$$

$$Y_i=(X_i)^{1.8}, \; i=1,2,\ldots,10$$

$$X_i=1+4(i-1)/9, \; i=1,2,\ldots,10$$

In the direct method formulation of the fourbar function generator, the crank, coupler and rocker link lengths are chosen as design variables. The ground link length is chosen to be unity. The actual value of the rocker angle is calculated as follow:

The rocker angle corresponding to the ith precision point is[41]:

$$(\Omega_a)_i = \arccos\left[(PQ/S^2) + \mathrm{sign}(\pi-\theta_i)[(PQ/S^2)^2-(P/S)^2-(Q/S)^2]^{0.5}\right]$$

where

$$P = X(2)^2 - 1 - X(3)^2 - X(1)^2 + 2X(1)\cos\theta_i$$

$$Q = 2X(3)[1-X(3)\cos\theta_i]$$

$$R = -2X(1)X(3)\sin\theta_i$$

$$S = (Q^2+R^2)^{0.5}$$

$$\mathrm{sign}(\pi - \theta_i) = \begin{cases} +1 & \text{if } 0 \lesssim \theta_i \lesssim \pi \\ -1 & \text{if } \pi \lesssim \theta_i \lesssim 2\pi \end{cases}$$

X(1) : crank length

X(2) : coupler length

X(3) : rocker length

Having calculated the rocker angle, the error E_i between the desired and calculated value at the ith precision point is

$$E_i = (\Omega_d)_i - (\Omega_a)_i$$

where

$(\Omega_d)_i$: the ith desired value of rocker angle

$(\Omega_a)_i$: the ith actual vale of rocker angle

Now the objective function can be formulated as the sum of the errors over the precision points. That is

$$F(\bar{X}) = \sum (E_i)^2$$

Table 2.2
The Initial Guesses and Optimal Values for
the Design Variables of the Function Generator
-- Direct Method --

	No.	Initial Value	Optimal Value
DESIGN VERIABLE	X(1)	1.	.8933
	X(2)	1.	1.8436
	X(3)	1.	1.1125
Objective Function		10.10	2.62

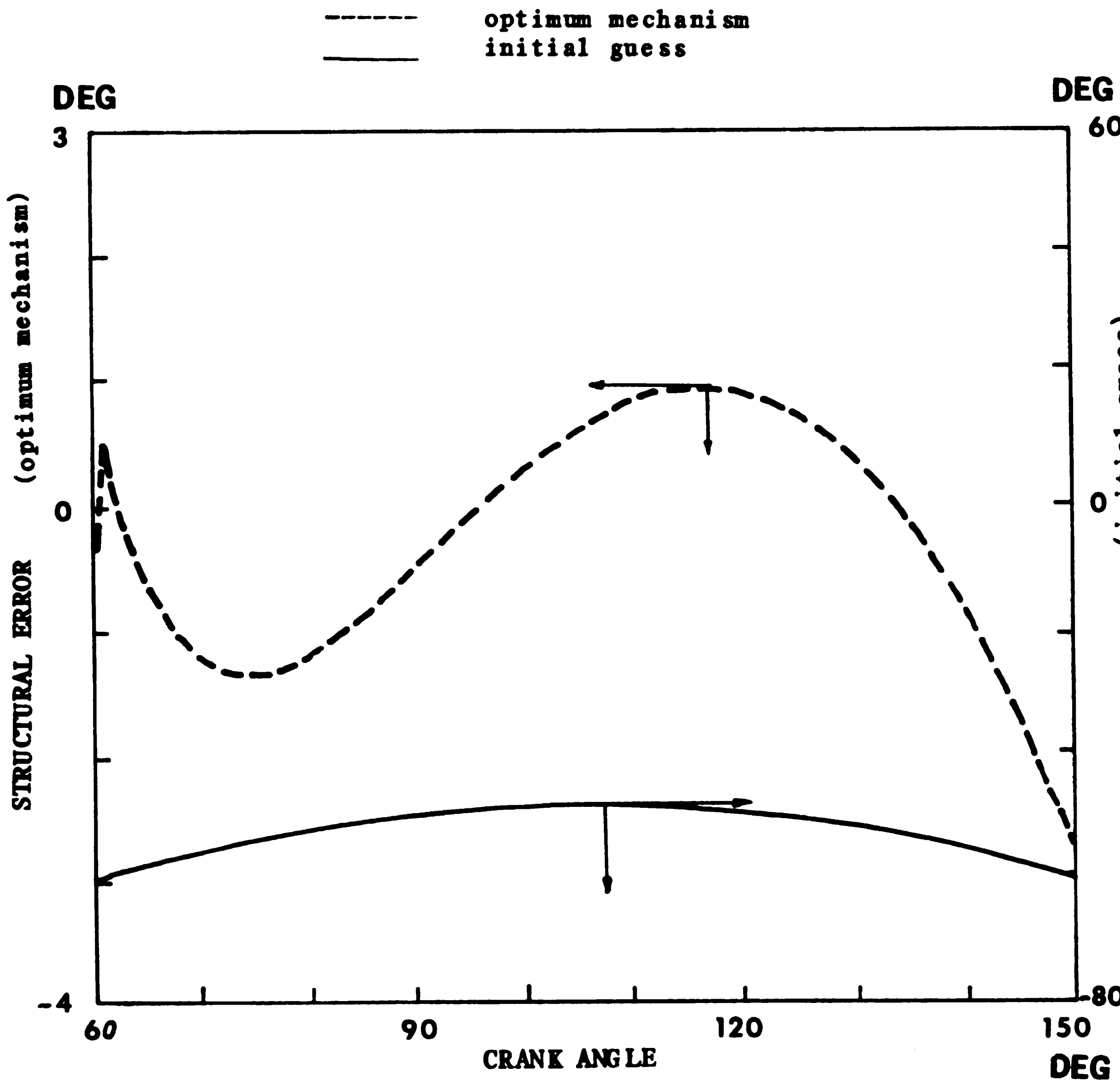

Figure 2.3 The Structural Error of the Rocker Angle From the Optimal Design of the Function Generator Using Direct Method

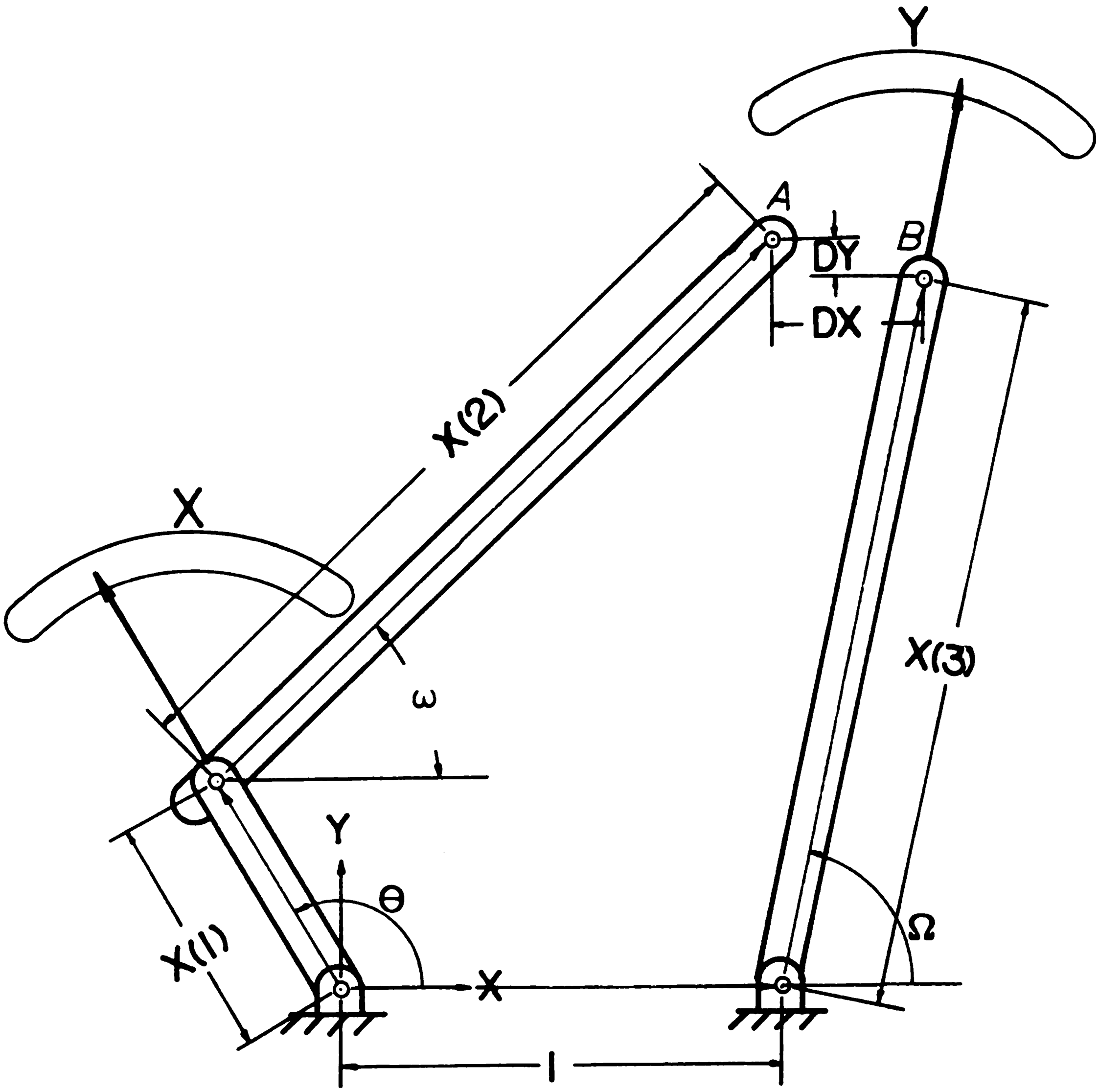

Figure 2.4 A Function Generator - Indirect Method

The following constraint is introduced to ensure having practical link lengths,

$X(2)-5*X(1) \lesssim 0,$

The rotatibility of the four bar mechanism is ensured by adding the following constraints,

$X(2)+X(1) \lesssim X(3)+1$,

where the crank and coupler are assumed to the shortest and longest link, respectively. The optimum as well as the initial guesses for the three link length design variables are shown in Table 2-2. The structural error which is the deviation of the calculated rocker angle from the desired value is shown in Fig.2-3.

For the indirect method, first the crank is positioned at the ith precision point. Then the rocker is moved to the corresponding desired position to satisfy the motion requirement. This is shown in Fig.2-4. In this problem, the joint connecting the coupler and rocker is broken. In order to calculate the distance between the two end points, the position of the end point on coupler should be calculated first. After imposing the motion requirement, the coupler angle remains unspecified. It is decided to choose the coupler angle at different precision points as design variables. Then the objective function for the indirect method can be calculated as follow:

At the ith precision point, the distance between the two ends of the broken joint is

$$D_i=(DX_i^{\,2}+DY_i^{\,2})^{0.5}$$

where

Table 2.3
The Initial Guesses and Optimal Values for
the Design Variables of the Function Generator
-- Indirect Method --

	No.	Initial value	Optimal Value
D	X(1)	1.	0.6443
E	X(2)	1.	1.5919
S	X(3)	1.	0.7809
I	X(4)	16.25	16.35
G	X(5)	13.24	13.40
N	X(6)	10.29	10.50
	X(7)	7.36	7.53
V	X(8)	4.39	4.47
A	X(9)	1.33	1.23
R	X(10)	-1.93	-2.29
I	X(11)	-5.54	-6.20
A	X(12)	-9.79	-10.55
B	X(13)	-15.29	-15.45
E	X(14)	-16.54	-20.98
Objective Function		9.1509	0.1087

Note: Unit of design variable 1-3 is in inch
Unit of design variable 4-14 is in degree
$X(3 + i) = \omega_i$, $i=1,2,\ldots,11$

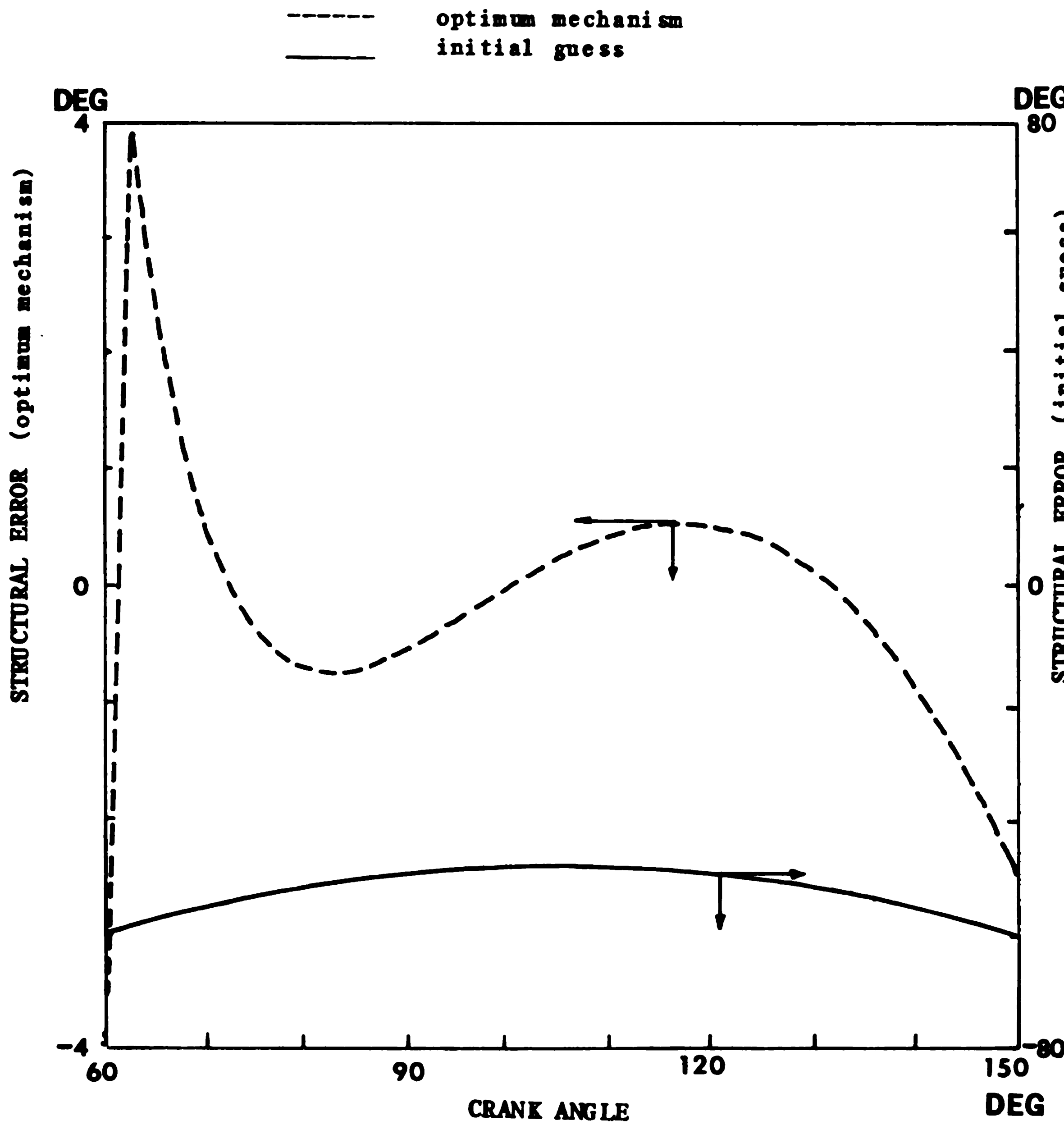

Figure 2.5 The Structural Error of the Rocker Angle From the Optimal Design of the Function Generator Using Indirect Method

$$DX_i = (X_A)_i - (X_B)_i$$

$$DY_i = (Y_A)_i - (Y_B)_i$$

$(X_A)_i$, $(Y_A)_i$, $(X_B)_i$ and $(Y_B)_i$ are the X- and Y- coordinates of the end points of the broken joint. They are determined as follow:

$$(X_A)_i = X(1)\cos\theta_i + X(2)\cos\omega_i$$

$$(Y_A)_i = X(1)\sin\theta_i + X(2)\sin\omega_i$$

$$(X_B)_i = 1 + X(3)\cos\Omega_i$$

$$(Y_B)_i = X(3)\sin\Omega_i$$

$X(1), X(2), X(3)$: the design variables of the length of crank, coupler and rocker

ω_i : the coupler angle corresponding to the ith precision point

and the objective function is

$$F(\bar{X}) = \sum D_i$$

The constraint equations are the same as those in the direct method. The initial guesses and final results for the design variables are shown in Table 2-3. Fig.2-5 shows the structural error which is the deviation of the calculated rocker angle from the desired value.

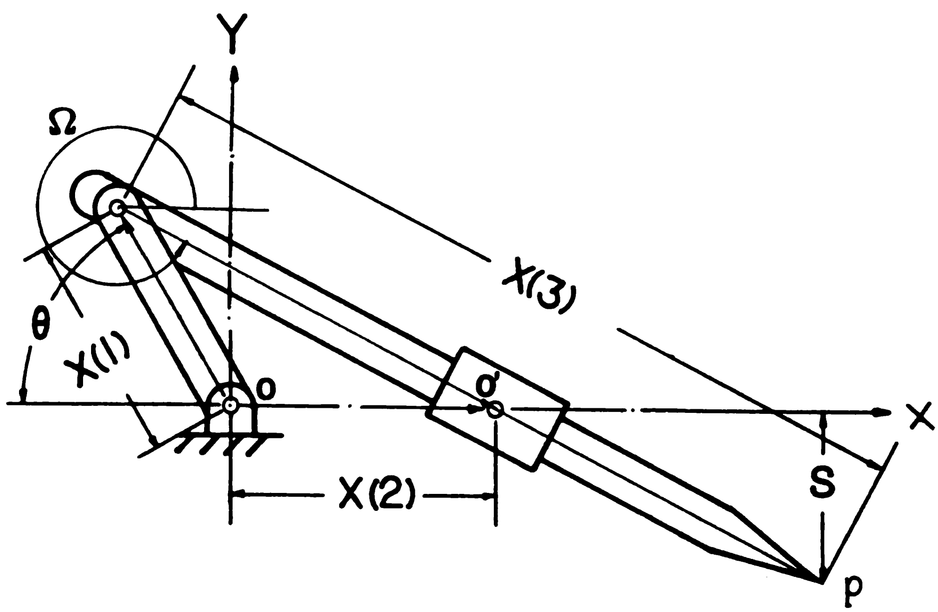

Figure 2.6 The Pen Recorder - Direct Method

2.4 Problem of Correlation of Crank Angle and Coupler Point – A Four Bar Pen Recorder

The inverted slider crank mechanism of Fig.2-6 is used as a pen recording device. It is required that the tip of the coupler follows a straight path for four inches above and below the X axis. The distance of the coupler tip from the X axis, S, should be a linear function of the crank angle θ. That is

$$S=K\theta$$

where

$$-90^\circ \lesssim \theta \lesssim 90^\circ$$

$$K=8/\pi$$

Since the motion of this mechanism is symmetric about the X-axis, the motion requirement will be imposed for $0^\circ \lesssim \theta \lesssim 90^\circ$ only. Ten precision points are selected, they are

$$\theta_i = 90^\circ - (i-1)10^\circ,\ i=1,2,\ldots,10$$

and the desired coordinates of the coupler tip are

$$(X_d)_i = 9,\ i=1,\ldots,10$$

$$(Y_d)_i = -K\,\theta_i,\ i=1,\ldots,10$$

The design variables in the direct method are crank, coupler and ground link lengths shown in Fig.2-6. The actual coordinate of the coupler tips is

$$(X_a)_i = X(1)\cos[\pi-\theta_i] + X(3)\cos\Omega_i$$

Table 2.4
The Initial Guesses and Optimal Values for the Design Variables of the Pen Recorder
-- Direct Method --

	No.	Initial Value	Optimal Value
DESIGN VARIABLE	X(1)	1.	0.8693
	X(2)	2.	1.6385
	X(3)	8.	9.9747
Objective Function		21.15	1.0372

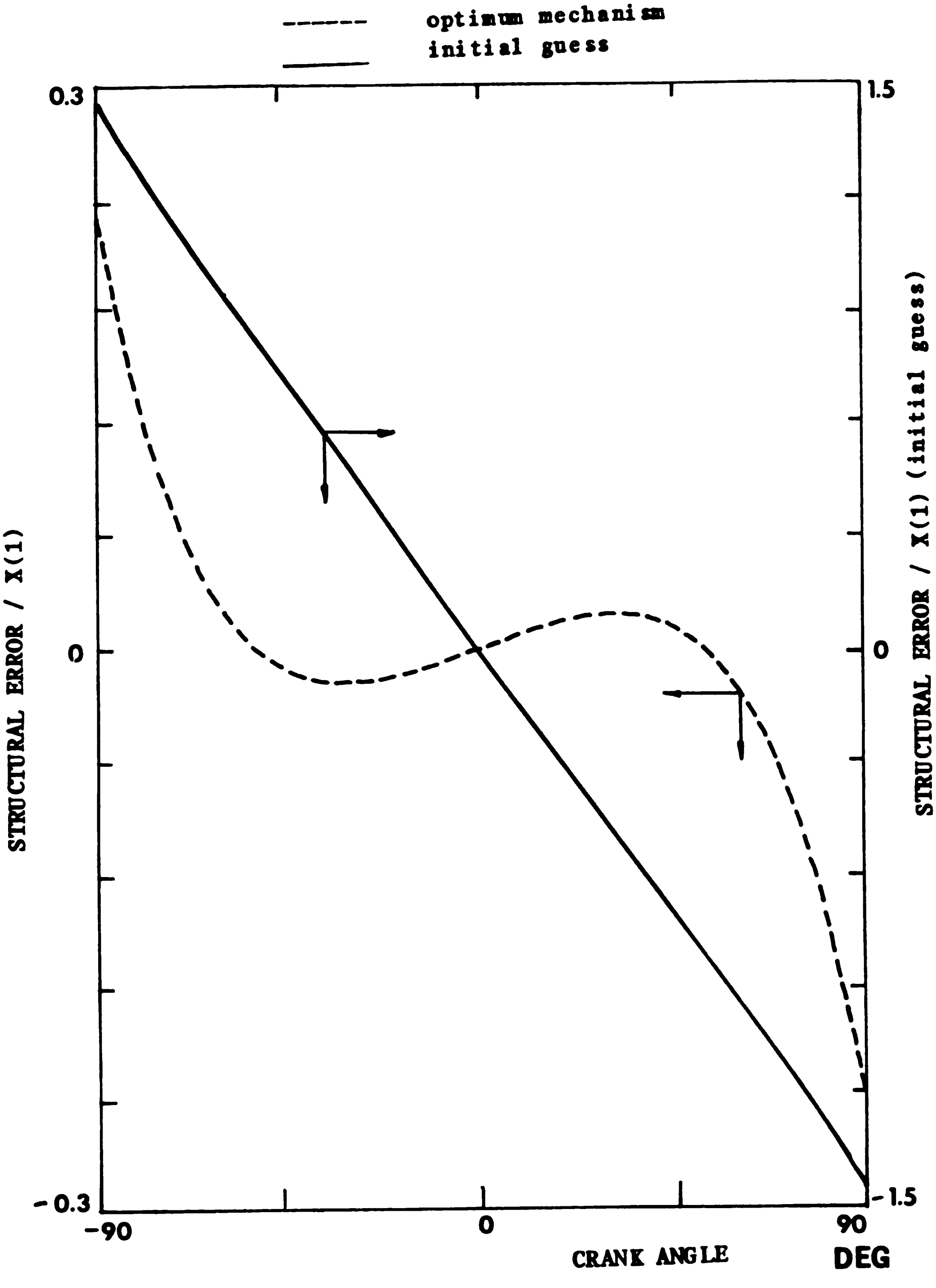

Figure 2.7 The Structural Error of the Coupler Path From the Optimal Design of the Pen Recorder Using Direct Method

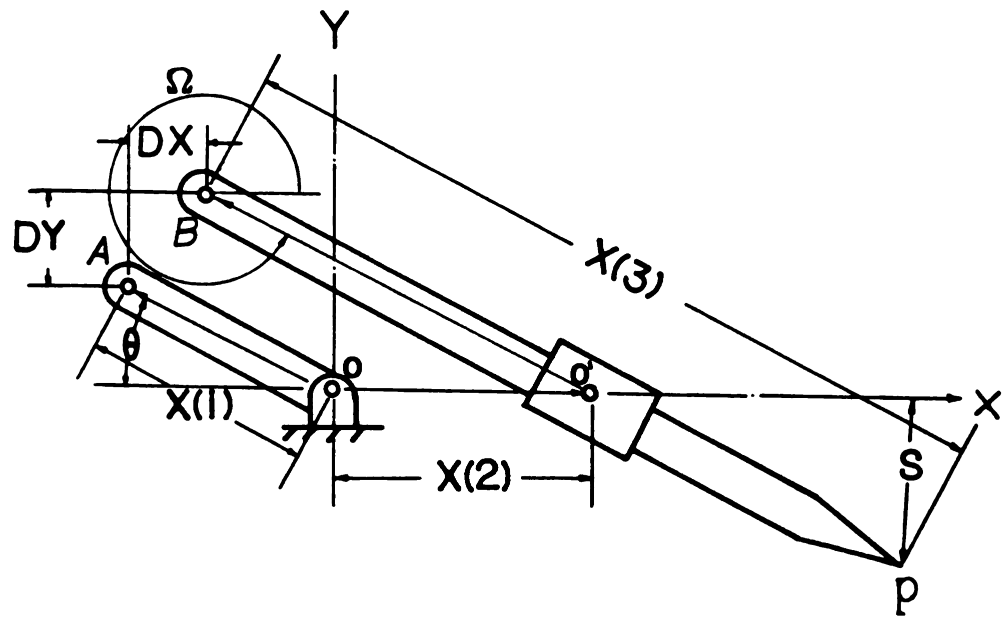

Figure 2.8 The Pen Recorder - Indirect Method

$(Y_a)_i = X(1)\sin[\pi-\theta_i] + X(3)\sin\Omega_i$

where

X(1),X(2),X(3) are crank, gound link and coupler lengths, respectively. Ω_i is the coupler angle at the ith precision point, it is

$\Omega_i = 2\pi - \arctan[X(1)\sin\theta_i/(X(2)+X(1)\cos\theta_i)]$

The error between the desired and the actual position of the coupler tip is

$E_i=[((X_a)_i - (X_d)_i)^2 + ((Y_a)_i - (Y_d)_i)^2]^{0.5}$

The objective function is then formed as

$F(\bar{x}) = \sum E_i$

Table 2-4 shows the initial guasses and the optimum solution. Fig.2-7 shows the structural error distribution which is the deviation of the calculated coupler curve of the optimum solution from the desired path.

In the indirect method, the joint connecting the crank and the coupler is broken as shown in Fig.2-8, the crank and the coupler tip are positioned at their ith desired precision point, then the coordinates of end point A (X_{Ai}, Y_{Ai}) of the broken joint are calculated as

$X_{Ai} = X(1)\cos[\pi-\theta_i]$

$Y_{Ai} = X(1)\sin[\pi-\theta_i]$

The coordinates of end point B (X_{Bi}, Y_{Bi}) are

$X_{Bi} = X(2) + W_i\cos\Omega_i$

$Y_{Bi} = W_i\sin\Omega_i$

where

W_i is the length of $\overline{OO'}$, it is

Table 2.5
The Initial Guesses and Optimal Values for
the Design Variables of the Pen Recorder
-- Indirect Method --

		Initial Value	Optimal Value
DESIGN VARIABLE	X(1)	1.	0.7538
	X(2)	2.	1.4271
	X(3)	8.	10.1783
Objective Function		22.6841	0.4246

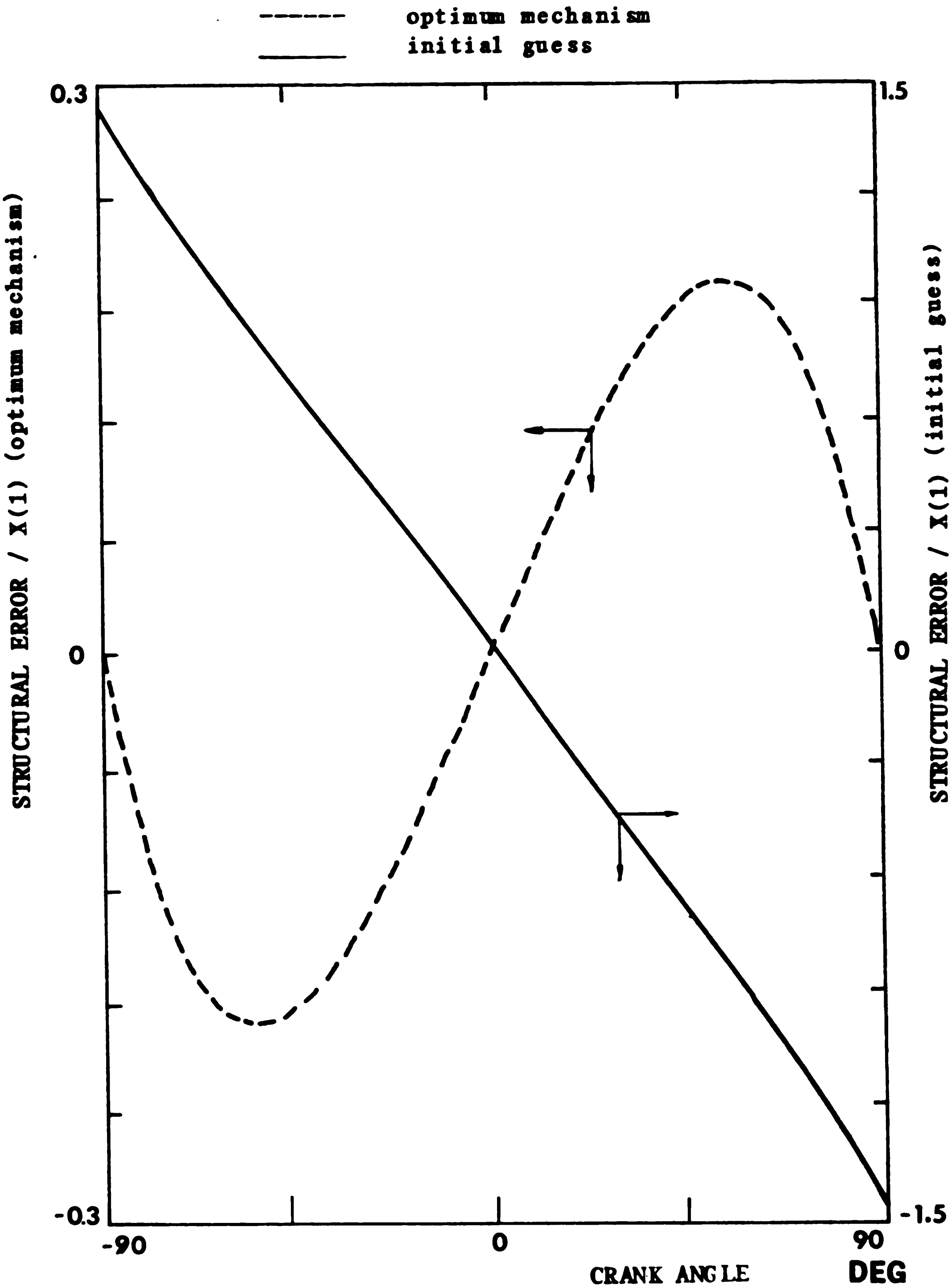

Figure 2.9 The Structural Error of the Coupler Path From the Optimal Design of the Pen Recorder Using Indirect Method

$W_i = X(3) - [(X_{di} - X(2))^2 + (Y_{di})^2]^{0.5}$,

Ω_i is the coupler angle at the ith precision point, it is

$\Omega_i = \pi - \arctan\left[Y_{di}/(X_{di} - X(2)) \right]$,

The distance between the ends of the broken joint at the ith precision point is

$$D_i = [(DX_i)^2 + (DY_i)^2]^{0.5}$$

DX_i, and DY_i are the X- and Y- components of the joining the ends of the broken joint at the ith precision point. That is

$$DX_i = X_{Ai} - X_{Bi}$$

$$DY_i = Y_{Ai} - Y_{Bi}$$

The objective function can be formed as

$$F(\bar{X}) = \sum D_i$$

The pen recording mechanism is synthesized using above the objective function. The results are reported in Table.2-5 and Fig.2-9.

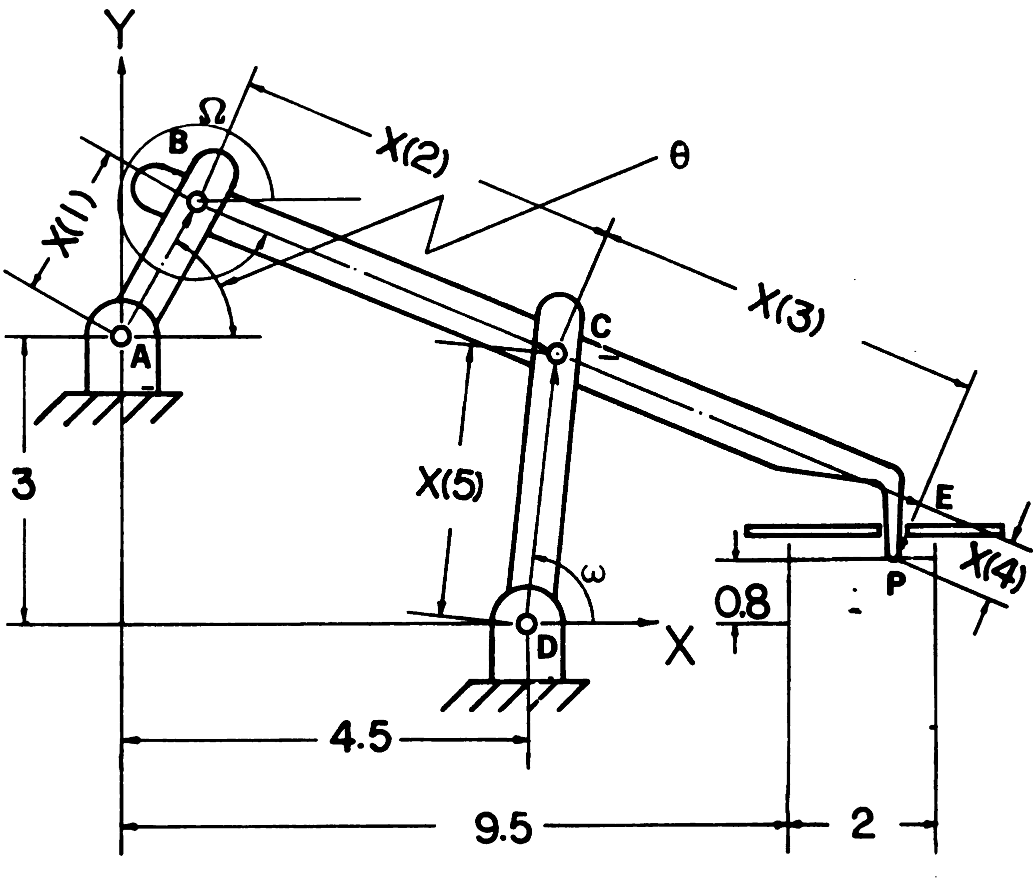

Figure 2.10 The Film Advancer - Direct Method

2.5 Problem of Guiding A Point Along A Specified Curve – A Four Bar Film Advancer

Consider the film advancer mechanism shown in Fig.2-10. The catcher is required to plug into the film hole, advance the film for two inches and finally release the film.

Refering to the Fig.2-10, the desired coordinate of point P is selected as

$X_{di} = 9.5 + 2(i-1)/9$, i=1,...,10

$Y_{di} = 0.8$, i=1,...,10

Since the crank angles corresponding to the desired position of the coupler point P are not specified, they are also selected as design variables. In order to avoid order problem[48] in the synthesized fourbar mechanism, the following constraints are introduced.

$\theta_{15} > \theta_{14} > . \quad . \quad . > \theta_6$

where

$\theta_6 - \theta_{15}$ are the crank angles at precision points.

For the direct method, the coordinate of coupler point P at the ith precision point can be calculated as follow:

The coupler angle Ω_i corresponding to the ith precision point is

$\Omega_i = \arccos\left[(PQ/S^2) + \text{sign}(\theta_i)[(PQ/S^2)^2 - (P/S)^2 + (Q/S)^2]^{0.5}\right]$

where

Table 2.6
The Initail Guesses and Optimal Values for
the Design Variables of the Film Advancer
-- Direct Method --

	No.	Initial Value	Optimal Value
D	X(1)	1.5	1.5746
E	X(2)	5.0	4.7657
S	X(3)	5.5	5.7905
I	X(4)	0.5	0.6168
G	X(5)	3.5	3.1372
N	X(6)	21.59	31.89
	X(7)	36.00	40.50
V	X(8)	46.80	48.78
A	X(9)	57.60	56.73
R	X(10)	68.40	64.43
I	X(11)	79.20	72.07
A	X(12)	90.00	79.73
B	X(13)	97.20	87.59
L	X(14)	104.40	95.91
E	X(15)	111.60	104.98
Objective Function		10.39	0.1548

Note: The unit of the design variable 1-5 is in inch
The unit of the design variable 6-15 is in degree
$X(5 + i) = \theta_i$, $i=1,2,\ldots,10$

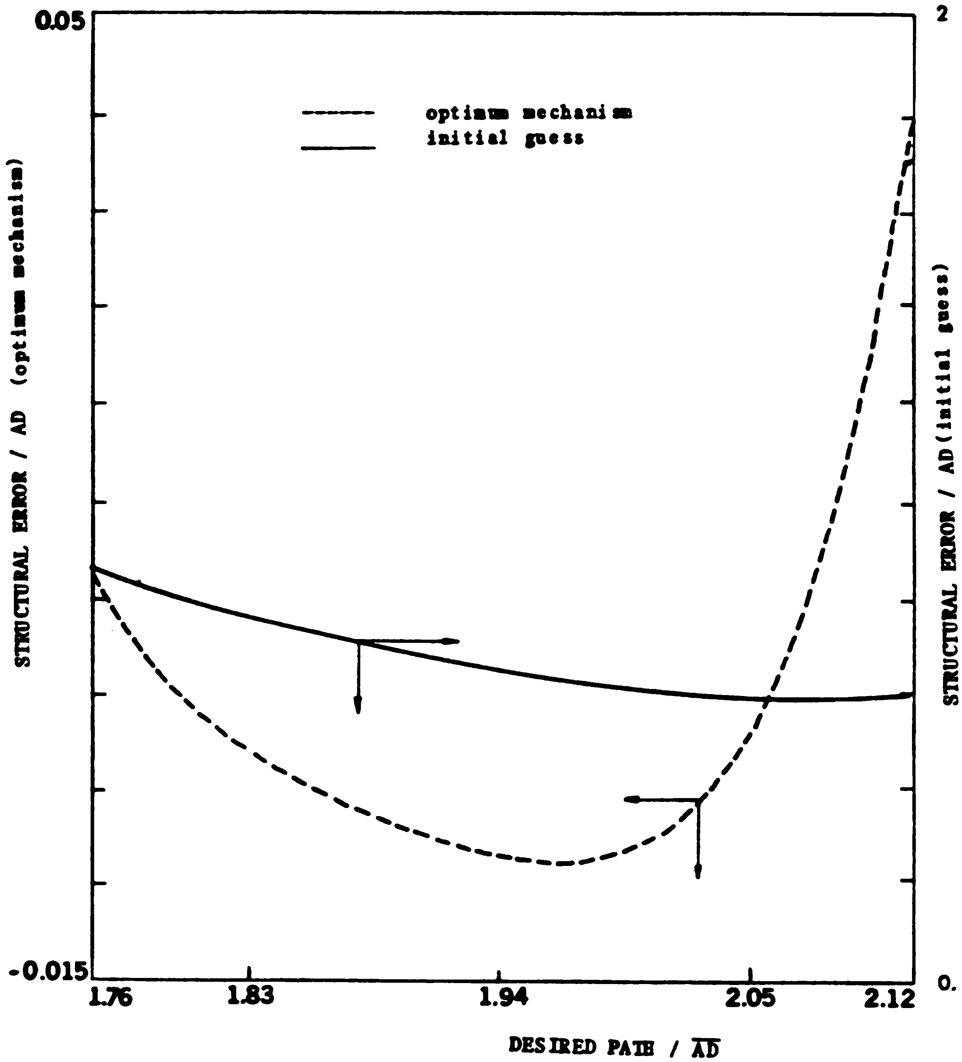

Figure 2.11 The Structural Error of the Copler Path From the Optimal Design of the Film Advancer Using Direct Method

$P = X(2)^2 - 29.25 - X(3)^2 - X(1)^2 + 9X(1)\cos\theta_i - 6X(1)\sin\theta_i$

$Q = X(3)[9 - 2X(1)\cos\theta_i]$

$R = -X(3)[6 + 2X(1)\sin\theta_i]$

$S = (Q^2+R^2)^{0.5}$

θ_i : the crank angle at ith precision point

$$\mathrm{sign}(\theta_i) = \begin{cases} +1 \text{ if } \pi-\arctan(3/4.5) \geq \theta_i \geq -\arctan(3/4.5) \\ \\ -1 \text{ if } 2\pi-\arctan(3/4.5) \geq \theta_i \geq \pi-\arctan(3/4.5) \end{cases}$$

The coordinate of the coupler point P (X_{pi}, Y_{pi}) can be determined as follow:

$X_{pi} = X(1)\cos\theta_i + W\cos\overline{\Omega}_i$

$Y_{pi} = 3 + X(1)\sin\theta_i + W\sin\overline{\Omega}_i$

where

$W = \left[[X(2) + X(3)]^2 + X(4)^2\right]^{0.5}$

$\overline{\Omega}_i$: angle of $\overline{BP}$

$= \Omega_i - V$, for $0 \leq \Omega_i \leq 2\pi - V$

$= 2\pi - \Omega_i + V$, for $2\pi - V \leq \Omega_i \leq 2\pi$

$V = \arctan\left[X(4)/[X(2)+X(3)]\right]$

The difference between the desired and calculated coupler point P is

$E_i = [(X_{pi}-Y_{di})^2 + (y_{pi}-Y_{di})^2]^{0.5}$

The objective function is then formed as

$F(\overline{X}) = \sum E_i$

The dimension of the optimum mechanism are shown in Table 2-6. Fig.2-11 shows the structural error which is the deviation of the actual coupler path from the desired path.

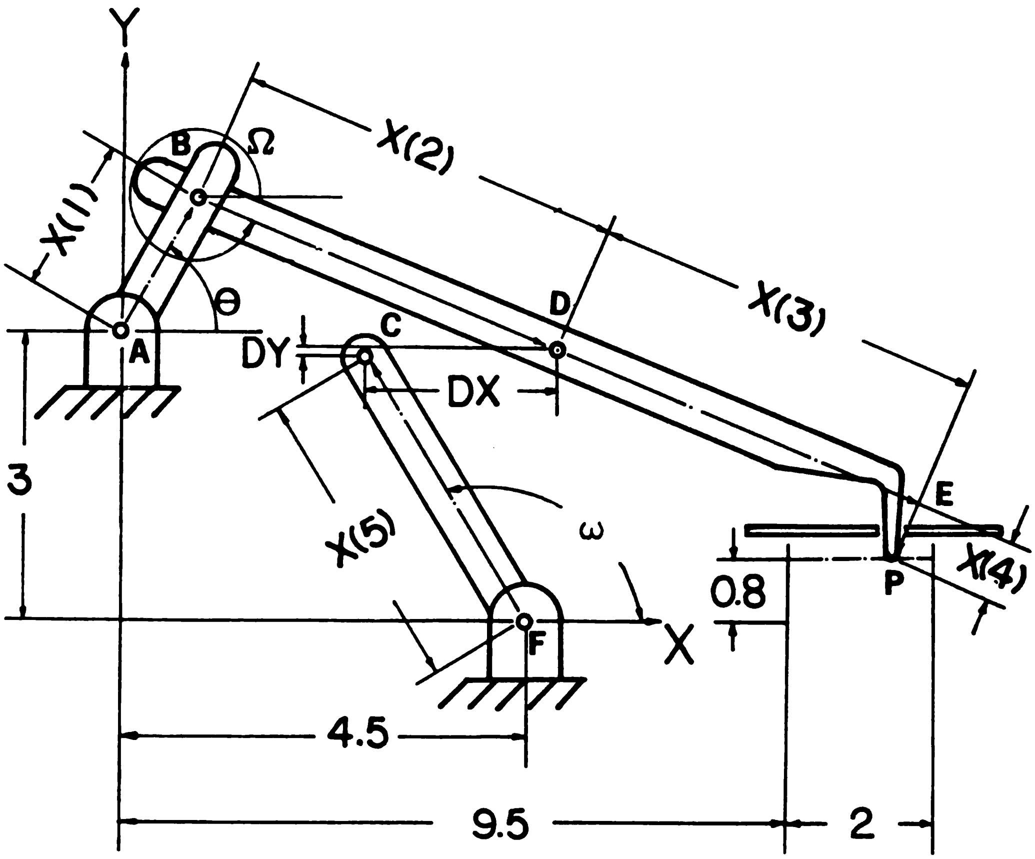

Figure 2.12 The Film Advancer - Indirect Method With One Broken

For the indirect method, the joint between the coupler and rocker is assumed to be disconnected as shown in Fig.2-12. Then the coupler point P is placed at the ith precision point. This fixes the crank angle Θ_i and the coupler angle Ω_i. They can be determined as follow:

$$\Theta_i=\arccos\left[IJ/L^2+[(IJ/L^2)^2-(I/L)^2+(K/L)^2]^{0.5}\right]$$

where

$$I=W^2-(X_{di})^2-(Y_{di})^2-9-X(1)^2+6Y_{di}$$

$$J=2X(1)[3-Y_{di}]$$

$$K=-2X(1)X_{di}$$

$$L=(J^2+K^2)^{0.5}$$

$$W=[X(2)^2+X(3)^2]^{0.5}$$

and the coupler angle is

$$\Omega_i = \begin{cases} \bar{\Omega}_i + V, \text{ when } 0 \le \bar{\Omega}_i \le 2\pi-V \\ \\ V - 2\pi + \bar{\Omega}_i, \text{ when } 2\pi-V \le \bar{\Omega}_i \le 2\pi \end{cases}$$

where

$$V = \arctan[X(4)/(X(2)+X(3)]$$

$\bar{\Omega}_i$ is the angle of $\overline{BP}$. It is

$$\Omega_i=2\pi- \arctan\left[[Y_{di}-3-X(1)\sin\Theta_i]/[X_{di}-X(1)\cos\Theta_i]\right],$$

The coordinate of end point D is

$$X_{Di} = X(1)\cos\Theta_i + X(2)\cos\Omega_i$$

$$Y_{Di} = 3 + X(1)\sin\Theta_i + X(2)\sin\Omega_i$$

and the coordinate of the other end point C is

$$X_{Ci} = 4.5 + X(5)\cos\omega_i$$

$$Y_{Ci} = X(5)\sin\omega_i$$

where

Table 2-7
The Initial Guesses and Optimal Values for the Design Variables pf the Film Advancer
-- Indirect Method with One Broken Joint --

	No.	Initial Value	Optimal Value
D	X(1)	1.5	1.6476
E	X(2)	5.0	4.7670
S	X(3)	5.5	5.5956
I	X(4)	0.5	0.6350
G	X(5)	3.5	3.1570
N	X(6)	53.93	57.46
	X(7)	59.12	61.51
V	X(8)	63.76	65.33
A	X(9)	68.84	69.06
R	X(10)	74.15	72.78
I	X(11)	79.56	76.51
A	X(12)	84.91	80.29
B	X(13)	88.37	84.13
L	X(14)	91.70	88.07
E	X(15)	94.85	92.12
Objective Function		5.1856	0.0881

Note: The unit of the design variable 1-5 is in inch
The unit of the design variable 6-15 is in degree
$X(5 + i) = \omega_i$, $i=1,2,\ldots,10$

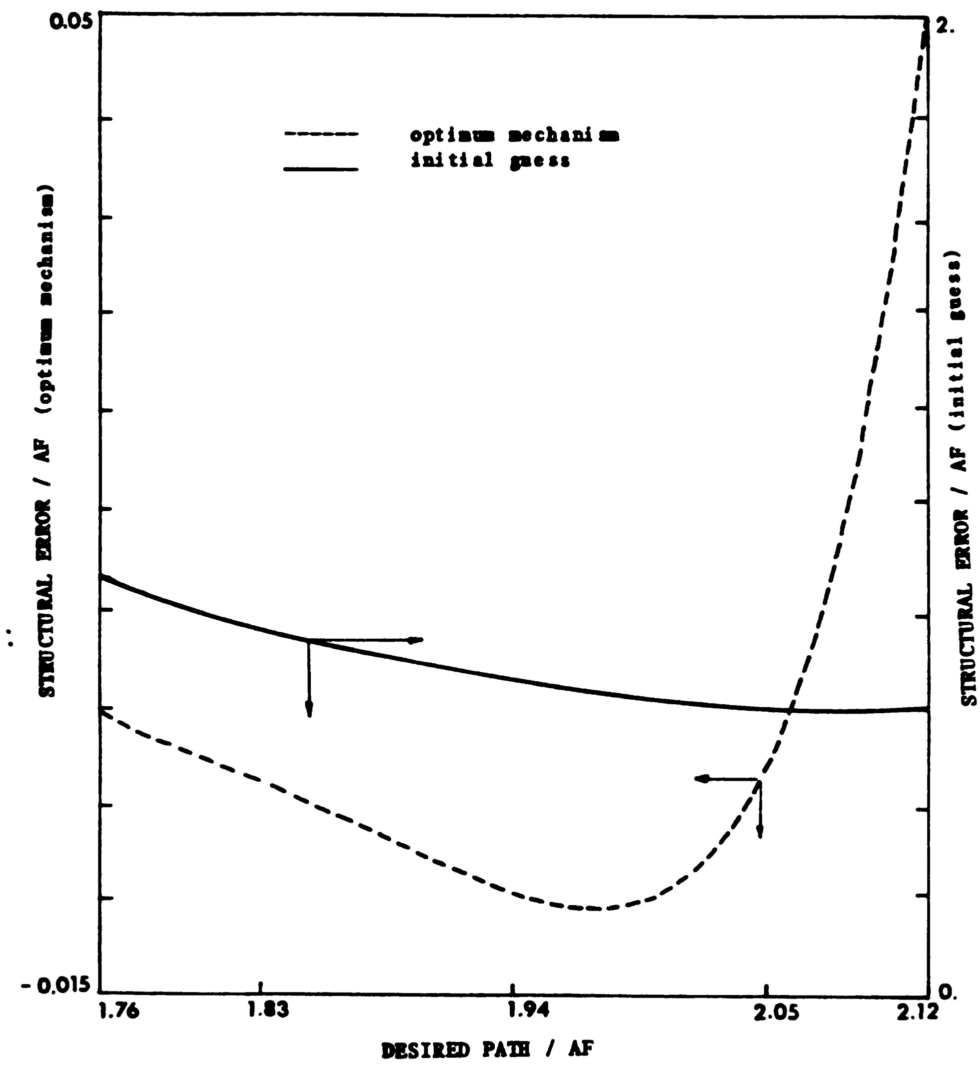

Figure 2.13 The Structural Error of the Coupler Path From the Optimal Design of the Film Advancer Using Indirect Method I

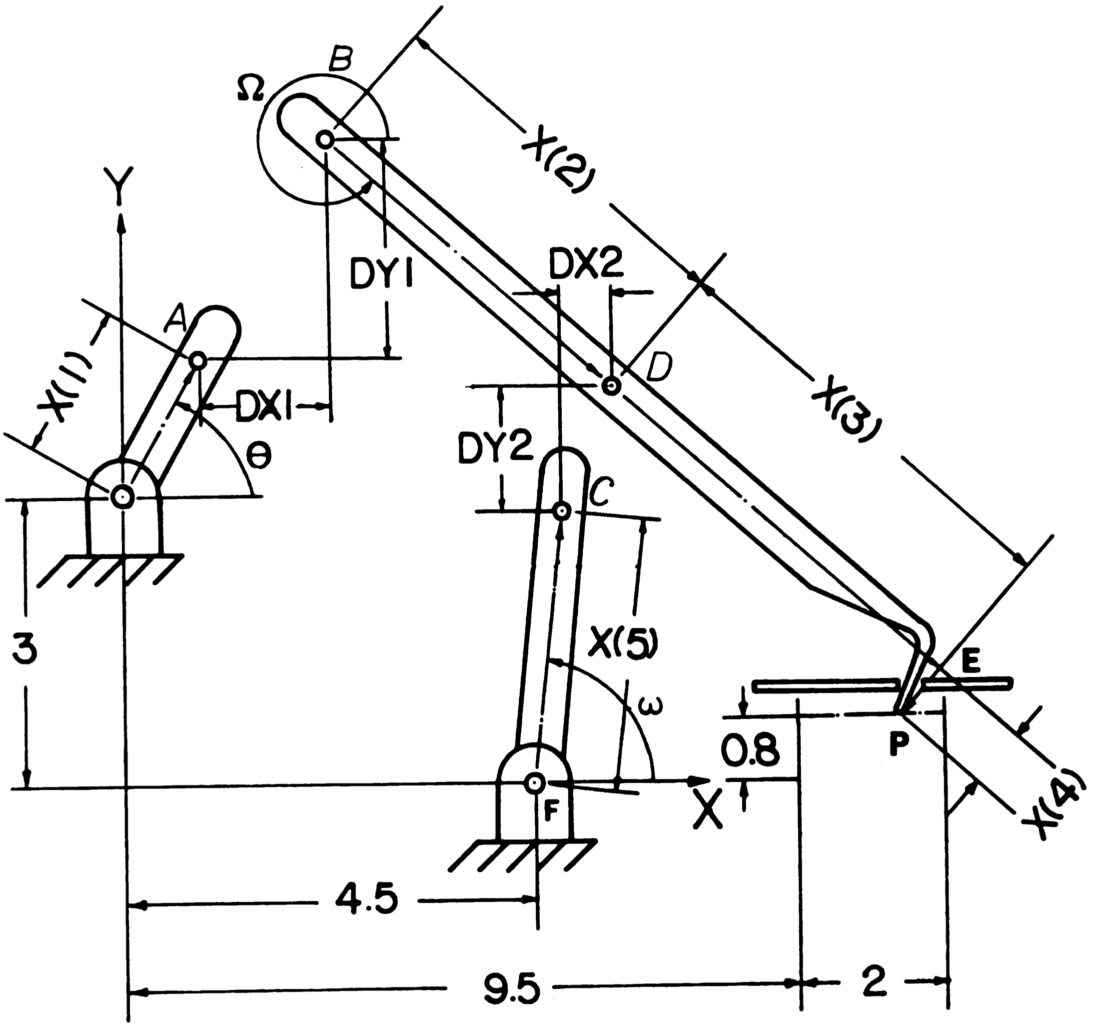

Figure 2.14 The Film Advancer - Indirect Method With Two Broken Joints

ω_i is the ith rocker angle.

Therefore, the distance at the broken joint is

$$D_i = [(DX_i)^2 + (DY_i)^2]^{0.5}$$

where DX_i and DY_i are the X- and Y- components of the distance at the broken joint. They are:

$$DX_i = X_{Ci} - X_{Di}$$

$$DY_i = Y_{Ci} - Y_{Di}$$

The objective function is then formed as the sum of the distance of the end of the broken joints over all the precision points. That is

$$F(\bar{X}) = \sum D_i$$

Table 2-7 shows the initial guesses and final results. Fig.2-13 shows the structural error which is the deviation of the actual coupler curve from the desired one.

An alternative, as shown in Fig.2-14, to synthesize this mechanism is to disconnect the coupler from the rest of the mechanism. Then point P is positioned at the ith precision point. Since the crank angle θ_i, coupler angle Ω_i and rocker angle ω_i are unspecified, they are chosen as the design variables. An appropriate objective function is the sum of the distances of the ends of the two broken joints.

The coordinate of point A is

$$X_{Ai} = X(1)\cos\theta_i,$$

$$Y_{Ai} = X(1)\sin\theta_i,$$

the coordinate of end point C is

$$X_{Ci} = 4.5 + X(5)\cos\omega_i,$$

$$Y_{Ci} = X(5)\sin\omega_i,$$

the coordinate of end point B is

$$X_{Bi} = X_{di} - W\cos\bar{\Omega}_i,$$

$$Y_{Bi} = Y_{di} - W\sin\bar{\Omega}_i,$$

where

$$W = [(X(2)+X(3))^2+X(4)^2]^{0.5}$$

$$\bar{\Omega}_i = \begin{cases} \Omega_i - V, & \text{for } 0 \lesssim \Omega_i \lesssim 2\pi - V \\ \\ V - 2\pi + \bar{\Omega}_i, & \text{for } 2\pi - V \lesssim \Omega_i \lesssim 2\pi \end{cases}$$

$$V = \arctan[X(4)/(X(2)+X(3))]$$

and the coordinate of end point D is

$$X_{Di} = X_{Bi} + X(2)\cos\Omega_i$$

$$Y_{Di} = Y_{Bi} + X(2)\sin\Omega_i$$

the distance of the two broken joints can be calculated as

$$D1_i = [(DX1_i)^2 + (DY1_i)^2]^{0.5}$$

$$D2_i = [(DX2_i)^2 + (DY2_i)^2]^{0.5}$$

where

$$DX1_i = X_{Ai} - X_{Bi}$$

$$DY1_i = Y_{Ai} - Y_{Bi}$$

$$DX2_i = X_{Ci} - X_{Di}$$

$$DY2_i = Y_{Ci} - Y_{Di}$$

The objective function is formed as

$$F(\bar{X}) = \sum [D1_i + D2_i]$$

The initial guesses and the optimum value for the design variables are shown in table 2-8. Fig.2-15 shows the structural error which is

Table 2.8
The Initial Guesses and Optimal Values for
the Design Variables of the Film Advancer
-- Indirect Method with Two Broken Joint --

	No.	Initial Value	Optimal Value
	X(1)	1.5	1.4678
	X(2)	5.0	5.0171
	X(3)	5.5	5.6068
	X(4)	0.5	0.6704
	X(5)	3.5	3.0971
	X(6)	320.90	320.91
	X(7)	342.12	325.55
	X(8)	340.98	341.79
D	X(9)	340.19	341.07
E	X(10)	339.71	340.51
S	X(11)	339.53	340.12
I	X(12)	339.67	339.90
G	X(13)	339.94	339.84
N	X(14)	340.37	339.99
	X(15)	340.94	340.20
V	X(16)	21.59	27.68
A	X(17)	36.00	40.75
R	X(18)	46.80	50.03
I	X(19)	57.60	63.16
A	X(20)	68.40	66.92
B	X(21)	79.20	75.09
L	X(22)	90.00	83.35
E	X(23)	97.20	91.94
	X(24)	104.40	101.05
	X(25)	111.60	109.81
	X(26)	53.93	57.74
	X(27)	59.12	65.54
	X(28)	63.76	65.54
	X(29)	68.84	69.33
	X(30)	74.15	73.15
	X(31)	79.56	77.02
	X(32)	84.91	80.96
	X(33)	88.37	85.00
	X(34)	91.70	89.30
	X(35)	94.85	93.45
Objective Function		7.0864	0.4651

Note: The unit of the design variable 1-5 is in inch
The unit of the design variable 6-35 is in degree
$X(5 + i) = \Omega_i$, $i=1,2,\ldots,10$
$X(15 + i) = \theta_i$, $i=1,2,\ldots,10$
$X(25 + i) = \omega_i$, $i=1,2,\ldots,10$

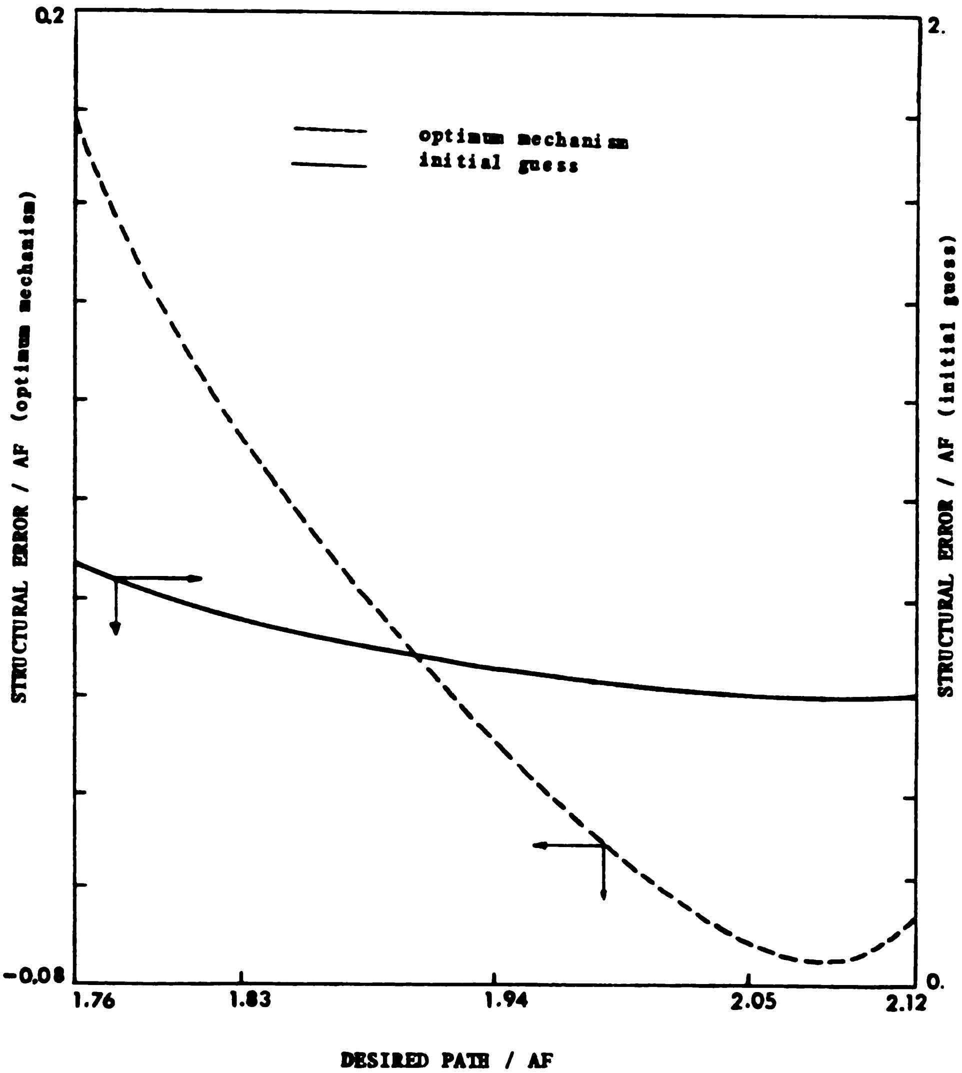

Figure 2.15 The Structural Error of the Coupler Path From the Optimal Design of the Film Advancer Using Indirect Method II

the deviation of the calculated coupler curve from the desired one.

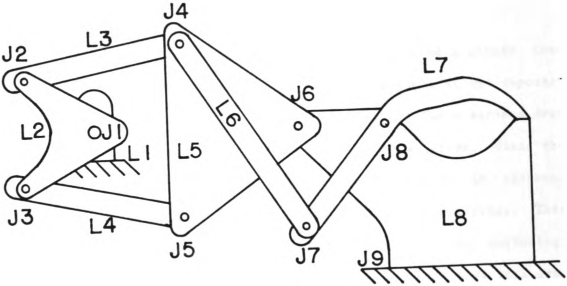

Figure 2.16 The Gripping Mechanism

2.6 The Problem of generating A Specified Motion - An Eight Bar Gripping Mechanism

Suppose a mechanism is required to perform the following task: it should stretch out, grasp a thin object, hold the object and then move back horizontally. A design to perform this task is depicted by Fig.2-16.

Since this is a multiloop mechanism, developement of a closed form relationship between the input and output is impractical if not impossible. Newton's method is an alternative. However, it has a serious draw back. Newton's method requires an estimate of the solution. Since the trial design generated by the the optimizer is not known in advance, there is a need for an intelligent guess to ensure convergence. This complication makes the whole procedure impractical. In the following, it is demonstrated that the indirect method does not have this draw back.

In order to convert a muiti-loop mechanism into a compound pendulum, the first step is to find the appropriate joints to be broken. Branins algorithm[50], which uses the interconnection of the mechanism to set up a tree, is used here to choose the broken joints. This algorithm first searches for the level 1 joints. A joint is said to be of level n if it is connected through n joints to the ground. Then the algorithm procedes to ascertain level 2, 3, . . . until the search is

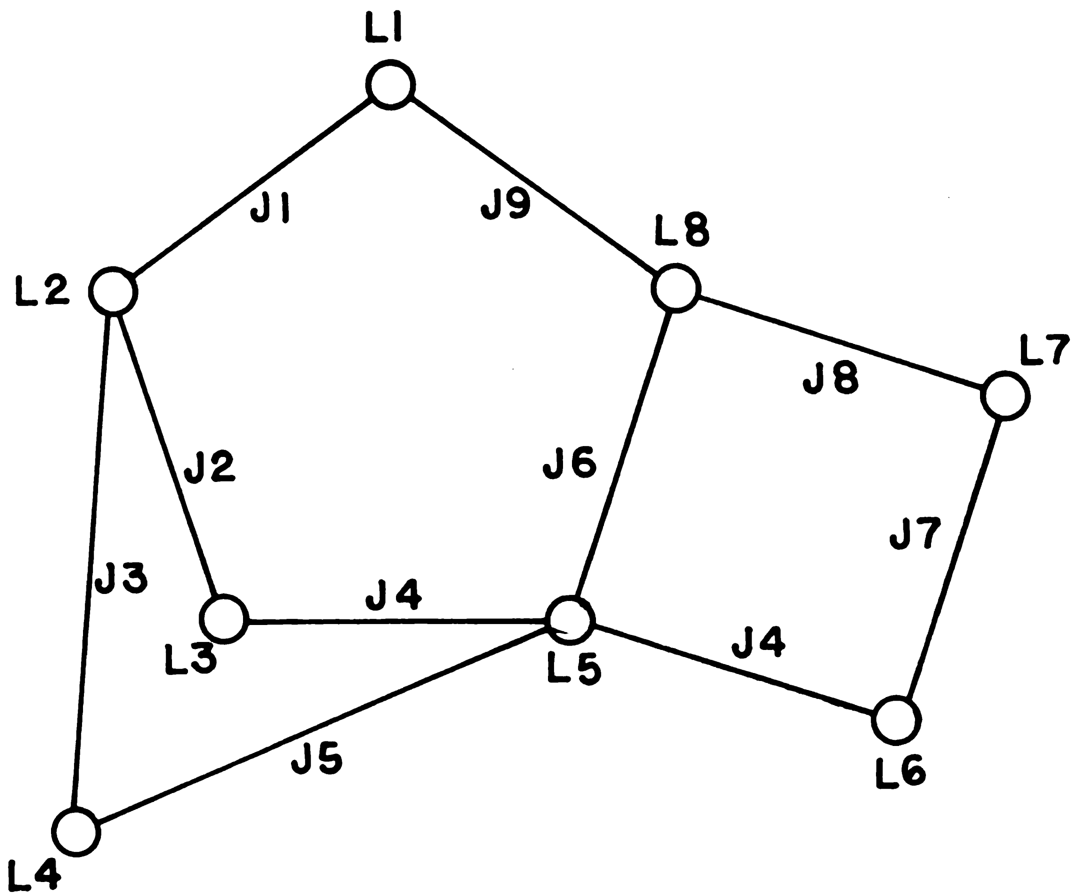

Figure 2.17 The Directed Graph of The Gripping Mechanism

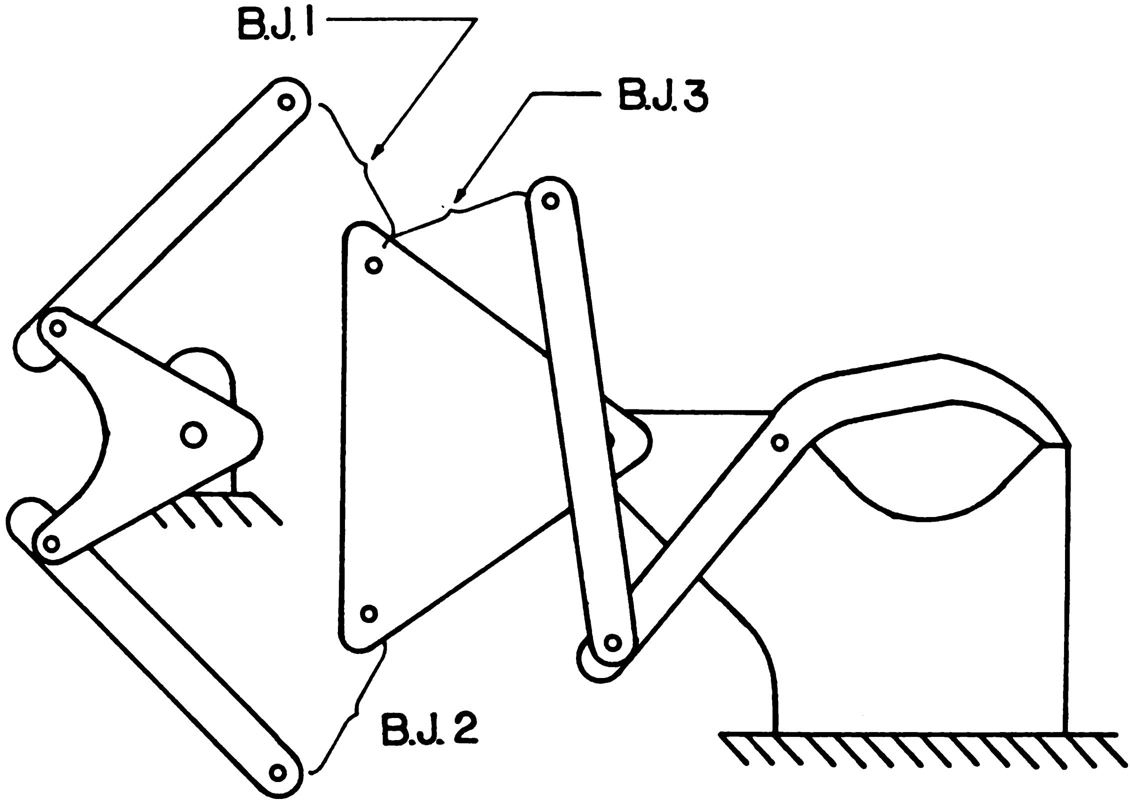

Figure 2.18 The Disconnected Graph of The Gripping Mechanism

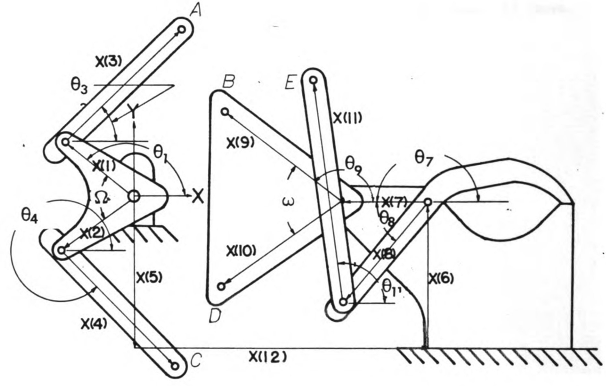

Figure 2.19 The Gripping Mechanism - Indirect Method

exhausted. Applying the Branin's algorithm to the graph of gripping mechanism(Fig.2-17), it is found that a tree can be obtained by removing edges 1,3 and 2. This implies that if the joints 1, 3 and 2 are broken, then a compound pendulum will be obtained(Fig.2-18).

The mechanism is required to grasp a thin object and move back. This implies that there should be no relative motion between the link 7 and the slider link 8 during the backward movement. Should link 7 remain stationary during the backward motion, then the angle of X(8) should take the same value as $\theta_7+\theta_8$. Therefore, at the ith precision point, the distance between the ends of the first broken joint, as shown in Fig.2-19, is

$$D1_i = [(DX1_i)^2 + (DY1_i)^2]^{0.5}$$

where

$DX1_i = X_{Ai}-X_{Bi}$

$DY1_i = Y_{Ai}-Y_{Bi}$

(X_{Ai}, Y_{Ai}): the ith coordinate of the end point A

(X_{Bi}, Y_{Bi}): the ith coordinate of the end point B

$X_{Ai} = X(1)\cos\theta_{1i}+X(3)\cos\theta_{3i}$

$Y_{Ai} = X(1)\sin\theta_{1i}+X(3)\sin\theta_{3i}$

θ_{1i}: the ith angle design variable for X(1)

θ_{3i}: the ith angle design variable for X(3)

$X_{Bi} = X(12)-(i-1)D+X(7)\cos\theta_7+X(9)\cos\theta_{9i}$

$Y_{Bi} = -X(5)+X(6)+X(7)\sin\theta_7+X(9)\sin\theta_{9i}$

θ_{9i}: ith angle design variable for X(9)

θ_7: angle of X(7)

The distance between ends of the second broken joint is

$$D2_i = [(DX2_i)^2 + (DY2_i)^2]^{0.5}$$

where

$$DX2_i = X_{Ci} - X_{Di}$$

$$DY2_i = Y_{Ci} - Y_{Di}$$

(X_{Ci}, Y_{Ci}): the ith coordinate of the end point C

(X_{Di}, Y_{Di}): the ith coordinate of the end point D

$$X_{Ci} = X(2)\cos[\theta_{1i}+\Omega]+X(4)\cos\theta_{4i}$$

$$Y_{Ci} = X(2)\sin[\theta_{1i}+\Omega]+X(4)\sin\theta_{4i}$$

Ω: angle between X(1) and X(2)

θ_{4i}: the ith angle design variable for X(4)

$$X_{Di} = X(12)-(r-1)D+X(7)\cos\theta_7+X(10)\cos[\omega+\theta_{9i}]$$

$$Y_{Di} = -X(5)+X(6)+X(7)\sin\theta_7+X(10)\sin[\omega+\theta_{9i}]$$

ω: angle design variable between X(9) and X(10)

and the distance between ends of the third broken joint is

$$D3_i = [(DX3_i)^2 + (DY3_i)^2]^{0.5}$$

where

$$DX3_i = X_{Bi} - X_{Ei}$$

$$DY3_i = Y_{Bi} - Y_{Ei}$$

(X_{Ei}, Y_{Ei}): the ith coordinate of the end point E

$$X_{Ei} = X(12)-(r-1)D+X(8)\cos[\theta_7+\theta_8]+X(11)\cos\theta_{11i}$$

$$Y_{Ei} = -X(5)+X(6)+X(8)\sin[\theta_7+\theta_8]+X(11)\sin\theta_{11i}$$

θ_{11i}: the ith angle design variable for X(11)

θ_8: angle of x(8)

having the distance between the ends of the three broken joints, then the objective function is formulated as

Table 2.9
The Initial Guesses and Optimal Values for the Design Variables of the Gripping Mechanism
-- Indirect Method --

	No.	Initial Value	Optimal Value	No.	Initial Value	Optiaml Value
	X(1)	1.0	0.3777	X(34)	151.1	131.7
	X(2)	1.0	0.7287	X(35)	154.6	132.7
	X(3)	1.0	0.8380	X(36)	158.0	133.4
	X(4)	1.0	0.7774	X(37)	347.8	359.0
	X(5)	1.0	0.8482	X(38)	366.0	374.5
	X(6)	1.0	1.1481	X(39)	384.0	387.5
	X(7)	1.0	1.1139	X(40)	402.0	400.0
D	X(8)	1.0	0.7663	X(41)	420.0	413.0
E	X(9)	1.0	1.0652	X(42)	438.0	426.7
S	X(10)	1.0	0.6687	X(43)	456.0	443.0
I	X(11)	2.0	1.8451	X(44)	474.0	464.0
G	X(12)	3.0	2.9637	X(45)	492.0	482.0
N	X(13)	60.0	59.5	X(46)	510.0	493.0
	X(14)	180.0	182.0	X(47)	50.3	54.1
V	X(15)	45.0	45.0	X(48)	53.5	58.4
A	X(16)	50.0	50.8	X(49)	56.8	61.2
R	X(17)	157.0	157.1	X(50)	60.0	62.7
I	X(18)	152.0	149.0	X(51)	63.3	63.4
A	X(19)	146.0	142.5	X(52)	66.6	65.7
B	X(20)	140.6	136.9	X(53)	69.9	63.1
L	X(21)	135.0	132.0	X(54)	73.2	63.4
E	X(22)	140.6	128.0	X(55)	76.4	67.4
	X(23)	146.0	126.0	X(56)	79.7	74.0
	X(24)	152.0	126.5	X(57)	313.7	325.6
	X(25)	157.0	128.4	X(58)	317.4	319.8
	X(26)	163.0	129.5	X(59)	321.0	317.3
	X(27)	154.6	149.0	X(60)	324.8	317.4
	X(28)	151.0	144.6	X(61)	328.5	320.0
	X(29)	147.8	140.8	X(62)	332.2	324.8
	X(30)	144.4	137.6	X(63)	336.0	292.7
	X(31)	141.0	134.9	X(64)	340.0	345.4
	X(32)	144.4	132.6	X(65)	343.3	356.8
	X(33)	147.8	131.3	X(66)	347.0	364.6
Objective Function					19.66	0.18

Note: The unit of the design variable 1-12 is in inch
The unit of the design variable 13-66 is in degree
$X(13) = \Omega$, $X(14) = \theta_7$
$X(15) = \theta_8$, $X(16) = \omega$
$X(16 + i) = \theta_{9i}$, $i=1,2,\dots,10$
$X(26 + i) = \theta_{11i}$, $i=1,2,\dots,10$
$X(36 + i) = \theta_{1i}$, $i=1,2,\dots,10$
$X(46 + i) = \theta_{3i}$, $i=1,2,\dots,10$
$X(56 + i) = \theta_{4i}$, $i=1,2,\dots,10$

note: The initial guess is not a rotatable mechanism

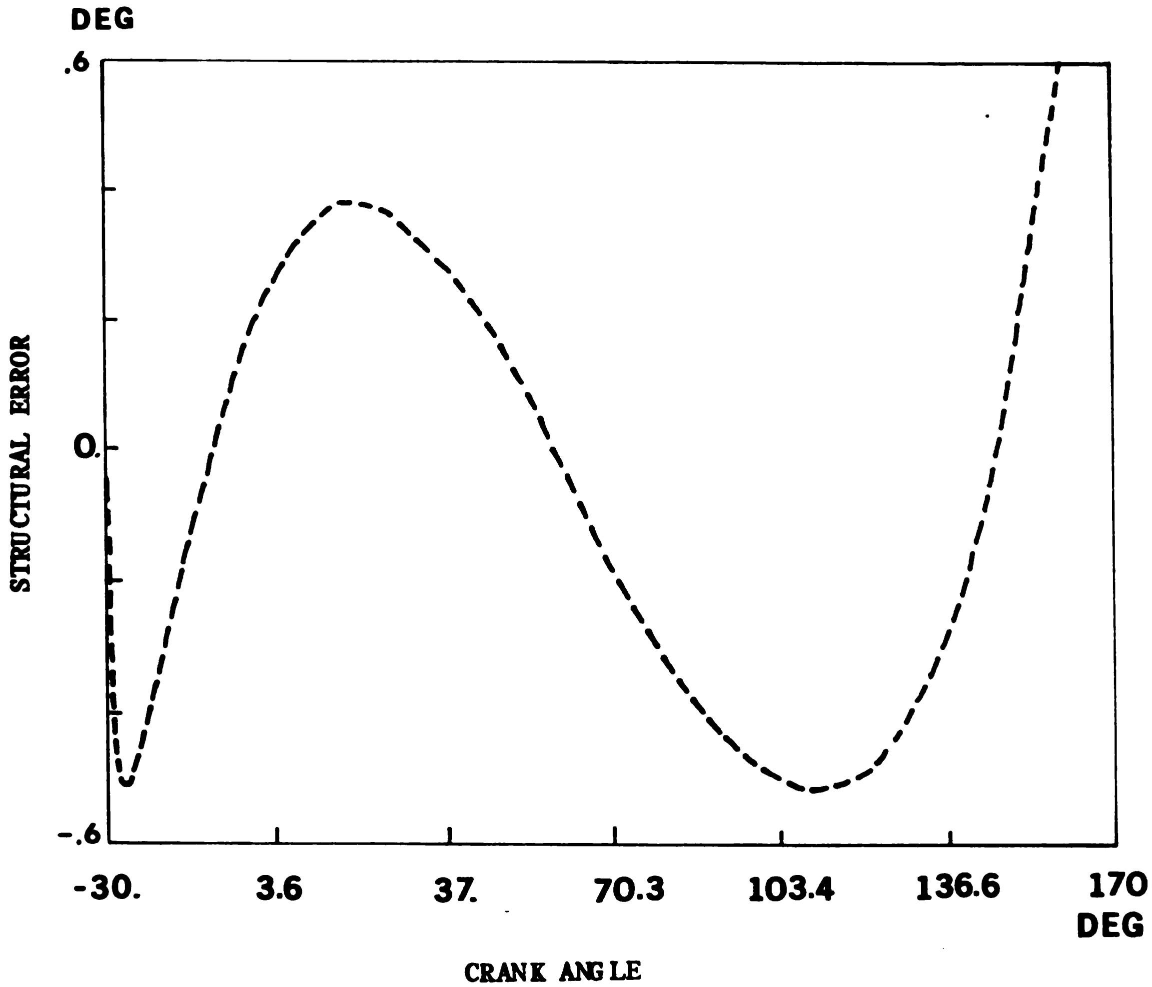

Figure 2.20 The Structural of The Link 8 From The Optimal Design of The Gripping Mechanism Uing Indirect Method

$$F(\bar{x}) = \sum D1_i + D2_i + D3_i$$

The initial and the optimal values for the design variables are shown in Table 2-9. The structural error which is the deviation of the calculated angle of link 8 from the desired value is plotted in Fig.2-20.

CHAPTER 3

DISCUSSION AND CONCLUSION

This work concentrated mainly on developing an indirect method for constructing an objective function for synthesis of planar mechanisms based on the examples presented in section 2-3 through section 2-6, the following conclusions can be drawn:

(i) The calculation and formulation of the objective function in the indirect method is simpler than in the direct method.

(ii) The application of the direct method to the multiloop mechanism, where the closed form solution is not available, is impractical. The indirect method is applicable to the multiloop mechanisms.

(iii) In the indirect method, since the unspecified angles are selected as design variables, the number of design variables is higher when compared with the direct method.

(iv) It is a good strategy to analyze the initial design and use the angles of the links of the mechanisms as the initial guesses for the angle design variables.

The basic steps of the indirect methods are: breaking a specified number of joints to generate a compound pendulum, positioning the input and output links to satisfy the motion requirements, and to calculate the coordinates of the end points of the broken joints to form the objective function. These steps can be automated. Generating a compound pendulum requires identification of a tree in the graph of a mechanism. Calculating the coordinates of the end points of the broken joints requires the identification of the paths in graph analysis and have been successfully implemented in automated procedures for analysis of mechanisms.

In solving the example problems, some angle were specified through imposing motion requirement. Others remain unspecified. The latter category are chosen as design variables. The angles of the vectors on given links differs by a constant. These observation should be considered to design the data structure for any automated procedure for synthesizing mechanisms.

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